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Curves of Geoheavy, Ascension Wave and Min Pointing Data for Hyperbolic Scenic

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15 December 1954

Curves of Shockwave, Expansion Wave and Skin Friction Data for Hypersonic Speeds

1.0 Summary

This report presents curves of shockwave, expansion wave and boundary layer data plotted for use at Mach numbers up to 20. Normal and oblique shockwave parameters, conical shock parameters, expansion wave parameters, some aerodynamic coefficients and compressible laminar and turbulent boundary layer skin friction formulas are given for Mach numbers up to at least 20. The use of these curves is briefly described in the text.

2.0 Symbols

C_D - wave drag coefficient = $\frac{\text{Wave Drag}}{qS}$

C_f - local skin friction coefficient = $\frac{\tau}{q}$

C_F - mean skin friction coefficient = $\frac{\text{Drag}}{qS}$

C_N - normal force coefficient = $\frac{\text{Normal Force}}{qS}$

C_p - coefficient of specific heat at constant pressure BTU/lb.°R

C_X - axial drag coefficient = $\frac{\text{axial drag}}{qS}$

g	- acceleration due to gravity	ft./sec.^{2}
h	- heat transfer coefficient	BTU/ft.^{2}hr.°R
H	- total pressure	lbs./ft.^{2}
J	- mechanical equivalent of heat	ft.-lb./BTU
k	- thermal conductivity	BTU/ft. hr.°R
l	- wetted length	ft.
l'	- effective starting length	ft.
M	- Mach number	
N	- Nusselt number = $\frac{hl}{k}$	
p	- static pressure	lb./ft.^{2}
P_r	- Prandtl number = $\frac{\mu C_p}{k}$	

Q	heat flux	BTU/hr. ft.^{2}
q	dynamic pressure	lb./ft.^{2}
R_e	Reynolds number = $\frac{\rho u l}{\mu}$	
S	reference area	ft.^{2}
T	static temperature	^oR
T'	reference temperature	^oR
T_i	recovery or inherent temperature	^oR
u	velocity	ft./sec.
x	axial length	ft.

Greek

α	angle of attack	degrees
β	shockwave angle	degrees
γ	ratio of specific heat	
δ	flow deflection angle, boundary layer thickness	degrees, ft.
θ	boundary layer momentum thickness	ft.
θ_v	cone semi-vertex angle	degrees
θ_w	cone shockwave half angle	degrees
μ	coefficient of viscosity	lb. sec./ft.^{2}
ν	Prandtl-Meyer angle	degrees
ρ	density	lb. sec.^{2}/ft.^{4}
τ	shear stress	lbs./ft.^{2}

Subscripts

1	initial or free stream
2	after the shock
s	surface of the cone
oo	sea level

w - at the wall

∞ - local

i - incompressible or location index

3.0 Discussion and Results

The need for a combined collection of basic supersonic aerodynamic data has long been evident to Convair Aerodynamicists working with high velocity bodies. The existing charts and tables (Reference 1, 2, 3) are either limited in some phase of Mach number and flow deflection angle or the data are not presented graphically (Reference 4). It was believed also that some convenient reference should be provided for the skin friction and heat transfer formulae associated with the reference temperature method explained in Reference 9. This report fulfills these needs.

The sections following will deal with the separate types of curves presented.

3.1 Inviscid Flow Parameters

The curves in this section are separated into one, two and three dimensional flow. The flow parameters presented, all versus initial Mach number, are Mach number after the shock, static pressure ratio, static temperature ratio, density ratio, velocity ratio, dynamic pressure ratio, Reynolds number ratio, total head ratio and shockwave angle. All ratios are quantities after the wave divided by the value of the same quantity before the wave.

C_N, $\frac{dC_N}{d\alpha}$ and C_X are given for cones in Newtonian flow (Reference 4, 5, 6).

The Reynolds numbers presented in Figure 1.6, 2.8 and 3.8 were found from

$$\frac{R_{e2}}{R_{e1}} = \frac{\rho_2 \, u_2}{\rho_1 \, u_1} \left(\frac{T_1}{T_2}\right)^{0.7}$$

where an exponential viscosity - temperature relation

$$\frac{\mu_2}{\mu_1} = \left(\frac{T_2}{T_1}\right)^{0.7}$$

was assumed.

The data for these curves were obtained from Reference 1 through Reference 6 or were computed for high Mach number-deflection angle combinations when they were not tabulated.

3.2 Viscous Flow Parameters By The Reference Temperature Method

The curves presented in this section are primarily for use with the reference temperature method of calculating heat transfer and skin friction. Figures 4.1 - 4.7 include: local and mean incompressible flow skin friction coefficients for laminar, transition and turbulent flow vs. Reynolds number (Reference 7); reference temperature ratio T'/T vs. Mach number; and reference temperature $(T')^{1.7}$ vs. T'. For use in the heat balance equation the inherent (recovery) temperature rise ΔT_i is given versus velocity. The theoretical laminary boundary layer stability curve (Reference 8) is shown in Figure 4.5 to indicate the type of boundary layer flow which might be expected for various H, T_w/T and Re combinations. Figures 4.6 and 4.7 present the NACA-NBS variation of C_p with T and variation of μ with T as given by the Sutherland viscosity rule.

The reference temperature method and its associated equations will be presented below in outline form. A complete derivation of these equations may be found in Reference 9, which also contains a critical evaluation of the method. The concept of an "effective length", needed for the application of this method to bodies other than flat plates, is discussed by Sieff (Reference 10) and Romig (Reference 11). Therefore the following sections are primarily on the use of the method in obtaining the heat transfer and skin friction.

The reference temperature method is based on the assumption that the use of a characteristic temperature in the compressible equations for drag and heat transfer will eliminate their outward dependence on temperature and Mach number, i.e., convert them to the incompressible, constant-property equation. Under this assumption the T' method is thus described as: if all temperature-dependent properties in the incompressible equations for heat transfer and drag are based on the reference temperature T' then the incompressible equations will yield the compressible friction or heat transfer coefficients. Use of this method therefore obviates solving the compressible laminar and turbulent boundary layer equations. The equations for the reference temperature are

$$T' = T_\infty \left[0.42 + 0.032 M_\infty^2 + 0.58 \frac{T_w}{T_\infty}\right], \quad M_\infty \leq 5.5$$

$$T' = T_\infty \left[0.70 + 0.023 M_\infty^2 + 0.58 \frac{T_w}{T_\infty}\right], \quad M_\infty \geq 5.5 \qquad (1)$$

where the ratio T'/T_∞ may be found in Figure 4.2 vs. Mach number for integral values of $1 \leq T_w/T_\infty \leq 4$. The eq. (1) are valid for both laminar and turbulent flow.

3.2.1 Basic Equations for Two-Dimensional Flow

The compressible skin friction may be obtained either as the ratio of compressible to incompressible, c_f / c_{f_i}, or explicitly as a function of the Reynolds number based on T'. For practical considerations explicit relationships will be developed first for the local and mean compressible friction.

Assume all air properties in the incompressible drag equations are based on f' so that according to the assumption stated in 3.2,

$$(\text{compressible})\, c_{f_\infty} q_\infty \equiv \tau \equiv (\text{incompressible})\, c_{f_i}' q'$$

where the primes denote that air properties are based on T'.

or
$$c_{f_\infty} = c_{f_i}' \frac{\rho'}{\rho_\infty}$$

Using the perfect gas equation at constant pressure,

$$\boxed{c_{f_\infty} = \frac{T_\infty}{T'} c_{f_i}'} \qquad (2)$$

where c_{f_i}' is given either by the Blasius formula

$$\text{LAMINAR} \quad c_{f_i}' = \frac{0.664}{\sqrt{\frac{\rho' u \ell}{\mu'}}} \qquad (3)$$

or the Karman equation

$$\text{TURBULENT} \quad \left(c_{f_i}'\right)^{-1/2} = 1.7 + 4.15 \log_{10} \frac{\rho' u \ell}{\mu'} c_{f_i}' \qquad (4)$$

These laminar and turbulent incompressible friction coefficients (Eq. 3 & 4) are plotted in Figure 4.1 versus Reynolds number. The Reynolds number to be used with Eq. (3) & (4) and hence in evaluating Eq. (2) is a function of T'. The expression for $R_{e_{T'}}$ is given as

$$R_{e_{T'}} = \frac{\rho' u \ell}{\mu'}$$

or, using the power law for viscosity, $\mu \propto (T)^{0.7}$ and using ambient conditions as a reference,

$$R_{e_{T'}} = \left(\frac{T_\infty}{T'}\right)^{1.7} R_{e_\infty} \tag{5}$$

If the perfect gas law is used and sea level (T = 518.4°R) conditions are used in the power law,

$$R_{e_{T'}} = 12.44 \times 10^4 \frac{\rho_\infty u_\infty l}{(T')^{1.7}} \tag{6}$$

Thus it is only necessary to enter Figure 4.1 with $R_{e_{T'}}$, computed by Eq. (5) or Eq. (6), locate the desired $C_{f_i'}$ and compute C_{f_∞} by Eq. (2). For convenience in computing $R_{e_{T'}}$, the function $(T')^{1.7}$ is plotted versus T' in Figure 4.3.

The mean skin friction coefficients may be obtained in much the same way from the drag equation. For both laminar and turbulent flow

$$C_{F_\infty} = \frac{T_\infty}{T'} C_{F_i'} \tag{7}$$

where $C_{F_i'}$ is given by the Blasius formula

$$\text{LAMINAR} \quad C_{F_i'} = \frac{1.328}{\sqrt{R_{e_{T'}}}} \tag{8}$$

and the Prandtl-Schlicting equation

$$\text{TURBULENT} \quad C_{F_i'} = \frac{0.455}{(\log_{10} R_{e_{T'}})^{2.58}} \tag{9}$$

Eqs. (8) and (9) are also plotted versus Reynolds number in Figure 4.1. The same procedure is used in evaluating the mean coefficient as was used for the local.

The compressible heat transfer rate can be obtained from the definition of the heat equation

$$Q = h(T_i - T_w) \tag{10}$$

where the heat transfer coefficient is defined as

$$h = \frac{Nu\, k}{\ell} \tag{11}$$

If the air properties in h are based on T' and the following incompressible relationships for Nusselt number are used,

$$\text{LAMINAR} \quad Nu_\infty = 0.332 \sqrt[3]{P_r'} \sqrt{R_{e_{T'}}} \tag{12}$$

and

$$\text{TURBULENT} \quad Nu_\infty = 0.0296 \left(R_{e_{T'}}\right)^{0.8} \tag{13}$$

then after considerable algebraic manipulation the following heat transfer equations are found

$$\text{LAMINAR} \quad h = 0.0074 \left[\frac{\rho_\infty u_\infty}{\ell}\right]^{0.5} \tag{14}$$

$$\text{TURBULENT} \quad h = \frac{0.02498 \left(\rho_\infty u_\infty\right)^{0.8}}{\ell^{0.2} \left(T'\right)^{0.51}} \tag{15}$$

It was assumed in Eq. (14) and (15) that the air properties varied in the following way

$$\frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty}\right)^{0.7}, \quad \frac{k}{k_\infty} = \left(\frac{T}{T_\infty}\right)^{0.85}, \quad P_r = 0.72 \neq p = \rho RT \tag{15a}$$

The validity of Eq. (14) in particular and the exponents in the power laws were substantiated in the investigations of Reference (9). It was found that these heat transfer equations are correct as long as the boundary layer air is not dissociated.

Curves of $\Delta T_i = T_i - T_1$, where T_1 is the free stream static temperature, are plotted versus free stream velocity in Figure 4.4a - 4.4c for use in eq. (10).

Thus to compute the compressible laminar or turbulent heat, it is only necessary to find the reference temperature and solve eq. (14) or (15). These values of h are used along with the temperatures obtained from Figure 4.4a - 4.4c in eq. (10) to find the heat rate Q.

3.2.2 Two Dimensional Flow With Discontinuities

The equations (1) - (15) written above apply in strict sense only to flat plate flow. Any condition which would tend to make the external flow discontinuous at a point, such as transition from laminar to turbulent flow, or passing through a shockwave or expansion, renders the equations inapplicable.

In order to use eq. (1) - (15) it is necessary, therefore, to somehow refer the boundary layer characteristics at the point of discontinuity to a flat plate boundary layer with identical characteristics but with no flow discontinuties. This can be accomplished by forcing, as it were, the momentum loss of the boundary layer flow to remain constant across any discontinuity. This momentum-loss method will give an effective starting length for the boundary layer characteristics to the point of discontinuity.

The momentum loss equations are discussed at length in Reference (10) and (11) and only a brief outline of them are given here. If the momentum loss through the boundary layer is defined as

$$\int_0^{\delta} \rho u (u_\infty - u)\, dy$$

then across any point of discontinuity it is assumed that

$$\int_0^{\delta_1} \rho u (u_1 - u)\, dy = \int_0^{\delta_2} \rho u (u_2 - u)\, dy \qquad (16)$$

This is equivalent to stating that.

$$\theta_1 q_1 = \theta_2 q_2 \qquad (17)$$

where θ is the boundary layer momentum thickness and q the stream dynamic pressure. The laminar or turbulent equations for θ allow (17) to be solved for ℓ_2, the effective starting length. For convenience in nomenclature ℓ_2 is designated as ℓ' hereafter.

Therefore let

$$\theta_{LAM} = \frac{1.328}{\left(\frac{\rho_\infty u_\infty \ell}{\mu_\infty}\right)^{1/2}} \left(\frac{C_F}{C_{F_i}}\right)_{LAM} \ell \qquad (18)$$

and

$$\theta_{TURB} = \frac{0.072}{\left(\frac{\rho_\infty U_\infty \ell}{\mu_\infty}\right)^{0.2}} \left(\frac{C_F}{C_{Fi}}\right)_{TURB} \ell \qquad (19)$$

where ℓ is the length along the body to the point of discontinuity. The turbulent expression for θ is based on the 1/7th power law for velocity. It is adequate for $5 \times 10^5 \leq Re \leq 10^7$. The more exact solution (Eq. (9)) would not be amenable for computation of ℓ'. These equations giving θ can be simplified by making the same assumptions used in the heat equations (14) and (15) in the skin friction derivation to get

$$\left(\frac{C_F}{C_{Fi}}\right)_{LAM} = \left(\frac{T_\infty}{T'}\right)^{0.15} \quad \& \quad \left(\frac{C_F}{C_{Fi}}\right)_{TURB} = \left(\frac{T_\infty}{T'}\right)^{0.66} \qquad (20)$$

Substitution of (20) into (18) and (19) and use of Eq. (15a) gives

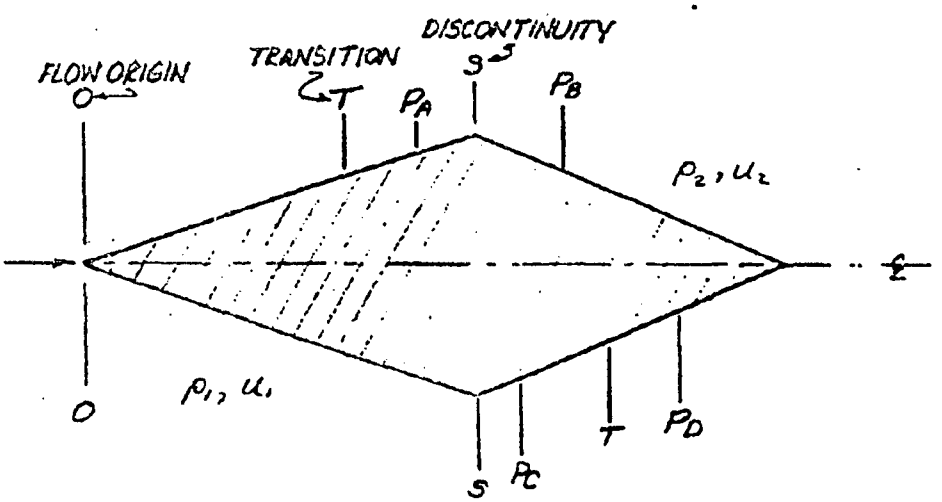
$$\theta_{LAM} = 0.003765 \left[\frac{\rho_\infty U_\infty}{(T_\infty)^{1.7}}\right]^{-0.5} \left(\frac{T_\infty}{T'}\right)^{0.15} \ell^{0.5} \qquad (21)$$

and

$$\theta_{TURB} = 0.006892 \left[\frac{\rho_\infty U_\infty}{(T_\infty)^{1.7}}\right]^{-0.2} \left(\frac{T_\infty}{T'}\right)^{0.66} \ell^{0.8} \qquad (22)$$

These equations can then be used in (17) to obtain ℓ'.

Two types of flow discontinuities can be encountered in two dimensional flow. These are (a) transition from laminar to turbulent flow and (b) shockwave or an expansion. The equations for ℓ' for combinations of these cases are obtained from Eq. (17) - (22) and are summarized in the following table for flow on an infinite double-wedge airfoil, a typical two-dimensional body.



The following effective starting lengths are given to the points P of interest

TO P_A $$\ell_A' = \overline{TP_A} + 0.4696\left[\frac{p_1 u_1}{(T_1')^{1.7}}\right]^{-0.375}(\overline{OT})^{0.625} \qquad (23)$$

TO P_B $$\ell_B' = \overline{SP_B} + (\overline{SP_A} + \ell_A')\frac{p_1}{p_2}\left(\frac{u_1}{u_2}\right)^{1.5}\left(\frac{T_2'}{T_1'}\right)^{0.825} \qquad (24)$$

TO P_C $$\ell_C' = \overline{SP_C} + \overline{OS}\,\frac{p_1}{p_2}\left(\frac{u_1}{u_2}\right)^{3}\left(\frac{T_2'}{T_1'}\right)^{0.3} \qquad (25)$$

TO P_D $$\ell_D' = \overline{TP_D} + (\ell_C' + \overline{P_C T})^{0.625} \cdot 0.4696\left[\frac{p_2 u_2}{(T_2')^{1.7}}\right]^{-0.375} \qquad (26)$$

Eq. (23) - (26) may then be used to evaluate the ℓ', which in turn can be used in Eq. (2) - (15) along with local conditions to evaluate the desired coefficients. Since Eq. (1) is independent of length this method does not involve iteration.

3.2.3 Three Dimensional Flow

Since the boundary layer at any point on a cone is much thinner than the boundary layer of a flat plate which has otherwise the same boundary layer characteristics as the cone, the Eq. (2) - (15) do not apply to conical flow unless some correction is made in order to correlate the boundary layer thicknesses. This correction can be made by working directly with the three-dimensional boundary layer equations in the laminar case. Mantzsche and Wendt (Reference 12) found that the conical equations transformed directly into the two dimensional equations if the factor 1/3 were inserted in the cone length. Therefore, for laminar flow, if

$$\text{LAMINAR: } l_{CONE} = \frac{1}{3} l_{FLAT\ PLATE} \tag{27}$$

then Eq. (2) - (15) (with slight modification in the mean friction coefficient) can be used. Van Driest (Reference 13) made the same type of investigation of the three-dimensional turbulent equations and found that the effective starting length for a cone in fully turbulent flow is

$$\text{TURBULENT } l_{CONE} = \frac{1}{2} l_{FLAT\ PLATE} \tag{28}$$

Summarized below, for convenience, are eq. (2) - (15) for conical flow. The length is used is the actual cone wetted length.

laminar $$R_{e_{T'}} = 4.147 \times 10^4 \frac{\rho_s u_s l}{(T')^{1.7}} \tag{29}$$

turbulent $$R_{e_{T'}} = 6.22 \times 10^4 \frac{\rho_s u_s l}{(T')^{1.7}} \tag{30}$$

Local skin friction (unchanged) $$c_{f\infty} = \left(\frac{T_\infty}{T'}\right) c_{f_c}'$$

mean laminar $$C_{F_\infty} = 2/3 \cot\theta_v \left(\frac{T_s}{T'}\right) C_{F_c}' \tag{31}$$

referred to base area of cone

mean turbulent $$C_{F_\infty} = 1.02 \cot\theta_v \left(\frac{T_s}{T'}\right) C_{F_c}' \tag{32}$$

referred to base area of cone
(using the 1/5 or 1/7 power law for gives identical results)

laminar $$c_f = \frac{0.01281}{0.1284} \left(\frac{\rho_s u_s}{l}\right)^{0.5} \tag{33}$$

and turbulent $$h = \frac{0.02866\,(\rho_s u_s)^{0.8}}{(\ell)^{0.2}\,(T')^{0.51}} \qquad (34)$$

Eq. (29) - (34) are then solved in the same manner as the equations (2) - (15) for two dimensional flow.

3.2.4 Three Dimensional Flow with Discontinuities

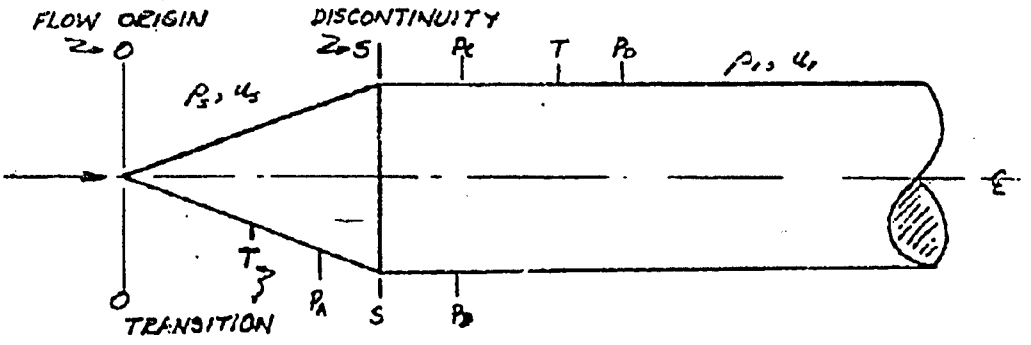
The constant momentum-loss method of section 3.2.2 can be applied to the case of three-dimensional flow. The only changes which need be made are in the equations for the momentum thickness where the factors 1/3 and 1/2 must be inserted in the laminar and turbulent δ, respectively. Making this change we have

$$\text{LAMINAR} \quad \theta_{cone} = 0.002174\left(\frac{\rho_s u_s}{(T_s)^{1.7}}\right)^{-.5}\left(\frac{T_s}{T'}\right)^{.15}\ell^{0.5} \qquad (35)$$

$$\text{TURBULENT} \quad \theta_{cone} = 0.003959\left(\frac{\rho_s u_s}{(T_s)^{1.7}}\right)^{-0.2}\left(\frac{T_s}{T'}\right)^{0.46}\ell^{0.8} \qquad (36)$$

These equations are used in Eq. (17) to find ℓ', the effective length, which is then used in Eq. (2) - (15) to obtain friction and heat transfer.

The types of discontinuities usually encountered in conical flow would be found, say, on a cone-cylinder. The equations for various starting lengths found from Eq. (17), (35) and (36) are summarized for that body in the following table:



ASSUMED PRESSURE GRADIENT ON THE CYLINDER

$$\text{AT } P_A: \quad \ell_A' = \frac{\overline{TP_A}}{2} + 0.4727\left(\overline{OT}\right)^{0.625}\left[\frac{\rho_s u_s}{(T_s')^{1.7}}\right]^{-0.375} \tag{37}$$

$$\text{AT } P_B: \quad \ell_B' = \overline{SP_B} + 0.57435\left(\ell_A' + \frac{\overline{SP_A}}{2}\right)\frac{\rho_s}{\rho_1}\left(\frac{u_s}{u_1}\right)^{1.5}\left(\frac{T_1'}{T_s'}\right)^{0.825} \tag{38}$$

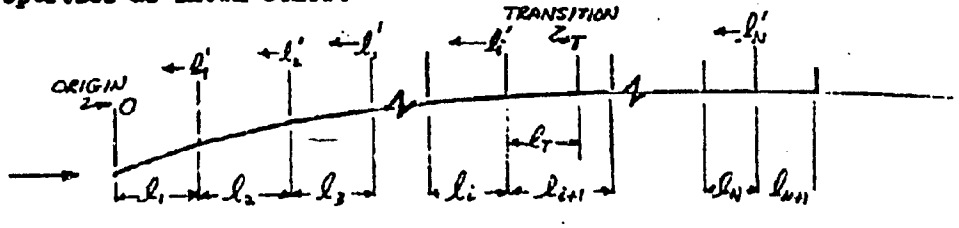
$$\text{AT } P_C: \quad \ell_C' = \overline{SP_C} + 0.5773\,\frac{\overline{OS}}{3}\,\frac{\rho_s}{\rho_1}\left(\frac{u_s}{u_1}\right)^{3}\left(\frac{T_1'}{T_s'}\right)^{0.3} \tag{39}$$

$$\text{AT } P_D: \quad \ell_D' = \overline{TP_D} + 0.4696\left[\ell_C' + \overline{TP_C}\right]^{0.625}\left[\frac{\rho_2 u_1}{(T_2')^{1.7}}\right]^{-0.375} \tag{40}$$

to be used in Eq. (2) - (15) to find the friction and heat transfer.

3.2.5 Flow With Streamwise Gradients

In section 3.2.4 the flow on the cylinder portion of the body investigated was assumed to be at constant pressure in order to obtain the simple equations (38), (39) and (40). For the case where a streamwise gradient is to be considered the body must be broken into small segments of constant flow properties. If this is done then the equations (17), (21) and (22) can be applied to each section and equivalent starting lengths can be found for any point on the body. For flow of such a type consider a semi-infinite biconvex airfoil broken into small segments of constant stream properties as shown below:



Each segment has the stream properties p_1, u_1; p_2, u_2; p_3, u_3; ...etc.

To derive the general equation for the effective length along the body at a point preceded by completely laminar flow, apply Eq. (21) and (22) to Eq. (17). For segment ℓ_1 ,

$$\theta_1 g_1 = \theta_2 g_2$$

So

$$0.003765 \left[\frac{p_1 u_1}{(T_1)^{1.7}}\right]^{-0.5} \left(\frac{T_1}{T_1'}\right)^{0.15} \ell_1^{0.5} g_1 = 0.003765 \left[\frac{p_2 u_2}{(T_2)^{1.7}}\right]^{-0.5} \left(\frac{T_2}{T_2'}\right)^{0.15} \ell_1'^{0.5} g_2$$

or

$$(\ell_1')^{0.5} = \ell_1^{0.5} \left(\frac{p_2 u_2}{p_1 u_1}\right)^{0.5} \left(\frac{T_1}{T_2}\right)^{0.5} \left(\frac{T_1}{T_2}\right)^{0.15} \left(\frac{T_2'}{T_1'}\right)^{0.15} \frac{g_1}{g_2}$$

$$\ell_1' = \ell_1 \left(\frac{p_2 u_2}{p_1 u_1}\right) \left(\frac{T_1}{T_2}\right)^2 \left(\frac{T_2'}{T_1'}\right)^{0.3} \left(\frac{g_1}{g_2}\right)^2$$

Apply the perfect gas law to the g ratio

$$\frac{g_1}{g_2} = \frac{\rho_1 u_1^2}{\rho_2 u_2^2} = \frac{p_1 u_1^2 T_2}{p_2 u_2^2 T_1}$$

and insert in the above equation to get

$$\ell_1' = \ell_1 \left(\frac{p_1 u_1^2}{p_2 u_2^2}\right)^2 \left(\frac{p_2 u_2}{p_1 u_1}\right) \left(\frac{T_2'}{T_1'}\right)^{0.3} \left(\frac{T_1}{T_2}\right)^2 \left(\frac{T_2}{T_1}\right)^2.$$

or $$\ell_1' = \ell_1 \frac{p_1}{p_2} \left(\frac{u_1}{u_2}\right)^3 \left(\frac{T_2'}{T_1'}\right)^{0.3}$$

This procedure can be repeated for each segment, i.e.,

$$\ell_2' = (\ell_2 + \ell_1') \frac{p_2}{p_3} \left(\frac{u_2}{u_3}\right)^3 \left(\frac{T_3'}{T_2'}\right)^{0.3}$$

or

$$\ell_2' = \ell_2 \frac{p_2}{p_3} \left(\frac{u_2}{u_3}\right)^3 \left(\frac{T_3'}{T_2'}\right)^{0.3} + \ell_1 \frac{p_1}{p_3} \left(\frac{u_1}{u_3}\right)^3 \left(\frac{T_3'}{T_1'}\right)^{0.3}$$

and in general

$$\ell_i' = \sum_{k=1}^{k=i} \ell_k \frac{p_k}{p_i} \left(\frac{u_k}{u_i}\right)^3 \left(\frac{T_i'}{T_k'}\right)^{0.3} \tag{41}$$

Let transition occur at some point T in segment ℓ_{i+1} at some distance ℓ_T from the beginning of the segment. Then the effective length to T is

$$\ell_T' = \left(\ell_T + \ell_i'\right)^{0.625} 0.469 \left[\frac{p_{i-1}\, u_{i+1}}{\left(T_{i+1}'\right)^{1.7}}\right]^{-0.375} \tag{42}$$

and for any general turbulent section thereafter

$$\ell_N' = \sum_{k=i+2}^{k=N} \ell_k \frac{p_k}{p_N} \left(\frac{u_k}{u_N}\right)^{1.5} \left(\frac{T_N'}{T_k'}\right)^{0.625} + \left(\ell_T' + \ell_{i+1} - \ell_T\right) \frac{p_{i+1}}{p_N} \left(\frac{u_{i+1}}{u_N}\right)^{1.5} \left(\frac{T_N'}{T_{i+1}'}\right)^{0.625} \tag{43}$$

These generalized formulas for ℓ' may then be used along with local conditions to evaluate the friction and heat transfer at any point from Eq. (2) - (15).

4.0 Conclusions

A collection has been made of aerodynamic flow characteristics for one, two and three dimensional hypersonic flow. The Mach number range is from $0 \leq M \leq 20$. The flow parameters are shown as functions of initial Mach number and deflection angle in sections 1, 2, 3; Figures 1.0 through 3.16.

Equations for compressible skin friction and heat transfer using the reference temperature method were presented for two and three dimensional flow, flow with discontinuities and flow with streamwise gradient. Figures necessary in using the formulae are in section 4, Figure 4.1 - 4.7.

5.0 References

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7. Von Karman, T. "Turbulence and Skin Friction", Journal of Aeronautical Sciences, January 1934.

8. Van Driest, E. R. "Calculation of the Stability of the Laminar Boundary Layer in a Compressible Fluid on a Flat Plate With Heat Transfer", Journal of Aeronautical Sciences, December 1952.

9. Romig, M. F. "The Reference Temperature Method for Calculating Skin Friction and Heat Transfer in Compressible Flow". Convair Memo A-Atlas-151, 3 December 1954.

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12. Hantzsche, W. and Wendt, H. "The Laminar Boundary Layer on a Cone in a Supersonic Air Stream at Zero Angle of Attack". Translation NAT-6, RAND Corporation, 1947.

13. Van Driest, E. R. "Turbulent Boundary Layer on a Cone in a Supersonic Flow at Zero Angle of Attack". Journal of Aeronautical Sciences, January 1952.

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Section 1. One-Dimensional Flow Characteristics

Plotted versus Initial Mach Number

Section 2. Two-Dimensional Flow Characteristics

A. Compression

Plotted versus Initial Mach Number with Wedge Angle a Parameter

B. Expansion

Section 3. Conical Flow

A. Flow Characteristics, Pointed Cone

Plotted versus Free Stream Mach Number with Cone Semi-Vertex Angle a Parameter

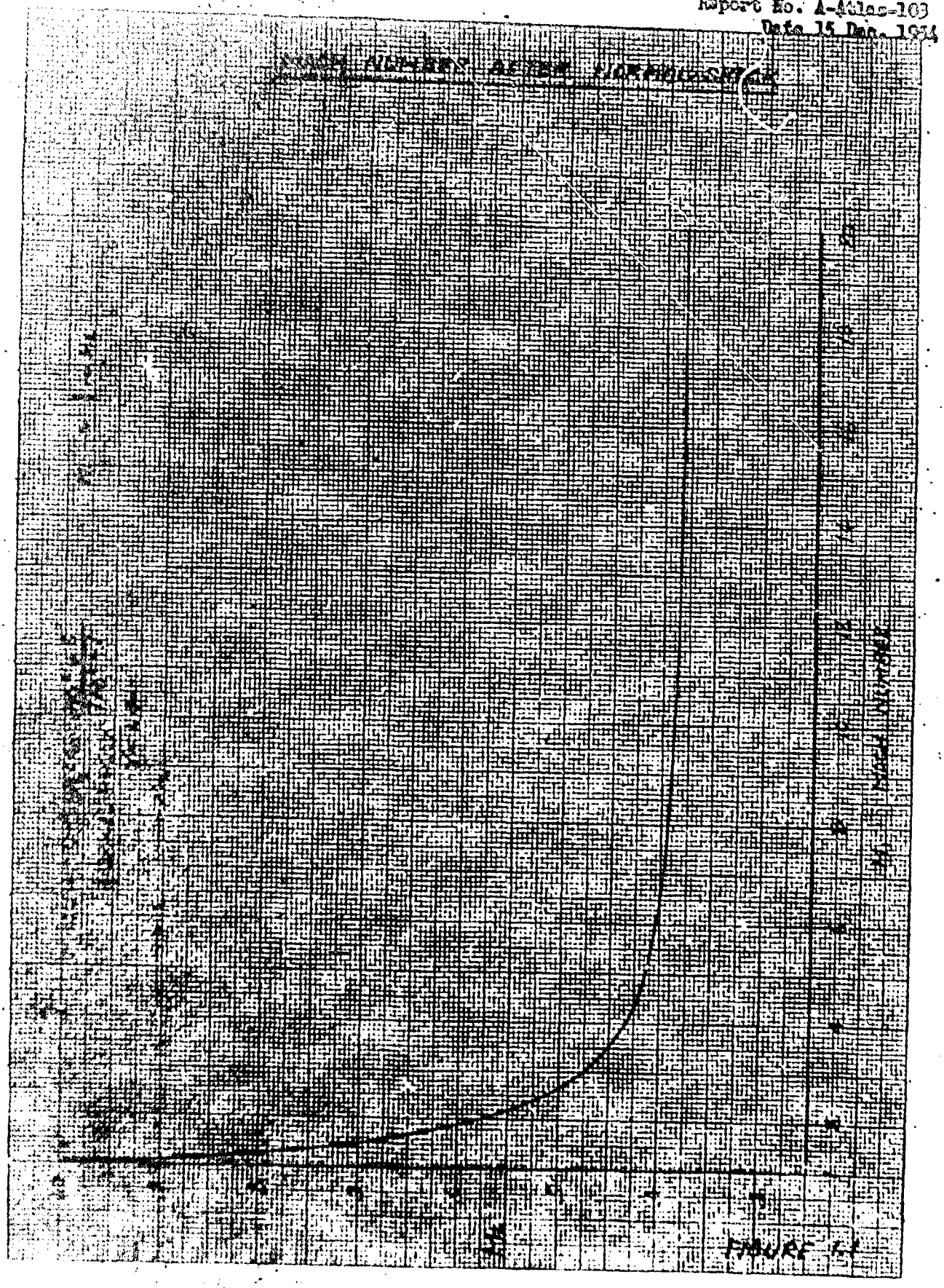


FIGURE 11

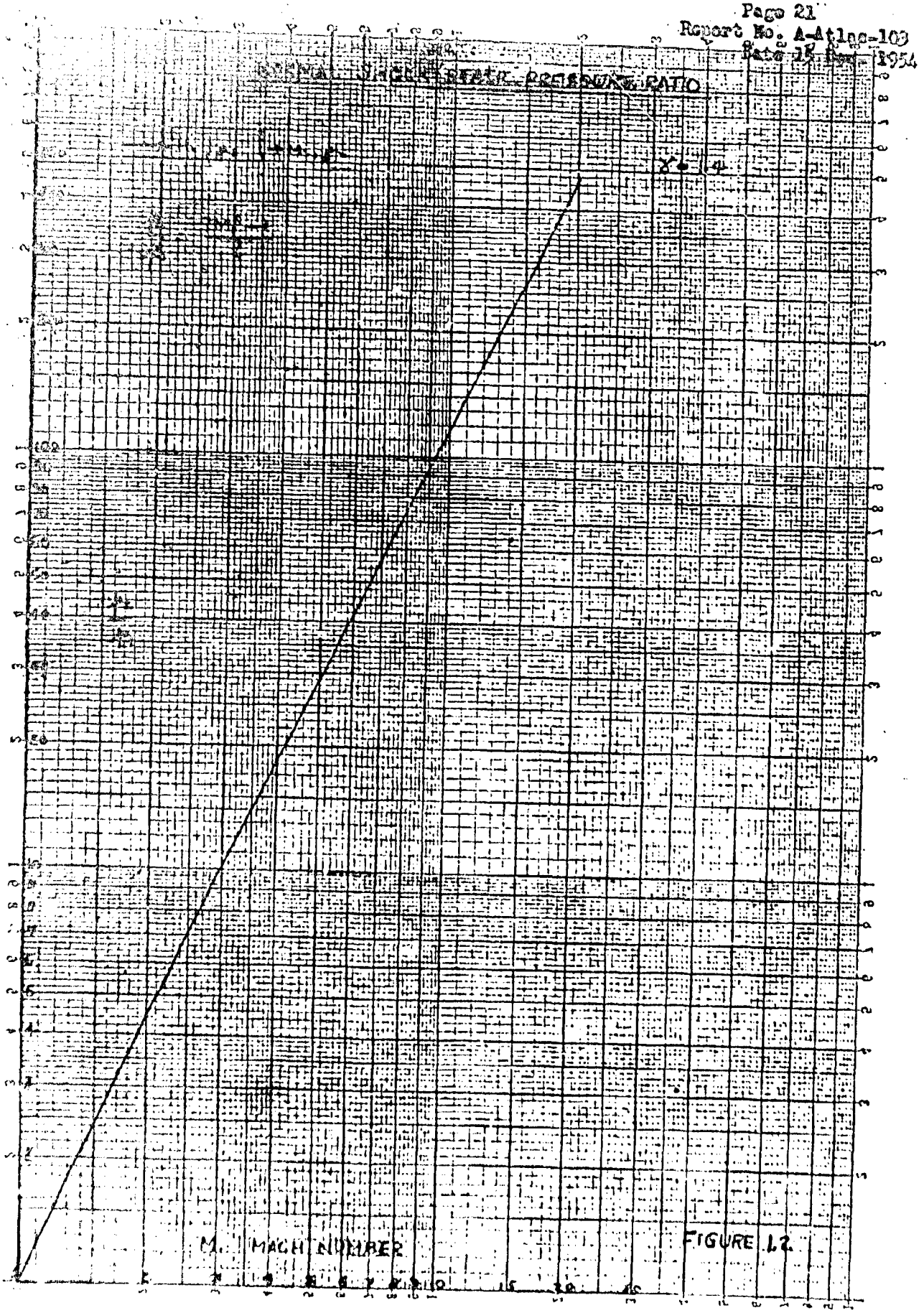


FIGURE 12

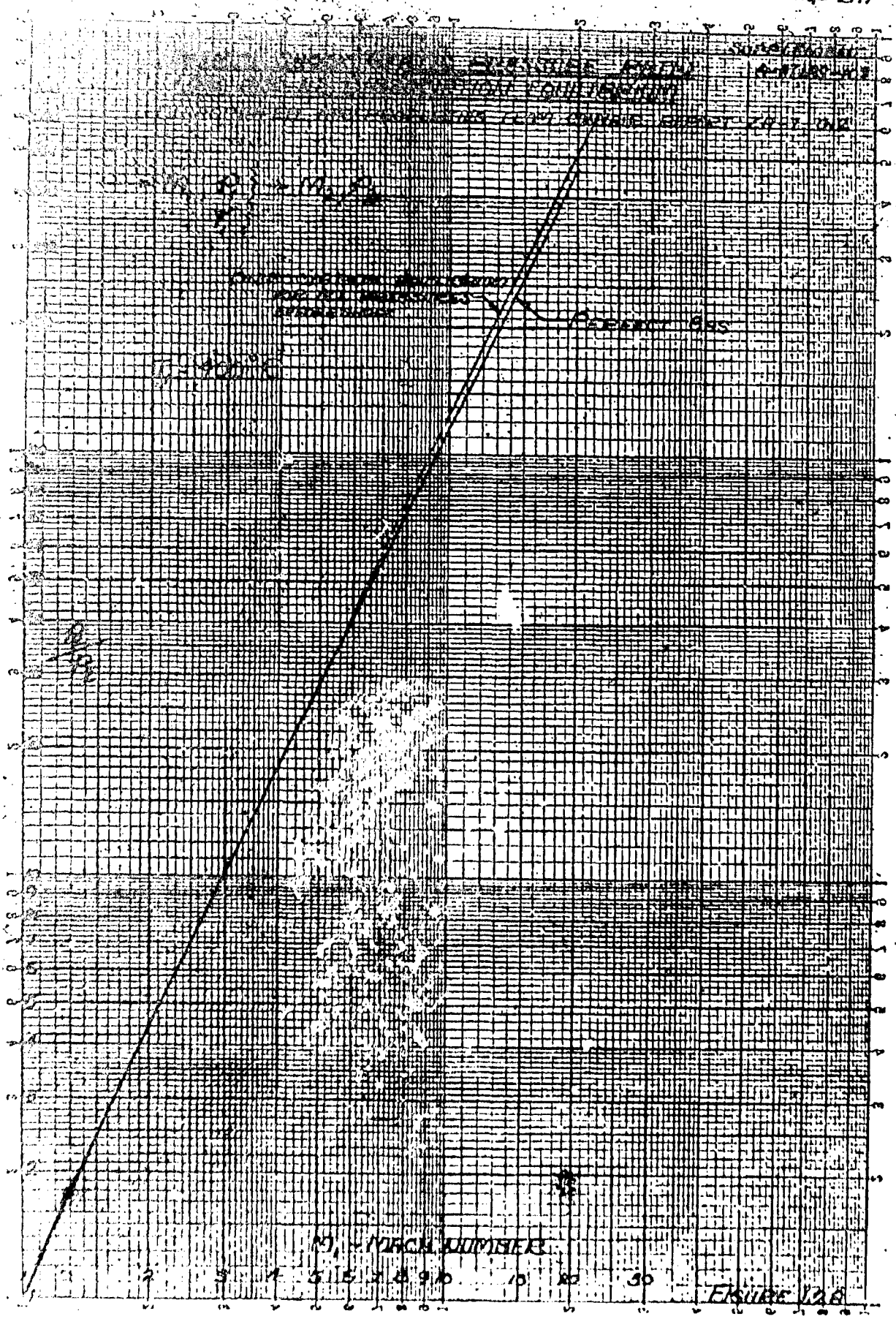


Figure 12B

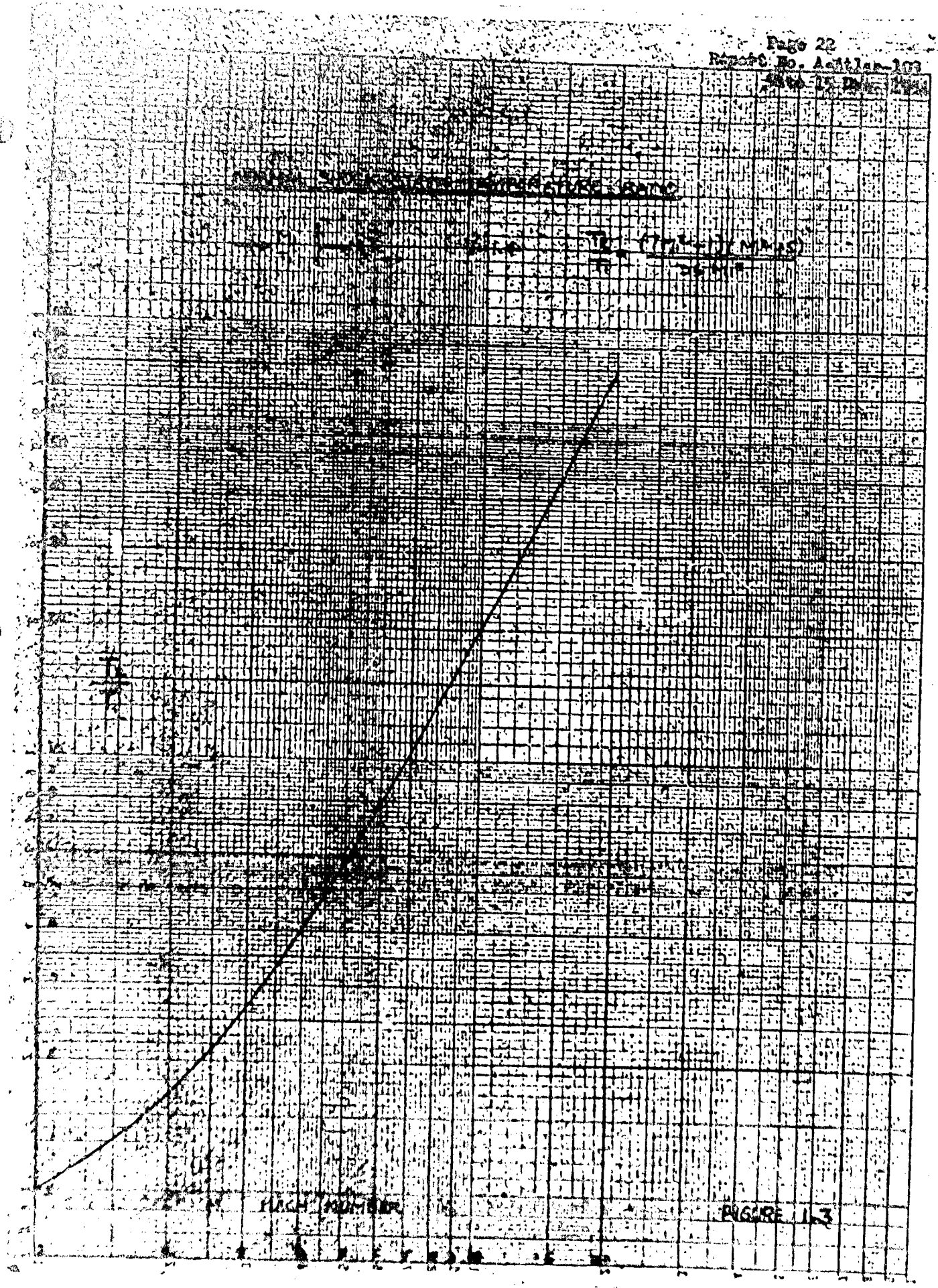


FIGURE 13

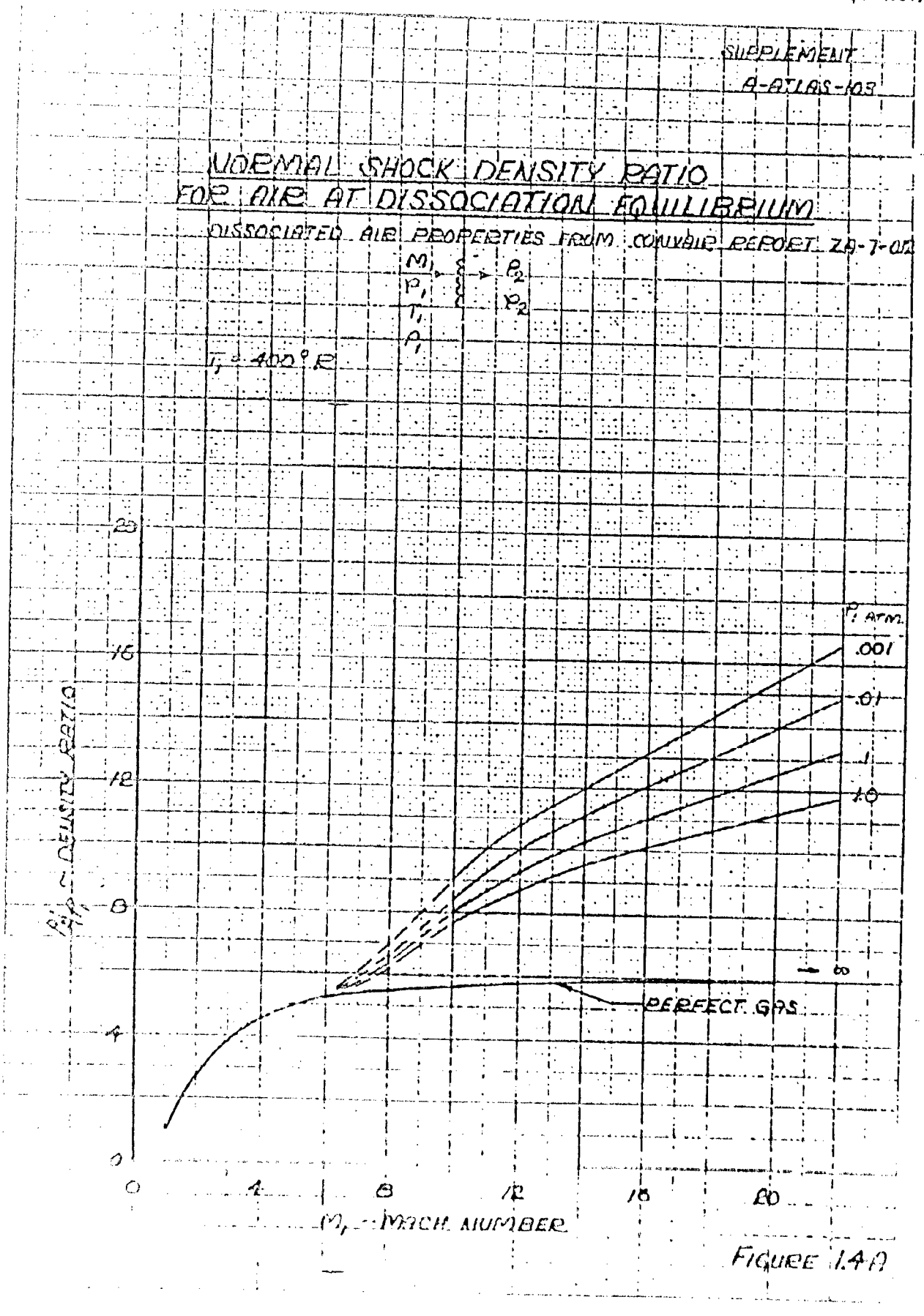

SUPPLEMENT
A-ATLAS-103
NORMAL SHOCK DENSITY RATIO
FOR AIR AT DISSOCIATION EQUILIBRIUM
DISSOCIATED AIR PROPERTIES FROM CONVAIR REPORT ZA-7-01
M1 P1 T1 ρ1 → ρ2 P2
T1 = 400° R
ρ2/ρ1 - DENSITY RATIO
M1 - MACH NUMBER
P ATM.
.001
.01
.1
1.0
∞
PERFECT GAS
0
4
8
12
16
20

FIGURE 1.4A

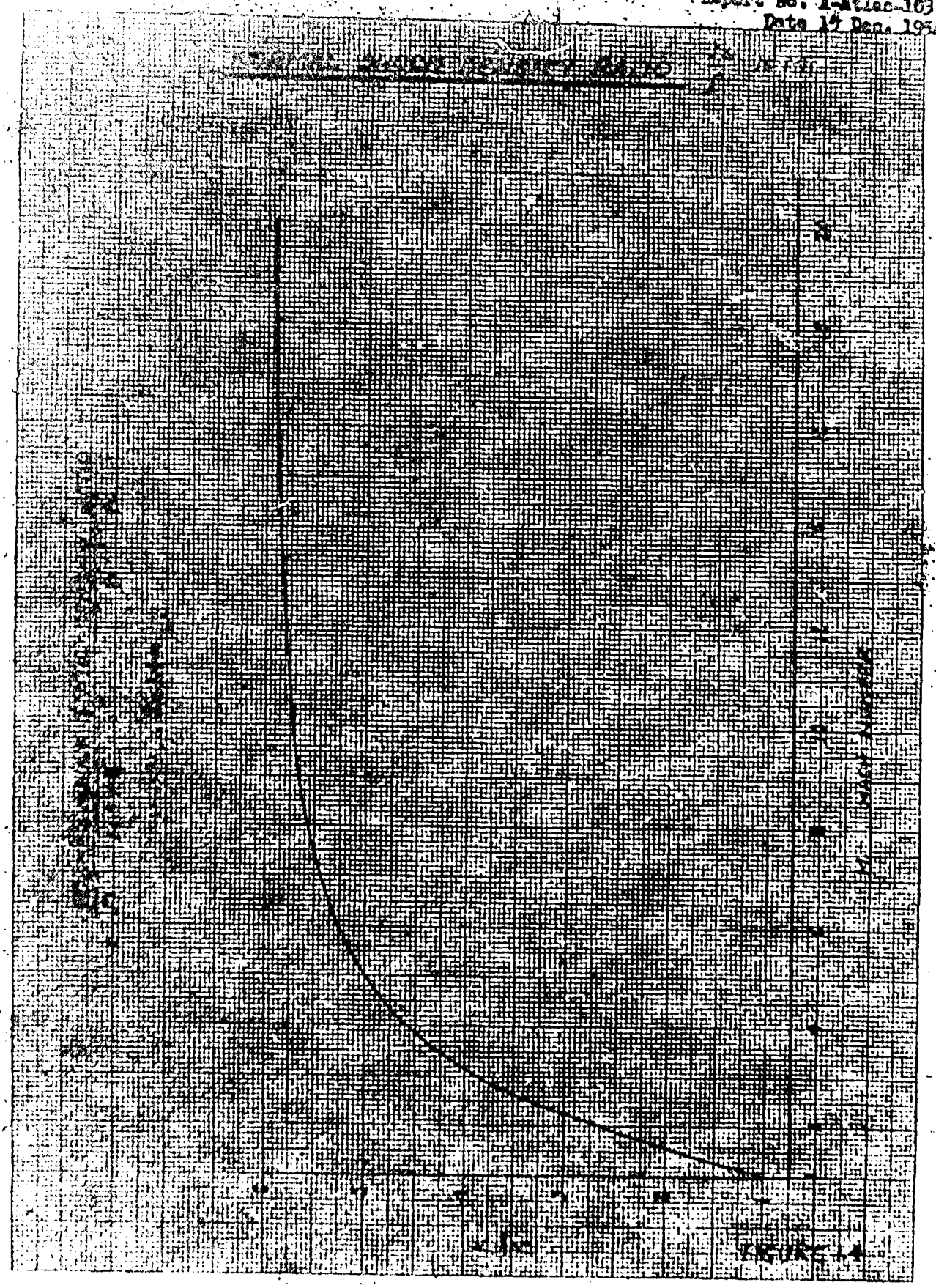


FIGURE 4

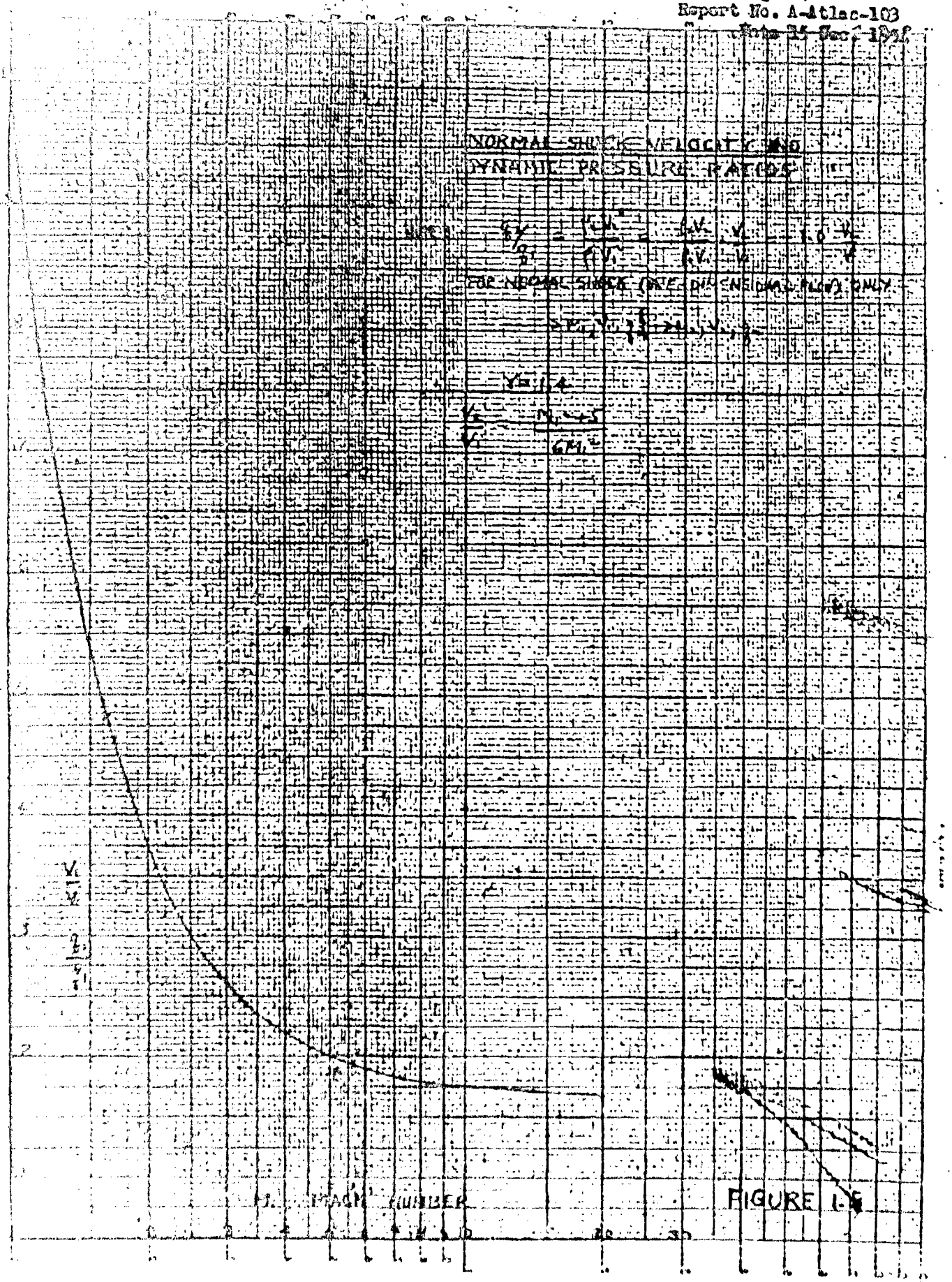

NORMAL SHOCK VELOCITY AND
DYNAMIC PRESSURE RATIOS
FOR NORMAL SHOCK (ONE DIMENSIONAL FLOW) ONLY
$\frac{V_1}{V_2}$
$\frac{q_2}{q_1}$
M, MACH NUMBER

FIGURE 1.5

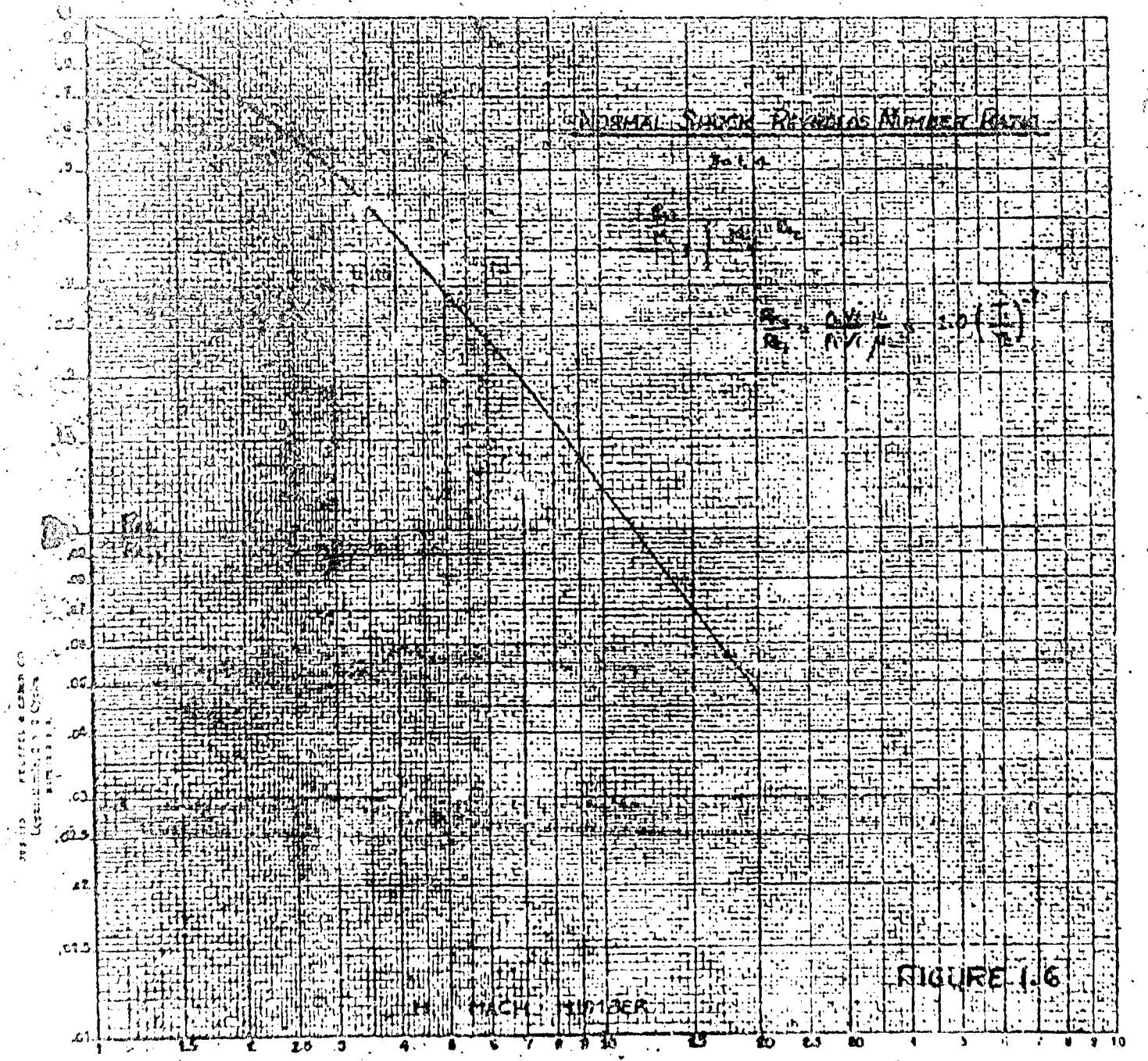


FIGURE 1.6

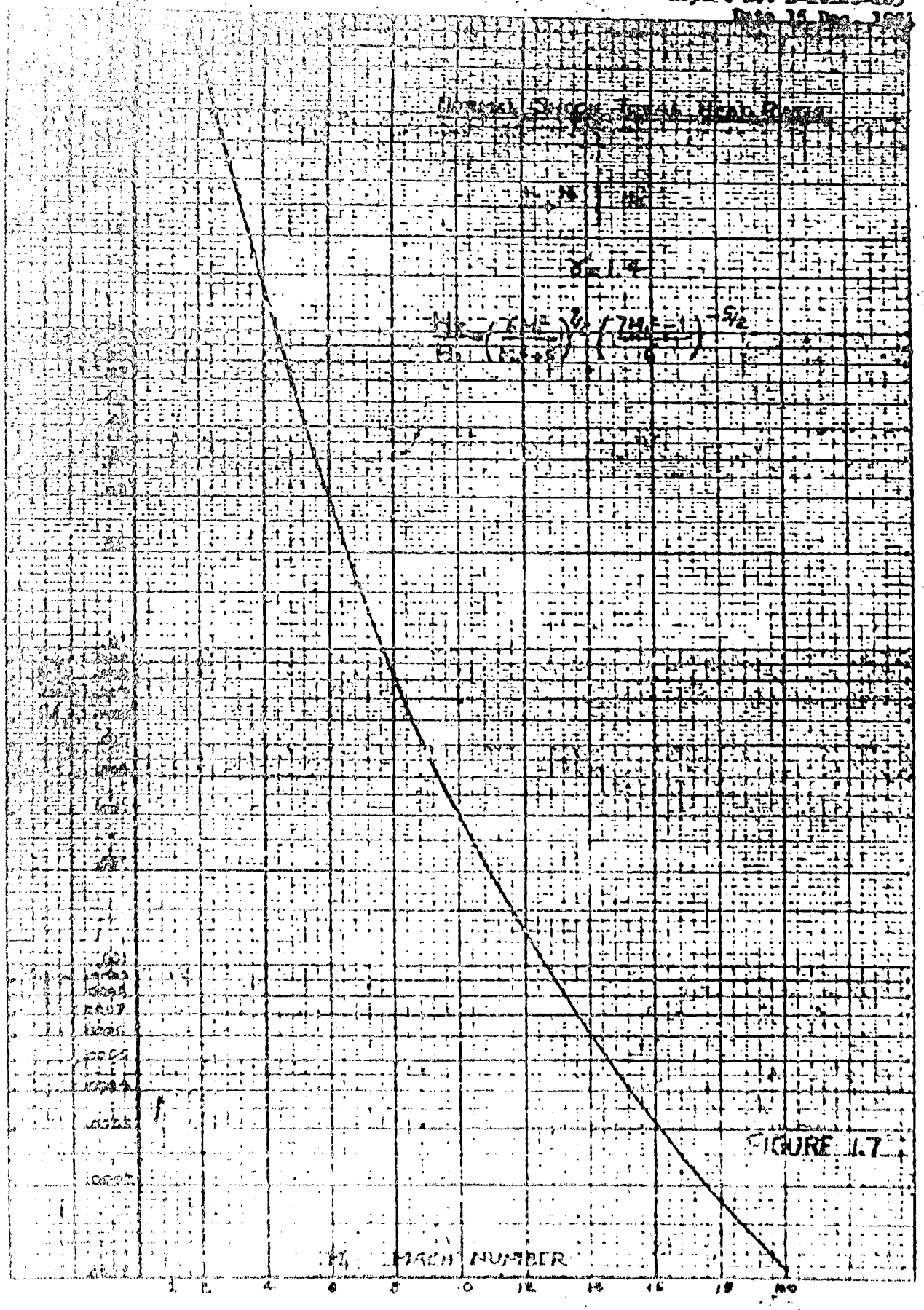

Normal Shock Total Head Ratio
$\gamma = 1.4$
$\frac{H_2}{H_1} = \left(\frac{\gamma+1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \left(\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{-1/(\gamma-1)}$
M_1 MACH NUMBER
1
2
4
6
8
10
12
14
16
18
20

FIGURE 1.7

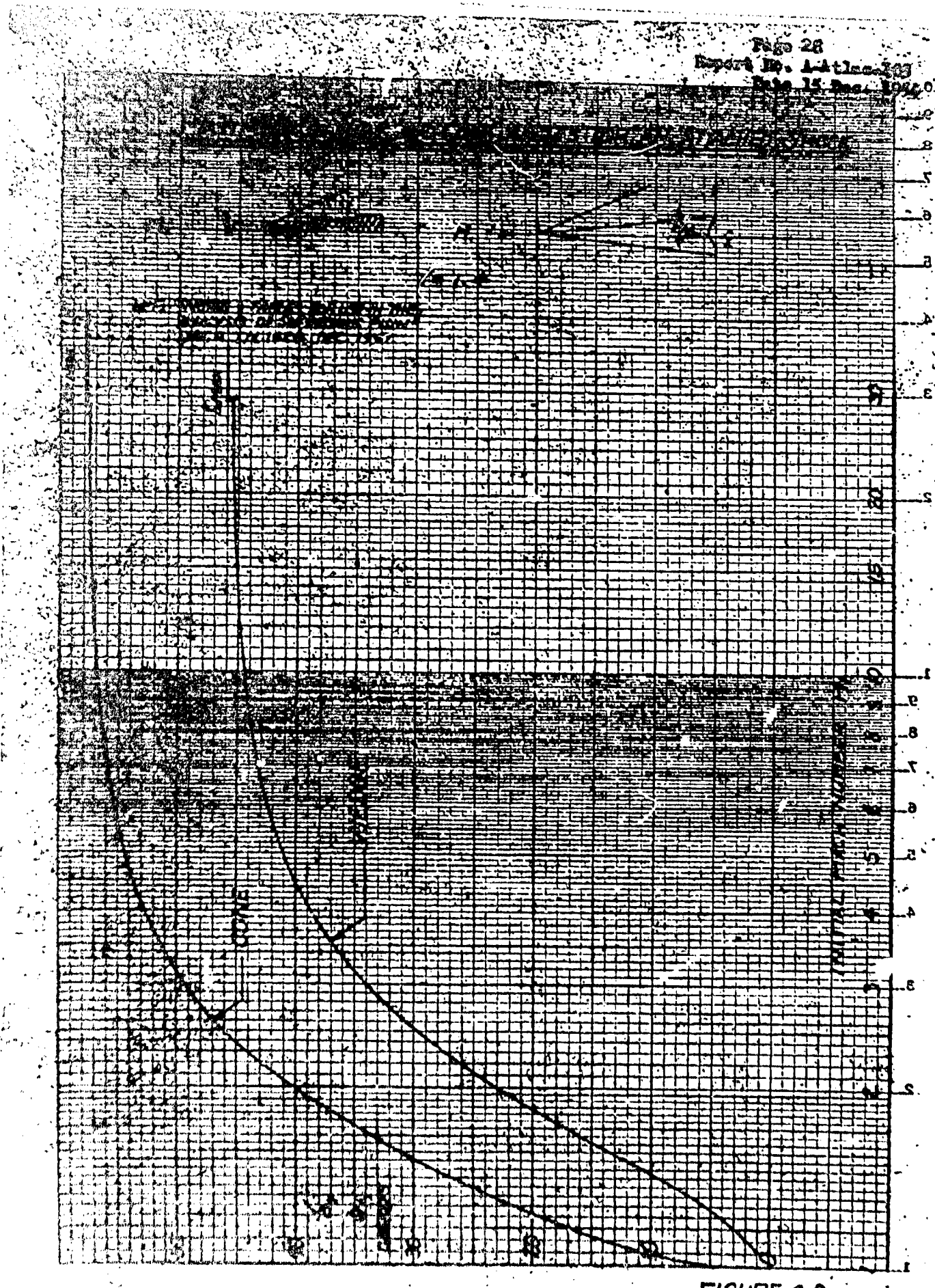
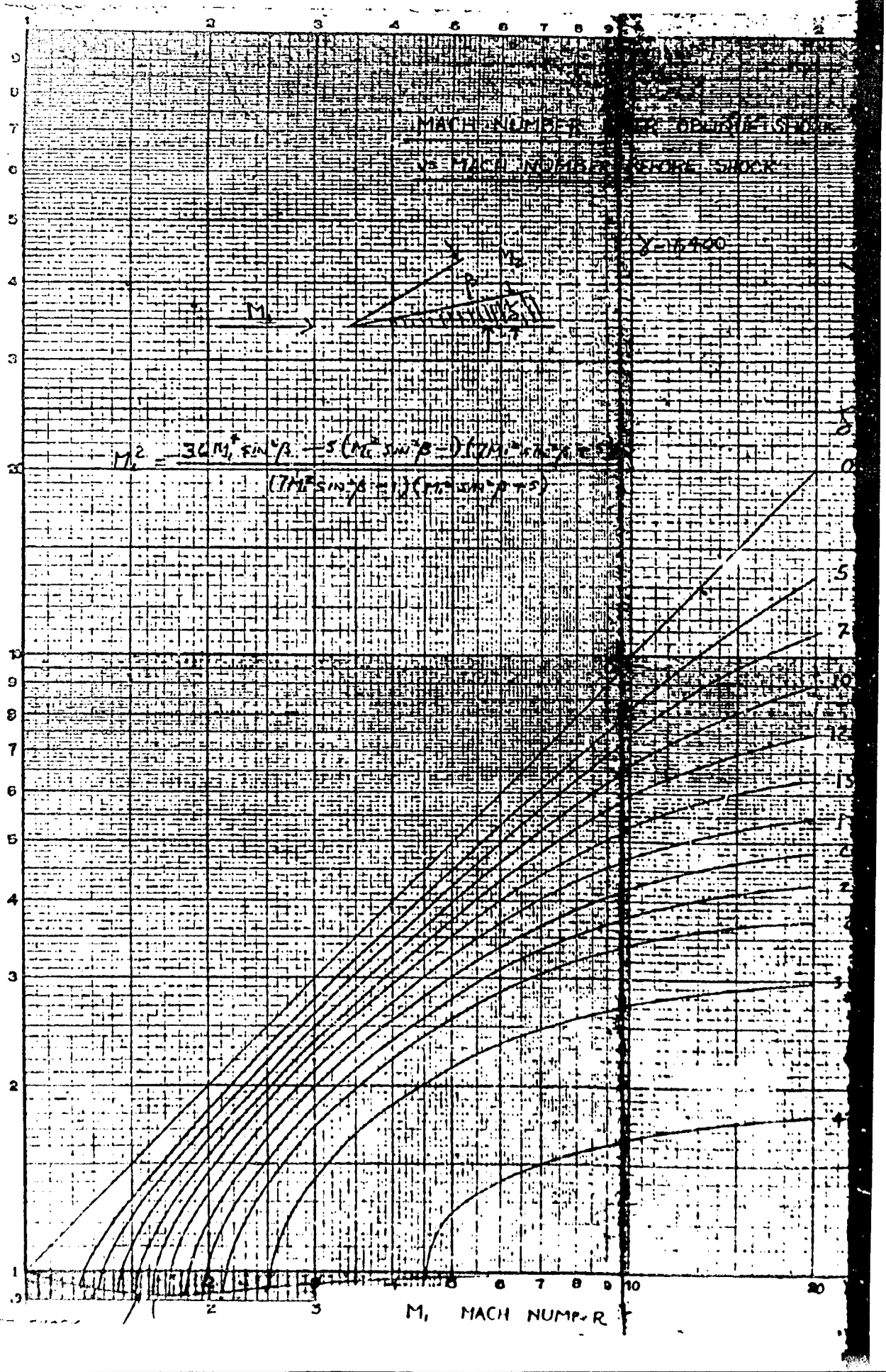


FIGURE 2.0


MACH NUMBER BEHIND OBLIQUE SHOCK
vs MACH NUMBER BEFORE SHOCK
γ = 1.400
$M_2^2 = \frac{3(M_1^2 \sin^2\beta - 5)(M_1^2 \sin^2\beta - 1)}{(7M_1^2 \sin^2\beta + 1)(M_1^2 \sin^2\beta + 5)}$
M2
M1, MACH NUMBER

M_2 vs M_1

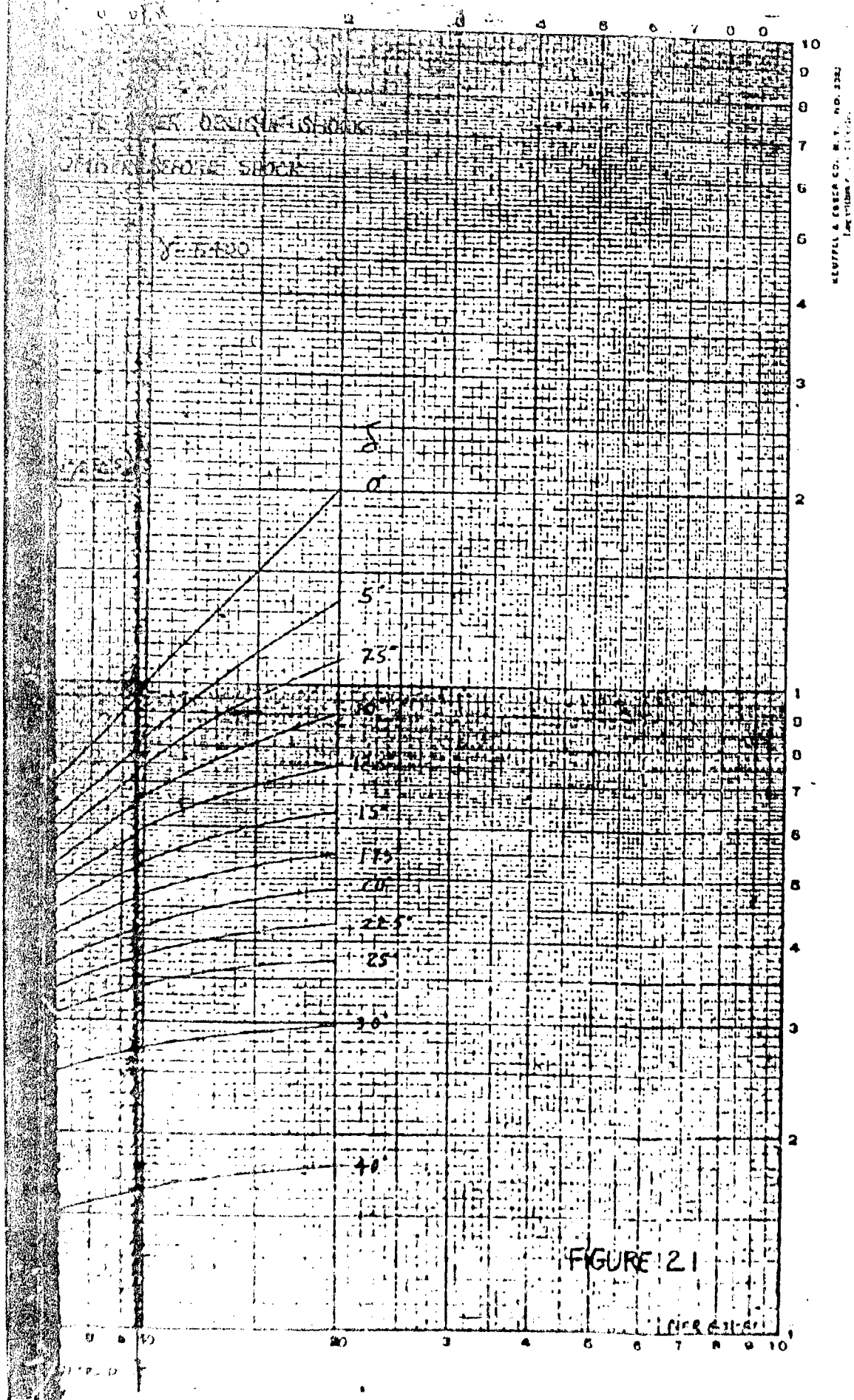


FIGURE 2.1

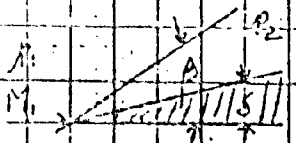
B

FIGURE 2.1

OBLIQUE SHOCK STATIC PRESSURE RA

VS MACH-NUMBER

$\gamma = 1.400$



$$\frac{p_2}{p_1} = \frac{7M_1^2 \sin^2\beta - 1}{5}$$

$\frac{T_2}{T_1}$

DETACHED SHOCK

M_1 MACH NUMBER

p_2/p_1 vs M_1

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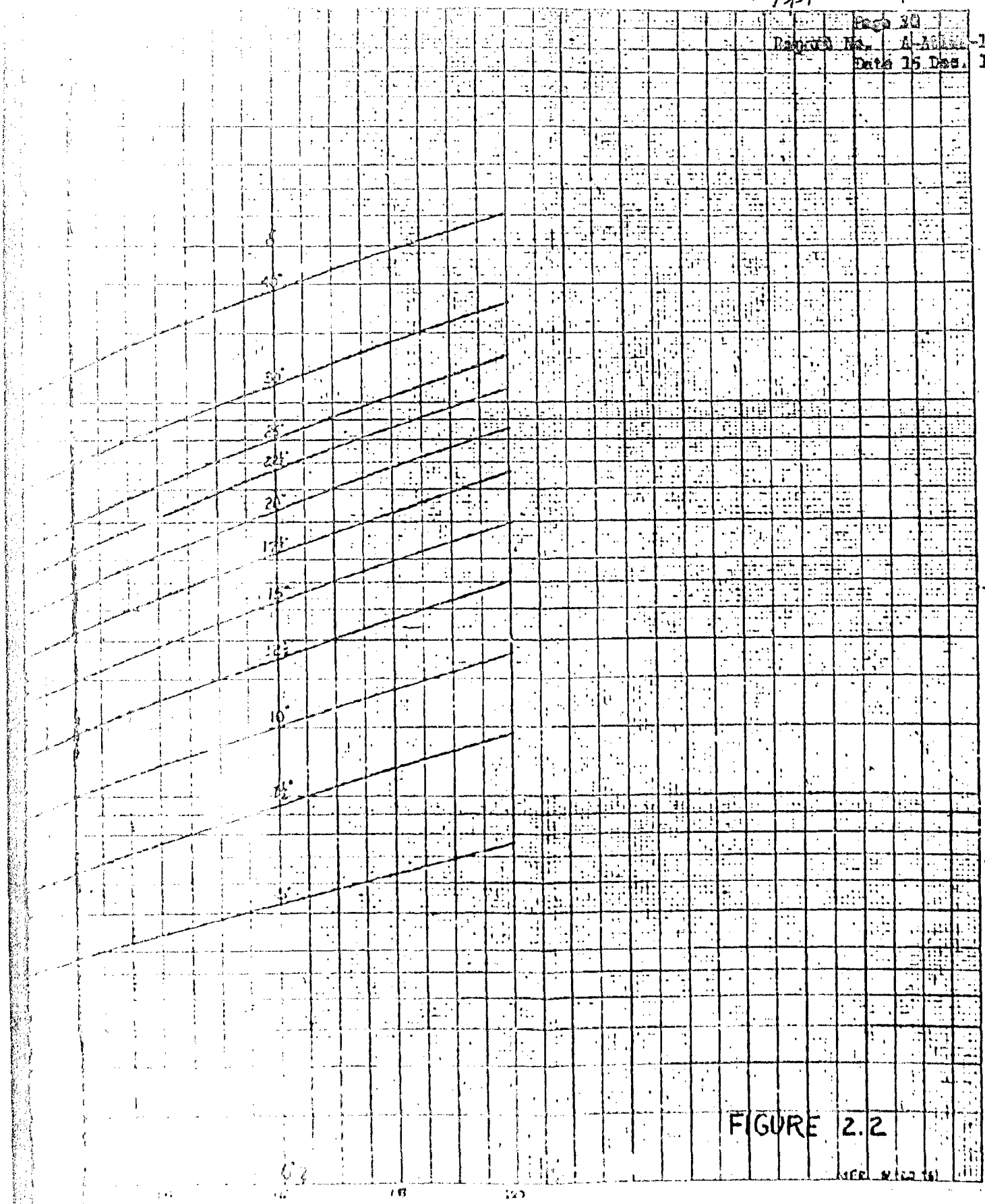
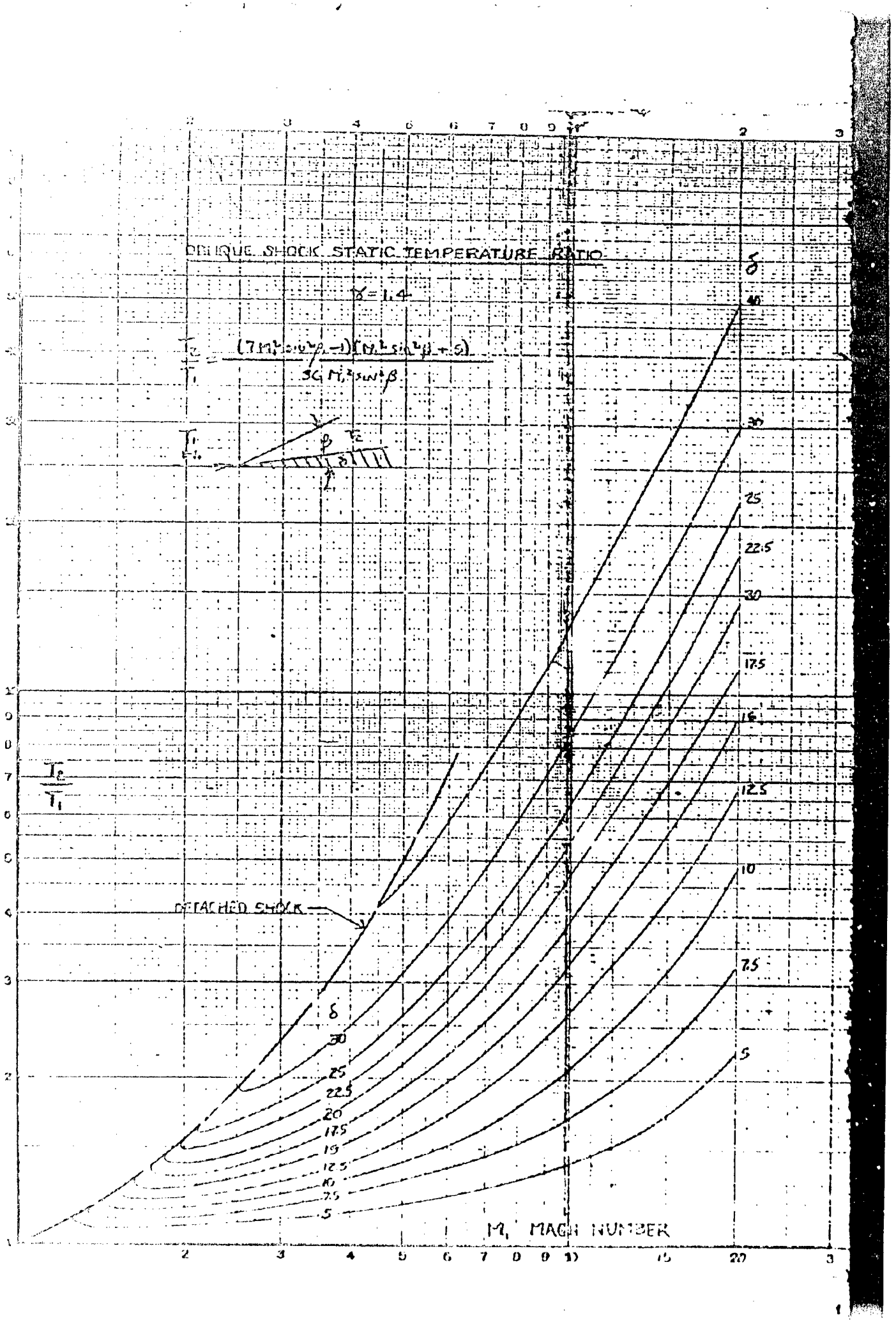


FIGURE 2.2

B


OBLIQUE SHOCK STATIC TEMPERATURE RATIO
γ = 1.4
$\frac{T_2}{T_1} = \frac{(7M_1^2\sin^2\beta - 1)(M_1^2\sin^2\beta + 5)}{36\,M_1^2\sin^2\beta}$
β
δ
T_2
T_1
$\frac{T_2}{T_1}$
DETACHED SHOCK
δ
40
30
25
22.5
20
17.5
15
12.5
10
7.5
5
M_1 MACH NUMBER

$\frac{T_2}{T_1}$ vs M_1

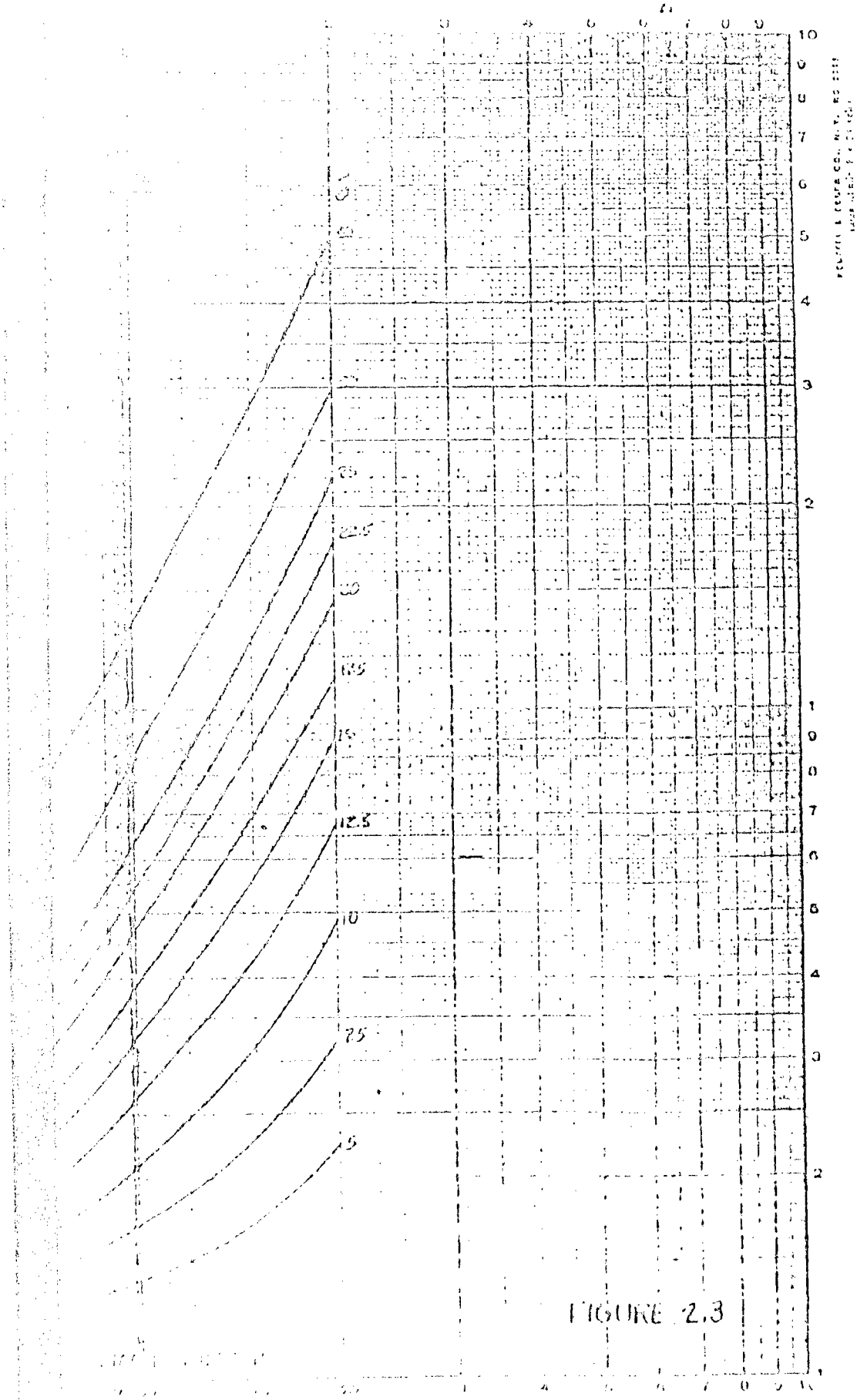


FIGURE 2.3

FIGURE 2.3

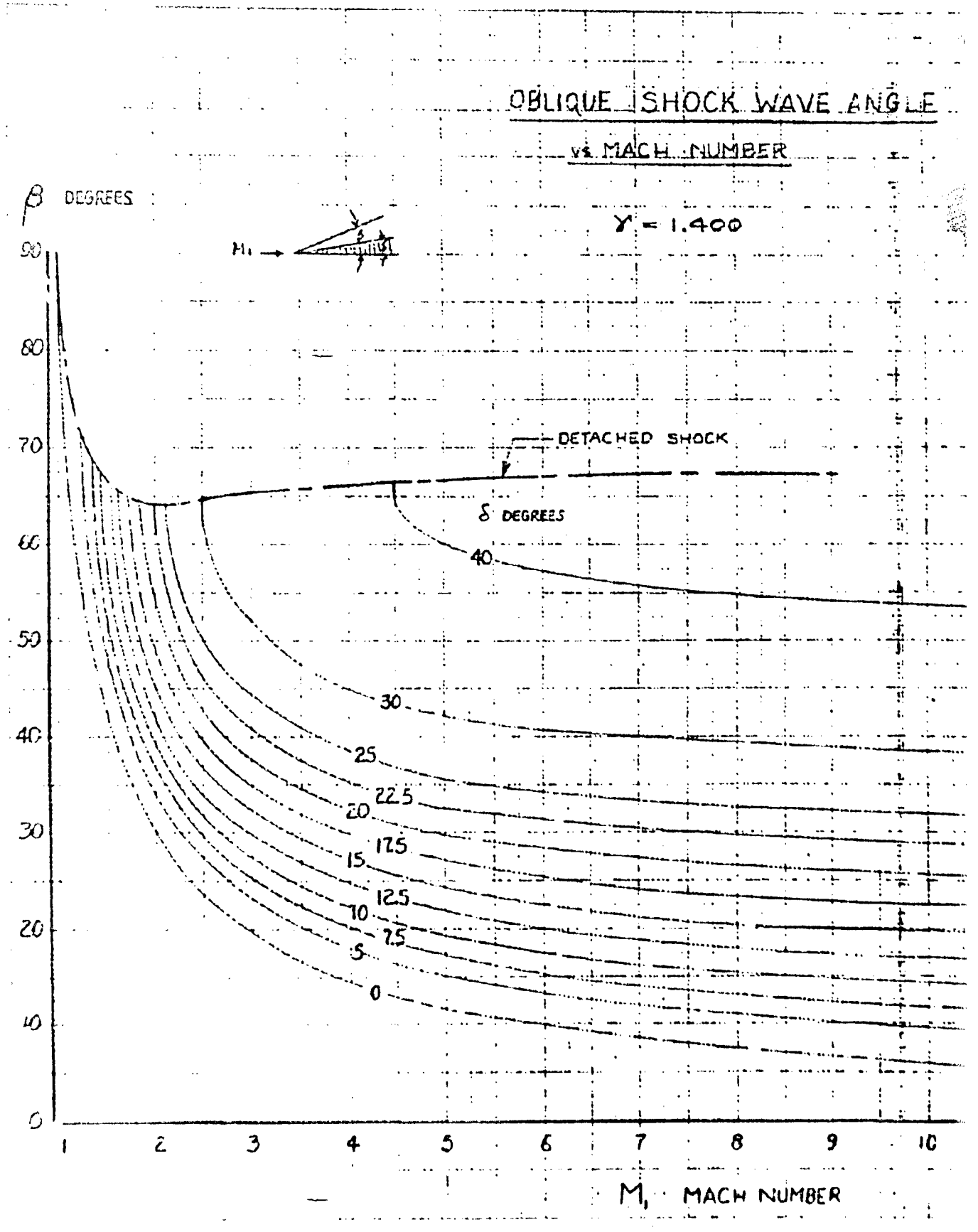
B

OBLIQUE SHOCK DENSITY RATIO
MACH NUMBER
M_1
DETACHED SHOCK
$\frac{\rho_2}{\rho_1}$

FIGURE 2.4

OBLIQUE SHOCK WAVE ANGLE

vs MACH NUMBER



ANGLE

DATA FROM NACA TN 1373

AND $\tan\delta = \dfrac{M_1^2 \sin 2\beta - 2\cot\beta}{2 + M_1^2(\gamma + \cos 2\beta)}$

δ DEGREES

40
30
25
22.5
20
17.5
15
12.5
10
7.5
5
0

10 11 12 13 14 15 16 17 18 ∞

FIGURE 2.5

B

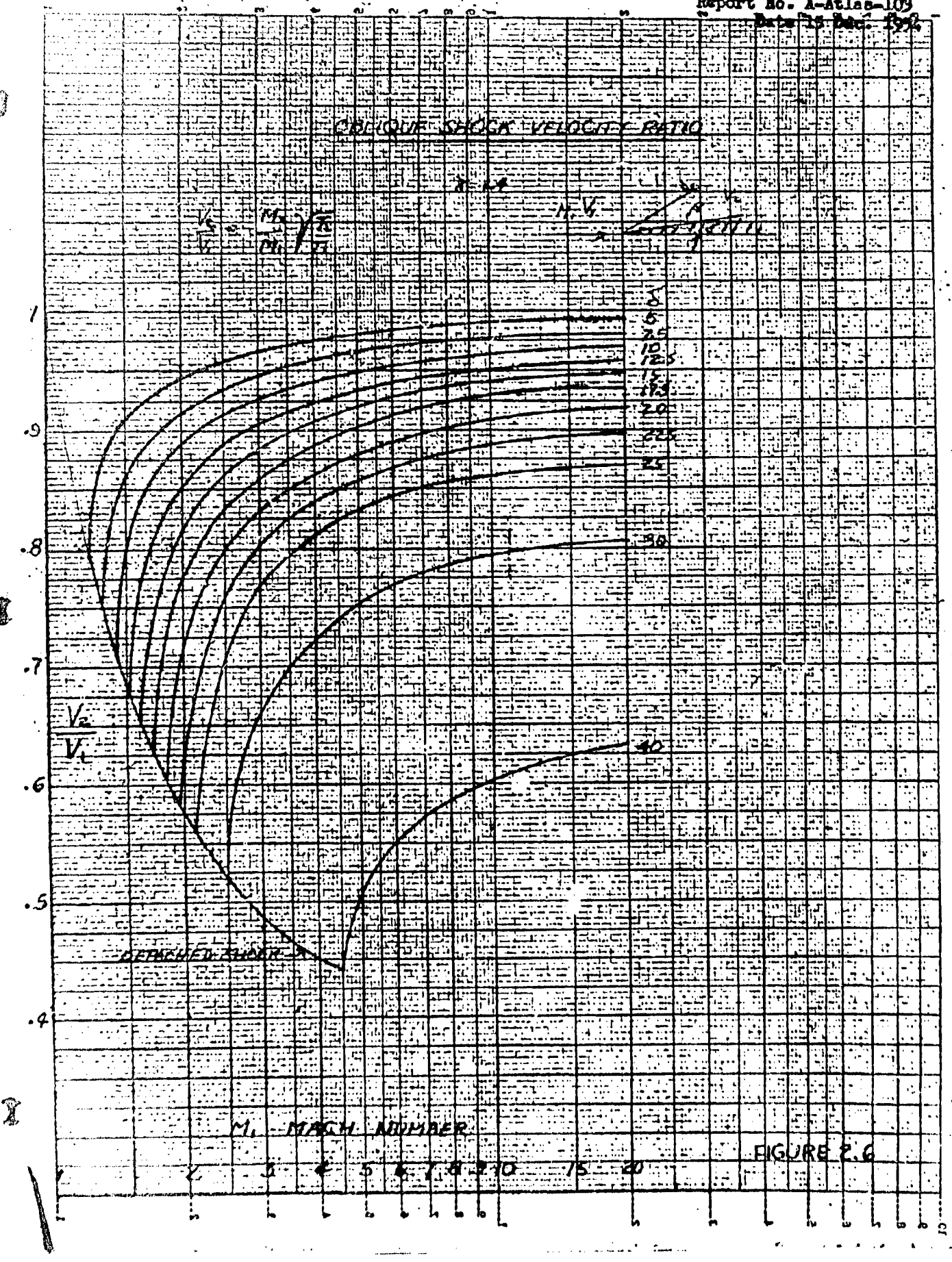

OBLIQUE SHOCK VELOCITY RATIO
$\frac{V_2}{V_1} = \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}$
M_1, V_1
$\gamma = 1.4$
$\frac{V_2}{V_1}$
1
.9
.8
.7
.6
.5
.4
δ
5
2.5
10
12.5
15
17.5
20
22.5
25
30
40
DETACHED SHOCK
M_1 MACH NUMBER
1
2
3
4
5
6
7
8
9
10
15
20

FIGURE 2.6

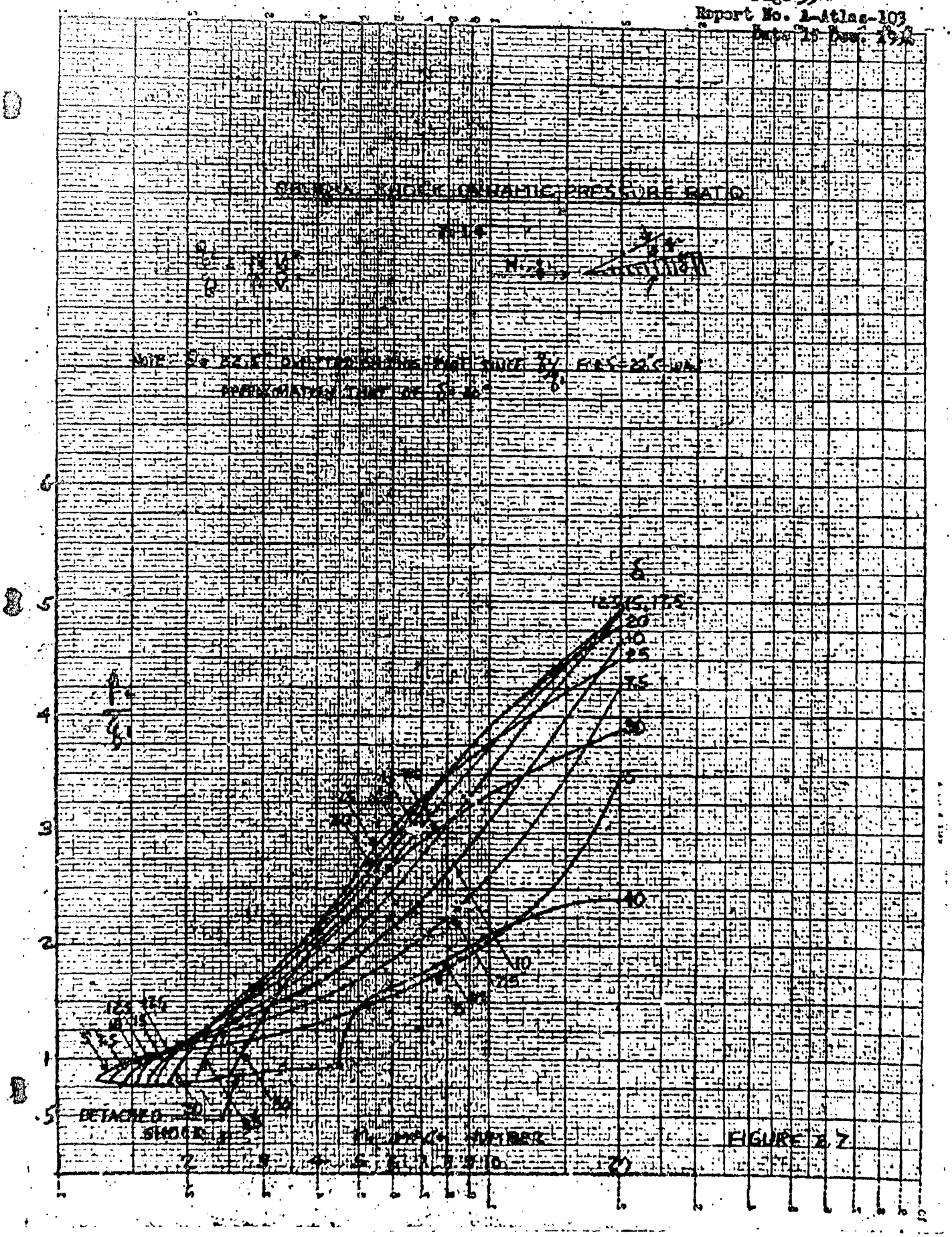

ORIFICE STATIC PRESSURE RATIO
DETACHED SHOCK
ATTACHED SHOCK

FIGURE 2.7

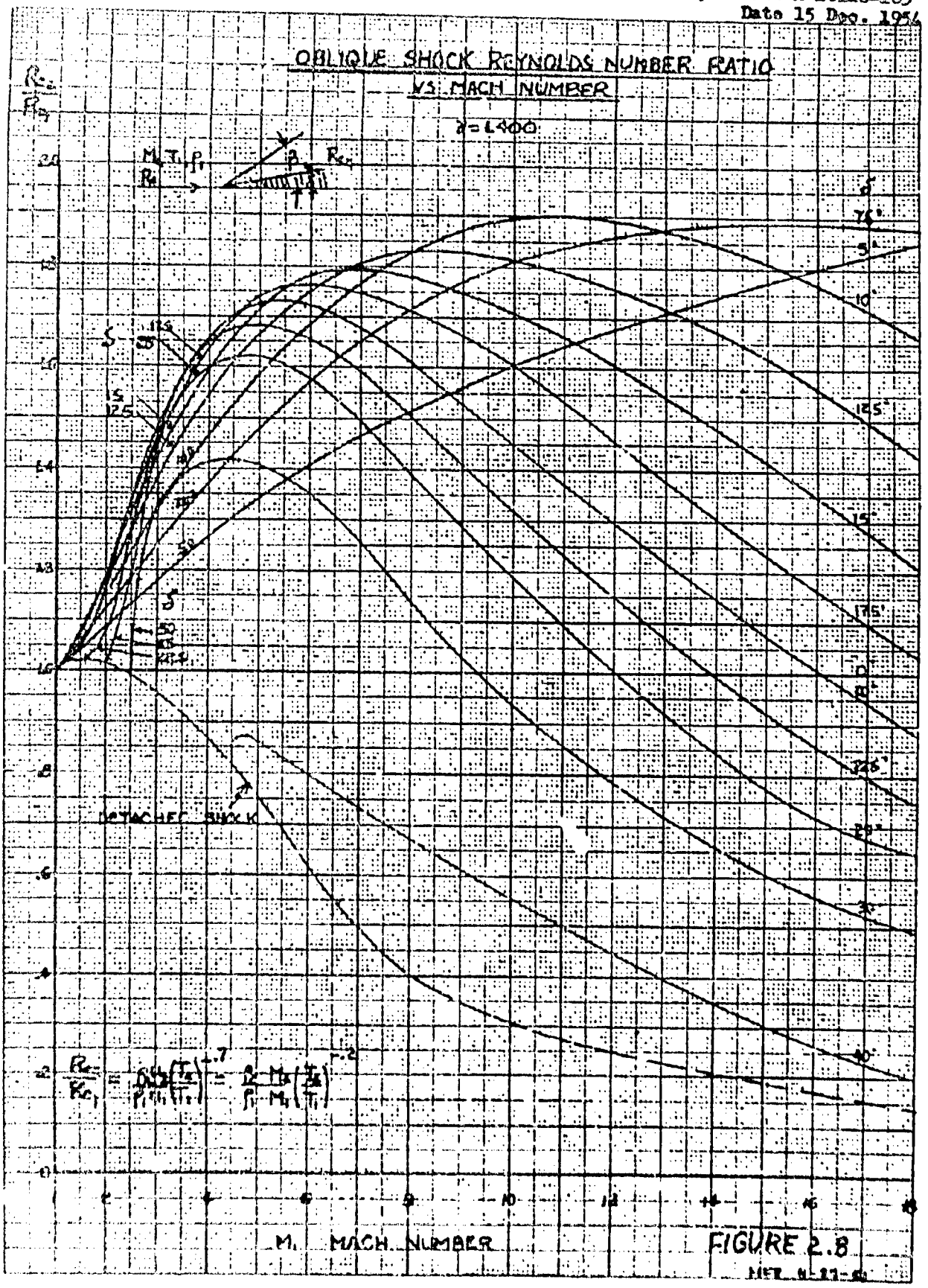

OBLIQUE SHOCK REYNOLDS NUMBER RATIO
VS. MACH NUMBER
γ = 1.400
$\frac{R_2}{R_1}$
M_1, T_1, p_1
R_1
R_2
β
DETACHED SHOCK
$\frac{R_2}{R_1} = \frac{p_2}{p_1}\frac{M_2}{M_1}\left(\frac{T_2}{T_1}\right)^{+.7} = \frac{p_2}{p_1}\frac{M_2}{M_1}\left(\frac{T_2}{T_1}\right)^{-.2}$
M, MACH NUMBER

FIGURE 2.8

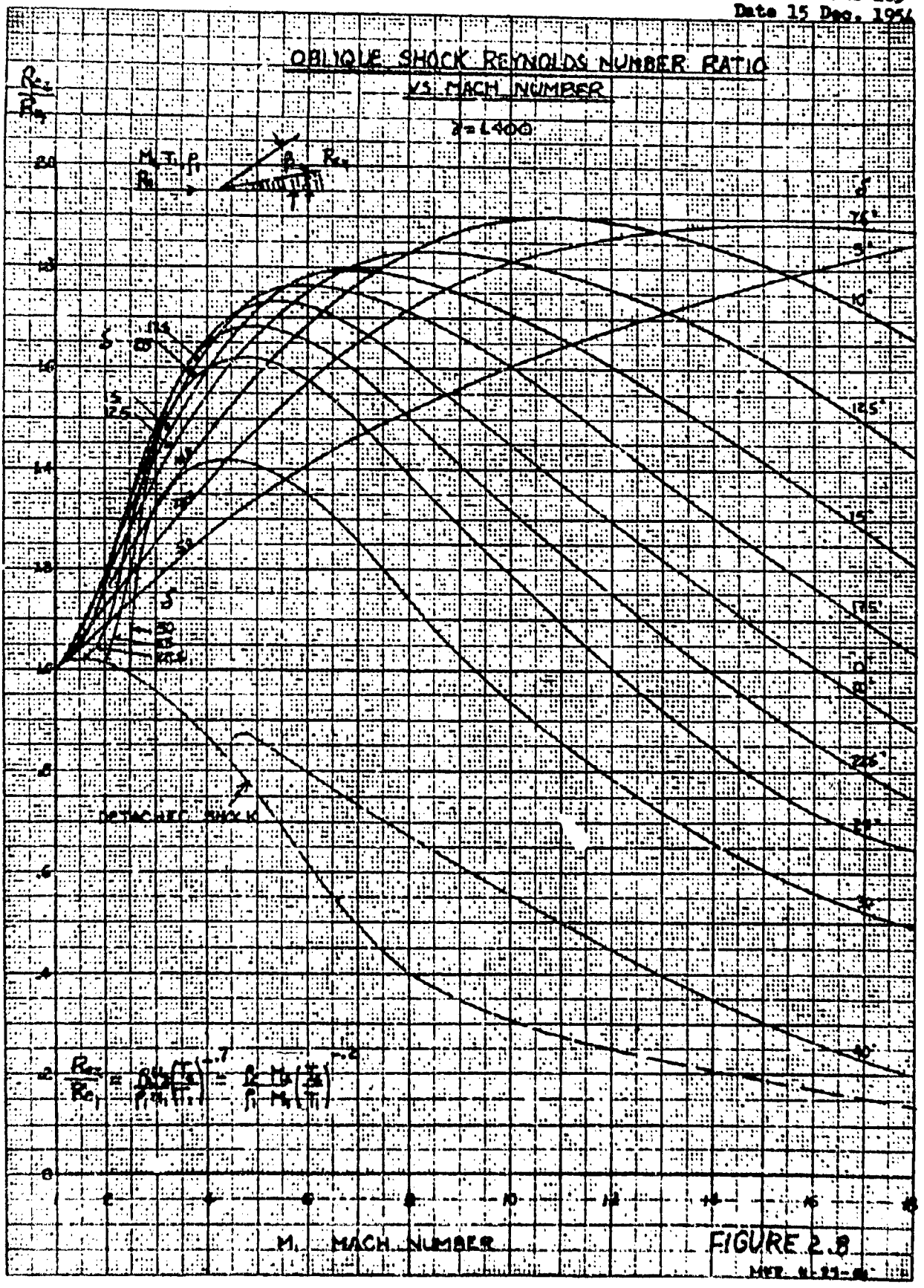

OBLIQUE SHOCK REYNOLDS NUMBER RATIO
VS MACH NUMBER
γ = 1.400
DETACHED SHOCK
M, MACH NUMBER

FIGURE 2.8

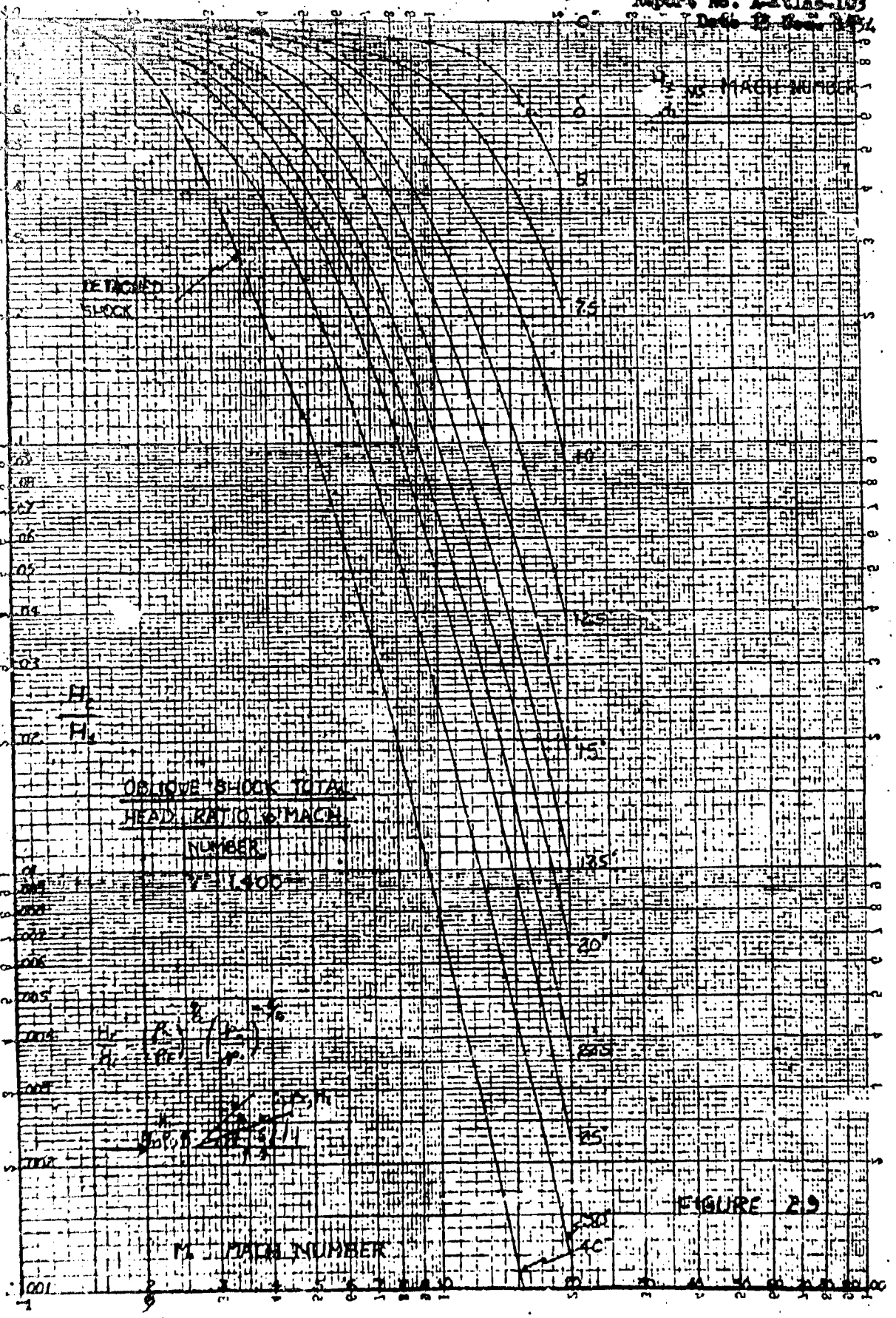
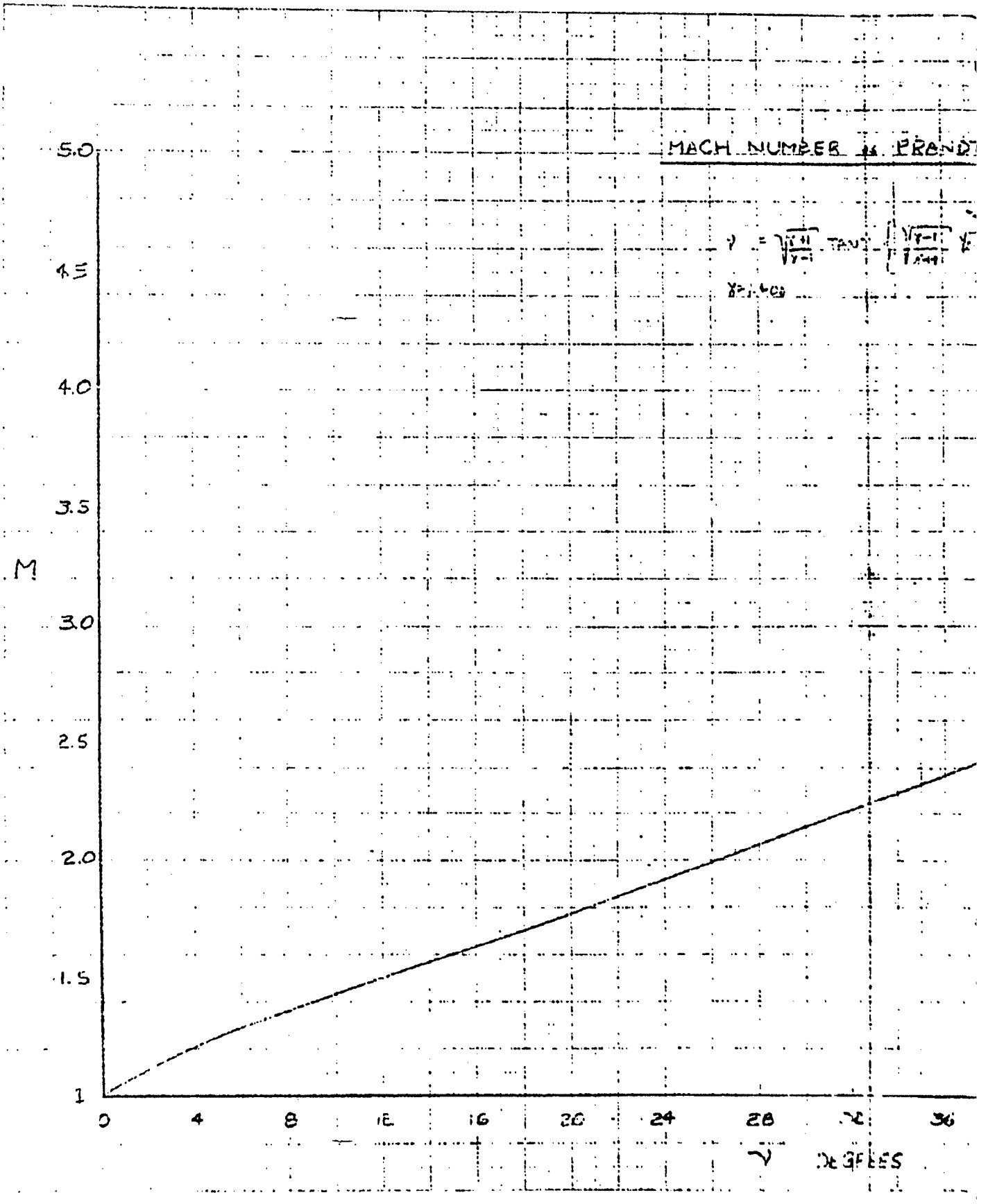
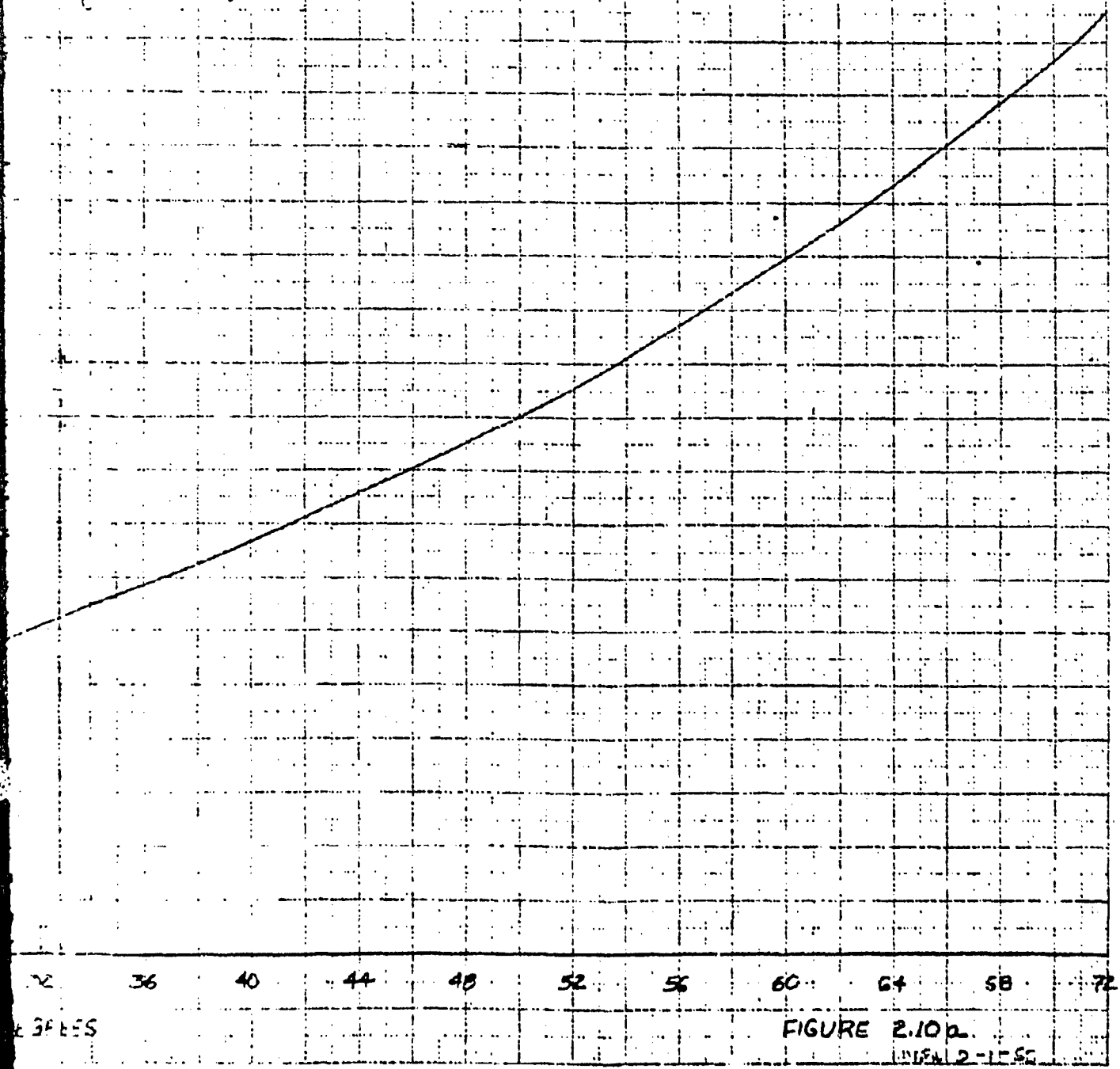

DETACHED SHOCK
M_s MACH NUMBER
$\frac{H_2}{H_1}$
ORDINATE: SHOCK TOTAL HEAD RATIO to MACH NUMBER
$\gamma = 1.400$
M_1 MACH NUMBER

FIGURE 29


MACH NUMBER vs PRAND
M
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1
0
4
8
12
16
20
24
28
32
36
DEGREES

EB. vs PRANDTL-MEYER ANGLE

$$\tan\nu = \left\{ \frac{\sqrt{\gamma+1}}{\tan\mu} \sqrt{\sin^2\mu - 1} \right\} = \tan^{-1}\sqrt{M^2-1}$$



DEGREES

FIGURE 2.10a.

B

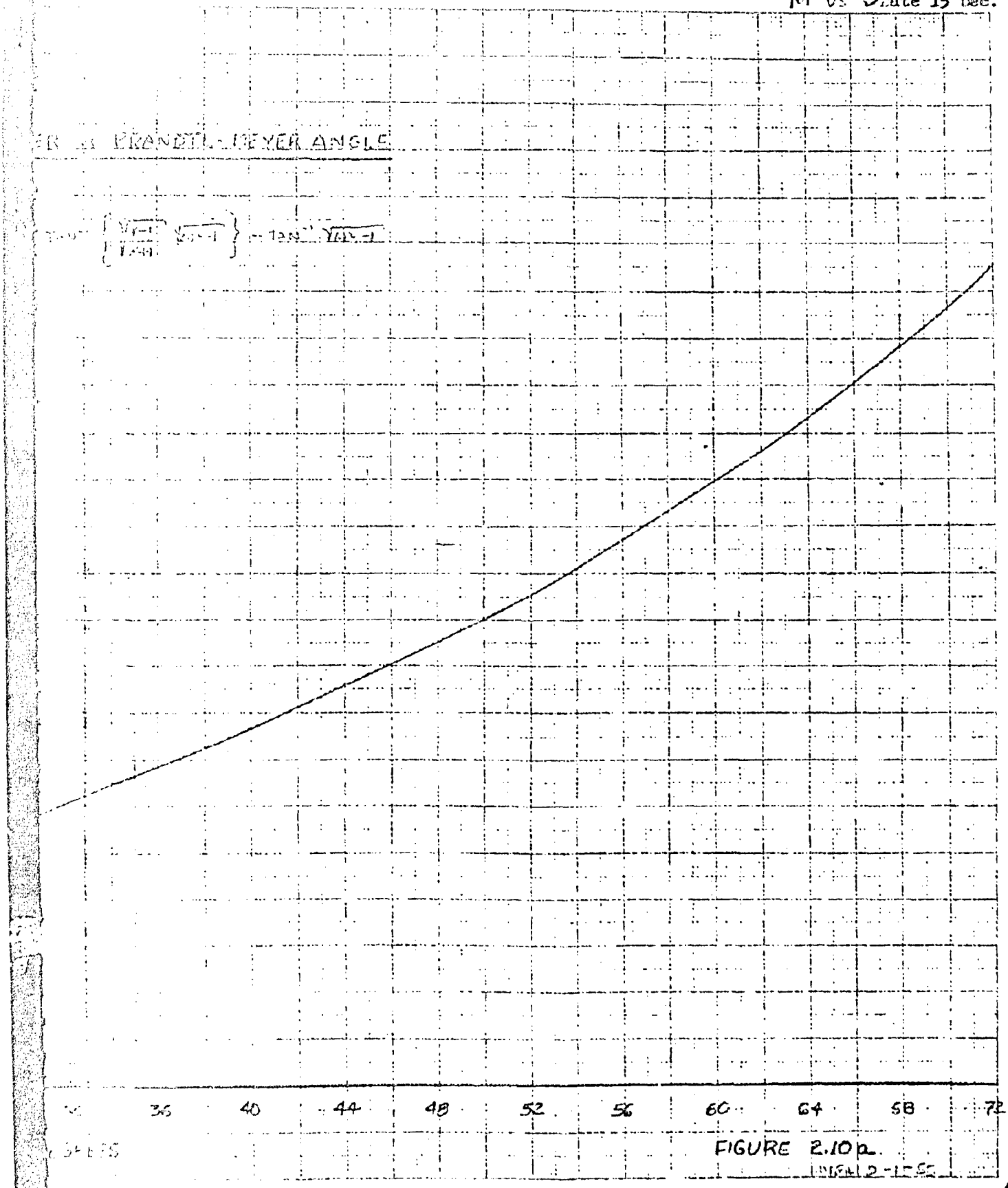
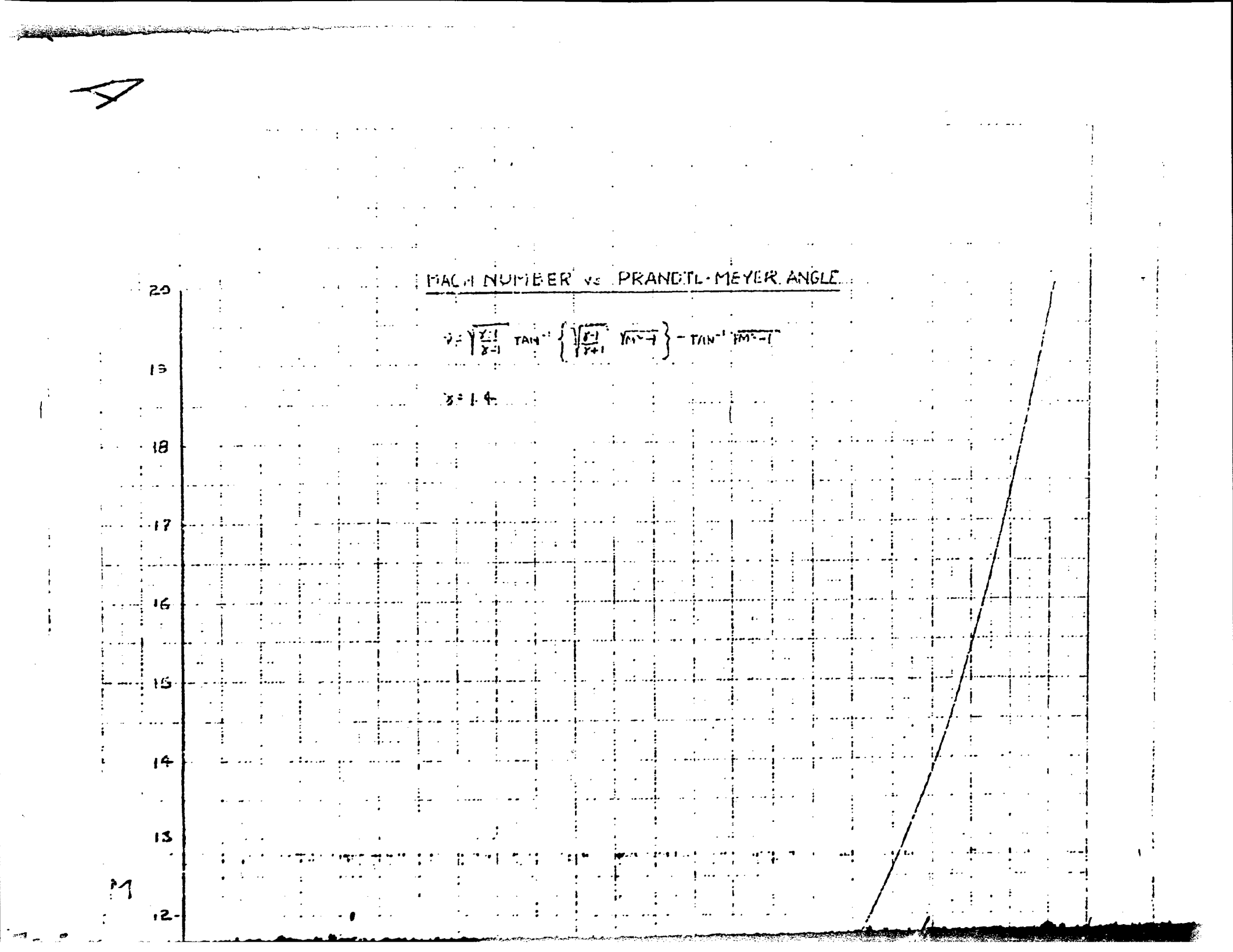


FIGURE 2.10a.


MACH NUMBER vs PRANDTL-MEYER ANGLE
$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}}\sqrt{M^2-1}\right\} - \tan^{-1}\sqrt{M^2-1}$
$\gamma = 1.4$
20
19
18
17
16
15
14
13
12
M

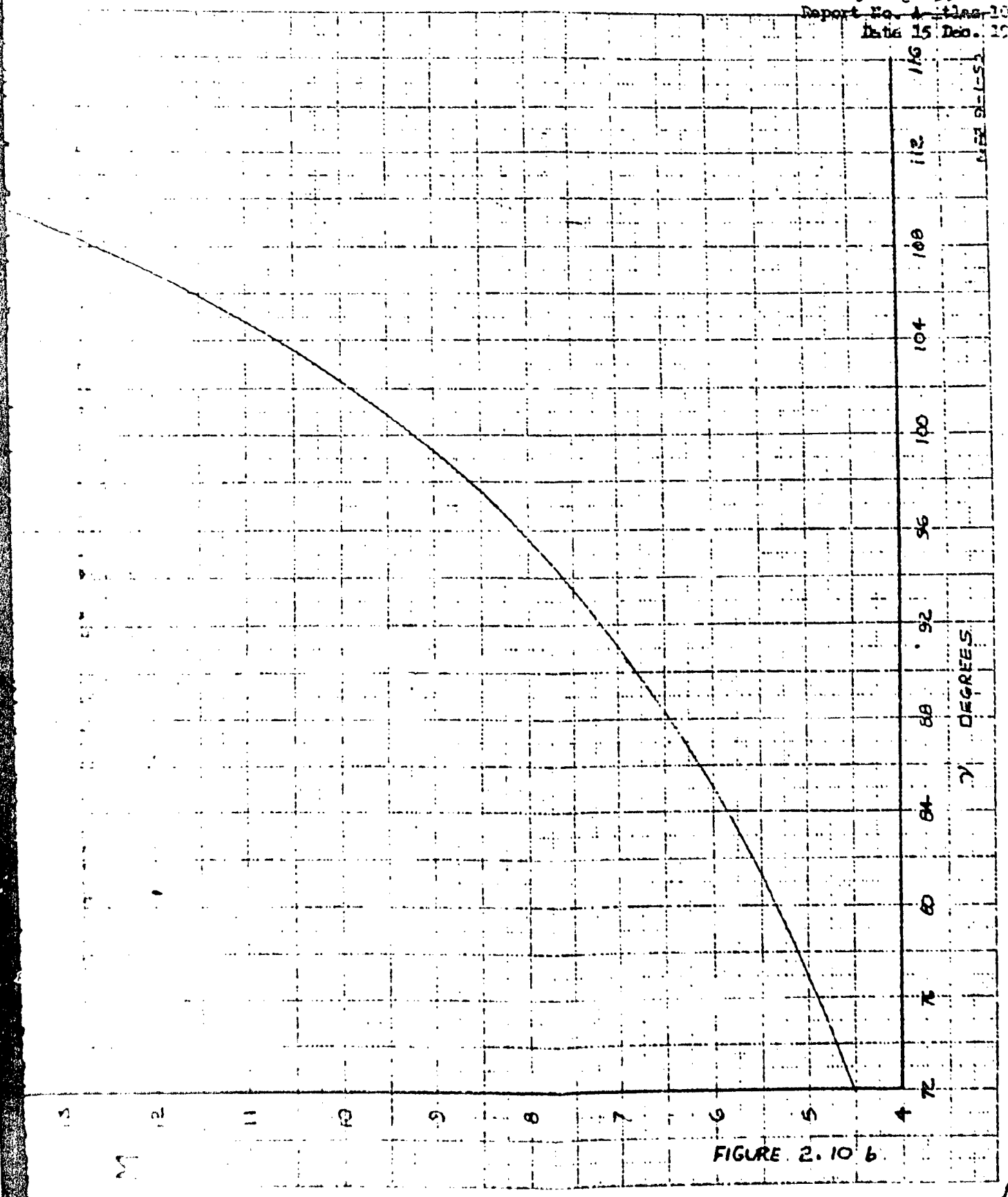
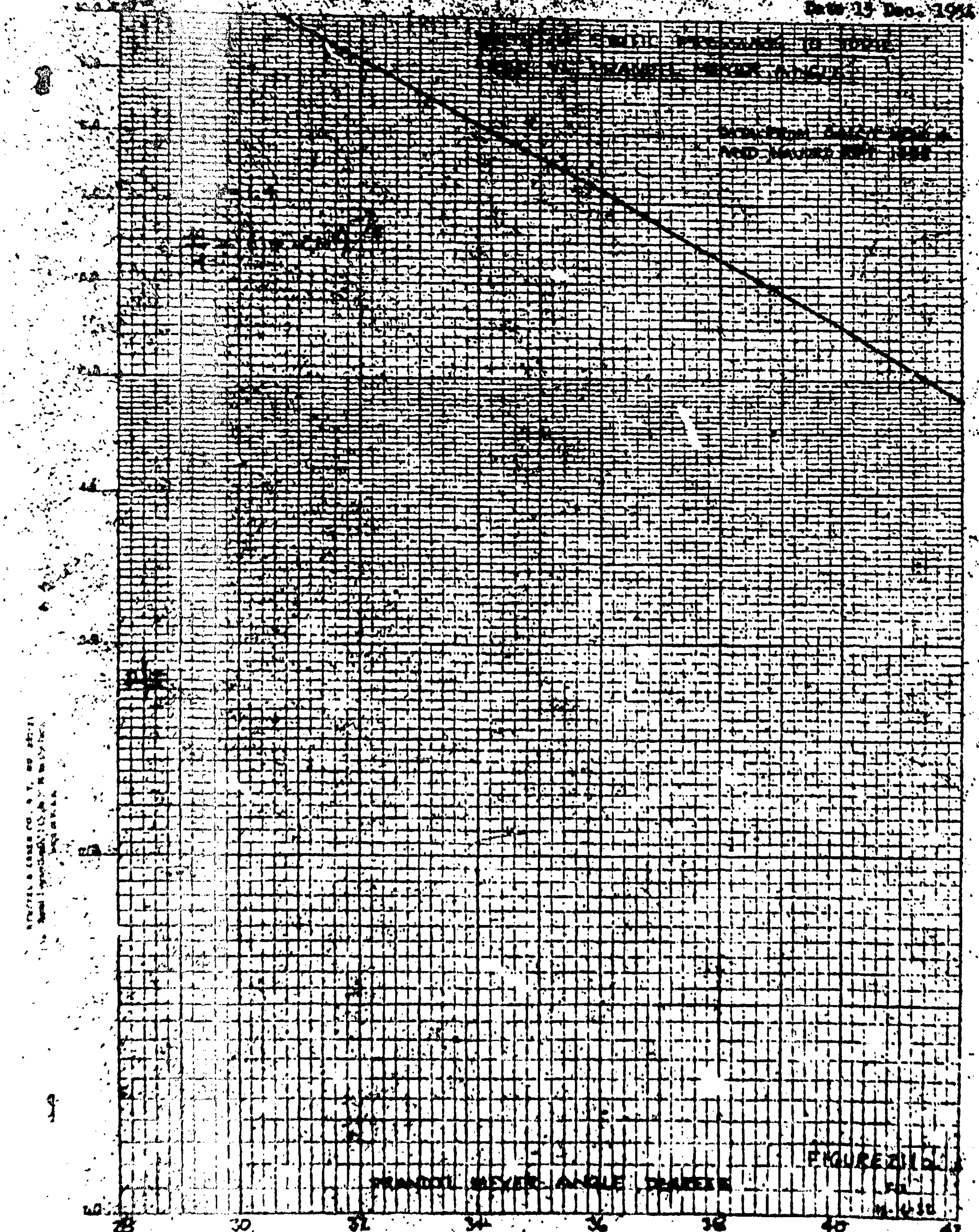
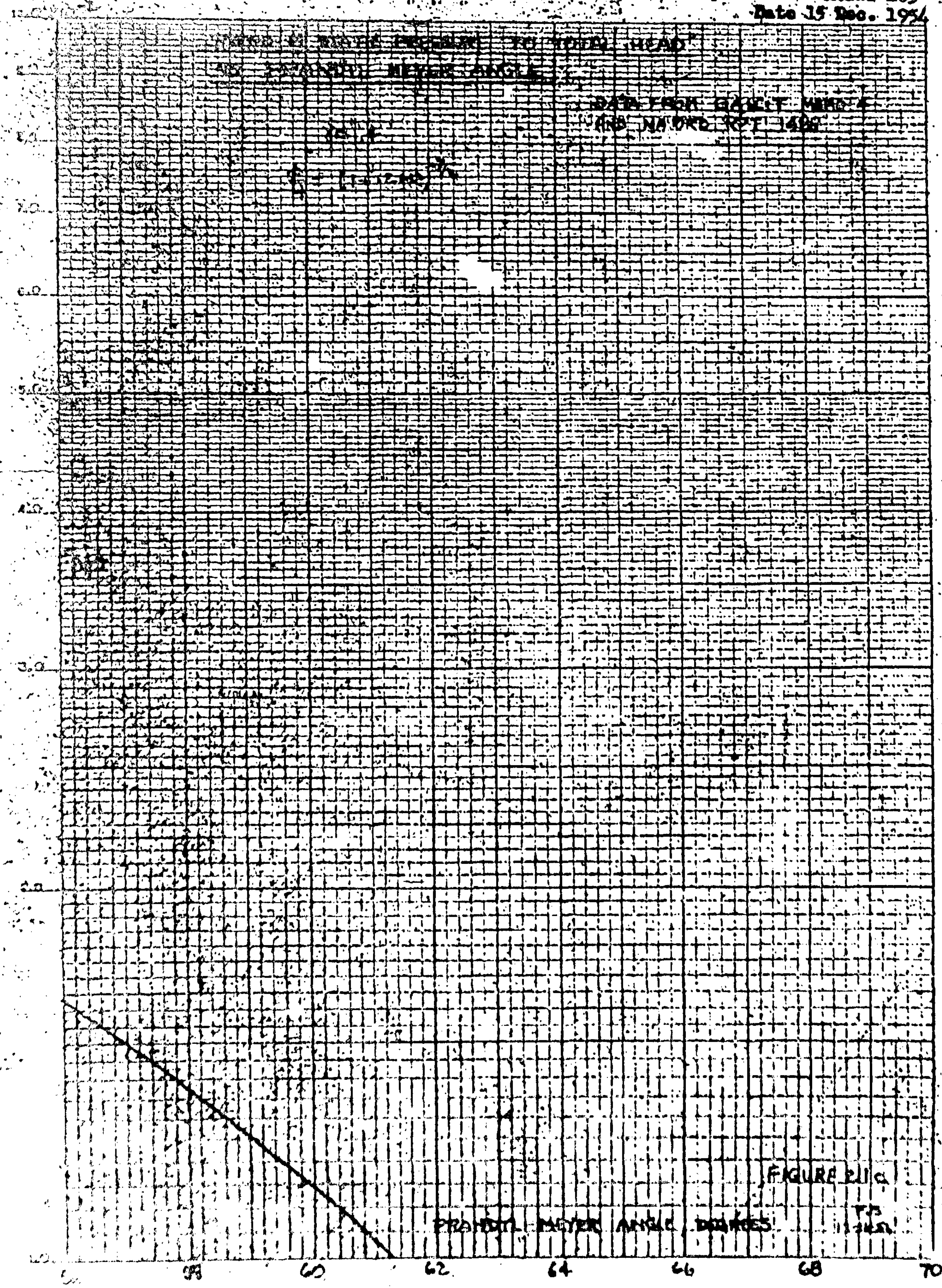
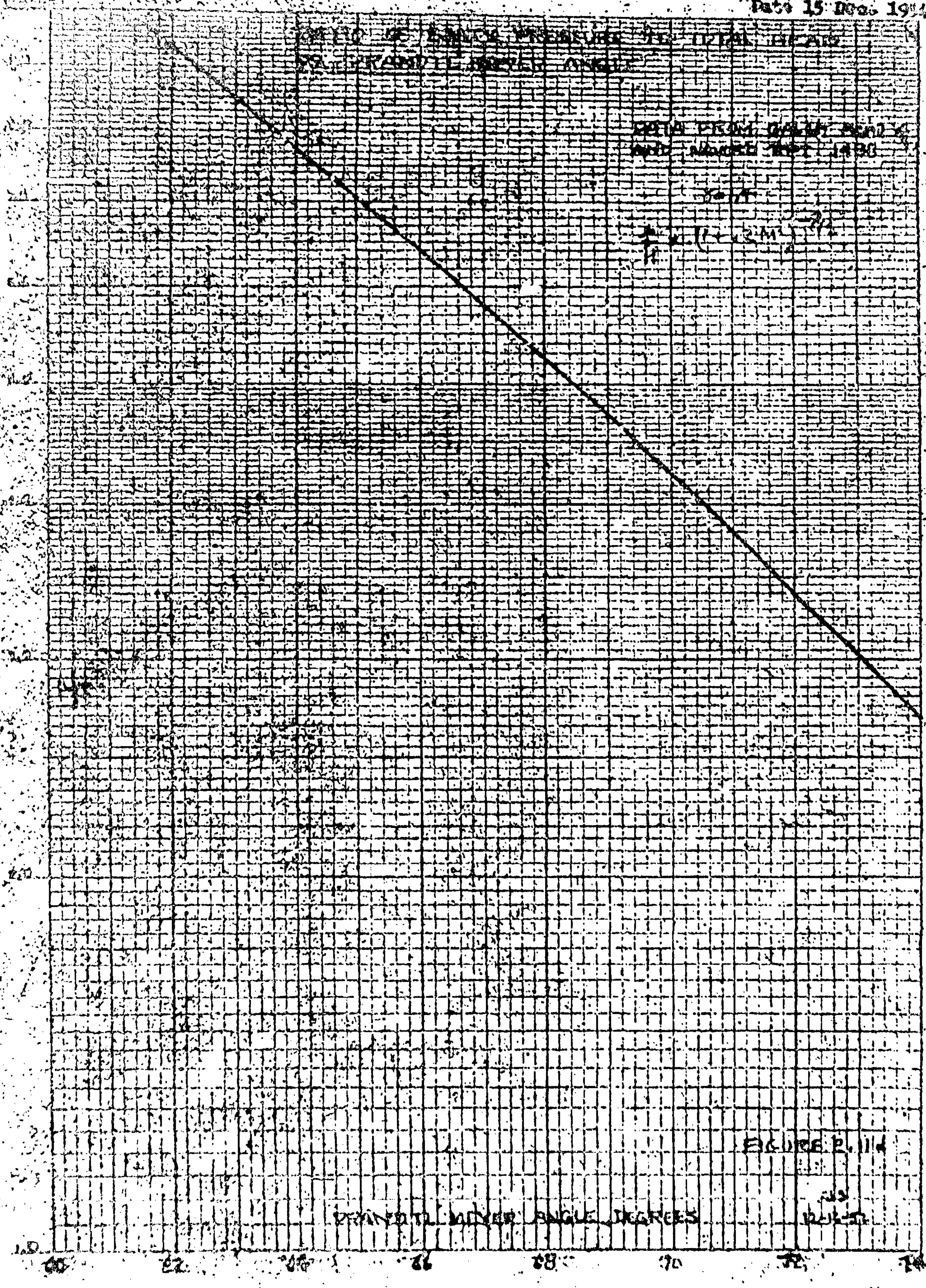


FIGURE 2.10 b

B


28
30
32
34
36
38
40
42


8.0
7.0
6.0
5.0
4.0
3.0
2.0
60
62
64
66
68
70
PRANDTL METER ANGLE DEGREES
FIGURE 21 a


60
62
64
66
68
70
72
74
DEGREES

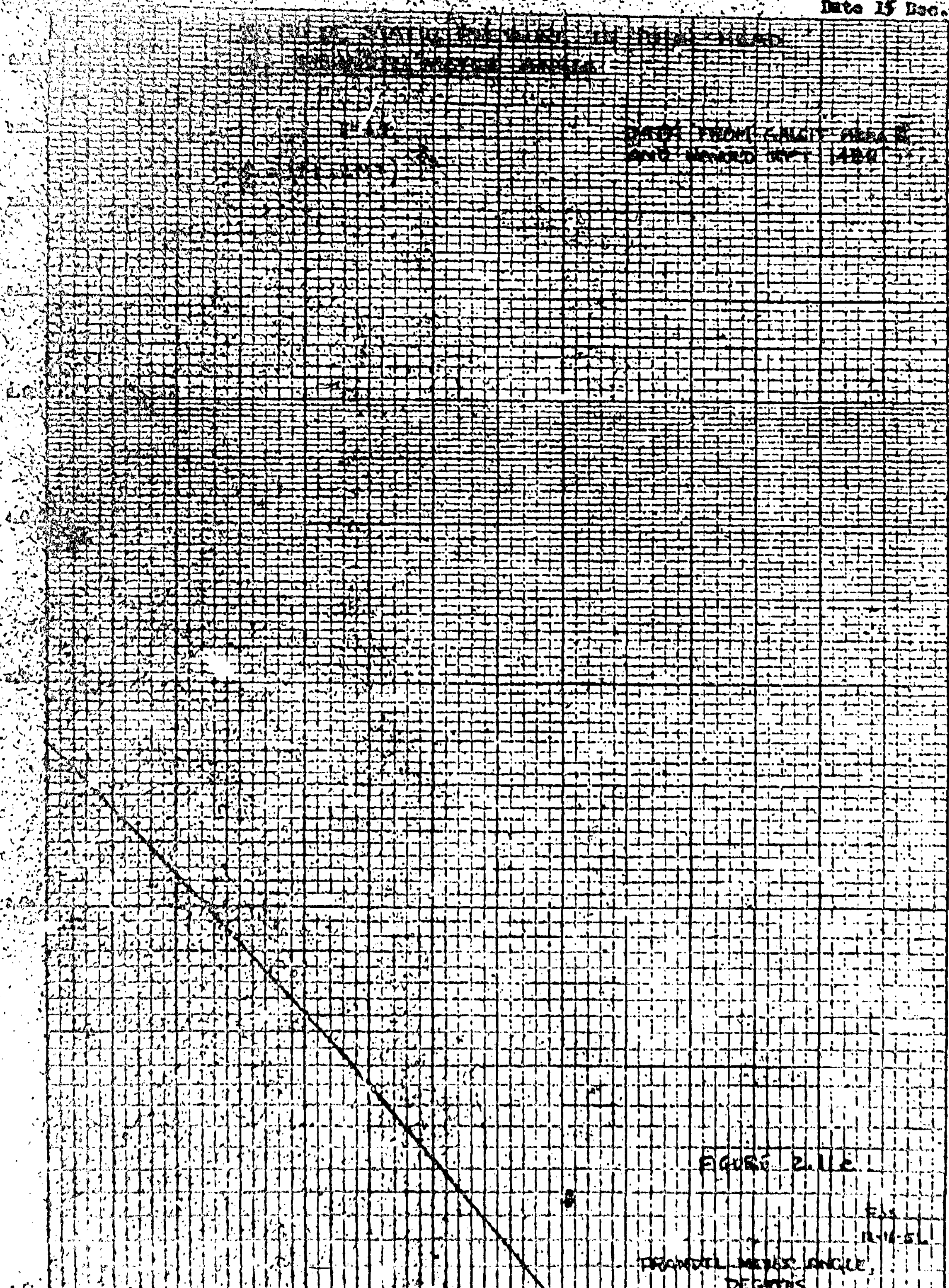
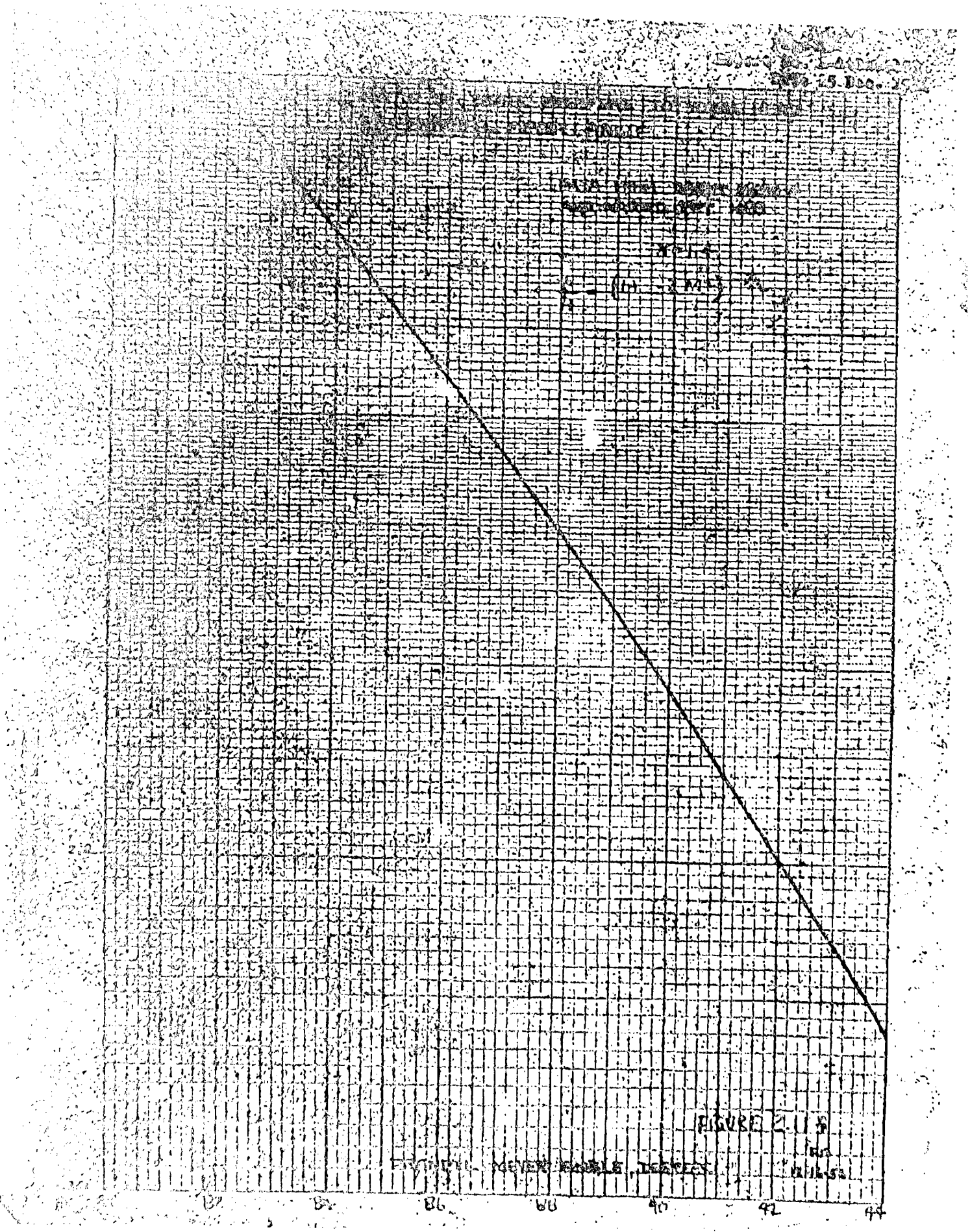


FIGURE 2.11c


84
86
88
90
92
94

RATIO OF STATIC PRESSURE TO TOTAL HEAD
VS PRANDTL-MEYER ANGLE

$$\frac{p}{H} = \left[\frac{1}{1 + \frac{\gamma-1}{2} M^2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}}\sqrt{M^2-1}\right\} - \tan^{-1}\sqrt{M^2-1}$$

$\gamma = 1.4$

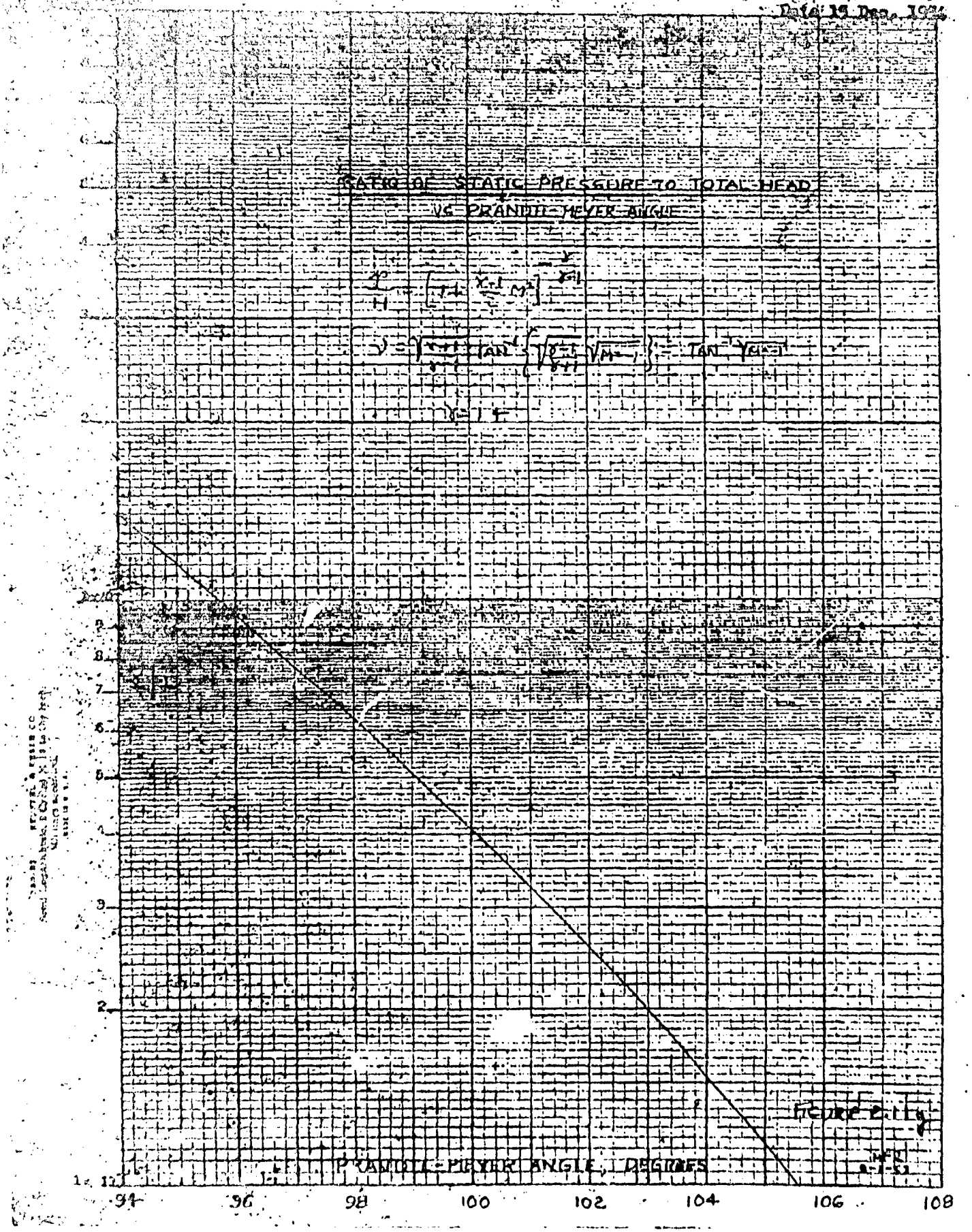


Figure C-11

A-1-53

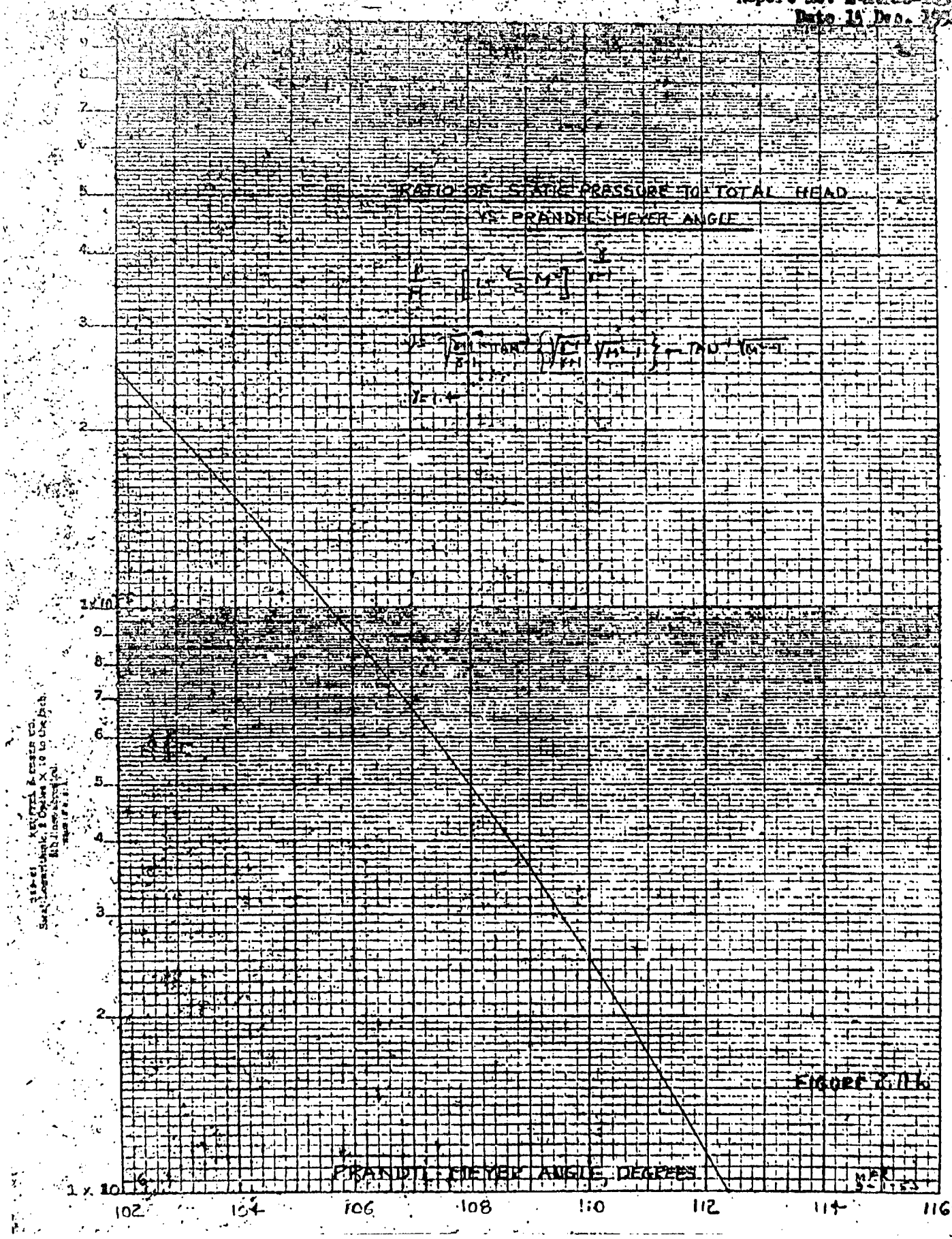
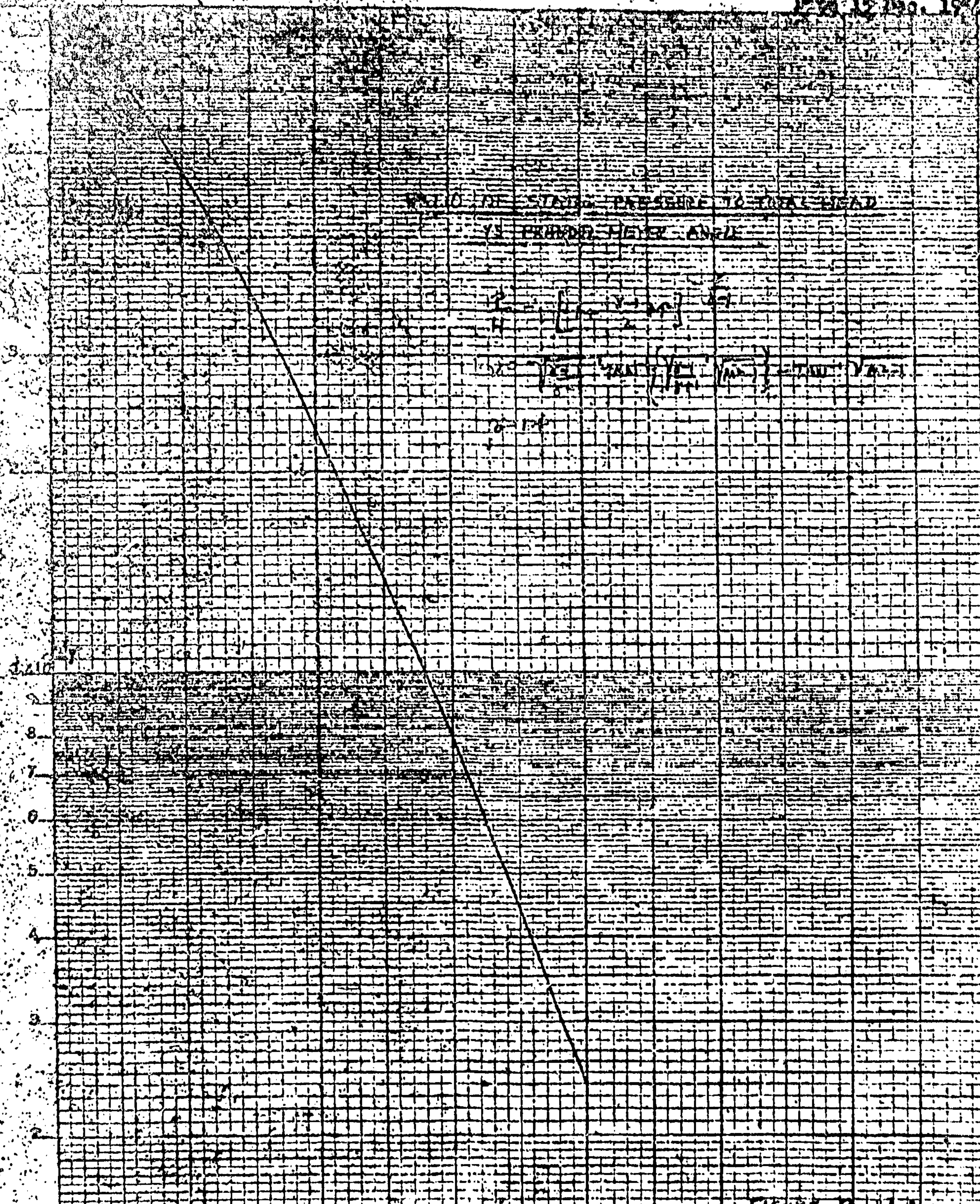

RATIO OF STATIC PRESSURE TO TOTAL HEAD
VS. PRANDTL-MEYER ANGLE
$\frac{p}{H} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{1-\gamma}}$
$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$
$\gamma = 1.4$
PRANDTL-MEYER ANGLE, DEGREES
102
104
106
108
110
112
114
116

FIGURE 2.11b


RATIO OF STATIC PRESSURE TO TOTAL HEAD
VS PRANDTL-MEYER ANGLE
γ = 1.4
PRANDTL-MEYER ANGLE, DEGREES
112
114
116
118
120
122

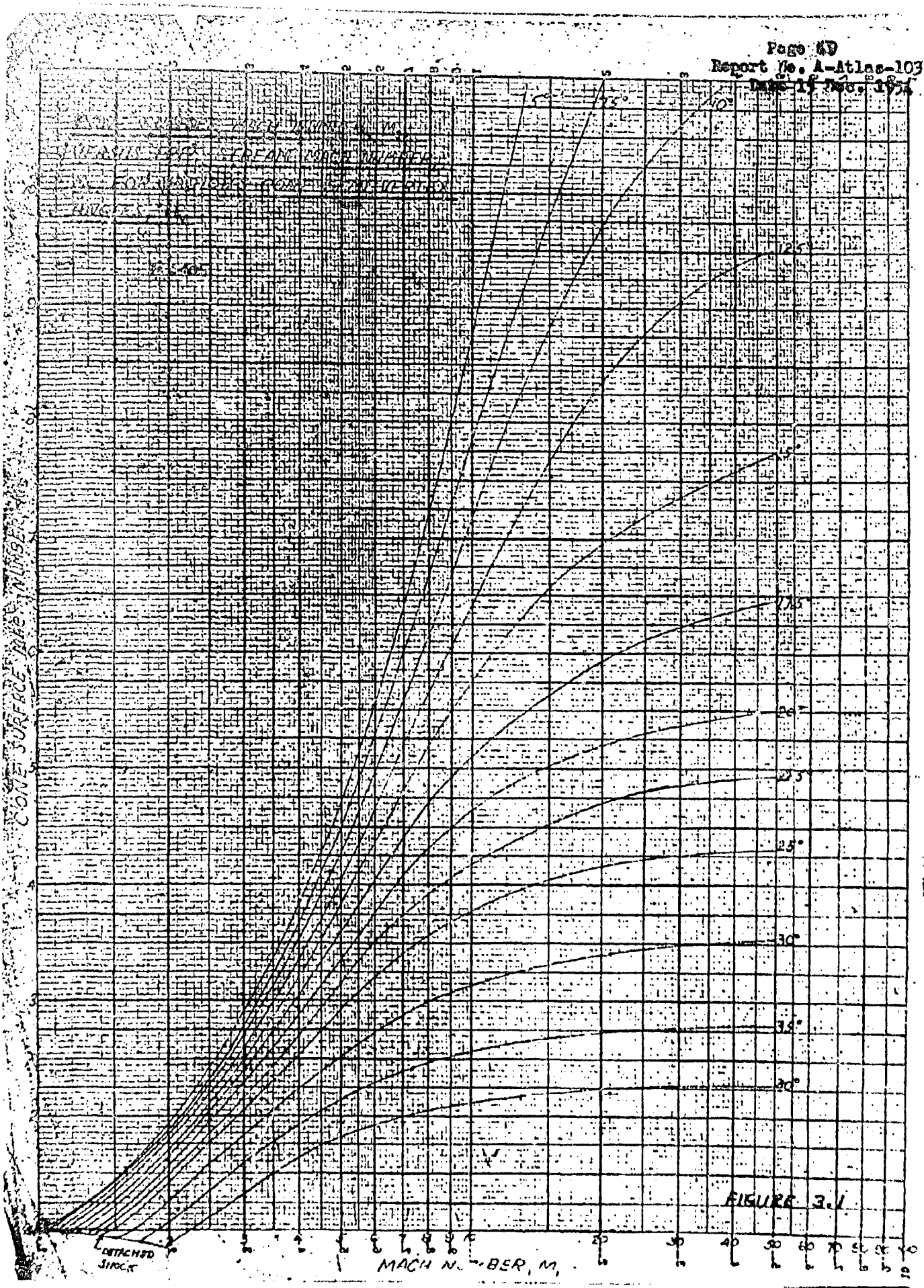

CONE SURFACE MACH NUMBER
MEAN CONE MACH NUMBER
FOR TURBULENT FLOW
ATTACHED SHOCK
MACH NUMBER, M,
5°
10°
12.5°
15°
20°
25°
30°
35°
40°

FIGURE 3.1

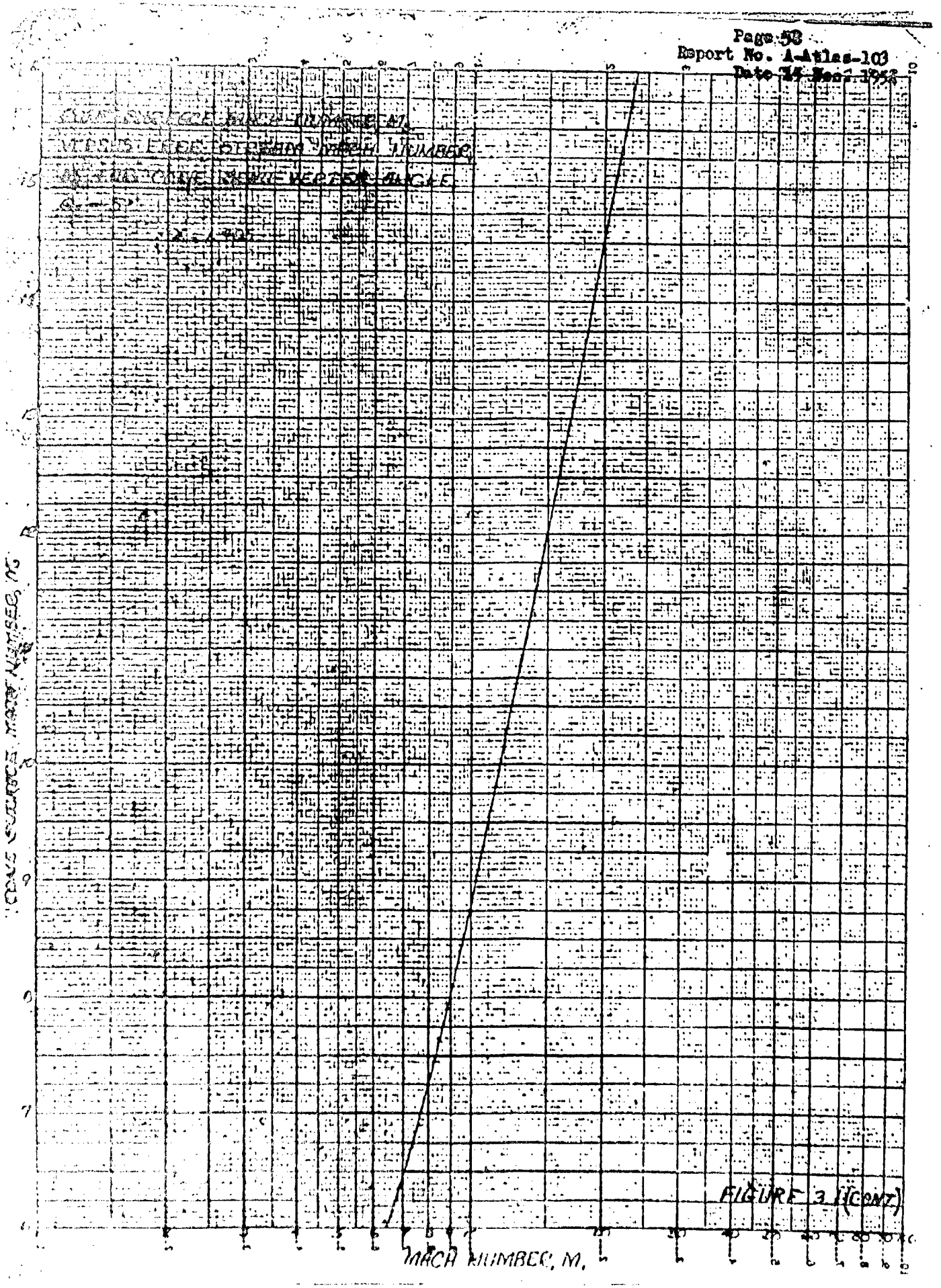


FIGURE 3 (CONT)

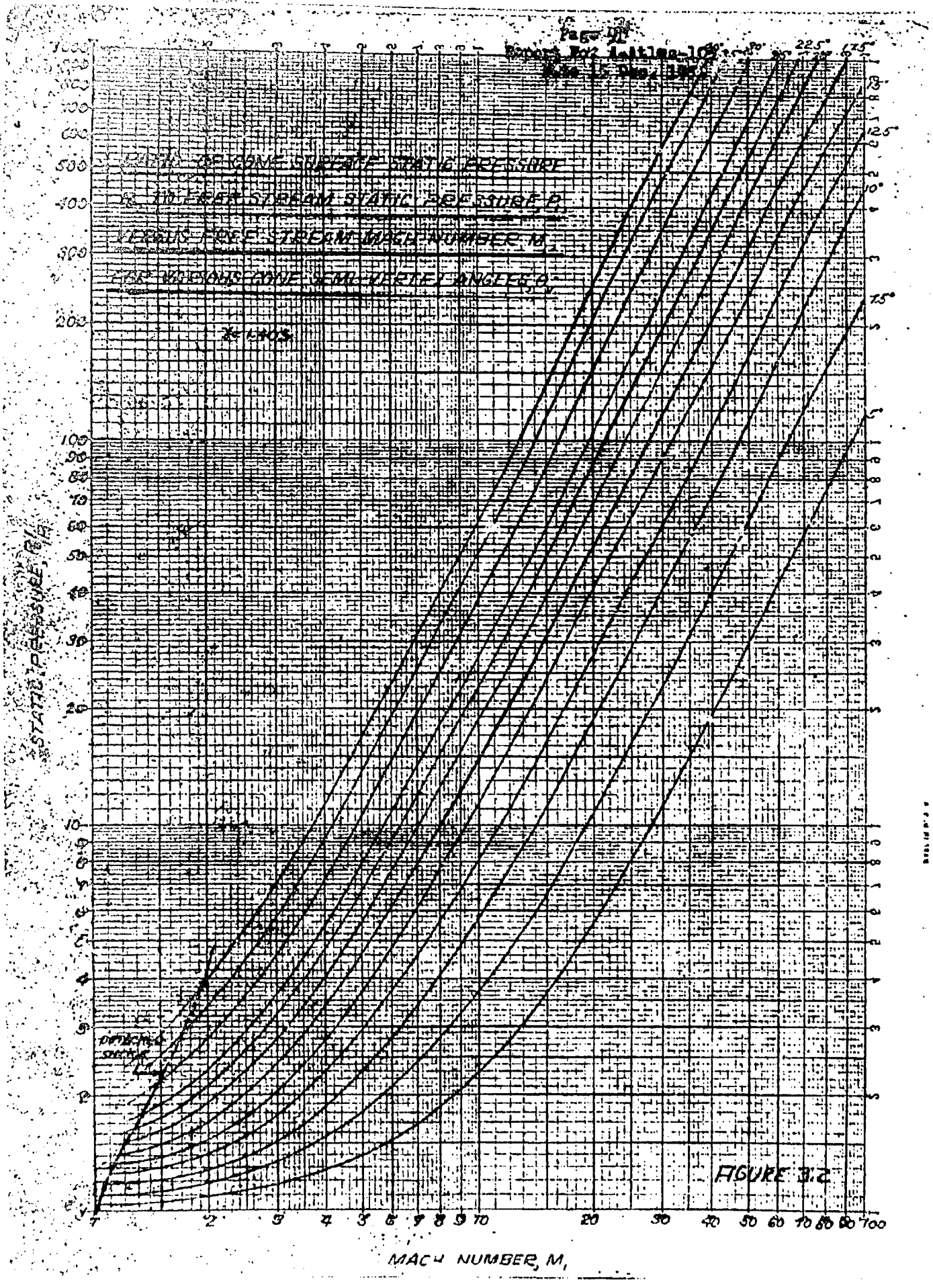

RATIO OF CONE SURFACE STATIC PRESSURE
TO FREE STREAM STATIC PRESSURE p/p1
FREE STREAM MACH NUMBER M1
HALF VERTEX ANGLE OF CIRCULAR CONE θc
γ = 1.405
STATIC PRESSURE, p/p1
MACH NUMBER, M1
DETACHED SHOCK

FIGURE 3.2

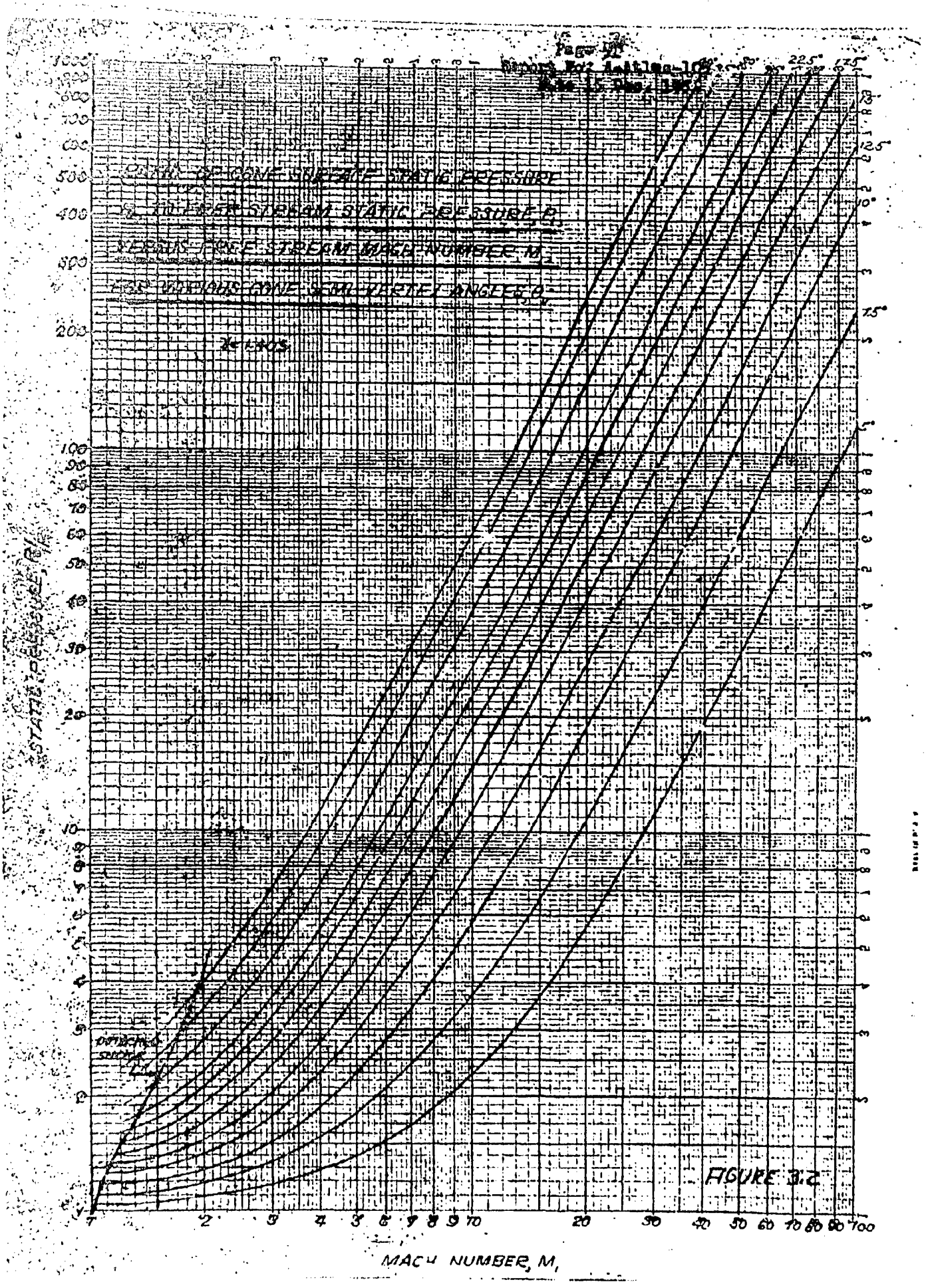


FIGURE 3.2

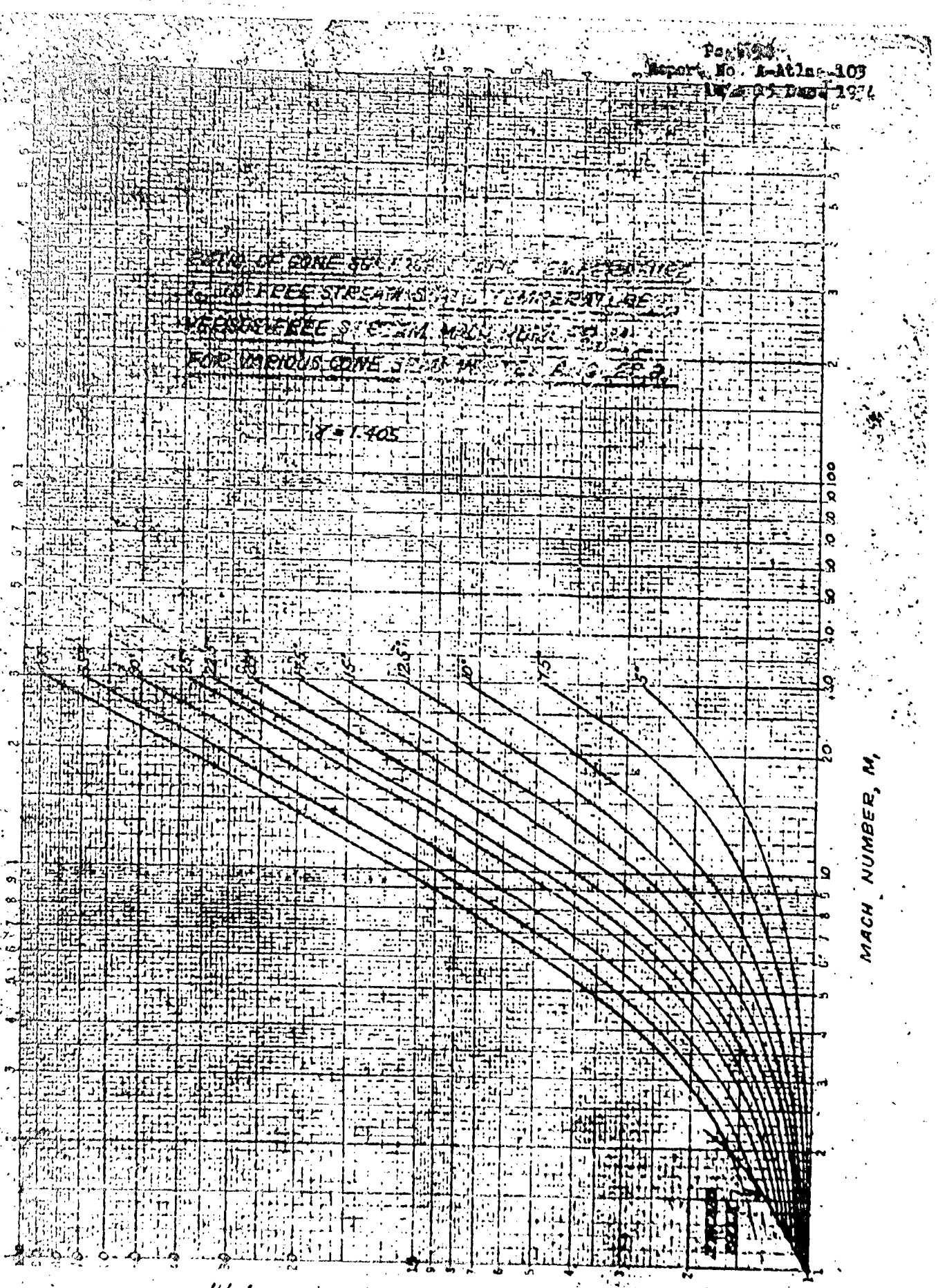


FIGURE 33

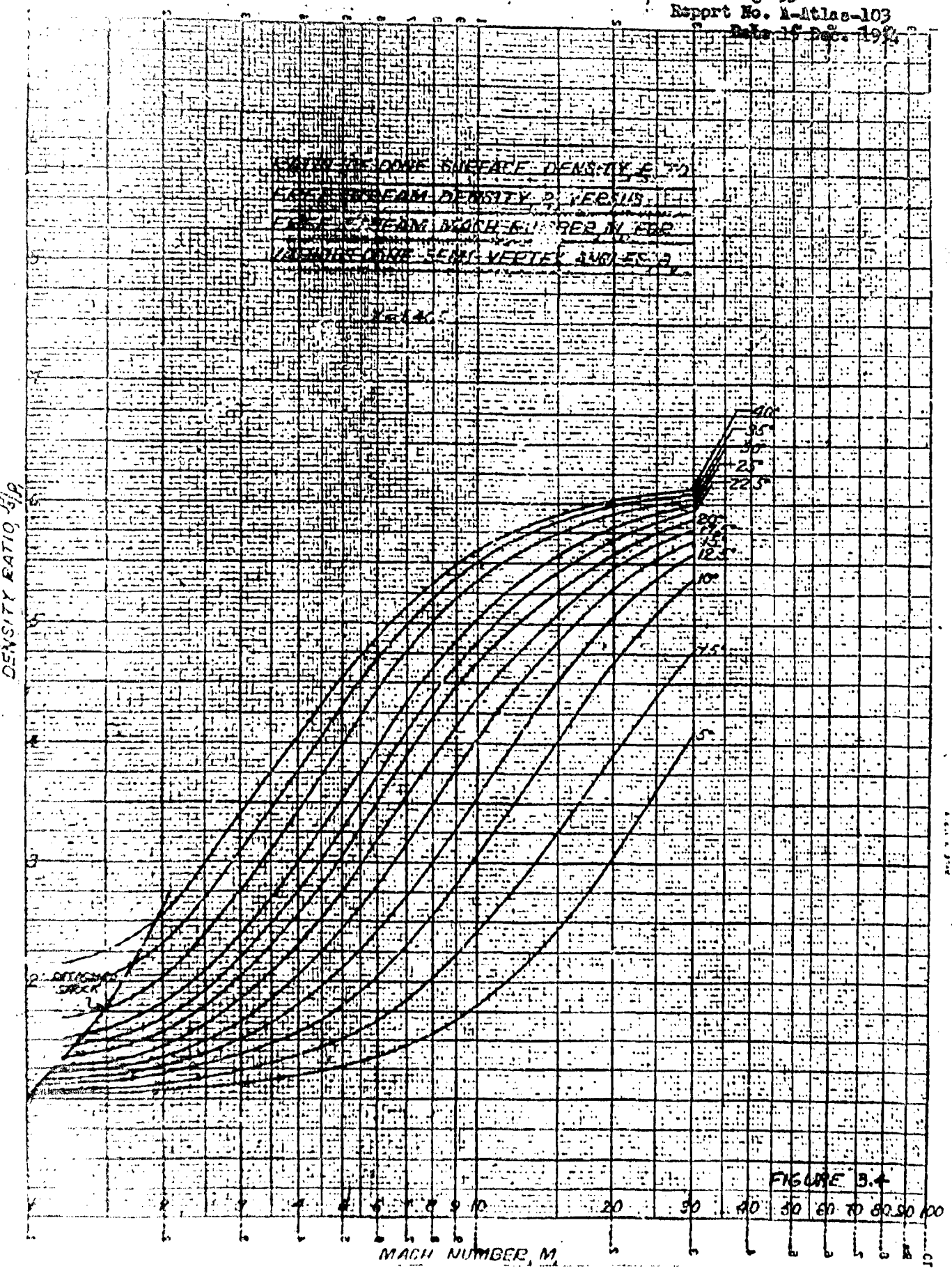

CONE SURFACE DENSITY
FREE STREAM DENSITY
FREE STREAM MACH NUMBER
CONE SEMI-VERTEX ANGLE
γ = 1.4
40°
35°
30°
25°
22.5°
20°
17.5°
15°
12.5°
10°
7.5°
5°
DENSITY RATIO
MACH NUMBER, M

FIGURE 3.4

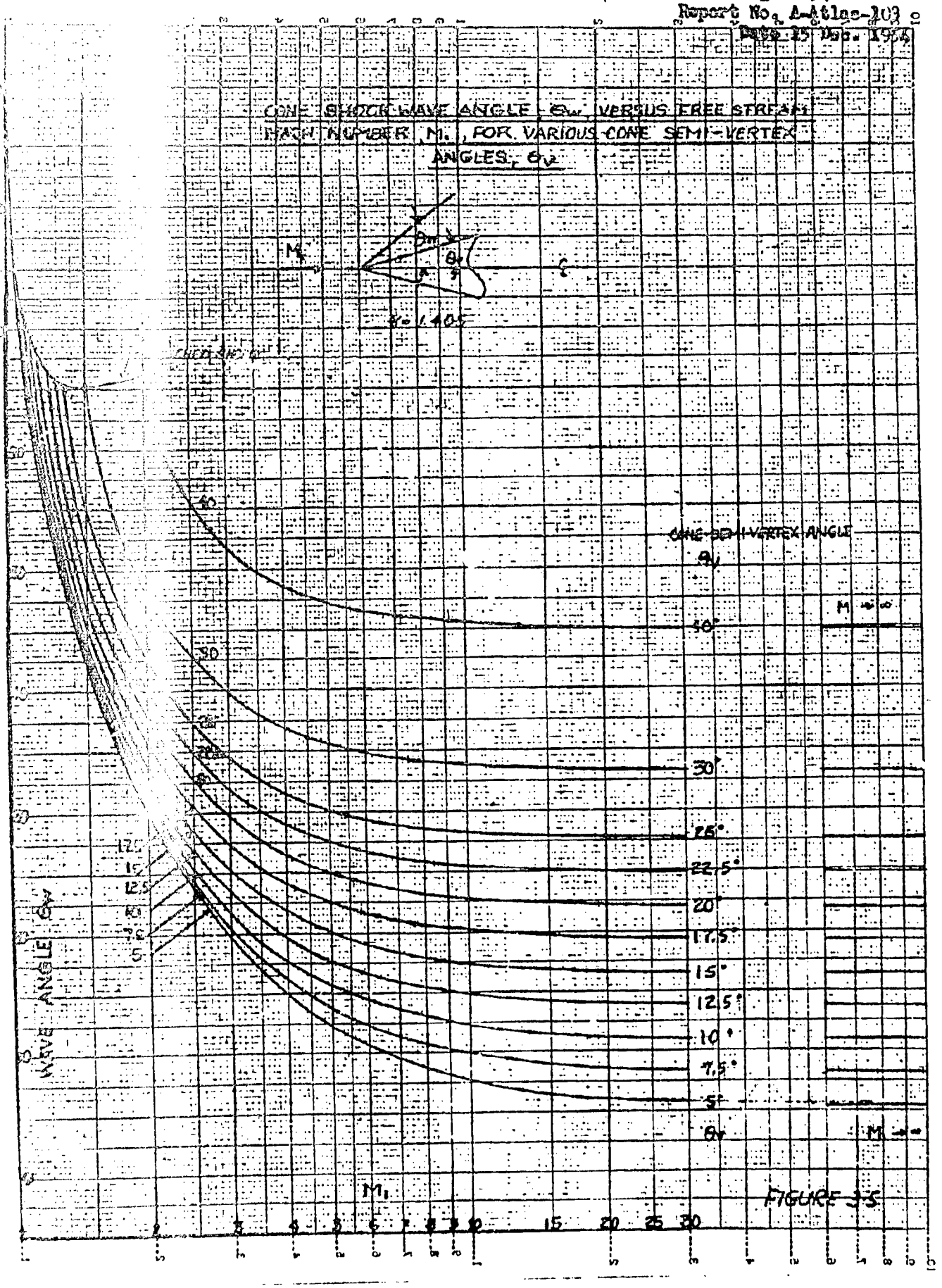
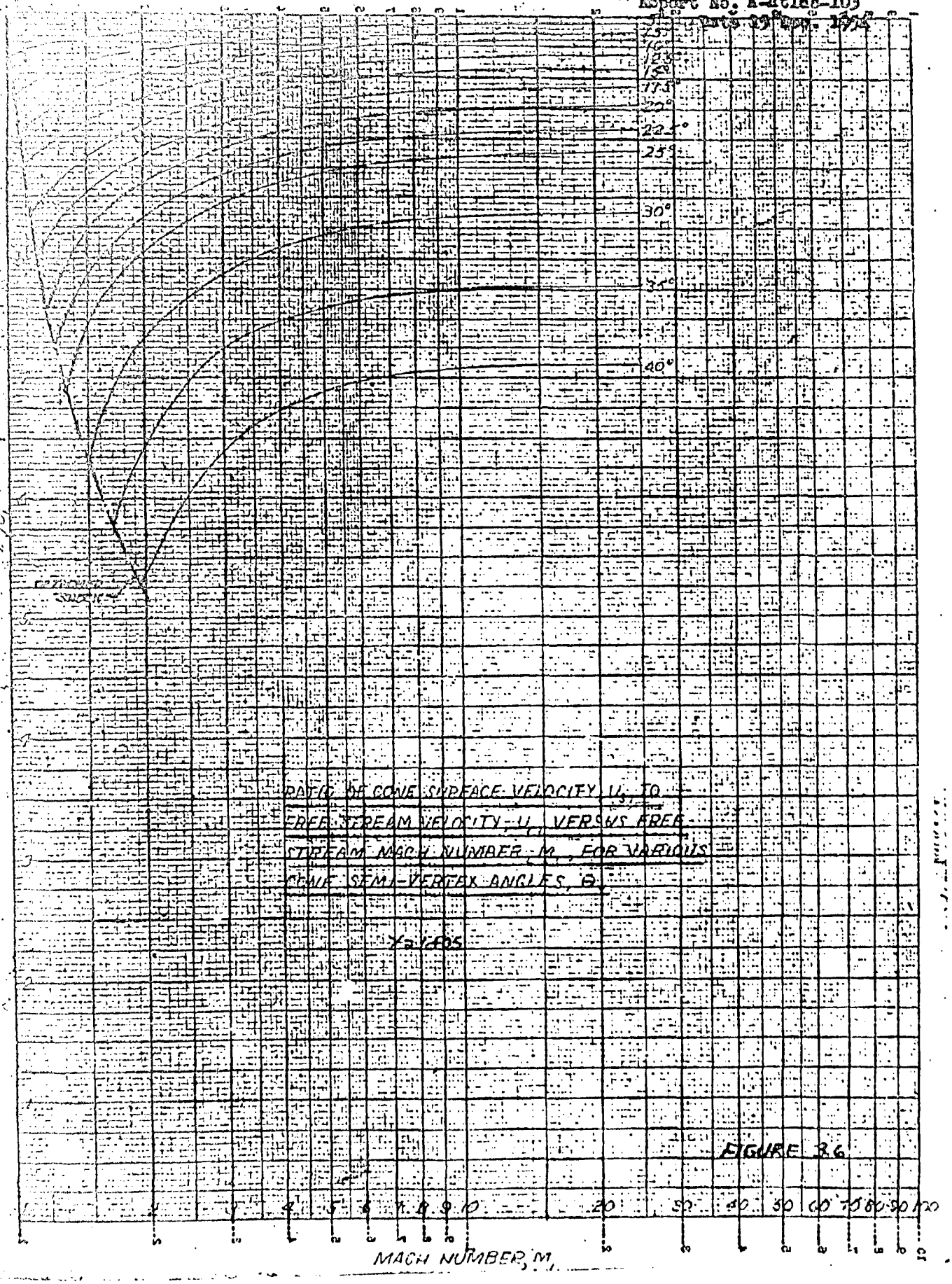

CONE SHOCK WAVE ANGLE - θ_w VERSUS FREE STREAM MACH NUMBER M_1, FOR VARIOUS CONE SEMI-VERTEX ANGLES, θ_v
M_1
θ_w
θ_v
$\gamma = 1.405$
CONE SEMI-VERTEX ANGLE θ_v
$M = \infty$
50°
30°
25°
22.5°
20°
17.5°
15°
12.5°
10°
7.5°
5°
θ_v
$M = 1$
SHOCK WAVE ANGLE θ_w
M_1
1 2 3 4 5 6 7 8 9 10 15 20 25 30

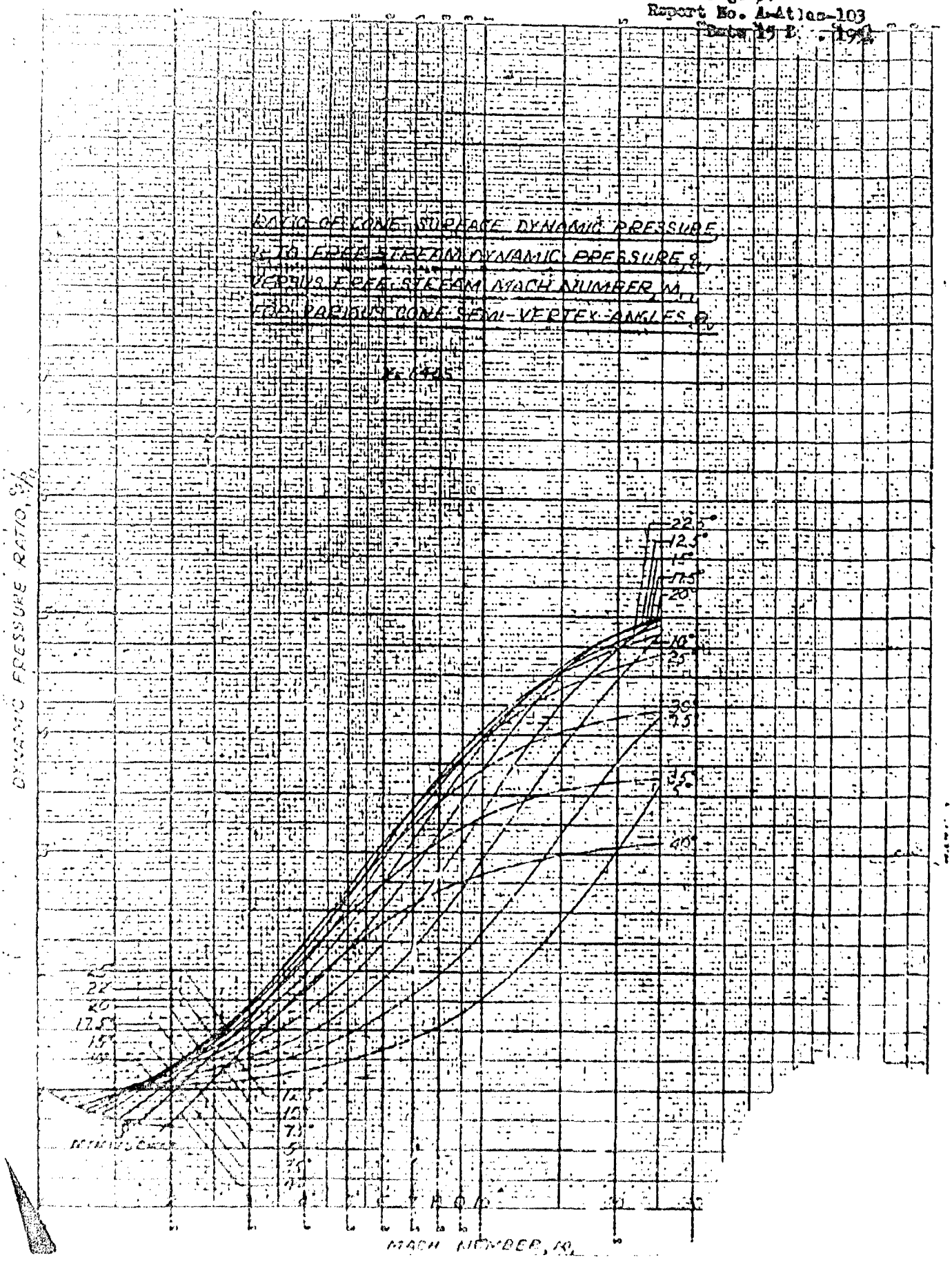
FIGURE 35

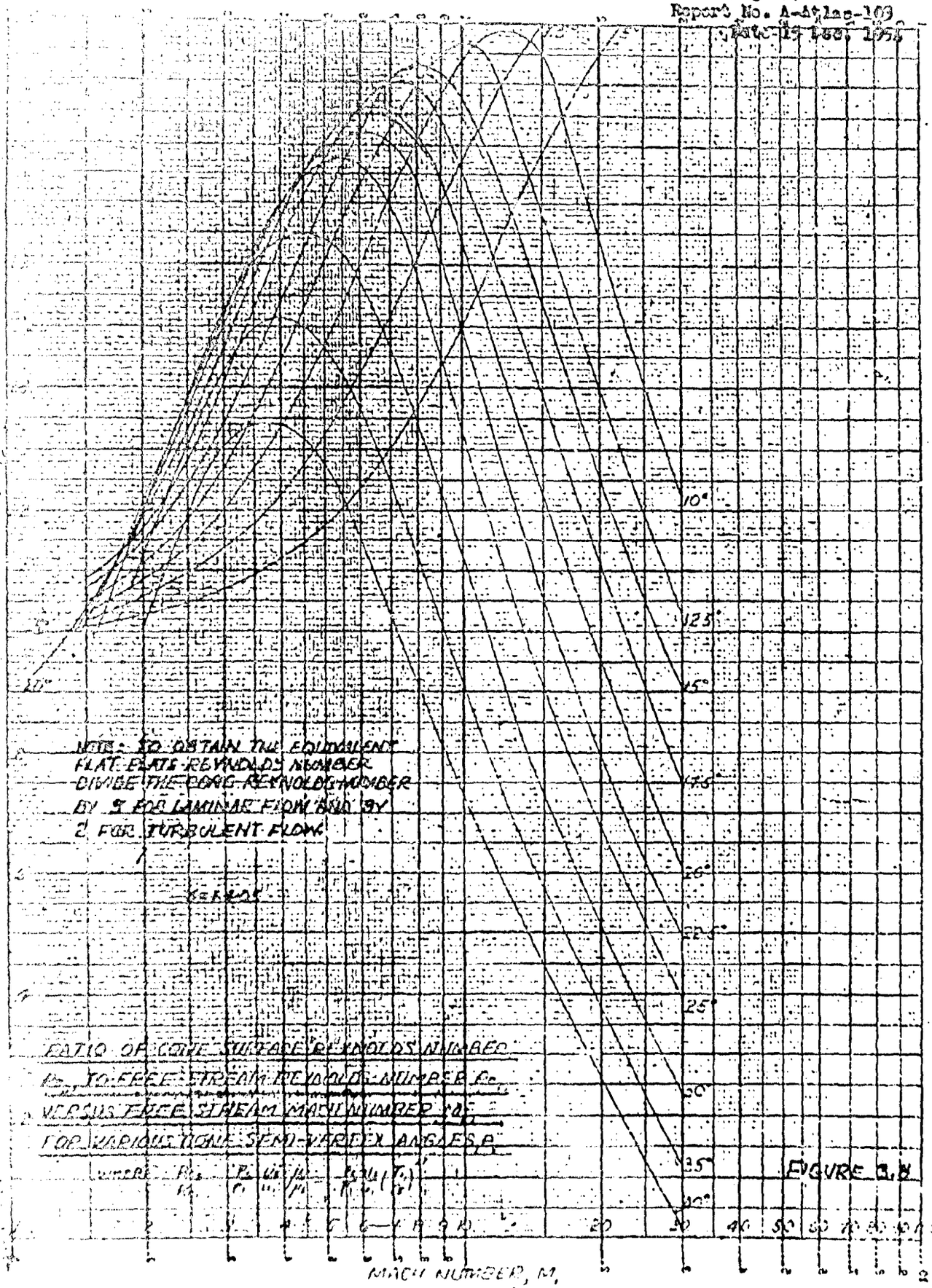


RATIO OF CONE SURFACE VELOCITY U_s TO FREE STREAM VELOCITY U_∞ VERSUS FREE STREAM MACH NUMBER M_∞ FOR VARIOUS CONE SEMI-VERTEX ANGLES, θ.

Y_a 1605

FIGURE 36


RATIO OF CONE SURFACE DYNAMIC PRESSURE
TO FREE STREAM DYNAMIC PRESSURE, q/q
VERSUS FREE STREAM MACH NUMBER, M
FOR PARTICULAR CONE SEMI-VERTEX ANGLES, θ
DYNAMIC PRESSURE RATIO, q/q
MACH NUMBER, M
22.5°
12.5°
15°
17.5°
20°
10°
25°
30°
32.5°
35°
40°



NOTE: TO OBTAIN THE EQUIVALENT FLAT PLATE REYNOLDS NUMBER DIVIDE THE CONE REYNOLDS NUMBER BY 3 FOR LAMINAR FLOW AND BY 2 FOR TURBULENT FLOW

RATIO OF CONE SURFACE REYNOLDS NUMBER R_{e_c} TO FREE STREAM REYNOLDS NUMBER R_{e_∞} VERSUS FREE STREAM MACH NUMBER M_∞ FOR VARIOUS CONE SEMI-VERTEX ANGLES θ_c

FIGURE C.8

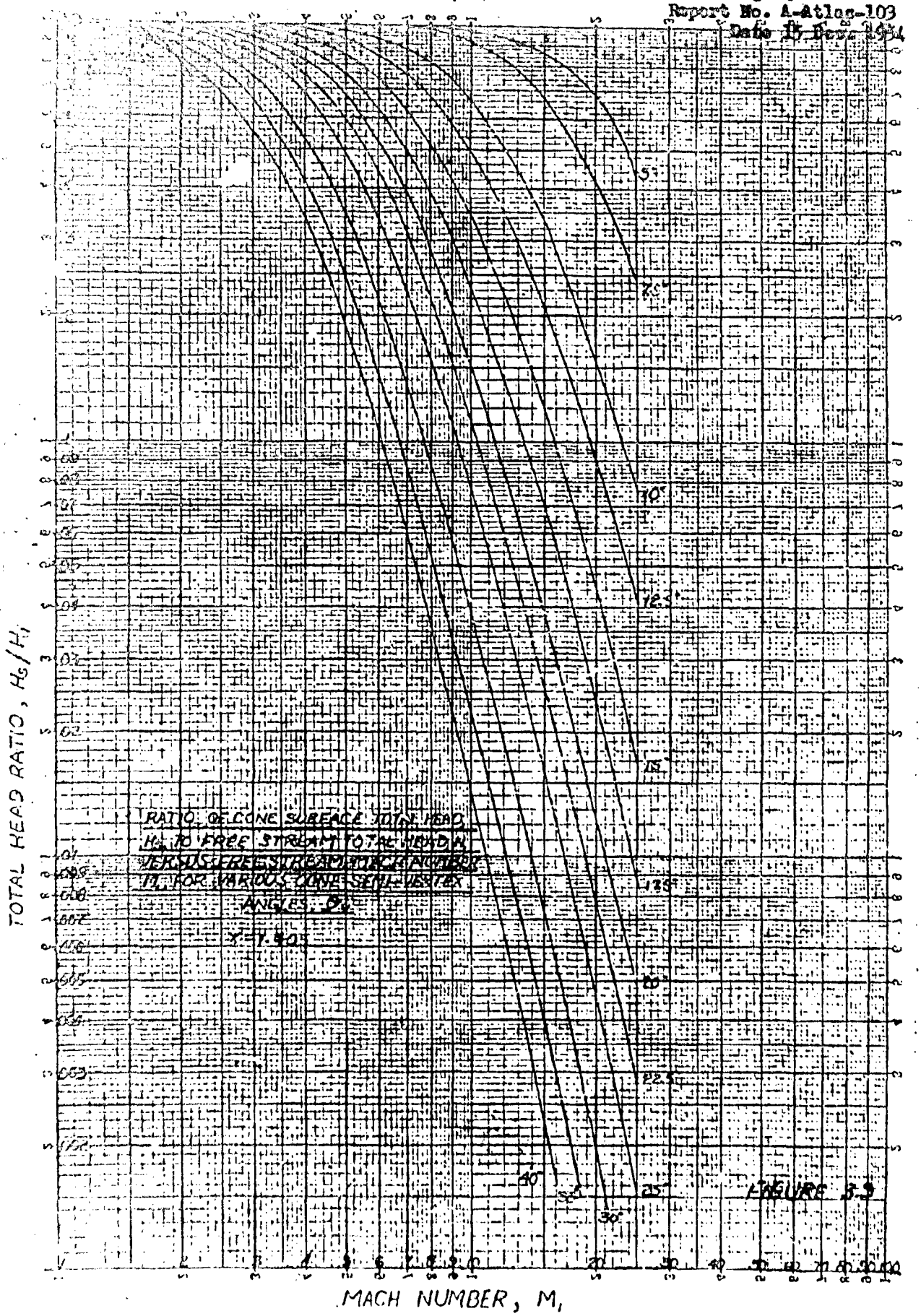

TOTAL HEAD RATIO, H_s/H_1
MACH NUMBER, M,
RATIO OF CONE SURFACE TOTAL HEAD H_s TO FREE STREAM TOTAL HEAD H_1 VERSUS FREE STREAM MACH NUMBER M_1 FOR VARIOUS CONE SEMI-VERTEX ANGLES, θ_c
$\gamma = 1.405$
5°
7.5°
10°
12.5°
15°
17.5°
20°
22.5°
25°
30°
35°
40°

FIGURE 5.3

WAVE DRAG COEFFICIENT OF A CONE
VS. CONE HALF-ANGLE

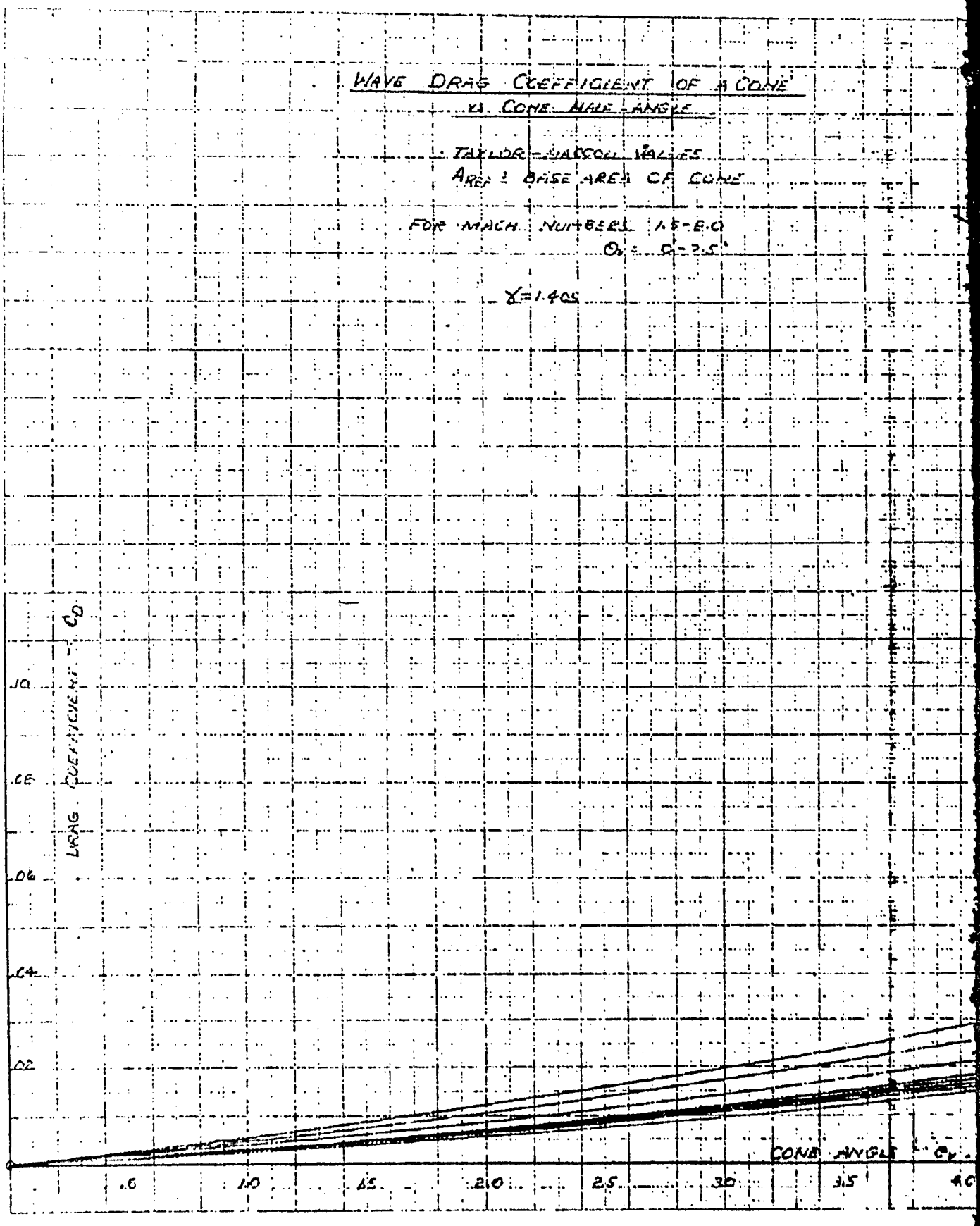
TAYLOR-MACCOLL VALUES

A_{REF} = BASE AREA OF CONE

FOR MACH NUMBERS 1.5 - 8.0

θ_c = 0 - 2.5°

$\gamma = 1.405$



A

C_D vs Θ_V

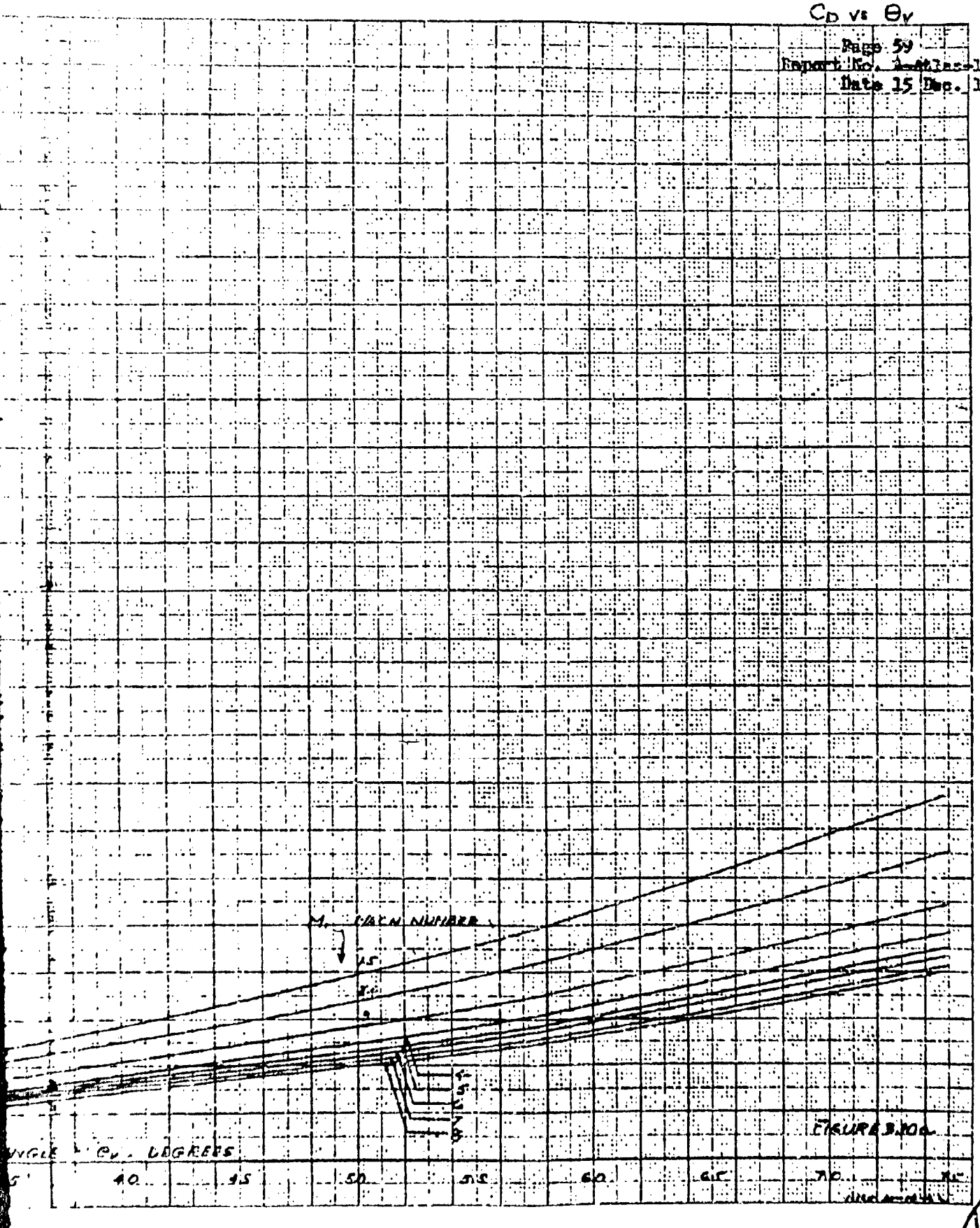


FIGURE 3.10c

B

C_D vs Θ_V

Report No. Aerolab-103
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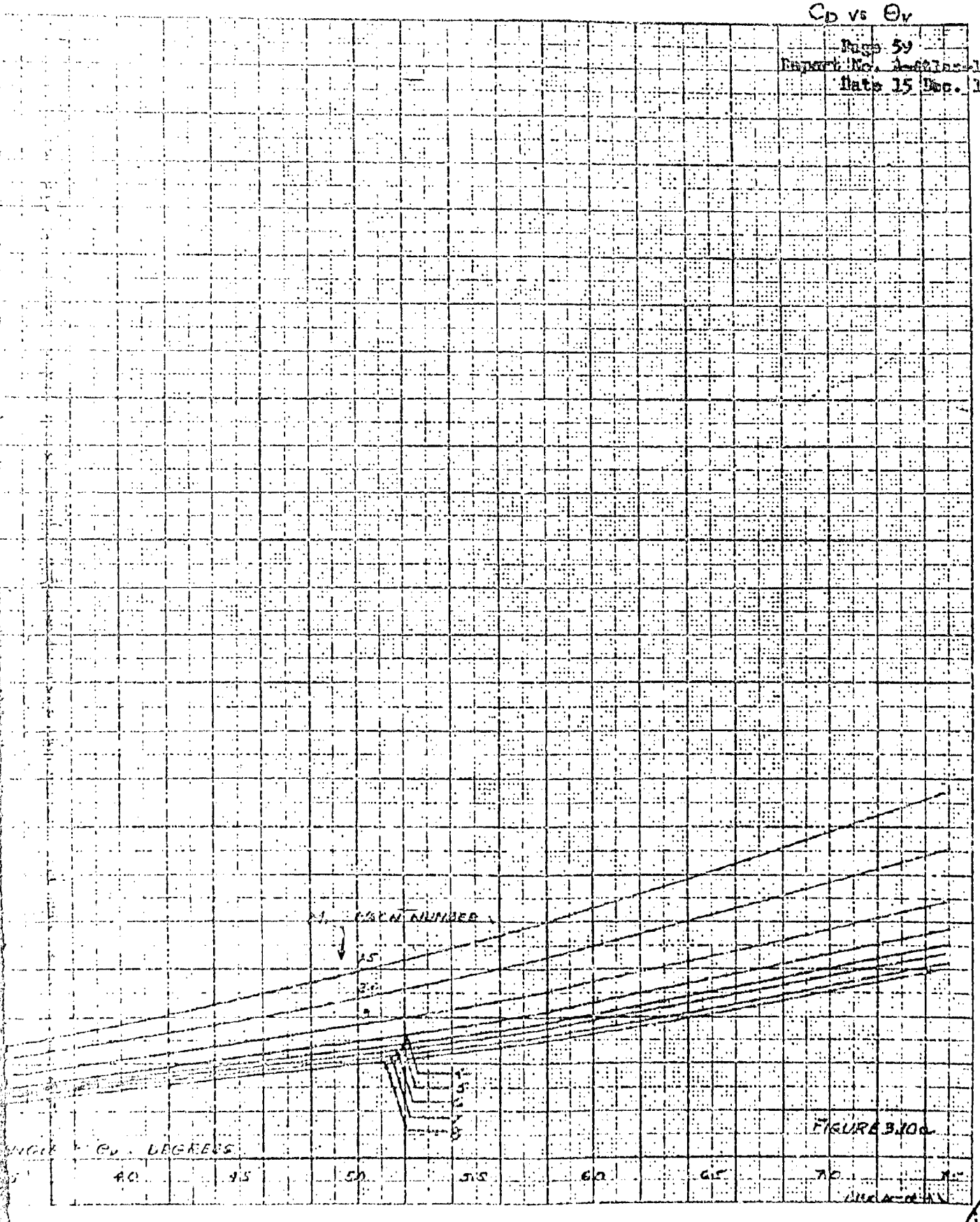
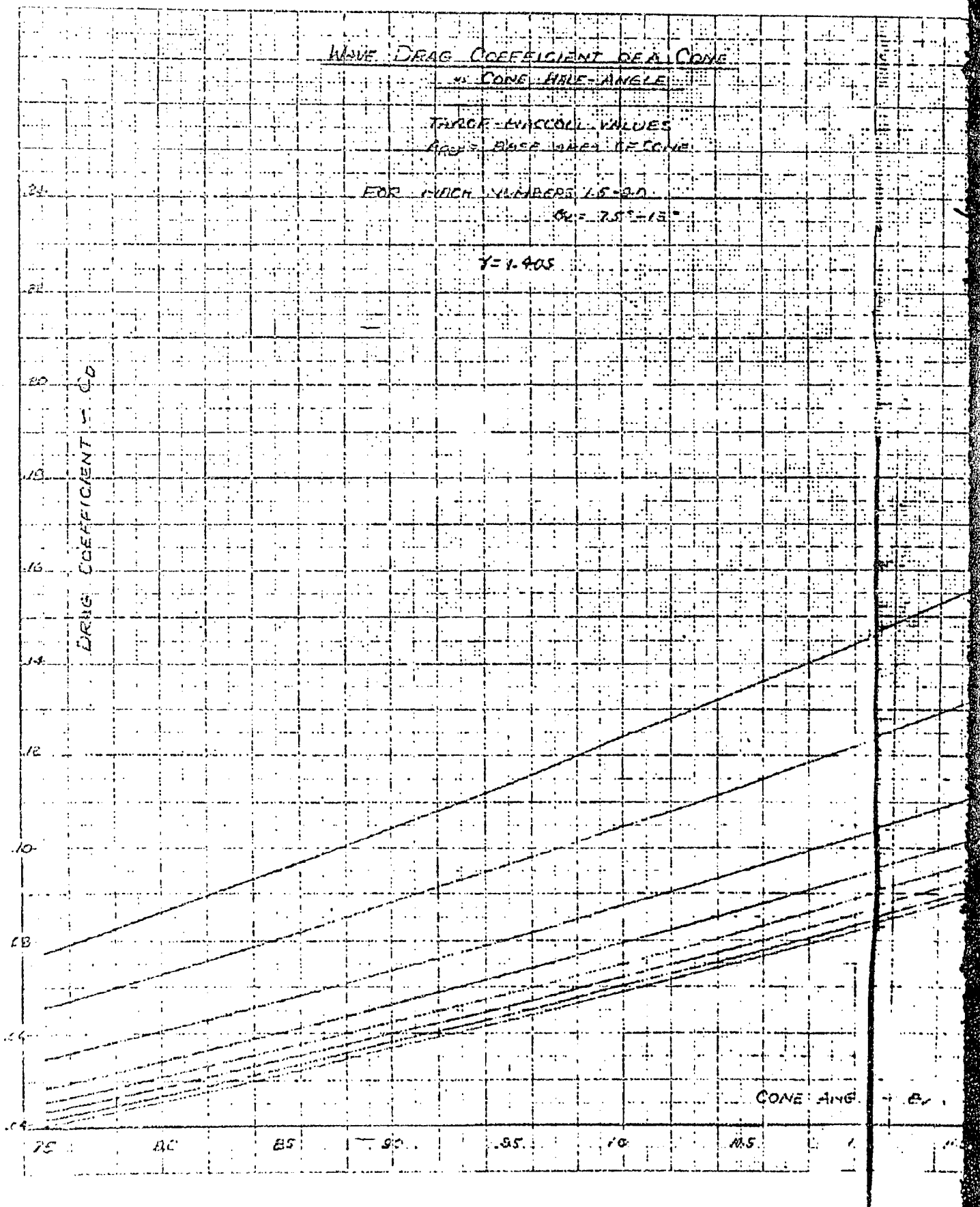


FIGURE 3.10c

B


WAVE DRAG COEFFICIENT OF A CONE
vs CONE HALF-ANGLE
THICK-TRUNCOLL VALUES
AREA = BASE AREA OF CONE
FOR MACH NUMBERS 1.5-2.0
θ_c = 7.5°-13°
γ = 1.405
DRAG COEFFICIENT = C_D
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
CONE ANG
.75
.80
.85
.90
.95
1.0
1.05
1

C_D vs θ_v

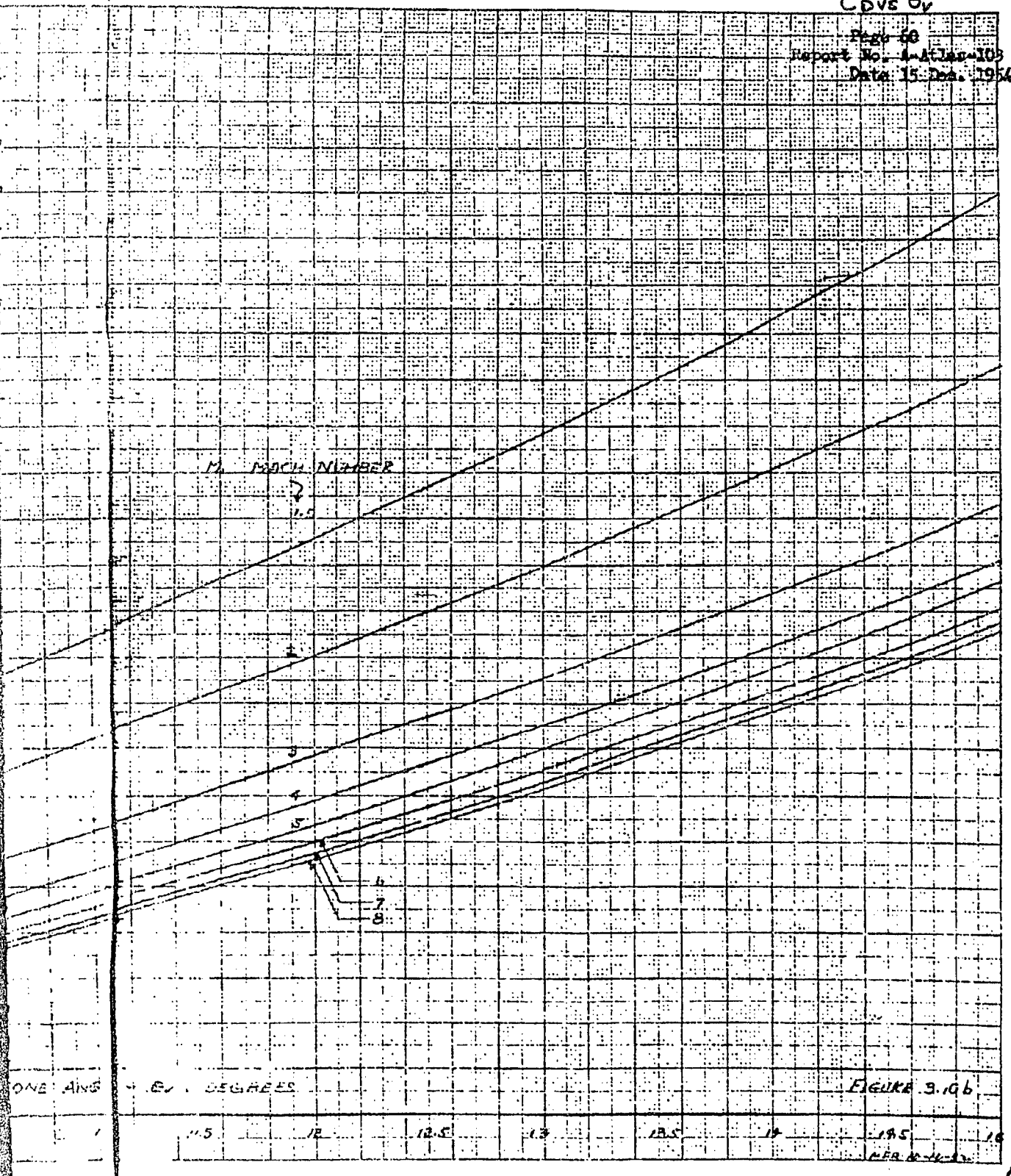
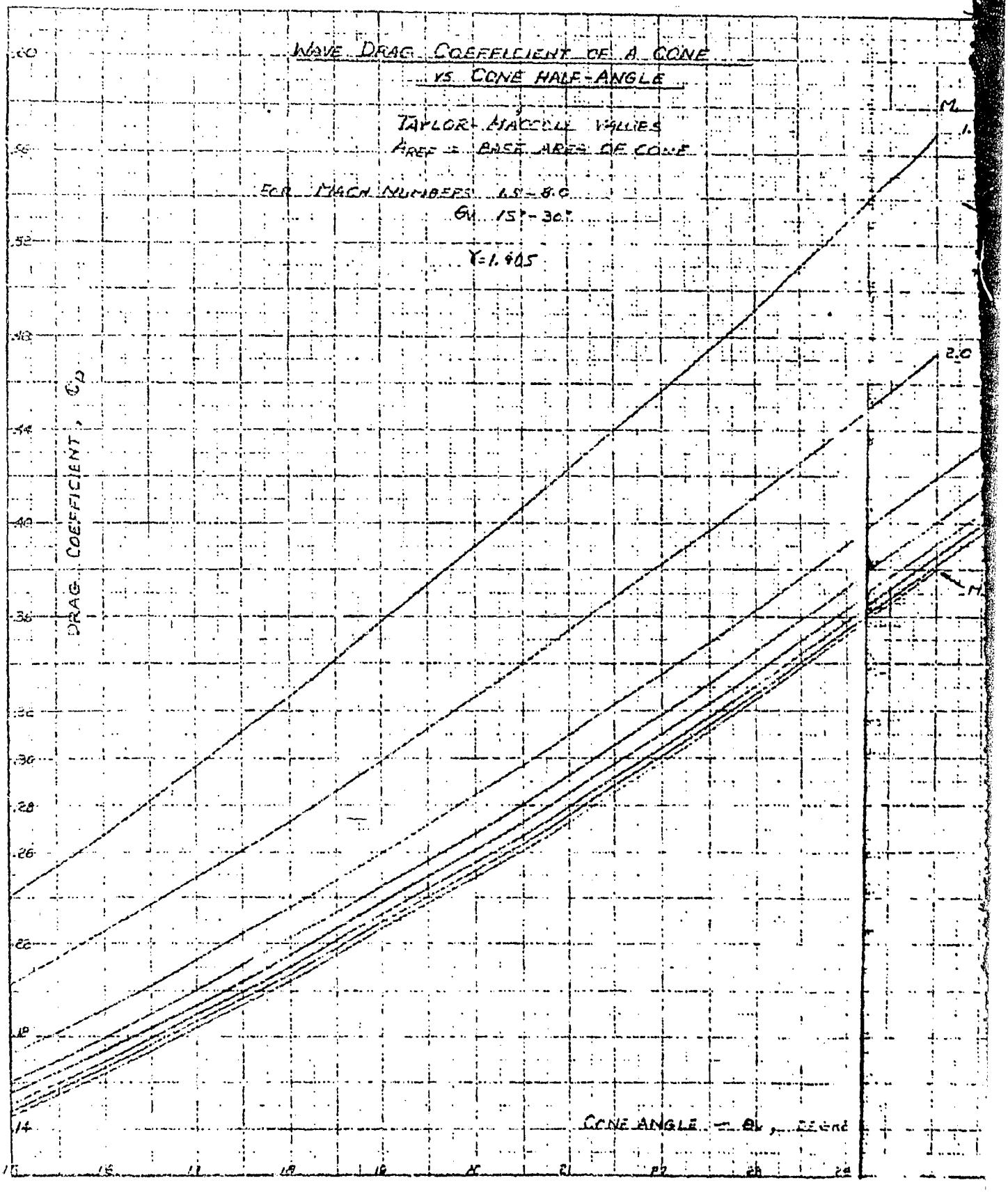


FIGURE 3.10b

B


WAVE DRAG COEFFICIENT OF A CONE
VS. CONE HALF-ANGLE
TAYLOR-MACCOLL VALUES
A_{REF} = BASE AREA OF CONE
FOR MACH NUMBERS 1.5 - 8.0
θ_c 15° - 30°
γ = 1.405
DRAG COEFFICIENT, C_D
M
1.
2.0
CONE ANGLE = θ_c, DEGREES

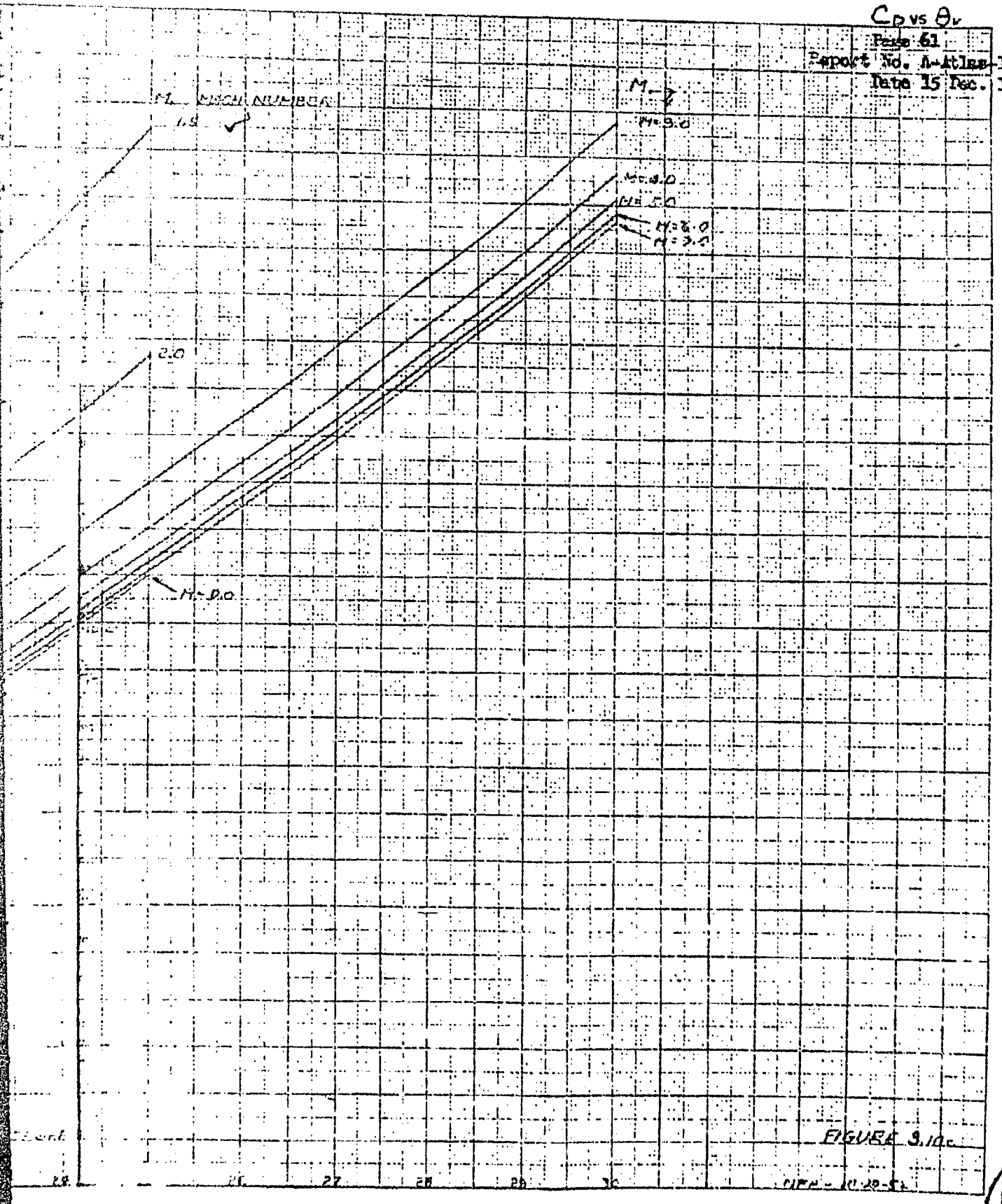
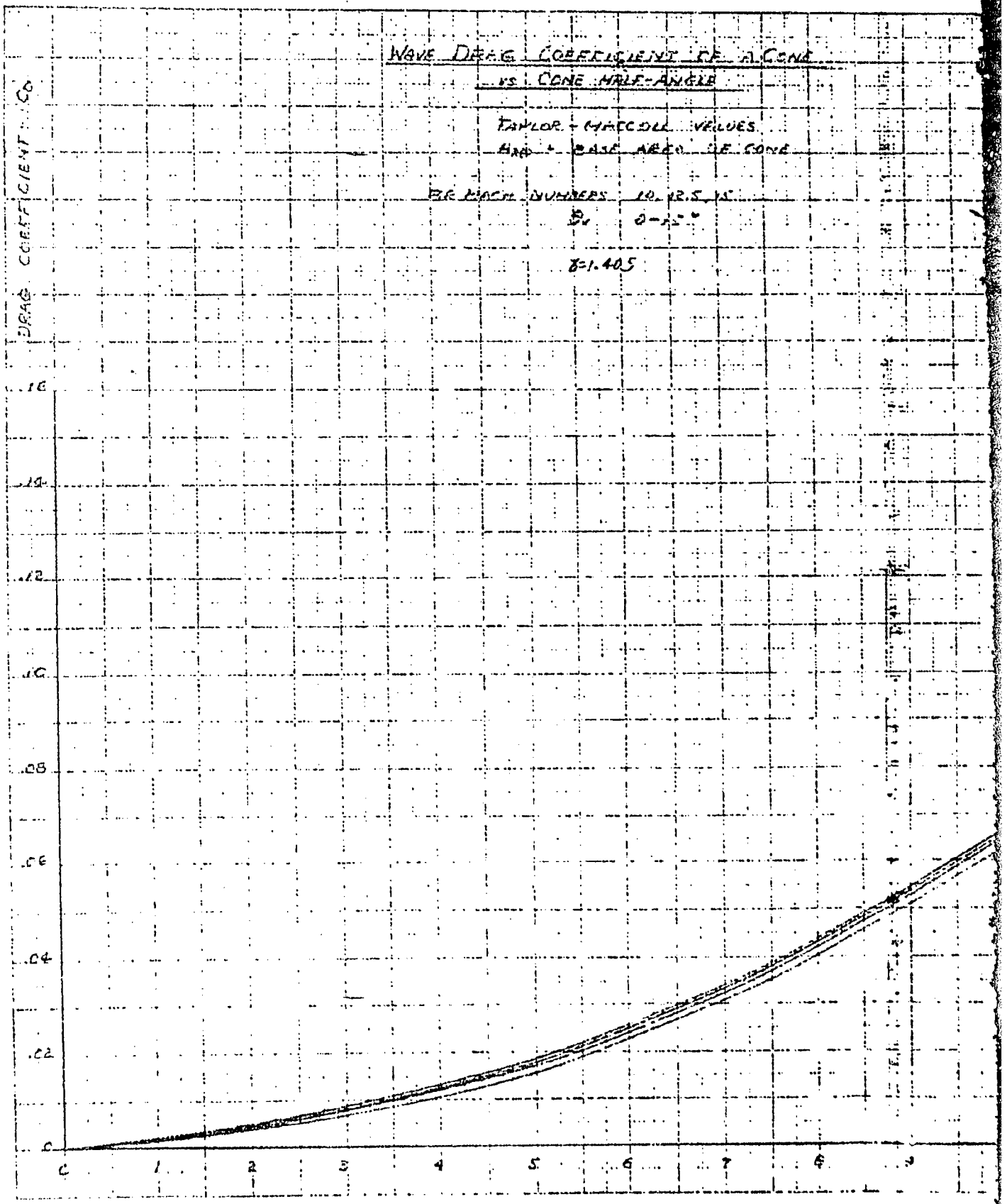


FIGURE 9.10c

B


WAVE DRAG COEFFICIENT OF A CONE
VS. CONE HALF-ANGLE
TAYLOR - MACCOLL VALUES
AND BASE AREA OF CONE
FOR MACH NUMBERS 10, 12.5, 15
δ_v 0 - 7.5°
γ=1.405
DRAG COEFFICIENT C_D
.16
.12
.08
.04
.02
0
0
1
2
3
4
5
6
7
8

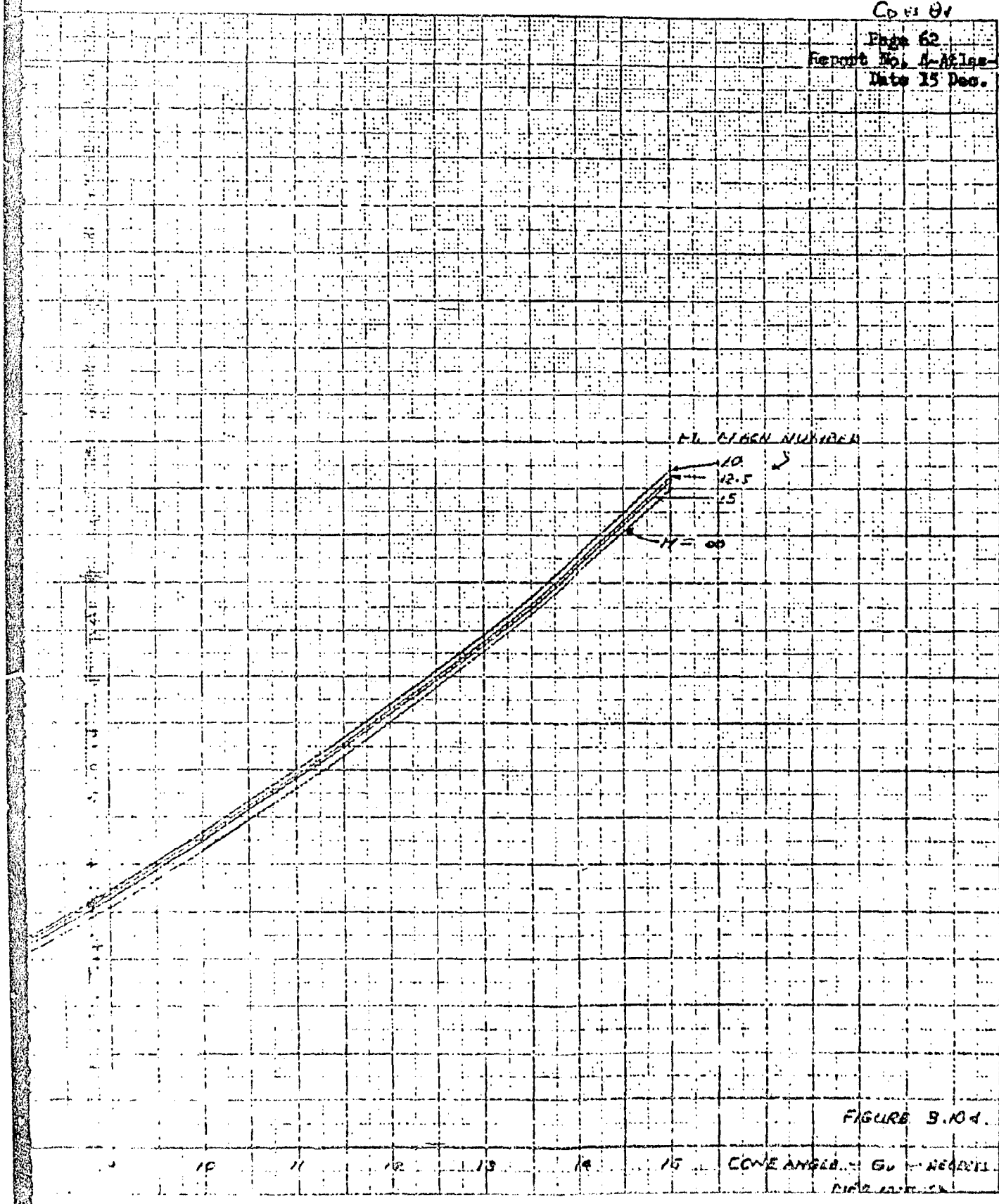
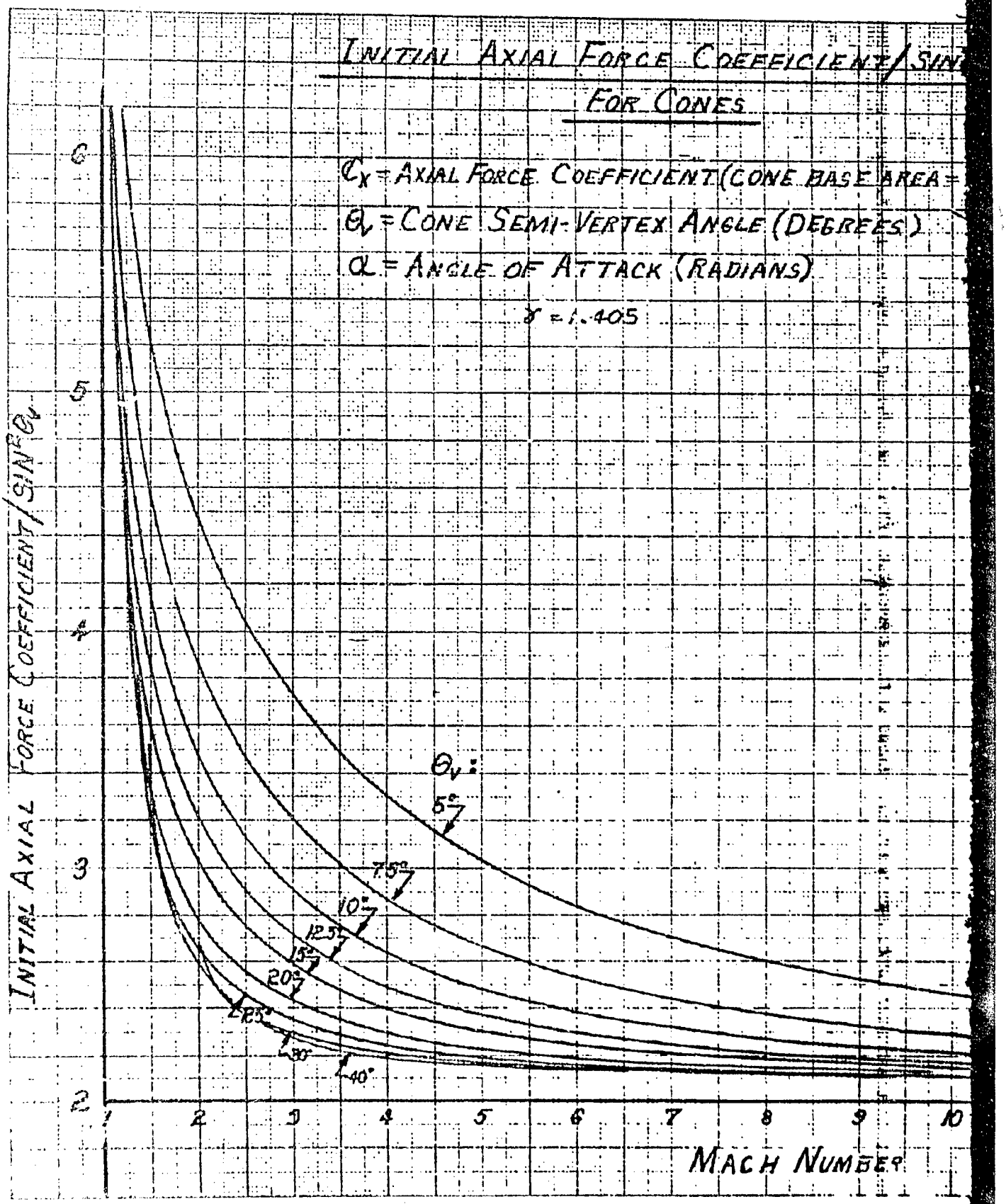


FIGURE 3.10 d.

B


INITIAL AXIAL FORCE COEFFICIENT/SIN
FOR CONES
C_X = AXIAL FORCE COEFFICIENT (CONE BASE AREA =
θ_v = CONE SEMI-VERTEX ANGLE (DEGREES)
α = ANGLE OF ATTACK (RADIANS)
γ = 1.405
INITIAL AXIAL FORCE COEFFICIENT/SIN$^2\theta_v$
MACH NUMBER
θ_v:
5°
7.5°
10°
12.5°
15°
20°
25°
30°
40°
2
3
4
5
6
1
2
3
4
5
6
7
8
9
10

C_x vs M_1

EFFICIENCY / $SIN^2\theta_1$ vs MACH NUMBER

(BASE AREA = REFERENCE AREA)

(DEGREES)

(VS)

NOTE: $C_{x_{\alpha=0}} / SIN^2\theta_1 = 2.0$ FOR NEWTONIAN FLOW

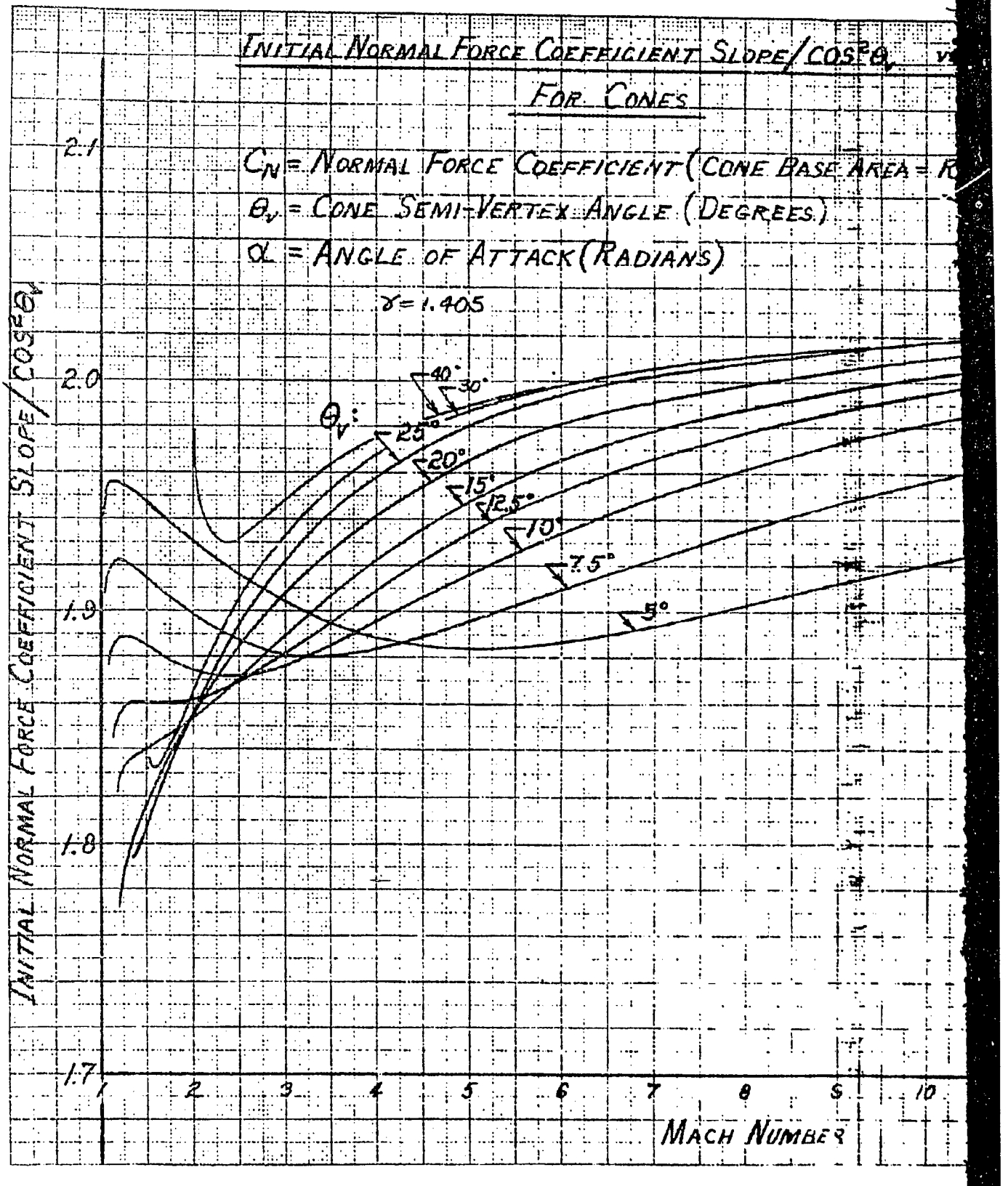
$\left[C_{x_{\alpha=0}} / SIN^2\theta_1\right]_{M=\infty} = 2.091$

9 10 11 12 13 14 15 16 17 18 19

NUMBER

FIGURE 3.11

FJD
5-22-53


INITIAL NORMAL FORCE COEFFICIENT SLOPE/COS²θv
FOR CONES
CN = NORMAL FORCE COEFFICIENT (CONE BASE AREA = R
θv = CONE SEMI-VERTEX ANGLE (DEGREES)
α = ANGLE OF ATTACK (RADIANS)
γ = 1.405
θv:
40°
30°
25°
20°
15°
12.5°
10°
7.5°
5°
INITIAL NORMAL FORCE COEFFICIENT SLOPE/COS²θv
2.1
2.0
1.9
1.8
1.7
1
2
3
4
5
6
7
8
9
10
MACH NUMBER

$\left(\frac{dC_N}{d\alpha}\right)_{\alpha=0} / \cos^2\theta_v$ vs M_1

$/\cos^2\theta_v$ vs MACH NUMBER

BASE AREA = REFERENCE AREA)

ES)

$\left[(dC_N/d\alpha)_{\alpha=0} / \cos^2\theta_v\right]_{M=\infty} = 2.037$

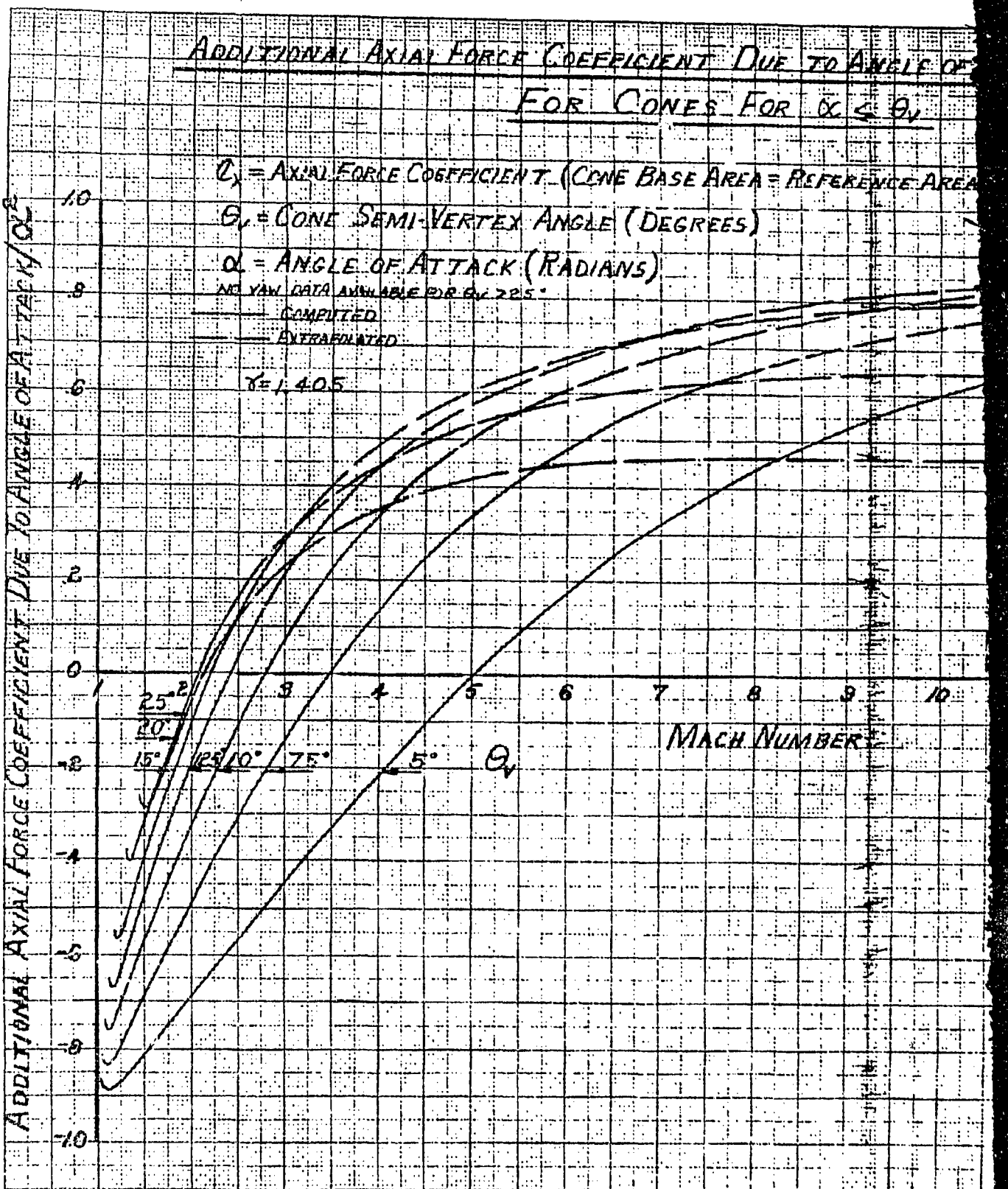
NOTE: $(dC_N/d\alpha)_{\alpha=0}/\cos^2\theta_v = 2.0$
FOR NEWTONIAN FLOW

9 10 11 12 13 14 15 16 17 18 19

BER

FIGURE 3.12 FID 9-22-55

B


ADDITIONAL AXIAL FORCE COEFFICIENT DUE TO ANGLE OF
FOR CONES FOR α ≤ θv
Cx = AXIAL FORCE COEFFICIENT (CONE BASE AREA = REFERENCE AREA
θv = CONE SEMI-VERTEX ANGLE (DEGREES)
α = ANGLE OF ATTACK (RADIANS)
NO YAW DATA AVAILABLE FOR θv > 25°
COMPUTED
EXTRAPOLATED
γ = 1.405
ADDITIONAL AXIAL FORCE COEFFICIENT DUE TO ANGLE OF ATTACK Cx/α²
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.0
1
2
3
4
5
6
7
8
9
10
MACH NUMBER
25°
20°
15°
12.5°
10°
7.5°
5°
θv

$\Delta C_x / \alpha^2$ vs M_1

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ZERO ANGLE OF ATTACK $\Delta C_x / \alpha^2$ VS MACH NUMBER

$\Delta \alpha < \theta_v$

NEWTONIAN FLOW:

$\Delta C_x / S \sin^2 \alpha = 1.35 \sin^2 \alpha$

(REFERENCE AREA)

θ_v = 5.0°, 7.5°, 10.0°, 12.5°, 15°, 20°, 25°

9 10 11 12 13 14 15 16 17 18 19

MACH NUMBER

FIGURE 3.13

B

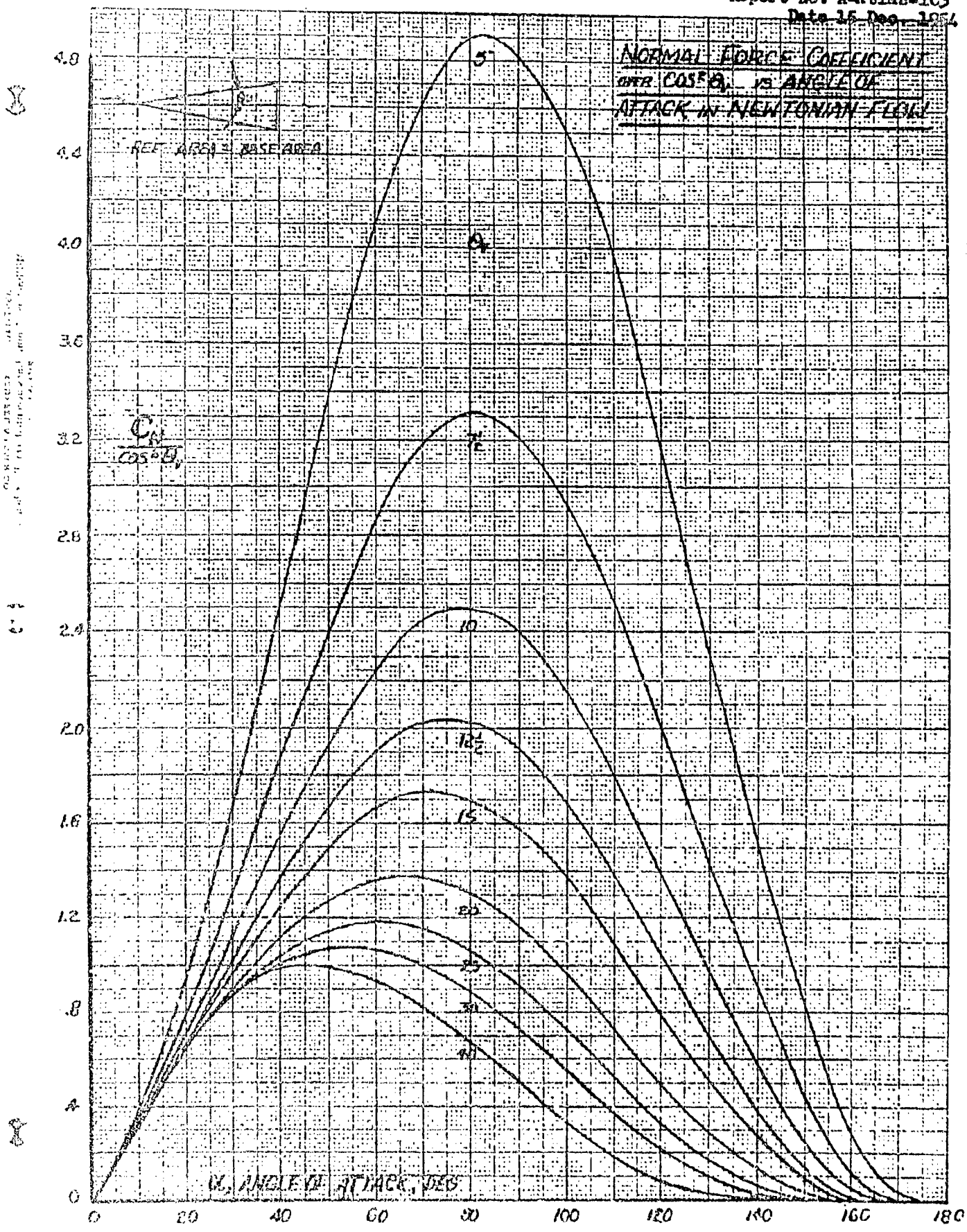

NORMAL FORCE COEFFICIENT WITH $\cos^2\theta_v$ AS ANGLE OF ATTACK IN NEWTONIAN FLOW
REF AREA = BASE AREA
$\frac{C_N}{\cos^2\theta_v}$
θ_v, ANGLE OF ATTACK, DEG
5°
7½
10
12½
15
20
25
30
40
4.8
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
.8
.4
0
0
20
40
60
80
100
120
140
160
180

FIGURE 3.14

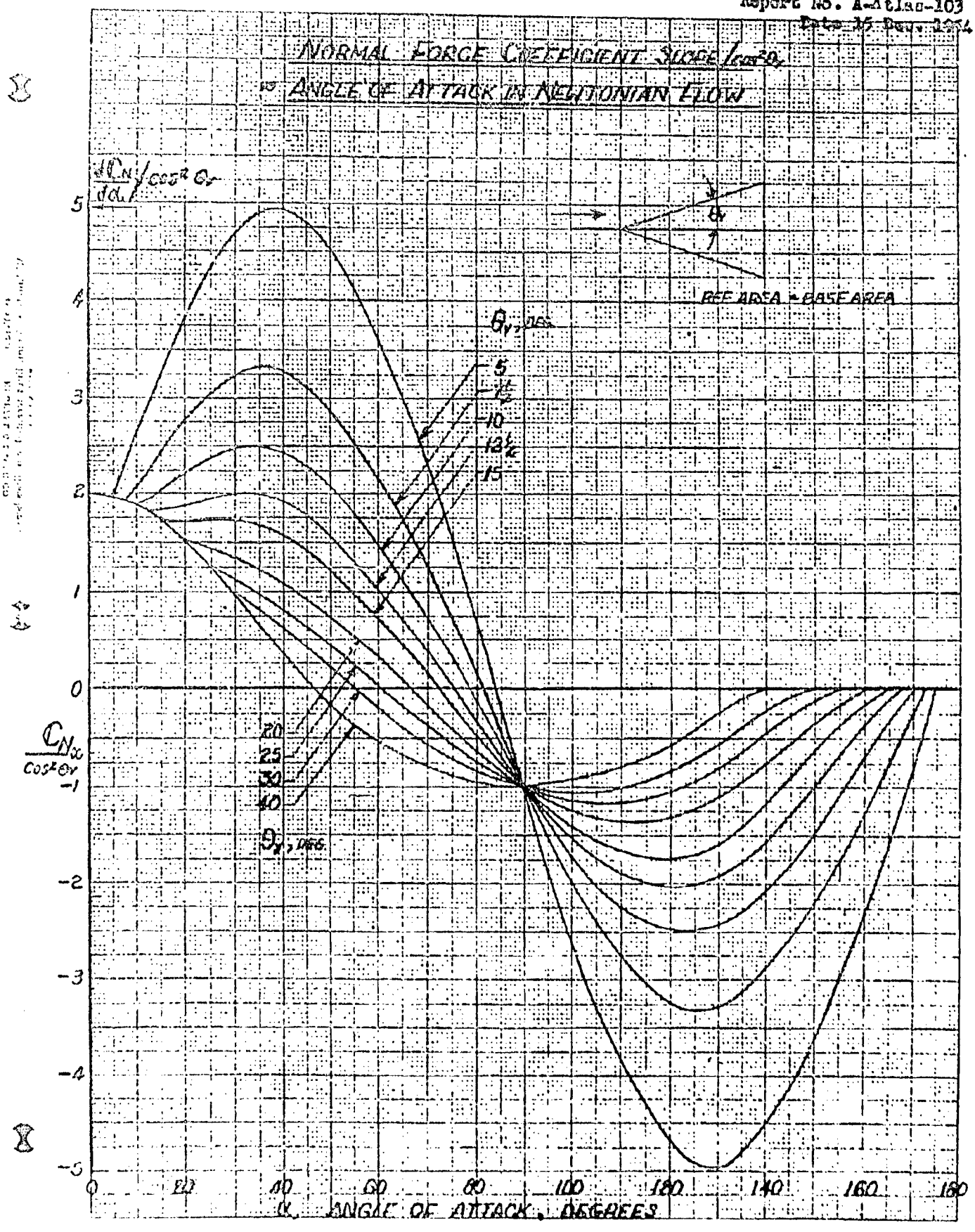

NORMAL FORCE COEFFICIENT SLOPE / cos²θv
vs ANGLE OF ATTACK IN NEWTONIAN FLOW
d(CN/cos²θv)/dα
θv, DEG
5
7½
10
12½
15
20
25
30
40
θv
REF AREA = BASE AREA
CNα/cos²θv
α, ANGLE OF ATTACK, DEGREES
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
20
40
60
80
100
120
140
160
180

FIGURE 3.15

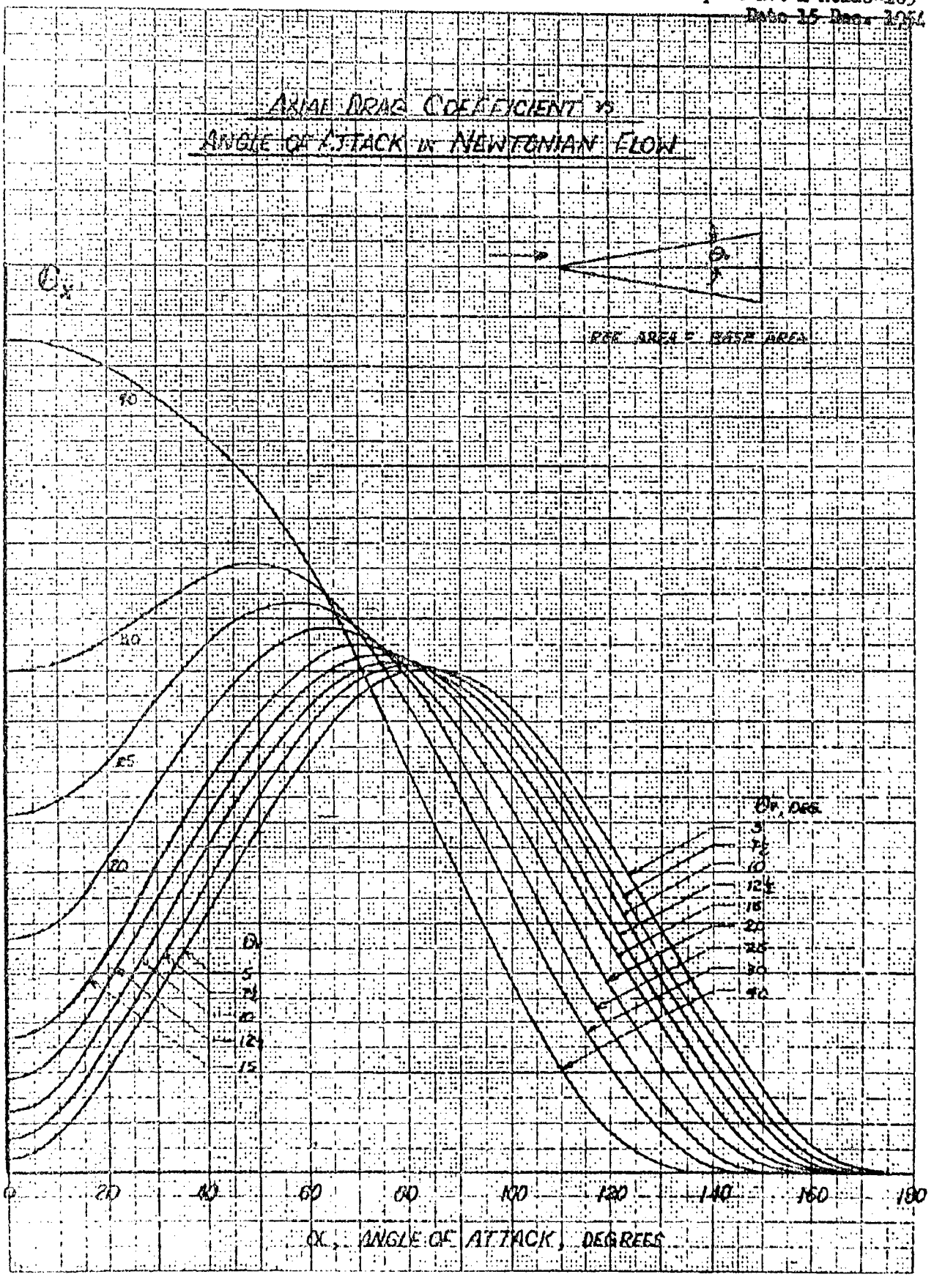
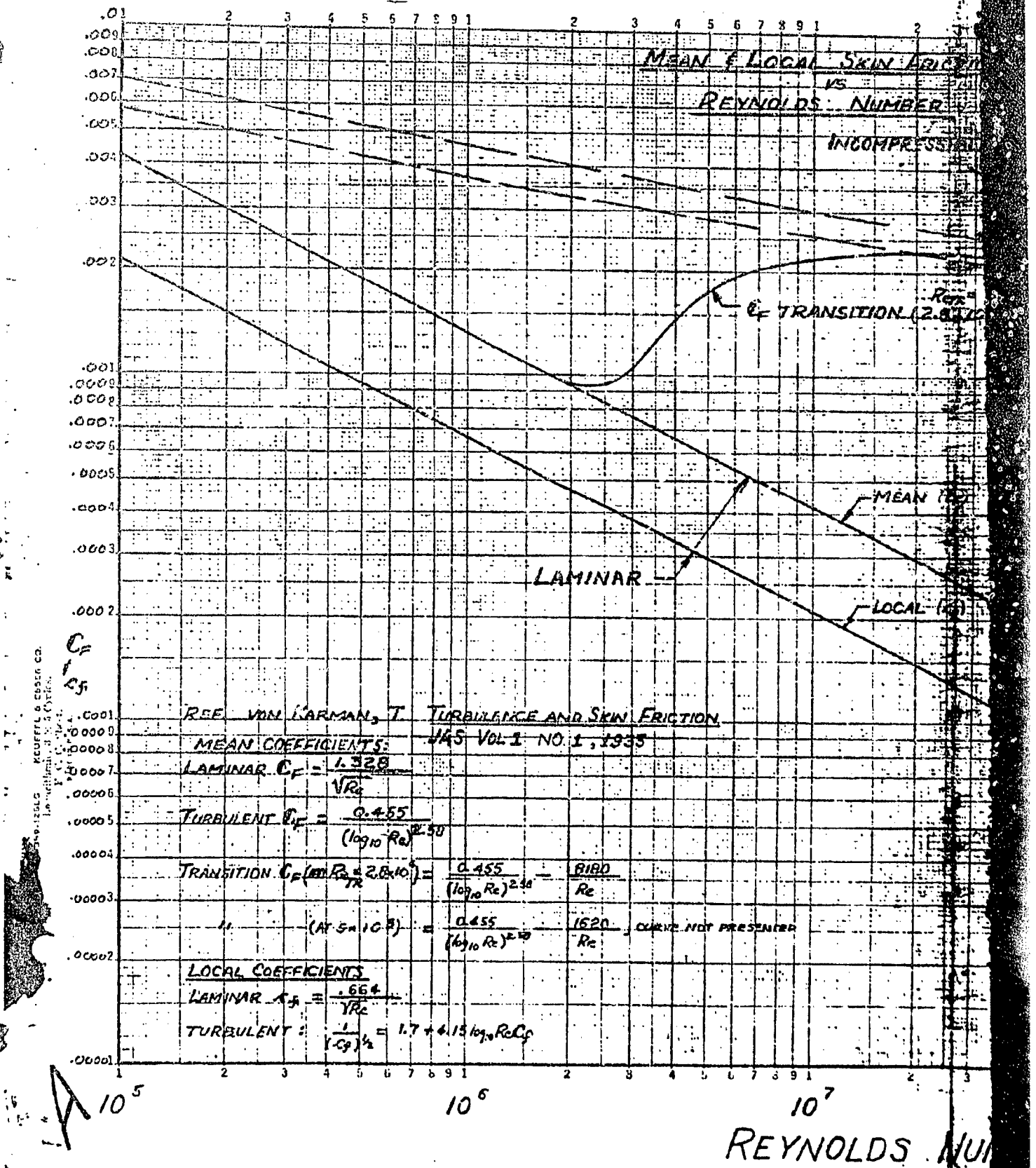

AXIAL DRAG COEFFICIENT VS
ANGLE OF ATTACK IN NEWTONIAN FLOW
C_x
PER AREA = BASE AREA
α
C_x
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
90
80
85
70
60
D_f, DEG
5
7½
10
12½
15
20
25
30
40
0
20
40
60
80
100
120
140
160
180
α, ANGLE OF ATTACK, DEGREES

FIGURE 3.16



REF VON KARMAN, T. TURBULENCE AND SKIN FRICTION
JAS VOL 1 NO 1, 1935

MEAN COEFFICIENTS:

LAMINAR $C_F = \frac{1.328}{\sqrt{R_x}}$

TURBULENT $C_F = \frac{0.455}{(\log_{10} R_x)^{2.58}}$

TRANSITION C_F (at $R_{x_{TR}} = 2.8 \times 10^5$) $= \frac{0.455}{(\log_{10} R_x)^{2.58}} - \frac{8180}{R_x}$

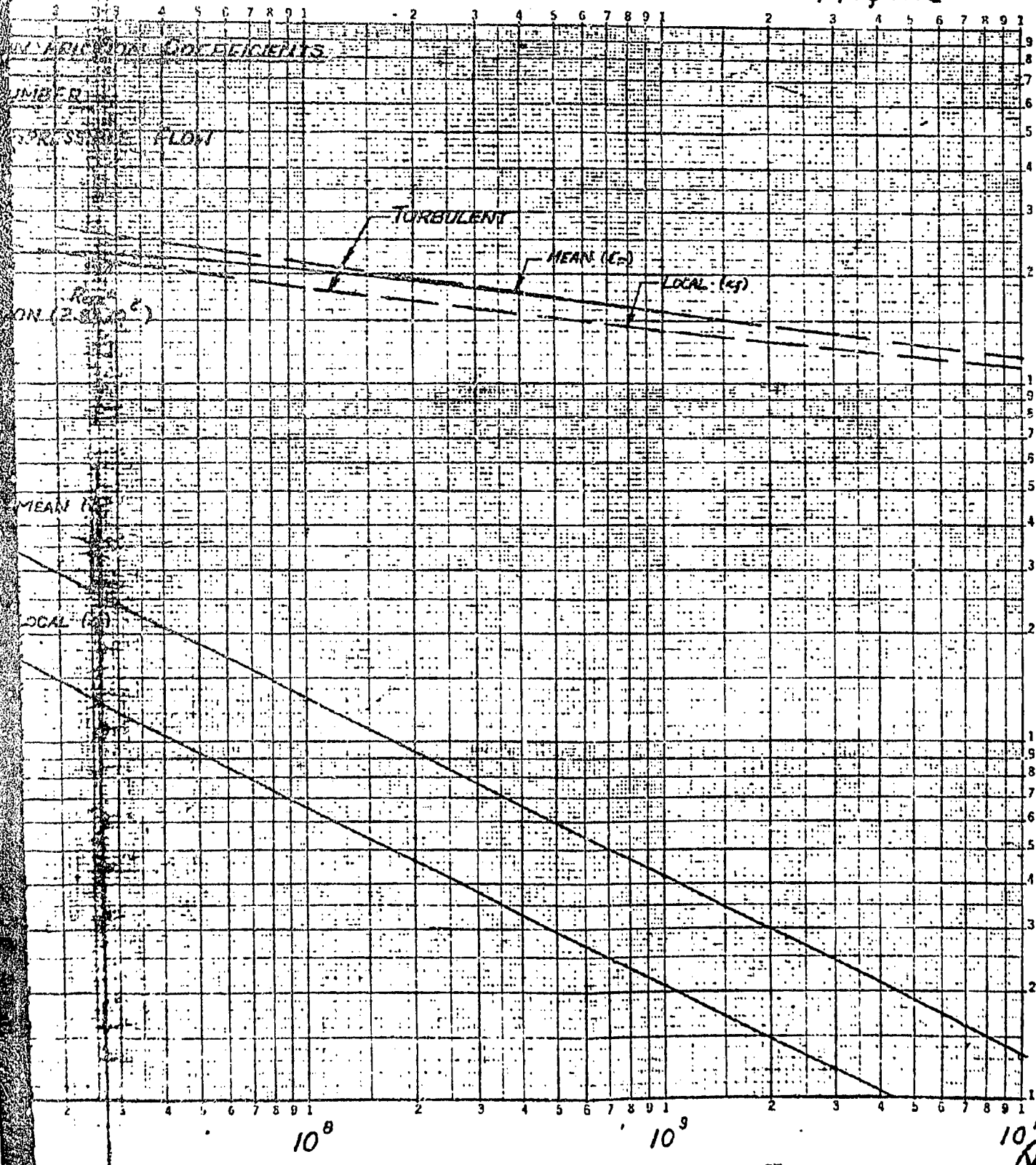
" (at 5×10^5) $= \frac{0.455}{(\log_{10} R_x)^{2.58}} - \frac{1620}{R_x}$ CURVE NOT PRESENTED

LOCAL COEFFICIENTS

LAMINAR $C_{f_x} = \frac{.664}{\sqrt{R_x}}$

TURBULENT : $\frac{1}{(C_f)^{1/2}} = 1.7 + 4.15 \log_{10} R_x C_f$

C_F & C_f vs R_e



...S NUMBER, R_e

FIGURE 4.1

B

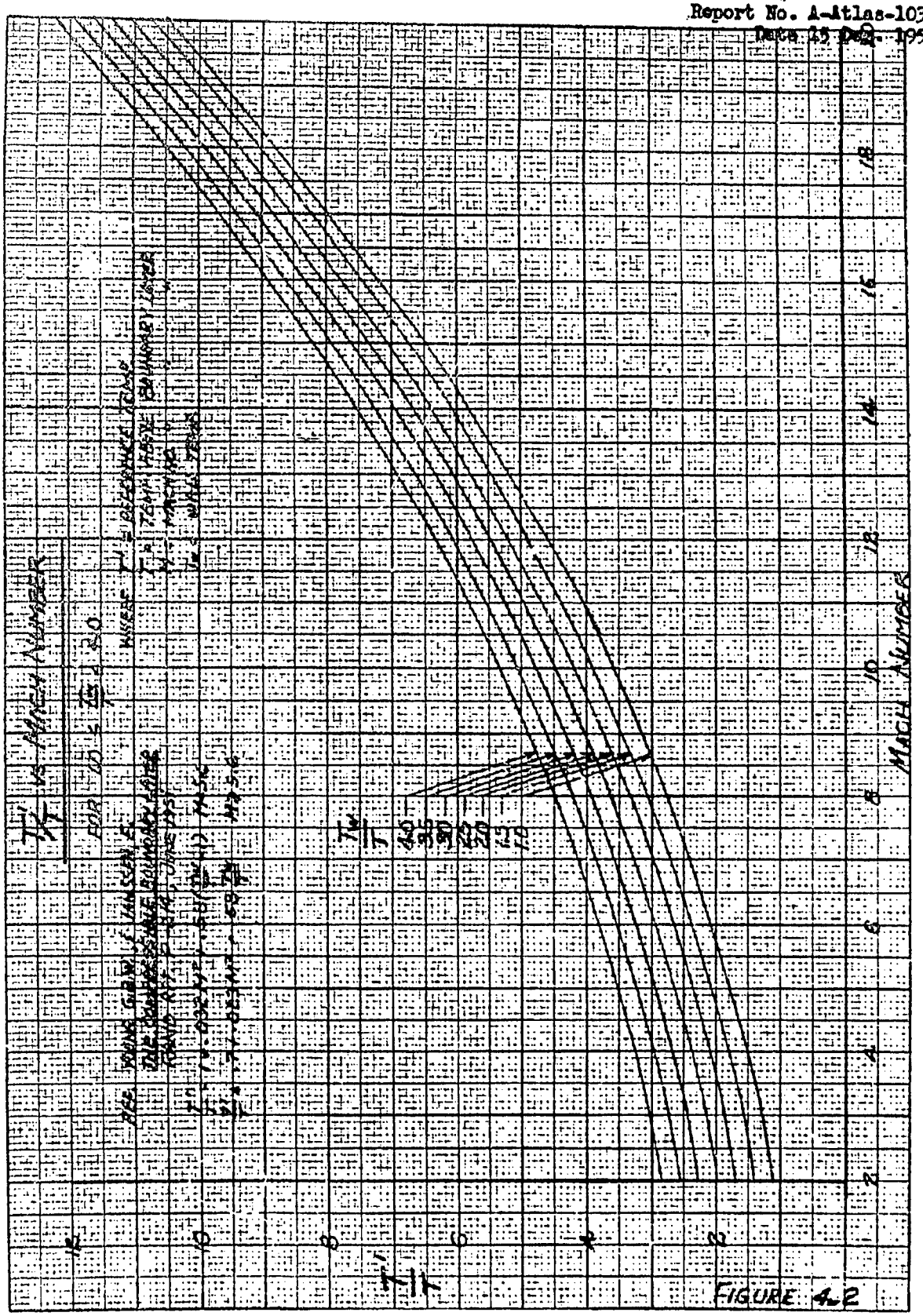
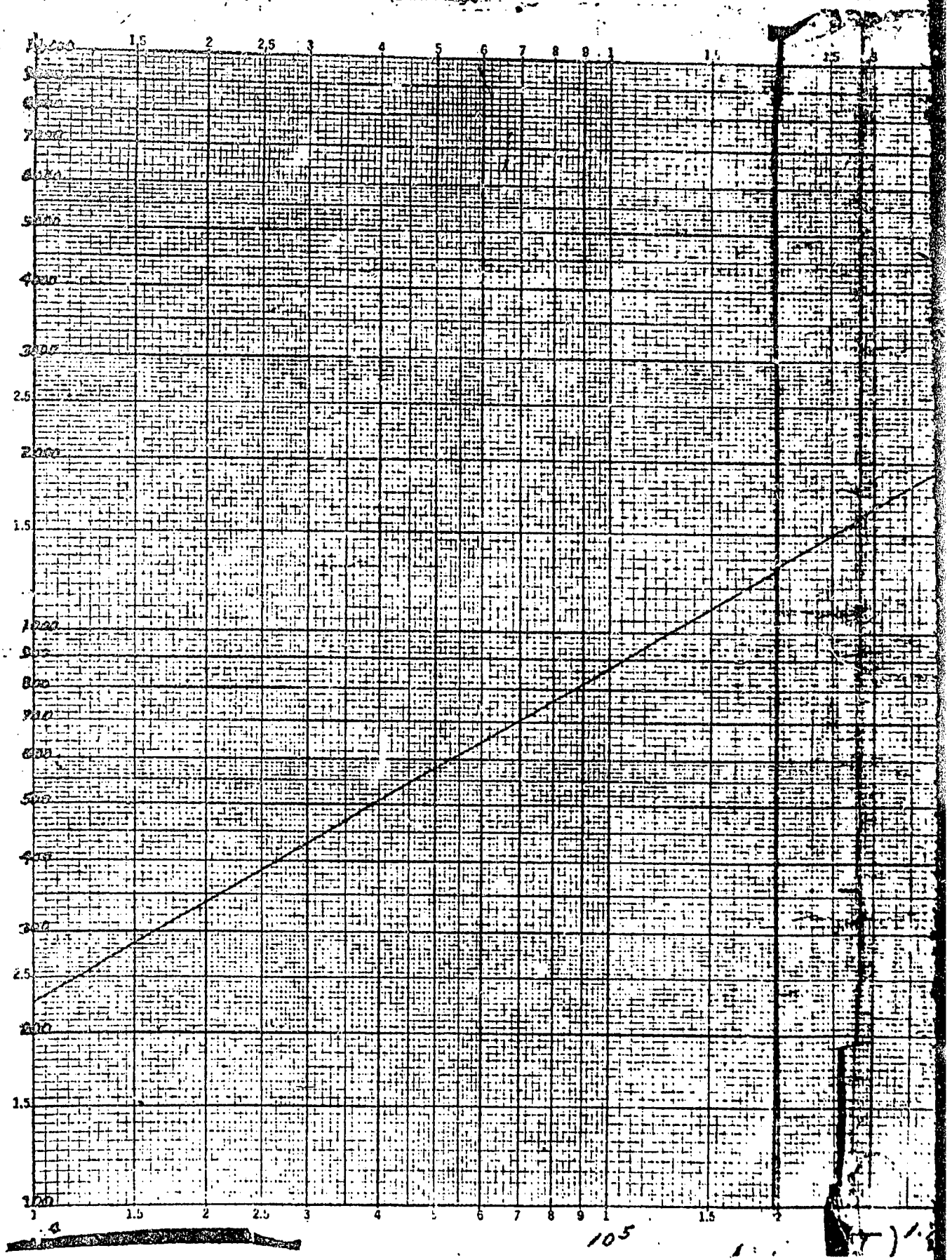
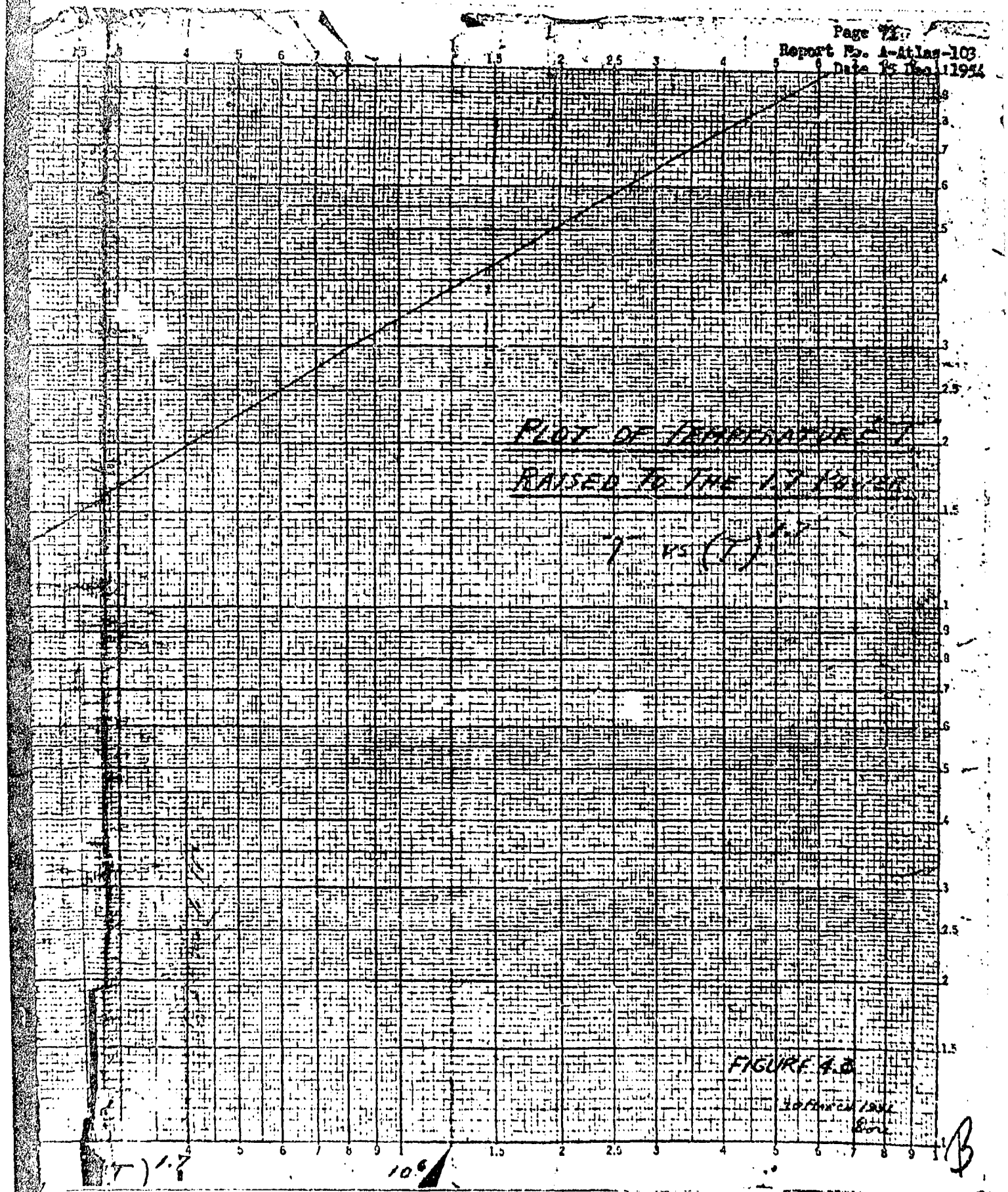


FIGURE 4.2


TEMPERATURE, T
105


PLOT OF TEMPERATURE T
RAISED TO THE 1.7 POWER
T vs (T)^1.7
FIGURE 4.8
(T)^1.7
10^6

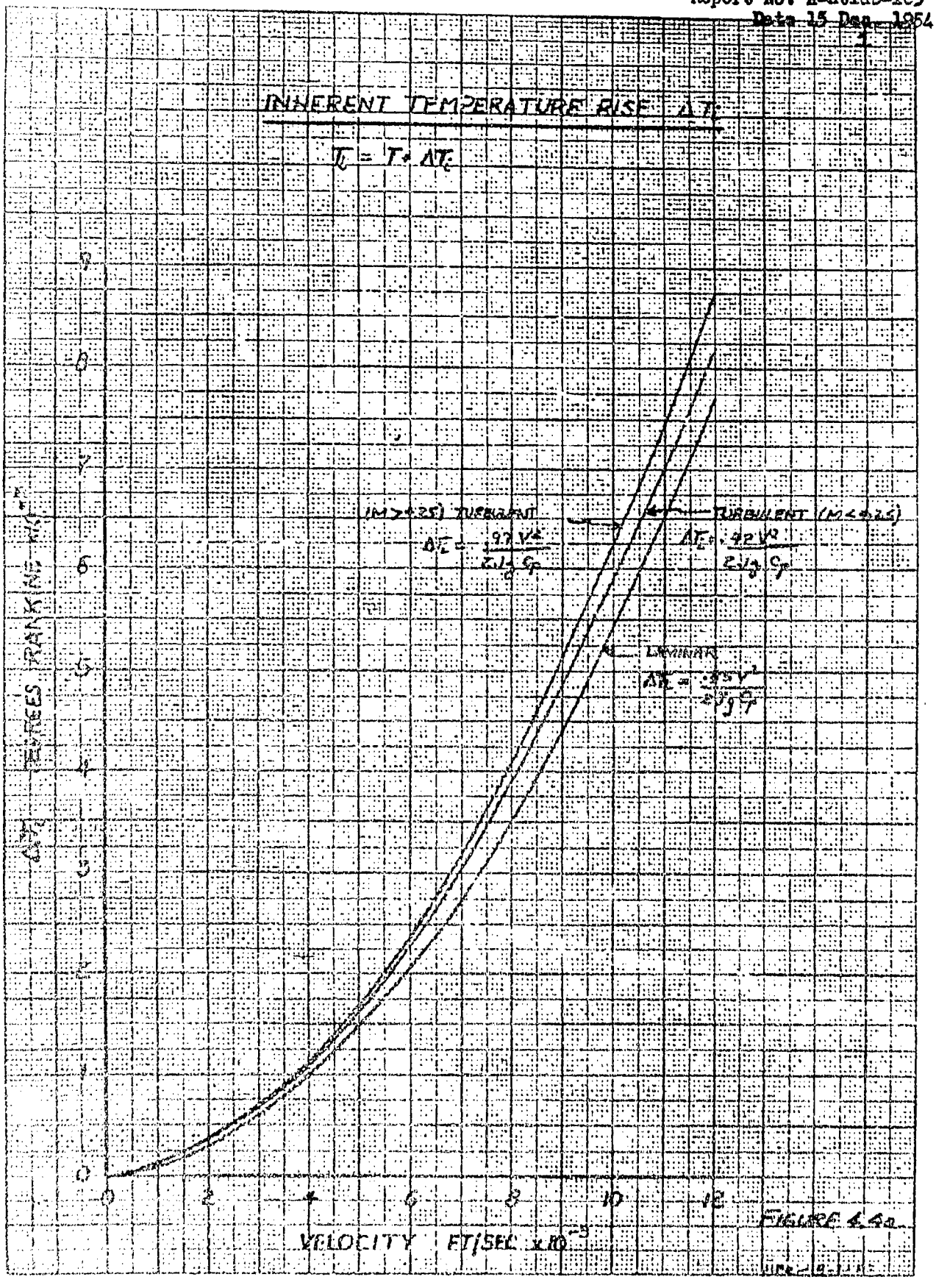

INHERENT TEMPERATURE RISE ΔT_i
$T_i = T + \Delta T_i$
DEGREES RANKINE - °R
ΔT_i
(M > 4.25) TURBULENT
$\Delta T_i = \frac{.97 V^2}{2 J_d C_p}$
TURBULENT (M < 4.25)
$\Delta T_i = \frac{.92 V^2}{2 J_g C_p}$
LAMINAR
$\Delta T_i = \frac{.85 V^2}{2 J_g C_p}$
0
2
4
6
8
10
12
VELOCITY FT/SEC x 10^{-3}

FIGURE 4.4a

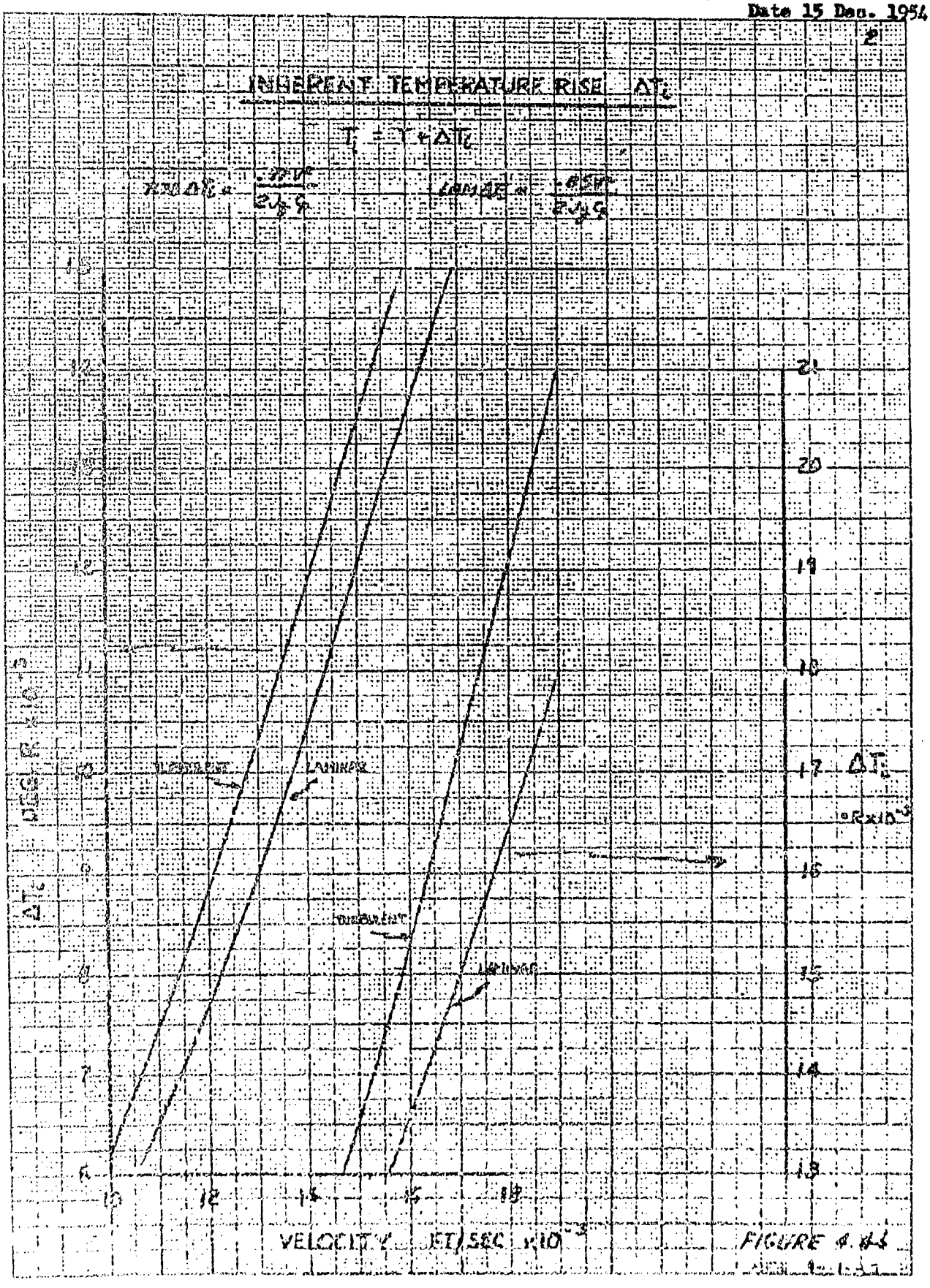

INHERENT TEMPERATURE RISE ΔT_i
T_i = T + ΔT_i
TURBULENT
LAMINAR
TURBULENT
LAMINAR
ΔT_i °F x10^-3
VELOCITY FT/SEC x10^-3

FIGURE 4.44

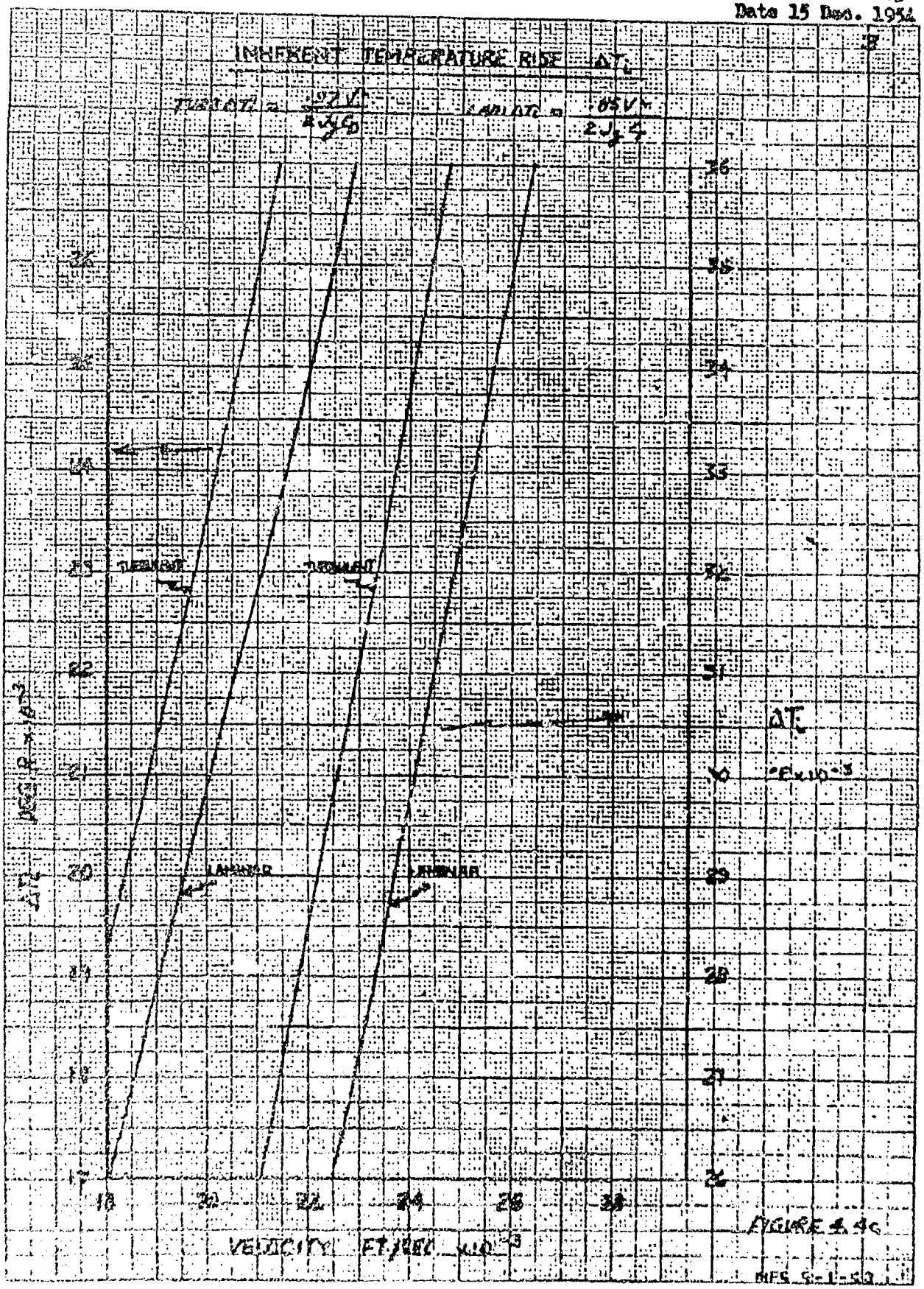
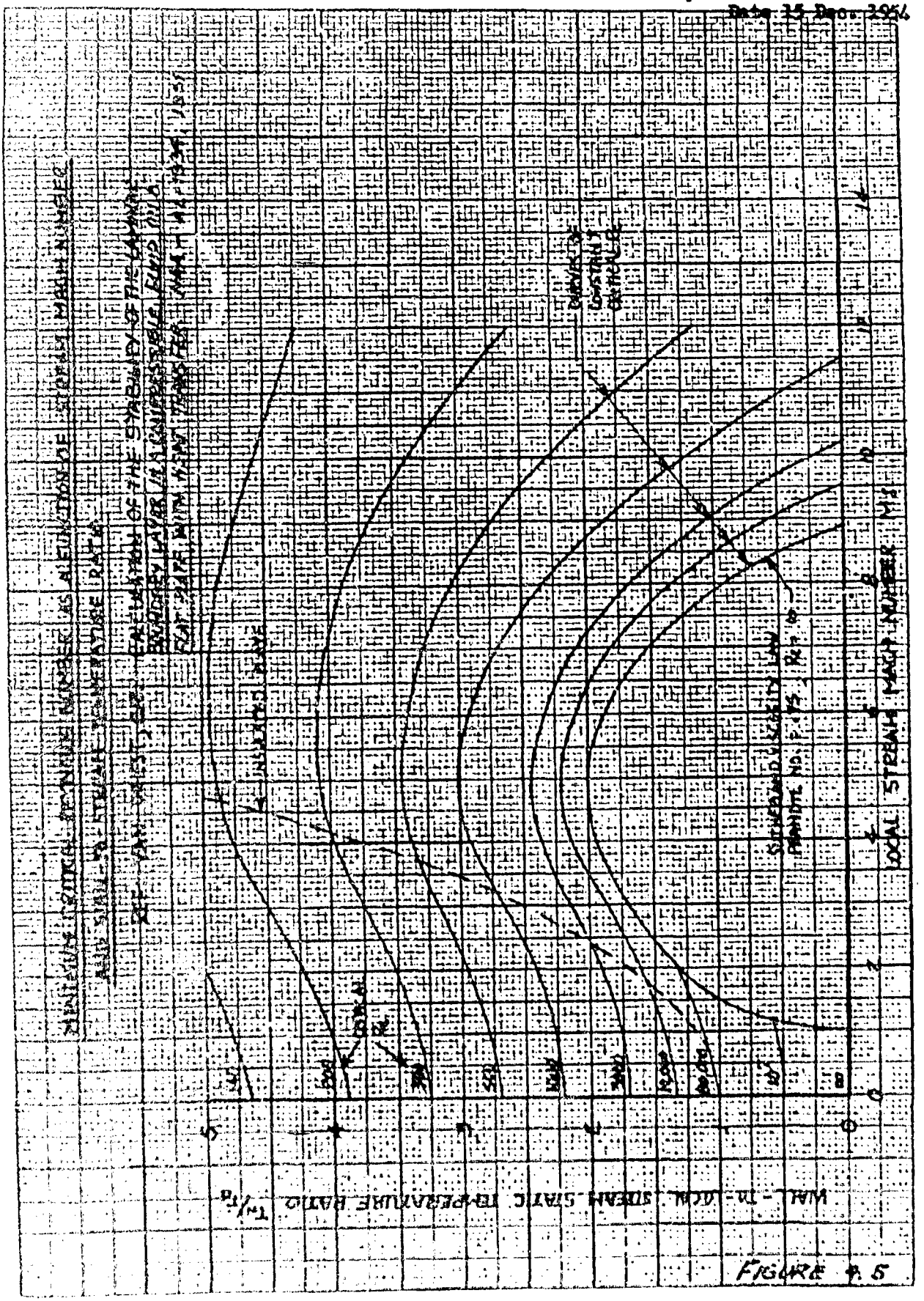


FIGURE 4.4c

MES 5-1-53


LOCAL STREAM MACH NUMBER M1
WAL - TA - ACM. STREAM STATIC TEMPERATURE RATIO Tw/Ta
FIGURE 4.5

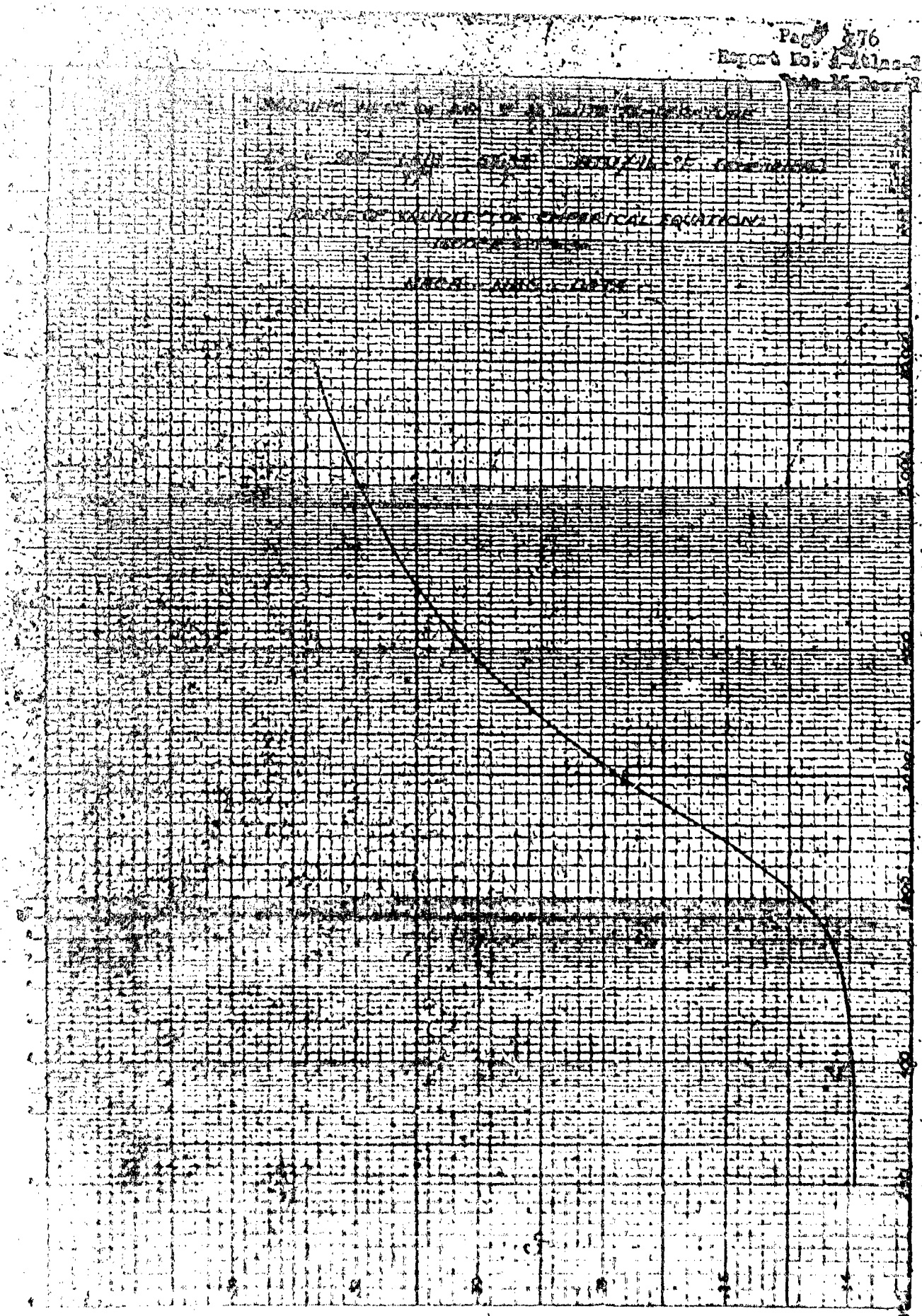
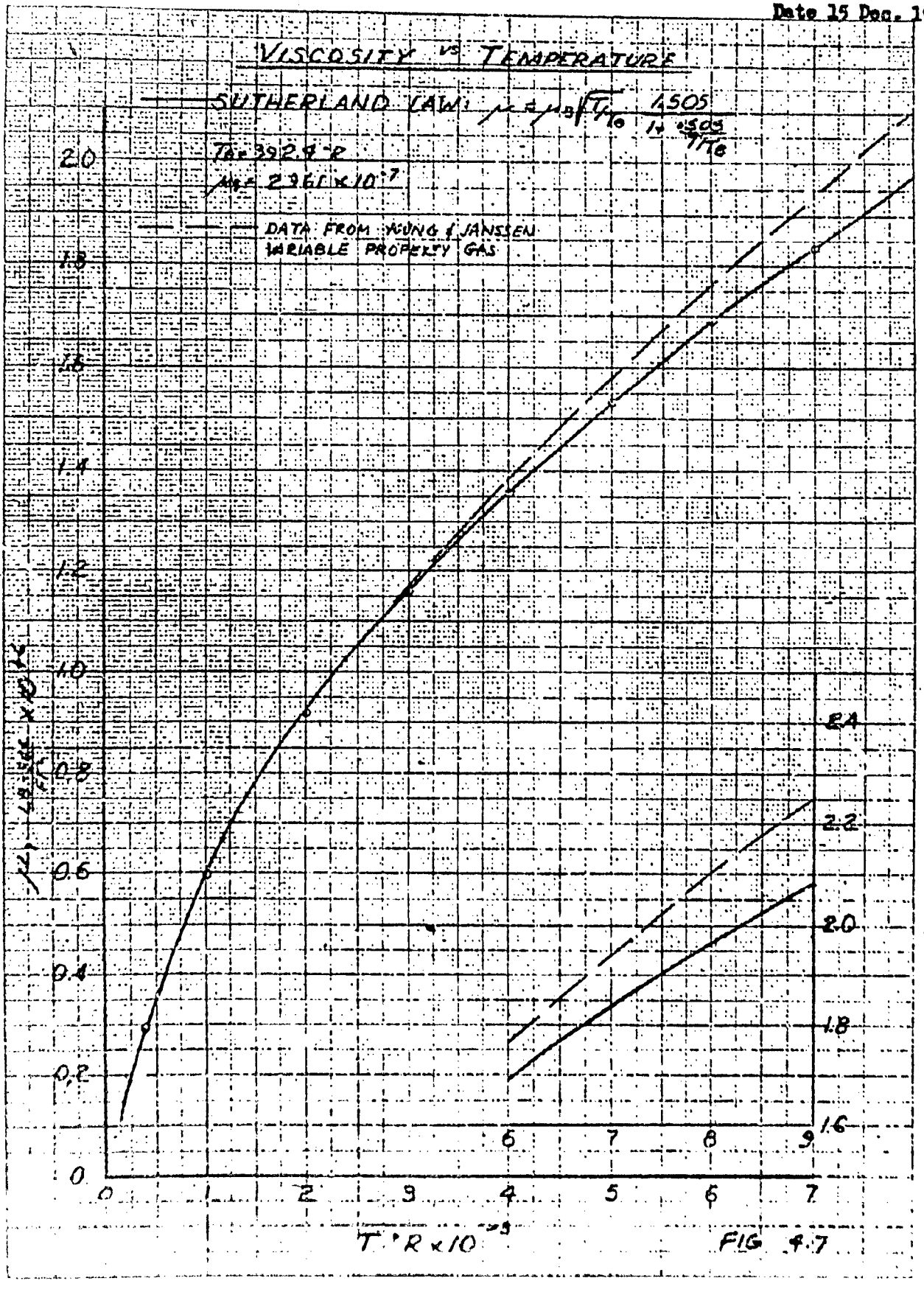


FIGURE 4.6


VISCOSITY VS TEMPERATURE
SUTHERLAND LAW: $\mu = \mu_0 \frac{T_0}{T} \frac{1+\frac{S}{T_0}}{1+\frac{S}{T}}$
$T_0 = 392.4°R$
$\mu_0 = 2.261 \times 10^{-7}$
DATA FROM KUNG & JANSSEN
VARIABLE PROPERTY GAS
$\mu, \frac{lb\ sec}{ft^2} \times 10^7$
$T°R \times 10^{-3}$
FIG 4.7