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MODELS AND TECHNIQUES FOR URBAN PLANNING

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FINAL REPORT

By: Douglass B. Lee, Jr.

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PREFACE

This report was begun while the author was employed by the Cornell Aeronautical Laboratory, Inc., in the summer of 1967, and was completed in the following academic year at Cornell University. The intent of the report was to provide an information base for primarily technically skilled people who might be interested in applying quantitative methodology to the modelling of urban phenomena. Thus we have tried to indicate how techniques might be applied, and also how they have been applied in the past to urban problems. Land use models were selected as the main area of interest, but the topic is extremely open-ended, and difficult to define, with the result that the report lacks a certain kind of elegance. This we regret.

Chapters 1-5 proceed from the general to the specific. Chapter 2 describes a number of abstract attributes of models which provide a framework for further discussion. Chapter 3 sketches out a variety of concepts and techniques that seem to be relevant to urban models, and attempts to relate these concepts to modelling of social activities. Chapter 4 is an overview of the modelling process, emphasizing those aspects that are critical to urban modelling. These three chapters abstract as much as possible of what can be said about urban modelling, with the 5th chapter being a description of the specifics of several applications. Finally, Chapter 6 gives a summary and suggested further work.

Valuable information and comment was received from a large number of people, but a particular few devoted considerable energies toward improving the report. Draft copies were reviewed and extensively noted upon by Robert Stevens, Edwin Roth, Sol Kaufman and Matt Rosenshine, all of Cornell Lab. Profitable discussions were also had with Fred Smith, of Cornell Lab., and Dr. Wilbur Steger, of Consad Research Corp. Ultimate responsibility, of course, lies with the author, and the opinions, omissions, and errors are my own.

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CHAPTER 1: INTRODUCTION

The intuitive appeal of using a model for urban planning purposes is the possibility of being able to test different policies and programs before they are implemented. In a manner analogous to the testing of a model airplane in a wind tunnel, a mathematical model of a city could be used to predict the consequences of different combinations of public actions and exogenous events. Public actions include urban renewal, taxation, transportation facilities, water, sewer, and other utilities, zoning, building code enforcement, mortgage and other credit availability, anti-discrimination efforts, employment opportunities, and so forth. Consequences of these actions which might be of interest are areas of population growth and physical development, the level of prosperity and the distribution of income, the level of employment, the quality of housing available to different subgroups of the population, the aesthetic quality of the environment, the responsiveness of the system to the needs of the people, the locations of sources of employment, the efficiency of the transportation system, the social costs of less-than-ideal conditions, etc. While there is little possibility of being able to answer many of these questions with present knowledge, the mathematical model seems to be the most likely vehicle if we are ever to answer them.

Direct effects of many programs are readily predictable, and provide the rationale for the program, but the programs often fail to achieve their aims or introduce new problems because of secondary effects. For example, a new expressway may decrease travel time by five minutes and require relocation of 3000 families, but what are the effects of the former on downtown sales volume and of the latter on housing quality? The replacement of a manufacturing firm employing 500 people by a central office employing 2000 results in a net increase of 1500 jobs, but what effect does the shift in occupational mix have on travel patterns? Many conflicting tendencies influence the outcome, and therefore a quantitative approach is necessary if results are to be obtained.

A. Planners

Among the large set of activities that might be called human behavior, there is one which is planning. Within the set of planning activities we may distinguish individual planning and group planning, and the latter set may be further divided into private (e.g., corporate) planning and public planning. Part of the planning done by public groups is for the purpose of maintaining the organization, and is thus similar to private group planning; other public planning is done for public groups, and this is social planning.

Social planning can be distinguished according to level, e.g., national, state, and local. National planning is typically economic and social, affecting such things as income, income distribution, interest rates and inflation, through the mechanisms of taxes, the money supply, the discount rate, and capital expenditures. At lower levels are regional and city planning activities, which operate through different mechanisms and are more physically or spatially oriented. Regional planning is a somewhat vague term, since it may mean anything from supranational planning to county planning, but we will reserve the term for single urban agglomerations and the surrounding area. Generally this includes a central city, its suburbs, and some of the adjacent sparsely settled area.

An urban planner is a person who, by virtue of special training and experience, is called upon to present the long-range interests of the community as a whole to those who make decisions for the community, presumably its elected representatives. To what extent the planner is justified in making this claim, and the means by which he should perceive and pursue community goals, has not been resolved, either within the profession or without, but this need not be clarified for our purposes.* At the very least, the planner is an information-processor, and it is his information-processing activities that we wish to consider.

* see, for example, Alan Altshuler, "The Goals of Comprehensive Planning," Journal of the American Institute of Planners, Vol. XXXI, No. 3 (August 1965), pp. 186-194.

Good decisions are likely to be based on good information. For example, the decision of where and when to begin an urban renewal project should be based upon the probable effects of the projects under consideration, and these should include a full range of alternatives. But the benefits gained by use of better information must be weighed against the costs of acquiring the information, including the time delay required. The information must also be in a reduced form so that it can be readily understood, but it should not be so distilled that important variation is suppressed. The amount of detail retained depends upon the decision to be made.

Gauging the effects of a project on a city means predicting the future, and doing so with enough accuracy so that different projects can be compared with each other and also with the course of events if no project is undertaken. That we cannot know the future is irrelevant; we can make reasonable estimates based on likely conditions, and hopefully our estimates will improve with experience. What is important to realize is that we can make better decisions by basing them on reasonable estimates, rather than on no estimates at all. Weather predictions are often wrong, but they are still useful in planning our activities. It is not idle curiosity that prompts people to ask what the weather report is.

The feature that allows us to attempt to predict the future, and is at the same time the source of our problems, is interdependence. If all events -- past, present, and future -- were unrelated, then there would be little information that could be obtained for prediction. The state of the environment could only be changed by direct action on the particular aspect to be changed. Planners have discovered, however, that everything depends upon everything else. Not only is it possible to affect aspects of the environment indirectly, there is a difficulty of affecting too much of the environment. Numerous governmental programs have achieved unexpected results, which were unexpected because of a failure to perceive relevant interrelationships.

The complexity created by the degree of interdependence between parts of the urban system requires the use of sophisticated techniques. The mind cannot retain, at one time, all the possible ramifications of even a single urban renewal project, let alone a comprehensive program. One approach is to consider relationships one by one, and do partial analyses which hold most factors constant. Once the relationships that describe the urban system are established, they can be linked together and some mechanical device, such as a computer, can trace out all the effects.

B. Models

A model is an abstraction of reality. The purpose of a model is usually to predict the consequences of a real-world action, and its success in this regard depends upon the relevance of the abstraction. An architectural model abstracts visual qualities of a proposed building, reduced in scale, to predict how the real building will look when it is built. Boyle's Law is a model that can be used to predict the pressure of a gas in a closed container as temperature increases. Ackoff suggests a three-way classification of models: Iconic, which are pictures or reproductions of physical aspects of reality; Analogue, which are representations of reality that are visually different from the reality, and Symbolic, which represent reality by means of mathematical symbols and equations. We are concerned only with symbolic models, and will use the term "model" to mean "symbolic model".

Models have two areas in which they are useful. The first is in the area of problem solving. Ackoff describes a "decision" model as

$$V = f(X, Y)$$

where V = the value of the outcome; a measure of desirability of the result,
 X = the set of variables over which the decision-maker has control,
 Y = the set of exogenous variables, or states of the environment,
and f = a function which relates the values of the variables to the
 value of the outcome.

Assuming that X is not a null set, the decision-maker seeks to find values of the X variables which will maximize V .

The second important area in which models are useful is in the development of theory. Research proceeds in three phases -- observation, generalization, and experimentation -- which are generally carried on either simultaneously or cyclically. Generalization is an inductive process linking observation to theory, and experimentation is a deductive process that extracts the implications that are inherent in a theory. Models are a means for stating theory in workable form, deriving hypotheses, and testing the hypotheses by comparison with observations of the real world. Often theory is already stated in model form, but the theory is not testable because the variables in the model are not measurable or at least not readily observable. Thus the value of a model is to state a theory in testable form.

If models are to be used, however, in the development of new urban theory, it is necessary to make some order out of chaos. At present there is almost no consistency in the terms used to describe models, the approaches taken in model construction, or the purposes to which they are put. Many of the participants in the scene seem to have prejudices that color their attitudes and make interpretation and evaluation of their contributions difficult. The jargon is obscure to those not well versed in the language of operations research, systems analysis and mathematical statistics. But knowing the jargon is not sufficient because there is little accepted pattern to the way the words are used in modelling discussions. This report is an attempt to survey the area in some depth and evaluate a number of pertinent efforts for their progress and their contributions.

CHAPTER 2 : THE TYPOLOGY OF MODELS

Almost inevitably when some kind of mathematical model is discussed in the literature, the writer describes the model in general terms, using an explicit or implicit typology. These typologies may extend a layer or two in depth, and the model is plugged into one of the resulting niches. Common terms used for this purpose are simulation, dynamic, normative, decision model, linear programming, iterative, probabilistic, ad hoc, regression, and econometric, to mention a relatively small number. A few writers, somewhat more introspective than the others, have found it necessary to explain the typology they are about to use, or generate a new one to help explain the model or models about to be discussed.

Many model typologies are redundant, parallel to other typologies, partially overlap other typologies, or cover only a small portion of the total set of models. In order for a typology or classification to be useful, it should have the following attributes:

- (1) The list of categories should be mutually exclusive. It should not be possible to place a single observation (model) in two categories simultaneously.
- (2) The list of categories should be exhaustive. Every observation regarded as being one of the major type being considered (e.g., a model) should fit into at least one category.

In the language of set theory, a set of classes that are disjoint and also span the space are said to partition the space. Thus any classification scheme should partition the set of objects that are within the domain of the sample space, or larger set. That this should be so is obvious in the abstract, but difficulties arise in the straightforward application of the partitioning notion.

In the process of partitioning, clearly recognizable levels may appear. The space of all money can be initially partitioned according to the nation issuing the money. The subset of United States currency can be further partitioned into paper money and coin, and the sub-subset of U.S. coins includes the set of nickels. This kind of partitioning produces a hierarchy, in that every observation can be placed first in the largest set and subsequently into subsets, each diminishing in size down to the lowest level of partition. In graph theory terms, this partitioning can be diagrammed as a tree. The classification of plants and animals follows a scheme of this kind. Figure 2.1 shows a Venn diagram of the money space and a tree diagram of a simplified biological taxonomy.

In some cases, an object can be completely described, for all practical purposes, by using a single partition. More often, however, it is desirable to classify objects according to several different partitions of the same space. The space of people can be decomposed according to nationality and according to sex, but it is no longer obvious which partition is the higher, i.e., comes first. Do we break the population into male and female, and then into subsets by nationality, or do we use the nationality breakdown first? The result can be represented by a tree or a Venn diagram either way, and the classification is also unambiguous, but there is an important difference between this and the hierarchy type. Once a partition is made, other relationships between the members of the subsets are ignored. Using a tree diagram, only one partition at the same level is made explicit; any others are obscured. The Venn diagram can illustrate several partitions at the same level, but it too has limitations.

The difficulty is that each classification that is made at the same level must be represented graphically by a new dimension. Topologically, more than three binary partitions can be represented on a sheet of paper, but it is awkward. The number of independent classificatory schemes already suggested for mathematical models is manifold, and consequently no device exists for graphically portraying the totality in a single space. The process of typing models must be done, in effect, by projecting the space of all models onto a plane or a line.

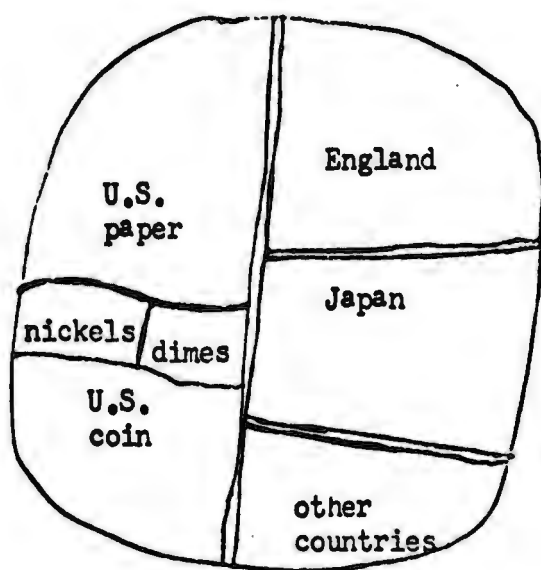


Figure 2.1a. Partition of money space.

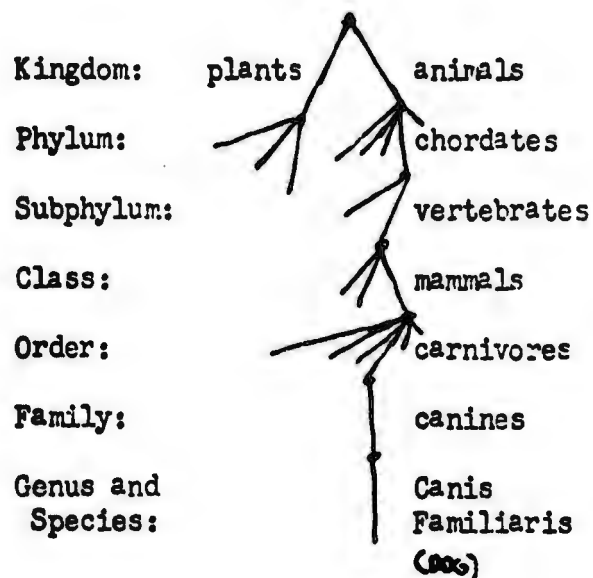


Figure 2.1b. Classification of Dog.

A typology is an artificial construct superimposed on a set of entities to create an illusion of order, and in most cases there is no unique or obvious way of doing this. Models are particularly difficult to classify for several reasons. First of all, models contain models as components, and these models may have lesser models; a classification that works at one level is likely not to work at another. Secondly, models have many attributes, and unless a typology excludes the majority of them there will be many observations that fall into one class according to some attributes and into another class according to other attributes. Models tend to lie along a continuum, no matter what characteristic is used, and thus the classification problem becomes one of finding clusters of observations to form the nuclei of different classes.

While the primary attributes of a good typology have been mentioned above, there are other less rigorous but also important properties. If the entities are to be classified along several scales, or dimensions, it is desirable to have these dimensions as independent and orthogonal as possible. If two schemes sort most or all entities into the same groups, then the schemes are parallel or redundant and a single scheme could be used. Another property, which might be called efficiency, reflects the desirability of having the entities spread out over classes in a fairly uniform way, without a lot of empty classes. For example, the four-way partition of households resulting from classification by number of children and marital state contains one subset -- unmarried households with children -- that has a relatively small number of members. This classification is inefficient, since it places most observations in three categories (unless, of course, the fourth category is of particular interest).

There will be no attempt here to construct a definitive typology of urban models, but considerable progress in this direction can be made. Figure 2.2 is a diagram of the modelling process. Its purpose is to call attention to the different points where important decisions are made regarding an urban model, and to provide a means of grouping the discussion of various

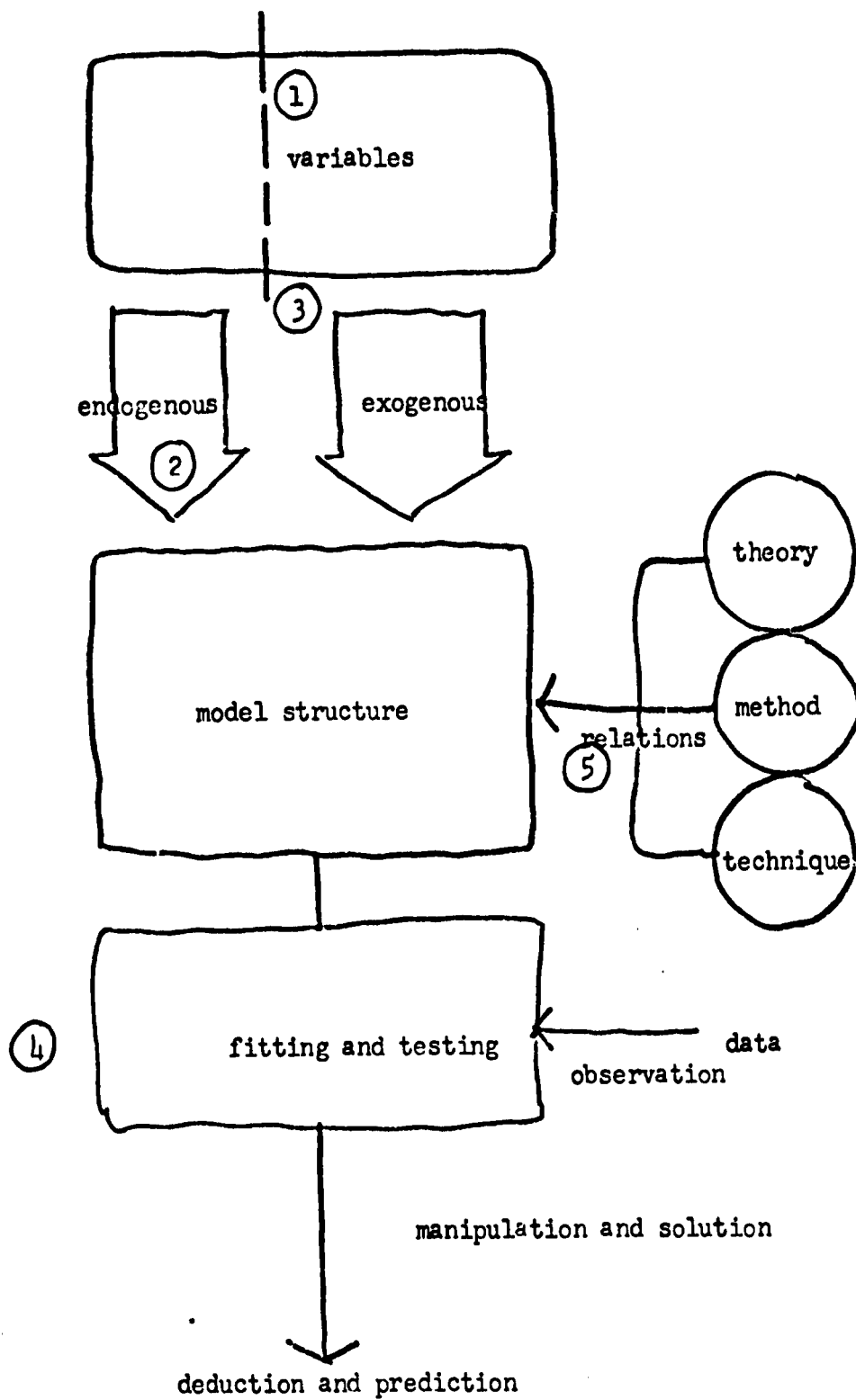


Figure 2.2. The Modelling Process

aspects of modelling. While the sequence of activities proceeds generally from top to bottom, there is so much feedback from later stages to earlier ones that the process can be considered as going on all at once.

Variables must be separated into endogenous (those which may be altered within the model) and exogenous (those accepted by the model as given). Linkages are established between the endogenous variables, and between the model and the exogenous variables, and theory, method, and technique can be drawn upon for this purpose. Observations of the real world are needed to establish and test specific relationships and to provide initial values for the variables, and then the model can be solved or manipulated for desired results. Each of these aspects will be described in more detail in subsequent chapters.

The remainder of this chapter is used to describe the primary dimensions along which models can be classified: comprehensiveness, aggregation, treatment of time, abstraction, and the descriptive-behavioral-normative scale. Urban models can be arrayed along each of these scales, and grouped into classes where it seems appropriate to do so. We have not tried to partition all five dimensions, but have been content to describe the dimensions themselves. Many other typologies have been presented in the literature, and a number of them are summarized in Appendix A for comparison with the scheme given here.

A. Dimensions

1. Comprehensiveness

The more comprehensive a model, the more relationships and variables from the real world that are included in it. While a model that used population by census tract might be considered more comprehensive ceteris paribus than one which used population by mile-square grid zones, we prefer to make this distinction on the basis of the level of aggregation rather than comprehensiveness. In general, the more economic sectors, population components, land uses,

activities, and movements, etc., that are endogenous to the model, the more comprehensive the model can be said to be. Whether a variable is a new one or part of a disaggregated old one may not be a very critical question, since either one tends to increase the complexity of the model and require more relationships. Thus there is some redundancy between comprehensiveness and aggregation, but the former refers primarily to breadth of relationships in the model, and the latter refers to depth and detail in relationships.

Equivalent to specifying the comprehensiveness of a model is the act of dividing the universe of variables into endogenous and exogenous sets, and this is shown in Figure 2.2 at ①. The model's window to the world is contained in the specification of its exogenous variables and the initial values given its endogenous variables. All exogenous variables left unspecified are therefore implicit, acting on the model through the explicit variables. It is often a very enlightening experience to place oneself inside a model, looking out through its window, and try to see what can be figured out knowing only what the model knows.

2. Disaggregation

No two individuals are alike, but it is necessary from the point of view of urban models to treat many individuals in groups, and assume that the individuals in any particular group are indistinguishable. Since the process usually starts with a single group of at least partly heterogeneous membership, and then divides the group into smaller and presumably different groups, the extent to which this is done is called disaggregation. Not only population, but land use, employment, mode of travel, etc., can also be disaggregated. Space and time are additional dimensions for disaggregating variables. A highly aggregative model might treat all households in the same way, while a highly disaggregated model might separate out a hundred or so different household types.

In the field of economics, theory is roughly divided into micro-economic and macro-economic. The former is built up from postulates of human behavior

and relates "small" units such as firms and households to each other, while the latter deals with large aggregates such as total income, consumption and investment in an institutional or accounting framework. While micro-economic theory is very abstract, it proceeds by aggregating or summing individuals to obtain group totals, and is regarded as being a behavioral area of study. On the other hand, macro-economic theory relies on properties of aggregate behavior that are not derived from individual behavior. For example, macro-economic theory postulates that, in the aggregate, households consume a fixed proportion of their income, but this does not preclude the possibility that some spend considerably more than their income in some time period. The simultaneous existence of two apparently conflicting consumption functions -- one from time series data and the other from cross-section or budget data -- has led to several different theories of individual behavior that might explain them.

The micro-macro dichotomy of economics is essentially the same as our level-of-aggregation scale, with a few minor differences that lead us to prefer the disaggregation term over the economic terms. One is that while the distinction between micro and macro in economics is often difficult, the difference is even less qualitative in the case of urban models. Micro-analytic models in economics tend to be axiomatic, behavioral and abstract, these traits do not apply particularly to their urban model counterparts. Another difference is that in economics the theory tends to cluster under the two categories, while the scale for urban models seems to be continuous. The model by Arthur D. Little, Inc., of the San Francisco residential market is considered to be micro-analytic because it handles over a hundred housing types separately, while the University of North Carolina model would be called macro-analytic (at least in this respect) since it makes no distinctions between different households. The Pittsburgh commercial and residential location model (TOMM) uses approximately half-a-dozen household types and is thus at an intermediate level of disaggregation.

3. Treatment of Time

The inclusion of time as a variable in a model may not be any different conceptually from the inclusion of some other variable. The comprehensiveness of a model could be increased by the addition of a time dimension or by the addition of a spatial dimension, and the discussion of which one to use, if either, would fall under the general question of comprehensiveness, i.e., the partitioning of the variable space into exogenous and endogenous sets. Time would simply be a special case. Traditionally, however, time has been important enough and special enough to warrant separate consideration, and this seems to be justified also for urban models.

The obvious static-dynamic dichotomy is enhanced by the addition of an intermediate category, both because the difference between static and dynamic is fuzzy at the boundary, and because many models that have been formulated would not be classifiable otherwise. The term for the intermediate group, from economic theory, is "comparative static," and provides for change over time without considering time as an explicit variable. Models of this type may be discrete approximations to a continuous-time model, or simply the comparison of two equilibria with an indeterminate amount of time between them. A discussion of the three temporal categories of models is easiest in the language of economic theory and equilibrium analysis.

A static equilibrium model requires only that there be a state of the system that satisfies all the conditions simultaneously. The process by which the system arrives at a state of equilibrium is not considered, nor are the consequences of disequilibrium. Since these questions are obviously important, particularly for models to be useful, very little economic analysis could be considered purely static. The first question is whether the equilibrium, when it exists, is stable or unstable, i.e., if disturbed, will the system return to equilibrium or diverge from it. The next step is to ask what happens if one of the variables or relationships

changes, and this is the core of the study of comparative statics. In many cases, however, comparative static questions cannot be answered without specifying the order in which adjustments are made (do suppliers respond to the old price and buyers to the new supply, or buyers to the old price and sellers to the new demand?). The next extension is to allow for responses in the present to be affected by several previous levels, i.e., more than one point in past history. Then the path the process has taken in arriving at the present becomes of interest to the model, and the result may be a difference equation model. Time is treated as occurring in discrete amounts, and the time reference is, in the abstract, an index taking on ascending integral values. This would now be considered a dynamic model. The logical extreme on what has been presented as the static-dynamic continuum is the continuous-time model in which the process is an explicit function of a single continuous variable (which should be equivalent to real time through a linear transformation). A differential equation is an example of a continuous-time model.

The Lowry model [327] and subsequent developments provide an example of how time is introduced into a model. Lowry's model, while basically of the static equilibrium type, explicitly considers how the process seeks an equilibrium and the order in which adjustments are made, with some slight potential for giving the mathematical convergence a time interpretation. It starts, however, with a blank map, and builds a complete metropolis in one shot. If the exogenous variables are changed, e.g., updated, a new metropolis will result without reference to any other previous model. A 1960 Pittsburgh could be generated which would never look like 1950 Pittsburgh in the process. Since the model starts from scratch each time, it must be regarded as a static model. Later development of Lowry's model, in the form of TOMM, allowed the model to begin at any previous known point and progress through discrete time periods on the basis of the starting distribution of activities and the predicted exogenous variables. Although the model settles at a new equilibrium each time period, it is a constrained equilibrium which approximates a continuous time process. The model could be formulated as a difference equation model, but it only regards a single point in the past at any particular moment, and it would be more appropriately called a comparative static model.

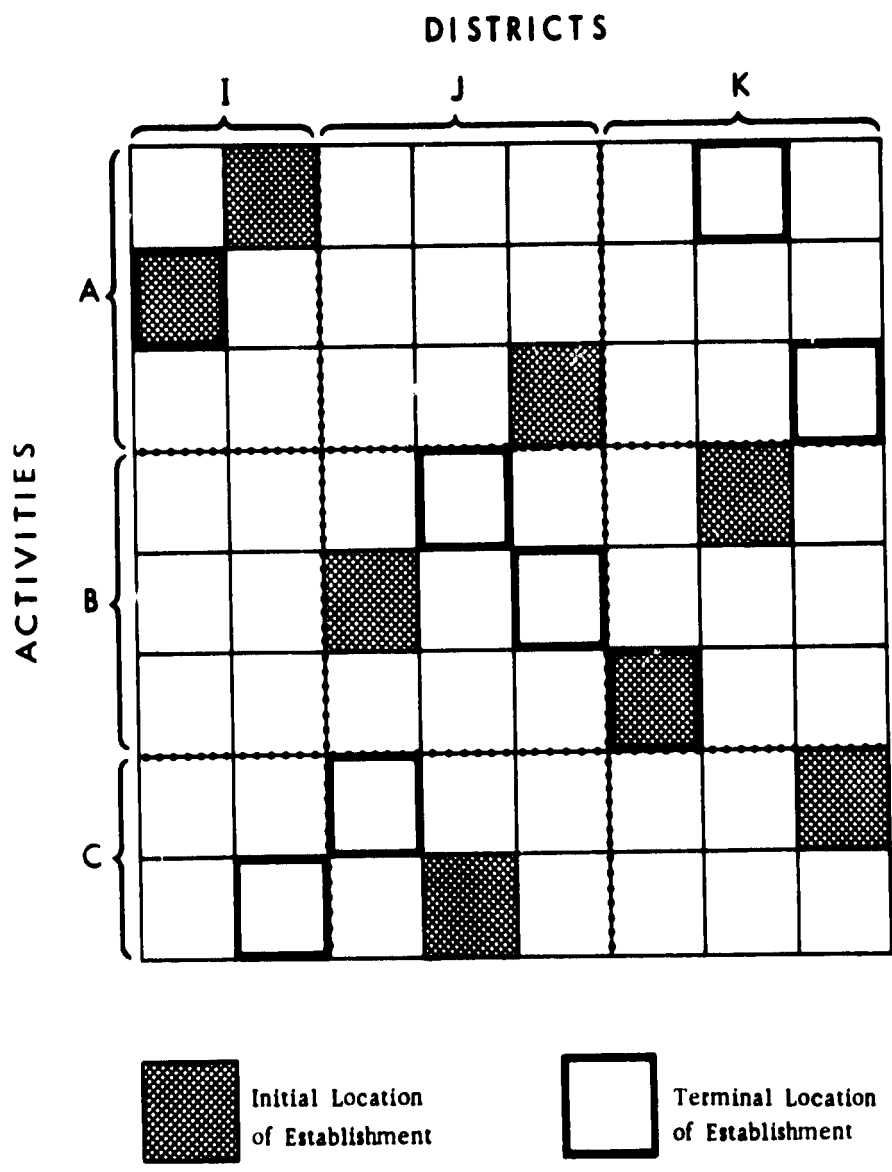


Figure 2.3 LOWRY'S PARADIGM

4. Abstraction

To say that a model is abstract is in one sense redundant -- all models are abstractions -- but the degree of abstraction is a quality that varies from one model to another. A high level of abstraction implies a high level of generality, which is to say that the model applies to many situations but not in great detail. Models of this type are also called theoretical. On the other hand, models at a low level of abstraction are likely to be applied to a specific situation. Conceptual models are those which have rather remote connections to reality, while models that actually work are called operational. Thus we have a number of labels for the same scale: general-specific, theoretical-applied, conceptual-operational, and level of abstraction. While distinctions between these terms can be made, they are not of great importance here.

At the present state of the art, abstraction and comprehensiveness tend to be somewhat parallel. Models that are comprehensive in scope are likely to be abstract because of the demands upon structural relationships, data and computational capacity which cannot yet be met, while operational models tend to be partial analyses for similar reasons. This relationship holds because of the difficulty of building operational comprehensive models, but it does not preclude the possibility of abstract partial models, of which there are many. This is a minor inefficiency in our typology which hopefully may be removed someday by being filled in.

5. Descriptive-Behavioral-Normative

This group of categories is the most widespread and commonly used means of labeling models. Although some models are bound to be combinations of structural relationships that fall into more than one category, the framework has generally been very workable and an excellent device for communication and discussion. The terms have also picked up a strong value connotation -- descriptive being the least desirable and normative the most -- that sometimes obscures the intent of the labels, but the damage is not serious.

Descriptive models are those which are based on an observed regularity without requiring an explanation for the regularity. The density gradient, or the gravity model, are examples of descriptive relationships. In many cases postulates of behavior are proposed in order to justify the use of a descriptive model, but usually there are a number of ways of doing so. Thus the descriptive regularity exists, and the attempts to explain it might be called honest rationalization or theory building. Urban theory at present is still largely in the descriptive stage.

Behavioral models are composed of relationships that purport to be cause-and-effect. A particular response is the result of a certain stimulus or set of stimuli. By knowing which variables cause which effects, the modeller can presumably predict the responses that will attend a different, and perhaps presently unobservable, set of stimuli. Thus predictive models, as opposed to simply projections, should be based on behavioral relationships.

A normative model establishes a pattern that is best or ideal on the basis of certain criteria. Underlying these models usually are some notions of completely rational behavior and simplifying assumptions about mobility and information, etc. Given the criteria for evaluating a solution and assumptions about relationships, the model describes an optimal strategy or allocation of resources. Although it would clearly be preferable, all other things being equal, to have a model that describes what should be done, as opposed to what will happen given conditions, or what actually is, the normative models are open to so many questions they do not presently have any particular advantage. Models which seek to maximize a normative criterion are also called "optimizing" models.

The first four dimensions pertain equally well to any level of model structure or any combination of models operating together. If two models are joined together to make one, the comprehensiveness of the resulting model must be at least as great as the comprehensiveness of either submodel, and is likely to be close to the sum of the submodels. The level of aggregation may vary between different components of a model, but

it is more likely to vary between sectors of the model. There is not much point in disaggregating land use into ten categories in one submodel if no other portion of the model can use more than three categories. Time must also be treated in a consistent manner throughout a group of interrelated models, and the level of abstraction or operationality is a characteristic of an entire set of models as well as a single model, in the same way that comprehensiveness is an overall attribute. The only dimension that lacks this property is the descriptive-behavioral-normative classification scheme, and here it is possible for each class to be present in different degrees even in fairly simple models.

B. Lowry's Paradigm

There is some urban theory, and a few general theories, but no General Urban Theory. Even if there were such a theory, it would have to be very well-developed to be able to provide much guidance to the urban modeller. Yet there is a need for some heuristic device to aid the modeller in making decisions about where to look for relevant variables and how to structure a model. While this device may not have the status of a theory, it can be called a paradigm. Lowry has suggested such a paradigm [8].

An urban area can be thought of as being composed of a number of sites, some of which have improvements on them. The population or activities of the urban area can be thought of as establishments (households, firms, government agencies), and the establishments occupy at least some of the sites. The value of the site to the establishment depends upon a number of factors which can be represented by an evaluation function which might have the form

$$p^{hi} = f(x_1^h, x_2^h, \dots; y_1^i, y_2^i, \dots; z^{1i}, z^{2i}, \dots z^{ni})$$

where p^{hi} is the value (price) of site i to establishment h , the X 's are attributes of the establishment, the Y 's are attributes of the site, and the Z 's are the locations of other establishments. It may be difficult to give empirical content to the function, f , but the variables might include income and number of children of a household, size and house on a lot, and access to shopping areas.

During each period of time, called a transaction period, each establishment evaluates all possible sites including his own as potential locations. The owners of each site consider the values, or demand prices, offered by each establishment, and award the site to the highest bidder. Bargaining may be required, and establishments that offer the highest price at two locations must decide which they want. Priorities may be used, i.e., commercial establishments select locations before residential. Eventually all establishments have sites (if there are at least as many sites as establishments), or all sites have establishments. Some establishments will have moved to new sites and some will have remained in place.

At the beginning of each transaction period a site owner may decide to make alterations on his site that may affect the demand prices of the other establishments. His decision is based upon an investment function written as

$$E_j^i = g(C_j^i, P_j)$$

where E_j^i is the expected gain from converting site i to state or condition j , C_j^i is the cost of making the conversion, and P_j is the current price for sites in condition j . The site owner will seek a conversion that will maximize his investment function.

Given the simplifications of the paradigm, it is still necessary to reduce the number of variables and relationships before a model can be constructed, and this done in two ways. First, sites and establishments are aggregated, the former into districts and the latter into activities. Secondly, the model focuses on a few aspects of the paradigm and lets the remainder occur as a residual or as a consequence of the primary decisions. Referring to Figure 2.3, a model oriented towards columns is concerned with land use patterns for a given point in time and land use succession over several periods of time. A model oriented towards rows is concerned with location patterns (where activities locate) for a single point in time, and migration of activities from one period to the next.

Lowry gives examples of models directed at land use (Chicago Area Transportation Study), land use succession (University of North Carolina), location (Boston's Empiric model), and migration (the Polimetric model, also originally for Boston). He also describes three models that do not fit the paradigm exactly: a hybrid (the Pittsburgh model), a market demand model (Penn-Jersey model), and a market supply model (for San Francisco). According to Lowry, these are ordered roughly according to increasing educational potential or elegance, and decreasing ease of construction and use for forecasting.

CHAPTER 3: CONTENT, FRAMEWORKS, AND COMPONENTS

Ackoff [2] distinguishes between method, technique, and tool in such a way that the three are hierarchical. Tools (instruments, mathematical notation) are employed by techniques (measurement of length, calculus), which are in turn employed in a research methodology. He also places fact, law, and theory on a particular-general continuum, with facts being particular observations and laws being specific statements of observed and verifiable relationships. Theories are the most general, and have additional qualities beyond laws that make theories capable of producing new laws as deductions.

The subjects we wish to discuss in this chapter can be grouped by using a modification of Ackoff's two schemata. Theories, laws, and empirical regularities which might be useful in urban modelling are placed in one group, which includes general equilibrium theory and the gravity model. A second group contains some general approaches that can be called method, that are above the level of specific technique and lack the content of a theory. While it would be inaccurate to call these methods "strategies" (which would probably be more in line with Ackoff's notion of method), they are means of organizing and utilizing theories and techniques. The third group are techniques, such as input-output, mathematical programming, and mathematics. The purpose of the groupings is to point out that some of the items are more alike than others, and that the reasons for using a particular item tend to vary with the group. Theories and laws are of interest for their substantive content, the techniques are useful for manipulation and for what is already known about them, and the methods are more eclectic, allowing for variations in model structure.

A. Theories, Laws, and Empirical Regularities

One purpose in grouping theories and observed regularities together is that it is then unnecessary to distinguish between the two. Whether a generalization deserves the status of theory or whether it is simply a

descriptive regularity is of considerably less importance to us than whether it is useful for modelling. The content of model relationships must be drawn largely from areas described in this section. Those presented have either been used already in urban models or else show promise of being usable in the near future.

Rent theory comes from classical economics, and Burgess' concentric zone theory is a sociological counterpart to von Thunen's rent theory. The density gradient and the gravity model are related aggregate descriptive regularities that have been heavily used in urban modelling. Supply and demand, general equilibrium theory, and activity analysis are theoretical frameworks that have supplied important ideas for modelling, and the frameworks themselves are gradually becoming more modellable. Generalizations and adaptations of the Keynesian multiplier notion have been widely used. The remaining topics in this section are relatively undeveloped from the modelling point of view, but eventually they must be incorporated.

Classical Rent Theory

An early attempt to explain the distribution of activities over the surface of the land resulted in the formulation of rent theory. Adam Smith and others studied the economic and social relationships of towns to their hinterlands, and contributed to rent theory in a variety of forms. One of the most complete of these theories was that of von Thunen. [165].

von Thunen posited a uniformly fertile plain, a single isolated settlement, and homogeneous and ubiquitous transportation. Given several kinds of undifferentiated goods, their prices at the point of production, their transportability, and the demand for the goods at the settlement, von Thunen then determined the locations of various agricultural activities as a function of distance from the point market. For a single product consumed at the market, at a single price, the cost of the goods was the cost of producing it (assumed to be uniform over the region) plus the cost of bringing it to market. If the market is cleared, the price of the goods will be

$$P = p_0 + r d_{\max}$$

where p_0 is the price at the source or production cost per unit of good, r is the transportation cost per unit of good per unit distance, and $d_{\max}$ is the maximum distance that the good can be transported to the market and still meet the equilibrium price.

Those producers operating closer to the market enjoy an advantage in that they have lower transport costs, but this advantage is offset by the increased amount that they must pay to the owners of the land. Thus a landowner at distance $d_{\max}$ from the settlement receives zero rent, and land closer to the market receives an increasing amount of rent, to a maximum at the center (the market). If we assume that a unit of land is that amount required to produce one unit of good, then rent per unit land as a function of distance can be diagrammed as in Figure 3.1.

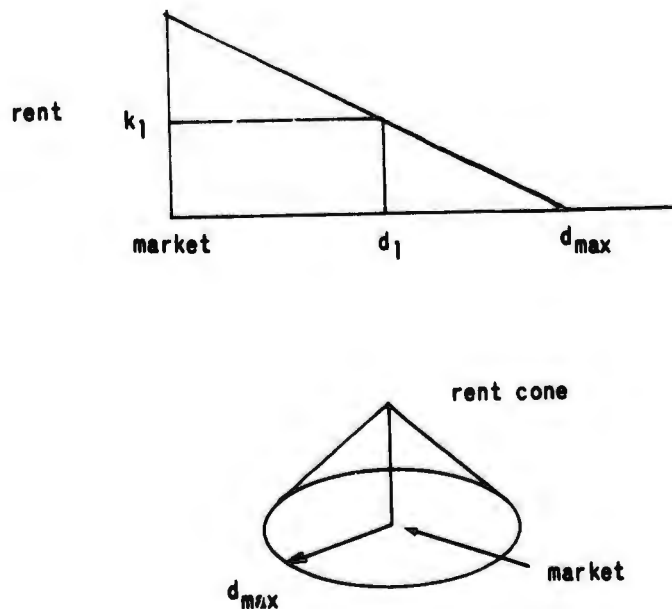


Figure 3.1. Rent cone.

A producer located at a distance d_1 could afford to pay a rent equal to k_1 . The rent function is a straight line because the transportation cost is a constant per unit distance, and the slope of the line equals r , the transportation rate. Since the distance may be measured out from the center in all directions, the diagram can be rotated about the market to form a rent cone. Different products will have different market values, depending upon demand and different transportation rates, and hence different rent cones. The activity that will prevail at a particular location will be that activity which can afford to pay the highest rent. If, for example, we have three activities, A, B, and C (apples, berries, and corn), with rent cones of the form shown in Figure 3.1, then activity A will be operated up to a distance d_1 from the market, activity B from there out to d_2 , and C from d_2 to d_3 . None of these activities can profitably be undertaken outside of d_3 , as seen in Figure 3.2.

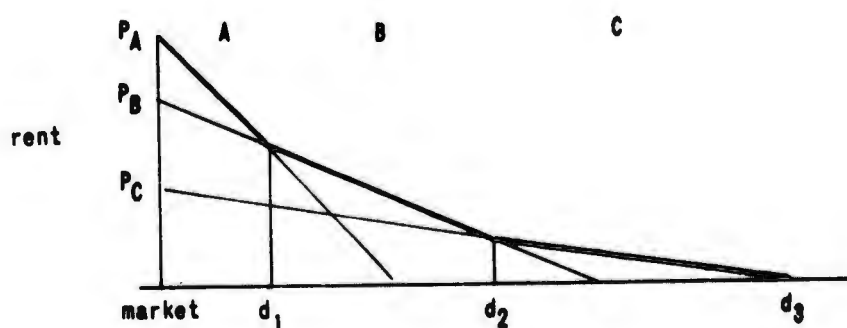


Figure 3.2. Rent cone for several products.

Thünen thus arrives at a model of urban form, with a spatial distribution of activities based on the cost of transporting goods to market. The pattern described is normative in that it both minimizes total (social) cost and maximizes consumer satisfaction, within the limitations imposed by the assumptions.

Burgess' Concentric Zone Theory

Burgess conceived his hypothesis from looking at Chicago, and without reference to von Thunen. The similarity of the two theories is remarkable, however; the concentric zone theory is an application of Thunen's theory to the spatial patterning that occurs within cities. Burgess' theory is also more sociological than Thunen's primarily economic formulation.

In addressing himself to two aspects of urban change -- extension and succession -- Burgess lists five zones, which are successive rings circling the center of the city. The first is the central business district (CBD), the second is called the zone in transition, and the outer three are the zone of workingmen's homes, the residential zone, and the commuters' zone. Burgess' description of the activities and inhabitants of each zone is quite thorough. The theory is depicted in Figure 3.3.

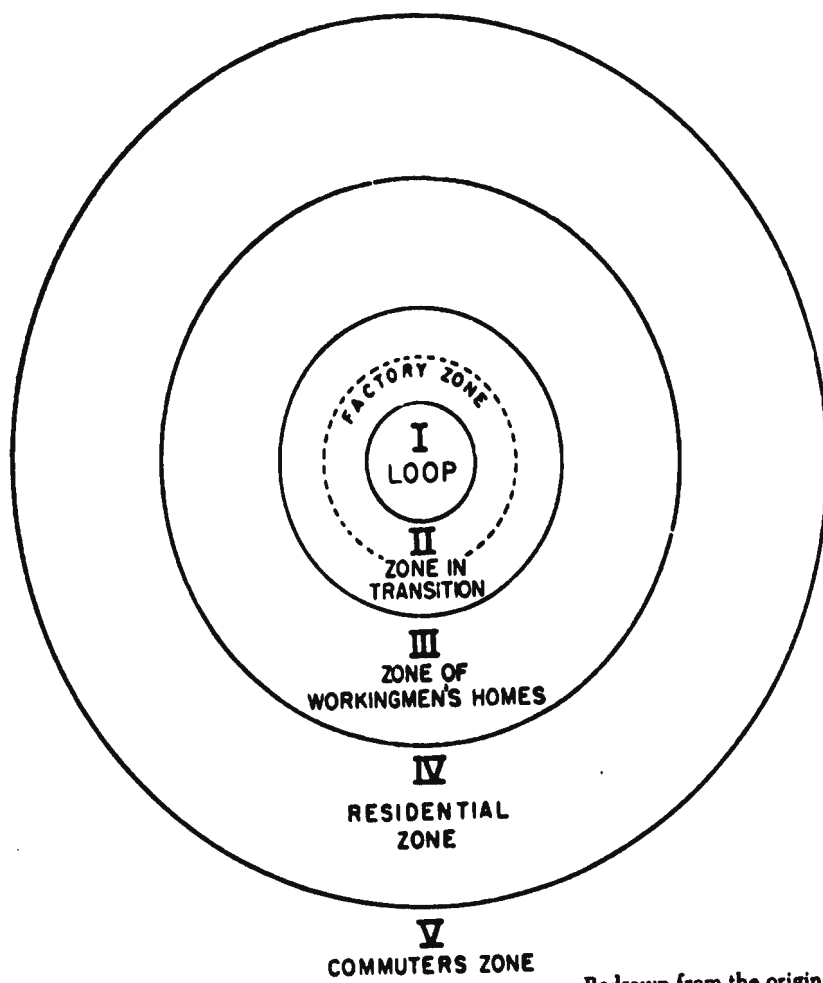
Burgess' theory is descriptive, in that the pattern is derived from observation, and Burgess does not claim for it the status of law. It is an approximation with some generality, and explicitly considers spatial dynamics.

The Density Gradient

Both von Thunen's and Burgess' theories imply that land is used more intensively towards the center of an agglomeration, and one way that this might be expressed is in residential density. Colin Clark [43] investigates the question of residential density, using the following two hypotheses:

"1. In every large city, excluding the central business zone, which has few resident inhabitants, we have districts of dense population in the interior, with density falling off progressively as we proceed to the outer suburbs.

2. In most (but not all) cities, as time goes on, density tends to fall in the most populous inner suburbs, and to rise in the outer suburbs, and the whole city tends to 'spread itself out.'" (p. 490)



Redrawn from the original.

Figure 3.3 BURGESS' CONCENTRIC ZONES

Clark acquired a good deal of data for cities all over the world, and found that a very good fit could be obtained by using an exponential decay function,

$$y = A e^{-bx}$$

where y equals density (population per square mile), A and b are empirical constants, and x is the distance from the center. The density gradient notion was widely used in the earlier transportation studies, and forms an important part of several current studies.

Gravity Models

Although the gravity concept results in an urban form very similar to those prescribed by rent theory or described by the density gradient, its origins and bases are considerably different. In the first place, it has a physical analog. As developed by Stewart [150] and others, the gravity principle is similar to certain portions of Newtonian physics, and the subject is sometimes referred to as "social physics." Secondly, as in the original statement by Carey, the gravity notion can be related to concepts of social interaction. Other writers have developed rationale to justify the gravity model, such as the journey to work or to shop, but its usefulness derives from the fact that it is an empirically observable regularity.

The gravity model has two forms -- an interaction form and a potential form -- and the second can be obtained from the first. Employing the physical analog for the gravity-interaction form, the interaction (force) between two points is directly proportional to the populations (masses) at the two points and inversely proportional to the distance between them. Mathematically,

$${}_i I_j = \frac{P_i P_j}{d_{ij}^b}$$

where ${}_i I_j$ is the interaction between points i and j , P_i and P_j are the populations at the respective points, and d_{ij} is the distance between the points. The distance factor can be raised to some exponent (two in the

physical model), represented by b in the above formulation.

Since interaction cannot be measured, at least directly, the statement given must be regarded as definitional, and then investigated for its empirical usefulness. Stewart has found that interaction correlates well with migration, retail trade, telephone calls, and auto trips, among other things. Most urban models, particularly transportation models, have made use of the gravity principle in some form.

To get from the interaction form to the potential form, we sum the interactions at one point to all other points.

$$\sum_j iI_j = \sum_j \frac{P_i P_j}{d_{ij}^b} = P_i \sum_j \frac{P_j}{d_{ij}^b}$$

The result is an index number that pertains to the single point, i . If we wish to control for the size of the observation unit, the sum of the interactions can be divided by the population at i , yielding per-capita potential, represented by iV . Then

$$iV = \frac{1}{P_i} \sum_j iI_j = \sum_j \frac{P_j}{d_{ij}^b}$$

Studies of the gravity model have looked into the questions of what value the exponent, b , should have (from .5 to 3.0), and how distance should be measured. Substitutes for airline distance mentioned are travel time, some general measure of access, over-the-road distance, or distance weights which may add social or economic obstacles to interaction.

The variable to be used for mass is also open to numerous interpretations. In a more general form, the potential model can be written as

$$iV = \sum_j \frac{w_j P_j}{d_{ij}^b}$$

where P_j may represent any statistical population (dollars, square feet, grocery stores, white protestants, etc.) and the w_j 's are weights which serve to differentiate the members of the population. Weights can be income, a location or status index, percent nonwhite, or other. Thus, as a descriptive tool, the gravity model has a great deal of flexibility; it can be used to delimit trade areas or otherwise describe aggregate populations, or it can be generalized to a point where it is primarily an equation for regression fitting.

A variation of the gravity model that has also seen extensive use is the opportunity model. Like the gravity model, the opportunity model postulates that a potential is exerted at a point in direct proportion to the attractiveness or mass of other points, and is decreased by the distance to the other points. In addition, however, the opportunity model suggests that intermediate points diminish the potential caused by more distant points. These intermediate points are called "intervening opportunities", and are considered to act upon a particle (person) passing near them. The model is stated by Stouffer [152] as,

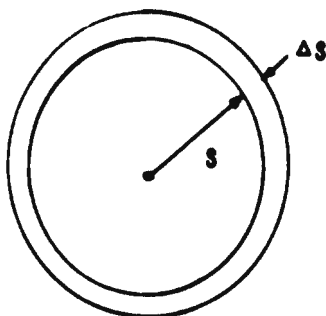
"...the number of persons going a given distance is directly proportional to the number of opportunities at that distance and inversely proportional to the number of intervening opportunities."
(p. 846)

The friction of space is therefore greater the more it is populated with attractive opportunities. Since opportunities, in order to be "intervening", must lie closer to the origin of the "mover" than his potential destination, the notion of distance enters the formulation explicitly. A difference here, however, is that the effect of distance in the formulation need only depend upon its ordinal properties.

Stouffer also gave the intervening opportunity model a mathematical formulation as

$$\frac{\Delta y}{\Delta s} = \frac{a}{x} \frac{\Delta x}{\Delta s}$$

where Δy = the number of persons moving from the origin to a circular band of width Δs ,
 x = the cumulative number of opportunities between the origin a distance s ,
 Δx = additional opportunities within the band of width Δs , and
 a = a scaling constant



The number of persons going a distance s is considered to be directly proportional to the percent increase in opportunities at that distance.

If we assume that a continuous function $x = F(s)$ exists, and reduce the above statement of the model to limitingly small quantities, then we have

$$\frac{dy}{ds} = \frac{a}{x} \cdot \frac{dx}{ds}$$

and the solution to this equation is

$$y = a \log x + c$$

or $y(s) = a \log F(s) + c$

which indicates that y , the cumulative number of movers within a distance s , is a linear function of the log of x , the cumulative number of opportunities at that distance.

As long as $F(s)$ is a function only of distance from the origin, then the distribution of opportunities is radially symmetric and the opportunity model will produce a radially symmetric "attractiveness" surface that is similar to that of the gravity model or the density gradient model in this respect. In applications of the opportunity model, opportunities are distributed in clusters (collected by zones or districts), and the mathematical formulation of the model is somewhat different. The derivations of these formulae are difficult to follow from the presentations in published sources, but the reader is referred to Schneider [140], Lowry [327], Ruiter [409], Lathrop, Hamburg and Young [92], Lathrop and Hamburg [391].

For the Upstate New York Transportation Study for Buffalo, the discrete version of the opportunity model was given as

$$A_j = A(e^{-kD} - e^{-k(D+D_j)})$$

where A_j is the amount of activity to be allocated to zone j , A is the aggregate amount of activity to be allocated, D is the set of intervening opportunities, and D_j is the set of opportunities in zone j . The constant k is the probability of a unit of activity being sited at a given opportunity. The form of this function is justified by Lathrop and Hamburg [391] by

"The use of the negative exponential formulation following an access search across an opportunity surface presumes that the settlement rate per unit of opportunity is highest at the point of maximum access or, most usually, the center of the region. This elementary presumption is consistent with both empirical observations and the bulk of the theory dealing with the economics of land use." (p.96)

In this instance the opportunity model was used for projecting land use.

In recent extensions, the Chicago Area Transportation Study has reformulated the opportunity model for use in small areas within a large

metropolitan region. Their trip generation function is

$$T_{ij} = \sum_{k=1}^m O_{ik} \left[e^{-L_k V_{jk}} - e^{-L_k V_{j+1,k}} \right]$$

where T_{ij} = trips from zone i to zone j (zonal interchange),

O_{ik} = trips originating in zone i of type k,

L_k = probability of type k trip destination being accepted if it is considered,

V_{jk} = volume of trips already considered, of type k, and

m = number of trip types.

The number of origins in a zone is assumed to equal the number of destinations, over a twenty-four hour period, so the subtended volume of trips, V_{jk} , is found by

$$V_{jk} = \sum_{n=1}^{j-1} O_{nk}$$

where O_{nk} 's are arranged by increasing travel time from zone i.

Supply and Demand

One of the most basic notions of economic theory is that of the equality of supply and demand, with price being the mechanism for adjustment. In the perfectly competitive static market, the amount of a good that suppliers will offer is considered to be a function of the price they can obtain for the good. It is assumed that the higher the price, the more that will be offered. On the other hand, the amount that buyers will remove from the market is also a function of price, but with less demanded at higher

prices than at lower. These two functions -- the supply function and the demand function -- should intersect at some point, and this point is the equilibrium price that will clear the market and satisfy all demand. A simple diagrammatic representation is shown in Figure 3.4.

It is a rare model that does not utilize this concept in some way, and very often it is used in many ways. The decisions that must be made in a model, such as the number of single family housing units built in one period, are quantitative rather than qualitative. It is not sufficient to say that more units will be built if there are more people, because we need to know how many units. Using the concept of supply and demand, the costs of supplying various amounts of housing can be determined, and the prices families are willing to pay for different types are computed independently of the supply. The schedules can be compared and adjustments made in the price in the present period and the number constructed in the next. This results in a dynamic model such as the cobweb model (see p.3-58). The supply-demand idea may be very complex in application, and it may be couched in different language, but the idea is pervasive in urban modelling.

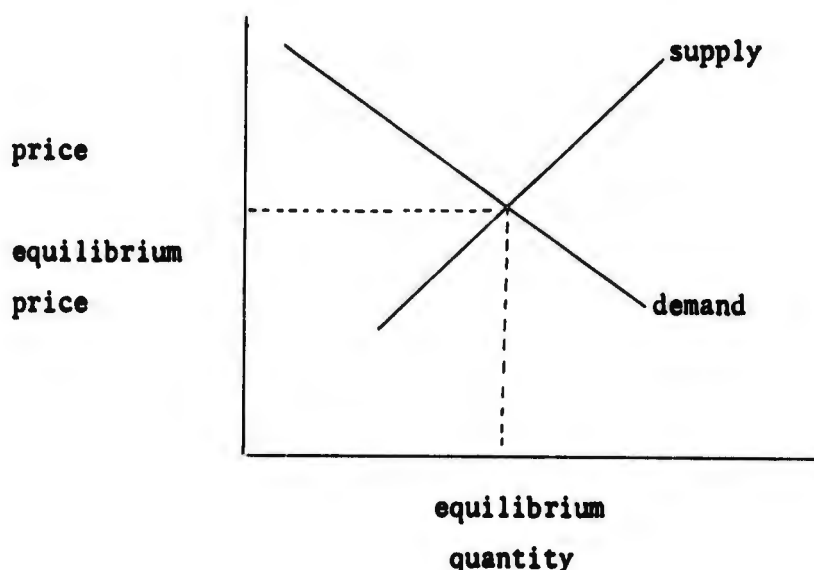


Figure 3.4 SUPPLY AND DEMAND

General Equilibrium Models

The notion of a general equilibrium is that of balancing supply and demand in a number of markets. If all prices are set such that in each market the amount supplied at that price will just equal the amount demanded at the price, then the system is in general equilibrium. In contrast, a partial equilibrium model would assume all markets in equilibrium but one or two, and study the process of adjustment towards equilibrium in them. Markets include goods and services, money, bonds, labor, capital, and foreign exchange. In each market two quantities are determined, the amount supplied (e.g., goods and services, loanable funds, exports) and the amount demanded, and in equilibrium these are equal. The variables which connect markets are prices. In addition, there is a set of exogenous variables which include policy parameters (e.g., government spending), tastes and preferences, technology, natural resources, capital, and population. Different areas within the field of economics can be described in terms of emphasis on markets and variables, e.g., economic growth is particularly concerned with capital stock and the capital market.

Notations for describing economic general equilibrium models vary widely, but we will adopt one which is reasonably common. Looking at the market for X (goods, say) there is the amount of X demanded, called " X ", and the amount of X supplied, called " $\bar{X}$ ". These amounts are determined by the set of all prices and the exogenous variables, and are equal in equilibrium.

$$X = X(P; E)$$

$$X \stackrel{e}{=} \bar{X}$$

$$\bar{X} = \bar{X}(P; E)$$

P is a vector of prices, E represents all exogenous variables, and $X()$ and $\bar{X}()$ are functions. In general there will be n markets, so there will necessarily be n prices, which when combined with the n demand variables and the n supply variables makes $3n$ variables. Since there are 3 equations to each market, we also have $3n$ equations for a general equilibrium, a necessary but not sufficient condition for a solution. In the expanded system,

$$\begin{array}{lll}
x_1 = x_1(p_1, p_2, \dots, p_n; E) & \bar{x}_1 = \bar{x}_1(p_1, p_2, \dots, p_n; E) & x_1 \stackrel{e}{=} \bar{x}_1 \\
x_2 = x_2(p_1, p_2, \dots, p_n; E) & \bar{x}_2 = \bar{x}_2(p_1, p_2, \dots, p_n; F) & x_2 \stackrel{e}{=} \bar{x}_2 \\
\dots & \dots & \dots \\
x_n = x_n(p_1, p_2, \dots, p_n; E) & \bar{x}_n = \bar{x}_n(p_1, p_2, \dots, p_n; E) & x_n \stackrel{e}{=} \bar{x}_n
\end{array}$$

A system such as the one above, which has all prices relevant in every market, is said to have "global" or "universal" consistency. If a single price is altered from the equilibrium price, then the entire system will be in disequilibrium. Systems that do not have this characteristic, i.e., in which some markets can be in equilibrium while others not, but which are still soluble, are said to be "equilibrium consistent." In practice, only very simple economic systems in economic theory have global consistency; in more complex systems it is unmanageable to handle all prices in all markets. Both the classical framework and the Keynesian formulation have equilibrium consistency but not global consistency. The opposite extreme would be to have all markets entirely independent, and thus there would be no "system." The notion of global consistency is important as a consideration rather than as an objective.

Two other considerations from the standpoint of economic theory tend to reduce the workload slightly. There is no conceptual difficulty in expressing prices in terms of one of the endogenous prices, and since one of the prices is that of money (usually), all prices can be expressed as money prices (e.g., dollars). In tandem with this simplification is the rule known as Walras' Law, which says that in a globally consistent system (and some others, too), a single market cannot be out of equilibrium while the rest are in equilibrium. Thus if all markets but one are in equilibrium, the last market can be ignored since it must also be in equilibrium.

The role of general equilibrium theory in economics is that of providing both a goal for analysis and a framework within which to operate. The more general the theory, the fewer the markets that are ignored and the fewer the variables held constant. At the same time, having the general framework allows the analyst to look objectively at the usefulness of his (necessarily) partial equilibrium model. The variables and relationships that are ignored are made as explicit as those that are included. A truly general model for urban planning would include economic, social, psychological, physical, political, etc., variables and relationships. A few models of this type have been conceived (Isard and Isard [91], but at present they do not offer much guidance to the applied urban modeller.

Activity Analysis

According to Baumol [29], the subject of activity analysis involves "...the applications of linear programming methods to general equilibrium theory." (p. 857) The primary value of activity analysis is as a synthesizing framework for economic theory that includes both input-output and linear programming as specific cases. While nothing new is added to the modeller's kit bag by this framework, it does offer a useful means of generalizing the techniques of linear programming and input-output and observing the parallels between them.

We postulate three activities -- construction, households, and gun production -- and two commodities -- houses and labor -- in our system. Each activity produces an output, in terms of the two commodities: "construction" requires one unit of labor (output of -1) and produces one unit of houses; "households" requires 3 units of houses and produces 4 units of labor; "guns" uses up one unit of labor. In tabular form, the system is represented by

commodities		1	2		
levels	u_1	1	-1	A_1	(construction)
	u_2	-3	4	A_2	(households)
	u_3	0	-1	A_3	(guns)
		r_1	r_2		
		(houses) (labor)			
		outputs			

The u_i are the levels at which respective activities are operated, i.e., if $u_1 = 2$, $u_1 A_1 = (2, -2)$ or 2 units of houses produced and 2 units of labor used up. One possible program, or set of levels, would be $(u_1=3, u_2=1, u_3=0)$, and the output of this program would be $u_1 A_1 + u_2 A_2 + u_3 A_3 = (r_1=0, r_2=1)$. Such a program would be called self-supporting, since there are only zero or positive outputs. We could also operate a program of $(3,1,1)$ which would make all outputs equal to zero, and this program would be called stationary because no resources would be added or depleted by the economy. A third program might be $(0,1,4)$ with the result that there would be a deficit of 3 units of houses each time the program were operated.

Input-output is a production-oriented technique that requires that demand be predicted exogenously, and the means for producing the necessary demand are determined by the model. Thus input-output describes an open system, while activity analysis is concerned with the general equilibrium of a closed system, and hence negative outputs. We have not mentioned how an activity analysis program would be determined, but presumably by the application of linear programming. The value of the outputs should be determined by welfare economics, and the outputs compared with the resources of the economy. In long-run equilibrium, all outputs should be zero, which means that all activities operated at a positive level should make zero profit and all other activities should be operated at zero level. The reasons for this can be derived from the dual of the linear program, discussed below. Which activities are operated is the economic problem to be solved.

Regional Multipliers and the Economic Base Concept

The notion of the multiplier, as embodied in Keynesian economics, is that when income is received, at least a portion of it is soon spent, and this creates income for someone else, who in turn also spends some of it. In this way the money goes round and round, gradually diminishing at each iteration until the effect gives out. Thus if \$100 of new income is generated exogenously and introduced into the system, and the average propensity to consume (APC) is .9 (90% of all income is spent), then the total amount of new income is the sum of the infinite series

$$\begin{aligned} & 100 + .9(100) + .9^2(100) + \dots + .9^n(100) + \dots \\ & = 100 (1 + .9^1 + .9^2 + .9^3 \dots) = 100 \left(\frac{1}{1-.9} \right) \quad \text{or} \quad \frac{1}{1-APC} \cdot \$100 \end{aligned}$$

In actuality, of course, there is some time lag before the income is spent at each round, so the theoretical total is never reached. Nor is the average propensity to consume easily determined; it varies from individual to individual and across time. A governmental policy question is to find the set of capital expenditures that will optimize the effect of the multiplier, as, for example, when the federal government wishes to stimulate a sluggish economy by "pump priming."

The use of the multiplier also seems appropriate at the regional level, and here, at least initially, the concern was with the effects of business cycles on the local economy. Not only does the multiplier have an expansionary effect in periods of rising incomes, it also has a depressing effect when incomes are falling. This in turn is related to the particular kinds of industries that exist in the region. Some industries are sensitive to economic forces outside the region, and others are affected because of their dependence on the first set. Those industries that produce goods primarily for distribution outside the region are labelled "basic industries", and those that deal primarily within the region are known as "service" or "nonbasic" or "local" industries. It is hypothesized that an increase in basic employment causes a corresponding increase in service employment, in the same way that the income multiplier operates.

By categorizing all regional employment into either basic or non-basic, the basic-service ratio or regional multiplier can be computed. The multiplier is then usually used for one of two purposes: (1) by studying the factors outside the region that affect employment in the regional basic industries, the consequences of cycles can be estimated and perhaps alleviated; and (2) by projecting employment in basic industry within the region for some future date, and then applying the multiplier, an estimate of future total employment can be obtained. From this it is a relatively simple task to estimate future total population. Business cycles have declined in their effects on the economy, in recent periods, but forecasting employment and population has become a subject of widespread interest.

The concept of the urban economic base, as outlined above, has been developed and widely applied in planning studies. Not all studies so labeled do, in fact, concern themselves with the basic-service dichotomy, but the term "economic base" is generally recognized as referring to activities that export their products or services outside the community. Considerable difficulty arises in measuring the amount of a community's output that is export, and in order to elucidate this difficulty we will temporarily drop the basic-service label and substitute "export-local" to mean the same thing. The reason for this is that economic activities can be classified into three groups -- primary (extractive, or using natural resources), secondary (processing), and tertiary (distribution and service) -- which do not necessarily match with the export-local breakdown. While tertiary or service activities are largely local, there are numerous exceptions. Andrews [25] has suggested a framework for pointing this out. He lists six categories of export activities: transfer to consumers outside the region of goods, services, and capital, and sales of these three items to consumers in the region who are from outside. Examples of these, in order, are sales of locally made products outside the region, sales of locally based insurance to outsiders, rent earned locally on property outside the region, local sales to shoppers from outside, haircuts sold to tourists, and local bank loans to outsiders. Thus many activities classified as "service" are not necessarily "local".

Many other problems are encountered in applying the economic base concept. There is the question of what measurement to use, i.e., number of employees, payroll, value added, or some other. Some local industries sell all or a portion of their output to export industries, and should therefore be counted in the economic base. Conceptually, linkages should be traced back to the point where all inputs are either made locally, are imported, or consist of labor, but this is seldom feasible. The extent of the geographic area that is considered local will affect the amount of output that is considered export. These problems, in turn, make data acquisition very difficult. Leven [102] has said,

"The sad truth seems to be that the only way to determine which firms engage in exporting and to what extent they do so is simply to ask them." (p.370)

The reason for mentioning the base concept and some of the problems in its implementation is that the basic-service dichotomy recurs constantly in urban models. In each case the intent is slightly different, and the problems somewhat different, but as an idea of how events are causally ordered in the real world it has seen widespread use. Typically, a set of industries called basic is located and the number of employees established, and then supporting activities such as retail are subsequently determined. If location is the primary concern, then basic industries are those which are located without reference to the location of the population (rather, through transportation access, land availability, site requirements, etc.) and service industries are those located to be near the population they serve. Although this breakdown parallels the export-local division, there will be some differences. For example, hospitals might be considered basic in the former and local in the latter.* Thus the economic base concept has relevance for both the linkages of a region with the rest of the world, and also for intra-regional economic structure.

* see Lowry [327], p. 83.

Other Theory from the Social Sciences

For a number of reasons, there has been little attempt to build operational models from the theories found in the behavioral sciences. An operational model, in this context, is one which predicts the outcome of a set of events in terms that are useful to a planner or some person who is in a position to affect one or more of the events. The behavioral sciences are broadly distinguished from the physical and natural sciences, and include anthropology, sociology, political science, and psychology, plus numerous sub-fields and combined disciplines, such as human ecology, social psychology, cybernetics, game theory, psychiatry, etc. The difference between behavioral and social sciences is not clear, since many of the above areas of study are included in both categories, but behavioral sciences are intended to include study of the individual and exclude portions of social sciences that are primarily descriptive. Economics, for example, is both a social and a behavioral science, but macroeconomics is less behavioral and has produced many operational models.

One reason for the absence of operational models from the behavioralists is the state of knowledge in these fields relative to the results we would call operational. A great deal of valuable work has emerged, in a relatively short life span, but the concepts have only recently taken on empirical content. The link between theory and observation, and the rigorous testing of hypotheses and development of new theory, is part of the present phase of activity. Considerable effort is being directed at operationalizing hypotheses and measuring theoretical variables, but there is need for much more effort.

Another reason for the lack of operational models may be the generally academic nature of the various fields involved. The desire to acquire knowledge primarily for the purpose of gaining greater understanding of the phenomenon being studied has outweighed the need to solve real problems, but behavioral scientists are now becoming interested in applying their theories to the decision process. Partly this is due to the increased perception of a need for considering social and cultural information in implementing policies such

as economic development, and partly it is the result of improvement in the theories.

A number of models have been constructed, however, and some of these are operational. A device that is often used for both teaching and research purposes is the heuristic or operational game, in which human players operate according to a set of rules (which may contain theory and assumptions). A single player represents either an individual or a group in the game, and the game may give the player a better intuitive grasp of a real situation, or the player is observed in his responses to game-generated stimuli. A slightly different kind of model simulates behavior by using decision criteria based on some kind of exogenously determined utility function. Models of these two types are surveyed (as of 1965) by Abt Associates [1].

There have been a few attempts to present theory in a systems analysis framework, which often means little more than drawing a diagram with boxes and arrows. Ultimately, however, if these are successful, the inputs and outputs of the model or components of the model can be measured, tested, and confirmed, rejected, or modified. Easton [59] has suggested a systems model in political science, and Smelser [145] and Homans [86] have developed sociological models. Finally, a major portion of social and behavioral science models might variously be called heuristic models, metatheories, or paradigms, since they do not have the generality or hypothesis-deducing powers of theory and they are not adequately stated symbolically.

While the social and behavioral sciences are hardly on the verge of being able to predict human behavior under a wide variety of conditions, the potential that exists for useful results in the near future is quite high. Unfortunately, there are a couple of prejudices in the conventional wisdom that would mark the effort as futile. First of all (they say), humans are capable of reasoning, discretion, decision, etc., and have a free will; therefore any thought of predicting human behavior is pointless and may even be tampering with God. Actually, there is a great deal of regularity in human

behavior, and predictions are routinely made about it, such as the number of persons using a state campsite on a weekend in July. A second item from the conventional wisdom states that humans will change their minds, behave irrationally, and make decisions based on whimsy. It may appear to some people that certain other people have tremendous power to accomplish change without reference to anyone else, or that seemingly small events have profound effects. While this is often true, these decisions are considerably more predictable than they appear to be at first glance, and, in fact, they may be highly regular over even a short period of time.

While it would be presumptuous for us to attempt to survey the ideas of the social and behavioral sciences, we may be able to mention a few starting points. The place of items such as political party, interest groups, minorities, social status, and bureaucracies in political science theory is subject to a great deal of discussion, but political scientists generally agree that the United States is neither a true democracy (each individual has equal influence) nor a strong oligarchy (power concentrated in a small elite). The term used to describe our society is pluralistic, and this means that power and influence are distributed unevenly but to a large number of people and groups. The formal means in which these groups are organized in the political process has some importance in analyzing their behavior, but it is seldom a determinant of their behavior. A political machine built from ethnic minorities may offend the sensibilities of upper middle class liberals, but it quite possibly is more responsive to the needs of the people than a reform government. Urban politics is represented by works of Dahl [47], Banfield and Wilson [26], Adrian [17], Altshuler [22], and Lowi [106], Hawley [84]. The impact of the political system on land use patterns may be indirect, but decisions concerning planning, zoning, building codes, and the locations of public facilities are both political and spatial.

Urban sociology and human ecology study the relationships of individuals to social groups, between groups, and between groups and the urban environment. Many of the concepts have only recently become operational or are still in the process, such as social class, differentiation, cohesion, latency, and succession, to name only a few. Some basic material can be found in Hawley [84], Bogue [34], Duncan and Reiss [56], March [110], Theodorson [159], Merton, Broom, and Cottrell [117], Lynd and Lynd [109], Warner [168], Glazer and Moynihan [72], and Rapkin and Grigsby [134]. A large body of literature has developed in the area of methodology much of which is directed at the analysis of qualitative variables.

Anthropology, psychology, and social psychology are not as directly applicable to urban problems and urban models as political science and sociology, but many of the questions being investigated in anthropology and psychology will eventually have important implications for urban studies. Some of the ideas in these fields deal with personal aspirations, expectations, conformity, organizational behavior, culture, perception, learning and reinforcement, role theory, attitudes, and personality. Among the more well-known results are the underdog effect, the bandwagon effect, the self-fulfilling prophecy, the Hawthorne effect, and the strategies used in advertising. A set of readings in anthropology has been collected by Kroeber [93], and social psychology has been surveyed by Deutsch and Krauss [49].

A survey of findings from a wide variety of behavioral disciplines is found in Berelson and Steiner [30]. Demography has not been mentioned up to this point, but the methods of population analysis are clearly of interest to planners. A basic text in this field is Barclay [27], and a survey can be found in Isard [236].

Most of the work that has been accomplished in the social and behavioral sciences must necessarily be omitted here, but much of it will be of great importance to urban planning. Studies that are presently being undertaken tend not to be committed to the boundaries of a particular discipline or a school of thought, and very often research teams are made up of scholars

with training in several of the fields. With sufficient inputs of time and talent, the social fields should produce substantial yields in the foreseeable future.

B. Methods

While modelling is itself an application of scientific method, three general areas of methodology can be mentioned. Econometric modelling is an eclectic and pragmatic approach originating in the field of economics but which seems to be appropriate for urban modelling. Social accounting provides a framework for disaggregation and a set of structural identities. Simulation is a more behavioral approach that developed in operations research. All three methods can and should be used in conjunction with each other, although the final product may not contain direct applications of each one.

Econometric Modelling

The field of econometrics has grown out of the synthesis of statistical analysis and economic theory. Primarily, the purpose of this combination has been to establish the validity of economic relationships by testing them with real data, and to use the relationships to aid in economic policy determination. Since not all economic relationships are stated in mathematical form, and even fewer are in a testable form, econometrics is concerned with the translation of economic theory into testable mathematical form. Data with which to test the relationships do not come from controlled experiments, but are contaminated by the effects of numerous unwanted variables and different kinds of error, so another part of econometrics deals with the application of probability theory and statistical inference to mathematical modelling. Thus two substantive areas exist within the field of econometrics: (1) the mathematical formulation of economic theory and the testing of it by statistical means, and (2) the development of procedures and methods of research for accomplishing (1).

Recent development has tended to be in the latter of these two areas, with the result that econometrics has been increasingly regarded as a methodology rather than a subject area within economics. The content of a model need not be economic theory for econometric techniques to be applicable, and so many models are labelled econometric which contain many noneconomic relationships. As a methodology, econometrics is particularly suited to urban modelling due to the similarity of the problems encountered. The construction of urban models requires experimenting with structural relationships, and fitting and testing these relationships with data that often have complex interrelationships within themselves. Many current attempts at urban modelling could properly be regarded as econometric modelling.

An econometric model might take the following form:*

$$X_d = A_0 + B_0 P + C_0 Y_0 + u$$

$$X_s = A_1 + B_1 P + C_1 W_0 + v$$

$$X_d = X_s$$

where X_d and X_s are quantities of a commodity X that are demanded and supplied, respectively, P is the price of the commodity, Y_0 and W_0 are exogenously determined income and wage level, and the coefficients $A_0, A_1, \dots, C_1$ are fitted by some form of regression. The letters u and v represent random terms that are due to the omission of influential variables or errors in measurement. Since X_d and X_s cannot normally be observed independently of each other, the variables Y_0 and W_0 are needed to insure that the equations are "identified", which means that the coefficients can be determined uniquely by statistical means. It is also necessary, of course, that the system of equations be logically sound, i.e., a solution can be obtained once the

*The example is similar to those in Brennan [179], Ch. 23.

parameters are known. Some other problems encountered in econometrics are autocorrelation (a variable which is correlated with itself over time), multicollinearity (independent variables which are correlated with each other), heteroscedasticity (the error term is not independent of the value of the independent variable), lagged variables (variables that are related to other variables but which require a time lag for the influence to be felt), and dummy variables (qualitative variables, such as sex, given a quantitative representation, e.g., zero or one).

Econometrics also considers the criteria to be used under different data-and-equation configurations that will yield the parameters of "best fit." Sometimes the equations are all fitted simultaneously, as in the model for Boston by the Traffic Research Corporation (Hill and Brand [376]). More commonly, a variety of techniques is used, as in the model by Niedercorn and Kain [465] or in Lowry's model [327]. The econometric method is an eclectic approach that allows for theory to be incorporated where it is available, and for experimentation and hypothesis testing where the theoretical basis for structural relationships is weaker.

Social Accounting

National income and product accounts -- Gross National Product and all its components -- have been with us for some time, and are part of a well-developed conceptual framework. Regional accounts are relatively new, and discussion is continuing with regard to their design, construction, and use. Starting with the division of activities into export and local, it is logical to introduce additional breakdowns, say, for capital and current expenditures, consumption and investment, and types of consumption, as is done in the national accounts. Differences, however, between national and regional accounts immediately emerge. The foreign trade sector of the national accounts has importance for balance-of-payments policy, but is not a major influence in GNP; for regional accounts, on the other hand, trade outside the region is of prime importance. Inventories are of minor significance for regional accounts, while some sectors of the local economy need

**Allocation of Employment Created by
Sales of Industry Groups to Seven Sectors and Local Industries
Los Angeles-Long Beach, California, S.M.S.A., 1960
(In Per cents)**

Industry From	Sectors & Local Ind. To		1	2	3	4	5	6	7	8
	Exports	Exports	Private	Govt.	Local	Invest.	Invest.	Govt.	Govt.	Local
					Cons.	- Bus	- Htg.	Current	Inv.	Inds.
1 Durable Mfrs.	30.5	35.3	.7	5.9	—	—	.2	.4	27.0	
2 Primary Mtls.	26.6	9.5	1.9	6.5	—	—	.4	1.1	53.8	
3 Fab. Metals	27.6	15.0	1.1	.5	—	—	.6	.3	54.8	
4 Non-Elec. Mchy.	66.5	8.5	—	17.9	—	—	—	.7	6.2	
5 Elec. Mchy.	26.1	46.8	.2	12.6	—	—	—	.4	13.9	
6 Transpn. Equip.	17.4	54.3	.1	2.4	—	—	—	—	25.9	
7 Instr. & Ord.	26.3	57.3	.5	.5	—	—	.2	.2	15.0	
8 Stone, Clay, Glass	65.6	1.4	2.7	—	—	—	.9	—	26.4	
9 Lumber Products	15.4	14.3	—	—	—	—	—	—	70.3	
10 Furn. & Fixtures	14.5	1.2	6.9	8.1	—	—	.8	1.9	36.5	
11 Misc. Mfrs.	50.1	—	.6	—	—	—	—	—	49.4	
12 Non-Durable Mfrs.	41.7	3.1	10.9	.6	—	—	.3	—	43.4	
13 Apparel	69.1	2.9	—	—	—	—	—	—	28.0	
14 Textile-Leather	47.4	2.1	1.1	—	—	—	—	—	49.5	
15 Paper	68.8	—	.7	.7	—	—	.7	—	29.2	
16 Printing	24.1	—	6.8	—	—	—	.3	—	68.8	
17 Chemicals	62.0	3.4	25.3	—	—	—	.9	—	18.5	
18 Petroleum	29.3	6.9	—	—	—	—	—	—	63.8	
19 Rubber	46.0	11.4	.4	5.5	—	—	—	—	36.7	
20 Foods & Bevgs.	21.5	1.4	28.9	—	—	—	.2	—	47.9	
21 Total Mfg.	33.7	26.0	3.7	4.4	—	—	.2	.3	31.7	
22 Agric., Forestry, Fisheries, Mining	7.8	3.4	—	—	—	—	—	—	88.8	
23 Contract Constrn.	—	.3	—	31.1	49.6	—	—	19.0	—	
24 Transpn., Comm. Public Utilities	33.2	2.3	27.8	—	—	—	3.0	—	33.7	
25 Wholesale Trade	14.5	.5	.7	—	—	—	.1	—	84.2	
26 Retail Trade	9.4	—	82.6	.1	—	—	.9	—	6.9	
27 Finance, Insur., Real Estate	16.1	.7	38.3	4.5	20.1	—	—	.5	19.8	
28 Services	12.6	.7	67.3	.9	2.8	1.0	.1	.1	14.8	
29 Government	—	7.6	—	—	—	91.1	1.3	—	—	
30 Total All Industries	17.8	9.2	30.1	3.7	4.6	10.1	1.4	23.0		

Notes: Blank entries indicate sales are either zero or negligible
Source: Same as Table 3.

Figure 3.5 EXAMPLE OF A REGIONAL SOCIAL ACCOUNT

to be broken out in great detail. The particular sectoral design used for a region should depend upon the mix of activities that exists within the region, and the purpose of the account.

A typical accounting framework is shown in Figure 3.5. The proportion of each industry group's sales that go to each sector are the entries in the table. Sectors, listed across the top, are exports by local industry to private outsiders, sales to nonlocal government, local consumption, local business investment, local investment in housing, sales to local government on current account, sales to local government on capital account, and sales to other local industries. The framework of social accounts is quite flexible and can be constructed in many ways, to fit local information needs and data availability. The more disaggregated the table, the more the information that can be obtained from it and the more it costs to build it. Portions of the main table can be extracted and expanded in greater detail in sub-accounts, if this is useful.

As a framework, social accounting allows for a great deal of flexibility in selecting sectors and linkages in the local economy to direct attention towards, while maintaining the constraints that all activities must be accounted for somewhere and no activities should be duplicated. While social accounting does not provide many specific instructions for the urban modeller, most urban models should have an accounts framework in the background. Some techniques, such as input-output and the basic-service multiplier, have developed from social accounting and are more directly useful in modelling.

Simul:

1. word "simulate" has recently acquired such a wide variety of meanings, depending upon the user, that its utility as a device for verbal communication has been seriously reduced. The dictionary definition of the word is "to assume or have the appearance or form of, without the reality." In current academic usage, simulate generally means "to make something behave as if it were something else," but numerous additional restrictions are often

placed on this definition to produce the multiplicity of meanings.

Cornell Aeronautical Laboratory has outfitted a propeller-driven airplane with mechanisms that make the plane handle, to the pilot, like one of a variety of quite different aircraft (such as a supersonic transport in low-speed flight). Pilot training and handling-qualities evaluation can be conducted in relative safety while simulating the behavior of the selected airplane. Models of many kinds can be used to simulate the appearance, operation, or some other aspect of interest, of many real-world objects and processes.

Simulation has been described as "... essentially a technique that involves setting up a model of a real situation and then performing experiments on the model" (Naylor [190]). The authors readily admit that this definition is too broad, since almost any model fits this definition if experimentation is broadly construed, but it suggests that the major purpose of simulation in the social sciences is to carry out controlled experiments without using real subjects. Naylor, et al., limit their discussion to simulations

1. Whose components are firms, industries, or some other economic unit,
2. that are logical and mathematical models,
3. that are designed for digital computers,
4. that are stochastic or dynamic and not analytically determinant, which excludes models operating under complete certainty and achieving a static equilibrium.

The first restriction pertains to the subject of simulation experiments, and is obviously not a general restriction. The second two apply to any computerized model we might be concerned with, so the important restriction is the last one. On the basis of these considerations, Naylor, et al., suggest the following:

"Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models that describe the behavior of a business or economic system (or some component thereof) over extended periods of real time." (p.3)

This definition seems satisfactory if two points can be cleared up: first, what are the "certain types" of models, and, secondly, how important is the stochastic or dynamic element.

Simulation is used to manipulate model systems that are too complex to be able to work out analytically. The approach is to describe the behavior of the components of the model (units, processes) in terms of their inputs and outputs, and to describe the relationships between the components. The model is then "operated" as if it were a real system, providing it with inputs and recording the status of the system at various stages and also its outputs. A simulation tends to be, but is not necessarily, micro-analytic (since it deals with relatively small components rather than aggregates) and behavioral. It is also sequential, in that events occur in a specified order, rather than simultaneous.

Whether a simulation is stochastic or not depends upon whether the behavior of the components is described deterministically or probabilistically; if the latter is true, and the basic units of the system are treated individually, the simulation is called a Monte Carlo simulation. An alternative to the Monte Carlo technique when relationships are probabilistic is to batch the basic units into groups, and apportion the groups according to the expected distribution. Whether the variability of a Monte Carlo approach is important to the model results or not is a decision to be made in the model construction stage.

It is also possible in a simulation to batch time, i.e., operate in discrete time, as long as the relationships are still sequential. The shorter the time periods, and the more there are of them, the more the model approximates continuous time and hence the more full-fledged a simulation it becomes. Models of a static or even comparative static type are generally not solved by simulation. Thus models which do not have a well-developed time dimension or whose outputs can be described analytically are generally not simulations.

Several variants of the simulation method can be mentioned. Operational gaming or game simulation is a means by which human players can participate in the model, and usually this is for the purpose of training the players or observing how they behave under simulated conditions. Another variant is Monte Carlo analysis, which has two forms. One has been described above, but the other is directed at obtaining numerical solutions to certain kinds of purely mathematical problems that are deterministic but difficult or impossible to solve in any but a probabilistic way.

C. Techniques

Some techniques, such as linear programming or input-output, are well-developed techniques for handling a particular class of problem, while other techniques, such as mathematics or statistics, are general tools for handling a wide range of problems. The advantage of using the more restricted technique associated with a standard problem is that all the results associated with the standard problem become available to the problem of interest. Many of these results took considerable time and effort to obtain, and are not likely to be rediscovered in the normal course of a modelling effort working only from basic techniques. Many problems, however, do not fit the standard problem formats, and then the problem must be solved with basic tools or the standard problem modified.

Input-Output

The input-output model combines the notions of social accounting and general equilibrium theory, resulting in a framework that is fairly comprehensive, provides guidance for the resolution of detailed problems that arise in its application, and is sufficiently well developed to be operationally feasible in many situations. Its primary focus is on production, by concentrating on the structural interrelationships between sectors producing goods and services. Demand is normally the driving force in the model, and such activities as household consumption, private investment, and trade with the rest of the world are usually presented as exogenous variables.

We will illustrate the technique with a simple example. Assume a region that is closed (self-sufficient) and has two kinds of local industry: manufacturing and services. The industrial sectors sell to each other and also to households (but they do not export); households pay for goods and services by selling labor to the local industries. All raw materials are assumed to be produced locally. The flow of goods and services in the region can be represented by dollar volume of sales in the following transactions table (in millions of dollars):

from \ to	manu- facturing	services	household consumption	total output
manu- facturing	2	5	3	10
services	4	3	5	12
labor	4	4	8	
total inputs	10	12	Gross Regional Product	

The 2x2 section of the table with manufacturing and services (outlined) is called the interindustry table, and shows the interrelationships between the producing sectors of the economy. The column headed "household consumption" is called final demand, and indicates the amount of production that is available after the producing sectors have used what they need. The total of final demand is a measure of the usable output of the economy, and is called total product or, in this case, Gross Regional Product. The total output column is not summed, since that would count the same output twice. In other words, some portion of the sales of services to households represents inputs from manufacturing, and adding up total output would include this twice. The table is designed in such a way that the value of the inputs to any sector must equal the outputs of the sector. This is simply double-entry bookkeeping, rather than the closure of our example economy.

A basic assumption of input-output is that the proportion of each input required to produce a unit of output of a particular industry is constant. In order to produce a dollar's worth of output of services, for example, inputs from manufacturing and services must be in the proportion of 5:3. Another way of looking at it is to say that a dollar of output of services requires 5/12 of a dollar's worth of manufacturing. This assumption is called "proportionality," and the proportion or fraction (5/12 in the example) is called a technical coefficient.

Standardizing the interindustry table so that it reads in terms of dollars of inputs per dollar of output, i.e., contains technical coefficients in place of dollar flows, is the first step in analyzing the information contained in an input-output table. We define a technical coefficient as

$$a_{ij} = \frac{t_{ij}}{X_j}$$

where t_{ij} is the volume of transactions between industries i and j , X_j is the total output of industry j , and a_{ij} is the unit of input of i per unit of output of j . Units may be measured in several ways, but dollars are most commonly used in this country. The model can now be stated in matrix form as

$$A \cdot X + Y = X$$

where $A = \{a_{ij}\}$ is the matrix of technical coefficients, $X = \{X_j\}$ is a vector of total outputs for each producing sector, and $Y = \{Y_j\}$ is a vector of final demand. This equation says that total output is composed of that portion of output used in the production of other goods plus the portion that goes to final demand. If we assume that A (the regional economic structure) and Y (desired final demand) are known, then the equation can be solved for X , the set of outputs that will exactly satisfy regional final demand. This will be

$$X = (I - A)^{-1} \cdot Y$$

where I is the identity matrix and "-1" indicates the inverse of the (I-A) matrix. In our example, the matrix of technical coefficients, found by dividing each entry in the transactions matrix by its column total, is given as

$$A = \begin{bmatrix} 2/10 & 5/12 \\ 4/10 & 3/12 \end{bmatrix}$$

and the inverse of (I-A) is

$$(I-A)^{-1} = \begin{bmatrix} 45/26 & 25/26 \\ 12/13 & 24/13 \end{bmatrix}$$

If our old final demand of (3,5) is multiplied by this matrix, the resulting total outputs will be those we started with, namely (10, 12). Postulating a new final demand of

$$Y = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

the transactions and total outputs that will achieve this final demand are

2.77	5.61	5	13.46
5.54	3.36	5	13.85
5.15	4.87		

By increasing only the final demand for manufacturing, we have still made many changes in the table. First, all transactions and outputs increased, since an increase in final demand for manufacturing requires additional inputs from services. Second, total output of manufacturing increased by 3.46 rather

than by 2.0, the increase in final demand. This means that \$1.46 million worth of manufacturing output went to supply other manufacturing activities and service activities necessary for manufacturing. Substantial support -- an additional \$1.54 million - was needed in the form of sales from services to manufacturing. Labor inputs, of course, increase to both sectors.

The above illustration shows one possible use for input-output analysis, but there are many others. The basic framework can be modified in many ways to incorporate special features of the local economy and to focus on aspects of the economy that are of interest. Sectors can be moved from final demand to the interindustry table, or vice versa, depending upon assumptions of the analysis and questions to be investigated. Nonlinearities in production functions and spatial attributes can be handled with some effort. A dynamic version of the input-output model can be obtained by constructing separate tables for current and capital transactions. Many difficulties arise in the application of input-output to regional analysis, but the technique has much flexibility and potential.

Mathematical Programming

The problem in mathematical programming is that of maximizing or minimizing some function subject to a set of constraints. In its most general form, the problem is stated mathematically as

Maximize $G(X_1, X_2, \dots, X_n)$

Subject to:

$$f_1(X_1, X_2, \dots, X_n) \leq b_1$$

$$f_2(X_1, X_2, \dots, X_n) \leq b_2$$

...

$$f_m(X_1, X_2, \dots, X_n) \leq b_m$$

where $G, f_1, f_2, \dots, f_m$ are functions of the variables $X_1, X_2, \dots, X_n$, and $b_1, b_2, \dots, b_m$ are constants.

No straightforward means are available for solving the problem in its general form, but many subsets of the general problem can be solved. These special cases depend upon somewhat more restrictive assumptions than the general problem. If the constraints were nonexistent or inoperative, and the function G were everywhere differentiable, then it can be maximized or minimized by setting its partial derivatives equal to zero and solving the resulting equations for the values of the variables. Constraints can be incorporated into the above case by means of the method of LaGrange multipliers. If all the functions are linear, then the methods of linear programming are appropriate. Some techniques for handling nonlinear functions (including discontinuous functions) have been developed (e.g., quadratic programming, convex programming, integer and mixed integer programming), but present computational algorithms and computer capacities place severe restrictions on the size of problem that can be solved by these techniques. Another approach to the programming problem that is applicable under some conditions and which reduces the search for an optimum by substantial amounts is dynamic programming. Linear programming and dynamic programming show the greatest potential for applications to urban modelling, and are described more fully in the following sections.

Linear Programming

Another framework which has conceptual elegance and high flexibility is linear programming. It is a natural for many of the resource allocation problems of economics, and has led to the recent development of activity analysis, described above. From work in operations research, the technique has been extended and refined, and numerous applications made to cost-minimization or profit-maximization problems in industry and in military situations.

The linear programming problem can be stated quite simply in its general form:

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

...

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

where a_{ij} , b_i , and c_j are given constants and the x 's are to be selected so as to maximize the objective function.

A small example problem would seem to be the best way to convey the notions of linear programming (the example will use terms that apply to real-world situations, but this should not imply that the example is realistic; the terms are used to make intuitive understanding easier). A municipal agency has two resources available, "land" and "service", which it may use in the production of two commodities, "housing" and "education." It is necessary that all costs and returns be measured on the same scale, usually dollars, but here we will be slightly more abstract and refer to "units", assuming that measurement problems have already been taken care of. The problem is stated in the following way: Two units of service and one unit of land are required to produce one unit of housing, while three units of service and zero units of land are needed for one unit of education; a unit of housing is considered to be worth twice that of a unit of education, and the sum of these two values is to be maximized subject to the constraints that only twelve units of service are to be used and three units of land. Mathematically,

$$\text{Maximize } Z = 2x_1 + x_2$$

Subject to

$$2x_1 + 3x_2 \leq 12$$

$$x_1 \leq 3$$

where x_1 is the amount of housing produced, and x_2 is the amount of education. Z , the total "worth" of the commodities produced, is called the "value of the program."

Since only two variables are used, the problem can be displayed in a two-dimensional diagram in which the solution will be obvious. Figure 3.6a is such a diagram. The slanting line shows the outer limit of the first constraint and the vertical line at $x_1 = 3$ is the outer limit of the second constraint. The shaded area contains all possible combinations of x_1 and x_2 that satisfy the constraints, and this is called the feasible set (the variables can only take on non-negative values). The dashed line gives a locus of points that have a constant value of the objective function, in this case $Z=8$. All other loci of constant Z will have the same slope, and those farther from the origin will have higher values of Z . The highest one that still contains a feasible point is the optimal solution, and so the point (3,2) on the line $Z=8$ is the optimum. The value of the program is 8, which is achieved by producing 3 units of housing and 2 units of education. Upon substituting these values into the constraints, we find that the resources are exactly used up.

Another problem can be derived from the first, and this second problem is called the dual. The first problem is then called the primal. Mechanically, the dual is formed by reversing everything: columns become rows, coefficients in the objective function are switched with the constraint values (b_i 's), and the inequalities are turned around. A new set of variables is defined, with one for each constraint of the primal. The result is

$$\text{Minimize } Z = 12y_1 + 3y_2$$

Subject to

$$2y_1 + y_2 \geq 2$$

$$3y_1 \geq 1$$

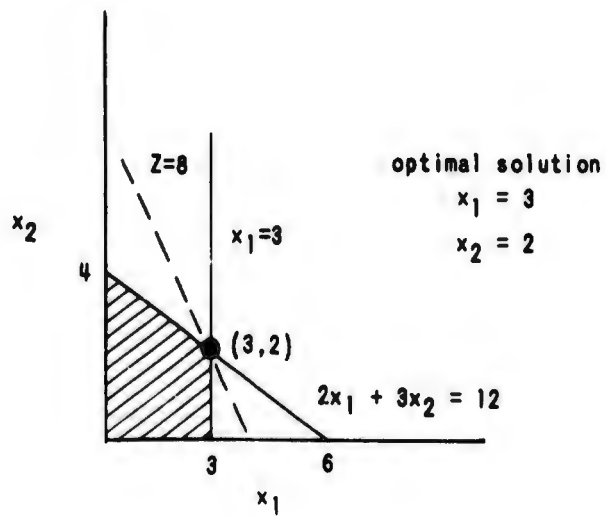


Figure 3.6a. The primal

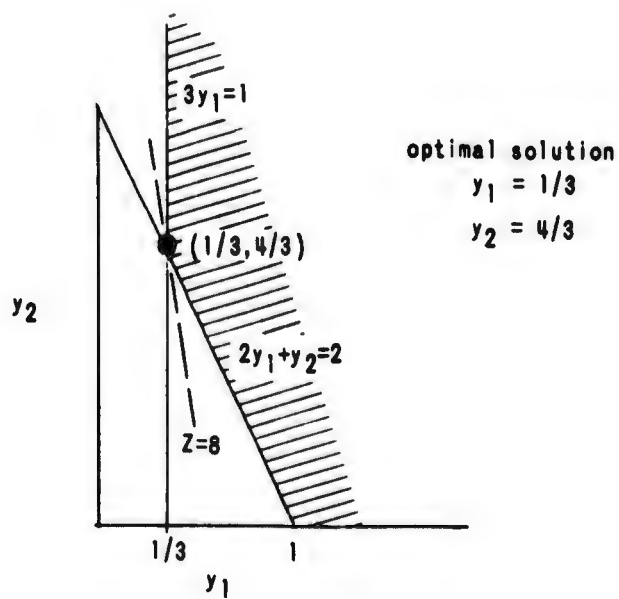


Figure 3.6b. The dual

Notice that the objective function is now to be minimized. Mathematically, the dual has many interesting relationships to the primal. For example, the dual of the dual is again the primal, so it is immaterial which way the labels are applied. Also, the optimal solution to the dual gives the optimal solution to the primal, and vice versa. The value of the program will be the same in both cases, but only when both solutions are optimal. Since again we have only two variables (there were two constraints in the primal), it is still possible to diagram the solution, and this is done in Figure 3.6b.

The question that now arises is what intuitive interpretation can be given to the dual problem? First of all, the coefficients in the objective function are quantities (units of resources in the primal) and it is reasonable to postulate that the dual variables are prices. In the constraints, then, the first one requires that the value of the resources used in producing x_1 (first column of the primal) be at least as great as the value of a unit of x_1 when it is done. The same is true for the resources used (only one) in producing x_2 . In the language of the primal, the municipal agency seeks to set prices on its resources (for sale to private developers) such that the prices will be as low as possible (to attract buyers) but high enough that the value of the inputs will not be less than the value of the outputs (no subsidy of the private market). The final result is that the social benefit to the community just equals the cost of the resources employed.

A realistic linear programming problem would have many more variables and constraints than the example given above, and could not be solved graphically. The mathematical technique used for this purpose is called the simplex method, and employs a systematic trial-and-error search for the solution that is well suited to computer implementation. In order to be solved, problems must fit one of the general formats described above, but there are many devices for making nonstandard looking problems fit the linear programming framework. Linear approximations to nonlinear functions can sometimes be used. There are

also several special cases which allow for more efficient solution techniques. One of these is called the transportation problem, which is phrased in terms of moving goods from warehouses to retail outlets but which may fit situations that do not involve any transportation. Another even simpler case is known as the assignment problem, which involves a one-for-one matching. Problems which fit into the transportation or assignment problem formats can achieve major computational economies.

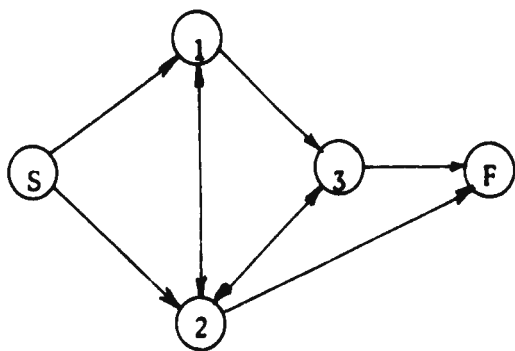
An attempt was made to utilize the linear programming framework in a residential location model for the Penn-Jersey transportation study (Herbert and Stevens [374]), but both computational limitations and data problems forced the study to abandon linear programming in favor of less elegant but more implementable techniques. Research with the original Penn-Jersey model is currently being carried on at the University of Pennsylvania (Harris [370]). Other land-use studies have considered using linear programming for various submodels, but none seem to be operational at present.

Dynamic Programming

Dynamic programming seems to have earned the label "dynamic" by virtue of the solution method, which proceeds in recursive "stages" that can be interpreted as time periods. A large part of dynamic programming problems, however, do not involve time, and other techniques (such as linear programming) can often handle time equally well. It would be more appropriate to view dynamic programming as a computational algorithm, specifically a branch-and-bound technique, rather than as a standard problem or model.

A simple, somewhat abstract example will serve to illustrate the nature of the dynamic programming style. Suppose there is a set of "tasks" associated with a certain project, and some but not necessarily all of the tasks must be undertaken before the project is completed. The order in which the tasks may be accomplished is restricted, and there is a "cost" attached to each task which depends upon which task preceded it. The problem is to

find a sequence of tasks that will minimize the total cost of the project. This problem is similar to finding the maximal flow in a network, determining the critical path in a work program, or solving the "stagecoach" problem of dynamic programming. Several algorithms could be used, including integer programming, but we will use a method that is basically dynamic programming. The problem can be diagrammed as a network:



"S" indicates "start", "F" is "finish", and numbered nodes mark completion of tasks. Arrows show possible sequences. The cost matrix is given by

	1	2	3	F
S	6	3		
1		1	4	
2	2		7	10
3		1		2

Blank entries mean that a particular pairwise ordering is not possible. For example, after completing task 3 it is possible to work on task 2 at a cost of 1, or to finish at a cost of 2, but task 1 cannot be worked on from 3. To solve this problem, we will systematically check all possible sequences, using the following rules:

- (i) Tasks will be considered from the finish backwards until we reach the start.
- (ii) Each sequence from the node being considered to the finish will be followed and costed until
 - a. A node that has already been considered is reached, or
 - b. The cost is higher than some previous sequence that reaches the finish, or
 - c. The finish is reached.
- (iii) When all sequences from a node to the finish have been investigated, the least-cost sequence will be marked and the node labelled with the cost. This means that at each stage (node being considered) the optimal sequence to the finish is determined given that the process arrives at that node. This is an approximate statement of Bellman's Principle of Optimality (Bellman and Dreyfus [200]).

The calculations are presented in the following table:

Task just completed	Possible sequences and costs				minimum, given node
3	3-F: 2	3-2-1x	3-2-3x	3-2-F: 11	2
2	2-F: 10	2-3,+2: 9	2-1-2x	2-1-3,+2: 8	8
1	1-2,+8: 9	1-3,+2: 6			6
S	S-1,+6: 12	S-2,+8: 11			11

Thus if one lands at node 3, the minimum cost from there is 2; working back to the first stage, the minimum cost sequence costs 11 and requires sequencing tasks in the order Start-2-1-3-Finish. While the example was very simple and could easily be solved by inspection, the rules can be used on larger problems, with substantial computational savings.

Birth and Death Processes

The growth of a population, the arrival of traffic at a toll booth, or the number of historic buildings existing in a city, are examples of real-world phenomena that can be modelled as birth and death processes. Like many of the models from operations research, the terminology used reflects the nature of early applications and the need to give special meanings to words. Theories of plant and animal evolution, and of the behavior of atomic particles, were studied with birth and death processes. A separate body of technique and language emerged from the study of queues, or waiting lines. At the elementary level there is no need to distinguish between different formulations of the same problem, since it is indifferent whether we call a problem a queuing problem or a birth and death problem. Only the language is different.

Let us assume that we are interested in studying the long-term implications of a historic preservation program. A birth, or arrival, takes place when a building of significant architectural merit is constructed, and a death, or service, occurs when such a building is torn down, destroyed by fire, or otherwise removed from the stock of historic buildings. We assume that the private market produces the new buildings, and generates the pressures for their removal, and that these events can be described probabilistically as functions of time and the number of buildings existing in the system. Thus

$\lambda_n(t)$ = birth or arrival rate, and

$\mu_n(t)$ = death or service rate

These rates give the likelihood of an event in some small unit of time, but may be expressed in any convenient metric, such as buildings per year. If we let $P_n(t)$ = Prob (n historic buildings exist in the system at time t), then

$$P_n(t+dt) = \mu_{n+1}(t)P_{n+1}(t)dt + \lambda_{n-1}(t)P_{n-1}(t)dt + [1-\mu_n(t)-\lambda_n(t)]P_n(t)dt$$

This says that the new probability of having n buildings alive at time $t+dt$ is the sum of the probabilities of three different ways of arriving at that state: having $n+1$ buildings alive at t and having a death in dt , having $n-1$ alive at t and having a birth, or having n buildings existing at t and not having either a birth or a death. Rearranging terms and dividing by dt , we get

$$\frac{P_n(t+dt) - P_n(t)}{dt} = \mu_{n+1}(t)P_{n+1}(t) + \lambda_{n-1}(t)P_{n-1}(t) - (\mu_n(t) + \lambda_n(t))P_n(t)$$

Taking the limit as $dt \rightarrow 0$, and substituting for n ($n=0,1,2,\dots$)

$$P'_0(t) = \mu_1(t)P_1(t) - \lambda_0(t)P_0(t)$$

$$P'_1(t) = \mu_2(t)P_2(t) + \lambda_0(t)P_0(t) - (\mu_1(t) + \lambda_1(t))P_1(t)$$

...

$$P'_n(t) = \mu_{n+1}(t)P_{n+1}(t) + \lambda_{n-1}(t)P_{n-1}(t) - (\mu_n(t) + \lambda_n(t))P_n(t)$$

Since we have set no bounds on the state space, the number of equations is infinite. We have also set $\mu_0 = 0$ since a death cannot occur if the system has no members. The system of differential equations given above is generally very difficult to solve, depending upon the functions used for arrival and service time distributions. An immense simplification can be achieved if we assume that both arrivals and services are Markovian, which means that the expected number of arrivals per unit time is a constant (Poisson inputs) and the expected length of service for each server is a constant (exponential holding times). The assumption has the same meaning for arrivals as services, i.e., the distribution of times between arrivals is exponential. In our example we would make the assumptions that the birth

rate was independent of the state of the system (number of buildings in it), and the number of deaths was proportional to the number in the system. Thus

$$\lambda_n(t) = \lambda \quad \text{and} \quad \mu_n(t) = n\mu$$

where λ and μ are now scalars (fixed parameters).

Placing these simplified rates in our system of equations, we can solve for the state distribution as a function of time and the initial state. If we are interested only in a stationary or equilibrium distribution (this does not mean the process is constant, only that the probabilities are constant), the system of equations is easily solved. The probability of zero buildings and the probability of n buildings are

$$P_0 = e^{-\frac{\lambda}{\mu}} \quad \text{and} \quad P_n = \frac{\left(\frac{\lambda}{\mu}\right)^n e^{-\frac{\lambda}{\mu}}}{n!}$$

The average number of buildings in the system is $\frac{\lambda}{\mu}$. We are particularly interested in the number of buildings that survive past a certain age, at which time public policy might go into effect. If this age is s , then the expected number of buildings older than s at any given moment is

$$\frac{\lambda}{\mu} e^{-\mu s}$$

We could also find the average lifetime of buildings or the expected length of time between periods with zero buildings, or several other measures of system behavior.

Queuing theory has been used extensively for studying traffic conditions at such potential bottlenecks as toll booths or street intersections, for the purpose of answering such questions as what is the average delay, probability of waiting more than five minutes, average time an individual spends in the system, average length of the queue, etc. Describing the system involves specifying the arrival and service distributions, and the number of channels for servicing (e.g., toll booths; in the building example we had an infinite-channel queuing system). Additional complications, such as priority queues and batch arrivals, may also be introduced, but theory becomes difficult extremely rapidly once the Markovian assumptions are relaxed.

Much of the theory of stochastic processes, including birth and death processes, is too micro-analytic to be of much use in large-scale urban planning models. As these models become more able to incorporate behavior of small units, and the stochastic models become more general and more tractable, they may see greater use. At present, the only feasible way of utilizing the more complicated stochastic models is through Monte Carlo simulation, and this effort is seldom justified for work other than research.

Markov Chains

A Markov process is a stochastic process in which the probability of moving from one state to another depends only upon the present state, the present time, and the time of interest. A stochastic process is a series of events which occur in time and are described only in a probability sense. A "state" is an abstract term which in some way describes the status of the process. The process might be inter-urban migration, and the states would be different cities. A change, or transition, might occur at any point in time. The important Markovian assumption is that the likelihood of a transition depends upon the present state, and not on any previous states or the time of arrival in the present state, or on any other attribute of the system or its past history.

If transitions occur only at specified intervals in time, and not continuously, then the process is a discrete-time Markov process, or a Markov chain. It is conceptually possible to have an infinite number of states, and some problems must be modelled this way, but we will reduce the demands for exposition by discussing only finite Markov chains. The theory of continuous-time stochastic processes and infinite state spaces is well developed, mathematically difficult beyond the basic level, and available elsewhere (Feller [218, 219], Karlin [239], Prabhu [256]). We will also eliminate almost all algebraic derivations and simply state results. Means of decomposing and classifying Markov chains will be described, and some examples given.

Consider a hypothetical four-state migration model. One state will be called The Farm. Most folks stay on the farm (if they are already there) but a few migrate off to New York. From New York a person might go to Paris (left bank), and once in Paris he may move to the right bank, or back to the left bank, but he never leaves Paris. This situation might be represented in a matrix as

$$T = \{p_{ij}\} = \begin{array}{c|cccc} & F & NY & PLB & PRB \\ \hline F & .9 & .1 & 0 & 0 \\ NY & .1 & .6 & .3 & 0 \\ PLB & 0 & 0 & .7 & .3 \\ PRB & 0 & 0 & .2 & .8 \end{array}$$

The matrix T is called the transition matrix, and gives the probabilities of moving from any state to any other (given that an individual is in some state), in one time period (say one year). Since an individual must either stay in some state or leave it, each row must sum to 1.0. For example, an individual in New York will either stay ($P=.6$), go to Paris, Left Bank ($P=.3$), or go back to The Farm ($P=.1$).

Suppose we are interested in where an individual will be after two time periods. If he starts on The Farm, he may still be on The Farm after two periods, or he may be in New York, or he may be in Paris. These probabilities can be obtained by multiplying T by itself.

$$T^2 = \begin{bmatrix} .82 & .15 & .03 & 0 \\ .15 & .37 & .39 & .09 \\ 0 & 0 & .55 & .45 \\ 0 & 0 & .3 & .7 \end{bmatrix}$$

Thus the probability of showing up in Paris after two years, starting from The Farm, equals .03. T^2 is called the two-step transition matrix. The n th power of the T -matrix is called the n -step transition matrix and gives the probability distribution of the process after n steps, or periods. If we let the process go on indefinitely, i.e., $n \rightarrow \infty$, then

$$\lim_{n \rightarrow \infty} T^n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & .4 & .6 \\ 0 & 0 & .4 & .6 \end{bmatrix}$$

This means that, no matter where the process starts, after a sufficiently long time it will be in state PLB with $P=.4$ and in PRB with $P=.6$. Whether an individual starts on The Farm or in New York, he will eventually go to Paris and never come back.

Since the probabilities of ultimately ending up in the states "The Farm" and "New York" approach zero, these states are called transient. If it is possible to return to a state after it has been left, then we can ask what is the probability of ever doing so, or what is the probability of returning in n time periods. If the probability of returning to a state after it has been left is unity, then the state is recurrent or persistent. The states PLB and PRB are persistent in the above example. We may then ask what is the average length of time before returning to the state, and this is called the mean recurrence time.

In some cases, return to a state may be possible only at certain equally-spaced intervals, in which case the state is called periodic. A trivial example will illustrate this point.

$$T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

An irreducible (ergodic) chain will always have a steady-state or stationary distribution. If, in our example, we drop off the transient states, we have a two-by-two transition matrix. A stationary distribution, if it exists, must have the property that repeating the process one more time period does not alter the distribution. Letting (π_1, π_2) be the stationary distribution,

$$(\pi_1, \pi_2) \cdot \begin{bmatrix} .7 & .3 \\ .2 & .8 \end{bmatrix} = (\pi_1, \pi_2)$$

The solution to this problem is the same as finding an eigenvector of the transition matrix whose elements sum to one. Such a vector is $(.4, .6)$, which corresponds to our earlier solution to the limiting distribution. In this case, we have found a unique initial distribution which is not changed by the process. The two questions (the limiting distribution and the stationary initial distribution) have the same answer.

From the above we can see that questions regarding the distributions within ergodic sets can be answered without reference to the chain in which they are placed. There are a number of questions we can ask, however, about chains which have several ergodic sets and one or more transient states.

(a) How long is it likely to be before the process is absorbed into the ergodic sets?

(b) Which sets are most likely to absorb the process?

(c) If a chain starts in state i , how long will it be before it is in state j ?

(d) What affect does the starting distribution have on the state distribution?

(e) What is the average number of times that the process will pass through a particular transient state before being absorbed? These questions

and many others can be answered, but the reader will have to rely on the references for the techniques for finding them.

Our discussion of Markov chains has been from the point of view of an individual passing through the process, and we have tried to make probability statements concerning his status. This approach is referred to as an "individual process." We can also regard the process from the point of view of a group, whose members are distributed according to the initial vector and who are subsequently allocated to different states by the transition matrix. The transition probability p_{ij} is the proportion of the group presently in state i that will be sent to state j in the next period. The San Francisco CRP employed a submodel of this type for describing the aging and deterioration of residential structures, with the states being different levels of building condition (Arthur D. Little [309]).

Differential and Difference Equations

Differential equations involve functions and their derivatives. Suppose, for example, we postulate a population that grows at a rate proportional to the number in the population (the larger the population, of fruit flies, say, the larger the number of offspring). If $P(t)$ is the size of the population as a function of time, then

$$\frac{d}{dt} P(t) = a P(t)$$

where a is an arbitrary constant.

Rearranging, we have

$$\frac{dP(t)}{P(t)} = a dt$$

Integrating both sides and taking antilogarithms, we get the solution

$$P(t) = P_0 e^{at}$$

where P_0 is the population at $t=0$.

This is an exponential growth or decay curve, depending upon whether a is positive or negative. Our hypothesis suggests that a is positive, in which case the population grows exponentially without bound.

The principle of the differential equation can be extended in many different ways, with some extensions being more difficult to solve than others. First of all, several different derivatives of the same function may appear in the equation, and the highest derivative determines the order of the equation. The equation solved above is a first order equation. Another complication which may occur is that the function or its derivatives may be raised to a power, or appear as products in the same term. If the equation is reducible to a form in which each member (function or derivative) is raised only to integral powers, the degree of the equation is the largest exponent or sum of exponents in any one term. The equation solved above is a first degree, or linear, equation. A third means of classifying differential equations depends upon whether the coefficients of the derivatives and the function are constant or variable (i.e., functions of the same variable as the primary function). A fourth broad classification is into partial and ordinary differential equations, with partial differential equations involving functions of more than one variable. Other means of distinguishing among various equations exist, but are mostly special cases of the classifications we have mentioned. To describe the equation we solved more completely then, we would call it a first-order linear ordinary differential equation with constant coefficients.

Ordinary linear differential equations with constant coefficients are relatively easy to solve. The general form for an n th order equation is

$$P^{(n)}(t) + a_{n-1}P^{(n-1)}(t) + \dots + a_1P'(t) + a_0P(t) = f(t)$$

where $P^{(n)}(t)$ represents the n^{th} derivative of the function, and the

constant term a_n has been eliminated by dividing the equation by it. If $f(t) = 0$, we have the homogeneous form of the equation. The solution to the general equation is composed of two parts: the solution to the homogeneous form and a (any) particular solution (a homogeneous equation will not have a particular solution). Letting $P_p(t)$ be the particular solution, and $P_h(t)$ the homogeneous solution, the general solution is

$$P(t) = P_h(t) + P_p(t)$$

The particular solution is obtained through methods best described as trial and error, employing solution forms that seem likely to work. The homogeneous solution always has the form

$$P_h(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \dots + c_n e^{r_n t}$$

where $c_1, c_2, \dots, c_n$ are constants which must be determined by initial conditions on the equation, and $r_1, r_2, \dots, r_n$ are the roots to the characteristic polynomial

$$r^n + a_{n-1} r^{n-1} + \dots + a_1 r + a_0 = 0$$

The completed general solution can be analyzed for its behavior over values of the independent variable.

The above discussion does not even hint at the variety of possibilities which may arise, but the techniques for solving these equations are well worked out in texts. Other kinds of equations are to varying degrees less tractable. A few partial differential equations can be solved for in closed form, and a few more in the form of infinite series. Nonlinear differential equations can be solved only under very limited conditions. Linear ordinary

equations with variable coefficients are quite often soluble in closed form, but this depends upon the functions. Elaborate numerical techniques have been developed, that take advantage of the speed and computational ability of the computer, to construct close approximations to solutions under specific circumstances. If only numerical results are needed, then the analytic techniques can be bypassed. The same numerical techniques are also applicable to systems of differential equations.

Differential equation methods have been very useful in the physical sciences, in analyzing heat flow, vibrations, particle movement, velocities in gases, etc. Applications in the social sciences have been few, however, and there are a number of possible explanations. We are at the stage in social science of searching for the relevant variables affecting variables of interest, and it seems inappropriate to develop elaborate models involving single variables. Measurements are also crude, and attempts to relate functions of derivatives higher than the second are likely to be overwhelmed by noise. Systems of equations may help to incorporate more interdependence into differential equation models, and theory may shed some light on the second problem, but present development is rudimentary. A differential equation model was designed by the Traffic Research Corporation (TRC [13]) for the Boston region, but was not actually used; the model, called POLIMETRIC, is being considered for further development by TRC.

The formulation and solution of difference equations utilizes techniques analogous to those for differential equations. The primary difference between the two forms is in the independent variable -- difference equations use a discrete integer variable while differential equations are functions of a continuous variable. It is not necessary that time be the independent variable, but it is a common one and heuristically useful. For difference equations, then, time is regarded as occurring in discrete lumps, or periods.

As an example of a simple difference equation, we will use a supply-demand model known as a "cobweb" model. Assume that the demand for a particular commodity is a linear function of the price of the commodity, and that supply is a (different) linear function of the price that prevailed in the market in the previous time period. Letting P_t be the price at time t ,

$$\text{Demand} = a_0 + a_1 P_t$$

$$\text{Supply} = b_0 + b_1 P_{t-1}$$

To clear the market, demand must equal supply, so

$$a_0 + a_1 P_t = b_0 + b_1 P_{t-1}$$

which is a linear first-order difference equation. Two questions can be answered from this equation: (a) whether an equilibrium exists and what it is, and (b) how price changes with time and whether it leads to equilibrium. If an equilibrium exists, then $P_t = P_{t-1}$. Assuming the equilibrium exists and representing it by P , we substitute this into the equation. Then

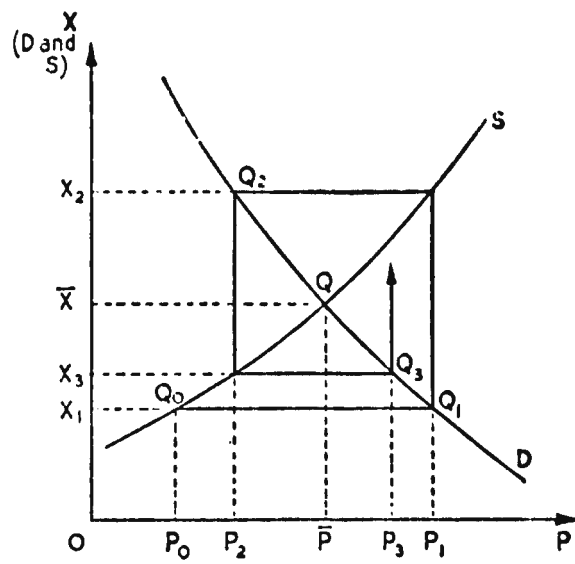
$$P = \frac{b_0 - a_0}{a_1 - b_1}$$

This is equivalent to the particular solution in solving differential equations. The rest of the solution is found by solving the characteristic equation of the homogeneous form, and in this case is

$$P_t = (P_0 - \bar{P}) \left(\frac{b_1}{a_1} \right)^t$$

where P_0 is the initial price at $t = 0$. The complete solution is then

$$P_t = \left[P_0 - \frac{b_0 - a_0}{a_1 - b_1} \right] \left(\frac{b_1}{a_1} \right)^t + \frac{b_0 - a_0}{a_1 - b_1}$$



Source: Allen [196], p.3

Figure 3.7 A COBWEB MODEL

We can see that if $\frac{b_1}{a_1}$ is less than 1, the first term in the right-hand side will eventually go to zero and the equilibrium price will prevail. If $\frac{b_1}{a_1}$ is greater than one, then the price will grow in absolute value until it explodes (actually it will probably oscillate in wider and wider sweeps). In this case, only if $P_0 = \bar{P}$ will an equilibrium dominate the solution, and it is highly unstable since any deviation will cause it to diverge. A convergent case is shown in Figure 3.7.

The general form of the linear difference equation with constant coefficients is

$$P_t + a_1 P_{t-1} + a_2 P_{t-2} + \dots + a_n P_{t-n} = f(t)$$

The characteristic equation is

$$r^n + a_1 r^{n-1} + a_2 r^{n-2} + \dots + a_{n-1} r + a_n = 0$$

and the complete solution is given by

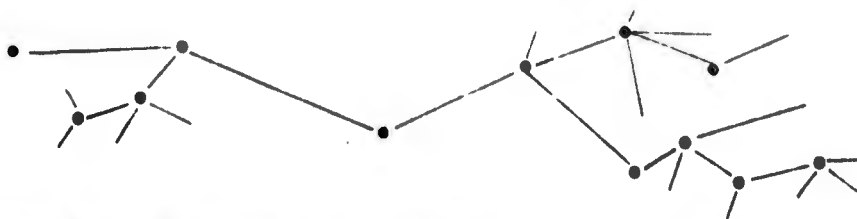
$$P_t = \alpha_1 r_1^t + \alpha_2 r_2^t + \dots + \alpha_n r_n^t + \bar{P}$$

where $\alpha_1, \alpha_2, \dots, \alpha_n$ are determined by specifying n initial conditions, usually P_t at $t=0, 1, \dots, n-1$. Depending upon the roots of the characteristic equation and the function $f(t)$, situations may arise that require special treatment.

While the difference equation model is very similar to the differential equation model, the difference equation model has a slight advantage in that it considers time as occurring in discrete periods. At the present state of urban modelling, comparative static models -- some quite complex -- are about as close to dynamic models as we have come. Although these models could be formulated as difference equations, rarely do they utilize information from more than one or two time periods and hence are not very sophisticated from the difference equation standpoint.

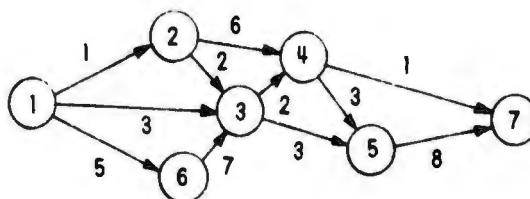
Networks

A graph is a set of points, called vertices, and a set of lines connecting the points, called edges. Although essentially a mathematical construct, many graphs can be represented on a sheet of paper (planar graphs) and many interesting problems of a wide variety can be translated into graphs, perhaps gaining insights and solutions in the process. Different kinds of graphs are described by their properties; a tree, for instance, is a connected graph (every vertex is accessible from every other, perhaps by going through other vertices) that has no circuits (it is not possible to return to any vertex without traversing the same edge twice). A tree is shown below.



The language of graph theory is not difficult, but we will not develop it further here.

A network is a directed graph (the edges have an arrow at one end) with quantitative values assigned to edges and/or vertices. In a network, "edges" are usually called "arcs", and "vertices" are called "nodes." Networks, more often than graphs, are direct analogs of the system they represent, such as a traffic network or a work schedule. A sample network with nodes numbered and attributes assigned to arcs is shown below.



One general type of network problem is to find a maximal flow through the network. For this purpose, one node is designated as the "source" (node 1 in the diagram) and another is called the "sink" (node 7). Flows might be electrons in an electrical circuit, water in pipes, or vehicles on streets. Nodes represent junctions or intersections, and the arc attributes are capacities. Using the diagram as a traffic network (single-headed arrows are one-way streets), the maximum volume of traffic that could be moved from the source to the sink would be 6, which is found by an algorithm similar to those used for traffic assignment in transportation models. If we allow all the streets to go in either direction, then the maximal flow is 7, and this solution is obtained by finding the "minimal cut". This is a set of arcs separating the source and the sink and having the least capacity, and in this case the minimal cut contains arcs 3-5, 4-5, and 4-7.

Using the same network, we might be concerned with finding the shortest path, and then the arc attributes would represent length or time required to traverse the arc. The shortest route through our network passes through nodes 1-2-3-4-7 in that order, and requires a total of 6 units. The attributes might also represent costs, with the objective being to find the minimum cost path.

If the network represents a work schedule, as in critical path or PERT analysis, then the nodes are activity completions and the arc constraints are time requirements for the associated activities. The network diagram depicts the ordering of the tasks, and all tasks must eventually be completed (some can be carried on concurrently). Of interest in such a network is the minimum time necessary to complete the project, or the longest path. The "critical path" in the diagram is 1-6-3-4-5-7 and requires 25 units (e.g., weeks) from start to finish. The consequence of this analysis might be to allocate more resources to tasks lying along the critical path.

Thus we have seen considerably different problems, with different solutions, that all used the same network. When the network is very small, as it was in this case, the solution to a problem can easily be determined

by inspection. More practical problems, on the other hand, will usually require large networks to describe them, and reliable and efficient algorithms for obtaining solutions. These algorithms are available for a large number of problems, and they are designed for implementation on a computer. Some of the problems can be formulated as transportation problems or as assignment problems, and the solution may be easier in the programming form or in network form. Dynamic programming is also an often useful means for attacking network problems. A number of Electrical Engineers have used graph and network theory in combination with analogs of physical systems to model social systems (Ellis and Milstein [215], Ellis and Van Doren [216], Grecco and Breuning [227], and Koenig [244]).

Statistics

Almost all models are statistical in the sense that in order to apply them to the real world it is necessary to adjust the parameters of the model by statistical means. It must be assumed that there are errors of approximation in the model and errors in the measurement of the variables, so models that fit the data exactly are usually out of the question. The best that can be done is to make the model fit the data in as optimal a way as is possible, using some criterion for "goodness of fit" such as minimizing squared deviations. Specifying a model that will replicate a portion of reality for a given time and place is, inherently, a statistical procedure. Thus one important use of statistics in modelling is in fitting or calibrating the model.

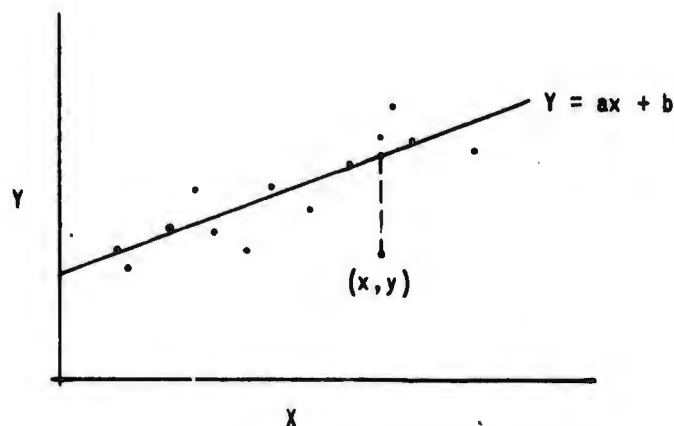
Statistics have another important use in modelling. All data can be thought of as having two components: a systematic component that can be explained (numerically) using relationships with other variables, and a random component that is called uniqueness or noise. The proportion of variation that is viewed as being random depends upon the state of our knowledge, and it is the object of research to reduce the proportion of variation that must be regarded as noise. Unavoidably, in most cases,

social data tend to measure several things at once, and seldom are they exactly what we are interested in. Statistics are used to sort out the systematic, explainable variation, and to control for unwanted variables in the data. Thus a second important use of statistics is in testing model relationships and structure.

In either case, the technique most commonly employed is regression. For example, assume our hypothesis is that the relationship between two variables is linear, i.e.,

$$Y = aX + b$$

where a and b are unknown constants to be estimated by regression. When numerical values are given to these constants, a particular line will have been selected out of the family of lines given above, and this line will have the property of minimizing the squared deviations of Y from the line. A typical data configuration with the fitted line is shown below.



Given a set of observations, $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, we wish to find a line which minimizes the squared deviations of the actual Y values from the line, i.e.,

$$\min M = \sum_{i=1}^n (y_i - ax_i - b)^2$$

The x's and y's are known values, and we are to find a and b. To do this, we take the partial derivatives of M with respect to a and b, and set them equal to zero.

$$\frac{\partial M}{\partial a} = 2 \sum_i (y_i - ax_i - b)(-x_i) = 0$$

$$\frac{\partial M}{\partial b} = 2 \sum_i (y_i - ax_i - b)(-1) = 0$$

These equations can be reduced to the normal equations,

$$\sum y_i = bn + a \sum x_i$$

$$\sum x_i y_i = b \sum x_i + a \sum x_i^2$$

which can be solved for a and b in terms of $n, \sum y_i, \sum x_i, \sum x_i^2$, and $\sum x_i y_i$. Since all these quantities are specific known numbers (from the data), we now have values for a and b such that, for any value of X, we can estimate a value for Y.

In addition to being able to make better estimates of Y by knowing the value of X, there are a number of other questions we can answer with regression tools. Depending upon our assumptions about the joint distribution of X and Y, we can find out how strong the linear association of X and Y is (the coefficient of correlation), how good our estimates are

likely to be if we use the estimating equation, and whether our coefficients (parameters, a and b) are significantly different from zero. We can also ask whether the linear relationship is meaningful in terms of the data, and this is the process of hypothesis testing.

The bivariate (two-variable) linear regression illustrated above is one simple example of a large variety of functions that can be fitted to data using the minimization of squared deviations, or "least squares," technique. Some other examples are shown below.

(i) The Y-intercept is zero (from theory, say). Then we fit

$$Y = aX$$

and solve for the single parameter, a (the slope).

(ii) The function is a polynomial in X. Then the function is

$$Y = a_0 + a_1X + a_2X^2 + \dots + a_nX^n$$

which results in a bivariate nonlinear regression.

(iii) Y is a function of several variables.

$$Y = a_0 + a_1X_1 + a_2X_2 + \dots + a_nX_n$$

In this case we have a multiple linear regression. The function fitted, instead of being a line, is a "hyperplane". Combinations of the above forms are also possible. The more complicated the function being fitted, the more strenuous the demands are upon data and the more difficult the function is to justify on theoretical grounds.

Linear regressions, including multiple regressions of fairly high order, are not hard to accomplish once the data are correctly punched on cards, or are on tape. Package computer programs for linear regression are commonly available. For this reason, it is often preferable to transform nonlinear hypotheses into linear ones, and this can readily be done.

(i) Fitting an exponential function,

$$Y = ae^{bX}$$

is most easily done by taking logarithms and fitting

$$\log_e Y = \log_e a + b \log_e X$$

instead. The latter function is a straight line.

(ii) Fitting a quadratic,

$$Y = a_0 + a_1X + a_2X^2$$

may be handled by letting $Z = X^2$ and fitting

$$Y = a_0 + a_1X + a_2Z$$

which is a multiple linear regression instead of a polynomial regression.

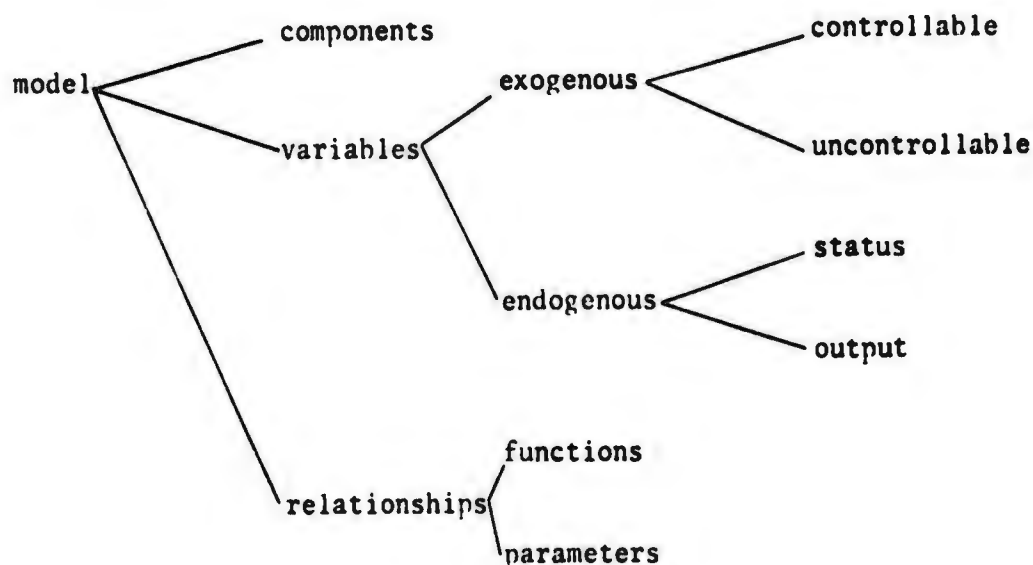
Some of the more direct uses of statistics have been mentioned above, but there are many more statistical techniques that may be of value at different stages in the research or modelling process. Some of these are factor analysis, component analysis, spectral analysis, canonical correlation, cluster analysis, latent structure and other nonmetric analysis, the theory of order statistics, nonparametric statistics and discriminant analysis. The literature in statistics, both theory and application, is extensive, and important advances are made at frequent intervals.

CHAPTER 4: MODEL CONSTRUCTION

A model which is useful for planning purposes must be operational, and many implications can be drawn from this statement. It means that the statements of the model can be manipulated and that all relevant sets of conditions can be handled without ambiguity. This usually takes the form of a working computer program that operates the model. Operational also means that the variables are measurable and have been measured, i.e., the data are available. In addition, the model must have clearly defined ties to reality, so that the results of model operation can be transformed into conclusions applicable to the real world. This is required in order that the model can be asked interesting questions, and also that it can be tested by comparison with the reality it is meant to model. Although these characteristics are not sufficient to produce a good model, they are certainly necessary. Nor is there any simple way of setting performance criteria for these characteristics, but if a model responds to data from the real world, operates on reasonable assumptions, produces information that is of interest to the planner or decision-maker, and can distinguish between significantly different alternatives, then the model can be called useful.

In this chapter we will attempt to synthesize the pieces that have been discussed up to this point, and carry through with the framework that was presented in chapter 2. The synthesis will be in terms of model elements, grouped into three categories: components, variables, and relationships. Components are the entities in the system, and might be automobiles, households, markets for goods, goods themselves, land uses, or any other subject of interest. Variables are attributes which describe the components, and might be number of households, age of head of household, volume of transactions in the market, value of goods, etc. Relationships are the linkages between the variables, and usually are represented as equations or sequences of operations in urban models.

Components may be grouped in different ways depending upon the subject of the model. Variables are grouped into two types, already mentioned, which are exogenous and endogenous; exogenous variables are either controllable (policy or decision variables) or uncontrollable (accepted as given), and endogenous variables may be separated into status variables and output variables. Status variables are those that describe the state of the system at any moment, and may become the output variables at the completion of model operation. Model relationships are functional relations between variables and parameters which establish specific relationships within families of functions. A tree-graph of the above breakdown is shown below.



Decisions in model construction are grouped in this chapter in steps or stages, but the difficulty with any stage theory is that it may be taken too seriously. The transition from one stage to another may not take the form of a clean break, and so entities will exhibit the characteristics of several stages. There will also be some feedback, in which entities move backwards through a stage (in planning, or in modelling, regression is called "feedback", and the word "regression" is retained only for its statistical meaning). If a stage theory is any help in studying some phenomenon, it is because the entities tend to cluster at the postulated levels or stages, and the amount of feedback is small compared to the forward flow and the tendency to remain in a stage. Our stage theory of model construction is probably marginal in this respect, but it seems important to attempt a description of the modelling process and so we have used it. The data problem does not fall into a stage, but exerts a constant pressure at all stages, so it is presented first.

A. Data

A model is simply a sophisticated information processor, and hence it is dependent upon the consumption of information for the production of information. Experience seems to indicate that the needs of models for quality and quantity of data are vastly underestimated, both by nonmathematical potential users and by modellers themselves. Data are needed for testing hypotheses concerning relationships, for fitting the model to the real world at a particular time and place, and for providing information to the model which it can operate upon.

Hence the quality of the inputs, and the strength and adequacy of the structural relationships determine the quality of the output. If, for example, a model uses census data based on a 25% sample, by census tract, and contains relationships that explain 70% of the variation between tracts,

the results of the model cannot possibly be very accurate for any particular census tract. Production of greater detail with those inputs is out of the question. That this should be true is obvious on the face of it, yet users and modellers alike seem compelled to claim unreasonable powers for their models. Very few models cannot be described as having sample data for inputs, and only partially explanatory relationships. Variability is part of the real world, and will always exist. A model does well if it does not add substantially to the variability in the information it receives, and it certainly cannot reduce it. To state that a model is no better than the information it is fed is to reiterate the truism about the weakest link in a chain, but it seems to need reiterating.

The difficulties of obtaining needed data are brought about by the complex interactions of the many facets to the problem, rather than any single problem. Sometimes a problem is acute only because of the number of times it occurs, and not because of any conceptual obscurity. Other problems result from the inadequate definition of apparently straightforward and easily measured variables. In many cases the problem is one of fixed resources which can be allocated in an almost infinite number of ways to attain competing objectives. A detailed discussion of some of the facets of the data problem may serve to make the difficulties real.

1. Multidimensionality

In urban planning we are concerned primarily with land use data, population data, and economic data, and particularly with those items of

information that have a spatial attribute, i.e., they are located in space. For our purposes we can conceive of these data as having roughly three dimensions: entities, attributes, and time. An entity may be defined differently for different purposes, but it is the basic unit of collection. Parcels, blocks, census tracts, cities, persons, and streets are common entities. Entities may then have attributes, e.g., parcels have houses and buildings, census tracts have square feet of commercial land use, cities have volume of sales from manufacturing to wholesaling, and street segments have sewer pipes. Census tracts can have people, which illustrates the point that the same object may be an entity at one point and an attribute at another. The label is primarily a conceptual aid. Finally, observations can be taken at any point in time, and in general will be different for different times.

The dimension problem can be readily seen in an example. If twenty attributes (e.g., no. of people, no. of males age 25-29, no. of people who work in hospitals and education, square footage in residential use, etc.) are recorded for the City of Pittsburgh in 1960 by census tract (189 tracts, averaging about 3000 persons per tract) for three different points in time (1940, 1950, 1960), then there are 11,340 items of information required at a very minimum. If, in order to obtain one of the twenty attributes, it is necessary to record two items and combine them (e.g., male plus female), then we have added 567 items to the total. Including all of Allegheny County in the analysis (487 census tracts) instead of just the city makes the total of 29,220 items for the twenty variables. Three time periods are not enough to generate a time series, census tracts still represent a fairly high level of aggregation, and twenty variables do a sketchy job of describing a city. Yet the job of recording these items, even from secondary sources, is sizeable.

2. Sectoral Disaggregation

A census tract has persons, white and nonwhite persons, male and female persons, male and female persons by five-year age groups by white and nonwhite, etc. Each use of the word "by" adds a dimension to the attributes

space. Without extensive testing, manipulating, and trial and error, it is difficult to know what variables are the most relevant and the best explainers. Consequently the investigator wants to try all variables that might possibly be relevant, with the consequence that he grabs too many, spends too much effort recording, tabulating, and removing errors, and not enough on analysis.

Once the variables are inside a computer, totals, percentages, ratios, logarithms, etc., can easily be computed at very little cost. Even here, however, poor planning can result in very inefficient operation. It can be a lengthy process to search through several magnetic tapes, pulling out one or two items of information from each two-hundred item record, but it is even worse when items of related information are on separate tapes. File design deserves a lot of attention in a computerized information system, but this discussion is beyond our scope.

Disaggregation is an important consideration in the use of secondary information, as, for example, from the census or from survey questionnaires. Some users may want detailed breakdowns by age, sex, and color, but others may want less disaggregation. It is awkward for someone who wants to know the number of elderly people by census tract to have to record and add together five age categories, two sex categories, and two color categories (twenty items) to get one variable. If a survey has recorded information at a level that turns out to be too disaggregated, there is little that can be done short of using it all. While overly disaggregated information may not be as bad as overly aggregated information, the choice is between the inaccessible and the unsatisfactory, which is not much of a choice.

Disaggregation has effects throughout the modelling process, and the costs of acquiring and processing more detailed information must be weighed against the benefits of more detailed output. A view widely held among modellers is that disaggregation produces stronger relationships, more or less automatically, but at the present state of knowledge the reverse is more often true. Disaggregation is likely to spread the data too thinly, disaggregated

entities are often so similar (due to poor sectoring) that relationships are simply diluted, and often less is known about any of the disaggregated relationships than was known about the aggregate relation. It is easy enough to say that households do not behave identically in their location preferences, but then quite difficult to define household types and obtain data for them that distinguish between types in a reliable way. In addition, the costs of disaggregation are high and typically underestimated, with the effect that sacrifices are made in other portions of the modelling effort.

3. Unit of Collection

Several hierarchies of spatial units exist from which to choose the most appropriate unit. Counties in New York State are divided into incorporated areas and towns; cities are divided up into wards, and census tract subdivisions usually follow ward lines. Within census tracts are blocks, and within blocks are parcels. Parcels may contain one or more structures. There are exceptions to this hierarchy, in that a structure may fall on two parcels, or a block may be in two census tracts, but usually the hierarchy is satisfactory for aggregation purposes. Thus blocks can be combined into tracts, etc., without difficulty.

An entirely different concept of spatial subdivision is the grid, an arbitrary (at least from the point of view of physical features on the land) system of orthogonal lines superimposed over the map. Normally a grid is keyed to a standard system, such as state-plane coordinates or the UTM grid system, but a grid can also be of some independent size or shape, and need not be oriented north and south. All data are then tabulated for the grid units, say one-quarter mile squares in the central area and one-mile squares on the fringes. The advantages of an arbitrary grid are ease of reference, ease of computations of centroids and distances between centroids, lack of need for extensive description of the unit itself, and known area for control totals on land use. The disadvantages are the awkwardness of splitting or allocating blocks, parcels, structures, rivers and other natural features,

difficulty of delineating homogeneous units, difficulty of conforming to political boundaries at the edges, and the difficulty of utilizing census data collected by tract. The grid block can be a very artificial area for collecting data, but it sometimes has an advantage, as, for example, when using airphotos.

Concomitant with the collection unit decision is the question of areal extent, since tradeoffs can be made between the size of the individual unit and the size of the total area covered. Ideally, the decision of what area is to be covered should depend upon the extent of the urban influences (market, newspaper circulation, etc.) that are of importance to the model, but unfortunately this is seldom the case. Data are available for cities, counties, and SMSA's, and intermediate-sized areas are often harder to handle than the larger area that contains it. Also, the political jurisdiction supporting the data collection may be unwilling to collect information pertaining to areas outside its boundaries.

4. Compatability

A number of problems can be called compatability problems. One is the difficulty of combining information collected by several different agencies. By far the greater portion of all data collection requires the slicing up of a continuous space into discrete lumps. Some slicing is conceptually easy, such as age, number of floors, previous residence; some slicing is obviously arbitrary, such as condition of structure. Most categorization is clear in intent but fuzzy at the edges. For example, what is a dwelling unit? Is a room with no facilities in a boarding house a dwelling unit, or a room in a private house occupied by a grandmother a dwelling unit? Should the definition depend upon the physical circumstances alone, or does it depend upon who occupies it? Similar questions arise in defining a household, a retail land use, or a basic industry. Information collected by the U. S. Census is standardized throughout the country, but the definitions settled upon are bound to be unsatisfactory for some users. Many municipal agencies collect data for their own needs, and in general the definitions will be different for every jurisdiction.

Another compatibility problem occurs through time. Time series data are useless if the same attribute of the same entity is not being measured at each point, but cities change in size and shape from one period to the next, and the base definitions may or may not change with them. The definition of the attribute may also change as needs and relative importance change. The number of categories of foreign persons declined in both 1950 and 1960 from previous censuses, and the definition changed from foreign born in 1950 to foreign stock in 1960, to reflect the declining proportion of total population represented by these categories.

A final kind of compatibility might be compatibility with the model. It is seldom that the variable conceived of by the modeller is the one that has actually been measured. Often the modeller's variable is not directly measurable, or a slightly different variable was measured because the collector of the data had a slightly different use in mind. Thus the modeller is frequently required to use surrogate variables to take the place of the ones he really had intended to use, or to use synthetic variables concocted from indices or factor analysis scores.

5. Accuracy

The broad topic of error in information is very big and can only be touched here. Error in measurement can be the result of the observer (building condition, or industry classification) or the respondent (people report annual incomes that are vastly different from their actual incomes, and without systematic bias; people report their ages incorrectly both intentionally and from forgetfulness. Negroes in the twenties age group are substantially underreported from fear of welfare and tax laws.). Errors may occur in recording information, both at the source (checking the wrong box) or at any other point where information is transferred from one form to another (coding, keypunching). Information obtained by expanding a sample is subject to wide variation from the true values. The census exercises caution in publishing information derived from too small a sample base, but even a good estimate from a 25% sample is going to vary from the true value. The alternative

is to measure fewer variables more completely, or perhaps get more variables with greater error. Unfortunately these difficulties are often overlooked when the data are printed in official-looking tables with apparent six-digit accuracy.

The data problem can be summarized by looking at it from another viewpoint. Theory suggests to the modeller a particular variable to use in a relationship, and this variable is incorporated into the model. But the model variable is not recorded in any data series, and so another variable is selected to represent the ideal one. Again, however, the variable recorded is not exactly the variable that exists in the real world. For example, the modeller may wish to know how many employees in each tract work in retail and service activities that are population-oriented in their location. He aggregates the appropriate categories, but inevitably he includes some activities he would rather not include, and excludes desirable ones. Then, the data are collected on a sample basis and consequently do not accurately measure the modified variable. Thus there are three levels to the problem: the model variable, the data variable, and the real world variable the data are intended to measure.

Almost any report of an empirical investigation contains some statement about data difficulties, and these statements are not made by inexperienced researchers. According to Kain (Kain [462]), "The principle impediment to incorporating more structure into [our] models was the lack of data pertaining to what we believe are the ultimate structural variables" (p. 155). Lowry (Lowry [327] has said, "I feel rather keenly the inadequacies of empirical work reported here. Somehow, one's theoretical structures always seem to demand more data and more elaborate analysis of data than can be supplied" (p. 57). The testimonials could be presented at length. Unfortunately, no amount of talk can convince the uninitiated of the magnitude of the difficulties. Like meeting a payroll, the proper respect can only be gained by being faced with the need for large quantities of data.

B. Model Objectives

Planning objectives should, in some way, determine model objectives. We will assume that planning objectives are known and that the needs of the planners are known to the modellers. This, in turn, assumes that modelling has a well-defined role within planning, and that planning itself has a well-defined role. Such assumptions are entirely contrary to fact. Planners and modellers have scanty knowledge of each other, and a great deal of effort will have to be made on both sides before modelling can take part in the public decision process.

Model objectives can be stated in terms of information desired. This depends upon which people are going to be using the model, what kinds of questions they will ask or are likely to ask, what level of detail they will accept, the amount of information they can digest, and the graphic aids, summaries, games, etc., that will be needed in order to communicate the appropriate information. Level of detail includes spatial disaggregation (e.g., blocks vs. mile-square grids), sectoral disaggregation (categories of retail activity, types of households), and temporal disaggregation (annual vs. five-year forecasts). Users tend to ask for more detail than they can possibly absorb, and consequently many summary measures at different levels are required, along with pictures.

This approach differs in two respects from current practice. First, it is much broader than the objectives normally considered. Modellers think in terms of the information output alone, or in vague notions of the "status of the system," with the image in their minds of a single user (themselves). The second difference is perhaps surprising, and that is that the desired end-state should determine the model objectives. It is what happens after the model is built that should influence how it is built. There is a strong tendency, however, for modellers to build what they are able to build and not what is needed. The intersection of these two sets may be small, but that is where work should begin.

As implied in the last paragraph, objectives must be balanced against constraints. The time available before the information is needed, the manpower and skills that can be employed, the resources available, data costs, etc., will be factors in conflict with the objectives. Since both constraints and objectives require estimates of future conditions that may be hard to predict, complete detail cannot be achieved at the start. It is nonetheless important that the attempt be made. Too many modelling efforts have expended their resources on data and had nothing left with which to tune and adjust the model or communicate its meaning.

C. Variables

In stage one the outputs were specified; in stage two the inputs are determined, while the model itself remains an undefined black box. Outputs are variables which we need, and inputs are variables which we have. Eventually the model will link them up. We can work backwards from the output variables, attempting to find other variables that the output variables depend upon, and we can also work forwards, starting from the present state of the system. It is not necessary to specify causal linkages between pairs of variables, or to describe functional forms. For example, it might be decided that the output variables $Z_1, Z_2, \dots, Z_n$ are functions of a set of intermediate variables $Y_1, Y_2, \dots, Y_m$, which are functions of input variables $X_1, X_2, \dots, X_k$. At the present state of the art, selecting an optimal set of input and intermediate variables is largely a matter of guesswork and intuition. Some compromise must be reached regarding variables which might or might not affect the outputs, since irrelevant or redundant variables are a burden that should be avoided in the early stages, yet important explanatory variables that are left out will weaken the model.

D. Structuring

At this point, the mechanical part of construction begins. The parts (elementary particles) must be hooked together until a complete structure is formed. Major design decisions -- conceptualizing, selecting methods and techniques, deriving hypotheses -- are made in the structuring stage.

An analogy to the construction of a model airplane seems appropriate here, both for where the analogy fits and where it doesn't. Performance objectives (certain flight characteristics) are achieved by cutting up and assembling small pieces (mostly straight sticks) into subunits (wings, fuselage), and then combining the subunits into a working model. For urban modelling, there are no kits with pre-cut pieces and detailed instructions for assembling them; it is necessary to estimate what materials are required, and how they may be used to produce the final objective. There are obviously many ways, some of which are better than others. In the end, there will be some pieces (variables, or sticks) left over, which did not get into the model, and some of the objectives will have been compromised.

What this means for model construction is that relationships must be placed in as simple a form as possible, and hypotheses derived which can then be tested. Those which are accepted can be used to assemble the subunits, and these, in turn, should also be tested. There is no sense in building a complete model almost entirely composed of untested parts. When the model doesn't work (and it won't), there is no way to find out which parts are responsible. It is very easy to build into a model some relationships that are both unintended and unbeknown, which may be the result of indiscriminately applied constraints or a mathematical quirk, for example.

Relationships can be of qualitatively different types. The simplest are identities, which usually state something to the effect that the whole is equal to the sum of its parts. Conceptually, they could be eliminated by expressing one variable in terms of the others, but the identity is usually easier to work with than the reduced set of equations. A second type of relationship is technological or of an engineering nature or of some other form which is fully determined outside the model. The third type, sometimes called behavioral relationships, includes everything else, and these usually require data before they can be made specific.

Most models eventually need a computer, and consequently a computer program. Computer programs are used routinely for statistical analysis, data processing, and for solving mathematical problems such as linear programming. In most cases, package programs are readily available and the modeller need do no programming of his own. For the model itself, however, a special program is usually required, and special considerations are applied to the writing of it. While the list that follows is more detailed than most of the present discussion, the important points relating to programming are about detail. The list is in descending order of priority.

Modular construction. This means that parts of the program can be pulled out and revised without affecting other parts of the program. Some efficiency is lost in doing this, but large benefits are obtained in ease of change and ease of understanding the program. Efficiency can reintroduced if the program ever reaches a state of being "finished," but it almost never does.

Information display. When information is read out, it should be clearly labelled (this often seems unnecessary at programming-writing time, but this is an illusion), tables should be neatly arranged, and pictures should be used as often as possible. Graphic displays can be done on the line printer or on a plotter.

Error detection. The modeller should spend a good deal of effort trying to conceive of ways in which the program or the model might behave incorrectly, and have this information displayed when it occurs. A good programmer includes numerous status checks, called debugging statements, that allow him to track down errors.

Connectivity. In a way similar to the modular construction of the program, the model must fit into other models (submodels). It should be a simple matter to plug one model into the next, and transmit the output of one into the other. Doing this means matching up output-input sets and keeping them from getting too voluminous.

Speed. Speed of execution of a program is critical where the program is used or repeated many times. While this is seldom the case for an urban model at present, more use should be made of models in the future. In fact, one of the present obstacles is the length of time some of these programs require on a computer. Routines that are used repetitively in a model can be streamlined to increase overall speed. Despite its importance, however, computational speed or efficiency can be largely ignored until it becomes a problem, as it would, say, in a Monte Carlo simulation.

E. Fitting and Testing

Fitting is a statistical process. A large part of this will have been done in the structuring stage, since the operation that tests an hypothesis also fits the relationship. Larger components of the model must then be fitted, and finally the entire model must be shaped to conform to reality. Sometimes entire sets of equations are fitted simultaneously, as in the advanced econometric methods. If the model is dynamic, it must be tuned or adjusted to the time dimension (this is referred to as calibration). The last step before using a model is sensitivity testing, akin to road testing an automobile.

"Sensitivity analysis is the measurement and study of changes in the solution to a problem as a result of changes in the conditions of the problem. For example, assume that a linear programming problem has been formulated and solved, and the optimal activity levels are known. The question which now arises is what change, if any, will there be in the optimal solution if one more unit of a resource is made available, or if one of the coefficients in the constraints is changed? The techniques for answering questions of this nature vary with the kind of problem and solution method. In some cases these techniques are well developed (e.g., linear programming, in which the dual solution is very helpful for sensitivity analysis. Parametric linear programming is a systematic way of investigating the consequences of changes in the parameters, i.e., coefficients), but more often, particularly in urban modelling, sensitivity analysis must be done mostly on an ad hoc basis, and usually gets slighted.

In measuring the sensitivity of urban planning models to changes in the inputs and parameters, the best way at present seems to be to change the values, rerun the model, note the differences, and make new changes. Although an expensive and time-consuming process, the models cannot be effectively used without detailed knowledge of what the model responds to. We are particularly interested in two kinds of stimulus-response patterns: (a) large changes in inputs or parameters that produce negligible changes in output, and (b) small changes in inputs or parameters that produce substantial changes in output.

To the extent that inputs fall into class (a), it is implied that either the model is too gross to detect differences in these parameters or variables, or that the relationships between these variables and the outputs are generally minor and tend to be suppressed in the real world, at least at the scale of the model. If the fault lies with the mathematical statement of the model, then the relationships should be re-examined and re-constructed; if the flaw is in the abstraction, then the variables and relationships should be eliminated and the remaining model re-examined. For example, the real balance (Patinkin-Pigou) effect in economic theory is not generally disputed, but its quantitative effect is so small that it would probably not be useful in a national income model. Thus an operational model should contain a minimum of low-productivity parameters and variables.

The question to ask regarding class (b) parameters is whether the real world responds in this way or not. Usually an input whose minor variation causes violent fluctuations in output is symptomatic of an unintended relationship introduced by the mathematical statement of the model, or simply an error in programming. Errors in program logic can be very subtle and difficult to detect without numerous systematic checks. High-productivity inputs should be retained in the model only after careful scrutiny has assured the modeller that the system being modelled actually performs in this way. In particular, it is nonsense to have a model that is overly sensitive to variations in inputs when the variation is smaller than the error tolerance of the input itself.

F. Manipulation and Solution

Once a model has been constructed, it must then be possible to "solve" it. Extracting from the model the information it was designed to produce becomes primarily a technical problem requiring the ingenuity of mathematicians, programmers, and computers. While, like all the other stages in model construction, solution techniques must be considered in conjunction with other aspects, the means for acquiring solutions is seldom dominant in modelling decisions. Most models can be solved, once completely stated, and questions concern whether the value of information obtained is worth the effort needed to get it. Often the solution technique is associated with the model technique, such as the simplex method of solving the standard linear program. Sometimes a number of techniques may be used to solve the same problem.

Two points need to be made regarding solution techniques. First, models are occasionally described by reference to the techniques used to solve them, and this is generally not very relevant information. While one attribute of a model is its solution method, this attribute seldom reveals anything about model content or structure. An example is the word "simulation," which may mean that the model simulates something (i.e., is a model), or uses Monte Carlo methods in its solution. Neither of these two meanings is very helpful. Another example is the transportation model, which is an algorithm for solving a particular form of linear program. Considerable confusion has been caused by using a solution technique to describe a specific model.

Also used to describe models are the two words "iterative" and "recursive". Both are applied to urban models, and both mean that the model operates by repeating something over again many times. The implication in both cases is that each time the cycle is repeated, the model works from the results of the previous cycle, until some specified goal is reached. The

termination criterion might be the number of iterations to be performed, a stated accuracy or tolerance level, or a boundary value to be attained. The words are used interchangeably.

The second point to be mentioned is that the end result of a model is not necessarily a specific value for each of the output variables. A range of possibilities exist for what we will call "precision" of the solution. At one extreme is the exact solution. This means that there is a single, unique number that can be completely specified, without any error if it is assumed that there is no error in the inputs. At the opposite extreme are stochastic solutions that are acquired by sampling. The model must be run a large number of times, and the value of the variable recorded at the end of each run. This results in an approximate distribution of possible solution values.

In between is a continuum of alternatives. At the high precision end (towards exact solutions) are solutions that are subject to representation error, e.g., the number .3 cannot be stored exactly in most computers. Next down the scale come solutions subject to rounding error, which may produce substantial or negligible effect. The solutions from exact through rounding error can be generally labelled "deterministic". The next group are techniques which approach a solution through successive approximations. These techniques cannot achieve an exact solution, but they can come arbitrarily close within the limits of other kinds of error such as rounding. Such techniques are called "numerical". Finally, there is a group of "probabilistic" techniques, which can only describe the solution by means of a probability distribution. The more precise of these give a specific probability distribution arrived at through analytic techniques; the less precise yield a frequency distribution from repeated model runs, or estimates of the parameters of an unknown or unspecified distribution. At this end are the Monte Carlo techniques. The precision continuum is arrayed in Figure 4.1.

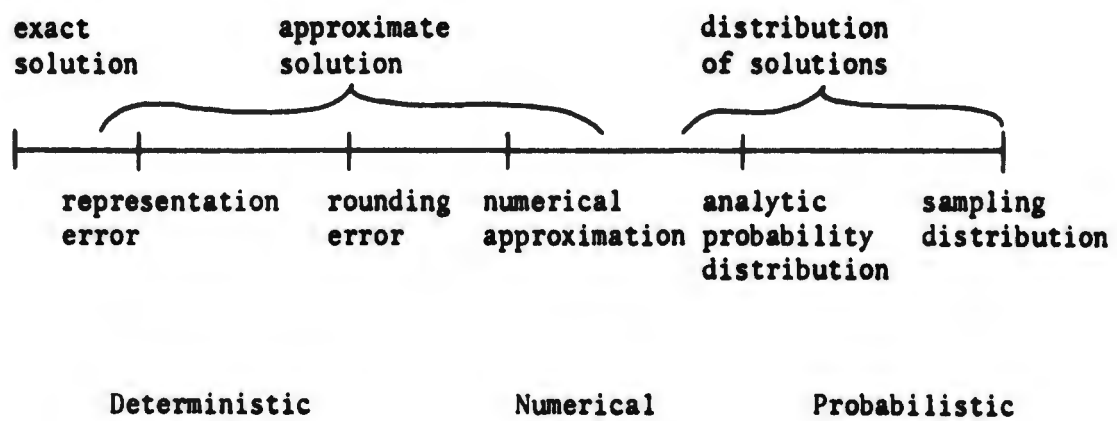


Figure 4.1. PRECISION IN MODEL SOLUTION

CHAPTER 5: SURVEY OF CURRENT MODELLING

In making a survey of this kind, the first problem is the two-part one of deciding which models to present and how much of each one. Some models can be described verbally in an adequate way, with most of the mechanical details of implementation being left out. Others require concentration on a few important mathematical relations that govern the performance of the model, and this means distinguishing between equations in the model that are simply identities or definitional relations and those that are technical or behavioral. Still others should be understood through the overall mathematical structure. The success of any applied model depends, to a large extent, on the quality of the data supplied to it, but this aspect is difficult to evaluate without direct contact with the model. Material chosen for this report reflects the author's judgement, personal biases, and experience with the Pittsburgh model.

Selecting specific models meant making decisions that were somewhat arbitrary. The primary area of interest was land use models, but these usually contain variables relating to transportation, employment, and population, and it is possible to get quite far into these other areas while still classifying the models as urban. The focus was also on single-region models, which tend to be more nearly operational than interregional models using such methods as interregional linear programming and interregional input-output. A number of national models -- mainly economic -- are omitted because they are too aggregative to be usable at the regional level and because they are oriented towards national objectives and policy variables. In the other direction, models which were very micro-analytic, such as psychological or political simulations of group interaction, were also excluded.

As much as possible, the typology developed in Chapter II is used to describe the models. Knowledge of the theories, components, methods, and techniques outlined in Chapter III, and the modelling process summarized in Chapter IV, is assumed in the following discussion. We have tried to

develop and use terminology in a precise and consistent manner, but much of the usage is not generally accepted and may conflict with both more or less specialized usages. While in most cases we have had to rely on published documentation of the models, we have attempted to interpret it and extrapolate from it rather than simply reproduce or summarize it.

Probably the dominant criterion used in selection was the degree to which the model objectives coincided with our own, i.e., comprehensive models of urban systems designed for planning purposes. Only one group, the Community Renewal Program models are directly aimed at this objective, and only one of these, the Pittsburgh set of models, is actually comprehensive. Since we are also interested in working models, a second criterion was the success achieved by the modelling effort in operational objectives. Here, the most satisfying group is the mainstream transportation study. These two groups -- the applicable but less successful, and the successful but less applicable -- have been placed first.

A third group of models is a blend of the first two, and these are recent transportation studies that are more comprehensive than the earlier ones. These might eventually go to the top of the list, but it is difficult at present to evaluate their success. The fourth group contains models that are somewhat more limited than the previous models, both because they are not forced to produce operational results and because they have fewer resources. A final group of models is a residual group that is spread thinly over a wide range of subjects that may or may not be of immediate importance to urban modelling efforts.

While the ordering of the groups is meant to imply a degree of applicability for the models surveyed, it does not necessarily imply relative importance or urgency of work. The comprehensiveness of the first group is a worthwhile ultimate goal, but not presently attainable in worthwhile form at a reasonable cost. On the other hand, the models in the last group are placed there because they are highly underdeveloped, not because

they are less relevant. There is also much to be gained from the limited-scope incremental approach of the transportation studies. The current modeller should be cognizant of the work done in CRP's or transportation studies, but need not focus on these areas himself.

It may appear that a new dimension of variation has been introduced here, beyond those listed in Chapter 2. Actually, a combination of the original five dimensions has been used, with the result that the models are grouped and ordered more or less according to purpose. They are also grouped by the agency exercising responsibility (e.g., the Pittsburgh model contains several models of widely varying characteristics), which is a property that is independent of the other five but not one of major interest.

Community Renewal Programs

Urban renewal programs have paralleled the development of transportation studies, and the present approach is a comprehensive urban redevelopment program called the CRP. The object of these studies is to provide a framework within which individual renewal projects could be placed, so as to maximize the long run effectiveness of each project, integrate the projects with each other and with other plans, ease the burdens of relocation and fiscal commitments which can easily occur if projects are not coordinated, and generally schedule urban renewal projects in time and space so as to maximize their favorable impacts and minimize the unfavorable.

Only two cities so far have attempted to apply modelling techniques in the preparation of their CRP's: Pittsburgh and San Francisco. The difficulties are considerable, both in the area of comprehensiveness (not only growth, but deterioration, rehabilitation, renewal, etc., in residential, commercial and industrial activities) and in the need for more specific information for small areas (household types, housing types, population characteristics, employment opportunities, etc., for areas of census tract

size or smaller). Many of the activities in question are in the process of undergoing profound structural changes which may have dramatic implications for future city forms (retailing, for example). The Pittsburgh operation was the more comprehensive attempt, and partly for that reason less successful.

The Pittsburgh Model

Conceptually, the Pittsburgh simulation exercise was designed as a large-scale comprehensive information processor to provide decision-makers with whatever they needed to undertake a Community Renewal Program. Highly computer oriented, the model could be interrupted at will in order to obtain status information or alter parameters or inputs. A complex set of submodels formed the components of the model, and each submodel could be operated independently of the main model, or in conjunction with related submodels. The submodels represented the various subsystems in the Pittsburgh urban environment, including such diverse activities as industrial relocation, population migration, and governmental revenues.

Components of the overall model ranged in complexity from straightforward data reduction or tabulation routines to complex iterative equilibrium models for allocating commercial and residential land use. In order to avoid making distinctions of this nature, we will refer to all component models and subroutines as simply "routines". Functionally, however, the approximately forty routines can be classified into several different types.

At the heart of the model are three routines that attempt to replicate the salient features of the structure of the city. The first, in order of operation, is an input-output interindustry model that produces basic employment by sectors (SIC codes), value added, and income. The basic employment data are fed into the second routine, an industrial location model, which matches up projected amounts of employment with suitable pieces of land in the city. The third routine uses the locations of basic employment to locate residential and commercial activities.

Two kinds of input routines are needed to supply the main models with sufficient data. One kind takes basic data for areal units, sectors, etc., and sorts, tabulates and reduces the data preparatory to input to the other routines. Data included are input/output data, industrial data, commercial data, residential data, and community facilities data. The other kind of data processing routine provides information about future plans, both public (urban renewal, capital improvement) and private, and about other exogenous variables that must be projected outside the model. These are total employment in basic industries and total population size.

Two more types of routines process data internally and provide linkages between components of the model. One type generates parameters for operating the main routines, and these are generally regression or factor analysis programs. The other type translates plan information into specific effects at specific times, in terms of costs and revenues, land use, structures, housing, population relocation, etc.

Finally, two more sets of routines prepare information for output and feedback. One set, called reconciliations, makes adjustments of inputs on the basis of other outputs (sometimes independent estimates of the same thing), and reconciles various totals. The other group of routines computes indices and other summary measures for output information and performance evaluation. These include commercial, industrial, and residential blight indices, the social level-of-living index, and the status of government revenues and expenditures. A list of the routines making up the model is shown in Figure 5.1. Figure 5.2, the "overview" diagram shows how they are hooked together.

A model as comprehensive as the Pittsburgh model could provide extremely current and useful information, in a highly sophisticated way, once built. Modification would not be difficult because of the plug-in nature of the submodels, and the generality of the framework. As an impact

Exogenous Inputs

- NMPLNS - Non-Municipal governmental plans
- URPLNS - Urban renewal plans
- CIPLNS - Capital improvement plans
- EXOPOP - Population (through cohort/ratio method and independent density estimates)
- BASEMP - Basic employment

Data Sources

- IODAT - Input/output inter-industry model
- INDDAT - Industrial-oriented data
- CFDAT - Community facilities data
- COMDAT - Commercial-oriented data
- RESDAT - Residential-oriented data

Data Analysis Results (coefficients and structural equations)

- DISANA - Distance-gradients analysis
- COMANA - Commercial data analysis
- RESANA - Residential data analysis
- INDANA - Industrial data analysis
- FACANA - Factor analysis

Reconciliations

- REcon 1 - Total employment, income, labor force and population
- REcon 2 - Commercial employment
- REcon 3 - Population spatial distribution

Figure 5.1 SUBROUTINE IN PITTSBURGH MODEL

Connection of Plans to Other Submodels

PLNTIM - Timing of specific events implied by plans
NCLCON - Matching attributes of plans to make efficient use of non-cash credits
RESRHB - Residential rehabilitation "direct" impact (upon households' actions) of plans
LNDIMP - Land (and structures) "direct" impact of plans
ACCIMP - Access impact
RELIMP - Relocation impact
POCIMP - Project-oriented capital improvements
POCOST - Project-oriented costs
POEMP - Project-oriented employment

Supply and Demand Submodels

IOIIM - Input/output inter-industry model
INIMP - Industrial spatial allocation model
TOMM - Time-oriented (residential and commercial) metropolitan
GOVPRO - Governmental revenues and expenditure projections
STANDA - Governmental expenditures' projections based upon standards
PROPRI - Project-oriented priorities
SOCIND - Social index projection
DECANA - Governmental decision analysis

Blight Indices

COMBLI - Commercial blight
INDBLI - Industrial blight
RESBLI - Residential blight

Figure 5.1 SUBROUTINE IN PITTSBURGH MODEL (Cont.)



Source: Steger and Douglas [334], p. 6

Figure 5.2 PITTSBURGH MODEL OVERVIEW

model, it could measure the effects of programs on a wide range of variables. It is an ambitious application of systems analysis and the operations research approach to the study of urban problems.

Unfortunately, it was too ambitious. In 1963, when the model was designed, there seemed to be some possibility that the detail problems in constructing the model could be worked out. At the present time, the task of building such a model would correctly be regarded as monumental, and the event is still some time in the future. In fact, computer systems of the third-generation, time-sharing type, adequate to handle the large storage, the billions of computations, and the accounting and transfer operations necessary for a large simulation, are not yet available. Nor have the data been found. Not least of the problems of such a venture is that of insufficient understanding of the phenomena being modelled. Our present knowledge of the interrelationships operating in an urban region does not justify the effort at extensive automation and fine tuning. Efforts can be better directed at limited-purpose models, for investigating the behavior of the models themselves and for testing hypotheses which are potentially useful in models.

Industrial Location Model. Estimates of future employment in thirty industry types were made by the input-output model for three different areas: the Western Pennsylvania region, Allegheny County, and the City of Pittsburgh. INIMP, the industrial location model, allocated the categories of basic industry to census tracts, for the city alone. It is thus an intra-urban location model. Models of this type may allocate employment on the basis of an index (which could be derived from a regression), or by matching firms with available and appropriate sites. This particular model uses both techniques.

A framework of the model is loosely articulated ("heuristic", as Putman [331] calls it) and hence very flexible. While it manages to get by on limited kinds of data, and mostly untested descriptive and behavioral

relationships, better data and different structural relationships could be easily incorporated. The limitations of the model are largely the consequence of insufficient information about the specific nature of the process, and not inherent limitations of the model. Some of the variables and relationships are the result of the area covered by the model (just the city) and different explanatory variables would probably be used in different settings.

Mathematically, the model is quite simple. The content of the model is primarily logic, and this is displayed in the flow chart, Figure 5.3. Each two-digit S.I.C. category of employment is allocated or located separately, and the flow chart does not show the cycling for different employment codes. If the total citywide change in a particular industry is not too great, either up or down, then the allocation procedure is used to spread the change around to the census tracts. The variables which were available from Department of City Planning surveys were

employment
 number of firms
 occupied building floor area
 number of buildings
 ground floor area
 site area
 building age and condition

which were given by two-digit code by census tract. After comparing different functional relations, methods, and variables, one variable was selected for use in the following equation:

$$\Delta E_{i,t}^k = \Delta E_{T,t}^k (E_{i,t-1}^k / \sum_j E_{j,t-1}^k)$$

The relation states that the change in employment in industry k in tract i between period t-1 and t equals the total change in the industry in the city, times the proportion of that industry that existed in the tract in period

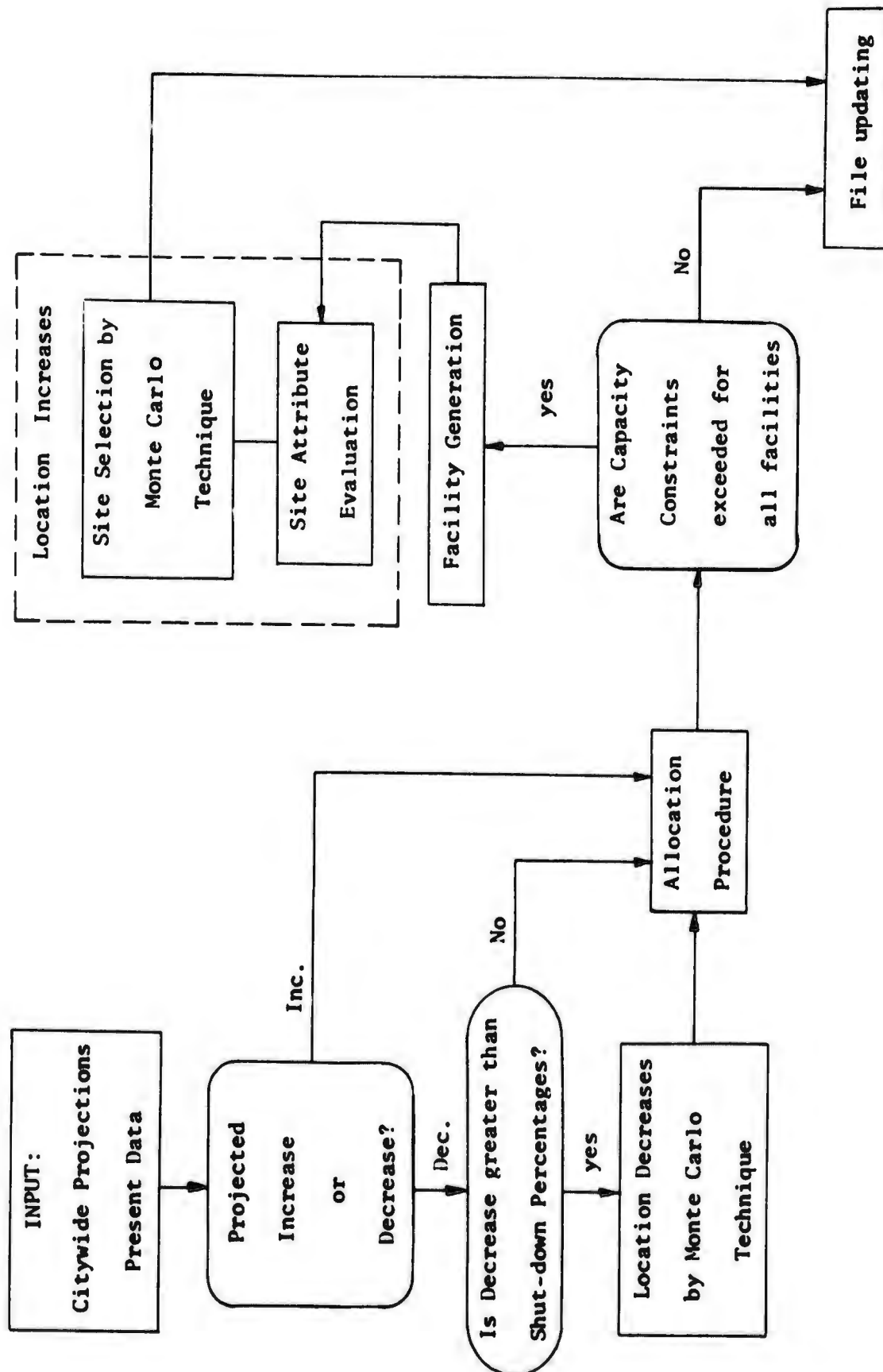


Figure 5.3. Flowchart of INIMP sub-sections for processing individual S.I.C.'s

t-1. The implication of the statement is that whatever change takes place will be distributed proportionately to all establishments. For example, if the citywide total for employment in electrical machinery shows a small increase, say 3%, then all electrical machinery firms will expand by 3%. The same is true for declines in total employment.

The allocation procedure, however, is only used for relatively small changes in employment, where the total number of firms remains unchanged. For larger changes, different routines are used. Decreases in employment are compared with "shut-down percentages", which are fractions of total employment that firms can absorb and still stay in business. If the projected decreases are larger than the allowable percentages, firms are selected for removal, and total employment in the category is decreased by the size of the firm. This critical percentage is considered to be size-specific, i.e., varies with the size (number of employees) of the firm, and also industry-specific. This necessitates grouping firms by size class, and seven categories were used. Since the procedure does not indicate where the shut-down firm is located, this is done randomly. After one firm is removed, the remaining decrease in employment (or increase, if the firm was large enough to offset the projected decrease) is allocated as before.

Increases are handled in a similar way. A maximum density constraint (employees per square foot) is generated for each industry for each tract (but not each size group), by randomly selecting a limit higher than that existing in the tract being considered. This selection is based on the distribution of employee-density ratios for tracts containing the industry. If the citywide increase in employment cannot be allocated and still stay within the employee-density constraints, then one or more new firms are generated and located. The size class of the new firm is selected randomly from among those equal to or smaller than the size needed, and if the firm selected is too small the process is repeated and both firms are located. Site selection is done according to four attributes of each census tract:

assessed value
available land
structural density
industrial clustering

plus a constraint attribute, which is used to insert public policy. The first three attributes are compared with a weighted average for the attribute for tracts with the relevant industry type, and each tract is scored 1 (yes) or 0 (no) on that attribute for that industry. The last attribute is based on nearness to like industries, and also comes out with a 0-1 score. A weighted sum of the attributes is formed for each tract, and the tract with the highest score (or a randomly selected tract from among those with highest scores) gets the industry.

Two features of the model are worth noting. One is the simplicity of its structural relations, particularly the allocation function. They tend to reinforce the status quo, i.e., they are "persistence" projections, but trends and shifts in emphasis in site requirements can be incorporated by adjusting the weights on the attributes, and by biasing the attribute criteria. It is encouraging to find that a model can produce reasonable results using so little information, but it would still be a good idea to find out what the relationships really are.

The other interesting feature is the number of times a Monte Carlo technique is introduced (one for every random selection). As was mentioned at one point, the reason for this is that no other criteria are available. Since this implies ignorance (insufficient information or lack of understanding of behavioral relationships), replacing the randomness should improve the model. Even if the only purpose of a random selection is to prevent unnatural "bunching up" (of a size class, say), other methods would be preferable. The framework is satisfactory, but needs filling out, which should be done by specific research.

For a number of reasons, the model is almost impossible to fit, calibrate or validate. No time series could be obtained, the various parameters (shut-down percentages, etc.) are based on reasonableness and informed opinion, and the changes occur slowly in the real world. In the few years since the model first operated, actual events seem to have conformed to the model, but nothing definitive can be said.

Residential and Commercial Location Model. The basic form of the model was developed by Ira S. Lowry [327] for the Economic Study of the Pittsburgh Region, a three-year study sponsored by the Pittsburgh Regional Planning Association and the Regional Industrial Development Corporation. From 1960 to 1963 (approximately), Lowry's work paralleled a more conventional economic analysis which was published separately. The model was admittedly experimental, and no operational results were expected (Lowry refers to it as a "first generation model"). In 1963, Lowry and the model moved to the RAND Corporation, and RAND published the final report in 1964. A large portion of the Pittsburgh simulation work was influenced by Lowry's model, and not just the residential and commercial submodels. Under the Consad Research Corporation and the Pittsburgh City Planning Department, the model became known as TOMM, but the development was continuous. Because the model is an important one, because it exhibits most of the characteristics of urban models, and because it is the model with which the author is most familiar, more space will be given to it in this report than will be given most other models.

Lowry's Model. Space-consuming activities that are to be located are divided into three types: Basic, Service, and Residential.

1. Basic activities are approximately those that would be classed as basic or export in an economic base framework. In this model, they are exogenously determined, i.e., by forces outside the region, and the model accepts them as given, in size, type, and location.
2. Service or retail activities also parallel the economic base breakdown, in that these exist in order to serve the population that serves the basic activities. Service employment is endogenous to the model, and is also

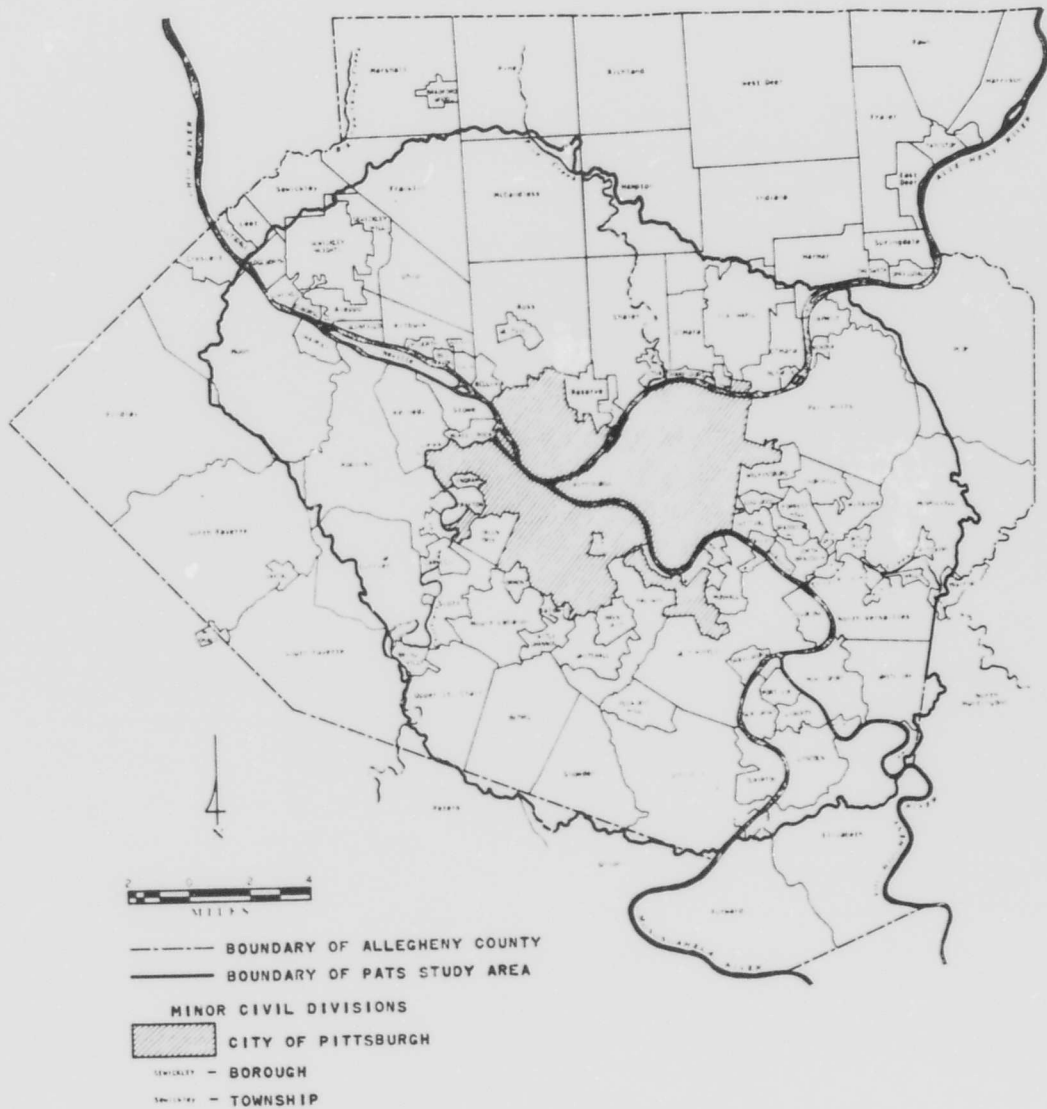
located by it.

3. Residential, or household activities are also endogenous.

The underlying motivation for the model is that basic activities determine the amount and location of residential activity, and indirectly the amount and location of service activity. Given the locations and numbers of employees of firms classed as basic, the residential population required to support the basic sector (workers and families) is located with reference to the locations of employment. The service sector is then located so as to serve the basic population, which in turn requires new population to fill more jobs. New residences are located, and subsequently more retail activities, in an iterative fashion, until the increments become too small to worry about and the process is said to have converged. Residences are given "trial" locations in each iteration, and they may be bumped out by commercial uses in the next iteration.

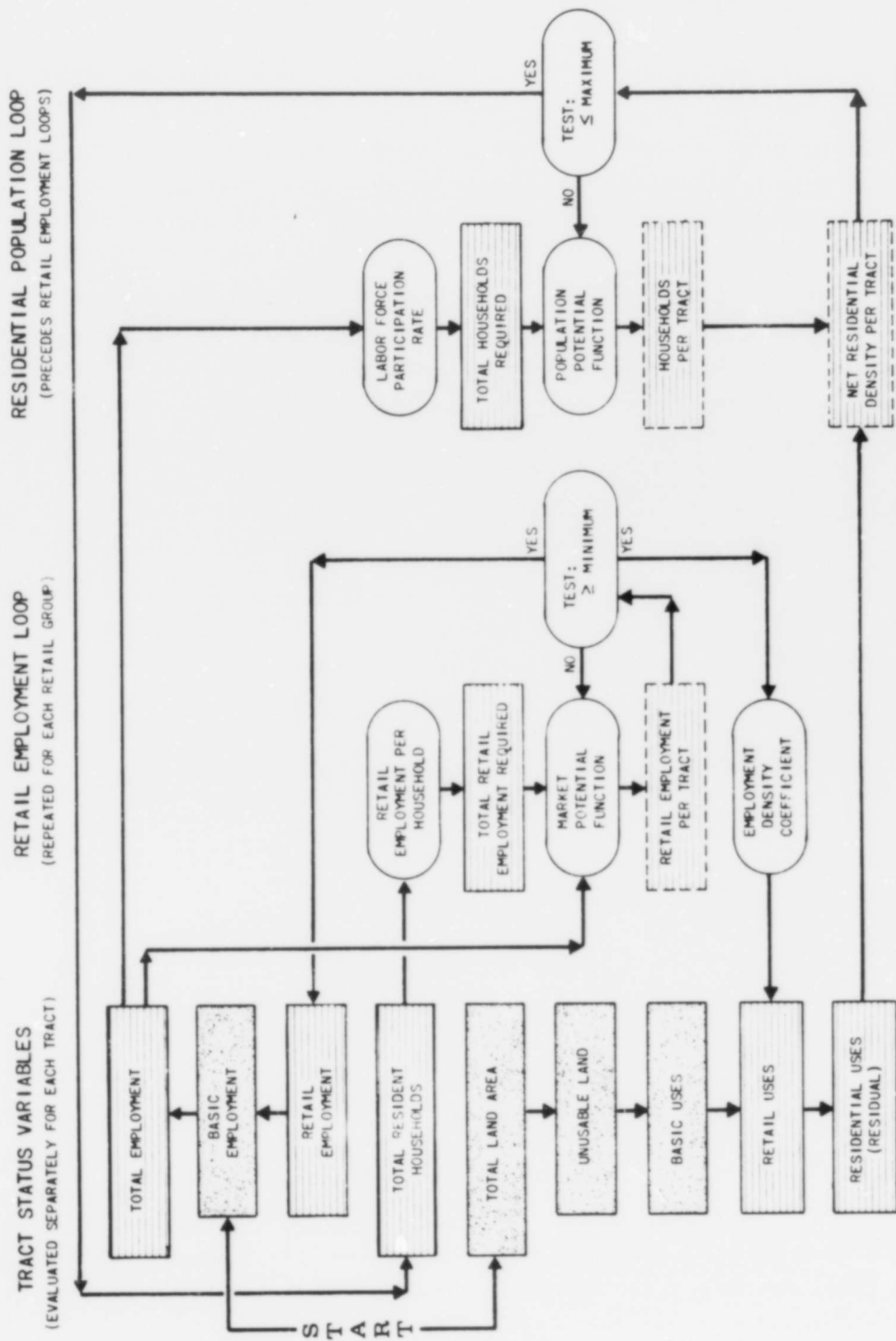
It is appropriate to call this model a gravity model. Each source of employment generates potential that radiates out from its location, and these potentials are summed to create an employment-potential surface. Households are allocated first to those areas with the largest potential. Households themselves then generate a potential surface for retail activities, which results in more employment potential and more households. Although the employment potential functions are monotonically decreasing functions of distance, and hence the location closest to the source is considered best, the retail-commercial potential functions peak at about a mile or two. This simply means that residences do not generate much retail potential in immediately adjacent areas.

Lowry fitted his model to an area somewhat smaller than Allegheny County but much larger than the City of Pittsburgh, which coincided with the data area for the Pittsburgh Area Transportation Study (see Figure 5.4). The basic areal units are mile square tracts. The total area of each tract (A_j) is divided into land for basic industry (A_j^B), unusable land (A_j^U),



Source: Rand Corp., RM-4035-RC, Aug. 64

Figure 5.4 MINOR CIVIL DIVISIONS OF THE PITTSBURGH AREA



Source: Rand Corp., RM-4035-RC, Aug. 64

Figure 5.5 INFORMATION FLOWS IN THE PITTSBURGH MODEL

commercial land (A_j^R), and residential land (A_j^H). Total, basic, and unusable land are inputs to the model, and commercial and residential land amounts are determined by the model. Also exogenous is basic employment by tract (E_j^B). A schematic flow chart appears in Figure 5.5.

The formal structure of the model consists of nine simultaneous equations and three inequalities, with many of these equations being repeated many times, e.g., one for each tract. The important features of the model, however, can be discussed in terms of two equations, the residential and the retail equations.

Household sector

The number of households in each tract, N_j , is a function of the tract's accessibility to places of employment, or, in other words, a function of the employment potential surface. This is expressed in the model as

$$N_j = g \sum_{i=1} \frac{E_i}{T_{ij}}$$

The summation is over all tracts, including j . The number of households assigned to j is positively related to the total employment in other tracts (E_i), but decreased by the difficulty of access to that tract from j . T_{ij} is called a trip distribution index, and increases with distance. Thus employment opportunities have less influence or potential at greater distances.

The trip distribution index could be simply distance, i.e., d_{ij} , or distance raised to some power, or some other function of distance, or some number specific to each pair of tracts which measured the resistance to travel between the two. Lowry used the data from the Pittsburgh Area Transportation Study (PATs) and decided that the power function

$$T = 43.90 r^{-1.330}$$

best fitted the data (see Figure 5.6). Lowry also determined that travel time and airline distance were sufficiently highly correlated that the latter could be used in place of the former.

The other component of the household location equation is a scale factor, g , that forces the total number of households located to equal the total determined by the available jobs in the region. This total is found by a simple expansion of total employment at each iteration i.e.,

$$N = h \sum_{j=1} E_j$$

where h is a constant (parameter) representing labor force participation, and equals (1/no. of employed persons per household).

Retail sector

Retail establishments are disaggregated into types (m in general; Lowry ultimately used 3), with a set of equations for each type. The form of the equations is

$$E_j^k = b^k \left[\sum_{i=1}^m \left(\frac{c^k N_i}{T_{ij}} \right) + d^k E_j \right]$$

where E_j^k is the employment in retail type k in tract j (all superscripts are indices, not exponents), N_i is the number of households in tract i , and E_j is the total employment (including retail) in tract j .

The trip distribution index is similar to the household-employment index, but now it is specific to each retail type. The general form, again determined empirically, is

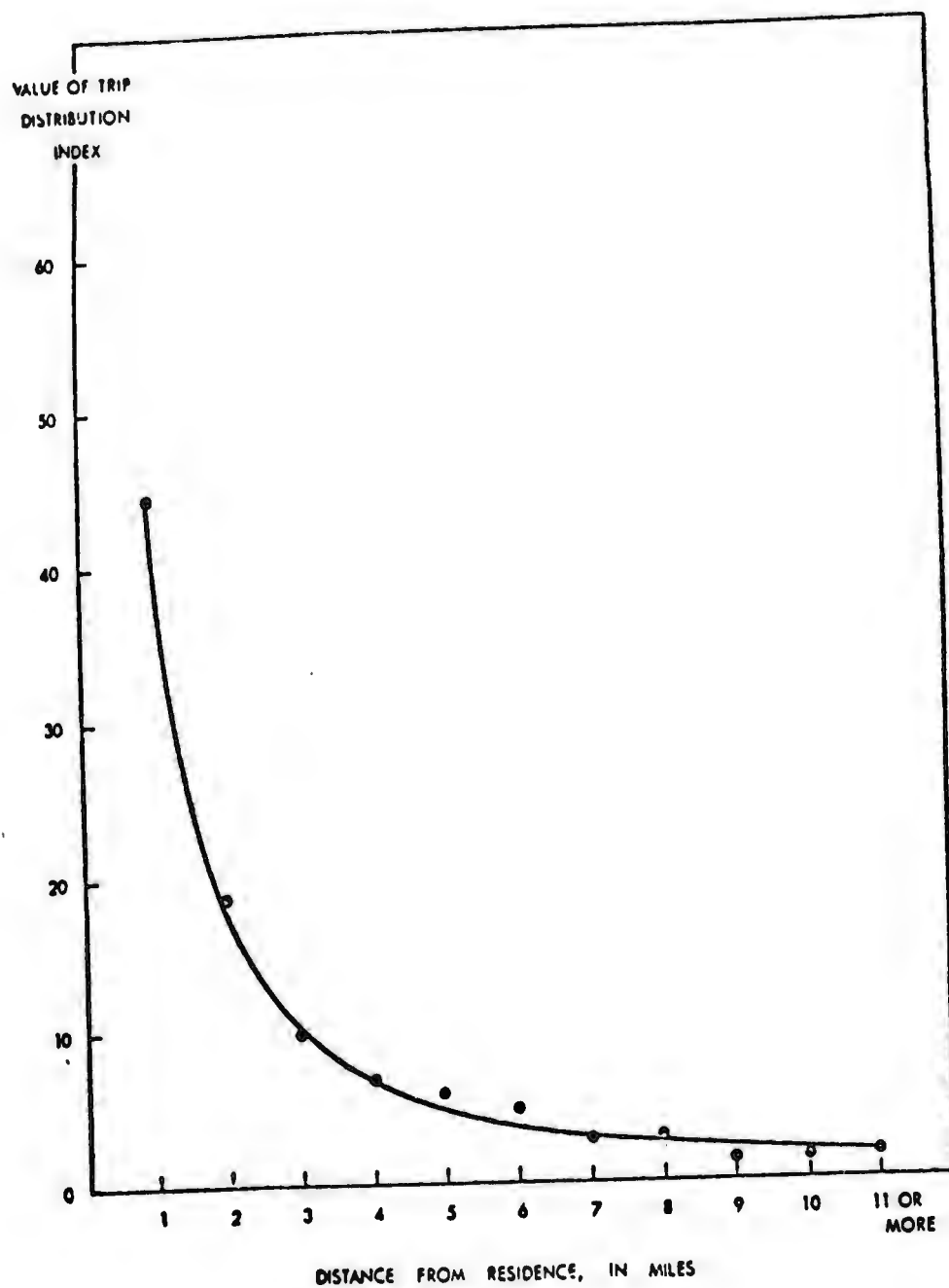


Figure 5.6 TRIP DISTRIBUTION INDEX FOR ALL OCCUPATIONS

$$T_{ij} = 2 \pi r (a - br + cr^2)$$

where r is the (radial) distance from i to j , and a , b , and c are constants determined by regression (see Figure 5.8).

The other empirically derived constants, c^k and d^k are also specific to each retail type, the former weighting the influence of households on potential for a particular retail type, and the latter weighting workplaces (in the tract) for the same type. This means that nearness to both households and places of work generates potential for retail activity, but the relative influences differ with different retail types. The remaining component, b^k , is again a scale factor that forces the totals allocated to equal the totals for the region (potential is a pure concept), and so one cannot say that "x potential = y households"; potential is used to allocate a fixed number of entities (jobs, households) among the set of possible locations. Thus potential is used almost exclusively for its ordinal properties.) The total employees in each retail category are determined in a manner analogous to the method for finding the total number of households,

$$E^k = a^k N$$

where E^k is the total regional employment in retail type k , and a^k is a constant relating the number of households to the number of retail employees of type k required to support the households.

Constraints

Without controls, the gravity model could very easily allocate all the population to a few tracts. In the real world this is constrained by competition for a limited amount of space, with the unsuccessful being forced to locate elsewhere. The intensity of the competition would be reflected in the density of use, but a limit still exists. The model imposes an exogenously determined maximum density on each tract (for residential population), expressed as

$$N_j \leq Z_j^H A_j^H$$

where Z_j^H is the maximum density for households in tract j , and A_j^H the amount of land available for households in the tract.

In order to prevent retail activities from being spread too thinly over the region, Lowry imposed a minimum size constraint on each retail type, in terms of the number of employees in a "retail cluster",

$$E_j^k \geq Z^k, \text{ otherwise } E_j^k = 0$$

where Z^k is the minimum number of employees in a cluster of type k .

The third kind of constraint prevents negative values for commercial land use,

$$A_j^R \leq A_j - A_j^U - A_j^B$$

The land given to basic uses is specified exogenously, and remains fixed for any given run of the model; unusable land is also exogenous, and total is, of course, fixed. The amount of retail land use in a tract is a scalar times the number of retail employees (square feet per employee, by type), and the residual is what is available for residential use.

Solution

Lowry counted up the variables in his model, subtracted those that were specified by input data, and this left $4n + mn + 2m + 2$ unknowns, where n is the number of tracts, and m is the number of retail types. For example, if $n = 600$ and $m = 3$ (approximately those used by Lowry), there will be 4,208 unknowns, which requires that there be 4,208 independent equations (Lowry determined that there would be). Most of these equations are simple, and many involve only a small number of unknowns (they are all linear), but

it is still a large number of equations to solve. To do so by means of a general matrix inversion is not presently feasible, and inversion routines taking advantage of special properties of the matrix (e.g., diagonality) might or might not work out. A matrix solution would most likely be numerical anyway, i.e., converge on the solution in an iterative fashion.

In addition, a solution might be either (a) not unique, which means that the equations are in some way redundant, (b) inconsistent, which means that the equations require some variable to take on two different values at the same time, or (c) fail to satisfy the constraints. Problems (a) and (b) generally do not occur with real data because the probability that an equation is not linearly independent is very small, but it is a question that cannot at present be answered very adequately for such large systems. Inconsistency would seem to be a flaw in the logical structure of the model, and should become readily apparent. Problem (c), however, must be dealt with explicitly. The addition of constraints suggests that the problem might then be a linear program, but this is not the case here because there is no objective function. Instead, the application of a constraint in effect requires that the problem be changed, and then a solution to the new problem worked out. This is done until a problem is formulated for which the solution satisfies all the constraints.

Since an iterative technique is apparently required anyway, the path by which iteration proceeds might just as well follow the logical structure of the model rather than being based solely on the mathematical properties of the problem. Lowry's program does this, and a great deal of effort was spent in working out bugs, devising efficient algorithms, and testing and calibrating the model. Allocations are made alternately to residential and retail land uses, with retail being allowed to preempt residential activities when a constraint on land is encountered. Substantial shifts may occur in this process, but the model converged satisfactorily in Lowry's runs.

Limitations of the model

Lowry discusses his model in a very thorough and objective manner. He refers to it as an "instant metropolis", and does not expect practical applications of his model as it stands. Most of the limitations we will mention are discussed by him; a few are not.

The major overall question is the adequacy of the gravity principle to explain residential and commercial location. Since all decisions within the model are based on this concept (disregarding the constraints which are exogenous), it is essential that this question be addressed. As originally formulated for a social context, the gravity model applied to people. Whatever potential was, it was generated by nearness to people. It was found that potential, measured in this way, correlated highly with a number of other phenomena, for example, land value. There is undoubtedly some circuitry in this reasoning, which in part explains why it is so difficult to give a behavioral interpretation to the gravity model, but as a descriptive regularity the gravity model has few rivals.

As used in modelling, the gravity model becomes simply a construct that can be used as a vehicle for numerous assumptions or relationships. At one extreme, the model can be used to describe very generally the decline in intensity of activity as distance from the central business district (CBD) increases. Used in this way it is highly aggregative and has little predictive power for small areas, but it is useful as a reference. At the other extreme, some kind of potential might be so limited in extent that it disappeared within a hundred yards, such as the amenity of a small fountain. What is important to note is that potential is simply a form for expressing distance relationships. All that is required is that the phenomenon being considered be expressible as a function of space (distance) with reference to a point source. A number of these can then be summed to generate a potential surface, as in the case of employment or retail potential in Lowry's model. The interpretation then depends upon the entities or attributes used to generate

the potential. The importance of the journey-to-work as a consideration in household location is used to justify employment potential. An entirely different model could be developed by postulating that households locate according to the quality of schools, amenities such as parks and shade trees, and the stability of the neighborhood. Given the right kinds of data, it would not be difficult to generate potential functions that expressed the attractiveness of any particular site in terms of nearness to these amenities. The model would still be a gravity model, but discussion of its merits vs. Lowry's model would turn on whether residential amenities are more important than the journey-to-work (or shop) in residential site selection, not on whether the model was based on the gravity principle or not. Thus the information that a model is a gravity model is of interest, but it is not a means of judging the sophistication of the model nor the relevance of its behavioral relations.

Another question concerns the disaggregation of households. Lowry's model treats all households as indistinguishable (as opposed to identical) in making spatial allocations. Although the aggregate nature of the gravity-potential functions allows for a great deal of variation between individual households while still fitting the overall pattern, there are clearly a number of difficulties in this assumption. In fitting the trip distribution index, Lowry disaggregated trips by occupation, with the following results:

managers, etc., and professional, etc.	$T = 35.69 r^{-1.080}$
clerical, etc., and sales	$T = 37.16 r^{-1.125}$
craftsmen, etc., and operatives	$T = 48.47 r^{-1.470}$
laborers, domestic and service workers	$T = 60.08 r^{-1.850}$

Figure 5.7 shows these functions plotted.

The functions vary consistently with status of occupation, which is a regularity both useful for modelling and awkward to suppress into one single function. High status types, as might be expected, are relatively

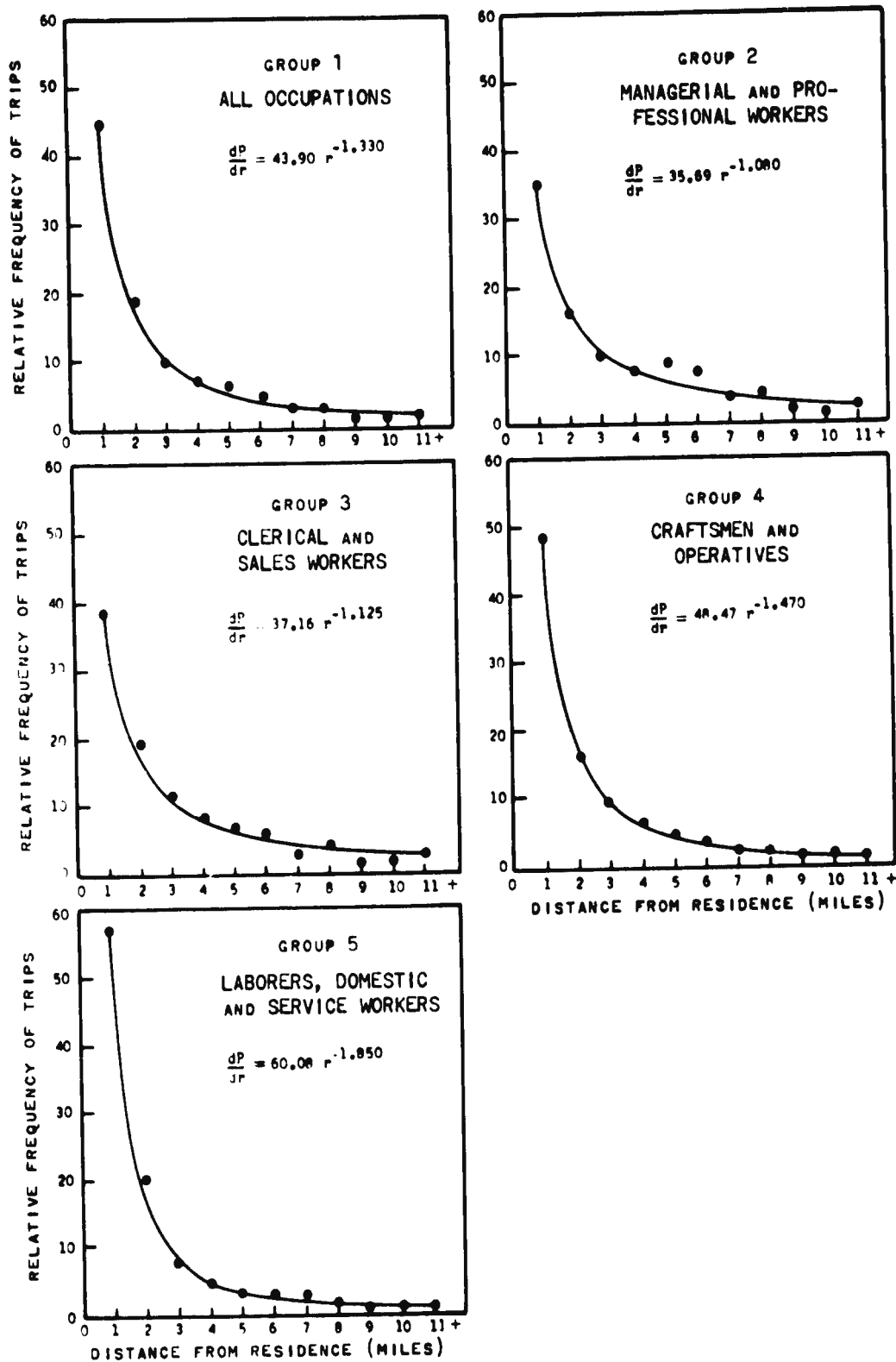


Figure 5.7 EMPIRICAL DISTRIBUTION OF WORK TRIPS
BY DISTANCE OF WORKPLACE: STANDARDIZED
CONDITIONS

unresponsive to the journey-to-work, while unskilled or semi-skilled workers appear to be much more so. (The faster T goes down with increasing distance, the greater the friction of distance. Thus it is the exponent that is of interest; the coefficient is a compensating scale factor and equals the height of the function at $D=1$. Lowry confuses the issue by calling both T and T_{ij} trip distribution indices, in different publications, when they are more nearly inverses of each other). This raises the questions of (1) what other disaggregations might distinguish households in a meaningful way, and (2) what high status types are sensitive to if they are not sensitive to the journey-to-work.

The same questions can be raised regarding the disaggregation of retail types. Lowry began operating the model with ten categories, including such types as food and drug stores; department, furniture, and appliance stores; and government offices and public buildings. These were distilled out of the PATS data as being types which were responsive to the locations of residences. In the course of testing the model, however, Lowry found that ten categories was too expensive in machine time, and cut the number to three, grouped along slightly different lines than the first ten. These were: neighborhood facilities, local facilities, and metropolitan facilities, in increasing order of size of market area. Each type formed a separate cluster of mixed retail trade and service activities. Several of the earlier type of retail function are shown in Figure 5.8.

A further problem is that of a time dimension. The model starts with a blank plane, records the locations and amounts of basic employment, basic land use, and unusable land, and then builds a city. The past distribution of activities other than basic is not considered, nor the time required to reach equilibrium. An alternative model of the city can be postulated that regards all decisions as operating at the margin, and in consideration (hypothetically) of all previous decisions. This continuous time model would appear to be in direct conflict with Lowry's instant metropolis, with damaging implications for the latter.

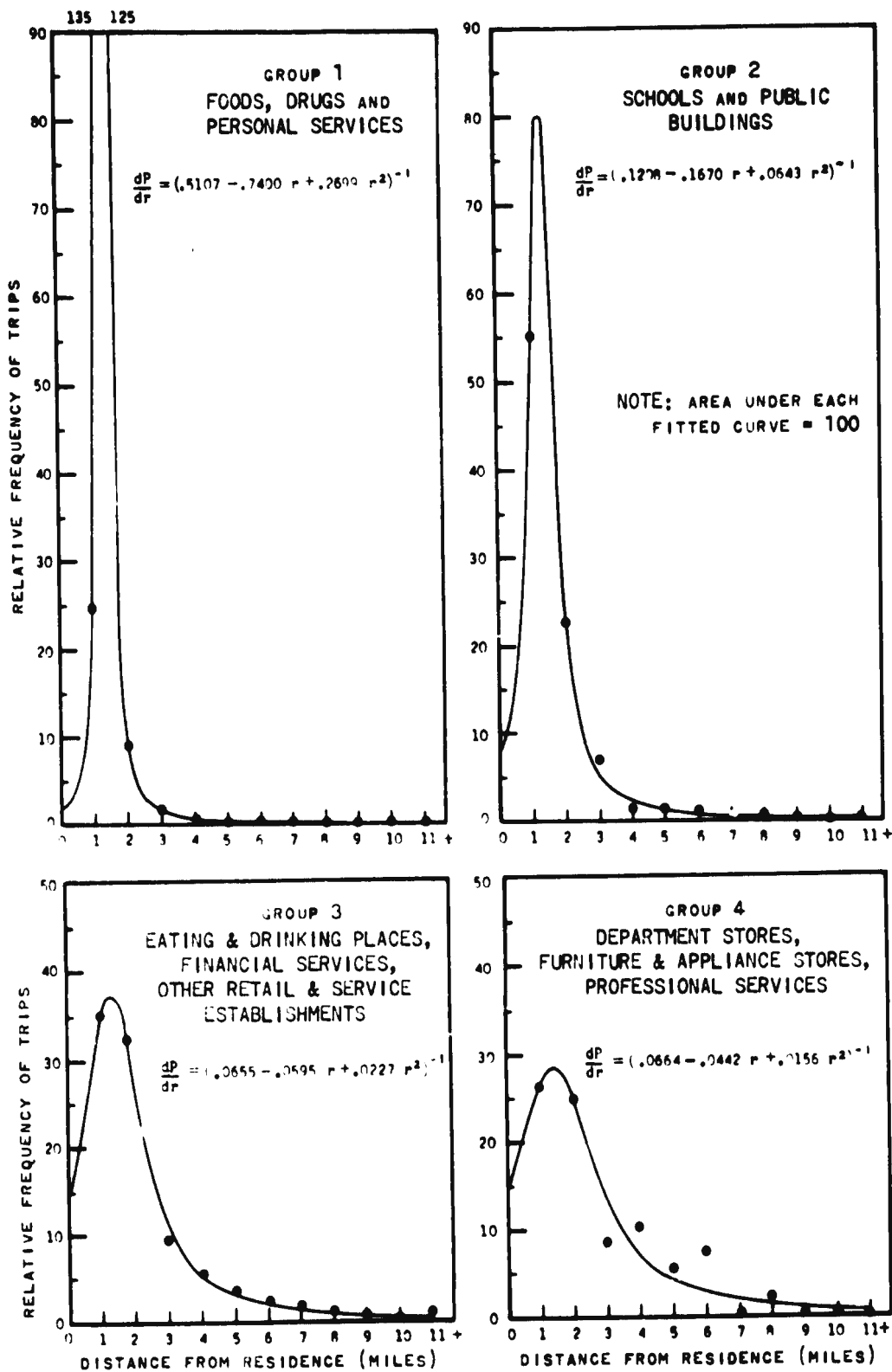


Figure 5.8

EMPIRICAL DISTRIBUTION OF RETAIL PATRONAGE
BY DISTANCE FROM RESIDENCE: STANDARDIZED
CONDITIONS

That Lowry's model can replicate (with qualifications) the 1958 distribution of activities in Pittsburgh is some indication of the speed of adaptation that occurs in a city, and the strength of the relationships in the model. The model is inaccurate in detail, but fairly good on a large scale. Considering the quality and quantity of the data, and the limitations of the relationships, the performance is as good as could be expected. The gross failures, such as an inability to "correctly" locate the CBD, can be explained by strong biases that exist in the real world that deviate the actual from the model equilibrium. However, for a metropolitan model to be practically useful, it must have a time dimension, and Lowry suggests ways in which this can be done.

Primarily, the means of making Lowry's model dynamic involve calibrating an iteration to a time dimension, and setting constraints on the operation that prevent complete distributions or redistributions. Some of his suggestions require that changes be made in the parameters or constraints after a certain number of iterations, to simulate some action in the real world. We will not describe these methods or other possibilities in detail because TOMM is a dynamic version of Lowry's model and the discussion is more appropriate there.

A final limitation of the Lowry model is the lack of detail due to output being for mile-squares. According to Lowry, these units are already too small for the quality of the data, while machine (running) time is another restriction on the number of areal units that can be incorporated.

TOMM. When the Pittsburgh CRP got under way, one of the major outputs of the Pittsburgh Regional Economic Study that was carried on by the City was Lowry's model. Although there were no transfers of personnel or data, the model itself became part of the Pittsburgh CRP Simulation effort. When the city picked up the model, three substantial changes were made in it: (1) the model was shifted from a static equilibrium to a dynamic equilibrium model, (2) households were disaggregated by type, and (3) the

total area covered by the model was reduced to the city boundaries. Other changes were relatively minor and did not affect the structure of the model, at least intentionally.

Conceptually, the dynamic aspects were handled by starting with some existing arrangement of activities, allowing a certain portion of the commercial and residential activities to relocate, and regarding the time required to reach a new (constrained) equilibrium as one time period. Thus time periods could be quite short if only a few activities were allowed to relocate, or longer if a larger proportion were free to move. Time calibration meant a simultaneous determination of real-world time periods and the "stable" function. Many of the equations could be regarded as first-order linear difference equations, e.g.,

$$N_{j,t}^H - N_{j,t-1}^H = w N_{j,t-1}^H + f(E_t)$$

which is a rough translation of one of the TCMM equations and says that the change in the number of households in one time period in tract j is a weight (scalar) times the previous total, plus some function of new employment opportunities. Solving the homogeneous part of this equation gives households as a function of time as

$$N_{j,t}^H = a (1+w)^t$$

where a is a constant determined by initial conditions. The particular solution depends upon the form of $f(E_t)$, but if w is greater than zero, as it would be, the partial solution given above would dominate any steady-state term in the complete solution. In that case the equation will not have a steady-state solution, for the number of households will increase without bound as t increases.

A solution of the above form is not very useful for several reasons. First of all, it ignores the interrelationships that we are trying to build into the model. Households in one tract depend only upon the previous number of households in the tract and the total employment, and in solution depend only upon time. This is too much simplification. Secondly, we are not really interested in an ultimate steady state condition sometime in the future. Conditions will alter the situation in the real world long before steady state is reached, and we are interested in the system's response to exogenous change. The difference equation formulation is of interest conceptually, but was not used in TOMM.

The difference equation formulation also points up another aspect of the dynamic model: the future depends upon the past only as it affects the present, i.e., only the previous time period need be known to determine the next. This is equivalent to the Markovian assumption, except that in the present case the model is not stochastic. Each state is uniquely determined by the previous state, rather than only in a probability sense. The alternative is the non-Markovian assumption, which means that the future depends not only on the present, but also on the path the process took in arriving at its present state. In difference equation form, this would require equations of order higher than one, and in an operating model such as TOMM it would require retaining information about several time periods in the past.

The second major change, household disaggregation, was accomplished within the model without great difficulty. Wherever the Lowry model had a household equation, TOMM replaced it with a set of equations, one for each household type. Household income seemed to be the most promising means for disaggregating households and classifying them as to type, with other possible variables being white-nonwhite and owner-renter.

Reducing the size of the area covered from the PATS data collection zone to the city boundaries was probably the result of two influences. For one thing, the CRP was concerned with renewal, not new development, and the

city was the major built-up area; there were many built-up areas outside the city, many of which needed renewing, but the city was the recipient of the Federal grant and the problems to be solved belong to the city. City politicians, administrators, and planners did not have much interest in collecting and processing a lot of information that did not pertain directly to their jurisdiction. The reduced-area decision was also probably influenced by the need for greater detail and the limitations on numbers of units in the model. Considering that the disaggregation of households added another computational dimension to the model, and there are 189 census tracts in the city of Pittsburgh, running times considerably in excess of Lowry's (17 minutes) could be expected. The census tract was the largest areal unit that could still provide sufficient detail for the information demands upon the model.

Partly for illustrative purposes and partly because the TOMM writeup (Crecine [313]) is somewhat difficult to obtain, the equations, in general form, are reproduced here in full. The notation is similar to Lowry's, with some changes due to the major changes mentioned above, and a few others. An additional category of land use, public, is introduced, and residential and commercial uses each have a stable (not free to relocate) and a mover component. Notation is as follows:

A,E,N	land area, employment households (quantities)
U,B,P,RS,HS,R,H	unusable, basic, public, stable retail, stable residential, movable retail, movable residential (superscripts)
t	time period (subscript)
i,j	tract index (subscript)
k	retail type index (superscript)
l	household type index (superscript)
a,b,c,d,e,f,g,p,w,r	constant coefficients or variable scale factors, which may be specific to household types, retail types, etc.

n, m, h	total number of tracts, retail types, and household types
Y_{ij}	trip distribution index, same as Lowry's T_{ij}

The asterisk symbol (*) on household variables means an absolute or cumulative total, with the absence of the asterisk denoting change in the variable from the previous time period. For example, equation (16) says the total number of households of type 1 in tract j at time t equals the total that existed at time $t-1$ plus the number added (or subtracted) in the period from $t-1$ up to t .

Only three of the equations will be discussed in detail; numbers 4, 11, and 14. A one-sentence description of all the equations is given below.

- 1 land area identity (sum of all uses equals total).
- 2 retail employment control totals, as a simple function of households (by type).
- 3 household identity (by type).
- 4 retail location equation.
- 5 retail identity (by type, i.e., the sum over all tracts for one type equals the city total for the type).
- 6 employment identity (by tract).
- 7 retail land use, as simple function of retail employees (by tract).
- 8 retail land use constraint.
- 9 updating identity for retail land (new total equals previous total plus change).
- 10 household control total as simple function of total employment.
- 11 household location equation.
- 12 updating identity for residential land.
- 13 household identity, city-wide total.
- 14 household change equation.
- 15 household identity (by tract).
- 16 updating identity for households (by type).

The equations themselves are presented in Figure 5.9.

Equation 4, which locates retail activities, is not quite the same as Lowry's retail equation. The inner sum is over household types (which the earlier model did not have), and is based on the assumption that different types of households exert different pressures on different retail activities. For example, a high-income suburban housewife tends to rely more on quality department stores and less on discount variety stores. It is interesting to note, however, that the distance "deflator" (Y_{ij}) does not discriminate between either households or retail types. A low income housewife will travel just as far as a banker, according to the model, and the ratio of near customers to far customers is the same for a major jewelry store as for a neighborhood grocer. To follow through on this line of thought, a 189×188 triangular matrix of access coefficients contains 17,766 entries; one of these for each retail and household combination brings the total number of empirically determined coefficients to 639,576. The purpose of mentioning this logical extreme is to make two points: (1) Disaggregating one class of entity may not be sufficient to incorporate desired behavioral relationships, and (2) Disaggregating more than one may be impossible.

What, then, do the various coefficients allow for in the way of improved relationships? The c^{kl} 's are weights which allow some households to generate more per-capita potential for a retail type than other households, and allow the relative weights to vary with retail type. The d^k 's suggest the hypothesis that noon-hour shoppers are selective in what kinds of retail activity they support, and the Y_{ij} 's are based on the assumption that the friction of movement between two tracts depends upon which two tracts are considered (and not on who or what is moving or why). These coefficients increase the flexibility of the model considerably, but whether a preferable set would include more or fewer is difficult to answer. At present, however, existing data bases cannot support more.

Equation Number	No. in Expanded System
1) $A_{j,t}^* = A_{j,t}^U + A_{j,t}^B + A_{j,t}^P + A_{j,t}^{RS} + A_{j,t}^{HS} + A_{j,t}^R + A_{j,t}^H$	n
2) $E_t^{Rk} = \sum_{l=1}^h a^{kl} N_t^{Hl*}$	m
3) $N_t^{Hl*} = \sum_{j=1}^n N_{j,t}^{Hl*}$	h
4) $E_{j,t}^{Rk} = b^k \sum_{i=1}^h \left[\frac{\sum_{l=1}^h (c^{kl} N_{i,t}^{Hl}) + d^k E_{i,t}}{Y_{ij}} \right]$	mn
5) $E_t^{Rk} = \sum_{j=1}^n b^k \sum_{i=1}^h \frac{\sum_{l=1}^h (c^{kl} N_{i,t}^{Hl}) + d^k E_{i,t}}{Y_{ij}}$	m
6) $E_{j,t} = E_{j,t}^B + \sum_{k=1}^m E_{j,t}^{Rk}$	n
7) $A_{j,t}^{R*} = \sum_{k=1}^m e^k E_{j,t}^{Rk}$	n
8) $A_{j,t}^R = \text{Max} \left(\sum_{k=1}^m e^k E_{j,t}^{Rk} - \sum_{k=1}^m e^k E_{j,t-1}^{Rk}, A_{j,t}^{RS} - \sum_{k=1}^m e^k E_{j,t-1}^{Rk} \right)$	n
9) $A_{j,t}^{R*} = A_{j,t}^R + A_{j,t-1}^{R*}$	n
10) $N_t^{H*} = \sum_{j=1}^n E_{j,t}$	one
11) $N_{j,t}^{H*} = g \sum_{i=1}^h \left[\frac{E_{i,t}}{Y_{ij}} \right] A_{j,t}^{H*}$	n
12) $A_{j,t}^{H*} = A_{j,t}^H + A_{j,t-1}^{H*}$	n
13) $N_t^{H*} = \sum_{j=1}^n N_{j,t}^{H*}$	one
14) $N_{j,t}^{Hl} = r_j \left(p_l N_{j,t-1}^{Hl*} + w_l \sum_{i=1}^h \left[\frac{E_{i,t}}{Y_{ij}} \right] \right)$	hn
15) $N_{j,t}^H = \sum_{l=1}^h N_{j,t}^{Hl}$	n
16) $N_{j,t}^{Hl*} = N_{j,t}^{Hl} + N_{j,t-1}^{Hl*}$	hn

Figure 5.9 THE TOMM EQUATIONS

Another minor difference between Lowry's equation and TOMM's is that Lowry included only the employment in the immediate tract, E_j (the noon-hour shoppers), while TOMM sums up potential from all tracts. The rest of equation 4, and its general form, are similar to Lowry, with b^k being a scale factor for conforming to control totals.

Equation 11 is the same as Lowry's household equation except that it is framed in terms of density rather than total households. The equation can be written as

$$\frac{N_{j,t}^{H*}}{A_{j,t}^{H*}} = g \sum_{i=1}^n \left[\frac{E_{i,t}}{Y_{i,j}} \right]$$

and this states that household density is an increasing function of employment potential. g is again a scale factor. The difference between using households or household density is not clear in its effect on the model, since Lowry's household constraints are in terms of density.

Equation 14 is another household equation and does not appear in Lowry. The content of this equation is that the number of new households of a certain type, in a tract, is related to the number of households of the same type that existed previously in the tract, and the accessibility to places of employment. The weights p_l and w_l are specific to each household type (the subscripts are "L's", not ones), and indicate whether the particular type is more concerned about living with similar families or getting to work. The employment potential variable appeared before, in equation 11, and again it is not clear what this redundancy implies. In the previous case, the dependent variable was (total) household density, while in the present case the dependent variable is change (from $t-1$ to t) in a household type. r_j is also a scale factor.

The relationship of TOMM to some of its exogenous inputs is shown in Figure 5.10. Adequate data with which to run the model have never been assembled, and around Pittsburgh the interest in the model is generally low. The City decided, after several years of work had gone by, not to continue the simulation effort, so most of the models have not been tested or used. A trickle of resources has gone into the model in the past few years, from the Department of City Planning and from Consad, but not enough to make the model usable. By indirect means, we are led to believe that John Crecine, the Author of TOMM, is currently associated with Richard Duke's METRO game at the University of Michigan, and that the model is operating as a part of the game.

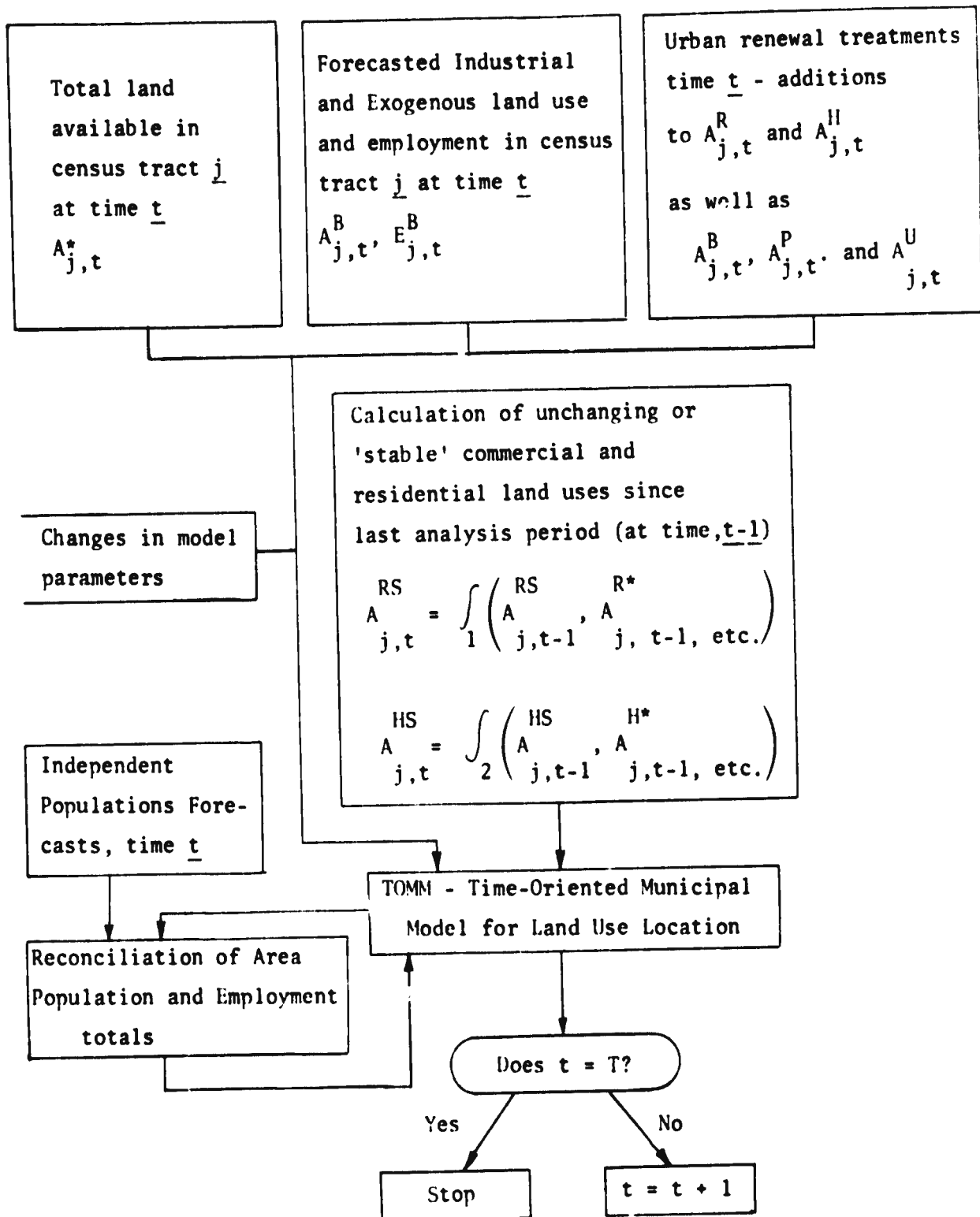
The San Francisco Model

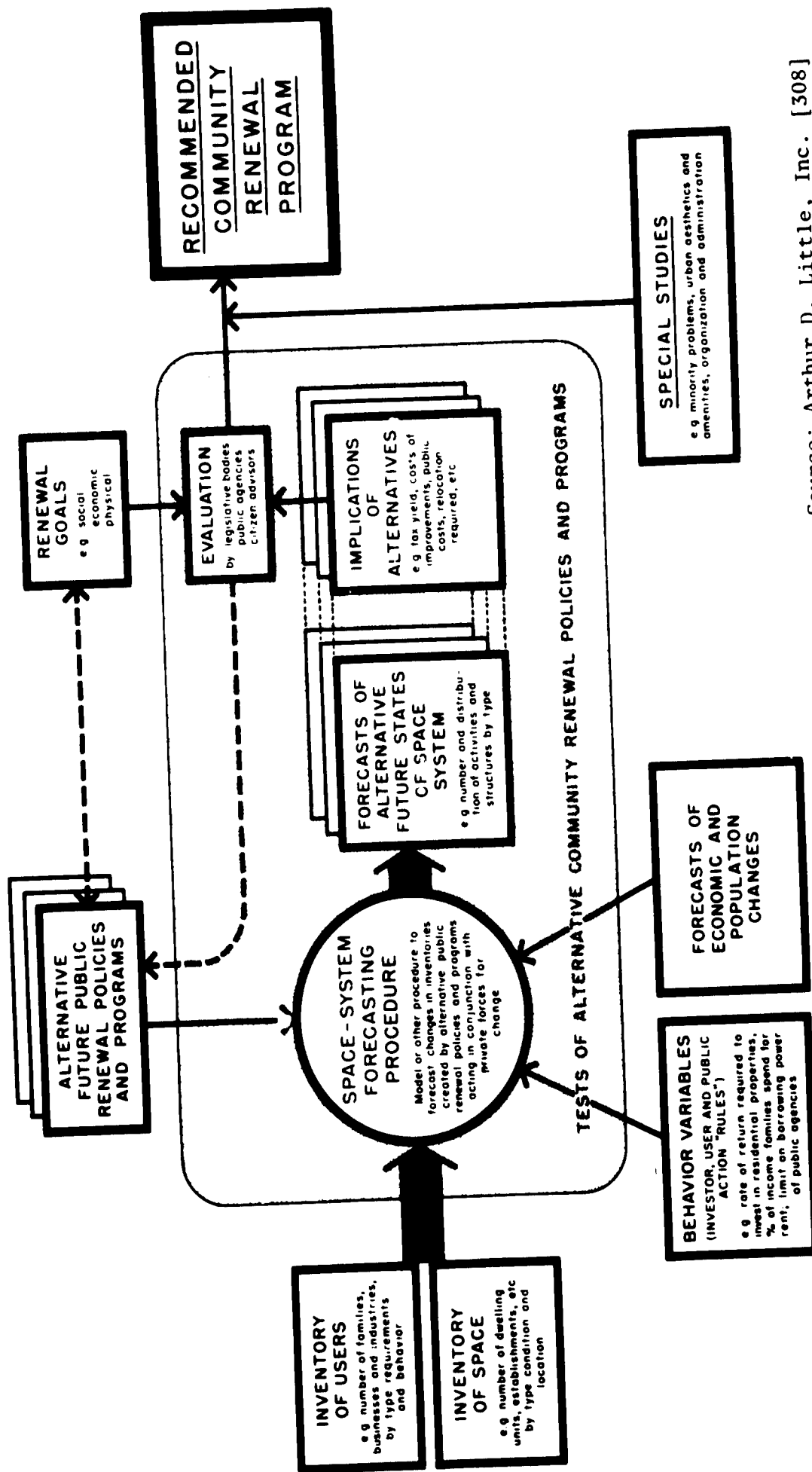
San Francisco started its CRP a year or so after Pittsburgh, and in the early stages it appeared that the San Francisco model would develop along similar lines. Whatever it was that caused Arthur D. Little, Inc., the principal consultants, to alter their course, the end result was a single residential model which had little of the Operations Research trappings. Compared to the original schematic diagram shown in Figure 5.11 -- comparable to Pittsburgh's but more generalized -- the flow chart for the final model (Figure 5.12) seems rather small and simple. The point is not that somebody or other failed to deliver, but that the job is harder than it looks. Both Pittsburgh and San Francisco are good examples of this problem, but the difference is that ADL narrowed its sights and produced a working model.

The model attempts to replicate the micro-economic actions of the residential market in responding to the supply of and demand for housing. Households are differentiated according to type, as are dwelling units, and the two sets are matched up by assigning housing to the households that will offer the highest prices for it. If excess demand exists, new housing may be added in the next time period, and other housing may be rehabilitated or

Figure 5.10.

INTEGRATION OF TOMM SIMULATION MODEL





Source: Arthur D. Little, Inc. [308]

Figure 5.11 PRELIMINARY SCHEMATIC DIAGRAM, RENEWAL PROGRAMMING PROCESS

otherwise altered to meet the demand. Structures are deteriorated or improved in response to the pressures of the market. The operation of the market may be modified by the introduction of government programs, and in this way the effects of alternative policies and programs can be measured.

Supply units (i.e., "residential space" to use ADL's language) are described according to the following attributes:

- (a) Housing type
 - single family or category of multiple family
 - number of rooms per dwelling unit
 - owner or renter occupied
 - rent or value
- (b) Condition
- (c) Location

Input to the model is a set of existing stocks of these various types, condition and location, plus certain land uses (such as vacant land) which may potentially become residential. Changes in the supply are generated internally by the model.

Demand for housing is generated by households, who are also differentiated in the market. Some of the attributes used to distinguish different households are:

- (a) Household type
- (b) Number of members in household
- (c) Race of head
- (d) Household income
- (e) Occupation of head
- (f) Rent-paying ability

The number of households in each category is predicted exogenously for each iterative period in the model. Associated with each household type is a

list of housing types ranked by preference (within ability to pay), and the housing assigned to the household type must be selected from this list.

Market operation is simulated by comparing the preference lists with the stock of available housing, starting with the highest (most expensive?) category. If the number of units available exceeds the number of units needed, then some housing is left vacant. On the other hand, if more units are demanded than are available, the price goes up and some households are bumped into their second choice category. The increase in price produces an increase in yield for that type of housing, and more is built, consequently, in the next period. Allowance is also made for a "bandwagon" effect, which assumes that builders will tend to build housing types that have been successful in the immediate past, even though demand for that type may have been satisfied and other types might be more profitable. Other than this feature, the model operates on the assumption that in the long run the private real estate market responds in a way so as to maximize yield on investment.

Public action can take several forms, which fall into three general categories:

(a) Direct operations in the market

- purchase of property by government
- selling of property by government
- maintaining or upgrading government property
- demolish (government?) building
- site improvements by government
- construction (for public or private use)
- lease property to private sector

(b) Indirect operations on the market (changing model parameters)

- restrict private occupancy changes (anti-discrimination)
- tax rate and assessment
- cash subsidies to households
- rent control

mortgage and loan insurance
direct reduction of the cost of financing housing
construction

- (c) Extra-market decisions (actions taken without reference to market)
code enforcement, in which structures are either brought up to standard condition or replaced.

Costs and revenues are computed not only for the private sector, but also for the public, to determine the direct costs to the government and both direct and indirect revenues.

Operation of the model in its computer program form can be simply described as bookkeeping. A few ratios and indices are calculated in order to determine priorities, and the various entities are maintained (ranked) on lists until they are allocated. Each "event" (comparison of supply and demand for a type) may have ramifications throughout the system, and these are reflected in changes in the indices, parameters, rankings, etc., which ultimately affect the supply and demand. Although the specific operation of the model is important to an evaluation of the model's behavioral characteristics, the key features are in the detailed workings that set priorities and order events. The model itself is primarily a framework which is neutral with respect to household behavior. The content of the model is determined by the ways used to disaggregate households and housing types, and the variables and weights employed in matching people with houses. Without warning, the reader might feel that a stage in the description was missing (the elegant overview) and that he was suddenly immersed in a sea of minor details.

The Entities and their Attributes

The City of San Francisco is divided, for model purposes, into approximately 100 neighborhoods, each containing a number of "fracts" (fractions of tracts?). A fract is a unit of land which is homogeneous with

respect to land use and housing type, condition, etc., and is of constant size (all fractas are the same size, approximately one large city block in area). The parcels making up a fracta are not necessarily contiguous, but they must be contained within the same neighborhood. A neighborhood can be located geographically, but a fracta is only located within a neighborhood.

Units of housing are differentiated on three dimensions: (1) Location category, (2) Housing type, and (3) Condition. The location category is based on three components, which are topography (hilly or level), modal structure type (single family, 2-4 families per structure, or over 5), and average rental (high, medium, or low). Some of the possible combinations are empty, resulting in 14 different categories. Housing type is also composed of three items: structure (single family, 2-4 families per structure, or over 5), tenure (owner or renter occupied), and number of rooms per dwelling unit (1-2, 3-4, 5-6, or over 7). Of the housing type combinations, 24 are non-empty. Housing condition may be one of four categories: sound, needs minor repairs, deteriorating, or dilapidated. The location category applies only to a neighborhood as a whole, while the housing type describes a fracta, which makes it possible, for example, to have a high density apartment fracta (over 5 dwelling units per structure, renter occupied, with 2 rooms per dwelling unit) in a level topography, single family, medium rental neighborhood. Two special categories are added to the list of housing types (public structures and vacant land) and there is also a third type of tenure (public housing). Again, not all combinations of housing and location (neighborhood) have entries, but still there are over 200 categories, not including condition.

On the demand side, households are grouped by using four variables: family type (no children, head under 60 years; no children, head over 60; and households with children), household size (3 or 4 categories), color (white, negro, oriental and other), and income (6 categories). The result is 114 kinds of households. In addition, rent paying ability for each household type covers a range of values rather than being set at one level, with an approximately normal distribution (sometimes skewed). This was to prevent sharp discontinuities in the demand for a housing type as rents changed for the type.

Each household type is given a preference list of housing types, indicating those types he would occupy first given the choice. These lists were imputed from data showing which household types presently occupy which housing types, and by a weighting scheme that implies that households would ultimately like to occupy the housing presently occupied by a similar household type but in a higher income bracket.

Locations of sources of employment or of shopping opportunities, either within the city or outside, are not considered to influence the location decisions of households within the city, except by preempting the land. This is equivalent to saying that any particular household will find housing that is adequately located for its purposes, if it finds any at all. If all low income negro employment were in the West end, and all housing available to the same group was in the East end, then some difficulty in the assumption disregarding the journey-to-work (or shop) might be expected. Apparently it is not important enough to deal with explicitly in the model. Although the model is generally micro-analytic in its approach, it is sufficiently aggregative to allow for those location factors not explicitly considered by the model to cancel out. Whether they actually do or not is difficult to say.

The basic time period is two years, with the total forecast interval designed for 18 years. Thus 9 complete iterations are required. Output can be generated for each period, or for just the full 18 years. Outputs include the following:

- overcrowding and excess demand, by household type
- distribution of minority groups
- total tax base
- distribution of substandard housing
- distribution of building activity
- rehabilitation activity
- total costs, deficits, expenditures, etc. rents, yields

Any of the above could be listed for any locational detail desired.

Model Operation

The computer program that operates the model has been summarized into nine steps by ADL, and the flow through those nine steps is a convenient means for describing the operation. A complete run of the model is nine two-year periods, with some steps being re-cycled within each two-year period. Figure 5.12 shows the flow chart.

(1) Initializing

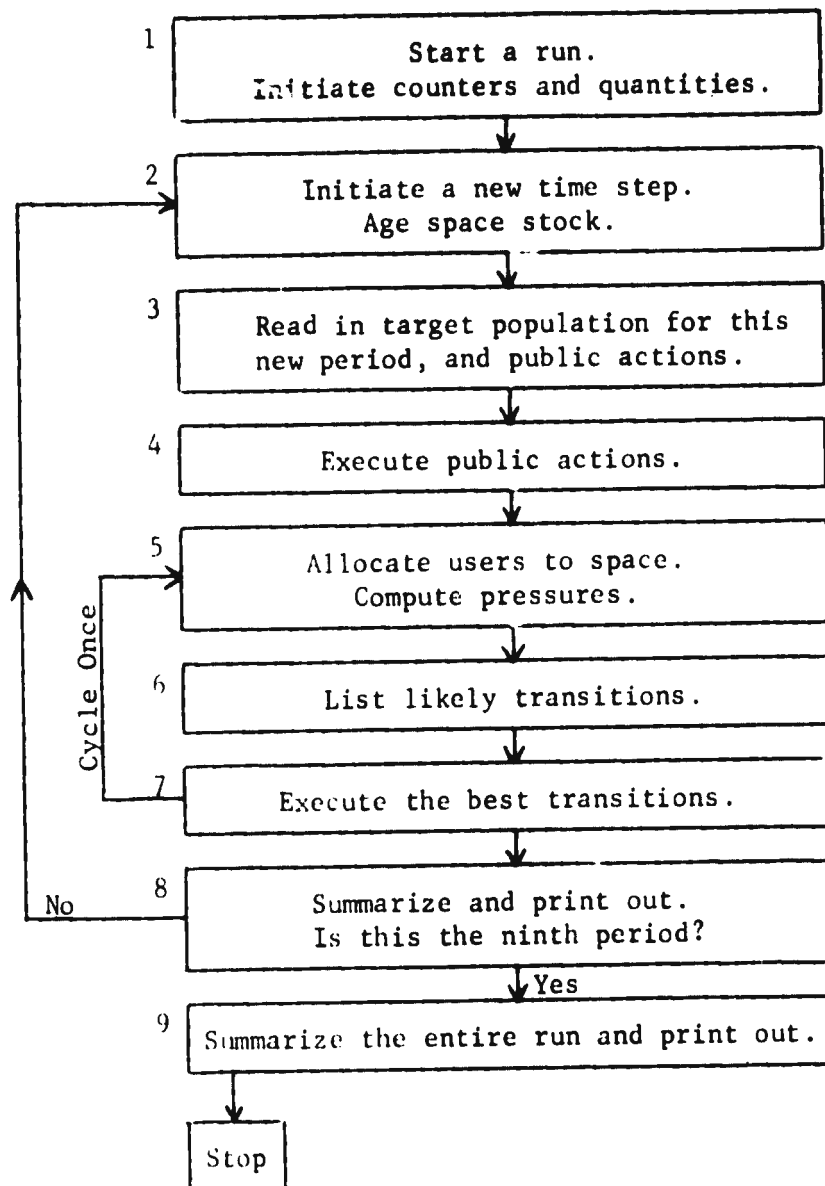
All models require some sort of initializing, which is simply the process of setting the model up in the computer. Registers to be used for counting or accumulating totals are cleared, input data describing the state of the system at time zero (the present) are entered, plus all the parameters, weights, etc., that govern the behavior of the model. In this case, inputs include current household types and their preference lists, housing types and location categories, information on construction costs, financing, etc., for computing yields, and indicators for specifying the information to be printed out.

(2) Aging

The stock of housing is run through a subroutine that may change the condition of the housing either upwards or downwards. This routine is an important one in understanding the behavioral characteristics of the model, and will be described separately.

(3) Enter period information

The target population by type, determined exogenously, plus the public action programs, are read in for the particular time period being considered.



Source: Arthur D. Little, Inc. [311]

Figure 5.12 MODEL OF SAN FRANCISCO HOUSING MARKET

(4) Execute public actions

The possible public actions are expressed to the model in different specific ways. The means for doing this are described below.

Zoning: Two numbers -- the maximum number of fract's that can be developed at high density, and the maximum number that can be developed at either high or medium density -- are associated with each neighborhood. These numbers might be changed at any time period to reflect changes in zoning policy.

Location category: The distribution of the housing supply can be altered by changing the location category of a neighborhood, which might be a consequence of public action (ADL says that this change reflects "amenities" and perhaps access, but how this is so is not clear).

Assessment: Any action that changes the normal yield of a housing type (neighborhood-housing-condition) can be entered by changing the normal yield parameter for the type.

Rental: The rental value for a housing type can be changed, perhaps reflecting rent control or some such.

Capital improvements: Either a location category or a specific neighborhood can be earmarked for public development, in which case the fract selected is no longer available for private residential use.

Code enforcement: Certain fract's may be designated for building or housing code enforcement. A randomly selected group of these that are also substandard (deteriorating or dilapidated) are upgraded to sound condition.

Construction costs: Changes in costs for specific transitions may be the result of subsidies or other financial inducements.

Household preferences: if for some reason it is anticipated that some household group will change its preferences at some point in the future, this can be entered by changing the order of housing types on the household's preference list.

Aging rates: If public policy operates directly on a particular housing category to change the aging rate, then this fact is reflected in the aging probability table for the type.

Rent-paying ability: Public subsidies to certain household groups may change their rent-paying ability.

Other direct tract action: The housing type category, condition, or any other information about the tract can be altered arbitrarily in any time period. Some of the tract information is in the form of indicators, called "flags" by computer-folk, which designate a tract as either having or not having a certain attribute. Since there are only two possible states for each attribute, a flag is said to be "on" or "off". There are six flags for each tract in the San Francisco model, which cover the following attributes:

1. Transition has (has not) occurred. Two transitions cannot occur in the same tract in the same time period, and this flag prevents it.

2. Tract is available (not available) for private market transitions. Public property is designated by this flag.

3. Tract is available (not available) for residential use. Public property may still be available for residential use either permanently or temporarily.

Construction costs: Changes in costs for specific transitions may be the result of subsidies or other financial inducements.

Household preferences: if for some reason it is anticipated that some household group will change its preferences at some point in the future, this can be entered by changing the order of housing types on the household's preference list.

Aging rates: If public policy operates directly on a particular housing category to change the aging rate, then this fact is reflected in the aging probability table for the type.

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1. Transition has (has not) occurred. Two transitions cannot occur in the same fract in the same time period, and this flag prevents it.

2. Fract is available (not available) for private market transitions. Public property is designated by this flag.

3. Fract is available (not available) for residential use. Public property may still be available for residential use either permanently or temporarily.

4. Code enforcement is (is not) applied.

5. Fract is (is not) in a rehabilitation area. This results in a ten percent discount on transition costs.

6. Fract is (is not) in a redevelopment area. This results in a twenty percent discount on transitions.

(5) Allocate households to housing

Households are ordered according to their ability to obtain the housing they desire, mostly on the basis of income (socio-economic status), and housing is distributed to household groups by referring to the preference lists of each household group in turn. A trial allocation is made in which the interaction of supply and demand does not affect the behavior of the market, e.g., excess households desiring a limited supply of a housing type do not drive the price up. After the trial allocation, pressures are computed. If pressures are sufficient to suggest a higher yield for a different use, then a transition may occur. Otherwise, excess demand or supply results in increased or decreased rentals.

ADL [311] states,

"Developers and property owners make their decision on the basis of many criteria, including economic and personal factors that a simulation model could not hope to replicate. We feel that in the long run, however, land-use decisions will respond more nearly to the value of economic yield than to any other individual indicator. By yield, we mean, roughly, the income of a property divided by either the amount of money currently invested or the market price of the property." (p.23)

For each household type there is associated a rental and a normal yield.

The normal yield is defined as

$$\text{Normal Yield} = \frac{\text{Rental}}{\text{Market Value}}$$

which is a form of return on investment. Normal yield is the return from the existing use; transition yield is the potential return from another use and is computed in one of two ways. If the transition involves rehabilitation, conversion, or merger, then

$$\text{Transition Yield} = \frac{\text{Rent}(\text{new condition}) - \text{Rent}(\text{old condition})}{\text{Cost}(\text{old to new condition})}$$

For new construction,

$$\text{Transition Yield} = \frac{\text{Rent}(\text{new housing type})}{\text{Cost}(\text{new construction, type}) + \text{Market value}(\text{old type})}$$

If the transition yield is greater than the normal yield for a particular project (and private action has not been pre-empted by government action), then the project is carried out.

As pressure (excess demand) increases, rents increase and more projects become worthwhile. As projects are built, the demand is satisfied and similar projects become less attractive. If there is excess supply, then rents will tend to drop, increasing demand for the type. The response of rents to excess demand (supply) differs depending upon whether demand is increasing or decreasing, and this relationship is shown in Figure 5.13. If the model were to seek an equilibrium for each time period, then it would keep iterating until no more projects (transitions) were feasible. Since transitions take some finite amount of time in the real world, the model replicates this by iterating only once (two allocation passes) through allocation, listing of transitions (steps 5, 6, and 7). This may leave excess

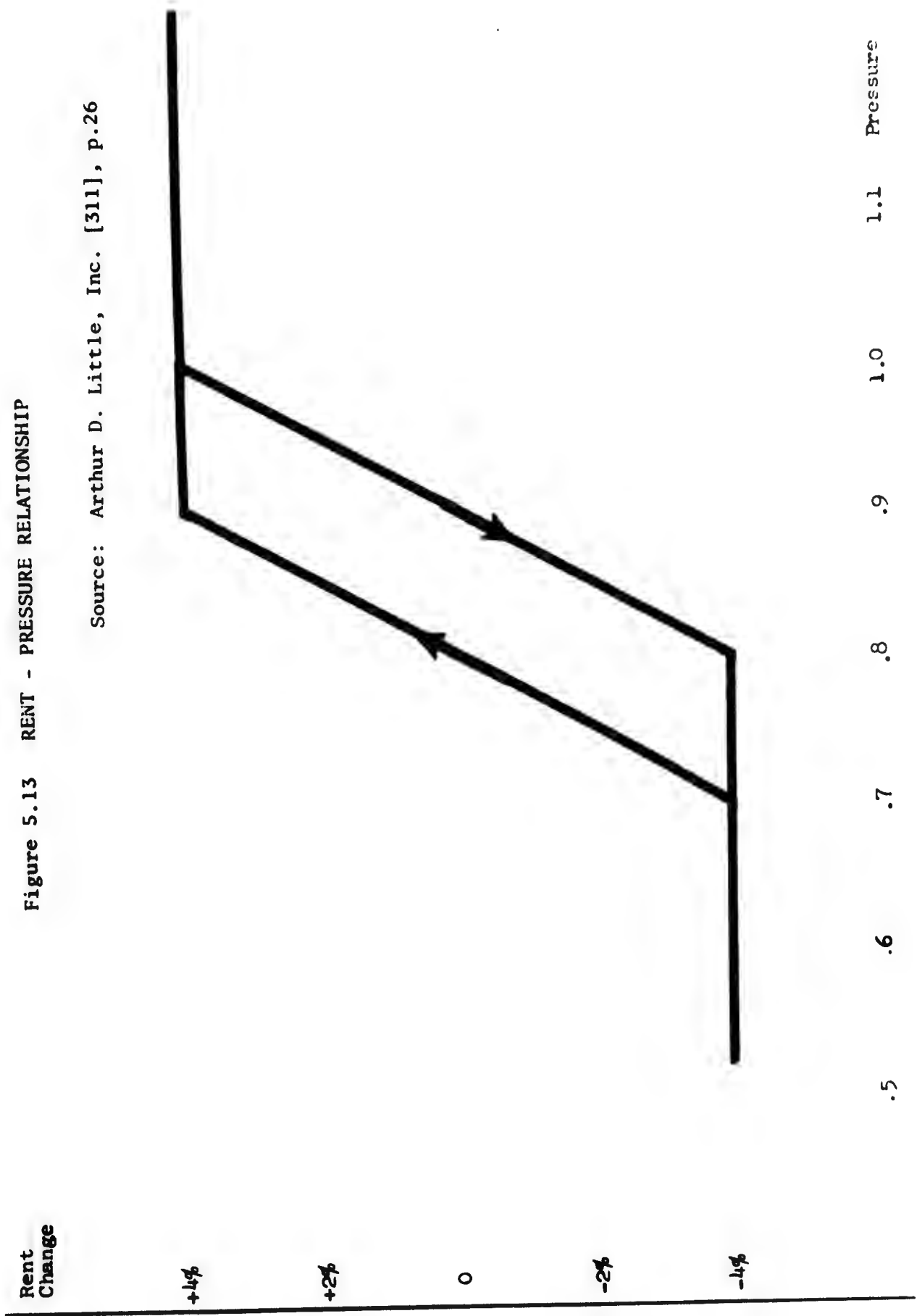


Figure 5.13 RENT - PRESSURE RELATIONSHIP

Source: Arthur D. Little, Inc. [311], p.26

demand, excess supply from previous periods, or an excess supply from overbuilding resulting from the bandwagon effect. At the end of step 5 we have a preliminary allocation of households to housing, the pressure of excess demand or supply, and the resulting rent changes.

(6) Assemble list of likely private market transitions

Household groups are selected in order of pressure exerted, and housing types taken from their preference lists to make up the list of potential projects. If the yield from a particular fract in a particular neighborhood for a particular housing type is greater than the yield for the existing type, then the fract is placed on the list of projects.

(7) Execute transitions

The list of projects (fracts) is ordered by yield, and a fixed number (80) is executed from the top of the list. Then steps 5, 6, and 7 are repeated one more time.

(8) Period information printout

The results of the current time period are printed out. Households which were unable to find acceptable space are either forced into overcrowding or into leaving the city, and this information is printed out. Other information pertaining to the current state of the system, such as housing stock, household allocation, pressures, rents, etc., is given at this point. Control then shifts back to step 2 for the next two-year time period, and the procedure repeated through step 8 until nine periods have been completed.

(9) Final output

Results of the entire run are assembled, digested, summarized, tabulated, and prepared for display.

The Aging Procedure

Structure condition can be improved through private market action, which occurs because the higher rents chargeable for better condition exceed the costs of improvement, through private market action induced by public subsidy (rehabilitation, for example), through private market replacement of the structure because it is profitable to do so, or through direct public action (government construction). In addition, there are two means by which structures may deteriorate to a lower condition, and these processes are called aging.

If it is profitable for a property owner to maintain his structure in its present condition (i.e., decreased costs from lower maintenance levels would not offset decreased rents obtainable), then it is assumed that the owner will give the structure normal routine maintenance. The property will then deteriorate at a normal rate, which will depend upon the structure, usage, and the neighborhood. On the other hand, if an owner can allow his property to deteriorate without serious loss in rentals, then it may be profitable for him to not maintain his structure. The first type of aging is called "normal aging" and the second is called "accelerated aging".

The model selected for the aging process subroutine was a Markov chain. To the four possible housing conditions (sound, minor repairs, deteriorating, dilapidated) a fifth state was added, which was an absorbing state for the purpose of removing a certain portion of the dilapidated structures. Three transition matrices were considered adequate to differentiate between the aging of different structures, and these are: tenant-occupied 1-4 family structure, owner-occupied 1-4 family structures, and 5 family and over structures. It was assumed that a structure could either stay in the same state from one period to the next, or deteriorate one condition, and hence the only non-zero transition probabilities are in the main diagonal and the superdiagonal immediately above it. The data seemed to indicate that, for some reason, the rate of aging from condition A to condition B (sound to minor repairs) increases sharply after eighteen

years, and so there are two transition probabilities for each of P_{AA} and P_{AB} in each matrix. The three normal aging matrices are shown in Figure 5.14. Since there is no apparent provision in the main model for recording the age of a fract, it is not clear how or whether the split values were used. The appropriate matrix for a particular neighborhood is applied to the fract within it, and rounding off, where necessary, is done with random numbers.

Under a certain set of conditions, a housing type (not fract or neighborhood) may receive an accelerated aging matrix, which is approximately ten percent higher than the normal matrix. The criteria for applying the accelerated matrix are, roughly,

- (1) There is excess demand by a set of household groups for housing within their rent-paying range, but high vacancy in the type at the next higher rental and next on their preference lists,

- (2) Vacancies do not exist in the higher rental type at a lower condition level, and

- (3) It is profitable for owners of the higher rental type to decrease maintenance and lower rents (demand is: elastic downwards). This in effect, says that where owners can increase profits substantially by tapping demand at lower rent levels, they will do so, if their units are heavily underutilized at the higher level.

The primary underlying assumption of a Markov chain is that the probability of transition from one state to another in one time period depends only upon the present state. The path the process took to arrive at the present state, the previous states it was in, the length of time that it has been in the present state, the number of times it has been in the present state, etc., have no effect on where it will go next. In the aging model, this means that the probability of a housing unit dropping to a lower condition level is independent of the age of the structure (except

MARKOV MATRICES

(6-Year Iteration Period)

Tenant-Occupied 1 - 4 Family

	A	B	C	D	E
A	.818	.182			
B	.677	.323			
C		.87	.13		
D			.90	.10	
E				.5	.5
					1

Owner-Occupied 1 - 4 Family

	A	B	C	D	E
A	.877	.123			
B	.753	.247			
C		.90	.10		
D			.90	.10	
E				.30	.70
					1

5-Family Structures

	A	B	C	D	E
A	.7901	.2099			
B	.6183	.3817			
C		.92	.08		
D			.80	.20	
E				.50	.50
					1

Source: Arthur D. Little, Inc. [309], p.11

Figure 5.14 MARKOV MATRICES FOR CONDITION AGING

in the one case for under- and over-eighteen), the number of structures in the class, the rate at which it has deteriorated in the past, or the deterioration of other structures. The model is aggregated to the extent that all structures in a fract deteriorate together, and all structures within one of the three classes are considered to be homogeneous for aging purposes.

A well-developed system exists for classifying Markov chains with respect to their properties. Since these properties pertain mostly to long-run or steady-state conditions, and our matrices are applied only once per period, the classification is not of great interest to us. Some features are, however, relevant. The first four states communicate in one direction only, which is downward to the next lower condition. This is sufficient in itself to label these states as transient states. One state, the absorbing state (removal), communicates only by receiving other states, with no way out. Thus the chain can be decomposed into one ergodic set (the absorbing state) and four transient states. In the long run, the entire process, i.e., all structures, would be absorbed in the fifth state, but this does not occur because new structures are introduced and ways exist exogenous to the Markov chain for improving condition. Thus the distribution of housing is revised between successive applications of the Markov matrices, and steady state is never reached.

Data

From the description of the model it should be apparent that success depends very heavily upon the quality of the data, and that the requirements for data are strenuous. Although the nature of the model is micro-economic and several behavioral assumptions are built in (such as the rent-demand relationship and the accelerated aging mechanism), the model still must be regarded as an accounting framework for organizing and processing data. As we pointed out earlier, all models are essentially limited to this function, but the San Francisco model stands much closer to the accounting side and consequently farther from the complex mathematical model or optimizing side. The model is not lacking in interdependence -- changes in any part of the system have an effect on the other parts -- but a relatively lower level of

complexity is reflected to some extent in the lack of any systems of equations and the absence of a necessity to iterate extensively to obtain a solution. This is not meant to imply that the model is somehow inferior (it may be the optimal level of complexity for the present state of the art), but that evaluation rests on knowing the quality of data which normally are difficult to obtain.

The basic data required to run the model come from special tabulations of the 1960 census data. Thus the information needed is collected by the census but not published in suitable form for this model. It was not possible to obtain the same data for any other period of time, 1950, for example. Approximately 15,000 items of data are needed to make a computer run, and this figure does not include the data necessary for fitting the parameters (entries in the Markov matrices, for example) or for tuning the completed model (e.g., actual housing construction). Much of the information can be used over again, in subsequent runs, but new data are necessary in those areas that represent different public policies or population forecasts. A large portion of the processing takes place before the information is ready for the model.

Beyond the basic data describing the present state of the system, predicted exogenous variables, and public programs, are the data necessary for calibration and tuning. Most of the data on household preference lists, the distribution of rent-paying abilities within a household group, vacancies by housing type, rentals, etc., can be imputed from the census data. The zoning table, the location table (neighborhood description), and land areas in tracts and neighborhoods must be transformed by hand but use readily available sources (maps, the zoning ordinance). Construction and rehabilitation costs are obtainable from "engineering" data (standard costing procedures. See Arthur D. Little [310]). Yields from each location-housing-condition type, while voluminous, can be derived from cost and rental data, with the addition of interest rates, amortization rates, financing, expected vacancy, maintenance costs, etc. Control data on actual construction, costs, transfers, and prices were obtained from building permits and (presumably)

real estate transactions. A few parameters, such as household group-housing type weightings and the maximum number of projects per iteration are apparently arbitrary. Most of the muddy detail problems of supply the model with working components are not mentioned or described in ADL reports, and so it is difficult to say how they were handled.

One problem that received special attention was the aging table. Using the Markovian assumptions (previously mentioned), ADL suggests a continuous-time aging model, which can be formulated as a birth-and-death process. If we let $P_1(t) = \text{Prob}(\text{structure is in sound condition at time } t)$, $P_2(t) = \text{Prob}(\text{structure needs minor repairs})$, ..., and $P_5(t) = \text{Prob}(\text{structure has been removed at time } t)$, then we can say that

$$P_n(t+dt) = \lambda_{n-1} P_{n-1}(t)dt + 0 \cdot P_{n+1}(t) + (1 - \lambda_n) P_n(t)dt$$

where λ_n is the death rate (per unit time) from state n . The equation says that the probability of being in state n at time $t+dt$ equals the probability of being in state $n-1$ at time t and having a death, or of being in state $n+1$ and going to state n (not possible), or of being in state n and having neither birth nor death. Since a death from state $n-1$ is a birth into state n , the first term is the probability of birth into n and the last term is the probability of not dying out of n . Rearranging terms and taking the limit

$$\frac{P_n(t+dt) - P_n(t)}{dt} = \lambda_{n-1} P_{n-1}(t) - \lambda_n P_n(t) = P'_n(t)$$

In steady-state, the derivatives all equal zero, i.e., $P'_n(t) = 0$, and substituting $n = 1, 2, \dots, 5$, we have

$$P'_1(t) = -\lambda_1 P_1(t)$$

$$P'_2(t) = \lambda_1 P_1 - \lambda_2 P_2$$

$$\dots$$

$$P_5'(t) = \lambda_4 P_4(t)$$

This system of differential equations can be solved sequentially for the functions $P_1, P_2, \dots, P_5(t)$. Taking the first,

$$P_1(t) = e^{-\lambda_1 t}$$

This says that the proportion of structures in sound condition is an exponential decay function of time, and should plot as a straight line on semi-logarithmic graph paper. In addition, assuming a building starts in sound condition, $P_1(0) = 1.0$. This leaves only the slope to be determined, and statistical means (least squares) can be used for this purpose. An example of the data and the fitted functions for one housing group is shown in Figure 5.15, with the line representing $P_1(t)$ taking a bend at eighteen years. Similar data are plotted in Figure 5.16 on arithmetic paper. The interpretation of the double-sloped line of Figure 5.15 implies two exponential functions for $P_1(t)$, which is labelled "good condition."

The other functions can also be solved, and parameters fitted, (ADL does not discuss the method for doing so). An interesting note is that the data used for fitting the transition matrices was recorded in 1939. According to Arthur D. Little [309],

"Relevant data on condition changes in structures is extremely hard to come by. The most comprehensive data available is that of the cross-sectional Real Property Survey carried out under the auspices of the WPA from 1937 to 1939. In this survey, teams of inspectors examined every structure and interviewers interviewed the occupants of all dwellings in the City. The WPA Survey includes data on the type of structure, age, date built, conversions, occupancy, repairs needed, and a structural estimate

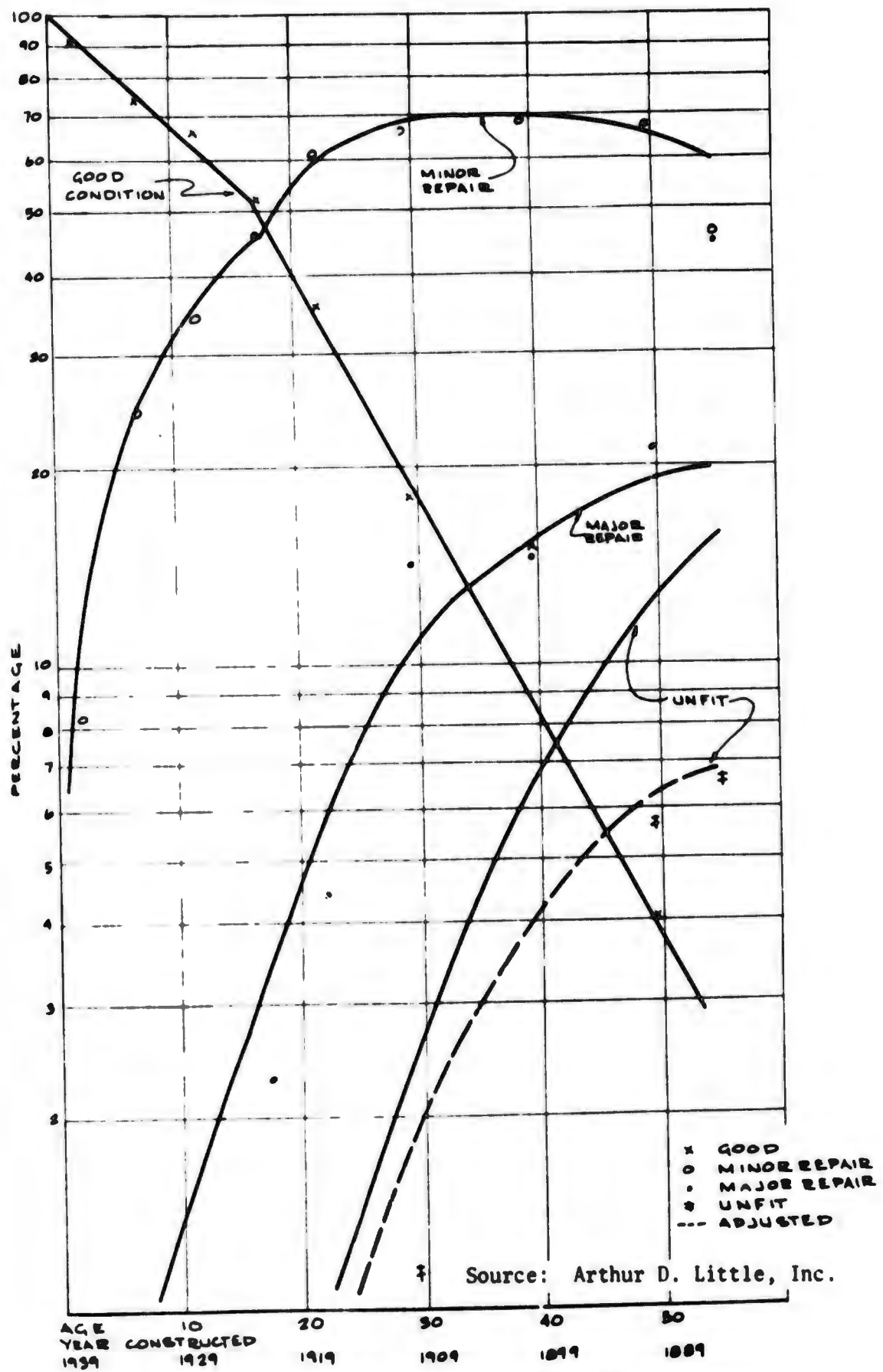
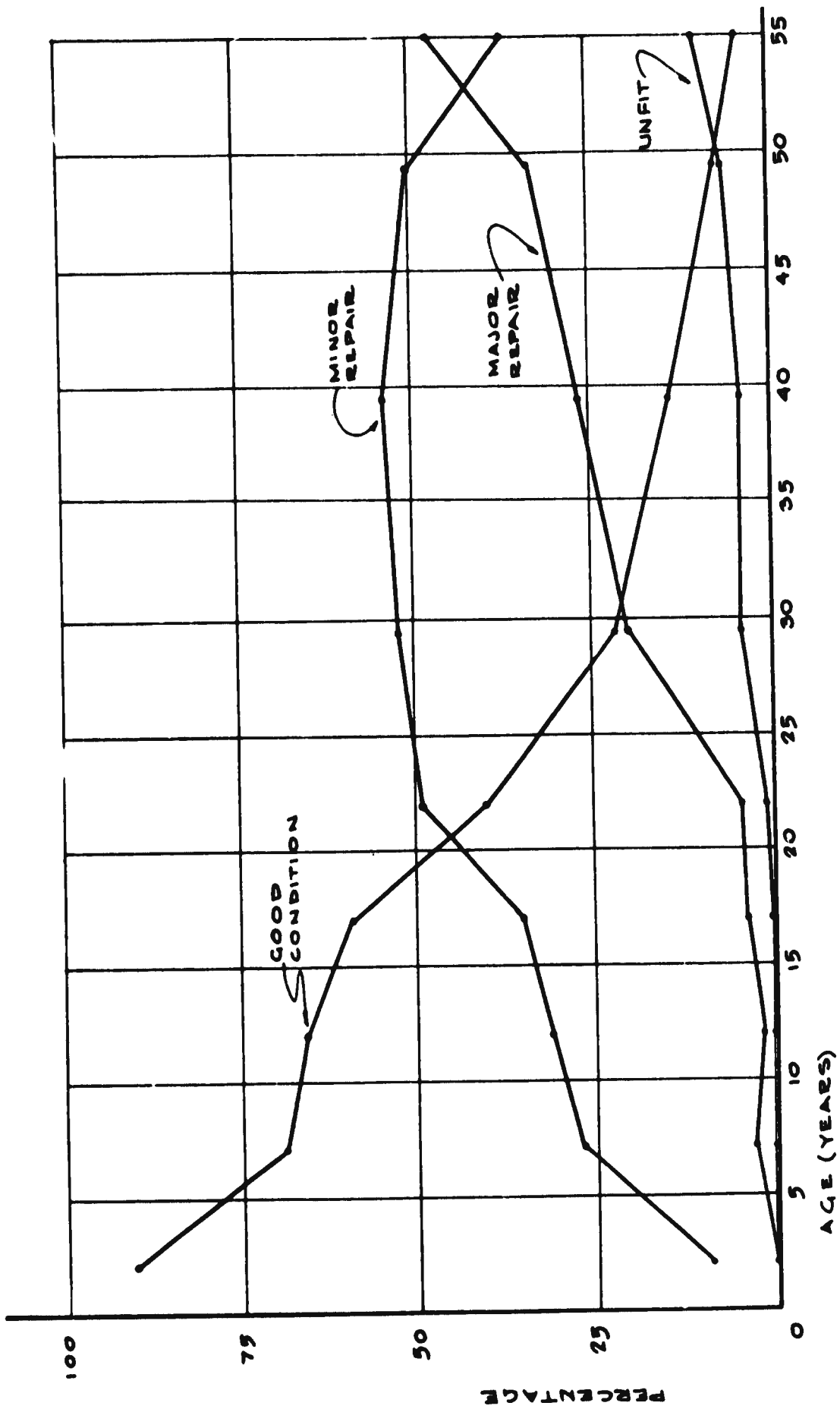


Figure 5.15 CONDITION AGING, FIVE-OR-MORE-FAMILY STRUCTURES



Source: Arthur D. Little, Inc. [309]

Figure 5.16 MODELS FOR CONDITION AGING OF RESIDENTIAL STRUCTURES

of substandardness based on well-defined criteria. As Hendricks* points out, the housing censuses are not nearly so valuable, and nothing has been done since 1939 comparable to the Real Property Survey. The work outlined in this report is mainly based, therefore, on data taken from the Real Property Survey of 1939 for the City of San Francisco." (p.2)

In another reference, ADL claims that the parameters were derived from 1960 census of housing (Arthur D. Little [311], p. 31). If so, this fact is a change from the earlier report.

Results

A full computer run (9 periods) takes about 2 hours on an IBM 7094 with disc file (probably 32K or so core storage). This equipment is as large and fast as that generally available at the time, and the running time is long but not unreasonable for a large experimental model. Most of the programming is in Fortran IV, a fairly high level compiler language. In order to understand and use the model effectively, it is necessary to make repeated runs which compare the effects of changes in the variables, parameters, and public policy, and this is when 2 hours per run starts to look big. ADL thinks this can be cut down.

Several tuning runs were made, with total quantity of new construction being the primary control (Figure 5.17). The distribution of housing to type was not unreasonable, compared with actual, but with some tendency to underestimate single family dwelling units (Figure 5.18). A production run was made (actually two runs) entering a program proposed for the San Francisco CRP. The results of this run were compared with a control run which was identical to the "public actions" run but lacked the public actions. This is probably the most reasonable way a model can be employed, given the present state of the art.

* "The Measurement of Housing Quality for Urban Renewal," Francis Howard Hendricks, Thesis submitted to the Graduate Division of the University of California, Berkeley, 1958.

Figure 5.17

NEW CONSTRUCTION
COMPARISON OF INVESTMENT

(Millions of Dollars)

	<u>Actual San Francisco</u>	<u>Simulation Runs</u>
Period 1 (1961-62)	\$121.4	\$121.7
Period 2 (1963-64)	155.7	156.6

Source: Arthur D. Little, Inc.

Figure 5.18

COMPARISON OF NEW UNITS CONSTRUCTED

	San Francisco Actual				Simulation Runs			
	<u>Single Family</u>	<u>2-4</u>	<u>5+</u>	<u>Total</u>	<u>Single Family</u>	<u>2-4</u>	<u>5+</u>	<u>Total</u>
Period 1 (1961-62)	1,415	756	5,891	8,062	428	828	6,886	8,142
Period 2 (1963-64)	855	750	8,240	9,885	798	560	7,830	9,188
Total	2,270	1,546	14,131	17,947	1,226	1,388	14,716	17,330

Source: Arthur D. Little, Inc.

Part of the public action program was a six-year period of housing code enforcement. The control run, without the code enforcement, predicted an increase in substandard units by 1,000, while the public program run predicted a decrease of about 4,000 in the number of substandard units. The model in effect says that the particular code enforcement program would make a difference of about 5,000 housing units, which would be upgraded from substandard to standard. Concomitant with this improvement in quality was a rise in rentals for substandard units, indicating that a code enforcement program should be accompanied by a program to provide low cost housing to low income groups.

The above example is a good illustration of the use of a model, but it is the only one mentioned. Whether other opportunities have occurred is not known. On the basis of their experience, ADL recommends some changes and suggests extensions of the model. The most interesting change proposed is to reduce disaggregation (re-aggregate), not because the data are too hard to get, but because the detail is not especially meaningful. The household categories contain some that are too small to be important and not behaviorally different enough to warrant different treatment. They also say that there are more neighborhoods than necessary. These changes would be important in reducing the time required for a single run.

In the nature of extensions, ADL advocates more research into some of the parameters, such as the rent-demand relationship, yield calculations, and the motivations for single-family housing starts (naturally). Finally, they indulge in the common pipe-dreams of wishing to incorporate more economic sectors, make the population projection and composition endogenous to the model, and expand the model to regional size.

Transportation Studies

The Mainstream

The era of the major transportation studies began in the early fifties and still continues. The first studies, for Detroit and Chicago, are regarded today as routine and mechanical (they are often labeled "ad hoc," apparently analogous to certain committees that lack formal structure), but they served their purposes and provided a base upon which to build more sophisticated methods. They utilized whatever automatic equipment was available at the time, but naturally they were much less computer-oriented than present studies. It is difficult to distinguish the contributions of each particular study, and consequently they have been lumped together into a series. Many of the studies have not only built upon previous studies conceptually, but have also utilized the same personnel, which is even more reason to treat them as a group.

A flow chart that is abstract enough to generalize the method followed by the mainstream studies, yet specific enough to distinguish them from other transportation studies, is not easy to come by, but a first approximation is given in Figure 5.19. Some boxes may be combined in some studies, and the pattern of feedbacks varies, but the flow chart is adequate for discussing the models involved. After the inventory of present facilities, land uses, population, etc., the first step is to project total population and employment for the region into the future as far as the target date for the transportation plan. These regional totals are then divided up among a number of subregions or zones, to produce future land use activity. Trips are considered to be generated by land uses, depending upon their respective characteristics, and the trips are broken down into automobile and public transit trips. This latter breakdown is called modal split, i.e., the decision as to what mode the trip will take. Trip distribution is the process of locating trip ends (formerly called destinations), and

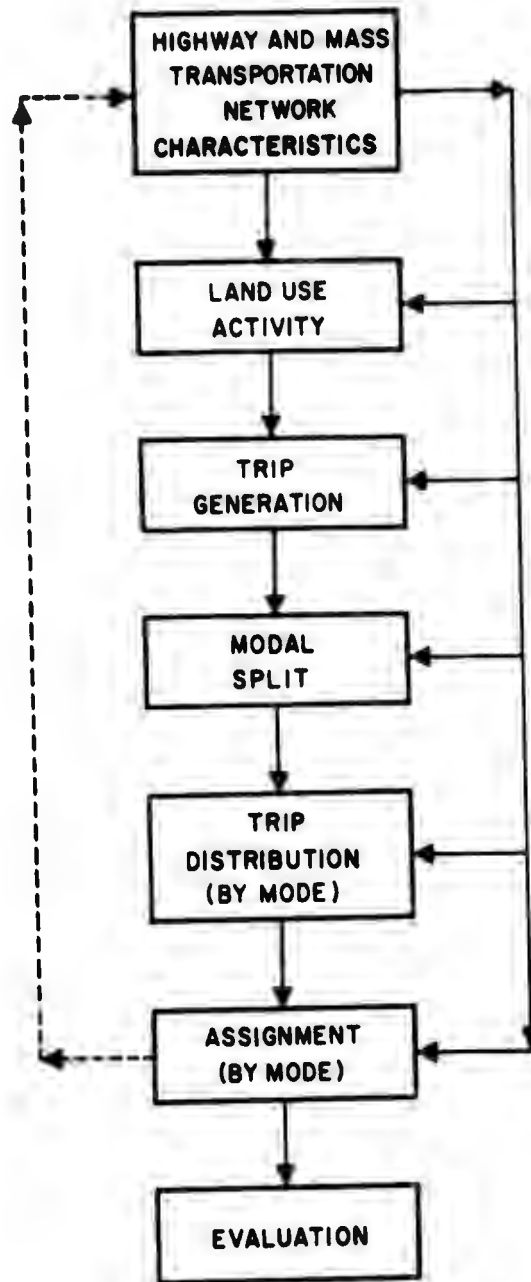


Figure 5.19 SIMPLIFIED CHART OF TRANSPORTATION FLOWS MODEL

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-
finally the trips are assigned to appropriate routes in the transportation plan, which is known as loading the network. If certain links in the network are overloaded by this process, then the network must be redesigned, reflecting a new transportation plan. The costs and benefits of various plans are compared and evaluated in this way.

Described in these terms, the transportation planning process is fairly comprehensive. Predicting future population and employment and then translating these into land uses requires a thorough study of the region in all aspects. In fact, transportation planning has not been quite as comprehensive as it sounds, particularly in the early stages, for a number of reasons. First of all, the process was arrived at backwards, i.e., by asking what information is needed to evaluate a transportation plan. The estimates of future land uses need be only sufficient to estimate trips from, so land use estimates were based on extrapolations of existing trends. More recent studies have incorporated means for testing different land use plans that are combinations of trends and policy decisions, and for measuring the feedback effects of the proposed transportation system on land use.

A second reason for limited comprehensiveness in transportation models is that they are usually aggregative enough so that they may focus entirely upon the growth characteristics of a region and neglect decline and redevelopment. In its simplest form, the land use projection was a matter of filling in the next suburban ring. Since the concern of the plan was new transportation facilities, growth was the subject of primary attention. For this reason, many of the land use projection models associated with transportation studies are referred to as growth allocation models.

The four-step sequence of trip generation, modal split, trip distribution, and assignment has been well developed and refined by the transportation planners, and is reported extensively in the pertinent literature. The methods are descriptive, mathematical, rely heavily on the regularities of aggregate behavior, and give good results. We will not consider them further, but it is comforting to know that the behavior of one component of the urban system of systems can be adequately described.

More variation exists in the other two areas of transportation planning: (1) the population and employment forecast, and (2) the land use forecast. Different studies have approached these problems with differing resources and policies, but generally the methods have become more systematic quantitatively and more elegant conceptually in progressing from the earlier to the later studies. Projecting population is largely a matter of picking which trends to extrapolate, and these may be aggregate trends, trends of population subgroups, or trends in migration rates and fertility rates. Which ones of the many available methods should be used is largely a pragmatic decision since there are many influences on the size and composition of future population other than the past history of the population, and thus a prediction based upon characteristics of the present population is not very reliable.

Economic studies have two purposes: (a) to provide a check on the population forecast, and (b) to provide estimates of future employment by sector. If the population projections and the economic forecasts agree, i.e., the predicted population can be supported with jobs, then the prediction has more weight and can be used for planning purposes. Estimates of future employment by sector can indicate industrial locational requirements, land use demands, trip potential, and even skill and age groups required from the population. Techniques for making economic forecasts range from simple extrapolation through economic base analysis to the more elegant methods of industrial complex, industrial locational advantage, and input-output analyses. This area is now the subject of regional science, and the methodology has been advanced substantially in recent years.

Methods for translating the population and employment estimates into a land use projection range from a reliance on judgment, experience, and a detailed knowledge of the urban area, to descriptive mathematical models. Some of the most ambitious attempts -- Boston, Southeastern Wisconsin, and Penn-Jersey -- are discussed separately in later sections.

Early studies, such as the Chicago Area Transportation Study (CATS), employed the notions of the density gradient and holding capacity. On the basis of existing densities and the amount of vacant land, each analysis zone was assigned a holding capacity, which was the amount of residential population it could contain if it were filled. Present density, as a percent of capacity, could be mapped or plotted in section, as is done in Figure 5.20a. The future capacity-gradient was determined by moving the existing gradient out, away from the CBD, until the future population was absorbed. Figure 5.20b shows this.

Later on, the gravity model was developed for transportation planning, and both the gravity model and its offshoot, the opportunity model, have been and are still used extensively in transportation models. Gravity and opportunity models have a great deal of flexibility and can be used for both land use forecasting and for trip distribution. Multiple regression models often contain gravity-potential variables. Considerable experience exists in the application of these techniques, but they are most successful for large aggregates and for large nodal regions; they do not work as well for smaller population groups, smaller zone sizes, and multi-nodal or truncated regions.

Transportation studies have proceeded step-by-step from relatively simple approaches to more sophisticated studies by using additions and modifications built upon past experience. Reliance has been on descriptive technique rather than behavioral concepts, and techniques that work rather than those which are elegant. The early studies were criticized by urban planners as being simply extrapolations of the status quo when they should have incorporated planning goals. Planners felt that transportation plans should be based upon land use plans, instead of land use projections, to which the transportation planners responded by asking what assurances existed that the plan (and not the projection) would prevail in the future. Although previous land use plans had created notably small impact, the planners countered by saying that the transportation system could affect the land use pattern. The recent transportation studies have built this idea into their models, but at present it is uncertain just how strong a determinant of land use transportation is.

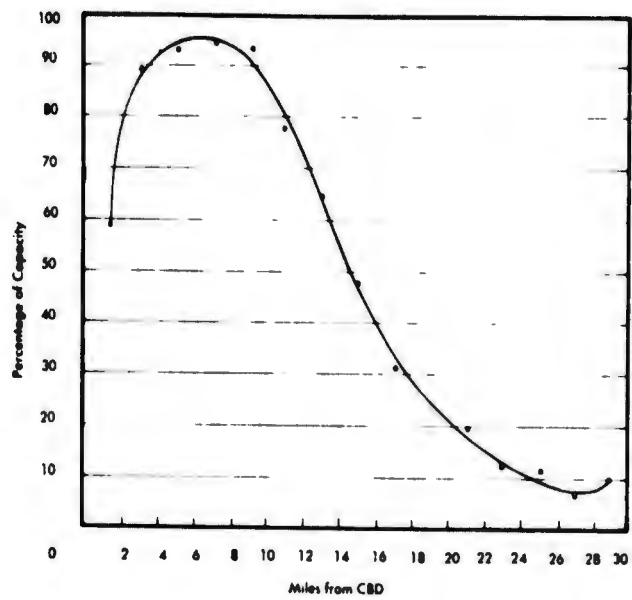


Figure 5.20a 1956 POPULATION AS A PERCENTAGE OF POPULATION CAPACITY

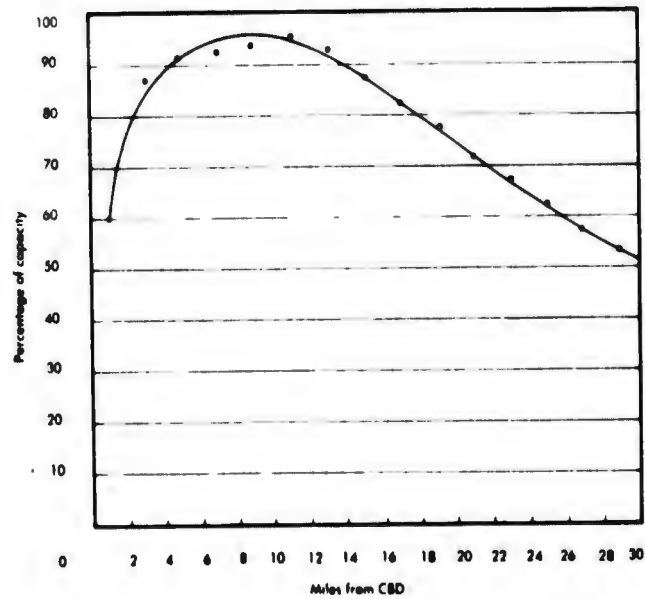


Figure 5.20b ESTIMATED PERCENTAGE OF POPULATION CAPACITY

Source: Chicago Area Transportation Study

A list of the major transportation studies in the United States in the past twenty years is shown in Figure 5.21. The information given is the study's acronym, city, starting date, and some of the study participants who are represented in the literature.

The Penn-Jersey (Delaware Valley) Models

The Penn-Jersey Transportation Study, which began sometime around 1960, was almost as comprehensive in its initial stages as the two CRP models previously discussed were in their initial stages. At some point in its long evolution, the project split into two parts, with the more operational transportation models being carried on by the Delaware Valley Regional Planning Commission, and the more elegant Herbert-Stevens model being carried on by Britton Harris at the University of Pennsylvania. We have chosen three points in this evolution to describe: (1) the Regional Growth Model, which is close to the original design, (2) the Activities Allocation Model, which is the operational part at the present time, and (3) the Herbert-Stevens model.

The Regional Growth Model. The model was to make one iteration every two to five years, which eventually settled at five years. Five groups of land uses were to be distributed during each period: heavy industry, residential, residential-serving activities, other industry, and transportation. The sequence of location is in the order listed. Heavy industries are those basic industries that are large employers and have specific locational requirements such as topography or transportation access. These activities are determined exogenously. Households are located after basic industry, followed by household-serving activities such as retail, personal services and schools. Other industries are those which locate according to neither exogenous influence nor nearness to population, e.g., wholesaling. Finally, a transportation network is designed to serve the distribution of activities, and which will presumably influence location in the next iteration.

Acronym	Place	Date Started	Authors
DATS	Detroit	1953	
MTS	Washington, D. C.	1955	
CATS	Chicago	1956	Carroll, Creighton, Hamburg
PATS	Pittsburgh (Now SPRPC: Southwestern Pennsylvania Regional Planning Commission)	1958	Carroll
Penn- Jersey	Philadelphia (Now Delaware Valley Regional Planning Commission)	1959	Herbert, Stevens, Harris
LARTS	Los Angeles	1959	
Puget Sound	Seattle- Tacoma	1960	
TCATS	Minneapolis- St. Paul	1960	
UNYTS	Buffalo (Formerly Upstate New York Transportation Study; now Department of Transportation)	1961	Creighton, Hamburg
Tri- State	New York City	1961	
Boston	Boston	1962	Hill, Brand, Hansen
SENRPC	Milwaukee	1962	Schlager
	Baltimore		Lakshmanan, Voorhees, Hansen
	Connecticut		Voorhees
BARTS	San Francisco- Oakland (Rapid transit only)		

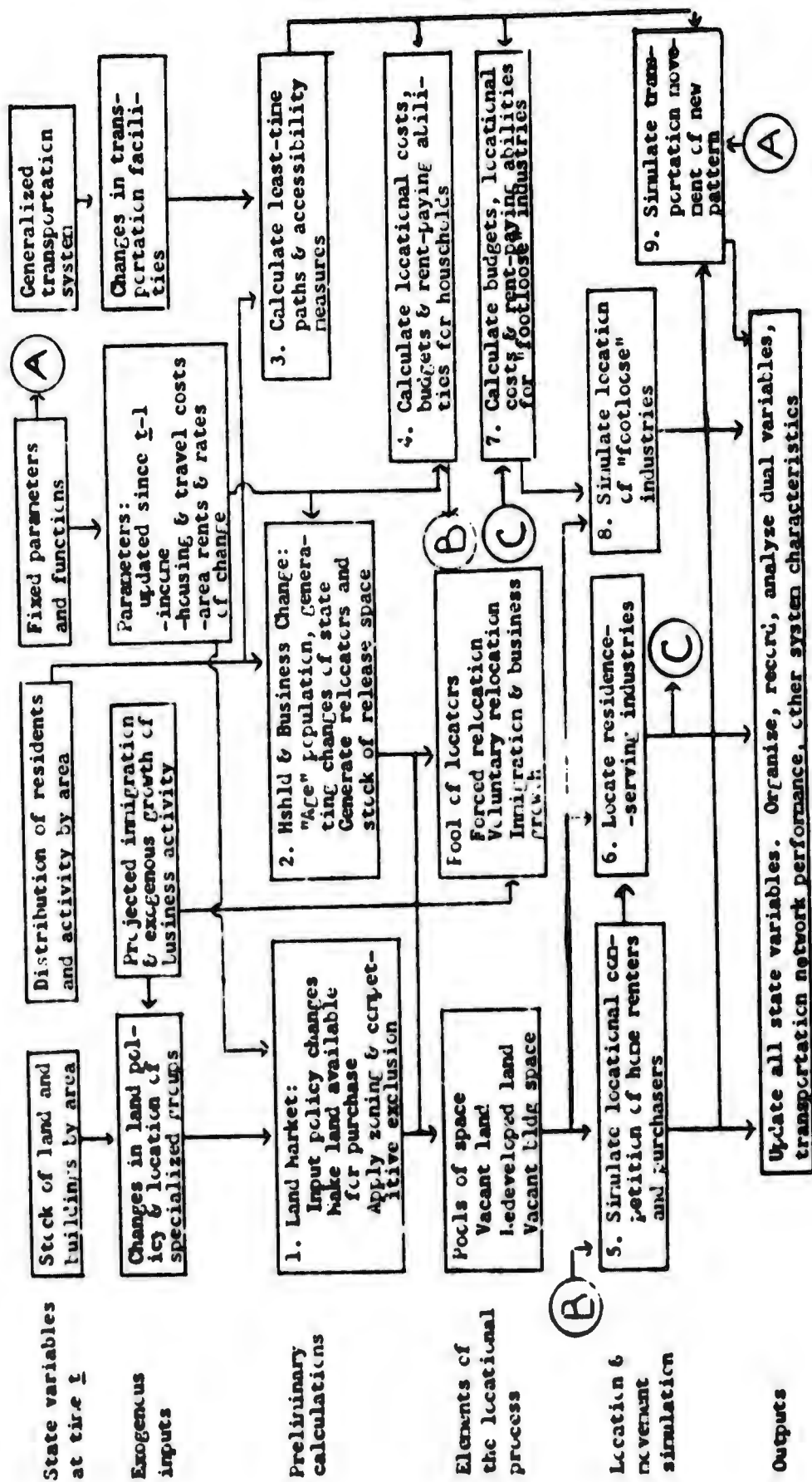
Figure 5.21
MAJOR TRANSPORTATION STUDIES

This sequence is similar to other urban models with one major difference. Most other studies have run into difficulty in dividing nonresidential uses into exogenously-located and population-oriented, with marginal cases being assigned by ad hoc procedures. Penn Jersey proposed to avoid this by using linear programming for most of their land use allocation. This meant that households and industries would compete in their respective markets for the sites they wanted, with decisions being based on optimizing some common reference criterion. The scale contemplated for this seemed to be economic rent, which was made explicit in the Herbert-Stevens model. While linear programming was retained for a considerable length of time in the residential location model, it seems to have been dropped fairly early for any other purposes.

A flow chart for the Regional Growth Model is shown in Figure 5.22. The Herbert-Stevens residential model is labelled 5. Block No. 2 is a routine that advances population cohorts in time, moving households from one type, income level, stage in the family cycle, etc., to another, and perhaps from one place to another. These changes then have effects on families being located and land available. Locators were to be broken also into renters and owners. Residences would generate a rent surface in the process of locating, and these rents would be used to locate population-serving activities and then the residual category of industry (called "footloose").

The Activities Allocation Model. It is a little ironic that the final result of a model that originally thought of itself as normative and behavioral should be a model that is unabashedly descriptive. The Activities Allocation Model is a set of seven submodels that are run in sequence for each five-year period. All of these models are combinations of single-equation linear and non-linear regression, and ratio or "index" allocations. The seven submodels are

- (1) The residential location model, which assigns population to districts by four income classes.



Source: Almendinger [338], p.16

Figure 5.22 FLOW CHART: ONE ITERATION OF THE REGIONAL GROWTH AND TRANSPORTATION MODEL

(2) The residential space consumption model, which assigns land to households.

(3) The manufacturing location model, which locates two categories of employment.

(4) The nonmanufacturing location model, which locates thirteen categories of employment.

(5) The manufacturing space consumption model.

(6) The nonmanufacturing space consumption model.

(7) A street area model, which computes the amount of land that will be devoted to streets and highways.

To call this set of allocation procedures a model is perhaps stretching the definition only slightly. As Seidman [417] points out, the projections obtained from the model "are probably about as good as projections obtainable from the best judgments of planners in the region." (p. 121) The model certainly lacks the glamour of the earlier Penn Jersey design, but it works and produces usable results. Two conclusions emerged from the study that are reinforced by similar results elsewhere. One is that the transportation system does not have a very substantial effect on the distribution of land uses. Under several widely varying policy assumptions the spatial response to changes in accessibility were on the order of five per cent or less at the district level. Secondly, the model predicted substantial outmigrations from central cities in the region. These two results are, of course, conclusions of the model structure and not necessarily of the real world, but if they are in fact model faults they would seem to be general to several kinds of models.

The Herbert-Stevens Model. Although first published in 1960, the Herbert and Stevens [374] residential location model designed for the Penn-Jersey Transportation Study never became operational. No doubt there were a number

of factors that led to its demise, but it would seem likely that the data problem was a major contributor. Despite the fact that the model was abandoned, there are several reasons why it is worth looking at. First of all, of course, we are interested in any attempt at an operational urban model; those that are unsuccessful provide information of a different kind. Secondly, the Herbert-Stevens model is a unique attempt at formulating the residential location problem as a linear program, which makes the model qualitatively different from others that will be reviewed.

For any given iterative time period, the model starts with the distribution of activities existing at the beginning of the time period, plus forecasts of the following information:

- a. Locations of "heavy" or basic industry that will begin operation during the period.
 - b. The number of households, by type, that will be locating during the period.
 - c. Desires of households, by type, for different "residential bundles."
 - d. The budget each household type has available for residential purposes.
 - e. The costs of providing each residential bundle, except for site rent (or annual cost of ownership).
 - f. The land available for residential use, for the period.
- The residential bundle has four components for each household: a housing unit, a travel pattern, an amenity level, and a site. The costs of providing these components will depend upon location (the desires do not), and it is assumed that the first three components of the bundle can be determined exogenously.

The difference between the residential budget, of each household, and the costs of the three components, at each site, is the amount of income that the household can spend for a site at the particular location. The output of the model is the number of households, by type, located in each subarea, and the site rent for the subarea.

The rationale for the linear programming formulation is based primarily upon economic theory. The quantity to be maximized is called "aggregate rent-paying ability," and is the total amount of income that can be spent for residential sites. Households compete with each other for the available sites, and the household (household group, actually) that can offer the highest price for the land, per unit area, gets the site. In mathematical form,

$$\text{maximize } \sum_{i=1}^n \sum_{K=1}^U x_i^K (b_i^K - c_i^K)$$

subject to:

$$\sum_{i=1}^n s_i^K x_i^K \leq L^K \quad K = 1, 2, \dots, U$$

$$\sum_{K=1}^U x_i^K = N_i \quad i = 1, 2, \dots, n$$

$$\text{and all } x_i^K \geq 0$$

where n = number of household types
 U = number of areal units
 b_i^K = residential budget of household type i in area K .
 c_i^K = cost of residential bundle, excluding site rent, for household type i in area K .
 s_i^K = units of land consumed by a type i household if it locates in area K .
 L^K = units of land available in area K .
 N_i = number of type i households to be located.
 x_i^K = number of type i households located in area K .

Only the x_i^K 's are variables; all the other letters represent known numbers at the beginning of any iterative period. The objective function is total rent-paying ability, and would be maximized (without constraints) by assigning all households with positive rent-paying ability to those areas where the difference between budget and cost was the greatest. The constraints require that the land consumed in each area not exceed the amount available, and that all households be located. There is one constraint of the first type for each area, and one of the second type for each household type.

Any linear programming problem has another problem associated with it, called its dual, and in this case the discussion of the model is greatly facilitated by reference to the dual. The dual, then, of the problem stated above is

$$\begin{aligned} &\text{minimize} \quad \sum_{K=1}^U r^K L^K + \sum_{i=1}^n v_i (-N_i) \\ &\text{subject to:} \\ &\quad s_i^K r^K - v_i \geq b_i^K - c_i^K \quad K=1, 2, \dots, U \\ &\quad \text{all } r^K \geq 0 \end{aligned}$$

where r^K = rent per unit of land in area K.

v_i = "subsidy" for households of type i.

Since the dual problem is a mathematical fact rather than an intellectual construct, interpretation of the dual can be awkward. Ignoring the dual variable labelled "subsidy," the other dual variable can be interpreted using the economic notion of imputing a value to a scarce resource. In this case, the resource is land and a price is to be assigned to it that will make the total value of resources the smallest yet still keep the value of the land at least as great as the rent-paying ability of households.

The linear programming framework can provide many conclusions about the solution without actually computing it. A household, for example, should have a choice of several residential bundles in any area, but the linear program will always pick the bundle which yields the highest rent-paying ability per unit of land.* Thus all choices save one can be eliminated for each household type in each area, and this is the reason that there are no summations over residential bundles (i.e., housing types).

Another characteristic of the manner in which a linear program seeks a solution is that once it is decided that a particular household type can bid the highest for a particular area, households will be assigned to the area until either (a) there are no more households of the type, or (b) there is no more space in the area. At one point PJ was talking in terms of 1000 areas and 20 household types, which means that at least 980 areas would each be filled by

* This statement, implied by Herbert and Stevens, is not true for at least two reasons. If a household is willing to pay more per unit of land for larger sites, then it is possible that aggregate rent-paying ability could be increased by crowding the household type into its second highest bid-rent sites. Even without this assumption, aggregate rent-paying ability might be increased by allowing some households to pay more total rent (less per unit) by spreading out onto vacant land. Harris [370] mentions some modifications to the model, of which one is to include a dummy household type (perhaps agriculture) that is always available as an alternate use, to take up the slack. By pricing the alternate high enough, households are prevented from spreading out too thinly.

single household type (recall, however, that we are only considering a single iteration and not all land in an area is available).

For areas in which the program allocates more than one household type, the dual program provides the insights. Since the rent variable is specific to an area, only one rent can be charged; yet the household group assigned second to the area has a lower rent-paying ability than the first. A mathematical theorem about primal-dual relationships says that for any activity operated at a nonzero level in the primal (e.g., $x_i^K \neq 0$) the associated constraint in the dual will be an equality. Thus the difference between the rent-paying abilities of two household types in the same area must be made up by the v_i , or subsidy, variable. In this sense, the dual subsidy variables are "slack" variables.

Another dual theorem says that dual variables associated with equalities in the primal are unconstrained in sign, and so the v_i 's can be either positive or negative. If some households are required to locate in areas in which they have a negative rent-paying ability, then the v_i would be positive (a subsidy). On the other hand, if a household group failed to completely fill an area, and there were no other households that needed locating, the economic rent for the area would necessarily be zero. The v_i would then be negative, and could be regarded as a tax on the households which would be transferred to the landowner to compensate for his lack of rent. The sign of the subsidy variable is arbitrary, i.e., positive values could be tax and negative values subsidy. Herbert and Stevens decided that they wanted positive values to be subsidies, and this is why the second set of constraints in the primal employ minus signs. They could be removed without affecting the problem.

As a final note on the subsidies, it should be pointed out that while rents are specific to areas, subsidies are specific to household types. Thus if a household group is subsidized in one area, it receives the same subsidy in all areas, which increases the overall rent-paying ability of the group.

Any other group which locates in the same area with the first group will then receive a subsidy to allow it to compete with the first group, unsubsidized, plus the first group's subsidy. In general, only one group will receive no subsidy, and all other groups will receive increasingly higher subsidies as their rent-paying ability decreases. This might be socially desirable were it not for two facts: First, low-income groups may have the highest rent-paying ability due to a willingness to crowd together, while the wealthy get the subsidies, and second, the subsidies are not necessarily paid, which means that the increased rents come from the household group's income. The subsidies, however, are an arbitrary construct, and result from the discrete nature of the areal units. Areas of double occupancy could be broken into two areas, and different rents charged in each one, which would eliminate the hierarchical characteristic of the subsidies.

In the above discussion it is assumed that in any run of the linear program there is more land available than required by the households. If there were less available than the minimum requirements, the program would not have a feasible solution. Although the space consumed by a household is fixed for any specific combination of type and area, the space-per-household can vary from area to area. Hence there would be some space constraint sets that would allow some configurations and not others. What sort of degeneracy this might lead to, we cannot say, but in the course of solution certain variables would be entered and then removed (households assigned to an area and then taken out). Another form of degeneracy arises because the matrix by which the problem can be represented is very sparse (many zeroes). According to Harris [369], the problem can be formulated as a transportation problem, which allows for a much more efficient solution than the simplex method.

Herbert and Stevens mention how the data might be acquired. The travel pattern can be derived from a typical transportation study traffic assignment model, and in this way the effects of access and the transportation system are incorporated. Data on household preferences and costs of amenity levels are more obscure. Since no results have ever come out of the model,

we do not know what kinds of interesting insights might have been obtained. The discussion about whether the model is normative or not, and whether this is good or bad, seems less important now than initially. In a nutshell, if it is felt that maximum rent-paying ability is good, then the model is normative; if one feels that households actually seek to maximize this quantity, then the model is behavioral or predictive.

Professor Harris and a small group are continuing work on the model. One test, for which data are being gathered, is on the New York City region under the auspices of the Regional Plan Association.

Boston

The only portion of the Boston Regional Planning Project model that seems to have received any exposure in the literature is the land use forecasting model. It is described as a growth allocation model, in that it takes forecasts of population and employment for the region as a whole and allocates these totals to a number of subregions. Two formats were developed for the model, one called "Empiric" and the other called "Polimetric," but only the empiric model will be described since it was the one used. The Polimetric model is a continuous-time differential equation framework, and is described in the references; the Empiric model is an econometric model that can be calibrated for different discrete time periods usually five years.

An outstanding characteristic of the model is the extent to which it is descriptive. It manages to squeeze maximum use out of the linear extrapolation of existing trends, which is a simple notion superficially but highly sophisticated in this application. Given the somewhat limited problem of allocating short-run growth and decline, this model seems to be very effective.

Five variables are to be allocated, and thus these are the dependent variables in the regressions. They are called "located" variables in the model.

1. White collar population
2. Blue collar population
3. Retail plus wholesale employment
4. Manufacturing employment (covered by the Massachusetts Division of Employment Security).
5. Other employment (including employment not covered by MPES). A maximum of 22 independent variables was used and these are grouped into three nested sets, as shown in Figure 5.23.

I: land use densities, zoning, and automobile accessibility

II: set I plus transit accessibilities

III: set II plus quality of water and sewage disposal services

These are called "locator" variables. A considerable amount of pre-testing of the interrelationships between the variables was done before the model was fitted, and the techniques used included graphic plots and factor analysis. The data were tabulated for 29 subareas of the Boston region, which can be seen in Figure 5.24.

The Empiric model is a system of linear equations of the form

$$\Delta R_i = \sum_{\substack{j=1 \\ j \neq i}}^N a_{ij} \Delta R_j + \sum_{k=1}^M b_{ik} (Z_k \text{ or } \Delta Z_k)$$

where ΔR_i = change in i^{th} located variable in one time period

Z_k = level of k^{th} locator variable at beginning of time period

ΔZ_k = change in k^{th} locator variable in one time period

a_{ij} and b_{ik} are regression coefficients on the variables, N is the number of dependent variables (5 in this case) and M is the number of independent variables (12, 18, and 22).

LOCATER VARIABLES FOR CALIBRATION OF EMPIRIC MODEL

No.	Name	Symbol of Normalized Variable
(a) Application I		
1	Difference between standardized ^a residential population density (at practical holding capacity) and gross residential population density at time t (serves as a zoning variable).	$\frac{STN}{L} \frac{PLR}{\sum PLR(t)} - \frac{PLR(t)}{\sum PLR(t)}$
2	Difference between standardized manufacturing employment density and the gross density at time t.	$\frac{STN}{L} \frac{MLR}{\sum MLR(t)} - \frac{MLR(t)}{\sum MLR(t)}$
3	Difference between standardized non-manufacturing employment density and the gross density at time t.	$\frac{STN}{L} \frac{NMLR}{\sum NMLR(t)} - \frac{NMLR(t)}{\sum NMLR(t)}$
4	Value of subregional gross residential population density at time t.	$\frac{1}{L} - \frac{PLR(t)}{\sum PLR(t)}$
5	Value of subregional gross manufacturing employment density at time t.	$\frac{1}{L} - \frac{MLR(t)}{\sum MLR(t)}$
6	Value of subregional gross non-manufacturing employment density at time t.	$\frac{1}{L} - \frac{NMLR(t)}{\sum NMLR(t)}$
7	Change over the time period in automobile accessibility to white collar residential population at time t.	$\frac{AV(t+1)PW(t)}{\sum AV(t+1)PW(t)} - \frac{AV(t)PW(t)}{\sum AV(t)PW(t)}$
8	Change over the time period in automobile accessibility to blue collar residential population at time t.	$\frac{AV(t+1)PB(t)}{\sum AV(t+1)PB(t)} - \frac{AV(t)PB(t)}{\sum AV(t)PB(t)}$
9	Change over the time period in automobile accessibility to total employment at time t (TE(t)).	$\frac{AV(t+1)TE(t)}{\sum AV(t+1)TE(t)} - \frac{AV(t)TE(t)}{\sum AV(t)TE(t)}$
10	Value of subregional automobile accessibility at end of period (t+1) to white collar residential population at time t.	$\frac{1}{L} - \frac{AV(t+1)PW(t)}{\sum AV(t+1)PW(t)}$
11	Value of subregional automobile accessibility at end of time period (t+1) to blue collar residential population at time t.	$\frac{1}{L} - \frac{AV(t+1)PB(t)}{\sum AV(t+1)PB(t)}$
12	Value of subregional automobile accessibility at end of period (t+1) to total employment at time t.	$\frac{1}{L} - \frac{AV(t+1)TE(t)}{\sum AV(t+1)TE(t)}$
(b) Application II		
1-12	Same as above.	Same as above.
13	Change over the time period in transit accessibility to white collar residential population at time t.	$\frac{AQ(t+1)PW(t)}{\sum AQ(t+1)PW(t)} - \frac{AQ(t)PW(t)}{\sum AQ(t)PW(t)}$
14	Change over the time period in transit accessibility to blue collar residential population at time t.	$\frac{AQ(t+1)PB(t)}{\sum AQ(t+1)PB(t)} - \frac{AQ(t)PB(t)}{\sum AQ(t)PB(t)}$
15	Change over the time period in transit accessibility to total employment at time t.	$\frac{AQ(t+1)TE(t)}{\sum AQ(t+1)TE(t)} - \frac{AQ(t)TE(t)}{\sum AQ(t)TE(t)}$
16	Value of subregional transit accessibility at end of time period (t+1) to white collar residential population at time t.	$\frac{1}{L} - \frac{AQ(t+1)PW(t)}{\sum AQ(t+1)PW(t)}$
17	Value of subregional transit accessibility at end of time period (t+1) to blue collar residential population at time t.	$\frac{1}{L} - \frac{AQ(t+1)PB(t)}{\sum AQ(t+1)PB(t)}$
18	Value of subregional transit accessibility at end of period (t+1) to total employment at time t.	$\frac{1}{L} - \frac{AQ(t+1)TE(t)}{\sum AQ(t+1)TE(t)}$
(c) Application III		
1-18	Same as above.	Same as above.
19	Change in quality of water supply over the time period.	$\frac{W(t+1)}{\sum W(t+1)} - \frac{W(t)}{\sum W(t)}$
20	Change in quality of sewage disposal service over the time period.	$\frac{S(t+1)}{\sum S(t+1)} - \frac{S(t)}{\sum S(t)}$
21	Value of subregional quality of water supply at time t.	$\frac{1}{L} - \frac{W(t)}{\sum W(t)}$
22	Value of subregional quality of sewage disposal service at time t.	$\frac{1}{L} - \frac{S(t)}{\sum S(t)}$

^aThis is the maximum net density of each pertinent activity in each subregion observed over the calibration period.

Source: Hill, Brand and Hansen [377]

Figure 5.23 LOCATER VARIABLES FOR CALIBRATION OF EMPIRIC MODEL

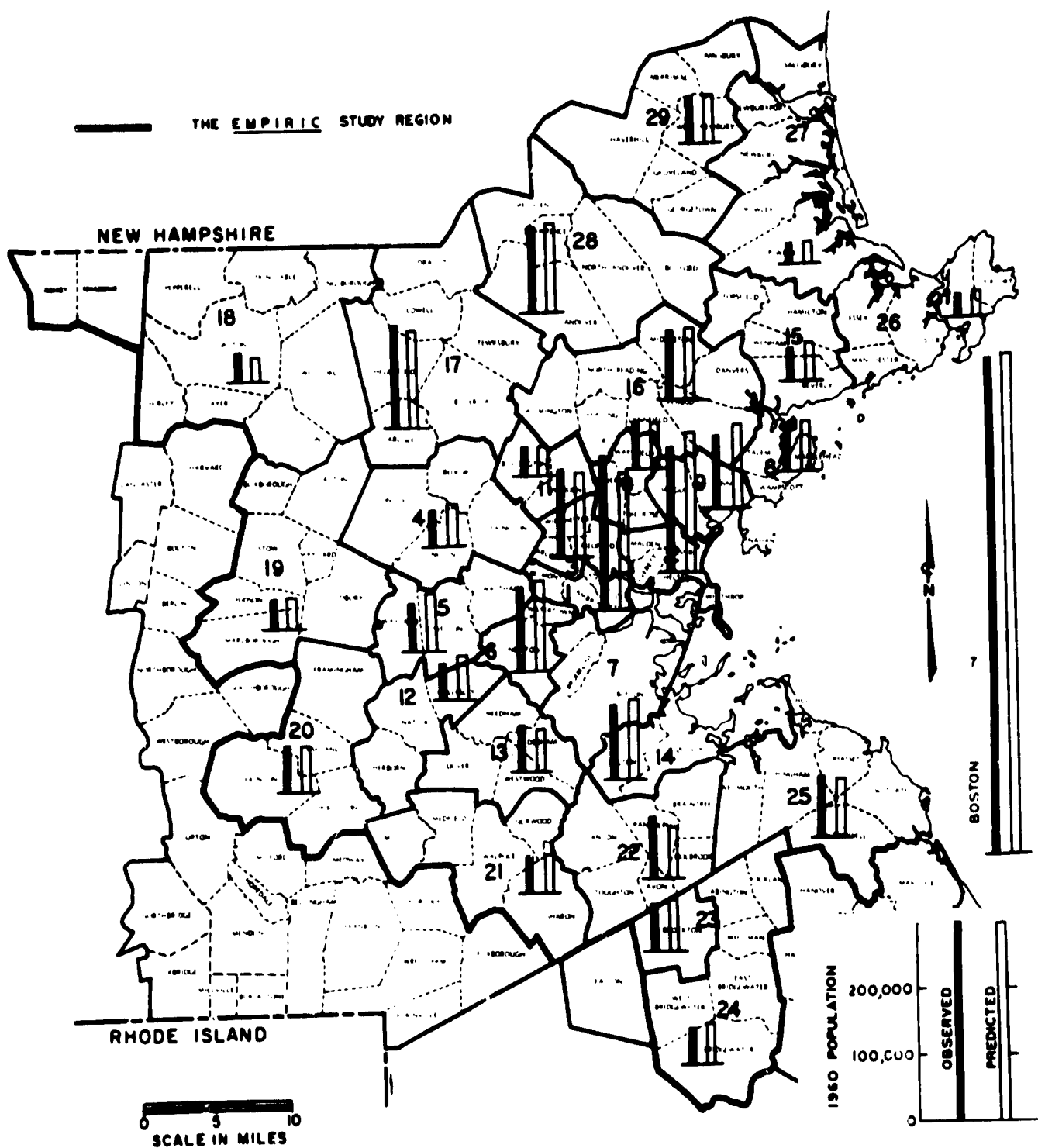


Figure 5.24 COMPARISON OF OBSERVED 1960 POPULATION LEVELS WITH THOSE PREDICTED BY THE EMPIRIC LAND USE MODEL

Source: Hill [375] p. 115

Including dependent variables on the right-hand sides of the equations (the ΔR_j 's) makes the approach somewhat novel for urban models but is a technique commonly used in econometrics. Were these variables not there, the equations could be fitted independently by ordinary least squares means. Leaving the dependent variables in all the equations requires solution by some simultaneous method, which determines the values of the coefficients all at the same time. The method selected is known as simultaneous least squares, and requires that N^2 (=25) of the coefficients be predetermined (the pattern must also meet certain requirements). The Traffic Research Corporation (TRC) chose to make all the b_{ii} 's equal to one, and $N(N-1)$ of the other b_{ij} 's equal to zero. Presumably these were selected on the basis of hypothesized or observed relationships.

Data from two time periods (1950 and 1960, here) were used to determine the coefficients, and then the values of the locator variables for each subregion can be inserted and the system of equations solved for the located variables for the subregion. The results of this process were quite successful for the Boston region, as seen in Figures 5.24 and 5.25. RMS error is the standard error of the estimate (root mean squared error), and RMS error ratio is the RMS error divided by the average (subregion) value of the located variable. R^2 is the coefficient of determination, the square of the multiple correlation coefficient.

These results must be interpreted with caution for several reasons. First of all, the final results were totals for the locator variables for each of the subregions for 1960, but the totals for 1950 were part of the input, and there is a high serial correlation between the values for 1950 and 1960. If someone were to take the values for 1950 and multiply them by a factor such that the totals would add to the 1960 totals, the correlation with the 1960 subarea values would be very high. Yet we would not call this a very sophisticated or subtle method. In essence this is what TRC has done, but in a highly refined way. Another contributing factor is that many of the independent variables used represented actual changes from 1950 to 1960, and may reflect changes in the dependent variables that had already occurred. None of this is meant to imply that the model is not a good one; we simply point out that the criteria presented for evaluating the model are irrelevant.

Summary Reliability Measures				
Model Application	Output Variable	RMS Error (people or jobs)	RMS Error Ratio	R ²
II	Total Population	3439	0.03	0.99
	White collar	3031	0.06	0.99
	Blue collar	1686	0.03	0.99
	Total Employment	1512	0.03	0.99
	Retail & wholesale	564	0.06	0.99
	Manufacturing	1128	0.08	0.99
	Other	955	0.05	0.99
I	Population	7825	0.07	0.99
	Employment	3631	0.08	0.99
III	Population	1259	0.01	0.99
	Employment	1306	0.03	0.99

Source: Hill [375]

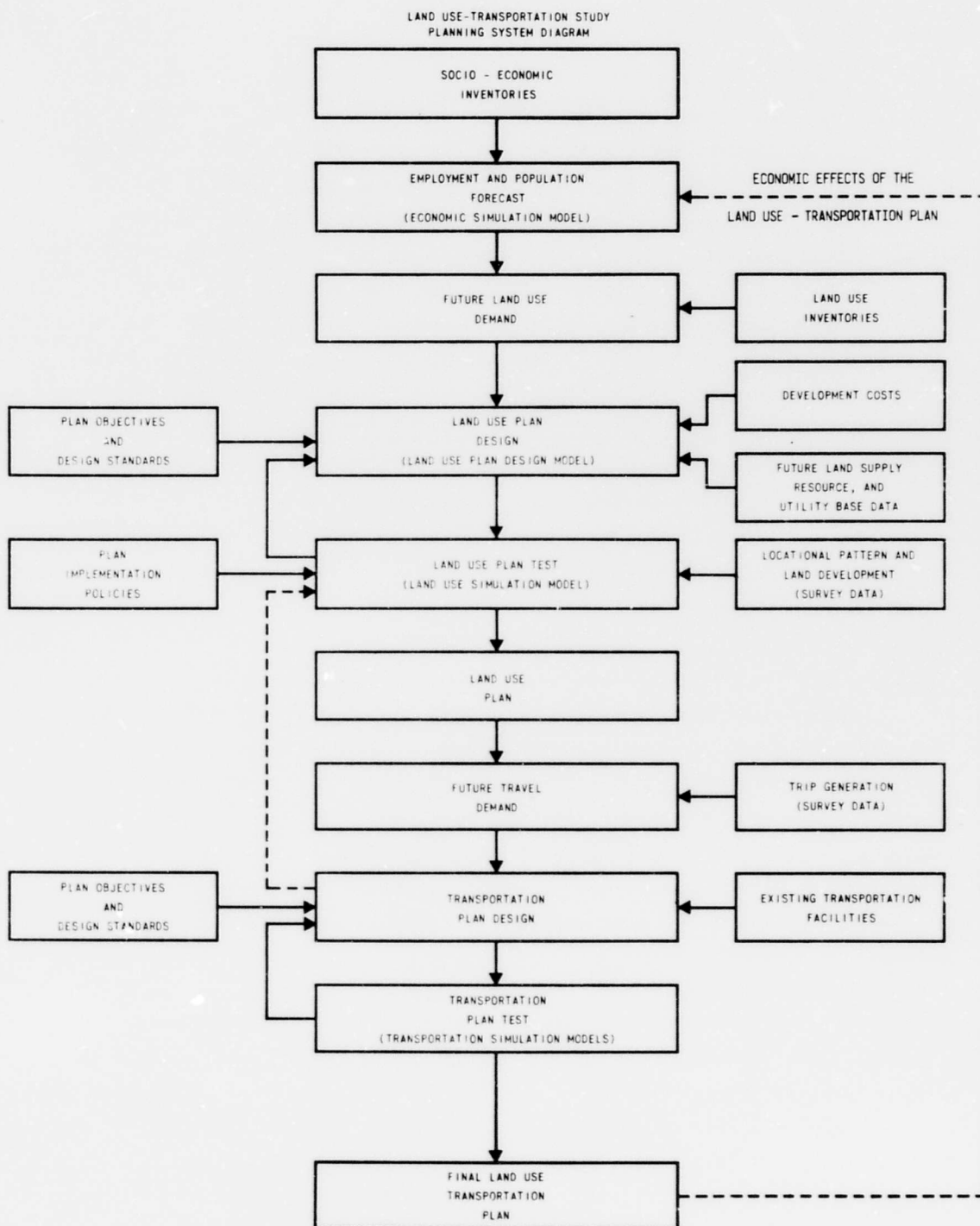
Figure 5.25 SUMMARY RELIABILITY MEASURES

The study is an example of what can be accomplished with highly descriptive techniques, sound statistical background, and thorough workmanship. Alternatives and hypotheses seem to have been methodically investigated at each step in the proceedings, and the results can hardly be questioned on methodological grounds. That the approach is so descriptive is perhaps an indication that this is the kind of effort that should be explored more deeply before clamoring for behavioral and normative relationships. The access measures used have not been described here, but are similar to those used in other transportation studies. The model was tested in a number of ways, including a 1950-1955 projection, a series of longer range projections comparing different time period calibrations, and a set of projections using greater disaggregations of subregions. The tests seem to confirm, as much as possible and more so than usually attempted, the validity of the model.

Southeastern Wisconsin

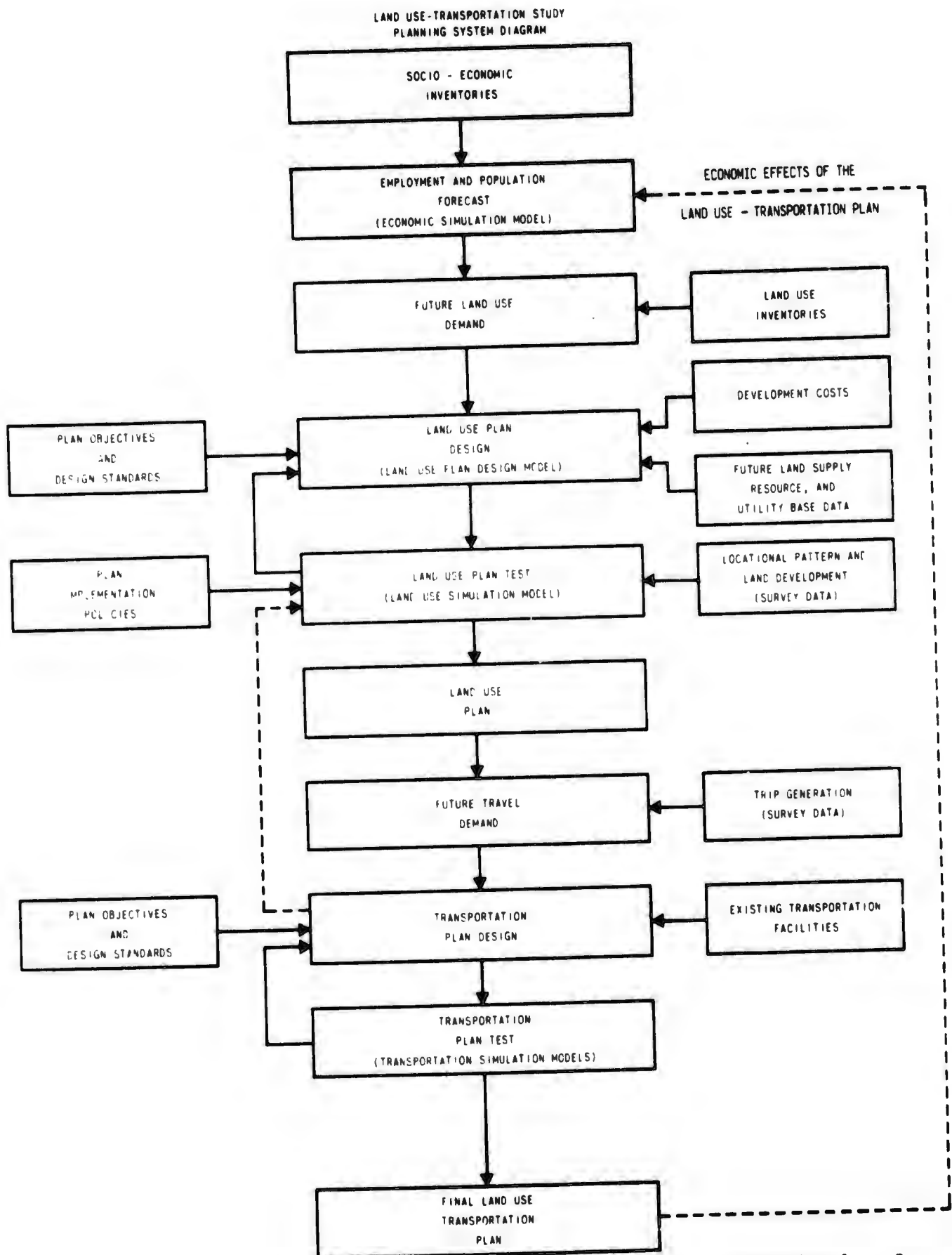
The Wisconsin model seems to be the most comprehensive attempt that shows signs of producing usable results, from two points of view. It includes the largest proportion of endogenous variables relative to exogenous variables (population age cohorts and capital investment, for example), and it is the program most strongly oriented towards policy variables. Although explicitly a transportation study, it is sponsored by the Southeastern Wisconsin Regional Planning Commission (SEWRPC) and to some extent this may explain the comprehensiveness of the approach.

Figure 5.26 shows the flow chart for the study. The economic simulation model forecasts employment and population in sufficient detail to provide input to the land use and transportation models. The land use plan design model constructs a land use plan based on demand for various uses, supply of land, and plan objectives and design standards, which act as constraints. The optimal plan is then tested by the land use simulation model, which implements the plan and determines its cost. These two stages are repeated



Source: SEWRPC [423], p.5

Figure 5.26 LAND-USE TRANSPORTATION STUDY PLANNING SYSTEM DIAGRAM



Source: SEWRPC [423], p.5

Figure 5.26 LAND-USE TRANSPORTATION STUDY PLANNING SYSTEM DIAGRAM

until a satisfactory plan is achieved, and then a transportation plan is designed to serve the land use plan. The combination land use-transportation plan is then tested by the transportation simulation models and the plans modified according to the results of the tests. Eventually, after whatever feedback is necessary, a final land use and transportation plan emerges.

Three major submodels are of interest in this framework:

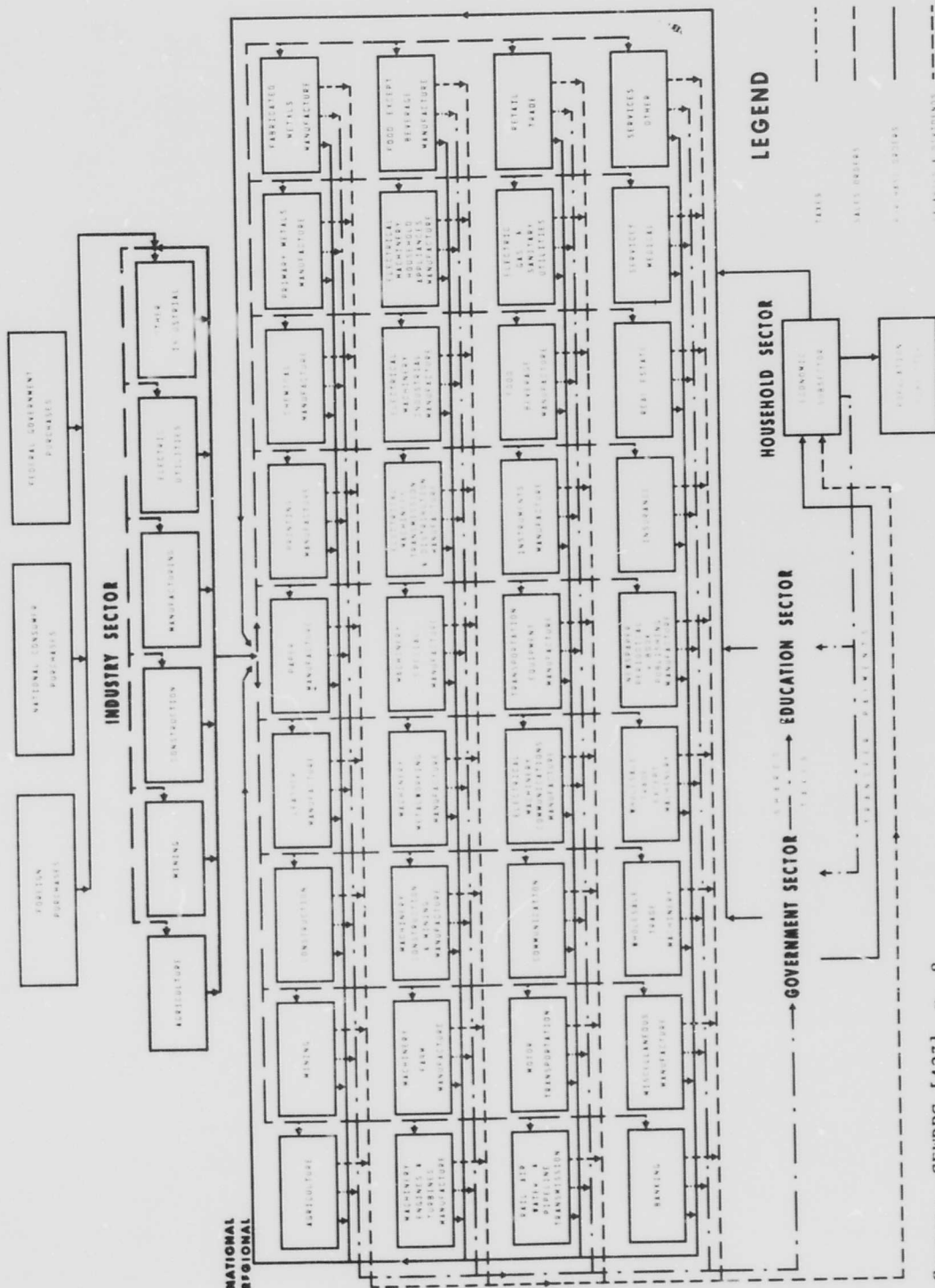
- (1) Economic Simulation Model
- (2) Land Use Plan Design Model
- (3) Land Use Simulation Model

The transportation simulation model does not seem to have been developed yet, but most likely it will follow current practice as exhibited in the mainstream transportation studies.

The economic simulation model is built on the dynamic input-output framework. Local industries are disaggregated into 30 sectors, with purchases on both current and capital accounts, which introduces the dynamic feature. Two inter-industry coefficient tables are required, one for each of current and capital purchases. Local households are included in the inter-industry tables as a 31st sector rather than as final demand, and local government is partially included. In addition, six sectors of national industry -- those that are major customers of regional industries -- are also appended to the table for the purpose of allocating national final demand for regional products among local industries. Final demand is completed with national consumption by households of local goods, federal government demand, and a foreign sector, plus local government demand for local capital goods. This last sector is considered to contain a set of important policy variables. The numbers of employees required in each sector is scaled from total output of the sector.

Figure 5.27 shows the flow chart published by SEWRPC, with the various interrelationships. Figure 5.28 is an attempt to place the model in standard input-output form. The equations for the model will not be presented in

REGIONAL ECONOMIC SIMULATION MODEL



Source: SEWRPC [423], p. 8

Figure 5.27 REGIONAL ECONOMIC SIMULATION MODEL

		current		HH		capital		HH		local government		national sectors		national HH		national government		foreign		total output	
HH	local Govt	wages and salaries (labor)		(a)		interest and dividends		investment by local households in housing		purchase of capital goods by govt		30x6									
		taxes (services)		(b)																	
		30x30		local consumption		purchase of local products		30x30													

any detail, but they are all linear and typically are products of coefficients and subtotals summed over a sector. For example, a "balance" equation for a local industry is given by

$$S_k = \sum_i F_{ki} P_i + \sum_i FC_{ki} C_i$$

where S_k is gross sales (output), P_i and C_i are current and capital purchases of industry sector i , respectively, and F_{ki} and FC_{ki} are input-output coefficients for current and capital purchases. These equations run horizontally across Figure 5.28 (output), but employ column totals in the summations (since purchases represent inputs). However, since there is no need for inputs to equal outputs in a dynamic input-output, the current purchases are scaled from total sales and the capital purchases are a function of sales, plant capacity, time, or some combination. For example, one possibility is

$$P_k = RFG_k S_k$$

and

$$C_k = COUT_k (S_k(t) - S_k(t-1))/TC_k$$

where RFG is the ratio of current purchases to sales in the sector, $COUT$ is the capital-output ratio, and TC is called the capital gestation time lag.

Although only sketched out roughly above, the economic simulation model is very interesting in form and apparently workable. The data requirements are prodigious, and these plus a number of the data reduction programs are described in the technical report. The model should presently be bearing fruit.

The land use plan design model finds the distribution of land uses that can be achieved at the least cost and still meet a set of constraints on the minimum sufficient quantities of each type, the maximum densities

or quantities for each analysis zone, and whatever relationships between land uses that can be specified in a general way. These constraints allow the designer to explore systematically the consequences of a number of design standards or policies, and as such represents something of an innovation. The plan is expressed by three sets of variables,

- (1) Type of land use (quality variables)
- (2) Density of land use (quantity variables)
- (3) Location

Location is described by using grid zones. Design requirements are stated in terms of the above variables and fall into two classes,

1. Within-zone requirements, such as maximum use or density (flood plain zoning, low density residential) or the prevention of the simultaneous development by two uses (residential and industrial).

2. Between-zone requirements, such as the provision of a school or shopping center within certain distances of residences.

The plan which meets all the requirements at least public and private cost is the one selected. In general, increasing the number of constraints or design standards can be expected to increase the total cost of the plan.

Two model frameworks are suggested for implementing the plan design model, and these are linear programming and dynamic programming. Linear programming requires linear constraints and relationships, and a linear objective function. These restrictions can be circumvented by using linear approximations for a nonlinear function, in some cases, but the computational costs are high if there are many of these nonlinearities. While linear programming has a well-developed body of technique, it is still somewhat restricted for general problems. Dynamic programming, on the other hand, is more flexible and less efficient computationally. A different algorithm is usually developed for each problem, and in some situations the solution is very awkward and time-consuming. SEWRPC developed the model first as a linear program, but discovered that indivisibilities due to the discrete nature of the problem required integer programming. Since integer programming is not computationally feasible for large systems, the linear programming approach was abandoned in favor of a dynamic program.

Finally, the land use simulation model determines whether the land use plan can be realized in the real world with the policy variables that are presently available. Five classes of land use are considered,

1. Residential. The residential market model is described below.
2. Industrial. At present the industrial sector is allocated to locations by means of the land use plan, with the possibility of a future endogenous industrial location model.
3. Services. These land uses include streets and schools and are scaled from primary land uses, such as residential.
4. Special. Also exogenous, these include such uses as highways, parks and other public facilities that the government can use to influence land use patterns.
5. Agricultural. This is treated as a residual category.

The residential sector is dealt with in a micro-analytic builder-household market in which decisions are ordered in the following way:

- a. Land developers subdivide land for residential use.
- b. Builders erect dwellings.
- c. Households rent or purchase dwellings.

This does not mean that households do not affect the subdivision process; rather, that they do so indirectly. Household types are composed of families of similar incomes, ages, size, and education and are thus matched to housing types desired, as in the San Francisco model. Households are also grouped, within those exercising demand for housing, into intraregional movers, in-migrants, and newly formed households. Since households are collected in age cohorts, they may progress through different categories with the passage of time, and as influenced by other factors.

Several formulations of the land use simulation exist in the literature, but we will use the one called "a recursive programming model". It can be stated as

minimize $C X(t)$

subject to

$$A_1 X(t) = B(t)$$

$$A_2 X(t) \leq D X(t-1)$$

$$A_3 X(t) \leq G$$

where $X(t)$ is a vector of the amount of land developed, by zone, and C is a cost vector. The first set of equations requires that the total amount of land developed, including service uses, equal the amount demanded. $B(t)$ is a forecast of demand based on previous demands by distributed lags. The second set of equations (constraints) is used to prevent builders from reacting fully to shifts in demand, i.e., a conservative bias. The final set expresses a number of fixed relationships such as zonal capacities and accessibilities.

The SEWRPC models have been described in a sketchy way in part because the published reports of the models are sketchy. Considerably more information can be obtained from the references, however, and the comments by Lowry following Schlager [413] are interesting. A great deal can be lost or gained in going from model framework to variables and numbers, and complete evaluation requires a rather thorough knowledge of the details. The SEWRPC program appears to have a good framework and excellent control of the data, but whether the completed model will relate usefully to the real world cannot yet be determined.

Other Comprehensive Empirical Models
University of North Carolina

The UNC model is relatively less comprehensive than most of the models mentioned above. It allocates residential use only, fails to differentiate between types of households, and starts with the total future residential land use already fixed. In fact, then, the model is a fairly limited residential allocation model, but it has features that are unlike other models and these features are worth describing.

Most noteworthy, it is the only urban model we know of that can rightfully be called a Monte Carlo simulation. Residential units are assigned to ninths (one ninth of a 1000 x 1000 foot grid square) by means of what may be conceptually regarded as a spinner or roulette wheel, where the colors or numbers on the wheel can be expanded or contracted at each spin. The deterministic model can be thought of as a special case of the Monte Carlo model in which the spinner has only one number on each spin. The North Carolina model is sufficiently random that the builders feel that at least 50 runs are necessary to be able to evaluate the output.

While the model is a representation of Greensboro, North Carolina, the purposes of the model are experimental and methodological rather than being directed at any specific use in Greensboro. It is also not oriented towards any particular academic discipline (such as economics) but is designed to be able to incorporate relationships or decision rules of any form. Presently, the model is a single computer program (outside of data preparation and statistical analysis), but the group intends to break it into submodels and also add other submodels at some future time. Thus the model is oriented towards theory and behavior but is firmly attached to a real-world situation, and it is a model being constructed by incremental growth and reconstruction, rather than proceeding from a grand design or attempting to assemble several loosely related models.

The mechanics of the model's operation can be summarized in four steps:

1. Initial status

There are approximately 4,000 of the 1000-foot grid cells, each of which is composed of nine subcells. The grid cells are located on the map, but the ninths are only associated with the cell, i.e., the ninths have no particular arrangement within the cell; they are simply the basic counting unit. Each ninth is listed as vacant open land, vacant subdivided land, or developed land.

2. Attractiveness scores

The attractiveness of each cell for residential development is computed, based on a number of variables determined from multiple regressions. Initially these values are actual assessed land values.

3. Reassessment

"Priming actions" are entered, such as schools and highways, and the cells are assigned new attractiveness scores. The weights on each of the priming variables are also derived from regressions. Additional constraints may be imposed at this time, such as zoning restrictions, open space policies, and new non-residential uses. The inventory of land available for residential development is then broken into ten density-value classes for each of the open and subdivided land categories.

4. Household Allocation

Households to be located during the current time period (usually taken at three years) are assigned to cells using a randomization procedure that makes those cells with high attractiveness scores more likely to receive development. Steps two, three, and four are then repeated with new exogenous

variables pertaining to the next period. At the end of the forecast period, say, twelve years, the entire process is repeated with the same inputs and priming actions, but with different assignments for development due to the randomization. After a sufficient number of runs, the results of all runs are evaluated and summarized.

Considerable effort went into selecting variables for computing attractiveness scores and determining priming actions. Fourteen variables were tested in a variety of combinations for their effect on two independent variables, total land in use and dwelling density. Although high correlations were not obtained, the objective of the regressions was to isolate important behavioral variables and eliminate most of the multicollinearity among them. At one stage, nine variables were being tested on three different cities in the Piedmont crescent, and the combinations are shown in Figure 5.29. In the model, vacant land, assessed value, and (sometimes) residential amenity were used as part of the attractiveness score, and access to workplaces, availability of sewerage, distance to street and distance to school were the major priming variables. With two of the most important variables being vacant land and distance to workplaces, it would not be stretching a point to call the North Carolina model a randomized gravity model. Many models generally regarded as gravity model applications have acquired the name by virtue of the incorporation of these two variables.

As is common with simulation models, a major object of effort on the part of the group was the computer programming, handled primarily by Donnelly. A good portion of the main program is written in an assembly language, which allows for substantially more efficient computation but requires much more programming effort and makes the program itself even more inaccessible to those who didn't write it. For package programs that many people will use and which also will seldom need to be changed, the effort is readily justifiable, but for an experimental program that is one step in a developmental process, the practice is questionable. Unfortunately, efficiency is required in order to obtain enough runs for statistically significant results in a reasonable amount of machine time.

Major Independent Variables Tested in Three Urban Centers

<i>Independent Variable</i>	<i>Greensboro</i>		<i>Winston-Salem</i>	<i>Lexington</i>
	<i>1948</i>	<i>1960</i>	<i>1960</i>	<i>1960</i>
Marginal land not in urban use	x	x	x	x
Travel distance to nearest major street	x	x	x	x
Availability of sewerage	x	x	x	x
Distance to nearest available elementary school	x	x	x	x
Zoning protection	x	x	x	x
Assessed value	x	—	—	—
Accessibility to work areas	x	x	x	x
Proximity to nonwhite areas	x	x	x	x
Proximity to blighted areas	x	x	x	x
Total travel distance to high value corner	x	x	x	x
Proximity to mixed uses	x	x	x	x
Distance to nearest playground or recreation area	x	x	x	—
Distance to nearest convenience shopping area	x	x	x	—
Residential amenity	—	x	—	—

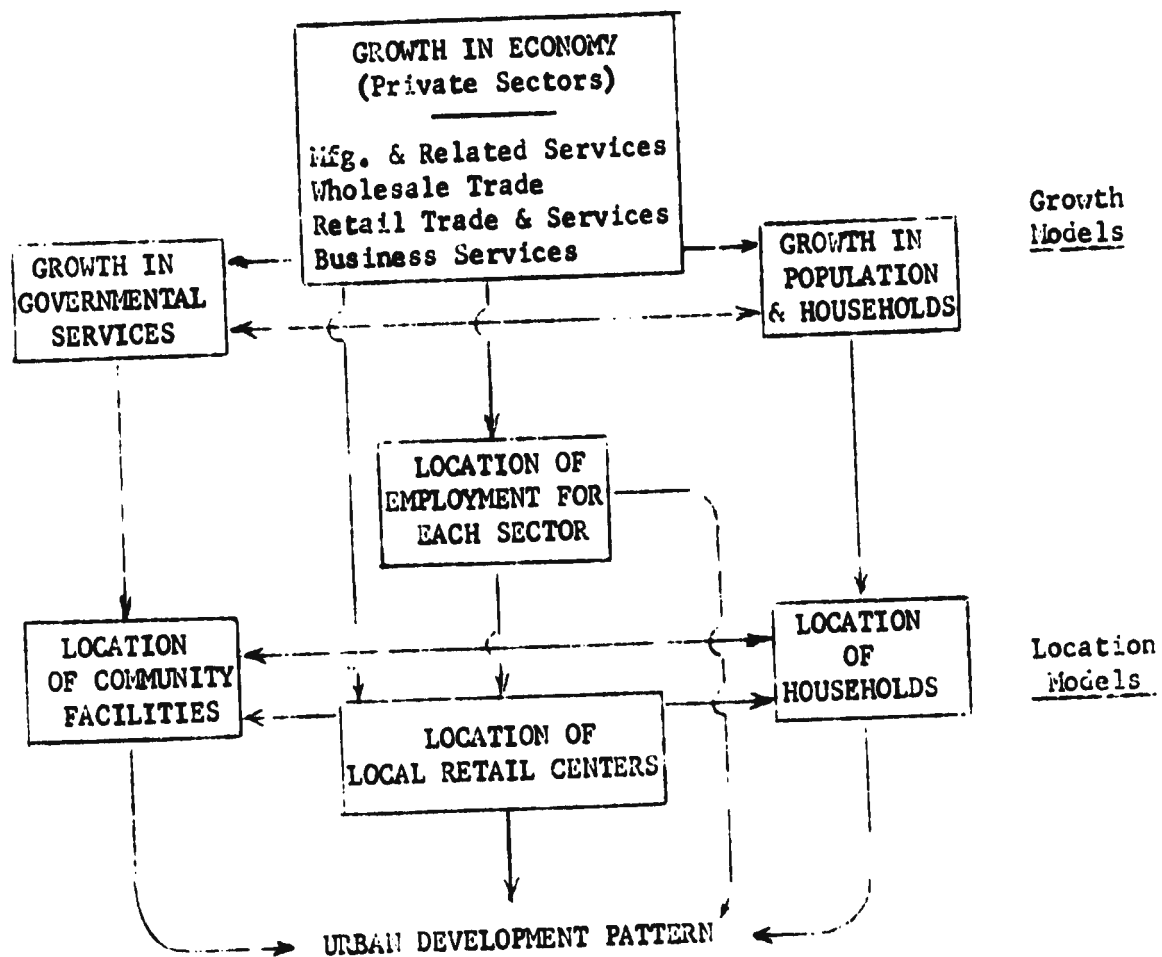
Figure 5.29 MAJOR INDEPENDENT VARIABLES TESTED IN THREE URBAN CENTERS

This raises the question of the value of the Monte Carlo technique, i.e., the randomization. The rationale would seem to be that there will always exist some portion of the development process that we will be unable to completely explain, and this portion must be thought of as random variation. Since the randomness will always be there, we are interested in knowing what effect this has on our forecasts. If we could describe the major part of the process with certainty, and had some chance of being able to describe the rest probabilistically, then Monte Carlo methods might make sense. Here, where regression is relied upon for relationships, and considerable residual variation already exists, the addition of more random variation would seem to be out of place. The magnitude of the effort needed to incorporate probabilistic elements into a model being what it is, the decision to do so should not be made without hard thought.

Chapin, Weiss, and Donnelly intended to expand their model by further disaggregation, by incorporating a wider range of variables, and by separating the residential model into two submodels -- one for housing supply, and one for demand. They also proposed a more comprehensive model along fairly conventional lines (Figure 5.30). Recent work, however, has tended to become more independent of the original model. A good deal of individual data have been collected on the temporal-spatial activities of households, and this investigation of "urban activity systems" has moved into new and unusual areas.

The RAND Model

The RAND Corporation, located in Santa Monica, was established to assist the Air Force in long-range planning. Around 1960, a group of economists and social scientists at RAND obtained a grant from the Ford Foundation to explore the possibilities of building a comprehensive computer simulation model of a generalized urban area for transportation planning purposes. Although the low probability of actually constructing such a model was acknowledged from the start, it was hoped that the work of several investigators working in the subject area could be assembled and developed coherently.



Source: Kain [458]

Figure 5.30 PROPOSEL MODEL EXPANSION

While the full model has not been finished, several submodels have evolved into operating tested components, and a number of specific studies have dropped out more or less as by-products. The Ford grant expired in 1963 and some of the work has been continued by RAND, but it is unlikely that they will complete the full model.

In its present form the work is best described as a set of loosely related studies and models, rather than a single model. The original flow chart, shown in Figure 5.31, is a reasonable framework for a general model. Exogenous inputs are employment by sector (industry mix) and the present state of the system. Sources of employment and other non-residential uses are located by the model, followed by residential uses. The results are used to operate a conventional traffic model and also to update the system. A variety of constraints, policy variables, and behavioral or descriptive relationships can be entered into the model where appropriate.

Most of the submodels that have been tested in some form could be called econometric. The fifteen-equation model reported by Niedercorn and Kain [465] is an intermediate version of the employment submodel, but is disaggregated spatially only into central city and ring (SMSA outside the central city). Employment is broken into four categories, manufacturing, wholesaling, service, and retailing. The model is recursive and operates on the present values of variables to produce annual changes in the endogenous variables. Census of manufactures data were used to fit the model to 39 SMSAs, and the results generally support the hypothesis that central city employment in manufacturing and retailing will continue to decline or remain fairly constant. A similar eleven-equation model is also reported by Niedercorn and Kain [466], with similar results. The flow chart for this model is shown in Figure 5.32.

Intentions of the RAND group were to develop the employment model into one whose equations would be of the form

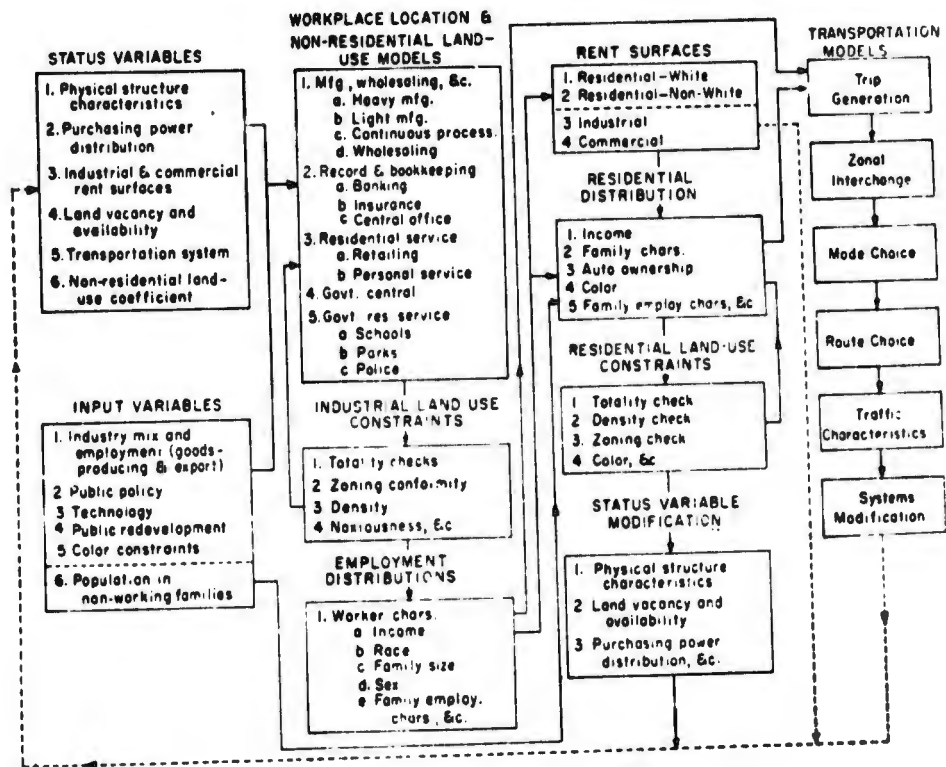
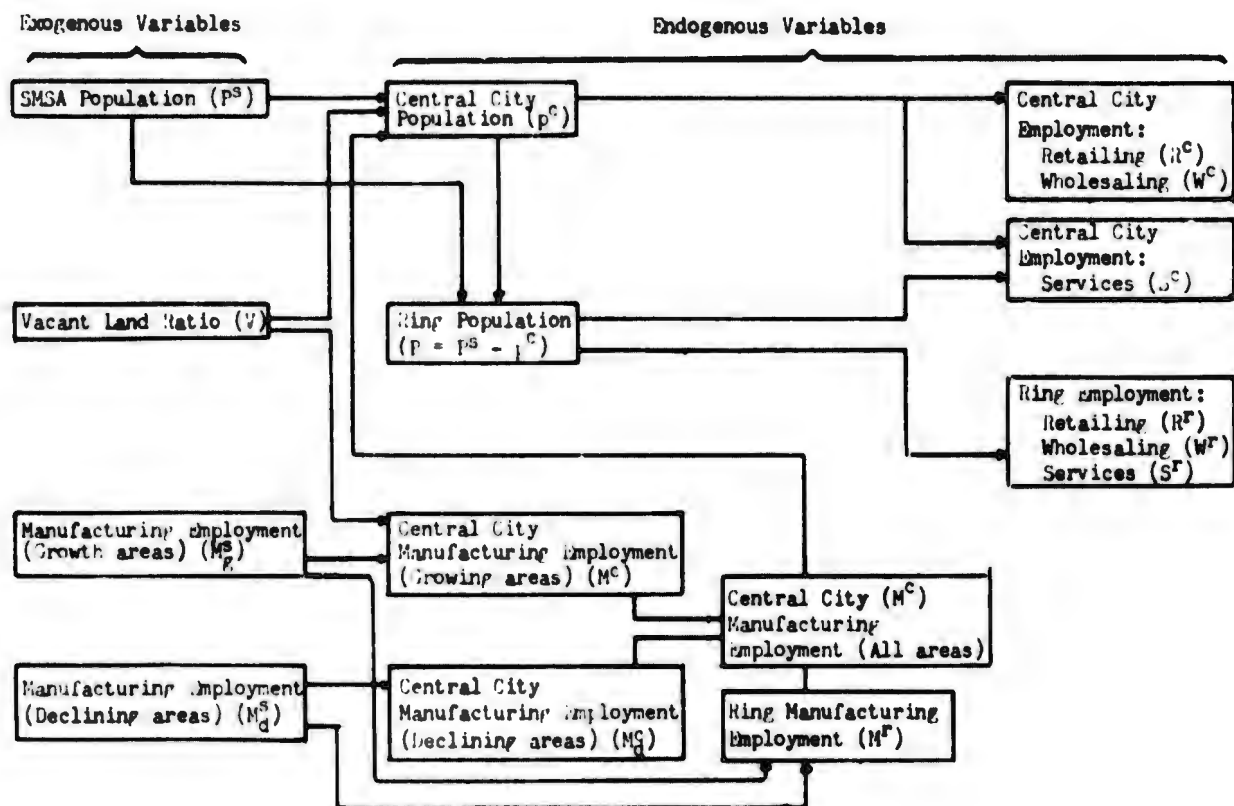


Figure 5.31 MODEL FOR STUDY OF URBAN TRANSPORTATION



Source: Niedercorn and Kain [466]

Figure 5.32 A MODEL OF POPULATION AND EMPLOYMENT CHANGES

$$\frac{w_j^k}{w^k} = F_1 (x_j^1, x_j^2, \dots, x_j^n)$$

where w_j^k/w^k is the proportion of the k^{th} industry's employment located in the j^{th} subarea, and x_j^1, x_j^2 , etc., are variables describing the j^{th} subarea. These include access to passenger and freight facilities, land values, characteristics of residential and non-residential structures, amount of vacant land, and others. Employment categories would be disaggregated further than in the earlier model.

Equations for the residential location submodel were to be of a form similar to the employment submodel equations, vis.,

$$\frac{w_{ij}^k}{w_j^k} = F_2 (x_1, x_2, \dots, x_m)$$

where w_{ij}^k is the number of workers of type k (defined by occupation, sex, race, family characteristics, etc.) residing in the i^{th} subarea and working in the j^{th} subarea. w_j^k is the number of type k workers employed in the j^{th} area, so the left-hand term is the proportion of j 's workers living in zone i . The variables $x_1, \dots, x_m$ describe characteristics of zone i and access to other zones. This model has probably not been developed, but several preliminary studies have emerged. The RAND group felt it necessary to incorporate constraints and relationships pertaining to ethnic minorities, and some of the studies are aimed at this purpose.

Kain [462] gives a fairly complete overview of the RAND transportation study group's efforts. Although Lowry is mentioned, his work at RAND was on the Pittsburgh model. A more recent work is the book by Meyer, Kain and Wohl [464], which Kain describes as "essentially a noncomputer completion of the initial model design". (p. 167) This book, while not a model in itself, is a careful study of relationships and current trends that are important considerations for modelling. It investigates a number of points contained

in conventional urban wisdom:

1. Urban transportation is in a mess which is steadily worsening.
2. The decline in central city populations and densities can be partially blamed on the decline in mass transit service.
3. Rail rapid transit is the most efficient solution, and far superior to the automobile as a means of handling rush hour volumes.
4. People will ride mass transit if it is available at competitive prices and service levels.
5. Transportation is a primary means of shaping urban form.

While the book is not an attack on the above views, the analysis indicates that one must disregard much of the evidence in order to hold to the conventional wisdom.

Miscellaneous Other Models

A number of other models have been displayed in the literature that are of widely varying levels of comprehensiveness, abstraction, and aggregation. Some are very broad conceptual frameworks stated in flow chart form. Others are very micro-analytic of some subsector of the economy or society. They may use data or not, they may be concerned with a technique (linear programming), a subject (regional economics), a problem (transportation), or general methodology (regional science), and they are difficult to categorize. Many are only marginally pertinent to the problem of urban planning models, for any of a number of reasons. We will mention some of them in approximately decreasing order of generality or comprehensiveness.

Isard and Reiner [457] present the broad framework of the regional scientists in a highly abstract synthesis. A general equilibrium model composed largely of a framework of social accounts is offered by Bramhall [440]. Wingo's model [471] is both comprehensive and detailed, and is oriented towards transportation planning. A general model suggested by Boyce and Goldstone [439] is developed in some detail and combines demographic and economic entities in a transportation simulation. A number of other, largely non-empirical, models of various kinds exist.

Models which use data to test the model hypotheses tend to be partial in scope. Czamanski [449] describes a macroeconomic model of urban growth. Hansen's model [454] is also oriented towards growth, using regression to explore spatial patterns of development. The model by David and Holt [450] is a national microeconomic analysis of the household sector. Ellis [452] describes a descriptive multivariate residential model, and a commercial model based on trip-potential has been done for Buffalo by Fidler [453]. A number of others can be found in the references.

CHAPTER 6: SUMMARY AND CONCLUSIONS

Present capability

Although the discussion has been limited to models that are intended to be used for urban planning purposes, we have nonetheless covered a wide range of types and sizes. Once more we can classify the models, this time according to purpose.

1. Location models

a. Residential location. The purpose of the model is to assign households to geographic subareas within an urban region. No other types of activity are located.

b. Nonresidential location. Usually manufacturing, retailing, or some other breakdown of activities which provide sources of employment, are treated by these models. Many models are a combination of several location types, including residential.

2. Nonlocation models

a. Traffic. The primary purpose of traffic models is to assign traffic by mode to different routes. It is necessary to know locations of origins and destinations, but the end result is a pattern of flows.

b. Economic. The term economic is used in a somewhat restricted sense to mean regional totals in various economic activities. An input/output model is an example, in that regional levels of employment in different sectors are the objects of analysis.

The purpose of the model will at least partly determine the type of model constructed with respect to the five dimensions described in Chapter 2, but

should not restrict the model to any one or set of the theories, methods, or techniques presented in Chapter 3.

Certain threads occur frequently in actual modelling efforts, and some of these, if broadly conceived, appear in almost all models. This persistence may be due to a lack of workable alternatives for use in modelling, the generality of the notion, the advantages resulting from an assumption, or some other reason. The threads which seem to be the most common are listed below.

1. Gravity notions. Originally derived from the analogies of social physics relating persons and distances between them, the label has come to apply to almost any relationship involving distance as a variable. For example, sources of employment are viewed as having circular potential fields around them that indicate the desirability of the area for residential use. Households and trip destinations may also generate potential fields. The units creating the potential can be weighted according to income, social rank, or some other attribute, and distance may also be weighted or transformed. These potential fields tend to have the characteristic of being highest at the source and decreasing monotonically in all directions away from the source. Potential may be accumulated from several sources to form a complex multi-peaked surface. The flexibility of the gravity tool has been widely exploited in urban modelling.

2. The basic-service multiplier. The multiplier concept seems to appear in many forms. As a regional multiplier, it is applied to forecasts of basic economic activities to expand them to levels of total activity. In land use models, the concept takes the form of locating basic activities first and then locating service activities on the basis of the locations of the basic activities. The input-output framework relies heavily on the stability of the ratio between different types of activities. Like the gravity concept, the multiplier concept is common to all models if it is sufficiently generalized. In this case, the multiplier becomes simply a factor applied to the value of a known variable to arrive at the value of another variable.

3. Supply and Demand. Another dichotomy widely used is that between the supply of a commodity or service (housing, transportation) and the demand for it. It is one of the more general behavioral relations that is presently available, and hence its appeal. It also allows for two independent measurement of the same quantity, since supply in equilibrium must equal demand, and hence a more reliable means for fitting and calibrating models. Supply-demand relationships have taken the form of matching industrial firms with appropriate sites (e.g., Pittsburgh's INIMP), matching the available supply of housing by type with demand from households by type (as in the San Francisco CRP model), and matching the total number of person-trips with an existing or proposed transportation network (traffic assignment models).

4. Linearity. At the present state of knowledge it is almost always impracticable to build large-scale nonlinear models. Mathematical difficulties grow rapidly once the linear assumption is left behind, and this feature, combined with the usual size and data requirements, tends to make a model linear. The restrictiveness of the linearity assumption varies, however, from model to model. In an econometric or regression model it is usually possible by means of transformations and other manipulations to make the model linear. Input-output models are basically linear, but introducing approximations to nonlinear functions is feasible if it is important enough to do so. In the case of linear programming, approximations to nonlinear functions are awkward at best, and practically impossible in many cases. Nonlinear mathematical programming is possible but extremely difficult in most situations. It is only in these last examples that nonlinearity is a serious problem, and the techniques mentioned are far less common than the regression methods. In addition, data are seldom available with which to test even slightly nonlinear hypotheses. It is an oft-repeated truism that the world is nonlinear, but it is also true that the world can be made to look sufficiently linear such that the difference is undetectable to the naked eye.

5. Analytic-synthetic models. The meaning of this term is that the problem or system to be modelled is broken down into a number of sub-problems or subsystems which are constructed and operated separately, and then the results put back together to serve the original purpose. Solutions to the subproblems are called submodels. Implied in the method is the idea that the components can be more accurately described than the system as a whole, and the final results after re-assembly will be more accurate than those achieved without decomposition. The notion is appealing because it emphasizes understanding of the behavior of parts of the system, whereas dealing with the system as a whole is often of a black box nature. While in general we subscribe to the analytic-synthetic approach, it has two inherent difficulties that should be mentioned.

First of all, it is not sufficient to say that submodels are necessary because summary statements cannot describe the behavior of disparate elements. It must also be true that more is known about each individual component than is known about the system as a whole; otherwise the summation of the small component errors will exceed the large single error resulting from a one-piece model. The problems of adequate descriptive and behavioral relationships and adequate supporting data are vastly increased by breaking a model into submodels and disaggregating components. Such a model places heavy demands upon the modeller, and unless these demands are satisfied, the model will be no better than a similar aggregative model, and probably a lot worse.

Secondly, there are practical limitations on the amount of interaction that can take place between submodels. Most submodels must be placed on the computer at different times, or at least occupy memory separately, and consequently less information can be exchanged between submodels and less frequently than is possible within a model. If a model is large enough to require breaking into submodels, then the division should be made in such a way that between-submodel interaction is as small as possible while still maintaining logical structure. Greater flexibility is obtained through the use of linked submodels, but at a cost in interaction between components.

All necessary information must be passed through the links between submodels, and the more independent the submodels the less information they can utilize.

6. Data. The unsatisfactory nature of data in general has been repeated and repeated ad nauseum. Very few reports of empirical studies do not contain specific reference to data problems. Those who have had experience with the acquisition of masses of data nod their heads knowingly; those who have not become tired of the repetition. It is a problem that concerns many people, yet it is almost impossible to discuss without being boring. The most hopeful outlook seems to be that better models will lead to demands for better data, and better data will allow the development of better models. In this fashion the two may progress forward incrementally in lock-step.

Problems Peculiar to Urban Models

Some of the difficulties of modelling in general and urban models in particular have been described at length in the previous chapters, but a summary of the difficulties that make urban planning models different from other modelling circumstances would be helpful at this point. The differences are primarily a shift of emphasis, but the shift is significant and has important implications for urban modelling efforts.

Before discussing differences, it might be interesting to mention some of the similarities between research in social problems and, say, research in the physical sciences. Research seeks to find abstract relationships that will explain the operation of a system, whether it be gas molecules under pressure, traffic at an intersection, or people renting housing. The relationships are always approximations, but if they are sufficiently general to be able to predict the behavior of the system under a wide range of conditions and accurate enough for all present purposes, then the relationship is called a law. Since people are familiar with the manifestations of human behavior and are aware of its complexity, while physical science is far removed from their everyday realm, the prevailing attitude is that laws exist in physics

but not in the social sciences. In truth, both subjects rely on abstract models that approximate reality; the physical sciences simply have more of them that work better. Research in both areas is a combination of trial and error, intuitive experimentation, prediction from theory, and serendipity.

Problems with social models. The purpose of a social model is to explain the behavior of groups of people, and the difficulties that are brought out by the social purpose generally fall into three areas. These areas are somewhat interrelated. The first is the problem of measurement. Many of the attributes of individuals, groups, and the environment that influences them, are impossible to measure directly and at least difficult to estimate by indirect means. There seems to be a fairly strong relationship between social class and background on the one hand, and the types of recreational activities preferred on the other, but measuring the attribute "social class" and then assigning an activity set to it is not easy. Before the physical sciences could make advancements it was necessary to develop devices for observing invisible phenomena, and some analogous work may be necessary in the social sciences before models will become useful.

A second area of difficulty is in the isolation of variables. The variables that seem to influence other variables often turn out to be quite surprising, while variables that one would expect to be related are not. Apparently minor variations in the composition in a variable or the city in which it is measured may affect the strength of a relationship significantly. To cover all the possible combinations of relationships and variations in the variables is usually impossible, even with computers doing the work, and consequently the investigator must use his judgment. Thus it may be hard to evaluate previous work for purposes of application in a different situation. A part of the difficulty lies in the fact that almost everything is changing at once, and it is only after a long period of observation over many units, and after highly sophisticated statistical techniques have been applied, that useful relationships can be identified.

The third area of difficulty is an extension of the second, and this is the near impossibility of performing controlled experiments. Even in small-group psychological studies it is extremely difficult to hold the rest of the world constant while altering a few variables. Different individuals react differently to the same situations, and why this should be so is far from completely understood. In addition, most social data are secondary, i.e., they are collected by somebody else; the researcher has no control over which variables are allowed to change and which are not. Our culture does not believe in permitting him to have control, either, except in a few cases and not for purposes of experimentation. The social modeller can only hope, at least at present, to be able to impute general relationships by obtaining large samples of large sets of variables at many points in time. The problems of confidentiality must be solved, also, before the social scientist will have satisfactory individual or small unit data available.

Problems with planning models. When a model is to be used for planning purposes, additional demands are placed upon it. First, it is more important that cause-effect relationships be used, since the model will be asked to predict the outcome of circumstances that may not have been observed previously. Behavioral relations are also easier to support politically. Discovering these relationships is compounded by the necessity to determine order (A causes B, B causes A, or both) and time lag. Second, planning models must be oriented towards policy variables as inputs (those variables which are subject to control by the government doing the planning) and towards goal variables as outputs. In other words, a planning model must be able to answer questions of whether proposed programs will achieve desired effects. Thus it must be possible to measure changes in variables reflecting goals, and it must be possible to determine which set of policy variables (i.e., which program) will achieve the desired ends most efficiently, say, at least cost. The planning modeller must include policy variables and performance variables in his model, and they must be related.

Another difficulty with planning models is that they are intended for use by people who did not take part in the construction, and often are not familiar with how models are constructed and may be distrustful of modellers. The politicians, developers, shop owners, taxpayers, etc., who may be involved in the decision process, are seldom skilled in the art of mathematical modelling. If planning models are to be used effectively, the essential features of the model must be communicated to nonquantitative consumers of the model. This includes the importance and reliability to be attached to various portions of the output, since the results are usually stated in exact terms while the modeller knows that they have some variability. Heuristic gaming seems to have considerable potential as an aid in the communication process.

The Next Stage.

While certain kinds of models have distinguished themselves in some areas, urban models in general have not made significant contributions to date to the solution of real-world problems. The one important exception is in the field of transportation planning, where traffic models have worked fairly successfully. In other areas, models have done as well as other methods in some cases and not as well in others. All this says, in effect, is that mathematical models have failed where others have failed; the problems remain unsolved.

The first wave of models raised a lot of furor. Planners who were pro-model made vast claims for their models (unconstructed, needless to say) that were undeliverable. Transportation planners, whose primary concern was transportation planning and not comprehensive planning, used models to help extrapolate the present. They were attacked by the comprehensive planners, who felt that emphasis should be on the control of future land use patterns and not on their prediction. Unfortunately, these latter planners held somewhat naive views about how the future could be influenced, and the costs of implementing their plans. Gradually, however, the transportation planners have been willing to incorporate more social variables in their models, the

comprehensive planners have become less naive, the claims made for models are becoming more realistic, some models are beginning to work in limited ways, and some of the conflicts and animosities have died down. Many people have been happy to forget all about models, while others have been equally happy to work on them in relative obscurity.

At the present time, the potential for models is very high. A general concern for urban problems has been building up, and along with it at least some awareness of the interrelatedness of the problems, the difficulties of solving them, and the necessity for a long term effort. The depth of the interest in urban problems is hard to assess, but it seems to be increasing. Yet it is clear that the massive task-force approach to the simultaneous and comprehensive solution of all urban problems will produce very little. In those areas where modelling has achieved the most success -- transportation planning -- model development has proceeded in small increments over a better-than ten year period. Reliance was primarily upon methods that would at least provide satisfactory results, with experiment and innovation injected in small doses. Workable descriptive methods were used in place of more glamorous but highly risky techniques. As a result, further development in modelling must, for the most part, start from the level to which it has been brought by the transportation planners.

Why not a general model? Theory and application in a field seem to move in cycles, from theory to empirical testing to application to inductive generalization and back to theory. Urban theory is unequal to supporting the further development of models, and existing empirical regularities have been thoroughly exploited. What is needed is a new body of theory, out of which may emerge a general model of urban phenomena, but before this can happen a good deal of empirical work is required. Observations must be made in the real world and regularities perceived. Secondary data are constantly improving, but the information must be made more pertinent to the variables of interest, it must be available more often on an individual basis, and

it must be more interrelated. Kain [462], in denying the need for a general model at the present time, states that, in addition to practical limitations on computer implementation,

"An even more serious shortcoming is the lack of adequate theory and specific quantitative information about urban phenomena. Although urban research has improved tremendously in recent years, there still is surprisingly little careful quantitative research [of the kind] required for econometric model building." (p. 150)

Many times in the previous pages the claim has been stated or implied that the demands made by an intellectually satisfying general model upon data sources and behavioral relationships cannot yet be met.

It is unnecessary to say that an interdisciplinary approach is required; the question is how this can be accomplished. Placing a group of engineers, sociologists, planners, psychologists, economists, mathematicians, etc., together will not produce a solution. The engineer must have an interest in and some feeling for social problems. The sociologist must desire to use his theory to solve real problems. The planner needs to know his way around computers and mathematics. All must realize that the solution lies outside his own particular domain, and that all disciplines are required. For even two of the above types to match up from different fields is rare, and to get a whole team together is almost impossible. Gradually, however, small groups of investigators will emerge who are willing to leave the traditional biases of their fields behind them to focus on the problems themselves.

On a broad scale, resources can be well utilized in education, research, and data development. Urban students should receive formal training from several disciplines and in some depth. In the long run, the major work must be done by researchers trained to be interdisciplinary, rather than members of traditional disciplines who are modified or converted. Research funds can be best employed by providing adequate facilities and support in numerous locations, rather than generating large chaotic interdisciplinary-sounding projects.

Funds should be maintained over a considerable period of time, and gradually increased where the receiving institution can absorb an additional load. Universities and research houses such as CAL are likely candidates. Much effort needs to be directed at making information, most of which is already collected, available to legitimate users at reasonable cost and in appropriate form.

A strategy for CAL. A good general policy to follow is to expand existing areas of competence towards modelling, and to add new skill incrementally as it becomes appropriate. This process is neither continuous nor monotonic, but it is closer to these than to a step function or to a single massive jump. Considerable time is required to build a group of personnel with the necessary background and experience, and one that is interdisciplinary in more than name. Small projects must be successfully completed to establish a reputation in the area and to open channels of communication, before larger ones can be attempted. Social systems models in general and urban models in particular are new enough so that there is no clear path towards building expertise in their construction.

Buffalo would appear to be a near-ideal laboratory for building and testing models. The city is not distorted by minor topographic variations nor by the influence of larger nearby metropolitan areas. The region is composed of what is basically a single urban area (Buffalo, Tonawanda, and Niagara Falls stretching along Lake Erie and the Niagara River) plus a rural hinterland. While the five-county Western New York region may ultimately be the complete analysis area, Erie and Niagara Counties would seem to be adequate for the kinds of urban studies proposed here. The two-county region is large enough to have most of the problems associated with large urban areas, but not so large that empirical studies would be overwhelmed simply by the problems of handling data.

In some areas, notably metropolitan employment, CAL has already done a good deal of work. Many of the important questions have been addressed, relationships perceived, variables isolated, and the necessary background information about the Buffalo region filled in. This work should be extended and translated into model form, which can be done through econometric studies. The most significant gap in the present work is the lack of spatial variables, which is tied to the problem of obtaining data for small areal units. Efforts should be made to establish a regional data library containing economic and social information, and CAL should participate in the design and management of such a facility when it can be created, but studies at the Laboratory should not be held up because of it. Thus CAL will have to expend some of its resources obtaining data. Demonstration studies using local data will hopefully hasten construction of the regional data bank.

Spatial information adds a considerable burden to data acquisition but it is necessary that all information gathered have a spatial dimension since location is of key importance to metropolitan planning and analysis. In this way social and demographic variables can be tied to economic variables and their spatial relationships studied. Economic studies completed or under way at CAL often include social variables, but they should be expanded so that they are covered systematically. For this purpose it is probably necessary for CAL to acquire a sociologist or demographer with analytic training. Studies of a primarily descriptive nature can then be undertaken to relate economic and social variables. Buffalo has an interesting ethnic mixture, particularly in light of the current interest in negro problems, and valuable information could be gained by careful study of the city.

While the transportation analysis that is being done at CAL is not as close to being ready for modelling as the economic studies, transportation is obviously an important area to add to any modelling effort. For example, the "urbmobile" study should include an analysis of the spatial impact of the system, the socio-economic characteristics of potential users, and the costs and benefits of the system versus alternative systems in terms of both economic

and noneconomic criteria. Such an analysis requires industrial, commercial, and residential location models, traffic assignment models, and some sophisticated analyses relating economic social, and psychological variables. A great deal of time and effort must be spent before these tools will become fully operational, but every opportunity should be taken to begin developing them.

There is no particular pattern to undertaking the above studies, and no framework presently exists for organizing them into an integrated comprehensive model. Progress will require constant review and revision to incorporate new information, new opportunities, specific interests and abilities of the laboratory staff, sources of funding, and the results of previous work. Interest in the solution of social problems should be encouraged, as opposed to the development or application of a specific set of techniques or the advancement of a specific discipline. The overall framework relating the various studies to each other will most likely have to be redesigned each time a new study is begun, and perhaps also in the course of the work.

CAL should continue to strengthen relations with state and local planning, development, and transportation agencies, for the purposes of helping to design projects that are both useful and supportable, and to exchange data and coordinate data acquisition. A wide variety of Federal programs should be investigated for possible support of projects. A final item that should be of interest is the nearness of the 1970 census. This census will contain much information that will be useful to modellers and it will be available for small areas, geographically coded. Test tapes will be ready as soon as 1968, and the census itself will start coming out in 1971. Organizations set up to receive this material will have a store of valuable and timely data.

APPENDIX A: OTHER MODEL TYPOLOGIES

Numerous classification schemes have been suggested for urban models, and some of these should be mentioned and compared with the framework presented in Chapter 2. Many of these schemes were designed for some other purpose than partitioning the space of models, and many must be imputed from their uses, i.e., they are not made explicit in the original sources.

Lowry [298]

The classification suggested by Lowry is based on sophistication (or alternatively, degree of difficulty of construction), and the type selected by the user should depend upon weighing the objectives of the model user against the resources available for construction. Along this continuum, Lowry lists three classes: descriptive models, predictive models, and planning models.

(1) Descriptive models aim at replicating the relevant features of the real world at some previous point in time. A successful model then indicates at least a partial verification of the assumptions or hypotheses upon which the model was based. Some measures of performance (in evaluating "success") are:

- a. ratio of input required to output produced.
- b. accuracy and cost of the output vs. direct observation of the variables of interest.
- c. generality of the model.

Although efficiency of input-to-output is desirable in a model, a very low ratio implies that a large number of decisions are based on a small amount of information. Since this is in general not the case in the real world, a descriptive model with a low ratio would be used as a rough approximation or norm from which to study deviations. On the other hand, a particularly high

ratio of input-to-output suggests not only inefficiency but an over-emphasis on a few measures of output.

(2) Predictive models must be able to describe a future state on the basis of present information, and perhaps also on the basis of future conditions predicted outside the model. This form is called a conditional predictive model, and is of prime importance to planners as a means of testing the impact of policies and programs. Emphasis on prediction requires a greater behavioral understanding of urban structure, and is consequently more demanding upon the modeller.

(3) Planning models carry beyond predictive models in that they evaluate the alternative being considered and select another one to investigate on the basis of the results obtained from the previous alternative. The steps in the process, given by Lowry, are:

- a. Specification of alternatives
- b. Prediction of the consequences of each alternative
- c. Scoring alternatives on goal-achievement
- d. Selecting the alternative with the highest score.

Mathematical programming techniques could provide a framework for constructing planning models. While models of this type are presently possible, their usefulness would be severely limited by inadequate knowledge of urban structure and by the absence of a satisfactory social welfare function for evaluating alternatives.

Steger [301]

In reviewing urban renewal forecasting models (those capable of assessing the impact of alternative renewal programs), Steger suggests a three-fold classification system:

(1) Controlled variation of independent variables

This type might be called controlled-experiment models, for observing changes in the system as a result of changes in selected variables. These are also called predictive-on-conditions models, or also impact models. The relationships in such a model would have to be largely behavioral, with descriptive relations useable over limited ranges. The model would also be micro-analytic, to be able to produce sufficient detail for urban renewal decisions, and either comparative static or dynamic.

(2) Simulation forecasting models

Corresponding to a higher level of sophistication, these models are better able to predict system-wide effects of renewal decisions. The major difference between this type and the previous one is that the simulation forecasting models are more comprehensive, probably employing a number of the controlled experiment type as submodels. Steger's categories are hierarchial, in that higher categories manifest all the characteristics of lower types, plus some additional characteristics.

(3) Mathematical optimizing models

The characteristic added at this level is a normative capability, which means in this case the ability to evaluate within the model the consequences of urban renewal decisions and select an optimal strategy in response. Models of this type require a much higher level of information quality than is now available, and a more sophisticated and reliable set of submodels than can presently be constructed. Thus these normative "decision models" are on the distant horizon at best.

The Lowry and Steger typologies, while very useful and based upon considerable insight, are attempts to collapse several dimensions of model typology into a single dimension. Lowry starts with the descriptive-behavioral-

normative category, adds a hint of a time dimension, plus emphasis on model purpose, to obtain the predictive category, and combines a specific blend of solution method, model framework, time orientation, and normative aspects to arrive at the third category, planning models. Steger starts with the predictive model category of Lowry's and ends with the same normative-decision model category, but moves in the direction of comprehensiveness for his second simulation category. These typologies might be viewed as a series of points in a three-dimensional space (or perhaps higher dimension), not necessarily lying on a line and not starting from the same spot, but approximately parallel.

The Abt Report [1]

A much wider range of model types was surveyed by Abt Associates, and the typology which they devised is aimed at a broader spectrum. Of interest are six dimensions of their scheme, shown in Figure A.1.

(1) Academic discipline originating model. The disciplines are arrayed from qualitative to quantitative, which is not particularly appropriate, and they placed all the models which we were also interested in under "economic" models (these include an input-output model, the San Francisco CRP and the Pittsburgh CRP models, and the Penn Jersey transportation model). This classification has low utility because almost all urban models will contain economic variables, and most will also contain demographic and social variables. Increasingly, political and psychological attributes are being represented.

(2) Size of population covered. Models range from international to individual on this continuum, and urban models are classed separately from regional models. Although strictly urban models will tend to be more disaggregated than regional models, the distinction is not one that we feel compelled to make. With that, all models with which we are concerned fall into one class.

(3) Micro-macro. This distinction in the level of aggregation is useful to us and is described above.

SPECTRA OF THE MODELS SURVEYED

(An overview of model characteristics)

SUBJECTS	Qualitative ↔ Qualitative + Quantitative ↔ Quantitative Psychology Politics Sociology Economics Engineering
SCOPE	Inter- — National — Regional — Urban — Small — Individuals national Industry Group Firm
DETAIL	Macro-variables (Large - scale, aggregated) Micro-variables (GNP, unemployment rates, votes) (Subjective decision criteria, perceptions)
OBJECTIVES	Education — Planning — Forecasting — Research — Analysis Training Evaluation Coordination
PROCESSING MODE	Human-player Games Man-Machine Simulation and Games Computer Simulation
"TRANS-PARENCY"	"Black box" _____ "White box" (only inputs and outputs represent process) (decision structure represented)
COSTS { Skills	Humanities Social Sciences Physical & Engineering Sciences History Sociology Political Economics Computer Math Science Programming
{ Time	10 years 3 months 50 man-years 3 man-months
{ Money	\$2000 \$2,000,000 \$2000/year \$400,000/year

Figure A.1. The Abt report typology.

(4) Objectives or purposes. The difficulty with this classification system is similar to that of the academic discipline originating the model, in that most modelling efforts are aimed at some mix of all the objectives listed. Although limiting the objectives of some models might improve their chances of success, the limiting should not be done by cutting out one of these arbitrary categories.

(5) "Processing Mode." These categories are more related to gaming models, and not particularly to planning models.

(6) Black box or "white box". This continuum is a combination of the descriptive-behavioral dichotomy and the micro-macro breakdown discussed under the topic of disaggregation. Large econometric models fitted to handfulls of variables by multiple regression methods are sometimes described as representing the "black box" approach.

Military

It is common for programs, operations, weapons, etc., in the military services to be labelled "strategic" or "tactical". A strategy is a broad-scale plan for achieving a set of objectives, and tactics are the means of implementing the strategy. The notion is somewhat useful for planning models, but the strict dichotomization of strategic-tactical destroys its value. Better descriptors are partial-general or project-oriented vs. comprehensive. In addition, military terms are ill-suited for civilian purposes that are as politically sensitive as social planning.

Zobel [15]

Three subjects are considered appropriate for regional analysis: the economic base (construed broadly), transportation, and land use. If we add social and demographic patterns, we have a fairly comprehensive list of subjects for urban models, but it is a rare model that doesn't include several

of them. Transportation studies project population, major industry, and several types of land use in order to evaluate a transportation network. Even relatively specialized submodels, such as a residential allocation model, require information from all four categories. The categories are useful if attached lightly to particular models, but they do not distinguish between models in a clear-cut fashion. The terms are used throughout this report, with the above caveat.

Traffic Research Corporation [13]

The Traffic Research Corporation selected 14 models for review, in their report to the Boston Regional Planning Project, and grouped them according to their degree of operationality -- operational or quasi-operational, research oriented, and conceptual. Although it is obvious that some models are more operational than others, the objections to the grouping are similar to those regarding the military terms, i.e., the groups obscure the nature of the continuum. There are many reasons why one model may be more operational than another. For example, an applied model may not be operational because the data are not available or because the study is not far enough along. Although our emphasis is on applied models or those which show potential for application in the near future, we will also cover some theoretical or conceptual models. The TRC breakdown essentially parallels the classification according to level of abstraction.

Tomazinis [12]

The following is the classification scheme proposed by Tomazinis:

1. Deterministic
 - a. analytical
 - b. numerical
2. Stochastic
 - a. analytical
 - b. numerical

3. Optimal

- a. plain
- b. twin-goal.

Optimal, or optimizing model, is a category that is often used, but it does not seem to be any different from normative in meaning. If there is a difference between the two words, normative is the broader in scope and the more oriented towards social goals. The first category under Optimal, plain optimal, refers to models of the linear programming type, in which the metric to be maximized or minimized (within the constraints) can be clearly stated in a single function. The twin-goal optimal model, or multigoaled, by extension, is analogous to the locus of Pareto-optimal points in the distribution of two commodities between two consumers (the contract curve of the Edgeworth Box, from economic theory). Such models are not remotely operational, and await resolution of the social utility function problem of welfare economics.

The other two categories -- deterministic and stochastic -- and their two subcategories can be made equivalent to our breakdown of solution techniques by equating "deterministic-analytical" with "deterministic" in our scheme, "deterministic-numerical" with "numerical," and both categories of "stochastic" with "probabilistic."

Allocation Models

Occasionally one runs across a model that is labelled in the literature as an Allocation model. What is meant by the term is a model that starts with a given, fixed amount of population or land use which is to be spread out among a number of subunits. A transportation land-use model will typically use a population projection pertaining to some target date in the future and allocate the future population to zones on the basis of an extrapolation of the density gradient (as in the early CATS model) or some other method. Allocation techniques vary from highly sophisticated weighted index measures and multiple correlations

to strictly ad hoc measures. The term "allocation" in the model context indicates a particular level of comprehensiveness, or break between exogenous and endogenous variables in which model totals are fixed and exogenous, while amounts (proportions, in effect) going to subunits or areas are computed endogenously.

Boyce and Cote [270]

Two possible dichotomies are proposed.

(1) incremental-total

The distinction seems to be between a general equilibrium solution (total) in which all variables are determined more or less simultaneously, and a partial equilibrium in which some variables are held constant or controlled. A recursive model that readjusts at each time period in response to exogenous stimuli would be of this second (incremental) type.

(2) nonstatistical vs. statistical

The issue here is whether the variables are fitted for each equation separately and outside the model (nonstatistical) or simultaneously for the entire model at once (statistical). This difference, while quite substantial from the construction standpoint, is minor from a general view. The authors originated the concept in order to discuss statistical verification of different models to establish some criteria for reliability.

Lamb [6]

According to Lamb, the scheme he proposes is not intended to be a classification system but simply a means for sorting out different mathematical techniques that are used in land use modelling. Nonetheless, the categories have at least some of the nature of a typology. He lists two basic approaches,

with categories within each.

(1) Trend models. Trend models extrapolate trends, and the following techniques are employed for the purpose:

- a. Graphic models
- b. Regression models
- c. Simultaneous equation models

(2) Logic models. These models "define how or why events occur," (p.4) and use this for prediction. There are two categories of this type.

- a. Linear Programming models
- b. Simulation models

Most models, he explains, use several of these techniques, with emphasis in the past being on trend methods. The basic breakdown -- trend vs. logic -- is equivalent to a descriptive-behavioral dichotomy, with normative models (linear programming) being thrown in with the behavioral. The subcategories are in the way of examples, since they are neither mutually exclusive nor exhaustive. The example given for the graphic model is a density gradient model, of the CATS type. Regression and simultaneous equations are methods for constructing models, and are normally used together. Linear programming is a specific optimizing framework, and simulation is a micro-analytic approach useful under some conditions.

Harris

Britton Harris' dimensions have much in common with our set, and were obtained via Lowry [8]. Harris' dimensions are listed as:

1. descriptive-analytic
2. holistic-partial
3. micro-macro
4. static-dynamic
5. deterministic-probabilistic
6. simultaneous-sequential

Each dimension is presented in terms of its two polar extremes. The descriptive-analytic dimension collapses two dimensions that we have listed separately, namely, disaggregation and descriptive-behavioral-normative. Descriptive models rely on data and calibration techniques, and are applied; analytic models are behavioral and theoretical. The next three of Harris' dimensions are the same as ours: holistic-partial equals comprehensiveness, macro-micro equals disaggregation, and static-dynamic equals treatment of time. Deterministic-probabilistic is discussed under solution techniques, and simultaneous-sequential seems to refer to model structure. In a pure sense, no events are determined simultaneously in a model, the dimension would appear to mean the extent to which a model operated recursively in time.

Kilbridge, et. al [5]

A research program, sponsored by the Harvard Business School and under the direction of Professor Kilbridge, has also surveyed the subject of mathematical and computer models in urban planning. Called the Urban Analysis Project, its object is to explore the parallels between the concepts of management science and the problems of urban planning. Their report is similar to this one in that they develop a framework for classifying and discussing models, and survey most of the major attempts in the field. The scheme they developed is shown in Figure A.2, and the main breakdowns are:

1. Subject, which may be land use, transportation, population, or economic activity.
2. Function, which may be projection, growth allocation, or the derivation of one subject area from another (e.g., traffic from land use).
3. Theory, which may be behavioral or based on growth forces.
4. Method, which includes econometric, mathematical programming, and simulation techniques.

This framework is largely descriptive, in that it summarizes model efforts to date, as opposed to Lowry's paradigm which we described in Chapter 2. Lowry postulates a framework and then fits models into it. The Kilbridge survey makes many distinctions that we tried to avoid (such as between land use and transportation models), but it contains a number of interesting abstractions and is a very succinct and informed survey of present capability.

A-13

Source: Kilbridge [5], p. 3

Figure A.2

REFERENCES

The list of references is broken into eight categories which conform to the outline of the text, but do not completely reproduce it:

- Surveys and Typologies (Ch. 2, App. A)
- Theories, Laws, and Empirical Regularities (Ch. 3A)
- Methods (Ch. 3B)
- Techniques (Ch. 3C)
- Model Construction (Ch. 4)
- Community Renewal Programs (Ch. 5, CRP's)
- Transportation Studies (Ch. 5, Transp.)
- Other Models (Ch. 5, Other Comprehensive, Misc. Other)

Each article or book is listed only once. Discussions and descriptions of theory, method or technique are grouped with references for Chapter 3, while applications and comparisons of models and techniques are placed under references for Chapter 5. Literature about the modelling process, data etc., are collected under Model Construction, and abstract discussions about models are under Surveys and Typologies. Since many of the materials fall into more than one group, the classification was sometimes arbitrary.

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SUPPLEMENTARY

INFORMATION

IDENTIFICATION	FORMER STATEMENT	NEW STATEMENT	AUTHORITY
AD-843 620 Cornell Aeronautical Lab., Inc., Buffalo, N. Y. Final rept. Rept. no. CAL-VY- 2474-G-1 Sep 68	For users' only.	No limitation	Cornell Aeronautical Lab Inc ltr, 10 Apr 69