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FROM A. Saastad

SUBJECT: Conversion from the Basic Measurements of a "Range Only" Tracking System to Rectangular Coordinates

DATE: 19 October 1959

INTRODUCTION

The method used here is that of obtaining the rectangular coordinates of three trackers with respect to a coordinate system whose x y plane is tangent to an ellipsoidal earth at the location of one of the trackers. Attention is called to the fact that the calculations for this are executed prior to any real time computations. After this is accomplished it is analytically simple to solve three distance equations in three unknowns to obtain the rectangular coordinates of the missile. Complete procedures are given in the text.

Given the geodetic latitudes and the longitudes of the trackers T_1 (L_1, λ_1), T_2 (L_2, λ_2), and T_3 (L_3, λ_3).

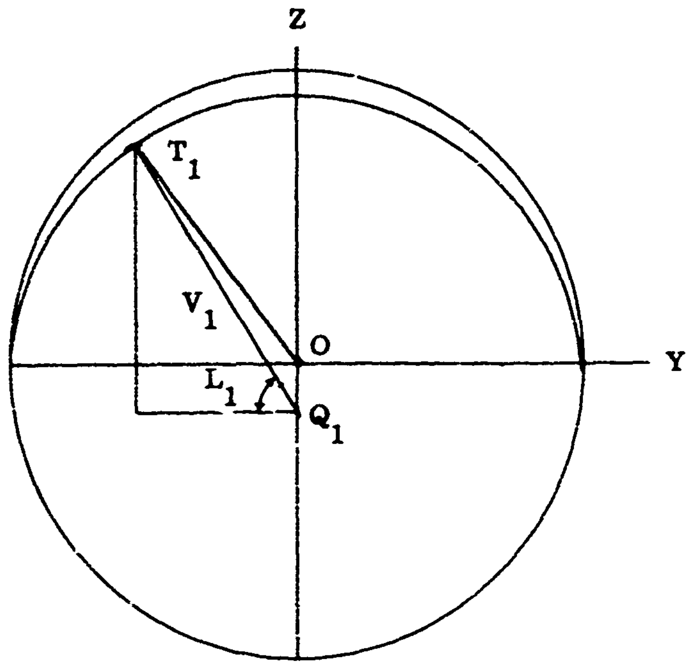

Figure 1

Referenced to an orthogonal right-handed Cartesian coordinate system with its origin at the center of the earth, its positive Y axis passing through the meridian whose longitude is $\lambda_1 + 180°$, and its positive Z axis passing through the north pole, the trackers will have the coordinates:

$$\left.\begin{aligned} X_j &= V_j \cos L_j \sin (X_1 - X_j) \\ Y_j &= V_j \cos L_j \cos (X_1 - X_j) \\ Z_j &= V_j (1 - e^2) \sin L_j , \end{aligned}\right\} \qquad (1)$$

where

$$j = 1,\ 2,\ \text{and } 3,$$

$$V_j = a\,(1 - e^2 \sin^2 L_j)^{-\frac{1}{2}},$$

a = semi major axis of the meridional ellipse,

$$e^2 = \frac{a^2 - b^2}{a^2}$$

$$1 - e^2 = \frac{b^2}{a^2},$$

b = semi minor axis of the meridional ellipse,

$$L_j = \tan^{-1} \frac{Z_j}{(1-e^2)\sqrt{X_j^2 + Y_j^2}}$$

V_j = distance from T_j along the radius of curvature extended to its point of intersection with the minor axis of the meridional ellipse.

The chordal distances $T_1 T_2$, $T_1 T_3$, and $T_2 T_3$ can be obtained using Euclidian distance formulas.

The distance

$$|0Q_1| = V_1 \sin L_1 - Z_1 .$$

Translating from 0 to Q_1 yields tracker coordinates

$$(X_{j_{Q_1}},\ Y_{j_{Q_1}},\ Z_{j_{Q_1}}),$$

where

$$X_{j_{Q_1}} = X_j,\ Y_{j_{Q_1}} = Y_j,$$

$$Z_{j_{Q_1}} = (V_1 \sin L_1 - V_j \sin L_j - Z_1 + Z_j) + Z_{j_{Q_j}} .$$

Next rotate through an angle of $90° - L_1$ about the $X_{j_{Q_1}}$ axis and then translate from Q_1 and T_1. The coordinates with respect to a coordinate system with its x y plane tangent to the earth at T_1, its positive z axis vertically upward, its positive y axis north, and its positive x axis east will then be given by:

$$x_1 = o, \quad y_1 = o, \quad z_1 = o. \tag{2}$$

$$\left.\begin{aligned} x_2 &= X_{2_{Q_1}}, \\ y_2 &= + Y_{2_{Q_1}} \sin L_1 + Z_{2_{Q_1}} \cos L_1, \\ z_2 &= - Y_{2_{Q_1}} \cos L_1 + Z_{2_{Q_1}} \sin L_1 \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} x_3 &= X_{3_{Q_1}}, \\ y_3 &= + Y_{3_{Q_1}} \sin L_1 + Z_{3_{Q_1}} \sin L_1 \\ z_3 &= - Y_{3_{Q_1}} \cos L_1 + Z_{3_{Q_1}} \sin L_1. \end{aligned}\right\} \tag{4}$$

The preceding work is all precomputational; i.e., none of it is done in real time.

To obtain the coordinates of the missile M write the distance equations:

$$x^2 + y^2 + z^2 = r_1^2, \tag{5}$$

$$x^2 + y^2 + z^2 - 2x_2 x - 2y_2 y - 2z_2 z + x_2^2 + y_2^2 + z_2^2 = r_2^2, \tag{6}$$

and

$$x^2 + y^2 + z^2 - 2x_3 x - 2y_3 y - 2z_3 z + x_3^2 + y_3^2 + z_3^2 = r_3^2. \tag{7}$$

Subtracting (6) from (5) yields

$$2x_2 x + 2y_2 y + 2z_2 z = r_1^2 - r_2^2 + x_2^2 + y_2^2 + z_2^2 \tag{8}$$

and subtracting (7) from (5) yields

$$2x_3 x + 2y_3 y + 2z_3 z = r_1^2 - r_3^2 + x_3^2 + y_3^2 + z_3^2 . \tag{9}$$

Solving (8) and (9) for x and y gives

$$x = \frac{f_2 y_3 - f_3 y_2 + 2(y_2 z_3 - y_3 z_2) z}{2(x_2 y_3 - x_3 y_2)} \tag{10}$$

$$y = \frac{f_3 x_2 - f_2 x_3 + 2(x_3 z_2 - x_2 z_3) z}{2(x_2 y_3 - x_3 y_2)} ; \tag{11}$$

where

$$f_2 = r_1^2 - r_2^2 + x_2^2 + y_2^2 + z_2^2 . \tag{12}$$

and

$$f_3 = r_1^2 - r_3^2 + x_3^2 + y_3^2 + z_3^2 . \tag{13}$$

Substituting the right members of (10) and (11) for x and y, respectively, in (5) and collecting like terms yields:

$$\left. \begin{aligned} &4\,[(x_2 y_3 - x_3 y_2)^2 + (y_2 z_3 - y_3 z_2)^2 + (x_3 z_2 - x_2 z_3)^2]\, z^2 \\ &+4\,[(f_2 y_3 - f_3 y_2)(y_2 z_3 - y_3 z_2) + (f_3 x_2 - f_2 x_3)(x_3 z_2 - x_2 z_3)]\, z \\ &+ (f_2 y_3 - f_3 y_2)^2 + (f_3 x_2 - f_2 x_3)^2 - 4r_1^2 (x_2 y_3 - x_3 y_2)^2 = 0 \quad . \end{aligned} \right\} \tag{14}$$

Solving this equation for z and rejecting the root bearing the negative sign yields:

$$z = \frac{-b + \sqrt{b^2 - 4ac}}{2a} , \tag{15}$$

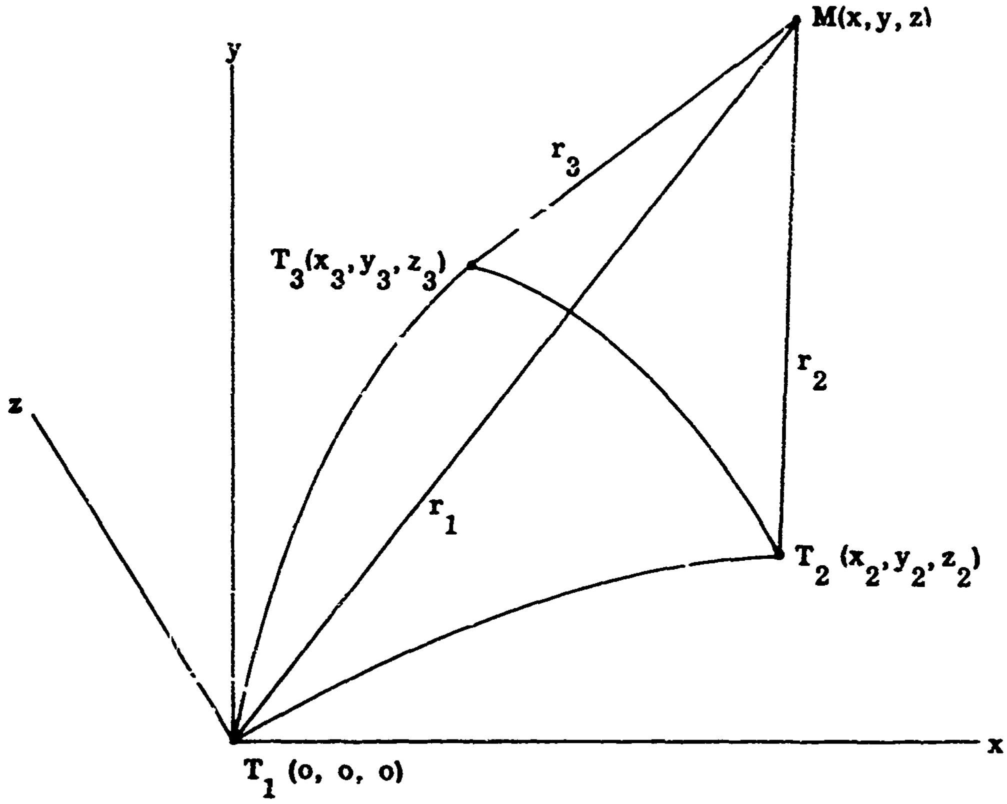

Figure 2

where, in (14)

a = the coefficient of z^2,

b = the coefficient of z ,

and

c = the sum of all other terms in the left member.

Substituting the right member of (15) in (10) and (11) yields:

$$x = \frac{a(f_2 y_3 - f_3 y_2) + (y_2 z_3 - y_3 z_2)(-b + \sqrt{b^2 - 4ac})}{a(x_2 y_3 - x_3 y_2)}, \tag{16}$$

$$y = \frac{a(f_3 x_2 - f_2 x_3) + (x_3 z_2 - x_2 z_3)(-b + \sqrt{b^2 - 4ac})}{a(x_2 y_3 - x_3 y_2)} \tag{17}$$

The x, y, and z coordinates of the missile can now be computed from (16), (17) and (15), respectively, in real time. It should be noted that in (15) the a is entirely precomputed, while the f_2 and the f_3 in the b and c are computed in real time from (12) and (13), although parts of these expressions are precomputed. Specifically any expression involving only one or more of the parameters x_2, y_2, z_2, x_3, y_3, z_3 is precomputed.

To obtain the Cartesian coordinates of the missile with respect to another point on the earth's surface, use transformations (42) and (44) from A. Saastad and C. F. Kettner, Screening and Processing of Mark I Azusa Data, Convair Astronautics Report AZN-26-055, 17 November 1958.

Reference: Bomford, B. C., Geodesy, Oxford U. Press. 1952.