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CORRELATION OF AMPLITUDE ANOMALIES AT LASA

9 JUNE 1967

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Washington, D. C.

By

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TELEDYNE, INC.

Under

Project VELA UNIFORM

Sponsored By

ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Test Detection Office
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CORRELATION OF AMPLITUDE ANOMALIES
AT LASA

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ABSTRACT

Peak-to-peak amplitudes recorded by subarray center instruments for independent events have been tested for correlation. Significant positive correlations were detected for events whose focuses originated close to each other. It is also shown that measurement errors tend to reduce the estimate of their coefficient of correlation; the distribution of the coefficient of correlation for small sample sizes are graphically presented.

1. INTRODUCTION

Certain consistencies have been shown to exist among the anomalies of short period teleseismic P-phase amplitudes when those anomalies are grouped according to geographical regions. In this report we will present evidence that the source path strongly influences the observed amplitude anomalies.

2. PROCEDURE

Eight earthquakes from the same geographical area, viz., the Fiji Islands, were selected for this investigation. All eight events occurred at 143° Az with respect to the center of the LASA and ranged from 9,500 km. to 10,500 km. distance. These events were ordered in relation to their distance from the LASA and were partitioned into pairs. The result of this grouping is a set of four pairs of events such that the elements within each pair have epicenters within eight kilometers of each other. We note here that although these epicenters are close together the focuses of the events have a considerably greater range. More will be said later regarding this aspect. We now list the paired events and some of their identifying characteristics.

TABLE 1

	<u>Pair 1</u>		<u>Pair 2</u>		<u>Pair 3</u>		<u>Pair 4</u>	
Event No.	152	253	219	254	197	247	171	342
Azimuth	243°	243°	243°	243°	243°	243°	243°	243°
Distance, km.	9569	9565	9586	9584	10170	10162	10423	10422
Depth, km.	33	35	127	33	338	320	525	511
Magnitude	5.0	5.1	4.9	5.2	4.8	5.5	5.1	5.4

Each pair consists of two independent earthquakes in the sense that all were separated in time by two to ten weeks. Each of the pairs was investigated for the presence of correlation between member events in the following manner. The peak to peak P-phases amplitudes recorded by the center elements of the sub-arrays were determined for each event. In this way, an ordered set of amplitudes indexed by the recording instruments was associated with each event. Thus, in order to detect the presence of correlation between any two events one merely tests for correlation between the two sets of amplitudes generated by the respective events. But before taking this step, it is important to note that an accurate interpretation of the correlation coefficient is, in general, possible "only when the underlying population is normally distributed."⁽²⁾ It has been demonstrated that a lognormal distribution describes the behavior of amplitude anomalies over LASA with remarkable precision.⁽¹⁾ Hence, by using the logarithms of the peak to peak amplitudes, we have characterized each event with a normally distributed set of measurements and can now apply the correlation theory. The estimate of the coefficient of correlation is defined by

$$r = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var } X \cdot \text{Var } Y}}.$$

The following table presents the calculated estimate of the coefficient of correlation along with the 5% and 1% points for the equal-tails test of the hypothesis.

TABLE 2

	<u>Pair 1</u>	<u>Pair 2</u>	<u>Pair 3</u>	<u>Pair 4</u>
r	0.613	0.257	0.409	0.624
5% Point	0.482	0.154	0.532	0.602
1% Point	0.606	0.641	0.661	0.735
Conclusion	Correlated	Not Correlated	Not Correlated	Incon- clusive

One pair appears to be highly correlated, two to be uncorrelated, and one to be a borderline case. Ranking these pairs from highest to lowest degree of correlation yields 1, 4, 3, 2. If we also rank the pairs from those with earthquakes which occurred closest together to those with earthquakes farthest apart, we get 1 (4.5 km), 4 (14.1 km), 3 (19.7 km), 2 (94 km); that is, the pairs have the same ordering in both cases. The events which occurred closest together had the highest correlation and those which occurred farthest apart were least correlated. This relationship is better illustrated in the accompanying drawing, Figure 1A, which depicts the distance between the focuses of the paired events and corresponding estimates, r .

To test further the apparent relationship between degree of correlation and physical proximity of events, all remaining twenty-four possible combinations of paired events were tested for correlation. Since the separation between events in these combinations (except for four pairs) is much greater than that of the four pairs discussed above, one would expect that there would be an absence of correlation in these cases. Table 3 in Appendix A lists the computed correlation coefficients.

The two pairs marked by an asterisk in Table 3 are consistent with the results illustrated in Figure 1A and in Table 2. Specifically, we find (152,254) has $r = 0.613$ and separation of 15 km and (253,254) has $r = 0.521$ and separation of 19.1 km. There still remain five pairs, three borderline and two correlated, which appear inconsistent with Figure 1A and Table 2. This may suggest that these events are, in some sense, "in phase". But also we note that there is one chance in twenty (at the 5% point) that we may compute a coefficient which implies correlation when none exists. Hence we should "expect" that at least one of the three borderline cases is not really correlated.

Table 4 (Appendix A) shows the approximate separation between all of the event combinations listed in Table 3. A graph of correlation coefficients versus separation of focuses is presented in Figure 1B for the four pairs in Table 1 and the four pairs in Table 4 whose focus separation was less than 100 km.

3. ADDITIONAL EVENTS

Similar tests were performed on three teleseismic events which occurred near the surface. It is believed that these three originated at the same location. All possible combinations of pairs were tested and in each instance a high positive correlation was found to exist. It is fairly certain that these events did not all occur at exactly the same azimuth and distance, and this may account for the variation in the computed coefficients. The coefficients were computed to be 0.900, 0.671, and 0.837 for the three combinations of pairs. Figures 2, 3, and 4 are plots of the logarithms of the subarrays center elements for all combinations of the three events.

4. ERRORS OF MEASUREMENT

Hald⁽³⁾ devotes a section to the influence of errors of measurement on the correlation coefficient. Briefly, assume (x_1, x_2) are two true properties whose relationship is being studied. If the errors of measurements are denoted (v_1, v_2) the observations are then $(y_1, y_2) = (x_1 + v_1, x_2 + v_2)$. Assuming these errors are stochastically independent and independent of (x_1, x_2) , the coefficients $\rho(v_1, v_2) = \rho(x_1, v_1) = \rho(x_1, v_2) = \rho(x_2, v_1) = \rho(x_2, v_2) = 0$. Assuming the means of v_1, v_2, x_1, x_2 are zero, we have

$$\text{Var}(y_i) = \text{Var}(x_i) + \text{Var}(v_i) = \sigma_{x_i}^2 + \sigma_{v_i}^2, \quad i = 1, 2 \text{ and by definition}$$
of Covariance,

$$\text{Cov}(y_1, y_2) = E[(x_1 + v_1) \cdot (x_2 + v_2)] - E(x_1 + v_1) \cdot E(x_2 + v_2).$$

Since $E(x_i) = E(v_i) = 0$, $i = 1, 2$, we have

$$\begin{aligned} \text{Cov}(y_1, y_2) &= E[(x_1 + v_1) \cdot (x_2 + v_2)] \\ &= E[x_1 x_2 + x_1 v_2 + x_2 v_1 + v_1 v_2] \\ &= E(x_1 x_2) + E(x_1 v_2) + E(x_2 v_1) + E(v_1 v_2) \end{aligned}$$

but by the assumption of independence, we note that

$$E(x_1 v_2) = E(x_1) \cdot E(v_2) = 0 \cdot 0 = 0;$$

$$E(x_2 v_1) = E(x_2) \cdot E(v_1) = 0 \cdot 0 = 0;$$

$$E(v_1 v_2) = E(v_1) \cdot E(v_2) = 0 \cdot 0 = 0;$$

and this leaves

$$\text{Cov}(y_1, y_2) = E(x_1 x_2). \text{ But recalling that}$$

$$E(x_1) = 0 \text{ and } E(x_2) = 0, \text{ we get}$$

$$\begin{aligned} \text{Cov}(y_1, y_2) &= E(x_1 x_2) - 0 \cdot 0 \\ &= E(x_1 x_2) - E(x_1) E(x_2) \end{aligned}$$

$$\text{Cov}(y_1, y_2) = \text{Cov}(x_1, x_2)$$

$$\text{Now by definition, } \rho(x_1, x_2) = \frac{\text{Cov}(x_1, x_2)}{(\sigma_{x_1})(\sigma_{x_2})} \quad (\text{I})$$

$$\text{or } (\text{Cov}(x_1, x_2)) = [\rho(x_1, x_2)] \cdot (\sigma_{x_1}) \cdot (\sigma_{x_2}) \text{ and finally}$$

$$\text{Cov}(y_1, y_2) = [\rho(x_1, x_2)] \cdot (\sigma_{x_1}) \cdot (\sigma_{x_2}).$$

The definition given by (I) then says

$$\rho(y_1, y_2) = \frac{\text{Cov}(y_1, y_2)}{\sqrt{(\text{Var } y_1)(\text{Var } y_2)}} = \frac{\sigma_{x_1} \sigma_{x_2} [\rho(x_1, x_2)]}{\sqrt{(\sigma_{x_1}^2 + \sigma_{v_1}^2)(\sigma_{x_2}^2 + \sigma_{v_2}^2)}}$$

$$\rho(y_1, y_2) = \frac{\rho(x_1, x_2)}{\left\{ \left[1 + \left(\frac{\sigma_{v_1}}{\sigma_{x_1}} \right)^2 \right] \left[1 + \left(\frac{\sigma_{v_2}}{\sigma_{x_2}} \right)^2 \right] \right\}^{1/2}} \quad (\text{II})$$

This says the correlation coefficient of the true properties plus errors of measurement is always less than the correlation coefficient of the properties themselves. Equation (II) shows that relatively large errors of measurement disguise existing correlations. Therefore, we may generally say that the computed coefficient is always going to be smaller than true ρ .

5. DENSITY FUNCTION OF r

Since we have been working with relatively small samples, we were interested in the behavior of the density function for r when n is small. R. A. Fisher is credited with discovering the distribution of the correlation coefficient.⁽⁴⁾ This rho distribution is a function of n , the sample size and ρ , the correlation coefficient. One expression for the density function is

$$f_n(r) = \frac{n-2}{\pi} (1-\rho^2)^{\frac{n-1}{2}} (1-r^2)^{\frac{n-4}{2}} \int_0^1 \frac{\chi^{n-2}}{(1-\rho r \chi)^{n-1}} \frac{d\chi}{\sqrt{1-\chi^2}}. \quad (\text{III})$$

The integral term can be simplified by setting $x = \sin \theta$, $dx = \cos \theta d\theta$ and correcting the limits to $[0, \pi/2]$. The integral becomes

$$\int_0^{\pi/2} \frac{\sin^{n-2} \theta \cos \theta d\theta}{(1-\rho r \sin \theta)^{n-1} \sqrt{1-\sin^2 \theta}}$$

Noting the identity

$$1 - \sin^2 \theta = \cos^2 \theta$$

we get
$$\int_0^{\pi/2} \frac{\sin^{n-2} \theta d\theta}{(1-\rho r \sin \theta)^{n-1}}$$

Not finding a closed form solution for this integral, a numerical solution using Romberg integration was devised by Dr. C. S. Duris.

When $n = 2$, it is obvious from (II) that $f_2(r) = \delta$ and therefore the coefficient of correlation is necessarily ± 1 . Graphs for $n = 3, 4, 6$ and 50 are presented Figures 5, 6, 7 and 8 in Appendix C along with a listing of the computer program.

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- (1) Broome, P. W., Frankowski, D. E., and Klappenberger, F. A., 9 January 1967, "Amplitude Anomalies at LASA", Report No. LL-4, Applied Research, Teledyne, Inc., Alexandria, Virginia.
- (2) Hald, A., 1952, "Statistical Theory with Engineering Applications", p. 613, New York, John Wiley and Sons, Inc.
- (3) op. cit., p. 615
- (4) Fisher, R. A., "Frequency Distribution of the Values of the Correlation Coefficient in Samples from an Indefinitely Large Population", Biometrika 10 (1915), p. 507.

A P P E N D I X A

Earthquake Data

PAIR 1



$$r = 0.613$$

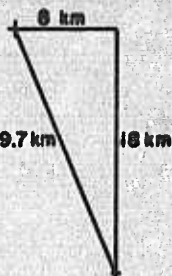
PAIR 2

2 km



$$r = 0.257$$

PAIR 3



$$r = 0.409$$

PAIR 4



$$r = 0.624$$

Figure 1A

Graph Of Correlations Between Events
Whose Focus Separation Is 100 KM.

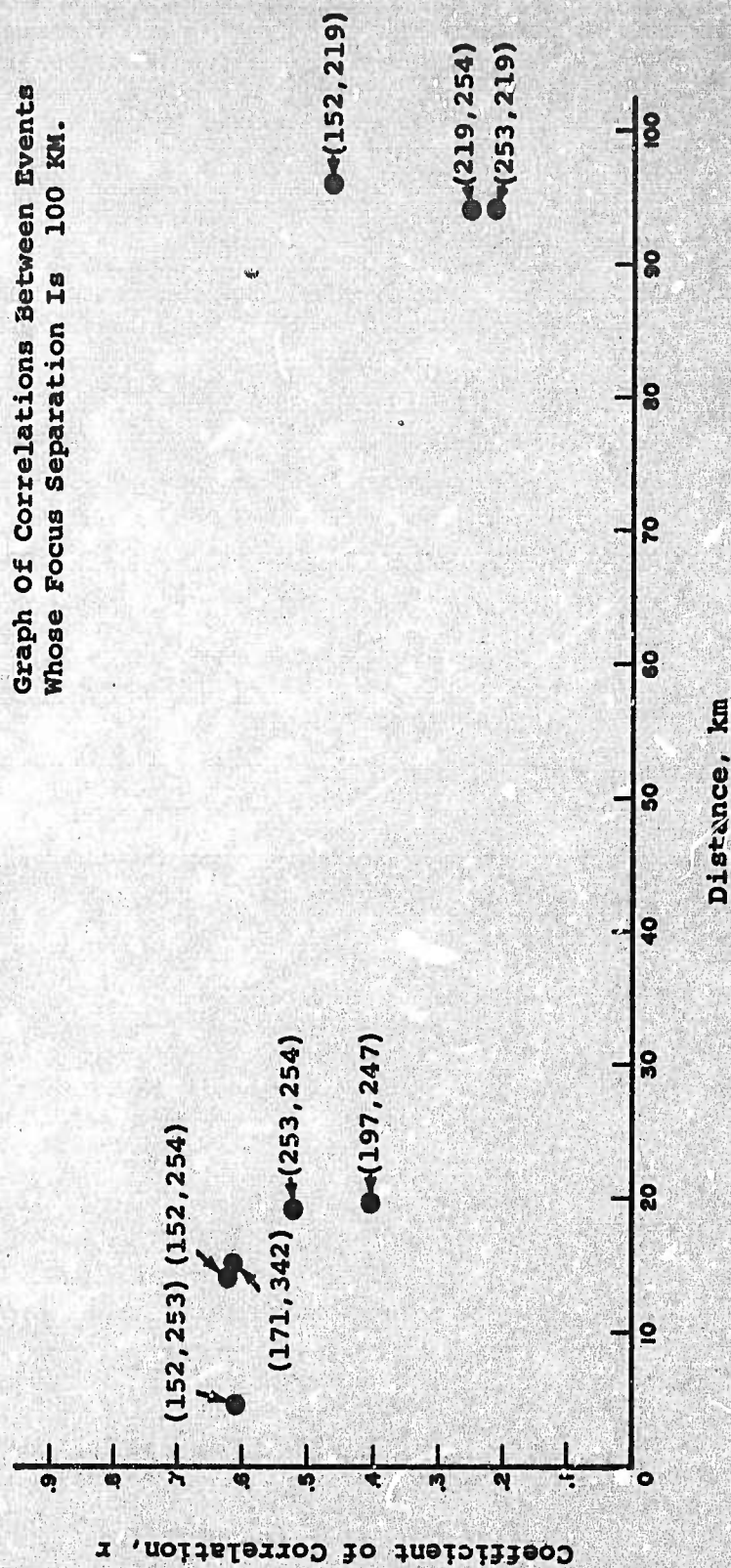


Figure 1B

TABLE 3

Paired Events	r	5% Point	1% Point	Conclusion
171,247	0.469	0.482		No Correlation
342,247	0.462	0.666		" "
171,197	0.178	0.532		" "
342,197	0.480	0.754		" "
152,219	0.468	0.497		" "
253,219	0.219	0.497		" "
197,219	0.300	0.532		" "
247,219	0.667	0.514		" "
171,219	0.402	0.482		" "
342,219	0.431	0.576		" "
152,254	0.613	0.497	0.623	Borderline (15 km)
253,254	0.521	0.497	0.623	Borderline (19.1 km)
197,254	0.241	0.553		No Correlation
247,254	0.050	0.497		" "
171,254	0.286	0.482		" "
342,254	0.216	0.666		" "
197,152	0.591	0.514	0.641	Borderline
247,152	0.708	0.497	0.623	Correlation
171,152	0.492	0.482	0.606	Borderline
342,152	0.366	0.666		No Correlation
197,253	0.397	0.532		" "
247,253	0.653	0.482	0.606	Correlation
171,253	0.543	0.482	0.606	Borderline
342,253	0.199	0.666		No Correlation

TABLE 4

COMBINATIONS	a (1) RELATIVE LATERAL DISPLACEMENT	b (2) RELATIVE DEPTH DISPLACEMENT	c (3) ABSOLUTE DISPLACEMENT
171-247	261	205	332
342-247	260	191	323
171-197	253	187	315
342-197	252	173	306
152-219	17	94	96
253-219	21	92	94
197-219	584	211	621
247-219	576	193	608
171-219	837	398	927
342-219	836	384	920
152-254	15	0	15
253-254	19	2	19
197-254	586	305	661
247-254	578	287	645
171-254	839	492	973
342-254	838	478	965
197-152	601	305	674
247-152	593	287	659
171-152	854	492	985
342-152	853	478	978
197-253	605	303	677
247-153	597	285	661
171-253	858	490	988
342-253	857	476	980

(1) Absolute difference of range
between focuses

(2) Absolute difference of depth
between focuses

$$(3) c = (a^2 + b^2)^{1/2}$$

(Separation of Focuses Presented in Table 3 Of The Text.)

A P P E N D I X B

Additional Event

30

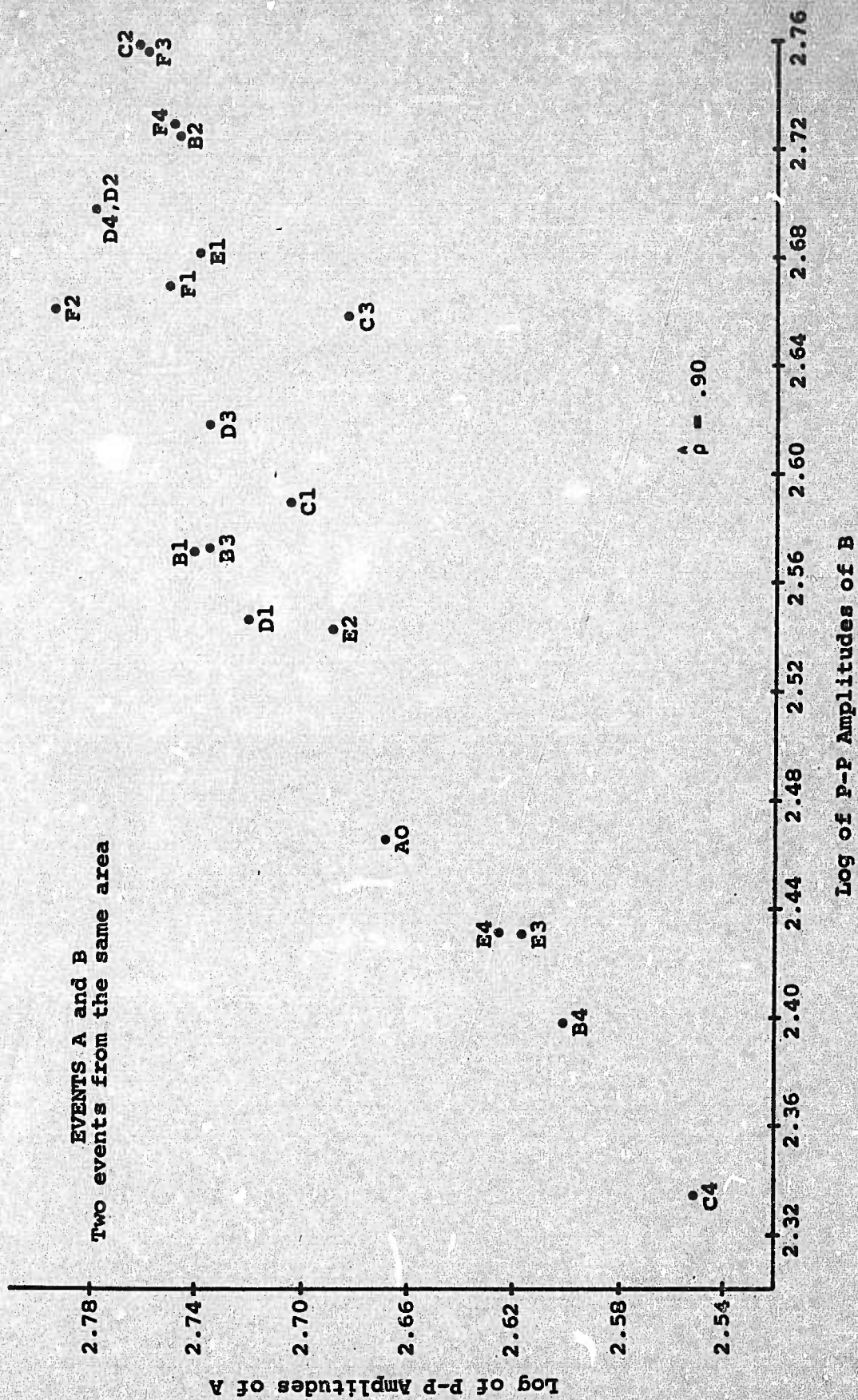


Figure 2

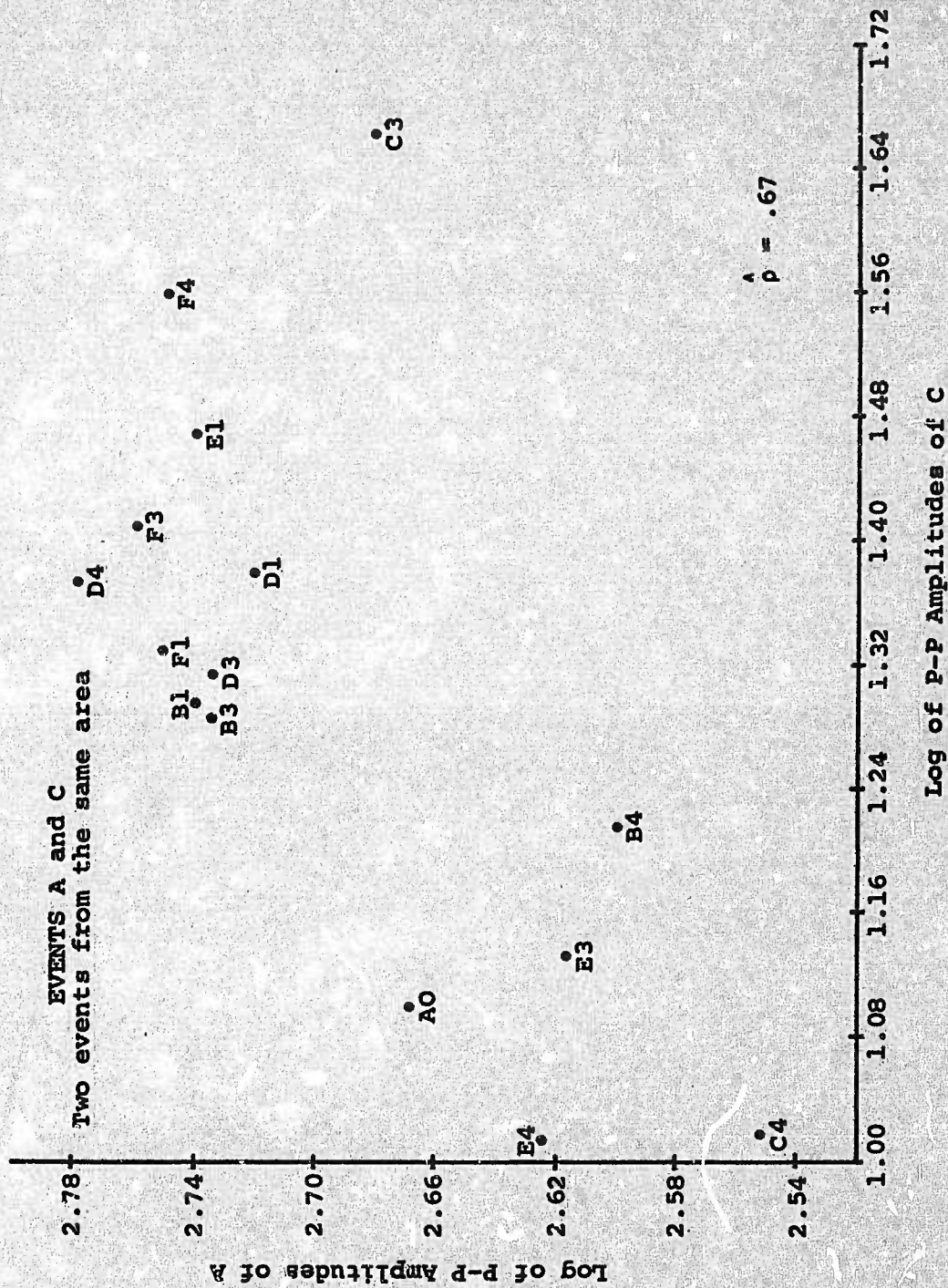


Figure 3

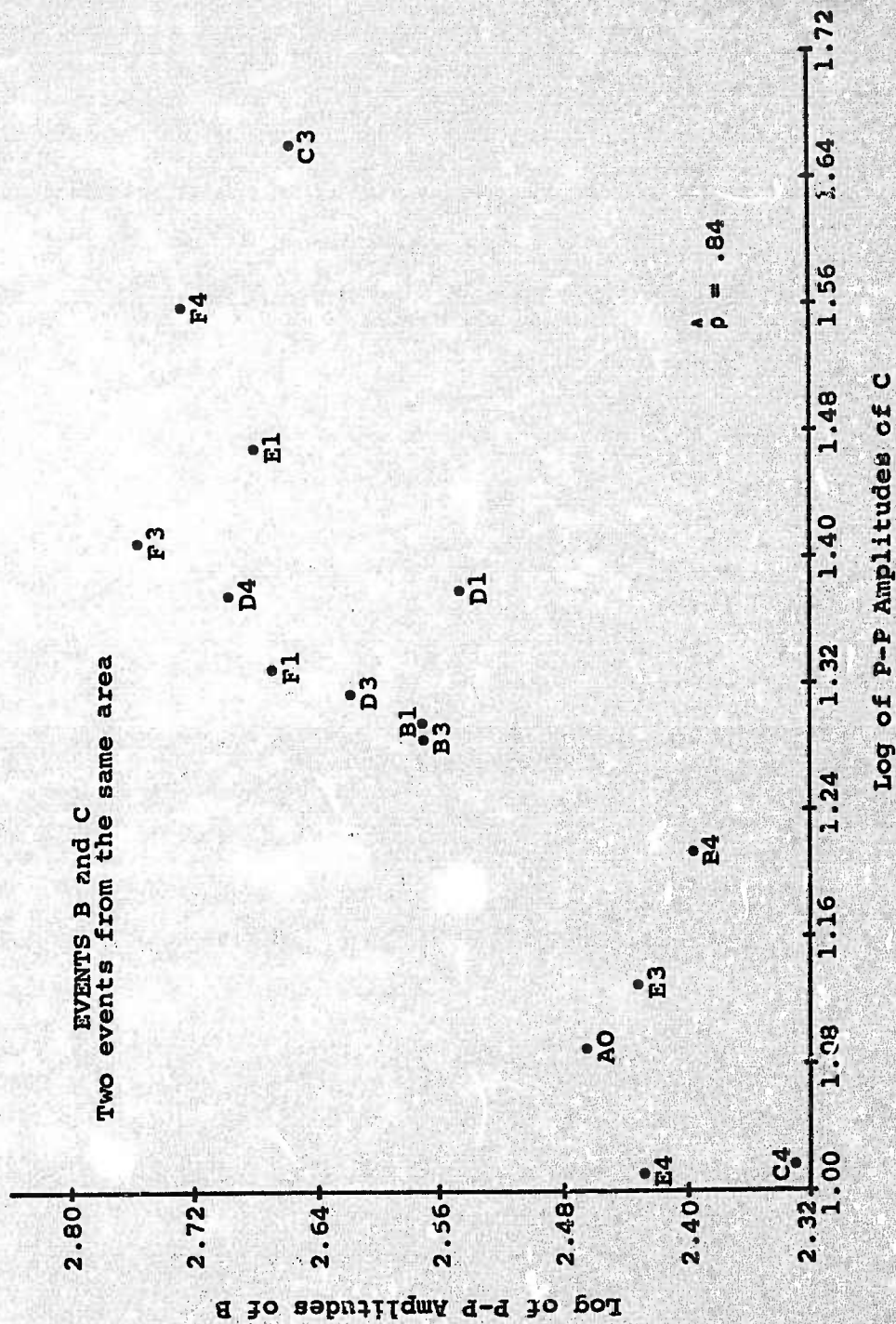


Figure 4

A P P E N D I X C

Density Functions of "r"

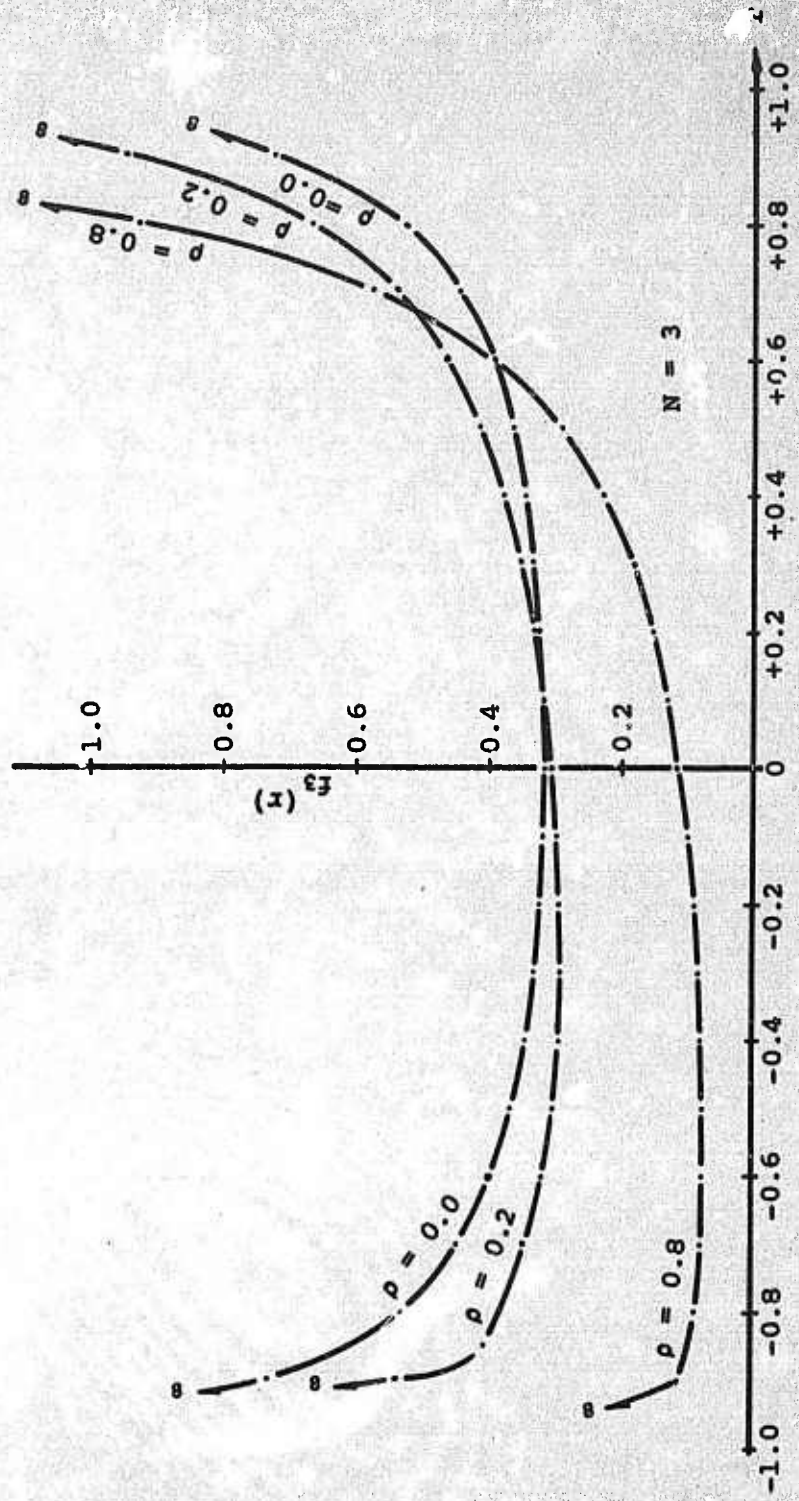


Figure 5

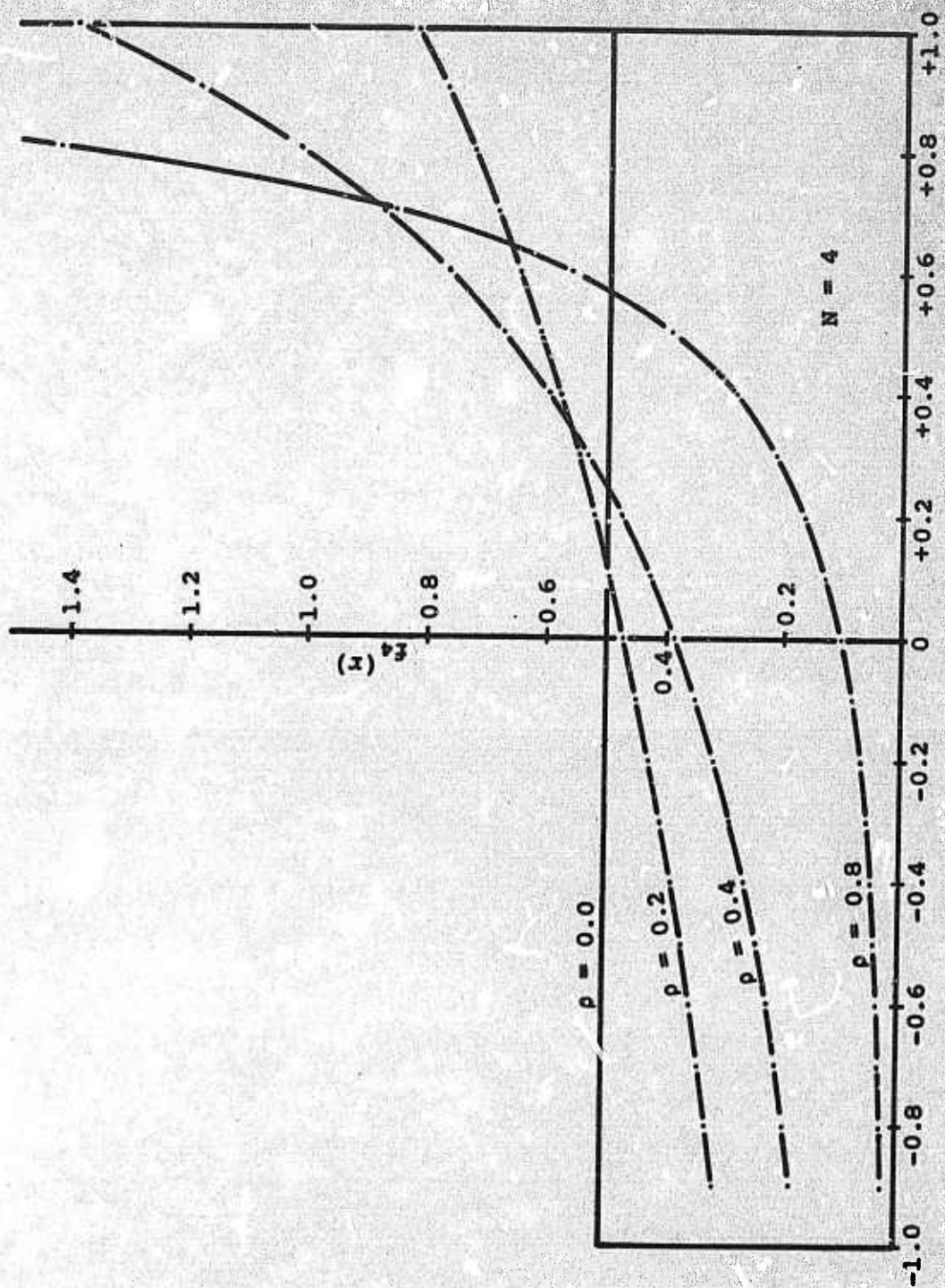


Figure 6

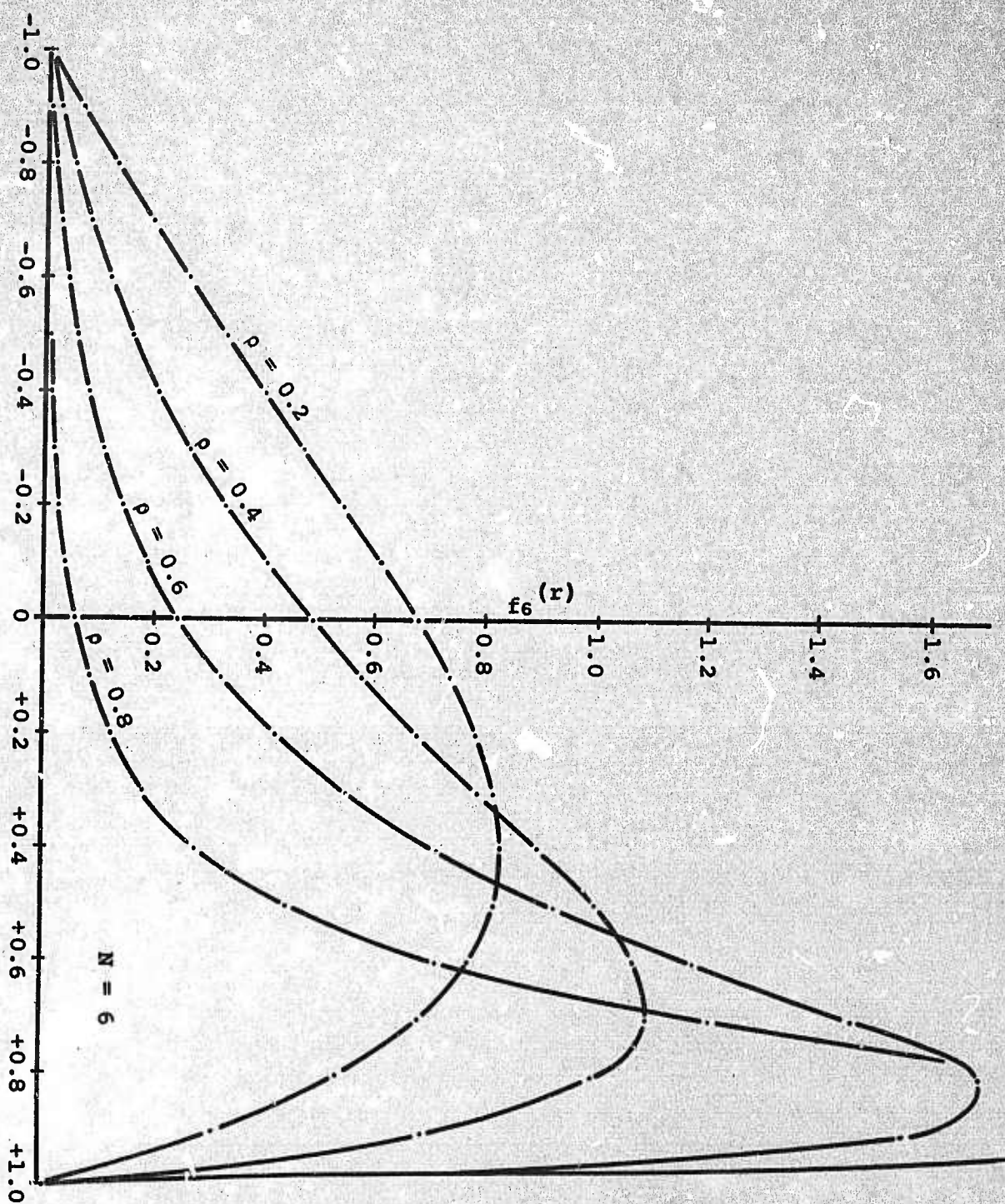


Figure 7

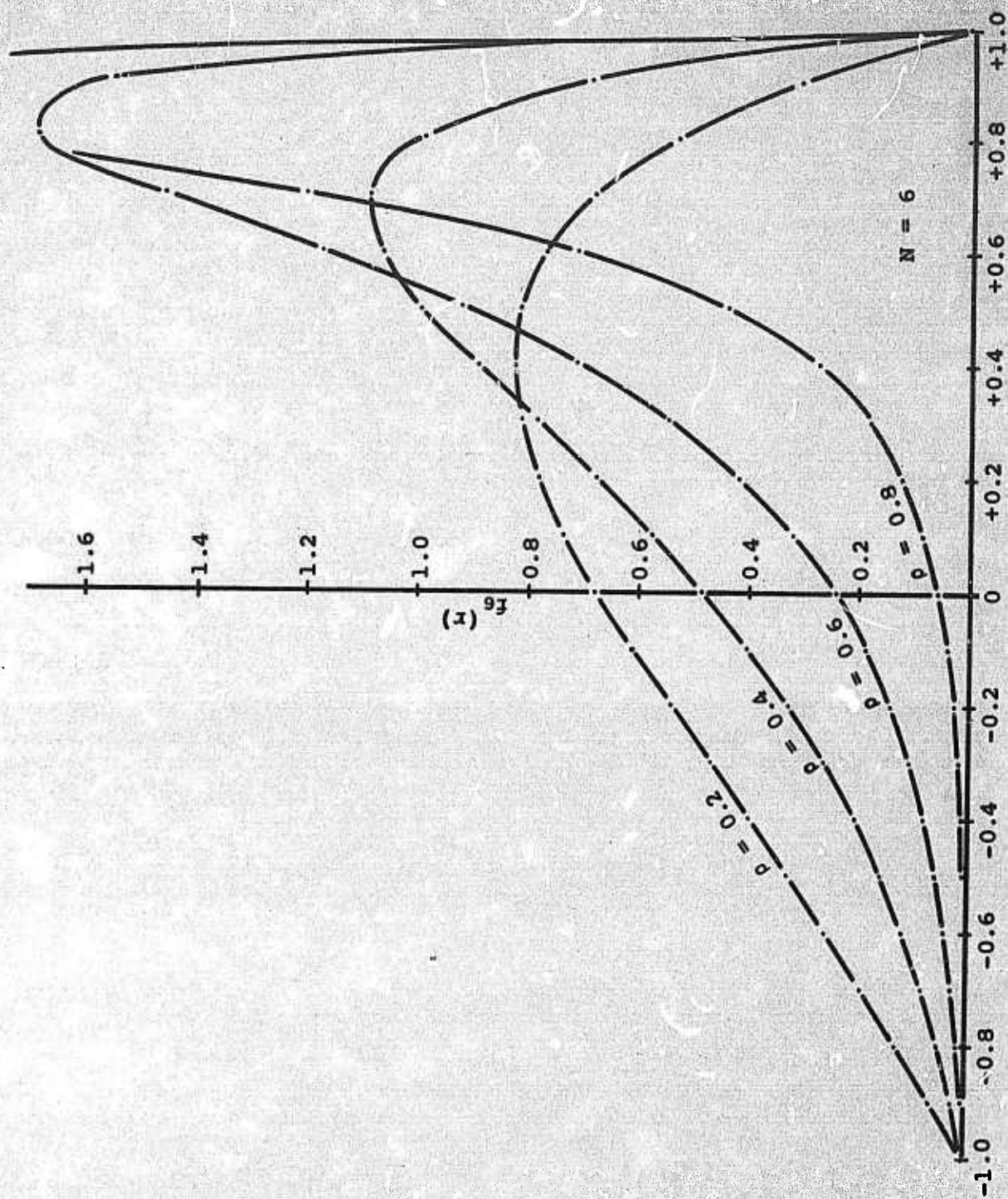


Figure 7

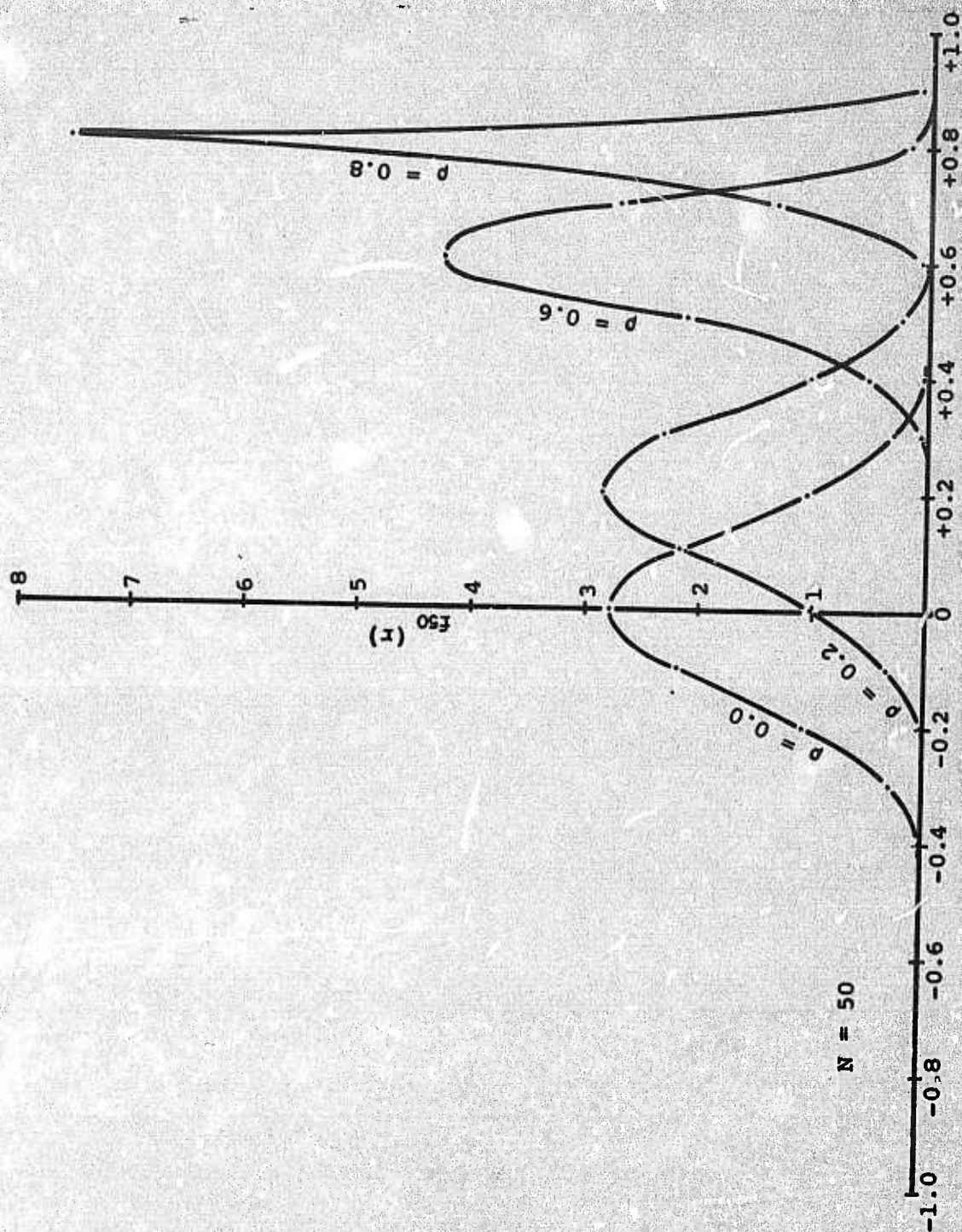


Figure 8

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14. KEY WORDS	LINK A		LINK B		LINK C	
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