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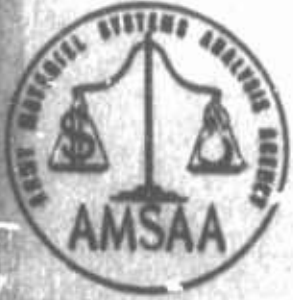
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TECHNICAL MEMORANDUM NO. 21

A LINEAR CLOSED LOOP SYSTEM ANALYSIS PROCEDURE USING LINE PRINTER PLOTS OF CHARACTERISTIC EQUATION ROOT LOCI

by

Harold H. Burke
Robert L. Payne, Jr.

November 1968



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U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
ARMY MATERIEL SYSTEMS ANALYSIS AGENCY
ABERDEEN PROVING GROUND, MARYLAND

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Weapon Systems Division

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ABERDEEN PROVING GROUND, MARYLAND

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TECHNICAL MEMORANDUM NO. 21

HHBurke/RLPayneJr/pbb
Aberdeen Proving Ground, Maryland
November 1968

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ABSTRACT

An existing program that determines the root locus of n 'th order polynomials has been modified to provide plots of these loci in the complex frequency plane using a standard line printer. A methodology that combines the computational capabilities of this root locus program with a variable scale graphical display of selectable regions of the complex frequency plane is presented. A listing of the Fortran IV source deck of the modified program and two examples are included.

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1. INTRODUCTION

The stability of linear closed loop systems is effectively determined through the use of root locus methods⁽¹⁾. For all but the most trivial cases, the polynomial representing the system's characteristic equation is high order. Manual calculation of the roots of this polynomial is laborious and time consuming. A Fortran IV program⁽²⁾ has been developed to expedite this process. The interpretation of a tabular display even when available is at best cumbersome. The purpose of this memo is to describe the use of a modified version of this basic program which provides selectable scaled graphical displays of the root loci as part of the tabular output.

Main features of the modified program are:

1. Programmed in Fortran IV. No machine oriented or object language.
2. No complex arithmetic.
3. No special graphical plotting equipment necessary.
4. Order of polynomial may be up to 100.
5. Number of variations of coefficient may be up to 100.

Main features of the graphical display are:

1. Log plot of third and fourth quadrants of complex frequency plane from 0 to 10,000 radians/second.
2. Linear plots of selected regions of the third and fourth quadrants of the complex frequency plane with arbitrary scales.

II. NATURE OF THE PROBLEM

Regardless of the complexity of a closed loop system its transfer function can be reduced to the equivalent form shown in Figure 1. For multiple loop systems, the G's and H's are readily expressed as sums of products of polynomials which are identified with individual elements making up the complete system.

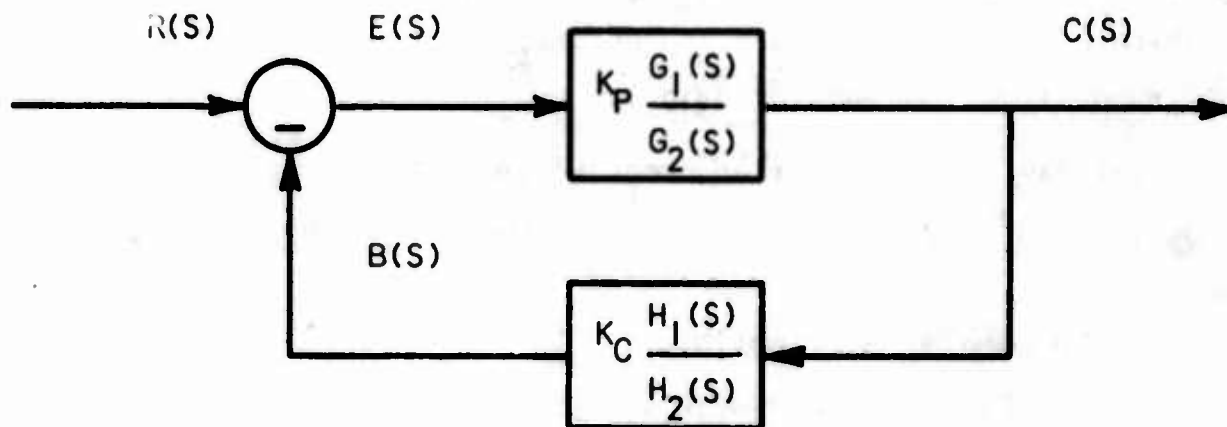


Figure 1. Linear Closed Loop System

where: $R(S)$ = System Input
 $C(S)$ = System Output
 $B(S)$ = System Feedback
 $E(S)$ = System Error
 K_p = Process Gain
 K_c = Controller Gain

The fractions $G_1(S)/G_2(S)$ and $H_1(S)/H_2(S)$ are equivalent transfer functions of the system and are represented by ratios of polynomials which upon expansion can be put in the form

$$\frac{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{s^{m+n} + b_1 s^{m+n-1} + b_2 s^{m+n-2} + \dots + b_{m+n}} = \frac{\sum_{u=0}^n a_u s^{n-u}}{\sum_{i=0}^{m+n} b_i s^{m+n-i}} \quad (1)$$

or in factored form

$$\frac{\prod_{u=1}^n (s + z_u)}{\prod_{i=1}^{m+n} (s + p_i)} \quad (2)$$

where the a's and b's are real, the poles and zeros ($-p_i$ and $-z_u$) can be either real or complex in conjugate pairs and $m \geq 1$.

The linear closed loop system transfer function is

$$\frac{C(S)}{R(S)} = \frac{K_p G_1(S) H_2(S)}{K_c K_p G_1(S) H_1(S) + G_2(S) H_2(S)} \quad (3)$$

The linear closed loop system's characteristic equation which determines the roots of the transfer function's denominator is

$$K_c K_p G_1(S) H_1(S) + G_2(S) H_2(S) = 0. \quad (4)$$

If we let $K_c K_p = + K^*$ (5)

$$G_1(S) H_1(S) = A(S)$$

$$G_2(S) H_2(S) = B(S)$$

then equation (4) can be written as

$$K^*A(S) + B(S) = 0 \quad (6)$$

and the linear closed loop system transfer function is

$$\frac{C(S)}{R(S)} = \frac{K_p G_1(S) H_2(S)}{K^*A(S) + B(S)} \quad (7)$$

The transfer function for a unit step in R(S) becomes

$$C(S) = \frac{K_p \prod_{u=1}^n (S + Z_u)}{S \prod_{i=1}^{m+n} (S + p_i)} \quad (8)$$

and the time response of the output, c(t) to the unit step is given by

$$c(t) = L^{-1}[C(S)] \quad (9)$$

where $L^{-1}(\)$ indicates the inverse Laplace transform

Expressing equation (8) in partial fraction form we have

$$C(S) = \frac{K_o}{S} + \sum_{i=1}^{m+n} \frac{K_i}{(S+p_i)} \quad (10)$$

where K_o and K_i are the residues at the respective poles of C(S).

Specifically

$$K_o = \frac{K_p \prod_{u=1}^n (Z_u)}{\prod_{i=1}^{m+n} (p_i)} \quad (11)$$

$$K_\ell = \frac{[K_p \prod_{u=1}^n (S + Z_u)]}{[S \prod_{i=1}^{m+n} (S + p_i)]} \Big|_{S=-p_\ell} \quad (12)$$

The inverse transform of (10) is therefore

$$c(t) = K_0 + \sum_{i=1}^{m+n} K_i e^{p_i t} \quad (13)$$

If a pole p_ℓ is complex, its corresponding residue, K_ℓ , is also complex. In this case there also exists a term of the form

$K_\ell e^{p_\ell t} + \bar{K}_\ell e^{\bar{p}_\ell t}$ where the bar denotes complex conjugate.

We then have

$$(K_\ell e^{p_\ell t} + \bar{K}_\ell e^{\bar{p}_\ell t}) = 2|K_\ell| e^{\sigma_\ell t} \cos(\omega_\ell t + \phi_\ell) \quad (14)$$

where:

$$p_\ell = \sigma_\ell + j\omega_\ell \quad (15)$$

$$\bar{p}_\ell = \sigma_\ell - j\omega_\ell$$

$$\phi_\ell = \tan^{-1} \frac{\omega_\ell}{\sigma_\ell}$$

Thus if j poles are real and $m+n-j$ poles are complex we arrive at the complete time solution for $c(t)$

$$c(t) = K_0 + \sum_{i=1}^j K_i e^{p_i t} + \sum_{\ell=j+1}^K 2|K_\ell| e^{\sigma_\ell t} \cos(\omega_\ell t + \phi_\ell) \quad (16)$$

where

$$K = \frac{m+n-j}{2}$$

As the value of K^* is varied, the roots of the system's characteristic equation change. Since $-\frac{1}{p_i}$ is the time constant for a single pole and $-\sigma_\ell$ is the damping coefficient associated with the frequency ω_ℓ , we can see that the locus of p_i and p_ℓ in the complex plane, as K^* varies, gives an indication of the stability of the system.

Although the stability of the system for a unit step function is generally adequate the response of the system $F(t)$ to an arbitrary input $R(t)$ can be found by the Duhamel integral

$$F(t) = \int_0^t R(u) c(t-u) du \quad (17)$$

where $c(t)$ is of the form given in equations (13) and (15). A locus of roots as K^* varies from K^*_{\min} to K^*_{\max} is called the root locus of the system. From (6) it is seen that when K_c is not present (no closed loop system), $K^*=0$ and the roots of the characteristic equation are the roots of $B(S)$. When K_c is present (closed loop system) and the value of K_c approaches infinity, K^* also approaches infinity. The roots of the characteristic equation are the roots of $A(S)$.

III. GRAPHICAL METHOD

The root loci of the system are the trajectories of the roots of equation (6) in the complex plane as K^* varies from 0 to ∞ . It will be recalled that since the coefficients of the characteristic equation are real the complex roots appear in conjugate pairs and the loci are symmetric about the σ axis. Hence investigation of the half plane reveals the nature of the entire set of loci. In our plots of the loci, we choose to display only the lower half on the complex frequency plane, i.e., the third and fourth quadrants which make up the left and right hand quadrants respectively. We include two options (1) a log plot and (2) an expanded scale linear plot.

In the log plot separate plots are made of the left and right hand quadrants. Each is partitioned into **decades** ranging from 0.01 rad/sec to 10,000 rad/sec. Figures 3 and 4 show the actual log plots for example 1, to be described later. Figures 5 and 6 show two actual linear expands for example 1, to be described later.

Detailed plotting accuracy is of no concern on the log plots. Selection of the proper scale will provide the desired accuracy on the linear expand plots. The left hand quadrant coordinates are minus sigma and j-omega. The right hand quadrant coordinates are plus sigma and j-omega. The dimensions of the coordinates in the complex frequency plane are radians/second.

Three different symbols are shown in the complex frequency plane log plot. The definition of these are

- * Roots of B(S)
- 0 Roots of A(S)
- Roots of K*A(S) + B(S)

Dependent on the proximity of any of the above roots priority is given to plotting first the roots A(S) then the roots of B(S) and finally the roots of K*A(S) + B(S). For all real roots which are located on the plus sigma or minus sigma axis, the roots of A(S) appear on the sigma axis. Any other root that may occur within the stepping increment of a root of A(S) i.e., other roots of A(S), B(S) or K*A(S) + B(S) will appear to the right. The same priority of graphical printout is maintained for the complex roots.

Rewriting (6) gives

$$K*A(S) + B(S) = 0 \quad (6)$$

Dividing (6) by B(S) and rearranging gives

$$1 - \frac{[-K*A(S)]}{B(S)}$$

where $\frac{-K*A(S)}{B(S)}$ = open loop transfer function of equivalent system $[TF]_{OL}$

The algebraic sign of the ratio of the lowest powered terms of open loop transfer function, $\frac{A(S)}{B(S)}$ will determine the regions on the \pm sigma axis where the roots of $\frac{B(S)}{B(S)} + \frac{K*A(S)}{B(S)}$ are located. For a positive $\frac{A(S)}{B(S)}$ the roots of $1 - [TF]_{OL} = 0$ are located on the zero

angle locus ⁽³⁾ and extend from + infinity on the positive sigma axis to the largest positive root of A(S) or B(S). From this most positive real root the locus of roots are located in the even numbered intervals between the roots of A(S) and B(S) to minus infinity on the sigma axis. For a A(S)/B(S) the roots of $K \cdot A(S) + B(S) = 0$ are located on the 180° angle locus and extend from the largest positive root of A(S) or B(S) on the Sigma axis leftward to the adjacent root and in the odd numbered intervals between the roots of A(S) and B(S) to minus infinity on the sigma axis.

Inspection of the log plot shown in Figures 3 and 4 demonstrate the real locus for a positive $\frac{A(S)}{B(S)}$, while Figures 5 and 6 demonstrate the real locus for a negative $\frac{A(S)}{B(S)}$. When applying this method a pencilled line parallel to the sigma axis to designate these closed loop root trajectories on the real axis is suggested.

The precise location of these roots is given in the numerically tabulated output. A detailed discussion of the complete input and output will be later. Mention is made here only to indicate that close continuity exists between the numerical and graphical output.

Regions of interest on the complex plane are obvious from the log plots. In order to study segments of the root loci more closely a linear expand plot of the complex frequency plane is used. Similar to the log plot, only one quadrant of the right and left hand planes are shown. The scaling of the linear expand plots is completely arbitrary. The regions of interest are determined from inspection of the log plots. As many regions as desired can be expanded. Figures 5 and 6 are one such linear expand on a specific region of the complex frequency plane.

IV. DATA FORMATS

A. Polynomial Multiplication and Root Locus Method

1. General Description.

a. Computes the numerator polynomial A, where A is the sum of the products of several sets of polynomials. Each set contains a variable number of polynomials and a variable number of sets form the sum.

b. Computes the denominator polynomial B, where B is the sum of the products of several sets of polynomials.

c. Computes the roots of A.

d. Computes the roots of B.

e. Computes the polynomial $K * A + B$, where K^* varies from K initial to K terminate in increments of ΔK , or particular values of K^* may be chosen.

f. Computes the roots of the $K * A + B$ polynomials.

2. Input.

<u>Description</u>	<u>Columns</u>	<u>Data</u>
a. Identification	1 - 80	Identification of run
b. Control card (Integers)	1 - 10	0 for ΔK N for number of input particular values of K
	11 - 20	Number of polynomial groups to be added in A(s)
	21 - 30	Number of polynomial groups to be added in B(s)
	31 - 40	Problem number
c. If Δk is used		
	1 - 10	K initial
	11 - 20	ΔK

2. Input. (Cont.)

<u>Description</u>	<u>Columns</u>	<u>Data</u>
c1. If A_k is used	21 - 30	K terminate
c2. If specific values of K are used,		
	1 - 10	K_1
	11 - 20	K_2
	21 - 30	K_3
	o	
	o	
	o	
	o	
	o	K_N
	(seven values per card)	
d. Numerator group count (Integers)		
	1 - 10	Number of polynomials in group 1 of numerator
	11 - 20	Number of polynomials in group 2 of numerator
	o	
	o	
	o	
	Etc.	
	(seven values per card)	

2. Input (Cont.)

<u>Description</u>	<u>Columns</u>	<u>Data</u>
e. Denominator group count (integers)		
	1 - 10	Number of polynomials in group 1 of denominator
	11 - 20	Number of polynomials in group 2 of denominator
	0	
	0	
	0	
	Etc.	
	(seven values per card)	

f. Polynomials. Seven values per card, with each polynomial starting on a new card. The first value of each polynomial is an integer, with the coefficients in ascending order in floating point:

1 - 10	Degree + 1 of the polynomial
11 - 20	Constant term
21 - 30	Coefficient of X
31 - 40	Coefficient of X ²
Etc.	

Load in polynomials in the order in which they appear in the fraction, with the numerator polynomials first.

3. Output.

The output consists of:

- a. Value of K* initial, increment K*, value of K* terminate.
- b. Number of polynomial groups added in A(S).
- c. Number of polynomial groups added in B(S).
- d. Number of polynomials in each group of A(S).
- e. Number of polynomials in each group of B(S).
- f. Coefficients of polynomials in each group of A(S) and B(S).

3. Output (Cont.)

- g. The degree, coefficients and roots of A.
- h. The degree, coefficients and roots of B.
- i. The degree, coefficients, roots, value of K^* for each $K^* A + B$ polynomial.

4. Special Considerations.

- a. The maximum order of A or B is 100.
- b. The maximum number of specific values of K is 100.

B. Root Locus Plot Subroutine

1. General Description. This is a subroutine designed to work with the basic polynomial multiplication and root locus program. This subroutine takes the roots of $K^*A + B$ calculated by the polynomial root locus program and plots them on the complex frequency plane with the following options:

Log plots of the third and fourth quadrants of the complex frequency plane from 0 to 10,000 rad/sec. Linear expand plots of the third and fourth quadrants of the complex frequency plane with selectable scale. The linear expand plots supplement the log plot. The region covered by any one linear expand plot can be designated by the analyst through the use of data cards. The sigma dimensions are determined by choosing a certain point in either the third or fourth quadrant and picking a percentage of this point to be the distance represented on the sigma axis. (See linear expand output of example). The J-omega dimensions are determined by choosing a certain point on the J-omega axis and picking a percentage of this point to be the distance represented on the J-omega axis.

2. Input.

<u>Description</u>	<u>Columns</u>	<u>Data</u>
Control Card Used to call the log plot subroutine	1 11 21 31 - 35	0 (zero) integer 0 (zero) integer 0 (zero) integer 10000(ten thousand) integer
Control Cards Used to call the expand option		
Number of Expands Desired	1 - 10	integer
For each linear Expand, data appears on one card	1 - 10 11 - 20 21 - 30 31 - 40	J-omega specific Real J-omega percent Real Sigma specific Real Sigma percent Real

Note: There must be as many data cards as there are number of expands desired.

3. Output.

Depending on the options specified the output consists of:

- A list of all of the roots plotted on the log plot.
- Two log plots of the roots calculated by the polynomial multiplication and root locus program.
- The specified number of linear expand plots of selectable regions in the complex frequency plane.

V. EXAMPLES

Two examples of the use of the techniques described above will be considered. The first is the realization of a typical control system and will be discussed in some detail. The second is included simply to indicate the results for a fairly large order system.

Example 1.

A typical control system is shown in Figure 2.

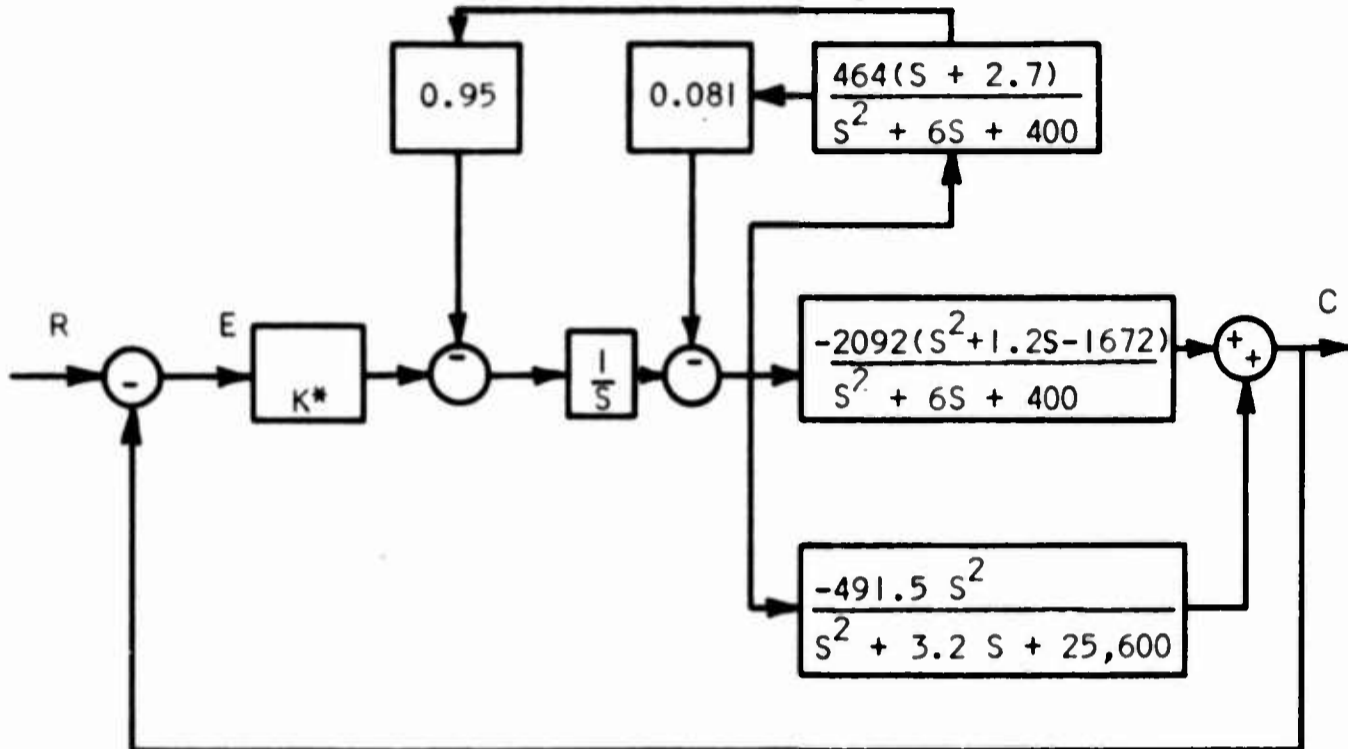


Figure 2. Linear Closed Loop Control System Block Diagram

The equivalent form of Figure 2 is shown in Figure 1 where $G_1(s) H_1(s) = A(s)$ and $G_2(s) H_2(s) = B(s)$ and $K^* = K_p K_c$.

The characteristic equation for the system shown in Figure 2 is

$$\begin{aligned}
 & -K^*[2092(S^2+1.2S-1672)(S^2+3.2S+25,600) + 491.5S^2(S^2+6S+400)] \\
 & + (0.081)(464)(S+2.7)(S^2+3.2S+25,600)S \\
 & + (0.95)(464)(S+2.7)(S^2+3.2S+25,600) \\
 & + S(S^2+6S+400)(S^2+3.2S+25,600) = 0
 \end{aligned}$$

Table I shows the input card format for this characteristic equation. Lines 1 through 20 apply to the polynomial multiplication and root locus method and lines 21 through 28 apply to the root locus plot subroutine.

Table 1. Input for Example 1

0	2	3	331
0.0	0.01	0.3	
3	2		
4	4	3	
3	25600.	3.2	1.
3	-1672.	1.2	1.
1	-2092.		
3	0.	0.	-491.5
3	400.	6.	1.
1	.081		
2	0.	464.	
2	2.7	1.	
3	25600.	3.2	1.
1	.95		
1	464.		
2	2.7	1.	
3	25600.	3.2	1.
2	0.	1.	
3	400.	6.	1.
3	25600.	3.2	1.
0	0	0	10000
6			
100.	100.	-50.	100.
100.	100.	100.	100.
50.	100.	-50.	100.
50.	100.	50.	100.
150.	20.	-5.	100.
150.	20.	5.	100.

POLYNOMIAL MULTIPLICATION
AND ROOT LOCUS PROGRAM

ROOT LOCUS
PLOT SUBROUTINE

The tabulated numerical output is shown in Tables 2 through 5. Table 2 mirrors the number of inputs values of K^* , ($0 = \Delta K$ increment, $N =$ number of particular input values of K^*), the number polynomial groups added in $A(S)$ and $B(S)$, the problem number, K^* initial, increment K^* , K^* terminate, the number of polynomial in each group of $A(S)$ and $B(S)$, and the coefficients of each of these polynomials proceeding from the left most term in the numerator to the final term in the denominator. Each polynomial group appears on a separate sheet. Table 3 defines the equivalent open loop $A(S)$ and $B(S)$ in single polynomial and root form. Table 4 defines $K^* A(S) + B(S)$ in polynomial and root form for values of K^* between K^* initial and K^* terminate. Table 5 is a tabulation of $K^* A(S) + B(S)$ roots which are to be plotted.

Figures 3 and 4 are the log plots for Example 1. The net sign for the lowest order coefficients is positive as seen in Table 4, hence the zero degree locus is plotted. The roots of $A(S)$ and $B(S)$ are

$$\begin{aligned}
 &A(S) \\
 &39.918653 \\
 &-41.162138 \\
 &-1.7304538 \pm j 145.22688
 \end{aligned}$$

$$\begin{aligned}
 &B(S) \\
 &-1.3440477 \\
 &-1.6000000 \pm j 159.99200 \\
 &-21.119976 \pm j 20.963084
 \end{aligned}$$

The locus of the $K^* A(S) + B(S)$ roots on the sigma axis as K^* varies from 0 to 0.3 in increments of 0.010 may be determined by inspection. It appears to the right of 39.918653 and between -1.3440477 and -41.162138. The complex locus has two branches. One branch comes from the complex pair at $-21.119976 \pm j 20.963084$ and the other from the complex pair at $-1.6000000 \pm j 159.99200$.

For K^* gains between 0 and 0.3 these two branches lie in the third and fourth quadrants. Inspection of this log plot in conjunction with the open loop roots of $A(S)$ and $B(S)$ given in Table 4 provides insight

into the regions of the complex plane where an expanded scale plot of the root locus is required. The region bounded by $-100 < \sigma < 200$ and $0 < \omega < 200$ is shown in Figures 5 and 6. The locus between sigma equal to -41.162138 and -1.3440477 is shown in detail. The locus between sigma equal to $-21.119976 \pm j 20.963084$ and the positive sigma axis roots $+ 39.918653$ and plus infinity is shown in detail. The locus in the $100 - 1000$ rad/second j-omega decade is shown in a compressed form in this plot. Figures 7 and 8 are another set of expanded scale plots of the region bounded by $-100 < \sigma < 100$ and $0 < \omega < 100$. This scale provides more sensitivity to that portion of the locus which progresses from -21.119976 ± 20.963084 to the positive sigma axis. The locus in the $100-1000$ rad/sec j-omega decade is not shown. Figures 9 and 10 are expanded scale plots in the region bounded by $-10 < \sigma < 10$ and $20 < \omega < 180$. The roots of $A(s)$ and $B(s)$ are in the third (left hand) quadrant while the roots of $K^*A(s) + B(s)$ are in the fourth (right hand) quadrant. The migration of these roots as K^* is varied is an indication of the requirement for this expanded scale plot of the complex frequency plane.

Example 2

Table 6 is the tabulated numerical output for a more complex multiloop system whose characteristic equation is of 24'th order. Figures 11 and 12 are log plots. The off axis printing on the sigma axis is shown. The net sign of the lowest order coefficients of $A(s)/B(s)$ is negative, hence the 180° locus is plotted. Inspection shows that the locus is between $+0.0040482270$ and -0.046149160 and then between each alternate root of $A(s)$ or $B(s)$ on the sigma axis. Inspection of Figures 11 and 12 indicates that several regions of the complex frequency plane are of interest. One such region is the region bounded by $-16.0 < \sigma < 16.0$ and $25. < \omega < 175$. which is shown in Figures 13 and 14.

TABLE 2

Mirror of Input, Example 1.

POLYNOMIAL MULTIPLICATION AND ROOT LOCUS
 DELTAK= 0 POLY.ADDED IN A(S)= 2 POLY.ADDED IN B(S)= 3 PROB.NO.= 331
 PROBLEM NO. 331

K-INITIAL= .0100000000 K-TERMINATE= .3000000000
 NUMBER OF POLY. IN GROUP 1 OF NUMERATOR= 3
 NUMBER OF POLY. IN GROUP 2 OF NUMERATOR= 2
 NUMBER OF POLY. IN GROUP 1 OF DENOMINATOR= 4
 NUMBER OF POLY. IN GROUP 2 OF DENOMINATOR= 4
 NUMBER OF POLY. IN GROUP 3 OF DENOMINATOR= 3
 C(1)= 25600.0000000000
 C(2)= 3.2000000000
 C(3)= 1.0000000000
 C(1)= -1672.0000000000
 C(2)= 1.2000000000
 C(3)= 1.0000000000
 C(1)= -2092.0000000000

TABLE 2
 Mirror of Input, Example 1. (Contd)

C(1)=	.0000000000
C(2)=	.0000000000
C(3)=	-.491.5000000000
C(1)=	400.0000000000
C(2)=	6.0000000000
C(3)=	1.0000000000

TABLE 2
 Mirror of Input, Example 1. (Contd)

C(1)=	.0810000000
C(1)=	.0000000000
C(2)=	464.0000000000
C(1)=	2.7000000000
C(2)=	1.0000000000
C(1)=	25600.0000000000
C(2)=	3.2000000000
C(3)=	1.0000000000

TABLE 2
Mirror of Input , Example 1 . (Contd)

C(1)=	.9500000000
C(1)=	464.0000000000
C(1)=	2.7000000000
C(2)=	1.0000000000
C(1)=	25600.0000000000
C(2)=	3.2000000000
C(3)=	1.0000000000

TABLE 2
 Mirror of Input, Example 1. (Contd)

C(1) =	0.000000000000
C(2) =	1.000000000000
C(1) =	400.000000000000
C(2) =	6.000000000000
C(3) =	1.000000000000
C(1) =	25600.000000000000
C(2) =	3.200000000000
C(3) =	1.000000000000

TABLE 3
Equivalent System Open Loop Polynomials and Roots, Example 1.

COEFFICIENTS ARE GIVEN IN ASCENDING ORDER

THE COEFFICIENTS OF POLYNOMIAL A (ORDER = 4)			
8.9544294E 10	-5.3073203E 07	-5.0262009E 07	-1.2153800E 04
			-2.5835000E 03
THE ROOTS OF A			
3.9918653E 01	+1	0.0000000E 00	+1
-1.7304538E 00	+1	1.4522688E 02	+1
			-1.7304538E 00
			+1
			-1.4522688E 02
THE COEFFICIENTS OF POLYNOMIAL B (ORDER = 5)			
3.0468096E 07	2.4126095E 07	1.1199558E 06	2.6681746E 04
			4.6784000E 01
			1.0000000E 00
THE ROOTS OF B			
-1.3440477E 00	+1	0.0000000E 00	+1
-1.6000000E 00	+1	1.5999200E 02	+1
			-2.1119976E 01
			-1.6000000E 00
			-2.1119976E 01
			+1
			-2.0963084E 01
			+1
			-2.0963084E 01

TABLE 4
Closed Loop System Polynomials and Roots, Example I.
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS

THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 0.000000E 00			
3.0468096E 07	2.4126095E 07	1.1199558E 06	2.6681746E 04
ROOTS OF K*A + B			
-1.3440477E 00	+ I	0.000000E 00	-2.1119976E 01
-1.600000E 00	+ I	1.5999200E 02	-1.600000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.000000E-02			
9.2591104E 08	2.3595363E 07	6.1733575E 05	2.6560208E 04
ROOTS OF K*A + B			
4.0494673E 00	+ I	3.3975966E 01	4.0494673E 00
8.7523407E-01	+ I	1.6024308E 02	8.7523407E-01
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 2.000000E-02			
1.8213540E 09	2.3064631E 07	1.1471566E 05	2.6438670E 04
ROOTS OF K*A + B			
-3.4675444E 01	+ I	0.000000E 00	1.6364826E 01
3.4158963E 00	+ I	1.5974276E 02	3.4158963E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 3.000000E-02			
2.7167969E 09	2.2533898E 07	-3.8790443E 05	2.6317132E 04
ROOTS OF K*A + B			
-3.6373738E 01	+ I	0.000000E 00	2.7830727E 01
5.7166419E 00	+ I	-1.5848945E 02	5.7166419E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 4.000000E-02			
3.6122399E 09	2.2003166E 07	-8.9052453E 05	2.6195594E 04
ROOTS OF K*A + B			
-3.7352153E 01	+ I	0.000000E 00	3.9472012E 01
7.4820644E 00	+ I	1.5663009E 02	7.4820644E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.000000E 00			
3.0468096E 07	2.4126095E 07	1.1199558E 06	2.6681746E 04
ROOTS OF K*A + B			
-1.3440477E 00	+ I	0.000000E 00	-2.1119976E 01
-1.600000E 00	+ I	1.5999200E 02	-1.600000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.000000E 00			
9.2591104E 08	2.3595363E 07	6.1733575E 05	2.6560208E 04
ROOTS OF K*A + B			
4.0494673E 00	+ I	3.3975966E 01	4.0494673E 00
8.7523407E-01	+ I	1.6024308E 02	8.7523407E-01
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.000000E 00			
1.8213540E 09	2.3064631E 07	1.1471566E 05	2.6438670E 04
ROOTS OF K*A + B			
-3.4675444E 01	+ I	0.000000E 00	1.6364826E 01
3.4158963E 00	+ I	1.5974276E 02	3.4158963E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.000000E 00			
2.7167969E 09	2.2533898E 07	-3.8790443E 05	2.6317132E 04
ROOTS OF K*A + B			
-3.6373738E 01	+ I	0.000000E 00	2.7830727E 01
5.7166419E 00	+ I	-1.5848945E 02	5.7166419E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.000000E 00			
3.6122399E 09	2.2003166E 07	-8.9052453E 05	2.6195594E 04
ROOTS OF K*A + B			
-3.7352153E 01	+ I	0.000000E 00	3.9472012E 01
7.4820644E 00	+ I	1.5663009E 02	7.4820644E 00

TABLE 4

Closed Loop System Polynomials and Roots, Example I. (Contd)

POLYNOMIAL MULTIPLICATION AND ROOT LOCUS

PROBLEM NO. 331

THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 5.0000000E-02			
4.5076828E 09	2.1472434E 07	-1.3931446E 06	-8.2391000E 01
ROOTS OF K*A + B			
-3.7993535E 01	0.0000000E 00	5.1675266E 01	4.7830316E 01
8.5170010E 00	-1.5445648E 02	8.5170010E 00	1.5445648E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 6.0000000E-02			
5.4031258E 09	2.0941702E 07	-1.8957647E 06	-1.0822600E 02
ROOTS OF K*A + B			
-3.8448009E 01	0.0000000E 00	6.4519180E 01	4.3289137E 01
8.8178250E 00	-1.5232117E 02	8.8178250E 00	1.5232117E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 7.0000000E-02			
6.2985687E 09	2.0410970E 07	-2.3983848E 06	-1.3406100E 02
ROOTS OF K*A + B			
-3.8787478E 01	0.0000000E 00	7.7857764E 01	3.2945263E 01
8.5664744E 00	-1.5048875E 02	8.5664744E 00	1.5048875E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 8.0000000E-02			
7.1940116E 09	1.9880238E 07	-2.9010049E 06	-1.5989600E 02
ROOTS OF K*A + B			
-3.905049E 01	0.0000000E 00	1.0137499E 02	0.0000000E 00
8.0065400E 00	1.4905381E 02	8.0065400E 00	-1.4905381E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 9.0000000E-02			
8.0894546E 09	1.9349506E 07	-3.4036250E 06	-1.8573100E 02
ROOTS OF K*A + B			
-3.9261499E 01	0.0000000E 00	6.4244710E 01	0.0000000E 00
7.3285547E 00	1.4798418E 02	1.4609118E 02	1.4798418E 02

TABLE 4

Closed Loop System Polynomials and Roots, Example 1. (Contd)

POLYNOMIAL MULTIPLICATION AND ROOT LOCUS

PROBLEM NO. 331

THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.0000000E-01			
8.9848975E 09	1.8818774E 07	-3.9062451E 06	-2.1156600E 02
			1.0000000E 00
ROOTS OF K*A + B			
-3.9433684E 01	0.0000000E 00	5.8581243E 01	0.0000000E 00
6.6421054E 00	1.4720164E 02	1.7913423E 02	1.0000000E 00
			-1.4720164E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.1000000E-01			
9.8803405E 09	1.8288042E 07	-4.4088652E 06	-2.3740100E 02
			1.0000000E 00
ROOTS OF K*A + B			
-3.9577150E 01	0.0000000E 00	5.5277448E 01	0.0000000E 00
5.9977661E 00	1.4662976E 02	2.0970517E 02	1.0000000E 00
			-1.4662976E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.2000000E-01			
1.0775783E 10	1.7757310E 07	-4.9114853E 06	-2.6323600E 02
			1.0000000E 00
ROOTS OF K*A + B			
-3.9698552E 01	0.0000000E 00	5.3041355E 01	0.0000000E 00
5.4135301E 00	1.4620872E 02	2.3906614E 02	1.0000000E 00
			-1.4620872E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.3000000E-01			
1.1671226E 10	1.7226578E 07	-5.4141054E 06	-2.3907100E 02
			1.0000000E 00
ROOTS OF K*A + B			
-3.9802630E 01	0.0000000E 00	5.1405678E 01	0.0000000E 00
4.8919037E 00	1.4589537E 02	2.6768414E 02	1.0000000E 00
			-1.4589537E 02
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.4000000E-01			
1.2566669E 10	1.6695846E 07	-5.9167255E 06	-3.1490600E 02
			1.0000000E 00
ROOTS OF K*A + B			
-3.9892853E 01	0.0000000E 00	5.0148663E 01	0.0000000E 00
4.4288433E 00	1.4565946E 02	2.9579242E 02	1.0000000E 00
			-1.4565946E 02

TABLE 4
Closed Loop System Polynomial and Roots, Example 1. (Contd.)

POLYNOMIAL MULTIPLICATION AND ROOT LOCUS

PROBLEM NO. 331

THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.5000000E-01									
1.3462112E 10	1.6165114E 07	-6.4193455E 06							
ROOTS OF K*A + B									
-3.9971821E 01 + I	0.0000000E 00	4.9148532E 01 + I	0.0000000E 00						
4.0182316E 00 + I	1.4547988E 02	3.2352783E 02 + I	0.0000000E 00						
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.6000000E-01									
1.4357555E 10	1.5634382E 07	-6.9219656E 06							
ROOTS OF K*A + B									
-4.0041520E 01 + I	0.0000000E 00	4.8331837E 01 + I	0.0000000E 00						
3.6533814E 00 + I	1.4534184E 02	3.5097892E 02 + I	0.0000000E 00						
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.7000000E-01									
1.5252998E 10	1.5103650E 07	-7.4245857E 06							
ROOTS OF K*A + B									
-4.0103495E 01 + I	0.0000000E 00	4.7651245E 01 + I	0.0000000E 00						
3.3282133E 00 + I	1.4523480E 02	3.7820682E 02 + I	0.0000000E 00						
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.8000000E-01									
1.6148441E 10	1.4572918E 07	-7.9272058E 06							
ROOTS OF K*A + B									
-4.0158964E 01 + I	0.0000000E 00	4.7074705E 01 + I	0.0000000E 00						
3.0373178E 00 + I	1.4515121E 02	4.0525562E 02 + I	0.0000000E 00						
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 1.9000000E-01									
1.7043884E 10	1.4042186E 07	-8.4298259E 06							
ROOTS OF K*A + B									
4.6579653E 01 + I	0.0000000E 00	-4.0208902E 01 + I	0.0000000E 00						
2.7760322E 00 + I	1.4508556E 02	4.3215818E 02 + I	0.0000000E 00						

TABLE 4
 Closed Loop System Polynomials and Roots, Example 1 (Contd.)
 POLYNOMIAL MULTIPLICATION AND ROOT LOCUS

PROBLEM NO. 331

THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K =				2.000000E-01
1.7939327E 10	1.3511454E 07	-8.9324460E 06	-4.6991600E 02	1.000000E 00
ROOTS OF K*A + B				
4.6149699E 01	+ I	0.000000E 00	+ I	-1.4503379E 02
2.5403847E 00	+ I	1.4503379E 02	+ I	0.000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K =				2.100000E-01
1.8834770E 10	1.2980722E 07	-9.4350661E 06	-4.3575100E 02	1.000000E 00
ROOTS OF K*A + B				
4.5772627E 01	+ I	0.000000E 00	+ I	-1.4499285E 02
2.3270093E 00	+ I	1.4499285E 02	+ I	0.000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K =				2.200000E-01
1.9730213E 10	1.2449990E 07	-9.9376862E 06	-5.2158600E 02	1.000000E 00
ROOTS OF K*A + B				
4.5439133E 01	+ I	0.000000E 00	+ I	-1.4496045E 02
2.1330558E 00	+ I	1.4496045E 02	+ I	0.000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K =				2.300000E-01
2.0625656E 10	1.1919258E 07	-1.0440306E 07	-5.4742100E 02	1.000000E 00
ROOTS OF K*A + B				
4.5141995E 01	+ I	0.000000E 00	+ I	-1.4493483E 02
1.9561081E 00	+ I	1.4493483E 02	+ I	0.000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K =				2.400000E-01
2.1521099E 10	1.1388526E 07	-1.0942926E 07	-5.7125600E 02	1.000000E 00
ROOTS OF K*A + B				
4.4875519E 01	+ I	0.000000E 00	+ I	-1.4491462E 02
1.7941126E 00	+ I	1.4491462E 02	+ I	0.000000E 00

TABLE 4
Closed Loop System Polynomials and Roots, Example 1. (Contd.)
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS PROBLEM NO. 331

THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 2.5000000E-01			
2.2416542E 10	1.0857794E 07	-1.1445546E 07	-5.9909100E 02
ROOTS OF K*A + B			1.0000000E 00
4.4635150E 01	0.0000000E 00	-4.0428064E 01	1.6453179E 00
1.6453179E 00	1.4489878E 02	5.9159328E 02	0.0000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 2.6000000E-01			
2.3311985E 10	1.0327062E 07	-1.1948167E 07	-6.2492600E 02
ROOTS OF K*A + B			1.0000000E 00
4.4417202E 01	0.0000000E 00	-4.0455148E 01	1.5082247E 00
1.5082247E 00	1.4488646E 02	6.1794750E 02	0.0000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 2.7000000E-01			
2.4207428E 10	9.7963297E 06	-1.2450787E 07	-6.5076100E 02
ROOTS OF K*A + B			1.0000000E 00
4.4218654E 01	0.0000000E 00	-4.0480303E 01	1.3815444E 00
1.3815444E 00	1.4487702E 02	6.4425956E 02	0.0000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 2.8000000E-01			
2.5102871E 10	9.2655977E 06	-1.2953407E 07	-6.7659600E 02
ROOTS OF K*A + B			1.0000000E 00
4.4037009E 01	0.0000000E 00	-4.0503728E 01	1.2641649E 00
1.2641649E 00	1.4486992E 02	6.7053439E 02	0.0000000E 00
THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER = 5) K = 2.9000000E-01			
2.5798113E 10	8.7348657E 06	-1.3456027E 07	-7.0243100E 02
ROOTS OF K*A + B			1.0000000E 00
4.3870184E 01	0.0000000E 00	-4.0525595E 01	1.1551225E 00
1.1551225E 00	1.4486475E 02	6.9677617E 02	0.0000000E 00

TABLE 4
 Closed Loop System Polynomials and Roots, Example I. (Contd.)
 POLYNOMIAL MULTIPLICATION AND ROOT LOCUS PROBLEM NO. 331

THE COEFFICIENTS OF POLYNOMIAL $K^*A + B$ (ORDER = 5) $K =$ 3.0000000E-01
 2.6893756E 10 0.2041336E 06 -1.3958647E 07 2.3035606E 04 -7.2826600E 02 1.0000000E 00

ROOTS OF $K^*A + B$
 4.3716425E 01 + I 0.0000000E 00 -4.0546057E 01 + I 0.0000000E 00 1.0535790E 00 + I -1.4486115E 02
 1.0535790E 00 + I 1.4486115E 02 7.2298847E 02 + I 0.0000000E 00

TABLE 5
Tabulation of Open and Closed System Roots, Example 1

THE FOLLOWING ROOTS ARE PLOTTED

SIGMA	J-OMEGA
39.918652815	.000000000
-41.162138411	.000000000
-1.730453835	-145.226876122
-1.344047696	.000000000
-21.119976152	-20.963084143
-1.600000000	-159.991999800
-1.344047696	.000000000
-21.119976152	-20.963084143
-1.600000000	-159.991999800
4.049467329	-33.975965742
-30.798402795	.000000000
.875234068	-160.243084341
-34.675444105	.000000000
16.364825726	-42.304285154
3.415896326	-159.742760839
-36.373738324	.000000000
27.830727273	-46.851800735
5.716641889	-158.489445208
-37.352152913	.000000000
39.472012057	-48.733259559
7.482064399	-156.630090123
-37.993534767	.000000000
51.675266342	-47.830315926
8.517001042	-154.456482021
-38.448009292	.000000000
64.519179607	-43.289137257
8.817825039	-152.321167086
-38.787477725	.000000000
77.857764477	-52.945263258
8.566474385	-150.488748933
-39.050948789	.000000000
101.374991629	.000000000
81.558877252	-149.053811987
4.006539954	.000000000
-39.261498996	.000000000
64.244210458	.000000000
7.328554678	-147.984179529
146.091179182	.000000000
-39.433683558	.000000000
58.581242789	.000000000
6.642105422	-147.201636726
179.134229926	.000000000
-39.577150069	.000000000
55.277448021	.000000000
5.997766078	-146.629758476
209.705169891	.000000000
-39.698552007	.000000000
53.041354920	.000000000
5.413330158	-146.208717669
239.066136792	.000000000
-39.802629544	.000000000
51.405674465	.000000000
4.891903727	-145.895365670
267.684143624	.000000000

TABLE 5
Tabulation of Open and Closed System Roots, Example 1. (Contd)

-39.892852699	.000000000
50.148663380	.000000000
4.428883308	-145.659455309
295.792422702	.000000000
-39.971820902	.000000000
49.148532277	.000000000
4.018231617	-145.479883461
323.527825392	.000000000
-40.041520391	.000000000
48.331836564	.000000000
3.653381446	-145.341835921
350.978920935	.000000000
-40.103495241	.000000000
47.651245034	.000000000
3.328213307	-145.234798142
378.206823593	.000000000
-40.158964097	.000000000
47.074705241	.000000000
3.037317836	-145.151210253
405.255623183	.000000000
46.579652970	.000000000
-40.208901728	.000000000
2.776032179	-145.085563619
432.158184400	.000000000
46.149698852	.000000000
-40.254097202	.000000000
2.540364711	-145.033791406
458.939628928	.000000000
45.772627107	.000000000
-40.295196145	.000000000
2.327009277	-144.992853581
485.619550483	.000000000
45.439132638	.000000000
-40.332731949	.000000000
2.133055800	-144.960450973
512.213487711	.000000000
45.141994575	.000000000
-40.367149172	.000000000
1.956108118	-144.934825743
538.733938362	.000000000
44.875518566	.000000000
-40.398821306	.000000000
1.794112605	-144.914620368
565.191077530	.000000000
44.635150405	.000000000
-40.428064441	.000000000
1.645317889	-144.898776716
591.593278258	.000000000
44.417202176	.000000000
-40.455147899	.000000000
1.508224688	-144.886462947
617.947496348	.000000000
44.218654218	.000000000
-40.480302574	.000000000
1.381544378	-144.877019957
644.259559601	.000000000
44.037009376	.000000000
-40.503727559	.000000000
1.264164870	-144.869921713
670.534388442	.000000000

TABLE 6
Equivalent System Open Loop Polynomials and Roots, Example 2.

COEFFICIENTS ARE GIVEN IN ASCENDING ORDER

THE COEFFICIENTS OF POLYNOMIAL A (ORDER = 20)					
-4.0799305E 19	6.1680749E 21	9.5684282E 23	2.2379228E 24	7.1031974E 23	2.3741745E 22
9.5066517E 19	8.7039308E 18	2.3363110E 18	7.5304025E 16	1.9097026E 15	4.1899455E 13
5.5180490E 11	7.7883807E 09	7.1958497E 07	6.3221235E 05	4.4817804E 03	2.2388089E 01
1.2305666E-01	2.8328917E-04	1.1650003E-06			
THE ROOTS OF A					
-1.0698598E-02	+I 0.000000E 00	-5.000000E-01	+I 0.000000E 00	4.0482270E-03	+I 0.000000E 00
-3.000000E 00	+I 0.000000E 00	-1.8661114E 01	+I -1.3795784E 01	-1.8661114E 01	+I 1.3795784E 01
1.7611750E 01	+I 1.5133260E 01	1.7611750E 01	+I -1.5133260E 01	-5.6548983E-01	+I 5.8604508E 01
-5.6548983E-01	+I -5.8604508E 01	-1.1383567E 01	+I 9.9133387E 01	-1.1383567E 01	+I -9.9133387E 01
-4.500000E 01	+I 0.000000E 00	8.7784732E 00	+I -1.0034827E 02	8.7784732E 00	+I 1.0034827E 02
-3.140000E 01	+I -1.8535922E 02	-3.140000E 01	+I 1.8535922E 02	-1.200000E 02	+I 0.000000E 00
-1.7100326E 00	+I -1.6740767E 02	-1.7100326E 00	+I 1.6740767E 02		
THE COEFFICIENTS OF POLYNOMIAL B (ORDER = 24)					
4.4350635E 18	2.7125958E 19	6.4758656E 20	1.6958813E 21	9.5767814E 20	3.3602128E 20
4.7035417E 19	3.1814755E 18	1.2920488E 17	3.9974714E 15	1.0012828E 14	1.9949174E 12
3.4265212E 10	4.7944633E 08	5.9492045E 06	5.8870857E 04	5.5411254E 02	3.8548769E 00
2.8002114E-02	1.3359248E-04	7.3606134E-07	2.2365276E-09	8.7580072E-12	1.3306326E-14
3.0508488E-17					
THE ROOTS OF B					
-4.6149160E-01	+I -2.1047476E-14	-1.2319939E-02	+I -8.5130501E-02	-1.2319939E-02	+I 8.5130501E-02
-1.4701442E 00	+I 2.1868848E 00	-1.4701442E 00	+I -2.1868848E 00	-1.7866417E 01	+I -3.0829112E-14
-1.250000E 01	+I 7.9241888E-14	-4.2279259E-01	+I -5.2021278E 01	-4.2279259E-01	+I 5.2021278E 01
-4.3533614E 01	+I -4.4791865E 01	-4.3533614E 01	+I 4.4791865E 01	-3.0696042E 01	+I 0.000000E 00
-6.2707067E-01	+I -6.0964654E 01	-6.2707067E-01	+I 6.0964654E 01	-1.5717536E 00	+I -1.1108758E 02
-1.5717536E 00	+I 1.1108758E 02	-2.7401987E 00	+I -1.3136153E 02	-2.7401987E 00	+I 1.3136153E 02
-1.9168424E 00	+I -1.5927933E 02	-1.9168424E 00	+I 1.5927933E 02	-3.1369099E 01	+I -1.8532335E 02
-3.1369099E 01	+I 1.8532335E 02	-1.0365000E 02	+I -3.2947940E 02	-1.0365000E 02	+I 3.2947940E 02

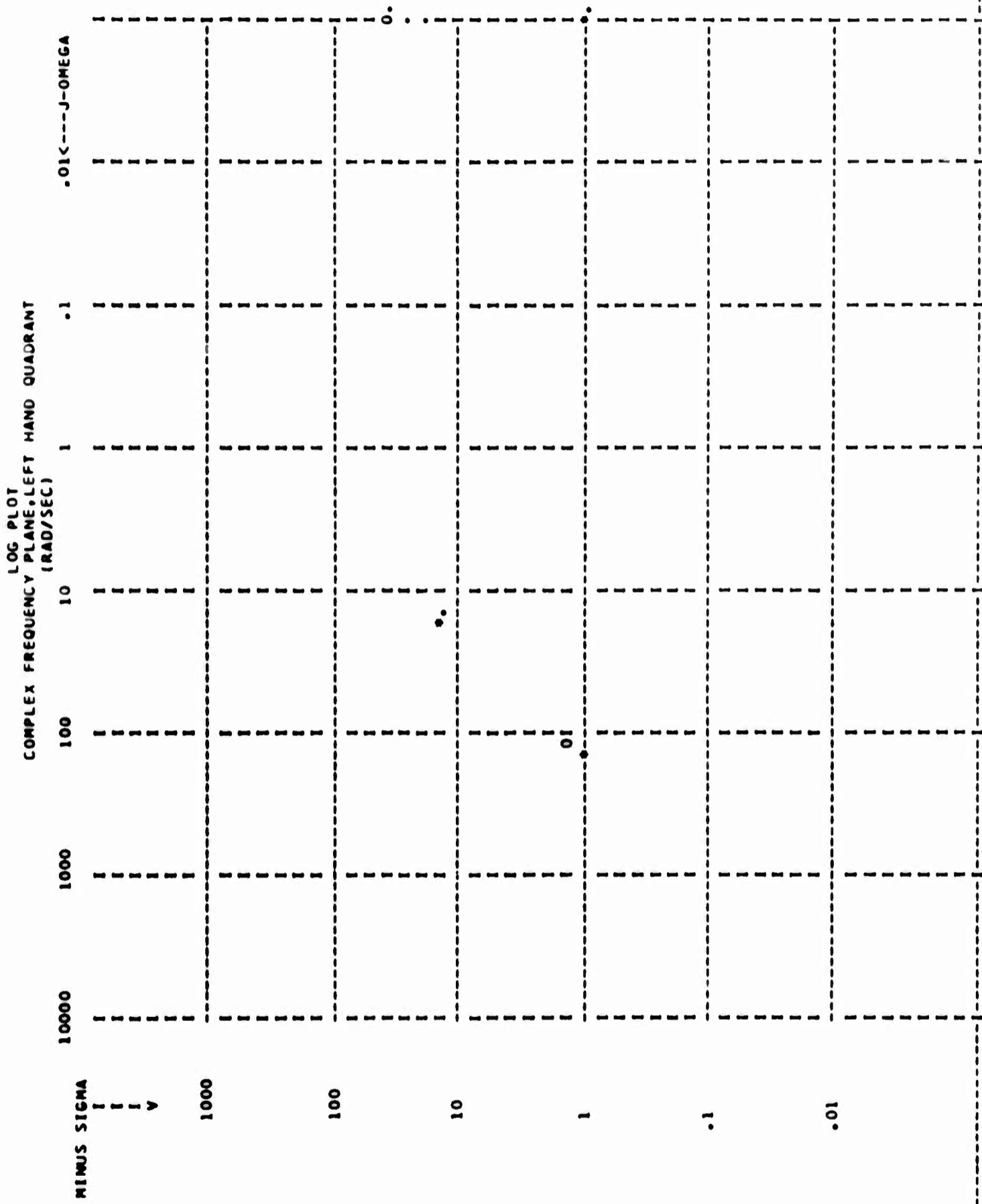


Figure 3. Graphical Display, Log Plot, Left Hand Quadrant, Example 1.

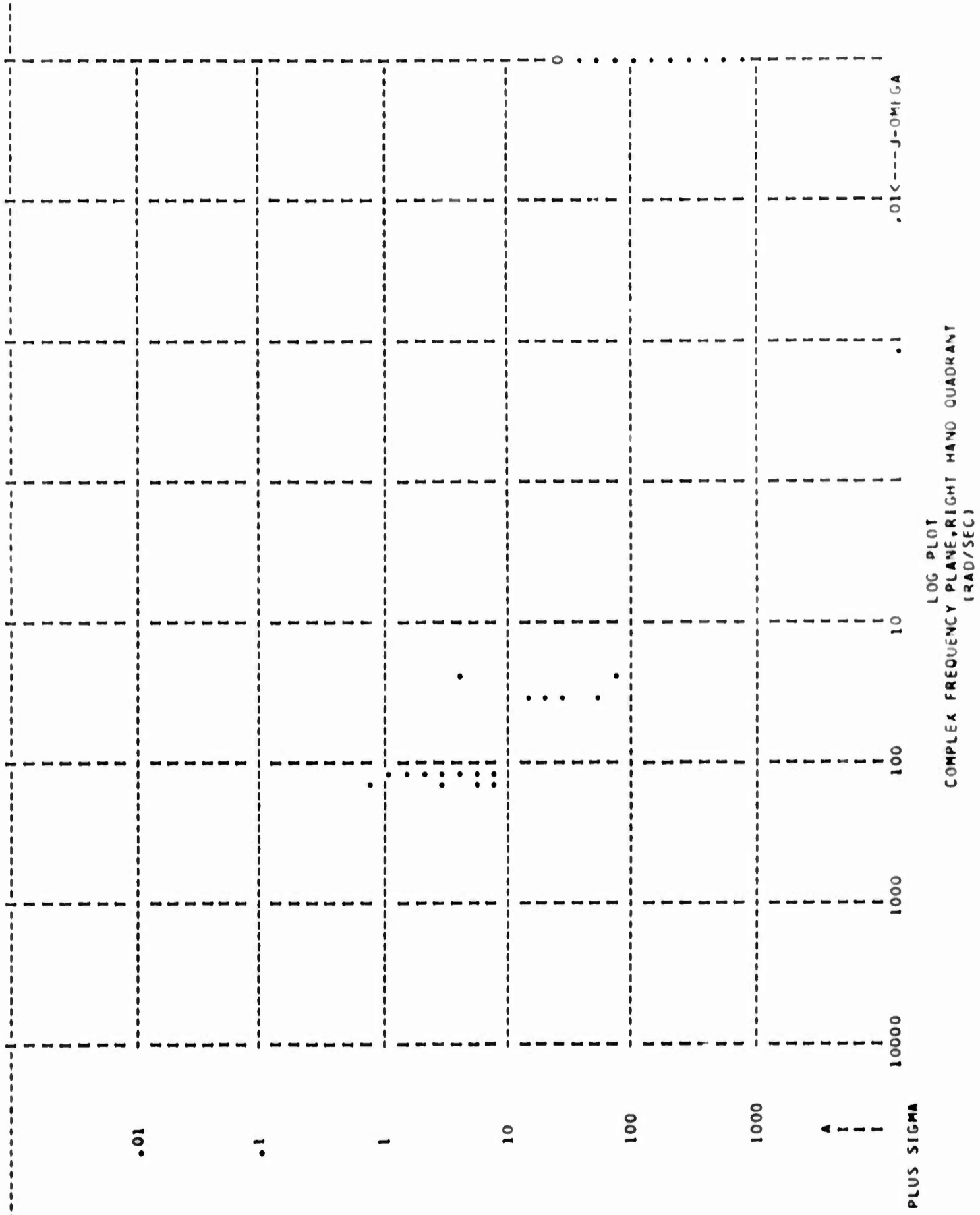


Figure 4. Graphical Display, Log Plot, Right Hand Quadrant, Example 1.

COMPLEX FREQUENCY PLANE, LEFT HAND QUADRANT

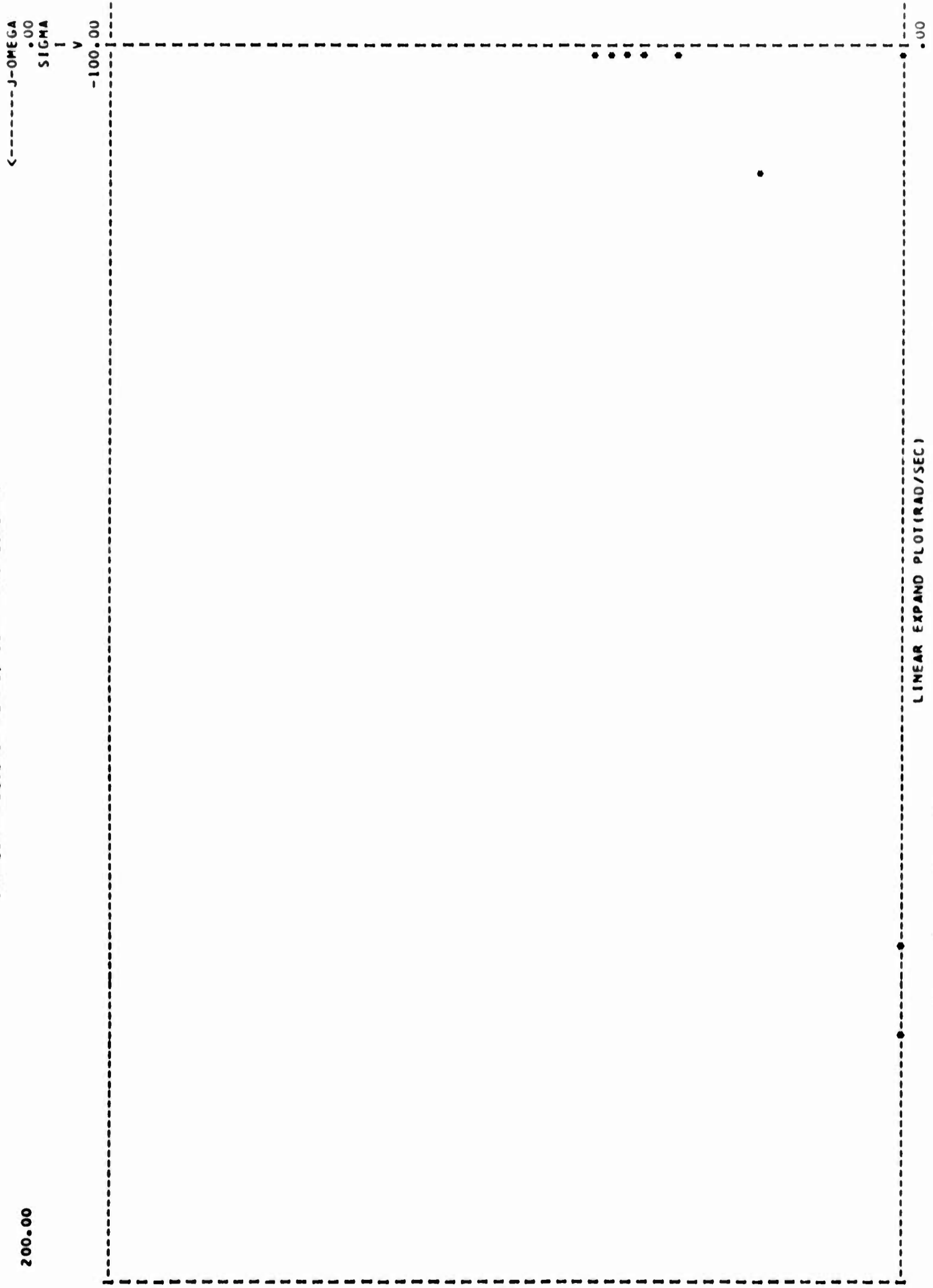


Figure 5. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example 1.

COMPLEX FREQUENCY PLANE, RIGHT HAND QUADRANT

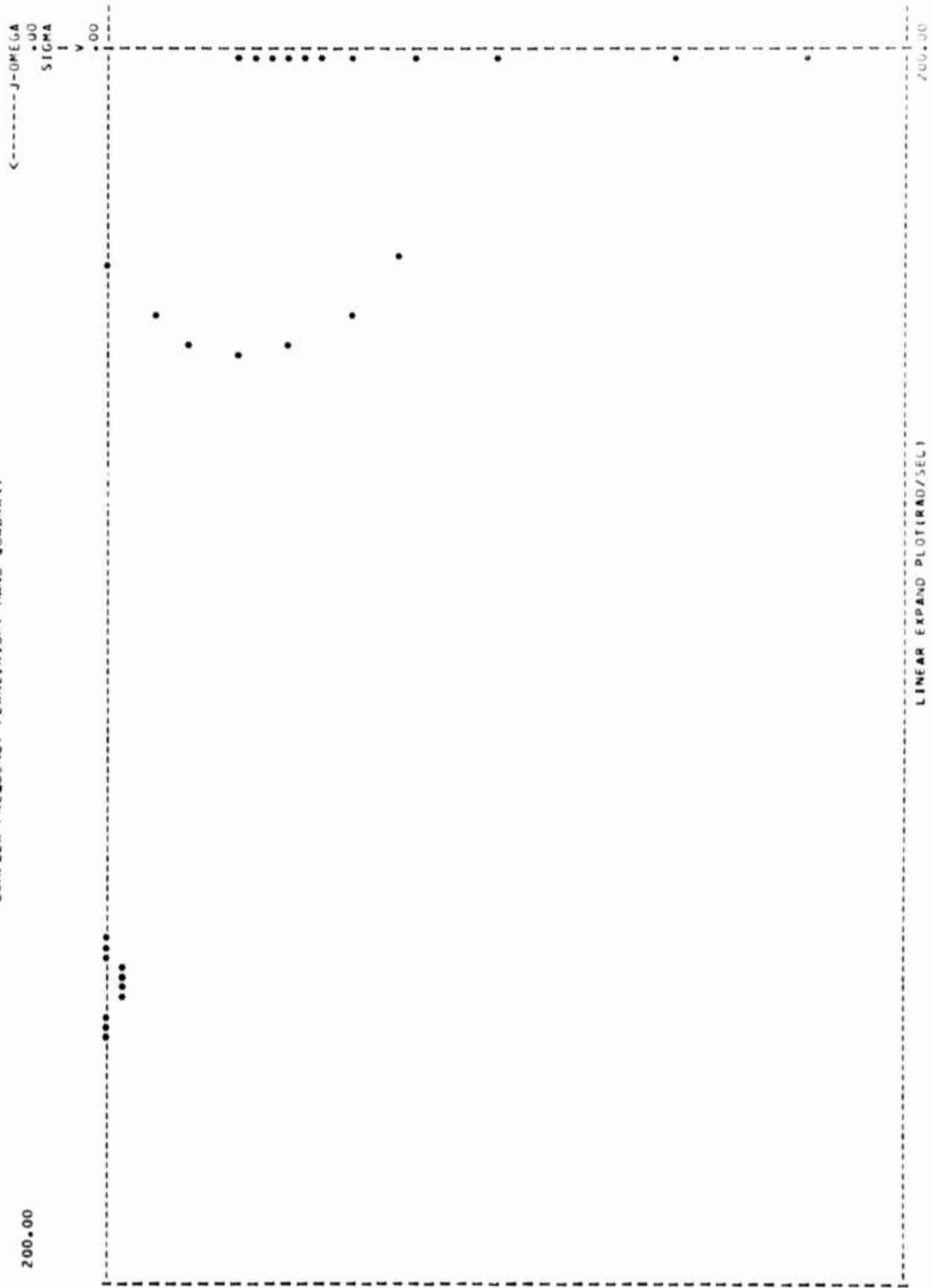


Figure 6. Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example 1.

COMPLEX FREQUENCY PLANE, LEFT HAND QUADRANT

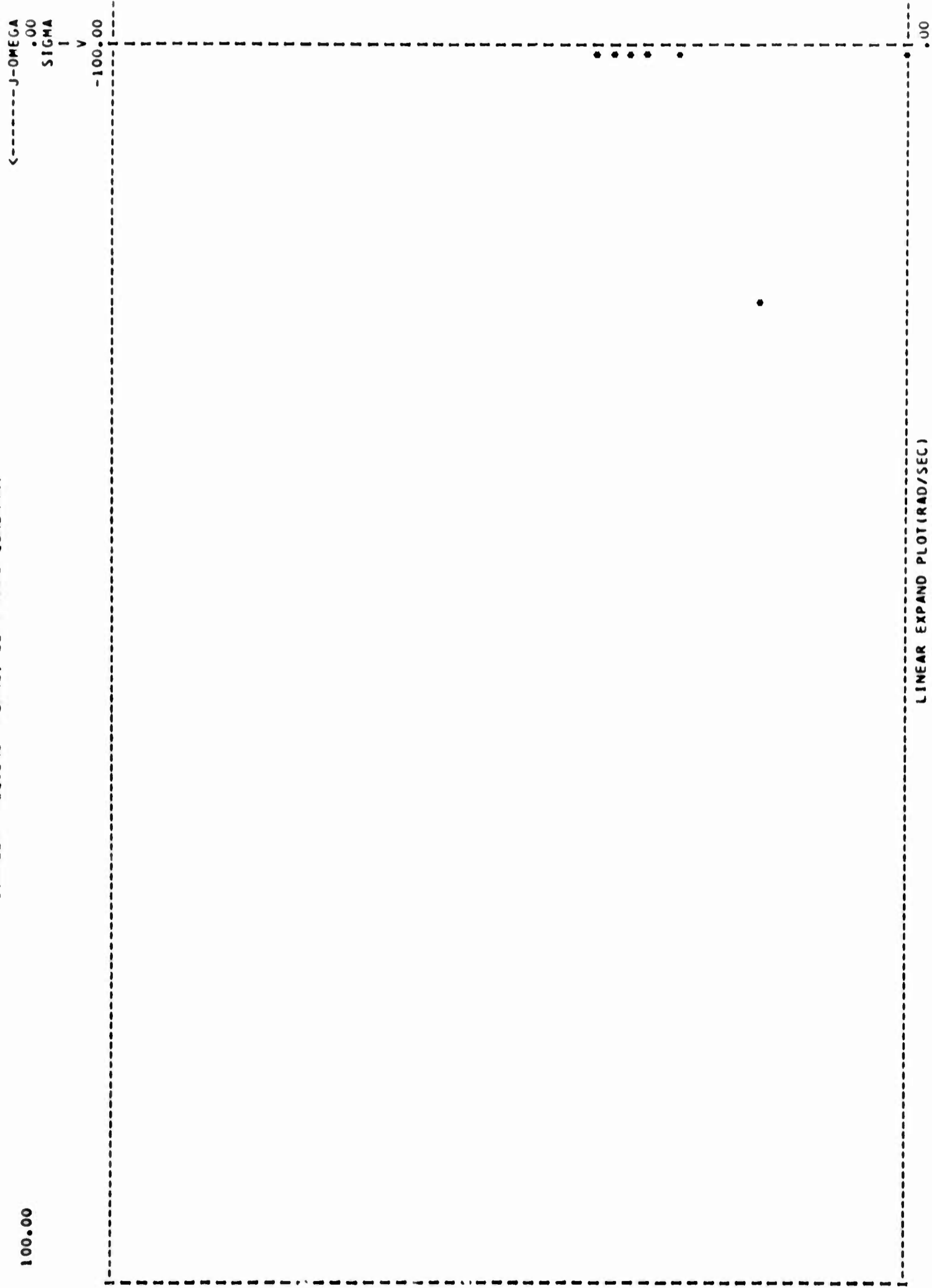
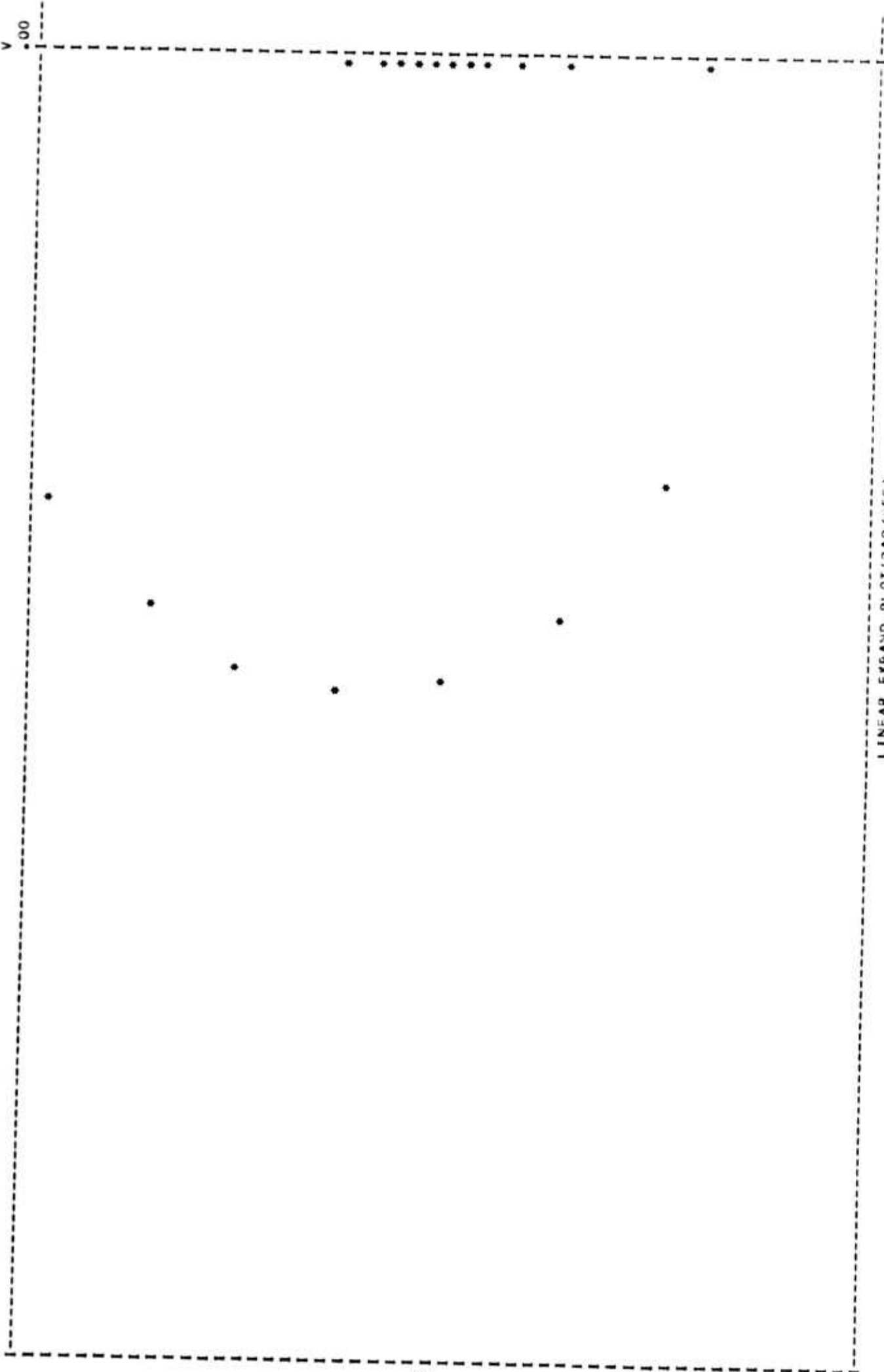


Figure 7. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example 1.

COMPLEX FREQUENCY PLANE, RIGHT HAND QUADRANT

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LINEAR EXPAND PLOT (RAD/SEC)

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Figure 8. Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example 1.

COMPLEX FREQUENCY PLANE, LEFT HAND QUADRANT

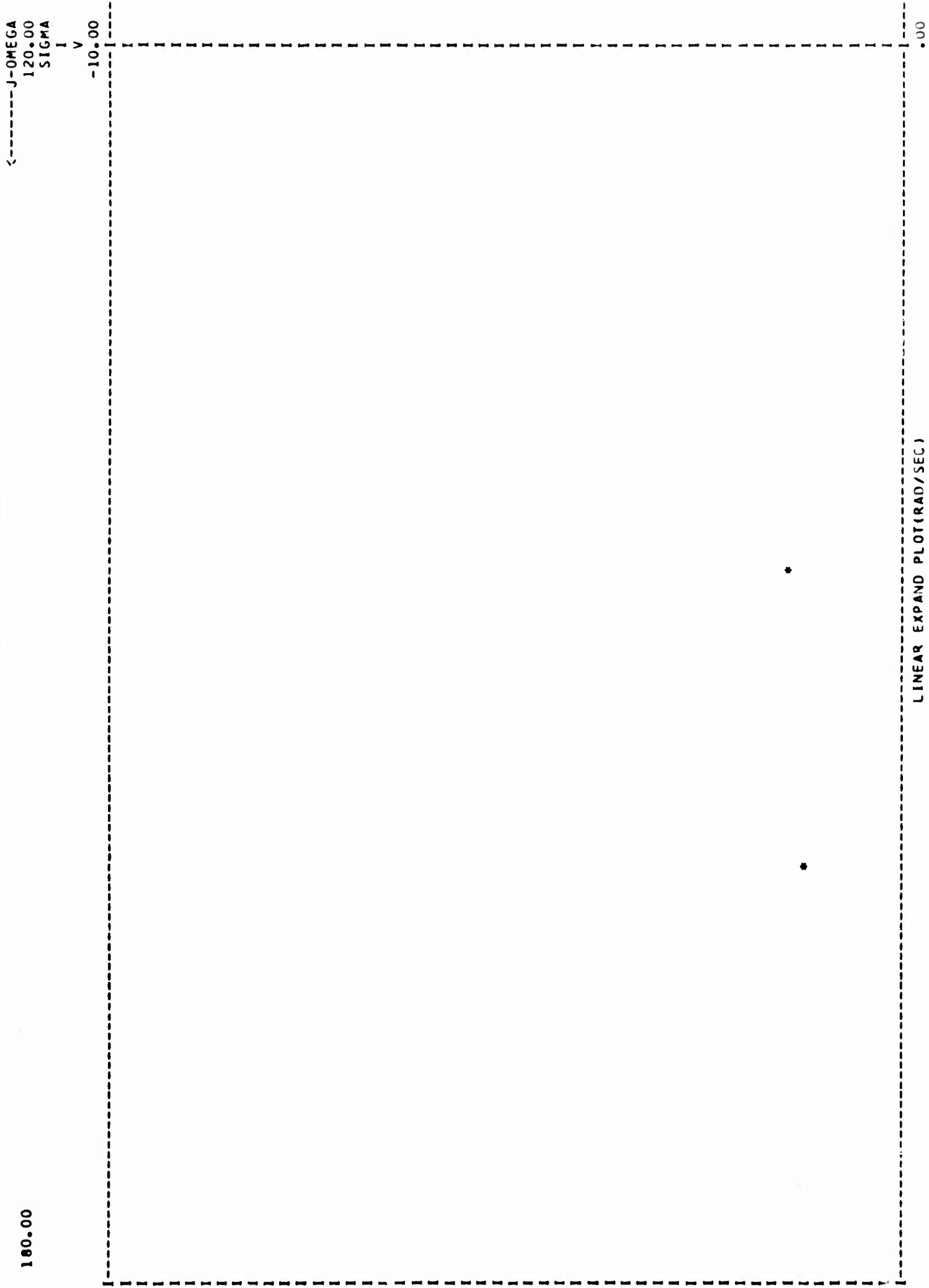


Figure 9. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example 1.

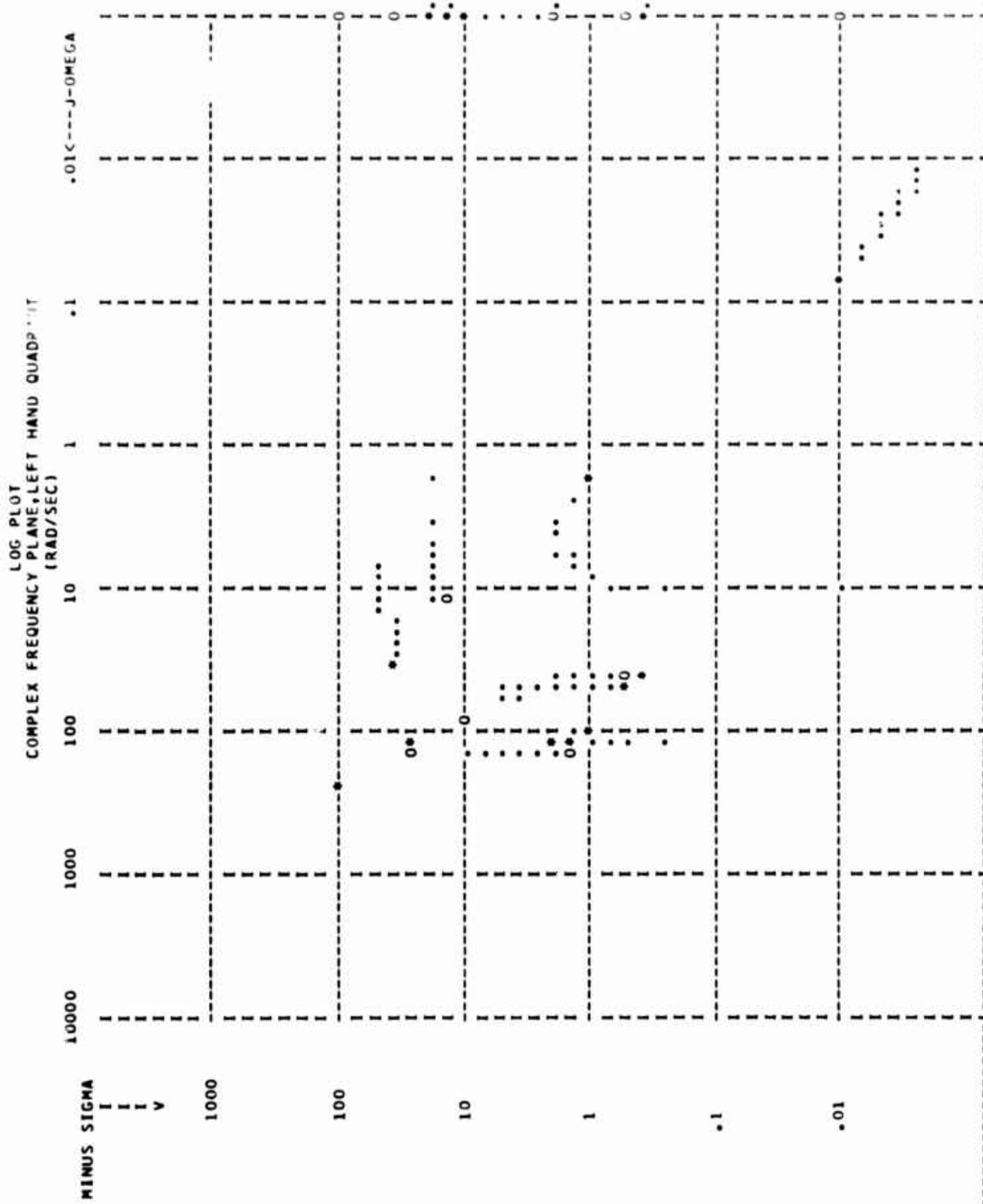


Figure 11. Graphical Display, Log Plot, Left Hand Quadrant, Example 2.

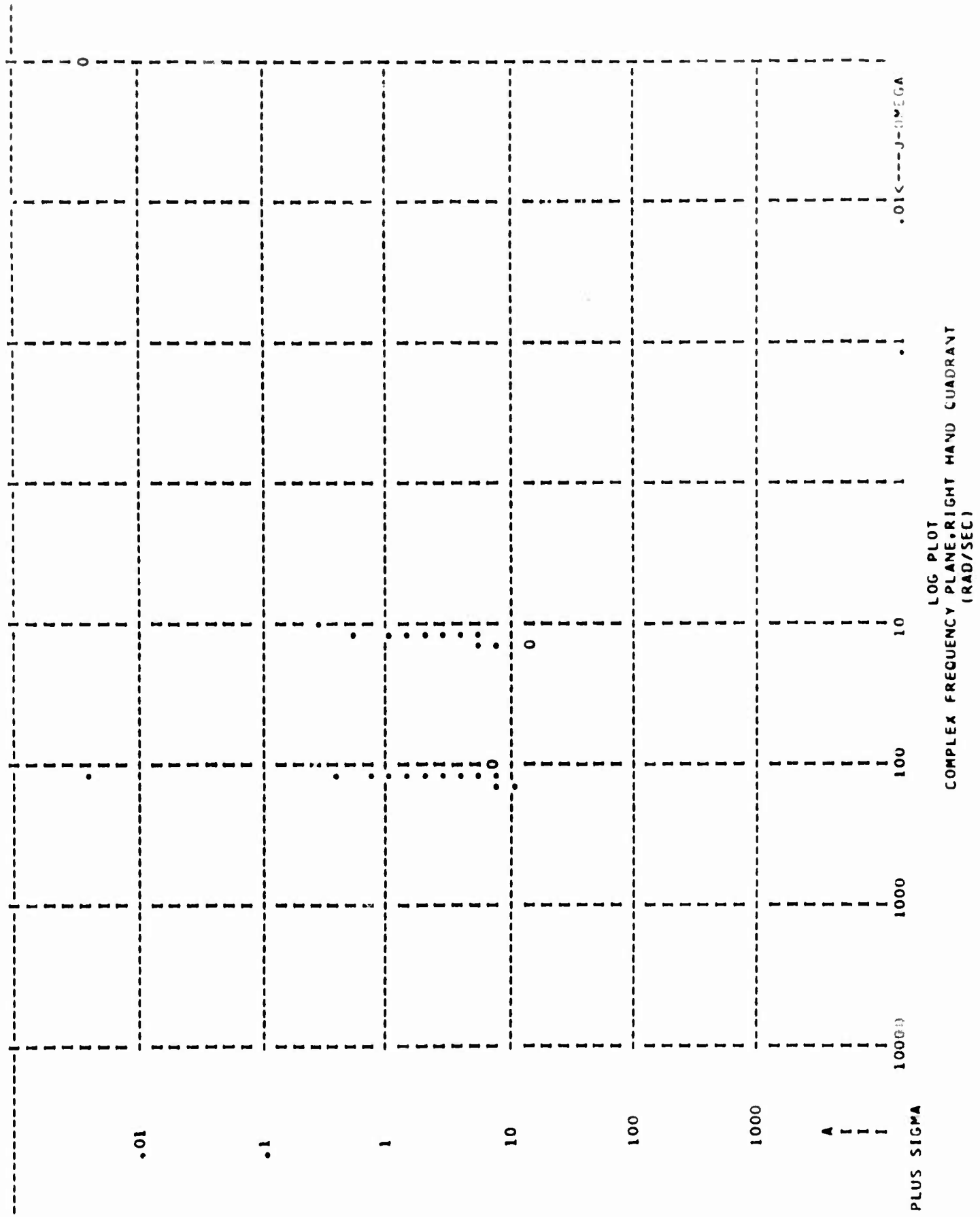


Figure 12. Graphical Display, Log Plot, Right Hand Quadrant, Example 2

COMPLEX FREQUENCY PLANE, LEFT HAND QUADRANT

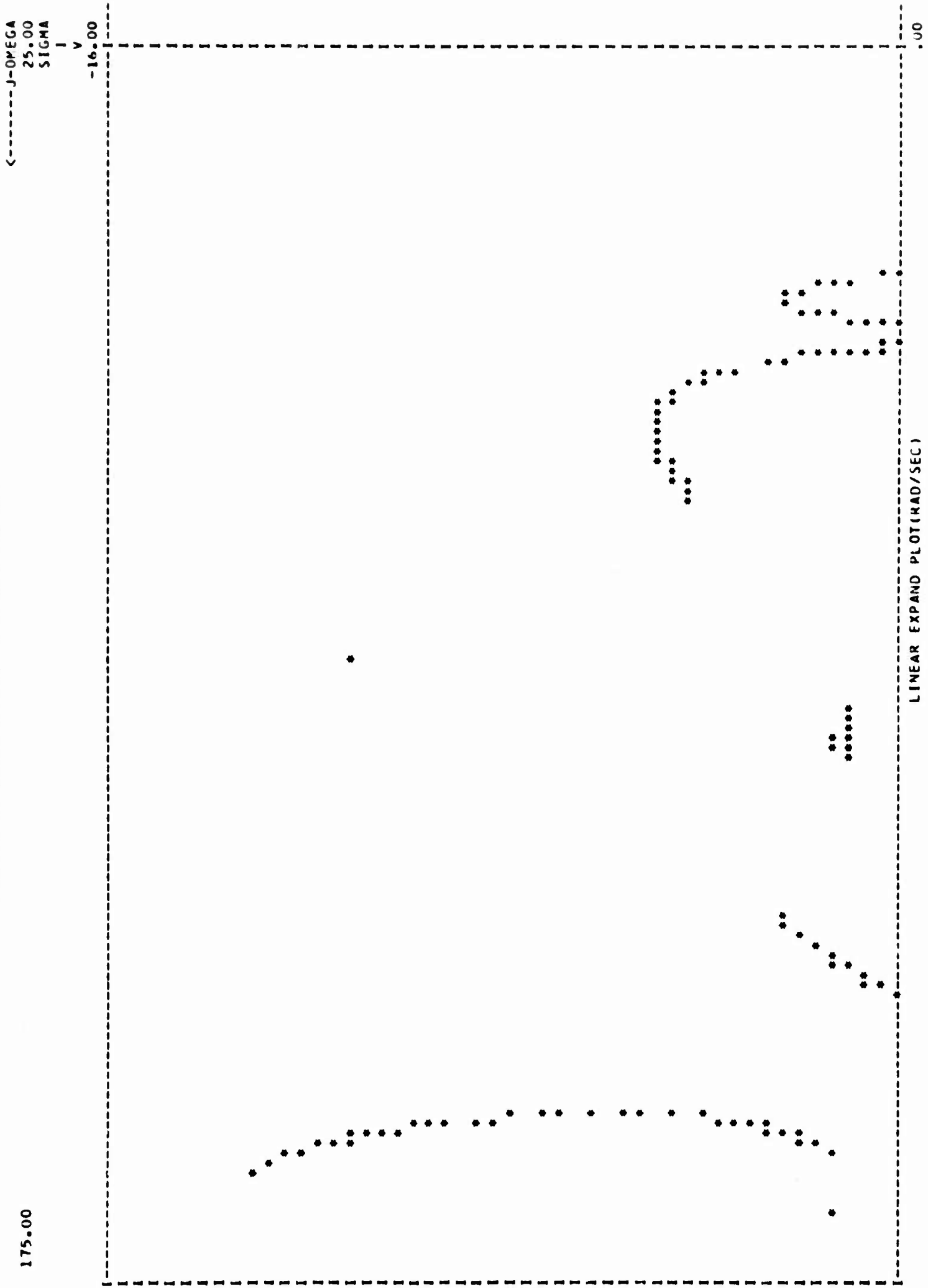
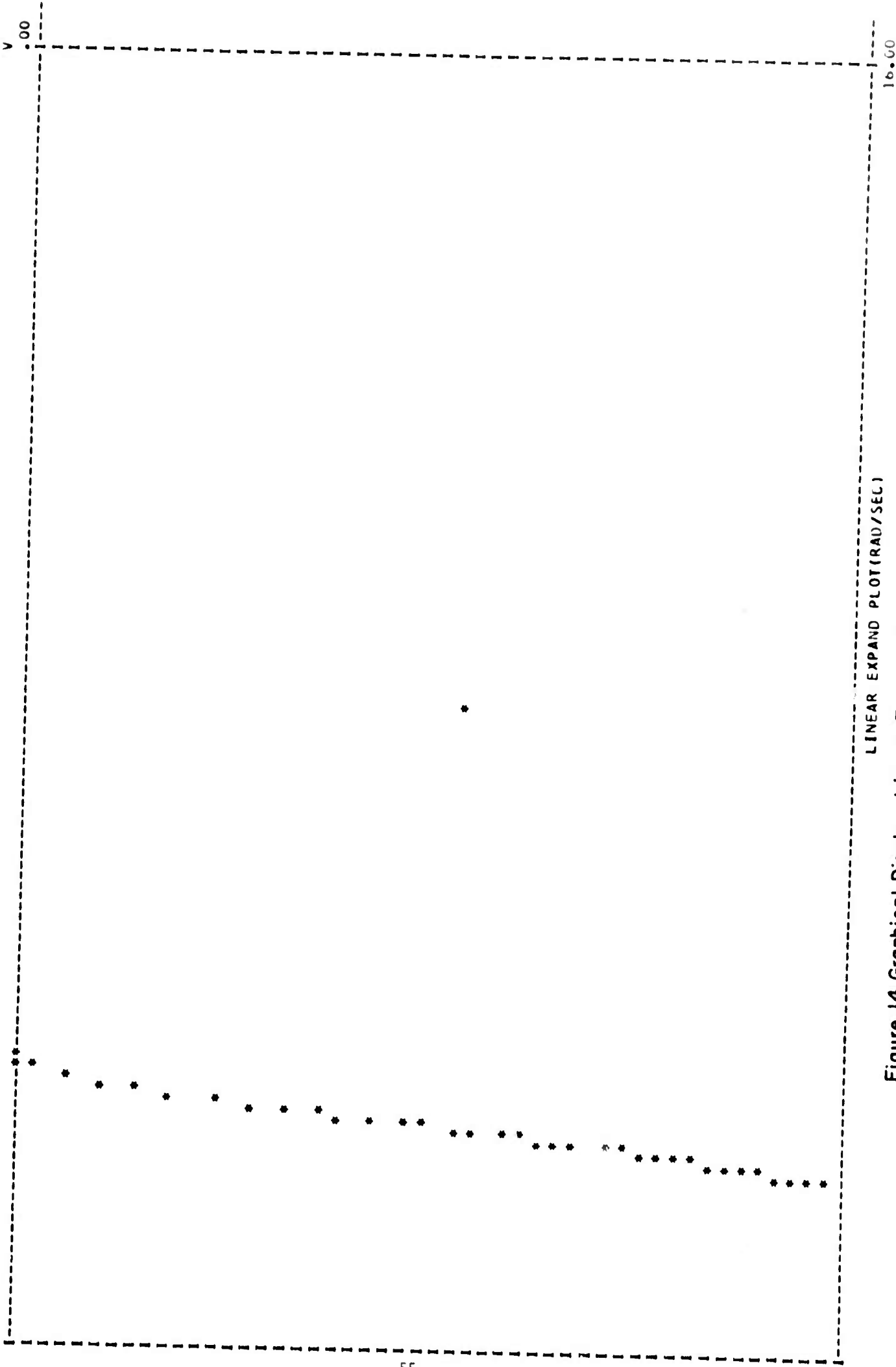


Figure 13. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example 2.

COMPLEX FREQUENCY PLANE, RIGHT HAND QUADRANT

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LINEAR EXPAND PLOT (RAD/SEC)

Figure 14 Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example 2.

VI. CONCLUSIONS

The method described in this memorandum for computing linear system root loci and plotting their trajectories on the complex frequency plane has the following merits.

1. Accepts input in the form of sums of products of polynomials.
2. Produces a log plot of the entire complex plane in two graphical displays.
3. Produces selectable scaled linear plots of regions in the complex frequency plane.
4. Relates the closed loop gain to the graphical display.

REFERENCES

1. Truxai, John G. "Control System Synthesis"; McGraw Hill, N. Y. 1955, pp 221-277.
2. Digital Computer Program "Polynomial Multiplication and Root Locus", Martin Marietta, Baltimore, Maryland.
3. Blakelock, John H. "Automatic Control of Aircraft and Missiles", John Wiley & Sons, Inc., New York, 1965, pp. 306-327.

APPENDIX

LISTING OF SOURCE DECK

CALL RTLOCS
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SUBROUTINE RTLOCS
DIMENSION WLOG(3000),PHILIN(3000),MDBLIN(3000),PHIPLN(3000),
*MPDBLN(3000)
DIMENSION SAVE1(100,100),SAVE2(100,100),XA(2000),XH(2000)
DIMENSION X(100),IGA(100),IGB(100),C(100),D(100),ANS(100),
1SAVE(100),ERA(100),AS(100),PS(100),A(100),B(100),ROOTR(100),
2ROOTI(100),ATK(100),CK(100),CKS(100)
INTEGER COMENT(20)
INTEGER STAR1,STAR2,STAR3,STAR
INTEGER ONE,ZERO,DOT,VEE,AYE
INTEGER POINT(130,100),DASH,BLANK,STAR,Q,SPOT(130,50)
REAL MDBLIN,MPDBLN
DATA ONE/1H1/,ZERO/1H0/,DOT/1H./,AYE/1HA/,VEE/1HV/
DATA BLANK/1H /,DASH/1H-/, Q/1HI/
DATA STAR1/1H0/,STAR2/1H*/,STAR3/1H./
LOGICAL PRNT
EQUIVALENCE (SAVE1(1,1),WLOG(1)),(SAVE2(1,1),PHILIN(1)),
1 (SAVE1(1,31),MDBLIN(1)),(SAVE2(1,31),PHIPLN(1)),
2 (SAVE1(1,61),MPDBLN(1))
EQUIVALENCE (SPOT(1,1),POINT(1,1))
EQUIVALENCE(STAR1,ZERO),(STAR3,DOT)
EQUIVALENCE(JACKIE,KJ,I),(JCANN,KK,J)
COMMON/FREEK/A,B
COMMON /BOB/ POINT
COMMON /INFO4/ Q,BLANK,DASH
COMMON /INFO5/ SAVE1,SAVE2
READ(5,11) COMENT
WRITE(6,13) COMENT
13 FORMAT(//////////,20X,20A4)
11 FORMAT(20A4)
DO277 IOU=1,2000
XA(IOU)=0.0
277 XB(IOU)=0.0
DO 401 KK=1,100
DO 401 KJ=1,130
401 POINT(KJ,KK)=BLANK
DO 402 KK=1,130
POINT(KK,51)=DASH
402 POINT(KK,50)=DASH
DO 403 KK=1,100
DO 403 KJ=34,125,13
403 POINT(KJ,KK)=Q
DO 404 KJ=34,124
DO 404 KK=7, 93,7
IF(KK.EQ.49) KK=58
404 POINT(KJ,KK)=DASH
DO 405 KK=7, 93,7
IF(KK.EQ.49) KK=58
405 POINT(25,KK)=ONE
DO 406 KK=26,28
406 POINT(KK,7)=ZERO
DO 407 KK=26,27
407 POINT(KK,14)=ZERO
POINT(26,21)=ZERO

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	DO 408 KK=35,65,7	61
	IF(KK.EQ.49) KK=58	62
408	POINT(24, KK)=DOT	63
	POINT(25,42)=ZERO	64
	POINT(25,58)=ZERO	65
	POINT(26,42)=ONE	66
	POINT(26,58)=ONE	67
	POINT(26,70)=ZERO	68
	POINT(26,79)=ZERO	69
	POINT(26,86)=ZERO	70
	POINT(27,86)=ZERO	71
	DO 409 KK=26,28	72
409	POINT(KK,93)=ZERO	73
	DO 420 KK=1,100	74
	IF(KK.EQ.4) KK=98	75
420	POINT(26, KK)=Q	76
	POINT(26,4)=VEE	77
	POINT(26,97)=AYE	78
	DO 7681 JACKIE=1,100	79
	DO 7681 JOANN=1,100	80
	SAVE1(JACKIE, JOANN)=0.0	81
7681	SAVE2(JACKIE, JOANN)=0.0	82
	NO=1	83
12	DO 5 I = 1,100	84
	A(I) = 0.0	85
5	B(I) = 0.0	86
	MI=0	87
	JZO=1	88
	K1=1	89
	III=1	90
	READ (5,101) N, IA, IB, IPROB	91
140	FORMAT(10X,7HDELTA=,110,5X,19HPOLY.ADDED IN A(S)=,110,5X,19HPOLY.	92
	*ADDED IN B(S)=,110,5X,9HPROB.NO.=,110)	93
	IF(IB.NE.0) GO TO 10	94
	IF(IA.NE.0) GO TO 10	95
	IF(N.NE.0) GO TO 10	96
	IF(IPROB.EQ.10000) GO TO 1016	97
101	FORMAT (7I10)	98
10	WRITE(6,1)	99
	FORK1=0.0	100
	FORK2 = 0.0	101
	FORK3 = 0.0	102
	WRITE (6 ,102)IPROB	103
102	FORMAT (1H1, 9X,40HPOLYNOMIAL MULTIPLICATION AND ROOT LOCUS,44X,11	104
	1HPROBLEM NO.,15//)	105
	IF (N) 21, 20, 21	106
20	READ(5,103)Y,DY,YT	107
	WRITE(6,140)N, IA, IB, IPROB	108
	WRITE(6,141)Y,DY,YT	109
141	FORMAT(1H0,5X,10HK-INITIAL=F20.10,5X,12HINCREMENT K=,F20.10,5X,12H	110
	*K-TERMINATE=,F20.10)	111
103	FORMAT (4F10.0)	112
	GO TO 25	113
21	READ(5,104)(X(I),I=1,N)	114
	WRITE(6,142) (I,X(I),I=1,N)	115
104	FORMAT(7E10.0)	116
25	READ (5 ,105)(IGA (I), I = 1, IA)	117
	WRITE(6,143) (I,IGA(I),I=1,IA)	118
143	FORMAT(10X,24HNUMBER OF POLY. IN GROUP, 13,14H OF NUMERATOR=,110)	119
142	FORMAT(10X,2HX(,13,2H)=,F20.10)	120

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	READ (5,105)(IGB(J), J=1, IB)	121
	WRITE(6,144) (J,IGB(J),J=1,IB)	122
	144 FORMAT(10X,24HNUMBER OF POLY. IN GROUP, 13,16H OF DENOMINATOR=,110	123
	*)	124
105	FORMAT(7I10)	125
300	DO 15 I = 1,100	126
15	SAVE(I) = 0.0	127
	ISDEG = 0	128
	JJ = 1	129
200	READ (5,106)MDEG,(C(I), I=1, 6)	130
	IF(MDEG.GT.6) GO TO 48	131
	WRITE(6,145) (I,C(I),I=1,MDEG)	132
	IF (MDEG - 6) 49, 49, 48	133
48	READ (5,104)(C(I), I = 7, MDEG)	134
	WRITE(6,145) (I,C(I),I=1,MDEG)	135
106	FORMAT(110,6E10.0)	136
49	IF (IGA(JJ)-1) 50, 51, 50	137
51	DO 56 I= 1, MDEG	138
56	ANS(I) = C(I)	139
	IADEG = MDEG	140
	IF (ISDEG - MDEG) 52, 52, 53	141
53	INDEG = ISDEG	142
	GO TO 68	143
52	INDEG = MDEG	144
	GO TO 68	145
50	READ (5,106)NDEG, (D(I), I =1, 6)	146
	IF(NDEG.GT.6) GO TO 54	147
	WRITE(6,145) (I,D(I),I=1,NDEG)	148
	IF (NDEG - 6) 55, 55, 54	149
54	READ (5,104)(D(I), I=7, NDEG)	150
	WRITE(6,145) (I,D(I),I=1,NDEG)	151
55	IADEG = NDEG + MDEG -1	152
	CALL POLMPY (C,MDEG,D,NDEG,ANS)	153
	IGA (JJ) = IGA(JJ) -1	154
	IF (IGA(JJ) -1) 65, 65, 64	155
64	DO 60 I = 1, IADEG	156
60	C(I) = ANS(I)	157
	MDEG =IADEG	158
	GO TO 50	159
65	IF (ISDEG - IADEG) 66,66,67	160
66	INDEG = IADEG	161
	GO TO 68	162
67	INDEG = ISDEG	163
68	CALL POLADD (SAVE,ISDEG,ANS,IADEG,ERA)	164
145	FORMAT(10X,2HC(, 13,2H)=,F20.10)	165
	WRITE(6,1)	166
1	FORMAT(1H1//)	167
6800	IF (ERA(INDEG))6803, 6802, 6803	168
6802	INDEG = INDEG - 1	169
	IF (INDEG) 6801, 6801, 6800	170
6801	INDEG = 1	171
6803	JJ = JJ + 1	172
58764	DO 70 I = 1, INDEG	173
60	SAVE(I) = ERA(I)	174
	ISDEG = INDEG	175
	IA = IA -1	176
	IF (IA) 201, 201, 200	177
201	IF (FORK1) 202, 202, 203	178
C	SAVE NUMERATOR.	179
202	DO 220 I = 1, ISDEG	180

220	A(I) = SAVE(I)	181
	IDA = ISDEG	182
	IA = IB	183
	FORK1 = 1.0	184
	DO 230 I=1, IA	185
230	IGA(I) = IGB(I)	186
C	START DENOMINATOR	187
	GO TO 300	188
C	SAVE DENOM.	189
203	DO 240 I = 1, ISDEG	190
240	H(I) = SAVE(I)	191
	IDB = ISDEG	192
	WRITE (6,109)	193
109	FORMAT (10X,41HCOEFFICIENTS ARE GIVEN IN ASCENDING ORDER/////)	194
339	IF (A(IDA)) 340, 341, 340	195
341	IDA = IDA - 1	196
	IF (IDA) 345, 345, 339	197
345	WRITE (6,120)	198
120	FORMAT (1H0,10X,20HPOLYNOMIAL A IS ZERO//)	199
	FORK2 = 1.0	200
	GO TO 410	201
340	IF (IDA - 2) 346, 347, 335	202
346	WRITE(6,121)A(I)	203
	STAR=STAR1	204
	PRNT=.TRUE.	205
	ANUMB1=A(I)	206
	ANUMB2=0.0	207
	CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III	208
	*,NO)	209
121	FORMAT (1H0,10X,28HPOLYNOMIAL A IS A CONSTANT =,1P1E16.7//)	210
	GO TO 410	211
347	ROOT = - A(1) / A(2)	212
	WRITE (6,133)A(1), A(2)	213
133	FORMAT (10X,21HTHE COEFFICIENTS OF A/1P2E20.7)	214
	WRITE (6,122)ROOT	215
	STAR=STAR1	216
	PRNT=.TRUE.	217
	ANUMB1=ROOT	218
	ANUMB2=0.0	219
	CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III	220
	*,NO)	221
122	FORMAT (1H0,10X,23HROOT OF POLYNOMIAL A IS,1P1E16.7//)	222
	GO TO 410	223
C	WRITE POLYS	224
335	ID1A = IDA - 1	225
	WRITE (6,107)ID1A,(A(I),I=1,IDA)	226
	K = IDA	227
	DO 800 I = 1, IDA	228
	AS(I) = A(K)	229
800	K = K-1	230
	IDP2A=IDA *2	231
	ID2A= 2 *ID1A	232
	CALL MULLER (AS, ID1A,ROOTR,ROOTI)	233
	DO 805 I = 1, ID1A	234
	SAM = 100. * AMAX1(ABS(ROOTR(I)),ABS(ROOTI(I)))	235
	IF (SAM + ABS(ROOTR(I)).EQ. SAM) ROOTR(I)= 0.0	236
	IF (SAM + ABS(ROOTI(I)).EQ. SAM) ROOTI(I)= 0.0	237
805	CONTINUE	238
400	WRITE (6,111) (RGOTR(I),ROOTI(I),I=1,ID1A)	239
	CALL ERCHEK(ROOTI, ID1A)	240

	PRNT=.TRUE.	241
	STAR=STAR1	242
	DO 2 III=1, ID1A	243
	ANUMB1=ROOTR(III)	244
	ANUMB2=ABS(ROOTI(III))	245
	CALL PLOTTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III	246
	*,NO)	247
	2 CONTINUE	248
410	IF(B(IDB)) 411, 412, 411	249
412	IDB = IDB - 1	250
	IF (IDB) 445, 445, 410	251
445	WRITE (6 ,123)	252
123	FORMAT (1H0,10X,20HPOLYNOMIAL B IS ZERO//)	253
	IF (FORK2)12,450,12	254
450	FORK3 = 1.0	255
	GO TO 698	256
411	IF (IDB - 2) 451, 452, 499	257
451	WRITE (6 ,124)B(IDB)	258
	STAR=STAR2	259
	PRNT=.TRUE.	260
	ANUMB1=B(IDB)	261
	ANUMB2=0.0	262
	CALL PLOTTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XR,MI,PRNT,STAR,III	263
	*,NO)	264
	PRNT=.FALSE.	265
	STAR=STAR3	266
124	FORMAT (1H0,10X,28HPOLYNOMIAL B IS A CONSTANT =,1P1E16.7//)	267
	GO TO 698	268
452	ROOT = -B(1) / B(2)	269
	WRITE (6 ,134)B(1), B(2)	270
134	FORMAT (10X,21HTHE COEFFICIENTS OF B/1P2E20.7)	271
	WRITE (6 ,125)ROOT	272
	STAR=STAR2	273
	PRNT=.TRUE.	274
	ANUMB1=ROOT	275
	ANUMB2=0.0	276
	CALL PLOTTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III	277
	*,NO)	278
	PRNT=.FALSE.	279
	STAR=STAR3	280
125	FORMAT (1H0,10X,23HROOT OF POLYNOMIAL B IS,1P1E16.7//)	281
	GO TO 698	282
107	FORMAT (10X,42HTHE COEFFICIENTS OF POLYNOMIAL A (ORDER = 13,1H)/ (283
	11P6E20.7))	284
499	ID1B = IDB -1	285
	WRITE (6,108)ID1B,(B(I),I=1,IDB)	286
108	FORMAT (////10X,42HTHE COEFFICIENTS OF POLYNOMIAL B (ORDER = 13,1H	287
	1)/ (1P6E20.7))	288
	K = IDB	289
	DO 801 I = 1, IDB	290
	BS(I) = B(K)	291
801	K = K-1	292
	IDP2B= IDB * 2	293
	ID2B = 2 * ID1B	294
	CALL MULLER (BS, ID1B,ROOTR,ROOTI)	295
	DO 806 I = 1, ID1B	296
	SAM = 100. * AMAX1(ABS(ROOTR(I)),ABS(ROOTI(I)))	297
	IF (SAM + ABS(ROOTR(I)).EQ. SAM) ROOTR(I)= 0.0	298
	IF (SAM + ABS(ROOTI(I)).EQ. SAM) ROOTI(I)= 0.0	299
806	CONTINUE	300

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500	WRITE (6,112)(ROOTR(I),ROOTI(I),I= 1, ID1B)	301
	CALL ERCHEK(ROOTI, ID1B)	302
	STAR=STAR2	303
	DO 3 III=1, ID1B	304
	ANUMB1=ROOTR(III)	305
	ANUMB2=ABS(ROOTI(III))	306
	CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III	307
	*,NO)	308
3	CONTINUE	309
	PRNT=.FALSE.	310
	STAR=STAR3	311
111	FORMAT (1H0,11X,14HTHE ROOTS OF A/ (1PE20.7,6H +I ,1PE14.7,1P1	312
	1E20.7,6H +I ,1PE14.7,1PE20.7,6H +I ,1PE14.7))	313
112	FORMAT (1H0,11X,14HTHE ROOTS OF B/ (1PE20.7,6H +I ,1PE14.7,1P1	314
	1E20.7,6H +I ,1PE14.7,1PE20.7,6H +I ,1PE14.7))	315
698	IF (FORK2)12,699,12	316
699	IF (FORK3)12,699,12	317
6991	WRITE (6 ,102)IPROB	318
	MSHEET = 5	319
C	START K CALCULATIONS	320
	IF (N) 702,702,533	321
533	DO 550 I= 1, N	322
	DO 541 J= 1, IDA	323
541	ATK(J) = X(I) * A(J)	324
C	COMPUTE ROOTS OF K * A + B	325
	IDC= MAX0(IDA, IDB)	326
	CALL POLADD (ATK,IDA,B, IDB,CK)	327
	IDS = IDC	328
554	IF (CK(IDS))555, 557, 555	329
557	IDS = IDS - 1	330
	IF (IDS) 558,558, 554	331
558	WRITE (6 ,129)X(I)	332
129	FORMAT (1H0,10X,35HPOLYNOMIAL K*A + B IS ZERO FOR K =,1PE16.7//)	333
	GO TO 550	334
555	IF (IDS - 2) 559, 560, 561	335
559	WRITE (6 ,130)CK(IDS), X(I)	336
130	FORMAT (1H0,10X,35HPOLYNOMIAL K*A + B IS A CONSTANT = ,1PE15.7,10	337
	1H FOR K = ,1PE14.7//)	338
	GO TO 550	339
560	ROOT = -CK(1) / CK(2)	340
	WRITE (6 ,131)ROOT, X(I)	341
131	FORMAT (1H0,10X,18HROOT OF K*A + B = ,1PE15.7,10H FOR K = ,1PE1	342
	14.7//)	343
	GO TO 550	344
561	K = IDS	345
	DO 803 J = 1,IDS	346
	CKS(J) = CK(K)	347
803	K = K - 1	348
	ID1C = IDS - 1	349
	IDP2C = IDS * 2	350
	ID2C = 2 * ID1C	351
	CALL MULLER (CKS,ID1C,ROOTR,ROOTI)	352
	DO 807 J = 1, ID1C	353
	SAM = 100. * AMAX1(ABS(ROOTR(J)),ABS(ROOTI(J)))	354
	IF (SAM + ABS(ROOTR(J)).EQ.SAM) ROOTR(J) = 0.0	355
	IF (SAM + ABS(ROOTI(J)).EQ.SAM) ROOTI(J) = 0.0	356
807	CONTINUE	357
	WRITE (6,808)ID1C,X(I),(CK(J),J=1,IDS)	358
808	FORMAT (///10X,48HTHE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =	359
	113,7H) K = 1PE16.7/(1P6E20.7)	360

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545	WRITE (6,115)(ROOTR(J),ROOTI(J),J=1,IDIC)	361
	CALL ERCHEK(ROOTI,IDIC)	362
	CALL SAVER(ROOTR,ROOTI,IDIC,SAVE1,SAVE2,JZO,K1)	363
115	FORMAT (1H0,9X,16HROOTS OF K*A + B/(1PIE20.7,6H + I ,1PIE14.7,1PI	364
	1E20.7,6H + I ,1PIE14.7,1PIE20.7,6H + I ,1PIE14.7))	365
5452	MSHEET = MSHEET - 1	366
	IF (MSHEET) 546, 546, 550	367
546	WRITE (6 ,102)IPROR	368
	MSHEET = 5	369
550	CONTINUE	370
	GO TO 12	371
702	DO 705 J = 1, IDA	372
705	ATK(J) = Y * A(J)	373
C	COMPUTE ROOTS OF K * A + B	374
	IDC= MAX0(IDA, IDB)	375
	CALL POLADD (ATK,IDA,B,IDB,CK)	376
	IDS = IDC	377
754	IF (CK(IDS))755, 757, 755	378
757	IDS = IDS - 1	379
	IF (IDS) 758, 758, 754	380
758	WRITE (6 ,129)Y	381
	GO TO 711	382
755	IF (IDS - 2) 759, 760, 761	383
759	WRITE (6 ,130)CK(IDS), Y	384
	GO TO 711	385
760	ROOT = -CK(1) / CK(2)	386
	WRITE (6 ,131)ROOT, Y	387
	GO TO 711	388
761	K = IDS	389
	DO 804 I = 1,IDS	390
	CKS(I) = CK(K)	391
804	K = K - 1	392
	IDIC = IDS - 1	393
	IDP2C = IDS * 2	394
	ID2C = 2 * IDIC	395
	CALL MULLER (CKS,IDIC,ROOTR,ROOTI)	396
	DO 809 I = 1, IDIC	397
	SAM = 100. * AMAX1(ABS(ROOTR(I)),ABS(ROOTI(I)))	398
	IF (SAM + ABS(ROOTR(I)).EQ. SAM) ROOTR(I)= 0.0	399
	IF (SAM + ABS(ROOTI(I)).EQ. SAM) ROOTI(I)= 0.0	400
809	CONTINUE	401
	WRITE (6,808)IDIC,Y,(CK(I),I=1,IDS)	402
	WRITE (6,115) (ROOTR(J),ROOTI(J),J=1,IDIC)	403
	CALL ERCHEK(ROOTI,IDIC)	404
	CALL SAVER(ROOTR,ROOTI,IDIC,SAVE1,SAVE2,JZO,K1)	405
711	Y = Y + DY	406
	IF (Y - YT) 712,712,12	407
712	MSHEET = MSHEET - 1	408
	IF (MSHEET) 713, 713, 702	409
713	WRITE (6 ,102)IPROB	410
	MSHEET = 5	411
	GO TO 702	412
58768	016 CALL PLOTTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XP,MI,PRNT,STAR,III	413
	*,NO)	414
	READ(5,22)L	415
22	FORMAT(I10)	416
	IF(L.EQ.0) GO TO 1017	417
	CALL EXPAND(L,XA,XB)	418
1017	RETURN	419
	END	420

C		421
C		422
C		423
C		424
	SUBROUTINE POLMPY (A,N,B,M,C)	425
	DIMENSION A(1),B(1),C(1)	426
	K = M+N	427
	DO 5 I=1,K	428
5	C(I) = 0.0	429
	DO 10 I=1,N	430
	L = I-1	431
	DO 10 J=1,M	432
	L = L+1	433
10	C(L) = C(L)+A(I)*B(J)	434
	RETURN	435
	END	436
C		437
C		438
C		439
C		440
	SUBROUTINE POLADD (A,N,B,M,C)	441
	DIMENSION A(1),B(1),C(1)	442
	IF (N-M) 1,1,2	443
1	NK = N	444
	GO TO 5	445
2	NK = M	446
5	DO 10 I=1,NK	447
10	C(I) = A(I)+B(I)	448
	NK = NK+1	449
	IF (N-M) 11,25,15	450
11	DO 20 I=NK,M	451
20	C(I) = B(I)	452
25	RETURN	453
15	DO 30 I=NK,N	454
30	C(I) = A(I)	455
	RETURN	456
	END	457
C		458
C		459
C		460
C		461
	SUBROUTINE MULLER(COE,N1,ROOTR,ROOTI)	HPRS 462
	DIMENSION COE(1),ROOTR(1),ROOTI(1)	463
	N2=N1+1	HPRS 464
	N4=0	HPRS 465
	I=N1+1	HPRS 466
19	IF(COE(I))9,7,9	HPRS 467
7	N4=N4+1	HPRS 468
	ROOTR(N4)=0.	HPRS 469
	ROOTI(N4)=0.	HPRS 470
	I=I-1	HPRS 471
	IF(N4-N1)19,37,19	HPRS 472
9	CONTINUE	HPRS 473
10	AXR=0.8	HPRS 474
	AXI=0.	HPRS 475
	L=1	HPRS 476
	N3=1	HPRS 477
	ALPIR=AXR	HPRS 478
	ALPII=AXI	HPRS 479
	M=1	HPRS 480

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	GOTO99	HPRS 481
11	BET1R=TEHR	HPRS 482
	BET1I=TEMI	HPRS 483
	AXR=0.85	HPRS 484
	ALP2R=AXR	HPRS 485
	ALP2I=AXI	HPRS 486
	M=2	HPRS 487
	GOTO99	HPRS 488
12	BET2R=TEHR	HPRS 489
	BET2I=TEMI	HPRS 490
	AXR=0.9	HPRS 491
	ALP3R=AXR	HPRS 492
	ALP3I=AXI	HPRS 493
	M=3	HPRS 494
	GOTO99	HPRS 495
13	BET3R=TEHR	HPRS 496
	BET3I=TEMI	HPRS 497
14	TE1=ALP1R-ALP3R	HPRS 498
	TE2=ALP1I-ALP3I	HPRS 499
	TE5=ALP3R-ALP2R	HPRS 500
	TE6=ALP3I-ALP2I	HPRS 501
	TEM=TE5*TE5+TE6*TE6	HPRS 502
	TE3=(TE1*TE5+TE2*TE6)/TEM	HPRS 503
	TE4=(TE2*TE5-TE1*TE6)/TEM	HPRS 504
	TE7=TE3+1.	HPRS 505
	TE9=TE3*TE3-TE4*TE4	HPRS 506
	TE10=2.*TE3*TE4	HPRS 507
	DE15=TE7*BET3R-TE4*BET3I	HPRS 508
	DE16=TE7*BET3I+TE4*BET3R	HPRS 509
	TE11=TE3*BET2R-TE4*BET2I+BET1R-DE15	HPRS 510
	TE12=TE3*BET2I+TE4*BET2R+BET1I-DE16	HPRS 511
	TE7=TE9-1.	HPRS 512
	TE1=TE9*BET2R-TE10*BET2I	HPRS 513
	TE2=TE9*BET2I+TE10*BET2R	HPRS 514
	TE13=TE1-BET1R-TE7*BET3R+TE10*BET3I	HPRS 515
	TE14=TE2-BET1I-TE7*BET3I-TE10*BET3R	HPRS 516
	TE15=DE15*TE3-DE16*TE4	HPRS 517
	TE16=DE15*TE4+DE16*TE3	HPRS 518
	TE1=TE13*TE13-TE14*TE14-4.*(TE11*TE15-TE12*TE16)	HPRS 519
	TE2=2.*TE13*TE14-4.*(TE12*TE15+TE11*TE16)	HPRS 520
	TEM = SQRT (TE1*TE1+TE2*TE2)	HPRS 521
	IF(TE1)113,113,112	HPRS 522
113	TE4 = SQRT (0.5 * (TEM - TE1))	HPRS 523
	TE3=.5*TE2/TE4	HPRS 524
	GO TO 111	HPRS 525
112	TE3 = SQRT (0.5 * (TEM + TE1))	HPRS 526
	IF(TE2)110,200,200	HPRS 527
110	TE3=-TE3	HPRS 528
200	TE4=.5*TE2/TE3	HPRS 529
111	TE7=TE13+TE3	HPRS 530
	TE8=TE14+TE4	HPRS 531
	TE9=TE13-TE3	HPRS 532
	TE10=TE14-TE4	HPRS 533
	TE1=2.*TE15	HPRS 534
	TE2=2.*TE16	HPRS 535
	IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205	HPRS 536
204	TE7=TE9	537
	TE8=TE10	538
205	TEM=TE7*TE7+TE8*TE8	HPRS 539
	TE3=(TE1*TE7+TE2*TE8)/TEM	HPRS 540

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TE4=(TE2*TE7-TE1*TE8)/TEM	HPRS 541
AXR=ALP3R+TE3*TE5-TE4*TE6	HPRS 542
AXI=ALP3I+TE3*TE6+TE4*TE5	HPRS 543
ALP4R=AXR	HPRS 544
ALP4I=AXI	HPRS 545
M=4	HPRS 546
GO TO 99	HPRS 547
15 N6=1	548
38 IF (ABS (HELL) + ABS (BELL) - 1.E-20) 18,18,16	549
16 TE7 = ABS (ALP3R - AXR) + ABS (ALP3I - AXI)	550
IF (TE7 / (ABS (AXR) + ABS (AXI)) - 1.E-7) 18,18,17	551
17 N3=N3+1	552
ALP1R=ALP2R	553
ALP1I=ALP2I	554
ALP2R=ALP3R	555
ALP2I=ALP3I	556
ALP3R=ALP4R	557
ALP3I=ALP4I	558
BET1R=BET2R	559
BET1I=BET2I	560
BET2R=BET3R	561
BET2I=BET3I	562
BET3R=TEHR	563
BET3I=TEHI	564
IF(N3-100) 14, 18, 18	565
18 N4=N4+1	566
ROOTR(N4)=ALP4R	567
ROOTI(N4)=ALP4I	568
M3=0	569
41 IF(N4-N1) 30, 37, 37	570
37 RETURN	571
30 IF (ABS (ROOTI(N4)) - 1.E-5) 10, 10, 31	572
31 GO TO(32, 10), L	573
32 AXR=ALP1R	574
AXI=-ALP1I	575
ALP1I=-ALP1I	576
M=5	577
GO TO 99	578
33 BET1R=TEHR	579
BET1I=TEHI	580
AXR=ALP2R	581
AXI=-ALP2I	582
ALP2I=-ALP2I	583
M=6	584
GO TO 99	585
34 BET2R=TEHR	586
BET2I=TEHI	587
AXR=ALP3R	588
AXI=-ALP3I	589
ALP3I=-ALP3I	590
L=2	591
M=3	592
99 TEHR=COE(1)	593
TEHI=0.0	594
DO100 I=1, N1	595
TE1=TEHR*AXR-TEHI*AXI	596
TEHI=TEHI*AXR+TEHR*AXI	597
100 TEHR= TE1+COE(I+1)	598
HELL=TEHR	599
BELL=TEHI	600

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42 IF(N4)102,103,102
 102 DO101I=1,N4
 TEM1=AXR-ROOTR(I)
 TEM2=AXI-ROOTI(I)
 TE1=TEM1*TEM1+TEM2*TEM2
 TE2=(TEMR*TEM1+TEMI*TEM2)/TE1
 TEMI=(TEMI*TEMI-TEMR*TEM2)/TE1
 101 TEMR=TE2
 103 GO TO(11,12,13,15,33,34),M
 END

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SUBROUTINE ERCHEK(X,I)
 DIMENSION X(100)
 DATA ERRLIM/0.1E-8/
 DO 10 J=1,I
 10 IF(ABS(X(J)).LT.ERRLIM) X(J)=0.0
 RETURN
 END

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SUBROUTINE EXPAND(ILOVE,X,Y)
 DIMENSION X2(2000),Y2(2000),X(2000),Y(2000)
 DIMENSION X1(2000),Y1(2000),Z(125),Z1(87)
 INTEGER SPOT(130,50),DASH,Q,BLANK,POINT(130,100)
 DATA Q/1H1/, DASH/1H-/,BLANK/1H /
 DATA APLUS/0.0/,BPLUS/0.0/,AMINUS/0.0/,BMINUS/0.0/
 COMMON /INFO4/ Q,BLANK,DASH
 COMMON /BOB/POINT
 COMMON /PAYNE/AM,AP,BM,BP
 COMMON /PJ/X1,Y1
 EQUIVALENCE(X1(1),Z(1)),(Y1(1),Z1(1))
 EQUIVALENCE (SPOT(1,1),POINT(1,1)),(I1,I2)
 EQUIVALENCE(AP,APLUS),(AM,AMINUS),(BM,BMINUS),(BP,BPLUS)
 DO 40 J=1,ILOVE
 DO 50 M=1,2000
 X1(M)=0.0
 50 Y1(M)=0.0
 DO 51 M=1,130
 DO 51 N=1,50
 51 SPOT(M,N)=BLANK
 DO 52 M=1,130
 SPOT(M,1)=DASH
 52 SPOT(M,50)=DASH
 DO 53 M=1,50
 SPOT(1,M)=Q
 53 SPOT(126,M)=Q
 DO 54 M=1,2000
 X2(M)=0.0
 54 Y2(M)=0.0
 READ(5,10) OMEGA,ENCRMT,SIGMA,DELTA
 OMEGA=ABS(OMEGA)
 10 FORMAT(4F10.0)
 A=ENCRMT
 PERCNT=A*0.01
 B=OMEGA

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B=B*PERCNT	661
APLUS=B+OMEGA	662
AMINUS=OMEGA-B	663
C=APLUS	664
D=AMINUS	665
I1=1	666
DO 41 L=1,2000	667
IF(Y(L).LT.D)GO TO 41	668
IF(Y(L).GT.C)GO TO 41	669
23 X1(I1)=X(L)	670
Y1(I1)=Y(L)	671
I1=I1+1	672
41 CONTINUE	673
A=DELTA	674
901 PERCNT=A*0.01	675
B=SIGMA	676
903 B=B*PERCNT	677
BPLUS=SIGMA+B	678
BMINUS=SIGMA-B	679
C=BPLUS	680
D=BMINUS	681
I2=1	682
910 DO 42 L=1,2000	683
IF(C.GT.0.0) GO TO 60	684
IF(X1(L).GT.D) GO TO 42	685
IF(X1(L).LT.C) GO TO 42	686
GO TO 25	687
60 IF(X1(L).GT.C) GO TO 42	688
IF(X1(L).LT.D) GO TO 42	689
25 X2(I2)=X1(L)	690
Y2(I2)=Y1(L)	691
I2=I2+1	692
42 CONTINUE	693
CALL SPLIT(X2,Y2,SPOT,APLUS,AMINUS,BPLUS,BMINUS)	694
40 CONTINUE	695
RETURN	696
END	697
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	701
SUBROUTINE SPLIT(X,Y,SPOT,APLUS,AMINUS,BPLUS,BMINUS)	702
DIMENSION X1(2000),Y1(2000)	703
DIMENSION X(2000),Y(2000),Z(125),Z1(87)	704
INTEGER SPOT(130,50)	705
COMMON /PJ/X1,Y1	706
EQUIVALENCE(X1(1),Z(1)),(Y1(1),Z1(1))	707
A=APLUS	708
B=AMINUS	709
C=A-B	710
D=BPLUS	711
E=BMINUS	712
G=ABS(D)	713
H=ABS(E)	714
F=G-H	715
DELTA=C/12%.0	716
DIFF=F/50.0	717
DO 11 J=1,124	718
Z(J)=A	719
11 A=A-DELTA	720

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SUBROUTINE RITEIT (SPOT)
COMMON /PAYNE/AM,AP,BM,BP
EQUIVALENCE (AP,APLUS), (AM,AMINUS), (BM,BMINUS), (BP,BPLUS)
INTEGER SPOT(130,50)
IF (BP.LT.BM) GO TO 40
A=BP
B=BM
WRITE(6,1)
1 FORMAT(1H1,40X,43HCOMPLEX FREQUENCY PLANE,RIGHT HAND QUADRANT )
GO TO 30
40 A=BM
B=BP
WRITE(6,2)
2 FORMAT(1H1,40X,43HCOMPLEX FREQUENCY PLANE, LEFT HAND QUADRANT )
30 WRITE(6,12)AP,AM
12 FORMAT(114X,15HC-----J-OMEGA,/1X,F8.2,112X,F8.2)
WRITE(6,14)B
14 FORMAT(124X,5HSIGMA,/126X,1HI,/126X,1HV,/121X,F8.2)
WRITE(6,11) SPOT
11 FORMAT(1X,130A1)
WRITE(6,15)A
15 FORMAT(60X,27HLINEAR EXPAND PLOT(RAD/SEC) ,34X,F8.2)
RETURN
END

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SUBROUTINE SAVER (ROOTR,ROOTI, IDIC,SAVE1,SAVE2, JZO,K1)
DIMENSION SAVE1(100,100),SAVE2(100,100),ROOTR(100),ROOTI(100)
IF (K1)30,9,10
9 K1=1
10 ID1=IDIC
IDIC=IDIC+(K1-1)
IF (IDIC.GE.100)GO TO 30
50 DO 40 IZAP=K1, IDIC
IZA=IZAP-(K1-1)
SAVE1(JZO,IZAP)=ROOTR(IZA)
SAVE2(JZO,IZAP)=ROOTI(IZA)
40 CONTINUE
K1=IDIC+1
GO TO 20
30 JZO=JZO+1
IDIC=ID1
GO TO 9
20 RETURN
END

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SUBROUTINE PLOT2R (SAVE1,SAVE2, ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,
*STAR,NBB,NO)
DIMENSION SAVE1(100,100),SAVE2(100,100),XA(2000),XB(2000)
INTEGER POINT(130,100),DASH,BLANK,STAR
LOGICAL PRNT
IF (PRNT) GO TO 43
DO 40 NAB=1,100
DO 40 NBB=1,100
IF (SAVE1(NAB,NBB))41,42,41
42 IF (SAVE2(NAB,NBB))41,40,41

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41 ANUMB1=SAVE1(NAB,NBB)
ANUMB2=SAVE2(NAB,NBB)
ANUMB2=ABS(ANUMB2)
43 AZZ=ANUMB2
AZZ=AZZ*100000.0
NZZ=AZZ
IF(NZZ.EQ.0) GO TO 45
50 NBB=NBB+1
45 CALL EXECUTE(ANUMB1,ANUMB2,POINT,XA,XB,MI,NO,STAR)
IF(PRNT)RETURN
40 CONTINUE
70 CALL WRITIT(XA,XB)
CALL PREPAR(POINT)
RETURN
END

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SUBROUTINE EXECUTE(ANUMB1,ANUMB2,POINT,XA,XB,MI,NO,STAR)
LOGICAL SKIP1,SKIP2
LOGICAL LESS
DIMENSION XA(2000),XB(2000)
INTEGER POINT(130,100),DASH,BLANK,STAR
DATA K1/10/,K2/100/,K3/1000/,K4/10000/,NEGONE/-1/
LESS=.FALSE.
I=0
J=0
CALL MEMORY(ANUMB1,ANUMB2,NO,XA,XB)
IF(ABS(ANUMB1).GT.10000.0) GO TO 50
IF(ABS(ANUMB2).GT.10000.0) GO TO 50
IF(ABS(ANUMB1).EQ.0.0) GO TO 500
IF(ABS(ANUMB2).EQ.0.0) GO TO 500
IF(ABS(ANUMB2).LT.0.001) GO TO 50
IF(ABS(ANUMB1).LT.0.001) GO TO 50
500 CALL SCALE1(K1,K2,K3,K4,NEGONE,I,ANUMB1,ICONS,LESS,SKIP1)
CALL SCALE2(K1,K2,K3,K4,J,JCONS,ANUMB2,SKIP2)
CALL WPOINT(J,JCONS,L,ANUMB2,SKIP2)
CALL SPOINT(I,ANUMB1,LESS,ICONS,L,POINT,SKIP1,NO,XA,XB,STAR)
50 RETURN
END

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SUBROUTINE SCALE1(K1,K2,K3,K4,NEGONE,I,ANUMB1,ICONS,LESS,SKIP1)
LOGICAL LESS
LOGICAL SKIP1
SKIP1=.FALSE.
ICONS=1
I=1
AKEEP=ANUMB1
21 NUMB1=ANUMB1
NUMB=ABS(NUMB1)
IF(NUMB.EQ.0)GOTO 12
IF(NUMB1)31,40,40
40 IF(NUMB1.GE.10)GOTO 11
GO TO 50
31 LESS=.TRUE.
XXX=-NUMB1

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NUMB1=XXX	901
GOTO 40	902
12 GOTO(1,2,3,4),I	903
1 ANUMB1=AKEEP	904
ICONS=K1	905
RK1=K1	906
ANUMB1=ANUMB1*RK1	907
I=2	908
GOTO 21	909
2 ANUMB1=AKEEP	910
ICONS=K2	911
RK2=K2	912
ANUMB1=ANUMB1*RK2	913
I=3	914
GOTO 21	915
3 ANUMB1=AKEEP	916
ICONS=K3	917
RK3=K3	918
ANUMB1=ANUMB1*RK3	919
I=4	920
GOTO 21	921
4 ANUMB1=AKEEP	922
ICONS=K4	923
RK4=K4	924
ANUMB1=ANUMB1*RK4	925
NUMB1=ANUMB1	926
IF(NUMB1.EQ.0) GO TO 51	927
GO TO 50	928
11 SKIP1=.TRUE.	929
GO TO (6,7,8,9),I	930
6 ANUMB1=AKEEP	931
ICONS=K1	932
RK1=K1	933
ANUMB1=ANUMB1/RK1	934
I=2	935
GOTO 21	936
7 ANUMB1=AKEEP	937
ICONS=K2	938
RK2=K2	939
ANUMB1=ANUMB1/RK2	940
I=3	941
GOTO 21	942
8 ANUMB1=AKEEP	943
ICONS=K3	944
RK3=K3	945
ANUMB1=ANUMB1/RK3	946
I=4	947
GOTO 21	948
9 ANUMB1=AKEEP	949
ICONS=K4	950
RK4=K4	951
ANUMB1=ANUMB1/RK4	952
GO TO 50	953
51 I=5	954
50 RETURN	955
END	956
	957
	958
	959
	960

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SUBROUTINE SCALE2(K1,K2,K3,K4,J,JCONS,ANUMB2,SKIP2)	961
LOGICAL SKIP2	962
SKIP2=.FALSE.	963
JCONS=1	964
J=1	965
ANUMB2=ABS(ANUMB2)	966
BKEEP=ANUMB2	967
20 NUMB2=ANUMB2	968
IF(NUMB2.EQ.0)GO TO 10	969
IF(NUMB2.GE.10)GO TO 11	970
GO TO 50	971
10 GOTO(1,2,3,4),J	972
1 ANUMB2=BKEEP	973
JCONS=K1	974
RK1=K1	975
ANUMB2=ANUMB2*RK1	976
J=2	977
GO TO 20	978
2 ANUMB2=BKEEP	979
JCONS=K2	980
RK2=K2	981
ANUMB2=ANUMB2*RK2	982
J=3	983
GO TO 20	984
3 ANUMB2=BKEEP	985
JCONS=K3	985
RK3=K3	987
ANUMB2=ANUMB2*RK3	988
J=4	989
GO TO 20	990
4 ANUMB2=BKEEP	991
JCONS=K4	992
RK4=K4	993
ANUMB2=ANUMB2*RK4	994
NUMB2=ANUMB2	995
IF(NUMB2.EQ.0) GO TO 51	996
GO TO 50	997
11 SKIP2=.TRUE.	998
GO TO (6,7,8,9),J	999
6 JCONS=K1	1000
ANUMB2=BKEEP	1001
RK1=K1	1002
ANUMB2=ANUMB2/RK1	1003
J=2	1004
GO TO 20	1005
7 JCONS=K2	1006
ANUMB2=BKEEP	1007
RK2=K2	1008
ANUMB2=ANUMB2/RK2	1009
J=3	1010
GO TO 20	1011
8 JCONS=K3	1012
ANUMB2=BKEEP	1013
RK3=K3	1014
ANUMB2=ANUMB2/RK3	1015
J=4	1016
GO TO 20	1017
9 JCONS=K4	1018
ANUMB2=BKEEP	1019
RK4=K4	1020

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	ANUMB2=ANUMB2/RK4	1021
	GO TO 50	1022
51	J=5	1023
50	RETURN	1024
	END	1025
C		1026
C		1027
C		1028
C		1029
	SUBROUTINE WPOINTIJ,JCONS,L,ANUMB2,SKIP2)	1030
	LOGICAL SKIP2	1031
	L=0	1032
	IDELTA=0	1033
	IF(ANUMB2.GE.9.2) IDELTA=12	1034
	IF(ANUMB2.GE.8.0) GO TO 200	1035
	IF(ANUMB2.GE.6.9) GO TO 201	1036
	IF(ANUMB2.GE.5.9) GO TO 202	1037
	IF(ANUMB2.GE.5.0) GO TO 203	1038
	IF(ANUMB2.GE.4.2) GO TO 204	1039
	IF(ANUMB2.GE.3.5) GO TO 205	1040
	IF(ANUMB2.GE.2.9) GO TO 206	1041
	IF(ANUMB2.GE.2.4) GO TO 207	1042
	IF(ANUMB2.GE.1.9) GO TO 208	1043
	IF(ANUMB2.GE.1.5) GO TO 209	1044
	IF(ANUMB2.GE.1.2) GO TO 210	1045
	GO TO 211	1046
200	IF(ANUMB2.LT.9.2) IDELTA=11	1047
201	IF(ANUMB2.LT.8.0) IDELTA=10	1048
202	IF(ANUMB2.LT.6.9) IDELTA=9	1049
203	IF(ANUMB2.LT.5.9) IDELTA=8	1050
204	IF(ANUMB2.LT.5.0) IDELTA=7	1051
205	IF(ANUMB2.LT.4.2) IDELTA=6	1052
206	IF(ANUMB2.LT.3.5) IDELTA=5	1053
207	IF(ANUMB2.LT.2.9) IDELTA=4	1054
208	IF(ANUMB2.LT.2.4) IDELTA=3	1055
209	IF(ANUMB2.LT.1.9) IDELTA=2	1056
210	IF(ANUMB2.LT.1.5) IDELTA=1	1057
211	IF(ANUMB2.LT.1.2) IDELTA=0	1058
	IDELTA=13-IDELTA	1059
	IF(J.EQ.5) GO TO 50	1060
	IF(SKIP2) GO TO 41	1061
	IF(JCONS.EQ.1)GOTO 1	1062
	IF(JCONS.EQ.10)GOTO 10	1063
	IF(JCONS.EQ.100)GOTO 100	1064
	IF(JCONS.EQ.1000)GOTO 1000	1065
	IF(JCONS.EQ.10000)GOTO 10000	1066
1	L=73+IDELTA	1067
	GO TO 40	1068
10	L=86+IDELTA	1069
	GO TO 40	1070
100	L=99+IDELTA	1071
	GO TO 40	1072
1000	L=112+IDELTA	1073
	GO TO 40	1074
5877000	L=125	1075
	GO TO 40	1076
41	KCONS=JCONS+1	1077
	SKIP2=.FALSE.	1078
	IF(KCONS.EQ.11)GOTO 11	1079
	IF(KCONS.EQ.101)GOTO 101	1080

	IF(KCONS.EQ.1001)GOTO 1001	1081
	IF(KCONS.EQ.10001)GOTO 10001	1082
	11 L=60+IDELTA	1083
	GO TO 40	1084
	101 L=47+IDELTA	1085
	GO TO 40	1086
	1001 L=34+IDELTA	1087
	GO TO 40	1088
	50 L=125	1089
	GO TO 40	1090
	10001 L=34	1091
	40 RETURN	1092
	END	1093
C		1094
C		1095
C		1096
C		1097
	SUBROUTINE SPOINT(I,ANUMB1,LESS,ICONS,L,POINT,SKIPI,NO,XA,XB,STAR)	1098
	DIMENSION XA(2000),XB(2000)	1099
	INTEGER POINT(130,100),DASH,BLANK,STAR,Q	1100
	INTEGER STAR1,STAR2	1101
	DATA STAR1/1H0/,STAR2/1H*/	1102
	DATA BLANK/1H /,DASH/1H-/,	Q/1H1/ 1103
	LOGICAL SKIPI ,LESS	1104
	AKEEP=ANUMB1	1105
	ANUMB1=ABS(AKEEP)	1106
	KOOL=0	1107
	IF(I.EQ.5)GO TO 1000	1108
	IF(ANUMB1.GE.7.1)KOOL=7	1109
	IF(ANUMB1.GE.5.0)GO TO 1011	1110
	IF(ANUMB1.GE.4.0)GO TO 1111	1111
	IF(ANUMB1.GE.3.1)GO TO 1211	1112
	IF(ANUMB1.GE.2.3)GO TO 1311	1113
	IF(ANUMB1.GE.1.6)GO TO 1411	1114
	IF(ANUMB1.GE.1.0)GO TO 1511	1115
	GO TO 4011	1116
1011	IF(ANUMB1.LT.7.1)KOOL=6	1117
1111	IF(ANUMB1.LT.5.0)KOOL=5	1118
1211	IF(ANUMB1.LT.4.0)KOOL=4	1119
1311	IF(ANUMB1.LT.3.1)KOOL=3	1120
1411	IF(ANUMB1.LT.2.3)KOOL=2	1121
1511	IF(ANUMB1.LT.1.6)KOOL=1	1122
4011	NCRMNT=8-KOOL	1123
	IF(LESS)GO TO 40	1124
	IF(SKIPI)GO TO 41	1125
	IF(ICONS.EQ.1)GOTO 1	1126
	IF(ICONS.EQ.10)GOTO 10	1127
	IF(ICONS.EQ.100)GOTO 100	1128
	IF(ICONS.EQ.1000)GOTO 1000	1129
1	KOOL=71+KOOL	1130
	GO TO 50	1131
10	KOOL=64+KOOL	1132
	GO TO 50	1133
100	KOOL=57+KOOL	1134
	GO TO 50	1135
1000	KOOL=50+KOOL	1136
	GO TO 50	1137
41	LCONS=ICONS+1	1138
	SKIPI=.FALSE.	1139
	IF(LCONS.EQ.11)GOTO 11	1140

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IF(LCONS.EQ.101)GOTO 101	1141
IF(LCONS.EQ.1001)GOTO 1001	1142
11 KOOL=78+KOOL	1143
GO TO 50	1144
101 KOOL=85+KOOL	1145
GO TO 50	1146
1001 KOOL=92+KOOL	1147
50 LESS=.FALSE.	1148
IF(POINT(L,KOOL).EQ.DASH) GO TO 51	1149
IF(POINT(L,KOOL).EQ.Q) GO TO 51	1150
14 IF(POINT(L,KOOL).EQ.STAR1) GO TO 15	1151
IF(POINT(L,KOOL).EQ.STAR2) GO TO 15	1152
IF(STAR.EQ.STAR1) GO TO 51	1153
IF(STAR.EQ.STAR2) GO TO 51	1154
IF(POINT(L,KOOL).NE.BLANK) GO TO 70	1155
51 POINT(L,KOOL)=STAR	1156
ANUMB1=AKEEP	1157
GO TO 70	1158
15 L=L+1	1159
IF(L.EQ.130) GO TO 51	1160
GO TO 14	1161
40 IF(SKIP1) GO TO 42	1162
IZ=ICONS+2	1163
IF(IZ.EQ.3)GOTO 3	1164
IF(IZ.EQ.12)GOTO 12	1165
IF(IZ.EQ.102)GOTO 102	1166
IF(IZ.EQ.1002)GOTO 1002	1167
3 KOOL=21+NCRMNT	1168
GO TO 50	1169
12 KOOL=28+NCRMNT	1170
GO TO 50	1171
102 KOOL=35+NCRMNT	1172
GO TO 50	1173
1002 KOOL=42+NCRMNT	1174
GO TO 50	1175
42 IY=ICONS+3	1176
SKIP1=.FALSE.	1177
IF(IY.EQ.13)GOTO 13	1178
IF(IY.EQ.103)GOTO 103	1179
IF(IY.EQ.1003)GOTO 1003	1180
IF(IY.EQ.10003)GOTO 10003	1181
13 KOOL=14+NCRMNT	1182
GO TO 50	1183
103 KOOL=7+NCRMNT	1184
GO TO 50	1185
1003 KOOL=NCRMNT	1186
GO TO 50	1187
10003 POINT(L,1)=STAR	1188
GO TO 70	1189
70 RETURN	1190
END	1191
	1192
	1193
	1194
	1195
SUBROUTINE PREPAR(POINT)	1196
INTEGER POINT(130,100),PRT	1197
DATA PRT /6/	1198
WRITE(6,1)	1199
1 FORMAT(1H1)	1200

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WRITE(6,11) 1201
11 FORMAT(///) 1202
WRITE(PRT,14) 1203
14 FORMAT(75X,8HLOG PLOT,/58X,42HCOMPLEX FREQUENCY PLANE,LEFT HAND QU 1204
*ADRANT,/75X,9H(RAD/SEC) ) 1205
WRITE(6,12) 1206
12 FORMAT(32X,5H10000,9X,4H1000,9X,3H100,10X,2H10,12X,1H1,11X,2H.1,10 1207
*X,14H.01<---J-OMEGA,/ 18X,11HMINUS SIGMA) 1208
13 FORMAT(32X,5H10000,9X,4H1000,9X,3H100,10X,2H10,12X,1H1,11X,2H.1,10 1209
*X,14H.01<---J-OMEGA,/ 18X,11H PLUS SIGMA) 1210
DO 50 I=1,130 1211
50 POINT(I,50)=POINT(I,51) 1212
WRITE(PRT,10)POINT 1213
10 FORMAT(1X,130A1) 1214
WRITE(6,13) 1215
WRITE(PRT,15) 1216
15 FORMAT(75X,8HLOG PLOT,/58X,43HCOMPLEX FREQUENCY PLANE,RIGHT HAND QU 1217
*UADRANT,/75X,9H(RAD/SEC) ) 1218
RETURN 1219
END 1220
C 1221
C 1222
C 1223
C 1224
SUBROUTINE MEMORY(ANUMB1,ANUMB2,NO,XA,XB) 1225
DIMENSION XA(2000),XB(2000) 1226
XA(NO)=ANUMB1 1227
XB(NO)=ABS(ANUMB2) 1228
NO=NO+1 1229
RETURN 1230
END 1231
C 1232
C 1233
C 1234
C 1235
SUBROUTINE WRITIT(XA,XB) 1236
DIMENSION XA(2000),XB(2000) 1237
WRITE(6,1) 1238
1 FORMAT(1H1) 1239
WRITE(6,11) 1240
11 FORMAT(5X,48HTHE FOLLOWING ROOTS ARE PLOTTED ON THE LOG PLOT,/ 1241
15X,99HROOTS AT THE ORIGIN ARE NOT PRINTED OR PLOTTED, ROOTS ON THE 1242
* J-OMEGA AXIS ARE NOT PLOTTED. 1243
2, //16X,5HSIGMA,25X,7HJ-OMEGA, //) 1244
DO 77 IZ0=1,2000 1245
XC=XB(IZ0) 1246
IF(XC.GT.0.0) XC=-XC 1247
IF(XC.NE.0.0) GO TO 22 1248
20 IF(XA(IZ0))22,40,22 1249
40 K1=IZ0 1250
K2=IZ0+12 1251
DO 50 K=K1,K2 1252
IF(XA(K).NE.0.0) GO TO 77 1253
50 IF(XB(K).NE.0.0) GO TO 77 1254
GO TO 60 1255
22 WRITE(6,10)XA(IZ0),XC 1256
10 FORMAT(5X,F20.9,10X,F20.9) 1257
77 CONTINUE 1258
60 RETURN 1259
END 1260

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13. ABSTRACT An existing program that determines the root locus of n'th order polynomials has been modified to provide plots of these loci in the complex frequency plane using a standard line printer. A methodology that combines the computational capabilities of this root locus program with a variable scale graphical display of selectable regions of the complex frequency plane is presented. A listing of the Fortran IV source deck of the modified program and two examples are included.			

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