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LARGE-ARRAY SIGNAL AND NOISE ANALYSIS

Special Scientific Report No. 14

WIENER NONTIME-STATIONARY PROCESSING

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ABSTRACT

In applying the Wiener multichannel time-series processing theory to seismic arrays for teleseismic signal extraction, the transient nature of the signal is not used. This report presents the theoretical development of optimum nontime-stationary signal-extraction filters which take advantage of the additional information of known signal-arrival time. The matrix inversion is made computationally practical by using an extension of the Levinson technique to determine prediction filters several points into the future. A sequence of programs to accomplish the filter design and application is described and illustrated by a sample problem. Significant improvement in mean-square error was not obtained for this sample problem compared to classical Wiener filters. Full evaluation of this technique requires additional data processing and examination with respect to criteria other than mean-square error.

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SUMMARY AND INTRODUCTION

In applying the Wiener multichannel time-series processing theory to seismic arrays for teleseismic signal extraction, the transient nature of the signal is not used. This report presents the theoretical development of optimum nontime-stationary signal-extraction filters which take advantage of the additional information of known signal-arrival time. The matrix inversion is made computationally practical by using an extension of the Levinson technique to determine prediction filters several points into the future. A sequence of programs to accomplish the filter design and application is described and illustrated by a sample problem. Significant improvement in mean-square error was not obtained for this sample problem compared to classical Wiener filters. Full evaluation of this technique requires additional data processing and examination with respect to criteria other than mean-square error.

Multichannel data from an array of seismometers are assumed to be

$$x_{i,t} = s_{i,t} + n_{i,t}$$

where

$s_{i,t}$ = seismic signal from seismometer i at time t

$n_{i,t}$ = noise from seismometer i at time t

$x_{i,t}$ = output of seismometer i at time t

It is desired to find the filters or linear combinations of the output for times $t = 1, \ldots, k+m$ which are best, in the mean-square-error sense, for estimating the signal $s_{i,t}$. In addition, we assume that the signal-arrival time is known to be at time $t = k+1$ for each seismometer.

The purpose of this report, therefore, is to evaluate and describe the signal-extraction process developed to use the additional information of known signal-arrival times. A series of programs has been written to perform the processing, and data recorded at LASA have been selected and processed. Results of the nontime-stationary processing have been compared with the usual MCF approach by theoretical mean-square-error comparisons and with actual data.

SECTION II

THEORETICAL DEVELOPMENT

The notation to describe the actual data processing is more difficult to follow than necessary for an explanation of the method. In this section, the method is explained generally; and the exact steps of the computation are given explicitly in Appendix A.

Let the row vector n_1 denote the multichannel pure noise data $(n_{i,t})$ $t = 1, \ldots, k$, which precedes the signal arrival. The vector $n_2 + s$ will denote the multichannel signal-plus-noise data $(s_{i,t} + n_{i,t})$ in the time interval $t = k+1, \ldots, k+m$. It is desired to find the least-mean-square estimate of s by filtering the data set $(n_1, n_2 + s)$.

Most results in this section can be easily seen in terms of the following lemma: Let a random vector x with zero-mean be partitioned in the form $x = (x_1, x_2)$ with covariance

$$\Omega = Ex^T x = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \tag{2-1}$$

where $Ex_i^T x_j = \Omega_{ij}$. Given x_1, then the least-mean-square estimates of x_2 are

$$\hat{x}_2^T = \Omega_{21} \Omega_{11}^{-1} x_1^T = \Gamma x_1^T \tag{2-2}$$

(assuming a mean of 0) with the filters Γ defined by

$$\Gamma = \Omega_{21} \Omega_{11}^{-1} \tag{2-3}$$

$$(\Gamma \Omega_{11})^T = \Omega_{21}^T$$

or

$$\Omega_{11} \Gamma^T = \Omega_{21}$$

The covariance matrix of the error is

$$E(x_2 - \hat{x}_2)^T (x_2 - \hat{x}_2) = \Omega_{22} - \Omega_{21} \Omega_{11}^{-1} \Omega_{12} = P \tag{2-4}$$

Equations (2-3) and (2-4) are often expressed in matrix form as

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} -\Gamma^T \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ P \end{bmatrix} \tag{2-5}$$

We now make the correspondence $x_2 = s$ and $x_1 = (n_1, n_2 + s)$ so that if $S = Es^T s$ and

$$E(n_1, n_2)^T (n_1, n_2) = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \tag{2-6}$$

with $En_i^T n_j = N_{ij}$, it follows from Equation (2-3) of the lemma that the optimum filters $F = (F_1 \; F_2)$ are given by

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} + S \end{bmatrix} \begin{bmatrix} F_1^T \\ F_2^T \end{bmatrix} = \begin{bmatrix} 0 \\ S \end{bmatrix} \qquad (2\text{-}7)$$

and the signal estimates are

$$\hat{s}^T = F(n_1, n_2 + s)^T \qquad (2\text{-}8)$$

with mean-square error given by

$$MSE_1 = S - F(0\ S)^T = S - F_2 S \qquad (2\text{-}9)$$

Although the problem is now solved, the solution is not in the best form for calculation. We observe that Equation (2-5) with $x_1 = n_1$ and $x_2 = n_2$ can be modified to

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} + S \end{bmatrix} \begin{bmatrix} -\Gamma^T \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ P + S \end{bmatrix} \qquad (2\text{-}10)$$

by adding S to N_{22} and to P. Then, by multiplying on the right by $(P + S)^{-1} S$, we obtain Equation (2-7). Thus, we have

$$F_2^T = (P + S)^{-1} S \qquad (2\text{-}11)$$

$$F_1^T = -\Gamma^T F_2^T$$

so that the nontime-stationary filter estimates are

$$\hat{s}^T = F_2(-\Gamma, I)(n_1, n_2 + s)^T \tag{2-12}$$

$$\hat{s}^T = F_2(n_2^T + s^T - \Gamma n_1^T)$$

In summary, the optimum filters to apply to $(n_1, n_2 + s)$ for signal estimation are equivalent to estimating n_2 from n_1 and subtracting the estimates from the observed data $n_2 + s$, then solving the smaller dimensioned problem of estimating s by the optimum filters operating on $s + n_e$.

Results given in a later section in terms of MSE or the conventional and nontime-stationary filters are more meaningful if we present some theoretical relations which are known to hold for the particular signal model used as the example in this report. Comparing MSE_1 from Equation (2-9) with conventional Wiener MSE provides an optimistic estimate, for certain signal models, of the expected improvement from using the filters F instead of the conventional filters. This comparison is optimistic because conventional Wiener MSE values are derived under the assumption that the data to be processed are $(n_1 + s_1, n_2 + s_2)$. The MSE_2 obtained when the Wiener filters are applied to $(n_1, n_2 + s)$ will be less than the conventional MSE. These statements can be made algebraically precise by again using the lemma. Wiener filters f are conventionally designed as the optimum filters applied to $n_1 + s_1$, $n_2 + s_2$ to estimate s_0, a scalar component of either s_1 or s_2. In the present comparison, we restrict s_0 to be a scalar component of s_2. We, therefore, employ the lemma by making the correspondence $x_2 = s_0$, $x_1 = (n_1 + s_1, n_2 + s_2)$ to obtain the filter

$$f = \gamma \Omega^{-1}$$

from Equation (2-2), where

$$E\, x_1^T\, x_1 = \Omega$$

$$(E\, s_0 s_1,\ E\, s_0 s_2) = (\gamma_1\ \gamma_2) = \gamma$$

$$\sigma_0^2 = E\, x_2 x_2$$

$$MSE = \sigma_0^2 - f\gamma^T$$

An equivalent problem is to find the filter f, minimizing the quadratic form

$$MSE = (1, -f) \begin{bmatrix} \sigma_0^2 & \gamma \\ \gamma^T & \Omega \end{bmatrix} (1, -f)^T$$

Now let

$$N_{ij} = E\, n_i^T\, n_j$$

and

$$S_{ij} = E\, s_i^T\, s_j$$

Then the Wiener filters f applied to the data $(n_1, n_2 + s_2)$ results in a mean-square error

$$MSE_2 = (1, -f) \left\{ \begin{bmatrix} \sigma_0^2 & \gamma \\ \gamma^T & \Omega \end{bmatrix} - \begin{bmatrix} 0 & \gamma_1 & 0 \\ \gamma_1^T & S_{11} & S_{12} \\ 0 & S_{21} & 0 \end{bmatrix} \right\} (1, -f)^T$$

so that

$$MSE_2 = MSE - (1, -f) \begin{bmatrix} 0 & \gamma_1 & 0 \\ \gamma_1^T & S_{11} & S_{12} \\ 0 & S_{21} & 0 \end{bmatrix} (1, -f)^T$$

If the signal model is such that $S_{12} = 0$, then since S_{11} is positive definite, one has the heirarchy $MSE_1 \leq MSE_2 \leq MSE$. Plotted values of MSE_1 and MSE are discussed in the next section for a specific example. The comparitive approprite term MSE_2 was not evaluated.

SECTION III

EVALUATION

A. DATA DESCRIPTION

Two adjacent noise samples recorded at LASA subarray B1 were selected for the evaluation of the nontime-stationary technique.

- Noise 1, 25 March 1966, 04:26:12.8 to 04:34:12.7
- Noise 2, 25 March 1966, 04:34:13.1 to 04:42:13.0

Both noise samples has been filtered with an antialiasing filter and resampled to 100 mils. To take advantage of the program restrictions, seven channels of data recorded at the center seismometer and two outer rings were selected (Figure III-1).

Another antialiasing filter was applied, and the data were resampled to 300 mils. Channel 1 of noise sample 1 was used to design a 21-point deconvolution filter, and the filter was applied to both noise samples. A correlation matrix was generated using the 300-mil whitened noise sample 1.

B. DISCUSSION OF PROCESSING RESULTS

The noise correlation matrix was used as input data to the sequence of programs described in Appendix B. Twenty-point multichannel prediction filters were designed to predict data vectors 1, 2, ..., 15 points ahead of the data. To provide a better evaluation of the theoretical quality of the multichannel filters, single-channel prediction filters were designed also. Predictability of each channel (100-percent minus percent of mean-square error) by the multichannel filters was plotted along with the single-channel predictability (Figures III-2 and III-3). Average multichannel predictability ranged from 48.7 percent at one point ahead to 16.3 percent at 15 points ahead, while single-channel predictability ranged from 12.7 to 6.6 percent over the same number of points. The multichannel predictability

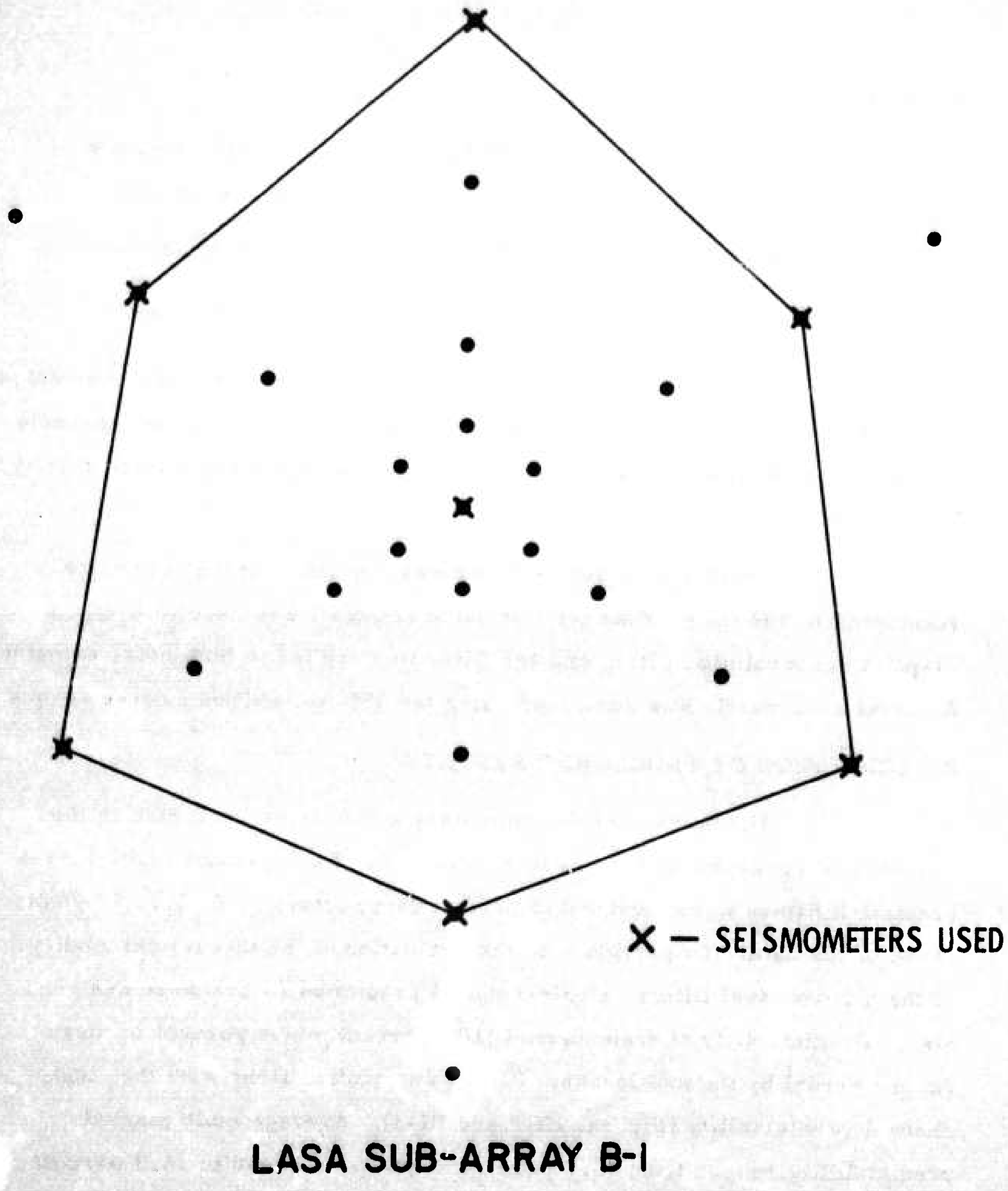

Figure III-1. LASA Subarray B1

decreases rapidly between M = 1 and M = 6; and from 6 points ahead, there is a consistent difference of about 10 percent between multichannel and single-channel predictability.

The single-channel predictability plots in Figure III-2 indicate that the 21-point deconvolution filter did not successfully prewhiten the data. The first 10 lags of the whitened correlation were plotted and appear to be behaving reasonably. It appears now that a shorter deconvolution filter would have been more successful. The expense of returning to the preprocessing stage and the absence of encouraging results made shorter prediction filters seem like the most efficient way to overcome this difficulty. Therefore, 5-point prediction filters to predict 1 to 4 points ahead were designed, using the same prewhitened correlations.

Single-channel and multichannel predictability are plotted in Figure III-3. The single-channel predictability on all channels has been significantly reduced by prewhitening, and channel 1 is approximately 0 as would be expected. The difference between multichannel and single-channel predictability appears to remain about the same.

An infinite-velocity signal model with gain slop was designed, using as the zero lag of the autocorrelation three times the zero lag of the noise autocorrelation and 0's for the other lags. The signal model and prediction filters were input to R687, and 10-point multichannel signal extraction filters were designed and written on the CPT for application. The values of MSE were computed as described in Section II and printed. The same signal model was added to the complete noise-correlation set, and a Wiener multichannel filter to estimate channel 1 was designed. The filter was designed to a length of 35 points, and the mean-square error was printed.

Since the data used in this experiment has not been normalized, the mean-square errors were divided by the zero-lag signal correlation for comparison. Plotted were the normalized MSE from the 35-point Wiener

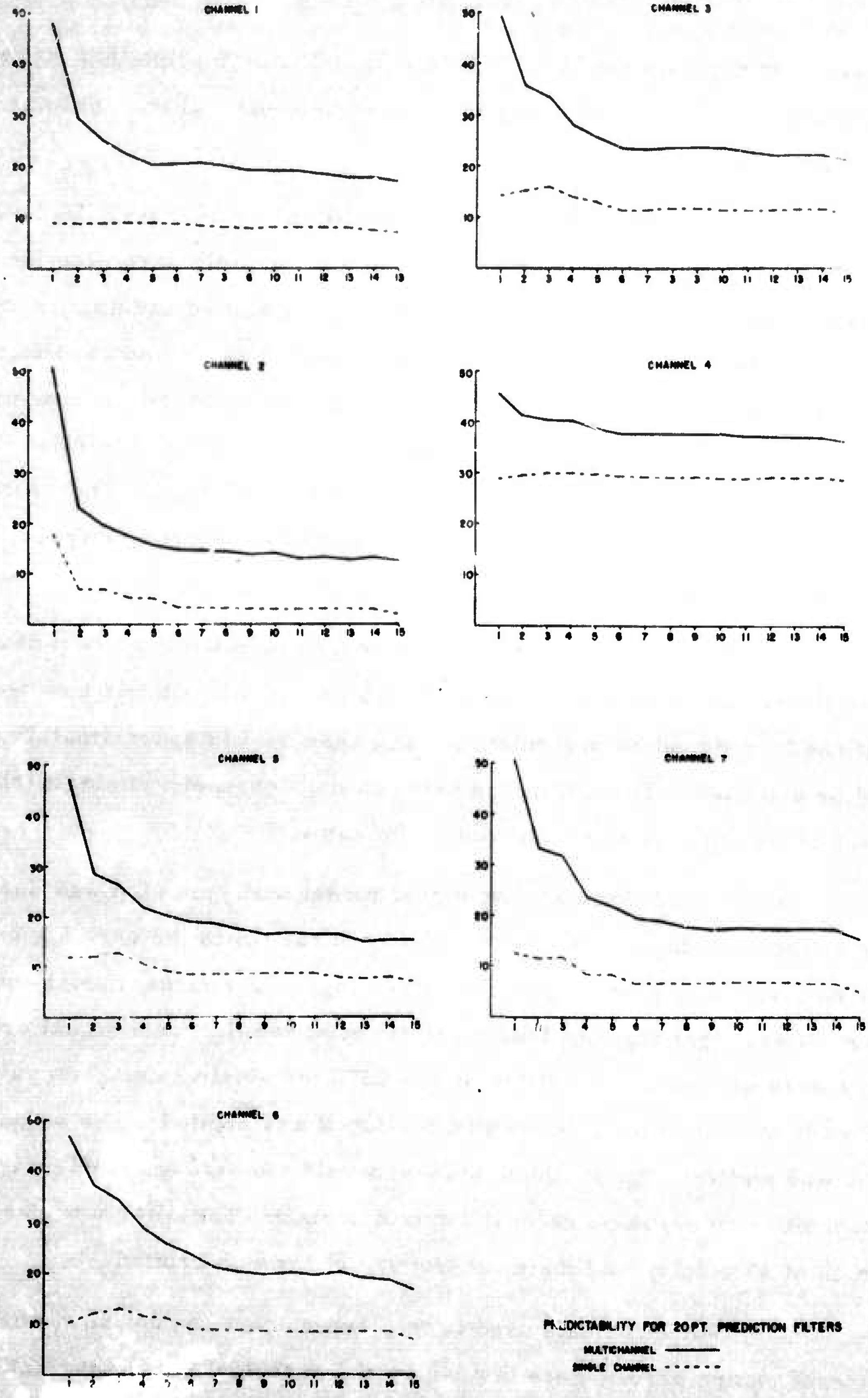

Figure III-2. Predictability for 20-Point Prediction Filters

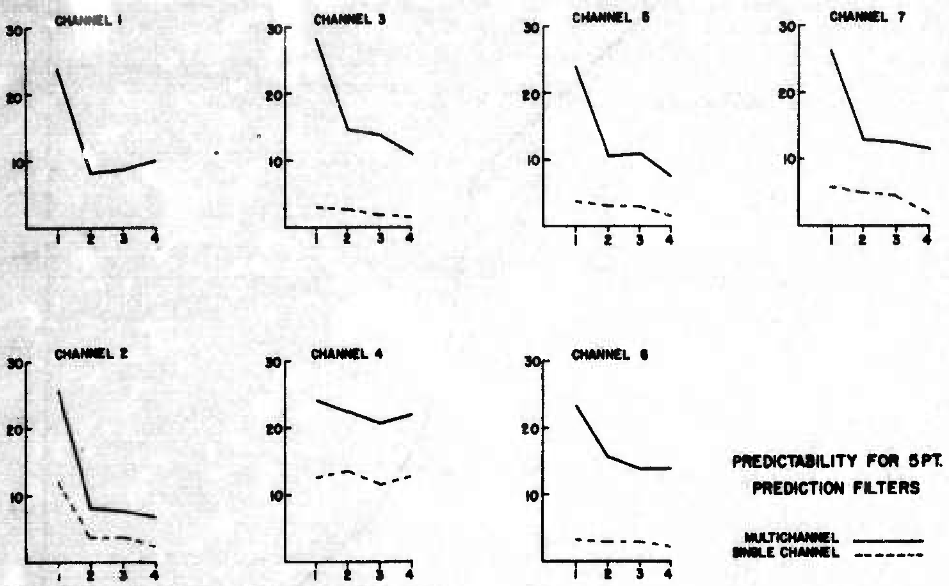

Figure III-3. Predictability for 5-Point Prediction Filters

filter design (as an x on Figure III-4) and the normalized MSE_1 for each point of channel 1 from the R687 output.

Once again, Wiener MCF's were designed for the purpose of comparing mean-square error. Filters of 9 points were computed, estimating each of the four signal points; results are shown in Figure III-4.

To provide a visual evaluation of the signal-extraction process, a signal was added to each channel of noise 2 at the same time interval. The time-aligned signal plus noise was input to R688 for application of the filters designed by R685 and R687. First, 10 of the noise-prediction filters were applied to the 20 points of noise just ahead of the inbedded signal to predict the noise in those 10 points of signal plus noise. The noise estimate was subtracted. Then the 10-point signal-extraction filters were applied to the signal-plus-noise prediction error. Figure III-5 shows a plot of the seven channels of data with a vertical axis to indicate the signal onset, the noise

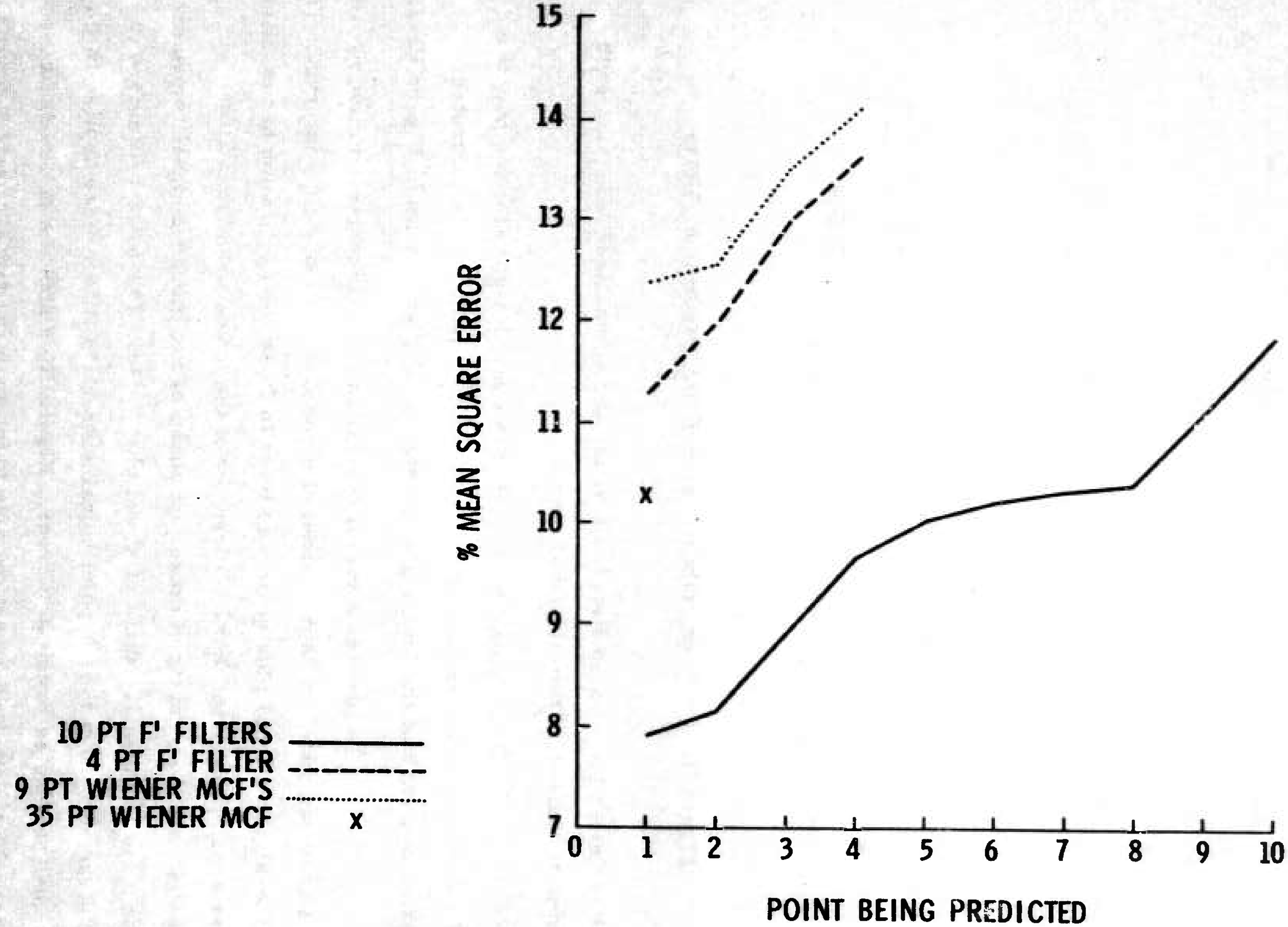

Figure III-4. Normalized MSE for Nontime-Stationary Filters

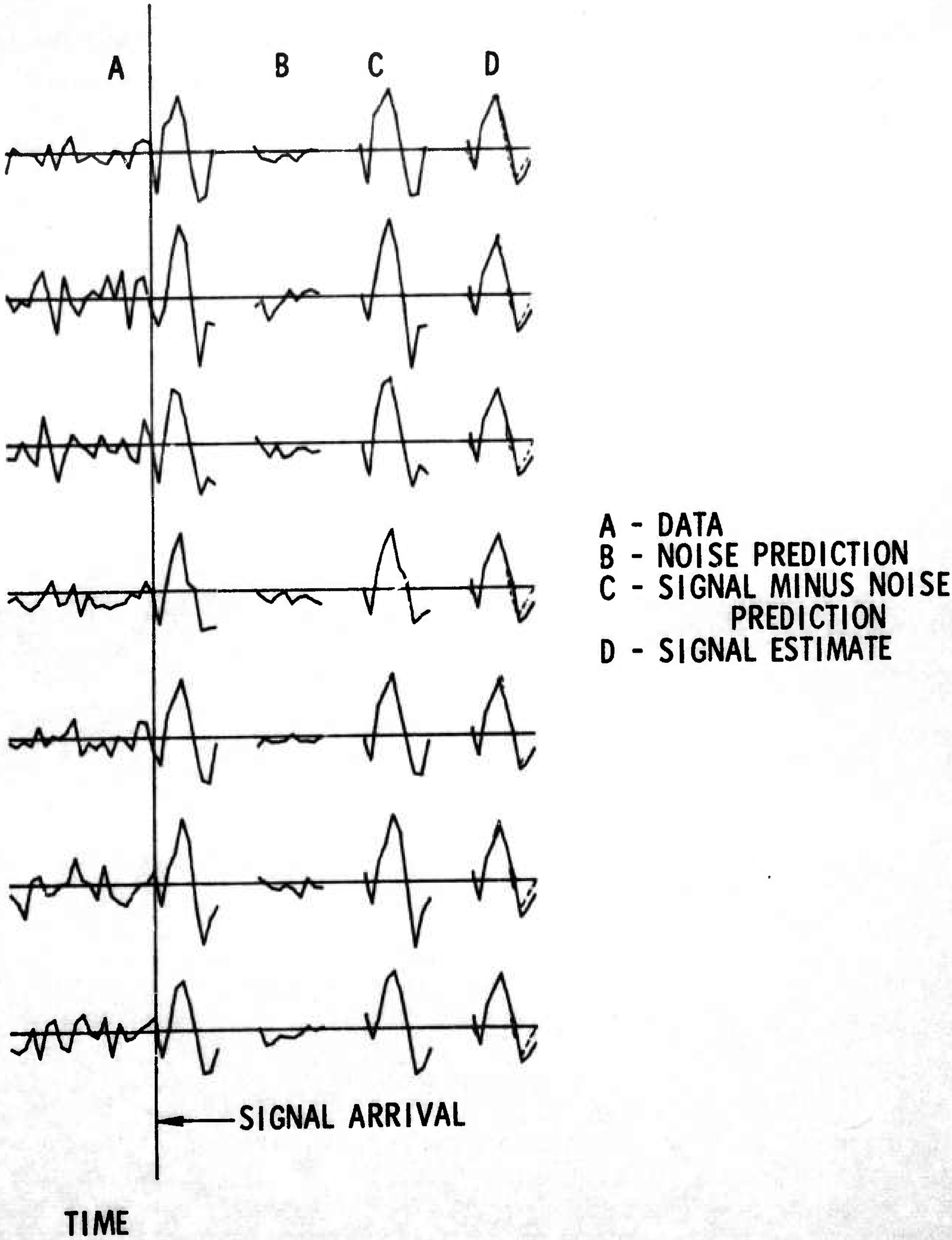

Figure III-5. Nontime-Stationary Signal Estimates prediction, the signal-plus-noise prediction error, and the signal estimate. Superimposed on the signal estimate is a broken-line image of the actual signal waveform.

SECTION IV

CONCLUSIONS

There was no significant improvement in mean-square error beyond the first four signal points. As a general signal-processing scheme, the nontime-stationary technique does not appear to offer any significant advantages. However, the slight improvement in the initial points suggests that the method may have value in determining first motion.

For determination of first motion, the mean-square-error criterion may have only limited significance. If such a study were desired, it would probably require the processing of several events of low signal-to-noise ratio.

APPENDIX A

PROGRAM SPECIFICATION

APPENDIX A
PROGRAM SPECIFICATION

A. NOISE PREDICTION, STEP 1

The problem is most easily solved when considered as a 2-step signal-extraction process. In the first step, the k-sample vectors of noise are used to predict the noise in the m-sample vectors of signal plus noise. Then, the noise prediction is subtracted from the signal plus noise, leaving signal-plus-noise prediction error. In the second step, optimum signal-extraction filters are designed and applied to the signal-plus-noise prediction error.

To implement the process, the first step is to design prediction filters capable of predicting noise vectors in the signal-plus-noise area. To express this problem in familiar notation, reorder the data so that

$$\bar{n}^T = \left(\bar{n}^T_{k+m}, \bar{n}^T_{k+m-1}, \ldots, \bar{n}^T_1\right)$$

and define

$$N_i = E\,\bar{n}_t\,\bar{n}^T_{t+i}$$

so that the variance-covariance matrix of $\bar{n}^T$ is

$$E\left[\bar{n}\,\bar{n}^T\right] = \begin{bmatrix} E\left[\bar{n}_{k+m}\,\bar{n}^T_{k+m}\right] & E\left[\bar{n}_{k+m}\,\bar{n}^T_{k+m-1}\right] & \cdots & E\left[\bar{n}_{k+m}\,\bar{n}^T_1\right] \\ E\left[\bar{n}_{k+m-1}\,\bar{n}^T_{k+m}\right] & E\left[\bar{n}_{k+m-1}\,\bar{n}^T_{k+m-1}\right] & \cdots & E\left[\bar{n}_{k+m-1}\,\bar{n}^T_1\right] \\ \vdots & & & \vdots \\ E\left[\bar{n}_1\,\bar{n}^T_{k+m}\right] & E\left[\bar{n}_1\,\bar{n}^T_{k+m-1}\right] & \cdots & E\left[\bar{n}_1\,\bar{n}^T_1\right] \end{bmatrix} = \begin{bmatrix} N_0 & N_{-1} & \cdots & N_{-k-m+1} \\ N_1 & N_0 & \cdots & N_{-k-m+2} \\ \vdots & & & \vdots \\ N_{k+m-1} & & \cdots & N_0 \end{bmatrix} = N^{(k+m)}$$

By partitioning $\bar{n}^T$ into

$$\bar{n}^T = \left\{ \bar{n}^T(1), \bar{n}^T(2) \right\}$$

where

$$\bar{n}^T(1) = \left\{ \bar{n}^T_{k+m}, \bar{n}^T_{k+m-1}, \ldots, \bar{n}^T_{k+1} \right\}$$

$$\bar{n}^T(2) = \left\{ \bar{n}^T_{k}, \bar{n}^T_{k-1}, \ldots, \bar{n}^T_{1} \right\}$$

so that

$$N^{(k+n)} = \begin{bmatrix} E\left[\bar{n}(1)\,\bar{n}^T(1)\right] & E\left[\bar{n}(1)\,\bar{n}^T(2)\right] \\ E\left[\bar{n}(2)\,\bar{n}^T(1)\right] & E\left[\bar{n}(2)\,\bar{n}^T(2)\right] \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

the noise prediction becomes equal to

$$N_{12} N_{22}^{-1} \bar{n}(2)$$

If the set of prediction filters is denoted by

$$X = \begin{bmatrix} X^T_{1,m} & X^T_{2,m} & \ldots & X^T_{k,m} \\ X^T_{1,m-1} & X^T_{2,m-1} & \ldots & X^T_{k,m-1} \\ \vdots & \vdots & & \vdots \\ X^T_{1,1} & X^T_{2,1} & \ldots & X^T_{k,1} \end{bmatrix}$$

then

$$X = N_{12} N_{22}^{-1}$$

and

$$X N_{22} = N_{12}$$

or

$$N_{22} X^T = N_{21}$$

which can be written as

$$N^{(k)} X^T = N_{21}$$

From this matrix equation, the particular system of equations for designing an NC-channel k-point prediction filter to predict r points ahead is given by

$$\begin{bmatrix} N_0 & N_{-1} & \cdot & \cdot & \cdot & N_{-k+1} \\ N_1 & N_0 & \cdot & \cdot & \cdot & N_{-k+2} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ N_{k-1} & N_{k-2} & \cdot & \cdot & \cdot & N_0 \end{bmatrix} \begin{bmatrix} X_{1r} \\ X_{2,r} \\ \cdot \\ \cdot \\ \cdot \\ X_{k,r} \end{bmatrix} = \begin{bmatrix} N_r \\ N_{r+1} \\ \cdot \\ \cdot \\ \cdot \\ N_{r+k-1} \end{bmatrix}$$

A recursive procedure for the numerical solution of these equations now will be developed. Using the Levinson forward and backward filters which predict one point, the procedure iterates to the filter predicting r + 1 points ahead from the filter predicting r points ahead. The k-point Levinson prediction filters

$$G^k = \begin{bmatrix} I \\ G_2^k \\ \cdot \\ \cdot \\ \cdot \\ G_k^k \end{bmatrix} \qquad G'^k = \begin{bmatrix} G_k'^k \\ \cdot \\ \cdot \\ \cdot \\ G_2'^k \\ I \end{bmatrix}$$

are given by

$$\begin{bmatrix} N_0 & N_{-1} & \cdot & \cdot & \cdot & N_{-k+1} \\ N_1 & N_0 & & & & N_{-k+2} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ N_{k-1} & N_{k-2} & \cdot & \cdot & \cdot & N_0 \end{bmatrix} \begin{bmatrix} \\ \\ G^k \\ \\ \\ \end{bmatrix} = \begin{bmatrix} P_k \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} N_0 & N_{-1} & \cdot & \cdot & \cdot & N_{-k+1} \\ N_1 & N_0 & & & & N_{-k+2} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ N_{k-1} & N_{k-2} & \cdot & \cdot & \cdot & N_0 \end{bmatrix} \begin{bmatrix} \\ \\ G'^k \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ P_k' \end{bmatrix}$$

where P_k and P'_k are the respective theoretical-error covariance matrices. Programs to compute G and G'^k from an efficient procedure already exist.

Using the notation

$$X_r^k = \begin{bmatrix} X_{1,r}^k \\ X_{2,r}^k \\ \cdot \\ \cdot \\ \cdot \\ X_{k,r}^k \end{bmatrix}$$

for the k-point prediction filter predicting r points ahead, it follows from the definitions of G^{k+1} and X_r^k that

$$X_1^k = \begin{bmatrix} -G_2^{k+1} \\ -G_3^{k+1} \\ \cdot \\ \cdot \\ \cdot \\ -G_{k+1}^{k+1} \end{bmatrix} \qquad \text{(A-1)}$$

which provides a starting point for the recursive procedure.

Recursion formulas to compute X^k_{r+1} when X^k_r is known will now be developed. Assume X^k_r is known. Observe that

$$X^k_r = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ X^{k-1}_{r-1} \end{bmatrix} + G^k X^k_{1,r}$$

holds when multiplying on the left by $N^{(k)}$ so that we can solve for

$$X^{k-1}_{i,r+1} = X^k_{i+1,r} - G^k_{i+1} \; X^k_{1,r} \qquad \text{for } i = 1, 2, \ldots, k-1$$

but

$$N^{(k)} \left\{ X^{k-1}_{r+1} + \left(G'^{k} \right) \right\} = \begin{bmatrix} N_{r+1} \\ N_{r+2} \\ \cdot \\ \cdot \\ \cdot \\ N_{r+k} \end{bmatrix}$$

so that

$$N_{r+k} = P'_k \; X^k_{k,r+1} + \sum_{j=1}^{k-1} N_j X^{k-1}_{k-j,r+1}$$

which implies that

$$X^{k}_{k,r+1} = (P'_k)^{-1}\left[N_{r+k} - \sum_{j=1}^{k-1} N_j X^{k-1}_{k-j,r+1}\right]$$

$$X^{k}_{i,r+1} = X^{k-1}_{i,r+1} + G'^{k}_{k-i+1} X^{k}_{k,r+1} \qquad \text{for } i = 1, 2, \ldots, k-1$$

But, using the above expression for $X^{k-1}_{r+1,i}$, these equations become

$$X^{k}_{k,r+1} = (P'_k)^{-1}\left[N_{r+k} - \sum_{j=1}^{k-1} N_j \left(X^{k}_{k-j+1,r} - G^{k}_{k-j+1} X^{k}_{1,r}\right)\right] \qquad \text{(A-2)}$$

$$X^{k}_{i,r+1} = X^{k}_{i+1,r} - G^{k}_{i+1} X^{k}_{1,r} + G'^{k}_{k-i+1} X^{k}_{k,r+1} \qquad \text{for } i=1, 2, \ldots, k-1 \qquad \text{(A-3)}$$

Equations (A-1), (A-2), and (A-3) establish the recursive procedure for computing the prediction filters.

The application of these filters to predict the noise ahead and the subtraction of the predicted noise from the signal plus noise result in signal-plus-noise prediction error. The variance-covariance matrix of the noise-prediction error can be computed by observing that the error n_e can be expressed as

$$n_e = \bar{n}(1) - \hat{n} = \bar{n}(1) - N_{12}N_{22}^{-1}\,\bar{n}(2) = \begin{bmatrix} I - N_{12}N_{22}^{-1} \end{bmatrix} \begin{bmatrix} \bar{n}(1) \\ \\ \bar{n}(2) \end{bmatrix}$$

so that the variance-covariance matrix is given by

$$N_e = \begin{bmatrix} I - N_{12}N_{22}^{-1} \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} I & \\ -N_{22}^{-1} & N_{21} \end{bmatrix} = N_{11} - N_{12}N_{22}^{-1}N_{21}$$

B. SIGNAL EXTRACTION, STEP 2

The data from the array of seismometers is now in the form

$$\bar{X}'_t = \bar{X}_t - \hat{n}_j = \left\{s_t + n_{i,t} - \hat{n}_{i,t}\right\} = \left\{s_t + n_{e,i,t}\right\}$$

for t = k+1, k+2, ..., k+m

and i = 1, 2, ..., NC

The second phase of the process is to design and apply these filters to the signal-plus-noise prediction error.

The random vectors are assumed to be zero mean; the signal and noise are assumed to be independent. The variance-covariance matrix of joint distribution of the random variables s and $s + n_e$ is

$$E(s, s+n_e)^T(s, s+n_e) = \begin{bmatrix} S & S \\ S & S+N_e \end{bmatrix}$$

Therefore, the best estimate of the signal s is given by

$$S(S+N_e)^{-1} = F' = \begin{bmatrix} F'_{1,m} & F'_{2,m} & \cdots & F'_{m,m} \\ F_{1,m-1} & F_{2,m-1} & \cdots & F_{m,m-1} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ F'_{1,1} & & \cdots & F'_{m,1} \end{bmatrix}$$

where the only difference between this equation and the usual Wiener equation is that N_e is not Toeplitz in form and a general matrix inversion will replace the Levinson iteration.

Two approaches to multichannel least-squares filtering of seismic data have been presented here. The signal estimate can be computed by using the general solution F as follows:

$$\hat{s}_p = \sum_{j=1}^{k+m} F_{jp}^T \overline{X}_{k+m-j+1}$$

or equivalently from the noise prediction solution by

$$\hat{n}_{k+p} = \sum_{i=1}^{k} X_{i,p}^T \overline{X}_{k-i+1}$$

and

$$\hat{s}_p = \sum_{j=1}^{m} F'_{j,p} \overline{X}'_{k+m-j+1}$$

APPENDIX B
PROGRAM DESCRIPTIONS

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PROGRAM DESCRIPTIONS

The nontime-stationary signal-extraction procedure is implemented by a series of three computer programs developed for use on the IBM 7044 processor. The first program, R685, designs the prediction filters; the second program, R687, designs signal-extraction filters using signal-plus-noise prediction error; the last program, R688, applies the filters developed in the first two programs to data. A more detailed description of the capabilities and limitations of these programs follows.

A. R685 - PREDICTION FILTER DESIGN

A noise correlation set is input, and prediction filters are designed. For

NC = number of channels
NFP = number of filter points
M = number of points to be predicted ahead
L = number of lags in correlation

the following restriction exists:

$$NC^2 * M \leq 2000$$

Since

$$L \geq M + NFP$$

the size of the filter and the number of filters designed are restricted. Of course, the restriction can be relaxed by a program modification if longer filters or more distant predictions appear to have merit.

The program provides a visual evaluation of the predictability of the data by generating, for each channel, a plot of the predictability percentage (Figure III-3).

B. R687 - SIGNAL EXTRACTION FILTER DESIGN

The prediction filters designed by R685 and a signal model are input, and signal extraction filters are computed. For

M = number of filter points

NC = number of channels

the following restriction exists:

$$(NC * M)^2 \leq 5500$$

The program also computes the diagonal elements of the variance-covariance matrix. Therefore,

$$MSE_1 = S - S(S + N_e)^{-1} S$$

for evaluation of the filters' theoretical effectiveness.

C. R688 - FILTER APPLICATION

The prediction filters, signal-extraction filters, and data set are input; and the filters are applied. The prediction filters are applied to the first data vectors, and the predicted noise is subtracted from each of the next vectors of signal plus noise. Then, the signal-extraction filters are applied to the signal-plus-noise prediction error. The input data, noise estimate, signal-plus-noise prediction error, and extracted signal then are plotted on the same set of axes.

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13. ABSTRACT

In applying the Wiener multichannel time-series processing theory to seismic arrays for teleseismic signal extraction, the transient nature of the signal is not used. This report presents the theoretical development of optimum nontime-stationary signal-extraction filters which take advantage of the additional information of known signal-arrival time. The matrix inversion is made computationally practical by using an extension of the Levinson technique to determine prediction filters several points into the future. A sequence of programs to accomplish the filter design and application is described and illustrated by a sample problem. Significant improvement in mean-square error was not obtained for this sample problem compared to classical Wiener filters. Full evaluation of this technique requires additional data processing and examination with respect to criteria other than mean-square error.

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