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Second Quarterly Report



REFLECTOR ANTENNA ZOOM TECHNIQUES

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Airborne Instruments Laboratory

TECHNICAL REPORT NO. RADC-TR-66-714

February 1967

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RADC (EMU), GAFB, N.Y. 13440

Rome Air Development Center
Research and Technology Division
Air Force Systems Command
Christie Air Force Base, New York

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FOREWORD

This report was prepared by the Airborne Instruments Laboratory, A Division of Cutler-Hammer, Inc., Deer Park, New York, under Contract AF 30(602)-4179, Project 04, Task 4506, and describes the work performed during the period from 22 June to 22 September 1966. The contractor's internal report number is 1031-I-2. Mr. Donald A. Hildebrand is the RADC Project Engineer on this contract. The authors report was submitted by the authors October 1966.

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This report has been reviewed and is approved.

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Approved:

Arthur J. Frohlich
ARTHUR J. FROHLICH
Chief, Techniques Branch
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ABSTRACT

This report describes the work performed during the second quarter of a study of a reflector antenna that provides zoom (variable beamwidth) and scan capability using controlled aperture amplitude and phase. The antenna consists of a primary reflector (paraboloid) and a secondary reflector/lens. It operates as a lens in conjunction with one feed for scanning in the receive mode and as a reflector in conjunction with another feed for zooming in the transmit mode. Switching between a zooming transmit mode and a scanning receive mode results in a versatile radar antenna with an inherent duplexing capability. The performance of this antenna system is being analyzed numerically with the aid of digital computers. This report describes the development of a computer program for solving the Fraunhofer aperture integral efficiently and with good accuracy.

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SECTION I INTRODUCTION

For several years the Rome Air Development Center (RADC) has sponsored two techniques that make use of some form of a reflector antenna in conjunction with aperture phase and amplitude control to provide zooming (variable beamwidth) and/or scanning.

One technique, developed by Airborne Instruments Laboratory (AIL), uses a cluster of feeds placed on a spherical surface concentric with the focal point of a paraboloidal reflector. Each feed is then independently controlled in phase and amplitude to provide zooming and/or scanning (reference 1). The other technique, developed by Blass Antenna Electronics, uses a flat reflector consisting of individual waveguide elements whose phase can be controlled by diode switches that change the position of the short circuit in the waveguide. Proper programming of the switches allows the beam from the reflector to be varied in width and/or scanned (reference 2). This system is commonly called a reflectarray.

The purpose of this study is to consider another technique, different from the two previously mentioned, that will use a reflector and result in a variable beamwidth and a scanning capability. The specific technique to be investigated attempts to use the advantages of the two previously mentioned systems in a single system called (for lack of a better name) a hybrid system. This hybrid system uses the optically fed phased array approach of the Blass system together with the beam magnification and phase correction advantages of the AIL system (Figure 1).

The hybrid system consists of a primary reflector (paraboloid) a secondary reflector acting as both a phase-controlled reflector and a phase-controlled lens (shape to be determined), and two feed systems.

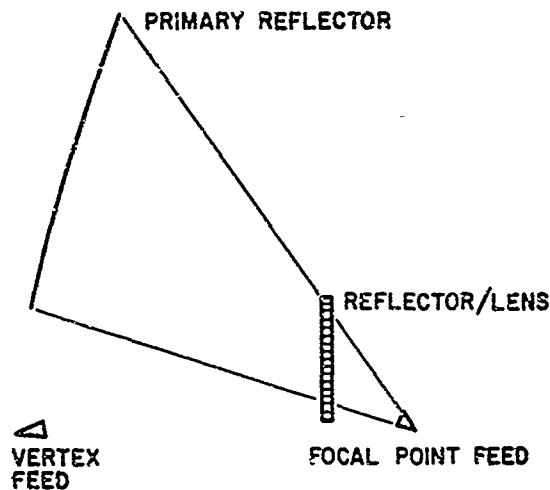


FIGURE 1. HYBRID ANTENNA SYSTEM

The focal-point feed system is used in the receive mode with the secondary reflector acting as a lens; the vertex feed system is used in the transmit mode with the secondary reflector acting as a reflector. Thus, the transmit mode uses a reflectarray as the secondary reflector of a two-reflector system and the receive mode uses a lens with a simple reflecting system. The basic premise on which this system is based is that the diode phase shifters, which comprise the phasing control of the secondary phasor, can be made to operate in both the reflectarray and lens modes. This is shown schematically in Figure 2

In the lens mode of operation (receive) the phasor is seen as a three-bit balanced hybrid transmission device providing 45, 90, and 180-degree increments in phase. The reflectarray mode (transmit) is obtained by properly biasing the 180 degree bit--that is, one diode is forward biased and the other is reverse biased. Under these conditions, the 180-degree bit becomes a short circuit and reflects all the

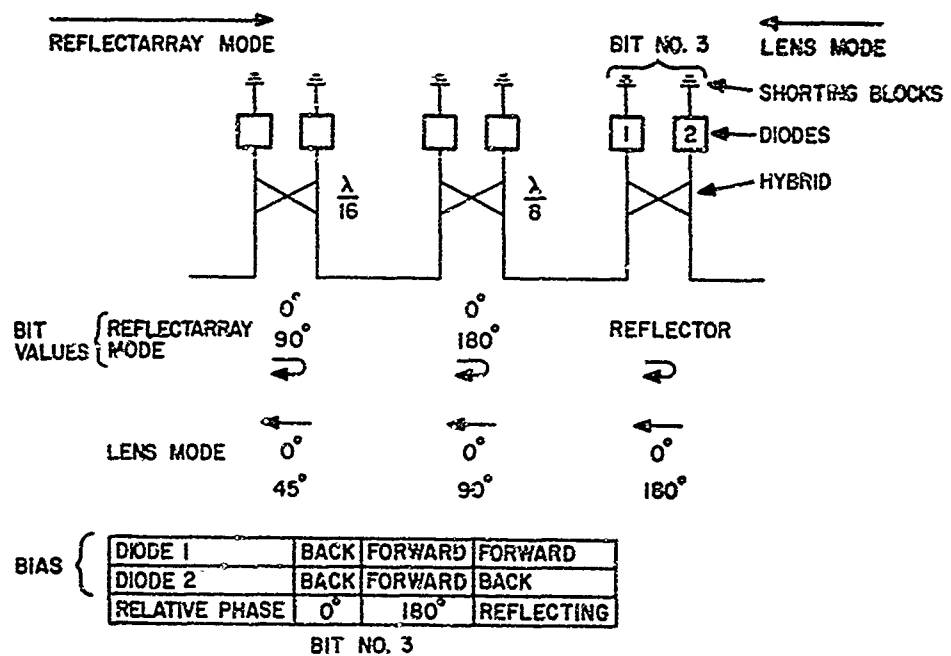


FIGURE 2. TRANSMISSION-REFLECTION PHASOR

energy back out of the input port. This provides a two-bit phasor having phase increments of 90 and 180 degrees since the energy passes each bit twice. In addition to providing mode switching, the 180-degree bit also serves as a duplexer between the transmit and receive modes.

On the basis of this simplified description, an engineering study is being performed to determine the design parameters for a system with the following general electrical characteristics:

TRANSMIT MODE

Frequency	3 Gc $\pm$ 5 percent
Half-power beamwidth	1 $\times$ 1 degree
Zoom factor	8
Side-lobe level	-25 db without zoom -20 db with maximum zoom

RECEIVE MODE

Frequency	3 Gc <u>+5</u> percent
Half-power beamwidth	1 x 1 degree
Scan angle	<u>+8</u> degrees
Side-lobe level	-25 db on boresite -20 db at maximum scan angle

In addition, the system should zoom and scan by electronic means and be feasible for implementation at UHF.

The effort during the first half of this contract has been directed toward the writing of a generalized computer program for computing the far-field radiation patterns in three dimensions of the hybrid system. This program will serve to define the necessary system geometry needed to obtain the electrical characteristics stated and to determine the number of bits needed in the secondary phasor to obtain the zooming, scanning, and side-lobe level.

SECTION II

SYSTEM ANALYSIS

Analysis of the hybrid system can be separated into two parts--determination of how energy propagates within the system, and calculation of the far-field radiation patterns using the results of the intrasystem analysis. This division is possible because the general characteristics of the relationship between an aperture excitation and the resulting far-field pattern are fairly well-known.

An experienced observer can relate the two functions sufficiently well for all preliminary design and the more refined calculations can be used for a final detailed analysis. The alternative procedure of determining the scanning and zooming patterns for each change in system geometry would be extremely time consuming.

The first quarterly report of this program contained a discussion of the intrasystem analysis and the computer programs needed to perform it. This report will be concerned primarily with the problem of obtaining the far-field radiation characteristic corresponding to an illumination function on a planar aperture--the output from the intrasystem analysis.

The relationship between the pattern function and the illumination function is given by the Fraunhofer integral

$$G(u, v) = \iint_A F(x, y) \exp(iux) \exp(ivy) dx dy$$

where

x, y = coordinates of planar aperture

$F(x, y)$ = illumination function

u, v = space variables $2\pi \sin \theta \sin \phi / \lambda$ and $2\pi \sin \theta \cos \phi / \lambda$

θ, φ = spherical coordinate angles,
 $G(u, v)$ = far-field pattern function

The obliquity factor has not been included. When $F(x, y)$ is a separable function of the coordinates the pattern function becomes the product of two integrals of the form,

$$\int_a^b f(x) \exp(iux) dx$$

In this case, the problem is not very difficult because each single integral can be solved rapidly and economically. When the illumination function is not separable, the integration is difficult and time consuming.

One approach is to require that the antenna systems aperture have a separable illumination function under all conditions. However, in complicated geometrical configurations, such as the hybrid system, the energy flow is quite complex and such a constraint would be unrealistic, leading to an unfair evaluation of the system's capabilities.

Most attempts to obtain a general method of numerically solving the Fraunhofer integral have approximated the planar aperture with a planar array. Solution of the array factor is then straightforward and the precision of the results is dependent on the number of array elements postulated. Evaluation of the total array factor is quite costly. Performance can be evaluated in several planes economically, but intermediate points can be conveniently solved only by changing the array grid structure or by introducing another approximation with an undetermined effect on precision of the results.

A more satisfactory and useful solution is found in Filon's method (reference 3, 4) of quadrature of definite integrals of the form

$$\int_a^b f(x) \sin ux \, dx \text{ and } \int_a^b f(x) \cos ux \, dx$$

The method involves dividing the interval (a, b) into a number of equal parts and evaluating sums of products of trigonometric functions and $f(x_0)$, where x_0 is one of the points defined in the interval (a, b) . The method is suggestive of that of evaluating array factors but the computer time needed is markedly less.

Successive application of Filon's quadrature method leads to an approximate solution of the Fraunhofer integral. A more complete discussion of this method of solving the double integral is included in the Appendix. The precision of the results from this method are primarily determined by the cube of the spacing between grid points in the aperture and by the fourth derivative of the illumination function. Since the real and the imaginary part of the illumination function must be evaluated separately; this means that precision degrades approximately as the fourth power of the scan angle and as the third power of the grid spacing. Best results are obtained when the aperture is symmetrically disposed about the system origin ($x = y = 0$).

A computer program was prepared (Appendix) for evaluating the double integral by Filon's technique. The program provided excellent boresight patterns that checked exactly with known results

for uniform, cosine, and cosine-squared aperture illumination functions and for combinations of these functions, for example, constant $\times$ cosine-squared. The accuracy degraded rapidly with scan angle.

The results of a study of the spacing required to obtain good precision for a diagonal (worst case) scan capability of 13 degrees ($\beta = 45$ degrees, $\theta = 13$ degrees) are shown in Figures 3, 4, and 5. Thirteen degrees was considered adequate for the program. An illumination function of cosine $\times$ cosine-squared on a square aperture was used. The illustrations show the principal plane patterns for the boresight condition and for the diagonal scan condition for spacing of 1, 1.5, and 2 wavelengths. For the least spacing, the patterns are almost identical, as they should be since the obliquity factor was not considered. As the spacing increases, the scanned pattern deteriorates until it loses almost all usefulness at the maximum spacing of two wavelengths. The computer printout for one wavelength spacing is shown in Figure 6.

A pair of principal plane patterns for boresight and for a 26-degree diagonal scan with half wavelength grid spacing is shown in Figure 7. The agreement is excellent. For scan angles above 30 degrees, pattern quality degrades rapidly for any spacing.

Considering the product of points in the aperture and the far field to be Z , the present cost of these calculations using the IBM 1620 facility at AIL is about $Z/4$ cents. Improved programming techniques and the use of a high-speed computer should cut this figure by more than an order of magnitude, probably by a factor of 50. So the present cost of an excellent quality 13-degree scanned pattern for a 60×60 wavelength array would be \$9.00/far-field point with improvement to less than \$0.25/far-field point obtainable. The cost for boresight conditions would be about 25 percent of the above.

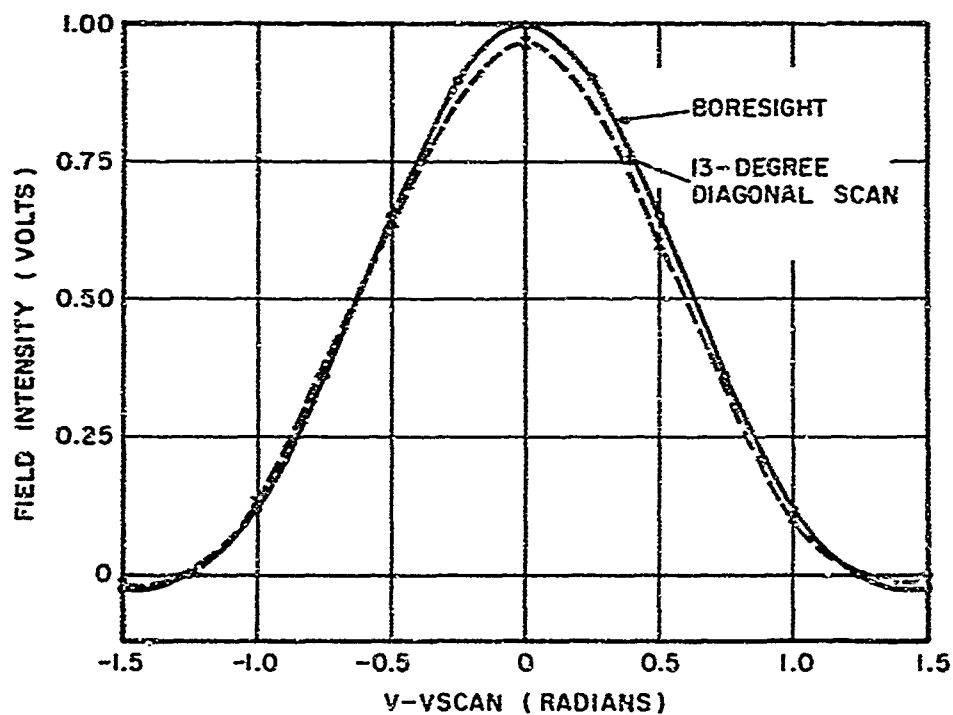
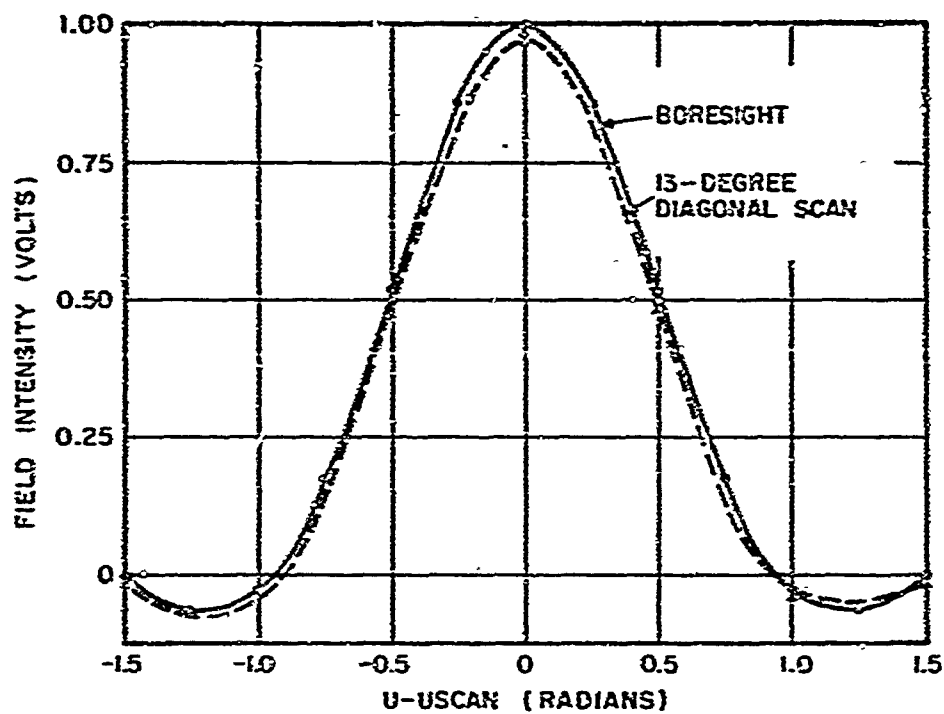


FIGURE 3. PRINCIPAL PLANE PATTERNS FOR ONE-WAVELENGTH SPACING, APERTURE OF 10×10 WAVELENGTH

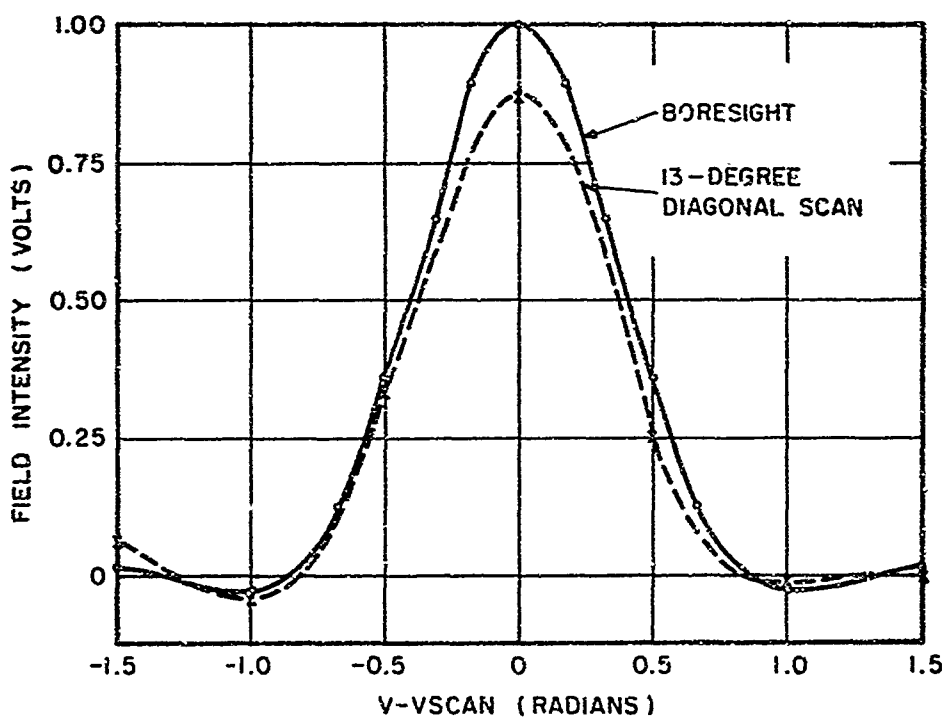
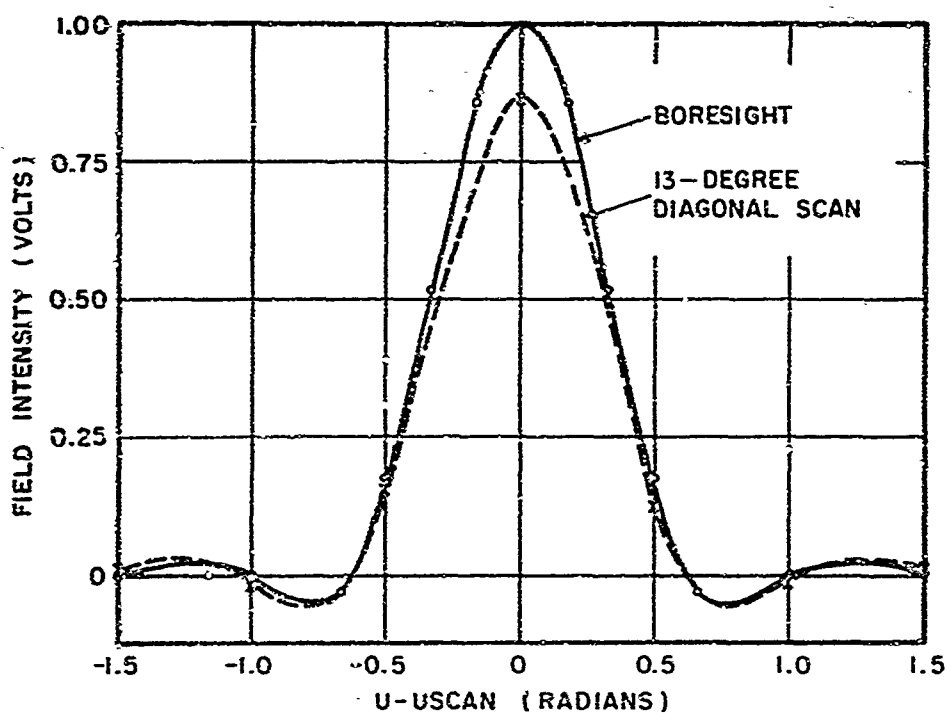


FIGURE 4. PRINCIPAL PLANE PATTERNS FOR 1.5-WAVELENGTH SPACING, APERTURE OF 15×15 WAVELENGTH

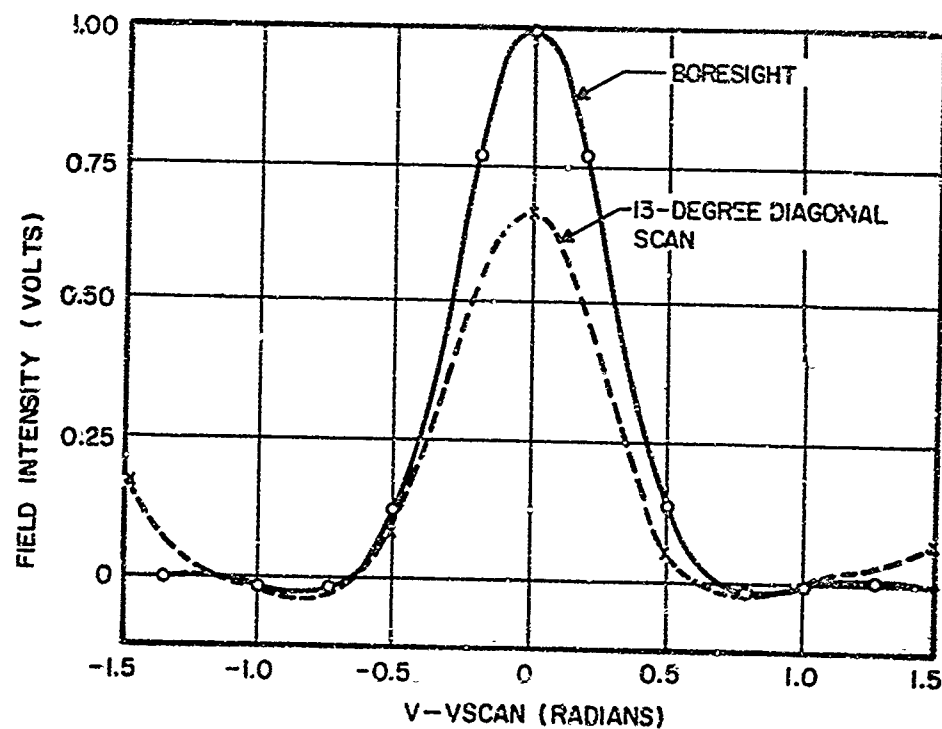
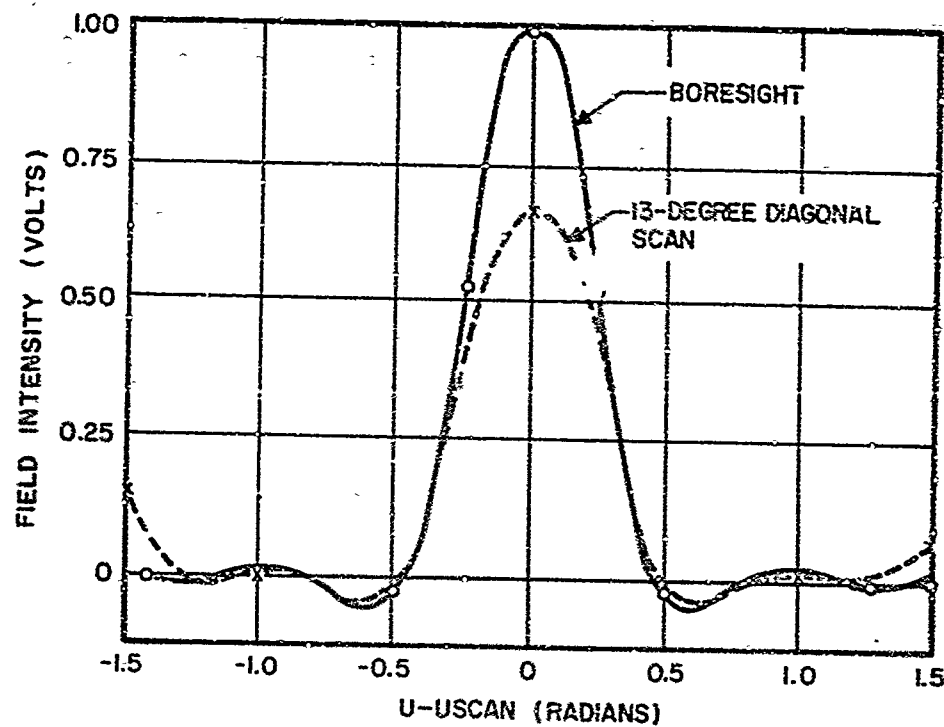


FIGURE 5. PRINCIPAL PLANE PATTERNS FOR TWO-WAVELENGTH SPACING, APERTURE OF 20×20 WAVELENGTHS

	V=	-.250	0.000	.250	.500	.750	1.000	1.250	1.500
U=	-.25	310.046	343.888	310.046	224.098	122.965	42.816	.614	9.162
U=	0.00	360.655	400.022	360.655	260.678	143.037	49.805	.714	10.658
U=	.25	310.046	343.888	310.046	224.098	122.965	42.816	.614	9.162
U=	.50	188.369	208.930	188.369	136.152	74.708	26.013	.373	5.566
U=	.75	62.893	69.758	62.893	45.458	24.943	8.685	.124	1.858
U=	1.00	11.175	12.395	11.175	8.077	4.432	1.543	.022	.330
U=	1.25	24.205	26.847	24.205	17.495	9.600	3.342	.047	.715
U=	1.50	5.698	6.320	5.698	4.119	2.260	.787	.011	.163

BORESIGHT

	V=	-.500	0.000	.500	1.000	1.500	2.000	2.500
U=	-.50	.179	.719	3.683	5.567	3.442	.582	.097
U=	0.00	.386	1.549	7.935	11.994	7.415	1.255	.210
U=	.50	6.630	26.581	136.152	205.789	127.227	21.532	3.610
U=	1.00	12.469	49.990	256.055	387.020	239.272	40.495	6.790
U=	1.50	6.235	24.998	128.046	193.538	119.653	20.250	3.395
U=	2.00	.344	1.382	7.079	10.700	6.615	1.119	.187
U=	2.50	.157	.631	3.235	4.889	3.023	.511	.085

13-DEGREE DIAGONAL SCAN

FIGURE 6. PRINTOUT OF FAR-FIELD PATTERN FUNCTION
FOR ONE-WAVELENGTH SPACING, APERTURE
OF 10 x 10 WAVELENGTHS

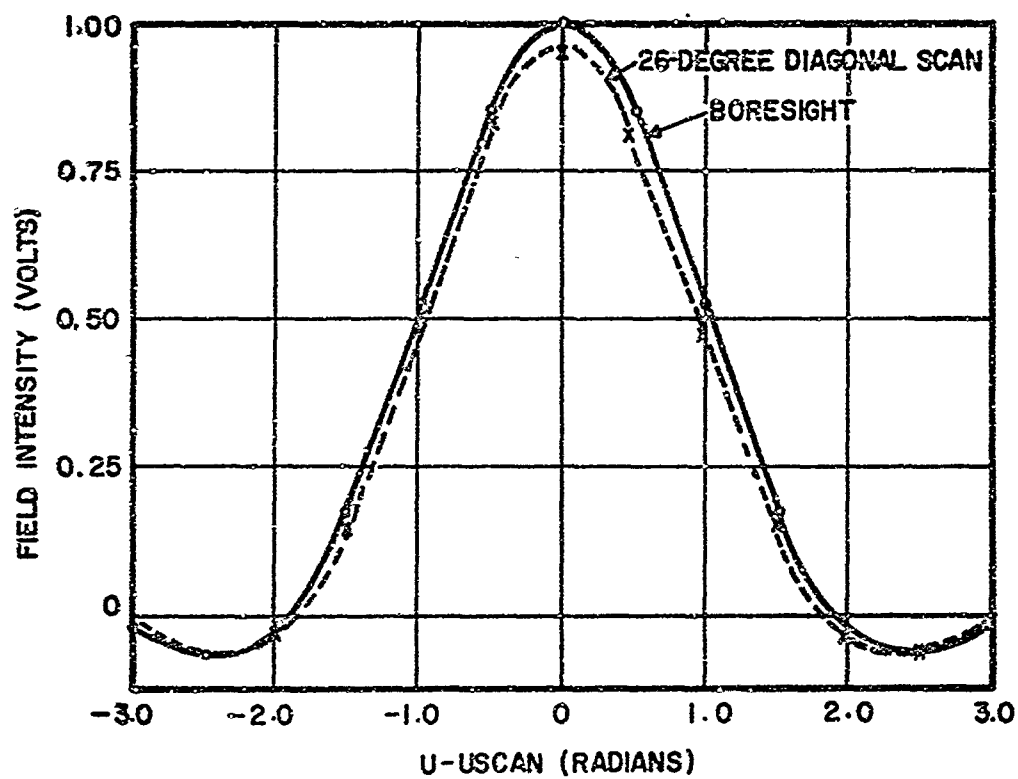


FIGURE 7. PRINCIPAL PLANE PATTERNS FOR 0.5 WAVE-LENGTH SPACING, APERTURE OF 5 x 5 WAVE-LENGTHS

SECTION III CONCLUSIONS

Analytic techniques for investigating the performance of the hybrid antenna systems have been studied and two computer programs have been prepared for calculating the essential relationships between the antenna configuration, the antenna aperture illumination, and the far-field radiation pattern. Both programs have been tested against known results and have shown good agreement with them.

SECTION IV

PROGRAM FOR NEXT INTERVAL

During the next quarter of this study, additional hybrid antenna configurations will be analyzed in light of their ability to provide the required pattern characteristics.

The computer programs that have been developed will be refined and put in proper form for conjoined use.

SECTION V

REFERENCES

1. L. K. DeSize, B. J. Musso, and D. W. Pride, "Zoom Antenna Techniques," RADC-TR-66-300, May 1966.
2. Blass Antenna Electronics Corporation, "Phased Backscatter Array," RADC-TDR-64-243, Vol II, October 1964.
3. L. N. G. Filon, "On a Quadrature Formula for Trigonometric Integrals," Proceedings of the Royal Society, Edinburgh, 1928.
4. Z. Kopal, "Numerical Analysis," first edition, Wiley, New York, 1955, p 408-410, 538-539.

APPENDIX

SOLUTION OF THE FRAUNHOFER INTEGRAL USING FILON'S METHOD

The aperture integral

$$G(u, v) = \int_c^d \int_a^b F(x, y) \exp(iux) \exp(ivy) dx dy$$

can be numerically solved by successive applications of Filon's quadrature method of solving oscillatory integrals of the form

$$\int_a^b f(x) \cos ux dx \text{ and } \int_a^b f(x) \sin ux dx$$

where $f(x)$ is continuous in the interval (a, b) .

The approximate solutions of these integrals are given by

$$\frac{1}{h} \int_a^b f(x) \begin{pmatrix} \cos ux \\ \sin ux \end{pmatrix} dx = \alpha \left[f(a) \begin{pmatrix} -\sin ua \\ \cos ua \end{pmatrix} + f(b) \begin{pmatrix} \sin ub \\ -\cos ub \end{pmatrix} \right] + \beta R_{2r} + \alpha R_{2r-1}$$

where

$$R_{2r} = \sum_{r=0}^m f(x_{2r}) \begin{pmatrix} \cos ux_{2r} \\ \sin ux_{2r} \end{pmatrix} - \frac{1}{2} \left[f(a) \begin{pmatrix} \cos ua \\ \sin ua \end{pmatrix} + f(b) \begin{pmatrix} \cos ub \\ \sin ub \end{pmatrix} \right]$$

$$R_{2r-1} = \sum_{r=1}^m f(x_{2r-1}) \begin{pmatrix} \cos ux_{2r-1} \\ \sin ux_{2r-1} \end{pmatrix}$$

The interval (a, b) has been divided into $2m$ segments, each of length h ; the segments are bounded by the points $x_0, x_1, \dots, x_{2m}$, with $x_0 = a$ and $x_{2m} = b$.

$$\alpha = \frac{1}{\theta} + \frac{\sin \theta}{2\theta^2} - \frac{2 \sin^2 \theta}{\theta^3}$$

$$\beta = 2 \left(\frac{1 + \cos^2 \theta}{\theta^2} - \frac{\sin 2\theta}{\theta^3} \right)$$

$$\gamma = -4 \left(\frac{\cos \theta}{\theta^2} - \frac{\sin \theta}{\theta^3} \right)$$

where $\theta = uh$. Expansions of these functions are used for small values of θ .

For the double integration, the interval (a, b) is divided into $2n$ segments of length h and the interval (c, d) is divided into $2m$ segments of length l . The integral must be solved twice, once for the real part of $F(x, y)$ and once for the imaginary part. Referring to either part as $Q(x, y)$ we have the double integral

$$\int_c^d \left[\int_a^b Q(x, y) \exp(iux) dx \right] \exp(ivy) dy$$

which leads to the single integral

$$h \int_c^d \left\{ \alpha_x \left[Q(a, y) (-\sin ua + i \cos ua) + Q(b, y) (\sin ub - i \cos ub) \right] - \right.$$

$$\left. \frac{\beta_x}{2} \left[Q(a, y) (\cos ua + i \sin ua) + Q(b, y) (\cos ub + i \sin ub) \right] + \right.$$

$$\beta_x \sum_{r=0}^n Q(x_{2r}, y) \left(\cos ux_{2r} + i \sin ux_{2r} \right) + - \\ \gamma_x \sum_{r=1}^n Q(x_{2r-1}, y) \left(\cos ux_{2r-1} + i \sin ux_{2r-1} \right) \Bigg\} \exp(ivy) dy$$

This expression must be integrated term by term with respect to y. An example is given:

$$\beta_x \int_c^d \sum_{r=0}^n Q(x_{2r}, y) \cos ux_{2r} \exp(ivy) dy =$$

$$\beta_x \cos ux_{2r} \sum_{r=0}^n \int_c^d Q(x_{2r}, y) \exp(ivy) dy =$$

$$i \beta_x \cos ux_{2r} \sum_{r=0}^n \left\{ \alpha_y \left[Q(x_{2r}, c) (-\sin vc + i \cos vc) \right. \right.$$

$$\left. Q(x_{2r}, d) (\sin vd - i \cos vd) \right] -$$

$$\frac{\beta_y}{2} \left[Q(x_{2r}, c) (\cos vc + i \sin vc) + Q(x_{2r}, d) (\cos vd + i \sin vd) \right] +$$

$$\beta_y \sum_{s=0}^m Q(x_{2r}, y_{2s}) \left(\cos vy_{2s} + i \sin vy_{2s} \right) +$$

$$\gamma_y \sum_{s=1}^m Q(x_{2r}, y_{2s-1}) \left(\cos vy_{2s-1} + i \sin vy_{2s-1} \right) \Bigg\}$$

Applying the same techniques to the other terms, we can express the double integral as

$$\int_c^d \int_a^b Q(x, y) \exp(iu'x) \exp(iv'y) dx dy =$$

$$I_1 + I_3 + \dots + I_{11} + I_{14} + I_{16} + \dots + I_{24} +$$

$$i(I_2 + I_4 + \dots + I_{12} + I_{13} + I_{15} + \dots + I_{23})$$

where

$$I_1 = -h\alpha_x \sin ua \int_c^d Q(a, y) \cos vy dy \equiv -h\alpha_x \sin ua (V_1)$$

$$I_2 = -h\alpha_x \sin ua \int_c^d Q(a, y) \sin vy dy \equiv h\alpha_x \sin ua (V_2)$$

$$I_3 = h\alpha_x \sin ub \int_c^d Q(b, y) \cos vy dy \equiv h\alpha_x \sin ub (V_3)$$

$$I_4 = h\alpha_x \sin ub \int_c^d Q(b, y) \sin vy dy \equiv h\alpha_x \sin ub (V_4)$$

$$I_5 = h\beta_x \sum_{r=0}^m \cos ux_{2r} \int_c^d Q(x_{2r}, y) \cos vy dy \equiv$$

$$h\beta_x \sum_{r=0}^m \cos ux_{2r} (V_5)$$

$$I_6 = h\beta_x \sum_{r=0}^m \cos ux_{2r} \int_c^d Q(x_{2r}, y) \sin vy dy \equiv$$

$$h\beta_x \sum_{r=0}^m \cos ux_{2r} (V_6)$$

$$I_7 = -\frac{h\beta_x}{2} \cos ua \int_c^d Q(a, y) \cos vy dy \equiv -\frac{h\beta_x}{2} \cos ua (V_7)$$

$$I_8 = -\frac{h\beta_x}{2} \cos ua \int_c^d Q(a, y) \sin vy dy \equiv -\frac{h\beta_x}{2} \cos ua (V_8)$$

$$I_9 = -\frac{h\beta_x}{2} \cos ub \int_c^d Q(b, y) \cos vy dy \equiv -\frac{h\beta_x}{2} \cos ub (V_9)$$

$$I_{10} = -\frac{h\beta_x}{2} \cos ub \int_c^d Q(b, y) \sin vy dy \equiv -\frac{h\beta_x}{2} \cos ub (V_{10})$$

$$I_{11} = h\gamma_x \sum_{r=1}^m \cos ux_{2r-1} \int_c^d Q(x_{2r-1}, y) \cos vy dy \equiv$$

$$h\gamma_x \sum_{r=1}^m \cos ux_{2r-1} (V_{11})$$

$$I_{12} = h\gamma_x \sum_{r=1}^m \cos ux_{2r-1} \int_c^d Q(x_{2r-1}, y) \sin vy \tilde{y} \approx$$

$$h\gamma_x \sum_{r=1}^m \cos ux_{2r-1} (V_{12})$$

$$I_{13} = h\alpha_x \cos ua(V_1)$$

$$I_{14} = h\alpha_x \cos ua(V_2)$$

$$I_{15} = -h\alpha_x \cos ub(V_3)$$

$$I_{16} = -h\alpha_x \cos ub(V_4)$$

$$I_{17} = h\beta_x \sum_{r=0}^m \sin ux_{2r} (V_5)$$

$$I_{18} = h\beta_x \sum_{r=0}^m \sin ux_{2r} (V_6)$$

$$I_{19} = -\frac{h\beta_x}{2} \sin ua(V_7)$$

$$I_{20} = -\frac{h\beta_x}{2} \sin ua(V_8)$$

$$I_{21} = -\frac{h\beta_x}{2} \sin ub(V_9)$$

$$I_{22} = -\frac{h\beta_x}{2} \sin ub(V_{10})$$

$$I_{23} = h\gamma_x \sum_{r=1}^m \sin ux_{2r-1}(V_{11})$$

$$I_{24} = h\gamma_x \sum_{r=1}^m \sin ux_{2r-1}(V_{12})$$

For each far field point these 24 terms must be evaluated and properly summed for the real part and for the imaginary part of the illumination function. These results are combined to give the field intensity at the point.

A printout of the computer program used to perform these operations is shown in Figure 8. A flow diagram of the program is shown in Figure 9. Improvements in the programming will have to be implemented before the program will operate near maximum efficiency. When this is done the integration program can be used in conjunction with the program for analyzing the energy flow within the system in order to obtain the total performance characteristic of any hybrid system.

FILON INTEGRATION IN TWO DIMENSIONS	0010
N AND R MUST BE ODD	
DIMENSION X(12,12),Y(12,12),Q(12,12),SH(30),SH1(30),CH(30),CH1(30)	
2,VK(30),FIELD(30),COY(30)	
RFAD 1000,M,N,UO,UL,DU,VO,VL,DV	0030
RFAD 1001,USCAN,VSCAN	
READ 1002,A,B,C,D	0050
1000 FORMAT (2I2,6F8.3)	2010
1001 FORMAT(2F8.3)	2011
1002 FORMAT(4F7.3)	2012
EM=N-1	
EN=N-1	
H=(B-A)/FM	0080
EL=(D-C)/FM	0090
Y2S=A	
DO 52 K=1,N	
COY(K)=COSF(3.14159*Y2S/(B-A))	
52 Y2S=Y2S+FL	
Y2S=A	
DO 31 K=1,N	
X2S=C	
DO 20 J=1,M	
CY= 3.14159*COSF(USCAN*X2S+VSCAN*Y2S)	
SY=-3.14159*SINF(USCAN*X2S+VSCAN*Y2S)	
X(J,K)=COY(J)*COY(K)*COY(K)*CY	
Y(J,K)=COY(J)*COY(K)*COY(K)*SY	
29 X2S=X2S+H	
31 Y2S=Y2S+FL	
N1=N+1	
M1=M+1	
DO 301 K=1,N1	
X(M1,K)=0.0	
301 Y(M1,K)=0.0	
DO 302 J=1,M1	
X(J,N1)=0.0	
302 Y(J,N1)=0.0	
12 U=UO	0100
NF=(VL-VO)/DV+1.	0330
V=VO	
DO13 K=1,NF	
VK(K)=V	
19 V=V+DV	
PRINT 1010,(VK(K),K=1,NF)	
1010 FORMAT (9X,2HV=,13F8.3)	

FIGURE 8. PRINTOUT OF FILON 2 PROGRAM LISTING
(SHEET 1 OF 6)

	MF=(UL-U0)/DU+1.	0110
	DO 100 I=1,NF	0120
14	TH=(U*H)	0130
	TTH=2.*TH	0140
	THSQ=TH*TH	0150
	THCU=THSQ*TH	0160
	THR=SQRTF(THSQ)	0170
	IF(THR-.25115,16,16	0180
15	ALX=(.04444444-(.006349206-.0004232804*THSQ)*THSQ)*THCU	
	BEX=.66666667+(.1333333-(.03809524-.003527337*THSQ)*THSQ)*THSQ	0200
	GAX=1.333333-(.1333333-(.004761905-.00008818342*THSQ)*THSQ)*THSQ	0210
	GO TO 17	0215
16	CTTH=COSF(TTH)	0220
	STTH=SINF(TTH)	0230
	CTH=COSF(TH)	0240
	STH=SINF(TH)	0250
	ALX=(1.+STTH/TTH-(1.-CTTH)/THSQ)/TH	0260
	BEX=(3.+CTTH-2.*STTH/TH)/THSQ	0280
	GAX=-4.*(CTH-STH/TH)/THSQ	0290
17	UA=U*A	0300
	UB=U*B	0310
	V=V0	0320
	DO 90 L=1,NF	0340
18	PH=(V*FL)	0350
	TPH=2.*PH	0360
	PHSQ=PH*PH	0370
	PHCU=PHSQ*PH	0380
	PHR=SQRTF(PHSQ)	0390
20	IF(PHR-.25121,22,22	0400
21	ALY=(.04444444-(.006349206-.0004232804*PHSQ)*PHSQ)*PHCU	
	BEY=.66666667+(.1333333-(.03809524-.003527337*PHSQ)*PHSQ)*PHSQ	0420
	GAY=1.333333-(.1333333-(.004761905-.00008818342*PHSQ)*PHSQ)*PHSQ	0430
	GO TO 23	0440
22	CTPH=COSF(TPH)	0450
	STPH=SINF(TPH)	0460
	CPH=COSF(PH)	0470
	SPH=SINF(PH)	0480
	ALY=(1.+STPH/TPH-(1.-CTPH)/PHSQ)/PH	0490
	BEY=(3.+CTPH-2.*STPH/PH)/PHSQ	0500
	GAY=-4.*(CPH-SPH/PH)/PHSQ	0510
23	VC=V*C	0520
	VD=V*D	0530
24	S1=SINF(UA)	0540
	S2=SINF(UB)	0550

FIGURE 8.
(SHEET 2 OF 6)

S3=SINF(VC)	0560
S4=SJNF(VD)	0570
C1=COSF(UA)	0580
C2=COSF(UB)	0590
C3=COSF(VC)	0600
C4=COSF(VD)	0610
S7=S3	
S8=S4	
C7=C3	
C8=C4	
25 Y2S=A	
Y2S1=A+EL	0630
26 DO 30 K=1,N,2	
K1=K+1	
SN(K)=SINF(V* Y2S)	0680
SN1(K1)=SINF(V*Y2S1)	0690
CN(K)=COSF(V*Y2S)	0700
CN1(K1)=COSF(V*Y2S1)	0720
Y2S=Y2S+2.*FL	0730
30 Y2S1=Y2S+FL	0760
DO 70 J=1,M	
DO 70 K=1,N1	
70 Q(J,K)=X(J,K)	
LN=0	0780
SUMX=0.	0790
SUMY=0.	0800
32 R1=-.5*(Q(1,1)*C7+Q(1,N)*C8)	0810
R11=0.	0820
R2=-.5*(Q(M,1)*C7+Q(M,N)*C8)	0830
R21=0.	0840
R4=-.5*(Q(1,1)*C7+Q(1,N)*C8)	0850
R41=0.	0860
R5=-.5*(Q(M,1)*C7+Q(M,N)*C8)	0870
R51=0.	0880
T1=-.5*(Q(1,1)*S7+Q(1,N)*S8)	0890
T11=0.	0900
T2=-.5*(Q(M,1)*S7+Q(M,N)*S8)	0910
T21=0.	0920
T4=-.5*(Q(1,1)*S7+Q(1,N)*S8)	0930
T41=0.	0940
T5=-.5*(Q(M,1)*S7+Q(M,N)*S8)	
T51=0	0960
34 DO 40 K=1,N,2	
K1=K+1	

FIGURE 8.
(SHEET 3 OF 6)

A-11

P1=R1+Q(1,K)*CN(K)	
R11=R11+Q(1,K1)*CN1(K1)	1010
R2=R2+Q(M,K)*CN(K)	1020
R21=R21+Q(M,K1)*CN1(K1)	1030
R4=R4+Q(1,K)*CN(K)	1040
R41=R41+Q(1,K1)*CN1(K1)	1050
R5=R5+Q(M,K)*CN(K)	1060
R51=R51+Q(M,K1)*CN1(K1)	1070
T1=T1+Q(1,K)*SN(K)	1080
T11=T11+Q(1,K1)*SN1(K1)	1090
T2=T2+Q(M,K)*SN(K)	1100
T21=T21+Q(M,K1)*SN1(K1)	1110
T4=T4+Q(1,K)*SN(K)	
T41=T41+Q(1,K1)*SN1(K1)	1130
T5=T5+Q(M,K)*SN(K)	1140
40 T51=T51+Q(M,K1)*SN1(K1)	1150
42 HL=H*FL*ALX	1160
V1=ALY*(-Q(1,1)*S3+Q(1,N)*S4)+BEY*R1+GAY*R11	1170
F11=-V1*HL*S1	1180
V2=ALY*(Q(1,1)*C7-Q(1,N)*C8)+BEY*T1+GAY*T11	1190
F12=-V2*HL*S1	1200
V3=ALY*(-Q(M,1)*S7+Q(M,N)*S8)+BEY*R2+GAY*R21	1210
F13=V3*HL*S2	1220
V4=ALY*(Q(M,1)*C7-Q(M,N)*C8)+BEY*T2+GAY*T21	
F14=V4*HL*S2	1240
46 HLR=H*FL*BEY	1250
V7=ALY*(-Q(1,1)*S3+Q(1,N)*S4)+BEY*R4+GAY*R41	1270
E17=-V7*HLB*.5*C1	1280
V8=ALY*(Q(1,1)*C3-Q(1,N)*C4)+BEY*T4+GAY*T41	1290
E18=-V8*HLB*.5*C1	1300
V9=ALY*(-Q(M,1)*S3+Q(M,N)*S4)+BEY*R5+GAY*R51	1310
F19=-V9*HLR*.5*C2	1320
V10=ALY*(Q(M,1)*C3-Q(M,N)*C4)+BEY*T5+GAY*T51	
E110=-V10*HLR*.5*C2	1340
E113=V1*HL*C1	1350
E114=-V2*HL*C1	1360
E115=-V3*HL*C2	1370
E116=V4*HL*C2	1380
E119=-V7*HLR*.5*S1	1390
F120=V8*HLR*.5*S1	1400
F121=-V9*HLR*.5*S2	1410
E122=V10*HLR*.5*S2	1420
48 F15=0	1430
E16=0	1440

FIGURE 8.
(SHEET 4 OF 6)

EI11=0	1450
FI12=0	1460
FI17=0	1470
EI18=0	1480
EI23=0	1490
EI24=0	1500
X2R=C	
X2R1=C+H	
DO 50 J=1,M,2	
J1=J+1	
R3=-.5*(Q(J,1)*C7+Q(J,N)*C8)	1530
R31=0	1540
R6=-.5*(Q(J1,1)*C7+Q(J1,N)*C8)	1550
R61=0	1560
T3=-.5*(Q(J,1)*S7+Q(J,N)*S8)	1570
T31=0	1580
T6=-.5*(Q(J1,1)*S7+Q(J1,N)*S8)	1590
T61=0	1600
DO 50 K=1,N,2	
K1=K+1	
R3=R3+Q(J,K)*CN(K)	1630
R31=R31+Q(J,K1)*CN1(K1)	
R6=R6+Q(J1,K)*CN(K)	1650
R61=R61+Q(J1,K1)*CN1(K1)	1660
T3=T3+Q(J,K)*SN(K)	1670
T31=T31+Q(J,K1)*SN1(K1)	1680
T6=T6+Q(J,K)*SN(K)	1690
50 T61=T61+Q(J1,K1)*SN1(K1)	1700
V5=ALY*(-Q(J,1)*S3+Q(J,N)*S4)+BEY*R3+GAY*R31	1710
EI5=EI5+HLB*V5*COSF(U*X2R)	1720
V6=ALY*(Q(J,1)*C3-Q(J,N)*C8)+BEY*T3+GAY*T31	1730
FI6=EI6+HLB*V6*COSF(U*X2R)	1740
HLG=H*EL*GAX	1750
V11=ALY*(-Q(J1,1)*S7+Q(J1,N)*S8)+BEY*R6+GAY*R61	1760
EI11=EI11+HLG*V11*COSF(U*X2R1)	1770
V12=ALY*(Q(J1,1)*C7-Q(J1,N)*C8)+BEY*T6+GAY*T61	1780
EI12=EI12+HLG*V12*COSF(U*X2R1)	1790
EI17=FI17+HLB*V5*SINF(U*X2R)	1800
EI18=FI18-HLB*V6*SINF(U*X2R)	1810
EI23=EI23+HLG*V11*SINF(U*X2R1)	1820
EI24=FI24-HLG*V12*SINF(U*X2R1)	1830
X2R=X2R+2.*H	1840
60 X2R1=X2R+H	1850
IF(LN)61,61,62	1860

FIGURE 8.
(SHEET 5 OF 6)

```

61 SUMX=F11+F13+E15+E17+F19+E111+F114+F116+E118+F120+E124+E122      1870
   SUMY=E12+E14+E16+F18+E110+E112+E113+E115+E117+E119+F121+E123      1880
   LN=1                                                                1890
   DO 71 J=1,M1
   DO 71 K=1,N1
71  Q(J,K)=Y(J,K)
   GO TO 32
62 SUMX=SUMX-E12-E14-E16-E18-F110-F112-E113-F115-E117-F119-E121-E123    1900
   SUMY=SUMY+E11+F13+E15+F17+E19+F111+F114+E116+E118+E120+F122+E124    1920
3000 FORMAT (2F8.3)
   FIELD(L)=4.*SQRTF(SUMX*SUMX+SUMY*SUMY)
   V=V+DV                                                                2020
   PRINT 1005,U,(FIELD(L),L=1,NF)
1005 FORMAT (/3H U=,F8.2,13F8.3)
100 U=U+DU                                                                2030
64 CONTINUE
   CALL EXIT
   FND

```

FIGURE 8.
(SHEET 6 OF 6)

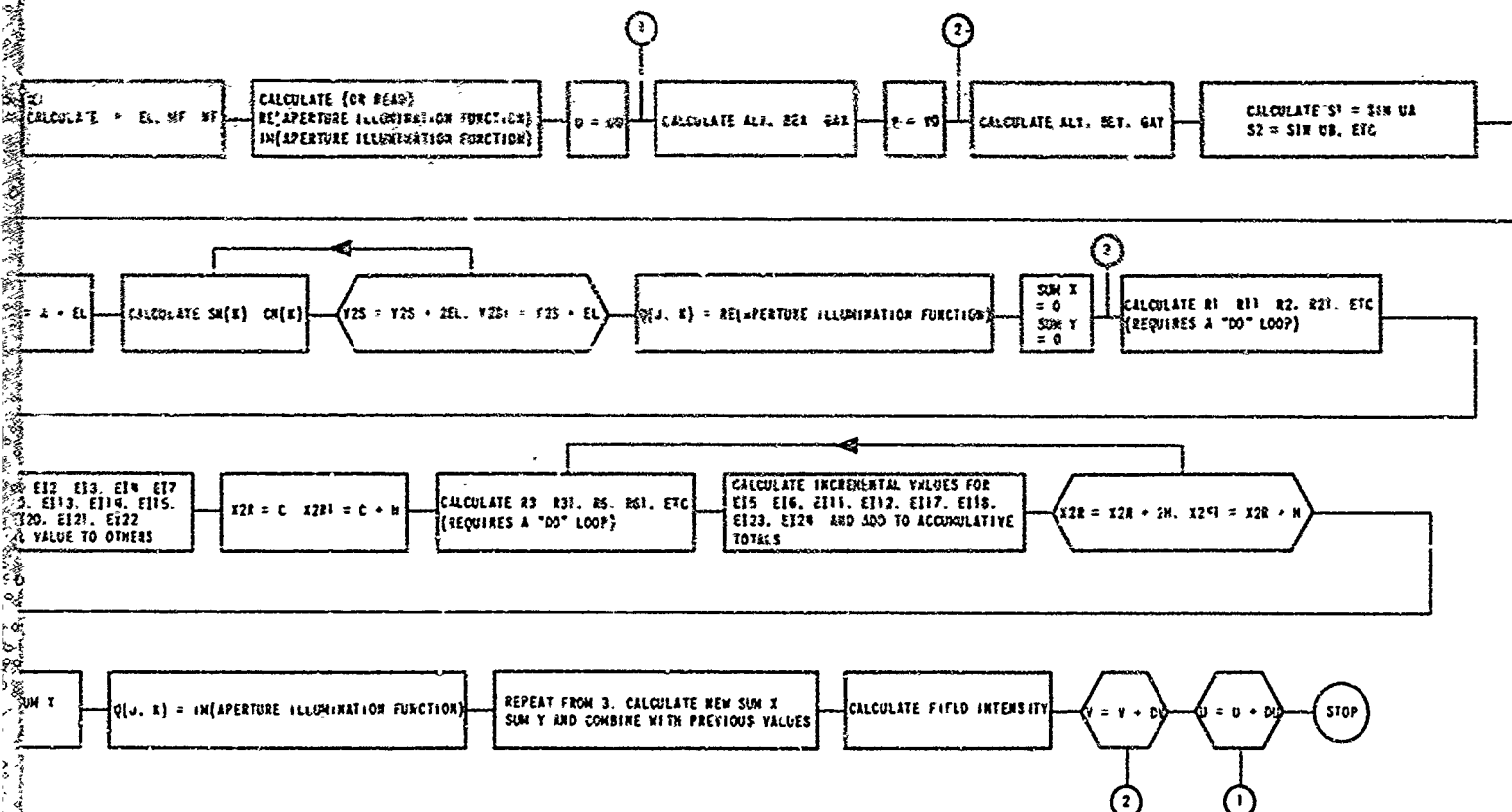


FIGURE 9. FLOW DIAGRAM FOR FILON 2

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13. ABSTRACT This report describes the work performed during the second quarter of a study of a reflector antenna that provides zoom (variable beamwidth) and scan capability using controlled aperture amplitude and phase. The antenna consists of a primary reflector (paraboloid) and a secondary reflector/lens. It operates as a lens in conjunction with one feed for scanning in the receive mode and as a reflector in conjunction with another feed for zooming in the transmit mode. Switching between a zooming transmit mode and a scanning receive mode results in a versatile radar antenna with an inherent duplexing capability. The performance of this antenna system is being analyzed numerically with the aid of digital computers. This report describes the development of a computer program for solving the Fraunhofer aperture integral efficiently and with good accuracy.		

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