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DESIGN CRITERIA FOR SYSTEM EFFECTIVENESS

Allen Chop
Lockheed Missiles & Space Company

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FOREWORD

This document is the Final Report submitted by Lockheed Missiles & Space Company, Sunnyvale, California, under Contract F30602-67-C-0182, Project 5519, Task 551907, with Rome Air Development Center, Griffiss Air Force Base, New York. Lockheed's report number is D050757. Jerome Klion, EMERS-2, was the RADC Project Engineer.

Distribution of the report is restricted under the provisions of the U.S. Mutual Security Acts of 1949.

This Technical Report has been reviewed and is approved.

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ABSTRACT

This report describes the methods and analytical techniques relative to Air Force system designs under a system effectiveness requirement, with the objective of providing design criteria at the subsystem and equipment levels. Presented are the relevant background concepts and general application considerations necessary for an understanding and implementation of a system effectiveness evaluation. The techniques for the objective apportionment of a system's figures of merit to their constituent parameters and accountable factors are described and illustrated in detail, with primary emphasis on the developed Lagrange multiplier with priority list solution method. The technical role and perspective of system functional transfer equations, methods for their application and use in the evaluation and apportionment of system effectiveness are rendered and demonstrated in detail. A plan for the dynamic monitoring and status reporting of system effectiveness progress during all phases of system development, and to provide management and design visibility on critical and sensitive problem areas, is outlined. Additionally, the methods and techniques are further illustrated and expanded in the Technical Supplement to this report.

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SUMMARY AND CONCLUSIONS

SUMMARY

- <u>Section 1 - Introduction and General Considerations</u>

  The basic objective and the framework to which the study is addressed are described. The objective is the preliminary development of formalized methodologies and analytical techniques relative to Air Force system designs under a system effectiveness requirement to provide design criteria for system effectiveness at the subsystem and equipment levels. Consistent with this objective, and to facilitate the immediate use of the methodologies as analytical tools, special emphasis is given to the important aspects of practicality, simplicity, general applicability, and a minimum need for data in addition to what is normally available during any system development program.

  The framework and its associated elements, to which the techniques, and methods are related, are shown in a system effectiveness evaluation network. This network describes the basic performance effectiveness evaluation process, which is characterized by the technical areas of effectiveness apportionment and dynamic monitoring and prediction. The report treats in detail the development results in these areas, which are vital to the implementation of system effectiveness analyses and the advancement of the technology.

- <u>Section 2 - System Effectiveness Evaluation Background</u>

  Presented in this section are the relevant background concepts and general application considerations necessary for an understanding and implementation of a system effectiveness evaluation. The technical aspects of defining figures of merit, combining figures of merit, and establishing accountable factors in system designs for effectiveness are examined and described.

Methods for the evaluation of system effectiveness are presented. The evaluation process is frequently performed in conjunction with an apportionment process when considering the technical problem of maximizing a figure of merit subject to possible constraints. These constraints may be imposed on the system by operational requirements and technological limitations.

An example of an evaluation of a figure of merit is given. This example relates to a squadron of missile systems for a multiple target mission, where each missile is assigned to a target. The figure of merit evaluated is the expected amount of damage inflicted on the target. Based upon the WSEIAC concepts, effectiveness is related to availability, dependability, and capability, and the evaluation techniques for these parameters illustrated. The structure of the dependability states is further expanded in Appendix A.

In evaluating a figure of merit, the quantitative and controllable subsystem accountable factors of the system are permitted to take on a range of values. Although some accountable factors are not precisely known, their possible values can be represented by probability distribution. Consequently, direct and Monte Carlo techniques are described for the translation of the means and standard deviations of the accountable factors to a calculated system figure of merit. These techniques are treated in detail in Appendix B.

The capability parameter for the illustrated missile system is related to, and evaluated for, various system parameters, including accuracy, warhead yield, hardness of target area, target density, and target distribution. The theory for evaluation of point and elliptical or circular target areas is presented and expanded in Appendices C and D. Additionally a numerical example of the theory and techniques is provided in Appendix E. Also, application of the evaluation theory and methods to other Air Force systems is indicated.

Finally, an illustration of an effectiveness evaluation based upon a spacecraft system with multiple figures of merit, which are combined into an overall figure or merit, is presented in Appendix F.

- Section 3 - System Effectiveness Apportionment

This section describes the analytical methods and application considerations for the objective apportionment of a system's figure(s) of merit to its constituent performance parameters and accountable factors, and for a further apportionment to subsystem design parameters. The analysis of three techniques is presented for the accomplishment of an optimum apportionment under the usual imposition of system constraints. Of the three techniques examined, namely direct search, dynamic programming, and Lagrange multiplier, emphasis is placed on the latter because of the fewer calculations involved.

With respect to the Lagrange multiplier technique, a description is provided of the mathematical formulation of the effectiveness and constraint functions, in terms of quantifiable accountable factors. Classical and iterative methods are described for the solution of the Lagrange equations. The priority list solution method, which was developed during the course of the study, is also included. The priority list method is shown to possess the capability for establishing the relative contribution of each accountable factor to overall effectiveness. This provides a priority rule for subsystem optimization alternatives for maximum system effectiveness. The procedure for the contruction of priority lists is also presented. Further, the technical circumstances under which all the described apportionment and solution techniques can be validly and favorably applied are indicated.

Three numerical examples are presented. Two examples illustrate the Lagrange multiplier with priority list solution method. The remaining example illustrates a graphical solution technique for determining the optimum value of each accountable factor.

The first example, detailed in Appendix G, provides a demonstration of the Lagrange multiplier with priority list solution technique. The step-by-step solution procedure is described, and the solution process illustrated. The example is addressed to the dependability parameter of effectiveness, and illustrates the optimization of the reliability parameter of a communications satellite through apportionment among various subsystems under multiple system constraints.

The second example, detailed in Appendix H, further demonstrates the logic of the Lagrange multiplier with priority list solution technique as applied to a more intensive analysis of the same communications satellite. The procedures illustrate the solution technique for determining the satellite design yielding the optimum effectiveness, where the figure of merit is expressed as an expected value. The interactions of all effectiveness parameters are detailed. The use of approximation formulas is also shown to solve the classical Lagrange equations.

The third example, detailed in Appendix I, demonstrates the apportionment of resources to a launch vehicle for a space payload. The apportionment technique illustrated is the classical method using the total derivative of the effectiveness function to identify optimums. Additionally, the use of graphical methods is illustrated to solve the equation systems at these optimums.

A computer program for an apportionment and trade-off analysis, detailed in Appendix J, is described for an effectiveness model of a specific form. The adaptation of this program to the apportionment of effectiveness based on the Lagrange multiplier with priority list method is further delineated.

- Section 4 - Transfer Functions for the Apportionment Process

The technical basis is presented for the existence, identification, and use of subsystem-to-system functional relationships. These are performance relationships used to objectively apportion the system's figure(s) of merit and its constituent performance parameters for maximum effectiveness. Such relationships, being expressable as physical laws, empirical relations, or probabilistic forms, are categorized as transfer functions in a general context. Special note is made of a subset of these transfer functions used for transformation of equation systems between coordinate systems. This use is to simplify the evaluation process. An example of such a transformation is presented for a missile system.

A perspective of representative and generalized transfer functions for the missile, spacecraft, and cargo aircraft classes of Air Force systems is provided. The functions which are detailed relate subsystem accountable factor performance to system performance. Specific emphasis is placed on the system capability parameter and its accountable factors because of the complexity of the equation systems used for the evaluation of this parameter.

With respect to missile system applications, transfer functions involving the laws of motion used in a six-degree-of-freedom dynamic simulation are developed. The application of these functions to generate simulated system performance outputs, by varying subsystem accountable factors is described. Considerations for a missile system simulation model are shown to include:

(1) Choice of reference frames

(2) Use of frame transformations

(3) Force and moment equations involved and their uses

A typical flow diagram of the trajectory simulation is shown.

To demonstrate a method which is useful for the evaluation of a key capability parameter, an accuracy error analysis is described and illustrated. This analysis applies to instances where small variations in values of the accountable factor are present. To facilitate the translation of an incremental change of system performance to an incremental change in design variables more suitable for guiding design actions, the use of sensitivity functions which evolve from the more complex transfer functions and the simulation results is described.

Various forms of sensitivity curves are illustrated to typify different effects of subsystem accountable factors on accuracy. Also, a list of subsystem accountable factors for which sensitivity functions can be generated is provided. A further expansion of three of these subsystem accountable factors to subtier design accountable factors, and a perspective of their functional relationships, are presented in Appendix K.

For a typical cargo aircraft system, representative accountable factors and performance parameters for the establishment of sensitivity relationships are shown. Also, generalized transfer functions for the capability parameters are described. These functions are indicative of the approach for generating the necessary total set of transfer functions to evaluate and optimize effectiveness for this type of Air Force system.

Representative spacecraft transfer functions based on orbital mechanics, and which are suitable for the establishment of sensitivity functions, are presented. These relationships allow for the apportionment of orbital positional errors as a function of velocity changes. Similar relationships for four basic on-orbit maneuvers are described.

Finally the use of transfer functions in the apportionment of a capability parameter is illustrated.

- Section 5 - System Effectiveness Monitoring Plan

A plan for the dynamic monitoring and status reporting of system effectiveness progress during all phases of system development is detailed. This plan is to be a necessary contractor-prepared document for Air Force management approval prior to implementation. Desirable features of such a plan are described. Also, methods for the achievement of these features are presented, including:

(1) Visibility on critical and sensitive problem areas early in, and during, development

(2) Prospectus of overall effectiveness growth and progress

(3) Focus of effort, and filtering of design changes based upon critical and sensitive parameters through identification and tracking

(4) Mechanism for timely corrective action response.

Alternate effectiveness management structures are examined. The composition and frequency of the status report to provide both Air Force and contractor management with visibility on overall effectiveness progress are presented.

Nine basic effectiveness task elements for the dynamic monitoring and prediction of current effectiveness levels are delineated. Task elements not covered in prior sections include the effectiveness task elements of mission definition and analysis, systems description, test plan analysis, and model exercise for parameter tracking. The technical aspects of these task elements are expanded to include:

(1) A typical mission profile with stress adjustment factors

(2) The translation of development test data into dynamic inputs for effectiveness monitoring

(3) An integrated approach for using data from normal development test programs.

Based upon test data, simulation results, and analytical determinations, methods for the tracking of the constituent parameters of effectiveness are indicated. One technique described is the use of parameter tracking charts to show the rate and magnitude of convergence to the apportioned requirements. A rating system to measure this convergence progress is also suggested.

CONCLUSIONS

- The apportionment and dynamic monitoring of technical activities of the system effectiveness technology are intimately related to other effectiveness technical activities. These associated activities are effectiveness evaluation and optimization, the delineation and analysis of critical and sensitive accountable factors, and the establishment of the functional relationships of effectiveness performance parameters to multi-accountable factors suitable to guide designers. Thus, in conducting an effectiveness apportionment or dynamic monitoring of effectiveness progress, a full technical perspective and proficiency in these interactive technical elements are required.
- The apportionment techniques described in the report have immediate and general applicability to a broad class of Air Force systems. This is particularly true of the Lagrange multiplier with either the developed priority list solution technique or the classical and iterative solution methods. Further, the solution process for the Lagrange equations can be computerized for complex applications.
- The apportionment of a system's figure(s) of merit effectiveness measure to its constituent performance parameters can be objectively and technically accomplished in an integrated and sufficient manner. In turn, the apportioned values for these performance parameters of availability, dependability, and capability are further apportionable to their technical accountable factors, and so on, cascading to the level of design parameters. Almost all of these design parameters are currently integral parts of subsystem design specifications. Thus, the necessary effectiveness design criteria will be familiar to, and controllable by, designers. Sensitivity functions can assist in providing the necessary design disciplines because they express cause and effect relationships.
- To establish the necessary effectiveness design criteria, only a minimum of data additional to what is normally generated during any system development program will be needed. All of the required data inputs are currently available, such as the results from dynamic simulation and other technical analysis activities. The techniques described herein are addressed to a more

deliberate, objective, and composite analysis of these data from a system performance effectiveness viewpoint. While the scope and frequency of the technical analysis activities will increase, the benefit is a potential corresponding increase in system effectiveness. From this total viewpoint, the techniques described in this report are considered to be practical and efficient.

- The techniques for apportionment and dynamic monitoring are relatively easy to understand. These techniques have a tradition of specialized applications in most system development programs. As such, their extended use should not pose a technical understanding barrier for implementation.
- A dynamic effectiveness monitoring and status plan can be readily implemented to provide necessary Air Force and contractor management visibility on current and projected effectiveness progress. Such a plan will provide a formalized and penetrating insight into critical effectiveness problem areas requiring corrective action early in, and during, development. As an integrating-type plan, it is non-duplicative of currently related plans.
- For complex systems, the classical Lagrange method is generally preferred in cases where the resulting effectiveness/constraint equation system can be solved readily by analytical or iterative techniques. Where such a ready solution is not possible, the use of the Lagrange multiplier with priority list technique can be profitably applied.
- Based upon the technical guidelines and perspective provided in this report, the necessary refinements, adaptations, improvements, and detailed stepwise procedures can evolve. This will occur as a result of direct applications to particular Air Force systems and missions.

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SECTION 1
INTRODUCTION AND GENERAL CONSIDERATIONS

1.1 INTRODUCTION

Efficient management of Air Force military development programs requires an optimum balance between resource allocations and performance. Sensitivity is such that, for future generations of Air Force systems and equipment, the delicate balance will be progressively more difficult to maintain.

Current technological advancements in existing strategic and tactical systems, often described with multiple superlatives, will become rudimentary and commonplace in comparison with those of the near future. Systems and equipment, though fewer in number, will be many times more complex and will possess multiple capabilities, and high effectiveness. To accomplish the planned Air Force missions of the future, a much more potent deterrent and striking force will evolve.

Yet, it is likely that the elastic limit of the national budget for such a planned and envisioned force, will probably not be increased beyond current levels, due to internal economic and political considerations. It is, therefore, mandatory that methods be devised which will permit system planners, developers, and command users, to evaluate the balance of resource allocations versus performance.

System effectiveness, as defined by the Weapon System Effectiveness Industry Advisory Committee (WSEIAC), is a measure of the extent to which a system or equipment may be expected to achieve a set of specific mission requirements. The measure is a function of Availability, Dependability, and Capability. In that it integrates the current spectrum of performance parameters individually and jointly specified for each system, it is an all-encompassing measure of the performance capabilities of a system or an equipment.

The Air Force using commands have always been concerned with and, in one form or another, have conducted effectiveness evaluations of targeting policies, maintenance practice, and the like. But a dichotomous situation has existed in that system developers have emphasized individual performance parameters of effectiveness without a formal, periodic evaluation of their interactive consequences as system performance knowledge was gained. The result, in some cases, was a compromise of the mission requirements after a military system was completely developed.

For many of the future generations of Air Force military systems, a first requirement will be system effectiveness evaluation. Formal system effectiveness analyses are being invoked for some of the large, complicated systems currently undergoing development.

With prime responsibility for research and development in system effectiveness technology, the Rome Air Development Center of AFSC has been developing the essential analytical tools, implementation procedures and documents, and the management science and monitoring programs necessary for a reasonably uniform but flexible implementation of system effectiveness concepts.

Air Force management, and the contractors, will thus be provided with the needed visibility of current and predicted weapon system effectiveness at all phases of system life. This report describes the results of one development area, namely the preliminary development effort of formalized methodologies and analytical tools relative to system design under a system effectiveness requirement for the following areas:

- Techniques and methodologies for the apportionment of a given system effectiveness requirement(s) to the subsystem and equipment level, in terms of meaningful specification requirement consistent with the effectiveness requirement(s) at the system level.
- Extension of these techniques and methodologies into a formal monitoring plan and format capable of dynamic application concurrent with the start of development, and continuing as the development progresses. Such a monitoring plan would, also, be capable of apprising the Air Force and contractor management of system effectiveness status at key points in the system's development cycle.

1.2 GENERAL CONSIDERATIONS

The techniques and methodologies described herein are intended to provide a framework for the establishment of design criteria at the subsystem and equipment level. The purpose is to guide designers, engineers, and system program managers, who must develop a system which will meet an effectiveness requirement, or objective.

Any design criteria, to be technically sufficient, must be capable of being evaluated at the subsystem and equipment level. Such an evaluation may be accomplished through a combination of direct, or indirect, test measurements and analytical procedures, including parametric studies and simulations, as applicable.

Design criteria must be directly relatable to the system performance parameters and, correspondingly, to the system effectiveness measure(s). This relationship is normally expressible in terms of physical laws, empirical associations, probabilistic functions, or mathematical analogs. But, design criteria must also be practical. This implies that they should be compatible, or hopefully coincidental, with many existing subsystem and equipment specification performance parameters. As a corollary, the analytical studies and measurements needed for assessment of these parameters should be concordant with those normally conducted during any development program.

1.2.1 Attributes of the Techniques and Methodologies Developed

As with the considerations for design criteria, the practicality and the simplicity of system effectiveness apportionment and monitoring techniques were prime considerations in their development, balanced against the need for their universal applicability which, normally, is an opposite forcing function. The techniques and methodologies described later in this report are responsive to these attributes. Furthermore, a minimum of data will be required to apply these techniques, in addition to those usually generated during any development program.

For broad and immediate applicability, the techniques are amenable to hand analyses, without need to resort to extensive computer programs. This characteristic of these techniques negates a major barrier for the subsystem and equipment manufacturers who may not possess computer facilities or capabilities comparable to those of the major system producers, and who are designing under a system effectiveness requirement.

1.2.2 System Effectiveness Evaluation Network

A system effectiveness evaluation network, characteristic of any effectiveness evaluation process, is shown in Figure 1-1 (of this section). It is essentially similar to that shown in [1]. The apportionment process flows from the overall weapon/military system effectiveness requirements to their constituent subsystem and equipment effectiveness requirements. Often, the later will be expressed in terms of different parameters or measures. The monitoring and prediction process flows in the reverse direction, generating predictions and measurements of weapon/military system growth based upon analysis, test, and maturity data on the equipments and subsystems.

To apply either the apportionment process or the monitoring and prediction process of the system effectiveness/evaluation network, an understanding of some of the fundamental areas of the system effectiveness technology is required. These include the technical areas of defining figures of merit, identification of accountable factors, and figure of merit evaluation. For this reason, background information needed for the practical application of a system effectiveness evaluation is presented in the next section (Section 2). Specific techniques and methods more directly associated with the apportionment process and the monitoring and prediction tasks are extensively developed and illustrated in Sections 3, 4, and 5 with equal emphasis on the availability, dependability, and capability parameters of effectiveness.

[1] RADC-TR-67-264, Vol. I, Final Report of Validation and Improvement of Effectiveness Evaluation Techniques, April 1967.

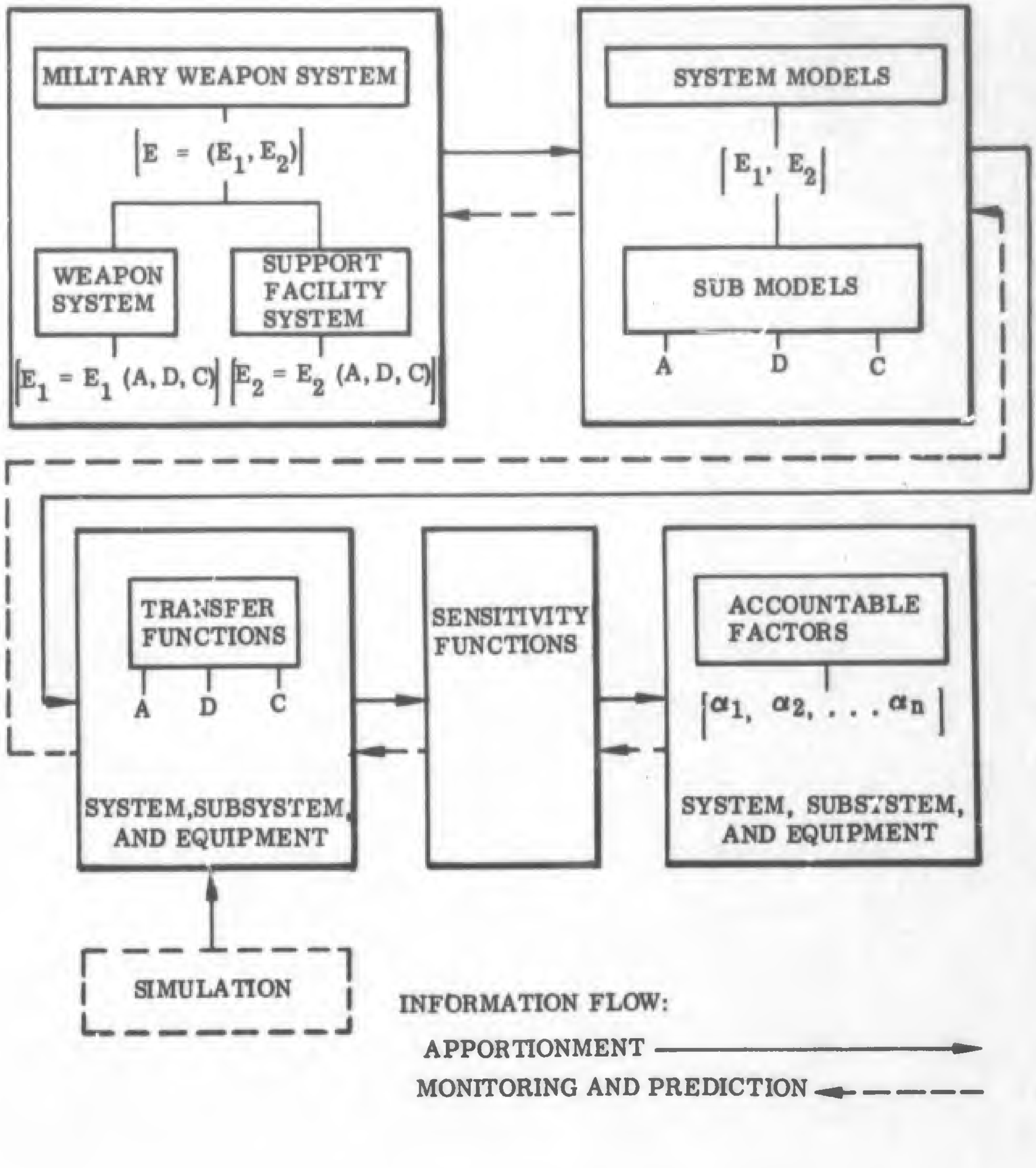

Figure 1-1 System Effectiveness Evaluation Network

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SECTION 2
SYSTEM EFFECTIVENESS EVALUATION BACKGROUND

This section describes the general methodologies useful for the specification of figure (s) of merit (FOMs), identification of accountable factors, and figure of merit evaluation. The methodologies are intended as unrestrictive application guidelines and background information. They are consistent with the WSEIAC concepts and are addressed to only the performance aspects of a system, as contrasted to the cost aspects.

2.1 DEFINING FIGURES OF MERIT

System effectiveness is a measure of the extent to which a system or an equipment may be expected to achieve a set of specific mission requirements, and is a function of the availability, dependability, and capability of the system. Figure(s) of merit (FOMs) are used to measure the system's effectiveness.

Figures of merit can be expressed as a measured physical quantity, such as range or payload, as a calculated quantity, such as mean time to failure, or as a predicted quantity, such as a probability of the system performing a required function. If the system has a number of missions to perform, each mission should correspond to a figure of merit. Thus, for a multi-mission system such as the Gemini Agena Target Vehicle, the first figure of merit could be the probability of docking during its first orbit, the second could be the probability of conducting docking practices, the third could be the probability of conducting extra-vehicular activity, etc. If the system has only one mission to perform, as with a missile system, there may be only one figure of merit, such as the probability of destroying or neutralizing a target.

In most cases, the system will have one overall performance requirement and numerous other independent performance parameters which are related and used to evaluate overall performance and are requirements in themselves. For example, with a cargo transport system, the overall requirement may be to deliver (X) number of tons over a distance of (Y) miles in (Z) number of days. This can be alternately expressed as a required ton-mile product over a fixed time period, the system's figure of merit. However, a separate requirement for cruise speed normally is not a figure of merit since the ton-mile figure of merit will be a function of cruise speed.

Many Air Force weapon and military systems are composed of two or more major systems - the total integration of which becomes the weapon or military system. For example, the weapon system may consist of two major systems, the weapon and the support facility systems. The weapon system and its constituent systems will each have a set of performance requirements appearing in a weapon system specification, as well as a subset of like or identical performance parameters appearing in each of the constituent system specifications. Correspondingly, figures of merit are prescribable for all constituent systems based upon the weapon system's figure(s) of merit.

As previously indicated, figures of merit may be expressed in a number of ways. The alternative of expressing a figure of merit as the probability of achieving some minimum required value has several advantages:

- It allows all the figures of merit to be expressed as common units of measure, i.e., as a probability, thus, facilitating their combination in isolated applications.
- It is a convenient method for translation of contractual specification requirements for effectiveness analyses. For example, if the system has a specification requirement of X ton-miles, conversion to the system effectiveness figure of merit may be the probability of the system's having a capability of at least X tons.
- It is convenient for the apportionment process.

2.2 COMBINING FIGURES OF MERIT

For the analysis of systems with multiple figures of merit, it may be convenient to express system effectiveness as a function of a vector of the specified figures of merit for the system. If k figures of merit are specified, then the vector is

$[E_1, \ldots, E_k]$ with E_i denoting a figure of merit, $i = 1, 2, \ldots, k$. This vector is not easy to interpret. It does not define a grand measure of the system's effectiveness nor is any priority placed on the E_i's in instances where a grand measure is not desirable. While the normal practice is not to combine figures of merit, in rare applications where a combination is desirable, the E_i's must be expressed in a common unit of measure, such as probabilities.

There are many possible ways to combine the E_i's, with the system being evaluated becoming the determining factor for the method to be used. For example, if the system has k figures of merit $E_1, \ldots, E_k$, all of which are considered necessary to achieve an effective system, then the product of the E_i's will give an overall effectiveness E:

$$E = \prod_{i=1}^{k} E_i$$

An example based on an airborne avionics system where this approach may be used was described in the WSEIAC Report, Task Force Group II, Volume II. In this example, the E_i's are subfunctions, all of which are required for the system to perform the mission..

Another method of combining figures of merit which may be applicable to systems called upon to perform any of k missions, is the expression of an overall effectiveness E as:

$$E = \sum_{i=1}^{k} p_i E_i$$

where p_i = the probability that the i^{th} mission will be chosen. An example of this type of system is a helicopter used for multiple missions such as reconnaissance, troop or cargo transport, retrieval, or combat support. In this case, the E_i's would represent the effectiveness of the system to perform each mission and p_i's would represent the probability of that mission being chosen. These p_i's can be estimated by the relative frequency of each mission having been chosen in the past.

If the system has several missions to perform, either concurrently or sequentially, then the E_i may be combined using a series of weighting factors, w_i such that:

$$E = \sum_{i=1}^{k} w_i E_i$$

where

$$\sum_{i=1}^{k} w_i = 1$$

The w_i are usually subjectively determined based upon the relative importance of each figure of merit. An example of this type of combination is a Gemini Agena Target Vehicle (GATV) system which is presented later as a detailed example. For this system, seven figures of merit can be identified, each of which is expressed as the probability of carrying out a mission, and with each mission rankable in terms of relative importance to the overall mission.

In a manned spacecraft such as the GATV system, crew safety is a primary figure of merit in that the total mission would be a failure if the crew could not return to earth safely. If E_1 denotes the probability of crew safety and $E_2, \ldots, E_k$ represent the figures of merit corresponding to the expected mission accomplishments, then,

$$E = E_1 \sum_{i=2}^{k} w_i E_i$$

where

$$\sum_{i=2}^{k} w_i = 1$$

Thus a low value of E_1 for crew safety will automatically result in a low effectiveness.

Obviously, a considerable flexibility may be employed in combining figures of merit. The guiding criterion is that the method selected should be a realistic physical analog of the factors affecting system effectiveness.

There are some disadvantages to combining the figures of merit, such as principally:

(1) Any combination may lack physical and mission realism. While the choice of the method for combining the E_i's is objective, the choice of the w_i's, if the weighting method is selected, will normally be subjective.

(2) It may not be convenient to express all the E_i's in the same unit of measure, i.e., as the probability of achieving a particular capability.

(3) One overall measure of system effectiveness may be insensitive to changes in key performance parameters, particularly if there are many figures of merit.

2.3 ACCOUNTABLE FACTORS IN SYSTEM DESIGN FOR EFFECTIVENESS

In initiating a system design to perform a defined mission, the overall system can be partitioned into a number of discrete, functional major systems, subsystems, and equipment. The overall system can be represented, for example, by block diagrams showing the functional relationship between these major systems, subsystems, and equipment.

Each major system, subsystem, or equipment is related to the overall weapon or military system by two types of accountable factors. One type represents the significant physical or functional parameters which influence the availability, dependability, and capability performance parameters of the overall system. Accountable factors of this type will have direct applicability in a numerical system effectiveness evaluation. Such factors include time, environmental and operating conditions, energy or power requirements, capacity, leakage, and similar design parameters.

Another type of accountable factors is represented by significant elements such as design reviews, maintenance schedules, and checkout procedures. Their importance and influences are used to compensate for deficiencies in the performance parameters of the major system, subsystem, and equipment, where such parameters cannot be measurably improved by altering the functional accountable factors.

Both types of accountable factors are necessary to a formal and thorough analytical evaluation of system effectiveness. They are also criteria for guiding the design and development of an overall system under an effectiveness requirement.

With a system so defined by mission requirements, functional diagrams, and accountable factors, a mathematical model consisting of an equation or sets of equations can be constructed which would define the effectiveness of the system in terms of the availability, dependability, and capability parameters. This model would express the functional relationships of the accountable factors to these parameters. Various analyses can be performed using this effectiveness model. One very important operation is the optimization of system effectiveness. A system is optimized by the proper apportionment of its requirements to the subsystem and equipment level, branching down to the unit level as necessary. The accountable factors often will be related to the effectiveness parameters using mathematical methods. Hence, effectiveness can be optimized by adding capacity, design features, quality, expendables, redundant units, etc. as necessary. The apportionment to optimize system effectiveness may be accomplished without constraints to achieve a specified level of effectiveness. Normally, however, in the real world systems, there will be constraints on one or more of the performance parameters and hence on the accountable factors.

In many cases, accountable factors will be apportioned subsystem and equipment design parameters which are directly suitable for guiding designers. Values are assigned to these subsystem design parameters when the system is blocked out. Typical design parameters may be stabilization accuracy, thrust accuracy and alignment, bandwidth, ballistic coefficient, electronic noise figure, etc. These design parameters will comprise the design specification for that particular subsystem or equipment. The subsystem or equipment engineer designs his unit to this specification.

When these designs are integrated into the total system, the system will meet the optimized effectiveness determined by the apportionment process. Further, degradation or gains in achieving subsystem and equipment accountable factors, including design parameters, can be monitored and evaluated as a basis for predicting the actual achievement of the optimized effectiveness during development of the system.

Thus, the basic steps in the design and development of an overall system in which effectiveness is to be optimized during design, and tracked for successful achievement during development, can be summarized as follows:

(1) The overall system is functionally partitioned in terms of its major systems, subsystems, and equipments.

(2) Each major system, subsystem, or equipment is specified, and tentative values are assigned to the design parameters through the apportionment process.

(3) The aggregate influence of accountable factors, including the design parameters, is to meet or exceed the required overall system characteristics and performance.

(4) For the overall system, a functional effectiveness relationship is defined, which analytically integrates the major systems, subsystems, and equipment, together with their pertinent accountable factors, into the availability, dependability, and capability parameters.

(5) The functional relationship established in (4) is analyzed to optimize effectiveness with or without constraints. This is accomplished by use of an apportionment technique to determine the optimum definition of the system.

(6) From the analysis in (5), the tentative values of accountable factors assigned in (2) are redetermined. This redetermined set of values will comprise the design specifications for the major systems, subsystems, and equipment, which are then designed to these specifications.

(7) If design specifications cannot be met in development, iterate steps (2) through (6) to reapportion system resources to again optimize system effectiveness.

2.4 FIGURE OF MERIT EVALUATION

A military system is designed to satisfy a purpose based on a military requirement. There is often more than one purpose, requiring the association of separate figures of merit. Additionally, there may be constraints that must be satisfied by the system, due to limitations such as resources, technical capability, design considerations, mission definition, or manner of deployment.

For example, a common purpose for a missile system is to cause damage to enemy sites, personnel, or property in designated target areas. However, it is also desirable that this purpose be accomplished at minimum cost and weight for such a system. Therefore, if E represents a function such as the expectant value of the amount of desirable damage inflicted on the targets, one or more alternative missile system designs could be compared according to which has the highest figure of merit, accounting for cost and perhaps weight penalties.

However, any system design for which specified constraints are not satisfied must be disqualified. The equations for the constraints will depend on accountable factors. Also, the equation for the figure of merit will depend on these or different accountable factors through many functional relationships on its performance parameters. These accountable factors may not only influence the figure of merit; they may determine whether or not the constraints are met. For example, an increase in some accountable factor, such as the weight of the payload in a missile system, would increase yield. However, because of an overall weight constraint, this would tend to reduce the weight available for subsystem redundancies and design simplifications, and hence, the availability and dependability of the mission. The dependency of the effectiveness E, on a set of accountable factors $\alpha_1, \alpha_2, \ldots, \alpha_m$, can be represented by the following, with the best value of the accountable factors being those that maximize:

$$E = E(\alpha_1, \ldots, \alpha_m)$$

with E a function of availability, dependability, and capability. Where a vector-matrix modeling approach is used, E may be expressed as

$$E_m = \sum_n \sum_k A_n D_{nk} C_{km}$$

for each E_m of the vector E. If the vector consists of a single value E, then the m subscript is dropped from the above expression.

The methodologies which follow presuppose this vector-matrix modeling approach. Also, the general evaluation methodologies are described in the context of applicability to a general missile system. However, this context is not a restrictive scenario, as the methods have universal applicability to most classes of Air Force systems.

2.4.1 General Methods for Evaluating Effectiveness

If the values of the accountable factors $\alpha_1, \alpha_2, \ldots, \alpha_m$ were known, the capability column vector could be calculated. Some accountable factors will also influence availability A and dependability D. A set of values $\alpha_1, \ldots, \alpha_m$, therefore, ultimately leads to a value for the effectiveness figure of merit E. Hence E_m, C_{km}, D_{nk}, and A_n can be considered as functions of all the accountable factors. Thus,

$$E_m = E_m(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_m),$$

= probability that the m^{th} level of target damage can be achieved

$$C_{km} = C_{km}(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_m),$$

= the probability that the m^{th} level of target damage can be achieved if the k^{th} dependability state applies.

etc.

$$E = E(\alpha_1, \alpha_2, \ldots, \alpha_m),$$

$$= \text{expected level of target damage}$$

$$C_k = C_k(\alpha_1, \alpha_2, \ldots, \alpha_m)$$

$$= \text{the expected level of target damage if the } k^{th} \text{ dependability state applies.}$$

etc.

In practice, however, most of the accountable factors are not known nor can they be controlled with ultimate precision. For example, at any time, thrust is known only approximately, but a mean expected value and a standard deviation from this mean can be determined for a particular mission. The more sufficient a missile system is designed, the smaller will be the standard deviation in a controlled accountable factor. An example of an uncontrolled accountable factor is the components of the wind vectors. With both a controllable accountable factor and an uncontrollable accountable factor, the mean and the standard deviation, but not the exact values, can be stated for any mission. The designer then attempts to alter the mean and standard deviations of the controllable factors in a way that will increase the figure of merit without violating the constraints.

To compute quantities such as E_m and C_{km}, when each accountable factor is not a fixed value but a random number having a probabilistic frequency function, $g(\alpha_1)$, $g(\alpha_2), \ldots, g(\alpha_m)$, the means and standard deviations of the accountable factors are used. The two most common distributions applicable to accountable factors are the normal and uniform distributions. Then the expectant values and probabilities of E_m, C_{km}, etc., are given as:

$$E = E^*\left(\mu_{\alpha_1}, \sigma_{\alpha_1}; \mu_{\alpha_2}, \sigma_{\alpha_2}; \ldots; \mu_{\alpha_m}, \sigma_{\alpha_m}\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} E(X_1, X_2, \ldots\ldots, X_m) \prod_{i=1}^{m} g\left[\alpha_i(X_i)\right] dX_1\, dX_2 \ldots dX_m$$

The same type of equation holds for E_m, D_{nk}, or C_{km}. The assumption is made that the accountable factors themselves are mutually independent. This assumption is reasonable because, if α_1 was the weight of propellant and α_2^* was the maximum thrust, and it was determined that α_2^* depended on α_1 according to

$$\alpha^*_2 = a\,\alpha_1 + b$$

where a and b are variables which do not depend on α_1, then the accountable factor a^*_2 could be deleted, and replaced by $\alpha_2 = a$ and $\alpha_3 = b$.

2.4.2 Postulated Typical Mission and Mission Considerations

Suppose a missile system has a maximum capacity of N missiles, designated for a hostile region having $\tilde{M}$ targets. Assume that each missile could be assigned to any target within the region, so the entire region is within range of each missile. Also assume that the number of missiles assigned to targets is the number that are available (i.e., deemed operable) at the time of launch, except that no more than M out of N missiles will be launched. The reasons that M might be less than N may include a desire to hold (N-M) missiles in reserve, and to put a ceiling on the total radioactivity from nuclear weapons if such were involved. A priority number from 1 to M, where $M \geq \tilde{M}$, is assigned to each target, and the target of the p^{th} priority is designated T_p. If fewer than M missiles are available at time of launch, they are assigned to targets of highest priority. If the number of targets $\tilde{M}$ is less than the number M of missiles launched, the possibility exists of launching more than one missile onto a target. For example, if to launch a missile onto a certain target has priority 3, and to launch a second missile onto the same target has priority 5, then both T_3 and T_5 refer to this same target.

Since a maximum of M missiles are to be launched, and each missile may or may not result in a detonation, there may be 2^M dependability states, i.e., 2^M possibilities of detonation and non-detonation with regard to the M missiles.

Corresponding to each dependability state k, calculation of a kill probability function is required. Such a probability function gives the relationship between each level of total destruction and, given that dependability is in state k, the probability q that total destruction is less than or equal to this level. The probability that said level of total destruction will be exceeded is then $1 - q$. The element $C_{k,m}$ of the capability matrix may be used to represent the probability that the m^{th} level of destruction will be exceeded if dependability is in state k. The unconditional probability for exceeding this level is E_k. If there is a single effectiveness value E, it may represent the expectant value of destruction. An element C_k of a one-column capability vector then could represent this expectant value, given that dependability was in the k^{th} state.

In certain cases, such as when more than one missile is assigned to the same target, several dependability states, as described above, may be similar in that the corresponding kill probability functions are the same. Suppose, for example, that T_3, T_4, and T_7 all refer to the same target. This means that this target has third, fourth, and seventh priorities, so that if there are seven or more missiles, three of them will be assigned to this one target. If it is known that exactly two missiles will detonate on the target, it is immaterial which two of the three missiles they are.

With this mission background, the general methodologies for the evaluation of the effectiveness parameters of availability, dependability, and capability now can be described.

2.4.3 Availability Evaluation

Availability A_n is the probability that exactly n missiles will be available. In a typical general situation, a maximum of N missiles are ready for M target assignments with $M \leq N$. Each of the N missiles can be used for any of the M assignments. Let a_m be the probability that any one missile is available. Then the probability that there will be exactly n of the N missiles available is given by

$$A_n = a_m^n (1 - a_m)^{N-n} \binom{N}{n}$$

The elements A_n may form an availability vector

$$\bar{A} = \left[A_1, A_2, \ldots\ldots, A_N \right]$$

If the targets are ranked in priority order, 1, 2, . . , p , . . , M, then the probability that a missile is available for the mission of p-th priority is the same as the probability A(p) that p or more missiles are available, and is given by:

$$A(p) = \sum_{n=p}^{N} A_n$$

$$= a_m^N \sum_{n=0}^{N-p} \left[\frac{1 - a_m}{a_m} \right]^n \binom{N}{n}$$

Assume that each missile consists of r equipment type $S_1, S_2, \ldots, S_i, \ldots, S_r$, each of which must be available in order for the missile to be available, and that each equipment type S_i consists of one or more parallel equipments $S_{i,1}, S_{i,2}, \ldots\ldots,$ $S_{i,j}, \ldots\ldots, S_{i,s(i)}$ such that if any one or more of the latter is available, S_i is available. The parallel equipments in each S_i are not necessarily identical and hence may have different failure rates. Then, the availability of an individual missile is given by

$$a_m = \prod_{i=1}^{r} \left(1 - \prod_{j=i}^{s(i)} \left[1 - a\,(S_{ij}) \right] \right) ,$$

where $a(S_{i,j})$ represents the probability that equipment $S_{i,j}$ is available. This probability is assumed to be of the form

$$a(S_{i,j}) = MTTF_{i,j}/(MTTR_{i,j} + MTTF_{i,j}),$$

where $MTTF_{i,j}$ is the mean time that equipment $S_{i,j}$ is constantly available without a failure, and $MTTR_{i,j}$ is the mean length of time from the instant equipment $S_{i,j}$ becomes unavailable until it is restored to availability by being repaired or replaced.

From the above it can be seen that the probability A_n of having n missiles available increases with the total number of missiles N, the number s of parallel equipment types, and the mean time to failure $MTTF_{i,j}$. This probability decreases with the increase in the number r of equipment types in a system and the mean time to repair $MTTR_{i,j}$, the latter of which depends on such consideration as the nearness and quantities of spare parts and frequency of inspection. It is probably harder to decrease the quantities r and MTTR than to increase N, s, and MTTF. The quantity MTTF may be increased by adding weight to an individual equipment and/or adding cost to its production. An increase in s or in N obviously increases both weight and cost through redundancy.

2.4.4 Dependability Evaluation

A missile squadron has been described as being composed of N missiles, from which up to M missiles are directed to target assignments $T_1, T_2, \ldots\ldots, T_M$ in order of descending priority of assignment. A missile is said to be dependable if it results in a detonation. There are 2^M different ways that M missiles can be dependable or not dependable. These correspond to 2^M dependability states. Then, the p^{th} dependability state is the condition in which the detonation of the missile of p^{th} priority is associated with a one as the p^{th} binary digit of k - 1. Thus,

$$b_p(k-1) = 1 \rightarrow \text{detonation}$$
$$b_p(k-1) = 0 \rightarrow \text{no detonation}$$

where b_p (k-1) is the p^{th} binary digit of k-1 expressed as an M digit binary number, so that

$$k - 1 = \sum_{p=1}^{M} \left| b_p \, (k-1) \right| 2^{M-p}$$

from which the binary number for the k^{th} dependability state is determined. The mechanism for the binary representation of the 2^M missile dependability states is given in Appendix A of this report.

A dependability matrix D with N rows and 2^M columns can be defined as follows: Denote the k^{th} element of the n^{th} row of D by $D_{n,k}$. This represents the probability that dependability will be in the k^{th} state, given that n missiles are available. These missiles are then assigned priorities one through n, and $D_{n,k} = 0$ if the k^{th} dependability state involves the dependability of a missile with priority $p > n$. Note that regardless of the number of missiles available, there are 2^M dependability states, and also that the last N - M + 1 rows of D will be identical.

To analyze in-flight dependability of the equipment of a missile, it can be assumed that the failure rate is constant, i.e., the small amount of ageing that takes place during a flight does not contribute significantly to the failure rate. Let $\lambda_{i,j}$ represent the failure rate per unit time for equipment $S_{i,j}$ during the time that equipment type S_i is operating. Then, if the equipment $S_{i,j}$ is available at the time of launch $(t = 0)$, and equipment type i is required to be in use for a length of time, t_p when used for target T_p, the probability that equipment $S_{i,j}$ is dependable for the required length of time for such a target assignment is

$$\begin{aligned} D_{p,i,j} &= D_{i,j}(t_p) \\ &= \exp\left[-\lambda_{i,j} \, t_p\right] \end{aligned}$$

The reason the length of time t_p, that an equipment type i will be needed, varies from one target assignment to another is that the different targets may be at unequal distances from the launch area.

For the situation under consideration, each equipment type S_i within a missile is assumed to be available if at least one equipment of this type is available. However, the dependability of the equipment type S_i will be reduced if some or all of the remaining parallel equipment of type S_i is unavailable. First, note that the probability $\hat{D}_p(S_{i,j})$, that equipment $S_{i,j}$ in missile p is dependable (without any knowledge of availability) is the product of the probability the equipment is available times the probability that, if available at time of launch, the equipment is dependable for the duration of the flight, that is

$$\hat{D}_p(S_{i,j}) = a(S_{i,j})\, D_{p,i,j}$$

$$= a(S_{i,j}) \exp\left[-\lambda_{i,j}\, t_p\right]$$

The probability that no equipment of type i in missile p is dependable (assuming failures are independent and also, as before, not presupposing availability) is equal to the probability that the necessary function to be performed in missile p by equipment of type i will not be performed, which in turn is

$$1 - \hat{D}_p(S_i) = \prod_{j=1}^{s(i)} \left(1 - \hat{D}_p(S_{i,j})\right)$$

With the same assumptions, the probability that each type will have at least one dependable equipment so that missile p will be dependable is

$$\hat{D}_p = \prod_{i=1}^{r} \hat{D}_p(S_i) = \prod_{i=1}^{r} \left[1 - \prod_{j=1}^{s(i)} \left(1 - \hat{D}_p(S_{i,j})\right)\right]$$

Since $\hat{D}_p$ is the probability that missile p is dependable, and a_m is the probability it is available (and it can be dependable only if available) then,

$$\hat{D}_p = a_m D^*_p,$$

or $$D^*_p = \hat{D}_p / a_m$$

where D^*_p is the probability that missile p is dependable, given that it

it is available. From the previous two equations, the following can be obtained:

$$D^*_p = \left\{ \prod_{i=1}^{r} \left[1 - \prod_{j=1}^{s(i)} \left(1 - \hat{D}_p (S_{i,j}) \right) \right] \right\} \Big/ a_m$$

$$= \left\{ \prod_{i=1}^{r} \left[1 - \prod_{j=1}^{s(i)} \left(1 - a (S_{i,j}) \exp \left[-\lambda_{i,j} t_p \right] \right) \right] \right\} \Big/ a_m$$

It is to be noted that the product of the vector $\overline{A}$ and the matrix D will be a one by 2^M vector, the k^{th} element of which represents the probability that missiles with priorities corresponding to ones in the k^{th} binary number, will be both available and dependable.

As previously indicated, $D_{n,k} = 0$ if $b_p(k-1) = 1$ for any $p > n$. Otherwise,

$$D_{n,k} = \prod_{p=1}^{M} \left[1 - D^*_p + (2D^*_p - 1)\, b_p(k-1)\right]$$

If D^*_p is independent of target assignment p, this reduces to

$$D_{n,k} = (D^*)^{M_o} (1 - D^*)^{M-M_o}$$

where M_o, the number of missiles which result in detonations, is given by

$$M_o = \sum_{p=1}^{M} b_p(k-1)$$

2.4.5 Capability Evaluation

The capability of a squadron of missiles is then the measure of what the squadron could be expected to accomplish if a given state of dependability was attained. Capability could represent the amount of damage that the missiles might inflict. Alternately, capability could represent the probability that damage would exceed a given level, assuming a definite state of dependability of the squadron of missiles. The latter representation is more general and informative since it gives capability in a C-matrix table depending on

- the amount of destruction required
- the level of dependability of the squadron of missiles

Given such a table, the alternate representation of capability, i.e., the expected amount of damage, can be determined.

For the squadron of missiles, capability may depend on two general areas of performance. These are: (1) the accuracy of the missiles, and (2) the lethal radius of each missile. In order for a missile to destroy a target, it must not only impact on or near the target, it must necessarily have a warhead capable of destroying the target. Each of these two areas of performance depends on accountable factors.

2.4.5.1 Accuracy Parameter

The accuracy of a missile is related to range error, the error in the downrange direction from the intended (nominal) target point, and error in track. If the missile overshoots, there will be a positive range error. If it undershoots, there will be a negative range error. If the missile falls to the right of the target as viewed from launch point, there will be a positive track error. If it falls to the left, the track error will be negative. The range and track errors will each depend on numerous accountable factors α_i. These accountable factors and the position of launch point and target, are inputs for a trajectory simulation program. The result of such a program is the error X_I in track and the error Y_I in range with the combined effect of all the equations of the trajectory simulation program represented as:

$$\Delta_T = X_I$$

$$= X_I(\alpha_1, \ldots\ldots, \alpha_m);$$

$$\Delta_R = Y_I$$

$$= Y_I(\alpha_1, \ldots\ldots, \alpha_m).$$

One simulation will yield one set of errors X_I and Y_I. When sufficient simulations have been accomplished, the following can be determined:

$$\overline{X}_I = \overline{X}_I(\mu_{\alpha_1}, \sigma_{\alpha_1}; \dots\dots\dots; \mu_{\alpha_m}, \sigma_{\alpha_m});$$

$$\overline{Y}_I = \overline{Y}_I(\mu_{\alpha_1}, \sigma_{\alpha_1}; \dots\dots\dots; \mu_{\alpha_m}, \sigma_{\alpha_m});$$

$$\sigma_{I(X)} = \sigma_{I(X)}(\mu_{\alpha_1}, \sigma_{\alpha_1}; \dots\dots\dots; \mu_{\alpha_m}, \sigma_{\alpha_m});$$

$$\sigma_{I(Y)} = \sigma_{I(Y)}(\mu_{\alpha_1}, \sigma_{\alpha_1}; \dots\dots\dots; \mu_{\alpha_m}, \sigma_{\alpha_m});$$

$$\sigma_{I(XY)} = \sigma_{I(XY)}(\mu_{\alpha_1}, \sigma_{\alpha_1}; \dots\dots\dots; \mu_{\alpha_m}, \sigma_{\alpha_m})$$

The above equations relate the means and standard deviations of the accuracy of the accountable factor inputs to the means and standard deviations of the range and track errors Y_I and X_I, as outputs. An example illustrating the use of these equations is presented in Section 4. The Monte Carlo and direct methods for evaluating these functional capability relationships are presented in Appendix B of this report.

2.4.5.2 Lethality of Warhead Parameters

The chief controllable accountable factor which influences lethality of a missile is the yield of its warhead, measured in equivalent tons of TNT. Lethal radius is compared to the distance between $(\overline{X}, \overline{Y})$, a point of the target, and impact. If the lethal radius is exceeded, the target is not destroyed at $(\overline{X}, \overline{Y})$, but, if not exceeded, the target is destroyed. Lethal radius, L_R, depends both on yield and hardness of target; that is, how well the target is protected against blast pressure. This dependence is expressed as:

$$\frac{L_R}{\text{feet}} \cong 204 \left(\frac{\text{yield}}{\text{tons of TNT}}\right)^{1/3} \left(\frac{\text{hardness of target at } (\overline{X}, \overline{Y})}{\text{lb/in.}^2}\right)^{-3/8} \qquad (2\text{-}1)$$

Equation (2-1) is of direct application to point targets having a single target hardness. However, its application can be extended to targets having an area distribution, such as the circular or elliptical bivariate normal distribution, and having variable hardness. For a bivariate normal distribution of targets, the target value per unit area is assumed to be greatest at the target center and to decrease as the distance from the center of the target increases. This bivariate normal distribution of the target concentration, i.e., the target density at a point (X, Y), can be expressed as:

$$V(X, Y) = \frac{V_\Omega}{2\pi\sigma_{V_1}\sigma_{V_2}} \exp\left[-\frac{\hat{V}_1^{\,2}}{2\sigma^2_{V_1}} - \frac{\hat{V}_2^{\,2}}{2\sigma^2_{V_2}}\right] \tag{2-2}$$

$$= \frac{V_\Omega}{2\pi\sqrt{M_2}} \exp\left[-\frac{1}{2M_2} - \left[\sigma^2_{V(Y)} X_2^2 - \sigma_{V(XY)} X_2 Y_2 + \sigma^2_{V(X)} Y_2^2\right]\right] \tag{2-3}$$

with

$$M_2 = \sigma^2_{V(X)}\,\sigma^2_{V(Y)} - \sigma^2_{V(XY)}$$

V(X, Y) = the concentration of target value at the point (X, Y)

V_Ω = the total value of the target, such as number of buildings destroyed, amount of retaliatory capability destroyed, military personnel disabled, etc.

The quantities σ_{V_1} and σ_{V_2} are measures of dispersion of the target. Mathematically, they are standard deviations of the target elliptical distribution along the major and minor axes. If the target is circular, σ_{V_1}, σ_{V_2}, $\sigma_{V(X)}$, and $\sigma_{V(Y)}$ are equal, and each then represents (1/2.4477) of the radius of the circle within which 95 percent of the target value lies. The coordinates $(\hat{V}_1, \hat{V}_2)$ represent the position of a point of the target in a transformed rectangular coordinate system. It is related to the (X, Y) coordinates by

$$\hat{V}_1 = (X_2 \cos\theta_V + Y_2 \sin\theta_V) ;$$

$$\hat{V}_2 = (-X_2 \sin\theta_V + Y_2 \cos\theta_V) ,$$

where:

$$\theta_V = 1/2 \arctan\left[2\sigma_{V(XY)}/(\sigma^2_{V(X)} - \sigma^2_{V(Y)})\right] ;$$

$X_2 = X - \overline{X}_V ; \; Y_2 = Y - \overline{Y}_V ; \; (X = \overline{X}_V, \; Y = \overline{Y}_V)$ is the center of the target;

and the σ's of Equation (2-3) defined as:

$$\sigma_{V(XY)} = \frac{1}{V_\Omega} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X, Y)\,(X - \overline{X}_V)\,(Y - \overline{Y}_V)\, dXdY ;$$

$$\sigma^2_{V(X)} = \frac{1}{V^2_\Omega} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X, Y)\,(X - \overline{X}_V)^2\, dXdY ;$$

$$\sigma^2_{V(Y)} = \frac{1}{V^2_\Omega} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X, Y)\,(Y - \overline{Y}_V)^2\, dXdY$$

The σ_{V_1} and σ_{V_2} dispersions in $(\hat{V}_1, \hat{V}_2)$ coordinates are related to $\sigma_{V(X)}$, $\sigma_{V(Y)}$, and $\sigma_{V(XY)}$, the dispersions in (X, Y) coordinates by:

$$\sigma_{V_1} = \sqrt{\frac{\sigma^2_{V(X)} + \sigma^2_{V(Y)}}{2} + \sqrt{\left(\frac{\sigma^2_{V(X)} - \sigma^2_{V(Y)}}{2}\right)^2 + \sigma^2_{V(XY)}}}$$

$$\sigma_{V_2} = \sqrt{\frac{\sigma^2_{V(X)} + \sigma^2_{V(Y)}}{2} - \sqrt{\left(\frac{\sigma^2_{V(X)} - \sigma^2_{V(Y)}}{2}\right)^2 + \sigma^2_{V(XY)}}}$$

Appendix C further explains the relationship between Equations (2-2) and (2-3).

For a target which is not bivariate normal, but of more general area concentration of value, the expected value of that portion of the target which is destroyed is given by

$$E_{kill} = E_k(p) = \iint_{\substack{\text{Set of all} \\ (X,\,Y)\text{ in} \\ \text{target area}}} P_k(X, Y)\ V(X, Y)\ dXdY \qquad (2\text{-}4)$$

$P_k(X, Y)$ is the probability that a target at (X, Y) will be destroyed, and is given by

$$P_k(\bar{X}, \bar{Y}) = \iint_{\xi^2 + \eta^2 \leq L_R^2} P_I(\bar{X} + \xi, \bar{Y} + \eta)\ dXdY, \qquad (2\text{-}5)$$

where $P_I(\bar{X} + \xi, \bar{Y} + \eta)$ is the probability density that the impact of the missile will occur at $(\bar{X} + \xi, \bar{Y} + \eta)$.

For a set of n point-targets for which a single explosion is intended, Equation (2-4) can be written in the form:

$$E_k(p) = \sum_{t=1}^{N} P_r(X_t, Y_t)\ V_t$$

where X_t and Y_t are the rectangular coordinates of the t^{th} point, and V_t is the value of the target at the t^{th} point.

2.4.5.3 Capability of a Squadron of Missiles

Equation (2-4) gives the expected amount of kill E_k as it pertains to the destruction rendered by a single missile, which is directed onto target p. The situation of interest is generally in C_k, the expected damage caused by all missiles in the squadron if dependability is in the k^{th} state. As previously indicated, mathematically, the k^{th} state of dependability is the condition under which the target assignment of the p^{th} priority results in a detonation if, and only if, b_p (k-1), the p^{th} digit from the left of k-1 expressed as an M digit binary number, is 1. Although two or more missiles may be assigned to the same target, assume that all the targets are widely separated such that no more than one missile is assigned to the same target. Then the total expectant amount of kill, if dependability is in the k^{th} state, is:

$$C_k = \sum_{p=1}^{M} E_k(p)\, b_p(k-1) \qquad (2\text{-}6)$$

where, as earlier stated, $E_k(p)$ is the expectant amount of kill on the p^{th} target, and $b_p(k-1) = 1$ if the k^{th} state of dependability implies that the p^{th} target has a detonation, and $b_p(k-1) = 0$ if the k^{th} state of dependability implies that the p^{th} target has no detonation.

To treat the possibility that more than one missile may detonate on the same target, and it is stipulated that if the j^{th} missile-to-target assignment (in order of priority) of those assigned to the i^{th} target is of p^{th} priority, then p can be designated as $p = p(i, j)$. For example, if there are three targets and five assignments, i.e., a maximum of five missiles assigned, the assignments can be ranked by priority to targets 1, 2, 3, 1, and 2 in that order. Then since the fourth assignment is the second one to target 1, we say $4 = p(1, 2)$. Also

$$1 = p(1, 1),\ 2 = p(2, 1),\ 3 = p(3, 1),\ 5 = p(2, 2).$$

Suppose there are $\widetilde{M}$ targets and each target i has a maximum of J (i) missiles assigned to it. Then, if the impact errors of all missiles are independent, and if, as is likely to be generally true, the missiles are aimed so the center of probable impact is the center of the target, the expectant amount of kill for the k^{th} state of dependability is

$$C_k = \sum_{i=1}^{\widetilde{M}} V_\Omega(i) \left(1 - \prod_{j=1}^{J(i)} \left[1 - E_k(p)\, b_p(k-1)/V_\Omega(i)\right]\right) \qquad (2\text{-}7)$$

where $V_\Omega(i)$ is the value of the i^{th} target and $p = p(i, j)$. Equation (2-6) is a special case of the above equation.

2.4.5.4 Warhead Impact and Expected Value of Kill

Each accountable factor has been indicated to be a random variable contributing to system miss distance in track X_I and range Y_I. The total system mean miss distance in X_I and Y_I is obtained by an algebraic sum of the mean errors resulting from the mean of each of such contributions. The system standard deviation of miss distance is obtained by a root-sum-square calculation of the standard deviations of each source error.

Since a nearly normal random variable results from the addition of a large number of independent random variables, the impact point can be considered to have approximately an elliptical bivariate normal distribution. Therefore, the impact point density $P_I(X, Y)$ has an identical form as the density $V(X, Y)$ for value of the target per area of Equations (2-2) and (2-3) with the notation changes of I for V, and the quantity $\hat{I}_1$ and $\hat{I}_2$ representing the coordinates of the point (X, Y) in a coordinate system based on the axes of the ellipse of concentration for the impact point probability distribution. With respect to the value of the target per area, the total value of the particular target is V_Ω. The distances from the center of the distribution along the major and minor axes are designated $\hat{V}_1$ and $\hat{V}_2$. The center of the target is the point where $X = \overline{X}_V$ and $Y = \overline{Y}_V$.

For the purposes of computing expectant amount of kill, it is useful to combine the normal distributions of impact and of target dispersion. That is, the location of a part of an area target is treated as a random variable having the same distribution as the value of the target. The random variables considered are then

$$X_{V_1} = X_V - X_I \text{ and } Y_{V_1} = Y_V - Y_I,$$

where (X_V, Y_V) is a random target point, and (X_I, Y_I) is the random impact point. Then, a target point can be viewed to be destroyed if and only if the vector $V_{F_1} = [X_{V_1}, Y_{V_1}]$ from impact has an absolute value not greater than the lethal radius, L_R. The mean and standard deviations of this vector are:

$$\bar{X}_{V_1} = \bar{X}_V - \bar{X}_I$$

$$\bar{Y}_{V_1} = \bar{Y}_V - \bar{Y}_I$$

$$\sigma_{V_1(X)} = \sqrt{\sigma^2_{V(X)} + \sigma^2_{I(X)}}$$

$$\sigma_{V_1(Y)} = \sqrt{\sigma^2_{V(Y)} + \sigma^2_{I(Y)}} ;$$

$$\sigma_{V_1(XY)} = \sqrt{\sigma_{V(XY)} + \sigma_{I(XY)}}$$

with the probability distribution of this vector being:

$$V_{F_1(X,Y)} = \frac{1}{2\pi\sqrt{M_3}} \exp\left[-\frac{1}{2M_3}\left(\sigma^2_{V_1(Y)} X_3^2 - 2\,\sigma_{V_1(XY)} X_3 Y_3 + \sigma^2_{V_1(X)} Y_3^2\right)\right] \qquad (2\text{-}8)$$

$$= \frac{1}{V\Omega} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(\xi + X, \eta + Y)\, P_I(\xi,\eta)\, d\xi\, d\eta$$

$$= \frac{1}{V\Omega} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X,Y)\, P_I(X-\xi, Y-\eta)\, dX dY$$

$$= \frac{1}{2\pi\tilde{\sigma}_X^2\, \tilde{\sigma}_Y^2} \exp\left[-1/2\, \frac{\tilde{X}^2}{\tilde{\sigma}_X^2} - 1/2\, \frac{\tilde{Y}^2}{\tilde{\sigma}_Y^2} \right], \qquad (2\text{-}9)$$

where

$$X_3 = X - \bar{X}_{V_1} \; ; \; Y_3 = Y - \bar{Y}_{V_1}$$

$$M_3 = \sigma^2_{V_1(X)}\, \sigma^2_{V_1(Y)} - \sigma^2_{V_1(XY)}$$

$$\tilde{\sigma}_X = \sqrt{\frac{\sigma^2_{V_1(X)} + \sigma^2_{V_1(Y)}}{2} + \sqrt{\left[\frac{\sigma^2_{V_1(X)} - \sigma^2_{V_1(Y)}}{2}\right]^2 + \sigma^2_{V_1(XY)}}}$$

$$\tilde{\sigma}_Y = \sqrt{\frac{\sigma^2_{V_1(X)} + \sigma^2_{V_1(Y)}}{2} - \sqrt{\left[\frac{\sigma^2_{V_1(X)} - \sigma^2_{V_1(Y)}}{2}\right]^2 + \sigma_{V_1(XY)}}}$$

$$\tilde{X} = X_3 \cos\theta + Y_3 \sin\theta,$$

$$\tilde{Y} = -X_3 \sin\theta + Y_3 \cos\theta,$$

$$\theta = 1/2 \arctan\left[2\,\sigma_{V_1(XY)} / (\sigma^2_{V_1(X)} - \sigma^2_{V_1(Y)}) \right]$$

The transition from Equation (2-8) to Equation (2-9) for $V_{F_1}(X,Y)$ represents a conversion from the (X, Y) coordinate system to a $(\tilde{X}, \tilde{Y})$ coordinate system, the origin of which is at the center of the probability distribution of V_{F_1}, and the axes of which follow the axes of the ellipse of concentration. This conversion involves a translation of the mean for the distribution of V_{F_1} and a rotation through an angle of θ.

The expectant value of kill is given by

$$E_k = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_k (X, Y) \, V (X, Y) \, dXdY$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \iint_{\xi^2 + \eta^2 \le L_R^2} \left[P_I (\bar{X} + \xi, \bar{Y} + \eta) \, d\xi \, d\eta \right] V (\bar{X}, \bar{Y}) \, dXdY$$

or

$$E_k = V_\Omega P_r$$

where $$P_r = \int_{\xi^2} \int_{\eta^2 \le L_R^2} V_F (\xi, \eta) \, d\xi d\eta$$

Tabulated values exist of P_r as a function of the following four quantities:

$$\mu_X = \text{center of } X = |\tilde{X}| / \tilde{\sigma}_X :$$

$$\mu_Y = \text{center of } Y = |\tilde{Y}| / \tilde{\sigma}_X ;$$

$$\sigma_Y = \tilde{\sigma}_Y / \tilde{\sigma}_X ;$$

$$R = \text{radius} = L_R / \tilde{\sigma}_X$$

The method for directly calculating values of $P_r (\mu_X, \mu_Y, \sigma_Y, R)$ is provided in Appendix D.

2.4.5.5 Kill of Point Target

For one-point targets, the damage evaluation is similar to that given for the elliptical bivariate normal distribution analysis of area targets. Equation (2-8) is applicable with

$$\sigma_{V_1(X)} = \sigma_{I(X)}; \; \sigma_{V_1(Y)} = \sigma_{I(Y)}; \; \sigma_{V_1(XY)} = \sigma_{I(XY)}$$

2.4.5.6 Area Targets of Variable Hardness

If a missile delivers one device onto a target, the expectant amount of damage has been shown to be

$$E_k = V_\Omega P_r$$

It is possible that the area target has structures of various hardnesses dispersed throughout the target area. For such a posture, the expectant amount of damage can be computed by calculating P_r for each level of target hardness. If target hardness has a continuum of values, then P_r is calculated for many possible values of target hardnesses $H_0, H_1, H_2, \ldots, H_p$, where H_0 is the lowest hardness, H_p is the greatest hardness, and each hardness $H_i = H_0 + i\Delta$, where $\Delta = (H_p - H_0)/p$. Let the P_r corresponding to any hardness H_i be called $P(H_i)$. This is based upon the lethal radius for that hardness, the area distribution of structures having that hardness, the accuracy of the flight impact (X_I, Y_I), and yield. Also denote by V_i the value of target with hardness H such that

$$H_i - 1/2\,\Delta \le H < H_i + 1/2\,\Delta$$

Then approximately,

$$E_k \cong \sum_{i=0}^{p} V_i\, P(H_i) \tag{2-10}$$

This approximation improves with a larger p. If V'(H) represents

$$\lim_{\epsilon \to 0} \frac{1}{2\epsilon} \left[\text{value of target with hardness between } H - \epsilon \text{ and } H + \epsilon\right],$$

then Equation (2-10) can be written in integral form as follows:

$$E_k = \int_{H_0}^{H_p} V'(H)\, P(H)\, dH \tag{2-11}$$

Equation (2-10) is an approximation for Equation (2-11). The total value of the target V_Ω is given by

$$V_\Omega = \int_{H_0}^{H_p} V'(H)\, dH$$

The possibility of directing more than one missile to the same target has been discussed. The value of C_k as given in Equation (2-7) can now be determined. The results are then used in the matrix representation of the effectiveness equation E to determine the system figure of merit, which is the total expectant value of kill of a squadron of missiles.

2.4.6 Examples

An example illustrating the methods described in this section is provided in Appendix E. Additionally, a less complex example illustrating the application of some of the general methodologies for system effectiveness evaluation to the Gemini Agena Target Vehicle (GATV) system for the manned Gemini XI rendezvous mission appears in Appendix F. The specific methodologies illustrated therein relate to the following:

- Establishment of a model for a multi-mission requirement involving a sequential performance of seven missions where a figure of merit was defined for each mission.
- Simplification of the state definition task by defining only critical failure states, and by using an effectiveness vector - matrix set, $\overline{A}'$ [D] $\overline{C}$, for each figure of merit instead of one set for the overall mission.
- Combination of the seven figures of merit into an overall system effectiveness measure E using weighting factors for each mission.
- Relationship of accountable factors to each figure of merit and to the combined measure E.

2.4.7 Total Applicability to Air Force Systems

The techniques described herein have applicability to most classes of Air Force systems. In particular, the accuracy parameter specified for most systems can be evaluated with the methods developed. For example, in the case of a spacecraft system or a tracking radar system, the analogous parameter to missile impact error (CEP) could be position and velocity error at a particular time t_o. These could be designated as (X_{IE}, Y_{IE}, Z_{IE}, $\dot{X}_{IE}$, $\dot{Y}_{IE}$, $\dot{Z}_{IE}$) where

$$X_{IE} = X(t_o)$$

$$= \text{intended value of } X(t_o)$$

$$\dot{X}_{IE} = \dot{X}(t_o)$$

$$= \text{intended value of } \dot{X}(t_o)$$

etc.

$X(t_o)$ is the X-coordinate at time t_o and $\dot{X}(t_o)$ is the time derivative of the X coordinate at time t_o. The mean and standard deviation of the position and velocity errors can be calculated in the same manner as for a missile system. Thus, each performance parameter becomes a function P_i of $(X_{IE}, Y_{IE}, Z_{IE}, \dot{X}_{IE}, \dot{Y}_{IE}, \dot{Z}_{IE})$ and other variables $(V_1, V_2, \ldots\ldots, V_n)$ not directly related to accuracy, but relating to the other requirements of the mission as reflected by the system's figure of merit measure.

Additionally, while this section describes the methods of evaluating a figure of merit for a generally simplified missile system and target posture, the contributions of other performance parameters to a more complex mission, and hence, a more complicated missile system, can be assessed. Performance requirements, such as vulnerability to enemy defense systems, penetration aids effectiveness, warhead spacing, maneuverability characteristics, and reaction time, which affect the expected amount of damage inflicted, can be integrated into the overall figure of merit and evaluated with the techniques described.

SECTION 3
SYSTEM EFFECTIVENESS APPORTIONMENT

3.1 INTRODUCTION

The primary technical problem associated with the full implementation of a system effectiveness requirement or objective is to establish detailed design criteria for specified numerical requirements (or objectives). Practical apportionment techniques are available and can be applied objectively and rationally to apportion these requirements to the overall system's constituent major systems, subsystems, or equipment. Based upon current technology, this apportionment can be accomplished in such a manner that:

(1) Requirements may be established for the availability, dependability, and capability parameters at the major system level.

(2) Requirements may be established at the subsystem or equipment level in the form of design criteria to guide designers and developers. This set of requirements may be in terms of accountable factors which are suitable for inclusion as design parameters and performance requirements in subsystem or equipment design specifications. As a corollary, the accountable factors are in terms of design variables which the designers can directly influence.

A necessary and sufficient criterion is that any technically acceptable apportionment technique must have the capacity to accomplish the apportionment of the overall military or weapon system effectiveness requirements to the availability, dependability, and capability parameters of its major systems. This is to accommodate the inclusion of the apportioned requirements into the top performance specification of each constituent major system of the overall system. Consider as an example an overall system such as a strategic bomber squadron which is composed of the bomber system and

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a separate support facility system. Each system will have a top performance specification into which can be included the apportioned requirements for the figures of merit as well as for its parameters of availability, dependability, and capability.

Also, the technique must have the capacity to accommodate a further apportionment of requirements from the major system to the subsystem level in instances where complex subsystems are involved. For example, it may be desirable to have apportioned availability, dependability, and capability parameters as well as apportioned figures of merit for a complex guidance subsystem.

The developed Lagrange multiplier with priority list apportionment technique as well as others described in this section provides the above capability. Furthermore, it will yield a unique or limited number of solutions for the consistent apportionment of achievable accountable factors, including design parameters. This technique can also be used for the initial establishment of preliminary figures of merit for an overall system.

3.2 GENERAL CONSIDERATIONS

The system effectiveness apportionment process, which is the inverse of the prediction process, is based on the system effectiveness equation and the functional relationships of the accountable factors to the effectiveness parameters. Proper apportionment of effectiveness requirements will allow a system to evolve which is optimum for performance. Conversely, if the system's accountable factors are optimized, these optimum values become the properly-apportioned values for a maximized effectiveness.

As indicated in Section 2, optimization can be accomplished with or without constraints on key system parameters which influence effectiveness. These constraints are essentially mandatory limitations and may be any combination of the following forms:

- physical constraints - weight; volume; etc.
- performance constraints - figures of merit and attendant availability, dependability, and capability parameters; operational requirements, defense posture, etc.

- resource constraints - cost, development time, technological barriers, etc.

It is also possible to have a set of non-optimum apportionment solutions which will meet a primary constraint such as a required figure of merit. In such cases, the final choice would be based on a trade-off of the physical and resource factors to arrive at an optimum combination without compromise of the primary constraint.

3.3 ACCOUNTABLE FACTORS

If there exist N_a accountable factors $\alpha_1, \alpha_2, \ldots, \alpha_{N_a}$, then each performance parameter r_i may be expressed by:

$$r_i = f_{r_i}\left(\alpha_1, \alpha_2, \ldots, \alpha_j, \ldots, \alpha_{N_a}\right) \text{with } i = 1, 2, \ldots, N_R \qquad (3\text{-}1)$$

In turn, each performance parameter r_i and correspondingly, each accountable factor α_j are related to system effectiveness E by:

$$\begin{aligned} E &= E^*\left(r_1, r_2, \ldots, r_i, \ldots, r_{N_R}\right) \qquad (3\text{-}2) \\ &= E\left(\alpha_1, \alpha_2, \ldots, \alpha_j, \ldots \alpha_{N_a}\right) \end{aligned}$$

There may be c^* constraints on one or more of the accountable factors, $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{N_a}$, such that each constraint is a function of one or more accountable factors, and may not exceed a limit value. This may be imposed by system constraint parameters C_k, with

$$C_k \geq f_k\left(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_j, \ldots, \alpha_{N_a}\right) \text{for } k = 1, 2, \ldots, c^* \qquad (3\text{-}3)$$

As with function theory, the functions $f_k\left(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_j, \ldots, \alpha_{N_a}\right)$ are not required to contain each of the accountable factors α_j as a variable.

3.4 METHODS OF OPTIMIZING EFFECTIVENESS

Methods of maximizing the effectiveness of Equation (3-2), observing the constraints stated by the relationship of Equation (3-3), include optimization by direct search, dynamic programming, the Lagrange multiplier with priority list method, and the classical Lagrange technique. The following describes these techniques.

3.4.1 Direct Search Method

The simplest method for optimizing the effectiveness of a system is to compute the effectiveness of each feasible design alternative, and to determine the system having the maximum effectiveness among those alternatives for which the constraint relationship of Equation (3-3) are valid. This is accomplished by examining the results of all combinations. This simple search method is very laborious because of the number of alternatives involved. For example, there exist $A = n_1 \times n_2 \times \cdots \times n_N$ alternatives for a system consisting of N subsystems with n_j different design choices for the j^{th} subsystem. The effectiveness for each of these alternatives must be computed when applying the method of direct search. For a large number of subsystems N and design choices n_j, the number of alternate configurations to be investigated will be extremely large.

Thus, if a system is composed of 10 subsystems, each of which can function or be constructed in four different ways, then the number of possible alternatives to be examined in the system using the direct search method will be $A = 4^{10}$ or greater than 1 million. Studying such a large number of alternatives is not practical even when the direct search method is computerized.

3.4.2 Dynamic Programming Method

A significant reduction in the number of alternatives to be examined may be obtained by applying the technique of dynamic programming developed by Bellman and associates at the Rand Corporation [2] for general decision problems of optimization. This method is based on the mathematical notion of recursion. It uses Bellman's principle of optimality, which states that:

> "An optimal policy has the property that, whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision."

Using the dynamic programming method, each subsystem is considered in turn, with effectiveness evaluated for each feasible alternative of this subsystem. The alternative which maximizes effectiveness without violating any constraints is then chosen. The process is continued in this step-by-step manner until the complete system is constructed, and hence optimized. An analogous situation would be to start with an optimized simple system; add one level of complexity; then optimize again; add another complexity level; and so forth until the final desired system is obtained. In some cases, the initial subsystem may be designed in an optimum manner without knowledge of how the remaining subsystems will be designed. When this occurs, each subsystem will require only one optimization. Then, with the dynamic programming method, approximately 1,000,000 alternatives indicated for the example used in the direct search method may be reduced to $A = n_1 + n_2 + \ldots + n_N = 4\,(10) = 40$. However, if the optimum choice for one subsystem depends upon the choices yet to be made on subsequent subsystems, then the optimization of the latter subsystems may involve a modification of the former choice. Consequently, the number of alternatives required to be investigated may be considerably more than 40. However, this number will be considerably less than 1,000,000 in the majority of cases.

The application of dynamic programming as an apportionment technique by optimizing system reliability under constraints is described in [3].

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[2] Bellman, Richard, Dynamic Programming, Princeton University Press, Princeton, 1957.

[3] G. E. Neuner and R N. Miller, Effectiveness Evaluation Using Dynamic Programming, Proceedings of the 1968 Annual Symposium on Reliability, January 16, 17, 18, 1968.

3.4.3 The Classical Lagrange Method

A method for further reducing the computation involved in optimizing the effectiveness of a system is through application of the classical Lagrange method. This method involves the subtraction from the effectiveness function of terms corresponding to the various constraints. The classical Lagrange method, when used in conjunction with priority lists is a powerful tool for apportioning effectiveness parameters to obtain optimum system effectiveness.

The general, well-known method of finding the extreme value of a function, i.e., a maximum or minimum, is to determine the total differential of the function and equate to zero. Since, for an optimum system, $E = E\left(\alpha_1, \alpha_2, \alpha_3, \cdots\cdots, \alpha_j, \cdots\cdots, \alpha_{N_a}\right)$ assumes an extreme value (a maximum),

$$dE = \sum_{j=1}^{N_a} \frac{\partial E}{\partial \alpha_j} \, d\alpha_j = 0 \qquad (3\text{-}4)$$

In the absence of constraints on the α_j's, an extreme value implies that dE must become zero for all the $d\alpha_j$ values. But this occurs only if the N_a coefficients of the $d\alpha_j$'s are zero. As a corollary then, if no constraints on the accountable factors exist, the N_a coefficients must be zero, namely:

$$\frac{\partial E}{\partial \alpha_j} = 0 \; ; \; j \text{ from } 1 \text{ to } N_a \qquad (3\text{-}5)$$

With constraints, and the α_j's having reached one or more constraints, some coefficients will not be zero.

Equation (3-5) is a system of N_a equations for the N_a unknowns, α_j, from which the α_j's may be determined for optimizing the effectiveness E. The conditions which must be fulfilled when E, found by this method, is indeed the maximum, becomes a function of the Air Force system under consideration. This knowledge is used to clarify whether the attained extreme value is a maximum or a minimum.

Now consider the case where it is required to maximize the measure of effectiveness, Equation (3-2), under the constraints stated by the relationship, Equation (3-3). Assume that the first c of the c^* constraints are satisfied with an equality, and that the remaining constraints are satisfied with a margin. This means that the maximum value of the effectiveness, subject to the constraints, occurs when the first c constraints are barely satisfied. Thus, the system of Equations (3-3) establishes restrictions on the $d\alpha_j$'s. Consequently, $dE = 0$ in Equation (3-4) no longer implies that all $\frac{\partial E}{\partial \alpha_j} = 0$ for j from 1 to N_a, because by restricting some of the $d\alpha_j$'s, the balance may not be chosen independently. The equations of (3-3) then will reduce the N_a degrees of freedom of Equation (3-4) by the number of constraints c.

With the Lagrangian technique, the first c constraint equations may be solved by differentiating Equation (3-3) to obtain:

$$dC_k = \sum_{j=1}^{N_a} \frac{\partial f_k}{\partial \alpha_j} d\alpha_j$$

Equating dC_k to zero, and multiplying the total differential by constant factors ζ_k will result in:

$$0 = \sum_{j=1}^{N_a} \zeta_k \frac{\partial f_k}{\partial \alpha_j} d\alpha_j \quad \text{for } k \text{ from } 1 \text{ to } c$$

The c values of ζ_k are the Lagrange multipliers. Subtracting the sum

$$\sum_{k=1}^{c} \sum_{j=1}^{N_a} \zeta_k \frac{\partial f_k}{\partial \alpha_j} d\alpha_j$$

from Equation (3-4), the following expression is obtained:

$$0 = \sum_{j=1}^{N_a} \left[\frac{\partial E}{\partial \alpha_j} - \sum_{k=1}^{c} \zeta_k \frac{\partial f_k}{\partial \alpha_j} \right] d\alpha_j \qquad (3\text{-}6)$$

This expression contains c coefficients, ζ_k, k from 1 to c, which may be used to reflect the dependency established by Equation (3-3). This fact now enables all of the $d\alpha_j$ to be independently chosen. Thus, since each coefficient of the $d\alpha_j$'s in Equation (3-6) must be zero in the case that the sum is zero,

$$\sum_{k=1}^{c} \zeta_k \frac{\partial f_k}{\partial \alpha_j} = \frac{\partial E}{\partial \alpha_j} \quad \text{for } j \text{ from } 1 \text{ to } N_a \qquad (3\text{-}7)$$

and from Equation (3-3) for the equality relationship,

$$C_k = f_k\left(\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_{N_a}\right) \quad \text{for } k \text{ from } 1 \text{ to } c \qquad (3\text{-}8)$$

The combined equation system, (3-7) and (3-8) consists of $(c + N_a)$ equations with the unknowns:

α_j, j from 1 to N_a

and

ζ_k, k from 1 to c

Thus, with $(N_a + c)$ equations and $(N_a + c)$ unknown values, the $\alpha_1, \cdots, \alpha_{N_a}$ accountable factor values which maximize the measure of effectiveness can be determined.

The procedure for using the Lagrange multiplier method in the general case is discussed in the illustration of Appendix G for constraints on three system parameters. The method has broad applicability to constraints of any number of system parameters.

3.4.4 Lagrange Theory

The classical Lagrange theory, which will lead to the same final results and the same working method, is as follows: The equation system (3-7), (3-8), the solution of which maximizes the effectiveness E under the constraints, may also be found by using the Lagrange expression:

$$L = E\left(\alpha_1, \alpha_2, \cdots\cdots, \alpha_{N_a}\right) - \sum_{k=1}^{c} \zeta_k \left[f_k\left(\alpha_1, \alpha_2, \cdots\cdots, \alpha_{N_a}\right) - C_k\right] \quad (3\text{-}9)$$

and computing the extreme value by the classical method of solving

$$\frac{\partial L}{\partial \alpha_j} = 0$$

$$\frac{\partial L}{\partial \zeta_k} = 0$$

from which the equation system (3-7), (3-8) immediately follows. Notice that the sequence of equations from (3-4) to (3-8) shows that the computation of the maximum of the effectiveness function, by determining the extreme value of the Lagrange expression, solves the optimization problem of system effectiveness.

Having obtained Equations (3-7) and (3-8), there remains the problem of their solution. If they are all linear equations, there are well known methods of solving systems of linear equations, such as Gaussian elimination. If an equation is not linear, a form of Newton's iteration may be used. This consists of: making a guess for $(\alpha_1, \cdots\cdots, \alpha_{N_a}, \zeta_1, \cdots\cdot, \zeta_c)$, replacing the functions $\partial f_k/\partial \alpha_j$, $\partial E/\partial \alpha_j$ in (3-7) and f_k in (3-8) with linear functions agreeing in value and slope at the guessed value of $(\alpha_1, \cdots\cdots, \alpha_{N_a}, \zeta_1, \cdots, \zeta_c)$ and solving the resulting system. The solution gives an improved $(\alpha_1, \cdots\cdot, \alpha_{N_a}, \zeta_1, \cdots, \zeta_c)$ which may serve as the next guess for a repetition of the procedure. Thus, in this method, the values of all the α's and ζ's may be improved with each iteration.

3.4.5 Lagrange Multiplier With Priority Lists

In another method, the Lagrange multiplier with priority lists, the value of only one α is improved at each step. The word "priority" connotes the fact that the method seeks the best choice of which variable α_j is to be improved at the next step. This choice is made by means of a method of "steepest ascent" for the effectiveness E.

Consider a case with one constraint only. By means of Equation (3-7),

$$\zeta_1 = \frac{\partial E}{\partial \alpha_j} \bigg/ \frac{\partial f_1}{\partial \alpha_j}$$

All possible values and combinations of ζ_1 may be computed and ordered according to their magnitudes in a list, setting the largest value of ζ_1 in the first rank of the priority list; the second largest ζ_1 in the second rank; etc. To find the solution for the optimum effectiveness of a system with one constraint by means of the priority list, the order of the priority list is followed. The contributions of the accountable factors to both the effectiveness and the constraint value are determined according to the rank of the priority list until the limit C_1 of the constraint parameter is reached.

The applicability of priority lists with more than one constraint is illustrated in Appendix G.

3.4.6 Difference Among Optimization Methods

- Direct Search

  The method of direct search would be used only when the number of possible and reasonable alternatives is small. Otherwise the amount of labor, even with the aid of an electronic computer, becomes prohibitive. Then, the method of dynamic programming is at least to be preferred to the method of direct search. The chief advantage of the direct search method, where few alternatives exist, is its simplicity, and hence its ease of understanding.

- Dynamic Programming

The method of dynamic programming will allow an optimization problem in n variables to be changed into n problems, each in one variable only. The fundamental relationship of the n variables may be continuous or discrete. The method is generally recommended when each accountable factor is limited to a small number of discrete possible values. If many possible values are permitted for each accountable factor, a preferred method of selecting the values of the accountable factors for an evaluation to compare E and f_k is by a method using Lagrange multipliers. Such methods, for example, might be the classical Lagrange method or the Lagrange multiplier with priority list method.

- Lagrange Multipliers

The Lagrange multiplier method applies the classical approach which uses differential calculus to determine the maximum of a function L. This function consists of the effectiveness function E minus the products of additional variables called Lagrange multipliers, times functions arising from the constraint relationships. In the strict classical sense, this method requires continuous effectiveness and constraint functions. However, by replacing differential terms with difference terms, the Lagrange multiplier method leads to engineering solutions even when the effectiveness function is meaningfully defined only at discrete values of the accountable factors. This means that, even though the functions E and f_k in Equations (3-7) and (3-8) are very complicated, so that in practice the determination of the derivatives involved is often impractical, the derivatives $\frac{\partial E}{\partial \alpha_j}$ and $\frac{\partial f_k}{\partial \alpha_j}$ may be estimated by the evaluation of E and f_k for two values of α_j, namely α_j^{**} and α_j^{*} which are fairly close, and using the approximations

$$\frac{\partial E}{\partial \alpha_j} \cong \frac{E\left(\alpha_1, \cdots, \alpha_j^{**}, \cdots, \alpha_{N_a}\right) - E\left(\alpha_1, \cdots, \alpha_j^{*}, \cdots, \alpha_{N_a}\right)}{\alpha_j^{**} - \alpha_j^{*}}$$

The same approximations may be used for f_k.

In most effectiveness optimization problems, the system of equations (3-7), (3-8) developed by the method of Lagrange multipliers converts immediately from a problem in n variables into n problems, each with one variable, as does the dynamic programming method by means of Bellman's principle of optimality. Thus, both the methods of dynamic programming and that of Lagrange multipliers are similar in that they treat a portion of the total problem at one time, and that they obtain essentially the same result for the same problem. However, the mathematical equations derived from the Lagrange multiplier method, Equations (3-7), (3-8), may be more easily formulated and solved in most effectiveness optimization problems than an approach using dynamic programming. Thus, of the two approaches, it is considered that the Lagrange multiplier method is to be recommended for general engineering use.

- Priority List and Classical Methods of Lagrange Equation Solution

If the method of Lagrange multipliers is to be used, the resulting Equations (3-7) and (3-8) may be solved by priority list or by another technique such as the previously mentioned Newton's method. There are systems of equations and initial guesses for which the use of Newton's method does not converge to a solution. In such cases, the priority list method could be used to approach a solution, and then Newton's method used for the final iterations.

The classical Lagrange method, with solution of the simultaneous equations by Newton's method or analytical means, is a more general technique than the Lagrange multiplier with priority list method for effectiveness apportionment. It will provide a full apportionment solution for complex systems, but, in all likelihood, will require the use of a computer for this solution. On the other hand, the less complex solution process of the alternative Lagrange multiplier with priority list method can be accomplished in many cases with manual calculation. The Lagrange multiplier with priority list method does have the following advantages:

(1) It provides system engineers with an analytical perspective of the step-by-step optimization process. As such, the major sensitivity areas of the system can be identified as the optimization process proceeds.

(2) It provides more understandable answers. In any event, both methods will provide the same solution.

(3) It converges to the solution in large steps.

It is to be noted that the priority list method will be most useful if the partial derivatives of the effectiveness and constraint functions with respect to an accountable factor α_j, namely $\frac{\partial E}{\partial \alpha_j}$ and $\frac{\partial f}{\partial \alpha_j}$, are functions of that α_j alone. In most applications, this condition will be met by the constraint functions, but not by the effectiveness function. Sometimes a simple transformation can cause this condition to be satisfied. Such a simple transformation is a logarithmic transformation in instances where an effectiveness function is a product expression. However, if the condition cannot be satisfied in the partial derivative functions, the priority list may still be used, but will require re-ordering during the solution process. A frequent re-ordering of the list is indicated if for each re-ordering, a radical change in the priority list results. The sole consequence of not re-ordering the list in the case of α dependencies is that more iterations would be required than otherwise needed.

If frequent re-orderings pose an inconvenience, or if the apportionment solution is extremely complicated, Newton's method or similar methods for non-linear equations are recommended for the solution of the simultaneous equations of (3-7) and (3-8). Where the solution is not converging with these methods, the priority list method may be used to aid the solution process. It can provide α values sufficiently close to the optimum solution to be in a convergence region for use with the classical iterative methods.

3.4.7 Application of the General Theory

To provide a more lucid basis of understanding of the developed Lagrange multiplier with priority list apportionment method, and to demonstrate its aspects, illustrative examples are provided in Appendices G, H, and I. The numerical values entered in these examples are typical ones for the systems being analyzed. This combination of details of the general theory and numerical results will also provide the reader and user with a basis for potential application to other systems.

With regard to other applications, in applying these approaches to optimize system effectiveness and to exercise the apportionment techniques outlined, it is necessary to perform two basic steps that are particularized to the system being analyzed. (These steps were previously described in Section 2.):

(1) Develop an overall functional relationship which analytically integrates the subsystems and system together with their pertinent accountable factors into a functional relationship for E. This relationship will be an equation or set of equations which include the A, D, and C parameters.

(2) Solve and analyze the functional relationships above and apportion E based on an optimization technique.

The accomplishment of (1) will require a thorough analysis of the system under consideration, together with an identification in detail of the mission, the accountable factors, system constraint parameters which apply, any incentive factors which apply, the system performance parameters which implicitly relate to subsystems and equipment, and all such system details. From this knowledge of the system and its mechanization, the analyst can develop the overall functional relationships expressed in mathematical form. Depending on the complexity of the system, the construction of this set of equations, including A, D, and C components, may require a specific amount of analytical modeling effort. With the advancements in the system effectiveness technology to date, this should not pose a problem.

The set of equations established in (1) may take various forms. They may be of closed or indefinite series form, or contain empirical terms. Almost always, the equations will contain probability distributions. This being the case, the solution of the equations and the application of the apportionment technique for an optimized effectiveness in (2) may entail computer application.

At this stage, while general steps can be specified for the application of the techniques described herein for the analysis and optimization of system effectiveness during design, and its tracking during development, it is not feasible to specify detailed procedural steps applicable to every type of system. However, as more systems are analyzed, it should be possible to reduce their application for each type of system to a specified set of procedural steps which may be applied on a routine basis.

3.5 ILLUSTRATION SUMMARY

The following is a summary of the application of the procedures in three examples. Two examples illustrate the Lagrange multiplier with priority list apportionment technique of optimizing system effectiveness. The third example illustrates the classical method of equating total derivatives to zero to identify optimums, and the use of graphical methods to solve equations at these optimums. Complete details on these illustrative examples and all mathematical steps in the procedures are contained in Appendices G, H, and I.

3.5.1 Example 1-Optimizing Dependability Within Constraints

This example applies the Lagrange multiplier with priority list apportionment technique to a problem which arose during the preliminary design phase of a communication satellite (Comsat) system. To emphasize simplicity in this initial example so that the apportionment technique would be better understood, the application was limited solely to the reliability parameter of the WSEIAC effectiveness model. The complete exposition of this example appears in Appendix G. Example 2 is a further expansion of this example to include the other two effectiveness parameters of availability and capability.

Problem

The Comsat is a spacecraft being designed and developed to relay communications via a satellite in synchronous orbit. In this particular application, the spacecraft is for commercial use, and the ability (effectiveness) of the system accurately to attain orbit, to maintain position, and to provide trouble-free service over a minimum required time of years is directly relatable to returned income. In addition to the minimum specified period of operation, there is an advantage in increased income resulting from maintaining position in orbit and maintaining a maximum number of parallel channels in operation during satellite life, over and above the minimum requirement.

In this Comsat system there are obvious constraints. The launch vehicle subsystem can deliver only so much payload to a given point in space, and the cost of the system has a direct bearing on net income. In this case, a maximum payload is specified for the spacecraft on the rising nodal crossing of the equator at synchronous altitude in a 28.5 degree inclined orbit. From this point, the various subsystems used for achieving orbit and for on-orbit operation must compete for weight and, to a degree, on cost based on their contribution to the overall system effectiveness. This means that the system weight and cost must be apportioned in such a manner as to obtain optimum system effectiveness. While the example is addressed to a commercial satellite, with minor changes in parameters, it will apply equally to a military Comsat.

Effectiveness Equation System

This example assumes that the overall system reliability may be increased by the application of redundancy to the subsystem which comprises the system and that increases in system effectiveness, E, are related only to improvements in the reliability performance of the dependability parameter. Specifically, the example considers that:

(1) The system consists of N subsystems in series, the failure of any one resulting in failure of the entire system.

(2) The i^{th} subsystem may consist of n_i subsystems (i.e., 1 subsystem with identical subsystems in redundancy).

(3) The redundancies for each subsystem are in standby, with 100 percent switching reliability assumed.

Then system reliability R_s is:

$$R_s = \prod_{i=1}^{N} R_i(n_i) \qquad i = 1, 2, \cdots\cdots, N \qquad (3\text{-}10)$$

For subsystems with constant failure rates, with the failure rate for the i^{th} subsystem being λ_i, the reliability of a single subsystem for a mission time t is

$$R_i(1) = \exp\left[-\lambda_i t\right] \tag{3-11}$$

and for each subsystem consisting of j standby redundant subsystems, the reliability is

$$R_i(j) = \exp\left[-\lambda_i t\right] \sum_{k=0}^{j-1} \frac{(\lambda_i t)^k}{k!} \tag{3-12}$$

$$j = 1, 2, \cdots\cdots, n_i$$

Applying Apportionment Within Constraint(s)

If there are no constraints on the system (i.e., no limits on the total mass, volume, cost, etc.), it is possible to continue adding redundant subsystems to achieve any desired level of reliability (dependability). For the system under consideration, however, constraints are imposed, and it is desired to apply apportionment techniques to optimize effectiveness (reliability) without exceeding these constraints.

Using the Lagrange multiplier with priority list technique, functional relationships may be identified which may be used to optimize system reliability within imposed constraints. This optimization is achieved by determining the optimum number of redundant units for each subsystem subject to the following constraints:

$$\sum_{i=1}^{N} n_i m_i \le M \tag{3-13}$$

$$\sum_{i=1}^{N} n_i v_i \le V \tag{3-14}$$

$$\sum_{i=1}^{N} n_i c_i \le C \tag{3-15}$$

Multiplying the constraint equations by the Lagrange multiplier, differentiating, and summing gives:

$$\sum_{i=1}^{N} \left(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i \right) d n_i$$

By subtracting the above from the derivative of the effectiveness function and equating to zero,

$$\sum_{i=1}^{N} \left[\frac{1}{R_i (n_i)} \frac{d R_i}{d n_i} - \left(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i \right) \right] d n_i = 0$$

Since each term in this equation must equal zero, the terms inside the bracket can be equated and integrated with respect to n_i to obtain:

$$\ln R_i (n_i) = \left(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i \right) n_i + \ln R_o$$

or

$$R_i (n_i) = R_o \exp \left[\left(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i \right) n_i \right]$$

from which it follows that the decision rule of selecting discrete n_i for any ζ_m, ζ_v, and $\zeta_c \geq o$ for optimum effectiveness, can be established to be:

$$\ln \frac{R_i (n_i)}{R_i (n_i - 1)} > \zeta_m m_i + \zeta_v v_i \quad \zeta_c c_i \geq \ln \frac{R_i (n_i + 1)}{R_i (n_i)} \qquad (3\text{-}16)$$

In Equations (3-13) through (3-16) M, V, and C, respectively, are the system constraints on mass m_i, volume v_i, and cost c_i of the i^{th} subsystem, and ζ_m, ζ_v, and ζ_c are the Lagrange multipliers relating to system mass, volume and cost. Confined to only a constraint on mass, Equation (3-16) can be written:

$$\frac{1}{m_i} \ln \frac{R_i(n_i)}{R_i(n_i - 1)} > \zeta_m \geq \frac{1}{m_i} \ln \frac{R_i(n_i + 1)}{R_i(n_i)} \qquad (3\text{-}17)$$

Using iteration techniques for these equations and the reliability (effectiveness) equations, the optimum number of redundant units for each subsystem can be determined for an optimized system reliability (Equation (3-10)) within the imposed constraint(s).

For the mass constraint, the relationship of Equation (3-17) suggests the computation of the priority value

$$q_{ij} = \frac{1}{m_i} \ln \frac{R_i(j)}{R_i(j-1)} \qquad (3\text{-}18)$$

$$> \zeta_m$$

for each subsystem i, i from 1 to N, beginning with j = 2 for the number of standby subsystems and continuing with 3, 4, etc. The q_{ij} are then ordered according to decreasing size, and tabulated in suitable formats for manual solution of the problem.

<u>Optimization Procedure and Results</u>

The results of one case example are reproduced herein from Appendix G.

Case A: <u>Constraints</u>
Weight $\leq$ 275 lbs.
Cost $\leq$ \$250,000.

The major steps in obtaining a solution for optimizing the system reliability are briefly summarized as follows:

(1) For the pertinent subsystems of the communication satellite, initially only the weight constraint was considered, and the calculated q_{ij}'s as shown in Table 3-1. The total accumulated system weights, Lagrange multipliers, etc., were computed and tabulated in a priority list of decreasing q_{ij}'s as shown in Table 3-2.

TABLE 3-1 PRIORITY LIST BASED ON WEIGHT CONSTRAINT ONLY

(5-year mission lifetime)

	λi	i	m_i *	q_{ij}		**
Subsystem	Conventional Reliability Failure Rate (hours)	Subsystem Designation	Weight (Pounds)	Priority Value q_{12}	Priority Value q_{13}	Priority Value q_{13}
Battery	1×10^{-6}	1	56	.000769	.000016	.000000
Charger & Instrument Box	$.3 \times 10^{-6}$	2	5.5	.002349	.000015	.000000
Roll H/S	5×10^{-6}	3	4	.049515	.004878	.000599
Mag. Coil	$.1 \times 10^{-6}$	4	5	.000797	.000002	.000000
Mag. Coil Electronics	$.014 \times 10^{-6}$	5	2	.000277	.000000	.000000
Flywheel	$.5 \times 10^{-6}$	6	52	.000418	.000004	.000000
Wheel Electronics	1.2×10^{-6}	7	4	.012914	.000325	.000006
Pitch H/S	5×10^{-6}	8	4	.049515	.004878	.000599
Pitch Jet Elect.	2×10^{-6}	9	4	.021090	.000873	.000025
Roll-Yaw Jet Electronics	2×10^{-6}	10	6	.014063	.000582	.000016
H/S Electronics	1×10^{-6}	11	4	.010767	.000226	.000003

* In these calculations it is assumed the same amount of additional mass is required to add the 1st, 2nd, and 3rd redundancies.

** j refers to the total number of i subsystem type. j=2 would be the basic subsystem plus one subsystem in standby redundancy; j=3 refers to two subsystems in standby redundancy; etc.

TABLE 3-2 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON WEIGHT CONSTRAINT ONLY

(1) Priority	(2) Subsystem Added i	(3) Priority Value q_{ij}	(4) Number of Redundant Subsystems n_i	(5) Additional Weight for Subsystem i (lbs) m_i	(6) Cumulative Total Weight M (lbs) $\sum_{i=1}^{N} n_i m_i$	(7) System Reliab. (%) R_s	(8) LaGrange Multiplier ζ_m	(9) Unit Cost ($1000) c_i	(10) Cumulative Total Cost C ($1000) $\sum_{i=1}^{N} n_i c_i$
		Basic System Only – No Redundancies			146.5	45			67.7
1	Roll Horizon Sensor, 1st Standby	$q_{3,2}=.049515$	$n_3=2$	4	150.5	54.8	.03	8	75.7
2	Pitch Horizon Sensor, 1st Standby	$q_{8,2}=.049515$	$n_8=2$	4	154.5	66.8	.03	8	83.7
3	Pitch Jet Electronics, 1st Standby	$q_{9,2}=.021090$	$n_9=2$	4	158.5	73.7	.015	4	87.7
4	Roll-Yaw Jet Electronics, 1st Standby	$q_{10,2}=.014063$	$n_{10}=2$	6	164.5	79.1	.013	6	93.7
5	Wheel Electronics, 1st Standby	$q_{7,2}=.012914$	$n_7=2$	4	168.5	83.3	.011	5	98.7
6	Horizon Sensor Electronics, 1st Standby	$q_{11,2}=.010767$	$n_{11}=2$	4	172.5	87.0	.005	6	104.7
7	Roll Horizon Sensor, 2nd Standby	$q_{3,3}=.004878$	$n_3=3$	4	176.5	88.7	.003	8	112.7
8	Pitch Horizon Sensor, 2nd Standby	$q_{8,3}=.004878$	$n_8=3$	4	180.5	90.4	.003	8	120.7

TABLE 3-2 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON WEIGHT CONSTRAINT ONLY (Continued)

(1) Priority	(2) Subsystem Added i	(3) Priority Value q_{ij}	(4) Number of Redundant Subsystems n_i	(5) Additional Weight for Subsystem i (lbs) m_i	(6) Cumulative Total Weight M (lbs) $\sum_{i=1}^{N} n_i m_i$	(7) System Reliab. (%) R_s	(8) LaGrange Multiplier ζ_m	(9) Unit Cost ($1000) c_i	(10) Cumulative Total Cost C ($1000) $\sum_{i=1}^{N} n_i c_i$
9	Charger, 1st Standby	$q_{2,2}$=.002349	n_2=2	5.5	186	91.6	.0009	6	121.3
10	Pitch Jet Electronics, 2nd Standby	$q_{9,3}$=.000873	n_9=3	4	190	92.0	.0008	4	125.3
11	Mag. Coil, 1st Standby	$q_{4,2}$=.000797	n_4=2	5	195	92.3	.00077	2.5	127.8
12	Battery, 1st Standby	$q_{1,2}$=.000769	n_1=2	56	251.0	96.4	.0006	5.6	133.4
13	Roll Horizon Sensor, 3rd Standby	$q_{3,4}$=.000599	n_3=4	4	255	96.6	.00059	8	141.4
14	Pitch Horizon Sensor, 3rd Standby	$q_{8,4}$=.000599	n_8=4	4	259	96.8	.00059	8	149.4
15	Roll-Yaw Jet Electronics, 2nd Standby	$q_{10,3}$=.000582	n_{10}=3	6	265.0	97.2	.0005	6	155.4
			INITIAL SOLUTION TO CASE A						
16	Flywheel, 1st Standby	$q_{6,2}$=.000418	n_6=2	52	317.0	97.9	.0004	20	175.4
17	Wheel Electronics, 2nd Standby	$q_{7,3}$=.000325	n_7=3	4	321.5	-	.00031	5	180.4

TABLE 3-2 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON WEIGHT CONSTRAINT ONLY (Continued)

(1) Priority	(2) Subsystem Added i	(3) Priority Value q_{ij}	(4) Number of Redundant Subsystems n_i	(5) Additional Weight for Subsystem i (lbs) m_i	(6) Cumulative Total Weight M (lbs) $\sum_{i=1}^{N} n_i m_i$	(7) System Reliab. (%) R_s	(8) LaGrange Multiplier ζ_m	(9) Unit Cost ($1000) c_i	(10) Cumulative Total Cost C ($1000) $\sum_{i=1}^{N} n_i c_i$
18	Mag. Coil Electronic, 1st Standby	$q_{5,2}$=.000277	n_5=2	2	323.5	-	.00023	2	182.4
19	Horizon Sensor Electronics, 2nd Standby	$q_{11,3}$=.000226	n_{11}=3	4	327.5	-	-	6	188.4

(2) Using the Lagrange multipliers, all redundancies with $q_{ij} > \zeta_m$ were accepted and all redundancies with $q_{ij} \leq \zeta_m$ were rejected. As an initial solution, $\zeta_m = .0005$ was selected. It was noted that this solution rejects all redundancies in the shaded portion of Table 3-1.

(3) Using this initial solution, a determination was made as to the closeness of the system to the weight constraint. This was necessary because the Lagrange multiplier with priority list method takes "large steps" in finding the optimum solution and therefore frequently "steps over" intermediate, but optimum points. Figure 3-1 shows an example of Lagrange multiplier with priority list solutions, other solutions along the optimum path, and other, but non-optimum solutions. It was noted that for the initial solution the total weight was 265 lbs., which leaves 10 lbs. that can still be allocated to redundancies. Referring to Tables 3-1 and 3-2, it was observed that 3 more redundancies can be added for these 10 lbs.

(4) Now that a solution was found within the weight constraint, the weight solution was examined to determine that the cost constraint had not been exceeded. This was determined from data in colums (9) and (10) of Table 3-2. In the aggregate, cost is $156,700 against an allowed constraint of $250,000. Therefore, the optimum solution is that itemized in Table 3-3. Note that the optimized solution shown in Table 3-3 remains at a considerable margin below the cost constraint. This suggest that advantage could be taken by improving the reliability of some subsystem at added cost but retaining the same weight characteristics - i.e., using "hi-rel" subsystems. This further optimization is detailed in Appendix G.

3.5.2 Example 2 - Apportionment of System Effectiveness Defined as a Continuous Function of Mission Time

This example further demonstrates the logic of the Lagrange multiplier with priority list technique by applying the method to a more intensive effectiveness analysis of the same communication satellite described in Example 1. A complete exposition of this example appears in Appendix H. In contrast to Example 1, however, the analysis is

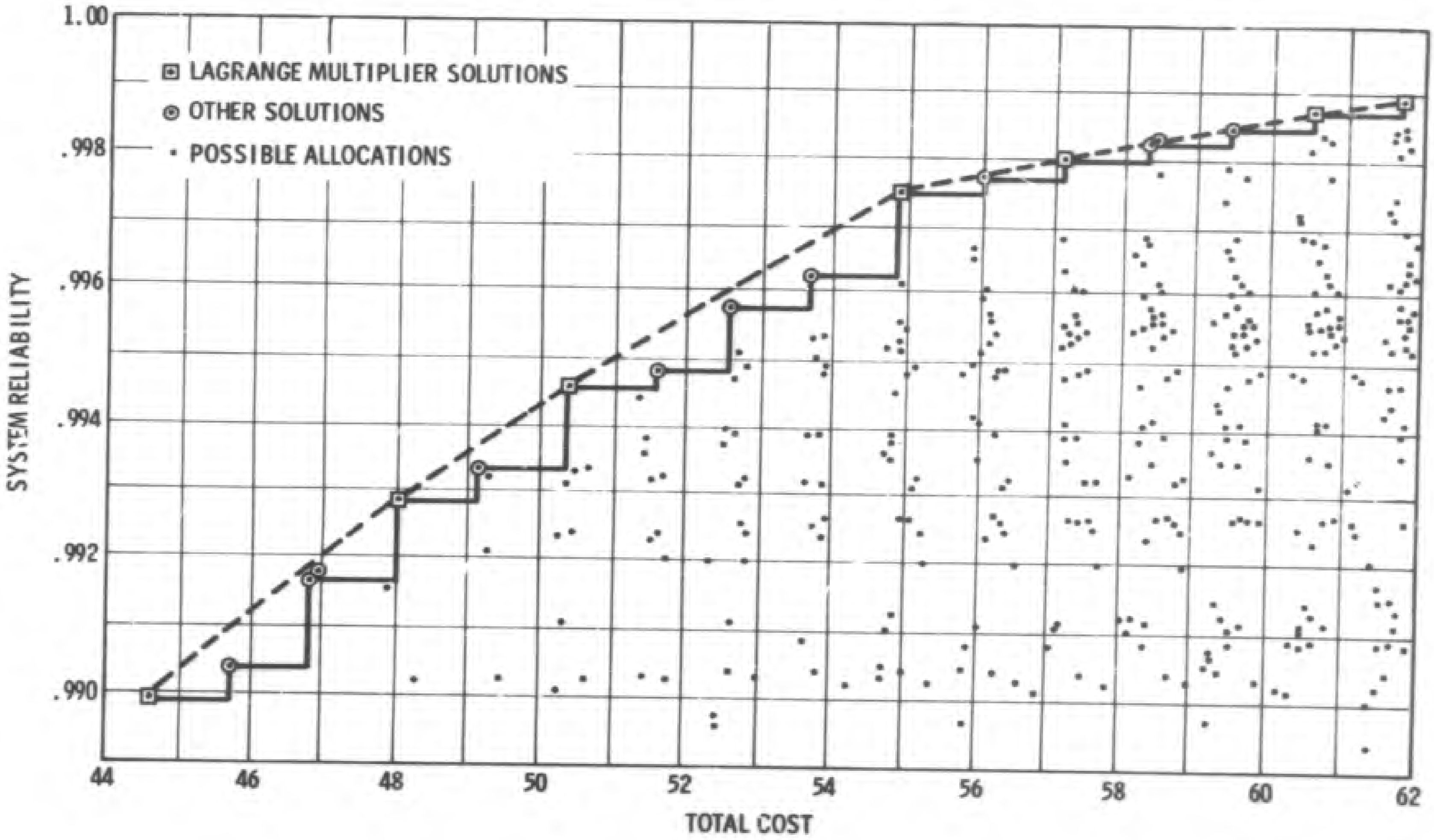

Figure 3-1 Illustration of Optimum-Seeking Routine

TABLE 3-3 SOLUTION TO CONSTRAINT CASE A

Constraints

$W \leq$ 275 lbs
$C \leq$ \$250,000

Subsystem	Number of Standbys in Initial Solution	Selection of Additional Standbys to Absorb Margin	Final Solution n_1
1	1		2
2	1		2
3	3		4
4	1		2
5	0	1st standby, 2 lbs	2
6	0		1
7	1	2nd standby, 4 lbs	3
8	3		4
9	2		3
10	2		3
11	1	2nd standby, 4 lbs	3
Total	W = 265 lbs	W = 10 lbs	C = \$156,700 W = 275 lbs

addressed to an evaluation of the satellite system from launch, through ascent, through establishment of its operating position, and during its intended operating life. Also, all the effectiveness parameters of availability, dependability, and capability are considered.

In the example, the system is briefly described, the system figure of merit, performance parameters, and accountable factors which influence the figure of merit are detailed, and the system effectiveness model developed. Then, the Lagrange multiplier technique with priority list is applied to determine the satellite design yielding the optimum effectiveness subject to a system constraint on weight. This is followed by a numerical example, and then by an optimization procedure in which the effect of various interest rates on the capital investment is considered in the light of the dollar revenue obtainable.

The functions and accountable factors which are considered in this problem are as follows:

- Ascent using a lower stage booster
- Second stage operation and stabilization subsystem operation
- Various orbit setup events
- Redundancy among satellite subsystems, to provide successful operation during a specified minimum period
- The matching of on-orbit expendables (station keeping propellant), to the probabilistic life of other subsystems
- The effect of interest rate of the capital investment on the design of the system for optimum effectiveness
- The lack of on-orbit repair capability, aside from built-in standby redundancies

The objective of the satellite is to achieve a maximum channel-time, which is the number of operating channels multiplied by the length of time the channels operate. The satellite must also be able successfully to attain its final orbital position and to maintain this position during its useful life with on-board expendable hydrazine propellant.

Effectiveness Equation System

The system figure of merit E is the average number of channel-years obtainable. In deriving the relationship for this measure, effectiveness was considered as a continuous function of mission time T. This mission time is equivalent to the planned expendable depletion time and may be divided into K time periods of ΔT length. During each time period, a different number of channels may remain operable. Further, the worth of the channels operational during any ΔT may be weighted by applying a discounting interest rate.

The effectiveness equation is defined as:

$$E = \beta \int_{o}^{T} \sum_{r=o}^{s} r \; M(t) \; dt$$

This measure includes the performance parameters influencing the amount of channel-time obtainable and is subject to a weight constraint relation:

$$W \geq c_1 \, s + f(c_2, \beta) + c_3 T + \sum_{i=1}^{N} n_i W_i$$

In these equations,

β = overall-all probability of successfully attaining orbit position. This includes the probability of success of boost to transfer orbit, the probability of success of shroud jettison, the probability of success of spacecraft separation, and the probability of success of remaining spacecraft phase of ascent. This is the availability parameter. It is a function of technical performance characteristics of the system, which are detailed in the Appendix.

$R_S(t)$ = the satellite's dependability parameter. This parameter is expressed as the reliability parameter depending on the included standby redundancies, which increase the satellite's expected lifetime. This reliability is exclusive of the subsystem for the operation of the channels, the communication subsystem, with

$$R_S(t) = \prod_{i=1}^{N} e^{-\lambda_i t} \sum_{j=0}^{n_i - 1} \frac{(\lambda_i t)^j}{j!}$$

and

N = number of independent subsystems i

n_i = number of standby redundant subsystems of the i^{th} subsystem type

λ_i = failure rate of the i^{th} subsystem

s = the number of channels. This is the capability parameter.

$M(t)$ = the probability that exactly r out of s channels is operating at any time t. The parameter p is the probability of survival of each channel with

$$p = e^{-\lambda_c t} \quad \text{and} \quad \lambda_c = \text{channel failure rate}$$

Further, for the system

c_1 = 20 lbs per communication channel

$$f(c_2, \beta) = \begin{cases} 10 \text{ lbs. for spin stabilized ascent design alternative} \\ 50 \text{ lbs. for active stabilization ascent design alternative} \end{cases}$$

$$= 10 + \left[(\beta - .8845)\, 40 / .0373 \right]$$

c_3 = 30 lbs./yr.

W_i = weight of the i^{th} subsystem

Solution and Results

As indicated, the communication satellite design has a weight constraint. Therefore, the solution of the effectiveness equation system requires that the available weight be apportioned in an optimum manner to the variables β, s, T, and R_S in order to maximize system effectiveness. Basically, this is accomplished by taking the total differential, dE, setting dE equal to zero, and introducing a Lagrange multiplier to provide another degree of freedom in the equation system. Because of the number of variables, iterations are performed on the length of mission (T = 5, 10, 7, 8, 9, and 8.9 years) to obtain the optimum solution at T = 8.9 years.

The iteration summary for the optimum weight allocation solution is shown in Table 3-4, with the values of the performance parameters shown in Table 3-5. Figures 3-2, 3-3, and 3-4 show the relationships between system effectiveness, the total satellite weight, and length of mission for the iterations.

System Effectiveness Including Effects of Interest Rate

In the analysis indicated, and in the optimum solution determined for system effectiveness, each increment of system effectiveness, dE, was equally weighted during the entire mission time. However, when the effect of interest rate on the return in investment is considered, there is less net return of income as mission time increases. Therefore a degrading effect is introducted in the value of dE as time increases.

The effects of different interest rates (6%, 12%, 18%) on the optimized satellite design are summarized in Table 3-6, and Figure 3-5. It will be noted from these summaries that the effect of increasing interest rate is to reduce the optimum satellite mission life. This is reflected in an increase in number of initial channels, in decreased weight allowance for on-board expendable propellant, and in a reduced allocation in weight for dependability redundancy.

TABLE 3-4 SUMMARY OF EFFECTIVENESS ALLOCATION

	Solution at T Years			
	T = 4.9	T = 7	T = 8	T = 8.9*
• Number of Channels	8	7	14	15
• Weight Allocated to Channels	160	240	280	300
• Weight Allocated to Hydrazine	147	210	240	266
• Weight Allocated to Redundancies	26	34	34	34
Roll H/S - 1 Standby	4	-	-	-
- 2 Standbys	-	8	8	8
Pitch H/S - 1 Standby	4	-	-	-
- 2 Standbys	-	8	8	8
Pitch Jet Electronics	4	4	4	4
Roll-Yaw Jet Electronics	6	6	6	6
Wheel Electronics	4	4	4	4
H/S Electronics	4	4	4	4
• Total Weight (lbs)	333	484	554	600
• Total Weight (lbs, including spin stabilization)	343	494	564	610
• Effectiveness (channel-years)	34	73	97	116

*Optimized Solution

TABLE 3-5 VALUES OF PERFORMANCE PARAMETERS FOR OPTIMUM EFFECTIVENESS

Availability, β:	88.45%
Dependability (Reliability) at End of Depletion Time, T:	82.4%
Capability (Initial Number of Channels), s:	15 Channels
Planned Expendable Depletion Time, T:	8.9 Years
Optimum Effectiveness:	116 Channel-Years

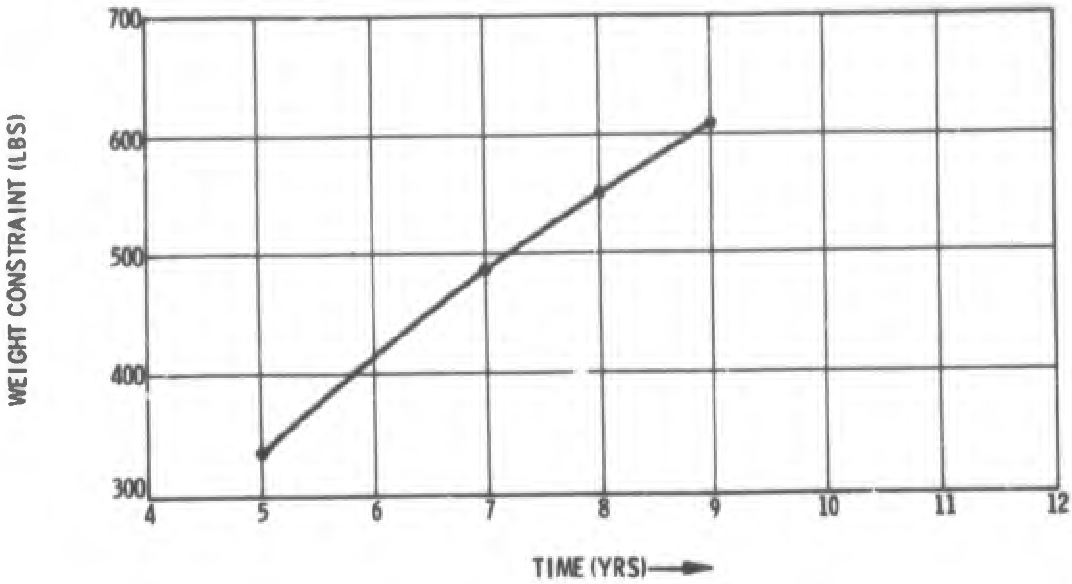

Figure 3-2 Weight Constraint vs Time

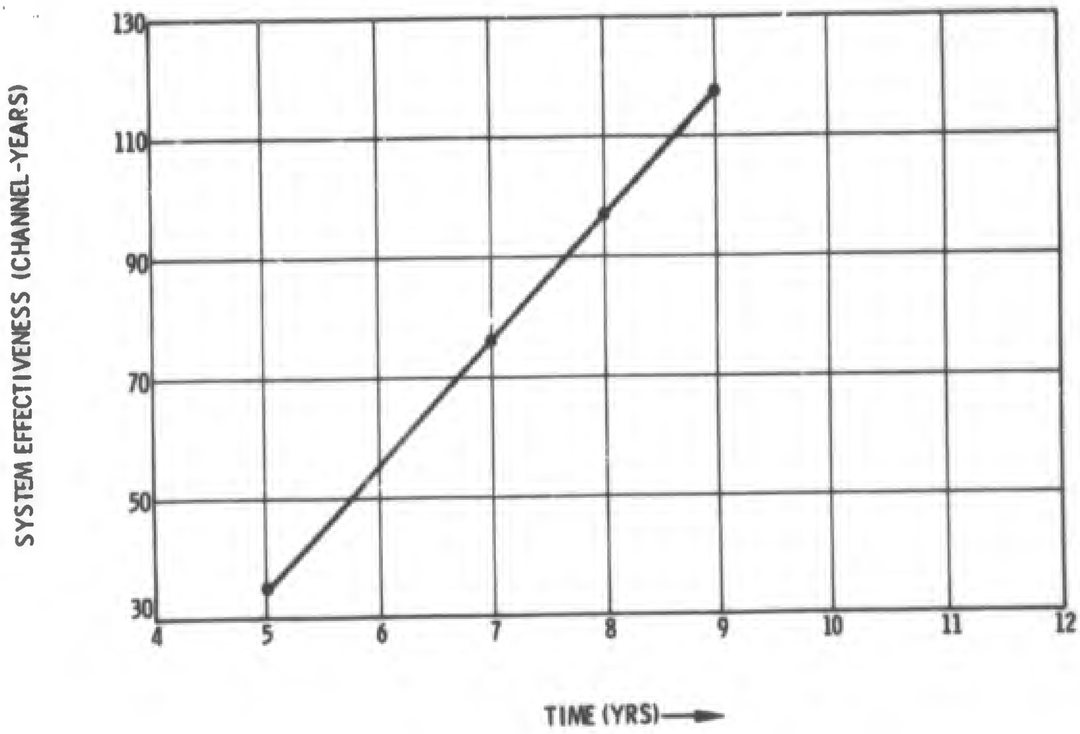

Figure 3-3 System Effectiveness vs Time

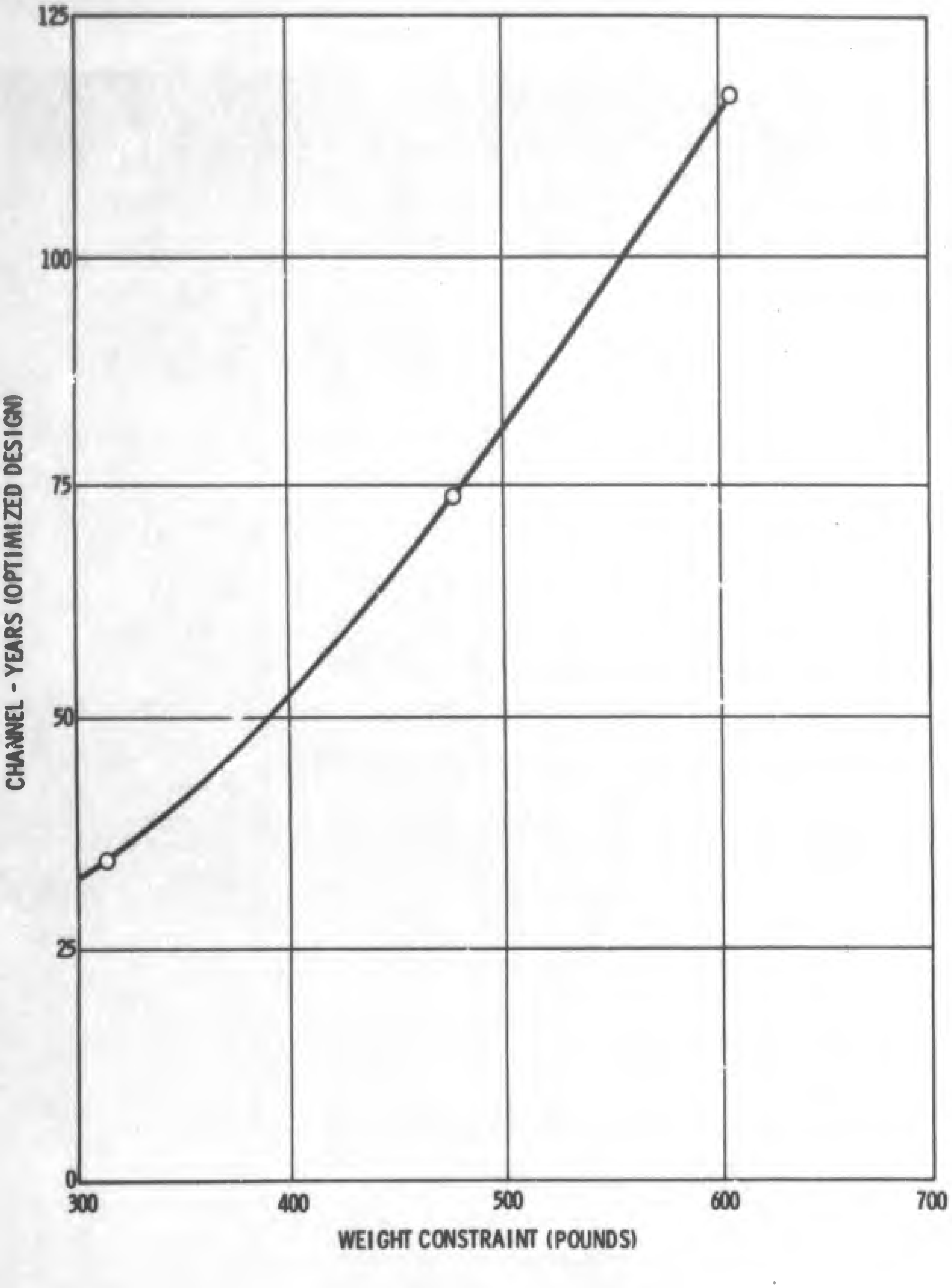

Figure 3-4 Optimized Effectiveness vs Weight Constraint

TABLE 3-6 OPTIMUM SATELLITE DESIGN WITH WEIGHT CONSTRAINT OF 610 LBS FOR VARIOUS INTEREST RATES

		Interest Rate	
	6%	12%	18%
• Number of Channels	17	18	19
• Planned Depletion Time (years)	7.53	6.87	6.47
• Weight of Spin Stabilized Ascent	10	10	10
• Weight of Channels	340	360	380
• Weight of Redundancies	34	34	26
Roll H/S - 1 Standby	-	-	4
- 2 Standbys	8	8	-
Pitch H/S - 1 Standby	-	-	4
- 2 Standbys	8	8	-
Pitch Jet Electronics	4	4	4
Roll-Yaw Jet Electronics	6	6	6
Wheel Electronics	4	4	4
H/S Electronics	4	4	4

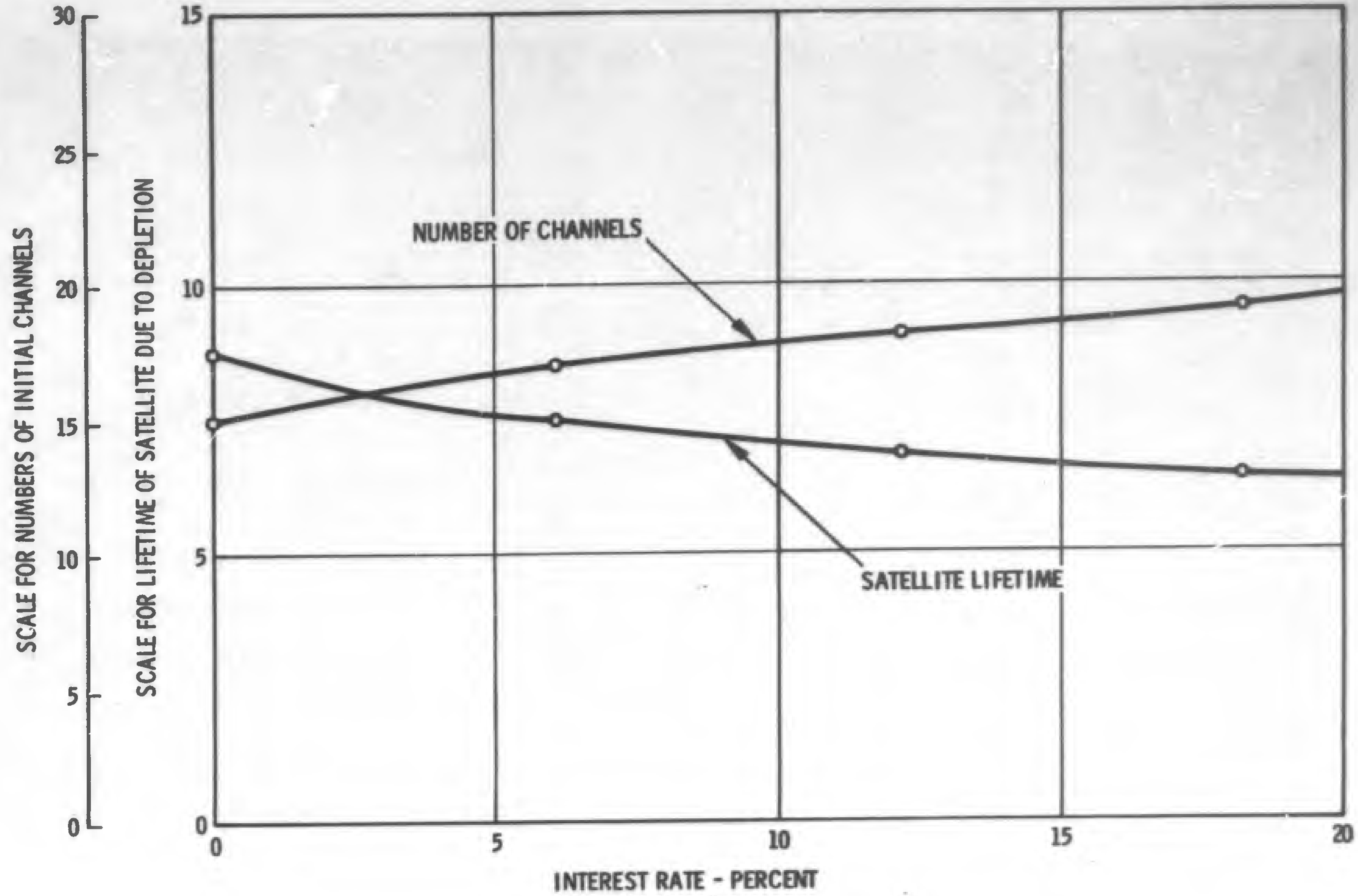

Figure 3-5 Number of Channels and Satellite Lifetime as a Function of Interest Rate

3.5.3 Example 3 - Optimization of the Effectiveness of a Spacecraft Launch Vehicle System

The previous examples illustrated the application of the Lagrange multipler with priority list technique to the problem of apportioning resources within constraints to optimize system effectiveness in the design of a communication satellite system. This third example, detailed in Appendix I, demonstrates apportionment of resources to a launch vehicle for a space payload, again within the framework of the WSEIAC effectiveness model. However, the apportionment technique illustrated is the classical method of taking the total derivative and setting it equal to zero to identify optimums. Additionally, graphical methods are used in certain cases to solve equation systems at these optimums because of the consideration of averages and equality relationships of constraints.

Problem

A launch vehicle system has the requirement of launching a given payload and achieving a selected orbit with a specified accuracy. The launch vehicle has an availability influenced by its dormant and checkout reliability, which may exclude or include redundancy, the mean time to repair malfunctions prior to launch, and the ability or non-ability to launch during a relatively short period of time (the launch window). Weight margin may be used to improve launch vehicle dependability by adding redundancy to one or more subsystems. The capability of the launch vehicle system is a function of its accuracy in delivering the payload into an orbit of specified altitude and the weight of the injected payload.

Effectiveness Equation System

The effectiveness E of the launch vehicle system is specified as the probability of successful launching and achieving a selected orbit within a prescribed accuracy. The measure of E is then defined as

$$E = (\text{Availability}) \times (\text{Reliability}) \times (\text{Capability})$$

$$= ADC$$

$$= A_o R_o \left(1 + r_R\right)^{-\frac{W_R}{\Delta W_R}} \left[1 + 2\left(\frac{\sigma_G}{c_A}\right)^2\right]^{-1/2} \left(\frac{W_S - W_{SC}}{W_{SL}}\right)^{\beta}$$

$$\cdot \exp\left\{-\left[\left(\frac{a_G}{\sigma_G}\right)^{\alpha} + \left(\frac{c_G}{\sigma_G}\right)^{\gamma} + N_R q_S \frac{W_R}{N_R \, \Delta W_R}\right]\right\} \quad (3\text{-}19)$$

with:

$$W_S \leq W_{SC} + W_{SL}$$

and

$$W_n + W_R + W_S \leq W_L$$

In the above three equations all parameters are constants, except σ_G, W_R, and W_{SL} which are variables. The values of these variables are to be apportioned for the optimization of system effectiveness, E.

In these equations, the parameters are defined as follows:

Weights:

W_n : Weight of subsystem of launch vehicle system with no redundancy applicability, including weight of propellant.

W_R : Weight of subsystem of launch vehicle system with redundancy applicability.

ΔW_R : Average weight of one subsystem of launch vehicle system with redundancy applicability.

W_{SC} : Weight of the subsystem of the spacecraft to be launched by the launch vehicle system without equipments for experiments and collection of other types of information (considered to be a fixed value).

W_S : Total weight of the spacecraft.

W_L : Maximum weight of launch vehicle system, including propellant plus spacecraft weight, which may be ejected into the intended orbit.

W_{SL} : Maximum weight of equipment for experiments and collection of other types of information which may be utilized in the mission of the ejected spacecraft.

Length parameters:

σ_G : Measure of precision of the guidance subsystem expressed by the standard deviation of the miss-distance of the altitude of the orbit.

a_G : Guidance characteristic relating the precision of the guidance to its maintainability.

c_A : Orbit characteristic of the spacecraft relating miss-distance of the altitude of the orbit of the spacecraft to the probability of successful accomplishment of the objective of the spacecraft.

c_G : Guidance characteristic relating the precision of the guidance to its reliability.

Each of these length parameters is relatable to subsystem performance characteristics which, in turn, are functions of detail design parameters of the subsystem. Examples of these relationships are shown in the Appendix.

Dimensionless parameters:

Of the following parameters, α, β, and γ are shaping constants. Their values are obtained by a curve-fitting procedure, utilizing empirical or theoretical data relating the applicable accountable factors described.

α :Exponent in the function relating the precision of the guidance to its maintainability.

γ :Exponent in the function relating the precision of the guidance to its reliability.

β :Exponent in the effectiveness function relating the weight in the spacecraft, available for equipment for experiments and collection of other types of information, to the capability measure of the launch vehicle system.

r_R :The quotient of the reciprocal of the average mean time to restoration of a failed subsystem occurring before launch of the system, to the average failure rate in the dormant and checkout state for each of the subsystems with redundancy in the launch vehicle system.

N_R :Number of subsystems of the launch vehicle system with redundancy applicability.

Probabilities:

q_S :Average probability of failure of each subsystem of the launch vehicle with redundancy applicability.

A_o :Availability of the subsystems of the launch vehicle system without redundancy applicability.

R_o :Reliability of the subsystems of the launch vehicle system without redundancy applicability.

Numerical Example

Values for the parameters of the effectiveness equation for a numerical example are specified in Table 3-7.

TABLE 3-7 NUMERICAL EXAMPLE VALUES

Weights		Lengths		Probabilities		Dimensionless Parameters	
Designation	Value	Designation	(Miles)	Designation	Value	Designation	Value
W_L	16000	a_G	0.100	q_S	0.025	N_R	8
W_n	13900	c_A	4	A_o	0.99	α	2
ΔW_R	62.5	c_G	0.020	R_o	0.98	β	1/3
W_{SC}	500	-	-	-	-	γ	1
W_{SL}	1000	-	-	-	-	r_R	0.0015

Solution and Results

It is shown in the detailed analysis of Appendix I that

$$E = E_o \cdot E\left(\sigma_G\right) \cdot E\left(W_R ; W_S\right) \tag{3-20}$$

where

$$E_o = A_o R_o \quad \text{with} \tag{3-21}$$

$$E\left(\sigma_G\right) = \left[1 + 2\left(\frac{\sigma_G}{c_A}\right)^2\right]^{-1/2} \exp\left\{-\left[\left(\frac{a_G}{\sigma_G}\right)^{\alpha} + \left(\frac{c_G}{\sigma_G}\right)^{\gamma}\right]\right\} \tag{3-22}$$

and

$$E\left(W_R ; W_S\right) = \left(1 + r_R\right)^{-\frac{W_R}{\Delta W_R}} \exp\left[-N_R q_S \frac{W_R}{W_R \Delta W_R}\right]\left(\frac{W_S - W_{SC}}{W_{SL}}\right)^{\beta}$$

The values of Table 3-7 are substituted into the equation for E_o to obtain

$$E_o = 0.970$$

The values of Table 3-7 are also used with the detailed procedures of Appendix I to determine optimum values for σ_G and $E\left(\sigma_G\right)$, namely σ_{GO} and $E\left(\sigma_{GO}\right)$ respectively. A graphic solution is shown for σ_{GO} in Figure 3-6, and a graphical representation for $E\left(\sigma_G\right)$ and $E\left(\sigma_{GO}\right)$ is shown in Figure 3-7. Similarly, a graphic solution for W_{RO1} and W_{SO1} is shown in Figure 3-8, and a graphical determination of W_{RO}, W_{SO} and for $E\left(W_{RO} ; W_{SO}\right)$ is indicated in Figure 3-9.

Substituting these optimum values, $E\left(\sigma_{GO}\right)$, $E\left(W_{RO} ; W_{SO}\right)$ and the value for E_o into Equation 3-20 gives

$$E_{Opt} = 0.812$$

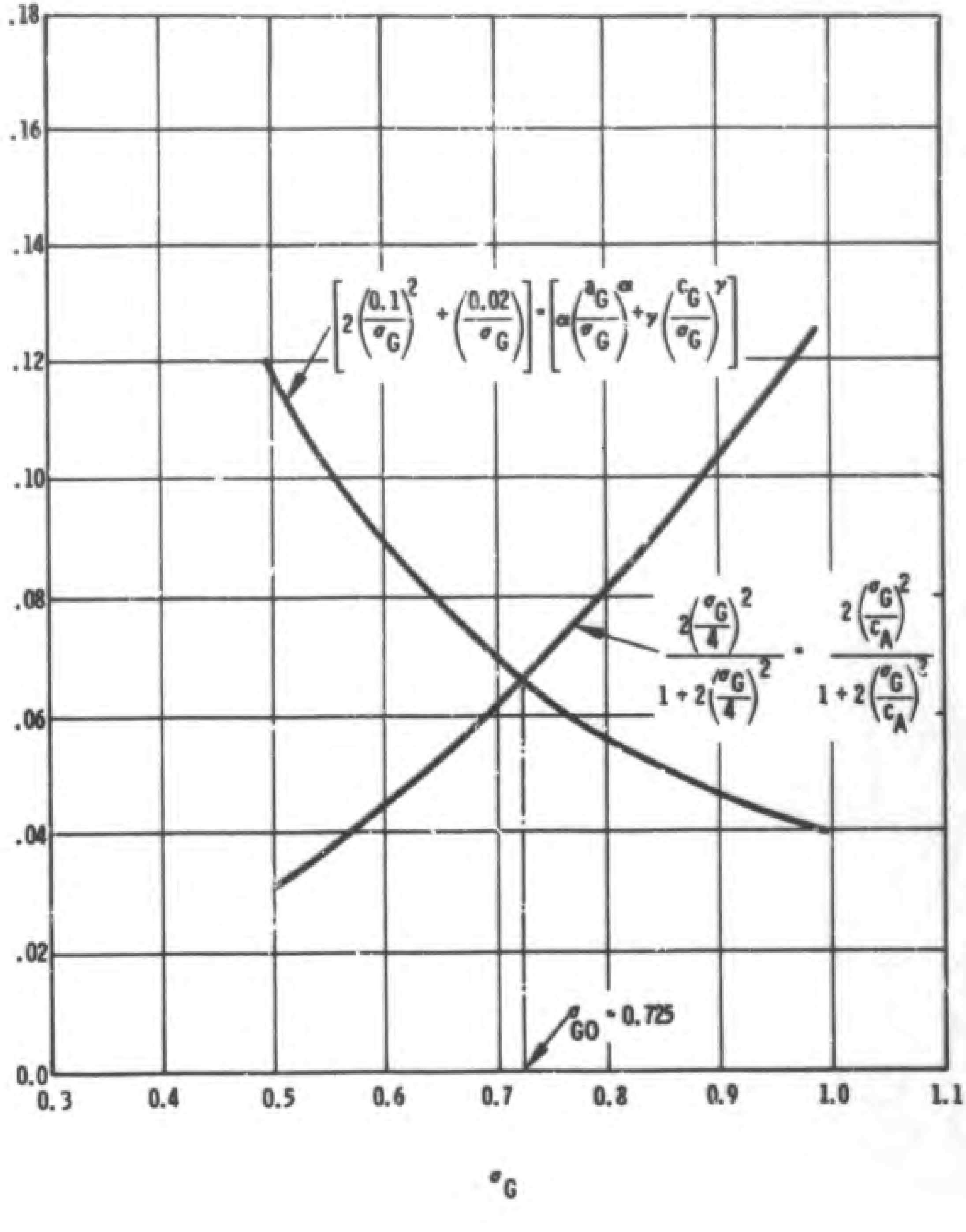

Figure 3-6 Graphical Determination of σ_{GO}

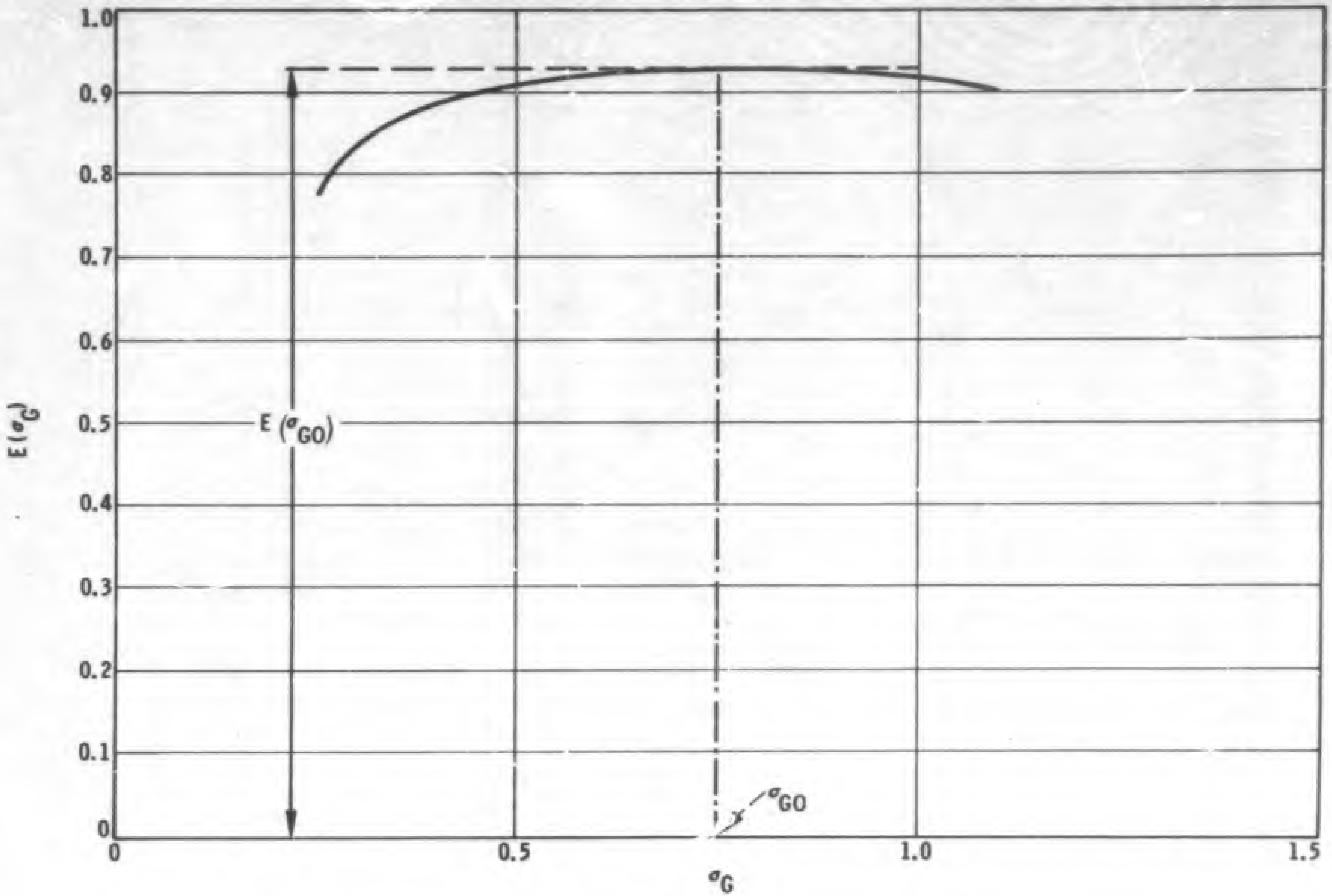

Figure 3-7 Graphical Representation of the Function E (σ_G)

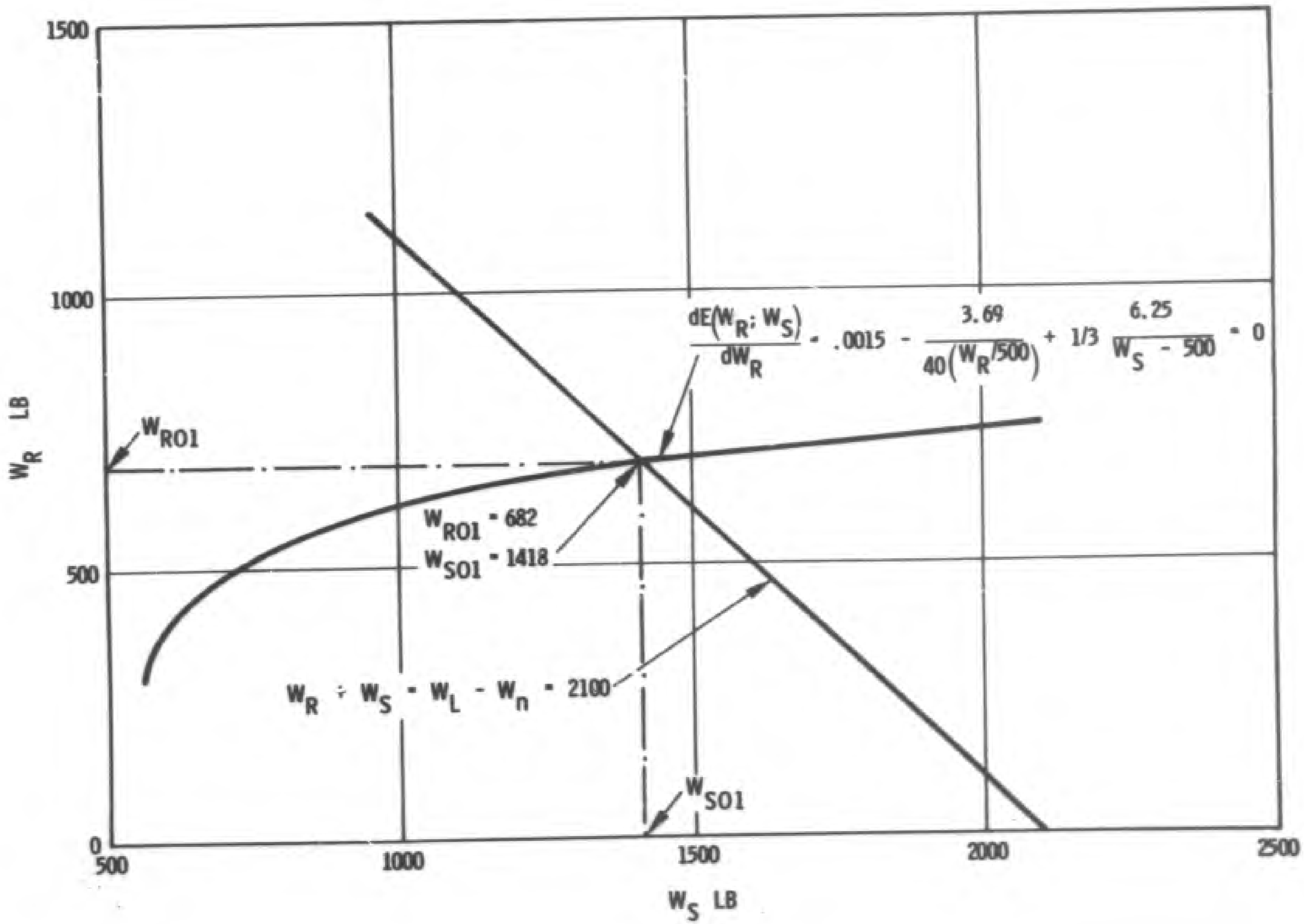

Figure 3-8 Graphical Determination of W_{RO1} and W_{SO1}

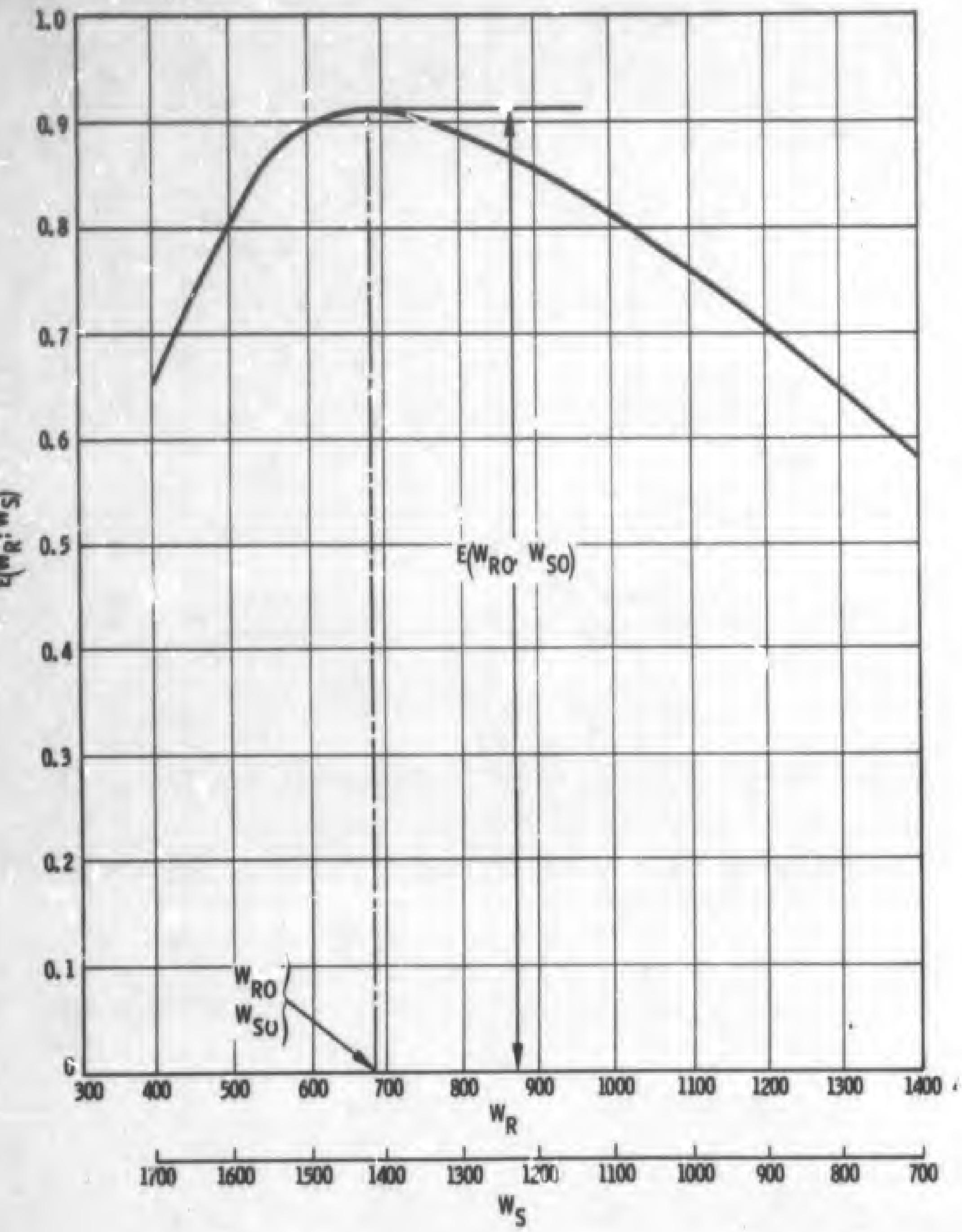

Figure 3-9 Graphical Determination of W_{RO}, W_{SO}, and $E(W_{RO}; W_{SO})$

The detailed analysis of Appendix I also shows that the A, D and C elements of Equation 3-19 are represented by:

$$A = A_o \left(1 + r_R\right)^{-W_R/\Delta W_R} \cdot \exp\left[-\left(a_G/\sigma_G\right)^{\alpha}\right] \qquad (3\text{-}23)$$

$$D = R_o \exp\left[- N_R \, q_S^{\;W_R/N_R \, \Delta W_R}\right] \cdot \exp\left[-\left(c_G/\sigma_G\right)^{\gamma}\right] \qquad (3\text{-}24)$$

$$C = \left[1 + 2\left(\frac{\sigma_G}{c_A}\right)^2\right]^{-1/2} \left(\frac{W_S - W_{SC}}{W_{SL}}\right)^{\beta} \qquad (3\text{-}25)$$

with

$$W_S \leq W_{SC} + W_{SL}$$

The derived optimum values and the other values from Table 3-7 can be substituted into Equations 3-23 through 3-25, and the value for optimum effectiveness can be verified as:

$$E_{Opt} = A_{Opt} \cdot D_{Opt} \cdot C_{Opt}$$

$$E_{Opt} = (0.965)\,(0.896)\,(0.938) = 0.812$$

Table 3-8 summarizes the system optimized values of the key accountable factors and performance parameters for this illustrative example of a launch vehicle system.

3.6 APPORTIONMENT AND TRADEOFF ANALYSIS COMPUTER PROGRAM

Appendix J presents the general theory and a description of a computer program for effectiveness apportionment and trade-off analyses of systems in which the effectiveness measure is the average number of experiments which can be performed per unit cost and may be expressed by the specific equation:

$$E = R_S WT/C$$

TABLE 3-8 VALUES OF KEY ACCOUNTABLE FACTORS AND PERFORMANCE PARAMETERS OPTIMIZING SYSTEM EFFECTIVENESS

Kind of Parameter	Function of Parameter	Designation	Value	Dimension
Optimizing Accountable Factors	Measure of precision of guidance	σ_G	0.725	miles
	Weight of subsystems with redundancy applied	W_R	682	lb.
	Total weight of spacecraft	W_S	1418	lb.
	Total number of redundant subsystems	N_R	3	-
Optimized Parameters	Availability	A_{Opt}	0.965	-
	Dependability	D_{Opt}	0.896	-
	Capability	C_{Opt}	0.938	-
	Measure of Effectiveness	E_{Opt}	0.812	-

where R_S is proportional to system reliability, W = weight available for experiments, T = time available for experiments, and C = cost. The constant of proportionality associated with R_S is determined by the units of measure of the variables W, T, and C, in order that their product will equal the actual expected number of experiments per unit cost, E. Variables other than W, T, and C may be used with equal facility in this type of analysis, provided only that they are related to the effectiveness measure in the same product form. The effectiveness model presented here will give an approximation to the effectiveness measure which is suitable for many simplified analyses. However, in cases where one of the factors such as R_S is a non-linear time-variant parameter, more accurate final results may be obtained by integration of the time-variant factors, in the manner used for the communications satellite analysis described previously in this section.

For such a model, the developed iterative Systems Effectiveness Tradeoff Analysis (SETA) Computer Program, performs the following four functions: (1) calculates the reliability, weight, cost, and expected maintenance time of a single-thread system; (2) optimizes the allocation of spares and redundancy for a single-thread system, and calculates initial and final reliability, weight, cost and expected maintenance time for each subsystem and system (reliability apportionment and reliability prediction or estimate); (3) computes the reliability, weight, cost, and expected maintenance time of a multithread system; (4) optimizes the allocation of spares and redundancy for a multithread system and calculates the initial and final reliability, weight, cost, and expected maintenance time for each multithread subsystem. Any combination of these four options can be performed in a single run.

This computer program is not based on the more efficient Lagrange multiplier with priority list apportionment method. However, it is advantageous in that the program exists. An adaptation of the theory for the apportionment of system effectiveness based on the Lagrange multiplier with priority list method also is presented in Appendix J. This method may be easily computerized.

Apportionment techniques for system effectiveness in the context of this specific model is the process by which the addition of on-board spares to a specific system design is optimized with respect to weight-carrying capability, maintenance time, cost, and reliability. Addition of a spare increases relisbility and capability, but incurs penalties in terms of additional weight or volume, maintenance time, and cost. The utility functions of the penalties are dependent upon system constraints.

The reliability improvement is a function of the reliability (failure rate) of the spare, the reliability of the item to be spared, and the method of employing the spare - which may be technically constrained. For any set of constraints and utility functions, there is an optimum number and allocation of spares to the items composing the system.

The system for which the model applies is normally weight- or volume-constrained. Typical systems include an aircraft with a maximum takeoff weight and a spacecraft which can be placed in a particular orbit with a known maximum weight. Further, such systems carry a payload; consume expendables in operating and transporting the payload; and contain equipment which is necessary to transport, operate, or monitor the payload and to perform housekeeping functions. Failure of housekeeping functions causes the system to fail. Increasing the reliability of the housekeeping function can only be accomplished by decreasing payload weight.

If the system is manned, then the man is assumed to have assigned duties, and maintenance activity decreases the time available for those duties. Cost may be a constraint in the sense that the model maximizes the pounds of payload and the hours of duty per dollar. The reliability and cost of the payload were not considered in this model.

Consider a simple example involving only the parameters of weight and reliability. With a given gross takeoff weight, the problem is to determine an optimum division of this weight between payload weight and housekeeping weight - so as to optimize the expected delivered payload weight. At a low reliability and a high payload weight, the expected delivered payload weight is low. As payload weight is decreased and housekeeping reliability is increased, the expected delivered payload weight rises to a maximum. Any further increase in reliability and decrease in payload weight now results in a reduced expected delivered payload capability weight.

The model optimizes a particular design - it does not provide discrimination as to the relative merits of alternative designs. However, the basic effectiveness equation can be used to evaluate competing designs.

The model is exercised by choosing estimates of the final payload weight available for performing experiments, a final system reliability value, the final working time available for experiments (man-working hours minus time required for maintenance), and the final cost for the program. The program computes the optimum effectiveness under the assumptions made for final payload weight, final system reliability, etc., and computes new final payload weight, final system reliability, etc. If assumed values and computed values agree with sufficient accuracy, the optimization process for the system effectiveness is finished. Otherwise, the computation must be repeated with better assumptions until sufficient accuracy is attained.

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SECTION 4

TRANSFER FUNCTIONS FOR THE APPORTIONMENT PROCESS

4.1 INTRODUCTION

The apportionment of a given system effectiveness numerical requirement to provide design criteria for equipment and subsystem designers can be accomplished objectively and validly only if system effectiveness can be related to subsystem or equipment effectiveness. This relationship is expressable in terms of system mathematical equations, which may be in a physical law, empirical relation, and/or in probabilistic form. These equations relate the specific subsystem or equipment design parameters to the system effectiveness performance parameters of availability, dependability, and capability. The subsystem design parameters are accountable factors which can be quantified.

System effectiveness apportionment requires the formulation and evaluation of these functional relationships of performance which, for the capability parameters of complex systems, may involve complex simultaneous differential equation systems. This section is addressed to a perspective of the nature of such functions, how they can be evaluated, and how they are used for the apportionment process. These functions exist and are available as the theoretical or empirical basis of designs, and for the purpose of evaluation and apportionment applications, they are called transfer functions. The apportionment techniques of Section 3 are applicable in the case of such complex transfer functions and may be applied in conjunction with direct or Monte Carlo simulation routines for the evaluation of system effectiveness.

Initially, some of the general considerations of transfer functions for system effectiveness applications are described in this section. The remainder of the section is then devoted to a perspective of the generalized performance relationships applicable to the missile, cargo aircraft,and spacecraft classes of Air Force systems, and a description of how such transfer functions may be used for apportionment.

Typical system transfer functions for subsystem accountable factors are described for each of these classes of systems, and methods indicated for their evaluation. The transfer functions described are limited to the capability parameters of effectiveness, primarily because these parameters are extremely complex, and not as well defined as those for the parameters of availability and dependability. Each system will have a unique set of capability transfer functions. Although a set of functions may be common to some systems, such as for missile and spacecraft systems, the necessary generality of such a set would make it inappropriate for direct use on a specific design evaluation. As a useful guide for capability evaluations and apportionment, however, the technical data and methodologies presented in this section on this subject are considered to fulfill this basic purpose.

4.2 GENERAL CONSIDERATIONS

Transfer functions, as used in circuitry and servo mechanism engineering, may be portrayed in their simplest form as follows:

Output = Transfer Function x Input

In linear relationships, if the transfer function is a matrix T, with elements T_{ij}, and the input and output are column vectors, with components e_i and r_i, respectively, then

$$r_i = \sum_j T_{ij} e_j$$

and for

$$C = T^{-1}$$

then

$$e_i = \sum_j C_{ij} r_j$$

where C is the inverse of the transfer function and is called the characteristic function.

Transfer functions of a similar form may be defined for the spectrum of system capability parameters. An example of the use of transfer functions of this form is in the computation of the trajectory of a guided missile from guidance time zero to impact. This type of transfer function is useful when transforming the basic equations from one defined coordinate frame into another coordinate frame for the computation of missile impact. Such coordinate frames include the missile body coordinate frame or M-frame, firing coordinate frame or F-frame, reference coordinate frame or R-frame, and inertial coordinate frame or I-frame. These frames are described in detail in paragraph 4.3.4.

The trajectory equations (such as force equations, moment equations, position rate equations, orientation rate equations, etc.) are often more easily derived in a specific frame than in the other frames. However, the equations must be expressed in the same coordinate system for the final evaluation of capability. Transfer functions used for such transformations may be formulated by matrix notations. For example, the forces acting on a missile may be expressed in the missile body coordinate frame, or M-frame, as a vector $\vec{F}_M$ with components F_{Mi}, i from 1 to 3.

For the inertial velocity vector $\vec{V}_F$ of the missile in the firing coordinate frame, or F-frame, the components V_{Fi}, i from 1 to 3 may be obtained from the solution of the following matrix differential equations based upon Newton's relationship

$$\frac{d}{dt}(MV) = F:$$

$$\left[T_{F\text{-to-}M}\right]\begin{bmatrix}\frac{d\left(mV_{F1}\right)}{dt}\\ \frac{d\left(mV_{F2}\right)}{dt}\\ \frac{d\left(mV_{F3}\right)}{dt}\end{bmatrix} = \left[T_{F\text{-to-}M}\right]\begin{bmatrix}F_{F1}\\ F_{F2}\\ F_{F3}\end{bmatrix} = \begin{bmatrix}F_{M1}\\ F_{M2}\\ F_{M3}\end{bmatrix} \quad (4\text{-}2)$$

where m is the mass of the missile at time t and $T_{F\text{-to-}M}$ is the transformation matrix from the F-frame to the M-frame, a transfer function. The elements of the matrix $T_{F\text{-to-}M}$ (transfer function) are:

$$T_{F\text{-to-}M} = \begin{bmatrix} \cos\nu_M \cos\psi_M; & \cos\nu_M \sin\psi_M \cos\phi_M + \sin\nu_M \sin\phi_M; & \cos\nu_M \sin\psi_M \sin\phi_M - \sin\nu_M \cos\phi_M \\ -\sin\psi_M; & \cos\psi_M \sin\phi_M; & \cos\psi_M \sin\phi_M \\ \sin\nu_M \cos\psi_M; & \sin\nu_M \sin\psi_M \cos\phi_M - \cos\nu_M \sin\phi_M; & \sin\nu_M \sin\psi_M \sin\phi_M + \cos\nu_M \cos\phi_M \end{bmatrix} \quad (4\text{-}3)$$

with ν_M, ψ_M, ϕ_M being the angles which relate the two frames to each other.

Since the forces acting on the missile are more likely to be known in the missile frame, a more convenient form of (4-2) may be

$$\begin{bmatrix} \frac{d(mV_{F1})}{dt} \\ \frac{d(mV_{F2})}{dt} \\ \frac{d(mV_{F3})}{dt} \end{bmatrix} = \begin{bmatrix} T_{M\text{-to-}F} \end{bmatrix} \begin{bmatrix} F_{M1} \\ F_{M2} \\ F_{M3} \end{bmatrix} \quad (4\text{-}4)$$

where $\left[T_{M\text{-to-}F}\right]$ is the inverse of $\left[T_{F\text{-to-}M}\right]$.

Although Eq. (4-2) is a differential equation expressed in matrix formulation, the form of this equation corresponds to that of Eq. (4-1). In circuit design, the input $\vec{e}$ and output $\vec{r}$ also may have components expressed by differential quotients and thus, Eqs. (4-1) and (4-2) completely correspond to each other in this case. The transfer function of Eq. (4-3) does not depend on the details of a specific missile system design and may be used in all guided missile systems and spacecraft launch vehicle systems. This generalization is true for all such matrices which transform an equation system from one coordinate frame to another.

The definition of a transfer function may be further generalized to include each function dealing with availability, dependability, and capability, and/or effectiveness in general. However, transfer functions which are extremely complex will usually occur in the formulation of the equation systems for the evaluation of the capability parameter.

The task of evaluating the capability parameters is usually connected to the task of computing the distribution of the components of the output vector when the distribution of the components of the input vector, and of the elements of the transfer matrix, are known. The methods for determining the distribution of the output vector are those previously described in Section 2 and detailed in Appendix B, namely:

(1) direct method, including simulation

(2) Monte Carlo simulation

Usually, the task of evaluating the elements of the capability matrix may not be accomplished by one equation of the form of Eq. (4-1), but by a system of such equations. Often, the final solution may be preceded by several steps in which the output vector of step n serves an input vector for step (n+1). In the case of relationships expressed by differential equations or integral equations, the Laplace transform may be applied to convert the equation system into algebraic linear relationships of the form of Eq. (4-1).

Apportionment of accountable factors for optimization of effectiveness suppose the existence of options of several design possibilities with varying capability within the constraints to be observed. The variation of the design may be discrete or continuous. The computation of the elements of the capability vector must be repeated for the total set of discrete design options, or for a selected set of continuous design options.

Obviously, the computation requirements may become laborious when computing the total set of options of the design. This is particularly true in the Monte Carlo simulation procedure where the computation of each element of the set of design options requires a large number of repeated computer runs and an evaluation of the results of these computations.

Clearly, well prepared computer programs will be required in order to execute the apportionment for an optimum effectiveness in many systems. Such optimization problems will require standardization of available transfer functions, subroutines, and even total computer programs in many systems. On the other hand, there will be systems where the optimization of their effectiveness may be accomplished without use of a mechanized method as demonstrated in the examples of Appendices H and I.

4.3 MISSILE SYSTEM TRANSFER FUNCTIONS

As with all complex systems, a missile system design represents the culmination of many diverse studies and trade-off analyses, establishing subsystem design requirements and nominal performance specifications. Analysis of total missile system performance requires a complex mathematical representation with both analog and digital simulation of the primary subsystem elements, and their contributions to the overall system performance. Some of the major missile system elements are:

Targeting Subsystem
Fire Control Subsystem — Aerospace Ground Equipment (AGE)
Launch Mechanisms

Propulsion
Flight Control
Guidance
Separation Mechanisms
Re-entry Vehicle

] — Flight System

Gross performance capability of a missile system in terms of maximum range may be determined from propulsion subsystem characteristics and known or assumed missile system weight statements. Gross estimates of damage expectancy may be prepared on the basis of yield, target vulnerability, and delivery accuracy. This capability of a missile system to inflict an expected amount of damage can be grossly determined in a preliminary sense for a given payload/yield and assumed CEP delivery accuracy. Optimization of a system capability requires that analyses be performed of the potential tradeoffs available, such as the substitution of a heavier, more accurate guidance subsystem, or a lighter, lower yield re-entry subsystem.

4.3.1 Subsystem Performance Relationship

Missile flight performance is simulated by mathematical representation of each contributing subsystem, including the facsimile of feedback and other forms of interdependence among subsystem functional elements. The input/output relationship of various subsystem elements is modified with respect to total missile system performance by the manner in which the particular subsystem operates on the vehicle dynamics. A simplified, missile system block diagram is shown in Figure 4-1. This figure indicates, for example, that the propulsion subsystem acts directly upon the vehicle, with this action modified by the guidance subsystem. In the event of less than nominal propulsion performance, such as minus one sigma performance, the guidance subsystem would counteract this effect by allowing a longer burning time, and this ultimately will provide the desired velocity and range. However, the converse of this situation does not hold. An input error for the range or velocity computations by the guidance subsystem will be propagated through the entire missile flight-to-impact sequence and will result in reduced capability.

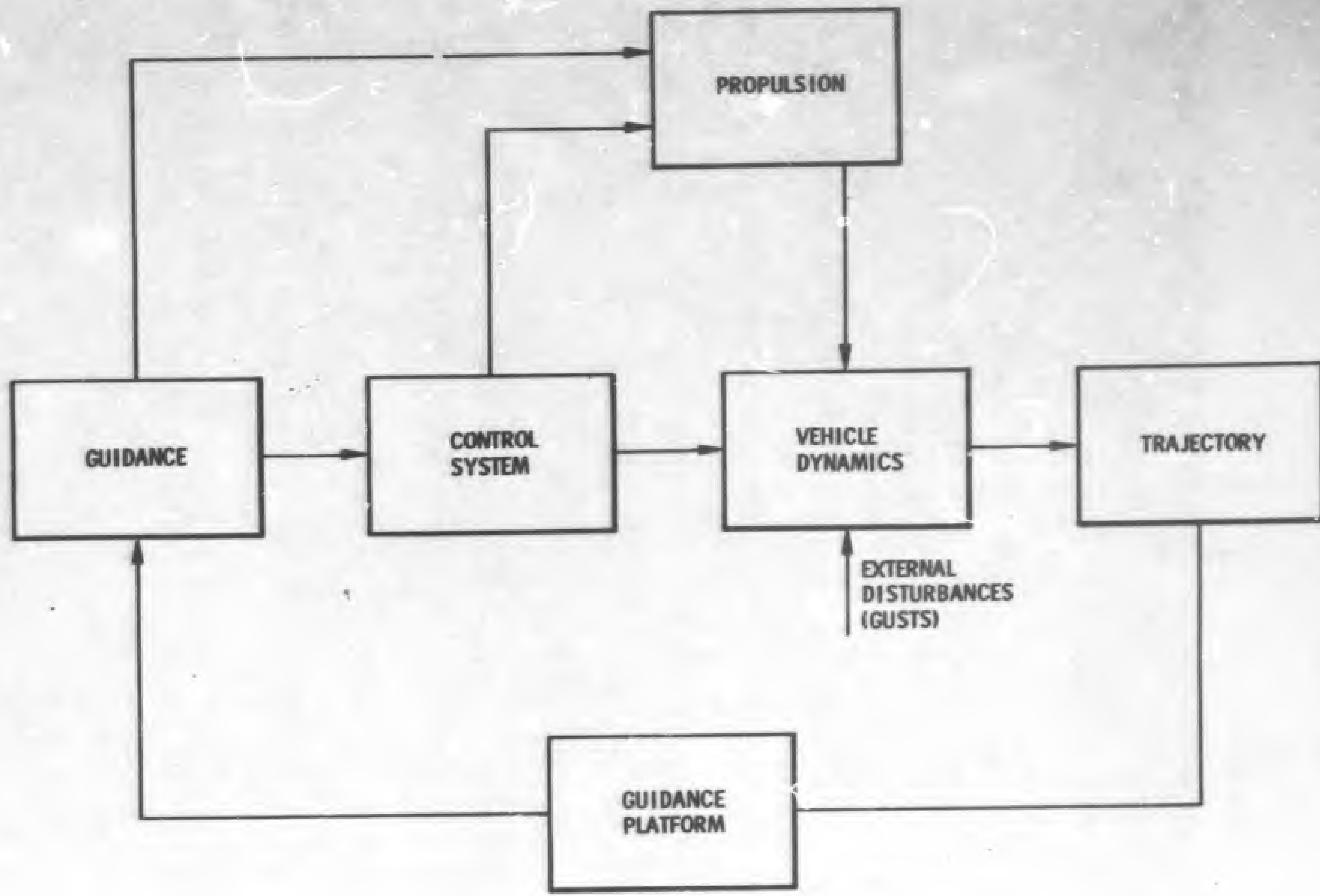


Figure 4-1 Simplified Missile Flight Block Diagram

Functional transfer equations relating subsystem performance to total missile performance can be derived for any subsystem interactive influence on the system. These equations form the basis for existing missile flight simulation programs and will allow for the investigation of the effects of one subsystem, or a combination of subsystems, on the total missile system performance. The proper allocation of subsystem design parameters may be achieved for optimum capability with complex simulation programs, by solving for desired missile system performance output distributions in an open loop, iterative fashion. Thus, the equations required for missile flight simulation, and the simulation itself, become the basic tool and form the methodology for establishing subsystem requirements. The simulation program contains the multiple transfer functions. Subsystem performance variations will be inputs to the computer simulation, in the manner of standard error analysis techniques, and the computer will generate total system performance as the related output.

During the conceptual design phase, the subsystem elements of a missile system must be postulated in a manner to meet the proposed missile system requirements. An air-to-ground missile, for example, will not require the types and levels of sophistication of guidance equipments required for a long range ballistic missile. Previous experience with similar missile systems will indicate what types of subsystems may be required, and the performance levels associated with these subsystems. Subsystem to total system performance is examined during the conceptual phase in the performance of trade-off and optimization studies. The tools and the methodology for performing these studies are similar to the methods used for that of a defined missile system. However, at this stage, the process is an iterative one in that a change in one subsystem might require changes to all, or some, of the remaining subsystems.

A transfer function is derived in the conceptual phase of design in the form of a simulation model based on postulated system elements and empirical performance data. The simulation model is tested to determine the performance of the postulated system. If the performance output of the simulation model does not meet the desired performance criteria, the postulated system is modified and a new simulation is tested. As each modification is made, the system performance response to these modifications is documented. When the relationship between modification and response is identified, the desired configuration and subsystems performance levels can be interpolated or extrapolated, and performance trade-off analyses made, both at the system and subsystem levels.

A constraint must be placed on this iteration process, based upon current and projected state of the art. Systems involving extraordinary improvements in the state of the art cannot be considered solutions to near-term design objectives and must be discarded. In this case the iteration process must be continued and the performance/modification sensitivities reexamined to find alternate approaches to obtain the desired performance objectives. Occasionally, system design objectives must be compromised or operational capability schedules lengthened, if no solutions can be found without violating state-of-the art constraints.

4.3.2 Capability Performance Criteria and Evaluation

In most systems, even the more complex, there exists a key capability performance parameter such as accuracy which, in turn, is a function of many other system parameters. For ballistic missile applications, the primary effectiveness figure of merit is usually associated with damage expectancy such as kill probability or the expectant amount of damage. This figure of merit depends on missile capability parameters such as range, payload yield, and delivery accuracy. In addition to missile capability parameters, defense characteristics or posture affect kill probability. For undefended targets, hardness is a key consideration as well as location and total numbers. In addition to these criteria, a defended target analysis must include the probability of successful penetration of the defense system. If penetration aid systems are used, the effectiveness of such systems against the defense must be estimated and this estimated effectiveness will modify the probability of successful penetration.

Accuracy is only one of a number of capability performance criteria associated with a ballistic missile system. Others include reliability, availability, range, vulnerability (in storage and in flight), re-entry angle and velocity, time of flight, arrival time, apogee, height of burst, and warhead yield. These parameters, plus the target parameters, are among considerations in determining the overall effectiveness of a missile system. Associated with each performance parameter is a set of transfer functions unique to that parameter.

In order to demonstrate the methodology for the analysis of missile performance parameters, the typical capability parameter of system accuracy will be used. The technique involved is similar to the method used in performing conventional error analysis studies. A simulation program is generated for the missile system under consideration. If the missile system is an existing item, or if design studies have progressed through the optimization stages to a "final" configuration, nominal and standard deviation data are available as design specification parameters for all subsystems. If the missile is in a conceptual stage, nominal subsystem design parameters are established in the manner discussed previously. Standard deviations are apportioned, based upon evaluations of the current state-of-the art.

The generated simulation program is used to determine the change in overall system performance, for changes in a given subsystem performance. For small (e.g., $\pm 1\sigma$) changes in a given subsystem, it is reasonable to assume that the nominal values of other subsystems will not change. Using dynamic flight simulations, the total system performance spectrum is obtained for varying subsystem performance. This relationship of subsystem performance to total system output is called the sensitivity characteristic of the subsystem. Sensitivity characteristics may be described by sensitivity functions, curves, or coefficients. The basic transfer functions involving the laws of motion are implemented in a dynamic flight simulation and resolved into sensitivity functions.

Missile system accuracy is defined in terms of the Circle of Equal Probability (CEP), the radius of a circle, centered at the target, within which 50 percent of all re-entry vehicles aimed at that target would impact. If the interest is in the weapon system CEP, then the missile system CEP must be added to those associated with launch errors (navigation, target location, fire control, etc.) which are mainly independent of the missile system itself.

Figure 4-2 shows the relationship of the error contributions, transfer functions, sensitivity characteristic functions, and an output system capability parameter. For the accuracy parameter, a sensitivity curve will show downrange and crossrange (track) miss-distances as a function of subsystem error magnitude. The curves are a function of range and trajectory, and are usually defined for a nominal range. The sensitivity curves then are graphical representations of an error analysis, and the techniques for deriving them are now described.

4.3.3 Trajectory Simulation By a Six-Degree-Of-Freedom Program

When forces and moments act on a body, the mass accelerates, and the integration of this acceleration by the inertia of the system results in a new position, orientation, and motion rates for the missile. A block diagram of this integration process is shown in Figure 4-3.

This motion can be represented mathematically by Newton's second law, for linear motion, and by the Euler equations, for motion of a rotating rigid body. Due to the instantaneous nature of the forces, moments, weights, accelerations, and environment, such as wind shears, air density profile, and gravity force changes, the output solution requires iteration processes and successive integration, and usually cannot be solved in a closed form. This leads to the use of high speed digital computers, which can perform many iterations in a fraction of a second and, using numerical integration methods, can achieve the same accuracy as a closed-form solution. A computer program which solves the equations of motion for a missile is called a trajectory simulation. Since there are six-degrees of dynamic freedom in space, 3 translational and 3 rotational relationships, a computer program which considers these dynamic effects is known as a 6-D program.

A block diagram of the trajectory simulation process is shown in Figure 4-4. Note that the block for the computer program simply replaces the block for the dynamic response of the vehicle in Figure 4-3 with a mathematical model.

For some design studies, certain dynamic responses may be set at zero to simplify the program, reduce computer time, and permit the generation of numerous tradeoff trajectories. Roll rate, for example, is one of the rotational responses quite often ignored, since it usually has small effect on overall system performance. These simplified simulations are known as 2, 3, 4, and 5-D programs.

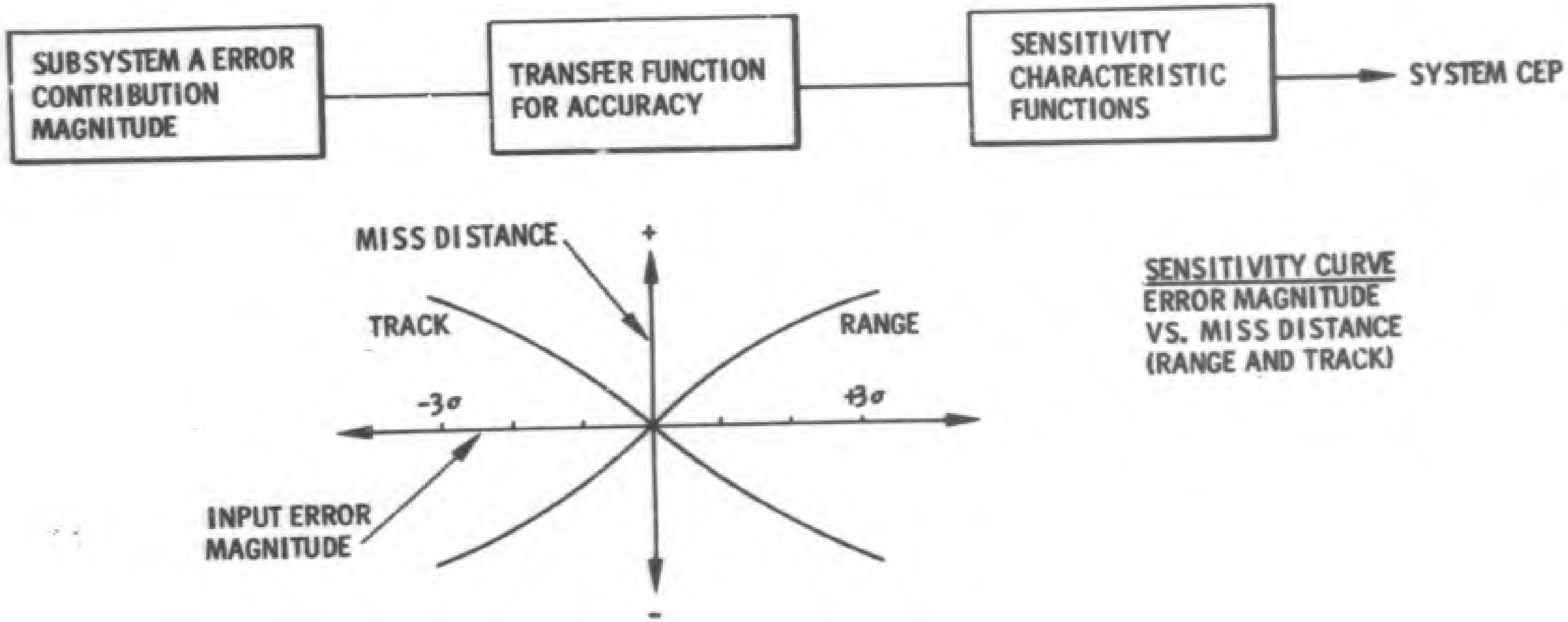


Figure 4-2 Sensitivity Curve for Accuracy

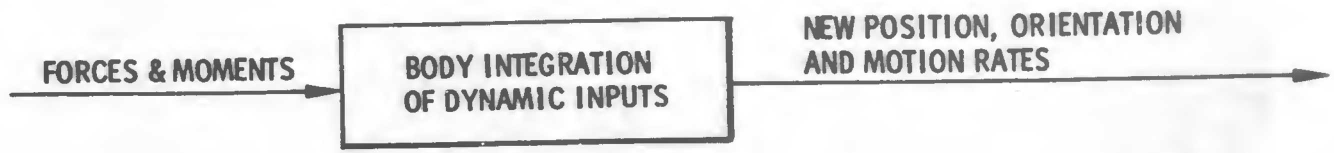


Figure 4-3 Trajectory Simulation Process

Figure 4-4 Computer Trajectory Simulation

4.3.4 Coordinate Frames

During the construction of a mathematical model for a trajectory simulation, the decision must be made as to the coordinate frames to be used. It is usually convenient to construct a number of reference frames, some fixed in inertial space, others which move with the missile. These frames are convenient:

- to preserve the required computation precision; specific mathematical operations can be performed more favorably in one frame over another.
- as certain inputs can be more easily expressed and evaluated in a particular frame, such as aerodynamic and thrust forces in the Missile (M) frame.
- because important subsystems (guidance, flight controls, etc.) have frames associated with their physical orientation.

The three most commonly used frames are the Inertial (I), Missile (M), and Reference (R) frames. The following is a brief description of these frames:

- I-Frame

  The Inertial (I) frame as shown in Figure 4-5 is a fixed inertial frame centered at the earth geocenter (geometric center of sphere or ellipsoid), with X_I axis parallel to local vertical (minus in the direction of gravity) and Z_I axis given an azimuth at launch, usually the azimuth of the target. The integration of the linear accelerations is accomplished in the I-frame.

- M-Frame

  The Missile body (M) frame as shown in Figure 4-6 is a right-handed coordinate system and is centered at the missile center of gravity, with X_M axis (roll) along the longitudinal axis (+ forward), Y_M axis (pitch) positive to the right when looking down on the missile, and Z_M axis (yaw) position down. This frame is used for integration of external forces and moments.

- **R-Frame**

  The Reference (R) frame as shown in Figure 4-7 is a right-handed coordinate frame. It is an inertial frame centered at the earth geocenter, with X_R axis in the zero longitudinal meridian plane, at time zero, and Z_R axis positive through the North Pole. This frame is used when integrating velocity to obtain inertial rectangular position coordinates.

In additional to these frames, there can be several others depending on guidance configuration and degree of simulation desired. Some examples are:

- Guidance subsystem frame, used to measure attitude errors, and hence generate guidance commands
- Local horizontal frame, used for relating wind and atmospheric profile, usually expressed in terms of local vertical
- Keplerian frame, used for point-mass computations (no forces present)
- Fire control frame, used during pre-launch phase for calculating initial guidance parameters.

4.3.5 Transformation of Frames

When values of dynamic factors such as forces, accelerations, and environmental characteristics must be expressed in a reference frame other than that in which they were originally expressed, it is necessary to utilize a transformation matrix as previously described. As an example, when a vector $\vec{V}$ expressed in Frame A must be expressed as a vector $\vec{W}$ in another frame B, use is made of an orthogonal 3 x 3 transformation matrix, $T_{A\text{-to-}B}$, to give:

$$\vec{W} = \left[T_{A\text{-to-}B}\right] \vec{V}$$

A typical example is when a rotation is made from the X_A, Y_A, Z_A axes to the X_B, Y_B, Z_B axes through three angular rotations ϕ_1, ϕ_2, ϕ_3. The rotations, taken sequentially, can be represented by

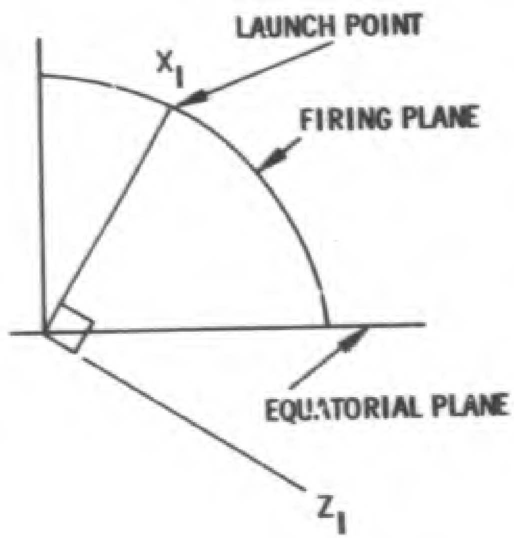


Figure 4-5 Inertial Frame (I)

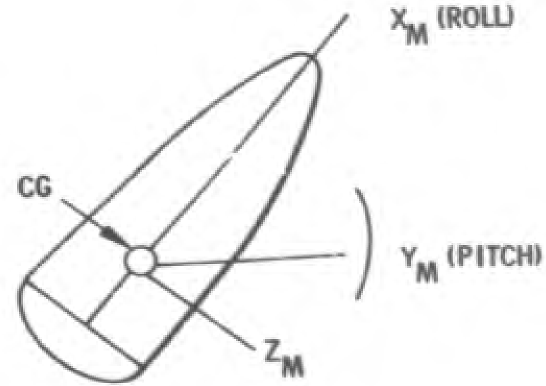


Figure 4-6 Missile Frame (M)

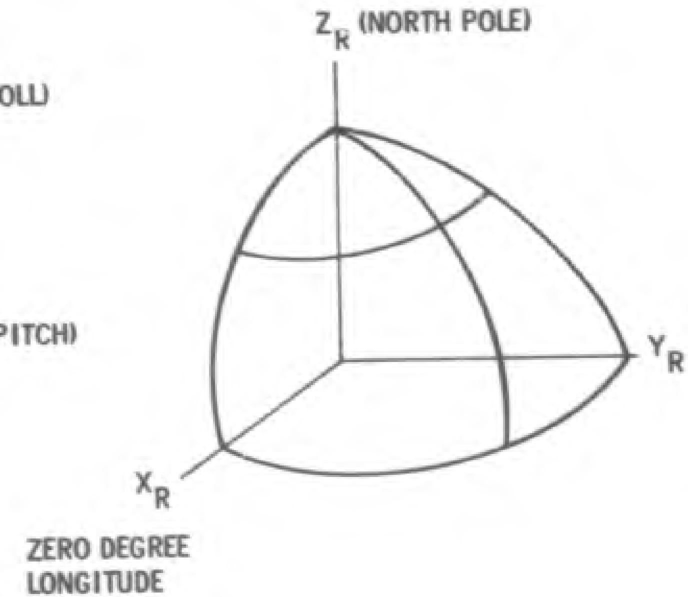


Figure 4-7 Reference Frame (R)

$$\text{Rotation } \phi_1 = \left[R_1\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}\,\phi_1 & \text{Sin}\,\phi_1 \\ 0 & -\text{Sin}\,\phi_1 & \text{Cos}\,\phi_1 \end{bmatrix}$$

$$\text{Rotation } \phi_2 = \left[R_2\right] = \begin{bmatrix} \text{Cos}\,\phi_2 & 0 & -\text{Sin}\,\phi_2 \\ 0 & 1 & 0 \\ \text{Sin}\,\phi_2 & 0 & \text{Cos}\,\phi_2 \end{bmatrix}$$

$$\text{Rotation } \phi_3 = \left[R_3\right] = \begin{bmatrix} \text{Cos}\,\phi_3 & \text{Sin}\,\phi_3 & 0 \\ -\text{Sin}\,\phi_3 & \text{Cos}\,\phi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then $\left[T_{A\text{-to-}B}\right] = \left[R_3\right] \left[R_2\right] \left[R_1\right]$

4.3.6 Force Equation

The translational motion of a body in space can be expressed in vector notation as:

$$\frac{d}{dt}\left(m \vec{\dot{r}}\right) = \vec{F} \tag{4-5}$$

where:

$\rightarrow$ = a vector

$\cdot$ = 1st derivative with respect to time

$\cdot\cdot$ = 2nd derivative with respect to time

F = total forces acting on the body

m = mass of the system

$\dot{r}$ = velocity of the body, expressed in terms of time rate of change of the radius vector $\vec{r}$, where $\vec{r} = \sqrt{x^2+y^2+z^2}$

After differentiating (4-5), the general expression becomes the more familiar:

$$\vec{F} = m\ddot{\vec{r}} + \dot{m}\dot{\vec{r}} \qquad (4-6)$$

where the term, $\dot{m}\ \dot{\vec{r}}$, should be included in the expression for total forces.

The summation of forces acting on the missile center of gravity (cg) can be represented by:

$$\vec{F} = \vec{F}_{aero} + \vec{F}_{thrust} + \vec{F}_{jet\ damping} + \vec{F}_{other}$$

where

$\vec{F}_{aero}$ = aerodynamic forces

$\vec{F}_{thrust}$ = thrust forces, including $\left(-\dot{m}\ \dot{\vec{r}}\right)$

$\vec{F}_{jet}$ = jet damping forces due to inertia of fuel

$\vec{F}_{other}$ = other forces, including gravitational attraction, winds, non-rigid body inertia effects, control forces, etc.

Assuming an X, Y, Z coordinate frame, the force $\vec{F}$ can be replaced by a column vector:

$$\vec{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad (4-7)$$

which represents the components of $\vec{F}$ along the orthogonal axes. These components can be converted to equivalent inertial acceleration by combining Equations (4-6) and (4-7) to give:

$$\frac{\vec{F}}{m} = \ddot{\vec{r}} = \begin{bmatrix} \frac{1}{m} F_x \\ \frac{1}{m} F_y \\ \frac{1}{m} F_z \end{bmatrix} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \qquad (4\text{-}8)$$

In this system, $r = \sqrt{x^2 + y^2 + z^2}$. If the translational velocity in an X, Y, Z frame is u, v, and w, Equation (4-8) becomes:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}$$

4.3.6.1 Example of Aerodynamic Forces

The aerodynamic force $\vec{F}_{aero}$, which has been shown as one of the forces acting on a missile's center of gravity represents the drag and lift forces on the missile body, conveniently expressed in the missile (M) axes. The components of $\vec{F}_{aero}$ can be expressed as:

$$\vec{F}_{aero} = \begin{bmatrix} \vec{F}_{x_{aero}} \\ \vec{F}_{y_{aero}} \\ \vec{F}_{z_{aero}} \end{bmatrix}$$

$$\vec{F}_{x_{aero}} = \bar{q}\, S\, C_A$$

$$\vec{F}_{y_{aero}} = \pm\left[F_N \text{ Sin } \phi\right]$$

$$\vec{F}_{z_{aero}} = \pm\left[F_N \text{ Cos } \phi\right]$$

where:

$\bar{q}$ = dynamic pressure = $1/2\ \rho\ v^2$

ρ = air density

v = velocity of body with respect to air mass

S = reference surface area

ϕ = angle that relates $F_{z_{aero}}$ to $F_{y_{aero}}$

C_A = drag coefficient

$F_N = \bar{q}\ S\ C_N$

C_N = lift coefficient

The drag and lift coefficients are complex functions involving a number of aerodynamic phenomena. If several of the less prominent terms are ignored, the following simplified formula is obtained for the drag coefficient:

$$C_A = (C_A)_{\delta = o} + (C_A)_{\delta_{pp}} \left|\delta_{pp}\right| + (C_A)_{\delta_{qq}} \left|\delta_{qq}\right| + (C_A)_{\delta_{rr}} \left|\delta_{rr}\right|$$

where:

$(C_A)_{\delta=0}$ = axial force for zero pitch, roll, yaw angles

$(C_A)_{\delta_{pp}}$ = axial force per degree roll

$(C_A)_{\delta_{qq}}$ = axial force per degree pitch

$(C_A)_{\delta_{rr}}$ = axial force per degree yaw

δ_{pp} = degree roll

δ_{qq} = degree pitch

δ_{rr} = degree yaw.

The normal force coefficient is a complex expression which, after neglecting terms with minor significance, reduces to:

$$C_N = (C_N)_{\delta=o} + (C_N)_{\delta_{qq}} \left| \delta_{qq} \right| + (C_N)_{\delta_{rr}} \left| \delta_{rr} \right|$$

where

$(C_N)_{\delta=o}$ = normal force for zero angle of attack

$(C_N)_{\delta_{qq}}$ = normal force per degree pitch

$(C_N)_{\delta_{rr}}$ = normal force per degree yaw

4.3.7 Moment Equation

The summation of moments about the missile cg can be expressed as:

$$\vec{L} = \vec{L}_{aero} + \vec{L}_{thrust} + \vec{L}_{jet\ damping} + \vec{L}_{other}$$

Some examples of other moment sources are the gyroscopic contribution of rotating machinery, the moment of momentum of burning propellant, and the moment due to flight control forces.

Again, if $\vec{L}$ is expressed in terms of vector components, the standard Euler equations for the angular acceleration of a rotating rigid body can be used to give:

$$\vec{L} = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix}$$

and

$$\dot{p}\, I_{xx} = L_x - p\, \dot{I}_{xx} - qr(I_{zz} - I_{yy})$$
$$\dot{q}\, I_{yy} = L_y - q\, \dot{I}_{yy} - pr(I_{xx} - I_{zz})$$
$$\dot{r}\, I_{zz} = L_z - r\, \dot{I}_{zz} - pq(I_{yy} - I_{xx})$$

where

I_{xx} = moment of inertia about x axis

I_{yy} = moment of inertia about y axis

I_{zz} = moment of inertia about z axis

p, q, and r = roll, pitch, and yaw angular velocities, respectively.

4.3.7.1 Example of Aerodynamic Moments

The aerodynamic moments, which have been shown as one of the moment sources about the missile's center of gravity, can be represented by the following:

$$\vec{L}_{aero} = \begin{bmatrix} L_{x_{aero}} \\ L_{y_{aero}} \\ L_{z_{aero}} \end{bmatrix}$$

$$L_{x_{aero}} = C_{1_{cg}} \bar{q} S d$$

$$L_{y_{aero}} = C_{m_{cg}} \bar{q} S d$$

$$L_{z_{aero}} = C_{n_{cg}} \bar{q} s d$$

$C_{1_{cg}}$ = rolling moment coefficient

$C_{m_{cg}}$ = pitching moment coefficient

$C_{n_{cg}}$ = yawing moment coefficient.

d = reference diameter

The moment coefficients lead to a lengthy mathematical formulation and will not be described herein. However, if higher order terms are neglected, the equation for C_m can be simplified to the following:

$$C_m = (C_m)_{\delta=o} + (C_m)_{\delta_{qq}} \left|\delta_{qq}\right| + (C_m)_{\alpha\,\delta_{qq}} \left|\delta_{qq}\right| + (C_m)_{\dot{\alpha}} \left(\alpha \frac{d}{2\nu}\right) + (C_m)_q \left(\bar{q} \frac{d}{2\nu}\right)$$

where

$(C_m)_{\delta=o}$ = pitching moment at zero angle of attack

$(C_m)_{\delta_{qq}}$ = pitching moment per degree due to pitch control

$(C_m)_{\alpha\delta_{qq}}$ = pitching moment gradient per $(\text{degree})^2$ due to pitch control

$(C_m)_{\dot{\alpha}}$ = damping derivative, pitch moment per degree/second

$(C_m)_q$ = damping derivative, pitch per degree/second

$\dot{\alpha}$ = rate of change of pitch angle of attack

α = pitch angle of attack, degree

ν = freestream velocity, ft/second

d = reference diameter, ft.

$\bar{q}$ = dynamic pressure

4.3.8 Utilization of Force and Moment Equations

The flow diagram of a typical trajectory simulation is shown in Figure 4-8 and is divided into 5 blocks. The following is a brief description of these blocks without the details of all the coordinate transformations necessary for actual computations.

- **Block (1)**

  The external forces acting on the missile body are added together and transformed to components in the R-Frame. Using Newton's basic law of motion, these components are converted to equivalent linear accelerations $(\ddot{x}, \ddot{y}, \ddot{z})$. These accelerations are integrated to get inertial velocity, which is then added to air mass velocity to obtain free stream velocity. This is used for the calculation of aerodynamic forces, which then become part of the new force data input (dotted line).

- **Block (2)**

  Inertial velocity is integrated once to get body position.

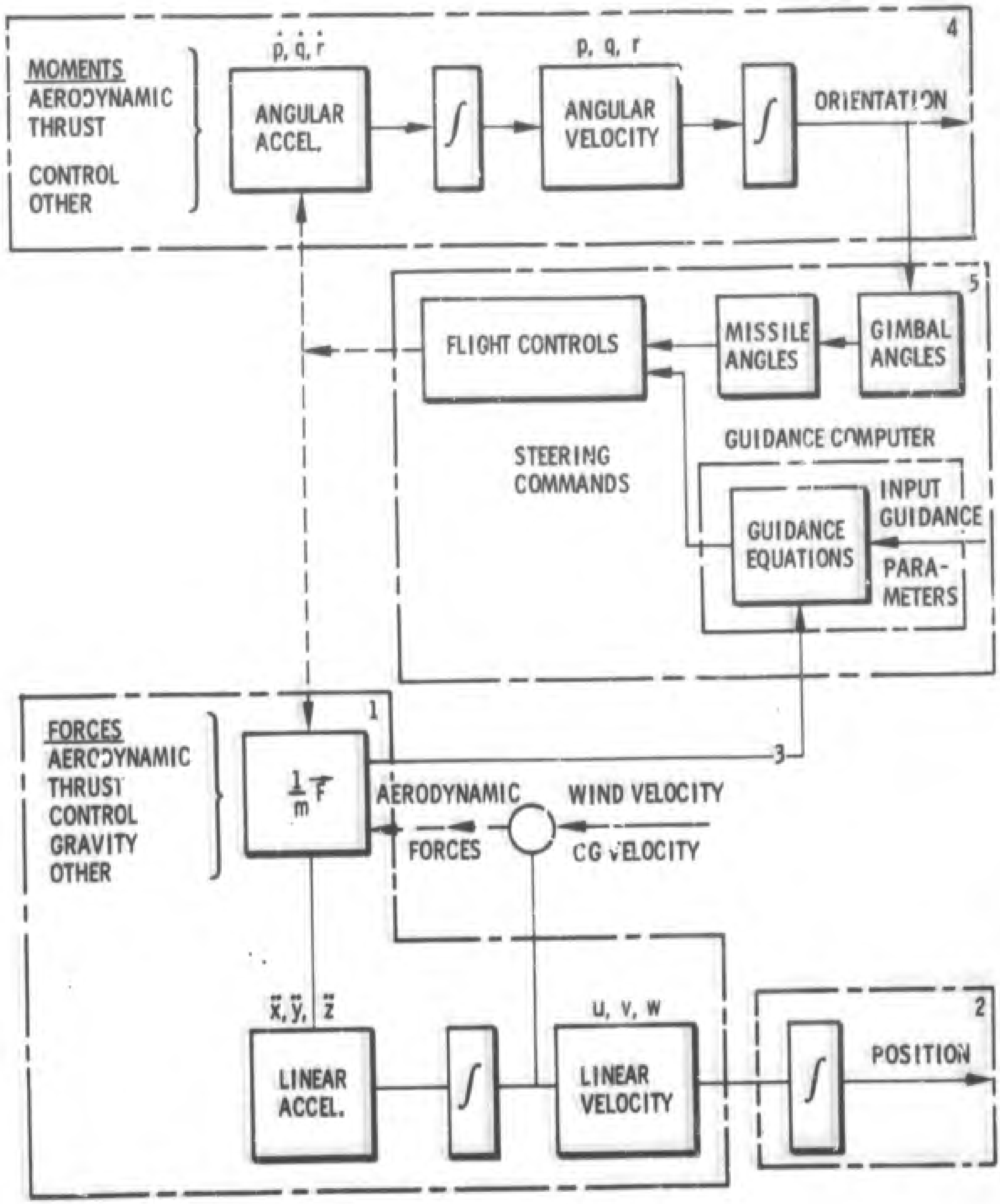


Figure 4-8 Flow Diagram for Digital Computer Trajectory Simulation

- <u>Block (3)</u>

  Missile body acceleration is added to relative guidance acceleration to obtain inertial acceleration of the guidance subsystem. This is used in the guidance and steering equations to determine necessary attitude, and thrust cut-off commands.

- <u>Block (4)</u>

  External moments are represented by the standard Euler equations of motion for a rotating rigid body. Angular acceleration ($\dot{p}$, $\dot{q}$, $\dot{v}$) is solved and then integrated twice to obtain angular velocity and orientation. The missile orientation is read into the guidance subsystem for use in the flight control mechanism.

- <u>Block (5)</u>

  The missile orientation is related to the guidance subsystem reference frame (for a gimballed guidance system, the angles between missile and gimbal axes are measured), and this relationship is used to generate attitude feedback signals. These signals are compared with guidance commands to produce the attitude error signals of the autopilot.

4.3.9 Sensitivity Function

As previously indicated, the transfer functions for a missile system are a system of equations related to the basic laws of motion, and are evaluated by an ordered series of dynamic trajectory simulations when analyzing capability parameters such as accuracy and range. One of the most useful outputs of these simulations is sensitivity functions.

Impact miss-distances can be related to subsystem errors by means of sensitivity functions or curves. A variation in a given source error will produce a variation in range and track miss-distances of the re-entry vehicle.

For a system under development where subsystem parameters are known, the nominal values of these parameters are used to determine the nominal impact point. The subsystem values are then perturbed by one, two, and three-sigma variations (error magnitudes) and the change in impact point is converted to range and track miss-distances. Sensitivity functions are generated which relate this subsystem effect on system performance. For example, the sensitivity functions may be expressed by:

$$\Delta R = S_R(E)$$

and

$$\Delta T = S_T(E)$$

where:

ΔR = down-range impact miss-distance from the aim point

ΔT = track (cross-range) miss-distance from the aim point

E = magnitude of a subsystem error

$S(E)$ = miss-distance of the missile system as a function of E.

The plot of sensitivity functions versus error magnitudes will be nonlinear in most cases, as shown in Figure 4-9, and can be approximated by a second or third order polynomial.

This type of sensitivity curve is the most common relationship for the various missile subsystems. For example, errors in the erection of the guidance platform would exhibit symmetrical error contributions in the same sense as the deviations. That is, if a plus 1 σ deviation produces either a positive or a negative error in range or track, then a minus 1 σ deviation will produce an opposite error.

For a spin-stabilized re-entry vehicle with a center-of-gravity offset, the sensitivity curve would show a different form of symmetry. Regardless of the sign of the deviation, any amount of center-of-gravity offset will cause angle of attack oscillations (coning) of the re-entry vehicle, and an attendant rise in drag. The increased drag will cause the vehicle to impact short of the intended target, and the sensitivity curve would exhibit the form shown in Figure 4-10.

SENSITIVITY CURVES

MISS DISTANCE ΔR, ΔT AS A FUNCTION OF SUBSYSTEM ERROR E

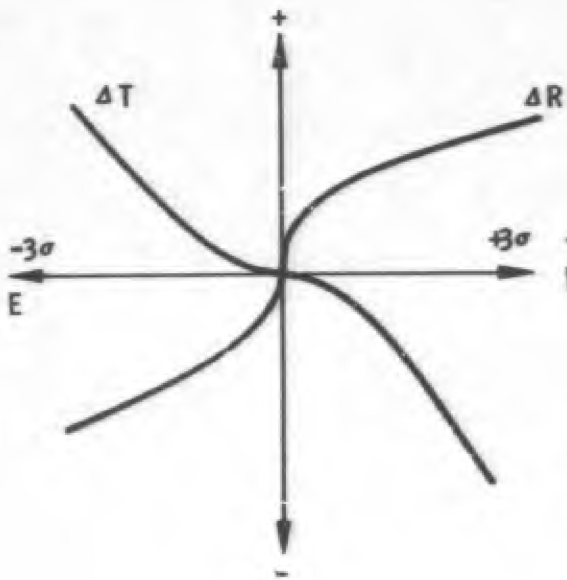


Figure 4-9
Guidance Platform
Erection Error

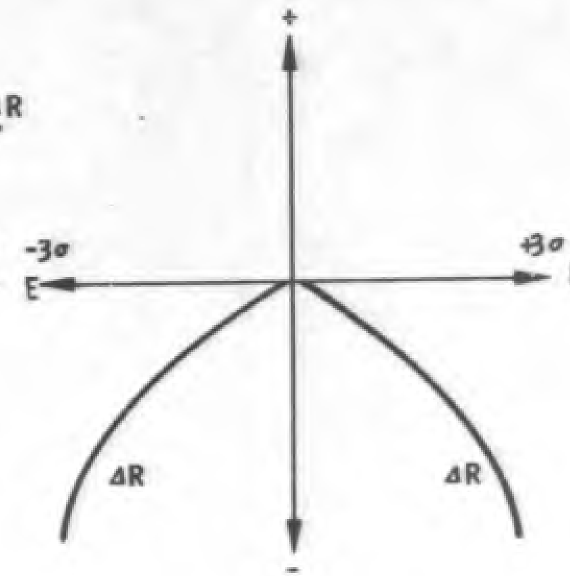


Figure 4-10
Center of Gravity
Offset Error

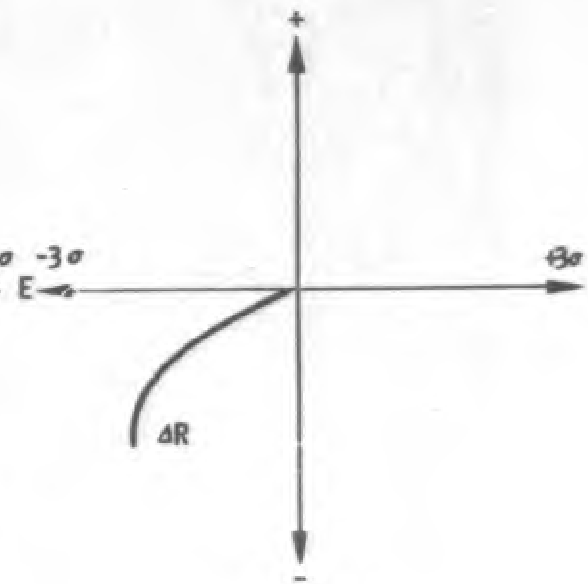


Figure 4-11
Propulsion Error

The sensitivity curve for a propulsion subsystem might exhibit the form shown in Figure 4-11. This curve reflects the condition of a motor with less than nominal loading (-1σ cold motor), producing a range decrement as a function of the range and available total impulse, with deviations on the positive side being cancelled out by nominal operation of the guidance cut-off signal.

Other sensitivity curves can be generated for each subsystem parameter in a similar fashion. Table 4-1 is a listing of candidate system parameters and subsystem design accountable factors for which sensitivity functions may be developed to guide designers for optimum capability performance. To illustrate the detail design accounable factors which are normally implicit to, or branching from, the level of subsystem accountable factors, a breakdown of the Total Impulse, Specific Impulse, and Thrust subaccountable factors is provided in Appendix K.

4.3.10 Accuracy Determination

If a normal probability distribution of the subsystem error is assumed, the mean μ and standard deviation σ of the system miss-distances will be:

$$\mu = \int_{-\infty}^{\infty} S(E)\ f(E)\ dE$$

where:

S(E) is the miss-distance of the missile system as a function of subsystem error E and f(E) is the probability distribution of E

$$\sigma = \sqrt{\left(\int_{-\infty}^{\infty} S^2(E)\ f(E)\ dE\right) - \mu^2}$$

The mean and standard deviation values for range and track miss-distances are used to determine overall system accuracy.

TABLE 4-1 LIST OF CANDIDATE SENSITIVITY FUNCTIONS

System Capability Parameters	Subsystem Accountable Factors	Units
Time of Flight	Pitch command	(degrees)
	Cutoff velocity	(f.p.s.)
	Aerodynamic coefficients	---
	Re-entry vehicle ballistic coefficients	---
Range	Payload	(pounds)
	Impulse	(pound-seconds)
	Pitch program	(degrees)
Re-entry Orientation	Altitude at cutoff	(miles)
	Velocity at cut-off	(f.p.s.)
	Flight path angle at cutoff	(degrees)
	Aerodynamic coefficients	---
	Total payload	(pounds)
Accuracy	Inert weight of each stage	(pounds)
	Computer clock	(seconds)
	Time of cut-off	(μ seconds)
	Accelerometer misalignment	(arc seconds)
	Thrust termination misalignment	(degrees)
	Total impulse	(pounds-seconds)
	Payload weight	(pounds)
	Moment of inertia	(slugs/ft^2)
	Thrust offsets, each stage	(inches)
	Center-of-gravity offset	(inches)
	Thrust termination time	(seconds)
	Ballistic coefficient	(pounds/ft^2)
	Atmospheric profile	(slugs/ft^3)
	Vibration effects	(g rms)
	Gas dynamics errors	(f.p.s.)
	Wind perturbations	(f.p.s.)
	Re-entry vehicle timing errors	(seconds)
	Accelerometer bias errors	(cm/sec^2)
	Guidance platform misalignment	(milli radians)
	Gyro drift errors	*(meru/g^2)
	Specific Impulse	(seconds)
	Thrust	(pounds)

*Mean earth rate unit.

The direct method of summing independent and small error contributions is useful for approximating overall system errors, and has the advantage of lending itself to hand analysis. Using this method, sensitivity functions can be derived which relate the impact error due to a small subsystem error magnitude (e.g., ±1σ). This procedure is repeated for all the subsystems to provide a mean μ and a standard deviation σ of the system's miss-distances in range and track. The system down range miss-distances μ and σ are then, approximately:

$$\mu_{\Delta R} = \mu_{\Delta R_1} + \mu_{\Delta R_2} + \cdot \cdot \cdot \cdot \cdot , + \mu_{\Delta R_n}$$

$$\sigma_{\Delta R} = \sqrt{\sigma^2_{\Delta R_1} + \sigma^2_{\Delta R_2} + \cdot \cdot \cdot \cdot \cdot \cdot + \sigma^2_{\Delta R_n}}$$

where:

ΔR_n = range error due to subsystem n

ΔR = system range error

$\mu_{\Delta R}$ = mean value of system range error

Similar mathematical relations exist for track (cross range) miss-distance.

The following example illustrates the direct method for determining the accuracy parameter:

Contributing Subsystem Accountable Factor	One Standard Deviation (1σ)	Miss-Distance (Yds.)			
		Mean Value μ		Std. Deviation σ	
		Range ΔR	Track ΔT	Range ΔR	Track ΔT
Ballistic Coefficient	3%	10	30	--	20
Inert Weight	10 lbs.	30	0	13.3	20
Impulse	20 lbs./sec.	40	0	12	20
Guidance Alignment	1 milli radian	0	30	16	20
		80	60		

Therefore,

$$\mu_{\Delta R} = 80 \text{ yds.}$$

$$\mu_{\Delta T} = 60 \text{ yds.}$$

$$\sigma_{\Delta R} = 24 \text{ yds.}$$

$$\sigma_{\Delta T} = 40 \text{ yds.}$$

With the values $\mu_{\Delta R}$, $\mu_{\Delta T}$, $\sigma_{\Delta R}$, and $\sigma_{\Delta T}$ so determined, the system accuracy can be calculated.

A reasonable assumption is that the impact pattern will exhibit a bivariate normal distribution. To obtain the system accuracy as measured by its CEP, a table of the integral of the bivariate normal elliptical distribution over offset circles is convenient. Since the CEP is the radius of a circle centered at the aim point within which 50 percent of all impact points will lie, and assuming an impact pattern defined by $\mu_{\Delta R}$, $\mu_{\Delta T}$, $\sigma_{\Delta R}$, and $\sigma_{\Delta T}$, then,

$$\text{Probability} = .50 = \int\int_S \frac{1}{2\pi\, \sigma_{\Delta R}\, \sigma_{\Delta T}} \exp\left[-\frac{1}{2}\left\{\left(\frac{x - \mu_{\Delta R}}{\sigma_{\Delta R}}\right)^2 + \left(\frac{y - \mu_{\Delta T}}{\sigma_{\Delta T}}\right)^2\right\}\right] dx\,dy$$

where S = coincident area.

The solution is to find the radius r, which generates an area S that will include 50 percent of the total volume of the bivariate distribution. A graphical view of this relationship is given in Figure 4-12. For the example presented, where $\mu_{\Delta R} = 80$, $\mu_{\Delta T} = 60$, $\sigma_{\Delta R} = 24$, $\sigma_{\Delta T} = 40$, and for a probability of 0.50, r can be determined to be 105 yards. For the case where $\mu_{\Delta R}$ and $\mu_{\Delta T} = 0$, the CEP is simplified to:

$$\text{CEP} = 1.1774\, \overline{\sigma} \text{ where } \overline{\sigma} = \frac{1}{2}\left(\sigma_{\Delta R} + \sigma_{\Delta T}\right)$$

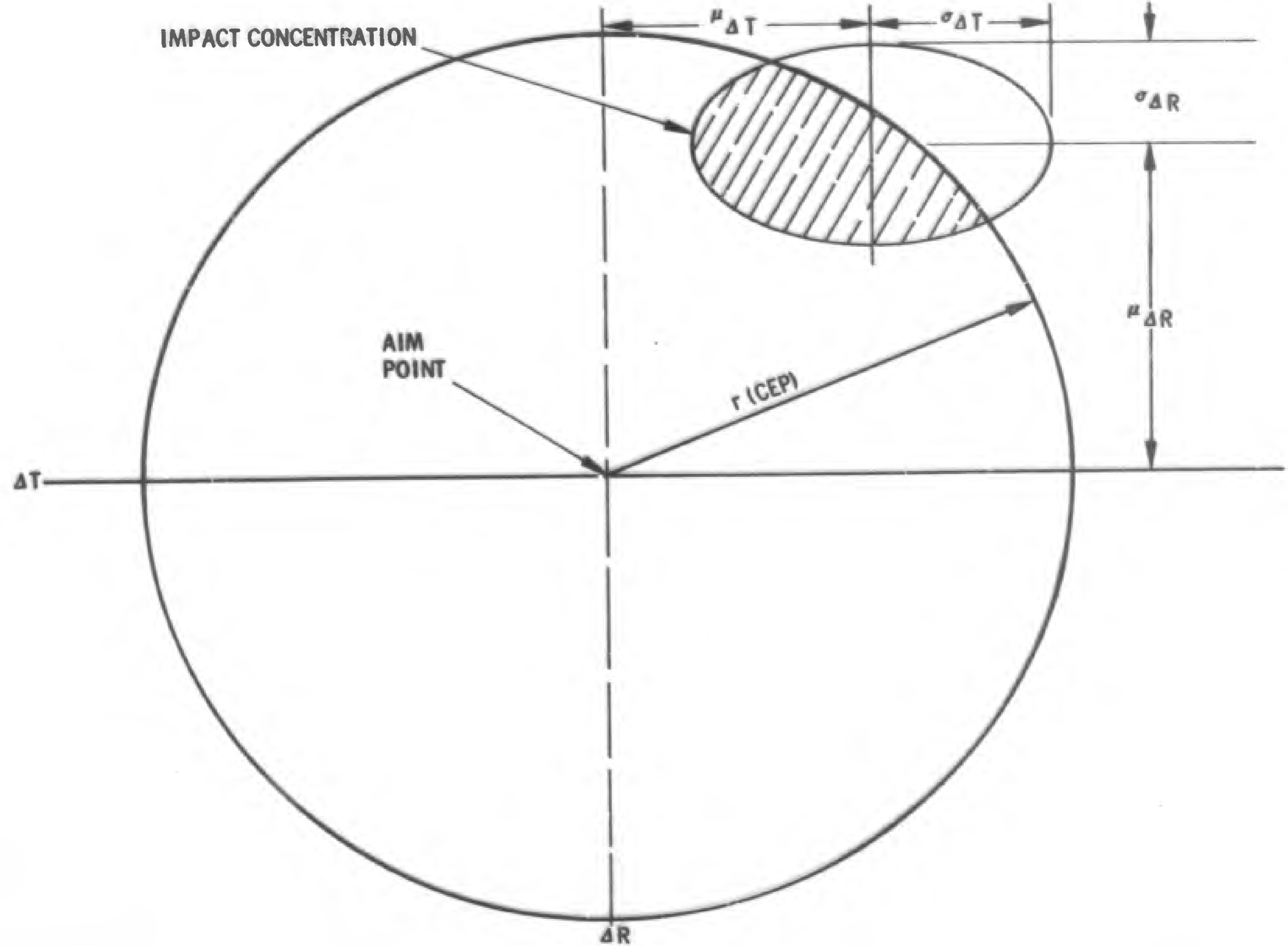


Figure 4-12 Aim Point vs Impact Concentration

4.4 CARGO AIRCRAFT TRANSFER FUNCTIONS

Depending on the basic objective of an aircraft system, different performance parameters become of paramount importance to the mission and, therefore, a measure of effectiveness. For an interceptor aircraft, items such as maneuverability, maximum rate of climb, speed, and fire power may be the most critical. However, for a large airlift cargo transport system, such as the C5A, the principal figure of merit or measure of effectiveness is related to the capability of transporting large, variable payloads quickly over long ranges. This capability could be described in terms of the average number of pounds of payload carried per aircraft per day or, alternately, could be expressed for a fleet of aircraft as the number of aircraft sorties required to complete a mission. For purposes of transfer function considerations, the fleet size can be assumed to be fixed, and effectiveness measured in units of payload pounds per aircraft per day.

4.4.1 Accountable Factors

The Loading factor will influence the size and type of cargo. A mathematical model is required which will use the definitions of the size and type of cargo, along with cargo compartment dimensions to provide optimized mixed loads. This optimized mixed load, when combined with payload limitations on various routing legs, allows for a determination of the number of flight sorties required to transport efficiently a complete military unit. The Routing factor will influence the average turn-around time for an aircraft sortie based on aircraft performance, base locations, enroute winds, and payload frequency distribution. The Utilization factor will influence an aircraft utilization value in terms of the average number of flight hours per aircraft per day, based on considerations of aircraft reliability, maintenance down-time, and logistic support policy. The Performance factor will relate to the effect of variations in aircraft general performance on the other accountable factors and on system effectiveness.

The outputs of these accountable factors can then be related to the capability parameters by sensitivity functions, as required, in order to have a common denominator for evaluating the effect of design changes on the performance parameters of effectiveness.

4.4.2 Sensitivity Functions

In order to assess the sensitivity of the measure of effectiveness to small parameter changes, sensitivity functions also are useful for a cargo aircraft system. This will allow for the translation of design parameter changes, measured in units the designers are accustomed to working with, into relative changes in effectiveness. These sensitivity functions approximate linear relationships and may be expressed as coefficients as long as the parameter changes are small. Typical performance parameters which are appropriate for the application of sensitivity coefficients and their influences on system effectiveness are:

	Parameter Change (Increase)	Units	Sensitivity Coefficient
(a)	Aircraft utilization	one minute per day	+ i pounds/aircraft/day
(b)	Empty weight	one pound	- j pounds/aircraft/day
(c)	Take-off thrust	one percent	+ k pounds/aircraft/day
(d)	Cruise thrust	one percent	+ 1 pounds/aircraft/day
(e)	Take-off drag	one count	- m pounds/aircraft/day
(f)	Cruise drag	one count	- n pounds/aircraft/day
(g)	Maximum lift coefficient	.01 change	+ o pounds/aircraft/day
(h)	Landing distance	one foot	- p pounds/aircraft/day

Sensitivity coefficients could be expressed directly in pounds or as a percent change from the effectiveness payload baseline. A change in utilization, item (a) above, will result in a direct corresponding change in effectiveness. The primary effect of changes in items (b) through (h) will be a corresponding change to the capability parameters of range, payload, or both.

A reasonably direct relationship is that of empty weight. One extra pound of empty weight will reduce the allowable payload by one pound for a specific range. Changes in cruise thrust or drag result in a change in cruise speed and/or fuel consumption, which results in a change in range or in the time to fly a specific route leg. Even more indirectly, changes in take-off thrust or drag/lift coefficient have a corresponding change in take-off characteristics such as distance, speed, and allowable weight, which in turn affects the range or time to fly a specific range. Changes in landing distance alter the basic lift/drag ratio which will have an effect on cruise drag. These relationships are complex in themselves since take-off involves several different aircraft configurations during ground operation and climb-out. Furthermore, the landing distance is a function of up to nine different aircraft configurations during let-down and ground operation.

Sensitivity coefficients must be evaluated periodically and updated, and are valid if parameter changes are small. However, for a large change such as a complete wing redesign, the sensitivity coefficients must be recomputed.

4.4.3 Capability Transfer Functions

Typically, the capability accountable factors, each concerned with different aspects of the total system, will relate to loading, routing, utilization, and performance. Typical examples of transfer functions in the Utilization and Performance areas are:

4.4.3.1 Utilization

Aircraft utilization depends on availability which, in turn, is a function of preventive maintenance and corrective maintenance. A capability transfer function for utilization (U) may be defined as:

$$U = kA$$

where

k = a constant reflecting the portion of aircraft availability that is directly attributable to aircraft flying time per day

$$A = 1440 \frac{MTBM}{MTBM + \overline{M}}$$

= aircraft availability

and for A,

1440 = minutes per day

MTBM = mean time between maintenance

$$= \frac{1}{\frac{1}{MTBM_c} + \frac{1}{MTBM_p}}$$

where the subscript c denotes corrective and p denotes preventive.

$$\overline{M} = \frac{\overline{M}_{ct}\,(fc) + \overline{M}_{pt}\,(fp)}{(fc + fp)}$$

= mean elapsed maintenance time (excluding servicing)

with

$\overline{M}_{ct}$ = mean elapsed corrective maintenance time

$\overline{M}_{pt}$ = mean elapsed preventive maintenance time

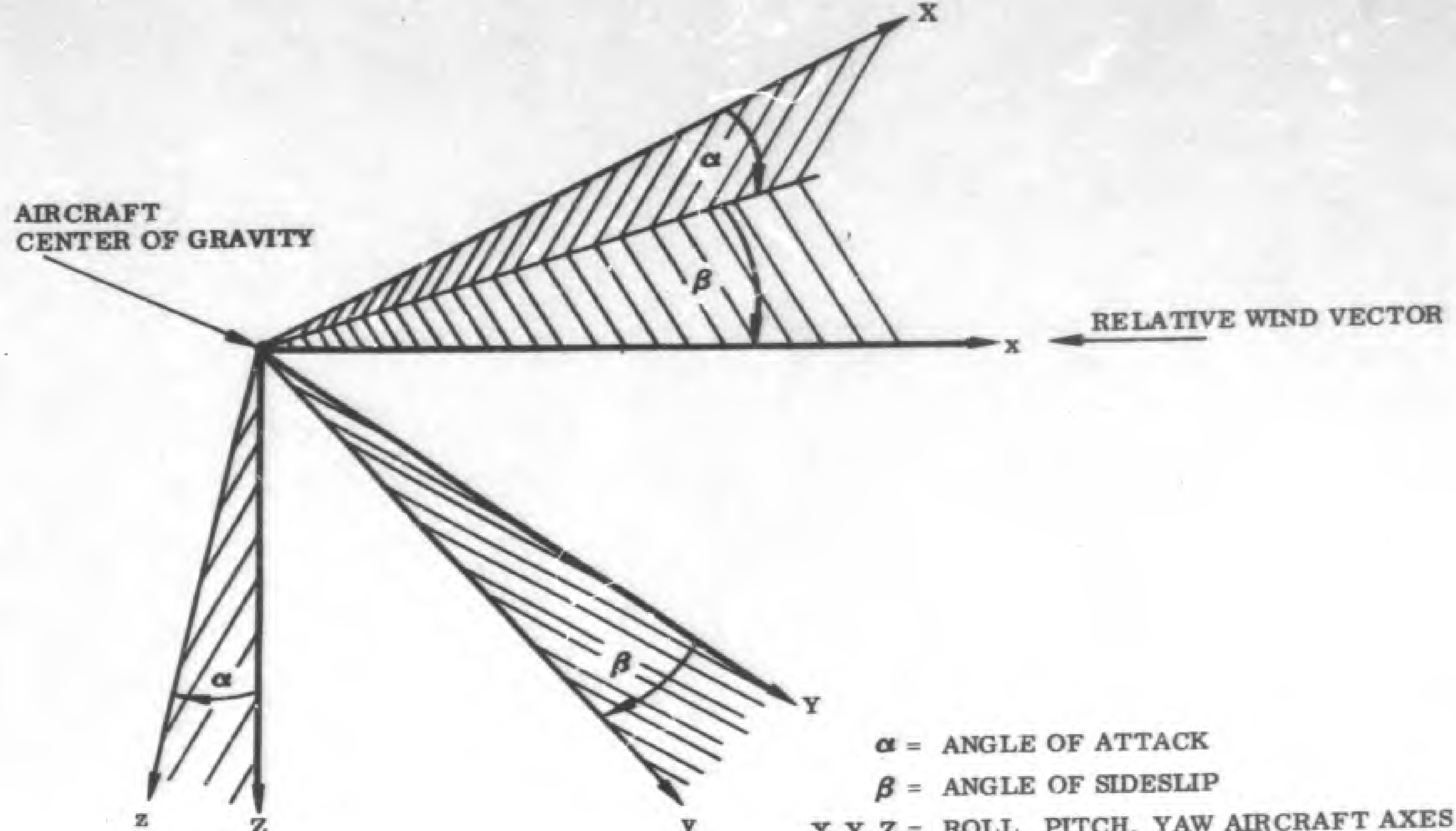
fc = number of corrective maintenance tasks

fp = number of preventive maintenance tasks

4.4.3.2 Performance

In order to evaluate the effect of variations in aircraft performance parameters on effectiveness, the primary transfer functions to be developed are in the form of a set of simultaneous equations having six degrees of freedom. These equations of motion and the reference axes are very similar to those used for a ballistic missile system.

The reference axes used in a missile system are the missile, inertial and reference frames. In an aircraft system, the equivalent of the missile frame would be the aircraft axis. Aerodynamic forces such as lift, drag, etc., are developed and summed in a new reference frame called the wind axis. The wind axis, originating at the aircraft center of gravity, is derived by rotating the aircraft axis first through an angle α (angle of attack) about the pitch (y) axis, and then through an angle β (sideslip angle) about the yaw (z) axis as shown in Figure 4-13.



α = ANGLE OF ATTACK
β = ANGLE OF SIDESLIP
X, Y, Z = ROLL, PITCH, YAW AIRCRAFT AXES
x, y, z = ROLL, PITCH, YAW WIND AXES

Figure 4-13 Wind Axis Orthogonal Coordinate System

A typical set of motion equations used for aircraft simulation is:

$$\dot{V} = \frac{g}{W}\left[\vec{F}_x\right]$$

$$\dot{\beta} = \frac{g}{WV}\left[\vec{F}_y\right] + p \operatorname{Sin}\alpha - r \operatorname{Cos}\alpha$$

$$\dot{\alpha} = \frac{g}{WV \operatorname{Cos}\beta}\left[\vec{F}_z\right] - \frac{1}{\operatorname{Cos}\beta}\left[p \operatorname{Cos}\alpha \operatorname{Sin}\beta - q \operatorname{Cos}\beta + r \operatorname{Sin}\alpha \operatorname{Sin}\beta\right]$$

$$\dot{p} = \frac{1}{I_{xx}}\left[\vec{M}_x + \left(I_{yy} - I_{zz}\right) qr + J_{xy}(\dot{q} - pr) + J_{xz}(\dot{r} + pq) + J_{yz}(q^2 - r^2)\right]$$

$$\dot{q} = \frac{1}{I_{yy}}\left[\vec{M}_y + \left(I_{zz} - I_{xx}\right) pr + J_{yz}(\dot{r} - pq) + J_{xy}(\dot{p} + qr) + J_{xz}(r^2 - p^2)\right]$$

$$\dot{r} = \frac{1}{I_{zz}}\left[\vec{M}_z + \left(I_{xx} - I_{yy}\right) pq + J_{xz}(\dot{p} - qr) + J_{yz}(\dot{q} + pr) + J_{xy}(p^2 - q^2)\right]$$

where:

V = total velocity

β = sideslip angle

α = angle of attack

$\rightarrow$ = a vector

p, q, r = aircraft roll, pitch and yaw rates, respectively

$\vec{M}_{X,Y,Z}$ = applied moments about aircraft X, Y, and Z axes.

$\cdot$ = first derivative

g = gravity

W = total weight

$\vec{F}_{x,y,z}$ = force components along x, y, z wind axes

$I_{xx,yy,zz}$ = aircraft moments of inertia

$J_{xy,xz,yz}$ = aircraft products of inertia

Each of the above equations has many accountable factors. For example, in the linear acceleration equation ($\dot{V}$), the primary variable $\vec{F}_x$ is a composite of four force components, namely aerodynamic, thrust, weight, and ground reaction forces. Furthermore, each accountable factor is a function of subtier accountable factors, such as the ground reaction force component being a function of gear configuration, brake characteristics, and aircraft Euler angle. Thus, for $\dot{V}$, the simplest of the equation set,

$$\vec{F}_x = \vec{F}_a + \vec{T}_x + \vec{W}_x + \vec{\tau}_x$$

where:

$\vec{F}_a = 1/2\, \rho V^2 S\, (C_y \text{ Sin}\,\beta - C_d \text{ Cos}\,\beta)$ = aerodynamic force component

$$\vec{T}_x = \sum_{i=1}^{n} \left[\vec{T}_{x_i} \text{ Cos}\,\alpha \text{ Cos}\,\beta + \vec{T}_{y_i} \text{ Sin}\,\beta + \vec{T}_{z_i} \text{ Sin}\,\alpha \text{ Cos}\,\beta\right]$$

= thrust component

$$\vec{W}_x = W\left[\text{Cos}\,\theta \text{ Cos}\,\phi \text{ Sin}\,\alpha \text{ Cos}\,\beta + \text{Cos}\,\theta \text{ Sin}\,\phi \text{ Sin}\,\beta - \text{Sin}\,\theta \text{ Cos}\,\alpha \text{ Cos}\,\beta\right]$$

= weight component

$\vec{\tau}_x$ = ground reaction force component and is a function of landing gear configuration, brake characteristics and aircraft Euler angle.

and,

ρ = air density

S = wing area

C_y = side force coefficient and is a function of gear, rudder, Mach number, aircraft configuration, etc.

C_d = drag coefficient and is function of α, flaps, Mach number, etc.

$\vec{T}_{x,y,z}$ = thrust components in aircraft axis

θ, ϕ, ψ = pitch, roll, and heading Euler angles

Additional transfer functions and simulations are now required to evaluate these equations. For example, in the $\vec{F}_x$ formula, additional functions are needed for the thrust $(\vec{T}_x)$ and weight $(\vec{W}_x)$ components.

Thrust $(\vec{T}_x)$ is simulated as a function of power setting, Mach number, altitude, and total thrust (T) available with respect to the aircraft reference frame, where

$$T = \sum_{i=1}^{n} \vec{T}_{m_i} \text{ Cos } \gamma_{m_i}$$

and

$\vec{T}_{m_i}$ = net thrust along engine axis for each engine i

γ_{m_i} = angle between engine axis and aircraft xy plane and the x axis for each engine i

Total aircraft weight (W) of aircraft throughout flight is also simulated as a function of power setting, Mach number, and altitude with fuel consumption during flight also considered. Thus,

$$W = W_{empty} + W_{payload} + W_{initial\ fuel} - \int \dot{W}_{fuel}\, dt$$

Also, transfer functions are required to define the Euler angles which relate body position to the inertial reference axes as follows:

$$\dot{\theta} = q \text{ Cos } \phi - r \text{ Sin } \phi$$

$$\dot{\phi} = p + \dot{\psi} \text{ Sin } \theta$$

$$\dot{\psi} = \frac{1}{\text{Cos } \theta} \left[q \text{ Sin } \phi + r \text{ Cos } \phi \right]$$

and to define the rate of climb $(\dot{h})$, where

$$\dot{h} = V \text{ Sin } \gamma$$

with

V = total velocity

$\gamma = \theta + \alpha$ = flight path angle

θ = heading Euler angle

α = angle of attack

With respect to the rate of climb ($\dot{h}$), typical accountable factors are the following:

- Gravity
- Aircraft weight
- Net engine thrust
- Sideslip angles
- Force components in wind axes
- Roll, pitch, and yaw rates
- Altitude and temperature
- Aerodynamic coefficients
- Aircraft trim

The transfer functions described in this paragraph (4.4) for a cargo aircraft system are representative of those which are required to evaluate and apportion effectiveness for this class of systems.

4.5 SPACECRAFT SYSTEM TRANSFER FUNCTIONS

4.5.1 Spacecraft Transfer Functions

In general the spacecraft transfer functions, which will allow for the allocation of errors, are simply an extension of the missile system transfer functions. However, the application of these functions is slightly different because of the mission differences. Generally, in a missile system, the objective is to establish a boost phase trajectory, placing the re-entry vehicle in an elliptical path which will intersect the earth's surface at or near the target point as illustrated in Figure 4-14. In this system, parametric errors in the boost phase are initially reflected in a velocity vector error. This velocity error causes a change to the elliptical path and will result in an impact miss-distance. This is called the trajectory positional error at impact. A spacecraft system is essentially the same basic system as the missile. An ascent to initial orbit, or boost phase, must still be accomplished. However, for a spacecraft system, an orbit is established which does not impact the earth and, generally, is an elliptical orbit as illustrated in Figure 4-15. All booster parameters result in changes in the initial orbit velocity and, therefore, result in orbital positional errors. Orbital position errors at any point in the orbit can be conveniently considered as radial position errors, in-track position errors, and cross-track position errors. Such errors can be directly related to the

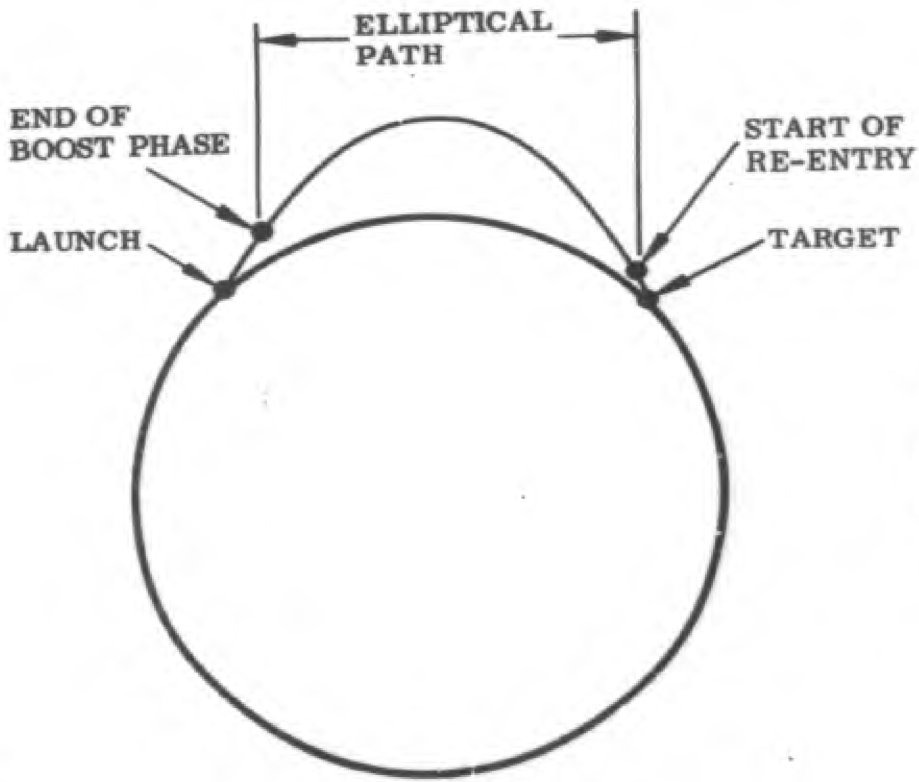


Figure 4-14 Missile Trajectory

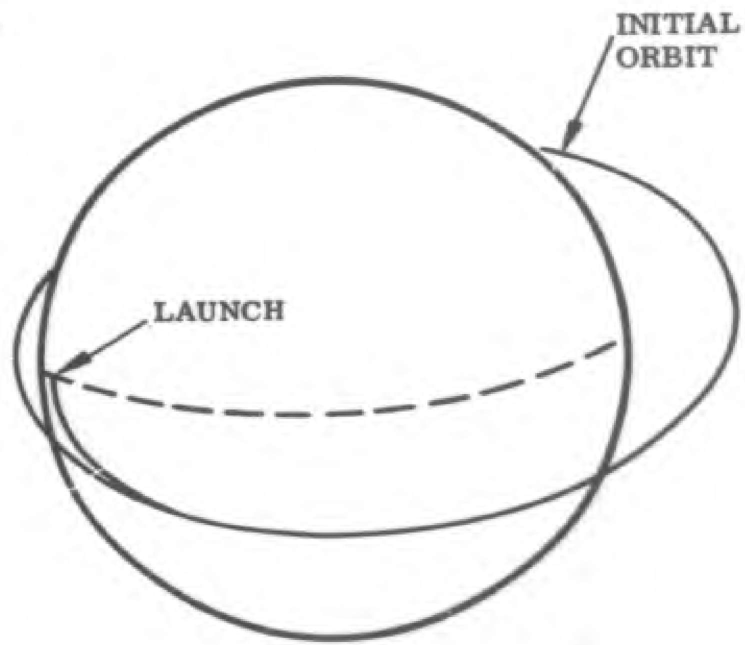


Figure 4-15 Spacecraft Orbit

velocity components in these respective directions. Figure 4-16 illustrates these velocity components for a circular orbit such as used in the Gemini Agena Target Vehicle.

The velocity and velocity changes are a continuous calculation of the motion equations as defined for the missile system, such as the evaluation of the three linear acceleration equations, their integration to velocity components, and the resolution of these velocity components from the missile frame of reference to an inertial frame of reference, and then to an orbital frame of reference. Inertial position can then be obtained through integration of the inertial velocity.

4.5.2 Orbit Maneuver Functions

Performance maneuvers for a spacecraft in an orbit involve adding velocity components to correct for an existing orbital error, or to modify the orbit to meet an objective, such as a plane change maneuver, phasing maneuver, or retro maneuver. The velocities are added by: (1) rotating the vehicle to the required orientation through control moment couples using the basic expressions for angular acceleration, as defined for the missile system, and (2) supplying sufficient impulse to add the required velocity determined by the integration of the acceleration equations.

Errors or desired changes in any orbit are generally associated with (1) size or shape of the orbit, and (2) misalignment of the orbital plane with respect to a reference. Figure 4-17 illustrates a change from an elliptical orbit to a circular orbit with the velocity changes made at the instant of intersection of the old and new orbits. Figure 4-18 illustrates the misalignment of orbit planes. An alternate illustration of this in terms of orbit velocities is shown in Figure 4-19, where:

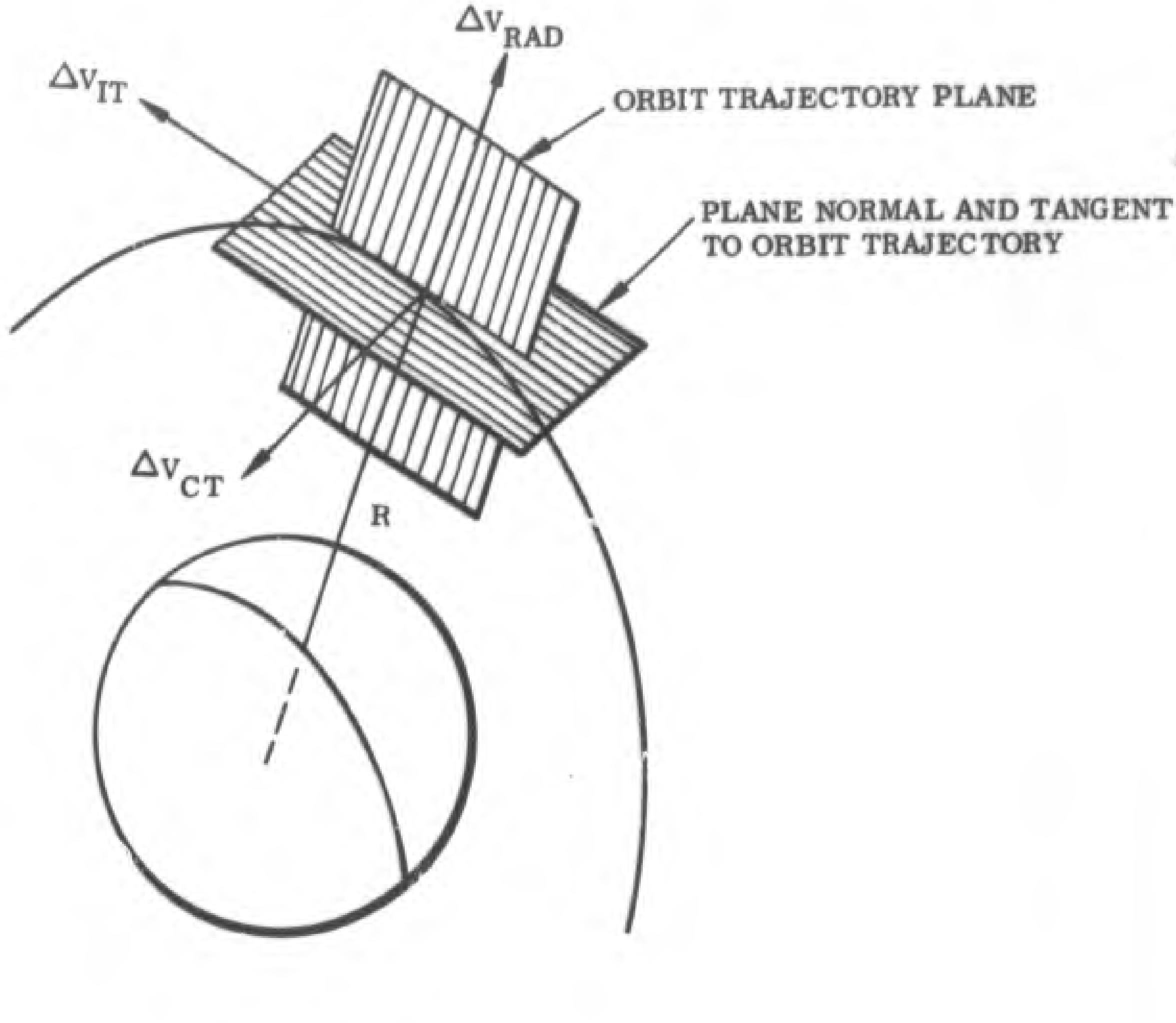


Figure 4-16 Position Error Velocity Components

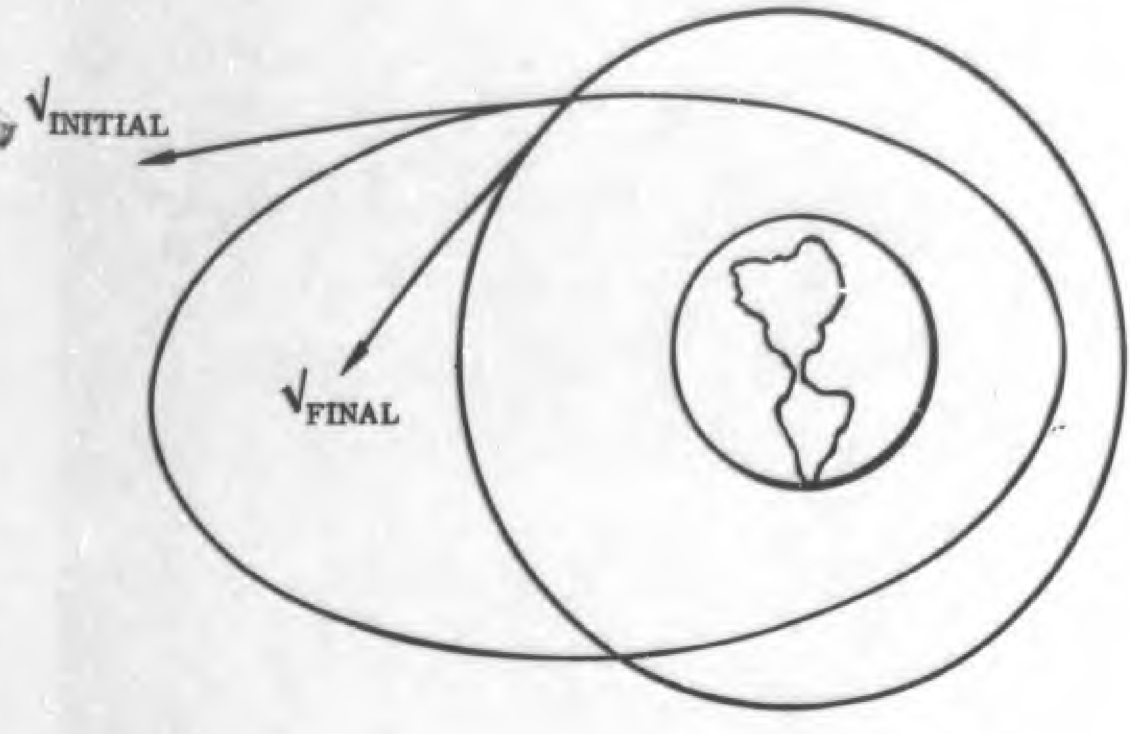


Figure 4-17 Orbit Correction Maneuver

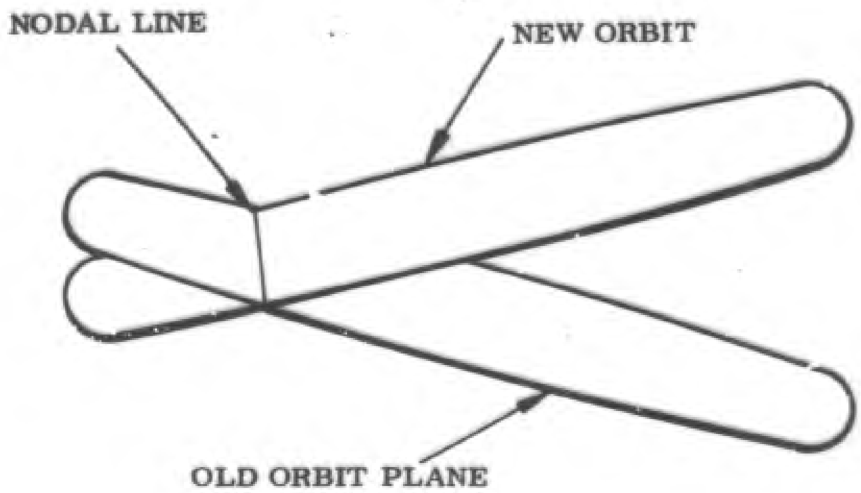
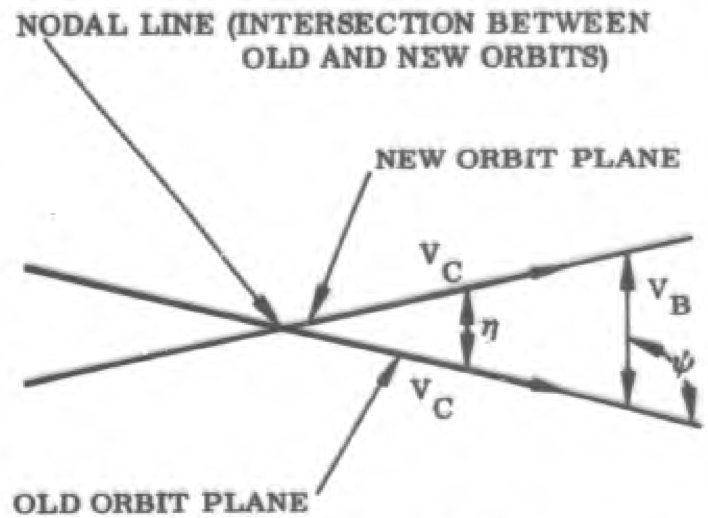


Figure 4-18 Orbit Plane Change



V_C = inertial orbit velocity

V_B = corrective boost velocity

ψ = corrective boost velocity attitude

η = nominal plane change angle

Figure 4-19 Plane Change Geometry

V_B is the corrective boost velocity magnitude to be added at the nodal line for a plane change

ψ is the corrective boost velocity attitude

V_C = inertial orbit velocity

$$V_B = 2\, V_C \operatorname{Sin} \frac{\eta}{2}$$

$$\psi = \pm\left(90^\circ + \frac{\eta}{2}\right)$$

η = nominal plane change angle

Two basic positional errors of a circular orbit can be delineated. One is that of periodic position errors which are harmonic and do not increase from one revolution to the next. The relationship in matrix form for determining the positional changes on an incremental basis is:

$$\begin{bmatrix} \frac{RAD}{R} \\ \frac{IT}{R} \\ \frac{CT}{R} \end{bmatrix} = \begin{bmatrix} (\operatorname{Sin} \delta_{\bar{u}}) & (2 - 2 \operatorname{Cos} \delta_{\bar{u}}) & 0 \\ (-2 + 2 \operatorname{Cos} \delta_{\bar{u}}) & (4 \operatorname{Sin} \delta_{\bar{u}}) & 0 \\ 0 & 0 & (\operatorname{Sin} \delta_{\bar{u}}) \end{bmatrix} \begin{bmatrix} \frac{\Delta V_{RAD}}{V_C} (1 + e \operatorname{Cos} \bar{u}) \\ \frac{\Delta V_{IT}}{V_C} (1 + e \operatorname{Cos} \bar{u}) \\ \frac{\Delta V_{CT}}{V_C} (1 + e \operatorname{Cos} \bar{u}) \end{bmatrix}$$

where:

RAD	=	radial position error
IT	=	in-track position error
CT	=	cross track position error
R	=	radius from center of the earth to the vehicle
$\delta_{\bar{u}}$	=	incremental central angle traversed where δ_u is the nominal central angle traversed from injection
ΔV_{RAD}	=	total radial velocity error
ΔV_{IT}	=	total in-track velocity error
ΔV_{CT}	=	total cross-track velocity error
V_C	=	inertial orbit velocity
e	=	eccentricity
u	=	central angle (measured from ascending node)

The second type of positional errors on an incremental basis are those which increase porportionally to the integer number of vehicle orbital revolutions. These errors are given by the following expressions:

$$\begin{bmatrix} \frac{RAD}{R} \\ \frac{IT}{R} \\ \frac{CT}{R} \end{bmatrix} = \begin{bmatrix} 0 & (-3\,e\,\delta_{\bar{u}}\,\mathrm{Sin}\,\bar{u}) & 0 \\ (-3e\delta_{\bar{u}}\,\mathrm{Sin}\,u) & (-3\,\delta_{\bar{u}}\,[1+2\,e\,\mathrm{Cos}\,\bar{u}+e\,\mathrm{Cos}\,u) & (1'\,\delta_u\,\mathrm{Cos}\,u_o) \\ 0 & 0 & (-q\,\delta_u\,\mathrm{Cos}\,u_o\,\mathrm{Cos}\,u) \end{bmatrix} \begin{bmatrix} \frac{\Delta V_{RAD}}{V_C}\,(1+e\,\mathrm{Cos}\,\bar{u}) \\ \frac{\Delta V_{IT}}{V_C}\,(1+e\,\mathrm{Cos}\,\bar{u}) \\ \frac{\Delta V_{CT}}{V_C}\,(1+e\,\mathrm{Cos}\,\bar{u}) \end{bmatrix}$$

where, in addition to the previous notation definitions,

$\bar{u}$ = nominal central angle from perigee at the time of comparison

i' = $6\,J_2$ (Sin i Cos i) where

i = the orbit inclination in degrees

J_2 = coefficient of the second harmonic in earth's gravitational potential

u_o = central angle at maneuver point

$$q = \frac{3}{2}\left(\bar{J}_2 \,\mathrm{Sin}^2\, i\right)$$

$$\bar{J}_2 = J_2 \left(\frac{REQ}{a\,(l-e)^2}\right)^2 \quad \text{where}$$

REQ = earth equatorial radius

a = orbit semi major axis length

A primary purpose of a spacecraft may be to conduct a rendezvous mission with another spacecraft. Four basic on-orbit maneuvers required for such a mission are: (1) the adjustment to the desired circular parking orbit; (2) the orbital plane change maneuver to make the orbits of the two rendezvousing vehicles coplanar; (3) a phasing maneuver to eliminate phase differences of the two vehicles; and (4) a retro maneuver to return one vehicle to the rendezvous orbit.

During all maneuvers, it is desired to have the velocity change occur instantaneously and to be applied in the exact direction. However, propulsion subsystem corrections require a finite reaction time during which continuous vehicle weight changes are occuring to create further errors. Also, pitch and yaw attitude errors are present because of guidance errors. Therefore, errors will result from any maneuver. Each of such errors may be evaluated with simulation routines for resultant orbital characteristic errors to determine sensitivity to the various parameters, and thereby provide an objective basis for error apportionments.

4.5.3 Parking Orbit

Once an initial parking orbit is established, small adjustments in eccentricity and inclination may be required to correct for injection errors causing orbital parameters of the parking orbit to exceed specified tolerances. The geometry for such in-plane and out-of-plane in-orbit maneuvers is shown in Figure 4-20. The in-track, cross-track, and radial velocity components of the velocity change with respect to the initial orbit are given by:

$$V_{IT} = V_B \cos\theta \cos\psi$$
$$V_{CT} = V_B \cos\theta \sin\psi$$
$$V_{RAD} = V_B \sin\theta$$

With respect to the final nominal orbit, these velocity components are given by the following relationships:

$$V_{IT} = V_B \cos\theta \cos(\psi - \eta)$$
$$V_{CT} = V_B \cos\theta \sin(\psi - \eta)$$
$$V_{RAD} = V_B \sin\theta$$

where

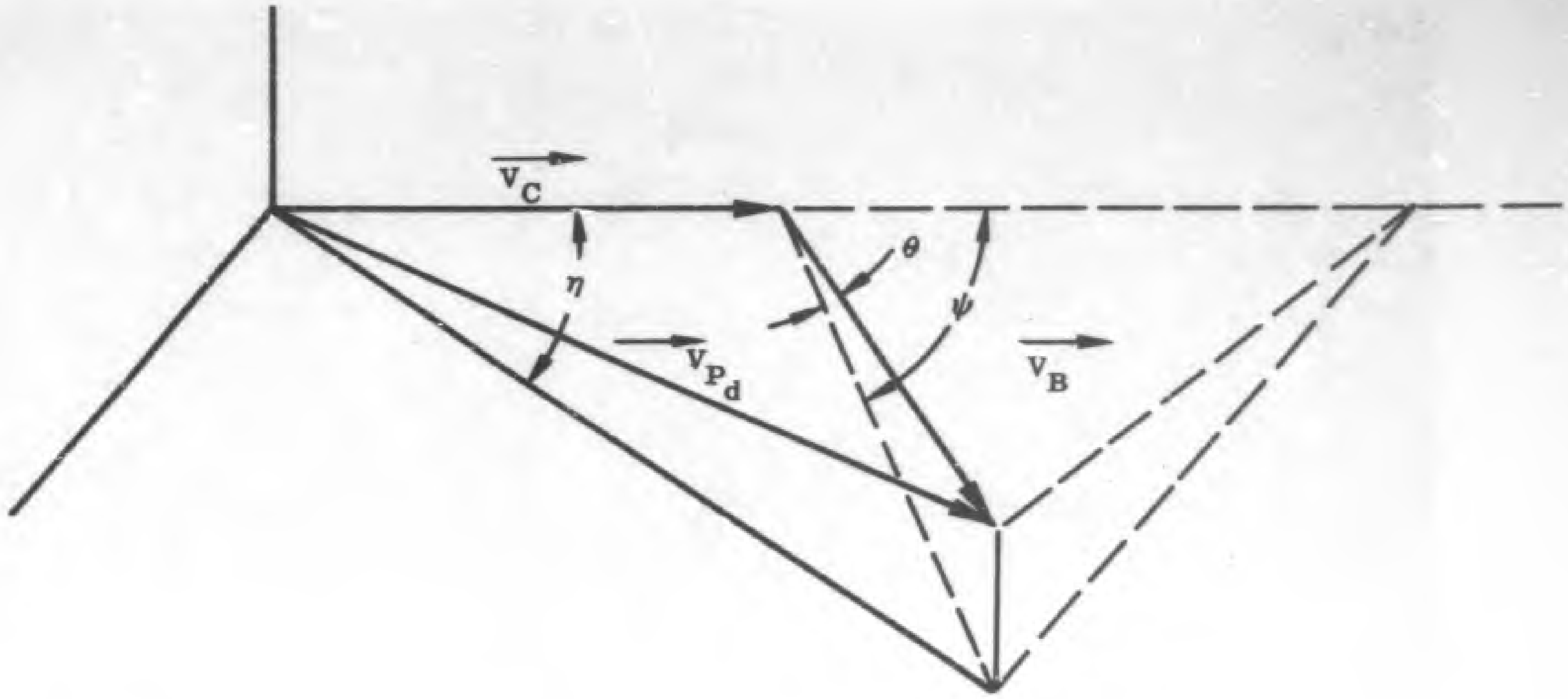
V_B is the nominal incremental boost velocity
η is the nominal plane change angle
ψ is the nominal yaw attitude
θ is the nominal pitch attitude.

Basic sensitivity functions may be obtained by differentiating the above equations with respect to the various parameters.

4.5.4 Orbital Plane Correction

After a parking orbit has been established within specified tolerances, the next maneuver is to correct for any orbital plane differences with the rendevous vehicle as previously indicated. The nominal plane change angle is defined by:

$$\tan\eta = \frac{\cos\theta \sin\psi}{\frac{V_C}{V_B} + \cos\theta \cos\psi}$$



$\vec{V_{P_d}}$ = desired perigee velocity

$\vec{V_B}$ = corrective boost velocity magnitude

$\vec{V_C}$ = inertial orbit velocity

ψ = corrective boost velocity attitude

η = nominal plane change angle

θ = nominal pitch attitude

Figure 4-20 Orbit Maneuver Geometry

The partial derivatives of Tan η may also be used to evaluate sensitivities, such as

$$\frac{\partial \eta}{\partial V_B} = \frac{\frac{V_C}{V_B^2} \operatorname{Sin} \psi}{\left(\frac{V_C}{V_B}\right)^2 + 2\left(\frac{V_C}{V_B}\right) \operatorname{Cos} \psi + 1} \quad \text{for small } \theta$$

Sensitivities to basic parameters, such as propulsion system characteristics or weight change, must be evaluated since they affect the incremental boost velocity, V_B, or orientation values.

4.5.5 Phasing Maneuver

Having established the desired orbit and orbit orientation, the next area of consideration is that of the phasing maneuver. The purpose of the phasing maneuver is to eliminate the angular position (phase) difference of two orbiting spacecrafts over a given number of revolutions. This is accomplished by transferring the maneuvering vehicle to an orbit having a period different than the other vehicle.

Assume that both vehicles are coplanar with similar orbits, and have a phase difference of ϕ degrees. The incremental boost requirements are defined by:

$$V_B = V_C \left(\sqrt{1+e} \; -1\right)$$

where

$$e = 1 - \left(1 + \frac{\phi/N}{360}\right)^{-2/3}$$

and

N is the number of revolutions in which the phase difference is to be eliminated.

The velocity requirements for a maneuver to correct errors incurred in the execution of a phasing maneuver can be related to that necessary to perform a required orbital period change, and is given by:

$$\Delta T_C = \frac{-N/A}{(N/A) - 1} \Delta T_E$$

where N is the total number of revolutions planned for the initial phasing maneuver, A is the revolution number at which the corrective maneuver is to be made, and ΔT_E is the magnitude of the phasing orbit period error.

The velocity requirement is determined by

$$\Delta V = \left(\frac{\partial V}{\partial T}\right)_{nominal} \left(\frac{\Delta T_C}{\Delta T_E}\right) \Delta T_E$$

where

$\frac{\partial V}{\partial T}$ is the rate of change of velocity with orbital period.

The orbital period may be evaluated by Kepler's third law, i.e.

$$T = 2\pi \sqrt{\frac{a^3}{\gamma}}$$

where

a is the elliptic semi-major axis

γ is a gravity-mass constant

4.5.6 Retro Maneuver

For final phase of the mission, that of the retro maneuver, the nominal retro velocity magnitude necessary to insert the spacecraft into the circular rendezvous orbit from the phasing orbit is a function of the phasing rate (ϕ/N) established in the previous phasing maneuver. The incremental boost velocity attitude is ψ = 180 degrees. Retro velocity may be approximated in many instances. As an example, for a 161 nautical mile circular rendezvous orbit, retro velocity can be approximated by $2.25\ (\phi/N) - .039\ (\phi/N)^2$. The retro velocity addition will generate errors which can be evaluated for the various parameters.

4.5.7 Evaluation of Performance

In general, all spacecraft performance can be evaluated in terms of basic orbital mechanics. The transfer functions presented are representative of spacecraft missions and are of convenient form for computerization. This allows for the efficient evaluation of all capability parameters over the spectrum of performance, as well as the interactions of all maneuvers. Once the relative sensitivity functions are established to reflect the important parameters, performance errors may be apportioned from the total allowable error.

4.6 USE OF TRANSFER FUNCTIONS FOR APPORTIONMENT

It has been described with considerable detail that, having identified key accountable factors, transfer functions may be used to calculate system capability and to test whether performance constraints are satisfied. Further, using the Lagrange multiplier or other optimization techniques as described in Section 3, values of these accountable factors can be determined which will maximize system effectiveness, or capability parameters, subject to the constraints. Thus, if a system performance constraint is a minimum value for a figure of merit, the set of values of the accountable factors which will allow the figure of merit to be exceeded will contain the potential apportioned values. That value for each accountable factor which will provide the maximum effectiveness is the optimum apportioned value. These accountable factors will invariably be design parameters and will be stated in terms of a nominal, or a nominal with a tolerance.

A general perspective of subsystem-to-system transfer functions and methods for their evaluation have been described in this section. Evaluating methods were also presented in Section 2. The following is an illustration of the specific use of a set of transfer functions in the apportionment of a capability parameter.

The capability parameter to be minimized is the time duration of the mission. Such a situation may apply to an anti-missile interceptor, to a system which is vulnerable to a particular hostile environment with lengthy exposure, to systems required to rendezvous, or to similar systems where a minimum time of operation is vital to mission success. Assume that for the system involved, the achievement of a minimum mission time is predicated upon minimizing the time $(\hat{t})$ for its rocket subsystem to reach a final velocity $\vec{V}(\hat{t})$. This final velocity $\vec{V}(\hat{t})$ must equal the system's prescribed final velocity $\vec{V}_f$. Simultaneous with achieving $\vec{V}_f$, it is required that the system's correct position $\vec{P}(t)$ at the end of flight be equal to the system's prescribed position $\vec{P}_f$. This joint achievement is to be accomplished with the constraints that:

(1) the total mass $m(0)$ of the rocket at time of launch $(t = 0)$, including propellant, is limited by:

$$m_\ell \leq m(0) \leq M_u \tag{4-9}$$

(2) the mass $m(\hat{t})$ of the rocket at the end of flight $(t = \hat{t})$ is greater or equal to the non-propulsive portion of the rocket. The rocket must be of sufficient structural strength and mass to support the propellant weight and withstand the maximum chamber pressure $p_c(t)$

Hence, there is a function f_1 such that the following constraint is satisfied:

$$m(\hat{t}) \geq f_1(m(0), \max p_c(t)) \geq 0 \text{ for } 0 \leq t \leq \hat{t} \tag{4-10}$$

Also, since the mass of the stage is constantly decreasing due to the expending of propellant, the mass flow rate of propellant, $\dot{w}(t)$ is:

$$\dot{w}(t) \equiv -\frac{dm(t)}{dt} \geq 0 \text{ for } 0 \leq t \leq \hat{t} \tag{4-11}$$

Since the velocity at end of rocket flight, $\vec{V}(\hat{t})$ is required to be equal to $\vec{V}_f$, simultaneous with achieving a position, $\vec{P}(\hat{t})$, which is required to be $\vec{P}_f$, the following equality constraints are implied:

$$\vec{V}(\hat{t}) = \vec{V}_f \,;\; \vec{P}(\hat{t}) = \vec{P}_f \tag{4-12}$$

At any time t^*, the rocket's velocity and position are related by:

$$\vec{V}(t^*) = \vec{V}(0) + \int_0^{t^*} \left[\frac{\vec{F}(t) + \vec{D}(t)}{m(t)} + \vec{G}(t) \right] dt \,; \tag{4-13}$$

$$\vec{P}(t^*) = \vec{P}(0) + \int_0^{t^*} \vec{V}(t)\, dt \,, \quad 0 \leq t^* \leq \hat{t} \tag{4-14}$$

The function $\vec{P}(t)$ has vector components $x(t)$, $y(t)$, and $z(t)$ in a coordinate system with the origin at the center of the earth. All vectors will be assumed to refer to this system. When the rocket is launched at $t = 0$, initial velocity $\vec{V}(0) = 0$ and the initial position $\vec{P}(0)$ are as prescribed for the mission. $\vec{F}(t)$ is the thrust force vector operating on the rocket at time t, $\vec{D}(t)$ is the drag force vector which is opposite in direction to the motion, $\vec{G}(t)$ is an acceleration vector pointed generally towards the earth's center, and $m(t)$ is the rocket mass at time t. Further,

(1) The controllable vector $F(t)$ has three components $F_x(t)$, $F_y(t)$, and $F_z(t)$ in the x, y, z directions. This vector is assumed to depend on the magnitude of thrust $\vec{F}(t)$ and the direction of thrust $\overrightarrow{NF}(t)$, with

$$\vec{F}(t) = F(t)\ \overrightarrow{NF}(t) \qquad (4\text{-}15)$$

and the sum of squares of the three components of any direction vector is such that $|\overrightarrow{NF}(t)| \leq 1$. The vector $\overrightarrow{NF}(t)$ is governed by a thrust vector control subsystem which determines direction of thrust.

(2) The drag force $\vec{D}(t)$ is in a direction opposite to the velocity and has a magnitude which increases with atmospheric density. This force is given by:

$$\vec{D}(t) = -\vec{V}(t)\ D_c(a(t)) \qquad (4\text{-}16)$$

where $\vec{V}(t)$, the velocity at time t, is given by Equation (4-13) with t replaced by $\hat{t}$, and D_c is the drag coefficient at the altitude $a(t)$ corresponding to a position $\vec{P}(t)$. Altitude $a(t)$ is given by:

$$a(t) = |P(t)| - R_e(x, y, z) \qquad (4\text{-}17)$$

with

$$|P(t)| = (x(t)^2 + y(t)^2 = z(t)^2)^{1/2}$$

$$R_e = \text{earth's radius along the line from } (0, 0, 0) \text{ to } (x, y, z)$$

(3) The gravity vector $\vec{G}(t)$ depends on $x(t)$, $y(t)$ and $z(t)$, and is approximately

$$\vec{G}(t) \cong -g M_e \vec{P}(t)/|P(t)|^3 \tag{4-18}$$

with g, the universal gravitation constant and M_e, the earth's mass.

(4) The mass m (t) of the rocket at terminal time $\hat{t}$ is given by:

$$m(\hat{t}) = m(0) - \int_0^{\hat{t}} \dot{w}(t)\, dt \tag{4-19}$$

with

$$\dot{w}(t) = A_b(t)\, \dot{r}(t)\, P_b, \tag{4-20}$$

where $A_b(t)$ is the burning area at time t, $\dot{r}(t)$ is the burning rate (e.g., units of thickness or length per time) at time t, and P_b is the propellant density.

(5) The force F (t) in Equation (4-15) is given by

$$F(t) = C_F\, a_t\, p_c(t) , \tag{4-21}$$

where C_F is the thrust coefficient, A_t is the nozzle throat area, and $p_c(t)$ is the chamber pressure at time t

Thus, final velocity $\vec{V}(\hat{t})$, as can be seen from Equation (4-13), depends upon intermediate controllable time parameters $\vec{F}(t)$, $\vec{D}(t)$, and m (t). Also, to a minor extent, the gravity vector $\vec{G}(t)$ can be considered a controllable parameter, because it depends upon position $\vec{P}(t)$, and hence upon the planned trajectory. In turn the quantities $\vec{F}(t)$, $\vec{D}(t)$, m (t) and $\vec{G}(t)$ depend, according to Equations (4-15) - (4-21), upon the more basic controllable factors and design parameters of:

$$\vec{NF}(t) ,\ D_c(a) ,\ C_F ,\ A_t ,\ p_c(t) ,\ A_b ,\ \dot{r}(t) ,\ \text{and } P_b . \tag{4-22}$$

As indicated, the direction of thrust $\overrightarrow{NF}(t)$ can be controlled by the use of the thrust vector control. The drag coefficient $D_c(a)$, a function of altitude a, is approximately of the form

$$D_c(a) \cong \rho(a)\ D_k , \tag{4-23}$$

where $\rho(a)$, the density of atmosphere at altitude a, is of the approximate form

$$\rho(a) \cong \rho(0)\ \exp(-a\,k_1) , \tag{4-24}$$

with k_1 as a constant coefficient and D_k a function of the rocket design.

The quantities and design parameters

$$\overrightarrow{NF}(t)\ ,\ D_k\ ,\ C_F\ ,\ A_t\ ,\ p_c(t)\ ,\ A_b(t)\ ,\ \dot{r}(t)\ ,\ \text{and}\ P_b \tag{4-25}$$

will be treated as basic accountable factors governed by the choice of propellant materials, propellant grain design, etc. There will be technological constraints on these accountable factors in addition to the performance constraints of Equations (4-9) - (4-12). This is due to the limitations in the types of materials and propellant available. In the evaluation process, the quantities in Equation (4-25) are assumed to be known, due to the type of propellant and other materials, grain configuration, etc. With respect to the technological limitations, these quantities can be represented by:

$$\begin{aligned}
D_k &\cong f_2 \quad \text{(construction limitations)}\\
C_F &\leq f_3 \quad \text{(material property constraint)}\\
A_t &\leq f_4 \quad \text{(construction limitation)}\\
p_c(t) &\leq f_5 \quad \text{(material property constraint)}\\
A_b(t) &\leq f_6 \quad \text{(construction limitations)}\\
\dot{r}(t) &\leq f_7 \quad \text{(material property constraint)}\\
P_b &\leq f_8 \quad \text{(material property constraint)}
\end{aligned} \tag{4-26}$$

Using Equations (4-15) - (4-21), the function $\vec{F}(t)$, $\vec{D}(t)$, $m(t)$, and $\vec{G}(t)$ may be computed. Then, using the expressions of Equations (4-13) and (4-14), a test can be made to determine the time $\hat{t}$ when both $\vec{V}(\hat{t}) = \vec{V}_f$ and $\vec{P}(\hat{t}) = \vec{P}_f$ are satisfied simultaneously. Equations (4-13) - (4-21) can be reduced to Equation (4-13) and the following recursive equation for the function $\vec{V}(t)$:

$$\vec{V}(t^*) = \vec{V}(0) + \int_0^{t^*} \left[\frac{C_F A_{t} \cdot {}_c(t)\, \overrightarrow{NF}(t) - \vec{V}(t)\, \rho(0)\, D_k \exp(-a k_1)}{m(0) - \int_0^t A_b(t_1) r(t_1)\, \rho_b\, dt_1} - g M_e \vec{P}(t) / |P(t)|^3 \right] dt \, , \tag{4-27}$$

where $a = |P(t)| - R_e$ as given by Equation (4-17).

The dependency of $\vec{V}(t)$ and $\vec{P}(t)$ upon the accountable factors in Equation (4-25), as defined recursively in Equations (4-14) and (4-27), may be represented in the functional form:

$$\begin{aligned} \vec{P}(t) &= P \left\{ \overrightarrow{NF}(t),\, D_k,\, C_F,\, A_t,\, p_c(t),\, A_b,\, \dot{r}(t),\, \rho_b \right\} ; \\ \vec{V}(t) &= V \left\{ \overrightarrow{NF}(t),\, D_k,\, C_F,\, A_t,\, p_c(t),\, A_b,\, \dot{r}(t),\, \rho_b \right\} . \end{aligned} \tag{4-28}$$

The time $\hat{t}$ at which the desired position $\vec{P}_f$ and velocity $\vec{V}_f$ is met simultaneously can be represented by

$$\hat{t} = h \left[\overrightarrow{NF}(t),\, D_k,\, C_F,\, A_t,\, p_c(t),\, A_b,\, \dot{r}(t),\, P_b \right] \tag{4-29}$$

If α is designated to be any of the quantities in the bracket term of Equation (4-29), then the derivative of $\hat{t}$ can be expressed as:

$$\frac{d\hat{t}}{d\alpha} = - \frac{\partial V_i(\hat{t})}{\partial \alpha} \Big/ \frac{\partial V_i(\hat{t})}{\partial \hat{t}} = - \frac{\partial P_i(\hat{t})}{\partial \alpha} \Big/ \frac{\partial P_i(\hat{t})}{\partial \hat{t}} \tag{4-30}$$

for

$$V_i = V_x,\ V_y,\ V_z\ ;$$

$$P_i = x,\ y,\ z$$

Unless the partials of Equation (4-30) are equal, the velocity and position constraints are not met. If they are equal, then Equation (4-30) gives the derivative of $\hat{t}$. The choice of values for the accountable factors of Equation (4-25) must be rejected if all the constraints as expressed by Equations (4-9) - (4-12) are not met. The acceptable choice of the values for the indicated accountable factors which would minimize $\hat{t}$ is the "best" choice.

An apportionment technique is a method of finding the "best" choice just mentioned. The solution of the problem for minimizing time of flight $\hat{t}$ as represented in Equation (4-29) can be found by equating the derivative of $\hat{t}$ with respect to the accountable factors to zero, that is

$$\frac{dh}{d\overline{A}} = 0\ , \tag{4-31}$$

where $\overline{A}$ is a column matrix, the transpose of which is:

$$\left[\overrightarrow{NF}(t),\ D_k,\ C_F,\ A_t,\ p_c(t),\ A_b,\ \dot{r}(t),\ P_b\right] \tag{4-32}$$

In this transpose, each function of t, such as $p(t)$, can be replaced by values. As an example, for some small ϵ, the values can be

$$p_c(0),\ p_c(\epsilon/\hat{t}),\ p_c(2\epsilon/\hat{t}),\ \ldots\ ,\ p_c(\hat{t}) \tag{4-33}$$

The matrix $\overline{A}_i$ is the i^{th} iterate of $\overline{A}$, and $\frac{dh}{d\overline{A}_{(i)}}$ is a column matrix, the k^{th} element of which is the derivative of h with respect to the k^{th} component of $\overline{A}_{(i)}$.

If a tentative value of each of the quantities in Equation (4-25) is given, these values can be improved iteratively to a minimum time $\hat{t}$ by use of one of the methods of Section 3, such as the method of Lagrange multipliers with Newton iteration.

To illustrate the Newton iteration procedure, consider the simple case where no constraints are present. The following iteration is used:

$$\bar{A}_{(i+1)} = \bar{A}_{(i)} - \theta \left[\frac{d^2 h}{d\bar{A}^2_{(i)}} \right]^{-1} \frac{dh}{d\bar{A}_{(i)}} \; ; \; i = 0, 1, 2, \ldots, \qquad (4\text{-}34)$$

where each term of this equation is a column matrix of the same order, so that the last term describes the numbers which would have to be subtracted from the individual elements of $\bar{A}_{(i)}$ to obtain $\bar{A}_{(i+1)}$. Thus, Equation (4-34) is the equivalent of many ordinary equations. The quantity θ is usually one, but may be altered to facilitate convergence.

The term $\frac{d^2 h}{d\bar{A}^2_{(i)}}$ is a matrix, the n^{th} element of the m^{th} row being the second derivative of h with respect to the m^{th} and n^{th} components of $\bar{A}$. This is a mixed derivative unless $m = n$.

Returning to the example, in iterating Equation (4-34), if any of the constraints of Equations (4-9) through (4-12) and (4-26) are violated, or if the constraints are equality constraints, then using the method of Lagrange multipliers, Equation (4-31) should be modified to the following system:

$$\frac{dh}{d\bar{A}} - \sum_{i=1}^{c} \zeta_i \frac{df_i}{d\bar{A}} = 0 \qquad (4\text{-}35)$$

where $f_i = 0$ represents the various constraints, and $c = 17$ for the example, with $f_i \geqslant 0$ or $f_i \leqslant 0$. For example, suppose that Equations (4-10) and (4-12) are the only constraints that must be imposed because they are the only ones that would otherwise be violated. Then equation system (4-35) becomes

$$\frac{dh}{dA} = \frac{d}{dA}\Big[\zeta_1 (V_x(\hat{t}) - V_{fx}) + \zeta_2 (V_y(\hat{t}) - V_{fy})$$

$$+ \zeta_3 (V_z(\hat{t}) - V_{fz}) + \zeta_4 (x(\hat{t}) - x_f) + \zeta_5 (y(\hat{t}) - y_f)$$

$$+ \zeta_6 (z(\hat{t}) - z_f) + \zeta_7 (m(\hat{t}) - f_1(m(0), \max p_c(t))\Big] \; ; \qquad (4\text{-}36)$$

$$V_x(\hat{t}) = V_{fx} \; ; \; V_y(\hat{t}) = V_{fy} \; ; \; V_z(\hat{t}) = V_{fz} \; ;$$

$$x(\hat{t}) = x_f \; ; \; y(\hat{t}) = y_f \; ; \; z(\hat{t}) = z_f \; ; \; m(\hat{t}) = f_1(m(0), \max p_c(t)).$$

The above system can be solved by Newton's iteration in a way similar to that in which Equation (4-31) is solved by Equation (4-32).

This example has illustrated how transfer functions may be used to apportion a system capability parameter. While the details presented are general in nature, their association with real systems and analyses are apparent. The Lagrange multiplier with Newton's iteration solution method was described to indicate how to apportion the values of the accountable factors of the rocket which will minimize the time for achieving a required velocity simultaneously with position. This apportionment will maximize capability and still satisfy performance constraints. Other apportionment solution methods could have been applied. If a computer is available, Newton's iteration method will be extremely efficient. If hand calculations are feasible, then the priority list solution method can be used.

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SECTION 5

SYSTEM EFFECTIVENESS MONITORING PLAN

5.1 INTRODUCTION

The achievement of a specified effectiveness requirement for a system is the result of an ordered series of deliberate and dynamic actions and reactions, executed in a timely manner and accomplished in accordance with a pre-established plan. This plan is the system effectiveness monitoring plan and should be a necessary document to be generated by the system development contractor for Air Force management approval prior to implementation. The plan prescribes the set of standards, guidelines, and methods for the accomplishment, by the prime contractor and associate contractors, of system effectiveness analyses, including cost analysis, for the assessment of effectiveness progress.

The system effectiveness monitoring plan is an integrating-type plan. The majority of its elements will consist of the normal technical activities associated with its constituent parameters. Thus, a subset of the outputs from the implementation of current reliability, maintainability, system engineering, and similarly related plans is utilized in the integrated effectiveness analyses for growth assessment and monitoring purposes.

The basic consequences of the system effectiveness monitoring plan are:

- To provide both Air Force and contractor management with a current prospectus of progress towards achievement of the effectiveness requirements and objectives.
- To provide visibility on critical or sensitive problem areas early in system development, and throughout the acquisition and operational phases, so that effective corrective actions may be accomplished.

- To define the technical task elements and methods to be used for the periodic evaluation and monitoring of system effectiveness parameters.

5.2 GENERAL CONSIDERATIONS

Characteristic of such a plan are fundamental considerations contributing to the plan's utility and acceptability. These are:

- Simplicity in implementation
- Technical sufficiency
- Flexibility for incorporation of changes
- Compatibility with existing Air Force and contractor systems, engineering procedures, tasks, and management practices
- Maximum utilization of information, data, and analyses from related system engineering and other current disciplinary efforts on the effectiveness parameters
- Non-duplication with other related plans, such as the system engineering, reliability, and maintainability plans.

The degree to which the principal figures of merit can be exceeded and maintained will be predicated upon the following effectiveness accomplishments:

- The early and exhaustive identification of design changes on sensitive or critical design and performance parameters which influence the principal figures of merit
- The sufficiency and timeliness of the effectiveness analyses to be performed, and the validity of techniques used
- The rapid and positive response to any necessary corrective actions, as indicated from the analyses, to upgrade or maintain an effective system configuration
- The proper management authority vested in the system effectiveness function.

- The timely and effective reporting of current status on system growth towards meeting the principal figures of merit, and the adequacy of the data system for the generation of such reports.

5.2.1 Identification of Design Changes

Engineering changes may have a major influence on the ability of the system to meet effectiveness requirements. With the multitude of expected equipment modifications during the Acquisition Phase, the basic effectiveness task is to properly focus the effort on only those changes influencing the system's effectiveness. The identification of this relevant subset of design changes with major or critical impact on the principal figures of merit can best be achieved as follows:

- Maintain current the listing of sensitive design and performance parameters and accountable factors. Disseminate this listing to all affected design personnel
- Filter engineering changes to isolate those which are critical or sensitive to the effectiveness parameters and accountable factors while these changes are in the proposal stage. The manner in which this may be efficiently performed is through participation in the preliminary and critical design reviews. The utilization of the results from such reviews, along with any necessary computer simulations and sensitivity analyses to augment these results, will allow the combined effectiveness influence on the system to be evaluated.

5.2.2 Timeliness of Analyses

To be of maximum usefulness, all effectiveness analyses must be accomplished in a timely manner such that sufficient development time remains for any required corrective action. Additionally, the timely execution of these analyses will prevent any change from proceeding to a formal incorporation with a negative influence on the principal figures of merit. The prior dissemination of the critical or sensitive parameter and accountable factor lists circumvents such occurrences to a large extent.

As a further act of prevention and toward establishing design discipline, it is desirable that a list of sensitivity functions (as described in Section 4) be provided designers in the form of coefficients, partials, or curves to serve as guidelines on the influence of key changes. Thus, the designer can be motivated, guided, and disciplined to make only positive effectiveness changes.

5.2.3 Corrective Action Response

There should be established a mechanism for the timely notification and execution of any necessary corrective action. This applies to instances where the rate of effectiveness growth is insufficient, as well as to instances where design changes have negative effects on the principal figures of merit.

5.2.4 Management Structure

It is vital that the management of the effectiveness function be established at an authoritative level and properly structured. The function is closely allied to the system engineering function because of its integrating aspects, and its involvement with the broad spectrum of system performance characteristics. There is, then, a natural tendency to consider the effectiveness function as a sub-function of system engineering. While the technical relationship may be satisfied and compatible, such a function must be provided the proper authority to make key design decisions which may influence the course of the development program and all related technical disciplines. This does not appear to be realizable under current organizational structures.

As an alternative, any of the other currently segmented activities of effectiveness, such as reliability or maintainability, could be charged with the system effectiveness function provided such organizations either currently have, or are given, commensurate authoritative posture in the development program.

Another alternative could be the placement of the analysis function in the system engineering area as a working group, but reserving the major decisions for a top-level effectiveness committee, chaired by a major technical organization and composed of management and technical specialists from the affected disciplines.

5.2.5 Status Reporting

Regular reports of effectiveness growth are required to provide contractor and procuring activity management with a current and projected status of system growth and maturity towards the achievement of the principal figures of merit. Effectiveness evaluations should be made during the acquisition phase as important configuration changes occur. However, such changes occur at an irregular frequency. A systematic and regular assessment of effectiveness activities and progress, and a formal reporting of the results, is mandatory for a well-disciplined effectiveness program. The attainment of the proper frequency of status reporting involves an achievement of a balance between consecutive reports which will reflect significant changes in effectiveness levels achieved and unduly excessive and uneventful reports. For major system development programs, a regular quarterly reporting period is an optimum balance.

The status report should provide the following data and information, among other elements:

- The significant results of all analyses performed during the report period
- The convergence of the sensitive and critical accountable factors and design and performance parameters to their apportioned values for optimum effectiveness
- Model refinements and exercises using analysis and test data.
- Significant corrective action taken and unresolved areas of concern
- Current level of growth with respect to contractual requirements.

5.2.6 Monitoring Methodology

Evaluation methodologies for monitoring system effectiveness basically involve an empirical approach or an analytical approach. An empirical methodology is involved with data collection on, and evaluation of, existing systems. An analytical methodology, on the other hand, is one that derives its results by inference, and uses a set of assumptions, weighting factors, and value judgments by technically qualified individuals. Early in the life cycle of the system, more weight should be given to the analytical method and, as realistic data become more available, the emphasis can be shifted towards the empirical method.

5.3 MONITORING SYSTEM EFFECTIVENESS

5.3.1 Concept Formulation and Concept Definition Phases

During the Concept Formulation Phase and Contract Definition Phase, a system will have been proposed to the procuring activity which will meet or exceed all system requirements with an optimum balance between cost and performance effectiveness. In connection with the studies to arrive at the best system for meeting the military objectives under consideration, many elements necessary for a dynamic effectiveness monitoring plan will have been established and defined. These elements include:

- Cost effectiveness comparisons
- A general mission analysis
- Unfriendly, neutralizing, and defensive threats
- Critical or sensitive design and performance parameters and accountable factors
- Trade-off analyses
- General system descriptions
- Survival potentials
- Maintenance concepts and criteria
- Quantitative and explicit descriptions of each performance characteristic requisite for the mission task.
- Simulation models

- Principal figures of merit
- System and cost effectiveness analyses
- Basic effectiveness analysis models
- Critical system status
- Statistical determination of the mean and standard deviations of accountable factors

To a considerable degree, much of the data used for these elements will be based upon information from similar and related systems, appropriately modified by engineering judgments and simulation results. As such, any effectiveness monitoring plan prepared during the Contract Definition Phase for implementation during the Acquisition Phase will require refinement of the detail methods to be used, and further descriptions and definitions of basic effectiveness elements to be monitored.

5.3.2 Acquisition Phase

During the Acquisition Phase, equipment and subsystem details and test data will initially become available to supplement and verify many engineering analyses performed previously. It is also during this Phase that the technical activity of system effectiveness monitoring is initiated. Basic monitoring tasks to be performed on a continuous basis, some of which involve refinements of details and updating, are the following:

(1) Mission definition and analysis

(2) System description

(3) Design and performance parameters and accountable factor updating and analysis, including reapportionment

(4) Model expansion to sub-model

(5) Effectiveness analysis

(6) Effectiveness parameter evaluation

(7) Test plan analysis

(8) Model exercise for effectiveness parameter tracking

(9) Status reporting

Only tasks (1), (2), (7), and (8) will be described herein since the technical aspects and composition of the other tasks have been fully described in this report.

5.3.3 Mission Definition and Analysis

The mission definition established during the Contract Definition Phase, and represented as one or more principal figures of merit, will require an expanded mission analysis to define fully the framework in which the performance parameters will be measured. The expanded mission analysis involves the following areas:

- Delineation of the mission parameters in detail
- Particularized development of the mission profile
- Establishment of stress adjustment factors to relate test data to actual environmental exposures

The mission parameters to be defined include, among others, the functional requirements, the success criteria, the mission phases (if any), the event sequences and times, the operating modes, and the environmental levels and duration. The mechanics of performing a mission analysis is aided by the preparation of a tabular chart for these items.

The mission profile is a summary of the information needed for converting test times into equivalent mission cycles. Typical fields of information required are shown in Table 5-1.

The stress adjustment routine illustrated in Table 5-2 combines the mission profile data with stress adjustment factors to be established from guidelines such as MIL-HDBK-217A and the RADC Reliability Notebook. The translation of test data

Table 5-1

Mission Profile

Functions/ Environments	Equipments	Mission Phases and Times		
		1	2	3
Functions		5 minutes *	20 minutes*	5 minutes*
Vehicle Control (Command Link)	Receiver	5	20	5
	Translator	5	20	5
	Programmer	5	20	5
	Baroswitch	1 cycle	3 cycles	0
Tracking	Pulse Generator	1	5	2
	Flashing Light	1	5	2
	Beacon	1	5	2
Vehicle Operation	Vehicle Controller	5	20	5
	Propulsion	5	20	5
Power Supply	Battery	5	20	5
Payload Operation	TV Camera	0	20	0
Monitoring	Sensors	5	20	0
	Multicoder	5	20	0
	Amplifier	5	20	0
	Transmitter	3	12	0
Environments				
High Temp.		1	20	2
Vibration		0	10	0

*Mission duration.

Table 5-2

Stress Adjustments

Subsystem	Mission Phase: Pre-Launch Check-out (0)	1st Stage Boost (1)	Coast (2)	2nd Stage Boost (3)	Coast & Reorient. (4)	3rd Stage Boost (5)	Coast (6)	During Re-entry (7)	Equiv. Mission Time
Guidance and Control	///	///							410
Inertial Reference and Attitude Control	///	///	///						416
Guidance	///	///	///	///					576
Functional Controller	///	///	///	///	///				596
Command and Control	///	///	///	///	///	///			676
Attitude Control	///						///		20
Experiment Package	///							///	93
Telemetry	///	///	///	///	///	///	///	///	769
Duration	10 min	5 min	6 min	2 min	20 min	1 min	10 min	5 sec	
Stress Factor	1	80	1	80	1	80	1	1000	

into total equivalent mission cycles for the various equipments and subsystems of the system can then be determined. The reciprocal of the total equivalent mission time becomes the factor for converting total test time into equivalent missions. The stress factors accommodate the usage of practically all test data during the Acquisition Phase.

5.3.4 System Description

The major equipment level is probably the lowest hardware level of the system against which data should be accumulated for monitoring purposes. These equipments will require full enumeration, and their functional requirements should be exhaustively defined. Also, all block diagrams of the system should now be refined to include details such as signal flows and redundancy. A fully sufficient system description is a requisite for any technically valid effectiveness evaluation and reapportionment of requirements.

5.3.5 Test Plan Analysis

The need for an integrated approach to planning and conducting of test programs is vital for any system being developed to an effectiveness requirement. The results of tests which are integrated can be used for multi-evaluation of many system performance requirements, including effectiveness. Thus, a maximum return of performance data proportionate with the resources applied can be obtained.

As the full details of the test program evolve, an analysis is required to ensure sufficiency in the following areas:

- The quantity of tests suitable for effectiveness evaluations
- The expected mission equivalent operating time or cycle in each environment. This will reveal the confidence to which performance values can be demonstrated in each environment for each of the effectiveness parameters during development.

- A reasonably balanced test program with respect to the equivalent mission time of each environmental exposure for each equipment.
- Test data documentation details for complete identification and integration into data processing systems.

An environmental test exposure versus test time matrix for each major equipment can be prepared and maintained.

5.3.6 Model Exercise, Availability and Dependability Tracking

The growth of system availability and dependability can be tracked based upon test data, augmented by simulations of deterministic events such as maintenance considerations. The probabilistic values for availability and dependability at regular points in the development program can be determined using test failure rate data. Assumptions of system additivity of stress effects and constant failure rate, where possible, can simplify the evaluations.

With respect to the additivity of stress effects, the failure rate induced by two simultaneously acting stresses can be considered as equal to the sum of the failure rates of the two stresses acting sequentially. This assumption permits adding of data from single environment tests to simulate actual mission exposure. Thus, the failure rate (λ) of the equipment for high temperature (h) and vibration (v) exposure can be considered as:

$$\lambda_{(h + v)} = \lambda_h + \lambda_v$$

Failure rates can be estimated from sample test data. If ΣX is a count of failures observed during test, and Σt is the total test duration in terms of equivalent mission cycle time, then:

$$\lambda = \frac{\Sigma X}{\Sigma t}$$

Under typical test conditions, where information becomes available on a piecemeal basis, and where many tests are stopped immediately upon failure and others are continued in the absence of failure, the failure rate estimate is subject to bias. This bias is most crucial during the early stages of a test program when data are fragmentary and will require correction.

5.3.7 Model Exercise, Capability Tracking

The tracking of the capability parameter is related to an assessment of the degree to which the spectrum of performance parameters can either meet established tolerances or exceed specified minimum levels. Both criteria are a function of accountable factor performance. During the Acquisition Phase, test data at the system level will be a premium. Thus, maximum use is required of equipment and subsystem test data to determine on a continuous and variables basis, the validity of the distribution assumptions and the current means and variances of the accountable factors. Through the defined transfer functions and sensitivity functions, the resultant capability achieved for each of the key system performance parameters can be estimated and monitored, using where necessary simulation exercises.

Capability parameter technical accomplishments can be displayed in the form of parameter tracking charts which show the parameters' convergence progress towards the system apportioned requirements. Where a set of critical or sensitive design parameters will contribute significantly to the capability parameters, then such contributions should be individually shown, as well as the composite influence on growth. The values achieved for the capability parameters can be rated on a continuous basis for the purpose of keying those critical areas for special management attention and development action. For instance, if desired, a rating factor of 10 may be given a parameter which has met or exceeded the apportioned requirement; a rating of 9, for one which is 90 to 99 percent achieved; a rating of 8, for an 80 to 89 percent achieved parameter; and so forth.

Most capability growth curves will reflect discrete improvement steps. This typifies the situation where a restraining accountable factor is eliminated by a change to an equipment design with greater precision, or to compensating design features.

Once the tolerance limits for the accountable factors and transfer functions have been fully defined, such that subsystem and equipment performance within these tolerances will ensure that the capability parameters will meet or exceed the requirements, then it may be appropriate to use the equivalent mission success concept described herein. This will provide a more convenient method for determining the probability that a capability parameter will be exceeded. Furthermore, the aspects of confidence statements should be considered at this time.

5.3.8 Effectiveness Monitoring

Finally, with the periodic evaluation of the constituent parameters of effectiveness, a corresponding exercise of the system model will provide the level of system effectiveness growth attained at the discrete status points.

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13. ABSTRACT

→ This report describes the methods and analytical techniques relative to Air Force system designs under a system effectiveness requirement, with the objective of providing design criteria at the subsystem and equipment levels. Presented are the relevant background concepts and general application considerations necessary for an understanding and implementation of a system effectiveness evaluation. The techniques for the objective apportionment of a system's figures of merit to their constituent parameters and accountable factors are described and illustrated in detail, with primary emphasis on the developed Lagrange multiplier with priority list solution method. The technical role and perspective of system functional transfer equations, methods for their application and use in the evaluation and apportionment of system effectiveness are rendered and demonstrated in detail. A plan for the dynamic monitoring and status reporting of system effectiveness progress during all phases of system development, and to provide management and design visibility on critical and sensitive problem areas, is outlined. ( ) Additionally, the methods and techniques are further illustrated and expanded in the Technical Supplement to this report.

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