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Flight Performance & Guidance Analysis

THE MARK II RE-ENTRY PROGRAM

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ENGINEERING DEPARTMENT

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THE MARK II RE-ENTRY PROGRAM

F. M. Chadwick, L. T. Gregg

ABSTRACT

The Mark II Re-entry Program computes zero angle of attack ballistic trajectories of vehicles which enter the earth's atmosphere. Forces acting on the vehicle due to the atmosphere, winds, and gravitational attraction of the oblate earth are computed. The resulting accelerations are summed and numerically integrated by means of Obrechkoff integration formulas to obtain vehicle position and velocity.

I. INTRODUCTION

The Mark II Re-entry Program was developed for the rapid, accurate simulation of freefall re-entry trajectories where the assumption of a zero angle of attack throughout re-entry is adequate. From the basic input of an initial position and velocity, the program computes the accelerations on the re-entry vehicle due to the atmosphere, winds, and the gravitational attraction of the oblate earth. These are summed and numerically integrated to obtain vehicle position and velocity.

The differential equations of vehicle motion are integrated by means of Obrechkoff formulas which employ the function and its derivatives at two points. Use of higher derivatives in the integration process decreases computer execution time by a factor of fifteen by reducing the number of integration steps required to obtain a given end point accuracy.

The program is coded in FORTRAN. As a basic subprogram in the VECTRAN system (1) it consists of a number of subroutines which are coded independently. The program may be employed either independently or as a subprogram in a larger powered flight simulation.

The main portion of the program was developed by the senior author and is based on earlier work done by the Flight Performance and Guidance Analysis Group. A description of the previous re-entry simulation may be found in reference (2). The program was coded and checked out by R. Bowen.

II. GENERAL DISCUSSION

A. Program Structure

A flow chart of the overall structure is given in Figure 1. As shown, the program consists of the following subroutines or blocks:

1. Oblate Gravity
2. Atmosphere and Winds
3. Aerodynamics
4. Integration
5. Time Step Control
6. Flight Path Parameters

A brief discussion of each subroutine will now be given. Complete details, with all equations and flow charts where applicable, will be found in Section III. In the Appendix a list of program constants and tables for atmospheric parameters are given.

1. Basic input consists of an initial position vector, $\bar{R}_o$, and velocity vector $\bar{v}_o$, in canonical units and referred to an earth-centered inertial coordinate system (defined later). Additional input data, such as nosecone coefficients, atmospheric model desired, etc., are described in the Appendix.
2. The gravitational acceleration, $\bar{g}_{ei}$, and its time derivative, $\dot{\bar{g}}_{ei}$ are computed in the Oblate Gravity block. Provision has been made for including the

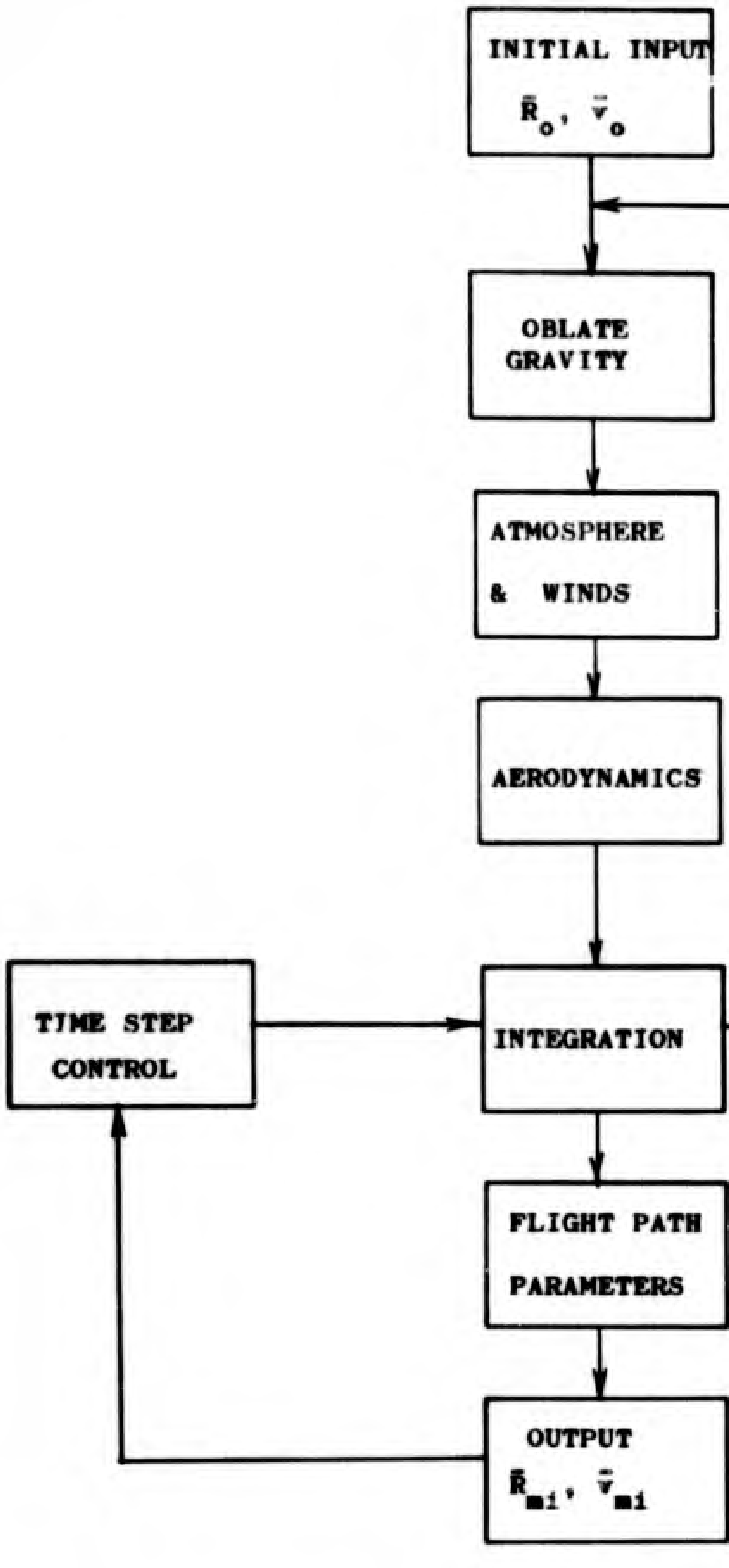


FIG. 1- Overall Program Structure

effects of the central force and the forces due to the second, third, and fourth harmonic terms.

3. A standard atmosphere and winds model is used to compute various atmospheric parameters such as Mach no., speed of sound, dynamic pressure, etc. and their time derivatives. These quantities are then used as input to the Aerodynamics block.

4. The acceleration due to the atmosphere and winds, $\bar{a}_{ci}$, and its time derivative, $\dot{\bar{a}}_{ci}$, are computed in the Aerodynamics block and used as input to the Integration block.

5. In the Integration block three operations are performed; summation, prediction, and numerical integration.

   a. The aerodynamic and gravitational accelerations and their time derivatives are summed to yield the total acceleration, $\bar{a}_{mi}$, and its time derivative, $\dot{\bar{a}}_{mi}$.

   b. The Time Step block is then entered and the integration time step, Δt, is computed.

   c. Control is returned to the Integration block where a linear prediction of $\bar{a}_{mi}$ at the next point on the trajectory is made; $\dot{\bar{a}}_{mi}$ at the next point is set equal to the previous $\dot{\bar{a}}_{mi}$.

d. The predicted values of $\bar{a}_{mi}$ and $\dot{\bar{a}}_{mi}$ are numerically integrated to yield position, $\bar{R}_{mi}$, and velocity, $\bar{v}_{mi}$. The integration is successively repeated by looping through the acceleration computations until a convergence is obtained. A comparison of the velocity magnitudes from succeeding iterations with a preset tolerance determines the convergence. Use of the Obrechkoff formulas, which employ higher derivatives, allows the use of large time steps without serious truncation error. For example, the number of integration steps necessary to compute a re-entry trajectory from an initial altitude of 285,000 feet has been reduced from 572 in an earlier simulation to 39, i.e., by a factor of fifteen.

6. The integration step size, Δt, is generated for each integration step by comparison of $|\hat{a}_{mi}|$ with control parameters which are based on a nominal re-entry trajectory. This allows smaller time steps to be used for periods of rapid deceleration.

7. The geocentric latitude, longitude, and flight path angle are computed and output at each integration step. Following these computations an advance to the next integration cycle is made.

8. The entire process described above proceeds until a test determines that $\bar{R}_{mi}$ lies within the earth. An impact routine then determines the negative time step which is used to integrate back to the earth's surface. Basic output of the program consists of $\bar{R}_{mi}$, $\bar{v}_{mi}$, $\bar{a}_{mi}$, and the time, t, at each integration step. These are printed out in standard British units: feet, pounds, and seconds. In addition, the flight path parameters and atmospheric parameters are computed and printed out at each step.

B. Units and Coordinates

The computations in the program are performed in canonical units and geocentric inertial coordinates, defined as follows:

1. Canonical Units

a. Unit of mass = mass of the earth, 0.4097570231 x 10^{24} slugs.

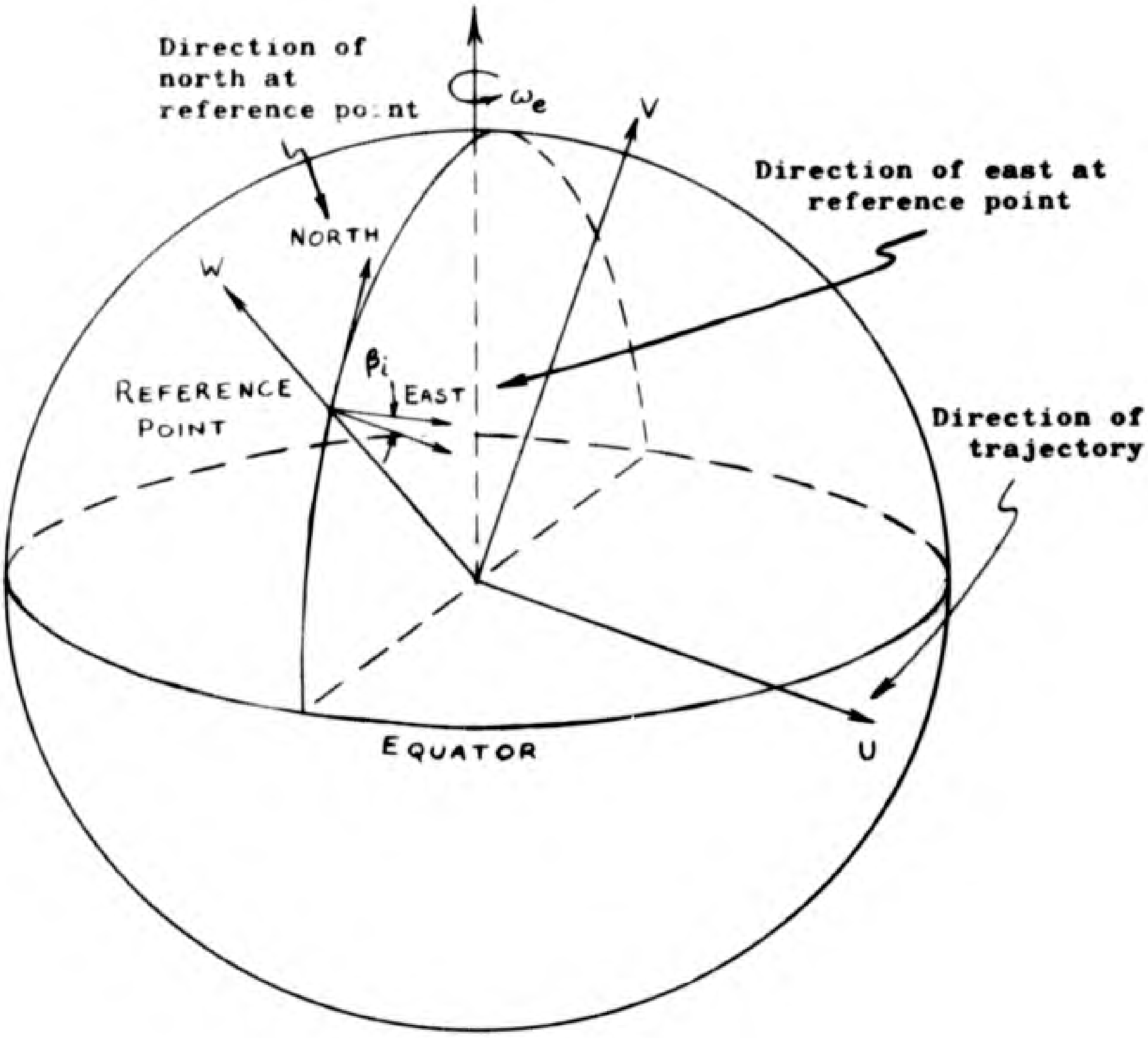
b. Unit of length = the mean equatorial radius of the earth, 20,926,010 feet (Ref. Army Map Service Ellipsoid).

c. The product of the universal gravity constant and the mass of the earth, GM_E, is set equal to one.

d. Unit of time = that necessary for Newton's Second Law of Motion ($\bar{F} = Km\bar{a}$) to be satisfied with a constant of proportionality, K, equal to unity. The canonical unit of time is equivalent to 806.8272479 seconds.

2. Inertial Coordinates

The geocentric inertial coordinate system is defined as follows: the origin is at the center of mass of the earth; the w axis is directed through the reference point, i.e., along the initial position vector; the u axis is in the plane formed by the initial position and velocity vectors, its direction making an angle β_i measured clockwise from the direction of due east at the reference point; and the v-axis is directed to form a right-handed orthogonal system. This set of axes is indicated in Figure 2. The system does not rotate with the earth, i.e., the direction of its axes are fixed in inertial space.



ω_e = Angular speed of rotation of earth

β_i = Initial azimuth

FIG. 2 - The Geocentric Inertial Coordinate System

III. DETAILS OF THE PROGRAM

In this section all details of each subroutine will be discussed. A detailed flow chart of the logic which connects the various subroutines into a re-entry simulation is also given. An additional subscript of (n) or (n-1) has been added in certain places, where (n) refers to a point on the trajectory and (n-1) refers to the previous point.

A. Oblate Gravity

This subroutine computes $\bar{g}_{ei}$, the gravitational acceleration of the oblate earth on the vehicle, and its time derivative, $\dot{\bar{g}}_{ei}$, as a function of position and velocity and the oblate spheroid parameters. Input consists of the vehicle position and velocity vectors $\bar{R}_{mi}$ and $\bar{v}_{mi}$ and the earth's angular velocity vector, $\bar{\omega}_{ei}$ (in uvw coordinates). The program constants are: the product of the universal gravitational constant and the mass of the earth, GM_E (equal to one in canonical units); the second, third, and fourth harmonic terms of the oblate earth gravitational potential, C_2, C_3, C_4; and the equatorial radius of the oblate spheroid, R_o (equal to one in canonical units).

The sequence of computations is as follows:

1. $\bar{I}_{\omega ei} = \dfrac{\bar{\omega}_{ei}}{|\bar{\omega}_{ei}|}$ (unit vector in the direction of $\bar{\omega}_{ei}$)

2. $\cos\theta = \bar{I}_{\omega ei} \cdot \dfrac{\bar{R}_{mi}}{|\bar{R}_{mi}|}$ (where θ = geocentric colatitude)

3. $$N' = \frac{-C_2(R_o)^2}{(R_{mi})^2} + \frac{15C_3(R_o)^3 \cos\theta}{8(R_{mi})^3} + \frac{C_4(R_o)^4(28 \cos^2\theta - 12)}{8(R_{mi})^4}$$

4. $$\Delta P' = N'R_{mi} \cos\theta - \frac{3\, C_3\, (R_o)^3}{8(R_{mi})^2}$$

5. $$Q' = \frac{C_2(R_o)^2(5 \cos^2\theta - 1)}{2(R_{mi})^2} - \frac{C_3(R_o)^3(35 \cos^3\theta - 15 \cos\theta)}{8(R_{mi})^3}$$

$$- \frac{C_4(R_o)^4(63 \cos^4\theta - 42 \cos^2\theta + 3)}{8(R_{mi})^4}$$

6. $$\bar{g}_{ei} = \frac{-GM_E}{(R_{mi})^3}\left[\bar{R}_{mi} - Q'\bar{R}_{mi} - \Delta P'\ \bar{I}_{\omega ei}\right]$$

7. $$\dot{Q}' = -\frac{C_2R_o^2}{(R_{mi})^3}\left[\frac{(10 \cos^2\theta - 1)\bar{R}_{mi}\cdot\bar{v}_{mi}}{R_{mi}} - 5(\bar{I}_{\omega ei}\cdot\bar{v}_{mi})\cos\theta\right]$$

$$+ \frac{15C_3R_o^3}{8(R_{mi})^4}\left[\frac{(14 \cos^3\theta - 4 \cos\theta)\bar{R}_{mi}\cdot\bar{v}_{mi}}{R_{mi}}\right.$$

$$\left. - (7 \cos^2\theta - 1)(\bar{I}_{\omega ei}\cdot\bar{v}_{mi})\right]$$

$$+ \frac{3C_4R_o^4}{2(R_{mi})^5}\left[\frac{(42 \cos^4\theta - 21 \cos^2\theta + 1)\ \bar{R}_{mi}\cdot\bar{v}_{mi}}{R_{mi}}\right.$$

$$\left. - (21 \cos^3\theta - 7 \cos\theta)\ (\bar{I}_{\omega ei}\cdot\bar{v}_{mi})\right]$$

8. $\Delta\dot{P}' = (\bar{I}_{\omega ei}\cdot\bar{v}_{mi})\ (N') + \dfrac{2C_2R_o^{\ 2}}{(R_{mi})^3}\ (\bar{R}_{mi}\cdot\bar{v}_{mi})\cos\theta$

$$- \frac{15C_3R_o^{\ 3}}{8(R_{mi})^4}\left[4\cos^2\theta(\bar{R}_{mi}\cdot\bar{v}_{mi}) - (\bar{I}_{\omega ei}\cdot\bar{v}_{mi})R_{mi}\cos\theta\right]$$

$$- \frac{C_4R_o^{\ 4}}{2(R_{mi})^5}\left[(42\cos^3\theta - 12\cos\theta)\ (\bar{R}_{mi}\cdot\bar{v}_{mi}) - 14(\bar{I}_{\omega ei}\cdot\bar{v}_{mi})\ (R_{mi})\cos^2\theta\right]$$

$$+ \frac{3C_3R_o^{\ 3}}{4(R_{mi})^4}\ (\bar{R}_{mi}\cdot\bar{v}_{mi})$$

9. $\dot{\bar{g}}_{ei} = \dfrac{GM_E}{(R_{mi})^3}\left[\bar{v}_{mi}(Q' - 1) + \dfrac{3(\bar{v}_{mi}\cdot\bar{R}_{mi})}{(R_{mi})^2}\left\{(1 - 5Q')\bar{R}_{mi} - \Delta P'\bar{I}_{\omega ei}\right\} + \bar{R}_{mi}\dot{Q}' + \Delta\dot{P}'\bar{I}_{\omega ei}\right]$

10. Output: $\bar{g}_{ei}$, $\dot{\bar{g}}_{ei}$

B. Atmosphere and Relative Winds

Computation of the vehicle altitude, h, and the atmosphere parameters a, λ, M, q, $\bar{V}_{Ri}$, and their time derivatives is accomplished in this subroutine. These quantities are required to compute the aerodynamic acceleration, $\bar{a}_{ci}$, and its time derivative, $\dot{\bar{a}}_{ci}$.

A test on the altitude determines if the re-entry vehicle is within the atmosphere. The limiting altitude of the atmospheric models employed is 300,000 ft.; however, this may be changed if desired. If the vehicle is not inside the atmosphere the remaining computations are bypassed; $\bar{a}_{ci}$ and $\dot{\bar{a}}_{ci}$ will be computed as zero in the Aerodynamics block.

There are two atmospheric models available for use in the program: the ICAO Standard Atmosphere and the WADC Tropical Atmosphere.[3] In each model the speed of sound, a, and the static pressure ratio, λ, are computed as functions of h from table look-ups (T.L.U.). At altitudes greater than 100,000 ft. (for λ) and 155,350 feet (for a) the models are identical.

The wind speed, V_{wi}, is obtained from a linear interpolation in a T.L.U. with h the argument. Values used are from the Sissenwine 1% Wind Profile.[4] They are subject to change as improved knowledge of the atmosphere is obtained. The wind angle, β_w, measured positive clockwise from local south (eq. 13 below), is considered constant throughout re-entry, except in post-flight simulations where exact values for an actual flight are available and are input as a T.L.U. Any desired value for βw may be specified for a pre-flight simulation. Note that when $\beta w = 0$, the wind is from north to south.

It should be noted that the re-entry simulation may be performed without the effect of winds; i.e., the winds computations may be bypassed.

Input consists of $\bar{R}_{mi}$, $\bar{v}_{mi}$, $\bar{a}_{mi}$ (either predicted or from the preceeding iteration), $\bar{\omega}_{ei}$ (the earth's angular velocity vector), $\cos\theta$ (θ = geocentric colatitude, computed in the Oblate Gravity block), and βw, the desired wind direction.

The sequence of computations is as follows:

1. $\bar{V}_{mi} = \bar{v}_{mi} - \bar{\omega}_{ei} \times \bar{R}_{mi}$ where $\bar{V}_{mi}$ = velocity relative to a rotating earth referred to inertial coordinates.

2. $R_s = \dfrac{AB}{\sqrt{B^2 + (A^2 - B^2)\cos^2\theta}}$, radius of the oblate spheroid, where A & B are the semi-major and semi-minor axes, respectively, of the oblate spheroid.

3. $h = |\bar{R}_{mi}| - R_s$, altitude of the vehicle

4. Test: $h > h_o$? If yes, output $\bar{v}_{mi}$, R_s, and h and exit the subroutine. If no, proceed.

5. $\dot{h} = \bar{v}_{mi} \cdot \frac{\bar{R}_{mi}}{|\bar{R}_{mi}|}$

6. $a = a(h) = C_o + C_1 h$ where a = speed of sound as a function of h from a Table Lookup (T.L.U.)

C_o, C_1 = constants

7. $\dot{a} = C_1 \dot{h}$ where $C_1 = \Delta a / \Delta h$

$\Delta a, \Delta h$ = differences from the T.L.U.

8. $k_1 = k_1(h)$ where k_1, k_2 = constants as a function of h from a T.L.U.

$k_2 = k_2(h)$

9. $\lambda = k_1 e^{k_2 h}$ Static pressure ratio

10. $\dot{\lambda} = k_2 \lambda \dot{h}$

11. $V_{wi} = V_{wi}(h)$ Wind speed, from a T.L.U. with h the argument.

12. $\dot{V}_{wi} = \frac{\Delta V_{wi}}{\Delta h} \dot{h}$

13. $\beta_w = \beta_w(h)$ Angle of wind with respect to a local XYZ system (defined in equation 15, below) from a T. L. U. or as input data, (in which case β_w is a constant).

14. $\dot{\beta}_w = \frac{\Delta \beta_w}{\Delta h} \dot{h}$

15. $\bar{I}_{wi} = -(\sin\beta_w \bar{I}_x + \cos\beta_w \bar{I}_y + 0\bar{I}_z)$, the unit vector in the direction of the wind,

where $\bar{I}_x = \frac{\bar{\omega}_{ei} \times \bar{R}_{mi}}{|\bar{\omega}_{ei} \times \bar{R}_{mi}|}$

$$\bar{I}_y = \frac{\bar{R}_{mi}}{\bar{R}_{mi}} \times \bar{I}_x$$

$$\bar{I}_z = \bar{I}_x \times \bar{I}_y$$

16. $\bar{V}_{wi} = V_{wi}\bar{I}_{wi}$ Velocity of the wind relative to the earth (inertial coordinates).

17. $\dot{\bar{I}}_{wi} = -\dot{\beta}_w(\cos\beta_w \bar{I}_x - \sin\beta_w \bar{I}_y) - (\sin\beta_w \dot{\bar{I}}_x + \cos\beta_w \dot{\bar{I}}_y)$,

where

$$\dot{\bar{I}}_x = \left[\frac{\bar{\omega}_{ei} \times \bar{v}_{mi}}{|\bar{\omega}_{ei} \times \bar{R}_{mi}|}\right] - \left[\frac{(\bar{\omega}_{ei} \times \bar{v}_{mi}) \cdot (\bar{\omega}_{ei} \times \bar{R}_{mi})}{|\bar{\omega}_{ei} \times \bar{R}_{mi}|^2}\right] \bar{I}_x$$

$$\dot{\bar{I}}_y = \frac{1}{R_{mi}} \left[\left\{\bar{v}_{mi} - \frac{\bar{R}_{mi}(\bar{v}_{mi} \cdot \bar{R}_{mi})}{(\bar{R}_{mi})^2}\right\} \times \bar{I}_x + (\bar{R}_{mi} \times \dot{\bar{I}}_x)\right]$$

18. $\dot{\bar{V}}_{wi} = \dot{V}_{wi}\,\bar{I}_{wi} + V_{wi}\,\dot{\bar{I}}_{wi}$

19. $\dot{\bar{V}}_{mi} = \bar{a}_{mi} - \bar{\omega}_{ei} \times \bar{V}_{mi}$

20. $\bar{V}_{Ri} = \bar{V}_{mi} - \bar{V}_{wi}$ velocity of vehicle relative to the wind (inertial coordinates)

21. $\dot{\bar{V}}_{Ri} = \dot{\bar{V}}_{mi} - \dot{\bar{V}}_{wi}$

22. $V_{Ri} = \left|\bar{V}_{Ri}\right|$

23. $\dot{V}_{Ri} = \dot{\bar{V}}_{Ri} \cdot \frac{\bar{V}_{Ri}}{V_{Ri}}$

24. $M = \frac{V_{Ri}}{a}$ Mach Number

25. $q = 1481.0\,\lambda\,M^2$ dynamic pressure

The dynamic pressure is not used per se in subsequent computations; however, it is computed and printed out as a useful performance parameter.

26. Output: R_s, h, a, $\dot{a}$, λ, $\dot{\lambda}$, V_{Ri}, $\bar{V}_{Ri}$, $\dot{V}_{Ri}$, $\dot{\bar{V}}_{Ri}$, M, q.

If the simulation is to be performed without the effect of winds, equations 11-18 are bypassed, and equations

20 and 21 reduce to:

20. $\bar{V}_{Ri} = \bar{V}_{mi}$

21. $\dot{\bar{V}}_{Ri} = \dot{\bar{V}}_{mi}$

C. <u>Aerodynamics</u>

The axial drag coefficient, C_D, the aerodynamic acceleration, $\bar{a}_{ci}$, and their time derivatives $\dot{C}_D$ and $\dot{\bar{a}}_{ci}$ are computed in this subroutine. A rational fraction in Mach No. is used to generate C_D and $\dot{C}_D$.

Input consists of: a, $\dot{a}$, λ, $\dot{\lambda}$, V_{Ri}, $\dot{V}_{Ri}$, $\bar{V}_{Ri}$, $\dot{\bar{V}}_{Ri}$, and M from the Atmosphere and Winds block (see p. 14 for definition of symbols); S, the maximum effective cross-sectional area of the re-entry vehicle; W, the weight of the vehicle; and the nose-cone coefficients, a_i and b_j.

The equations are as follows:

1. $\bar{I}_{VRi} = \dfrac{\bar{V}_{Ri}}{|\bar{V}_{Ri}|}$, unit relative velocity vector

2. $$C_D = \frac{\sum_{i=o}^{6} a_i M^i}{\sum_{j=o}^{7} b_j M^j}$$

3. $$\dot{C}_D = \left[\frac{\dot{V}_{Ri} - \dot{a}M}{a}\right] \left[\frac{\sum_{i=1}^{6} i a_i M^{i-1} - C_D \sum_{j=1}^{7} j\, b_j M^{j-1}}{\sum_{j=o}^{7} b_j M^j}\right]$$

4. $$\bar{a}_{ci} = -\left[\frac{1481\, \lambda\, M^2\, S\, C_D}{W/g_o}\right] \bar{I}_{VRi}$$ where g_o = weight-to-mass conversion factor, defined in the Appendix.

5. $$\dot{\bar{a}}_{ci} = \frac{-1481\ S}{Wa^2/g_o} \left[(\lambda V_{Ri} \dot{C}_D + \lambda \dot{V}_{Ri} C_D + \dot{\lambda} V_{Ri} C_D - \frac{2\dot{a}}{a} \lambda V_{Ri} C_D)\, \bar{V}_{Ri} + (\lambda C_D V_{Ri})\, \dot{\bar{V}}_{Ri} \right]$$

6. Output: $\bar{a}_{ci}$, $\dot{\bar{a}}_{ci}$

D. <u>Integration</u>

Three functions are performed in this subroutine: prediction, summation, and numerical integration.

1. Prediction Formulae

Input consists of $\bar{a}_{mi_{(n-1)}}$ and $\dot{\bar{a}}_{mi_{(n-1)}}$ from the preceding integration, and the time step, Δt. The prediction is made as follows:

(a) $\dot{\bar{a}}_{mi_{(n)}} = \dot{\bar{a}}_{mi_{(n-1)}}$

(b) $\bar{a}_{mi_{(n)}} = \bar{a}_{mi_{(n-1)}} + (\dot{\bar{a}}_{mi_{(n-1)}}) \Delta t$

2. Total Acceleration

The total acceletation, $\bar{a}_{mi}$, and its time derivative $\dot{\bar{a}}_{mi}$, on the vehicle due to both aerodynamic and gravitational forces are summed in this block. Input consists of $\bar{a}_{ci}$ and $\dot{\bar{a}}_{ci}$ from the Aerodynamics subroutine, and $\bar{g}_{ei}$ and $\dot{\bar{g}}_{ei}$ from the Oblate Gravity subroutine. The equations are:

$$\bar{a}_{mi} = \bar{a}_{ci} + \bar{g}_{ei}$$

$$\dot{\bar{a}}_{mi} = \dot{\bar{a}}_{ci} + \dot{\bar{g}}_{ei}$$

3. Obrechkoff Integration

Input consists of $\bar{R}_{mi_{(n-1)}}$, $\bar{v}_{mi_{(n-1)}}$, $\bar{a}_{mi_{(n-1)}}$, and $\dot{\bar{a}}_{mi_{(n-1)}}$ from the preceeding integration; $\bar{a}_{mi_{(n)}}$ and $\dot{\bar{a}}_{mi_{(n)}}$ from either the prediction block or the total acceleration block, and the integration step size, Δt. The equations are:

$$\bar{v}_{mi_{(n)}} = \bar{v}_{mi_{(n-1)}} + \frac{\Delta t}{2}\left[\bar{a}_{mi_{(n)}} + \bar{a}_{mi_{(n-1)}}\right] + \frac{\Delta t^2}{12}\left[\dot{\bar{a}}_{mi_{(n-1)}} - \dot{\bar{a}}_{mi_{(n)}}\right]$$

$$\bar{R}_{mi_{(n)}} = \bar{R}_{mi_{(n-1)}} + \frac{\Delta t}{2}\left[\bar{v}_{mi_{(n-1)}} + \bar{v}_{mi_{(n)}}\right]$$

$$+ \frac{\Delta t^2}{10}\left[\bar{a}_{mi_{(n-1)}} - \bar{a}_{mi_{(n)}}\right] + \frac{\Delta t^3}{120}\left[\dot{\bar{a}}_{mi_{(n-1)}} + \dot{\bar{a}}_{mi_{(n)}}\right]$$

Further description of these formulas may be found in reference (5).

E. Time Step Control

As seen in the flow chart, Fig. 3, this subroutine is almost entirely a series of tests on $\left|\dot{\bar{a}}_{mi_{(n-1)}}\right|$ to determine the step size, Δt. The control parameters A, B, C, D, E, F are based on a nominal case of an atmospheric re-entry, and are listed in the Appendix. Values for Δt range from 1 to 20 seconds. Since the time step is a function of the preceding $\left|\dot{\bar{a}}\right|$, smaller integration steps are used for periods of rapid deceleration.

It should be noted that Δt, as determined from the series of tests, is generally not the "optimum" time step. Further in the program a comparison is made of $\left|\hat{a}_{mi_{(n)}}\right|$ and $\left|\dot{\bar{a}}_{mi_{(n-1)}}\right|$, and if necessary, the time step is halved and a new $\bar{a}_{mi_{(n)}}$ and $\dot{\bar{a}}_{mi_{(n)}}$ are predicted from the new time step. For the nominal trajectory from which the control parameters are taken, Δt is near-optimum; for non-nominal trajectories it is slightly off optimum. The difference is small and yields suitable results with a simple scheme which has the advantage of greater reliability over more sophisticated prediction schemes.

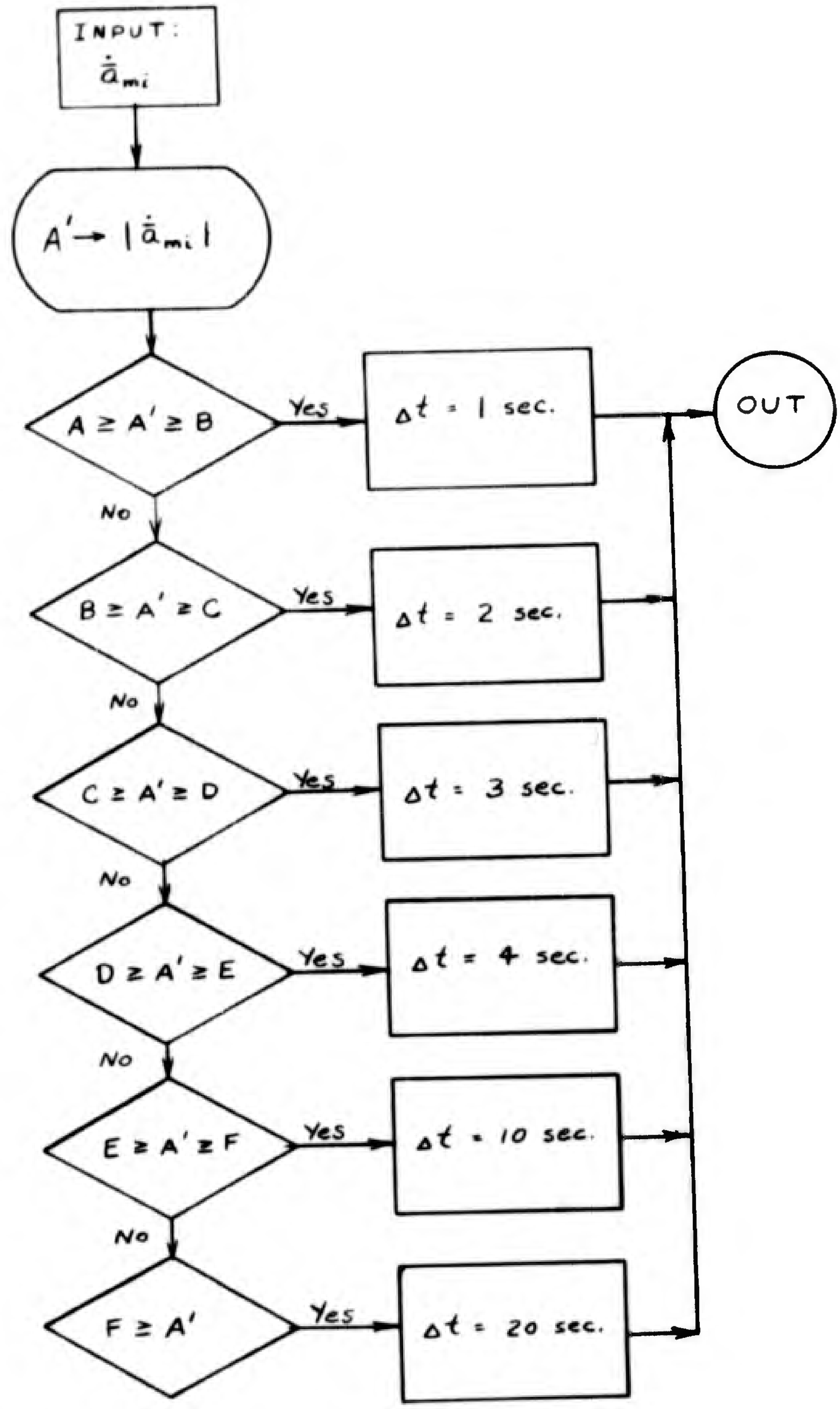


FIG. 3 - Time Step Control Subroutine

F. <u>Flight Path Parameters</u>

This subroutine provides continuous in-flight computations of the geocentric latitude, longitude, and flight path angle of the re-entry vehicle. Input consists of the position vector, $\bar{R}_{mi}$, velocity vector, $\bar{v}_{mi}$, the earth's angular velocity vector $\bar{\omega}_{ei}$, the time, t, as measured from the beginning of the trajectory, and the longitude, λ_o, of the reference point.

1. <u>Flight Path Angle</u>

$$\gamma = \sin^{-1}\left[\frac{\bar{v}_{mi} \cdot \bar{R}_{mi}}{|\bar{v}_{mi}|\,|\bar{R}_{mi}|}\right]$$

measured positive for the velocity vector above the local horizontal.

2. <u>Geocentric Latitude</u>

a. $$\phi' = \sin^{-1}\left[\frac{\bar{R}_{mi} \cdot \bar{\omega}_{ei}}{|\bar{R}_{mi}|\,|\bar{\omega}_{ei}|}\right]$$

b. Test: $\sin \phi'$ positive? If yes, position is in the Northern Hemisphere. If no, position is in the Southern Hemisphere.

3. <u>Longitude</u>

a. $$\bar{I}_{R_{mi}} = \frac{\bar{R}_{mi}}{|\bar{R}_{mi}|} \; ; \; \bar{I}_{\omega ei} = \frac{\bar{\omega}_{ei}}{|\bar{\omega}_{ei}|} \; ; \; \bar{I}_{w} = (0,0,1)$$

b. $\Delta\lambda = \cos^{-1}\left[\frac{\bar{I}_{\omega ei} \times \bar{I}_{Rmi}}{\left|\bar{I}_{\omega ei} \times \bar{I}_{Rmi}\right|} \cdot \frac{\bar{I}_{\omega ei} \times \bar{I}_{w}}{\left|\bar{I}_{\omega ei} \times \bar{I}_{w}\right|}\right]$

c. Test sign of u -component of $\bar{R}_{mi}$: if positive, position is in the 1st or 2nd quadrant. If negative, position is in the 3rd or 4th quadrant.

d. Compute $\Delta\lambda'$. If position is in 1st or 2nd quadrant, $\Delta\lambda' = -\Delta\lambda$. If position is in 3rd or 4th quadrant, $\Delta\lambda' = \Delta\lambda$.

f. Compute λ_I, the longitude of the position vector:

$$\lambda_I = \lambda_o + \Delta\lambda' + \omega_e t$$

where ω_e = earth's angular speed.

g. Test: is λ_I positive? If yes, longitude is west of Greenwich. If no, longitude is east of Greenwich.

G. <u>Program Logic</u>

The flow chart in Figure 4 illustrates the way that the various subroutines are joined to form a re-entry simulation. As indicated by the dashed lines and Roman numerals, the logical structure may be broken down into four principle sections. Note the change of subscripts from (mi) to (n) and (n-1), where (n) refers to a point on the trajectory and (n-1) refers to the previous point. In the flow chart the subscripts (k) and (k +1) refer to successive iterations within one iteration cycle.

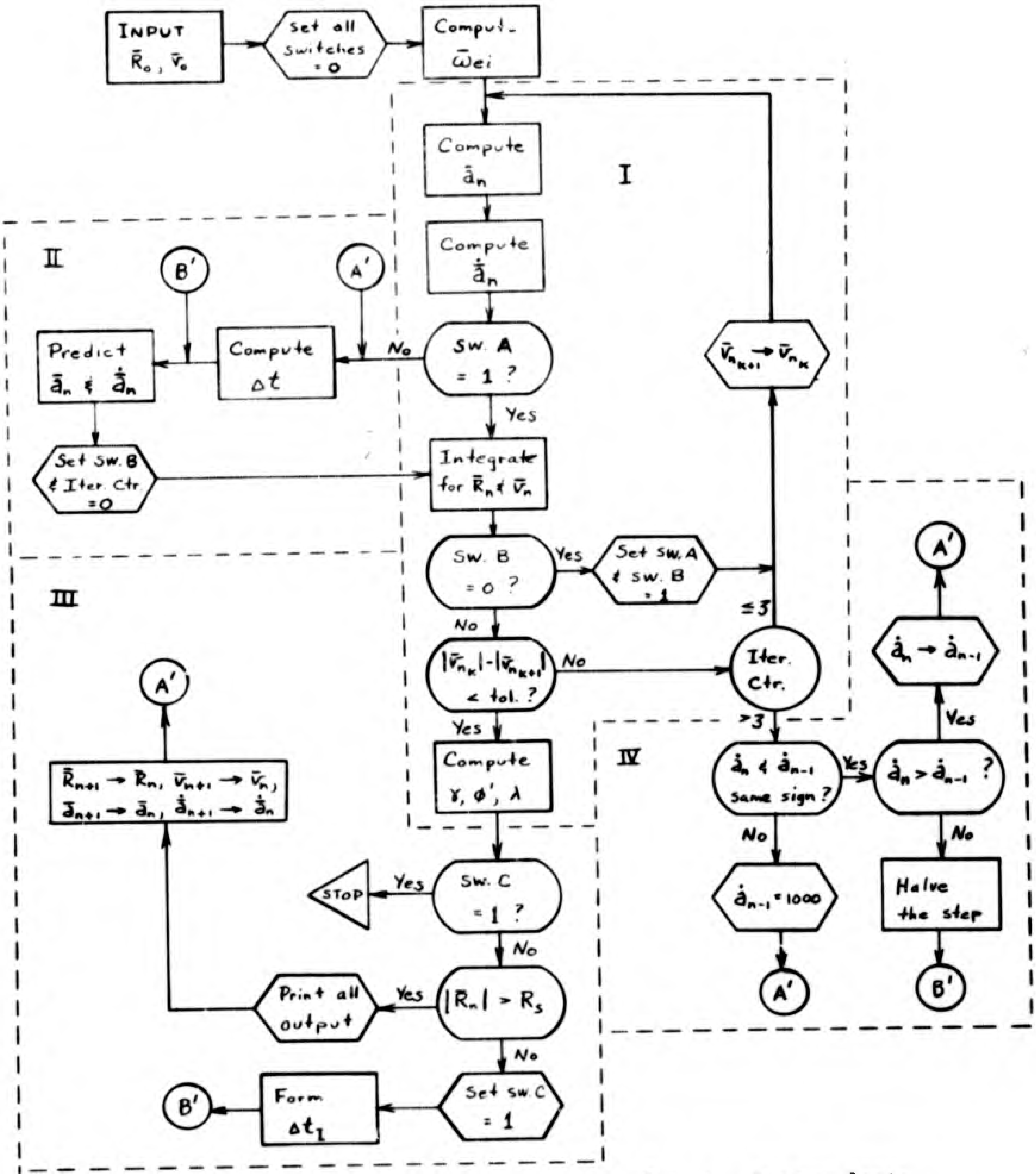


Fig. 4 - Program Logic

1. Section I - Main Control Block.

This block accomplishes the following:

a. The start procedure, or setting up of initial conditions from the input data for the first integration. The basic input of $\bar{R}_o$ and $\bar{v}_o$ is used to compute $\bar{a}_o$ and $\dot{\bar{a}}_o$ (the acceleration and its time derivative at the initial point) by use of the oblate gravity, atmosphere and winds, and aerodynamics subroutines.

b. Switching control to the time step and prediction subroutines.

c. Control of the integration process, which involves an iteration. The iteration is continued until a test indicates that convergence has been obtained. The flight path parameters are then computed and an advance is made to the next integration cycle.

d. Switching control to the Time Step Limiter, Section IV, by means of an iteration counter.

e. Control of the output. Output is printed only when the convergence test is satisfied.

2. Section II - Time Step and Predictor

This section is entered at A' when an advance to the next integration step is made, or at B' when the logical tests in Section IV indicate that the time step,

Δt must be adjusted or when impact has been achieved as determined by the test on $|\bar{R}_n|$ in Section III.

3. Section III - Stopping Control

When the convergence is reached, a test is made on $|\bar{R}_n|$ to determine if impact has been achieved. If it has not an advance to the next integration cycle is made. If it has, the time of impact is linearly predicted by:

$$t_I = t_n - \frac{\partial t}{\partial R}\Delta R = t_{n-1} + \left[\frac{|\bar{R}_n| - \frac{1}{2}(R_{sn-1} + R_{sn})}{|\bar{R}_{n-1}| - |\bar{R}_n|}\right]\Delta t_s$$

where Δt_s = time step from time t_{n-1} to time t_n.

The second term in the above expression, i.e., $(\frac{\partial t}{\partial R}\Delta R)$, is the time step which is used to integrate back to the earth's surface. The iteration for impact is performed as described above.

4. Section IV-Time Step Limiter

If more than three iterations are required to converge, as determined by a counter, tests on $\dot{a}_n$ and $\dot{a}_{n-1}$ are made and the time step is adjusted accordingly, where a_n and $\dot{a}_{n-1}$ are signed magnitudes.

IV. REFERENCES

1. C. E. Herrick, "Programming Manual for the Preliminary VECTRAN System", General Dynamics/Astronautics, Report No. ERR-AN-054, dated 6 July 1961.
2. W. F. Monroe and H. W. Sorenson, "IBM 704 Flight Performance Simulation from Launch to Impact", Convair/Astronautics, Report No. Z N-7-305, dated 8 November 1958. Confidential.
3. R. A. Minzner, K.S.W. Champion, "The ARDC Model Atmosphere, 1959, Air Force Surveys in Geophysics, Report No. 115, August, 1959.
4. N. Sissenwine, "Windspeed Profile, Wind Shear, and Gusts for Design of Guidance Systems for Vertical Rising Air Vehicles," Air Force Surveys in Geophysics, Report No. 57, November, 1954.
5. F. B. Hildebrand, Introduction to Numerical Analysis, McGraw-Hill, 1956, pp. 231-232.

V. APPENDIX

A. Input Data

The following quantities are required as input to the Mark II Re-entry Program:

1. $\bar{R}_o$, $\bar{v}_o$ = initial vector position and velocity, in inertial (uvw) coordinates and canonical units.
2. t_o = initial time, in canonical units.
3. ϕ_o', λ_o = geocentric latitude and longitude of the initial (reference) position, in radians. Latitude north of the equator is considered positive. Longitude west of Greenwich to 180° is considered positive; longitude east of Greenwich to 180° is considered negative.
4. β_i = initial orientation azimuth, measured positive clockwise from due east at the initial position, in radians.
5. W, S = weight, in lbs., and maximum effective cross-sectional area, in square feet, of the re-entry vehicle.
6. a_i, b_j, $(i = 0,1,\ldots 6)$ $(j = 0,1,\ldots 7)$ = empirical coefficients for the rational fraction expression of C_D and $\dot{C}_D$, dependent upon the type of re-entry vehicle (nose cone).

7. Designation of atmospheric model desired: either ICAO Standard or WADC Tropical.

8. β_w = direction of desired wind angle (except for post-flight simulation), in radians.

9. h_o = re-entry altitude, in canonical units.

B. <u>Program Constants</u>

1. Time Step Control Parameters

A = ∞	Δt = 1.0 seconds
B = 4000.0	= 2.0 seconds
C = 1000.0	= 3.0 seconds
D = 100.0	= 4.0 seconds
E = 34.0	= 10.0 seconds
F = 6.0	= 20.0 seconds

2. Weight to mass conversion factor.

g_o = 32.17398 ft/sec^2

3. A = semimajor axis of the oblate spheroid

= 1.0000000 C.U.L.

= 20,926,010 ft.

B = semiminor axis of the oblate spheroid

= 0.99663230 C.U.L.

= 20,855,552 ft.

(Ref. AMS Ellipsoid)

4. Harmonic Coefficients of Earth's Oblateness

C_2 = Second harmonic = 0.0032755879

C_3 = Third harmonic = 0.00000000

C_4 = Fourth harmonic = 0.000010729476

5. Angular speed of the earth about its axis of rotation.

ω_e = 0.058834778 radians / C.U.T.

$\bar{\omega}_{ei} = \omega_e \left(-\sin\theta \sin\beta_i \, \bar{i}_u + \sin\theta \cos\beta_i \, \bar{i}_v + \cos\theta \, \bar{i}_w \right)$

Where θ = Geocentric colatitude, in radians

β_i = initial azimuth, in radians

$\bar{i}_u$, $\bar{i}_v$, $\bar{i}_w$ = unit vectors along the u,v,w, axes of the inertial coordinate system.

= (1,0,0), (0,1,0), (0,0,1).

C. Tables

Two atmospheric models are available for use in the program: the ICAO Standard and the WADC Tropical. The tabular values for λ and a as functions of altitude h are as follows:

(1) Static Pressure Ratio $= k_1 e^{k_2 h} + k_3 h + k_4$

(a) ICAO Standard

Interval, Ft.	Constants
$-2 \times 10^4 < h \leq 10^4$	$k_1 = 1.001429229$
	$k_2 = -0.3736458900 \times 10^{-4}$
	$k_3 = k_4 = 0$
$10^4 < h \leq 3 \times 10^4$	$k_1 = 1.053370708$
	$k_2 = -0.4175843227 \times 10^{-4}$

$3 \times 10^4 < h \leq 10 \times 10^4$	$k_1 = 1.257496100$
	$k_2 = -0.4784240876 \times 10^{-4}$
$10 \times 10^4 < h \leq 20.5 \times 10^4$	$k_1 = 0.56191219$
	$k_2 = -0.39598283 \times 10^{-4}$
$20.5 \times 10^4 < h \leq 26.5 \times 10^4$	$k_1 = 3.0246341$
	$k_2 = -0.46817963 \times 10^{-4}$
$26.5 \times 10^4 < h \leq 33.5 \times 10^4$	$k_1 = 2.4039739$
	$k_2 = -0.46562699 \times 10^{-4}$
$33.5 \times 10^4 < h \leq 500 \times 10^4$	$k_1 = 0.065723072$
	$k_2 = -0.35776307 \times 10^{-4}$

(b) WADC Tropical

$-2 \times 10^4 < h \leq 2 \times 10^4$	$k_1 = 1.013816987$
	$k_2 = -0.3701555416 \times 10^{-4}$
	$k_3 = k_4 = 0$
$2 \times 10^4 < h \leq 4 \times 10^4$	$k_1 = 1.150277975$
	$k_2 = -0.4323494264 \times 10^{-4}$
$4 \times 10^4 < h \leq 10 \times 10^4$	$k_1 = 1.553021783$
	$k_2 = -0.5060730613 \times 10^{-4}$
$10 \times 10^4 < h$	Identical with ICAO

(2) Speed of sound $\qquad a = C_o + C_1 h$

(a) ICAO Standard

$-2 \times 10^4 < h \leq 3.615 \times 10^4$	$C_o = 1118.0$
	$C_1 = -41.10 \times 10^{-4}$
$3.615 \times 10^4 < h \leq 8.238 \times 10^4$	$C_o = 969.5$
	$C_1 = 0.0$

$8.238 \times 10^4 < h \leq 15.535 \times 10^4$	$C_o = 814.3$
	$C_1 = 18.85 \times 10^{-4}$
$15.535 \times 10^4 < h \leq 17.505 \times 10^4$	$C_o = 1107.7$
	$C_1 = 0.0$
$17.505 \times 10^4 < h \leq 25.575 \times 10^4$	$C_o = 1494.6$
	$C_1 = -22.10 \times 10^{-4}$
$25.575 \times 10^4 < h \leq 27.800 \times 10^4$	$C_o = 928.3$
	$C_1 = 0$

(b) WADC Tropical

$-2 \times 10^4 < h \leq 5.550 \times 10^4$	$C_o = 1152.9$
	$C_1 = -42.86 \times 10^{-4}$
$5.550 \times 10^4 < h \leq 6.650 \times 10^4$	$C_o = 741.3$
	$C_1 = 31.25 \times 10^{-4}$
$6.650 \times 10^4 < h \leq 15.535 \times 10^4$	$C_o = 828.0$
	$C_1 = 18.0 \times 10^{-4}$
$15.535 \times 10^4 < h$	Identical with ICAO

(3) Wind Speed, V_{wi}

A linear interpolation for V_{wi} is used in the following table which has for argument the altitude, h:

h in ft x 10^4	V_{wi} in ft/sec
-2.0	18.0
0.0	18.0
1.10	54.0
2.35	150.0
3.40	258.0

3.60	300.0
3.75	250.0
4.20	178.0
5.10	139.0
8.10	78.0
9.80	89.0
11.50	133.0
17.50	310.0
27.80	0.0
40.00	0.0