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SF 011-05-11, Task 11292

Lab. Project 940-105, Progress Report 5

A Numerical Technique to Determine The Thermal Histories of Two-Dimensional Solids in the Nuclear Environment

M. L. Cohen

J. E. Koch

R. J. Heilferty

Physical Sciences Division

U. S. NAVAL APPLIED SCIENCE LABORATORY

BROOKLYN, NEW YORK

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2 JAN 1968

M. L. Cohen
J. E. Koch
R. J. Heilferty

PHYSICAL SCIENCES DIVISION
J. M. McGreevy, Head

Approved: [redacted]
E. J. JEHLE
Technical Director

Approved: [redacted]
T. T. McGILLICUDDY, CAPTAIN, USN
Commanding Officer and Director

U. S. NAVAL APPLIED SCIENCE LABORATORY
Flushing and Washington Avenues
Brooklyn, New York 11251

Lab. Project 940-105
Progress Report 5

ABSTRACT

The mathematical techniques necessary to evaluate the transient temperature distribution and histories in two-dimensional solids subject to a time varying radiant flux have been developed. In particular the finite difference form of the equations to be utilized in evaluating the thermal histories in solids of both rectangular and circular cross-sections are presented. The analyses allow for an arbitrary time dependent radiant flux to impinge upon the solids under investigation, and further assumes that the solids may exchange energy to space via convective and radiation processes. The solids are considered to be homogeneous, with temperature dependent thermal properties.

SUMMARY

The calculation of transient temperature distributions in three-dimensional structures is a complex problem requiring extensive computer facilities. The complexity of the problem can be reduced by breaking the structure up into its individual elements, and subjecting each to a two-dimensional analysis. This can be done by considering each element to be insulated at the ends, so that heat flow will only occur in two directions. This technique gives an accurate temperature history for each element, and permits a reasonably accurate estimate to be made of the temperatures at each interface of adjoining structural elements.

In this report, the mathematical techniques necessary to evaluate the transient temperature distribution and histories in two-dimensional solids subject to a time varying radiant flux are presented. Equations have been developed, in finite difference form, to evaluate the thermal histories in solids of both rectangular and circular cross-sections. The analysis takes into account the temperature dependence of the thermal properties, and heat losses due to radiative and convective processes. These mathematical techniques will form the basis of a computer program to evaluate temperatures in structural elements in the nuclear warfare environment.

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ADMINISTRATIVE INFORMATION

As outlined in the U. S. Naval Applied Science Laboratory Program Summary dated 1 November 1967, SF 011-05-11, Task (Work Unit) 11292, the Laboratory is engaged in a research and development program to study the effects of extreme thermal environments on advanced design naval vessels. This report covers the recent effort by North Eastern Research Associates, Queens Village, N. Y. in support of this task.

ACKNOWLEDGMENTS

The work reported herein was conducted by personnel of North Eastern Research Associates under contract to the U. S. Naval Applied Science Laboratory (NASL). R. J. Heilferty, NASL Principal Investigator for this task, served as project monitor. The overall NASL program is under the supervision of W. L. Derksen, Senior Task Leader, and under the general direction of A. C. Clark, Acting Head, Physics Branch. The Naval Ship Systems Command Program Manager is L. E. Sieffert, SHIPS 03541, and the Naval Ship Engineering Center Project Engineer is Y. Park, Code 6364B.

DISCUSSION

The objective of Task 11292 is to determine from theoretical, analytical and experimental studies the nature and extent of thermal radiation damage to shipboard structures and systems exposed to the thermal radiation and blast environment associated with a nuclear detonation. Studies will also be conducted where required, with the objective of preparing bases for material specifications for advanced designs and for hardening existing installations.

There is a requirement for the Navy to maintain the operational capabilities of combat and support vessels during nuclear attack. This involves the development of ships and critical structures which can sustain the effects of thermal radiation without functional degradation. Prerequisite to the development of the above, is the necessity to determine vulnerability to the thermal radiation damage and to develop means of hardening ships and critical structures to combined thermal and blast effects. Past work on thermal radiation effects is of limited application to the problem of vulnerability. Past experimental programs have been concerned mainly with direct effects such as ignition of susceptible materials or ablative effects on resistant materials. The work in this program will encompass both analytical and experimental approaches for determination of strength and failure of materials and structures under thermal loading.

To evaluate the vulnerability of an exposed topside shipboard structure to the thermal radiation released by the detonation of a nuclear warhead, it is first necessary to calculate the temperature distributions induced in the structure by the thermal pulse. The calculation of temperatures in complex three-dimensional structures is a difficult task requiring extensive computer facilities. At this phase of the study, the approach being taken is to break up the structure into its elements, and treating these elements as one- or two-dimensional solids. This greatly reduces the effort required to evaluate the temperatures induced by the transient thermal pulse, and gives an accurate picture of temperature distribution throughout most of the structure.

A previous report presented the equations and a computer program to evaluate temperatures in one-dimensional solids (Ref. 1). This report presents the mathematical techniques necessary to evaluate the transient temperature distribution and histories in two-dimensional solids subject to a time-varying radiant flux. In particular, the finite difference form of the equations to be utilized in evaluating the thermal histories in solids of both rectangular and circular cross-sections are presented.

FUTURE WORK

A report is currently being prepared which describes a computer program based on the equations presented herein. This program will permit the calculation of temperature in beams of arbitrary rectangular cross-sections subject to the appropriate boundary conditions. A second program for the calculation of temperatures in beams of circular cross-section is currently being programed for NASL's computer facility.

REFERENCES

1. "Equations and Computer Program to Calculate the Temperature History of a Dual-Layered Plate Subject to the Thermal Pulse of a Nuclear Weapon," J. E. Koch, M. L. Cohen, NASL Project 9400-105, Progress Report 3, SF 011-05-11, Task 11292, December 1966.

NORTH EASTERN RESEARCH ASSOCIATES
211.35 JAMAICA AVENUE
QUEENS VILLAGE, NEW YORK 11428

"A Numerical Technique to Determine the Thermal Histories of Two Two-Dimensional Solids Exposed to the Thermal Pulse of a Nuclear Weapon"

by

M. L. Cohen
J. E. Koch

Research Report No. 4 30 June 1967

Prepared for: U. S. Naval Applied Science Laboratory
Code 941
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Contract No.: N00140-67-C-0423

N.E.R.A. No. RR-NA-4

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List of Symbols

A	Area
c	Specific heat
$\bar{h}$	Film coefficient
H^t	Time dependent radiant flux
k	Thermal conductivity
r_i	Radius at point i
Δr	Radial increment
S	Spatial division
x, y, z	Spatial coordinates
α	Absorptivity
β	Pseudo-temperature
ϵ	Emissivity
θ	Temperature
ϑ	An angle
$\Delta\phi$	Angular increment
σ	Stefan Boltzmann constant
τ	Time increment

Subscripts and Superscripts

i	Spatial index, initial index
j	Spatial index
k	Time index
s	Space
f	Free stream

Subscripts and Superscripts

H	Horizontal
V	Vertical
0, 1, 2, ... etc.	NODE Numbers

I. Introduction

The work described in this report was performed to satisfy the requirements of Task No. 4 of the scope of work of contract N00140-67-C-0423, "The Analysis of Thermal Stresses and Transient Temperature Distributions in Certain Shipboard Structures Subject to the Thermal Pulse of a Nuclear Weapon". The primary objective of the work phase covered by this report is to provide the mathematical techniques necessary to evaluate the transient temperature histories and profiles in two-dimensional solids. In particular, those equations necessary to determine the thermal history in both rectangular and round semi-infinite rods exposed to the thermal pulse of a nuclear weapon will be developed.

The development of the mathematical techniques necessary to evaluate the transient temperature histories and profiles in two-dimensional solids will necessarily begin with the solution to Fourier's Equation in two-dimensions, subject to the appropriate boundary and initial conditions. Classically solutions of this type have been accomplished using analytical, closed form, techniques. Unfortunately, these methods do not lend themselves to the most general problem classes involving complex geometry, thermophysical properties which vary as arbitrary functions of temperature, and complex boundary conditions. These difficulties may be overcome by using open-form solution techniques, which while giving up some of the generality obtained for the problem classes which can be treated by closed form methods, may be applied to a much wider range of problems without the necessity of restrictive problem assumptions. The open form methods to be used in this report are similar to those developed in Reference 1, fcr use in one dimensional cases.

II. Problem Statement

The specific problem to be treated in the present analysis may be stated as follows: Develop the mathematical techniques, suitable for digital computer programming, so that the thermal histories and profiles in (a) a semi-infinite rectangular rod and (b) a semi-infinite round rod may be determined when these rods are subject to a time varying radiant flux. The thermal conductivity, specific heat, and density of both rods will be considered to be temperature dependent. All exterior surfaces will be considered as non-grey, and will radiate to arbitrarily fixed space environments. Further, all exterior surfaces will be considered to be in contact with a convective film whose heat transfer coefficient varies as a function of surface temperature. Figure 1 indicates both physical models and their boundary conditions.

III. The Rectangular Semi-Infinite Rod

A. General Solution

The general equation, in cartesian coordinates, for the conduction of heat in an isotropic homogeneous solid whose thermal conductivity and specific heat are temperature dependent, is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial \theta}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial \theta}{\partial z}\right)=\rho c(\theta)\frac{\partial \theta}{\partial \tau} \qquad \ldots 1$$

Equation 1 may be simplified by introducing a pseudo-temperature, β, defined as:

$$\beta = \frac{1}{k_i}\int_{\theta_i}^{\theta} k(\theta)\,d\theta \qquad \ldots 2$$

where:

$$k_i = k(\theta_i) \qquad \ldots 3$$

hence, equation 1 may be written as:

$$\nabla^2 \beta = \frac{\rho c(\theta)}{k(\theta)}\frac{\partial \beta}{\partial \tau} \qquad \ldots 4$$

Equation 4, together with the appropriate boundary conditions, are sufficient to completely define the heat transfer problem.

B. The General Solution in Open Form

In the two dimensional case, equation 4 reduces to:

$$\frac{\partial^2 \beta}{\partial x^2}+\frac{\partial^2 \beta}{\partial y^2}=\frac{\rho c(\theta)}{k(\theta)}\frac{\partial \beta}{\partial \tau} \qquad \ldots 5$$

The temperature at any point in the interior of the two dimensional solid may be obtained by placing equation 5 in its finite difference form. That is, taking central differences for the left hand side of equation 5 and forward differences for the right hand side of equation 5, yields the explicit form of the finite difference equation:

$$\frac{(\beta^k_{i+1,j} - \beta^k_{i,j}) - (\beta^k_{i,j} - \beta^k_{i-1,j})}{\Delta x^2} + \frac{(\beta^k_{i,j+1} - \beta^k_{i,j}) - (\beta^k_{i,j} - \beta^k_{i,j-1})}{\Delta y^2} = \frac{\rho c(\theta)}{k(\theta)} \frac{(\beta^{k+1}_{i,j} - \beta^k_{i,j})}{\tau} \quad \ldots 6$$

Assuming $\Delta x = \Delta y$, equation 6 may be simplified as:

$$\frac{1}{\Delta x^2}\left[\beta^k_{i+1,j} + \beta^k_{i-1,j} + \beta^k_{i,j+1} + \beta^k_{i,j-1} - 4\beta^k_{i,j}\right] = \frac{\rho c(\theta)}{k(\theta)\tau}\left(\beta^{k+1}_{i,j} - \beta^k_{i,j}\right) \quad \ldots 7$$

When applied to the interior region of a solid subdivided into n square elements of side s(see Figure 2), equation 7 will yield the general recurrence relationship for the pseudo-temperature, β .

$$\beta^{k+1}_{i,j} = \frac{k(\theta)\tau}{\rho c(\theta) s^2}\left\{\beta^k_{i,j-1} + \beta^k_{i+1,j} + \beta^k_{i,j+1} + \beta^k_{i-1,j} + \left[\frac{s^2 \rho c(\theta)}{k(\theta)\tau} - 4\right]\beta^k_{i,j}\right\} \quad \ldots 8$$

Stability of the explicit solutions obtained by using equation 8 is assured provided the following stability criterion is met, i.e. -

$$\frac{s^2}{\tau} \geq \frac{4k(\theta)}{\rho c(\theta)} \quad \ldots 9$$

Equation 9 infers that minimal stability is assured provided that the coefficient of $\beta^k_{i,j}$ is positive, or at least non-negative. It can be shown that convergence becomes more rapid as the size of the coefficient of $\beta^k_{i,j}$

increases. The rapidity of convergence is limited however by the fact that increases in the size of the coefficient of $\beta^{n}_{i,r}$ require proportional reductions in the size of the time increment τ. Obviously then some compromise must be reached between solution convergence and solution time. This optimization is usually performed prior to actual calculations being made, and can be tailored to the needs of the particular computer program and size of computer being employed for the calculation.

C. Boundary Conditions

The general recurrence relationship when combined with the appropriate finite difference equations for the elements bounding the rectangular solid can be used to determine the thermal history of the solid. For the rectangular solid there are two general types of surface elements that must be considered; the exterior corner, and the flat surface. The finite difference equations for these elements are obtained by taking heat balances about the elements. The particular radiation and convective boundary conditions imposed on the solid are taken into consideration at this point in the mathematical formulation. Figure 3 illustrates the general model positions for an arbitrary exterior corner and a flat surface. Obviously the equations developed for these illustrated cases can be applied to any other surface or exterior corner element.

1. The Flat Surface

The solid region under consideration is assumed to be subject to a time varying radiant flux H^{t}. The angle between the solid surface and a line normal to the incoming radiation is taken as ϑ, where $0^{\circ} \leq \vartheta \leq 90^{\circ}$. The solid reradiates energy to an arbitrarily fixed space environment of temperature θ_s . Further, energy may enter or leave the solid via convection at the solid surface. Here the free stream temperature is denoted by θ_f . Energy leaves the node under consideration (0) via conduction to the adjacent nodes (1, 2, and 3). Therefore, by performing an energy balance about the surface node we obtain:

$$\left[H^t \cos\vartheta \cdot \alpha \cdot A\right] - \left[\bar{h} A(\theta_0^k - \theta_f^k) + \epsilon A \sigma (\theta_0^{k^4} - \theta_s^4)\right) + \frac{k_i A}{S}(\beta_0^k - \beta_2^k)$$

$$+ \frac{k_i A}{2S}(\beta_0^k - \beta_1^k) + \frac{k_i A}{2S}(\beta_0^k - \beta_3^k)\Big] = \frac{\rho c(\theta) S A}{2} \frac{k_i}{k(\theta)} \frac{(\beta_0^{k+1} - \beta_0^k)}{\tau} \qquad \ldots 10$$

After some algebraic manipulation and simplification, equation 10 may be solved to yield the pseudo-temperature β at node (0) at time $k+1$:

$$\beta_0^{k+1} = \frac{2\tau k(\theta)}{\rho c(\theta) S^2} \left\{ \frac{S}{k_i}\left[H^t \alpha \cos\vartheta + \bar{h}(\theta_f^k - \theta_0^k) + \epsilon\sigma(\theta_s^4 - \theta_0^{k^4})\right] \right.$$

$$\left. + \frac{1}{2}(2\beta_2^k + \beta_1^k + \beta_3^k) + \beta_0^k\left[\frac{\rho c(\theta) S^2}{2\tau k(\theta)} - 2\right] \right\} \qquad \ldots 11$$

In this case the stability criteria to be followed becomes:

$$\beta_0^k\left[\frac{\rho c(\theta) S^2}{2\tau k(\theta)} - 2\right] - \frac{S\theta_0^k \bar{h}}{k_i}\left[1 + \frac{\epsilon\sigma\theta_0^{k^3}}{\bar{h}}\right] \geq 0 \qquad \ldots 12$$

2. Exterior Corners

In order to obtain the difference equation for the temperatures in an exterior corner subject to a time variant heat flux, a heat balance is obtained about the corner element in the nodal grid. In particular, the numbering scheme indicated in Figure 3 for the nodal elements will be followed. It is noted that in this case the portion of the radiant flux striking the horizontal surfaces of the exterior corner is given by $H^t \cos\vartheta_H$, while that striking the vertical surfaces is given by $H^t \cos\vartheta_V$. Again ϑ_H and ϑ_V must not exceed 90° in magnitude if the corresponding surface "sees" the radiation. Taking a heat balance about the corner node in the same manner as previously employed yields:

$$\left[H^t \cos\vartheta_V \cdot \alpha \cdot \frac{A}{2} + H^t \cos\vartheta_H \cdot \alpha \cdot \frac{A}{2}\right] - \left[\bar{h} A(\theta_0^k - \theta_f^k) + \epsilon A \sigma (\theta_0^{k^4} - \theta_s^4)\right.$$

$$\left. \frac{k_i A}{2S}(\beta_0^k - \beta_1^k) + \frac{k_i A}{2S}(\beta_0^k - \beta_1^k)\right] = \frac{\rho c(\theta) S A}{4} \cdot \frac{k_i}{k(\theta)} \cdot \frac{(\beta_0^{k+1} - \beta_0^k)}{\tau} \qquad \ldots 13$$

and on simplifying and solving for the pseudo-temperature at node (0) at time $k+1$, we obtain:

$$\beta_0^{k+1} = \frac{2\tau k(\theta)}{\rho c(\theta) S^2} \left\{ \frac{2S}{k_i} \left[\frac{H^t}{2} \alpha \cos\vartheta_V + \frac{H^t}{2} \alpha \cos\vartheta_H + \bar{h}(\theta_f^k - \theta_0^k) \right.\right.$$

$$\left.\left. + \sigma\epsilon(\theta_s^4 - \theta_0^{k^4}) \right] + \beta_1^k + \beta_2^k + \beta_0^k \left[\frac{\rho c(\theta) s^2}{2\tau k(\theta)} - 2 \right] \right\} \qquad \ldots 14$$

In this case the stability criteria to be followed is:

$$\beta_0^{k}\left[\frac{\rho c(\theta) s^2}{2\tau' k(\theta)} - 2\right] - \frac{2 S \theta_0^{k} \bar{R}}{k_i}\left[1 + \frac{\epsilon \sigma \theta_0^{k^3}}{\bar{h}}\right] \geq 0 \qquad \ldots 15$$

D. Compilation of the Governing Equations

Referring to Figure 1(a), it is seen that the temperature history throughout the rectangular solid, at any time, can be determined through the use of the proper combinations of equations 8, 11, and 13. These equations are perfectly general, and hence will have to be modified for actual use in any particular problem case by the deletion of appropriate terms. For example, in the case of a surface element shaded from the incoming radiant flux, the general governing equation, 11, is modified by the deletion of the radiant flux term $H^{t} \cos\delta$. Similarly, any other boundary variation occurring from point to point about the perimeter of the solid can be taken into consideration.

The stability criterion used when performing numerical calculations is obtained by using the most restrictive (i.e., the largest) of equations 9, 12, or 15. Strictly speaking it is possible to apply each of the three stability criteria in the region for which they were obtained. This however leads to complications in grading the nodal network in each of the regions, and is hence usually avoided.

IV. The Cylindrical Semi-Infinite Rod

In dealing with the finite difference solutions to the transient temperature distribution in solids of cylindrical cross-section, two general methods of approach are open to investigation. In the first, a rectangular nodal net is constructed, and is fitted to the circular cross-section. Obviously error will be introduced into any numerical results due to the boundary mismatch. This error is reduced as the size of the nodal mesh is reduced. Hence, as the mesh size becomes small (with a concommitant increase in calculation time), this method of approach becomes attractive.

The second general method of attack is to divide the cylindrical section into a series of wedge shaped segments of width $\Delta\varphi$ and length Δr (see Figure 4), and to apply a finite differencing scheme to the resultant mesh. Since the first method of attack has been illustrated in section III, the second general technique will be investigated here.

A. General Solution

The expression for the pseudo-temperature, equation 4, may be transformed into polar coordinates, and the resultant expression becomes:

$$\frac{\partial^2 \beta}{\partial r^2} + \frac{1}{r}\frac{\partial \beta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \beta}{\partial \varphi^2} = \frac{\rho\, c(\theta)}{k(\theta)}\frac{\partial \beta}{\partial \tau} \qquad \ldots 16$$

The pseudo-temperature at any interior node (see Figure 5) may be obtained by placing equation 16 in its finite difference form. Proceeding as before (section IIIb), we obtain:

$$\frac{\beta_1^k + \beta_2^k - 2\beta_0^k}{(\Delta r)^2} + \frac{\beta_1^k - \beta_2^k}{(2\Delta r) r_i} + \frac{\beta_4^k + \beta_3^k - 2\beta_0^k}{(r_i)^2 (\Delta\phi)^2} = \frac{\rho c(\theta)}{k(\theta)} \frac{\beta_0^{k+1} - \beta_0^k}{\tau} \qquad \ldots 17$$

and on solving for the pseudo-temperature at node (0) at time $k+1$, the general recurrence relationship is obtained as:

$$\beta_0^{k+1} = \frac{k(\theta)\tau}{2(\Delta r)\rho c(\theta) r_i} \left\{ \beta_1^k \left(1 + \frac{2r_i}{\Delta r}\right) + \beta_2^k \left(\frac{2r_i}{\Delta r} - 1\right) + \frac{2\Delta r}{r_i (\Delta\phi)^2}\left(\beta_3^k + \beta_4^k\right) + \beta_0^k \left[\frac{2(\Delta r)\rho c(\theta) r_i}{k(\theta)\tau} - \frac{4(r_i)^2(\Delta\phi)^2 + (\Delta r)^2}{r_i (\Delta\phi)^2 \Delta r} \right] \right\} \qquad \ldots 18$$

The stability of equation 18 may be assured provided that the following is satisfied:

$$\left[\frac{2(\Delta r)\rho c(\theta) r_i}{k(\theta)\tau} - \frac{4(r_i)^2(\Delta\phi)^2 + (\Delta r)^2}{r_i (\Delta\phi)^2 \Delta r} \right] \geq 0 \qquad \ldots 19$$

It is noted that the nodal element at the center of the cylinder, $r=0$, is exceptional. Here the terms $\frac{1}{r}\frac{\partial\beta}{\partial r}$ and $\frac{1}{r^2}\frac{\partial^2\beta}{\partial r^2}$ of equation 16, and the corresponding terms in equation 17, are indeterminite. This mathematical difficulty is avoided by examining a center element of circular cross-section and radius $\frac{\Delta r}{2}$, exchanging heat with the n wedge shaped nodes comprising its surroundings (see Figure 6). A heat balance taken about this central node yields:

$$k_i\left(\frac{\Delta r \Delta\phi}{2}\right)_1 \frac{(\beta_1^k - \beta_0^k)}{\Delta r} + \cdots + k_i\left(\frac{\Delta r \Delta\phi}{2}\right)_n \frac{(\beta_n^k - \beta_0^k)}{\Delta r} = \rho c(\theta) \frac{\pi(\Delta r)^2}{4} \frac{(\beta_0^{k+1} - \beta_0^k)}{\tau} \frac{k_i}{k(\theta)} \qquad \ldots 20$$

Noting that $\left(\frac{\Delta r \Delta \varphi}{2}\right) = \frac{\pi \Delta r}{n}$, equation 20 becomes:

$$\frac{k_i \pi \Delta r}{n \Delta r} \sum_{i=1}^{n} (\beta_i^k - \beta_o^k) = \frac{\rho c(\theta) \pi (\Delta r)^2}{4 \tau} \frac{k_i}{k(\theta)} (\beta_o^{k+1} - \beta_o^k) \qquad \ldots 21$$

On solving equation 21 for the temperature at the central node at time $k+1$ we obtain:

$$\beta_o^{k+1} = \frac{4 \tau k(\theta)}{\rho c(\theta)(\Delta r)^2} \left[\frac{1}{n} \sum_{i=1}^{n} \beta_i^k + \beta_o^k \left(\frac{\rho c(\theta)(\Delta r)^2}{4 \tau k(\theta)} - 1 \right) \right] \qquad \ldots 22$$

The stability of equation 22 is assured if:

$$\frac{(\Delta r)^2}{\tau} \geq \frac{4\, k(\theta)}{\rho c(\theta)} \qquad \ldots 23$$

B. Bourdary Conditions

Elements on the surface of the cylinder will be subject to the boundary conditions discussed in section III. That is, surface elements will be exposed to a time varying flux of radiant energy H^*; these elements will reradiate to some arbitrary space temperature θ_s , and will be subject to a convective transfer of energy, through a film of coefficient $\bar{h}$, to a surrounding temperature θ_f. The recurrence relations for these surface elements are obtained by performing heat balances on the surface nodes, and solving the resultant equations for the future temperature at the surface node in terms of the present temperatures of the surroundings. Figure 7 illustrates a typical surface element and its boundary conditions. Performing the indicated heat balance about the surface node yields:

$$\left[H^{t}\alpha \cos\vartheta\, r\Delta\varphi\right] - \left[\bar{h} r\Delta\varphi(\theta_0^{k} - \theta_f^{k}) + \epsilon r\Delta\varphi\sigma(\theta_0^{k^4} - \theta_s^{4})\right] + \frac{k_i r_m \Delta\varphi}{\Delta r}(\beta_3^{k} - \beta_0^{k})$$

$$+ \frac{k_i \Delta r}{2 r_m \Delta\varphi}(\beta_1^{k} + \beta_2^{k} - 2\beta_0^{k}) = \frac{\rho c(\theta) r\Delta\varphi\Delta r}{2}\,\frac{k_i}{k(\theta)}\,\frac{(\beta_0^{k+1} - \beta_0^{k})}{\tau} \qquad \ldots 24$$

where: $r_m = r + \frac{\Delta r}{2}$

If we assume that the difference between r and r_m is negligibly small, then equation 24 may be solved for β_0^{k+1} as follows:

$$\beta_0^{k+1} = \frac{2\tau k(\theta)}{\rho c(\theta)(\Delta r)^2}\left\{\frac{\Delta r}{k_i}\left[H^{t}\alpha\cos\vartheta + \bar{h}(\theta_f^{k} - \theta_0^{k}) + \epsilon\sigma(\theta_s^{4} - \theta_0^{4})\right]\right.$$

$$\left. + \left[T_3 + \frac{(\Delta r)^2}{2r^2(\Delta\varphi)^2}(T_1 - T_2)\right] + T_0\left[\frac{\rho c(\theta)(\Delta r)^2}{2\tau k(\theta)} - 1 - \frac{(\Delta r)^2}{r^2(\Delta\varphi)^2}\right]\right\}$$

$$\ldots 25$$

The stability criteria to be followed when using relations of the form of equation 25 is:

$$\beta_0^{k}\left[\frac{\rho c(\theta)(\Delta r)^2}{2\tau k(\theta)} - \frac{(\Delta r)^2}{r^2(\Delta\varphi)^2} - 1\right] - \frac{(\Delta r)\bar{h}\theta_0^{k}}{k_i}\left(1 + \frac{\epsilon\sigma\theta_0^{3}}{\bar{h}}\right) \geq 0 \qquad \ldots 26$$

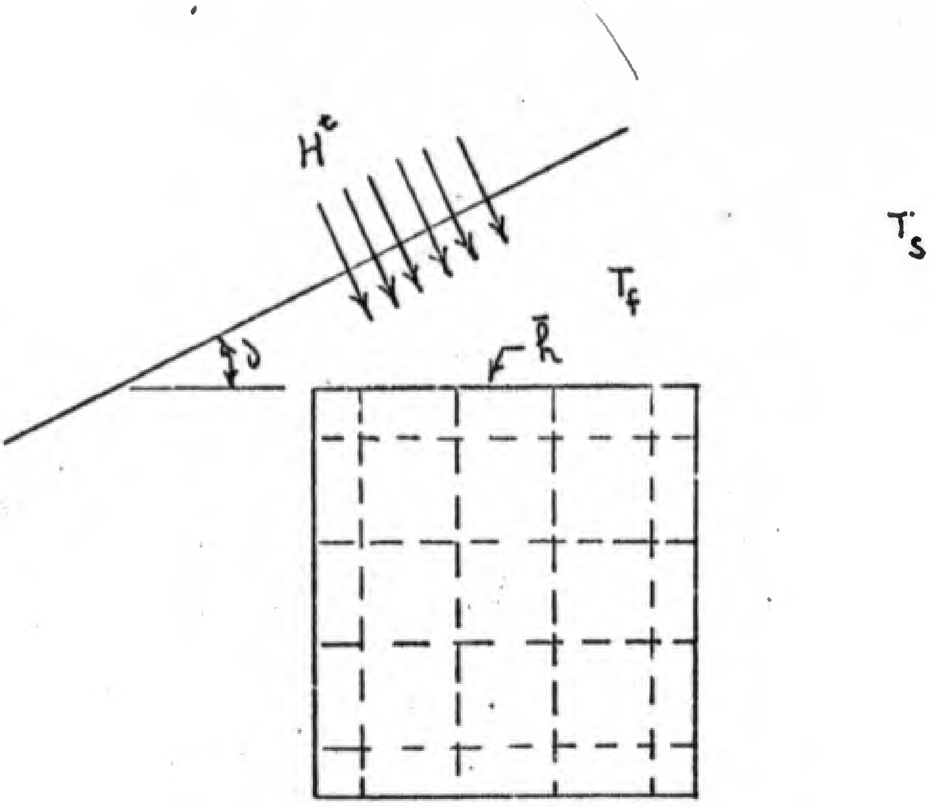
C. Use of the Cylindrical Equations

The temperature history throughout the cylindrical solid (Figure 1b), at time τ, can be determined by using the proper combinations of equations 18, 22, and 25. These equations will, of course, have to be modified for use in any particular case, by the deletion of appropriate terms. Stability of the numerical computations is obtained by using the most restrictive of equations 19, 23 or 26.

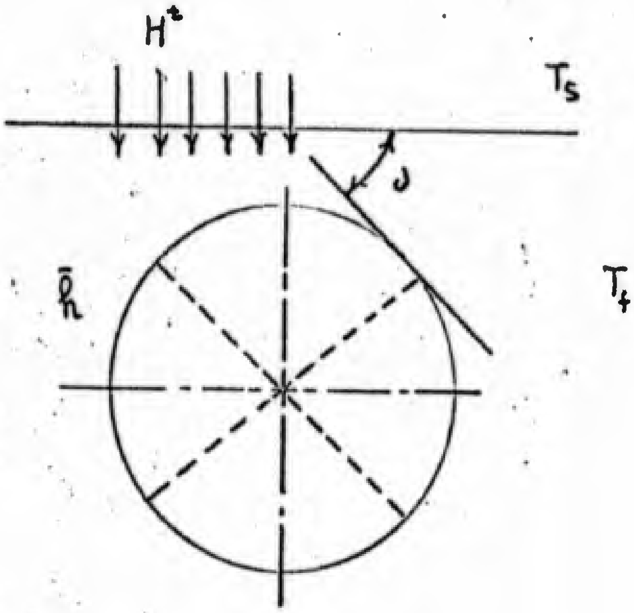
V. Concluding Remarks

Some general comments may now be made concerning the use of the equations developed in this report. The finite difference form of the equations describing the flow of heat in two-dimensional solids have been developed so that a computer program, based upon these equations, may be written to facilitate the evaluation of the thermal histories of these solids. Of the equations, those developed in section III, for the rectangular semi-infinite rod, are the most general. These equations may, with little difficulty, be applied to irregular cross-sections as well as rectangular shapes. Since the mis-match error incurred when fitting a rectangular mesh to a non-rectangular cross-section decreases as the mesh size decreases, the error involved in applying the two-dimensional rectangular equations developed in section III for these shapes also decreases.

The equations developed in section IV for use with circular cross-section solids, are strictly applicable only to circular solids, and hence have application limited only to these solids. Their obvious advantage over the system of equations written in rectangular coordinates is in the minimization of errors. While somewhat more restrictive, it would be most beneficial to develop a computer code based on this equation system.



a) The Rectangular Semi-Infinite Rod



b) The Circular Semi-Infinite Rod

Figure 1 - The Physical Models of the Two Dimensional Solids

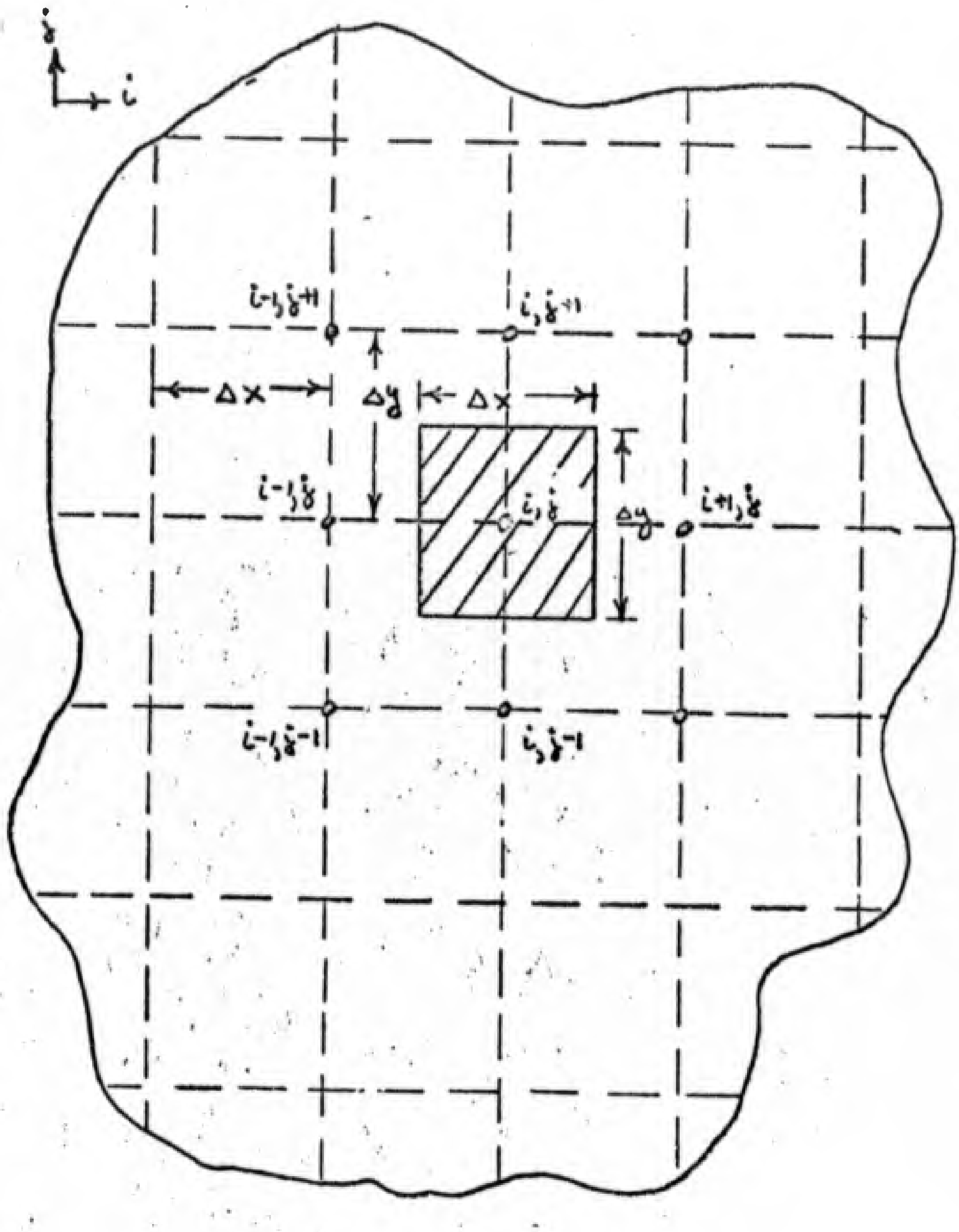
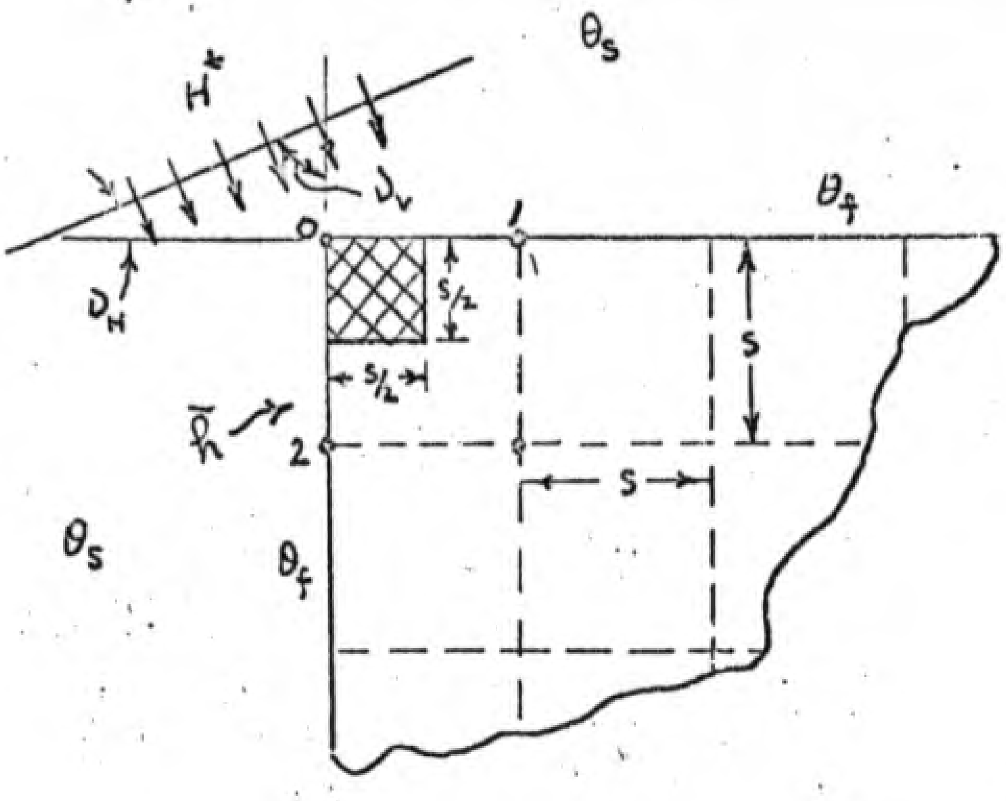
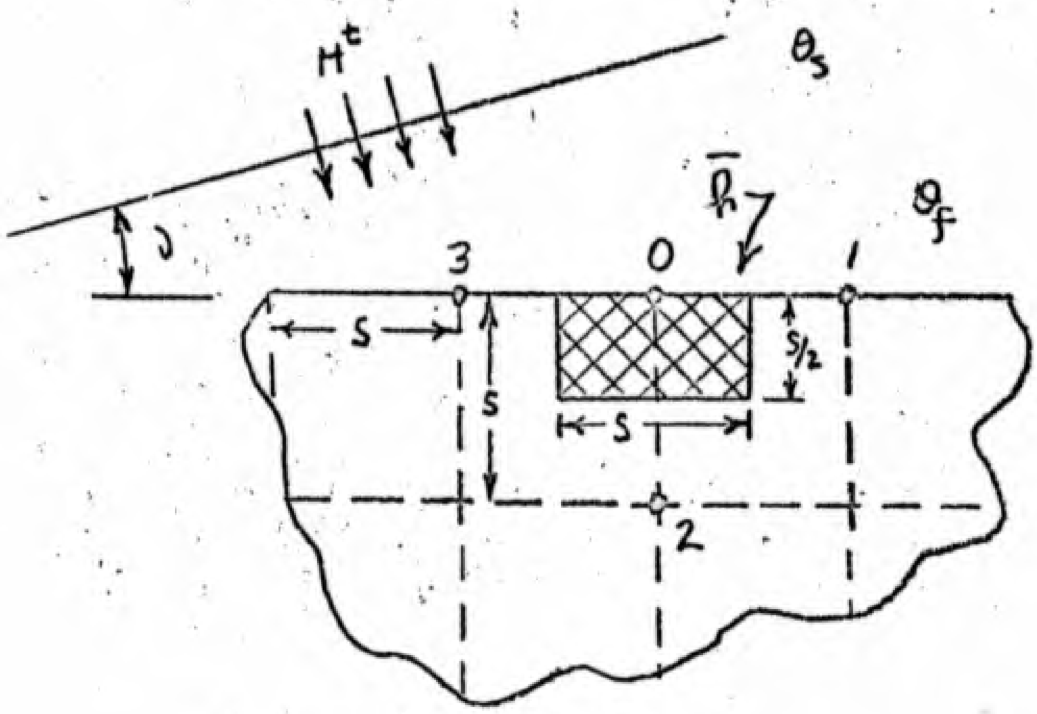


Figure 2 - The Nodal Scheme for the Rectangular Solid



a) Corner Elements



b) Surface Elements

Figure 3 - The Boundary Nodal Elements for a Rectangular Solid

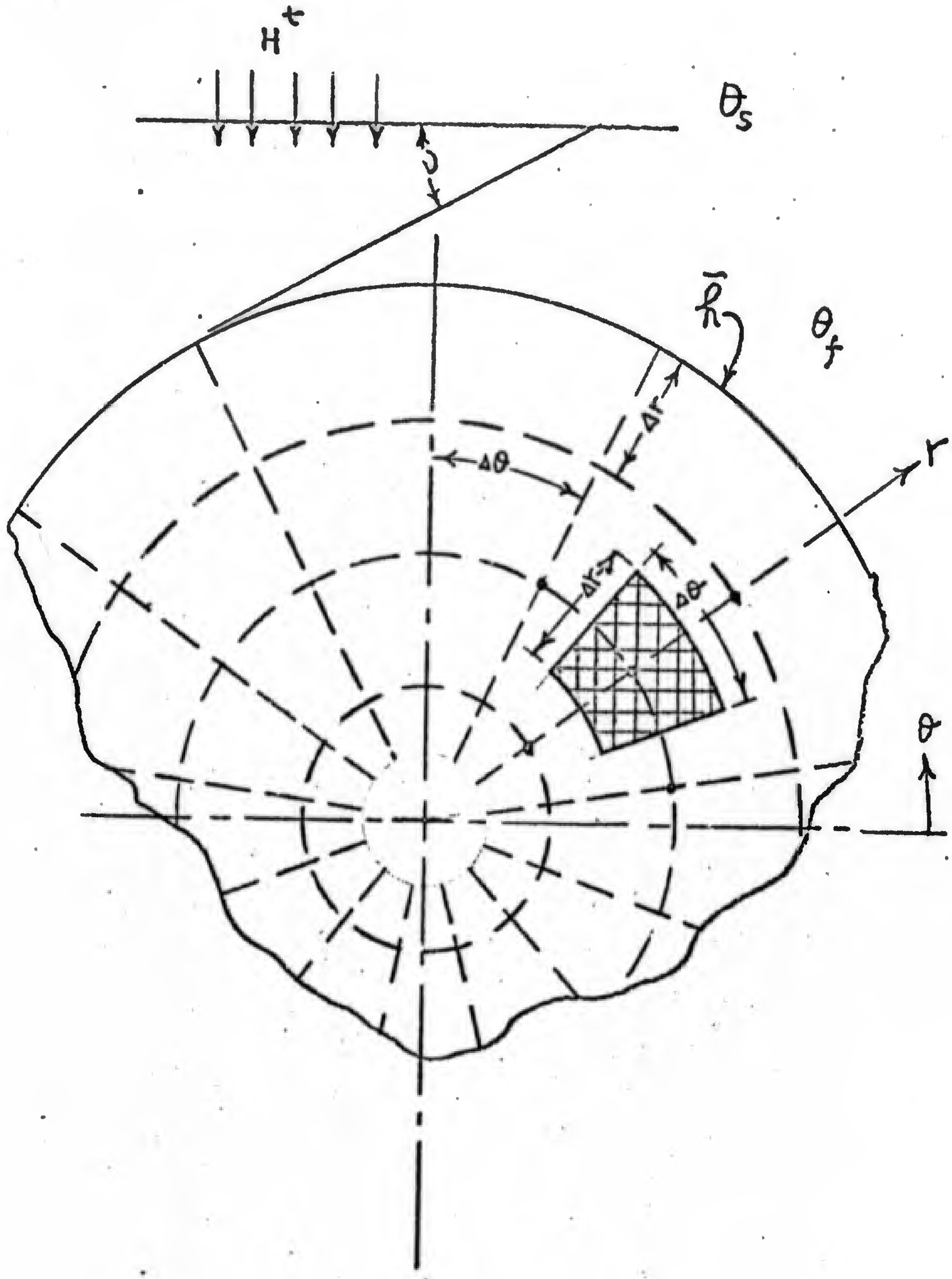


Figure 4 - Nodal Scheme for the Cylindrical Rod

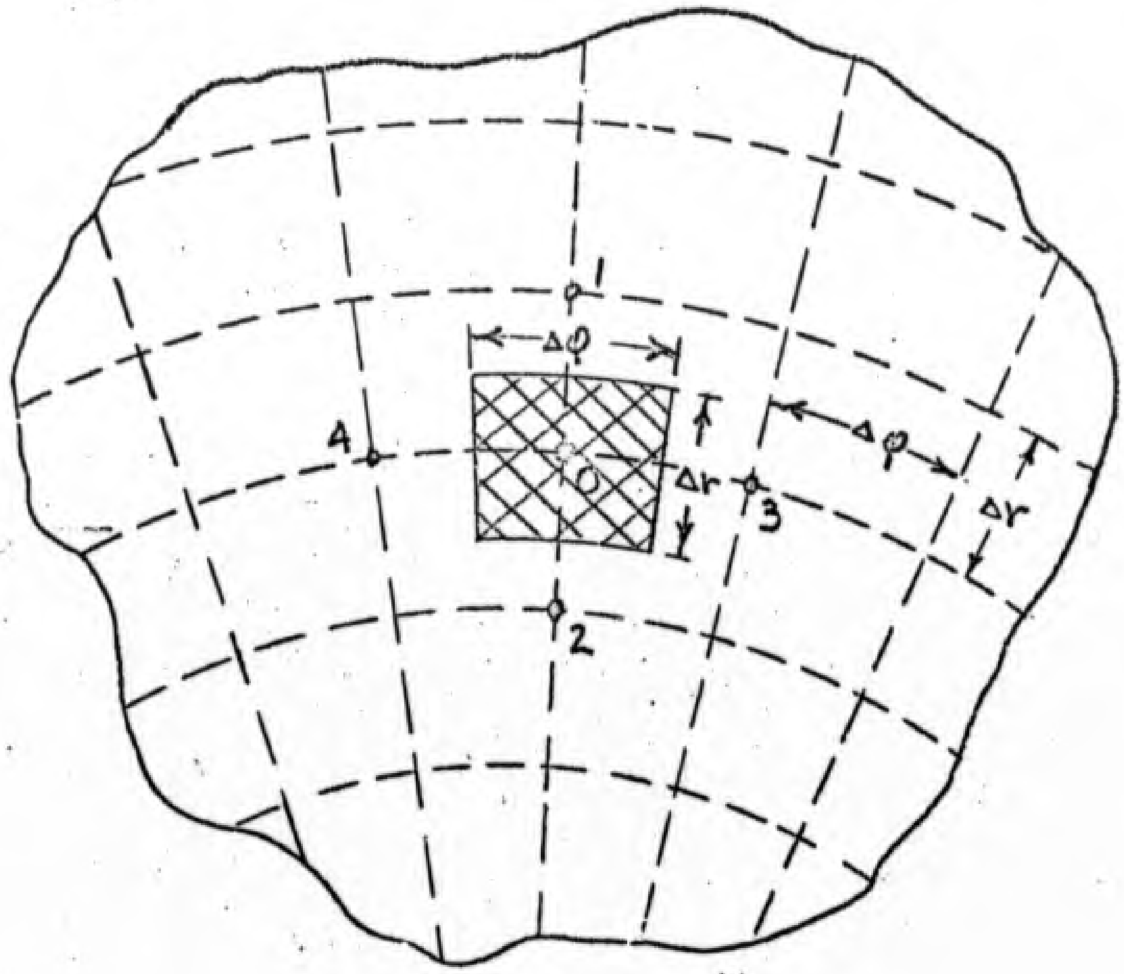


Figure 5 - An Interior Cylindrical Node

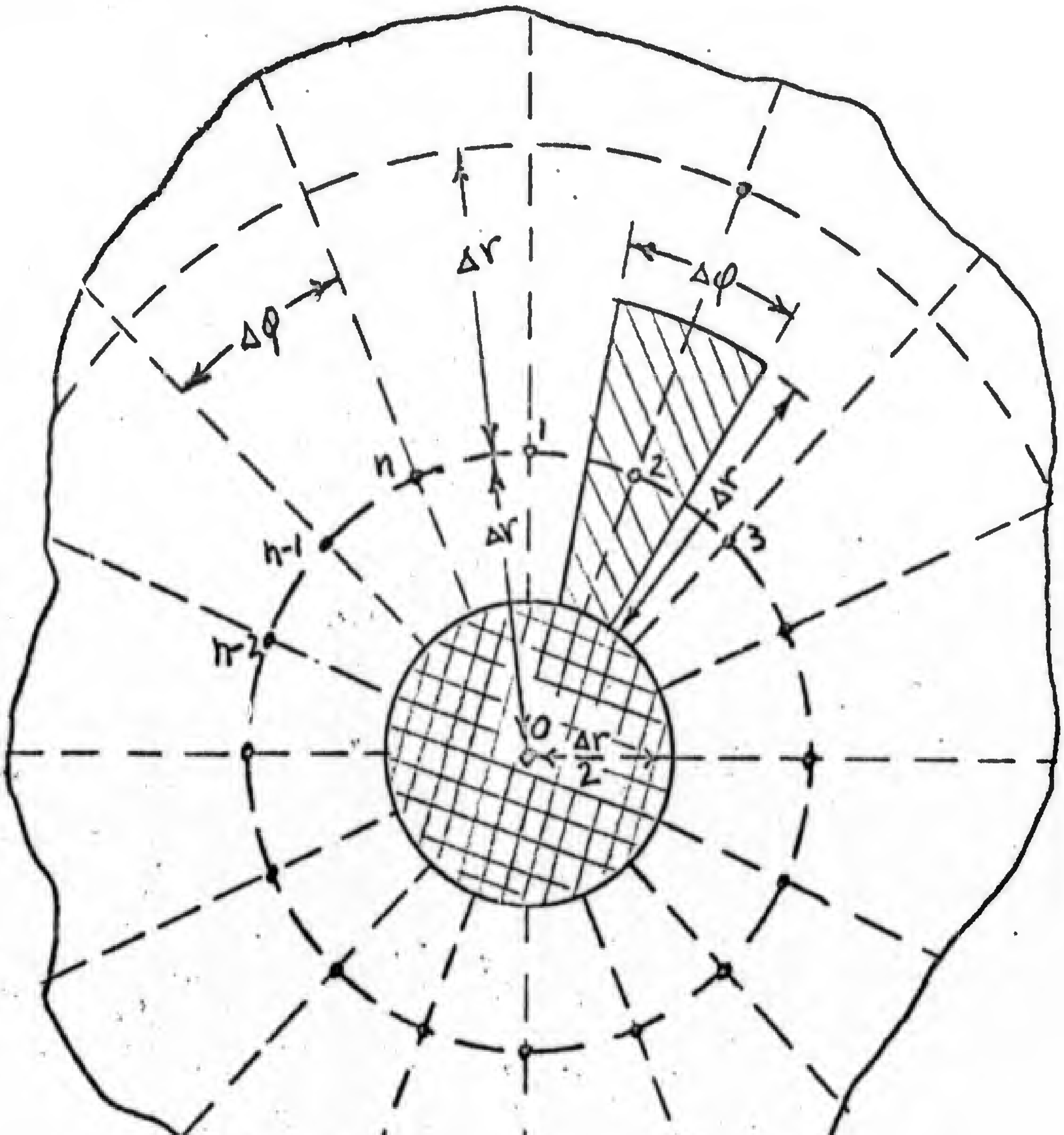


Figure 6 - The Central Node in a Rod of Circular Cross-Section

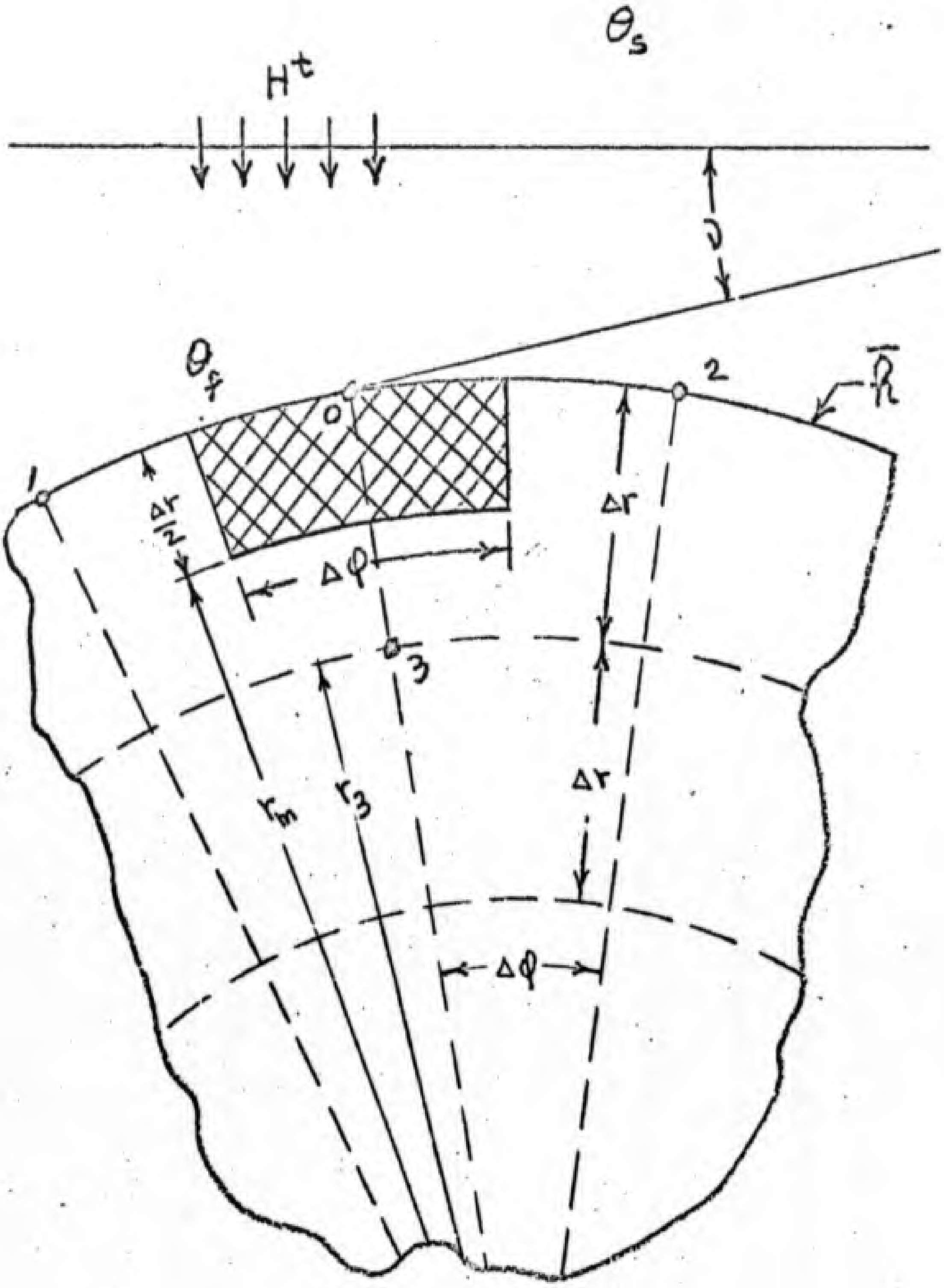


Figure 7 - A Typical Element on the Cylindrical Surface

References

1. Koch, J. F. and M. L. Cohen, "Equations and Computer Program to Calculate the Thermal History of a Dual-Layered Plate Subject to the Thermal Pulse of a Nuclear Weapon", N.E.R.A. Report No. RR-NA-1, 15 September 1966.

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John E. Koch
Myron L. Cohen
Robert J. Heilferty

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13. ABSTRACT

The mathematical techniques necessary to evaluate the transient temperature distribution and histories in two-dimensional solids subject to a time varying radiant flux have been developed. In particular the finite difference form of the equations to be utilized in evaluating the thermal histories in solids of both rectangular and circular cross-sections are presented. The analyses allow for an arbitrary time dependent radiant flux to impinge upon the solids under investigation, and further assumes that the solids may exchange energy to space via convective and radiation processes. The solids are considered to be homogeneous, with temperature dependent thermal properties.

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