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FTD-HT-23-911-67

FOREIGN TECHNOLOGY DIVISION

THE NUMERICAL SOLUTION OF ONE CLASS OF NONLINEAR INTEGRAL EQUATIONS OF DISPERSED TYPE

by

I. Nedyalkov and G . Penchev

EDITED TRANSLATION

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English pages: 9

SOURCE: Bulgarska Akademiya na Naukite. Fizicheski Institut. Izvestiya na Fizicheskiya Institut s Aneb (Bulgarian Academy of Sciences. Institute of Physics. News of the Institute of Physics and the Atomic Energy Scientific Research Foundation). Vol. 12, Nos. 1 and 2, 1964, pp. 143-151.

Translated under: Contract F33657-67-C-1455

BU/2503-64-012-1/2

TT7002090

ITIS INDEX CONTROL FORM

01 Acc Nr	68 Translation Nr	65 X Ref Acc Nr	76 Reel/Frame Nr
TT7002090	FTD-HT-23-911-67	AT5004299	1882 0680

97 Header Clas	63 Clas	64 Control Markings	94 Expansion	40 Ctry Info
UNCL	UNCL, 0	0		BU

02 Ctry	03 Ref	04 Yr	05 Vol	06 Iss	07 B. Pg.	45 E. Pg.	10 Date
BU	2503	64	012	1/2	0143	0151	NONE

Transliterated Title: VURKHU CHISLENOTO RESHENIE NA EDNA KLASA NELINEYNI INTEGRALNI URAVNENIYA OT DISPERSIONEN TIP

09 English Title: THE NUMERICAL SOLUTION OF ONE CLASS OF NONLINEAR INTEGRAL EQUATIONS OF DISPERSED TYPE

43 Source: BULGARSKA AKADEMIYA NA NAUKITE. FIZICHESKI INSTITUT. IZVESTIYA NA FIZICHESKIYA INSTITUT S ANEB (BULGARIAN)

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98 Document Location:

47 Subject Codes: 12

39 Topic Tags: nonlinear integral equation, singular integral equation, adiabatic process

ABSTRACT: The shortcomings of the N/D method and of the Chew-Low-Salzman method for the numerical solution of singular nonlinear integral equations of the Low type, utilizing the preliminary regularization of those equations, are analyzed. The method is presented for the direct numerical solution (without preliminary regularization) of one particular class of Low type integral equations--the Chew-Low equation, which describes the P-wave resulting from the pion-nucleon interaction under the assumption that the nucleon is fixed. By utilizing certain substitutions of variables, the Chew-Low equation is transformed to a form for which solutions are derived by the method of successive approximations. Measures for eliminating the instability from the iterative process are indicated. The authors explain how the proposed method can be used to obtain adiabatic and resonance solutions of the Chew-Low equation. It is shown that the adiabatic solution ceases to be analytic when the coupling constant $f^2 = 0.07$. Some numerical results obtained by authors at the Computing Center of the Joint Institute of Nuclear Research at Dubna are presented.
Orig. art. has: 2 figures and 11 formulas. English translation: 9 pages.

THE NUMERICAL SOLUTION OF ONE CLASS OF NONLINEAR INTEGRAL EQUATIONS OF DISPERSED TYPE

I. Nedyalkov and G. Penchev

This work presents one method of numerical solution of nonregularized Low integral equations. Low equations are systems of N, nonlinear integral equations of dispersed type (which include Cauchy integrals).

For various values of N, particular cases of Low equations obtain. Thus, for example, where N is equal to 1 or 2, we have Castilejo, Dalitz and Dyson equations, where N=2 -- Wanders equations, where N=3, Chew and Low equations [1], Chew and Mandelstam equations [3], Shirkov equations [5], etc.

In modern investigations of the numerical solution of Low equations, two methods are used: the ND-method and the similar Chew-Low-Salzman method [2]. It is characteristic for both methods that first the Low equations are regularized, then solved by iteration. An algebraic method essentially different from these two was recently suggested, but we will not discuss it [6].

It is known from the theory of singular integral equations [8] that the solution to regularized equations may not correspond to the solution to the initial equations and due to this, in each individual case, detailed investigation is required. This is also true of the two methods mentioned above. Due to the fact that the denominators $D_u(z)$ and $h_a(z)$ may have zeros in the complex equation sometimes solutions are produced which do not satisfy the original equations. Since in practical calculations the zeros in the denominator are not sought out, the usage of these two methods involves a certain risk.

The purpose of the present work is to present a numerical method for solution of Low equations which does not have the defects of the ND-method or the Chew-Low-Salzman method. Just as in the case of linear singular integral equations [9, 10, 11], the procedure involves direct solution of the initial equations without preliminary regularization. Due to the presence of nonlinearities in the integral equations, the difficulties of numerical solution are much greater than those in [9, 10, 11].

Since the difference in the individual form of the equations is inessential from the point of view of the method, throughout the following we will concern ourselves with a single special type of Low equation -- the Chew-Low equation [1], using which one can describe the P-wave of pion-nucleon scattering on the assumption that the nucleon is fixed.

In section 1 below we give formulas for replacement of variables and outline the primary operational formulas; in sections 2 and 3 we present the necessary explanation concerning the application of the method

of sequential approximations to produce adiabatic and resonant solutions, and in section 4 a few numerical results which we produced using an electronic computer in the computer center of the Joint Institute of Nuclear Research in Dubna are presented.

1. Conversion of the Equation

The Chew-Low equation can be written in the form

$$h_a(z) = \frac{\lambda_a}{z} + \frac{1}{\pi}\int_1^\infty d\omega' p^3(\omega') v^2(p') \left[\frac{|h_a(\omega')|^2}{\omega' - z} + \sum_{\beta=1}^{3} \frac{A_{a\beta} |h_\beta(\omega')|^2}{\omega' + z} \right], \quad (1)$$

where h_{a_z} are unknown functions, analytic in plane z with cross-section $(-\infty, -1)$ and $(+1, +\infty)$ with the exception of point z=0, where they have simple poles with residuum λ_a; $z \to \omega + i\varepsilon$ where $\omega > 1$, $z \to \omega - i\varepsilon$ where $\omega < -1$,

$$\lambda_a = \frac{2}{3} f^2 \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix}, \quad A_{a\beta} = \frac{1}{9} \begin{pmatrix} 1 & -8 & 16 \\ -2 & 7 & 4 \\ 4 & 4 & 1 \end{pmatrix}, \quad (2)$$

$p = (\omega^2 - 1)^{1/2}$ is the impulse of the pion in the system at the inertial center, ω is the total energy of the pion in the same system, f^2 is the coupling factor, $v(p) = 1/(1 + p^2 a^2)$ is the formation function (a=0.27) with the given h, c and rest mass of the pion in the position equal to unity, and index a has the value

$$a = 1 \to \left(\frac{1}{2}, \frac{1}{2}\right),$$

$$a = 2 \to \left(\frac{1}{2}, \frac{3}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right),$$

$$a = 3 \to \left(\frac{3}{2}, \frac{3}{2}\right).$$

The first index in the parentheses is the total isotropic spin of the pion-nucleon system, the second index -- is the total moment of the system.

From (1) and (2), certain properties of function $h_a(\omega)$ follow immediately:

$$\frac{\sin\delta_a(l)\cos\delta_a(l)\cdot l^2}{(1-l^2)^{3/2}v^2(l)} = \lambda_a - \frac{1}{\pi}P\int_0^1 d\tau\ \frac{\sin^2\delta_a(\tau)\cdot\tau^2}{(1-\tau^2)^{3/2}v^2(\tau)(\tau-l)} \tag{7}$$

$$+\frac{1}{\pi}\int_0^1 d\tau\ \frac{\sum_{\beta=1}^{3}A_{a\beta}\sin^2\delta_\beta(\tau)\cdot\tau^2}{(1-\tau^2)^{3/2}v^2(\tau)(\tau+l)}.$$

This can be written as follows:

$$X_a(l)[L(l)-X_a(l)] = \lambda_a - \frac{1}{\pi}P\int_0^1 d\tau\ \frac{X_a(\tau)}{\tau-l} \tag{8a}$$

$$+\frac{1}{\pi}\int_0^1 d\tau\ \frac{\sum_{\beta=1}^{3}A_{a\beta}X_\beta(\tau)}{\tau+l},$$

in which

$$X_a(l) = \frac{\sin^2\delta_a(l)\cdot l^2}{(1-l^2)^{3/2}v^2(l)},\quad L(l) = \frac{l^2}{(1-l^2)^{3/2}v^2(l)}. \tag{9}$$

If we represent this portion of equation (8a) as $Q_a(t)$, obviously

$$X_a(l) = \frac{L(l)\pm\sqrt{L^2(l)-4Q_a^2(l)}}{2}. \tag{8b}$$

Finally, multiplying (7) by tg $\delta_a(t)$ and using the representation of (9), we produce

$$X_a(l) = \operatorname{tg}\delta_a(l)\left[\lambda_a - \frac{1}{\pi}P\int_0^1 \frac{X_a(\tau)}{\tau-l}\,d\tau\right. \tag{10}$$

$$\left.+\frac{1}{\pi}\int_0^1 d\tau\ \frac{\sum_{\beta=1}^{3}A_{a\beta}X_\beta(\tau)}{\tau+l}\right].$$

$$R_e h_a(\omega) = \frac{\lambda_a}{\omega} + \frac{1}{\pi} P \int_1^\infty d\omega' p^3 v^3 \frac{|h_a(\omega')|^2}{\omega' - \omega} + \frac{1}{\pi} \int_1^\infty d\omega' p^3 v^3 \frac{\sum A_{a\beta} h_\beta^2}{\omega' + \omega}, \quad (3a)$$

$$J_m h_a(\omega) = p^3(\omega) v^3(p) |h_a(\omega)|^2 \quad \text{when} \quad 1 < \omega < \infty, \quad (3b)$$

$$h_a(-\omega) = \sum_{\beta=1}^{3} A_{a\beta} h_\beta(\omega) \quad \text{when} \quad -1 > \omega > \infty \quad \text{and} \quad 1 < \omega < \infty. \quad (3c)$$

Functions $h_a(\omega)$ are connected with the phases of scattering $\delta_a(\omega)$ of the P-wave by relationship

$$h_a(\omega) = \frac{\exp |h_a(\omega)| \sin \delta_a(\omega)}{p^3(\omega) v^3(\omega)} \quad \text{when} \quad \omega = 1. \quad (4)$$

To find functions $R_e h_a(\omega)$ and $J_m h_a(\omega)$ $(\omega > 1)$ it is necessary to solve equations (3a) and (3b) simultaneously, which produces a system of three nonlinear singular integral equations and three algebraic equations. We will use the relationship (4):

$$\frac{\sin \delta_a(\omega) \cos \delta_a(\omega)}{p^3(\omega) v^3} = \frac{\lambda_a}{\omega} + \frac{1}{\pi} P \int_1^\infty \frac{d\omega' \sin^2 \delta_a(\omega')}{(\omega' - \omega) p^3 v^3(p')} \quad (5)$$

$$+ \frac{1}{\pi} \int_1^\infty d\omega' \frac{\sum_{\beta=1}^{3} A_{a\beta} \sin^2 \delta_\beta}{(\omega' + \omega) p^3 v^3}.$$

Now let us introduce the boundaries of integration by replacement of variables

$$\omega = \frac{1}{t}, \quad \omega' = \frac{1}{t'}. \quad (6)$$

For simplicity after the replacement of variables, we introduced new symbols for the functions h_a, δ_a, v and p.

Substituting (6) in (5), we produce the system

In working for an adiabatic solution, the sign before the square root in (8b) must be negative, and in the third phase (a=3) of the resonant solution it was proven that beginning with the minus on even numbered paths the same sign changes [4]. In the case which we have analyzed, the sign of two paths changed.

In [5] it is shown that in another type of equation (the equation describing pion-pion scattering) three types of solutions are possible for various asymptotic behaviors with increasing energy ($1/\ln \omega$, $1/\omega$ and $1/\omega^2$). We will seek the solution which approaches zero at infinity.

2. Adiabatic Solution

We note the following fact. It is obvious that systems (8) and (10) are equivalent, i.e. that if calculations are performed with sufficiently high accuracy, the solutions produced will be the same regardless of the system used. However, due to limitations on the accuracy, the systems may be found inadequately effective. We believe, for example, that the accuracy with which calculations are performed in this work is insufficient for an adiabatic solution to the problem where $f^2 \geq 0.05$, since near the maximum of function $X_1(t)$ the expression under the radical in (8b) becomes negative. Due to this, we will seek the adiabatic solution by solving system (10) and using the first relationship in (9). To use the method of sequential approximations we must know the coupling factor f^2 and use the first approximation of amplitude δ_a (a=1,2,3). Since the speed of convergence of the iterational process depends on the suitability of the first approximation, usually a program is written which, even though the convergence may not be too rapid, has advantages as concerns systematic ordering. This sort of program is required in correspondence with the well known concept of the adiabatic interaction in which at $f^2=0$ all phase differences are equal to zero. Then the first approximation with f^2 can be produced using (8b), assuming $Q_a=\lambda_a$. The subsequent approximations with the f^2 selected are produced by solving system (10). After this, varying f^2 with sufficiently small increment Δf^2 with the value of f^2 as the zero approximation, we use the solutions produced corresponding to the preceding f^2.

3. Resonant Solution

Our calculations have shown that the adiabatic solution cannot be coordinated with experimental results even qualitatively as calculated in this work; we therefore set ourselves the task of seeking the resonant solution. However, since there can be an unlimited number of such

solutions, we sought the primary resonant solution, corresponding closely to the experimental curve. This means that we must find a solution in which the first two phases are small in comparison with the third phase, in which the resonance is located. The program which we used in producing the adiabatic solution is not suitable for producing the resonant solution. With the given values of coupling factor as the first approximation of amplitudes δ_1 and δ_2 we can use $\delta_1(t)\equiv 0$ and $\delta_2(t)\equiv 0$.

The first approximation of the third phase is produced from

$$\operatorname{ctg}\delta_3(\omega)=\frac{3\omega^*(\omega_0-\omega^*)}{4f_0^2\omega_0 p^3(\omega)\tau^2(p)} \qquad (11)$$

where $\omega_0=2.17$, $f^2_0=0.087$, $p=\sqrt{\omega^2-1}$, M=6.67 (mass of the nucleon) and $\omega^*=\omega+\sqrt{\omega^2+M^2}-1-M$.

This last formula, as is shown in the work by Layson [7], gives good correspondence with the results produced for δ_3 experimentally.

As was mentioned in section 2, systems (8) and (10) may be found inadequately effective. This appears particularly strong in an attempt to find the resonant solution. The numerical investigations which we performed on our electronic computer showed that it is impossible by iteration to produce the resonant solution using equations (8a) and (8b).

We have already noted that the first two phases require the minus sign before the square root, and that δ_3, due to the resonance, requires two paths with variable sign. It was found that the points where the sign changes (where t=0.4 and t=0.57) discontinuities appear which disrupt the smooth course of the curve. Equation (10) is essentially inapplicable to the resonant solution, since it does not contain tg δ_a, and at $t=0.5(\omega=2)\delta_3=\pi/2$. Due to this, we decided to use a combined method which consisted of the following: at the beginning of the interval, up to t=0.45, the calculation was conducted according to (10) and the first formula in (9), then in the interval from t=0.45 to t=0.55 -- according to (8a), (8b) and (9), in which in (8b) the minus sign was used for δ_1 and δ_2, the plus sign for δ_3; in the next interval calculations were performed once more according to (10) and the first formula of (9). This was done in order to eliminate the deficiencies in systems (8) and (10). In the calculation process it was found that it was necessary to artificially smoothe function δ_3 in those places where the equations were exchanged, since with increasing number of iterations at these points (t=0.45 and t=0.55) discontinuities gradually appear.

4. Numerical Results

Calculations were conducted using an electronic computer and the methods described in sections 1, 2 and 3. The interval from 0 to 1 for t

was divided into 100 subintervals. Also, to improve the accuracy, the integrals were calculated with automatic selection of the step; at points other than those selected, interpolation was performed. The integrals were calculated with a relative accuracy of $5 \cdot 10^{-4}$, and iteration continued until the modulus of the difference between two successive iterations was less than $5 \cdot 10^{-3}$. Approximately five minutes of machine time was required for each complete cycle.

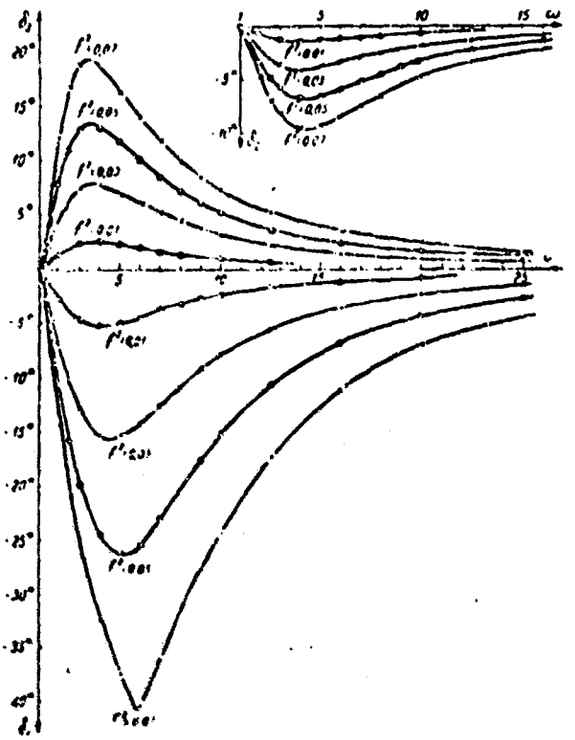

Figure 1.

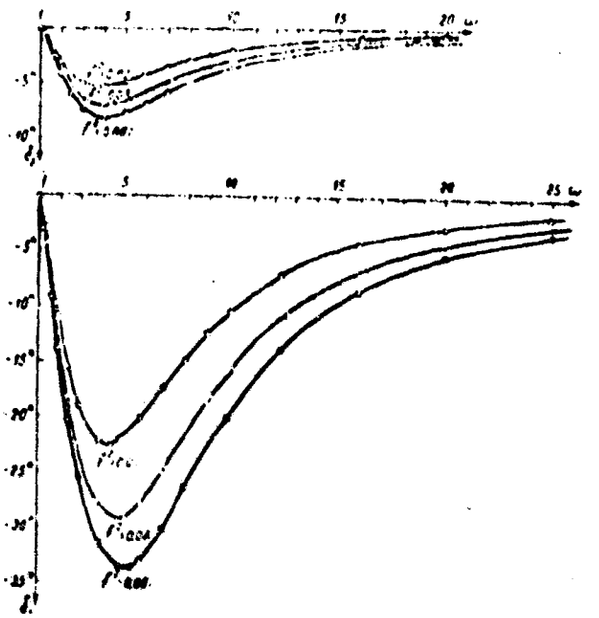

Figure 2.

In the calculation of the adiabatic solution, the coupling factor varied by $\Delta f^2=0.01$. The iterations converged where $f^2 \leq 0.07$. In this case, when f^2 increased, the number of iterations also increased, increasing from seven iterations at $f^2=0.01$ to 21 iterations at $f^2=0.07$. We must also note that where $f^2=0.07$ the following phenomenon was observed: up to to the thirteenth iteration the process converged normally, while between the fourteenth and eighteenth iterations two peaks appeared in the function δ_1 (at t=0.14 and t=0.22) after which they converged to a single peak at

t=0.18 and the result after the twenty-second iteration approached the solution of the problem. When the iterational process was continued beyond $f^2=0.07$, the same phenomenon was repeated and as was to be expected at $f^2=0.08$ the sequential approximations no longer converged.

All of the functions produced and subjected to checking approximately satisfied the initial equation. It was found that in the adiabatic case the solutions which we determined satisfied crossing symmetry (3) with an accuracy of 0.01%, while the resonance solution showed an accuracy of about 0.1%. We present below some of our solutions for various values of coupling factor. Figure 1 shows $\delta_a(\omega)(a=1,2,3)$ in degrees for the adiabatic case, figure 2 -- $\delta_1(\omega)$ and $\delta_2(\omega)$ for the resonant solution; $\delta_3(\omega)$ for the same solution is shown in table 1.

The authors express their gratitude to I. Todorov and R. Genshev for useful discussion.

Table 1

ω	$\delta_3(\omega)$ where $f^2=0.07$	$\delta_3(\omega)$ where $f^2=0.08$	$\delta_3(\omega)$ where $f^2=0.087$
1,00	0	0	0
1,11	1°09'	1°12'	°18'
1,25	4°25'	4°31'	4°54'
1,43	12°09'	12°18'	13°04'
1,67	34°16'	33°21'	32°44'
2,00	97°02'	96°18'	94°05'
2,50	141°32'	147°34'	145°44'
3	157°12'	164°46'	165°43'
4	170°56'	178°06'	180°26'
5	176°57'	182°55'	185°05'
7	180°56'	184°40'	186°12'
10	181°38'	184°39'	184°39'
20	180°45'	181°21'	181°41'

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