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SHOCK AND VIBRATION TECHNICAL DESIGN GUIDE

FOR UNITED STATES ARMY ELECTRONIC EQUIPMENT PACKAGES

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SHOCK AND VIBRATION TECHNICAL DESIGN GUIDE

BOOK 2 OF 2

• VOLUME III – RELATED TECHNOLOGIES

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VOLUME III

RELATED TECHNOLOGIES

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ABSTRACT:

The objective of this volume is to present a complilation of the various Mechanical Engineering disciplines that make up the experience of the Structural Engineer. This compendium relates these diverse areas of technical knowledge to the general problem of dynamic integrity in the equipment system.

For convenience, the eleven chapters of Volume III are organized into three categories; those subjects dealing with the analytical aspect of structural dynamics; those subjects concerned with the validation of the equipment system and the accumulation of dynamic data; and those subjects dealing with the execution of the equipment package.

Each chapter is a complete treatise, having its own terminology, selected bibliography, and handbook-type information to support the technology. Each chapter also relates to the general problem of shock and vibration, and illustrates the utility of the speciality in improving structural reliability. In all chapters, the information is presented in the language of the Mechanical Engineer and stresses the structural design and execution aspects of the equipment packaging problem.

VOLUME III

RELATED TECHNOLOGIES

Chapter 1 - Basic Mechanics

Chapter 2 - Natural Frequency

Chapter 3 - Mechanical Impedance

Chapter 4 - Stress Concentration

Chapter 5 - Fatigue

Chapter 6 - Dynamic Simulation

Chapter 7 - Instrumentation

Chapter 8 - Fragility

Chapter 9 - Dynamic Attenuation

Chapter 10 - Materials and Processes

Chapter 11 - Packaging Design Techniques

CHAPTER 1 — BASIC MECHANICS

VOLUME III

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Chapter 1 - Basic Mechanics

ERRATA SHEET

Page	Paragraph	Line	Correction
1.1-2	4	6	lbf = $(\frac{1}{g_o})$ lbm (ft/sec^2)
1.2-2	1	2	provided by the principles...
1.3-2	3	8	moment
1.3-3	4	2	horizontal
1.3-4	2	2	varies
1.5-2	3	Top Eqn.	W/60 EIL^2
1.5-2	3	Last Eqn.	Add "at x = o"
1.5-11	2	Last Eqn.	$e \approx \frac{b}{2 + h/3b}$
1.5-11	2	Add note	Note also that (t) drops out of the approximate equation for (e). Shear center location is thus nearly independent of the thickness of an open section having thin flanges.
1.5-18			Change Xi (ξ) to Zeta (ζ)

VOLUME III - CHAPTER 1
RELATED TECHNOLOGIES

Section		Page
1	INTRODUCTION	1.1-0
	• Idealizations Used in Engineering Mechanics . .	1.1-0
	• Basic Concepts of Engineering Mechanics . . .	1.1-2
2	ANALYSIS TECHNIQUES	1.2-0
	• Equilibrium Methods in Analytical Mechanics Problems	1.2-0
	• Variational Methods for the Solution of Equilibrium Problems	1.2-2
	• Equilibrium Methods of Analysis for Common Engineering Structures	1.2-4
3	EVALUATION OF STRESS EFFECTS	1.3-0
	• The Effect of Material Behavior on Design of Structures	1.3-0
	• Shear and Bending Stresses in Beams	1.3-2
	• The Shear Center Concept in Beam Loading . . .	1.3-4
	• Torsional Stresses	1.3-6
	• Geometric Considerations in Determination of Stress Levels	1.3-8
	• The Deflection of Structural Members Due to Load	1.3-12
4	DESIGN CONSIDERATIONS	1.4-0
	• Design Criteria and the Theory of Elastic Failures	1.4-0
	• Lateral Buckling of Beams	1.4-2
	• The Response of Single-Degree-of-Freedom, Undamped Systems	1.4-4
	• The Response of Single-Degree-of-Freedom Systems with Viscous Damping	1.4-6

CONTENTS (Continued)

Section		Page
5	APPENDIX	1.5-0
	• Bibliography	1.5-0
	• General Method of Solution for Equilibrium Problems	1.5-1
	• Elastic Characteristics of Some Common Beam Configurations	1.5-2
	• Material Characteristics	1.5-4
	• Column End Fixity Coefficients for Various End Conditions	1.5-5
	• Summary of Beam Columns	1.5-6
	• Beam Column Coefficients	1.5-7
	• The Calculation of Horizontal Shear Stress in Beams	1.5-8
	• The Calculation of the Shear Center for a Channel	1.5-11
	• The Calculation of Torsional Stress in Non-Circular Sections	1.5-12
	• An Application Using Mohr's Circle	1.5-14
	• The Calculation of the Response of Damped Systems	1.5-16
	• Shear and Moment Diagrams for a Simple Beam	1.5-22

VOLUME III - CHAPTER 1

BASIC MECHANICS

SECTION 1 - INTRODUCTION

- Idealizations Used in Engineering Mechanics
- Basic Concepts of Engineering Mechanics

IDEALIZATIONS USED IN ENGINEERING MECHANICS

Engineering mechanics is based on the representation of the physical world by a hypothetical, highly simplified model. The restrictions involved in the use of these simplifications must be understood in order to allow their application without loss of meaning.

Analytical mechanics like all analytical sciences is based on the representation of the physical world by a hypothetical, highly simplified model. The concepts involved in these simplifications are reviewed in order to clarify the restrictions involved in their use.

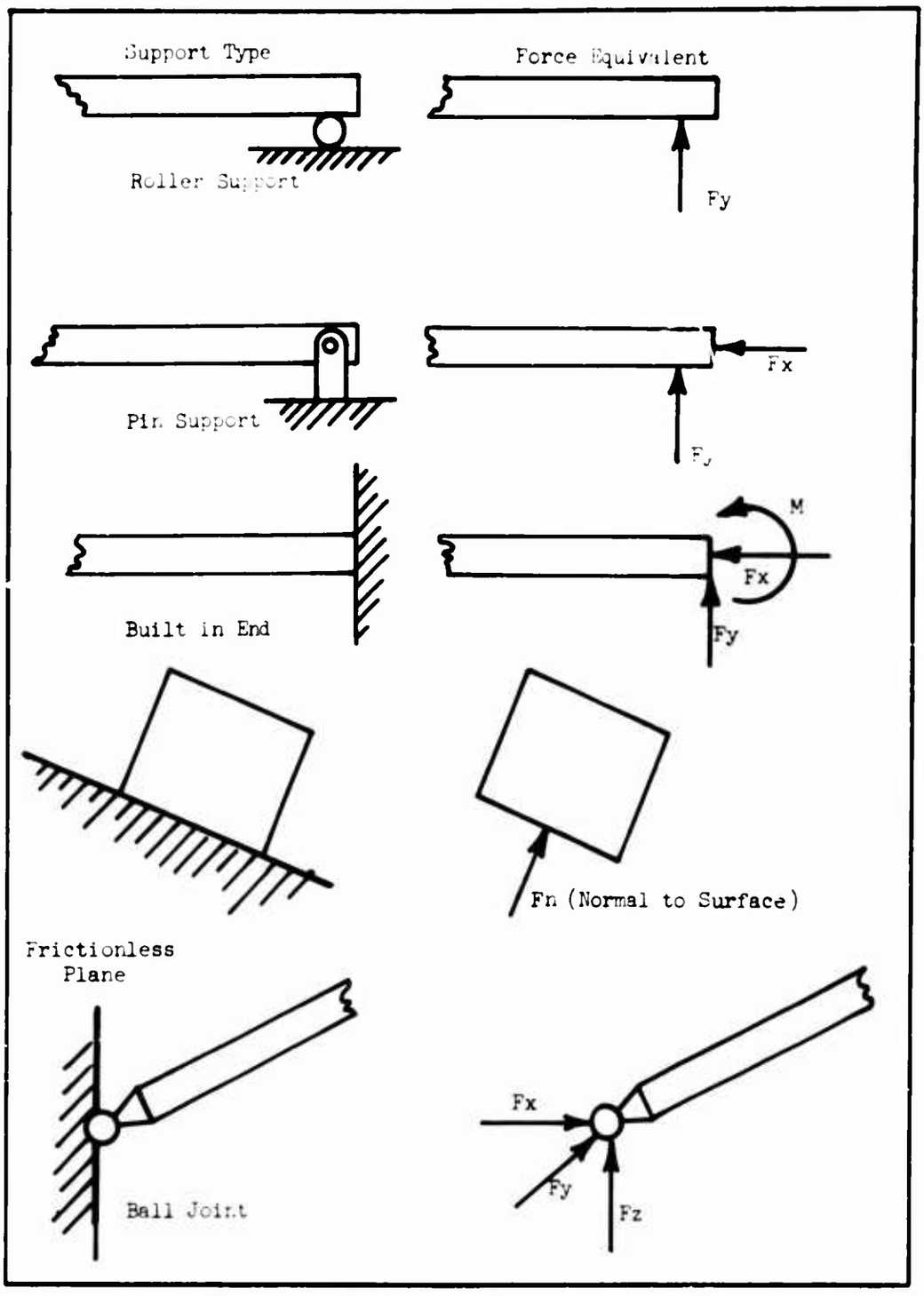
The Continuum: In problems where only the average measurable reactions of bodies are of interest the bodies may be assumed to consist of a continuous distribution of matter, rather than an association of elementary particles.

The Rigid Body: In problems that involve the determination of the forces acting on a body in response to some applied force, it is often possible to neglect the deformation of the body under the applied load. If the deformation of a body is of such extent that the final orientation of the loads applied is unknown, this assumption is invalid.

The Particle: For use in mechanics problems, a particle is defined as a body that has mass but not size. This assumption is valid and useful in problems that involve the action of a body under the influence of body forces, such as gravity.

The Point Force: Any force acting on a real body through a point of contact will cause local deformation resulting in a finite area of contact. In many problems this contact area may be ignored and the force considered as acting at a point.

Free Body Diagrams: A free body diagram is a sketch of a body or portion of a body, with all interacting bodies removed and replaced with equivalent forces. In order to construct a free body diagram, the forces equivalent to various types of supports must be known. The figure shown at the right gives the force equivalents of common types of supports.



FREE BODY DIAGRAMS: An equivalent force system representing a portion of a complex system is the basis of most mechanical analysis.

BASIC CONCEPTS OF ENGINEERING MECHANICS

The formulation and solution of engineering mechanics problems consists of the application of the basic laws within a standard set of dimensions and units.

The solution of problems in analytical mechanics depends on the logical application of the laws of mechanics in a consistent manner. This involves the definition and use of a standard set of dimensions and units, and understanding of the basic laws themselves.

Dimensions and Units: The choice of independent basic dimensions to be used in mechanics is somewhat arbitrary. The two most common English systems used are Force, Length, and Time (FLT); and Mass, Length, and Time (MLT). Of these two, the second is preferred on philosophical grounds as Mass is a property of matter whereas Force is a defined quantity. In either case, the three dimensions constitute the set of independent dimensions for mechanics problems and all other quantities are defined in terms of them. The units attached to the basic dimensions to be used in this handbook (MLT) are the following:

Mass:	pounds
Length:	feet
Time:	seconds

In the alternative (FLT) system, slugs would be used as the mass unit. This has the advantage of eliminating the gravitational constant from the Force-Mass relationship. In this text the pound mass (lbm) unit will be used on the grounds that it is a more familiar unit for most engineers. With the lbm chosen as the mass unit, the force unit (pound force, lbf) is defined as:

$$\text{lbf} = g_o \text{lbm} \ (\text{ft/sec}^2)$$

Where g_o is the gravitational attraction of 1 lbm under standard conditions, the quantity g_o is then the proprotionality constant relating force and mass.

$$g_o = \text{lbm ft/lbf sec}^2 = 32.2$$

Dimensional Homogeneity: Any valid analytical formulation of a physical law must be independent of the system of units used. This is apparent from the fact that a change in the system of units does not affect the phenomena described. The consequence is that all terms in an equation must have the same dimension when reduced to basic dimensions. This serves as a check on the validity of any derived relationship.

The Laws of Mechanics: Analytical mechanics is based on a relatively few basic laws. These consist of Newton's three laws of motion, the law of gravitational attraction, and the parallelogram law. A summary of these laws is given at the right. Newton's first law of motion may be interpreted as defining the coordinate systems in which the second law is valid. These systems, called inertial reference systems, consist of those which are fixed or in uniform motion with respect to the fixed stars. A coordinate system fixed to the earth's surface is not an inertial system. However, for many engineering problems the second law may be assumed to be valid in this coordinate system. The errors introduced are generally negligible except when the displacements involved are large.

1. Every body continues in a state of rest or uniform motion in a straight line unless acted upon by external forces.

2. The acceleration of a body is proportional to the force acting on it and in the direction of the force.

3. To every action there is always opposed an equal reaction.

Law of Gravitational Attraction

Two particles are attracted toward each other along their connecting line with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Parallelogram Law

The equivalent of two forces applied at a point is their vector sum.

NEWTON'S LAWS: The basic laws of mechanics and dynamics have been summarized by Newton.

VOLUME III - CHAPTER 1

BASIC MECHANICS

SECTION 2 - ANALYSIS TECHNIQUES

- Equilibrium Methods in Analytical Mechanics Problems
- Variational Methods for the Solution of Equilibrium Problems
- Equilibrium Methods of Analysis for Common Engineering Structures

EQUILIBRIUM METHODS IN ANALYTICAL MECHANICS PROBLEMS

Application of the laws of mechanics to the determination of unknown forces acting on a body frequently involves the consideration of equilibrium conditions only.

The first and second laws of motion state that for a body in equilibrium (one that is at rest or in uniform motion with respect to an inertial reference) the sum of the forces acting on the body must equal zero.

In solving for the unknown forces acting on a body, the equilibrium equations are applied at a point, generally the center of the coordinate system. In order to apply the equilibrium equations at this point, the forces and moments acting on the body must be replaced by an equivalent set of forces and moments acting at the point. For rigid body problems an equivalent system of forces is defined as one which results in the same total force and moment vectors on the body. The following rules define the development of equivalent force systems:

1. Moment vectors are free vectors; they may be moved to any point in space providing the magnitude and direction of the vector is not changed.

2. Force vectors are transmissible vectors; they may be moved along their line of action.

3. If a force vector is moved to a position parallel to its original position, it must be supplemented by a moment vector equal in magnitude to the force-distance product.

Summarizing the equivalent force-moment system in vector notation:

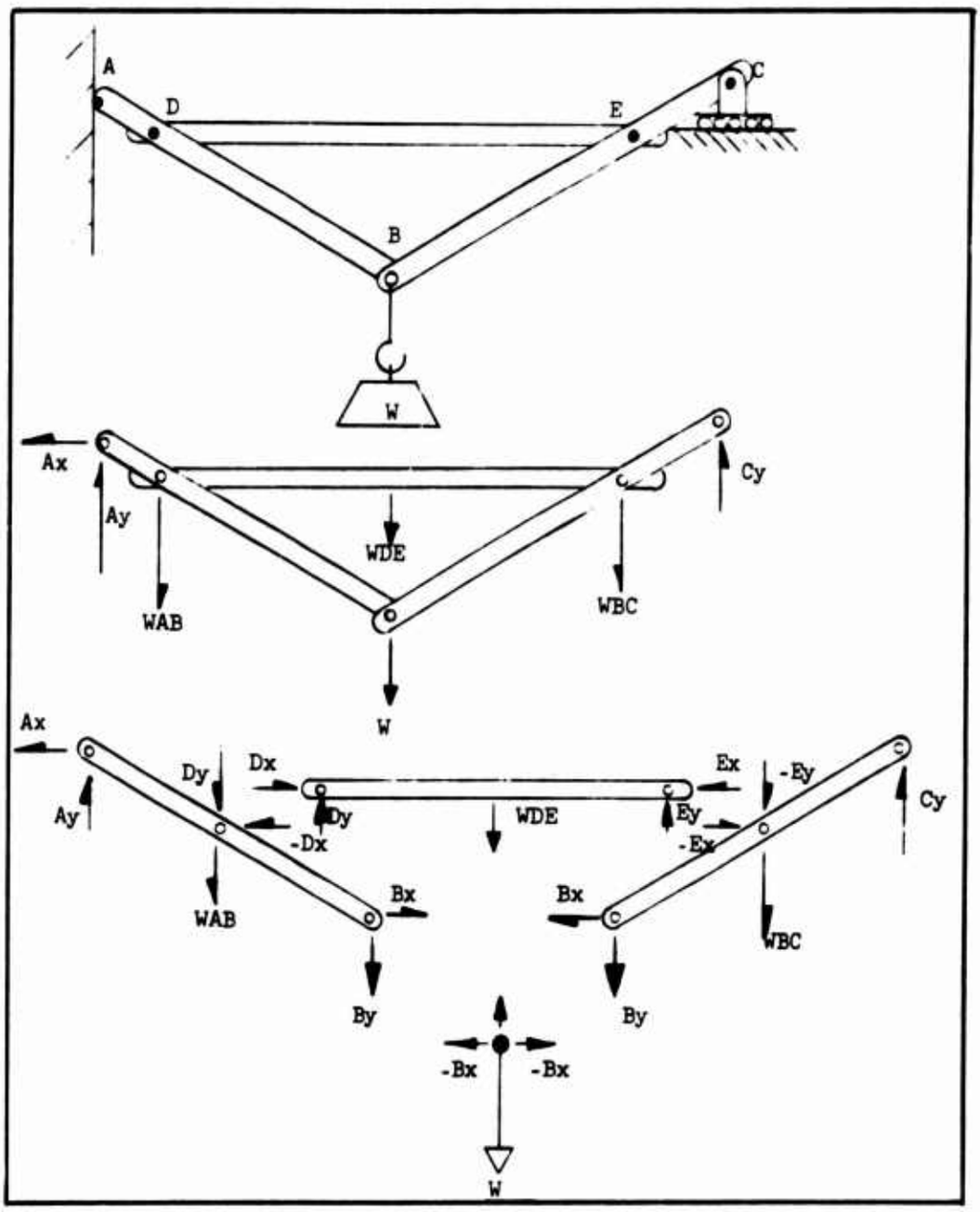
$$\vec{F}_e = \sum_{i}^{n} \vec{F}_i \; ; \; \vec{M}_e = \sum \vec{R}_i \times \vec{F}_i + \sum \vec{C}_i$$

The equilibrium equations for a body are therefore:

$$\sum \vec{F}_i = 0 \; ; \quad \sum \vec{R}_i \times \vec{F}_i + \sum \vec{C}_i = 0$$

In solving for unknown forces acting on a system of structural members the first step is to construct a free body diagram of the entire system. The equilibrium equations are expanded to scalar form and applied to the system free body diagram. Simultaneous solution of this set of equations determines the unknown forces if the number of unknowns is now more than the number of unknowns. If there are more unknowns than equations, the system free body diagram must be broken down into component free body diagrams and the equilibrium equations applied to the individual diagrams. The adjacent Figure illustrates the development of a free body diagram for a system of linkages and for the component parts. Note that in assuming a direction for the forces at a link connection, the forces on the interacting bodies must be assumed in opposite directions in accordance with the third law of motion. The weight attached to the linkage in the figure is assumed to transfer its load through the pin at B. This pin is shown as a separate free body. An alternative method of diagramming the same problem would be to cut the entire system at some plane and show the free body diagram in terms of the

internal forces in the links at the cutting plane. In any case, if when a free body diagram has been constructed for each of the component parts, there are still more unknowns than equations of equilibrium, the system is statically indeterminate and determination of the unknown forces will involve the consideration of deflections.



EQUILIBRIUM DIAGRAMS: The decomposition of structure into free body diagrams of component parts is a fundamental tool of force-system analysis.

VARIATIONAL METHODS FOR THE SOLUTION OF EQUILIBRIUM PROBLEMS

An alternative to direct application of the equations of equilibrium to the determination of unknown forces in equilibrium systems is provided for the principles of virtual work and minimum potential energy.

In developing the principle of virtual work, the forces acting on a body are considered in two classes, the active forces (external) (K_i) and the constraining forces. The constraining forces are those representing the interaction of the body with a rigid constraining surface in space. If the body is given a hypothetical infinitesimal displacement (δr) along the constraining surface, the work done will be equal to the dot product of the displacement and the forces acting on the body:

$$\text{Work} = \delta_r \left[\Sigma K_i + N\right] \qquad \text{Work = Forces Acting x Displacement}$$

From the equilibrium equations, the quantity $\Sigma K_i + N = 0$, therefore, an alternative statement of the equilibrium condition is:

$$\Sigma \delta r_i \, (K_r)_i = 0$$

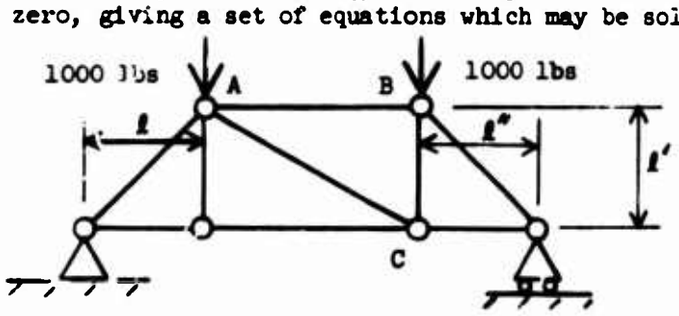
The principle of virtual work may be extended to include frictional forces acting at the constraining surfaces if these forces are considered as active forces opposing the impending motion, that is, the direction of the frictional forces is independent of the virtual displacement. Similarly, the constraints on the system may be removed by considering virtual displacements normal to the constraining surfaces if the surface forces are then treated as active forces. In effect, a degree of freedom is being added to the system with the removal of the constraint.

For conservative system, that is, those without frictional effects and with forces expressible as the gradient of a scalar function, another alternative statement of equilibrium may be formulated. This is based on the fact that for a conservative system the work done and, consequently, the change in potential energy, in displacement along any path is independent of the path and a function of the end points only. If the change in potential energy of such a system is written in terms of an expansion of the force field it can be shown that for first order effects the potential energy variation for a small displacement from a position of equilibrium is zero. The equilibrium conditions for the system may consequently be expressed in the following form:

$$\frac{\partial p}{\partial q_i} = 0$$

Where: p = potential energy
q_i = an independent variable

In solving problems by this method, the potential energy of the system is written in terms of a convenient set of independent variables and the derivatives of the potential energy with respect to each of the variables set equal to zero, giving a set of equations which may be solved for the unknown forces.

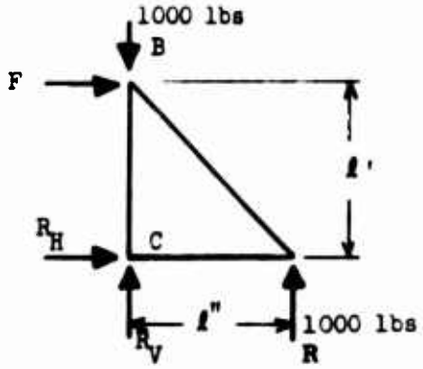


GEOMETRY OF ILLUSTRATIVE PROBLEM

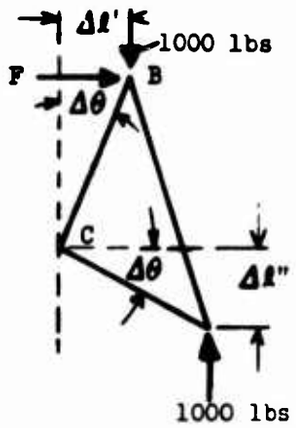
Illustrative Problem: To determine the force in member AB.

Solution: By symmetry the support reactions are 1000 pounds each.

A free body diagram shown below holds the body in equilibrium. The desired force is (F). The reactions at (C) are (R_H) and (R_V). The reaction at the roller is (R).



Now permit a virtual angular displacement of the free body about point (C). The displacement angle ($\Delta\theta$) is arbitrarily small.



The infinitely small displacement angle results in infinitely small displacement distances ($\Delta\ell'$) at point (B) and ($\Delta\ell"$) at the roller reaction. There is a work contribution by the roller reaction (R) and the force (F) but the contribution by the 1000 lbs weight at (B) is zero since the displacement is perpendicular to the weight. Hence,

$$F\,(\ell'\Delta\theta) = R\,(\ell"\Delta\theta)$$

which gives

$$F = R\,\frac{\ell"}{\ell'} = 1000\,\frac{\ell"}{\ell'}$$

For $\ell" = \ell'$ as given, F = 1000 lbs (compression) the compression arises from considering moments about point (C). The force (R) exerts a CCW moment hence the force (F) must exert a CW moment. For this the force (F) must represent a compression in member (AB).

EQUILIBRIUM METHODS OF ANALYSIS FOR COMMON ENGINEERING STRUCTURES

Determination of the internal forces in structures is simplified by classifying the structures according to function and loading, such as trusses and beams, and by using appropriate methods of analysis for each category.

Trusses: A truss is a system of uniform members constructed to support loads. It may have welded, pinned, or riveted joints. In analyzing trusses as rigid bodies, it is assumed that all joints are pinned or joined with ball and socket joints and that all loading is at the joints with the members themselves weightless. Trusses may be "just rigid," that is, statically determinate, or they may be over-rigid. Over-rigid trusses (i.e., those whose rigidity is not destroyed by the removal of one or more of the members) are statically indeterminate and deformation must be considered in their analysis. The necessary criteria for the determination of a "just rigid" truss is:

$m = 3j - 6$ for three-dimensional trusses
$m = 2j - 3$ for two-dimensional trusses

Where:

m = the number of members
j = the number of joints

The criteria specified is necessary but not sufficient; that is, it is possible to construct a truss which satisfies these criteria but is not rigid. Forces in the members of a truss are determined by applying the equations of equilibrium to a free body consisting of a portion of the truss. Generally the free bodies chosen consist either of the pins or sections through the members.

Beams: A beam is defined as a member that is subjected to a transverse load. Because of the transverse loading, the equivalent force system at a cut section of the beam consists of both a force and a moment. The force is considered in terms of its normal or shear axial components. The sign convention for shear and bending moments is defined so that a net vertical force upward on the left free body when the beam is split is positive and so that a bending moment which would cause the beam to deform concave upward is positive. Sign conventions are illustrated in the adjacent figure. Shear and bending moment diagrams are graphical plots of the shear and moment forces along a beam. Analysis of a beam segment shows that shear and moment relations at any point are as follows:

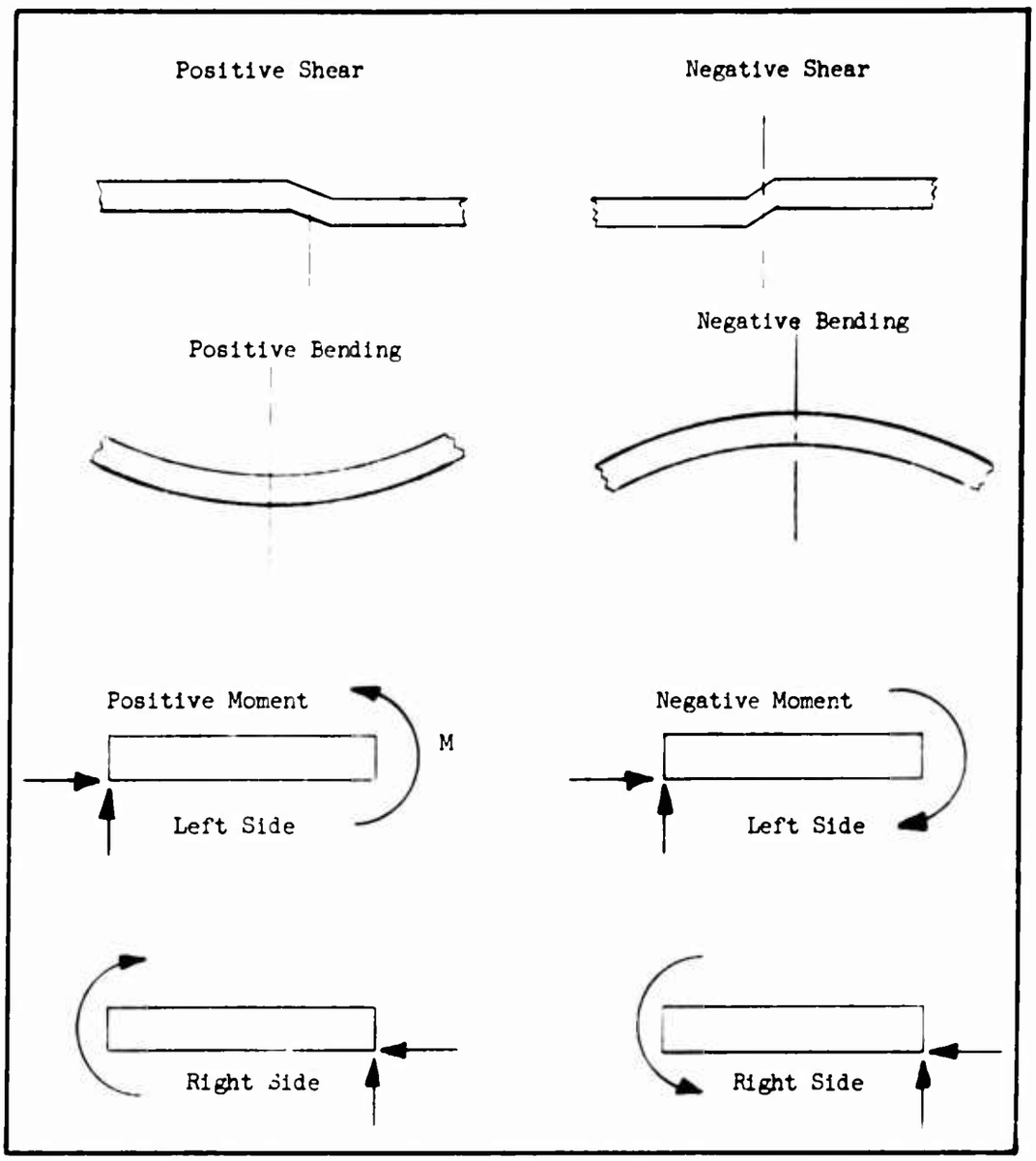
$$\frac{dV}{dx} = w$$

$$\frac{dM}{dx} = V$$

Where:

V = Shear
M = Moment
w = Load
x = Axial Dimension

Shear and Moment Diagrams are illustrated in the Appendix on page 1.5-22.



SIGN CONVENTIONS: An understanding of the sign convention of the shear and moment directions in beams is necessary for proper numerical evaluation.

VOLUME III - CHAPTER 1

BASIC MECHANICS

SECTION 3 - EVALUATION OF STRESS EFFECTS

- The Effect of Material Behavior on Design of Structures
- Shear and Bending Stresses in Beams
- The Shear Center Concept in Beam Loading
- Torsional Stresses
- Geometric Considerations in Determination of Stress Levels
- The Deflection of Structural Members Due to Load

THE EFFECT OF MATERIAL BEHAVIOR ON DESIGN OF STRUCTURES

The behavior of structural members under load is a function of the properties of the material used, and will generally constitute the limiting design criteria.

The determination of unknown forces in structures discussed in the previous paragraphs was based on the consideration of the structural members as rigid bodies. The distribution of internal forces in members and their effect on the material forming the members was ignored. In most structural design problems, these effects will constitute the limiting design criteria and consequently must be considered.

The stress in structural members at a cutting plane is considered in terms of the stress or force per unit area distributed over the cross section. The internal forces in a member are divided into forces normal and parallel to the cutting plane employed in the free body section. Stresses are defined in terms of the normal and parallel forces per incremental area as:

$$\text{Normal Stress:} \quad \sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

$$\text{Parallel (shear) Stress:} \quad \tau = \lim_{\Delta A \to 0} \frac{\Delta V}{\Delta A}$$

Where: F = normal force

V = parallel force

The resultant force at the plane determined by equilibrium conditions is equal to the integral of the stress over the plane area:

$$F = \int_A \sigma \, dA$$

In order to have a uniform distribution of stress at a free body cutting plane, the resultant force vector must act through the centroid of the cutting plane area.

The elastic strain in a material under load is defined as the deformation per unit length resulting from stress.

$$\text{Strain (e)} = \frac{\Delta L}{L}$$

The relation between stress and strain is independent of the size or length of the material involved. Plots of stress versus strain for many structural materials show a linear relationship for the initial portion of the stress-strain diagram. This is known as Hooke's Law. The proportionality constant relating stress and strain over the linear portion of the curve is called the Elastic Modulus or Young's Modulus of the material. Ductile materials such as mild steel will show a definite change in the slope of the stress-strain curve at the point where the linear Hooke's Law ceases to apply. (Typical stress-strain curves are shown in the Appendix, page 1.5-4. This point is defined as the Yield Point of the material. In very ductile materials, the curve of stress versus strain may have a flat spot at this

point with an increase in strain occuring at the constant stress level. For brittle materials without a well defined limit of stress-strain proportionality the Yield Point or Elastic Limit of the materials is defined in terms of a percentage of permanent deformation. Until the Elastic Limit is reached, the material will return to its original dimensions when the load on it is removed. This property is called Elasticity. The Yield Point represents the Elastic Limit of the material and increases in stress above the Yield Point stress will result in permanent deformation. Most structural materials show the same stress-strain relationship for compression loading as for tension. Some exceptions are cast iron and cast magnesium, both which are weaker in tension than in compression.

When a member is stressed in a specific direction by the application of a force, it undergoes a strain proportional to the stress in the direction of the loading, as discussed. In addition, the material undergoes an expansion or contraction in the direction normal to the load direction. This deformation or strain may be considered as an attempt by the material to maintain constant volume. The axial and normal strains are related in the elastic range of the material by a proportionality constant known as Poisson's Ratio:

$$\text{Poisson's Ratio } (\mu) = \frac{\text{lateral strain}}{\text{axial strain}}$$

where the strains are the result of a uni-axial stress only. Poisson's Ratio for most steels is about 0.3. The lateral strain due to the Poisson Effect does not involve additional stresses in the material unless the transverse deformation is prevented by some restraint.

Consideration of shear stress on a two-dimensional element shows that the shear stresses existing on the elements surfaces must be equal and directed toward opposite corners to satisfy equilibrium conditions. The element will undergo an angular deformation through an angle (γ) as a result of the stresses imposed. The angle of deformation is related to the applied shear stress by a constant of proportionality, called the Modulus of Rigidity (G) or shear modulus of elasticity.

$$\tau = G\gamma$$

Where:

τ = Shear Stress (psi)
G = Modulus of Rigidity (psi)
γ = Deformation Angle (radians)

It can be shown that the modulus of Elasticity (E), the modulus of Rigidity (G), and Poisson's Ratio are related as follows:

$$G = \frac{E}{2(1 + \mu)}$$

SHEAR AND BENDING STRESSES IN BEAMS

Shear and bending stresses in beams are basic to the understanding of the mechanisms of flexure and torsion.

Shear and bending stresses in beams involve geometric properties other than area at the section and are consequently less intuitive. The determination of these stresses is, therefore, considered in more detail before methods of combining stresses are discussed.

Bending Stresses in Beams: The analysis of deflections of beams in pure bending is based on the assumption that plane sections taken normal through the beam remain plane after bending and that Hooke's Law applies. The normal stresses in the beam due to bending, consequently, vary linearly as the distance from the neutral axis of the section. Application of the equations of equilibrium to a symmetrical beam section with external moment loading in a plane parallel to either principal axis is given by the following relation between applied mement and beam stress,

$$\sigma = My/I$$

Where:

M = externally applied moment
y = distance from internal axis
I = moment of inertia of section.

The same equilibrium conditions show that the netural axis of the beam passes through the centroid of the cutting plane area. The peak stress in the beam due to bending will, therefore, occur at the maximum distance from the centroid. This derivation is based on the assumption of a homogenous beam material. Beams of more than one material may be treated in the same manner by first reducing the problem to an equivalent single material beam. This is done by changing the dimensions perpendicular to the axis of symmetry by the ratio of the elastic moduli of the materials.

Shearing Stresses in Beams: In the consideration of beams in the section on statics it was stated that $dM/dx = V$. Therefore, at an interval of beam where there is no shear loading there is no change in the bending moment. The axial force in a beam due to the bending moments is, from the equation given above;

$$\frac{dF}{dA} = \frac{My}{I} \quad \text{or} \quad F_x = \int_A \frac{My}{I}\, dA$$

differentiating with respect to x:

$$\frac{dF}{dA} = \int \frac{dM}{dx}\frac{y}{I}\, dA = V\int \frac{y}{I}\, dA$$

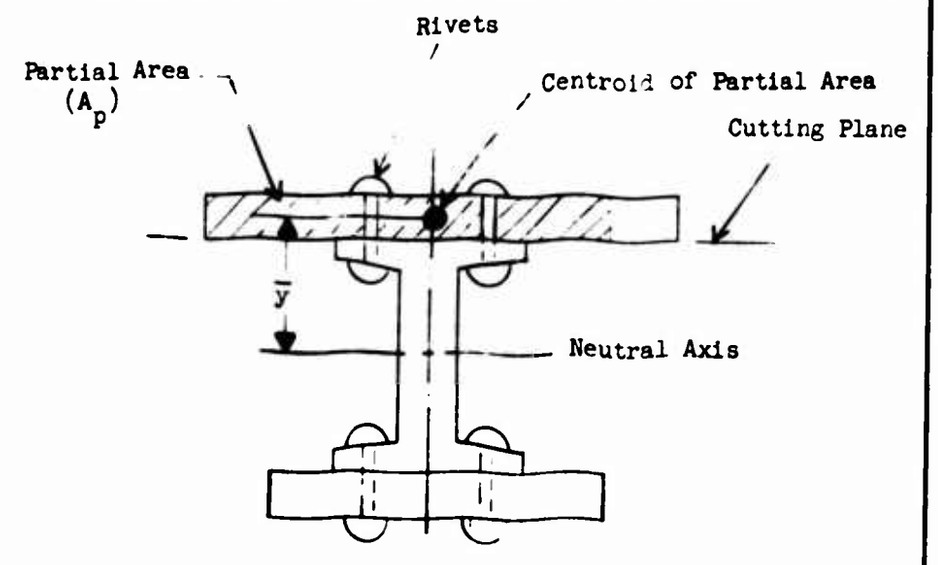
This quantity is defined as the Shear Flow (q):

$$q = \frac{dF}{dx} = \frac{V}{I}\bar{y}A = \frac{VQ}{I}$$

where (Q) is the value of the integral $\int y\, dA$ for the partial area between the axial cutting plane and the outer fibers of the beam. The resultant shear stress for a thin member with thickness (t) at the cutting plane is given by:

$$\tau = \frac{1}{t}\frac{dF}{dx} = \frac{VQ}{It}$$

The maximum value of the shear stress will occur at the neutral axis of the beam. For a beam of rectangular section the maximum value of the shear stress will occur at the neutral axis of the beam. The expression for shear flow is of interest in the determination of required fastener strength in composite beams. An illustration is given in the figure below.*



$$q = \frac{V}{I}\bar{y}A_p\left(\frac{lb}{in.}\right) \qquad q = \text{shear force per unit length}$$

CROSS SHEAR: The horizontal shear stress in the attachment of a composite beam is a function of the vertical shear and the section properties of the beam parts.

*A sample problem further illustrating the calculation of rivet stresses in composite beams due to horixontal shear is given in the Appendix on Page 1.5-8.

THE SHEAR CENTER CONCEPT IN BEAM LOADING

Applied loads on a beam must act through the shear center of the beam in order to avoid twisting moments about the beam axis.

Application of the shear flow equation to the symmetrical I beam shows that the shearing stress in the flanges varies linearly from zero at the outer edge to a center-line value. The zero value at the outer fiber is required from the boundary condition for a free surface. Similarly, the value of the shear stress in the top surface must be zero. If an element in the lower surface of the upper flange is considered in terms of the same boundary conditions, a contradition is apparent. The shear stress in the lower surface of the flange must equal zero from the free boundary condition, however, the shear stress in the web of the beam at this level is not zero. A discontinuity in shear stress will, therefore, exist at the point where the lower surface of flange intersects the web. This is in violation of the requirement that shear stresses on opposite sides of a cutting plane be equal and opposite. Resolution of this contradition involves mathematical methods beyond the scope of an introductory treatment of the subject. The formulas derived in this chapter are of sufficient accuracy for most design purposes, however, the shear stresses in the flange of a beam will generally be small and the stresses in the web are given by the derived formula with reasonable accuracy. The existence and direction of the shear stress in cutting planes which are not parallel to an axis of symmetry of a beam are of concern in the determination of loading points for beams which are nonsymmetrical in the loading plane. Inconsistency in the determination of stresses in these cases may be avoided by restricting the analysis to beams of thin section and considering properties at the section centerline only. The accompanying figure illustrates the shear stresses in a U-shaped beam. The elements shown at the top and bottom corners indicate the shear stress directions. The shear in the upper flange is directed to the left of the picture and that in the lower flange to the right. As a result, there is a moment at the section due to the shear forces which will tend to twist the beam about its axis. This moment may be avoided by positioning the applied loads so that an equal and opposite moment is developed. The point through which the external force must act in order to cancel this moment is called the shear center of the section. For the beam shown in the figure, the shear center will be located on the horizontal axis of symmetry a distance (e) from the back surface of the beam. The distance (e) may be determined by equating the moment due to the shear stress in the flanges to that developed by the externally applied force. The average shear stress in the flanges will be equal to one-half the value at the flange-web intersection. The moment due to the flange shear stress will be:

$$M_s = \left(\frac{\tau}{2}\right) bth \quad \text{(internal moment on beam section)}$$

The moment due to the externally applied force (P) will be:

$$M_e = P_e \quad \text{(external moment on beam section)}$$

Where P must equal the vertical shear force at the section.

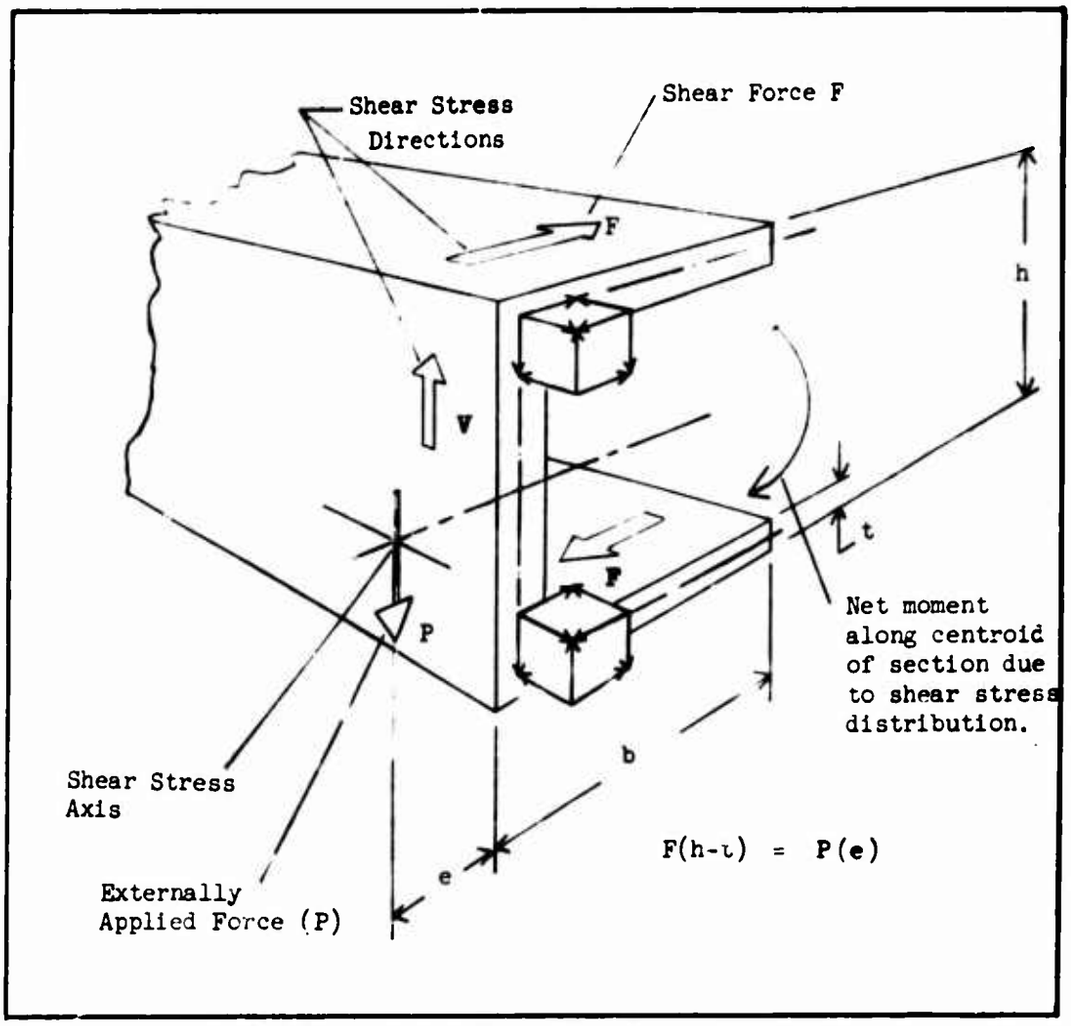
$$P = V$$

Applying $M_s = M_e$ with $P = V$

$$e = \frac{\tau bth}{P} = \frac{bth}{2P}\frac{VQ}{It} = \frac{bth}{2P}\frac{Vbt(h/2)}{It} = \frac{b^2h^2t}{It}$$

When any externally applied transverse force acts through the shear center, the member does not twist. The shear center is also commonly called the "Center of Twist".

An illustrative example dealing with the concept of shear center is given in the Appendix on page 1.5-1[illegible].



SHEAR CENTER: The equilibrium point of the moments due to the lateral shearing forces in a beam locates the beam shear center.

TORSIONAL STRESSES

Shearing stress due to torsional loading may be evaluated by a general equation for circular and non-circular cross-sections.

The determination of the shear stress in a member due to torque loading about the long axis of the member is based on the use of the equations of equilibrium to find the torque which must exist at a section in order to balance the external loading. The distribution of stresses at the section must be such that the integral of the shear stress over the area of the section equals the external torque. For the circular or tubular sections most generally used for torque loaded members, the internal stresses are assumed to vary linearly with distance from the neutral axis, and deformation is assumed to consist of rotation only. The use of these assumptions leads to the following relationship:

$$\tau = \frac{T\rho}{J}$$

Where:

τ = shear stress
T = torque loading
ρ = radius from neutral axis
J = polar moment of inertia for the section

The peak stress will, therefore, occur at the outer surface of the material, at the maximum distance from the neutral axis. For noncircular sections these assumptions are invalid. More involved methods of analysis may be used to show that in the case of noncircular sections the maximum stress occurs at the points on the outer boundary which are nearest to the centroid of the section. It may also be demonstrated that for a section consisting of a singly connected area of given cross section the torsional rigidity increases with decreases in the polar moment of the section. For a given amount of material a circular shaft will, therefore, have the largest torsional rigidity. Noncircular sections which are thin-walled and open, that is, where the walls of the member do not form a closed curve, may be treated by approximate methods which result in the following expression:

$$T = \frac{G\Phi}{L} \sum \frac{c_i {}^3 d_i}{3}$$

Where:

T = external torque
G = torsional modulus of the material
Φ = angular displacement over length L
c & d = section dimensions as illustrated in the adjacent figure.

This expression is of the same form as the corresponding equation for a circular shaft. The similarity is enhanced by defining an equivalent polar moment of inertia for the section as:

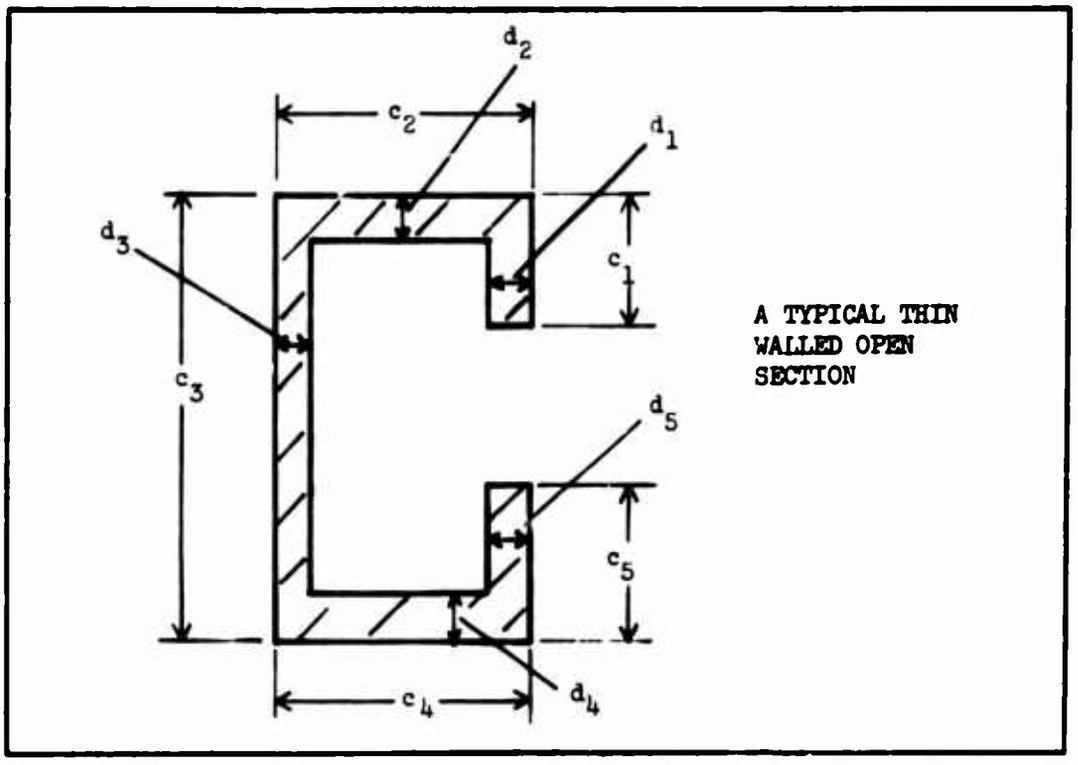
$$J_e = \frac{1}{3} \sum c^3 d$$

The resulting expression for the maximum shear stress in a particular rectangle is given by:

$$S_s = \frac{Tc}{J_e}$$

Application of the conditions of equilibrium to an element in a plane parallel to the axis of a member in torsion shows that the shear stress in an axial direction is equal in magnitude to the tangential stress. In the application of non-isotropic materials, such as cast iron, the lowest value of the shear stress should be used as the design criterion.

Common applications of open section used in torsion include channels and I beams. Several typical sections are analyzed in Appendix, page 1.5-12.



A TYPICAL THIN WALLED OPEN SECTION

TORSIONAL STRESS: Shear stress may be calculated in non-circular members by use of an equivalent polar moment of inertia concept. The above figure illustrates the approximate solution for a thin-walled, open section.

GEOMETRIC CONSIDERATIONS IN DETERMINATION OF STRESS LEVELS

The shear and normal stresses at a point in a structural member will depend on the orientation of the stressed element considered, and on the local geometry of the member.

Determination of the actual maximum values of the shear and normal stresses at a point in a member involves consideration both of the effect of combined stresses and of the stress concentrations caused by abrupt changes in cross section.

Combined Stresses: The total stress at a point in a structural member may be calculated by determining the stresses induced by each type of loading and combining the results in appropriate form. Normal stresses due to axial loading are determined directly by the average force per unit area at the cutting plane. Beam bending and shear stresses and stresses due to torsional loading have been discussed in the preceding paragraphs. These various types of stresses may be simultaneously present at a section of a loaded member. Combination of similar types of stresses at a section is based on the principle of superposition which states that the stresses may be added vectorially. The underlying assumption is that the stresses are linearly related to the forces causing them so that the stress addition is equivalent to force addition. This in turn implies that the total stress is within the elastic limit of the material. Within these restrictions, normal stresses may be added to normal stresses and shear stresses to shear stresses, whatever the source. The transformation equations for combining shear and normal stresses in a plane are developed by considering an infinitestimal element in equilibrium under the action of shear and normal stresses. The element remains in equilibrium when sliced at an arbitrary angle (θ). The stresses on the new surface (σ_θ) and (τ_θ) may then be written in terms of the known stresses on the other two faces from equilibrium conditions. The resulting values of equivalent shear and normal stresses at an angle (θ) from the x and y axis are given by:

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \left[\cos 2\theta\right] + \tau \sin 2\theta$$

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2} \left[\sin 2\theta\right] + \tau \cos 2\theta$$

The maximum values of shear and normal stresses may be found by differentiating the above expressions with respect to (θ) and equating to zero, giving:

$$\sigma_{\theta \max,\min} = 1/2\ (\sigma_x + \sigma_y) \pm \sqrt{\left[\frac{1}{2}\ (\sigma_x - \sigma_y)\right]^2 + \tau^2}$$

commonly referred to as the principal stresses, where

$$\tau_\theta = 0$$

with $$\tan 2\theta = \frac{\tau}{\frac{1}{2}\ (\sigma_x - \sigma_y)}\ ,$$

where θ corresponds to $\sigma_{\max,\min}$

and

$$\tau_{max,min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

at θ where

$$\tan 2\theta = -\frac{1}{2}\frac{(\sigma_x - \sigma_y)}{\tau}$$

and with

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2}$$

where θ corresponds to $\tau_{max,min}$

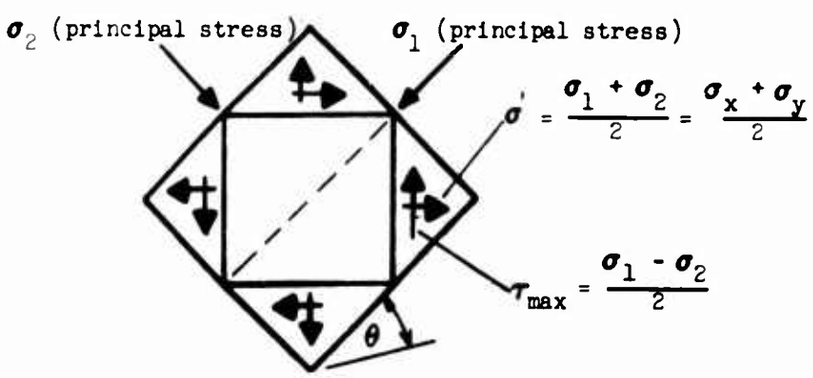
If (σ_x) and (σ_y) are the principal stresses then

$$\tau_{max,min} = \frac{\sigma_1 - \sigma_2}{2}$$

since $\tau = 0$ on planes which are associated with principal normal stresses.

It is noted that the tangents of the angles corresponding to maximum and minimum normal stresses (principal stresses) and maximum and minimum (tangential stresses) are the negative reciprocals of each other. Since these are double angle functions the angle between the principal stresses and the maximum and minimum tangential stresses must be 45°.

Once the principal stresses and associated elemental volume are determined from Mohr's circule, the construction below may be made; the correct directions for maximum tangential stress and the associated normal stresses on these faces are readily determined. Note is taken of the fact that τ_{max} points to the shear diagonal and that this shear diagonal always coincides with the algebraically greater principal stress. Also, the magnitude and sign of the normal stresses is given by the distance from the coordinate origin to the circle center.



GEOMETRIC CONSIDERATIONS IN DETERMINATION OF STRESS LEVELS (Continued)

The equations for shear and normal stresses at an arbitrary angle (θ) represent the parametric equations for a circle. If the equations are squared, added, and simplified, the result is:

$$\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_\theta^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2$$

This equation represents a circle of radius

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

with center at

$$x = \frac{\sigma_x + \sigma_y}{2}$$

$$y = 0$$

The determination of stresses at an angle (θ) is simplified by plotting this equation. The plot, generally referred to as Mohr's Circle, is developed below. The sign convention in the use of Mohr's Circle is established as follows. Figure (a) shows the normal and shear stresses on a typical elemental volume to be investigated. The stresses in the directions shown are all regarded as positive. In addition, the normal stresses shown represent tensile stresses. Figure (b) represents Mohr's Circle corresponding to the elemental volume. It has a radius and position calculated from the equations above. For the axes shown, stresses to the right or upward are positive. Stresses to the left or downward are negative. Note that since this particular circle lies to the right of the origin that all normal stresses must be tensile. This agrees with the elemental volume for which the circle was drawn. Had compressive stresses been associated with the elemental volume then by convention the circle would appear to the left of the origin.

Points (A) and (B) on the circle correspond to faces (A) and (B) on the elemental volume. At these points stresses are shown all positive. Had the shear stresses or the elemental volume been opposite in direction to those shown then faces (A) and (B) on the volume would be represented by points A' and B' on the circle.

The right hand face is usually taken as the reference face although this is not essential. It merely helps in being consistent. From point (A) on the circle construct lines to (σ_2) and (σ_1) as shown in Figure (e). The new orientation of the elemental volume is as shown in Figure (d). The principal stresses (σ_2) and (σ_1) are associated with the faces (A) and (B) as shown. (σ_1) is perpendicular to the line ($\overline{A\sigma_1}$) and (σ_2) is perpendicular to line ($\overline{A\sigma_2}$). Note that with respect to the origin of the coordinate system (σ_1) is positive and (σ_2) is negative. On the basis of the convention established in Figure (a) these are introduced accordingly in Figure (d).

Note must be taken of the fact that in the figures as drawn below, the stress on face (A) is larger than that on (B). This of course is not always the case. The construction is however always drawn from point (A) on the circle whether it is largest or smallest, positive or negative. An example in the Appendix on page 1.5-14 shows a more general case for clarity.

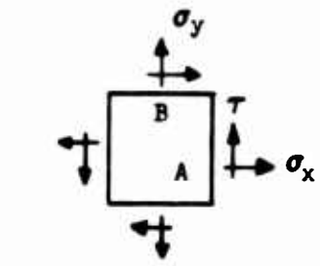


Figure (a)

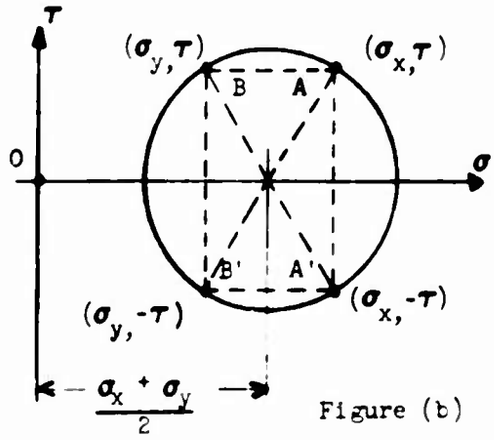


Figure (b)

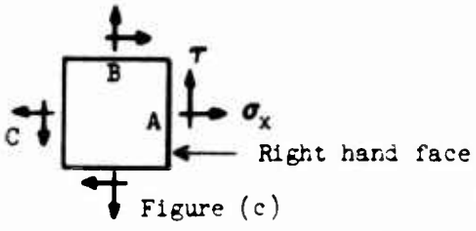


Figure (c)

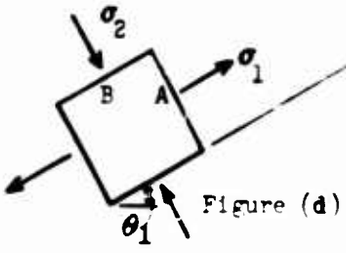


Figure (d)

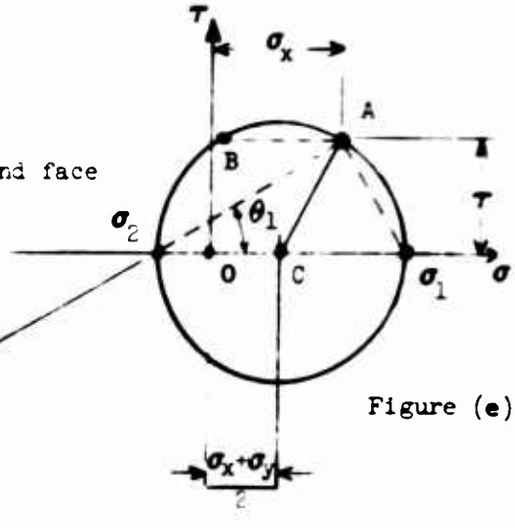


Figure (e)

Stress Concentration: The shear and normal stresses defined by the average force per unit area at a section are approximately correct for uniform material sections remote from the point of force application. Determination of the actual stress distribution within a geometrically complex member by analytical methods is extremely difficult. The generally accepted alternative is to use the average value of the stress and correct with a stress concentration factor (K) which has been experimentally determined. In cases that involve static loading of ductile materials, local yielding will tend to equalize stresses and reduced stress concentration factors may be used. The stress concentration effect in a dynamic situation, however, is more pronounced and more complex. A detailed treatise on this dynamic stress amplification problem is presented in Volume III, Chapter 7, "Stress Concentration."

THE DEFLECTION OF STRUCTURAL MEMBERS DUE TO LOAD

The deflection of a structural member is a function of the type of loading and may be calculated from the known loading conditions.

Determination of the compression or extension of a member under the effect of an axially applied load follows directly from the stress-strain relationship. Deflection due to other types of loading is discussed in the following paragraphs.

Deflection of Beams: When a beam deflects laterally under load the fibers on one side of the beam will be compressed and those on the opposite side stretched. Between these two extremes, there will be a plane of no deformation in length. The stress relations developed in previous paragraphs indicated that the neutral axis of a beam section was the point of zero stress in bending; consequently, it will be the undeformed axis or elastic curve of the beam. The strain of any other fiber will be proportional to the local stress: $\epsilon = \sigma/E$. The strain of a fiber located a distance (y) from the neutral axis of the beam may also be written as: $\epsilon = y/\rho$ based on the fact that the increase in length over a section of beam which bends through an angle $d\theta$ will be $(yd\theta)$ and the length on the neutral axis will be given by $(\rho\, d\theta)$ where ρ is the radius of curvature of the beam. Equating those expressions for the strain gives: $\sigma = Ey/\rho$, the stress at a distance (y) from the neutral axis due to bending. From the flexure formula previously derived, this stress is equal to My/I, where M is the moment causing the beam deflection. The relation between moment and curvature is, therefore, given by, $1/\rho = M/EI$. From analytic geometry the radius of curvature may be written in terms of the local derivatives of the elastic curve as:

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

which for small values of dy/dx may be approximated by

$$\frac{1}{\rho} = \frac{d^2y}{dx^2}$$

Substituting for moment in terms of curvature gives:

$$M = EI\,\frac{d^2y}{dx^2}$$

But it has been shown that $dM/dx = V$ and $dV/dx = W$, so the differential equations relating deflection, slope, moment, shear and loading on the elastic curve may be summarized:

y = deflection

$\frac{dy}{dx} = \theta$ = slope of elastic curve

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{d^3y}{dx^3} = \frac{V}{EI}$$

$$\frac{d^4y}{dx^4} = \frac{W}{EI}$$

Where in general (EI) may be a function of (x).

Boundary conditions for the equations will be dependent on the type of support. For roller or pin supports the deflection will be zero. For a fixed support both deflection and rotation (slope) at the support will be 0.

In complicated problems the beam may have to be considered by sections. An additional boundary condition between sections is then provided by the requirement of continuity of the elastic curve.

$$y_i = y_{ii}$$

$$\frac{dy}{dx_i} = \frac{dy}{dx_{ii}}$$

Shown in the Appendix on page 1.5-6 are deflections for common beam loading conditions. If a given problem is statically determinate the support reactions may be determined from a free body diagram of the beam and elastic curve equation solved separately. If the problem involves a statically indeterminate system the deflection equations must be written in general form and solved simultaneously with the equations of static equilibrium.

If a beam is loaded in more than one plane the deflections in a particular plane may be determined in terms of the loads in that plane and the results superimposed for the total deflection. This is possible, of course, only within the elastic limit of the material.

Deformation of Circular Members in Torsional Loading: From the definition of the Modulus of Rigidity (G) the angular deformation of a torque loaded element is given by $\gamma = \tau/G$. Consequently the angle of twist of a torque loaded member over a length AB will be:

$$\phi = \int_A^B \frac{Tdz}{JG}$$

or for a shaft with uniform loading and constant geometric properties:

$$\phi = \frac{TL}{JG}$$

For open, thin wall non-circular sections the same expression may be used with the equivalent polar moment of inertia.

VOLUME III - CHAPTER 1

BASIC MECHANICS

SECTION 4 - DESIGN CONSIDERATIONS

- Design Criteria and the Theory of Elastic Failures
- Lateral Buckling of Beams
- The Response of Single-Degree-of-Freedom, Undamped Systems
- The Response of Single-Degree-of-Freedom Systems with Viscous Damping

DESIGN CRITERIA AND THE THEORY OF ELASTIC FAILURES

In the design of load carrying structures the requirements for a member may be determined by strength criteria, by the allowable deflections, or by conditions of stability.

In using the design criteria based upon limiting strength, the maximum combined stress, either normal or shear, is determined as a function of the size of the member selected for the limiting values of allowable stresses. In designing for deflection the limiting condition is the displacement of the structure from its unloaded condition, where the displacement is proportional to stress but the stresses involved are within the elastic limit of the material and failure due to over-stressing is not a concern. The third possible design criteria is the stability of the structure. This may be considered as a special case of designing for limited deflection; however, in stability failures the deflections past some point are not proportional to the stress in its members and may increase catastrophically under constant loading with stresses well below the yield point of the materials. This type of failure is referred to as an elastic failure, resulting from an elastic instability of the structural geometry.

Examples of designs which are generally limited by elastic failure considerations are thin shells loaded externally, such as submarine hulls, and compression members having large ratios of length to cross section, commonly referred to as columns. As externally loaded shells are not usually found in electronic equipment this discussion of elastic failures will be limited to columns. When a column (with a marginal "slenderness ratio") is loaded compressively with increasing loads, slight increases in compression may be measured. However, no large deflections will be observed until some limiting load value, the critical force (P_{cr}), is reached. For load values equal to or greater than P_{cr} any lateral force on the column will result in buckling. The fact that no gradually increasing deflection warns of the impending failure makes instability failures dramatic. Such elastic failures are the most common type of structural failure under static loading, due to the design of members for strength criteria without stability considerations.

The critical force for a column may be determined from the equations of the elastic curve by considering a column deformed so that the axially applied compressive load is displaced a distance (y) from the neutral axis of the column at some point. The moment in the column at that point will then be proportional to (y) and the elastic curve may be written in differential form as:

$$\frac{d^2y}{dx^2} + k^2y = 0,$$

where k is defined as: $\sqrt{\frac{P}{EI}}$

The general solution to this equation is of the form $y = A \sin kx + B \cos kx$ where A and B are boundary condition constants. Application of the boundary conditions for a pin ended beam gives: $y = 0 = A \sin kl$, where the solution given by $A = 0$ corresponds to the case where no buckling exists. For any other solution $kL = n\pi$. Taking $n = 1$ as the minimum values gives:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Similar analysis for other end conditions gives the following formulas:

One end fixed, one free:

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

One end fixed, one pinned:

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

Both ends fixed:

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

A general expression for the critical load may be written as:

$$P_{cr} = \frac{c\pi^2 EI}{L^2}$$

Where (c) is a coefficient which depends on the column end conditions. Alternately, an effective length may be defined as:

$$\frac{1}{\sqrt{c}} L = L_G$$

in terms of the above equation. The critical load formula is then:

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

Values of the end coefficient and the reciprocal of its square root are summarized in the Appendix on page 1.5-5.

In applying the formulas listed to a non-symmetrical member, the minimum value of the moment of inertia is to be used. In applications where the end conditions for the column do not correspond exactly to any of the given conditions, pin-ended conditions are frequently assumed. A noteworthy feature of the critical force formulas is the fact that the only material property to appear is the modulus of elasticity. The critical force formulas may be converted to critical stress form by substituting $I = Ar^2$, where A is the cross sectional area of the column and r is the minimum radius of gyration. For the pin ended case this results in:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

The quantity (L/r) is defined as the slenderness ratio of the column. The Appendix, page 1.5-4 gives plots of σ_{cr} as a function of slenderness ratio of some typical engineering materials.

LATERAL BUCKLING OF BEAMS

Beams may exhibit elastic failure in a plane other than the principal loading plane if the difference in moment of inertia between the two planes is large, or if the beam is unusually long and slender.

Lateral Buckling of Beams: Beams which have a large ratio of vertical to lateral moment of inertia may exhibit elastic failure under load. The failure consists of buckling in the lateral plane due to loads in the vertical plane. If compressive loads are present, as in beam columns, the problem is intensified. For the case of loading in the vertical plane only, the following expression for the critical buckling stress has been derived:

$$S_{cr} = \frac{\pi^2 E}{2\left(\frac{L}{d}\right)^2} \sqrt{\left(\frac{I_y}{2I_x}\right)^2 + \frac{KI_y}{2(1+\mu)I_x^2}\left(\frac{L}{\pi d}\right)^2} \qquad (1)$$

Where:

S_{cr} = critical buckling stress, psi

E = modulus of elasticity, psi

L = length of span, laterally unsupported, in.

d = depth of beam, in.

I_x = moment of inertia about neutral axis for bending in plane of web, $in.^4$

I_y = moment of inertia about neutral axis for bending in the lateral direction, $in.^4$

K = torsional constant, $in.^4$

μ = Poisson's ratio.

If I_x is much larger than I_y, as in many standard I beams, the formula may be reduced to:

$$S_{cr} = \left(\frac{18.83 \times 10^6}{\frac{Ld}{bt}}\right)\left(\frac{1}{FS}\right)\left(\frac{E_{Alloy}}{E_{Steel}}\right) \qquad (2)$$

In which (b) is the total width of the compression flange, (t) its thickness, and (FS) is the desired factor of safety.

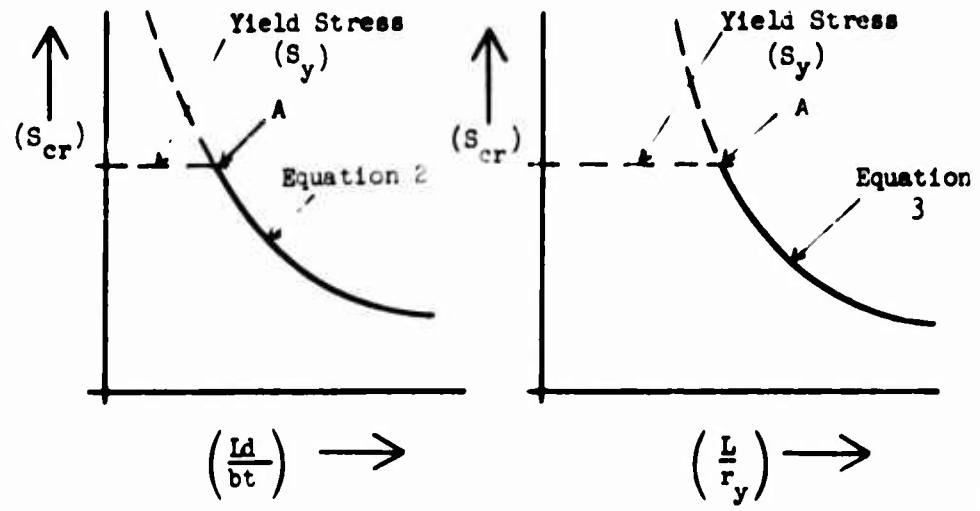
In the opposite case, where I_y is much larger than I_x, the formula reduces to:

$$S_{cr} = \frac{447.2 \times 10^6}{(L / r_y)^2} \qquad (3)$$

The theory of elastic failure criteria beam-columns is considered beyond the scope of this discussion. The reader is referred to the bibliography for more information on the subject. However, a summary of parameters for some common beam-column situations is given in the Appendix, page 1.5- .

CRITICAL STRESS FOR LATERAL BUCKLING OF BEAMS

(Equations 2 and 3)



LIMITING VALUES of Ld/bt AND L/r_y AT REFERENCE POINT A. (8)

S_y (ksi)	$\frac{Ld}{bt}$ Ratio	$\frac{L}{r_y}$ Ratio
33	571	116
45	418	100
50	377	95
55	342	90

BEAM BUCKLING: Lateral buckling in beams due to a torsional instability is a frequent and unexpected failure mode.

THE RESPONSE OF SINGLE-DEGREE-OF-FREEDOM, UNDAMPED, SYSTEMS

Prediction of the response of a structure to dynamic force inputs may be estimated on the basis of relatively simple analysis.

This review of dynamic systems is limited to those topics which it is felt will be of concern to the designer of electronic equipment. The cases considered consist of the application of Newton's laws of motion to systems involving the response of mass particles to applied forces. Newton's second law may be written as:

$$F = ma = m\,\frac{d^2x}{dt^2}$$

Successive integrations give the expressions for velocity and displacement:

$$\frac{dx}{dt} = V = \int_o^t \frac{F(t)dt}{m} + C_1$$

$$x = \int_o^t \left(\int_o^\eta \frac{F(t)}{m}\,dt + C_1 \right) d\eta + C_2$$

where force is assumed to be a function of time only.

The response of a mass particle in the case where the force is proportional to displacement from some equilibrium position ($F = -kx$) is of basic interest in the design of any dynamically loaded structure as an approximation to the deflection of a compact mass under the influence of some disturbing force. In this case, the second law of motion may be written as:

$$\frac{d^2x}{dt^2} + \frac{k}{m}\,x = 0$$

The general solution to this equation is given by:

$$x = C_1 \cos\sqrt{\frac{k}{m}}\,t + C_2 \sin\sqrt{\frac{k}{m}}\,t$$

where:

C_1 and C_2 are constants of integration

With initial conditions taken as $x = x_o$ and $V = V_o$, the values of the constants will be:

$$C_1 = x_o \qquad \text{and} \qquad C_2 = \frac{V_o}{\sqrt{\frac{k}{m}}}$$

or for a system initially at rest:

$$x = x_o \cos\sqrt{\frac{k}{m}}\,t$$

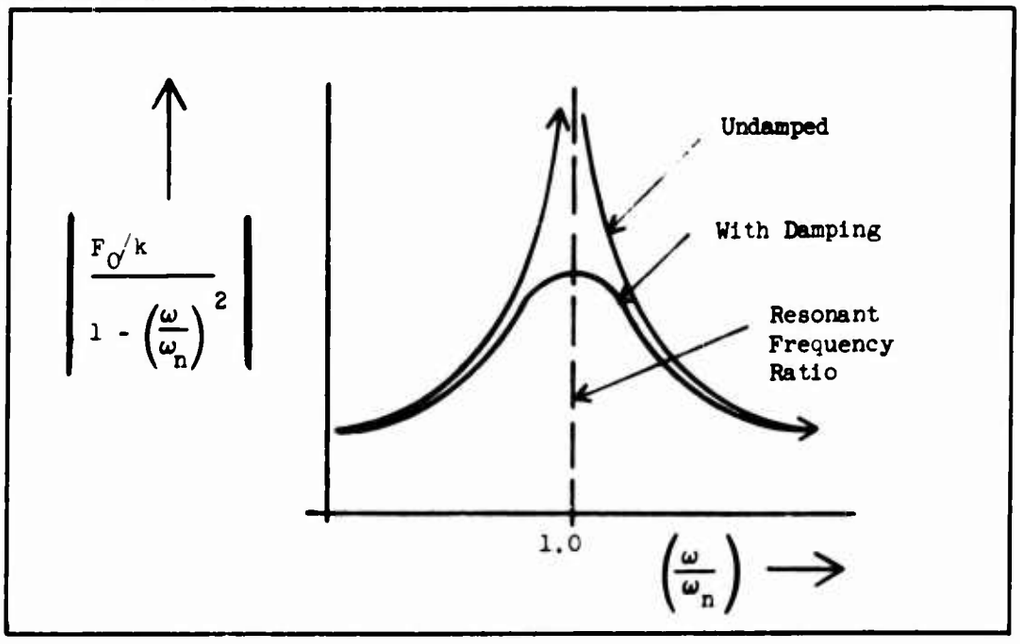
These equations describe what is called simple harmonic motion. The particle oscillates through an amplitude of x_0 at a frequency of $\sqrt{k/m}$, defined as the natural frequency of the system. In the case where the same system is excited by a continuously varying external force rather than being displaced and released, the equation of motion will be:

$$m \frac{d^2x}{dt^2} = -kx + F_0 \sin \omega t$$

where the external force is assumed to vary sinusoidally. The displacement solution in this case is given by:

$$x = C_1 \sin\sqrt{\frac{k}{m}}\, t + C_2 \cos\sqrt{\frac{k}{m}}\, t + \frac{F_0/m}{\frac{k}{m} - \omega^2} \sin \omega t$$

Variation of the ratio of driving frequency to the natural frequency of the system changes the amplitude of the system response, as shown in the figure below. A plot of the variation of the coefficient of the last term in the above expression with the frequency ratio at the point where the driving frequency equals the natural frequency of the system shows that the amplitude of the displacement goes to infinity. This condition is known as resonance.



VISCOUS DAMPING: The time rate of decay of a vibrating system is a measure of the degree of damping present in the system.

THE RESPONSE OF SINGLE-DEGREE-OF-FREEDOM SYSTEMS WITH VISCOUS DAMPING

The inclusion of a damping term in the equations of motion provides more realistic modeling of an actual system; the frictionless model was discussed in the previous topic.

A more realistic representation of a physical system is provided by the inclusion of damping force in the system equations. The most generally assumed type of damping is viscous damping. This represents the case where a force proportional to the mass velocity resists the system displacement ($f = -c\,dx/dt$). The equation for system motion will be:

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin\omega t$$

The displacement solution consists of a transient complementary solution which approaches zero as time increases and a particular solution consisting of harmonic motion at the driving frequency with amplitude less than the case where no damping is present. The system amplitude will remain finite when $\omega = \omega_n$. The steady state displacement is given by:

$$x_p = \frac{\frac{F_0}{m}(\omega^2 - \omega_n)^2 \sin\omega t}{(\omega^2 - \omega_n)^2 + \left(\frac{\omega c}{m}\right)^2} + \frac{F_0 \omega c/m^2}{(\omega^2 - \omega_n^2)^2 + \left(\frac{\omega c}{m}\right)^2}\cos\omega t$$

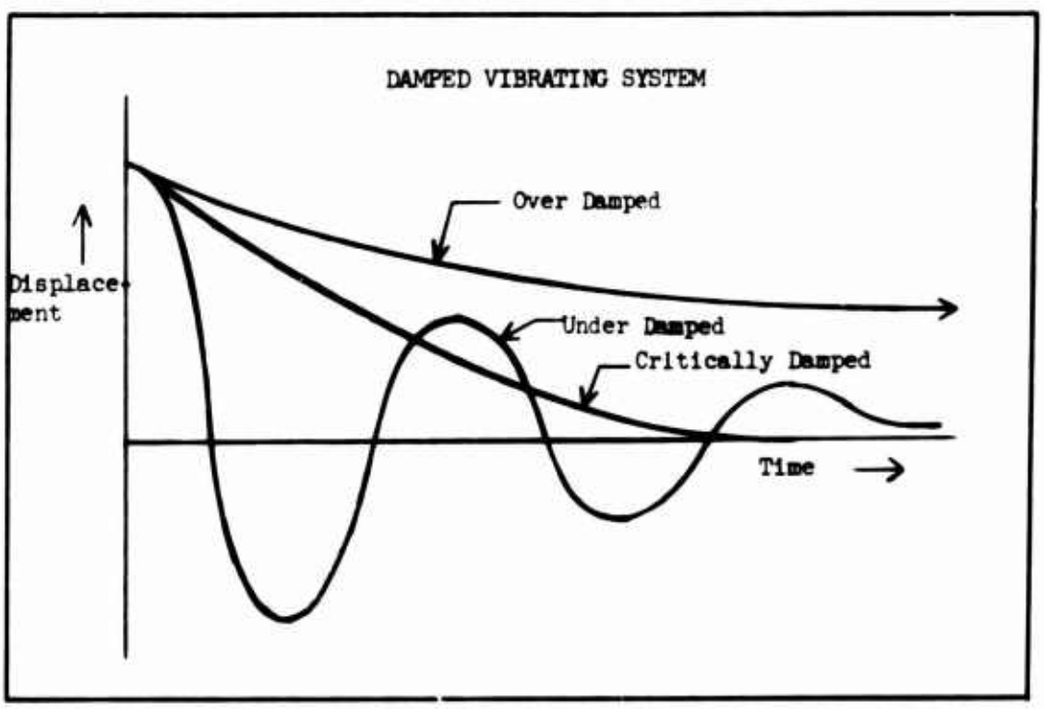
The complementary solution is the system response for the case where no driving force is present. The displacement expression

$$x_c = e^{-\left(\frac{c}{2m}\right)t}\left[C_1 e^{\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\,t} + C_2 e^{-\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\,t}\right]$$

A plot of this equation as a function of time (see figure) shows that for values of $C<2\sqrt{km}$, the motion will consist of exponentially decreasing amplitude without oscillation. For $C>2\sqrt{km}$ the system will oscillate with exponentially decreasing amplitude at a frequency less than the free undamped natural frequency. The value of $C = 2\sqrt{km}$ is referred to as the critical damping constant.

The analysis of systems consisting of more than a single degree of freedom is conducted in a similar fashion but will involve the simultaneous solution of as many equations as the number of degree of freedom.

Several examples on damping in simple systems are given in the Appendix starting on page 1.5-16.



VISCOUS DAMPING: The time rate of decay of a vibrating system is a measure of the degree of damping present in the system.

VOLUME III - CHAPTER 1

BASIC MECHANICS

SECTION 5 - APPENDIX

- Bibliography
- General Method of Solution for Equilibrium Problems
- Elastic Characteristics of Some Common Beam Configurations
- Material Characteristics
- Column End Fixity Coefficients For Various End Conditions
- Summary of Beam Columns
- Beam Column Coefficients
- The Calculation of Horizontal Shear Stress in Beams
- The Calculation of the Shear Center for a Channel
- The Calculation of Torsional Stress in Non-Circular Sections
- An Application Using Mohr's Circle
- The Calculation of the Response of Damped Systems
- Shear and Moment Diagrams for a Simple Beam

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3. Popov, E. P., Mechanics of Materials, Prentice-Hall, 1958

4. Higdon, A. and Stiles, W. B., Engineering Mechanics, Prentice-Hall, 1957

5. Long, R. R., Engineering Science Mechanics, Prentice-Hall, 1963

6. Housner, G. and Hudson, D., Applied Mechanics, Dynamics, D. Van Nostrand Company, Inc., 1959

7. Timoshenko, S., Theory of Elastic Stability, McGraw-Hill, 1936

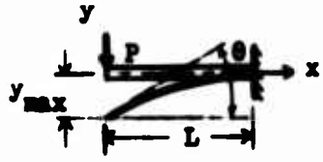
8. Priest, H. M., Design Manual for High Strength Steels, U. S. Steel Corp., 1956

9. Murphy, G., Advanced Mechanics of Materials, McGraw-Hill, 1946

GENERAL METHOD OF SOLUTION FOR EQUILIBRIUM PROBLEMS

- List known data and results to be determined.
- Construct a free body diagram of the members on which the unknown forces to be determined are acting, replacing interacting members with equivalent force systems.
- Determine the number of independent equilibrium equations available for the system, compare with the number of unknowns on the free body diagram.
- If there are as many independent equations available as the number of unknowns, solve the equations for the unknowns.
- If there are more unknowns than equations for the free body diagram, subdivide the free body diagram and repeat the procedure.
- If free body diagrams have been constructed for each body in the problem and there are still more unknowns than equations, the problem is statically indeterminate and deflections must be considered in the solution.

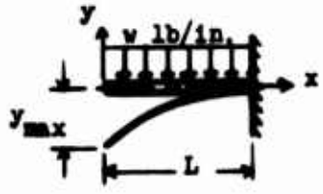
ELASTIC CHARACTERISTICS OF SOME COMMON BEAM CONFIGURATIONS



$$y = -\frac{P}{6EI}(2L^3-3L^2x+x^3)$$

$$\theta = +\frac{PL^2}{2EI}$$

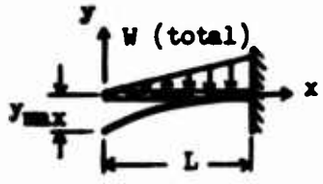
$$y_{max} = -\frac{PL^3}{3EI} \text{ at } x = 0$$



$$y = -\frac{w}{24EI}(x^4-4L^3x+3L^4)$$

$$\theta = +\frac{wL^3}{6EI}$$

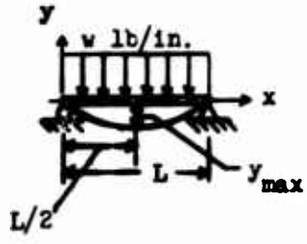
$$y_{max} = -\frac{wL^4}{8EI} \text{ at } x = 0$$



$$y = -\frac{w}{60EIL^2}(x^5-5L^4x+4L^5)$$

$$\theta = +\frac{WL^2}{12EI}$$

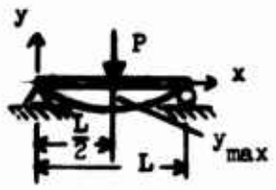
$$y_{max} = -\frac{WL^3}{15EI}$$



$$y = -\frac{wx}{24EI}(L^3-2Lx^2+x^3)$$

$$\theta = \frac{-wL^3}{+24EI}$$

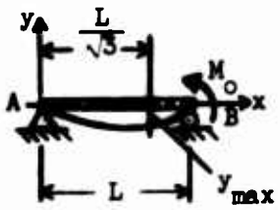
$$y_{max} = -\frac{5wL^4}{384EI} \text{ at } x = L/2$$



$$y = -\frac{Px}{16EI}(3L^2-4x^2)$$

$$\theta = -\frac{PL^2}{16EI}$$

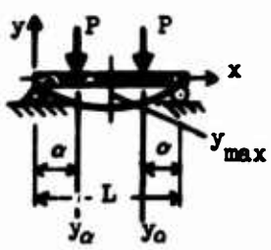
$$y_{max} = -\frac{PL^3}{48EI} \quad \text{at } x = L/2$$



$$y = +\frac{M_o x}{6EIL}(x^2-L^2)$$

$$\theta_A = -\frac{M_o L}{6EI} \; ; \; \theta_B = +\frac{M_o L}{3EI}$$

$$y_{max} = -\frac{ML^2}{9\sqrt{3}EI} \quad \text{at } x = L/\sqrt{3}$$



$$y_a = -\frac{Pa^2}{6EI}(3L-4a) \quad \text{at } x = a$$

$$y_{max} = -\frac{Pa}{24EI}(3L^2-4a^2) \quad \text{at } x = L/2$$

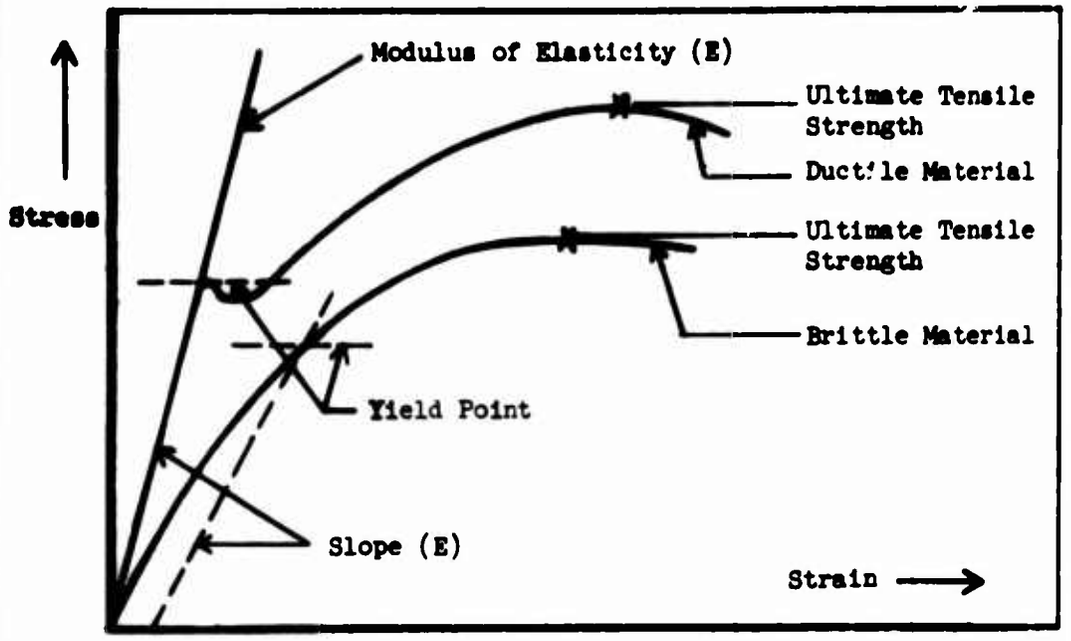
$$\theta_{ends} = -\frac{Pa}{2EI}(L-a)$$

Where y = f(x) is the equation of elastic curve (upward deflection positive)

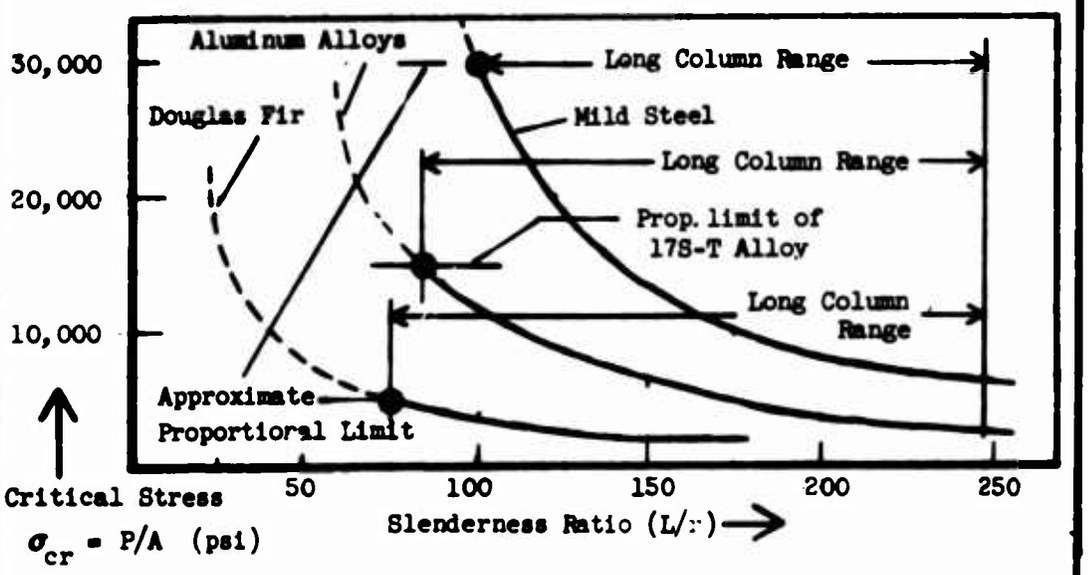
θ = Slope at end

y_{max} = Maximum deflection

MATERIAL CHARACTERISTICS

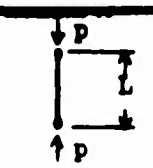
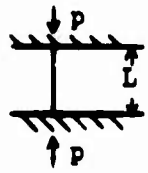
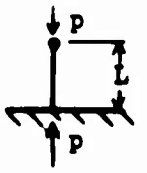
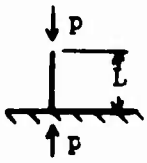
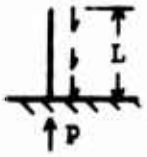
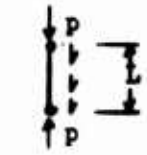
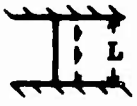
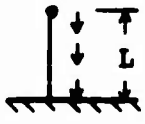


THE CLASSIC STRESS-STRAIN CURVE FOR A DUCTILE AND BRITTLE MATERIAL.

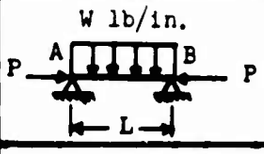
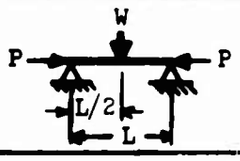
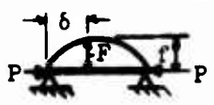
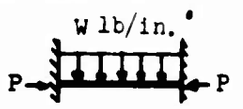
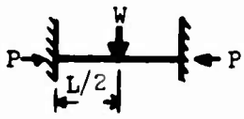
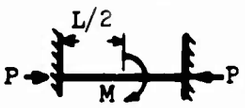
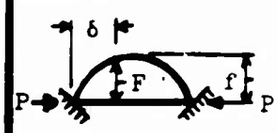
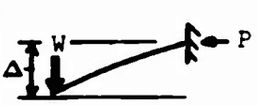


VARIATION OF THE CRITICAL COLUMN STRESS WITH THE SLENDERNESS RATIO FOR THREE DIFFERENT MATERIALS.

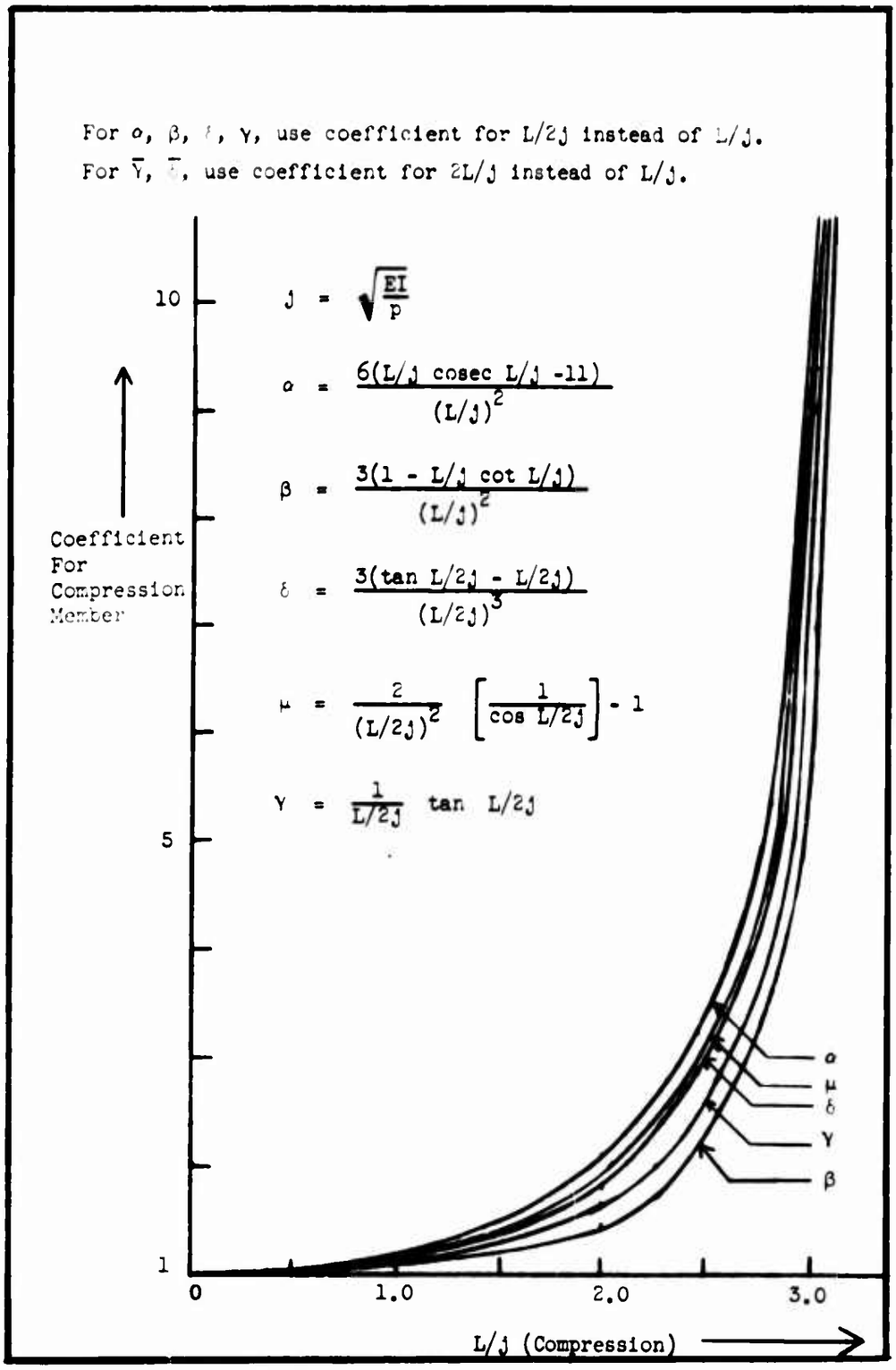
COLUMN END FIXITY COEFFICIENTS FOR VARIOUS END CONDITIONS

Column Shape and End Conditions		End Fixity Coefficient
	Uniform column, axially loaded, pinned ends.	$c = 1$ $\frac{1}{\sqrt{c}} = 1$
	Uniform column, axially loaded, fixed ends.	$c = 4$ $\frac{1}{\sqrt{c}} = 0.5$
	Uniform column, axially loaded, one end fixed, one end pinned.	$c = 2.05$ $\frac{1}{\sqrt{c}} = 0.70$
	Uniform column, axially loaded, one end fixed, one end free.	$c = 0.25$ $\frac{1}{\sqrt{c}} = 2$
	Uniform column, distrivuted axial load, one end fixed, one end free.	$c = 0.794$ $\frac{1}{\sqrt{c}} = 1.12$
	Uniform column, distributed axial load, pinned ends.	$c = 1.87$ $\frac{1}{\sqrt{c}} = 0.732$
	Uniform column, distributed axial load, fixed ends.	$c = 7.5$ $\frac{1}{\sqrt{c}} = 0.365$
	Uniform column, distributed axial load, one end fixed, one end pinned.	$c = 3.55$ (approx) $\frac{1}{\sqrt{c}} = 0.530$

SUMMARY OF BEAM COLUMNS

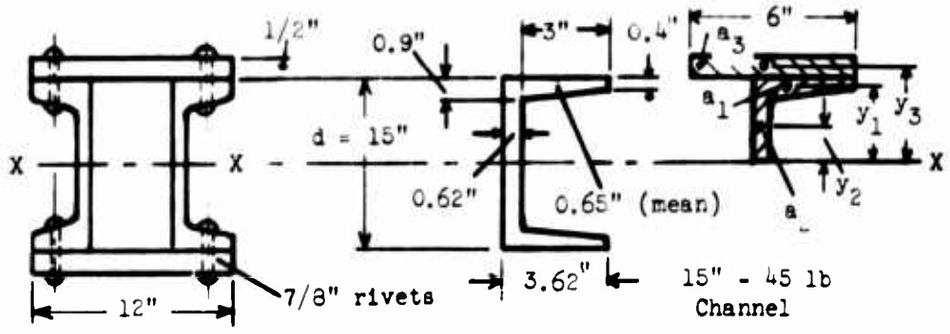
Loading	Moment Over Support	Moment At Center	Deflection At Center
	$M_A = 0$ $M_B = 0$	$M_C = \frac{wL^2}{8}\mu$	$\Delta = \frac{5}{384}\frac{wL^4}{EI}\delta$
	$M_A = 0$ $M_B = 0$	$M_C = \frac{WL}{4}\gamma$	$\Delta = \frac{WL^3}{48EI}\delta$
	$M_A = 0$ $M_B = 0$	$M_C = Pf\mu$	$\Delta = \frac{5PfL^2}{48EI}\delta$
	$M_A = M_B$ $M_A = M_B$	$M_C = \frac{M_A + M_B}{2\cos\frac{L}{2j}}$	$\Delta = \frac{(M_A + M_B)L^2}{16EI}\mu$
	$M_A = M_B$ $= -\frac{WL^2}{12}\beta''$	$M_C = \frac{wL^2}{24}\alpha''$	$\Delta = \frac{1}{384}\frac{wL^4}{EI}\delta$
	$M_A = M_B$ $-\frac{WL}{8}\gamma''$	$M_C = \frac{WL}{8}\gamma''$	$\Delta = \frac{WL^3}{192EI}\delta''$
	$M_A = M_B$ $= \frac{M}{4}\frac{\alpha''}{\beta''}$	$M_C = \pm\frac{M}{2}$	$\Delta = 0$
	$M_A = M_B$ $= \frac{2}{3}Pf\beta''$	$M_C = \frac{Pf}{3}\alpha''$	$\Delta = \frac{1}{48}\frac{PfL^2}{EI}\delta''$
	$M_B = -WL\bar{Y}$		Deflection at A $\Delta = \frac{WL^3}{3EI}\delta$

BEAM COLUMN COEFFICIENTS



THE CALCULATION OF HORIZONTAL SHEAR STRESS IN BEAMS

Compute the rivet spacing for the simply supported composite beam shown below if the allowable shear force on the rivets is 4,500 pounds. (A structural steel rivet has an allowable shear stress of 15,000 lb/in.^{2}) Consider two loading cases. Case I, a concentrated load of 100,000 lb applied at center of span. Case II, A uniformly distributed total load of 100,000 pounds between supports. Span between supports is 12 feet. Neglect the weight of the beam.



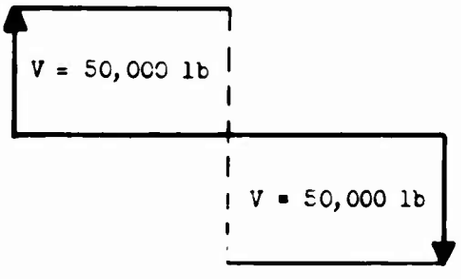
Solution, Case I:

$$I_x \text{ (channel)} = 375 \text{ in.}^4, \text{ a - } 13.2 \text{ in.}^2 \text{ - from handbook data}$$

$$I_x \text{ (cover plate)} = 12 \times \frac{1}{2} \times 7.75^2 = 360 \text{ in.}^4$$

$$I_x \text{ (total section)} = 2\,(375 + 360) = 1470 \text{ in.}^4$$

The shear diagram for concentrated load is shown below. Note that the vertical shear equals 50,000 pounds magnitude throughout the entire span except at the transition point where it crosses through zero.



The shear flow is given by

$$q = \frac{V\bar{y}A_p}{I_x}$$

pounds per inch span, where the quantity ($\bar{y}$ A_p) represents the centroid of the area above the plane of interest for horizontal shear. In this case the range of interest is the boundary between the cover plate and the channel flanges since the shear force on the rivets is desired. The section along the beam does not vary and since the vertical shear (V) is also constant the shear flow is constant. This implies a fixed rivet spacing throughout the entire length of span.

The rivet spring at any point along the beam is given by allowable shear force/shear flow which equals 4500/q for this beam system.

Using

$$V = 50,000 \text{ lb}$$

$$I_x = 1470 \text{ in.}^4$$

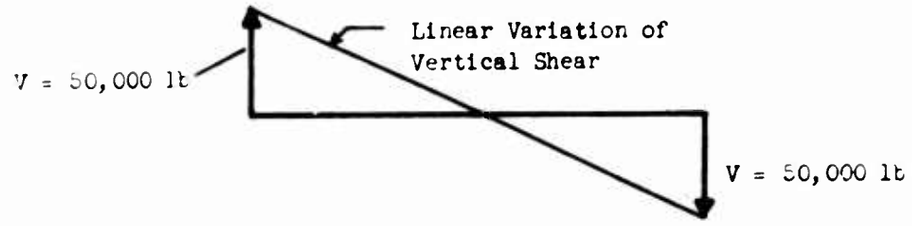
$$\bar{y}A_p = 7.75 \times (6 \times 1/2) = 23.25 \text{ in.}^3$$

yields q = 790 lb/in. which results in a rivet spacing of 4500/50,000 x 23.25/1470 = 5.7 in. (maximum). Twenty-eight rivets spaced at 5.25 in. intervals is a satisfactory arrangement.

Solution, Case II:

The method of solution here is basically the same as that for the previous case but the distributed load introduces a varying shear flow which results in a varying rivet spacing.

The shear diagram for the distributed load is shown below. Note that the vertical shear varies along the beam length.



The shear flow is given by

$$q = \frac{V\bar{y}}{I_x} A_p$$

but now (V) is a variable and hence the shear flow is variable.

Starting from some arbitrary station along the beam span at which a rivet is placed, the spacing to the next rivet must be such that

$$\int_n^{n+1} q \, d\ell = 4500 \text{ lb}$$

Taking the center of the span as station (0) the first rivet is placed at station 1, the second at station 2, etc., until the entire span is nearly covered. The last rivet must not be placed closer to the end of the beam than 2-1/2 times rivet diameter from rivet center. This is to prevent tearing or crushing of the channel or plate in the margin.

THE CALCULATION OF HORIZONTAL SHEAR STRESS IN BEAMS (Continued)

If the first attempt results in a last rivet much further away than 2-1/2 diameter, then the spacing must be decreased to allow an extra rivet. Minimum total rivets in a half span equal

$$\frac{\int_0^{L/2} q\, d\ell}{9{,}000} = \frac{\bar{y} A_p}{4500 I_x} \int_0^{L/2} V\, d\ell$$

where from the shear diagram $V = \frac{50{,}000}{72}\ell$. This gives total rivets = 6.32

which is just half of the minimum rivets per half span of Case I. This latter value was 72 /5.7 = 12.64. Six rivets is the nearest integral and this will be the basis for a trial calculation.

The general form for spacing is given by performing

$$\int_n^{n+1} q\, d\ell = 4500 \text{ lb}$$

(n is the rivet "station" from mid-span)

This results in:

$$\ell^2 \Big|_n^{n+1} = 819 \text{ in.}^2$$

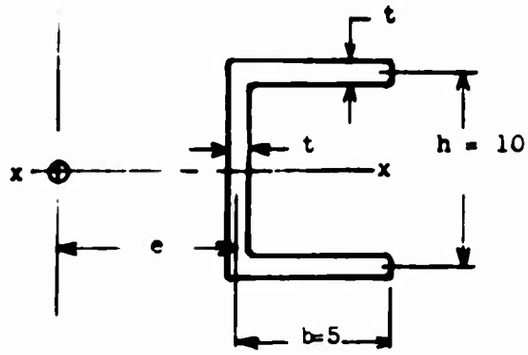
A tabulation of (ℓ) starting with n=0 at mid-span results in the following spacing. Distances (ℓ) are measured in inches from mid-span.

28.6, 40.5, 49.7, 57.3, 64.1, 70.1

Theoretically the beam section past 70.1 in. no longer acts as an integral composite beam but as the sum of the channels plus plates since there is no rivet past 70.1 in. to take up horizontal shear between plate and channels acting as an integral section.

THE CALCULATION OF THE SHEAR CENTER FOR A CHANNEL

Illustrative Problem: Find the approximate location of the shear center for the channel shown below:



Solution: The previously derived result for the distance (e) to the shear center was

$$e = \frac{b^2 h^2 t}{4I}$$

Where I represents the section inertia about the axis x-x. Frequently, the flange thickness is small with respect to the distance (h) and an approximate form may be used for (I). This form neglects the contribution of the flanges about their own axes.

Thus,

$$I \approx I_{WEB} + (Ad^2)_{Flanges} = \frac{th^3}{12} + 2bt\left(\frac{h}{2}\right)^2$$

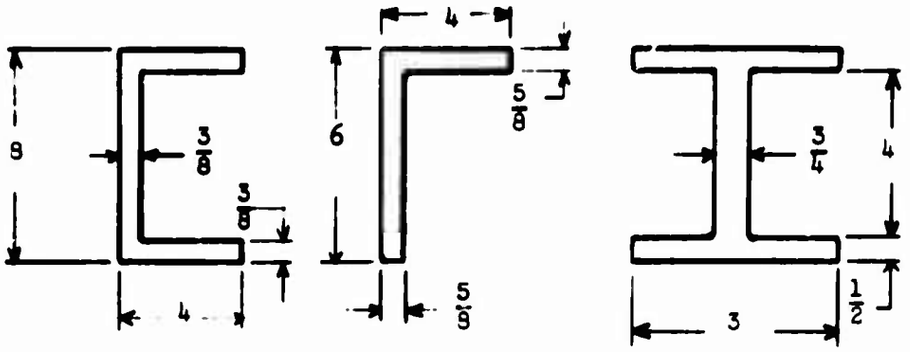
substitution into the formula for (e) results in

$$e \approx \frac{b}{2 + \frac{h}{36}}$$

which results in e = 1.87". Note that for (b) very large (e) approaches (b/2) and for (h) very large (e) approaches (0).

THE CALCULATION OF TORSIONAL STRESS IN NON-CIRCULAR SECTIONS

Illustrative Problem: Calculate the maximum shearing stress for each of the following sections using an approximate method. The twisting moment is 12,000 in.-lbs.



Solution: For sections of this type where thickness is small compared to lengths an approximate maximum shearing stress will be given by

$$S_s = \frac{Th'}{\frac{1}{3}\sum bh^3}$$

where h' is the greatest thickness of a member in the section.

For the channel

$$\frac{1}{3}\sum bh^3 = \frac{1}{3}\left[8 \times \left(\frac{3}{8}\right)^3 + 2 \times 3\frac{5}{8} \times \left(\frac{3}{8}\right)^3\right] = 0.206 \text{ in.}^4$$

For the angle

$$\frac{1}{3}\sum bh^3 = \frac{1}{3}\left[6 \times \left(\frac{5}{8}\right)^3 + 3\frac{3}{8} \times \left(\frac{5}{8}\right)^3\right] = 0.764 \text{ in.}^4$$

For the I Beam

$$\frac{1}{3}\sum bh^3 = \frac{1}{3}\left[2 \times 3 \times \left(\frac{1}{2}\right)^3 + 4 \times \left(\frac{3}{4}\right)^3\right] = 0.811 \text{ in.}^4$$

Solving for the maximum shear stress gives

$$S_s \text{ (Channel)} = \frac{12{,}000 \times \frac{3}{8}}{0.206} = 21{,}800 \text{ psi}$$

$$S_s \text{ (Angle)} = \frac{12{,}000 \times \frac{5}{8}}{0.764} = 9{,}820 \text{ psi}$$

$$S_s \text{ (I Beam)} = \frac{12{,}000 \times \frac{3}{4}}{0.811} = 11{,}100 \text{ psi}$$

Note that each section has almost the same area (about 6 in.^{2}).

Problem: Compare the maximum shearing stress of a circular section whose area equals 6 in.^{2} with the stress of the sections in the previous problem.

Solution:

$$S_s = \frac{TC}{J}$$

where C is the radius of the cylinder and J is it's polar moment of inertia

$$C = \sqrt{\frac{6}{\pi}} = 1.38 \text{ inches}$$

$$J = \frac{1}{2}\pi r^4 = \frac{1}{2} \times \pi \times \frac{36}{2} = \frac{18}{\pi} \text{ in.}^4$$

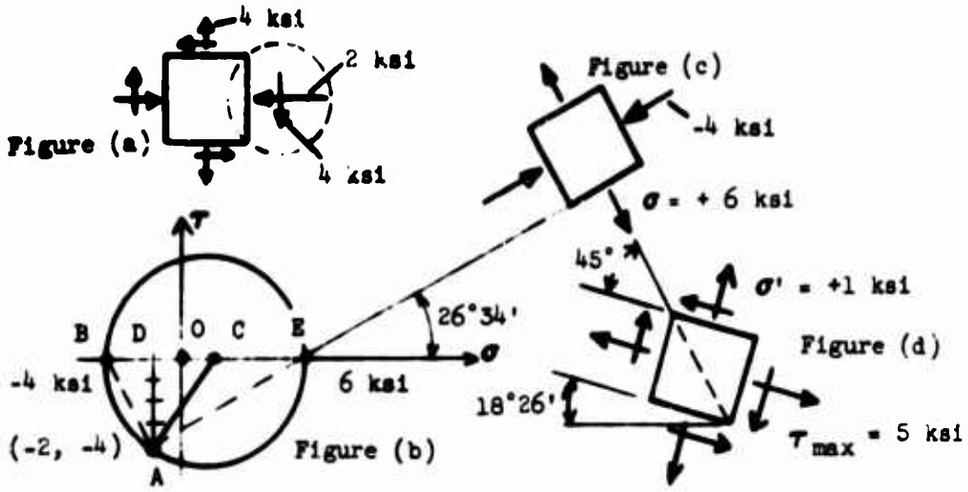
Therefore:

$$S_s = \frac{12{,}000 \times 1.38 \times \pi}{18} = 2{,}890 \text{ psi}$$

The above is a demonstration of the practicality of using condensed sections where torsion is concerned.

AN APPLICATION USING MOHR'S CIRCLE

Illustrative Problem: Given the state of stress shown in Figure (a), transform it into the principal stresses, and into the principal shearing stresses and the associated normal stresses. Show the results for both cases on properly oriented elements.



Solution: The co-ordinate axes are set up in Figure (b). The center C of the circle is at 1/2(-2,000 + 4,000) = +1,000 ksi on the σ-axis. From the right-hand face of the element, the required values for plotting the the point A on the circle are

$$\sigma_s = -2{,}000 \text{ psi}$$

$$\tau = -4{,}000 \text{ psi}$$

Thus the distances CD and DA are 3,000 psi and 4,000 psi, respectively, and the radius of the circle is equal to

$$CA = \sqrt{CD^2 + DA^2} = 5{,}000 \text{ psi}$$

Hence from the diagram, τ_{max} = 5,000 psi, and the associated normal stress is represented by the distance OC, i.e., σ' = 1,000 psi. The principal stresses are given by the intercepts E and B; they are respectively +6,000 psi and -4,000 psi.

The angle

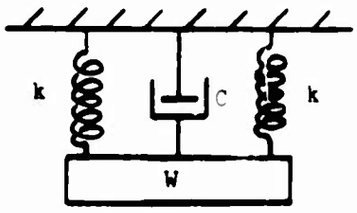
$$DEA = \tan^{-1} \frac{AD}{DE} = \tan^{-1} \frac{4{,}000}{8{,}000} = 26°34'$$

An element with its sides parallel to the lines AB and AE is shown in Figure (c). Since the faces of this element intersect at right angles, several other angles (not shown) may be used to specify its orientation. The principal stress given by the E intercept acts normal to the line EA. The principal stress given by the B intercept acts normal to the line BA.

An element oriented with its planes parallel to the maximum shearing stresses is shown in Figure (d). The maximum shearing stresses act toward the positive shear diagonal, which coincides with the direction of the algebraically larger principal stress. The associated normal stresses are also shown on the diagram. All of these are the same, and all of them are tensile in character.

THE CALCULATION OF THE RESPONSE OF DAMPED SYSTEMS

Illustrative Problem: The system shown below is initially at rest at $t = 0^-$. At $t = 0$ a velocity of 4 in./sec is given to the mass. Find the subsequent displacement and velocity of the mass. The damping coefficient is 0.85 lb-sec/in. The spring constant is 25 lb/in. and the weight is 40 lb.



Solution: The differential equation of motion for this system is

$$\frac{W}{g}\frac{d^2x}{dt^2} + C\frac{dx}{dt} + 2kx = 0$$

The solution of which is

$$x = e^{-\zeta\omega_n t}\ (A \cos \omega_d t + B \sin \omega_d t)$$

$$= D\, e^{-\zeta\omega_n t} \sin\ (\omega_d t + \phi)$$

Where

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{(Natural Frequency)}$$

$$\zeta = \frac{c}{2m\omega_n} \quad \text{(Damping Factor)}$$

$$\omega_d = \sqrt{1 - \zeta^2}\,\omega_n \quad \text{(System Frequency)}$$

$$D = \sqrt{A^2 + B^2}$$

$$\phi = \tan^{-1}\frac{A}{B}$$

introducing the given quantities and solving yields,

$$\omega_n = 22 \text{ rad/sec}$$

$$\zeta = 0.181$$

$$\omega_d = 21.6 \text{ rad/sec}$$

Substituting these values into the displacement equation gives

$$x = e^{-3.98t} (A \cos 21.6t + B \sin 21.6t)$$

differentiating results in

$$\frac{dx}{dt} = v = -3.98\, e^{-3.98t} (A \cos 21.6t + B \sin 21.6t) + 21.6\, e^{-3.98t} (-A \sin 21.6t + B \cos 21.6t)$$

Substitution of the initial conditions yields the values for (A) and (B)

At

$$t = 0$$

$$x = 0$$

then

$$A = 0$$

At

$$t = 0$$

$$\frac{dx}{dt} = v = 4$$

then

$$B = \frac{4}{21.6} = 0.185$$

Finally

$$x = 0.185\, e^{-3.98t} \sin 21.6t$$

$$\frac{dx}{dt} = v = e^{-3.98t} (4 \cos 21.6t - 0.737 \sin 21.6t)$$

or

$$v = 4.08\, e^{-3.98t} \cos (21.6t + 9.5^\circ)$$

Displacement and velocity may subsequently be calculated for any particular time (t) by evaluating the above equations for x and v.

THE CALCULATION OF THE RESPONSE OF DAMPED SYSTEMS (Continued)

Illustrative Problem: For the above problem calculate the transient and steady state response if an excitation force F = 10 sin 15t is applied to the mass. The initial conditions for the previous problem again apply.

Solution: The transient response is the complimentary solution of the differential equation of motion already solved in the previous example. It is,

$$x_c = D\, e^{-3.98t} (\sin 21.6t + \phi)$$

where

$$\phi = \tan^{-1} \frac{A}{B}$$

Initial values will be introduced later to solve (A) and (B).

The steady state response is the particular solution of the differential equation of motion, which from the theory of differential equations takes the form

$$x_p = (A \sin\omega t + B \cos\omega t)$$

Taking the first and second order derivatives of this equation, and substituting into the original differential equation of motion yields

$$\frac{W}{g} \frac{d^2x}{dt^2} + C \frac{dx}{dt} + 2kx = F(t) = 10 \sin\omega t, \text{ yields,}$$

after algebraic manipulation

$$A = \frac{F\,(k - m\omega^2)}{(k - m\omega^2)^2 + (c\omega)^2}$$

$$B = \frac{-F\,c}{(k - m\omega^2)^2 + (c\omega)^2}$$

Substitution gives,

$$x_p = \frac{F_0}{(k - m\omega^2)^2 + (c\omega)^2} \left[(k - m\omega^2) \sin\omega t - c\omega \cos\omega t \right]$$

letting

$$\psi = \tan^{-1} \frac{c\omega}{k - m\omega^2} = \frac{2\xi(\omega/\omega_n)}{1 - (\omega/\omega_n)^2}$$

Finally gives

$$x_p = \frac{F_0}{(k-m\omega^2)^2 + (c\omega)^2} \sin(\omega t - \psi)$$

Substitution of the given values arrives at

$$x_p = 0.337 \sin(15t - 28^\circ)$$

Note that there is no introduction of initial value conditions for the steady state solution.

The total solution is the sum of the complimentary and particular solutions

$$x = x_c + x_p$$

$$= D e^{-3.98t} \sin(21.6t + \phi) + 0.337 \sin(15t - 28^\circ)$$

In order to determine (D) and (ϕ) the initial values must be introduced.

Solving for

$$t = 0$$

$$x = 0$$

and

$$t = 0$$

$$\frac{dx}{dt} = 4$$

Results in

$$x = 0.176 e^{-3.98t} \sin(21.6t + 65^\circ) + 0.337 \sin(15t - 28^\circ)$$

Which is the total response as a function of time. The first term on the right is the transient response; the second term is the steady state response.

THE CALCULATION OF THE RESPONSE OF DAMPED SYSTEMS (Continued)

Illustrative Problem: A 25 pound weight is suspended from a spring with a constant of 10 lb/in. The dashpot in the system has a resistance of 0.1 lb at a velocity of 2 in./sec and remains fairly constant in value. Find the damped frequency of the system, the critical damping constant, the ratio of successive amplitudes, the amplitude 10 cycles later if the initial displacement before being released is 3/4 inch.

Solution: The equation expressing the vibration is

$$y = y\, e^{-\zeta \omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \phi\right)$$

(This is the form for under-damping.)

The oscillation curve is tangent to the exponential envelope whose form is

$$y\, e^{-\zeta \omega_n t}$$

However, the tangents are not horizontal, and the points of tangency appear slightly to the right of the point of maximum amplitude. The assumption will be now made that the amplitude at the point of tangency equals the maximum amplitude of the oscillation, for the particular point in consideration. The logarithmic decrement is defined as the natural logarithm of the ratio of any two successive amplitudes It is equal to

$$\delta = \ln \frac{y_1}{y_2} = \ln \frac{e^{-\zeta \omega_n t_1}}{e^{-\zeta \omega_n (t_1 + \tau)}} = \ln e^{\zeta \omega_n \tau} = \zeta \omega_n \tau$$

Since the period of oscillation is equal to

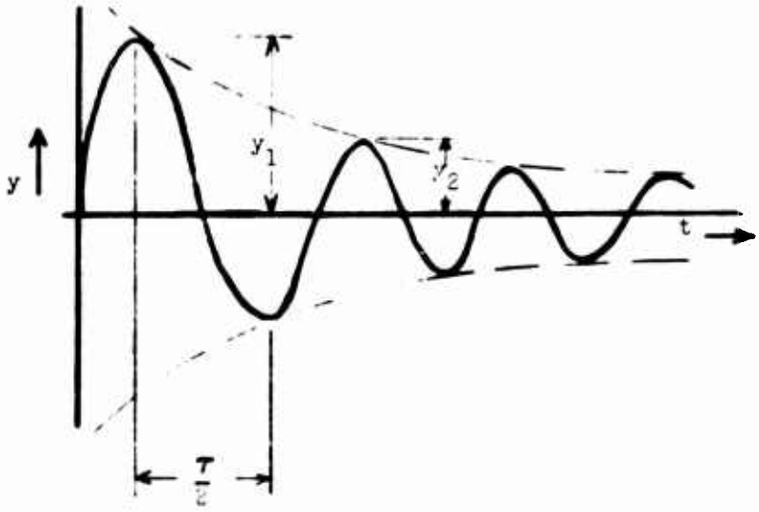
$$\tau = \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

the decrement can also be expressed as

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{\pi c}{m \omega_{nd}} = \frac{\pi c}{m\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}}$$

For damping factors equal to and less than 0.4, the decrement can be made

$$\delta \approx 2\pi\zeta$$



$$C = \frac{F}{v} = \frac{0.1}{2} = 0.05 \frac{\text{lb-sec}}{\text{in.}}$$

$$\omega_n(\text{Damped}) = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{\frac{10x386}{25} - \left(\frac{0.05x386}{2x25}\right)^2} = 12.42 \frac{\text{rad}}{\text{sec}}$$

$$\delta_n(\text{Damped}) = \frac{\omega_{nd}}{2\pi} = 1.98 \text{ Hz}$$

$$C_c = 2\sqrt{mk} = 2\sqrt{\frac{25x10}{386}} = 1.61 \frac{\text{lb-sec}}{\text{in.}}$$

C_c is the critical damping constant

$$\delta = \frac{c}{m\omega_{nd}} = \frac{\pi x 0.05x386}{25x12.42} = 0.195 = \ln \frac{y_0}{y_1} = \ln \frac{y_n}{y_{n+1}}$$

Therefore the ratio of two successive amplitudes

$$\frac{y_n}{y_{n+1}} = e^{\delta} = 1.216$$

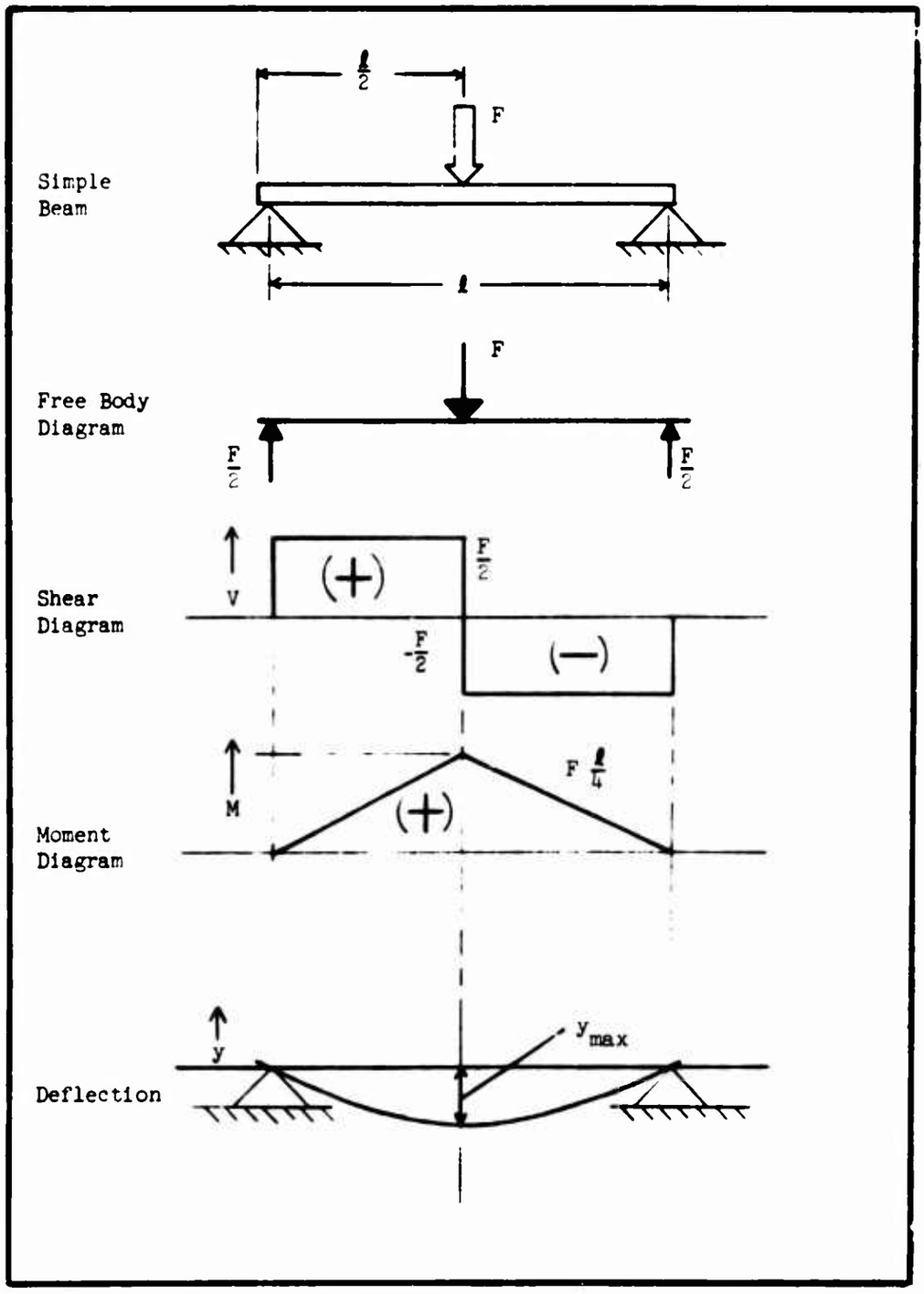
$$\tau = \frac{2\pi}{\omega_{nd}} = \frac{2\pi}{12.42} = 0.506 \text{ seconds (period)}$$

Time for 10 cycles $= 10\tau = 5.06$ seconds. Since,

$$\frac{y_n}{y_{n+1}} = e^{n\delta} = (e^{\delta})^n$$

$$y_{10} = y_0 (e^{-\delta})^{10} = \frac{3}{4}\left(\frac{1}{1.216}\right)^{10} = 0.106 \text{ inches}$$

SHEAR AND MOMENT DIAGRAMS FOR A SIMPLE BEAM



CHAPTER 2 – NATURAL FREQUENCY

VOLUME III - RELATED TECHNOLOGIES

CHAPTER 2
NATURAL FREQUENCY

ABSTRACT:

Natural frequency is perhaps the most important single structural parameter that the design engineer will have to evaluate and manipulate in the successful evolution of equipment structure. The frequency at which resonant amplification occurs and subsequent attenuation beyond the resonant range are response phenomena directly related to the natural frequency parameter.

The factors of mass distribution, weight, support geometry, and stiffness are all design-relevant constraints which the packaging engineer will control in the design and execution of the equipment package. These factors also define the natural frequency of the structure. This chapter relates these constraints with the response of the equipment system to shock and vibration influences.

The application of the natural frequency concept to some practical design situations is discussed in detail. The use of natural frequency in absorption and isolation design problems, shock response, and the improvement of joints and interfaces are covered in depth.

Methods for measuring, estimating, and calculating natural frequency are outlined, including computer techniques and electrical analogies. Visual and analytical estimation methods are also discussed, as well as the use of transducers on full scale laboratory models.

Chapter 2 - Natural Frequency

ERRATA SHEET

Page	Paragraph	Line	Correction
2.1-3	3		DAMPER
2.1-5	Upper sketch		Add "y" direction
2.1-5	Caption	2	... a simple means...
2.2-0	Thesis	2	Delete "response"
2.2-0	6	3	experimentally
2.3-0	2	5	... is 160 Hz.
2.3-0	5	5	deflection
2.3-0	6	6	Delete "and"
2.3-2	1	4	... is proportional to m,
2.3-3	Sketch		drop height = h
2.4-3	Sketch, max force		F_o
2.5-3	Equation (2)		$k = \frac{W}{d_W}$
2.5-3	Left column		Weight (top figure) is W
2.5-4	4	5 & 7	a_i
2.5-4	7	3	classical
2.5-5	1	4	For example,
2.5-9	Caption	3	represent
Section 6	Divider	9	Nomogram
Section 6	Divider	10	...Standard Shock Pulses
2.6-0	7	1	"Vibration Frequency...
2.6-0	8	1	"Detecting Resonant Frequencies...
2.6-1	-	20	T Torque
2.6-1	-	29	Damping ratio
2.6-2	7	4	in the system.
2.6-3	Example B	2	(exponentially)
2.6-9	2	1	rotational
2.6-10	3rd Column		Frequency, (Hz)
2.6-13 thru -18	All (cps) notations		(Hz)

VOLUME III – CHAPTER 2
NATURAL FREQUENCY

CONTENTS

Section		Page
1	INTRODUCTION	2.1-0
	• Natural Frequency: What is it, Why is it Important?	2.1-0
	• Springs, Masses and Dampers	2.1-2
	• The Single-Degree-of-Freedom System	2.1-4
2	IMPORTANCE OF NATURAL FREQUENCY	2.2-0
	• The Transmissibility Curve	2.2-0
	• Amplification of Resonances	2.2-2
	• Modes of Vibration	2.2-4
	• Natural Frequency and Fatigue	2.2-6
	• Natural Frequency and Shock Spectra	2.2-8
3	APPLICATIONS OF NATURAL FREQUENCY	2.3-0
	• Shock and Vibration Isolation	2.3-0
	• Natural Frequency and Drop Shock	2.3-2
	• Effects of Joints on Natural Frequency	2.3-4
4	MEASURING NATURAL FREQUENCY	2.4-0
	• Visual Methods for Studying Natural Frequency .	2.4-0
	• Using Transducers to Detect Natural Frequency .	2.4-2
5	ESTIMATING AND CALCULATING NATURAL FREQUENCY . . .	2.5-0
	• Build a Model to Simplify Analysis	2.5-0
	• Static Deflection and Natural Frequency	2.5-2
	• Determining Natural Frequency Analytically . . .	2.5-4
	• Using Electrical Analogies to Study Natural Frequency	2.5-6
	• Digital Computer Techniques	2.5-8

CONTENTS (Continued)

Section		Page
6	APPENDIX	2.6-0
	• Bibliography	2.6-0
	• Symbology	2.6-1
	• Glossary	2.6-2
	• Examples on Sweep Rate of Vibratory Frequency	2.6-3
	• Examples on Natural Frequency and Fatigue	2.6-4
	• Examples Showing the Relationship Between Fatigue Life and Natural Frequency	2.6-6
	• Example on Natural Frequency and Drop Shock	2.6-8
	• Example on Effects of Joints on Natural Frequency	2.6-9
	• Nomogram for Harmonic Motion	2.6-10
	• Shock Spectra for Standard Shock Pulses	2.6-11
	• Vibration Frequency Charts	2.6-12

VOLUME III - CHAPTER 2

NATURAL FREQUENCY

SECTION 1 – INTRODUCTION

- Natural Frequency: What is it, Why is it Important?
- Springs, Masses and Dampers
- The Single Degree of Freedom System

NATURAL FREQUENCY: WHAT IS IT, WHY IS IT IMPORTANT?

All elastic structures oscillate when excited. The frequencies at which they oscillate are predictable and measurable, and greatly influence structural response.

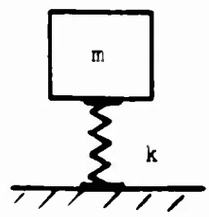
What is Natural Frequency?

An elastic system which is displaced from its equilibrium (at rest) position and then released will oscillate at distinct frequencies. These frequencies are the natural frequencies of the system. For a single degree of freedom (s.d.f.) system, the natural frequency is given by Equation 1. The natural frequency of the s.d.f. system is proportional to the square root of the spring stiffness divided by the mass of the body mounted on that spring (See the section on the s.d.f. system). For a multiple degree of freedom system, more than one natural frequency exists and the equations are more complex. In simplified terms, all natural frequencies are proportional to the square root of some function characteristic of the stiffness of the system divided by some function characteristic of the mass of the system. The degrees-of-freedom of the system are the minimum number of independent coordinates needed to describe the motion of that system.

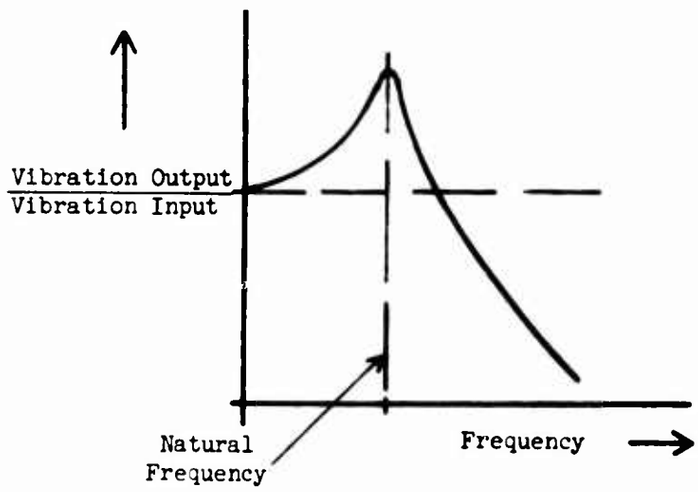
Why is Natural Frequency Important?

The natural frequencies of a structure determine the manner in which the structure will respond to shock, vibration, acoustics and other time dependent loads (see the adjacent transmissibility curves). The natural frequencies of systems must be properly matched with the usage environment in order to avoid failures caused by excessive response. This chapter discusses the influence of natural frequency on response, as well as methods for predicting and controlling natural frequency.

SINGLE DEGREE OF FREEDOM SYSTEM



$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (1)$$



NATURAL FREQUENCY: The natural frequency parameter greatly influences the dynamic response of a structure.

SPRINGS, MASSES AND DAMPERS

Natural frequency is a function of the stiffness, the mass capabilities and the energy absorption of a system.

A spring is that element of a structure which tends to restore the system to its original position when that system is displaced. The stiffer the spring, the more quickly it tends to restore the system, and, as one would expect, the stiffer the spring, the higher the natural frequency. The stiffness of a spring is described in terms of units of force per unit of displacement; for example, the units of a tension-compression spring can be lbs/in and the units of a torsion bar can be lb-in/degree. The linear spring, the type most commonly encountered in analytical studies, deflects as a linear function of the force applied. For example, a linear spring with a stiffness of 10 lbs/in will deflect 1 inch if 10 lbs of force are applied, 3 inches if 30 lbs of force are applied, etc. A plot of force vs deflection for a linear spring is a straight line and the slope of that line is the stiffness of the spring. Equation 1 is the stiffness of a linear system. Many types of non-linear springs occur in actual structures (see the references), but for small deflections, the slope of the force-displacement curve can be considered constant, even for some non-linear springs.

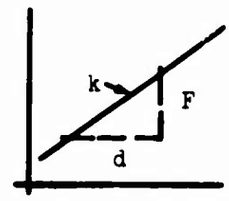
Mass can be considered as the multiplying factor which relates the acceleration of a body to the force applied to that body, as in Equation 2. Mass is related to weight (Equation 3) in that weight is the force exerted on a body by the acceleration of gravity. The weight of a body changes as gravity changes but the mass of a body is a constant. (Some readers may comment that mass changes as the speed of light is approached; although this is true, it is also true that no cases of structure failure have ever been attributed to this effect.) The units of mass are such that when mass is multiplied by acceleration, the result is in units of force. For example, if the system involves force in lbs and acceleration in in/sec^2, mass will be in units of $lbs\text{-}sec^2/in$. In general, the greater the mass of a system, the lower the natural frequency because it will take longer for a spring to return the heavier mass to its original position. For a torsional system the effect of mass on the system is expressed in terms of rotary inertia, the applied force is a torque and acceleration is angular acceleration (Equation 4).

Damping is that element of a system which absorbs, or dissipates energy. For purposes of analysis, the mechanics of this dissipation can be represented in several ways. All methods of representing damping refer to a system in motion. The most common method for representing damping is with viscous friction. With viscous damping, energy is dissipated by doing work against a viscous fluid. The force associated with viscous damping is directly proportional to velocity (Equation 5). The units of damping are such that when c is multiplied by velocity, the result is in units of force. Increased damping has the effect of slightly (insignificantly, for small values of damping) lowering the natural frequency. However, the major effect of damping is its influence on the maximum displacement of a resonating system. The smaller the coefficient of damping, the larger the resonant displacement. The qualitative aspects of damping are discussed further in the chapter on "Dynamic Attenuation."

SPRING

Symbol (Linear Spring)

$$k = \frac{F}{d} \quad \text{or} \quad \frac{T}{\theta} \qquad (1)$$



The stiffness (k) of a spring is the slope of the Force-Displacement curve.

MASS

Symbol

$$F = ma \qquad (2)$$

$$W = mg \qquad (3)$$

$$T = I\alpha \qquad (4)$$

DAMPING

Symbol

Viscous Damping

$$F = cv \qquad (5)$$

SPRING, MASS AND DAMPER: The basic elements of the vibratory system.

THE SINGLE DEGREE OF FREEDOM SYSTEM

Many types of structures may be approximately represented by the single degree of freedom system. This allows simple analysis procedures and generalized graphs and charts to be used.

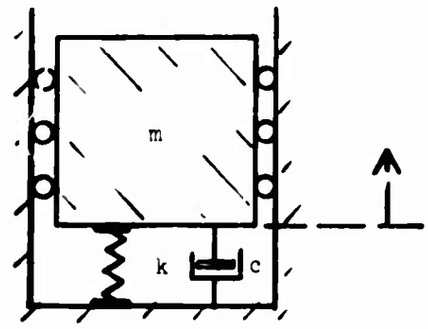
A single degree of freedom (s-d-f) system consists of a mass, spring, and damper restrained to move in such a manner that all motion can be described in terms of a single coordinate (linear or angular displacement). The spring has no mass, the mass is completely rigid, the damper is both rigid (except for the viscous fluid) and without mass, and the imaginary restraint is frictionless. The single degree of freedom system has only one natural frequency.

For analysis purposes, various structures may be represented by a single degree of freedom system. Any structure which can be reasonably represented by a massless spring and a concentrated mass can be analyzed as a single degree of freedom system. In addition to the illustrated structures, it should be noted that any uncoupled mode of vibration (i.e., any natural frequency whose motion can be described by a single coordinate) can be represented by a single degree of freedom. For example, a beam or a plate vibrating at one of its natural frequencies will deflect according to a definite pattern or mode shape. The deflections of all moving points on the body are directly proportional to any single moving point on the body (assuming a linear system). Therefore, if a point is selected, its motion can be defined by a single degree of freedom system and the motion of any other point can be defined by simply using a proportional constant (for the mode being considered).

The single degree of freedom system is a convenient analytical tool because the equations involved are fairly simple and the results can be expressed in general terms. General transmissibilities, phase angle relationships, damping effects and shock responses for the single degree of freedom are covered extensively in the literature. These items are discussed further in other sections.

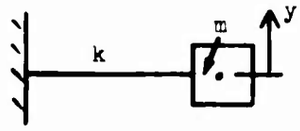
Structures which contain two or more spring-connected masses, structures which have widely distributed mass, and structures whose masses are free to move in several directions require analysis by more complex methods. As the degrees of freedom of a system become greater, it becomes increasingly difficult to express the problem in general terms. Multiple degree of freedom systems require analyses which look at the specific problem rather than use the general approach. The relative merits of the single degree of freedom system and more complex structures are discussed further in the subsection on Estimating and Calculating Natural Frequency.

The single degree of freedom system will be used in most of the discussions in this chapter because it is generally applicable and easier to illustrate and understand.

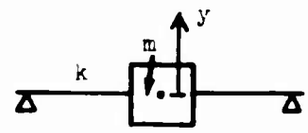


THE SINGLE DEGREE OF FREEDOM SYSTEM

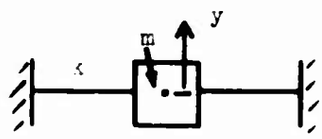
Structures which are single degree of freedom system.



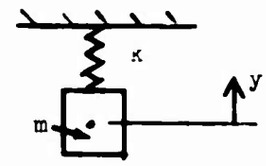
Cantilevered beam with mass at end.



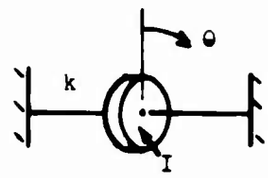
Simply supported beam with mass in center.



Restrained beam with mass in center.



Mass on a spring.



Inertia on torsion bar.

THE SINGLE-DEGREE-OF-FREEDOM SYSTEM: The single-degree-of-freedom system provides a single means for studying various types of structures.

VOLUME III - CHAPTER 2

NATURAL FREQUENCY

SECTION 2 – IMPORTANCE OF NATURAL FREQUENCY

- The Transmissibility Curve
- Amplification of Resonances
- Modes of Vibration
- Natural Frequency and Fatigue
- Natural Frequency and Shock Spectra

THE TRANSMISSIBILITY CURVE

The transmissibility curve indicates how a system will respond to a vibratory response excitation.

The transmissibility curve is vibratory response amplitude divided by the input amplitude, as a function of frequency.

A single degree of freedom system is considered here for illustrative purposes. Two types of excitation are possible: the mass (or force) excited system and the base (or displacement) excited system. Figure 1 shows the mass excited system; the force transmissiblity, T_F, is the ratio of the force at the base (response) to the force applied at the mass. Figure 2 shows the base excited system; the displacement transmissibility, T_D, is the ratio of the absolute displacement of the mass (response) to the displacement of the base (input).

The equations for T_F and T_D are identical; see Equation 1. The transmissibility is a function of f_N (the undamped natural frequency, Equation 2) and ζ (the damping ratio). ζ is the ratio of the damping factor, c, to the critical damping c_c (Equation 3) the critical damping for a s.d.f. system is given by Equation 4; this is the minimum damping which will cause the system to return to its equilibrium position, without oscillation, after being displaced and released.

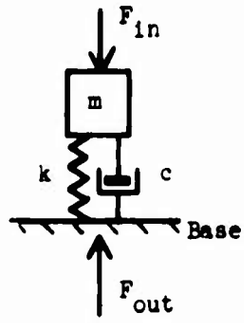
Figure 3 is a plot of T_F and T_D as a function of frequency. The curve is generalized by plotting the frequency axis as the ratio of the applied frequency, f, to the undamped natural frequency, f_n. It can be seen that when the frequency ratio $(f/f_n) = \sqrt{2}$, or approximately 1.4, the transmissibility is 1, i.e., the output equals the input. For values of f/f_n between 0 and 1.4 the transmissibility is greater than 1 and therefore the input is amplified. For values of f/f_n greater than 1.4 the input is attenuated because the transmissibility is less than 1. The degree of amplification or attenuation is dependent on ζ, the damping ratio. Increased damping reduces the amplification, but also increases the value of transmissibility in the attenuation range.

The damped natural frequency, f_n, (see Equation 5) occurs at the maximum values of transmissibility. As seen from Figure 3 and Equation 5, the damped natural frequency decreases slightly with increased damping.

For multiple degrees of freedom systems additional peaks will occur in the transmissibility curve; for complex systems the transmissibility can be established experimentally.

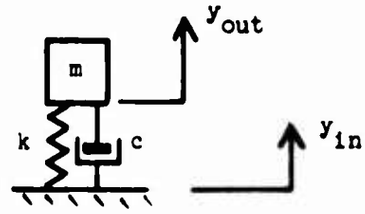
In general, the transmissiblity of a system should be such that attenuation (or if attenuation is not possible, small amplification) occurs at critical frequencies. For further discussions, see the sections on isolation in this chapter and the chapter on Mechanical Impedance.

FIGURE 1. MASS EXCITED SYSTEM



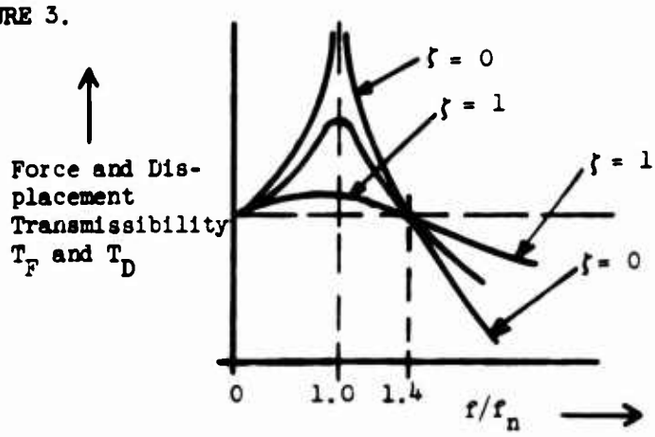
Bass fixed, input force on mass.

FIGURE 2. BASE EXCITED SYSTEM



Base motion, no force on mass.

FIGURE 3.



$$T_F = T_D = \sqrt{\frac{1 + \left(2\frac{f}{f_N}\zeta\right)^2}{\left(1 - \frac{f^2}{f_N^2}\right)^2 + \left(2\frac{f}{f_N}\zeta\right)^2}} \qquad (1)$$

$$f_N = \frac{1}{2\pi}\sqrt{k/m} = \text{undamped natural frequency} \qquad (2)$$

$$\zeta = \frac{c}{c_c} \qquad (3)$$

$$c_c = 2m f_N \qquad (4)$$

$$f_n = f_N\sqrt{1 - \zeta^2} \qquad (5)$$

TRANSMISSIBILITY: The resonant response of a simple system may be expressed analytically by a series of basic equations.

AMPLIFICATION OF RESONANCES

The maximum sinusoidal vibration response of a system is dependent upon the damping in the system and the sweep rate of vibratory frequency.

The transmissibility at resonance, Q, is a maximum response for a system, and is dependent upon the damping in a system. For a single degree of freedom system with viscous damping, Q is given by Equation 1 and plotted in Figure 1. It can be seen from Figure 1 that for small values of (less than 0.1) Q is found from Equation 2.

Figure 1 shows the Q for a s.d.f. system which has reached its steady ζ state response. However, laboratory test conditions do not always allow a system to reach its steady state response. Consider a test in which the sinusoidal input frequency is swept from a minimum frequency, f_o, to maximum frequency, f_f. The sweep rate, K, is exponential and is in units of octaves per minute. The input frequency, f_{in}, after t minutes of testing is given by Equation 3. The less damping in a system, the longer it takes to reach its steady state response. If a natural frequency is swept through rapidly it will reach a smaller maximum response than if it is swept through slowly. Figure 2 shows the percen of steady state response achieved as a function of damping, sweep ra e, and natural frequency. If the test sweep rate is too fast, the test is not severe enough even though the input levels are correct.

Another effect of the swept frequency test is to cause an apparent shift in the maximum response frequency. This occurs when the steady state transmissibility is greater than the actual response achieved when the natural frequency has been passed. As the sweep rate increases, the maximum response occurs further away from the natural frequency. This is illustrated by Figure 3. Therefore, when the natural frequency is determined from laboratory transmissibility tests, the sweep rate must be sufficiently slow to minimize this shift in maximum response frequency and to allow steady state response to occur.

Several examples employing Equation 3 are given in the Appendix on Page 2.6-3.

FIGURE 1

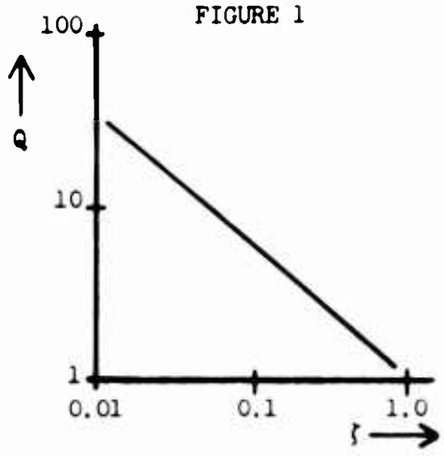


FIGURE 2

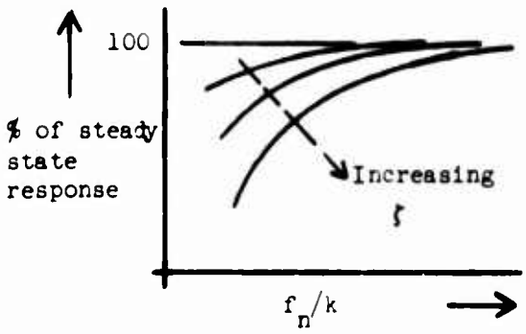
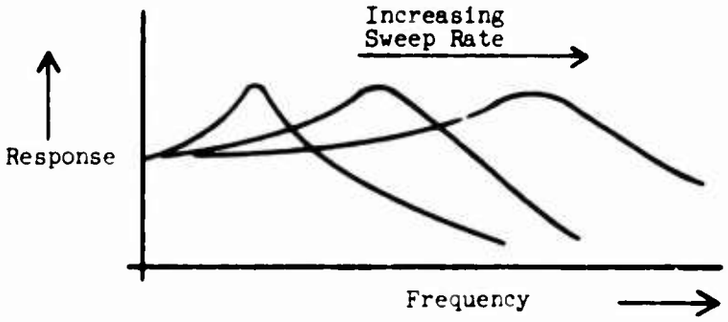


FIGURE 3



$$Q = \frac{4\zeta^2}{\sqrt{16\zeta^4 - 8\zeta^2 - 2 + 2\sqrt{1 + 8\zeta^2}}} \qquad (1)$$

$$Q = \frac{1}{2\zeta} \qquad (2)$$

$$f_{in} = f_o\, 2^{Kt} \qquad (3)$$

RESONANT RISE: The amplification at resonance is affected by damping and sweep rate.

MODES OF VIBRATION

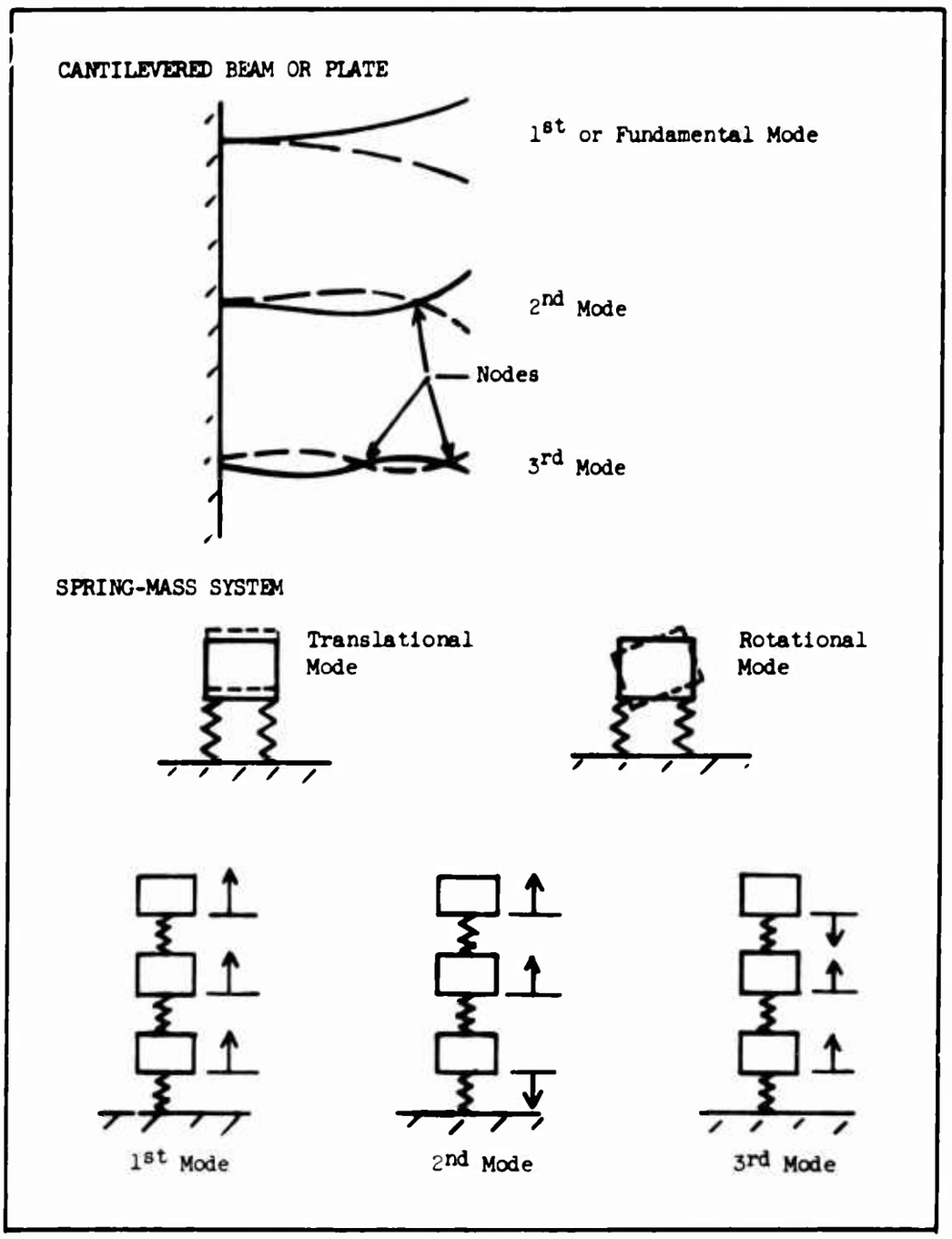
Structures vibrate in definite patterns at their natural frequencies. These patterns are useful in studying the dynamics of a system.

A vibration mode is a pattern of motion that a system assumes when undergoing vibration. When a system is vibrating in one of its modes, all points in the system are moving at the same frequency, with a constant relationship between their amplitudes. Some points in the mode have zero motion; these are node points or node lines (see the adjacent figure). A system may vibrate in many of its modes simultaneously. For example, a broad band vibration spectra will excite all of the modes in that band; a shock applied to a system will usually cause all of the modes of vibration to respond. The extent to which a mode responds depends on the nature of the excitation.

The first three typical mode shapes for a cantilevered beam (or plate) are shown in the figure at the right. The first (or fundamental) mode has its maximum amplitude at the tip with a continuous decrease as the support is approached. Amplitude at the support (for all modes) is zero, unless the support itself is in motion. The second mode of the cantilevered beam has large amplitudes at the tip, decreases in amplitude toward the support until a mode occurs at some intermediate position along the beam; increases again to a second large amplitude; and finally decreases to zero at the base. Similarly, the third mode has two nodes between the tip and the base. The exact positions of the nodes and the relative amplitudes of the mode shape are dependent upon the mass distributions and stiffness distributions in the system.

The cantilevered beam has more than three modes; a system has as many modes of vibration as degrees of freedom. (The degrees of freedom are the number of independent coordinates needed to completely define the vibratory motion, and hence position, of a system.) A single mass on a system of springs can have six degrees of freedom and therefore six modes of vibration. These modes consist of translational and pitching motion in each of three mutually perpendicular planes.

The response of items mounted on vibrating systems can be minimized by selecting attachment points near the nodes of the system. Usually the lower modes of vibration are the most significant; however, higher modes are important if they occur at the sensitive frequencies of the mounted item. In addition, stresses in the system are dependent on the mode shapes. Characteristic mode shapes also aid in the identification of natural frequencies determined by test.



MODE SHAPE: Structures vibrate in characteristic patterns.

NATURAL FREQUENCY AND FATIGUE

Changing the natural frequency of a structure can improve the fatigue strength. A number of factors affect the fatigue damage; these factors must be considered along with the frequency change.

A simple model is used here to illustrate the effect of changing natural frequency on the fatigue damage to a structure. The model consists of a mass m mounted on a massless column of length l, cross-section area A and stiffness k. The structure is excited with an input acceleration a_{in}; response acceleration a and response displacement d are relative to the base. The system is free to oscillate in the vertical direction only. The fatigue strength of the structure is assumed to be represented by the S-N curve shown, and damage is assumed to occur according to Miner's Rule. (See the chapter on Fatigue.*) The system is vibrated for time t, and the fatigue damage is D. (Failure occurs when D = 1). A factor, K_s, is introduced which represents the stress per unit of response displacement. Equation 1 is an expression for the fatigue damage (D_2) in the changed structure, in terms of the fatigue damage (D_1) in the original structure. The derivation of Equation 1 is given in the Appendix on page 2.6-4 and 2.6-5. The material properties (E, C, α) are assumed to remain unchanged; however, this approach can be expanded to include material changes.

Equation 1 states that the changes in the fatigue damage is dependent upon changes in the following:

1. The natural frequency, f_n.
2. The vibration time, t.
3. The input acceleration at the natural frequency, a_{in}.
4. The stress/displacement factor, K_s.

For Equation 2 the test time and input acceleration is considered to remain constant. Three examples are given in the appendix where the natural frequency is increased by; increasing the cross-sectional area; decreasing the column length; and increasing the mass. The fatigue damage change is not only a function of the natural frequency increase, but also a function of the manner in which the natural frequency is changed.

For the model shown, an increase in the natural frequency will increase the fatigue life if f_n is increased by increasing the area or reducing the mass. Alternatively, if f_n is increased by a decrease in length, then the fatigue life decreases.

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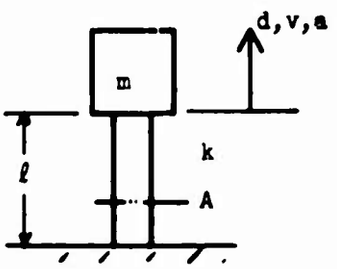
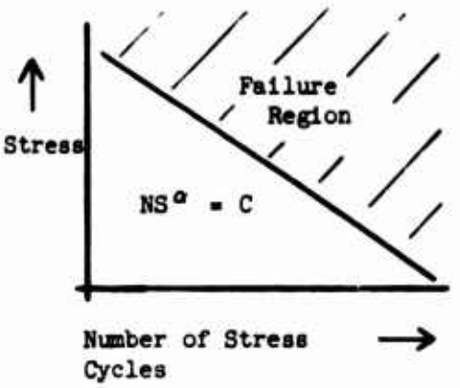
*See Volume III, Chapter 5, Section 2 - "Fatigue".





$$D_2 = D_1 \left[\frac{\left(\frac{t_2}{t_1}\right)\left(\frac{a_{in_2}}{a_{in_1}}\right)^{\alpha}\left(\frac{k_{s2}}{k_{s1}}\right)^{\alpha}}{\left(\frac{f_{n2}}{f_{n1}}\right)^{2\alpha-1}} \right] \quad (1)^{*}$$

$$D_2 = D_1 \left[\frac{\left(\frac{K_{s2}}{K_{s1}}\right)^{\alpha}}{\left(\frac{f_{n2}}{f_{n1}}\right)^{2\alpha-1}} \right] \quad (2)$$

For the illustrated model:

$$K_s = \frac{k}{A} \quad (3)$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (4)$$

$$k = \frac{AE}{\ell} \quad (5)$$

FATIGUE AND NATURAL FREQUENCY: Natural frequency is a major factor in determining the fatigue damage to a structure.

* See the Appendix pages 2.6-4/-5 for the derivation of equation (1). Examples on the application of the other equations above are also given in the Appendix, pages 2.6-6/-7.

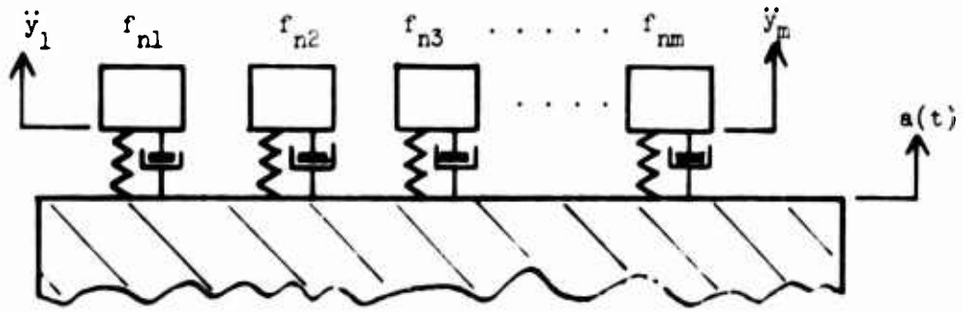
NATURAL FREQUENCY AND SHOCK SPECTRA

Natural frequency is a determining factor in the response of a system to shock inputs.

Shock Spectra: Shock spectra provide a convenient approach to studying the effects of a shock on a system and for comparing the severity of shocks with different pulse shapes. The shock spectrum is a plot of the maximum response of a single degree of freedom system plotted as a function of frequency. Physically, this can be viewed as many s.d.f. systems, each with slightly different natural frequencies, attached to a rigid base (Figure 1a). A shock, a(t), (Figure 1b) is applied at this rigid base and the s.d.f. systems respond ($\ddot{y}$) at their natural frequencies. The maximum response for each system is noted and plotted as a function of natural frequency (Figure 1c); this is a form of shock spectra. It is important to note that shock spectra is not a time plot; it is a frequency plot showing only maximum response values. The appendix shows the shock spectra for some typical simple shock pulses. The plot is generalized by dividing the maximum response acceleration ($\ddot{y}_{max}$) by the maximum input acceleration (a_{max}). The frequency axis is generalized by multiplying f_n by the pulse duration, τ. The response is plotted for s.d.f. systems with no structural damping; systems with damping are given in Reference 4.

Natural Frequency and Shock Response: It can be seen from the shock spectra plots in the appendix that if τf_n is less than approximately 0.3 (depending on the type of shock) the response factor, A_m, is less than 1. This means that the maximum response is less than the maximum input; the shock is attenuated. For example: if some part of an electronic component experienced a 1/2 sine input shock (curve b) with a 0.010 second duration (τ), the shock would be attenuated if the f_n of the part were less than 30 cps ($f_n = \tau f_n/\tau = 0.3/0.010 = 30$). The most severe response ($1.8a_{max}$) would occur if f_n were equal to approximately 80 cps ($f_n = \tau f_n/\tau = 0.8/0.10 = 80$). For f_n above 240 cps, the response is approximately equal to the input (actually, response is slightly higher). Therefore, for this example, it is desirable to avoid natural frequencies between 30 and 240 cps, with frequencies below 30 cps most desirable. Restrictions based on shock response have to be balanced against vibration response limitations and deflection restrictions.

FIGURE 1a.



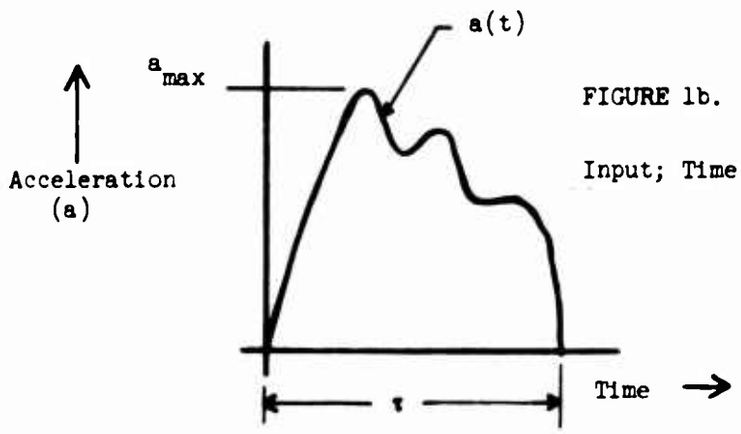


FIGURE 1b.

Input; Time Plot

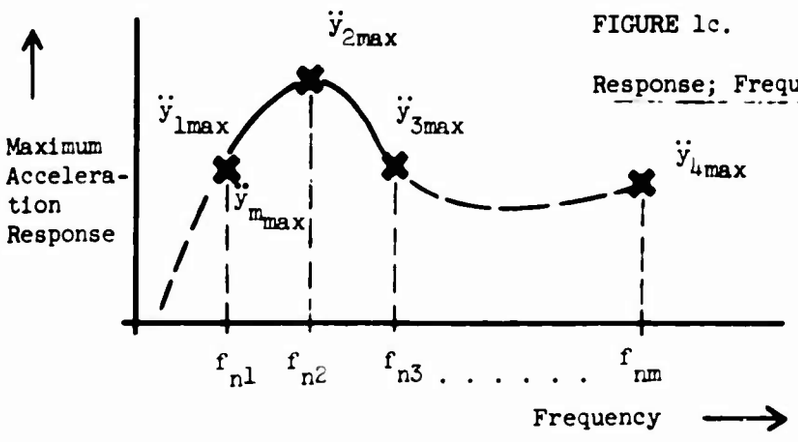


FIGURE 1c.

Response; Frequency Plot

SHOCK SPECTRA: The natural frequency of a structural element is used to define the shock response of that element.

VOLUME III - CHAPTER 2

NATURAL FREQUENCY

SECTION 3 – APPLICATIONS OF NATURAL FREQUENCY

- Shock and Vibration Isolation
- Natural Frequency and Drop Shock
- Effects of Joints on Natural Frequency

SHOCK AND VIBRATION ISOLATION

The natural frequency of a system will strongly influence the response of that system to a given shock or vibration input. It is necessary to balance design restrictions with reduction in response.

For illustrative purposes, consider a single degree of freedom system with a natural frequency of 25 Hz and a Q of 10. The system is subjected to a sinusoidal base (force) excitation of 1g from 5 Hz to 50 Hz, at a sufficiently slow sweep rate to allow full response to occur. In addition, the base of the system is subjected to a 1/2 sine, 10g, 0.002 second duration shock pulse. Assume the system is an electronic component that will fail if its acceleration response exceeds 5g, or if deflection response exceeds 1 inch. It can be seen from the illustration that the system will have a 10g response due to the vibration and failure will occur.

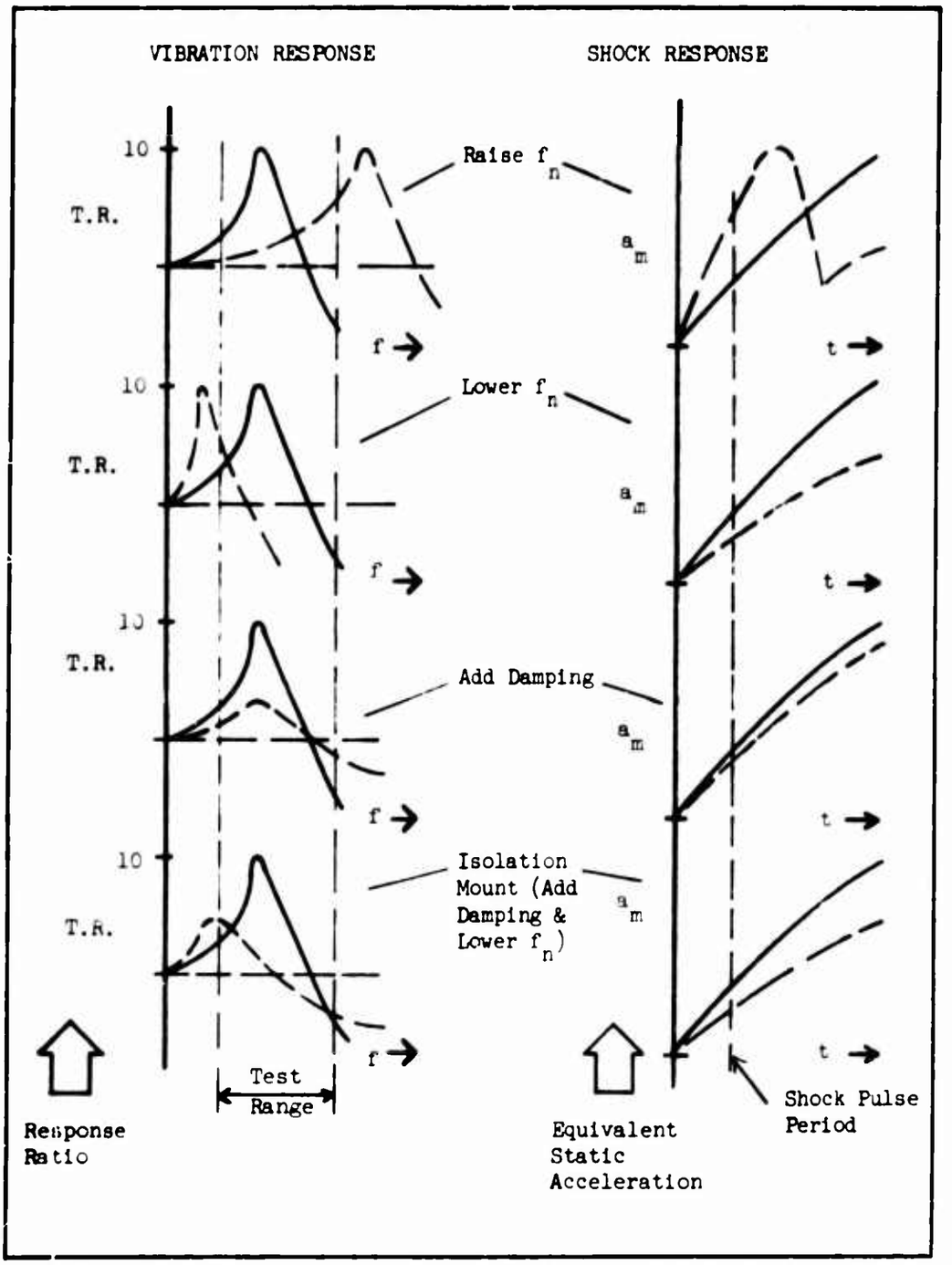
If the natural frequency is raised beyond the vibration test range (assume no vibration input occurs beyond 50 cps) then the system will have a vibration response below 5g at all frequencies. Failure will not now occur due to vibration; the shock response however is another matter. Assume the new natural frequency is 160 cps. In this case the shock response would be 15g and now failure would be caused by the input shock.

If the natural frequency is low red to 4 Hz, the shock response is lower and the vibration response is down to a maximum of approximately 2g at 5 Hz. (Assume no inputs occur below 5 Hz.) However, deflection at 5 Hz is now approximately 1.6 inches and failure will occur. A nomograph for displacement, velocity, acceleration, and frequency is given in the Appendix, page 2.6-10.

If damping is added to the 25 cps system, the vibration response is reduced considerably, but the effect on shock response is small.

If the system is put on an isolation mount, both the damping and the natural frequency is affected. In most cases a soft mount or isolator will add damping to the system and reduce the natural frequency. This reduces both shock and vibration response (in terms of g) but again it is important to keep an eye on reflection. In the illustrated system, the deflection does not exceed 1 inch. However, if the isolated natural frequency were 5 Hz, with a Q of 3, the deflection during vibration would exceed 1 inch.

It is not always necessary to use an isolation system (i.e., add damping and lower the natural frequency) to obtain satisfactory reductions in response. Many problems can be resolved by strengthening the system, raising or lowering the natural frequency or merely adding damping. A good design maintains the proper balance of such factors as functional requirements, shock response, vibration response, cost, weight, and size of the assembly and availability of parts and material.



FACTORS INFLUENCING RESPONSE: Raising or lowering natural frequency or damping changes the response of a structural system.

NATURAL FREQUENCY AND DROP SHOCK

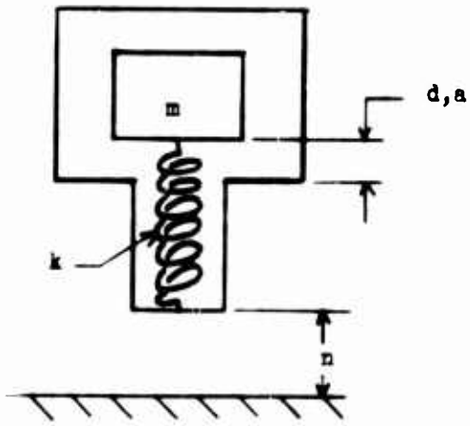
Lowering the natural frequency of a package will decrease its response acceleration when dropped; however, the response displacement will increase. A good design will balance allowable acceleration with allowable displacement.

The Model: A very simple model is used here to illustrate the effect of natural frequency on the shock caused by dropping a package. The model is a single degree of freedom system consisting of a mass, m, and spring, k. The weight of the package is represented by m, and k indicates the stiffness of the structure which deflects during impact. The package drops from a height, h, and experiences a maximum acceleration, a_{max}, and a maximum deflection, d_{max}.

If the maximum deflection of the mass exceeds the allowable deflection, d_a, internal impact (a failure) will occur. If the maximum acceleration exceeds the allowable acceleration, failure will occur.

Design for Response: As the system impacts, the spring deflects until it reaches d_{max} as given by Equation 1. At the same time the system experiences its maximum acceleration as given by Equation 2. The natural frequency of the system must be low (small in value) enough such that a_{max} does not exceed a_a. On the other hand, th natural frequency must be high enough to avoid internal impact (impac occurs when d_{max} exceeds d_a). These restrictions limit the natural freq ency to a band between f_{min} and f_{max} as expressed by Equations 3 and 4. Additional restrictions on the natural frequency may be imposed by vibration requirements.

Limitations: This is a simplified model used to illustrate the facts that, during drop impact, the acceleration response varies directly with the natural frequency and the deflection response varies inversely with the natural frequency. Most real systems will have several degrees of freedom, different shock restrictions for different components, and structural damping. Some real systems will have nonlinear deflections or experience plastic deformation. These systems would have to be considered as individual cases. Reference 3 goes into greater detail for designing packages for drops. The model presented here can be used as a first-cut quick check on the adequacy of a simple design.



$$d_{max} = \frac{1}{2\pi f_n} \sqrt{2gh} \qquad (1)$$

$$a_{max} = 2\pi f_n \sqrt{2gh} \qquad (2)$$

$$f_{min} = \frac{1}{2\pi d_a} \sqrt{2gh} \qquad (3)$$

$$f_{max} = \frac{a_a}{2\pi\sqrt{2gh}} \qquad (4)$$

$$k_{min} = \frac{2ghm}{d_a^2} \qquad (5)$$

$$k_{max} = \frac{a_a^2 m}{2gh} \qquad (6)$$

Where:

$$\frac{k_{max}}{k_{min}} \geq 1$$

$$d_a \leq d_{max}$$

$$a_a \leq \varepsilon_{max}$$

For a given system the natural frequency must fall between the limits of (f_{min}) and (f_{max}) or where (k) is variable. The desig value must fall between the limits of (k_{min}) and (k_{max}) if a_a is expressed in units of g (A_a), then $A_a d_a/(2h) \geq 1$ for safe operation.

An illustrative example is given in the Appendix on page 2.6-8.

EFFECTS OF JOINTS ON NATURAL FREQUENCY

The manner in which sections of a structure are joined has a major influence on the natural frequency of the structure.

The stiffness across a joint can have a major influence on the natural frequency of a structure. For example, consider a cantilevered beam which is fastened to the support by means of a joint with very small stiffness (small with respect to the stiffness of the beam). In this case the stiffness of the beam will have little effect on the fundamental frequency; the first natural frequency will depend almost completely on the stiffness of the joint and the mass distribution of the beam.

The illustration shows joint stiffnesses for thin-skinned tube structures. The joint rotation constant is the angular displacement across a joint (in bending) for an applied moment. An excellent joint (high stiffness) has a low value for the joint rotation constant. A loose joint has a high value for the joint rotation constant. The joint constants are plotted as a function of tube diameter. The joints discussed below have joint constants which fall in bands around the lines shown rather than exactly on the lines.

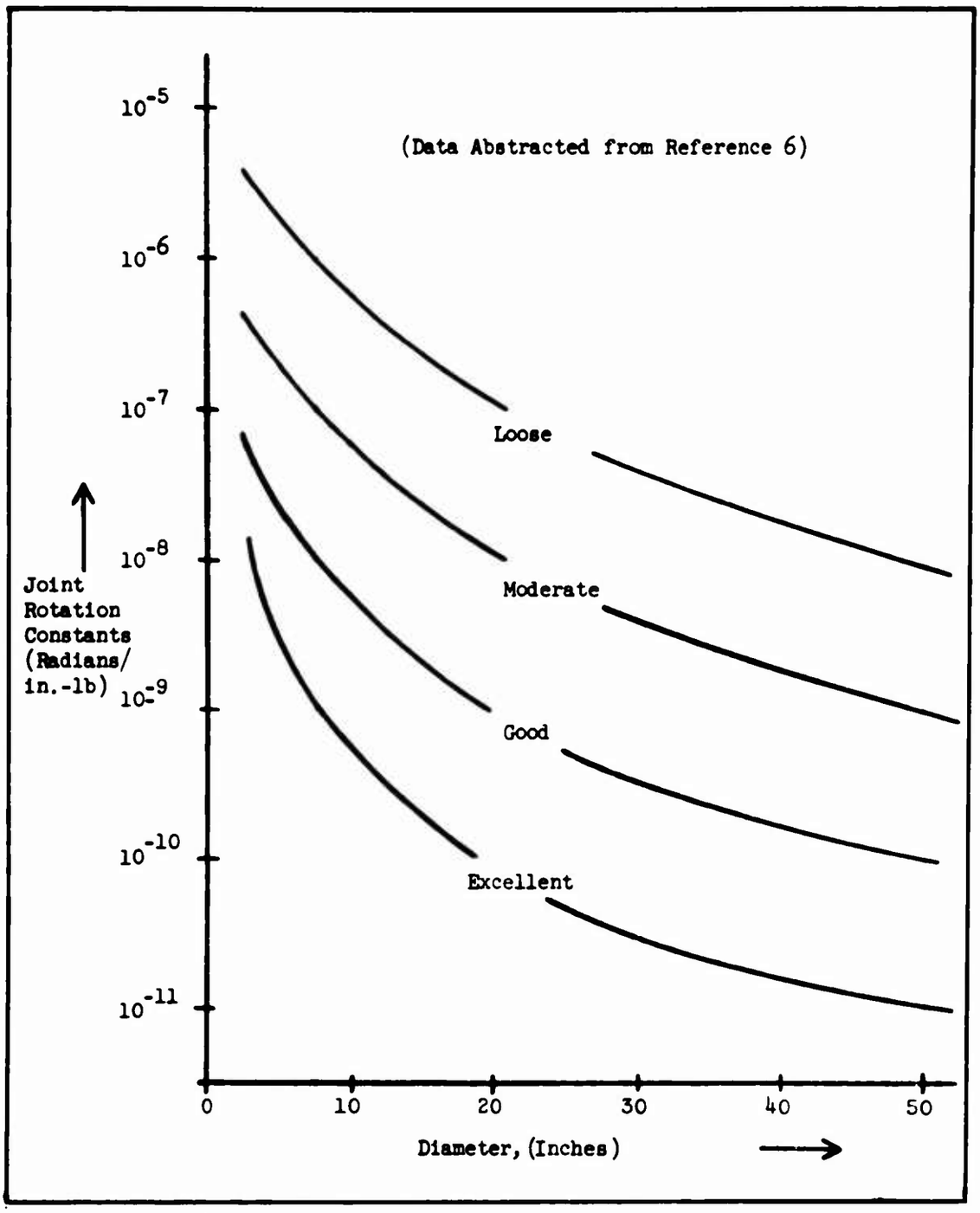
Excellent Joints: These joints include butt welded skins and heavy bolted, preloaded joints.

Good Joints: This category includes heavy flange bolted joints, threaded sections with a butt, and Marman bands.

Moderate Joints: Some joints of moderate stiffness include: riveted, lapped; riveted to inner ring; and thread section without a butt.

Loose Joints: This includes light flanges and lapped joints with screw fasteners.

A typical example is given in the Appendix on page 2.6-9.



JOINT STIFFNESS: Joints have a major effect on the stiffness of a structural system.

VOLUME III - CHAPTER 2

NATURAL FREQUENCY

SECTION 4 – MEASURING NATURAL FREQUENCY

- Visual Methods for Studying Natural Frequency
- Using Transducers to Detect Natural Frequency

VISUAL METHODS FOR STUDYING NATURAL FREQUENCY

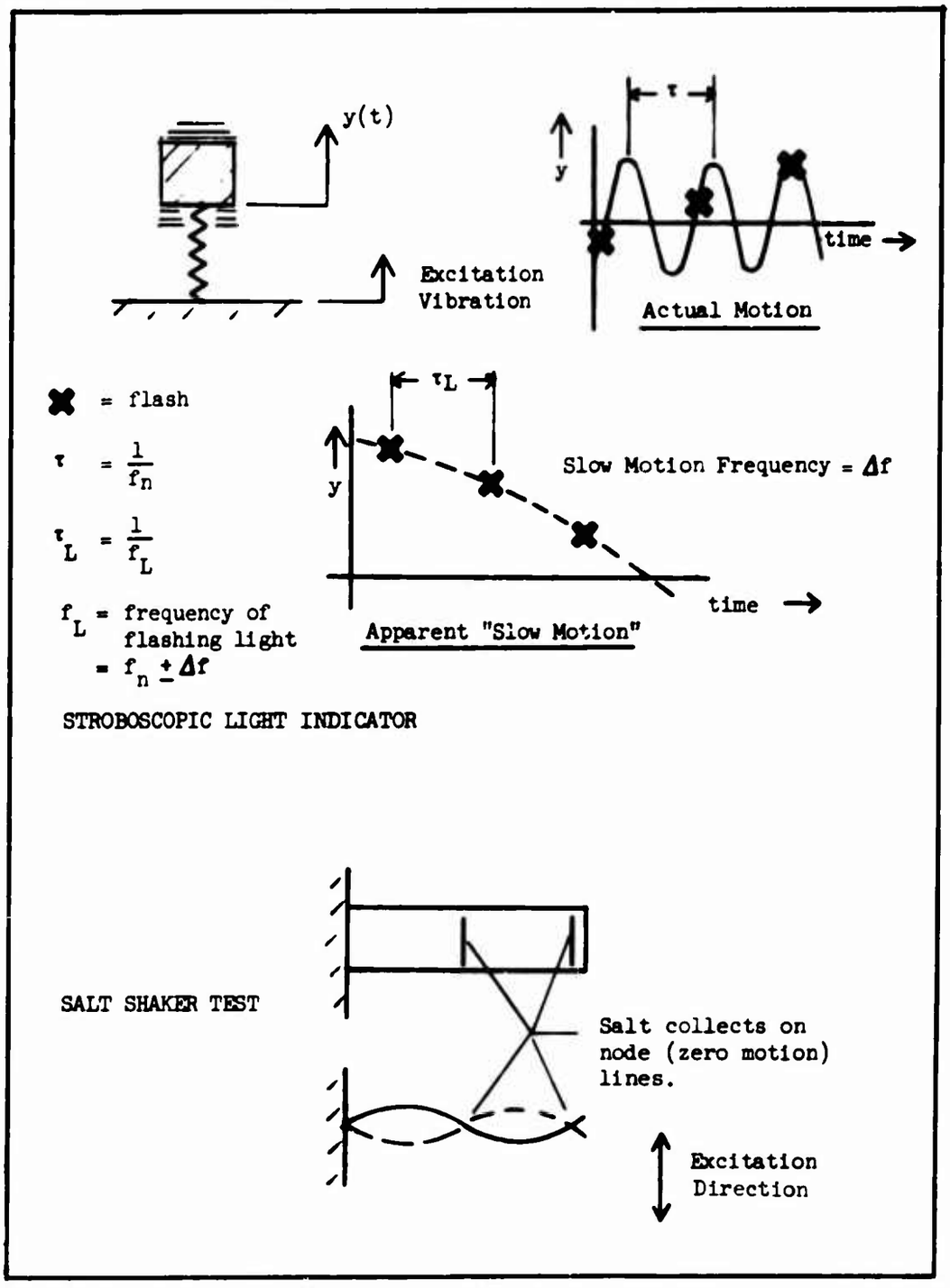
Natural frequencies can be found by visually noting the modes of vibration.

When a system has a very low natural frequency, which implies large deflections when excited, its natural frequency can be seen with the naked eye. For example, in the case of a low frequency system consisting of a weight hanging on a soft spring, the natural frequency can be determined by disturbing the weight and counting the number of free oscillations per unit time. In most practical systems, however, the deflections of a resonating system are too small (and the motion too rapid) to be detected with the naked eye. This section discusses two of the methods used to visually study oscillatory motion; the section that follows discusses the use of transducers in studying natural frequency.

Stroboscopic Light: A stroboscopic light produces a bright flashing light at a selected frequency and is used to give the appearance of "slow motion" to a moving object. Natural frequency is detected by exciting the system (using an electromagnetic shaker) at a given frequency and flashing the light on the system at a slightly different frequency. The excitation frequency (as well as the slightly different flash frequency) is slowly increased until a resonant mode of vibration appears in slow motion. The excitation frequency causing maximum deflection is noted and thus a natural frequency has been found.

The slow motion effect is explained by the accompanying figure. With each flash the light catches the motion in a slightly different position, and the eye makes the motion seem continuous. This system can also be used with a specially synchronized motion picture camera.

Sprinkled Salt Test: Another method for detecting natural frequency is the "salt shaker test." In this approach, salt (or sand, sugar, etc.) is sprinkled over the vibrating surface. (This approach is limited by the fact that the vibrating surface must be in the horizontal plane and must be fairly flat.) The excitation frequency is changed until the salt collects in distinct patterns indicating the node lines (see the adjacent figure). The salt is thrown off the vibrating surface, but collects at the node lines where zero (or very small) motion occurs.



VISUAL DETECTION OF NATURAL FREQUENCY: Natural frequency may be found by observing the modes of vibration.

USING TRANSDUCERS TO DETECT NATURAL FREQUENCY

The acceleration, velocity, deflection, and force relationships of a system can be detected and used in the study of natural frequency.

Transducers are devices which provide electrical signals proportional to the motion of (or forces on) the area to which they are mounted. This section discusses the use of acceleration, velocity, deflection, force, and strain transducers in studying natural frequency. These devices can be used to get transmissibility and response plots. For simple structures, the natural frequencies will occur at the frequencies which have maximum transmissibilities. For more complex structures, response peaks also occur at other frequencies in addition to natural frequencies. Therefore, it is useful to study another important characteristic of the dynamics of a vibrating system, that is, the phase angle between force and displacement. The phase angle, ϕ, represents the time lag between peaks, as shown in the adjacent figure for the single degree of freedom system.

At resonance the phase angle between force and displacement is 90° ($\pi/2$ radians). Measuring this phase angle (or similar force relationships with velocity or acceleration) provides another indication of resonance.

In addition to response and phase angle, transducers are used to find natural frequency with mechanical impedance methods (see the chapter on impedance).

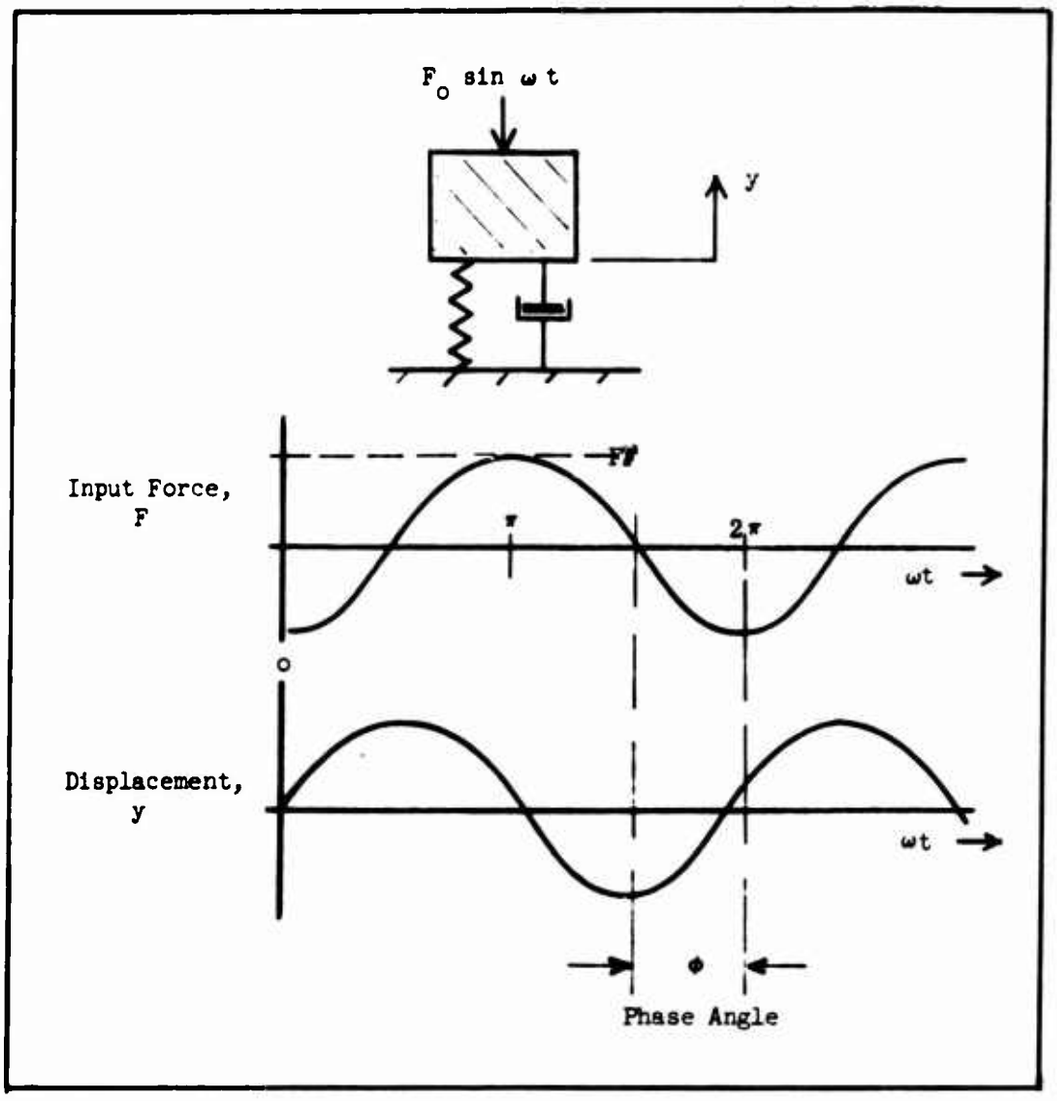
Accelerometers: Two general classes of accelerometers are used; the piezoelectric and strain gage types. The piezoelectric type uses a crystal which produces an electrical signal proportional to the dynamic force applied to it. The strain gage type gives a signal proportional to the strain in a flexible mass-support. The piezoelectric type is used mostly where small transducer weight and size are important, and the strain gage type where sensitivity and low frequency response are governing factors.

Velocity Meters: Devices which measure velocity directly, and usually do so by breaking lines of magnetic flux. Sometimes it is more convenient to integrate an acceleration signal to obtain velocity.

Displacement Devices: For large deflection studies, a potentiometer can be used to detect deflection. Another device, the proximity gage, senses the change in electrical capacitance between the sensor and the vibrating surface. An advantage of the proximity gage is that the device does not contact the vibrating structure.

Force Transducer: Both piezoelectric and strain gages type transducers are used to sense force. The strain gage type will sense both static and dynamic forces. The piezoelectric devices sense only dynamic force.

Strain Gages: The resistance of a strain gage (and thus the voltage drop across it) changes as the surface to which it is mounted is strained. This is an indication of the stress and loads (forces) in a structure.



PHASE ANGLE INDICATES RESONANCE. The phase angle between force and displacement is 90° when the system is vibrating at its natural frequency.

VOLUME III - CHAPTER 2

NATURAL FREQUENCY

SECTION 5 – ESTIMATING AND CALCULATING NATURAL FREQUENCY

- Build a Model to Simplify Analysis
- Static Deflection and Natural Frequency
- Determining Natural Frequency Analytically
- Using Electrical Analogies to Study Natural Frequency
- Digital Computer Techniques

BUILD A MODEL TO SIMPLIFY ANALYSIS

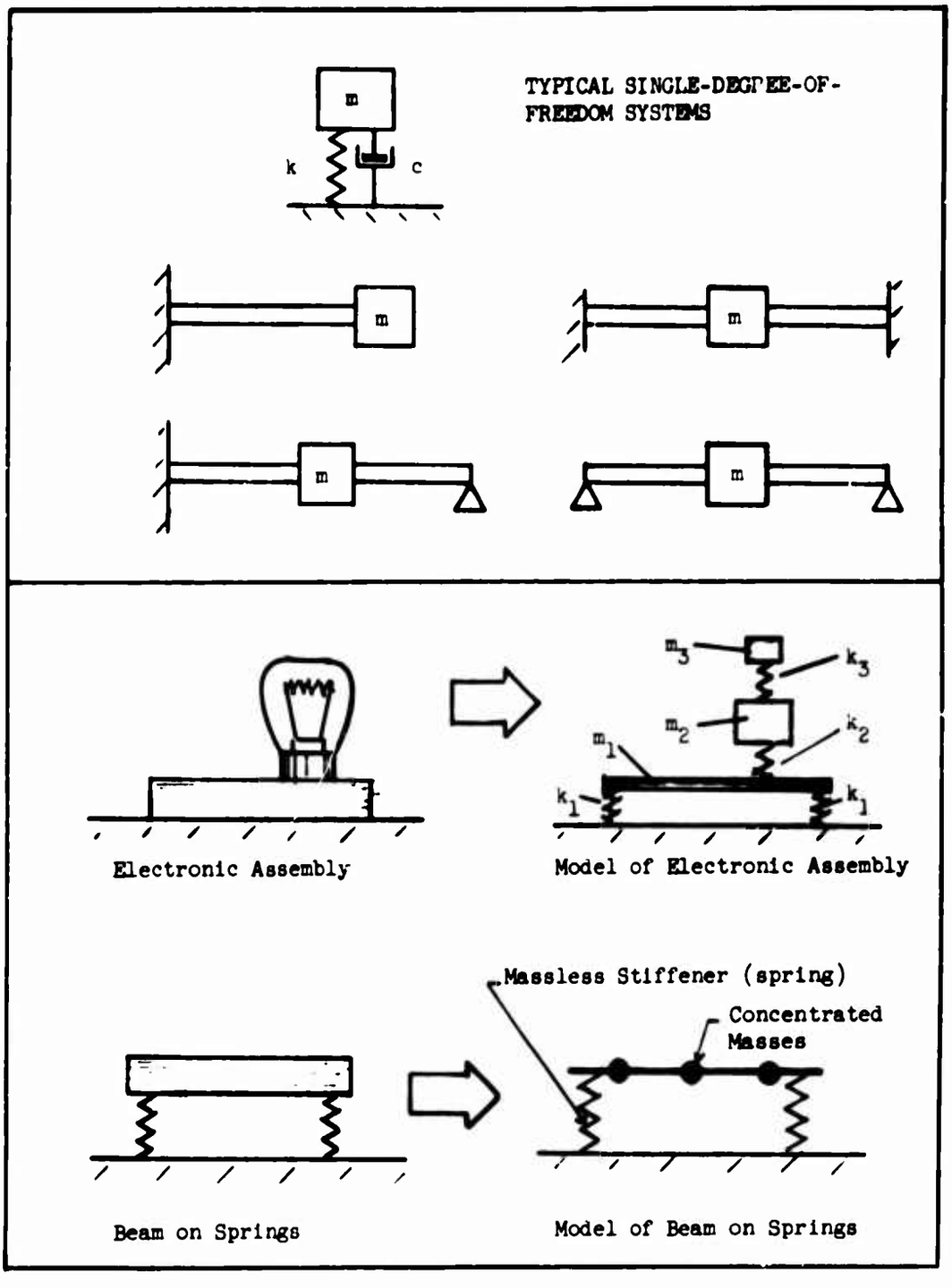
All structures must be represented by an analytical model in order to calculate their natural frequencies.

A simple structure can be modeled as a single degree of freedom system. For example, the structures shown at the right have a concentrated mass on a light-weight beam. These can be represented by a single degree of freedom system with mass m and stiffness (at the mass) k; stiffness is dependent on the support conditions.

For more complex structures, springs, masses and dampers are used as the building blocks for the model. The vacuum tube mounted on a chassis can be considered as a system of point masses connected by springs and dampers. As shown in the adjacent figure, k_1 represents the stiffness of the chassis, m_1 the mass of the chassis, k_2 the stiffness between the tube body and the chassis, m_2 the mass of the tube body, and k_3 and m_3 represent the filament structure. This reduces the structure (which is actually quite complex) to a form which will give usuable answers for the response of the sensitive item, i.e., the filament.

If a more detailed model were needed to give proper response, additional mass and spring breakdowns can be made. For example, a more detailed model of the top of the chassis may be needed. This can be obtained by modeling the chassis top as a beam on springs. The beam is broken down into concentrated masses connected by massless stiffness. The natural frequencies of these systems are found by using the methods discussed in the topic "Determining Natural Frequency Analytically."

The complexity of the model depends on the objectives of the analysis. In determining the vibratory stress in a simple structure, a single degree of freedom model may often be sufficient. For a structure which must have carefully controlled displacements (e.g., the mounting surface for an alignment device), a complex model yielding many modes may be needed. The limitations of the model should not be exceeded, nor should an overly complex model be used when not needed.



STRUCTURAL MODELS: The model puts the structure in a form that can be readily analyzed.

STATIC DEFLECTION AND NATURAL FREQUENCY

The static deflection of a structure provides a simple means for finding the fundamental frequency.

Static deflection (δ_{st}) is the displacement of a structure (at rest) under its own weight. For a single degree of freedom system, δ_{st} is the displacement of the mass due to gravity. For other structures (see the illustrations), δ_{st} is the maximum deflection.

The relationship between natural frequency and static deflection is easily shown with the single degree of freedom system. The natural frequency is given by Equation 1. The stiffness, k, is equal to a force, W, divided by the displacement d_w, caused by W (if W is the weight of the system, the displacement is δ_{st} - Equation 2). Substituting the expression for stiffness (in terms of δ_{st}) into Equation 1 yields Equation 3, the natural frequency of a single degree of freedom system in terms of its static displacement. The first column at the right shows a few simple structures which can be represented as single degree of freedom systems and therefore have a natural frequency as described by Equation 3. All structures in the first column consist of a point mass (with weight W in pounds) on a weightless beam of length L in inches, area moment of inertia I in inches^4, and modulus of elasticity E in pounds per square inch.

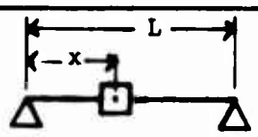
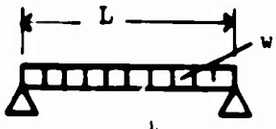
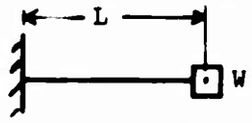
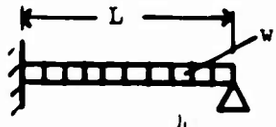
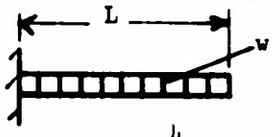
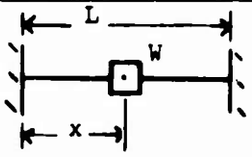
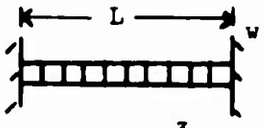
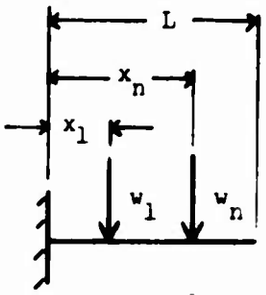
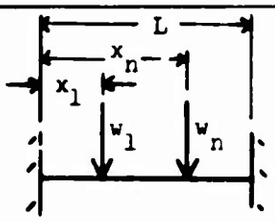
The equations relating fundamental frequency (lowest natural frequency) to static deflection are given in the second and third columns for several other types of structures; these structures consist of uniform loads, w (lbs/in), or several point loads, W_1, W_2, etc., on massless beams. The static deflections for these systems are the maximum at-rest deflections when the structures are horizontal, as shown. The fundamental modes, for a given static deflection, are higher in frequency for a multiple mass system than for the single degree of freedom system. In the single degree of freedom system, all the mass vibrates at the maximum amplitude of the system. In multiple-mass systems, only the mass at the point of maximum deflection for the fundamental mode vibrates at the maximum amplitude of the system. All other masses are vibrating at some amplitude less than the maximum. This can be viewed as introducing an "equivalent mass" concept which is less than the physical mass of the system. (This may also be shown rigorously by using energy methods.) This "equivalent mass" is the value of m which causes Equation 1 to yield the correct frequency for a multiple-mass system (k is still expressed by Equation 2). The lower value for "equivalent mass" results in a higher natural frequency and this accounts for the constant in Equations 4 and 5 being larger than 3.13.

Illustrative Example: Find the natural frequency of a cantilevered beam with the weight concentrated at the tip (the second figure in Column 1).
W = 3 pounds, L = 20 inches, $E = 10 \times 10^6$ psi, $I = .08$ in^4

$$\delta_{st} = \frac{WL^3}{3EI} = \frac{3\,(20)^3}{3(10^7)\,(.08)} = 0.01 \text{ inch}; \quad f_n = \frac{3.13}{\sqrt{\delta_{st}}} = \frac{3.13}{\sqrt{.01}} = 31.3 \text{ cps}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (1)$$

$$k = \frac{W}{dw} = \frac{mg}{\delta_{st}} \qquad (2)$$

$f_n = \frac{3.13}{\sqrt{\delta_{st}}}$ (3)	$f_n = \frac{3.55}{\sqrt{\delta_{st}}}$ (4)	$f_n = \frac{3.89}{\sqrt{\delta_{st}}}$ (5)
 $\delta_{st} = \frac{W}{48EI}(3L^2-4x^2)$	 $\delta_{st} = \frac{5wL^4}{394EI}$	
 $\delta_{st} = \frac{WL^3}{3EI}$	 $\delta_{st} = \frac{wL^4}{185EI}$	 $\delta_{st} = \frac{wL^4}{8EI}$
 $\delta_{st} = \frac{Wx^2}{48EI}(3L-4x)$	 $\delta_{st} = \frac{wL^3}{384EI}$	 $\delta_{st} = \sum_{i=1}^{n} \frac{W_i x_i^2}{6EI}(3L-x_i)$
	 $\delta_{st} = \sum_{i=1}^{n} \frac{W_i x_i^2}{48EI}(3L-4x_i)$	

STATIC DEFLECTION: The fundamental frequency is equal to a constant divided by the square root of the static deflection.

DETERMINING NATURAL FREQUENCY ANALYTICALLY

Several mathematical methods are available for determining natural frequency from the analytical model.

The following approximate methods are commonly used in calculating frequencies and mode shapes.

Rayleigh Method: This method is based on the conservation of energy concept. In a system vibrating with simple harmonic motion, the potential energy (stored in the spring at maximum deflection) is equal to the kinetic energy (function of velocity as the mass passes the equilibrium position). Since the deflection characteristics of the system are necessary for determination of these energies, the method involves assumption of the normal deflection curve. The static deflection curve usually results in a fairly accurate frequency value. If greater accuracy is necessary, the curve may be repeatedly improved.

For a beam of length, l, m mass/unit l, the potential energy U and kinetic energy T reduce to the form in Equation 1. Equating energies and solving for frequency yields Equation 2. This method calculates only the fundamental (first) frequency.

Rayleigh - Ritz Method: The Rayleigh method effectively increases the stiffness of the system through the difference in the actual mode shape and that assumed. The Ritz extension of the Rayleigh method represents the deflection as a function of several variables in the form of a series. (See Equation 3.) The magnitude of A_1 chosen to reduce w to a minimum. (See Equation 4.) Manipulation of equations yields a set of linear homogeneous equations in A_1 (one equation for each A_1 used in the series. For a nontrivial solution to exist, the determinant must equal zero. The resulting characteristic equation is then solved to determine the frequencies and modes.

Holzer Method: This method is tranditionally used for torsional vibrations. A frequency is assumed and the deflection calculated. When the calculated deflection curve satisfies the boundary conditions, the assumed frequency is the natural frequency and the deflection is the mode shape.

Mykelstad Method: This is similar to the Holzer analysis in that the frequency is assumed and the deflection is calculated. It is better adapted to bending problems than the Holzer method. As with the Holzer Method, higher frequencies (above fundamental) may be determined.

Eigenvalue Solution: Manipulation of the system equations of motion expressed in matrix form leads to Equations 5 and 6, which is the the classifical eigenvalue problem. $1/w^2$ values are called eigenvalues and the relationships among the amplitudes are called eigenfunctions. The nontrivial solution exists if the determinant of the ϕ coefficients, $|[D] - [\lambda I]|$ is equal to zero. Expansion of the determinant leads to a higher degree polynomial in λ called the frequency equation. Solution of this equation for its roots yields the frequency values and the corresponding mode shapes are computed by substitution of λ values into Equation (6) and computing any column of the adjoint of the resulting matrix. This column is proportional to the mode shape.

See Reference 5 in the Appendix on page 2.6-0 for more information.

Influence Coefficients: The stiffness or flexibility information about the structure necessary for formulation of the (D) matrix may be determined as influence coefficients. An influence coefficient is simply a deflection at one point due to a load at another point. Example, consider a beam on which four points are marked. The influence coefficients are generated by calculating the deflection at each of the points due to a unit load applied successively at each point; i.e. δ_{ij} = deflection at point i due to a load at j. Apply the load at point 4 and calculate δ_{14}, δ_{24}, δ_{34}, and δ_{44}. Then move the load to point 3 and calculate δ_{13}, δ_{23}, δ_{33}, and δ_{43}, continue until the load has been applied to each point. Only half of the deflections need be calculated since Maxwell's Reciprocal theorom which holds for any elastic structure states that $\delta_{ij} = \delta_{ji}$.

RAYLEIGH METHOD

$$u_{max} = \frac{1}{2}\int_o^l EI\left[\frac{d^2y}{dx^2}\right]^2 dx \qquad (1)$$

$$T_{max} = \frac{w^2}{2}\int_o^l my^2\,dx$$

$$\omega^2 = \frac{\int_o^l EI\left[\frac{d^2y}{dx^2}\right]^2 dx}{\int_1^l my^2\,dx} \qquad (2)$$

RAYLEIGH - RITZ METHOD

$$y = a_1 f_1(x) + a_2 f_2(x) + \ldots + a_n f_n(x) \qquad (3)$$

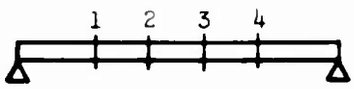
$$\frac{\partial}{\partial a_i}\left[\frac{\int_o^l EI\left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_o^l my^2\,dx}\right] \qquad (4)$$

EIGEN VALUE SOLUTION

$$[D]\{\phi\} = \lambda\{\phi\} \qquad (5)$$

$$[D] - \lambda[I]\{\phi\} = 0 \qquad (6)$$

INFLUENCE COEFFICIENTS



ANALYTICAL METHODS: There are several effective methods for calculating fundamental frequency.

USING ELECTRICAL ANALOGIES TO STUDY NATURAL FREQUENCY

The equations of motion of an elastic structural system can be duplicated by analogy with an electrical system.

Two types of analog systems are presented in this section: the direct and the indirect computers.

The direct analog computer is an electrical network consisting of resistors, capacitors and inductors. The equations of the electrical system are analogous to the equations for the mechanical system. The equation of motion for the single degree-of-freedom system is of the same form as the equation of a simple R-C-L circuit. The applied force, F(t), is analogous to the applied voltage, E(t). The displacement, x, (and its time derivatives) is analogous to charge, q, (and its time derivatives). Inductance, resistance, and the inverse of capacitance are analogous to mass, viscous damping, and stiffness, respectively. In a similar manner, analogous electrical system can be made for multiple degrees-of-freedom systems.

The indirect analog computer forms the equations of motion by using an electrical system which performs integration, multiplication and summing. The heart of the system is the operational amplifier. The illustration at right shows integrating amplifiers, a multiplying amplifier, and potentiometers combined to form the equation of motion for a single degree-of-freedom system. A characteristic of the amplifiers shown here is that the sign (+ or -) of the output is the opposite of the sign of the input. The multiplying amplifier is used here only as a device for changing a negative value to a positive value.

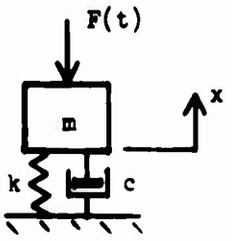
The equation of motion is rearranged to solve for $\ddot{x}$, the acceleration. The quantities on the right hand side of the equation are summed at point (1) to form $\ddot{x}$. $\ddot{x}$ goes into the first integrating amplifer point (2) and the output is $-\dot{x}$. $-\dot{x}$ is fed back to point (1) through a potentiometer used to set the coefficient c/m; this supplies the quantity $(-c/m)\,\dot{x}$ to point (1). $-\dot{x}$ is also fed into the second integrating amplifier point (3) to yield the output, x. $-\dot{x}$ is obtained by using a multiplying amplifier. The quantity $(-k/m)\,x$ is fed back to point (1) by putting -x through a potentiometer used to set the coefficient k/m. This completes the system. The quantities shown are voltages proportional to the values in the mechanical system. Amplitude and time-scaling considerations for this type of analog computer is beyond the scope of this section; the reader is referred to the literature for more detailed descriptions of analog computers.

For more information see References 5, 9, and 10 in the Appendix on page 2.6-0.

DIRECT ANALOG COMPUTER

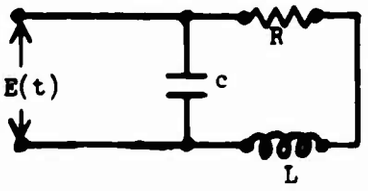
Mechanical System

$$\text{Force} = F(t) = m\ddot{x} + c\dot{x} + kx$$

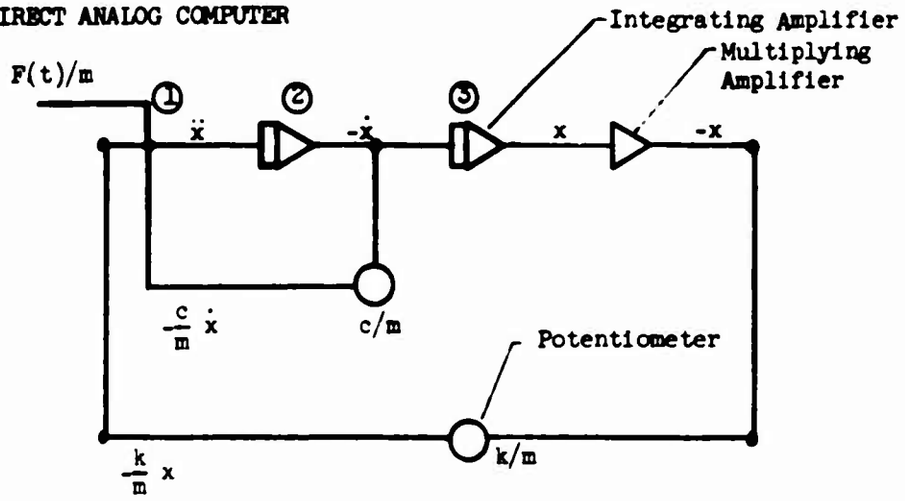


Electrical System

$$\text{Voltage} = E(t) = L\ddot{q} + R\dot{q} + \frac{1}{c}q$$



INDIRECT ANALOG COMPUTER



$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x + \frac{F(t)}{m}$$

ANALOG COMPUTERS: Electrical systems are used to simulate the mechanical equations of motion.

DIGITAL COMPUTER TECHNIQUES

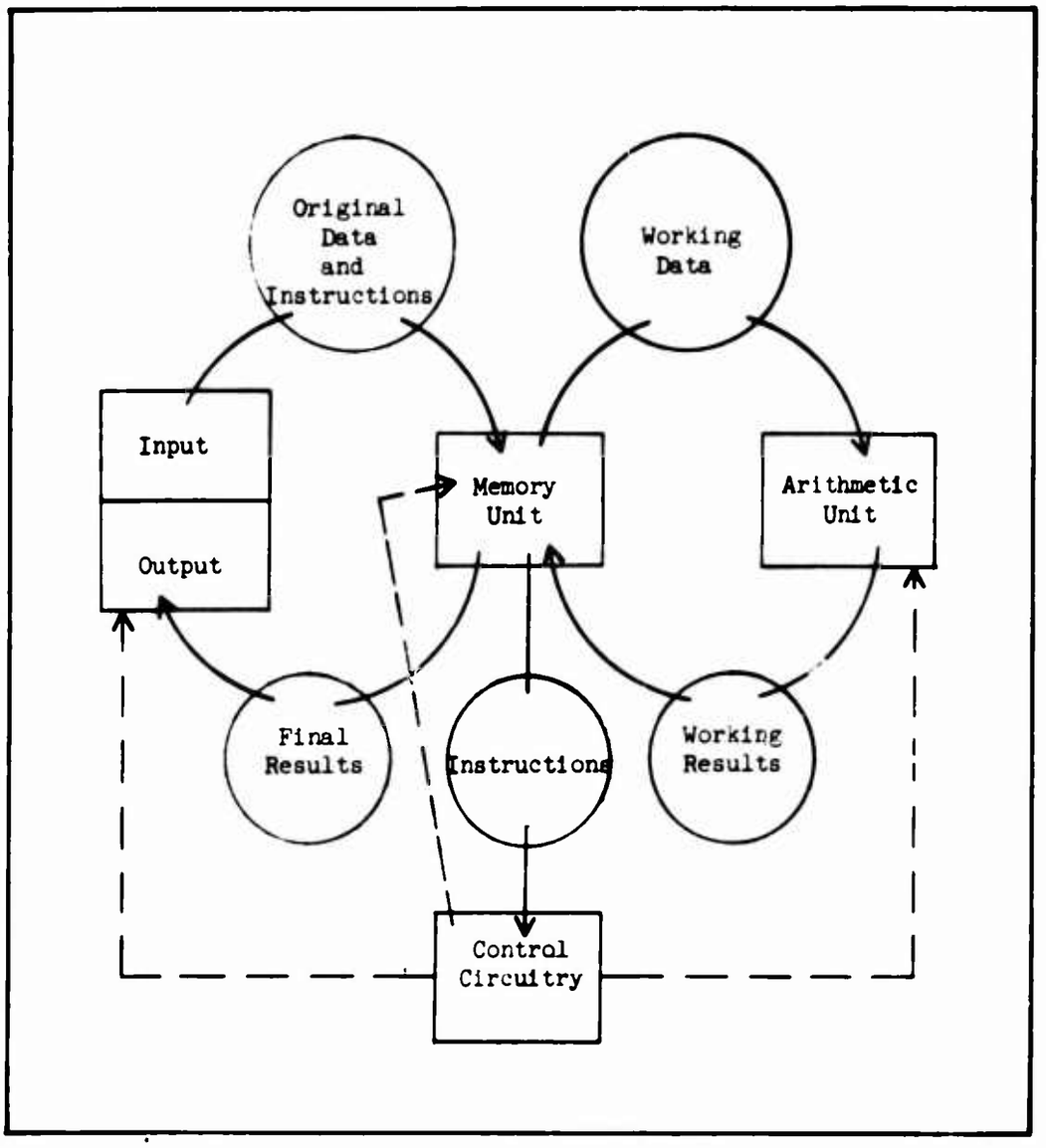
The digital computer rapidly handles laborious calculations and allows the use of complex or lengthy analysis procedures.

The digital computer is capable of handling vast amounts of data and performing calculations so rapidly that it is now possible to employ solutions which were previously too lengthy to be practical. The high-speed computer has placed new emphasis on numerical methods of problem solution. Methods utilizing matrix techniques are particularly well adapted to computer solutions.

Computing methods may be classified as either direct or iterative. A direct method gives a solution after a finite number of steps whereas an answer is obtained from an iterative method as a limit of an infinite number of successive approximations. Iterative methods are useful in solving a variety of problems even when specific techniques are available but are awkward to use; i.e. solving for roots of a polynomial, solving systems of equations, etc.

The eigenvalue problem may be solved by a matrix iteration procedure applied to the equation (1). Select any values for the (q) column matrix, premultiply by $[D]$, normalize the resulting matrix $\{q\}$, the normalizing factor is the first approximation of $\frac{1}{w^2}$. Normalizing is accomplished by dividing each element of $\{q\}$ by the value of the largest element. $\{q\}$ is then taken as the trial column and the procedure repeated to obtain $\{q\}_2$ and so on until the normalizing factor and resulting $\{q\}$ converge to some value. The values thus obtained are the fundamental frequency and its associated normalized mode shape. Certain constraints are then placed on the $[D]$ matrix and the iterative process is repeated until convergence on the second mode is obtained. In this manner successively higher modes are calculated. This procedure is much too lengthy (for even a simple problem) to be accomplished by hand with a desk calculator but takes only seconds to accomplish with a digital computer.

Equation 1: $$[D]\ \{q\} = \frac{1}{w^2}\ \{q\}$$



DIGITAL COMPUTERS: Illustrated is a simplified block diagram of an electronic digital computer. Square boxes represent parts of the machine itself, while circles represents material upon which the machine performs a function.

VOLUME III - CHAPTER 2

NATURAL FREQUENCY

SECTION 6 - APPENDIX

- Bibliography
- Symbology
- Glossary
- Examples on Sweep Rate of Vibratory Frequency
- Examples on Natural Frequency and Fatigue
- Examples Showing the Relationship Between Fatigue Life and Natural Frequency
- Example on Natural Frequency and Drop Shock
- Example on Effects of Joints on Natural Frequency
- Nonogram for Harmonic Motion
- Shock Spectra for Standard Shock and Pulses
- Vibration Frequency Charts

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SYMBOLOGY

A_m	Shock response factor
a	Acceleration
C	Coefficient of viscous damping
C_c	Critical damping coefficient
d	Displacement
F	Force
f	Forcing frequency
f_n	Damped natural frequency
f_N	Undamped natural frequency
g	Acceleration of gravity
h	Drop height
I	Rotary inertia
K	Sinusoidal sweep rate
k	Stiffness
m	Mass
N	Number of stress cycles to failure
n	Number of applied stress cycles
Q	Transmissibility at resonance
S	Stress
TT	Torque
T_D	Displacement transmissibility
T_F	Force transmissibility
v	Velocity
$\ddot{y}$	Response acceleration
α	Angular acceleration
δ_{st}	Static deflection
θ	Angular displacement
τ	Shock pulse duration
ζ	Damping ration = C/C_c

GLOSSARY

Attenuation - The dissipation of energy with time or distance; damping.

Cycle - The complete sequence of values of a periodic quantity that occur during a period.

Degrees-of-Freedom - The number of degrees-of-freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the positions of all parts of the system at any instant of time.

Frequency - The number of times that a periodic function repeats the same sequence of values during a unit variation of time The unit is the cycle per second which equals one Hertz (Hz).

Mode of Vibration - In a system undergoing vibration, a mode of vibration is a characteristic pattern assumed by the system in which the motion of every particle is simple harmonic with the same frequency. Two or more modes may exist concurrently in a multiple degrees-of-freedom system.

Response - The motion (or other output) of a system or device resulting from an excitation.

Shock - Nonperiodic excitation (e.g., a motion of the foundation or an applied force) of a mechanical system that is characterized by suddeness and severity, and visually causes significant relative displacements is the system.

Stiffness - The ratio of change of Force (or torque) to the corresponding change in translational (or rotational) deflection of an elastic element.

Stress - Internal force exerted by either of two adjacent parts of a body upon the other across an imagined plane of separation.

Transmissibility - Non dimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio may be one of forces, displacements, velocities, or accelerations.

EXAMPLES ON SWEEP RATE OF VIBRATORY FREQUENCY

Example A:

How many octaves are between 10 Hz and 200 Hz?

Solution:

$$f_{in} = 200$$

$$f_o = 10$$

$$\frac{f_{in}}{f_o} = 20 = 2^{Kt}$$

$$Kt = 4.32 \text{ octaves}$$

Example B:

What is the sweep rate if the frequency changes from 5 Hz to 500 Hz (exponentially in 7.5 minutes?

Solution:

$$f_{in} = 500$$

$$f_o = 5$$

$$\frac{f_{in}}{f_o} = 100 = 2^{Kt}$$

$$Kt = 6.64 \text{ octaves}$$

$$t = 7.5 \text{ min}$$

$$K = \frac{6.64}{7.5} = 0.885 \text{ octaves/min}$$

Example C:

In example B, what is the frequency after 5 minutes of sweep?

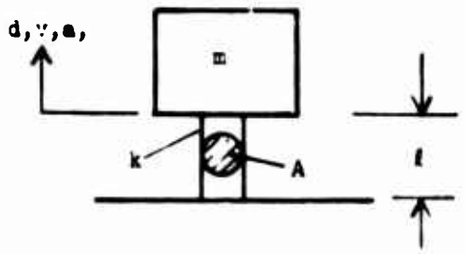
Solution:

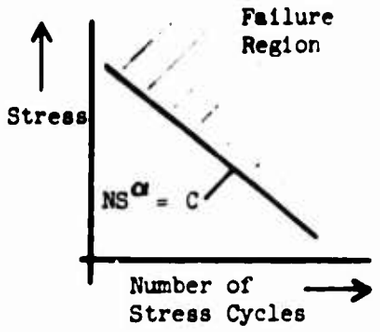
$$f_{in} = (5)2^{Kt} = (5)2^{(0.885)(5)} = (5)2^{4.425} =$$

$$5(21.5) = 107.5 \text{ Hz}$$

EXAMPLES ON NATURAL FREQUENCY AND FATIGUE

Derivation of the equation which relates natural frequency and fatigue damage. (See Chapter 2, page 2.2-6.)





$$\text{Stress} = \frac{kd}{A} = \frac{Ed}{\ell} \tag{1}$$

$$d = \frac{a}{(2\pi f_n)^2} = \frac{Qa_{in}}{(2\pi f_n)^2} \tag{2}$$

$$S = \frac{kQa_{in}}{A(2\pi f_n)^2} \tag{3}$$

$$D = \frac{n}{N} \tag{4}$$

$$n = t\, f_n \tag{5}$$

$$NS^{\alpha} = C \qquad N = \frac{C}{S^{\alpha}} \tag{6}$$

$$D = \frac{t\, f_n\, S^{\alpha}}{C} = \frac{t\, f_n\, (kQa_{in})^{\alpha}}{C\, A^{\alpha}\, (2\pi f_n)^{2\alpha}} \tag{7}$$

Let $K_s = k/A^*$, and Q, C and α remain constant (same material)

*K_s = geometric structural characteristic relating stress to response displacement, d. For this model, $K_s = (k/A)$(psi/in.).

$$\frac{D_2}{D_1} = \frac{t_2 \, f_{n2} \, (a_{in_2})^{\alpha} \, (K_{S2})^{\alpha} \, (f_{n1})^{2\alpha}}{t_1 \, f_{n1} \, (a_{in_1})^{\alpha} \, (K_{S1})^{\alpha} \, (f_{n2})^{2\alpha}} \tag{8}$$

$$D_2 = D_1 \left[\frac{\left(\frac{t_2}{t_1}\right)\left(\frac{a_{in_2}}{a_{in_1}}\right)^{\alpha}\left(\frac{K_{S2}}{K_{S1}}\right)^{\alpha}}{\left(\frac{f_{n2}}{f_{n1}}\right)^{2\alpha-1}} \right] \tag{9}$$

EXAMPLES SHOWING THE RELATIONSHIP BETWEEN FATIGUE LIFE AND NATURAL FREQUENCY

Example 1: Increase the area, A. The natural frequency of the model shown is increased by a factor of R. This increase is accomplished by increasing the area. How does this affect the fatigue damage, if nothing else is changed?

Since $f_{n2} = R f_{n1}$, then $k_2 = R^2 k_1$ and therefore $A_2 = R^2 A_1$ (Equations 4 and 5).

$$K_{s2} = \frac{k_2}{A_2} = \frac{R^2 k_1}{R^2 A_1} = \frac{k_1}{A_1} = K_{s1} \therefore \frac{K_{s2}}{K_{s1}} = 1$$

$$D2 = \frac{D1}{R^{2\alpha-1}} \qquad \text{(from Equation 2)}$$

Conclusion: For this example, fatigue damage is reduced by increasing the area, A.

Example 2: Decrease the length, l. f_n is increased by a factor R; this is accomplished by decreasing l. What is the effect on the fatigue damage if nothing else is changed?

Since $f_{n2} = R f_{n1} = R^2 k_1$. A is unchanged. (Equations 4 and 5)

$$K_{s2} = \frac{k_2}{A_2} = \frac{R^2 k_1}{A_1} = R^2 K_{s1}$$

$$D_2 = D_1 \frac{R^{2\alpha}}{R^{2\alpha-1}} = D_1 R \qquad \text{(from Equation 9)}$$

Conclusion: For this example, fatigue damage is increased by reducing the length, l.

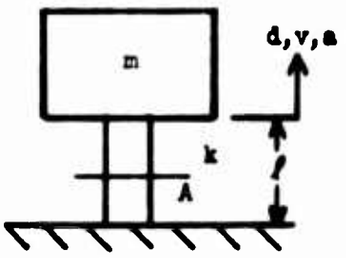
Example 3: Decrease the mass, m. f_n is increased by a factor R; this is accomplished by decreasing the mass, m.

Since $f_{n2} = R f_{n1}$, therefore $m_2 = (m_1)/R^2)$. K_s is unchanged.

$$D_2 = \frac{D_1}{R^{2\alpha-1}}$$

Conclusion: For this example, fatigue damage is reduced by reducing the mass, m.

Example 4: For the model shown (top of following page) the natural frequency (f_n) is increased by increasing the cross-sectional area (A) by a factor of 1.5. What is the fatigue damage (D) if the input levels and test time remain constant? Assume $\alpha = 6.5$.



$$A_2 = 1.5\,A_1$$

$$k_2 = \frac{A_2 E}{\ell}$$

$$k_1 = \frac{A \cdot E}{\ell}$$

$$\frac{K_{s2}}{K_{s1}} = \frac{k_2 A_1}{k_1 A_2} = \frac{(1.5\,k_1)(A_1)}{k_1\,(1.5A_2)} = 1$$

$$f_{n1} = \sqrt{\frac{k_1}{m}} \qquad f_{n2} = \sqrt{\frac{k_2}{m}}$$

$$f_{n2} = \sqrt{1.5}\,f_{n1} \qquad \frac{f_{n2}}{f_{n1}} = \sqrt{1.5}$$

Whereby, using Equation 2 *

$$D_2 = D_1 \frac{(1)^{6.5}}{\left(\sqrt{1.5}\right)^{12}} = 0.0877\,D_1$$

Therefore, in this example, increasing the natural frequency (by increasing the area by a factor of 1.5) reduces the fatigue damage by a factor of 0.0877.

Example 5: For the above example what is the fatigue damage if the input acceleration is changed from 1.3 g to 2.5 g and the test time is reduced from 3 hours to 2 hours. Using Equation 1, *

$$D_3 = D_2 \left(\frac{t_3}{t_2}\right)\left(\frac{a_{in,3}}{a_{in,2}}\right)^{\alpha} = D_2 \left(\frac{2}{3}\right)\left(\frac{2.5}{1.3}\right)^{6.5} \qquad 46.7\,D_2$$

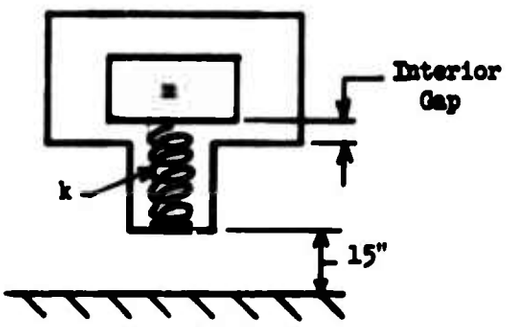
From the results of Example 4 * $D_2 = 0.0877\,D_1$. Therefore, $D_3 = 46.7\,(0.0877\,D_1) = 4.1\,D_1$.

* Note: All equation numbers refer to the equations listed on page 2.2-7.

EXAMPLE ON NATURAL FREQUENCY AND DROP SHOCK

Illustrative Example: Determine for the case of the simple model shown below the allowable range of spring constant (k) to protect the equipment against a 15 inch drop, if the interior gap is not to close by more than 2 inches and the maximum acceleration is not to exceed 20g's.

The weight of the spring mass is assumed to be 10 pounds.



Solving Equation 5; (from page 2.3-3)

$$k_{min} = \frac{2ghm}{d_a^2} = \frac{2hn}{d_a^2} = \frac{2x15x10}{4} = 75 \text{ lb/in.}$$

Solving Equation 6; (from page 2.3-3)

$$k_{max} = \frac{a_a^2 m}{2gh} = \frac{A_a^2 W}{2h} = \frac{400x10}{30} = 133 \text{ lb/in.}$$

Note that,

$$\frac{k_{max}}{k_{min}} = \frac{A_a^2 W}{2h} \frac{d_a^2}{2hW} = \left(\frac{A_a d_a}{2h}\right)^2 = \left(\frac{20x2}{30}\right)^2 \geq 1$$

which is a necessary condition in order that both maximum acceleration and maximum gap closure limits be satisfied. Since if

$$\left(\frac{A_a d_a}{2h}\right)^2 \geq 1$$

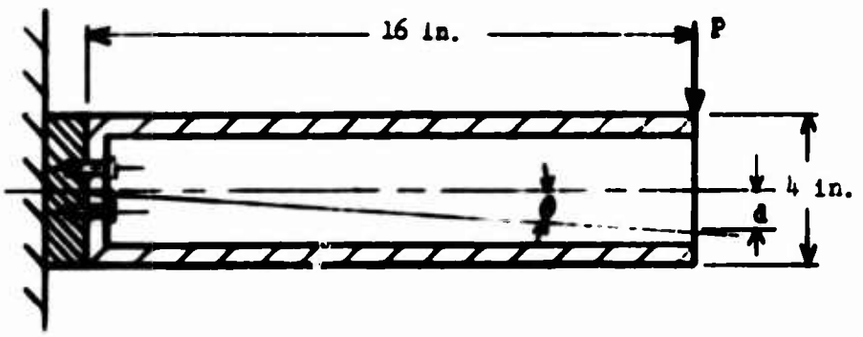
Then

$$\frac{A_a d_a}{2h} > 1$$

the latter condition is sufficient.

EXAMPLE ON EFFECTS OF JOINTS ON NATURAL FREQUENCY

Illustrative Example: A cantilever beam 4 inches in diameter and 16 inches long is attached to it's foundation with heavy bolts. What is the joint rotation constant? What is the bending stiffness (k) at the end of the beam if the beam itself is considered rigid. Assume the joint to be excellent.



Solution: From the curves the potational constant equals 10^{-8} radians/in.-lb for an excellent joint with a 4 inch diameter beam.

$$\theta = M(10^{-8}) = 16P\,(10^{-8}) \text{ Radians}$$

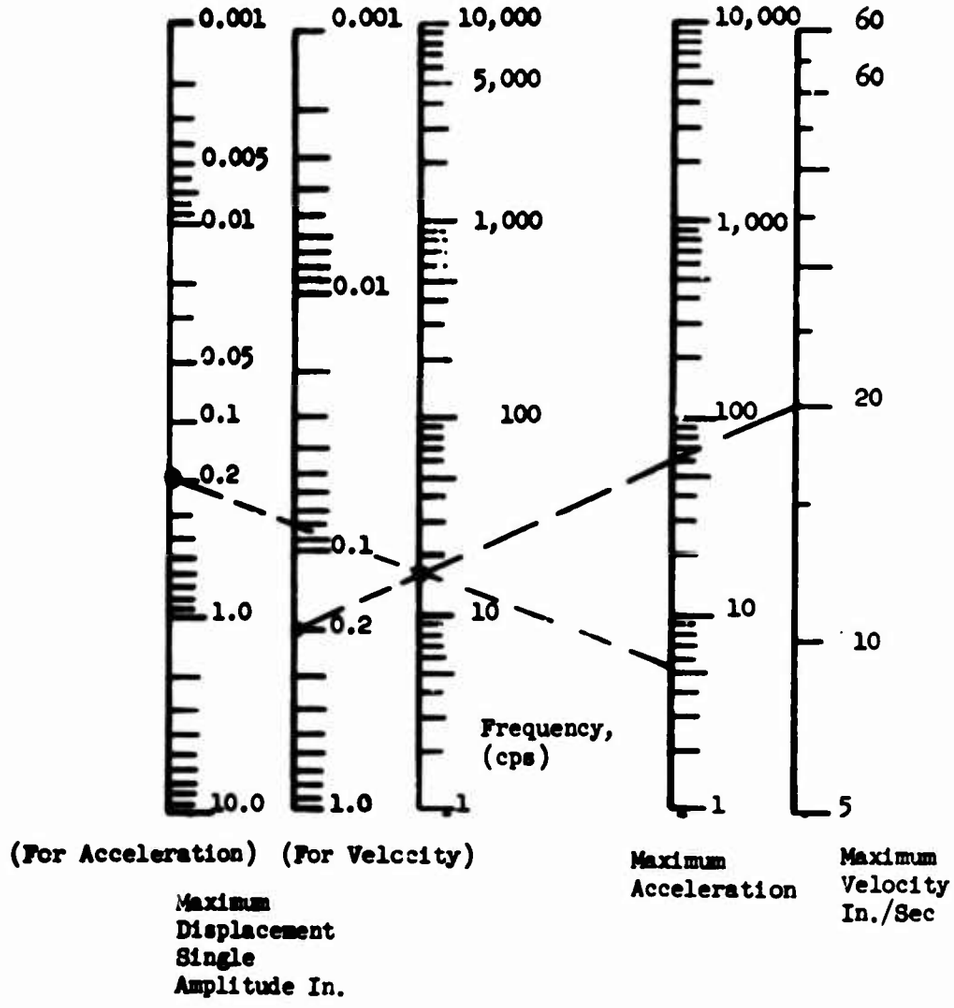
$$d = 16\theta = 256P \times 10^{-8} \text{ inches}$$

and since

$$k = \frac{P}{d}$$

$$k = \frac{P}{256P \times 10^{-8}} = 3.9 \times 10^{5} \text{ lbs/inch}$$

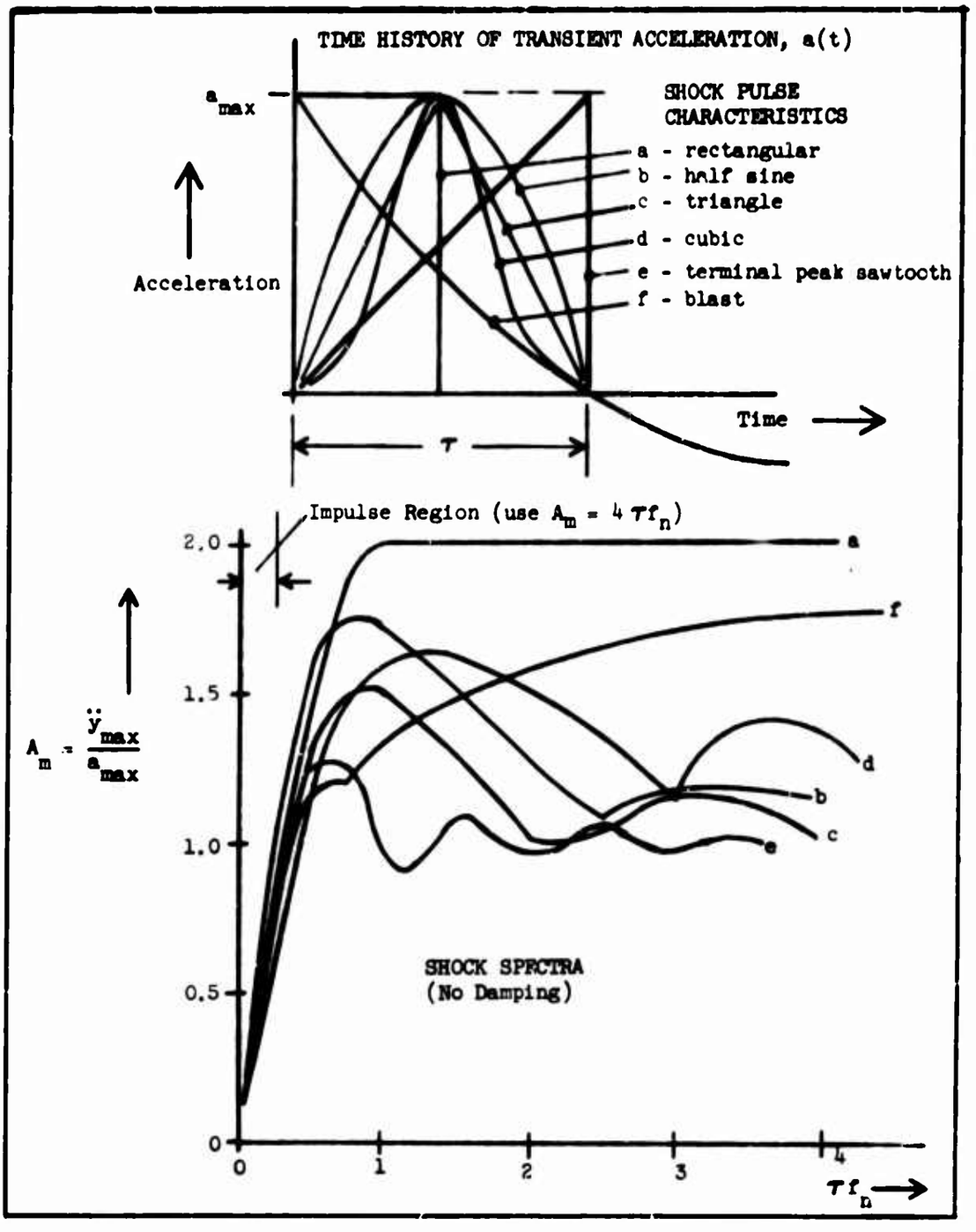
NOMOGRAM FOR HARMONIC MOTION



Nomogram for simple harmonic motion giving relationship between frequency, acceleration, and displacement. Typical problems are shown by the dashed lines. For example motion with a maximum displacement of 0.2 in. (0.4 in. peak-to-peak) at a frequency of 16 Hz. has a maximum velocity of 20 in./seconds and a maximum acceleration of 5.2 g's.

Slide rule type calculations are available to perform the same function as the nomograph above. Examples are the Ling Calculator by Ling Electronics of Anaheim, California, and The Lord Calculator by Lord Manufacturing Co. of Erie, Pennsylvania.

SHOCK SPECTRA FOR STANDARD SHOCK PULSES



VIBRATION FREQUENCY CHARTS

The following charts, figures, and nomographs present a method for estimating the natural frequency parameter for a variety of beams, plates, rings, membranes, and other common structural configurations. This material was organized by J.N. Macduff and R.P. Felgar for a paper presented at the ASME Annual Meeting, November, 1956, which was also reprinted in Machine Design, February, 1957.

The data for the tables was edited from available references by Mr. Macduff and Mr. Felgar, and has since become a standard among stress analysts as a quick procedure for estimating natural frequency of uniform and non-uniform beams and plates. This information is presented here in edited form.

METHOD OF SOLUTION: The method is based on the use of a frequency constant defined as follows for the different types of structural elements:

Beams	$C = fL^2/k$
Square and rectangular plates	$C = fa^2/h$
Circular plates	$C = fr^2/h$

Symbols are defined for each case in Tables 1 to 10, appearing on the following pages, which give values of frequency constant C, or corresponding frequency function, for various mechanical structures and several modes of vibration. These tabulated values of C are based on the characteristic density and Young's modulus for steel.

The nomograph in Fig. 1 may be used with the proper frequency constant C and the characteristic dimensions to determine natural frequency directly. Nomographs in Figs. 2 and 3 present an alternate method for determining the natural frequency by first determining the value of L^2/k, a^2/h or r^2/h from the nomograph in Fig. 2 and then entering the nomograph in Fig. 3 with this item and the frequency constant. Figs. 1 to 3 are to be used with Tables 1 to 8 in which the values of the frequency constants are tabulated.

For materials other than steel, the material correction factor is obtained from the table in Fig. 4. With this factor, and the natural frequency, f_s, of a steel member of the same dimensions, the nomograph in Fig. 4 may then be used to determine the natural frequency.

Some of the less common mechanical structural members, such as membranes, have a frequency relation which is not defined by the foregoing nomographs. In such cases, numerical or slide rule calculation is necessary. The frequency constants for these members are given in Tables 9 and 10.

EXAMPLE: Determine the fundamental natural frequency of a circular titanium plate, fixed at the center. Plate material is Ti-75A titanium; plate radius, r = 3 in.; plate thickness, h = 0.090-in.; and estimated temperature of operation is 400°F.

From Table 7, $C/10^4 = 3.649$. From Fig. 1 then, the natural frequency of a steel plate of the same dimensions is $f_s = 370$ Hz .

This result is also arrived at by the alternate procedure, Figs. 2 and 3. From Fig. 2, for r = 3 and h = 0.09, $r^2/h = 100$, and from Fig. 3, for a steel plate of the same dimensions, $f_s = 370$ Hz.

Since plate material is titanium, final solution will be given by Fig. 4. From the table in Fig. 4, material correction factor for Ti-75A titanium at 400°F is $K_m = 0.910$. From the nomograph then, for this value of K_m and $t_s = 370$ Hz, frequency of the titanium plate is f = 325-350 Hz.

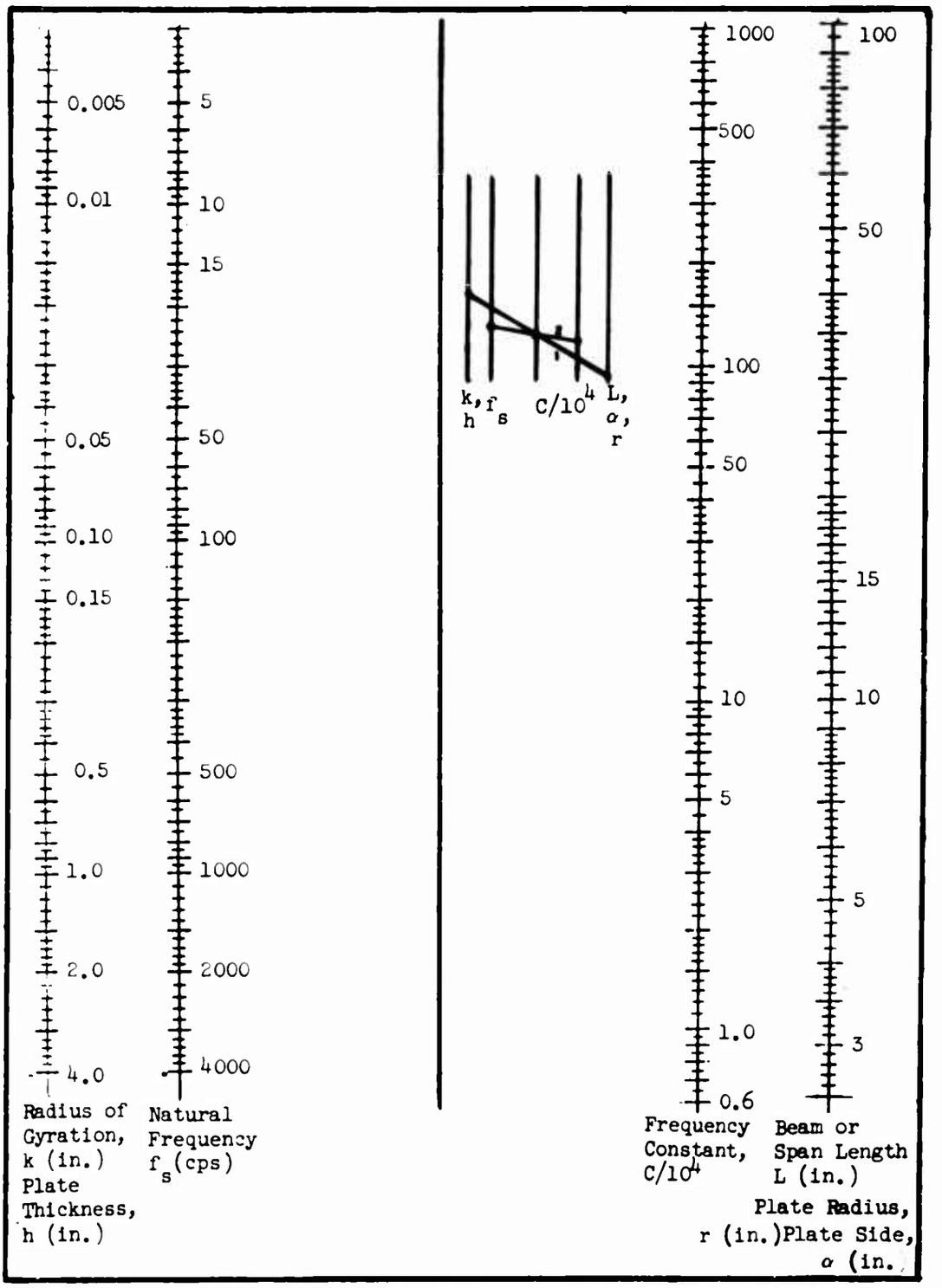


FIGURE 1: Nomograph for determination of natural frequency (f_s), from frequency constant (C), in Tables 1 to 8.

VIBRATION FREQUENCY CHARTS (Continued)

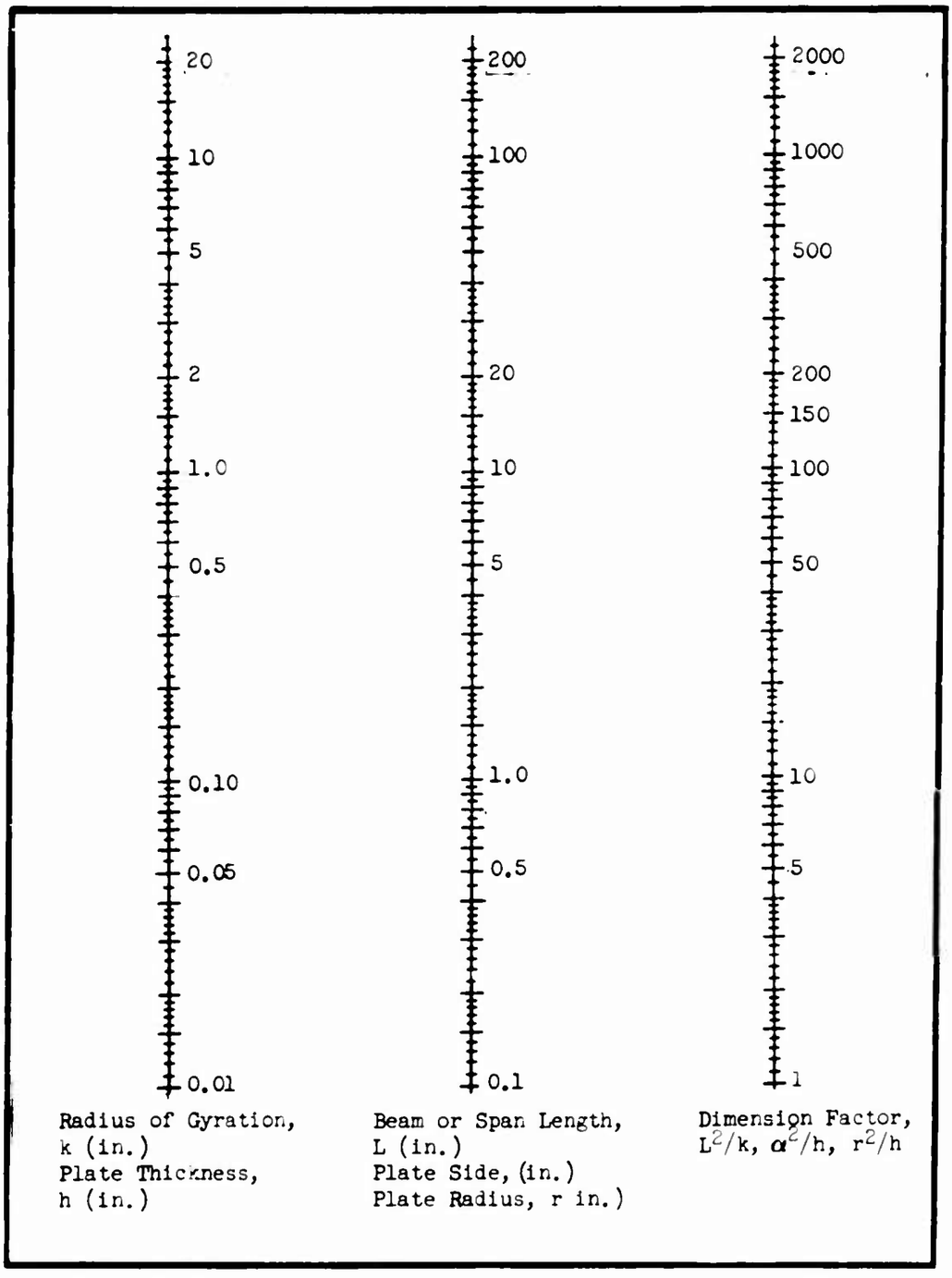


FIGURE 2: Nomograph for determination of dimension factors, L^2/k, α^2/h, and r^2h in frequency constant equations of Tables 1 to 8.

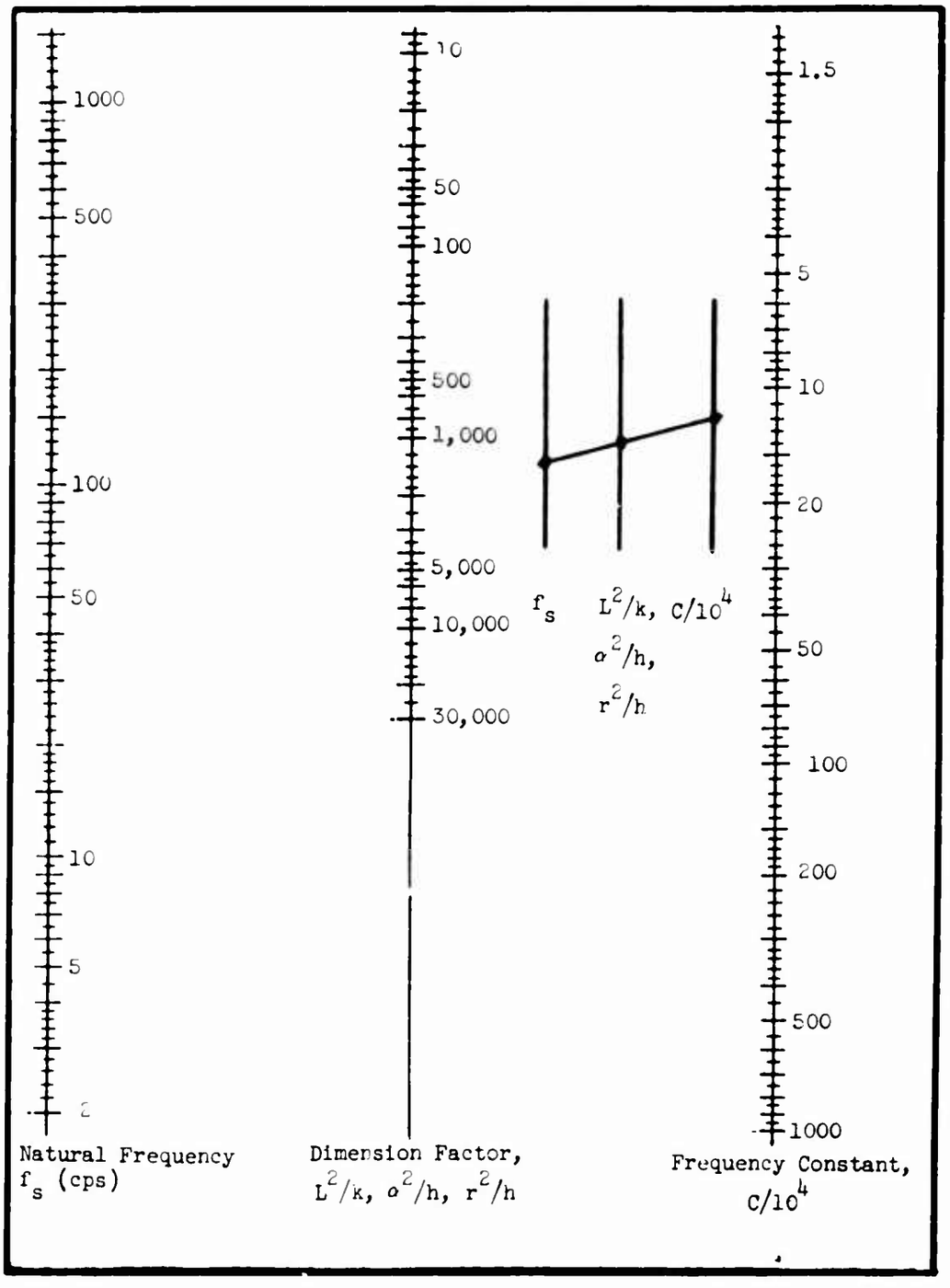
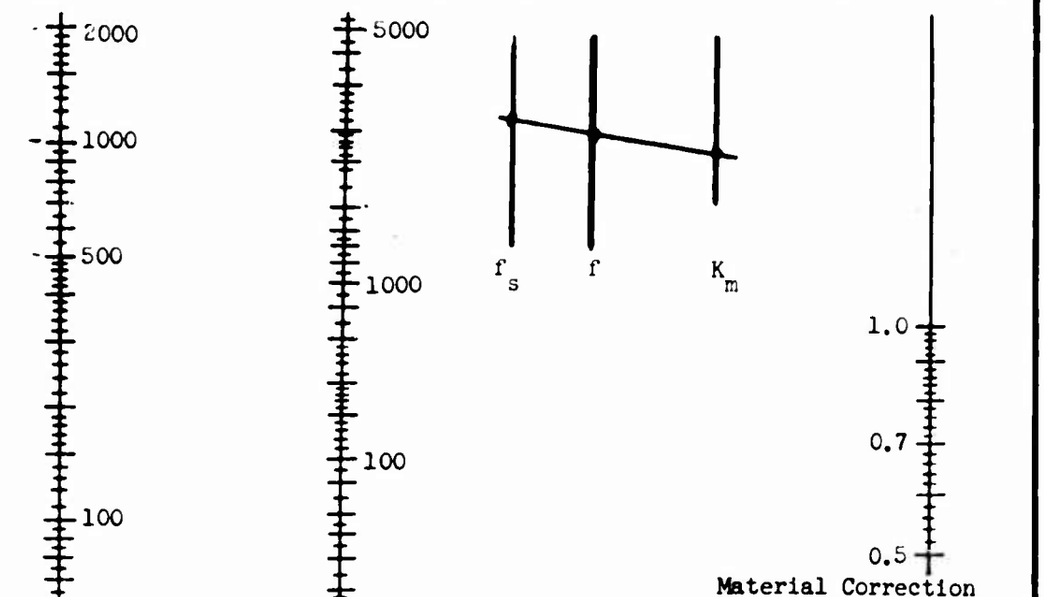


FIGURE 3: Nomograph for alternate solution of natural frequency (f_s) from frequency constant (C) and dimension factor determined from Figure 2.

VIBRATION FREQUENCY CHARTS (Continued)



MATERIAL CORRECTION FACTOR

E = Young's modulus for material, psi
E_s = Young's modulus for steel, (30×10^6) psi
e = Mass density of material, lb-sec^2/in.^{4}
e_s = Mass density of steel (735×10^6), lb-sec^2/in.^{4}
K_m = Material correction factor

$$K_m = \sqrt{\frac{Ee_s}{eE_s}}$$

Steel	1
Aluminum alloys (1100, 3003, 3004, 2017, 2024, 2025, 6051, 5052)	0.935
Brass, bronze	0.673
Nickel	0.940
Monel Metal	0.872
Magnesium	0.965
Titanium (listed below)	

Temperature (F)	Tl-50A	Tl-75A
80	0.985	0.975
200	0.966	0.945
400	0.932	0.910
600	0.896	0.873
800	0.866	0.835
1000	0.828	0.784

FIGURE 4: Correction table and nomograph for determination of correction factor (K_m) and natural frequency (f_s) for nonsteel structures.

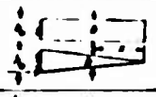
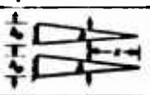
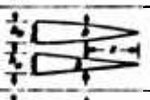
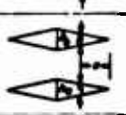
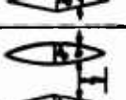
Table 1—Uniform Steel Beams

C = Frequency constant L = Beam length, in.
f_n = Natural frequency, cps M = Vibration mode number
$k = (I/A)^{1/2}$ = Radius of gyration, in.

Beam Structure	$C/10^4 = (f_nL^2/k)/10^4$				
	$M=1$	$M=2$	$M=3$	$M=4$	$M=5$
Clamped-Clamped / Free-Free	71.96	199.29	388.73	642.69	960.94
Clamped-Free	11.30	70.88	198.29	388.73	642.69
Clamped-Hinged / Free-Hinged	49.87	169.66	338.17	573.29	874.66
Clamped-Guided / Free-Guided	17.96	97.18	238.96	446.26	718.96
Hinged-Hinged / Guided-Guided	31.73	126.93	285.69	507.73	793.39
Hinged-Guided	7.83	71.60	199.29	388.73	642.69

Table 2—Variable-Section Steel Beams

C = Frequency constant L = Beam length, in.
f_n = Natural frequency, cps M = Vibration mode number
$k = (I/A)^{1/2}$ = Radius of gyration, in.

Beam Structure	b/b_0	h/h_0	$C/10^4 = (f_nL^2/k)/10^4$		
			$M=1$	$M=2$	$M=3$
	1	a/L	17.00	48.00	96.07
	a/L	a/L	26.00	68.00	133.04
	$(a/L)^{1/2}$	a/L	22.30	58.18	109.00
	a/L	1	16.20	77.76	206.07
	1	a/L	21.21* 20.00†	66.07	
	a/L	a/L	22.72* 60.00†	76.07	
	$(a/L)^{1/2}$	a/L	20.00* 42.00†	66.00	

*Symmetric. †Antisymmetric.

Table 3—Continuous Uniform Steel Beams

C = Frequency constant L = Span length, in.
f_n = Natural frequency, cps M = Vibration mode number
$k = (I/A)^{1/2}$ = Radius of gyration, in. n = Number of spans

Beam Structure	n	$C/10^4 = (f_nL^2/k)/10^4$				
		$M=1$	$M=2$	$M=3$	$M=4$	$M=5$
Extreme Ends Simply Supported	1	31.73	126.94	285.61	507.76	793.37
	2	31.73	49.50	126.94	199.66	285.61
	3	31.73	49.53	99.06	126.94	243.06
	4	31.73	37.02	49.99	63.99	126.94
	5	31.73	34.99	44.19	55.29	66.72
	6	31.73	34.33	49.53	49.50	59.06
	7	31.73	33.07	38.46	45.70	53.63
	8	31.73	33.02	37.02	42.76	49.99
	9	31.73	33.02	38.06	49.53	49.46
	10	31.73	33.02	34.99	39.19	44.19
	11	31.73	32.37	34.33	37.76	41.97
	12	31.73	32.37	34.33	37.02	49.53
Extreme Ends Clamped	1	72.30	199.34	388.73	642.63	960.90
	2	49.50	72.30	199.66	199.34	335.30
	3	49.53	99.56	72.30	243.06	178.25
	4	37.02	49.99	63.99	72.30	137.30
	5	34.99	44.19	55.29	69.72	72.45
	6	34.33	49.53	49.99	59.56	67.65
	7	33.07	38.49	45.70	53.63	62.30
	8	33.02	37.02	42.76	49.54	56.95
	9	33.02	38.06	49.53	49.46	53.81
	10	33.02	34.99	39.19	44.19	49.99
	11	32.37	34.33	37.76	41.97	47.23
	12	32.37	34.33	37.02	49.53	44.94
Extreme Ends Clamped-Supported	1	49.99	199.66	335.2	573.21	874.66
	2	37.72	63.99	137.30	185.85	301.66
	3	34.33	49.50	87.65	132.07	199.66
	4	33.02	42.76	56.96	69.51	129.49
	5	33.02	39.19	49.50	61.31	76.43
	6	32.37	37.02	44.94	54.46	63.99
	7	32.37	36.46	41.97	49.50	57.94
	8	32.37	34.99	39.81	45.70	53.63
	9	31.73	34.33	38.46	43.44	49.50
	10	31.73	33.67	37.02	41.26	46.46
	11	31.73	33.67	36.33	39.81	44.19
	12	31.73	33.02	36.06	39.19	42.76

Table 4—Square Steel Plates

a = Plate side, in. h = Plate thickness, in.
C = Frequency constant M = Vibration mode number
f_n = Natural frequency, cps

Plate Structure*	$C/10^4 = (f_na^2/h)/10^4$					
	$M=1$	$M=2$	$M=3$	$M=4$	$M=5$	$M=6$
	3.60	8.33	20.00	20.71	30.33	
	6.77	22.43	28.97	46.75	61.44	
	13.73	19.00	33.36	34.00	60.03	63.47
	19.30	46.00	76.03	80.01	124.03	163.20

VIBRATION FREQUENCY CHARTS (Continued)

Table 4 (Cont.)

Plate Structure*	$C/10^3 = (f_n a^2/h)/10^3$					
	$M=1$	$M=2$	$M=3$	$M=4$	$M=5$	$M=6$
S S S S	39.01	84.28	87.66	88.79	97.89	156.13
C C S S	35.36	63.29	87.64	98.68	99.43	129.69
C C C C	35.91	71.48	103.86	128.68	128.71	168.73

*F = free, S = supported, C = clamped.

Table 5—Rectangular Steel Plates (First Vibration Mode)

$C/10^3 = (f_n a^2/h)/10^3$

a = Plate side, in. f_n = Natural frequency, cps
C = Frequency constant h = Plate thickness, in.

Plate Structure*	a/b	$C/10^3$	a/b	$C/10^3$
S S S F	1.0	19.20		
	1.5	13.67		
	2.0	12.00		
	2.5	11.16		
	3.0	10.67		
	∞	9.89		
C S S F	1.0	23.01	1.0	23.01
	1.5	16.20	1.5	16.19
	2.0	14.00	2.0	13.87
	2.5	13.10	2.5	11.43
	3.0	12.53	3.0	10.04
	∞	12.01	∞	9.89
C S C F	1.0	28.16	1.0	28.16
	1.5	24.27	1.5	16.00
	2.0	23.17	2.0	13.22
	2.5	22.64	2.5	11.60
	3.0	22.37	3.0	11.08
	∞	21.76	∞	9.89
C C C F	1.0	30.00		
	1.5	26.27		
	2.0	25.00		
	2.5	25.13		
	3.0	25.06		
	∞	21.76		

*F = free, S = supported, C = clamped.

Table 6—Cantilever Steel Plates

a = Plate side, in. h = Plate thickness, in.
C = Frequency constant M = Vibration mode number
f_n = Natural frequency, cps θ = Skew angle, deg

Plate Structure*	a/b	$C/10^3 = (f_n a^2/h)/10^3$				
		$M=1$	$M=2$	$M=3$	$M=4$	$M=5$
Rectangular Cantilever Plate	½	3.41	8.33	21.36	0.86	34.18
	1	3.49	8.33	20.86	36.71	56.29
	2	3.52	14.83	21.82	61.82	67.30
	3	3.56	23.79	39.94	660.66	160.66

Skewed Cantilever Plate	θ	$M=1$	$M=2$
	15	3.50	8.62
	30	3.55	9.91
	45	4.09	13.56

*F = free, C = clamped.

Table 7—Circular Steel Plates

C = Frequency constant r = Plate radius, in.
f_n = Natural frequency, cps m = Number of nodal circles
h = Plate thickness, in. n = Number of nodal diameters

Plate Structure*	$C/10^3 = (f_n r^2/h)/10^3$			
	n	$n=0$	$n=1$	$n=2$
Circular Plate Clamped at Boundary	0	9.936	38.713	88.016
	1	20.861		
	2	39.866		
Circular Plate With Free Boundary	0		8.933	37.687
	1		19.979	88.289
	2	8.116	34.298	
	3	11.988	81.491	
Circular Plate Clamped at Center		3.649	20.340	60.883	116.600

Table 8—Steel Ring Vibrating in Its Own Plane

C = Frequency constant n = Number of full waves around periphery
f_n = Natural frequency, cps r = Mean ring radius, in.
h = Ring thickness, in.

Ring Structure	$C/10^3 = (f_n r^2/h)/10^3$				
	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
	2.81	7.10	13.6	22.2	32.2

Table 9—Circular Steel Membranes

C_f = Frequency function m = Number of nodal circles
f_n = Natural frequency, cps n = Number of nodal diameters
h = Membrane thickness, in. s = Tension of periphery, lb/in.

Membrane Structure	m	$C_f = f_n r/(s/h)^{1/2}$					
		$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
	1	14.68	22.49	30.12	37.48	44.58	51.53
	2	32.41	41.22	49.44	57.30	64.94	72.72
	3	50.79	59.71	64.94	76.15	84.05	91.18
	4	69.28	78.08	86.90	95.12	103.34	111.66
	5	87.48	96.86	105.68	113.91	122.13	130.35
	6	106.27	115.66	123.89	132.09	140.91	149.13
	7	126.47	133.87	142.66	150.99	159.70	167.92
	8	145.36	153.97	160.58	169.68	178.49	186.77

Table 10—Longitudinal Vibration of Steel Beams

C_l = Frequency function L = Beam length, in.
f_n = Natural frequency, cps n = Number of halfwaves along beam length

Beam Structure	$C_l/10^3 = f_n L/10^3$					
	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
Clamped-Free	5.65	15.15	25.25	35.35	45.45	55.55
Clamped-Clamped		15.15	30.30	45.45	60.61	75.76

CHAPTER 3 — MECHANICAL IMPEDANCE

CHAPTER 3
MECHANICAL IMPEDANCE

ABSTRACT:

Mechanical impedance relates force and velocity in a manner which completely describes the characteristics of a dynamic system. Procedures are outlined in this chapter for applying the mechanical impedance methodology to practical problems. In support of this thesis, the terminology and symbology of the impedance technique are presented in the language of the Mechanical Engineer.

Chapter 3 - Mechanical Impedance

ERRATA SHEET

Page	Paragraph	Line	Correction
3.1-1	3	3	acceleration
3.1-1	4	4	... are operating on the ...
3.2-0	1	3	electrical
3.3-0	5	4	acceleration
3.5-2	5	1	acceleration

VOLUME III - CHAPTER 3
MECHANICAL IMPEDANCE

CONTENTS

Section		Page
1	INTRODUCTION	3.1-0
	• The Mechanical Impedance Concept	3.1-0
2	FOUR POLE PARAMETER TECHNIQUES	3.2-0
	• The Mechanical System as Represented by the Four Pole Parameter Concept	3.2-0
	• Ground Rules for Connecting Mechanical Four Poles	3.2-2
3	EXPERIMENTAL TECHNIQUES	3.3-0
	• Measuring and Plotting Impedance	3.3-0
	• Measuring Four Pole Parameters	3.3-2
4	APPLICATIONS	3.4-0
	• Information Contained in the Mechanical Impedance Plot	3.4-0
	• Impedance Model for Vibration Isolation of a Mechanical System	3.4-2
	• Vibration Isolation Techniques and Procedures . .	3.4-4
5	APPENDIX	3.5-0
	• Bibliography	3.5-0
	• Symbology	3.5-1
	• Glossary	3.5-2
	• A Typical Impedance Plot	3.5-3
	• An Example of Forced Vibration With Damping Employing Rotating Phasor Techniques	3.5-4
	• Applying the Rules of Four Pole Parameters . . .	3.5-6
	• Determination of Four Pole Parameters for a Mass, a Spring, and a Damper	3.5-10
	• Example on System Response Versus Frequency Using Mechanical Impedance Concepts and Plots . .	3.5-12

VOLUME III - CHAPTER 5

MECHANICAL IMPEDANCE

SECTION 1 - INTRODUCTION

- The Mechanical Impedance Concept

THE MECHANICAL IMPEDANCE CONCEPT

Mechanical impedance relates a sinusoidal force applied to a point on a structure to the velocity of a point on that structure.

Some Preliminary Considerations

A sinusoidal function can be represented by a rotating phasor on the complex plane. This is very commonly used in electrical engineering. For instance consider the voltage $v = V\cos\omega t$. The graphical description of this is as shown in Figure 1. Alternatively this may be expressed mathematically as $v = V\cos\omega t + jV\sin\omega t$ which on the complex plane represents a rotating phasor with magnitude (V) rotating at a constant angular velocity of (ω) in the counter-clockwise direction as shown in Figure 2. The instantaneous voltage is the projection of the rotating phasor on the real axis ($V\cos\omega t$). Note also that the angular position is given by (ωt).

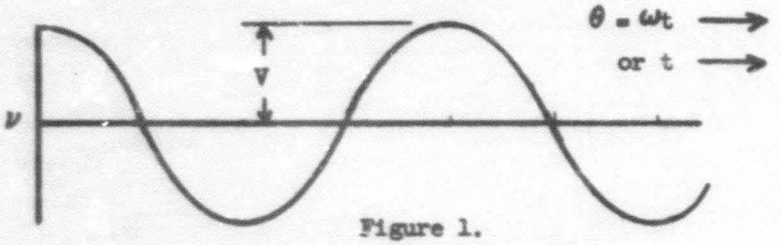


Figure 1.

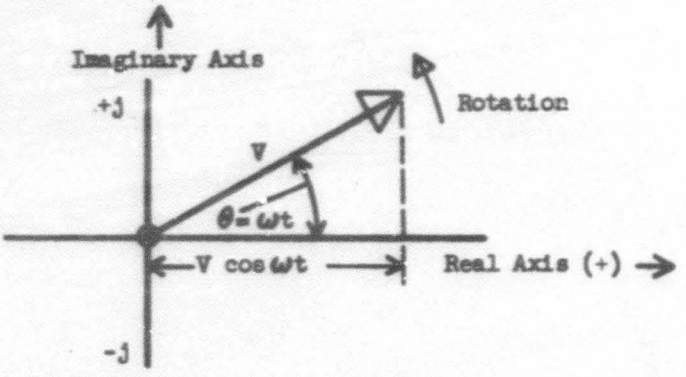


Figure 2.

The differential equations of vibratory motion are analogous to the differential equations of linear, bilateral electric circuits. Just as there are phase relationships between electrical quantities, such as voltage and current, there are phase relationships (angle or time lag between phasors) of mechanical quantities such as force, velocity, and acceleration.

For example, assuming the forcing function (F) to be equal to $F_o \cos\omega t + j F_o \sin\omega t$, there results the general form for the velocity (ν) which is

$$\nu = \nu_o \left[\cos(\omega t \pm \phi) + j\sin(\omega t \pm \phi)\right] = \nu_o e^{j(\omega t \pm \phi)}$$

where (ϕ) is the angular displacement or phase difference between (F) and (ν).

The displacement distance (x) may be expressed in the form $x = (\nu)/(j\omega)$ when (ν) is of the form $\nu_o e^{j\omega t}$ and it is noted that $x = \int \nu dt$. Since acceleration is $(d\nu)/(dt)$ the phasor form becomes $a = j\omega\nu$. Figure 3 shows a typical phasor diagram relating forcing function, displacement, velocity, and acceleration. An example employing rotating phasor techniques is given in the Appendix on page 3.5-4.

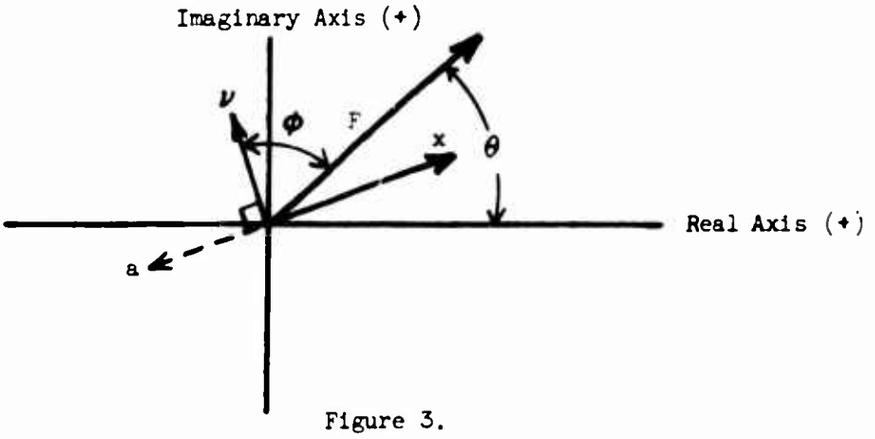


Figure 3.

What is Mechanical Impedance?

Mechanical impedance is the ratio of force to velocity during simple harmonic motion. This ratio is a complex quantity; it is expressed as a function of frequency in terms of both magnitude and phase angle (Equation 1). When the force and velocity are in the same point and in the same direction, Z is the point impedance. Z is transfer impedance when F and V are at different points or in different directions. The discussions in this chapter will be limited to point impedance.

THE MECHANICAL IMPEDANCE CONCEPT (Continued)

Mechanical admittance (mobility) is a similar relationship, but is the complex ratio of velocity to force (Equation 2). Some problems lend themselves more readily to the admittance concept than to impedance, and vice versa. This chapter presents the impedance concept, but it should be kept in mind that the generalities also apply to admittance.

$$Z(\omega) = \frac{F}{V}(\cos\phi + j\sin\phi) = \frac{F}{V}e^{j\phi} \qquad (1)$$

$$Y(\omega) = \frac{V}{F}e^{j\phi} \qquad (2)$$

Note must be taken of the fact that as defined here, impedance is taken as the 'velocity' impedance. There also exists concepts of displacement and acceleration impedances, although these are less widely used. Further discussion along these lines appears later in the text.

Impedance of Physical Elements

Ideally mechanical elements are represented by lumped parameters which are linear. These are mass, spring constant, and damping constant.

Consider the removal of a spring from a system as in Figure 4. The forces (F_P) and (F_Q) acting at the ends of the spring must be equal in magnitude since the mass of the spring is considered zero. The net change in spring length is given by Δx which equals $x_2 - x_1$. Note that the end points have relative displacements with respect to some arbitary point in space. The constant of proportionality relating the net change in spring length to the applied force is given by

$$\Delta x = x_2 - x_1 = \frac{F}{k}$$

where (k) is the spring constant. Fixing point (Q) and applying a force $F_P = F\,e^{j\omega t}$ results in

$$x_1 = \frac{F\,e^{j\omega t}}{k} = x\,e^{j\omega t}$$

Using

$$Z_k(\omega) = \frac{F(\omega)}{v(\omega)} \text{ from Equation (1) with}$$

$$v(\omega) = \frac{dx(\omega)}{dt} \text{ results in}$$

$$Z_k(\omega) = -j\frac{k}{\omega}$$

This is the mechanical impedance of a spring.

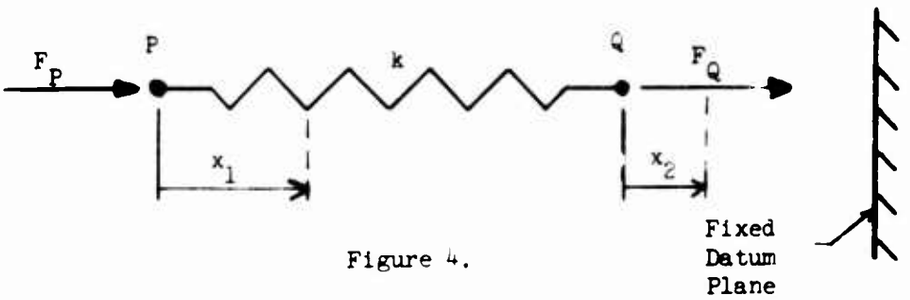


Figure 4.

Consider the removal of a damping element from a system as in Figure 5 and assume the damping to result from viscous friction (dashpot). The end points have relative velocities with respect to the arbitrary fixed point in the system and the dashpot is massless. The constant of proportionately relates the velocity (ν) and the force (F) as follows

$$\nu = (\nu_2 - \nu_1) = \frac{F}{c}$$

where c is called the damping constant. Points (P) and (Q) may move relative to a fixed point in the system, or one point could be fixed. Consider point (Q) fixed and apply

$$F_P = F\, e^{j\omega t}$$

This results in

$$\nu_P = \frac{F\, e^{j\omega t}}{c} = \nu\, e^{j\omega t}$$

Using Equation 1 there results

$$Z_c(\omega) = \frac{F}{\nu} = c$$

Thus the mechanical impedance of a dashpot is simply (c).

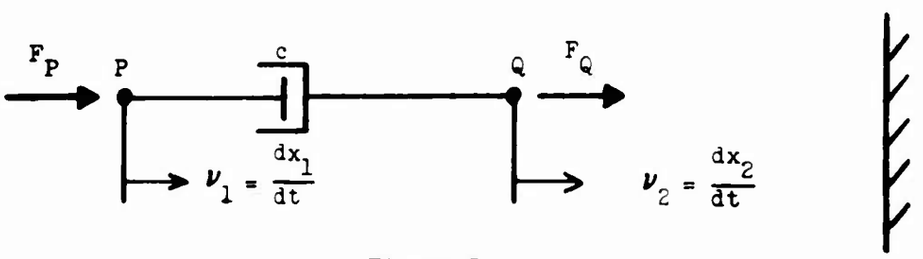


Figure 5.

THE MECHANICAL IMPEDANCE CONCEPT (Continued)

Consider the removal of a mass element from a system as shown in Figure 6. Since the mass is considered rigid the accelerations (a_1) and (a_2) are equal. Further, the resultant applied external force is the sum of (F_P) and (F_Q) which is simply (F_R). Let $F_R = F\,e^{j\omega t}$. Since $F = ma$ and $v = \int a\,dt$, the mechanical impedance from Equation 1 is given by

$$Z_m = \frac{F_R}{v} = \frac{F\,e^{j\omega t}}{F\,e^{j\omega t}/j\omega m} = j\omega m$$

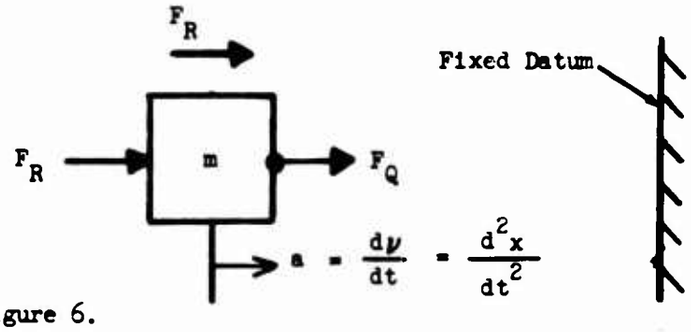


Figure 6.

The impedances derived above may be represented on a complex plane similar to electric circuit impedances where the damping constant is analogous to resistance; mass is analogous to inductance; and the spring constant is analogous to capacitance. The angle by which the displacement vector lags the forcing function is given by

$$\phi = \tan^{-1}\frac{c}{\omega m - \frac{k}{\omega}}$$

The plot is shown in Figure 7.

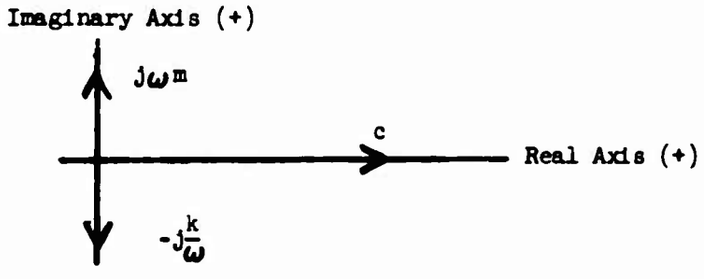


Figure 7.

Mobility is merely the inverse of impedance. These are given below. The complex plane representation is shown in Figure 8.

Mass - $M_m = \frac{1}{j\omega m}$

Damping - $M_c = \frac{1}{c}$

Spring - $M_k = \frac{j\omega}{k}$

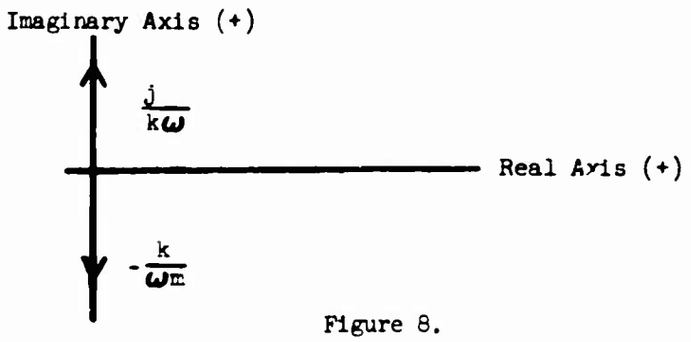


Figure 8.

Systems and System Simplification

The most general case consists of elements in series parallel arrangements similar to electric circuit arrangements. In fact, information desired at various points in the 'network' is obtained resorting to methods used in electric circuit problems. Included are system simplification techniques combining series and parallel elements, the superposition and reciprocity theorems, Thevenin's and Norton's equivalent circuits, and in general any of the techniques used for solving electric circuits. Reference (10) in the Appendix covers very adequately these various techniques and theorems. These constitute an indispensable aid in the solving of more complex vibration problems.

VOLUME III - CHAPTER 3

MECHANICAL IMPEDANCE

SECTION 2 – FOUR POLE PARAMETER TECHNIQUES

- The Mechanical System as Represented by the Four Pole Parameter Concept
- Ground Rules for Connecting Mechanical Four Poles

THE MECHANICAL SYSTEM AS REPRESENTED BY THE FOUR POLE PARAMETER CONCEPT

Four pole parameters provide a convenient method for studying elastic systems in block diagram form.

Four pole parameters greatly facilitate the use of block diagrams in the study of linear structures. The term four pole parameter arises from the development of these methods for electricap applications. The electrical "black box" with a single pair of input poles (terminals) and a single pair of output poles is referred to as a four pole network. The input and output relationships of the analogous mechanical system are given by Equations 1 and 2. The four pole parameters are the α's. The figure shows a general block diagram and notations. The relationship is given in matrix form in Equation 3. (See Appendix for a definition of the matrix.) It can be seen from dimensional considerations that α_{11} and α_{22} are dimensionless, α_{12} is in terms of F/V (impedance) and α_{21} is in terms of V/F (admittance).

Four Pole Parameters for Mechanical Elements

The basic elements of the mechanical system considered here are the rigid mass, the massless spring and the viscous damper. The derivations of these parameters, based on the equations of motion, are given in Reference 1. The derivations assume a linear system subjected to sinusoidal motion and utilize standard complex number notations. The table lists the four pole parameters for the mass, spring, and damper. Graphic representation of the elements is given in the figure. Note that $V_1 = V_2$ for a mass, and $F_1 = F_2$ for a spring or damper; m is in mass units, k in force per unit length and c in force per unit velocity.

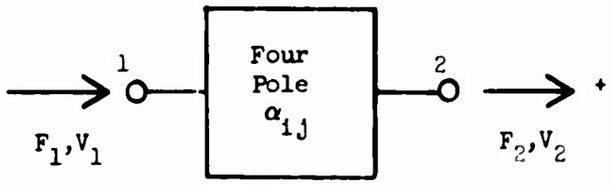
These basic elements can be coupled to form systems. The parameters for any four pole is independent of what precedes or follows it and this allows a "black box" approach to the analysis of the system.

$$F_{input} = \alpha_{11} F_{output} + \alpha_{12} V_{output} \tag{1}$$

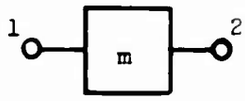
$$V_{input} = \alpha_{21} F_{output} + \alpha_{22} V_{output} \tag{2}$$

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \tag{3}$$

THE MECHANICAL FOUR POLE

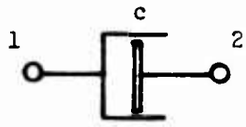


MASS, SPRING, AND DAMPER



Rigid Mass

Massless Spring



Viscous Damper

FOUR POLE PARAMETERS

	α_{11}	α_{12}	α_{21}	α_{22}
Mass, m	1	$m\omega j$	0	1
Spring, k	1	0	$\omega j/k$	1
Damper, c	1	0	1/c	1

FOUR POLE PARAMETERS. Mechanical Four Poles are used to study systems in block diagram form.

GROUND RULES FOR CONNECTING MECHANICAL FOUR POLES

Simple connecting rules allow complex systems to be represented by one four pole model.

The general rules for combining four pole parameters may be organized into the following categories:

1. General:

   a. The sum of the output forces of all the four poles driving a junction is equal to the sum of all the input forces of all the four poles driven by that junction.

   b. All the four poles points connected to a given junction have the same velocity.

2. Series Connections: four poles connected in series can be combined to form a composite four pole. The four pole parameters for the composite are given by the matrix product of the component four pole parameters (Equation 1).

3. Parallel Connections: four poles connected in parallel can be combined to form a composite four pole. The four pole parameters for the composite are found by using Equation 2. Four poles are said to be connected in parallel when the following conditions are met:

   a. All the input points are connected to a common junction.

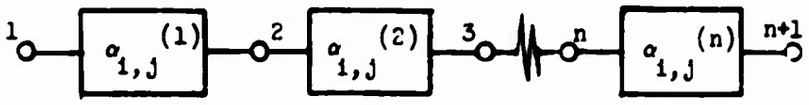
   b. All the output points are connected to a common junction.

   c. The input force of the composite four pole is the sum of input forces of the component four poles.

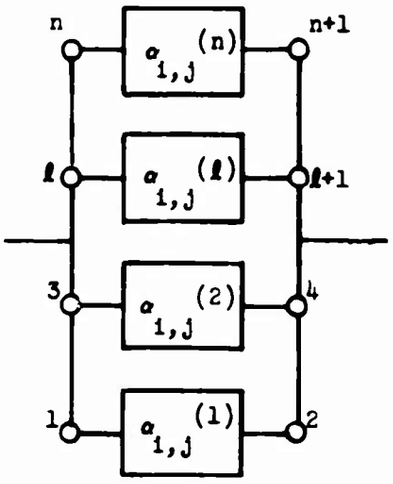
   d. The output force of the composite four pole is the sum of the output forces of the component four poles.

An example of the application of the above rules is given in the Appendix, where the four pole parameters for a single degree-of-freedom system are derived. More detailed descriptions of four pole parameters are contained in the references, particularly Reference (1), page 3.5-0.

SERIES CONNECTION



PARALLEL CONNECTION



$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{(1)} & \alpha_{12}^{(1)} \\ \alpha_{21}^{(1)} & \alpha_{22}^{(1)} \end{bmatrix} \times \begin{bmatrix} \alpha_{11}^{(2)} & \alpha_{12}^{(2)} \\ \alpha_{21}^{(2)} & \alpha_{22}^{(2)} \end{bmatrix} \times \cdots \tag{1}$$

$$\cdots \times \begin{bmatrix} \alpha_{11}^{(n)} & \alpha_{12}^{(n)} \\ \alpha_{21}^{(n)} & \alpha_{22}^{(n)} \end{bmatrix} \begin{bmatrix} F_{n+1} \\ V_{n+1} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} F_{n+1} \\ V_{n+1} \end{bmatrix}$$

$$\alpha_{11} = \frac{A}{B}, \quad \alpha_{12} = \frac{AC}{B} - B, \quad \alpha_{21} = \frac{1}{B}, \quad \alpha_{22} = \frac{C}{B}$$

$$A = \sum_{\ell=1}^{\ell=n} \left(\frac{\alpha_{11}^{(\ell)}}{\alpha_{21}^{(\ell)}} \right), \quad B = \sum_{\ell=1}^{\ell=n} \left(\frac{1}{\alpha_{21}^{(\ell)}} \right), \tag{2}$$

$$C = \sum_{\ell=1}^{\ell=n} \left(\frac{\alpha_{22}^{(\ell)}}{\alpha_{21}^{(\ell)}} \right)$$

CONNECTING FOUR POLE PARAMETERS. These formulas are used to facilitate connection of the four pole model.

VOLUME III - CHAPTER 3

MECHANICAL IMPEDANCE

SECTION 3 - EXPERIMENTAL TECHNIQUES

- Measuring and Plotting Impedance
- Measuring Four Pole Parameters

MEASURING AND PLOTTING IMPEDANCE

Force and velocity information are needed to plot the impedance characteristics of a structure.

The information needed to define the impedance at a point on a structure includes the ratio of the force applied to that point compared to the velocity of the point, as a function of frequency. In addition, the phase angle between force and velocity as a function of frequency must be known. Both F/V and the phase angle, ϕ, must be known to completely designate the impedance at a point.

The Impedance Plot

As shown in the graphs illustrated in the Appendix, impedance is conventionally plotted as log impedance versus log frequency. Lines of constant mass and constant stiffness are included on the same plot. The constant mass lines increase with frequency at 6 dB/octive and the constant stiffness lines decrease with frequency at 6 dB/octive. The impedance versus frequency plot gives the impedance magnitude; the phase versus frequency plot completes the picture by presenting the phase angle relationships. The mathematical representation is given in Equation 1.

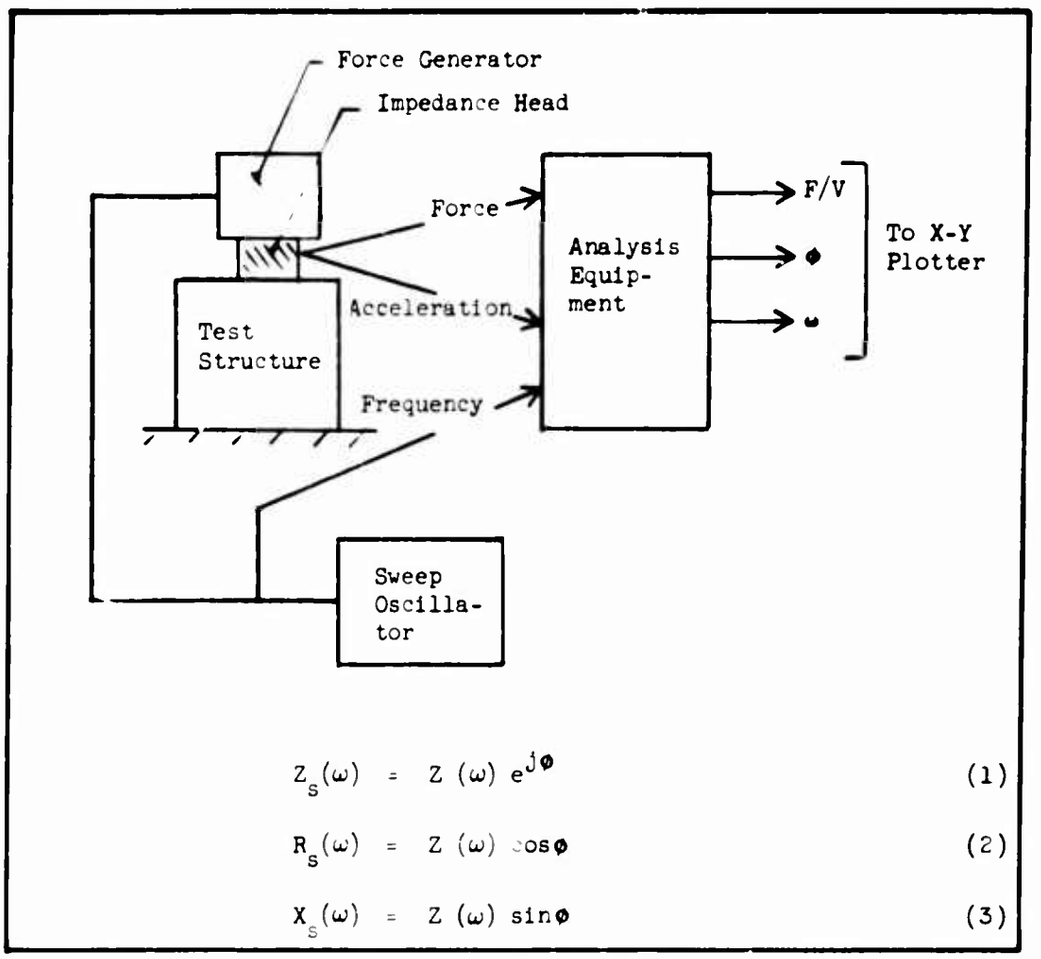
For some applications, it is more convenient to plot the real and imaginary parts of impedance as a function of frequency. In this case, both R_s, the real part, and X_s, the imaginary part, would be plotted on the graph (Appendix). The real and imaginary parts are given by Equations 2 and 3.

Measuring Impedance

The following items are required to determine mechanical impedance:

1. A force generator
2. A force transducer
3. A motion (displacement, velocity, or acceleration) transducer
4. An instrument for measuring phase angle
5. Equipment for plotting the data as a function of frequency

The necessary data is conveniently obtained by using a commercially available "impedance head" and force generator (Reference 15). Basically, the impedance head consists of a highly sensitive, combined force and accelerometer transducer. The force generator consists of a small permanent magnet, spring-mounted in a case. The magnet, which serves as a reaction mass, is driven by an electrical coil rigidly mounted in the case. The force generator is driven by a variable frequency signal generator. The force generator is attached to the impedance head which is in turn attached to the test structure. The force and acceleration signals are put into electronic equipment which integrates the acceleration into velocity, cancels the effect of the force generator and impedance head mass, and provides output signals of F/V, ϕ and ω. This equipment is discussed in Reference (4).



$$Z_s(\omega) = Z(\omega)\, e^{j\phi} \qquad (1)$$

$$R_s(\omega) = Z(\omega) \cos\phi \qquad (2)$$

$$X_s(\omega) = Z(\omega) \sin\phi \qquad (3)$$

PLOTTING MECHANICAL IMPEDANCE: Force and acceleration transducers are used to measure the impedance of a structure.

MEASURING FOUR POLE PARAMETERS

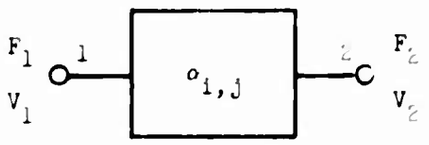
The four pole parameter representation of a system is determined when a connection point is "blocked" or "free".

The mechanical four pole parameters shown in the adjacent figure are physically defined by the following:

1. α_{11} is the ratio of F_1 to F_2 when point 2 is "blocked" ($V_2 = 0$).
2. α_{12} is the ratio of F_1 to V_2 when point 2 is "free" ($F_2 = 0$).
3. α_{21} is the ratio of V_1 to F_2 when point 2 is "blocked" ($V_2 = 0$).
4. α_{22} is the ratio of V_1 to V_2 when point 2 is "free" ($F_2 = 0$).

Reference (1) defines the relationships between impedance and four pole parameters by Equations 1 through 4. The subscript (oc) refers to a "blocked" junction and the subscript (sc) refers to a "free" junction. Z_{1oc} is the impedance at point 1 when point 2 is "blocked"; the other impedances are similarly defined. In electrical systems the analogy is, (sc) refers to output short circuited, and (oc) refers to output open-circuited, (α_{11}) and (α_{22}) are the force and velocity transfer functions, (α_{12}) is impedance and (α_{21}) is a mobility.

An example in the determination of the four-pole parameters is given in the appendix on page 3.5-10.



$$F_1 = \alpha_{11} F_2 + \alpha_{12} V_2$$

$$V_1 = \alpha_{21} F_2 + \alpha_{22} V_2$$

$$\alpha_{21} = \frac{1}{\pm \left[Z_{2oc} \cdot (Z_{1oc} - Z_{1sc}) \right]^{1/2}} \qquad (1)$$

The radical is chosen such that α_{21} has a positive real part.

$$\alpha_{11} = Z_{1oc} \cdot \alpha_{21} \qquad (2)$$

$$\alpha_{22} = Z_{2oc} \cdot \alpha_{21} \qquad (3)$$

$$\alpha_{12} = Z_{1sc} \cdot Z_{2oc} \cdot \alpha_{21} \qquad (4)$$

THE MECHANICAL FOUR POLE: The four pole parameters are related to mechanical impedance by the above equations.

VOLUME III - CHAPTER 3

MECHANICAL IMPEDANCE

SECTION 4 – APPLICATIONS

- Information Contained in the Mechanical Impedance Plot
- Impedance Model for Vibration Isolation of a Mechanical System
- Vibration Isolation Techniques and Procedures

INFORMATION CONTAINED IN THE MECHANICAL IMPEDANCE PLOT

The impedance plot contains physical information about the system under study.

In addition to defining mechanical impedance as a function of frequency, the impedance plot contains physical information about the system. This information can include:

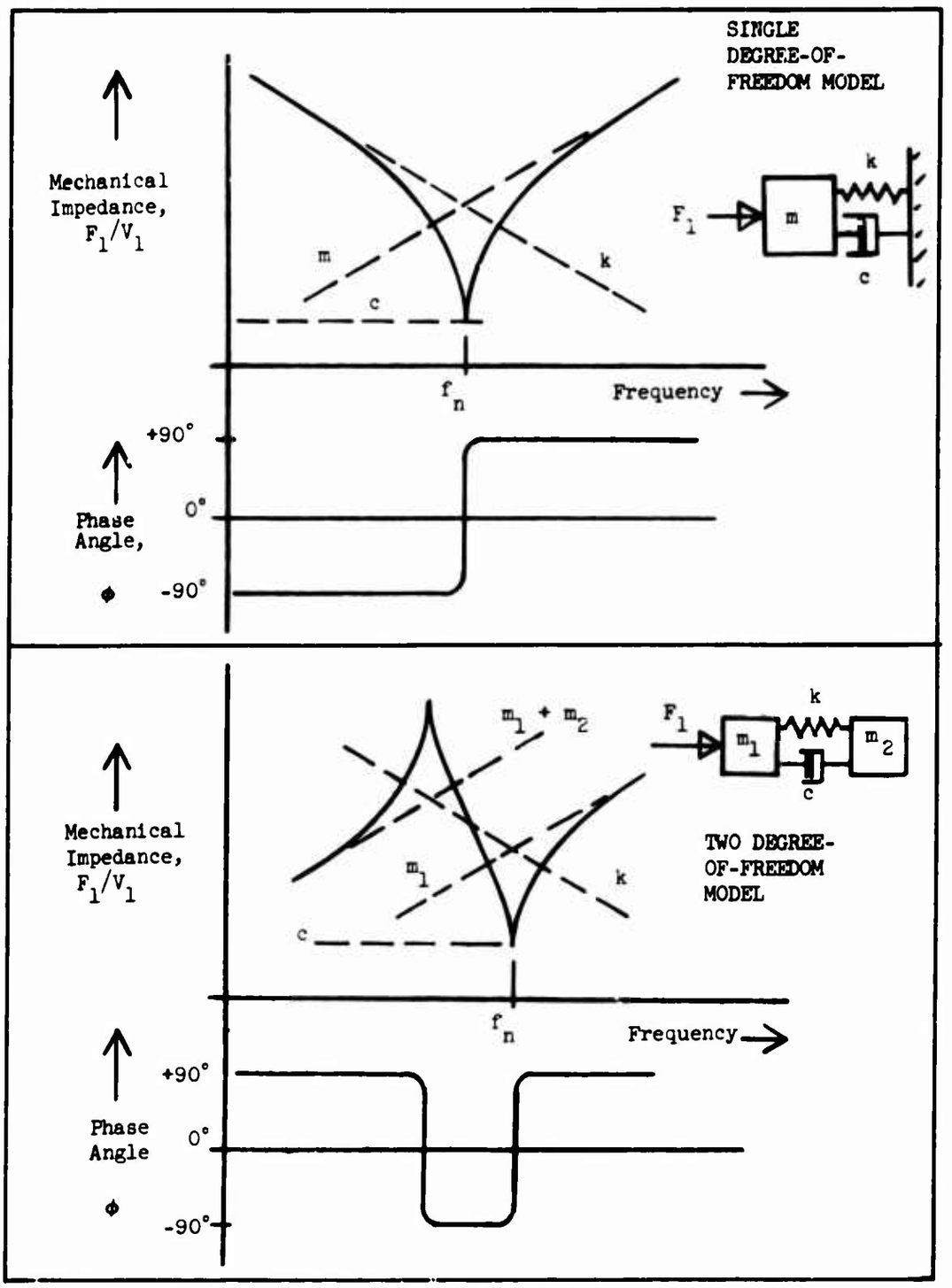
1. Static weight (mass)
2. Stiffness characteristics
3. Damping properties
4. Resonant frequencies
5. Anti-resonant frequencies.

Illustrated is a sample impedance plot for a single degree-of-freedom system driven by a force (F_1) acting on the mass (m). At low frequencies the plot follows the constant stiffness line representing the spring constant, (k). At higher frequencies, the effect of the mass dominates and the plot follows the constant mass line representing the mass of the system (m). At the resonant frequency, the impedance goes to its minimum value and the phase angle experiences a 180° shift. The value of impedance at resonance is the damping of this sytem, (c). Note that the dimensions of both impedance and damping are force per velocity.

Also illustrated in the impedance plot for two free masses connected by a spring and damper in parallel. The system is driven by the force (F_1) acting on mass (m_1). This is similar to a base-driven single degree-of-freedom system where (m_1) is the mass of the base. At low frequencies the spring can be viewed as acting as a rigid bar and thus the impedance plot follows the constant mass line representing ($m_1 + m_2$). This sytem has an anti-resonant frequency; at this frequency the impedance is a maximum and the phase goes through a 180° shift. The resonant frequency is characterized by the minimum impedance and a 180° phase shift. At high frequencies the effect of the spring becomes small and the impedance plot follows the constant mass line representing the drive mass (m_1). Again, the impedance at resonance represents the inherent damping characteristics of the system.

Similar information can be obtained from impedance plots of multiple degree of freedom systems. Resonances can be identified as minimum impedance points where the phase angle experiences a 180° phase shift. The impedance at resonance is the damping of that mode of vibration. The anti-resonances are identified as maximum impedance points where the phase angle experiences a 180° phase shift. At frequencies where mass dominates the impedance, the plot will approximately follow a constant mass line and the phase angle will be +90°. Similarly, when a spring dominates, the plot will approximately follow a constant stiffness line and the phase angle will be -90°.

An illustrative example for a single-degree-of-freedom system is given in the Appendix, page 3.5-12. Reference (16) in the appendix gives a wealth of practical information on use of mechanical impedance and the plots for single and multiple degrees of freedom.



IMPEDANCE PLOTS: Mass, stiffness, damping, and resonant frequency are found in the impedance and phase angle plots for two simple systems.

IMPEDANCE MODEL FOR VIBRATION ISOLATION OF A MECHANICAL SYSTEM

Mechanical impedance is used to study the effects of mounting equipment on a mechanical isolator which is mounted to a flexible vibrating structure.

The System

The system considered here consists of a flexible structure which is vibrating, an isolation mount and piece of equipment for which vibration isolation is needed. The flexible, vibrating structure is modeled as the mechanical source, the isolator as a four pole and the equipment as a mass (see adjacent figures). The source consists of a mechanical generator and an elastic system; the isolator consists of a spring, k, and a damper, c, connected between points 1 and 2 in parallel.

Assumptions

1. The excitation is sinusoidal vibration.
2. The mounting structure is elastic.
3. The responding masses inside the equipment do not affect the input and therefore the equipment can be represented as a rigid mass.
4. The system consists of linear springs, viscous dampers, and rigid masses.
5. Only translational motion is significant.
6. The isolator is a center-of-gravity suspension system which allows the three mutually perpendicular axes to be considered separately.
7. An equipment fragility curve exists which consists of a single curve of sinusoidal amplitude versus frequency. Points below the curve have a very small probability of causing failure. It is assumed that this fragility curve is exceeded if no isolator is used (i.e., point 2 is mounted directly to point 1).

Information Needed

In the lower figure the system is shown in block diagram form, with the parameters needed to perform the analysis. The information needed is:

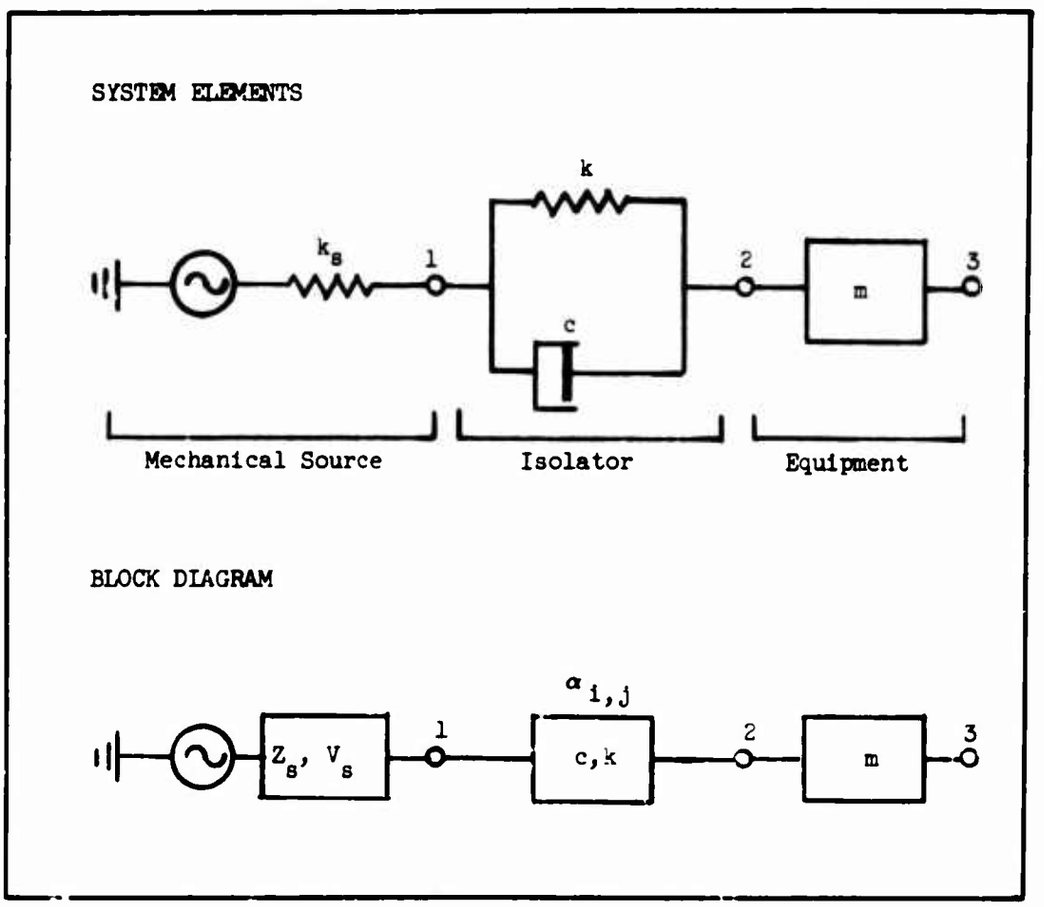
1. The fragility curve for the equipment, V_f
2. The mass of the equipment, m
3. The unloaded vibration of the mechanical source, V_s
4. The mechanical impedance of the source, Z_s
5. The vibration response of the hard mounted equipment, V_o

6. The maximum allowable displacement for the equipment

7. The stiffness, k, and damping, c, of the isolators being considered.

Note that V_f, V_s, Z_s, and V_o are all functions of frequency.

The procedures for designing the isolator are outlined in the following section.



VIBRATION ISOLATION MODEL: The elements of the mechanical system are shown in physical and block diagram format.

VIBRATION ISOLATION TECHNIQUES AND PROCEDURES

Procedures are outlined for the design of an isolator using combined test and analysis methods.

Procedure

1. Establish the fragility level, V_f, of the equipment by subjecting it to sinusoidal vibration tests.

2. Measure the unloaded vibration, V_s, of the mounting area (the mechanical source). Obtain this information by recording the signals from transducers mounted in this area during the desired operating conditions. The masses of the transducers are assumed to be so light that they do not affect the vibration.

3. Determine the response, V_o, of the equipment when it is hard mounted (no isolator) to the vibrating structure. V_o can be determined by:

   a. Calculating the response, or

   b. Measuring the response under actual conditions.

   If b. is used, it is advisable to mount a dummy simulating the equipment to avoid possible damage. If V_o is less than V_f for all frequencies, no further study is indicated.

4. If V_o exceeds V_f at any frequency, mechanical isolation is needed. Additional information is required. Determine the mechanical impedance of the source, Z_s, using a mechanical impedance head and a force generator.[5] For this study, it is convenient to express Z_s in terms of its real and imaginary parts (Equations 1, 2 and 3).

5. It is now necessary to determine the characteristics of the isolator. The first cut is made by utilizing the transmissibility (T) for the single degree of freedom system (base excited). This approximation applies exactly if the source has infinite impedance. The mass m is the mass of the equipment and a spring and damper are selected such that T times V_s is less than V_f for all frequencies. The flexibility of the isolator is limited by the maximum allowable deflection for the equipment (space limitations) and the damping is limited by the isolator material properties.

6. The design is now refined by considering the flexibility of the source. We now define the insertion ratio, η, which is equal to the ratio of the response velocity before isolation (V_o) to the response velocity after isolation (V_1). See Equation 4. The absolute value of η is the ratio of the transmissibility before isolation to the transmissibility after isolation, and is an indicator of the effectiveness of isolation. The absolute value of η is given by Equation 5. The isolator design is refined by varying values of c and k.

7. The isolator design must result in values of V_i less than V_f for all frequencies, even when c and k are varied to allow for tolerances and temperature changes.

Equation 5 provides a convenient method for determining the effectiveness of an isolator (described in terms of k and c) as a function of the mass being isolated (m), the impedance of the mounting surface (R_s, X_s) and frequency (ω). Although Equation 5 can be solved by hand for each frequency of interest, it is more easily handled by the computer.

$$Z_s(\omega) = R_s(\omega) + j\, X_s(\omega) \tag{1}$$

$$R_s(\omega) = |Z_s(\omega)| \cos\phi \tag{2}$$

$$X_s(\omega) = |Z_s(\omega)| \sin\phi \tag{3}$$

where

ϕ = phase angle of velocity behind force

$$\eta = V_o/V_i = \text{insertion ratio} \tag{4}$$

$$|\eta| = \left|\frac{V_o}{V_i}\right| = \left\{\frac{\left[R_s - M(R_s k + X_s \omega c)\right]^2 + \left[(\omega m + X_s) + M(R_s \omega c - X_s k)\right]^2}{R_s^2 + (\omega m + X_s)^2}\right\}^{\frac{1}{2}} \tag{5}$$

where

$$M = \omega^2 m/(\omega^2 c^2 + k^2)$$

Note that V_f, V_s, Z_s, V_o, V_i, R_s, X_s, and η are all functions of frequency, ω.

ISOLATION PROCEDURES: Equations are presented for the step-by-step procedure for the design of an isolation system.

VOLUME III - CHAPTER 5

MECHANICAL IMPEDANCE

SECTION 5 - APPENDIX

- Bibliography
- Symbology
- Glossary
- A Typical Impedance Plot
- An Example of Forced Vibration With Damping Employing Rotating Phasor Techniques
- Applying the Rules of Four Pole Parameters
- Determination of Four Pole Parameters for a Mass, Spring and a Damper
- Example on System Response Versus Frequency Using Mechanical Impedance Concepts and Plots

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SYMBOLOGY

a	Linear Acceleration
c	Viscous Damping
dB	Decibels
e	2.17...
f	Frequency, Cycles per Second
f_n	Natural Frequency
F,f	Force
j	$\sqrt{-1}$
k	Spring Constant; Stiffness
m	Mass
M	Mobility
R_s	Real Part of Impedance
T	Transmissibility
V,ν	Velocity
x	Linear Displacement
X_s	Imaginary Part of Impedance
Y	Mechanical Admittance
Z	Mechanical Impedance
α	Four Pole Parameter
σ	Phasor Displacement Angle
η	Insertion Ratio
Φ	Phasor Displacement Angle
$\cup$	Phasor Displacement Angle
ω	Frequency, Radians per Second

GLOSSARY

Damping - The dissipation of energy with time or distance.

Degrees-of-Freedom - The number of degrees-of-freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the positions of all parts of the system at any instant of time.

Fragility - The maximum dynamic response to which a system should be exposed, based on the strength of the equipment.

Frequency - The number of times that a periodic function repeats the same sequence of values during a unit variation of time. The unit is the cycle-per-second which equals one Hertz (Hz).

Impedance Head - A highly sensitive, combined force and accelerometer transducer.

Isolation - A reduction in the capacity of a system to respond to an excitation, attained by the use of a resilient support. In steady-state forced vibration, isolation is expressed quantitatively as the complement of transmissibility.

Phase of a Periodic Quantity - The fractional part of a period (for a particular value of the independent variable) through which the independent variable has advanced, measured from an arbitrary reference.

Mechanical Admittance (Mobility) - The complex ratio of velocity to force.

Mechanical Impedance - Ratio of force to velocity during simple harmonic motion.

Resonance - Resonance of a system in forced vibration exists when any change, however small, in the frequency of excitation causes a decrease in the response of the system.

Resonant Frequency - Vibrating frequency at which resonance occurs.

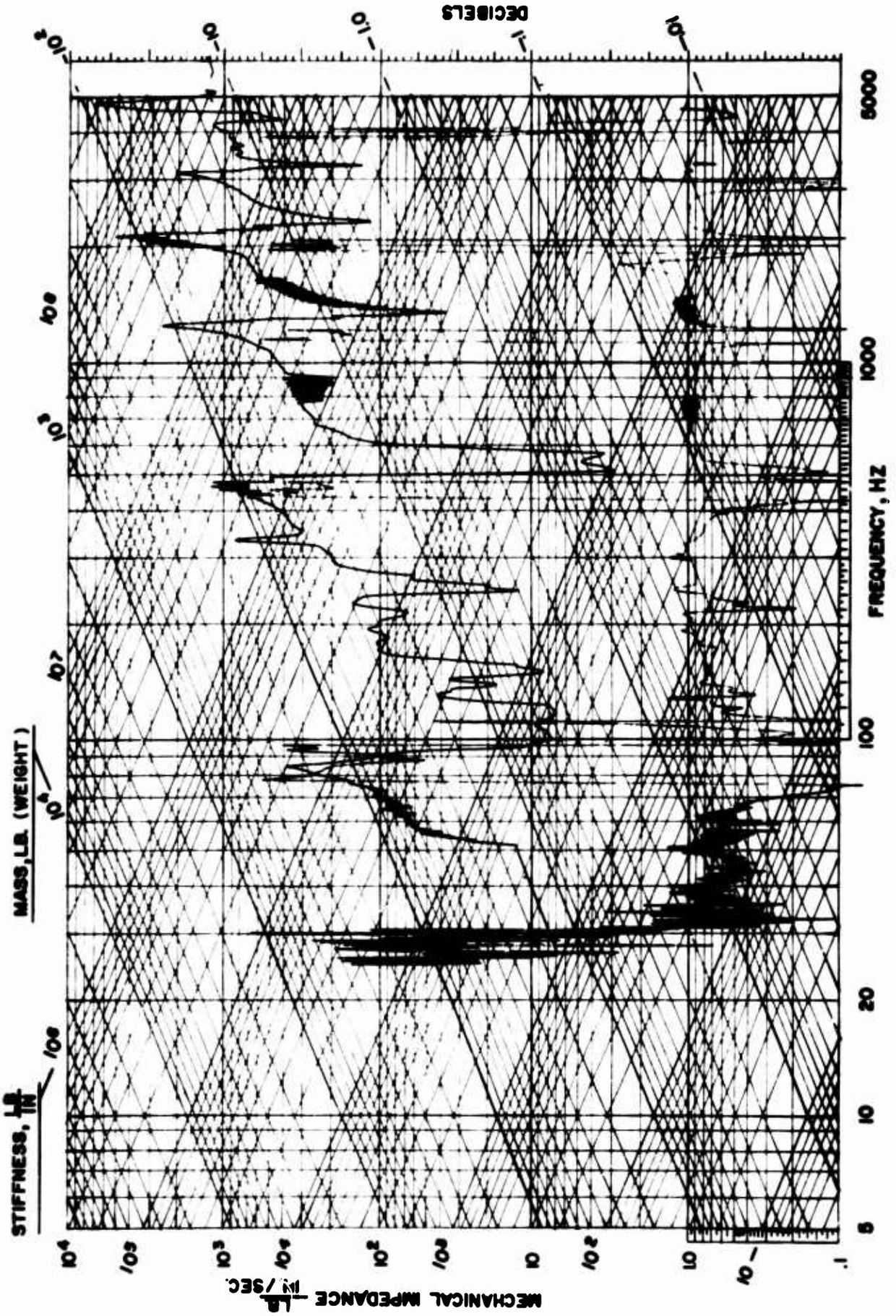
Response - The motion (or other output) of a system or device resulting from an excitation.

Stiffness - The ratio of change of force (or torque) to the corresponding change in translational (or rotational) deflection of an elastic element.

Transducer - A device for translating faithfully the changing magnitude of one kind of quantity into corresponding changes of another kind of quantity. A dynamic transducer translates a shock pulse into an electric current output.

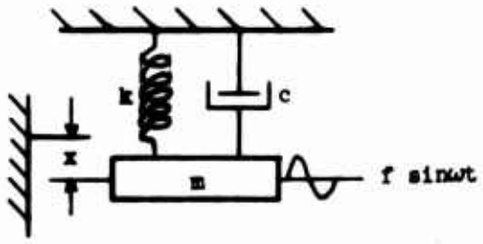
Transmissibility - Non-dimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio may be one of forces, displacements, velocities, or accelerations.

A TYPICAL IMPEDANCE PLOT (10)



AN EXAMPLE OF FORCED VIBRATION WITH DAMPING EMPLOYING ROTATING PHASOR TECHNIQUES

Illustrative Example: For the system shown below find the steady state displacement of the mass using rotating phasors.



Solution: The differential equation of motion for this type of system is

$$f \sin\omega t = m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx$$

Since $\bar{F} = f\, e^{j(\omega t)} = f(\cos\omega t + j \sin\omega t)$ only the imaginary part of the solution would apply if $\bar{F}$ is used in place of $f \sin\omega t$ in the differential equation.

The displacement vector will be represented by $\bar{x} = x\, e^{j(\omega t - \psi)}$ and correspondingly the velocity vector is $(j\omega\bar{x})$ and the acceleration vector is $(-\omega^2 x)$.

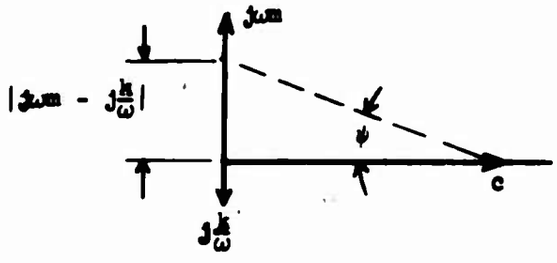
After substitution into the original equation and algebraic manipulation there results,

$$xe^{-j\psi} = \frac{f}{k - m\omega^2 + j\omega c} = x(\cos\psi - j \sin\psi)$$

The angle of lag (ψ) may be determined from the plot of the impedance on the complex plane shown in Figure 8 on page 3.1-5. It is given by

$$\psi = \tan^{-1}\frac{c}{\omega m - \frac{k}{\omega}}$$

or, graphically, this is shown as



obtaining (sin ψ) from the previous relationship for (ψ) and substituting this into the form. For x(cos ψ-j sin ψ) and taking only the imaginary part, results in the displacement.

Solution:

$$\text{Imag } \bar{x} = \frac{f}{\sqrt{(c\omega^2)^2 + (k-m\omega^2)^2}} \sin(\omega t - \psi)$$

Attention is called to the fact that when not using the rotating phasor techniques the lag angle (ψ) is obtained by substituting into the original equation of motion the known form of the solution to this type equation, the the form $x = A\sin\omega t + B\sin\omega t$. After some algebraic manipulation there is obtained the identical solution shown immediately above with

$$\psi = \tan^{-1} \frac{c\omega}{k-m\omega^2}$$

APPLYING THE RULES OF FOUR POLE PARAMETERS

MATRIX PRODUCT

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

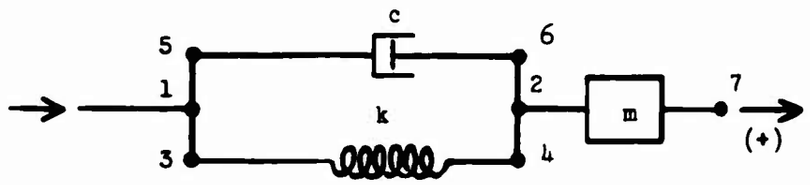
$$c_{11} = (a_{11} \times b_{11}) + (a_{12} \times b_{21})$$

$$c_{12} = (a_{11} \times b_{12}) + (a_{12} \times b_{22})$$

$$c_{21} = (a_{21} \times b_{11}) + (a_{22} \times b_{21})$$

$$c_{22} = (a_{21} \times b_{12}) + (a_{22} \times b_{22})$$

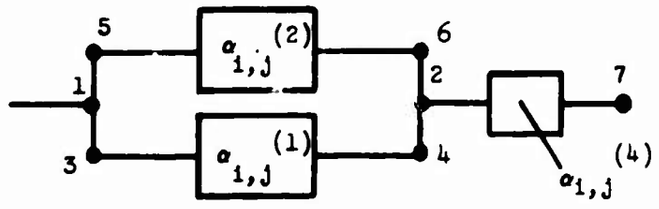
FOUR POLE PARAMETERS FOR THE SINGLE-DEGREE-OF-FREEDOM SYSTEM



Spring (k) and damper (c) are connected in parallel; this parallel connection is in series with mass (m).

Points 1, 3, and 5 form a common junction, as do points 2, 4 and 6.

APPLYING THE RULES OF FOUR POLE PARAMETERS FOR A PARALLEL CONNECTION



The spring $\left[\alpha_{i,j}^{(1)}\right]$

and damper $\left[\alpha_{i,j}^{(2)}\right]$

are combined to form a composite four pole $\left[\alpha_{i,j}^{(3)}\right]$.

$$\alpha_{i,j}^{(1)} = \begin{bmatrix} 1 & 0 \\ \frac{\omega j}{k} & 1 \end{bmatrix} \qquad \alpha_{i,j}^{(2)} = \begin{bmatrix} 1 & 0 \\ \frac{1}{c} & 1 \end{bmatrix}$$

$$\alpha_{i,j}^{(3)} = \begin{bmatrix} \alpha_{11}^{(3)} & \alpha_{12}^{(3)} \\ \alpha_{21}^{(3)} & \alpha_{22}^{(3)} \end{bmatrix} \qquad \alpha_{i,j}^{(4)} = \begin{bmatrix} 1 & m\omega j \\ 0 & 1 \end{bmatrix}$$

$$\alpha_{11}^{(3)} = \frac{A}{B}$$

$$\alpha_{12}^{(3)} = \frac{AC}{B} - B$$

$$\alpha_{21}^{(3)} = \frac{1}{B}$$

$$\alpha_{22}^{(3)} = \frac{C}{B}$$

$$A = \sum\left(\frac{\alpha_{11}}{\alpha_{12}}\right) = C + \frac{k}{j\omega}$$

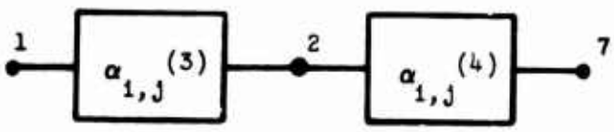
$$B = \sum\left(\frac{1}{\alpha_{21}}\right) = C + \frac{k}{j\omega}$$

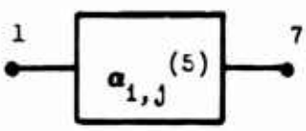
$$C = \sum\left(\frac{\alpha_{22}}{\alpha_{21}}\right) = C + \frac{k}{j\omega}$$

$$\alpha_{i,j}^{(3)} = \begin{bmatrix} 1 & 0 \\ \frac{1}{c + \frac{k}{j\omega}} & 1 \end{bmatrix}$$

APPLYING THE RULES OF FOUR POLE PARAMETERS (Continued)

APPLYING THE RULES OF FOUR POLE PARAMETERS FOR A SERIES CONNECTION





The four poles $\alpha_{i,j}^{(3)}$ and $\alpha_{i,j}^{(4)}$

are combined to form a composite four pole $\alpha_{i,j}^{(5)}$

$$\alpha_{i,j}^{(5)} = \begin{bmatrix} \alpha_{11}^{(5)} & \alpha_{12}^{(5)} \\ \alpha_{21}^{(5)} & \alpha_{22}^{(5)} \end{bmatrix} = \begin{bmatrix} \alpha_{11}^{(3)} & \alpha_{12}^{(3)} \\ \alpha_{21}^{(3)} & \alpha_{22}^{(3)} \end{bmatrix} \times \begin{bmatrix} \alpha_{11}^{(4)} & \alpha_{12}^{(4)} \\ \alpha_{21}^{(4)} & \alpha_{22}^{(4)} \end{bmatrix}$$

$$\alpha_{i,j}^{(5)} = \begin{bmatrix} 1 & 0 \\ \frac{1}{c + \frac{k}{j\omega}} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & m\omega j \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & m\omega j \\ \frac{1}{c + \frac{k}{\omega j}} & 1 - \frac{m\omega^2}{c\omega j + k} \end{bmatrix}$$

THE FOUR POLE EQUATIONS FOR THE SINGLE-DEGREE-OF-FREEDOM SYSTEM

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & m\omega j \\ \dfrac{1}{c + \dfrac{k}{\omega j}} & 1 - \dfrac{m\omega^2}{c\omega j + k} \end{bmatrix} \begin{bmatrix} F_7 \\ V_7 \end{bmatrix}$$

$$F_1 = F_u + V_7\, m\omega j$$

$$V_2 = \frac{F_7}{c + \dfrac{k}{\omega j}} + V_7 \left(1 - \frac{m\omega^2}{c\omega j + k}\right)$$

DETERMINATION OF FOUR POLE PARAMETERS FOR A MASS, A SPRING, AND A DAMPER

In the equations below the α terms are to be determined. These are the four pole parameters.

$$F_1 = \alpha_{11} F_2 + \alpha_{12} V_2$$

$$V_1 = \alpha_{21} F_2 + \alpha_{22} V_2$$

For the mass, since the body is considered rigid the velocities at input and output are equal. Therefore $V_1 = V_2$. It is immediately seen from the above equations that $\alpha_{21} = 0$ and $\alpha_{22} = 1$. With the output clamped $F_1 = F_2 = \alpha_{11} F_2$ at $V_2 = 0 = V_1$ therefore, $\alpha_{11} = 1$.

Further, since at (sc) $F_1 = 0 + \alpha_{12} V_2$, $V_1 = V_2$, and $\alpha_{12} = (F_1)/(V_2)$ at $F_2 = 0$, α_{12} must equal $(F_1)/(V_1) = j\omega m = Z_m$.

For the spring the input force and the output force are equal ideally since the spring is considered massless. Further the relative velocity relationship $V_1 - V_2 = (j\omega F)/(k)$. Hence, $F_1 = F_2$ and

$$V_1 = \frac{j\omega}{k} F_2 + V_2$$

To satisfy the original equations for (F_1) and (V_1) $\alpha_{11} = 1$, $\alpha_{22} = 1$, $\alpha_{12} = 0$, $\alpha_{21} = (j\omega)/(k)$.

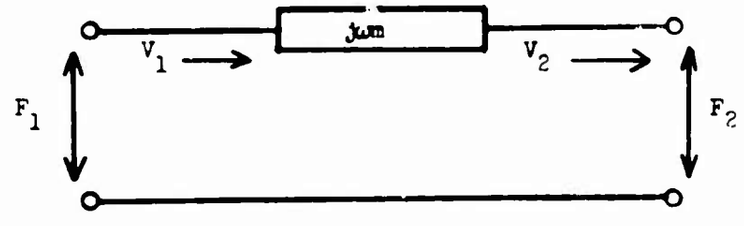
For the damper $F_1 = F_2$ and the relative velocity is expressed as $V_1 - V_2 = (F_2)/(c)$. Therefore, $F_1 = F_2$ and

$$V_1 = \frac{F_2}{c} + V_2$$

To satisfy the original equations for (F_1) and (V_1) $\alpha_{11} = 1$, $\alpha_{22} = 1$, $\alpha_{12} = 0$, $\alpha_{21} = (1)/(c)$.

The above results may also be obtained by resorting to electric circuit considerations.

For the mass, the following electric circuit analogy applies;



For open circuit (oc)

$$F_1 = F_2$$

$$V_1 = V_2 = 0$$

and since

$$F_1 = \alpha_{11} F_2 + \alpha_{12} V_2$$

$$\alpha_{11} = 0$$

Further, since

$$V_1 = \alpha_{21} F_2 + \alpha_{22} V_2$$

$$\alpha_{21} = 0$$

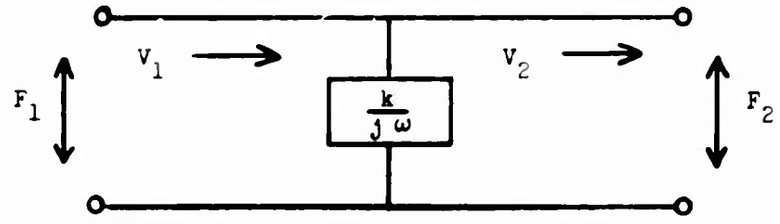
For short circuit (sc)

$$V_1 = V_2$$

$$F_2 = 0$$

and again using the above equations for (F_1) and (V_1) results in $\alpha_{12} = j\omega m$ and $\alpha_{22} = 1$.

For the spring the electric circuit analogy is



For open circuit (oc)

$$F_1 = F_2$$

$$V_2 = 0$$

$$\frac{F_1}{\frac{k}{j\omega}} = V_1$$

For short circuit (sc)

$$V_1 = V_2$$

$$F_2 = 0 = F_1$$

Applying these results to the original equation leads to $\alpha_{11} = 1$, $\alpha_{12} = 0$, $\alpha_{21} = (j\omega)/(k)$, $\alpha_{22} = 1$.

For the Damper the electric circuit is the same as for the spring except that the impedance is c. There results; $\alpha_{11} = 1$, $\alpha_{12} = 0$, $\alpha_{21} = (1)/(c)$, $\alpha_{22} = 1$.

EXAMPLE ON SYSTEM RESPONSE VERSUS FREQUENCY USING MECHANICAL IMPEDANCE CONCEPTS AND PLOTS

The system shown below in Figure 1, will be investigated for response with variable frequency for the forcing function.

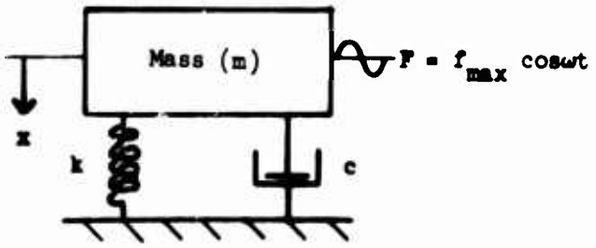


Figure 1.

The associated phasor diagram is as shown below in Figure 2.

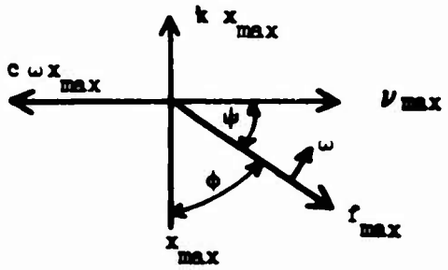


Figure 2.

The velocity mobility of the system is given by

$$M = \frac{1}{\Sigma Z} = \frac{1}{c - \frac{jk}{\omega} + j\omega m}$$

which, after algebraic manipulation yields,

$$M = \frac{c\omega^2 + j\omega(k - \omega^2 m)}{(c\omega)^2 + (k - \omega^2 m)^2}$$

Note that this is in a form which readily shows the complex number representation A+jB.

The magnitude of (M) is given by

$$|M| = \frac{v_{max}}{f_{max}} = \frac{\omega}{\sqrt{(c\omega)^2 + (k - \omega^2 m)^2}}$$

with the displacement angle given by

$$\tan \varphi = \frac{k-\omega^2 m}{c\omega}$$

Note that $|M|$ and (φ) may be readily found as (ω) varies.

Since $(|M|)$ is of the form $(\nu_{max})/(f_{max})$, the maximum forces on the mass, spring, and damper may readily be found. For example, since the velocity mobility of the system is $(\nu_{max})/(f_{max})$, ν_{max} may be found and applied to the mobility of each element for determination of f_{max} for each element.

In Figure 3 (on the following page), the velocity mobility curve of the system is asymptotic to the spring mobility line in the low frequency range and is asymptotic to the mass mobility line in the high frequency range. This means that at low frequencies the spring predominates and at high frequencies the mass predominates. For forcing function frequencies below natural frequency (f_n) the system is spring controlled and the angle (φ) is between 0 and +90°. For above the natural frequency the angle is between 0 and -90° and the system is mass controlled.

Typical units used are:

- Frequency (f) - cycles per second (Hz)
- Mass (m) - pounds
- Compliance $\left(\frac{1}{k}\right)$ - microinches per pound
- Velocity Mobility (M) - microinches per pound second

Note that the spring mobility line is parallel to the (1/k) system of lines, that the mass mobility line is parallel to the (m) system of lines, and that the damper mobility lines are parallel to the (M) system of lines (shown in the following chart).

In conjunction with the mobility plot the displacement plot may be made. This is merely a plot of the displacement (maximum) of each element in the system versus frequency. A typical plot is shown in Figure 4, (on page 3.5-15). This is for the type of Spring-Mass-Damper System considered for the mobility plot.

EXAMPLE ON SYSTEM RESPONSE VERSUS FREQUENCY USING MECHANICAL IMPEDANCE CONCEPTS AND PLOTS (Continued)

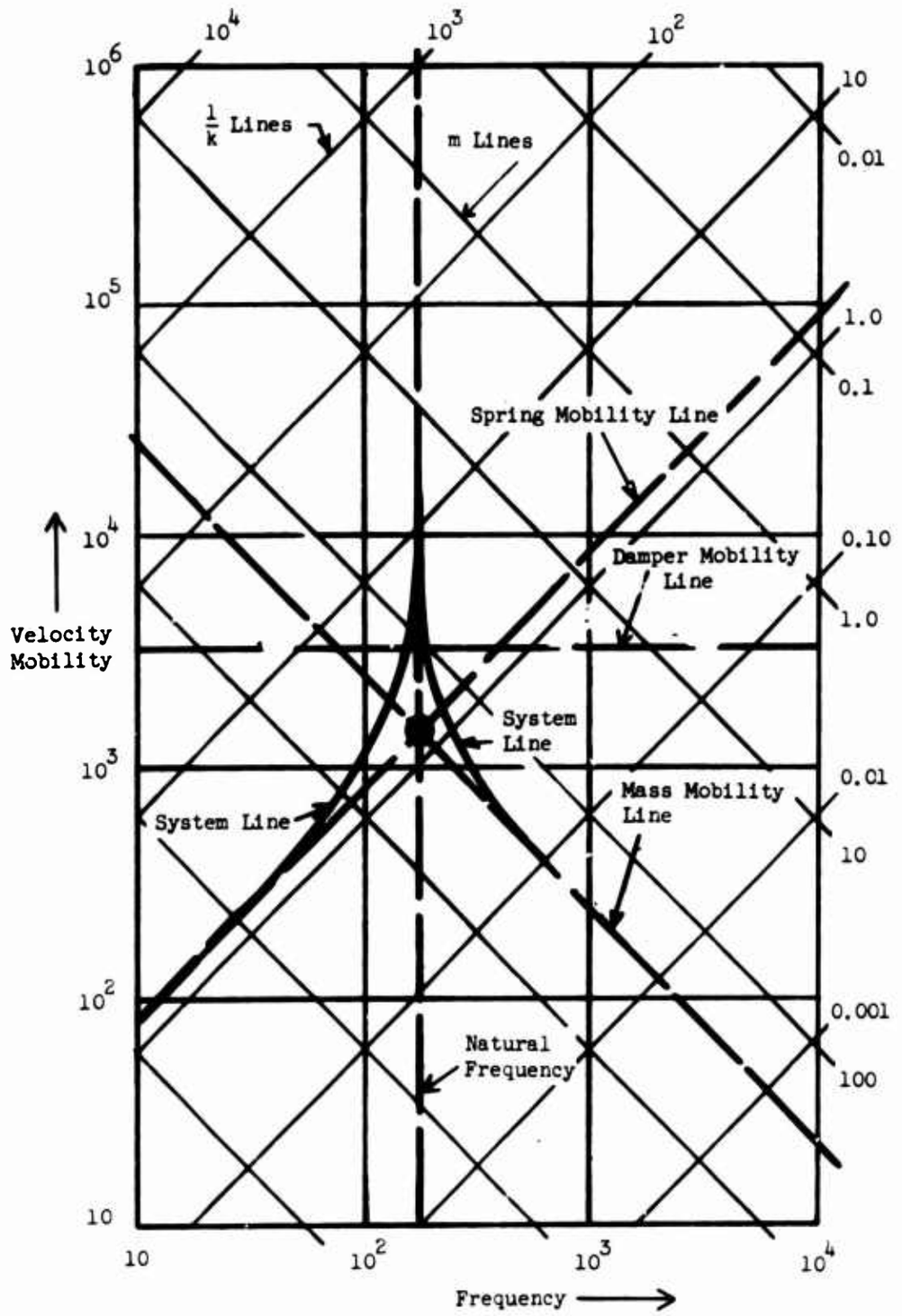


Figure 3.

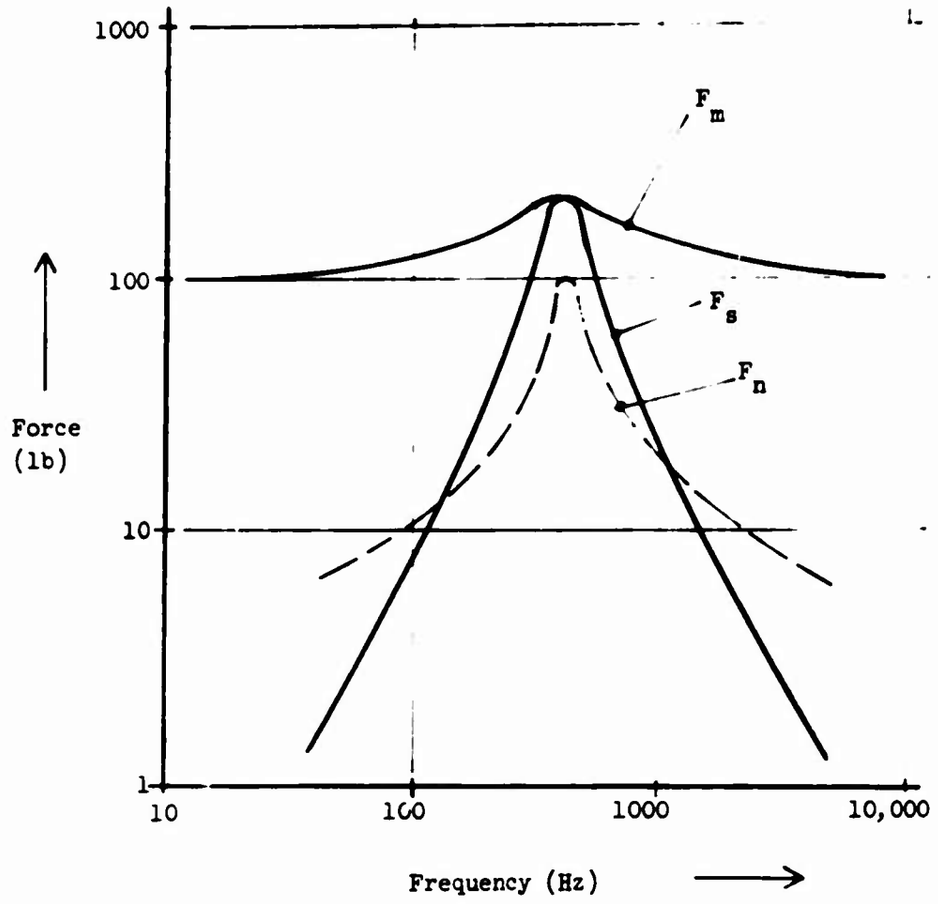


Figure 4.

CHAPTER 4 – STRESS CONCENTRATION

CHAPTER 4
STRESS CONCENTRATION

ABSTRACT:

Stress concentration effects are known to cause a sharp increase in stress at the local area near a notch or discontinuity. This effect is particularly important to structure loaded repetitively since the stress raiser acts as an incipient crack, which may be ultimately manifest as a fatigue failure. What is not so clearly understood is the stress-amplification effect of discontinuities in equipment structure from component access requirements, cooling provisions, mounting interfaces, and similar functional necessities. Each of these interruptions in the structural continuity has the capability of causing a drastic decrease in service life.

This chapter outlines the parameters used in the analysis of stress amplification effects including theoretical factors, fatigue factors, and material notch sensitivity effects. Procedures are discussed for the analysis of static, repetitive, and impact loading conditions.

Some practical suggestions are offered on the approaches a designer may employ to minimize the stress concentration effect in new designs as well as fixes for stress raisers in existing equipment structure, short of a complete redesign. A summary of important stress concentration factors is presented in the appendix, with a selected bibliography on the subject.

Chapter 4 - Stress Concentration

ERRATA SHEET

Page	Paragraph	Line	Correction
4.1-0	6	1	material's
4.1-3	1	2	fatigue-resistant
4.2-2	1	7	Classically
4.2-3	Graphic	Lower Title	TYPICAL STEEL ALLOY
4.3-2	1	5	is calculated ...
4.3-2	2	5	Margin of Safety = [(Safety Factor)-1] x 100
4.5-9	Graphic	Dotted Curve	r = h
4.5-12	Graphic	4-(b)	... shaft tension
4.5-15	Graphic	L.H. Sketch	(7) Equal Spaces

VOLUME III - CHAPTER 4
STRESS CONCENTRATION

CONTENTS

Section		Page
1	INTRODUCTION	4.1-0
	• The Importance of Stress Concentration to the Packaging Designer	4.1-0
	• The Influence of Stress Concentration on Dynamic Structural Integrity	4.1-2
2	EVALUATING THE CONCENTRATION EFFECT	4.2-0
	• The Theoretical Stress Concentration Factor	4.2-0
	• The Stress Concentration Factor for Fatigue	4.2-2
3	STRESS AMPLIFICATION IN STRUCTURAL MEMBERS	4.3-0
	• Some Factors Affecting the Severity of the Stress Concentration	4.3-0
	• Strength Calculations for Members Subjected to Static and Impact Loads	4.3-2
	• Strength Calculations for Members Subjected to Repeated Loads	4.3-4
4	MINIMIZING THE STRESS AMPLIFICATION EFFECT	4.4-0
	• Using Force Flow Lines to Visualize Stress Concentration	4.4-0
	• Optimizing Material Characteristics, Heat Treatment, and Surface Protection	4.4-2
	• Some Suggestions for Improving Stress Concentration Effects in Existing Structure	4.4-4
	• Reducing Stress Concentration in Welded Joints	4.4-6
5	APPENDIX	4.5-0
	• Bibliography	4.5-0
	• Glossary	4.5-2
	• Geometric Effects - Some Typical Stress Concentration Factors	4.5-4
	• Examples of Designs to Minimize the Stress Raiser Effect	4.5-10
	• Examples of Design Oversight Which Cause High Stress Concentration and Service Failure	4.5-14
	• Conditions Which Reduce Dynamic Structural Integrity	4.5-16

CHAPTER 4

STRESS CONCENTRATION

SECTION 1 – INTRODUCTION

- The Importance of Stress Concentration to the Packaging Designer
- The Influence of Stress Concentration on Dynamic Structural Integrity

THE IMPORTANCE OF STRESS CONCENTRATION TO THE PACKAGING DESIGNER

Stress amplification effects due to stress concentration may greatly increase the calculated or nominal stress in a structural member. This is a design fact-of-life which the Electronic Equipment Packaging Engineer must handle analytically and structurally.

Stress concentration in an elastic material (and most of the structural materials of interest to the Packaging Engineer fall roughly into this category) is an abrupt increase in stress intensity, localized within a relatively small region. This phenomenon exists apart from and in addition to the nominal stress variation across a section resulting from a distributed load, such as flexure.

Theoretical stress concentration is characterized by two qualitative groups: geometric factors, which include abrupt changes in section, grooves, and fillets; and stress raisers, which cover a multitude of engineering and fabrication sins, such as scratches, burrs, fastener holes, threads and metallurgical anomalies. This useful categorization of stress concentration is developed in greater detail later in this section.

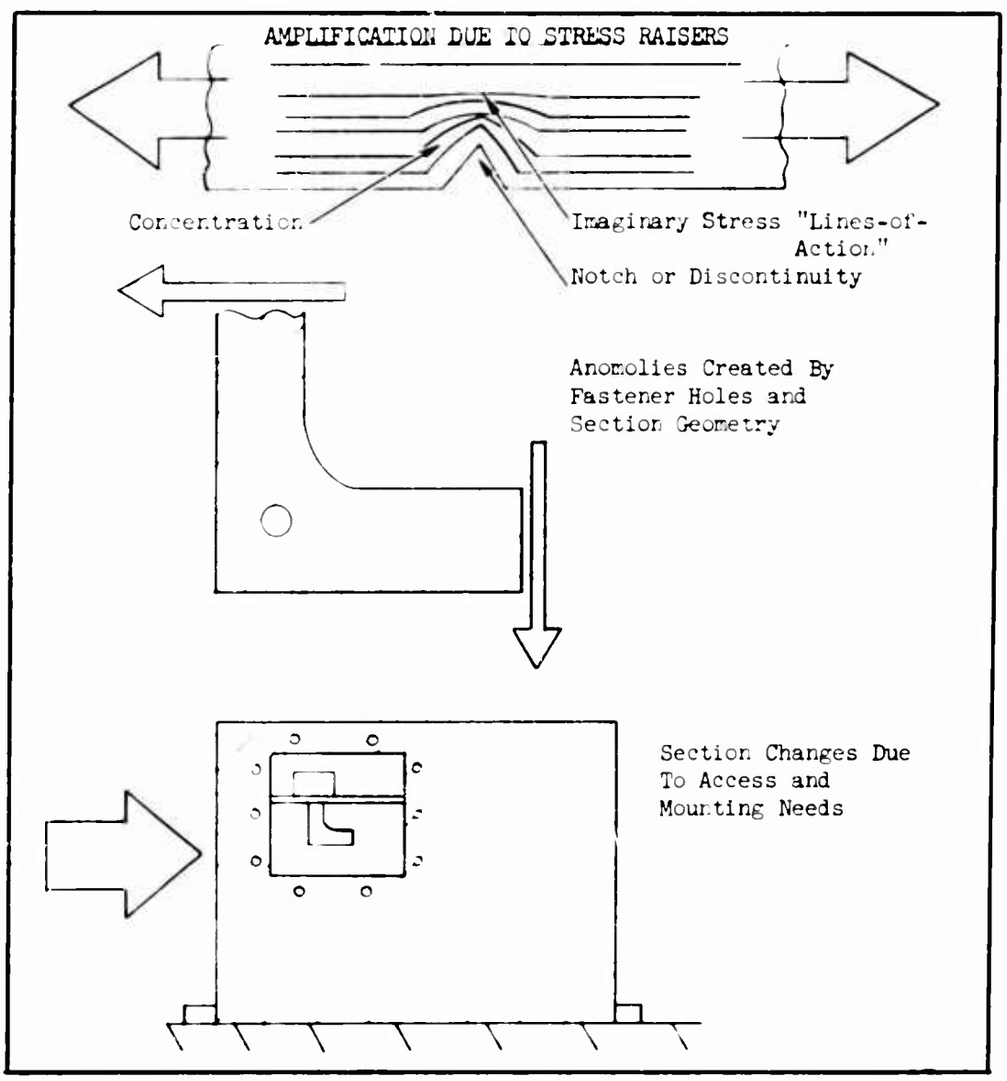
Stress amplification due to geometric discontinuity is handled analytically by multiplying the appropriate factor times the stress calculated from simple theory (such as P/A, Mc/I, Tc/J). The stress concentration factor thus acts as a correction factor to modify the calculated stress to more closely reflect the true situation. Almost invariably, the true stress exceeds the nominal stress; occasionally, as for a repetitive loading situation, this amplification is disastrous.

The practical significance of the stress magnification effect depends to a large degree on the circumstances: the category of the stress concentration effect, and the response of the material to the effect. The high local stress intensity created by stress raisers such as nicks and scratches is often relieved by local yielding of the material. This is a characteristic of a ductile material. For such materials exhibiting much plastic region capability (i.e., ductility), stresses induced by static loading rarely undergo much of a magnification due to stress raisers. Brittle materials such as cast iron, however, are much more sensitive to stress raisers due to their inability to deform plastically in the region of a discontinuity.

The situation is much more serious under dynamic conditions, such as vibration and "ringing" created by a shock disturbance. The end fracture due to repetitive load, such as a vibration, is a fatigue failure. Fatigue failures almost exclusively originate from a region of stress concentration, even in very ductile materials. Materials will vary widely in their susceptibility to stress raisers under dynamic conditions, an effect known as "notch sensitivity".

Ductility is not a particularly good measure of a material's immunity to stress concentration effects in fatigue. Thus, a new set of factors, referred to as the fatigue stress concentration factors, must be created to reflect this stress anomaly. These factors are defined as the ratio of the endurance limit in a plain specimen to the endurance limit in a notched specimen. The factors, therefore, measure the material's notch sensitivity.

Stress concentration is present in an electronic equipment package for three basic reasons: (1) as a design necessity resulting from fasteners, access areas, mounting arrangements, and changes in section geometry; (2) accidentally as a result of tool marks, scratches, burrs, and surface deterioration due to corrosion; and (3) inherently due to metallurgical anomalies such as inclusions, slip planes, quenching cracks and surface decarburization. The preceding list is not exhaustive. Any experienced Packaging Designer could undoubtedly add many other examples of the effect in equipment elements. The problem to which this chapter is directed is the recognition of this stress amplification effect and some of the qualitative and quantitative approaches available to the Designer for their control.



STRESS CONCENTRATION: Stress amplification is frequently manifest in equipment packages as an accidental or inherent discontinuity, or as a functional necessity.

THE INFLUENCE OF STRESS CONCENTRATION ON DYNAMIC STRUCTURAL INTEGRITY

Stress amplification effects are critically important to structural life and integrity. Stress concentration must be considered when designing packages subjected to both shock and vibration, but for a separate set of criteria.

The stress amplification effect must be treated in two discrete manners when designing structure for shock and vibration. The treatment of stress raisers in impact situations is roughly similar to the static analysis procedure; stress concentration in repetitively loaded structures, however, mainly reflects the material characteristics or notch sensitivity.

Experience with impact loading has shown that the stress raiser effect is just as pronounced and important in shock as it is under vibratory disturbances. Material ductility appears to be vital for the mitigation or redistribution of stress in the vicinity of a stress discontinuity. The maximum stress also varies directly with the square root of the material's elastic modulus. (16) It follows that maximum inpact resistance will be exhibited by the material with the lowest elastic modulus, provided that the elastic limit is not exceeded. The effect of loading speed (strain rate) has an important influence on structure subjected to impact loads. The response of a resonant system to an increasing strain rate has been shown to decrease dramatically. (17) In other words, the faster the loading rate, the less the resonant response. In addition, test data indicates that the apparent strength of typical materials increases with increased rate of load application. (16) As a fortunate consequence of these two effects, the analysis of structure under impact loading at a stress raiser represents a conservative approach if strain rate is assumed static.

Although fatigue is generally associated with a vibrating disturbance, the effect is nonetheless important in certain cases of load iteration resulting from impact. Often, the accelerations associated with a shock pulse are relatively high. If the impacted structure subsequently "rings", or resonates, at the high stress level, then an early fatigue failure is possible at an extremely low number of cycles of stress. This situation is compounded by the stress concentration effect since most impact fractures will originate at a stress raiser or discontinuity. As indicated previously, the best defense is a ductile material with a low elastic modulus.

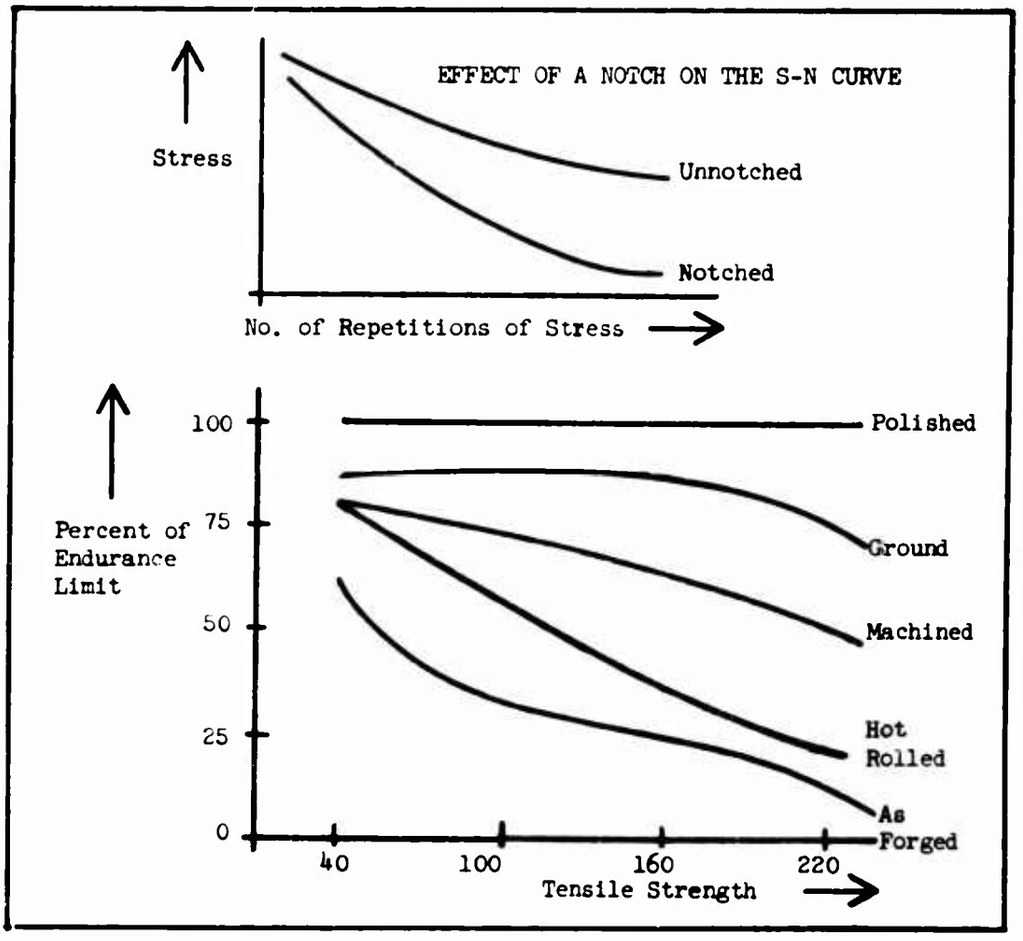
The most important aspect of the stress concentration effect to the designer is the influence of stress raisers on the fatigue life of structural materials, a loading situation induced by repetitive distrubances (vibration) or resonating structural elements (shock induced). Stress concentration under static loading may reduce the ultimate strength of the structure to half the value calculated by simple theory. The same discontinuity in a vibrating system could reduce the endurance limit of the material to 10 percent of its unnotched value. This dramatic loss of structural life is illustrated at the right for notched and unnotched specimens of the same material subjected to the same repeated loadings.

Surface finish is known to play an important role in the integrity of a vibrating structure since many fatigue failures originate at the outer fiber of a loaded member. Thus the manufacturing process is important as it affects surface. (18) It follows that machine finish, surface protection, and corrosion sensitivity which are potential sources of pits,

scratches, and surface blemishes are highly important design considerations for fatigue-resistnat structure. (See the lower figure.)

Fatigue failures emanating from stress raisers are not entirely confined to the surface, however. Spalling failures, which occur as a flaking off of large chunks of case material from a contact surface such as a gear tooth face, are the result of a rapid change in hardness. The interior or core material has insufficient fatigue strength to support the harder outer case, which eventually cracks and spalls off.

Designing structure for dynamic loadings subject to stress concentration effects appears to be an exercise in moderation. Moderation is required in the fairing of abrupt changes in section; in the use of ample radii to blend corners; in avoiding materials that are ultra-hard and strong, but are also very notch sensitive; in careful attention to eliminating scratches, sharp corners, and burrs; in selecting a material that exhibits good core strength even though it may not be indicated on the basis of static strength requirements.



STRESS RAISERS AND STRENGTH: The presence of notches and discontinuities dramatically reduces the strength capacity of materials subjected to dynamic loading.

CHAPTER 4

STRESS CONCENTRATION

SECTION 2 – EVALUATING THE CONCENTRATION EFFECT

- The Theoretical Stress Concentration Factor
- The Stress Concentration Factor for Fatigue

THE THEORETICAL STRESS CONCENTRATION FACTOR

Stress concentration effects may be represented analytically by a series of theoretical correction factors designed to bring the calculated stress more in line with the true stress in the region of a discontinuity.

The theoretical or geometric stress concentration factor is the most popular method of expressing the effect of stress raisers due to abrupt changes in section, notches, sharp discontinuities, and a variety of scratches, burrs, and tool marks. The discontinuities often occur as a design necessity such as holes, threads, and changes in a stressed section; or they may occur accidentally as a result of poor workmanship, such as scratches and sharp corners.

The theoretical factor (known as K_t) may be defined as the ratio of the actual elastic stress at the point of stress concentration to the nominal elastic stress at the same point (assuming no stress concentration), as calculated by simple theory.(3) The most successful attempts to derive K_t on the basis of Theory of Elasticity considerations have been accomplished by Neuber (6) and Timoshenko(5). The great source of data on this class of stress raisers comes, however, from empirical results. The effect of amplification of stress in the vicinity of holes and patterns of holes has been explored in depth by Savin(19) and Griffel(12). Many other standard works on geometric factors are reported authoritatively by Peterson (13), Gouza (15), and Roark (1) to cite just a few. It remains for the Design Engineer to model his particular stress concentration situation after one of those reported in the literature, select the range of parameters affecting the geometry, and pick the appropriate stress factor. Since the factor is defined as a ratio,

$$K_t = \frac{\text{actual stress}}{\text{nominal stress}} ,$$

the approximation of true stress near the stress raiser is simply the stress calculated by conventional theory multiplied by K_t. An example is shown in the adjacent figure. If a hole in a thin plate occurred such that h/R = 2.5, then the stress concentration factors would be approximately 0.8, 1.6, and 1.8 at the three critical locations indicated around the hole. The most significant stress then would be calculated by,

$$S_t = K_t \left(\frac{\text{tensile load}}{\text{net area}}\right) \text{ at}$$

location (b). Classically, a hole in the center of a wide plate in tension exhibits a stress amplification factor of 3.0, which is also the asymptotic value of the factor at (c) and (b) as the ratio h/R becomes large.

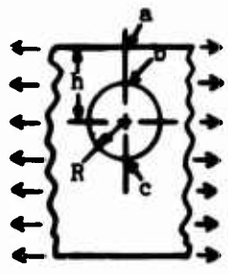
The theoretical factor K_t depends primarily on the geometry of the statically stressed element. Its derivation also assumes that the stressed material is homogeneous and isotropic, and behaves as a perfectly elastic material. The loading rate is assumed to be static.

Roark (1) has introduced the concept of a factor for stress concentration at rupture to more adequately cover those materials that exhibit brittle characteristics. This factor (K_r) is denoted as the ratio of the computed stress at rupture for a plain specimen to the computed stress at rupture for a specimen containing the stress raiser. K_r is used analytically much the same as K_t, as it is a ratio of stresses. Both factors are a measure

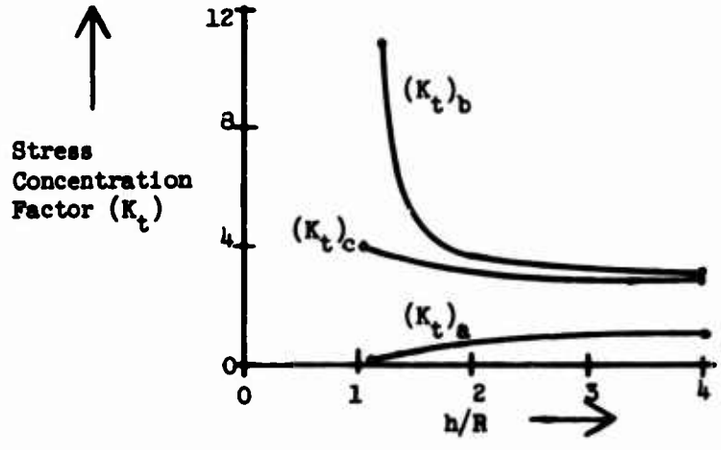
of the strength-reducing effect of stress concentration. Both are useful in analyzing the equivalent static effect of a shock-induced dynamic load.

The geometric stress concentration factor is affected by scale; the factor determined for a small specimen may not be extrapolated directly to larger structures. The larger the model, the more the amplification factor. (1)

The geometric factor may be reduced by attention to the placement of notches, holes, and other discontinuities. A series of holes is less detrimental than a single, large hole. Changes in section should be gradual and faired smoothly with generous radii. Surface blemishes should be avoided, and good workmanship is imperative, particularly in regions of high stress. Metallurgical considerations of proper alloy and heat treatment are critical, surface treatment and fabrication process are also important, as well as corrosion resistance.



TYPICAL PLOT OF STRESS CONCENTRATION AROUND A HOLE FOR A SECTION IN TENSION (3)



STRESS CONCENTRATION: A quantitative representation of stress concentrations exerted on a configuration may be evaluated in terms of the stress factor, (K_t) and a dimensional ratio.

THE STRESS CONCENTRATION FACTOR FOR FATIGUE

The stress amplification effect is most severe for loading conditions involving fatigue. Unlike the geometric factor, the fatigue factor is affected by a range of variables relating to the notch sensitivity of the material, as well as the geometry of the loaded element.

The stress concentration effect in members subjected to repeated loading may be represented by a fatigue strength reduction factor, (K_f). This factor is defined as the ratio of the endurance limit (or fatigue strength) for an unnotched structural member to the endurance limit (or fatigue strength) of the same member without the notch. (3) (13) In order to evaluate this factor, the designer must understand the concept of endurance limit and fatigue strength. Classically, endurance limit is the highest stress a material can be subjected to and still sustain repetitive loading indefinitely without failure. Since some common materials (notably the aluminum alloys) do not exhibit an indefinite life, the endurance limit is further defined at some finite number of stress cycles, usually many million. The fatigue strength, then, is the maximum stress that the member may endure for a given number of stress cycles without failure. The fatigue factor (K_f) tends to reduce both the endurance limit and the fatigue strength.

The fatigue factor (K_f) is affected adversely by a broad range of variables, some of which are not particularly important to the geometric factor, (K_t). Cummings (20) has reported a list of variables having an influence on (K_f), which may be summarized as follows: form factors, including elements of notch form and type of discontinuity; stress factors, including the type of loading, the amount of residual stress at the notch, and the degree of stress reversal; metallurgical considerations, such as material notch sensitivity, hardness, grain size, and grain direction; and certain environmental influences, such as temperature and corrosion. As is the case with the geometric factor, scale has a pronounced affect on (K_f). A factor which is not apparent, however, is the complete lack of correlation between (K_f), tensile strength, and the different types of stressing.

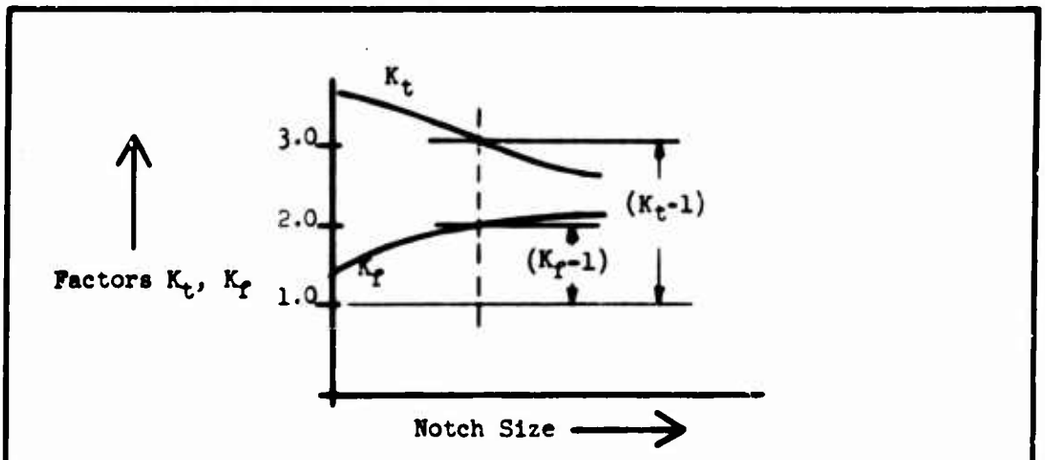
A parameter which relates (K_t) and (K_f) to the material characteristics most influential to fatigue strength, is the notch sensitivity factor, q. Notch sensitivity may be defined in these terms as:

$$q = \frac{K_f - 1}{K_t - 1} \quad (13)$$

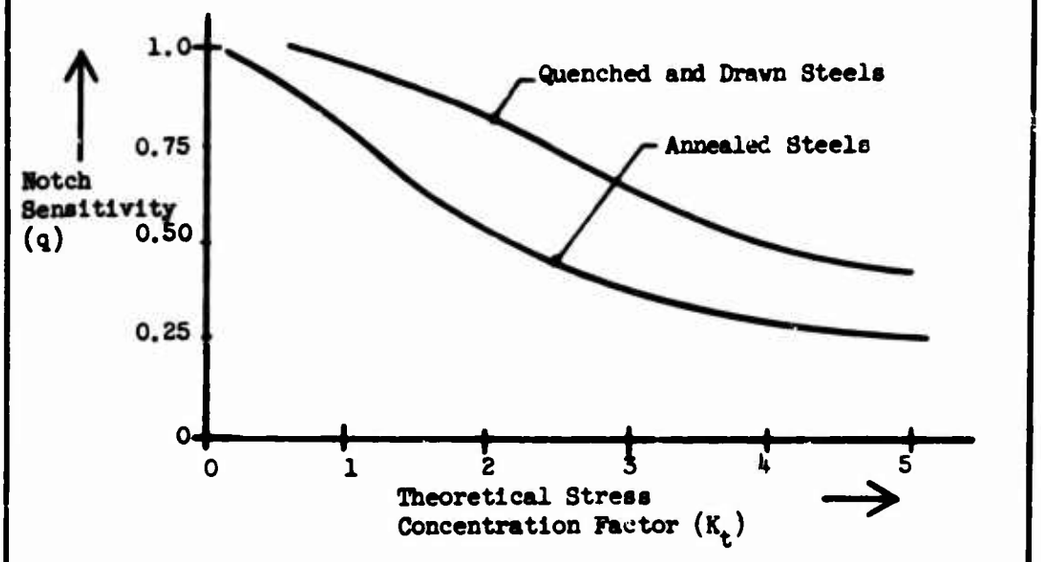
it may be seen that q = 0 when (K_f) = 1.0 (or no apparent reduction of endurance limit due to a notch) indicating no notch sensitivity. A plot of (K_t), (K_f), and q for a range of notch sizes is presented in the adjacent figure. The second figure presents some useful data on the variation of notch sensitivity and theoretical stress concentration factor for the case of a typical structural steel in the hardened and drawn versus the annealed condition. The curves represent average values for a variety of stress raisers, including vee notches, holes, fillets, and grooves.

The concept of notch sensitivity is also important in shock calculations since materials vary in their response to impact in the vicinity of a notch, much the same as they do under repeated loading. These impact sensitivity factors are evaluated empirically by a range of tests including tension impact, bending impact, and torsional impact. (8) Endurance limits and notch sensitivity factors for fatigue are commonly determined by bending,

tension-compression, and torsional tests. Of importance to the Designer is the knowledge that much of the data is obtained from highly polished specimens under laboratory conditions, and thus represents an upper value. The deleterious effect of the environmental conditions and size variations of the real world of structural packages should be evaluated when using these numbers in a stress calculation.



REPRESENTATION OF NOTCH SENSITIVITY FACTOR (q)



NOTCH SENSITIVITY OF TYPICAL STRESS

FATIGUE STRENGTH REDUCTION: The effect of stress raisers on structure subjected to repeated loading may be evaluated in terms of (K_t), (K_f), and (q)

CHAPTER 4

STRESS CONCENTRATION

SECTION 3 – STRESS AMPLIFICATION IN STRUCTURAL MEMBERS

- Some Factors Affecting the Severity of the Stress Concentration
- Strength Calculations for Members Subjected to Static and Impact Loads
- Strength Calculations for Members Subjected to Repeated Loads

SOME FACTORS AFFECTING THE SEVERITY OF THE STRESS CONCENTRATION

The state of stress in a dynamically loaded element is adversely affected by design necessities such as fasteners and changes in section, by accidental stress raisers such as scratches and tool marks, and by inherent factors such as material characteristics and heat treatment effects.

Stress concentration has the effect of significantly amplifying the actual surface stress in the vicinity of geometric discontinuities. If the equipment package could be a smooth, continuous, uninterrupted flow of structure made of an optimum material, with components bonded in place by large areas without fasteners, then stress concentration would be an academic consideration. Obviously, this is not the case in the electronic equipment package. By outlining some of the more important factors influencing notch severity, it is hoped that the designer will be better equipped to cope with the reality of the stress concentration effect.

The independent variable most often encountered in the selection of a concentration factor is form, or geometry, of the discontinuity. Theoretical stress concentration factors are available in abundance from the literature on virtually every type of notch, hole, groups of holes, fillet radii, changes in section, and more. The selected bibliography in the appendix to this chapter provides a good starting point in the designer's evaluation of the structural model in question. The range of this form effect may vary from only slightly over 1.0 for a generous radius in a shaft shoulder subjected to a static load or an impact load that can be expressed in terms of equivalent static damage potential, to a K_t factor of 10 or more for a severe notch subjected to a repeated, fully reversed loading. The works of Peterson,[13] Gomza,[15] Savin,[19] Griffel,[12] and Roark[1] provide ample documentation for stress amplification factors. It remains for the designer to determine the type of loading occurring at the critical section (tensile, flexural, shear, torsional), to estimate the geometry of the discontinuity, and to select a factor from the literature which best fits the situation.

The notch effect discussed above is mostly design related and falls under the category of designing to accommodate a geometric condition which may be dictated by functional or mechanical necessities. Stress concentration effects resulting from manufacturing techniques, heat treatment, and environment may not be quite so apparent to the designer, but nonetheless just as important. Tool marks, rough machined surfaces, sharp edges and corners, undressed welds, and pitting or blow-out of material adjacent to a weld may reduce the element's endurance limit from 10% to 75%. This effect is accidental or the result of poor shop procedure. It does, however, exist in most structure and must either be accounted for analytically or eliminated by rigorous inspection.

Fastening techniques which apply local pressure to the parent structure are subject to galling and fretting, stress amplification mechanisms that may reduce the apparent strength of a member by 75%. Similarly, residual tensile stresses induced locally by mechanical assembly, grinding, or cold forming, have an adverse effect on fatigue strength. Reduction of dynamic integrity due to improper heat treatment or material selection is a common pitfall. Surface hardening for better wear resistance may cause under-surface defects that ultimately precipitate fatigue failures. Surface deterioration due to heat treatment is not uncommon. Metallurgical anomalies such as quenching cracks, slip planes, hard inclusions, and surface decarburization may form

the nucleous for a fatigue failure. Ductility is known to be a poor measure of fatigue resistance, but is beneficial in mitigating a stress raiser by plastic deformation in the region of high stress in members subjected to static loading. Very hard materials are known to be more notch sensitive, and hence more susceptible to fatigue damage. Attention to the factors that contribute to the stress concentration effect will aid the designer in eliminating or accommodating them.

SUMMARY OF FACTORS AFFECTING STRESS CONCENTRATION
(See Appendix For Details)

- Small Fillets
- Tool Marks
- Surface Machining Marks
- Fretting and Galling
- Corrosion
- Plating
- Heat Treatment
- Size
- Speed
- Shape
- Inclusions

REDUCED STRUCTURAL INTEGRITY: Stress concentration leads to reduced structural adequacy, and is affected by a variety of physical causes.

STRENGTH CALCULATIONS FOR MEMBERS SUBJECTED TO STATIC AND IMPACT LOADS

Straightforward analytical techniques are available to the designer to evaluate the effects of stress raisers in structural members subjected to impact and static loading.

The state of stress in a structural member is evaluated on the basis of a load (or moment or torque) divided by an area (or section modulus or polar moment) or some combination of these loading situations as dictated by the free body diagram of the element. The load portion of these basic equations are calculated on the basis of an input disturbance factored by a structural response. The details of this input and response are the subject of Volume II, "Analytical Procedures." Once the load has been estimated and section characteristics evaluated, then the stress calculation may be made, corrected by the stress concentration factor (K_t). The expressions for static or impact stress resulting from three elemental loading conditions are:

$$S = \frac{\text{load}}{\text{area}} (K_t) \qquad \text{for tension, compression, or direct shear}$$

$$S = \frac{\text{bending moment}}{\text{section modulus}} (K_t) \qquad \text{for bending}$$

and

$$S = \frac{\text{torque}}{\text{polar section modulus}} (K_t) \qquad \text{for torsion.}$$

The subsequent comparison of the stress resulting from the loading conditions divided into the "allowable stress" yields a safety factor, or safety margin.

$$\text{Factor of Safety} = \frac{\text{allowable stress}}{\text{actual stress}} \text{, and}$$

$$\text{Margin of Safety} = \frac{(\text{Safety factor}) - 1}{100} \text{ in percent.}$$

Stress calculations involving static loads or impact loads which have been reduced to equivalent static force may be made directly using a value for K_t which best represents the geometry of the discontinuity. The K_t factor is a handbook value relating to form and loading method, as outlined previously. The resulting stress calculation is conservative for shock since increasing strain rate is beneficial to material strength and structural response.

Perhaps the use of the geometric stress concentration factor may best be illustrated by example. Suppose we were faced with the design of a stepped shaft loaded in tension as shown in the figure. Also shown is a typical plot of the K_t factor in terms of the geometric parameters of the shaft.[16] If we assume a mild steel material with an ultimate tensile strength of 120 ksi (kips per square inch), and yield strength of 80 ksi, and further assume an axial load of 10 kips (10,000 lbs), we are in a position to calculate a safety margin for this shaft. For the geometry shown in the sketch, we may calculate the r/d and h/r ratios to be 0.25 and 4.0 respectively. Projecting the h/r = 4.0 curve, we see that K_t will be about 1.55.

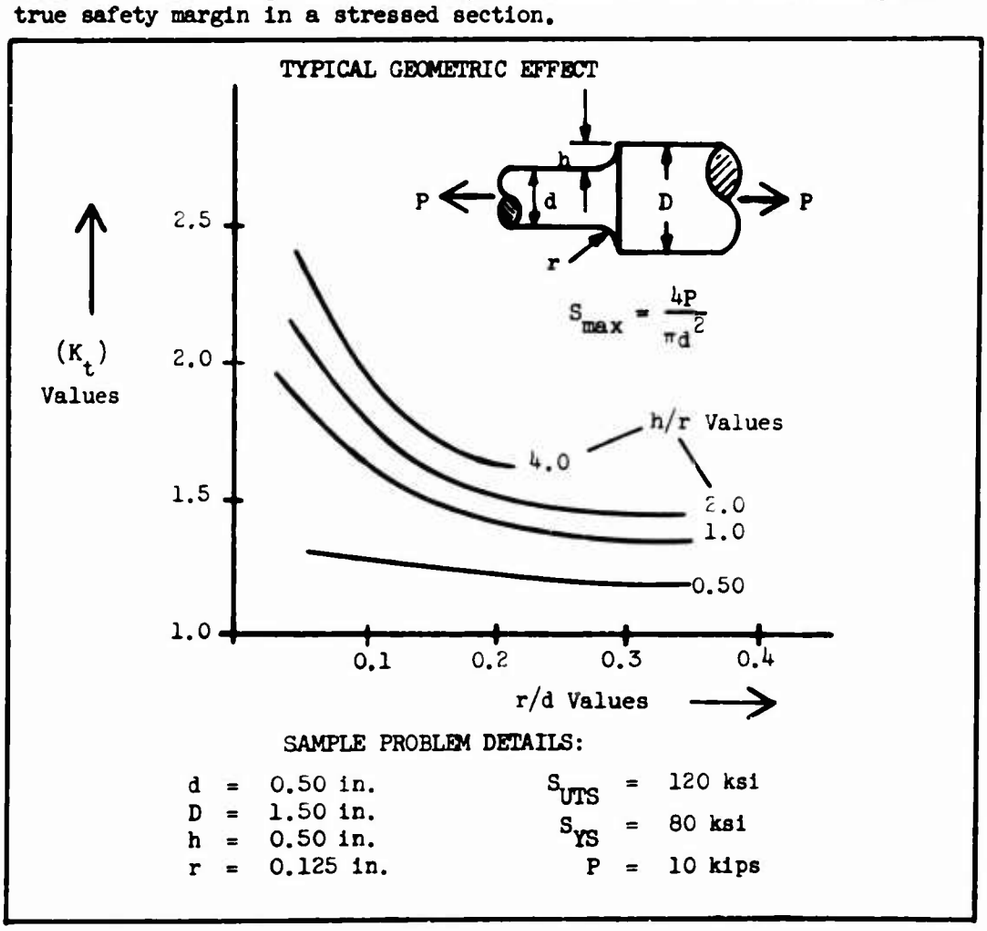
The stress calculation for the axial load will be;

$$S_t = \frac{P}{A} (K_t) \quad \text{or 79 ksi.}$$

The safety factor may then be calculated;

$$F.S. = \frac{120 \text{ ksi}}{79 \text{ ksi}} = 1.52 \text{ based on the ultimate strength.}$$

The safety margin (on ultimate) will be 52%. The margin based on the material yield strength will be virtually zero. If the stress concentration factor had been ignored, we would have calculated a safety factor of 1.57 on yield and 2.36 on ultimate, a margin which we might find acceptable. The true stress at the shaft shoulder which we calculate with the K_t factor, shows us that yielding will probably occur at the radius, a situation which may not be acceptable. The designer must realize the importance of considering the stress amplification effect in evaluating the true safety margin in a stressed section.



SOLID CIRCULAR SHAFT IN TENSION: The stress in a shaft with a shoulder may be magnified by 20% to 150% for common geometric situations.

STRENGTH CALCULATIONS FOR MEMBERS SUBJECTED TO REPEATED LOADS

The strength calculation for fatigue situations involves the fatigue reduction factor K_f and the material characteristic q. Further, the par value for endurance limit is deteriorated by load category, member size, and surface finish, effects which may be evaluated by efficiency factors.

Stress calculations for repeated loading conditions must involve the fatigue reduction factor K_f. Unlike K_t, K_f is a strength reduction factor used to modify the material fatigue strength, or endurance limit. Most endurance limit data is measured under laboratory conditions on small, highly polished specimens, loaded in reverse bending. The value thus collected is generally higher than a value for the same material under actual service conditions. The factor K_f must therefore, be modified by a set of efficiency factors which tend to further reduce the usable endurance limit of the material.

The first of these efficiency factors compensates for the loading method. Endurance limit data is classically taken in bending. Axial and torsional loadings are known to reduce the limit by 15% and 42% respectively.(16) A reasonable efficiency factor relating loading condition then, may be taken as; bending, 100%; axial, 85%; and torsional, 58%.

A second deleterious effect is member size. Stress concentration has been shown to increase with size, particularly in bending and torsion; axial loading, however, has a negligible effect. A reasonable estimate of the size efficiency factor for bending and torsion is 85%. (16)

The effect of surface finish on endurance limit is known to be severe, and further, is known to vary with strength for typical structural materials. Illustrated is the reduction of endurance limit for a sampling of manufacturing methods, for a range of material tensile strengths. The surface efficiency factor may then be picked from the plot to match the particular conditions.

The best estimate of K_f is obtained in the lab by duplicating the size, material, finish, and geometry of the actual element, and conducting a fatigue test with the same loading conditions. Without this alternative, the designer may estimate by calculating K_f in terms of K_t and q, and reduce the endurance limit by the other efficiency factors. A value for the material notch sensitivity (q) may be taken from the figure, which normalizes material hardness and notch sensitivity. K_t, the geometric stress concentration factor, may be taken from the handbooks as previously discussed. K_f may then be calculated from the expression:

$$K_f' = 1.0 + (K_t - 1)\, q, \quad (3)\ (16)$$

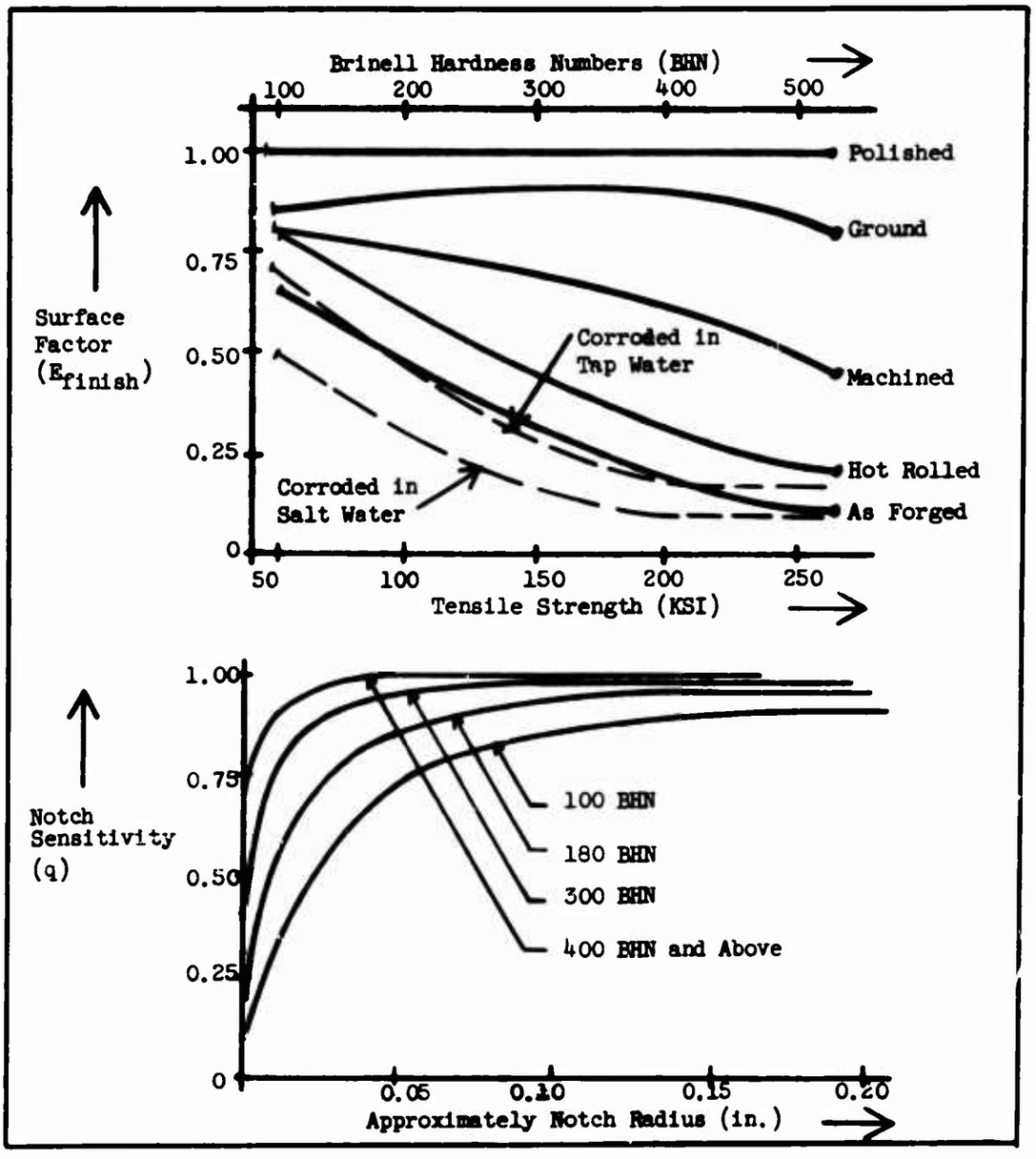
which yields an estimate of the strength reduction factor relative to a laboratory specimen. The K_f' factor must then be corrected for the difference between the laboratory specimen and the service member in terms of loading category, size, and finish, as follows:

$$\text{Corrected } K_f = K_f' \,(E_{load})\,(E_{size})\,(E_{finish}).$$

The factor of safety may then be evaluated:

$$F.S. = \frac{\text{endurance limit}}{\text{calculated stress}} \quad (K_f).$$

It is important for the designer to note that the fundamental difference between calculations involving K_t and K_f is that K_t operates on the stress calculated to be present in the loaded member, while K_f modifies the material characteristic of fatigue strength.



NOTCH SENSITIVITY: The effect of material sensitivity to notches and discontinuities is factored into the stress calculation.(16)

CHAPTER 4

STRESS CONCENTRATION

SECTION 4 – MINIMIZING THE STRESS AMPLIFICATION EFFECT

- Using Force Flow Lines to Visualize Stress Concentration
- Optimizing Material Characteristics, Heat Treatment, and Surface Protection
- Some Suggestions for Improving Stress Concentration Effects in Existing Structure
- Reducing Stress Concentration in Welded Joints

USING FORCE FLOW LINES TO VISUALIZE STRESS CONCENTRATION

Stress amplification from notches, holes, access panels, and other design necessities may be effectively visualized by force flow lines, analogous to fluid streamlines. Once the stress concentration influence is qualitatively estimated, design steps may be taken to minimize their deleterious effects.

It is apparent that all stress concentration effects may not be eliminated in any practical equipment package. The real design task is one of learning to evaluate their effect in a concise manner and take steps to live with the amplified stress or reduced material capability. A thorough knowledge of the factors influencing the stress concentration effect will of course be a valuable aid in accommodating the effect. Obviously, the more rigorously the designer is able to calculate the reduced margin, the closer he may work to the ultimate strength or endurance limit.

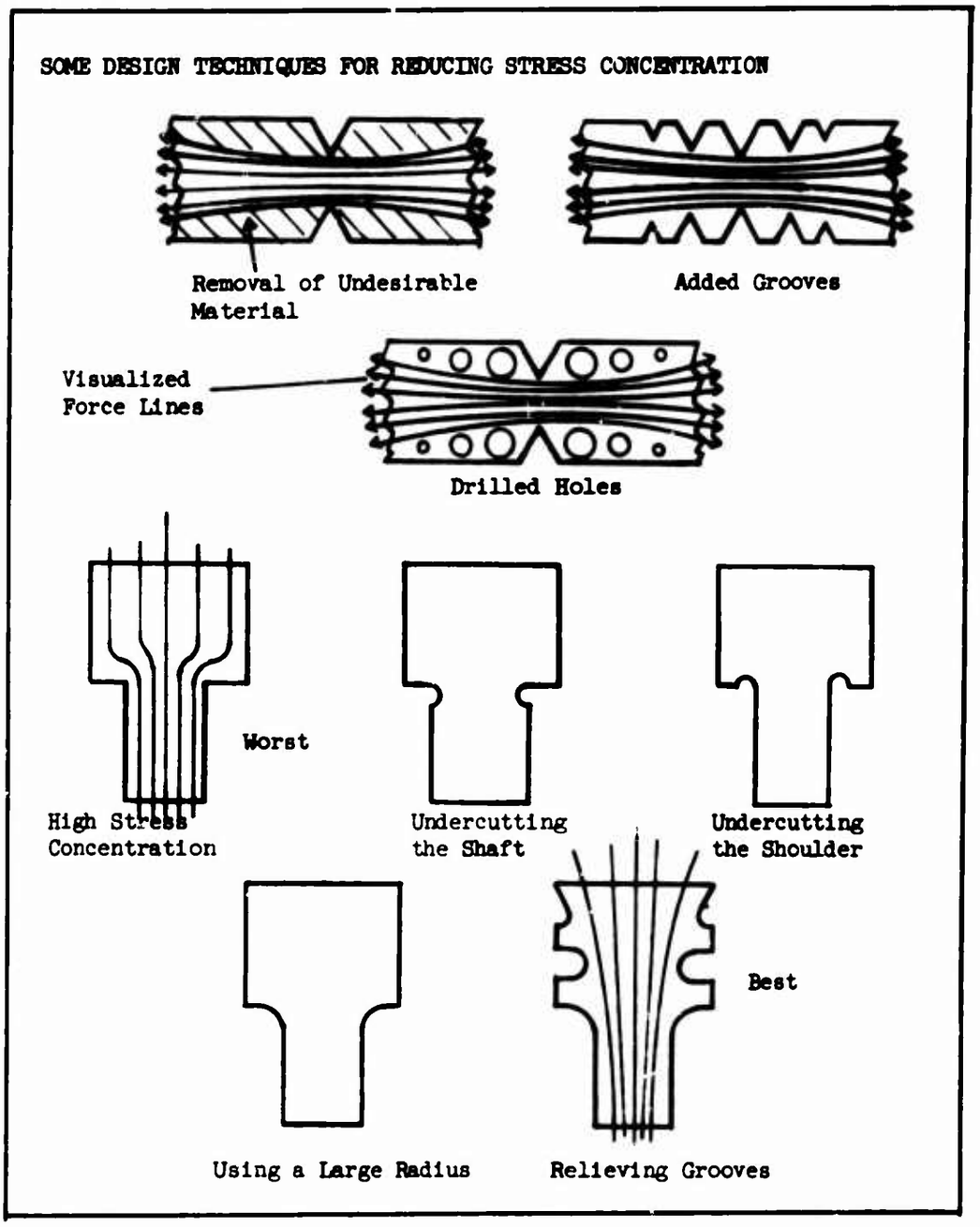
Structural intuition is the most important tool available to the designer at the outset of a packaging problem. This intuitive judgement may be strengthened by visualizing the lines of force flow in the region of a stress raiser. These flow lines are roughly analogous to fluid streamlines of flow existing in a vessel having the same geometry as the structure in the vicinity of the discontinuity. The fluid flow lines tend to remain straight unless impeded by an obstruction, where they are throttled together as they pass the obstruction. Similarly, lines of force are crowded together at the immediate vicinity of a notch. The elongation necessary for the force line to remain within the section causes the distortion of the line to extend into the stressed section. Thus the stress amplification is produced, extending into a member for a finite distance from the discontinuity, and originating at the outer fiber of the structure. Since flexural and torsional stress are maximum at the outer fiber of the section, and most scratches and accidental stress raisers are predominantly surface effects, the combination of the two influences causes a marked decrease in strength.

The designer must choose the optimum location for a necessary discontinuity with care, basing his decision on force lines through the area. Fasteners, for example, are necessary evils which can cause much less stress amplification if they are located in areas of minimal stress. Placing a holddown bolt at the apex of a reentrant corner is a common design error; locating the same bolt a few diameters away from the corner can achieve the same functional effect while avoiding the compound aspects of two stress raisers. The important point here is this: don't double up on stress discontinuities, since their effect will be additive. Visualize the lines of force flow through the area when choosing the location of holes and openings; allow at least four diameters between discontinuities.

The concept of force flow lines may also be useful to the designer when a pattern of holes or notches is required. Similarly, any needed change in geometry should be faired, and accomplished as smoothly as possible. These ideas are illustrated in the accompanying figures. The unneeded material (shown at the right) may be isolated by adding adjacent grooves or holes, thus mitigating the notch by reducing the abruptness of the change in direction of the force lines. Similarly, the familiar problem of a shoulder in a shaft may be lessened by the series of material removal operations illustrated in the figure.

Force flow lines may be further investigated by photoelastic modeling techniques. This method utilizes the interference characteristics of

polarized light rays traveling through transparent models under stress. Regions of stress concentration are indicated by close spacing of the interference fringes.



FORCE FLOW LINES: Visualization of the force line patterns by analogy to fluid flow in a stressed section is a powerful design tool.(16)

OPTIMIZING MATERIAL CHARACTERISTICS, HEAT TREATMENT, AND SURFACE PROTECTION

Fatigue strength may be enhanced and notch sensitivity reduced by careful application of heat treatment and surface hardening techniques.

The elimination or mitigation of the geometric stress concentration effect is largely one of operating on the stress raiser itself, or at least evaluating its effect. Conversely, the fatigue strength reduction factor K_f relates to those material characteristics that influence the notch sensitivity of the structural alloy. For this reason, the physical metallurgy of the material, its heat treatment, and particularly its surface treatment, have a pronounced impact on the strength reduction factor for repeated loading conditions.

The heat treatments most commonly specified for structural alloys are those that affect the entire section, such as precipitation hardening, quench hardening, annealing, or tempering; or those that treat the surface of the material such as nitriding, carburizing, or anodizing. The tensile strength of most structural alloys is improved by heat treatment; fatigue strength, on the other hand, does not improve at the same rate. Indeed, some alloys show a decrease in fatigue strength above a critical hardness value. It is apparent that only uncertain correlation exists between tensile strength and fatigue strength, and thus the factors that influence and improve material hardness and tensile properties are of minor importance to the fatigue problem. A summary of the available data indicates that the endurance limit for most structural alloys ranges from 25% to 50% of the ultimate tensile strength.(3)

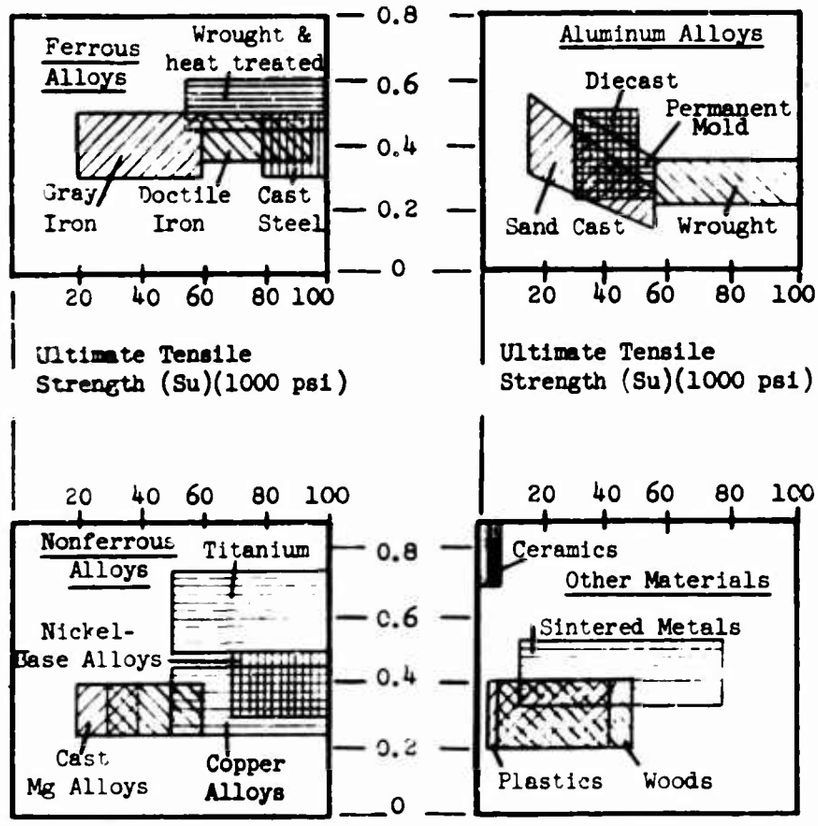
Grain size is indeterminate as it affects stress concentration. It is generally accepted in the literature that fine grained materials have better fatigue resistance, and subsequently less notch sensitivity, than do coarse grained materials at comparable strength levels. The design criteria for grain size would involve the specification of a fine grained material if possible, and would protect the grain characteristics that are inherent in the alloy by avoiding those heat treatment operations that are conducive to grain growth. (Some heat treatments, such as prolonged soaking at elevated temperatures, cause grain coarsening.)

Microscopic inclusions in structural alloys generally have a detrimental effect on the material fatigue strength. Nonmetallic inclusions, slag, and other nonmalleable microscopic particles are in themselves stress raisers as they interfere with the stress flow lines of the section. Precipitation hardened alloys, such as some of the aluminum alloys, exhibit hard microscopic particles that are rejected from the atomic matrix. During cooling, these particles contract at a different rate than the parent material, causing a residual stress situation, tending to counteract the stress raiser effect.

Surface hardening techniques are often helpful in raising fatigue strength, but are not always predictable, and are sometimes harmful. Occasionally, the discontinuity or nucleus for a fatigue crack will exist in the area between the case and core material in a surface-hardened member. Conversely, notched specimens have been shown to exhibit improved fatigue strength after nitriding, provided the nitriding operation was accomplished after the structure was notched. Deep carburizing and subsequent hardening and

tempering can provide even greater fatigue strength improvement than nitriding. The danger lies in the potential overloading of the core material rather than the hardened case. Local yielding could then occur under the case, causing subsequent fatigue failure from spalling. In general, a through-hardening alloy is more consistent and more predictable a material for basic structure subjected to dynamic loads.

RANGES OF FATIGUE STRENGTH FOR VARIOUS MATERIAL



ENDURANCE LIMITS: The fatigue resistance of many common structural alloys may be estimated (with care) on the basis of heat treatment or ultimate tensile strength. (11)

SOME SUGGESTIONS FOR IMPROVING STRESS CONCENTRATION EFFECTS IN EXISTING STRUCTURE

Reducing stress concentration in existing structure without an extensive rework is a formidable design task. There are a few effective approaches available to the engineer which may solve some marginal situations.

Frequently, the packaging engineer is confronted with the problem of designing a fix for a member which has failed under dynamic load where the failure has progressed from a stress raiser. Stress concentrations have a way of compounding a critical stress situation either by accident or analytical oversight. Designing around this handicap without ordering a major rework or redesign is a formidable task which will tax the designer's ability.

Many fatigue failures result from stress raisers that are more severe than estimated, or are being magnified by other discontinuities in the area such as tool marks. A first fix in this case would involve the smoothing, fairing, and polishing of the stressed area to minimize the stress amplification as much as practical. If the geometry of the member is such that the notch must remain, then additional approaches may be indicated. The introduction of a residual compressive stress in the region of the concentration will generally raise the effective fatigue limit. This may be accomplished by shot peening, work hardening, local hardening, or a complete surface hardening process such as nitriding. Thus it is usually helpful to mechanically work fillets and grooves, roll machine cut threads, ream drilled holes, and generally work-harden the member. This is particularly effective in materials that have a high capacity for strain hardening.

The removal of material adjacent to a stress raiser may fair out the force lines to the extent that amplification is reduced to an acceptable level. Sharp corners and shoulders often may be relieved by removing some material from the shoulder or undercutting the face to smooth out the abrupt change.

The material properties may frequently be improved by judicious use of heat treatment. Surface treatments may be effective, such as nitriding or flame hardening in the vicinity of the notch.

The presence of corrosion may be the unexpected by-product of the working environment. The deterioration of the surface of a member, particularly one loaded in flexure, will often serve to magnify already existing stress raisers. The solution here is one of insulating the sensitive surface from the corrosive effects of the environment by a hard finish such as anodizing, or an inert coating material.

SOME SUGGESTIONS FOR IMPROVING STRESS CONCENTRATION EFFECTS IN EXISTING STRUCTURE.

1. REDUCE COMPOUND STRESS RAISER EFFECTS

- Add gussets and fairing material where practical
- Smooth surfaces in region of critical stress
- Remove tool marks, scratches, and other surface blemishes
- Remove surface decarburized material
- Look for quenching cracks

2. MITIGATE STRESS RAISER BY MATERIAL REMOVAL

- Add notches of diminishing size adjacent to principal stress raiser
- Remove material to reduce abrupt section changes
- Undercut shoulders with generous radii
- Dress weld beads at critical locations

3. HEAT TREAT TO IMPROVE MATERIAL CHARACTERISTICS

- Surface treatment, particularly for members in bending and torsion
  - Light case, such as anodize or nitride
  - Deep case, such as carburize or aerocase
- Through section treatment to improve strength, ductility, sensitivity
  - Anneal or normalize to relieve residual tensile stresses
  - Harden and temper to upgrade strength and fatigue limit
  - Selected heat treat to reduce notch sensitivity

4. STRAIN HARDENING BY MECHANICAL WORK

- Shot peening, rolling, hammering
- Work harden critical fillets, grooves, radii
- Roll critical machine-cut threads
- Ream or broach critical drilled holes
- Burnish rough surfaces
- Develop residual compressive stress wherever practical

5. PROVIDE ENVIRONMENTAL PROTECTION OR LIMITATION

- Corrosion protection by coating, plating, painting, cladding
  - Reduce surface and intergranular effects
  - Insulate from water attack, particularly salt
- Temperature insulation
  - Avoid creep-fatigue interaction
  - Above 300°F temperatures become significant

6. IMPROVEMENT OF CRITICAL ELEMENTS ONLY

- Select optimum material and heat treatment
- Design in more fillet and radii material
- Improve fastener pattern
- Reduce load concentration...more bearing area
- Eliminate rubbing, scoring, fretting
- Improve lubrication
- Redistribute high stresses away from discontinuities
- Use direct load paths....avoid bending and torsion

REDUCING STRESS CONCENTRATION IN WELDED JOINTS

Correct welding techniques are essential to minimize stress concentrations in a welded connection. Some practical suggestions are offered for welded steel members subjected to repeated loading.

Geometrical Effects and Design Considerations*

DO

- Change sections gradually and avoid re-entrant corners. Where welding joins different thickness of plates, or sizes of plates or sections, provide a gradual transition.
- Grind butt welds flush and smooth.
- Use butt joints instead of lap joints.
- Extend cover plates on beams well beyond theoretical cutoff points.
- Align parts to avoid eccentricity.
- Join parts of unequal stiffness with a gradual transition.
- Streamline fillet-welded joints.
- Give preference to structures with multiple load paths, in which a fatigue crack in any one of several key members is not likely to precipitate sudden collapse of the whole structure.
- Plan welding sequence to avoid shrinkage cracks and other weld defects as well as excessive deformation.
- Locate welded joints where fatigue conditions are not severe.

DON'T

- Use plug or slot welds; instead, use a hole or slot of adequate size with a fillet weld all around.
- Use joints having large local variations in ability to deform under the applied load. For example, don't butt-weld a beam to an unstiffened column flange; the edges of the column flange can deform easily, causing the part of the butt weld opposite the flange to be overstressed.
- Introduce high restraint in localized regions.
- Attach fittings, handles, and bosses, or make openings at locations of high stress.
- Specify excessive welding.

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* Prepared by the Subcommittee on the Fatigue of Welded Joints, Committee of the Welding Research Council.(10)

Metallurgical and Fabrication Considerations*

DO

- Use welding procedures and methods that will eliminate internal defects, such as microcracks, gas pockets, and slag inclusions, as well as excessive surface ripples or roughness. In some cases, it may be advisable to use low-hydrogen electrodes or submerged arc welding, or to pre-heat or administer a suitable post-heating treatment. Some steels should be welded only with low-hydrogen electrodes on submerged-arc.
- In welded joints, avoid undercutting, cracks, spatter, and other surface imperfections which might serve as stress raisers.
- Machine or otherwise dress the weld at critical locations, if necessary, to obtain satisfactory smoothness.
- Provide proper maintenance, including adequate protection against corrosion, wear, abuse, overheating, improper lubrication, and repeated overloading.
- Use multipass cascade welding in making fillet welds on thick material.

DON'T

- Leave incomplete root penetration.
- Use intermittent welding.
- Leave end defects in fillet welds.
- Peen the first layer of a weld. Welds on steels with no pronounced yield point, such as ASTM A 514, may be peened after the first layer; others should not.

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* Prepared by the Subcommittee on the Fatigue of Welded Joints, Committee of the Welding Research Council.(10)

CHAPTER 4

STRESS CONCENTRATION

SECTION 5 – APPENDIX

- Bibliography
- Glossary
- Geometric Effects – Some Typical Stress Concentration Factors
- Examples of Designs to Minimize the Stress Raiser Effect
- Examples of Design Oversight Which Cause High Stress Concentration and Service Failure
- Conditions Which Reduce Dynamic Structural Integrity

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GLOSSARY

Stress Concentration - In an elastic material is the abrupt increase in stress intensity within a highly localized region, apart from distributed stress variation such as flexure.

Stress Raisers - The form irregularities, discontinuities, surface defects, metallurgical anomalies, et. al., which cause the highly localized stress concentration effect.

Theoretical (Geometric) Stress Concentration Factor - A multiplying factor which corrects the stress calculated by simple theory to more closely agree with the true stress in the area of the stress raiser. It is the ratio of the true maximum stress to the stress calculated by ordinary mechanics using the net section, but ignoring the artificial change in stress distribution due to the notch.

Elastic Material - It has the property of being able to sustain a stress without permanent deformation. The material further has the characteristic of conformity to the law of stress-strain proportionality, within the elastic limit.

Elastic Modulus or Young's Modulus - The rate of change of stress with respect to strain within the proportional limit. The property is thus measured as the slope of the elastic portion of the stress-strain diagram of the material.

Elastic Limit- The least stress which will cause permanent deformation in in an elastic material.

Proportional Limit - The maximum stress which a material can sustain within the law of stress-strain proportionality, i.e., remain elastic.

Yield Strength - The stress at which a material exhibits a specified permanent set. Yielding in a material is the plastic deformation of material in the region of stress which has exceeded the elastic limit.

Ultimate Strength - Tensile strength, or ultimate tensile strength, is the maximum stress that the material can sustain without fracture, calculated on the basis of load/original cross-sectional area.

Ductility - A material property reflecting the amount of deformation (usually plastic) that a specimen can sustain before fracture. This characteristic is proportional to the extension at rupture from the stress-strain diagram.

Creep - The continuous increase in deformation under the action of a sustained or decreasing stress. In common structural materials, the phenomenon is usually associated with stresses at elevated temperatures, where the effect is more pronounced.

Fatigue - The tendency of materials to fracture from progressive cracking due to many repetitions of stress.

Fatigue Strength - The maximum stress that a material may sustain without failure for a specified number of load repetitions.

Endurance Limit – The stress which a material may sustain for an indefinite number of load repetitions without failure.

Stress Concentration Factor in Fatigue – (Or fatigue strength reduction factor) The ratio of a material's fatigue strength without a notch to the same material with a notch. It is thus a measure of the effect of the notch in reducing the strength of the material under repeated load.

Notch Sensitivity – The material characteristic which relates the effect of a notch or discontinuity to the material's physical properties.

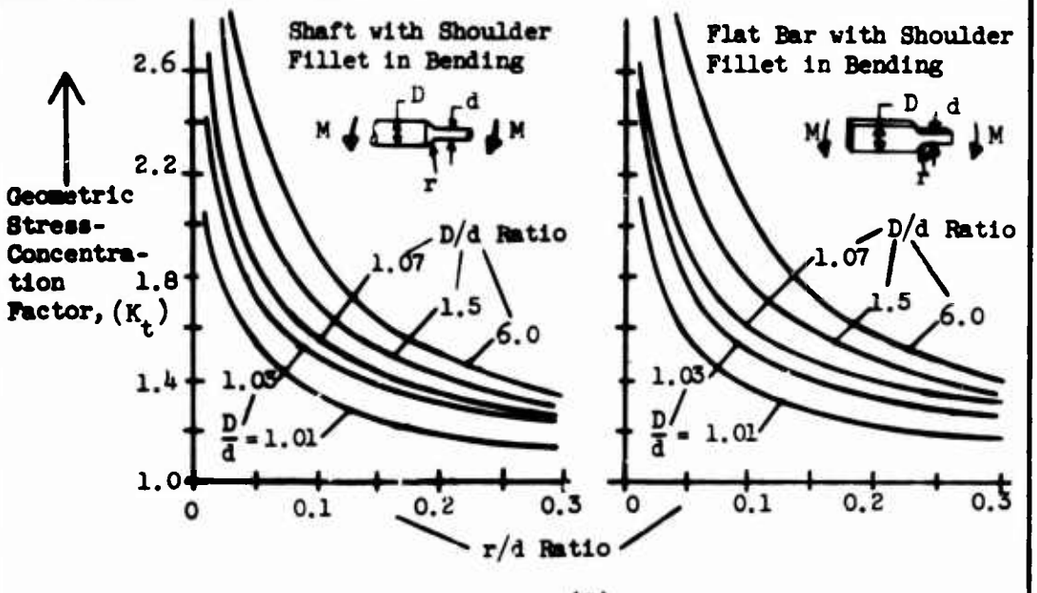
Isotropic – The material characteristic of exhibiting the same physical properties in all directions. The term is commonly applied to both strength levels and elastic characteristics.

Resilience – The ability of a material to store and release energy. The property is analogous to the area under the elastic portion of the stress-strain diagram.

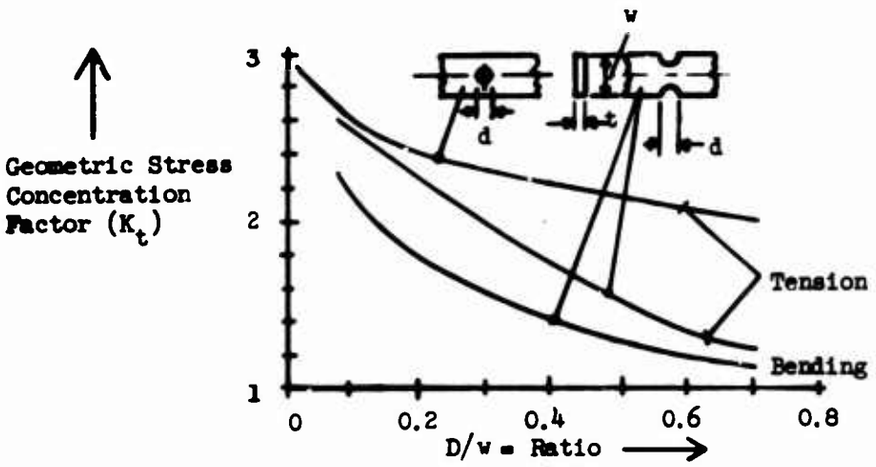
Stress – The internal force exerted by either of two adjacent parts of a body upon the other across an imaginary plane of separation. Shear, compressive, and tensile stresses, respectively, resist the tendency of the parts to mutually slide, approach, or separate under the action of applied forces.

Strain – Is manifest as any forced change in dimension of a body under the action of a force or unit stress.

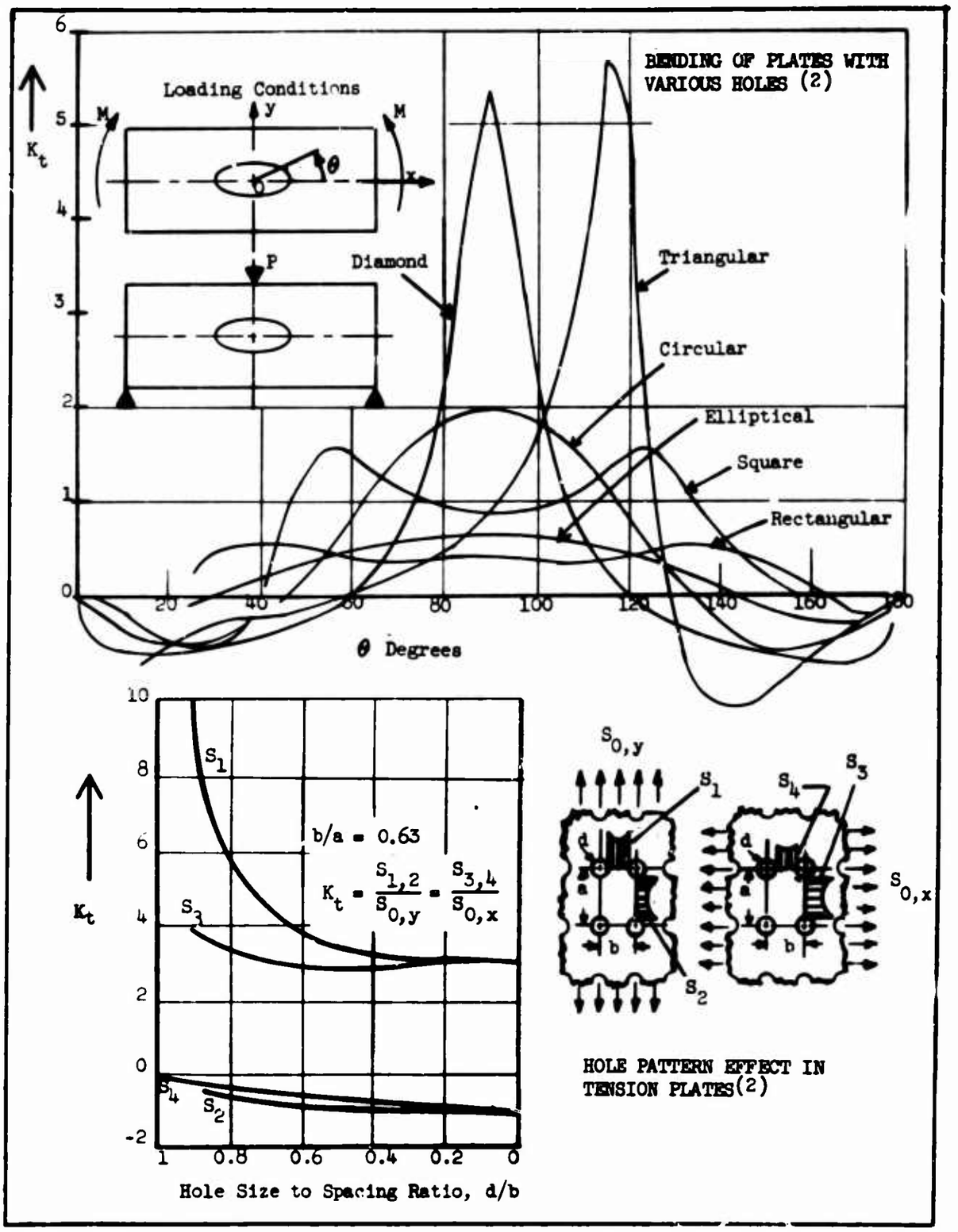
GEOMETRIC EFFECTS - SOME TYPICAL STRESS CONCENTRATION FACTORS



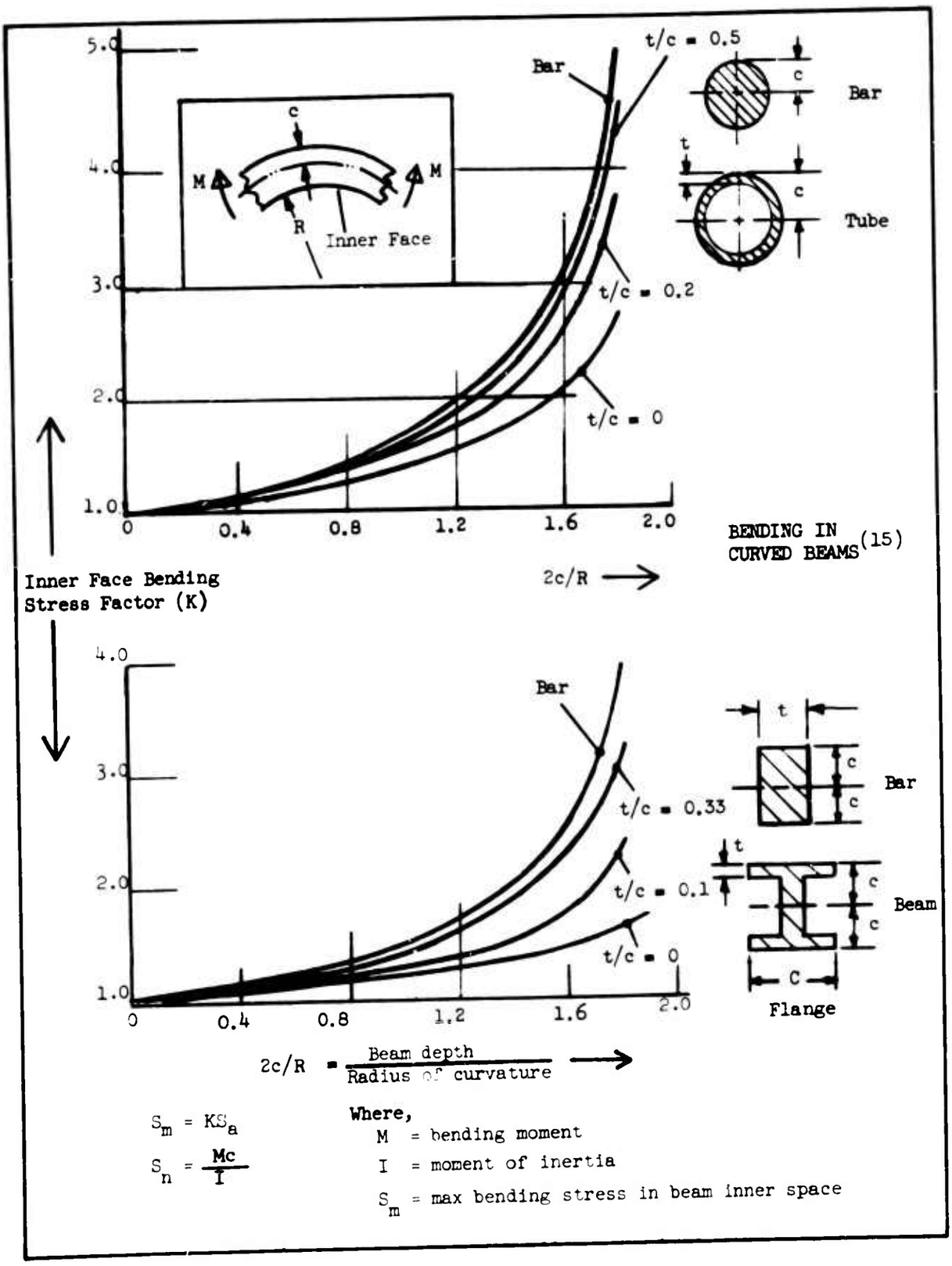
SHAFTS AND BARS IN BENDING(3)

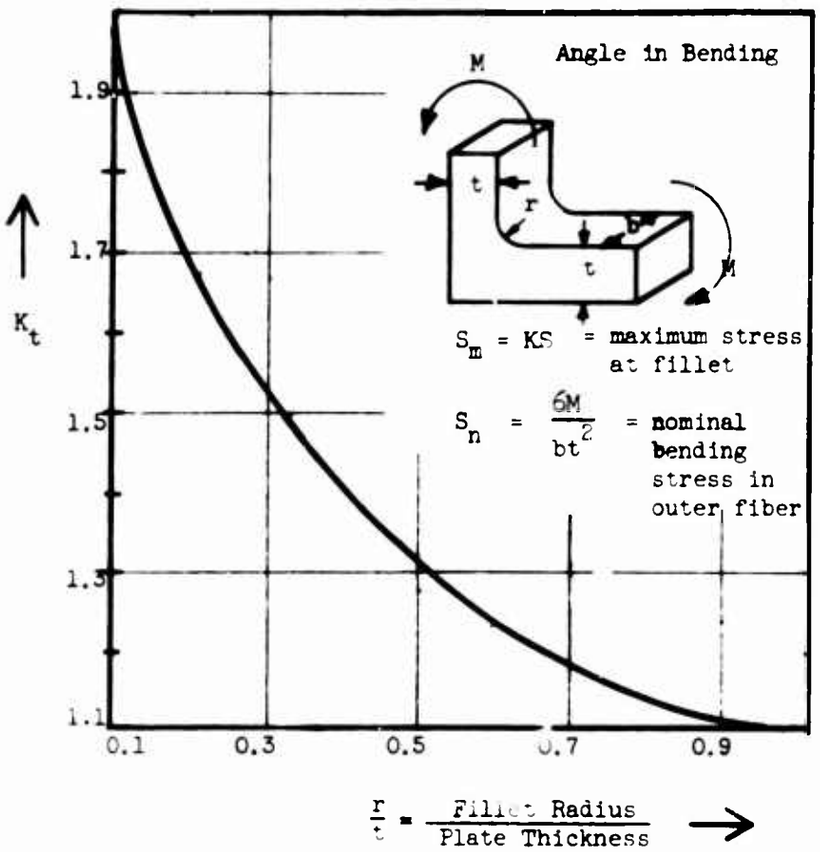


NOTCHED PLATES IN BENDING AND TENSION (18)


BENDING OF PLATES WITH VARIOUS HOLES (2)
Loading Conditions
M
M
y
x
θ
P
K_t
Diamond
Triangular
Circular
Elliptical
Square
Rectangular
θ Degrees
S_1
S_3
S_4
S_2
b/a = 0.63
$K_t = \frac{S_{1,2}}{S_{0,y}} = \frac{S_{3,4}}{S_{0,x}}$
Hole Size to Spacing Ratio, d/b
$S_{0,y}$
$S_{0,x}$
HOLE PATTERN EFFECT IN TENSION PLATES(2)

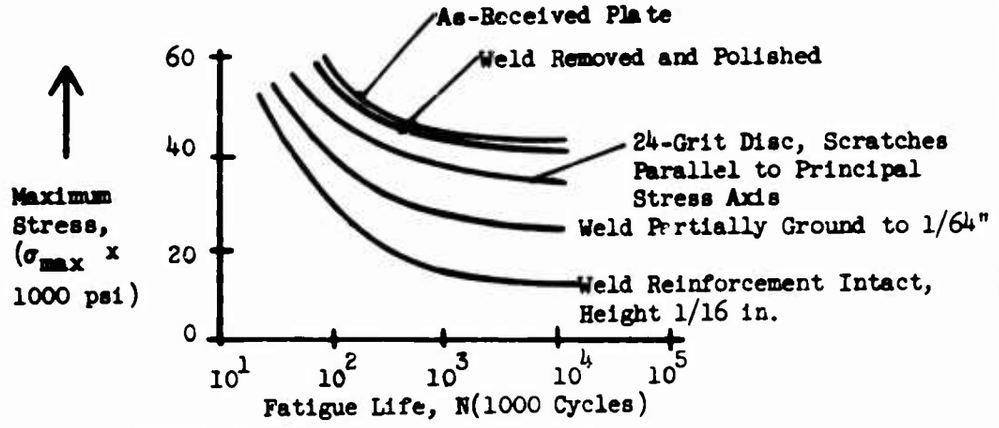
GEOMETRIC EFFECTS (Continued)



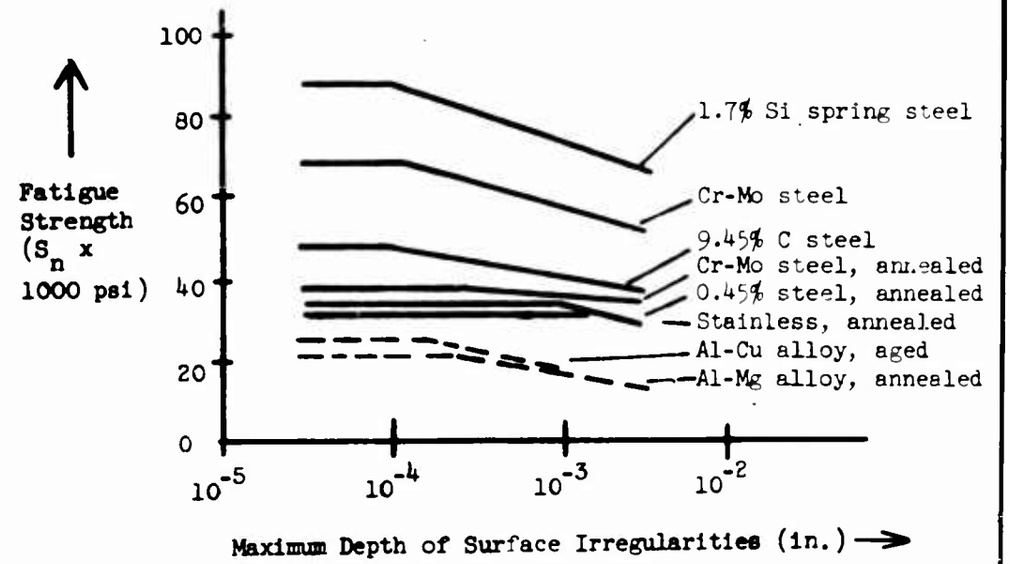


ANGLE SECTION IN BENDING[15]

GEOMETRIC EFFECTS (Continued)

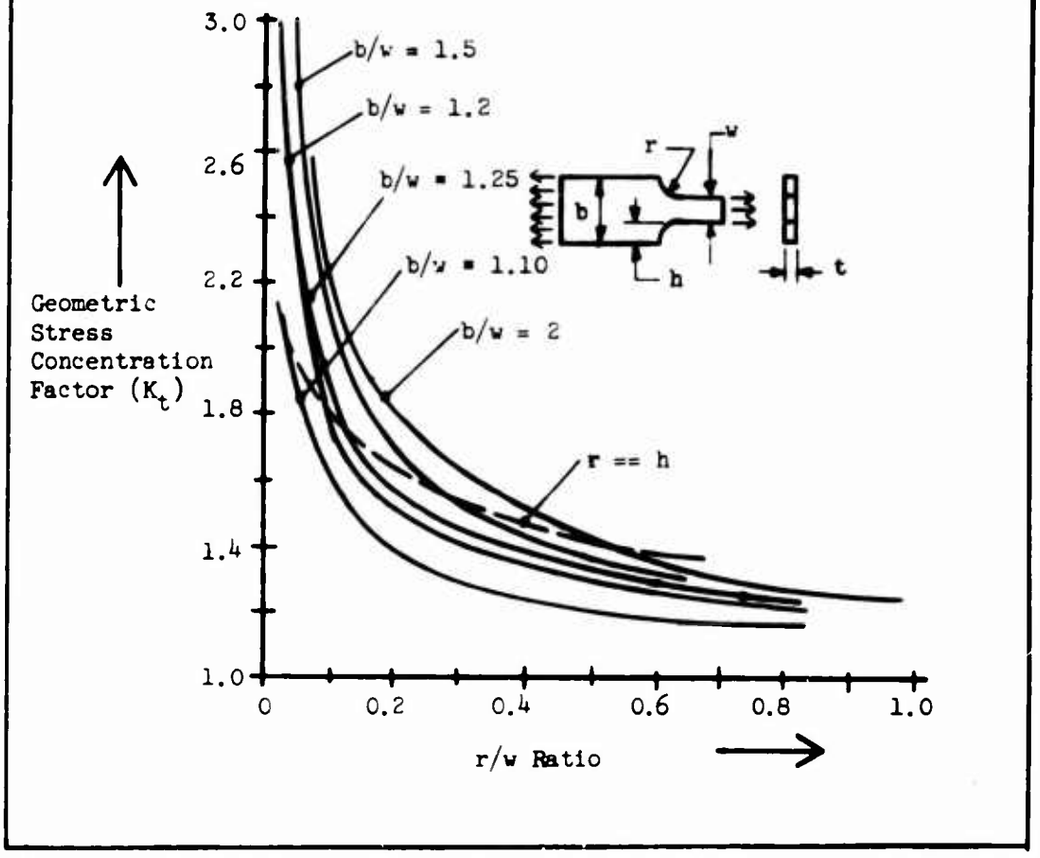


EFFECT OF REMOVAL OF WELD REINFORCEMENT ON FATIGUE LIFE OF TRANSVERSELY BUTT-WELDED 1/4-IN. USS "T-1" (ASTM A 514, GRADE F) STEEL PLATES WITH R = -1. (10)

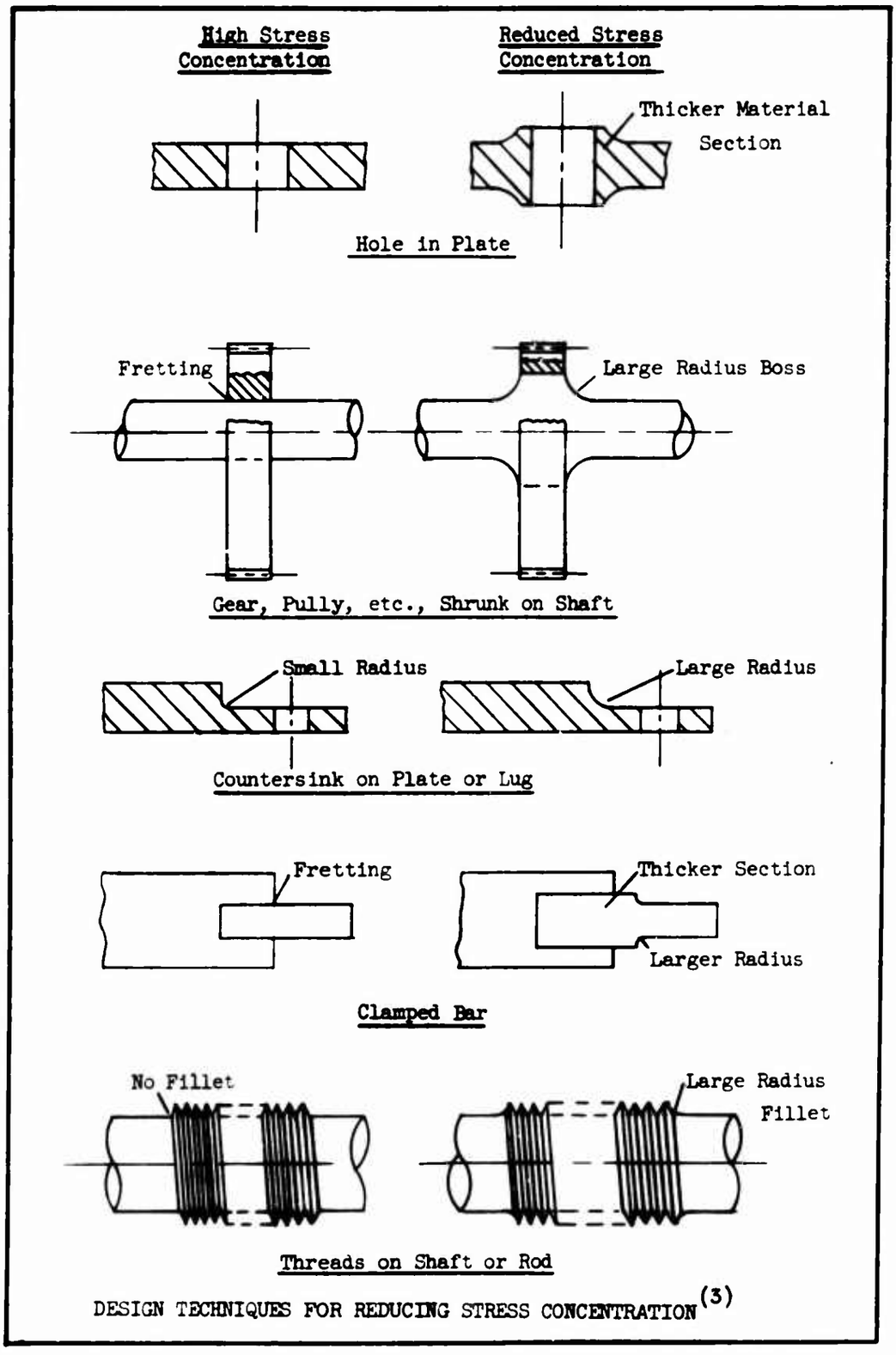


EFFECT OF SURFACE FINISH ON FATIGUE STRENGTH OF VARIOUS METALS.(11)

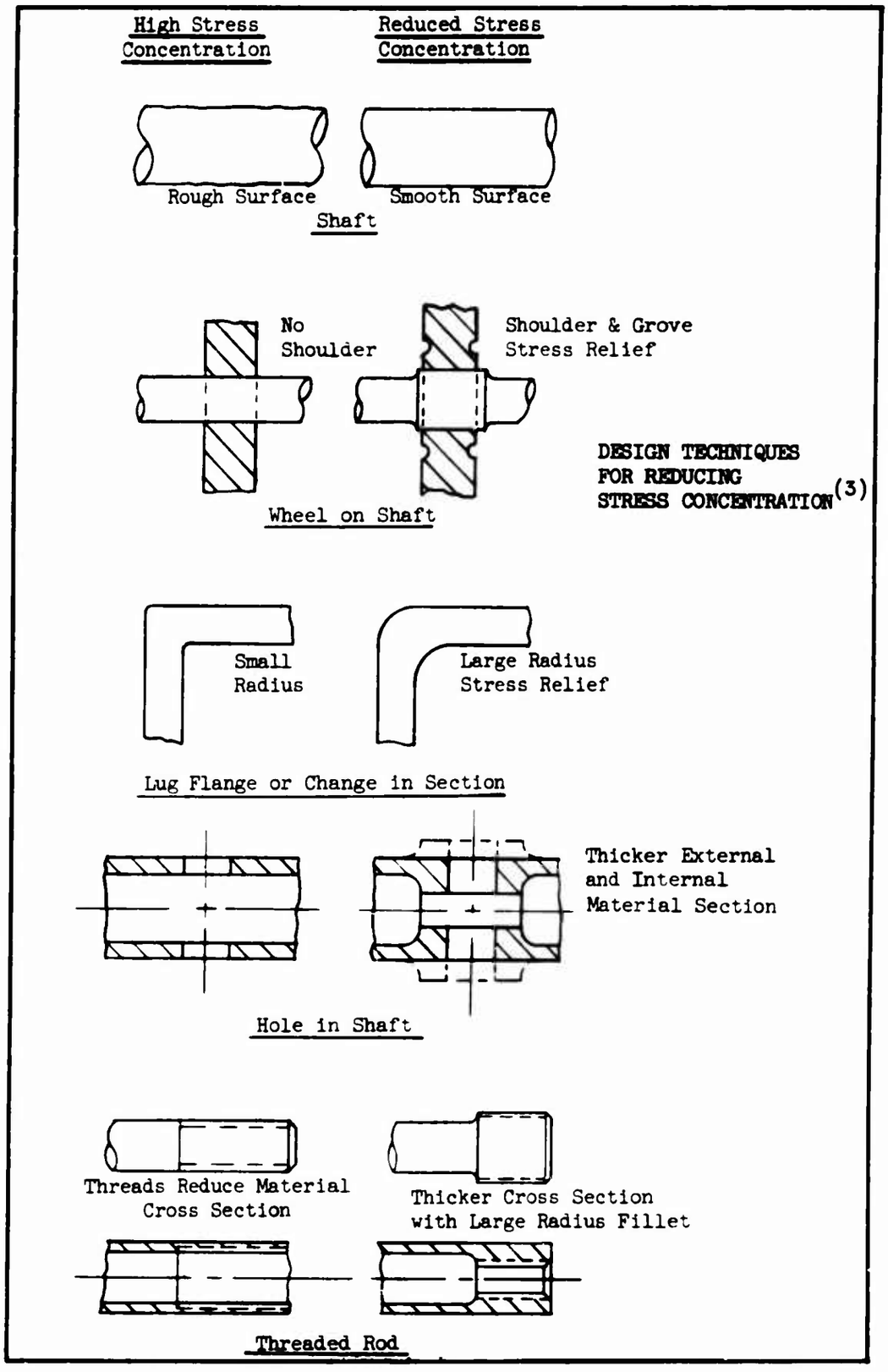
STATIC STRESS-CONCENTRATION FACTORS FOR FLAT PLATES IN TENSION OF COMPRESSION.(18)



EXAMPLES OF DESIGNS TO MINIMIZE THE STRESS RAISER EFFECT

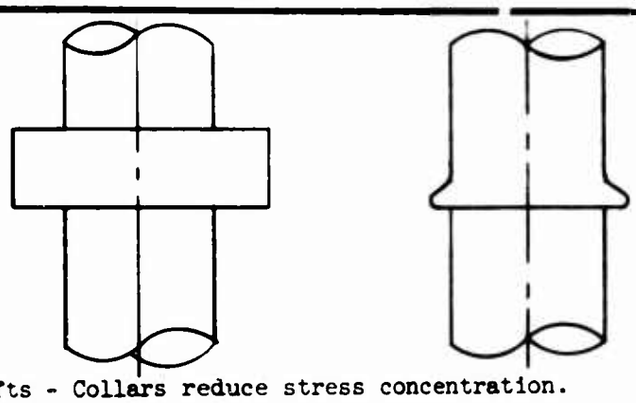


DESIGN TECHNIQUES FOR REDUCING STRESS CONCENTRATION (3)

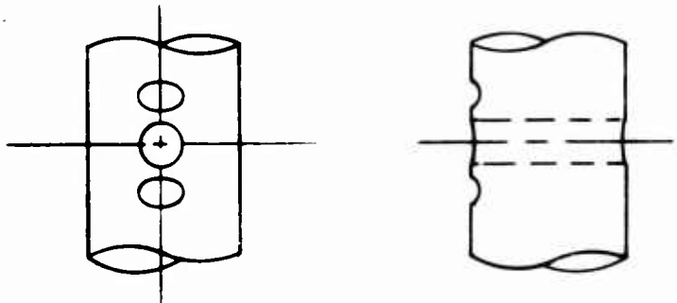

High Stress Concentration
Reduced Stress Concentration
Rough Surface
Smooth Surface
Shaft
No Shoulder
Shoulder & Grove Stress Relief
Wheel on Shaft
DESIGN TECHNIQUES FOR REDUCING STRESS CONCENTRATION(3)
Small Radius
Large Radius Stress Relief
Lug Flange or Change in Section
Thicker External and Internal Material Section
Hole in Shaft
Threads Reduce Material Cross Section
Thicker Cross Section with Large Radius Fillet
Threaded Rod

EXAMPLES OF DESIGNS TO MINIMIZE THE STRESS RAISER EFFECT (Continued)

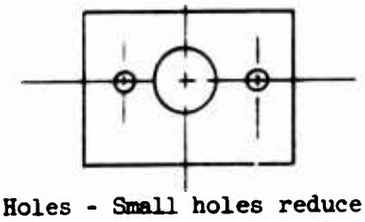
DESIGN TECHNIQUES FOR REDUCING STRESS CONCENTRATION (16)



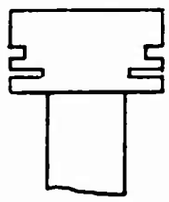
Shafts - Collars reduce stress concentration.



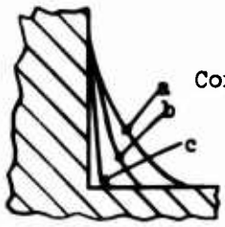
Shafts - Relieving groves reduce stress concentration around radial hole.



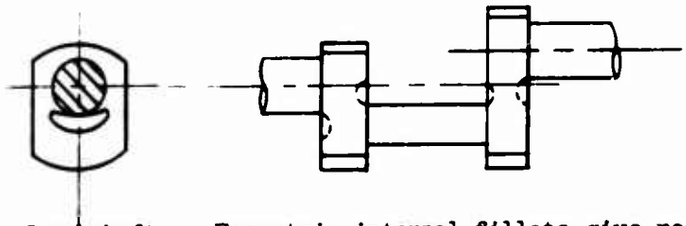
Holes - Small holes reduce stress concentration around large hole.



Groves reduce stress concentration from sudden dimensional change.

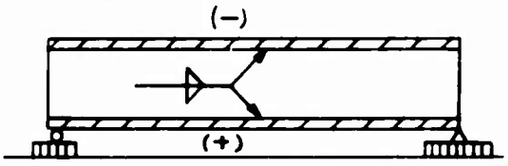
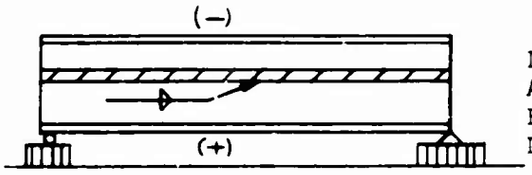
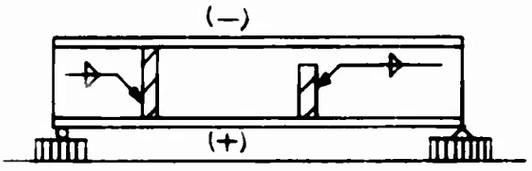
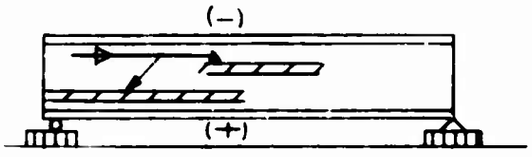
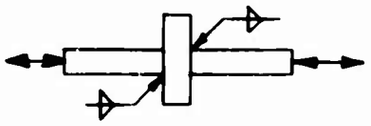
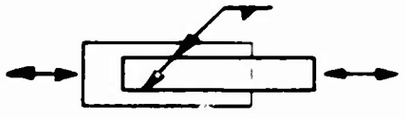


Corners - "Streamlined" fillets for various types of loading: (a) Plate in tension or compression
(b) Plate in bending and shaft tensio
(c) Shaft in bending or tension, such fillets help reduce stress concentration factors.



Crankshafts - Eccentric internal fillets give reduced stress concentration.

WELDING TECHNIQUES FOR STRESS CONCENTRATION REDUCTION(10)

Joint	Stress Ratio, R	Reduction* (percent)
 Longitudinal Flange-to-Web	0	0
	1	0
 Longitudinal Attachment, Both Ends in Low Stress Regions	0	0
	1	0
 Traverse Attachment, Full Width on One End in Compressive-Stress Region	0	3
	1	8
 Longitudinal Attachment, One or Both Ends in High Stress Region	0	41
	1	25
 Traverse Tee Joint. No Complete-Penetration Welds†	0	51
	1	50
 Lap Joint§	0	59

* Compared to traversely butt-welded joints, weld reinforcement in place.

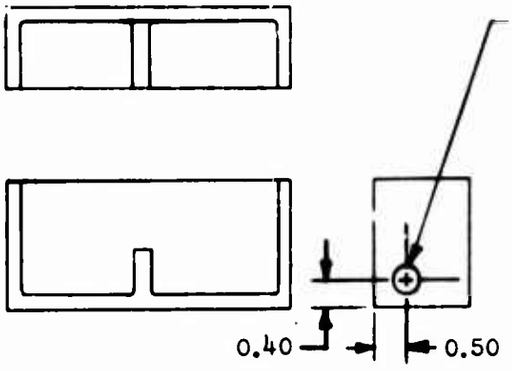
† Plate carrying main stress is interrupted by a perpendicular plate in such a way that main stress must be transferred through the joint.

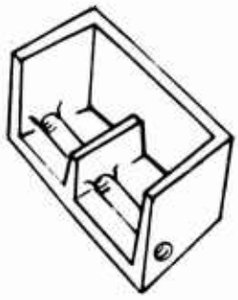
§ Welded only on sides of plate parallel to direction of stress in plates.

EXAMPLES OF DESIGN OVERSIGHT WHICH CAUSE HIGH STRESS CONCENTRATION AND SERVICE FAILURE (9)

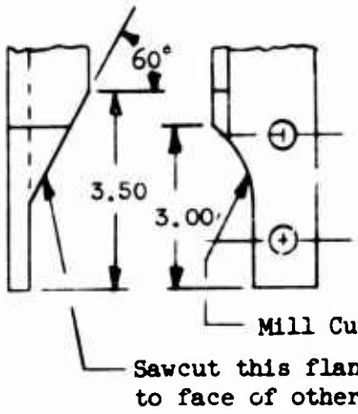
Oversight

Result

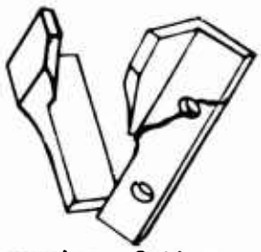




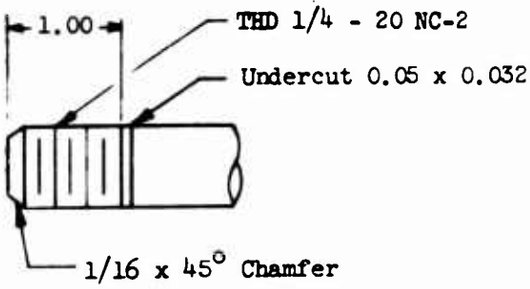
In service, fatigue cracks developed at feather edges.



In drafting, 2 views did not show complexity of intersection of compound cuts.



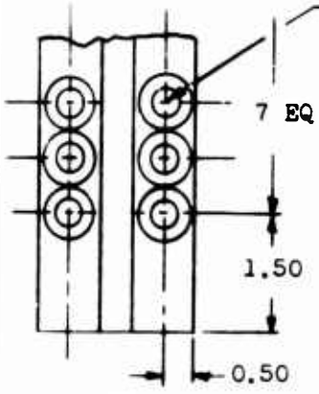
In service, fatigue cracks developed at feather edges.



In drafting, the thread runout at undercut did not show.

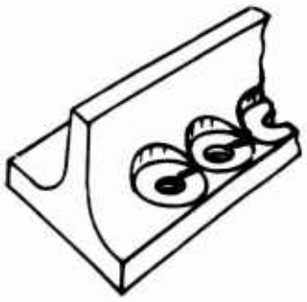
In service, fatigue cracks developed at feather edges.

Oversight

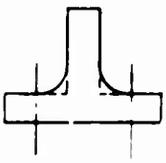


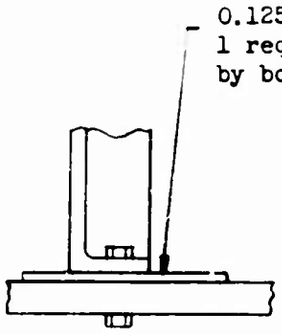
In drafting, typical short-cut method provided all necessary information for fabrication, allowing for bolt-head clearance in next assembly as well, but ..

Result

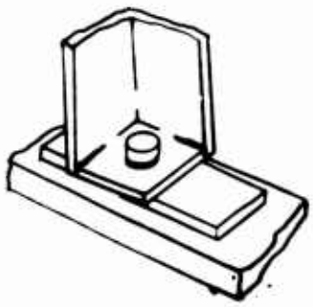


In service, fatigue cracks developed at feather edges caused by intersecting spot faces.

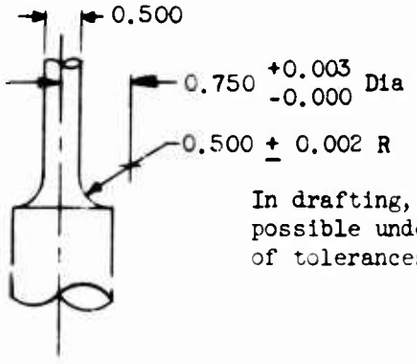




In drafting, only one view of assembly was shown. Filler of minimum allowable width for sufficient edge distance and bearing contact under normal loads was called for.



In service, fatigue cracks developed around bearing web due to lack of solid practical contact across face.



In drafting, at least one possible undesirable combination of tolerances was overlooked.

In service, fatigue crack developed at feather edge.

CONDITIONS WHICH REDUCE DYNAMIC STRUCTURAL INTEGRITY

Stress concentration is often increased because of design necessity, accident, or poor technique, or factors inherent in the material of the structure.(3)

Source	Importance
Stress concentration due to improper design (small fillet radii in shafts, changes in sections, etc.)	Reduction in fatigue strength depends on the geometric factor and the sensitivity of the material to notches. Reduction can be as high as 75 percent.
Stress concentrations due to improper manufacturing (file marks, rough machined surfaces, etc.)	Difficult to evaluate since geometry of notches is usually non-standard. Fifty percent loss in fatigue strength is not uncommon.
Residual tensile surface stresses induced by grinding, cold forming, or assembly method.	Improper grinding may introduce very high tensile stresses, causing a loss of fatigue strength of 10 to 15 percent. Although beneficial compressive stresses can be introduced by cold forming, tensile stresses are sometimes produced, causing a loss of fatigue strength. Tensile stresses induced by mechanical assembly methods usually have an adverse effect on fatigue strength.
Fretting or galling of surfaces that are simultaneously submitted to fatigue stresses.	Fretting or galling can result in a fatigue-strength loss of up to 80 percent. Most clamped, bolted, or riveted joints are subject to this condition.
Corrosion	Corrosion caused by moisture or liquids can reduce fatigue strength by as much as 75 percent. Most metals require an adequate surface protection.
Plating	Plating usually reduces the fatigue strength of a part, the amount depending on the type of plating, the thickness of the plate, the method of plating, and the embrittlement-relief treatment. Chromium plating in some instances can cause a large loss in fatigue strength.

Source	Importance
Surface conditions introduced by heat treatment (oxide penetration, decarburization., etc.)	Only by the most careful control can the surface be protected during the heat-treating process. These anomalies often form the nucleous for a fatigue failure.
Size	Most published fatigue data on materials are based on small laboratory specimens which do not adequately evaluate the fatigue strength of large parts. Large parts may be weaker than small test specimens by more than 10 percent.
Speed	Although strain rate ordinarily has only a minor effect on fatigue life, very high or very low speeds usually reduce fatigue strength.
Shape	It has been found that the shape of a part has some influence on its fatigue strength. Rectangular specimens may be up to 30 percent weaker than round ones.
Inclusions in Materials	Nonmetallic inclusions in high strength steels may reduce fatigue strength to a point considerably below that of relatively inclusion-free steels.

CHAPTER 5 – FATIGUE

CHAPTER 5
FATIGUE

ABSTRACT:

Fatigue life largely determines the useful life of most structural packages subjected to repeated loadings. The usual structural fracture associated with prolonged vibration (and in some cases repeated light shock impulses) is a fatigue failure. Certain types of short term overload also have a deleterious effect on fatigue life, and hence equipment life.

The intent of this chapter is to outline the nature of the fatigue phenomena and define the engineering terms normally used to describe it. Analytical methods are also presented to model the structural fatigue damage potential during sine and random excitations.

The major portion of this chapter is devoted to the design aspects of fatigue resistance. The discussion includes the influence of size, stress concentration, improved relative stiffness, improved damping and natural frequency parameters, and the general design engineering approaches to increased fatigue strength and reduced fatigue stress. Design methods are discussed for reducing the effects of fretting, scoring, corrosion, and stress concentration in structural systems.

Chapter 5 - Fatigue

ERRATA SHEET

Page	Paragraph	Line	Correction
Abstract	2	2	phenomenon
5.3-1	Graphic	Equation (6)	$f_u = f_n \left(1 + \frac{1}{2Q}\right)$
5.3-2	1	9	Rayleigh distribution
5.3-5	1	2	delete second "A = 10"
5.3-5	1	3	a_{eq} = ... 1.93g
5.4-9	Caption	1	DYNAMIC RESPONSE:
5.4-14	2	2	ratio S_a/S_N
5.4-18	4	1	Corrosion fatigue
5.4-24	2 & 3	-	delete space between paragraphs
5.5-2	1	15	K_f
5.5-9	Graphic	Lower Plot	R = -0.5 (center curve)

VOLUME III - CHAPTER 5
FATIGUE

CONTENTS

Section		Page
1	INTRODUCTION	5.1-0
	• The Meaning of Fatigue in Electronic Equipment Package	5.1-0
2	THE NATURE OF FATIGUE	5.2-0
	• The S-N Curve and the Constant Life Diagram	5.2-0
	• Loading Conditions and Fatigue Strength	5.2-2
	• Cumulative Damage - Miner's Rule	5.2-4
3	ANALYTICAL METHODS	5.3-0
	• A Model for Fatigue Damage During Swept Sine Vibration	5.3-0
	• A Model for Fatigue Damage During Random Vibration	5.3-2
	• Sine - Random Equivalence	5.3-4
4	DESIGN METHODS FOR PREVENTING FATIGUE FAILURE	5.4-0
	• The Design Choice: Fatigue Stress vs Fatigue Strength	5.4-0
	• The Influence of Size on the Fatigue Strength of Critical Sections	5.4-2
	• Improved Fatigue Strength Through Reduced Stress Concentration	5.4-4
	• The Effects of Redistributed Stress and Improved Relative Stiffness	5.4-6
	• The Influence of Increased Natural Frequency, Lateral Stiffness, and Damping	5.4-8
	• The Influence of Reduced Mean Stress and Improved Stress Flow in Critical Sections	5.4-10
	• The Effect of Increased Bulk Strength on Fatigue Life	5.4-12
	• Improving Fatigue Life Through Increased Local Strength	5.4-14
	• The Importance of Fretting Effects to Fatigue Life	5.4-16
	• Eliminate Scoring and Corrosion to Enhance Fatigue Life	5.4-18

CONTENTS (Continued)

Section		Page
4	(Continued)	
	• Eliminate Sharp Corners and Improve Fatigue Life	5.4-20
	• Improve Surface Finish to Improve Fatigue Strength	5.4-22
	• Improved Ductility and Impact Strength Also Improves Fatigue Life	5.4-24
5	APPENDIX	5.5-0
	• Bibliography	5.5-0
	• Glossary	5.5-1
	• Symbology	5.5-2
	• Fillets and Notches	5.5-3
	• Common Factors Influencing Fatigue Strength . .	5.5-4
	• Values of I(A)	5.5-6
	• Problem on Fatigue Damage During Random Vibration	5.5-7
	• A Typical Application of the S-N Diagram . . .	5.5-8
	• Using the Modified Goodman (Constant-Life) Diagram	5.5-10
	• Calculating a Safety Factor in a Repetitively Loaded Member	5.5-12

VOLUME III - CHAPTER 5

FATIGUE

SECTION 1 - INTRODUCTION

- The Meaning of Fatigue in Electronic Equipment Package

THE MEANING OF FATIGUE IN ELECTRONIC EQUIPMENT PACKAGES

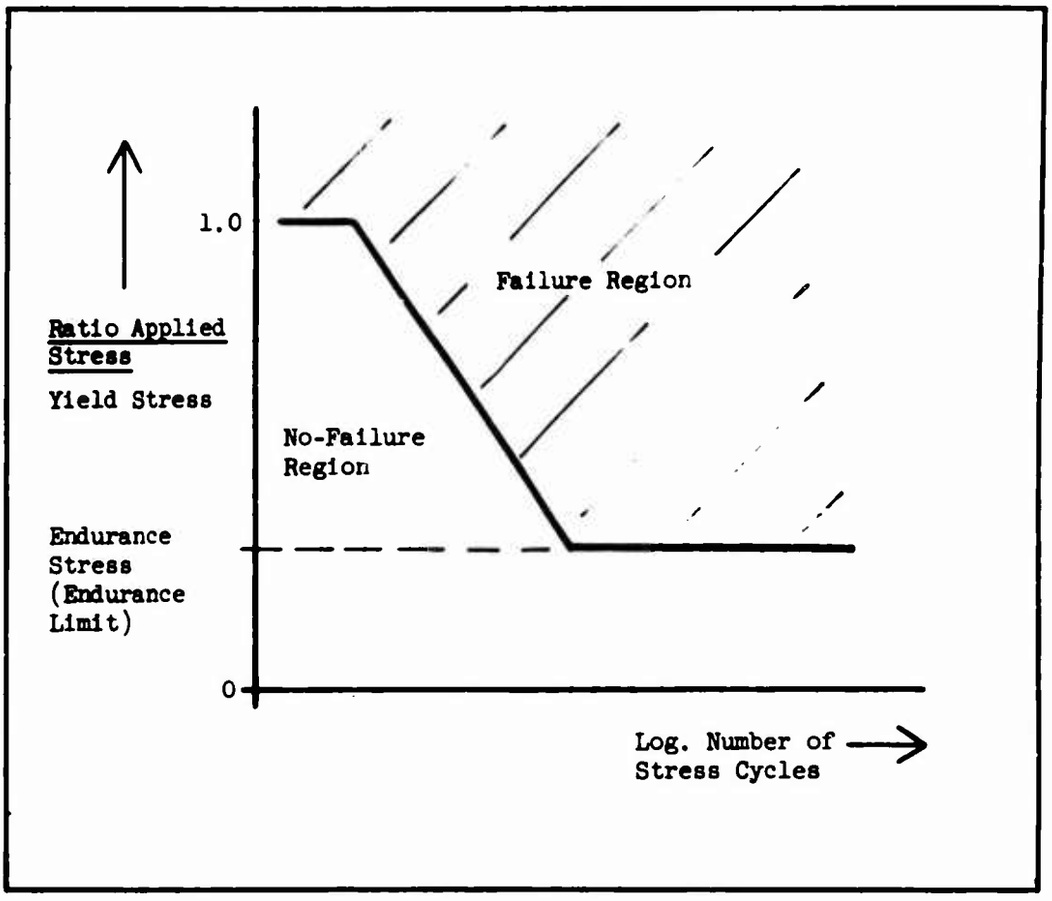
Potential areas of fatigue failure need attention early in the design phase.

What is fatigue? A structure which is subjected to varying loads can fail at stress levels well below those which cause static failure. The mechanism of fatigue failure is beyond the scope of this guide. Basically, the fatigue failure starts as a tiny crack (or cracks) which grows with successive alternating loads and thus reduces the strength of the part. Eventually the strength is reduced to the point where the loads present cause the remaining material to rupture.

Why is fatigue important? A fatigue failure is particularly dangerous because it gives the appearance of occurring without warning after the structure has apparently functioned successfully for some period of time. Any structure which is subjected to repeated loads (such as loading and unloading, repeated shocks, or vibration) is a potential candidate for fatigue failure.

What can be done about fatigue? The time to worry about fatigue failure is during the design phase. The mistake of allowing a fatigue problem to get into the finished product can, at best, be expensive and in the worst extreme, be disastrous.

The general design practices outlined in this chapter can help to eliminate most fatigue problems. Where potential fatigue problems exist in critical components, the designer is advised to take the additional precaution of seeking the assistance of the fatigue, stress, and metallurgical experts.



THE STRUCTURAL FATIGUE CURVE: Fatigue strength is represented by the classic S-N curve.

VOLUME III - CHAPTER 5

FATIGUE

SECTION 2 - THE NATURE OF FATIGUE

- The S-N Curve and the Constant Life Diagram
- Loading Conditions and Fatigue Strength
- Cumulative Damage - Miner's Rule

THE S-N CURVE AND THE CONSTANT LIFE DIAGRAMS

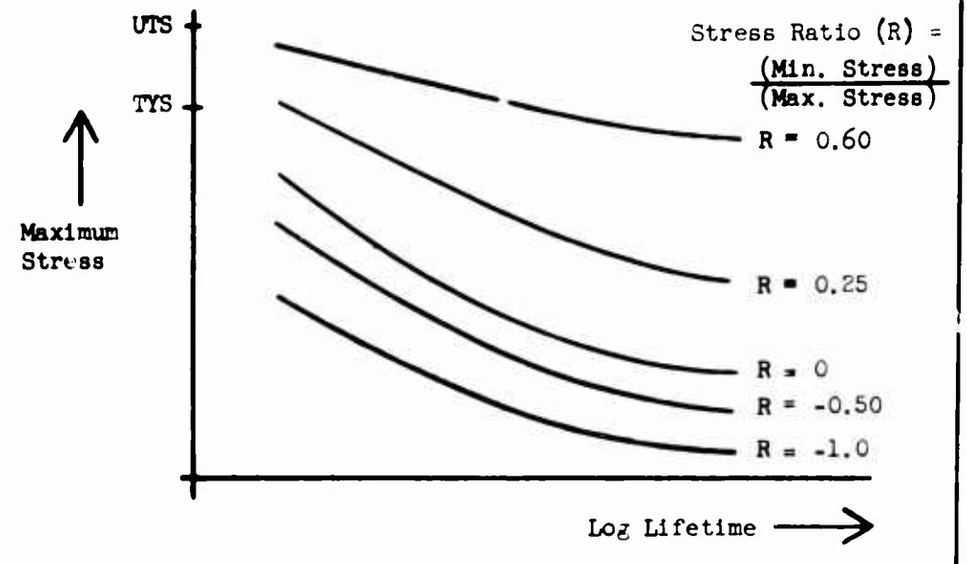
The fatigue characteristics of an engineering material subjected to repetitive loadings may be conveniently defined in terms of an S-N curve and constant life diagrams.

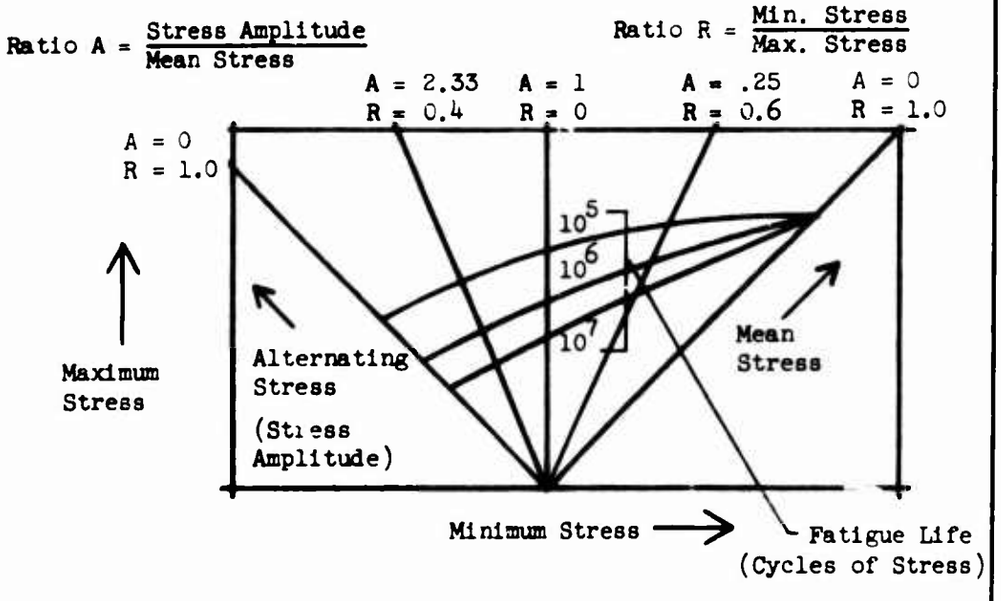
The S-N Curve: The results of a typical fatigue test are presented in the S-N curve in the adjacent figure. Curves for various materials under several test conditions are also available in the referenced literature. It is important for the designers to select an S-N curve whose conditions closely match those to which the design item will be subjected. For example, it can be seen that the stress ratio (R) has a severe effect on the fatigue life. The fatigue life under given conditions is designated by the line on the S-N curve. Conditions to the right of the line (and above it) are expected to result in failure. The fatigue strength line usually represents the average fatigue life to fracture; it should be noted that there is considerable scatter in the fatigue life of a material, even under identical test conditions. Therefore, it is necessary to provide enough of a safety factor to assure adequate strength. The designer is advised to consult the fatigue expert when using S-N curves for design purposes.

The Constant-Life Diagram: The constant-life diagram (or the modified Goodman diagram) is a summary plot of fatigue data, shown in the lower figure. The data lines represent the number of cycles to fatigue failure. If the point designating the stress conditions is above the line designating the number of stress cycles to be encountered, then the specimen is expected to fail. Again, the statistical uncertainty of most of the data available must be kept in mind. Consult an expert.

The convenience of these two fatigue life diagrams in defining the parameters of a repetitively loaded structural model is illustrated in the appendix. Sample problems are given for the application of both the S-N curve (page 5.5-8) and the constant-life diagram (page 5.5-10).

CHARACTERISTIC S-N CURVE SHOWING VARIOUS STRESS RATIOS (6)





CONSTANT LIFE DIAGRAM FOR THE S-N CURVES SHOWN ABOVE (6)

FATIGUE PLOTS: The important engineering properties showing the characteristics of fatigue in structural materials are visible in the S-N and Constant Life plots.

LOADING CONDITIONS AND FATIGUE STRENGTH

The conditions for applying the varying load to a structure greatly affect the fatigue strength.

The fatigue strengths of materials are obtained by subjecting test samples to varying loads and noting the conditions which cause failure. The test conditions of importance are as follows:

1. The method of loading
2. The stress levels
3. The number of stress cycles
4. The test temperature
5. Complete description of the test specimen.

The Method of Loading: As shown in the accompanying figure, the varying load can result in a constant amplitude, completely reversed stress ⓐ, a constant amplitude plus a steady stress ⓑ, a random amplitude stress ⓒ, and a random amplitude plus steady stress ⓓ. Test specimens are rarely subjected to the random stresses and they will not be discussed here. See the discussion of Cumulative Damage for more information on random loading.

The Stress Levels: The relationship between the mean (or steady) stress and the oscillating stress is designed in terms of the stress ratio, R, (the ratio of the minimum stress to the maximum stress) or in terms of A (the ratio of the stress amplitude to the mean stress). See Equations 1 and 2.

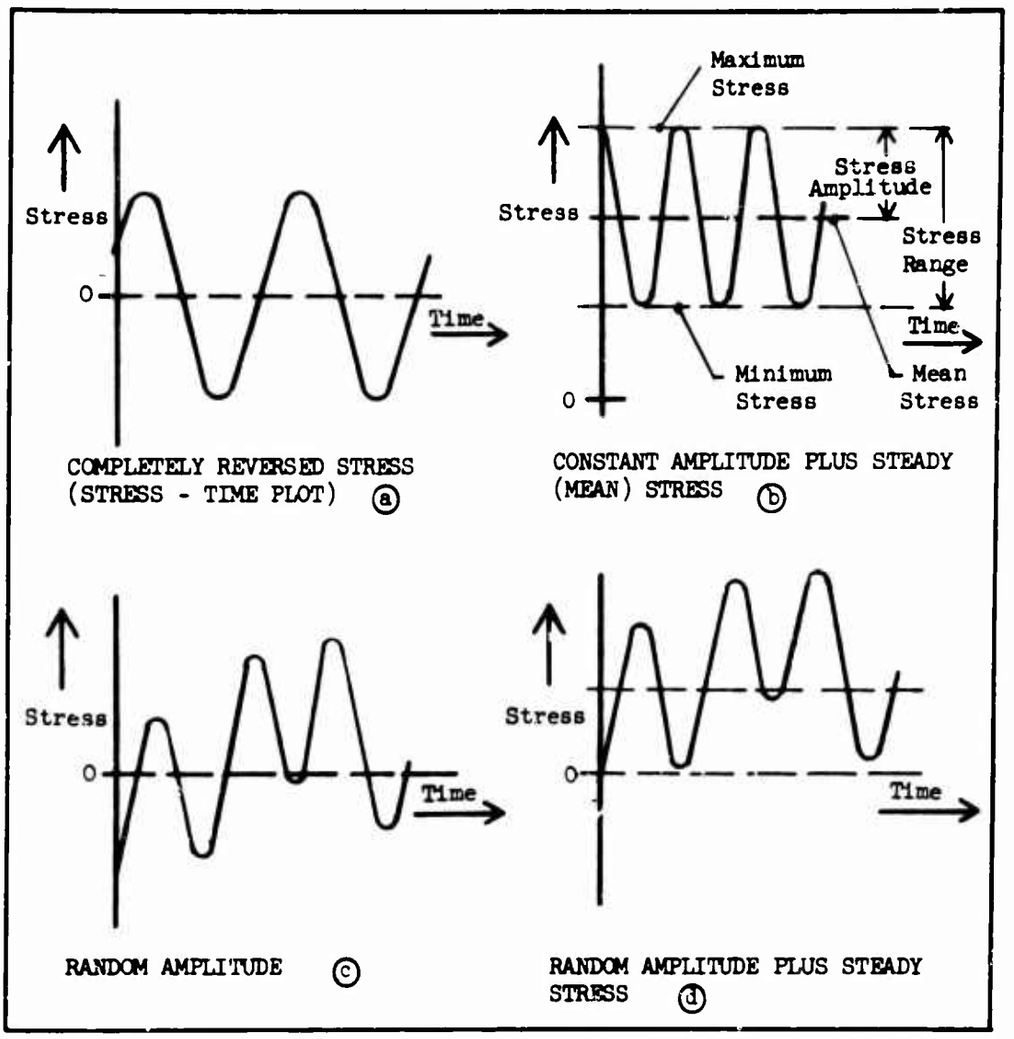
The Number of Stress Cycles: A stress cycle occurs when the stress goes from the mean to the maximum, then to the minimum and back to the mean (illustrated at right). The number of cycles a specimen has undergone at a given test condition is n, and the fatigue life at that condition is N.

Other Conditions: The results of a fatigue test will often include such information as the test temperature, the forming process and heat treatment of the specimen, the surface condition, the static strength properties and the rate of loading.

$$R = \frac{\text{Minimum Stress}}{\text{Maximum Stress}} \qquad (1)$$

$$A = \frac{\text{Stress Amplitude}}{\text{Mean Stress}} \qquad (2)$$

for completely reversed stress, $R = -1$ and $A = \infty$.



TYPES OF FATIGUE LOADS: The manner in which the repetitive load is applied to the structure greatly affects its fatigue strength

CUMULATIVE DAMAGE - MINER'S RULE

Miner's Rule provides a simple method for evaluating the fatigue damage caused by stress occurring at different levels.

Miner's Rule hypothesizes that fatigue damage accumulates in a linear manner. As shown in Equation 1, the damage index (D) is equal to the summation of the ratios of the number of applied cycles (n_i) to the theoretical number of cycles to failure (N_i) at stress (S_i).

For loads at only one stress level, failure is expected when n = N and the damage index is equal to 1. For loads at various stress levels (see accompanying figure), D will have a range of values. According to Reference (7), when there is more than one discrete stress level, the index D can vary over a range as large as 0.3 to 3.0, depending on the methods of testing. This variation becomes smaller when the discrete loads are applied by increasing and decreasing the levels or when the loads are applied randomly. Reference (7) states that with various stress distributions the damage index ranges between 1/5 and 2/3, with most of the data near 1/2.

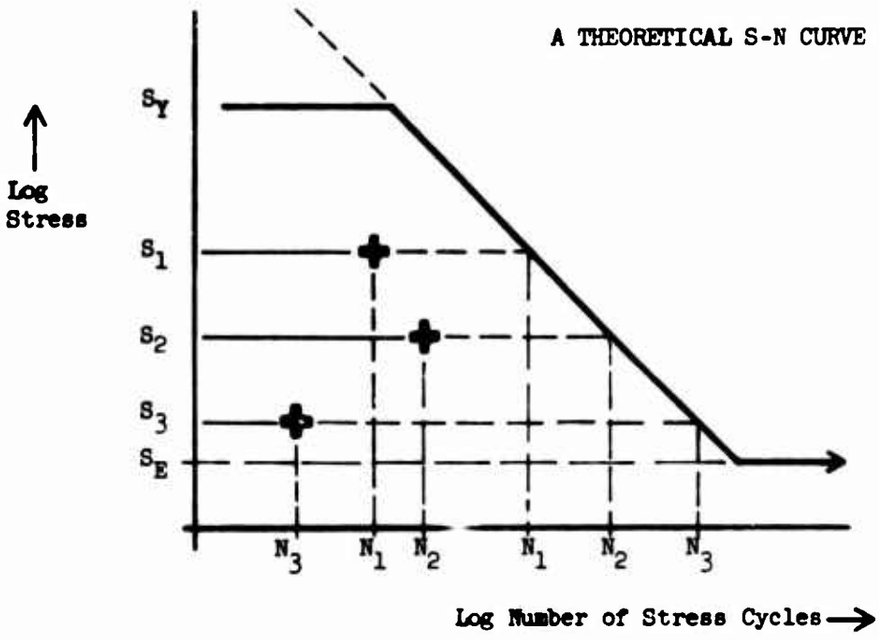
Two lines on the S-N curve are of particular interest. One is the endurance stress (S_E), also known as the endurance limit. Below this stress, a material can experience an infinite number of stress cycles without failure. Some materials do not have an endurance limit. The other level is the yield stress (S_Y); the S-N curve becomes horizontal at (or slightly above) the yield stress for less than (approximately) 10,000 stress cycles.

The adjacent table presents a typical example of multi-leveled stress applications. S is the applied stress (if the S-N curve is a plot of peak stress, S in this case must be peak stress), n is the number of cycles applied at that stress level and N is the number of cycles to failure at that stress level. If we assume for this example that a damage index greater than 1/2 will result in failure, it can be seen that (by applying Equation 1) no failure is expected.

Miner's Rule should not be applied without recognizing its limitations. Uncertainties exist in the S-N curve, in structural variations from one assembly to another, and in the actual loading environment. Adequate design strength should not be assumed unless a large safety factor is shown, and, even then, it is recommended that critical items be subjected to comprehensive structural tests.

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots = \sum \frac{n_i}{N_i} \qquad (1)$$

A THEORETICAL S-N CURVE



AN EXAMPLE OF CUMULATIVE FATIGUE DAMAGE

	S, psi	n, cycles	N, cycles
1	60,000	20,000	130,000
2	55,000	40,000	370,000
3	40,000	10,000	7,500,000

$$\sum \frac{n}{N} = \frac{2\times10^4}{13\times10^4} + \frac{4\times10^4}{37\times10^4} + \frac{10^4}{750\times10^4} = .2573 = D$$

$.257 < .5$, Therefore no Failure

MINOR'S RULE: Fatigue strength is determined for an illustrative example using Minor's Rule.

VOLUME III - CHAPTER 5

FATIGUE

SECTION 3 - ANALYTICAL METHODS

- A Model for Fatigue Damage During Swept Sine Vibration
- A Model for Fatigue Damage During Random Vibration
- Sine - Random Equivalence

A MODEL FOR FATIGUE DAMAGE DURING SWEPT-SINE VIBRATION

A simple model can be used to obtain an estimate of the fatigue damage of a structure subjected to the types of vibration encountered in the laboratory. Sinusoidal vibration is considered here.

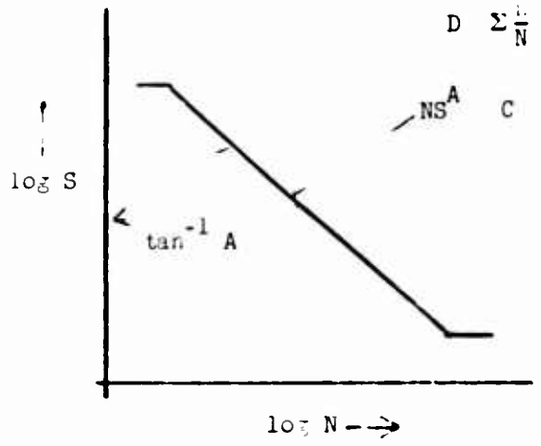
This model assumes that the structure can be idealized as a linear single degree-of-freedom system, that damage occurs according to Miner's Rule (Equation 1), and that segments of the S-N curve can be expressed as shown in Equation 2 and the accompanying figure. Further, it is assumed that the stress at resonance can be expressed by Equation 3; the stress is a linear function of the input vibration. The stress at the critical section per unit acceleration is given by the constant, C_s.

The most common laboratory vibration tests consist of either a swept sine or a random input. The sine vibration case is studied below and the random case is given in the next section.

Sinusoidal Vibration: In this case, the input vibration consists of sinusoidal vibration which sweeps from the minimum frequency to the maximum and back to the minimum. The sweep rate is logarithmic and is expressed by Equation 4; f_2 is the higher frequency, f_1 is the lower frequency, K is the sweep rate in octaves per minute and t is the elapsed time in minutes. The significant vibration is considered to occur only between the resonant half-power points. The frequencies at the half-power points are given in Equations 5 and 6. It is assumed that Q, the transmissibility at resonance, is greater than 5. Combining Equations 4, 5 and 6 yields Equation 7. Equation 7 is solved for t_R, the time at resonance during one sweep. The total number of significant stress cycles, n, is equal to t_R times M (the total number of cycles) times f_n (the natural frequency) as shown in Equation 8. One sweep occurs when the frequency goes from minimum to maximum or vice versa. Note that the sweep rate must be sufficiently slow for the resonance to reach steady state response. The fatigue damage may now be found by solving Equation 1.

Note: Several simplifying assumptions are used to arrive at this model. Actual test conditions will probably yield results different from this analysis. This approach should not be used if stresses are above the yield stress or below the endurance stress. This model is intended as a first estimate of the fatigue damage to a structure.

Example: Assume that the design in question can be represented by a single degree-of-freedom system with a natural frequency of 100 Hz and a Q of 10. The input vibration of 2g is swept from 5 to 500 to 5 Hz at the rate of 1 octave per minute. Stress at the critical area is such that C_s is 1250 psi/g. Find the fatigue damage if C is $(75,000)10$, A is 10 and D greater than 1 will cause failure. In this case, based on the calculations given, failure is expected.



$$D \sum \frac{n}{N} \quad (1)$$

$$NS^A = C \quad (2)$$

$$S = C_S Q g_{in} \quad (3)$$

$$f_2 = f_1 2^{Kt} \quad (4)$$

Lower half-power point: $f_L = f_n \left(1 - \frac{1}{2Q}\right)$ (5)

Upper half-power point: $f_u = f_n \left(1 - \frac{1}{2Q}\right)$ (6)

$$\frac{2Q+1}{2Q-1} = 2^{Kt_R} \quad (7)$$

$$n = 60\, t_R M\, f_n \quad (8)$$

EXAMPLE: Assume $g_{in} = 2\ g$, K = 1 oct/min, f_n = 100 Hz, Q = 10, C_s = 1250 psi/g, C = $(75,000)^{10}$, A = 10

Solving (3) S = (1250)(10)(2) = 25,000 psi

Solving (7) $\frac{20+1}{20-1} = 2^{(1)t_R}$ t_R = 0.144 Min

Solving (8) n = 60 (0.144)(100)(100) = 8.64×10^4 cycles

Solving (2) $NS^{10} = (75,000)^{10}$ N = 5.9×10^4 cycles

Solving (1) $D = \frac{8.64 \times 10^4}{5.9 \times 10^4} = 1.45$ **THEREFORE, fatigue failure is expected.**

FATIGUE DAMAGE: The cumulative damage from a swept-sine input may be evaluated using Minor's Rule.

A MODEL FOR FATIGUE DAMAGE DURING RANDOM VIBRATION

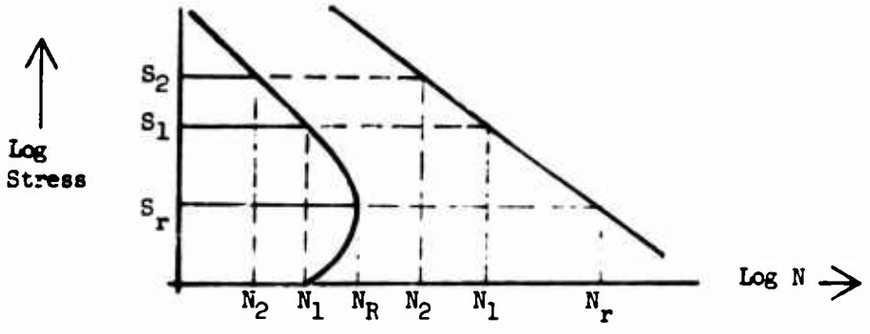
Using simplifying assumptions, a model is derived which is used to obtain an estimate of the fatigue damage caused by random vibration.

This model, as in the case of sinusoidal vibration, assumes that the structure can be represented by a single degree-of-freedom system, that damage occurs according to Miner's rule (Equation 1), that the S-N curve can be expressed by Equation 2, and that the stress is proportional to the response acceleration (Equation 3). S_r is the rms stress level and g_r is the rms response g level. In addition, it is assumed that F(f), the input random vibration, is constant over a sufficiently wide band (in the area of the natural frequency) to be considered constant for an infinite bandwidth. Finally, it is assumed that the response peaks have a Rayleight distribution (Equation 4). In order for this approach to be valid, $3S_r$ must be below the yield stress of the material. It should be noted that the analysis given below is conservative for loss-stress levels because it assumes that there is no endurance limit.

A single degree-of-freedom model responds to random vibration at its natural frequency and with a random amplitude. The number of cycles with peak amplitude S_1 is equal to the total number of cycles, n_T, multiplied by the probability of occurrence of peak S_1. For this approach, the system should have a Q greater than 5 such that successive peaks and minimums vary by a small amount. The fatigue damage is the summation of the number of cycles at the various stress levels, each divided by the number of cycles to failure at that stress level (figure below and Equation 5). The total number of cycles is the test time, t_T (in minutes) multiplied by the natural frequency, f_n (Equation 6). In order to put Equation 5 in terms of stress, we solve Equation 2 for N_i, put this in Equation 5 and get Equation 7. Let X equal the ratio S/S_r (Equation 8) and substitute Equation 4 into Equation 7: this yields Equation 9. Calling the integral I (A) and substituting Equation 6 yields Equation 10, the fatigue damage due to the random vibration of a single degree-of-freedom system. Values for I(A) are given in the appendix.

If the response, g_r, is not known, it can be derived from Equation 11. F(f) is constant in the area of f_n.

A tabulation of I(A) is given in the Appendix on page 5.5-6. A sample problem on fatigue damage is given in the Appendix on page 5.5-7.



$$D = \sum \frac{n}{N} \tag{1}$$

$$NS^A = C \tag{2}$$

$$S_r = C_s g_r \tag{3}$$

Rayleigh Distribution:

$$p(S) = \left(\frac{S}{S_r}\right) e^{-\frac{1}{2}\left(\frac{S}{S_r}\right)^2} \tag{4}$$

$$D = \sum \frac{n}{N} = \frac{n_T p(S_1)}{N_1} + \frac{n_T p(S_2)}{N_2} + \ldots$$

$$\ldots + \frac{n_T p(S_i)}{N_i} = n_T \sum \frac{p(S_i)}{N_i} \tag{5}$$

$$n_T = 60 t_T f_n \tag{6}$$

$$D = \frac{n_T}{C} \sum S_i^A \, p(S_i) \tag{7}$$

$$X = \frac{S}{S_r} \quad \text{or} \quad S_i^A = X^A S_r^A \tag{8}$$

$$D = \frac{n_T}{C} \sum X^A S_r^A X \, e^{-\frac{X^2}{2}} = \frac{n_T S_r^A}{C} \sum X^{A+1} e^{-\frac{X^2}{2}} =$$

$$\frac{n_T S_r^A}{C} \int_0^\infty X^{A+1} e^{-\frac{X^2}{2}} \, dX \tag{9}$$

$$D = \frac{S_r^A}{C} 60 t_T f_n I(A) = \frac{\left(C_s g_r\right)^A}{C} 60 t_T f_n I(A) \tag{10}$$

$$g_r = \sqrt{\frac{\pi}{2} F(f) f_n Q} \tag{11}$$

RANDOM VIBRATION EFFECTS: Equations are given for determining fatigue damage caused by random vibration.

SINE-RANDOM EQUIVALENCE

Using simplifying assumptions, a sinusoidal vibration input can be derived which will cause the same fatigue damage as a random input.

The following assumptions are used to develop a model which is used to derive sine-random equivalence:

1. The system in question can be represented by a linear, single degree-of-freedom system.
2. Miner's rule (Equation 1) for fatigue damage applies.
3. The S-N curve is represented by Equation 2.
4. The stress at the critical section is a linear function of the g level (Equation 3).
5. The random input, F(f), is constant over a sufficiently wide bandwidth to be considered infinitely wide.
6. The random response peaks have a Rayleigh distribution.
7. $3S_r$ is below the yield stress.
8. The criterion for failure during random vibration, D_1, is considered to be 1/2 and the criterion for failure during sine vibration, D_2, is considered to be 1.

A sinusoidal input at the system's natural frequency is sought which will cause the same damage as a given random input. For a given test time, the number of cycles of random vibration and sine vibration will be equal because the system responds at its natural frequency.

The damage caused by random vibration, D_1, was derived in the preceding section (Equation 5). The damage caused by the sine input is the number of applied stress cycles divided by the number of stress cycles to failure (Equation 6). Since $D_1 = 1/2$ for failure and $D_2 = 1$ for failure, we get Equation 7. The equivalent stress, S_{eq}, is introduced by using Equation 2, and by substitution we arrive at Equation 8. The stress is a linear function of the g level (Equation 3), therefore we can substitute g for S (Equation 9). Equation 9 is in terms of the response of the single degree-of-freedom system. To determine equivalency on the basis of inputs, we utilize Equations 10 and 11 to arrive at Equation 12. Values of I (A) are listed in the Appendix.

Note: This approach to sine-random equivalence should not be used indiscriminately. The equations given are restricted by the assumptions used. In actual situations, it is possible to have several natural frequency and various materials. The sine equivalent for one component could be a severe undertest or overtest for a different component.

Illustrative Example: f_n = 100 Hz, F(f) = 0.04g^2/Hz from 5 to 1000 Hz, Q = 10, A = 10, A = 10; find the equivalent sinusoidal input at 100 Hz which causes the same fatigue damage.

$$Aeq = \sqrt{\frac{\pi(0.04)100}{2(10)}}\left[2(3.84 \times 10^3)\right]^{1/10} = 1.93 \text{ g (to peak)}$$

$$D = \Sigma(n/N) \quad (1)$$

$$NS^A = C \quad (2)$$

$$S = C_S Q\, g_{in} \quad \text{or} \quad S_r = C_s\, g_r \quad (3)$$

$$n_{eq} = n_T = 60 t_T f_n \quad (4)$$

$$D_1 = \frac{n_T S_r^A}{C} I(A) \quad (5)$$

$$D_2 = \frac{n_{eq}}{N_{eq}} = \frac{n_T}{N_{eq}} \quad (6)$$

$$D_2 = 2D_1 = \frac{2n_T S_r^A}{C} I(A) = \frac{n_{eq}}{N_{eq}} \quad (7)$$

Rewriting Equation 2; $N_{eq} S_{eq}^A = C$ or $N_{eq} = \frac{C}{S_{eq}^A}$

$$S_{eq} = S_r \left[2I(A)\right]^{1/A} \quad (8)$$

$$g_{eq} = g_r \left[2I(A)\right]^{1/A} \quad \text{Response Levels} \quad (9)$$

$$g_r = \sqrt{\frac{\pi}{2} F(f) f_n Q} \quad (10)$$

$$g_{eq} = Q\,(a_{eq}) \qquad a_{eq} = \text{Input g Level} \quad (11)$$

$$a_{eq} = \sqrt{\frac{\pi F(f) f_n}{2Q}} \left[2I(A)\right]^{1/A} \quad \text{Input Levels} \quad (12)$$

SINE-RANDOM EQUIVALENCE: The damage potential of a random excitation may be represented by an equivalent sinusoidal input.

VOLUME III - CHAPTER 5

FATIGUE

SECTION 4 - DESIGN METHODS FOR PREVENTING FATIGUE FAILURE

- The Design Choice: Fatigue Stress Vs Fatigue Strength
- The Influence of Size on the Fatigue Strength of Critical Sections
- Improved Fatigue Strength Through Reduced Stress Concentration
- The Effects of Redistributed Stress and Improved Relative Stiffness
- The Influence of Increased Natural Frequency, Lateral Stiffness, and Damping
- The Influence of Reduced Mean Stress and Improved Stress Flow in Critical Sections
- The Effect of Increased Bulk Strength on Fatigue Life
- Improving Fatigue Life Through Increased Local Strength
- The Importance of Fretting Effects to Fatigue Life
- Eliminate Scoring and Corrosion to Enhance Fatigue Life
- Eliminate Sharp Corners and Improve Fatigue Life
- Improve Surface Finish to Improve Fatigue Strength
- Improved Ductility and Impact Strength Also Improves Fatigue Life

THE DESIGN CHOICE: FATIGUE STRESS VS FATIGUE STRENGTH

The design approach for avoiding fatigue failure is based on a simple premise: stress must not exceed strength. When it does, the designer has two choices: decrease the fatigue stress or increase the fatigue strength.

The best time to design against fatigue failure is at the preliminary design stage, when the proposed geometry of each component can be studied to establish where fatigue failure may occur. Potentially critical sections are easily recognized: fatigue cracks almost always originate at notches such as holes, fillets, grooves, keyways, and splines; at sharp corners and feather edges; or at surfaces damaged by fretting, scoring, or corrosion.

Nominal fatigue stresses are decreased by redistributing loads and stresses as uniformly as possible among components. Local fatigue stresses are reduced by decreasing stress concentrations; that is, the local stresses are redistributed more uniformly in the vicinity of stress raisers. There are several ways of redistributing stresses, as listed below:

1. Increase size of critical sections
2. Reduce stress concentrations
3. Redistribute stress
4. Increase natural frequency
5. Reduce mean stress
6. Change shape of critical section

When decreasing fatigue-causing stress will not eliminate a possible fatigue failure, the only alternative is to increase the strength of the part or system. The increased strength approach has analogies in the stress analysis methods. Nominal stress corresponds to bulk (average) strength; local stress corresponds to local strength. Local strength depends on micro-structural changes resulting from forging, casting, or heat-treating. Thus, local strength involves a strength distribution analogous to stress distribution.

The approaches to increasing fatigue strength are as follows:

1. Increase bulk strength
2. Increase local strength
3. Eliminate fretting, scoring and corrosion
4. Eliminate sharp corners
5. Improve surface finish
6. Improve ductility and impact strength

The methods listed above are discussed in detail in the sections that follow.

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*Much of the material presented in this section was edited with permission from a series of articles entitled "How to Prevent Fatigue Failure", by Robert E. Little of the University of Michigan. The articles appeared in MACHINE DESIGN, June and July, 1967, and are copyright 1967 by The Penton Publishing Co., Cleveland, Ohio.

Areas in Which Stress May be Decreased	Areas in Which Strength May be Increased
Critical Sections	Bulk Strength
Stress Concentration	Local Strength
Distributions	Surface Defects
Natural Frequency	Material Properties
Mean Stress	

STRESS VS STRENGTH: Fatigue failures may be minimized by increasing strength in combination with decreased stress.

THE INFLUENCE OF SIZE ON THE FATIGUE STRENGTH OF CRITICAL SECTIONS

Fatigue stress can be reduced by increasing the size of critical sections.

The simplest way to decrease fatigue stress is to increase the size of critical sections. Yet this method is seldom acceptable because larger components obviously cost more and weigh more. Also, size alone is a poor measure of good fatigue proportions; other fatigue design parameters are just as important. The relationship between size and fatigue proportions is discussed in this section.

In a component with good strength proportions, failure is equally likely to occur at any of the critical sections, whatever the mode of failure. The objective, then, is to size components by trading useless material in noncritical components for stronger critical sections in highly stressed components. Maximum reliability is attained when all components have the same factor of safety whatever the modes of failure.

In practice, the factor of safety for each component and each mode of failure should be adjusted slightly up or down from the overall (nominal) design factor, depending upon the relative cost and importance of the potential failure. For example, factors of safety for fatigue are generally somewhat higher than for other modes of failure because fatigue failure occurs abruptly and often leads to considerable system damage.

Since fatigue failure is commonplace in high performance equipment, fatigue proportions are usually the best criterion for establishing the size of components. For the simple situation in which all mean stresses are zero, Equation 1 applies. Good fatigue proportions require that both S_f and K_f be fixed, and that nominal alternating-stress amplitudes be adjusted such that the ratio of alternating stress to fatigue strength is a constant (Equation 2) for all components. Thus, size is based on the ratio of nominal alternating-stress amplitude to the fatigue strength of the unnotched component. Fatigue strength can be estimated from laboratory data, using correction factors for important fatigue factors such as mode of loading, surface finish, temperature, etc. (see Equation 3). In most preliminary designs, S_{Nt} can be substituted directly for S_N in equation 2 provided that care is taken that the fatigue data pertains to the appropriate mode of loading.

When one or more mean stresses are not zero, the corresponding values of S_N must be estimated from a fatigue working-stress diagram.

This topic is further illustrated by example, which appears in the appendix on page 5.5-12.

$$S_f = \frac{S_N}{K_f S_a} \quad \text{(Mean Stresses are Zero)} \tag{1}$$

$$\frac{S_a}{S_N} = \frac{1}{S_f K_f} = \text{Constant} \tag{2}$$

$$S_N = S_{Nt} \prod_{i=1}^{N} K_i \tag{3}$$

Where:

S_{Nt} = Fatigue strength obtained from laboratory tests

S_f = Fatigue safety factor

S_N = Fatigue strength, psi

K_f - Fatigue-strength reduction factor

S_a = Nominal Alternating stress, psi

DECREASE FATIGUE STRESS: Increasing the size of critical sections is effective first step toward reduced stress.

IMPROVED FATIGUE STRENGTH THROUGH REDUCED STRESS CONCENTRATION

Another approach to reducing fatigue stress is to keep the stress concentration factor as small as possible.

Most fatigue failures in service can be attributed to poor fatigue proportions. In the majority of cases the problem is that the bulk strength of the material is not used effectively because the fatigue strength reduction factor K_f is too large. This means that stress-concentration factor K_t must be kept as small as possible. Ideally, K_t should be such that the corresponding K_f values are constant.

Various types of notches can have markedly different effects on low-cycle fatigue strength. The notches have similar stress-concentration factors, yet the circumferentially grooved shaft (solid curves) is noticeably better for low-cycle fatigue. But regardless of how "benign" the notch may seem, reducing stress concentrations is always beneficial when required fatigue life is more than 10^5 cycles.

Fatigue-strength reduction factor is shown in Equation 2, where notch-sensitivity index q can be estimated from appropriate curves shown at the right. Values of K_f predicted from these curves have been found accurate within 20 percent. All curves end at a notch radius of 0.02 in. because smaller radii offer very poor fatigue performance. For materials other than those given in the plot, specific notch-sensitivity data should be used. In the absence of such data, the following comments are helpful in estimating q:

1. Wrought copper, nickel, magnesium, and titanium alloys have roughly the same notch sensitivities as wrought steels of the same ultimate strength.

2. Wrought metals are more notch-sensitive than cast metals. But wrought metals tested in the transverse direction have approximately the same notch sensitivity as cast metals of the same ultimate strength.

3. Cast metals are normally notch-sensitive; but q can be as low as 0.1 for gray cast irons and porous aluminum sand castings.

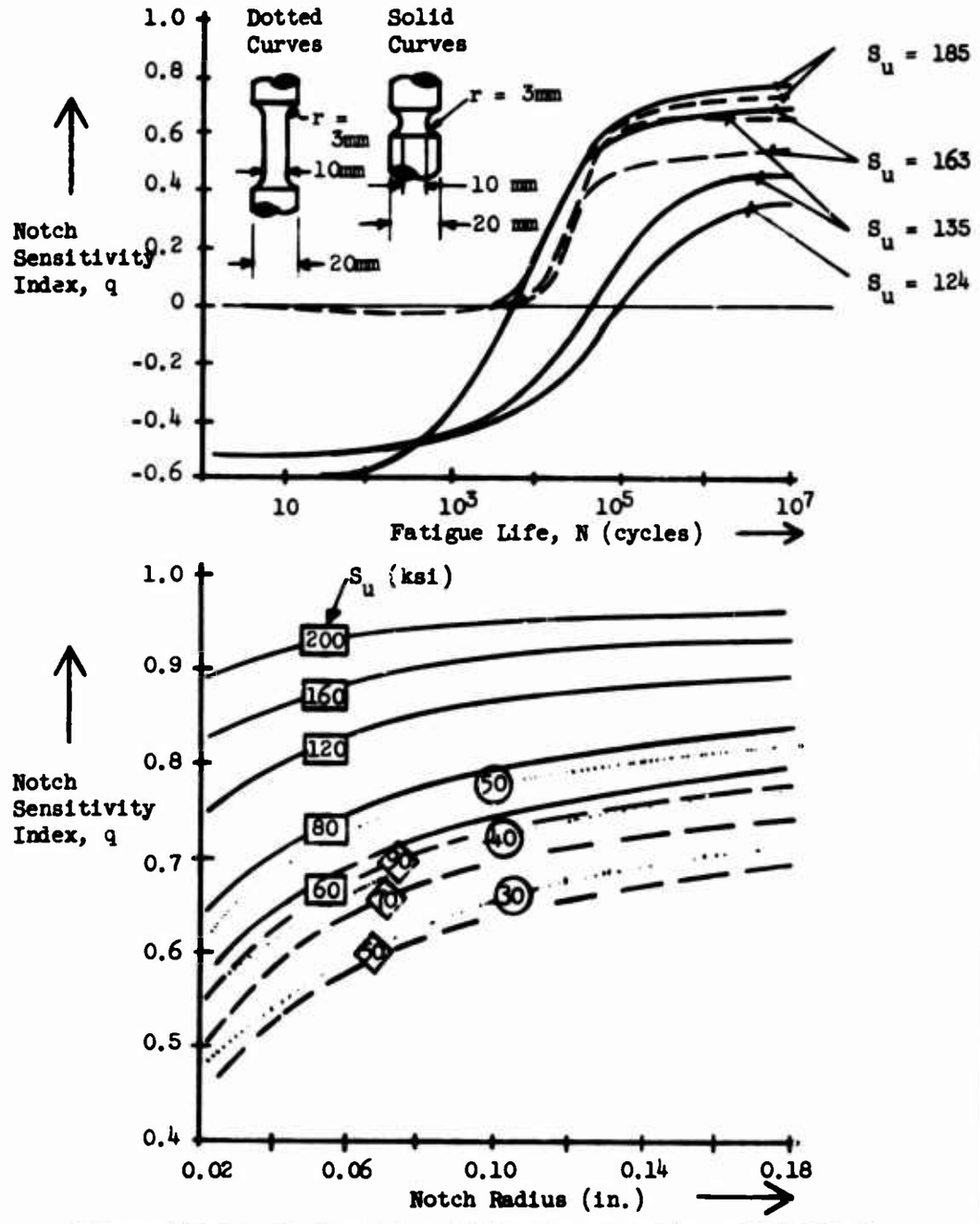
4. The only structural materials practically insensitive to notches are reinforced plastics.

The best way to reduce stress concentrations is to increase notch radii. However, in many cases consideration must also be given to the form of the notch. The appendix material on fillets and notches shows examples of the influence of notch design on fatigue strength.

$$\frac{S_a}{S_N} = \frac{1}{S_f K_f} = \text{Constant} \qquad (1)$$

$$K_f = 1 + (K_t - 1)q \qquad (2)$$

EFFECT OF TWO DIFFERENT NOTCHES ON STATIC AND FATIGUE STRENGTH OF AN ALLOY STEEL (S_u = Ultimate Strength)



NOTCH SENSITIVITY CURVES FOR STEELS (SOLID LINES), HEATTREATED ALUMINUM ALLOYS (DASHED LINES), AND STRAIN-HARDENED ALUMINUM ALLOYS (DOTTED LINES).

NOTCH SENSITIVITY: Notch design has a major influence on fatigue strength.

THE EFFECTS OF REDISTRIBUTED STRESS AND IMPROVED RELATIVE STIFFNESS

The relative stiffness of members in a system must be balanced in order to effectively distribute stresses.

A system or machine can be thought of as consisting of members that behave as a network of elastic springs. Hence, the stiffer a member the greater the share of the total load it carries. In designing for fatigue strength, relative stiffness can be so arranged that each member carries its share of the load.

The key to balancing stress distribution in a system lies in tearing force flow between members in much the same manner as stress flow within members. Construction of approximate force-flow diagrams for complex structures is particularly helpful. These diagrams usually show which members are too stiff or too flexible. Force flow can be improved by adding tapered sections, extra rivets, brackets, ribs, or cutouts.

Force-flow diagrams also show that:

1. The smaller the member, the better it can distribute the stress.

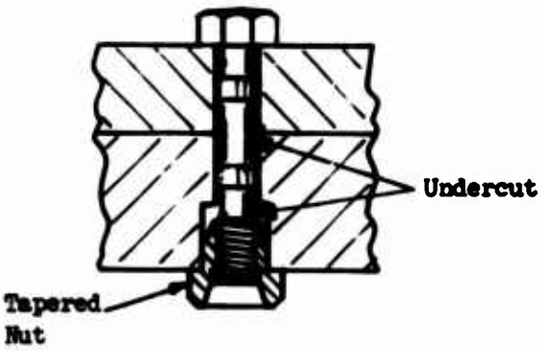
2. Loads can be carried more efficiently by members in direct tension or compression than by members subjected to bending and/or torsion.

3. Members should be sized and positioned to carry distributed loads, rather than to have individual members carry concentrated loads.

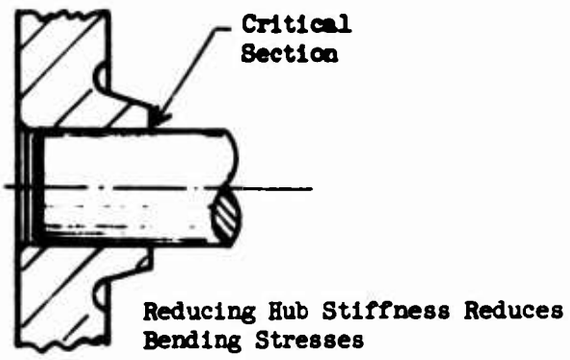
A common example of changing relative stiffness to redistribute stresses occurs in the bolted joint. As shown in the accompanying figure, the simplest way to reduce the stiffness of a bolt is to undercut the shank. The tapered nut balances the stiffness of the nut and the threaded portion of the bolt so that each mating thread carries its share of the total load. Reducing the elastic modulus of the nut by using aluminum or magnesium has the same effect as tapering a steel nut. The objective is to make the stiffness of the bolt as low as possible relative to the stiffness of the clamped members; then the bolt absorbs only a small portion of the imposed fatigue loading.

Another case where relative stiffness is important is in the design of bolted, riveted, and glued joints subjected to shearing stresses. In such cases, there has been widespread use of scarfing, tapered splice plates (or auxiliary splice plates), and attention to number and spacing of fasteners. The "relative-stiffness principle" is also used to relieve stresses imposed by shrinking a hub on shaft. When the hub is too stiff, a disproportionately large share of the bending stress is concentrated at the edge of the hub. This problem can be mitigated by reducing the stiffness of the hub, illustrated in the lower figure at right.

EXAMPLE OF RELATIVE STIFFNESS IN BOLTED JOINTS



METHOD OF MODIFYING RELATIVE STIFFNESS IN SHRINK-FIT HUB/SHAFT ASSEMBLY



STRESS DISTRIBUTION: The relative stiffness of common joints may be improved to improve fatigue resistance.

THE INFLUENCE OF INCREASED NATURAL FREQUENCY, LATERAL STIFFNESS, AND DAMPING

The dynamic response characteristics of a structural system can be modified to improve fatigue strength.

Stress analyses based on force-flow concepts are valid only as long as robust harmonics of the imposed (exciting) forces act at frequencies well below the lowest natural frequency of any of the members analyzed. Otherwise, one or more of these members may be excited at or very near its natural frequency, with the corresponding dynamic amplification of nominal stresses eventually leading to resonance fatigue failure.

Resonance fatigue failure is most likely in high-speed, high-performance equipment. In such cases, fatigue failure is generally attributable to bending stresses caused by large lateral vibrations. Accordingly, lateral stiffness of all members should be as large as practical.

Three effective ways to increase the lateral stiffness of structural members are:

1. Increase moment of inertia of the cross-section by using beads, ribs, and flanges; if possible, use I, round, or square sections.

2. Modify mounting conditions by using angle braces to simulate built-in supports rather than simple (pivotal) supports.

3. Reduce effective length of the member by mounting struts parallel to the direction of vibratory motion.

In designing against resonance fatigue, it is necessary to consider not only each member but also groups of members acting as an assembly.

Resonance fatigue of mechanical origin can also be minimized by using vibration-isolation techniques to reduce the magnitude of transmitted forces. In certain cases vibration absorbers are also effective.

If component natural frequencies cannot be increased substantially, and if the magnitude of the transmitted forces cannot be adequately reduced, the only remaining alternative is to use damping - either external or internal.

External damping involves mechanical slip between mating surfaces; energy is dissipated by coulomb friction associated with rubbing on dry surfaces, or by hysteresis associated with cyclic shear strain in viscoelastic gaskets and adhesives. For example, riveted joints can be used in place of welded joints to increase the damping capacity of a structure.

Internal damping is also effective in reducing dynamic amplification of resonance. The damping material should have a high specific damping energy; this generally means use of a magnetic material such as 403 stainless to take advantage of magnetoelastic hysteresis. Special high-damping materials and laminates are also being used more and more.

It must be noted that it is possible for situations to exist where decreasing the natural frequency will increase the fatigue life. This

will depend on the nature of the input vibration. In all cases it is necessary to maintain the proper balance between the stiffnesses of the various components of a system (See "Redistribute Stress"). More detailed information on using the natural frequency of a structure to control the loads can be found in the chapter on Natural Frequency and the chapter on Shock Absorption and Vibration Attenuation Techniques.

METHODS FOR MODIFYING DYNAMIC RESPONSE

- Change the Weight of the System
- Change the Mass Distribution
- Change the Stiffness of the System
- Change the Material Properties
- Add Structural Damping
- Use Dynamic Attenuation Techniques

DYNAMIC RESPONE: The natural frequency of a structural system can be easily modified to improve the fatigue life.

THE INFLUENCE OF REDUCED MEAN STRESS AND IMPROVED STRESS FLOW IN CRITICAL SECTIONS

The mean stress and the shape of critical sections of a member can be modified to increase fatigue life.

In most cases (especially where there is axial loading) tensile mean stress decreases fatigue strength; compressive mean stress increases it. Accordingly, it is often advantageous to superimpose compressive mean stress on critical sections by preloading.

Some form of heat treating is the most common means of inducing compressive residual stresses at the surface of components. But such stresses can be induced by localized plastic flow at the surface of the material. When this flow is caused mechanically by overloading the component until yielding occurs, the mechanism is termed overstraining. The best known example of overstraining is "scragging" of helical compression springs. In this process the spring is compressed well beyond its elastic limit (often to solid height) so that it takes on a residual stress distribution such as shown in the upper figure at right when unloaded.

Variations of overstraining are found in rolling of fillets and grooves, shot-peening of stress concentrators and surfaces, and in processes such as vapor blasting, tumbling, and burnishing. In each case, the surface layers of the material are plastically deformed while the core material behaves elastically. The depth of localized yielding (penetration) and the resulting surface finish obtained from the difference processes differ markedly.

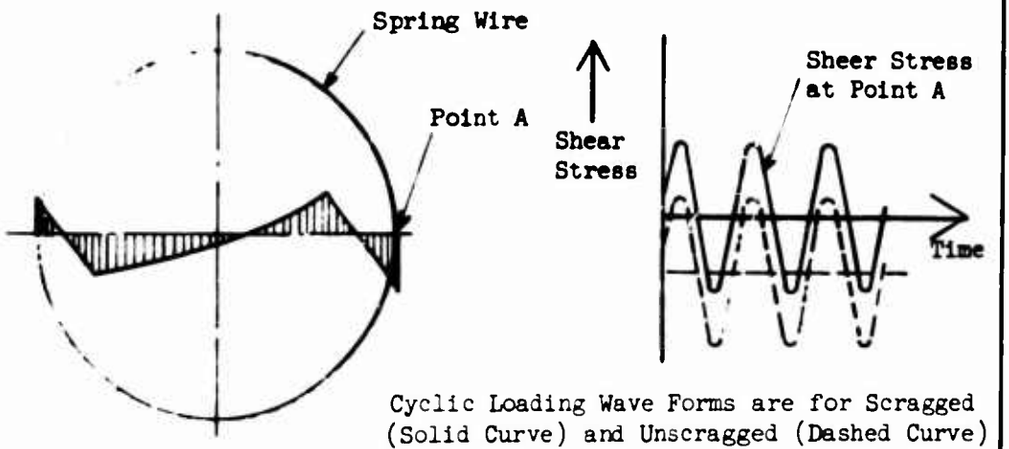
Residual stresses generally fade over a period of time when components are subjected to cyclic loading. The improvement in fatigue strength therefore depends upon the ability of the material to prevent fading during service operation. Occasional overloads are often beneficial in reestablishing residual stresses. In fact, occasional proof stressing of critical components not only rids the system or machine of deteriorating components, it enhances the strength of sound components.

Improved stress flow means that stress flow lines are more streamlined in the vicinity of notches; it can also result in more strealined stress flow lines throughout the critical section. In this case, the objective is to decrease the nominal stress. The lower figure shows the increase in the fatigue strength of a crankshaft obtained by improving the stress flow from the journal to the crank pin. In this case, fillet geometry was not changed; rather, the overlap at the critical section was modified such that the transition from one cross section to the next is much smoother.

Improved stress flow is especially effective in increasing fatigue strengths (by reducing nominal stresses) where, in spite of complex component geometry, there is still freedom to modify cross-sectional shapes - for example, in cast and forged components. Tips on designing common components for fatigue life are in the literature.

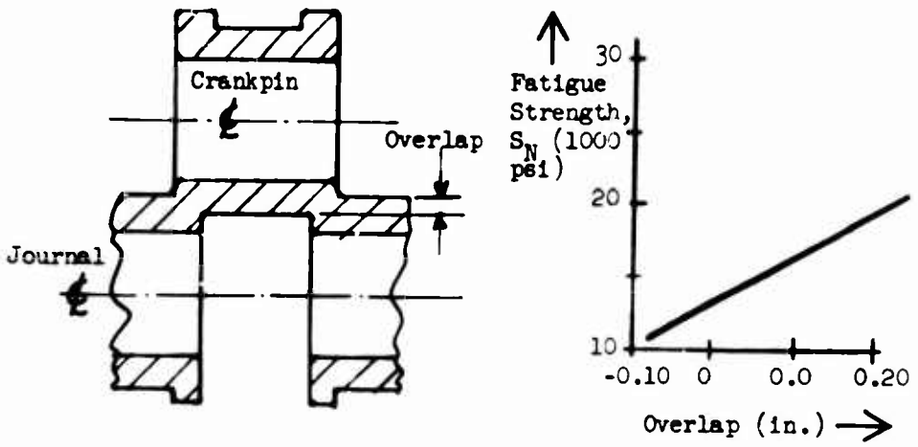
A word of caution: The above methods are general and do not apply to all situations. If applied improperly, these methods will result in reduced fatigue life. As in all fatigue problems, it is advisable to consult the fatigue, stress and materials experts.

RESIDUAL STRESS DISTRIBUTION IN SCRAGGED COIL SPRINGS.



Cyclic Loading Wave Forms are for Scragged (Solid Curve) and Unscragged (Dashed Curve) Springs

EXAMPLE OF IMPROVED FATIGUE STRENGTH OF CRANKSHAFT OBTAINED BY SMOOTHING STRESS FLOW FROM JOURNAL TO CRANK PIN



Improvement was Obtained by Modifying the Overlap at the Critical Section. Effect of the Amount of Overlap is shown in the Graph.

MEAN STRESS: Fatigue life can be improved by modification of the stress flow characteristics in critical sections.

THE EFFECT OF INCREASED BULK STRENGTH ON FATIGUE LIFE

The fatigue life of a member is increased by increasing its bulk strength. Bulk strength is affected by the geometry of the component, the composition of the material and metallurgical considerations.

Equation 1 shows that increasing bulk fatigue strength of a material is equivalent to decreasing the nominal alternating stress (shown in the upper figure). For the stress and strength distribution (and probable failure) shown in ⓐ, the strength may be increased, ⓑ, or the stress may be decreased by making the part larger, ⓒ. Increasing bulk strength by changing material or process is usually much cheaper and simpler than decreasing nominal alternating stress by changing component size or loading. However, this does not necessarily hold true in preliminary design.

Several alternative component geometries should be considered during preliminary design. Equation 1 can be used to estimate the bulk strength required for each proposed geometry. Next, a table can be made listing materials with appropriate bulk strength, for each geometry. On the basis of material cost and relative volume, the best geometry and material can then be chosen.

Composition: Fatigue strength generally increases in proportion to the increase in ultimate strength (or hardness). Thus, a 10 percent increase in bulk fatigue strength generally requires a 10 percent increase in ultimate strength. Increases of this order are easily obtained for most materials by minor changes in composition or processing.

This proportional relationship is valid for fatigue life as low as 10^5 cycles. But for shorter life - below 10^4 cycles - other factors are as important as ultimate strength or hardness in establishing fatigue strength.

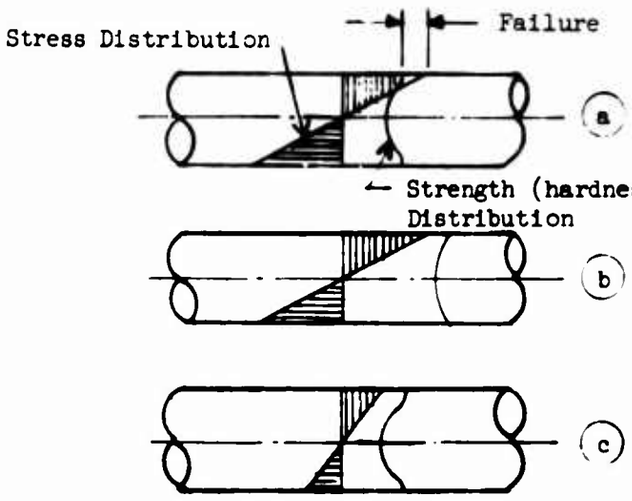
The relationship between material fatigue strength and ultimate strength must be used with care for materials with high tensile strength or hardness. This is especially true when there are considerable differences in microstructure at different strength levels. Fatigue strengths for commercially heat-treated steels peak out at very high values of hardness, shown in the lower figure. The microstructure at high hardness levels is essentially as-quenched martensite. This very brittle microstructure may have an S_N/S_U ratio as low as 0.20 to 0.25.

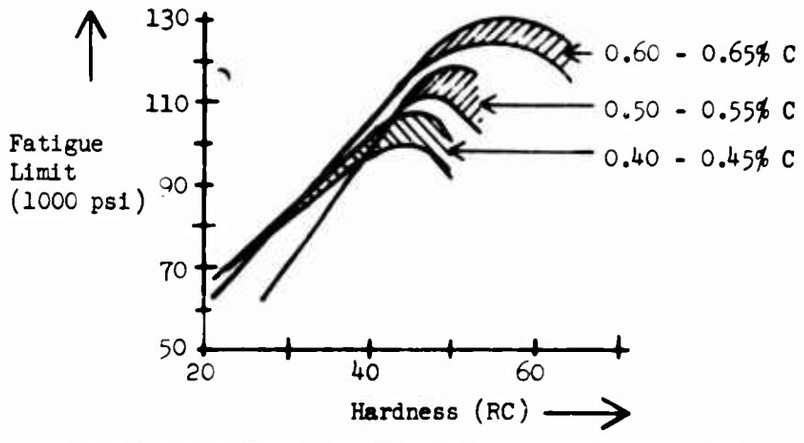
Several metallurgical mechanisms are available to increase the bulk strength of engineering materials. The most effective hardening mechanisms are: grain-size refinement, work or strain hardening, dispersion hardening, and solid-solution hardening, although some of these are not effective for certain materials. As an example, for mild steels, work hardening and dispersion hardening are very effective in increasing bulk strength. Thus, shifting from hot-rolled to cold-rolled stock increases fatigue strength.

Changing from a coarse to a fine pearlite also increases fatigue strength. Often it is possible to simply change to a low-alloy steel, retaining the same processing. In some cases, increasing the "effective" bulk strength of the existing material may require only that cyclic stresses act in the longitudinal rather than the transverse direction.

$$S_a/S_N = 1/Kf\ Sf \qquad (1)$$

TWO BASIC METHODS OF AVOIDING FATIGUE FAILURE





RELATIONSHIP BETWEEN HARDNESS AND FATIGUE LIMIT FOR VARIOUS HEAT-TREATED STEELS

BULK STRENGTH: Fatigue life can be improved by modifying the geometry or material composition of a structure.

IMPROVING FATIGUE LIFE THROUGH INCREASED LOCAL STRENGTH

Another approach to greater fatigue strength is to increase the local strength of a member. The fatigue strength is affected by hardenability, the amount of local strengthening and the manufacturing process.

It is often difficult to distinguish precisely what is meant by the terms local and bulk strength. (The same problem occurs in describing stress distributions.) For this reason, strength, like stress, should always be evaluated at the point of interest - for example, at the surface of the component, or at the root of a stress concentration. Accordingly, hardness readings usually are more meaningful in fatigue design analysis than tensile test data.

From the standpoint of strength, the best use of a material occurs when ration S_a/S_N (S_a = local alternating stress, psi, S_N = local fatigue strength, psi) has a constant value throughout the component, or at least across the section which is critical in fatigue. Thus, axial loading of normalized and annealed steels represents the best balance of stress and strength distributions. However, when the stress gradient is large, part of the material experiences negligible stress. This material contributes to excessive cost and weight. Balancing stress and strength throughout the component always helps in choosing the best materials and processing methods.

As was illustrated in the previous topic, high-hardenability steels seldom justify their expense for large stress gradients. However, it should be remembered that typical Jominy curves tend to overestimate the effect of hardenability on strength distributions, unless attention is given to the effect of tempering. Jominy curves for the appropriate tempering temperature should be used to predict strength distributions, rather than curves for as-quenched microstructures.

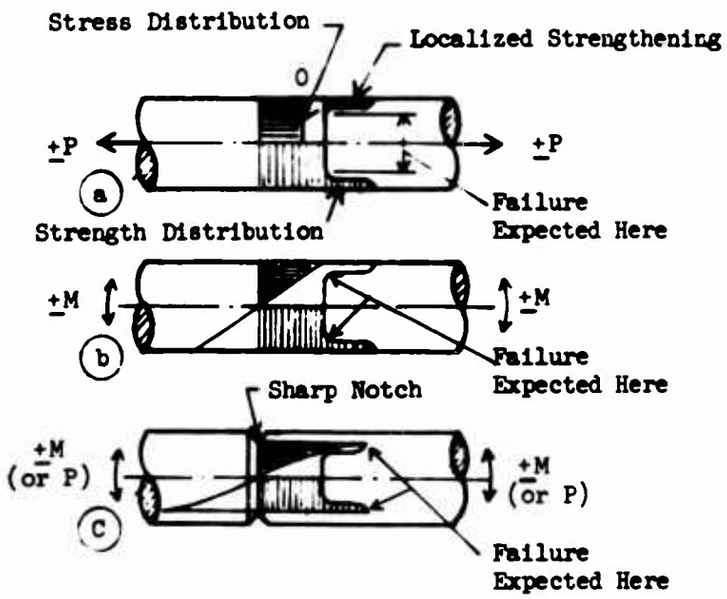
For good fatigue proportions, the product of the fatigue strength reduction factor and the fatigue safety factor should be a constant (Equation 5). For best results, K_f should be a constant; that is, all stress raisers should be designed such that Equation 1 applies, where K_t = stress concentration factor; q = notch-sensitivity index.

However, it is generally impossible to attain the conditions established by Equation 2 for all components. Localized strengthening then should be used for highly stressed components to reduce the effect of excessively large stress raisers. The amount of local strengthening required can be estimated from Equation 3.

Effect of Process: Any manufacturing process which produces a non-uniform strength distribution can be used to increase local strength. Casting and forging are versatile in this regard. Also widely used are various heat-treating methods - flame and induction hardening, carburizing, and nitriding. Surface rolling and shot peening have found wide application in increasing local fatigue strength. The adjacent figure illustrates the beneficial effect of local strengthening in terms of stress and strength distributions. In ⓐ, localized strengthening is ineffective; fatigue life may even decrease because of tensile residual stresses at the center of the bar. In ⓑ, strengthening is moderately effective, depending on the size of the stress gradient; the larger the gradient, the greater the

increase in life. In ©, localized strengthening is very effective. In this case, for less-sharp notches, fatigue cracks may originate at points shown in ⓑ; thus, life may be further improved by increasing case depth.

Removal of decarburized surface layers (or white layers in nitriding) also increases local strength. Similarly, certain platings and coatings increase local strength by resisting corrosion or wear acting simultaneously with cyclic stresses.



EFFECT OF STRESS DISTRIBUTION ON FATIGUE LIFE, FOR A GIVEN STRENGTH DISTRIBUTION

$$K_f S_f = \text{Constant} \qquad (1)$$

$$1 + (K_t - 1)q = K_f = \text{Constant} \qquad (2)$$

$$\frac{K_f \text{ (actual)}}{K_f \text{ (desired)}} = \frac{S_N \text{ (local)}}{S_N \text{ (bulk)}} \qquad (3)$$

LOCAL STRENGTH HARDENABILITY: Local strength and the manufacturing process affect the fatigue life.

THE IMPORTANCE OF FRETTING EFFECTS TO FATIGUE LIFE

Surface damage such as fretting, greatly reduces fatigue strength. Fretting damage can be minimized by proper design practices.

The effects of fretting, scoring, and corrosion on fatigue strength are similar in several respects. The simultaneous action of these mechanisms combined with cyclic stressing is far more serious than their action preceding fatigue stressing. With this simultaneous action, failure will inevitably occur - it is only a question of time. Adequate surface protection and control of surface damage, however, can often restore the fatigue strength, but at a reduced level.

Fretting fatigue can occur whenever any component experiencing cyclic stresses rubs against a mating component. Press fits and bolted or riveted joints are the most common problem areas.

Rubbing motions as small as 10^{-6} in. can cause surface damage. This damage causes fatigue cracks to be initiated at comparatively low stress amplitudes.

The decrease in fatigue strength with simultaneous fretting increases markedly with increasing hardness and may be so large that all materials in a given class have approximately the same fretting fatigue strength regardless of their ultimate strengths. It is not unusual that the fatigue strength of a bolted or riveted joint is not noticeably affected by changing the material. In certain cases, fretting damage so completely dominates fatigue strength that the effect of mean stress is negligible.

Fretting fatigue can be minimized by:

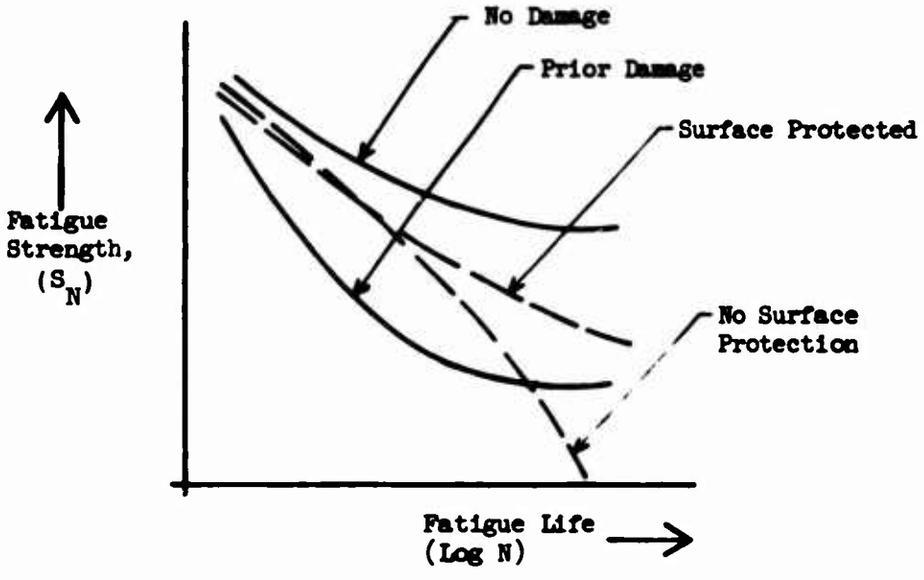
1. Eliminating rubbing motion; in some cases clamping pressure can be increased sufficiently to eliminate rubbing. In press fits, clamping pressure is increased by increasing the nominal interference or decreasing the nominal area of contact. Increased pressures in bolted joints can be attained by using more bolts and higher prestressing. Coarser surface finishes may also be effective. The complete elimination of the rubbing motion, however, is difficult to attain. Thus this approach is less effective than the redidstribution of stresses.

2. Inhibiting local welding; fretting damage can be reduced by lubricants which act as antifluxes. Molybdenum disulphide is most often suggested, but its effectiveness is erratic. The basic problem is to find a lubricant that is not squeezed out by the clamping pressure. This problem has led to the widespread use of platings and coatings. Several investigators have shown that electroplating with the cadmium, chromium,nickel, or zinc increases fatigue strength. Phosphating is also effective in certain cases.

3. Separating rubbing surfaces; mating surfaces are often separated by shims and inserts of rubber, plastics,and asbestos-reinforced resins. In some cases, use of elastic flex-arms to isolate moving components from stationary components is the only sure remedy.

The only technique of fairly consistent effectiveness in increasing fretting fatigue strength is to introduce compressive residual stresses into one or both of the rubbing surfaces. Flame and induction hardening, case hardening by carburizing or nitriding, surface rolling, and shot peening have been used successfully.

Selecting relative hardnesses of mating surfaces to concentrate surface damage in the least critical component is worthwhile whenever practical.



TYPICAL S-N CURVES FOR SPECIMENS SUBJECTED TO SIMULTANEOUS FRETTING, SCORING, AND CORROSION COMBINED WITH CYCLIC STRESS

FRETTING: The fatigue strength of most engineering materials is significantly reduced by fretting fatigue.

ELIMINATE SCORING AND CORROSION TO ENHANCE FATIGUE LIFE

The designer must evaluate the environmental conditions which can cause surface damage. Scoring and corrosion will greatly reduce the fatigue life of a structure.

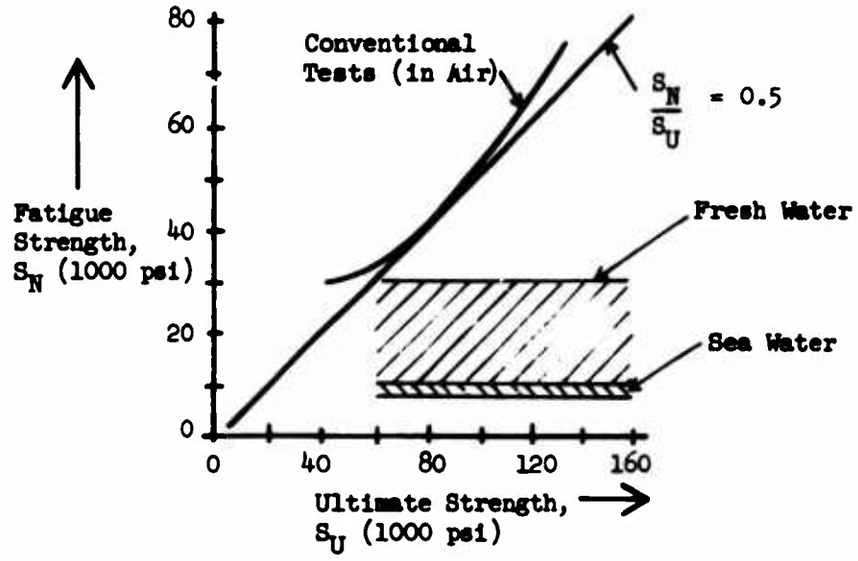
In scoring, wear debris is flushed from between the rubbing surfaces by the motion of the lubricant or the components themselves. The compatibility of metal pairs with respect to localized welding, as well as the tenacity of the surface films, are important factors in both fretting and scoring.

Surface damage is generally greatest when materials of similar hardness are mated. For example, in one experiment the fretting fatigue strength of hard steel specimens (Rc 40) was reduced to 91, 81, 59 and 50% of the unfretted fatigue strength when clamping pads of aluminum, soft steel, brass and hard steel respectively were used.

Metals that develop strong, tough, oxide layers resist continued surface damage. The same effect is obtained artificially by surface coatings and platings.

Corrision fatigue, like fretting fatigue, can be severe. As shown in the adjacent figure, for moderate and severe corrosive environments, the fatigue strength may be, for all practical purposes, independent of material hardness. Thus, minor changes in composition or processing often provide no noticeable improvement in fatigue strength. (Even austenitic stainless steels are only about 50 percent stronger than common structural steels in sea water.) Only surface treatments and coatings are consistently effective in improving corrosion fatigue strength.

Case hardening usually protects a component against serious reduction in fatigue strength due to corrosion, even when the case is only a few thousandths of an inch. Mechanical work hardening by surface rolling also provides effective protection. However, the damaging effect of simultaneous corrosion and cyclic stress depends on time as well as on number of stress cycles. Thus it is usually worthwhile to coat surface-hardened components. Tables for the effects of surface hardening, surface rolling, and plating on corrosion fatigue strength are given in the Appendix.



RANGES OF CORROSION FATIGUE STRENGTH FOR COMMON CONSTRUCTIONAL STEELS

CORROSION: Surface damage caused by corrosion or scoring will significantly reduce fatigue strength.

ELIMINATE SHARP CORNERS AND IMPROVE FATIGUE LIFE

Sharp edges are significant sources of incipient fatigue cracks. Good practice demands elimination of this surface defect.

Good fatigue design always requires provision for generous fillets at both internal and external corners. The need for a large fillet is obvious when stress concentrations exist; but generous fillets are also important even when there is no stress concentration, to reduce the possibility that fatigue cracks will originate along sharp edges.

Under a microscope, sharp edges are seen to be very jagged, particularly when the direction of machining is perpendicular to the edge. Thus, a sharp edge is in effect a series of small notches, each increasing the chance of local failure. Improving the surface finish only produces more notches of more uniform depth. For coarse surface finishes, such as commonly result from production machining, edge notches may be so severe that each may realistically be viewed as an incipient fatigue crack.

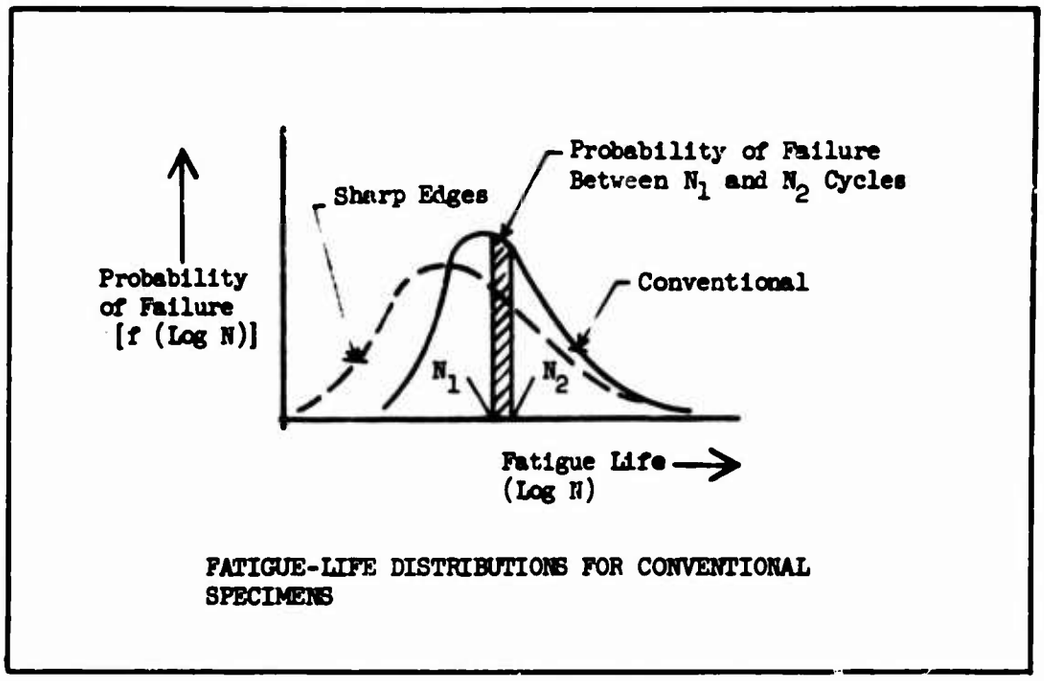
Chamfers are especially effective in improving fatigue strength, when they are machined along sharp edges so that they eliminate perpendicular machining marks. However, it must be remembered that chamfering is simply replacing one sharp edge by two moderately sharp edges. Thus the improvement in fatigue strength for chamfering is seldom as great as when a sharp corner is carefully broken or when a fillet is machined along the edge.

Sharp edges left by shearing sheet metal are especially detrimental to fatigue strength; the problem is compounded when forming (bending or stretching) follows a shearing operation. Since it is seldom possible to insure that stamped components are not highly stressed in a direction parallel to their sheared edges, subsequent operations such as grinding, milling, or peening to eliminate burrs and sharp edges in local areas are often necessary. Sometimes, fatigue problems can be eliminated simply by providing a hemmed flange, made by folding the flange back on itself.

The effect of sharp edges on fatigue strength is often confused with shape-effect. Shape-effect data is generally explained in terms of volume of highly stressed material, cyclic dependent material behavior, localized inelastic behavior, etc. None of these factors normally has as much influence on fatigue strength as sharp edges, especially for steels between Rockwell "C" scale 45-60. For these hardnesses, rectangular specimens may be only one-half as strong in fatigue as circular specimens.

The severity of the effect of sharp edges on fatigue life is shown in the adjacent figure. Mean life is decreased; in a specimen with sharp edges, the test data also displays much more pronounced scatter. Assuming that only a small percentage of failures is acceptable, the detrimental effect of sharp edges is magnified because of increased scatter.

In areas of high stress, feather edges are even more detrimental to fatigue strength than sharp edges. In such cases, a feather edge almost guarantees a fatigue problem.



FATIGUE-LIFE DISTRIBUTIONS FOR CONVENTIONAL SPECIMENS

SHARP CORNERS: The fatigue life of specimens with sharp corners is significantly reduced from the norm.

IMPROVE SURFACE FINISH TO IMPROVE FATIGUE STRENGTH

The depth and direction of surface-finish marks are important factors in the fatigue strength of a part.

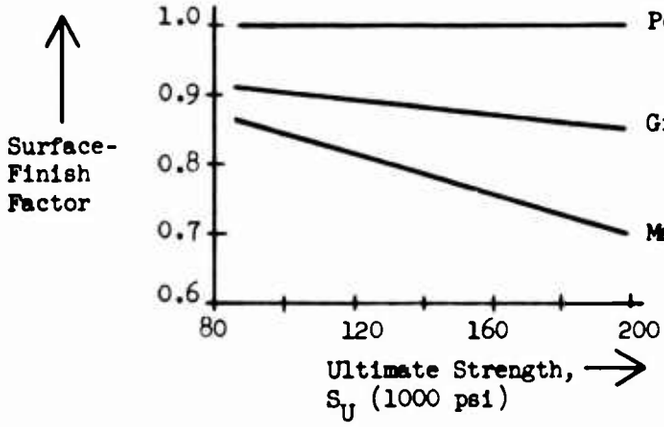
Surface finish is one of the most complex areas of fatigue analysis because residual stresses cannot be disassociated from true surface-finish effects. It is important therefore that data from curves such as those in the upper figure at the right be used with caution. Experience has shown that "direction" of surface finish is as important as the finish itself. The effect of surface finish on components produced by such processes as shaping, broaching, and milling depends largely on the number and size of the deepest surface-finish marks. Deep surface scratches should not lie perpendicular to the direction of stressing. Drawings should specify direction of machining for components critical in fatigue.

Most components are highly susceptible to failure at deep tool marks in fillets and other transition sections adjacent to hardened surfaces. The problem often arises because less care is taken in rough machining when stock is to be left for subsequent grinding. Deep tool marks are especially detrimental to fatigue strength in transition areas if the material is hardened and therefore more notch-sensitive, or if the material has been somehow weakened. Simple remedies to this problem are: greater care in machining transition areas, and routine grinding of troublesome transition areas.

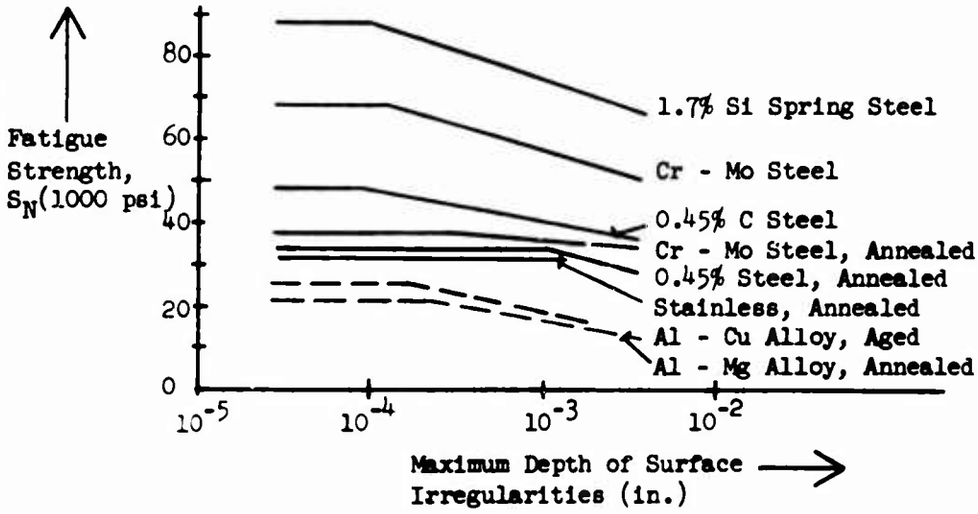
Assuming that deep tool marks are eliminated, there is a point of diminishing return in further improving surface finish, as shown in the lower figure. Thus, gentle grinding (finishing with a moderately light cut) will give the best results. Precise rms values are difficult to state because the effect of surface finish depends on the hardness of the material, and residual stresses are generally erratic in their effect on surface finish.

Gentle grinding has a secondary beneficial effect in that ground surface finishes tend to be more uniform from component to component. This reduces the magnification factor (as discussed in the section on sharp corners). Abrasive tumbling or shot peening is often beneficial in improving the fatigue strength of ground surfaces.

EFFECT OF SURFACE FINISH ON ULTIMATE STRENGTH OF MILD STEEL SPECIMENS



EFFECT OF SURFACE FINISH ON FATIGUE STRENGTH OF VARIOUS METALS



SURFACE FINISH: Tool marks and scratches caused by finish machinining affect the fatigue strength.

IMPROVED DUCTILITY AND IMPACT STRENGTH ALSO IMPROVES FATIGUE LIFE

High ductility and good impact strength are important considerations in selecting materials for finite-life fatigue applications.

High ductility (particularly notch ductility) and good impact strength are important considerations in selecting materials for finite-life fatigue applications. On the other hand, these parameters have little effect on fatigue limits. Conversely, material parameters such as hardness and tensile strength are important in establishing fatigue limits but may or may not be important in finite-life fatigue, depending on the mode of cyclic straining.

Fatigue life is dictated by the range of strain regardless of the alloy when cyclic strains fluctuate between fixed limits (see adjacent figure). As a rule of thumb, a range of strain of 0.02 inches/inch (± 0.01 inches/inch) i.e., 2 percent, gives a fatigue life of about 500 cycles for unnotched specimens of various structural alloys. Accordingly, the universal remedy for low-cycle fatigue applications (whether the material is notched or not) is to reduce the range of cyclic strain. Since the total range of strain is governed by Equation 1:

(where $\Delta\epsilon_t$ = total range of strain, in./in.; $\Delta\epsilon_E$ = elastic range of strain; $\Delta\epsilon_p$ = plastic range of strain) this reduction can be accomplished in two ways; by selecting materials with higher elastic moduli, and higher cyclic yield strengths. Cyclic strain hardening materials are preferable to cyclic softening for applications involving constant-amplitude loads. With the strain hardening material, the actual range of strain will diminish during fatigue stressing.

Notch ductility is not particularly important in long-life fatigue because nominal stress amplitudes are relatively small. At these stress amplitudes the bulk material experiences only elastic stresses. This means that the stress-strain behavior of the very small volume of highly stressed material at the root of a notch is governed by the elastic behavior of the bulk material. Accordingly, the material at the root of the notch appears to behave elastically with regard to range of strain, even though localized yielding may have taken place. Thus, fatigue strengths are not markedly influenced by the type of notch; that is, only the magnitude of K_t has primary importance.

For large nominal stresses, such as are imposed during low-cycle fatigue, much more material at the root of the notch is highly stressed. In addition, the bulk material itself may even exhibit some plastic deformation. Local strains are much larger than elastic strains computed without considering yielding. These strains may be so large that fatigue strength is dominated by the static strength of the notched material.

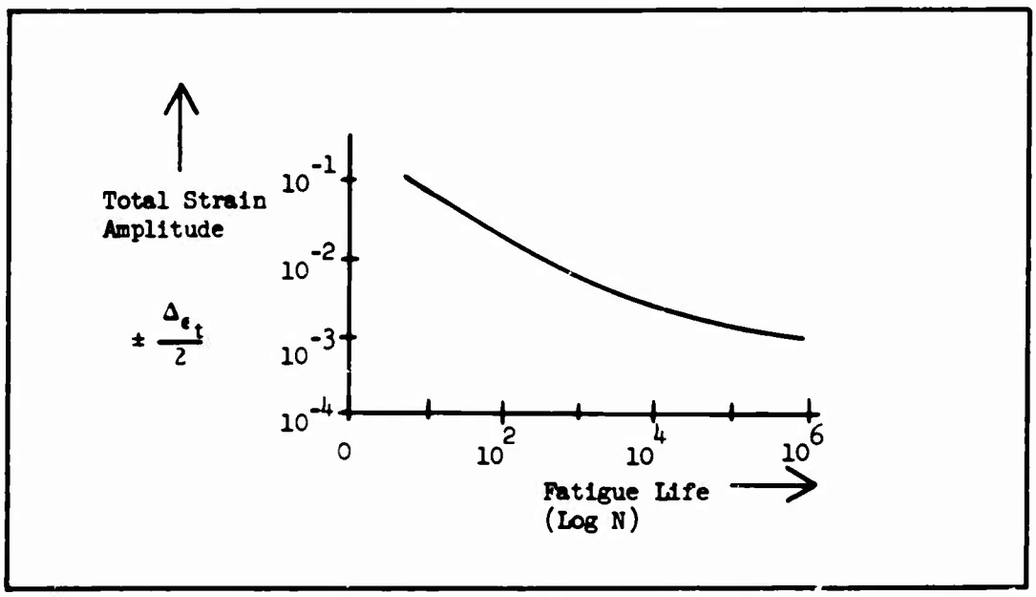
This means that the type of notch is very important in determining strength. For sharp circumferential notches the resulting three-dimensional state of stress results in notch strengthening. But for sharp two-dimensional notches (for example, V-notches in sheet stock)

the notched strength may be reduced to well below the yield strength of the unnotched sheet. Thus, sharp two-dimensional stress raisers should be avoided. When they cannot, a material should be selected with high notch ductility.

High notch ductility is even more important after a material has initiated a fatigue crack during service loading. Ideally, the presence of a fatigue crack only decreases the static strength in proportion to the cracked cross-sectional area. However, even materials with high nominal ductility are seriously weakened by fatigue cracks.

The influence of impact strength on selecting materials to obtain good finite-life fatigue characteristics is similar to that of notch ductility for cracked specimens, except that the cracked component is now viewed as a nonconventional impact specimen. The (potential) presence of a fatigue crack is sufficiently serious to warrant specifying good impact strength. Materials with sharp ductile-brittle transition temperatures should be avoided, especially where low service temperatures occur from time to time.

$$\Delta_{\epsilon_t} = \Delta_{\epsilon_E} + \Delta_{\epsilon_P} \qquad (1)$$



DUCTILITY: The relationship between total strain amplitude and fatigue life is illustrated.

VOLUME III - CHAPTER 5

FATIGUE

SECTION 5 - APPENDIX

- Bibliography
- Glossary
- Symbology
- Fillets and Notches
- Common Factors Influencing Fatigue Strength
- Values of I(A)
- Problem on Fatigue Damage During Random Vibration
- A Typical Application of the S-N Diagram
- Using the Modified Goodman (Constant-Life) Diagram
- Calculating a Safety Factor in a Repetitively Loaded Member

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GLOSSARY

Cycle - The complete sequence of values of a periodic quantity that occur during a period.

Degrees of Freedom - The number of degrees-of-freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the positions of all parts of the system at any instant of time.

Ductility - The property of undergoing considerable permanent deformation when tested in tension. This property is correlated with ability to be drawn into a wire. Ductility is measured by percent elongation and percent reduction of area from the tensile test.

Elastic Limit - The greatest stress which a material is capable of withstanding without permanent deformation upon release of stress.

Endurance Limit - The maximum stress to which a material may be subjugated many millions of times without failure. Ten million cycles without failure is generally regarded as indicating a stress below the endurance limit for steel.

Failure - An irreversible process of operation outside of specified tolerances, upon removal of the environmental load, the equipment remains inoperative or out of tolerance.

Fatigue - Progressive fracture of a member by means of a crack which spreads under repeated cycles of stress. Resistance to fatigue is often expressed in terms of "Endurance Limit," but this term pertains to a specific test generally.

Natural Frequency - The frequency of free vibration of the system. For a multiple degree-of-freedom system, the natural frequencies are the frequencies of the normal modes of vibration.

Random Vibration - Vibration whose instantaneous magnitude is not specified for any given instant of time. The instantaneous magnitudes of a random vibration are specified only by probability distribution functions giving the probable fraction of the total time that the magnitude (or some sequence of magnitudes) lies within a specified range. Random vibration contains no periodic or quasi-periodic constituents. If random vibration has instantaneous magnitudes that occur according to the Gaussian distribution, it is called "Gaussian Random Vibration."

Stiffness - The ratio of change of force (or torque) to the corresponding change in translational (or rotational) deflection of an elastic element.

Stress - Internal force exerted by either of two adjacent parts of a body upon the other across an imagined plane of separation.

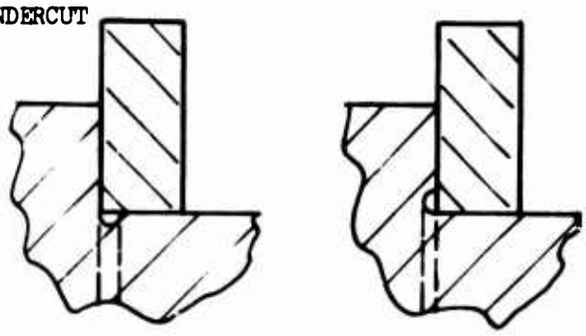
Transmissibility - Non-dimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio may be one of forces, displacements, velocities, or accelerations.

SYMBOLOGY

a	Characteristic of the varying load, the ratio of stress amplitude to mean stress
A	Characteristics of the S-N curve, where $NS^A = C$
a_{eq}	Equivalent sinusoid input level, g
C	Fatigue constant, $NS^A = C$
C_s	Stress factor, psi/g $S = C_s Q g_{in.}$
D	Damage Index, $D = \Sigma(n/N)$
f	Frequency, Hz
f_n	Natural frequency, Hz
$F(f)$	Input random vibration, g^2/Hz
g	Acceleration amplitude, 0 to peak, g
g_r	rms g level (response), g
g_{eq}	Equivalent sine level, response, g
$I(A)$	Integral which is a function of A
K	Sweep rate, octaves per minute
Kf	Fatigue strength reduction factor
M	Number of sine sweep cycles
n	Number of applied stress cycles
n_t	Total number of stress cycles
N	Number of stress cycles to failure
$P(S)$	Probability of occurrence of stress level S
Q	Amplification factor at resonance
R	Ratio of minimum stress to maximum stress
S	Stress, psi
S_a	Nominal alternating stress, psi
S_f	Fatigue safety factor
S_n	Fatigue strength, psi
S_{nt}	Fatigue strength from laboratory tests
S_r	rms stress level, psi
S_u	Ultimate strength, psi
t	Test time, minutes
t_R	Test time at resonance, minutes
t_T	Total Test Time

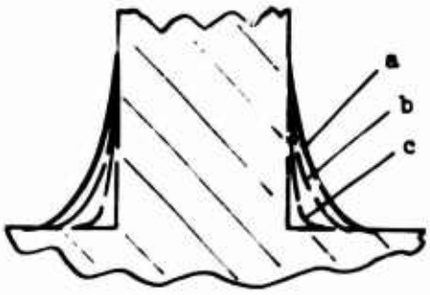
FILLETS AND NOTCHES

PROPER WAY TO UNDERCUT A FILLET



This not only decreases stress concentration, but also decreases nominal stress at the critical section.

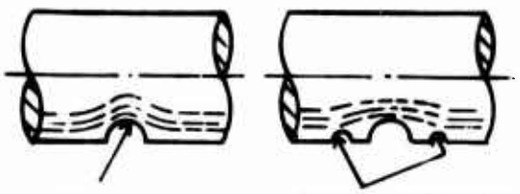
"STREAMLINED" FILLETS FOR VARIOUS TYPES OF LOADING



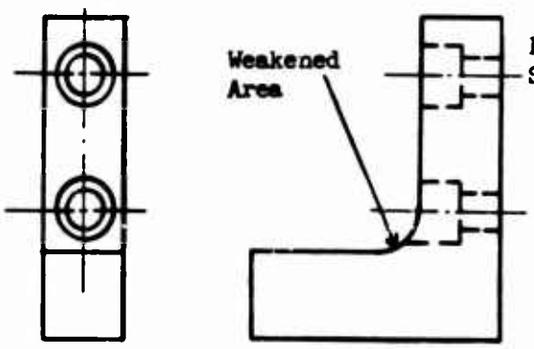
a) Plate in tension or compression
b) Plate in bending, and shaft in tension
c) Shaft in bending or torsion

Such fillets help lower stress-concentration factors.

EFFECT OF STRESS-RELIEVING NOTCHES ON STRESS FLOW LINES



Because stress concentrations are smaller with these notches flow lines become less abrupt.



EFFECT OF SUPERIMPOSED STRESS CONCENTRATIONS

In this case, running the spot-face into the fillet of the bearing support bracket can cause failure.

COMMON FACTORS INFLUENCING FATIGUE STRENGTH

Table 1 - Effect of Surface Hardening on Corrosion Fatigue Strength* of Steel

Material and Treatment	Fatigue Strength (% of original)		
	Air	Fresh Water	3% NaCl
1045 Steel (100% = 36,300 psi)			
Normalized, no surface protection	100	65	39
Short-period nitriding	140	142	77
Shot peening	116	...	79
Induction hardening	187	187	140
Induction hardening with subsequent zinc coating	...	...	177
Alloy Steel (100% = 78,100 psi)			
Nitrided	100	100	...
Alloy Steel (100% = 104,600 psi)			
Nitrided	100	81	...

*All steels have fatigue limits of 10^7 cycles; nitrided alloy steel with 104,600 psi fatigue strength has fatigue limit of 10^5 cycles.

Table 2 - Effect of Surface Rolling on Corrosion Fatigue Strength* of Various Steels

Ultimate Tensile Strength (1000 psi)	Corrosion Fatigue Strength			
	In Air (1000 psi)	In Air After Surface Rolling (%)	In Water (%)	In Water After Surface Rolling (%)
45.5	31.9	102	77	95
63.3	29.9	131	88	107
65.4	33.4	132	73	101
98.1	44.9	119	53	98
101.0	42.7	129	47	106
137.5	57.7	118	48	90
139.4	54.0	111	55	84

*Bending fatigue limit = 2×10^6 cycles.

Table 3 - Effect of Platings and Coatings on Corrosion Fatigue Strength of 1050 Steel

Type of Surface Protection	Corrosion Fatigue Strength (% of original)			
	Cold-Drawn (100% = 54,900 psi)		Normalized (100% = 36,700 psi)	
	Air	Salt Spray	Air	Salt Spray
None	100	14	100	24
Enamel	93	44	105	68
Hot galvanizing	101	95	90	101
Zinc electroplating	100	87	98	90
Cadmium electroplating	93	77	93	84
Cadmium plating and enamel	95	72	96	82
Phosphating and enamel	93	44	108	79
Spray metallizing with Al	105	80	...	...
Spray metallizing with Al and enamel	103	93	...	...

VALUES OF I (A)

$$I(A) = \int_0^\infty X^{A+1} e^{-\frac{x^2}{2}} dx$$

When A is an odd integer: $I(A) = [1, 3, 5, \ldots (A)]\sqrt{\pi/2}$

When A is an even integer: $(A) = 2, 4, 6, \ldots (A)$

A	I (A)	A	I (A)	A	I (A)
2	2	10	3.84×10^3	18	18.58×10^7
3	3.756	11	13.01×10^3	19	81.97×10^7
4	8	12	46.08×10^3	20	37.16×10^8
5	18.78	13	16.92×10^4	21	17.21×10^9
6	48	14	64.51×10^4	22	81.75×10^9
7	131.46	15	25.38×10^5	23	39.59×10^{10}
8	384	16	103.2×10^5		
9	1.183×10^3	17	43.14×10^6		

A is a parameter in the equation for an S-N curve, where $NS^A = C$.

PROBLEM ON FATIGUE DAMAGE DURING RANDOM VIBRATION

Illustrative Example:

Consider the following illustrative example (the equations evaluated are taken from page 5.3-3, Section 3).

The material has no endurance limit, the total test time is 30 minutes.

$$f_n = 100 \text{ Hz}$$

$$F(f) = 0.04 \text{ g}^2/\text{Hz} \text{ from 5 to 1000 Hz}$$

$$C_s = 1250 \text{ psi/g}$$

$$C = (75,000)^{10}$$

$$Q = 10$$

$$A = 10$$

$$I(A) = 3.84 \times 10^3 \quad \text{(See adjacent chart.)}$$

Solving Equation (10);

$$D = \frac{(1250\ g_r)^{10}}{(75,000)^{10}} (60)(30)(100)(3.84 \times 10^3)$$

Simplifing Equation (10);

$$D = \left(\frac{1250 \times g_r}{75,000}\right)^{10} 6.91 \times 10^8$$

From Equation (11);

$$g_r = \sqrt{\frac{\pi}{2} (0.04)(100)(10)} = 7.91$$

Completing Equation (10);

$$D = \left(\frac{9900}{75,000}\right)^{10} (6.91 \times 10^8) = 1.53$$

Therefore, failure is expected.

A TYPICAL APPLICATION OF THE S-N DIAGRAM

Assume a prismatic bar which is to be subjected to an alternating stress (load/cross-sectional area) of +60,000 psi (tension) maximum and -30,000 psi (compression) minimum. At this stress ratio (R), what is the anticipated fatigue life of the bar for the typical material shown in the S-N plot at the right?

$$\text{Stress Ratio (R)} = \frac{\text{Minimum Stress}}{\text{Maximum Stress}}$$

$$R = \frac{-30{,}000 \text{ psi}}{+60{,}000 \text{ psi}} = -0.5$$

Examination of the S-N plot will show that the worst case (lowest fatigue life for a particular value of maximum stress) will occur when R = -1.0. This represents the completely reversing stress situation. Most fatigue data is taken in the materials laboratory by specimens loaded in this manner; a rotating beam type specimen with a load in the center and supports at the ends.

Also of interest from the S-N plot is the relative increase in fatigue life (for a given maximum stress value) as the value of the stress ratio (R) becomes larger positively. As the value of (R) becomes very large, the maximum stress approaches the ultimate strength of the material.

For R = -0.5

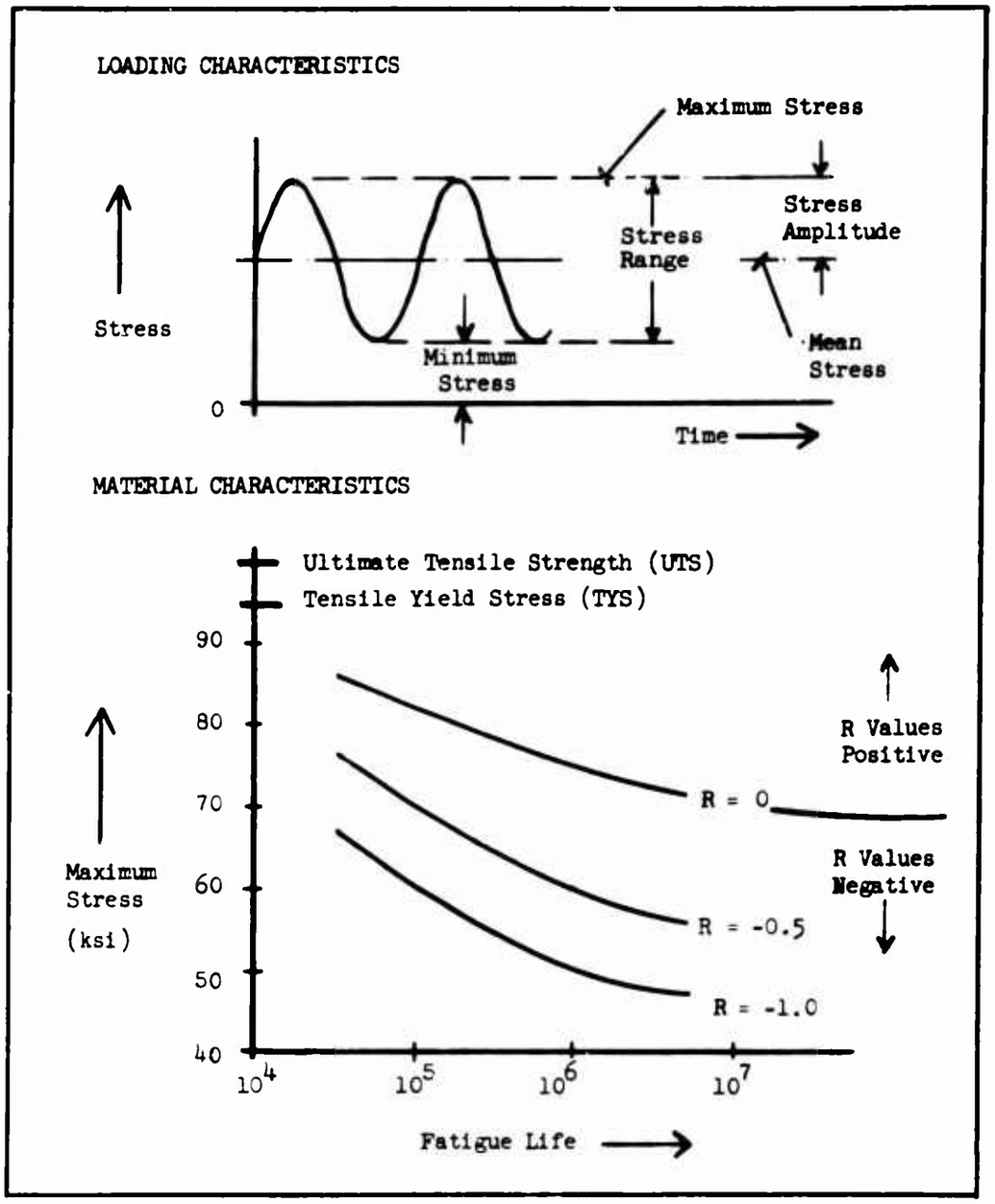
and Maximum Stress = +60,000 psi

Fatigue Life = 10^6 cycles

If the maximum stress were increased to 70,000 psi, at the same stress ratio R = -0.5 (minimum stress also increased to -35,000 psi), then the anticipated fatigue life will reduce to 10^5 cycles.

Similarly, if the designer can find a way to make the stress ratio more positive by altering the support geometry, then the expected fatigue life may be increased dramatically for the same maximum stress.

The designer should also note that most fatigue data, similar to that shown, is a plot of the mean experience at that particular set of parameters. In other words, 50 percent of the specimens will fail at this value and 50 percent will not. The data scatter is log-normally distributed, and adheres to the laws of the Gaussion function. The importance is in the potential erosion of the safety factor for a particular set of design parameters.



THE S-N DIAGRAM: Illustrated are families of fatigue curves for varying stress ratios.

USING THE MODIFIED GOODMAN (CONSTANT-LIFE) DIAGRAM

The Goodman plot synthesizes a theory of failure for loading situations where an alternating stress is superimposed on a steady stress. This plot (shown at right) defines a failure surface that is conservative; that is, most failure points (stress ratio parameters) fall outside of the arbitrary straight line. The Gerber failure surface is perhaps more realistic, but the Goodman theory represents a conservative approach and is easy to use.

The classic Goodman diagram has recently been modified into a constant-life diagram, which has utility for the stress analyst in manipulating the fatigue parameters of loading conditions and material properties, for a given or desired fatigue life. Constant life plots are assembled in recent handbooks (e.g., Reference 6) for a variety of engineering materials.

Shown at right in the lower plot, is the modified Goodman plot for a 7075-T6 wrought aluminum alloy. The test specimen had a polished (900 grit) surface finish, and exhibited ultimate and yield tensile strengths of 82 ksi and 72 ksi respectively. The loading direction was axial (tension and compression) for the constant-life diagram shown.

Assume that a service life of 10^5 cycles was required for a given situation using this alloy. If the steady-state stress (mean stress) was known, (say for example 20 ksi) what alternating stress or stress amplitude could safety be superimposed on this structure and still maintain the 10^5 cycle life requirement? Moving into the diagram at a mean stress value of 20 ksi, the alternating stress intercept at 10^5 cycles can be seen as 30 ksi. This corresponds to a loading situation where R = -0.2 and A = +1.5. The factor R as before, is

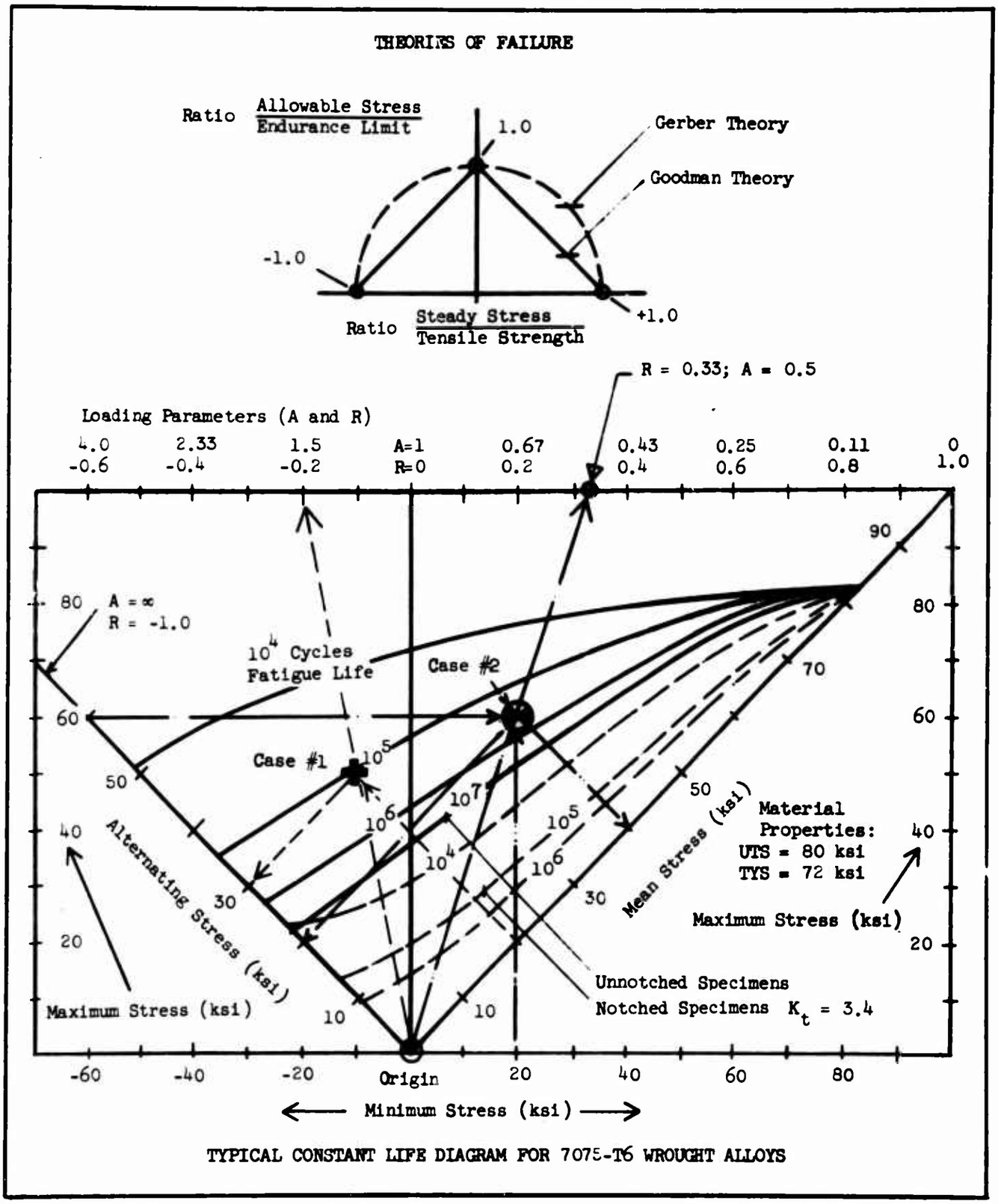
$$R = \frac{\text{Minimum Stress}}{\text{Maximum Stress}}$$

and

$$A = \frac{\text{Stress Amplitude}}{\text{Mean Stress}} \quad .$$

The loading parameters are determined by construction of a line through the origin, upwards to the A and R axis; A = +1.5 and R = -0.2 for this case. The importance of a change in stress ratio to fatigue life for a given alloy may be easily assessed from the diagram.

Assume another example to exercise the plot; for a maximum stress of 60 ksi and a minimum stress of 20 ksi, what would be the anticipated fatigue life for this alloy? Plotting S_{min} and S_{max} indicates an alternating stress of 20 ksi and a mean stress of 40 ksi. The loading parameters are R = 0.33 and A = 0.5, which checks with the loading situation. The anticipated fatigue life lies between 10^5 and 10^6 cycles.



TYPICAL CONSTANT LIFE DIAGRAM FOR 7075-T6 WROUGHT ALLOYS

CALCULATING A SAFETY FACTOR IN A REPETITIVELY LOADED MEMBER

Illustrative Problem: A rectangular steel bar with a circular hole in it is subjected to an alternating bending stress of 35,000 psi. The mean stress is 0 (that is, R = minimum stress/maximum stress = -1). The fatigue strength is 45,000 psi. The factor of stress concentration for repeated stress (the fatigue-strength reduction factor) is 1.20. What is the fatigue safety factor?

$$S_a = 30{,}000 \text{ psi}$$

$$S_N = 45{,}000 \text{ psi}$$

$$K_f = 1.2$$

Evaluating Equation (1): (from Section 4, page 5.4-3)

$$S_f = \frac{45{,}000}{(1.2)\ (30{,}000)} = 1.8$$

A more complete discussion of this topic appears in Chapter 4, Volume III, entitled "Stress Concentration." The topic illustrating the strength calculation for members subjected to repeated loads (page 4.3-4) treats this subject in detail. In summary, it may be said that the geometric stress concentration factor and material notch sensitivity may be used to estimate the fatigue life reduction factor. Further, the factor may be corrected for loading type, size effect, and surface finish. The corrected factor is then close to the actual structural situation and yields good results.

For additional information see R. J. Roark, "Formulas for Stress and Strain" McGraw-Hill Book Company, Articles 8, 9, 10 and Table XVII.

CHAPTER 6 – DYNAMIC SIMULATION

CHAPTER 6
DYNAMIC SIMULATION

ABSTRACT

The science of testing to simulate the dynamic environments has several useful functions for the equipment engineer. First, the required Quality Assurance Tests represent the first-line of design loads criteria upon which the strength of the package is scaled. The successfull completion of these tests also provides a level of assurance of the dynamic structural integrity of the equipment package. Secondly, the same test experiences may be used as developmental tests to establish dynamic parameters of the equipment support structure. Some of the test procedures may also be useful as procurement test standards for components and small equipment elements.

This chapter reviews some of the qualitative details of the Army's spectrum of dynamic tests, the machines used to accomplish these tests, and some insight into problems of fixturing, instrumentation, and failure-acceptance criteria. Most importantly, the dynamic excitations resulting from each test experience are delineated in the frequency domain, and the design implications of the experience discussed.

Chapter 6 - Dynamic Simulation

ERRATA SHEET

Page	Paragraph	Line	Correction
6.1-1	Graphic		Use Roman Numerals for Equipment Class Designations
6.1-4	4	8	MIL-S-901C
6.1-4	4	10	... onto the fixture and specimen.
6.2-4	2	11	... to the floor.
6.2-7	Lower Figure	Title	Longidudinal
6.2-8	5	8	... 30 millisec ...
6.2-9	1	6	... 50 millisec ...
6.3-8	1	3	... to over 30,000 lbs.
6.3-8	2	5	diameter

VOLUME III - CHAPTER 6
DYNAMIC SIMULATION

CONTENTS

Section		Page
1	INTRODUCTION	6.1-0
	• The Importance of Developmental and Quality Assurance Tests to Equipment Development Programs	6.1-0
	• Shock, Vibration, and Complex Excitations Resulting From the Quality Assurance Tests	6.1-2
	• Laboratory Methods for Dynamic Environment Simulation	6.1-4
2	SHOCK TESTING	6.2-0
	• Shock Test Requirements for Army Electronic Equipment Elements	6.2-0
	• A Physical Description of the Rail Car Coupling Test	6.2-2
	• Dynamic Excitations Resulting From the Humping Impact	6.2-4
	• Humping Impact and Its Effect on Dynamic Structural Integrity of Equipment Packages	6.2-6
	• Characteristics of the Shaped Pulse Shock Tests	6.2-8
	• The Light Weight, High Impact Hammer Test for Ballistic Shock	6.2-10
	• Excitations and Design Implications of the Ballistic Impact Test	6.2-12
	• Details of the Bench Handling Drop Test	6.2-14
	• Shock Loads Resulting From the Transit Drop Tests	6.2-16
3	VIBRATION TESTING	6.3-0
	• Vibration Test Requirements for Army Electronic Equipment	6.3-0
	• Some Vibration Testing Techniques and Procedures	6.3-2
	• Random Vibration Testing Procedures	6.3-4
	• Details of the Mechanical Vibration Test Machines	6.3-6
	• Details of the Electrodynamic Vibration Test Systems	6.3-8
	• Details of the Hydraulic Vibration Test Systems	6.3-10

CONTENTS (Continued)

Section		Page
4	TESTING FOR COMPLEX ENVIRONMENTS	6.4-0
	• Qualification for Road Mobility - The Munson Road Test	6.4-0
	• Dynamic Excitations Resulting From the Road Mobility Tests	6.4-2
	• Random Excitations Resulting From the Bounce Tests	6.4-4
5	APPENDIX	6.5-0
	• Bibliography	6.5-0
	• Glossary	6.5-1

VOLUME III - CHAPTER 6

DYNAMIC SIMULATION

SECTION 1 - INTRODUCTION

- The Importance of Developmental and Quality Assurance Tests to Equipment Development Programs
- Shock, Vibration, and Complex Excitations Resulting From the Quality Assurance Tests
- Laboratory Methods for Dynamic Environment Simulation

THE IMPORTANCE OF DEVELOPMENTAL AND QUALITY ASSURANCE TESTS TO EQUIPMENT DEVELOPMENT PROGRAMS

The equipment designer has two categories of dynamic tests to deal with; the developmental test to investigate load paths and structural responses; and the Quality Assurance Provisions which are contractually required.

There are two important sources of design loading criteria for Army equipment package; the service environment which the equipment is constrained to operate within, and the Quality Assurance provisions which the equipment is contracturally required to meet. In general, the equipment designer is mainly concerned with meeting the contractural test requirements delineated in paragraph 4.0 of the equipment procurement specification. The degree to which the Quality Assurance tests represent the actual service environment is the responsibility of the procuring agency; the equipment MUST pass the specified dynamic tests.

The range of Quality Assurance tests normally employed by the Army are of the demonstrational type, that is the equipment is required to demonstrate its ability to survive a group of random dynamic tests such as bounce, road mobility, and railroad humping. The exact spectra of tests normally specified for Army equipment systems are tabulated by equipment class in the accompanying figure. The equipment must survive the test experiences without structural failure, or without functional degradation, depending upon the specified acceptance criterion; operating or non-operating.

Since the equipment specimen experiences some go-no-go type test experiences, there has been a tendency on the part of the packaging designers to fabricate an equipment package purely on a functional basis, and gloss over the structural analysis in favor of the Quality Assurance test, which will pass or fail the equipment. The obvious weakness in this approach is the late awareness of structural problems (which are usually solved on a crash basis late in the program, at great expense), or the gross overdesign of the structure resulting in a weight penalty. Neither approach is totally satisfactory. The answer to this problem is a better definition of the transfer characteristics of the dynamic energy passing through the equipment.

The input excitations presented in this chapter are representative of the range of loads to be expected from given test requirements. These input loads are for SYSTEM inputs; that is, the loads are those which may be expected at the interface of the test specimen and the test machine. The input excitations must be used carefully during preliminary design, since differences in equipment stiffness and weight (or natural frequency), and damping characteristics (which affects resonant rise) will have a feedback effect on the test machine. In some cases, this effect may change the magnitude and frequency characteristics of the input load.

The best approach to define firm input loads and transfer functions in complex equipment packages is a well conceived developmental test program. Developmental tests are conducted on structurally similar models during the preliminary stages of an equipment program to determine response characteristics of the support structure and bracketry. As the equipment takes shape, the same tests may be repeated on more representative hardware to establish confidence that the equipment will pass the Quality Assurance tests. In this manner, more efficient structure may be generated for the equipment package since the dynamic loads and energy transfer characteristics will be firmly established.

The equipment designer thus has two categories of tests to apply to the equipment package structure; the developmental test which aids in the definition of loads, load paths and responses; and the Quality Assurance provisions which he must contracturally pass within prescribed limits, and which generally dictate the design loads criteria.

EQUIPMENT CLASS SUMMARY

1. Transported as loose cargo – Man-packed
2. Vehicular Mounted – Non-operating
3. Vehicular Mounted – Operating
4. Tracked Vehicular Mounted – Non-operating
5. Tracked Vehicular Mounted - Operating
6. Airborne – Operated or Transported

QUALITY ASSURANCE TEST REQUIREMENTS SUMMARY

		← Class →					
		1	2	3	4	5	6
SHOCK	Ballistic				X	X	
	Bench Drop	X	X	X	X	X	
	Shipping Drop	X	X	X	X	X	
	Shaped Pulse						X
	Railroad Hump		X		X		
VIBRATION	Survey	X	X		X		X
	Dwell						X
SPECIAL	Cargo Bounce	X					
	Vehicular Bounce		X	X	X	X	
	Road Mobility		X	X	X	X	

EQUIPMENT CLASS AND QUALITY ASSURANCE TESTS: Army equipment systems are normally required to pass a set of dynamic tests, as dictated by the equipment class.

SHOCK, VIBRATION, AND COMPLEX EXCITATIONS RESULTING FROM THE QUALITY ASSURANCE TESTS

The dynamic tests required by Army Quality Assurance provisions may be categorized into three groups according to characteristics of the excitation; shock, vibration, and complex.

The dynamic test environments discussed in this chapter will be categorized into three groups; vibration, shock, and special test requirements. Each of these test categories and all of the individual tests included within them are required by Quality Assurance Provisions of the equipment contract.

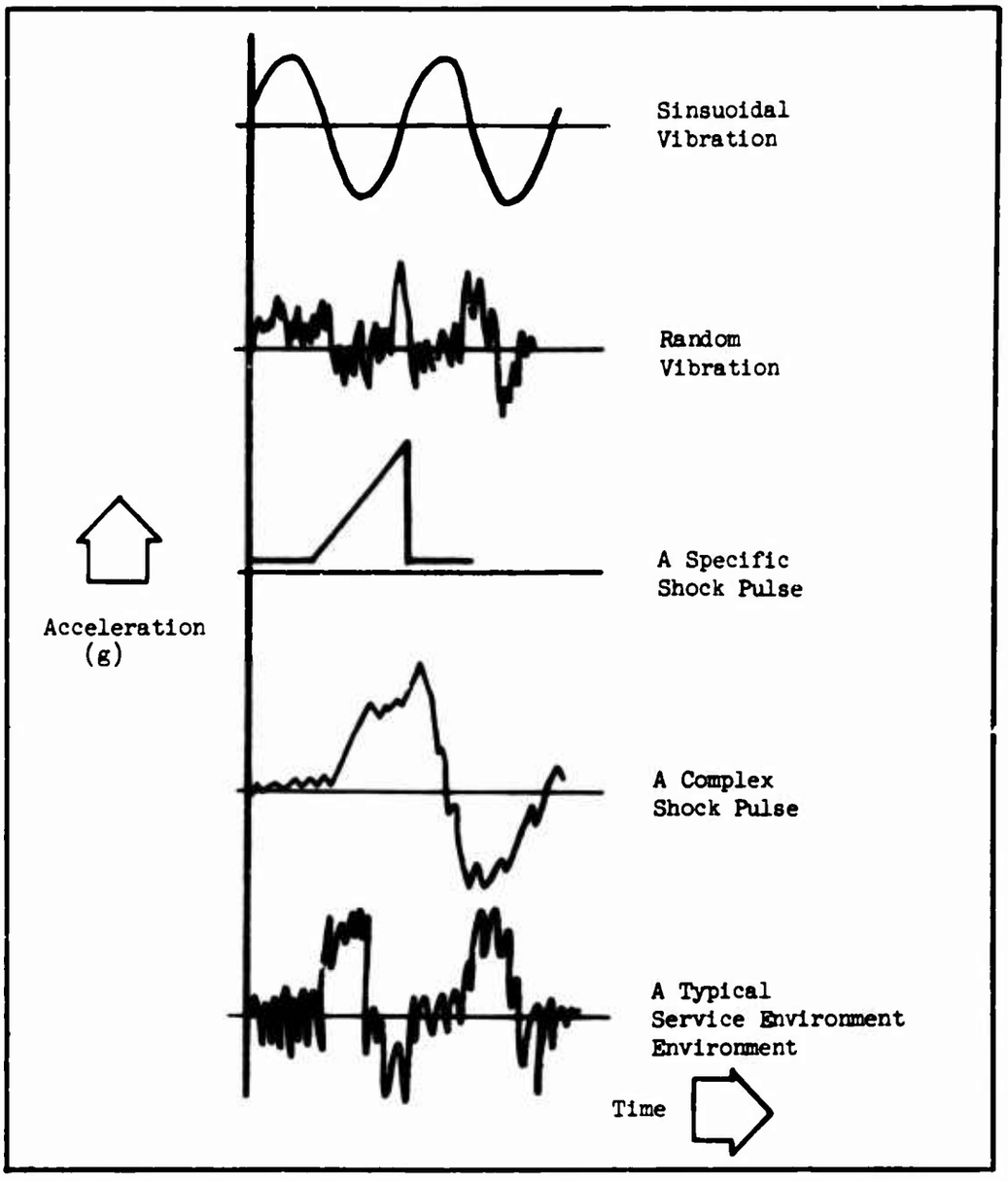
Vibration is a mechanical oscillation featuring a displacement about a central point or equilibrium datam level. Vibration is inherent in most modes of transport, particularily transport by aircraft or ship, as well as ground vehicles. Failure of equipment structure as a result of prolonged vibration may take two basic modes; a fatigue of the structural material which is manifest as a crack leading ultimately to fracture; and/or an excessive excursion of the equipment elements resulting from a resonant rise excited by repetitive loading at a frequency numerically near the natural frequency of the equipment. Vibration inputs encountered in the range of Quality Assurance tests may be sinusoidal, as in the case of the resonant survey, or they may be random as typified by vehicular bounce.

Shock is characterized by a sudden, relatively large, displacement within a short time period which tends to develop significant forces within a system. Shock loads occur in a variety of ways during the service environment. Bounce and drop during transport as well as ballistic impact of a near-miss are examples of shock loadings common to Army equipment. Failure of equipment exposed to shock energies may be manifest as initial fracture from the overload resulting from dynamic amplification; overexcursion of the equipment elements due to the impact causing collision of adjacent elements; or in rare cases a fatigue failure resulting from the excitation of structural elements at their natural frequency due to the shock impact. Shock loads resulting from the specified Quality Assurance tests may be complex in their time characteristics (e.g., railroad humping) or they may be tailored to a specific pulse such as the crash safety requirement for airborne equipment. The shock information presented for design criterion in this section is given in frequency spectra format, a style that is readily adapted to structural analysis.

Certain of the special Quality Assurance tests contain elements of both shock and vibration excitations. Bounce tests and road mobility tests are examples of this category of dynamic excitation. The dynamic disturbances into the base of the equipment elements are random and complex and are most conveniently displayed as power spectral density plots. Failure of equipment elements due to this special environment may follow the form of either the vibration or shock failure modes. This test category is used to demonstrate endurance of equipment packages, in lieu of a sinusoidal vibration dwell. The excitations, being complex, are quite representative of the actual service environment and thus are valuable demonstrations of dynamic structural integrity of field equipment.

The demonstration of equipment integrity by dynamic testing demands the best example of the equipment execution. Quality Assurance tests are frequently aborted by such avoidable mistakes as loose nuts, incomplete assemblies, and loose debris left in electronic chassis which may cause

malfunction. The specimen presented for Quality Assurance evaluation should be the best example of the equipment package. Similarily, developmental tests should reflect the best representation of the equipment masses and their orientation, if valid results are to be anticipated.



DYNAMIC EXITATIONS: The shock and vibration environment imposed on Army equipment systems may be quite complex.

LABORATORY METHODS FOR DYNAMIC ENVIRONMENT SIMULATION

The Quality Assurance test provisions employed to qualify Army equipment for service may be conveniently classified into shock, vibration, and special categories.

The preceding topic outlined the arbitrary categorization of the Army Quality assurance tests by test type; shock, vibration, and special tests. It remains for the test engineer to implement these excitations by test machine, fixturing, and operational procedures consistent with the Quality Assurance requirements. There are several significant choices which must be made, the details of which are developed in the following paragraphs.

Shock: Shock has been defined in terms of a sudden positional change, significantly high accelerations, relatively short pulse time durations, and a non-periodic excitation characteristic. The shock pulse is created in the laboratory by one of two fundamental means; the specimen is allowed to impact against a prepared surface or body, or a high impact is generated by allowing the test implement to impact against the specimen.

The first shock category (i.e., specimen impacting against a stationary body) is typified by the drop tests. Bench drop, for example, employs no fixturing and is simply a drop onto a prescribed surface from a given height. Similarily, shipping drop imparts a shock representative of that encountered in shipping handling and accidental drop. The shaped pulse test is also accomplished by dropping the specimen from a predetermined height (to accomplish the desired peak acceleration) onto a prepared impacting surface. The surface characteristics determine the shape of the shock pulse, as plotted on an acceleration-time scale.

Ballistic shock and railroad coupling (humping) impact are examples of the shock testing category where the specimen is impacted with an energetic test implement. Ballistic shock for Army equipment is designed to simulate the shock resulting from a near miss shell impact, or nearby explosion. The impact pulse is sharp and intense, that is, a relatively high acceleration for a short time duration. Ballistic shock is simulated with the Light Weight, High Impact Hammer, a Navy testing device for shock testing in accordance with MIL-STD-901C. The hammer may impact the specimen and fixture in a horizontal direction (after swinging from a predetermined height), or may drop vertically onto the specimen.

Railroad humping impacts are designed to simulate the shock experienced as the rail cars are coupled. Coupling may occur at speeds up to 20 mph; the Army criterion is 7 mph which represents an average experience. The specimen is attached firmly to the bed of the rail car, backed up by another loaded car, and coupled at 7 mph by a third car. The resulting shock is severe and complex and represents a formidable design criterion. There is a great deal of variability of loading from the rail hump test due to variations of blocking and tiedown stiffnesses, as well as differences in the response characteristics of the specimen.

Vibration: Vibration, as it is used to qualify Army equipment, is a sinusoidal, steady state experience. The vibratory excitation is a mechanical oscillation about a reference point of equilibrium. The acceleration-time plot of the excitation is intended to be as close to a sine wave as the physical restrictions of the test machine allow.

There are three widely used methods for generating the desired vibratory oscillation; mechanical, hydraulic, and electromagnetic. Each of these types of vibration testing machines have their own peculiarities and limitations. The characteristics of the machines, their operational advantages and disadvantages, fixturing requirements, and other details are discussed in depth in a following topic. The intent of the vibration investigations for Army equipment qualification is to determine the main resonant frequencies of the equipment, and to establish the extent of amplification at these resonances. The dwell or sustained aspect of the oscillating environment is left to the bounce and mobility tests for verification. An exception is the airborne equipments, Class VI, which require a resonant dwell during qualification.

Special: An important group of tests which are hybrids from the previous shock and vibration categories are the bounce and road mobility test requirements. These test excitations contain elements of both shock and vibration and cause a random disturbance in the specimen.

Both the cargo bounce and vehicular bounce tests are performed on a bounce test machine of suitable capacity; 1000 lb and 5000 lb capacity machines are common. The bed of the machine is oscillated by a motor driven eccentric, which imparts the bouncing motion to the test package. The object of this test is to simulate the experience that a loose package would feel as it is being transported by ground vehicle in the field. Some differences between the vehicular bounce and cargo bounce tests are:

1. The specimen is mounted to a steel base plate for the vehicular bounce test and is loose for the cargo test.
2. The input is controlled by machine speed for the cargo test while an input range is specified for the vehicular bounce test.
3. Bumpers are employed for the vehicular test while wooden fences are used to restrict the specimen during the cargo bounce test.

One of the most effective and environmentally realistic of the Quality Assurance provisions are the road mobility tests. In these tests, the equipment is installed in its transport vehicle which in turn is driven over the Munson and Perryman road mobility test courses. The Munson course is a collection of rough road experiences which represent most road conditions to be found in service. The usual mobility requirement is five trips around the Munson circuit, which includes six-inch washboard, belgian block, radial washboard, spaced bump and two-inch washboard roadways. The vehicle speeds used for each road segment are somewhat dependent upon the vehicle capabilities. The Perryman course is a collection of cross-country terrains ranging from a moderately rough experience on a substantial roadbed, to a track of extremely rough terrain including marshy areas. Normal procedure calls for equipment to be transported over the Perryman courses for 200-300 miles during qualification. Vehicles designed for Army service, by contract, are driven over the Perryman courses until failures are precipitated.

Some other tests which are occasionally used to qualify Army equipment include the vibration isolation test and the acceleration test, which are demonstrational, and do not normally constitute a severe design loading criterion.

VOLUME III - CHAPTER 6

DYNAMIC SIMULATION

SECTION 2 - SHOCK TESTING

- Shock Test Requirements for Army Electronic Equipment Elements
- A Physical Description of the Rail Car Coupling Test
- Dynamic Excitations Resulting From the Humping Impact
- Humping Impact and Its Effect on Dynamic Structural Integrity of Equipment Packages
- Characteristics of the Shaped Pulse Shock Tests
- The Light Weight, High Impact Hammer Test for Ballistic Shock
- Excitations and Design Implications of the Ballistic Impact Test
- Details of the Bench Handling Drop Test
- Shock Loads Resulting From the Transit Drop Tests

SHOCK TEST REQUIREMENTS FOR ARMY ELECTRONIC EQUIPMENT ELEMENTS

Shock tests are required for all Army electronic equipment classes. Shock excitations are simulated in the laboratory by impacting the specimen with an energetic object, or allowing the specimen to impact against a prepared surface.

Shock tests, in support of the Quality Assurance test phase of an Army equipment program, are required for all classes of equipment. Normally, shock tests of one or more types are constrained to each of the equipment classes, as outlined in the test matrix presented in a previous topic. For example, Classes I - V equipments generally are required to pass bench handling and shipping drop tests; Class IV equipment requires a ballistic impact test; Classes II and IV equipments require a railroad humping demonstration; and Class VI equipment is constrained to survive a shaped pulse impact simulating an aircraft crash condition.

For descriptive convenience, the various shock tests may be categorized into four groups, as follows:

1. Drop tests, which include the bench handling requirements for equipment elements taken apart for repair or servicing, and shipping drop which simulates the impact an equipment system might experience in the field as a result of an inadvertant fall during handling.

2. Shaped pulse tests, where the equipment (or component) is required to survive a specific pulse (either terminal sawtooth or half-sine shaped) of given intensity and duration.

3. Ballistic shock tests, which are high intensity hammer impact tests conducted on a light weight, hammer drop shock test machine, intended to approximate the near-miss impact of a shell or explosion.

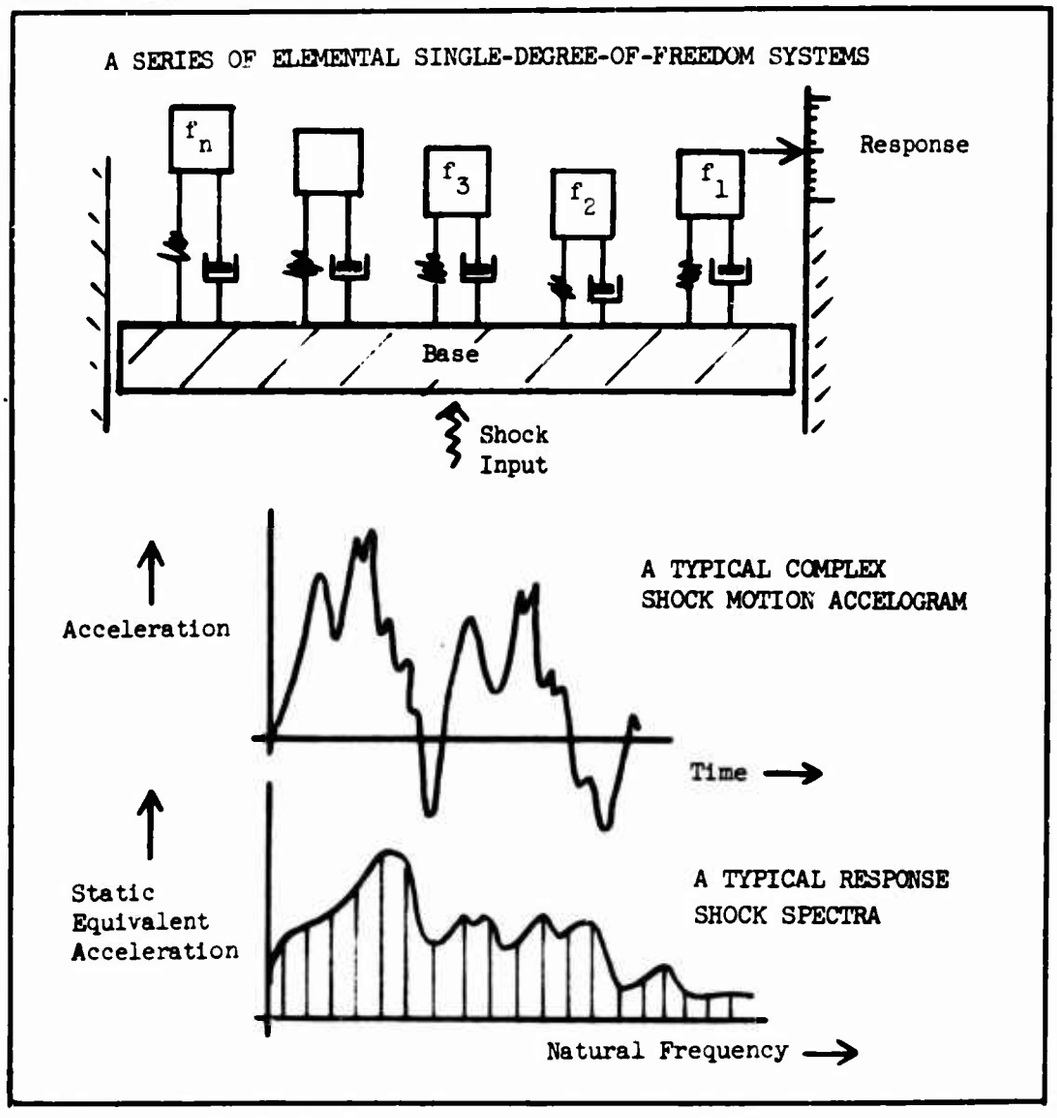
4. Railroad humping impact, which simulates the impact generated by coupling impact of rail cars. This test is usually conducted with actual rolling rail equipment, and is constrained to those equipments normally shipped by rail without a shipping container, such as a heli-hut module filled with electronic gear.

Shock, as it is used in dynamics, is characterized by sudden positional changes, short duration energy impact, and relatively large displacements which often create significant internal damage in electronic equipment systems. Shock is simulated in the laboratory by one of two basic methods, regardless of the shock category; a stationary test specimen is impacted by some energetic object, or the specimen itself is allowed to impact into a stationary surface at a predetermined velocity.

The shock environment constrained to Army equipment systems is inherently complex. The time-history records, accelograms, resulting from the specified shock tests are complex functions which necessitate some practical data reduction method to extract useful design parameters. The response shock spectra has come to be a useful analytical tool for the dissemination of the shock experience. Essentially, the method will indicate the predominate frequencies existing in an equipment structure in response to a given shock impulse. This impulse function may be a measured input to a

complete system, or may be the response of a specific element within the system.

Roughly defined, the shock spectra represents the maximum response of a series of single degree-of-freedom systems to a given shock experience, as a function of the fundamental frequency of the elemental systems. The shock spectra plot is given in terms of static equivalent acceleration vs. natural frequency, and is the recommended method of analysis presented in Volume II of this design guide. The shock spectra may also be expressed in units of displacement or velocity; it is often desirable to use all three forms.



SHOCK RESPONSE: The complex excitations imposed on Army equipment by the shock tests may be conveniently defined by the shock spectra.

A PHYSICAL DESCRIPTION OF THE RAIL CAR COUPLING TEST

The rail humping test is usually conducted with standard rolling stock, equipped with standard draft gear.

Railroad "humping" is a test requirement pertaining to Classes III, IV, and VI equipments; Classes III and IV deal with equipments that are shelter mounted or vehicular mounted, while Class VI equipments are armored vehicle mounted. Each of these classes could potentially be shipped by rail. The equipment would usually not be expected to function while in transit or motion, but rather would be required to operate as intended upon reaching the destination after rail transport. The humping procedure is a term derived from the switching or coupling operation associated with a group of rail cars. The switching "hump" is a carry-over term describing the raised portion of track in a switching yard. The cars to be coupled are moved to the peak of the hump and then released, imparting a velocity sufficient to activate the coupling mechanism. These velocities average about 5 mph and range to 12 mph as an upper limit. The Army humping requirement is designated at 7 mph impacting speed, a good practical number toward the upper end of the operational average. (3,4)

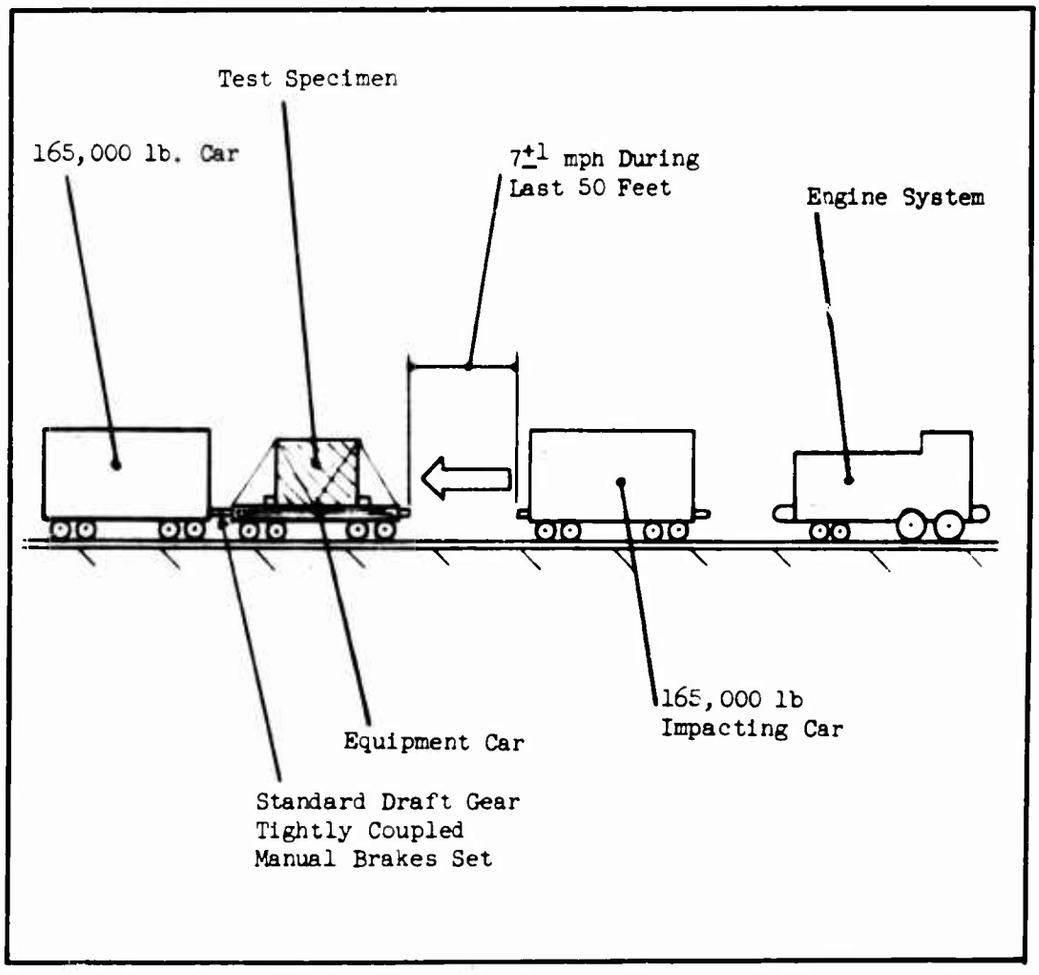
The humping test is usually performed with actual rolling stock, although recent developments include a ramp and abuttment setup to simulate the spectrum of frequencies and accelerations encountered in the actual coupling impact.(5) The humping test is conducted with two cars, each with a gross weight of 165,000 lbs., and a flat car (or other typical transport rail car) with the test equipment mounted to its floor. The mounting and stabilizing techniques must be in accordance with the "Standard Railway Freight Mounting Procedures" associated with cargo of the type under test. Generally, the larger modules are blocked and cabled in position, with the blocking tied laterally into the flatcar stake pockets through 6" x 6" wood beams. The blocking thus provides a certain amount of compliance which tends to soften the impact. Any failure in the blocking or cabling is considered an inadequate test and must be repeated after repairing the gear.

The impact velocity is specified at 7 ± 1 mph measured in the last 50 feet before the impacting car collides with the equipment car. The equipment car must be backed up by a separate 165,000 lb. car, similar to the impacting car. Both stationary cars must have their manual brakes set, and must be coupled together such that no play exists between them. All the rail cars must be equipped with American Association of Railroads standard draft gear. Some special draft gear currently available will substantially attenuate the humping shock and thus are not representative of standard gear likely to be encountered during military rail operation anywhere in the world.

The equipment car is impacted from each end while the test equipment is mounted both longitudinally and laterally, for a total of four coupling impacts. An exception is made in the case of an equipment that cannot physically be rotated on the railcar bed; the test is then conducted twice on each end, for a total of four impacts, as before. Upon completion of the four impacts, the equipment is constrained to meet the normal

operational requirements under ambient conditions without significant performance degradation. The rail humping test is always conducted under ambient environmental conditions.

Although the humping test is of the go/no-go variety, instrumentation is usually desirable to enhance the backlog of empirical knowledge on this extreme test. Accordingly, a straight level stretch of track at least 1/4 mile long is indicated, with an adjacent area or road for an instrumentation car or shelter. In addition, the prime mover must be able to switch ends of the car-test setup, necessitating a shunt track available to the engine.



RAILROAD HUMPING: A severe shock test for cargo to be shipped by rail, simulating the impact associated with coupling of standard draft gear rail cars.

DYNAMIC EXCITATIONS RESULTING FROM THE HUMPING IMPACT

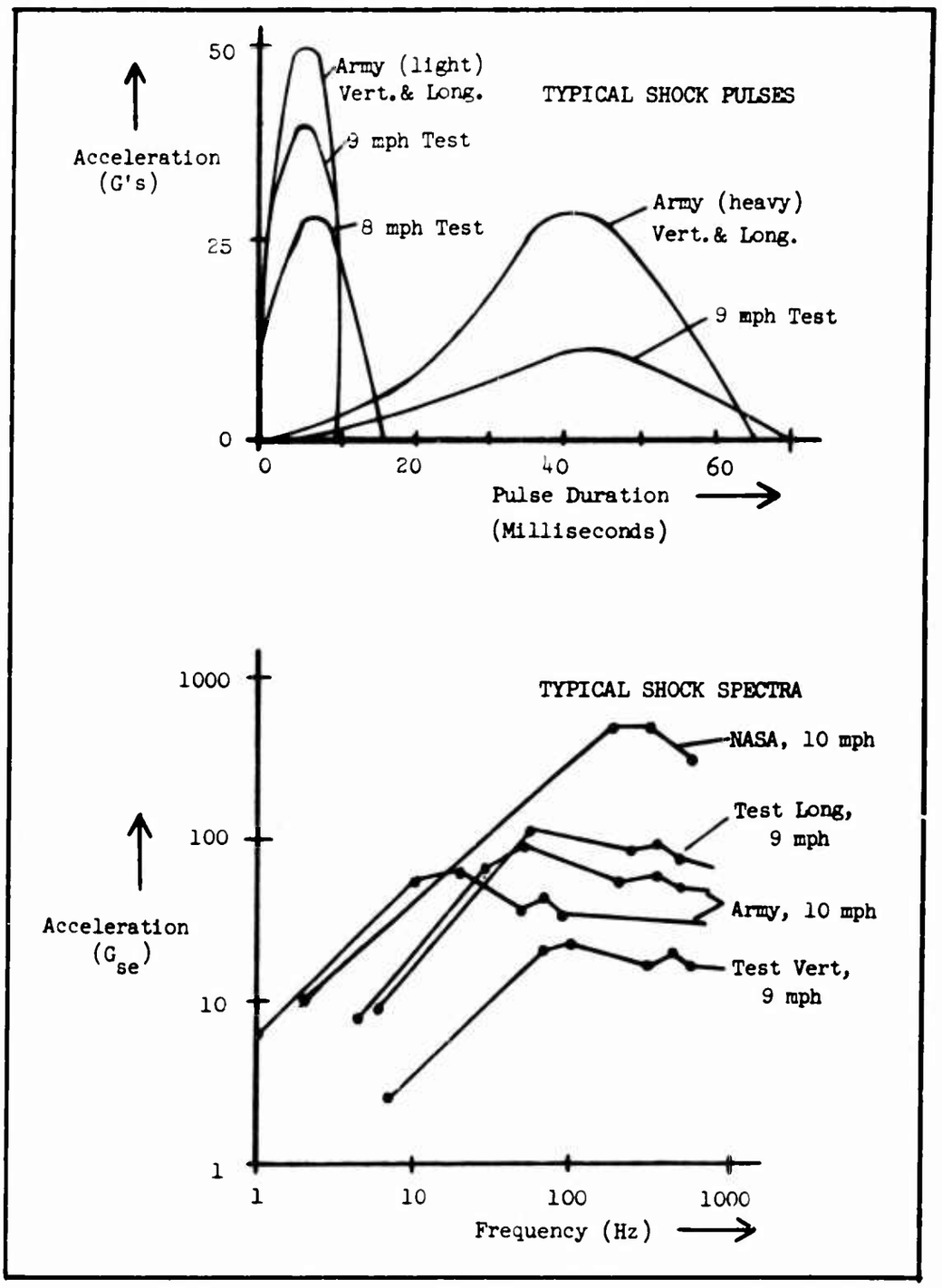
The shock pulse resulting from the railroad humping impact may vary substantially. The net effect of the coupling impact is, in all cases, a severe design constraint.

The humping requirement is perhaps the most severe of all the Army dynamic quality assurance procedures. The accelerations encountered are extreme (to 100 G's and higher at certain frequencies) and the pulse duration long (to 50 milliseconds) which together may manifest as a critical damage potential to sensitive equipment. To further complicate the issue, there is a pronounced variability in the input pulse to the equipment under test. Most of this variation is due to differences in rail cars and their draft gear as well as variations in track alignment, spacing, and lineup of the coupling devices. Other differences may be traced to the manner in which the data was acquired and transformed and a feedback effect on the test bed inputs resulting from impedance variations of the test specimen, particularly weight, natural frequency, and the amount of damping present.

The adjacent figure presents a time plot of various shock pulses attributed to rail humping by several sources, including some test data on actual equipments. The most significant pulse for Army equipment requirements was taken from Army transport criteria.[6] Other pulse information is presented from a NASA document on transportation criteria [3] and tests conducted on a range of equipments.[7,8,9] The Army criteria is given for coupling impacts at 10 mph, and is adjusted artificially for the 7 mph test speed requirement. A pulse is given for two types of cargo; "heavy" cargo which presents a significant footprint on the railcar bed and has a C.G. some distance above the floor; "light-dense" cargo which is very stiff and mounts close to the deck. It is significant to note that the equal criteria presented for both the vertical and longitudinal directions implies that excitations are equally damaging longitudinally as well as vertically, a fact that is not always apparent in this type of impact.

The second adjacent figure presents the shock spectra taken from several sources as well as some test validation. The spectra of those impacts judged to be most pertinent to the Army requirement lead to an envelope (in the frequency domain) which represents a reasonable design criteria for equipment to be subjected to the humping impact at 7 mph.

The shock spectra are transforms of the half-sine pulses pictured in the upper figure shown on the adjacent page. This approximation may be made by standard analytical transforms such as that presented in Reference 13. The response spectra are representative of a single-degree-of-freedom model with no damping subjected to a half sine shock pulse. Obviously, this acceleration plot will suffice as a first-cut approximation only, and should be substantiated by test data obtained from a structurally similar model of the equipment under consideration.



HUMPING IMPACT: There is a great deal of variability in the input shock resulting from the rail humping experience.

HUMPING IMPACT AND ITS EFFECT ON DYNAMIC STRUCTURAL INTEGRITY OF EQUIPMENT PACKAGES

The shock spectra resulting from the railroad humping impact excites most major structural frequencies. The pulse is bi-directional and extremely destructive.

The accelerations and frequencies presented in the preceding plot of a design spectrum for railroad humping impact is at least a very imposing requirement. Only very rugged equipment can be expected to survive the shock, a consideration which the equipment designer must address early in his design concepting. In fact, the accelerations are such that serious consideration should be given to a weight tradeoff study to determine whether beef-up and internal isolation is as efficient as a shipping support system, designed specifically to attenuate some of the coupling impact.

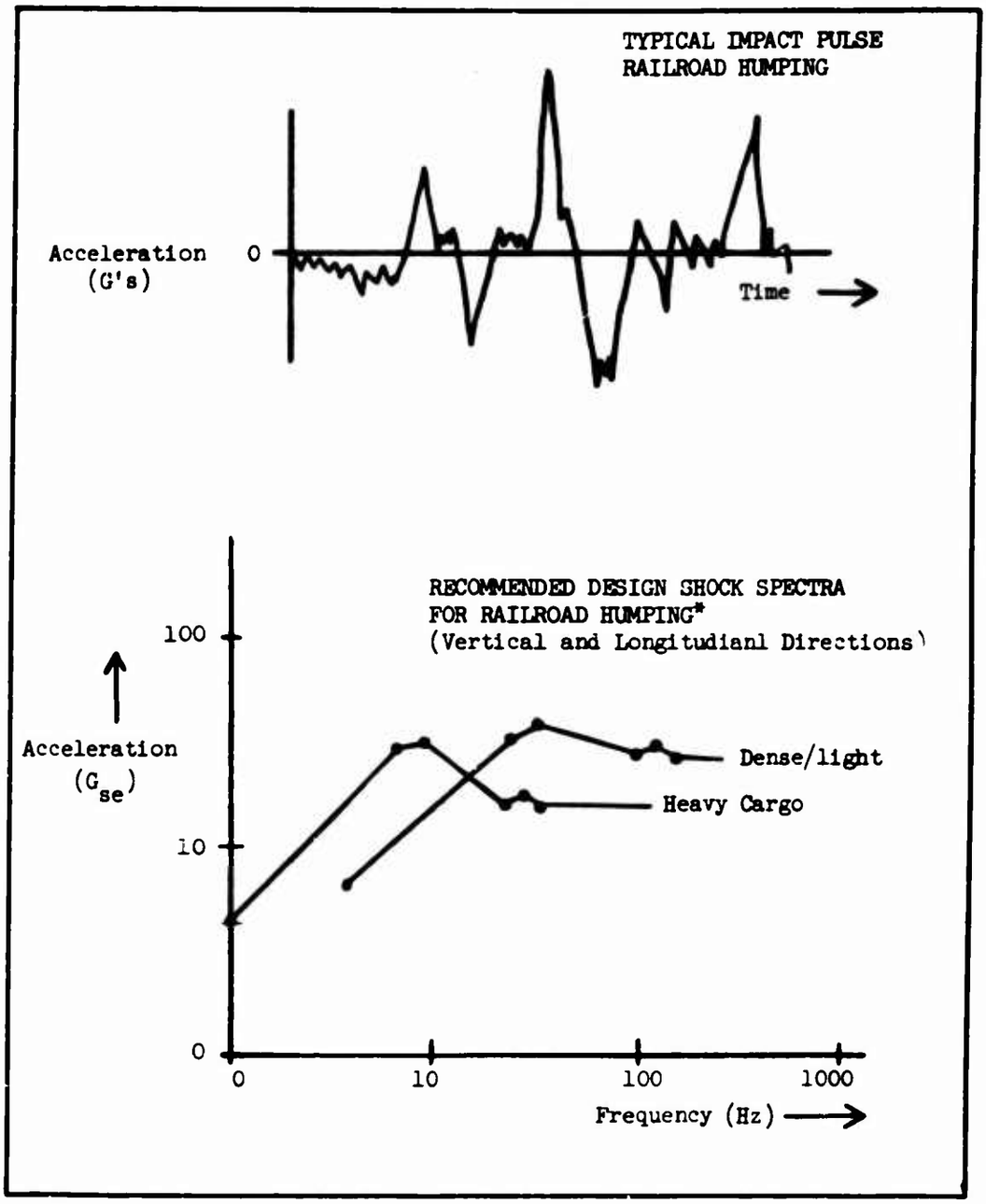
The impact pulse must be considered to be bi-directional; that is, the load will rebound to a negative value as severe as the initial positive peak. The designer must view the forces as completely reversing, from fully positive to fully negative. In addition, the forces resulting from the energy pulse must be combined vectorially since excitations will exist in two orthogonal planes simultaneously. This occurrence is not always fatal, as long as the designer is aware of the possibility and provides structure and fasteners in both planes, loaded concurrently.

The first-cut design criterial must address the possibility that vertical loads could equal loads in the direction of impact. There is some test data to the contrary, however, and an early dynamic model of the system is strongly indicated to definitize the actual humping accelerations for the particular system under consideration. Accelerations could easily be reduced in order of magnitude from the given peak criteria. It should be noted that recommended design criteria reflects the summation of many humping experiences on a variety of equipment geometries, some of which may be too conservative for the problem at hand.

Another hazard to the structure not readily apparent is the possibility of fatigue damage and subsequent failure resulting from the humping impact. Ringing iterations of the initial pulse could number as many as 50 per hump; four humping impacts thus could impart a considerable damage potential at the high end of the fatigue curve.

Damping has a considerable effect on the intensity of the shock spectrum, particularily at the higher frequencies. At frequencies in excess of 100 cps, introduction of 5 percent damping could reduce the longitudinal acceleration to one-fifth the original. Similarly, the vertical accelerations could be reduced by one-half with a 5 percent damped system. A lesser effect will be experienced under 100 cps.(3)

Certain generalities may be drawn from testing experience with complex structures subjected to the humping impact. The impact pulse in the vertical direction may transfer at virtually the same acceleration as the input, while the pulse duration often is increased, exciting the lower range of natural frequencies to a higher value. The longitudinal pulse however, is often reduced to some 60 percent of the input value. The pulse duration is usually increased, as before. This response is probably due to the difference in stiffness, and thus compliance, of the structure in shear as compared to tension and compression.



COUPLING SHOCK: The Army rail humping requirements may be conservatively enveloped by two shock spectra; heavy cargo and dense/light cargo equipment configurations.

*These shock spectra were transformed from the half-sine pulse given in Reference 6, and adjusted for 7 mph impact speed. The transform method employed[13] assumes a single-degree-of-freedom model with no damping.

CHARACTERISTICS OF THE SHAPED PULSE SHOCK TESTS

The shaped pulse shock test is required for Class VI equipments. This test category is also useful as a component procurement standard, and as a simplified method for simulating a complex shock acceleration history.

The shaped pulse shock test is an impact test where the plot of the acceleration-time accelogram is contrained to a given shape, within operational tolerances. The most common pulse shape of interest to designers of Army equipment are the terminal sawtooth and half-sine pulse. The designated pulse is a plot of acceleration vs. time and is defined by pulse height (peak acceleration) and pulse width (pulse duration), in addition to the pulse shape.

Each of the pulse shapes may be transformed readily into shock spectra, a fact which is useful in comparing the simple pulse with complex acceleration histories.[13] This approximation is often employed as a procurement test requirement for electronic components. The assumption is that the simplified pulse (which may be readily simulated in the lab) will demonstrate a minimum level of structural adequacy for the component, such that when the element is assembled into the equipment system, there will be a reasonable change of survival. The design choice involves the careful selection of peak acceleration and pulse duration to effectively reproduce the shock energy within the system complex.

The shaped pulse test is also employed as a Quality Assurance test provision for Class VI (airborne) equipments. This test is required to demonstrate the structural integrity of the equipment mounting facility, when the system is subjected to a crash situation. The equipment may experience some distortion and permanent set, but must not come adrift, to satisfy the crash safety requirements. A similar test such as high intensity impact may be employed with different pulse parameters and acceptance criteria, as noted in the equipment procurement specification. Both test categories may be conducted on identical test machines. The basic source of information for these tests is available in MIL-STD-810.

The shaped pulse test owes its pulse characteristic to the deformation qualities of the impacted material. Changes in impact block material and geometry are made by the test engineer to tailor the shock to the desired shape. Thus early setup impacts are usually accomplished with dummy weighted platforms, while monitoring the shock input for waveform. The tests are then usually conducted in each of the orthagonal directions, repeated as required.

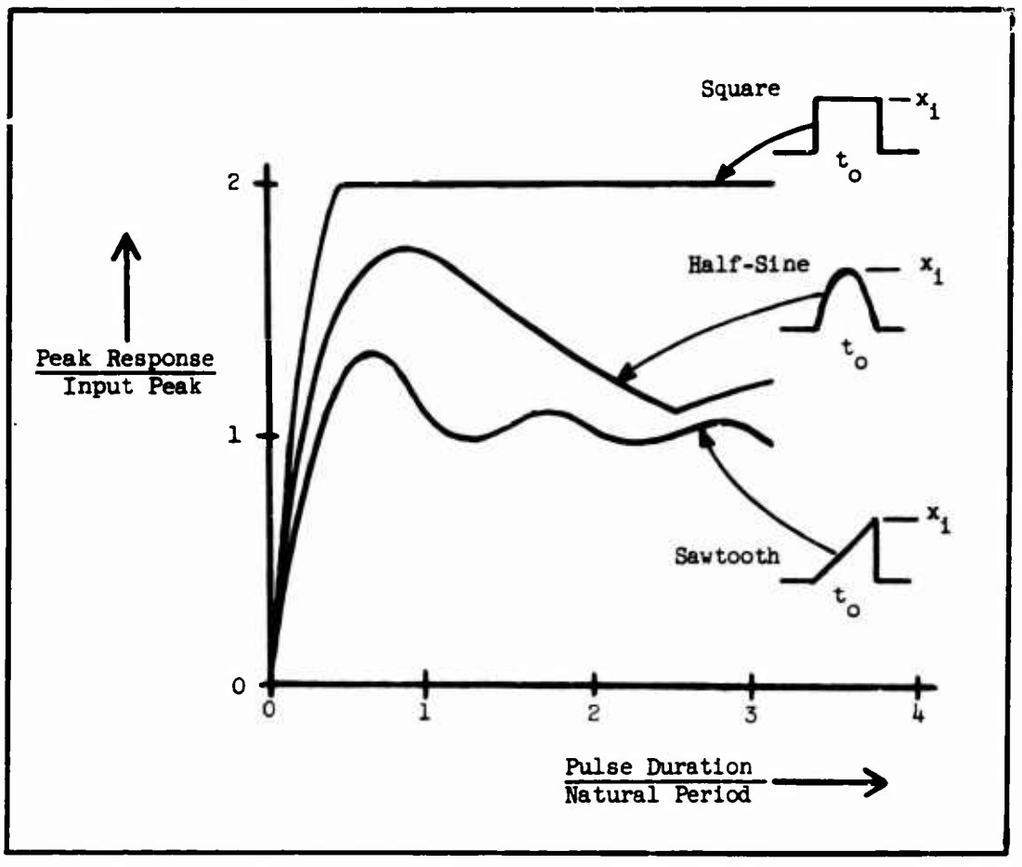
This category of test is usually conducted on a drop or ram type machine, with a programmable impact surface. Some important impact test machines available to the test engineer are:

1. The sand-drop system - a scheme which relies on the variable stiffness of the base platform to tailor the shock pulse. The specimen and platform complex is dropped from a variable height onto a sand surface. The resulting pulse may range to 60 g peak and up to 30 μsec duration. The machine produces a ragged pulse somewhat difficult to duplicate, with an approximate half-sine shape.

2. The controlled pulse drop system - an improved version of the sand drop machine, featuring more rigid structure to minimize spurious excitations within the machine. The impact surface is also variable, and is capable of reproducing accurate, repeatable pulses with a minimum of hash and distortion. The pulse parameters range to several hundred g's and 50 μsec for average test specimens.

3. Ram impact machines - which rely on a pneumatic or hydraulic system with a high specific impulse to impact a ram against the specimen base. The pulse shape is usually controlled by metering the pressurized fluid. Machines of this type are capable of producing shocks up to several hundred g's on small, relatively rigid, test specimens.

Other shock test machines, such as air guns and high energy slings, are not usually employed on Army equipments. They do find application on small, stiff, components when a procurement standard is required.



SHAPED SHOCK PULSES: The various shock pulse shapes imposed on equipment and components may be easily converted into shock spectra plots.

THE LIGHT-WEIGHT, HIGH IMPACT HAMMER TEST FOR BALLISTIC SHOCK

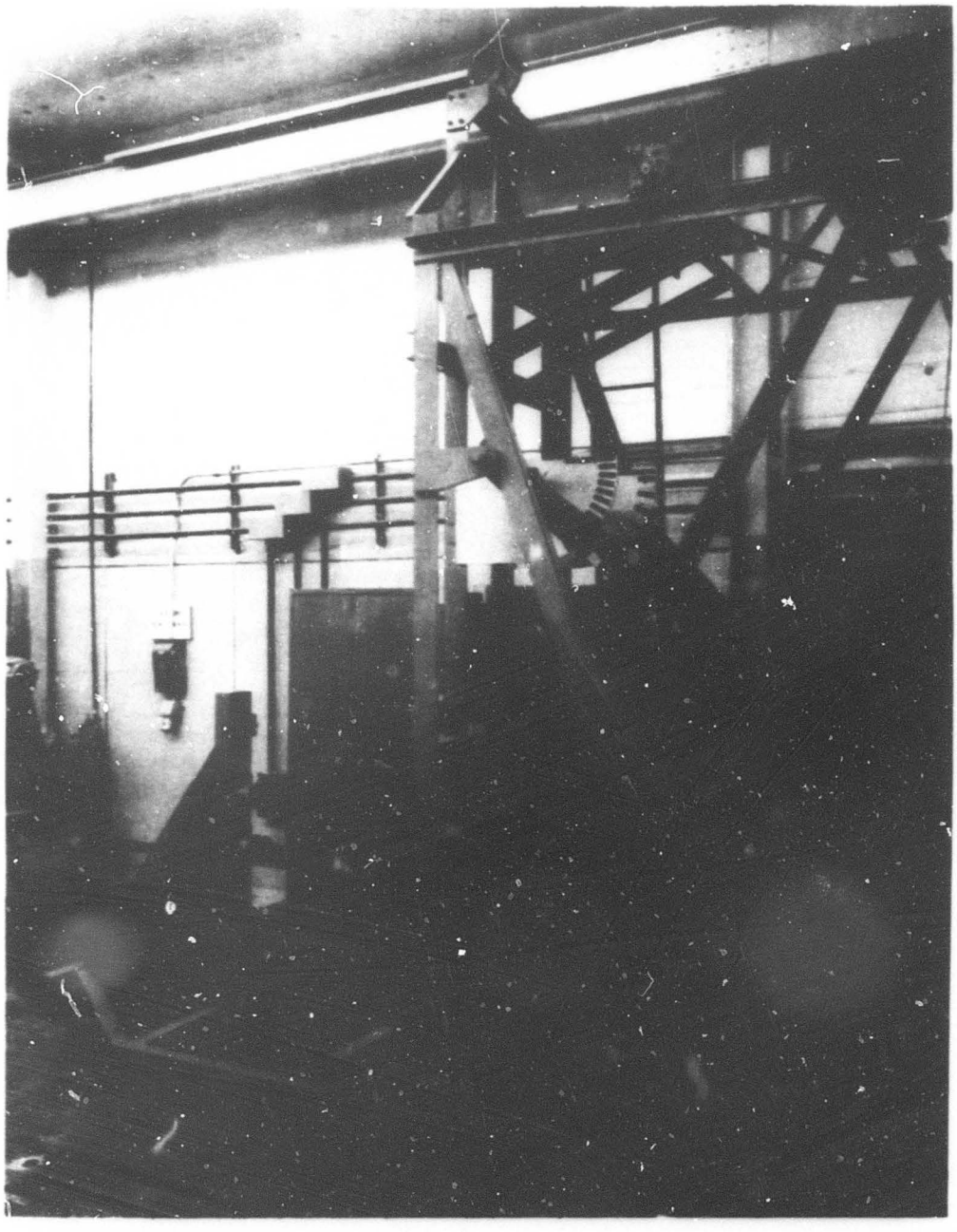
The ballistic shock test required for equipment Classes IV and V is conducted on the light weight, high impact hammer drop machine patterned after MIL-S-901C procedures.

Ballistic shock tests are normally required for Classes IV and V equipments, an equipment category which includes items installed in tracked vehicles. The equipment may or may not be required to operate during impact, as specified in the detail procurement documents. The impact characteristics of the ballistic tests are designed to simulate the shock effects of an explosion created by a near-miss shell impact, or the firing of the vehicle's own weapon systems.

The machine normally used for this demonstration is the High Impact Shock Testing Machine, designed originally for Navy equipment under specification MIL-S-901C. The machine has two hammers weighing about 400 lbs., which are capable of impacting the test specimen and platform from the three orthagonal directions. The energy of impact is obtained by raising the hammers to a specified height, and allowing it to impact against a special plate. One hammer swings through a vertical arc to accomplish two impact directions, while the other hammer is dropped onto an anvil for the vertical impact direction. Generally, the shock spectra repeatability of the swinging hammer is superior to that of the dropped hammer.

The specimen is secured to a standard plate which receives the impact of the hammer. The test complex is snubbed with springs and retaining bolts to restrict the excursion of the specimen after impact. The resulting rebound condition creates a unique ringing response which results in high transient acceleration levels. In addition, the resonant characteristics of the mounting plate and test specimen also have a feedback effect on the input shock. The net effect is considerable variability in input excitation, from specimen to specimen, and test to test.

The light weight machine is limited to specimens under 250 lbs, and has a mounting surface approximately 24 in. square. Each of the 400 lb hammers may be raised to a maximum height of five ft above the impact position, to deliver a maximum energy of 2000 ft-lbs at contact. The test sequence normally requires blows from three increasing heights, along all three axes. No tightening or adjustment of the test specimen is allowed during the test, which highlights the necessity for good assembly practice on the test article. Finger tight bolts usually loosen during this test.



BALLISTIC SHOCK: Highly energetic shock pulses required for equipment Classes IV and V are generated by a hammer drop test machine, the High Impact Shock Testing Machine for Light Weight Equipment, patterned after MIL-S-901C shock test procedure.

EXCITATIONS AND DESIGN IMPLICATIONS OF THE BALLISTIC IMPACT TEST

The ballistic shock test imparts severe accelerations to equipment constrained to survive the test. The available data on shock response from this test experience should be used with caution because of the variability that exists between different structural configurations.

The ballistic impact test creates a formidable design criterion for equipments constained to survive or operate within this shock environment. Typical shock spectra data indicates that the static equivalent acceleration which may be experienced during a back blow to the mounting plate will be several hundred g's for natural frequencies around 10 Hz, and will exceed 1000 g's for natural frequencies above 100 Hz. Since support structure for electronic equipment often exhibits fundamental frequencies in this 10-100 Hz range, the design implications of several hundred g's input are extreme.

The ballistic shock test is normally mounted to a plate fixture and impacted in all three principal directions. Both the vertical drop and swinging hammer are employed to accomplish a back impact, an edge impact, and an end impact. The geometry of the plate and its support creates a differing stiffness situation, which results in significantly different shock spectra for the back vs. end (or edge) experiences. There is characteristically a peak at 100 Hz for the back blows for most specimens, while the edge blow generally peaks at natural frequencies in the range of 350 Hz. In general the worst case accelerations result from the five foot hammer height impact, and is considered to be a conservative design criterion.

Another variable in the excitation resulting from the ballistic impact is that of weight/mass distribution, natural frequency, and damping characteristics of the test specimen. Since the impedance characteristics of the specimen are the same order of magnitude as the test plate, then feedback effects from the specimen become important. The differences in shock input between rigid specimens of different weights are apparent in the adjacent figure. In either case however, natural frequencies in the 100 Hz range should be avoided.

It is apparent from the diversity of shock data in the literature on the ballistic impact test that much variability in the input shock spectra exists. For this reason, the designer should exercise great care in establishing the loads criteria that he will use for basic design. The best approach for evaluating the dynamic structural necessities of an equipment to be subjected to the ballistic impact test may be summarized as follows:

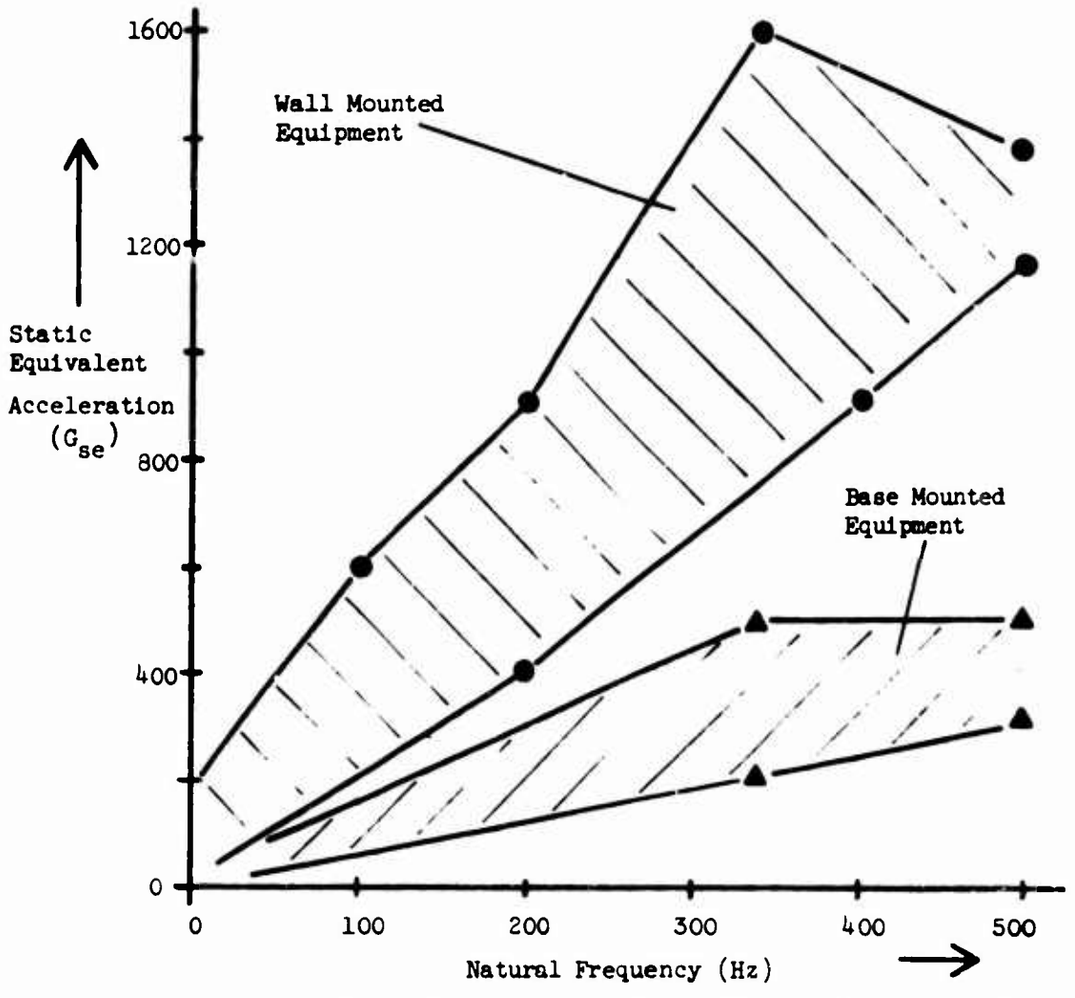
1. Use the input loading criteria presented in this design guide and in the literature with caution. The acceleration levels should be considered as a point of departure for basic design.

2. Construct a structurally accurate model of the equipment element as early as possible in the equipment development program.

3. Conduct well instrumented developmental tests on the model, monitoring the response characteristics at critical locations within the equipment and at the mounting interface.

4. Evaluate the damage potential of the impact on the particular structural geometry under consideration.

SUMMARY OF SHOCK SPECTRA FOR BALLISTIC SHOCK TESTS ON VARIOUS SPECIMENS TO 250 LBS. ALL BLOWS FROM FIVE FOOT HEIGHT INCLUDING, BACK, EDGE, AND TOP ORIENTATION.



BALLISTIC SHOCK: A typical acceleration time history and shock spectra of the ballistic impact indicates the severity of this hammer-drop test.

DETAILS OF THE BENCH HANDLING DROP TEST

The bench drop test is a shock experience which is representative of the rough handling that a chassis might receive during maintenance and repair.

The bench handling drop tests are conducted to prove the structural adequacy of an equipment component to survive accidental impacts encountered during routine servicing. Accordingly, the test specimen is a chassis or subassembly with front panel or other accoutrements removed, in the configuration for normal bench servicing.

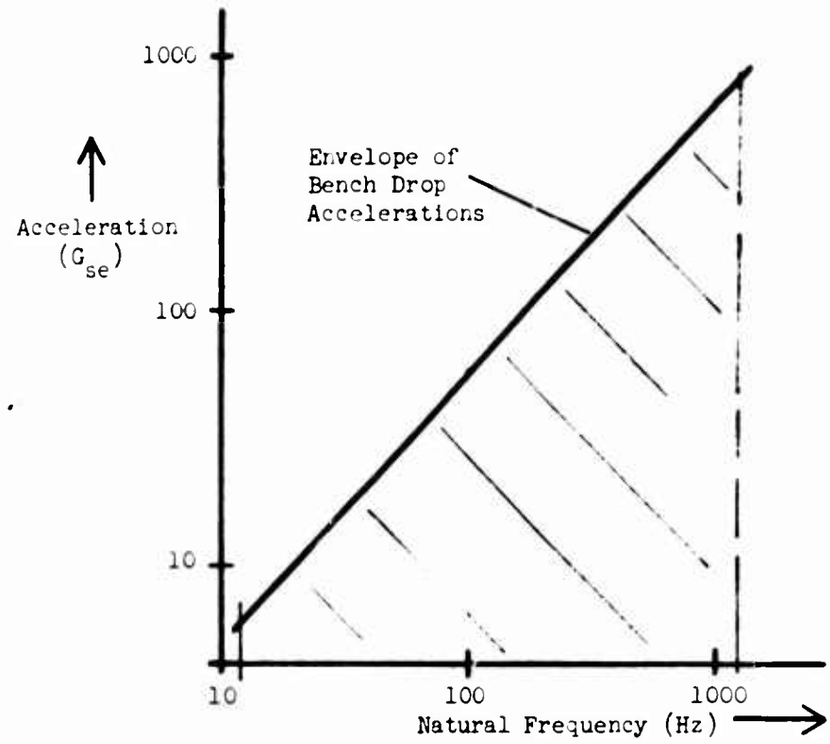
The drop is performed on a solid wooden bench, at least 1-5/8 in. thick. The specimen is placed on the bench in the manner simulating its normal servicing placement. Each of the horizontal edges that may conceivably be used for elevating during servicing, are required for pivot edges during the test. The chassis is then lifted about that edge until its base forms a 45° angle, the lifted edge is four in. off the bench, or the lifted edge is just below the point of balance on the elements whichever geometry occurs first.

The chassis is then allowed to drop freely upon the horizontal bench surface. The test is repeated for the other potential edges, for a total of four drops. The test may be repeated for other horizontal faces of the element, if the face represents a potential servicing configuration. Acceptance criterion is usually normal operation after the bench drop experience without functional degradation. Certain inspection procedures are also normally described in the procurement specification.

No structural help is allowed for the element during test. The unit must be represented in its exact servicing configuration. Instrumentation is not usually employed during this test, since the test is of the go/no-go variety.

The shock excitations resulting from the experience are usually not too severe. Failures of the chassis are unusual as a result of the bench drop. It is possible however, to develop accelerations up to 50 g's under certain conditions which could represent a significant design constraint. The element is usually excited at its natural frequency from the shock. The input energies are also established in both the + and - directions, as a result of rebound. Accelerations in both the horizontal and vertical directions are not uncommon. The sharpness of the resulting shock response spectra is dependent upon the natural frequency and damping characteristics of the specimen.

The bench drop test is quite representative of the rough handling environment which may be encountered in the field, and as such is a practical demonstration of integrity. Local isolation and strengthening of components that are loosened during the drop may usually be handled without great design difficulty.



ASSUMPTIONS

1. Two inch center of gravity drop, no rotation.
2. One Degree-of-Freedom System
3. All potential energy transformerd kinetic energy, manifest as stored energy in spring.
4. Rigid floor, no plastic deformation of specimen.

BENCH DROP IMPACT: This demonstrational test develops accelerations which reflect the natural frequency and damping characteristics of the test specimen.

SHOCK LOADS RESULTING FROM THE TRANSIT DROP TESTS

The transit or shipping drop test series, are designed to approximate the impacts resulting from inadvertant drop of the equipment package during load and unload operations in the field.

All classes of ground systems equipments are normally required to pass a series of transit drop tests, to complete their Qualification Test spectrum. An exception are the Class VI equipments which are not normally shipped by ground vehicle, and hence not constrained to the test.

Transit drop, or shipping drop as it is sometimes known, is an arbitrary impact test of the equipment package, complete with its transport case, as prepared for field use. The intent of the test series is to duplicate the shock loads which might be induced into the equipment through inadvertant drop of the package while loading or unloading. For this reason, the equipment package is dropped onto a floor or barrier of wood backed by concrete, or concrete alone for the large equipments over 1000 lbs gross weight.

The drop test series is performed on face, edge, and corner, as outlined in the test category table below. The independent variable of drop height is determined by the equipment package weight and largest dimension of the package. The exact details of the individual drop setup including the number of drop iterations required are usually noted in the equipment procurement specification.

The excitations resulting from this impact experience will of course be dependent upon the potential energy developed during the test, as given by the drop height. The same limitations occur with the shipping drop as have been noted for the bench drop impacts. The exact characteristics of the resulting shock spectra input is dependent upon the impedance feedback of the specimen itself. The curves presented in the accompanying figure are idealizations representing the maximum accelerations that could occur if all of the limiting conditions were met. Some of the important assumptions leading to this plot are:

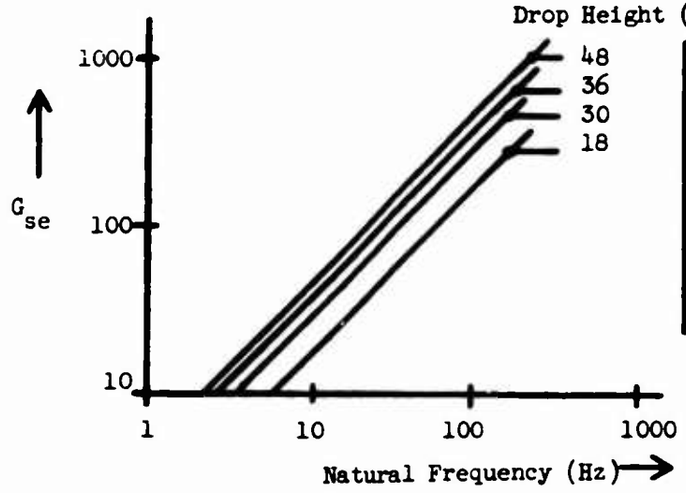
1. The impact floor is assumed to be rigid and completely elastic.
2. No permanent deformation or damping occurs in the specimen.
3. The specimen acts as a one-degree-of-freedom spring.
4. No rotation occurs in the specimen during drop and impact.
5. All the stored potential energy is converted to kinetic energy at impact.

As may be noted, the extreme accelerations may be critical to the equipment system if some protection is not designed into the equipment package or shipping container. The presence of damping or some permanent deformation characteristics in the package will tend to decrease the severity of the impact.

TRANSIT DROP TEST PARAMETERS

Weight of Equipment Including Case	Largest Dimension (in.)	Height of Drop (in.)	No. of Drops
Under 100 lb Man-packed and Man-portable	Under 36	48	Drop on each face, edge, and corner total of 26 drops.
	36 and over	30	
100 to 200 lb inclusive	Under 36	30	Drop on each corner. Total of 8 drops.
	36 and over	24	
Over 200 to 1000 lb inclusive	Under 36	24	
	36 to 60	36	
	Over 60	24	
Over 1000 lb	No limit	18	4 edgewise drops. 2 corner drops.

SHIPPING DROP SHOCK SPECTRA



$G_{se} \cong K f_n$

Height	K
48"	π
36"	2.72
30"	2.48
18"	1.92

SHIPPING DROP: Preliminary design loads criteria may be taken from an idealized spectra of accelerations for a family of drop heights.

VOLUME III - CHAPTER 6

DYNAMIC ISOLATION

SECTION 3 - VIBRATION TESTING

- Vibration Test Requirements for Army Electronic Equipment
- Some Vibration Testing Techniques and Procedures
- Random Vibration Testing Procedures
- Details of the Mechanical Vibration Test Machines
- Details of the Electrodynamic Vibration Test Systems
- Details of the Hydraulic Vibration Test Systems

VIBRATION TEST REQUIREMENTS FOR ARMY ELECTRONIC EQUIPMENT

Vibration Quality Assurance tests for Army ground equipment is intended to define and evaluate the resonant characteristics of the equipment structure. Vibration tests for airborne equipment is more rigorous.

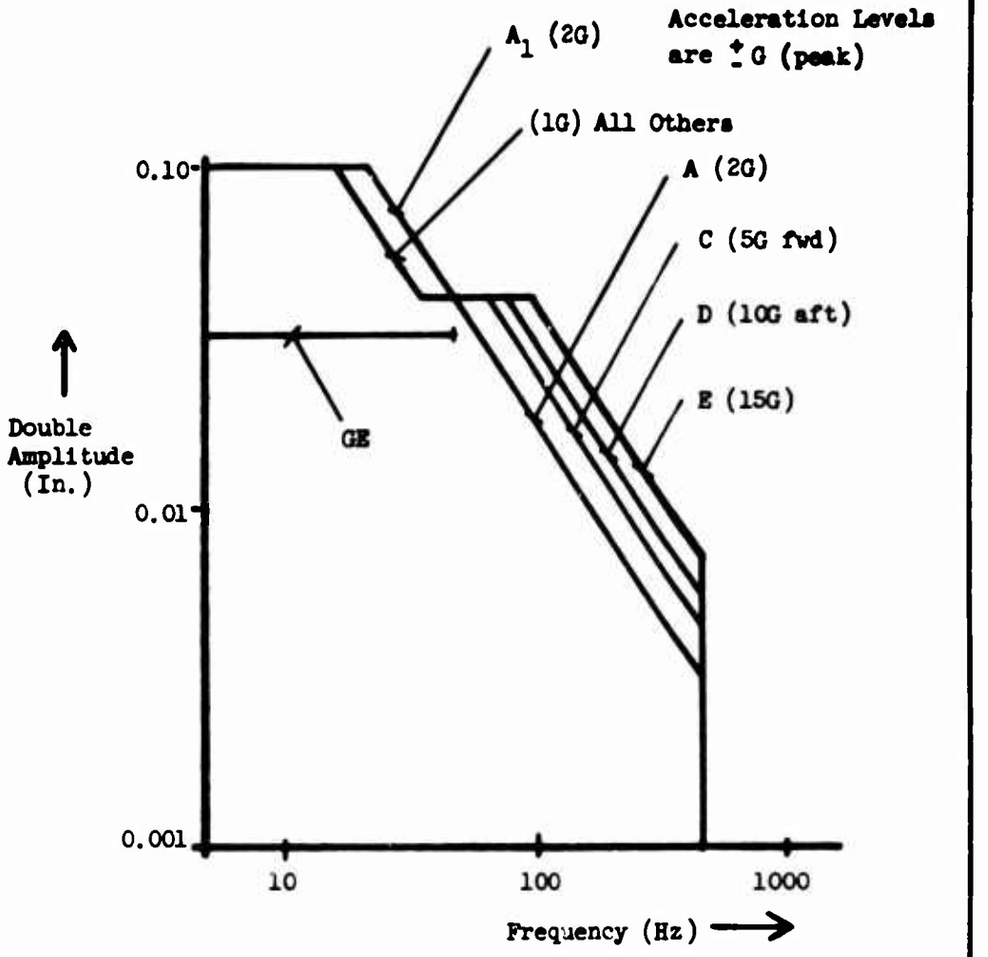
Vibration excitations of interest to the qualification and investigation of Army equipment systems is of the sinusoidal or steady state variety. The intent of the vibration Quality Assurance tests contrained to ground equipment is the evaluation of the response characteristics of the equipment structure as opposed to a test of the ultimate capabilities of the structure. The vibration dwell requirements for airborne equipment are more rigorous. Equipments in Classes I, II, and IV are normally subjected to a vibrational survey in the 10-55 Hz range; equipments in Class VI are normally subjected to a 5-500 Hz survey and resonant dwell. In either case, the vibrational excitation is nominally sinusoidal. The random aspect of the service environment is left to the bounce and mobility tests.

The required objectives of the vibration survey for Classes I, II, and IV equipments are the definition of resonant frequencies within the equipment complex, and the evaluation of the extent of the resonant rise. The test requires the excitation of the equipment (with isolator mounts blocked or removed) through the stated frequency range with 0.03 inches double amplitude input from the vibration exciter. The acceptance criterion for this resonant search is a maximum resonant gain of two through the equipment structure. Resonant rise in excess of two must be justified. The equipment is excited successively in each of the three mutually perpendicular axes. The sweep interval is one cycle per second, with a 10 second dwell at each discrete cycle integral. Vibrational amplitudes are then recorded from critical locations by optical or other effective means, as long as the response of the equipment is unaffected by the measurement.

The vibration test for isolated assemblies is conducted in a similar manner. The natural frequency and maximum response is measured for the first fundamental mode. Acceptance criterion in this case is a fundamental frequency of the isolated assembly within the range of 22-27 Hz, with a maximum transmissibility value across the isolation mounts of five.

The test procedures and frequency-amplitude requirements for airborne equipment, Class VI, are more rigorous, and reflect the needs of the equipment with respect to the mounting area within the aircraft. The test procedures are detailed in MIL-STD-810, and are roughly similar to that required for ground equipment. The frequency range of interest varies from 5 to 500 Hz. The inputs however are acceleration oriented, which dictates a different type of test machine. The capabilities and limitations of the various vibration machines which may be used for these tests are outlined in a later topic.

The essence of the vibration test for airborne equipment entails a resonant search and significant dwell at the major resonant frequencies. This dwell requirement represents the main functional departure from the vibration test requirements for ground equipment. The frequency-amplitude-acceleration requirements for the various equipment classes are outlined in the adjacent figure.



Curve	Equipment Description
A	Equipment installed on Vibration isolated panels and racks when the panel or rack is not available for test.
A_1	Hard mounted helicoptor equipment.
C	Equipment in forward half of fuselage or equipment in wing areas of aircraft with engines at rear of fuselage, and isolated helicoptor equipment.
D	Equipment in rear half of fuselage or equipment in wing areas of aircraft with wing mounted engines.
E	Equipment located in the engine compartment or pylon.
GE	Ground Equipment

VIBRATION TEST CURVES: Illustrated are the vibration parameters for the various classes of Army equipment.

SOME VIBRATION TESTING TECHNIQUES AND PROCEDURES

Fixturing is important to the investigation and validation of a test equipment, since the driving excitations must transmit through this structure. The determination of resonant frequency is the fundamental purpose of most vibration surveys. These two test functions have a heavy influence on the success of a vibration test.

Two important aspects of vibration testing of interest to the structural engineer are the fixturing requirements to accomplish the test and the determination of major resonances within the equipment during the test. The fixturing has a heavy influence on the response characteristics of the equipment, since the fixture transmits the imposed energy into the equipment from the test machine. The determination and evaluation of resonances within the test equipment is also fundamental to the evaluation of the dynamic adequacy of the equipment, whether the test is developmental or required by Quality Assurance provision.

The task of fixture design is most heavily influenced by the structural parameter of stiffness. Since the test machines usually have some sort of physical limitation, it is usually desirable for the fixture to transmit the machine energy to the specimen as efficiently as possible. For this reason, it is usual practice to design the fixture as stiff as practical, which in turn implies a high resonant frequency. A good rule of thumb is.....design the fixture for a natural frequency at least twice the highest anticipated test frequency range. This is practical if the upper limit of test frequency does not exceed 55 Hz.

Some structural design tips to improve fixture stiffness are:

1. Use the stiffest materials available. (Material stiffness is measured by the elastic modulus.) Steel is about three times as stiff as aluminum, although it is also three times as heavy.

2. Stress is usually not a limiting consideration in fixture design. Stiffness (hence resistance to deflection) is more important, and the design should reflect this need.

3. Use structural members in direct load configuration. Members designed into tension and compression (such as a truss) are usually stiffer and lighter than an equivalent bending member. Avoid bending members where possible, particularly the cantilever beam.

4. Use fasteners and weldments in shear wherever possible, since they are stiffer and more reliable when stressed in this manner.

An alternative to maximum stiffness in the fixture is the near approximation of the stiffness of the actual mounting structure when the equipment is in service. In either case, it is wise to place the control transducer at the interface of fixture and specimen, and adjust the machine activity to achieve the desired test input at that point. The machine input to the fixture-specimen complex may then be programmed to adjust for any response variations imposed by the fixture. This approach is particularly important when the fixture resonant frequency falls within the test frequency spectrum. Some degree of damping may also be employed to reduce the resonant response of the fixture.

The determination of resonance in the equipment during vibration survey is accomplished by a variety of methods in the laboratory. Most of these methods, however, take advantage of the human senses of the test engineer; sight, sound, and feel. The main problem in identifying a structural resonance is that of observation. This is a severe limitation of a transducer which measures response, since the measuring element must be placed close to the physical point of resonance. In cases where the maximum response can be estimated, accelerometers and direct reading "Vee-Scopes" may be used. Resonance appears then, when the ratio of output to input is maximum.

Resonance may be found in certain cases by noting a functional anamoly, such as a relay chatter, which may indicate an extreme excitation of the support structure. The sound of a chatter is occasionally evident at resonance when elements bang together from extreme displacement. Other high excursions may sometimes be located by touch; that is, lightly touching the structure with the fingertips, and noting the frequency where the maximum excitation occurs.

The most effective method for resonant survey is probably the technique of stroboscopic investigation. When the strobe light is adjusted 1-2 Hz "out of sync" with the vibrating frequency, the visual image of the structural element may be made to slowly oscillate back and forth. At resonance, this activity is maximum.

- Use a high elastic modulus material
- Keep Stress low (high bulk)
- Use tension and compression members
- Avoid bending members, particularily cantilever
- Use fasteners and welds in shear
- Use ample gusseting to minimize joint rotation

FIXTURE STIFFNESS: Stiff fixtures for vibration testing are generally desirable. The chart above lists some techniques for increasing this parameter.

RANDOM VIBRATION TESTING PROCEDURES

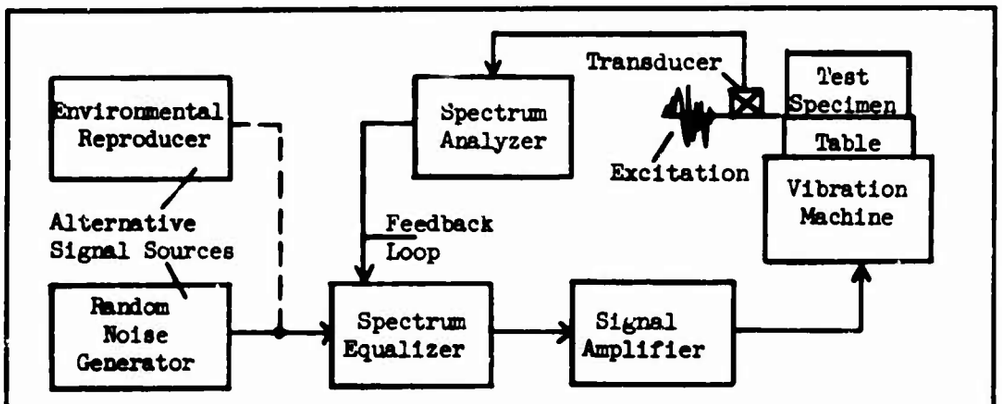
The techniques of random vibration testing are becoming increasingly important to the qualification of Army equipment, as the service environment becomes less periodic and unpredictable with time.

Historically, vibration testing has been accomplished with a steady-state, sinusoidal excitation. The random aspects of the ground environment were left to the phenominological tests, such as vehicular and loose cargo bounce, and the road mobility experiences. As the Army equipment requirements became more versatile, equipment transport accomplished more and more by air, and the equipment application more universal (such as airborne equipment), the need for more random vibration qualification became more urgent.

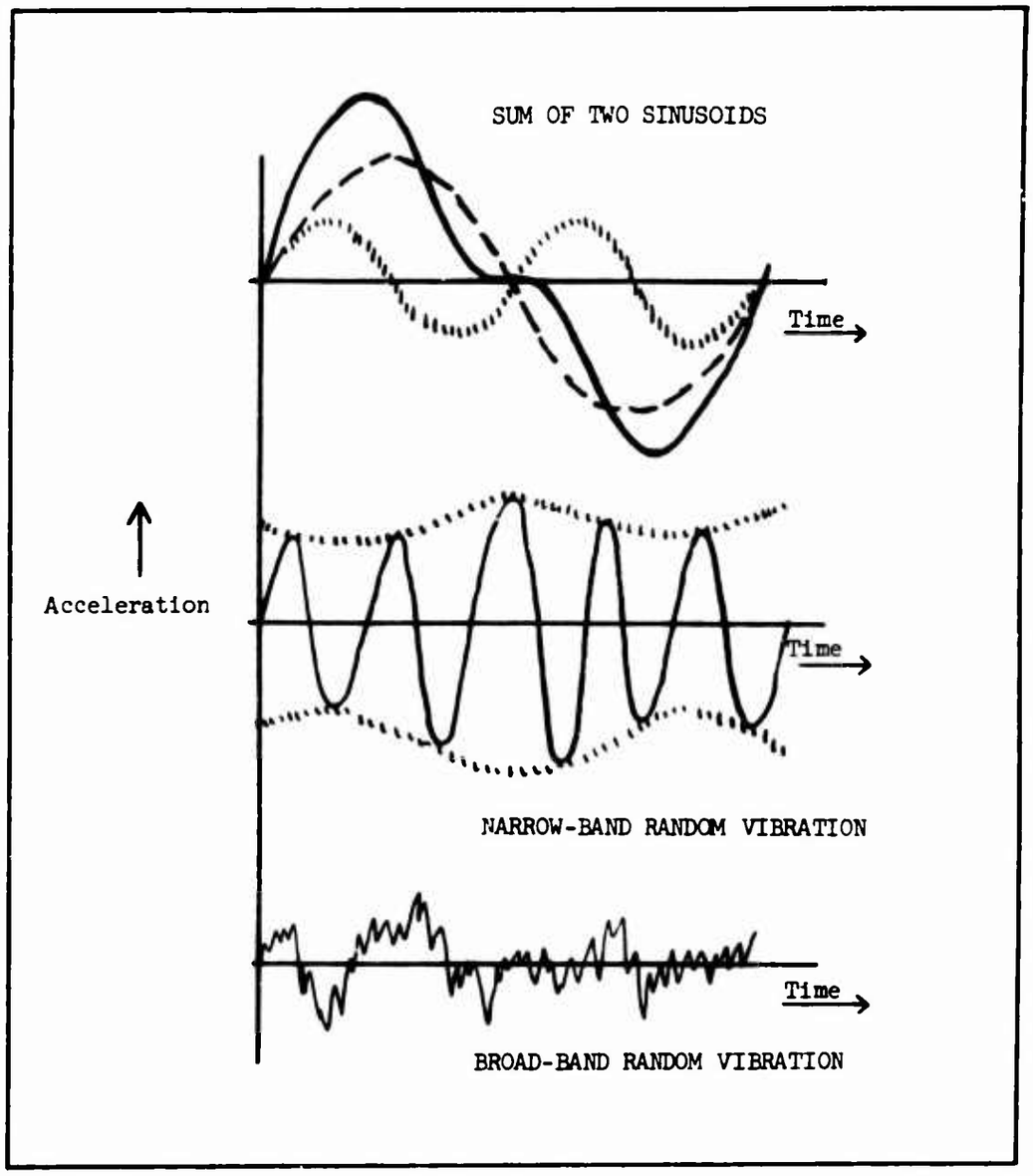
Excitations in the service environment that exhibit displacements which vary in an unpredictable manner with time, are said to be random. Often, the vibrational disturbance is a super-position of many steady state excitations, which result in a complex variation with time, but is stationary or repetitive in interval. This stationary vibration may be found in the helicopter environments. It is complex, but not truly random. The duplication of both types of complex vibration in the laboratory is accomplished in the same manner.

The application of random vibration testing became more prevalent following War II, with the advent of high amplification drive systems for the shaker machines. Thus, by properly inserting a signal into the drive servo control loop of the vibration test system, virtually any vibrational profile may be duplicated. Thus, the shaker systems that are controlled by high gain amplification schemes are useful in random vibration testing. These systems are known as programmable, and are typified by the electrodynamic and electro-hydraulic vibration test systems.

The random vibration test is accomplished by amplification of a random signal, created either by a noise generator, or reproduced from an actual environmental experience. The results at the shaker table may then be compared with the input characteristic to form a closed loop servo system. This concept is illustrated in the block diagram below.



FUNCTIONAL DIAGRAM OF A RANDOM VIBRATION EXCITATION SYSTEM



RANDOM VIBRATION: Typical time histories of complete vibration excitations show the nonperiodic nature of this environment.

DETAILS OF THE MECHANICAL VIBRATION TEST MACHINES

The mechanical vibration machine is well suited for testing large, bulky specimens. An important limitation of the mechanical exciter system however, is the constant amplitude sweep capabilities.

Vibration tests and investigations are normally accomplished by firmly fixing the test specimen to the bed of a vibration exciter. The test machine is usually required to maintain the excitation to within ± 10%, and is usually large with respect to the test specimen. Some investigational work is done with small exciters which "tickle" the equipment structure sufficient to excite the major frequencies, while structural response is noted. The vibration Quality Assurance Tests however, are universally performed on machines that have a large specific impulse and are able to drive the test specimen through the necessary displacements.

Vibration test machines in current use fall generally within one of three categories; mechanical, electro-mechanical, and hydraulic. Each of these machine types have unique features, advantages, and limitations which are important to the test engineer when implementing the vibration test.

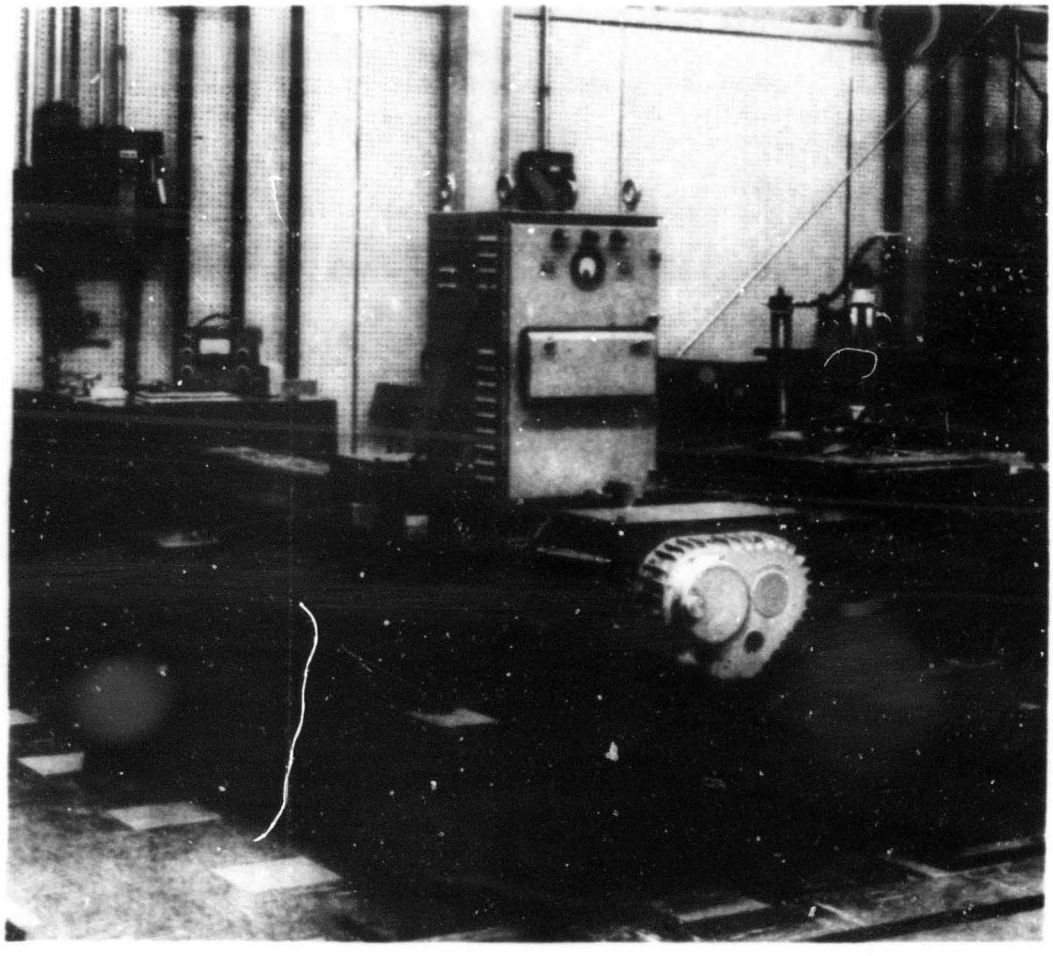
Mechanically actuated vibration machines rely upon some direct mechanical linkage to excite the machine table. Included in this category are the mechanical reaction type, which are cam or eccentric driven and react against the earth or a seismic mass. These machines are known as the "brute force" variety, since the exciting energy is supplied directly to the test machine without assistance from a suspension system or diaphram. These machines are constant displacement devices and are capable of sweeping through only the low frequency range.

A more prevalent group of mechanical vibration machines are those that are mechanically actuated, but suspended on a low natural frequency system such as a soft spring or torsion bar. The nominal displacement of the machine is preset, and remains constant through a frequency sweep. Since the vibrating system is a two degree-of-freedom model (the machine bed plus the test specimen), feedback effects become important. Thus, the input accelerations exerted upon the specimen will vary as the specimen is swept through its natural frequency. This effect is small for specimens that are small relative to the machine mass. Larger specimens however, occasionally require an adjustment in the driving eccentrics to maintain test tolerances.

The mechanical type vibration machines are available in a wide range of capacities which adds to their versatility. In general, these machines are used to test equipments larger than the capabilities of the electro-machanical types. They are particularily well suited to large, bulky specimens, large displacements, and frequencies up to 60 Hz.

Rocking of the test assembly occasionally becomes a problem when the C.G. of the test specimen is high, and the specimen mass significant with respect to the total machine. Counterbalancing is indicated under these conditions when performing a horizontal test sequence. The use of flexures which restrict the vertical motion while permitting a horizontal excursion, is also indicated in extreme cases to react the overturning moment of the test complex. Again, the intent is to input a relatively pure horizontal excitation, without rocking.

The mechanically actuated vibration machines are capable of a constant amplitude sweep through the lower frequency ranges. If a constant acceleration sweep is needed, or higher frequency investigations required, then an electro-mechanical or hydraulic shaker system is indicated.



MECHANICAL SHAKER: The mechanical vibration test machine is well suited for testing large, bulky, specimens through the low frequency range.

DETAILS OF THE ELECTRODYNAMIC VIBRATION TEST SYSTEMS

Electrodynamic shaker systems are versatile, programable devices capable of providing excitation to about 2000 Hz. External support of the specimen weight is usually required, particularly for heavy, bulky equipments.

The electrodynamic shaker systems are capable of providing vibratory excitations from 5 Hz to 2000 Hz approximately, with force capability ranging to 30,000 lbs. The systems are versatile, programable, and capable of performing many vibration testing jobs within certain limitations.

The electrodynamic systems operate on a principle similar to the familiar audio speakers. The vibratory excursion is created by a controlled coil which excites an armature, which in turn is mechanically linked to the test specimen. The armature is supported on a low frequency spring system called flexures, which allows the armature to excurse within the physical limits of the springs. In general, as the specimen weight approaches that of the armature, the mechanical system must be supported by external means. This is the fundamental limitation of these shaker systems; they are primarily excitation devices, and are not capable of supporting heavy specimens as are the mechanical and hydraulic systems. This external support may take the form of springs, bungee cord, pneumatic devices, or other low natural frequency systems, depending upon the ingenuity of the test engineer.

Most electrodynamic systems have self-contained shaker heads which have the capability of being rotated in a trunnion through 90°, to provide horizontal as well as vertical excitations. The mounting face of these machines provide bolt patterns for specimen attachment, and range in size to about three feet in diamter. The shaker head and specimen support portion of the system are commonly used with a bank of electrical gear which provides power amplification, excitation control, and positional feedback for the system.

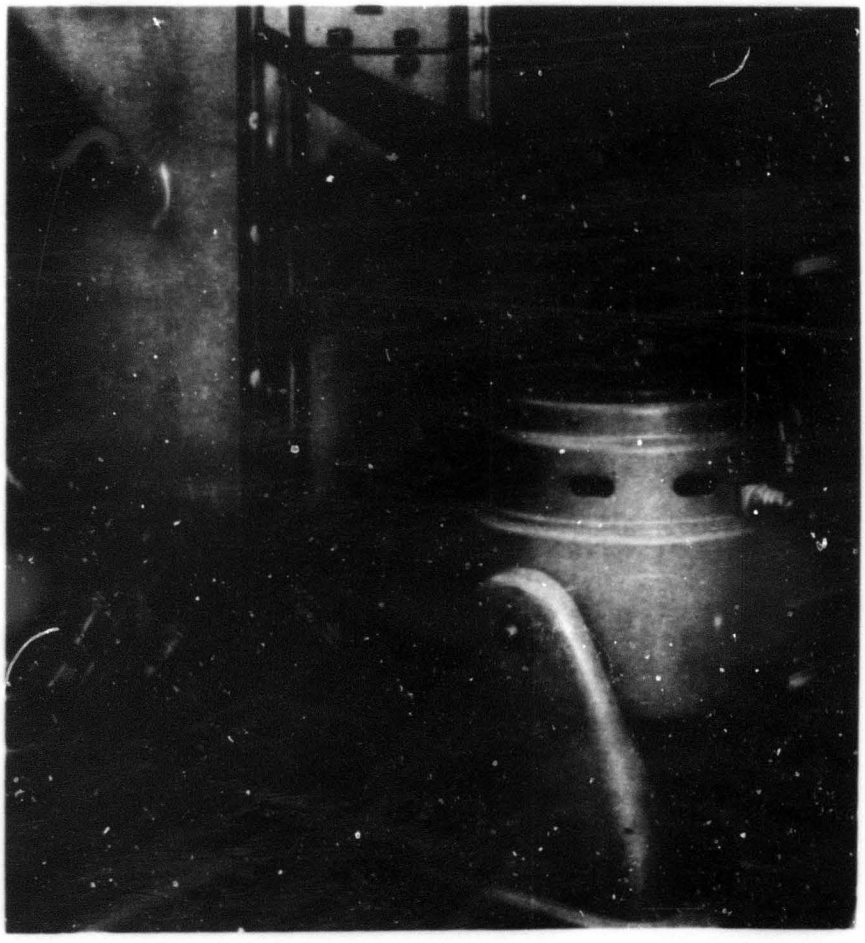
The shaker heads may be ganged together and synchronized to provide excitational energy for large specimens. These schemes have the usual problems of synchronizing multiple units, but in general prove successful provided external support of the specimen weight is considered. This type of multiple system competes directly with the mechanical and hydraulic shaker systems, with the additional capability of high frequency (to 2000 Hz.) range.

The electrodynamic system has a velocity cutoff in the range 100 in./sec, which limits the acceleration capability at high frequency. Nevertheless, this type of machine is the only effective method of vibrating specimens above 200 Hz, the practical limit of the mechanical-hydraulic systems. The usual Army requirements of 10-55 Hz vibrational surveys for Class I through V equipments may be met with all three types of shaker systems. The higher test frequencies required for airborne equipments, Class VI, are limited to the electrodynamic systems for test execution.

The degree of control inherent in the electrodynamic systems is often helpful to the test engineer. Both frequency and amplitude may be controlled within the machine limitations. Amplitude may be varied while the machine is in motion; thus, an intermediate resonant frequency may be explored without the necessity of exciting the lower frequencies by

driving to the desired frequency at zero amplitude. The required acceleration range may then be achieved by slowly raising the excursion amplitude, while the machine is vibrating at the desired frequency.

Virtually any waveform may be programmed into these systems, a direct result of the versatility of the control electronics. Due to the directional rigidity of the armature guides, the electrodynamic systems have the least amount of cross-axis excitation of the shaker systems.



ELECTRODYNAMIC SHAKERS: These vibration test systems are easily programmed, and are versatile machines for high frequency investigations.

DETAILS OF THE HYDRAULIC VIBRATION TEST SYSTEMS

The modern hydraulic shaker system is a velocity limited, high force capability excitation system which is ideally suited for testing large, heavy, bulky specimens through the low and intermediate frequency range.

As the Quality Assurance Provisions are constrained to increasingly large electronic systems, the need arises for high dynamic force capability vibration test machines. The hydraulic excitation systems have been developed to fill this need, and are ideally suited for vibration investigations of large, heavy, or bulky equipment systems.

The hydraulic shaker system combines the features of the electro-dynamic machines with the high specific impulse capabilities of a hydraulic system. The electro-magnetic element is used to excite a hydraulic servo valve, which in turn activates a hydraulic cylinder. The usual features of frequency and amplitude control (of the electro-magnetic system) are inherent capabilities of this combined electro-hydraulic system.

The hydraulic shaker systems are programable for frequency and amplitude, much the same as the electro-magnetic systems. The output of the hydraulic system is monitored by a displacement transducer to provide positional information and close the servo-control loop. The hydraulic servo valve, which controls the flow of hydraulic energy to the actuating cylinder, acts as an amplifier of the signal generated by the electro-dynamic exciter. The excitation is thus generated, amplified, used to drive the specimen, and fed-back to provide closed-loop control.

The chief advantage of the hydraulic shaker system, as previously noted, is the high dynamic force capability in the low and intermediate frequency ranges. The hydraulic systems are capable of providing relatively large force-pounds of excitation from virtually d.c. (around one hz) to 250 Hz, and in some instances up to 500 Hz.

The hydraulic systems are velocity limited, which implies a cutoff of force capability at high frequencies, but also permits very large excursions at the lower end of the frequency spectrum. Thus, it is possible to conduct constant acceleration tests at the low frequency end without the displacement cutoff associated with other vibration systems.

The hydraulic shakers are quite useful as a programable testing tool. This advantage results from the inherently responsive characteristics of the hydraulic system. The input waveform to the specimen however, is subject to some feedback effects from the specimen, and thus is generally somewhat distorted from the traditional sine wave.

The principal operational limitations of this type of excitation system stem from the hydraulic system itself, and reflect the usual problems of hydraulic mechanisms. There are more moving parts associated with these shakers, and generally closer tolerances are required. A high capacity hydraulic supply module is required with a high degree of filtering for system cleanliness.

Once the operational difficulties are mastered, the hydraulic vibration system is a valuable addition to the dynamic test laboratory.

HYDRAULIC EXCITATION SYSTEMS: The electro-hydraulic shaker machines offer a high force, low to intermediate frequency range, vibration test machine with great versatility.

VOLUME III - CHAPTER 6

DYNAMIC SIMULATION

SECTION 4 - TESTING FOR COMPLEX ENVIRONMENTS

- Qualification for Road Mobility - The Munson Road Test
- Dynamic Excitations Resulting From the Road Mobility Tests
- Random Excitations Resulting From the Bounce Tests

QUALIFICATION FOR ROAD MOBILITY; THE MUNSON ROAD TEST

Road mobility tests simulate the true service environment and represent a rigorous design criterion for electronic equipment scheduled to be ground transported.

The Munson Road Tests are a series of specific road course experiences designed to qualify certain classes of electronic equipment for road mobility. Although the Munson Course does not exactly simulate all possible mobility environments that an Army equipment might experience anywhere in the world, the course does however represent each of the various road experiences of Army equipment. Thus, a finite number of trips around the course will expose the equipment to excitations representative of ground travel by transport vehicle. The frequency of the road test disturbance is encountered more often than the actual service experience, thus providing an accelerated life test of the equipment. The course is designed to excite the frequencies susceptible to this mode of vehicular transport in the equipment structure under qualification.

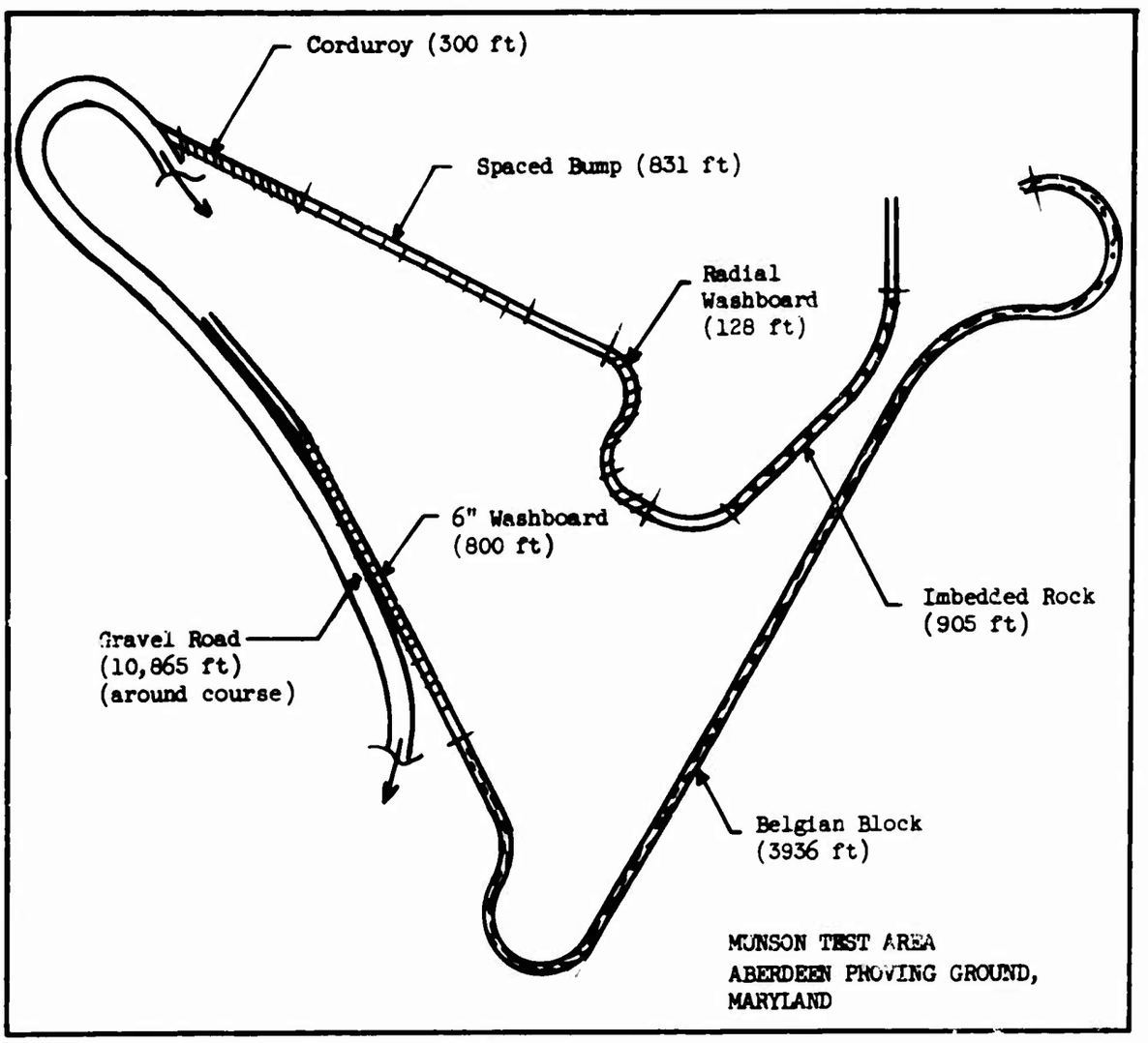
Equipments that will be normally transported by ground vehicles, both tracked and wheeled, are usually constrained to qualify on the Munson Course. Specifically, equipment classes III and V are normally required to survive the road course complex with the equipment operating, while equipment Classes II and IV are only transported by ground vehicle and thus are constrained only to survive the experience.

The road mobility tests, or Munson tests, are unique among the Army quality assurance provisions, in that the equipment is secured upon the actual transport vehicle with which the equipment class will be mobilized. Thus the contributing factors of tire pressure, spring characteristics, and chassis compliance are accurately represented during the test in a manner identical to the actual operational environment.

The Munson course is a collection of road conditions of varying length and severity, arranged in a closed loop. The usual Army test requirements include traverse of the six-inch washboard, Belgian block, radial washboard, spaced bump, and two-inch washboard, for a total of five circuits. A smooth track to accommodate an instrumentation vehicle is usually maintained adjacent to the test course. The most widely used road course facility is the Munson course at the Army Automotive Engineering Laboratories, Aberdeen Proving Grounds, Maryland.[11] Recently however, other qualified courses have been constructed to meet the increased emphasis on road mobility and the associated shock and vibration environment. An example on the West Coast is the Munson Course at Hughes-Fullerton, California.[12]

Cross-country simulation by ground transport is accomplished on the Perryman Test Area, or its equivalent. The Perryman course is a collection of cross-country terrains, which include a moderate course with loops and curves on a substantial roadbed; a moderately irregular terrain; a rough course of native soil; and an extremely rough course including marshy terrain. The usual Army requirements for cross-country qualification of electronic equipment are 200-300 miles of transport over the Perryman terrain. Army vehicles, conversely, are driven over the course until failures occur, to uncover any weaknesses in the transport vehicles.

There are no set fixturing requirements for the Munson Road Tests, other than the hold-down provisions needed to transport the test equipment upon its transport vehicle. The test acceptance criterion for this requirement is the ability of the equipment to meet its operational constraints after completion of the test with no evidence of physical damage in the case of Equipment Classes II and IV. Class III and V equipments are required to operate during the test experience without functional degradation or physical damage.



THE MUNSON COURSE: This test course is a collection of road conditions representing the conditions which might be experienced during road and cross-country mobilization.

DYNAMIC EXCITATIONS RESULTING FROM THE ROAD MOBILITY TESTS

The input load characteristic of the Road Mobility Tests varies greatly between wheeled and track-laying vehicles. Accordingly, the design approach employed must reflect this varying condition.

The Munson Road Test provides a dwell or time exposure for the test equipments under varying loads that simulates the service environment very closely. Other Quality Assurance Provisions, such as the vibration search tests, do not completely cover the fatigue damage potential present in the real service environment. Thus, one function of the Road Mobility Test and the bounce tests, is the simulation of a sustained excitation to the test equipment of a random nature. Accordingly, these excitations are highly variable and thus difficult to represent for input loads design criteria.

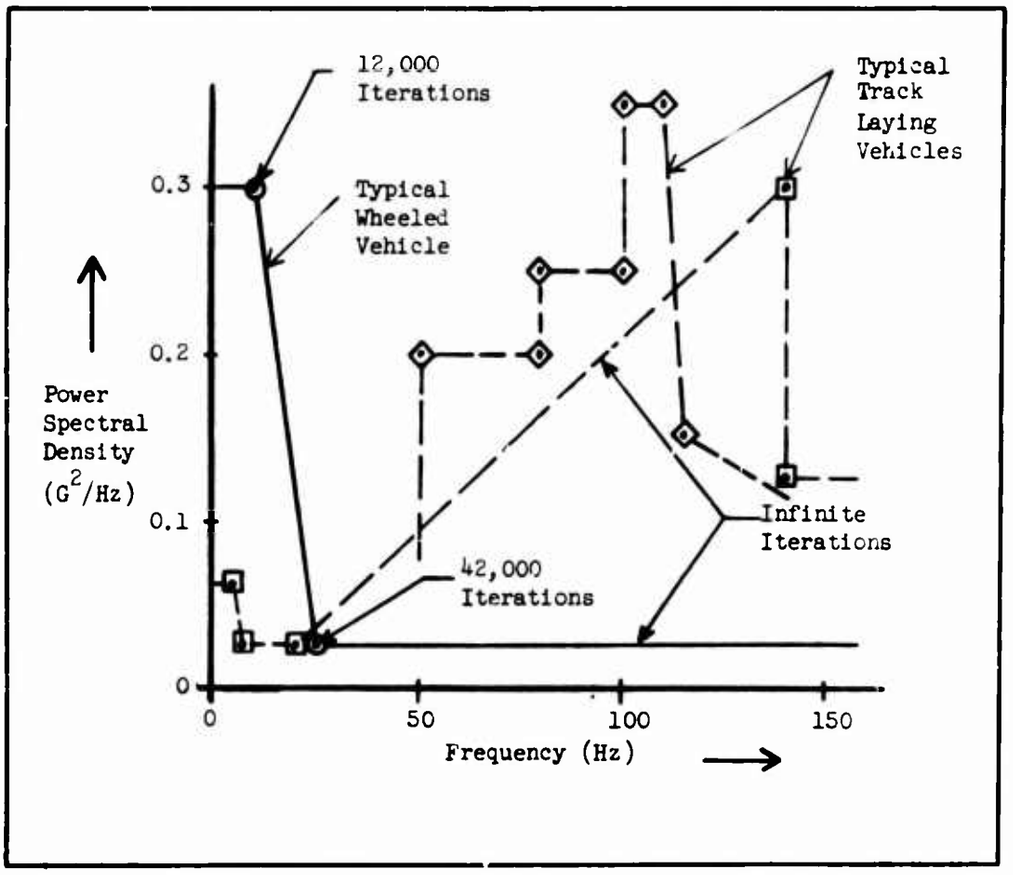
Useful information on the characteristics of the road test excitations may be abstracted from reference (11), and is summarized at the right of this section. The spectral envelopes differ greatly between wheeled and track-laying vehicles; thus it is necessary to determine in advance which type of vehicle will be used to qualify the particular test equipment. The spectral plot for wheeled vehicles reveals a high PSD input for the low frequency end of the environment. This response results from the individual disturbances of the course in the frequency range below 30 Hz. The M-35 vehicle generally exhibits the worst excitation, and should be used for design load criterion in lieu of a specific vehicle designation.

The track-laying vehicles reported also exhibit a similar phenomenological response in the frequency range below about 30 Hz. A generally higher input is evident for frequencies above 50 Hz for tracked vehicles, a response which is speed-related, resulting from track-roller-wheel disturbances created during transport over smooth terrain. Thus the most serious excitations resulting from transport by track-laying vehicles will occur at high frequencies. If the test vehicle may be either wheeled or track-laying, then characteristics of the differing spectra must be considered in structural design due to the frequency anomaly.

When designing for the road test criteria, the effect of potential fatigue damage must be evaluated. In the case of transport by wheeled vehicle, an estimate may be made on the number of iterations of load resulting from the experiences below 30 Hz, based upon an assumed exposure time over the course. Thus, the number of stress iterations was estimated between 12,000 and 42,000 cycles, a first cut fatigue life figure based upon 1500 seconds of excitation. For the frequencies above 30 Hz, the input loads should be considered to be iterated an infinite number of times, and the material endurance limit used as a basis for design.

Transport by track-laying vehicle exhibits excitations peaking through the higher frequency range. The loads should be considered to be iterated indefinitely since the disturbance emanates from smooth road travel; again, the material endurance limit should be used for the design allowable load. The excitations below 30 Hz are probably not critical in this case, since higher loads are developed by wheeled vehicles.

Since most of the dynamic activity resulting from transport by wheeled vehicle exists below 30 Hz, it is imperative that natural frequencies of equipment structure be designed to be above this level. In addition, rotational accelerations will exist in this test environment. Pitching accelerations may be expected up to 15 rad/sec and rolling accelerations of 30 rad/sec are common. These rotational disturbances will be present up to 30 Hz, similar to the vertical linear acceleration. The application of damping is indicated for resonant structure below 30 Hz to limit the amount of resonant rise during test.



MUNSON COURSE LOADINGS: Spectral envelope of input excitations for wheeled and track-laying vehicles over all Munson courses at various speeds.(11)

RANDOM EXCITATIONS RESULTING FROM THE BOUNCE TESTS

The loose cargo and vehicular bounce test requirements are intended to simulate the random vibrational experiences of transport by ground vehicle over rough terrain.

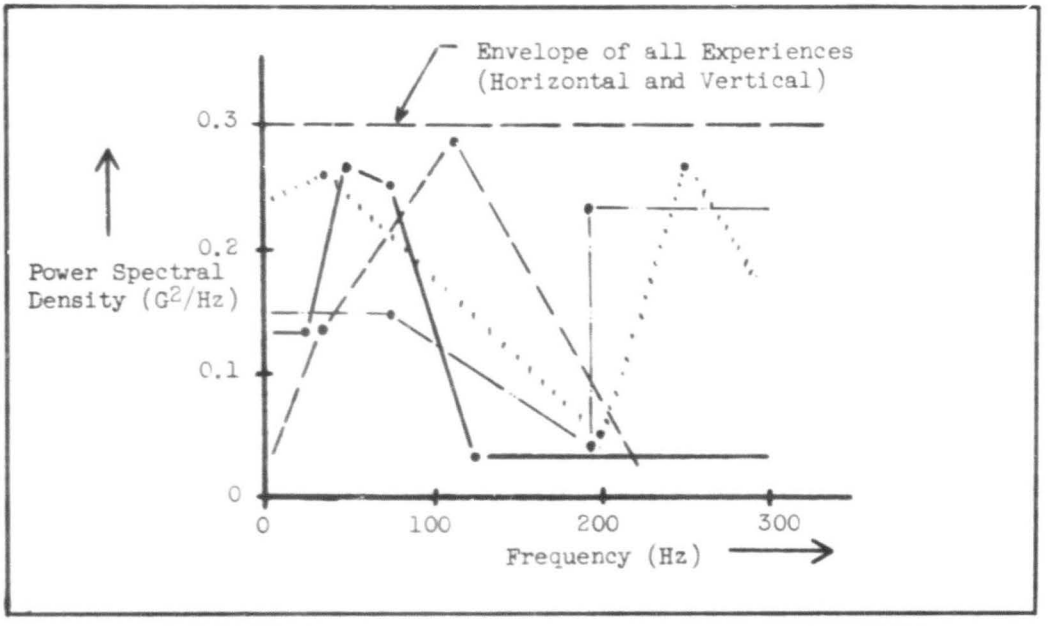
The Army requirements for dwell or sustained excitation which might be experienced during transport are covered by a random exposure on a package bounce test machine. Where the vibrational requirements are interested in surveying resonances in equipment structure by sinusoidal excitation, the bounce tests subject the package to a sustained random excitation quite similar to the excitations experienced on the cargo area of a transport vehicle traveling over rough terrain.

A bounce test is normally required for each of the equipment classes, with the exception of Class VI, airborne. In addition, the bounce tests are divided into two categories; bounce tests for cargo normally transported loose in the cargo area of the transport vehicle, and vehicular bounce for equipments normally shipped on a base or platform which tends to keep the equipment upright on its base. Both categories of bounce tests are conducted on a special machine which duplicates the gyrating environment of a ground vehicle traveling over rough terrain. The machine most often employed for these tests is the L.A.B. bounce tester, available in 1000 and 5000 lb capacity sizes.

The vehicular bounce test has the equipment specimen mounted to a base plate which restricts its activity during excitation. The specimen thus tends to stay upright with the random excitations delivered through the base. Chassis and equipment sub-assemblies are typical of equipments shipped in this manner. The machine table is driven by eccentrics which impart a bouncing motion to the specimen. The speed of the drive motor is adjusted to create input peak random accelerations which average 7.5 g's. The maximum anticipated peak accelerations may reach 10 g's. The specimen-plate complex is turned in a horizontal plane to expose all four base directions. The bounce is required to continue for a total experience of three hours, and the equipment is required to function without degradation upon completion.

The loose cargo bounce tests are conducted on the same machines without the base plate restriction. In addition, the specimen is turned onto all six box surfaces, and exposed for a total of three hours. The machine speed is adjusted to 284 rpm.

Both types of bounce experiences impart a random excitation to the specimen. The input to the base of the specimen is also dependent upon the response characteristics of the equipment structure. For these reasons, there is a great deal of excitation variability between tests, specimens, and test machines. The plot (right) shows envelopes of random spectral densities for some typical bounce test experiences. A conservative approximation for first-cut design loads appears to be 0.30 g^2/Hz power spectral density constant throughout the investigated frequency range. Further, these experiences should be considered indefinite in duration; the material endurance limit should be used for design allowable strength levels. The severity of the horizontal and vertical components of the excitation appear to be equal in intensity.



BOUNCE TESTS: A gyrating platform is used to simulate the random vibrational experiences of equipment transported by ground vehicle.

VOLUME III - CHAPTER 6

DYNAMIC SIMULATION

SECTION 5 - APPENDIX

- Bibliography
- Glossary

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GLOSSARY

Accelogram - A pictorial plot showing acceleration levels versus time which a test specimen experiences as a result of an input excitation.

Amplitude - The maximum value of a sinusoidal quantity.

Cycle - The complete sequence of values of a periodic quantity that occur during a period.

Damping - The dissipation of energy with time or distance.

Degrees-of-Freedom - The number of degrees-of-freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the positions of all parts of the system at any instant of time.

Dissemination - Damping caused by spreading out and redistribution energy throughout a system.

Fatigue - Tendency of materials to fracture under many repitiotions of a stress considerably less than the ultimate static strength.

Frequency - The number of times that a periodic function repeats the same sequence of values during a unit variation of time. The unit is the cycle-per-second which equals one Hertz (Hz).

Impedance - The ratio of a force-like quantity to a velocity-like quantity when the arguments (distance, etc.) of the quantities increase linearly with time.

Impulse - The product of a force and the time during which the force is applied.

Natural Frequency - The frequency of free vibration of the system. For a multiple degree-of-freedom system, the nautral frequencies are the frequencies of the normal modes of vibration.

Phenomenological - Relating to the formal structure of phenomena in abstraction from interpretation or evaluation.

Response - The motion (or other output) of a system or device resulting from an excitation.

Resonance - Resonance of a system in forced vibration exists when any change, however small, in the frequency of excitation causes a decrease in the response of the system.

Snubber - A device used to increase the stiffness of an elastic system (usually by a large factor) whenever the displacement becomes larger than a specified value.

Stiffness - The ratio of change of force (or torque) to the corresponding change in translational (or rotational) deflection of an elastic element.

Stress - Internal force exerted by either of two adjacent parts of a body upon the other across an imagined plane of separation.

Transmissibility - Nondimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude. The ratio maybe one of forces, displacements, velocities, or accelerations.

CHAPTER 7 — INSTRUMENTATION

VOLUME III - RELATED TECHNOLOGIES

CHAPTER 7
INSTRUMENTATION

ABSTRACT:

Proper execution of present data acquisition methods will afford technical insight into the problem of structural dynamic integrity. The instrumentation and data reduction techniques presented in this chapter will yield information that is vital to the design engineer for the initiation and validation of electronic equipment elements.

The first section of this chapter introduces the tie-in of instrumentation with structural design. Sections 2 and 3 present the various methods of data acquisition and practical applications of many types of shock and vibration measuring equipment and techniques. The topics in these sections will acquaint the reader with the technical information necessary in the selection of a particular equipment for a particular job.

Chapter 7 - Instrumentation

ERRATA SHEET

Page	Paragraph	Line	Correction
Chapter Table/ Contents	2	11	Optimizing
Section 2 Table of Contents		11	Optimizing
7.2-10	2	14	contrast
7.2-13	3	6	5 Hz to 2000 Hz
7.3-2	1	4	Nyquist
7.3-3	Graphic	Plot Ordinate	g^2/Hz
7.4-2	4	1	Impedance
7.4-2	4	5	$\|Z\| = (R^2 + X^2)^{\frac{1}{2}}$
7.4-2	8	2	specifically
7.4-6	Graphic	Titles	Switch Titles in Graphic

VOLUME III - CHAPTER 7
INSTRUMENTATION

CONTENTS

Section		Page
1	INSTRUMENTATION AND ELECTRONIC EQUIPMENT PACKAGING	7.1-0
	• Enhancement of Structural Design Through Test Instrumentation Discipline	7.1-0
2	DYNAMIC TEST DATA ACQUISITION	7.2-0
	• Organization of Test Data Collection	7.2-0
	• Current Graphic Data Recording Methods and Presentation Formats.	7.2-2
	• Improving the Usefulness of Dynamic Response Data by Means of Magnetic Tape	7.2-4
	• Refining the Pretest Planning and Equipment Selection Process	7.2-6
	• Direct Reading Vibration and Shock Measurement Instrumentation	7.2-8
	• Current Trends in Data Recorders	7.2-10
	• Optiminizing Transducer Characteristics for Dynamic Measurements	7.2-12
	• Transducer Calibration Methods	7.2-14
	• Transducer Mounting Methods and Techniques	7.2-16
	• Improving the Signal Conditioning System	7.2-18
3	REDUCTION OF DYNAMIC TEST DATA	7.3-0
	• A Discussion of Oscillogram Interpretation Techniques	7.3-0
	• Application of Amplitude Spectrum Analysis Equipment	7.3-2
4	APPENDIX	7.4-0
	• Bibliography	7.4-0
	• Glossary	7.4-1
	• Typical Dynamic Transducers	7.4-4
	• Data Recording by Oscilloscope	7.4-6

VOLUME III - CHAPTER 7

INSTRUMENTATION

SECTION 1 - INSTRUMENTATION AND ELECTRONIC EQUIPMENT PACKAGING

- Enhancement of Structural Design Through Test Instrumentation Discipline

[redacted] OF STRUCTURAL DESIGN THROUGH TEST INSTRUMENTATION DISCIPLINE

When properly implemented, instrumentation presently available to the packaging engineer provides a useful complement to dynamic environmental testing.

The instrumentation engineer and the mechanical designer should determine at an early stage, the data requirements of the equipment design, and outline it in great detail. Much too often, excessive data is acquired which is of no real benefit to the equipment designer. It is primarily the responsibility of the instrumentation engineer to research out the practical data needs. With regard to this, it is suggested that the equipment designer merely define his requirements and allow sufficient freedom for the instrumentation engineer to determine how the task of acquiring the necessary data is to be accomplished. Such an arrangement will insure the acquisition of only the meaningful data.

The techniques of data reduction that are to be applied must be thought out well in advance of any testing. Since the primary purpose for prototype testing is the measurement of product performance, the performance subsequent to the test must be measured in terms that are valid. For instance, vibration and shock can be measured in terms of acceleration displacement or velocity. What to measure, where to measure, how to measure, and where the transducers are to be placed and the type to be used, should all be decided long before testing begins. The identification and evaluation of possible secondary effects is also an important aspect of instrument selection.

When all of the aforementioned principles are employed, the acquisition of the dynamic test data will aid significantly in the design of equipment that will eventually have a high probability of meeting the test, as well as the tactical, environmental requirements.

INSTRUMENTATION AND STRUCTURAL DESIGN

- Acquisition Of Meaningful Data
- Data Format Which Is Useful To The Mechanical Engineer
- Data Accumulation and Reduction Techniques That Fit The Problem
- Proper Placement Of Transducers
- Effective Pre-Test Planning
- Data To Initiate Changes and Improvement

INSTRUMENTATION TECHNIQUES: The effective use of dynamic instrumentation and data reduction procedures will provide greater insight into the problems of dynamic integrity.

VOLUME III - CHAPTER 7

INSTRUMENTATION

SECTION 2 - DYNAMIC TEST DATA ACQUISITION

- Organization of Test Data Collection
- Current Graphic Data Recording Methods and Presentation Formats
- Improving the Usefulness of Dynamic Response Data by Means of Magnetic Tape
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- Optimizing Transducer Characteristics for Dynamic Measurements
- Transducer Calibration Methods
- Transducer Mounting Methods and Techniques
- Improving the Signal Conditioning System

ORGANIZATION OF TEST DATA COLLECTION

Definition of the data format prior to any test is of paramount importance, since the ultimate usefulness of the data is greatly dependent upon this definition.

Definition of the test objectives is the first step toward a methodical test data collection process. The objectives should be stated in clear and concise terms; the technician must be made aware by the responsible test engineer of the nature of the test. Is it to be a developmental or a qualification test? The test objectives will necessarily be different for the two cases and will therefore affect the data collection methods.

Once the nature of the test has been established, the method of data collection may be determined. It is advisable to prepare a checklist outlining the objectives and critical measurements. A "Test Proceedings Journal" is recommended to fill this need (illustrated on the opposite page) which lends itself to the efficient recording of these data by the technician.

The general format of this Test Proceedings Journal will be quite similar for both the developmental and the qualification tests, however greater emphasis on the documentation of test data must be placed in the case of the latter. For both cases, a chronological record of the test must be maintained. It is essential that the acquired test data will enable an engineer at a later date to write a detailed test report without having to rely on any written or verbal information beyond that contained in the Test Proceedings Journal.

Possible failures of the test specimen must be recorded in an objective manner; the emphasis here must be placed on acquisition of test data only, not on analysis on the spot. Data must be provided to the engineer for a detailed evaluation of the problem.

Test facilities and associated instrumentation must be described in sufficient detail to determine its accuracy status. In particular, any equipment which is subject to periodic calibration must be supported with backup documentation relating its traceability to the National Bureau of Standards.

CHECK LIST (TO DEFINE TEST OBJECTIVES)

- Design Data
- Qualification Data
- Outline Important Parameters
- Acceleration, Velocity, or Displacement Outputs?
- Static or Dynamic?
- Reproducible Format Desired?

TEST PROCEEDINGS JOURNAL (TO RECORD DATA CHRONOLOGICALLY)

- Photo-Media References
- Record Transducer Locations
- Record Transducer Sensitivity Axes
- Phasing Orientation

CALIBRATION CERTIFICATE (TO SUPPORT EQUIPMENT ACCURACY)

- Description of Calibration Methods
- Clear Substantiation of NBS Traceability

ELEMENTS OF THE DATA COLLECTION TASK: Careful attention to the details of the task, a rigorous chronology of the test events, and clear documentation of the equipment calibration will result in greater usefulness of test data.

CURRENT GRAPHIC DATA RECORDING METHODS AND PRESENTATION FORMATS

Graphic data recording methods and the presentation formats currently obtainable provide a very useful adjunct to the data collection process in shock and vibration tests.

The basic structural response variables with which the test engineer is confronted during the course of shock and vibration tests are acceleration, displacement, and velocity. In the majority of test cases, displacement and velocity data can be derived from the acceleration data by various data processing methods. Thus, only the recording and presentation of acceleration data will be discussed in detail at this point.

The graphical presentation of acceleration as a function of real time (accelogram) is a basic prerequisite for meaningful analysis of the test specimen's response to the imposed environment. To accomplish this, two basic equipment categories are available; i.e. various chart recorders and the cathode-ray oscilloscope/camera combination. It must be stated at this point that either method will provide a useful accelogram. Neither method, however, provides data which could be used as a possible signal source to recreate the test environment at a future time. Data acquisition methods which do provide this capability are called "live data", and these will be discussed in detail later.

Perhaps the most common chart recorder in use today is the direct-writing, light sensitive, oscillographic recorder. This type of recorder utilizes a light-sensitive photographic paper as the recording media. The heart of this recorder is a D'Arsonval galvanometer. The galvanometer consists of an armature coil suspended within a permanent magnet structure. Attached to this coil is a tiny mirror. The armature and mirror are free to rotate within a defined arc. A high-intensity light beam is focused onto the mirror and the reflection in turn is focused through a complex mirror and optical prism system onto the light sensitive emulsion of the photographic recording paper.

The signal voltages derived from the accelerometer (after proper level conditioning) are used to deflect the galvanometer armature/mirror assembly and in turn record the history of acceleration vs time. Current recorders of this type yield a readable record within 5-15 seconds after recording. This type of record will however be subject to gradual "fading" if exposed to direct light for extended periods. Chemical processing must be employed if permanent records are a requirement; this process takes from 15-20 minutes.

The final data is presented with high contrast on white background with sufficient time resolution. The frequency response is limited primarily by paper speed and the galvanometer used. Current recorders and galvanometers provide DC to 10 KHz response, which is adequate for most shock and vibration data acquisition needs. Direct writing oscillography has without question established itself as the most useful method for reliable "quick-look" shock and vibration data acquisition and display.

Where frequency response in excess of about 10 KHz is required the cathode-ray oscilloscope/camera combination can be used to obtain acceleration vs time histories.

The principle of operation is quite similar. The amplified signal from the accelerometer is utilized to deflect the electron beam of the cathode-ray tube, a standard photographic camera is mounted to the display screen to photograph the display. The principal disadvantage of this system is that only a relatively short real time history is presented on the resultant picture; typically 10-15 milliseconds. However, the practical upper frequency response is limited only by the oscilloscope's electronics, and is therefore much higher than even the most stringent requirements of a shock test.

To obtain consistantly reliable data with the oscilloscope/camera combination, the problem of scope triggering must be treated in considerable detail. This is true in each data acquisition case; vibration, shock, or other non-periodic data.

In order to trigger the scope's sweeping electron beam, it is important to generate a voltage spike of predictable amplitude and lead time ahead of the data to be recorded. The simplified diagram shown on the appendix (7.4-3) illustrates a typical method which has been used with consistent success. Similar methods can be arranged for other requirements.

Recent developments and improvements in the area of oscilloscopes with long retention time capabilities on the cathode-ray display screen make this instrument desirable for presentation of relatively short pulses, such as shock data. Several companies produce instruments with excellent storage capacity retention in the order of several hours. The storage scope is treated in a manner identical with the common oscilloscope.

IMPROVING THE USEFULNESS OF DYNAMIC RESPONSE DATA BY MEANS OF MAGNETIC TAPE

Magnetic tape yields a data format which cannot be duplicated by any other present day recording media. Advantages such as instant replay of the environment and ease of storage for future use are only two of its many attributes.

For years shock and vibration engineers have complained strenuously about the vast amounts of data being collected during dynamic testing without much thought being given as to how this data would ultimately be put to use.

In the majority of the cases, data was collected by some form of direct writing recorder (pen or light beam sensitive type). Although this yields a data format somewhat better than the indication obtained from a panel meter, it does not provide a live data format; one which provides for essentially instant replay of the test environment.

Let us now examine what advantages magnetic tape has as a recording media. Magnetic recording tape has a coating of a permanent magnetic material deposited onto a synthetic film base. Initially the magnetic material is in a demagnetized state. Recording heads in the form of electromagnets are arranged in such a fashion that their tiny air gap is bridged over the magnetic material coated side of the tape as it is drawn across at a constant velocity. As the tape leaves the trailing edge of the recording head gap, each particle in the coating is left permanently magnetized. The intensity of magnetization is proportional to the instantaneous signal current in the recording head winding at the instant the particle left the gap.

The reproduce (replay) head is an electromagnet similar in construction and arrangement to the record head. When the magnetized particles are bridging the gap of the reproduce head, a magnetic flux is established in the core. As the tape is drawn past this gap, variations in the flux induce a voltage in the reproduce head winding proportional to: the amplitude of magnetic intensity of the tape particles; and the tape velocity. The resultant voltage recreates the original input signal.

The magnetic tape recording process thus described, uses what is called the "Direct Recording Process". Several other methods are available to the user and each have their particular advantages and disadvantages; these will be treated in detail in sections to follow. The advantages of magnetic tape as a recording media may be defined as follows, regardless of the selected process:

1. Reproduces original dynamic signals in electrical form. Permits further study by chart recorder, X-Y plotter, oscilloscope, wave analyzer, A/D conversion and by many other display and investigating instruments without the necessity of manual data plotting and reduction.

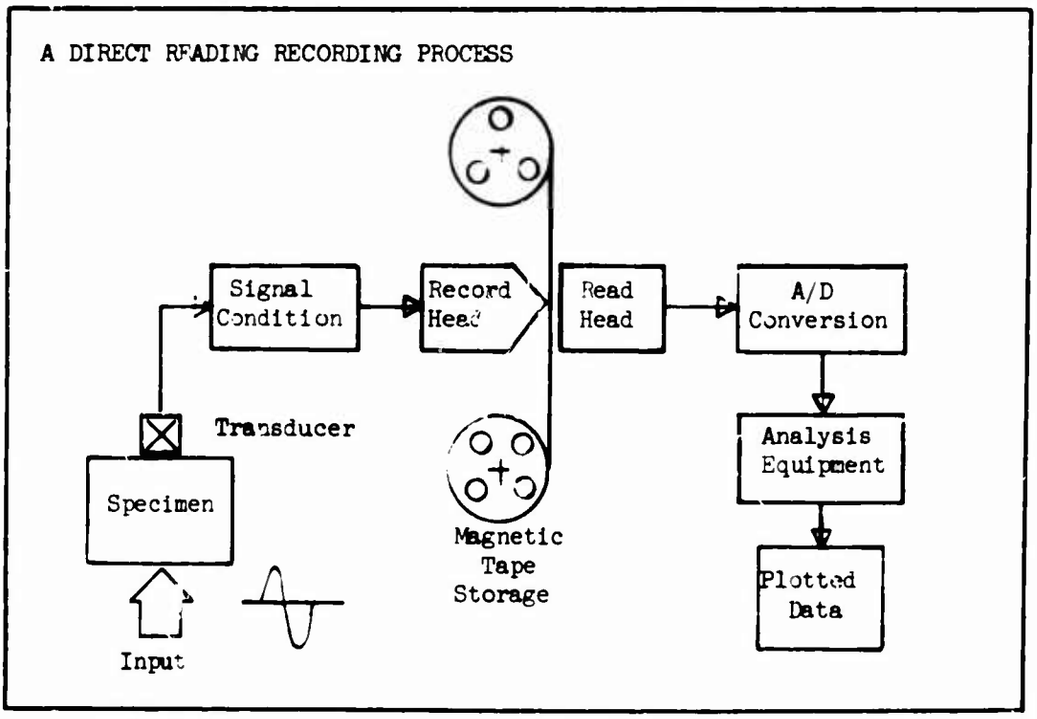
2. Provides a frequency range much wider than other recording methods.

3. Allows the speed of events and frequencies to be changed. Thus rapidly occurring events may be reproduced and analyzed at reduced speeds. This enables the user to utilize the more common low frequency analyzers and associated data processing equipment. Alternatively, long time recordings of slow events or low frequencies may be speeded up for study by simply reproducing the signal at a faster speed than was used during recording.

4. Time and phase relationships between numerous signals are accurately preserved. This permits identification of the sequence of parallel events and is especially valuable where shock propagations through a test specimen are to be determined.

5. Has great inherent flexibility in reproduction. The record (tape) may be reproduced immediately without special processing. Copies can be made for possible parallel use or for a variety of other purposes.

6. The process is economical because the magnetic tape may be erased and reused many times. This makes it practical to freely record many signals of interest at a low cost. Only data which proves to be of significant interest need be preserved or transferred to a data library for storage.



MAGNETIC TAPE: Dynamic phenomena may be permanently recorded on tape for subsequent use or analysis. The signal may be plotted, analyzed, or converted into other formats without compromizing the originally recorded data.

REFINING THE PRETEST PLANNING AND EQUIPMENT SELECTION PROCESS

A detailed examination of the measurement and equipment requirements prior to the start of any test is a prerequisite to efficient data collection, which will ultimately prove useful to the equipment design engineer.

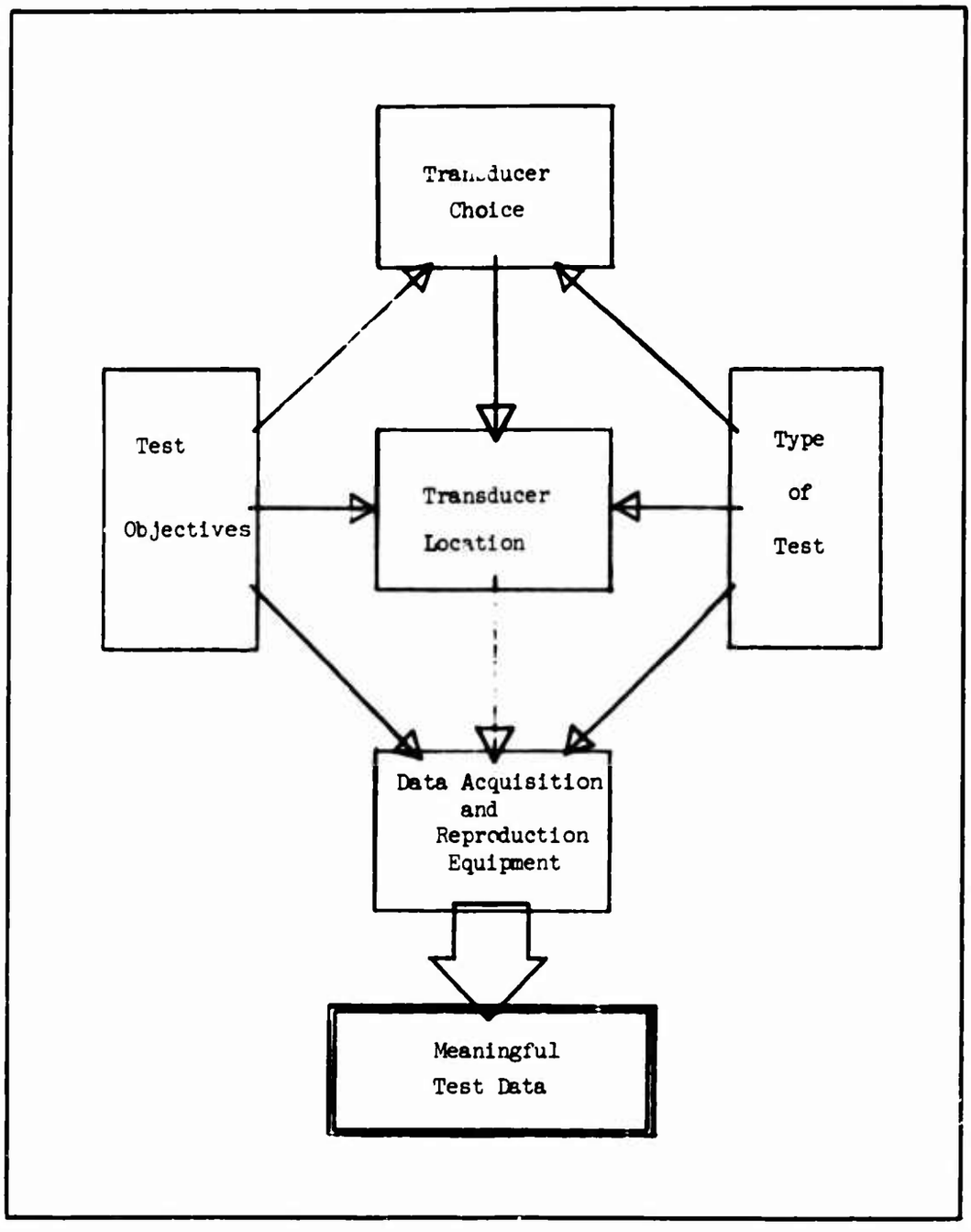
Since the instrumentation which is used to acquire the desired test data will vary with what is to be measured, it is important that the parameters be established well in advance of the test. The test engineer should discuss with the instrumentation engineer his measurement OBJECTIVES in great detail. Quite often a test is "over-instrumented", yet little if any data of real value to the equipment design engineer is obtained. The solution to this problem in most instances is a precise delineation of the measurement and objectives by the equipment design engineer, since he knows best what data he needs in the design of his product. The instrumentation engineer is now equipped to select the best methods and equipments to accomplish the task. The tabulation of the parameters to be measured can be of great value in organizing the test data collection process.

Consideration must also be given to the type of test to be performed. In the case of a qualification test, for example, the requirements for test data recording may be minimal, whereas in the case of a developmental test, data acquisition requirements may call for far more sophisticated equipments and methods.

The location of transducers on the specimen is one of the most important steps in the process of meaningful test data collection. Data obtained from locations chosen rather arbitrarily can be misleading and at times is worse than no data at all. In most cases a near optimum location can be determined from data which was obtained during a similar test or perhaps during the developmental test phase of the same test specimen.

Even though the meaningfulness and ultimate usefulness of the test data to the equipment design engineer must be used as the major guideline in the choice of methods and equipments, the factor of cost must also be considered. Relative cost comparisons can be made on this basis.

Another basic decision which must be made in the early test planning stage is the choice of direct reading vs data recording (storage) type instrumentation to be used. The details, advantages and disadvantages of both will be discussed in succeeding topics.



PRETEST PLANNING: The careful consideration of test objectives, test type, and transducer location (including interaction) will aid in the optimization of the instrumentation support equipment.

DIRECT READING VIBRATION AND SHOCK MEASUREMENT INSTRUMENTATION

Direct measuring and indicating shock and vibration instrumentation when properly utilized provides a means of rapid and economical on-line data collection and display.

In today's demanding pace of shock and vibration instrumentation, the trend seems to be to ever increasing equipment sophistication. This trend has indeed resulted in advancements which allow the instrumentation engineer to provide the structural designer with data in engineering terms he can use to better design his equipment.

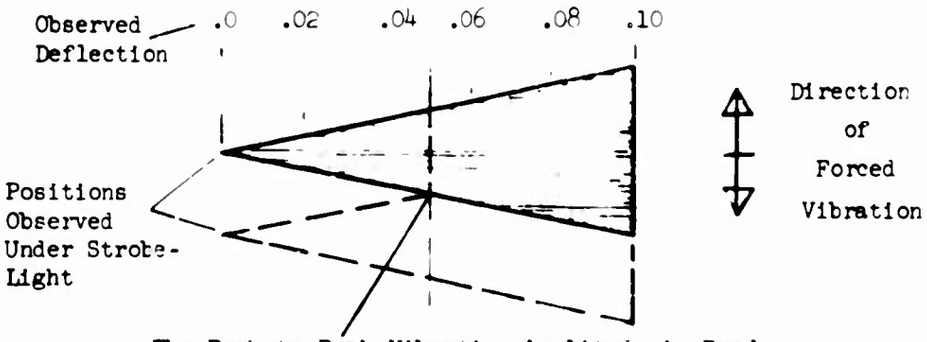
The instrumentation engineer however is sometimes tempted to over-instrument the test; the result being a lot of sophisticated data often useless to the equipment designer. The choice of instrumentation equipment must be based on the actual data acquisition needs for the particular job at hand.

In many test instances, a collection of the basic variables (acceleration, velocity and displacement) can still be accomplished reliably and economically with some of the time proven, direct measurement methods. One of the more popularly known methods is the use of a gage to determine the amplitude of vibration. As a rule, this method is somewhat limited to simple harmonic motion rather than complex vibration or shock. However, this method presents a means of rapid data presentation in terms of amplitude. A black-colored wedge is attached on a white background, and thence to the specimen. While the test specimen undergoes the vibration cycle, the wedge is viewed under stroboscopic light. The trigger source is derived from the vibration machine; synchronization is such that the stroboscopic light source is approximately four cycles above or below the frequency of the forced vibration. The result is that the wedge appears stationary to the observer's eye. By holding a linear scale against this wedge, one can directly measure the vibration displacement. As the amplitude of vibration decreases, movement of the wedge becomes increasingly difficult to measure. By using a simple magnifying lens, one can extend the range of measurement possibly by a factor of 10 to 1. Even so, this method does not fall in the area of sophisticated instrumentation. It presents an economical and rapid method of determining the vibration amplitude in gross terms. The approximate range of useful amplitude measurements of this method is limited to about .02 inches.

The hand held vibration pickup is another excellent method for obtaining quick on-line vibrational data in terms of amplitude and frequency. These pickups consist of an extremely light moving coil element which produce an electrical signal proportional to either velocity or displacement. The obvious advantage of this method is the capability of reading the variables directly on the face of a meter without complicated recording techniques and equipments.

In most cases direct readings are available in terms of vector velocity in inches per second, or vector acceleration in g units. A sweep through the frequency of interest, holding a constant level of input at the exciter table and monitoring the response of the test specimen, will readily indicate the resonant frequency.

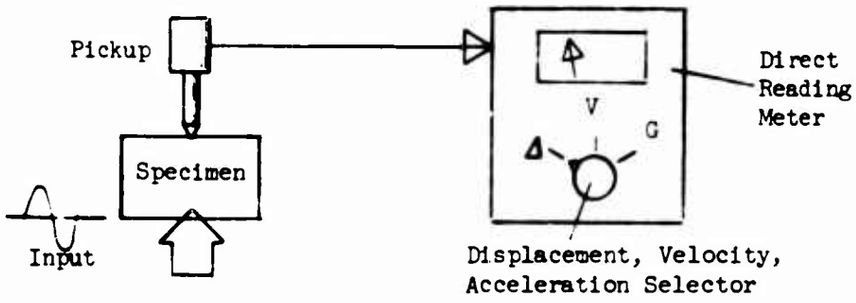
TYPICAL WEDGE INDICATOR



Some Advantages:

- Ease of Location Change
- Low Cost - No Supporting Meters Needed

TYPICAL HAND-HELD PICKUP



Some Advantages:

- Direct Read-Out of Three Variables
- Built-In Electronics for Derivation of Two Variables from One Measured Parameter
- Ease of Operation
- Self-Calibrating

DIRECT READING METHODS: The stroboscopic wedge and hand-held probe are two familiar methods for direct measurement of dynamic excitations.

CURRENT TRENDS IN DATA RECORDERS

Recording of variables as a function of time in a form which can be stored and reproduced must be evaluated with respect to other available data collection methods prior to the test start.

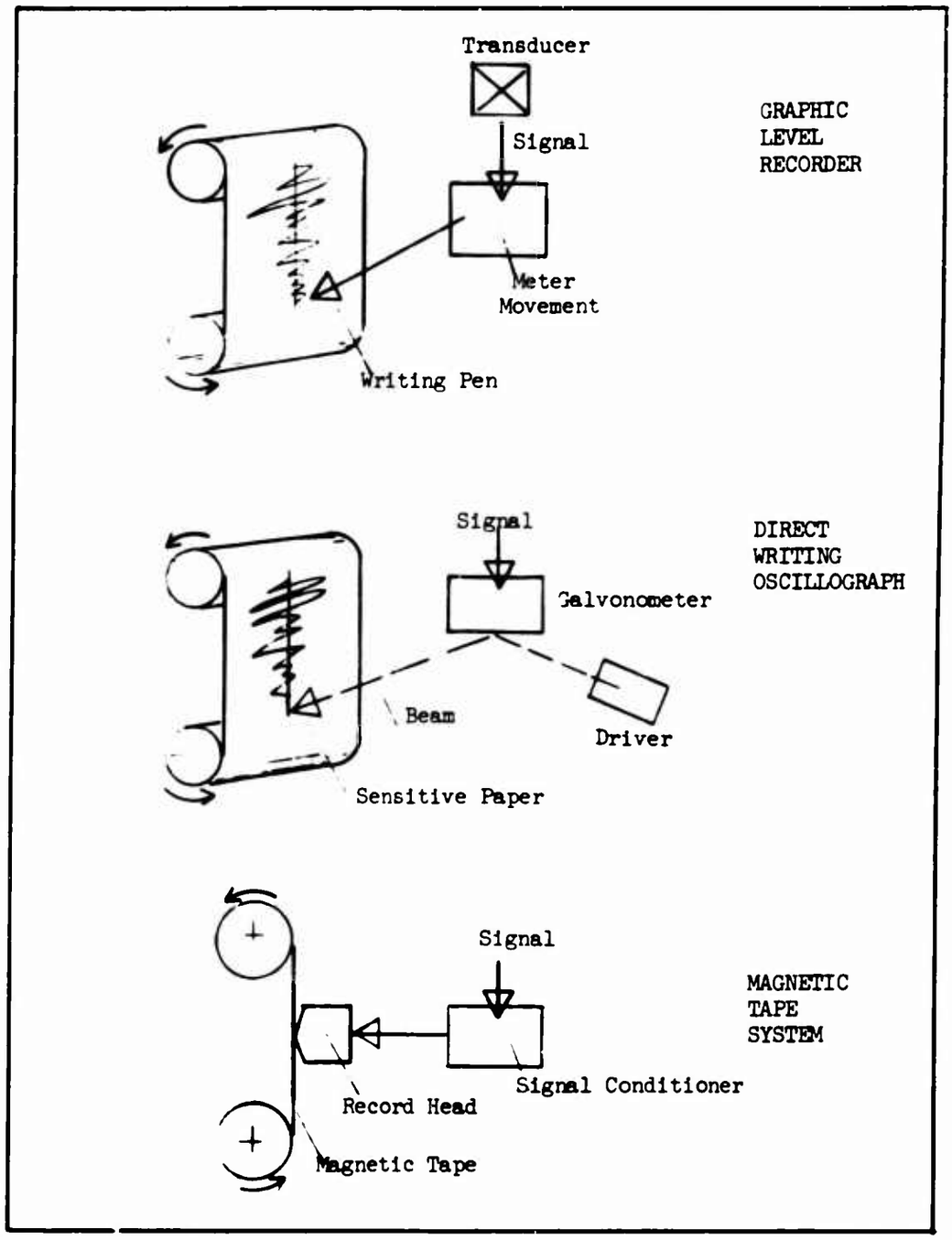
The recording equipment available on today's instrumentation market can be subdivided into three categories:

1. Graphic Level Recorder
2. Direct Writing Oscillograph
3. Magnetic Tape Recorder will be discussed

Each of the three categories will be discussed in greater detail. The graphic level recorder was for years the most common recording method for variables such as acceleration, velocity and displacement. Its main attribute is the low cost of initial acquisition plus the on-line readout capability of the acquired data. Basically, this type of recorder consists of a D'Arsonval meter movement to which a writing element is attached with either a hot-tip or an ink-filled cartridge which writes onto common recording paper. The writing-pen is deflected by the electrical current flowing through the armature. The resultant trace on the recording paper is a history of the amplitude and frequency to which the transducer is subjected to. Its greatest disadvantage is the fact that this type of recorded data cannot be reproduced in a live format. That is, it cannot be used as a signal source for a vibration machine in order to recreate the vibration environment.

The direct writing light-beam oscillograph in contrast to the graphic level recorder, utilizes a light beam galvanometer as the writing element. This method consists of a tiny mirror attached to the armature of the D'Arsonval movement. A collimated light beam is directed against this mirror. The reflection of this light beam is focused directly onto a light-sensitive photographic recording paper. This recording paper is drawn past the light beam at a constant velocity, impressing a time history of the vibration. The resultant trace becomes an acceleration versus time history of the phenomenon under observation. One distinct advantage is the higher frequency response realized. Frequencies from DC to well above 5000 Hz can easily be recorded by this method. However, the data is not reproducible in an electrical form. This method does, however, present a quick one-line readout of the data. It has a distinct advantage over the graphic level recorder in its higher contract and higher frequency response capability.

The third recording category is magnetic tape. This is by far the most commonly used method in today's field of instrumentation. The basic advantage is that the electrical analog received from various sensors such as accelerometers is recorded in a live format which can be replayed many times over. The signal derived from the magnetic tape in the playback process also lends itself to use as a signal source for electrodynamic vibration machines. A major advantage of this method is the industry standardization on tape speeds, frequency response, center frequencies, and bandwidths. This allows greater data communication between test laboratories.



DATA RECORDERS: Currently, dynamic data is recorded by one of three means; graphic level recorders, direct writing oscillographs, or magnetic tape recorders.

OPTIMIZING TRANSDUCER CHARACTERISTICS FOR DYNAMIC MEASUREMENTS

By simple definition, a transducer is a device which converts energy from one form to another. Transducers used in shock and vibration instrumentation provide a means of converting mechanical forces into proportional electrical currents which can be amplified for measurement and evaluation.

For a basic understanding of the transducer principle, one might think of a phonograph pickup as an example. In this application, mechanical forces sensed by the phonograph needle are converted into small electrical signals which are then amplified for practical use in the form of sound. Just as there are different principles employed in phonograph pickups (i.e. crystal, magnetic), there are several types of transducers used in shock and vibration measurement.

The shock and vibration measurement transducers discussed here represent three of the most commonly used types. These are: the piezoelectric accelerometer, the velocity pickup, and the unbonded strain gage. The selection of the type of transducer for a particular application depends upon the nature of the mechanical force to be measured with respect to the magnitude and complexity of the shock or vibration signal, and upon range limitations of the monitoring instrumentation.

Present-day technology places many demands on transducer performance criteria. Of prime consideration, is the ability of the accelerometer to be useful over a broad frequency range, a wide dynamic range, and its ability to faithfully reproduce the dynamic acceleration which it senses. In addition, since dynamic tests are often combined with climatic environments, the transducer must also be capable of performing within its tolerance limits under extreme conditions of temperature and humidity.

The most versatile and widely used transducer for both shock and vibration applications is the piezoelectric accelerometer. The piezoelectric theory relates to a pressure-sensitive property of the crystalline material used in the accelerometer. That is, when the material is subjected to a force or compression, an electrical charge is produced. The electrical charge is proportional to the quantity of force applied and, therefore, provides a practical means for measuring that force. The crystalline element of the piezoelectric accelerometer responds with high sensitivities to measurement of shock impacts of up to 10,000 g's and sinusoidal vibrations of up to 2000 g's at frequencies of up to 15,000 Hz. Adding to its versatility, is its ruggedness in design and its compactness in size which facilitate handling and mounting.

The velocity pickup is an induction type transducer which uses a principle similar to that of the variable-reluctance (or magnetic) phonograph pickup. Its output voltage is developed by the movement of a permanent bar-magnet within a magnetic field. The voltage developed is proportional to the displacement velocity and can be monitored directly. For a velocity of about 10 centimeters per second, the output voltage is in the order of 10 millivolts. However, the velocity pickup is limited only to use in the measurement of shock motions having relatively small displacements.

Another transducer of interest is the unbonded strain gauge type where a resistive principle is used. The strain sensing element consists of four

sets of strain-sensitive wires connected to form a Wheatstone bridge (a balanced resistive circuit used for accurate resistance or voltage measurements). The wires are mounted under some pre-stress between a frame and a movable element. As the element moves about a pivot point, strain is created which changes the wire resistance, thereby causing an unbalance of the bridge. If an external voltage is connected to the bridge circuit, the output of the strain gage will be zero when the bridge is balanced (no movement); a force acting upon the movable element of the strain gage will unbalance the bridge, giving rise to an output voltage which is proportional to the displacement.

Although their accuracy can be better than 0.1 percent, a disadvantage is the susceptibility of the wire materials to changes in temperature, making it necessary to restrict their use to measurement under constant temperature conditions, unless temperature compensating measures are applied.

In summary, it can be stated that generally the transducer types discussed fall into three basic categories:

1. Piezoelectric (Crystal) - Shock excitations and others above 5 Hz. Data acquisition.

2. Velocity (Moving Coil or Magnet) - Vibration and other periodic motion data in the range 5 Hz to 2 Hz.

3. Unbonded Strain-Gage - Vibration and shock situations (less than 100g) in the frequency range DC - 500 Hz.

The most common form of signal conditioning used today for category No. 1 is the change type amplifier. It provides for more rapid and reliable procedures during field calibration. Various voltage amplifiers and readout instruments can be used with the category No. 2 type transducer; the most common is the self-contained vibration meter (e.g, MB, CEC).

The category No. 3 transducer requires either a DC or an AC excitation voltage coupled with some method for balancing. Numerous commercial equipment is available(e.g, Endevco, CEC, Honeywell).

Each of these transducer categories is illustrated in the Appendix, page 7.4-4.

TRANSDUCER CALIBRATION METHODS

In order to guarantee the accuracy and validity of any shock or vibration measurement, it is essential that the calibration of the transducer be verified. Various methods for accomplishing this have been employed; the most common is the "comparison" method, which is rapid, accurate, and easy to perform.

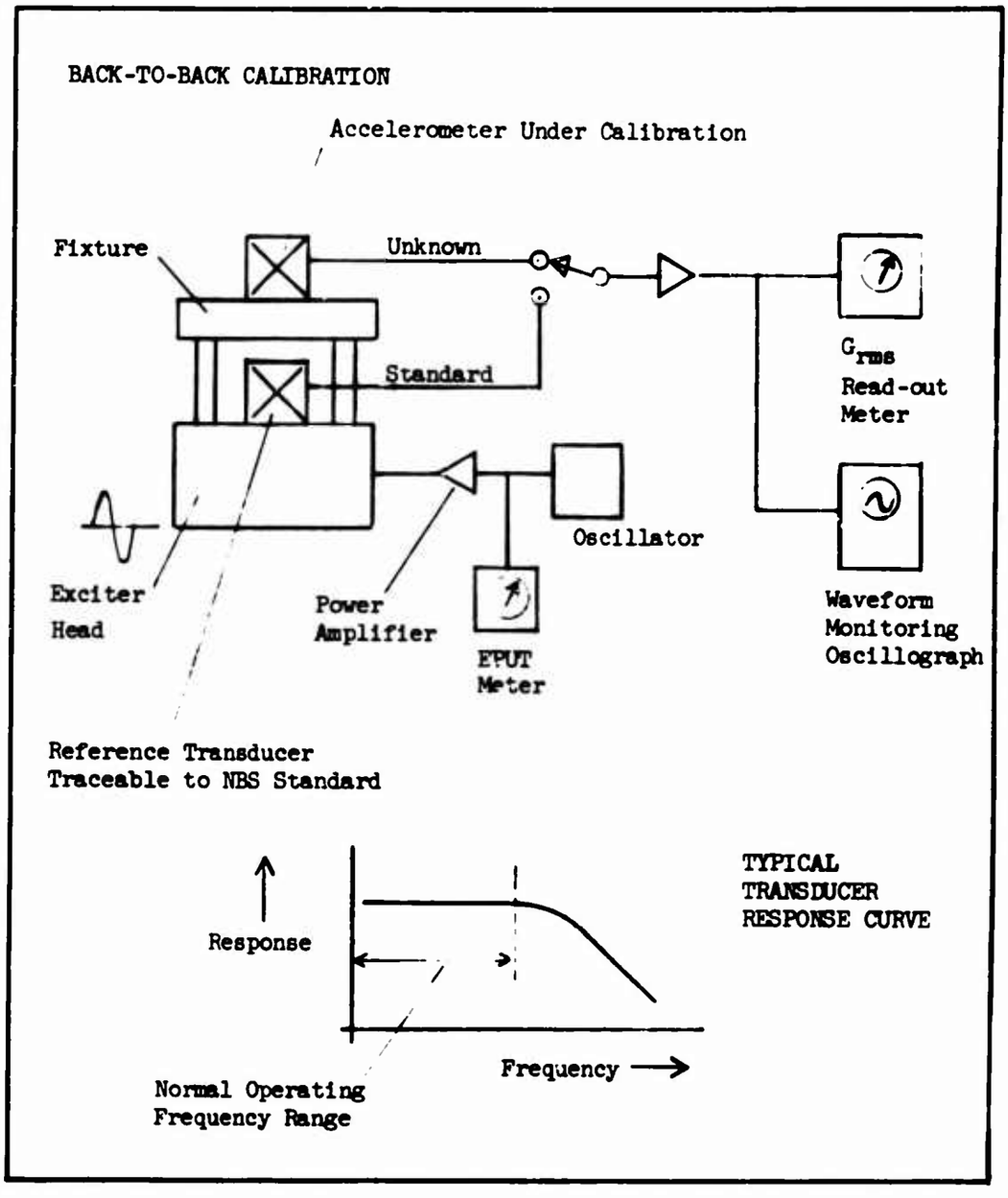
Determining the basic sensitivity of a transducer, that is, the ratio of change in electrical output to a change in mechanical input, requires accurate measurement of both the mechanical input and the resultant electrical output. Initial sensitivity calibration of each transducer, using the comparison method, is performed by the manufacturer. The same method is also employed by the shock or vibration laboratory as a reliable and rapid means of verification. In this method, the calibration of the transducer to be verified is compared to that of a reference transducer whose accuracy has been established and is traceable to the National Bureau of Standards (NBS). In performing such a comparison, both the calibrated transducer and the transducer to be calibrated are mounted in a "back-to-back" configuration as shown, enabling both transducers to be subjected to the same input motion and their output signals to be monitored simultaneously.

Factory sensitivity calibrations are performed using the instrumentation shown in the block diagram. While the shaker table is vibrated at 50 cps and 10 G's, the voltage divider is adjusted with the switch in either position. The ratio indicated on the decade voltage divider is then used to compute sensitivity; the sensitivity of the standard transducer divided by that ratio, yields the sensitivity of the transducer being calibrated. The degree of accuracy obtained using this system is better than 1 percent.

To eliminate errors which might be introduced by a transducer-controlled drive system, the amplitude of vibration is determined optically by means of a high resolution microscope viewing an illuminated light spot on the shaker table and measuring table displacement in terms of inches double amplitude (peak-to-peak displacement).

Acceleration for any given frequency can be calculated using the following formula: $G = 0.0511\ Df^2$, where 0.0511 is a constant, D is double amplitude, or peak-to-peak displacement in inches, and f is frequency in cycles per second. Other useful formulae are available for determining the sensitivity of a transducer when a reference standard is not available for comparison. Calibration data supplied with each transducer includes the necessary parametric values for such calculations.

The graph illustrated on the opposite page represents a typical sensitivity calibration curve. The linear portion of the curve represents an acceleration of 10 G's. Sensitivity is given in peak millivolts per peak G (if read on an rms voltmeter, this value is then converted to rms millivolts per rms G). This information, along with a calibration certificate, is documented by the manufacturer and accompanies each transducer.



CALIBRATION: Sensitivity calibration of dynamic transducers is accomplished by comparison with a known transducer

TRANSDUCER MOUNTING METHODS AND TECHNIQUES

The transducer mounting and coupling techniques must be considered and evaluated for each test case since they form an important and integral part of the overall data acquisition system.

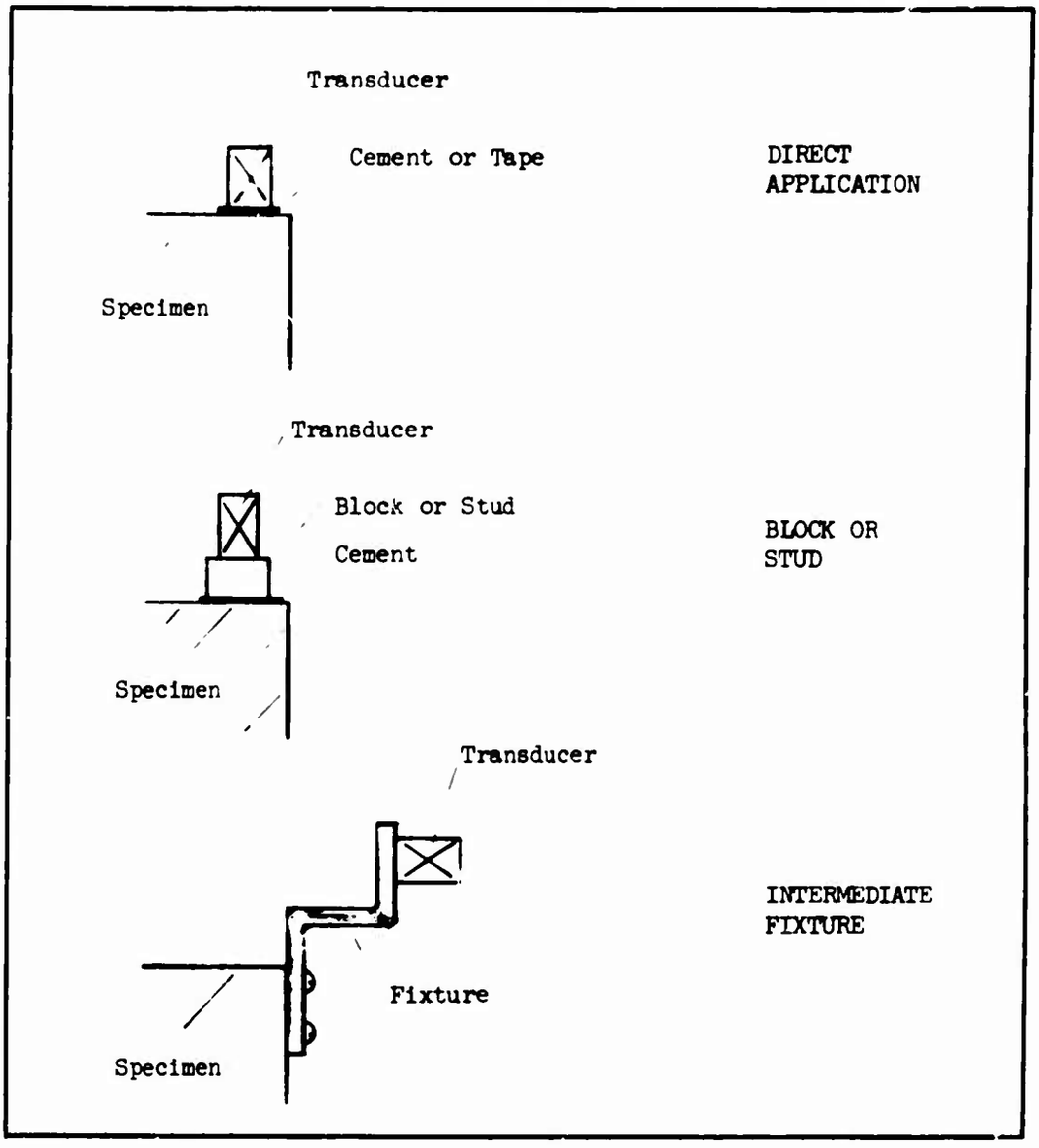
In order for an accelerometer to generate accurate and useful data, it must be properly coupled to the equipment under investigation. The methods of attachment should not introduce any di[illegible]ion. This requires that the accelerometer mounting method be rigid [illegible]he frequency range of interest. It is necessary that the bonding meth[illegible]mployed does not introduce either an amplification or filtering effec[illegible] to the stimuli that is acting upon the accelerometer.

A popular method of mounting accelerometers of relatively low weight is that of cementing the accelerometer and an intermediate mounting stud to the test specimen surface. In rare instances it may be desirable to apply the cement directly to the accelerometer to reduce the effective accelerometer weight. To insure adhesion sufficient for efficient energy transfer at higher frequencies, it is recommended that an oil film or a silicone base jelly be applied between the accelerometer mounting surface and that of the specimen.

Another common mounting method for lightweight accelerometers is pressure sensitive tape. The tape has an adhesion layer on both sides, and is placed between the accelerometer and the surface of the test specimen. This method should be restricted to frequencies below 500 Hz; higher frequencies may result in an alteration of the response of the accelerometer.

At times fixtures may be required to attach an accelerometer to a test specimen in order to transfer the energy from one plane into another plane. Prior to application of this method, one should investigate possible alteration effects which the fixture might have on the accelerometer's response. In order to properly design such a fixture, the exact nature of its use must be known. Frequency range, G levels and, of course, the mechanical specification of the test object must be well defined. A common rule is; the more complex the test fixture, the more points will have to be examined since the probability of this fixture having several resonance points is quite high. Some ground rules for use of a mechanical fixture are: the major resonant frequency of the fixture should be well above or below the frequency test range; the test fixture should be as light as possible in order to minimize undesirable loading effects of the test specimen.

The cable which transfers the signal from the accelerometer to the signal conditioning equipment must be properly supported. It is recommended that the cable be fixed with tape or similar methods every three inches over its length. Failure to do so may not only result in spurious outputs but it can behave as a noise generator. This is especially true in cases where the acceleration level is relatively low which may result in an erroneous indication of the test specimen's behavior.



TRANSDUCER MOUNTING: The effect of the transducer mounting technique on the specimen response and signal distortion characteristics should be carefully evaluated.

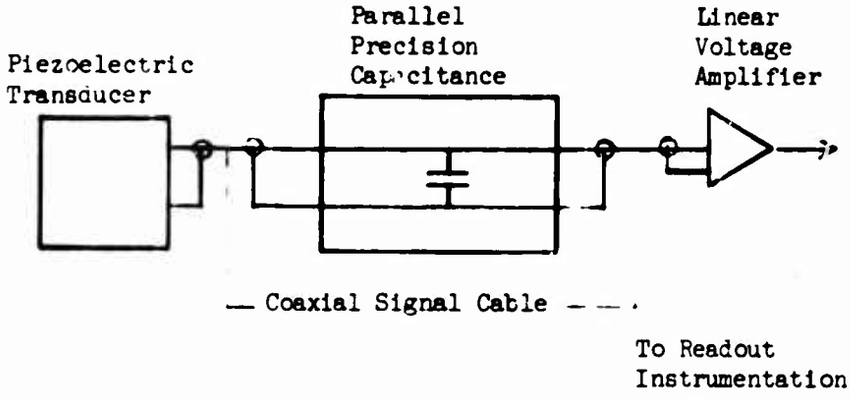
IMPROVING THE SIGNAL CONDITIONING SYSTEM

The equipment used to condition a transducer output signal into a recordable level must be evaluated with respect to the expected and non-expected situations which might occur.

Since any transducer is a device which converts mechanical motion into an electrical signal having a linear relationship to the mechanical motion, it is important to amplify this electrical with "high fidelity".

A typical signal conditioning system is illustrated in block diagram. The output from the crystal accelerometer, normally in millivolts per G of acceleration, is transmitted to the high impedance input of a voltage amplifier by a special low-capacity cable having low noise characteristics. The high input impedance (in the order of several hundred megohms) of the amplifier provides a matched termination for the high internal source impedance of the accelerometer. This results in a relatively long RC time constant, assuring good low frequency response. The sensitivity of the accelerometer can be attenuated by simply lengthening the interconnecting cable between the accelerometer and the amplifier input. However, in most cases, it is desirable to utilize a separate capacitive attenuator to facilitate the control of the attentuation value. Usually, a decade capacitor is used for this purpose. Precisely known amounts of capacity are placed at the output of the accelerometer and in parallel with the input to the amplifier. A new value for the effective sensitivity of the accelerometer may be calculated from the circuit parameters when this attentuation method is used. The attenuated output thus introduced to the input of the voltage amplifier is then amplified to a level suitable for further processing. This amplifer is normally a linear-voltage type.

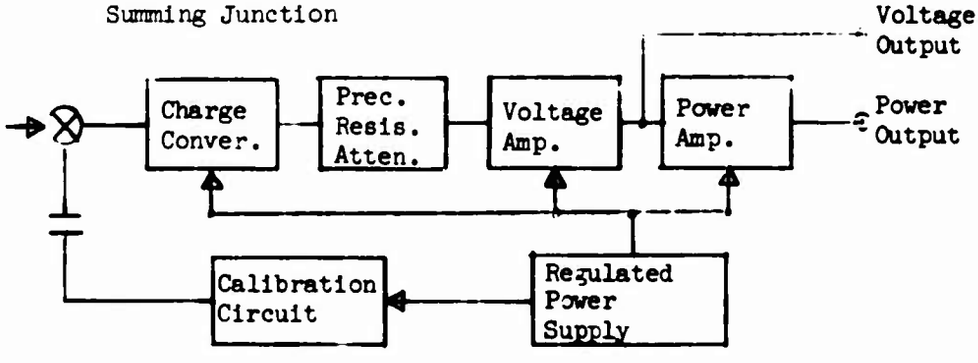
In many instances, it may be desirable to subject the data signal to some filtering in order to separate spurious signals from the basic wave shape. Normally, the filtering is accomplished subsequent to the amplification of the transducer signal; that is, at the output of the amplifier. When filtering is applied, care should be taken to avoid applying the technique indiscriminately. An analysis should first be made as to whether the high frequency content of the basic sine wave is actually a part of the data being received or is noise which is introduced somewhere between the sensor and the read-out device. Once this has been determined, the type of filtering can then be considered. One must be fully aware that if filtering is applied, the apparent acceleration levels will be decreased. As a general rule, if the high frequency cut-off is five times the basic frequency of a half sine wave signal of the vibration frequency, this will not produce undue distortion of the signal. Ratios as low as one and one-half times the signal frequency are being applied and have been quoted as acceptable. In all cases, the filter should have a sharp roll-off at its cut-off frequency. No recognized commercial filter is available, within the budget range of most users, which has a sharp enough roll-off to be used at the lower filter cut-off ratio.



TYPICAL VOLTAGE ACCELEROMETER/AMPLIFIER SYSTEM

FEATURES:

- Transducer Generates a Voltage Proportional to Acceleration
- Precision Capacitance Decade Box May be Used to Attenuate the Signal to Desired Level
- Linear Voltage Amplifiers Provide the Relatively High Input Impedance (≥10 MΩ) Required for Low Frequency Response and Also Provide Current to Drive the Readout Instrumentation



TYPICAL CHARGE AMPLIFIER SYSTEM

FEATURES:

- Output from Piezoelectric Charge Accelerometer is a Charge (Number of Picocoulombs per g) Proportioned to the Acceleration Signal Which is Converted in the Charge Converter to a Conventional Voltage Proportioned to the Acceleration Signal; From Here on, It is Treated as the Voltage System

VOLUME III - CHAPTER 7

INSTRUMENTATION

SECTION 3 - REDUCTION OF DYNAMIC TEST DATA

- A Discussion of Oscillogram Interpretation Techniques
- Application of Amplitude Spectrum Analysis Equipment

A DISCUSSION OF OSCILLOGRAM INTERPRETATION TECHNIQUES

In order to transform oscillograms and other chart records into data which can be effectively reduced and analyzed, a well defined methodology and technique must be developed.

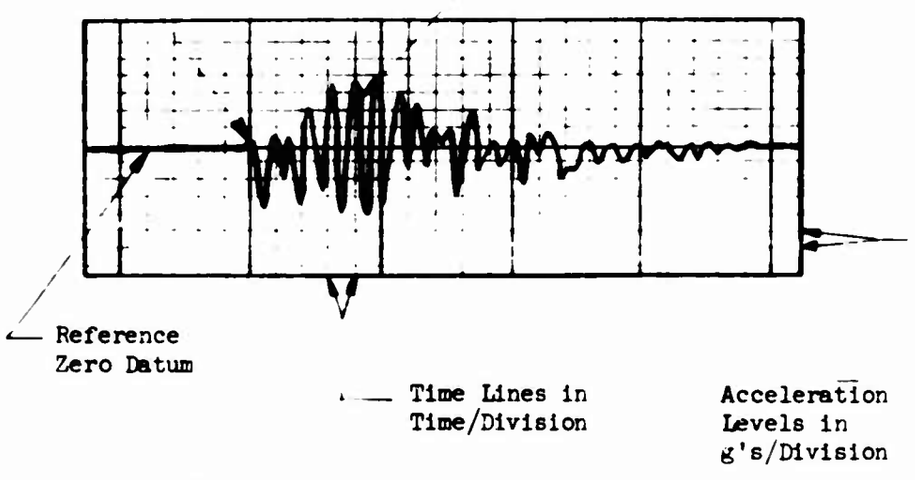
In spite of the increased applications of magnetic tape as a recording media, the wide use of the oscillograph in dynamic data acquisition still prevails. For practical design purposes, it is necessary to be able to extract from the oscillogram the meaningful data required. Some of the more commonly used methods of oscillogram analysis will be discussed in this section. This discussion also includes oscillogram reduction, oscillogram to magnetic tape transformation, oscillogram waveform analysis, and interpretation of shock data oscillograms.

Examining the illustrated shock waveform, the number of g's per unit of trace deflection is determined from the oscillograph calibration employed during the data acquisition phase. Similarly, the paper-speed established determines the real-time to recording-paper length relationship. Once these quantities have been established, the recording can be edited by simple visual means. Parameters such as acceleration peaks and time and frequency of occurrence can now be evaluated. Another parameter of importance to the equipment designer, is the comparison of acceleration propagation time throughout the structure under test. This is determined from the time required for the shock pulse to propagate from the first point of impact to the various accelerometer locations on the equipment structure. Knowing the frequency response of the recording galvanometer, the degree of damping, if any, within the structure can also be determined.

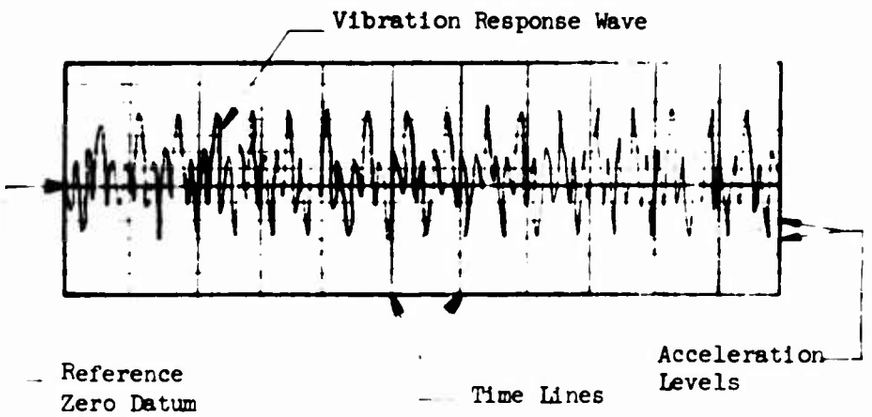
When analyzing shock data, the characteristics of the particular accelerometer used during the test must also be taken into consideration. Some crystal accelerometers experience a change in these characteristics when subjected to repeated blows. This will manifest itself as an apparent low-frequency base-line shift of the recording trace. Also, when a "ringing" of the trace is noted, one should investigate the possibility of the accelerometer having been excited near its natural frequency.

In order to approach oscillogram interpretation from an analytical point of view, it becomes necessary to transform the graphic data into a live format; that is, to transform the traces into electrical analogs. Recent developments in oscillogram reading equipment have made this possible. The equipment illustrated transforms the oscillograph trace into voltage levels which are in turn recorded on a digital magnetic tape system. The final data format of this conversion process is a digital presentation of the original analog oscillograph trace on magnetic tape. This format lends itself to a number of methods for further processing. One of the most useful is its use in conjunction with a shock spectrum analyzer. If the tape is properly formatized, it can also be interfaced directly with present-day computers. Punched card or tape is still another application for further analysis.

TYPICAL OSCILLOGRAM OF A SHOCK PULSE



TYPICAL OSCILLOGRAM OF VIBRATION RESPONSE



OSCILLOGRAM RECORDS: Oscillograms are still widely used as permanent records of shock and vibration phenomena.

APPLICATION OF AMPLITUDE SPECTRUM ANALYSIS EQUIPMENT

A significant portion of shock and vibration test data analysis can be performed with relatively simple, yet reliable, amplitude spectrum analysis equipment and techniques.

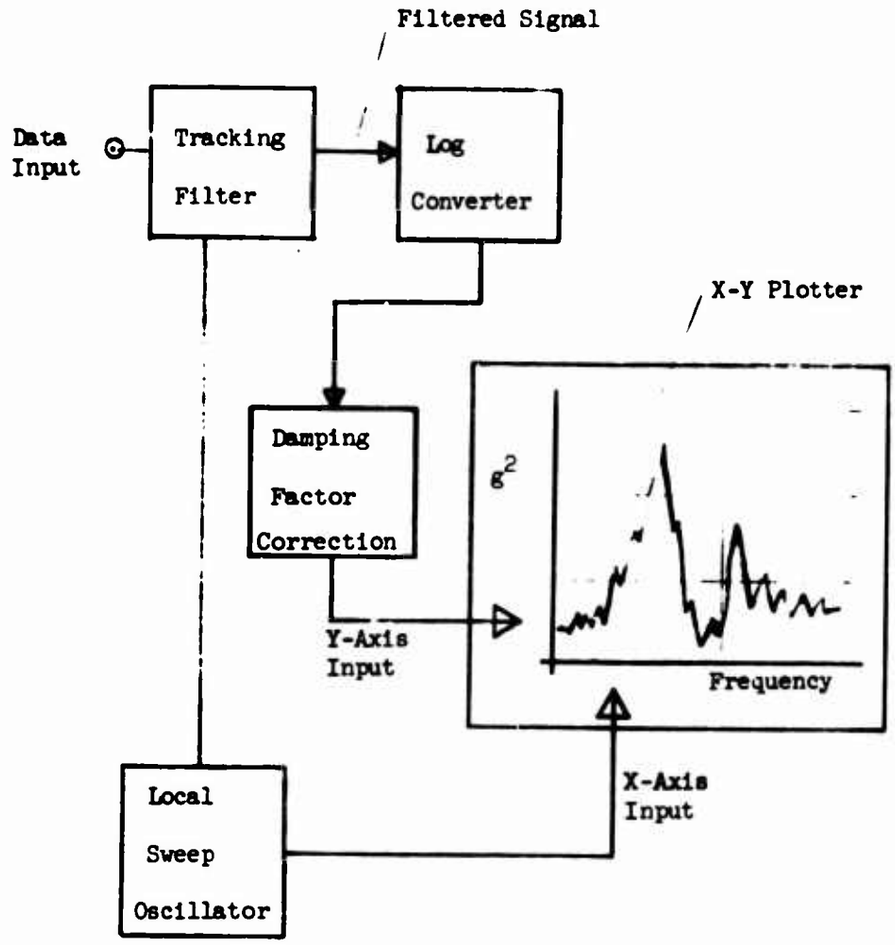
Analyzing data in which the signals are essentially sine waves, presents no real problem. Analyzers for determining signal amplitude and phase (vector representation) are readily available and perform this function satisfactorily. The plot of such a vector is referred to as a Hyquist diagram. When amplitudes are plotted against frequency on a separate chart, the familiar Bode charts are used. However, analysis becomes more complicated when signals are not repetitive, or are somewhat noise-like, and their inter-relationships more involved.

Most Mechanical Engineers prefer the power type over the amplitude type spectrum analyzer. The accuracy of the indications of a typical power spectrum analyzer depends greatly on the amount of time spent on the analysis and on the bandwidth of the filter used. For example, the use of a narrowband filter implies an analysis limited to part of the spectrum, requiring less time to accomplish, but with a net result of lower confidence.

A common system of filtering employed in the analysis of a spectrum uses a set of comb filters which operate simultaneously and are in parallel with the data signal. Generally, the spacing of the center frequency of each of the comb filters is one-third of an octave or a full octave apart. Each filter requires a separate detector; thus, the parallel outputs that are provided require the use of multi-channel recorders for output communication.

Another commonly used analyzer employs a single filter in a heterodyne system. In this system, the data input signal beats with a local oscillator signal, and a resultant sum or difference signal is fed to the filter. The advantage of this system over the type previously discussed is that a single filter of exceptionally good characteristics may be used, thereby requiring the use of only a single detector or recorder. With a pair of such analyzers, transmissibility ratios which so often have to be plotted from typical vibration and shock data can easily be derived. Also, scalar ratios of the output spectrum versus the input spectrum can be plotted as a function of frequency, where the input spectrum could be that derived from an accelerometer at the base of the equipment under test; the second channel could be any one of several accelerometers located within the equipment structure. The results can be used for automatically analyzing resonances and losses in mechanical structural integrity. The final data format of this type of data reduction will usually be represented by an acceleration-squared versus frequency plot such as the one illustrated. This plot provides the Mechanical Design Engineer with data presented in engineering units useful for practical application.

TYPICAL TRACKING/X-Y PLOTTER SYSTEM



FEATURES:

- Provides Power Spectral Density Plots
- Can Be Reconfigured to Show Frequency Distribution
- Determines Phase and Amplitudes of Periodic Motion Data
- Similar Systems are Available for Analysis of Non-Periodic (Non-Stationary) Data

TRACKING SYSTEM: Now data from a dynamic test can be input into a tracking complex to provide a graphical plot of power spectral density.

VOLUME III - CHAPTER 7

INSTRUMENTATION

SECTION 4 - APPENDIX

- Bibliography
- Glossary
- Typical Dynamic Transducers
- Data Recording by Oscilloscope

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GLOSSARY

Accelogram - A pictorial plot showing acceleration levels versus time which a test specimen experiences as a result of an input excitation.

Accuracy - The capability of an instrument to follow the true value of a given phenomenon. Often confused with "inaccuracy", which is the departure from the true value into which all cases of error are lumped - including hysteresis, nonlinearity, drift and temperature effect.

Amplitude - The magnitude of variation in a changing quantity from its zero value. The word must be modified with an adjective such as "peak", "R.M.S.", or "maximum" which designates the specific amplitude in question.

Amplitude Response - The maximum output amplitude obtainable at various points over the frequency range of an instrument operating under rated conditions.

Analog - An adjective which has come to mean continuous, cursive, or having an infinite number of connected points. The instrumentation industry uses the words analog and digital where the more precise language would be continuous and discrete.

Attenuation - Reduction or division of signal amplitude while retaining the characteristic waveform. It implies deliberately throwing away or discarding a part of the signal energy for the sake of reduced amplitude.

Bandwidth - The range of frequencies over which a given device is designed to operate within specified limits.

Calibration - The process of comparing a set of discrete magnitudes or the characteristic curve of a continuously varying magnitude with another set or curve previously established as a standard. Primary calibration is a calibration procedure in which the instrument output is observed and recorded while input stimulus is applied under precise conditions, usually from a primary external standard traceable directly to the U. S. Bureau of Standards. Deviation of indicated values from their correction (or calibration) curve for inferring true magnitude from indicated magnitude.

Calibration Curve - The path or locus of a point which moves so that its coordinates on a graph are corresponding values of input signals and output deflections. Also the plot of error versus input (or output).

Distortion - An unwanted change in waveform. Principal forms of distortion are inherent nonlinearity of the device, nonuniform response at different frequencies, and lack of constant proportionality between phase-shift and frequency. (A wanted change would be called modulation.)

Electromagnetic - Pertaining to the mutually perpendicular electric and magnetic fields associated with the movement of electrons through conductors, as in an electromagnet.

Feedback - In a control system, a short form of the expression "closed loop feedback control." In such a system, either the forward or the feedback path includes an active sensor whose output is mixed with incoming signal.

GLOSSARY (Continued)

Thus, the system "throughput" is always the sum of sensor output and raw incoming signal. This throughput is sometimes called an error signal, which enables the system to govern its own behavior. In an amplifier, the return of a portion of the output from any stage to the input of that stage or of a preceding stage.

Frequency - The number of times that a periodic function repeats the same sequence of values during a unit variation of time. The unit is the cycle-per-second which equals one Hertz (Hz).

Frequency Response - The portion of the frequency spectrum which can be passed by a device as it produces an output within specified limits of amplitude error.

Impedance - An indication of the total opposition that a circuit or device offers to the flow of alternating current at a particular frequency. A combination of resistance R and reactance X at a designated frequency (all expressed in ohms):

$$|Z| = (R^2 + X^2)^{\frac{1}{2}}$$

Linearity - The "straight-lineness" of the transfer curve relating an input to output; that condition prevailing when output is directly proportional to input.

Noise - Any unwanted electrical disturbance or spurious signal which modifies the transmission, measurement, or recording of desired data.

Piezoelectric - The interaction between the electric charge and the deformation of certain asymmetric crystals having piezoelectric qualities. Piezoelectric transducers subjected to excitation give out an electric current proportional to the severity of excitation.

Reluctance - The opposition offered by a magnetic substance to magnetic flux; specifically, the ratio of the magnetic potential difference to the corresponding flux.

Repeatability - The maximum deviation from the mean of corresponding data points taken from repeated tests under identical conditions. Also, the maximum difference in output for any given identically repeated stimulus with no change in other test conditions. Also, the maximum difference in output for any given identically repeated stimulus with no change in other test conditions.

Response - The motion (or other output) of a system or device resulting from an excitation.

Rolloff - The condition (and its magnitude) which describes an intentional or desired attenuation at frequencies above (or below) a certain point. Thus, a low-pass filter is designed to provide rolloff at high frequencies, and a high pass filter is designed to provide rolloff at low frequencies. Commonly rated in db per octave. Unintentional rolloff is more properly called decay.

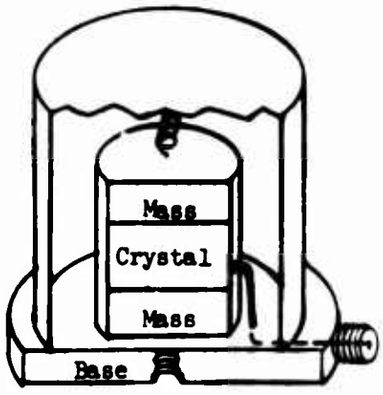
Scale Factor - The amount by which a quantity being measured must change in order to produce unit pen deflection. Also, the ratio of real to analog values.

Sensitivity - The property of an instrument which determines scale factor. As commonly used, the word is often short for "maximum sensitivity", or the minimum scale factor with which an instrument is capable of responding.

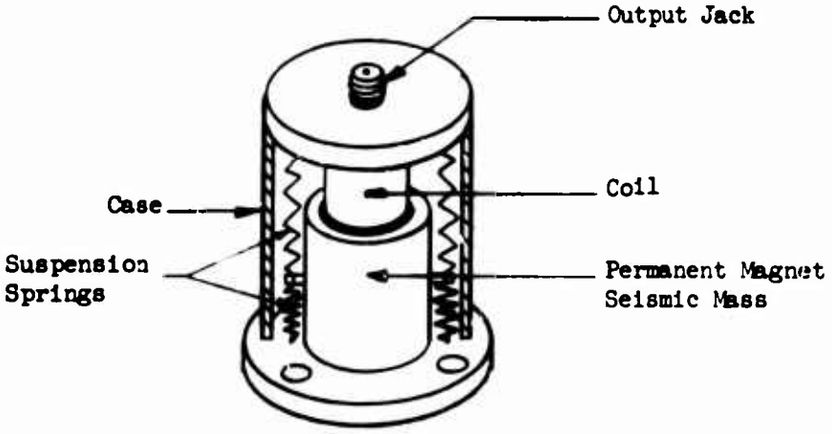
Stroboscope - An instrument consisting of a light source which blinks off and on at a desired frequency. Permits visual slow motion observation of maximum test specimen deflections caused from an input vibration.

Transducer - A device for translating faithfully the changing magnitude of one kind of quantity into corresponding changes of another kind of quantity. A dynamic transducer translates a shock pulse into an electric current output.

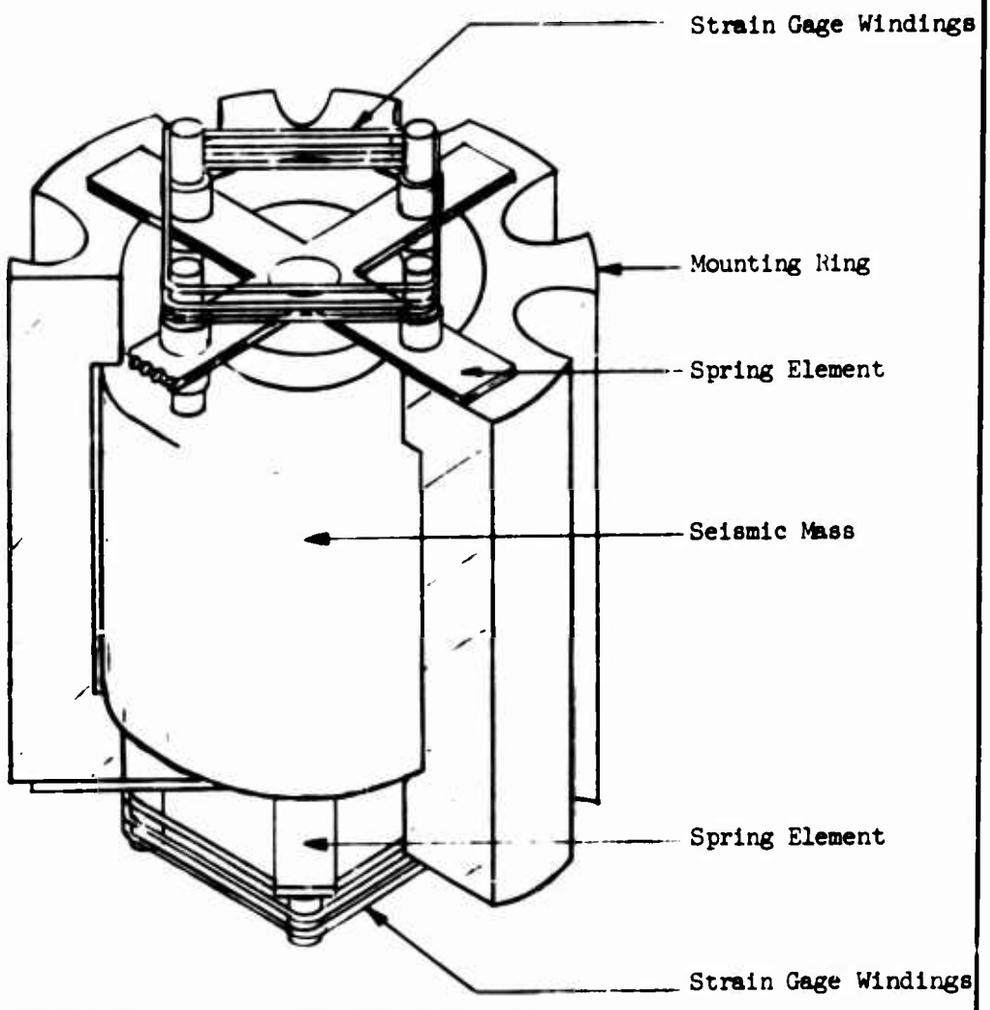
TYPICAL DYNAMIC TRANSDUCERS



T[illegible] PIEZOELECTRIC ACCELEROMETER
(Cr[illegible]ectional View, Typical for
Com[illegible]n Type Design)



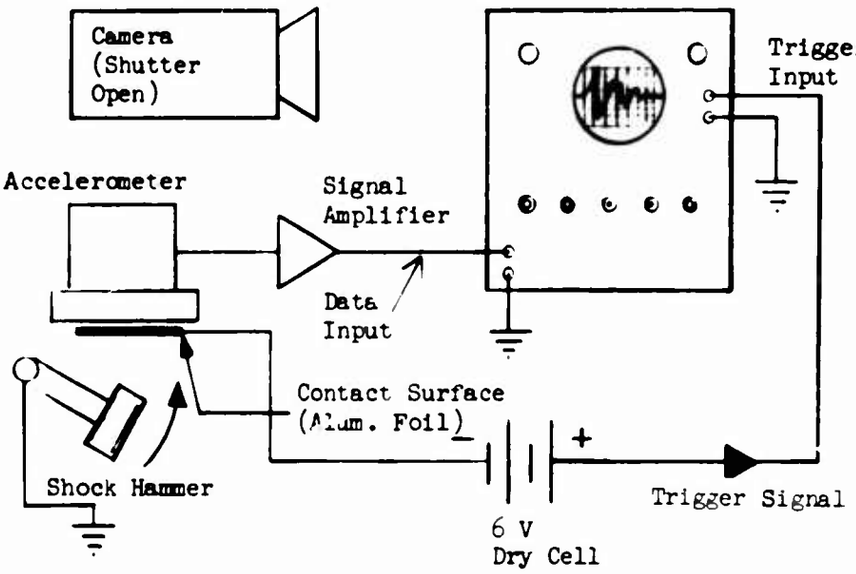
TYPICAL VELOCITY VIBRATION PICK-UP
(Permanent Moving Magnet Principle)



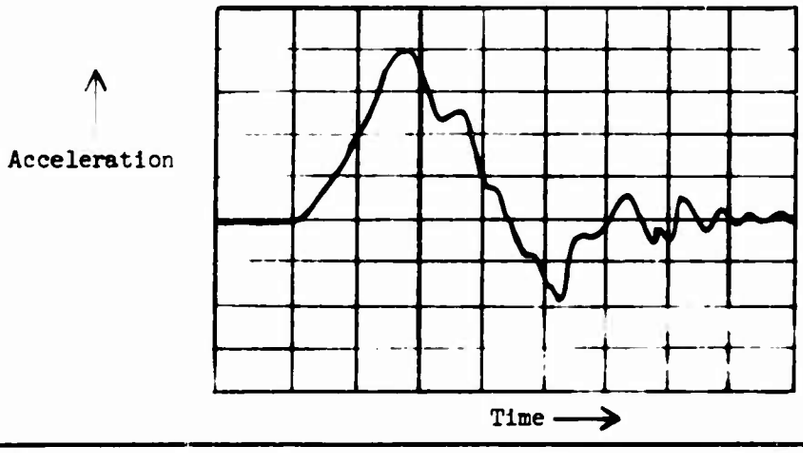
UNBONDED STRAIN GAGE TYPE ACCELEROMETER
(Cross-sectional View of Typical Accelerometer)

DATA RECORDING BY OSCILLOSCOPE

TYPICAL ACCELERATION VS TIME HISTOGRAM



A TYPICAL SCOPE/CAMERA COMBINATION



CHAPTER 8 – FRAGILITY

VOLUME III - RELATED TECHNOLOGIES

CHAPTER 8
FRAGILITY

ABSTRACT:

The concept of fragility is defined and its application to the design of equipment discussed. The methods used to determine fragility are reviewed and the application of the various methods during the course of a design development outlined. The influence of failure definition and measurement on the determination of equipment fragility by test is discussed. The influence of various modes of failure on the limiting fragility surface is considered and the possible failures considered by type. Considerations involved in the use of high strength, light weight structures and the effects on failure modes are presented. Common types of electronic equipment failures are discussed at component, sub-chassis, and console levels.

VOLUME III – CHAPTER 8
FRAGILITY

CONTENTS

Section		Page
1	INTRODUCTION	8.1-0
	• The Use of the Concept of Fragility in Electronic Equipment Design	8.1-0
2	THE CONCEPT OF FRAGILITY	8.2-0
	• The Role of the Failure Concept in Fragility Determination	8.2-0
	• The Determination of Fragility in Equipment Design	8.2-2
3	MODES OF FAILURE	8.3-0
	• Causes and Types of Failure	8.3-0
	• Chassis and Console Failures	8.3-2
	• Assembly Failures at Sub-Chassis Level	8.3-4
	• Failure Modes at Component Level	8.3-6
	• The Influence of Fracture on Equipment Design	8.3-8
	• Analysis of Service Failures	8.3-12
4	APPENDIX	8.4-0
	• Bibliography	8.4-0
	• Glossary	8.4-1

VOLUME III - CHAPTER 8

FRAGILITY

SECTION 1 - INTRODUCTION

- The Use of the Concept of Fragility in Electronic Equipment Design

THE USE OF THE CONCEPT OF FRAGILITY IN ELECTRONIC EQUIPMENT DESIGN

The definition of the fragility of an electronic equipment is a requirement for the prediction of the equipment's probability of survival in a given environment.

In predicting the probability of survival for an electronic equipment in a specific dynamic environment, use is made of the concept of fragility. Fragility is a measure of the dynamic excitation that an equipment can experience with a 50 percent chance of survival. It is a function of both the frequency of excitation and the number of stress cycles. Fragility is generally expressed as the allowable amplitude of an excitation for a specific frequency and given duration.*

Alternatively, it may be specified in terms of acceleration, frequency and time. A typical fragility surface is shown in the adjacent figure.

Fragility requirements are sometimes specified by a two dimensional curve giving allowable amplitude or acceleration as a function of frequency. The assumption is that in these cases the fragility is not time dependent. The time axis intersections of the fragility surface are therefore straight lines parallel to the frequency-time plane. When the fragility is time dependent, the value to be used in selecting or specifying components or assemblies is the value corresponding to the intended service life of the equipment.

When design criteria are to be specified for an equipment which is not directly exposed to the system external dynamic environment, the system inputs must be modified by appropriate transfer functions. The survival probability of the equipment will then be determined by a comparison of the system input times the transfer function to the fragility of the unit under consideration. If the designer knows the system level dynamic inputs and the transfer functions modifying the inputs for a given unit location, the required fragility of the unit may then be stated as part of the unit specifications.

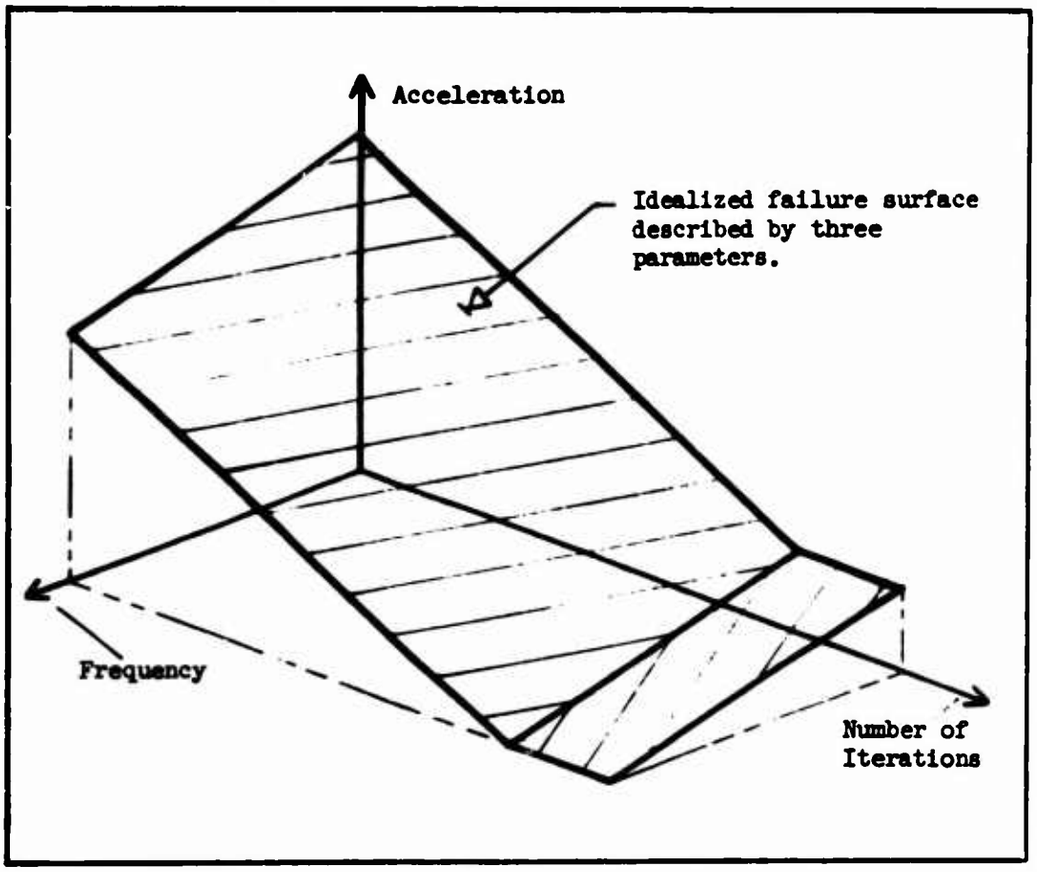
Alternatively, if the fragility of the component units of an equipment and the transfer functions for the unit to system interfaces are known, the fragility surfaces for the equipment may be defined in terms of allowable input. The equipment fragility surface will be made up of the minimum value envelope of the unit fragility curves divided by their respective transfer functions. Comparison of the equipment fragility surface with the system dynamic environment specification will then show where alteration of either component fragility or transfer function is necessary.

*This definition is an expansion of the simplified fragility concept introduced in Volume II. The concept used there assumes that failure will occur at the fragility surface; no statistics are employed to explain the distribution of test values about the failure surface. This distribution is similar to the scatter encountered in the S-N fatigue curve, discussed in Chapter 5.

Similarily, other factors which may influence the fragility surface (such as the number of stress iterations, as illustrated) are also ignored for simplicity in the analytical procedures outlined in Volume II.

The permissible values of the transfer functions will influence the mechanical design of the equipment in such areas as material selection, location and type of fasteners, and resonance points of the structure. Modification of the effective fragility of a particular area in an equipment by structural changes which affect the transfer function to the area will frequently be required in improvement of existing equipment in order to eliminate excessive failure rates in sub-units. The effective fragility in an area may be altered through the use of dynamic attenuators. The use of fragility in the selection of attenuators is discussed in detail in Chapter 8 of this volume.

A primary advantage to be gained by the specification of component or sub-assembly requirements in terms of the actual fragility required lies in the possibility of reducing the severity of the environmental requirements. Components of an assembly which will be subjected to severe dynamic environments may not be required to survive the assembly level environmental extremes. If the transfer function from the assembly to the component interface is low, (for those frequencies at which the component has low fragility) the component requirements will be reduced.



THE FRAGILITY ENVELOPE: A description of the acceleration response limits within which the equipment element has a 50 percent probability of survival. Beyond this surface, the element will experience failure 50 percent of the time.

VOLUME III - CHAPTER 8

FRAGILITY

SECTION 2 – THE CONCEPT OF FRAGILITY

- The Role of the Failure Concept in Fragility Determination
- The Determination of Fragility in Equipment Design

THE ROLE OF THE FAILURE CONCEPT IN FRAGILITY DETERMINATION

The determination of the fragility of an equipment by test will depend on the definition and detection of failure.

In defining the fragility of an assembly by either complete or partial assembly testing, decisions must be made as to the definition of a failure and the methods to be used for failure detection. Gross structural failures due to over-stressing or fatigue are generally easy to define and to detect. Operational failures or malfunctions are more difficult to define, particularly in test of partial assemblies where complete duplication of interface conditions may be impossible. In partial assembly testing a critical question involved in interpretation of the test data is the accuracy of the transfer functions used to determine the dynamic input to the equipment under test. Fragility levels determined for an inaccurate model of the local environment are of little value due to the importance of inputs at specific frequencies on the performance of the equipment. Whenever possible the dynamic inputs at lower assembly levels should be determined by dummy loading of the higher level structure and recording the shock and vibration spectral data in-situ. Malfunctions which are a result of inappropriate dynamic test inputs may not be considered as failures for fragility determination.

Many of the shock and vibration tests defined in military specifications as acceptance tests for equipment are non-operating tests. That is, the equipment is checked for electrical performance, exposed to the specified dynamic environment in non-operating configuration, and then re-energized for verification of electrical operation. This type of test will not reveal equipment weaknesses which involve transient failure modes. An example of a common type of electrical failure which will not be exposed by this type of test is the shorting of adjacent electrical conductors due to transient motion in a shock or vibration environment. The testing used to determine fragility level of an equipment will more usually be developmental level testing rather than military acceptance tests; however, the functions monitored in any case must be capable of exposing all failure modes. The most reliable data will be developed from the dynamic testing of complete assemblies or systems in operational configuration and energized.

1. Definition of Failure Criteria
   - Structural
   - Operational
2. Selection of Dynamic Test Inputs
   - Exterior Environments
   - Transfer Functions to Sub-assemblies
3. Selection of Failure Detection Methods for Test
   - Structural
   - Operational

FAILURE AND FRAGILITY: Decisions involved in determination of equipment fragility by test.

THE DETERMINATION OF FRAGILITY IN EQUIPMENT DESIGN

The methods used to determine the fragility of an equipment design will vary during the course of the design effort.

The concept of fragility is certainly of value to the equipment designer. The information necessary for its application however, is usually difficult to obtain. The methods that are used to determine the fragility surface for an equipment include the following: dynamic testing of the complete equipment, testing of partial assemblies with simulation of interface conditions, mathematical analysis of the structure, and estimates based on the personal experience of the designer.

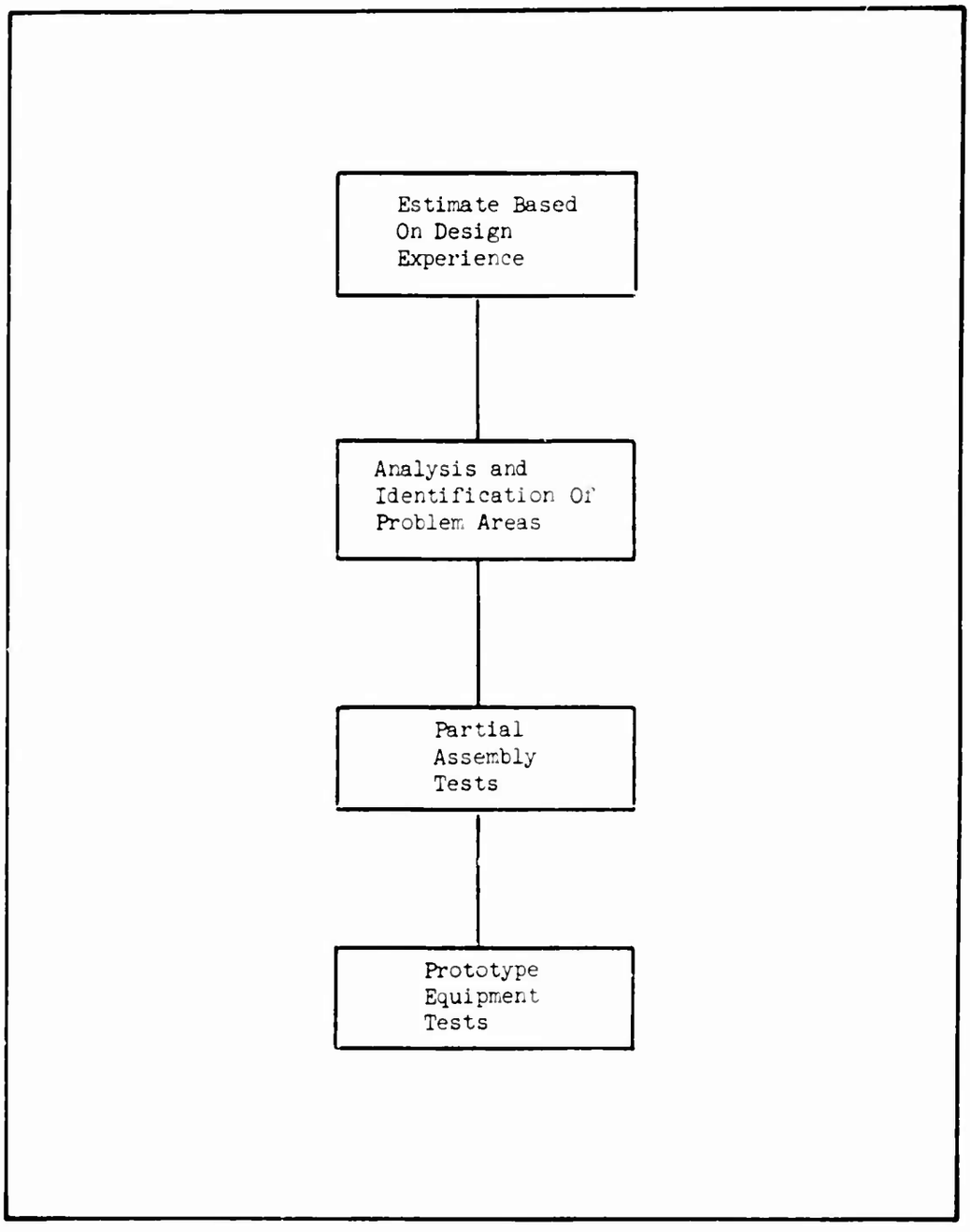
Testing of the entire equipment is the most accurate method available but also the most expensive and time consuming. In order for an equipment to be tested it must first be built. Correction of any design deficiencies revealed by the test program will therefore involve redesign and modification of the equipment. The ideal procedure to be used for determination of the equipment fragility should allow an initial design which will require a minimum of redesign.

The most efficient determination of equipment fragility will involve all of the methods listed above, as indicated in the figure in reverse order. That is, conceptual design decisions involving fragility are initially based on the designer's personal experience. The conceptual design will then be reviewed, analysis conducted in any questionable areas and resultant design changes incorporated. As the development of the equipment progresses, tests of partial assemblies will be conducted. Finally the prototype equipment will be verified by complete assembly testing.

The earlier fragility determinations provided by this sequence will be less accurate than the data taken from the final equipment test. The initial estimates however, should be sufficiently accurate to permit identification of critical areas and prediction of required fragility levels for sub-assemblies.

Both the final equipment testing and the testing of partial assemblies involve the simulation of the dynamic inputs that the unit under test will see at its interfaces during actual use conditions. For equipment level tests, these dynamic inputs will generally be defined by an appropriate Quality Assurance test specification. For partial assembly tests and undefined equipment tests the interface inputs must be determined from field data or analysis.

After the actual dynamic inputs that the equipment will see in service or qualification have been determined, a method of simulating these inputs in the laboratory must be selected. The subject of dynamic simulation is discussed in detail in another chapter of this volume. The dynamic inputs used in test may consist of sinusoidal or random vibration or may involve various shocks. The most commonly used excitation is sinusoidal vibration as the information resulting from this dynamic input is the easiest to evaluate while still yielding adequate information. An advantage of equipment level tests is that not only the fragility of the specimen but also the structural transfer functions may be determined.



FRAGILITY DETERMINATION: Fragility values for equipment elements are usually determined by a combination of experience, analysis, and tests.

VOLUME III - CHAPTER 8

FRAGILITY

SECTION 3 – MODES OF FAILURE

- Causes and Types of Failure
- Chassis and Console Failures
- Assembly Failures at Sub-Chassis Level
- Failure Modes at Component Level
- The Influence of Fracture on Equipment Design
- Analysis of Service Failures

CAUSES AND TYPES OF FAILURE

The shape of the limiting fragility surface is determined by a number of different types of failure. When the fragility surface is being constructed by analysis rather than complete equipment test all possible failure modes must be considered.

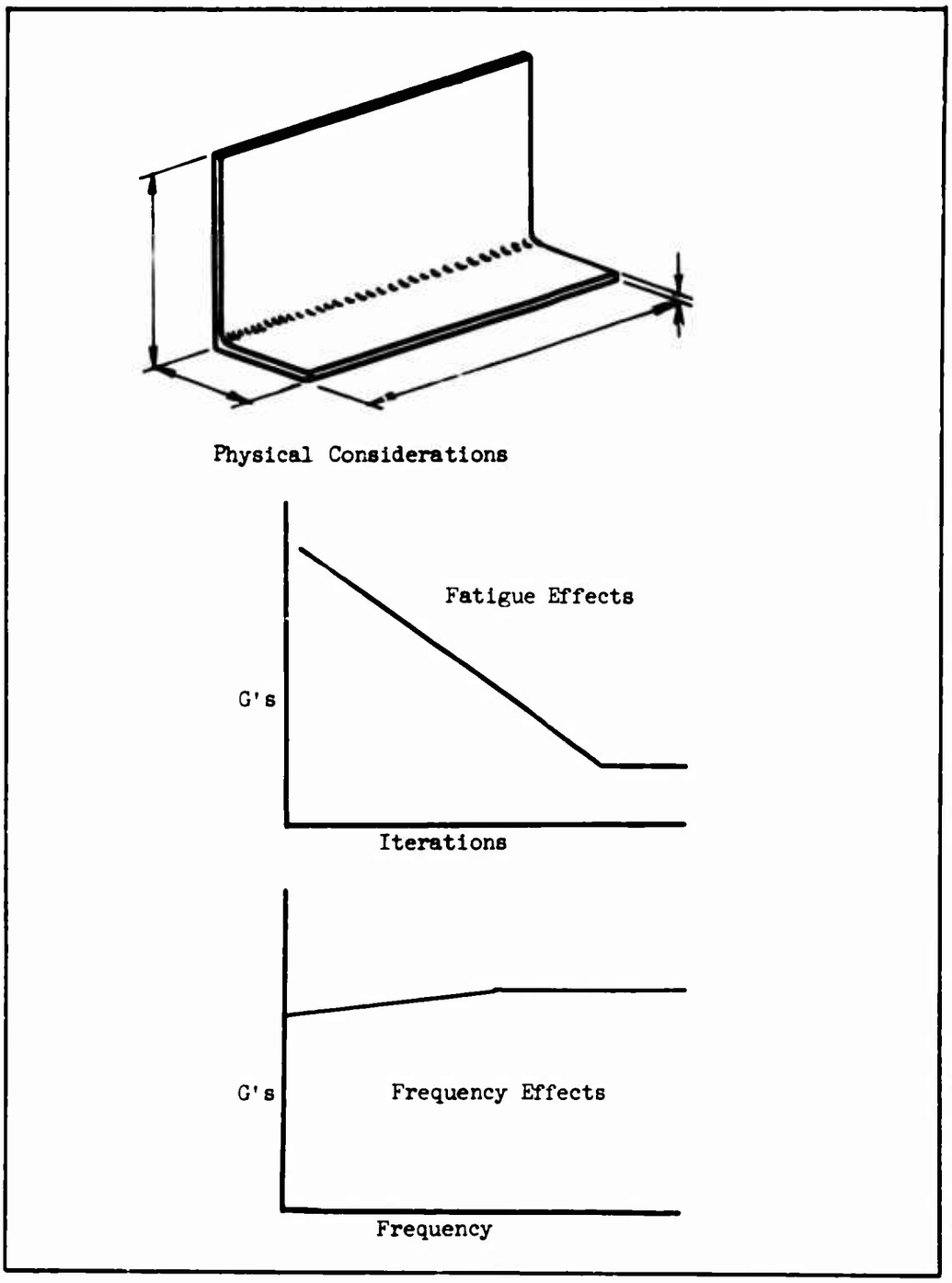
In constructing the fragility surface for an equipment by analytical methods, the effects of all possible failure modes must be considered. The common types of failure modes include the following:

First Iteration Failures: First iteration failures, corresponding to the zero time intersection of the fragility surface, are generally overload failures caused by excessive stress levels in some component or failures due to excessive deformation. Overload failures will be independent of frequency; however, failures due to excessive deformation will be frequency dependent. First iteration failures are generally associated with shock loading.

Failure with Repetition: Many of the failures resulting from both shock and vibration environments can be attributed to fatigue of materials. The shock spectra presented in the Dynamic Simulation chapter illustrate the fact that shock can excite vibration. Shock induced vibration may exceed the magnitude of the specified vibration tests for an equipment. Both shock induced vibration and the specified vibration can cause fatigue failures if the accumulated number of iterations is greater than the endurance limit for a part of the equipment. The allowable acceleration response will decrease as the total number of iterations increases.

It is possible to change the shape of the fragility surface by altering the resonant frequency of portions of the structure or of components. This is illustrated in the figure which shows test data for several sizes of capacitors and resistors. Increasing the resonant frequency will generally raise the equipment fragility by decreasing the internal transfer functions.

Typical fatigue failures are illustrated in the figure. As indicated in the adjacent figures, the fatigue failure points are those of high stress concentration - areas around holes, sharp corners, scratches, and similar stress raisers.



FRAGILITY DETERMINATION: Fragility may be defined by calculation and estimation as well as by testing and measurement.

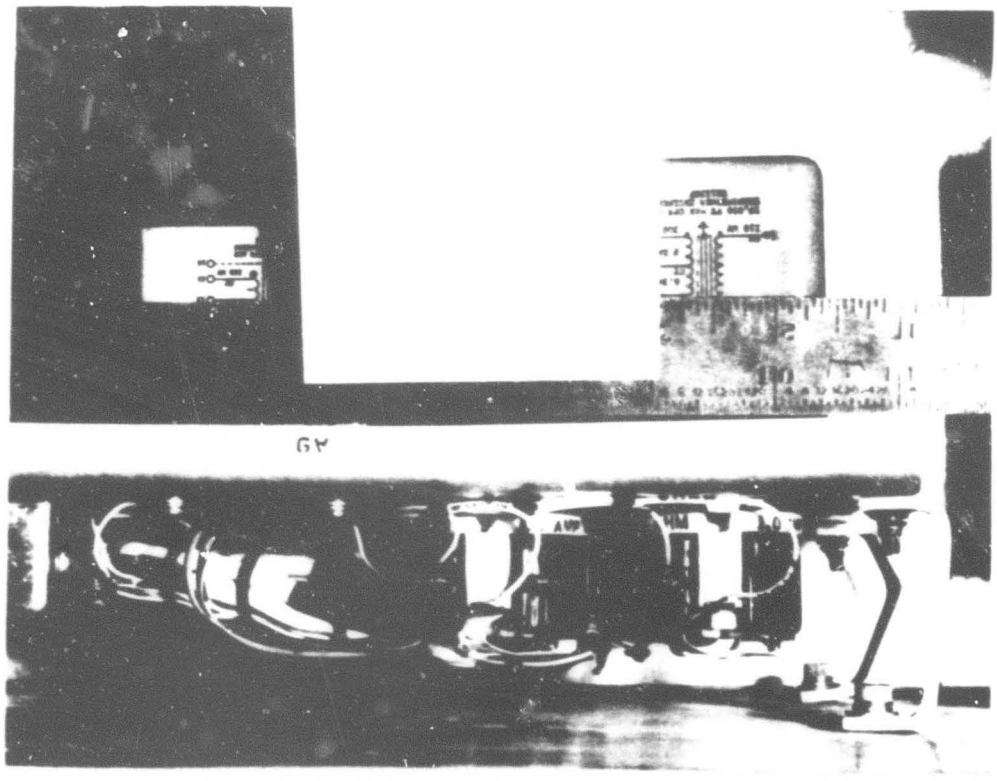
CHASSIS AND CONSOLE FAILURES

While the possible variation in designs at chassis or console level prohibits failure mode classification by assembly type, the use of certain general design guidelines will improve the fragility of the units.

The majority of console level failures observed during dynamic testing are caused by either lack of stiffness in the structure or areas of high stress concentration.

Shock tends to cause permanent deformation of the chassis or console structure although, according to M. E. Gurin, (4) some deformation may also result from vibration excitation. Typical structural deformation resulting from a shock test is illustrated at the right. It should be noted that as the weight of the components increases, the importance of their location increases. If the designer must cantilever a chassis off it's front panel, heavy components should be placed near the supported end. This raises the natural frequency of the assembly and thus raises the relative height of the fragility surface. Resonant frequencies can be raised in chassis panels by dimpling the panel. (4)

A major design consideration for chassis and console design is the natural frequency of the basic structure. The natural frequency should be outside the vibration test range, preferably outside on the high end. If it is not, then the structure is not only subjected to unnessarily high repeated loadings but the components are also subjected to these high loadings. If it is impossible to design the system with an adequately high resonant frequency, then this should be compensated for by modifying the fragility envelope for this area.



SHOCK DAMAGE: Distortion and fracture are common in poorly designed chassis. Bending and unsupported masses should be avoided.

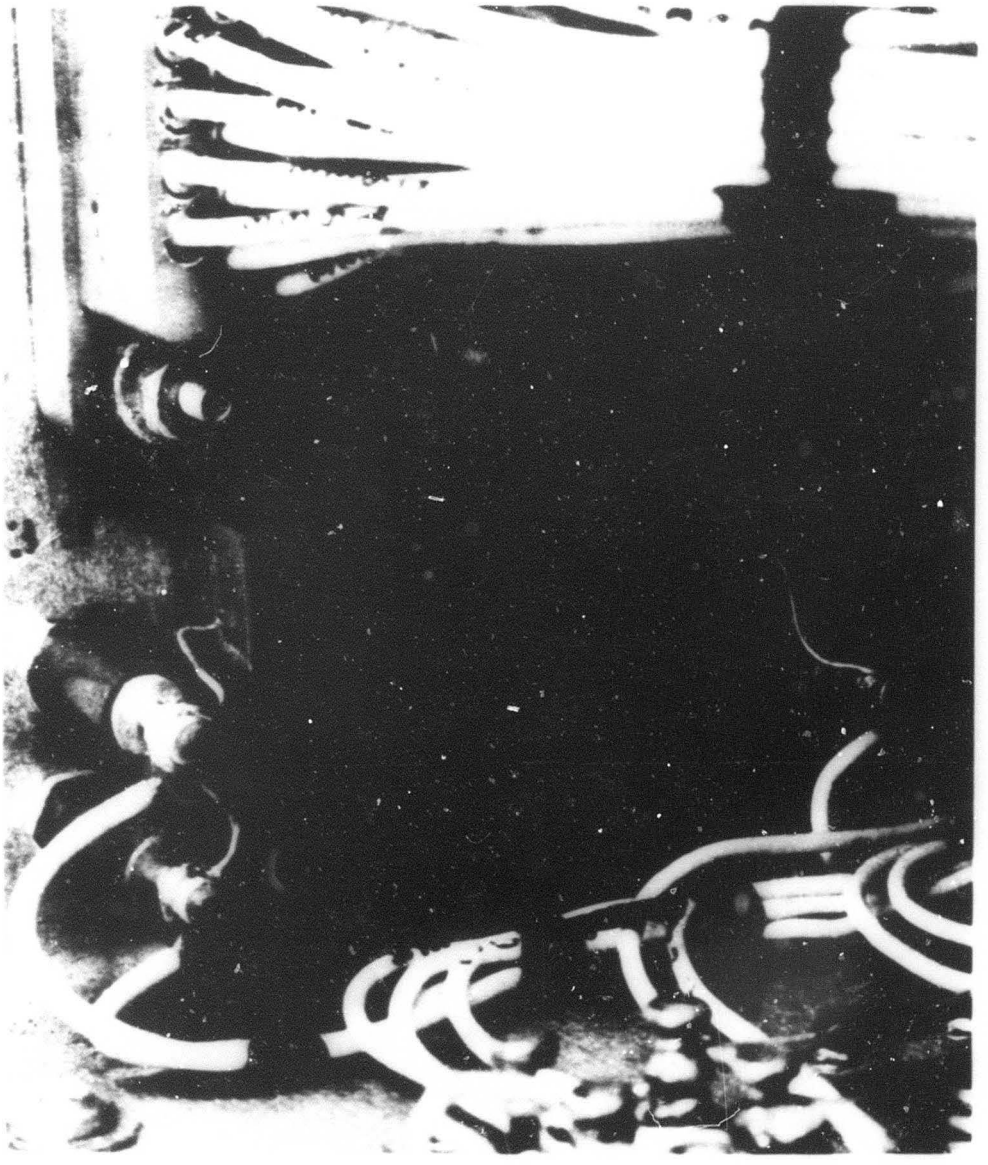
ASSEMBLY FAILURES AT SUB-CHASSIS LEVEL

Failure modes of some sub-chassis level assemblies may be considered by assembly type.

The equipment assembly sub-divisions considered in the chapter on Dynamic Attenuation are: component, sub-chassis, chassis, and console levels of assembly. Failure modes for the lowest of these, component level, have been discussed in the previous paragraph. Failure modes for the next level (sub-chassis), like the component level, may be classified by assembly type for some commonly used sub-chassis units. Some of the failure modes, such as those involving wiring and fasteners, will be the same as those considered at the component level.

Assemblies that might be considered as belonging to the sub-chassis level are meters and indicators. The U.S. Naval Research Laboratory (7) has investigated the operation of moving-coil galvanometer units, bourdon tube and drive-type synchro units either in various dynamic environments and found that, in the most cases, the indicator needle needed either balancing or damping for satisfactory operation in vibration environments. The bourdon tube and synchro indicators had some erratic performance and zero shift difficulties. The "ruggedized" meters that are on the market performed quite well.

Other assemblies that may be classified as a sub-chassis level are cable connectors. In general they have a very high damage resistance.(5) When failure does occur it is usually due to loose or bent pins. These failures occur when the plastic discs separating the pins become worn allowing excursion of the pins. Experience has shown that wires which are soldered into many-pin connectors are difficult to inspect and are thus prone to failure as a result of either cold solder joints or insufficient solder. Cables of this type are very difficult to trouble-shoot as an electrical discontinuity is usually intermittent.



SUB-CHASSIS FAILURES: Unsupported electrical wire leads are particularly susceptible to failure from shock and vibration excitations.

FAILURE MODES AT COMPONENT LEVEL

Components are frequenty tested to determine their fragility and failure mode. These tests indicate the effect of orientation, mounting and other related considerations.

Many types of components have undergone extensive testing to determine their fragility and failure modes. Typical failure modes for various types of components and methods of increasing the critical fragility level are discussed below:

1. Relays are prone to failure to hold position during shock and to chatter during vibration.(3) This is due to mechanical problems such as imbalance, weak springs, weak coils or flexible armatures. According to M. E. Gurin,(4) relays withstand vibration best when energized.

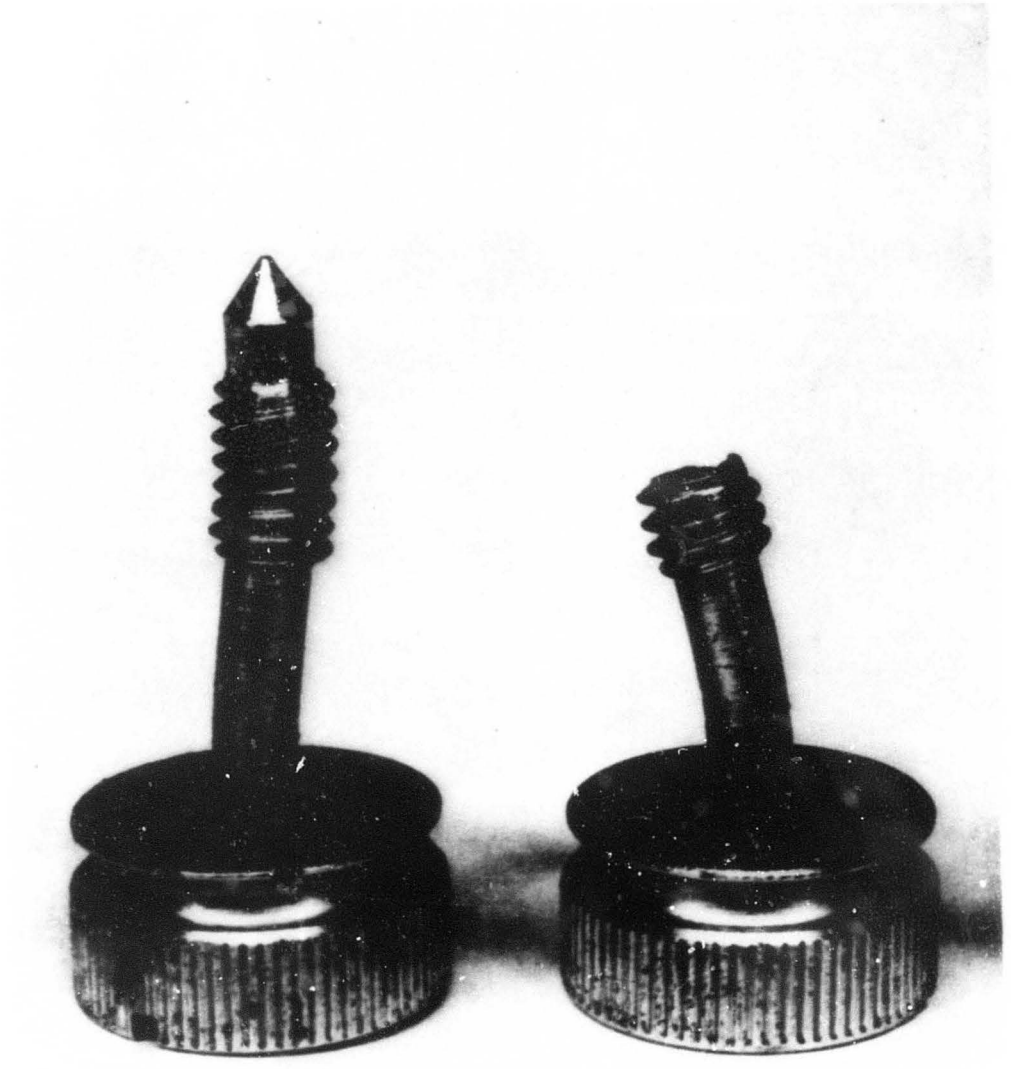
2. Capacitor failures are due mostly to low resonant frequency mounting results in a fatigue failure of the electrical leads. This failure mode is also common in resistors and similar components and can be corrected by restraining the components with, as an example, a conformal coat which will raise the resonant frequency of the assembly.

3. The vibration resistance of potentiometers may be increased by either reducing the mass of the shaft or increasing the shaft stiffness. Either change decreases the transfer function of the shaft which increases the effective level of critical fragility. Potentiometers are least susceptible to vibration in planes perpendicular to the shaft axis.(4)

4. Connector wire flexibility is detrimental to shock and vibration resistance as it increases the possibility of tearing and fatiguing of the wire system. Malfunctions are very difficult to locate. Unsupported leads fatigue most rapidly. Solid conductor wire and plastic cable clamps are not usually desirable; sufficient slack should be provided to allow for the relative motion of subassemblies. The wire should be looped around terminals to provide mechanical support in addition to the solder.

5. Fasteners (bolts with nuts or elastic stop nuts) loosen easily under both shock and vibration.(3) In addition most fasteners are susceptible to fracture or bending. In design review, fasteners should be checked to insure that their selection is based on the dynamic environment and required fragility rather than on static loading.

The application of components should always be reviewed in light of the test and operating environments. Components having low values of critical fragility, such as cathode ray tubes, should be isolated from the dynamic environment. The use of friction fits or spring clips for the retention of components should be avoided in high shock environments, as should the use of electrical connectors for mechanical retention of tubes or circuit boards.



COMPONENT FAILURES: Failures of components and other small elements can be minimized by good design review habits.

THE INFLUENCE OF FRACTURE ON EQUIPMENT DESIGN

In design involving highly stressed, high strength materials, elementary stress analysis methods will not accurately predict failure.

In designing structures that are not highly stressed, the normal design procedure consists of selecting sections and materials such that the yield strength of the material is not exceeded. If local stress raisers such as holes, bosses, or small flaws in the material exist, it is assumed that local yielding will permit plastic deformation to redistribute the concentrated stress. When high strength materials are used this assumption may not be valid. The stress concentration may cause propagation of the flaws and result in failure of the structure at stress levels considerably below the predicted yield point. Use of stress concentration factors to account for the local conditions is of little use if the design is to efficiently utilize the strength of the materials. This is due to the fact that the stress concentration factor is a geometric correction which does not take into account the fracture resistance of the materials. If this material property is to be considered in an attempt to insure efficient design, a new parameter, the stress intensity factory, must be used. The stress intensity factory is a measure of a materials resistance to unstable crack propagation. It is independent of the geometry of either the flaw or the part and of the method of loading.

Development of the stress intensity factor is based on the assumption that a crack of less than some particular size exists in the material. The size is determined by the inspection procedure. If the inspection method used can detect flaws larger than a specific size, cracks of just less than the detectable size are assumed to exist in the material. The stress intensity at the crack tip is proportional to a scalar quantity which is designated as the stress-intensity factor K. An unstable fracture is assumed to occur when the stress intensity factor is greater than a critical value called the fracture toughness. When crack geometry and applied stress are known, K is determined from the equations given in the table at the right.

Fracture toughness (K_C) is a mechanical property that places an upper limit on the value of K. The fracture toughness is established by test, using specimen types, procedures, and data analysis methods, which result in a K_C factor which is independent of crack and specimen geometry and external loading. Reference (8) goes into some detail concerning the determination of K_C. It is known that fracture toughness varies with specimen thickness; the thicker the specimen, the lower the toughness and thus the lowest value (minimum intrinsic fracture toughness) is designated as K_{IC}. In practical situations, the critical toughness is not expected to be lower than the intrinsic critical value and hence the latter is the basic index of crack toughness.

Basic Form of Stress Intensity Factor

$$K^2 = Q\ \sigma^2\ \pi\ a$$

where σ = stress

a = crack radius for circular cracks

or a = length of semiminor axis for elliptical cracks

and Q is a function of the geometry.

$Q = 1$ for through crack in infinite plate

$Q = \frac{4}{\pi^2}$ for internal circular crack

$Q = \frac{1}{\Phi^2}$ for internal elliptical crack

$Q = 1.2$ for long shallow surface crack

$Q = 1.2\ \Phi^2$ for elliptical surface crack,

where $$\Phi = \int_0^{\pi/2} \sqrt{1 - \left(\frac{c^2 - a^2}{c^2}\right)\ \sin^2\theta}\ d\theta$$

c = length of semimajor axis of ellipse.

STRESS INTENSITY FACTOR: A design parameter reflecting a material's resistance to unstable crack propagation may be calculated.

THE INFLUENCE OF FRACTURE ON EQUIPMENT DESIGN (Continued)

For thin sections, the strain in the thickness direction is unsuppressed and as a result considerable plastic flow is associated with the cracking phenomena especially for ductile materials. Since considerable plastic flow occurs, linear elastic fracture mechanics is not applicable and cannot be used. Although experimentation is being carried on in this area, and mathematical modeling is being worked on, most of the present effort is directed towards the more needed and more easily interpreted plane-strain fracture such as is experienced with thicker sections of ductile material, or more importantly, thin sections of brittle material.

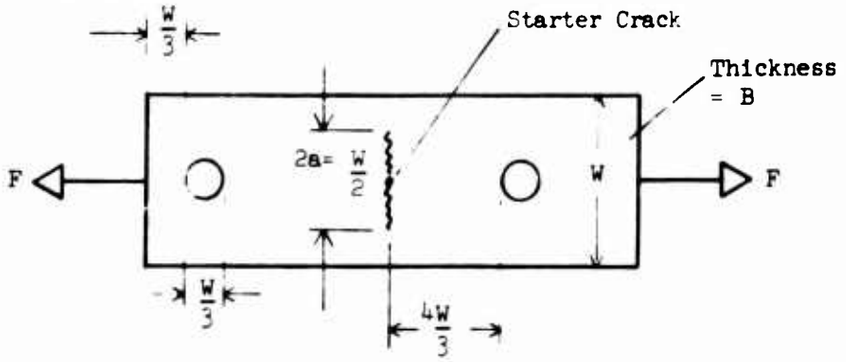
Although the field is relatively new some progress has been made in relating behavior of test specimens to the design of structural parts. Test specimens are used which contain a sharp crack which is induced by crack starters subject to fatigue loading. The crack starter may be in form of a sharp notch as a prime example. The crack is extended sufficiently so that the particular geometry of the starter crack does not influence the stress field at the crack tip. Reference (8) in the Appendix brings out the importance of the effect of the maximum fatigue stress used in fatigue cracked specimens as related to the "sharpness" of the induced crack. The "sharpness" being an important consideration in the determination of the fracture toughness (K_{IC}) obtained through testing specimens. The reference indicates that fatigue cracks should be induced at the lowest possible level of stress.

The size of the specimen also is an important consideration. The accuracy with which the experimentally determined (K_{IC}) describes fracture behavior depends on how closely the stress intensity factor represents the conditions of stress and strain inside the fracture phenomena region. Practically, if the plastic region at the crack front is small with respect to the region around the crack for which the stress intensity factor is a good approximation, then the accuracy of the experimentally obtained (K_{IC}) is sufficient. The present state-of-the-art suggests that the characteristic parameter of the plastic region useful in choosing minimum characteristic specimen dimensions is the square of the ratio of (K_{IC}) to tensile yield strength. Recent tests suggest that 2.5 times this characteristic parameter is satisfactory for minimum characteristic specimen dimensions. The characteristic dimensions are specimen thickness, crack length, and uncracked length.

The top two figures on the opposite page show a tensile specimen and a bend specimen. The details of the starter cracks are not shown. As an example, if a material which is to be used in simple bending has a (K_{IC}) of 160 ksi/in.^{2} and a yield strength of 180 ksi then the width of the specimen should be $2.5 \times (160/180)^2 \approx 2$ inches with the crack length equal to 2 inches and the uncracked length equal to another 2 inches.

To determine specimen dimensions for a new material, it is necessary to have an indication of the highest $(K_{IC}/\sigma_{yld})^2$ that the material is likely to exhibit. The bottom figure may be used for this purpose. It represents a plot for steels but the reference claims that all non-ferrous results known to the author fall within the indicated region. More detailed information on testing is given in the reference.

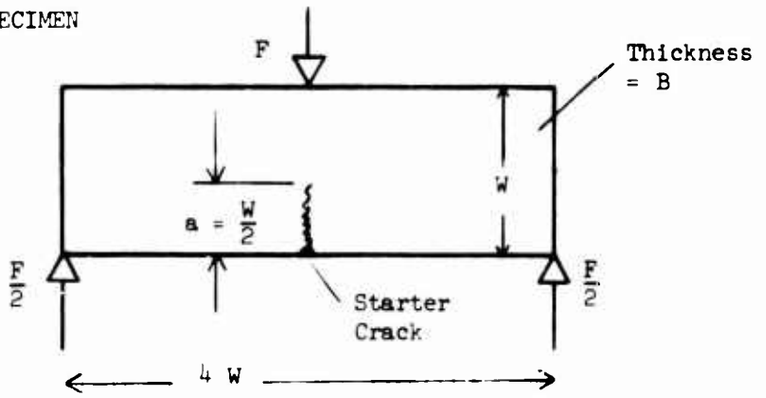
TENSILE SPECIMEN



$$K = Y\left(\frac{P\sqrt{a}}{B\,W}\right)$$

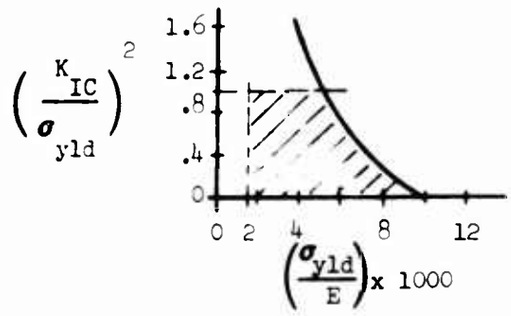
Where: $Y = 1.77 + 0.227\left(\frac{2a}{W}\right) - 0.51\left(\frac{2a}{W}\right)^2 + 2.7\left(\frac{2a}{W}\right)^3$

BENDING SPECIMEN



$$K = Y\left(\frac{G\,M\sqrt{a}}{B\,W^2}\right)$$

Where: $Y = 1.92 - 3.1\left(\frac{a}{W}\right) + 14.5\left(\frac{a}{W}\right)^2 - 25\left(\frac{a}{W}\right)^3 + 25\left(\frac{a}{W}\right)^4$



ANALYSIS OF SERVICE FAILURES

Identification of common modes of material failure by their characteristic appearance will aid in elimination of the failure cause.

In examining failed equipment in order to determine the cause of failure, characteristic symptoms of various failure modes should be carefully checked. Service failures which occur after considerable exposure to operating environments will involve a greater variety of failure modes than verification test failures which occur in the laboratory. General causes of failure which should be considered in the examination of equipment include the following:

- Wear
- Overload
- Corrosion and Stress Corrosion Cracking
- Heat Treatment and Phase Change Failures
  - Quench Cracking
  - Transformation Stress Failures
  - Flaking
  - Hydrogen Embrittlement

Wear: Failures due to wear of operating parts are among the most easily recognized types of failure due to the dimensional changes involved. Wear failures which occur before the design obsolesence point will generally be due to overloads or lubrication failure.

Overload: Service failures which occur after successful completion of design verification testing of a prototype system should be examined for evidence of improper use of materials or material treatments or of operator error.

Corrosion: Failures due to corrosion may be difficult to identify without laboratory analysis. Stress corrosion cracking is a brittle type of failure even in normally ductile materials; little or no plastic deformation accompanies the cracking. Visual evidence of corrosion may not be apparent without microscopic examination which will reveal grain boundary attack. Corrosion failures may be minimized by the use of protective coatings.

Quench Cracking: Quench cracking is caused by the austenite to martensite transformation which involves an increase in volume. Recognizable characteristics of quench cracks include the fact that the cracks run from the surface straight into the center of mass. No decarburization will be evidenced, the fracture surface will show a fine crystalline texture. Quench cracking can be reduced by elimination of design stress raisers and proper process control in heat treatment.

Transformation Stress Failures: Quench cracking is one type of transformation stress failure, that is, material failures caused by a change in physical properties due to a phase change. Other transformation stress failures involve the austenite to pearlite transformation in steels. Characteristics of the failures are similar to those described for quench cracks.

Flaking: Flakes are small internal fissures found in heavy sections after forging. They are not visible from the surface of the material and therefore require specialized testing methods for location. The term flake comes from the appearance of the fracture surface which has a glistening appearance similar to snowflakes. Flaking is an internal form of quench crack. The occurrence of flaking can be reduced by the same procedures recommended for the avoidance of quench cracks.

Hydrogen Embrittlement: The presence of relatively small quantities of hydrogen in some metals will cause a major reduction in ductility. Elastic behavior of the material usually will not be greatly affected, the embrittling effects are limited to the plastic region of the stress-strain diagram. Fractures due to hydrogen embrittlement will usually show bright round spots known as "fish eyes" on the fracture surfaces.

Probable causes of hydrogen embrittlement include welding and plating processes.

Wear

Overload

Corrosion and Stress Corrosion Cracking

Heat Treatment and Phase Change Failures

- Quench Cracking
- Transformation Stress Failures
- Flaking
- Hydrogen Embrittlement

COMMON SERVICE FAILURES: Reduction and elimination of service failures may be accomplished by careful examination of the failure mode.

VOLUME III - CHAPTER 8

FRAGILITY

SECTION 4 - APPENDIX

- Bibliography
- Glossary

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GLOSSARY

Cycle - The complete sequence of values of a periodic quantity that occur during a period.

Excursion - The excursion of a harmonic vibration is double the amplitude or peak to peak magnitude of displacement.

Failure - An irreversible process of operation outside of specified tolerances. Upon removal of the environmental load, the equipment remains inoperative or out of tolerance.

Fatigue - Tendency of materials to fracture under many repetitions of a stress considerably less than the ultimate static strength.

Flaking - Flakes are small internal fissures found in heavy sections after forging. Flaking is an internal form of quench crack. The occurrence of flaking can be reduced by the same procedures recommended for avoidance of quench cracks.

Fracture Toughness - A mechanical property that places an upper limit on the value of the stress intensity factor.

Fragility - A measure of the dynamic excitation that an equipment can experience with a 50 percent chance of survival.

Hydrogen Embrittlement - The presence of relatively small quantities of hydrogen in certain metals which cause a major reduction in ductility.

Malfunction - A reversible process outside of specified tolerances. Upon removal of the environmental load, the equipment will return to operation within tolerance.

Overload - Service failures which occur after successful completion of design verification testing of a prototype system.

Quench Cracking - Quench cracking is caused by the austenite to martensite transformation which involves an increase in volume. The cracks run from the surface straight into the center of mass.

Resonance - Resonance of a system in forced vibration exists when any change, however small, in the frequency of excitation causes a decrease in the response of the system.

Resonant Frequency - Vibrating frequency at which resonance occurs.

Stiffness - The ratio of change of force (or torque) to the corresponding change in translational (or rotational) deflection of an elastic element.

Stress - Internal force exerted by either of two adjacent parts of a body upon the other across an imagined plane of separation.

Stress Corrosion Cracking - A brittle type of failure even in normally ductile materials; little or no plastic deformation accompanies the cracking.

GLOSSARY (Continued)

Stress Intensity Factor - A measure of a material's resistance to unstable crack propagation. It is independent of the geometry of either the flaw or the part and of the method of loading.

Wear - Wear failures will occur before the design obsolesence point is reached and is generally due to overloads or lubrication failure.

Transfer Function - The quantitative description of a system's dynamic characteristics which relates the dynamic inputs to the dynamic responses.

CHAPTER 9 – DYNAMIC ATTENUATION

VOLUME III - RELATED TECHNOLOGIES

CHAPTER 9
DYNAMIC ATTENUATION

ABSTRACT:

There are two basic approaches available to the structural engineer for the control of the imposed dynamic environment; provide equipment elements of sufficient structural integrity to withstand the raw environment without external help, or reduce the environmental intensity that is felt by the fragile component to a level which it may sustain without failure. The latter alternative, attenuation, is the subject of this chapter.

An attenuator selection procedure is outlined in general terms, which reviews the specification, the design limitations, and features the first approximation of a working transmissibility curve which will protect the fragile equipment elements.

A major portion of the chapter is devoted to a discussion of the various materials and configurations currently in use as attenuation devices. Included are the coulomb damped devices, the fluid devices, elastomers, composites, and the old standby, the spring.

Some practical design suggestions are offered on the application of attenuation devices to common dynamic situations. Techniques are offered for the attenuation of equipment elements with respect to equipment level, a concept developed to categorize the various elements of Army equipment packages. These levels begin at the component division and extend to console and module level of equipments.

Chapter 9 - Dynamic Attenuation

ERRATA SHEET

Page	Paragraph	Line	Correction
Table of Contents 2	Sec. 5	4	Manufacturers
9.1-0	4	7	Absorb
9.2-8	5	7	Means
9.3-0	2	11	Absorbers
9.3-3	Caption	2	Resonant
9.3-4	3	6	...resistance and
9.3-7	Caption	1	These devices represent a recent...
9.3-10	Thesis	1	...which make
9.3-14	5	3	Per ^{o}F at 75^{o}F
9.3-19	Upper table-note		...low coefficient...
9.3-20	5	3	...application to the attenuation problem...
9.3-21	Lower figure-ordinate		(Hz)
9.3-22	4	4	By forcing them
9.3-23	Caption	2	damping
9.4-6	Thesis	1	Complement
9.4-6	1	4	backing
9.4-6	2	3	isolator (no quotation marks)
9.4.8	1	6	from excitation by....
9.4-8	2	2	resonating
9.4-9	Caption	1	These techniques are adaptations...
9.4-10	3	3	aperture
9.5-13	Figure Ordinate		"1", not 0

VOLUME III - CHAPTER 9
DYNAMIC ATTENUATION

CONTENTS

Section		Page
1	CONTROLLING THE DYNAMIC ENVIRONMENT	9.1-0
	• The Alternatives of Structural Improvement and Dynamic Attenuation	9.1-0
2	ATTENUATOR SELECTION PROCEDURES	9.2-0
	• Summary of the Procedures for Selecting an Attenuator	9.2-0
	• Review of the Applicable Specification and Definition of the Environment	9.2-2
	• Review of Design Limitations	9.2-4
	• Evaluating Fragile Components	9.2-6
	• Establishing the Transmissibility Envelope	9.2-8
	• Finding a Suitable Transmissibility Curve . .	9.2-10
3	MATERIALS COMMONLY USED FOR ATTENUATION DEVICES	9.3-0
	• A "Response Spectrum" for Classifying Attenuation Systems	9.3-0
	• Characteristics of "Dry Friction" Devices . .	9.3-2
	• Isolating With Woven Metal	9.3-4
	• Isolating With Stranded Wire	9.3-6
	• Usefulness of Fluid Devices	9.3-8
	• Properties of Elastomers	9.3-10
	• Response Characteristics of Elastomers	9.3-12
	• General Characteristics of Foam	9.3-14
	• Response Characteristics of Foam	9.3-16
	• Properties of Natural and Synthetic Fiber Felts	9.3-18
	• Characteristics of Other Special Composite Materials	9.3-20
	• A Summary of Spring Attenuators	9.3-22

CONTENTS (Continued)

Section		Page
4	A REVIEW OF ATTENUATION TECHNIQUES	9.4-0
	• Classification of Electronic Equipment by Level	9.4-0
	• Attenuation at the Component Level	9.4-2
	• Absorption at the Sub-Chassis Level	9.4-4
	• Isolation at the Sub-Chassis Level	9.4-6
	• Techniques of Chassis Absorption	9.4-8
	• Isolation of Chassis With Highly Damped Devices	9.4-10
	• Isolation of the Chassis With Predominantly Resonant Devices	9.4-12
	• Use of Padding at the Console Level	9.4-14
	• Isolation Devices for the Console Level . . .	9.4-16
5	APPENDIX	9.5-0
	• Bibliography	9.5-0
	• Glossary	9.5-2
	• Attenuator Manufactures	9.5-4
	• Illustrative Problem on Attenuation	9.5-6

VOLUME III - CHAPTER 9

DYNAMIC ATTENUATION

SECTION 1 – CONTROLLING THE DYNAMIC ENVIRONMENT

- The Alternatives of Structural Improvement and Dynamic Attenuation

THE ALTERNATIVES OF STRUCTURAL IMPROVEMENT AND DYNAMIC ATTENUATION

Structural improvement and dynamic attenuation are the two alternatives available to the designer for the protection of electronic equipment from shock and vibration inputs

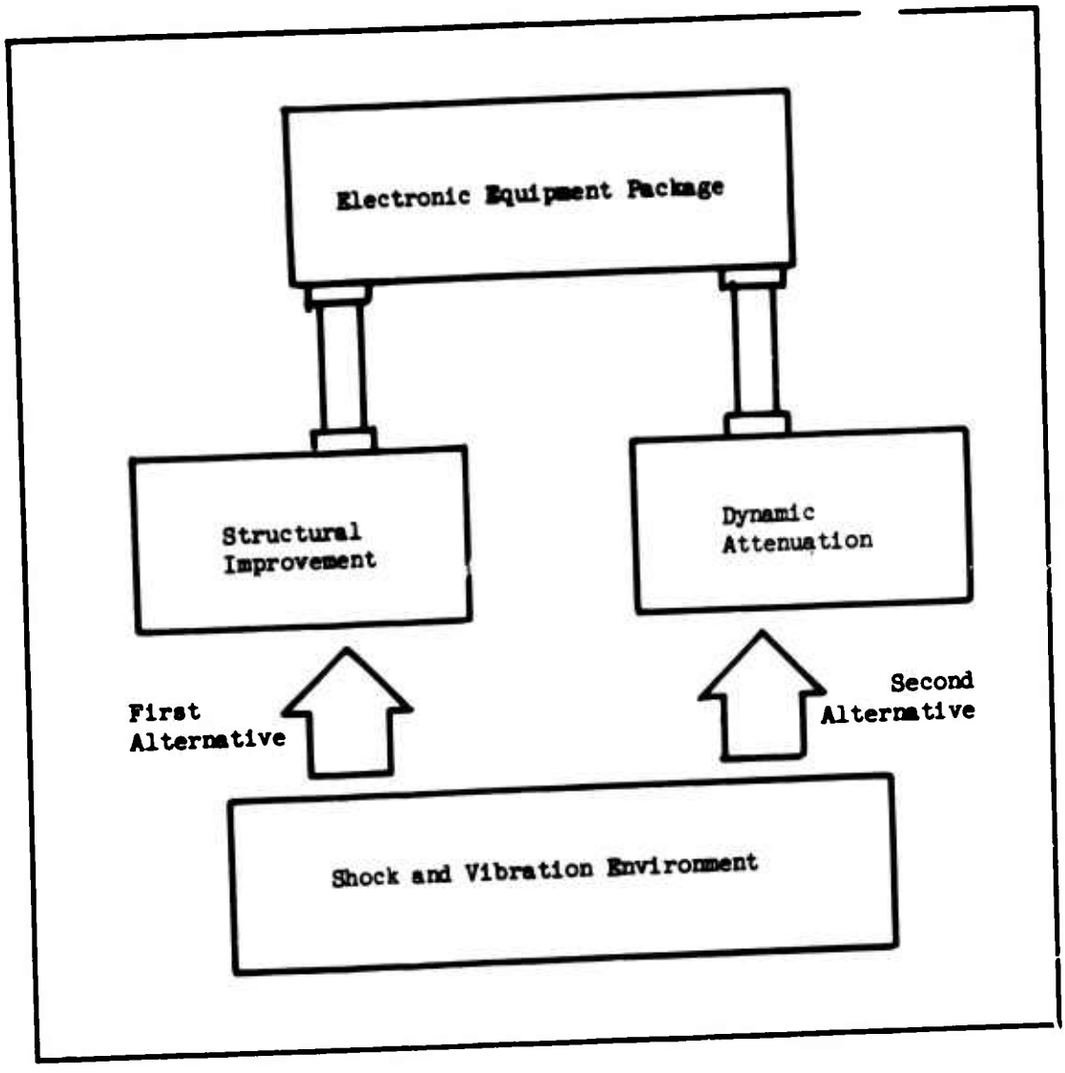
The Packaging Designer is faced with the problem of designing equipment to survive a specific shock and vibration environment and must decide upon a constructive course of action. If, through intuition or as a result of preliminary testing, the designer does not believe his equipment capable of surviving these environments, he must select one of two alternatives: structural improvement or dynamic attenuation.

Structural improvement of the electronic equipment can be accomplished in many ways, discussed in detail in other chapters of this design guide, and reviewed here for reference. One tool often utilized by the designer is the technique of substituting materials to change resonant frequencies or masses. It is often feasible to select a material which will do an adequate job but which has a resonant frequency which is outside the excitation region and, therefore, unaffected by the excitation.

The most drastic changes in resonant frequency can be obtained by changing the geometry of the electronic equipment package or some of its components. The designer may accomplish this geometric change through a variety of avenues which include the relocation or reorganization of the components in a trouble area or modification of critical area. The designer must first try to obtain a rigid structure which will survive the anticipated shock and vibration environment. Only after this alternative has been pursued in its fullest should the designer consider the second alternative, dynamic attenuation.

Dynamic attenuation is the technique of utilizing a group of devices to reduce the effect of the imposed environment. This group of devices consists of shock absorbers, dampers, foams, isolators or resonant devices. Through the application of dynamic attenuation techniques, the designer may incorporate devices which will either reduce the amount of energy actually getting to the electronic equipment by isolation of the equipment or he may utilize devices which will abosrb the energy once it is within the electronic equipment. These techniques are the subject of this chapter.

Included in this chapter are: a general procedure for selecting an attenuator; a discussion of the materials utilized in attenuators in both isolation and absorption applications; and some information and suggestions regarding the applications of these materials.



EFFECTIVE DYNAMIC PROTECTION: Electronic equipment can be designed to survive the required shock and vibration excitations by employing the proper choice of design alternatives which include structural improvement and dynamic attenuation.

VOLUME III - CHAPTER 9

DYNAMIC ATTENUATION

SECTION 2 – ATTENUATOR SELECTION PROCEDURES

- Summary of the Procedures for Selecting an Attenuator
- Review of the Applicable Specification and Definition of the Environment
- Review of Design Limitations
- Evaluating Fragile Components
- Establishing the Transmissibility Envelope
- Finding a Suitable Transmissibility Curve

SUMMARY OF THE PROCEDURES FOR SELECTING AN ATTENUATOR

The comparison of the environment with the element's fragility will establish an allowable transmissibility envelope. Comparision of that envelope with the actual hardware capability results in an effective selection procedure for an attenuation device.

The procedure for selecting a dynamic attenuator has been arranged in a sequence which will lead the designer to an accurate and rapid conclusion. This procedure will require slight modification for each individual application but the general philosophy will still apply.

As illustrated in the adjacent block diagram, the first information which must be determined is the environment to which the equipment will be subjected. This is usually specified in the Quality Assurance provision section of the procurement specification. These provisions tell the designer which shock and vibration tests must be performed, the parameters of the tests, and how many specimens must be subjected to the tests. Unfortunately, this basic consideration, the required envirionment, is often neglected by the designer. In addition, the basic inputs of some of these environments, such as ballistic shock, have been neglected to the point where little research has been done or published. The packaging engineer thus has a formidable task in defining the design criteria.

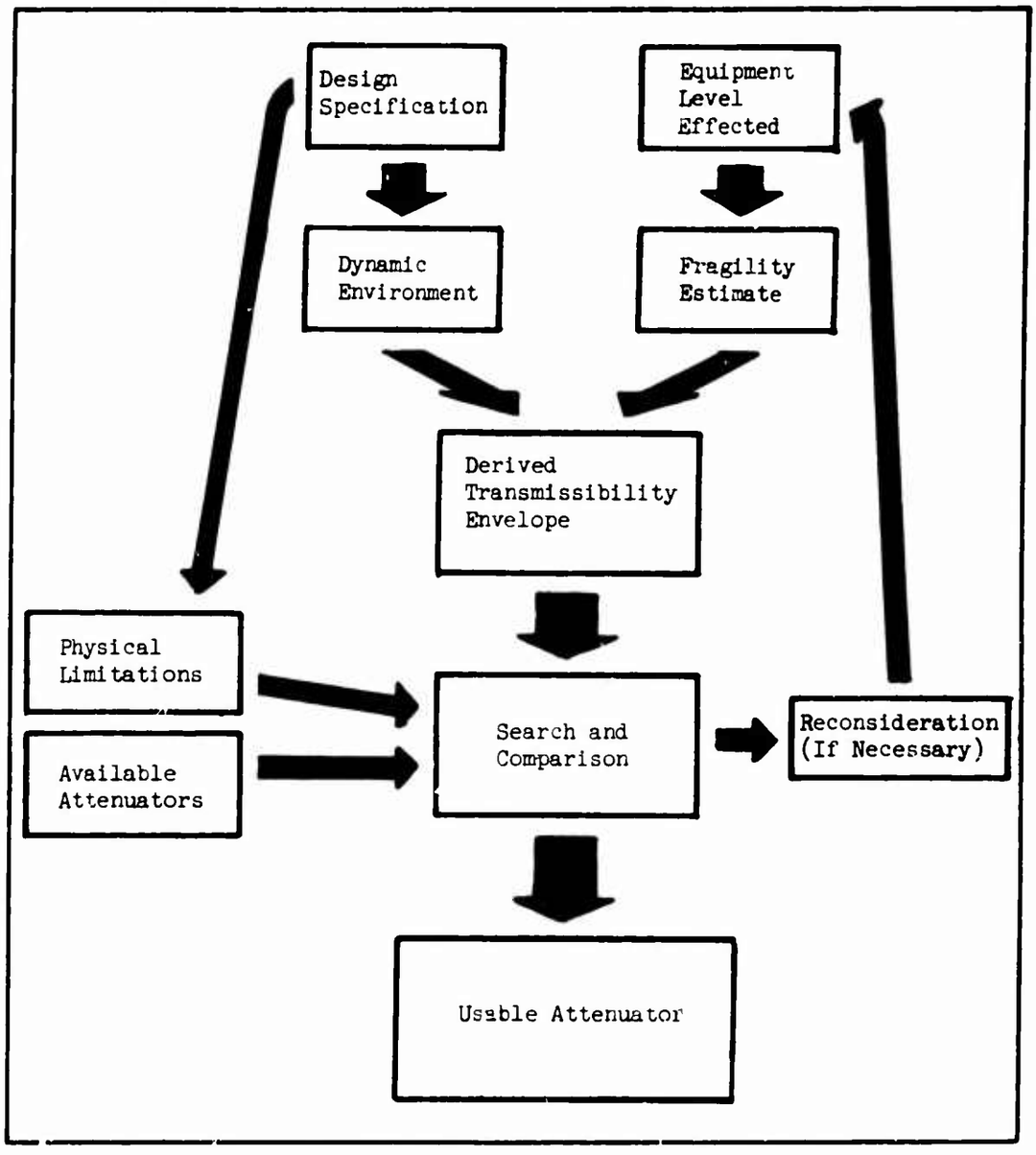
It is next necessary to consider what level of assembly will be affected by the environment and estimate how the electronic equipment will most probably fail. The load tolerance, or "fragility," of each level should be occasionally estimated to establish those parts needing attenuation. Occasionally it is advisable to isolate the entire system. The knowledge of the failure or malfunction modes of the equipment elements is also useful during preliminary design stages.

Next a transmissibility envelope is computed by comparision of the environmental input data with the fragility data. This transmissibility envelope is such that any attenuator with a transmissibility curve everywhere less than the envelope, will be acceptable. It will be obvious that several attenuation devices will appear suitable.

Additional consideration must be given to the physical properties of the electronic equipment package and where it is to be mounted. These considerations include the weight at each mounting point, physical size, any mounting restrictions, the inertia of the package, the amount of excursion the package is allowed, the stiffness of the adjacent mounting structure, the location of the package's center of gravity, and accessibility requirements to name just a few.

Consideration must be given to the presence of extreme temperature, active chemicals, or other adverse environmental influences which can affect the life of the attenuator selected.

After the primary design constraints have been defined, the designer will select an attenuator by comparing the transmissibility curves of actual attenuators with the theoretical envelope. It is also possible that an ideal attenuator will not be apparent. In that instance, it will be necessary to repeat the sequence changing the assembly level considered for attenuation or adjusting the fragility restrictions through structural improvement of the electronic equipment package.



ATTENUATOR SELECTION PROCEDURE: Environmental severity, element fragility and other considerations influence the choice of an attenuation device.

REVIEW OF THE APPLICABLE SPECIFICATION AND DEFINITION OF THE ENVIRONMENT

The first step in the selection of a shock and vibration attenuator is the study of the related specification to determine the dynamic environment which must be met.

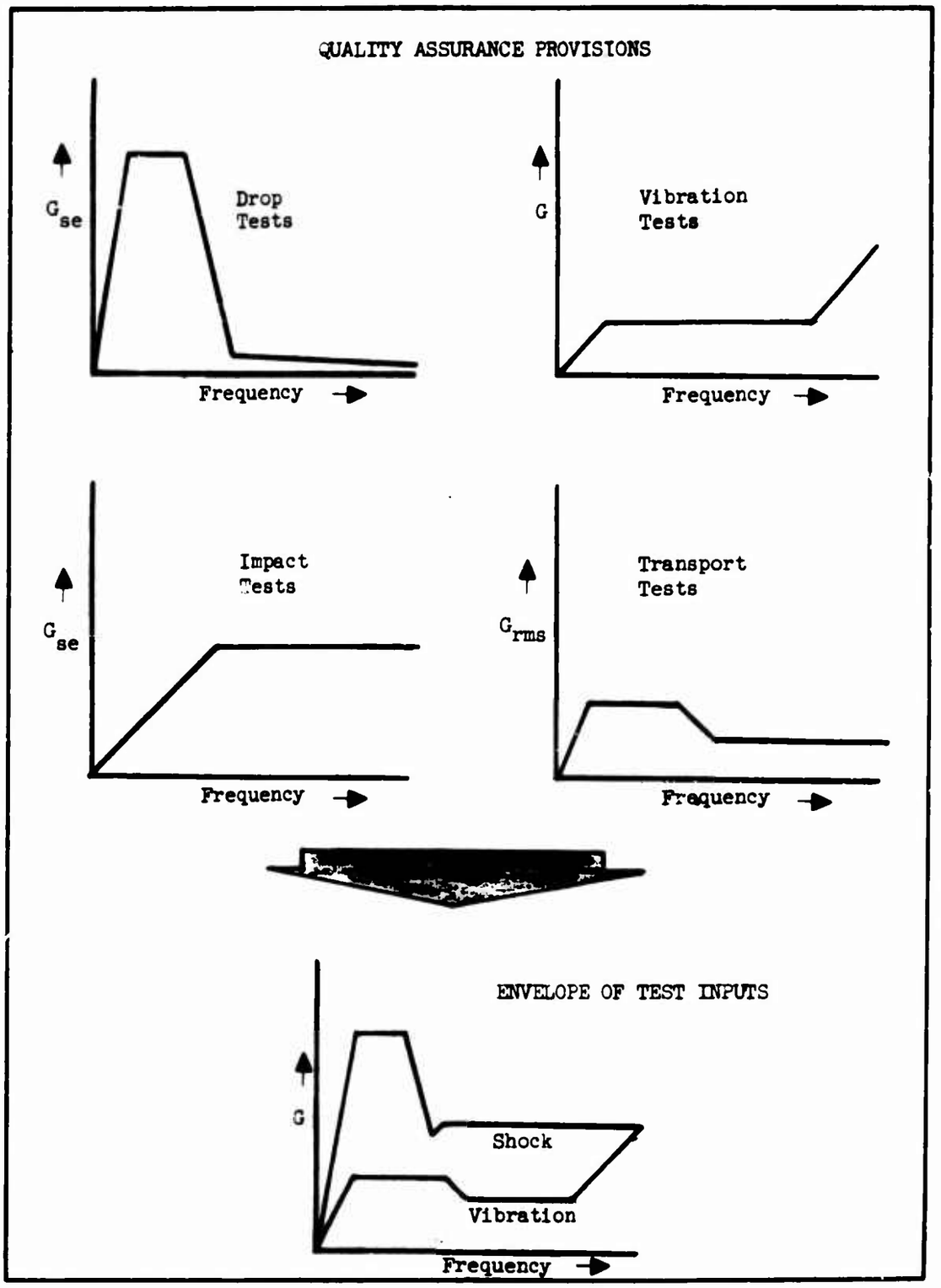
When selecting an attenuator, one of the preliminary pieces of information to be accumulated is a knowledge of the dynamic environment that the equipment package will be subjected to and within which the equipment package will be expected to survive and function. This shock and vibration environment is specified in the Quality Assurance Provisions Section (usually Section 4.0) of the procurement specification.

According to the American Standards Association, shock occurs when "the position of a system is significantly changed in a relatively short time in a non-periodic manner. It is characterized by suddenness and large displacement, and developes significant internal forces in the system."(1) There are several kinds of shock of interest to the designer which are required of specific equipment classes. These include ballistic shock, shaped pulse, bench drop, shipping drop and railroad humping. Related to these are cargo bounce and vehicle and road bounce. A complete description of each of these excitations with their characteristics can be found in the chapter on "Dynamic Simulation" and are summarized as well in Volume II of this Design Guide.

Vibration, on the other hand, is defined by Wayne Tustin as "a mechanical oscillation or motion about a reference point of equilibrium."(2) As with shock, vibration test requirements vary and are specified for particular classes of electronic equipment. These include sinusoidal, random, forced, deterministic, and others. A further discussion of these can also be found in the chapter on "Dynamic Simulation."

Since shock is characterized by large accelerations and large displacements, a relatively stiff attenuator which will limit equipment over-excursion is occasionally indicated. Vibration, however, is many repetitions of smaller acceleration and displacement which sometimes requires a soft suspension system to take advantage of the inertia effects of the equipment to be protected. Unfortunately, the exact opposite is also true, due to the resonant effects of the attenuation. This conflict between shock and vibration will obviously require a compromise on the part of the designer. The problem is eased somewhat by the development of combined isolator/absorbers for some levels of equipment. These products will be discussed later in this chapter.

For the purposes of analysis, the environment should be plotted as acceleration versus frequency as shown in the adjacent figure and these individual plots then combined into an overall environment plot which is extremely usable.


QUALITY ASSURANCE PROVISIONS
G_{se}
Drop Tests
Frequency
G
Vibration Tests
Frequency
G_{se}
Impact Tests
Frequency
G_{rms}
Transport Tests
Frequency
ENVELOPE OF TEST INPUTS
G
Shock
Vibration
Frequency

REVIEW OF DESIGN LIMITATIONS

Consideration must be given to the imposed design constraints and other physical restrictions which may limit the attenuation selection process.

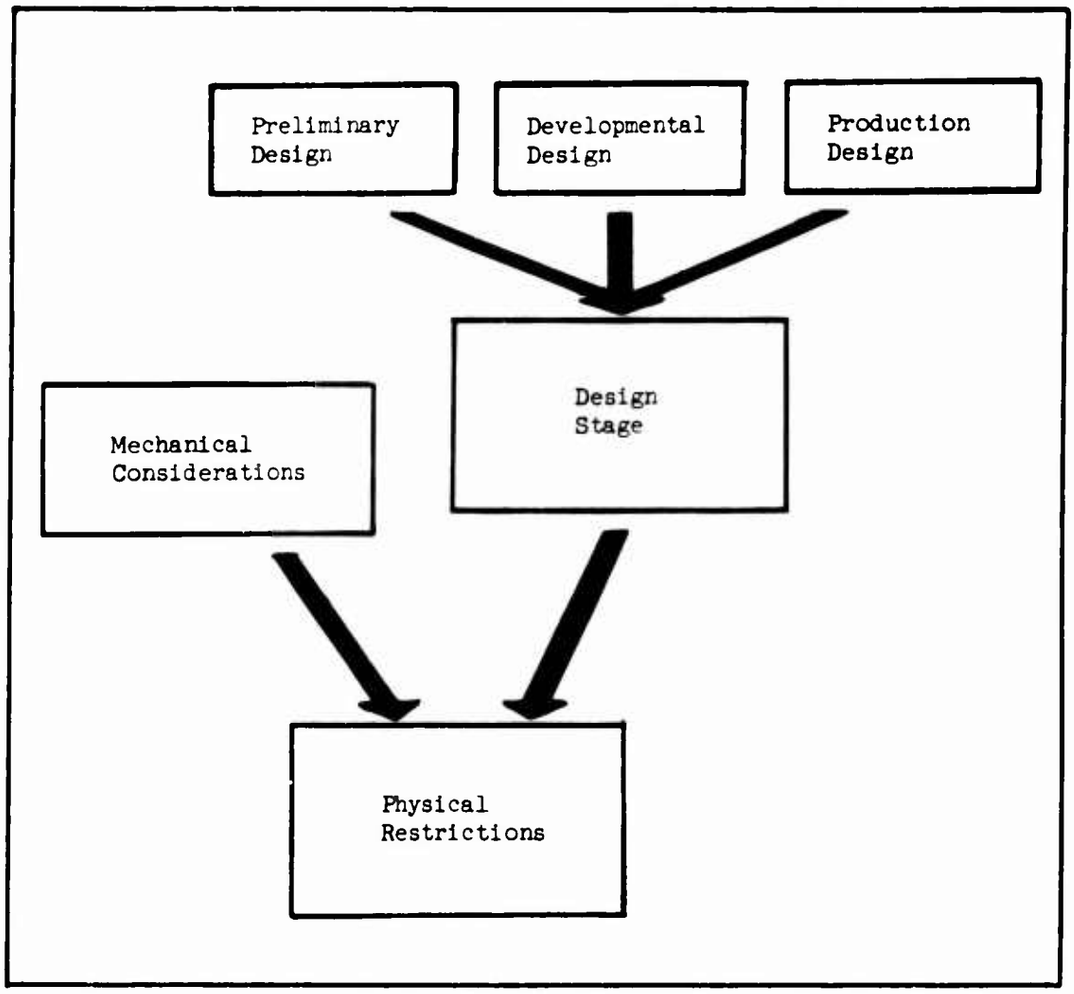
The selection of an attenuator can be restricted by the design itself or by the design specification. The stage of the design, whether the design is in a preliminary, developmental, or production phase, has a bearing on which attenuation method can be adapted. It should be obvious that if the equipment has been completed it will be impossible to optimize the location of isolators between the components and subassemblies. If the philosophy of dynamic attenuation is incorported at an early enough stage, many more avenues are available to the designer. The components and subassemblies chosen by the designer will ultimately establish the equipment size and weight (the design specification will probably establish maximum totals for these) as well as the location of the center of gravity, the moments of inertia of the electronic equipment and accessibility of the components.

The design specification will probably mention such things as the rigidity of the supporting structure, the amount of sway and other excursion space available, and the required life of the electronic equipment package. It can be seen that many of these considerations diminish in magnitude when one considers which components need protection. It is much easier to treat the part rather than the whole.

The designer has the fundamental choice of whether to isolate the equipment from the input or absorb the energy within the equipment; isolation devices are external to the equipment being protected and thus increase the volume occupied by the equipment. Absorption devices are added internally to the package and thus increase the object's density. Both have advantages and disadvantages and will be discussed in detail in this chapter.

The designer should also consider how the equipment will be used and whether or not it will be subjected to negative accelerations. If negative accelerations are anticipated, then an isolator must be chosen which will be capable of sustaining this reverse deformation. During ballistic shock, some electronic equipment has been seen (through high speed photography) to jump six inches or more laterally. This excursion can completely dislocate it relative to its isolators. When the equipment slams back onto the base support, the results are usually disastrous.

A calculation will be required to determine the portion of the total weight which will apply at each isolator, should isolators be selected. This individual weight will be utilized in the final selection of the isolator.



THE SECOND STEP: Review the design limitations and allowances.

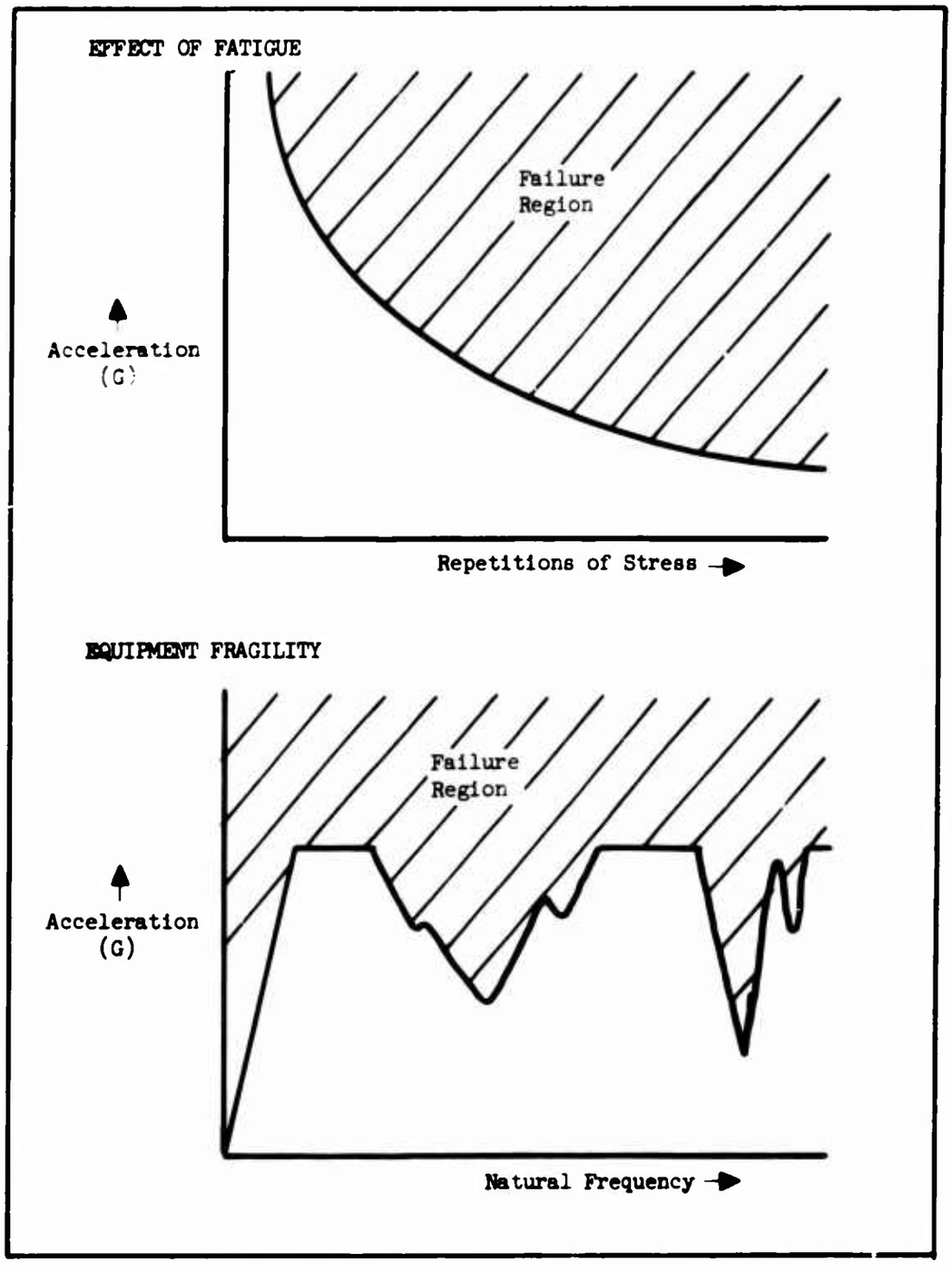
EVALUATING FRAGILE COMPONENTS

It is necessary to visualize how different areas of the equipment will be affected by both short and long term dynamic excitations to determine which areas need protection.

The designer must now establish which portion of this electronic equipment package will be most critically affected by the dynamic environment. The designer does this by estimating the fragility of the portions of his equipment under consideration. Fragility is defined by R. D'Agostino as a "qualitative index of the 'G' load limit that the equipment can sustain without damage."(3) An example may best illustrate this concept: if a radio has an even chance of surviving a 20 "G" acceleration at 30 Hz, then this is said to be its "fragility" for this frequency. If, for some reason, it is subjected to a slightly higher acceleration at this frequency, it will probably be damaged. If this radio must (as a design requirement) survive in a 30 "G," 30 Hz environment, then attenuation is mandatory to reduce the acceleration the radio "sees" to at least 20 "G's." Not all levels or portions of the equipment will have the same fragility curve and the curve usually varies with frequency. The determination of this fragility parameter is discussed in detail in another chapter of this Design Guide.

Since both shock and vibration are normally part of the dynamic environment, the designer is interested in which of them will have the most severe effect on the equipment. The usual mode of failure associated with these two disturbances varies as does the excitation itself. Shock, for example, usually causes failure due to the first impact of load, either as a direct fracture, or as an overextension or excessive excursion of an equipment element. Vibration failures are typically fatigue fractures or operational malfunctions due to chatter. A low level shock disturbance, such as bounce, may also excite the equipment into some resonant activity and subsequent fatigue problems. The decision is, therefore, one of evaluating the extent of the damage potential, whether it is due to initial or repetitive excitation, and relating this information to both the fragility of the element and the "safe loading level" or endurance limit of the equipment. These properties are illustrated in the adjacent figure. Detailed information on the design of structure for failure resistance is discussed in other chapters.

An overall fragility curve should be plotted as acceleration versus frequency and on the same scale as the environment to facilitate later use. This plot may range from a straight line limit which might be found in a procurement specification, to an undulating characteristic which might evolve from empirical test data.



FRAGILITY: The elements of the equipment complex must be determined and their sensitivity to either repeated excitations or initial impact evaluated.

ESTABLISHING THE TRANSMISSIBILITY ENVELOPE

A calculation may be performed to determine the maximum transmissibility envelope for the attenuator location for both shock and vibration by factoring the environment with the fragility of the system.

Now that the preliminary work has been accomplished, the designer is ready to determine the transmissibility envelope for the particular attenuator application. A convenient definition of transmissibility is the ratio of the energy transferred through an object to the energy originally applied to the object. The electronic analogy is "gain." The region of the curve where this ratio is greater than unity is called the amplification region. This is a characteristic of a resonant system which is excited at a frequency near its natural frequency. It must be realized that most attenuation devices have such a region and it is imperative to choose an attenuator whose amplification region is below any anticipated excitation since this is a frequency-related factor. When the output to input ratio is less than unity, the attenuator is reducing the effect of the input or the device is behaving like an attenuator. As can be seen at the right, this curve has one fundamental peak (though it may have many other peaks present) whose height is the maximum amplification at the system's fundamental resonant frequency. The other peaks often represent the resonant frequencies of the subassemblies within the system.

The frequency of the fundamental (and any other resonance, for that matter) may be changed by altering the mass/spring relationship of the resonating body. The amplitude of the peak is an indication of the damping properties of the system. Both the peak frequency and the peak amplitude may be modified intentionally or by chance when the mass or damping properties of the system are changed.

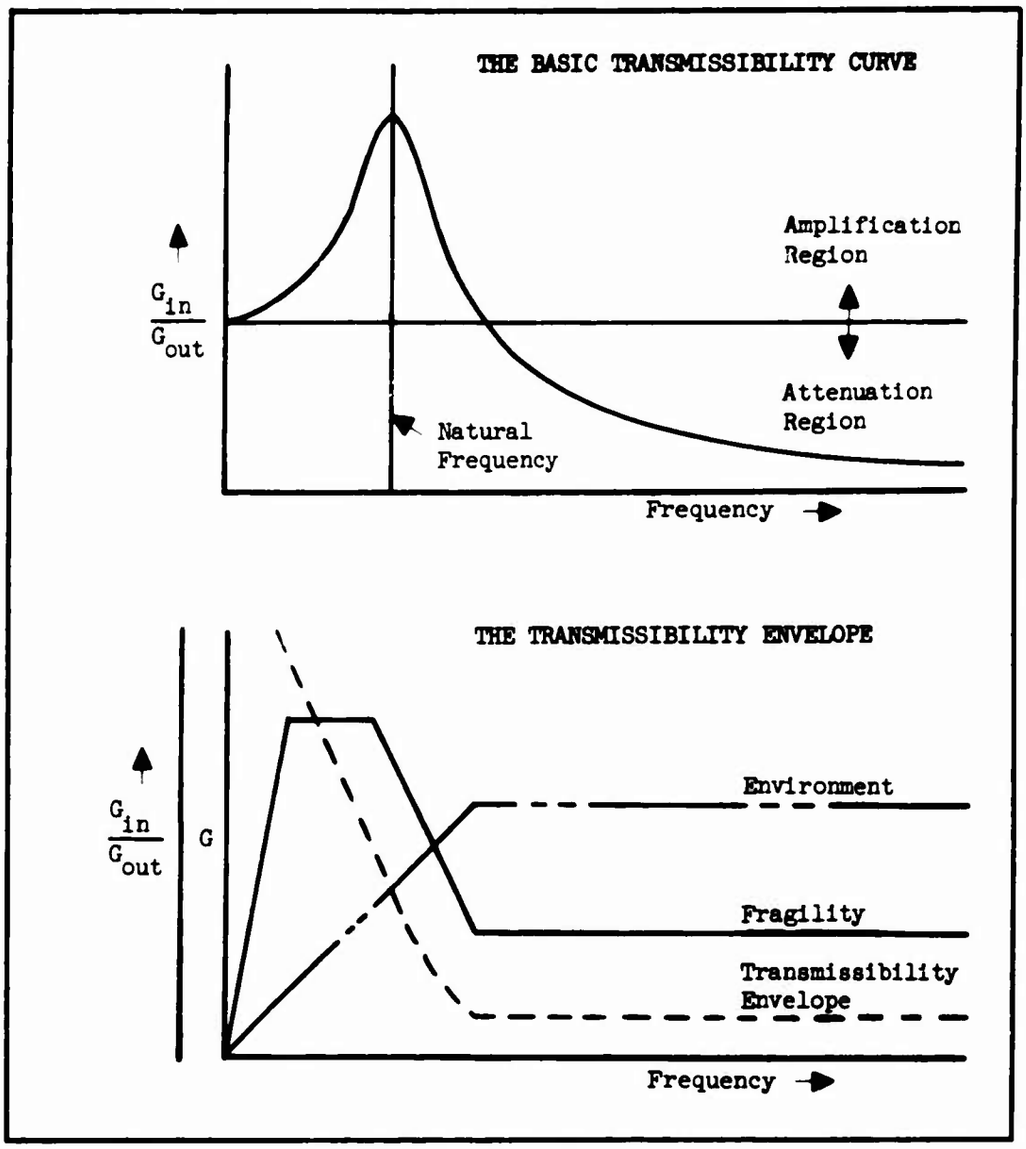
The envelope developed will be the locus of the maximum allowable (without failure) transmissibilities for the span of frequencies of interest. Since this envelope is a locus of maximums, any actual curve which lies everywhere below this envelope will be acceptable for the applicatio...

The evolution of the envelope may be described in the following manner:

$$T_f = \frac{F_f}{E_f}$$

where: T = Transmissibility at a given frequency (f)
F = Fragility level in G's at a given frequency (f)
E = Environmental input in G's at the given frequency (f).

This is shown in graphical form at the right. An example is the hypothetical case where, at a common frequency, an electronics package can withstand 25 G's (fragility) but the environment is applying 50 G's (perhaps due to shock). The maximum transmissibility would then be 0.5 and an isolator with a transmissibility of 0.45 at this frequency would be acceptable. The designer should be alert to the fact that he is not working with single frequencies, but rather a span of frequencies which menas he is exercising two curves to establish a third.



TRANSMISSIBILITY CURVES: The amplification and attenuation regions as well as the limit for usable attenuator may be defined in the frequency domain.

FINDING A SUITABLE TRANSMISSIBILITY CURVE

The selection is complete when an attenuator is found which possesses a transmissibility characteristic that lies within the theoretical curve and has the correct physical requirements.

Once the transmissibility envelope has been established, the task of locating a suitable attenuator remains. Some shortcuts are available to the designer, however. The weight of the equipment will restrict the size of the isolator or absorber; the location or excursion restrictions will also eliminate many attenuators. As a result of this and other thinning, the designer will have only a few attenuators whose curves will have to be compared with the allowable envelope. The actual selection is done by fitting the actual curves to the theoretical. In some instance it may be necessary to combine two different attenuators (such as an isolator and an absorber) to obtain the desired result.

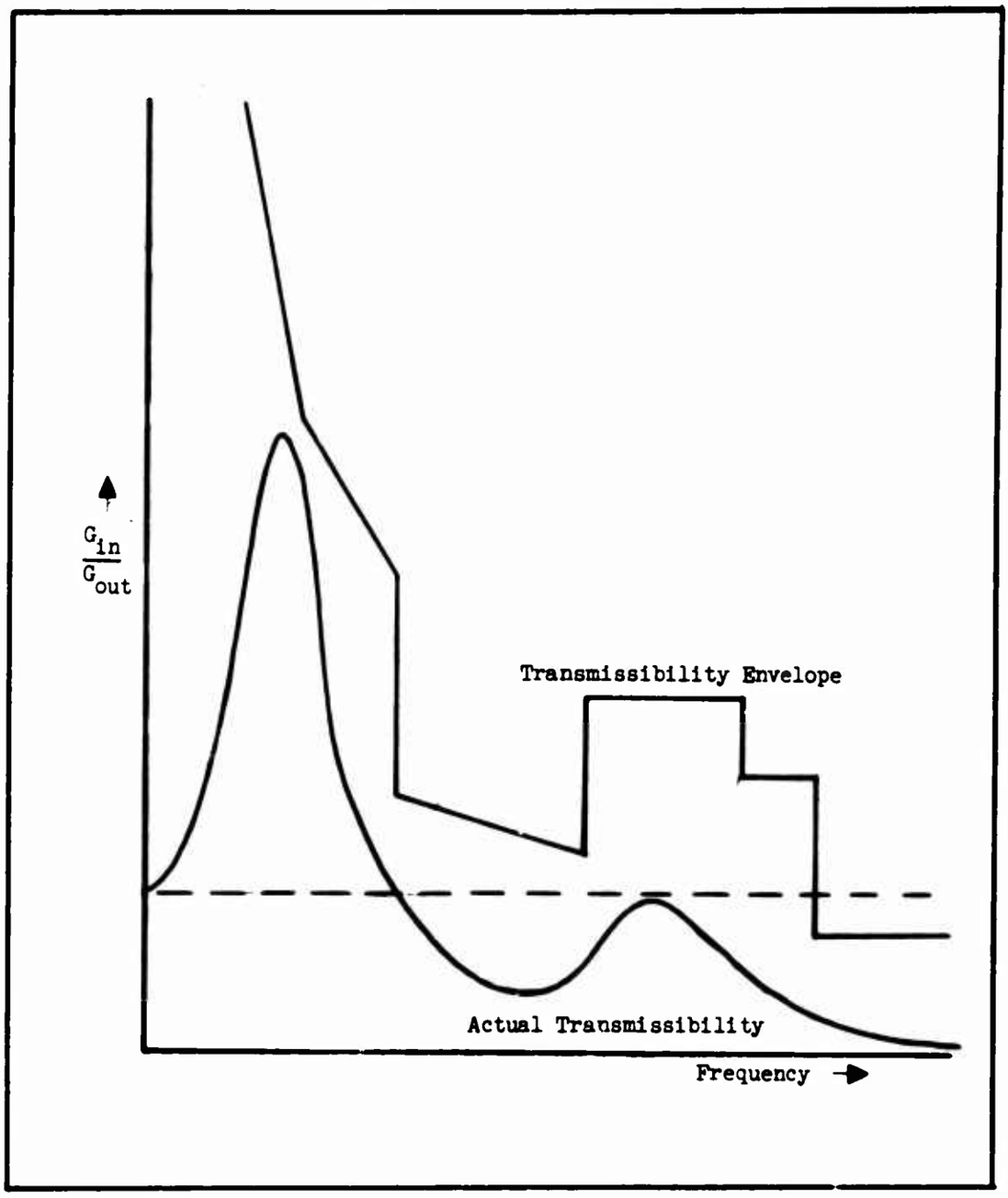
Should the end result be unsatisfactory for some reason, it will be necessary to return to the selection procedure and alter some of the assumptions made. In such a case it is possible to let the selection procedure indicate what assumptions should be changed. If the designer finds an attenuator or combination of attenuators that are suitable in every way but transmissibility, then he can generate a transmissibility envelope which will enclose the desired attenuator. The basic equation

$$T_f = \frac{F_f}{E_f}$$

can be rearranged into

$$T_f E_f = F_f$$

which, if the input environment is a fixed requirement, will produce a new fragility curve for the equipment. This fragility envelope will assist the designer in altering the electronic equipment structurally.



ATTENUATOR SELECTION: The attenuator should possess a transmissibility characteristic that lies within the established envelope.

VOLUME III - CHAPTER 9

DYNAMIC ATTENUATION

SECTION 3 - MATERIALS COMMONLY USED FOR ATTENUATION DEVICES

- A "Response Spectrum" for Classifying Attenuation Devices
- Characteristics of "Dry Friction" Devices
- Isolating With Woven Metal
- Isolating With Stranded Wire
- Usefulness of Fluid Devices
- Properties of Elastomers
- Response Characteristics of Elastomers
- General Characteristics of Foam
- Response Characteristics of Foam
- Properties of Natural and Synthetic Fiber Felts
- Characteristics of Other Special Composite Materials
- A Summary of Spring Attenuators

A "RESPONSE SPECTRUM" FOR CLASSIFYING ATTENUATION SYSTEMS

Dynamic attenuation materials can be arranged into a spectrum in which their interrelated properties become apparent. This attenuator spectrum replaces the familiar damper, isolator, and absorber terminology.

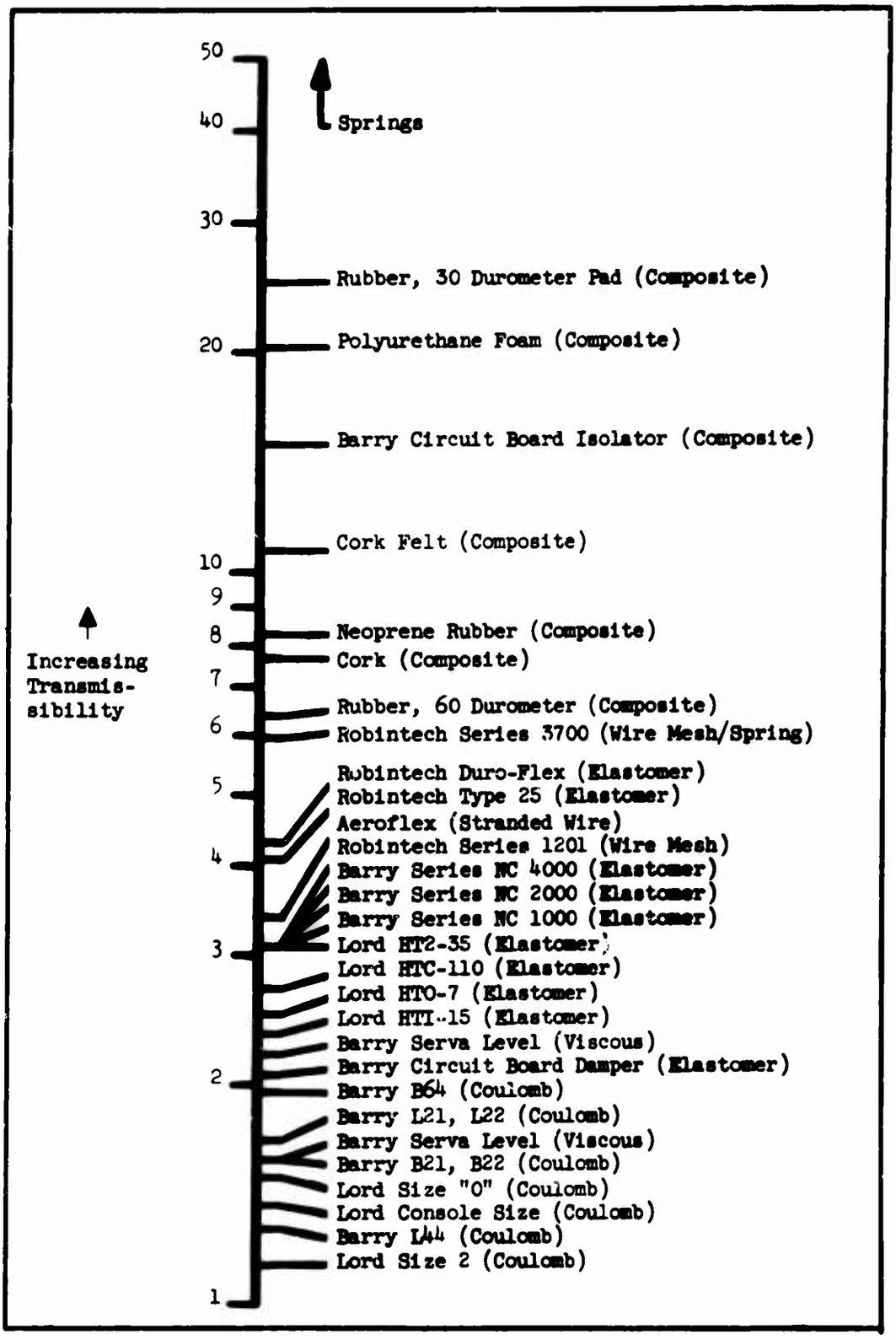
There are many kinds of attenuation devices ranging from automotive shock absorbers, to springs, to felt padding, to elastomeric vibration isolators. This section presents a cross section of these devices arranged in order of their general location within the spectrum shown in the adjacent figure. The search for a comparative base has resulted in the attenuators being presented as part of this continuous spectrum of devices.

In order to evolve this spectrum, it must be realized that there are really only two properties of a transmissibility curve which actually are necessary for the description of the basic curve; the properties of resonant frequency and amplification or "gain." It is possible to normalize the frequency axis of the transmissibility curves with the resonant frequency as unity. This normalization removes the dependence on mass or loading and relegates the curve to a point, the maximum transmissibility for the primary resonant frequency which is now unity. The maximum transmissibility is a characteristic of the attenuator alone and ranges from 1.0 for a critically damped system having no resonant rise, to infinity for a solely resonant system with no damping. All attenuation systems (i.e., isolators, abosrbers, dampers) may be charted within this spectrum.

The available data on contemporary attenuators was plotted and is presented at the right. As would be expected, considerable overlapping does exist. Five major divisions of material, however, tend to group themselves within the spectrum. These are, in order of increasing magnitude: coulomb damped devices, viscous devices, elastomers, composite materials, and springs. These devices are discussed herein in the order mentioned.

This "spectrum of devices" concept greatly simplifies the actual selection of an attenuator mentioned previously in that once the transmissibility envelope is known and the resonant frequency of the new system (the electronic equipment including the attenuator) estimated, then the transmissibility envelope can be normalized.

The transmissibility at this resonant frequency then becomes the maximum allowable for the application. Any device located lower on the spectrum than this limit is acceptable in the application, provided it also meets the other physical restrictions imposed.


50
40
30
20
10
9
8
7
6
5
4
3
2
1
Springs
Rubber, 30 Durometer Pad (Composite)
Polyurethane Foam (Composite)
Barry Circuit Board Isolator (Composite)
Cork Felt (Composite)
Neoprene Rubber (Composite)
Cork (Composite)
Rubber, 60 Durometer (Composite)
Robintech Series 3700 (Wire Mesh/Spring)
Robintech Duro-Flex (Elastomer)
Robintech Type 25 (Elastomer)
Aeroflex (Stranded Wire)
Robintech Series 1201 (Wire Mesh)
Barry Series MC 4000 (Elastomer)
Barry Series MC 2000 (Elastomer)
Barry Series MC 1000 (Elastomer)
Lord HT2-35 (Elastomer)
Lord HTC-110 (Elastomer)
Lord HTO-7 (Elastomer)
Lord HTI-15 (Elastomer)
Barry Serva Level (Viscous)
Barry Circuit Board Damper (Elastomer)
Barry B64 (Coulomb)
Barry L21, L22 (Coulomb)
Barry Serva Level (Viscous)
Barry B21, B22 (Coulomb)
Lord Size "0" (Coulomb)
Lord Console Size (Coulomb)
Barry L44 (Coulomb)
Lord Size 2 (Coulomb)
Increasing Transmis-sibility

CHARACTERISTICS OF "DRY FRICTION" DEVICES

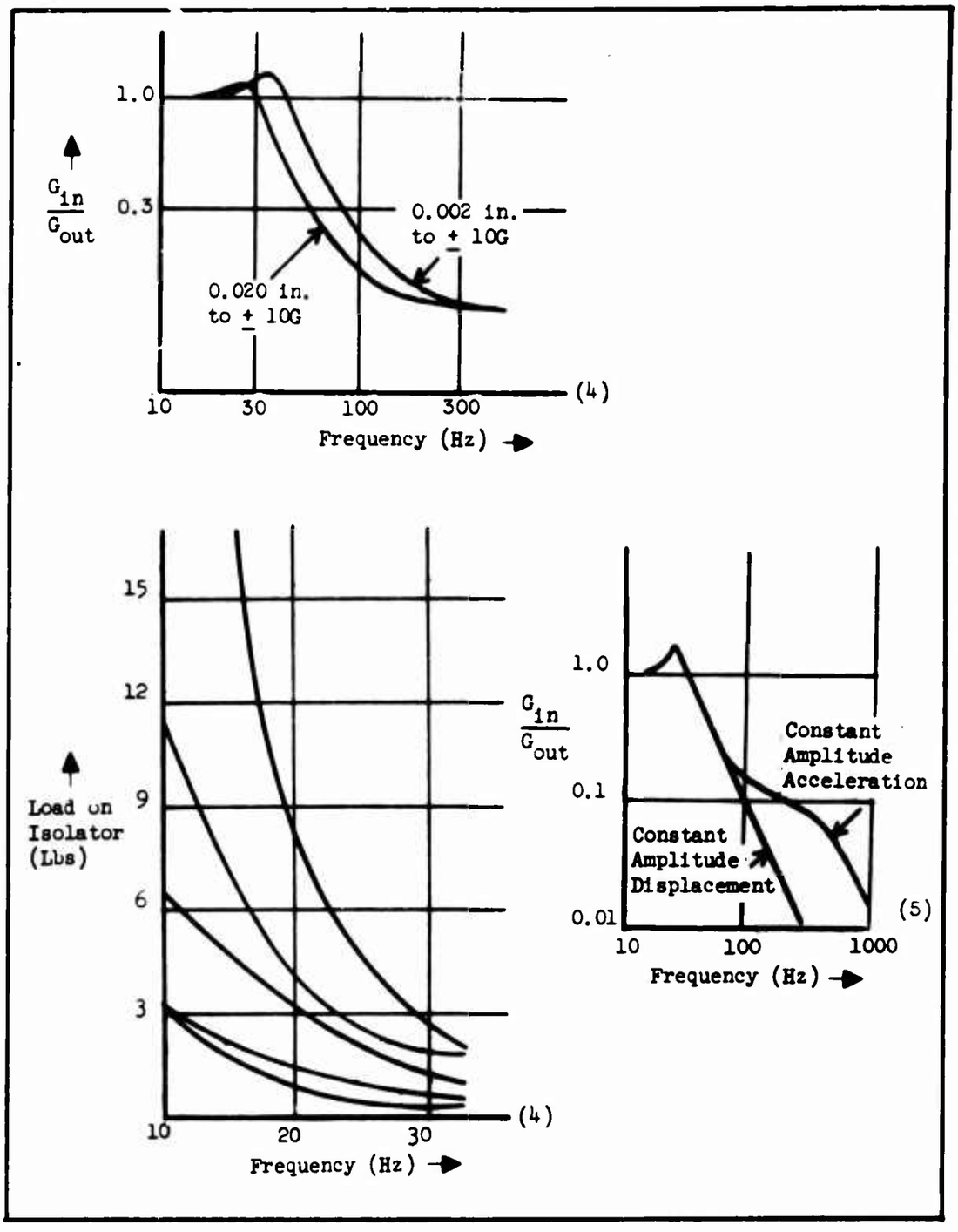
Coulomb damped (dry friction) devices combine high damping capability with low resonant response characteristics and dominate the low end of the spectrum.

Dry friction dampers are attenuation devices which dissipate energy only, i.e., no energy can be stored as potential energy as in springs. These damping elements dissipate energy through sliding friction and exert a force which is independent of velocity. In addition, these devices are dependent only upon relative displacement, i.e., they do not exhibit restoring forces from stored potential energy such that they return to the original equilibrium position. These dampers are very useful for providing additional damping to a suspension system without adding resilient properties since the only force they generate is related to equipment motion. These devices are operable through a wide temperature range and are only slightly affected by dirt, sand, and dust. Their performance is deteriorated, however, by lubricants such as water, oil, or gasoline which tend to destroy their frictional properties.

Sophisticated devices are now available which combine dry friction with spring properties. These devices are available both in series and in parallel and are excellent for specific applications.

The combination-type device would be located more toward the middle of the spectrum depending on the ratio of damping to resonance. Once the decision has been made to employ a combination device, it is best for the designer to contact the component manufacturer for transmissibility data on these systems.

It should also be noted by the designer that the transmissibility curves for these devices indicate that the devices do not isolate as effectively at the higher frequencies as well as some other classes of attenuators.



COULOMB DAMPED SYSTEMS: "Dry Friction" type devices generally exhibit high damping characterisitcs coupled with a low frequency resonatn response.

ISOLATING WITH WOVEN METAL

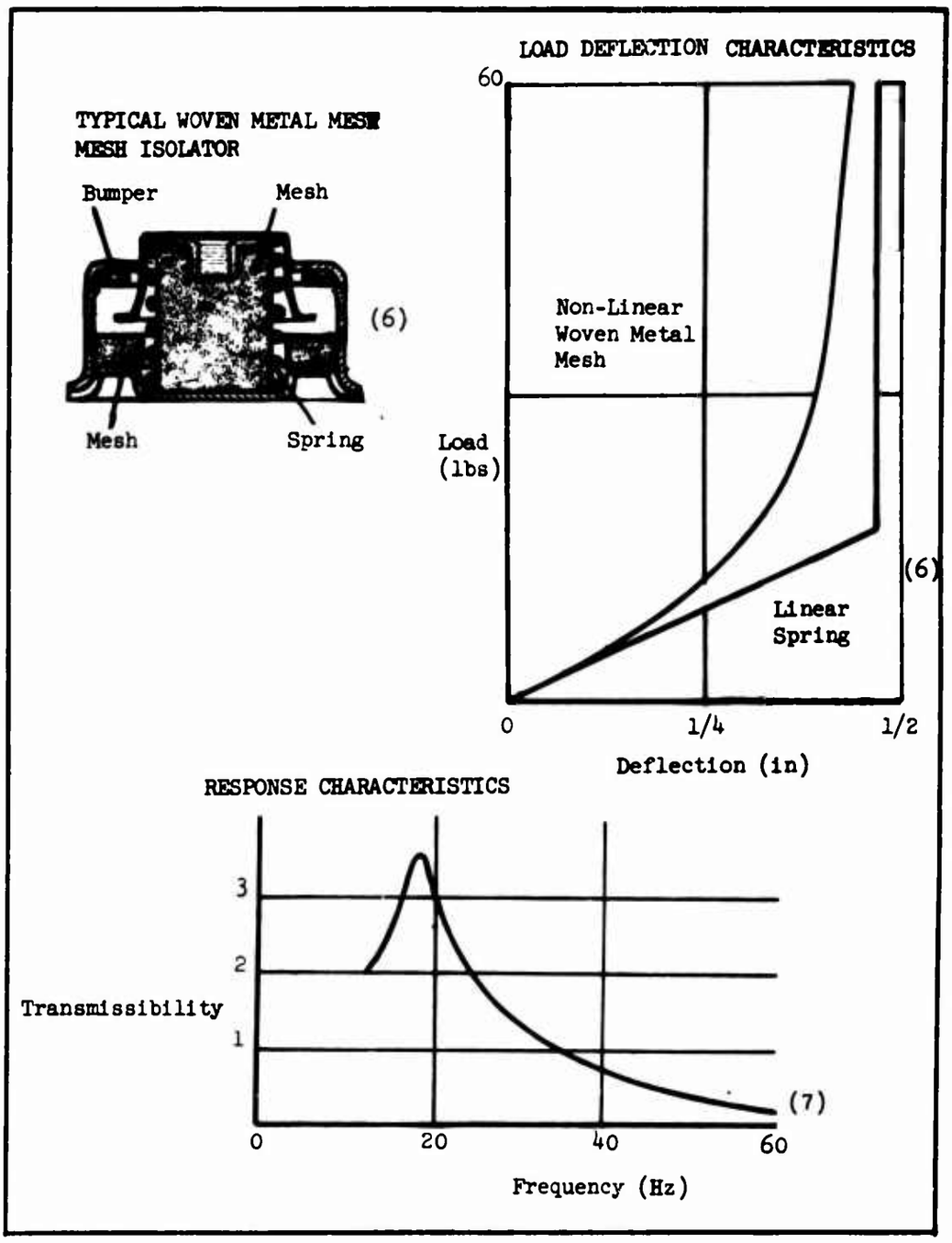
In certain applications, wire mesh devices have a distinct advantage over other isolation systems.

A typical design for a woven metal mesh attenuation device is shown at the right. As can be seen in the illustration, it consists primarily of woven metal in combination with a spring. The metal mesh provides damping while the spring supplies resonant characteristics and load support. Buffers are also shown, but these only come into play in the event of over-travel or bottoming of the primary attenuation material.

Also shown is a typical transmissibility plot for these devices. Since this is a combination device, the peak is toward the middle of the attenuator spectrum as might be anticipated. It may also be noted that the transmissibility drops off much more rapidly than the dry friction devices, with a similar increase in frequency.

The metal mesh in these devices mechanically damps motion by dissipating energy through friction losses due to the rubbing of the mesh strands against one another. As such, the operational efficiency is little affected by temperature changes. By proper selection of metal used for the mesh, the temperature extremes which the isolator will endure may be altered. Metal alloy selection may also facilitate corrosion resistance non-magnetic properties. The energy from the shock or vibration excitation is converted into heat which is readily dissipated by the mesh. The stress within the woven metal mesh is usually far enough below the fatigue limit of the material to preclude a fatigue failure. These devices usually exhibit excellent life characteristics.

Woven metal devices are presently available only as isolators. There are several configurations on the market, each having specific advantages for given applications.



WOVEN METAL ISOLATORS: There are times when these devices have an advantage over other attenuation systems.

ISOLATING WITH STRANDED WIRE

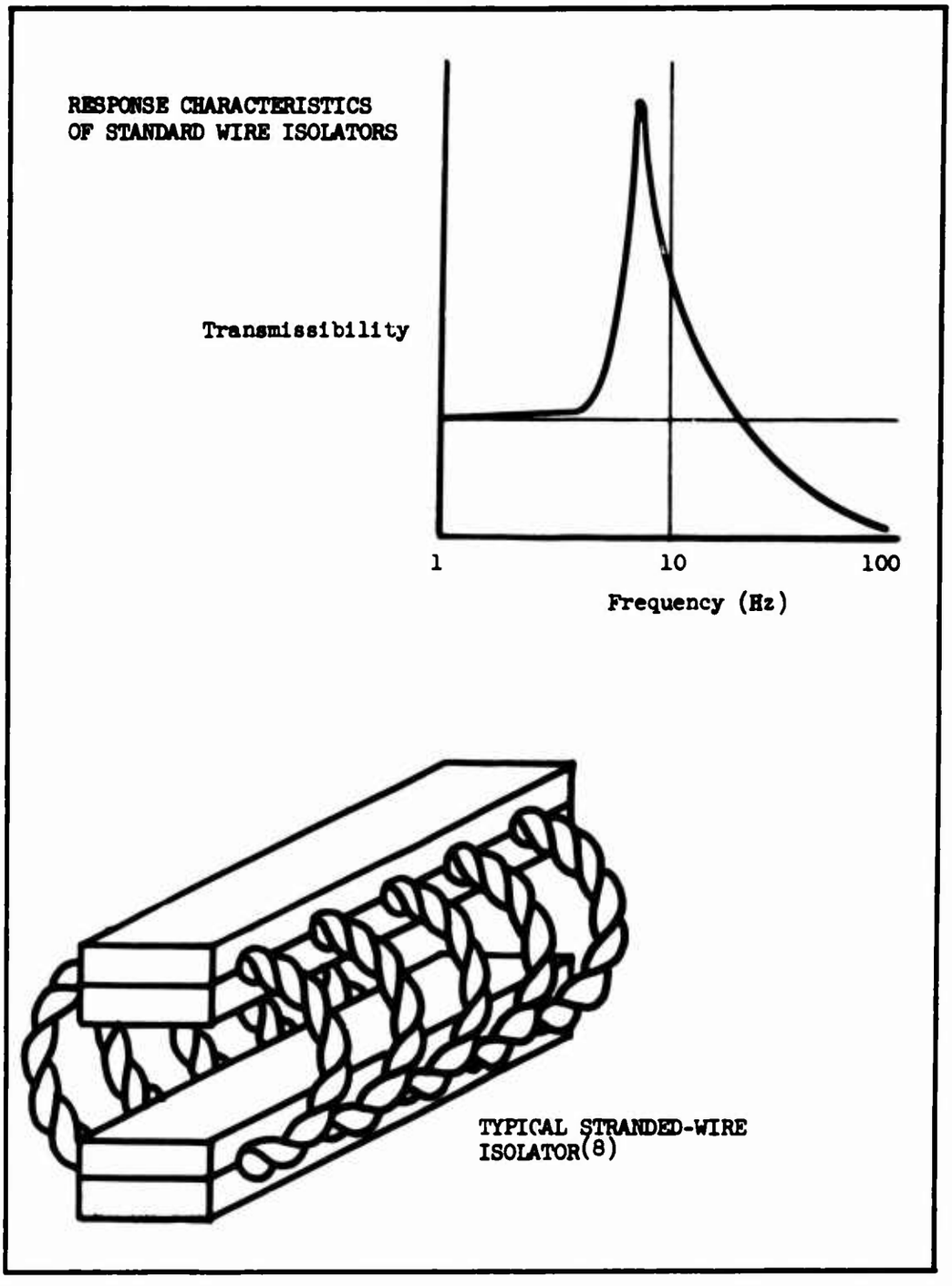
Cable isolators are recent developments within the range of dry frication devices which exhibit great versatility of application.

Cable isolators are a recent development in the field of isolators that are capable of attenuating both shock and vibration. Typical configurations of this device are as illustrated at the right. These devices are classified as coulombic since dynamic energy is dissipated due to friction between the individual strands of wire in each cable. This dissipation process is similar to that exhibited in the woven metal isolators. In this case, however, no additional spring is required since the cable is sufficiently stiff to support loads ranging from four ounces to several thousand pounds (depending upon the size of the isolator).

These cable isolators can operate in all environments, but some instances of fatigue problems have been reported. The all-metal construction results in electrical and thermal conductivity as well. Their function is relatively unaffected by temperature changes.

As with "Dry Friction" devices, cable isolators are affected by oils, lubricants, or abrasives which tend to alter the coefficient of friction within the cables. The damping generated by these mounts is related to the stress applied, which equates to large damping from large displacements and small damping (or predominant resonance isolation) at small amplitudes.

The response characteristics (resonant frequency, damping, displacement) may be varied by altering cable diameter, number of strands per cable, cable length, cable twist or lay, and the number of cables per isolator. This latter variation, the number of cables per isolator, is easily reduced by clipping, which results in a lowering of the resonant frequency of the system with an accompanying increase in the excursion for a given shock excitation.



STRANDED WIRE ISOLATORS: A recent development in the coulomb damped category of isolators having good versatility.

USEFULNESS OF FLUID DEVICES

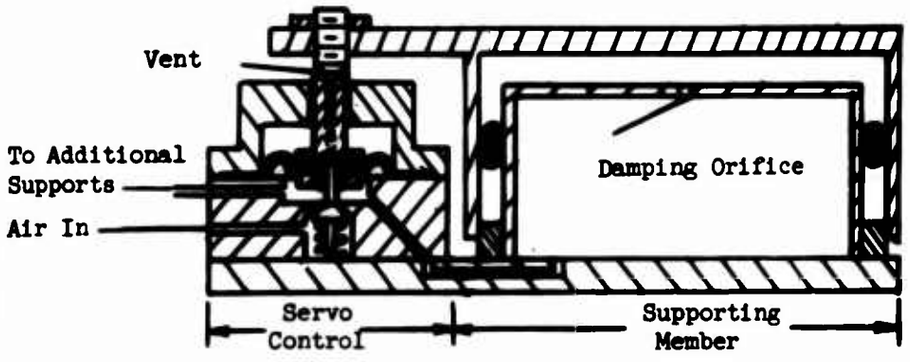
Fluid devices have long been used for shock absorption, but are now finding additional application as vibration isolators. Their attenuation capability is manifest by the metering of fluid through an orifice or by the utilization of the elastic properties of the fluid.

Fluid devices are very versatile in resisting the shock and vibration environment. They may be used with components as small as transistors, or modules as large as a room. Fluid devices operate primarily on two basic principles: the compressibility of a fluid and the flow of a fluid through an orifice.

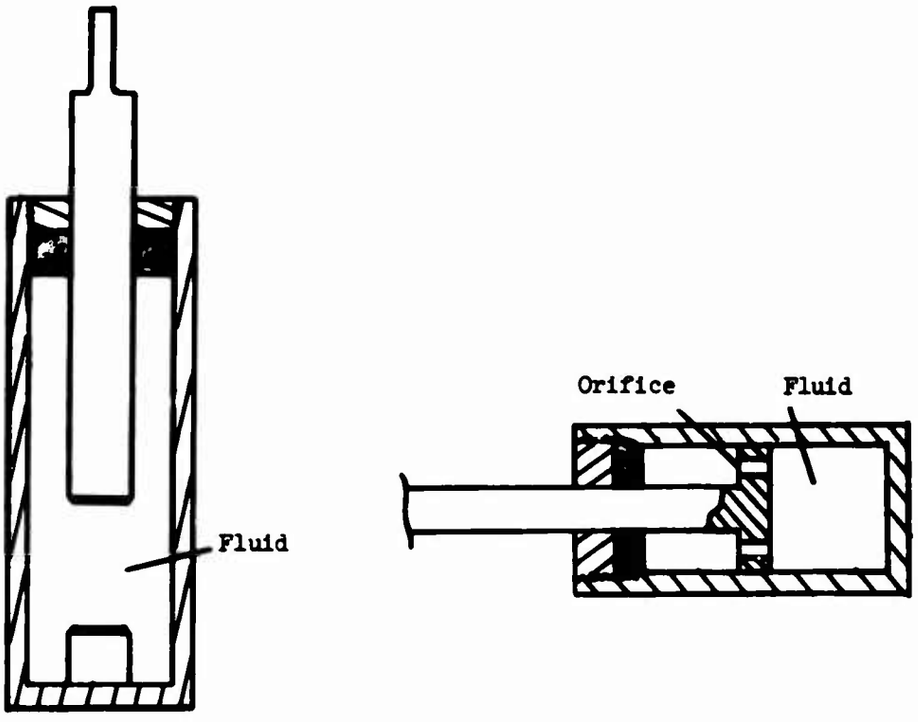
A technique which employs both of these principles of operation may be illustrated by the insertion of a small amount of grease under such components as transistors or integrated circuits. This technique provides a higher capacity path to the heat sink but it also allows for some shock and vibration absorption by coupling the component to the mounting base.

Examples of fluid devices which operate primarily on the compressibility of a fluid are the liquid springs, either servo-controlled or fixed air bags, and automobile tires. Liquid springs are essentially a filled and sealed hydraulic cylinder with the shaft in place, but without a piston. As the shaft moves in and out, the volume of the fluid within the cylinder changes. This change in volume manifests as a change in pressure within the cylinder. Air bags and servo-controlled air bags are commonly available; vibration isolators have even been made successfully from automobile tires and inner tubes. The tire may be used as a container for the tube. After all excess rubber is removed from the carcass, a very low frequency system at low cost results.

Familiar to all of us is the common shock absorber used on motor vehicles. These devices are hydraulic cylinders with an orifice (leak) through the piston. Fluid, being viscous, is restricted in its flow through this orifice; the energy of the shock pulse is thus converted to heat at the orifice. These shock absorbers need the assistance of a spring (liquid, mechanical, or elastomeric) to support a load, as the restoring force is not displacement related. In this way, they are similar to the dry friction shock absorbers discussed earlier.



A TYPICAL SERVO CONTROLLED PNEUMATIC SYSTEM(11)



TYPICAL LIQUID SPRINGS(10)

FLUID ATTENUATION SYSTEMS: There are a variety of attenuation systems whose characteristics depend on the viscosity and compressibility of the fluid.

PROPERTIES OF ELASTOMERS

Elastomeric compounds have elasticity and formability characteristics which makes them particularly adaptable to shock and vibration attenuation.

The elastomer is basically a rubber. It may be either natural or synthetic and can range from extremely pliant to a very rigid material. This diversity is due to the formulation or "blend" and results in many different chemical and physical properties. Elastomers may range from encapsulants to self-adhering sheets, to isolators, to ship dock fenders.

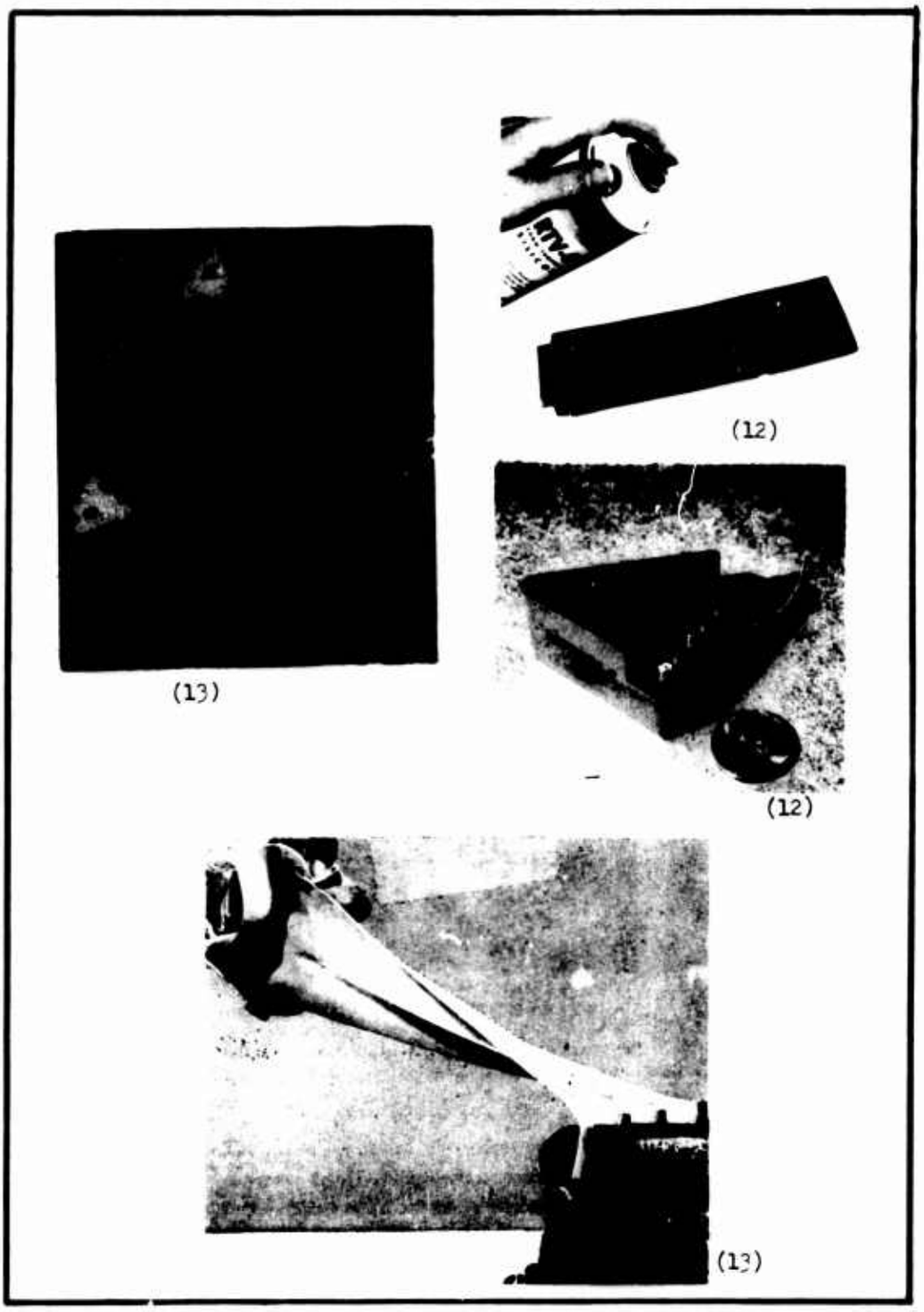
The energy originating from a shock or vibration environment in some instances may be controlled by absorption and subsequent conversion to heat within an elastomer. This conversion is usually accomplished through the internal friction properties of the attenuator. Elastomers are most efficient at this transfer of energy when the applied stress manifests itself as shear within the device. There are many available attenuators which are specifically designed to take advantage of this shear characteristic.

Encapsulants are elastomeric compounds which are available in a full range of hardness. Almost all are cast in place for the best attenuation effect. While they are available in either an opaque or transparent state, filters may be added to the encapsulant for a desired optical effect. Some encapsulants may be stripped off for repair of failed units and subsequently "patched" back in place.

Epoxy material is an example of the elastomer used primarily to raise the resonant frequency of the assembly above the anticipated excitations. The molds used for this encapsulation may be either rigid metal or pliant R. T. V. (room temperature vulcanizing) silicone rubber.

R. T. V. silicone rubber is also an encapsulant often used with small electronic assemblies. It must be employed with discretion since it has poor adhesion qualities to the rigid parts being potted. It will easily peel off connectors if the connector is handled often. A primer is suggested when using R. T. V. silicone rubber encapsulants, but its effect is small. Silicone rubber is very useful where a short term, high temperature environment is expected (as on a launch pad) as it forms a crust which provides a good thermal insulator. Because the silicone rubbers exhibit a poor adhesion to the part they encapsulate, the encapsulant may be easily cut away and the part repaired. After the repair, the cavity may then be filled with new silicone rubber.

When only a limited amount of damping is required along with environmental protection, a "conformal coat" may be employed. This method lends itself to production as the assemblies are dip-coated without molds. The parts of the assembly can, in most cases, be repaired and the coating patched. This is a thinner coat than the other encapsulants which results in an overall cost saving.



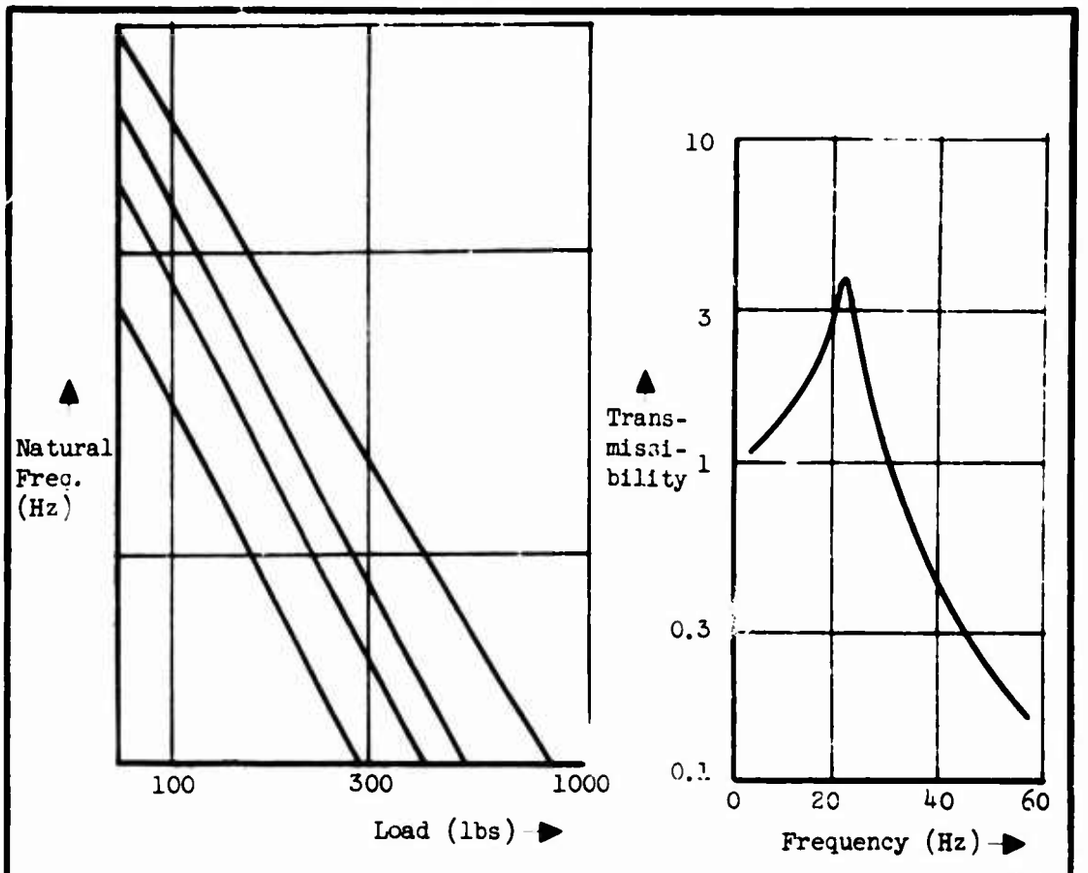
THE ADAPTABLE ELASTOMER: Elastomeric compounds are available in forms ranging from sheet materials to sprays.

RESPONSE CHARACTERISTICS OF ELASTOMERS

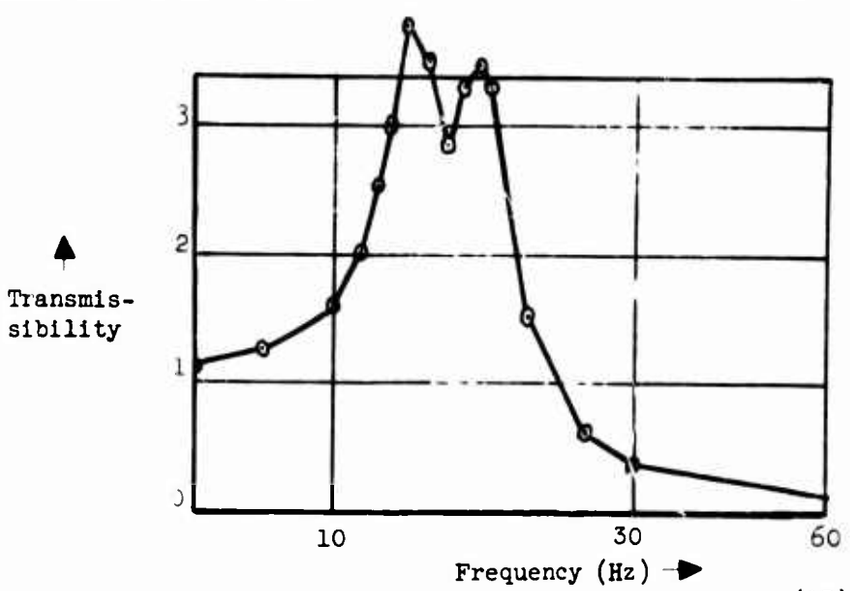
The transmissibility and load curves for elastomers va ver a broad range depending upon composition, temperature, and the mechanical confi ration of the elastomer.

It may be seen in the attenuator spectrum presented earlier that the attenuator manufacturers have taken advantage of the versatility of elastomers to produce a wide variety of absorbers and isolators. Elastomers are available in sheets which may be used as absorbers by bonding to a vibrating panel, or as an isolator by placing it between the subject component and the damaging excitation. When applied to a vibrating panel, the absorption properties may be enhanced by forming a sandwich with the elastomer between sheets of metal since elastomers absorb energy best in the shear mode. This type of sandwich material is commercially available.

Isolators are available in virtually all shapes and sizes from a variety of manufacturers. The response curves presented in the adjacent figure are representative and do not apply to any particular configuration. The amount of amplification and attenuation depends upon the configuration of the isolator; the resonant frequency depends upon the load applied. Some of the elastomers used are subject to aging and ozone attack which results in cracking and decomposition of the elastomer. Others are affected by gasoline or oil. The designer should carefully review the other environmental influences on the equipment to optimize his choice of elastomeric material. To aid in the selection procedure, a list of manufacturers is provided in the appendix.



LOAD/FREQUENCY AND TRANSMISSIBILITY OF A TYPICAL ELASTOMERIC ISOLATOR(7)



TRANSMISSIBILITY OF A TYPICAL SILICONE RUBBER(15)

RUBBER ATTENUATORS: Characteristics of elastomeric devices vary with composition, temperature and configuration.

GENERAL CHARACTERISTICS OF FOAM

Foamed materials have many unique properties that make them desirable shock and vibration attenuators for certain applications.

The composites are considered to be mixtures of two or more elements. Foamed materials have been included in the composite group because they are considered as a mixture of two elements, one of which is a gas. The gas may be entrapped as if it were in many small baloons (closed cell) or may be free (open cell). It is the crushing of these cells with its associated displacement of gas that makes the foam useful as an attenuator.

The density of foamed materials can be varied by restricting the amount of gas (blowing agent) that is trapped in the cells of the foam. Most of the foams discussed here will be of the closed cell variety which also have the most utility as attenuators. How the natural frequency varies with density is unknown at this time and is an area where further study would be informative. Foams can be classified as to whether they are rigid or pliant or whether they are precast or foamed-in-place. For the purpose of electronic enclosures and modules, the last two divisions will be discussed.

Precast foams are available in blocks, sheets, and rods in both rigid and pliant form. These foams are easily worked to obtain any specific shape and can be attached with adhesive. Precast foams are used as both an isolator and an absorber on all sizes of package from missiles to transistors. Some of the rigid-type precast foams are permanently deformed by shock loading when used as an isolator but still make excellent packaging material for shipment of electronic equipment. These foams are often used to absorb acoustic energy in electronic equipment and may be applied to large unsupported sheets to dampen panel resonances.

Either a natural cavity or one formed by temporary dikes may be filled with foam-in-place material. These materials have the same ingredients as the precast, before foaming. This technique results in excellent bonding, a complete filling of the void, and an increase in the structural integrity of the assembly. Accessibility and repairability are poor, however. This technique of potting can be used to fill a wall cavity as well as potting a coil assembly, to illustrate the size-versatility of the composite.

Due to their dead air spaces, foams tend to be good thermal insulators (urethane foams have an effective thermal conductivity of 0.10 to 0.15 BTU per hour per sq ft per F° at 75°F).[19] Some of the plastics used to make the foams will melt at moderate temperature. A hot tool is thus an excellent way to machine these foams. The effect of various atmospheres and solvents will obviously depend upon the plastic in the foam. Most of these foams may be glued in place. The solvents in some glues, however, will dissolve some foams. An example is the effect of M. E. K. or polyester resin on polyurethane foam. The foam vanishes leaving only a very small amount of melted plastic residue. Before gluing, it is always advisable to experiment with some scrap material.

Properties of Foamed Materials [17]

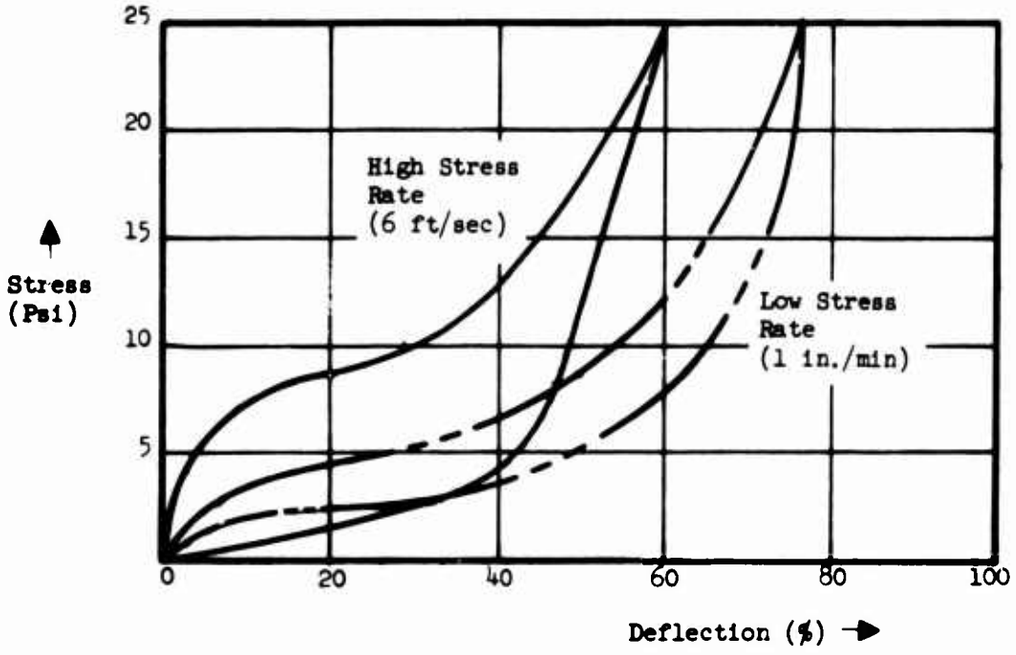
Type	Density	Preformed	Foam-in-Place	Shear Str (PSI)	Compression Str (PSI)	Uses
Epoxy	1.8-2.1		X	21	13-17	Embedding & Potting
Natural Rubber	6-7	X		-	0.15-2.4	Shock Absorption
Neoprene	8-10	X		-	0.15-2.4	Shock Absorption
Phenolic	2-5	X	X	13-40	22-85	Void Filling, Potting
Silicone	12-16		X	-	200	Vibration Isolation
	10		X	-	1-3	Vibration Isolation
Urethane	1.5-2.5		X	20-25	-	Embedding & Potting
	4-7		X	60-120	60-120	Vibration Isolation
	1.5-2.0	X		-	0.3-0.5	Shock Absorption
	4.0-5.0	X		-	0.9-20	Shock Absorption

FOAMED MATERIALS: Properties of some of the more common foamed materials.

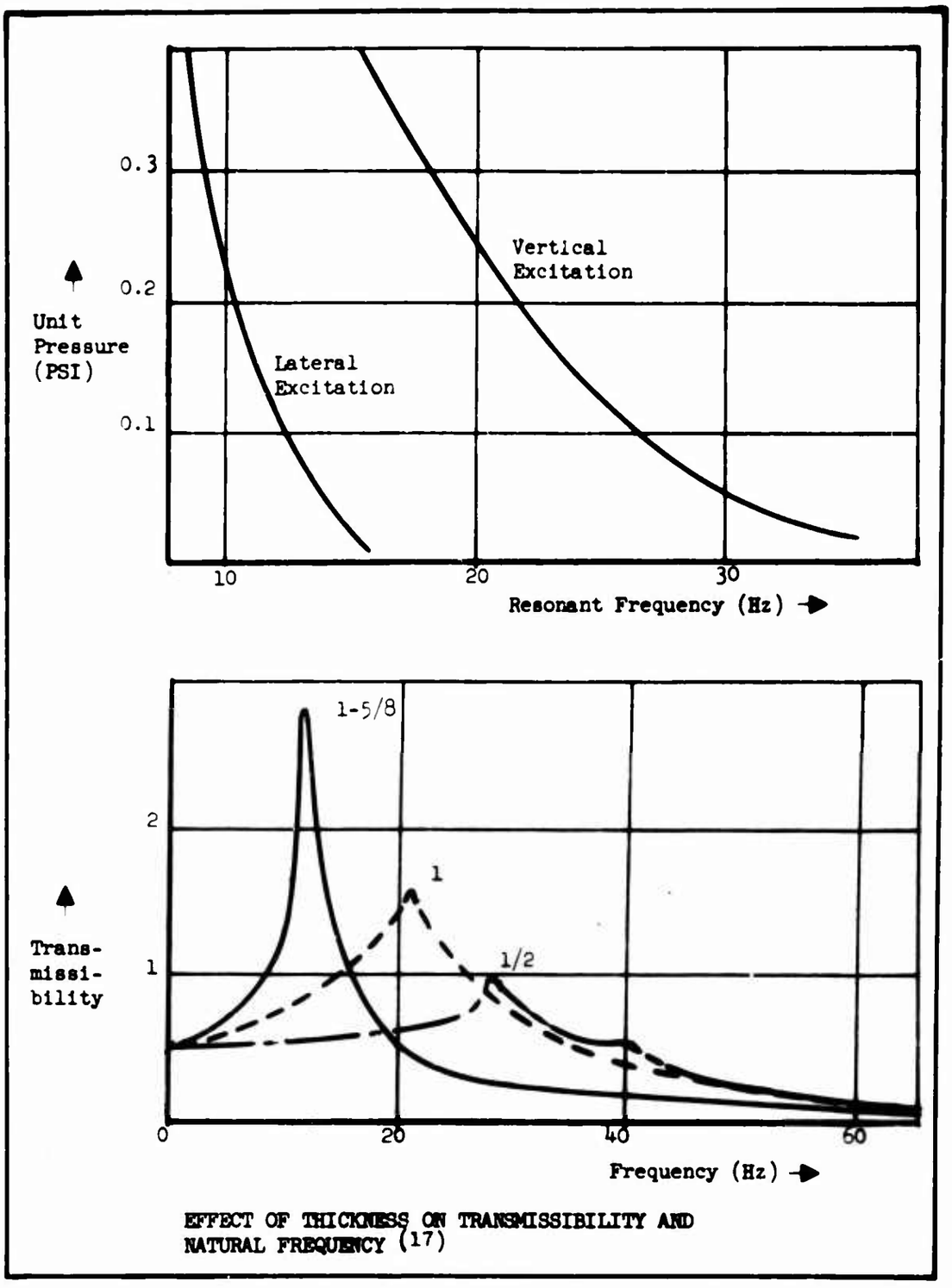
RESPONSE CHARACTERISTICS OF FOAM

Much data is available on the properties of foam materials. Their application must be carefully tailored to the problem.

The data presented here applies to the more common foams which are used for isolation and absorption. A recent development is an adhesive-backed resilient foam sheet which may be used for the isolation of light components. It was developed primarily for acoustic deadening. When using adhesives it is wise to verify the aging properties of the bonding material as well as the solvent effect of the adhesive on the foam.



COMPRESSION LOAD DEFLECTION CHARACTERISTICS OF POLYURETHANE FOAM[18]



EFFECT OF THICKNESS ON TRANSMISSIBILITY AND NATURAL FREQUENCY (17)

FOAMED MATERIALS: Useful composites for both absorption and isolation.

PROPERTIES OF NATURAL AND SYNTHETIC FIBER FELTS

Natural and synthetic fiber felts damp in a manner similar to open cell foam and are used as absorption devices as well as damping materials for resonanting panels.

Both natural and synthetic fiber felts find application as electronic equipment attenuators. Their normal usage, however, is intended as acoustic energy absorbers.

Natural fiber felts are not affected by sunlight or oxidation as are some foams. They are also very stable in oil. All natural fiber felts are usable between the temperatures of -80°F and +200°F. All exhibit a very low (or negligible) coefficient of thermal expansion.

Synthetic fiber felts may also survive service temperatures above 200°F; some have the added safety feature of being fire retardant. Most synthetic fiber felts merely shrink or melt when subjected to excessive heat.

Like foam, felts have a low thermal conductivity and actually behave much like an open-cell foam. When felt is used as a damper on panels, the damping properties can be improved by the addition of a metal sheet to the exposed side of the felt. The barrier enhances the damping action by converting compressive stress into shear stress and by restricting the air flow through the felt. This technique is applicable to foams and elastomeric sheet as well.

Various types of felt are frequently seen in commercial applications such as interior panels of television sets and office typewriters. Another example is the felt pad that is often used under some office machines to minimize noise amplification. All of the applications cited are primarily for acoustic noise reduction. This is the reverse of the problem presented throughout this Design Guide; the electromechanical equipment is generating the disturbance and the surrounding are to be protected. It should be noted that all of the attenuators can be employed in this way and that the selection procedure still defines the decision parameters.

Natural Fiber Felts (Wool)				
Grade	Specific Gravity	Compress (10% Def) PSI	Coef of Noise Reduction	Uses
9R3	0.181	3	0.64	Sound Deadening
12R1	0.262	6	0.58	Shock Damping
12R2	0.262	6	0.58	Vibration Mounts
12R3X	0.256	-	0.58	Sound Deadening
12S1, 12S2, 12S3, 12S4	0.256	18	0.58	Dampers
16R2	0.342	21	0.50	Vibration Mounts
16S1, 16S2, 16S3, 16S4	0.342	32	0.50	Vibration Mounts
Not effected by sunlight or oxidation. Excellent stability in oil. Very low coef of thermal expansion.				

Synthetic Fiber Felts				
Type	Specific Gravity	Compress (10% Def) PSI	Coef of Noise Reduction	Uses
Dacron Polyester	0.08-0.25	1.3-5.3	0.59-0.69	Sound Deadening
Polypropylene	0.13	4.5	0.61	↓
Rayon Viscose	0.08-0.25	2.0-4.2	0.59-0.68	↓
Acrylic	0.08-0.25	2.1-5.0	0.59-0.68	↓
Nylon	0.08-0.20	1.5-3.2	0.62-0.68	Sound Deadening
Teflon	0.40	6.7	0.43	Vibration Mounts
Some are fire retardant, most shrink and melt.				

FELTS: Felts have many applications in vibration attenuation.

CHARACTERISTICS OF OTHER SPECIAL COMPOSITE MATERIALS

There are many special composite materials that can be applied to the shock and vibration attenuation problem.

Cork is one of the composite materials that have been utilized as an attenuator for a number of years. It consists of granules of pure cork (a wood tissue) that have been compressed and baked under pressure to form blocks or boards. Since these sheets are entirely of wood, they can be worked with normal woodworking tools. The flammability of this material can be reduced through proper chemical treatment. Cork is resistant to sunlight, normal temperature extremes, oil, and water.

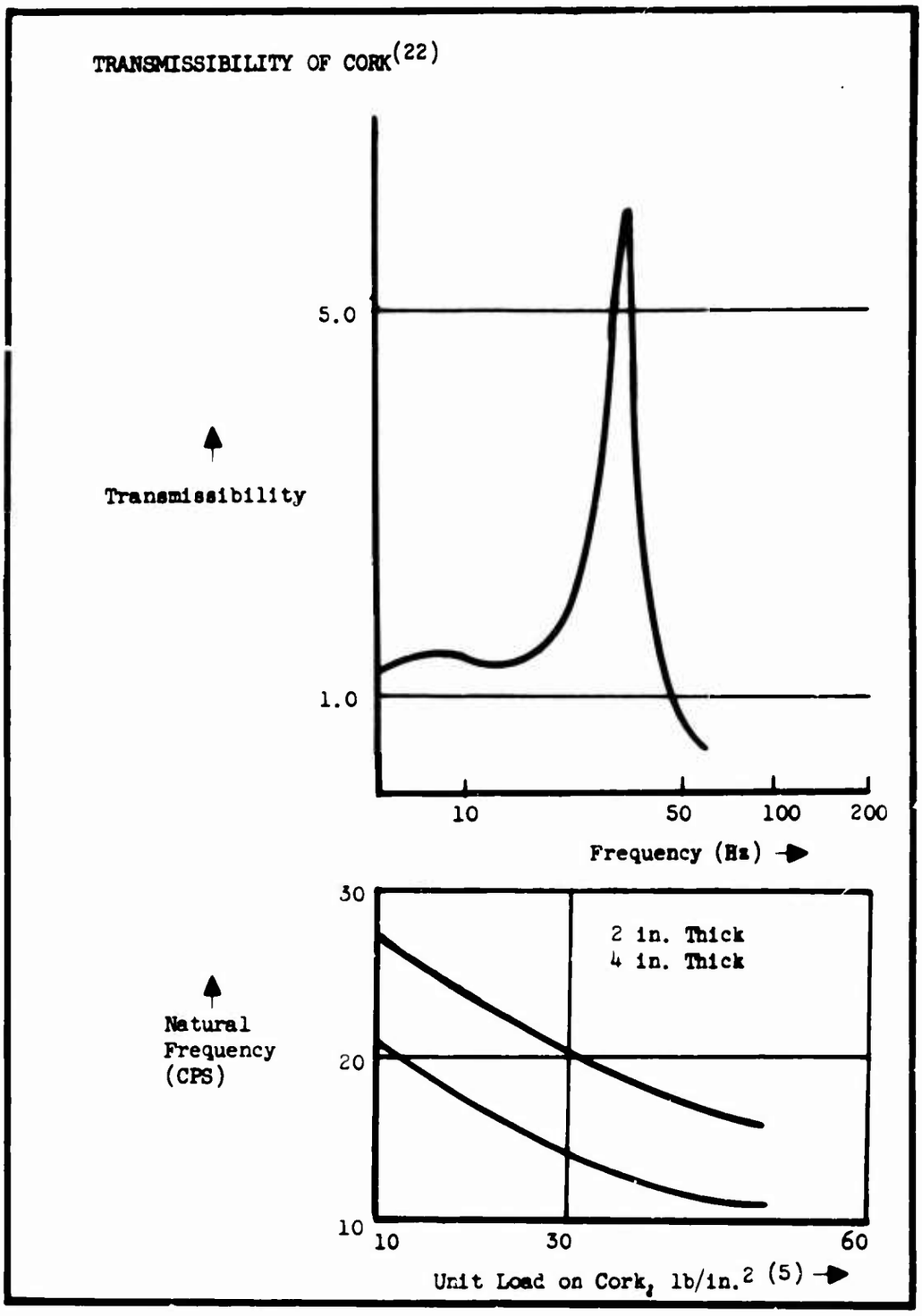
Cork is best employed in permanent installations since it has a tendency to crumble. It works best with the "heaviest permissible loading on the greatest practical thickness of cork."(20)

The load natural frequency and transmissibility curves for cork are illustrated in the accompanying graphs.

Granular cork may also be utilized as an in-transit shipping material by placing it around the equipment to be protected. Other materials mentioned in this chapter, such as the rigid and pliant foams, may also be used in this manner.

A recently developed material which may prove useful in shock attenuation is a foam made entirely of glass or metal. It is presently difficult to obtain and has seen little application to problem as yet. From the nature of the material, it should provide excellent thermal insulation as a bonus.

A fairly recent release in acoustic absorbers is an adhesive-backed flocking. This material appears best suited as an acoustic coating for panels.



SPECIAL COMPOSITES: There are unique composite materials possessing suitable properties for dynamic attenuation.

A SUMMARY OF SPRING ATTENUATORS

Spring devices attenuate dynamic excitation primarily from their resonant response characteristics, with only a small associated damping effect.

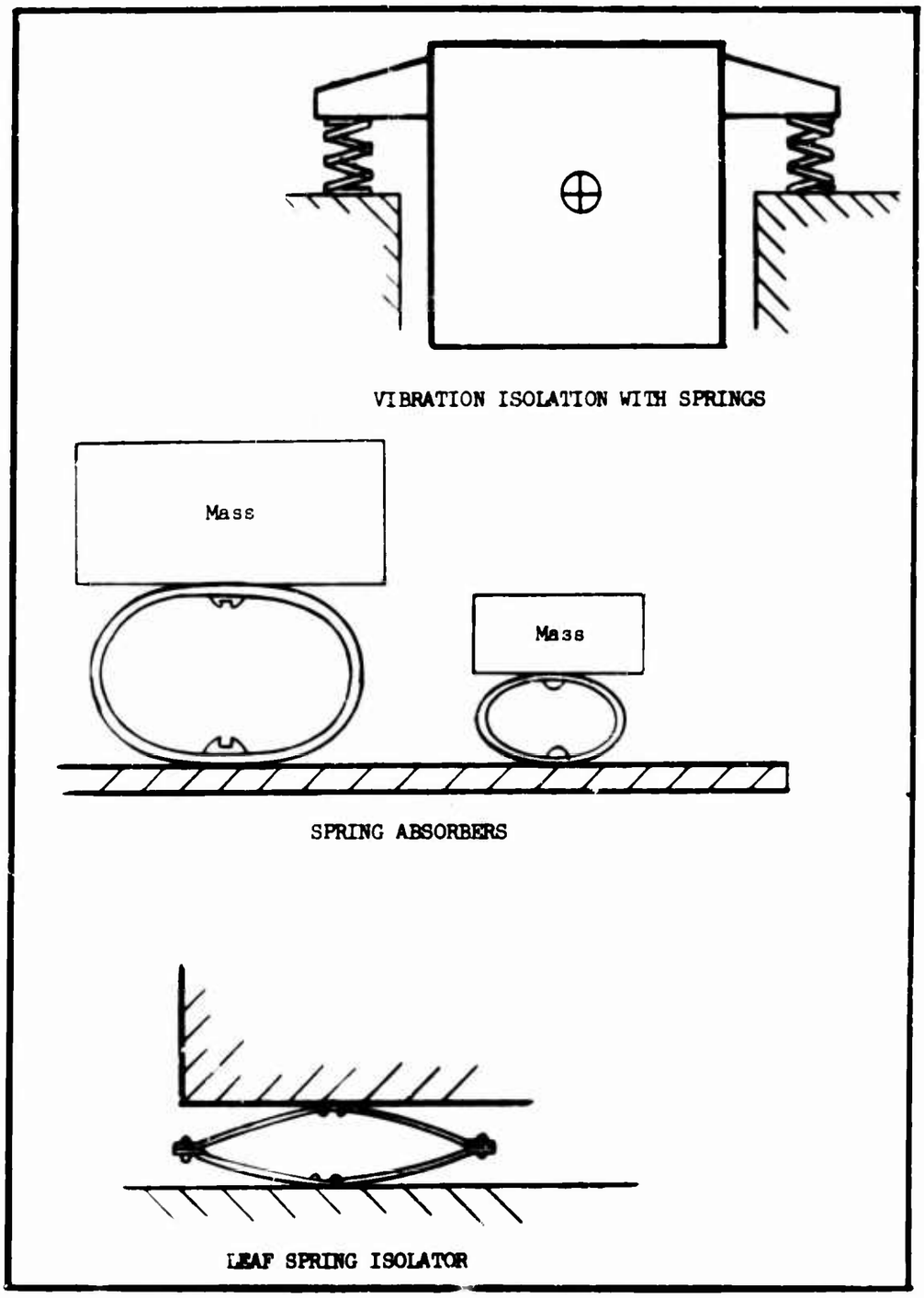
The spring is primarily a mechanism which stores the energy applied to it and releases this energy at a later time. Some small losses are encountered due to hystersis damping within the material of the spring.

Because of extremely low damping properties, spring isolation systems exhibit extremely large amplitudes at resonance. This characteristic does not detract from the spring's usefulness as a shock isolator, but does warrant careful attention when using this type of element as a vibration isolator.

When spring attenuators are employed for vibration isolation, it is necessary to choose the primary resonant frequency of the system (equipment and attenuators) well below (or above) any anticipated excitations. A poor choice will manifest as a highly amplified input, causing excessive excursion and acceleration due to the resonant effect of the device. This decision factor is reflected in the single-degree-of-freedom response illustrated in the adjacent figure.

Occasionally the designer finds equipment containing panels that "drum" or resonate unnecessarily at certain frequencies. A designer may reduce this condition with tuned absorbers. These devices are merely masses on the end of a spring (illustrated in the accompanying sketch). By forcing into intentional resonance, they absorb the energy from the parent structure at discrete frequencies near the resonant frequency of the device. The energy is then dissipated as non-destructive excursion of the spring-mass complex.

Many configurations of springs are presently available to the packaging designer. These attenuation systems have become commonplace as isolation devices. The materials employed are as diverse as the way the final device is utilized. Recent developments employ glass and plastics to complement metal alloys. Springs are commercially available to isolate entire rooms; spring clips small enough to hold and isolate electronic components are examples of the other end of the size range.



SPRING DEVICES: Good for shock but poor for vibration because of low samping.

VOLUME III - CHAPTER 9

DYNAMIC ATTENUATION

SECTION 4 - A REVIEW OF ATTENUATION TECHNIQUES

- Classification of Electronic Equipment by Level
- Attenuation at the Component Level
- Absorption at the Sub-Chassis Level
- Isolation at the Sub-Chassis Level
- Techniques of Chassis Absorption
- Isolation of Chassis With Highly Damped Devices
- Isolation of the Chassis With Predominantly Resonant Devices
- Use of Padding at the Console Level
- Isolation Devices for the Console Level

CLASSIFICATION OF ELECTRONIC EQUIPMENT BY LEVEL

The electronic equipment may be divided arbitrarily by level. The application of attenuation devices will be discussed as they apply to each level grouping.

When discussing the application of attenuators by groups or "levels," it is best to reference the Packaging Design Handbook[22] and utilize, as a basis of division, the levels of electronic equipment established therein. These levels are illustrated in the figure at the right and may be defined in ascending order of assembly, as follows:

Level 1 - The lowest level of mechanical assembly generally consisting of several parts grouped on a card, chassis, or module. Typically, this level includes circuit cards, cordwood modules, terminal boards, subchassis, etc. This level is often described in terms of a manufacturing process.

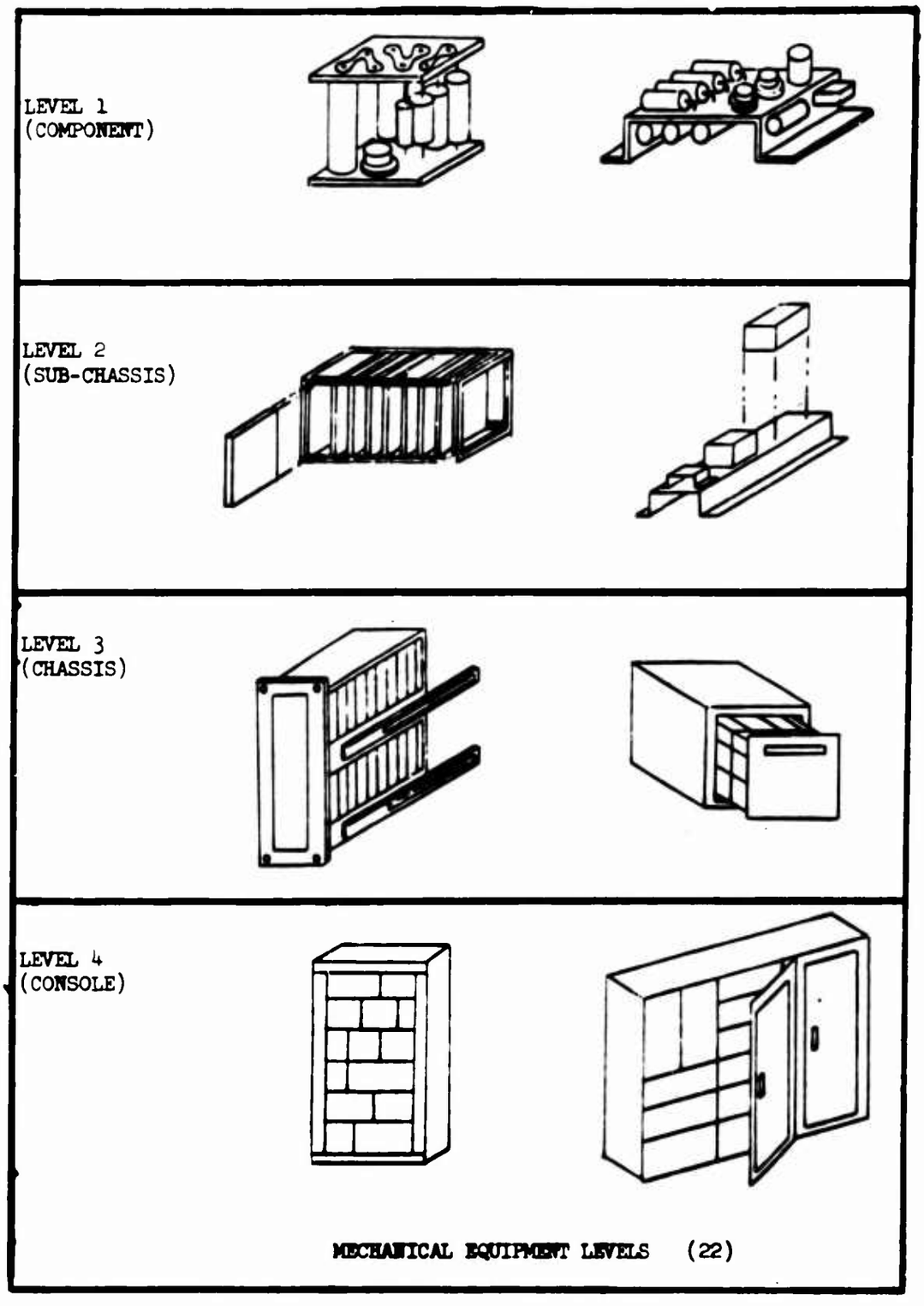
Level 2 - The second level of assembly consisting of groups of "Level 1" assemblies. Typically, this level includes card-mounted modules, card baskets or chassis, and assemblies or subchassis on a chassis.

Level 3 - The third level of assembly consists of groups of "Level 2" assemblies into portable cases, drawers, or hinged doors.

Level 4 - The fourth level of assembly is the highest level of fabrication normally used prior to installation of equipment for operational use. It consists of groups of "Level 2" and/or "Level 3" assemblies and typically includes bays, cabinets, and racks.

It should be realized that based on these definitions, some equipment will overlap definitions. This causes no great complication however, when selecting and applying attenuators. For ease of reference, these levels have been labeled, respectively: component, subchassis, chassis, and console levels.

As a result of the variation of volumes and weights involved, some attenuation materials are practical only at certain levels. An encapsulant, for example, is recommended for the chassis level and console level of assembly in only extreme cases. On the other hand, an elastomeric isolation device would probably not be applicable to many components.



MECHANICAL EQUIPMENT LEVELS (22)

ATTENUATION AT THE COMPONENT LEVEL

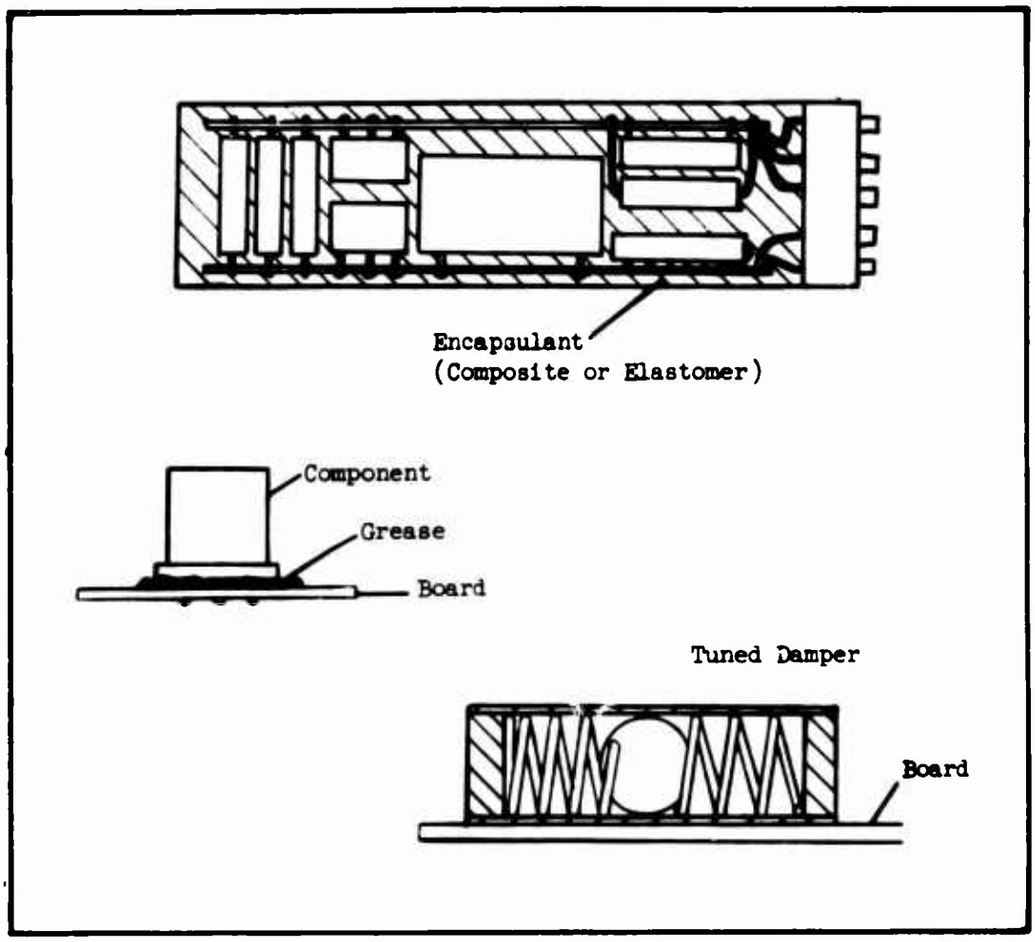
Attenuation is accomplished at the component level primarily by absorption. Improved structural integrity through encapsulation is the most important method of absorbing a dynamic disturbance.

The component level lends itself readily to the application of encapsulants for dynamic attenuation due to the small average size of the parts involved. This encapsulation can involve foam-in-place composites, cast-in-place elastomers, a conformal coat, or just plain grease.

The foam-in-place composites and cast-in-place elastomers are readily adaptable to "Level 1" components since the volumes involved are not usually very large. Their use greatly enhances the structural integrity and stiffness of the component in addition to reducing the probability of damage due to handling. The repairability of the encapsulated component, for example, will depend upon the hardness of the encapsulant used and the ease with which it may be removed. The presence of encapsulant will also change the cooling mode of the component from convection to conduction. This can be important if the design has inadequate heat sinking or is relying upon a blast of air for cooling the components.

A technique that has been in use for some time is the placement of silicone grease under transistors and other small components. The presence of this viscous fluid hydraulically couples the component to its mounting thus increasing the unit's structural rigidity. This procedure also increases the thermal conductivity to the heat sink.

It is possible to design a tuned damper which will absorb some energy at the component level. Cost, however, makes this technique impractical in all but a few cases. It is usually easier and more effective to encapsulate the component or isolate the next assembly group, the sub-chassis level.



COMPONENT ATTENUATION: At this level of assembly the primary means of attenuation is absorption.

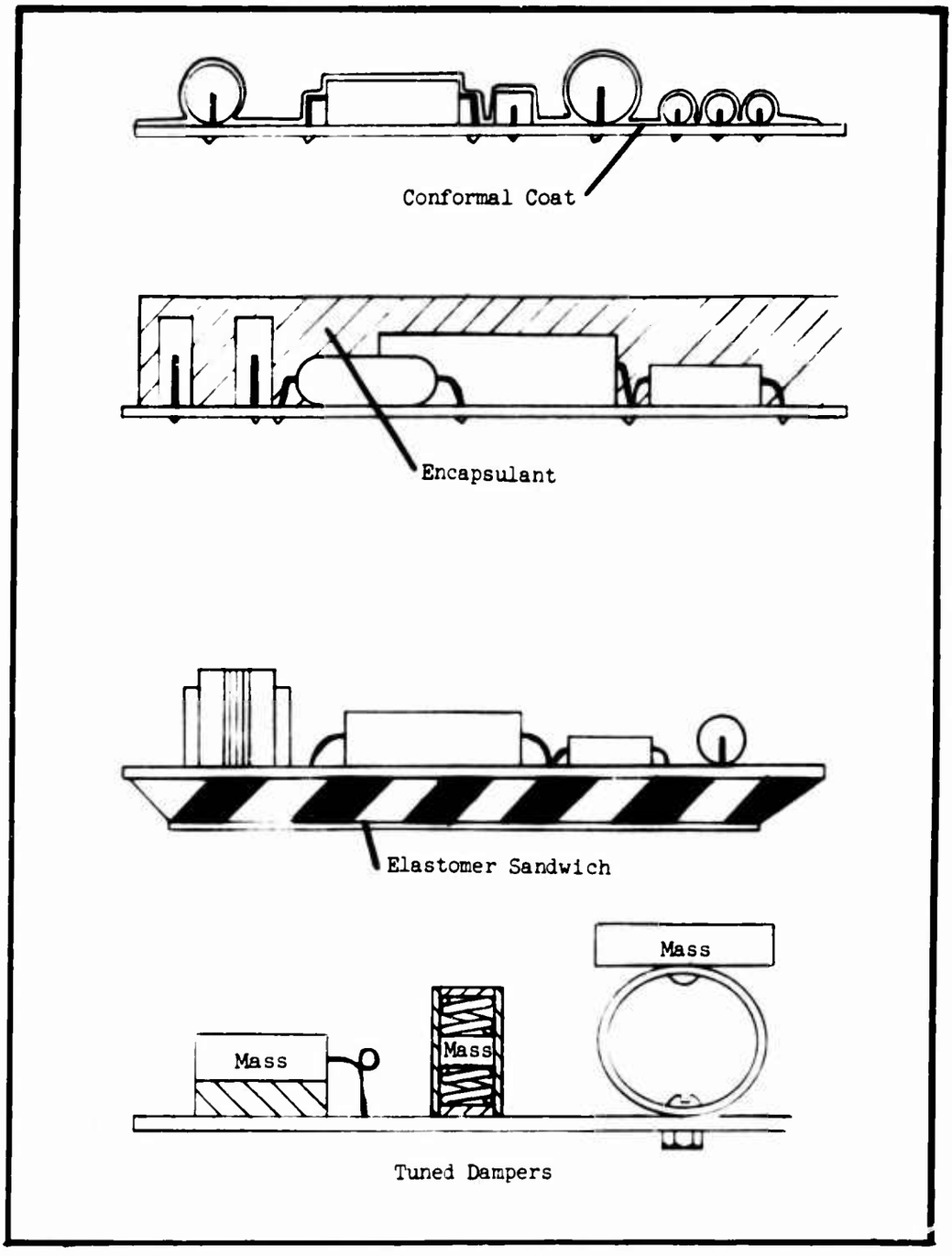
ABSORPTION AT THE SUB-CHASSIS LEVEL

Several effective methods of absorption are available to the packaging designer at the sub-chassis level of assembly including conformal coating, encapsulants, small isolators, and tuned dampers.

Many of the techniques that are effective at the component level may also be applied to the sub-chassis group (or Level 2). Again either a foam-in-place composite or a cast-in-place elastomer may be efficiently applied to encapsulate the assembly. A conformal coat may also prove useful for environmental protection and attenuation. The problem of repairability of the encapsulated element remains roughly the same as it was for the component level. Structural reinforcement is still a significant factor due to encapsulation, but the volume of material involved in encapsulating at this level may be prohibitive.

The application of "elastomeric sandwiches" and pads is physically practical for Level 2 absorption. Circuit board dampers are also commercially available for this level of assembly.

Another device which may be useful is the tuned damper. This approach can consist of a mass on a springy medium, such as an elastomer or spring. The resonating mass can be a component in certain cases if the electrical leads are adequately protected.



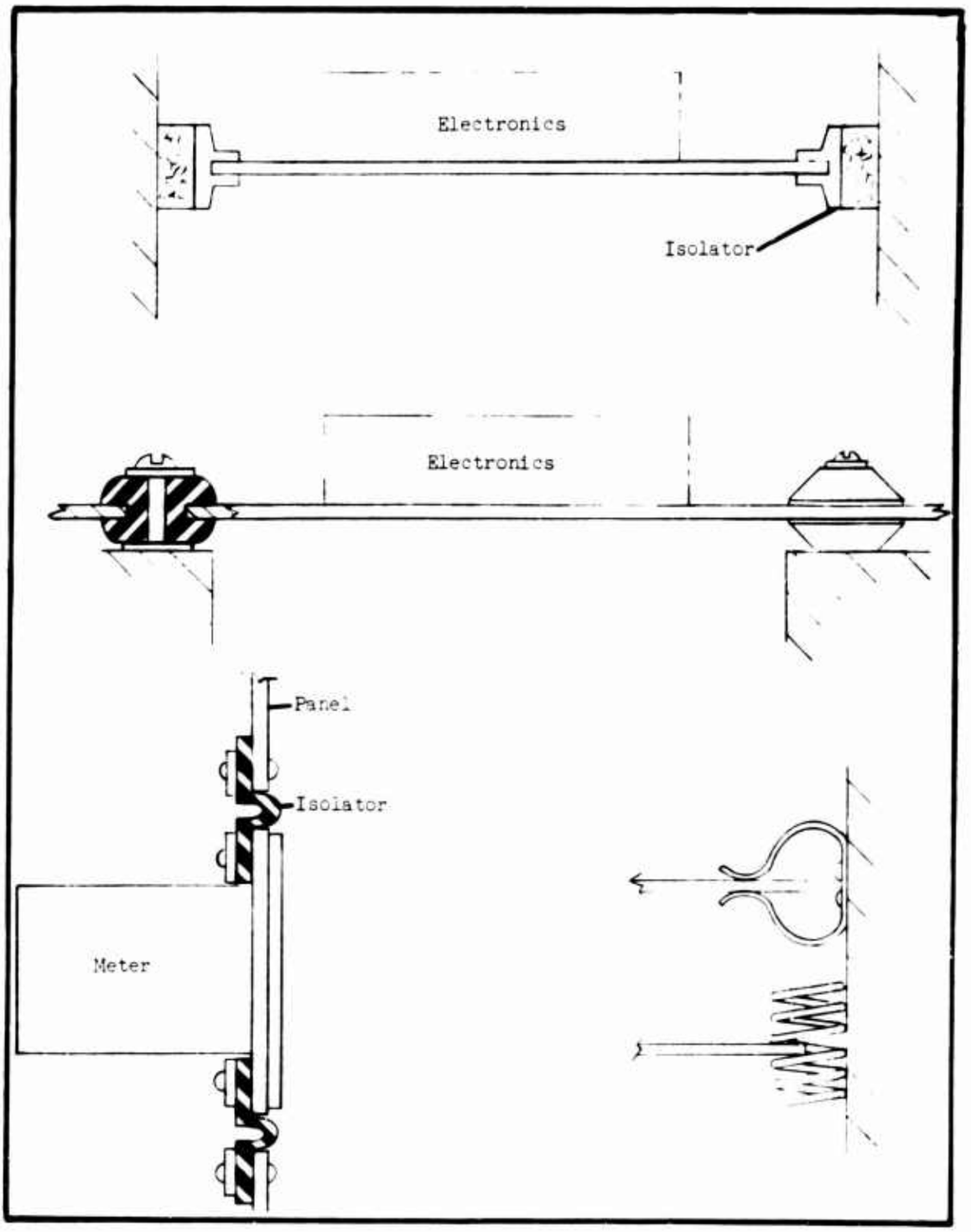
SUB-CHASSIS ABSORPTION TECHNIQUES: Several methods of energy absorption at the board level are available to the equipment designer.

ISOLATION AT THE SUB-CHASSIS LEVEL

Isolation at the sub-chassis level may compliment the absorption system previously described or may be used alone to combat the dynamic environment.

Several types of isolators are applicable to equipment elements near the size of the sub-chassis level. When the sub-chassis is the size and shape of a circuit board, it is possible to isolate it by: applying a foam-backed (circuit board isolator); mounting the board between springs; mounting it to an elastomeric isolator; utilizing a woven metal isolator; applying a small friction or viscous isolator which is available for light loads; or utilizing a combination of these devices.

Occasionally, sub-chassis elements are rather heavy masses bordering on the chassis level. Many of the above-mentioned techniques, however, still apply, with the exception of the "foam-in-shear" circuit board isolator" which may be too weak for the weight to be supported. As the mass increases it becomes increasingly important to orient the isolators on an axis through the center of gravity of the assembly. This orientation reduces the roll, pitch and yaw modes of oscillation so that only the translatory modes remain. The designer should also note that a combination device (external shock absorber, external vibration isolator and/or internal energy absorber) may be effectively employed to attain similar results. Frequently, more than one approach is indicated to reduce the excitation to an acceptable level.



SUB-CHASSIS ISOLATION: Isolators generally applied to the sub-chassis level range from foam materials to various spring devices.

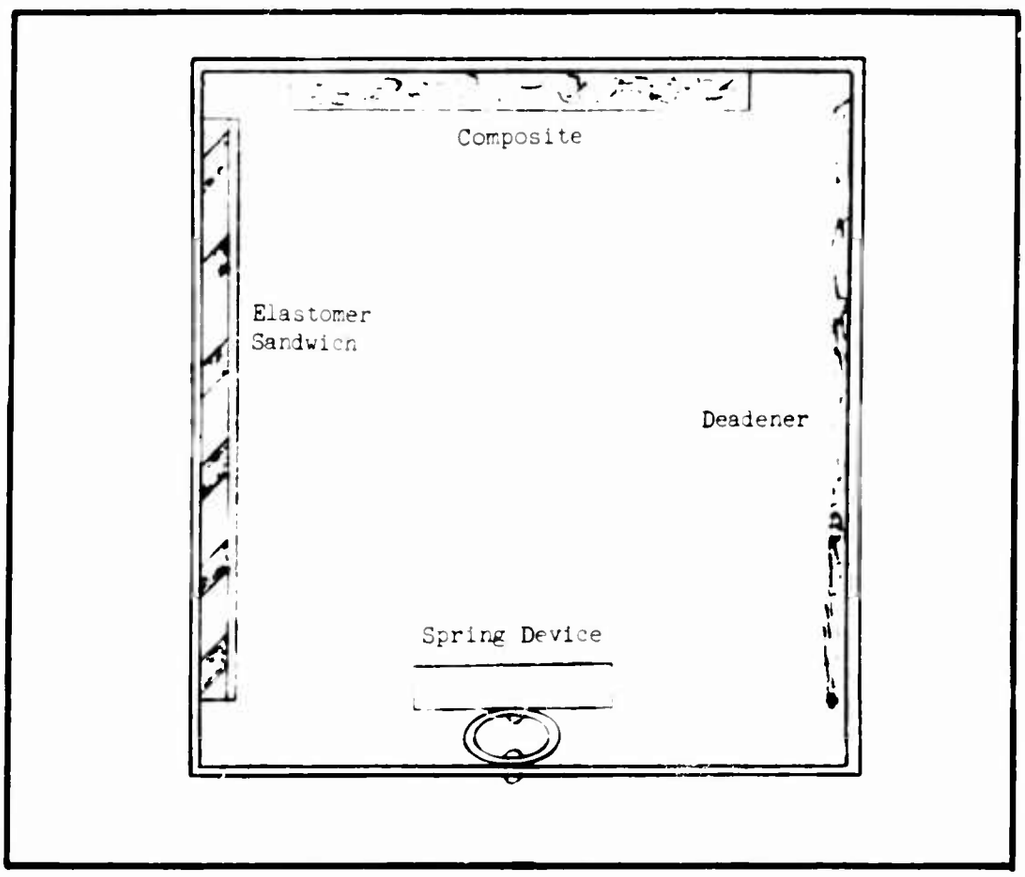
TECHNIQUES OF CHASSIS ABSORPTION

Many of the chassis level absorption devices available to the designer originate from attenuators used in acoustical insulating and air-conditioning practice.

The sizes and masses usually associated with the chassis level add a new range of problems to those discussed previously. The resonant frequency of the structures involved is sufficient to cause both acoustic and fatigue problems. The electronic equipment at the chassis level is of sufficient size that panels may occasionally be in resonance (drumming) with frequency excited by the dynamic environment. This problem has been defined in air conditioning systems for some time; many of the solutions and techniques are applicable here. These solutions involve the use of foam, felt, cork, elastomeric sheet and similar materials used alone, or as the center of a sandwich to damp the vibrating "drumhead." The sandwich assembly will absorb more energy because it subjects the medium to more shear than the absorption medium alone would receive. Careful selec ion of an elastomeric media will result in such bonuses as fire retardance and fungus resistance. Another material frequently used is undercoat. These absorbers usually have an asphaltic base which can result in other problems when used in military electronic equipment.

The simple spring mass system can be effectively employed to damp out resonanting panels. The spring may be composite, elastomeric or metallic in nature and the mass may often be a component from the electronic equipment. Resonant absorbers for this application are easily designed and tuned for specific applications.

If the chassis is not to be accessible for repair, it is then possible to fill with a foam-in-place material that may range from rigid to pliant. The resultant package would show an increase in resonant frequency and effective mass through stiffening. Some disadvantages are an increase in weight and additional difficulty in repairing the unit. Convection cooling effect is eliminated, which may result in additional heating problems.



CHASSIS ABSORPTION: Adaptations from the field of acoustics and air conditioning.

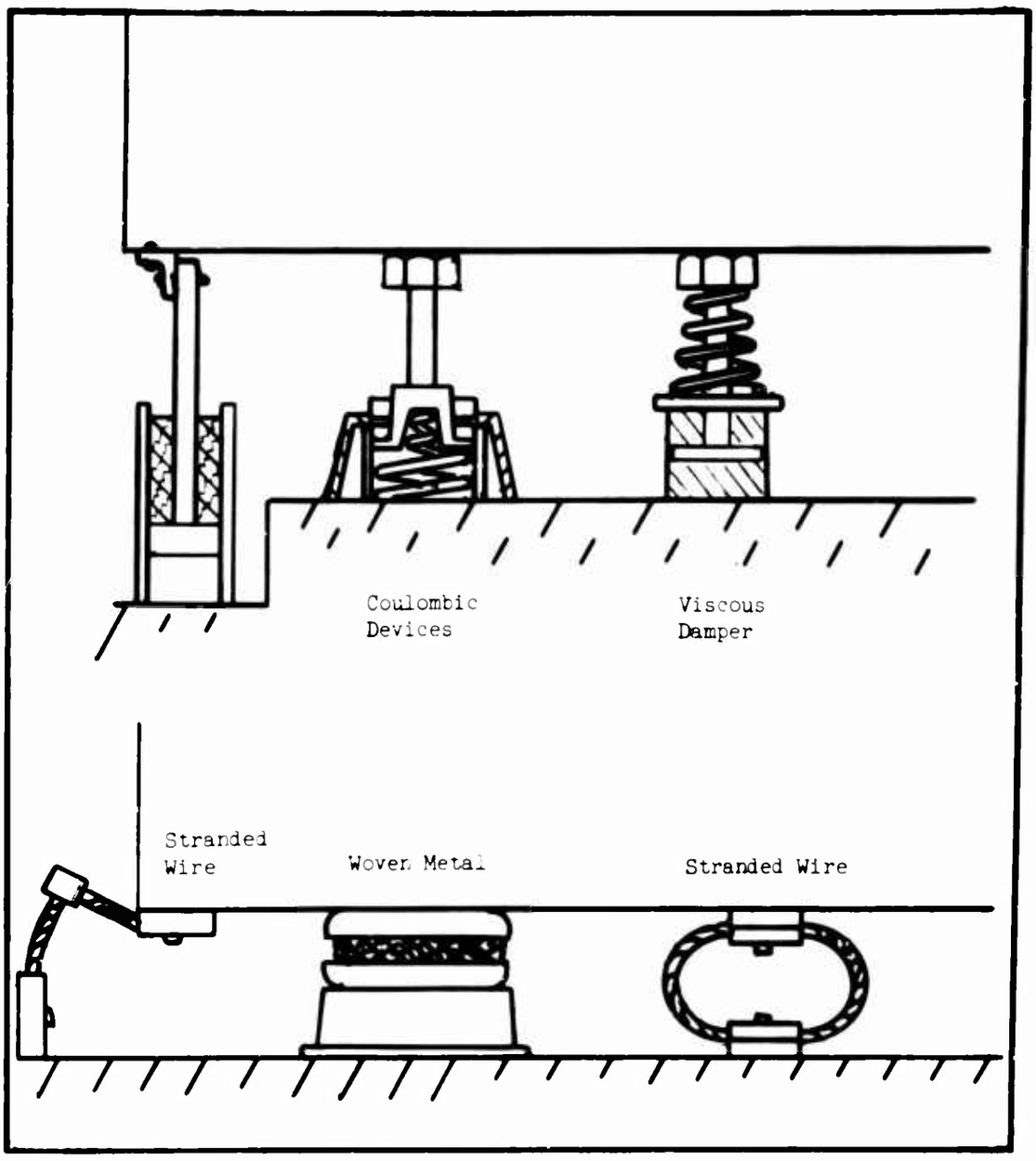
ISOLATION OF CHASSIS WITH HIGHLY DAMPED DEVICES

Chassis may be isolated with coulombic or viscous devices as long as certain restrictions and limitations are observed.

The chassis level of assembly is usually of such bulk and weight to warrent extra care in the application of various coulombic and viscous devices. The coulombic devices include the common dry-friction absorbers, the woven metal isolators, and the relatively new stranded-wire isolators. It must be noted that the dry friction devices will probably require the addition of a load bearing spring if one is not incorporated into the isolator design.

The woven metal and stranded-wire devices usually have a slightly higher resonant frequency but are usually as easy to employ as the dry-friction isolator. There is also a larger selection available than with the dry friction devices. The stranded wire has the advantage of being "tunable" by clipping the strands to lower the resonant frequency or softening the isolator.

Many viscous devices are available to isolate an assembly of bulk approximately the size of a chassis. The energy absorption capability of cylinders containing liquids can be altered by varying aperature size or by incorporating tapered pin constrictors within the aperture. It may be necessary to add a load spring to some soft isolators to prevent "sagging" of the load. In other instances, the springs are internal to the isolator. Other fluid devices which can be utilized are the pneumatic systems such as air bags, balloons, and inner tubes. These devices can be obtained in various degrees of sophistication and with varying amounts of damping.



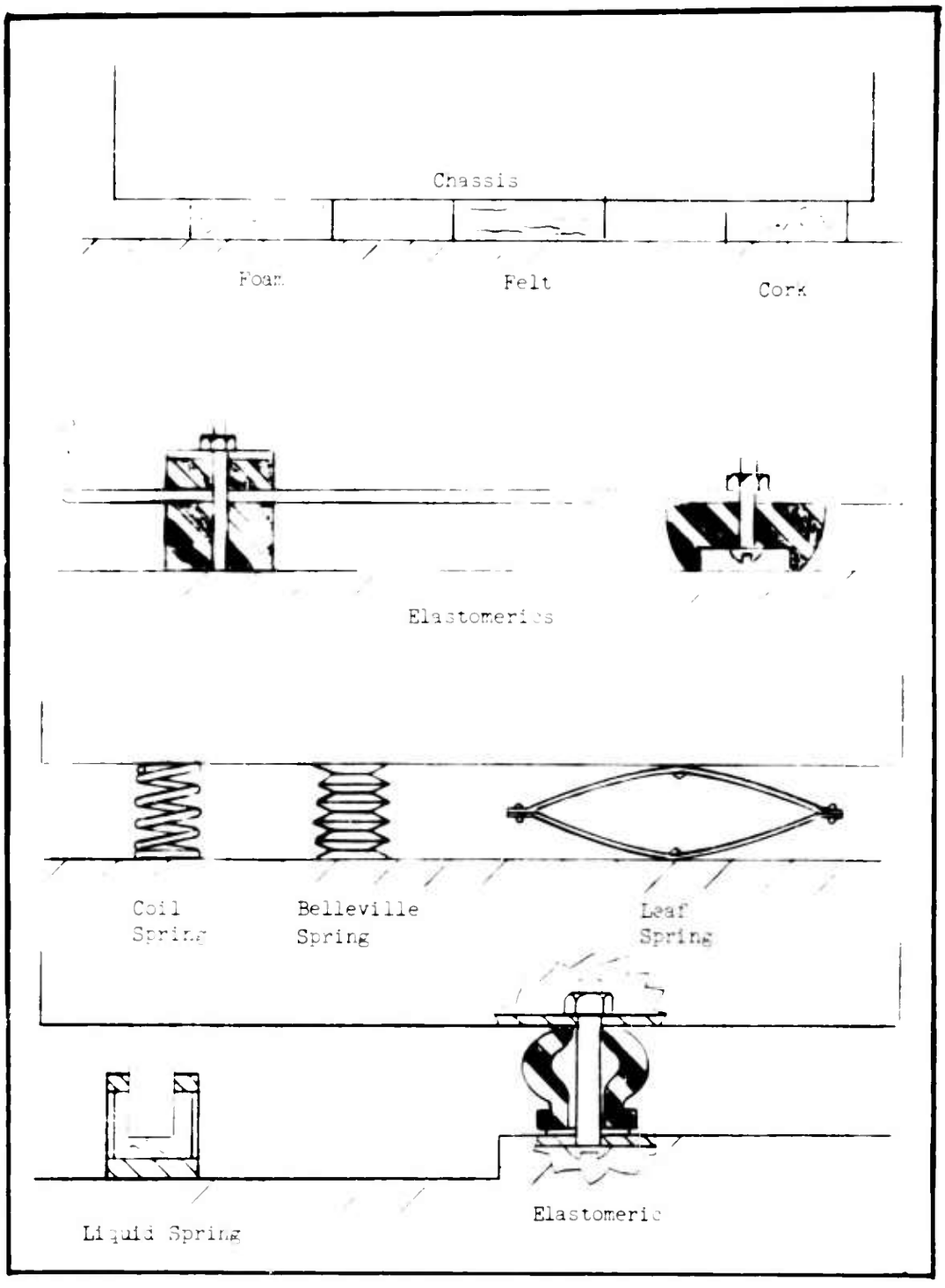
CHASSIS LEVEL ISOLATION: These devices are examples of the application of predominately damped isolators.

ISOLATION OF THE CHASSIS LEVEL WITH PREDOMINANTLY RESONANT DEVICES

The chassis level of assembly may also be effectively isolated with resilient devices exhibiting only small degrees of damping. The major attenuation results from the resonant characteristics of the system.

In addition to the coulombic and viscous damping devices previously mentioned, the other three classifications of attenuators contain hardware that can be utilized to varying degrees to this level of assembly. When the decision has been made to isolate some electronic equipment, then thought must be given concerning how the equipment will be used; what "class" the equipment is. In most cases the equipment will be subjected to negative "G" loading due to rebound during shock environments. Excursion must be restricted for these negative loadings. Should either elastomeric or composite pads be employed, then they must be firmly anchored. They must act in both tension and compression during excitation. An additional pad may be added that will receive the rebound compressive load. On the other hand, if the designer is only concerned with the isolation of the equipment after it has been placed in a structure where negative loads are not anticipated, the double mounting is not required. These last three classifications of attenuators rely more on the resonant properties of the system. Resonance often implies rebound.

Another problem which is present at this assembly level is space. The type of isolator selected and the load capacity of the attenuator determine the size of the device. This size adds to the volume occupied by the electronic equipment. Care should be exercised to insure that the overall volume does not exceed the maximum allowed.



CHASSIS LEVEL ISOLATION: The chassis can be isolated with devices which are predominantly resonant in nature.

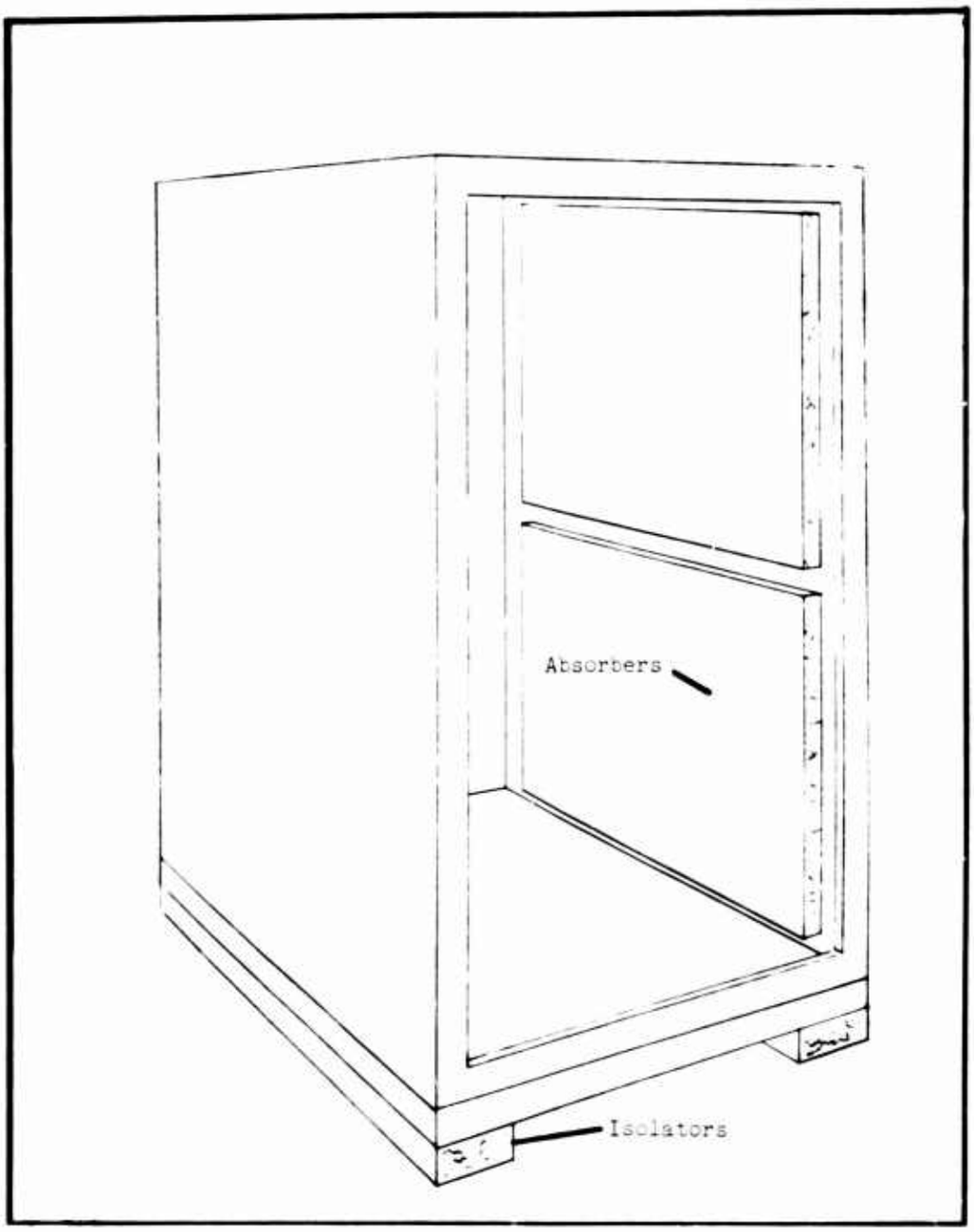
USE OF PADDING AT THE CONSOLE LEVEL

Several effective padding materials are available to the designer to reduce the dynamic energy transfer at the console level.

The large resonating panels sometimes present in conventional consoles may best be deadened by the use of composite padding, elastomeric sheet, or sandwiches containing combinations of these. The sandwich will absorb the most energy per unit weight, but weight or space limitations often restrict their selection. Some absorption is possible through the application of flocked foam, precast foam, cork, or undercoat. These approaches are usually incorporated for acoustic reasons rather than structural.

Composite and elastomeric pads can be employed externally as well as internally. Precast foams, cork, felt, and elastomeric pads are all available to the designer and are well worth consideration for electronic equipment isolation. Some of the materials, such as rigid, closed-pore foam, will crush under shock load and have to be replaced. Nevertheless, in some applications this solution might be advisable.

The high roll moment inherent in the taller (higher center of gravity) consoles will usually necessitate the use of a stabilizer toward the top of the console. This requirement is also present when the other kinds of isolators are used; that is, the centroid of the lines of action of the individual isolators should coincide as close as possible with the center of gravity of the equipment element.



THE USE OF PADS: Methods of reducing extraneous noise and dynamic excitations at the console level.

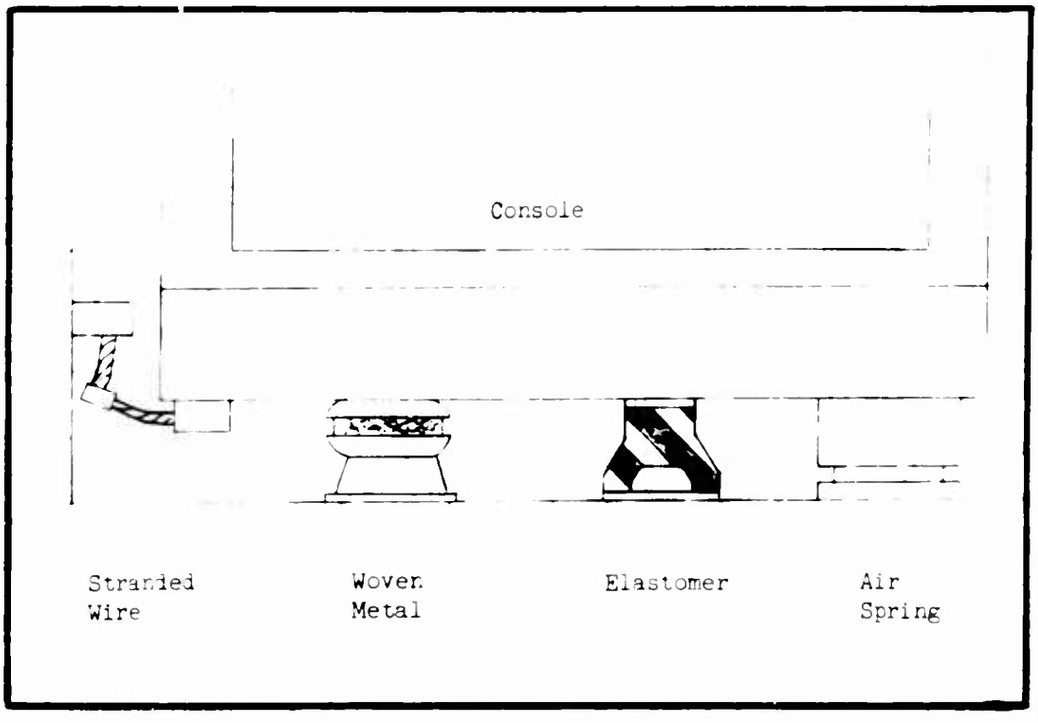
ISOLATION DEVICES FOR THE CONSOLE LEVEL

A great range of hardware is available for isolating consoles. The designer must choose the one which best fulfills his needs.

The designer may choose many from among the mechanical devices which will isolate a console, in addition to the composite and elastomeric pads previously mentioned. Due largely to the number of manufacturers making similar products, the first that draws attention is the elastomeric device. Smaller versions of this device have been mentioned in connection with isolation of subassemblies and sub-chassis which make up the console. When used at the console level, however, the elastomeric devices are essentially scaled-up versions of the previously discussed hardware. Since they are generally located outside the console, they are subjected to more dirt, oil, and other contaminants than those located on the subassemblies within the enclosure. This applies to any isolator used in this location.

Also of interest to the console level are the stranded wire isolators and woven metal devices. Caution should be exercised when applying the stranded wire devices as they are subject to fagigue. This effect, however, can be reduced by deliberately underloading them. With the larger consoles, an air spring can lower the resonant frequency of the system to a very few cycles. Additional damping may be added in parallel to reduce resonant excursion. Once again, if the console is tall and rocking modes can be visualized, some form of stabilizer will be indicated.

The old standby, the spring, is quite useful when combined with an appropriate damper. Care must be used by the designer to insure that the resonant frequency of the system differs from any excitation range. It must be remembered that the amplification at resonance is quite high unless sufficient damping is employed in the system. The rocking mode of oscillation may also be important when this type of attenuator is used. A stabilizer may be needed, as before, to minimize the overturning effect.



ISOLATING THE CONSOLE: The devices are similar to those normally used to isolate chassis level elements.

VOLUME III - CHAPTER 9

DYNAMIC ATTENUATION

SECTION 5 - APPENDIX

- Bibliography
- Glossary
- Attenuator Manufacturers
- Illustrative Problem on Attenuation

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GLOSSARY

Absorb – To receive the shock and vibration energy and convert a portion of the energy to heat e.g. a shock absorber.

Attenuate – To reduce the amount of energy transferred through a vibrating system by either absorption or isolation of the input disturbance.

Damping – The checking or lessening of a dynamic environment usually through the utilization of a contrivance.

Excursion – Dynamic displacement of a system.

Fatigue – The action which takes place in material, especially metals, causing deterioration and failure after a repetition of applied stress.

Fragility – The qualitative index of the acceleration load limit that equipment can withstand without damage.

Isolate – To prevent the shock or vibration energy from passing through as with a vibration isolator.

Resonance – The phenomenon shown by a vibrating system which responds with maximum amplitudes when excited by an applied force whose frequency is the same as the natural frequency of the vibrating system.

Resonant Frequency – The frequency at which a vibrating system resonates.

Shock – When the position of a system is significantly changed in a relatively short time in a non-periodic manner. It is characterized by suddenness and large displacement, and develops significant internal forces in the system. (1)

Structural Improvement – The redesign of electronic equipment so as to improve the survivability and reliability of the equipment when subjected to dynamic environments.

Transmissibility – The ratio of the energy transferred through an object to the energy applied. If the ratio is greater than one, amplification is occurring; if the ratio is less than one, attenuation is occurring.

Vibration – A mechanical oscillation or motion about a reference point of equilibrium. (2)

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ATTENUATOR MANUFACTURERS

AIR-LOCK PRODUCTS
FRANKLIN, MASS.
Vinyl-bonded cork & sizal sheet, pads

AMERICAN FELT COMPANY
GLENVILLE, CONN.
Felt pads, wool and synthetic fiber felt

ARNO ADHESIVE TAPES, INC.
MICHIGAN CITY, IND.
Low-density urethane foam tape

BARRY CONTROL
WATERTOWN 72, MASS.
Elastomer mounts, springs with friction air springs

BLOCKSON & COMPANY
MICHIGAN CITY, IND.
Bonded curled hair, sheet & molded

BUSHING INC.
ROYAL OAK, MICH.
Elastomer bushings & mounts

CLEVITE HARRIS PRODUCTS
MILAN, OHIO
Elastomer mounts with foam damping; tubular mounts and bushings

CONSOLIDATED KINETICS CORP.
COLUMBUS 15, OHIO
Glass fiber sheet, pads & mounts

DOW CORNING CORP.
MIDLAND, MICHIGAN
Elastomer sheet, liquid, foam

FILTERS CO., UNISORB DIV.
BOSTON 11, MASS.
High-density felt, sheet and pads

FIRESTONE INDUSTRIAL PROD. CO.
NOBLESVILLE, INC.
Air springs, elastomer mounts

GENERAL ELECTRIC CO.
SILICONE PRODUCTS DEPT.
WATERFORD, N. Y.
Elastomer sheet

GENERAL TIRE & RUBBER CO.
LOGANSPORT, IND.
Bonded and unbonded elastomer bushings & mounts; air springs

GIRDER PROCESS INC.
HACKENSACK, N.J.
Sponge rubber sheet & tape

HAMILTON KENT MFG. CO.
KENT, OHIO
Neoprene shear pads

HENRITE PRODUCTS CORP
MORRISTOWN, TENN.
Bonded plate, ring & tubular mounts

INTEGRATED DYNAMICS INC.
BUFFALO, N. Y.
Viscous shock absorbers

K. W. JOHNSON & CO., INC.
DAYTON 4, OHIO
Steel spring mounts, wire mesh damping

KORFUND DYNAMICS CORP.
WESTBURY, N. Y.
Cork & elastomer pads & mounts, steel springs with friction and wire mesh

LORD MANUFACTURING CO.
ERIE, PA.
Bonded elastomer plate, sandwich & tubular mounts; steel springs with friction

MB ELECTRONICS
NEW HAVEN 8, CONN.
Rubber & Neoprene pads; elastomer plate & shear mounts; steel springs

OHIO RUBBER COMPANY
WILLOUGHBY, OHIO
Bonded elastomer plate, sandwich & tubular mounts

ROBIN TECH INC.
BURBANK, CALIF.
Knitted mesh, steel spring with mesh & elastomer snubber, elastomer mounts

TECHNICAL WIRE PRODUCTS
CRANFORD, N. J.
Metal mesh

VIBRATION ELIMINATOR CO.
LONG ISLAND CITY 1, N. Y.
Cork, rubber & steel spring rails, bases

VIBRATION MOUNTING AND CONTROLS INC.
CORONA, N. Y.
Cork & bonded elastomer pads &
mounts. Steel springs with snubbing

VIBRO DYNAMICS CORP.
BROOKFIELD, ILL.
Elastomeric isolators

WESTERN FELT WORKS
CHICAGO 23, ILL.
Felt pads, wool & synthetic fiber felt

ZERO MANUFACTURING CO.
BURBANK, CALIFORNIA/MUNSON, MASS.
Polyurethane Foam Cushioning

ILLUSTRATIVE PROBLEM ON ATTENUATION

PROBLEM: Consider an electronics package as shown in Figure 1 with the indicated fragility curve which has been empirically determined. This package is to be used as Class III or Class VI (helicopter) equipment, depending upon the specific application chosen by various users. The environments which will be imposed upon this package are shown in Figures 2 and 3. The equipment designer's problem is to choose from those isolators available to him (for problem simplicity the two isolators of Figure 4) the proper isolator such that the equipment package will never experience a loading which exceeds the fragility curve limits.

SOLUTION: For ease of illustration, the example problem is limited to one-degree-of-freedom and the input excitations on the following pages are applied in the vertical direction. Likewise, this problem compares only two isolation systems, whereas in actual problems one may have to consider a much larger group of possible isolators.

From Figures 1 and 4, one can immediately calculate the basic parameters describing the two candidate isolation systems. Using four isolators with the C.G. symmetrically located gives:

$$f_{n1} = \frac{1}{2\pi}\sqrt{\frac{4(200)386}{80}} = 9.9 \text{ Hz}$$

$$f_{n2} = \frac{1}{2\pi}\sqrt{\frac{4(800)386}{80}} = 19.8 \text{ Hz}$$

In addition, from the transmissibility curves (Figure 4) we calculate:

$$Q_1 = 7.0 \quad \text{or} \quad \zeta_1 = 7.15 \text{ percent}$$

$$Q_2 = 5.0 \quad \text{or} \quad \zeta_2 = 10.0 \text{ percent}$$

Having the above parameters which describe the isolation systems, we proceed accordingly to the theory outlined in Section 3 of Volume II. The first step is to determine the composite (most severe) envelope for each type of excitation. Note that Class III does not have a sinusoidal environment specified nor does Class VI have a random input specification: therefore, the composite envelopes are made up from the random excitation of Class III (Figure 2), the sinusoidal inputs of Class VI (Figure 3 - helicopter), and a superposition of the shock spectra from Class III and VI which is shown in Figure 5.

The next calculations pertain to the actual determination of the anticipated responses of the two isolation systems to the imposed environment as given in Figures 2, 3, and 5.

Sinusoidal Response: In Section 3 of Volume II it is shown that the transmissibility of an isolator may be written (for the one-degree-of-freedom model) as;

$$T_f = \sqrt{\frac{1 + (2\zeta\omega/\Omega)^2}{\left[1 - (\omega/\Omega)^2\right]^2 + \left[2\zeta(\omega/\Omega)\right]^2}}$$

This equation, in fact, describes the two transmissibility curves given in Figure 4. Multiplying the transmissibility curve (point by point) times the sinusoidal input from Figure 3 determines the sinusoidal vibration level imposed on the package by the isolation systems (shown in the table below as values of E_s). These two results are superimposed on the same plot with the fragility curves, shown in Figure 1(b).

Freq. (Hz)	6	8	10	12	14	16	18	20	22	28	34	40
TR_1	1.57	2.75	6.93	2.03	1.30	0.63	0.45	0.34	0.27	0.15	0.10	0.08
TR_2	1.10	1.20	1.33	1.56	1.93	2.61	3.96	4.95	3.27	1.02	0.54	0.35
E_s	0.2	0.32	0.48	0.7	1.0	1.2	1.4	1.6	2.2	2.2	2.2	2.2

Random Response: Using Equation (1), Section 3, Volume II, Page 3-4, 3σ responses are:

$$G_{rms_1} = \sqrt{\pi/2\ (9.9)\ (7.0)\ (0.3)} = 5.7$$

$$\therefore G_{se_1}(3\sigma) = 3 \times 5.7 = 17.1 \text{ g's}$$

$$G_{rms_2} = \sqrt{\pi/2\ (19.8)\ (5.0)\ (0.3)} = 6.8$$

$$\therefore G_{se_2}(3\sigma) = 20.5 \text{ g's}$$

Note in Section 3 that the 3σ levels were taken to be the design peak levels.

Shock Response: From the shock spectra plot given in Figure 5 one finds that the response of these two systems to the specified shock input will be:

$$G_{se_1} = 11.5 \text{ g's} \quad \text{at } f_{n_1} = 9.9 \text{ Hz}$$

$$G_{se_2} = 53.5 \text{ g's} \quad \text{at } f_{n_2} = 19.8 \text{ Hz}$$

Moreover, converting these responses to pseudo response displacements (relative isolator displacement) gives:

$$X_1 = \frac{G_{se_1}}{0.1022\ f_n^2} = \frac{11.5}{0.1022\ (9.9)^2} = 1.15 \text{ in.}$$

$$X_2 = \frac{53.5}{0.1022\ (19.9)^2} = 1.32 \text{ inches}$$

Note that with system B the isolator will bottom out under shock which is not acceptable, however, the travel for isolator A is acceptable. It should be noted that the problem of isolator selection is most critical in in this particular area - allowable travel without bottoming.

Problem Summary: Having determined the response of the two isolation systems to the three environments, the remaining comparison is to determine if the fragility limits are exceeded in any instance. This comparison is shown in Figure 1(b). Note that isolator A will limit the excitations to the package below exceptable limits for all three environments. Isolator B, however, performs as desired only in the sinusoidal vibration mode with the random and shock responses being in excess of the allowable limits. Therefore, isolator A provides the acceptable system.

FIGURE 1(a) - EQUIPMENT PACKAGE

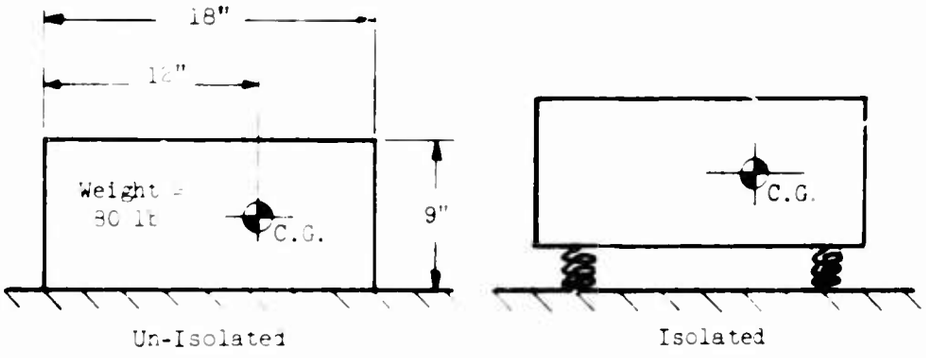
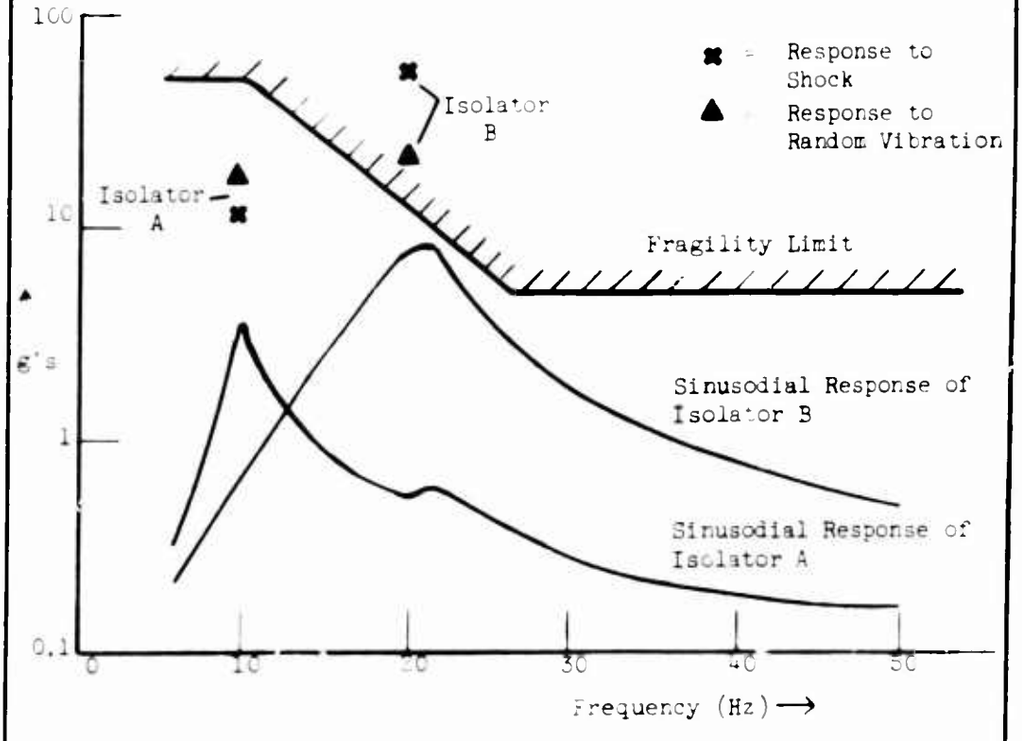


FIGURE 1(b) - FRAGILITY CURVE FOR PACKAGE



ILLUSTRATIVE PROBLEM ON ATTENUATION (Continued

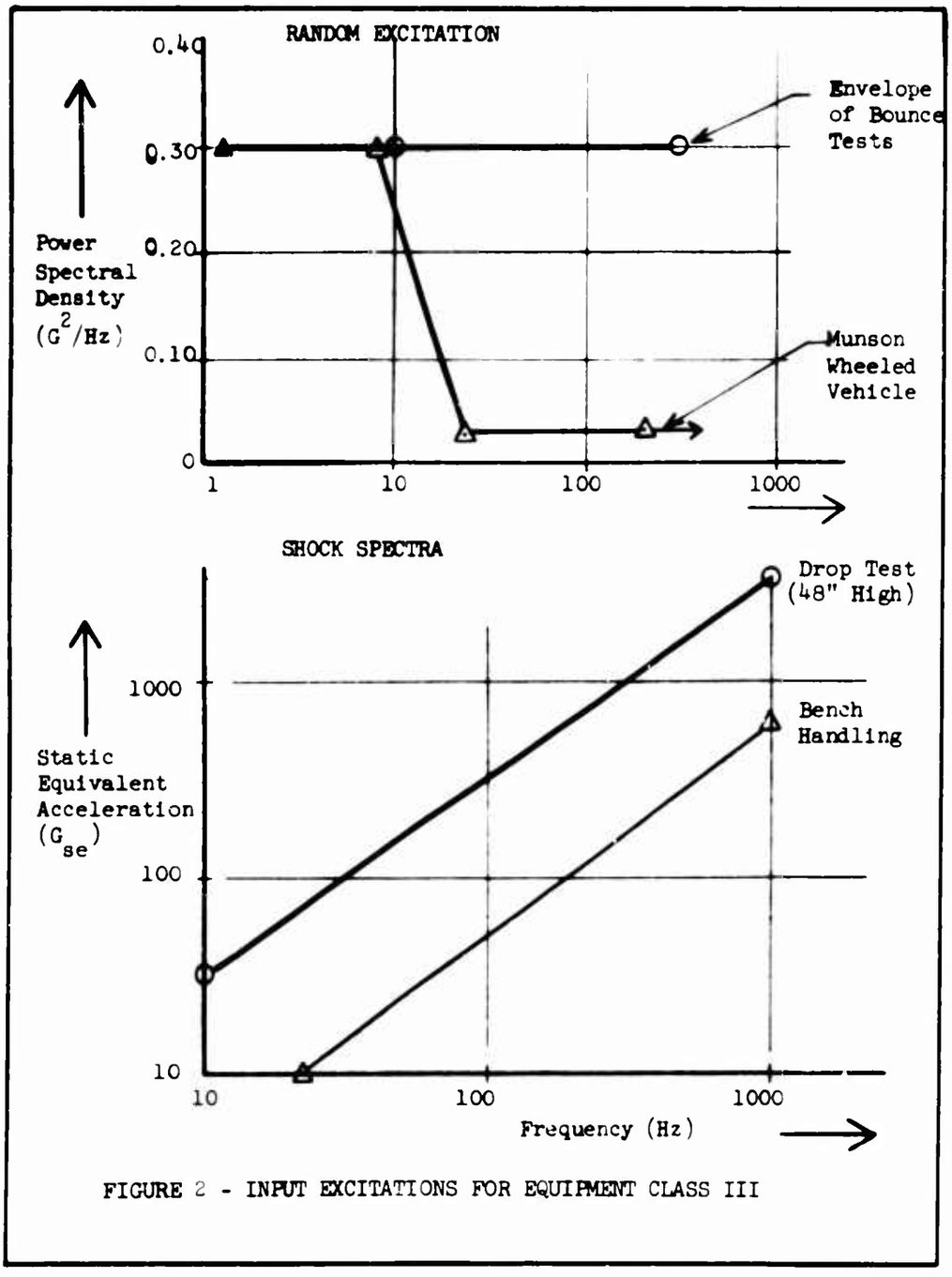


FIGURE 2 - INPUT EXCITATIONS FOR EQUIPMENT CLASS III

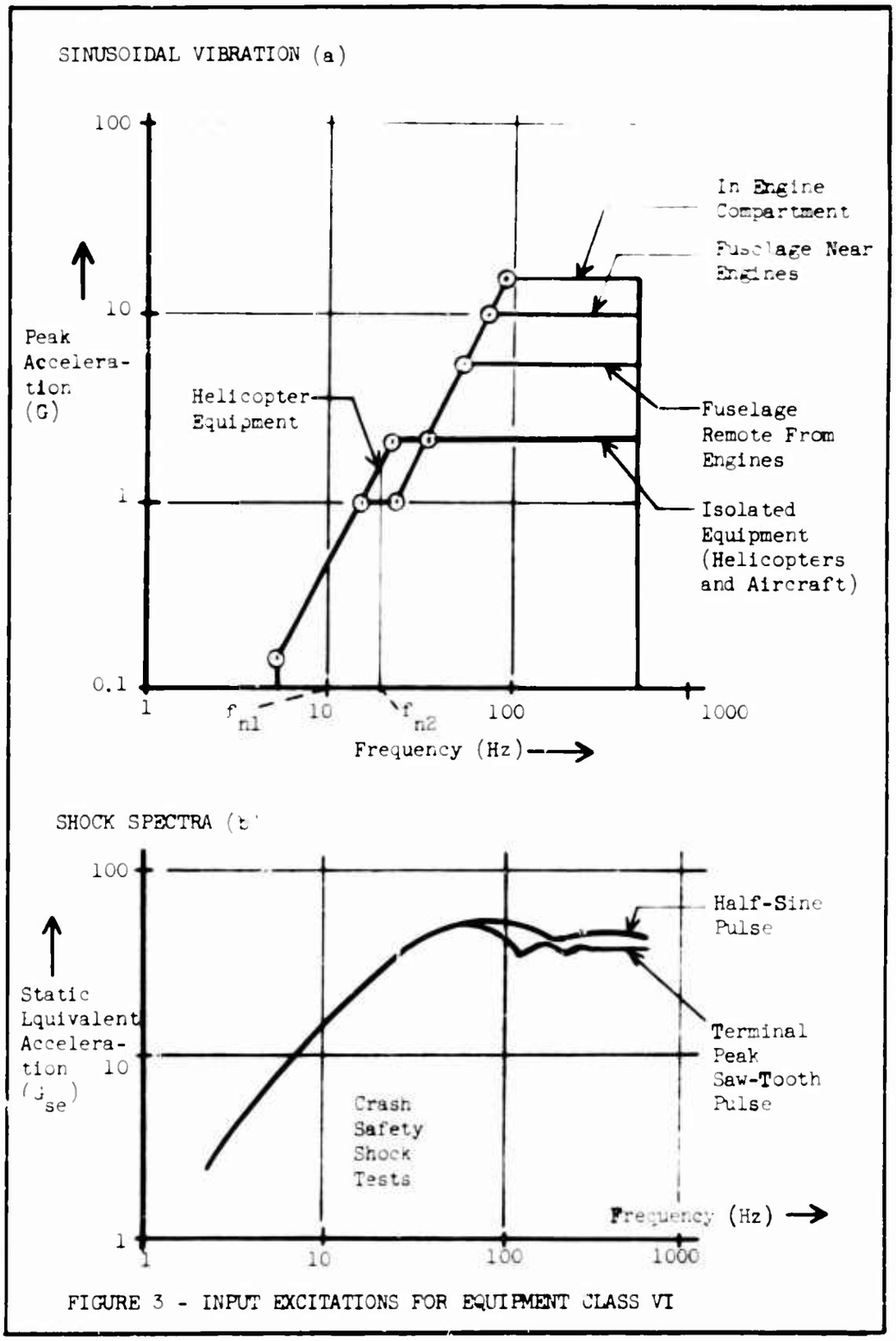
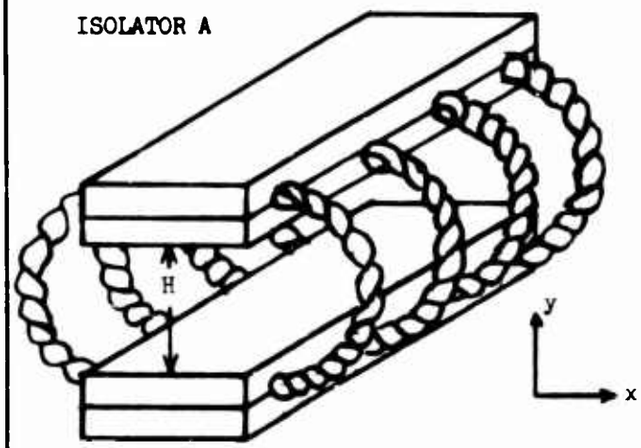


FIGURE 3 - INPUT EXCITATIONS FOR EQUIPMENT CLASS VI

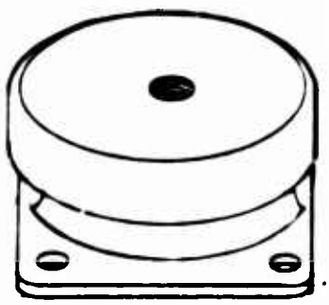
ILLUSTRATIVE PROBLEM ON ATTENUATION (Continued)



Aeroflex Steel Cable Type

k = 200 lb/in.
Clearance (H) = 1.5 in.

ISOLATOR B



Barry Cup Type

k_x = 800 lb/in.
Clearance (Vertical) = 7/16 in.

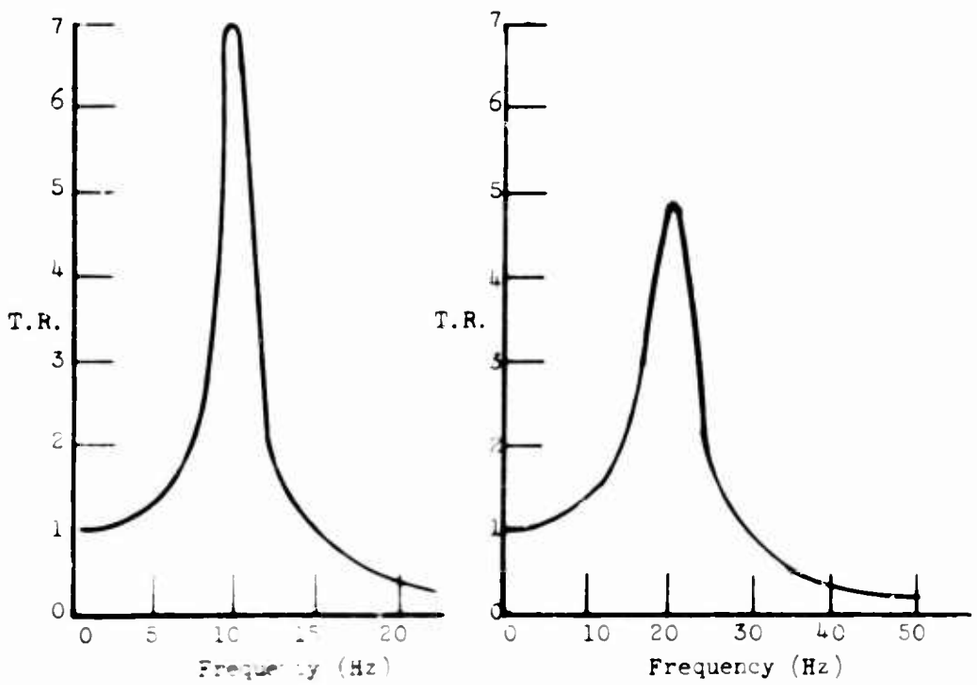


FIGURE 4 - ISOLATOR CHARACTERISTICS

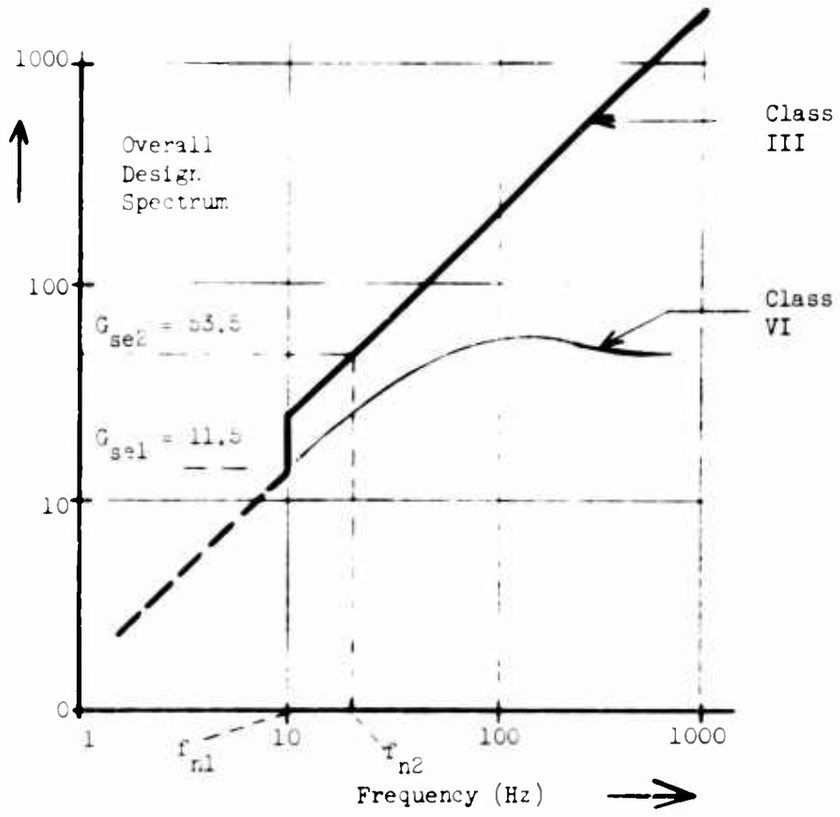


FIGURE 5 - SHOCK SPECTRA ENVELOPE

CHAPTER 10 – MATERIALS AND PROCESSES

CHAPTER 10
MATERIALS AND PROCESSES

ABSTRACT:

Balanced structural design of equipment constained to survive the Army shock and vibration environments involves the consideration of a material's resistive capability in the light of the environmental pressure; that is, the balance of environmental stress with dynamic structural integrity. Dynamic integrity of dynamic strength as it is used here, is the inherent load carrying capability of the structural material as modified by the particular manufacturing process employed to accomplish the package geometry. This dynamic strength quality of the engineering material also encompasses the properties of toughness and resilience; properties which are modified by the material processes employed. This chapter discusses some of the important aspects of this interrelationship - material capability-manufacturing procedures-material processes.....and their influence on the dynamic structural integrity of the equipment package.

Chapter 10 - Materials and Processes

ERRATA SHEET

Page	Paragraph	Line	Correction
Abstract	1	1	constrained
Abstract	1	5	...integrity or dynamic...
10.1-0	4	4	Failure in an...
10.1-0	4	5	Failure in a...
10.1-5	Caption	2	...heat treatment and cold work.
10.2-0	3	7	moduli
10.2-0	5	4	...high toughness
10.3-3	Graphic	4	...Environmental Reaction
10.4-0	2	9/10	Delete "as shown in the adjacent diagram."
10.4-1	3	1	Tungsten increases hardness,
10.4-1	4	3	...degree than does
10.4-1	7	2	as in the case of aluminum.
10.4-2	6	3	...may be accomplished by...
10.4-4	7	6	Delete "materially"
10.4-9	3	5	but require special...
10.4-10	1	3	Plastics are also referred...
10.4-11	Graphic	Various	Degree marks omitted (^{o}F or ^{o}C)
Sec 5 Divider	2	12	Plotting and Encapsulation
10.5-0	2	4	...to mechanical fastening...
10.5-4	Thesis	1	...the effect of...
10.5-8	2	1	Adhesion is accomplished by...
10.5-8	5	7	Little post-bonding finishing...
10.5-10	Thesis	2	...the as-cast material
10.5-10	1	3	antedates
10.5-10	2	8	amount and detail
10.5-12	1	2	...for situations which require...
10.5-13	1	3/4	(where the new surface oxidizes)
10.5-13	Graphic	8 & 10	Laminar
10.5-14	4	3/4	(like toothpaste...tube).
10.5-15	1	3	which exhibit...
10.5-16	3	3	titanium,
10.5-23	Graphic	Column 4	Flexural Strength (1000 psi)
10.5-23	Caption	2	...encapsulating materials for...
10.6-0		Ref. (12)	Delete "also see yearly, etc."

Chapter 10 - ERRATA SHEET - Cont.

Page	Paragraph	Line	Correction
10.6-0		Ref.(13)	Reference redundant with (19)
10.6-3	2	3/4	(True of certain ferrous metals.)
10.6-5	3	2	...held <u>at</u> temperature
10.6-7	Title		Add "CHARACTERISTICS OF COMMON CASTING PROCESSES"

VOLUME III - CHAPTER 10
MATERIALS AND PROCESSES

CONTENTS

Section		Page
1	INTRODUCTION	10.1-0
	• Strength as a Function of Material Characteristics and Fabrication Processes	10.1-0
	• Design Implications of the Physical Properties of Materials	10.1-3
	• The Importance of Fabrication Techniques to Effective Equipment Packaging	10.1-4
2	THE PHYSICAL PROPERTIES OF ENGINEERING MATERIALS	10.2-0
	• The Classic Stress-Strain Curve as an Indicator of Material Properties	10.2-0
	• Material Properties Derived from the Fatigue Test	10.2-2
	• Design Properties of Engineering Materials Developed from Impact Tests	10.2-4
	• Metallurgical Mechanisms and Their Importance to Material Properties	10.2-6
	• The Influence of Surface Effects on Material Properties	10.2-8
	• Considerations of Wear and the Application of Engineering Materials	10.2-10
3	MATERIAL CHARACTERISTICS DEFINITION	10.3-0
	• A Summary of Important Physical Properties . . .	10.3-0
	• Desirable Metallurgical Characteristics for Materials Used in Dynamic Applications	10.3-3
	• The Influence of Alloying Elements on Metallic Material Properties	10.3-4
4	MATERIAL SELECTION CRITERIA	10.4-0
	• Alloying Elements in the Ferrous Materials . . .	10.4-0
	• Physical Properties of the Ferrous Alloys . . .	10.4-2
	• Characteristics of the Aluminum Alloys	10.4-4
	• Mechanical Properties of the Magnesium Alloys. .	10.4-6
	• Characteristics of Beryllium and Titanium Alloys	10.4-7
	• Characteristics of Copper and Its Alloys	10.4-8
	• Properties of Some High Temperature Alloys . . .	10.4-9
	• Characteristics of the Common Plastics and Fabricated Materials	10.4-10

CONTENTS (Continued)

Section		Page
5	MANUFACTURING PROCESSES AND PROCEDURES	10.5-0
	• Manufacturing Techniques Applied to Equipment Structure	10.5-0
	• Joining by Mechanical Fabrication	10.5-2
	• Welding: A Fusion Joining Process	10.5-4
	• Brazing and Soldering	10.5-6
	• Structural Joining by Adhesive Bonding	10.5-8
	• Forming by Casting	10.5-10
	• Forming Procedures Involving Material Removal. .	10.5-12
	• Structural Forming by Material Deformation . . .	10.5-14
	• Heat Treatment: A Material Process to Improve Mechanical Properties	10.5-16
	• Surface Hardening Techniques	10.5-18
	• Plating and Chemical Finishing	10.5-20
	• Potting and Encapsulation	10.5-22
6	APPENDIX	10.6-0
	• Bibliography	10.6-0
	• Glossary	10.6-2
	• Review of Principal Casting Processes	10.6-7
	• Steel Alloys	10.6-8
	• Stainless Steel Alloys	10.6-10
	• Aluminum Alloys	10.6-12
	• Special Aluminum Applications	10.6-16
	• Magnesium Alloys	10.6-17
	• Copper Alloys.	10.6-20
	• Nickel Alloys	10.6-22
	• Miscellaneous	10.6-23
	• Plastics	10.6-25

VOLUME III - CHAPTER 10

MATERIALS AND PROCESSES

SECTION 1 - INTRODUCTION

- Strength as a Function of Material Characteristics and Fabrication Processes
- Design Implications of the Physical Properties of Materials
- The Importance of Fabrication Techniques to Effective Equipment Packaging

STRENGTH AS A FUNCTION OF MATERIAL CHARACTERISTICS AND FABRICATION PROCESSES

The evaluation of the dynamic structural integrity in an electronic equipment package is the process of comparing imposed stress and energy with inherent strength and toughness. The upper limit of dynamic strength is the ultimate resistive capability of the structural material; the fabrication process may enhance or diminish these values.

Balanced design for dynamic structural integrity in an electronic equipment element is a comparative process. That is, the evaluation of the loads induced within the equipment from the dynamic environment with respect to the inherent resistance capability of the structural package. The dynamic stress induced in the structure is a function of the input excitations as modified by the structural response of the package. Response is known to be particularly sensitive to natural frequency and damping characteristics. In addition, some residual locked-in stresses often result from thermal differences encountered during fabrication.

The strength capability of the equipment package conversely, is determined by the intrinsic resistance of the material to dynamic loads, as influenced by the fabrication technique employed to construct the package. The designer's task then, is the comparison of strength capability with environmental input and evaluation of the resultant safety margin. Obviously, equal attention must be given both sides of the equation to gain a high probability of success.

Volume II, "Analytical Procedures" treats the problem of inputs, transfer functions, and structural responses. This chapter will outline the important details of the material and fabrication processes as relates to their toughness and strength characteristics.

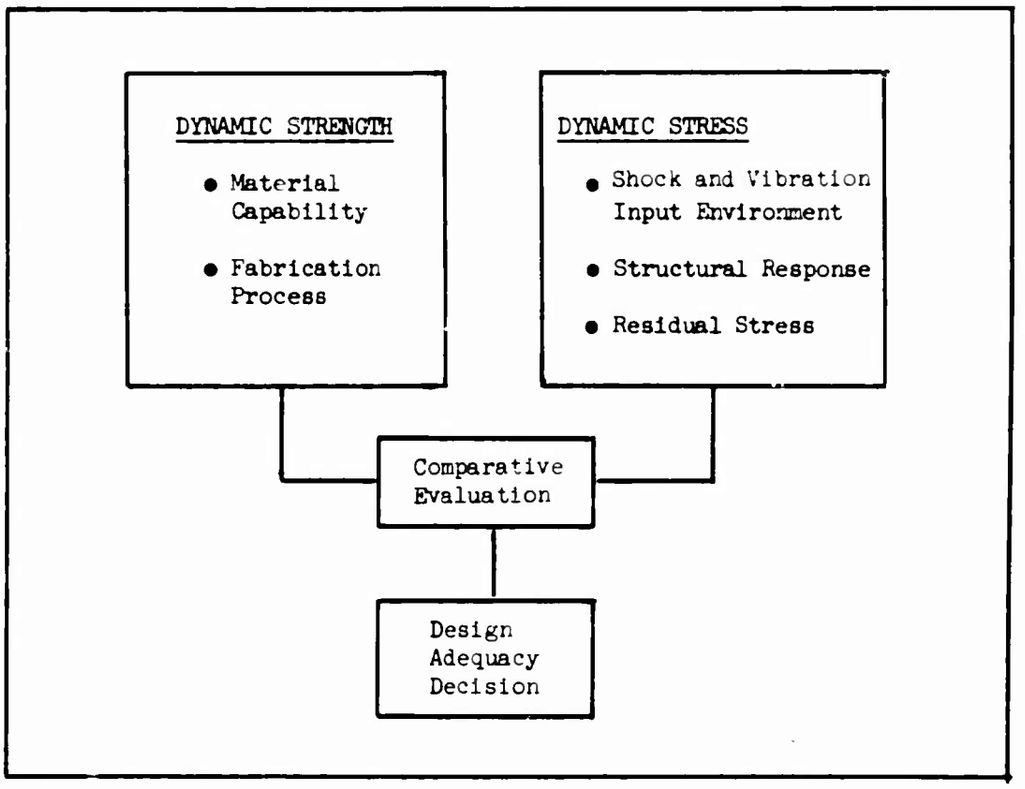
Strength may be considered to be the ultimate or highest load resistance capability of a material at the instant just prior to failure. Thus, strength must be defined, or at least bounded, by a firm definition of failure. Failure is an electronic equipment package is often a functional interruption. Failure in mechanical system may be the fracture of the material; an excessive excursion or elastic deformation; or a plastic deformation which places the components out of orientation.

The material characteristics of importance to the electronic equipment packaging Engineer are all those characteristics which influence the material's resistance to failure. These characteristics are classically measured in the Materials Laboratory by tensile tests, fatigue tests, various impact tests, and assessment of certain physical properties such as conductivity, hardness, micro-structure, and chemical composition.

The fabrication techniques that influence dynamic structural integrity are those which have an effect on the inherent fracture resistance of the basic structure. In simple approximation, most fabrication processes tend to diminish the basic material strength or toughness; an example of this type of fabrication procedure is a riveted connection. The material in the joint is removed to permit installation of the fastener. The discontinuity also creates a load magnification due to stress concentration. The net effect is then generally deleterious to the parent material, and some increase in material bulk is indicated to compensate for the increased stress.

Fabrication techniques may also have an important influence on dynamic integrity where failures get built into the structure, such as a non-penetrated weld or a heat effected zone. Proper balance in the selection of fabrication process for a given situation can, however, improve the long-time material strength and toughness.

Material finish affects both equipment strength and appearance. An example is the coating or surface protection afforded by various cases, clads, and finishes used in the electronic packaging industry. The effect is to insulate the parent material from the corrosive environment, and hence protect the surface and minimize fatigue failures emanating from the surface. It is apparent that surface finish considerations are critical to proper selection of the fabrication process, particularly in fatigue situations.



DYNAMIC STRESS AND STRENGTH: Material capabilities, factored by the limitations of the fabrication process, are the basis of dynamic strength which must compare favorably with dynamic stress.

DESIGN IMPLICATIONS OF THE PHYSICAL PROPERTIES OF MATERIALS

A working knowledge of the physical properties of engineering material is the basis for good design. These physical characteristics of importance are derived from a series of standard tests as well as the assessment of the material's metallurgical properties.

The discipline of good material selection habits for the structural design engineer begins with a thorough understanding of the more important physical characteristics of the materials themselves. Important, that is, to the structural dynamic integrity of equipment package and its support structure. The design task is one of matching those desirable characteristics known to have an influence on the material response to the dynamic environment, with the proper choices available to the designer. These characteristics include selection of the best material group, the selection of the optimum alloy within that group, the choice of optimum heat treatment for the alloy, consideration of its susceptibility to other environmental influences, designing the most efficient method of joining and fabricating the structure from the alloy, and the choices of surface finish. The degree of skill that the designer can muster in dealing with these important choices (and there are many more than are listed) will to a large extent, dictate the ultimate capability of the package to insulate the dynamic environment away from its fragile components.

The traditional stress-strain plot which yields many of the familiar physical material properties, results from a rather straightforward tensile test; a testing technique familiar to the metallurgist for many years. Parallel information is also available from an assortment of simple tests, similar to the tensile test. Load-deflection plots obtained from standard shear, bending, torsion, and compression tests heavily influence the selection of the manufacturing technique for a given material.

The importance of these physical properties to the equipment packaging designer lies in their influence on the material resistance to dynamic loadings. The correlation of static tensile properties with fatigue resistance in structural materials for example, is not always linear. The comparison of impact resistance with these static properties, however, is more direct, and a conservative analysis may be made on the basis of equivalent static load or acceleration, using the static properties from the stress-strain curve.

There is a range of other tests for physical properties worthy of note to the structural engineer, many of which are directed toward specific dynamic characteristics. Fatigue tests for example are the basis of an entire science of physical metallurgy of materials subjected to repetitive loads. Properties obtained from the fatigue tests include endurance limit, fatigue strength for a finite number of load iterations, notch sensitivity, and life expectancy.

Many of the standard impact tests yield data on material strength directly applicable to the shock impact problem. Energy-to-failure information may be obtained from the Charpy-Izod type impact tests for specimens with various notches(8). Other impact tests have been devised to obtain material resistance data for a variety of loading situations, such as torsion, tension, and bending (9). The design task is one of matching the type of impulsive loading anticipated in a member with the available test data, for a given engineering material.

Many of the other important characteristics of interest to the designer are inherent metallurgical properties of the material. Data on damping, isotrophy, creep resistance, corrosion resistance, hardness and hardenability, among many others, must be factored into the final decision as to be used in a given equipment packaging situation. This chapter will outline the qualitative aspects of this decision matrix, and provide some ground rules for the application of common engineering materials and fabrication processes to the equipment packaging problem.

PERTINENT LABORATORY TESTS FOR MATERIAL CHARACTERISTICS

- Tension, compression
- Bending, torsion
- Impact, fatigue
- Wear, corrosion
- Hardness, hardenability

PHYSICAL PROPERTIES OF IMPORTANCE TO THE DYNAMIC INTEGRITY OF A STRUCTURE

- Toughness, strength, ductility
- Stiffness, resilience
- Isotropy, homogeniety
- Environmental resistance
- Creep resistance
- Damping ability
- Heat treat capability

MATERIALS TESTING: Laboratory tests conducted under controlled conditions yield useful data on a material's resistance to impulsive and repetitive loadings.

THE IMPORTANCE OF FABRICATION TECHNIQUES TO EFFECTIVE EQUIPMENT PACKAGING

The fabrication processes employed in an equipment package serve to limit the inherent material capability. Fabrication techniques include joining, forming and surface treatment and are closely related to material properties and heat treatment selection criteria.

Effective structural design for electronic equipment packages is the proper balance of material characteristics and optimum fabrication processes. If material properties may be likened to heredity in human development, then the fabrication procedure is analogous to the environmental limitations. The construction technique may at best develop only the inherent strength capability of the basic material; at worst, the process may limit the strength of the fabricated structure to a small fraction of its potential. The fabrication technique employed to package electronic components must be selected to enhance and protect the base material as much as practical. The designer must match the two characteristics to optimize the structural integrity of the package.

Fabrication processes include the functions of forming and joining base materials into an an acceptable package. The technique also interfaces the discipline of component location due to function, cooling, access, fragility, and other system constraints. Fabrication processing also includes consideration for material heat treatment and surface finish, and appearance of the final product.

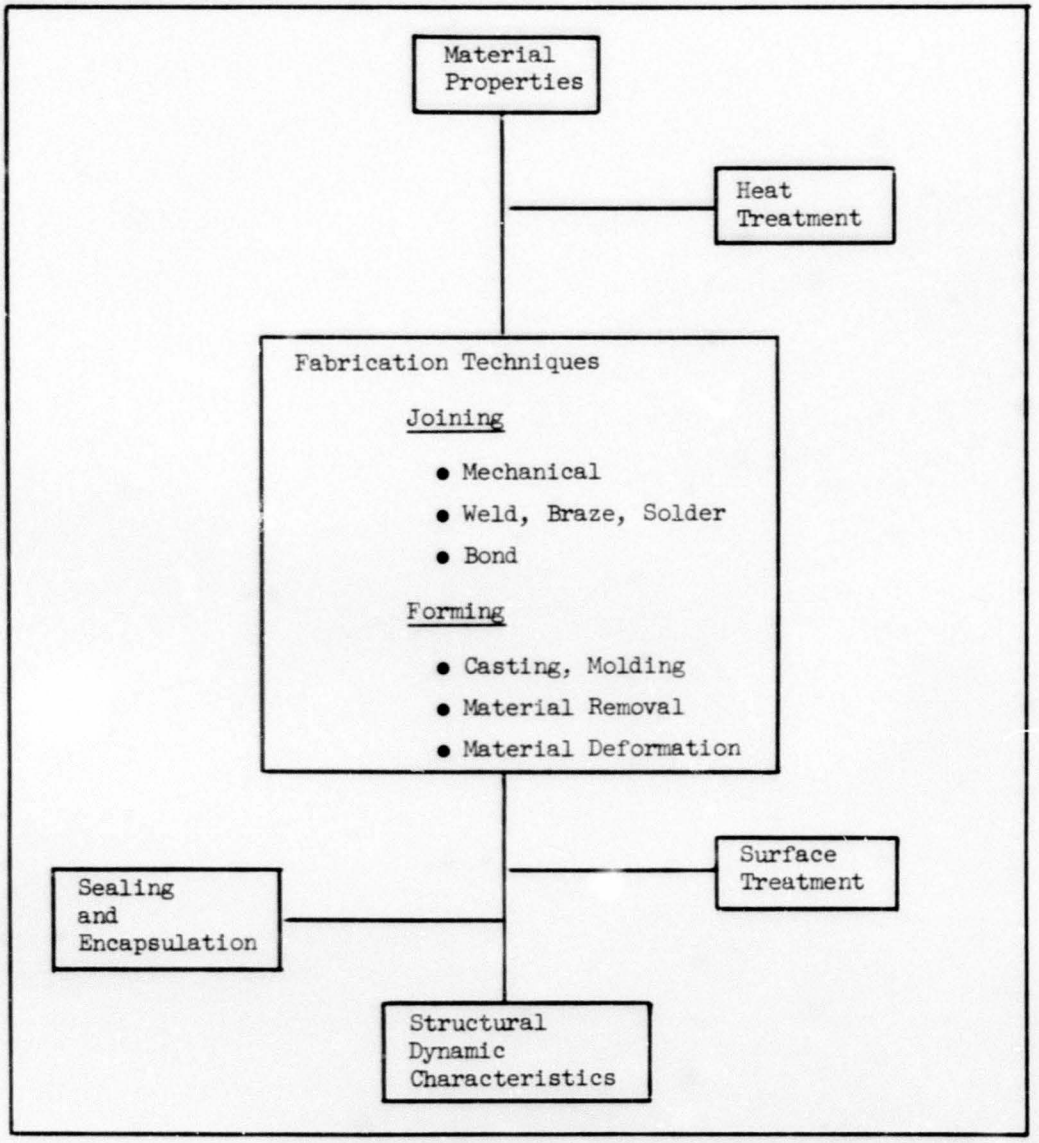
Joining, as a subdivision of the fabrication processes, may be further broken down into three main categories: joining by mechanical means; joining by welding, brazing, or soldering; and joining by adhesives.

Forming, the second fabrication process category, may also be subdivided into three groups: forming by casting and molding; forming by material removal; and forming by material deformation.

Design decisions concerning the fabrication processes of joining and forming are incomplete without due consideration for the material's inherent capability to have the process performed upon it. Obviously, some materials are more readily machined, or cast, or welded, than others. In fact, great differences in fabrication ease exist between alloys of the same material group. A case in point is the range of weldability of the aluminum alloys. Thus, consideration for the formability and joinability of the material contemplated for use in a given structure must be evaluated before the final decision on fabrication technique can be made. These characteristics are really material properties, but serve to emphasize the necessary balance between materials and processes in good design.

Surface finish is another example of how material properties heavily influence the characteristics of the manufacturing technique. The surface properties of the finished structure are vitally important to dynamic integrity since most fatigue failures emanate from surface discontinuities or anamolies. The susceptibility of the material to scratches, blemishes, pits and surface decarburization will dictate to the designer the optimum material for a given manufacturing process. Similarly, the ability of the material to be formed without tearing or chipping will influence the selection of best forming and joining process for a given engineering material.

Surface protection is also important to the equipment structure in the service environment. The designer must select a surface finishing process that will protect and enhance the inherent physical properties of the structural material. This is particularly true of corrosive environments where surface pitting may provide the nucleus for a fatigue crack. Material removal by corrosion may cause a functional failure in service long before the design life of the equipment has been reached. Proper surface treatment thus serves to protect the properties inherent in the material.



DESIGNING FOR STRUCTURAL INTEGRITY: Strength begins with the inherent physical properties of the material as developed by heat treatment. The fabrication process, sealing provisions, and surface treatment enhance and protect these properties.

VOLUME III - CHAPTER 10

MATERIALS AND PROCESSES

SECTION 2 - THE PHYSICAL PROPERTIES OF ENGINEERING MATERIALS

- The Classic Stress-Strain Curve as an Indicator of Material Properties
- Material Properties Derived From the Fatigue Test
- Design Properties of Engineering Materials Developed From Impact Tests
- Metallurgical Mechanisms and Their Importance to Material Properties
- The Influence of Surface Effects on Material Properties
- Considerations of Wear and the Application of Engineering Materials

THE CLASSIC STRESS-STRAIN CURVE AS AN INDICATOR OF MATERIAL PROPERTIES

The physical properties derived from the tensile stress-strain plot have long been used as a measure of the important material characteristics. Some of these characteristics correlate well with the dynamic properties of interest to the packaging designer.

The standard tensile test is performed on specimens with a two-inch gage length, and a diameter of about one half inch. The specimens are stressed in tension and the independent variable is the increment of applied tensile force. The extension over the two inch length is measured for each force interval, and subsequent stress-strain plot made. The stress is classically calculated on the basis of original cross-sectional area. The adjacent figure illustrates a typical stress-strain plot for a ductile material tested in tension, and delineates the physical characteristics which may be measured from it.

Many of the familiar material physical properties are derived from the stress-strain plot. These material characteristics include stiffness (measured by the elastic modulus, or slope of the elastic portion of the plot); toughness (proportional to the area under the entire stress strain curve); resilience (a measure of a material's ability to store and release energy, proportional to the area under the elastic portion of the curve); ductility (measured by the total extension of the test specimen to failure); and a variety of strength values, such as ultimate strength, elastic limit, yield strength and proportional limit.

Stiffness, as measured by the elastic modulus, is perhaps the most important of the tensile properties. Since natural frequency of a structural member is proportional to the square-root of the spring constant of the system, a significant range of values is available to the designer. The elastic modulus is the spring rate of the material since it relates the rate of deflection of the material to the stressing force. A glance at a table of elastic modulii for the common structural materials (5) reveals;

Magnesium alloys	$E = 6.5 \times 10^6$ psi
Cast irons and steels	$E = 10\text{-}20 \times 10^6$ psi
Titanium alloys	$E = 16 \times 10^6$ psi
Aluminum alloys	$E = 10 \times 10^6$ psi
Steel alloys	$E = 30 \times 10^6$ psi

Thus the designer may vary the natural frequency of a structure by $\sqrt{30/6.5}$, or about two, merely by sutstituting steel for magnesium and without changing the geometry of the member.

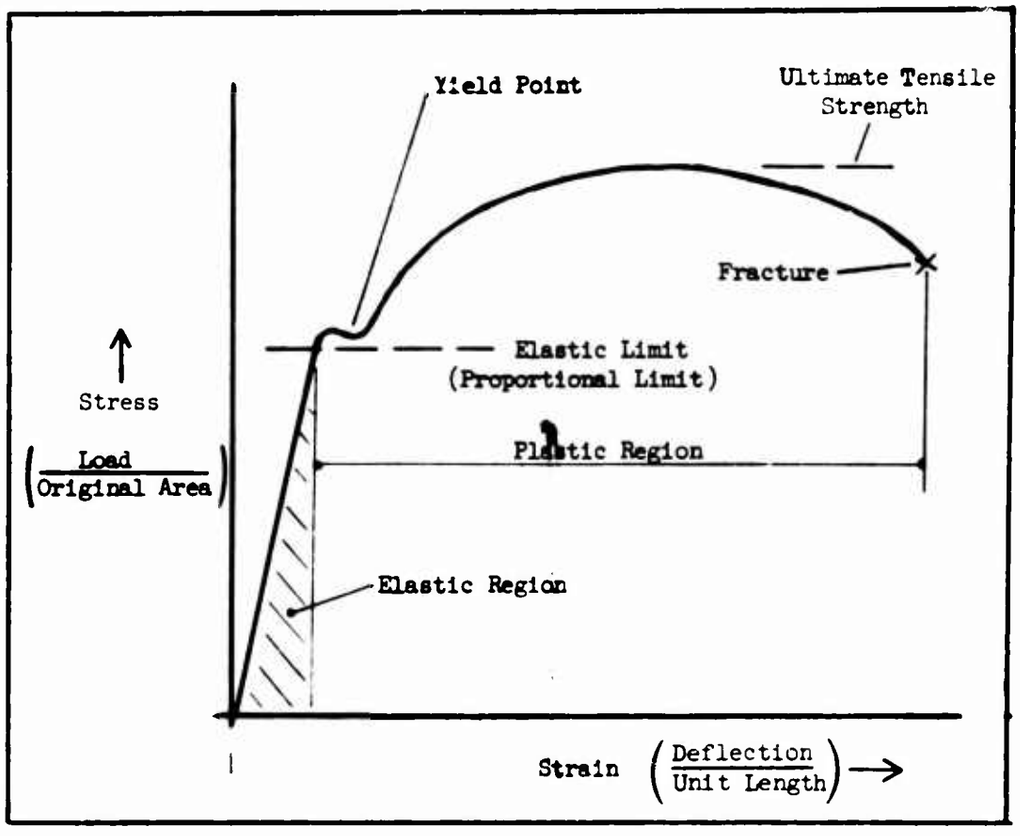
The modulus of resilience also reflects the stiffness parameter, as well as the yield strength of the material, since it is proportional to the area under the elastic portion of the curve. Since most acceptance criterion requires no appreciable permanent set after stressing, a high modulus of resilience is desirable in most structural applications.

The importance of ductility and toughness to structural packages is often not completely understood. Both of these material parameters are dependent upon the extension of the material at rupture. A high percent elongation means high ductility and high touchness modulus (for the same strength levels). The deformation involved is almost completely plastic; thus, nominal failure of the package may have already occurred due to excessive plastic excursion.

The toughness property does, however, provide a measure of the energy required to cause rupture of the material, a factor of importance to structure constrained to remain intact after impact.

The relationship of material tensile strength to fatigue strength is not always linear. Comparisons of fatigue strength with ultimate tensile strength for most steel alloys appears to be quite predictable, while the same is not true for the common aluminum alloys (3). The magnesium alloys and the copper, copper-base, and nickel-base alloys exhibit a linear relationship between fatigue and tensile strengths, but with a large scattering of results.

In general, tensile strength (implying toughness) is a reasonable estimate of resistance to impact loads. For most common structural materials, the variation is direct. In fact, increases in loading rate or strain rate are known to increase the apparent yield and ultimate strength of common materials (7). This is particularly true of the ductile alloys, such as copper, magnesium, aluminum alloys, irons and most steel alloys.



THE STRESS-STRAIN CURVE: The physical properties derived from this classic plot define many important material characteristics.

MATERIAL PROPERTIES DERIVED FROM THE FATIGUE TEST

Fatigue test data, cumulative and probable damage criteria, and early crack detection are the most effective design tools for dealing with repetitive stress situations.

Material failure from vibration excitations is usually a fatigue fracture which propagates from a scratch, crack, joint or area of high stress concentration due to a geometric discontinuity. Other types of fatigue failures may be caused by resonant ringing of structure which has been excited by impact. In either case, the physical characteristics which describe a material's ability to withstand these repetitive excitations are fatigue properties and relative values may be extracted from fatigue test data.

Fatigue tests are commonly conducted on specimens containing varying notches and surface finishes. The familiar rotating beam test provides stresses that are completely reversed in bending; the outer fiber of the specimen experiences stresses ranging from 100 percent tension to 100 percent compression. A variation of this test is also available where the amount of stress reversal is investigated through a range of stress ratios. Thus data is available on bending fatigue for virtually any mean stress and fluctuating stress level. The concept of stress ratio is illustrated in the accompanying figure.

Other fatigue tests have been devised to duplicate a variety of loading modes, such as torsion, axial loads, or combinations of stressing methods. In addition, standard tests are conducted on notched and unnotched specimens to evaluate the effect of stress raisers in the test material. This gives rise to the notch sensitivity parameter, which is the ratio of endurance limits for plain versus notched specimens. This is a very important material characteristic which correlates material properties with the geometric stress concentration factor and fatigue strength reduction factor(5).

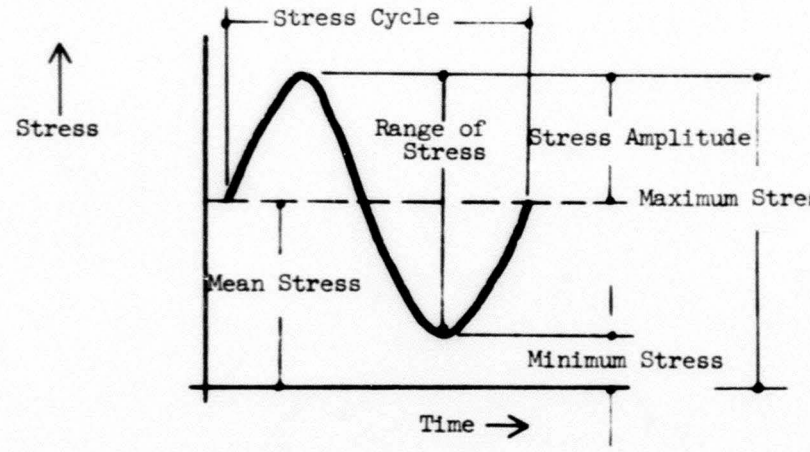
The data collected during a fatigue test is customarily plotted into a stress versus number-of-stress-iterations format. This is the familiar S-N curve, illustrated in the adjacent figure, and is the basis for the physical parameters assocated with fatigue.

Two concepts of interest to the designer are probable damage and cumulative damage of engineering materials subjected to repetitive stress. The probable damage region of the S-N curve, illustrated at right, means that any stress cycles experienced within that region will probably result in some degradation of the material's mean fatigue curve, and hence endurance limit. Cumulative damage theory states that the summation of overstress (i.e., above the material endurance limit) experiences is predictable. This theory, first stated by Minor(10), may be written analytically:

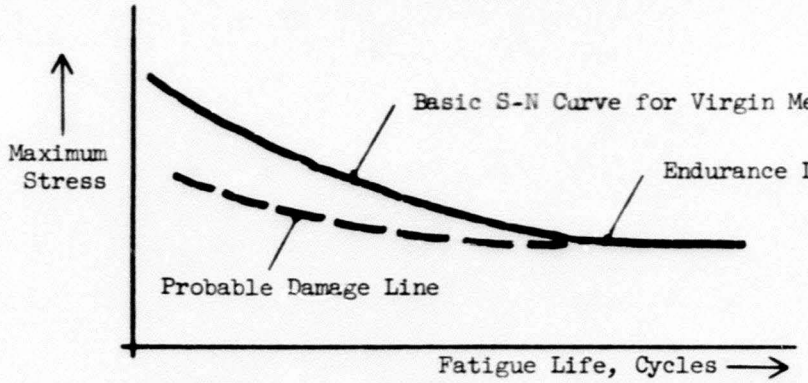
$$\Sigma_i \left(\frac{n_i}{N_i} \right) = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} \quad . \; . \; . \; = 1$$

where n_1, n_2, etc. represent the number of stress cycles at specific overstress levels, and N_1, N_2, etc. represent the mean fatigue life at these stress levels. This concept enables the designer to closely approximate the anticipated experiences of repeated stress in the equipment package, even if the summation is not exactly unity.

A major drawback in the evaluation of fatigue problems in existing structures is the early detection of fatigue cracks. Fatigue failures, unlike elastic fracture, will give warning of an incipient fatigue fracture only if intense and continuous observation of the structure is practical. There are a number of non-destructive test methods available for the early detection of the presence of surface nucleii from which a fatigue crack could propagate. Thse methods include critical visual, X-ray and ultrasonic investigation, magnetic and fluorescent penetrants, caustic etching, anodizing, and surface plating.



A TYPICAL FATIGUE STRESS CYCLE



A TYPICAL S-N CURVE

FATIGUE DATA: Stress vs Number of Cycles (S-N) data may be applied directly in design, with careful attention to the loading and material parameters.

DESIGN PROPERTIES OF ENGINEERING MATERIALS DEVELOPED FROM IMPACT TESTS

Impact tests are an important source of material design data when packaging for the shock environment. Temperature and strain rate variations exert a heavy influence on impact strength of most structural materials.

Much attention in the technical literature has been directed towards material properties associated with static tensile and fatigue strength characteristics. Some material properties of interest to the packaging designer are best described, however, by an impact test, particularly for equipment constrained to survive a shock or impulsive load environment. It is a well established metallurgical fact that many materials are subject to a complete loss of impact strength below certain critical temperatures. This temperature-impact strength interaction is solved by early definition of the service environment and use of the proper alloying elements in the structural material.

Impact strength is most often measured from a simple pendulum impact test. The value of input energy required to fracture a standard notched specimen is the basis of both the Charpy and Izod impact tests. The principal difference lies in the mode of test specimen support. The Charpy test is conducted on a simply supported beam, with a specified notch arranged away from the impact. The Izod impact test is conducted on a notched cantilevered specimen. Both tests yield a relative impact energy to fracture, usually noted in units of energy per unit area at the notch. Thus, the additional variable of notch sensitivity is factored into the test, and results are often referred to as notch toughness or notched impact strength.

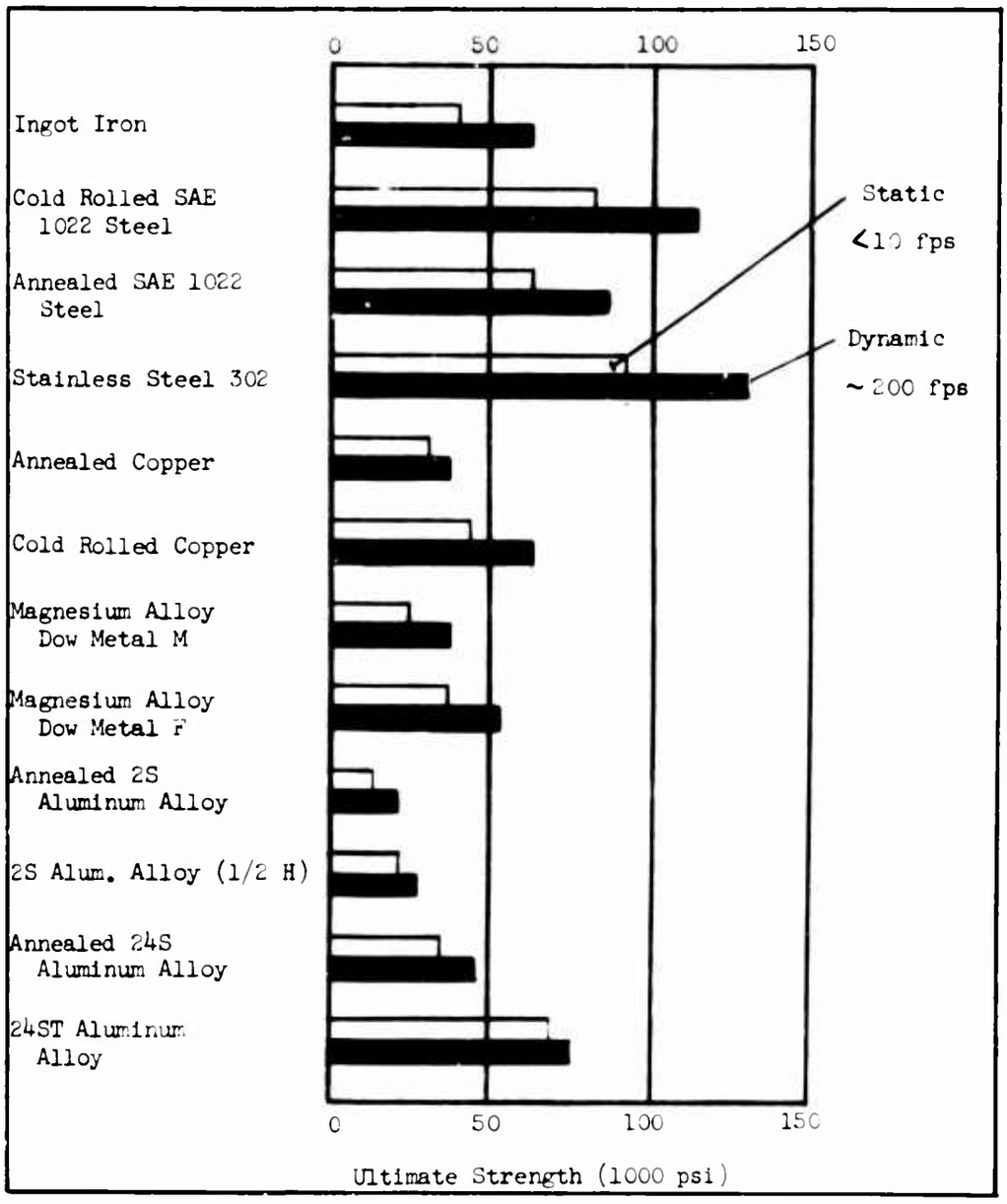
Other types of impact tests may be important to the designer as the impact mode and specimen support parameters may more closely duplicate the method of loading in the actual equipment. Impact tests on tension members, as well as torsionally loaded specimens yield useful data on material behavior for these loading conditions.

As is usual for members subjected to dynamic loads, the stress concentration effect should be kept to a minimum. This stress magnification influence may be tolerated during impulsive loading of the general stress level is kept low and uniform across a section, and if sufficient material ductility is available from the alloy. Material ductility has the effect of mitigating the high local stresses in the region of a notch or discontinuity by local yielding of the material and redistribution of the stress. Since maximum stress varies directly with the square root of the material elastic modulus,(5) then low modulus is desirable for impact load design. Sufficient ultimate strength is mandatory to preclude failure and the designer should recognize that low modulus means equivalently low natural frequency, usually contra-indicated in vibration design.

Laboratory tests have shown that strain rate, or speed of impulsive loading, has a pronounced effect on material strength and other dynamic properties. Ultimate and yield strength increase dramatically with increasing strain rate, andyield tends to close with ultimate at high load rates(5). This combination happily aids the designer, since structure may successfully withstand impact loads unknown to the analyst at the inception of design.

Many other physical characteristics differ widely under impact loading from the same properties under static conditions. Steels are known to exhibit a

wide variation in impact strength with variations in carbon content, manufacturing process, and degree of cold work to name a few (8). The designer is directed to standard texts on physical metallurgy to assess the problem of marginal designs and how changes in process and composition may help. (See References 11, 12 and 13.)



DYNAMIC STRENGTH: There are corallary factors of static ultimate strength and dynamic strength for some engineering materials, for intermediate impact velocities.(7)

METALLURGICAL MECHANISMS AND THEIR IMPORTANCE TO MATERIAL PROPERTIES

The designer should consider the material properties defined metallurgically in optimizing the structural material selection. These considerations include hardness, creep, wear and corrosion resistance.

Many of the most important physical characteristics of materials are delineated by special tests, other than those previously outlined, as well as close examination of the physical metallurgy of the material microstructure. The designer, by optimizing those physical properties of the structural material influencing the dynamic integrity of the equipment package, may greatly improve the equipment's chances of surviving the intended dynamic environment. The great advantage lies in the fact that some of these mechanisms, such as heat treatment, may strengthen the structure without an increase in sectional bulk.

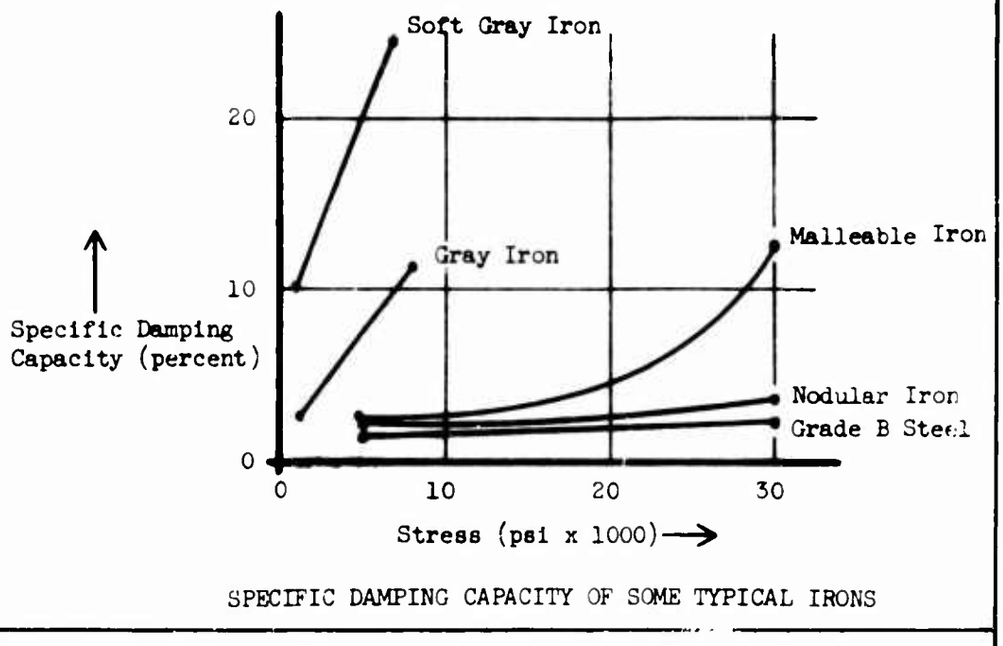
Some other physical tests of importance to dynamic integrity in a material include creep, hardenability, hardness, damping, wear and corrosion is a continuing phonemena which causes a deterioration in the geometry of a stressed member. The influence of creep on shock and vibration resistant structure lies in the amplification or fluctuation of the resonant effect due to the combined action of fatigue, impact, and creep. The situation is generally to be avoided in good design.

Hardenability relates the tensile strength of a material to resistance to penetration (hardness) as dictated by the inherent ability of the material to be heat treated. The importance lies in the fact that impact and fatigue strength vary directly with tensile strength for many engineering alloys.

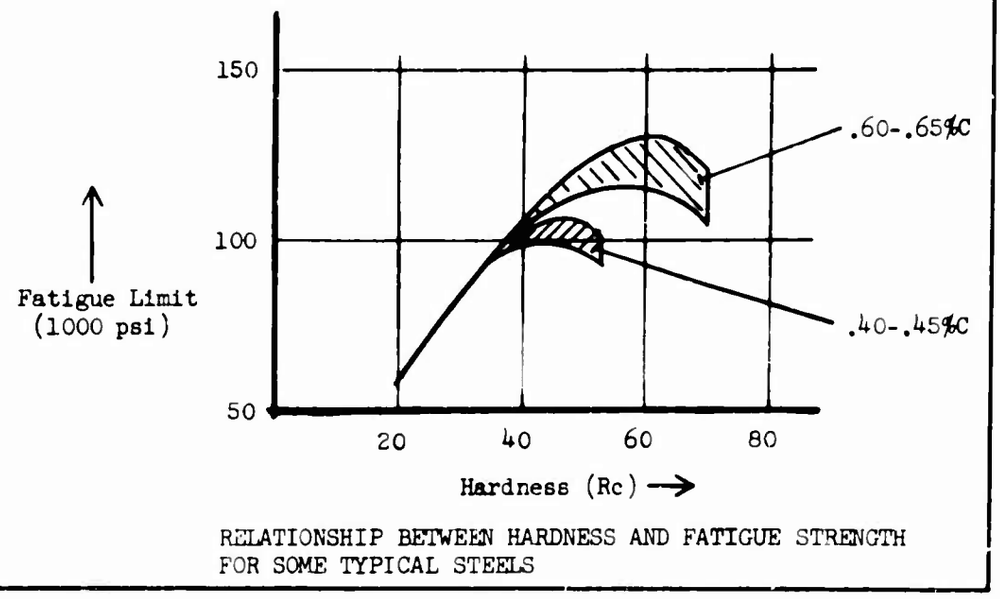
Damping within the structural alloy tends to absorb a portion of the dynamic energy due to internal hysteresis. It is generally desirable to employ high damping material in support structure to resist both shock and vibration influences. Each case should be analyzed individually, however, with respect to the location of major resonances within the enviornmental vibration spectrum.

The material properties defined by microstructural investigation and chemical analysis include the material's ability to age or stabilize, resist temperature and corrosion effects, retain a small grain size, have the ability to be thermally or strain (work) hardened, and have a microstructure that is hogeneous and istropic. These factors among others, may be defined to optimize the alloy characteristics best suited for the environment.

The method of manufacture of the structural alloy has a decided influence on the material impact and fatigue strength. The presence of scale and surface decarburization may seriously decrease fatigue strength of a member. The skin of a casting may protect the base material from the environment, yet provide the nucleii for a fatigue crack. Residual stress set up by the manufacturing process may frequently relieve a service stress. Forging, rolling, or burnishing can strengthen metals susceptible to strain hardening, particularly around stress-raisers. Conversely, rolling or other directional working may set up patterns of discontinuities which could cause directional weaknesses in the finished structure. In general, residual compressive stresses are considered helpful in combating fatigue, while residual tensile stresses are to be avoided. Examples of this approach are peening, sand blasting, and other surface stressing.



SPECIFIC DAMPING CAPACITY OF SOME TYPICAL IRONS



RELATIONSHIP BETWEEN HARDNESS AND FATIGUE STRENGTH FOR SOME TYPICAL STEELS

METALLURGICAL MECHANISMS AND DYNAMIC STRENGTH: Hardness and hardenability (and by implication, tensile strength) as well as damping capacity are important criteria in the selection of an optimum material for dynamically strong structure.

THE INFLUENCE OF SURFACE EFFECTS ON MATERIAL PROPERTIES

The importance of surface treatment to fatigue strength must be considered when specifying heat treat, cold work, machine finish, and plating in equipment package.

Material characteristics associated with surface conditions have recieved a great deal of attention in the literature. It has long been recognized that most fatigue failures are proprogated from some type of surface anamoly; hence, much research has been directed at surface characteristics with generally good results. Surface conditions are most important to structure subjected to repeated loads. Impact loads are only secondarily affected by surface conditions, mostly with respect to material removal such as corrosion. Fatigue on the other hand, is particularly surface sensitive since most structure is loaded in flexure or torsion to the extent that some critical stress is present near the surface. The combination of high stress and built-in stress raisers is the crux of the surface problem. The fact that corrosion and other environmental effects are most prevalent on the exterior of an equipment package only adds to the severity of the problem.

A convient classification of the influence of surface effects on fatigue life is given by Gordon, Grover and Jackson (3). The first involves the existence of surface roughness caused by the mechanical finishing process; the second relates the difference in strength between the outer shell and the core material; and the last deals with the differences in stress level as a result of a residual stress.

Surface considerations of importance to the designer also include the heat treat, cold work, coating and plating, and corrosion resistance provisions of the intended structure. For example, the dramatic variation of fatigue strength for various types of surface protection techniques in a typical steel is illustrated in the accompanying figure. Also shown are charts summarizing the effects of surface hardening, and surface plating on corrosion fatigue strength for various steels(4).

The most commonly used mechanical or metallurgical processes to improve surface conditions for which information is available to the designer, may be categorized into three groups (5); cold working, which includes shot peening, cold rolling, stretching, and burnishing (cold work is done at temperatures below the material recrystallization temperature); surface hardening, covering carburizing, nitriding, cyaniding, flame hardening, and induction hardening; plating, which includes chromium, zinc, and cadmium coatings.

The effect of these processes on fatigue life varies with each category, as well as varying within processes. In general, fatigue life may be improved up to 100 percent by shot peening and other cold working operations. This type of material improvement may be accomplished after forming the structural member, without a geometric change.

Surface hardening by heat treatment will result in a strong outer case of hard material, varying from a few thousandths to one-quarter inch in thickness. The most pronounced effect occurs when the treatment is made after the material has been notched. Nitriding, for example, can cause an increase of 300 percent in the notched fatigue strength of mild steels. The design constraint lies in the balance of core and case strength; sufficient core integrity is required to prevent failure below the case at the interface with the parent material.

Plating as a surface treatment, is primarily aimed at corrosion protection, to minimize the combined action of fatigue and surface deteriorization. Plating and coating may actually decrease the basic fatigue strength of many alloys. The rate of strength reduction due to corrosion may be more rapid, however, and the net result may well be positive, particularly in the presence of fresh and salt water environments.

EFFECT OF PLATINGS AND COATINGS
ON CORROSION FATIGUE STRENGTH OF 1050 STEEL

Type of Surface Protection	– Corrosion Fatigue Strength (% of original) – Cold-Drawn (100% = 54,900 psi) Air	Salt Spray	Normalized (100% = 36,700 psi) Air	Salt Spray
None	100	14	100	24
Enamel	93	44	105	68
Hot galvanizing	101	95	90	101
Zinc electroplating	100	87	98	90
Cadmium electroplating	93	77	93	84
Cadmium plating and enamel	95	72	96	82
Phosphating and enamel	93	44	108	79
Spray metallizing with Al	105	80	..	..
Spray metallizing with Al and enamel	103	93	..	..

EFFECT OF PLATINGS AND COATINGS
ON CORROSION FATIGUE STRENGTH OF 1050 STEEL

Material and Treatment	Fatigue Strength (% of original) Air	Fresh Water	3% NaCl
1043 Steel (100% = 36,300 psi)			
Normalized, no surface protection	100	65	39
Short-period nitriding	140	142	77
Shot peening	116	...	79
Induction hardening	187	187	140
Induction hardening with subsequent zinc coating	...	...	177
Alloy Steel (100% = 78,100 psi) Nitrided	100	100	...
Alloy Steel (100% = 104,600 psi) Nitrided	100	81	...

* All steels have fatigue limits of 10^7 cycles; nitrided alloy steel with 104,600 psi fatigue strength has fatigue limit of 10^4 cycles.

SURFACE TREATMENT: The effects of some typical surface operations on the corrosion-fatigue strength of steel are illustrated.

CONSIDERATIONS OF WEAR IN THE APPLICATION OF ENGINEERING MATERIALS

Wear is a surface or contact phenomenon which may cause an incipient stress-raiser or crack from which a fatigue failure may propagate. Wear considerations thus constitute a basic material criteria.

Wear has an important influence on equipment packages, as the dynamic characteristics will undergo a resulting change, often to the detriment of the structure. A comprehensive introduction to the concepts of wear is contained in the work of Professor Lipson on "Wear Considerations in Design" (14), and should be required reading for the structural designer.

Wear may be defined as the deteriorization of a surface due to use; thus the impact on dynamic integrity. Wear is often the major limiting factor on the life of an equipment package. Wear manifests iteself whenever there is load and motion. Furthermore, wear is usually a combination of several elementary forms such as galling, abrasion, and corrosion. Material characteristics should be selected on the basis of their ability to resist wear and surface deteriorization, as a fundamental criterion.

Failure of the lubricating film between two stressed surfaces caused by excessive pressures, sliding velocity, or temperatures cause a basic type of wear; galling, scuffing, scoring, and seizing are examples of this category. The differences within this group vary as to the severity of the action. The group is typically an adhesive form of wear caused by a welding and fracture of mating surfaces.

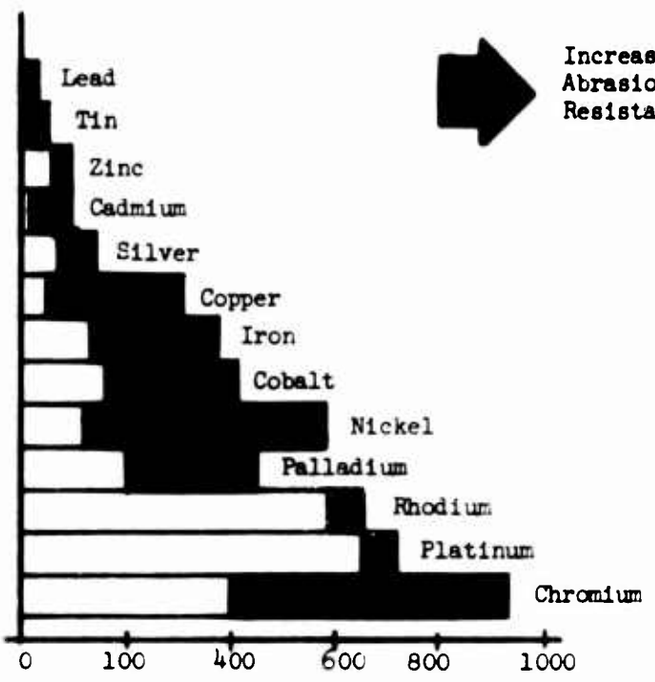
Abrasion is a cutting type wear which takes place when a hard foreign particle is present between two rubbing surfaces. The particle may be a metal grit, a hard oxide, or a contaminant from the environment, which penetrates the metal surface and subsequently tears off metallic particles. Abrasion may take the form of scratching or gouging of material, depending upon the severity.

Local surface fatigue or high contact stresses may result in pitting type wear such as case-crushing, spalling or stress corrosion. Cyclic loading manifest as a high sliding or contact stress is the usual ingredient of this wear category. A crack resulting from the high stress causes a particle of material (or a segment of the case material) to separate from the parent material and spall off. Some forms of incipient pitting may be healed or erased by subsequent wearing action.

A wearing phenomena associated with press fitted assemblies subjected to varying loads is known as fretting. Other wear of this type is called frettage, stress corrosion, or chafing fatigue. The worn area has the appearance of being corroded, or may seem to be heavily galled. The surface is deteriorated by the action of a reciprocating force at the contact region. Damage may be varied from erosion of large chunks of material, to simple discoloration of the surface. Fretting may cause a local stress raiser in the region of an existing highly stressed area, from which a subsequent fatigue failure could propagate.

Erosion and corrosion are examples of a wear category which occurs without the presence of a second surface or high contact stress. Galvanic corrosion is the exchange of ions between metals of dissimilar contact potential (or occupying different positions on the galvanic scale). Erosion is the mechanical or chemical removal of material from an exposed surface.

Wear is usually combatted by lubrication, by insulation with surface protection, or by the use of inherently self-lubricating materials such as cast iron. Wear is important to structure subjected to repetitive load because of material removal and the presence of discontinuities which may ultimately precipitate a fatigue crack.



RELATIVE ABRASION RESISTANCE OF VARIOUS HARD SURFACING DEPOSITS(14)

CONSIDERATIONS: Serious loss of fatigue strength is a common outgrowth of wear. Some detrimental wear effects may be decreased by the application of hard surface deposits.

VOLUME III - CHAPTER 10

MATERIALS AND PROCESSES

SECTION 3 - MATERIAL CHARACTERISTICS DEFINITION

- A Summary of Important Physical Properties
- Desirable Metallurgical Characteristics for Materials Used in Dynamic Applications
- The Influence of Alloying Elements on Metallic Material Properties

A SUMMARY OF IMPORTANT PHYSICAL PROPERTIES

Material selection is aided by the balance of stress with strength. Desirable material properties may be conveniently categorized for easy comparison of characteristics.

Material selection is the process of matching the known desirable material characteristics with the anticipated service and test environment. The skill with which the designer makes this choice will constrain the equipment life as much as any other factor. A convenient method of summarizing material properties is a categorization by major groups, as follows: tensile properties defined by the stress-strain curve; properties derived from the fatigue test; properties defined by impact tests; properties defined by the physical metallurgy of the alloy; characteristics involving surface phenomena; and considerations of wear.

In general, factors that influence the static strength of an engineering alloy also have a parallel effect on the fatigue and impact strength of that alloy. For example, most of the strength parameters measured from the static tensile test are, as a first approximation, directly relatable to strength in fatigue and impact. There are many limitations to this approximation; an increase in tensile strength does not necessarily reflect a parallel increase in fatigue strength. In fact, most alloys become increasingly notch sensitive at the high heat treat levels.

High ductility, particularly high notch ductility, is an important property for material subjected to both shock and vibration. This ductility will allow a certain amount of plastic flow in the region of a stress raiser, which mitigates the net stress effect. High ductility also implies high toughness, assuming equal strength. A high elastic modulus (manifest as a higher natural frequency) is usually desirable in a resonating structure. This implies increased stiffness and resilience. This characteristic is contraindicated for high impact resistance, however. The shock strength of a low modulus material is superior to that of a higher modulus material for the same strength and ductility levels.

Fatigue properties are directly applicable to materials which will be subjected to repetitive loads. In fact, fatigue data exists for all of the recognised loading modes, and many of the common geometries and engineering materials. The designer should use the data which most closely approximates his structural system. Some extrapolation of data may be made with good engineering judgement. Sometimes transference of properties is sufficient, but this is the prime purpose of the consulting materials and processes engineer.

The application of fatigue data to impact situations is not too valid. In general, low notch sensitivity in fatigue also means low sensitivity in impact. That is, good notch ductility is a desirable characteristic in a material experiencing both shock and vibration.

The properties derived from impact tests, of course, are directly relatable to materials to be subjected to impulsive loads. High toughness and impact resistance, and low notch sensitivity reflect generally superior material qualities for both shock and vibration applications. Materials with sharp ductile-brittle transition temperatures should be avoided, particularly when low service temperatures are anticipated.

Impact tests on most engineering materials are characterized by a great deal of scatter in the resulting data. The designer must exercise care in extrapolating the data to his problem, particularly as concerns specimen size. In addition, materials that are known to exhibit a marked yield point are also subject to high strain rate sensitivity.(15)

High Values

- Resilience
- Ductility
- Toughness
- Fatigue Strength
- Impact Strength
- Notch Ductility

Low Values

- Elastic Modulus (sometimes)
- Notch Sensitivity
- Strain-Rate Sensitivity
- Ductile-Brittle Transition Temperature

DESIRABLE MATERIAL PROPERTIES: Good material selection habits for the Design Engineer begin with an understanding of those characteristics that are most effective in shock and vibration applications.

DESIRABLE METALLURGICAL CHARACTERISTICS FOR MATERIALS USED IN DYNAMIC APPLICATIONS

The important metallurgical properties to high structural reliability and dynamic integrity include stability, high bulk strength, homogeneity, low sensitivity, and good wear and surface characteristics.

Metallurgical mechanisms of importance to the dynamic strength of materials range through all of the principal physical properties. In a qualitative sense, some of the more apparent and useful properties may be summarized as follows:

Stability - An alloy that exhibits a flat response through a range of environmental influences is generally more useful in dynamic applications. High resistance to aging, creep, corrosion and wear are all desirable traits in an engineering material.

High Bulk Strength - This property implies high hardenability since increased tensile strength generally indicates high fatigue and impact strength, within certain limits. Some of the hardening mechanisms are strain or work hardening, grain size refinement (or at least an inherent resistance to grain growth), quench hardening, precipitation hardening, dispersion hardening, and order-disorder transformation. A good discussion of these processes is available in reference (12).

Homogeneity - The avoidance of incipient stress raisers in a material for dynamic application, is always a good ground rule. This characteristic implies a uniform microstructure, more uniformity of strength between case and core material, a degree of work on a cast alloy to break up and disperse the dendritic structure or planes of weakness in the section, and an avoidance of non-isotropic materials. Since many dynamic disturbances are random, it is often difficult for the designer to arrange the directional properties to his advantage, hence it is safer to use uniform material. Generally, a material with a high degree of inclusion or one that is subject to rapid surface deteriorization is to be avoided. If a material "skin" resulting from casting or co -rolling is unavoidable, then care should be taken to leave it intact.

Sensitivity - Materials which exhibit low notch sensitivity, a broad ductile-brittle temperature transition range, good creep resistance, good damping characteristics, are unimpaired by temper embrittlement and adverse aging phenomena, and generally are insensitive to exterior influences, are the best choice for dynamic applications. Low sensitivity minimizes the effect of a stress raiser and hence lessens the material reaction to the excitation.

Surface characteristics and wear consideration are parallel criteria since they both relate to the outer skin of the material, or very close to it. Surface finish is always important to dynamic integrity; materials that finish easily and without directional properties are usually better in dynamic applications. Materials that respond to cold work (such as peening and burnishing) in the area of an unavoidable stress raiser are desirable.

Surface properties may be enhanced by proper surface heat treatment, plating, or coating. These processes may also improve the wear and corrosion resistance of the alloy. Fatigue strength is often improved and notch effect reduced by surface treatments such as nitriding or cyaniding, particularly after the notch has been formed.

A wealth of handbook data is available on the mechanical properties of certain engineering materials. A good summary of these characteristics as well as descriptive material on ferrous and non-ferrous alloys is contained in the "Metals Reference Issue" of Machine Design.(20)

It must be remembered that all of the varied characteristics of materials should be considered if designs are to be optimized. The designer who considers only yield stress is liable to be embarrassed.

- Stability – Flat Response and High Environmental Resistance
- Bulk Strength – High Hardenability
- Homogeniety – Uniform Microstructure
- Sensitivity – Low Environmental Relation
- Surface Qualities – High Wear Resistance and Fatigue Resistance

METALLURGICAL PROPERTIES: Some of the desirable material properties for dynamic applications are heavily influenced by metallurgical mechanisms, some of which may be enhanced by proper selection.

THE INFLUENCE OF ALLOYING ELEMENTS ON METALLIC MATERIAL PROPERTIES

Alloying elements are added to engineering materials to improve the physical properties of the base metal. For convenience, these structural materials are divided into two groups: ferrous and non-ferrous alloys.

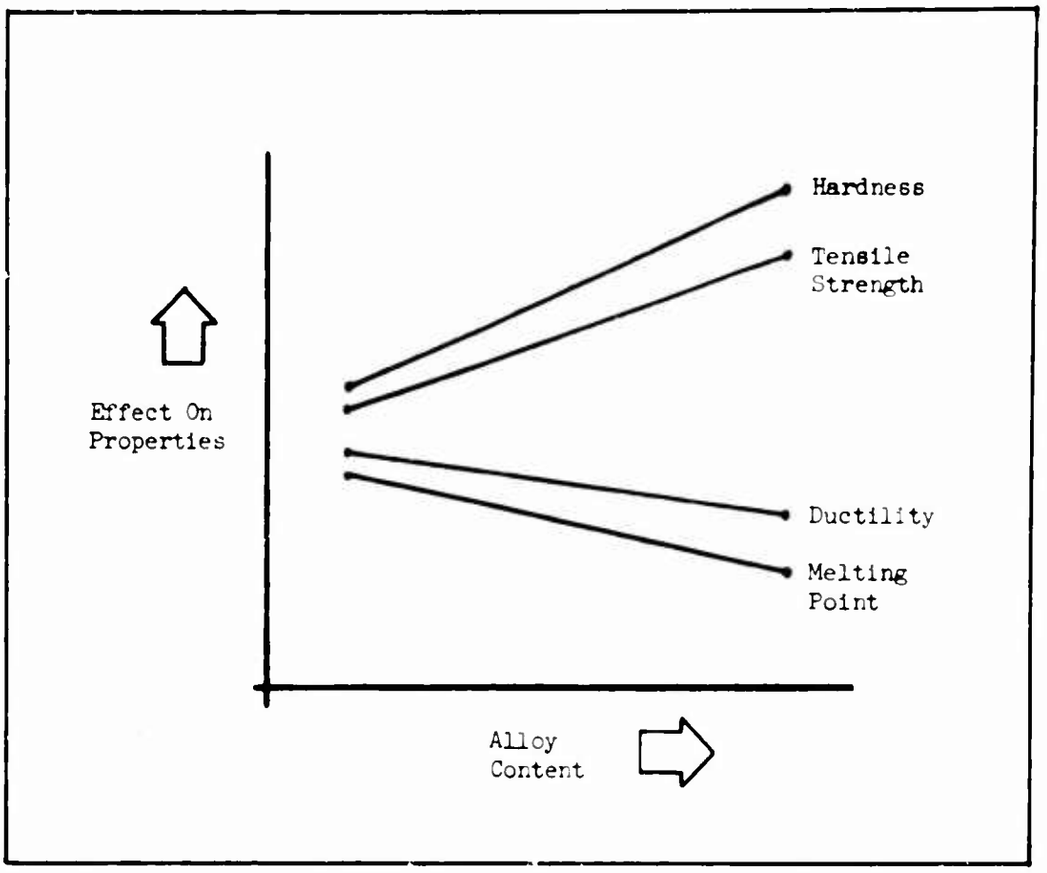
The alloying elements used in the common engineering materials have a profound effect on physical properties of the material. For example, copper may be made very malleable or may be used as a cold chisel depending on minor alloy additions and processing. The following paragraphs summarize qualitatively, the physical impact of these elements on those material properties important to dynamic structural integrity. The reader is directed to some of the referenced discussions on alloying elements for deeper study (1) (16) (17) (18).

Alloying elements are added to materials to change the physical properties of the base material. Alloying elements will combine with the base metal in one of three mechanisms: (a) mechanical mixtures, or heterogenous combinations of metals that retain their original identity, and are essential insoluable in the parent metal in the solid state, (b) intermetallic compounds, usually a combination of metals well dispersed through the microstructure, and (c) solid solutions where the solute atoms occupy a position in the space lattice of the solvent matrix, that is the alloy is completely dissolved in the base metal. This mechanism makes certain stable material conditions usable.

Alloying elements also differ in their effect on individual materials. An alloy may dramatically strengthen one material and have little effect on another. These alloying elements will be presented in two groups; alloying elements in ferrous alloys, and alloying elements in non-ferrouse alloys. The ferrous alloys will cover irons and steels, while the non-ferrous alloys will be limited to the commonly encountered structural materials.

Ferrous alloys make up the most commonly used class of engineering materials today. Although various forms of iron are still being used, the emphasis has shifted to steels which achieve their desirable properties by the addition to iron of a small amount of carbon (usually less than one percent) and other alloying elements such as chromium, nickel, molybdenum, manganese, silicon, and tungston. These are a few of the elements employed in the alloying of steel to improve hardness, ductility, corrosion resistance, machinability, weldability, dimensional stability, elevated and sub-zero temperature properties.

Non-ferrous alloys are those engineering materials that do not contain iron as the primary alloying ingredient. The total output of the non-ferrous alloys represents about one-tenth of the tonnage production of iron and steel. The most commonly used non-ferrous alloys are the aluminum, copper, magnesium, and nickel alloys. Some newer and more exotic alloys which are becoming increasingly important structurally, include beryllium, zirconium, and titanium.



ALLOYING ELEMENTS: Tensile strength, hardness, ductility, and melting point are among the important physical properties affected by alloy content in engineering materials.

VOLUME III - CHAPTER 10

MATERIALS AND PROCESSES

SECTION 4 - MATERIAL SELECTION CRITERIA

- Alloying Elements in the Ferrous Materials
- Physical Properties of the Ferrous Alloys
- Characteristics of the Aluminum Alloys
- Mechanical Properties of the Magnesium Alloys
- Characteristics of Beryllium and Titanium Alloys
- Characteristics of Copper and Its Alloys
- Properties of Some High Temperature Alloys
- Characteristics of the Common Plastics and Fabricated Materials

ALLOYING ELEMENTS IN THE FERROUS MATERIALS

Alloying elements are added to iron to improve the physical characteristics of the resulting alloy. Some knowledge of the qualitative aspects of alloys in engineering materials enhances an equipment design.

The effect of proper amounts of alloying elements on a given steel is usually greater than the sum of their individual effects. Thus, alloying elements are added to steels and irons in varying combinations to attain a particular set of physical properties. The most common alloying elements of iron and steels are carbon, nickel, chromium, molybdenum, vanadium, tungsten, manganese, copper, and boron. Some of the qualitative aspects of their impact on the ferrous alloys may be summarized as follows:

Alloying Elements in the Ferrous Material

Carbon: Carbon is the most widely used alloying element in steels and irons and by definition is what separates irons from steels. Low carbon steels are produced in larger quantities than all other ferrous alloys combined(19). Steel alloys are often classified by their carbon content, as low, medium or high carbon steels. Iron carbon alloys below 0.05 percent carbon are known as wrought irons, while alloys above 1.30 percent carbon are referred to as cast irons. Carbon has a pronounced effect on virtually all of the physical properties of steel. Carbon heavily influences strength, hardness, machinability, and melting temperature of steels, as shown in the adjacent diagram. This change in mechanical properties is true for both annealed and hardened steels. An important effect of carbon on steel is the improved ability of the alloy to be martensitic in microstructure after quenching from an elevated (austenitic) temperature. Transformation diagrams (time-temperature plots) are readily available for steel alloys showing this austenite to martensite transition. It is usually desirable to employ a microstructure of martensite (after tempering to the required ductility) for any given stress application. This is the fundamental general rule for the selection of a serviceable steel alloy.

A good summary of the qualitative effects of the other important alloying elements in steels is presented by Lindberg(1) and is excerpted here by element, as follows:

Nickel: Nickel increases toughness and resistance to impact, particularly at low temperatures, and lessens distortion in quenching. It lowers the critical phase-transition temperature of steel and widens the range of successful heat treatment. Nickel steels are particularly good for case-hardened parts. Such steels provide strong, tough, wear-resistant cases and also ductile core properties. Nickel does not unite with carbon to contribute to hardness, but it does help provide a tough core and does lower the transition temperature of the case permitting a milder quench for equivalent strength.

Chromium: Chromium, unlike nickel, joins with the carbon of the steel to form chromium carbides, thus adding improved resistance to abrasion and wear. Chromium also makes the transformation more sluggish and thus allows depth hardenability. Of the common alloying elements, chromium ranks near the top in hardenability. Chromium steels are relatively stable at high temperatures because tenacious chromium oxides provide good surface barriers and chromium carbides resist solution at high temperature.

Molybdenum: Molybdenum, like chromium, promotes hardenability of steel. It has a strong tendency to hamper grain growth prior to quenching, thus making the steel fine grained and extra tough at the various hardness levels. It is also used to increase tensile and creep strength at high temperatures. Alloys of both chromium and molybdenum develop high strength at elevated temperatures.

Vanadium: The grain-growth-inhibiting effect of vanadium promotes a fine-grained structure over a fairly broad quenching range, thus imparting strength and toughness to heat-treated steel. Vanadium steel would be used for items requiring impact and fatigue resistance.

Tungsten: Tungsten increases the hardness, promotes a fine grained microstructure, and resists heat. Tungsten has a crystalline lattice array that allows it to dissolve in both alpha and gamma iron, forming tungsten carbides. These carbides are very hard and quite stable.

Manganese: Manganese is one of the basic alloying components in steel. In fact, all alloys contain manganese to some extent. Manganese contributes markedly to strength and hardness, but to a lesser degree then does carbon. Fine-grained manganese steels attain unusual toughness and strength. They are almost impossible to machine except by grinding, but they can be cast and rolled.

Copper: Copper is added to steel in varying amounts, generally 0.2 to 0.5 percent. It is used primarily to increase resistance to atmospheric corrosion, but also acts as a strengthening agent.

Boron: Boron is used in steel for one purpose - to increase its hardenability, or the depth to which the steel will harden when it is quenched. Its use is recommended for steels with carbon content of 0.60 percent or less.

Other elements, such as sulphur, exist as impurities in steel; some are employed as deoxidizing agents, in the case of aluminum.

PHYSICAL PROPERTIES OF THE FERROUS ALLOYS

Ferrous alloys include the irons, steel, stainless steels, and nickel steels, each with a unique set of physical characteristics. These properties result from the metallurgical mechanism of allotropy.

Allotropy is the characteristic of an element which enables an element to exist in more than one chrystalline form. It is allotropy in iron, for example, that explains the unparalleled range of properties associated with modern alloys of irons and steels. The main reason for adding an alloying element to iron is to improve its physical properties, particularly the properties of hardness and toughness, through control of this allotropy.

Ferrous alloys may be divided into the following categories: iron alloys, carbon steel, alloy steel, stainless steel and special alloys. Iron alloys (other than steel) are usually used in cast or sintered forms except in electrical applications and the classic wrought iron.

Carbon steels are low in cost due to the large production advantages and are suitable for many uses. They may receive organic or chemical finishes or be cadmium, nickel, zinc or otherwise plated for corrosion resistance. Those with carbon content above 0.30 percent are commonly heat treated to improve strength. Alloy steels are available in a wide range of hardenabilities through heat treatment, and are finished to provide corrosion resistance when necessary.

Stainless steels are alloyed to give increased corrosion resistance and provide an attractive finish without painting or plating. Some classes of stainless steel may be heat treated to very high strength levels, but are characterized by less corrosion resistance than the non-heat treatable grades. Austenitic stainless steels are often cold worked to high strength levels. They are essentially non-magnetic when annealed but become progressively magnetic upon cold working due to coversion of austenite to martensite.

Special alloys include the iron-nickel alloys with various magnetic and electrical properties and other iron-nickel alloys with very low coefficients of thermal expansion.

Ferrous alloys may be fabricated by all common methods including forming, machining, sawing, grinding, welding, brazing, and adhesive bonding. Finishing may be by polishing, chemical treatment, organic finishes or plating systems as dictated by appearance, function, or corrosion resistance requirements.

Ferrous alloys are generally divided into two categories: wrought and cast. Cast alloys of iron have little application to packaging, while wrought alloys are very basic. Information on mechanical properties are presented for the various wrought alloys of steel and stainless steel.

ENHANCEMENT OF MECHANICAL PROPERTIES

- Increase in strength of steel as manufactured.
- Increase in toughness or plasticity in steel at any minimum hardness or strength.
- Increase of allowable maximum section which may be quench-hardened to desired properties.
- Decrease in quench-hardening capacity.
- Increase in rate of hardening with cold work.
- Decrease in plasticity at given hardness in the interest of machinability.
- Increase in abrasion resistance or cutting capacity.
- Decrease in warping and cracking in development of desired hardness.
- Improvement of physical properties at either high or low temperatures.

ENHANCEMENT OF MAGNETIC PROPERTIES

- Increase in initial permeability and maximum induction.
- Decrease in coercive force, hysteresis and watt loss (magnetically "soft" iron).
- Increase in coercive force and remanence (permanent magnets).
- Decrease of all magnetic responses.

ENHANCEMENT OF CHEMICAL INERTNESS

- Decrease of rusting in moist environment.
- Decrease of attack at elevated temperature.
- Decrease of attack by chemical reagents.

ALLOYING ELEMENTS IN STEEL: The marked effect of alloying elements on material physical properties may be illustrated by their influence on the ferrous materials.(18)

CHARACTERISTICS OF THE ALUMINUM ALLOYS

The aluminum alloys are readily available in a variety of shapes and strength ratings. Their lightness and high resilience make the aluminums desirable structural materials for dynamic loading situations.

Aluminum is the most abundant of the metallic elements found in the earth's crust. Very pure aluminum is quite soft and weak, and thus has little application to structural problems. Alloyed aluminums exhibit greatly improved mechanical strength properties which are enhanced (or degraded if misapplied) by various combinations of heat treatment and strain hardening.

The strength to weight ratio of the aluminums, coupled with good corrosion resistance due to a passive oxide film, are important engineering criteria for equipment applications. Cost, availability, ease of fabrication, and many other desirable assets are favorable to aluminum from a manufacturing standpoint.

Pure aluminum possesses high reflectivity and high thermal and electrical conductivity. Alloyed aluminums are commercially available in strength ranges approximating that of mild steel, at about one-third of the weight. The toughness of the hardened alloys, however, is generally low compared to steel. The inherent stiffness of the aluminums is also about one-third that of steel, as measured by the elastic modulus. The tenaceous oxide film generally tends to interfere with welding operations, and presents special problems in joining operations.

Aluminum alloys are readily available in a spectrum of strength levels, ranging from 10 ksi for the 1100 series alloys, to nearly 80 ksi for the 7000 series alloys. In general, the lower strength alloys possess relatively high formability and corrosion resistance. The higher strength alloys are usually connected by mechanical attachment due to poor welding, brazing, and forming characteristics and are subject to brittle fracture. Some of the more important characteristics and applications of the aluminum alloys are summarized in the appendix.

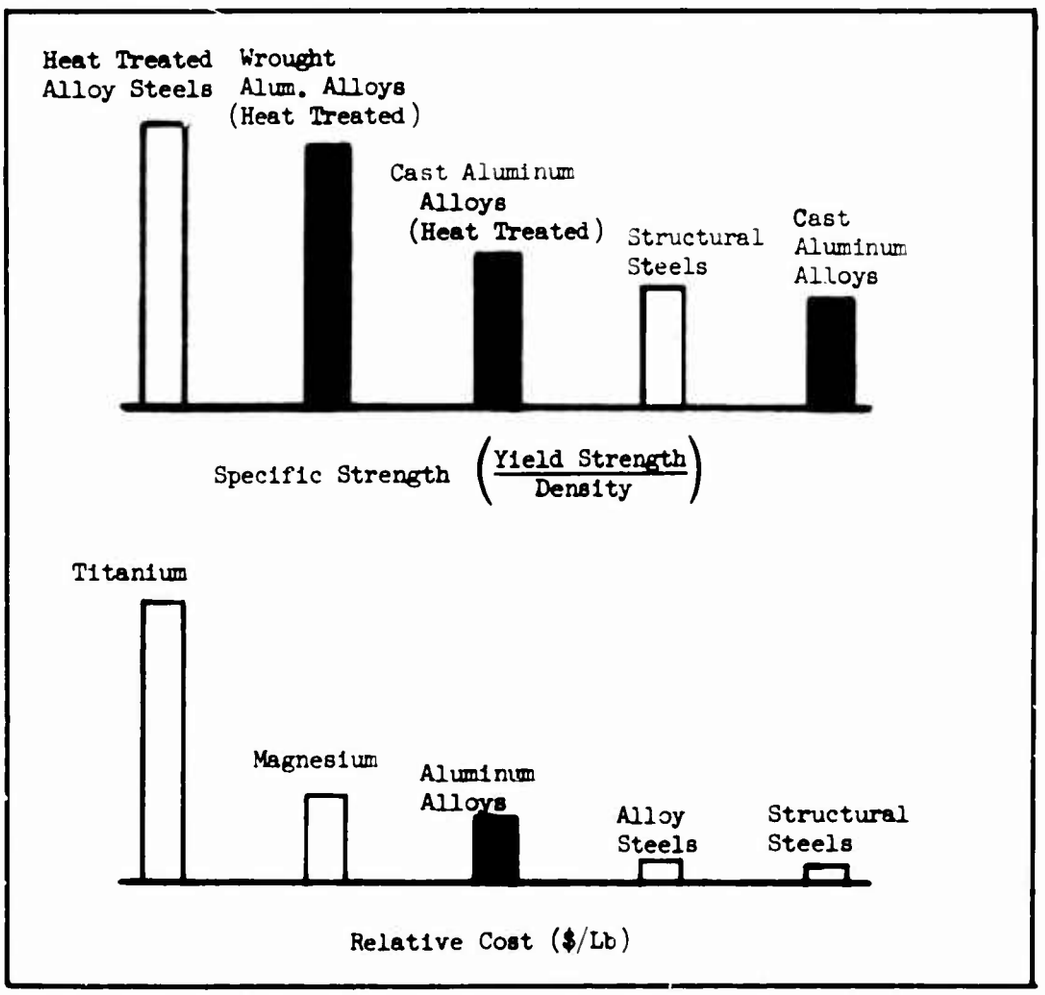
The upper temperature limit at which most aluminum alloys have useful mechanical properties is 300°to 400°F, although special powdered metal aluminum alloys retain a high porportion of its room temperature properties up to 700°F.

Aluminum alloys are produced in the form of foil, sheet, plate, bar, wire, extruded shapes, impact extrusions, hand and die forgings, die castings, investment castings, permanent mold castings, and sand castings. Except for higher strength alloys such as 7075 and 2024, which are not ordinarily fusion welded, aluminum alloys are weldable by all fusion and resistance welding processes.

Special controls are generally required, however, because the aluminum alloys as a class are just not as weldable as the ferrous alloys. Certain of the aluminum alloys, such as 6061, can be brazed by dip, furnace, or torch methods. The higher strength or more highly alloyed groups have poorer corrosion resistance than pure aluminum because the hardening ingredients cause anodic cells and the materially stressed structure is in a high energy state. Where maximum corrosion resistance is required, the higher strength alloys in sheet and plate are customarily clad with a higher purity aluminum.

Anodizing and other chemical conversion coatings are used to improve corrosion resistance or to provide a base for organic finishes. Where wear resistance is required, hard anodized coatings as thick as 0.002 inch are used. Higher strength properties may be obtained in most aluminum alloys by cold work, heat treatment, or some combination of the two.

Aluminum is commonly alloyed with copper, silicon, manganese, magnesium, zinc, and nickel. Aluminum alloys owe their mechanical properties to heat treatment and cold working. Heat treatment is the process of solution treating at elevated temperature, followed by quenching and then aging at lower temperature. This process is known as precipitation heat treatment. Cold work of these alloys is accomplished by rolling, forging, stretching, extruding, or straightening.



PROPERTIES OF STRUCTURAL ALLOYS: A comparison of strength-weight ratios and cost for some typical engineering materials.

MECHANICAL PROPERTIES OF THE MAGNESIUM ALLOYS

The magnesium alloys exhibit moderate physical properties for structural applications, combined with extreme low density.

Magnesium alloys find their greatest use in applications where their low density is an important constraint. Moderate mechanical properties at room and elevated temperatures become very desirable when considered in conjunction with potential weight savings. (Magnesium alloys are approximately one-fourth as heavy as steel.) Lightweight structures exhibiting good strength, excellent rigidity, good fatigue properties at low stress levels, and useful strength at temperatures to 800°F are possible with magnesium. Magnesium also possesses the useful attribute of high damping capacity, making it an ideal material for dynamic applications where a low resonant response is desirable.

Both wrought and cast magnesium alloys are characterized by excellent dimensional stability at temperatures up to 200°F. Another important feature in the fabrication of magnesium components is the excellent machineability of the alloy. Cutting speeds up to ten times of that used for steel is not uncommon. Welding of these alloys is accomplished by fusion or resistance methods. Fusion welding can produce joints with load efficiencies up to 90 percent. Structural magnesium materials are usually alloyed with aluminum, zinc, zirconium, maganese, thorium, and certain of the rare earth materials.

A summary of mechanical properties of the magnesium alloys is presented in the appendix; included are the wrought, forged, and cast alloys.

CHARACTERISTICS OF BERYLLIUM AND TITANIUM ALLOYS

The beryllium and titanium light metal alloys offer some unique physical properties for structural applications. There are some important limitations in cost, fabrication, and impact strength.

Beryllium and titanium alloys are generally classified in the light metals category, having densities comparable with aluminum and magnesium. Their strength capabilities, however, exceed these of the aluminums and compare favorably with some heat treated steels. Although many of the processing problems of the titanium alloys have been solved, the processing of beryllium is still in the developmental stage.

Titanium has a high strength-weight ratio and good corrosion resistance, even to exposure to salt water. Its mechanical properties lie between steel and aluminum in ductility, density, elasticity, strength and serviceability. Titanium has yet to reach a useage plateau near its potential due to high manufacturing costs stemming from difficulties with fabrication. Titanium alloys are available in sheet and bar form, and in strengths up to 130 ksi.

Beryllium is one of the lightest of the stable metals, with strengths comparable to heat treated aluminum alloys, combined with a high modulus. Its extremely high elastic modulus gives it a rigidity superior to any of the structural materials. Machining beryllium presents some special problems, however, and cold forming is not practical. Adhesive bonding, soldering, welding, and brazing are possible under controlled conditions. Beryllium alloys are extremely costly to produce and fabricate at this time, and are applicable only to cases of high weight penalty. Characteristically, beryllium alloys are extremely brittle and their inability to absorb strain energy make the alloys of limited applicability in impact siutations even if the economics were favorable.

CHARACTERISTICS OF COPPER AND ITS ALLOYS

The copper alloys find their principal application in electrical circuitry where good conductivity is essential. Some of the brass and bronze alloys offer modest mechanical properties for structural applications.

Pure copper is characterized by low strength, good ductility, high electrical and thermal conductivity, and good corrosion resistance. However, by the addition of one or more alloying elements, a wide varieity of properties may be obtained.

The copper category includes copper alloys of tellurim, zirconium, cobalt, chromium, arsenic, lead, phosphorus, sulfur, or beryllium. Brasses are basically alloys of copper and zinc which may also include manganese, aluminum, arsenic, silicon, and lead.

The greatest single application of the coppers is in electrical circuitry due to their high electrical conducitivity plus ductility and corrosion resistance.

The corrosion resistance of copper is relatively good for specific corrosive media; certain copper alloys give even better protection than pure copper. Unfortunately, exposed copper surfaces will change color as they acquire a protective oxide film. This film may be detrimental for decorative or pschological reasons even though the metal properties are not affected. In addition, copper in contact with dissimilar metals must be separated from the environment primarily to protect the other more anodic metals of the combinations. For these reasons, copper is often given a plating of cadmium, nickel, silver or gold, or a black oxide finish.

The mechanical properties of copper alloys cover a wide range from the deep draw formability of cartridge brass to the very hard spring materials such as beryllium copper and phosophor bronze. Many copper alloys are also available in leaded versions to improve the rather poor machineability typical of most of these materials.

PROPERTIES OF SOME HIGH TEMPERATURE ALLOYS

Some alloys of nickel, cobalt, and molybdenum are capable of retaining their moderate physical properties at elevated temperatures.

There are several special purpose alloys of only passing interest to the packaging engineer, which round out the materials list. Some of the high temperature application alloys of commercial importance are the nickel, cobalt, and molybdenum alloys.

Nickel alloys, and also the cobalt alloys, generally exhibit good corrosion resistance and mechanical strength at temperatures up to 2000°F, depending on the alloy. Nickel alloys are available in bar, sheet, plate, and wire forms. Their strength is developed by precipitation heat treatment and work hardening.

Molybdenum alloys exhibit good mechanical properties at elevated temperatures. They also have excellent rigidity and low thermal expansion. Oxidation protection is usually indicated for these alloys, particularly at high temperatures. The molybdenums may be fabricated by brazing and welding, but requires special techniques for machining.

CHARACTERISTICS OF THE COMMON PLASTICS AND FABRICATED MATERIALS

Plastics and composite structural materials offer a wide spectrum of physical properties for packaging applications.

The classification of the many plastic materials may be made according to form or shape, processing requirements, mechanical and electrical properties, and application. Plastics are usually referred to by type, either thermoplastic or thermosetting. Thermoplastic resins soften when heated and may then be formed or shaped. This process is reversible and may be repeated as necessary. Thermosetting resins are reacted (cured) to infusible solids in conjunction with the shaping process by the action of heat and/or chemical catylization, occasionally accompanied by elevated pressure. No further forming can take place after the material is fully cured. Thermoplastics find their most useful applications in small parts and components; thermosetting plastics have application to basic structure and transport cases.

Thermoplastics are usually formed without fillers, while thermosetting plastics find their greatest use as binders in laminated or filled forms and as adhesives. Laminated plastics employ layers of paper, textiles, glass fibers, and glass fabric as a reinforcing material. Asbestos, cellulose fibers, graphite, and metal and plastic fibers are also used as fillers in reinforced plastics. Strength, flexibility, coefficient of thermal expansion, conductivity, specific gravity, abrasion resistance, and environmental resistance can all be controlled over wide ranges by the judicious use of fillers and reinforcements.

Composite structural materials are now being fabricated from dissimilar raw stock to the advantage of the individual materials. The honeycomb panel for example, is produced in a variety of geometries, with differing facing and filler material. In concept, although not in appearance, the honeycomb sandwich is similar to any other laminated panel where layers of three or more materials are bonded together to produce a structure of greater strength than that of the sum of individual layers. The honeycomb sandwich is usually made up of layers, with two outer sheets, or facings, bonded to a comparatively thick internal honeycomb core.

High strength-to-weight ratio is probably the best known quality of the honeycomb sandwich. However, honeycomb panels have other superior qualities such as resistance to heat transfer and vibration, which can be just as important as strength to weight ratios.

In general, any metal that can be made into a foil then welded, brazed, or adhesive bonded can be made into honeycomb. In a honeycomb sandwich, the facings generally are the prime load carrying members. The main function of the core is to provide an essentially continuous support and stabilization for the facings, which are under compression and/or tension when the panel is loaded. At the same time, the core itself must withstand shear and compression stresses resulting from the loading.

MECHANICAL PROPERTIES OF ALLYLIC PREPOLYMERS[35]

	Diallyl Phthalate	Diallyl Isophthalate
Tensile Strength (psi)	-	4300
Flexural Strength (psi)	7000 to 9000	7400 to 8300
Flexural Modulus (10^6 psi)	0.5	0.5
Impact Strength, Izod (ft-lb/in. of notch)	0.2 to 0.3	0.2 to 0.3
Deflection Temp (C)		
at 264 psi	155	238
at 546 psi	-	184 to 211
Hardness, Rockwell	M114 to M116	M119 to M121
Specific Gravity at 25 C	1.27	1.264
Refractive Index at 25 C	1.571	1.569
Moisture Absorption 24 hr, 25 C (per cent)	0.0 to 0.2	0.1
Dielectric Constant, at 60 Hz	9.9	3.4
Dissipation Factor, at 60 Hz	0.005	0.008
Volume Resistivity at 25 C (ohm-cm)	1.7×10^{16}	3.9×10^{17}
Surface Resistivity at 25 C (ohm-cm)	9.7×10^{15}	8.4×10^{12}
Dielectric Strength step by step, v/mil	450	422
Arc Resistance (sec)	118	123 to 128

TYPICAL PROPERTIES OF EPOXY – GLASS-CLOTH LAMINATES[35]

At 73 F

Flexural Strength (psi)	76,000
Flexural Modulus (psi)	3.3×10^6
Tensile Strength (psi)	55,000
Compressive Strength, edgewise (psi)	52,000
Water Absorption, 24 hr (per cent)	+0.1
After 30 days water immersion:	
Flexural Strength (psi)	66,500
Flexural Modulus (psi)	3.2×10^6
Tensile Strength (psi)	52,000
Compressive Strength (psi)	48,000

At 160 F

Flexural Strength (psi)	70,000
Flexural Modulus (psi)	3.2×10^6

At 300 F

Flexural Strength (psi)	55,000
Flexural Modulus (psi)	2.9×10^6

At 400 F

Flexural Strength (psi)	40,000
Flexural Modulus (psi)	2.8×10^6

At 500 F

Flexural Strength (psi)	25,000
Flexural Modulus (psi)	2.5×10^6
Tensile Strength (psi)	45,000
Compressive Strength(psi)	19,000

At 500 F (after 192 hr at 500 F)

Flexural Strength (psi)	40,000
Flexural Modulus (psi)	2.5×10^6

STRUCTURAL PLASTICS: Allylics and epoxies are typical of the thermosetting plastics that have application in electronic packages.

VOLUME III - CHAPTER 10

MATERIALS AND PROCESSES

SECTION 5 - MANUFACTURING PROCESSES AND PROCEDURES

- Manufacturing Techniques Applied to Equipment Structure
- Joining by Mechanical Fabrication
- Welding: A Fusion Joining Process
- Brazing and Soldering
- Structural Joining by Adhesive Bonding
- Forming by Casting
- Forming Procedures Involving Material Removal
- Structural Forming by Material Deformation
- Heat Treatment: A Material Process to Improve Mechanical Properties
- Surface Hardening Techniques
- Plating and Chemical Finishing
- Plotting and Encapsulation

MANUFACTURING TECHNIQUES APPLIED TO EQUIPMENT STRUCTURE

Most manufacturing procedures may be categorized into divisions of joining or forming operations. This categorization provides a convenient organizational method.

The preceding section dealt with the physical properties of materials, how they are determined, and how they are applied to a design situation. A design decision of equal importance is the selection of the fabrication process that is to be employed, as well as decisions relating to the material processes to be used. The breakdown of procedures and processes that will be presented in subsequent sections, are organized as shown in the adjacent figure.

Mechanical joining methods include riveting, bolting, or screwing, and thus are limited to those materials that can tolerate a hole being drilled or punched without extreme degradation of the base material. The obvious disadvantage to mech-fastening is the necessity for material removal and the accompanying problems of stress concentration and crevice corrosion.

Welding is the intimate union of elements by fusion of the base material. The material is heated until a molten or plastic state is reached and fusion occurs, occasionally with the addition of mechanical pressure. The use of a fluxed filler rod to build up the weld area is common, and frequently welding is done in the presence of an inert gas to preclude oxidation. Welding is applicable to members constructed of the same metal composition, as well as dissimilar materials under certain conditions.

Brazing and soldering are metal joining processes in which another metal or alloy is used to fuse base elements. The fusing alloy usually has a substantially lower melting point than the base material, which allows the original material to retain more of its mechanical properties after joining. Brazed joints usually exhibit considerable strength and are able to withstand higher temperature environments than the soldered joint. Soldering, however, is less detrimental to the basic material characteristics.

Adhesives and other special cements are capable of holding materials together by surface attachment, without thermal processes and without reduction of structural sections. Adhesives have the great advantage of joining dissimilar materials without complicated metallurgical considerations. Adhesives generally tend to have inferior strength when compared with the strength of the base materials available. Since most bonding agents are organic, adhesives tend to weaken rapidly with a rise in temperature.

Forming procedures are those manufacturing processes that configure a structure by material deformation, material removal, or the molding of cast material. The casting and molding category includes sand casting, permanent mold casting, shell molding, die casting centrifugal casting, and investment casting. Also grouped in this category are the special processes of sintering and powder metallurgy.

Forming by material removal includes all of the conventional machining operations such as milling, turning, and hole preparation, as well as the surfacing and finishing operations such as grinding, broaching, and honing. This category also includes some of the exotic forming processes such as chemical milling and ultrasonic machining.

Forming by material deformation processes is characterized by the working of the structural material, without chips. Forming processes are accomplished either hot or cold, depending on the relationship of operational temperature and material recrystallization temperature. The material deformation operations include forging, extruding, cold heading, stamping, spinning, drawing, cold rolling, and high energy forming processes.

Material processing, although more a characteristic of materials rather than a manufacturing procedure, is of primary importance in developing and enhancing basic material properties. An example of these processes is heat treatment, which is employed to improve the mechanical properties of engineering materials. Heat treatments for ferrous alloys include quench and temper, anneal, normalize, and other processes that can enhance the physical properties of a material prior to service or other fabrication operations. Other alloys of interest to the structural designer are strengthened by precipitation heat treatment, a process used with the aluminum alloys.

Surface conditions and strength are vital to the integrity of a dynamically loaded structure. Surface hardening techniques of interest to the packaging engineer include flame and induction hardening, carburizing, nitriding, and cyaniding.

Surface finishing processes of plating and chemical finishing are customarily used by the structural engineer to improve environmental protection and appearance. Anodic plating is an effective corrosion barrier for certain engineering materials, as are chemical conversion coatings. Primers and paints enhance appearance and provide a measure of environmental protection.

Potting, sealing, and encapsulation are useful packaging techniques for preventing or absorbing dynamic excitations as well as providing good electrical and environmental insulation. These processes include embedment, potting, encapsulation, impregnation, molding, and conformal coating.

Joining Procedures	Mechanical Fastening Welding Brazing and Soldering Bonding and Adhesives
Forming Procedures	Casting and Molding Material Removal – Machining Material Deformation – Hot and Cold Work
Material Processing	Heat Treatment Surface Hardening Plating and Finishing Potting, Sealing, and Encapsulation

MANUFACTURING PROCEDURES: The fabrication operations having impact on material physical properties include joining, forming, and material processes.

JOINING BY MECHANICAL FABRICATION

Mechanical joining usually involves a fastener which precludes a variety of design problems. A thorough knowledge of fasteners and detail of their application is needed for good design.

Mechanical fabrication procedures invariably involve some type of fastener, and consequently, a thorough knowledge of fasteners is imperative to their proper application. There are literally thousands of fasteners manufactured by as many different companies available to the structural designer. Fasteners are manufactured from a variety of materials, ranging from soft brass to a highly alloyed, ultra-high strength steel. Some of the common materials used for contemporary fasteners are alloys of steel, aluminum, brass, copper, nickel, stainless steel, and many non-metallic materials as well. A variety of finishes and coatings are applied to fasteners, primarily to improve appearance and corrosion resistance. These include mechanical platings, electro-deposited coatings, chemical-conversion coatings, and hot dip coatings. Thus, the designer is faced with the formidable task of selecting the optimum fastener design, fastener material (and its heat treatment), and fastener coating, to suit a given environment. A proven approach in organizing this wealth of information is to maintain an up-to-date library of fastener details for convenient and trusted manufacturers, plus some general discussions on fasteners such as reference (21).

The category of fasteners includes bolts, screws, rivets, nuts and washers, studs, clinching devices, inserts, retaining devices and a range of quick-operating mechanisms.

Proper use of fasteners in a mechanical joint involves attention to detail by the designer. Rather than discuss the minutae of fasteners, most of which is better covered in the literature, this section will offer some design tips on the application and implementation of the mechanical joint.

Threads in Shear: This is a common mistake in the application of a threaded fastener to shear situations. Threads are fastening devices to be used in tension (along the length of the fastener) and are satisfactory in this direction since the load spreads out along the thread length. Threads in shear, however, are perfect sources of incipient fatigue cracks, even for lightly loaded joints. Always provide a thread grip in excess of the joint thickness in shear, and washer out past the thread start for the nut.

Combined Loads: This is more of an analysis shortcoming than a fastener problem. Often fasteners are loaded simultaneously in shear and tension. Tension in a shear joint may occur inadvertently from bending in the fastener or from an unsuspected "kick" load due to a misalignment of force and resistance. Shear often occurs in a joint designed for pure tension by accidental pick up of load due to a tight fit of the fastener. The rules are: construct a good free body diagram of the fastened member, to include all kick loads acting (it helps keep the line of action of the forces within the base of the joint); make the shear joints stiff enough laterally to preclude rotation, and subsequent tension in the fasteners; provide clearance for tension fasteners so they will not load up in shear; be aware of the shear-tension interaction curves for common fasteners.

Multiple Fastener Joints: The important criterion here is the match up of holes such that all the fasteners are loaded. Poorly fitted groups will

sometimes cause failure of the tight fastener, and a subsequent "domino" type failure of the rest. Even in the best matched multiple shear fastener attachments, the first row of fasteners is subjected to more load than the last row of the series. At best, multiple fasteners will fail in a mode characteristic of fatigue.

Spacing: Improper edge distance may greatly decrease the efficiency of a group of fasteners. One and one-half diameters is a good rule for most ductile materials, with a four diameter spacing between fasteners to minimize stresses between the holes. In addition, the thickness of the shear material may decrease the loading efficiency of the fastener, particularly for thin sheets.

Balanced Stress: High stress in the base material coupled with low stress in the fastener is generally to be avoided in dynamic applications. Failure due to stress corrosion or fretting fatigue is common in such situations. Any unusually high local stress is a potential source of fatigue failure.

Joint Compliance: A good method of reducing transmissibility across a joint is to intentionally introduce some compliance at the material interface. This may be accomplished with a spongy material layer within the joint, arranged to load the compliant material in shear (subject to functional limitations). A riveted joint also has this capability due to minute slippage across the joint.

Preload: Preload in a tensile fastener is used to reduce the severity of load reversal; that is, the ratio of maximum to minimum tension in the fastener. The more closely these two values are to 1.0, the least affect results from repeated stress. This, of course, is subject to the strength limitation of the fastener. In general, preload should be used to oppose the anticipated service stresses. Preload in a fastener is usually accomplished by torque-up of the nut on a threaded fastener.

Nut Retention: The prime feature of a nut under a repetitive loading situation is the need to retain the joint preload torqued in when the joint is first made. Thus, various schemes have been devised to maintain the nut makeup during vibration. The most reliable of these approaches is the cotter pin or lock wire. These devices are used in primary shear joints, where loosening of the pin (or bolt) would cause catastrophic failure. Other locking devices include metal and plastic inserts that resist turning in the threaded joint; crimped nuts which resist turning by gripping the thread; mechanical staking, a practice generally to be avoided in fatigue situations because of the incipient notch; and chemical staking which uses the bonding capability of a fluid material to lock the nut in place. A positive feature of some nut-retention ideas is the ability of the joint to be made, loosened, and remade a few times with the same components without loss of the locking feature.

Contact Potential: Check for galvanic corrosion; the intimate contact of dissimilar metals may cause galvanic attack of the less-noble material, particularly in the presence of a corrosive media and a high local contact stress. To avoid this, seal the joints with polysulfide, RTC, or chromate primer.

WELDING: A FUSION JOINING PROCESS

Depending upon the effort of the allotropic and metastable transitions, all metals are weldable by all processes. The design decision is one of matching the optimum procedure to the material and its application.

Welding, brazing, and soldering are all methods of joining metals by fusion of the base material, or the fusion of a filler material. The differences are largely those of fusing temperature.

When the process involves localized temperatures exceeding the melting point of the base metal, the process is known as welding. If a filler material is used in the welding operation, it usually is different in composition but related by class to the base material.

Fusion welding is the process generally used for structural applications and is capable of providing strength nearly equal to that of the base metal, but with reduced ductility and toughness. Heat treatable alloys are usually not welded after heat treating because of decreased strength in the heat-affected zone and cracking in certain alloys. Work hardened alloys will also be weakened by fusion welding. Heat treatable aluminum alloys except 2219 and 6061 are usually not welded by fusion methods because the alloy ingredients that cause hardening also cause incipient fusion in adjacent areas with consequent embrittlement. Resistance welding utilizes the resistance of the base metals to the flow of electric current to heat the metals in a very small area and produce a weld under the combined effects of fusion and pressure. Systems for providing multiple spot welds are called seam welds. This process may be used with many alloys that are not fusion weldable, but when employed on these materials, precautions should be taken to assure that the reduction in the size of the heat affected zone is adequate to provide reliability.

Welding processes are often classified as to their effect on the base material. Welding of many steels, for example, causes the formation of a hard microstructure, martensite, in the region of the weld which must be controlled by adjustment of the temperature-time gradients. Welding problems are thus minimized in steels having low hardenability, or cases where the joined members are slowly cooled after welding. Weld cracking is often aggravated by the presence of hydrogen, which causes embrittlement.

Non-ferrous alloys are also greatly affected by the welding process. Solution treated alloys will be re-treated in the region of the weld, and may age naturally. Often, a filler rod of higher physical properties is used to upgrade the weld strength. A compromise solution to weld-induced problems is the re-heat treatment of the assembly after welding. A better solution is to maintain the parent metal relatively soft and tough and the weld relatively strong. This forces the structure to hinge in the parent metal. Welding materials that have been previously hardened by cold working has the effect of softening the material in the region of the weld. This is why materials hardenable by cold work are generally more weldable than those hardened by metastable transitions.

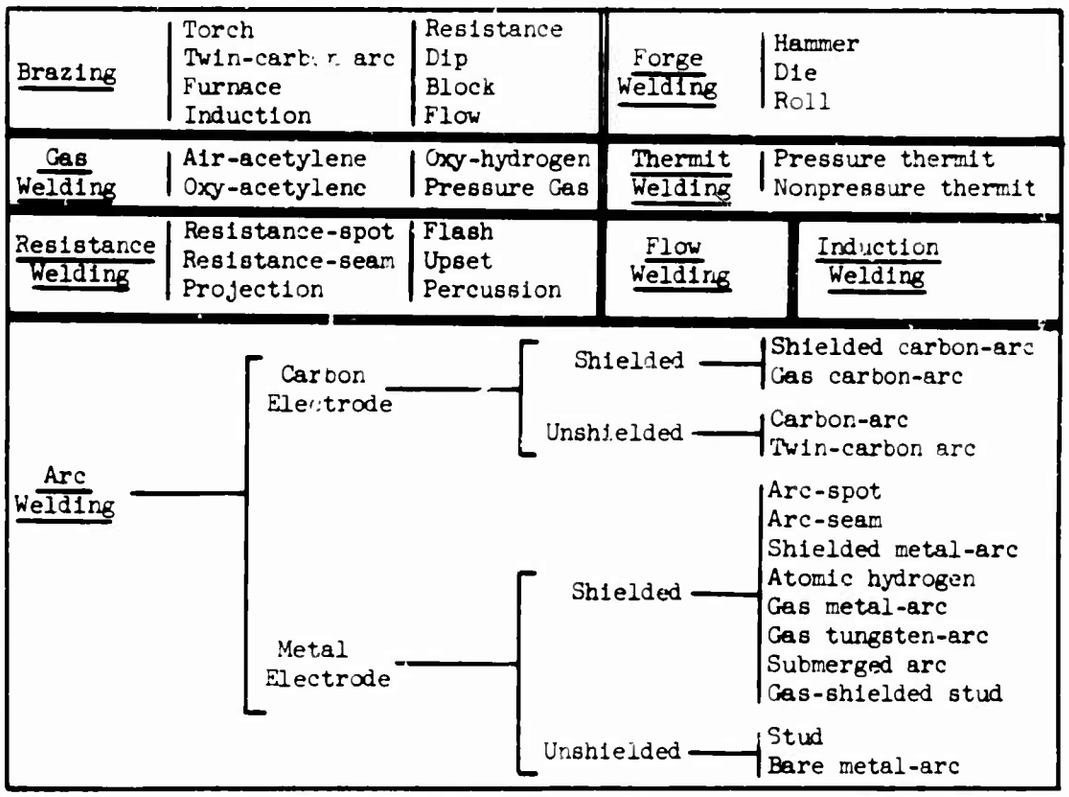
The welding process is adversely affected by flaws resulting from the operation, such as porosity, cracking, and inclusions of oxide and slag. Many of these flaws are the result of poor technique, while some (such as hot shortness cracking) are inherent in the base alloy. Gas porosity and inclusions

are serious problems to be avoided in welded joints subjected to vibration, since they are potential sources of fatigue cracks.

The low carbon steels are weldable by most common processes. Alloy steels, where the formation of martensite is a critical consideration, is customarily welded by processes which permit good control of heating and cooling rates. Care must be taken in evaluating the preheat and postheat treatment requirements when welding the steel alloys.

Depending on the effect of allotropic and metastable transitions, all metals are weldable by all processes. Nickel and nickel alloys may be welded by the arc processes as well as oxyacetylene techniques. Austenitic stainless steels are weldable by shielded arc, gas metal arc, and gas tungsten-arc processes. The aluminum alloys are usually welded by inert gas-arc processes. Gas welding is possible, but generally has poorer post weld properties, primarily as a secondary effect of the fluxes which must be used. Weld defects of cracking, incomplete fusion and penetration because of the particular refractory and tenacious character of the surface oxides, are common drawbacks.

The most important welding process for the magnesium alloys now in use is the gas tungsten-arc process, although other techniques are feasible, under specific conditions.



WELDING PROCESSES: A chart of the various welding procedures as compiled by the American Welding Society.

BRAZING AND SOLDERING

Brazing processes are often used in structural applications where joint stress is modest. Soldering is primarily a contacting procedure across a structural joint.

Brazing and soldering are fusion joining techniques that differ by the process temperature. When the fusing temperatures are below the melting point of the base metal but are above 800°F, the process is known as brazing. The filler material used in brazing is usually an alloy of metals dissimilar to the base metal. When the process temperatures are below 800°F, the operation is called soft soldering and the filler materials are generally alloys of lead, tin, indium, zinc or bismuth.

Brazing is used where moderate strength is needed, but welding is impractical because of joint configuration, heat treatment constraints, or metallurgical incompatibility. In the brazing process, the filler material flows into the joint by capillary action and wets the surface by metallic interaction. Thus, proper clearances between mating surfaces are indicated; this will develop optimum strength in the joint since the bulk properties of the filler material is usually inferior to the mating materials.

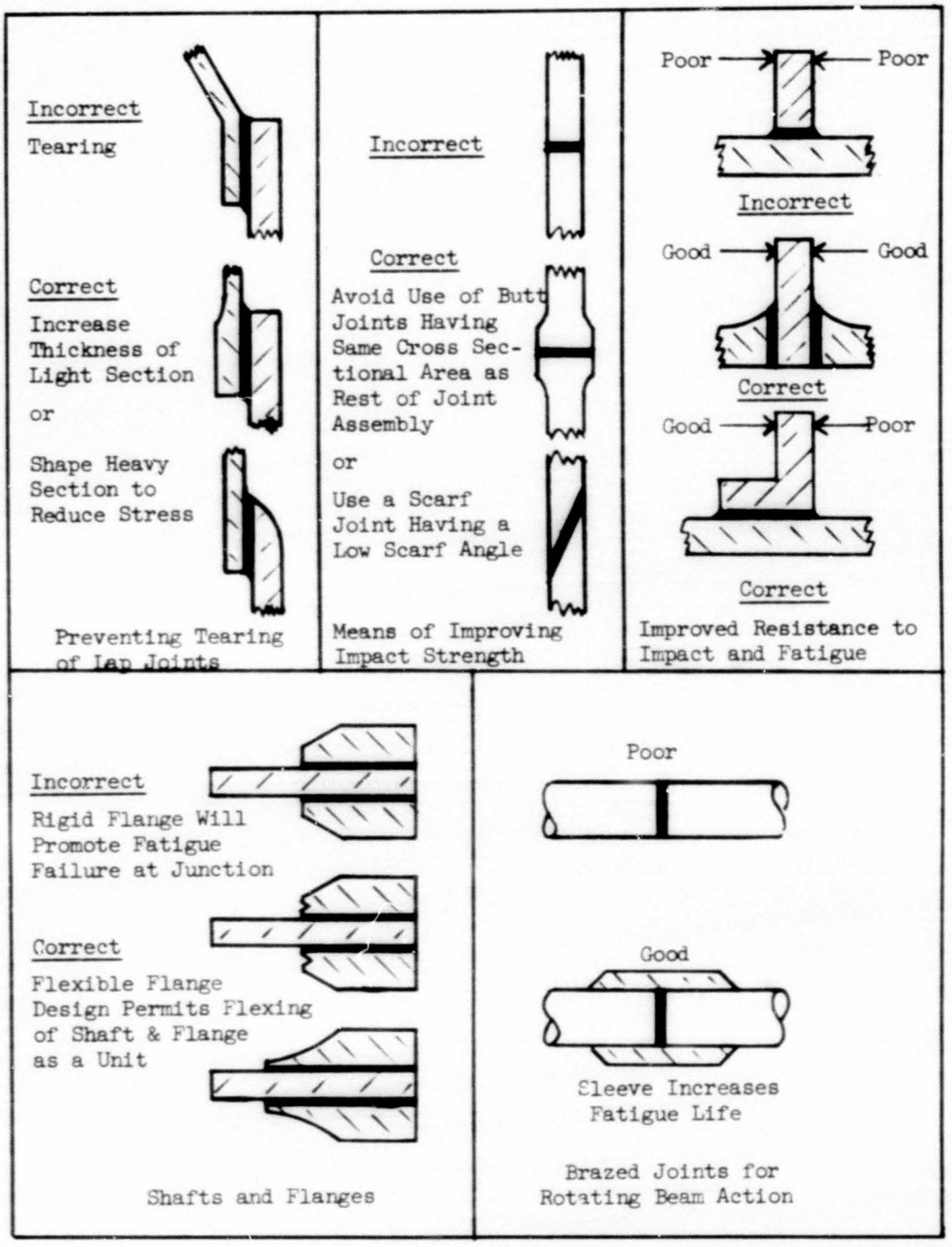
Brazing is suited to mass production techniques using continuous furnaces. The process has many other advantages for structural assembly, including:

- Joining of dissimilar metals is practical.
- Little post-joining finishing is required.
- Assemblies may be joined in a reduced stress condition.
- Complex assemblies may be joined by using filler materials with progressive lower melting temperatures.

Most of the common structural alloys may be joined by brazing. It is the task of the metallurgist to choose the proper filler material, flux material, and brazing process. Good operational technique includes joint cleaning and preparation, and post brazing cleanup and inspection. The designer is most interested in the mechanics of the joint itself. The two most important joints for brazing are the butt and lap joints. Of these procedures, the lap joint, which places the filler material in shear, is the most desirable. In practice, the filler material is either pre-placed in the joint, or is allowed to flow into the joint by wetting capillation, aided by gravity.

The brazing process is occasionally done in an inert or reducing atmosphere to minimize oxidation of the metal surfaces. There are many techniques for accomplishing the brazed joint. Almost any source of heat that is neutral or reducing will be excellent. Fluxes can overcome the effects of an oxidizing environment. Post braze heat treatment may be accommodated: (a) when the filler material possesses a melting temperature in excess of the required soaking temperature for heat treat; (b) when quenching requirements don't overstress the joints, and (c) when some deformation is tolerable.

Soldering processes have less importance as a joining technique for support structure, but have the advantage of lower heat and consequently less distortion. The soldering process should not be expected to supply any structural integrity to a joint. The designer should rely on some other positive technique and use the soldered material for contacting purposes only. Examples are interlocked joints, wire-wrap, riveting, and edge reinforcement.



BRAZED JOINTS: The joint should be designed to utilize to full advantage the available contact area.

STRUCTURAL JOINING BY ADHESIVE BONDING

Adhesive bonding is an important method for joining odd shaped or dissimilar materials. Modest strength is possible when severe environments can be avoided.

Adhesive bonding of structural members is becoming increasingly important in the designer's repertoire of joining processes. While this process has some limiting characteristics for the designer, the advantages of the procedure are attractive, particularly for vibration situations.

Adhesion is due to the molecular attraction between the adhesive material and the bonding surfaces. In certain very porous materials, some mechanical bonding or interlocking is also helpful in maintaining joint strength. Generally, adhesive processes may be divided into two groups; structural and non-structural. Structural bonding is used in the same manner as fusion welding, brazing, or mechanical fastening. In certain optimum applications, adhesive bonds will not fail until yield strength of the substrate material is reached. The yielding of the metal will then cause the rigid adhesive to be weakened, cracked and lead to subsequent failure.

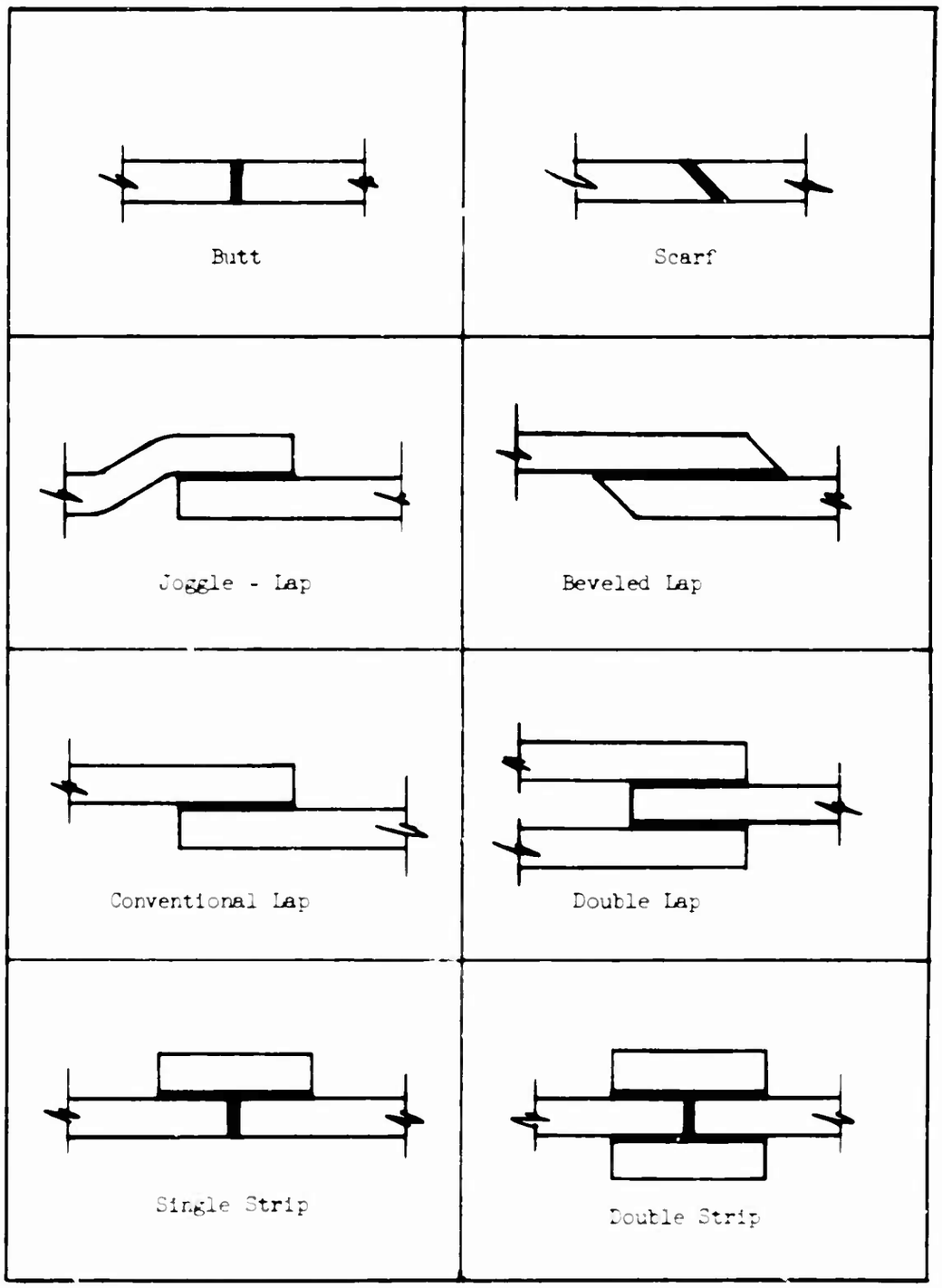
Non-structural adhesive bonding is of great importance to equipment packaging. The process is useful in sealing, filling, densifying and providing a damping barrier to resonant effects.

Structural adhesives are generally thermosetting materials and, hence, the liquid-solid transformation is not reversible. All adhesives are subject to some deterioration from heat and other environments because all polymers tend to over-polymerize. The designer must evaluate each material on this basis.

There are substantial advantages to the designer in the use of structural adhesives. There is no great difficulty in bonding of dissimilar materials. Materials can be attached to very thin members, too thin to be fabricated by any other type of joining process. Adhesive bonding usually requires little heat to accomplish the joint. The load distribution across the joint is favorable, with no interruptions and magnifications due to holes and abrupt changes in section. Little post bonding, finishing or cleanup is required, and few specialized personnel are needed. The adhesive layer provides a good insulation barrier, and may exhibit superior fatigue strength.

Some of the more important limitations to the bonding process include the instability of the adhesives in extreme environments. The material tends to deteriorate with time, heat and exposure. The finished joints are difficult to inspect and test nondestructively. Strong, reliable joints require good surface preparation; a substantial loss of expected strength can occur with poorly cleaned interfaces. Bonded joints are susceptible to transient or unsuspected loads such as peeling.

Generally, the strongest joints are made with the thinnest adhesive film, although many adhesives seem to be relatively insensitive to moderate variations in film thickness. The designer should strive to use as much adhesive surface as possible in shear. Some methods for improving common joints are shown in the adjacent figures.



JOINTS: Some common types of adhesive joints are illustrated. The maximum use of surface area is desirable.

FORMING BY CASTING

Most engineering materials may be molded by one or more of the casting procedures. The amount of post-solidification machining, and the strength of the as-cost material are limiting considerations.

A casting may be simply described as the product of pouring a molten material into a prepared cavity and allowing it to solidify. The process of casting dates to antiquity, and probably antidates forging as a method of material forming. Virtually any material may be cast by one or another process, and almost any shape and complexity of structure may be reproduced by some casting process.

The casting process consists of first preparing a cavity for the molten material. The cavity may be constructured of sand, metal, plaster, wax, or any material which will retain its shape while the cast material solidifies, and still allow a certain degree of permeation or gas escape, while solidification proceeds. The cavity must be formed by copying a pattern, removal of the pattern, and pouring of the molten material. After solidification, the casting must be cleaned and may require machining into the desired final dimensions. The amount anddetail of these operations constitute the limiting cost factors in the use of castings.

Since most casting procedures involve the removal of a pattern from the mold or casting from a permanent mold, draft angle and parting line are then legitimate design prerogatives. However, all draft angles may be eliminated by some method if it is economical. The particular casting process employed for a given situation will depend upon many factors, including cost, production quantity, the cast alloy itself, the complexity of the casting, the desired dimensional accuracy, the degree of subsequent machining and heat treatment, just to name the most obvious.

Casting processes currently in use for the structural materials of interest to packaging engineers, are many and varied. The most common includes sand casting (both green sand and dry sand), shell molding, centrifical casting, permanent mold, die casting, plaster molding, and investment casting. There are some hybrid processes whose characteristics are close to casting, such as sintering and powder metallurgical techniques. Casting processes are generally classified by either the type of mold or the pressure used to fill the mold.

Sand casting is the process of molding sand (with a binder) around a pattern, removing the pattern, and filling the cavity with molten material. Shell molding is a similar process except that the sand is coated with a thermosetting resin, hardened around the pattern, stripped away from the pattern, supported, and poured. Permanent mold castings are poured in a metallic mold whose cavity has been machined into the mold. Plaster molded castings are those in which the mold has been constructed of a hardened plaster material. In all of the preceding cases the casting material is usually gravity-fed into the mold.

Investment casting is the process whereby the pattern is entrapped within the mold after setting, and is subsequently melted out. After pouring, the mold is usually broken away from the casting.

Centrifugal casting relies upon the forces induced by spinning the mold to hold the molten material in place while the material soldifies. The liquid is thrown out against the wall of the cavity during the hardening process.

Die casting procedures are typically those where the molten material is injected into a closed steel mold under pressure. When the material has solidified, the die mold is opened and the casting ejected.

Powder metallurgy, which is the compaction of alloy powders into a die and simultaneous or subsequent heating until bonded, is not a usual packaging procedure, and hence is only noted here.

Casting Process	METALS USED IN CASTING PROCESSES (2)												
	Irons	Steels	Heat and C.R. Alloys	Aluminum Alloys	Copper Alloys	Lead Alloys	Magnesium Alloys	Nickel Alloys	Precious Metals	Refractory Metals	Tin Alloys	Titanium Alloys	Zinc Alloys
Sand Castings	M	M	M	M	M	O	M	M	X	X	O	X	O
Shell Mold Castings	M	O	O	M	M	X	X	O	X	X	X	X	X
Permanent Mold Castings	M	O	X	M	O	O	M	O	X	X	O	X	O
Die Castings	X	X	X	M	M	X	X	X	X	X	X	X	X
Plaster Mold Castings	X	X	X	M	M	X	X	X	X	X	X	X	X
Investment Castings	X	M	O	M	M	X	X	O	X	X	X	X	X
Centrifugal Castings	M	M	M	O	O	X	X	O	X	X	X	X	X

M = Materials most frequently used.

O = Other materials currently being used.

X = Not used, or cast with great difficulty.

CASTING: Most of the common engineering materials may be cast by one process or another. The casting process should be optimized for the material being used.

FORMING PROCEDURES INVOLVING MATERIAL REMOVAL

Forming by machining may be accomplished on virtually all of the engineering materials. Machined elements are characterized by high dimensional accuracy, and good surface smoothness.

Forming operations by material removal or machining are probably the most often employed manufacturing processes for those that require a high degree of dimensional accuracy or surface smoothness. Material removal processes often are used as direct substitutes for the other forming procedures (such as casting and material deformation) because cooling requirements are minimized. The design criteria for this selection include cost, ease of the forming operation, and the suitability of the procedure to produce a product that is environmentally insensitive.

A convenient categorization of the machining processes may be made on the basis of the operational procedures involved, the amount of material removed, and its effect on the finished surface.

1. Milling and turning operations include lathe-work, planning, shaping, sawing, filing, hobbing, shaving, spotfacing, and all the milling and routing procedures. Chip removal and machinability of the material are major considerations in the application of these processes. In general, a relatively large amount of material is removed by these machining operations, and surface finishes vary from quite rough for coarse planning work to very fine for slow-feed milling and turning operations. A major consideration in production milling and turning is the influence of material properties on tool life.

2. Hole preparation involves a modest amount of material removal, a wide range of surface finishes, and is subject to the same considerations of machinability, chip removal, and tool wear as the milling operations. This category includes drilling, reaming, boring, counterboring, tapping, and spotfacing (virtually a milling operation).

3. Surfacing or sizing operations are characterized by generally small amounts of material removal and a relatively fine surface finish. The degree of cold work accomplished by the surfacing procedures are usually beneficial to the member under fatigue service conditions. The surfacing category includes grinding, burnishing, broaching, honing, lapping, sizing, ballizing, buffing and polishing.

4. Material removal by exotic or unusual processes is becoming increasingly important in the fabrication of large structures. They are characterized by small cutting forces, or no contact by conventional cutting tool, and are used extensively on brittle materials, curved surfaces, and unusually thin or complex shapes. These operations include chemical milling, ultransonic machining, electro-discharge and electro-chemical milling, and cutting and welding by electron and laser beams.

Most metals, alloys and structural materials can be machined by some or all of the preceding operations. Some materials are machined with ease, and others with great difficulty. The material characteristic which reflects this facility is machinability, or the ease of material removal. This term implies that the forces acting against the cutting tool will be relatively low, that the chips will come away from the cutting surface easily and will be broken up, and that the desired surface finish will result.

The basic material properties desirable in machining are, in many cases, the opposite of those desired for normal serviceability. For example: softness allowing easy cutability is often desirable, and reactivity where the new surface oxidizes rapidly minimizes galling, cold welding, and tool build-up.

FACTORS AFFECTING MACHINABILITY OF METALS

	Factors That Increase Machinability	Factors That Decrease Machinability
Structure	Uniform microstructure	Nonuniformity
	Small, undistorted grains	Presence of abrasive inclusion
	Spheroidal structure in high-carbon steels	Large, distorted grains
	Lamellar structure in low- and medium-carbon steels	Spheroidal low- and medium-carbon steels
		Lamellar high-carbon steels
Treatment	Hot working of alloys that are hard, such as medium- and high-carbon steels	Hot working of low-carbon steels
	Cold working of low-carbon steels	Cold working of higher-carbon steels
	Annealing, normalizing, tempering	Quenching
Composition	Small amounts of lead-manganese, sulfur, phosphorus	Carbon content below 0.30% or above 0.60%
	Absence of abrasive inclusions such as Al_2O_2	High alloy content in steels

MACHINING: Material removal or machining is the most widely used forming process because of accuracy and control.

STRUCTURAL FORMING BY MATERIAL DEFORMATION

Forming operations by material deformation is a chipless process involving the plastic (permanent) movement of material to accomplish the desired end-shape.

Forming techniques which employ the process of material deformation to accomplish structural shapes may generally be characterized as chipless. As one old machining buff put it, "it's easier to move it than remove it." Often, it is also better from the standpoint of surface strength, toughness and hence fatigue strength, to do work on the material in forming it, particularly at lower temperatures.

Forming procedures are accomplished either hot or cold. Hot forming is plastic deformation which is done generally above the material's recrystallization temperature, and usually in a temperature range where the material is more plastic. Cold work which increases hardness, strength, and resilience, but lowers ductility, is a plastic deformation occurring at room temperature.

Hot forming procedures are typified by the forging and extruding operations. Forging is the process of squeezing, hammering or otherwise plastically deforming a material into a desired shape with generally good mechanical properties. The material may be in the form of a billet, ingot, bar, or powder-metal shape. Virtually all ductile materials may be forged. Forgings usually exhibit strongly directional characteristics, a fact that may be used to advantage in certain situations.

Extruding is the process of forcing a hot material through a die opening to form a shape having the geometry of the die like toothpaste squeezed out of a tube. Tooling costs may be high for an extruded shape, but quantity runs are highly feasible. Most of the ductile structural materials can be extruded, although the higher melting point alloys are the most difficult. Extrusion sizes (cross-sectionally) are limited by the extruding machine capacity, usually about 6" in diameter. The length of the extrusion is determined by the volume of material that may be accommodated in front of the ram, and is usually large with respect to the net diameter.

Drawing, cold extruding, impact extruding, and cold forging are examples of processes commonly applied to form deep-drawn, container type geometries. The work is accomplished cold, such that the material mechanical properties are improved. Resultant surface finishes and dimensional tolerances are generally good, and shapes may be produced without draft relief.

Cold heading is the process of upsetting a shape on a material blank, usually larger in diameter than the shaft (like a head on a nail). Most of the structural materials that can be cold-worked without tearing may be cold-headed.

Stamping is a forming operation that is usually limited to sheet or coin type material shapes. This process includes the operations of shearing, punching, piercing, coining, many forming procedures, drawing, ironing and striking. Most structural materials may be stamped, if they have sufficient ductility and may be plastically formed without tearing.

Metal spinning, deep drawing, and roll forming are example of forming procedures that are capable of producing relatively thin walled structures, that exhibit good surface fatigue strength. Most of the ductile materials may be formed by these processes, and many complex or multi-operation forming steps are possible.

Some of the newer innovations in the forming field include: high energy forming (HERF) procedures which supply the deforming energy by controlled explosion or similar source; magnetic forming where high energy magnetic impulses act as the deforming force; and cold rolling operations used to finish threads, gears, and other highly stressed parts subjected to repeated loading.

RELATIVE FORGEABILITY INDEX OF METALS

Material	Index
Low-Carbon Steel	1.0
Brass	
Medium-Carbon Steel	1.1
Copper	
Ductile Titanium Alloys	
Lower Alloys of Cr-Ni-Mo	
High-Carbon Steel	1.2
6061 Aluminum	
Higher Alloys of Cr-Ni-Mo	1.3
Hy Tuf	
AMS 6407, AMS 6427	
	1.5
400 Series Stainless	
2014 Aluminum	
Most Magnesium Alloys	
300 Series Stainless	
Tricent	
Vascojet 1000	1.75
Thermold J	
7075, 7079 Aluminum	
17-4 PH, 17-7 PH	
	2.0
AM 350, AM 355	
A-286	
Discaloy	
Stainless W, X	2.5
16-25-6	
19-9-DL	
N-155	3.0
Hastelloy C	
Molybdenum	
Udimet 500	
Inco 700, 718	
	3.5
Rene 41	
M-252	
Waspaloy	
Tough Titanium Alloys	
Columbium	
Tungsten	5.0

*Higher index numbers indicate less forgeability.

FORGING: Forging is the most widely used method of forming by material deformation. Some materials are inherently more forgeable than others.

HEAT TREATMENT: A MATERIAL PROCESS TO IMPROVE MECHANICAL PROPERTIES

A knowledge of the mechanisms and results of the common heat treatment processes is an important design skill.

Heat treatment of metals and alloys is employed to accomplish a variety of objectives. It may be used to increase mechanical strength, relieve residua stresses, improve fabrication properties, to reduce electrical resistivity or increase permeability and magnetic properties. Conversely, the process may be employed to soften metals for machining or increase electrical resistance where needed.

The most common use of heat treatment is to improve the mechanical properties of an alloy. The usual heat treatment used to strengthen steels consists of hardening and tempering. Hardening is accomplished by first heating the alloy to above its critical temperature which causes a phase transformation to austenite. Subsequent rapid cooling (quenching) produces a hard brittle structure known as martensite which has high internal stresses. By heating at moderately elevated temperatures, this structure is tempered (drawn) to a more ductile, tough, condition while retaining most of its tensile strength and hardness. By varying the tempering temperature, a spectrum of mechanical properties can be obtained. The range of strengths obtainable by the quench and temper process is directly related to the type and amount of alloying elements present in the material.

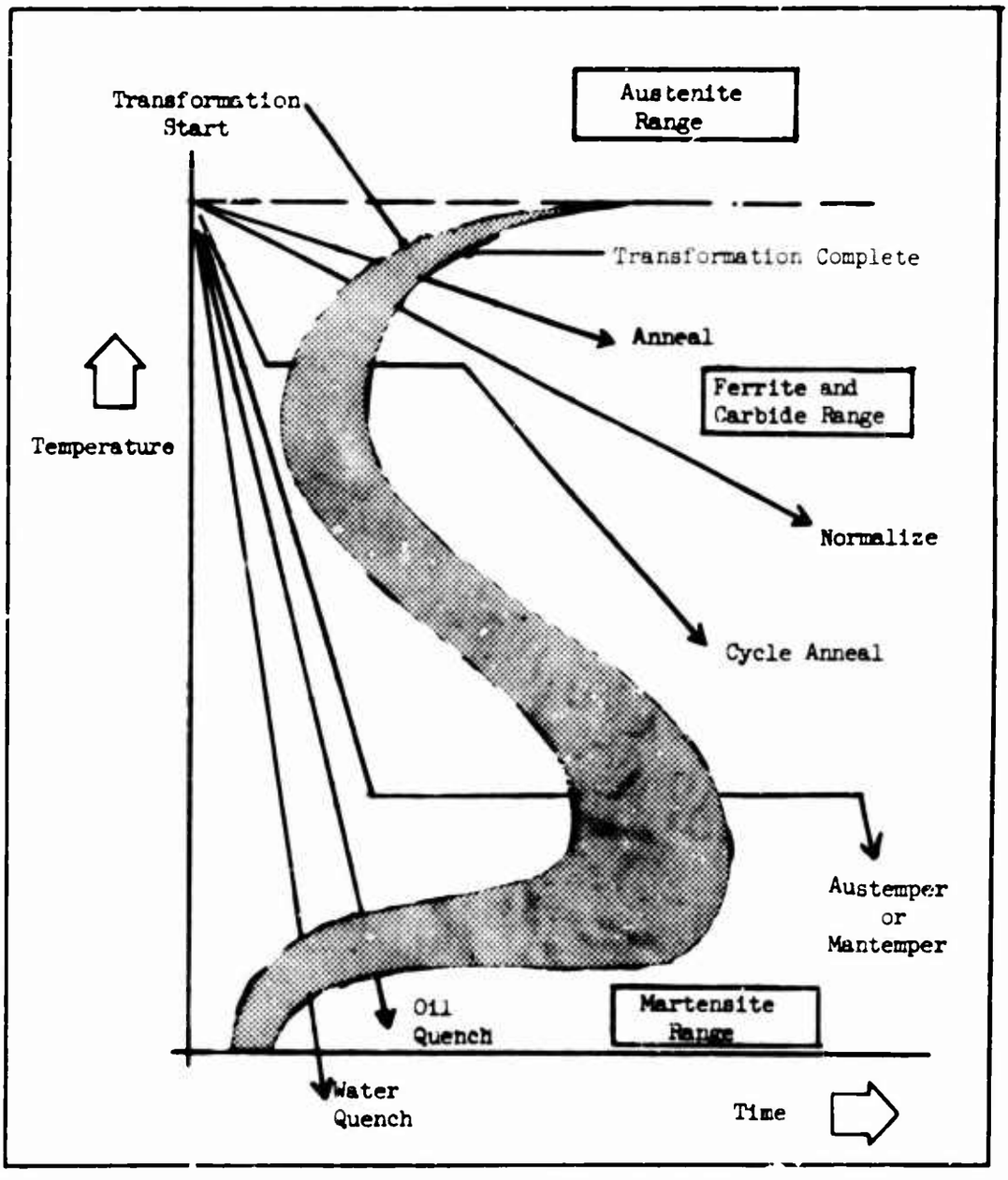
Another well known heat treatment process used to improve mechanical properties in certain structural alloys is precipitation hardening. This process is used on the aluminum, magnesium, titaneium, and nickel alloys, beryllium copper, and some steels. The material is first heated to an elevated temperature, where the constituents are dissolved into solid solution, and quenched rapidly to room temperature. This treatment leaves a microstructure that is somewhat unstable since some of the constituents were retarded from precipitating during the rapid quench. As the constituents precipitate with time, or are artificially aged at moderate temperature, the alloy properties are improved. A degree of cold work is often combined with this heat treatment to develop optimum mechanical properties in the alloy. Some alloys harden by a combination of martensitic and precipitation mechanisms.

Parts which have been severely formed, welded, brazed, or machined may contain internal stresses which can impair their usefulness. These stresses are minimized by subsequent heat treatment, can be reduced by stress relieving at lower temperatures, or can be mechanically aligned by such work procedures as stretching.

Various heat treatments are used with different alloys to improve their machining properties. Solution heat treatment and annealing are used to facilitate forming, both as a pretreatment and, if necessary, as an intermediate process between forming operations.

A knowledge of the various heat treatment mechanisms is useful to the designer in specifying a heat treatment to achieve a desired result. In steels, a convenient method of classifying these procedures may be made by superimposing the time-temperature plots of the processes on a typical transformation diagram, illustrated in the adjacent figure. The optimum hardening

process is one that escapes the knee of the s-shaped transformation diagram, resulting in a modified martensitic microstructure. The slower cooling rate causes a more orderly transformation of microstructure, and generally results in a softer material. Higher hardenability results when the knee of the transformation curve is moved to the right, generally because of the addition of certain alloying elements.



HEAT TREATMENT: The "S" shaped time-temperature transformation plot is a helpful device for understanding the cooling products of steel alloys.

SURFACE HARDENING TECHNIQUES

A significant improvement in the strength, wear resistance, and environmental insulation may be accomplished by judicious application of surface hardening procedures.

There are numerous effective surface hardening techniques available to the structural designer, ranging from local quench hardening procedures to surface strain hardening by cold work. Actually, many of the plating and chemical finishing processes discussed in the following section provide a measure of surface hardening effect along with environmental protection. Similarly, the surface hardening techniques discussed here also provide a measure of environmental protection to complement the strengthening and wear resistance improvement imparted to the material surface. This is particularly true of the cold finishing procedures.

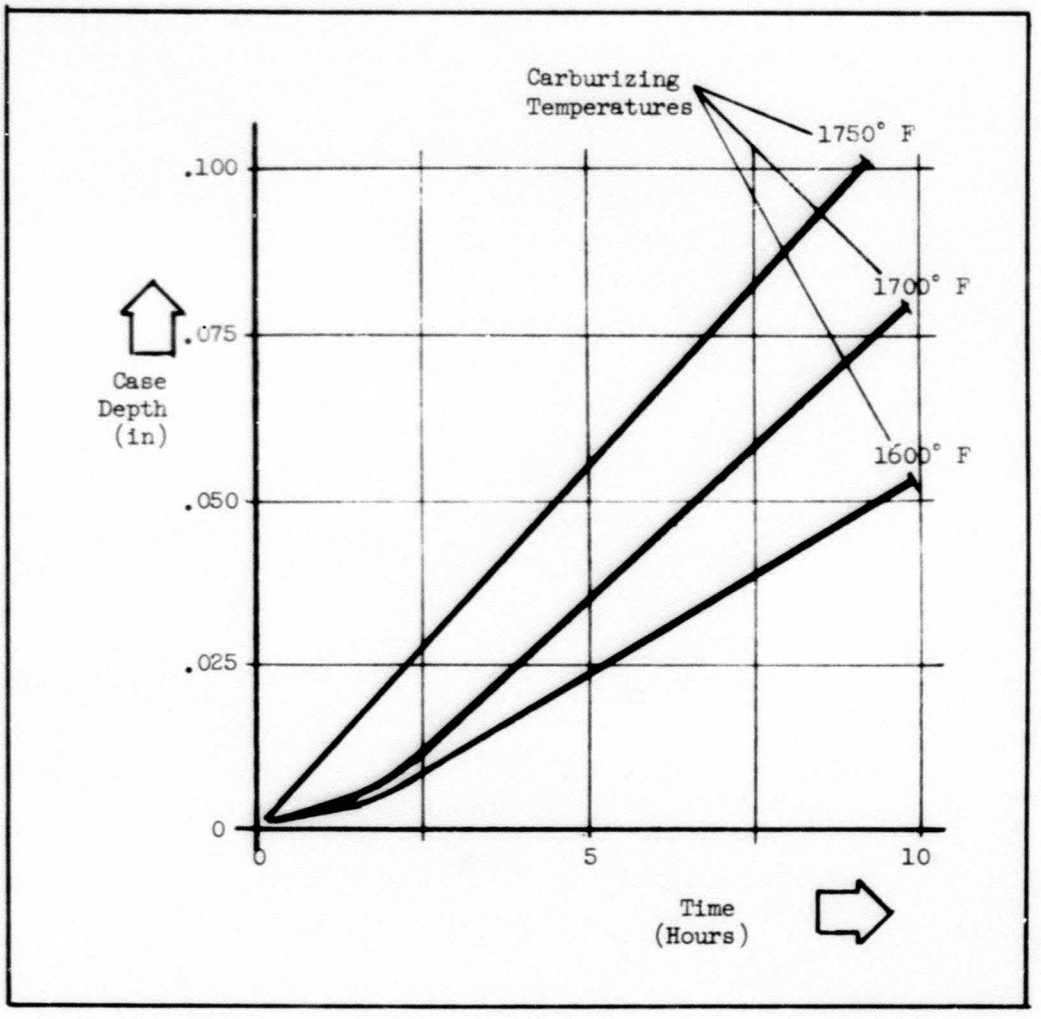
A widely used local hardening technique is the flame hardening process, which takes advantage of the inherent hardenability of the base material. No additional alloying elements are imparted into the surface, as in other processes. The local area is flame heated and quenched to obtain a martensitic microstructure. Hardened depths up to 1/4 inch are possible by the technique, depending upon heating capability and material characteristics. This local hardening process is most effective on large structures or members where general heating is impractical.

Carburizing is the process by which carbon is introduced into the surface of a ferrous material while the metal is heated in the presence of a carbonaceous material, either solid, liquid, or gas. The time of exposure at this elevated temperature determines the depth to which the penetration occurs. The carburizing process is usually followed by quench and temper to attain the desired strength level in the case. Carburizing grade steels are usually low enough in hardenability that the core material is unaffected by this process. Care must be taken in the carburizing procedures to minimize grain growth due to prolonged soaking at relatively high temperatures.

Although generally limited to shallow case hardened depths, cases up to .020 inch are possible by cyaniding and carbonitriding processes. The cases, which contain both nitrogen and carbon, are produced in solid cyanide salts, liquid baths, or gas atmospheres. The exposure temperatures are generally lower than those for the carburizing process, which, coupled with a shorter exposure time, results in a thinner case with less distortion. The extreme hardness in these thin cases is obtained by subsequent quench from elevated temperature followed by a temper at lower temperature.

Nitriding is a method of casehardening steels by treating the surfaces with a nitrogenous material, such as ammonia. Nitrides are formed near the surface of the steels, which possess high inherent hardness; no following quench of the material is required to develop the desired surface characteristics but a quench may be desirable to develop maximum properties in the core. Nitriding temperatures in the range of 1000°F are common (although higher temperatures may be used) and exposures range up to 100 hours. Nitriding produces a white layer of material that must be removed before the member is put in service. Fatigue strength is generally improved by nitriding, as is corrosion resistance in certain steels. A notable exception is the stainless steel group of alloys.

Strain hardening by cold work is an effective method of improving local hardness in susceptible materials, particularly after the member is formed and no other heat treatment can be tolerated. Burnishing, rolling, buffing, honing, lapping, and polishing are helpful in strengthening an area in the region of a discontinuity, as well as reducing the effect of local surface blemishes.



CARBURIZING: The introduction of carbon into the surface of ferrous alloys followed by heat treatment is a common surface hardening technique. The case depth depends upon exposure time and carburizing temperature.

PLATING AND CHEMICAL FINISHING

A significant improvement in the environmental resistance and mechanical properties of the surfaces of engineering materials may be accomplished with proper plating and chemical finishing techniques.

Plating and chemical finishes on metals can be used to meet many engineering design requirements. The most frequent applications of these processes are the improvement of:

- Corrosion protection,
- Electrical properties,
- Mechanical properties, and
- Appearance.

One of the most common uses for finishes is corrosion protection. There are two broad categories of plating systems for corrosion protection. The first involves the use of a less noble (anodic) metal which is plated on a more noble cathodic base metal. The plating will corrode preferentially to protect the base metal, and where thin scratches or imperfections occur in the plating, the base metal will be protected until the plating metal becomes depleted in these immediate areas.

A second type of plating protection involves the use of platings which are more noble than the base metal which is to be protected. Since the base metal is less noble than the plating, it will corrode if exposed; thus, it is necessary for the plating to be as free of pinholes as possible. However, pitting of the base metal will eventually occur in the sites of scratches and unavoidable porosity.

Chemical finishes of either the anodic or conversion type can also be used to provide corrosion protection. They are frequently used because they serve as a good base for paint and because their tightness slows the corrosion processes. Chromate coatings offer protection for aluminum, magnesium, cadmium and zinc. Stainless steels are generally passivated (oxidized). Alloy steels are oxide or phosphate treated.

Platings offer a conductive surface as well as corrosion protection and in some cases (such as silver, gold, and copper plating) may be more conductive than the base metals. Both plating and anodic finishes may provide a hard, wear and abrasion resistant coating.

Other platings, such as rhodium or chromium, are used as gall-resistant surfaces. Porous platings are useful as minute oil pockets. Other platings may improve soldering characteristics.

Metal platings do not remove scratches and surface defects. In fact, electroplates generally magnify these surface defects. Thus consideration must be given to the removal of surface blemishes by burnishing or buffing before final plating.

Organic finishes, like plating and chemical finishing, are used on equipment structure to enhance appearance and corrosion resistance. Organic finishes may be divided into categories by type or purpose. These processes include primers, epoxies, enamels, lacquers, and special finishes.

Primers are used as a pretreatment coating to provide better adhesion of the final coating, as well as additional corrosion resistance. They often react with the substrate. Epoxy paints can provide a good combination of wear and abrasion resistance combined with excellent corrosion resistance as they do not require the evaporation of a carrier but harden merely by reaction. The use of epoxies, however, is justified only where their protective qualities are essential due to the high material cost.

Enamels and lacquers are generally used as decorative finishes, although they may provide a good resistance to moderate environments. Enamels and lacquers are not as resistant to harsh environments and abrasion as the epoxy materials.

Some of the special finishes of note are decorative (such as black wrinkle and epoxy splash), but many are functional as well. Proper manipulation of these finishes can provide a range of conductivity, reflectivity, thermal control, and good environmental resistance.

CATHODIC (Most Noble)

Platinum
Gold
Rhodium
Silver

300 Series CRES (18-8)
Titanium
Chromium
Copper-Nickel Alloys
Nickel and Alloys
Silver Solder
Copper and Alloys
400 Series CRES (12 percent Cr)
Tin
Lead
Lead-Tin Solder

Iron and Steel
Aluminum Alloys (Over 2 percent Copper)
Cadmium
Aluminum Alloys (Low or no Copper)

Zinc

Magnesium and Alloys

ANODIC (Least Noble)

THE GALVANIC SERIES: Base metals are subject to galvanic corrosion by an exchange of ions. Metals widely separated on the galvanic scale are most susceptible and must be protected from one another by plating or coating.

POTTING AND ENCAPSULATION

Electronic equipment elements are often encased in non-metallic materials to provide electrical insulation, stress protection, and environmental isolation.

There are many processes by which magnetic and electronic components may be encased in plastic or elastomeric resins for electrical insulation and protection from environmental conditions and mechanical damage. The following definitions will help in distinguishing among the many similar processes.

Embedment:	A process by which circuit components are encased in a dielectric material. It includes both potting and encapsulation.
Potting:	An embedment process in which the container (can) used in embedment remains as part of the completed assembly.
Encapsulation:	An embedment process in which the resin is cast in removable molds.
Impregnation:	A process by which the assemblies are filled with resin, usually by means of a vacuum-pressure cycle.
Casting, Molding:	Processes by which plastic parts are formed in a mold from a liquid, powder, or granulated resin.
Conformal Coating:	A thick coating which generally follows the contours of the assembly, providing resistance to mechanical shock and environmental conditions.

The selection of embedment process is usually dependent on anticipated service environment. Temperature, humidity, vacuum, thermal shock, and corrosive chemicals and solvents can all have serious effects on components and encapsulating resins. Electrical requirements include dielectric strength, dielectric constant, insulation resistance, loss factor, and corona suppression. Mechanical requirements might be impact resistance, flexibility or rigidity, weight, and strength. Shrinkage of resins during cure can cause mechanical stresses in components and connections. In many cases, shrinkage can cause microcracks requiring secondary impregnation to create an effective seal.

Tooling is an especially important consideration in the design of encapsulating assemblies and the selection of appropriate processes. In designing for potting or encapsulation, at least 0.050 inch of resin should cover the components, connections and other surfaces near the exterior of the completed assembly. Eutectic metal slush molds are used when up to 75 pieces are to be encapsulated. Vacuum-formed molds are used for up to 125 pieces. Silicone rubber molds may be used instead of the vacuum formed type, but they are short-lived and not as reliable dimensionally.

Resins used in these processes include epoxies, silicones, polyamides, polysulfides, polyethylenes and polyurethanes. The thermal, mechanical, and electrical properties can be controlled and adjusted by the use of various fillers.

Properties Required: Excellent electrical resistance in low to medium frequencies. High strength and impact properties, good fatigue resistance, and heat resistance. Good dimensional stability at elevated temperatures.
Suitable Plastics: Allylics, alkyds, aminos, epoxies, phenolics, polycarbonates, polyesters, and silicones.
Other Suitable Materials: Ceramics and glass.
Consider Plastics When: 1. Shock loadings are high. 2. Minimum weight is important. 3. Dimensional accuracy must be close. 4. Complex integral conductor-insulator parts are needed (printed circuitry and slip-ring assemblies).
Consider Other Materials When: 1. Extreme temperatures are encountered. 2. Compressive loadings are high.
Property Summary: Polycarbonates for transparent parts requiring high impact strength. Cast epoxies for encapsulating electric or electronic assemblies for maximum environmental resistance. Molded epoxies for uses which require dimensional stability over wide temperature ranges. Melamines for hardness. Silicones for high heat resistance. Aminos for low cost. Phenolic laminates for punched, stamped parts.

	Cost, Base Resin (¢ per cu in)	Impact Strength (ft-lb/in of notch) Range	Impact Strength (ft-lb/in of notch) Typical	Flexural Strength (1000 Hz) Range	Flexural Strength (1000 Hz) Typical	Dielectric Strength (v per mil) Range	Dielectric Strength (v per mil) Typical	Remarks
Allylics	4.5 to 22	3 to 6	..	10 to 20	20	375 to 400	350	Dielectric properties little affected by moisture; has excellent dimensional stability.
Alkyds	3 to 4.5	0.3 to 12	2.5	7 to 17	10 to 15	300 to 350	350	Excellent dimensional accuracy and uniform, low shrinkage during cure.
Aminos	1 to 3	0.3 to 12	7	10 to 23	14	320 to 430	360	Hard scratch-resistant surfaces; retains whiteness.
Epoxies	2.4 to 5	0.4 to 30	8 to 15	12 to 60	20 to 26	375		Outstanding adhesion to metallics or nonmetallics, excellent chemical resistance, low shrinkage rate in encapsulation.
Phenolics	1.5 to 3	0.3 to 27	3.4	10 to 45		300		Available in casting or molding compounds.
Polycarbonates	4.5	2 to 16	12 to 16	11 to 13	12	400 to 440	410	Transparent.
Polyesters	0.9 to 3.2	1.5 to 24	..	6 to 25	13 to 20	345 to 420	..	Available in rigid or flexible forms, readily colorable, transparent to radio waves of radar frequency.
Silicones	13 to 25	0.3 to 10	6.5	7 to 18	12	350 to 400	350	Retains strength and electrical properties after prolonged exposure to heat.

POTTING MATERIALS: The table illustrates the physical properties of the common encapsulating material for application in electro-structural elements. (31)

VOLUME III - CHAPTER 10

MATERIALS AND PROCESSES

SECTION 6 - APPENDIX

- Bibliography
- Glossary
- Review of Principal Casting Processes
- Steel Alloys
- Stainless Steel Alloys
- Aluminum Alloys
- Special Aluminum Applications
- Magnesium Alloys
- Copper Alloys
- Nickel Alloys
- Miscellaneous
- Plastics

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GLOSSARY

Strength – The measure of a material's ability to resist load without rupture, collapse, or undue distortion.

Stress – The application of load to a member caused by the intensity of the internal resisting force.

Strain – A dimensional change produced by stress. Hooke's law states that stress is proportional to strain (within the proportional limit).

Proportional Limit – The stress above which Hooke's law ceases to apply.

Modulus of Elasticity – The ratio of stress to accompanying strain provided the stress does not exceed the elastic limit.

Stiffness – The property measured by the modulus of elasticity. Stiffness implies high resistance to elastic deformation.

Elasticity – The property of recovering original shape and dimensions upon removal of a deforming force.

Plasticity – The property of being permanently deformed without rupture.

Ductility – The property of undergoing considerable permanent deformation when tested in tension. This property is correlated with ability to be drawn into wire. Ductility is measured by percent elongation and percent reduction of area from the tensile test.

Malleability – The measure of a material's ability to be hammered into sheets.

Toughness – The property of requiring a large amount of work (force x distance) to produce rupture. Tough materials have high strength combined with high ductility. Toughness is proportional to the area under the stress strain diagram.

Brittleness – The property of requiring a small amount of work to produce rupture. Brittle materials may have high tensile strength but always have low ductility.

Resilience – The ability to store and release energy upon application and removal of a load by deformation within the elastic range.

Modulus of Resilience – The amount of energy absorbed when one cubic inch of material is stressed to its elastic limit. The modulus of resilience is proportional to the area under the elastic portion of the stress-strain diagram. Materials having a high modulus of resilience are capable of undergoing high impact without permanent injury.

Ultimate Strength or Tensile Strength – The maximum stress exerted in the tensile test. The stress is computed on the basis of the original cross sectional area of the tensile specimen and is expressed in force per unit area transverse to a uniaxial load.

Hardness – A complex, non-linear, non-dimensional measure of the resistance to permanent deformation by indentation.

Elastic Limit – The greatest stress which a material is capable of withstanding without permanent deformation upon release of stress.

Yield Point – The stress at which there is a marked increase in strain with no further increase in stress. At the yield point the stress-strain curve becomes horizontal or drops. (Generally applied to ferrous metals.)

Yield Strength – The stress at which a material exhibits a specified permanent set. This stress is determined by drawing a line parallel to the elastic portion of the stress-strain curve and passing through the point on the strain axis marking the specified set, usually 0.2% elongation. The stress given by the point of intersection of this line with the stress-strain curve is the yield strength. (Generally applied to non-ferrous metals.)

Creep – A continuing change in the deformation or deflection of a stressed member. Creep of metals is the growing deformation under continuous load. Ordinary plastic deformation requires a constantly increased stress if it is to be continuous. The flow occurring in creep will continue indefinitely at constant stress. The rate at which creep occurs depends on load and temperature. Creep can occur at a given temperature at loads less than the proportional limit of the metal as determined at the given temperature. The creep resistance of materials is expressed in terms of their "creep limits".

Creep Limit – The maximum stress to which a material may be subjected without having the inelastic deformation exceed a specified amount after a specified time at a specified temperature.

Fatigue – Progressive fracture of a member by means of a crack which spreads under repeated cycles of stress. Resistance to fatigue is often expressed in terms of "endurance limit", but this term pertains to a specific test generally.

Endurance Limit – The maximum stress to which a material may be subjected many millions of times without failure. Ten million cycles without failure is generally regarded as indicating a stress below the endurance limit for steel.

Endurance Ratio - The ratio of endurance limit to ultimate strength. For most ferrous materials this value lies between .4 to .6.

Recrystallization Temperature – The temperature at which a work hardened metal will be restored to its soft condition as a result of a recrystallization which does not involve a phase change.

Cold Work – Plastic deformation of a metal at a temperature below the recrystallization temperature.

Hot Working – Plastic deformation performed at a temperature above the recrystallization temperature.

GLOSSARY (Continued)

Annealing – Originally a term implying heating to some elevated temperature, holding, and slow cooling to put the metal in a soft condition. At present the term has several meanings as defined below.

Annealing (Following Cold Work) – The heating of cold worked metal to a temperature above its recrystallization temperature, holding at temperature, and cooling as desired. The purpose of this treatment is the softening of strain hardened metal and the restoration of ductility and formability.

Process Annealing – Heating of cold worked, low carbon steel to 1000°F - 1250°F, holding, and cooling as desired. The purpose of this treatment is to produce softening by recrystallization of the ferrite.

Full Annealing – Heating of steel to above its critical range, holding and slow cooling.

Cycle Annealing – Heat treatment in which steel is quenched from the austenitic range in molten salt at a temperature between 1000°F, and the lower critical, and held at temperature to bring about isothermal transformation to a microstructure which contains pearlite.

Normalizing – Heating steel to approximately 100°F above the upper critical, holding, and cooling in still air. The microstructure produced by this treatment in any given steel is dependent on section size.

Tempering – Reheating a quenched steel to a temperature in the range of 400°F to 1300°F, holding and cooling as desired.

Drawing – Term used synonymously with tempering.

Spheroidizing – A heat treatment for steel in which the steel is held very close to the lower critical temperature for a time sufficient to produce a microstructure consisting of globular particles of cementite in a matrix of ferrite, followed by cooling as desired.

Austempering – Heat treatment in which steel is quenched from the austenitic region in a salt bath at a temperature in the range of 400° - 1000°F, followed by holding at constant temperature for a time sufficient to transform the austenite to the dark-etching acicular structure, Bainite.

Martensite – The microstructure in steel produced by cooling from the austenitic condition at a rate exceeding a certain critical rate. When viewed under the microscrope martensite appears as a mass of needle-like crystals in a light etching background. Martensite is the hardest and strongest structure which can be developed in a given steel, the actual hardness being dependent on the carbon content of the steel.

Tempered Martensite – The microstructure produced by heating martensite to an intermediate temperature range. Martensite is hard and strong, but is brittle. Reheating martensite (tempering or drawing) lowers the hardness, strength, and elastic limit but raises the ductility and toughness.

Solution – A state of matter defined by the following characteristics:

1) no definite proportion by weight
2) homogeneity
3) no settling or separation on standing.

Steel – An alloy of iron and carbon containing less than 1.7% carbon, and produced by casting from the liquid state into an ingot which is malleable.

Martempering – Heat treatment in which steel is quenched from above the critical range in a salt bath at approximately 450°F, and held a temperature for a period insufficient to transform the austenite, but sufficient to equalize the temperature in the steel, followed by cooling as desired to room temperature. The microstructure produced by this treatment is martensite.

Hardenability – The property which determines the depth to which a steel will transform to martensite when quenched from the austenitic condition. An inverse relationship exists between the hardenability of a steel and its critical cooling rate. Steels high in hardenability have low critical cooling rates, and as a result have a deep penetration of the effect of quenching.

Inherently Coarse Grained Steel – A steel in which the austenite grains undergo an appreciable increase in size as the temperature is raised from the upper critical to 1700°F. Steel of this type will show an A.S.T.M. grain size in the range of 1 to 5 when subjected to the McQuaid Ehn test.

Inherently Fine Grained Steel – A steel in which the austenite grains do not coarsen appreciably as the temperature is raised from the upper critical to 1700°F. Inherently fine grained steels will show an A.S.T.M. grain size of 5-8 when subjected to the McQuaid Ehn test.

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	Advantages	Limitations
Sand casting	Almost any metal can be used; almost no limit on size and shape of part; extreme complexity possible; low tool cost; most direct route from pattern to mold	Some machining always necessary; castings have rough surfaces; close tolerance difficult to achieve; long thin projections not practical; some alloys develop defects
Shell-mold casting	Rapid production rate; high dimensional accuracy; smoother surfaces; uniform grain structure; minimized finishing operations	Some metals cannot be cast; requires expensive patterns, equipment, and resin binder; size of part limited
Plaster-mold casting	High dimensional accuracy; smooth surfaces; almost unlimited intricacy; low porosity; plaster mold is easily machined if changes are needed	Limited to nonferrous metals and relatively small parts; mold-making time is relatively long
Investment casting	High dimensional accuracy; excellent surface finish; almost unlimited intricacy; almost any metal can be used	Size of part limited; requires expensive patterns and molds; high labor costs
Permanent-mold casting	Good surface finish and grain structure; high dimensional accuracy; repeated use of molds (up to 25,000 times); rapid production rate; low scrap loss; low porosity	High initial mold costs; shape size and intricacy limited; high-melting-point metals such as steel unsuitable
Die casting	Extremely smooth surfaces; excellent dimensional accuracy; rapid production	High initial die costs; limited to nonferrous metals; size of part limited
Centrifugal and centrifuge casting	Centrifugal force helps fill mold completely. Gases and impurities are concentrated nearest center of rotation. Solid good outer surface, gates and risers can be kept to a minimum.	Alloys of separable compounds may not be evenly distributed. Castings must be symmetrical. Centrifuge – generally limited to small intricate castings.

STEEL ALLOYS

STEEL SHEET AND PLATE

SPECIFICATION INFORMATION

DRAWING CALLOUT			MIN TENSILE PROPERTIES			SIZE RANGE, INCHES	APPLICATIONS
[1]ALLOY AND SPEC	COND	FORM	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.		
1005-1020 QQ-S-698	CR#4 CR#5 HRAR	Strip Strip Sheet	--- --- ---	--- --- ---	-- -- --	--- --- ---	LOW CARBON STEEL. Low strength with fair to good ductility for cold forming. Fair machinability. Suitable for welding or brazing. Specify finish (No. 3 for CR, AR for HR)
A373 QQ-S-741		Plate	58,000	32,000	24	All	LOW CARBON STEEL. Available in Type II, Class 1, for welded structures. Medium strength and good weldability.
FS-1045 QQ-S-635		Plate	---	---	--	---	MEDIUM CARBON STEEL. Composition (FS-1045) must be specified. Furnished in as rolled condition and finish. Moderate strength. Good machinability. Weldable.
1095 MIL-S-7947	A H	[2]Sheet	[3,4]80,000 [4]220,000	--- ---	-- --	All All	HIGH CARBON STEEL. Used principally for springs. If forming is required, specify condition A and heat treat after forming.
4130 MIL-S-18729	A N A N	[2]Sheet Plate	[3]85,000 95,000 95,000 95,000 [3]85,000 90,000 90,000 90,000	--- 75,000 75,000 75,000 --- 70,000 70,000 70,000	-- 10 12 15 -- 17 18 20	All up to 0.062 0.063-0.125 0.126-0.187 All 0.188-0.249 0.250-0.749 0.750-1.500	CHROME-MOLYBDENUM ALLOY STEEL. Used where heat treatment is desired to get high strength and toughness in moderately thick sections. Good machinability. Can be welded with special precautions.
18-7-5 HMS 6-1404	A A H H H H	All All Sheet Sheet Plate Plate	[5] I [3]159,000 II [3]168,000 I 240,000 II 280,000 I 240,000 I 280,000	--- --- 230,000 270,000 230,000 270,000	-- - 1-4 1-4 4 3.5	--- --- --- --- --- ---	MARAGING STEEL. Nickel-cobalt-molybdenum steel. Simple heat treatment with minimum distortion. Weldable. Good formability and machinability. Specify Class 1 (Vacuum Arc Remelt) or Class 2 (Air Arc).

[1]Alloy callout is commercial designation, not specification callout.

[2]Data for sheet applies equally to strip.

[3]Maximum.

[4]Figure given is obtained by conversion of equivalent hardness value required by specification.

[5]Specification callout must include Type as indicated.

TYPICAL CALLOUT

.049 SHEET, 1095 STEEL, MIL-S-7947, COND A

STEEL BAR, SHAPES, AND FORGINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		FORM	MIN TENSILE PROPERTIES			APPLICATIONS
COMP AND SPEC	COND		ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
C1008-C1020 QQ-S-633	CF	Bar	---	---	--	LOW CARBON STEEL. Low strength steel with good ductility for cold forming. Fair machinability. Suitable for welding or brazing. Composition must be specified.
C1045 QQ-S-633	TGP	Bar	---	---	--	MEDIUM CARBON STEEL. Moderate strength and ductility. Good machinability. May be welded with special procedures. Responds to conventional heat treatment.
C1137 QQ-S-633	CF	Bar	---	---	--	FREE-MACHINING STEEL. Best machinability. Not usually welded. Moderate strength. Poor cold-forming characteristics. May be brazed successfully by copper furnace brazing. Heat treatable.
ASTM A-7		Shapes or Bar	60,000	33,000	24	LOW CARBON STEEL. Standard alloy for structural angles, channels, tees, etc. Readily welded by all standard methods. Fair machinability.
MIL-S-6709	F1	Bar or Forging	---	---	--	NITRIDING STEEL (NITRALLOY 135). Moderate strength steel for use where surface or case hardening by nitriding is desired. Weldable by atomic, hydrogen and flash welding methods.
MIL-S-7493 Comp A4620	B1 B4	Bar Bar	--- ---	--- ---	-- --	CARBURIZING STEEL (A4620). Moderate strength steel for use where surface or case hardening by carburizing is desired. Weldable. Good machinability. Good impact properties. Composition (A4620) must be specified.
4130 MIL-S-6758	D1 D4 F4	Bar Bar Bar	--- --- 125,000	--- --- 100,000	-- -- 17	CHROME-MOLYBDENUM ALLOY STEEL (4130). Heat treatable by conventional methods up to 180,000 psi. Weldable by fusion methods with special precautions. Tough. Good machinability.
4340 MIL-S-5000	C1 C4	Bar Bar	[1,2] 125,000 [1,2] 125,000	--- ---	-- --	CHROME-NICKEL-MOLYBDENUM ALLOY STEEL (4340). Heat treatable by conventional methods to 200,000 psi and suitable for thicker sections than 4130. Welding possible but difficult. Fair machinability. Good ductility and excellent toughness.
18-7-5 HMS 6-1404	A A H H	All All All All	[3] [2] 159,000 II [2] 168,000 I 240,000 II 280,000	--- --- 230,000 270,000	-- -- 6 5	MARAGING STEEL. Nickel-cobalt-molybdenum steel. Simple heat treatment with minimum distortions. Weldable. Good formability and machinability. Specify Class 1 (Vacuum Arc Remelt) or Class 2 (Air Arc)
D6AC HMS 6-1464	H	All	225,000	200,000	10	ALLOY STEEL. Intended for use in rocket motor cases. Good machinability and weldability.

[1] Maximum.

[2] Value represents conversion from hardness required by specification.

[3] Specification callout must include Type as indicated.

TYPICAL CALLOUT

BAR, ALLOY STEEL 4340, MIL-S-5000, COND C

STAINLESS STEEL ALLOYS

STAINLESS SHEET AND PLATE

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			SIZE RANGE INCHES	APPLICATIONS
CLASS AND SPEC	COND	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.		
[1]Class 301 QQ-S-766	1/4 hard	125,000	75,000	25	All	AUSTENITIC STAINLESS STEEL. Develops high strength through cold working. Not generally fusion welded or brazed due to carbide precipitation and reduction in strength. May be resistance welded. Excellent ductility decreases as work hardening increases. Slightly magnetic. Good corrosion resistance. Good toughness and low temperature properties. Do not use above 700° F.
	1/2 hard	150,000	110,000	15	To 0.015	
		150,000	110,000	18	Over 0.015	
	3/4 hard	175,000	135,000	10	To 0.015	
		175,000	135,000	12	Over 0.015	
	Full hard	185,000	140,000	8	To 0.030	
		185,000	140,000	9	Over 0.030	
[1]Class 302 QQ-S-766	A	75,000	30,000	40	To 0.015	AUSTENITIC STAINLESS STEEL. Used in the annealed condition. Properties similar to 301. May be fusion welded without loss of corrosion resistance if followed by annealing. Excellent formability. Fair machinability. Very good corrosion resistance.
		75,000	30,000	45	0.016-0.030	
		75,000	30,000	50	Over 0.030	
[2]Class 304 and 316 QQ-S-766	A	75,000	30,000	40	All	AUSTENITIC STAINLESS STEEL. Similar to 302 but with somewhat better corrosion resistance and available also in plate. 316 similar with excellent corrosion resistance and has better properties than 304 at elevated temperatures.
[2]Class 304L and 316L QQ-S-766	A	70,000	----	40	All	AUSTENITIC STAINLESS STEEL. Same as basic classes (304, 316) except reduced carbon to allow fusion welding and high service temperatures without carbide precipitation.
[2]Class 321 and 347 QQ-S-766	A	75,000	30,000	40	All	AUSTENITIC STAINLESS STEEL. Stabilized to allow fusion welding and high service temperatures. Higher strength at elevated temperatures than 304L. Very good corrosion resistance. Fair machinability. Excellent formability.
[2]Class 410 QQ-S-766	A	70,000	35,000	20	To 0.050	MARTENSITIC STAINLESS STEEL. Capable of heat treatment to high strength levels (200,000 psi) after forming, welding, etc. Moderate corrosion resistance. Magnetic. Fair machinability. Can be welded.
		70,000	35,000	22	Over 0.050	
[2]MIL-S-25043	A	[3]150,000	[3]55,000	20	All	PRECIPITATION HARDENING STAINLESS STEEL (17-7PH). Capable of heat treatment to high strength levels (180,000 psi) by precipitation hardening. Easily formed in A condition. May be welded in any condition with suitable precautions and accompanied by some strength loss if in heat treated condition.
[1]MIL-S-8840	H	[3]180,000	[3]75,000	20	All	PRECIPITATION HARDENING STAINLESS STEEL (AM 350). Capable of heat treatment to high strength levels (185,000 psi) by precipitation hardening. Easily formed in H condition. May be welded or brazed in H condition and then heat treated with 90 percent joint efficiency. May be resistance welded in any condition.
[1]AMS 5525	Sol. Tr.	[3]105,000	----	25	Over 0.004	PRECIPITATION HARDENING STAINLESS STEEL (A-286). Precipitation hardenable to high strength (145,000 psi). Good formability, fair machinability. Non-magnetic in all conditions. Maintains good mechanical properties to 1300° F. Good impact strength. Poor weldability.

[1]Sheet and strip

[2]Sheet, strip, and plate

[3]Maximum

TYPICAL CALLOUT

.040 SHEET, 301 CRES, QQ-S-766, CL 301, COND 1/4 HARD

STAINLESS BAR, SHAPES, AND FORGINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			SIZE RANGE, INCHES	APPLICATIONS
[1]CLASS AND SPEC	[2]COND	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.		
303 Se QQ-S-763	A(CF) B(CF)	75,000 125,000 115,000 105,000 100,000	30,000 100,000 80,000 65,000 50,000	35 12 15 20 28	All Up to 3/4 Over 3/4 to 1 Over 1 to 1-1/4 Over 1-1/4 to 1-1/2	FREE-CUTTING AUSTENITIC STAINLESS STEEL. Alloyed with selenium for excellent machinability. Not usually welded. Corrosion resistant but less than type 304. Non-magnetic in the annealed condition.
[3]304 QQ-S-763	A(CF) A(HF)	90,000 75,600 75,000	45,000 30,000 30,000	35 40 40	Up to 1/2 Over 1/2 All	AUSTENITIC STAINLESS STEEL. Very good corrosion resistance. Must be annealed after fusion welding. Good formability. Fair machinability. Non-magnetic in annealed condition.
321 and 347 QQ-S-763	A(CF) A(HF)	90,000 75,000 75,000	45,000 30,000 30,000	35 40 40	Up to 1/2 Over 1/2 to 2 Over 1-3/4	AUSTENITIC STAINLESS STEEL. Stabilized to allow welding without need for subsequent annealing. Good corrosion resistance. Fair machinability. Non-magnetic when annealed.
17-4 PH AMS 5643	Sol. Tr.	[4]190,000	[4]170,000	[4]10	All	PRECIPITATION HARDENING STAINLESS STEEL. Precipitation-hardenable to high strength levels. Excellent weldability. Very good machinability. No heat treat distortion.
AM 350 AMS 5745	Equal.	[4]165,000	[4]135,000	[4]10	All	PRECIPITATION HARDENING STAINLESS STEEL. Precipitation-hardenable to high strength martensitic structure. Good ductility for forming. Good welding characteristics. Easily brazed. Machines best in overtempered condition.
416 QQ-S-763	A(CF)	70,000 120,000	40,000 90,000	16 12	All All	MARTENSITIC STAINLESS STEEL. May be hardened by conventional heat treatment to high strength levels. Excellent machinability. Good formability. Not usually welded. Magnetic. Fair corrosion resistance.
440C QQ-S-763	A	---	---	--	All	MARTENSITIC STAINLESS STEEL. Heat treatable to highest strength levels of any standard stainless steel (280,000 psi). Superior wear resistance. Good corrosion resistance. Poor machinability.
[5]AMS 5734 [5]AMS 5737 AMS 5736 AMS 5735	Annl. Ht. Tr. Sol. Tr. Ht. Tr.	[6]--- 140,000 [6]--- 130,000	--- 95,000 --- 85,000	-- 12 -- 15	All All All All	PRECIPITATION HARDENING STAINLESS STEEL (A-286). Hardenable to high strength. Good formability, fair machinability. Non-magnetic in all conditions. Maintains good mechanical properties to 1300°F. Good impact strength and fatigue resistance. Weldable in thin sections.

[1]Class must be specified on drawing where applicable.

[2]CF = cold finished, HF = hot finished.

[3]Also available in shapes.

[4]After precipitation heat treatment.

[5]Melted from consumable electrode.

[6]Required to be heat treatable to values given for heat treated stock.

TYPICAL CALLOUT

BAR, 416 CRES, QQ-S-763, CL. 416, COND A (COLD FINISHED)

ALUMINUM ALLOYS

BAR, ROD, WIRE AND SHAPES

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			SIZE RANGE INCHES	APPLICATIONS
ALLOY AND SPEC	[2]TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.		
2014 QQ-A-225/4	O T4 T6	[1]35,000 55,000 65,000	-- 32,000 55,000	12 16 8	-8.000 -6.750 -6.750	High strength structural use. Fair corrosion resistance. Fair weldability and formability.
2017 QQ-A-225/5	O T4 T451	[1]35,000 55,000 55,000	-- 32,000 32,000	16 12 12	-8.000 -8.000 0.500-8.000	High strength. Good machinability. Fair corrosion resistance and formability. Poor weldability and brazeability.
2024 QQ-A-225/6	O T4 T351	[1]35,000 62,000 62,000	-- 40,000 40,000	16 10 10	-8.000 -6.500 0.500-6.500	High strength. Fair corrosion resistance and formability. Not weldable by fusion methods.
3003 QQ-A-225/2	O F	[1]19,000 --	-- --	25 --	All 0.375-	Moderate strength. Good formability and weldability. Very good corrosion resistance.
5052 QQ-A-225/7	O F	[1]32,000 --	-- --	25 --	All 0.375-	Moderate strength. High fatigue strength. Good formability, weldability and corrosion resistance.
6061 QQ-A-225/8	O T6	[1]22,000 42,000	-- 35,000	18 10	-8.000 -8.000	Good strength, formability, weldability and corrosion resistance. Brazeable. Heat treatable.
7075 QQ-A-225/9	O T6 T651	[1]40,000 77,000 77,000	-- 66,000 66,000	10 7 7	-8.000 -4.000 0.500-4.000	Poor formability and weldability. High strength. Fair corrosion resistance. Not brazeable.

[1]Maximum

[2]Where a choice exists (i.e. T4 versus T451) the simple designation (T4) covers only sizes under 0.500 inch.

TYPICAL CALLOUT

BAR, AL ALLOY 2024-T4, QQ-A-225/6, TEMP T4

EXTRUSIONS

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	[2]TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
2014 QQ-A-200/2	O T4, T4510 T6, T6510	[1]30,000 50,000 60,000	[1]18,000 35,000 53,000	12 12 7	High strength structural use. Fair corrosion resistance. Fair weldability and formability.
2024 QQ-A-200/3	O T4, T3510	[1]35,000 57,000	[1]19,000 42,000	12 8-12	High strength. Fair corrosion resistance and formability. Not weldable by fusion methods.
3003 QQ-A-200/1	O F	[1]19,000 --	-- --	25 --	Moderate strength. Good formability and weldability. Very good corrosion resistance.
6061, 6062 QQ-A-200/8	O T4, T4510 T6, T6510	[1]22,000 26,000 38,000	[1]16,000 16,000 35,000	16 16 10	Good strength, formability, weldability and corrosion resistance. Brazeable. Heat treatable.
6063 QQ-A-200/9	O T4 T5 T6	[1]19,000 18,000 21,000 30,000	-- 9,000 15,000 25,000	18 14 8 8-10	Good strength, formability, weldability and corrosion resistance. Brazeable. Heat treatable.
7075 QQ-A-200/11	O T6, T6510	[1]40,000 78,000	[1]24,000 68,000	10 6-7	High strength. Fair corrosion resistance. Poor formability and weldability. Not brazeable.

[1]Maximum

[2]Where a choice exists (i.e. T6 versus T651C) the simple designation (T6) covers only sizes under 0.500 inch.

TYPICAL CALLOUT

BAR, AL ALLOY 7075-T6, QQ-A-200/11, TEMP T6

SHEET AND PLATE

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES				
ALLOY AND SPEC	TEMP	ULTIMATE PSI	YIELD PSI	[1]% ELONG IN 2 IN.	[1] SIZE RANGE INCHES	APPLICATIONS
1100 QQ-A-250/1	0 H14	11,000 16,000	3,500 14,000	15-30 1-10	0.006-3.000 0.009-1.000	Low to moderate strength. Excellent weldability, formability and corrosion resistance.
2024 QQ-A-250/5	0 T3	[2]32,000 64,000	[2]14,000 42,000	12 12-15	0.010-1.750 0.010-0.249	High strength structural use. Not weldable by fusion methods. Fair corrosion resistance and formability.
2219 MIL-A-8920	0 T31 T351 T37	[2]32,000 46,000 39,000 43,000	[2]16,000 28,000 25,000 34,000	12 10 8 4	0.040-2.000 0.040-0.249 0.250-6.000 0.040-5.000	Good strength at temperatures to 400°F obtainable by heat treatment. Fair corrosion resistance and formability. Not brazeable, but may be welded by standard procedures.
3003 QQ-A-250/2	0 H14	14,000 20,000	-- --	14-25 1-10	0.006-3.000 0.009-1.000	Moderate strength. Good weldability, formability and corrosion resistance.
5052 QQ-A-250/8	0 H32 H34	25,000 31,000 34,000	-- -- --	15-20 4-12 3-10	0.006-3.000 0.017-2.000 0.009-1.000	Highest strength for non-heat-treatable alloys. Good weldability and formability. Very good corrosion resistance.
6061 QQ-A-250/11	0 T4 T6	[2]22,000 30,000 42,000	[2]12,000 16,000 35,000	14-18 14-18 8-10	0.010-3.000 0.010-3.000 0.010-2.000	Good corrosion resistance, weldability and formability. Heat treatable and brazeable.
7075 QQ-A-250/12	0 T6 T651	[2]40,000 76,000 70,000	[2]21,000 65,000 60,000	10 7-8 3-8	0.015-2.000 0.015-0.499 0.249-3.000	High strength structural use. Fair corrosion resistance. Poor formability and not weldable by fusion methods.

[1] Larger value for elongation corresponds to thinnest sheet of size range. Smaller value corresponds to thickest plate or sheet.

[2] Maximum.

TYPICAL CALLOUT

.032 SHEET, AL ALLOY 6061-T4, QQ-A-327 TEMP T4

PERMANENT MOLD CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
113 QQ-A-596 Class 1	F	24,000	--	--	General use.
B195 QQ-A-596 Class 4	T4 T6 T7	33,000 35,000 33,000	-- -- --	4.5 2.0 3.0	General use where good strength and high ductility are required. Good casting and machining properties. Welding is not recommended. Very good machinability.
43 QQ-A-596 Class 7	F	21,000	--	5.0	Maximum corrosion resistance. Excellent casting properties. Fair machining qualities, weldable, brazeable.
356 QQ-A-596 Class 8	T6 T7	33,000 29,000	-- --	3.0 4.0	High strength and resistance to corrosion. Excellent casting and welding properties. Good machining qualities.
319 QQ-A-596 Class 11	F T6	29,000 40,000	-- --	2.5 2.0	High strength. Excellent casting properties.
Ternalloy 5 QQ-A-596 Class 13	T5	30,000	--	10.0	High strength and ductility, good corrosion resistance, and pressure tightness.
Ternalloy 7 QQ-A-596 Class 14	T5 T7	42,000 45,000	-- --	4.0 3.0	High strength and ductility, good corrosion resistance, and pressure tightness.

TYPICAL CALLOUT

113-F AL ALLOY PERM MOLD CSTG, QQ-A-596, CL 1, TEMP F

ALUMINUM ALLOYS (Continued)

SAND CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
43 QQ-A-601	F	17,000	--	3	Excellent casting and corrosion resistance properties. Thin walled and complicated shapes. Can be brazed or welded. Pressure [illegible] strength and corrosion resistance.
356 QQ-A-601	T4 T51 T6	25,000 23,000 32,000	-- -- 20,000	3 -- 3	Excellent casting properties. Good machinability and weldability.
195 QQ-A-601	T4 T6 T62 T7	29,000 32,000 36,000 29,000	-- 20,000 -- --	6 3 -- 3	High strength, ductility, and resistance to shock. Good casting properties. Very good machinability. Do not weld.
214 QQ-A-601	F	22,000	--	6	Superior resistance to corrosion. Good strength and elongation as cast.
142 QQ-A-601	T21 T571	23,000 29,000	-- --	-- --	Strength and hardness at elevated temperature. Good machinability.
122 QQ-A-601	T2 T61	23,000 30,000	-- --	-- --	Strength and hardness at elevated temperature. Good machinability. May be welded or electroplated.
355 QQ-A-601	T51 T6 T7 T71	25,000 32,000 35,000 30,000	-- 20,000 -- --	-- 2 -- --	High strength and corrosion resistance.
220 QQ-A-601	T4	42,000	22,000	12	Maximum strength, elongation, and resistance to shock. Fair casting properties. Excellent machinability. Do not weld.
40E QQ-A-601	F T5	34,000 34,000	25,000 25,000	4 4	Requires special foundry practice. High strength, ductility and resistance to shock.
Almag 35 QQ-A-601	F T4	35,000 35,000	18,000 18,000	9 9	Excellent shock resistance and corrosion resistance. Excellent machinability.

TYPICAL CALLOUT

356-T51 AL ALLOY SAND CSTG, QQ-A-601, ALLOY 356, TEMP T51

PRECISION INVESTMENT CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		[1]MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
356 HMS 1-1034	T51 T6	23,000 30,000	-- 20,000	-- 3	Moderately stressed small parts.
TENS-50 HMS 1-1142	T6	42,000	32,000	2	High strength.
C612 HMS 1-1270	T5	28,000	18,000	4	Weldable and brazeable.

[1]Separately cast test bars.

TYPICAL CALLOUT

356-T6 AL ALLOY INVESTMENT CSTG, HMS 1-1034, TEMP T6

FOIL

SPECIFICATION

HMS 1-1259

APPLICATIONS

Aluminum foil is used industrially for shielding and as vapor and corrosion barriers. Direct contact with other metals in the presence of moisture can lead to galvanic corrosion.

CLASSIFICATION

TEMPER	SURFACE CONDITION
0	Dry-annealed
	Slick-annealed
H-14	Washed
H-18	Washed
H-19	Washed

Finishes: Matte one side, Bright two sides, Extra Bright two sides, Satin.

Alloys: 1100, 1145, 1180, 1188, 1199, 1235, 3003, 5052, 5056.

TYPICAL CALLOUT

.0020 FOIL, AL ALLOY COMP 1100, TEMP 0, SLICK ANNEALED, MATTE ONE SIDE

FORGINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		[1] MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
2014 QQ-A-367	T4 T6	55,000 65,000	30,000 55,000	11 7	Moderately high strength.
2017 QQ-A-367	T4	55,000	30,000	11	Small hydraulic system forgings. Good machinability.
6061 QQ-A-367	T6	38,000	35,000	7	For intricate parts difficult to forge in high strength alloys. Good corrosion resistance.
7075 QQ-A-367	T6	75,000	65,000	7	Small and medium sized forgings where maximum weight saving is essential. Do not use for forgings over 3 inches thick.
7079 QQ-A0367	T6	72,000	62,000	7	Large forgings where maximum weight saving is essential. Use for forgings over 3 inches thick.
7178 HMS 1-1283	T6 T652	80,000 74,000	74,000 67,000	7 7	Used for maximum strength. T652 only is used where section thickness exceeds 1 inch.

[1] Test specimen parallel to forging flow lines. Properties are based on die forgings. Properties for hand forgings will depend on cross-sectional area.

TYPICAL CALLOUT

FORGING, AL ALLOY 2014-T4, QQ-A-367 ALLOY 2014, TEMP T4

SPECIAL ALUMINUM APPLICATIONS

HONEYCOMB CORE

SPECIFICATION
MIL-C-7438

APPLICATIONS

This honeycomb core material is available in a number of configurations to give desired strength, weight, and size. Features of the honeycomb are specified in a coded number which also includes the manufacturer's trade name (see Typical Callout). The specification provides curves for required flatwise compressive strength, core shear modulus, and sandwich shear strength for any required density of material. In making up a sandwich structure using this core, the facing material and adhesive used must be separately specified.

CLASSIFICATION

Designated Nominal Density	1.6 to 10.0 p.c.f. (pounds per cubic foot)
Cell Size	1/8 to 3/8 inch
Nominal Foil Thickness	07 to 50 ten-thousandths of an inch
P or N	Perforated or non-perforated core
Alloy	3003 or 5052 (commercial designation)

TYPICAL CALLOUT

HONEYCOMB CORE, AL ALLOY PER MIL-C-7438 (TRADE NAME) 4 3-1/4-2OP(3003)

DIE CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		TYP TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN	
A13 QQ-A-591	As Cast	42 000	19 000	3 5	Excellent casting properties. Good mechanical qualitites and corrosion resistance. Welding is not recommended
43 QQ-A-591	As Cast	33 000	14 000	9 0	General purpose castings. Machines well and is weldable and brazeable. Good resistance to corrosion. Excellent casting properties
218 QQ-A-591	As Cast	45 000	28 000	5 0	Best combination of strength, ductility, resistance to corrosion. Difficult to cast
A360 QQ-A-591	As Cast	46 000	24 000	3 5	More resistant to corrosion than alloy A380. Good strength
A380 QQ-A-591	As Cast	47 000	23 000	3 5	Use for high strength castings and where resistance to severe corrosion is not required

TYPICAL CALLOUT

43 AL ALLOY DIE CSTG, QQ-A-591, ALLOY 43

MAGNESIUM ALLOYS

SHEET AND PLATE

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			SIZE RANGE INCHES	APPLICATIONS
ALLOY AND SPEC	TEMPER	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN		
AZ31B QQ-M-44	O H24 H24	30,000 39,000 34,000	--- 29,000 20,000	[1]10-12 6 8	0.016-2.000 0.016-0.249 0.250-2.000	Good forming properties. May be welded. Good flatness and ductility in H24 temper. Dent resistant.
HK31A MIL-M-26075	H24 H24	34,000 33,000	24,000 25,000	4 4	0.016-0.250 0.251-3.000	Good forming properties. Excellent weldability. For applications up to 600° F or for short times at higher temperatures.
HM21A MIL-M-8917	T8	30,000	18,000	6	All	Retains useful strength at elevated temperatures (11,000 psi tensile ultimate at 600° F). Good weldability.

[1] Lower elongation refers to thicker gages of material

TYPICAL CALLOUT

040 SHEET MAG ALLOY AZ31B-H24 QQ-M-44 TEMP H24

BAR, ROD AND SHAPES

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			SIZE RANGE INCHES	APPLICATIONS
ALLOY AND SPEC	TEMPER	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN		
AZ31B QQ-M-31	F	32,000	21,000	7	0.249-2.499	Good extrusion characteristics. May be welded. Moderate strength and ductility
AZ61A QQ-M-31	F	38,000	16,000	7	0.249-2.499	Slightly better mechanical properties than AZ31B at room and elevated temperatures. May be welded
AZ80A QQ-M-31	F T5	43,000 47,000	28,000 30,000	6 4	0.249-2.499 0.249-2.499	Best mechanical properties of alloys listed. May be welded. Good elevated temperature properties in F temper
HM31A MIL-M-8916	T5	37,000	26,000	4	Up to 4.000	For extrusions. Good properties at elevated temperatures (17,000 tensile ultimate at 600° F)
ZK60A QQ-M-31	F T5	40,000 45,000	28,000 36,000	5 4	All All	Used for good strength and toughness. Difficult to fusion weld. Resistance welding ability is excellent

TYPICAL CALLOUT

BAR, MAG ALLOY AZ61A-F, QQ-M-31, COMP AZ61A, TEMP F

MAGNESIUM ALLOYS (Continued)

DIE CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		[1]TYP TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMPER	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN	
AZ91A QQ-M-38	F	29,000	24,000	3	Least expensive for high production runs. Good dimensional accuracy. Excellent surface finish.

[1]Separately cast test specimens.

TYPICAL CALLOUT

AZ91A-F MAG ALLOY DIE CSTG, QQ-M-38, TEMP F

PERMANENT MOLD CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		[1]MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMPER	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN	
AZ63A QQ-M-55	F T4 T5 T6	24,000 34,000 24,000 34,000	10,000 10,000 11,000 18,000	4 7 2 3	Limited to relatively simple shapes and sizes. Prone to porosity and segregation.
AZ81C QQ-M-55	F T4 T5 T6	18,000 34,000 20,000 34,000	10,000 10,000 11,000 18,000	- 7 2 3	Maximum strength and hardness consistent with good ductility and good casting properties. Good weldability.
EZ33A QQ-M-55	T5	20,000	14,000	2	Use where soundness is essential. Has good properties up to 500° F including good creep strength.

[1]Separately cast test specimens.

TYPICAL CALLOUT

AZ63A-T6 MAG ALLOY PERM MOLD CSTG, QQ-M-55, ALLOY AZ63A, TEMP T6

SAND CASTINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		[1]MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TEMPER	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN	
AZ63A QQ-M-56	F T4 T5 T6	26,000 34,000 26,000 34,000	11,000 11,000 12,000 16,000	4 7 2 3	Limited to relatively simple shapes and sizes. Prone to porosity and segregation
AZ91C QQ-M-56	F T4 T5 T6	23,000 34,000 23,000 34,000	11,000 11,000 12,000 16,000	- 7 2 3	Maximum strength and hardness consistent with good ductility and good casting properties. Good weldability
EZ33A QQ-M-56	T5	20,000	14,000	2	Use where soundness is essential. Has good properties up to 500 F including good creep strength
HK31A QQ-M-56	T6	27,000	13,000	4	Use to 500 F for short times. Excellent weldability and pressure tightness
ZK51A QQ-M-56	T5 T6	39,000	26,000	5	Maximum strength and toughness. Limited weldability
QE22A AMS 4418	T6	28,000	20,000	2	Best properties at elevated temperatures to 400 F or short times at higher temperatures

[1]Separately cast test specimens

TYPICAL CALLOUT

AZ91C-T6 MAG ALLOY SAND CSTG, QQ-M-56, COMP AZ91C, TEMP T6

FORGINGS

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	CONDITION	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN	
AZ61A QQ-M-40	F	38,000	22,000	6	Relatively easy to weld. Suitable for press forging of intricate parts
AZ80A QQ-M-40	F T5	42,000 42,000	26,000 28,000	5 2	Maximum strength. Not recommended for large and intricate forgings
ZK60A QQ-M-40	T5	42,000	26,000	7	Best combination of strength and ductility. Excellent press forging characteristics. Not usually fusion welded

TYPICAL CALLOUT

FORGING, MAG ALLOY ZK60A-T5, QQ-M-40, COMP ZK60A, TEMP T5

COPPER ALLOYS

SHEET AND PLATE

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			
SPEC AND COMPOSITION	CONDITION	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	APPLICATIONS
[1]QQ-C-576	CR, light CR, 1/2 hard CR, soft anl HR HR, annealed	32,000 37,000 ---- 30,000 30,000	---- ---- ---- ---- ----	-- -- -- -- --	COPPER. Best formability in annealed condition. High electrical and thermal conductivity. Low strength. Specify oxygen-free if brazing will be used. Specify tinned one side or both sides for soldering. Poor machinability.
[1]QQ-B-613 Comp 1	Annealed 1/4 hard 1/2 hard Hard Spring	---- 49,000 55,000 68,000 86,000	---- ---- ---- ---- ----	-- -- -- -- --	YELLOW BRASS. Low electrical and thermal conductivity. Moderate strength. Fair machinability. Excellent formability. May be brazed or soldered.
[1]QQ-B-613 Comp 2	Annealed 1/2 hard Spring	---- 57,000 91,000	---- ---- ----	-- -- --	CARTRIDGE BRASS. Better formability than yellow brass. Electrical and thermal conductivity and joining characteristics are similar to yellow brass.
[1]QQ-B-613 Comp 4	Annealed 1/2 hard	---- 51,000	---- ----	-- --	RED BRASS. Electrical and thermal conductivity better than yellow brass. Fair machinability. May be brazed or soldered. Excellent formability.
[1]QQ-B-613 Comp 24	1/4 hard 1/2 hard Hard Extra hard	49,000 55,000 68,000 79,000	---- ---- ---- ----	-- -- -- --	LEADED BRASS. Excellent machinability. Fair formability. May be soft soldered or brazed. Low electrical and thermal conductivity.
[1]QQ-B-639 Comp 1	1/2 hard	60,000	35,000	20	NAVAL BRASS. Good formability. May be soldered or brazed. Fair machinability. Low cost for high strength.
[2]QQ-C-533 Comp 172	A 1/4 H 1/2 H H AT 1/4 HT 1/2 HT HT	60,000 75,000 85,000 100,000 165,000 175,000 185,000 190,000	-- - ---- ---- ---- 140,000 150,000 160,000 165,000	35 10 5 2 3 2.5 1 1	BERYLLIUM COPPER. May be bought to A or -H conditions and, after fabrication, heat treated to AT or -HT conditions. Low electrical conductivity. Machines best in hardened condition. Best formability in "A" condition. May be hot worked, welded, soldered, or brazed.
[1,3]QQ-C-591 Comp 655	Soft Hard	50,000 80,000	15,000 60,000	35 5	HIGH SILICON BRONZE. Excellent cold working properties. Fair machinability. Very low electrical conductivity. Suitable for structural parts.
[1,3]QQ-B-750 Comp A	Hard Spring	72,000 91,000	---- ----	-- --	PHOSPHOR BRONZE. Excellent formability. High resistance to fatigue. Low friction coefficient. Poor machinability.

[1] Includes also strip and flat bar stock with slit, sheared, edge rolled, sawed or machined edges.
[2] Includes also strip.
[3] Includes also strip and flat bar with finished edges.

TYPICAL CALLOUT

.020 STRIP, BERYLLIUM COPPER, QQ-C-533, COND ½H

SPECIFICATION INFORMATION

BAR, ROD AND SHAPES

DRAWING CALLOUT				MIN TENSILE PROPERTIES			
SPEC AND COMPOSITION	CONDITION	[1]FORM	DIMENSION INCHES	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	APPLICATIONS
QQ-C-502	Hard	[2]Rod	Up to 0.250	50,000	----	--	COPPER. High electrical and thermal conductivity. Moderate strength. Fair machinability. Brazing will reduce strength.
			0.250-0.375	45,000	----	10	
			0.376-1.000	40,000	----	12	
			1.001-2.000	[3]35,000	----	15	
			2.001-3.000	33,000	----	15	
	Hard	[4,5]Bar	0.188-0.375	42,000	----	10	
			0.376-0.500	40,000	----	12	
			0.501-2.000	33,000	----	15	
			2.001-4.000	32,000	----	15	
QQ-B-626 Comp 1	1/2 Hard	[2]Rod	Up to 0.500	57,000	----	15	YELLOW BRASS. Low electrical and thermal conductivity. Moderate strength. Fair machinability. Strength will be reduced by welding.
			0.501-1.000	55,000	----	20	
			1.001-2.000	50,000	----	25	
			Over 2.000	45,000	----	30	
		[4,5]Bar	Up to 0.500	50,000	25,000	10	
			0.501-2.000	45,000	17,000	20	
			Over 2.000	40,000	15,000	20	
QQ-B-626 Comp 22	1/2 Hard	[2]Rod	Up to 0.500	57,000	25,000	7	CARTRIDGE BRASS. Similar to yellow brass with slightly higher electrical and thermal conductivity. Strength will be reduced by brazing.
			0.501-1.000	55,000	25,000	10	
			1.001-2.000	50,000	20,000	15	
			Over 2.000	45,000	15,000	20	
		[4,5]Bar	Up to 0.500	50,000	25,000	10	
			0.501-2.000	45,000	17,000	20	
			Over 2.000	40,000	15,000	20	
QQ-B-637 Comp 1	1/2 Hard	[4]All	Up to 0.500	60,000	27,000	22	NAVAL BRASS. Fair machinability. Good strength for low cost. May be brazed but strength will be reduced.
			0.501-1.000	60,000	27,000	25	
			1.001-2.000	58,000	26,000	25	
			2.001-3.000	54,000	25,000	25	
			3.001-4.000	54,000	22,000	27	
			Over 4.000	54,000	22,000	30	
	Hard	[4]All	Up to 1.000	67,000	45,000	13	
			1.001-2.000	62,000	37,000	18	
QQ-B-637 Comp 3	1/2 Hard	[4]All	Up to 1.000	60,000	27,000	12	LEADED NAVAL BRASS. Similar to naval brass with improved machinability.
			1.001-2.000	58,000	26,000	20	
			2.001-3.000	54,000	25,000	20	
			Over 3.000	54,000	22,000	20	
QQ-B-679 Comp 1	Stress Relieved	[4]All	Up to 0.500	80,000	40,000	15	ALUMINUM BRONZE (BETA). Good machinability. May be brazed but strength will be reduced.
			0.501-1.000	75,000	37,500	15	
			1.001-3.000	72,000	35,000	20	
QQ-B-728 Class B	1/2 Hard	[2]All	All	105,000	60,000	7	MANGANESE BRONZE. High strength. Tough and wear resistant.
	Hard	[2]Rod	All	115,000	68,000	5	
QQ-B-750 Comp A	Hard	Flats	Up to 0.375	60,000	----		PHOSPHOR BRONZE. Excellent formability. High resistance to fatigue. Low friction coefficient. Poor machinability.

[1] For data on forms not listed, consult applicable specification.

[2] Rod is defined as round, hexagonal, or octagonal solid section furnished in straight lengths.

[3] Maximum.

[4] Includes bar with finished edges only. For bar with slit, edge-rolled, sawed, sheared, or machined edges, see under sheet and plate.

[5] Bar is defined as rectangular or with two parallel surfaces and two finished regular edges.

TYPICAL CALLOUT

BAR, NAVAL BRASS, QQ-B-637, COMP 1, ½ HARD

NICKEL ALLOYS

NICKEL ALLOYS - BAR

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			APPLICATIONS
COML NAME	SPEC NUMBER	ULTIMATE PSI	YIELD PSI	% ELONG IN 4 DIA	
RENÉ 41	AMS 5712	[1]170,000	[1]130,000	[1] 8	Intended for use at elevated temperatures. High strength to 1600 F. Oxidation resistance to 1800 F. May be fusion welded in solution treated condition with good joint strength and ductility. May be brazed. Good corrosion resistance. Better machinability in fully aged condition. Generally used in heat treated condition. Supplied in solution treated condition.
INCONEL X750	AMS 5667	[2]165,000	[2]105,000	[2]20	Intended for long service life at temperatures from 800 F to 1100 F. Good resistance to oxidation and corrosion. Weldable in solution treated condition. Good impact strength to -320 F. Solution heated condition is easier to machine, but heat-treated material gives best machined finish. Generally used in heat treated condition. Supplied in equalized condition.

[1]Properties after precipitation heat treatment per AMS 5712

[2]Properties after aging per AMS 5667

TYPICAL CALLOUT

BAR, NICKEL ALLOY, RENÉ 41, AMS 5712

NICKEL ALLOYS - SHEET AND PLATE

SPECIFICATION INFORMATION

DRAWING CALLOUT		SIZE RANGE INCH	MIN TENSILE PROPERTIES			APPLICATIONS
COML NAME	SPEC NUMBER		ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN	
RENE 41	AMS 5545	0.010 to 0.187 Over 0.187 All Up to 0.018 0.019 - 0.024 Over 0.024	[1]170,000 [1]195,000 [2]170,000 [2,3]130,000 [2,3]135,000 [2,3]140,000	[1]100,000 [1]140,000 [2]130,000 [2,3]110,000 [2,3]110,000 [2,3]110,000	30 20 [2]10 [2,3] 3 [2,3] 3 [2,3] 3	Intended for use at elevated temperatures. High strength to 1600 F. May be fusion welded in solution-treated condition with good joint strength and ductility. May be brazed. Good corrosion resistance. Good formability. Generally used in heat treated condition.
INCONEL X750	MIL-N-7786	Under 0.025 0.025 to 0.125 0.126 to 0.250 All	[1]130,000 [1]130,000 [1]130,000 [4]155,000	--- [1] 60,000 [1] 65,000 [4]100,000	-- 40 40 [4]20	Intended for long service life at temperatures to 1000 F. Good resistance to oxidation and corrosion. Weldable in solution treated condition by fusion or resistance methods. Good formability. Generally used in heat treated condition. Supplied in cold rolled and annealed condition.

[1]Maximum

[2]After precipitation heat treatment per AMS 5545

[3]Tested at 1400 F

[4]After aging per MIL-N-7786

TYPICAL CALLOUT

031 SHEET, NICKEL ALLOY INCONEL X750, MIL-N-7786

MISCELLANEOUS ALLOYS

MOLYBDENUM ALLOY - BAR

SPECIFICATION
AMS 7813

DESIGN DATA

SIZE RANGE INCHES	MIN TENSILE PROPERTIES		
	ULTIMATE MIN. PSI	YIELD MIN. PSI	% ELONG IN 2 IN.
0.188 to 0.875	105,000	90,000	10
0.876 to 1.125	100,000	85,000	8
1.126 to 1.875	90,000	75,000	5
1.876 to 2.875	80,000	70,000	4
2.876 to 3.500	75,000	65,000	3
3.501 to 4.500	70,000	60,000	2

TYPICAL CALLOUT

BAR, MOLYBDENUM ALLOY, AMS 7813

MOLYBDENUM ALLOY - SHEET AND PLATE

SPECIFICATION
AMS 7811

DESIGN DATA

SIZE RANGE INCHES	ORIENTATION TO ROLLING DIRECTION	MIN TENSILE PROPERTIES		
		ULTIMATE MIN. PSI	YIELD MIN. PSI	% ELONG IN 2 IN.
Up to 0.1875	Parallel	100,000	90,000	0
	Perpendicular	110,000	95,000	4
0.188 to 0.500	Parallel	95,000	85,000	3
	Perpendicular	105,000	95,000	3
0.501 to 1.500	Parallel	90,000	82,000	2
	Perpendicular	100,000	90,000	2

TYPICAL CALLOUT

.030 SHEET, MOLYBDENUM ALLOY, AMS 7811

TITANIUM ALLOYS - SHEET

SPECIFICATION INFORMATION

DRAWING CALLOUT			MIN TENSILE PROPERTIES			APPLICATIONS
ALLOY AND SPEC	TYPE	COMP	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	
70 MIL-T-9046	I	B	80,000	70,000	15.0	UNALLOYED TITANIUM. Not hardenable by heat treatment. Excellent corrosion resistance. Difficult to cold-form but may be hot formed above 500° F. Readily welded with welds also exhibiting excellent corrosion resistance.
6Al 4V MIL-T-9046	III	C	[1]160,000	[1]145,000	[1,2]5.0	ALPHA-BETA TITANIUM ALLOY. Hardenable by heat treatment after forming and welding have been completed. May be used to 1000° F. Readily welded. Difficult to cold-form. May be hot-formed at 900-1200° F. Specify SOLUTION TREATED AND AGED if material is required to be bought in the heat-treated condition.

[1] Solution treated and aged condition.

[2] Elongation given is for 0.050 inch and heavier sheet. Elongation is 4.0% minimum from 0.033 to 0.049 inch and 3.0% minimum for 0.032 inch and below.

TYPICAL CALLOUT

.063 SHEET, TI ALLOY 6 AL 4V, MIL-T-9046, TYPE III, COMP C, SOL TR AND AGED

Appendix

MISCELLANEOUS ALLOYS

BERYLLIUM SHEET AND PLATE

SPECIFICATION
AMS 7902

DESIGN DATA

Property	Value
Tensile ultimate strength (up to 0.250 inch thick)	55,000 psi, minimum
Tensile yield strength (up to 0.250 inch thick)	45,000 psi, minimum
Elongation, in 1 inch (up to 0.250 inch thick)	5 % minimum
Modulus of elasticity	44×10^6 psi, typical

TYPICAL CALLOUT

046 SHEET, BERYLLIUM, AMS 7902

STOCK SIZES

This material is available on special order only and any gage from 0.001 to 0.500 inch may be ordered. Tolerances generally run about ±10 percent of ordered thickness.

BERYLLIUM BAR AND ROD

SPECIFICATION
AMS 7901

DESIGN DATA

Property	Value
Tensile ultimate strength	40,000 psi, minimum
Tensile yield strength	30,000 psi, minimum
Elongation in 1 inch	1 % minimum
Modulus of elasticity	44×10^6 psi, typical

TYPICAL CALLOUT

BAR, BERYLLIUM, AMS 7901

TITANIUM ALLOY - BAR

SPECIFICATION INFORMATION

DRAWING CALLOUT		MIN TENSILE PROPERTIES			[1] IMPACT STRENGTH	
ALLOY AND SPEC	CLASS	ULTIMATE PSI	YIELD PSI	% ELONG IN 2 IN.	MIN, FT-LB	APPLICATIONS
6Al 4V MIL-T-9047	5	[2] 160,000	[2] 150,000	[2] 8	10	ALPHA-BETA TITANIUM ALLOY. Hardenable by heat-treatment. May be fusion welded. Supplied in annealed condition. Usable strength to 750 F oxidation resistance to 1000 F. Fair machinability and produces good surface.

[1] Charpy V notch

[2] Solution treated and aged per HP 1-20.

TYPICAL CALLOUT

BAR, TI ALLOY, 6AL 4V, MIL-T-9047, CL 5

THERMOSETTING FILLED MOLDING MATERIAL

SPECIFICATION INFORMATION

SPECIFICATION	TYPE	[1] COMPRESSIVE STRENGTH MIN. PSI	[1] FLEXURAL STRENGTH MIN. PSI	[1] IMPACT STRENGTH MINIMUM FT. LB. IN.	[1] TENSILE STRENGTH MIN. PSI	APPLICATIONS
MIL-M-19833	GDI-30 GDI-30F	20,000 20,000	10,000 10,000	2.75 2.75	4,500 4,500	DIALLYL PHTHALATE, GLASS FIBER FILLER. Type GDI-30F is flame retardant. Low moisture absorption. Excellent electrical properties. High impact strength.
MIL-M-14	MDG	18,000	6,800	0.28	4,000	DIALLYL PHTHALATE, MINERAL FILLER. Low shrinkage. Good dielectric properties. Easier to mold than GDI types.
MIL-M-14	SDG	16,000	9,000	0.30	4,500	DIALLYL PHTHALATE, GLASS FIBER FILLER. Good moisture resistance and tensile strength.
MIL-M-14	SDI-5	18,000	8,000	0.60	3,500	DIALLYL PHTHALATE, ACRYLIC FIBER FILLER. Good dielectric properties. Low shrinkage. Good moisture resistance. Moderate impact strength.
MIL-M-14	MME	25,000	6,000	-	4,200	MELAMINE, MINERAL FILLER. Arc and flame resistant. Excellent dimensional stability.
MIL-M-14	CFI-5	23,000	8,000	0.48	5,700	PHENOLIC, CELLULOSE FILLER. Low cost. General purpose.
MIL-M-14	MFE	15,000	8,000	-	4,200	PHENOLIC, MINERAL FILLER. Good electrical properties.
MIL-M-14	MFG	15,000	8,000	0.64	4,500	PHENOLIC, ASBESTOS FILLER. Heat resistant.
MIL-M-14	MFH	15,000	7,000	0.25	4,200	PHENOLIC, MINERAL FILLER. More heat resistance than MFG.
MIL-M-14	GPI-100	20,000	15,000	10.0	4,500	PHENOLIC, GLASS FIBER FILLER. Impact resistant. Good electrical properties.
MIL-M-14	MAI-60	18,000	12,000	6.0	3,500	POLYESTER, GLASS FIBER FILLER. Impact resistant. Good dielectric properties and arc resistance.
MIL-M-14	MSG	15,000	6,500	0.25	2,500	SILICONE, MINERAL FILLER. Good dielectric properties. Excellent heat resistance.
MIL-M-14	MSI-30	10,000	7,000	3.2	2,000	SILICONE, GLASS FIBER FILLER. Impact resistant and heat resistant. Electrical properties inferior to Type MSG.

[1] Conditioned 48 hours at 50°C plus 96 hours at 23°C and 50% RH.

TYPICAL CALLOUT

PLASTIC MOLDING PER MIL-M-14, TYPE MFE

THERMOSETTING RIGID FOAM

SPECIFICATION INFORMATION

DRAWING CALLOUT		THICKNESS RANGE INCHES	RESIN TYPE	APPLICATIONS
SPECIFICATION AND TYPE	DENSITY LB CU FT			
HMS 16-1287 Type II	2 4 6 8 10 12 15 20	Not applicable	Polyurethane	For foam-in-place or encapsulation use. Requires elevated temperature cure (200°F minimum). Maximum service temperature is 400°F. Two component mixture.
HMS 16-1287 Type III	1 2 5 8 10 18	Not applicable	Polyurethane	For foam-in-place or encapsulation use. Room temperature cure. Maximum service temperature is 200°F. Two component mixture.
HMS 16-1287 Type IV	2 3 4 6 8 10 12 16	Not applicable	Polyurethane	For foam-in-place or encapsulation use. Room temperature cure. Maximum service temperature 200°F. Two component mixture.
HMS 16-1317	3 4 5 6 8 10 14	1 8 to 4	Polyurethane	Same material as HMS 16-1287, Type II except that this material is supplied in the cured condition as sheets or blocks.
MIL-C-18345	[1]6-7		Cellulose acetate	For use as low density core material for laminates. Maximum service temperature is 325°F. Machinable using conventional woodworking tools.

[1]Not necessary to specify density for MIL-C-18345.

TYPICAL CALLOUT

PLASTIC FOAM, HMS 16-1287, TYPE II, DENSITY 6

THERMOSETTING MOLDING MATERIAL, LAMINATED

SPECIFICATION INFORMATION

The specifications tabulated below cover requirements for plastic laminated materials consisting of plastic resins and reinforcement material. Each specification for laminated material specifies a separate resin, or several similar resins. The resins are classified as to Type and Class depending on various properties. Pages 6-14 through 6-16 contain additional data on these resins. The reinforcement material is either glass fabric or glass fiber mats as indicated in the table. The glass fabrics are classified in types based on yarn size and spacing. The glass fiber mat is classified in types based on binder material. Specifications for reinforcement materials are covered on page 6-17. Drawing callouts should specify resin type and class, when applicable, and reinforcement material desired. Only rarely are either resins or reinforcements specified separately without reference to the specification on the laminated material. The selection of glass fabric reinforcement requires careful judgment based on experience and the assistance of the Plastics Section of the Materials Technology Department should be requested.

SPECIFICATION	TYPE	DESCRIPTION	[1]RESIN	[1]FILLER	DESCRIPTION
MIL-P-25421	I II	General purpose Heat resistant	MIL-R-9300, Type I MIL-R-9300, Type II	MIL-M-15617 MIL-M-15617	EPOXY RESIN-GLASS FIBER MAT. Class 1 - Non-electrical; Class 2 - Radio Frequency; Class 3 - Radar Frequency.
MIL-P-25515	I II	General purpose Heat resistant	MIL-R-9299, Type I MIL-R-9299, Type II	MIL-C-9084 or MIL-M-15617	PHENOLIC RESIN-GLASS FIBER MAT OR CLOTH. Class 1 - 10 to 50 psi; Class 2 - 50 to 300 psi.
MIL-P-8013	I II III	General purpose Radio frequency Radar frequency	MIL-R-7575, GRA CL0 MIL-R-7575, 2 MIL-R-7575, GRA CL3	MIL-C-9084 or MIL-M-15617	POLYESTER RESIN-GLASS FIBER MAT OR CLOTH.
MIL-P-25395		Heat resistant	MIL-R-25642	MIL-C-9084 or MIL-M-15617	POLYESTER RESIN-GLASS FIBER MAT OR CLOTH.
MIL-P-25518	I II III	General purpose Radio frequency Radar frequency	MIL-R-25506, Type I MIL-R-25506, Type II MIL-R-25506, Type III	MIL-M-15617	SILICONE RESIN-GLASS FIBER MAT.

[1]Resins and fillers are described on the following pages.

[2]The resin specification does not provide for meeting radio frequency requirements.

TYPICAL CALLOUT

LAMINATED PLASTIC MIL-P-25515, TYPE II, CL 1,
USING ONE LAYER OF GLASS CLOTH MIL-C-9084, TYPE VIII

THERMOPLASTIC RIGID FOAM

SPECIFICATION INFORMATION

DRAWING CALLOUT			
SPECIFICATION	CLASSIFICATION	DESCRIPTION	APPLICATIONS
MIL-P-40619	Type I Type II Class 1 Class 2 Grade A Grade B	Thermal insulation Buoyant Non-fire retardant Fire retardant Low density High density	CELLULAR POLYSTYRENE. Recommended for service temperatures from -100°F to +158°F. The higher density material has higher compressive strength.

TYPICAL CALLOUT

FOAM PLASTIC, MIL-P-40619, TYPE I, CL 2, GR B

CHAPTER 11 – PACKAGING DESIGN TECHNIQUES

VOLUME III - CHAPTER 11
PACKAGING DESIGN TECHNIQUES

ABSTRACT:

In the final analysis, it is the degree of excellence that the packaging engineer can muster in the execution of the equipment structure, that will largely determine the operational suitability of the system. Most of the environmental parameters are influenced or mitigated by the equipment package itself. If the package sustains the equipment in the field, then the system is a success. If the equipment is unable to survive and perform as intended in the field, then the system is a failure, regardless of the functional capabilities which have been designed into it.

This chapter discusses some of the more important decisions that the designer must make during the design of the equipment package. These topics include the definition of the true design criteria, the evaluation of the important constraints, the interaction of other environmental stresses, and a discussion of the damage potential inherent in the dynamic environment.

Some procedures are outlined for dealing with joints and interfaces, material compatibility, configuration, component location, and other design oriented packaging problems.

Chapter 11 - Packaging Design Techniques

ERRATA SHEET

Page	Paragraph	Line	Correction
11.1-9	2	6 & 7	... faces which permit ...
11.1-9	2	8	... slipping joint
11.2-0	3	2	intersecting
11.2-2	4	6	compression

VOLUME III - CHAPTER 11
PACKAGING DESIGN TECHNIQUES

CONTENTS

Section		Page
1	INTRODUCTION	11.1-0
	• Functional Requirements and Design Criteria . .	11.1-0
	• Evaluating Principal Design Constraints	11.1-2
	• Support Versus Capture in Structural Design . .	11.1-4
	• Dynamic Stresses and Their Damage Potential . .	11.1-6
	• Interacting Effects of Other Environmental Stresses	11.1-8
2	ATTACHMENT OF EQUIPMENT ELEMENTS	11.2-0
	• Homogenous Joining Techniques	11.2-0
	• Consideration of Tension and Shear in the Selection of Fasteners	11.2-2
	• Some More Design Tips on the Use of Fasteners	11.2-4
3	IMPORTANT DESIGN CONSIDERATIONS	11.3-0
	• Designing for Material Compatibility	11.3-0
4	APPENDIX	11.4-0
	• Bibliography	11.4-1

VOLUME III - CHAPTER 11

PACKAGING DESIGN TECHNIQUES

SECTION 1 - INTRODUCTION

- Functional Requirements and Design Criteria
- Evaluating Principal Design Constraints
- Support Versus Capture in Structural Design
- Interacting Effects of Other Environmental Stresses

VOLUME III - CHAPTER 11
Section 1 - Introduction

FUNCTIONAL REQUIREMENTS AND DESIGN CRITERIA

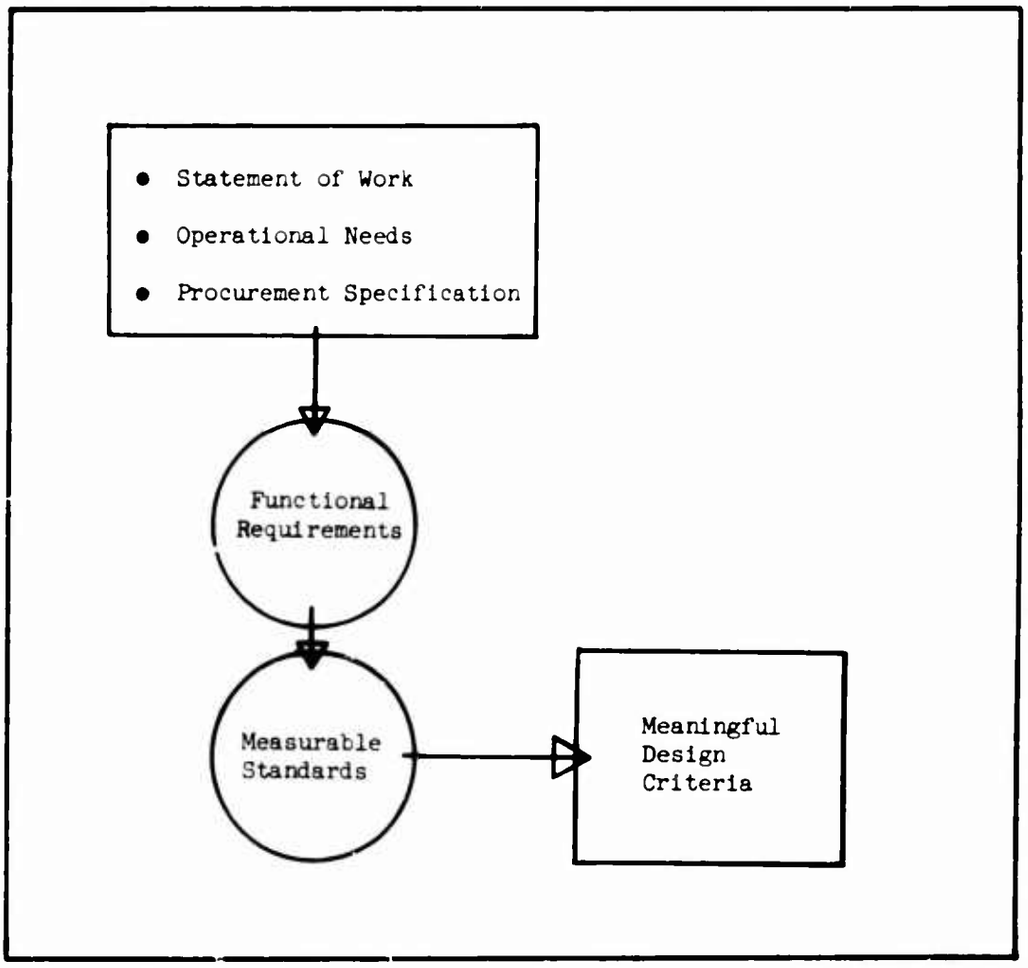
Designers are obliged to think clearly about functional requirements early in the equipment design phase and identify the meaningful design criteria.

Functional requirements are those operational and maintenance requirements which are imposed on the design of the product by the authorized user or the originator of the request for the design. When the equipment designer is confronted with a specified functional requirement, he must assume that it is valid, that there have been tradeoff studies made, and that operations analysis and other techniques were used to establish the true operational and maintenance requirements. Making all such previous studies available to the equipment designer usually helps him to clearly identify the real functional requirements. It is an important factor of effective design that the real functional requirements of any system be reflected in its individual components, equipments, and subsystems. Ultimately, this integration ought to extend into the vehicle, or tactical assembly of vehicles, and on up through the Army's command hierarchy. An equipment which is not responsive to the real functional requirements cannot be considered a good design, no matter what other redeeming features it may possess. Designers are obliged to think clearly about the functional requirements, to assess as accurately as possible the operational environment in which the equipment is to be used, then set out to design equipment which is, first of all, producible, and which is functionally capable and serviceable. Designers must insure that the equipment can be installed by people who are going to be available in the operating environment to install it; that it can be operated within the skill level of people who will be assigned to operate the equipment in the field; and that maintenance, fault isolation, troubleshooting, and other functions which are subordinate to the operating mode can be performed in the removal, refurbishing, restoration, reinstallation, and a host of other constraints. These are some of the real functional requirements which must constitute the body of criteria used to judge or trade off one design for another.

There is a logical connection between functional requirements and design criteria. Functional requirements come to the design area either in a statement of work, in operating requirements, or in a procurement specification. A variety of descriptors are used to identify what the originator has in mind when he talks about the product he wants designed. These we identify as a statement of functional requirement. For each functional requirement there logically follows a specific criteria against which the excellence of the design will have to be judged. That is, for every functional requirement, there is a quantified or measurable criteria against which the design must be measured to see if it satisfies that requirement. The other design criteria are standards of measure used to evaluate how closely the designer has come to satisfying the requirement.

The first responsibility of the designer is to sift out the real requirements. These must be identified by the designer before he begins the task of mechanical, system, or equipment design. One way he can do this is to make a serious study of extracting functional requirements from the formal document. Usually this document comes to him in the form of a specification containing some quantitative information and some information which is not quantitative. If it could all be quantified in the

beginning, it would simplify the task for everyone. Instead, the specification must be researched and reasoned out and a careful analysis made of the requirements. Functional requirements usually have to be derived since they are sometimes not plainly stated in the customer's outline of design tasks. As these requirements become more identifiable, a point is approached where specifics can be assigned; a quantity is derived and numerical values can now be assigned to these requirements. From this point, the designer who proceeds without checking back to see that these requirements are, in fact, what the customer had in mind, may send himself into a long and fruitless work cycle. The earlier the specific information can be derived, the less the chances of wasted motion due to misinterpretation of the real requirements.



DESIGN CRITERIA: Functional requirements may be quantified as measureable standards upon which the system design excellence will be judged; these are the real design criteria.

EVALUATING PRINCIPAL DESIGN CONSTRAINTS

The designer must carefully consider the environment in which the equipment is going to be used and avoid the use of costly and useless design cliches.

Design constraints are an extension of the functional criteria which identify and describe the functional requirement for which the machine, equipment, or system is to be responsive. Primarily, the equipment must be capable of being produced. That is, processes, tooling, sequences of operation, etc., required to assemble a particular item must be within the realm of feasibility and, more specifically, within the capability of the particular organization that will produce the equipment.

Transportability might be included as one of the constraints described in the design specification. The introduction of transportability, portability, modularity, etc., in order that the equipment may be moved in sections less than its whole, generally results in an increase in cost. Anything that can be done to reduce the number of pieces into which an equipment must be broken to be transported will directly affect the cost.

Installation procedures, installation sequences, tools, and requirements for the accessibility of the equipment while it is being installed are all to be taken into serious consideration when designing a system. The problem of accessibility brings with it a tendency toward the arbitrary use of familiar design cliches. Often, maintenance access is conceived as drawers on slides, when fixed panels, roll-up screens, doors, or any number of other configurations might be more appropriate than the conventional drawer and slide. Frequently we find that equipment with a designed-in ease of maintenance which costs the designer additional time to create, increased manufacturing costs and, although exquisite and ingenious, cannot be identified as being responsive to any functional maintenance requirement. In many cases, military electronic equipment mounted in a slide device would rapidly deteriorate from continuous exposure to the normal operating environment. The designer must, therefore, know what kind of environment the equipment is going to be used in and what degree of maintenance is expected while in that environment.

Every time that access is provided to the inside of a machine, the reliability of that machine is necessarily degraded. Almost without exception, the opening of an equipment to provide maintenance access significantly compromises the structural continuity, the internal cooling capability, the air contaminant protection system, the RFI shield, and the personnel shock hazard. Therefore, before the designer starts to consider how he is going to provide access for maintenance, he should determine whether that maintenance must be performed in the particular environment in which the equipment will be operated. Also, tradeoff studies should be made in each system to determine the point at which it is no longer feasible to make field repairs. Maintenance philosophy thus will materially affect the structural design of the equipment.

AVOIDABLE DESIGN CLICHES. . . .

- Unnecessary access provisions
- Unneeded field maintenance provisions
- Unused modularity
- Oversimplification of operational procedures
- Features that do not satisfy the real design criteria

OFTEN LEAD TO INCREASED . . . .

- Design time
- Fabrication complexity
- Overall system cost
- Weight and bulk
- Failure rate from environmental intrusion

DESIGN CLICHES: An early assessment of the real functional needs and design criteria will result in a more efficient system concept, to the exclusion of avoidable design cliches.

SUPPORT VERSUS CAPTURE IN STRUCTURAL DESIGN

Environmental stresses are usually omnidirectional, and usually transfer through equipment from any quadrant. The designer is more concerned with "capture" than "support" in deriving a structural geometry.

The use of the word "support" in structural design denotes a limited design approach. The word "capture" much more appropriately describes the task to be accomplished in tying together equipment components and assemblies. The equipment we are faced with designing will eventually find itself in a rough-and-tumble world, where environmental stress may emerge from virtually any quadrant.

The military equipment designer is concerned with grasping each individual component, part, or subassembly in such a way as to ensure that, no matter from what direction dynamic stresses might be applied, he has provided a sufficient means of tie-down. The integrity of the tie-down, or capture, of components and assemblies is probably as important as any single factor in the survival of equipment in shock and vibration environments.

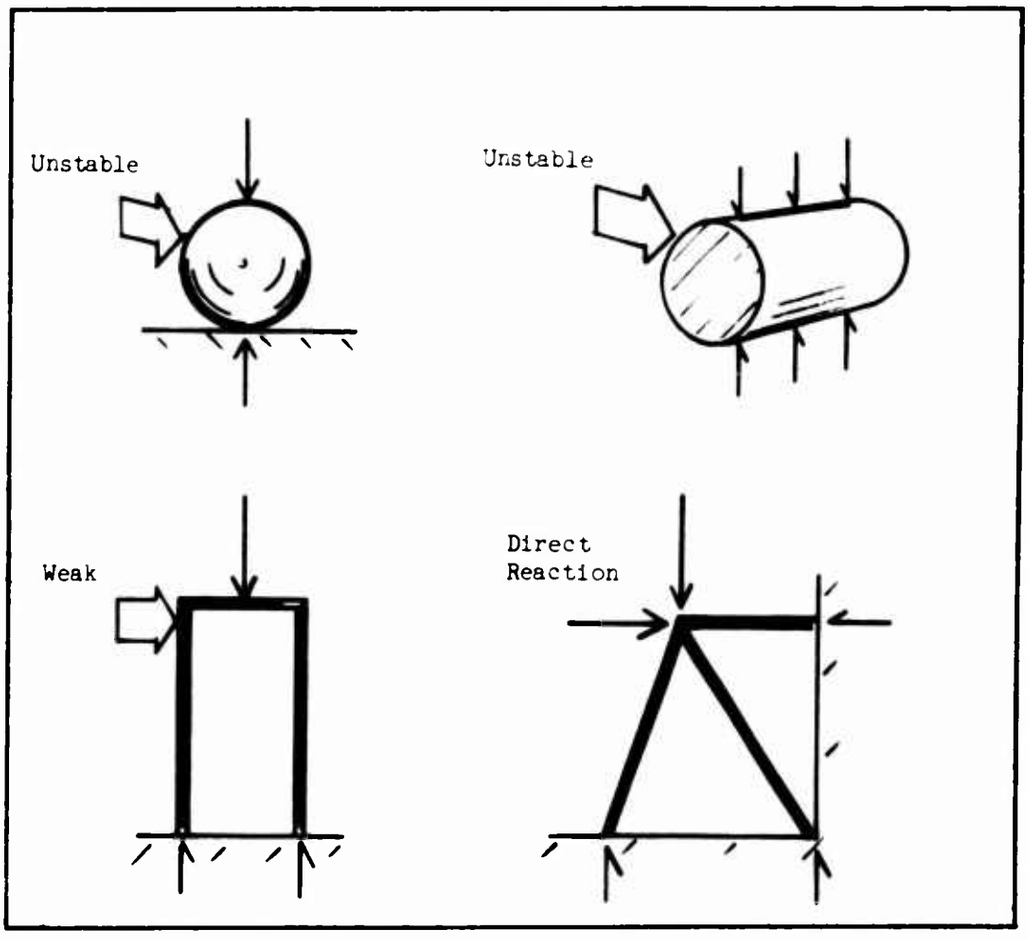
The consideration of removal for maintenance or replacement is of almost equal importance in capturing individual parts or subassemblies. For example, if a part is not destined for replacement in the field, it is unnecessary to provide a quick disconnect, or easy removal means of tying it down. Therefore, the maintenance level functional requirement must be identified. The complexity and vulnerability of devices which permit easy removal should be avoided wherever they are not required to satisfy specific functional requirements. There are basic geometric forms which are somewhat more stable in dynamic loads than others. For instance, the sphere, which is an extremely economical way to contain space, offers a maximum strength for minimum material, but is not suitable for externally applied single point-load application. It is unstable when supported by a single plane or the intersection of two planes. By definition, a force which goes anywhere other than normal to the plane of support will tend to cause the sphere to roll. It takes at least a three-point support to capture a sphere.

Contrasting the sphere with a tetrahedron, there are four equilateral triangles comprising the sides; the simplest enclosure of space, using flat planes. Within the angle of the sides, a highly stable configuration is obtained, where loads applied at any apex of the triangle will be transmitted as column loads through the intersection of the opposing faces. If the structure were configured with struts, wires, or columns, a load applied at any one of the apexes would follow a compressive path to the opposite force applied by the face or base of the tetrahedron.

A cylinder is a form of cone when we look at it in terms of its inherent stability. The cone should possibly be looked at first. From the standpoint of apex loading, the cone is similar to the tetrahedron with the exception that the base is circular instead of triangular. Loads applied at the apex find a resistive path through the surface of the cone for load analysis. A cylinder is essentially two inverted cones. Loads applied at the top of the cylinder must be within the same conical displacement or orientation as the base of the cone inverted.

The cube, or the rectangular solid, is made from intersecting parallel planes. It is a highly unstable device, with load applications from almost any direction. The tendency to parallelogram, to skew, rotate, or shear, are always present in the rectangular form. Consequently, some basic things must be done to the popular rectangular form to stiffen it. The first thing that is usually done is to gusset the corners, superimposing on the corners the kind of stability that is characteristic of the tetrahedron. Individual equipments will not likely ever be packaged in the tetrahedron, but a series of interlocking, or nesting, pyramid or tetrahedron shapes need not sacrifice any space.

Structural shapes and sections which provide stiffness, strength, and lightness, use box or tubular sections for beams and columns, ribbed panels and edge-flanging for flat plates, edge-flanging for lightening-holes, gussets and brackets for end connections of beams and columns, and stiffeners for large load-supporting plates or surfaces.



WHY CAPTURE? The real world in which an Army equipment system must operate is a complex of omnidirectional stresses which combine in unpredictable patterns. Further, these stress patterns are almost always oriented in unconventional ways.

DYNAMIC STRESSES AND THEIR DAMAGE POTENTIAL

It is apparent that some dynamic stresses are omnidirectional; some are repetitive, and some are constant and predictable in direction. Their damage potential comes in the form of eventual failure from fatigue, excessive stress of supporting structures, and excessive deflection of parts.

Shock, as an example, is an unscheduled, omnidirectional dynamic stress which generally enters the equipment through whatever face of the structure is attached to the next assembly. For example, a box sitting in a truck is exposed to shock through its base. However, if not poperly secured, the box may experience subsequent shocks from virtually every direction as it bounces and collides with other equipment. Equipment mounted to the bulkhead of an equipment shelter will experience shock forces from any direction when it is airdropped from an airplane or helicopter.

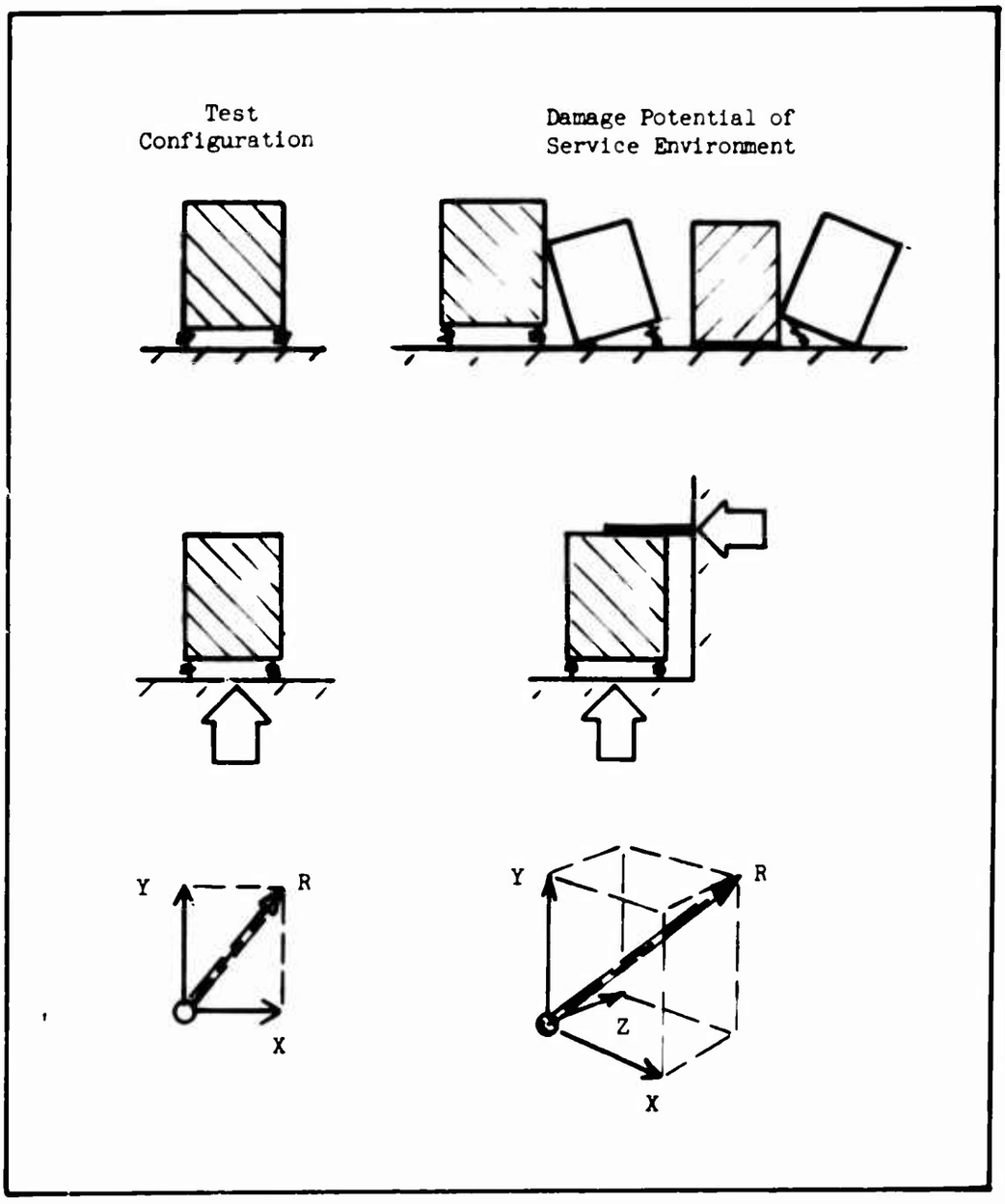
Conversely, vibration is usually imparted to a system or equipment in the direction permitted by the attachment of the item to the next assembly. The excitation axis may be anticipated or limited, but generally it can be predicted as relating to one of the conventional faces of the equipment. This means that vibration resistance in a design may not necessarily be an omnidirectional resistance. Resistance to vibration implies the use of one of two basic techniques for isolating or protecting an object from the raw environment; adequately isolating the fragile equipment, or adequately securing it to the forcing structure. The same rules imply specifically that an isolation system designed to separate a fragile equipment, subassembly, or component from the vibration stresses in the dynamic environment, must be capable of providing adequate separation or isolation from all anticipated frequencies, in all directions from which the stress may be applied. We have frequently observed conditions where an equipment is to be base-mounted, and the laboratory simulated environment is constrained to the three principle axes: vertical, horizontal, and front-to-back. It is quite possible to satisfy these three specific situations successfully and fail to solve the real problem of actual frequencies or amplitudes of vibration which may be present in vectors other than the three principle axes.

In an electronic assembly made up of a combination of individual masses, each of these masses may have one or more natural frequencies. If these frequencies are within the range of frequencies imposed by the dynamic environment, the various individual components will resonate at various points within this range of excitation. Failure may occur through fatigue, excessive single stress, or excessive deflection of parts.

Although fatigue failure results from a large number of stress cycles, the time required for these stress to accumulate is short when a component is vibrating at hundreds of cycles per second.

Excessive single stress may cause brackets or other supporting structures to yield or fracture.

Excessive deflections of parts may result in their hitting one another with resultantly high impacts.



TRUE SERVICE STRESS: The designer must evaluate the operational requirements of the equipment systems and be aware of the potential differences with the Quality Assurance Tests.

INTERACTING EFFECTS OF OTHER ENVIRONMENTAL STRESSES

The combined effects of temperature, corrosion, and dynamic stresses upon a structural interface should be considered if the part must function in a combination of environments.

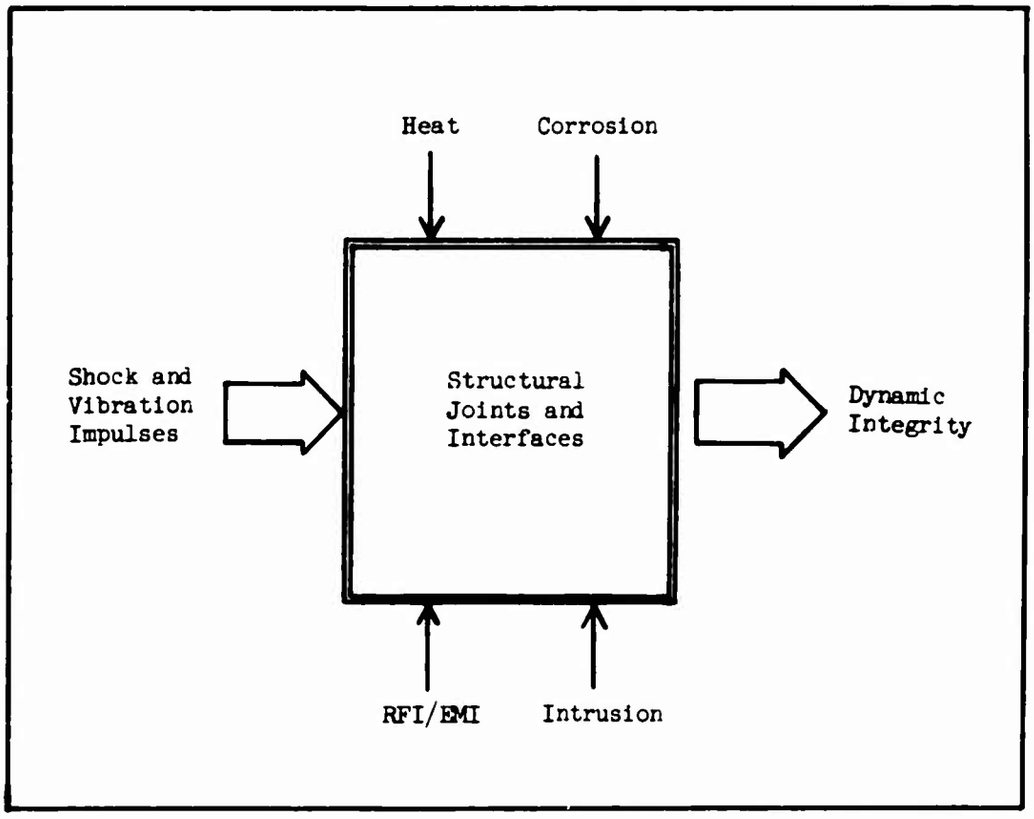
Considerations for RFI and EMI are complex when a combination of environmental stresses are applied simultaneously upon opposing faces of a structural connection which must be held in fixed positions throughout all the dynamic environments. If the connection is one which permits even limited motion between the two opposing faces, the gasketing interference, labyrinths, or whatever might be used to secure an RFI closure, may fail to provide that closure during shock or vibration excitation. It is also possible that the effective closure will progressively degrade, as motion between the two faces is permitted to continue. In effect, we have a successful RFI shield during the early period of the equipment's existence, but after prolonged exposure to an environment which causes the two opposing faces to work with respect to one another, the RFI shield may ultimately have no shielding effectiveness.

Heat is identified as one of those conditions which is frequently assumed to be passed through a structural interface. Whether this can be done by conduction, or whether it must be done by radiating from one surface to the other, makes a substantial difference in the effective heat path. A structural interface usually presents a barrier to the passage of heat, regardless of the condition, roughness, color, or material of the two opposing surfaces. Some heat will pass by conductive means from one surface to the other where close contact is maintained. In the case of components specifically intended to transfer heat into the next assembly, this close fit is imperative. Often this is not a true conductive path because a significant air space is permitted between the faces. It is essentially a radiator and an absorber looking at one another over a short distance; the distance is significant. Convection currents may become a factor in transferring heat over structural interruptions which have substantial air gaps.

Corrosion is another of the environmental stresses we must be aware of when we encounter a structural interface in our design. Specifically, the two opposing faces which are not in complete physical contact with each other, create atmospheric eddies, and consequently do not breathe normally. Any cavity which is large enough to permit sweating or the collection of condensing moisture, but is not large enough to admit normal air currents, presents this problem. Any component or part which, because of its orientation with respect to the rest of the equipment may not have the opportunity to breathe and periodically get drying air circulating over all of its faces, is similarly vulverable. These conditions are always potential degradations to the RFI barrier between the two parts or surfaces, since the corrosive oxides are generally electrical insulators rather than conductors. Cavities, unfortunately, are often located in areas where either externally or internally condensed moisture flows to and collects. Entrapments, therefore, degrade the RFI integrity, interfere with the passage of heat, and tend to make the structural material vulnerable to corrosive action. There are numerous preservation techniques which may inhibit the corrosive action of entrapped moisture, but most of these also interfere with the passage of electrical current and heat. One, for example, is the use of hard anodizing to obtain a

high corrosion resistance in an aluminum alloy which may be exposed to a cavity or an entrapment. The hard anodize, however, is an excellent corrosive inhibitor, but an almost perfect insulator with respect to both heat and RFI.

Usually, those materials which are good conductors of electrical current generally are prone to corrosive action. Paint treatments, coating, plating treatments, and the selection of basic materials all are of substantial concern in the design of a structural interface, where we intend to have a continuous path to make an RFI shield or a continuous path through which to pass heat to a sink. Two opposing faces which are permitted limited movement with respect to one another will periodically wipe or clean the corrosion products. The design of a slopping joint which permits the motion between two parts to keep insulating oxides from developing, is probably not justified to improve the RFI shielding. The fact remains that a continuous mechanical scraping of one surface against the other tends to keep the surfaces conductive for RFI purpose and generally tends to prevent the formation of more than superficial corrosion.



SYSTEMS APPROACH TO DYNAMIC INTEGRITY: The integrity of a structural interface is dependent upon its ability to withstand the combination of all the environmental stresses. The design must reflect this combination of other, less apparent, factors.

VOLUME III - CHAPTER 11

PACKAGING DESIGN TECHNIQUES

SECTION 2 - ATTACHMENT OF EQUIPMENT ELEMENTS

- Homogenous Joint Techniques
- Consideration of Tension and Shear in the Selection of Fasteners
- Some More Design Tips on the Use of Fasteners

HOMOGENEOUS JOINING TECHNIQUES

Structural interfaces may be joined by a variety of homogenous joining techniques. If the interfaces are to be joined permanently, welding is the technique which produces the fewest shock and vibration problems.

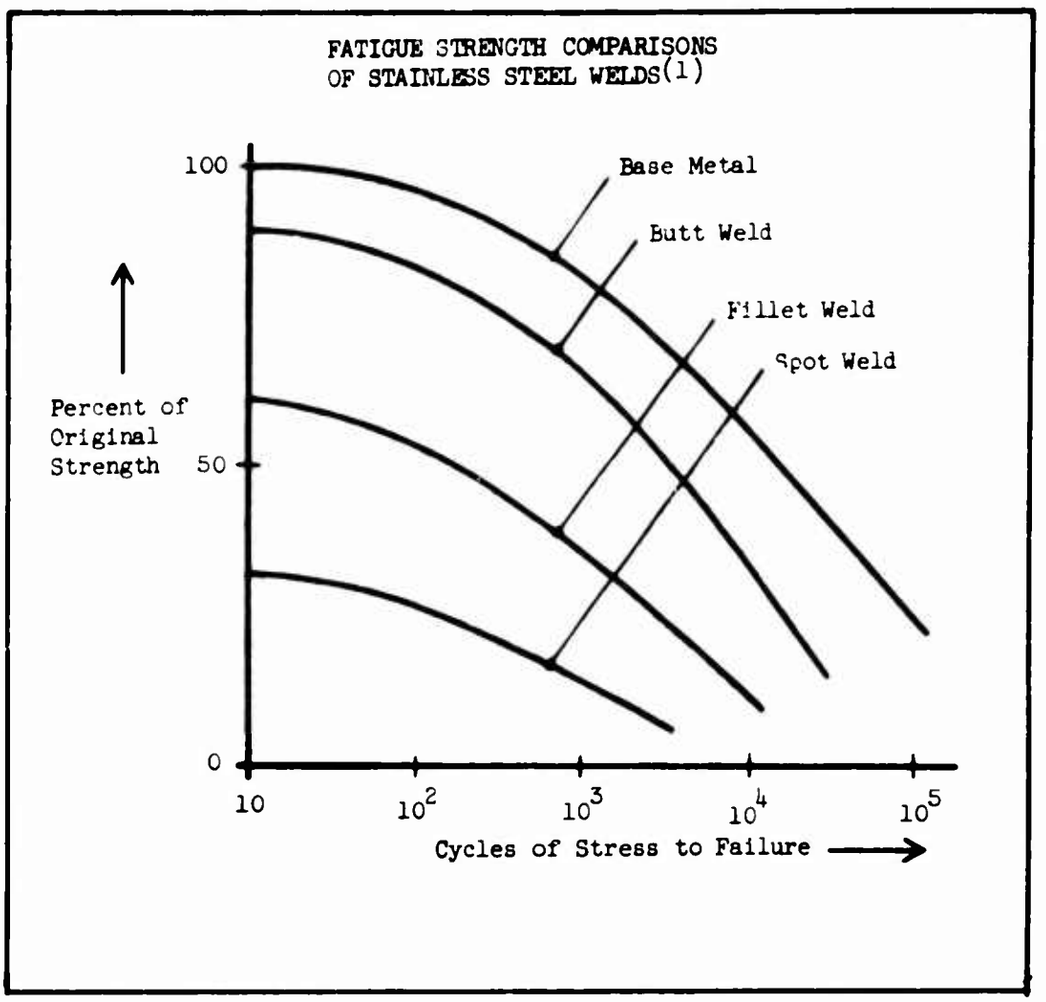
Increasing the friction between mating parts is one means of preventing their movement. An adhesive between two parts will increase the shear resistance across the interface, with the thin membrane sticky surfaces acting as a shear-imparting connection between the two joined pieces. It need not be a glue or a pressure-sensitive, sticky surface; merely texturing the surfaces or interposing a high friction material, such as a rubber blanket, or even a double-sided piece of sandpaper-like material, will supply the required shear resistance up to and slightly beyond the point where motion between the two parts could be induced by environmental stress.

The extremely smooth, polished, flat surface is another excellent high static friction source. By word of caution, it should be emphasized that any kind of friction material which is actually permitted to move, introduces a rapidly deteriorating joint, in that each subsequent motion tends to grind itself into a situation where it will move easier next time and progressively destroy the shear capability entirely. However, within or beneath the limits of the force that it takes to move a particular configuration of high friction surfaces, this would be considered a successful shear interface design. One of the chief reasons for considering this technique is economy.

Point-loading of parts, whenever possible, ought to be avoided. Weldments of intersection planes or parts offer a more appropriate approach to a homogenous (or more than point contact) method of joining two or more pieces in a structural entity. Here, again, the weld should be regarded as a method of putting two or more parts in shear. In the tension mode, the best weld will be most vulnerable to fracture and failure. In such techniques as weldments, the placing of components and the configuration of the structure should be such that the components of the structure end up in shear across their interfaces.

Stress concentrations in weldments can be avoided by making full-depth welds and by ensuring good fusion at the bottom of the weld. Short, intermittent welds are undesirable. Welded joint strengths can also be increased by heat treatment to relieve any residual stresses. Spotwelds should be avoided. Spotweld damage is aggravated by the fact that a high stress concentration exists in the junction between the two bonded materials and, under repeated loadings, this increases the possibility of damage.

A truly homogenous joint is available in dip brazing, atmospheric soldering, or impact fusion. It is possible with these techniques, to obtain a tension characteristic across the joining of two pieces which is equivalent to the parent material strength.



JOINTS AND INTERFACES: Joints always cause an increase in weight and manufacturing complexity due to the extra layer of base material. Welding is probably the best compromise method for unavoidable interface.

CONSIDERATION OF TENSION AND SHEAR IN THE SELECTION OF FASTENERS

The limitations of homogenous joining techniques across an interface necessitate the use of other joining devices. In selecting these devices, the designer must consider their application with regard to modes of stress to be encountered; namely, tension and shear.

Devices that create a homogenous or continuous material bond, such as glues, laminations, welds, dip brazes, etc., are most desirable as an interface joining technique. These devices usually obviate the requirement to do something special across the interface to obtain an RFI shield or establish a good heat transfer path, as well as limit excursion or motion between the two faces. Although connections of this type are more able to resist omnidirectional stresses and strains, there are other devices which must be used when the homogenous techniques cannot be applied. These are separated in terms of whether they are a tension or a shear device.

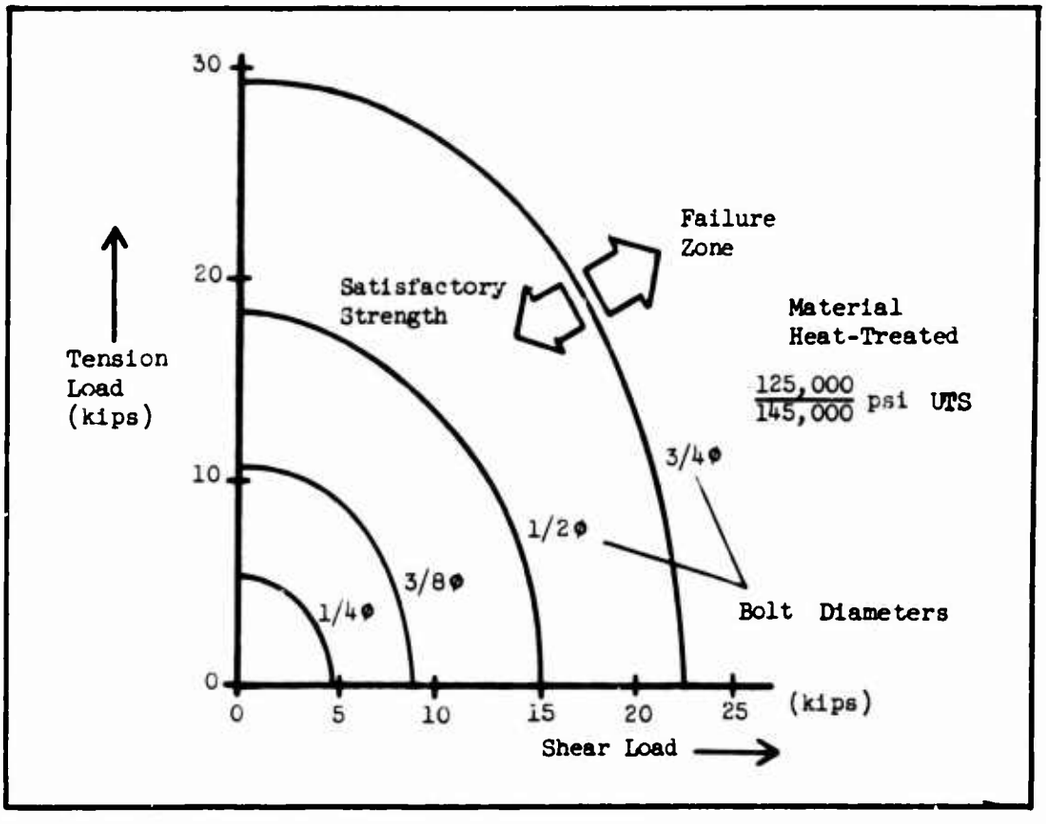
A shear device, such as a pin in a bushing, can be used to load one part with respect to another in a direction normal to the major axis of the pin. The effectiveness of a shear pin and bushing combination as a joining device is largely a function of two things: (1) the diameter of the pin with respect to the kinds of materials involved in both interfacing parts and the pin, and (2) the amount of tolerance or clearance allowed between the maximum diameter of the pin and the minimum diameter of the corresponding bushings or holes in the mating parts. In military electronic equipment, high strength shear pins of reasonably small diameter (1/4" and 3/8") are used for joining two pieces of aluminum. Shear devices should only be considered where the parts will be permanently attached. There should be a high pre-load in the directions in which the loads can be applied dynamically in the plane perpendicular to the major axis of the pin. Therefore, an undersize hole or an oversize pin, or both, are needed. The use of dry ice to reduce the diameter of the pin to minimum, and then a drive or press fit into the two opposing parts accomplishes the kind of extreme pre-load that is necessary. The force to be transmitted across a pin and bushing combination in shear is not added to the internal forces exerted by the pin and the bushing.

One good way to provide high expanding force in the pin or radially compressive force on the bushings is to provide a Collet-type system. In these devices, the pin tends to fully engage the hole in the bushing. Examples are the split or expanding bushings commonly found in the chucks of power tools.

Another technique which can be used alone or in connection with split devices is a lead angle, or taper, on the pins. This requires a compressing force which tends to drive the pin continually into the corresponding tapered cone of the bushing. This has at least one unique advantage over cylindrical forms. No matter how well pinned the cylindrical form, the conical form can be arranged so that the application of compressive external to the pin permits periodic or continuous retightening or resetting. It is common to apply a continuous spring force which is sufficient to keep the pin fully engaged in the cone through all of the limits of shock and vibration that the equipment might be subjected to. A cylinder is essentially capable of being a shear device in any direction in the one plane which is normal to its major axis. Tapered pins can be configured so that they tend to be gripped or seized by the receiving bushing.

Tension devices should be limited to the tension mode; specifically, a bolt should not be used in shear unless it is a shear bolt. The stripper type bolt or the shear bolt, has a cylindrical body which is not threaded. This shear body will provide a tight fit as well as a tension path over the shear length. The application of shear forces over threads loaded in tension creates a situation which probably causes most bolt failures.

A bolt is essentially a tension device, while a rivet is essentially a shear device. The bolt can be retorqued; normally rivets cannot be conveniently reset. So long as a shear load is imparted across the rivet up to the limits of the material, there exists a best case positive connective joint; the upset rivet having swelled outward to fill the cavity of the hole in the two joined parts. The bolt, however, does not swell. In fact, a bolt contracts slightly as the tension forces approach the elastic limit. The bolt will remain secure in tension so long as it does not encounter a shearing force. A bolt torqued well below its maximum limit may fail with the addition of very minor shearing forces applied across the bolt. The shearing forces impart very high tension forces at the point of the application of the shear force. The use of spring-loaded locking devices usually work to compromise the tension imparting characteristics of a bolt.



SHEAR-TENSION INTERACTION: The necessity for combined shear and tension in a fastener must be carefully evaluated for the stress interaction.

SOME MORE DESIGN TIPS ON THE USE OF FASTENERS

Fasteners offer the designer a wide variety of mechanical alternatives. Proper attention to the details of their applications will add to their effectiveness.

The designer should be concerned with the possible replacement necessity of the joint. Some fasteners, such as rivets, must be destroyed in order to be removed. If joint takedown is required, this must be taken into consideration when selecting the type of fastener.

Rivets are ideal for multiple fastener joints where close hole tolerances are impractical. The filling tendency of the rivet when it is bucked is beneficial in spreading the service load among all the fasteners. A profusion of small fasteners has better strength potential than one large fastener, particularly in thin sheet material. Mechanical fasteners presently available offer virtually an infinite number of types, sizes, and characteristics to match the wide range of design requirements. Some more items to consider when selecting a fastener are:

1. corrosive condition
2. elevated temperatures
3. weight
4. type of load
5. magnetic or non-magnetic
6. high vibrations or cyclic load
7. heat or electrical conductivity
8. cost
9. removability

Some features that should be considered in the design of a joint are:

1. The strength of the fastener should balance the strength of the joining members.

2. Locking devices should be used, e.g. lockwire, self-locking screws, self-locking nuts, and locking inserts.

3. There is a large variation in the torque-load relations when using torque values to gauge the fastener preload.

4. Consider preload when determining the capacity of the fastener.

5. During installation, the torque load and induced tension load should not produce yielding. Care should be exercised in using the thread root area rather than the shank area.

For retaining at least partial effectiveness when a bolt is elongated by shock, the use of a split-ring washer is recommended. If the backing-off of the bolt is to be prevented, then a star, or tooth, type washer would be more appropriate. Since the teeth usually cut through anodic and other insulating coatings, they are also used when good electrical contact is required.

A preferred method for holding a screw or bolt and nut secure is by means of a locknut. The locknut develops friction between the bolt threads and

the nut. In preventing loosening of bolts under vibration stresses, locknuts are quite effective. Under shock stresses, they maintain joint tightness and are not easily damaged.

Failure to provide the necessary joint tightness may be due to insufficient tightening of the fastener and locking device. To avoid this, specifying the use of a torque wrench or pre-load indicating washer is recommended for proper application of the specified torque.

FASTENER HEAD STYLES

- Binding - Undercut binds and eliminates fraying of wire in electrical work.
- Button - Used for bolts and cap screws.
- Fillister - Available with slotted or Phillips driving recess for machine screws, tapping screws and cap screws.
- Flat Fillister - Used in counterbored holes that require a flush screw.
- Flat, 82° - Use where flush surface is desired.
- Flat, 100° - Larger head than 82° design.
- Flat Undercut - Permits flush assemblies in thin stock.
- Headless - Used for set screws only.
- Hex - Standard head for machine bolts and screws.
- Hex Washer - Used for machine screws and tapping screws.
- Oval - Similar to standard flat head.
- Pan - For use instead of round head screws.
- Round - Used for bolts, machine screws, cap screws and drive screws.
- Round - Countersunk - Used for bolts only.
- Round Washer - Used for tapping screws only.
- Square (bolt) - Generous bearing surface for wrench tightening.
- Square (set-screw) - Can be tightened to higher torque.
- Truss - For covering large diameter clearance holes in sheet metal.
- Twelve-point - Available for bolts and machine screws.

COMMON NUT STYLES

- Hex Jam
- Hex Thick
- Hex Slotted
- Hex Castle
- Hex Flange
- Track Bolt
- Square
- Acorn
- Twelve Point

CLASSIFICATION OF METAL WASHERS

Group	Type
• Flat	ASA Type A
• Plain	ASA Type B
• Spring	Conical Belleville Serrated Edge Wave Ramp Conical Helical Spring
• Tooth	Standard Special

USING FASTENERS: The geometric configuration of bolts, screws, nuts, and washers should be carefully reviewed when selecting a fastener.

VOLUME III - CHAPTER 11

PACKAGING DESIGN TECHNIQUES

SECTION 3 - IMPORTANT DESIGN CONSIDERATIONS

- Designing for Material Compatibility

DESIGNING FOR MATERIAL COMPATIBILITY

The designer should be interested in matching, in so far as he can, the total mechanical and physical characteristics of all of the contributing members, parts, and assemblies of the structural system.

Galvanic couples and electrolysis are probably the chief reasons for avoiding the placement of dissimilar metals adjacent to one another. Crevices, cavities, and entrapments usually are places where galvanic corrosion is manifested and obviously very difficult to detect. It is usually not detected until it has gone so far that the part has completely deteriorated. The effect of the galvanic couple may be transmitted through a succession of parts until it comes to that part which will be sacrificed. This may be incidental or it may be critical to the functioning of the system.

The techniques available for preventing galvanic coupling are fairly classical and known to most designers. The point to remember is, virtually no applied finish plating or material used as an insulating membrane will be as successful as choosing non-coupling materials at the outset. At least one of the acceptable devices in this regard is to provide a sacrificial element; one which is designed to be anodic, and which will, in fact, lose its ions while preventing all the rest of the parts of the equipment from changing their composition. This is an acceptable technique and requires periodic maintenance and inspection of the sacrificial element so that it can be replaced, once the couples have commenced to operate. In addition to the corrosive effects already mentioned, there is a need for compatibility and consistency of materials within the structural system. In non-structural elements such as circuit components, it is necessary to use specific materials for corrosion resistance, capacity, or conductivity. In the case of the structural materials, we would do well to try to arrange the materials which are in the same general region of thermal expansion and contraction to be adjacent to one another. It is difficult to make a lamination of steel and aluminum with the expectation that under widely varying temperatures it will not set up some rather severe stresses due to dissimilarities in thermal expansion characteristics.

Fatigue limits also should be a consideration in selecting materials for a structure. In selecting structural metals for environments of shock and vibration, the most desirable properties are high ductility and high yield point.

Temperature effects on rubber and thermoplastic materials in electronic equipment must also be considered. These materials become more brittle at low temperatures, as do almost all materials. Conversely, at higher temperatures, their stiffness decreases. Therefore, if shock or vibration is imposed upon electronic equipment exposed to temperature extremes, it is necessary to consider these changes in the physical properties of these materials. For example, the change in the stiffness of wire insulation will result in a change in resonant frequency, or a change in stiffness of potting material may cause it to flow at high temperatures.

THE GALVANIC TABLE

Anodic
(active)

- Magnesium
- Magnesium Alloys
- Zinc
- Galvanized Steel
- Aluminum 6053
- Alclad
- Cadmium
- Aluminum 2024
- Mild Steel
- Wrought Iron
- Cast Iron
- 18-8 Stainless Steel (active)
- Lead
- Tin
- Naval Brass
- Nickel (active)
- Yellow Brass
- Red Brass
- Copper
- Silver Solder
- Nickel (passive)
- Titanium
- 18-8 Stainless Steel (passive)
- Silver
- Graphite
- Gold
- Platinum

Cathodic
(noble)

SPECIFIC STRENGTH

High

- Titanium and Its Alloys
- Martensitic Stainless Steels
- Ultra High Strength Steels
- Alloy Steels
- Aluminum Alloys (hard)
- Carbon Steels (h&t)
- Magnesium Alloys
- Nickel and Its Alloys
- Carbon Steels (hot rolled)
- Carbon Steels (cold worked)
- Aluminum Alloys (annealed)
- Plain Brasses (hard)
- Malleable Irons
- Precious Metals
- Lead and Its Alloys

Low

THERMAL EXPANSION

High

- Zinc and Its Alloys
- Magnesium Alloys
- Aluminum and Its Alloys
- Tin and Aluminum Brasses
- Silver
- Stainless Steels
- Coppers
- Nickel-Base Superalloys
- Nickel and Its Alloys
- Alloy Steels
- Alloy Steels (cast)
- Malleable and Wrought Irons
- Martensitic Stainless Steels
- Ferritic Stainless Steels
- Gray Irons (cast)
- Molybdenum and Its Alloys

Low

MATERIAL COMPATIBILITY: The designer should select materials that are similar in their reaction to the imposed environmental stresses.

VOLUME III - CHAPTER 11

PACKAGING DESIGN TECHNIQUES

SECTION 4 - APPENDIX

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