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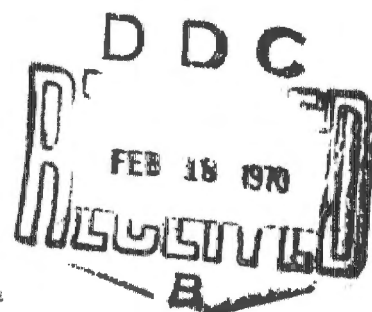
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NONEQUILIBRIUM CONTINUUM THEORY
OF SPHERICAL ELECTROSTATIC PROBES
AT LARGE DEBYE NUMBER

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by
Francis Fendell



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**NONEQUILIBRIUM CONTINUUM THEORY OF
SPHERICAL ELECTROSTATIC PROBES AT LARGE DEBYE NUMBER**

**By
Francis Fendell**

**Prepared for
Bell Telephone Laboratories, Inc.
Whippany, New Jersey
and
U.S. Army Research Office
Durham, North Carolina**

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Continuum theory is used to describe the steady spherically symmetric state of an unbounded expanse of slightly ionized gas about a perfectly catalytic conductor. Far from the sphere the gas is in chemical equilibrium, but at finite distances finite-rate ionization and recombination may occur according to a direct one-step reversible reaction in which either a neutral or an electron (the negative-charge carrier) participates as the third body in the recombination process. The limit of large Debye number appropriate to the use of small electrostatic probes in slightly ionized gases (where the Debye number is the ratio of the Debye length to the probe radius) is examined. In this limit the case of nearly frozen chemistry near the sphere is of greatest practical interest; modification of current collected from the value given by Su and Lam owing to incipient reaction effects is determined for negative potential bias applied at the probe.

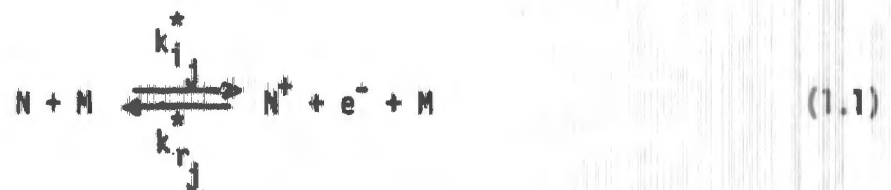
1. INTRODUCTION

Electrostatic probes have recently been used to examine the electrical properties of flames occurring in aerothermochemical flows for which continuum analysis is applicable (Turcotte and Friedman, 1965). The electrical properties are, of course, due to the generation of ions by exothermic oxidation reactions. The successful use of

electrostatic probes in flames requires an understanding of the effects of finite-rate ionization and recombination on the current-collecting characteristics of a conductor with a known applied potential bias. A study of the effect of nonequilibrium electronic chemistry for the small-Debye-number limit appropriate to larger probes in a somewhat more highly ionized gas has been carried out elsewhere (Carrier and Fendell, 1969). Here some brief comments are presented on the effect of nonequilibrium electronic chemistry on the large-Debye-number limit appropriate to smaller probes in a very slightly ionized gas.

A spherical perfectly catalytic conductor is immersed in an unbounded expanse of quiescent slightly ionized gas. The probe has a zero or negative bias relative to the ambient gas, taken to be in chemical equilibrium. The gas has zero net macroscopic velocity; it is modelled as an isobaric constant-density constant-property multicomponent continuum in which the ratio of (constant) ion and electron temperatures is of order unity. The diffusion coefficient for the positive ion and ionizable neutral are taken as identical; the negative charge carrier is taken to be an electron.

The ionization and recombination is taken to occur according to the following direct one-step reversible reaction (radiative effects are neglected):



where M is either an electron (for more highly ionized cases) or a populous non-ionized neutral of some species other than N (for less highly

ionized cases). The ionization rate k_{ij}^* and the recombination rate k_{rj}^* are both primarily functions of temperature; k_{ij}^* displays a classical Arrhenius dependence while k_{rj}^* has inverse-power-law form. The subscripts j on the specific rate constants k_{rj} and k_{ij} acknowledge that these factors have different values when $j=0$ (populous neutral as third body for recombination) and when $j=1$ (electron as third body for recombination).

The nonlinear two-point boundary-value problem describing the flow in the absence of magnetic field effects is:

$$-e^2 \left(\frac{d^2 \psi}{dr^2} + \frac{2}{r} \frac{d\psi}{dr} \right) = n_i - n_e, \quad (1.2)$$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dn_n}{dr} \right) = D_j (-n_i n_e + S_j n_n) n_e^j, \quad (1.3)$$

$$-\frac{1}{r^2} \frac{d}{dr} \left[r^2 \left(\frac{dn_e}{dr} - n_e \frac{d\psi}{dr} \right) \right] = \beta D_j (-n_i n_e + S_j n_n) n_e^j, \quad (1.4)$$

$$-\frac{1}{r^2} \frac{d}{dr} \left[r^2 \left(\frac{dn_i}{dr} + \tau n_i \frac{d\psi}{dr} \right) \right] = D_j (-n_i n_e + S_j n_n) n_e^j, \quad (1.5)$$

subject to

$$r \rightarrow \infty: n_e \rightarrow 1, n_i \rightarrow 1, n_n \rightarrow S_j^{-1}, \psi \rightarrow 0 \quad (1.6)$$

$$r = 1: n_i = n_e = 0, \psi = \psi_p \text{ (given)}, \frac{d}{dr} (n_i + n_n) = 0 \quad (1.7)$$

Here $r = r^*/a^*$ where r^* is the spherical radial coordinate and a^* , the probe radius; $n_i = n_i^*/n_0^*$, $n_e = n_e^*/n_0^*$, $n_n = n_n^*/n_0^*$ where n_i^* , n_e^* , n_n^* are the number densities of (positive) ions, electrons, and (singly ionizable) neutrals of species N , respectively, and n_0^* is the number of electrons per unit volume of the electrically neutral ambient fluid; the degree of ionization of species N is given by $S_j = n_0^*/n_n^* (r^* \rightarrow \infty) =$

$k_{ij}^*/k_{rj}^* n_0^*$ (specified); $\tau = T_e^*/T_i^*$ where T_e^* is the (constant) electron temperature and T_i^* , the (constant) ion temperature; $\psi = e^* \phi^*/k^* T_e^*$ where e^* is the magnitude of the electronic charge, k^* is the Boltzmann constant, and ϕ^* is the electrostatic potential (the radial component of the electric field $E_r^* = -d\phi^*/dr^*$). The Debye number $\epsilon = \lambda_D^*/a^*$ where the Debye length $\lambda_D^* = (k^* \epsilon_0^* T_e^*/e^{*2} n_0^*)^{1/2}$ and ϵ_0^* is the permittivity of free space. The first Damköhler number $D_j = (n_m^*)^{1-j} a^{*2} k_{rj}^* n_0^{*2}/D_i^*$ where D_i^* is the ion diffusion coefficient and $n_m = n_m^*/n_0^*$ where n_m^* is the (constant) number density of species M. The diffusion coefficient of the neutral species $D_n^* \approx D_i^*$, but $D_i^*/D_e^* = \beta \neq 1$. Einstein's relations (for example, Baum and Chapkis, 1969) have been used in the course of the derivations of the governing equations.

The net current collected by the sphere (the net charge per unit time, suitably nondimensionalized) is

$$J_T = \frac{J_T^*}{4\pi a^* n_0^* e^* D_e^*} = \left(\frac{dn_e}{dr} - \beta \frac{dn_i}{dr} \right)_{r=1} \quad (1.8)$$

The variance of J_T with ψ_p , ϵ , and D_j is sought.

This formulation of the kinetics is widely adopted (see, for example, Turcotte and Ong, 1968). Here attention is confined to the "thick sheath" limit $\epsilon \gg 1$ (small probe in a slightly ionized gas). Su and Lam (1963) first treated this limit for frozen chemistry ($D_j = 0$); later, in a more systematic approach, Baum and Chapkis (1968) continued the solution to higher order. Sonin (1967) has considered the effect of forced convection on the $\epsilon \gg 1$ solution; Sonin emphasized that the nearly frozen limit $D_j \ll 1$ is of primary practical interest, but undertook no analysis of recombination-ionization processes.

It is here taken that $\tau = O(1)$, $S_j \geq O(1)$, and $\beta = o(1)$. That is, the ratio of ion to electron diffusion coefficient is small, ion and electron temperatures are comparable, and the ionizable neutral may be a trace species or the background gas. The results in Section II will be uniformly valid over a wide range of applied negative potentials, $\psi_p \leq 0$.

II. NEAR-FROZEN THICK-SHEATH CASE ($\epsilon \gg 1$, $D_j \ll 1$)

Since $\epsilon \sim a^{*-1}$ and $D_j \sim a^{*2}$, the so-called thick-sheath case $\epsilon \gg 1$ is most frequently associated with nearly frozen chemistry ($D_j \ll 1$). Thus, the perturbation about frozen chemistry is examined first, although the near-equilibrium case ($D_j \gg 1$) for a thick sheath is briefly treated in Section III.

Because only the zero- and first-order solutions will be sought, one need specify only that $1 \gg D_j \gg \epsilon^{-2}$ (as opposed to the limit $1 \gg \epsilon^{-2} \gg D_j$ implicitly treated in earlier works). Matched asymptotic expansions are adopted; for the inner region

$$n_i(r, D_j, \epsilon) = n_{iI}(r) + D_j^{1/2} n_{i1}(r) + \dots, \quad (2.1)$$

$$n_e(r, D_j, \epsilon) = n_{eI}(r) + D_j^{1/2} n_{e1}(r) + \dots, \quad (2.2)$$

$$n_n(r, D_j, \epsilon) = n_{nI}(r) + D_j^{1/2} n_{n1}(r) + \dots, \quad (2.3)$$

$$\psi(r, D_j, \epsilon) = \psi_I(r) + D_j^{1/2} \psi_1(r) + \dots; \quad (2.4)$$

for the outer region, if $\rho = D_j^{1/2} r$

$$n_i(r, D_j, \epsilon) = 1 + D_j^{1/2} n_{iM}(\rho) + \dots, \quad (2.5)$$

$$n_e(r, D_j, \epsilon) = 1 + D_j^{1/2} n_{eM}(\rho) + \dots, \quad (2.6)$$

$$n_n(r, D_j, \epsilon) = S_j^{-1} + D_j^{1/2} n_{nM}(\rho) + \dots, \quad (2.7)$$

$$\psi(r, D_j, \epsilon) = D_j^{1/2} \psi_M(\rho) + \dots, \quad (2.8)$$

The lowest-order inner equations obtained by substituting (2.1)-(2.4) in (1.2)-(1.5) are, if L denotes the spherically symmetric Laplacian operator $[(d^2/dr^2) + 2r^{-1} (d/dr)]$,

$$L\psi_I = Ln_{iI} = (L + \psi_p r^{-2} \frac{d}{dr})n_{eI} = (L - \tau \psi_p r^{-2} \frac{d}{dr})n_{iI} = 0. \quad (2.9)$$

The solutions, uniformly valid in r and ψ_p , are

$$\psi_I = \psi_p/r, \quad n_{eI} = [\exp(\psi_p/r) - \exp(\psi_p)]/[1 - \exp(\psi_p)], \quad (2.10)$$

$$n_{iI} = \{\exp[\tau \psi_p(r-1)/r] - 1\} / [\exp(\tau \psi_p) - 1], \quad (2.11)$$

$$n_{nI} = S_j^{-1} + \{\tau \psi_p / [\exp(\tau \psi_p) - 1]\} r^{-1}. \quad (2.12)$$

The first nontrivial outer equations, obtained by substituting (2.5)-(2.8) in (1.2)-(1.5), are, if $L_p = (d^2/d\rho^2) + 2\rho^{-1} (d/d\rho)$,

$$L_p \psi_M = L_p (n_{eM} + \beta n_{nM} - \psi_M) = L_p (n_{iM} + n_{eM} + \tau \psi_M) = 0, \quad (2.13)$$

$$(L_p - S_j)n_{nM} = -(n_{iM} + n_{eM}) . \quad (2.14)$$

The solutions satisfying the outer boundary conditions are

$$\psi_M = s\rho^{-1}, \quad \beta n_{nM} + n_{eM} - \psi_M = P\rho^{-1}, \quad n_{nM} + n_{iM} + \tau \psi_M = T\rho^{-1}, \quad (2.15)$$

$$n_{nM} = X\rho^{-1/2} K_{1/2}[(S_j + 1 + \beta)^{1/2} \rho] + \frac{-s(\tau - 1) + T + P}{S_j + 1 + \beta} \rho^{-1}, \quad (2.16)$$

where s , P , T , and X are constants of integration to be determined by matching.

The next-to-lowest-order inner equations still describe a locally frozen flow:

$$L\psi_1 = Ln_{n1} = \frac{d}{dr} \left[r^2 \left(\frac{dn_{e1}}{dr} - n_{e1} \frac{d\psi_1}{dr} - n_{e1} \frac{d\psi_1}{dr} \right) \right] = 0, \quad (2.17)$$

$$\frac{d}{dr} \left[r^2 \left(\frac{dn_{i1}}{dr} + \tau n_{i1} \frac{d\psi_1}{dr} + \tau n_{i1} \frac{d\psi_1}{dr} \right) \right] = 0. \quad (2.18)$$

The solutions satisfying the boundary conditions at the probe are

$$\psi_1 = A_1 [1 - \exp(\psi_p)] (1 - r^{-1}), \quad n_{n1} = B_1 + K_1 r^{-1}, \quad (2.19)$$

$$n_{i1} = H_1 \frac{\exp(-\tau \psi_p/r) - \exp(-\tau \psi_p)}{1 - \exp(-\tau \psi_p)} + \frac{\tau A_1 [1 - \exp(\psi_p)]}{\exp(\tau \psi_p) - 1} \left[\frac{\exp[\tau \psi_p (1 - r^{-1})]}{r} - 1 \right] \quad (2.20)$$

$$n_{e1} = J_1 \left[\frac{\exp(\psi_p/r) - \exp(\psi_p)}{1 - \exp(\psi_p)} \right] + A_1 \left[\exp(\psi_p) - \frac{\exp(\psi_p/r)}{r} \right], \quad (2.21)$$

where

$$K_1 = H_1 \frac{\tau \psi_p \exp(-\tau \psi_p)}{1 - \exp(-\tau \psi_p)} + \frac{\tau A_1 (\tau \psi_p - 1) [1 - \exp(\psi_p)]}{\exp(\tau \psi_p) - 1} \quad (2.22)$$

and A_1 , B_1 , H_1 , and J_1 are constants of integration.

Matching of the expansions is carried out by defining an intermediate variable $\bar{r} = \Sigma (D_j^{1/2}) r$ where, as $D_j \rightarrow 0$, $1 \gg \Sigma \gg D_j^{1/2}$. Then for \bar{r} fixed and $D_j \rightarrow 0$, $r = (\bar{r}/\Sigma) \rightarrow \infty$ while $\rho = (D_j^{1/2} \bar{r}/\Sigma) \rightarrow 0$. When the inner and outer expansions are rewritten in terms of \bar{r} , compatibility requires

$$s = \psi_p, \quad T = A_1 = 0, \quad -B_1 = H_1 = \beta^{-1} J_1 = \left(\frac{\pi}{2}\right)^{1/2} (S_j + \beta + 1)^{1/4} X, \quad (2.23)$$

$$P = \psi_p \left[\frac{\exp(\psi_p)}{1 - \exp(\psi_p)} - \frac{\beta \tau}{1 - \exp(\tau \psi_p)} \right], \quad (2.24)$$

$$\left(\frac{\pi}{2}\right)^{1/2} \frac{X}{(S_j + 1 + \beta)^{1/4}} = \frac{\tau \psi_p}{1 - \exp(\psi_p)} + \frac{\psi_p}{S_j + 1 + \beta} \left[\frac{\tau [1 - \exp(\psi_p)] - 1}{1 - \exp(\psi_p)} + \frac{\beta \tau}{1 - \exp(\tau \psi_p)} \right] \quad (2.25)$$

The vanishing of the $O(D_j^{1/2})$ perturbation to the potential in the inner region and the attendant simplifications are noteworthy.

The current is (Figures 1 - 3 present numerical examples)

$$J = - \frac{\psi_p \exp(\psi_p)}{1 - \exp(\psi_p)} (1 + D_j^{1/2} \beta H_1) - \beta \left[\frac{\tau \psi_p \exp(-\tau \psi_p)}{1 - \exp(-\tau \psi_p)} \right] (1 + D_j^{1/2} H_1) \quad (2.26)$$

where explicitly

$$H_1 = (S_j + 1 + \beta)^{1/2} \psi_p \left\{ \frac{\tau}{1 - \exp(\psi_p)} + \frac{1}{S_j + 1 + \beta} \left[\frac{\tau (1 - \exp(\psi_p)) - 1}{1 - \exp(\psi_p)} + \frac{\beta \tau}{1 - \exp(\tau \psi_p)} \right] \right\} \quad (2.27)$$

The following characteristics of the result may be worth emphasizing. Because reaction effects enter the inner region [which extends out to distances of $O(D_j^{-1/2})$] to $O(D_j)$ only, the ion and electron current densities individually are unmodified through $O(D_j^{1/2})$ near the probe. If small perturbations in the ion and electron number densities and in the electrostatic potentials are permitted to account for only small perturbations in the net current collected by the probe, one must require not only that $D_j \ll 1$, but also $|\psi_p| \beta D_j^{1/2} \ll 1$. However, since for electrons $\beta = o(1)$, rather large negative applied bias is permitted before the domain of validity of the solution is exceeded.

Near plasma (ambient) potential, i.e., for $1 \gg \beta D_j^{1/2} \gg |\psi_p|$,

$$J \doteq 1 - \beta + (\psi_p/2) \left\{ 1 + \beta \tau + \frac{\beta D_j^{1/2} (1 + \tau)}{(S_j + 1 + \beta)^{1/2}} \left[1 - \beta - \tau(S_j + 1 + \beta) \right] \right\} \quad (2.28)$$

For large negative bias, i.e., for $|\psi_p| \gg 1 \gg D_j^{1/2} |\psi_p|$, one formally extracts

$$J \doteq \beta \tau \psi_p \left\{ 1 + D_j^{1/2} \psi_p [(S_j \tau - 1) + 2\tau (1 + \beta)] \right\} \quad (2.29)$$

to exponentially small error. Since J characteristically decreases monotonically from the order-unity positive (mainly electron) current described by (2.28) to the slightly negative (mainly ion) current given by (2.29) as ψ_p becomes increasingly more negative, the floating (zero current) potential occurs for a negative ψ_p of order unity in magnitude. However, (2.26) and (2.27) readily yield an only implicit relation for the floating potential. Nevertheless, one can see that the sensitivity

of J to changes in τ for $\tau = 1$ or slightly larger is slight for small ψ_p . The variation of J with ψ_p is more rapid for $\psi_p = 0(1)$ than for $|\psi_p| \gg 1$; in this sense, the probe saturates for large negative bias.

At any fixed negative potential, incipient ionization and recombination (which enter together, first in the far zone) reduce in magnitude both the ion and the electron current from their values under frozen conditions provided $\tau > (S_j + 1 + \beta)^{-1}$ [sufficient condition for $H_1 < 0$]. Since $\beta > 0$, $S_j > 0$, and $\tau \geq 1$ in cases of interest here, the criterion is fulfilled. Figures 1 to 3 give results for $\tau \approx 1$, $S_j \leq 0(1)$, $\beta = 10^{-3}$; computation reveals that the validity of the perturbational approach is generally limited in such cases to $D_1 \leq 0(10^{-3})$ if the entire range $0 \leq (-\psi_p) \leq 10$ is to be described. If attention is limited to $0 \leq (-\psi_p) \leq 1$, then $D_1 \leq 0(10^{-2})$ is acceptable.

Figure 1 is typical in form for all cases computed; in fact, for $(-\psi_p) < 5$ the current-potential plots are indistinguishable on the scale used in Fig. 1. The cases diverge as the floating potentials are approached. The floating potential is increased appreciably as τ is reduced, moderately as D_j is increased, and slightly as S_j is increased. [When S_j is small, a slight degree of ionization of a species whose molecules are mostly electrically neutral is under study; when S_j is order unity, an appreciably ionized trace species is under consideration. In either case the analysis is limited to a multicomponent fluid which is only slightly ionized.]

III. NEAR-EQUILIBRIUM THICK-SHEATH CASE ($\epsilon \ll 1$, $D_j \gg 1$)

For $\epsilon \gg 1$, $\psi_p = 0$, and $(D_j \beta) \ll 1$,

$$\psi = 0, \quad n_n + n_i = S_j^{-1} + 1, \quad n_e = 1 - r^{-1}. \quad (3.1)$$

For a small probe at plasma potential in which the negative charge carrier is an electron, the boundary-value problem becomes

$$\frac{d^2 n_n}{dr^2} + \frac{2}{r} \frac{dn_n}{dr} + D_j (1 - r^{-1})^j [-(1 - r^{-1}) (S_j^{-1} + 1 - n_n) + S_j n_n], \quad (3.2)$$

where

$$n_n(r = 1, D_j) = S_j^{-1} + 1, \quad n_n(r \rightarrow \infty; D_j) = S_j^{-1}. \quad (3.3)$$

Since characterization for $D_j \ll 1$ was discussed above, attention here is limited to near-equilibrium conditions, $D_j \gg 1$, although the restraint $D_j \beta \ll 1$ is retained.

1. Neutral as the Third Body for Recombination ($j = 0$)

Matched asymptotic expansions are adopted in the form

$$n_n(r, D_0) = \begin{cases} N_{n0}(r) + D_0^{-1} N_{n1}(r) + \dots \\ n_{n0}(\epsilon) + D_0^{-1/2} n_{n1}(\epsilon) + \dots \end{cases} \quad (3.4)$$

where $\epsilon = D_0^{1/2} (r - 1)$. Substitution of (3.4) in (3.2) and (3.3) yields

$$N_{n_0} = S_0^{-1} \frac{(1 + S_0)(r - 1)}{(S_0 + 1)r - 1}, \quad n_{n_0} = S_0^{-1} (S_0 + 1) \exp[-S_0^{1/2} \epsilon], \quad (3.5)$$

$$n_{n_1} = \frac{S_0^{-2} (S_0 + 1)}{4} \epsilon [4 - (S_0^{1/2} \epsilon + 1 + 4 S_0) \exp(-S_0^{1/2} \epsilon)]. \quad (3.6)$$

A diffusional boundary layer of thickness $O(D_0^{-1/2})$ lies contiguous to the probe; in it the flow is out of equilibrium.

2. Electron as the Third Body for Recombination ($j = 1$)

The lowest-order outer solution N_{n_0} is again as given in (3.5) but the inner variable is now $\zeta = D_1^{1/3} (r - 1)$, so the diffusion layer is thicker. The lowest-order inner solution, which again is of order unity, involves the Airy function, tabulated by Antosiewicz (1964):

$$n_{n_0} = S_1^{-1} \left(\frac{S_1 + 1}{.355} \right) \text{Ai} (S_1^{1/3} \zeta). \quad (3.7)$$

Curiously, the Airy function also arises in the $\epsilon \ll 1$ case for small ψ_p (Bush and Fendell, 1969).

3. The Current Collected at the Probe

Whenever the effect of ionization and recombination on electrons is neglected because $\beta D_j \ll 1$, for $\epsilon \gg 1$ the potential in general is given by $\psi = \psi_p/r$ to lowest order and

$$n_e = 1 - \frac{1 - \exp(\psi_p/r)}{1 - \exp(\psi_p)}. \quad (3.8)$$

For consistency the $O(\beta)$ contribution of the ion current to the total current is neglected for $D_j \gg 1$, and

$$J = - \psi_p \frac{\exp(\psi_p)}{1 - \exp(\psi_p)} \quad (3.9)$$

The current given by (3.9) is obviously independent of τ . When $\psi_p \rightarrow 0$, $J \rightarrow 1 + O(\psi_p)$, the current appropriate for the plasma-potential cases 1 and 2 examined earlier in Section III.

The probe potential has been taken as $O(1)$; for $(-\psi_p) \gg 1$ the electronic contribution is no longer the dominant contribution to the total current. For $S_j = O(1)$ and $\psi_p < 0$ and $D_j \gg 1$, according to cases 1 and 2 (which can be rough guides only) when $\psi_p \exp(\psi_p) = O(\beta D_j^{1/(j+2)})$, one must begin to inspect closely whether the ionic current becomes comparable to the electronic current. Analytic derivation of the current under such conditions would entail extensive manipulation.

IV. CONCLUSIONS

A small spherical perfectly catalytic electrostatic probe in a very slightly ionized flow has been characterized by large Debye number and small first Damköhler number in most cases. This characterization has been shown to imply that the region near the probe is free of space charge and chemical effects to good approximation. Space-charge and incipient ionization-recombination effects enter at distances of $O(D_j^{-1/2})$, $D_j \ll 1$ from the probe, distances so far from the sphere that a linearization about the ambient state suffices as a quantitative description. An almost uniformly valid result for the variation of current collected by the probe with negative bias applied is readily derived. The effect of reaction is to increase algebraically the current (where an electron current is taken as positive) and to decrease algebraically the value at which the floating potential occurs.

In the unlikely event that large Damköhler number occurs in the large Debye number limit, then the bulk of the gas is nearly in chemical equilibrium (as opposed to the nearly frozen state that usually occurs). Only a narrow region near the probe is in chemical nonequilibrium.

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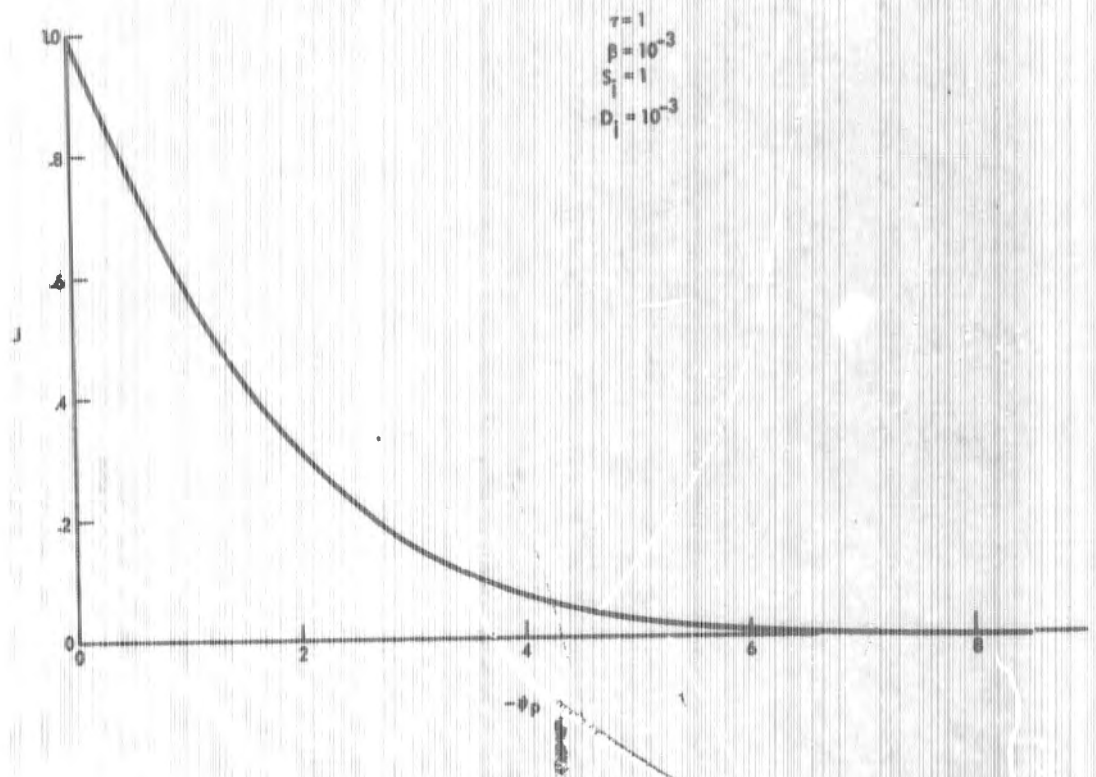


Figure 1. This curve is typical of how the net current collected at the probe J varies with increasingly negative probe bias v_p for nearly frozen flow at large Debye number.

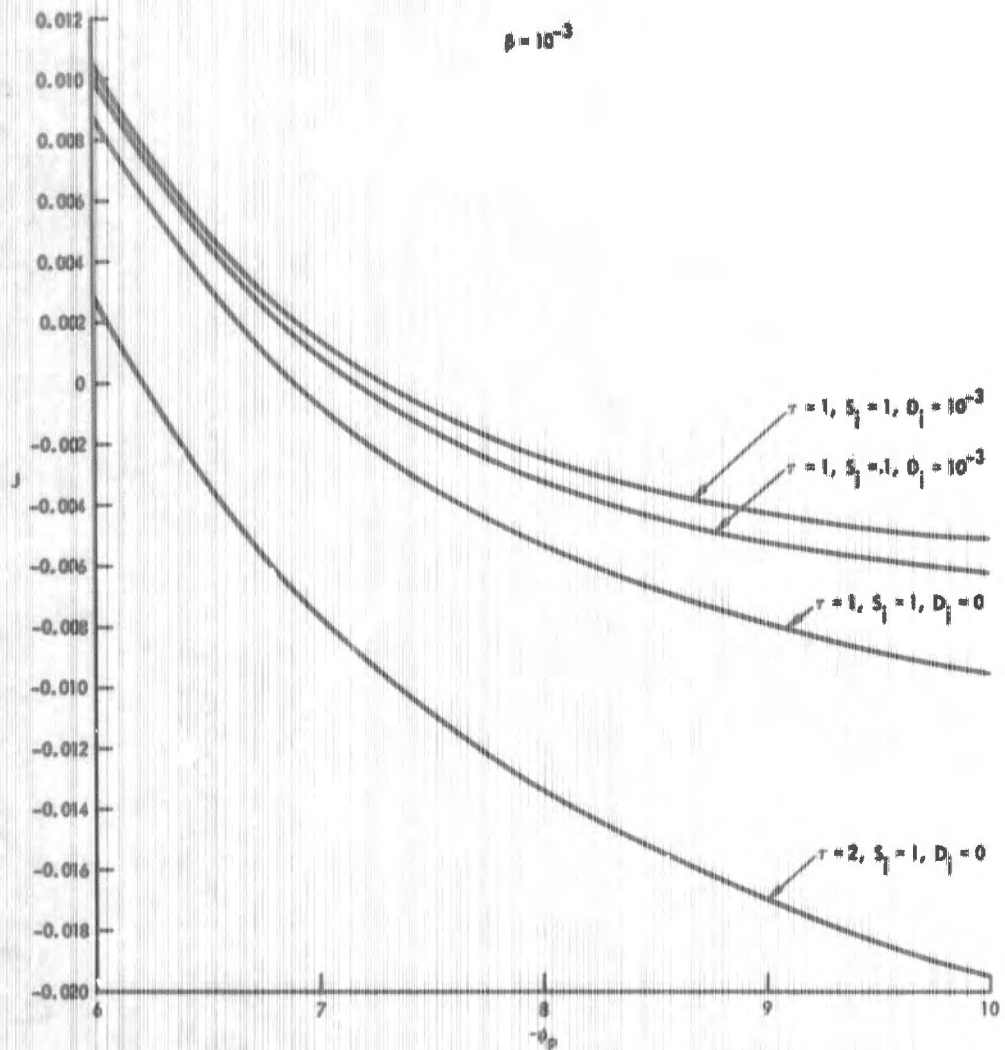


Figure 2. The magnitude of the so-called saturated electron current (negative J , slowly varying with increasing negative probe bias ϕ_p) is reduced for small departures from completely frozen flow ($0 < D_j \ll 1$).

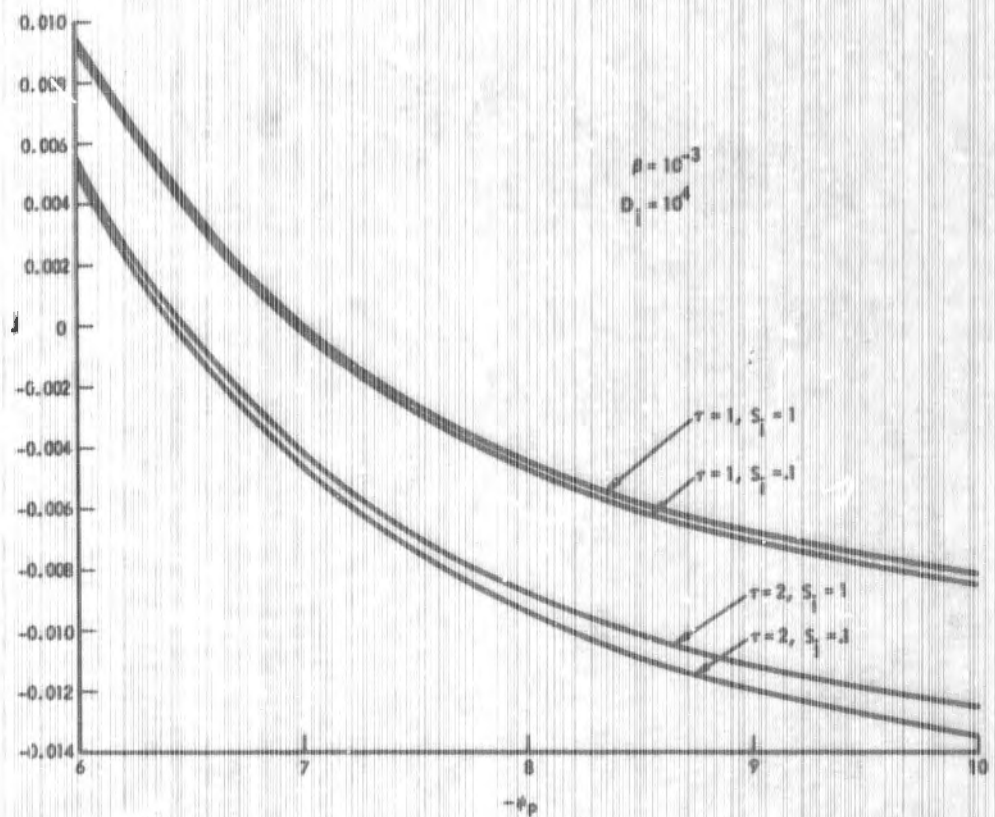


Figure 3. For constant (but not necessarily equal) ion and electron temperatures in nearly frozen flow at large Debye number, the current collected at the probe is more sensitive to the ratio of electron to ion temperatures τ than to the degree of species ionization under ambient conditions S_j .