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Won-Hwa Chu and H. Norman Abramson

Technical Report
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SwRI Project 02-2311

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Approved:

H. Norman Abramson, Director
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ABSTRACT

Some new calculations of the flutter speed of the SwRI fully submerged subcavitating hydrofoil model are presented. Variations in lift curve slope and in center of pressure location are found to have a most profound influence on both flutter speed and frequency. When variations in these parameters are combined with a relaxation of the Kutta condition (proposed previously), excellent agreement with the measured flutter speed is obtained.

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INTRODUCTION

The catastrophic structural failure of the SwRI flutter model as reported by Abramson and Ransleben (Ref. 1) has been the focal point of much discussion and the subject of a variety of calculations and analyses. The conditions at which catastrophic failure of this model occurred were a velocity of 48.1 knots and a frequency of 17.5 cps (corresponding to a reduced velocity of 1.48). However, in making any comparison or correlations with these data, it should be quite clearly noted that uncertainties exist even here (Refs. 2, 3), so that exact agreement should be viewed with as much skepticism as would wide disagreement.

Initial attempts to calculate the flutter velocity of this model by two-dimensional and Reissner-Stevens theories failed to yield reasonable results (Refs. 1, 4), although the model had an aspect ratio of five. This difficulty was not unanticipated, however, as large differences between experimental data and computed results for various flutter studies at low mass density ratios had previously been encountered and had been the subject of much controversy (Refs. 2, 5). It therefore has now become rather widely accepted that, for the range of parameters pertinent to hydrofoil applications the more-or-less classical formulations of the unsteady hydrodynamic loads are not adequate. However, even the evidence available here is somewhat clouded: for example, measured data on oscillatory lift and moment distributions (Refs. 6, 7) employed directly in a flutter analysis of the SwRI flutter model yielded a substantially overconservative prediction (Refs. 1, 4); unfortunately, the accuracy of this measured data is also open to some question (Ref. 3), and the analytical techniques employed in the analysis may not have been adequate.

A somewhat different approach was also taken quite early in all of these studies by considering some relaxation of the Kutta condition (Refs. 8, 9); but, again, reasonable flutter predictions were not readily forthcoming (Ref. 4) and could only be obtained from this semiempirical procedure by using rather large arbitrary shifts in the phase of the circulation function.

Several years ago, Yates began the development of a modified strip-analysis method employing arbitrary spanwise distributions of lift curve slope and center of pressure location (Refs. 10-14). This method can be based on the idea of employing measured data for these parameters or, alternatively, employing values calculated from lifting surface theory. Yates has applied this basic method to the SwRI flutter model (Ref. 15), obtaining a flutter velocity of approximately 38.5 knots, with a structural damping coefficient of g = 0.02 cps and a flutter frequency of 21.3 cps. Some further refinements and modifications appear to have yielded somewhat better agreement, but these have not yet been reported in detail.

Yet another approach has relied upon the rather full application of lifting surface theories (Refs. 16-18). The method proposed by Rowe (Ref. 18) provides for some variation in satisfaction of the Kutta condition and inherently generates data on lift curve slope and center pressure location.

It is the purpose of the present report to present yet another series of results. These are all based essentially upon strip theory, but involving various modifications to account for Kutta condition, lift curve slope, center of pressure location, etc.

ANALYSIS

Basic Equations

For combined pitching and heaving oscillations of a hydrofoil, represented as a uniform unswept beam, the equations of motion may be put into the nondimensional form (Ref. 19)

$$-\mu\gamma_h^2 h_0^* - \mu\gamma_{ah}x_a a_0 + \mu\gamma_h^2 r_\omega^2 \frac{\omega_a^2}{\omega^2}(1 + ig)\, h_0^* = A_{hh}^* h_0^* + A_{ha}^* a_0 \tag{1a}$$

$$-\mu\gamma_{ah}x_a h_0^* - \mu r_a^2 \gamma_a^2 a_0 + \gamma_a^2 \frac{\omega_a^2}{\omega^2}[1 + ig]\, \mu r_a^2 a_0 = A_{ah}^* h_0^* + A_{aa}^* a_0 \tag{1b}$$

where

$$\gamma_h^2 = \int_0^1 f_h^2(y^*)\, dy^* \quad , \quad \gamma_a^2 = \int_0^1 f_a^2(y^*)\, dy^* \quad , \quad \gamma_{ah} = \int_0^1 f_a(y^*)\, f_h(y^*)\, dy^* \tag{2a, b, c}$$

f_a, f_h are the pitching and heaving mode shapes, respectively, i.e.,

$$a = a_0 f_a(y^*) \quad \text{positive nose-up} \tag{3a}$$

$$\frac{h}{b} = h_0^* f_h(y^*) \quad \text{positive downward} \tag{3b}$$

$$y^* = \frac{y}{sb} \quad s = \text{aspect ratio, } b = \text{semichord} \tag{4a}$$

$$\mu = \frac{M}{\pi\rho b^2} \quad , \quad r_a^2 = \frac{I_a}{Mb^2} \tag{4b, c}$$

$$x_a = \frac{S_a}{Mb} \quad , \quad r_\omega^2 = \frac{\omega_h^2}{\omega_a^2} \tag{4d, e}$$

The (nondimensional) generalized loadings are

$$A^*_{hh} = \int_0^1 L^*_h f^2_h(y^*)\, dy^* \tag{5a}$$

$$A^*_{h\alpha} = \int_0^1 L^*_\alpha f_\alpha(y^*)\, f_h(y^*)\, dy^* \tag{5b}$$

$$A^*_{\alpha h} = \int_0^1 M^*_h f_h(y^*)\, f_\alpha(y^*)\, dy^* \tag{5c}$$

$$A^*_{\alpha\alpha} = \int_0^1 M^*_\alpha f^2_\alpha(y^*)\, dy^* \tag{5d}$$

The force L* is positive downward; the moment M* is positive nose-up.

The determinant of the coefficients must vanish at the critical (flutter) speed; in this case,

$$a_2 z^2 + a_1 z + a_0 = 0 \tag{6}$$

where

$$z = \frac{\omega_\alpha^2}{\omega^2}(1 + ig) \tag{7}$$

and

$$a_2 = \mu^2 r_\alpha^2 r_\omega^2 \gamma_\alpha^2 \gamma_h^2 \tag{8a}$$

$$a_1 = -\mu r_\omega^2 \gamma_h^2 (\mu r_\alpha^2 \gamma_\alpha^2 + A^*_{\alpha\alpha}) - \gamma_\alpha^2 \mu r_\alpha^2 (\gamma_h^2 \mu + A^*_{hh}) \tag{8b}$$

$$a_0 = (\gamma_h^2 \mu + A^*_{hh})(\mu r_\alpha^2 \gamma_\alpha^2 + A^*_{\alpha\alpha}) - (\mu x_\alpha \gamma_{\alpha h} + A^*_{h\alpha})(\mu x_\alpha \gamma_{\alpha h} + A^*_{\alpha h}) \tag{8c}$$

The standard procedure employed for determining the flutter speed from Eq. (6) is to assume a value for the reduced frequency $k = \frac{\omega b}{V}$ and then to calculate z and $\frac{\omega_\alpha}{\omega}$. The value of g may then be found and a plot made of V vs. g; the value of V at which g = 0 then gives the flutter velocity.

<u>Case 1. Reissner-Stevens Method (Ref. 20)</u>

Following the convention of Reference 19, the expressions for force and moment corresponding to pitch and heave are, respectively,

$$L_h^* = \frac{\frac{L_h'}{h^*}}{\pi\rho\omega^2 b^3} = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\left[C(k) + \sigma_h\right]\right\} = \bar{L}_h^* e^{i\alpha_{Lh}} \tag{9a}$$

$$L_\alpha^* = \frac{\frac{L_\alpha'}{\alpha}}{\pi\rho\omega^2 b^3} = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[C(k) + \sigma_\alpha\right]\right\}$$

$$= \bar{L}_\alpha^* e^{i\alpha_{L\alpha}} \tag{9b}$$

$$M_h^* = \frac{\frac{M_h'}{h^*}}{\pi\rho\omega^2 b^4} = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - \left(a + \frac{1}{2}\right) ik\left[C(k) + \sigma_h\right]\right\} = \bar{M}_h^* e^{i\alpha_{Mh}} \tag{9c}$$

$$M_\alpha^* = \frac{\frac{M_\alpha'}{\alpha}}{\pi\rho\omega^2 b^4} = -\frac{2}{k^2}\left\{\frac{1}{2} ik\left(\frac{1}{2} - a\right) - \frac{1}{2}k^2\left(\frac{1}{8} + a^2\right) - \left(a + \frac{1}{2}\right)\right.$$

$$\left.\cdot\left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[C(k) + \sigma_\alpha\right]\right\} = \bar{M}_\alpha^* e^{i\alpha_{M\alpha}} \tag{9d}$$

The procedure required to calculate the finite span correction factors σ_h and σ_α are given in Reference 20 (for two-dimensional theory, $\sigma_h = \sigma_\alpha = 0$).

The lift and moment calculated for the SwRI flutter model are found to be in agreement with those obtained previously with a different computer program (Ref. 1). The V-g curve is given in Figure 1; the flutter speed lies between two previously calculated values, all of which are based upon certain data read from curves given in Reference 20, so that the basic computer program to be employed in the calculation of the curves to follow is confirmed to be correct.

The measured mode shapes and characteristics of the SwRI flutter model, as employed in these calculations, are given in Table 1.

<u>Case 2. Calculation Using Experimental Data for Steady-State Center of Pressure Location</u>

The expressions for lift and moment are now written as

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\left[C(k) + \sigma_h\right]\right\} \tag{10a}$$

$$L_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[C(k) + \sigma_\alpha\right]\right\} \tag{10b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - (a - x_{cp})\, ik\left[C(k) + \sigma_h\right]\right\} \tag{10c}$$

$$M_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2} ik\left(\frac{1}{2} - a\right) - \frac{1}{2}(k^2 + a^2) - (a - x_{cp})\left[1 + ik\left(\frac{1}{2} - a\right)\right] \cdot \left[C(k) + \sigma_\alpha\right]\right\} \tag{10d}$$

where x_{cp} is the location of the center of pressure, dependent upon the spanwise positions, or obtained from the experimental data of Reference 6. In the moment expressions, Eqs. (10c) and (10d), the term $a - x_{cp}$ has been used in place of the term $\frac{1}{2} + a$ as the lift force is to be taken as acting at x_{cp} rather than at the quarter-chord point $\left(-\frac{1}{2} b\right)$.

The V-g curve is given in Figure 2, indicating a flutter speed of approximately 67 knots (compared with the experimental value of 48 knots).

Case 3. Calculation Using Experimental Data for Oscillating Center of Pressure Location

The forces will be considered to be the same as those of the classical theory of Case 1, but the moments will be taken as

$$M_h^* = \bar{M}_h^* e^{i\alpha_{Mh}} - \bar{M}_h^* \cos(\alpha_{Mh} - \alpha_{Lh})\, e^{i\alpha_{Lh}} + x_{cp_h}^* L_h^* e^{i\alpha_{Lh}} \quad (11a)\dagger$$

$$M_\alpha^* = \bar{M}_\alpha^* e^{i\alpha_{M\alpha}} - \bar{M}_\alpha^* \cos(\alpha_{M\alpha} - \alpha_{L\alpha})\, e^{i\alpha_{Lh}} + x_{cp_\alpha}^* L_\alpha^* e^{i\alpha_{Lh}} \quad (11b)$$

The unsteady center of pressure locations are taken as $x_{cp_h}^*$, $x_{cp_\alpha}^*$ in these expressions and defined (Ref. 6) so that

$$L_\alpha^* x_{cp_\alpha}^* = \bar{M}_\alpha^* \cos(\alpha_{Mh} - \alpha_{Lh}) \quad (12a)$$

$$L_h^* x_{cp_h}^* = \bar{M}_h^* \cos(\alpha_{M\alpha} - \alpha_{L\alpha}) \quad (12b)$$

where

$$x_{cp_h}^* = 2\tilde{x}_{cp_h} - \frac{1}{2} - d \quad (13a)$$

$$x_{cp_\alpha}^* = 2\tilde{x}_{cp_\alpha} - \frac{1}{2} - d \quad (13b)$$

and $\tilde{x}_{cp_h}$, $\tilde{x}_{cp_\alpha}$ are the oscillatory center of pressure locations corresponding to heave and pitch motions, respectively, in fractions of chord measured from the leading edge; d is the distance between the moment axis and the quarter-chord point. For the SwRI flutter model

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†M_h^*, M_α^*, α_{Mh}, $\alpha_{M\alpha}$, L_h^*, L_α^*, α_{Lh}, $\alpha_{L\alpha}$ are given by Eqs. (9a-d) with $\sigma_h = \sigma_\alpha = 0$.

$$\frac{1}{2} + d = 0.31 \tag{14}$$

Values of $x^*_{cp_h}$, $x^*_{cp_\alpha}$ are given in Table 2.

The V-g curve is given in Figure 3, indicating a flutter speed of approximately 70 knots.

Case 4. Calculation Using Modified Values of Lift Curve Slope and Center of Pressure

It is well known, in steady flow, that the actual lift curve slope of a foil is somewhat less than the theoretical value of 2π, thus having the effect of reducing the value of the circulation. Denoting the ratio of actual to theoretical lift curve slopes by β, the Theodorsen function may be multiplied by this ratio (no additional corrections are to be applied to the Reissner-Stevens three-dimensional factor). Then, considering the lift force to act at the center of pressure, the expressions for lift and moment become

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\left[\beta C(k) + \sigma_h\right]\right\} \tag{15a}$$

$$L_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[\beta C(k) + \sigma_\alpha\right]\right\} \tag{15b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - (a - x_{cp})\, ik\left[\beta C(k) + \sigma_h\right]\right\} \tag{15c}$$

$$M_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2} ik\left(\frac{1}{2} - a\right) - \frac{1}{2}k^2\left(\frac{1}{8} + a^2\right) - (a - x_{cp})\left[1 + ik\left(\frac{1}{2} - a\right)\right] \cdot \left[\beta C(k) + \sigma_\alpha\right]\right\} \tag{15d}$$

Various values of C_{l_a} and x_{cp} are given in Table 3. The data in Table 3a are derived from experiments (Ref. 6) while those in Table 3b are derived from lifting surface theory (Ref. 15).

V-g curves calculated by using these two sets of C_{l_a} and x_{cp} values are given in Figures 4a and 4b. The first, employing measured foil data, gives a flutter speed of approximately 100 knots while the second, employing calculated foil data, gives a flutter velocity of approximately 76 knots. Comparing these results directly with those of Case 2, it would mean that the effect of modifying the lift curve slope in this manner is to raise the flutter velocity.

<u>Case 5. Calculation Using Yates' Modified Strip Theory (Ref. 10)</u>

According to Yates (Ref. 10), the constant factor $\frac{1}{2}$ - a in the downwash terms of a conventional strip theory analysis could be replaced by

$$\lambda = \beta + x_{cp} - a \tag{16}$$

which varies along the span of the foil. Since this modification includes three-dimensional effects, σ_a and σ_h are to be taken as zero. The expressions for lift and moment then become

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ikC(k)\right\} \tag{17a}$$

$$L_a^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + (1 + ik\lambda)\,C(k)\right\} \tag{17b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - \left(\frac{1}{2} + a\right) ikC(k)\right\} \tag{17c}$$

$$M_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2}ik\lambda - \frac{1}{2}k^2\left(\frac{1}{8}+a^2\right) - \left(a+\frac{1}{2}\right)(1+ik\lambda)\,C(k)\right\} \tag{17d}$$

Two subcases are now considered. Case 5a is based on the use of the values derived by Yates (Ref. 15) and given in Table 3b. The V-g curve corresponding to this calculation is shown in Figure 5a and gives a flutter velocity of approximately 30 knots with $g \cong 0$ and 32 knots with $g = 0.02$; this compares with a value of 38.5 knots with $g = 0.02$ as computed by Yates himself (Ref. 15). Note, however, that the slope of the critical V-g curve for small values of g is so flat that only very slight changes in the computed points would be necessary to alter the flutter velocity by a very large amount. Further, Yates employed six calculated modes (two bending and four torsion) in his analysis, while the present calculations are based on two measured mode shapes (one bending and one torsion)*. In any event, it would appear that the modified strip theory proposed by Yates (and as employed in this report) is overconservative by a significant degree.

Case 5b employs the measured steady state values shown in Table 3a. The corresponding V-g curve is shown in Figure 5b, giving a flutter speed of approximately 26 knots.

Case 6. Calculations Based on Appliedation of the Generalized Kutta Condition

As proposed a number of years ago (Refs. 4, 8, 9, 21), a generalization of the Kutta condition can be formulated in terms of a factor λ_1 applied to the

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* Some rather simple considerations show that our use of the two measured modes should give reasonably accurate results. Any discrepancies arising between using calculated and measured modes is the result of differences in the dominant terms rather than in an insufficient number of modes.

trailing edge tangential velocity. Defining $\lambda_1 = 1 - \eta e^{i\phi_0}$, and replacing the constant factor $\frac{1}{2} + a$ by $a - x_{cp}$ as was done in Case 2, the lift and moment become:

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\eta e^{i\phi_0}\left[C(k) + \sigma_h\right]\right\} \tag{18a}$$

$$L_a^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\eta e^{i\phi_0}\left[C(k) + \sigma_a\right]\right\} \tag{18b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - (a - x_{cp})\, ik\eta e^{i\phi_0}\left[C(k) + \sigma_h\right] - \frac{i\lambda_1 k}{2}\right\} \tag{18c}$$

$$M_a^* = -\frac{2}{k^2}\left\{\frac{1}{2} ik\left(\frac{1}{2} - a\right)\eta e^{i\phi_0} - \frac{\lambda_1}{2} - \frac{1}{2}k^2\left(\frac{1}{8} + a^2\right)\right.$$

$$\left. - (a - x_{cp})\left[1 + ik\left(\frac{1}{2} - a\right)\right]\eta e^{i\phi_0}\left[C(k) + \sigma_a\right]\right\} \tag{18d}$$

We shall assume that

$$\eta = \frac{\beta}{\cos\phi_0} \tag{19a}$$

$$\beta = \frac{C_{\ell_a}}{2\pi} \tag{19b}$$

and shall select specific values of ϕ_0.

Case 6a is based on the use of measured data for C_{ℓ_a} and x_{cp} (Table 3a), with V-g curves corresponding to several values of ϕ_0 shown in Figure 6a. The value of $\phi_0 = 0$ gives a flutter speed of approximately 42 knots, compared with the experimental value of 48.1 knots; a calculation using the value of

$\phi_0 = 10°$ gives almost exactly 48 knots. It is interesting to note that the Kutta condition modification alone (e.g., without any simultaneous correction on C_{ℓ_α} and x_{cp}) gives computed results that agree with the actual hydrofoil flutter velocity of 48 knots only for the somewhat unrealistic value of $\phi_0 = -30°$ (Ref. 4). Even for $\phi_0 = 0$, however, these new results are quite good, especially if one considers that we have spoken only of the flutter speed corresponding to $g = 0$; in fact, based on zero forward speed tests, the damping of the SwRI flutter model was estimated to be $g_h = 0.016$ and $g_\alpha = 0.042$ in air, and is roughly estimated to be $g_h = 0.070$ and $g_\alpha = 0.047$ in water.† Calculated data corresponding to the critical frequency are also shown in Figure 6a, giving a value of $\frac{\omega}{\omega_\alpha} = 0.82$ and $\frac{1}{k} = 1.34$ (for $g = 0$) which are in good agreement with the measured values.

Case 6b is based on the use of Yates' calculated data for C_{ℓ_α} and x_{cp} (Table 3b), with V-g curves corresponding to several values of ϕ_0 shown in Figure 6b, along with data for critical frequency. The value of $\phi_0 = 0$ (for $g = 0$) gives a flutter velocity of 45 knots and $\frac{\omega}{\omega_\alpha} = 0.82 \left(\frac{1}{k} = 1.43\right)$. These are again in very good agreement with the measured values.

Finally, Case 6c differs from Case 6a only in that the center of pressure is taken as $x_{cp} = -\frac{1}{2}$, with the results being shown in Figure 6c. Again, with $\phi_0 = 0$ and $g = 0$, the flutter speed is 45 knots and $\frac{\omega}{\omega_\alpha} = 0.82$ $\left(\frac{1}{k} = 1.43\right)$.

†Data provided by Mr. Guido E. Ransleben, Jr.

DISCUSSION AND CONCLUSIONS

A summary of the results obtained from the various cases studied is given in Table 4. Of these, only Case 6 gives results that are closely comparable with the measured values of flutter speed and frequency.

There can be no question that variations in lift curve slope and in center of pressure location, as suggested by Yates (Ref. 15), have a most profound influence on the flutter speed and frequency of a foil having parameters similar to those of the SwRI flutter model (Ref. 1). While the calculated results presented in this report are not in exceptionally close agreement with those of Yates, it appears this may be the consequence, in part, of sensitivity of the computations and also of the use of measured modes in the present calculations and of calculated modes by Yates. Both sets of results, however, lead one to conservative predictions, and this in itself is a significant advance over practically all previous results. The difference between using the measured values of lift curve slope and the center of pressure location and that of using those calculated from lifting surface theory is apparently not large. Yates' method certainly deserves further study and application.

The semiempirical method based on generalization of the Kutta conditions (Refs. 8, 9, 21), and used here in conjunction with lift curve slope and center of pressure modifications, appears to give excellent results (Case 6). As with the Yates' method, the differences arising from various C_{ℓ_α} and x_{cp} values are small. An essential point is again that the flutter predictions are conservative, as well as being in close agreement with the measured flutter speed and frequency. It would mean that the applications of the procedures employed in Case 6 to other hydrofoil flutter studies would be quite useful and revealing, if done prudently.

The role of structural damping could be significant; in Case 6, the inclusion of small positive damping would serve to improve even further the agreement between calculated and measured flutter speed.

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TABLES

TABLE 1a. SwRI FLUTTER MODEL-MEASURED MODE SHAPES (IN AIR)

	y/L	f_h	f_α
Root	0	0	0
	0.1	0.025	0.213
	0.2	0.075	0.403
	0.3	0.158	0.570
	0.4	0.260	0.710
	0.5	0.370	0.825
	0.6	0.488	0.915
	0.7	0.610	0.966
	0.8	0.738	0.990
	0.9	0.870	0.999
Tip	1.0	1.000	1.000

TABLE 1b. SwRI FLUTTER MODEL CHARACTERISTICS (Ref. 1)

Model Parameter	Measured Value
Aspect ratio	5.00
Semichord	0.50 ft
Elastic axis location, a	-0.50
Center of gravity location, x_α (from a.e.)	0.524
Radius of gyration, r_α^2	0.512
Bending stiffness*, EI	3.40×10^6 lb-in^2
Torsional stiffness*, GJ	0.973×10^6 lb-in^2
Frequency ratio, ω_h/ω_α	0.490
Torsional frequency, ω_α	20.5 cps
Total weight (wing only)	121.2 lb

*The calculated ω_α and ω_h are about 18.7 cps and 11.2 cps. The calculated ω_h/ω_α is about 0.60. However, the measured frequencies are possibly more accurate than the calculated values based on measured EI and GJ.

TABLE 2. VALUES OF OSCILLATORY CENTER OF PRESSURE LOCATIONS IN SEMICHORDS (Ref. 6)

$1/k$	$\bar{k}$	$2\tilde{x}_{cph}$	$2\tilde{x}_{cp\alpha}$	x^*_{cph}	$x^*_{cp\alpha}$
0.4	2.5	0.85	0.92	0.54	0.61
0.8	1.25	0.630	0.76	0.32	0.45
1.0	1.00	0.560	0.70	0.25	0.39
1.5	0.667	0.480	0.58	0.17	0.27
1.8	0.556	0.48	0.53	0.17	0.22
2.0	0.5	0.48	0.50	0.17	0.19

TABLE 3a. EXPERIMENTAL VALUES FOR STEADY STATE LIFT CURVE SLOPE C_{ℓ_α} AND CENTER OF PRESSURE, x_{cp} (Ref. 6)

y*	C_{ℓ_α} (rad.)	x_{cp} (semichords)
0	4.01	-0.522
0.1	4.01	-0.522
0.2	4.01	-0.536
0.3	4.01	-0.540
0.4	4.01	-0.544
0.5	4.01	-0.552
0.6	4.01	-0.564
0.7	3.724	-0.580
0.8	3.438	-0.596
0.9	2.58	-0.632
1.0	0	-0.640

TABLE 3b. THEORETICAL VALUES FOR STEADY STATE LIFT CURVE SLOPE C_{ℓ_a} AND CENTER OF PRESSURE x_{cp} FOR THE SwRI FLUTTER MODEL

y*	C_{ℓ_a} (rad.)	x_{cp} (semichords)
0	4.85	-0.531
0.1	4.84	-0.531
0.2	4.80	-0.533
0.3	4.71	-0.535
0.4	4.60	-0.536
0.5	4.44	-0.542
0.6	4.21	-0.550
0.7	3.86	-0.563
0.8	3.36	-0.578
0.9	2.54	-0.605
1.0	0	-0.646

TABLE 4. SUMMARY OF RESULTS

Case No.	Brief Description	Flutter Speed, V_f (knots)	Critical Frequency, ω/ω_α
Experiment	SwRI flutter model ($b = 0.5$ ft, $\omega_\alpha = 20.5$ cps)	48.1	0.854
1	Reissner-Stevens Theory	108	--
2	Exp. (steady) x_{cp}, $C_{\ell_\alpha} = 2\pi$; $\sigma_j \cdot j = \alpha, h$	67	--
3	Unsteady $x^*_{cp_\alpha}$, $x^*_{cp_h}$	70	--
4a	Exp. (steady) x_{cp}, C_{ℓ_α}; $\sigma_j = 0$	100	--
4b	Exp. (steady) x_{cp}, C_{ℓ_j}; σ_j	76	--
5a	Yates Theory; Yates x_{cp}, C_{ℓ_α}	30	--
5b	Yates Theory; exp. (steady) x_{cp}, C_{ℓ_α}	26	--
6a	Exp. (steady) x_{cp}, C_{ℓ_α}; Generalized Kutta condition, $\phi_0 = 0$	42	0.82
6b	Yates x_{cp}, C_{ℓ_α}; Generalized Kutta condition, $\phi_0 = 0$	45	0.82
6c	Exp. (steady) C_{ℓ_α} ; $x_{cp} = -1/2$ Generalized Kutta condition, $\phi_0 = 0$	45	0.82

ILLUSTRATIONS

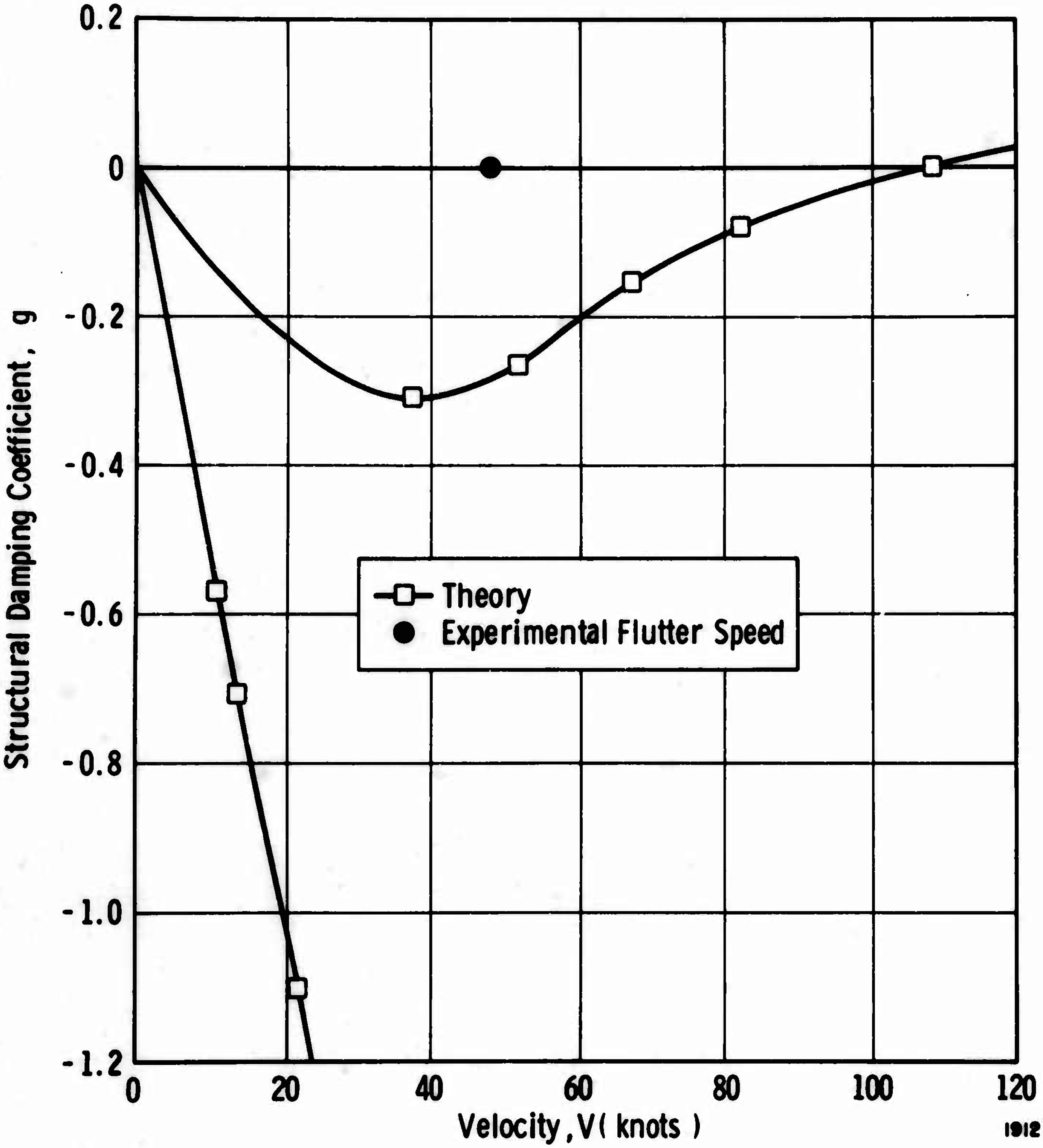

Figure 1. V-g Curves For Case 1 (Reissner - Stevens Theory)

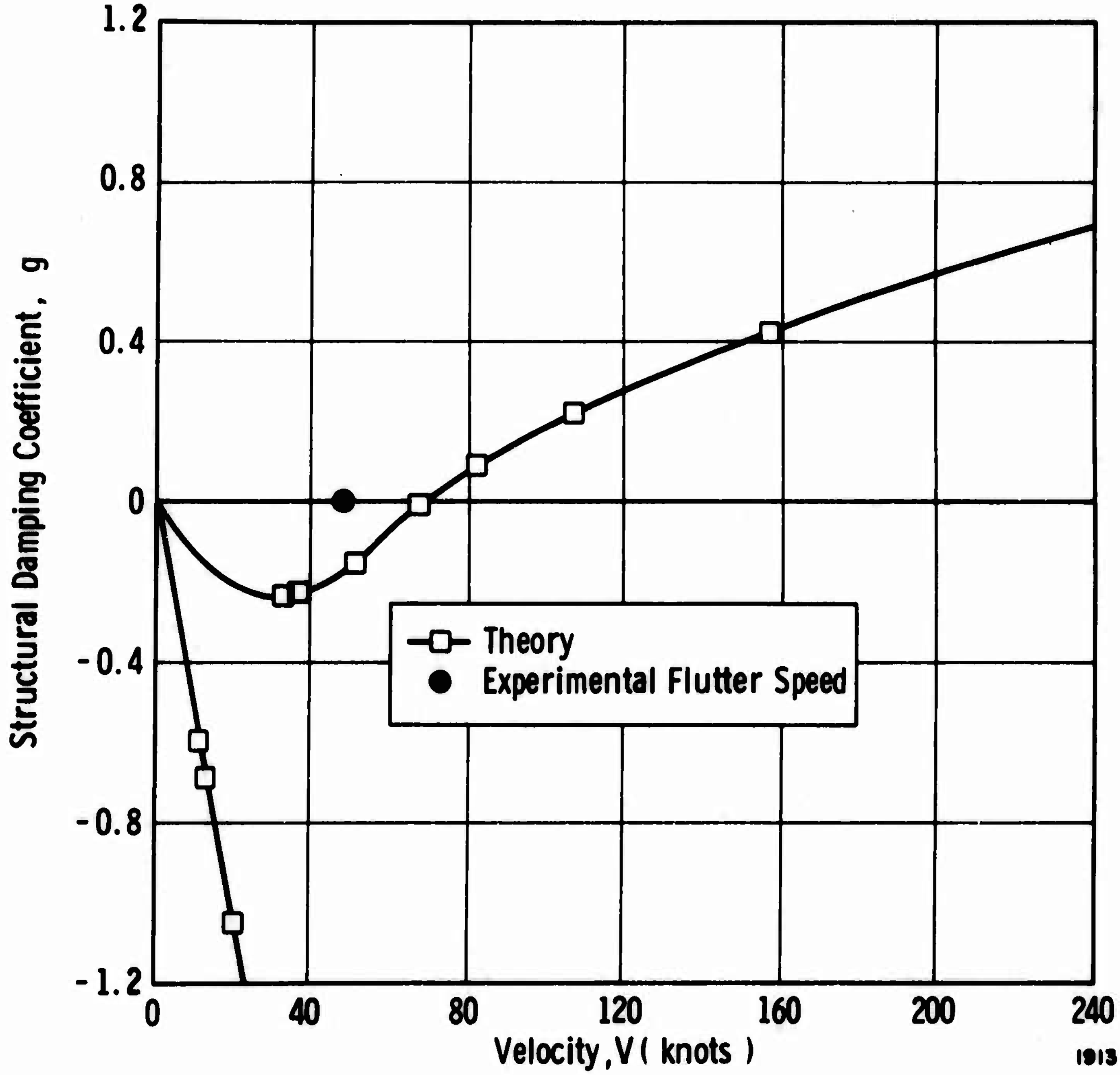

Figure 2. V-g Curves For Case 2

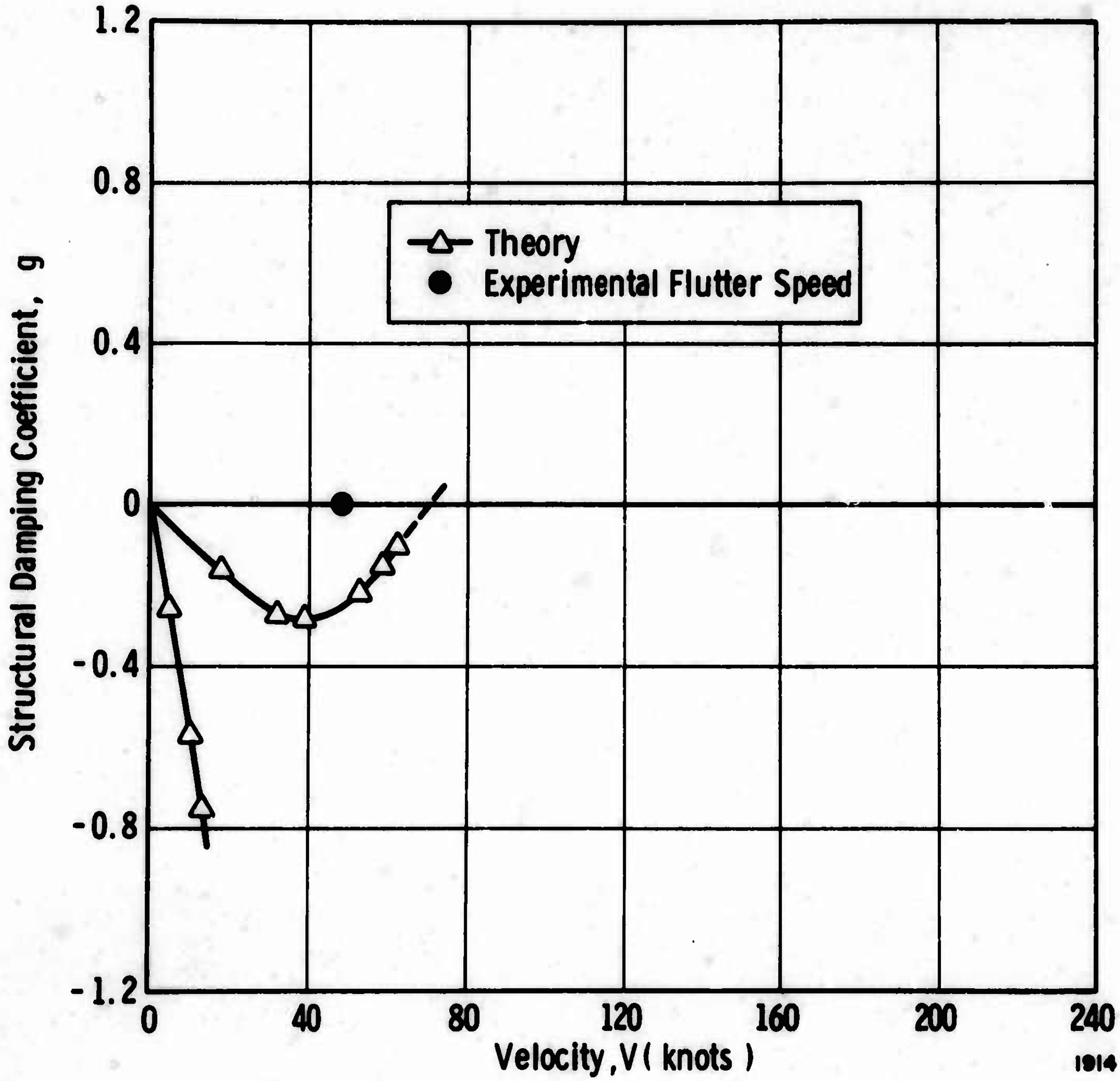

Figure 3. V-g Curves For Case 3

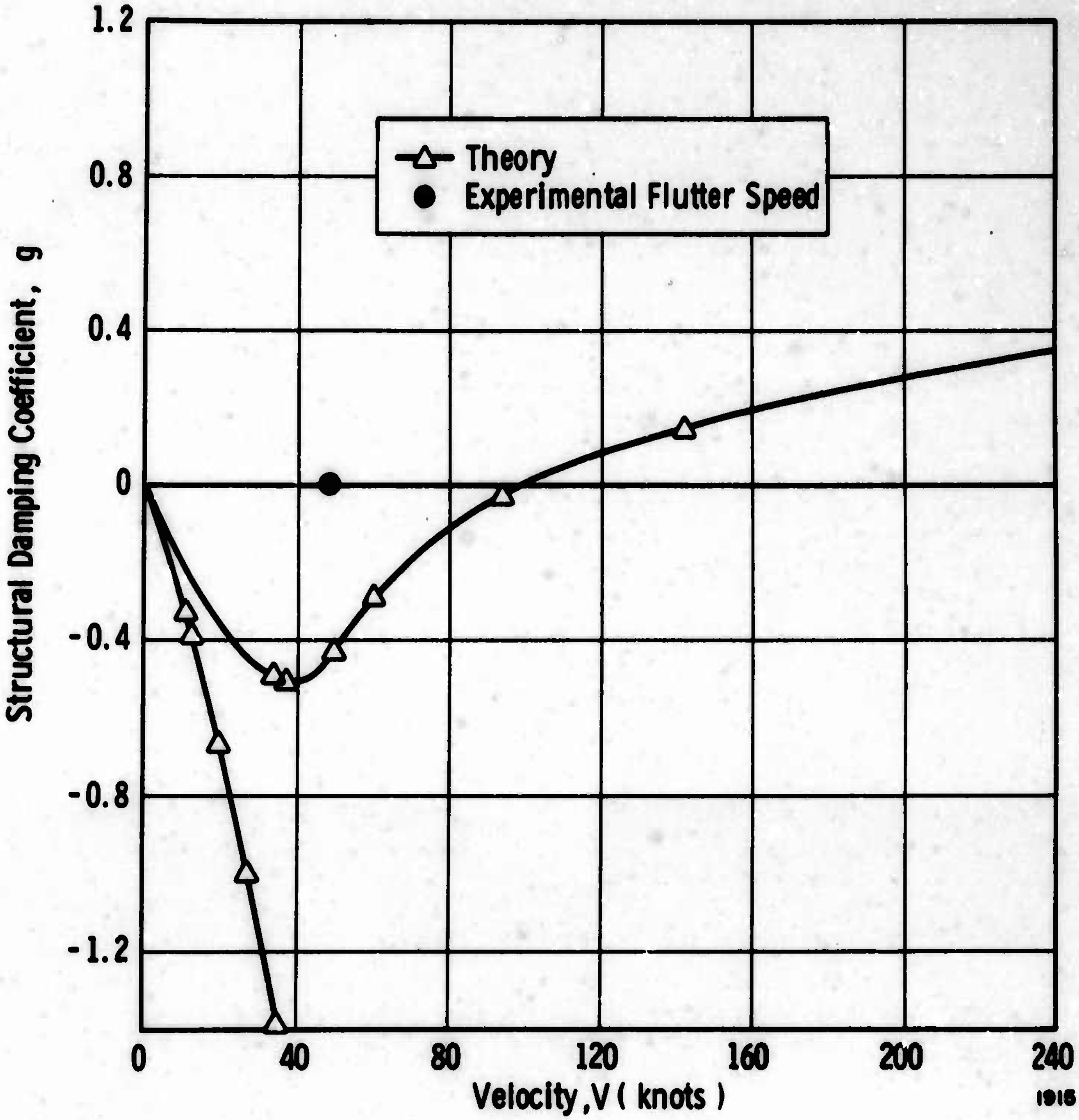

Figure 4a. V-g Curves For Case 4a

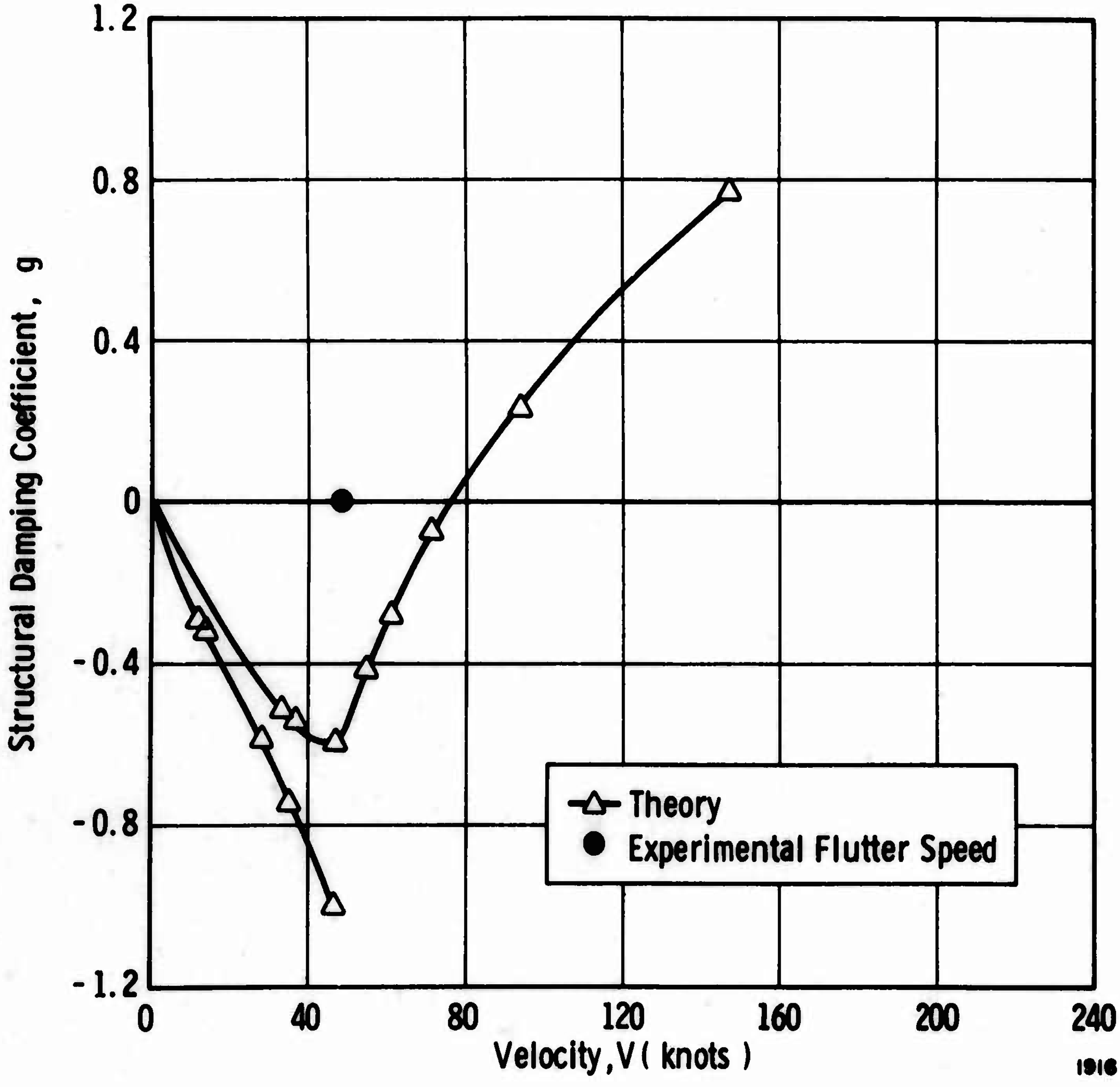

Figure 4b. V-g Curves For Case 4b

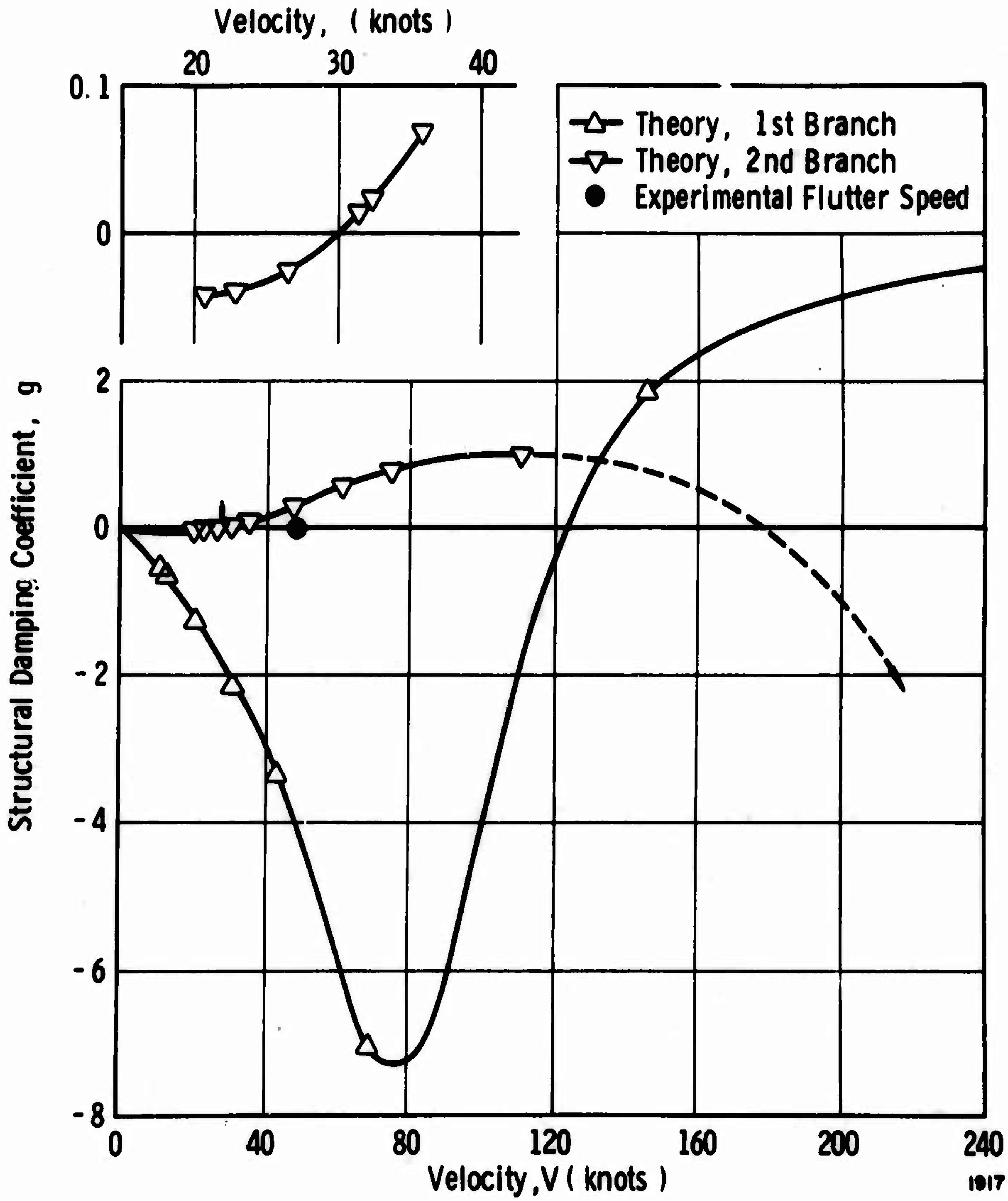

Figure 5a. V-g Curves For Case 5a

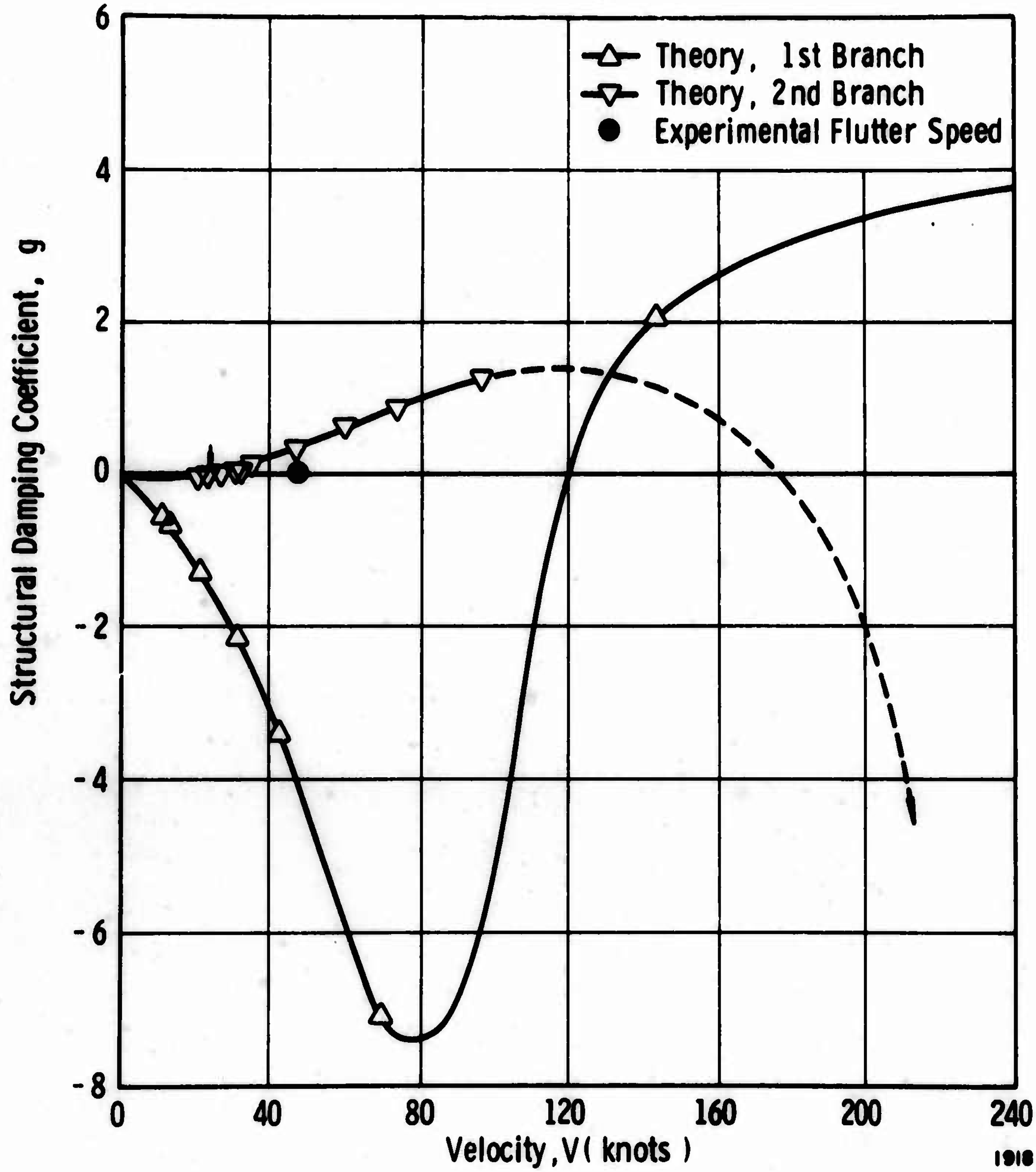

Figure 5b. V-g Curves For Case 5b

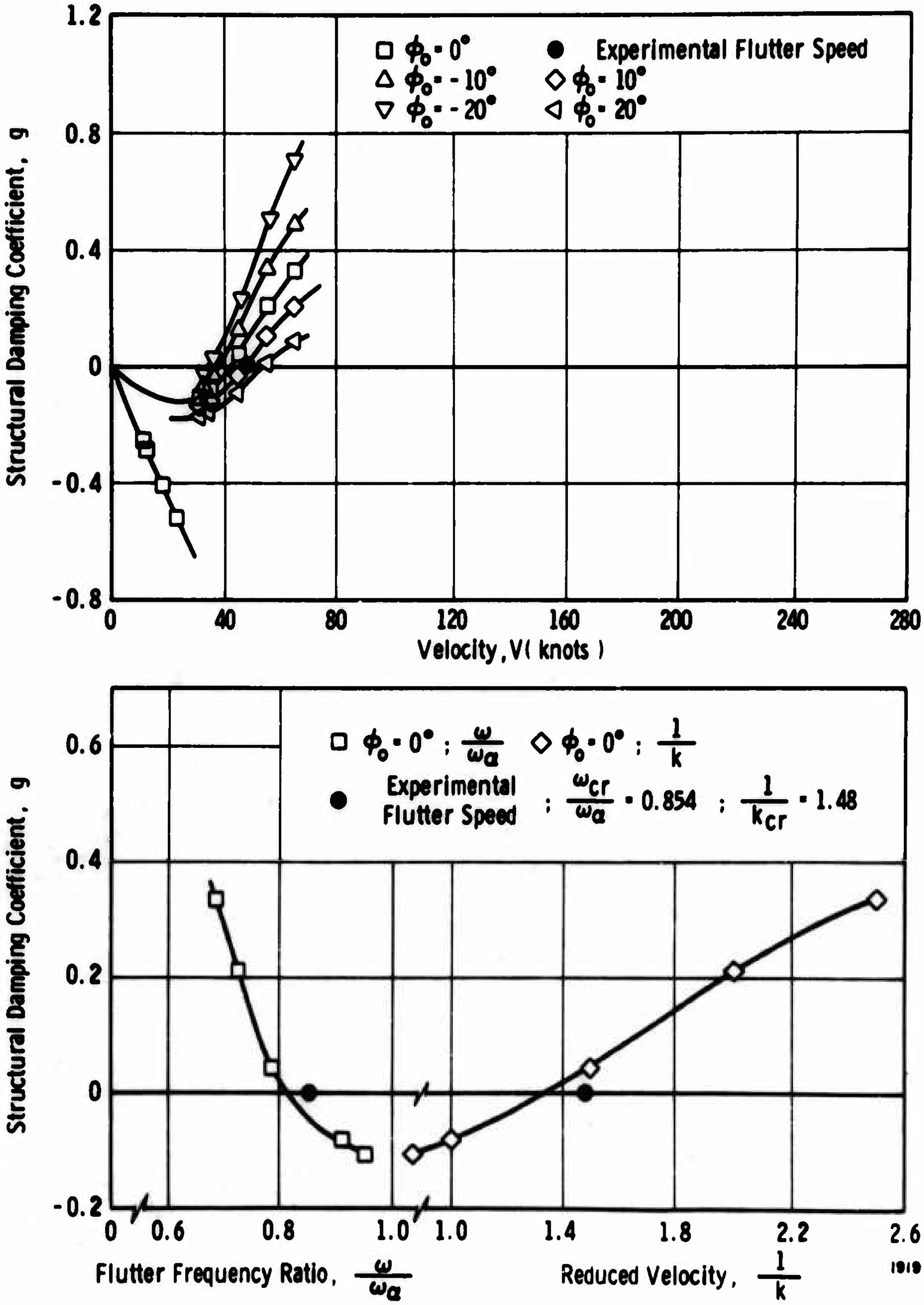

Figure 6a. V-g Curves, $\frac{\omega_{cr}}{\omega_\alpha}$ And $\frac{1}{k_{cr}}$ Curve For Case 6a

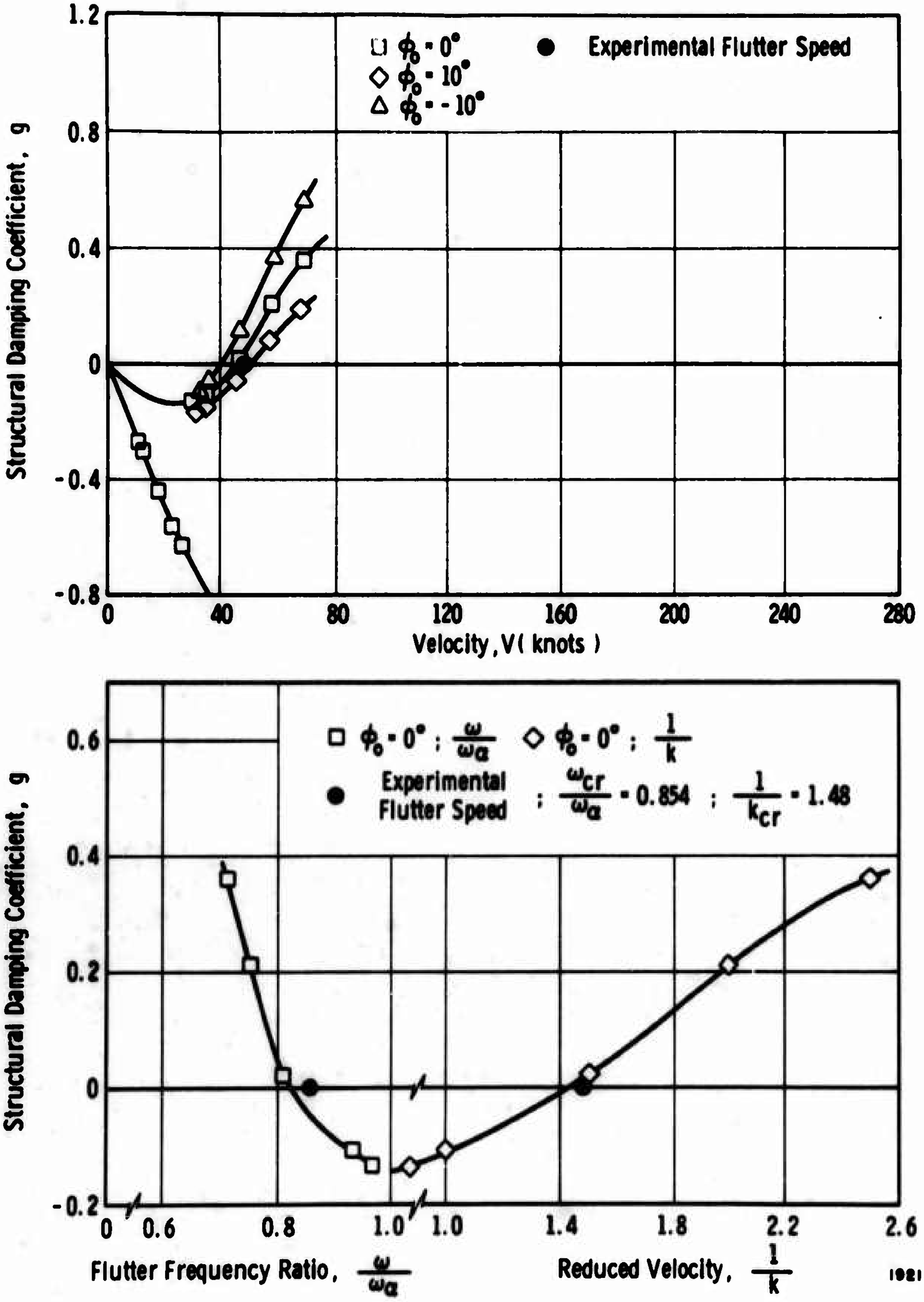

Figure 6b. V-g Curves, $\frac{\omega_{cr}}{\omega_\alpha}$ And $\frac{1}{k_{cr}}$ Curve For Case 6b

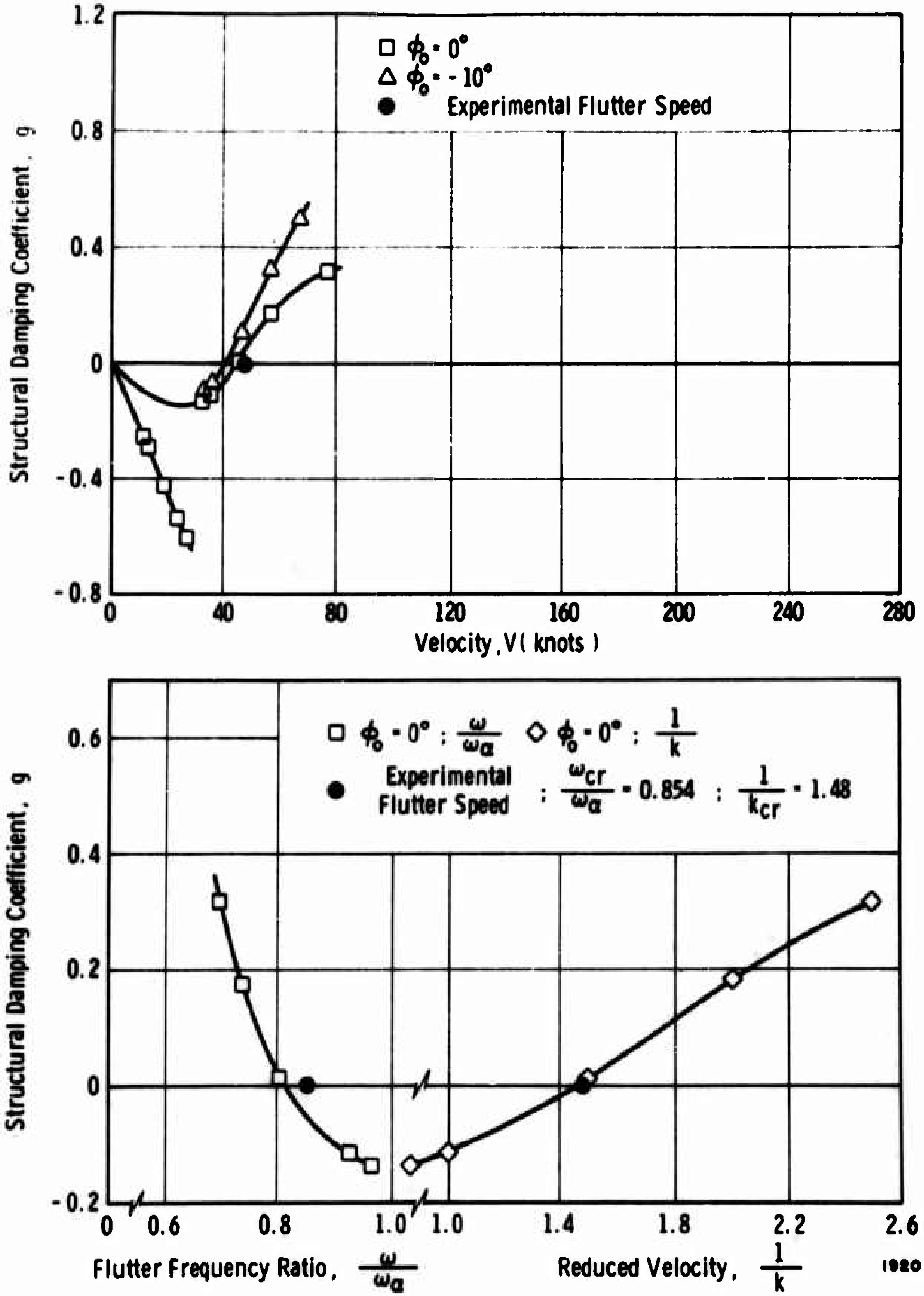

Figure 6c. V-g Curves, $\frac{\omega_{cr}}{\omega_\alpha}$ And $\frac{1}{k_{cr}}$ Curve For Case 6c

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13. ABSTRACT

Some new calculations of the flutter speed of the SwRI fully submerged subcavitating hydrofoil model are presented. Variations in lift curve slope and in center of pressure location are found to have a most profound influence on both flutter speed and frequency. When variations in these parameters are combined with a relaxation of the Kutta condition (proposed previously), excellent agreement with the measured flutter speed is obtained.

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