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The Propagation of Long Electric Waves in Magnetized Plasma and Their Passage Through Plasma Layers

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TRANSLATION OF

THE PROPAGATION OF LONG ELECTRIC WAVES IN MAGNETIZED PLASMA AND THEIR PASSAGE THROUGH PLASMA LAYERS

(Über die Ausbreitung langer elektrischer Wellen in magnetisierten Plasmen und ihren Durchgang durch Plasmaschichten)

by

Winfried Otto Schumann

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THE PROPAGATION OF LONG ELECTRIC WAVES IN MAGNETIZED PLASMA AND THEIR PASSAGE THROUGH PLASMA LAYERS

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Summary

The propagation of electromagnetic waves in damped plasmas at low frequencies, i.e., $\omega << \nu$ (ν= number of collisions) is discussed. The plasma acts as a conductor when there is no magnetic field and propagation occurs transverse to the magnetic field. The plasma also behaves as a conductor for $\Omega << \nu$, with propagation along the magnetic field. On the other hand, for $\Omega >> \nu$, i.e., $\omega << \Omega$, both of the familiar wave types appear: the ordinary one has a negative dielectric constant, while the extraordinary one has a positive dielectric constant and only slight damping. Both dielectric constants are very high at low frequencies. In this situation, electron motion transverse to the magnetic field resembles that in static fields in a vacuum. If $\Omega_J >> \nu_J$ also holds for the ions, electrons and ions oscillate synchronically with the same amplitude in the same direction, so that the plasma (like the vacuum) has an effective dielectric constant of 1 for frequencies $\omega << \Omega_J$. Waves with $\omega << \Omega_e$ (but not with $\omega << \Omega_J$) along a magnetic field are almost completely reflected at low frequencies and inconstant transitions from air to plasma, owing to the very high dielectric constant of the latter. In the case of a gradual transition, they can penetrate almost without reflection at low damping, if the wavelength is equal to or less than the layer thickness or the increase of the layer's dielectric coefficient to the maximum.

The propagation of electric waves in bounded plasmas was discussed in [1] and [2]. For the sake of simplicity, damping was disregarded in these papers. However, since the shock absorption of the electrons exerts a very significant influence, especially in the case of long waves, we shall discuss it in what follows, with special emphasis on the case of an external homogeneous magnetic field. If the magnetic field B runs in the z direction, the dielectric deflections (see, e.g., [3, 4]) become

$$\left.\begin{aligned} D_x &= \varepsilon_{xx} E_x + \varepsilon_{xy} E_y \qquad & \varepsilon_{xx} &= \varepsilon_{yy} \\ D_y &= \varepsilon_{yx} E_x + \varepsilon_{yy} E_y \qquad & \varepsilon_{xy} &= -\varepsilon_{yx} \\ D_z &= \varepsilon_z E_z, & & \end{aligned}\right\} \tag{1}$$

where

$$\left.\begin{aligned} \varepsilon_{xx} &= \varepsilon_0 + \frac{\varepsilon_0 \omega_0^2}{j\omega} \cdot \frac{\nu + j\omega}{\Omega^2 + (\nu + j\omega)^2} \\ \varepsilon_{xy} &= \frac{\varepsilon_0 \omega_0^2}{j\omega} \cdot \frac{\Omega}{\Omega^2 + (\nu + j\omega)^2} \\ \varepsilon_z &= \varepsilon_0 + \frac{\varepsilon_0 \omega_0^2}{j\omega} \frac{1}{\nu + j\omega}. \end{aligned}\right\} \tag{2}$$

In these equations, ϵ_0 is the dielectric constant of the empty space $\frac{1}{4\pi \times 9 \times 10^9} \frac{F}{m}$, and ν is the average number of collisions per second between electrons and ambient gas molecules; $\Omega = \frac{e}{m} B$ is the rotational frequency of the electrons in magnetic field B; $\omega = 2\pi f$ is the impressed frequency, and $\omega_0^2 = Ne^2/\epsilon_0 m$ is the resonance frequency of the plasma, with N as the number of electrons/m^3.

When $\nu \ll \omega$, these formulas become the familiar ones for undamped or weakly damped plasma [4]. If, on the other hand, $\nu \gg \omega$, we have

$$\left.\begin{aligned} \varepsilon_{xx} &= \varepsilon_0 + \frac{\varepsilon_0 \omega_0^2}{j\omega} \cdot \frac{\nu}{\Omega^2 + \nu^2} \\ \varepsilon_{xy} &= \frac{\varepsilon_0 \omega_0^2}{j\omega} \cdot \frac{\Omega}{\Omega^2 + \nu^2} \\ \varepsilon_z &= \varepsilon_0 \left(1 - \frac{\omega_0^2}{\nu^2} + \frac{\omega_0^2}{j\omega\nu}\right). \end{aligned}\right\} \tag{3}$$

For the ionospheric plasma with $\nu \approx 10^6$ sec^{-1} at the lower interface, $\omega \gg \nu$ everywhere above an impressed frequency of ~ 1 MHz. For frequencies of ~ 10 kHz and less, however, $\nu \gg \omega$, at least in the D and E layers. $\nu \approx 10^8$ sec^{-1} for a gas discharge plasma of ~ 5×10^{-3} torr and small currents, so that $\nu \ll \omega$ only at frequencies above 100 MHz, while $\nu \gg \omega$ at 1 MHz.

In most cases, the real portions of ϵ_{xx} and ϵ_z should be disregarded for $\nu \gg \omega$ if Ω is not extremely large. This is related to the large values of ω_0. Hence, ω_0^2/ω^2 must be much greater than ν/ω, and $\Omega^2/\omega_0^2 << \nu/\omega$. The plasma is then characterized only by "conductances;" and $\epsilon_{xx}/\epsilon_{xy} = \nu/\Omega$. In the ionosphere f_0 rises as high as $\sim 10^7$ Hz, while in discharge plasmas it rises to more than 10^9 Hz, corresponding approximately to $N = 10^{10}$ electrons/cm^3.

The effect of the magnetic field also depends on Ω/ν. If, for example, we make B = 0.6 gauss, and $\Omega \approx 10^7$ Hz, such a field is already strong ($\Omega >> \nu$) for the ionospheric plasma and affects the phenomena to a very great degree. A gas discharge plasma would have to have B = 60 gauss in order to produce the same effect (see also [8]).

I. Propagation of Plane Waves Transverse to the Magnetic Field (Perpendicular to the z Direction)

Of the two wave types possible in this case, the one whose electric fields runs in the z direction is not at all affected by the magnetic field. The effective dielectric constant is ϵ_z. The other type, whose electric field is perpendicular to z, is affected by the magnetic field and the effective dielectric constant for this wave is [4]:

$$\varepsilon_q = \varepsilon_{xx}\left(1 + \frac{\varepsilon_{xy}^2}{\varepsilon_{xx}^2}\right). \tag{4}$$

If we substitute the values from equation (3) and disregard ϵ_0 in ϵ_{xx}, we obtain

$$n_q^2 \cdot \varepsilon_0 = \varepsilon_q = -j\frac{\varepsilon_0 \omega_0^2}{\omega \nu} \tag{5}$$

regardless of whether $\Omega \gtrless \nu$, i.e., ϵ_z according to equation (3) if we disregard $\epsilon_0(1 - \frac{\omega_0^2}{\nu^2})$ in it. This is also understandable, since even without shock absorption, $\nu = 0$ for $\omega^2 << \omega_0^2$ and Ω^2 is not significantly larger than ω_0^2, the dielectric constant $\epsilon_q = \epsilon_0[1 - \frac{\omega_0^2}{\omega^2} \times \frac{\omega_0^2 - \omega^2}{\Omega^2 + \omega_0^2 - \omega^2}]$ already becomes negative, so that no propagation at all occurs without damping. If the dielectric constant is negative, propagation with large wavelengths and very large phase velocity occurs with damping at a low conductance κ; at a high conductance, the familiar propagation occurs, as in metals with high conductance, but the waves can be damped sharply at a low conductance, owing to the negative dielectric constant. Hence, the plasma behaves like a conductor having the conductance $\kappa_q = j\omega\epsilon_q = \epsilon_0 \frac{\omega_0^2}{\nu}$, which has the value of ~ 4 mho/m for a discharge plasma (as above), while the conductance in the ionosphere is very much less.

II. Propagation of Plane Waves along the Magnetic Field (in the z Direction)

The effective dielectric constant of the two possible waves is now given (from [4]) by $\epsilon_{l_{1,2}} = \epsilon_{xx} \pm \epsilon'_{xy}$, $\epsilon'_{xy} = -j\epsilon_{xy}$, from which

$$\varepsilon_{l_{1,2}} = \varepsilon_0 \left[1 - \frac{\omega_0^2}{\omega} \frac{1}{(\omega \mp \Omega) - j\nu}\right] \tag{6}$$

follows according to equations (2). According to equations (3), disregarding ϵ_0 in ϵ_{xx} once again, we obtain

$$n_l^2 = \frac{\varepsilon_{l_{1,2}}}{\varepsilon_0} = \frac{\omega_0^2}{\omega} \frac{1}{\mp \Omega + j\nu}. \tag{7}$$

If $\Omega << \nu$, this dielectric constant also changes into ϵ_z. The two waves coincide and the plasma behaves like a dc conductance. If, on the other hand, $\Omega >> \nu$, the plasma acts as a dielectric, having a positive dielectric coefficient for one wave and a negative dielectric coefficient for the other wave; at low frequencies, these dielectric coefficient values can become very large. The negative dielectric coefficient leads us to expect that this type of electric wave can be propagated longitudinally by a plasma beam surrounded by air from a frequency of 0 to a definite upper frequency (see [5] and [2]).

If we compare the above values of $\epsilon_{l_{1,2}}$ with the values of the ideal plasma having $\nu = 0$,

$$n_l^2 = \frac{\varepsilon_{l_{1,2}}}{\varepsilon_0} = 1 - \frac{\omega_0^2}{(\omega \mp \Omega)\omega}. \tag{a}$$

holds there for the index of refraction [4, p. 98]. For $\omega << \Omega$,

$$n_l^2 = \frac{\varepsilon_{l_{1,2}}}{\varepsilon_0} \approx \frac{\omega_0^2}{\omega} \cdot \frac{1}{\mp \Omega}. \tag{b}$$

Here, too, there is a dielectric propagation without damping for the extraordinary wave, in contrast to section I, so that the damping merely modifies the propagation. Thus, in the ideal situation ($\nu = 0$) we obtain the same values for $\omega \ll \Omega$ as in our approximation for $\omega \ll \nu$, if $\Omega \gg \nu$, and the results from [1, 2] can also be used for the plasma with losses under these conditions. The following propagation constants for the plane wave follow from equation (7):

$$\beta_{1,2}^2 = \frac{\omega^2}{c^2} n_l^2 = \frac{\omega_0^2}{c^2} \cdot \frac{\omega}{\mp \Omega + j\nu}. \qquad \text{(c)}$$

From this it follows, as we know, that for $\Omega \gg \nu$ the "ordinary" wave in the plane situation cannot propagate, but is a standing damped wave, while the "extraordinary" wave is weakly damped and travels with the phase velocity $v_p = \frac{c}{n_l} = \frac{c}{\omega_0}\sqrt{\omega\Omega}$, the wavelength $\lambda = 2\pi \frac{c}{\omega_0}\sqrt{\frac{\Omega}{\omega}}$, and the group-velocity $v_g = \frac{1}{2} v_p$. The dc resistances of the two waves are

$$W = \frac{1}{n_l} \cdot W_0, \quad W_{1,2} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{1}{\omega_0} \sqrt{\omega(\mp \Omega + j\nu)}. \qquad \text{(d)}$$

For low frequencies, $W_2 \ll W_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$; hence, the dc resistance W_2 is very small relative to that of air and the electric fields are also very small relative to the magnetic ones. This is the same wave type suggested by Storey [11] for whistler propagation.

III. Electron Motions in Long Waves

The pattern of electron motion supplements the preceding discussion.

If we assume a constant B field in the z direction, the periodic electron velocity in the xy plane perpendicular to it is

$$v = E_0 \frac{e}{m} \frac{1}{(\nu + j\Omega)^2 + \omega^2} [(\nu + j\Omega) \cos \omega t + \omega \sin \omega t]. \qquad \text{(e)}$$

with $E = E_0 \cos \omega t$, in the xy plane as well ([4, p. 93]). If E oscillates in the x direction, the imaginary portion represents an oscillation in the y direction perpendicular to it. If we assume $\omega \ll \nu$, we have

$$v \approx E_0 \frac{e}{m} \frac{1}{\nu + j\Omega} \cdot \cos \omega t \qquad \text{(f)}$$

which also holds at the limit for $\omega \rightarrow 0$ (static condition). If $\Omega \ll \nu$, $|v_y| \ll |v_x|$ and we have $v_x = E_0 \frac{e}{m} \frac{1}{\nu} \cos \omega t$, a velocity in the direction of the field with the "mobility" $b = \frac{e}{m\nu}$. On the other hand, if $\Omega \gg \nu$, i.e., $\Omega \gg \omega$ as well, we have

$$v = -jE_0 \frac{e}{m} \frac{1}{\Omega} \cos \omega t + E_0 \frac{e}{m} \frac{\nu}{\Omega^2} \cos \omega t. \qquad (g)$$

v_x, the velocity in the direction of the field, is very low and proportional to ν. However, v_y (the velocity perpendicular to the field direction) is much larger than v_x and is

$$v_y = -j \frac{E_0}{B} \cos \omega t. \qquad (h)$$

It has the value $\frac{E_0 \cos \omega t}{B}$ perpendicular to the electric and magnetic fields, as it also appears in vacuum in static E and B fields with $\omega = 0$ and is independent of the charge and mass of the particles. The influence of damping has vanished. We also obtain the same result without damping ($\nu = 0$) if $\Omega \gg \omega$. This is because in our case the accelerative forces opposing the frictional forces vanish ($\omega \ll \nu$); since $\Omega \gg \nu$, these frictional forces are completely negligible by comparison with the magnetic forces at the same velocities in both forces. From the equations for motion in the x and y directions,

$$\begin{aligned} m\nu v_x + m \frac{dv_x}{dt} &= eE_x + v_y B_z e, \\ m\nu v_y + m \frac{dv_y}{dt} &= -v_x B_z e \end{aligned} \qquad (i)$$

it follows that v_x becomes much less than v_y; hence, the electric and magnetic forces must nearly compensate themselves in the x direction, from which it then follows that $v_y = -E_x/B$. ϵ_{xx} and ϵ_{xy} with $|\epsilon_{xx}/\epsilon_{xy}| = |v_x/v_y|$ then follow from the values of v_x and v_y. If we assume that the same laws of motion hold for the ions, an ion would have exactly the same velocities v_y, and the identical pulsations of positive and negative charges of equal density would not produce a current in the dielectric constant, i.e., the plasma would be practically non-existent for such a wave and would have the dielectric constant $\epsilon_p/\epsilon_0 = 1$. It would have only a weak damping effect. Unfortunately, this is not the case in the ionosphere, at least for the lower layers: for example, $\Omega_J \approx 300 \text{ sec}^{-1}$ for 1/2 gauss in the case of 0^+ ions, but the number of ion collisions in the D and E layers would be $\sim 3 \times 10^3$ to $\sim 3 \times 10^4$ sec^{-1}, i.e., much greater than Ω_J, so that the ions would oscillate with very much smaller amplitudes practically in the direction of the field. Then they would not be able to compensate the electron motions, and would have practically no effect on the propagation of waves. This would seem to be possible only in the high F layers with a very small ν, which could eventually be penetrated at night, when no D or E layers were present. During the day, the D and E layers would still offer a separate layer to be penetrated which, according to the foregoing, must have a pronounced damping and reflecting effect. Then, e.g., waves produced in the atmosphere with $\omega << \Omega_J$ would be reflected in the daytime, but would be able to penetrate the ionosphere more or less freely at night. This may help account for the fact that the 9 Hz frequency generated by lightning is so very much weaker at night than in the day [12]. This would only hold, however, for these low frequencies $\omega << \Omega_J$.[†)]

IV. Passage of Long Waves from Air into Plasmas

After seeing in section II that (generally speaking) a propagation in the direction of the magnetic field is possible only at low frequencies (later we shall deal with the participation of positive ions discussed in section III), let us examine a case in which we assume (for the sake of simplicity) that the waves impact perpendicularly against the interface after they have passed from the air into the plasma. The two possible wave types involved in the propagation in the direction of the magnetic field (z direction) are given in [4, p. 98]:

$$
\begin{aligned}
H_y &= H_1 e^{-j\beta_1 z} + H_2 e^{-j\beta_2 z}, \\
H_x &= jH_1 e^{-j\beta_1 z} - jH_2 e^{-j\beta_2 z}, \\
E_y &= -jW_1 H_1 e^{-j\beta_1 z} + jW_2 H_2 e^{-j\beta_2 z}, \\
E_x &= W_1 H_1 e^{-j\beta_1 z} + W_2 H_2 e^{-j\beta_2 z} \\
W_1 &= \sqrt{\frac{\mu_0}{\varepsilon_{xx} + \varepsilon'_{xy}}}, \quad W_2 = \sqrt{\frac{\mu_0}{\varepsilon_{xx} - \varepsilon'_{xy}}}.
\end{aligned} \qquad (j)
$$

---

†) Dr. D.-H. Poeverlein has informed me that he has also encountered this possibility of penetration of the ionosphere (cf. his paper soon to appear in the <u>Journal of Atmospheric and Terrestrial Physics</u>).

a) Passage through a simple interface (fig. 1)

When a linearly polarized E_{x_a}, H_{y_a} wave arrives in the air, the amplitudes of the waves that continue in the plasma are

$$H_1 = H_{y_a} \frac{W_0}{W_1 + W_0}, \qquad H_2 = H_{y_a} \frac{W_0}{W_2 + W_0} \tag{k}$$

which must still be multiplied by W_1 or W_2 for the respective E_y and E_x. There are also two reflected waves (H_{y_r}, E_{x_r}) and (H_{x_r}, E_{y_r}), in which

$$H_{x_r} = j H_{y_a} W_0 \frac{W_2 - W_1}{(W_1 + W_0)(W_2 + W_0)},$$
$$H_{y_r} = H_{y_a} \frac{W_0^2 - W_1 W_2}{(W_1 + W_0)(W_2 + W_0)}. \tag{ℓ}$$

Not only the penetrating, but also the reflected wave is elliptically polarized, while the plane of polarization in the plasma rotates as the wave continues, in accordance with the Faraday effect. Without losses, we have

$$\varepsilon_{l_{1,2}} = \varepsilon_{xx} \pm \varepsilon'_{xy} = \varepsilon_0 \left[1 - \frac{\omega_0^2}{(\omega \pm \Omega)\,\omega}\right]. \tag{m}$$

for these waves. Depending on the frequency used, $\epsilon_{\ell_{1,2}}$ has values of zero and infinity [4, p. 100], where either W_1 or W_2 tends toward ∞ or 0. If one W becomes ∞ or 0 and the other remains finite, only a circularly polarized wave enters the plasma (i.e., for $\omega = \Omega$). In the case of the low frequencies ($\omega << \Omega$) discussed in section II, both W_1 and W_2 become very small relative to W_0 at a sufficient ω_0, and we have nearly complete reflection. In this case, the electric fields in the plasma become extremely small relative to the magnetic ones.

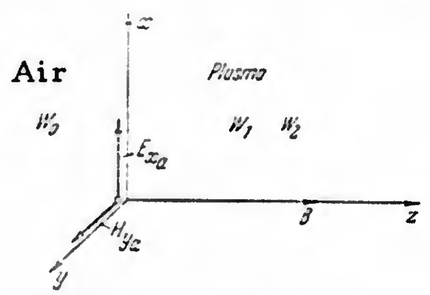

Figure 1

Passage of waves through an interface.

b) <u>Passage through a finite layer (fig. 2)</u>

If we also have a linearly polarized wave (E_{x_a}, H_{y_a}), the following holds for the emergent waves H_{x_d} and H_{y_d} :

$$
\left.\begin{aligned}
H_{y_d}\cdot e^{-j\beta_0\delta} &= H_{y_a}\left\{\frac{1}{2\cos\beta_1\delta + j\left(\frac{W_0}{W_1}+\frac{W_1}{W_0}\right)\sin\beta_1\delta} + \right. \\
&\qquad \left. + \frac{1}{2\cos\beta_2\delta + j\left(\frac{W_0}{W_2}+\frac{W_2}{W_0}\right)\sin\beta_2\delta}\right\}, \\
-jH_{x_d}\cdot e^{-j\beta_0\delta} & \\
&= H_{y_a}\left\{\frac{1}{2\cos\beta_1\delta + j\left(\frac{W_0}{W_1}+\frac{W_1}{W_0}\right)\sin\beta_1\delta}\right. \\
&\qquad \left. - \frac{1}{2\cos\beta_2\delta + j\left(\frac{W_0}{W_2}+\frac{W_2}{W_0}\right)\sin\beta_2\delta}\right\},
\end{aligned}\right| \qquad (8)
$$

$$\beta_{1,2} = \frac{\omega}{c}\, n_{l_{1,2}}, \qquad W_{1,2} = \frac{W_0}{n_{l_{1,2}}}, \qquad n_{l_{1,2}}^2 = \frac{\epsilon_{l_{1,2}}}{\epsilon_0}.$$

Without magnetic coupling, $B = 0$, $\epsilon_{xy} = 0$, W_1 becomes equal to W_2 and $\beta_1 = \beta_2$ and $\frac{H_{y_d} \times e^{-j\beta_0\delta}}{H_{y_a}} = d$ changes into the usual equation

$$d = \frac{1}{\cos\beta\delta + j\,\frac{1}{2}\left(\frac{W_0}{W}+\frac{W}{W_0}\right)\sin\beta\delta}. \qquad (n)$$

H_{x_d} then becomes zero (see also [6]).

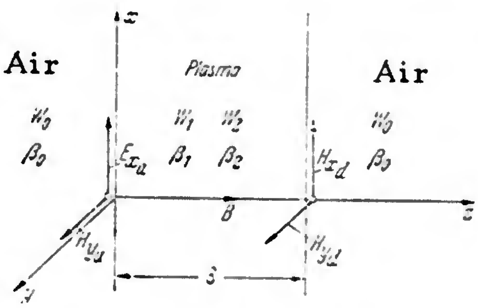

Figure 2

Passage of waves through a finite layer.

In the general case, the emergent wave is elliptically polarized. In the undamped case, $\nu = 0$ for $\omega = \Omega$, $\epsilon_{l_2} \to \infty$, $\beta_2 \to \infty$, $W_2 \to 0$. Then $H_{y_d} = -jH_{x_d}$. As we know, the emergent wave is circularly polarized in this case [7]. If, on the other hand, $W \to \infty$, $\epsilon_l \to 0$, $\beta \to 0$ and the corresponding term becomes

$$\frac{1}{2 + j\frac{\delta}{\lambda_0} \cdot 2\pi}, \tag{o}$$

if λ_0 represents the vacuum wavelength of the frequency ω at which $n \rightarrow 0$. Depending on whether the thickness of the layer is large or small relative to this wavelength, this term lies between 1/2 and 0 and, therefore, contributes either a great deal or a little to the penetrating field strength (cf. also [6]). Finally, the effect of the layer thickness appears in the terms $\cos \beta\delta$ and $\sin \beta\delta$. If $\beta\delta = g \times \pi$, the effect of the dc resistance vanishes and the corresponding term has the value $\pm$ 1/2. This requires

$$\delta = \frac{g\pi}{\beta} = g \cdot \frac{\lambda}{2}, \tag{p}$$

where λ is the wavelength of the corresponding frequency in the plasma. The thickness must include a whole number of half wavelengths. At greater thicknesses, where g is a large number, the slightest variation in thickness will disturb this interference effect, which becomes fully apparent only in the absence of damping, then becomes small and vanishes as damping increases.

Finally, according to section II, both W's become very small at low frequencies ($\omega << \Omega$) with a sufficiently high ω_0 and not excessively large Ω; the W for the ordinary wave becomes imaginary. Both β tend toward infinity. Both terms of equation (8) then become very small, and practically nothing passes through the layer. If n_ℓ^2 is to equal 1 in this range, Ω must be equal to ω_0^2/ω. This would require an Ω of $2\pi \times 10^{10}$ Hz for the ionospheric plasma, with $\omega_{0\,max} = 2\pi \times 10^7$ Hz, and for $\omega = 2\pi \times 10^4$ Hz, i.e., a magnetic field of $\sim$ 3500 gauss. At low frequencies, the requirement is even much higher, i.e., n_ℓ^2 is extremely large relative to 1 in all cases in the ionospheric plasma at frequencies below 10 kHz, even in the E layer. In a discharge plasma with still greater ω_0, magnetic fields greater by a factor of 10,000 and more are required.

c) Passage through a layer having a variable dielectric constant

Having shown in sections IVa and IVb that slow waves are almost completely reflected at inconstant layer interfaces, owing to the large ω_0 at a moderate Ω, we shall examine briefly the case of layers whose electron density is constant. We shall consider only the case of the extraordinary wave which, in passing through a medium at n_L, rises from 1 to a very high value, then gradually sinks to 1 again.

The passage of electromagnetic waves through a layer with variable electron density can also be calculated for the case of a dielectric constant which rises to a very high positive value of ϵ_ℓ/ϵ_0 and then falls again to $\epsilon_\ell = \epsilon_0$, after Rawer [9]. Although Rawer deals with the case of a dielectric constant that decreases within the layer, basing his work on Epstein's method [10], one can obtain a dielectric pattern of the form

$$\frac{\varepsilon_l}{\varepsilon_0} = \varepsilon_r = 1 + N \frac{e^{\kappa x}}{1 + e^{\kappa x}} + M \frac{4 e^{\kappa x}}{(1 + e^{\kappa x})^2} \qquad \text{(q)}$$

by substituting negative values of the constants M and N in equations (3) and (4) of [9]. The case M = 0 describes the gradual rise of the dielectric constant from the value 1 (at sufficiently negative values of x) to 1+ N (for sufficiently large positive values of x); this is a monotonic transition. The case N = 0 describes the symmetric layer, in which ϵ_r increases from 1 (at large positive and negative x) to the value $\epsilon_{r_{max}} = 1 +$ M (at x = 0) (see fig. 3). Hence, the "relative layer thickness" $S = 2\frac{k}{\kappa}$ ($k = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$) and $\lambda_0 S$ corresponds to the "total layer thickness" D.

1) Symmetric layer

In this case, the reflection factor for a wave impinging vertically on the layer without damping is

$$R R^* = \frac{\cos^2 \pi d_2}{\cos^2 \pi d_2 + \sinh^2 \pi S} \qquad \text{(r)}$$

and the transmission coefficient is

$$F F^* = \frac{\sinh^2 \pi S}{\cos^2 \pi d_2 + \sinh^2 \pi S} \qquad \text{(s)}$$

$$d_2 = \frac{1}{2} \sqrt{1 + 4 S^2 \varepsilon_{r_{max}}}$$

where $M = \epsilon_{r_{max}} - 1 \approx \epsilon_{r_{max}}$. As damping increases, the transmissivity decreases and $|F|$ becomes approximately proportional to $e^{-S\nu}$ (cf. [9, p. 408]). $RR^* + FF^* = 1$ necessarily follows from the formulas given above. Since $\cos \pi d_2$ lies between + 1 and 0, reflection is slight if $\sin h^2 \pi S >> 1$. If we regard $\sin h\rho = 2$, $\rho = 1.44$ as a sufficient condition, $\pi \frac{D}{\lambda_0}$ must be ≈ 1.44 and D must be $\approx \frac{\lambda_0}{2}$ with $RR^* \leq 1/5$. Assuming $D \approx$ 800 km for the ionosphere, λ_0 must be less than 1600 km, corresponding to a frequency $\approx$ 190 Hz. Greater frequencies (down to 190 Hz) may also penetrate the layer to a significant degree with slight damping. On the other hand, $\lambda_0 = 30,000$ km for '0 Hz, which is much too long. Hence, with the exception of the improbable case $\pi d_2 = (2g + 1) \frac{\pi}{2}$ with a layer thickness

$$D = \frac{\lambda_0}{2\sqrt{\varepsilon_{r_{max}}}} \cdot \sqrt{(2g+1)^2 - 1} \tag{t}$$

only a very small percentage of the energy passes outward through the layer at 10 Hz, if the above-mentioned ion effect is neglected. λ_0 must be equal to or less than 2D, assuming that is it to be reflected only slightly, at which time the value of $\epsilon_{r_{max}}$ has no effect.

2. Monotonic transition layer

Without damping, we have

$$RR^* = \frac{\sinh^2\left[\frac{\pi S}{2}(1 - \sqrt{\varepsilon_{r_{max}}})\right]}{\sinh^2\left[\frac{\pi S}{2}(1 + \sqrt{\varepsilon_{r_{max}}})\right]} \tag{u}$$

If the arguments of the sin h functions are large relative to 1, where

$$\sqrt{\varepsilon_{r_{max}}} \gg 1, \tag{v}$$

then

$$\frac{D}{\lambda_0} \gg \frac{2}{\pi\sqrt{\varepsilon_{r_{max}}}} \tag{w}$$

is assumed, and we have

$$|R| = e^{-\pi S} = e^{-\pi \frac{D}{\lambda_0}} \tag{x}$$

Thus, the thickness of the transition layer must be approximately the same as, or larger than the vacuum wavelength, if the reflection is to be small and independent of the value of $\epsilon_{r_{max}}$.

If, on the other hand, the arguments of the sin h function are very small, $S \to 0$ and $|R| = \frac{1 - \sqrt{\epsilon_{r_{max}}}}{1 + \sqrt{\epsilon_{r_{max}}}}$, i.e., the result is the usual reflection condition in an inconstant transition.

3. Layers with other distribution

When the layer follows the pattern shown in fig. 3, it should be transparent up to ~ 800 km for frequencies > about 200 Hz, but not for lower ones. However, the question arises whether the electron density profile in the ionosphere corresponds to this figure or whether the density has a more stepwise increase - first in the D and E layers and finally in the F layer. Should the latter be correct, it would be better in each case to use the formula for the monotonic transition layer with each increase. For example, if we assume an increase 100 km long up to the maximum for the F layer, approximately $\lambda \leq 100$ km would hold as the transmission condition, with a frequency $f \geq$ about 3 kHz. In the E layer, there would be a much more rapid rise in ~10 km, which would require a frequency of at least 30 kHz for penetration.

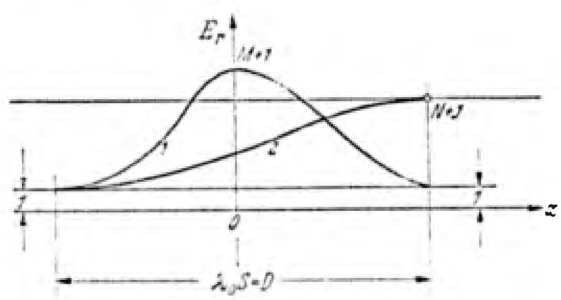

Figure 3

Curve 1: symmetric Epstein layer.
Curve 2: monotonic transition.

The largest dielectric coefficient in this instance would be ~ 1300, with $\epsilon_{r_{max}} = \frac{\omega_{0}^{2} m}{\Omega \omega}$ with $\omega_{0_{max}} \approx 2\pi \times 3.5 \times 10^{6}$ and $\Omega \approx 10^{7}$ Hz at $\omega = 2\pi \times 3 \times 10^{4}$; i.e., it is always large relative to 1 at 30 kHz. Thereafter, the E layer would be a much greater impediment to lower frequencies than would the F layer.

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Literature Cited

1. Schumann, Winfried Otto. "Über Wellenausbreitung im Plasma zwischen zwei unendlich gut leitenden Ebenen in Richtung eines aufgeprägten äusseren Magnetfeldes" (Wave propagation in plasma between two infinitely well conducting surfaces in the direction of an impressed external magnetic field), Zeitschrift für angewandte Physik, 8 (10): 482-485, 1956.

2. Schumann, Winfried Otto. "Über die Ausbreitung elektrischer Wellen längs einer dielektrisch begrenzten Plasmaschicht mit einem longitudinalen Magnetfeld" (Propagation of electric waves along a plasma layer in a magnetic field), Zeitschrift für angewandte Physik, 10 (1): 26-31, 1958.

3. Schumann, Winfried Otto. "Der Einfluss des Erdmagnetfeldes auf die Ausbreitung elektrischer Längstwellen" (The influence of the earth's magnetic field on the propagation of very long electric waves), Zeitschrift für angewandte Physik, 7 (6): 284-290, 1955.

4. Schumann, Winfried Otto. Elektrische Wellen, eine Einführung in die räumliche Ausbreitung elektromagnetischer Vorgänge (Electric waves, an introduction to the spatial propagation of electromagnetic processes). Munich, Hanser, 1948. 340 pp. N. B. p. 93.

5. Schumann, Winfried Otto. "Hochfrequente Schwingungserscheinungen in inhomogenen Plasmen" (High frequency oscillation phenomena in inhomogeneous plasma), Bayerische Akademie der Wissenschaften, Sitzungsberichte, 257: 281-290, 1948.

6. Schumann, Winfried Otto. "Über den Durchgang elektromagnetischer Wellen durch Plasmaschichten endlicher Dicke unterhalb der Grenzfrequenz" (The passage of electromagnetic waves through plasma layers of finite thickness below the critical frequency), Archiv der elektrischen Übertragung, 4 (4): 173-174, 1950.

7. Goldstein, L. Advances in Electronics and Electronphysics, Vol. 7: 487, 1955.

7a. Goldstein, L. "Nonreciprocal electromagnetic wave propagation in ionized gaseous media," Institute of Radio Engineers, Transactions, Vol. MTT-6 (1), 19-29, 1958.

8. Alfvén, Hannes. Cosmical electrodynamics. Oxford, Clarendon Press, 1950. 237 p. N. B. p. 41.

9. Rawer, Karl. "Elektrische Wellen in einem geschichteten Medium: zur Frage der partiellen Reflexion und zur Berechnung

der scheinbaren Höhe von Ionosphärenschichten" (Electric waves in a stratified medium: the problem of partial reflection and calculation of the apparent height of ionospheric layers), Annalen der Physik, 35 (5): 385-416, 1939.

10. Epstein, Paul S. "Reflection of waves in an inhomogeneous absorbing medium," National Academy of Sciences, Washington, D. C., Proceedings, 16 (10): 627-637, 1930.

11. Storey, L. R. O. "An investigation of whistling atmospherics," Royal Society of London, Philosophical Transactions, Series A, 246 (905): 113-141, 1953.