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DESIGN CRITERIA FOR SYSTEM EFFECTIVENESS

Volume II Technical Supplement

Allen Chop
Lockheed Missiles & Space Company

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BINARY REPRESENTATION OF DEPENDABILITY STATES

A missile squadron is composed of N missiles from which up to M missiles are directed to target assignments T_1, T_2, ..., T_M in order of descending priority of assignment. An assignment is said to be completed if the missile involved results in a detonation. There are 2^M different ways that M assignments can be completed or not completed. These correspond to 2^M dependability states. The following describes the representation of the states by binary numbers.

Consider two lines of numbers. On the first line are the numbers from 1 to M. On the second line is a number 0 or 1 corresponding to each number in the first line. The number appearing on the second line under 1 on the first line will be 1 if, and only if, the missile with the assignment of highest priority results in a detonation. Likewise, the second number on the second line under the 2 on the first line will be 1 if, and only if, the missile with the assignment of second highest priority results in a detonation. This process is continued until the second line is completed. The 2^M ways in which the second line may appear when complete correspond to the 2^M dependability states of the missile system. It is convenient to denote each of these states with a different number ranging from 1 to 2^M. In the first such state, each digit of the second line is zero; in the second state, each digit is zero except the last digit on the right, which is one; etc. An alternate representation of these 2^M states is to number the states from 0 to 2^M-1.

The representation of states may be accomplished by writing the second line as a binary number, and to consider each 0 or 1 on this line as a digit of this binary number. The value of a binary number is then the sum of all numbers 2^{p-1} such that a 1 appears as the p^{th} digit from the right of the binary number. For example, 1 0 1 0 0 1 is binary for $2^{1-1} + 2^{4-1} + 2^{6-1} = 2^0 + 2^3 + 2^5 = 1 + 8 + 32 = 41$.

The letter k corresponding to a dependability state is 1 more than the binary number on the second line, as previously described, corresponding to that state. Hence, if the maximum number of assignments M is 6, and if only the assignments of 1st, 3rd, and 6th highest priority result in detonations, the second line will be 1 0 1 0 0 1, and the number k of the dependability state will be 41 + 1 = 42. Thus, the k^{th} dependability state is the condition in which, for any integer p, the target assignment of p^{th} priority is completed to detonation, if, and only if, the p^{th} binary digit of k-1 is 1. Thus:

$$b_p\,(k-1) = 1 \longrightarrow \text{detonation}$$

$$b_p\,(k-1) = 0 \longrightarrow \text{no detonation}$$

where (b_p) is the p^{th} binary digit of k-1 expressed as an M digit binary number, so that

$$k-1 = \sum_{p=1}^{M} \left[b_p\,(k-1) \right] 2^{M-p}$$

from which the number of the k^{th} dependability state is determined.

APPENDIX B

METHODS FOR EVALUATING THE CAPABILITY PARAMETER

The Monte Carlo Method

The two common distributions usually assumed for missile system accountable factors are the normal and uniform distributions. In the Monte Carlo method, random numbers are generated for a normal distribution or a uniform distribution as appropriate for the accountable factors. By a suitable transformation, a random variable having the same distribution as any accountable factor α_i can be generated. Each random variable should be independent of all other random variables generated.

In the evaluation of the conditional overall kill probabilities, C_{km} or the expectant amount of kill C_k, and the subsequent calculation of a figure of merit E, a large number n of Monte Carlo simulations may be required. For each simulation, the value for each accountable factor α_i is replaced by a random number generated in such a manner that the random numbers have the same probability frequency distribution as α_i. Then a full calculation, using a flight trajectory program, followed by a calculation of computed amount of kill, and consequently the C matrix, and the effectiveness figure of merit is accomplished. This constitutes one simulation. Often, updated simulations are performed, with only small changes in the α_i, based upon the interpolation of results obtained in previous simulations.

The method for computing the amount of kill for any one simulation is as follows. The amount of target killed in each target hit is given by (for the n^{th} simulation):

$$E_{k,n} = \iint_{\left(X - X_{I,n}\right)^2 + \left(Y - Y_{I,n}\right)^2 \leq L_{R,n}^2} V(X, Y)\, dX dY \tag{1}$$

where $\left(L_{R,n}\right)$ is the lethal radius for the n^{th} simulation, and is given by the equation:

$$\frac{L_R}{\text{feet}} \cong 204 \left(\frac{\text{yield}}{\text{tons of TNT}}\right)^{1/3} \left(\frac{\text{hardness of Target at } (\overline{X}, \overline{Y})}{\text{lb./in.}^2}\right)^{-3/8}$$

$\left(X_{I,n}, Y_{I,n}\right)$ is the impact point for the n^{th} simulation by either the trajectory program or an approximation thereof, and $V(X, Y)$ is the concentration of target value. If a point target is involved, $E_{k,n}$ is equal to the target value V_{Ω} or 0 according to whether or not

$$\left(X_v - X_{I;n}\right)^2 + \left(Y_v - Y_{I,n}\right)^2 \leq L_{R,n}^2$$

where $\left(X_v, Y_v\right)$ is the target point. If the target is a bivariate normal distribution, the following substitutions are made:

$$\sigma_{V_1(X)} = \sigma_{V(X)}$$

$$\sigma_{V_1(Y)} = \sigma_{V(Y)}$$

$$\sigma_{V_1(XY)} = \sigma_{V(XY)}$$

Then the probability for any event occurring is approximately the number of simulations for which it occurs divided by N, the number of simulations. This approximation tends to improve as N gets larger. For example, if a total kill of 90% or more was calculated in .95 N of the N simulations, then, it can be estimated that the probability that the 90% level will be reached or exceeded is .95.

As another example, suppose an estimation of the expectant amount of total kill is of interest. If $E_{k,n}$ represents amount of kill according to the n^{th} of the N simulations, then the estimated value of kill is

$$E^*_k = \frac{1}{N} \sum_{n=1}^{N} E_{k,n}$$

The standard deviation of expected kill can be determined by:

$$\sigma^*\left(E_k\right) = \left[\frac{1}{N-1} \sum_{n=1}^{N} \left(E_{k,n} - E^*_k\right)^2\right]^{1/2}$$

The root mean square error in E^*_k, i.e., the estimated difference between E^*_k and E_k, the expectant value of kill for an unlimited number of simulations, is:

$$\Delta\left(E_k\right) = \sigma^*\left(E_k\right) / \sqrt{N}$$

Therefore, the deviation from the value E_k of the expectant value of kill E^*_k, as calculated from the Monte Carlo simulations and which would be obtained if an unlimited number of simulations could be run, is proportional to $1/\sqrt{N}$. Hence accuracy improves slowly with the number of simulations. A rough approximation with about 10% σ error can be expected from 100 simulations. It is important to note that $C_{k,m}$, E, and all other probabilities and expectant values, such as errors in range and track, may be estimated by the above described methods.

Direct Methods

In a direct method, the sensitivity of capability parameters such as the accuracy error X_I in track and the error Y_I in range to small errors in the accountable factors α_i can be determined. This is accomplished by performing many calculations (simulated flights) by means of a trajectory program of

$$X_I = X_I\left(\alpha_1, \alpha_2, \ldots\ldots, \alpha_m\right)$$

and

$$Y_I = Y_I\left(\alpha_1, \alpha_2, \ldots\ldots, \alpha_m\right)$$

In the first such flight, all α_i are held at their nominal value, i.e., $\alpha_i = \mu_{\alpha_i}$, the mean expectant value for the given design and conditions. For each of the m accountable factors α_i, more simulated flights may be calculated, with a simulation performed with α_i equal to its nominal value plus or minus one or more multiples of the standard aleviation σ_{α_i} (normally $\pm 1\sigma_{\alpha_i}$, $\pm 2\sigma_{\alpha_i}$, and $\pm 3\sigma_{\alpha_i}$). In each such simulation all other accountable factors are held at their nominal values. If $\Delta\alpha_i$ is designated as the deviation from the nominal value of the accountable factor α_i, then

$$\Delta\alpha_1 = \alpha_1 - \mu_{\alpha_1}$$

Further, if X^*_I and Y^*_I are the values of impact errors X_I and Y_I measured from the position of impact which results if all accountable factors have their nominal values, then

$$X^*_I\left(\Delta\alpha_1, \ldots\ldots, \Delta\alpha_m\right) = X_I\left(\alpha_1, \ldots\ldots, \alpha_m\right) - X_I\left(\mu_{\alpha_1}, \ldots\ldots, \mu_{\alpha_m}\right)$$
$$Y^*_I\left(\Delta\alpha_1, \ldots\ldots, \Delta\alpha_m\right) = Y_I\left(\alpha_1, \ldots\ldots, \alpha_m\right) - Y_I\left(\mu_{\alpha_1}, \ldots\ldots, \mu_{\alpha_m}\right)$$

Normally these simulations would be sufficient to describe range and track errors for small deviations from the nominal values, $\Delta\alpha_i$, except that the influence of perturbing some accountable factors are not mutually additive. Thus, for $X^*_I\ (0, 0, \ldots, 0) = 0$, it is not always true that:

$$X^*_I\left(\Delta\alpha_1, \Delta\alpha_2, 0, \ldots\ldots, 0\right) = X^*_I\left(\Delta\alpha_1, 0, 0, \ldots\ldots, 0\right) + X^*_I\left(0, \Delta\alpha_2, 0, \ldots\ldots, 0\right)$$

If the above is not true for practical purposes, then the factors $\Delta\alpha_1$ and $\Delta\alpha_2$ are coupled, and a simulated flight is required to determine

$$X^*_I = X^*_I\left(\Delta\alpha_1, \Delta\alpha_2, 0, \ldots\ldots, 0\right)$$
$$Y^*_I = Y^*_I\left(\Delta\alpha_1, \Delta\alpha_2, 0, \ldots\ldots, 0\right)$$

for the four cases:

(1) $\Delta\alpha_1 = 3\sigma_{\alpha_1}, \quad \Delta\alpha_2 = 3\sigma_{\alpha_2}$

(2) $\Delta\alpha_1 = 3\sigma_{\alpha_1}, \quad \Delta\alpha_2 = -3\sigma_{\alpha_2}$

(3) $\Delta\alpha_1 = -3\sigma_{\alpha_1}$, $\Delta\alpha_2 = 3\sigma_{\alpha_2}$

(4) $\Delta\alpha_1 = -3\sigma_{\alpha_1}$, $\Delta\alpha_2 = -3\sigma_{\alpha_2}$

This should be performed for all pairs $\left(\Delta\alpha_i , \Delta\alpha_j\right)$, except those known not to be coupled.

After X_I and Y_I are calculated for all the variations in accountable factors indicated above, coefficients $b_{k,s}$, $b^*_{k,s}$, $c_{k,s}$, $c^*_{k,s}$, $d_{k,s}$, $d^*_{k,s}$, $e_{k,s}$, and $e^*_{k,s}$, can be determined such that the following is true, at least for values of α for which a trajectory has been computed:

$$X^*_I = \sum_{k=1}^{m} \left[b_{k,s} \left(\Delta\alpha_k\right) + c_{k,s} \left(\Delta\alpha_k\right)^2 + d_{k,s} \left(\Delta\alpha_k\right)^3 \right] \tag{2}$$

$$+ \sum_{k=1}^{m-1} \sum_{p=k+1}^{m} \left[e_{k,p,s} \left(\Delta\alpha_k\right) \left(\Delta\alpha_p\right) \right],$$

and

$$Y^*_I = \sum_{k=1}^{m} \left[b^*_{k,s} \left(\Delta\alpha_k\right) + c^*_{k,s} \left(\Delta\alpha_k\right)^2 + d^*_{k,s} \left(\Delta\alpha_k\right)^3 \right] \tag{3}$$

$$+ \sum_{k=1}^{m-1} \sum_{p=k+1}^{m} \left[e^*_{k,p,s} \left(\Delta\alpha_k\right) \left(\Delta\alpha_p\right) \right]$$

where for any k and p

$$s = \begin{cases} 1 \text{ if } \Delta\alpha_k \geq 0 \text{ and } \Delta\alpha_p \geq 0 \\ 2 \text{ if } \Delta\alpha_k < 0 \text{ and } \Delta\alpha_p \geq 0 \\ 3 \text{ if } \Delta\alpha_k \geq 0 \text{ and } \Delta\alpha_p < 0 \\ 4 \text{ if } \Delta\alpha_k < 0 \text{ and } \Delta\alpha_p < 0 \end{cases} ,$$

and for purposes of the first summation of Equations (2) and (3) where α_p does not apply, s = 1 or 2.

Equations (2) and (3) may be utilized so that given the mean μ_{α_i} and the standard deviation σ_{α_i} for each accountable factor perturbation $\Delta\alpha_i$, the following means and standard deviations can be determined

$$\mu\,(X^*_{I,k})\ ;\ \ \mu\,(X^*_{I,k,p})\ ;\ \ \mu\,(Y^*_{I,k})\ ;\ \ \mu\,(Y^*_{I,k,p}),$$

$$\sigma\,(X^*_{I,k})\ ;\ \ \sigma\,(X^*_{I,k,p})\ ;\ \ \sigma\,(Y^*_{I,k})\ ;\ \ \sigma\,(Y^*_{I,k,p})$$

where $X^*_{I,k}$, $X^*_{I,k,p}$, $Y^*_{I,k}$, and $Y^*_{I,k,p}$ refer to the four expressions in square brackets in Equations (2) and (3). From these quantities, the mean values of impact errors $\overline{X}_I$ and $\overline{Y}_I$ and the standard deviations can be determined to be:

$$\overline{X}^*_I = \sum_{k=1}^{m}\left[\mu\,(X^*_{I,k}) + \sum_{p=k+1}^{m}\mu\,(X^*_{I,k,p})\right]\ ; \tag{4}$$

$$\overline{Y}^*_I = \sum_{k=1}^{m}\left[\mu\,(Y^*_{I,k}) + \sum_{p=k+1}^{m}\mu\,(Y^*_{I,k,p})\right]\ ; \tag{5}$$

$$\sigma_{I(X)} = \left(\sum_{k=1}^{m}\left[\sigma^2\,(X^*_{I,k}) + \sum_{p=k+1}^{m}\sigma^2\,(X^*_{I,k,p})\right]\right)^{1/2}; \tag{6}$$

$$\sigma_{I(Y)} = \left(\sum_{k=1}^{m}\left[\sigma^2\,(Y^*_{I,k}) + \sum_{p=k+1}^{m}\sigma^2\,(Y^*_{I,k,p})\right]\right)^{1/2}; \tag{7}$$

$$\sigma_{I(XY)} = \sum_{k=1}^{m}\left[\mu\left((X^*_{I,k} - \overline{X}^*_{I,k})\,(Y^*_{I,k} - \overline{Y}^*_{I,k})\right) + \sum_{p=k+1}^{m}\mu\left((X^*_{I,k,p} - \overline{X}^*_{I,k,p})\,(Y^*_{I,k,p} - \overline{Y}^*_{I,k,p})\right)\right] \tag{8}$$

where

a bar (-) denotes the mean value.

Equation (8) may be expressed as:

$$\sigma_{I\,(XY)} = \sum_{k=1}^{m} \left[c_k \; \sigma(X_{I,k}) \; \sigma(Y_{I,k}) + \sum_{p=k+1}^{m} c_{k,p} \, \sigma(X_{I,k,p}) \; \sigma(Y_{I,k,p}) \right]$$

where the c's are correlation coefficients ($-1 \le c_k \le +1$; $-1 \le c_{k,p} \le +1$). Each c is generally near +1 or -1. If $\partial X_I / \partial(\Delta\alpha_k)$ always has the same sign as $\partial Y_I/\partial(\Delta\alpha_k)$, then $c_k > 0$. If the reverse is always true, then $c_k < 0$. If $\partial X_I/\partial\left((\Delta\alpha_k)(\Delta\alpha_p)\right)$ always has the same sign as $\partial Y_I/\partial\left((\Delta\alpha_k)(\Delta\alpha_p)\right)$, then $c_{k,p} > 0$. If the reverse is always true, then $c_{k,p} < 0$.

The equations (4) through (8) give the mean and standard deviation of track X_I and range Y_I with respect to nominal impact, i.e., where all accountable factors have their nominal value and $X^*_I = Y^*_I = 0$. To convert to coordinates in X_I and Y_I:

$$X_I = X_n + X^*_I$$

$$Y_I = Y_n + Y^*_I$$

$$\overline{X}_I = X_n + \overline{X}^*_I$$

$$\overline{Y}_I = Y_n + \overline{Y}^*_I$$

where (X_n, Y_n) represents the nominal impact point.

Thus, the means and standard deviations of track X_I and range Y_I of the accuracy parameter can be determined from the means and standard deviations of the accountable factors.

Combination of Monte Carlo and Direct Method

Sometimes it is desirable to combine the usage of the direct method with the Monte Carlo (MC) method. The advantage of the direct method is that generally fewer calculations are needed, as compared with MC, to achieve an adequate precision and confidence in the results. However, in the direct method, the assumption is made that the effects of the various accountable factors, or even of pairs of accountable factors, are additive, which is not always the case.

For example, suppose that, for the k^{th} state of dependability, the distributions of the impact of each of M_o missiles resulting in detonation are determined by the direct method as described, and that each such distribution is thereby known to be of the form:

$$P_I(X, Y) = \frac{1}{2\pi\sigma_{I_1}\sigma_{I_2}} \exp\left[-\frac{\hat{I}_1^2}{2\sigma_{I_1}^2} - \frac{\hat{I}_2^2}{2\sigma_{I_2}^2}\right] \qquad (9)$$

with

$$\hat{I}_1 = \left(X - \bar{X}_I\right) \text{Cos}\ \theta_I + \left(Y - \bar{Y}_I\right) \text{Sin}\ \theta_I \qquad (10)$$

$$\hat{I}_2 = -\left(X - \bar{X}_I\right) \text{Sin}\ \theta_I + \left(Y - \bar{Y}_I\right) \text{Cos}\ \theta_I \qquad (11)$$

$$\theta_I = 1/2 \text{ arctan}\left[2\sigma_{I(XY)} \Big/ \left(\sigma^2_{I(X)} - \sigma^2_{I(Y)}\right)\right] \qquad (12)$$

$$\sigma_{I_1} = \sqrt{\frac{\sigma^2_{I(X)} + \sigma^2_{I(Y)}}{2} + \sqrt{\left(\frac{\sigma^2_{I(X)} - \sigma^2_{I(Y)}}{2}\right)^2 + \sigma^2_{I(XY)}}} \qquad (13)$$

$$\sigma_{I_2} = \sqrt{\frac{\sigma^2_{I(X)} + \sigma^2_{I(Y)}}{2} - \sqrt{\left(\frac{\sigma^2_{I(X)} - \sigma^2_{I(Y)}}{2}\right)^2 + \sigma^2_{I(XY)}}} \qquad (14)$$

Then a number of MC simulations can be performed, as follows. First, two random, normally distributed variables, $r_{1,n}$ and $r_{2,n}$, with mean 0 and standard deviation 1 are generated for each impacting missile n, for $n = 1, 2, \ldots, M_o$. Then the impact point $\left(X_{I,n}, Y_{I,n}\right)$ of each $\left(n^{th}\right)$ impacting can be calculated in terms of $\left(\sigma_{I(XY)}\right)_n$, $\left(\sigma_{I(X)}\right)_n$, $\left(\sigma_{I(Y)}\right)_n$, $\bar{X}_{I,n}$, $\bar{Y}_{I,n}$, $r_{1,n}$, and $r_{2,n}$ as follows:

Calculate:

(a) $\theta_{I,n}$ using Equation (12)

(b) $\sigma_{I,1,n}$ and $\sigma_{I,2,n}$ using Equations (13) and (14) (15)

(c) $\hat{I}_{1,n} = r_{1,n}\,\sigma_{I,1,n}$; $\hat{I}_{2,n} = r_{2,n}\,\sigma_{I,2,n}$

(d) $$\begin{bmatrix} X_{I,n} \\ Y_{I,n} \end{bmatrix} = \begin{bmatrix} \bar{X}_{I,n} \\ \bar{Y}_{I,n} \end{bmatrix} + \begin{bmatrix} \text{Cos}\ \theta_{I,n} & -\ \text{Sin}\ \theta_{I,n} \\ \text{Sin}\ \theta_{I,n} & \text{Cos}\ \theta_{I,n} \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix}$$

Sometimes, due to influences such as wind biases on two or more missiles, say 1 and 2, the impact points may not be independent. For this situation, the impact point for each can be considered to be the resultant sum of a vector common to both 1 and 2, say $\left[X_{I,1,2}, Y_{I,1,2}\right]^*$ plus a different independent vector, $\left[X_{I,1}, Y_{I,1}\right]^*$ and $\left[X_{I,2}, Y_{I,2}\right]^*$, for each missile, i.e.

$$X_{I,1} = X^*_{I,1} + X^*_{I,1,2}\ ;\ X_{I,2} = X^*_{I,2} + X^*_{I,1,2}$$
$$Y_{I,1} = Y^*_{I,1} + Y^*_{I,1,2}\ ;\ Y_{I,2} = Y^*_{I,2} + Y^*_{I,1,2} \qquad (16)$$

In such a case, $\left[X_{I,1,2}, Y_{I,1,2}\right]^*$, $\left[X_{I,1}, Y_{I,1}\right]^*$, $\left[X_{I,2}, Y_{I,2}\right]^*$ are then all calculated by the direct method and Equation (15) separately, and then added according to Equation (16). After the impact position of each missile is simulated, the total amount of target destroyed, $\left(\sum E_k\right)$, can be calculated by adding the value of all target locations destroyed, or by integrating or summing over the area destroyed. A target point $\left(X_T, Y_T\right)$ is interpreted as being destroyed if it is within the lethal radius of any impact point, namely if for any $n = 1, 2, \ldots, N$,

$$L^2_{R,n} \geqslant \left(X_T - X_{I,n}\right)^2 + \left(Y_T - Y_{I,n}\right)^2,$$

where $L_{R,n}$ is the lethal radius for the n^{th} missile, depending on its payload and yield, and is given by:

$$\frac{L_{R,n}}{\text{feet}} \cong 204 \left(\frac{\text{yield}}{\text{tons of TNT}}\right)^{1/3} \left(\frac{\text{hardness of target at } \left(X_T, Y_T\right)}{\text{lb/in}^2}\right)^{-3/8}$$

After Z simulations are accomplished, the probability can be estimated that the total amount of damage, $\sum E_k$, will exceed a specified limit value. This probability is $\frac{1}{Z}$ times the number of simulations in which $\sum E_k$ exceeds this limit value.

This quantity is the element $C_{k,m}$ of the capability matrix, and is the probability, given the k^{th} dependability state, that the m^{th} level of destruction limit will be exceede... When all the elements $C_{k,m}$ are calculated as described, it is possible to calculate the figure of merit E, and to test whether any system constraints are satisfied.

APPENDIX C

RELATIONSHIP OF ALTERNATE FORMULAS FOR THE CONCENTRATION OF TARGET VALUE V(X, Y)

The concentration of the target value at the point (X, Y) was designated in Section 2 as V(X, Y) with the following two alternate formulas shown:

$$V(X,Y) = \frac{V_{\Omega}}{2\pi\sigma_{V_1}\sigma_{V_2}} \exp\left[-\frac{\hat{V}_1^2}{2\sigma_{V_1}^2} - \frac{\hat{V}_2^2}{2\sigma_{V_2}^2}\right] \qquad (1)$$

$$= \frac{V_{\Omega}}{2\pi\sqrt{M_2}} \exp\left[-\frac{1}{2M_2}\left(\sigma^2_{V(Y)} X_2^2 - \sigma_{V(XY)} X_2 Y_2 + \sigma^2_{V(X)} Y_2^2\right)\right] \qquad (2)$$

where

V_{Ω} = total value of target

σ_{V_1} = standard deviation of elliptical target distribution along the major axis

σ_{V_2} = standard deviation of target distribution along minor axis

L_R = lethal radius of missile

$(\bar{X}_V, \bar{Y}_V)$ = a point of distance L_R from the impact point of the missiles

$\sigma^2_{V(X)} = \frac{1}{V_{\Omega}^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X,Y)(X - \bar{X}_V)^2 \, dX\, dY$

= variance of target value along major axis

$\sigma^2_{V(Y)} = \frac{1}{V_{\Omega}^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X,Y)(Y - \bar{Y}_V)^2 \, dX\, dY$

= variance of target value along minor axis

$\sigma_{V(XY)} = \frac{1}{V_{\Omega}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(X,Y)(X - \bar{X}_V)(Y - \bar{Y}_V) \, dX\, dY$

= covariance of target value

$M_2 = \sigma^2_{V(X)}\sigma^2_{V(Y)} - \sigma^2_{V(XY)}$

The following is the relationship between Equations (1) and (2):

The transformation from the (X, Y) to the $(\hat{V}_1, \hat{V}_2)$ coordinate system is given by

$$\begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} \text{Cos } \theta_V & \text{Sin } \theta_V \\ -\text{Sin } \theta_V & \text{Cos } \theta_V \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix}$$

where

$$X_2 = X - \bar{X}_V$$

$$Y_2 = Y - \bar{Y}_V$$

and θ_V is the angle of rotation for the transformation. The standard deviations of $\hat{V}_1$ and $\hat{V}_2$, each of which has zero mean, satisfies

$$\begin{bmatrix} \sigma_{V_1} & 0 \\ 0 & \sigma_{V_2} \end{bmatrix} = \text{mean} \left\{ \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} \begin{bmatrix} \hat{V}_1 & \hat{V}_2 \end{bmatrix} \right\}$$

$$= \text{mean} \left\{ \begin{bmatrix} \text{Cos}\theta_V & \text{Sin } \theta_V \\ -\text{Sin } \theta_V & \text{Cos}\theta_V \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \begin{bmatrix} X_2 & Y_2 \end{bmatrix} \begin{bmatrix} \text{Cos } \theta_V & -\text{Sin } \theta_V \\ \text{Sin } \theta_V & \text{Cos } \theta_V \end{bmatrix} \right\}$$

$$= \begin{bmatrix} \text{Cos } \theta_V & \text{Sin } \theta_V \\ -\text{Sin } \theta_V & \text{Cos}\theta_V \end{bmatrix} \text{mean} \left\{ \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \begin{bmatrix} X_2 & Y_2 \end{bmatrix} \right\} \begin{bmatrix} \text{Cos } \theta_V & -\text{Sin } \theta_V \\ \text{Sin } \theta_V & \text{Cos } \theta_V \end{bmatrix}$$

$$= \begin{bmatrix} \text{Cos } \theta_V & \text{Sin } \theta_V \\ -\text{Sin } \theta_V & \text{Cos } \theta_V \end{bmatrix} \begin{bmatrix} \sigma^2_{V(X)} & \sigma_{V(XY)} \\ \sigma_{V(XY)} & \sigma^2_{V(Y)} \end{bmatrix} \begin{bmatrix} \text{Cos } \theta_V & -\text{Sin } \theta_V \\ \text{Sin } \theta_V & \text{Cos } \theta_V \end{bmatrix} \quad (3)$$

The angle θ_V is to be chosen such that Equation (3) holds. For this to be true, it is necessary that $\sigma_{V_1}^2$ and $\sigma_{V_2}^2$ be eigenvalues of the second to the last matrix in Equation (3). That is, the following determinant should be zero if $\lambda = \sigma^2_{V_1}$ or $\lambda = \sigma^2_{V_2}$:

$$\begin{vmatrix} \sigma^2_{V(X)} - \lambda & \sigma_{V(XY)} \\ \sigma_{V(XY)} & \sigma^2_{V(Y)} - \lambda \end{vmatrix}$$

From this determinant and the quadratic formula,

$$\lambda = \frac{\sigma^2_{V(X)} + \sigma^2_{V(Y)}}{2} \pm \sqrt{\left(\frac{\sigma^2_{V(X)} - \sigma^2_{V(Y)}}{2}\right)^2 + \sigma^2_{V(XY)}}$$

with σ_{V_1} and σ_{V_2} being the positive square roots of the two solutions of λ .

From the relationship of Equation (3), then

$$\sigma^2_{V(X)} \text{Sin } \theta_V \text{ Cos } \theta_V + \sigma_{V(XY)} (\text{Sin}^2 \theta_V - \text{Cos}^2 \theta_V) - \sigma^2_{V(Y)} \text{Sin}\theta_V \text{ Cos}\theta_V = 0$$

$$\sigma_{V(XY)} \text{Tan}^2 \theta_V + \left(\sigma^2_{V(X)} - \sigma^2_{V(Y)}\right) \text{Tan } \theta_V - \sigma_{V(XY)} = 0$$

$$\text{Tan } (2\theta_V) = 2\sigma_{V(XY)} \Big/ \left(\sigma^2_{V(X)} - \sigma^2_{V(Y)}\right)$$

Therefore,

$$\theta_V = 1/2 \text{ arctan} \left[2\sigma_{V(XY)} \Big/ \left(\sigma^2_{V(X)} - \sigma^2_{V(Y)}\right) \right]$$

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APPENDIX D

DIRECT CALCULATION OF THE EXPECTANT PROPORTION OF TARGET DAMAGE

The following provides the method for the direct calculation of the expectant proportion of target damage, $P_r\ (\mu_X, \mu_Y, \sigma_Y, R)$

$$P_r\ (\mu_X, \mu_Y, \sigma_Y, R) = \frac{1}{2\pi\sigma_Y} \iint_{S'} \exp - \left[\frac{1}{2}\left((X - \mu_X)^2 + (Y - \mu_Y)^2 / \sigma^2_Y\right)\right] dXdY$$

where

S' is the region of (X, Y) such that $X^2 + Y^2 \leq R^2$ and $\sigma_X = 1$.

Let

$$X^* = X - \mu_X,\ Y^* = (Y - \mu_Y) / \sigma_Y$$

Then

$$P_r = \frac{1}{2\pi} \iint_{S'} \exp \left[-\frac{1}{2}(X^{*2} + Y^{*2})\right] dX^* dY^*,$$

where

S' is the region such that

$$(X^* + \mu_X)^2 + (Y^* \sigma_Y + \mu_Y)^2 \leq R^2$$

Let

$$r = (X^{*2} + Y^{*2})^{\frac{1}{2}},$$

$$\theta = \cos^{-1} (X^*/r) = \sin^{-1} (Y^*/r) = \tan^{-1} (Y^*/X^*).$$

Then

$$P_r = \frac{1}{2\pi} \iint_{S'} \exp\left[-\frac{1}{2} r^2\right] r \, dr \, d\theta$$

$$= \frac{1}{2\pi} \int_{\text{boundary of } S'} \left(1 - \exp\left[-\frac{1}{2} \left[r(\theta)\right]^2\right]\right) d\theta,$$

or

$$P_r = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \phi_1(\theta) \, d\theta$$

where

$\phi_1(\theta)$ is determined for any θ as follows:

$(r \cos\theta + \mu_X)^2 + (r\sigma_Y \sin\theta + \mu_Y)^2 = R^2$

To solve for r,

$$r = \left[-b \pm \sqrt{b^2 - 4ac}\right] / 2a$$

$$a = 1 + (\sigma^2_Y - 1)\sin^2\theta,$$

$$b = 2\left[\mu_X \cos\theta + \sigma_Y \mu_Y \sin\theta\right]$$

$$c = \mu^2_X + \mu^2_Y - R^2$$

The solutions for r are designated r_1, r_2 with $|r_1| \le |r_2|$. There are three cases:

(1) if there are no real solutions, then $\phi_1(\theta) = 0$;

(2) if r_1 and r_2 are of the same sign, then $\phi_1(\theta) = \exp\left[-\frac{1}{2} r_1^2\right] - \exp\left[-\frac{1}{2} r_2^2\right]$;

(3) if r_1 and r_2 are of different sign, then $\phi_1(\theta) = 2 - \exp\left[-\frac{1}{2} r_1^2\right] - \exp\left[-\frac{1}{2} r_2^2\right]$.

APPENDIX E

EXAMPLE OF FIGURE OF MERIT EVALUATION

This example illustrates the calculations for the expectant amount of damage rendered by a squadron of ten missiles on seven targets. The system figure of merit is that this expectant amount of damage shall be 85% or greater.

Suppose there are seven targets within the region covered by a missile squadron, and that these targets are sufficiently removed so that there is no duplication of kill between targets. Assume that five targets are area targets, and the other two are point targets, with the five area targets all having bivariate normal distributions and dispersions $\sigma_{V(X)}$ and $\sigma_{V(Y)}$. For the two point targets, $\sigma_{V(X)} = \sigma_{V(Y)} = 0$. When $\sigma_{V(X)} = \sigma_{V(Y)}$ for an area target, it is circular, with approximately 95% of the target value within 2.4477 $\sigma_{V(X)}$ of the target center $(\overline{X}, \overline{Y})$. If $\sigma_{V(X)}$ and $\sigma_{V(Y)}$ are not equal, the target is elliptical in shape. The value of each target is designated V_{Ω}. Each target is assigned one or more numbers to indicate the target priority, with 1 indicating the highest priority. A maximum of 10 missiles is to be sent to the 7 targets. Also, assume the target parameters given in Table E-1 with $\sigma_{V(XY)} = 0$.

Each missile has equipments $S_{i,j}$ as indicated in Table E-2, together with the given probability $a(S_{i,j})$ that the equipment is available (ready) at time of launch. Also, the equipments have failure rates $\lambda_{i,j}$ during the operation phase, and are required to operate for 500 seconds. The following calculations do not assume that equipment $S_{i,j}$ is necessarily identical with $S_{i,j+1}$, or that $\lambda_{i,j} = \lambda_{i,j+1}$.

<u>Calculation of Availability and Dependability</u>

The quantity $\hat{D}_p(S_{i,j})$, the dependability of equipment $S_{i,j}$ with no presupposition of availability is independent of p (namely, which missile is assigned to the p^{th} target) because flight times are equal. This quantity is given by the following formula and is tabulated in Table E-2.

$$\hat{D}_p(S_{i,j}) = a(S_{i,j})\, D_{p,i,j} = a(S_{i,j}) \exp\left[-\lambda_{i,j}\, t_p\right]$$

TABLE E-1 ASSUMED TARGET PARAMETERS

Target (i)	$\sigma_{V(X)}$ (feet)	$\sigma_{V(Y)}$ (feet)	Structures In Target V_Ω	Priority p (i, j)	Effective Hardness (lb. /in^2)
1	10,000	15,000	20,000	6	3.0
2	10,000	20,000	85,000	2, 10	2.0
3	18,000	18,000	43,000	4	3.0
4	0	0	12,000	7	3.0
5	0	0	21,000	5	3.0
6	10,000	15,000	666,000	1,8,9	1.5
7	6,000	6,000	58,000	3	2.0

TABLE E-2 CALCULATED AVAILABILITY AND DEPENDABILITY VALUES FOR MISSILE EQUIPMENT

Equipment $S_{i,j}$	$a(S_{i,j})$	$\lambda_{i,j}$ (per 10^6 sec.)	$\exp[-\lambda_{i,j} t_p]$	$\hat{D}_p(S_{i,j})$
$S_{1,1}$	.999	40	.980	.979
$S_{1,2}$	.998	24	.988	.986
$S_{1,3}$	.997	52	.974	.971
$S_{2,1}$	.990	32	.984	.974
$S_{2,2}$	.998	20	.990	.988
$S_{3,1}$	.997	48	.976	.973
$S_{3,2}$	.990	28	.986	.976
$S_{3,3}$	.999	24	.988	.987
$S_{4,1}$	.996	52	.974	.970
$S_{5,1}$	.999	28	.986	.985
$S_{5,2}$	.998	40	.980	.978
$S_{5,3}$	.996	32	.984	.980
$S_{5,4}$	.992	20	.990	.982

Then a_m, the availability of each of the ten missiles, and D_p^*, the dependability of each of the ten missiles given it was available, is calculated based on the following formulas, resulting in $a_m = .9960$ and $D_p^* = .9737$

$$a_m = \prod_{i=1}^{r} \left(1 - \prod_{j=1}^{s(i)} \left[1 - a(S_{i,j})\right]\right),$$

$$a(S_{i,j}) = MTTF_{i,j} \;/\; (MTTR_{i,j} + MTTF_{i,j}) \;.$$

Since the probability that exactly n of N missiles are available is given by:

$$A_n = a_m^n \,(1 - a_m)^{N-n} \binom{N}{n}$$

then, $$A = \left[0, 0, 0, 0, 0, 0, 0, .0007, .0385, .9608\right]$$

The probability D_p^* that missile p will be dependable if it is available is calculated by use of the formula

$$D_p^* = \left\{ \prod_{i=1}^{r} \left[- \prod_{j=1}^{s(i)} \left(1 - \hat{D}_p(S_{i,j})\right)\right]\right\} \Big/ a_m$$

The eighth, ninth, and tenth elements, respectively, of the vector A represent the probabilities that eight, nine, or ten missiles will be available. Since these elements total unity, it is practically certain there will be at least eight missiles available. Hence the elements $D_{n,k}$ of the dependability matrix for which $n < 8$, the first seven rows of the matrix, need not be determined for this example. The remaining elements of D (as defined in Section 2 of this report) are those for which $n = 8, 9, 10$ and $k = 1, 2, \ldots, 2^M$. Recalling from Section 2 that $D_{n,k}$ represents the probability of the k^{th} dependability state given that n missiles were available, the element $D_{n,k}$ is calculated according to:

$$D_{n,k} = \prod_{p=1}^{M} \left[1 - D_p^* + (2D_p^* - 1)\, b_p(k-1)\right]$$

or, if D_p^* is designated D^* because all missiles are independent of p,

$$D_{n,k} = (D^*)^{M_o} (1-D^*)^{M-M_o}$$

where $M = 10$, the number of missiles, and M_0 = number of missiles dependable.
If $b_p(k-1) = 1$ for any $p > n$, then $D_{n,k} = 0$.

For this example, the probability that the missiles are dependable during the mission, $D_p^* = .9737$, is assumed, regardless of target. As an example, the elements $D_{n,k}$ for which $n = 8, 9, 10$ and $k = 1016, 1017, \ldots, 1024$ are displayed in Table E-3.

The elements of $D_{n,k}$ are zero to four decimal places, when $n = 8, 9$, and 10, except as follows:

$D_{8,1021} = .8080$

$D_{8,k} = .0218$ for $k = $ 1017, 1013, 1005, 989, 957, 893, 765, and 509.

$D_{8,k} = .0006$ for $k = $ 1009, 1001, 985, 953, 889, 761, 505, 997, 981, 949, 885, 757, 501, 973, 941, 877, 749, 493, 925, 861, 733, 477, 829, 701, 445, 637, 381, and 253.

$D_{9,1023} = .7868$

$D_{9,k} = .02125$ for $k = $ 1021, 1019, 1015, 1007, 991, 959, 895, 767, and 511.

$D_{9,k} = .0006$ for $k = $ 1017, 1013, 1005, 989, 957, 893, 765, 509, 1011, 1003, 987, 955, 891, 763, 507, 999, 983, 951, 887, 759, 503, 975, 943, 879, 751, 495, 927, 863, 735, 479, 831, 703, 447, 639, 383, and 255.

$D_{10,1024} = .7661$

$D_{10,k} = .0207$ for $k = $ 1023, 1022, 1020, 1016, 1008, 992, 960, 896, 768, and 512.

$D_{10,k}$ = .0006 for k = 1021, 1019, 1015, 1007, 991, 959, 895, 767, 511, 1018, 1014, 1006, 990, 958, 894, 766, 510, 1012, 1004, 988, 956, 892, 764, 508, 1000, 984, 952, 888, 760, 504, 976, 944, 880, 752, 496, 928, 864, 736, 480, 832, 704, 448, 640, 384, and 256.

TABLE E-3 SOME ELEMENTS OF THE DEPENDABILITY MATRIX D

n \ k	1016	1017	1018	1019	1020	1021	1022	1023	1024
8	0	.0218	0	0	0	.8080	0	0	0
9	0	.0006	0	.02125	0	.02125	0	.7868	0
10	.0207	.0000	.0006	.0006	.0207	.0006	.0207	.0207	.7661

Each value represents the probability of a dependability outcome in state k if n missiles are available, and is the element $D_{n,k}$ of the dependability matrix.

Calculation of Capability

To determine the capability of the squadron of 10 missiles, given that the dependability outcome for the missiles is in state k, assume that there are four accountable factors, $\alpha_1, \alpha_2, \alpha_3,$ and α_4 which determine the position of burst. They may be measurements of guidance errors, maximum thrust, average wind velocities, and peak gusts. Let

$$\Delta\alpha_j = \alpha_j - \mu_{\alpha_j} \quad \text{for } j = 1,2,3, \text{ and } 4$$

where μ_{α_j} is the expectant value of the accountable factor α_j, and $\Delta\alpha_j$ is the difference between the actual value and the expectant value. Assume that the accountable factor α_j, and hence $\Delta\alpha_j$, have standard deviations σ_{α_j} of 1,2,3, and 1 units, respectively, for j = 1,2,3, and 4.

Also assume that for the following coefficients (as previously defined in Appendix B) it is stipulated that for each s,

$$b_{1,s} = 100 \text{ ft.}, \; b^*_{1,s} = 200 \text{ ft.}, \; e_{1,2,s} = -10 \text{ ft.},$$

$$b_{2,s} = 50 \text{ ft.}, \; b^*_{2,s} = 300 \text{ ft.}, \; e^*_{1,2,s} = 10 \text{ ft.},$$

$$b_{3,s} = 100 \text{ ft.}, \; b^*_{3,s} = 250 \text{ ft.}, \; e_{2,3,s} = 5 \text{ ft.},$$

$$b_{4,s} = -100 \text{ ft.}, \; b^*_{4,s} = -200 \text{ ft.}, \; e^*_{3,4,s} = -15 \text{ ft.},$$

and all other values of $e_{k,p,s}$ and $e^*_{k,p,s}$, are 0.

Given these conditions and the impact errors as defined in Appendix B,

$$\mu(X^*_{I,k}) = \mu(X^*_{I,k,p}) = \mu(Y^*_{I,k}) = \mu(Y^*_{I,k,p}) = 0$$

for all k and p; and also

$$\sigma(X^*_{I,1}) = \left|b_{1,s}\right| \sigma_{\alpha_1} = (100)\ 1 = 100,$$

$$\sigma(X^*_{I,2}) = \left|b_{2,s}\right| \sigma_{\alpha_2} = (50)\ 2 = 100,$$

$$\sigma(X^*_{I,3}) = \left|b_{3,s}\right| \sigma_{\alpha_3} = (100)\ 3 = 300,$$

$$\sigma(X^*_{I,4}) = \left|b_{4,s}\right| \sigma_{\alpha_4} = (100)\ 1 = 100,$$

$$\sigma(Y^*_{I,1}) = \left|b^*_{1,s}\right| \sigma_{\alpha_1} = (200)\ 1 = 200,$$

$$\sigma(Y^*_{I,2}) = \left|b^*_{2,s}\right| \sigma_{\alpha_2} = (300)\ 2 = 600,$$

$$\sigma(Y^*_{I,3}) = \left|b^*_{3,s}\right| \sigma_{\alpha_3} = (250)\ 3 = 750,$$

$$\sigma(Y^*_{I,4}) = \left|b^*_{4,s}\right| \sigma_{\alpha_4} = (200)\ 1 = 200.$$

Since

$$\sigma(X^*_{I,k,p}) = \left[e^2_{k,p,s}\sigma_{\alpha_k}\sigma_{\alpha_p}\right]^{1/2}, \text{ and}$$

$$\sigma(Y^*_{I,k,p}) = \left[e^{*2}_{k,p,s}\sigma_{\alpha_k}\sigma_{\alpha_p}\right]^{1/2}$$

then,

$$\sigma(X^*_{I,1,2}) = 100\sqrt{2}\ ,\ \sigma(Y^*_{I,1,2}) = 100\sqrt{2},$$

$$\sigma(X^*_{I,2,3}) = 50\sqrt{6}\ ,\ \sigma(Y^*_{I,2,3}) = 0\ ,$$

$$\sigma(X^*_{I,3,4}) = 0\ ,\ \sigma(Y^*_{I,3,4}) = 150\sqrt{3}.$$

$$\sigma(X^*_{I,k,p}) = \sigma(Y^*_{I,k,p}) = 0, \text{ except as above}$$

and,

$$\overline{X}^*_I = \overline{Y}^*_I = 0$$

$$\sigma_{I(X)} = 50\sqrt{62} = 393.7$$

$$\sigma_{I(Y)} = 100\sqrt{109} = 1044.0$$

$$\sigma_{I(XY)} = 345000.$$

The combined effect of impact and target dispersions is $\bar{X}_{V_1} = \bar{Y}_{V_1} = 0$ for all targets, with the calculated combined dispersions as shown in Table E-4.

The quantities $\tilde{\sigma}_X$ and $\tilde{\sigma}_Y$ represent the dispersions in the canonical coordinate system, the axes of which are those of the ellipse of dispersion. These are given in Table E-5.

If 1,000,000 tons of TNT (1 megaton) is the yield for each missile warhead, then the lethal radius of each missile is as indicated in Table E-5. These data are derived from the knowledge of yield and hardness for each target, as listed in Table E-1, and are calculated according to the equation:

$$\frac{L_R}{\text{feet}} \approx 204 \left(\frac{\text{yield}}{\text{tons of TNT}}\right)^{1/3} \left(\frac{\text{hardness of target at } (\bar{X}, \bar{Y})}{\text{lb/in}^2}\right)^{-3/8}$$

Assume each missile is aimed so that expectant impact is at center of target, such as to maximize expectant amount of damage. Hence $\tilde{X}$, $\tilde{Y}$, μ_X, and μ_Y in the equations below are zero.

$$\mu_X = \text{center of X} = \left|\tilde{X}\right| / \tilde{\sigma}_X \; ;$$

$$\mu_Y = \text{center of Y} = \left|\tilde{Y}\right| / \tilde{\sigma}_X \; ;$$

$$\sigma_Y = \tilde{\sigma}_Y / \tilde{\sigma}_X$$

$$R = \text{radius} = L_R / \tilde{\sigma}_X$$

TABLE E-4 CALCULATED COMBINED IMPACT AND TARGET DISPERSIONS

Target	$\sigma_{V_1}(X)$ (ft.)	$\sigma_{V_1}(Y)$ (ft.)	$\sigma_{V_1}(XY)$ (ft.^{2})
1	10,007.7	15,036	345,000
2	10,007.7	20,027	345,000
3	18,004.3	18,030	345,000
4	393.7	1,044	345,000
5	393.7	1,044	345,000
6	10,007.7	15,036	345,000
7	6,012.8	6,090	345,000

TABLE E-5 COMBINED IMPACT AND TARGET DISPERSIONS
IN A CANONICAL COORDINATE SYSTEM

Target	σ_X (ft.)	σ_Y (ft.)	L_R Lethal Radius (ft.)
1	15,036	10,494	13,512
2	20,027	10,008	15,730
3	18,033	18,000	13,512
4	1,097	204	13,512
5	1,097	204	13,512
6	15,036	10,494	17,523
7	6,099	6,003	15,730

The quantities σ_Y and R are calculated, and P_r determined, using appropriate tables. Then, the value of V_Ω from Table E-1 is used to calculate the expectant amount of kill, E_k, if one missile explodes on target, according to the relation $E_k = P_r V_\Omega$. The result of this calculation is given in Table E-6.

The quantity E_k gives the expectant kill on each target if one missile detonates on the target. If J equally identical and directed missiles detonate on the same (i^{th}) target, the expectant kill due to these missiles is

$$V_{\Omega}(i) \left| 1 - \left[1 - E_k / V_{\Omega}(i)\right]^J \right| = V_{\Omega}(i) \left| 1 - \left[1 - P_r\right]^J \right| .$$

For any i, J is also the number of values of j from 1 to J (i) for which $b_{p(i,j)} \cdot (k-1) = 1$. Hence, the expectant kill on target 2 is 58,140 if two missiles detonate on the target. The kill on target 6, if two or three missiles detonate on it, is 565,221 or 626,797, respectively. The i^{th} term in the summation of the equation below represents the kill on the i^{th} target, where, for this example, $\tilde{M} = 7$:

$$C_k = \sum_{i=1}^{\tilde{M}} V_{\Omega}(i) \left(1 - \prod_{i=1}^{J(i)} \left[1 - E_k(p) b_p(k-1) / V_{\Omega}(i)\right]\right)$$

According to this equation, the k^{th} element of the capability matrix C can be determined as follows. The expression $b_p(k-1)$ represents the p^{th} digit of the binary representation of k-1. Then $C_k = 8{,}720\, b_6(k-1) + \left|0, 37{,}230, \text{ or } 58{,}140\right.$, corresponding to $\left. b_2(k-1) + b_{10}(k-1) = 0, 1, \text{ or } 2 \right| + 10{,}535\, b_4(k-1) + 12{,}000\, b_7 \cdot (k-1) + 21{,}000\, b_5(k-1) + \left|0, 406{,}926, 565{,}221, \text{ or } 626{,}797\right.$, corresponding to $\left. b_1 \cdot (k-1) + b_8(k-1) + b_9(k-1) = 0,1,2, \text{ or } 3\right| + 55{,}970\, b_3(k-1)$.

According to the equation

$$E = \sum_{n=1}^{N} \sum_{k=1}^{2^M} A_n D_{nk} C_k,$$

or by computation of the destruction for each target individually, E = 812,550. This means that the expectant amount of total destruction on all targets under the given conditions is the equivalent of about 812,550 structures destroyed. There are the equivalent of 905,000 structures in the combined target areas. Hence the expectant destruction is about 89.8% of total destruction under the assumed conditions. This value exceeds the figure of merit requirement.

In this example, the calculation of effectiveness E could be simplified by computing dependability and capability of each target separately and combining the results.

TABLE E-6 EXPECTED AMOUNT OF KILL, E_k

Target	c_Y	R	P_r	V_Ω	E_k
1	0.6979	0.8986	0.436	20,000	8,720
2	0.4997	0.7854	0.438	85,000	37,230
3	0.9982	0.7493	0.245	43,000	10,535
4	0.1856	12.317	1.000	12,000	12,000
5	0.1856	12.317	1.000	21,000	21,000
6	0.6979	1.1654	0.611	666,000	406,926
7	0.9843	2.5789	0.965	58,000	55,970

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APPENDIX F

AN ILLUSTRATIVE EXAMPLE OF THE GENERAL METHODOLOGIES FOR SYSTEM EFFECTIVENESS EVALUATION

PURPOSE

The purpose of this example is to illustrate how the general methodologies for system effectiveness evaluation may have been applied to the Gemini Agena Target Vehicle (GATV) system. The example also shows some of the special problems associated with model construction, as well as their solution.

MISSION DESCRIPTION

The Gemini program consisted of a series of flights, each one accomplishing increasingly difficult tasks. Each flight in the series had a specific set of objectives. Of the five Gemini flights which had a rendezvous mission, Gemini XI was considered to be the most difficult. Gemini XI was the ninth manned mission and the fifth rendezvous mission of the Gemini Program.

The GATV was the in-orbit Agena less all droppable equipment. It was the rendezvous and docking target for the Gemini Spacecraft. Figure F-1 is a pictorial explanation of the terminology and relationship of the separate and combined Gemini Vehicle hardware.

Gemini XI Mission

The primary objective of the Gemini XI mission was:

(1) Rendezvous and dock during the first orbit.

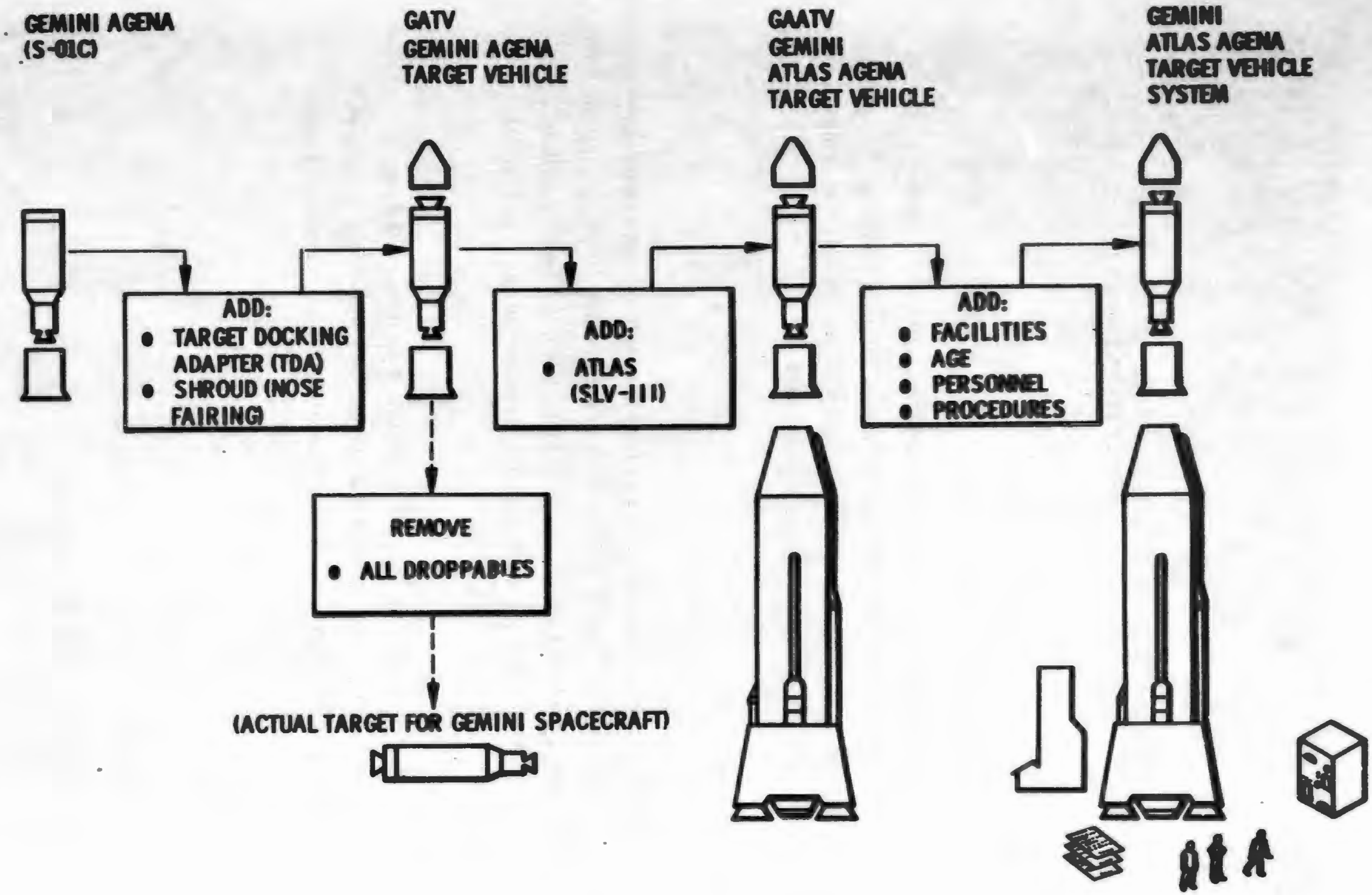


Figure F-1 Gemini Program Hardware – Terminology and Relationships

The secondary objectives were as follows:

(2) Conduct docking practice

(3) Conduct extravehicular operations

(4) Conduct experiments

(5) Conduct high apogee maneuvers

(6) Conduct tethered vehicle experiments

(7) Park Gemini Agena Target Vehicle

The mission is outlined in Figure F-2 which shows both the planned and the actual mission activities.

Gemini Agena Target Vehicle (GATV) Mission

The flight objectives of the Gemini XI GATV were as follows:

a. Primary objectives:

(1) Ascent phase

- Boost itself, after separation from the Atlas, into a 161-nm orbit with an inclination angle of 28.87 deg., with one primary propulsion subsystem firing and with a significant velocity increment capability remaining after injection.

(2) On-orbit phase

- Maintain a stable attitude for a nominal 5-day active orbital life.
- Provide a safe environment during the rendezvous, and participate in docking and undocking operations with the Gemini spacecraft.
- Provide a safe environment in the docked configuration while operating the primary and secondary propulsion systems.
- In response to commands from ground stations or Gemini spacecraft, change orbital elements in the undocked or docked configuration.

b. Secondary Objective:

- Determine GATV performance with the use of telemetry data.

The example illustration herein is addressed solely to the GATV performance with respect to the set of Gemini XI mission objectives listed under Mission Definition.

The GATV performed its missions by the following sequential phases:

- Pre-launch phase
- Boost phase - where the Atlas booster motor performs
- Orbit-injection phase - where the GATV is separated from the booster and injected into orbit
- Orbit phase - one phase for each of the seven mission objectives

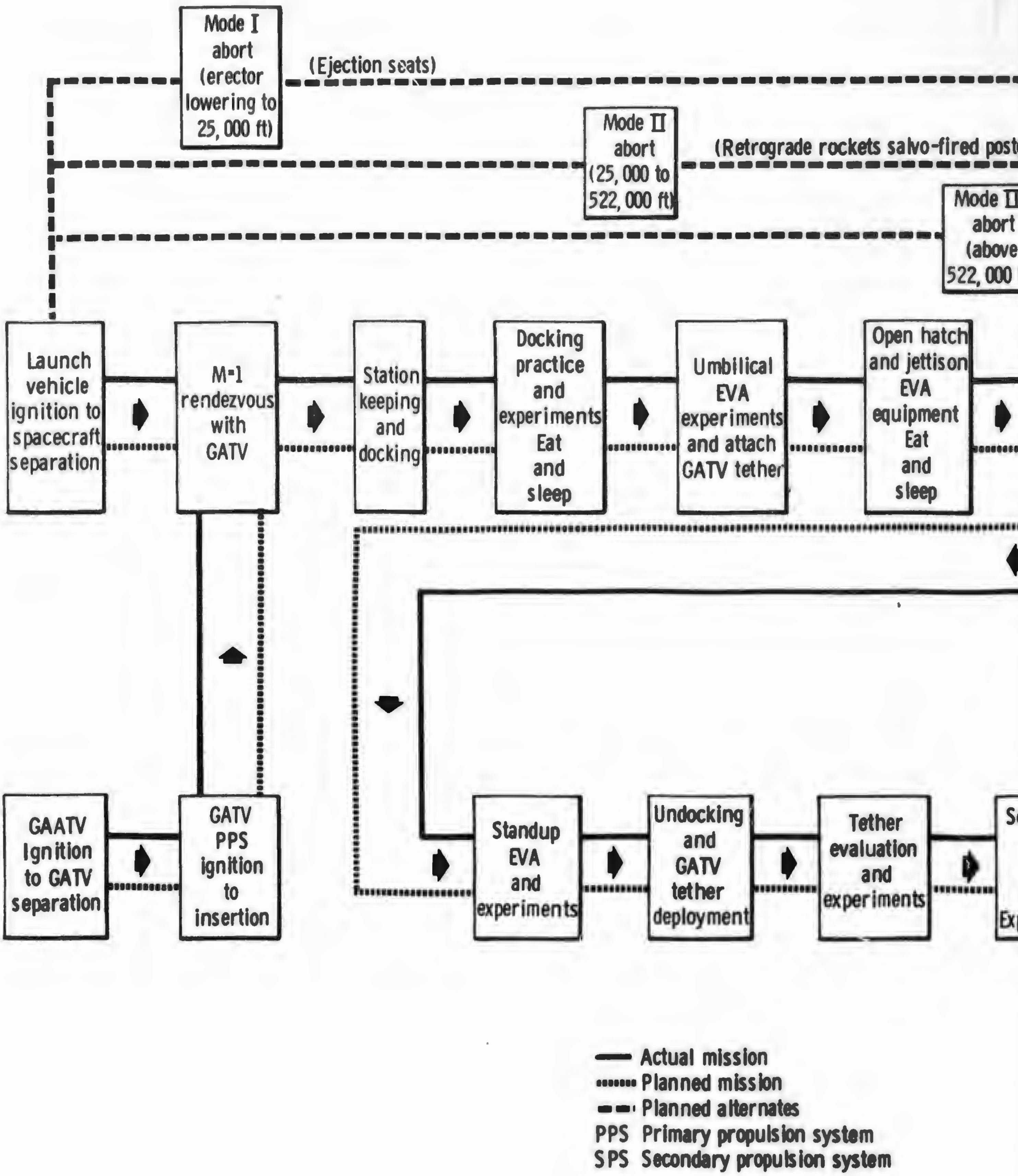
SYSTEM DESCRIPTION

The systems or group of equipment that comprise the GATV are airframe, primary propulsion, secondary propulsion, electrical power, guidance and flight control, tracking, command, telemetry, and a target docking adapter which includes a status display panel. A block diagram of the vehicle is presented in Figure F-3. Equipments or subsystems associated only with the ascent phase are not shown.

SPECIFICATION OF FIGURES OF MERIT

Seven figures of merit can be defined corresponding to the extent that the GATV can be expected to achieve each of the seven mission objectives. The seven figures of merit are:

E_1 = probability of successful rendezvous and dock during the first orbit
E_2 = probability of successfully conducting docking practice
E_3 = probability of successfully conducting extra-vehicular operations
E_4 = probability of successfully conducting experiments
E_5 = probability of successfully conducting high apogee maneuvers
E_6 = probability of successfully conducting tether vehicle experiments
E_7 = probability of successfully GATV parking


Mode I abort (erector lowering to 25,000 ft)
(Ejection seats)
Mode II abort (25,000 to 522,000 ft)
(Retrograde rockets salvo-fired post
Mode II abort (above 522,000
Launch vehicle ignition to spacecraft separation
M=1 rendezvous with GATV
Station keeping and docking
Docking practice and experiments Eat and sleep
Umbilical EVA experiments and attach GATV tether
Open hatch and jettison EVA equipment Eat and sleep
GAATV Ignition to GATV separation
GATV PPS ignition to insertion
Standup EVA and experiments
Undocking and GATV tether deployment
Tether evaluation and experiments
S
Ex
Actual mission
Planned mission
Planned alternates
PPS Primary propulsion system
SPS Secondary propulsion system
A

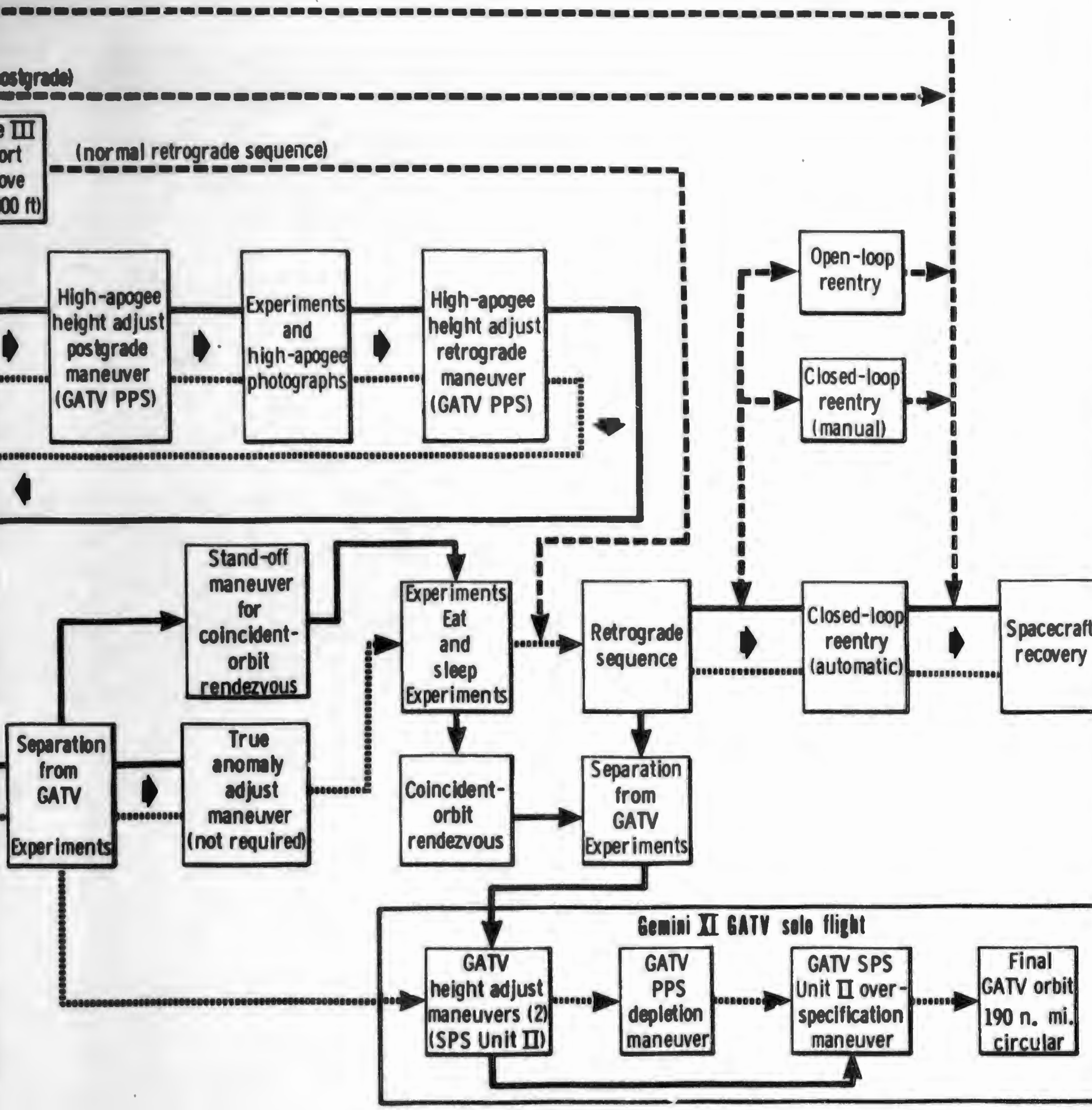


Figure F-2 Gemini XI Mission – Planned and Actual

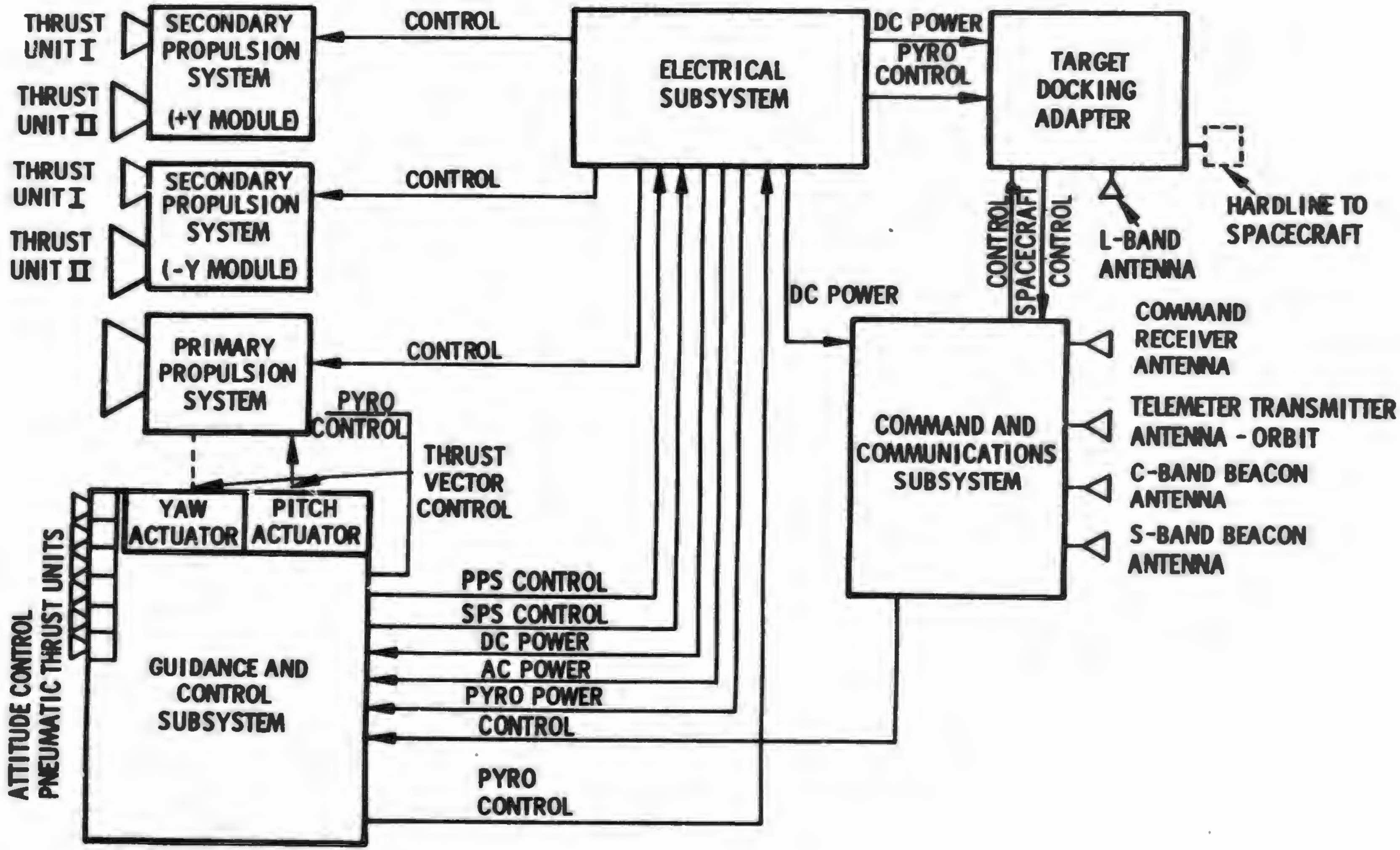


Figure F-3 Gemini Agena Target Vehicle – Block Diagram

SYSTEM STATES

Four critical failure conditions influencing the performance of the missions were defined, including:

(1) Catastrophic failure which completely aborts the mission and all subsequent missions

(2) Primary propulsion failure which partially affects missions 2, 5, and 7. This failure is defined as late shut-down resulting in consumption of extra fuel and orbit location error.

(3) Docking mechanism failure which affects missions 1, 2, and 5

(4) "Long burn" failure which affects missions 5, 6, and 7. This may occur during the orbit injection phase only and result in extra consumption of fuel reserves due to additional correction maneuvers.

States (2), (3), and (4) all cause partial degradation in performance.

MODEL CONSTRUCTION

A weighting factor, w_i, proportional to the relative importance of each mission was assigned, and the figures of merit for each mission combined into a single effectiveness measure

$$E = \sum_{i=1}^{7} w_i E_i \qquad \text{where} \quad \sum_{i=1}^{7} w_i = 1$$

and $w_1 = .50$; $w_2 = w_3 = w_4 = .10$; $w_5 = w_6 = .09$; and $w_7 = .02$

A convenient method for evaluating effectiveness is to compute each of the seven figures of merit with a different set of A, D, C matrices, defined differently for each figure of merit. The system will be considered available for each mission (figure of merit) if it is operable through the boost phase. Dependability will be evaluated differently. The system will be considered dependable for the i^{th} mission if it performs dependably in the i^{th} mission and was dependable throughout the $(i - 1)^{th}$ mission. The capability matrix for each figure of merit will be a one column vector. By evaluating each figure of merit using a separate set of availability, dependability, and capability matrices, the number of states required to be defined was reduced. An alternate approach would have been to evaluate the GATV in terms of an overall mission. However, many states would be required to reflect every possible combination of the four failure conditions. This is to account properly for the varying influences on overall mission performance due to failures occurring at different times in the mission. In any event, both methods will yield the same results.

The following is the formulation of the model for each figure of merit. Representative capability accountable factors for each figure of merit are developed at the GATV system level and are related to the listed subsystem accountable factors which may be evaluated by simulation routines.

FIGURE OF MERIT (1)

This is the probability that the GATV can successfully rendezvous and participate in docking and undocking operations with the Gemini manned spacecraft under a safe environment.

<u>System States</u>

1. No failure
2. Docking mechanism failure
3. Catastrophic failure before the first mission is complete

<u>Availability</u>

<u>Availability Vector</u>

$$\overline{A}'_1 = [A_1, A_2, A_3]$$

<u>Availability Elements</u>

A_1 = probability of no failures in boost phase
A_2 = probability of docking mechanism failure during boost phase
A_3 = probability of catastrophic failure before or during boost phase

Probability Terms

P_D = probability of a docking mechanism failure. $\overline{P}_D = 1 - P_D$

P_C = probability of a catastrophic failure. $\overline{P}_C = 1 - P_C$

Accountable Factors

P_D: Ejection of L band transponder fairings

P_C: (1) Atlas-Agena separation

(2) Sequence timer performs within specifications

(3) Ejection of horizon sensor fairings

Element Evaluation

$A_1 = \overline{P}_D \cdot \overline{P}_C$

$A_2 = \overline{P}_C \cdot P_D$

$A_3 = P_C$

Dependability

Dependability Matrix

$$D_1 = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform the mission, given it was in the i^{th} state after the boost phase.

Probability Terms

R_D = probability of a docking mechanism failure during orbit injection phase or the first mission. $\overline{R}_D = 1 - R_D$

R_{C1} = probability of a catastrophic failure during orbit injection phase or the first mission. $\overline{R}_{C1} = 1 - R_{C1}$

Accountable Factors

R_D = L band transponder and docking system performance within specifications.

R_{C1} = Propulsion, pitch program, horizon sensor and gyro performance within specifications.

Element Evaluation

$D_{11} = \bar{R}_D \cdot \bar{R}_{C1}$	$D_{21} = 0$	$D_{31} = 0$
$D_{12} = R_D \cdot \bar{R}_{C1}$	$D_{22} = \bar{R}_{C1}$	$D_{32} = 0$
$D_{13} = R_{C1}$	$D_{23} = R_{C1}$	$D_{33} = 1$

Capability

Capability Vector

$$\bar{C}_1 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix}$$

Capability Elements

C_{11} = probability the Agena capsule will rendezvous and dock during first revolution, given no failures

C_{12} = same probability, given a docking mechanism failure

C_{13} = same probability, given a catastropic failure = 0

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
(a) Control gas pressure ≥ 97 pounds	Attitude Control	Pneumatic system pressure ≥ 97 PSIG. No leakage in pressure system or control valves.
(b) Electric power ≥ 1800 ampere hours	Electrical	Agena batteries = 28 volts ±10% Electric Bus Voltage = 25 volts ± 0.5 volts.
(c) Radial distances between vehicles ≤ 50 nautical miles	Guidance and Control	Velocity meter bias < .0004 feet/second Programmer uncertainty < .05 degrees Gyro drift rate < .002 degrees/second

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
(d) Specified orbit achieved	Airframe	Actuator misalignment $< .15$ degrees Actuator null uncertainty $< .05$ degrees Center of gravity offset $< .16$ degrees Center of thrust offset $< .04$ degrees Initial vehicle weight nominal ± 30 pounds Hydraulic actuator gain ≈ 1.0 degrees/degree Vehicle response to torque $\approx .02$ degrees/foot pound.
	Guidance and Control	Horizon sensor/gyro gain $\approx .02$ seconds^{-1} Horizon sensor uncertainty $< .4$ degrees Velocity meter resolution $\leq .20$ feet/second
	Propulsion (Primary and Secondary)	Boost velocity error ≤ 3 feet/second Engine burn time $= 30$ seconds ± 0.5 seconds Propellant specific impulse ≈ 300 seconds Propellant weight flow ≈ 60 pounds/seconds Impulse tailoff uncertainty ± 400 pound seconds

Element Evaluation

Capability element probabilities are determined by evaluating the effect of each failure state on the accountable factors.

Effectiveness, FOM (1)

$$E_1 = \bar{A}_1' \left[D_1 \right] \bar{C}_1$$

FIGURE OF MERIT (2)

The second mission figure of merit is the probability of successful completion of the GATV docking practice maneuvers.

System States

1. No failures
2. Docking mechanism failure only
3. Primary propulsion system failure only
4. Both docking mechanism and primary propulsion failures
5. Catastropic failure before second mission is complete

Availability

Availability Vector

$$\bar{A}_2' = [A_1, A_2, A_3]$$

Availability Elements

A_1, A_2, and A_3 are the same as those elements used to evaluate the first figure of merit. These elements correspond to system states 1, 2 and 5 above which are the only possible system states at the end of the boost phase.

Dependability

Dependability Matrix

$$D_2 = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform mission (2), given it was in i^{th} state after mission (1).

Probability Terms

R_D = probability of docking mechanism failure during orbit injectiod phase or the first mission. $\bar{R}_D = 1 - R_D$.

R_{C1} = probability of a catastropic failure during orbit injection phase or the first mission. $\bar{R}_C = 1 - R_{C1}$.

R_{P1} = probability of primary propulsion system failure before second mission is complete. $\bar{R}_{P1} = 1 - R_{P1}$.

R_{C2} = probability of catastropic failure during second mission. $\bar{R}_{C2} = 1 - R_{C2}$.

Accountable Factors

R_{P1}: Primary propulsion shutdown within time specifications. Tolerance tighter than required for R_{C1}.

R_{C2}: Primary propulsion specific impulse within specifications

Element Evaluation

$D_{11} = \bar{R}_D \cdot \bar{R}_{P1} \cdot \bar{R}_{C1} \cdot \bar{R}_{C2}$

$D_{12} = R_D \cdot \bar{R}_{P1} \cdot \bar{R}_{C1} \cdot \bar{R}_{C2}$

$D_{13} = \bar{R}_D \cdot R_{P1} \cdot \bar{R}_{C1} \cdot \bar{R}_{C2}$

$D_{14} = R_D \cdot R_{P1} \cdot \bar{R}_{C1} \cdot \bar{R}_{C2}$

$D_{15} = 1 - \left(\bar{R}_{C1} \cdot \bar{R}_{C2}\right)$

$D_{21} = 0$

$D_{22} = \bar{R}_{P1} \cdot \bar{R}_{C1} \cdot \bar{R}_{C2}$

$D_{23} = 0$

$D_{24} = R_{P1} \cdot \bar{R}_{C1} \cdot \bar{R}_{C2}$

$D_{25} = 1 - \left(\bar{R}_{C1} \cdot \bar{R}_{C2}\right)$

$D_{31} = D_{32} = D_{33} = D_{34} = 0$

$D_{35} = 1$

Capability

Capability Vector

$$\bar{C}_2 = \begin{bmatrix} C_{21} \\ C_{22} \\ C_{23} \\ C_{24} \\ C_{25} \end{bmatrix}$$

Capability Elements

C_{2i} = probability of successful completion of docking practice maneuvers, given system is in i^{th} state when it must perform mission, where $i = 1, \ldots, 5$

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
(a) Control gas pressure ≥ 75 pounds	Attitude Control	Pneumatic system ≥ 75 PSIG No leakage in pressure system or control valves
(b) Electrical power ≥ 1200 ampere hours	Electrical	Agena batteries = 28 volts ±10% Electric bus voltage = 25 volts ±.5 volts

Effectiveness, FOM (2)

$$E_2 = \bar{A}_2' \left[D_2 \right] \bar{C}_2$$

FIGURE OF MERIT (3)

The third mission figure of merit is the probability of successful completion of extra-vehicular activities.

System States

1. No failures
2. Catastrophic failure before third mission can be performed.

Availability

Availability Vector

$$\bar{A}_3' = \left[A_1, A_2\right]$$

Availability Elements

$$A_1 = \bar{P}_C = 1 - P_C$$

$A_2 = P_C$ (probability of catastrophic failure)

Accountable Factors

P_C: Sequence timer function within specifications

Dependability

Dependability Matrix

$$D_3 = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform mission (3), given it was in the i^{th} state after mission (2).

Probability Terms

R_{C1} = probability of catastrophic failure during orbit injection phase or the first mission. $\bar{R}_{C1} = 1 - R_{C1}$.

R_{C2} = probability of catastrophic failure between first and second mission. $\bar{R}_{C2} = 1 - R_{C2}$

R_{C3} = probability of catastrophic failure between second and third missions.

$$\bar{R}_{C3} = 1 - R_{C3}$$

Accountable Factors

R_{C3}: Roll, pitch and yaw stability within spec.

Element Evaluation

$D_{11} = \bar{R}_{C1} \cdot \bar{R}_{C2} \cdot \bar{R}_{C3}$

$D_{12} = 1 - D_{11}$

$D_{21} = 0$

$D_{22} = 1$

Capability

Capability Vector

$$\bar{C}_3 = \begin{bmatrix} C_{31} \\ C_{32} \end{bmatrix}$$

Capability Elements

C_{31} = probability of successful completion of extra-vehicular activities.

$C_{32} = 0$

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
Maintain stabilized vehicle attitude while docked.	Guidance and Control	Horizon sensor resolution ≤ .4 degrees Velocity meter resolution ≤ .2 feet/second Gyro programmer uncertainty = .05 degrees Gyro drift rate = .0002 degrees/second

Effectiveness, FOM (3)

$$E_3 = \overline{A}_3' \cdot \left[D_3\right] \cdot \overline{C}_3$$

FIGURE OF MERIT (4)

The fourth mission figure of merit is the probability of successfully conducting experiments.

System States

1. No failures
2. Catastrophic failure before fourth mission can be performed.

Availability

Availability Vector

$$\overline{A}_4' = \left[A_1, A_2\right]$$

Availability Elements

$A_1 = \overline{P}_C$

$A_2 = P_C$ (probability of catastrophic failure)

Accountable Factors

P_C: Sequence timer function within specifications

Dependability

Dependability Matrix

$$D_4 = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform mission (4), given it was in the i^{th} state after mission (3).

Probability Terms

R_{C1} = probability of catastrophic failure during orbit injection phase or the first mission. $\bar{R}_{C1} = 1 - R_{C1}$

R_{C2} = probability of catastrophic failure between first and second missions. $\bar{R}_{C2} = 1 - R_{C2}$

R_{C3} = probability of catastrophic failure between second and third missions. $\bar{R}_{C3} = 1 - R_{C3}$

R_{C4} = probability of catastrophic failure between third and fourth missions. $\bar{R}_{C4} = 1 - R_{C4}$

Accountable Factors

R_{C4}: Roll, pitch and yaw stability within spec. for mission (4).

Element Evaluation

$D_{11} = \bar{R}_{C1} \cdot \bar{R}_{C2} \cdot \bar{R}_{C3} \cdot \bar{R}_{C4}$

$D_{12} = 1 - D_{11}$

$D_{21} = 0$

$D_{22} = 1$

Capability

Capability Vector

$$\bar{C}_4 = \begin{bmatrix} C_{41} \\ C_{42} \end{bmatrix}$$

Capability Elements

C_{41} = probability of successful completion of fourth mission experiments

$C_{42} = 0$

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
Maintain stabilized vehicle attitude while docked	Guidance and Control	Horizon sensor resolution ≤ .4 degrees Velocity meter resolution ≤ .2 feet/second Gyro programmer uncertainty = 0.5 degrees Gyro drift rate = .0002 degrees/second

Effectiveness, FOM (4)

$$E_4 = \bar{A}'_4 \cdot \left[D_4\right] \cdot \bar{C}_4$$

FIGURE OF MERIT (5)

The fifth mission figure of merit is the probability of successful completion of the high apogee maneuvers.

System States

1. No failures
2. Docking mechanism failure only
3. Primary propulsion failure only
4. Long burn failure only
5. Docking mechanism and primary propulsion failures
6. Docking mechanism and long burn failures
7. Primary propulsion and long burn failures
8. Primary propulsion, long burn and docking mechanism failures
9. Catastrophic failure before fifth mission can be performed.

Availability

Availability Vector

$$\bar{A}'_5 = \left[A_1, A_2, A_3\right]$$

Availability Elements

A_1 = probability of no failures through boost phase

A_2 = probability of docking mechanism failure during boost phase

A_3 = probability of catastrophic failure before or during boost phase

Probability Terms

P_D = probability of docking mechanism failure. $\bar{P}_D = 1 - P_D$

P_C = probability of catastrophic failure. $\bar{P}_C = 1 - P_C$

Accountable Factors

P_D: Ejection of L band transponder fairings

P_C: (1) Atlas-Agena separation

(2) Sequence timer performs within specifications

(3) Ejection of horizon sensor fairings

Dependability

Dependability Matrix

$$D_5 = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} & D_{17} & D_{18} & D_{19} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} & D_{27} & D_{28} & D_{29} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} & D_{37} & D_{38} & D_{39} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform mission (5), given it was in i^{th} state after mission (4).

Dependability Terms

R_D = probability of docking mechanism failure during orbit injection phase or the first mission. $\bar{R}_D = 1 - R_D$

R_{P1} = probability of primary propulsion system failure before second mission is completed. $\bar{R}_{P1} = 1 - R_{P1}$

R_{C1} = probability of catastrophic failure during orbit injection phase or the first mission. $\bar{R}_{C1} = 1 - R_{C1}$

R_{C2} = probability of catastrophic failure between first and second missions. $\bar{R}_{C2} = 1 - R_{C2}$

R_{C3} = probability of catastrophic failure between second and third missions. $\bar{R}_{C3} = 1 - R_{C3}$

R_{C4} = probability of catastrophic failure between third and fourth missions. $\bar{R}_{C4} = 1 - R_{C4}$

R_{C5} = probability of catastrophic failure between performance of fourth and fifth missions. $\bar{R}_{C5} = 1 - R_{C5}$

R_{L1} = probability of long burn failure before mission (5). $\bar{R}_{L1} = 1 - R_{L1}$

$\bar{R}^*_5 = \bar{R}_{C1} \cdot \bar{R}_{C2} \cdot \bar{R}_{C4} \cdot \bar{R}_{C5}$; $R^*_5 = 1 - \bar{R}^*_5$

Accountable Factors

R_{C5}: Primary propulsion ignition, specific impulse and shutdown within specifications.

R_{L1}: (1) Adequate fuel supply available

(2) Adequate electric power available

(3) Thrust time within specifications

(4) Vehicle center of gravity and engine center of thrust alignment within specifications.

Element Evaluation

$D_{11} = \bar{R}_D \cdot \bar{R}_{P1} \cdot \bar{R}_{L1} \cdot \bar{R}^*_5$

$D_{12} = R_D \cdot \bar{R}_{P1} \cdot \bar{R}_{L1} \cdot \bar{R}^*_5$

$D_{13} = \bar{R}_D \cdot R_{P1} \cdot \bar{R}_{L1} \cdot \bar{R}^*_5$

$D_{14} = \bar{R}_D \cdot \bar{R}_{P1} \cdot R_{L1} \cdot \bar{R}^*_5$

$D_{15} = R_D \cdot R_{P1} \cdot \bar{R}_{L1} \cdot \bar{R}^*_5$

$D_{16} = R_D \cdot \bar{R}_{P1} \cdot R_{L1} \cdot \bar{R}^*_5$

$D_{17} = \bar{R}_D \cdot R_{P1} \cdot R_{L1} \cdot \bar{R}^*_5$

$D_{18} = R_D \cdot R_{P1} \cdot R_{L1} \cdot \bar{R}^*_5$

$D_{19} = R^*_5$

$D_{21} = 0$

$D_{22} = \bar{R}_{P1} \cdot \bar{R}_{L1} \cdot \bar{R}^*_5$

$D_{23} = 0$

$D_{24} = 0$

$D_{25} = R_{P1} \cdot \bar{R}_{L1} \cdot \bar{R}^*_5$

$D_{26} = \bar{R}_{P1} \cdot R_{L1} \cdot \bar{R}^*_5$

$D_{27} = 0$

$D_{28} = R_{P1} \cdot R_{L1} \cdot \bar{R}^*_5$

$D_{29} = R^*_5$

$D_{31} = D_{32} = \ldots = D_{38} = 0, \; D_{39} = 1$

Capability

Capability Vector

$$\bar{C}_5 = \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_9 \end{bmatrix}$$

Capability Elements

C_i = probability of successful completion of high apogee maneuvers, given the system is in the i^{th} state for $i = 1, \ldots, 9$

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
(a) Increase vehicle velocity by 920 feet/second for 25 seconds	Primary Propulsion System	Boost velocity error ≤ 3 feet/second Engine burn time = 25 seconds Propellant specific impulse ≈300 seconds Propellant weight flow ≈ 60 pounds/second Impulse tailoff uncertainty ±400 pound seconds

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
	Guidance and Flight Control	Velocity meter resolution ≤ .20 feet/second. Horizon sensor resolution < .4 degrees. Horizon sensor/gyro gain ≈ .02 seconds^{-1}.
(b) Decrease vehicle velocity by 920 feet/second for 25 seconds.	Primary Propulsion System Guidance and Flight Control	Accountable factors same as in (a) above.

Effectiveness, FOM (5)

$$E_5 = \bar{A}'_5 \cdot [D_5] \cdot \bar{C}_5$$

FIGURE OF MERIT (6)

The sixth mission figure of merit is the probability of successfully conducting the tethered vehicle experiments.

System States

1. No failures
2. Long burn failure
3. Catastrophic failure before 6th mission can be performed

Availability

Availability Vector

$$\bar{A}'_6 = [A_1, A2]$$

Availability Elements

$$A_1 = \bar{P}_C = 1 - P_C$$

$A_2 = P_C$ (probability of catastrophic failure)

Accountable Factors

P_C: Sequence timer function within specifications

Dependability

Dependability Matrix

$$D_6 = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform mission (6), given it was in the i^{th} state after mission (5).

Probability Terms

R_{C1} = probability of catastrophic failure during orbit injection phase or the first mission. $\bar{R}_{C1} = 1 - R_{C1}$

R_{C2} = probability of catastrophic failure between performance of the first and second missions. $\bar{R}_{C2} = 1 - R_{C2}$

R_{C3} = probability of catastrophic failure between performance of the second and third missions. $\bar{R}_{C3} = 1 - R_{C3}$

R_{C4} = probability of catastrophic failure between performance of the third and fourth missions. $\bar{R}_{C4} = 1 - R_{C4}$

R_{C5} = probability of catastrophic failure between performance of the fourth and fifth missions. $\bar{R}_{C5} = 1 - R_{C5}$

R_{C6} = probability of catastrophic failure between performance of the fifth and sixth missions. $\bar{R}_{C7} = 1 - R_{C7}$

R_{L1} = probability of long burn failure before mission five

R_{L2} = probability of long burn failure during performance of the fifth mission

$$\bar{R}^*_6 = \bar{R}_{C1} \cdot \bar{R}_{C2} \cdot \bar{R}_{C3} \cdot \bar{R}_{C4} \cdot \bar{R}_{C5} \cdot \bar{R}_{C6} : R^*_6 = 1 - \bar{R}^*_6$$

Accountable Factors

R_{C6} : Roll, pitch and yaw stability within specifications for mission (6).

R_{L2} : (1) Thrust time within specifications
(2) Vehicle center of gravity and engine center of thrust alignment within specifications.
(3) Primary propulsion specific impulse within specifications

Element Evaluation

$D_{11} = \bar{R}_{L1} \cdot \bar{R}_{L2} \cdot R^*_6$

$D_{12} = (1 - \bar{R}_{L1} \cdot \bar{R}_{L2}) \bar{R}^*_6$

$D_{13} = R^*_6$

$D_{21} = 0$

$D_{22} = 0$

$D_{23} = 1$

Capability

Capability Vector

$$\bar{C}_6 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

Capability Elements

C_{6i} = probability of successfully conducting tethered vehicle experiments, given system is in the i^{th} state when it must perform mission, where $i = 1,2,3$

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
Vehicle must induce two specified rotational rates to the tethered vehicles	Attitude Control	Pneumatic system pressure ≥65 PSIG. No leakage in pressure system or control valves.
	Electric Power	Power to electric bus ≥ 1400 ampere hours

Effectiveness, FOM (6)

$$E_6 = \overline{A}'_6 \cdot \left[D_6 \right] \cdot \overline{C}_6$$

FIGURE OF MERIT (7)

The seventh mission figure of merit is the probability of successfully parking the GATV.

System States

1. No failures
2. Primary propulsion system failure only
3. Long burn failure only
4. Both primary propulsion and long burn failures
5. Catastrophic failure before seventh mission can be performed.

Availability

Availability Vector

$$\overline{A}'_7 = [A_1, A_2]$$

Availability Elements

$$A_1 = \overline{P}_C = 1 - P_C$$

$A_2 = P_C$ (probability of catastrophic failure)

Accountable Factors

P_C: Sequence timer function within specifications

Dependability

Dependability Matrix

$$D_7 = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} \end{bmatrix}$$

Dependability Elements

D_{ij} = probability the system will be in the j^{th} state when it must perform mission (7), given it was in the i^{th} state after mission (6).

Probability Terms

R_{C1} = probability of catastrophic failure during orbit injection phase or the first mission. $\bar{R}_{C1} = 1 - R_{C1}$

R_{C2} = probability of catastrophic failure between performance of the first and second missions. $\bar{R}_{C2} = 1 - R_{C2}$

R_{C3} = probability of catastrophic failure between performance of the second and third missions. $\bar{R}_{C3} = 1 - R_{C3}$

R_{C4} = probability of catastrophic failure between performance of the third and fourth missions. $\bar{R}_{C4} = 1 - R_{C4}$

R_{C5} = probability of catastrophic failure between performance of the fourth and fifth missions. $\bar{R}_{C5} = 1 - R_{C5}$

R_{C6} = probability of catastrophic failure between performance of fifth and sixth missions. $\bar{R}_{C6} = 1 - R_{C6}$

R_{C7} = probability of catastrophic failure between performance of the sixth and seventh missions. $\bar{R}_{C7} = 1 - R_{C7}$

R_{P1} = probability of primary propulsion system failure before second mission is completed. $\bar{R}_{P1} = 1 - R_{P1}$

R_{P2} = probability of primary propulsion system failure during performance of the fifth mission. $\bar{R}_{P2} = 1 - R_{P2}$

R_{L1} = probability of long burn failure before the fifth mission. $\bar{R}_{L1} = 1 - R_{L1}$

R_{L2} = probability of long burn failure during performance of the fifth mission. $\bar{R}_{L2} = 1 - R_{L2}$

$\bar{R}_7^* = \bar{R}_{C1} \cdot \bar{R}_{C2} \cdot \bar{R}_{C3} \cdot \bar{R}_{C4} \cdot \bar{R}_{C5} \cdot \bar{R}_{C6} \cdot \bar{R}_{C7}; R_7^* = 1 - \bar{R}_7^*$

Accountable Factors

R_{C7}: (1) Primary power supply shutdown within specifications.

(2) Primary power supply specific impulse within specifications.

(3) Primary power supply ignition time within specifications.

R_{P2}: Primary power supply shutdown within specifications.

Element Evaluation

$$D_{11} = \bar{R}_{P1} \cdot \bar{R}_{P2} \cdot \bar{R}_{L1} \cdot \bar{R}_{L2} \cdot \bar{R}_7^*$$

$$D_{12} = (R_{P1} + \bar{R}_{P1} \cdot R_{P2}) \cdot \bar{R}_{L1} \cdot \bar{R}_{L2} \cdot \bar{R}_7^*$$

$$D_{13} = (R_{L1} + \bar{R}_{L1} \cdot R_{L2}) \cdot \bar{R}_{P1} \cdot \bar{R}_{P2} \cdot \bar{R}_7^*$$

$$D_{14} = (R_{P1} + \bar{R}_{P1} \cdot R_{P2}) \cdot (R_{L1} + \bar{R}_{L1} \cdot R_{L2}) \cdot \bar{R}_7^*$$

$$D_{15} = R_7^*$$

$$D_{21} = \ldots = D_{24} = 0$$

$$D_{25} = 1$$

Capability

Capability Vector

$$\bar{C}_7 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}$$

Capability Elements

C_{7i} = probability of successfully parking the GATV, given the system is in the i^{th} state where i = 1, . . . , 5.

Accountable Factors

System Accountable Factors	Contributing Subsystem	Equipment Accountable Factors
Perform orbit correction maneuvers	Secondary Propulsion	Boost velocity error ≤ 3 feet/second Propellant specific impulse ≈300 seconds Propellant weight flow ≈ 60 pounds/second
	Guidance and Control	Horizon sensor resolution ≤ .4 degrees Velocity meter resolution ≤ .20 feet/second
Control gas pressure	Attitude Control	Pneumatic system pressure ≤60 PSIG No leakage in pressure system or control valves.
Electric power ≥ 600 ampere hours	Electrical	Power to electrical bus ≥ 600 ampere hours

Effectiveness, FOM (7)

$$E_7 = \bar{A}_7' \cdot \left[D_7\right] \cdot \bar{C}_7$$

APPENDIX G

LAGRANGE MULTIPLIER METHOD FOR OPTIMIZING DEPENDABILITY OBSERVING MULTICONSTRAINTS

INTRODUCTION

System availability, reliability, and capability performance may be increased by adding weight, volume, and dollars for which constraints may exist. For example, the payload capability of the Atlas launch vehicle limits the weight of the Agena system. In this example, optimizing the reliability parameter under constraints will be illustrated. By limiting the discussion to the reliability parameter of the system effectiveness parameters, a simple demonstration of the steps of the Lagrange multiplier method for apportionment can be accomplished without the complexity of the interactions of other system effectiveness parameters.

The method may be easily adapted to include the availability and the capability parameters into the investigation as demonstrated by the examples in Appendices H and I.

PROCEDURE

The procedures followed in this example differ only in their addition of detail from those specified in Section 3, and include the following steps:

- Step one was to define the mission for which the effectiveness of the resulting design is to be assessed.
- Step two was to establish the measure of effectiveness (figure of merit) of the resulting system design. This measure was functionally related to the accountable factors, with relationships determined so that the measure may be readily

computed for all values of these accountable factors. The FOM chosen here was the reliability of the system. The example was designed to be only illustrative of the apportionment technique, which can be applied to most missions where maximum reliability is the principal figure of merit. In the example, the reliability is a figure of merit used to evaluate the dependability parameter of a larger system.

- Step three was to identify the constraints placed upon the system, in the absence of which effectiveness could be made arbitrarily high. The constraints chosen for the example were mass, volume, and cost.

- Step four was to select an optimization technique which would maximize the figure of merit selected. The technique used to apportion reliability among N different subsystems was the Lagrange multiplier technique with a priority list. This technique was developed to obviate the need for solution of the N + K simultaneous equations which arise from the classical Lagrange multiplier technique applied to N subsystems, with K constraints. The classical theory, however, yields a direct optimum apportionment whenever these equations can be solved without difficulty.

- Step five was to differentiate the effectiveness and constraining functions, to obtain expressions for the computation of the Lagrange multipliers and priority list values. The computation of these values was continued until the value of the multiplier exceeded that of the priority number, or until one of the constraints was violated.

- Step six was to establish a decision rule to exclude those critical points which would result in only local optimum, or minimum values of the effectiveness measure. A direct means to accomplish this is to evaluate the effectiveness measure at each of the resulting one or more critical points (solutions) obtained, and merely select the maximum such value. A simple decision function, less cumbersome than the effectiveness function, was so developed in in this example to take advantage of the discrete nature of the main accountable factor.

Regardless of which decision rule is used, however, the effectiveness function of one of the computed critical points will indicate the optimum system design.

GENERAL FORMULAS FOR THE LAGRANGE MULTIPLIER METHOD

Assume there are N subsystems in a system, all of them subject to reliability improvement. The reliability improvement will usually be accomplished by the application of redundancy. Alternatively, this improvement may be achieved by using highly reliable subsystems, which are normally more costly than conventional ones.

Assume that n_i is the number of redundancies of the i^{th} subsystem and its reliability R_i is a function of n_i. If failures of the N subsystems occur independently, the system reliability R_s is:

$$R_s(t) = \prod_{i=1}^{N} R_i(n_i, t) \qquad (1)$$

Since t is a constant value and depends only on the specific subsystem or mission under consideration, this variable can be deleted from Equation (1) for this general explanation. Taking the natural logarithm which converts the product into a sum,

$$\ln R_s = \sum_{i=1}^{N} \ln R_i(n_i) \qquad (2)$$

If the i^{th} subsystem has a mass m_i, a volume v_i, and a cost c_i, and if the total mass $\sum_{i=1}^{N} m_i n_i$ of the system is limited to a mass constraint M, the total volume $\sum_{i=1}^{N} v_i n_i$ to a volume constraint V, and the total costs $\sum_{i=1}^{N} c_i n_i$ to a cost constraint C, then the following constraint relations exist:

$$\sum_{i=1}^{N} n_i m_i \leqslant M \qquad (3)$$

$$\sum_{i=1}^{N} n_i v_i \leq V \tag{4}$$

$$\sum_{i=1}^{N} n_i c_i \leq C \tag{5}$$

If ln R_s becomes a maximum, R_s becomes a maximum. Thus, to obtain the maximum of ln R_s, Equation (2) is differentiated to get:

$$\frac{dR_s}{R_s dn_i} = \sum_{i=1}^{N} \frac{dR_i(n_i)}{R_i(n_i)\, dn_i}$$

In the case of a maximum of R_s, the quotient $\frac{dR_s}{R_s}$ becomes zero. Thus,

$$o = \sum_{i=1}^{N} \frac{dR_i\ (n_i)}{dn_i R_i(n_i)} dn_i \tag{6}$$

Assume that the equal signs apply for the constraint inequalities of (3), (4), and (5). The deviation from this assumption will be illustrated later. Since, M, V, C are system constants, by differentiation:

$$\sum_{i=1}^{N} \zeta_m\, m_i dn_i = 0 \tag{7}$$

$$\sum_{i=1}^{N} \zeta_v\, v_i dn_i = 0 \tag{8}$$

$$\sum_{i=1}^{N} \zeta_c c_i dn_i = 0 \tag{9}$$

Notice that the sums are multiplied by ζ_m, ζ_v, ζ_c, the Lagrange multipliers, in order to obtain more degrees of freedom in the mathematical operations. Adding Equations (7), (8), and (9), gives

$$\sum_{i=1}^{N} \left(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i \; dn_i\right) = 0 \tag{10}$$

and subtracting Equation (10) from Equation (6), results in

$$\sum_{i=1}^{N} \left[\frac{dR_i(n_i)}{R_i(n_i)\, dn_i} - \left(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i\right)\right] dn_i = 0$$

Equating the terms inside the brackets gives:

$$\frac{dR_i(n_i)}{R_i(n_i)} = (\zeta_m m_i + \zeta_v v_i + \zeta_c c_i) \; dn_i$$

or, by integrating,

$$R_i(n_i) = R_o e^{(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i) \, . \, n_i} \tag{11}$$

Each combination of ζ_m, ζ_v, ζ_c with the condition

$$\zeta_m \geqslant o \; ; \; \zeta_v \geqslant o \; ; \; \zeta_c \geqslant o$$

will generate an optimized system reliability for a specific M, V, and C set of constraints. ζ_m, ζ_v, ζ_c must be zero or larger because of the subtraction of Equation (10) from Equation (6). Since the n_i values are integers, Equation (11) requires modification for this example.

The decision rule of selecting n_i for any set of ζ_m, ζ_v, ζ_c is

$$\frac{R_i(n_i)}{R_i(n_i-1)} > e^{(\zeta_m m_i + \zeta_v v_i + \zeta_c c_i)} \geqslant \frac{R_i(n_i+1)}{R_i(n_i)} \qquad (12)$$

The conversion of Equation (11), developed for continuous n_i, into Equation (12) when the n_i's are integers, may be easily verified.

<u>The Case of One Constraint Only</u>

The evaluation procedure becomes significantly simple for the case of one constraint only, a case occurring in many of the actual situations. For space systems, the mass of the space vehicle is limited because of the limited payload capability of the launch vehicle.

The relationship of Equation (12) may be expresses as

$$\ln\frac{R_i(n_i)}{R_i(n_i-1)} > (\zeta_m m_i + \zeta_v v_i + \zeta_c c_i) \geqslant \ln\frac{R_i(n_i+1)}{R_i(n_i)} \qquad (13)$$

In the case of a mass constraint only, ζ_v and ζ_c are zero. Dividing (13) by m_i,

$$\frac{1}{m_i}\ln\frac{R_i(n_i)}{R_i(n_i-1)} > \zeta_m \geqslant \frac{1}{m_i}\ln\frac{R_i(n_i+1)}{R_i(n_i)} \qquad (14)$$

Substituting j for n_i and q_{ij} for the left member of (14) gives

$$q_{ij} = \frac{1}{m_i} \ln \frac{R_i(j)}{R_i(j-1)} \quad \text{and} \quad q_{ij} > \zeta_m \tag{15}$$

for each subsystem i, i from 1 to N, beginning with j equal 2 and continuing with 3, 4, etc.

Upon computation of the q_{ij}'s, which are the priority values, the procedure for determining the redundancy n_i of each subsystem i for optimum reliability is:

(1) List the redundancy candidate subsystems and their corresponding mass m_i in decreasing numerical order of their q_{ij} values.

(2) Successively cumulate the mass m_i associated with each redundancy added to the basic nonredundant mass of the system. Continue this process until the mass constraint M is reached.

(3) Determine the largest j for each subsystem which was cumulated. This is the n_i value of each subsystem for optimum reliability. Any excluded subsystems at the M cutoff point will not be redundant.

This listing, which has been generated, is called a priority list and, for this case, gives the redundancy to be applied as a function of system mass. By means of the relationship in Equation (14) and the last q_{ij} included, the Lagrange multiplier ζ_m may be computed. Also, curves may be drawn of the Lagrange multiplier ζ_m as a function of the mass constraint.

The previously described method applies equally to other constraints which may be imposed on the system, such as volume or cost constraints, by changing the m symbol to v or c respectively, in the Equations (14) and (15).

The Multi-Constraint Case

In the case of more than one constraint, a separate priority list is generated for the mass constraint, for the volume constraint, and for the cost constraint. Thus, the following cases are considered:

Mass Constraint only:

$$M(\zeta_m ; 0 ; 0) = M_c \quad \text{(0 denoting no constraint)} \tag{16}$$

Volume Constraint only:

$$V(0 ; \zeta_v ; 0) = V_c \tag{17}$$

Cost Constraint only:

$$C(0 ; 0 ; \zeta_c) = C_c \tag{18}$$

Investigate

$$M(\zeta_m ; 0 ; 0) = M_c$$

If in addition $V(\zeta_m ; 0 ; 0) \leq V_c$

$C(\zeta_m ; 0 ; 0) \leq C_c$

then $R_s(\zeta_m ; \zeta_v = 0 ; \zeta_c = 0)$ is the optimized system reliability under the given constraints.

If the above inequalities cannot be met, then investigate

$$V(0 ; \zeta_v ; 0) = V_c$$

If in addition

$$M(0\ ;\ \zeta_v\ ;\ 0) \leq M_c$$

$$C(0\ ;\ \zeta_v\ ;\ 0) \leq C_c$$

then $R_s\ (0\ ;\ \zeta_v\ ;\ 0)$ is the optimized system reliability under the given constraints.

If the above inequality also cannot be met, then investigate

$$C(0\ ;\ 0\ ;\ \zeta_c) = C_c$$

If in addition

$$M(0\ ;\ 0\ ;\ \zeta_c) \leq M_c$$

$$V(0\ ;\ 0\ ;\ \zeta_c) \leq V_c$$

then $R_s\ (0\ ;\ 0\ ;\ \zeta_c)$ is the optimized system reliability under the given constraints. If the last inequalities are also not met, then the procedure is to restrict the priority lists to members up to the constraints M_c, V_c, and C_c, respectively. Members which are common in all of the three restricted priority lists are also members of the final solution.

Assume that the mass, volume, and cost of the members common in all three restricted priority lists are

M_r, V_r, and C_r.

Then, the remaining members can be investigated by determining the constraints on these members as follows:

$$M_{c-1} = M_c - M_r$$

$$V_{c-1} = V_c - V_r$$

$$C_{c-1} = C_c - C_r$$

Thus, the final solution process is simplified considerably. The final solution is then found by a systematic investigation of the following cases:

Investigate

$$M(\zeta m \;;\zeta v \;;\; 0) = M_{c-1}$$

$$V(\zeta m \;;\zeta v \;;\; 0) = V_{c-1}$$

If

$$C(\zeta m \;;\zeta v \;;\; 0) \leq C_{c-1} \qquad (19)$$

R_s $(\zeta m \;;\; \zeta v \;;\; 0)$ is the optimized system reliability under the given constraints.

If Equation (19) cannot be met, by means of Equation (12), find

$$M(\zeta m \;;\; 0 \;;\zeta c) = M_{c-1}$$

$$C(\zeta m \;;\; 0 \;;\zeta c) = C_{c-1}$$

If

$$V(\zeta m \;;\; 0 \;;\zeta c) \leq V_{c-1} \qquad (20)$$

R_s $(\zeta m \;;\; 0 \;;\; \zeta_c)$ is the optimized system reliability under the given constraints.

If Equation (20) cannot be met, find

$$V(0 \;;\; \zeta v \;;\zeta c) = V_{c-1}$$

$$C(0 \;;\; \zeta v \;;\zeta c) = C_{c-1}$$

If

$$M(0 \;;\; \zeta v \;;\zeta c) \leq M_{c-1} \qquad (21)$$

R_s $(0; \zeta_v; \zeta_c)$ is the optimized system reliability under the constraints.

Finally, if Equation (21) cannot be met, find

$$M(\zeta_m \;;\zeta_v \;;\zeta_c) = M_{c-1}$$

$$V(\zeta_m \;;\zeta_v \;;\zeta_c) = V_{c-1}$$

$$C(\zeta_m \;;\zeta_v \;;\zeta_c) = C_{c-1}$$

R_s $(\zeta_m; \zeta_v; \zeta_c)$ is the system reliability optimized under the constraints of

For simple systems, the solution of the multi-constraint problem by the described method usually can be obtained without a computer program.

RELIABILITY OPTIMIZATION BASED ON HIGH RELIABILITY SUBSYSTEMS

The procedures described so far are applicable to the apportionment of the reliability parameter for an optimum effectiveness based on the application of redundancy. Parallel and standby redundancies are two forms which may be applied. For subsystems of constant failure rates, with the failure rate of the i^{th} subsystem being λ_i, the reliability $R_i(j)$ of a single subsystem type for the mission time t is $R_i(1) = e^{-\lambda_i t}$. Then, for a system of j parallel subsystems,

$$R_i(j) = 1 - (1 - e^{-\lambda_i t})^j$$

Therefore

$$\frac{R_i(j)}{R_i(j-1)} = 1 + \frac{e^{-\lambda_i t}(1 - e^{-\lambda_i t})^{j-1}}{1 - (1 - e^{-\lambda_i t})^{j-1}}$$

and Equation (15) becomes for parallel redundancy,

$$q_{ij} = \frac{1}{m_i} \ln \left[1 + \frac{e^{-\lambda_i t}(1-e^{-\lambda_i t})^{j-1}}{1 - (1-e^{-\lambda_i t})^{j-1}} \right] \quad (22)$$

For a system of j standby redundant subsystems,

$$R_i(j) = e^{-\lambda_i t} \sum_{k=0}^{j-1} \frac{(\lambda_i t)^k}{k!} \quad (23)$$

Therefore,

$$\frac{R_i(j)}{R_i(j-1)} = 1 + \frac{\dfrac{(\lambda_i t)^{j-1}}{(j-1)!}}{1 + \lambda_i t + \dfrac{(\lambda_i t)^2}{2!} + \dfrac{(\lambda_i t)^3}{3!} + \ldots + \dfrac{(\lambda_i t)^{j-2}}{(j-2)!}}$$

and Equation (15) becomes for standby redundancy,

$$q_{ij} = \frac{1}{m_i} \ln \left[1 + \frac{\dfrac{(\lambda_i t)^{j-1}}{(j-1)!}}{1 + \lambda_i t + \dfrac{(\lambda_i t)^2}{2!} + \ldots + \dfrac{(\lambda_i t)^{j-2}}{(j-2)!}} \right] \quad (24)$$

The reliability of a system also may be increased by the simultaneous addition of redundancy and high reliability subsystems. Usually, a high reliability subsystem will have the same mass and volume as the conventional one, but the costs will be higher. To optimize a system by the joint application of redundant and high reliability subsystems the following steps are required:

- Solve for conventional subsystems using the procedures described so far,
- If the total costs of the solution with conventional subsystem is equal to the cost constraint, this solution is the final one.
- If the total costs of the solution with application of conventional subsystems is lower than the cost constraint C, and it is desirable to replace some conventional subsystems of the system by high reliability subsystems, the compute

$$q_{ij_o} = \frac{1}{C_{i_o} - c_i} \ln \frac{R_i(n_i\,;\,j_o)}{R_i(n_i\,;\,j_o-1)} \quad (25)$$

where:

c_i = cost of the conventional subsystem i

C_{i_o} = cost of the high reliability subsystem i

j_o = number of redundant high reliability subsystems in subsystem i (which consists of n_i total redundancies)

λ_i = failure rate of the i^{th} conventional subsystem

λ_{i_o} = failure rate of the i^{th} high reliability subsystem

- List the q_{ij_o} in decreasing order and select from this priority sequence high reliability subsystems until the cost constraint limit is reached. Both standby redundant conventional subsystems and high reliability subsystems may be used in a system in an arbitrary sequence.

If $\lambda_{i_o} < \lambda_i$, the reliability $R_i(n_i ; j_o)$ of n_i redundant subsystems i, of which j_o are high reliability subsystems, may be computed in the case of parallel redundancy by,

$$R_i(n_i ; j_o) = 1 - (1-e^{-\lambda_{i_o} t})^{j_o} (1-e^{-\lambda_i t})^{n_i - j_o} \qquad (26)$$

and in the case of standby redundancy

$$R_i(n_i ; j_o) = \frac{\lambda_i^{n_i - j_o} \lambda_{i_o}^{j_o}}{(\lambda_i - \lambda_{i_o})^{n_i}} \left[\sum_{k=1}^{j_o} (-1)^{j_o + k} \binom{n_i - 1 - k}{n_i - j_o - 1} \right.$$

$$\frac{(\lambda_i - \lambda_{i_o})^k}{\lambda_{i_o}^k} e^{-\lambda_{i_o} t} \sum_{\ell=0}^{k-1} \frac{(\lambda_{i_o} t)^\ell}{\ell !} +$$

$$+ \sum_{k=1}^{n_i - j_o} (-1)^{j_o} \binom{n_i - 1 - k}{j_o - 1} \frac{\left(\lambda_i - \lambda_{i_o}\right)^k}{\lambda_i^k} \cdot$$

$$\left. e^{-\lambda_i t} \sum_{\ell=0}^{k-1} \frac{(\lambda_i t)^\ell}{\ell !} \right]$$

The mathematical expressions of the form $\binom{u}{r}$ in the above equation are binominal coefficients defined by

$$\binom{u}{r} = \frac{u!}{(u-r)!\ r!}$$

This equation may be converted into a form more convenient for numerical evaluation by using the relation:

$$\sum_{\ell=0}^{k-1} \frac{(\lambda_i t)^\ell}{\ell !} = e^{\lambda_i t} - \sum_{\ell=k}^{\infty} \frac{(\lambda_i t)^\ell}{\ell !}$$

and observing that $R_i(n_i \ ; \ j_o) = 1$ for $t = 0$, the following expression is obtained:

$$R_i(n_i \ ; \ j_o) = 1 - \frac{\lambda_i^{n_i - j_o} \lambda_{i_o}^{j_o}}{(\lambda_i - \lambda_{i_o})^{n_i}} \left[e^{-\lambda_{i_o} t} \sum_{k=1}^{j_o} (-1)^{j_o + k} \cdot \right.$$

$$\binom{n_i - 1 - k}{n_i - j_o - 1} \frac{(\lambda_i - \lambda_{i_o})^k}{\lambda_{i_o}^k} \sum_{\ell=k}^{\infty} \frac{(\lambda_{i_o} t)^\ell}{\ell !} +$$

$$e^{-\lambda_1 t}\sum_{k=1}^{n_1 j_0} (-1)^{j_0}\begin{pmatrix} n_1-1-k \\ j_0-1 \end{pmatrix}\frac{(\lambda_1-\lambda_{1_0})^k}{\lambda_1^k}\sum_{\ell=k}^{\infty}\frac{(\lambda_1 t)^\ell}{\ell!}\Bigg] \qquad (27)$$

Since $R_1(n_1 \,;\, j_0)$ is usually close to one Equation (27) may be more convenient to use.

For parallel redundancy application using both conventional and high reliability subsystems, q_{1j_0}, Equation (25) becomes

$$q_{1j_0} = \frac{1}{C_{1_0} - c_1}\ln\left[1 + \frac{\left(e^{-\lambda_{1_0} t} - e^{-\lambda_1 t}\right)\left(1-e^{-\lambda_{1_0} t}\right)^{(j_0-1)}\left(1-e^{-\lambda_1 t}\right)^{(n_1-j_0)}}{1-\left(1-e^{-\lambda_{1_0} t}\right)^{(j_0-1)}\left(1-e^{-\lambda_1 t}\right)^{(n_1-j_0+1)}}\right] \qquad (28)$$

For standby redundancy Equation (25) may not be advantageously simplified. Table G-1 gives the formulas for the reliability of n_1 standby redundant subsystems with $(n_1 - j_0)$ conventional subsystems of failure rate λ_1, and j_0 high reliability subsystems of failure rate λ_{1_0} and tabulated $R_1\,(n_1 \,;\, j_0)$. All cases for $n_1 = 2, 3,$ and 4 were considered. Replacing λ_{1_0} by λ_1, and λ_1 by λ_{1_0} in the corresponding expressions for $R_1\,(n_1 \,;\, j_0)$ of Table G-1 will give the reliability of n_1 standby redundant subsystems in the case of conventional subsystems only, and in the case of high reliability subsystems only, respectively.

NUMERICAL EXAMPLE

The example selected to illustrate this method of optimizing system effectiveness is that of a space craft with a required mission lifetime of 5 years. It is assumed that the spacecraft has the single mission of providing a communications link and that no

TABLE G-1 RELIABILITY WITH BOTH HIGH RELIABILITY AND CONVENTIONAL SUBSYSTEMS

The Reliability $R_1(n_1; j_o)$ of n_1 Standby Redundant Subsystems with

$(n_1 - j_o)$ Conventional Subsystems

λ_1 = Failure Rate of Conventional Subsystems

λ_{1_o} = Failure Rate of High Reliability Subsystems

n_1	j_o	$R_1(n_1 ; j_o)$
2	1	$1 - \frac{(\lambda_1 t)(\lambda_{1_o} t)}{2!}\left[1 - \frac{(\lambda_1 + \lambda_{1_o})t}{3} + \frac{(\lambda_1^2 + \lambda_1 \lambda_{1_o} + \lambda_{1_o}^2)t^2}{3 \cdot 4} - \frac{(\lambda_1^3 + \lambda_1^2 \lambda_{1_o} + \lambda_1 \lambda_{1_o}^2 + \lambda_{1_o}^3)t^3}{3 \cdot 4 \cdot 5} + \ldots - \ldots\right]$
3	1	$1 - \frac{(\lambda_1 t)^2(\lambda_{1_o} t)}{3!}\left[1 - \frac{(2\lambda_1 + \lambda_{1_o})t}{4} + \frac{(3\lambda_1^2 + 2\lambda_1 \lambda_{1_o} + \lambda_{1_o}^2)t^2}{4 \cdot 5} - \frac{(4\lambda_1^3 + 3\lambda_1^2 \lambda_{1_o} + 2\lambda_1 \lambda_{1_o}^2 + \lambda_{1_o}^3)t^3}{4 \cdot 5 \cdot 6} + \ldots - \ldots\right]$
3	2	$1 - \frac{(\lambda_1 t)(\lambda_{1_o} t)^2}{3!}\left[1 - \frac{(\lambda_1 + 2\lambda_{1_o})t}{4} + \frac{(\lambda_1^2 + 2\lambda_1 \lambda_{1_o} + 3\lambda_{1_o}^2)t^2}{4 \cdot 5} - \frac{(\lambda_1^3 + 2\lambda_1^2 \lambda_{1_o} + 3\lambda_1 \lambda_{1_o}^2 + 4\lambda_{1_o}^3)t^3}{4 \cdot 5 \cdot 6} + \ldots - \ldots\right]$
4	1	$1 - \frac{(\lambda_1 t)^3(\lambda_{1_o} t)}{4!}\left[1 - \frac{(3\lambda_1 + \lambda_{1_o})t}{5} + \frac{(6\lambda_1^2 + 3\lambda_1 \lambda_{1_o} + \lambda_{1_o}^2)t^2}{5 \cdot 6} - \frac{(10\lambda_1^3 + 6\lambda_1^2 \lambda_{1_o} + 3\lambda_1 \lambda_{1_o}^2 + \lambda_{1_o}^3)t^3}{5 \cdot 6 \cdot 7} + \ldots - \ldots\right]$
4	2	$1 - \frac{(\lambda_1 t)^2(\lambda_{1_o} t)^2}{4!}\left[1 - \frac{(2\lambda_1 + 2\lambda_{1_o})t}{5} + \frac{(3\lambda_1^2 + 4\lambda_1 \lambda_{1_o} + 3\lambda_{1_o}^2)t^2}{5 \cdot 6} - \frac{(4\lambda_1^3 + 6\lambda_1^2 \lambda_{1_o} + 6\lambda_1 \lambda_{1_o}^2 + 4\lambda_{1_o}^3)t^3}{5 \cdot 6 \cdot 7} + \ldots - \ldots\right]$
4	3	$1 - \frac{(\lambda_1 t)(\lambda_{1_o} t)^3}{4!}\left[1 - \frac{(\lambda_1 + 3\lambda_{1_o})t}{5} + \frac{(\lambda_1^2 + 3\lambda_1 \lambda_{1_o} + 6\lambda_{1_o}^2)t^2}{5 \cdot 6} - \frac{(\lambda_1^3 + 3\lambda_1^2 \lambda_{1_o} + 6\lambda_1 \lambda_{1_o}^2 + 10\lambda_{1_o}^3)t^3}{5 \cdot 6 \cdot 7} + \ldots - \ldots\right]$

capability for on-orbit repair exists. It is also assumed that full mission value is accrued if the satellite operates for 5 years, its design lifetime, and that no partial value can be accrued for less than 5 years of operation. In this example, the dependability and capability parameters are considered to be identical with the system reliability of providing a communications channel. The program phase under consideration is the design phase. The basic problem is to select the optimum spacecraft design, in the sense of establishing a reliability apportionment which will maximize mission success and be within the available budget for spacecraft weight and cost.

The satellite subsystems which are subject to random failures that are catastrophic to mission performance are listed in Table G-2. For the purpose of this example, satellite subsystem failures which are not catastrophic to mission accomplishment are considered to be 100% reliable. Also, in this example, subsystem failures which do not cause mission failure are: (1) failure of redundant solar cells in the solar array panels, (2) failure of channels in the satellite transponder, since there are redundancies in the channels and the presence of ground-commanded switching capability among the individual communications channels.

Example of Multi-Constraint Case

Initially, consider the example given as a case of one constraint only. This will be expanded to the multiple constraint case and the determination of the optimum solution under the following different conditions of constraints:

Case A: Weight $\leq$ 275 lbs
Cost $\leq$ \$250,000

Case B: Weight $\leq$ 325 lbs
Cost $\leq$ \$175,000

Case C: Weight $\leq$ 295 lbs
Cost $\leq$ \$155,000

TABLE G-2 ACCOUNTABLE FACTORS OF SATELLITE SUBSYSTEMS

(based on conventional reliability subsystems)

i	Subsystem	Failure rate (hrs.)	Cost (Dollars)	Weight (Pounds)
1	Battery	1×10^{-6}	5,600	56
2	Charger & Inst. Box	$.3 \times 10^{-6}$	600	5.5
3	Roll H/S*	5×10^{-6}	8,000	4
4	Mag. Coil	$.1 \times 10^{-6}$	2,500	5
5	Mag. Coil Electronics	$.014 \times 10^{-6}$	2,000	2
6	Flywheel	$.5 \times 10^{-6}$	20,000	52
7	Wheel Electronics	1.2×10^{-6}	5,000	4
8	Pitch H/S*	5×10^{-6}	8,000	4
9	Pitch Jet Electronics	2×10^{-6}	4,000	4
10	Roll-Yaw Jet Electronics	2×10^{-6}	6,000	6
11	H/S* Electronics	1×10^{-6}	6,000	4

*H/S = Horizon Sensor

Constraint Case A

First consider Case A and the weight constraint only. The q_{ij}'s, computed by formula (15), are shown in Table G-3. These q_{ij}'s are then arranged in decreasing order to form the priority list given in Table G-4. For the case of a weight constraint only, columns (9) and (10) of Table G-4 should be ignored. The cumulative system weight (Column 6) indicates that only the first 15 redundancies can be accepted.

For each step or priority number, the Lagrange multiplier ζ_m has been computed (Column 8) to yield the solution given in that step. If the Lagrange multiplier is used, rather than a known weight constraint, the procedure is to accept all redundancies with $q_{ij} > \zeta_m$ and reject all redundancies with $q_{ij} \leq \zeta_m$. A $\zeta_m = .0005$ corresponds to the solution of $W \leq 275$ lbs; therefore, all redundancies in the shaded portion of Table G-3 ($q_{ij} \leq \zeta_m$) were rejected.

In this initial solution, the proximity to the weight constraint must be considered. This is necessary because the principle of Lagrange multipliers involves finding the optimum solution in large steps. As such, intermediate but optimum points are frequently bypassed as illustrated in Figure G-1, which gives an example of Lagrange multiplier solutions, other solutions along the optimum path, and all possible solutions.

For the initial solution, the weight is 265 pounds and the constraint is 275 pounds which gives a 10 pound weight margin. By referring to Tables G-3 and G-4, three redundancies, which equals 10 pounds, can be added to absorb the weight margin.

Now consider Case A with both the weight and cost constraints. As previously indicated, the recommended method for determining the optimum solution for multiple constraints is to find the solution considering only one constraint. The weight constraint was considered first and the Lagrange multiplier ζ_m, which corresponds to that solution, was calculated. Columns (9) and (10) of Table G-4 are constructed to evaluate whether the cost constraint has been violated by the weight solution obtained. In this case, since the cost is considerably below $250,000, the cost constraint is satisfied and the weight solution obtained is optimum (see Table G-5).

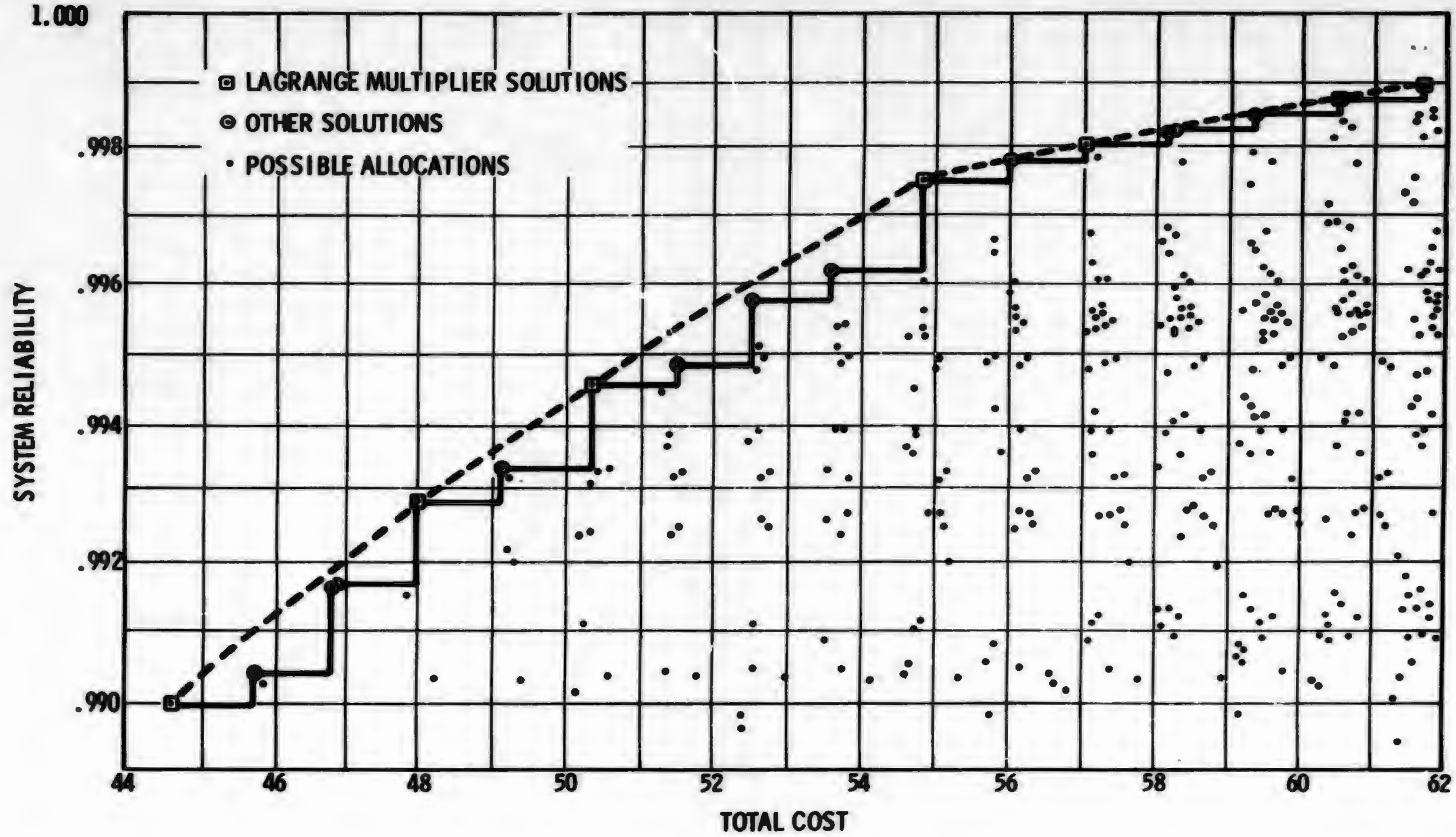


Figure G-1 Illustration of Optimum-Seeking Routine

TABLE G-3 PRIORITY LIST BASED ON WEIGHT CONSTRAINT ONLY

(5-year mission lifetime)

Subsystem	λi Conventional Reliability Failure Rate (hours)	i Subsystem Designation	m_i * Weight (Pounds)	q_{ij} ** Priority Value q_{i2}	Priority Value q_{i3}	Priority Value q_{i4}
Battery	1×10^{-6}	1	56	.000769	.000016	.000000
Charger & Instrument Box	$.3 \times 10^{-6}$	2	5.5	.002349	.000015	.000000
Roll H/S	5×10^{-6}	3	4	.049515	.004878	.000599
Mag. Coil	$.1 \times 10^{-6}$	4	5	.000797	.000002	.000000
Mag. Coil Electronics	$.014 \times 10^{-6}$	5	2	.000277	.000000	.000000
Flywheel	$.5 \times 10^{-6}$	6	52	.000418	.000004	.000000
Wheel Electronics	1.2×10^{-6}	7	4	.012914	.000325	.000006
Pitch H/S	5×10^{-6}	8	4	.049515	.004878	.000599
Pitch Jet Elect.	2×10^{-6}	9	4	.021090	.000873	.000025
Roll-Yaw Jet Electronics	2×10^{-6}	10	6	.014063	.000582	.000016
H/S Electronics	1×10^{-6}	11	4	.010767	.000226	.000003

* In these calculations it is assumed the same amount of additional mass is required to add the 1st, 2nd, and 3rd redundancies.

** j refers to the total number of i subsystem type. j=2 would be the basic subsystem plus one subsystem in standby redundancy; j=3 refers to two subsystems in standby redundancy; etc.

TABLE G-4 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON WEIGHT CONSTRAINT ONLY

(1) Priority	(2) Subsystem Added i	(3) Priority Value $q_{i,j}$	(4) Number of Redund. Subsystems n_i	(5) Additional Weight for Subsystem i (lbs) m_i	(6) Cumulative Total Wt. M (lbs) $\sum_{i=1}^{N} n_i m_i$	(7) System Reliab. (%) R_s	(8) Lagrange Multiplier ζ_m	(9) Unit Cost ($1000) c_i	(10) Cumulative Total Cost C ($1000) $\sum_{i=1}^{N} n_i c_i$
		Basic System Only - No Redundancies			146.5	45.	–		67.7
1	Roll Horizon Sensor, 1st Standby	$q_{3,2}=.049515$	$n_3=2$	4	150.5	54.8	.03	8	67.7
2	Pitch Horizon Sensor, 1st Standby	$q_{8,2}=.049515$	$n_8=2$	4	154.5	66.8	.03	8	83.7
3	Pitch Jet Electronics, 1st Standby	$q_{9,2}=.021090$	$n_9=2$	4	158.5	73.7	.015	4	87.7
4	Roll-Yaw Jet Electronics, 1st Standby	$q_{10,2}=.014063$	$n_{10}-2$	6	164.5	79.1	.013	6	93.7
5	Wheel Electronics, 1st Standby	$q_{7,2}=.012914$	$n_7=2$	4	168.5	83.3	.011	5	98.7
6	Horizon Sensor Electronics, 1st Standby	$q_{11,2}=.010767$	$n_{11}=2$	4	172.5	87.0	.005	6	104.7
7	Roll Horizon Sensor, 2nd Standby	$q_{3,3}=.004878$	$n_3=3$	4	176.5	88.7	.003	8	112.7
8	Pitch Horizon Sensor 2nd Standby	$q_{8,3}=.004878$	$n_8=3$	4	180.5	90.4	.003	8	120.7

TABLE G-4 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON WEIGHT CONSTRAINT ONLY (Continued)

(1) Priority	(2) Subsystem Added i	(3) Priority Value $q_{i,j}$	(4) Number of Redund. Subsystems n_i	(5) Additional Weight for Subsystem i (lbs) m_i	(6) Cumulative Total Wt. M (lbs) $\sum_{i=1}^{N} n_i m_i$	(7) System Reliab. (%) R_s	(8) Lagrange Multiplier ζ_m	(9) Unit Cost ($1000) c_i	(10) Cumulative Total Cost C ($1000) $\sum_{i=1}^{N} n_i c_i$
9	Charger, 1st Standby	$q_{2,2}$=.002349	n_2=2	5.5	186	91.6	.0009	6	121.3
10	Pitch Jet Electronics, 2nd Standby	$q_{9,3}$=.000874	n_9=3	4	190	92.0	.0008	4	125.3
11	Mag. Coil, 1st Standby	$q_{4,2}$=.000797	n_4=2	5	195	92.3	.00077	2.5	127.8
12	Battery, 1st Standby	$q_{1,2}$=.000769	n_1=2	56	251.0	96.4	.0006	5.6	133.4
13	Roll Horizon Sensor, 3rd Standby	$q_{3,4}$=.000599	n_3=4	4	255	96.6	.00059	8	141.4
14	Pitch Horizon Sensor, 3rd Standby	$q_{8,4}$=.000599	n_8=4	4	259	96.8	.00059	8	149.4
15	Roll-Yaw Jet Electronic, 2nd Standby	$q_{10,3}$=.000582	n_{10}=3	6	265.0	97.2	.0005	6	155.4
			INITIAL SOLUTION TO CASE A						
16	Flywheel, 1st Standby	$q_{6,2}$=.000418	n_6=2	52	317.0	97.9	.0004	20	175.4
17	Wheel Electronics, 2nd Standby	$q_{7,3}$=.000325	n_7=3	4	321.5		.00031	5	180.4

TABLE G-4 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON WEIGHT CONSTRAINT ONLY (Continued)

(1) Priority	(2) Subsystem Added i	(3) Priority Value $q_{i,j}$	(4) Number of Redund. Subsystems n_i	(5) Additional Weight for Subsystem i (lbs) m_i	(6) Cumulative Total Wt. M (lbs) $\sum_{i=1}^{N} n_i m_i$	(7) System Reliab. (%) R_s	(8) Lagrange Multiplier ζ_m	(9) Unit Cost ($1000) c_i	(10) Cumulative Total Cost C ($1000) $\sum_{i=1}^{N} n_i c_i$
18	Mag. Coil Elec., 1st Standby	$q_{5,2}$=.000277	n_5=2	2	323.5		.00023	2	182.4
19	Horizon Sensor Electronics, 2nd Standby	$q_{11,3}$=.000226	n_{11}=3	4	327.5			6	188.4

TABLE G-5 SOLUTION TO CONSTRAINT CASE A

Constraints

$W \leq$ 275 lbs
$C \leq$ \$250,000

Subsystem	Number of Standbys in Initial Solution	Selection of Additional Standbys to Absorb Margin	Final Solution n_i
1	1		2
2	1		2
3	3		4
4	1		2
5	0	1st standby, 2 lbs	2
6	0		1
7	1	2nd standby, 4 lbs	3
8	3		4
9	2		3
10	2		3
11	1	2nd standby, 4 lbs	3

Total	W = 265 lbs	W = 10 lbs	C = \$156,700 W = 275 lbs

Constraint Case B

Next, consider Case B with the constraints of weight 325 pounds and cost $175,000. As before, only the mass constraint is initially considered and ζ_m calculated. In Table G-4, ζ_m = .00023 and the first 18 redundancies may be added. However, the total cost of $182,400 shown in column (10) violates the cost constraint. Therefore, the cost constraint must be evaluated to determine ζ_c. To obtain the q_{ij} based on cost considerations only, the factor $\frac{1}{n_i}$ is replaced by $\frac{1}{c_i}$ in the q_{ij} equation for mass only to give:

$$q_{ij} = \frac{1}{c_i} \ln \frac{R_i(j)}{R_i(j-1)}$$

Table G-6 shows the calculated q_{ij} based on cost considerations. As before, q_{ij}'s are ordered to obtain the priority listing given in Table G-7. Referring to Table G-7, ζ_c = .00028 corresponds to the solution of a cost $\leq$ $175,000. The total system weight as given in column (10) is 311.5 pounds, which satisfies the constraint of weight $\leq$325 pounds.

The initial solution has a cost of $174,800 which leaves a cost margin of $200. However, no subsystems can be added for that price and, therefore, the initial solution of the first 15 redundancies as shown on Table G-7 is the final solution.

Constraint Case C

Case C with the constraints of weight $\leq$ 295 pounds, and of cost $\leq$ $155,000, is now examined. In Table G-4, ζ_m = .0005 and satisfies the weight, $\leq$ 295 pounds, but the $155,400 cost violates the cost constraint of $155,000. In Table G-7, ζ_c = .0006 satisfies cost $\leq$ $155,000, but the weight = 297.5 pounds violates the weight constraint of 295 pounds.

The procedure in this case is to accept the members common to each solution. The members not common are then examined to determine the subset which yields the highest reliability and, at the same time, satisfies both constraints. Column (3) of

TABLE G-6 CALCULATIONS BASED ON COST CONSIDERATIONS ONLY

(5-yr. mission lifetime)

Subsystem	i	Failure Rate (hrs)	c_i ($1000)	q_{12}	q_{13}	q_{14}
Battery	1	1×10^{-6}	5.6	.007692	.000162	.000002
Charger	2	$.3 \times 10^{-6}$	.6	.000216	.000001	.000000
Roll H/S	3	5×10^{-6}	8.	.024757	.002439	.000300
Mag. Coil	4	$.1 \times 10^{-6}$	2.5	.001594	.000004	.000000
Mag. Coil Elect.	5	$.014 \times 10^{-6}$	2	.000235	.000009	.000000
Flywheel	6	$.5 \times 10^{-6}$	20	.001088	.000010	.000000
Wheel Elect.	7	1.2×10^{-6}	5	.01033	.000260	.000005
Pitch H/S	8	5×10^{-6}	8	.024757	.002439	.000300
Pitch Jet Elect.	9	2×10^{-6}	4	.021090	.000874	.000025
Roll-Yaw Jet Elect.	10	2×10^{-6}	6	.014088	.000383	.000016
H/S Elect.	11	1×10^{-6}	6	.007192	.000151	.000002

TABLE G-7 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON COST CONSTRAINT ONLY

(1) Priority	(2) Subsystem i Added	(3) q_{ij} (Based on Cost Only)	(4) No. of Redund. Subsys. n_i	(5) Unit Cost ($1000) c_i	(6) Cumulative Total Cost ($1000) $C=\sum_{i=1}^{N} n_i c_i$	(7) System Reliab. (%) R_s	(8) Lagrange Multiplier ζ_c	(9) Additional Wt. for Subsystems (lbs) m_i	(10) Cumulative Total Weight $M=\sum_{i=1}^{N} n_i m_i$
0	None - Basic System				67.7				146.5
1	Roll H/S 1st Standby	$q_{3,2}$=.024757	n_3=2	8	75.7	54.8	.022	4	150.5
2	Pitch H/S 1st Standby	$q_{8,2}$=.024757	n_8=2	8	83.7	66.8	.022	4	154.5
3	Pitch Jet Elect. 1st Standby	$q_{9,2}$=.021090	n_9=2	4	87.7	73.7	.015	4	158.5
4	Roll Yaw Jet Elect. 1st Standby	$q_{10,2}$=.01[illegible]088	n_7=2	5	98.7	83.3	.008	4	168.5
5	Wheel Elect. 1st Standby	$q_{7,2}$=.01033	n_7=2	5	98.7	83.3	.008	4	168.5
6	Battery 1st Standby	$q_{1,2}$=.007692	n_1=2	5.6	104.3	87.0	.0075	56	224.5
7	H/S Elect. 1st Standby	$q_{11,2}$=.007192	n_{11}=2	6	110.3	90.8	.003	4	228.5
8	Roll H/S 2nd Standby	$q_{3,3}$=.002439	n_3=3	8	118.3	92.6	.0016	4	232.5
9	Pitch H/S 2nd Standby	$q_{8,3}$=.002439	n_8=3	8	126.3	94.4	.0016	4	236.5
10	Mag. Coil 1st Standby	$q_{4,2}$=.001594	n_4=2	2.5	128.8	94.8	.0011	5	241.5
11	Flywheel 1st Standby	$q_{6,2}$=.001088	n_6=2	20	148.8	96.9	.0009	52	293.5

TABLE G-7 PRIORITY OF INCLUDING ADDITIONAL REDUNDANCIES BASED ON COST CONSTRAINT ONLY (Continued)

(1) Priority	(2) Subsystem i Added	(3) q_{ij} (Based on Cost Only)	(4) No. of Redund. Subsys. n_i	(5) Unit Cost ($1000) c_i	(6) Cumulative Total Cost ($1000) $C=\sum_{i=1}^{N} n_i c_i$	(7) System Reliab. (%) R_s	(8) Lagrange Multiplier ζ_c	(9) Additional Weight for Subsystems (lbs) m_i	(10) Cumulative Total Weight $M=\sum_{i=1}^{N} n_i m_i$
12	Pitch Jet Elect. 2nd Standby	$q_{9,3}$=.00874	n_9=3	4	152.8	97.2	.0006	4	297.5
13	Roll-Yaw Jet Elect. 2nd Standby	$q_{10,3}$=.00583	n_{10}=3	6	158.8	97.6	.0004	6	303.5
14	Roll H/S 3rd Standby	$q_{3,4}$=.000300	n_3=4	8	166.8	97.8	.00028	4	307.5
15	Pitch H/S 3rd Standby	$q_{8,4}$=.000300	n_8=4	8	174.8	98.02	.00028	4	311.5
			INITIAL SOLUTION CASE B						
16	Mag. Coil Elect. 1st Standby	$q_{5,2}$=.000275	n_5=2	2	176.8	98.1	.00027	2	313.5
17	Wheel Elect. 2nd Standby	$q_{7,3}$=.000260	n_7=3	5	181.8	98.3	.00022	4	317.5
18	Charger 1st Standby	$q_{2,2}$=.000216	n_2=2	6	182.4	99.5	.0002	5.5	323
19	Battery 2nd Standby	$q_{1,3}$=.000162	n_1=3	5.6	188.0	99.6	.00016	5.6	379
20	H/S Elect.	$Q_{11,3}$=.000151	n_{11}=3	6	194.0	99.7		4	383

Table G-8 gives the members in common, which can be accepted as part of the final solution. The mass, M_r, and cost, C_r, for the common members are:

$$M_r = 245.5 \text{ pounds}$$

$$C_r = \$132,800$$

Thus:

$$M_c - M_r = 295 - 245.5 = 49.5 \text{ pounds} = \text{weight margin}$$

$$C_c - C_r = \$155,000 - \$132,800 = \$22,200 = \text{cost margin}$$

The next step is to determine which combination of the members not in common (Column 4) will give the maximum R_s and satisfy both weight and cost constraints. By inspection of the dollar and weight combination of uncommon members in Column 5, the flywheel can be eliminated from consideration since its weight is >49.5 pounds. This leaves only the following four members to examine:

A: Charger - 1st standby

B: Roll H/S - 3rd standby

C: Pitch H/S - 3rd standby

D: Roll-Yaw Jet Electronics - 2nd standby

Again, by inspection of column (5), any combination of three of the above four members has a weight of 49.5 pounds and cost of \$22,200. Next, examine the R_s for the 4-member combinations which are: $R_{s(ABC)}$, $R_{s(ABD)}$, $R_{s(BCD)}$, and $R_{s(CDA)}$. The maximum numerical value of ln R_s is obtained (which gives max. R_S) and is:

$$R_{s(ABD)} = R_{s(CDA)} = .9955$$

Since there are two optimum combinations, and the weight and cost of both are identical, (ABD) is arbitrarily selected as the optimum. The total weight of this system is 261.0 pounds (this leaves a margin of 34 pounds) and the total cost is \$147,400 (leaving a margin of \$7600).

TABLE G-8 SOLUTION – CASE C

(1) Sub-system	(2) No. of Standby (Initial Solution Based on Weight)	(3) No. of Standby (Initial Solution Based on Dollars)	(4) Common Members	(5) Uncommon Members	(6) Dollars and Weight of Uncommon Members	(7) Common Members Plus Selection from Uncommon Members	(8) Selection of Subsystem to Absorb Margin	(9) Final Solution (No. of Standby Plus 1) n_i
1	1	1	1st Standby	-	-	1st Standby	-	2
2	1	0	-	1st Standby**	C=$600 W=5.5	1st Standby	2nd Standby	3
3	3	2	1st and 2nd Standby	3rd Standby**	C=$8000 W=4	3rd Standby	-	4
4	1	1	1st Standby	-	-	1st Standby	-	2
5	0	0	-	-	-	-	1st Standby	2
6	0	1	-	1st Standby	C=$20,000 W=52	-	-	1
7	1	1	1st Standby	-	-	1st Standby	2nd Standby	3
8	3	2	1st and 2nd Standby	3rd Standby	C=$8000 W=4	2nd Standby	-	3
9	2	2	1st and 2nd Standby	-	-	2nd Standby	-	3
10	2	1	1st Standby	2nd Standby**	C=$6000 W=6	2nd Standby	-	3
11	1	1	1st Standby	-	-	1st Standby	-	2
			C=$132,800 W=245.5 lb	49.51 lbs & $22,200 can be applied.		C=$147,400 W=261 Margin of 34 lbs $7600.		C=$155,000 W=272.5 lb

**Combination Yielding Max R.

With margins of 34 pounds and $6500, additional redundancies may be selected from the following list:

	C	W	q_{ij}	Standby
Charger	$ 600	5.5	.000001	2nd
Mag. Coil	2500	5	.000004	2nd
Mag. Coil Electronics	2000	2	.000275	1st
Wheel Elect.	5000	4	.000260	2nd
Pitch Jet Electronics	4000	4	.000025	3rd
Roll-Yaw Jet Electronics	6000	6	.000016	3rd
H/S Elect.	6000	4	.000151	2nd

The q_{ij}'s based on cost (Table G-6) indicate that the following may be added:

Mag. Coil Electronics	-	1st standby
Wheel Electronics	-	2nd standby
Charger	-	2nd standby

This gives the solution presented in Column (8) of Table G-8 of a total weight of 272.5 pounds and a total cost of exactly $155,000 for the Case C constraints of weight $\leq$295 pounds and cost $\leq$ $155,000.

Determination of Constraints and/or Lagrange Multipliers

This example concentrated on finding the optimum selection using Lagrange multipliers, or the constraints (limits) on system parameters. In most practical problems, the weight constraint is known, but the cost constraint is usually not known. The 'marginal utility" of increasing reliability by applying dollars $\frac{\Delta R}{\Delta \$}$, can be derived from renewal theory considerations. That is, it can be determined at what point, or value of $\frac{\Delta R}{\Delta \$}$, it is no longer profitable to increase reliability but, instead, should rely on a complete replacement.

Example of Using Both Conventional and High Reliability Subsystems

In constraint Case A a weight constraint of 275 pounds and a cost constraint of \$250,000 were assumed. From the results as presented on Table G-6, the solution, considering conventional subsystems only, is to add the first 15 standby redundancies. This gives a satellite with 128.5 pounds of redundancies at an incremental cost of only \$89,000. Since \$250,000 has been allocated for the satellite cost, and the cost for conventional subystems as given in Table G-4 is \$156,700, the cost surplus of \$93,300 can be used to replace conventional with high reliability subsystems. A rapid calculation indicates that it would cost \$224,400 to replace all units with high reliability units. Thus, the selection criterion may be used to determine which units, when replaced with high reliability units, will give the maximum gain for the allotted money.

The failure rates and costs of the high reliability subsystems are presented in Table G-9. The computational procedure for determining which high reliability subsystems to include is described in detail below.

Calculating the q_{ij} Priority Values

The q_{ij} or "priority value" may be computed in accordance with the formulas given below. The results are presented in Table G-10.

Calculations for Subsystems with $n_i = 1$

First, for those subsystems with $n_i = 1$ (subsystem No. 6 shown in Table G-10), the q_{ij} is computed for replacing the basic conventional subsystems with a high reliability subsystems. q_{ij} is calculated by means of Equation (25), which in this case, becomes:

$$q_{ij} = \frac{1}{\Delta C_o} \ln \frac{R_i(1, 1_o)}{R_i(1, 0_o)}$$

TABLE G-9 COST OF CONVENTIONAL AND HIGH RELIABILITY SUBSYSTEMS

i	Subsystem	Conventional Reliab. Subsystem		Hi-Rel Subsystems		ΔC_o *
(1)	(2)	(3) Failure Rate (hrs)	(4) ($1000)	(5) Failure Rate (hrs)	(6) ($1000)	(7) ($1000)
1	Battery	1×10^{-6}	$5.6	1×10^{-6}	$5.6	0
2	Charger and Instrument Box	$.3 \times 10^{-6}$	$ .6	$.1 \times 10^{-6}$	$1.8	$1.2
3	Roll H/S	5×10^{-6}	$8	3×10^{-6}	$24	$16
4	Mag. Coil	$.1 \times 10^{-6}$	$2.5	$.05 \times 10^{-6}$	$7.5	$5
5	Mag. Coil Electronics	$.014 \times 10^{-6}$	$2	$.005 \times 10^{-6}$	$5	$3
6	Flywheel	$.5 \times 10^{-6}$	$20	$.5 \times 10^{-6}$	$20	0
7	Wheel Electronics	1.2×10^{-6}	$5	$.4 \times 10^{-6}$	$10	$5
8	Pitch H/S	5×10^{-6}	$8	3×10^{-6}	$24	$16
9	Pitch Jet Electronics	2×10^{-6}	$4	$.6 \times 10^{-6}$	$10	$6
10	Roll-Yaw Jet Electronics	2×10^{-6}	$6	$.6 \times 10^{-6}$	$15	$9
11	H/S Electronics	1×10^{-6}	$6	$.6 \times 10^{-6}$	$12	$6

* ΔC_o = Col. (6) - Col. (4)

TABLE G-10 CALCULATIONS FOR INCLUDING HIGH RELIABILITY SUBSYSTEMS

i Subsystems Designations	n_i, Total No.	No. of Standby Redundancies	$q_{ij} \times 10^{-4}$			
			Replacing Basic Subsystems With Hi-Rel Subsystems	Replacing 1st Standby With Hi-Rel Subsystems	Replacing 2nd Standby With Hi-Rel Subsystems	Replacing 3rd Standby With Hi-Rel Subsystems
1	2	1	No Hi-Rel Subsystem	No Hi-Rel Subsystem	-	-
2	2	1	.47484	.15869	-	-
3	4	3	.01984	.01171	.00730	.00446
4	2	1	.00956	.00478	-	-
5	2	1	.00040	.00033	-	-
6	1	0	No Hi-Rel Subsystem	No Hi-Rel Subsystem	-	-
7	3	2	.03083	.0104	.00349	-
8	4	3	.01984	.01171	.00730	.00446
9	3	2	.12166	.03698	.01137	-
10	3	2	.08111	.02466	.00754	-
11	3	2	.00897	.00540	.00326	-

where

$$\Delta C_o = C_{i_o} - c_i$$

$R_i(1, 1_o)$: denotes 1 unit only, and this unit is a high reliability unit.

$$= e^{-\lambda_{i_o} t}$$

o : denotes high reliability

$R_i(1, 0_o)$: denotes 1 unit only, and this is a conventional unit

$$= e^{-\lambda_i t}$$

In this case

$$\ln \frac{R_i(1, 1_o)}{R_i(1, 0_o)} = \ln \frac{e^{-\lambda_{i_o} t}}{e^{-\lambda_i t}} = t \left[\lambda_i - \lambda_{i_o}\right]$$

<u>Subsystem With $n_i = 2$</u>

Next, compute the q_{ij} for replacing the basic unit, then the standby unit for those cases where $n_i = 2$ (subsystems 2, 4, and 5 of Table G-10). For replacing the basic unit:

$$q_{ij} = \frac{1}{\Delta C_o} \ln \frac{R_i(2, 1_o)}{R_i(2, 0_o)}$$

where

$R_1(2, 1_o)$ is the first formula given on Table G-1

$R_1(2, 0_o)$ is computed by using $\lambda_{1_o} = \lambda_1$ in the first formula given on Table G-1. It should be noted that formula (23) should not be used since, in most cases, this formula cannot be computed with sufficient accuracy.

For replacing the first (and only) standby unit:

$$q_{ij} = \frac{1}{\Delta C_o} \ln \frac{R_1(2, 2_o)}{R_1(2, 1_o)}$$

where

$R_1(2, 2_o)$ is also computed by using the first formula on Table G-1 with $\lambda_1 = \lambda_{1_o}$

Subsystems with $n_1 = 3$

The q_{ij} for subsystems with 2 standby units (subsystems 7, 9, 10, and 11 of Table G-10) are computed as follows:

Replacing the basic unit:

$$q_{ij} = \frac{1}{\Delta C_o} \ln \frac{R_1(3, 1_o)}{R_1(3, 0_o)}$$

Replacing the first standby:

$$q_{ij} = \frac{1}{\Delta C_o} \ln \frac{R_1(3, 2_o)}{R_1(3, 1_o)}$$

Replacing the second standby:

$$q_{ij} = \frac{1}{\Delta C_o} \ln \frac{R_i(3, 3_o)}{R_i(3, 2_o)}$$

Subsystems with $n_i = 4$

The q_{ij} for subsystems #3 and #8 (of Table G-10) are computed by means of similar formulas.

<u>Final solution to Constraint Case A</u>

Using the q_{ij}'s from Table G-10, the priority list is constructed as shown in Table G-11. The first ten subsystems on this list can be replaced with a high reliability unit; replacing the 11th subsystem would exceed the costs limitation. However, by excluding the 11th subsystem, the two additional members listed at the bottom of Table G-11 could be added to absorb the cost "margin".

Table G-12 presents the final solution for constraint Case A. From the mathematical equations given, it makes no difference which of the units is labeled as high reliability. For example, with subsystem #5 (Mag. Coil Electronics) the mathematical computation of the system reliability would be the same whether the high reliability subsystem was applied to the basic unit or to the standby unit. However, from a practical standpoint, the high reliability subsystems are listed first since a failure could

(a) induce secondary failures

(b) be diagnosed improperly, or the switching mechanism may not operate properly.

TABLE G-11 PRIORITY OF INCLUDING HIGH RELIABILITY SUBSYSTEMS

Priority	Subsystem	ΔC_o ($1000)	$\Sigma\Delta C_o$ ($1000)	Total Cost ($1000)
0				156.7
1	Charger - Basic	1.2	1.2	157.9
2	Charger - 1st Standby	1.2	2.4	158.1
3	Pitch Jet Elect. - Basic	6	8.4	164.1
4	Roll-Yaw Elect. - Basic	9	17.4	173.1
5	Pitch Jet Elect. - 1st Standby	6	23.4	179.1
6	Wheel Elect. - Basic	5	28.4	184.1
7	Roll-Yaw Jet Elect. - 1st Standby	9	37.4	193.1
8	Roll H/S - Basic	16	52.4	209.1
9	Pitch H/S - Basic	16	68.4	225.1
10	Roll H/S - 1st Standby	16	84.4	241.1
11	Pitch H/S - 1st Standby	16	(Not Included*)	
12	Wheel Electronics - 1st Standby	5	89.4	246.1
	Mag Coil Elect. - Basic**	3	92.4	249.1

* Inclusion of this hi-rel subsystem exceeds cost constraint.

**Only remaining hi-rel subsystem which can be included to absorb cost margin.

TABLE G-12 FINAL SOLUTION CONSTRAINT CASE A

For Weight $\leq$ 275 lbs. (1) Cost $\leq$ \$250,000 (2)

Subsystem	n_i	Basic Subsystem	First Standby	Second Standby	Third Standby
Battery	2	Conv. Rel.	Conv. Rel.	-	-
Charger & Inst. Box	2	Hi-Rel.	Hi-Rel.	-	-
Roll H/S	4	Hi-Rel.	Hi-Rel.	Conv. Rel.	Conv. Rel.
Mag. Coil	2	Conv. Rel.	Conv. Rel.	-	-
Mag. Coil Elec.	2	Hi-Rel.	Conv. Rel.	-	-
Flywheel	1	Conv. Rel.	-	-	-
Wheel Electronics	3	Hi-Rel.	Hi-Rel.	-	-
Pitch H/S	4	Hi-Rel.	Conv. Rel.	Conv. Rel.	Conv. Rel.
Pitch Jet Electronics	3	Hi-Rel.	Hi-Rel.	Conv. Rel.	-
Roll-Yaw Jet Electronics	3	Hi-Rel.	Hi-Rel.	Conv. Rel.	-
H/S Electronics	3	Conv. Rel.	Conv. Rel.	Conv. Rel.	-

(1) Weight of basic subsystem = 146.5 lbs.; actual weight of redundancies = 128.5 lbs; and Total Weight = 275 lbs.

(2) Cost of basic subsystem (conventional reliability) = \$87,700; actual cost of redundancies + hi-rel subsystem = \$161,400; and Total Cost = \$249,100.

APPENDIX H

APPORTIONMENT OF SYSTEM EFFECTIVENESS DEFINED AS A CONTINUOUS FUNCTION OF MISSION TIME

1.0 INTRODUCTION

This example further demonstrates the logic of the Lagrange multiplier and priority list apportionment methods with a more intensive effectiveness analysis of the same communications satellite described in Appendix G. All the effectiveness parameters of availability, dependability, and capability are considered in this example. Initially, the system is treated with a constraint on one of the system constraint parameters, that of weight. The example is then expanded to include also the constraint of cost, with the effectiveness measure expanded to that of a cost utility or cost effectiveness measure.

In this example, the system is described briefly. Then there is discussion of the system figure of merit performance parameters, with the accountable factors which influence the figure of merit; and the system effectiveness model is developed. The Lagrange Multiplier technique is then illustrated, with the derived priority list for determination of the satellite design yielding the optimum effectiveness. There follows a numerical exercise addressed to the weight constraint situation only, and then to an optimization procedure in which cost is also considered.

The example selected is that of a communications satellite without on-orbit repair capability. The objective of the satellite system is to obtain the most channel-time possible for the given weight constraint arising from the booster which had been selected. In a later part of this example, the cost revenue analysis obtainable for channel-time is described. To maximize channel-time per satellite, consideration must be given to the number of channels to include in the design, and the lifetime expected to be achieved with each satellite. To maximize this lifetime, the length of time that each satellite operates must be considered, as well as the probability that each satellite successfully achieves orbit.

The satellite separates from the booster while still in the transfer orbit. It must rely on on-board equipments for stabilization while coasting to apogee and to provide the thrust for final orbit circularization and error adjust. The satellite relies on a hydrazine station-keeping subsystem. When this subsystem is out of fuel or depleted, the satellite rapidly drifts and becomes useless for communications.

2.0 PERFORMANCE PARAMETERS

In this example, the performance parameters which influence the amount of channel-time obtainable with each satellite are:

(1) β, the probability of successfully attaining orbit and completing satellite set-up events. This is the availability parameter and is a function of technical performance characteristics of the system as described in paragraph 4.3.

(2) T, the planned expendable depletion time which is determined by the amount of hydrazine propellant loaded.

(3) $R_S(t)$, the satellite's dependability parameter expressed as the reliability parameter depending on the included standby redundancies which increase the satellite's expected lifetime. This reliability is exclusive of the equipment for the operation of the channels, the communications subsystem.

(4) s, the number of channels. This is the capability parameter.

The mission time T is equivalent to the planned expendable depletion time. If T is divided into ΔT intervals, then K different time intervals may be defined for the transmission of information by a number of operating channels, with

$$K = \frac{T}{\Delta T}$$

The channels used may differ for each time interval. Thus, there may be defined an average channel capability for the ΔT intervals, the sum of which is a measure of overall effectiveness. The summation is transformed to an integral as $\Delta T \rightarrow 0$.

The system figure of merit will be the expected channel-time or expected number of channels usable over the length of time each channel was available. The major effectiveness performance parameters of availability, dependability, and capability are included in the system effectiveness model.

3.0 SYSTEM EFFECTIVENESS MODEL

The effectiveness measure is:

$$E = \beta \int_0^T \sum_{r=0}^{s} r\, M(t)\, R_S(t)\, dt \qquad (1)$$

where:

β = probability of successful orbit achievement

$R_S(t)$ = the probability that the satellite, exclusive of the communications subsystem, operated reliably at any time t

$M(t)$ = the probability that exactly r out of s channels will be operating at time t

T = expendable depletion time

$\sum_{r=0}^{s} r\, M(t)$ = expected number of channels operating at any time t, and is the capability parameter.

3.1 Number of Channels in Communications Subsystem

Figure H-1 shows the communications subsystem and the simplifying assumptions used in constructing the mathematical model. Further, it is assumed that the s independent channels each exhibit the same failure rate λ_c, and each weigh 20 pounds. Also, any failure of the power system is included as a catastrophic failure of the main bus.

Due to the simplicity of each channel, the design does not include any redundancy in the components comprising the channels. If any component in a channel fails, the entire channel is lost since it lacks restoration capability.

At time t, each channel has the same probability of being operative, and thus, the distribution of the number of operating channels $M(t)$ may be considered to be binomial, with

$$M(t) = P_r(r, s, p)$$

$$= \binom{s}{r} p^r (1-p)^{s-r}$$

= the probability that exactly r out of s channels is operating at any time t. The parameter p is the probability of survival of each channel with $p = e^{-\lambda_c t}$ and λ_c = the channel failure rate.

Thus, the effectiveness becomes

$$E = \beta \int_0^T \left[\sum_{r=0}^{s} r \binom{s}{r} p^r (1-p)^{s-r} \right] R_S(t)\, dt$$

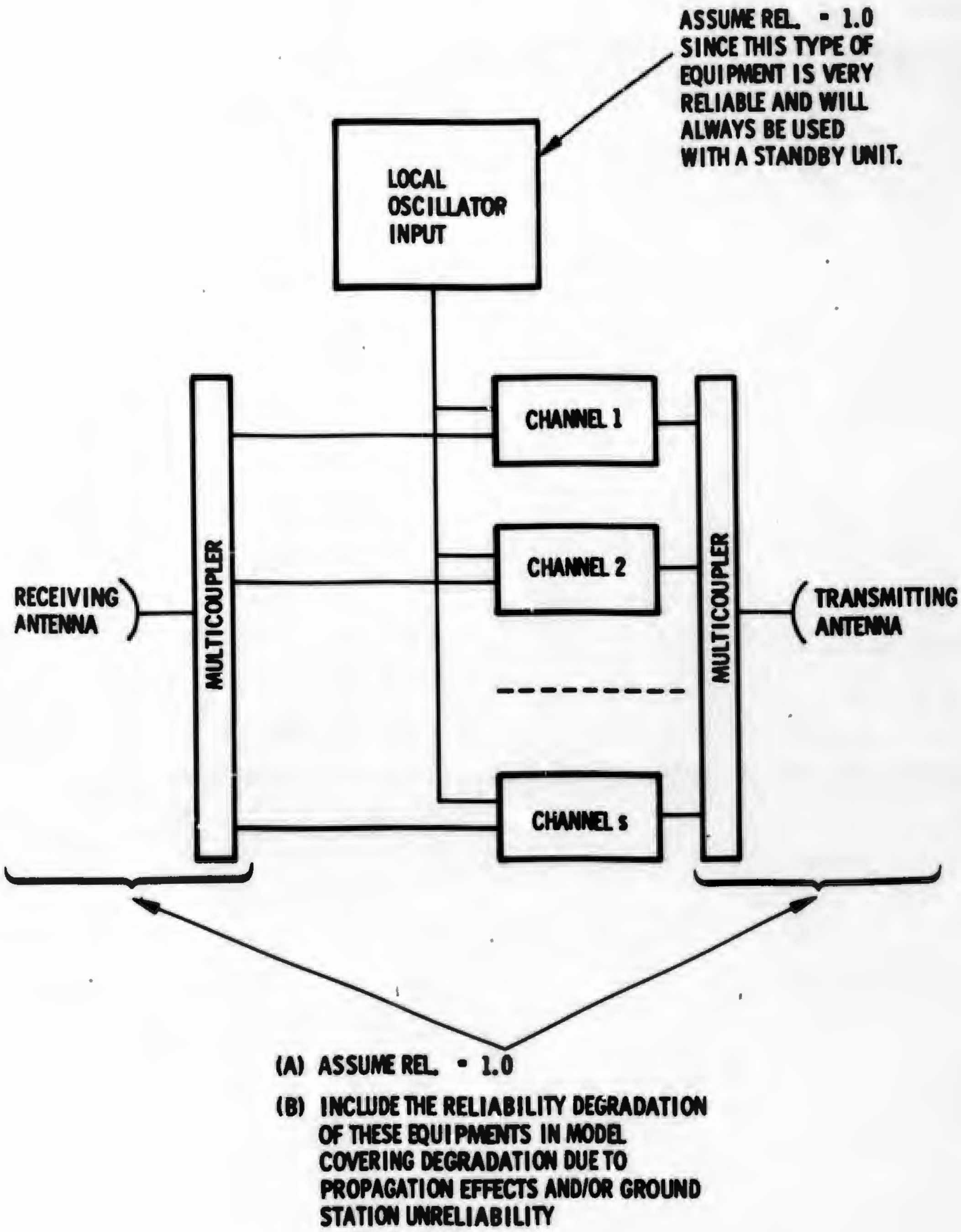


Figure H-1 Communications Subsystem

and, since

$$\left[\sum_{r=o}^{s} r \binom{s}{r} p^{r} (1-p)^{s-r}\right] = sp$$

$$= se^{-\lambda_c t}$$

E may then be expressed as:

$$E = \beta s \int_{0}^{T} e^{-\lambda_c t} R_S(t)\, dt \qquad (2)$$

The satellite, exclusive of the communications subsystem, consists of N subsystems in series. The failure of any one of the N subsystems will result in failure of the entire system. The reliability of the satellite, $R_S(t)$, is the product of the reliabilities of the N subsystems, where the i^{th} subsystem may have n_i redundancies:

$$R_S(t) = \prod_{i=1}^{N} R_i(n_i, t)$$

Since only standby redundancies are considered with 100 percent reliable switching assumed,

$$R_i(n_i, t) = e^{-\lambda_i t} \sum_{j=0}^{n_i - 1} \frac{(\lambda_i t)^j}{j!}$$

and thus

$$R_S(t) = \prod_{i=1}^{N} e^{-\lambda_i t} \sum_{j=0}^{n_i - 1} \frac{(\lambda_i t)^j}{j!} \qquad (3)$$

3.2 Derivation of Effectiveness Measure

For the k^{th} time interval of Δt duration, there may be defined a $(\Delta E)_k$, the channel time available during the k^{th} time interval, with k taking on values from 1 to k and, as previously defined, with $K = T/\Delta T$. Thus, $(\Delta E)_k$ may be expressed as:

$$(\Delta E)_k = \sum_i \sum_j A_i D_{ij} C_{jk}$$

Since only successful orbit attainment is of interest in this example, the i^{th} element, A_i, of the availability vector may be defined as

$$A_i = \begin{cases} \beta & \text{for } i = 1 \\ 0 & \text{for } i \neq 1 \end{cases}$$

The elements D_{ij} of the dependability matrix which are a function of the reliability of the satellite's subsystems exclusive of the communications subsystem, may be defined as

$$D_{ij} = \begin{cases} R_S(j\Delta T) & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

The elements of the capability matrix, which are a function of the number of operating channels during the k^{th} time interval may be defined as

$$C_{jk} = \begin{cases} \Delta T \sum_{r=0}^{a} r M(k\Delta T) & \text{for } j = k \\ 0 & \text{for } j \neq k \end{cases}$$

Thus,

$$(\Delta E)_k = \beta R_s(k\Delta T)\, \Delta T \sum_{r=0}^{s} r\, M(k\Delta T)$$

and if the $(\Delta E)_k$ are equally weighted,

$$E = \sum_{k=1}^{K} (\Delta E)_k$$

$$= \sum_{k=1}^{T/\Delta T} \beta \left[\sum_{r=0}^{s} r\, M(k\Delta T) \right] R_s(k\Delta T)\ \Delta T$$

If $\Delta T \rightarrow dt \rightarrow 0$ and $k\Delta T \rightarrow t$, then

$$E = \beta \int_0^T \left[\sum_{r=0}^{s} r\, M(t) \right] R_s(t)\, dt \qquad (4)$$

4.0 RELATIONSHIP OF SYSTEM PARAMETERS TO WEIGHT CONSTRAINT W

4.1 The Relation $\Delta T/\Delta W$

The hydrazine subsystem must include a fixed amount of propellant for ascent orbit adjust and initial precision placement. 30 pounds additional is required for each year of on-orbit operation; e.g., if the planned operational span is 5 years, 150 pounds of hydrazine is required for the on-orbit phase. The rate of expendable depletion for the anticipated operational span can vary as much as ±10 percent, depending on the season and the year. However, for the purposes of this example, assuming a linear time factor (i.e., 1 year/30 pounds) is sufficiently accurate.

4.2 <u>The Relation $\Delta s/\Delta W$</u>

Each channel in the initial design weighs 20 pounds, including weight for power equipment. The failure rate of each channel, λ_c, is 1.75×10^{-2} per year. Weight increases to improve λ_c are not practical.

4.3 <u>The Relation $\Delta\beta/\Delta W$</u>

The ascent phase consists of the booster phase into the transfer orbit, shroud jettison, and spacecraft separation. It also includes the remaining portion of the ascent phase carried out by the spacecraft itself, consisting of coasting to the selected apogee, thrusting at apogee to transfer into a near-circular equatorial orbit, drifting on-orbit to attain the desired satellite station (Earth longitude position), and final orbit trim to attain as close to a 24-hour orbit as possible over the desired Earth longitude.

The major alternatives under consideration for accomplishing the spacecraft coast phase up to apogee thrust, which could be several days duration, are: (1) spin stabilization and (2) active stabilization. The spin stabilization method during coast requires 10 pounds of satellite weight and gives a $\beta_S = .95$. Active stabilization requires 50 pounds of weight and gives a $\beta_S = .99$.

The availability parameter β is the probability of success for all ascent events, with

$$\beta = (.95)(.99)(.99)(\beta_S)$$

and

.95 = prob. of success of boost to transfer orbit

.99 = prob. of success of shroud jettison

.99 = prob. of success of spacecraft separation

β_S = prob. of success of spacecraft coast phase up to apogee thrust.

S, in turn, is a function of the availability, dependability, and capability of the booster system, together with the capability and dependability parameters of the subsystems required to accomplish the necessary set-up events of stabilization, deployment of solar arrays, etc.

The availability of the booster system is a parameter of relatively minor consequence, since the mission is not dependent upon precise launch time. The booster must, however, be capable of injecting the satellite into the desired orbit, which involves its capability for a particular payload and orbit, together with its dependability in performing properly during the ascent phase.

Success of all ascent events may involve, for example, the following technical characteristics:

- Countdown success.
- Ignition of all motors within the prescribed time.
- Prescribed level of thrust in the prescribed time.
- Release of vehicle from pad within the time tolerance after thrust build-up.
- Dependability of rate gyros and circuitry to control motor gimbals, thrust levels, and engine shut-down time within specified thrust-time envelope.
- Separation of first stage within specified time from boost burnout.
- Ignition of vernier orientation rockets to achieve desired alignment prior to second stage ignition.
- Ignition of second stage at correct time as calculated from thrust-time envelope and repeat of last three characteristics for second and third stages.
- Jettisoning satellite shroud upon receipt of a signal when satellite injected into desired orbit.
- Actuation of horizon sensors upon shroud jettison and detect horizon and/or star within prescribed time, and to specified tolerance.
- Correct firing of vernier alignment rockets based upon output from horizon sensors/star tracker.

- Actuation of motors to deploy solar arrays (if not automatically deployed upon shroud ejection).
- Operation of relays to connect batteries with solar arrays.
- Operation of stabilization subsystem when batteries are connected.
- Activation of transmit and receive antennas.

Each of these performance characteristics is a function of accountable factors at the design level. The functional relationships to this level, and to the booster-satellite system level, are expressible by transfer functions, which may be evaluated by direct or Monte Carlo simulation methods.

For the alternatives for accomplishing the spacecraft phase of ascent, two discrete design choices are:

(1) spin stabilization with $\beta = .8845$ and weight of 10 pounds, or

(2) active stabilization with $\beta = .9218$ and a weight of 50 pounds,

and with $\Delta\beta/\Delta W = .0373/40$ pounds for an active stabilization design versus the spin stabilization design.

4.4 The Relation $\Delta R_S/\Delta W$

The subsystems aboard the satellite have been divided into the following categories for this analysis:

(a) Subsystems which, for most practical purposes, can be considered to have no failures (e.g., structures and solar array panels with inherent redundancies). The consideration of a failure rate greater than zero for these subsystems would, in most cases, complicate the model and would not increase its utility as a decision tool. Thus, these subsystems have been assumed to be 100 percent reliable.

(b) Subsystems which exhibit failures catastrophic to the mission. Subsystems in this category are listed in Table H-1 and are included in the redundancy model to be discussed.

TABLE H-1

SATELLITE SUBSYSTEMS EXHIBITING CATASTROPHIC RANDOM FAILURES

Subsystem	Weight (lbs.)	Failure Rate (per yr.)
Battery	64	$.9 \times 10^{-2}$
Charger & Inst. Box	5.5	$.26 \times 10^{-2}$
Roll Horizon Sensor	4	4.5×10^{-2}
Pitch Horizon Sensor	4	4.5×10^{-2}
Mag. Coil	5	$.09 \times 10^{-2}$
Mag. Coil Elect.	2	$.012 \times 10^{-2}$
Flywheel	52	$.44 \times 10^{-2}$
Wheel Electronics	4	1.0×10^{-2}
Pitch Jet Elect.	4	1.75×10^{-2}
Roll-Yaw Jet Elect.	6	1.75×10^{-2}
Horizon Sensor Electronics	4	$.9 \times 10^{-2}$

5.0 APPLICATION OF THE LAGRANGE MULTIPLIER METHOD

For the illustration, effectiveness can be considered to improve by adding:

- weight to increase the initial number of channels, $\left(\frac{\Delta s}{\Delta W}\right)$
- weight to increase the ascent availability, $\left(\frac{\Delta \beta}{\Delta W}\right)$
- weight to increase the planned depletion time, $\left(\frac{\Delta T}{\Delta W}\right)$
- weight to improve reliability by redundancy application, $\left(\frac{\Delta R_S}{\Delta W}\right)$

As previously shown in Equation (2), the system effectiveness measure is:

$$E = \beta s \int_0^T e^{-\lambda_c t} R_S(t)\, dt$$

with

$$R_S(t) = \prod_{i=1}^{N} R_i(n_i, t)$$

and

$$R_i(n_i, t) = e^{-\lambda_i t} \sum_{j=0}^{n_i - 1} \frac{(\lambda_i t)^j}{j!}$$

The achievable effectiveness of the system is subject to the constraint relation:

$$W \geq c_1 s + f(c_2, \beta) + c_3 T + \sum_{i=1}^{N} n_i W_i \tag{5}$$

where

c_1 = 20 lbs. per channel

$$f(c_2, \beta) = \begin{cases} 10 \text{ lbs. for spin stabilization ascent design} \\ 50 \text{ lbs. for active stabilization ascent design} \end{cases}$$

$$= 10 + \left| (\beta - .8845)40/.0373 \right|$$

c_3 = 30 lbs./year

W_i = weight of the i^{th} subsystem

W = system weight

T = expendable depletion time

In order to find the value of the variables s, $f(c_2, \beta)$, T, and n_i which will maximize E, the system effectiveness, the total differential of Equation (2) is taken and equated to zero, with the following expression obtained:

$$\Delta E = \frac{\Delta s}{s} E + \frac{\Delta \beta}{\beta} E + \beta s e^{-\lambda_c T} R_S(T) \Delta T$$

$$+ \beta s \sum_{i=1}^{N} \Delta n_i \int_0^T e^{-\lambda_c t} \frac{R_S(t)}{R_i(n_i, t)} \frac{\partial R_i(n_i, t)}{\partial n_i} dt = 0 \qquad (6)$$

where

$$\frac{\partial R_i(n_i, t)}{\partial n_i} = e^{-\lambda_i t} \frac{(\lambda_i t)^{n_i - 1}}{(n_i - 1)!}$$

Next, the total differential of Equation (5) is computed and the result multiplied by the Lagrange multiplier to obtain another degree of freedom. Assuming W is a constant and replacing the inequality with an equal sign, the following is obtained:

$$0 = \Delta W = \zeta c_1 \Delta s + \zeta c_2 \Delta \beta + \zeta c_3 \Delta T + \zeta \sum_{i=1}^{N} W_i \Delta n_i \qquad (7)$$

Subtracting Equation (7) from Equation (6) and noting this difference must be equal to zero, which means that the coefficient of each "term" must therefore be equal to zero, the following set of equations is obtained

$$\left(\frac{E}{s} - \zeta c_1\right)\Delta s = 0$$

$$\left(\frac{E}{\beta} - \zeta c_2\right)\Delta\beta = 0$$

$$\left(\beta s e^{-\lambda_c T} R_S(T) - \zeta c_3\right)\Delta T = 0 \tag{8}$$

$$\left\{\left[\beta s \sum_{i=1}^{N} \int_0^T e^{-\lambda_c t} \frac{R_S(t)}{R_i(n_i, t)} e^{-\lambda_i t} \frac{(\lambda_i t)^{n_i - 1}}{(n_i - 1)!} dt\right] - \zeta \sum_{i=1}^{N} W_i\right\}\Delta n_i = 0$$

The coefficient of each term is then solved for ζ to yield the following set of equations:

$$\zeta = \frac{E}{c_1 s}$$

$$= \frac{\beta}{c_1} \int_0^T e^{-\lambda_c t} R_S(t)\; dt$$

$$\zeta = \frac{E}{c_2 \beta} = \frac{s}{c_2} \int_0^T e^{-\lambda_c t} R_S(t)\; dt \tag{9}$$

$$\zeta = \frac{\beta s e^{-\lambda_c T} R_S(T)}{c_3}$$

$$\zeta = \frac{\beta s}{W_i} \int_0^T \frac{R_S(t)}{R_i(n_i, t)} e^{-(\lambda_c + \lambda_i)t} \frac{(\lambda_i t)^{n_i - 1}}{(n_i - 1)!} dt$$

for $i = 1, \ldots\ldots, N$

The solution is the value of $f(c_2, \beta)$, s, T, and n_i which will simultaneously satisfy the set of equations (9). To make the problem computationally tractable, some assumptions will be required to provide a first approximation to the numerical value of these equations.

In most situations, the functions $R_S(t)$ and $R_i(n_i, t)$ in the integrals of Equation (9) may be approximated as follows:

$$R_S(t) = e^{-\lambda_S t}$$

$$R_i(n_i, t) = 1$$

where λ_S is the sum of the failure rates of all subsystems with no standby redundancies. Such nonredundant subsystems will usually contribute significantly to the unreliability of the system and may be determined after an iteration has been made at some value of T. On the other hand, subsystems with standby redundancies will not contribute significantly to the unreliability of the system. Their failure rates will be assumed to be zero and are not included in the failure rate sum λ_S. It should be noted that λ_S cannot be computed until the redundancies have been determined for the value of T being computed.

Substituting

$$\lambda = \lambda_c + \lambda_S$$

the first 3 equations for the set of equations of (9) become:

$$\zeta = \frac{\beta}{c_1} \int_0^T e^{-\lambda t}\, dt$$

$$= \frac{\beta}{\lambda c_1} \left[1 - e^{-\lambda T}\right] \tag{10}$$

$$\zeta = \frac{s}{c_2} \int_0^T e^{-\lambda t}\, dt$$

$$= \frac{s}{\lambda c_2} \left[1 - e^{-\lambda T}\right] \qquad (11)$$

$$\zeta = \frac{\beta s e^{-\lambda T}}{c_3} \qquad (12)$$

and the last equation of (9) becomes:

$$\zeta = \frac{\beta s}{W_i} \int_0^T e^{-(\lambda + \lambda_i) t} \frac{(\lambda_i t)^{n_i - 1}}{(n_i - 1)!}\, dt \quad \text{for } i=1, \ldots, N$$

$$= \frac{\beta s}{W_i} \frac{\lambda_i^{\,n_i - 1}}{(\lambda + \lambda_i)^{n_i}} \frac{1}{(n_i - 1)!} \int_0^T \left[(\lambda + \lambda_i)t\right]^{n_i - 1} e^{-(\lambda + \lambda_i)t} (\lambda + \lambda_i)\, dt$$

Letting $z = (\lambda + \lambda_i)\, t$ and using the relation:

$$\frac{1}{(n_i - 1)!} \int_0^Z z^{n_i - 1} e^{-z}\, dz = 1 - e^{-Z} \sum_{j=0}^{n_i - 1} \frac{Z^j}{j!}$$

then:

$$\zeta = \frac{\beta s}{W_i} \frac{\lambda_i^{\,n_i - 1}}{(\lambda + \lambda_i)^{n_i}} \left\{1 - e^{-(\lambda + \lambda_i)T} \sum_{j=0}^{n_i - 1} \frac{\left[(\lambda + \lambda_i)T\right]^j}{j!}\right\}$$

Further, using the relation

$$e^{(\lambda + \lambda_i)T} = \sum_{j=0}^{n_i -1} \frac{\left[(\lambda + \lambda_i)T\right]^j}{j!} + \sum_{j=n_i}^{\infty} \frac{\left[(\lambda + \lambda_i)T\right]^j}{j!}$$

ζ may now be expressed as:

$$\zeta = \frac{\beta s}{W_1} \frac{\lambda_i^{n_i -1}}{(\lambda + \lambda_i)^{n_i}} \left\{ e^{-(\lambda + \lambda_i)T} \sum_{j=n_i}^{\infty} \frac{\left[(\lambda + \lambda_i)T\right]^j}{j!} \right\}$$

Finally, this expression may be written in the following form which is computationally tractable since the higher order terms can be dropped:

$$\zeta = \frac{\beta s e^{-(\lambda + \lambda_i)T} (\lambda_i T)^{n_i}}{W_i \lambda_i (n_i)!} \left\{ 1 + \frac{(\lambda + \lambda_i)T}{n_i + 1} + \frac{\left[(\lambda + \lambda_i)T\right]^2}{(n_i + 1)(n_i + 2)} + \ldots \right\} \quad (13)$$

Using equations (10) and (12),

$$s = \frac{c_3}{c_1 \lambda} (e^{\lambda T} - 1) \quad (14)$$

and using equations (12) and (13):

$$\frac{1}{c_3} \leq \frac{e^{-\lambda_i T} \lambda_i^{n_i -1} T^{n_i}}{W_i (n_i!)} \left\{ 1 + \frac{(\lambda + \lambda_i)T}{(n_i + 1)} + \frac{(\lambda + \lambda_i)^2 T^2}{(n_i + 1)(n_i + 2)} + \ldots \right\} \quad (15)$$

Furthermore, with these assumptions, the effectiveness measure of Equation (2) reduces to:

$$E = \frac{\beta s}{\lambda}(1 - e^{-\lambda T})$$

or (16)

$$E = \beta s T\left[1 - \frac{\lambda T}{2} + \frac{(\lambda T)^2}{6} - \frac{(\lambda T)^3}{24} + \ldots\right]$$

Equation (16) expresses the effectiveness E as the expected number of channel-years obtainable from each satellite (average number of operating channels times the average number of years).

5.1 Computational Algorithm

As previously indicated, the two discrete design alternatives for $f(c_2, \beta)$ are

Spin stabilization with β = .8845 for 10 pounds

Active stabilization with β = .9218 for 50 pounds

The choice between these two design alternatives will be the one which maximizes the effectiveness as expressed by Equation (16). Therefore, the most efficient algorithm is to determine which β will optimize effectiveness and, using this method of ascent stabilization, to then determine the weight allocation to the remaining system accountable factors.

The total weight budget is 610 pounds which can be allotted to: (1) redundancies, (2) ascent stabilization, (3) on-orbit hydrazine gas, and (4) channels. This means that for the case of a spin-stabilized vehicle, there will be 600 remaining pounds to allocate to R_S, T, or s. For the case of active stabilization, 560 pounds remain.

5.2 <u>Determination of Ascent Stabilization Design</u>

The selection of the ascent design can be made by examining the set of equations in (9) which defines the optimized design. The first two equations of (9) may be interpreted as follows:

$$\zeta = \frac{E}{c_1 s_{optimum}}$$

$$\zeta \leq \frac{E}{c_2 \beta_i} \qquad i = 1, 2$$

Thus,

$$\frac{E}{c_2 \beta_i} \geq \frac{E}{c_1 s_{optimum}}$$

$$c_2 \beta_i \leq c_1 s_{optimum}$$

$$s_{optimum} \geq \frac{c_2 \beta_i}{c_1}$$

Designate the two values of β_i as β_1 = .8845 and β_2 = .9218. A gain of $\Delta\beta$ = 0.0373 is achieved with the addition of 40 pounds of weight, that is:

$$c_2 = \frac{40 \text{ lb}}{0.0373}$$

Substituting the values of β_1 and β_2 into the expression for $s_{optimum}$ yields values of 47.4 and 49.4, respectively. This indicates that the 10 lb spin stabilization system will optimize the effectiveness if the number of channels exceeds 47, and for the heavier system, when the number exceeds 49. However, 30 is the maximum number of channels (at 20 lb per channel) which can be specified because of the weight constraint of 600 lb. Therefore it can be concluded that, of the two design choices, the 10 lb stabilization system, requiring fewer channels, will provide the greater effectiveness.

5.3 Determination of Optimized Design

Since spin stabilization is the optimum choice for the ascent mode, there remains a weight of 600 lb to allocate to R_S, T, and s, the values of which must be determined to simultaneously satisfy equations (14) and (15) and meet the weight constraint. This is accomplished by iterating on T with equations (14) and (15) until the weight constraint is met. The optimum is isolated in five steps as follows:

Step 1 : T = 5 years

Step 2 : T = 10 years

Step 3 : T = 7 years

Step 4 : T = 8 years

Step 5 : T = 9 years

The computations for each step in iterating to the optimum are discussed in more detail below.

Step 1

First, determine by using inequality of (15), which redundancies to add. That is, start with $\lambda = \lambda_c$ and compute the redundancies to add at T = 5 years. Then, update $\lambda = \lambda_S + \lambda_c$ (where λ_S = the sum of the failure rates of only those subsystems which do not have redundancies added in the first try). See if any more redundancies can be accepted. In this example, for all values of T that were used, no updating of the redundancy list was necessary.

Recalling from (15) that

$$\frac{1}{c_3} \leq \frac{e^{-\lambda_i T} \lambda_i^{n_i - 1} T^{n_i}}{w_i (n_i!)} \left\{ 1 + \frac{(\lambda + \lambda_i)T}{(n_i + 1)} + \frac{(\lambda + \lambda_i)^2 T^2}{(n_i+1)(n_i+2)} + \ldots \right\}$$

and letting the right side of this inequality $= q_{i,\, n_i}$, then using logarithms for computational ease;

$$-3.4 \leq \ln (q_{i,\, n_i})$$

Next, construct a priority list based on $\ln (q_{i,\, n_i})$ and accept all redundancies which satisfy the above inequality. Table H-2 gives the calculated values of $\ln (q_{i,\, n_i})$. In Table H-3, the redundancies are listed in the priority of adding them to the system. The above inequality indicates the acceptance of only the first six redundancies, for a total weight allotment of 26 pounds to redundancies.

The priority list of Table H-3 is extended beyond the first six redundancies, in order to compare the results with those obtained in Appendix G. This comparison is made to establish the equivalence of the two methods for adding redundancies and, also, to determine the suitability of using (15) with $\lambda = \lambda_c$ to derive a priority list. As can be

TABLE H-2

CALCULATIONS FOR PRIORITY LIST BASED ON WEIGHT CONSTRAINT
(Mission Lifetime = 5 years)

Subsystem	i	λ_i Failure Rate (per year)	W_i (1) weight (lbs.)	ln (q_{i2})	ln (q_{i3})	ln (q_{i4})
Battery	1	$.9 \times 10^{-2}$	56.0	-6.15	-10.35	-14.83
Charger & Instrument Box	2	$.26 \times 10^{-2}$	5.5	-5.04	-10.49	-16.20
Roll Horizon Sensor	3	4.5×10^{-2}	4	-2.09	-4.68	-7.54
Mag. Coil	4	$.09 \times 10^{-2}$	5	-6.00	-12.51	-19.28
Mag. Coil Electronics	5	$.012 \times 10^{-2}$	2	-7.28	-15.98	-24.95
Flywheel	6	$.44 \times 10^{-2}$	52	-6.77	-11.69	-16.87
Wheel Electronics	7	1.0×10^{-2}	4	-3.41	-7.51	-11.88
Pitch Horizon Sensor	8	4.5×10^{-2}	4	-2.09	-4.68	-7.54
Pitch Jet Electronics	9	1.75×10^{-2}	4	-2.89	-6.43	-10.24
Roll-Yaw Jet Elect.	10	1.75×10^{-2}	6	-3.30	-6.84	-10.64
Horizon Sensor Electronics	11	$.9 \times 10^{-2}$	4	-3.51	-7.72	-12.19

(1) In these calculations, it is assumed the same amount of additional weight is required to add a redundant subsystem as for the initial subsystem.

TABLE H-3

PRIORITY LIST BASED ON WEIGHT CONSTRAINT (Mission Lifetime 5 years)

Priority	Subsystem	n_i	$\ln q_{i,n_i}$	W_i (lbs.)	ΣW_i
1	Roll H/S - 1st standby	$n_3 = 2$	-2.09	4	4
2	Pitch H/S - 1st standby	$n_8 = 2$	-2.09	4	8
3	Pitch Jet Electronics - 1st standby	$n_9 = 2$	-2.89	4	12
4	Roll-Yaw Jet Elect. - 1st standby	$n_{10} = 2$	-3.30	6	18
5	Wheel Electronics - 1st standby	$n_7 = 2$	-3.41	4	22
6	H/S Electronics - 1st standby	$n_{11} = 2$	-3.51	4	26
	- - - - - - - - Accept redundancies to here - - - - - - - -				
7	Roll H/S - 2nd standby	$n_3 = 3$	-4.68		
8	Pitch H/S - 2nd standby	$n_8 = 3$	-4.68		
9	Charger - 1st standby	$n_2 = 2$	-5.04		
10	Mag. Coil - 1st standby	$n_4 = 2$	-6.00		
11	Battery - 1st standby	$n_1 = 2$	-6.15		
12	Pitch Jet Electronics - 2nd standby	$n_9 = 3$	-6.43		
13	Flywheel - 1st standby	$n_6 = 2$	-6.77		
14	Roll-Yaw Jet Electronic - 2nd standby	$n_{10} = 3$	-6.84		
15	Mag. Coil Electronic - 1st standby	$n_5 = 2$	-7.28		
16	Wheel Electronics - 2nd standby	$n_7 = 3$	-7.51		
17	Roll H/S - 3rd standby	n_3 -4	-7.54		
18	Pitch H/S - 3rd standby	$n_8 = 4$	-7.54		
19	H/S Electronics - 2nd standby	$n_{11} = 3$	-7.72		

H/S: Horizon Sensor

seen by comparing the priority list in Table H-3 with that of Appendix G, the approximation of $\lambda = \lambda_c$ for this illustration is valid. The two priority lists are identical for the first 9 redundancies, with a difference in order for only one subsystem from the 9th to the 12th redundancies. Thereafter, the ordering is fairly different, although the members included on both lists are identical up to 19 redundancies.

Next, Equation (14) is solved for integer s to obtain the value of T closest to 5 years.

$$s = \frac{c_3}{c_1 \lambda} (e^{\lambda T} - 1)$$

where:

$$\lambda = \lambda_c + \lambda_s = .0345$$

$$c_3 = 30 \text{ pounds/year}$$

$$c_1 = 20 \text{ pounds/channel}$$

which yields:

s = 8 channels, which require 160 pounds.

T = 4.9 years, which require 147 pounds of hydrazine.

To verify progress towards the optimum, the effectiveness given by Equation (16) is calculated next,

$$E = \beta s T \left[1 - \frac{\lambda T}{2} + \frac{(\lambda T)^2}{6} - \frac{(\lambda T)^3}{24} + \dots \right]$$

where

$$\beta = .8845$$

$$s = 8$$

$$T = 4.9$$

$$\lambda = \lambda_c + \lambda_S = .0345$$

which yields

$$E = 34 \text{ channel-years}$$

The result indicates the weight allocation as 333 pounds at $T = 4.9$ years. Since there is a total weight budget of 600 pounds, this solution is not the final one. Table H-4 summarizes this weight allocation as well as those in Steps 3, 4, and 5 to follow.

Step 2

Next, set $T = 10$ years and solving for Equation (14),

s = 18 channels, which require 360 pounds.

T = 10 years, which require 300 pounds.

These two allocations give a total weight of 660 pounds, which exceeds the weight constraint.

Step 3

Next, set $T = 7$ years. Then,

s = 12 channels, requiring 240 pounds.

T = 7 years, requiring 210 pounds.

TABLE H-4 SUMMARY OF EFFECTIVENESS ALLOCATION

	Solution at T Years			
	T = 4.9	T = 7	T = 8	T = 8.9*
• Number of Channels	8	7	14	15
• Weight Allocated to Channels	160	240	280	300
• Weight Allocated to Hydrazine	147	210	240	266
• Weight Allocated to Redundances	26	34	34	34
Roll H/S - 1 Standby	4	---	---	---
- 2 Standby	---	8	8	8
Pitch H/S - 1 Standby	4	---	---	---
- 2 Standby	---	8	8	8
Pitch Jet Elect.	4	4	4	4
Roll - Yaw Jet Electronics	6	6	6	6
Wheel Electronics	4	4	4	4
H/S Electronics	4	4	4	4
• Total Weight (lbs)	333	484	554	600
• Total Weight (lbs, including spin stabilization)	343	494	564	610
• Effectiveness (channel - years)	34	73	97	116

* Optimized Solution

which yields a total weight of 484 pounds and an effectiveness of 73 channel-years.

Step 4

Next, set T = 8 years. Then,

s = 14 channels, requiring 280 pounds.

T = 8 years, requiring 240 pounds.

which gives a total weight of 554 pounds and 97 channel-years.

Step 5

To converge to the optimum, let T = 9 years, which requires 270 pounds. The solution, using Equation (14), gives s = 15 which requires 300 pounds. The weight allocation to redundancies is again 34 pounds. This gives a total weight of 604 pounds, so T is adjusted to 8.9 which decreases the weight allocation for hydrazine to 266 pounds.

The optimized effectiveness solution for a weight constraint of 610 pounds, including weight for the ascent stabilization mode, is 97 channel-years.

5.4 System Effectiveness Related to Weight Constraint

Furthermore, the optimum design for any weight constraint can be determined as shown in Figure H-2. This figure also depicts the weight versus T for each of the solutions as given in Table H-4. Figure H-3 depicts effectiveness for each of these solutions. For any weight constraint, the design which maximizes system effectiveness is shown in Figure H-4.

Table H-5 supplements Table H-4 by presenting the values of the effectiveness parameters for the optimum solution.

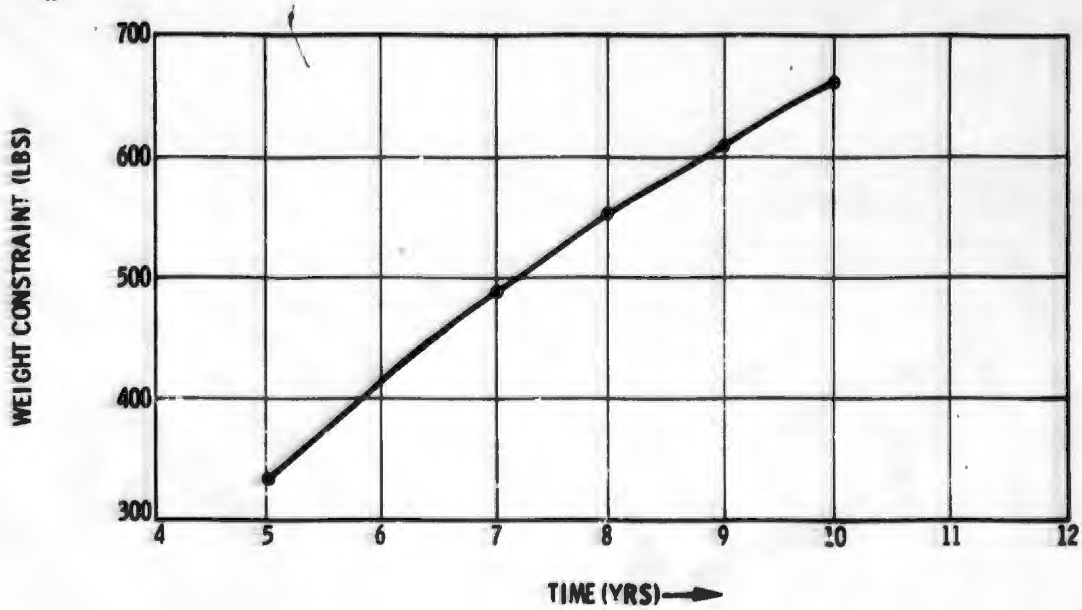


Figure H-2 Weight Constraint vs Time

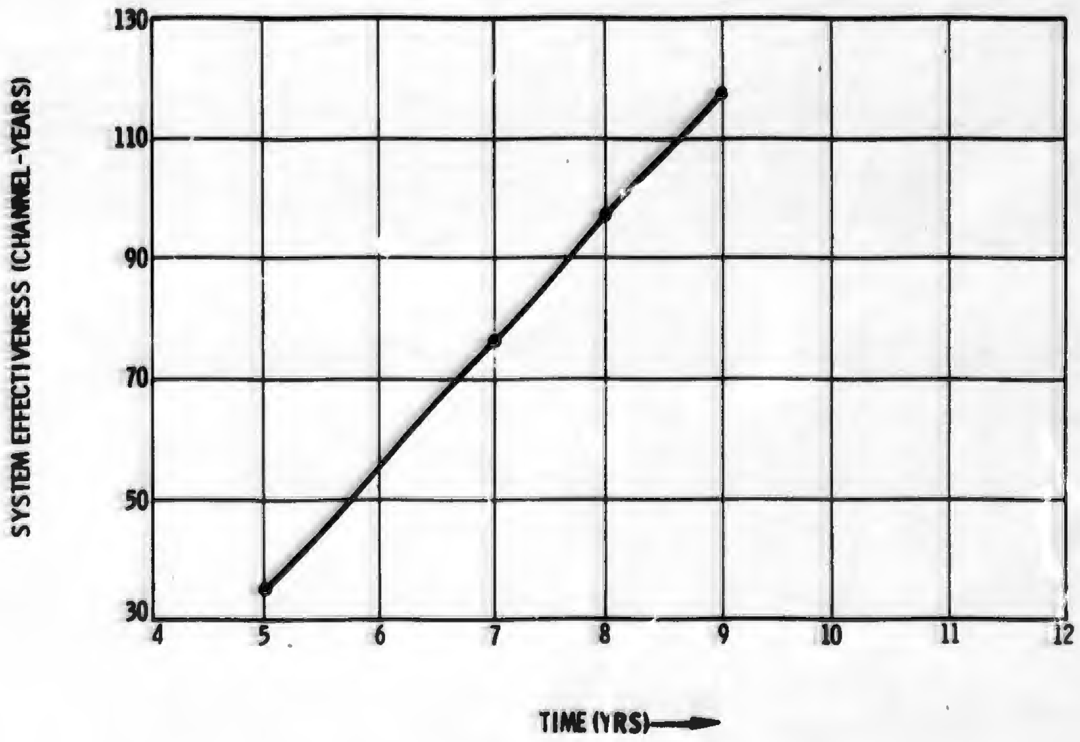


Figure H-3 System Effectiveness vs Time

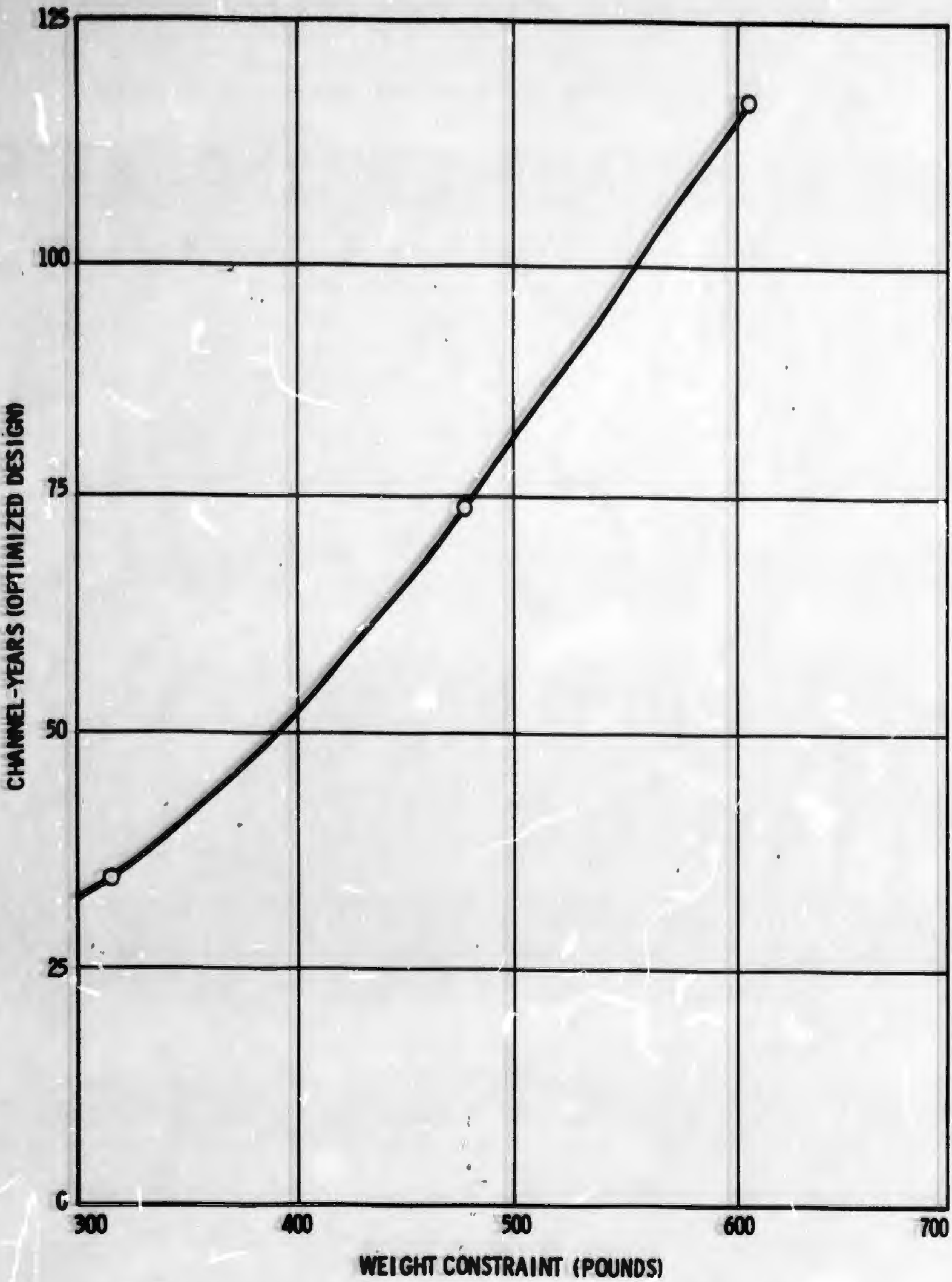


Figure H-4 Optimized Effectiveness vs Weight Constraint

TABLE H-5 VALUES OF PERFORMANCE PARAMETERS FOR OPTIMUM EFFECTIVENESS

Availability, β :	88.45%
Dependability (Reliability) at end of Depletion Time, T:	82.4%
Capability (Initial Number of Channels), s:	15 Channels
Planned Expendable Depletion Time, T:	8.9 Years
Optimum Effectiveness:	116 Channel-Years

6.0 EFFECTIVENESS, INCLUDING COST/UTILITY

In the basic illustration, each $(\Delta E)_k$ was weighted equally for each of the K time intervals. But, a discounting factor based on money interest rates must be included in order to determine the future worth, or value, of the dollar revenue obtainable from each channel over its operational time span. This is to cover the interest rate, or cost of the money, which would have been earning otherwise had it not been invested in the satellite for the purpose of obtaining revenue for use in supplying communications during some future time. Therefore, in this section, a weighted effectiveness measure is introduced. This is recognition of the fact that any money invested now for dollar returns in the distant future, t, must be devalued or weighted depending on how far into the future, t, the return on the investment is expected.

This devaluation factor may be designated as W(t) and described in terms of the interest rate percent p:

$$W(t) = \frac{1}{\left(1 + \frac{p}{100}\right)^t}$$

Thus, the alternate effectiveness measure of Equation (4) may be expressed as

$$U = W(t)E$$

$$= \beta \int_{o}^{T} W(t) \left[\sum_{r=o}^{s} r\, M(t) \right] R_S(t)\, dt \qquad (18)$$

where dU designates the value in terms of dollars returned (utility) of an increment of channel-years, corresponding to a time increment dt.

Substituting from Equations (2) and (17) above, Equation (18) then becomes

$$U = \beta s \int_{o}^{t} e^{-\lambda_c t} R_S(t) \Big/ \left(1 + \frac{p}{100}\right)^{t} dt. \qquad (19)$$

Designating $q = \ln\left(1 + \frac{p}{100}\right)$. (20)

$$U = \beta s \int_{o}^{T} e^{-(\lambda_c + q)t} R_S(t) dt.$$

$$U = \beta s \int_{0}^{T} e^{-(\lambda_c + q)t} R_S(t)\,dt$$

and subtracting from this equation the initial total cost C, where C is expressed as,

$$C = C_o + \gamma_1 s + \gamma_2 \beta + \gamma_3 T + \sum_{i=1}^{N} \gamma_i n_i$$

with

C_o = initial investment costs (booster + satellite)

γ_1 = cost of each channel

γ_2 = ratio of cost to availability of ascent alternative

γ_3 = cost of hydrazine

γ_i = cost of each redundant subsystem

the utility function is obtained:

$$U = \left[\beta s \int_{0}^{T} e^{-(\lambda_c + q)t} R_S(t)\,dt\right] - \left[C_o + \gamma_1 s + \gamma_2 \beta + \gamma_3 T + \sum_{i=1}^{N} \gamma_i n_i\right]$$

The change of the terms

$$-\left(\gamma_1 s + \gamma_2 \beta + \gamma_3 T + \sum_{i=1}^{N} \gamma_i n_i\right)$$

for various designs of the satellite will be small compared to the total utility $\dot{U}$. Thus, these terms may be assumed to be a constant for the optimization of U. And, since subtracting the remaining constant C_o does not change the optimization process, U becomes an optimum when U_o is optimized, where:

$$U_o = \beta s \int_o^T e^{-(\lambda_c + q)t} R_s(t)\, dt \qquad (21)$$

Thus, the optimization process applied in the first part of this illustration represents the case of $q = o$, i.e., where the interest rate is o. The optimization of the utility is mathematically equivalent to an increase of the failure rate in Equation (2) from λ_c to $(\lambda_c + q)$. To maximize utility, a simple change to Equations (14), (15), and (16) is made to give:

$$s = \frac{c_3}{c_1(\lambda + q)} \left[e^{(\lambda + q)T} - 1 \right] \qquad (22)$$

$$\frac{1}{c_3} \leqslant \frac{e^{-\lambda_1 T} \lambda_1^{n_1 - 1} T^{n_1}}{W_1 (n_1 !)} \left[1 + \frac{T(\lambda + q + \lambda_1)}{(n_1 + 1)} + \frac{(\lambda + q + \lambda_1)^2 T^2}{(n_1 + 1)(n_1 + 2)} + \dots \right] \qquad (23)$$

$$U_o = \beta s T \left[1 - \frac{(\lambda + q)T}{2} + \frac{(\lambda + q)^2 T^2}{6} + \dots \frac{(-1)^n (\lambda + q)^n T^n}{(n+1)!} \right] \qquad (24)$$

The last column of Table H-4 presented the optimized satellite design assuming p=0, or 0% interest rate. The effects of interest rates of 6%, 12%, and 18% will now be investigated. By Equation (20), q may be determined for the three interest rates:

p:	6%	12%	18%
q:	0.0558	0.1134	0.1655

Using the computed q-values and Equations (22), (23), and (24), the optimization problem for p = 6%, 12%, and 18% may be solved in a manner similar to that for the case p = 0. The computation results are tabulated in Table H-6. Comparing the weights allocated to redundancy for the four investigated interest rates, equal redundancy allocations of a total weight of 34 pounds is appropriate for interest rates from 0% to 12%. In the case of an interest rate of 18%, the redundancy allocation diminishes to a total weight of 26 pounds.

Figure H-5 presents the number of channels and the lifetime decay (due to depletion) of the considered system as a function of the interest rate.

TABLE H-6 OPTIMUM SATELLITE DESIGN WITH WEIGHT CONSTRAINT OF 610 POUNDS FOR VARIOUS INTEREST RATES

	Interest Rate		
	6%	12%	18%
• Number of Channels	17	18	19
• Planned Depletion Time (years)	7.53	6.87	6.47
• Weight of Spin Stabilized Ascent	10	10	10
• Weight of Channels	340	360	380
• Weight of Redundancies	34	34	26
Roll H/S - 1 Standby	---	---	4
- 2 Standbys	8	8	---
Pitch H/S - 1 Standby	---	---	4
- 2 Standbys	8	8	---
Pitch Jet Electronics	4	4	4
Roll-Yaw Jet Electronics	6	6	6
Wheel Electronics	4	4	4
H/S Electronics	4	4	4

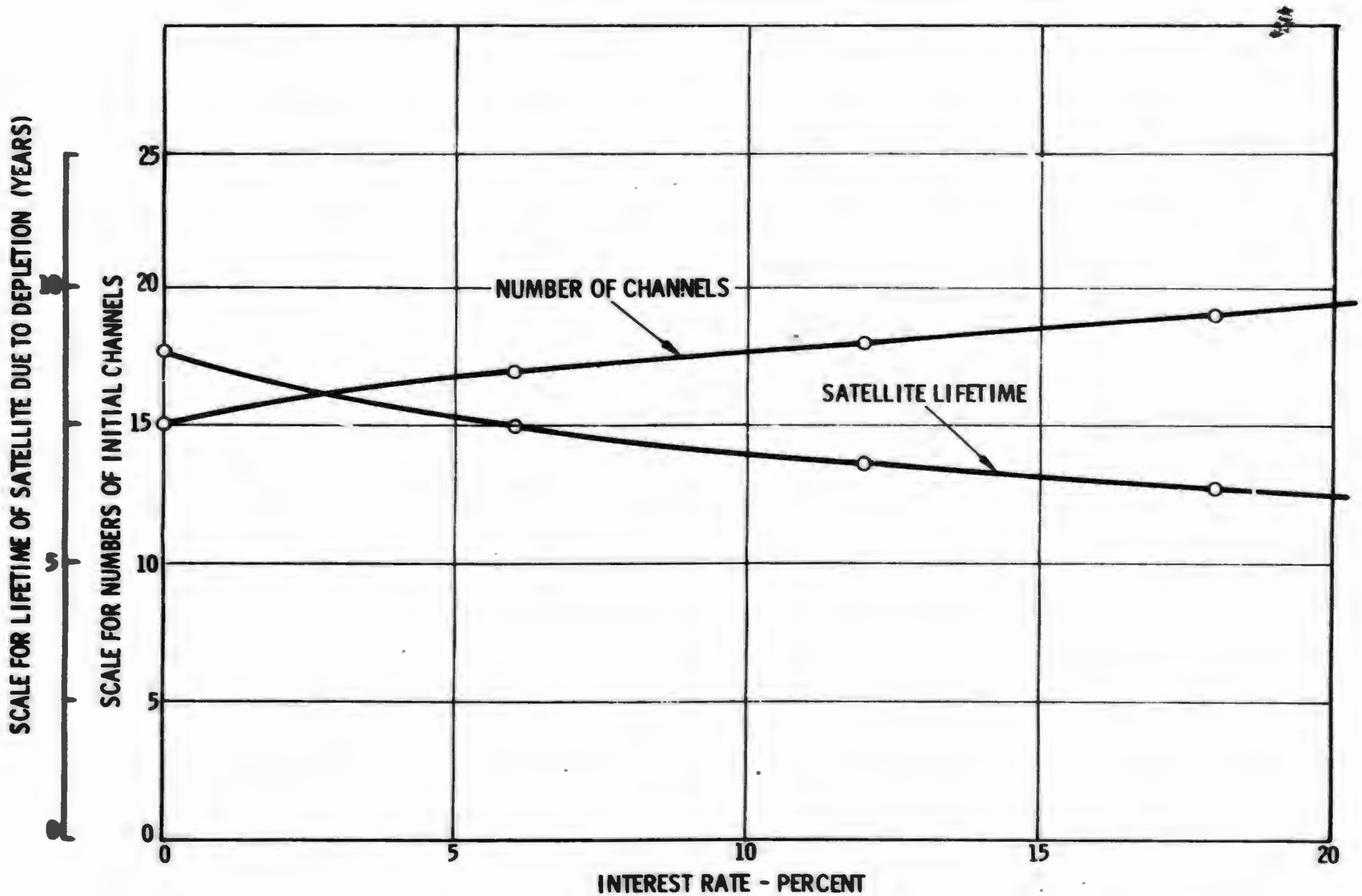


Figure H-5 Number of Channels and Satellite Lifetime as a Function of Interest Rate

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APPENDIX I
AN ILLUSTRATIVE EXAMPLE OF THE EFFECTIVENESS OPTIMIZATION OF A SPACECRAFT LAUNCH VEHICLE SYSTEM

INTRODUCTION

This example is intended to illustrate and demonstrate apportionment techniques by means of a spacecraft launch vehicle system with objectives closely related to missions of Agena type systems.

THE MISSION

The launch vehicle system is to achieve an orbit of a spacecraft (payload) at a specified launch window with a specified accuracy. Phase I of the design study is concerned with conceptual design without detailed definition and selection of subsystems. The problem is to apportion the accountable factors in such a manner that the effectiveness of the system is optimized.

It is assumed that functional relationships exist among the accountable factors obtained by experience with design, test, and operation of former launch vehicle systems. These functions represent the average values gained by this past experience. Thus, there exist best estimates for the performance of the conceptual design. Specific functional relationships were assumed, the constraints of which were computed by the least squares method.

A launch window is assumed of a short known period of time during which the launch has to occur. A non-occurrence of the launch during the launch window is identical with the effectiveness zero. Thus, the availability is a basic factor of the effectiveness model.

Two main parameters contribute to the dependability of the launch vehicle system. These are the system reliability and the probability of successful stabilization of the system.

The main capability parameter of inaccuracy is considered to be the deviation of the altitude at orbit ejection from the specified value. Other parameters, such as geodetic latitude, longitude, and azimuth are considered of second order influence and may be ignored. The velocity inaccuracy is considered to be a part of the altitude inaccuracy; thus, the velocity inaccuracy is covered by stating the altitude inaccuracy. Only in the case of altitude inaccuracy of zero is the achievement of the orbit considered to be 100 percent. Otherwise, the probability of achievement of the orbit is a function of the altitude inaccuracy Δh, decreasing with increasing Δh. The weight of the spacecraft injected into orbit is assumed to be composed of a required minimum weight, plus an additional weight which may be used for experiments and gathering of other information during the space mission.

There is a useful maximum weight for experiments and information gathering applications up to which the capability of the spacecraft is assumed to increase. When this maximum weight is reached, no further gain in effectiveness may be attained. Thus, the capability of the launch vehicle consists of two main parameters: (1) the accuracy in achieving an orbit of specified altitude and (2) the weight of the spacecraft which may be put into orbit at the specified altitude.

APPORTIONMENT PROCEDURE

The Lagrange multiplier with priority list method will not be used in this example for apportionment of effectiveness. The procedure will be to use the classical method of determining extreme values of functions. The use of this simple method is possible because of the consideration of the average number of redundant spares, thus reducing the complexity of the problem. Another reason is that the constraints are given by equality relations rather inequalities.

ACCOUNTABLE FACTORS

The accountable factors influencing the performance parameters of the launch vehicle system under investigation are:

Availability

- The failure rates during the dormant state of the spacecraft launch vehicle system.
- The mean times to restore the various subsystems of the system.
- The failure rates during the launch countdown state of the system.
- The amount of weight for redundancy applications.
- Guidance complexity and accuracy influences.

Dependability

- The amount of weight for redundancy and non-redundancy applications.
- The guidance complexity.

Capability

- The weight for experiment equipment in the spacecraft placed into orbit at a specified altitude.
- The precision σ_G of the guidance subsystem which is the standard deviation of the altitude error distribution of the injected spacecraft orbit.

PERFORMANCE PARAMETERS

The performance parameters influencing the effectiveness of the launch vehicle system are:

- The probability that the vehicle is available during a launch window of a known, short period of time.
- The probability that the launch vehicle system is dependable during flight, including stabilization.

- The probability of achieving a specified orbit accuracy with a specified payload.

The key system performance parameters, and their accountable factors usually consist of many accountable factors at the subsystem design level for which average values may be defined.

SYSTEM EFFECTIVENESS MODEL

The WSEIAC effectiveness model may be applied for computation of the effectiveness of the investigated system. Thus,

$$E = ADC$$

However, as will be shown, the simplicity of representation of the availability A, of the dependability D, and of the capability C and their relationship to the effectiveness E of the system (one figure of merit) makes it expedient not to use the A, D, C vector-matrix approach in the present example.

Availability

The space launch vehicle is assumed to consist of some subsystems with no redundancy application, and of other subsystems with redundancy application to its equipment. Thus availability may be increased by addition of weight. Furthermore, it is assumed that the guidance package has a characteristic different from the other subsystems in that its accuracy may be increased in exchange for reliability and maintainability. The following may be distinguished:

- Weight W_n for subsystems where the availability value A_o cannot be improved by weight additions.
- Weight W_R for subsystems where availability may be increased by increasing their weight (usually by means of redundancy). This weight is composed of a weight W_B, the minimum weight for the basic subsystems required, and of the variable weight W_V, the potential weight addition. Thus,

$$W_R = W_B + W_V$$

In conceptual design of the system, an average weight ΔW_R may be defined for each of N_R subsystems with redundancy applicability and an average number $\bar{N}_R$ of redundant subsystems for each of the N_R subsystem types. The number $\bar{N}_R$ is not necessarily an integer. Therefore,

$$W_B = N_R \, \Delta W_R$$

and

$$W_V = N_R \, \bar{N}_R \, \Delta W_R$$

or

$$W_R = N_R \, \Delta W_R \, (1 + \bar{N}_R) \qquad (1)$$

The values N_R and ΔW_R are constants for the system type under investigation, and $\bar{N}_R$ is a variable with a specific value in the case of optimum effectiveness.

Defining an average dormant and countdown failure rate λ_D for each of the N_R subsystems, and an average mean time to restore of $\frac{1}{\lambda_R}$ for a malfunction which occurs in the dormant and the countdown state, an availability, A_R, of the presently considered system may be expressed as:

$$A_R = \left(\frac{1}{1 + \frac{\lambda_R}{\lambda_D}} \right)^{N_R(\bar{N}_R + 1)}$$

By substituting

$$\frac{\lambda_R}{\lambda_D} = r_R$$

A_R reduces to:

$$A_R = (1 + r_R)^{-\frac{W_R}{\Delta W_R}}$$

where r_R and ΔW_r are system characteristics. Furthermore, it is necessary to distinguish the availability A_G of the guidance subsystem, which may be designed with any precision value σ_G without significant change of its weight. The meaning of σ_G is discussed later. If it is assumed that the more accurate the guidance package is, the more difficult is its maintenance, then availability A_G of the guidance package may be expressed by

$$A_G = \exp\left[-(a_G/\sigma_G)^{\alpha}\right]$$

where a_G and α are known characteristics of the guidance subsystem. The availability A of the system may then be expressed as the product of A_o, A_R, and A_G, namely

$$A = A_o(1 + r_R)^{-W_R/\Delta W_R} \exp\left[-(a_G/\sigma_G)^{\alpha}\right] \qquad (2)$$

Dependability

The dependability of the system is assumed to consist of one value only, the system reliability which is composed of factors equivalent to those described for the availability parameter. The reliability of the subsystems with no redundancy application and thus of fixed weight W_n is designated by R_o. The reliability of subsystems which can be improved by increasing their weight may be approximately expressed by

$$R_R = \exp\left[-N_R\, q_S^{(1 + \bar{N}_R)}\right] \qquad (3)$$

where q_S is the system failure characteristic and N_R, $\bar{N}_R$ are as previously defined for Equation (1). This functional relationship may be derived by considering that for a short-time operating system such as the one for the present example, redundancy is approximately of parallel type, with the N_R subsystems each having an average of $\bar{N}_R$ redundancies. Assuming an average probability of failure, q_S, for each subsystem, then

$$R_R = \left(1 - q_S^{(1 + \bar{N}_R)}\right)^{N_R}$$

or, in a logarithmic form,

$$R_R = \exp\left[N_R \ln\left(1 - q_S^{(1 + \bar{N}_R)}\right)\right] \qquad (4)$$

Using the approximation

$$\ln\left(1 - q_S^{(1 + \bar{N}_R)}\right) = -q_S^{(1 + \bar{N}_R)}$$

and substituting in Equation (4), Equation (3) is obtained.
Using Equation (1),

$$R_R = \exp\left[-N_R\, q_S^{W_R/(N_R\, \Delta W_R)}\right]$$

Finally, the reliability of the guidance subsystem R_G, which is a function of the guidance precision σ_G, can be assumed to have the following functional relationship:

$$R_G = \exp\left[-(c_G/\sigma_G)^{\gamma}\right]$$

where c_G, γ are known characteristics of the guidance package.

The dependability D, being identical with the reliability of the system in this example, is the product of R_o, R_R, and R_G:

$$D = R_o \exp\left[-N_R\, q_S^{W_R/(N_R\, \Delta W_R)}\right] \cdot \exp\left[-(c_G/\sigma_G)^{\gamma}\right] \qquad (5)$$

Capability

As previously indicated, the capability of the spacecraft launch vehicle system will be a function of two main parameters: accuracy of achieving the orbit and the weight of the spacecraft injected into orbit.

Accuracy of achieving an orbit is a direct function of the extent to which the specified apogee altitude, perigee altitude, and inclination angle parameters are met. These parameters are directly related to the major and minor axes, eccentricity, inertial ascent node, period, and velocity at injection into orbit.

For simplicity, it is assumed that the accuracy of achieving an orbit is related to one accountable factor only, the precision σ_G, which may be interpreted as the standard deviation of the altitude error distribution of the orbit for the injected spacecraft. Thus, σ_G has a length dimension. Assuming a normal distribution of the altitude error (Δh), the following familiar density function may be defined:

$$\phi(\Delta h) = \frac{1}{\sqrt{2\pi}\ \sigma_G} \exp\left[-\frac{(\Delta h)^2}{2\sigma_G^2}\right] \tag{6}$$

There will be a relationship between the probability P_o of achieving the capability objective and the altitude error (Δh) of the orbit. Assume that this relationship may be expressed by the function

$$P_o = \exp\left[-(\Delta h)^2/c_A^2\right] \tag{7}$$

where c_A is a system constant and P_o possessing the necessary characteristics that

$$P_o = 1 \text{ for } \Delta h = 0$$

$$P_o = 0 \text{ for } \Delta h \rightarrow \infty$$

By means of Equations (6) and (7), the probability P_C that an orbit is achieved with sufficient accuracy to attain the objectives of the injected spacecraft may then be expressed by:

$$P_C = \int_{-\infty}^{\infty} P_o \phi(\Delta h) d\Delta h$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_G} \exp\left[-\frac{(\Delta h)^2}{2}\left(\frac{2\sigma_G^2 + c_A^2}{\sigma_G^2 c_A^2}\right) d(\Delta h)\right]$$

$$= \frac{1}{\sqrt{1 + 2\frac{\sigma_G^2}{c_A^2}}} \qquad (8)$$

Now consider the relationship to capability of the weight W_S of the spacecraft injected into orbit. This weight may be viewed as being the sum of a fixed weight W_{SC} and a variable weight W_{SU} utilized in the spacecraft for experiments and collection of other types of information. Assume that W_{SU} is limited to a value W_{SL} beyond which no utilization of additional weight for experiments and/or collection of information is possible. Then, reasonably, the capability C may be defined by the equation:

$$C = \begin{cases} P_C \left(\dfrac{W_S - W_{SC}}{W_{SL}}\right)^{\beta} & \text{if } (W_S - W_{SC}) \le W_{SL} \\ P_C & \text{if } (W_S - W_{SC}) > W_{SL} \end{cases}$$

where β is a spacecraft characteristic.

Using Equation (8),

$$C = \begin{cases} \dfrac{1}{\sqrt{1 + 2\left(\dfrac{\sigma_G}{c_A}\right)^2}} \left(\dfrac{W_S - W_{SC}}{W_{SL}}\right)^{\beta} & \text{if } \left(W_S - W_{SC}\right) \le W_{SL} \\ \dfrac{1}{\sqrt{1 + 2\left(\dfrac{\sigma_G}{c_A}\right)^2}} & \text{if } \left(W_S - W_{SC}\right) > W_{SL} \end{cases}$$

The last equation system may also be formulated as

$$C = \left[1 + 2\left(\frac{\sigma_G}{c_A}\right)^2\right]^{-1/2} \left(\frac{W_S - W_{SC}}{W_{SL}}\right) \tag{9}$$

with

$$W_S \le W_{SC} + W_{SL} \tag{10}$$

<u>The Effectiveness Equation System</u>

Equations (2), (5), (9), and the relationship (10) form the effectiveness relationship to be applied when optimizing the effectiveness of the hypothetically considered launch vehicle system. This effectiveness measure is expressed as the probability of injecting a spacecraft into orbit at a specified launch window within a specified accuracy, and is defined as E, where

$$E = ADC$$

$$= A_o R_o (1 + rR)^{-\frac{W_R}{\Delta W_R}} \left[1 + 2 \left(\frac{\sigma G}{c_A} \right)^2 \right]^{-1/2} \left(\frac{W_S - W_{SC}}{W_{SL}} \right)^{\beta} \cdot$$

$$\exp \left\{ - \left[\left(\frac{a_G}{\sigma G} \right)^{\alpha} + \left(\frac{c_G}{\sigma G} \right)^{\gamma} + N_R q_S^{\frac{W_R}{N_R \Delta W_R}} \right] \right\} \qquad (11)$$

with $W_S \leq W_{SC} + W_{SL}$ from Equation (10).

Additionally, to accommodate the weight restriction of the selected propulsion launch vehicle system, a further restriction applies for E, namely:

$$W_n + W_R + W_S \leq W_L \qquad (12)$$

In the equation system, (11) and (12), A_o, R_o, r_R, c_A, a_G, c_G, N_R, W_n, W_{SC}, ΔW_R, α, β, γ, and q_S are constant characteristics and σ_G, W_R, and W_{SL} are variables which are to be apportioned by optimizing the measure of effectiveness E under the inequalities of Equations (10) and (12). To summarize the significance of these parameters,

Weights:

W_n : Weight of subsystems of launch vehicle system with no redundancy applicability

W_R : Weight of subsystems of launch vehicle system with redundancy applicability.

ΔW_R : Average weight of one subsystem of launch vehicle system with redundancy applicability.

W_{SC} : Weight of the subsystems of the spacecraft to be launched by the launch vehicle system without equipments for experiments and collection of other types of information (considered to be a fixed value).

W_S : Total weight of the spacecraft.

W_L : Maximum weight of launch vehicle system, including propellant plus spacecraft weight, which may be ejected into the intended orbit.

W_{SL} : Maximum weight of equipment for experiments and collection of other types of information which may be utilized in the mission of the ejected spacecraft.

Length parameters:

σ_G : Measure of precision of the guidance subsystem expressed by the standard deviation of the miss-distance of altitude of the orbit. Factors influencing standard deviation of miss-distance are:

- Gyro drift resulting in incorrect flight path angle output to computer
- Gyro drift resulting in incorrect accelerometer measurement of thrust time history
- Incorrect start time, burn duration, or shut-down time of one or more engines due to malfunction in electrical control system, turbopump assembly, gas generator assembly, start tank and valve assemblies, main oxydizer valve, fuel control valve.

- Incorrect thrust level due to malfunction or defect in helium sphere, helium control valve, fuel valve, oxydizer valve, main propellant tanks, oxydizer tanks, propellant isolation valves, turbopump assembly, gas generator, or thrust chamber
- Failure in electrical power supply resulting in inoperative sequence timer, electrically controlled valves, gyro motors, computer, etc.
- Malfunction in horizon sensor resulting in inoperation of sensor or angular error in flight path angle
- Leak causing pressure loss in hydraulic system controlling pitch and yaw actuators resulting in incorrect thrust vector angle.

a_G : Guidance characteristic relating the precision of the guidance to its maintainability. The measure a_G is a function of the standard deviation of the output parameters of the guidance subsystem, with a small standard deviation for these outputs being associated with a highly complex, sophisticated design. Correspondingly, this will result in higher dormant and checkout failure rates, longer restoration time, and longer checkout time, all of which will influence the system availability.

c_A : A characteristic of the spacecraft and mission relating miss-distance of the orbit altitude of the spacecraft to the probability that the spacecraft will successfully accomplish its objective. Examples of a decrease in capability due to an altitude error are:

- A satellite which fails to achieve a stationary orbit, and thus will have a closed-curve ground track, the cause of which is attributable to a lack of orbit adjustment capability. The consequence of this may be an antenna mis-alignment with the ground stations which will result is a reduction in received power.
- A satellite which has a vernier stabilization subsystem. The amount of fuel used for altitude adjustment purposes will affect the usable fuel available for its mission requirements.
- A satellite which is to rendezvous and where an altitude error will reduce its capability to accomplish this mission.

The measure c_A is obtained by determining the factors, and their effects on capability as a function of altitutde error. A curve can then be plotted of capability versus altitude error. The curve is then approximated by the function P_o, the probability of achieving the mission objective, as defined in this Appendix. From this equation, c_A can be obtained.

c_G : Guidance characteristic relating the precision of the guidance to its reliability. A highly reliable and precise guidance subsystem implies that the guidance output parameters will be within narrow tolerances, with small standard deviations for these output distributions. Correspondingly, the system miss-distance will be small and will result in a small altitude error standard deviation. The following steps may be used to determine c_G:

- Out-of-tolerance conditions of the guidance outputs affecting miss-distance are isolated, and their effects determined.
- The frequency distribution of these out-of-tolerance conditions, together with their effects, is used to determine the distribution of miss-distance, which in turn, defines σ_G.
- The guidance subsystem reliability then can be plotted as a function of σ_G, from which C_G and γ can be obtained using the guidance reliability experssion, R_G in this Appendix.

Dimensionless parameters:

Of the following parameters, α, β, and γ are shaping constants. Their values are obtained from empirical or theoretical data relating the applicable accountable factors described.

α : Exponent in the function relating the precision of the guidance to its maintainability.

γ : Exponent in the function relating the precision of the guidance to its reliability.

β : Exponent in the effectiveness function relating the weight in the spacecraft, available for equipment for experiments and collection of other type of information, to the capability measure of the launch vehicle system.

r_R : The quotient of the reciprocal of the average mean time to restoration of a failed subsystem occurring before launch of the system, to the average failure rate in the dormant and checkout state for each of the subsystems with redundancy in the launch vehicle system.

N_R : Number of subsystems of the launch vehicle system with redundancy applicability, with the average being $\overline{N}_R$

<u>Probabilities:</u>

q_S : Average probability of failure of each subsystem of the launch vehicle system with redundancy applicability.

A_o : Availability of the subsystems of the launch vehicle system without redundancy applicability.

R_o : Reliability of the subsystems of the launch vehicle system without redundancy applicability.

OPTIMIZATION OF THE EFFECTIVENESS

Equation (11) may be composed as the product of three terms:

$$E = E_C \cdot E(\sigma_G) \cdot E(W_R ; W_S) \tag{13}$$

with E_C the constant term of the effectiveness, and

$$E_C = A_o R_o \tag{14}$$

$E(\sigma_G)$ is the term of the effectiveness equation which depends on the value of the precision parameter of the guidance, σ_G, and is expressed by

$$E(\sigma_G) = \left[1+2\left(\frac{\sigma_G}{c_A}\right)^2\right]^{-\frac{1}{2}} \exp\left\{-\left[\left(\frac{a_G}{\sigma_G}\right)^{\alpha} + \left(\frac{c_G}{\sigma_G}\right)^{\gamma}\right]\right\} \tag{15}$$

$E(W_R ; W_S)$ is the term of the effectiveness which depends on both the values of the weight W_R of subsystems of the launch vehicle system with redundancy applicability and of the weight W_S of the spacecraft, with

$$E(W_R; W_S) = (1+r_R)^{\frac{W_R}{\Delta W_R}} \exp\left[-N_R q_S^{\frac{W_R}{N_R \Delta W_R}}\right]\left(\frac{W_S - W_{SC}}{W_{SL}}\right)^{\beta} \quad (16)$$

subject to the conditions of Equations (10) and (12) of

$$W_S \leq W_{SC} + W_{SL}$$

$$W_R + W_S \leq W_L - W_n$$

Equation (13) suggests that the maximum of $E(\sigma_G)$ and $E(W_R ; W_S)$ may be separately determined in order to optimize the effectiveness E.

<u>The Maximum of $E(\sigma_G)$:</u>

$E(\sigma_G)$ is not involved in any limit condition; thus the classical method of determining the maximum of a function may be applied. Differentiating $E(\sigma_G)$ with respect to σ_G and equating to zero gives

$$\left(\frac{dE(\sigma_G)}{d\sigma_G}\right)_{\sigma_G = \sigma_{GO}} = 0$$

where σ_{GO} is the optimum σ_G, or

$$\alpha\left(\frac{a_G}{\sigma_{GO}}\right)^{\alpha} + \gamma\left(\frac{c_G}{\sigma_{GO}}\right)^{\gamma} = \frac{2\left(\frac{\sigma_{GO}}{c_A}\right)^2}{1+2\left(\frac{\sigma_{GO}}{c_A}\right)^2} \quad (17)$$

The Maximum of E $(W_R\ ;\ W_S)$

To compute the maximum of $E(W_R\ ;\ W_S)$, the constraints stated by the inequalities of Equations (10) and (12) must be considered. By means of the technical situation, the equal sign applies in Equation (12) in the case that $E(W_R\ ;\ W_S)$, Equation (16) is a maximum. The maximum of E $(W_R\ ;\ W_S)$ under the constraints is determined by two steps:

In Step 1, the constraint of Equation (10) is irrelevant. The natural logarithm of Equation (16) is taken to give:

$$-\ln E(W_R\,;W_S) = W_R\,\frac{\ln(1+r_R)}{\Delta W_R} + N_R q_S^{\frac{W_R}{N_R \Delta W_R}} - \ln\frac{W_S - W_{SC}}{W_{SL}}$$

Differentiating this expression and Equation (12) with respect to W_R and W_S results in:

$$-\frac{dE(W_R\ ;\ W_S)}{E(W_R,\ W_S)\,dW_R} = \frac{\ln(1+r_R)}{\Delta W_R} + q_S^{\frac{W_R}{N_R \Delta W_R}}\,\frac{\ln q_S}{\Delta W_R} - \beta\,\frac{1}{W_S - W_{SC}}\,\frac{dW_S}{dW_R}$$

and $$1 + \frac{dW_S}{dW_R} = 0 \quad \text{(for the equality case)}$$

In the case of $E_{max}(W_R\ ;\ W_S)$, $dE(W_R\ ;\ W_S)/dW_R$ is zero. Thus,

$$\ln(1+r_R) + q_S^{\frac{W_{R01}}{N_R \Delta W_R}}\,(\ln q_S) + \beta\,\frac{\Delta W_R}{W_{S01} - W_{SC}} = 0 \qquad (18)$$

and $$W_{RO1} + W_{SO1} = W_L - W_n \quad (19)$$

with 01 denoting optimum for step 1

Equations (18) and (19) determine the values of W_{RO1} and W_{SO1}, since all other values in these equations are known. If the inequality of Equation (10) is to be satisfied, a determination has to be made of W_{RO} and W_{SO}. In the case of $W_{SO} > (W_{SL} + W_{SC})$ as determined by the procedure in Step 1, then:

$$W_{SO2} = W_{SL} + W_{SC} \quad \text{if} \quad W_{SO1} > W_{SL} + W_{SC} \quad (20)$$

with W_{RO2} to be determined as in the next step.

In Step 2, investigate

$$W_{RO2} = W_L - W_n - W_{SL} - W_{SC} \quad (21)$$

The optimum values for W_R and W_S as determined by using Equations (18), (19), and (20) will provide the solution for the optimum of $E(W_R ; W_S)$. This optimum $E(W_R, W_S)$, which may be designated by $E_{Opt}(W_R, W_S)$, is :

$$E_{Opt}(W_R, ; W_S) = (1+r_R)^{-\frac{W_{RO1}}{\Delta W_R}} \exp(-N_R q_S^{\frac{W_{RO1}}{N_R \Delta W_R}}) \cdot \left(\frac{W_{SO1} - W_{SC}}{W_{SL}}\right)^{\beta} \quad \text{if} \quad W_{SO1} \leq W_{SC} + W_{SL} \quad (22)$$

where W_{SO1}, W_{RO1} is computed by Equations (18) and (19). If the last relationship of Equation (22) is not satisfied, then

$$E_{Opt}(W_R;W_S) = (1+r_R)^{-\frac{W_{RO2}}{\Delta W_R}} \exp\left[-N_R q_S^{\frac{W_{RO2}}{N_R \Delta W_R}}\right] \quad (23)$$

and

$$W_{SO1} > W_{SL} + W_{SC} \quad .$$

where W_{RO2} is given by Equation (21)

Thus, the optimum effectiveness of the presently investigated system has been determined by finding specific values of the three parameters σ_G, W_R, W_S.

NUMERICAL EXAMPLE

The example selected has the parameters listed in Table I-1. The definitions of the various parameters are those previous described.

TABLE I-1 NUMERICAL EXAMPLE VALUES

Weights		Lengths		Probabilities		Dimensionless Parameters	
Designation	Value (lb)	Designation	Value (miles)	Designation	Value	Designation	Value
W_L	16000	a_G	0.100	q_S	0.025	N_R	8
W_n	13900	c_A	4	A_o	0.99	α	2
ΔW_R	62.5	c_G	0.020	R_o	0.98	β	1/3
W_{SC}	500	–	–	–	–	γ	1
W_{SL}	1000	–	–	–	–	r_R	0.0015

As discussed, a solution is required for the maximum value of the two effectiveness functions $E(\sigma_G)$ and $E(W_R; W_S)$ given in equations (15) and (16). Substituting the values of Table I-1,

$$E(\sigma_G) = \frac{1}{\sqrt{1+2\left(\frac{\sigma_G}{4}\right)^2}} \exp\left\{-\left[\left(\frac{0.10}{\sigma_G}\right)^2 \left(\frac{0.02}{\sigma_G}\right)\right]\right\} \tag{24}$$

$$E(W_R; W_S) = \begin{cases} e^{-\frac{W_R}{41700}} \exp\left[-\frac{8}{40^{W_R/500}}\right] \sqrt[3]{\frac{W_S - 500}{1000}}, & \text{if } W_S \le 1500 \\ e^{-W_R/41700} \exp\left[-\frac{8}{40^{W_R/500}}\right], & \text{if } W_S > 1500 \end{cases} \tag{25}$$

and $W_R + W_S = 2100$

The overall effectiveness of the system, Equations (13) and (14) is:

$$E = 0.970 \cdot E(\sigma_G) \cdot E(W_R; W_S)$$

Determination of σ_{GO} and $E(\sigma_{GO})$

The optimum of σ_G, σ_{GO} is determined by means of Equation (17). Substituting the known values,

$$2\left(\frac{0.10}{\sigma_{GO}}\right) + \left(\frac{0.020}{\sigma_{GO}}\right) = \frac{2\left(\frac{\sigma_{GO}}{4}\right)^2}{1+2\left(\frac{\sigma_{GO}}{4}\right)^2}$$

Figure I-1 graphically determines σ_{GO} as being equal to .725 miles.

Substituting into Equation (24), $E(\sigma_{GO}) = .923$

The function of $E(\sigma_G)$ is graphically represented in Figure I-2. The sensitivity of $E(\sigma_G)$ in the vicinity of σ_{GO} is low in that the decrease of $E(\sigma_G)$ is comparatively small when σ_{GO} is increased or decreased.

Determination of W_{RO} and $E(W_{RO}; W_{SO})$

W_{RO} is computed by means of Equation (18) and (19). Substituting the values from Table I-1:

$$0.0015 - \frac{3.69}{40^{W_{R01}/500}} + \frac{1}{3}\,\frac{62.5}{W_{S01}-500} = 0$$

$$W_{R01} + W_{S01} = 2100$$

Figure I-3 graphically determines W_{R01} and W_{S01} to give

$$W_{R01} = 682 \text{ lb.}$$
$$W_{S01} = 1418 \text{ lb.}$$

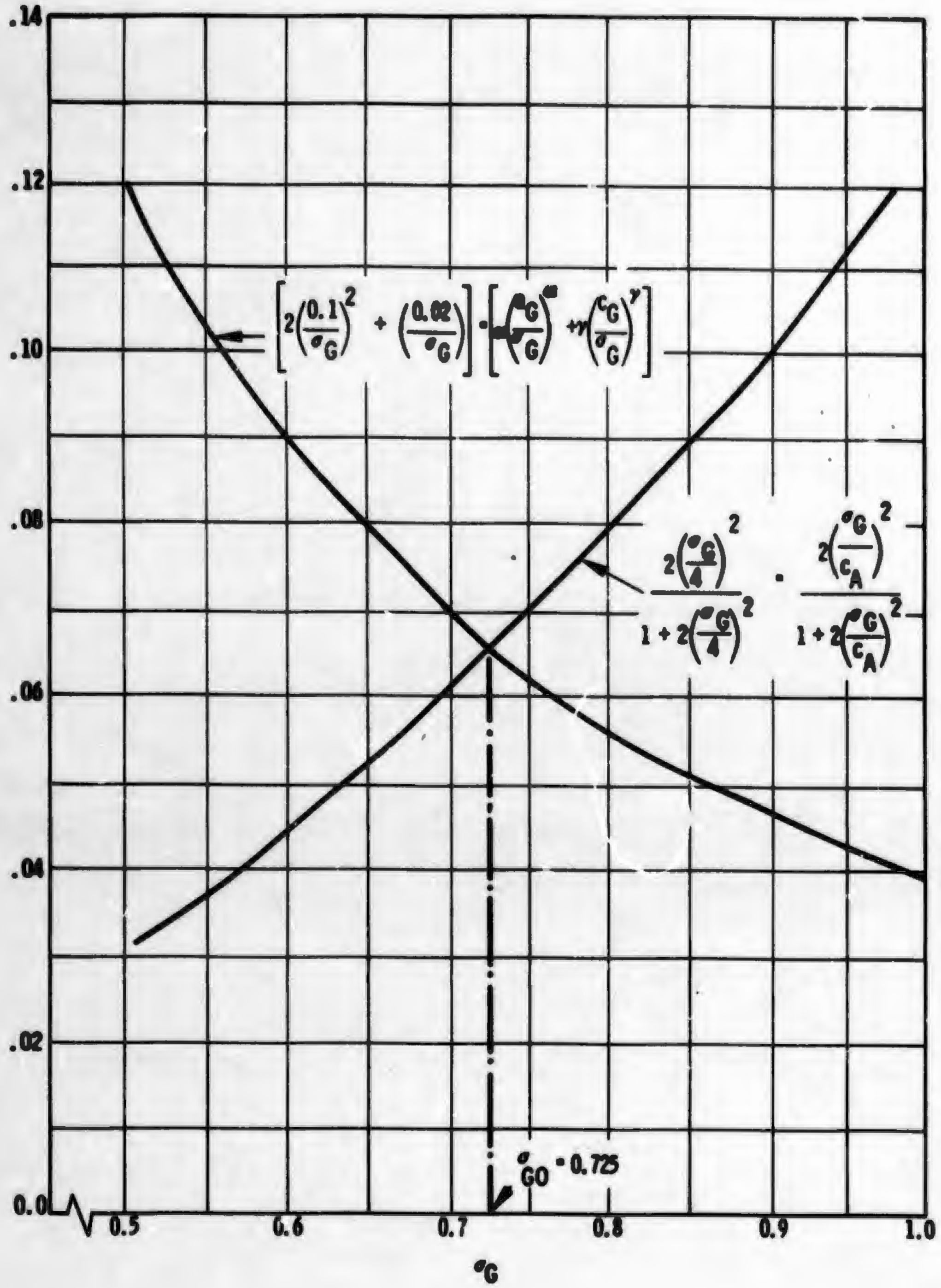

Figure I-1 Graphical Determination of σ_{GO}

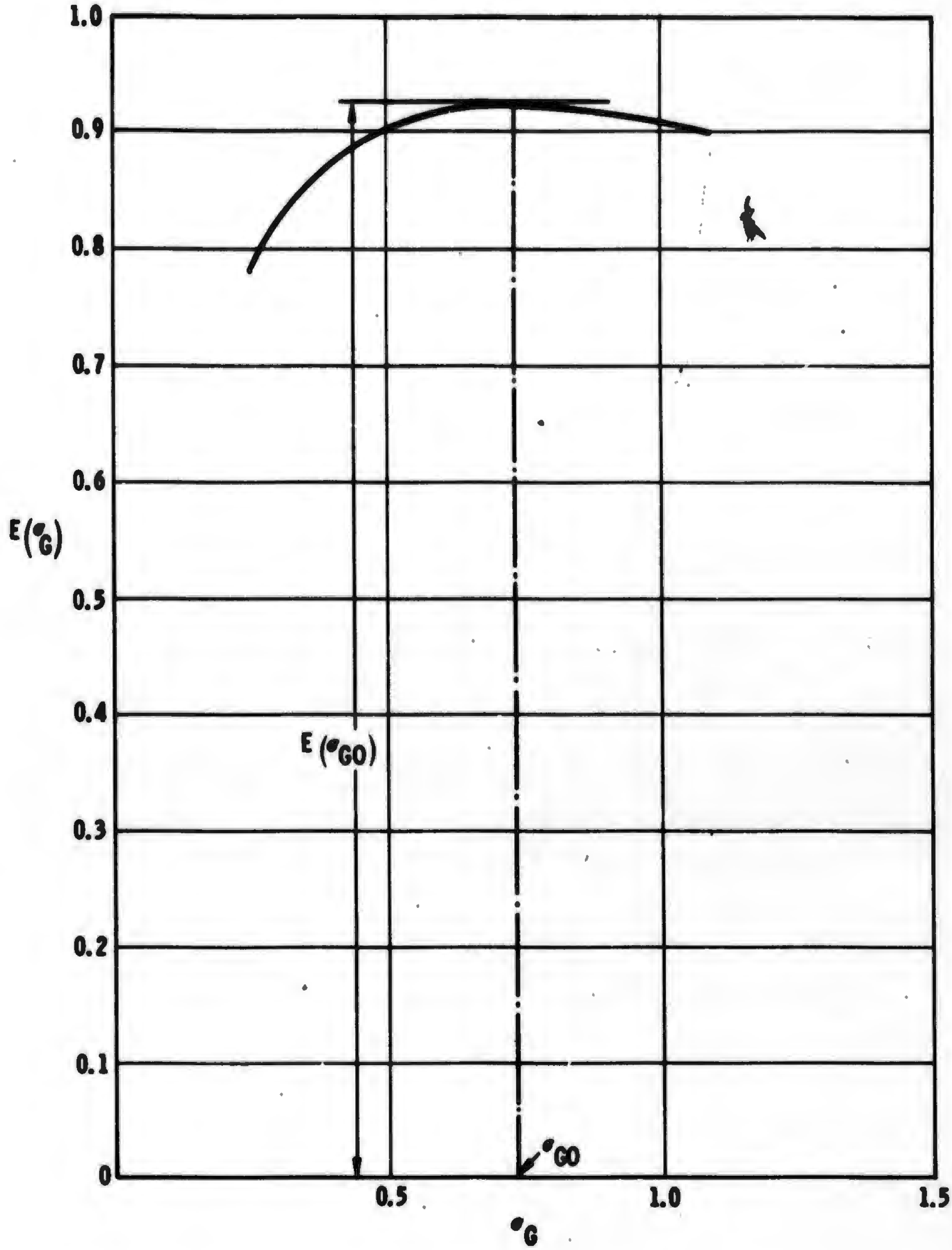

Figure I-2 Graphical Representation of the Function E (σ_G)

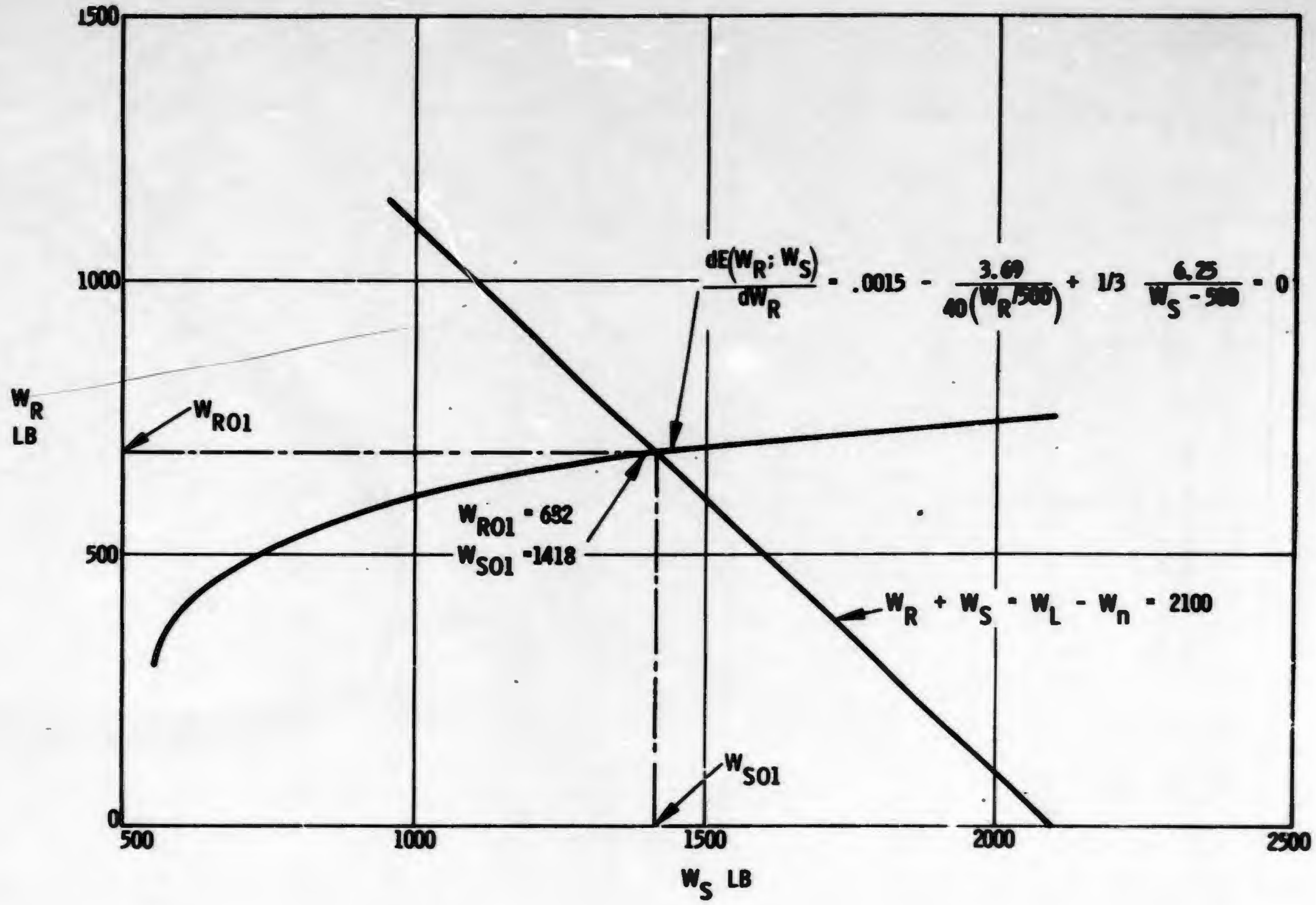

Figure I-3 Graphical Determination of W_{RO1} and W_{RO2}

Because the inequality of Equation (22) is satisfied, namely

$$W_{SO1} < W_{SC} + W_{SL}$$
$$(1418 < 1000 + 500)$$

W_{RO1} and W_{SO1} are the final values.

Since

$$W_{RO} = 682 \text{ lb}$$
$$W_{SO} = 1418 \text{ lb}$$

by means of Equation (25)

$$E(W_{RO}; W_{SO}) = 0.907$$

Finally, a graphical determination of W_{RO}, W_{SO}, and $E(W_{RO}; W_{SO})$ is shown in Figure I-4 with both scales: W_R and W_S.

Optimized Effectiveness

By means of Equation (13), the optimum effectiveness may be computed to be:

$$E_{Opt.} = E_C \, E(\sigma_{GO}) \, E(W_{RO} ; W_{SO})$$
$$= (0.97) \; (0.923) \; (0.907)$$
$$= 0.812$$

The availability A_{Opt} at the optimum value of the effectiveness may be computed by means of Equation (2). The result is

$$A_{Opt.} = 0.965$$

The dependability D_{Opt} at the optimum value of the effectiveness may be obtained using Equation (5). The result is

$$D_{Opt.} = 0.896$$

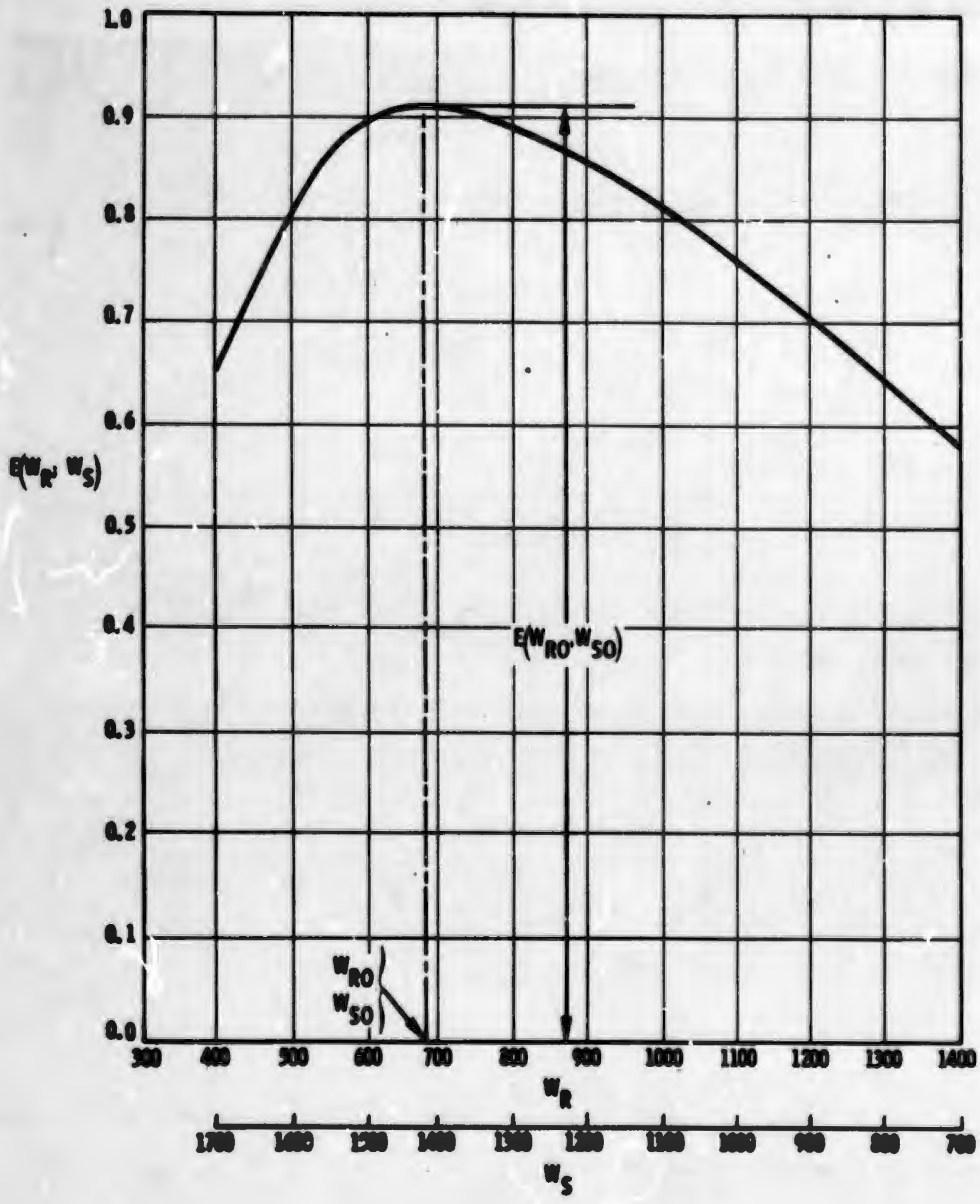

Figure I-4 Graphical Determination of W_{RO}, W_{SO}, and $E(W_{RO}; W_{SO})$

The capability C_{Opt} at the optimum value of the effectiveness may be computed by means of Equation (9). The result is

$$C_{Opt.} = 0.938$$

Thus, $E_{Opt.}$ is again 0.812 with

$$E_{Opt.} = (A_{Opt.})\ (D_{Opt.})\ (C_{Opt.})$$

$$= 0.812$$

Furthermore, the average number of redundant subsystems $\bar{N}_R$ for each of the 8 subsystem - type with applicability of redundancy is:

$$\bar{N}_R = \frac{W_{RO}}{N_R \quad W_R}$$

$$= \frac{662 \text{ lbs}}{8(62.5) \text{ lbs}}$$

$$= 1.32$$

Thus, 32 percent of the 8 subsystems have one redundancy or approximately three subsystems have one redundancy each and five subsystems have no redundancies. Thus, $N_R = 3$.

Finally, Table I-2 summarizes the values of the accountable factors optimizing the effectiveness measure for the spacecraft launch vehicle system, and the values of the optimized performance parameters.

TABLE I-2 VALUES OF THE KEY ACCOUNTABLE FACTORS AND PERFORMANCE PARAMETERS OPTIMIZING SYSTEM EFFECTIVENESS

Kind of Parameter	Function of Parameter	Designation	Value	Dimension
Optimizing Accountable Factors	Measure of precision of guidance	σ_G	0.725	miles
	Weight of subsystems with redundancy applied	W_R	682	lb.
	Total weight of spacecraft	W_S	1418	lb.
	Total number of redundant subsystems	N_R	3	-
Optimized Parameters	Availability	A_{Opt}	0.965	-
	Dependability	D_{Opt}	0.896	-
	Capability	C_{Opt}	0.938	-
	Measure of Effectiveness	E_{Opt}	0.812	-

APPENDIX J

THEORY AND GENERAL DESCRIPTION OF A COMPUTER PROGRAM FOR EFFECTIVENESS APPORTIONMENT AND TRADEOFF ANALYSIS

INTRODUCTION

This Appendix contains a general procedure based on the Lagrange multiplier with priority list method for the effectiveness apportionment of a system which is normally weight- or volume-constrained. Typical systems include an aircraft with a maximum takeoff weight and a spacecraft which can be placed in a particular orbit with a known maximum weight. Further, such systems carry a payload; consume expendables in operating and transporting the payload; and contain equipment which is necessary to transport, operate, or monitor the payload and to perform housekeeping functions. Failure of housekeeping functions causes the system to fail. Increasing the reliability of the housekeeping function can be accomplished only by decreasing payload weight.

Also detailed is the general theory for the apportionment and evaluation of the dependability parameter of reliability for such complex systems which are characterized by standby redundant operating equipment and paths. The theory is primarily related to the development of general reliability equations which are suitable for reduction to a computer program. Following this is a description of an iterative computer program to accomplish the desired apportionment of the reliability and other system effectiveness parameters.

PROCEDURE FOR OPTIMIZING THE EFFECTIVENESS OF THE CONSIDERED SYSTEM TYPE

The following is the procedure for the optimizing of systems using the Lagrange multiplier with priority list method where the effectiveness is the average number of experiments which can be performed per unit cost, and is represented by:

$$E = R_S WT/C = R_S(W_C - W_H)(T_C - T_H)/C \qquad (1)$$

R_S is proportional to system reliability, and where W_C is the maximum weight of the spacecraft system, constrained by the selected launch vehicle system, and W_H is the weight of the equipment for housekeeping functions. Thus, $(W_C - W_H)$ is the weight available for conducting experiments. T_C is the working time available when no maintenance actions are required and T_H is the time required for maintenance during the mission.

A spare, for the purpose of increasing the system reliability for housekeeping functions, requires an increment of weight, ΔW_H, an increment of maintenance time, ΔT_H, and an increment of cost, ΔC.

Equation (1) may be written

$$\ln E = \ln R_S + \ln (W_C - W_H) + \ln (T_C - T_H) - \ln C \qquad (2)$$

Differentiating and factoring,

$$\frac{\Delta E}{E} = \frac{\Delta R_S}{W_{Exp}} \left[\frac{W_{Exp}}{R_S} - \left(\frac{\Delta W_H}{\Delta R_S} + \frac{W_{Exp}}{T_{Exp}} \frac{\Delta T_H}{\Delta R_S} + \frac{W_{Exp}}{C} \frac{\Delta C}{\Delta R_S} \right) \right] \qquad (3)$$

with

$$W_{Exp} = W_C - W_H \; ; \; T_{Exp} = T_C - T_H \qquad (4)$$

Equation (2) and the weight constraint W_C for the weight of the total spacecraft suggest application of a priority list with

$$\frac{\Delta E}{E} = \zeta \qquad (5)$$

where ζ is a Lagrange multiplier for the present problem with one constraint.

The computation steps for optimization by means of a priority list are:

(1) Start with the initial system (no redundancy) ;

thus, $W_{Exp} = W_C$; $T_{Exp} = T_C$; $C = C_{Initial}$

$R_S = R_{S\text{ - no redundancy}}$

(2) Compute ζ for each component for which spares may be applied.

(3) Select the spare for which ζ becomes a maximum.

(4) Compute W_{Exp}, T_{Exp}, C, R_S for initial system plus first spare.

(5) Compute ζ for each component with spare possibility with W_{Exp}, T_{Exp}, C, R_S as obtained in step (4), and add spare with largest Lagrange multiplier to the system.

(6) Recompute, W_{Fxp}, T_{Exp}, C, and R_S for the new system.

(7) Continue the process.

In this process, ζ, being initially positive, becomes smaller from step to step (addition of spare with largest ζ). The optimized system is found by finally adding the spare having the smallest positive ζ. When ζ becomes negative, the efficiency of E declines according to Equation (2).

In the computer program, whenever ΔE and $\frac{\Delta R_S}{W_{Exp}} > 0$, the relation of inequality

$$\frac{W_{Exp}}{R_S} \geq \frac{\Delta W_H}{\Delta R_S} + \frac{W_{Exp}}{T_{Exp}} \frac{\Delta T_H}{\Delta R_S} + \frac{W_{Exp}}{C} \frac{\Delta C}{\Delta R_S} \tag{6}$$

obtained from Equation (3) is solved, having initially estimated W_{Exp}, T_{Exp}, C, R_S for the optimized system. If assumed values and computed values for W_{Exp}, T_{Exp}, C, R_S agree with each other, the solution is found. In another case, the computation must be repeated until sufficient agreement is obtained.

COMPUTER PROGRAM DESCRIPTION

The System Effectiveness Tradeoff Analysis (SETA) Program is written in Fortran IV for use on the Univac 1108 Computer and generates a plot tape for the SC 4020 Plotter.

- Calculates and writes subsystem reliability, cost, weight, and maintenance time with given redundancy
- Calculates and writes subsystem multithread optimal redundancy, calculates and writes multithread reliability with given redundancy, or calculates and writes a combination of these two
- Selects the multithread path and path element which will result in the maximum reliability gain with minimum penalty
- Calculates the factorial N!
- Averages path failure rates and adjusts element failure rates for paths with equal or near-equal failure rates
- Identifies paths with equal failure rates and identifies lowest and highest powers of the Laplace terms

- Multiplies the coefficients of the equal-failure-rate paths and adds the powers of the Laplace terms
- Calculates the failure density function of each path in coefficients and powers of Laplace terms (SOLVE)
- Sets up the multinomial in SOLVE (RESULT)
- Solves the multinomial in RESULT
- Calculates the factorial N! in double precision
- Multiplies term-by-term over all paths to obtain terms of the multithread system
- Separates the Laplace terms
- Integrates the terms of the multithread system to obtain the reliability contribution of the term
- Sets up an SCM 4020 plot tape to plot reliability block diagrams of the systems requiring this subroutine (PLOT)
- Generates characters and circles for use by PLOT

The user can select one, or any combination, of the following options:

(a) Calculation of the initial (single-thread) and final reliability, weight, cost, and expected maintenance time of a subsystem with specified standby, active, phantom, or binomial redundancies

(b) Calculation of the initial (single-thread) and optimal (redundant elements assigned by the computer according to the system effectiveness equation) reliability, weight, cost, and expected maintenance time of a subsystem with type of redundancy (standby, active, phantom, or binomial) specified for each element

(c) Calculation of the initial and optimal (redundant elements assigned by the computer according to the system effectiveness equation) or final (specified redundant elements) reliability, weight, cost, and expected maintenance time of a multithread subsystem ($m \leq 5$ threads maximum) with elements in standby redundancy (r threads can be optimized with specified redundancy in the remaining $m - r$ threads), with printed failure rates adjusted for duty cycle

(d) System totals for $n \leq 10$ subsystems; a subsystem can be divided into subsystems to use options (a), (b), and (c)

(e) An optimized or final subsystem block diagram for specified subsystems

APPENDIX K

ACCOUNTABLE FACTORS FOR THRUST, SPECIFIC I

Thrust coefficient (C_F)

C_F = f | specific heat ratio (k)
chamber pressure to nozzle exit pressure ratio $\left(\frac{p_1}{p_2}\right)$
ambient pressure (p_o)
nozzle area ratio (ϵ)

Nozzle throat area (A_t)

A_t = f | chamber pressure (p_c)
nozzle expansion ratio (ϵ)
hot gas weight flow ($\dot{w}$)

Chamber pressure (p_c)

p_c = f | characteristic exhaust velocity (c^*)
hot gas weight flow ($\dot{w}$)
throat area (A_t)
acceleration of gravity (g)
desired burning rate (r)

Characteristic exhaust velocity (c^*)

c^* = f | chamber pressure (p_c)
throat area (A_t)

Exposed burning surface (A_b)

A_b = f | specific heat ratio (k)
nozzle throat area (A_t)
chamber pressure (p_c)
propellant density (ρ_b)
propellant burning rate (r)
gas properties (R, M)

Propellant linear burning rate (r)

r = f | chamber pressure (p_c)
initial grain temperature
propellant formulation
grain geometry

Propellant density (ρ_b)

ρ_b = f | propellant formulation
grain size and geometry
inhibitor type and geometry

Effective propellant weight (w)

w = f | hot gas weight flow rate ($\dot{w}$)
burning time (t)
residual propellant
thrust buildup inefficiencies

Thr
Thrust c
Nozzle th
Chamber

Spe
Thrust (
Hot gas
$\dot{w}$ = A

Tot
Average
Effective
I_t =
or
Specific
Effective
I_t = w

APPENDIX K

S FOR THRUST, SPECIFIC IMPULSE, AND TOTAL IMPULSE

Thrust (F)

Thrust coefficient (C_F) $= \frac{F}{p_c A_t}$

Nozzle throat area (A_t) $= \frac{F}{p_c C_F}$

Chamber pressure (p_c) $= \frac{c^* \dot{w}}{A_t g}$

Specific Impulse (I_s)

Thrust (F)

Hot gas weight flow ($\dot{w}$)

$\dot{w} = A_b \, r \, \rho_b$

Total Impulse (I_t)

Average effective thrust ($\bar{F}$)

Effective burning time (t)

$I_t = \bar{F} \, t$

or

Specific Impulse (I_s)

Effective propellant weight (w)

$I_t = w \, I_s$

Overall Rocket Performance

Thrust (F)

$F = C_F A_t \, p_c$

Specific Impulse (I_s)

$I_s = \frac{F}{\dot{w}}$

Total Impulse (I_t)

$I_t = \int_0^t F \, dt$

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13. ABSTRACT

This report describes the methods and analytical techniques relative to Air Force system designs under a system effectiveness requirement, with the objective of providing design criteria at the subsystem and equipment levels. Presented are the relevant background concepts and general application considerations necessary for an understanding and implementation of a system effectiveness evaluation. The techniques for the objective apportionment of a system's figures of merit to their constituent parameters and accountable factors are described and illustrated in detail, with primary emphasis on the developed Lagrange multiplier with priority list solution method. The technical role and perspective of system functional transfer equations, methods for their application and use in the evaluation and apportionment of system effectiveness are rendered and demonstrated in detail. A plan for the dynamic monitoring and status reporting of system effectiveness progress during all phases of system development, and to provide management and design visibility on critical and sensitive problem areas, is outlined. Additionally, the methods and techniques are further illustrated and expanded in the Technical Supplement to this report.

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