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THEORETICAL INVESTIGATION OF ACOUSTIC SURFACE WAVES
ON PIEZOELECTRIC CRYSTALS

by

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Contract No. F19628-69-C-0132
Project No. 5635
Task No. 563503
Work Unit No. 56350301

FINAL REPORT

Period Covered: 1 December 1968 through 30 November 1969

4 December 1969

Contract Monitor: Andrew J. Slobodnik, Jr., 1/Lt, USAF
Microwave Physics Laboratory

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Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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ABSTRACT

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.

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I. INTRODUCTION

This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.

The physical configurations of the propagation problems considered in the sequel are shown in Figure 1 and are enumerated below:

- (1) Surface wave propagation on a piezoelectric half space in the presence of an infinitesimal electric or "magnetic" conductor located at an arbitrary but fixed distance h above the crystal surface.
- (2) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to a perfect isotropic elastic conductor (e.g. gold or aluminum) of arbitrary thickness h .
- (3) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to a perfect fluid half space.
- (4) Surface wave propagation on a piezoelectric or pure elastic half space contiguous to an isotropic elastic layer of arbitrary thickness h .

The following section contains the details of the analyses of the propagation problems described above including special degenerate cases which are encountered. These cases correspond to conditions of surface wave propagation wherein one or more components of displacement vanish or the electric and mechanical fields become decoupled (in the general piezoelectric case the surface wave contains all components of displacement and is coupled via the piezoelectric constants to the electric field). In practice degenerate cases have been found to occur when the sagittal plane lies either in a plane of symmetry of the crystal under consideration, in the basal plane of crystals of class 6 mm, or in the principal plane(s) of cubic crystals.

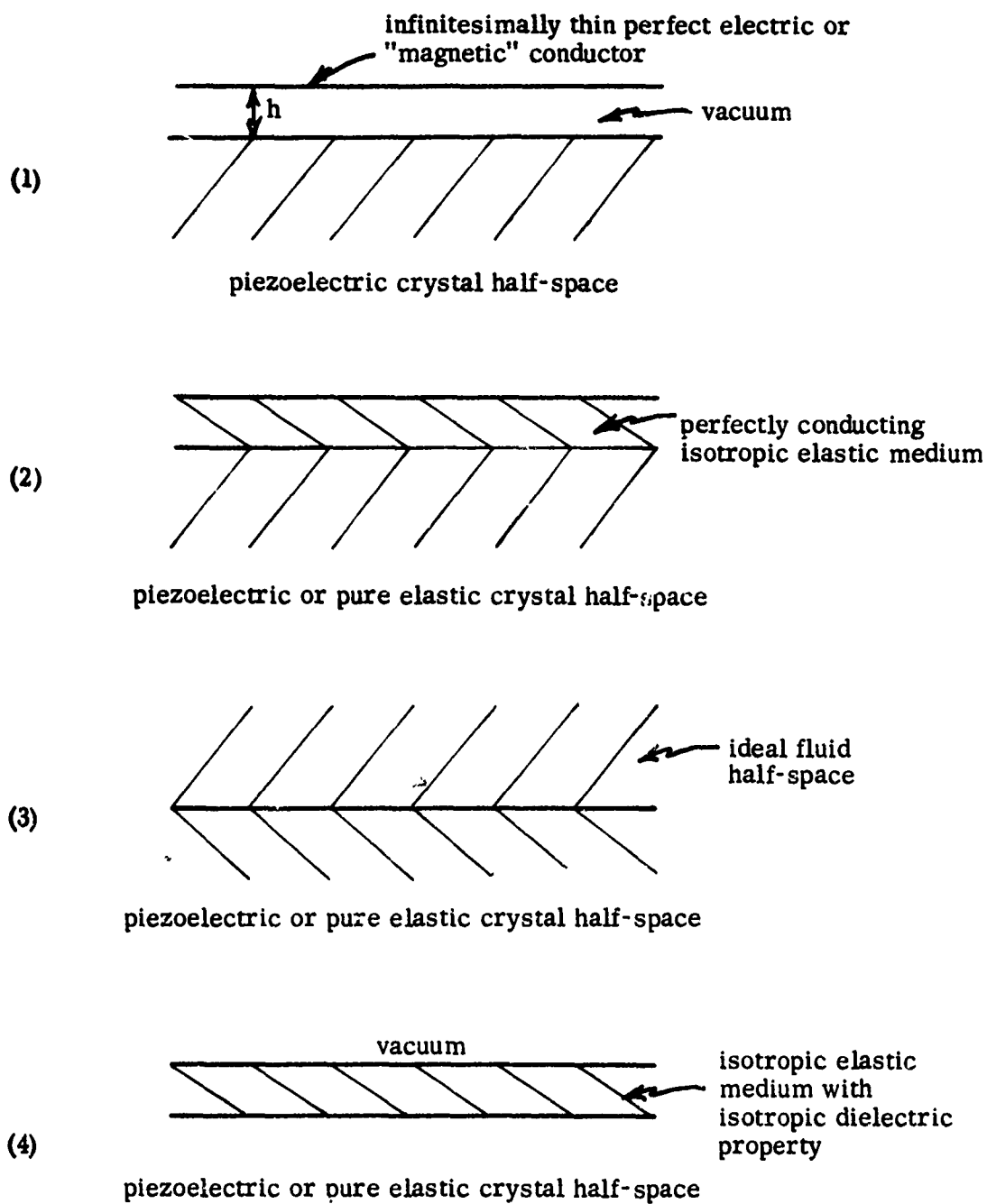


Figure 1

The third section presents an outline of the analysis pertaining to the excitation of surface and bulk wave by an electric current line source located above a piezoelectric half space. This problem was originally undertaken as an approach to the study of interdigital electric transducers but was ultimately abandoned due to lack of funds and the fact that inordinant amounts of computation would be required to extract useful data regarding the efficiency of excitation of surface waves.

Finally, Section II provides detailed descriptions of the computer programs written to implement the numerical analyses of the surface wave propagation problems described in Section II. The material presented in this section provides the reader with sufficient information for the use of the computer programs and the comprehension of the programming methods employed therein. Source deck listings for the various programs are also provided.

II. PROPAGATION OF SURFACE WAVES ON PIEZOELECTRIC SUBSTRATES

1. Surface Waves on Piezoelectric Crystals in the Presence of Infinitesimally Thin Electric and "Magnetic" Conductors

In this section the formal analysis is presented for the propagation characteristics of surface waves on a general piezoelectric crystal surface in the presence of perfect electric and "magnetic" conductors. The geometries under consideration are depicted in Figure 2.

A rectangular coordinate system is chosen with the x_3 axis normal to the crystal surface and the x_1 axis in the direction of propagation. Arbitrary orientations of the crystal surface with respect to the crystal axes are considered. This is accomplished by means of a coordinate rotation through the Euler angles from the crystal axes to the desired x_1, x_2, x_3 coordinate system. The matrix defining such a rotation is given by

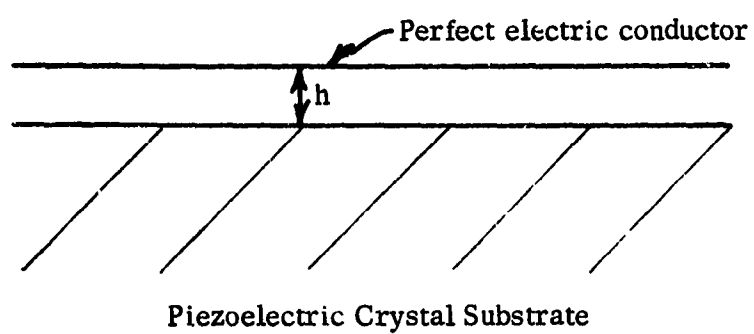
$$V = \begin{pmatrix} \cos \alpha \cos \gamma - \sin \alpha \cos \beta \sin \gamma & \sin \alpha \cos \gamma + \cos \alpha \cos \beta \sin \gamma & \sin \beta \sin \gamma \\ -\cos \alpha \sin \gamma - \sin \alpha \cos \beta \cos \gamma & -\sin \alpha \sin \gamma + \cos \alpha \cos \beta \cos \gamma & \sin \beta \cos \gamma \\ \sin \alpha \sin \beta & -\cos \alpha \sin \beta & \cos \beta \end{pmatrix}$$

where α , β , and γ are the Euler Angles. Since the x_1, x_2, x_3 coordinate system is relative to the crystal surface and direction of propagation, the form of the differential equations for the mechanical displacements and electric potentials in this coordinate system is independent of the surface under consideration. Only the values of the coefficients change with the surface orientation relative to the crystal axes. This is also true of the boundary conditions. Different cuts are thus distinguished only through the transformed tensor quantities involved in coefficients of the differential operators.

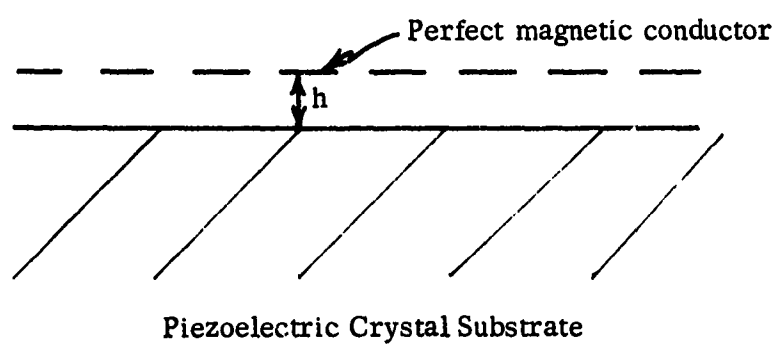
In terms of the Euler transformation matrix V , the tensor quantities of interest; viz. the elastic constants (C_{ijkl}), the piezoelectric constants (e_{ijk}), and the dielectric constants (ϵ_{ij}) are transformed as follows:

$$C'_{ijkl} = \sum_{r,s,t,u=1}^3 V_{ir} V_{js} V_{kt} V_{lu} C_{rstu} \quad , \quad (1)$$

$$e'_{ijk} = \sum_{r,s,t=1}^3 V_{ir} V_{js} V_{kt} e_{rst} \quad , \quad (2)$$



(a)



(b)

Figure 2

$$\epsilon'_{ij} = \sum_{r,s=1}^3 V_{ir} V_{js} \epsilon_{rs}, \quad (3)$$

where the primed quantities refer to a rotated coordinate system and the unprimed correspond to the coincidence of the $x_1 x_2 x_3$ coordinate system and the crystal axes, i.e., when V is the identity matrix. The elastic and piezo-electric constants can be reduced to 2 index symbols in the usual fashion, viz. $C'_{ijkl} \rightarrow C_{pq}$ and $e'_{ijk} \rightarrow e_{ip}$ where p or $q = 1, 2, 3, 4, 5, 6$ are equivalent to 11, 22, 33, 23, or 32, 13 or 31, and 12 or 21 respectively.

The differential equations for the components U_i , $i = 1, 2, 3$, of the mechanical displacement and electric potential φ are given by

$$\left. \begin{aligned} C'_{ijkl} U_{k,li} + e'_{kij} \varphi_{,ki} &= \rho \ddot{U}_j \\ e'_{ikl} U_{k,li} - \epsilon'_{ik} \varphi_{,ki} &= 0 \end{aligned} \right\} \quad \begin{matrix} j = 1, 2, 3 \\ x_3 > 0 \end{matrix} \quad (4)$$

$$\nabla^2 \varphi = 0 \quad -h \leq x_3 \leq 0. \quad (5)$$

In the above equations, indices preceded by a comma denote differentiation with respect to space coordinates. The summation convention for repeated indices is employed as is the dot notation for differentiation with respect to time.

As indicated above, the surface waves under consideration are assumed to be traveling in the x_1 direction along a surface whose normal is in the x_3 direction. The displacements and potentials are considered to be independent of the x_2 coordinate. Consequently, traveling wave solutions of the form

$$U_i = \beta_i e^{-\alpha \omega x_3 / v_s} e^{j\omega(t - x_1 / v_s)}$$

and

$$\varphi = \beta_4 e^{-\alpha \omega x_3 / v_s} e^{j\omega(t - x_1 / v_s)}$$

are sought. When these surface waveforms (as identified by an exponential decay into the crystal) are substituted into the differential equations for $x_3 > 0$ a linear homogeneous system of four equations in the unknowns $\beta_1, \beta_2, \beta_3, \beta_4$

results. The determinant of the coefficients of the unknowns in these equations must be zero in order that a non-trivial solution exist, that is

$$\det \begin{pmatrix}
 \begin{array}{c} C_{55}\alpha^2 + 2C_{15}j\alpha \\ - C_{11} + \rho v_s^2 \end{array} & \begin{array}{c} C_{45}\alpha^2 + [C_{14} + C_{56}]j\alpha \\ - C_{16} \end{array} \\
 \begin{array}{c} C_{45}\alpha^2 + [C_{14} + C_{56}]j\alpha \\ - C_{16} \end{array} & \begin{array}{c} C_{44}\alpha^2 + 2C_{46}j\alpha \\ - C_{66} + \rho v_s^2 \end{array} \\
 \begin{array}{c} C_{35}\alpha^2 + [C_{13} + C_{55}]j\alpha \\ - C_{15} \end{array} & \begin{array}{c} C_{34}\alpha^2 + [C_{36} + C_{45}]j\alpha \\ - C_{56} \end{array} \\
 \begin{array}{c} e_{35}\alpha^2 + [e_{15} + e_{31}]j\alpha \\ - e_{11} \end{array} & \begin{array}{c} e_{34}\alpha^2 + [e_{14} + e_{36}]j\alpha \\ - e_{16} \end{array}
 \end{pmatrix} \quad (6)$$

$$\begin{pmatrix}
 \begin{array}{c} C_{35}\alpha^2 + [C_{13} + C_{55}]j\alpha \\ - C_{15} \end{array} & \begin{array}{c} e_{35}\alpha^2 + [e_{15} + e_{31}]j\alpha \\ - e_{11} \end{array} \\
 \begin{array}{c} C_{34}\alpha^2 + [C_{36} + C_{45}]j\alpha \\ - C_{56} \end{array} & \begin{array}{c} e_{34}\alpha^2 + [e_{14} + e_{36}]j\alpha \\ - e_{16} \end{array} \\
 \begin{array}{c} C_{33}\alpha^2 + 2C_{35}j\alpha \\ - C_{55} + \rho v_s^2 \end{array} & \begin{array}{c} e_{33}\alpha^2 + [e_{13} + e_{35}]j\alpha \\ - e_{15} \end{array} \\
 \begin{array}{c} e_{33}\alpha^2 + [e_{13} + e_{35}]j\alpha \\ - e_{15} \end{array} & \begin{array}{c} - e_{33}\alpha^2 - 2e_{13}j\alpha \\ + e_{11} \end{array}
 \end{pmatrix} = 0$$

Evaluation of the above determinant results in an eighth order polynomial in α of the form

$$A_8\alpha^8 + jA_7\alpha^7 + A_6\alpha^6 + jA_5\alpha^5 + A_4\alpha^4 + jA_3\alpha^3 + A_2\alpha^2 + jA_1\alpha + A_0 = 0 \quad (7)$$

with the coefficients A_n , $n = 0, 1, \dots, 8$, purely real. Since the fields must be bounded, or go to zero as $x_3 \rightarrow \infty$, only the roots with non-negative real parts are allowed. If the unknown in equation (7) is considered to be $j\alpha$ instead of α then the polynomial in $j\alpha$ has purely real coefficients. Thus, either the roots $j\alpha$ are real or occur in conjugate pairs, e.g.

$$j\alpha_1 = a + jb$$

$$j\alpha_2 = a - jb$$

whence

$$\alpha_1 = b - ja$$

$$\alpha_2 = -b - ja$$

Therefore, the roots α are either pure imaginary or occur in pairs with positive and negative real parts.

In the range of velocities where generally surface waves can exist (i.e. velocities below the lowest bulk wave velocity in the direction of propagation under consideration) the roots occur such that four with positive real parts can be selected. However, if for a given velocity four such roots are not found the possibility of the existence of a degenerate* surface wave remains and must be considered. These special waves are discussed in detail in the section on degenerate cases. Upon obtaining the admissible values of α , corresponding values of β_1 (to within a constant factor) can be found for each α .

In addition to the equations for $x_3 > 0$, the differential equation (5) for $-h \leq x_3 \leq 0$ must be satisfied together with appropriate boundary conditions at $x_3 = 0$ and $x_3 = -h$. Assuming that the crystal surface is stress free ($T_{3j} = 0$ at $x_3 = 0$), the mechanical boundary conditions at each point of the surface of the crystal are

*The term degenerate is used to signify that certain components of displacement and/or the electric potential vanish identically.

$$T_{3j} \Big|_{x_3=0} = C'_{3jk} U_{k,i} + e'_{k3j} \varphi_{,k} \Big|_{x_3=0} = 0, \quad (8)$$

$$j = 1, 2, 3.$$

For the electric wall case (Figure 2a) the boundary conditions on the electric potential are the continuity of φ at $x_3 = 0$ and, without loss of generality, $\varphi(-h) = 0$. Also the normal component of electrical displacement must be continuous across the surface of the crystal.

The total fields (mechanical displacement and potential) may be expressed as a linear combination of the fields associated with the admissible values of α for $x_3 > 0$, namely,

$$U_i = \sum_{l=1}^4 B^{(l)} \beta_i^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}, \quad i = 1, 2, 3, \quad (9)$$

$$\varphi = \sum_{l=1}^4 B^{(l)} \beta_4^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}. \quad (10)$$

In the region $-h \leq x_3 \leq 0$ the potential is a solution of Laplace's equation (5). A solution satisfying the continuity condition at $x_3 = 0$ and vanishing at $x_3 = -h$ is

$$\varphi = \sum_{l=1}^4 B^{(l)} \beta_4^{(l)} \operatorname{csch} \left(\frac{\omega h}{v_s} \right) \sinh \left(\frac{\omega}{v_s} (x_3 + h) \right) e^{j\omega(t-x_1/v_s)}, \quad (11)$$

$$-h \leq x_3 \leq 0.$$

Finally, the normal component of $\vec{D}$ (viz. D_3) must be continuous across the surface $x_3 = 0$. Inside the crystal the electrical displacement is given by $D_i = e_{ikl} U_{k,l} - e_{ik} \varphi_{,k}$ ($i = 1, 2, 3$), while in the region $-h \leq x_3 \leq 0$, $\vec{D} = -\epsilon_0 \nabla \varphi$.

Substituting the waveforms (9), (10) in equation (8) and expressing the continuity of D_3 at $x_3 = 0$ in terms of equations (9), (10), and (11) yields the following set of homogeneous equations for the amplitudes $B^{(l)}$, namely,

$$\sum_{l=1}^4 \left[\beta_1^{(l)} [jC_{15} + \alpha^{(l)} C_{55}] + \beta_2^{(l)} [jC_{56} + \alpha^{(l)} C_{45}] + \beta_3^{(l)} [jC_{55} + \alpha^{(l)} C_{35}] + \beta_4^{(l)} [je_{15} + \alpha^{(l)} e_{35}] \right] B^{(l)} = 0 \quad , \quad (12)$$

$$\sum_{l=1}^4 \left[\beta_1^{(l)} [jC_{14} + \alpha^{(l)} C_{45}] + \beta_2^{(l)} [jC_{46} + \alpha^{(l)} C_{44}] + \beta_3^{(l)} [jC_{45} + \alpha^{(l)} C_{34}] + \beta_4^{(l)} [je_{14} + \alpha^{(l)} e_{34}] \right] B^{(l)} = 0 \quad , \quad (13)$$

$$\sum_{l=1}^4 \left[\beta_1^{(l)} [jC_{13} + \alpha^{(l)} C_{35}] + \beta_2^{(l)} [jC_{36} + \alpha^{(l)} C_{34}] + \beta_3^{(l)} [jC_{35} + \alpha^{(l)} C_{33}] + \beta_4^{(l)} [je_{13} + \alpha^{(l)} e_{33}] \right] B^{(l)} = 0 \quad , \quad (14)$$

$$\sum_{l=1}^4 \left[\beta_1^{(l)} [je_{31} + \alpha^{(l)} e_{35}] + \beta_2^{(l)} [je_{36} + \alpha^{(l)} e_{34}] + \beta_3^{(l)} [je_{35} + \alpha^{(l)} e_{33}] - \beta_4^{(l)} \left[je_{13} + \alpha^{(l)} e_{33} + \epsilon_0 \coth \left(\frac{uh}{v_s} \right) \right] \right] B^{(l)} = 0 \quad . \quad (15)$$

The transcendental equation obtained by setting the determinant of the matrix ($\hat{L}$) of coefficients of this system equal to zero determines the surface wave velocities.

In the limiting case ($h \rightarrow 0$) the region $-h \leq x_3 \leq 0$ disappears and the boundary conditions on the electric potential and normal component of displacement in the crystal are replaced by $\varphi(0) = 0$. In this case equation (15) above reduces to

$$\sum_{l=1}^4 \beta_4^{(l)} B^{(l)} = 0 \quad . \quad (16)$$

In addition to the electric wall problem, the magnetic wall case (Figure 2b) also has been considered. The only change in the formulation of this problem is the boundary condition at $x_3 = -h$. In this case the solution for the potential in the region $-h \leq x_3 \leq 0$ assumes the form

$$D_3 = \sum_{l=1}^4 B^{(l)} \beta_4^{(l)} \operatorname{sech}\left(\frac{uh}{v_s}\right) \cosh\left(\frac{\omega}{v_s}(x_3 + h)\right) e^{j\omega(t-x_1/v_s)} \quad (17)$$

This function satisfies the condition that the normal component of electrical displacement (D_3) vanish at the magnetic wall $x_3 = -h$.

Equation (15), the continuity of D_3 at $x_3 = 0$, is modified for the magnetic wall case by replacing $\coth(uh/v_s)$ with $\tanh(uh/v_s)$. Otherwise the equations (12)-(15) remain unchanged. The limiting case $h \rightarrow 0$ requires no special change as it did in the electric wall case since the continuity condition on D_3 at $x_3 = 0$ now simply becomes $D_3 = 0$ and $D_3|_{x_3=0} \sim \tanh(uh/v_s)$ (outside the crystal), which goes to zero as $h \rightarrow 0$. Thus equation (15) with the $\tanh(uh/v_s)$ term needs no modification for the limiting case.

Once a surface wave velocity has been found the partial field amplitudes $B^{(j)}$, $j=1, 2, 3, 4$ may be calculated to within a constant factor. Consequently, $B^{(4)}$ is chosen as unity (except for certain degenerate cases described later) and $B^{(1)}$, $B^{(2)}$, $B^{(3)}$ are found from equations (12)-(14). These amplitudes are used to evaluate the displacement components (eq. (9)), electric potential (eq. (10)), the components of stress, strain, electric displacement, electric field, and the time average power flow as functions of ωx_3 . The explicit forms of the components of the aforementioned physical quantities are:

Stress

$$\begin{aligned} \frac{T_{11}}{\omega} &= \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} C_{11} - \frac{\alpha^{(l)}}{v_s} C_{15} \right] \right. \\ &\quad \left. + \beta_2^{(l)} \left[\frac{-j}{v_s} C_{16} - \frac{\alpha^{(l)}}{v_s} C_{14} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} C_{15} - \frac{\alpha^{(l)}}{v_s} C_{13} \right] + \beta_4^{(l)} \left[\frac{-j}{v_s} e_{11} - \frac{\alpha^{(l)}}{v_s} e_{31} \right] \right\} \\ \frac{T_{1z}}{\omega} = \frac{T_{21}}{\omega} &= \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} C_{16} - \frac{\alpha^{(l)}}{v_s} C_{56} \right] \right. \\ &\quad \left. + \beta_2^{(l)} \left[\frac{-j}{v_s} C_{66} - \frac{\alpha^{(l)}}{v_s} C_{46} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} C_{56} - \frac{\alpha^{(l)}}{v_s} C_{36} \right] + \beta_4^{(l)} \left[\frac{-j}{v_s} e_{16} - \frac{\alpha^{(l)}}{v_s} e_{36} \right] \right\} \end{aligned}$$

$$\frac{T_{13}}{\omega} = \frac{T_{31}}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} C_{15} - \frac{\alpha^{(l)}}{v_s} C_{55} \right] \right. \\ \left. + \beta_2^{(l)} \left[\frac{-j}{v_s} C_{56} - \frac{\alpha^{(l)}}{v_s} C_{45} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} C_{55} - \frac{\alpha^{(l)}}{v_s} C_{35} \right] + \beta_4^{(l)} \left[\frac{-j}{v_s} e_{15} - \frac{\alpha^{(l)}}{v_s} e_{35} \right] \right\}$$

$$\frac{T_{22}}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} C_{12} - \frac{\alpha^{(l)}}{v_s} C_{25} \right] \right. \\ \left. + \beta_2^{(l)} \left[\frac{-j}{v_s} C_{26} - \frac{\alpha^{(l)}}{v_s} C_{24} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} C_{25} - \frac{\alpha^{(l)}}{v_s} C_{23} \right] + \beta_4^{(l)} \left[\frac{-j}{v_s} e_{12} - \frac{\alpha^{(l)}}{v_s} e_{32} \right] \right\}$$

$$\frac{T_{23}}{\omega} = \frac{T_{32}}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} C_{14} - \frac{\alpha^{(l)}}{v_s} C_{45} \right] \right. \\ \left. + \beta_2^{(l)} \left[\frac{-j}{v_s} C_{46} - \frac{\alpha^{(l)}}{v_s} C_{44} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} C_{45} - \frac{\alpha^{(l)}}{v_s} C_{34} \right] + \beta_4^{(l)} \left[\frac{-j}{v_s} e_{14} - \frac{\alpha^{(l)}}{v_s} e_{34} \right] \right\}$$

$$\frac{T_{33}}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} C_{13} - \frac{\alpha^{(l)}}{v_s} C_{35} \right] \right. \\ \left. + \beta_2^{(l)} \left[\frac{-j}{v_s} C_{36} - \frac{\alpha^{(l)}}{v_s} C_{34} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} C_{35} - \frac{\alpha^{(l)}}{v_s} C_{33} \right] + \beta_4^{(l)} \left[\frac{-j}{v_s} e_{13} - \frac{\alpha^{(l)}}{v_s} e_{33} \right] \right\}$$

Strain

$$\frac{S_{11}}{\omega} = \sum_{l=1}^4 \frac{-j}{v_s} B^{(l)} \beta_1^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{22}}{\omega} = 0$$

$$\frac{S_{33}}{\omega} = \sum_{l=1}^4 \frac{-\alpha^{(l)}}{v_s} B^{(l)} \beta_3^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{12}}{\omega} = \frac{S_{21}}{\omega} = \frac{1}{2} \sum_{l=1}^4 \frac{-j}{v_s} B^{(l)} \beta_2^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{13}}{\omega} = \frac{S_{31}}{\omega} = \frac{1}{2} \sum_{l=1}^4 B^{(l)} \left[\frac{-\alpha^{(l)}}{v_s} \beta_1^{(l)} - \frac{j}{v_s} \beta_3^{(l)} \right] e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

$$\frac{S_{23}}{\omega} = \frac{S_{32}}{\omega} = \frac{1}{2} \sum_{l=1}^4 \frac{-\alpha^{(l)}}{v_s} B^{(l)} \beta_2^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

Electric Field

$$\frac{E_1}{\omega} = \frac{j}{v_s} \sum_{l=1}^4 B^{(l)} \beta_4^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} = \frac{j}{v_s} \varphi$$

$$\frac{E_3}{\omega} = \frac{1}{v_s} \sum_{l=1}^4 \alpha^{(l)} B^{(l)} \beta_4^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

Electric Displacement

$$\begin{aligned} \frac{D_1}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} & \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} e_{11} - \frac{\alpha^{(l)}}{v_s} e_{15} \right] \right. \\ & + \beta_2^{(l)} \left[\frac{-j}{v_s} e_{16} - \frac{\alpha^{(l)}}{v_s} e_{14} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} e_{15} - \frac{\alpha^{(l)}}{v_s} e_{13} \right] - \beta_4^{(l)} \left[\frac{-j}{v_s} e_{11} - \frac{\alpha^{(l)}}{v_s} e_{13} \right] \left. \right\} \end{aligned}$$

$$\begin{aligned} \frac{D_2}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} & \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} e_{21} - \frac{\alpha^{(l)}}{v_s} e_{25} \right] \right. \\ & + \beta_2^{(l)} \left[\frac{-j}{v_s} e_{26} - \frac{\alpha^{(l)}}{v_s} e_{24} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} e_{25} - \frac{\alpha^{(l)}}{v_s} e_{23} \right] - \beta_4^{(l)} \left[\frac{-j}{v_s} e_{21} - \frac{\alpha^{(l)}}{v_s} e_{23} \right] \left. \right\} \end{aligned}$$

$$\begin{aligned} \frac{D_3}{\omega} = \sum_{l=1}^4 B^{(l)} e^{-\alpha^{(l)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} & \left\{ \beta_1^{(l)} \left[\frac{-j}{v_s} e_{31} - \frac{\alpha^{(l)}}{v_s} e_{35} \right] \right. \\ & + \beta_2^{(l)} \left[\frac{-j}{v_s} e_{36} - \frac{\alpha^{(l)}}{v_s} e_{34} \right] + \beta_3^{(l)} \left[\frac{-j}{v_s} e_{35} - \frac{\alpha^{(l)}}{v_s} e_{33} \right] - \beta_4^{(l)} \left[\frac{-j}{v_s} e_{31} - \frac{\alpha^{(l)}}{v_s} e_{33} \right] \left. \right\} \end{aligned}$$

Power Flow

The flow of complex mechanical power at any point in the piezoelectric medium is given in component form as follows:

$$P_i = -\frac{1}{2} \sum_{l=1}^3 T_{il} \dot{U}_l^*$$

The real part of this expression represents the time average power flow at a point.

Since all fields decay exponentially in the x_3 direction there is no net flow of real power in this direction. Thus, only P_1 and P_2 need be considered.

The components of the total time-average power flow are as follows:

$$P_1^t = \int_0^\infty \text{Re} [P_1] dx_3 = \text{Re} [P_1^t]$$

$$P_2^t = \int_0^\infty \text{Re} [P_2] dx_3 = \text{Re} [P_2^t]$$

where $\text{Re} [P_{1,2}]$ means the real part of $P_{1,2}$. The final expressions for the complex mechanical power flow P_1^t and P_2^t are

$$\begin{aligned} \frac{P_1^t}{\omega} = \frac{1}{2} \sum_{l=1}^4 \sum_{k=1}^4 \frac{1}{[\alpha^{(l)} + \alpha^{*(k)}]} B^{(l)} B^{*(k)} & \left\{ \beta_1^{*(k)} [\beta_1^{(l)} (C_{11} - j\alpha^{(l)} C_{15}) \right. \\ & + \beta_2^{(l)} (C_{16} - j\alpha^{(l)} C_{14}) + \beta_3^{(l)} (C_{15} - j\alpha^{(l)} C_{13}) + \beta_4^{(l)} (e_{11} - j\alpha^{(l)} e_{31})] \\ & + \beta_2^{*(k)} [\beta_1^{(l)} (C_{16} - j\alpha^{(l)} C_{56}) + \beta_2^{(l)} (C_{66} - j\alpha^{(l)} C_{46}) + \beta_3^{(l)} (C_{56} - j\alpha^{(l)} C_{36}) \\ & + \beta_4^{(l)} (e_{16} - j\alpha^{(l)} e_{36})] \\ & + \beta_3^{*(k)} [\beta_1^{(l)} (C_{15} - j\alpha^{(l)} C_{55}) + \beta_2^{(l)} (C_{56} - j\alpha^{(l)} C_{45}) + \beta_3^{(l)} (C_{55} - j\alpha^{(l)} C_{35}) \\ & \left. + \beta_4^{(l)} (e_{15} - j\alpha^{(l)} e_{35})] \right\} \end{aligned}$$

$$\begin{aligned}
\frac{P_2^t}{\omega} = \frac{1}{2} \sum_{l=1}^4 \sum_{k=1}^4 \frac{1}{[\alpha^{(l)} + \alpha^{*(k)}]} B^{(l)} B^{*(k)} \left\{ \beta_1^{*(k)} [\beta_1^{(l)} (C_{16} - j\alpha^{(l)} C_{56}) \right. \\
+ \beta_2^{(l)} (C_{66} - j\alpha^{(l)} C_{46}) + \beta_3^{(l)} (C_{56} - j\alpha^{(l)} C_{36}) + \beta_4^{(l)} (e_{16} - j\alpha^{(l)} e_{36})] \\
+ \beta_2^{*(k)} [\beta_1^{(l)} (C_{12} - \alpha^{(l)} C_{25}) + \beta_2^{(l)} (C_{26} - j\alpha^{(l)} C_{24}) + \beta_3^{(l)} (C_{25} - j\alpha^{(l)} C_{23}) \\
+ \beta_4^{(l)} (e_{12} - j\alpha^{(l)} e_{32})] \\
+ \beta_3^{*(k)} [\beta_1^{(l)} (C_{14} - j\alpha^{(l)} C_{45}) + \beta_2^{(l)} (C_{46} - j\alpha^{(l)} C_{44}) + \beta_3^{(l)} (C_{45} - j\alpha^{(l)} C_{34}) \\
\left. + \beta_4^{(l)} (e_{14} - j\alpha^{(l)} e_{34})] \right\} .
\end{aligned}$$

The flow of electromagnetic power (Poynting Vector) in a piezoelectric medium requires a knowledge of the magnetic field as well as the electric field. It is a common belief (although an incorrect one) that the complex Poynting vector $\mathbf{E} \times \mathbf{H}^*$ reduces to $\phi \dot{\mathbf{D}}^*$ when the electric field is approximately derivable from a scalar potential function ϕ . This mistaken notion is based upon the following derivation for energy flow out of a closed surface.

Maxwell's equations are as follows for fields derivable approximately from a scalar potential function (i.e. where $\vec{E} \approx -\nabla\phi$) in a non-conducting medium.

$$\begin{aligned}
\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t} \approx 0 & \qquad \nabla \cdot \mathbf{B} = 0 \\
\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \dot{\mathbf{D}} & \qquad \nabla \cdot \mathbf{D} = 0
\end{aligned}$$

Now the flow of power out of any closed surface with a surface normal element $d\vec{s}$ is

$$P = \frac{1}{2} \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot d\vec{s}$$

but

$$\iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot d\vec{s} = \iiint_V \nabla \cdot (\mathbf{E} \times \mathbf{H}^*) dv$$

where the second integral is over the volume enclosed by the surface S . Using

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) = \mathbf{H}^* \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H}^* \approx -\mathbf{E} \cdot \nabla \times \mathbf{H}^*$$

it follows that

$$\frac{1}{2} \iiint_V \nabla \cdot (\mathbf{E} \times \mathbf{H}^*) \, dv \approx -\frac{1}{2} \iiint_V \mathbf{E} \cdot \nabla \times \mathbf{H}^* \, dv = \frac{1}{2} \iiint_V (\nabla \phi \cdot \dot{\mathbf{D}}^*) \, dv$$

Furthermore

$$\nabla \cdot (\phi \dot{\mathbf{D}}^*) = \nabla \phi \cdot \dot{\mathbf{D}}^* + \phi \nabla \cdot (\dot{\mathbf{D}}^*) = \nabla \phi \cdot \dot{\mathbf{D}}^*$$

may be used to infer that

$$\begin{aligned} \frac{1}{2} \iiint_V \nabla \cdot (\mathbf{E} \times \mathbf{H}^*) \, dv &\approx \frac{1}{2} \iiint_V \nabla \cdot (\phi \dot{\mathbf{D}}^*) \, dv \\ &= \frac{1}{2} \iint_S \phi \dot{\mathbf{D}}^* \cdot d\vec{a} \end{aligned}$$

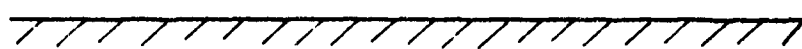
Therefore

$$P = \frac{1}{2} \iint_S (\mathbf{E} \times \mathbf{H}^*) \cdot d\vec{s} \approx \frac{1}{2} \iint_S \phi \dot{\mathbf{D}}^* \cdot d\vec{a}$$

This equation states that the surface integral over a closed surface of $\mathbf{E} \times \mathbf{H}^*$ is equal to that of $\phi \dot{\mathbf{D}}^*$ or alternately that $\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) = \nabla \cdot (\phi \dot{\mathbf{D}}^*)$, consequently $\mathbf{E} \times \mathbf{H}^* = \phi \dot{\mathbf{D}}^* + \vec{A}$ where $\vec{A}$ is some vector whose divergence is zero ($\nabla \cdot \vec{A} = 0$).

It seems a natural assumption to assume that $\vec{A} = 0$ and that $\mathbf{E} \times \mathbf{H}^* = \phi \dot{\mathbf{D}}^*$. However, this is not necessarily so as an illustrative example from electro-magnetic theory shows.

Consider a surface wave propagating in the free space region



Impedance
Sheet

A solution of the following form will exist.

$$H_z = e^{-j\beta x} e^{-\sqrt{\beta^2 - k_0^2} y}$$

$$E_x = \frac{-\sqrt{\beta^2 - k_0^2}}{j\omega\epsilon} H_z$$

$$E_y = \frac{\beta}{\omega\epsilon} H_z$$

The wave impedance of this wave is

$$Z = \frac{E_x}{H_z} = \frac{+j\sqrt{\beta^2 - k_0^2}}{\omega\epsilon}$$

Assume an impedance sheet with surface impedance jX_s . Then we find

$$\frac{\sqrt{\beta^2 - k_0^2}}{\omega\epsilon} = X_s$$

determining the value of β . If $k_0 \ll \beta$ (low frequency limit) then we have

$$\beta \approx \omega\epsilon X_s$$

and the field quantities assume the form

$$H_z \approx e^{-j\beta x} e^{-\beta y},$$

$$E_x \approx j \frac{\beta}{\omega\epsilon} H_z, \quad \text{and} \quad E_y = \frac{\beta}{\omega\epsilon} H_z.$$

This is the quasi-static case ($k_0 \ll \beta$) and an approximate expression for the electric field is derivable from a scalar potential. Indeed, the scalar potential may be taken to be

$$\varphi = \frac{H_z}{\omega\epsilon} \approx \frac{e^{-j\beta x} e^{-\beta y}}{\omega\epsilon}$$

with the result $\vec{E} \approx -\nabla\varphi$.

For this approximation the x component of the Poynting vector ($\vec{E} \times \vec{H}^*$)

reduces to $N_x = E_y H_z^* = \beta / \omega \epsilon e^{-2\beta y}$ while

$$\begin{aligned}\phi \dot{D}_x^* &= \frac{H_z}{\omega \epsilon} \epsilon \dot{E}_x^* \\ &= -\frac{\beta}{\omega \epsilon} e^{-2\beta y},\end{aligned}$$

and clearly $\vec{N} \neq \phi \dot{D}^*$.

Summing up, the electromagnetic power flow ($E \times H^*$) at any point requires a full solution of Maxwell's equations and cannot be obtained from the scalar potential method. Only the total power flow out of a closed surface can be obtained from the scalar potential method since

$$\iint_S E \times H^* \cdot d\vec{a} = \iint_S \phi \dot{D}^* \cdot d\vec{a}$$

where S is closed. A more thorough investigation into the electromagnetic power flow will be taken up during the transducer study part of the study. For now it can be said that the electromagnetic power flow is expected to be much smaller than the mechanical power flow based on computer runs where $\phi \dot{D}^*$ was used as an order of magnitude estimate of the Poynting vector.

2. Surface Waves on Non-Piezoelectric (Pure Elastic) Media

Surface wave propagation on a free surface of a non-piezoelectric elastic medium can be accounted for by appropriate modifications of the foregoing analysis. In this case the piezoelectric constants e_{ijk} are identically zero and the electric field and mechanical displacements are decoupled. Consequently, the fourth order matrix appearing in equation (6) reduces to the third order matrix obtained by deleting the last row and column. Subject to this modification equation (7) reduces to a sixth order equation in α which in general will have three roots with positive real parts. The boundary conditions at the free surface are given by equation (8) with the coefficients $e'_{k3j} \equiv 0$. Inasmuch as there is no coupling to an electric field the additional boundary conditions applicable thereto are unnecessary.

The relative amplitudes of the component of displacement $\beta_k^{(l)}$, $k = 1, 2, 3$ are evaluated for each $\alpha^{(l)}$ with positive real part in a manner identical to that used in the general piezoelectric case. Upon evaluation of these quantities the boundary conditions are invoked and the equations (12-14) result with the piezoelectric constants equal to zero and the sums only over the indices 1, 2, and 3. The characteristic equation for the determination of surface wave velocities again obtains from the condition that the determinant of the matrix coefficients associated with the linear system (12-14) vanishes.

The stresses, strains, power flow, etc. may be calculated as before by using the appropriate evaluations of $\beta_k^{(l)}$, $k = 1, 2, 3$, $B^{(k)}$, $k = 1, 2, 3$ and setting the β_4 's, $A^{(4)}$, and e_{lp} equal to zero in the equations for these quantities given previously.

3. Degenerate Cases -- Piezoelectric Medium

The modes of surface wave propagation described as degenerate cases arise when the four coupled partial differential equations (4) which govern the mechanical components u_1 , u_2 , and u_3 and the electric field (via a scalar potential φ) reduce to two independent sets of coupled equations, assuming traveling wave motions independent of the coordinate normal to the sagittal plane.

With the coordinate system chosen assuming no variation in the x_2 direction, the equations of motion (4) for the displacement components and potential may be written in the operator form (assuming $e^{j\omega t}$ time dependence)

$$L_{11}U_1 + L_{12}U_2 + L_{13}U_3 + L_{14}\varphi = 0$$

$$L_{21}U_1 + L_{22}U_2 + L_{23}U_3 + L_{24}\varphi = 0$$

$$L_{31}U_1 + L_{32}U_2 + L_{33}U_3 + L_{34}\varphi = 0$$

$$L_{41}U_1 + L_{42}U_2 + L_{43}U_3 + L_{44}\varphi = 0 \quad ,$$

where

$$L_{11} = C_{55} \frac{\partial^2}{\partial x_3^2} + 2C_{15} \frac{\partial^2}{\partial x_3 \partial x_1} + C_{11} \frac{\partial^2}{\partial x_1^2} + w^2 \rho$$

$$L_{12} = L_{21} = C_{45} \frac{\partial^2}{\partial x_3^2} + (C_{14} + C_{56}) \frac{\partial^2}{\partial x_3 \partial x_1} + C_{16} \frac{\partial^2}{\partial x_1^2}$$

$$L_{13} = L_{31} = C_{35} \frac{\partial^2}{\partial x_3^2} + (C_{13} + C_{55}) \frac{\partial^2}{\partial x_3 \partial x_1} + C_{15} \frac{\partial^2}{\partial x_1^2}$$

$$L_{14} = L_{41} = e_{35} \frac{\partial^2}{\partial x_3^2} + (e_{15} + e_{31}) \frac{\partial^2}{\partial x_3 \partial x_1} + e_{11} \frac{\partial^2}{\partial x_1^2}$$

$$L_{22} = C_{44} \frac{\partial^2}{\partial x_3^2} + 2C_{46} \frac{\partial^2}{\partial x_3 \partial x_1} + C_{66} \frac{\partial^2}{\partial x_1^2} + w^2 \rho$$

$$L_{23} = L_{32} = C_{34} \frac{\partial^2}{\partial x_3^2} + (C_{36} + C_{45}) \frac{\partial^2}{\partial x_3 \partial x_1} + C_{56} \frac{\partial^2}{\partial x_1^2}$$

$$L_{24} = L_{42} = e_{34} \frac{\partial^2}{\partial x_3^2} + (e_{14} + e_{36}) \frac{\partial^2}{\partial x_3 \partial x_1} + e_{16} \frac{\partial^2}{\partial x_1^2}$$

$$L_{33} = C_{33} \frac{\partial^2}{\partial x_3^2} + 2C_{35} \frac{\partial^2}{\partial x_3 \partial x_1} + C_{55} \frac{\partial^2}{\partial x_1^2} + w^2 \rho$$

$$L_{34} = L_{43} = e_{33} \frac{\partial^2}{\partial x_3^2} + (e_{13} + e_{35}) \frac{\partial^2}{\partial x_3 \partial x_1} + e_{15} \frac{\partial^2}{\partial x_1^2}$$

$$L_{44} = - e_{33} \frac{\partial^2}{\partial x_3^2} - 2e_{13} \frac{\partial^2}{\partial x_3 \partial x_1} - e_{11} \frac{\partial^2}{\partial x_1^2}$$

The elastic c_{ij} , piezoelectric e_{ij} , and dielectric ϵ_{ij} constants refer to the transformed (from the crystal coordinate system) quantities and are represented in terms of the abbreviated double subscript notation.

If the elastic and piezoelectric constants are such that the operator matrix $[L_{ij}]_{i,j=1,2,3,4}$ is appropriately sparse, the equations (1) decouple and the possibility of degenerate cases is encountered.

For example, in the case reported by Bleustein⁽⁴⁾ the elastic and piezoelectric constants are such that $L_{12} = L_{14} = L_{23} = L_{34} \equiv 0$ and the equations of motion decouple into two independent systems; one system governing u_2 and φ and the other characterizing the behaviors of u_1 and u_3 . This is an example of one of the two general degenerate cases which has been found to exist for a number of crystals on particular cuts and directions of propagation.

A second degenerate case which also has been reported and which appears with some regularity is manifest by the conditions $L_{12} = L_{23} = L_{24} \equiv 0$. In this case the equations of motion decouple into a coupled system of partial differential equations for the displacement components u_1 , u_3 and the potential φ , and a single partial differential equation for the displacement component u_2 . This particular degenerate case has been studied extensively for surface wave propagation on the basal plane of hexagonal crystals⁽⁷⁾ and it has been shown that a surface wave solution with the displacement component u_2 alone cannot exist. It should be noted that the latter observation carries over to the general case, independent of the crystal class, surface cut, and direction of propagation.

There are other degenerate cases that can be considered. For example,

$$L_{12} = L_{13} = L_{14} \equiv 0$$

or

$$L_{31} = L_{32} = L_{34} \equiv 0$$

These cases were considered in the analysis leading to the present study but numerical examples of these cases have not been found.

The conditions $L_{41} = L_{42} = L_{43} \equiv 0$ lead to a complete decoupling of the electric and mechanical fields and has not been found to occur for surface wave propagation on specific surfaces of any piezoelectric crystal considered thus far.

The occurrence of degenerate cases can also be described in terms of the linear equations for the relative amplitudes, β_l , $l = 1, 2, 3, 4$, of the mechanical displacement and electric potential. The determinant of the coefficients of this system of equations is given by equation (6), wherein the elements of the determinant correspond to evaluations of the operators L_{ij} with

$$\frac{\partial}{\partial x_1} = -j \frac{\omega}{v_s} \quad \text{and} \quad \frac{\partial}{\partial x_3} = - \frac{\alpha \omega}{v_s} .$$

For the sake of the following discussions let the linear system of equations for the determination of the β 's be denoted

$$\sum_{l=1}^4 A_{il} \beta_l = 0 \quad , \quad i = 1, 2, 3, 4 \quad . \quad (18)$$

The possible combinations of the elastic and piezoelectric constants which caused the equations of motion to decouple lead to the decoupling of the linear equations in the exact same fashion. Inasmuch as the linear equations (18) are employed in the numerical analyses, the various cases of decoupling are discussed again in more detail below.

The following degenerate cases have been found to exist and are accounted for in the computer program which implements the numerical analysis. They are denoted by representing the matrix $\hat{A} = \{A_{il}\}_{i,l=1,2,3,4}$ with its appropriate zeros displayed, viz.,

$$\begin{array}{cc} \text{Case (1)} & \begin{vmatrix} A_{11} & 0 & A_{13} & A_{14} \\ 0 & A_{22} & 0 & 0 \\ A_{13} & 0 & A_{33} & A_{34} \\ A_{14} & 0 & A_{34} & A_{44} \end{vmatrix} \end{array} \quad \begin{array}{cc} \text{Case (2)} & \begin{vmatrix} A_{11} & 0 & A_{13} & 0 \\ 0 & A_{22} & 0 & A_{24} \\ A_{13} & 0 & A_{33} & 0 \\ 0 & A_{24} & 0 & A_{44} \end{vmatrix} \end{array}$$

In Case (1) we note that β_2 decouples from β_1 , β_3 , and β_4 . The determinant of $\hat{A}$ is zero if $A_{22} = 0$ (as a function of α) or if the determinant

$$A_1 = \begin{vmatrix} A_{11} & A_{13} & A_{14} \\ A_{13} & A_{33} & A_{34} \\ A_{14} & A_{34} & A_{44} \end{vmatrix} = 0 \quad .$$

The condition $A_{22} = 0$ leads to a quadratic equation in α . If $A_{22} = 0$ the system of equations yields a solution $\beta_1 = \beta_3 = \beta_4 = 0$ while β_2 can be chosen as an arbitrary constant.

If $A_1 = 0$, giving a sixth order equation in α , the system of equations requires a solution $\beta_2 = 0$ while either β_1 , β_3 , or β_4 may be chosen arbitrarily and the remaining two β 's calculated from any two of the three equations not involving A_{22} .

In case (2) we note that β_1 and β_3 decouple from β_2 and β_4 . The determinant of $\hat{A}$ goes to zero if the determinant

$$A_2 = \begin{vmatrix} A_{11} & A_{13} \\ A_{13} & A_{33} \end{vmatrix} = 0$$

or the determinant

$$A_3 = \begin{vmatrix} A_{22} & A_{24} \\ A_{24} & A_{44} \end{vmatrix} = 0.$$

Both the equations $A_2 = 0$ and $A_3 = 0$ lead to quartic equations in α . If $A_2 = 0$ the system yields the solution $\beta_2 = \beta_4 = 0$ while β_1 or β_3 may be arbitrarily chosen and the remaining β calculated from either the first or third equation of the system. If $A_3 = 0$ the system yields the solution $\beta_1 = \beta_3 = 0$ while β_2 or β_4 may be arbitrarily chosen and the remaining β calculated from either the second or fourth equation of the system.

Let the coefficients of the amplitudes $B^{(l)}$ (equations (12)-(16)) be considered elements of the matrix $\hat{L}$. In case (1) $\hat{L}$ takes the form

$$\begin{vmatrix} 0 & L_{12} & L_{13} & L_{14} \\ L_{21} & 0 & 0 & 0 \\ 0 & L_{32} & L_{33} & L_{34} \\ 0 & L_{42} & L_{43} & L_{44} \end{vmatrix}.$$

If the determinant

$$L_1 = \begin{vmatrix} L_{12} & L_{13} & L_{14} \\ L_{32} & L_{33} & L_{34} \\ L_{42} & L_{43} & L_{44} \end{vmatrix} = 0$$

(considered as a function of the velocity v_s) then the following solution is found: $B^{(1)} = 0$ while $B^{(2)}$, $B^{(3)}$, or $B^{(4)}$ can be chosen arbitrarily and the remaining B's calculated from any two of the three equations not involving L_{21} . This situation corresponds to a wave with displacement components U_1 , U_3 and potential φ and U_2 is identically zero.

If L_{21} is zero we would be led to a solution where U_1 , U_3 , and φ are zero while only U_2 would be present in the wave. However, it can be shown that L_{21} can not be equal to zero and therefore such a mode does not exist.*

In case (2) $\hat{L}$ takes the form

$$\begin{vmatrix} L_{11} & L_{12} & 0 & 0 \\ 0 & 0 & L_{23} & L_{24} \\ L_{31} & L_{32} & 0 & 0 \\ 0 & 0 & L_{43} & L_{44} \end{vmatrix}$$

If the determinant

$$L_2 = \begin{vmatrix} L_{11} & L_{12} \\ L_{31} & L_{32} \end{vmatrix} = 0$$

then the following solution is found: $B^{(3)} = B^{(4)} = 0$ while $B^{(1)}$ or $B^{(2)}$ can be arbitrarily chosen and the remaining B can be calculated from the first or third equation of the system (equations (12) or (14)). This situation corresponds to a wave with displacement components U_1 and U_3 while U_2 and φ are identically zero.

If the determinant

$$L_3 = \begin{vmatrix} L_{23} & L_{24} \\ L_{43} & L_{44} \end{vmatrix} = 0$$

*This case was considered in detail for surface wave propagation in the basal plane of hexagonal piezoelectric crystals by Tseng and White⁽⁷⁾.

then the following solution is found: $B^{(1)} = B^{(2)} = 0$ while $B^{(3)}$ or $B^{(4)}$ can be arbitrarily chosen and the remaining B can be calculated from the second or fourth equation of the system (equations (13) or (15)). This situation corresponds to a wave with displacement component U_2 and potential φ while U_1 and U_3 are identically zero.

It should be noted that in paragraph 1.1 it was stated that four α 's with positive real parts can be found when in the range of velocities below the slowest bulk wave velocity in the direction of propagation being considered. However, in the degenerate cases surface waves may exist when there are less than four such α 's provided the appropriate α 's have positive real parts. For example, in case (1) only three roots of A_1 must have positive real parts for the existence of a surface wave. The roots of $A_{22} = 0$ are not required to assume any particular form. That is, both roots corresponding to $A_{22} = 0$ may be purely imaginary but if three of the six roots corresponding to $A_1 = 0$ have positive real parts a surface wave may still exist. Similarly, for case (2) it is possible to have a solution with only two α 's with positive real parts provided that both of the α 's come from the same equation (i.e. both come from $A_2 = 0$ or both from $A_3 = 0$). An example of the latter situation has been reported in the literature⁽⁴⁾ and corresponds to a wave with displacement component U_2 and potential φ . It may be noted that the (necessarily) degenerate waveforms that occur with higher velocities than the bulk waves alluded to above appear to be the non-attenuated limits of leaky or pseudo-waves and have been described for non-piezoelectric crystals by Lim and Farnell.⁽⁵⁾

One further case of a peculiar nature will be mentioned here although it does not fit into the category of degenerate cases. On a surface of a hexagonal crystal, solutions of the algebraic equations for the decay coefficients $\alpha^{(k)}$ exist which cause the minors of the last row and column of $\hat{A}$ to vanish. Since all the minors of the elements of $\hat{A}$ are not new the rank of the determinant is not reduced but the component β_4 is forced to be zero. Thus only β_1 , β_2 , and β_3 exist for such an α . The total wave however is not an example of a degenerate case since the other α 's are needed for a surface wave solution. This behavior is a manifestation of the fact that one of the general bulk wave solutions is decoupled from the electric field for all directions of propagation in a piezoelectric hexagonal crystal.

4. Degenerate Cases – Non-piezoelectric (Pure Elastic) Medium

For surface wave propagation in a pure elastic medium the system of equations discussed in the preceding paragraph takes the form

$$\sum_{\ell=1}^3 A_{i\ell} \beta_{\ell} = 0 \quad i = 1, 2, 3 \quad (19)$$

where the $A_{i\ell}$ are the same as in the piezoelectric case but involve only the elastic constants (the piezoelectric constants are zero and the dielectric constants do not enter the problem). The equation for α is now sixth order. Generally three roots with positive real parts are required for a surface wave solution.

Only one degenerate case occurs for a pure elastic medium. In this case the $\hat{A}$ matrix assumes the form

$$\begin{pmatrix} A_{11} & 0 & A_{13} \\ 0 & A_{22} & 0 \\ A_{13} & 0 & A_{33} \end{pmatrix}.$$

As before the condition $A_{22} = 0$ leads to a quadratic equation in α but this case is of no interest inasmuch as it would lead to a solution with U_2 the only component of displacement and this is impossible for the same reason as in the piezoelectric case (viz. L_{21} cannot equal zero for the form of the decay coefficient α required for a surface wave solution). If

$$A_1 = \begin{vmatrix} A_{11} & A_{13} \\ A_{13} & A_{33} \end{vmatrix} = 0$$

we obtain the solution $\beta_2 = 0$, $\beta_1, \beta_3 \neq 0$, and either β_1 or β_3 can be chosen arbitrarily with the other β being calculated from one of the two remaining equations of the system. In this case the $\hat{L}$ matrix assumes the form

$$\begin{vmatrix} 0 & L_{12} & L_{13} \\ L_{21} & 0 & 0 \\ 0 & L_{32} & L_{33} \end{vmatrix} = 0,$$

and the boundary conditions can be satisfied if

$$L_1 = \begin{vmatrix} L_{12} & L_{13} \\ L_{32} & L_{33} \end{vmatrix} = 0.$$

The above form of the linear system requires that $B^{(1)} = 0$ while either $B^{(2)}$ or $B^{(3)}$ can be chosen arbitrarily and the remaining amplitude calculated from either of the equations

$$L_{12}B^{(2)} + L_{13}B^{(3)} = 0$$

or

$$L_{32}B^{(2)} + L_{33}B^{(3)} = 0.$$

This solution corresponds to a wave with displacement components U_1 and U_3 while U_2 is identically zero, i.e., a Rayleigh wave.

5. Surface Wave Propagation in an Isotropic Elastic, Perfectly Conducting Film on a Piezoelectric Substrate

An additional problem that has been considered is that of a finite thickness layer of isotropic elastic conductor on a piezoelectric substrate. When the displacement component waveforms

$$U_i = \beta_i e^{-\alpha u x_3 / v_s} e^{j\omega(t - x_1 / v_s)}, \quad i = 1, 2, 3,$$

are substituted into the equations of motion for an isotropic elastic medium, a linear system of equations for the relative amplitudes $\beta_1, \beta_2, \beta_3$, of the displacement components is obtained. The determinant of the system, set equal to zero, yields the equation for the determination of the exponents α , namely

$$\det \begin{pmatrix} \mu\alpha^2 - (2\mu + \lambda) + \rho v_s^2 & 0 & j\alpha[\lambda + \mu] \\ 0 & \mu\alpha^2 - \mu + \rho v_s^2 & 0 \\ j\alpha[\lambda + \mu] & 0 & (2\mu + \lambda)\alpha^2 - \mu + \rho v_s^2 \end{pmatrix} = 0 \quad (20)$$

where λ, μ are the Lamé constants of the medium. The polynomial form of (18) is of order six and, inasmuch as the medium is of finite thickness, the solution corresponding to all six roots is needed to satisfy the boundary conditions.

The assumed forms of the solutions in the piezoelectric medium with those employed in Section II.1, namely,

where λ, μ are the Lamé constants of the medium. The polynomial form of (18) is of order six and inasmuch as the medium is of finite thickness, the solutions corresponding to all six roots are needed to satisfy the boundary conditions.

The assumed forms of the solutions in the piezoelectric medium are identical with those employed in Section II.1, namely,

$$U_i^p = \sum_{\ell=1}^4 A^{(\ell)} \beta_{pi}^{(\ell)} e^{-\alpha_p^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \quad (21)$$

, $i = 1, 2, 3$,

$$\varphi^p = \sum_{\ell=1}^4 A^{(\ell)} \beta_{pi}^{(\ell)} e^{-\alpha_p^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \quad (22)$$

In the elastic conductor the total displacements assume the form

$$U_i^c = \sum_{k=1}^6 B^{(k)} \beta_{ci}^{(k)} e^{-\alpha_c^{(k)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} , \quad i = 1, 2, 3 , \quad (23)$$

and the potential function is identically zero. Thus there are 10 unknown amplitude coefficients $A^{(\ell)}$, $B^{(k)}$, $\ell = 1, 2, 3, 4$, $k = 1, \dots, 6$ to be determined.

The boundary conditions applicable in this problem are as follows:

Continuity of displacement at $x_3 = 0$	$U_i^p(x_1, 0) = U_i^c(x_1, 0)$	$i = 1, 2, 3$
Continuity of stress components at $x_3 = 0$	$T_{3j}^p(x_1, 0) = T_{3j}^c(x_1, 0)$	$j = 1, 2, 3$
Vanishing of stress components at the free surface at $x_3 = -h$	$T_{3j}^c(x_1, -h) = 0$	$j = 1, 2, 3$
Vanishing of potential at $x_3 = 0$	$\varphi^p(x_1, 0) = 0$	

Applying these ten conditions to the above solutions yields a system of ten homogeneous algebraic equations in the ten unknown amplitude coefficients $A^{(i)}, B^{(j)}$. From this point the solution for surface wave velocities and field distributions proceeds as before except that there are now ten equations instead of (4). The explicit form of the determinant of the system stemming from the boundary conditions is given in Appendix I.

6. Surface Wave Propagation at the Interface between a Piezoelectric Substrate and a Semi-Infinite Fluid Medium

The physical problem considered in this section is that of a surface wave propagating along the interface between a piezoelectric crystal and a semi-infinite fluid. Again a rectangular coordinate system is chosen with the x_3 axis normal to the crystal surface and the x_1 axis in the direction of propagation as in the preceding problems. The fields in the crystal and fluid media are assumed to be independent of the x_2 direction. Arbitrary orientations of the crystal surface with respect to the crystal axes are handled by means of an Euler Transformation as before and the differential equations in the crystal medium are the same.

The elastic properties of the fluid medium are described in terms of a single elastic constant λ (modulus of compression); the effect of viscosity is ignored. Consequently, the differential equations in the fluid are

$$\begin{aligned}
 U_{1,11} + U_{3,13} &= \rho_f \ddot{U}_1 / \lambda \\
 U_{1,13} + U_{3,33} &= \rho_f \ddot{U}_3 / \lambda, \quad x_3 < 0, \\
 \nabla^2 \varphi &= 0,
 \end{aligned}
 \tag{24}$$

wherein the derivatives with respect to x_2 are taken to be zero in keeping with the uniformity of the field in this direction and ρ_f is the density of the fluid medium.

In the crystal medium ($x_3 > 0$) traveling wave solutions of the form

$$\begin{aligned} U_i &= \beta_i e^{-\alpha_c \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}, \quad i = 1, 2, 3, \\ \varphi &= \beta_4 e^{-\alpha_c \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \end{aligned} \quad (25)$$

are sought and all analytical considerations pertaining to the crystal medium are identical with those discussed in Sections II.1 and II.2.

In the fluid ($x_3 < 0$) the particle displacements and electric potential are decoupled and are assumed to have the forms

$$\begin{aligned} U_1 &= \gamma_1 e^{-\alpha_f \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}, \\ U_3 &= \gamma_3 e^{-\alpha_f \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}, \\ \varphi &= C e^{\omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}. \end{aligned} \quad (26)$$

Substitution of these displacement and potential waveforms into the differential equations (24) leads to the following equation for α_f in terms of the velocity v_s , namely

$$\det \begin{pmatrix} \rho_f v_s^2 - \lambda & j\alpha_f \lambda \\ j\alpha_f \lambda & \lambda \alpha_f^2 + \rho_f v_s^2 \end{pmatrix} = 0, \quad (27)$$

from which it follows that

$$\alpha_f = \pm \sqrt{\frac{\lambda - \rho_f v_s^2}{\lambda}}. \quad (28)$$

The relative amplitudes γ_1 and γ_3 are obtained from the homogeneous linear system of equations whose coefficient matrix appears in equation (7), viz.,

$$\gamma_1 = \frac{j\alpha_f \lambda}{\lambda - \rho_f v_s^2} \gamma_3 \quad (29)$$

The sign of α_f in equation (28) is determined by the condition that the surface wave is bounded as $x_1 \rightarrow +\infty$.

The total field in the crystal is expressed as a linear combination of the "partial" fields associated with the allowed values of α_c , namely

$$U_i = \sum_{\ell=1}^4 B^{(\ell)} \beta_i^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)} \quad (30)$$

$$\varphi = \sum_{\ell=1}^4 B^{(\ell)} \beta_4^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}$$

In the fluid medium the total displacements and potential are given by equation (26).

The amplitude coefficients $B^{(1)}$, $B^{(2)}$, $B^{(3)}$, $B^{(4)}$, C and γ_3 are determined by the boundary conditions:

$$U_3 \text{ continuous at } x_3 = 0$$

$$\varphi \text{ continuous at } x_3 = 0$$

$$D_3 \text{ continuous at } x_3 = 0 \quad (\text{electric displacement})$$

$$T_{33} \text{ continuous at } x_3 = 0$$

$$T_{31} = 0 \text{ at } x_3 = 0$$

$$T_{32} = 0 \text{ at } x_3 = 0$$

The components of the electric displacement vector $\vec{D}$ in the crystal are given by

$$D_i = e_{ik\ell} U_{k,\ell} - \epsilon_{ik} \varphi_{,k}, \quad i = 1, 2, 3;$$

in the fluid medium, $\vec{D} = -\epsilon_f \nabla \varphi$, where ϵ_f is the dielectric constant of the fluid.

Application of the boundary conditions to the total field solutions leads to a set of six homogeneous equations in the unknown amplitudes $B^{(\ell)}$ $\ell = 1, 2, 3, 4$, C and γ_3 . The coefficient matrix of this system of equations, $M = [M_{ik}]_{j,k=1,\dots,6}$, assumes the form*

*The explicit equations for the elements of M are given in Appendix II.

$$M = \left(\begin{array}{ccccc|c} M_{11} & M_{12} & M_{13} & M_{14} & 0 & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & 0 \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} & 0 & M_{46} \\ M_{51} & M_{52} & M_{53} & M_{54} & 0 & 0 \\ M_{61} & M_{62} & M_{63} & M_{64} & 0 & 0 \end{array} \right) \quad (31)$$

The characteristic equation for the surface wave velocity v_s is obtained from the condition for the existence of a non-trivial solution of the aforementioned homogeneous system, namely, $\det M = 0$.

The complex solutions to the equation $\det M = 0$ can be obtained in a straightforward fashion using the iterative scheme described in the programming sections and such a procedure has been built into the computer program for the fluid problems. However such a procedure is time consuming and does not fully exploit prior work on piezoelectric surface wave propagation problems. Inasmuch as a computer program exists to calculate piezoelectric surface wave characteristics under a variety of conditions, in particular, when the surface of the crystal is traction free and the adjacent half space is a massless, non elastic dielectric, it is desirable to make maximum use of this program. This can be done for a wide range of parameter values for the fluid medium by making use of a perturbation scheme for obtaining the roots of $\det M = 0$ which utilizes the results of this program and requires, in addition, only the evaluation of a few determinants at specified velocities.

The implementation of the perturbation procedure is based on the fact that a particular sub-matrix N of the matrix M (as indicated by the partitioned matrix in equation (31)) is the coefficient matrix of the linear system corresponding to the boundary conditions at the surface of a crystal in contact with a medium whose elastic properties are those of a vacuum but whose dielectric constant is that of the fluid medium. Consequently, the equation $\det N = 0$ is the characteristic equation for the velocity of the surface waves

which can propagate in this configuration, and the roots of this equation can be found using the computer program for the first problem with $u h = \infty$ and the dielectric constant of a vacuum ϵ_0 replaced by the dielectric constant of the fluid (ϵ_f).

The perturbation procedure is based on the assumption that the complex velocity which satisfies $\det M = 0$ corresponds to a small perturbation on the real velocity solution of $\det N = 0$, i.e. that the mechanical loading of the substrate by the fluid medium is quite small.

Formally the perturbation scheme is derived as follows. Let v_{s0} be the velocity such that $\det N(v_{s0}) = 0$ and assume that there exists a complex perturbation Δv_s such that the $\det M(v_s) = \det M(v_{s0} + \Delta v_s) = 0$ and $|\Delta v_s / v_{s0}| \ll 1$. For $|\Delta v_s / v_{s0}| \ll 1$

$$\det M(v_s) = \det M(v_{s0}) + \frac{d}{dv_s} [\det M(v_s)] \Big|_{v_s = v_{s0}} \cdot \Delta v_s + O[(\Delta v_s)^2] = 0, \quad (32)$$

whereupon neglecting terms $O[(\Delta v_s)^2]$ yields

$$\Delta v_s = - \frac{\det M(v_{s0})}{\frac{d}{dv_s} (\det M(v_{s0}))}. \quad (33)$$

Expanding $\det M(v_{s0})$ about the last column (which has only two elements) gives

$$\Delta v_s = \frac{M_{46} K}{M_{16} (\det N)' - M_{46} K' - M_{46}' K} \quad (34)$$

where the quantity K in (34) is the minor of the element M_{46} in the matrix M , the primes denote differentiation with respect to v_s , and all quantities are evaluated at v_{s0} . In obtaining (34) explicit use has been made of the fact that $\det N(v_{s0}) = 0$.

The derivatives of the determinants employed in the perturbation procedure are calculated numerically. The derivative of the matrix element M_{46} was obtained analytically.

In both methods of obtaining the complex velocity v_s the functions involved (matrix elements and minors of the matrix M) contain α_f as an independent variable which in turn is a dependent variable with argument v_s . Equation (28) shows that there is an ambiguity in the sign of α_f . The resolution of this ambiguity leads to the particular character of the piezoelectric surface wave.

The variation of the surface wave in the direction of propagation is assumed to be bounded in the positive ($x_1 \rightarrow +\infty$) direction of propagation. This assumption imposes the requirement that $\text{Im}[v_s] \geq 0$. Consequently, the sign of α_f must be chosen such that this condition is satisfied.

Since λ and ρ_f are positive real it can be shown that

$$\text{Re} \left[-\frac{\alpha_f^{(\pm)}}{v_s} \right] \geq 0 ,$$

where $\alpha_f^{(\pm)}$ denotes to the values of α_f from equation (28) corresponding to the positive and negative signs of the radical. Consequently, if $\alpha_f^{(-)}$ is required to obtain a root v_s such that $\text{Im}[v_s] \geq 0$ the corresponding surface wave is of the leaky type. On the other hand, if $\alpha_f^{(+)}$ is required to obtain a solution of the determinantal equation the surface wave is evanescent in character. In all numerical cases considered the surface wave was a leaky wave.

In the program described in the following section, two values of input velocity are required depending on the program option used. If the perturbation scheme is used, a very accurate value (at least 6 place accuracy) of velocity must be input. This value is to be computed from the existing surface wave program wherein the dielectric constant of the fluid medium is substituted for that of free space (outside the crystal medium). On the other hand, if the root finding scheme is employed only a reasonable estimate of the complex velocity is required.

A final word of caution is in order regarding the use of the computer program. In checking out the various options available with the program, it was found that if the leaky wave velocity is a small perturbation on the surface wave velocity in the absence of the fluid medium, then the use of the perturbation scheme led to more reliable results than the root finding option. On the other hand, if the fluid medium significantly loaded the substrate material, the root

finding scheme gave good results whereas the perturbation scheme (as would be expected) gave erratic results in some cases. The former differences stem from the fact that the change in the velocity due to the air loading is on the order of the errors incurred in the root finding scheme while the latter are due to the approximations inherent to the perturbation procedure.

7. Surface Wave Propagation in an Isotropic Elastic Film on a Piezoelectric Substrate

This section gives a brief description of the theoretical analysis of surface wave propagation on a semi-infinite piezoelectric substrate with a contiguous isotropic dielectric-elastic layer, as shown in Figure 3.

The substrate is assumed to be a completely general piezoelectric (or non-piezoelectric) crystal medium with arbitrary surface normal direction relative to the crystal axes of the medium. The material layer adjacent to the substrate is assumed to be a general isotropic elastic medium with isotropic dielectric properties. Only pure modes of propagation are considered, that is, leaky surface waves or evanescent (or cut off) modes of propagation have not been accounted for in the computer program.

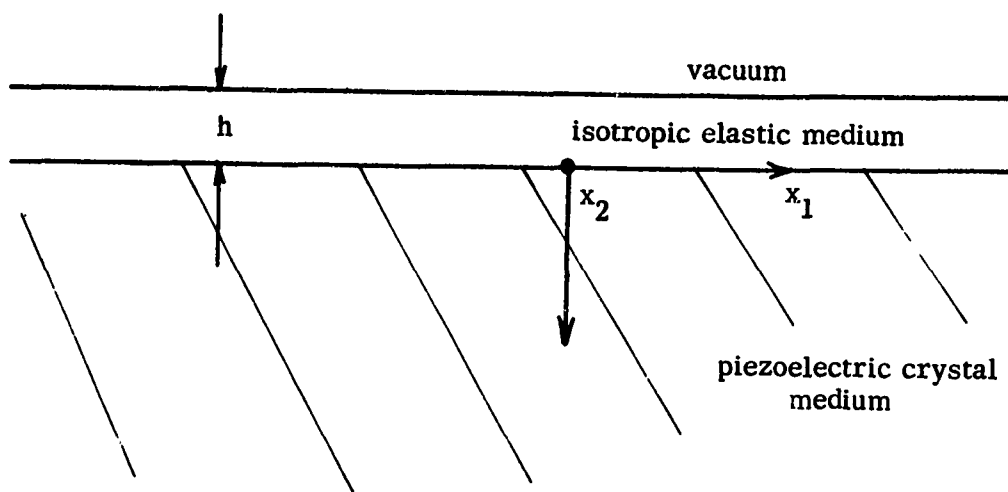


Figure 3. Semi-infinite Piezoelectric Substrate with a Contiguous Isotropic Elastic Layer.

The coordinate system employed in the analysis is illustrated in Figure 3. The piezoelectric crystal medium occupies the region $x_3 > 0$ and the direction of propagation is assumed to be in the x_1 direction. The fields in both the crystal and layer are assumed to be independent of the x_2 coordinate. Arbitrary orientations of the crystal surface with respect to the crystal axes are considered as before by an Euler Transformation. Inasmuch as the dielectric layer is isotropic, both from an elastic and an electromagnetic point of view, the quantities characterizing the medium are invariant under coordinate transformations.

The analysis pertaining to the crystal or "substrate" medium is identical to that described in Sections II.1 and II.2.

The elastic properties of the dielectric layer are described in terms of two elastic constants, λ_d , the modulus of compression or Lamé's constant, and μ_d the shear modulus. Inasmuch as the layer is non-piezoelectric the differential equations for the mechanical displacements and electric potential decouple and assume the form

$$\mu_d \nabla^2 \vec{U} + (\lambda_d + \mu_d) \nabla (\nabla \cdot \vec{U}) = \rho_d \ddot{\vec{U}} \quad , \quad -h < x_3 < 0 \quad , \quad (35)$$

and

$$\nabla^2 \phi = 0 \quad , \quad (36)$$

where $\vec{U} = (U_1, U_2, U_3)$ and ρ_d is the density of the dielectric medium.

In the dielectric medium the assumed displacement waveforms may be expressed as

$$U_i = \beta_{d_i} e^{-\alpha_d \omega x_3 / v_s} e^{j\omega(t - x_1 / v_s)} \quad , \quad i = 1, 2, 3 \quad . \quad (37)$$

Substitution of these waveforms into the differential equations (35) yields a linear system of homogeneous equations in the unknowns β_{d1} , β_{d2} and β_{d3} . The existence of non-trivial solutions requires that the determinant of the coefficients of the system vanish thus leading to the following classical equations for normalized transverse wave numbers α_d in terms of the velocity v_s , namely,

$$\alpha_d^{(1,2)} = \alpha_d^{\pm} \text{ (shear)} = \pm \sqrt{\frac{\mu_d - \rho_d v_s^2}{\mu_d}} \quad (38)$$

$$\alpha_d^{(3,4)} = \alpha_d^{\pm} \text{ (compressional)} = \pm \sqrt{\frac{\lambda_d + 2\mu_d - \rho_d v_s^2}{\lambda_d + 2\mu_d}} \quad (39)$$

and

$$\alpha_d^{(5,6)} = \alpha_d^{(1,2)} ,$$

since the shear mode is degenerate for an isotropic elastic medium.*

Finally, in the dielectric medium, the two independent solutions of (36), assuming $e^{-j\omega x_1/v_s}$ variation in the x_1 direction, are

$$\varphi_{1,2} = C_{1,2} e^{\pm \omega x_3/v_s} e^{j\omega(t-x_1/v_s)} \quad (40)$$

In the "free space" region $-\infty < x_3 < -h$ there are no mechanical displacements but a potential function exists and must satisfy the differential equation (36). In addition, the requirement that the potential be bounded as $x \rightarrow -\infty$ is imposed. Therefore, the form of the potential is taken to be

$$\varphi_s = C_3 e^{\omega x_3/v_3} e^{j\omega(t-x_1/v_s)} \quad (41)$$

The total displacement and potential waveforms in the piezoelectric crystal are expressed as linear combinations of the "partial" fields associated with the allowed values of α_c . Denoting these values $\alpha_c^{(l)}$, $l = 1, 2, 3, 4$, the displacement components and potential may be expressed as

*The term degenerate is used here in the sense that in the characteristic equation for the normalized transverse wave numbers α (for example, corresponding to the determinantal equation (3) for the general piezoelectric crystal) the roots $\alpha^{(1)}$ and $\alpha^{(2)}$ are double roots and hence two linearly independent eigenvectors can be defined for each distinct value.

$$U_i^{(c)} = \sum_{\ell=1}^4 B^{(\ell)} \beta_{Ci}^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}, \quad i = 1, 2, 3, \quad (42)$$

$$\varphi = \sum_{\ell=1}^4 B^{(\ell)} \beta_{Ci}^{(\ell)} e^{-\alpha_c^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}.$$

In the dielectric medium the total displacement components assume the form

$$U_i^{(d)} = \sum_{\ell=1}^6 D^{(\ell)} \beta_{di}^{(\ell)} e^{-\alpha_d^{(\ell)} \omega x_3 / v_s} e^{j\omega(t-x_1/v_s)}, \quad i = 1, 2, 3, \quad (43)$$

while the total potential is given by

$$\varphi^d = \left(C_1 e^{\omega x_3 / v_s} + C_2 e^{-\omega x_3 / v_s} \right) e^{j\omega(t-x_1/v_s)} \quad (44)$$

In the free space region the total potential is given by equation (41).

The as yet unspecified amplitude coefficients $B^{(\ell)}$, $\ell = 1, 2, 3, 4$; $D^{(\ell)}$, $\ell = 1, 2, \dots, 6$; C_1 ; C_2 ; and C_3 are determined, to within a constant, together with the surface wave velocity v_s , by the following continuity and boundary conditions:

- (i) U_1 , U_2 , and U_3 continuous at $x_3 = 0$
- (ii) φ continuous at $x_3 = 0$ and $x_3 = -h$
- (iii) Continuity of the normal component of electric displacement at $x_3 = 0$ and $x_3 = -h$
- (iv) Continuity of shear and normal stresses (T_{31} , T_{32} , T_{33}) at $x_3 = 0$
- (v) The surface $x_3 = -h$ is stress free ($T_{31} = T_{32} = T_{33} = 0$).

The components of the electric displacement vector $\vec{D}$ are given by

$$D_i = \epsilon_{ik\ell} U_{k,\ell} - \epsilon_{ik} \varphi_{,k}, \quad i = 1, 2, 3 \quad x_3 > 0,$$

$$\vec{D} = \epsilon_d \nabla \varphi \quad -h < x_3 < 0,$$

and

$$\vec{D} = \epsilon_0 \nabla \varphi \quad x_3 < -h .$$

The components of stress T_{31} , T_{32} , and T_{33} are given by

$$\begin{aligned} T_{3j} &= C_{3jk} U_{k,l} + e_{k3j} \varphi_{,k} , & j = 1, 2, 3, \quad x_3 > 0 . \\ T_{31} &= \mu_d (U_{1,3} + U_{3,1}) \\ T_{32} &= \mu_d U_{2,3} \\ T_{33} &= (\lambda_d + 2\mu_d) U_{3,3} + \lambda_d U_{1,1} \end{aligned} \quad \left. \vphantom{\begin{aligned} T_{31} \\ T_{32} \\ T_{33} \end{aligned}} \right\} -h < x_3 < 0 .$$

Application of the continuity and boundary conditions (i)....(v) to the total displacements and potentials (40), (41), (42), (43), and (44) leads to a system of thirteen linear homogeneous equations in the thirteen unknown amplitudes $B^{(\ell)}$, $\ell = 1, 2, 3, 4$, $D^{(\ell)}$, $\ell = 1, \dots, 6$, C_1 , C_2 and C_3 . The equation for the surface wave velocity v_s is obtained from the condition for the existence of a non-trivial solution of this system of equations, namely, that the determinant of the system vanish. The explicit forms of the coefficients L_{ij} , $i, j = 1, \dots, 13$, of this system are contained in Appendix III where the appropriate boundary conditions represented by each row of the matrix are indicated.

If the substrate is non-piezoelectric, modifications of the foregoing analyses identical to those described in Section II.1 are required. In this case, the characteristic equation for the surface wave velocity is the determinant of a (9×9) matrix comprised of the coefficients of the amplitudes $D^{(\ell)}$, $\ell = 1, \dots, 6$, and $B^{(\ell)}$, $\ell = 1, 2, 3$ in the homogeneous system of 9 equations in 9 unknowns derived from the boundary conditions given above upon neglecting the electric field and setting the piezoelectric constants equal to zero.

Degenerate Cases (Piezoelectric Substrate)

The same degenerate cases arise as those considered in the preceding sections and the selection of β 's proceeds as before.

For case (1) (Section II.2) solutions are sought wherein U_1 , U_3 and φ only exist in the crystal. This type of solution uses only the α values which

lead to $\beta_2 = 0$ and β_1, β_3 , and β_4 non-zero. Also in this problem solutions where U_2 only exists in the crystal must be considered (e.g. Love waves or the piezoelectric perturbations thereof). This solution stems from the root α which leads to non-zero β_2 and zero β_1, β_3 , and β_4 .

As in the previously considered degenerate cases the determinant of the boundary condition matrix $\hat{L}$ factors into the product of two determinants. The determinant which corresponds to the U_1, U_3, φ solutions is denoted M and assumes the form,

$$M = \begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{18} & L_{19} & L_{1,10} & L_{1,11} & L_{1,12} & L_{1,13} \\ L_{31} & - & - & - & - & - & - & - & - & - \\ L_{41} & - & - & - & - & - & - & - & - & - \\ L_{61} & - & - & - & - & - & - & - & - & - \\ L_{71} & - & - & - & - & - & - & - & - & - \\ L_{91} & - & - & - & - & - & - & - & - & - \\ L_{10,1} & - & - & - & - & - & - & - & - & - \\ L_{11,1} & - & - & - & - & - & - & - & - & - \\ L_{12,1} & - & - & - & - & - & - & - & - & - \\ L_{13,1} & - & - & - & - & - & - & - & - & L_{13,13} \end{vmatrix}$$

The solution for the U_2 case depends upon existence of zeros of a determinant N where N is a (3×3) determinant.

$$N = \begin{vmatrix} L_{25} & L_{26} & L_{27} \\ L_{55} & L_{56} & L_{57} \\ L_{85} & L_{86} & L_{87} \end{vmatrix}.$$

For case (2) (Section II.2) solutions wherein only U_1 and U_3 exist are considered. This type of solution uses only those α 's which lead to zero β_2 and β_4 and non-zero β_1 and β_3 . Solutions where only U_2 and φ exist also must be considered. This solution employs the α 's which give non-zero β_2 and β_4 but zero β_1 and β_3 .

The solution for the U_1, U_3 case depends upon existence of zeros of a determinant P (6×6), where,

$$P = \begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{17} & L_{18} \\ L_{31} & - & - & - & - & - \\ L_{41} & - & - & - & - & - \\ L_{61} & - & - & - & - & - \\ L_{71} & - & - & - & - & - \\ L_{81} & - & - & - & - & - \end{vmatrix}$$

The solution for the U_2, U_4 case depends upon existence of zeros of a determinant Q (7×7), which assumes the form

$$Q = \begin{vmatrix} L_{25} & L_{26} & L_{29} & L_{2,10} & L_{2,11} & L_{2,12} & L_{2,13} \\ L_{55} & - & - & - & - & - & - \\ L_{85} & - & - & - & - & - & - \\ L_{10,5} & - & - & - & - & - & - \\ L_{11,5} & - & - & - & - & - & - \\ L_{12,5} & - & - & - & - & - & - \\ L_{13,5} & - & - & - & - & - & - \end{vmatrix}$$

Degenerate Cases (Non-Piezoelectric Substrate)

The degenerate cases U_1, U_3 , only or U_2 only involve one α with zero β_1, β_3 , and non-zero β_2 and two α 's with zero β_2 and non-zero β_1, β_3 .

The U_1, U_3 case requires the investigation of the roots of a determinant of the same form as P except for relabeling the columns due to a relabeling of the α 's. This determinant is designated R and has the form,

$$R = \begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{18} & L_{19} \\ L_{31} & - & - & - & - & - \\ L_{41} & - & - & - & - & - \\ L_{61} & - & - & - & - & - \\ L_{71} & - & - & - & - & - \\ L_{91} & - & - & - & - & - \end{vmatrix}$$

Solutions with U_2 only (Love waves) lead to the consideration of the roots of a determinant which is identical to N .

III. ANALYSIS OF AN ELECTRIC CURRENT LINE SOURCE ABOVE A PIEZOELECTRIC HALF-SPACE

In this section a study is made of the excitation of piezoelectric waves by means of an interdigital electrode transducer. Arbitrary crystals and crystal orientations are considered as in the preceeding chapter. The problem is treated from a field theory point of view and is case in the formalism of a Green's function solution. Due to the complexity of the problem it is expediant to make some simplifying assumptions before the analysis is attempted. Consequently, it is assumed that the coupling between the individual strips can be neglected and that the current on the strips can be approximated by an assumed current distribution. Furthermore it is assumed that only current flow normal to the array is of importance in exciting the piezoelectric waves and that the strips of the array can be considered to be of infinite extent thus reducing the problem to a two dimensional one.

With these assumptions in mind the Green's function sought is one for an infinitesimal two dimensional electric dipole above a piezoelectric substrate as illustrated in Figure 4.

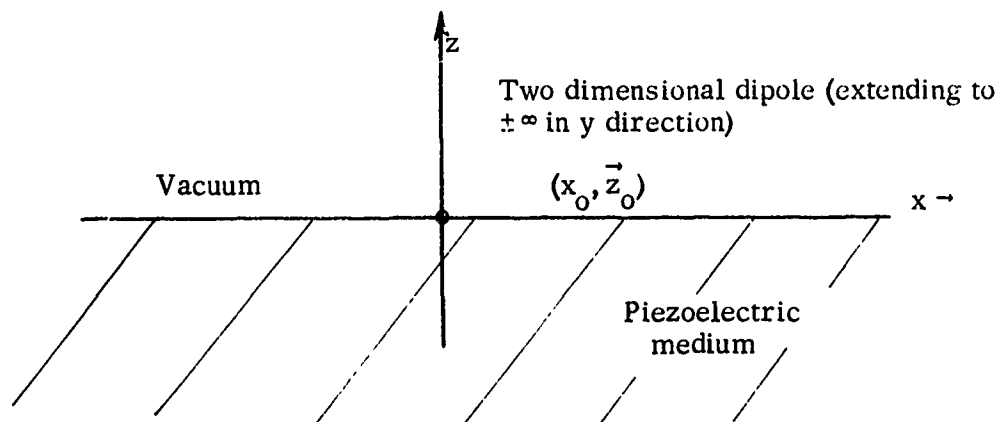


Figure 4

The dipole is located at x_0, z_0 and extends to $\pm\infty$ in the y direction. It is oriented in the x direction. The crystal fills the region $z \leq 0$ while a vacuum exists in the region $z > 0$. Assuming an $e^{j\omega t}$ time dependence, the equation for the electric field in the vacuum is as follows:

$$\nabla \times \nabla \times \vec{E} = j\omega\mu_0 \vec{J} + \omega^2\mu_0 \vec{D} \quad (45)$$

For the two dimensional dipole $\vec{J} = \vec{U}_x \delta(z-z_0) \delta(x-x_0)$ and equation (45) reduces to

$$\nabla^2 \vec{E} + k_0^2 \vec{E} = -j\omega\mu_0 \left[1 + \frac{\nabla\nabla}{k_0^2} \right] \cdot \vec{J}$$

where 1 is the unit dyadic, $k_0 = 2\pi/\lambda_0$, and λ_0 is the free space wavelength. Setting $\vec{E} = (1 + \nabla\nabla/k_0^2) \cdot \vec{G}$ it is easily seen that

$$\nabla^2 \vec{G} + k_0^2 \vec{G} = -j\omega\mu_0 \delta(z-z_0) \delta(x-x_0) \vec{U}_x \quad (46)$$

A particular solution to equation (46) is

$$\vec{G} = -\vec{U}_x \frac{\omega\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{e^{j\sqrt{k_0^2 - k_x^2} |z-z_0|}}{\sqrt{k_0^2 - k_x^2}} e^{jk_x(x-x_0)} dk_x \quad (47)$$

A particular solution for $\vec{E}$ (viz. $\vec{E}_p$) is derivable from $\vec{G}$ and in the region $0 < z < z_0$ the following expression results, namely,

$$\begin{aligned} \frac{\vec{E}_p}{k_0} &= \frac{1}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \left\{ -\vec{U}_x \int_{-\infty}^{\infty} \sqrt{1-K_x^2} e^{-j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \right. \\ &\quad \left. -\vec{U}_z \int_{-\infty}^{\infty} K_x e^{-j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \right\} \\ &= \frac{E_{px}}{k_0} \vec{U}_x + \frac{E_{pz}}{k_0} \vec{U}_z \end{aligned} \quad (48)$$

where $K_x = k_x/k_0$, $\bar{x} = k_0 x$, and $\bar{z} = k_0 z$. A solution for the total electric field may be obtained as a superposition of $\vec{E}_p$ and a general solution of the homogeneous equation $\nabla^2 \vec{E} + k_0^2 \vec{E} = 0$. That is, in the region $0 < z < z_0$ $\vec{E}$ may be written in the form

$$\begin{aligned}
\frac{E_x}{k_0} &= \frac{E_{px}}{k_0} + \int_{-\infty}^{\infty} B_0(K_x) e^{j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \\
\frac{E_y}{k_0} &= \int_{-\infty}^{\infty} A_0(K_x) e^{j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \\
\frac{E_z}{k_0} &= \frac{E_{pz}}{k_0} - \int_{-\infty}^{\infty} B_0(K_x) \frac{K_x}{\sqrt{1-K_x^2}} e^{j\sqrt{1-K_x^2}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x
\end{aligned} \tag{49}$$

where $A_0(K_x)$ and $B_0(K_x)$ are functions of K_x which are determined through the application of the boundary conditions on the total fields at $z = 0$.

In the crystal medium ($z < 0$) the mechanical displacement fields and the electric fields may be expressed as follows:

$$\begin{aligned}
U_i &= \int_{-\infty}^{\infty} U_i(K_x) e^{-jK_z(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \quad i = 1, 2, 3 \\
\frac{E_i}{k_0} &= \int_{-\infty}^{\infty} \psi_i(K_x) e^{-jK_z(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \quad i = 1, 2, 3
\end{aligned} \tag{50}$$

When the above integral representations are substituted into the differential equations for the crystal, viz.

$$\begin{aligned}
C_{ijkl} U_{k,li} - e_{kij} E_{k,i} &= \rho \ddot{U}_j \quad j = 1, 2, 3 \\
\nabla_x \nabla_x \vec{E} &= \omega^2 \mu_0 \vec{D}
\end{aligned} \tag{51}$$

there results a linear system of homogeneous equations for the amplitudes $U_i(K_x)$ and $\psi_i(K_x)$. The determinant of the coefficients of U_i and ψ_i must vanish for a non-trivial solution to exist, namely,

$-C_{11}K_x^2 + 2C_{15}K_xK_z$	$-C_{16}K_x^2 - C_{45}K_z^2$	$-C_{15}K_x^2 - C_{35}K_z^2$	$-e_{11}jk_x$	$-e_{21}jk_x$	$-e_{31}jk_x$
$-C_{55}K_z^2 + C^2\rho$	$+(C_{14} + C_{56})K_xK_z$	$+(C_{13} + C_{55})K_xK_z$	$+e_{15}jk_z$	$+e_{25}jk_z$	$+e_{35}jk_z$
$-C_{16}K_x^2 - C_{45}K_z^2$	$-C_{66}K_x^2 + 2C_{46}K_xK_z$	$-C_{56}K_x^2 - C_{34}K_z^2$	$-e_{16}jk_x$	$-e_{26}jk_x$	$-e_{36}jk_x$
$+(C_{14} + C_{56})K_xK_z$	$-C_{44}K_z^2 + C^2\rho$	$+(C_{36} + C_{45})K_xK_z$	$+e_{14}jk_z$	$+e_{24}jk_z$	$+e_{34}jk_z$
$-C_{15}K_x^2 - C_{35}K_z^2$	$-C_{56}K_x^2 - C_{34}K_z^2$	$-C_{55}K_x^2 + 2C_{35}K_xK_z$	$-e_{15}jk_x$	$-e_{25}jk_x$	$-e_{35}jk_x$
$+(C_{13} + C_{55})K_xK_z$	$+(C_{36} + C_{45})K_xK_z$	$-C_{33}K_z^2 + C^2\rho$	$+e_{13}jk_z$	$+e_{23}jk_z$	$+e_{33}jk_z$
$C^2\mu_o [e_{11}jk_x - e_{15}jk_z]$	$C^2\mu_o [e_{16}jk_x - e_{14}jk_z]$	$C^2\mu_o [e_{15}jk_x - e_{13}jk_z]$	$C^2\mu_o \epsilon_{11}$	$C^2\mu_o \epsilon_{12}$	$C^2\mu_o \epsilon_{13}$
$C^2\mu_o [e_{21}jk_x - e_{25}jk_z]$	$C^2\mu_o [e_{26}jk_x - e_{24}jk_z]$	$C^2\mu_o [e_{25}jk_x - e_{23}jk_z]$	$C^2\mu_o \epsilon_{21}$	$C^2\mu_o \epsilon_{22}$	$C^2\mu_o \epsilon_{23}$
$C^2\mu_o [e_{31}jk_x - e_{35}jk_z]$	$C^2\mu_o [e_{36}jk_x - e_{34}jk_z]$	$C^2\mu_o [e_{35}jk_x - e_{33}jk_z]$	$C^2\mu_o \epsilon_{31}$	$C^2\mu_o \epsilon_{32}$	$C^2\mu_o \epsilon_{33}$
			$-K_xK_z$		$-K_x^2$

$\det$
 $= 0$
 (52)

In the above equation C is the velocity of light. Expanding this determinant leads to a 10'th order algebraic equation for K_z as a function of K_y .

For a given K_x in the range $-\infty < K_x < \infty$ there will be 10 values of K_z satisfying the determinental equation but only 5 will be admissable (representing field solutions that are bounded as $z \rightarrow -\infty$ and have the form of out going waves in the region $z < 0$). For each usable K_z it is necessary to solve the homogeneous system for the corresponding field amplitudes U_i, ψ_i .

Thus U_i and E_i can be expressed as follows:

$$U_i = \int_{-\infty}^{\infty} \sum_{n=1}^5 A_n(K_x) U_i^{(n)}(K_x) e^{-jK_z^{(n)}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x \quad (53)$$

$$\frac{E_i}{k_0} = \int_{-\infty}^{\infty} \sum_{n=1}^5 A_n(K_x) \psi_i^{(n)}(K_x) e^{-jK_z^{(n)}(\bar{z}-\bar{z}_0)} e^{jK_x(\bar{x}-\bar{x}_0)} dK_x .$$

where $A_n(K_x)$ are unknown amplitude coefficients to be determined by an application of the boundary conditions. The magnetic field in the crystal medium and in the vacuum can be written in a similar fashion and is derivable from the equation

$$\nabla \times \vec{E} = j\omega\mu_0 \vec{H} .$$

The boundary conditions imposed on the field solutions at $\bar{z} = 0$ are as follows:

Continuity of $T_{3j} \quad j = 1, 2, 3$

Continuity of E_1 and E_2

Continuity of H_1 and H_2 .

The imposition of these conditions leads to the following set of equations in the amplitude coefficients A_0, B_0, A_n . The limiting case $\bar{z}_0 \rightarrow 0$ has been taken in the following since the electrodes will be located on the crystal surface at $\bar{z} = 0$.

Continuity of T_{3j} $j = 1, 2, 3$

$$\begin{aligned}
 & \sum_{n=1}^5 \{ [jK_x C_{15} - jK_z^{(n)} C_{55}] U_1^{(n)} + [jK_x C_{56} - jK_z^{(n)} C_{45}] U_2^{(n)} \\
 & \quad + [jK_x C_{55} - jK_z^{(n)} C_{35}] U_3^{(n)} - e_{15} \psi_1^{(n)} - e_{25} \psi_2^{(n)} - e_{35} \psi_3^{(n)} \} A_n = 0 \\
 & \sum_{n=1}^5 \{ [jK_x C_{14} - jK_z^{(n)} C_{45}] U_1^{(n)} + [jK_x C_{46} - jK_z^{(n)} C_{44}] U_2^{(n)} \\
 & \quad + [jK_x C_{45} - jK_z^{(n)} C_{34}] U_3^{(n)} - e_{14} \psi_1^{(n)} - e_{24} \psi_2^{(n)} - e_{34} \psi_3^{(n)} \} A_n = 0 \quad (54) \\
 & \sum_{n=1}^5 \{ [jK_x C_{13} - jK_z^{(n)} C_{35}] U_1^{(n)} + [jK_x C_{36} - jK_z^{(n)} C_{34}] U_2^{(n)} \\
 & \quad + [jK_x C_{35} - jK_z^{(n)} C_{33}] U_3^{(n)} - e_{13} \psi_1^{(n)} - e_{23} \psi_2^{(n)} - e_{33} \psi_3^{(n)} \} A_n = 0
 \end{aligned}$$

Continuity of E_1, E_2

$$\sum_{n=1}^5 \psi_1^{(n)} A_n - B_0 = -\frac{1}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{1-K_x^2} \quad (55)$$

$$\sum_{n=1}^5 \psi_2^{(n)} A_n - A_0 = 0$$

Continuity of H_1, H_2

$$\sum_{n=1}^5 jK_z^{(n)} \psi_2^{(n)} A_n + j \sqrt{1-K_x^2} A_0 = 0$$

$$\sum_{n=1}^5 [jK_z^{(n)} \psi_1^{(n)} + jK_x \psi_3^{(n)}] A_n + \left[j \sqrt{1-K_x^2} + j \frac{K_x^2}{\sqrt{1-K_x^2}} \right] B_0 = \frac{j}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \quad (56)$$

In practice the solution of equations (54), (55), and (56) would be performed numerically on a computer as a function of K_x .

The integral expressions for the total mechanical displacements and electromagnetic field components follow from the solution of the system of equations described above. An asymptotic evaluation of the resulting integrals may be employed to obtain formal expressions for the physical quantities of significant interest such as the surface wave fields, power in the surface wave, total power input to the crystal by the transducer, and the bulk wave scattered amplitude pattern. The expressions for the aforementioned quantities are very formidable and would require an extensive amount of numerical computation to obtain even limited information. Consequently, it was decided to abandon this approach and the numerical implementation of the theoretical analysis was not carried out.

IV. COMPUTER PROGRAM OUTLINES

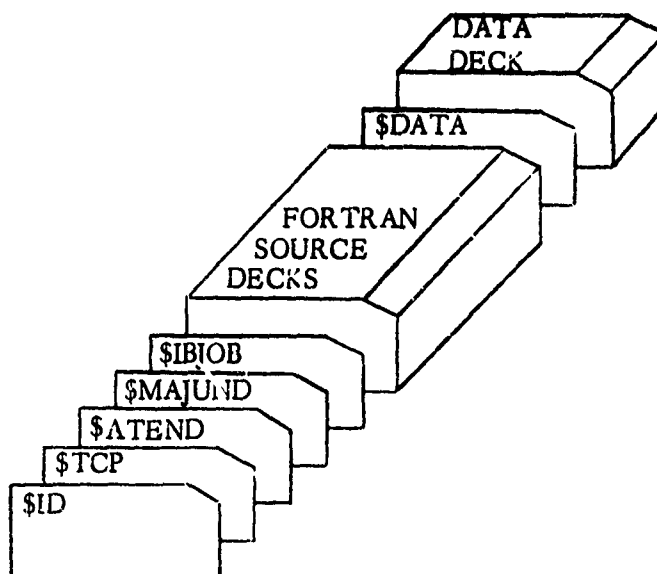
This section describes the use and programming format of the computer programs which were written to implement the numerical analysis of the various surface wave propagation problems described in Section II.

1. Surface Waves on Piezoelectric Crystals in the Presence of Infinitesimally Thin Electric and "Magnetic" Conductors

This computer program is divided into two parts: Part A is concerned with an isotropic elastic conductor (such as gold) of finite thickness above a piezoelectric substrate (such as lithium niobate); Part B is concerned with an infinitesimally thin electric or magnetic conductor above a piezoelectric substrate. All information necessary for the operation of the program is described below. For example, an initial guess of the surface wave velocity is required. From this information, the program refines the initial guess, resulting in a velocity accurate to input specifications.

The program is set up to run on an IBM 7094, and the form of input is FORTRAN Namelist input. Although it is discussed in this document, it is suggested that those not familiar with Namelist input read the appropriate sections in a Fortran manual.

Deck Setup



The \$ID and \$TCP control cards must be supplied by the user; the remaining control cards are already in the program deck. The data deck (i.e., the input data for the program) utilizes Namelist input. Two input sections are required: the first describes the parameters of the substrate crystal; the second provides the remainder of the information necessary for the execution of the program.

The first data set is called CONST. This set includes the piezoelectric, elastic and dielectric constants. Column 2 of the data card contains a dollar sign (\$) and columns 3-7 contain the letters CONST. The constants begin in column 9 of the first card and continue up to column 72; they then continue to columns 2-72 of each succeeding card for as many cards as needed. The general form of the data to be input is:

Variable name = 1st value, 2nd value, ... , last value .

For example, the piezoelectric constants (e_{ip}), called P in the program, could be input as:

P = 0., 0., 0., 0., 3.7, -2.5, ..., 0. ,

There are 18 piezoelectric constants and they should be input in the following order:

e_{11}

e_{12}

e_{13}

e_{14}

e_{15}

e_{16}

e_{21}

e_{22}

e_{23}

e_{24}

e_{25}

$$e_{26}$$

$$e_{31}$$

$$e_{32}$$

$$e_{33}$$

$$e_{34}$$

$$e_{35}$$

$$e_{36}$$

i.e. $P = e_{11}, e_{12}, \dots, e_{36},$

In the program the transformed piezoelectric constants e'_{ij} are printed out as

$$E_1 = e'_{11}$$

$$E_2 = e'_{13}$$

$$E_3 = e'_{14}$$

$$E_4 = e'_{15}$$

$$E_5 = e'_{16}$$

$$E_6 = e'_{31}$$

$$E_7 = e'_{33}$$

$$E_8 = e'_{34}$$

$$E_9 = e'_{35}$$

$$E_{10} = e'_{36}$$

$$E_{11} = e'_{12}$$

$$E_{12} = e'_{32}$$

$$E_{13} = e'_{21}$$

$$E_{14} = e'_{23}$$

$$E_{15} = e'_{24}$$

$$E_{16} = e'_{25}$$

$$E_{17} = e'_{26}$$

The transformed constant e'_{22} is never used and therefore is not printed out.

The elastic constants (C_{pq}), called G in the program, are next. Immediately following the comma (,) behind the last piezoelectric constant (excluding blanks), print:

G =

followed by the 21 values of the elastic constants in the following order, separating each variable by a comma:

C_{11}

C_{22}

C_{33}

C_{12}

C_{13}

C_{14}

C_{15}

C_{16}

C_{23}

C_{24}

C_{25}

C_{26}

C_{34}

C_{35}

C_{36}

C_{44}

C_{45}

C_{46}

C_{55}

C_{56}

C_{66}

In the program the transformed elastic constants C'_{ij} are printed out as:

$$C_1 = c'_{11}$$

$$C_2 = c'_{13}$$

$$C_3 = c'_{14}$$

$$C_4 = c'_{15}$$

$$C_5 = c'_{33}$$

$$C_6 = c'_{34}$$

$$C_7 = c'_{35}$$

$$C_8 = c'_{36}$$

$$C_9 = c'_{44}$$

$$C_{10} = c'_{45}$$

$$C_{11} = c'_{46}$$

$$C_{12} = c'_{55}$$

$$C_{13} = c'_{56}$$

$$C_{14} = c'_{66}$$

$$C_{15} = c'_{16}$$

$$C_{16} = c'_{12}$$

$$C_{17} = c'_{25}$$

$$C_{18} = c'_{26}$$

$$C_{19} = c'_{24}$$

$$C_{20} = c'_{23}$$

The transformed constant c'_{22} is not used and therefore not printed out.

The dielectric constants (ϵ_{ij}), called EPS in the program, are the last constants to be entered. They should be entered following the comma after the last value of the elastic coefficients, as

EPS =

followed by 9 values of EPS in the following order, separating each variable by a comma:

ϵ_{11}

ϵ_{12}

ϵ_{13}

ϵ_{21}

ϵ_{22}

ϵ_{23}

ϵ_{31}

ϵ_{32}

ϵ_{33}

In the program the transformed dielectric constants ϵ'_{ij} are printed out as:

$T_1 = \epsilon'_{11}$

$T_2 = \epsilon'_{13}$

$T_3 = \epsilon'_{33}$

$T_4 = \epsilon'_{21}$

$T_5 = \epsilon'_{23}$

The transformed constant ϵ'_{22} is never used and therefore is not printed out.

After the last value of EPS, namely ϵ_{33} , print a dollar sign (\$) instead of a comma. That is,

$$\text{EPS} = \epsilon_{11}, \epsilon_{12}, \epsilon_{13}, \epsilon_{21}, \epsilon_{22}, \epsilon_{23}, \epsilon_{31}, \epsilon_{32}, \epsilon_{33} \$ \quad .$$

This signals the end of the first data set.

The second data set is called "INPUT." \$INPUT must be printed in columns 2-7 of the next card (following the EPS data). Then each input parameter should be entered, followed by a comma (except the last value, which should be followed by a dollar sign, \$). The following is a definition of each input parameter (unless otherwise stated, the input parameters will refer to both Part A and Part B):

<u>Input Name</u>	<u>Equation Names</u>	<u>Definition</u>
MUA	μ (Part A)	Lame's constants for elastic conductor
LAMDAA	λ (Part A)	
RHOA	ρ	Mass density of elastic conductor
LAMDAB	λ (Part B)	Euler Angles
MUB	μ (Part B)	
NUB	ν (Part B)	
RHOB	ρ (Part B)	Mass density of crystal
VS	v_s	Initial guess to a velocity. This initial value will be used to find a final velocity, $\bar{v}_s$ such $ f(\bar{v}_s) < \epsilon$, where ϵ is input.
KS	k_s	Can take on two values: $k_s = 0$ for Part B $k_s = 1$ for Part A
EPSLON	ϵ	A positive number used as a convergence criterion. When $ f(v_s) < \epsilon$, then v_s is assumed to be the root required.
WH	wh	Normalized height of conducting wall or magnetic wall (Part B) Normalized thickness of elastic conductor (Part A). To input $wh = \infty$, set $WH > 10^{10}$.
WXA	ux_a (Part A)	Normalized distance into elastic conductor
WXB	ux_b (Part A)	Normalized distance into crystal
KL	K_L (Part B)	K_L is normally 0. However, if the electric wall case is being run (see K_M) and if $wh = 0$, then K_L should be set to 1.
KM	K_M (Part B)	This can take on two values: $K_M = 0$ electric wall $K_M = 1$ magnetic wall

<u>Input Name</u>	<u>Equation Names</u>	<u>Definition</u>
MAX	--	Since an iteration scheme is used for convergence for a final root v_s , there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 10). If MAX is set to zero (MAX = 0) the determinant $ f(\bar{v}_s) $ will be evaluated for the particular v_s value input – the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which v_s lies.
ICHECK	--	A logical parameter which controls the use of a checkout option. If ICHECK = .FALSE., all FINAL ANSWERS* are computed in addition to the evaluation of the determinant $ f(\bar{v}_s) $. If ICHECK = .TRUE., FINAL ANSWERS are not computed – evaluation of the determinant only. This option was included for use when MAX = 0.
DVS	Δv_s	Increment to be used for v_s when ICHECK = .TRUE. (DVS ≥ 0 .)
VSMAX	$v_{s\max}$	Maximum value of v_s to be used when DVS $\neq 0$.
EPSO	ϵ_0	Permittivity of free space
WX	ux (Part B)	Normalized distance into crystal
DNU	Δv	If the user wishes to vary v (NUB) from some initial value, v_i , to some final value, $v_{\max}$, in steps of Δv , then set DNU equal to the steps desired; also, see NUMAX.
NUMAX	$v_{\max}$	The maximum value of v (see DNU). $v_{\max}$ is only used when DNU $\neq 0$.
DWX	Δux	An increment for ux , similar to DNU. If DWX = 0, then ux is not incremented.

*The FINAL ANSWERS consist of the partial field relative amplitudes (Eta), stress components, strain components, time average power flow, electric and mechanical displacements, electric potential, and electric field.

<u>Input Name</u>	<u>Equation Names</u>	<u>Definition</u>
WXMAX	ω_{max}	The maximum value of ωx (see DWX). ω_{max} is only used when DWX $\neq$ 0.
TITLE	--	An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example TITLE = 6HQWARTZ
REPEAT	--	REPEAT is a logical variable and in its usage, can take only one value: .TRUE. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input as .TRUE. This means that the \$CONST data will have to be input again (in the other cases above, \$CONST would not have to be input again).
HXAGNL	--	Parameter which controls the calculation of betas (β's) for a hexagonal crystal (such as zinc oxide) .TRUE. hexagonal crystal (use special technique) .FALSE. non-hexagonal crystal (use normal procedure)
VSINC	--	VSINC = .TRUE. -- New estimates of initial velocity (v_s) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ..., NUMAX) VSINC = .FALSE. -- The same initial estimate of velocity is used for all values over the specified range of NUB.

The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

ROOTS	Print the roots of the polynomial each time they are calculated.
COEFF	Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.
DETERM	Print the L matrix and the value of the determinant.
POLY	Print the coefficients of the 8'th order polynomial.
BETA	Print the values of β_{ij} .
ALPHA	Print α_A 's, Part A.
ALL	Print all of the above.

The manner in which the above listed parameters in the \$INPUT data set are input is best illustrated by an example (assume Part B is being run):

```
$INPUT      MUB = 90., LAMDAB = 90., NUB = 100., RHOB = 4700.,
            VS = 3400., KS = 0, EPSLON = 1.E-11, WH = 0, KL = 1,
            KM = 0, WX = 0., DWX = 10., WXMAX = 100.,
            title = 15HLITHIUM NIOBATE
```

wh is zero in the above example. To input $wh = \infty$, set $wh > 10^{10}$. Note that some of the values discussed in the list are not present in the above example. This is because either they are not required or the program assumed nominal values. A nominal value is a value that a parameter will take on if no other value is input. In the above example, MAX, EPSO, and DNU take on their nominal values of 10, 8.85×10^{-12} , and 0, respectively. It is not necessary to input NUMAX since DNU = 0; all parameters referring to Part A are not necessary since Part B is being run; and all the logical parameters take on their nominal value of false. The following is a complete list of nominal values:

<u>Parameter</u>	<u>Nominal Value</u>
MUA	2.85×10^{10}
LAMDAA	1.5×10^{11}
RHOA	1.888×10^4
VS	3000
EPSLON	1×10^{-11}
KL	0
KM	0
MAX	10
EPSO	8.85×10^{-12}
DNU	0
DWX	0
DVS	0
TITLE	15LITHIUM NIOBATE
ROOTS	.FALSE.
COEFF	
DETERM	
POLY	
BETA	
ALL	
REPEAT	
ALPHA	

Sample Data Decks:

The following sample data deck, illustrated on the attached code sheet, gives an example of three data runs: the first is a 90-90-100 degree cut of lithium niobate. The second is the same, except for a new value of μh . The third is a 0, 0, -90 cut of quartz (note that REPEAT is set to true in the second case, just prior to the case when new coefficients are to be input).

FORM 100-REV. 3-66

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1. $Y = 1.5$ 2. $Y = 1.5$ 3. $Y = 1.5$ 4. $Y = 1.5$ 5. $Y = 1.5$ 6. $Y = 1.5$ 7. $Y = 1.5$ 8. $Y = 1.5$ 9. $Y = 1.5$ 10. $Y = 1.5$ 11. $Y = 1.5$ 12. $Y = 1.5$ 13. $Y = 1.5$ 14. $Y = 1.5$ 15. $Y = 1.5$ 16. $Y = 1.5$ 17. $Y = 1.5$ 18. $Y = 1.5$ 19. $Y = 1.5$ 20. $Y = 1.5$ 21. $Y = 1.5$ 22. $Y = 1.5$ 23. $Y = 1.5$ 24. $Y = 1.5$ 25. $Y = 1.5$ 26. $Y = 1.5$ 27. $Y = 1.5$ 28. $Y = 1.5$ 29. $Y = 1.5$ 30. $Y = 1.5$ 31. $Y = 1.5$ 32. $Y = 1.5$ 33. $Y = 1.5$ 34. $Y = 1.5$ 35. $Y = 1.5$ 36. $Y = 1.5$ 37. $Y = 1.5$ 38. $Y = 1.5$ 39. $Y = 1.5$ 40. $Y = 1.5$ 41. $Y = 1.5$ 42. $Y = 1.5$ 43. $Y = 1.5$ 44. $Y = 1.5$ 45. $Y = 1.5$ 46. $Y = 1.5$ 47. $Y = 1.5$ 48. $Y = 1.5$ 49. $Y = 1.5$ 50. $Y = 1.5$ 51. $Y = 1.5$ 52. $Y = 1.5$ 53. $Y = 1.5$ 54. $Y = 1.5$ 55. $Y = 1.5$ 56. $Y = 1.5$ 57. $Y = 1.5$ 58. $Y = 1.5$ 59. $Y = 1.5$ 60. $Y = 1.5$ 61. $Y = 1.5$ 62. $Y = 1.5$ 63. $Y = 1.5$ 64. $Y = 1.5$ 65. $Y = 1.5$ 66. $Y = 1.5$ 67. $Y = 1.5$ 68. $Y = 1.5$ 69. $Y = 1.5$ 70. $Y = 1.5$ 71. $Y = 1.5$ 72. $Y = 1.5$ 73. $Y = 1.5$ 74. $Y = 1.5$ 75. $Y = 1.5$ 76. $Y = 1.5$ 77. $Y = 1.5$ 78. $Y = 1.5$ 79. $Y = 1.5$ 80. $Y = 1.5$ 81. $Y = 1.5$ 82. $Y = 1.5$ 83. $Y = 1.5$ 84. $Y = 1.5$ 85. $Y = 1.5$ 86. $Y = 1.5$ 87. $Y = 1.5$ 88. $Y = 1.5$ 89. $Y = 1.5$ 90. $Y = 1.5$ 91. $Y = 1.5$ 92. $Y = 1.5$ 93. $Y = 1.5$ 94. $Y = 1.5$ 95. $Y = 1.5$ 96. $Y = 1.5$ 97. $Y = 1.5$ 98. $Y = 1.5$ 99. $Y = 1.5$ 100. $Y = 1.5$																																																																																																			

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HUGHES AIRCRAFT COMPANY
7094 FORTRAN CODING SHEET

PAGE _____ OF _____

The following is a description of the program flow diagram provided at the end of this section.

(1)
First, the nominal data values are set up in the program. These values are assumed by certain parameters in the program unless new values are specified. Following this the program reads in the elastic (G), piezoelectric (P), and dielectric (EPS) constants (CONST DATA) of the substrate medium. Finally, the remaining input data is read in (INPUT).

Next, subroutine SETCTE is called to perform the Euler transformation to obtain the elastic (CC), piezoelectric (CE), and dielectric (CT) constants relative to the input coordinate system as specified by the constants λ , μ , and ν . At this point subroutine ROOT is called to perform the calculations leading to the evaluation of the determinant of the boundary condition matrix (L matrix of the analysis). The determinant is referred to as $F(VS)$ since it is evaluated as a function of velocity (VS).

There is an option in the program to use a root finding scheme to minimize $|F(VS)|$ or simply to increment VS in steps of DVS and calculate $F(VS)$ at each value. To perform these various calculations at a particular velocity (VS) ROOT calls subroutine F which is described in detail below (in Determination of $F(VS)$).

After exiting from ROOT and returning to the main program logical checks are made to establish the type of case considered in ROOT. Depending upon the results of these checks the values of the amplitudes of the partial surface wave fields are computed ($A^{(i)}$ of analysis section). In the program these are called ETA(1), ETA(2), etc. The program now proceeds to compute the magnitude (MAG U(I)) and phase (PHASE U(I)) of the mechanical displacements ($\bar{U}(I)$, $I=1, 2, 3$) and electric potential ($\bar{U}(4)$). Next subroutine PIFUN is called to compute the time average flow (P1M, P2M) followed by the computation of the stress components (TW31, TW32, TW33, TW11, TW12, TW22) in subroutine TFUN. Subroutine SFUN then implements the calculation of the strain components (S11, S33, S12, S13, S23). Finally the electric field (E1, E3) and electric displacement (D1, D2, D3) are computed. All the above quantities are evaluated as a function of normalized distance (WX) into the crystal. They can be computed at incremented values of WX for any specified initial and final values.

The velocity (VS) can be incremented if desired (up to some specified maximum value, VSMAX) and the steps in ROOT and that which follows are repeated for each new velocity. Thus it is possible to plot the determinant as a function of velocity. After VSMAX is reached there is an option to increment the third Euler angle (NU) and repeat the steps from SETCTE on. When this has been completed the program returns to read in new CONST DATA if the crystal is being changed or to read in new INPUT DATA if the crystal is to remain unchanged but the orientation is to be changed. After all data from both sources has been exhausted the program stops.

Determination of F(VS)

Subroutine F calls subroutine STRIP to compute the coefficients of the eighth order polynomial equation in α .^{*} Next subroutine CROOT calculates the 8 roots (ALFA(I), I = 1, 8) of the polynomial equation by Muller's method. If the medium is non-piezoelectric the solution for the roots involves two extraneous roots which are rendered useless by setting them equal to $-10-10j$. The roots with positive real part are chosen (ALFAB(I), I=1, K).

If the medium is piezoelectric and the number of roots with positive real part (K) is equal to 4 the program proceeds to calculate the relative amplitudes ($\beta_i^{(t)}$ of the analysis section) of the displacement and potential corresponding to each α . These amplitudes are referred to as BETAB(I,J) in the program. The matrix (ACAP, $\hat{A}$ for simplicity) of coefficients of the amplitudes ($\beta_i^{(t)}$) is set up for each α . If the crystal is not hexagonal non-degenerate cases are solved by setting $\beta_4^{(t)} = 1$ and solving the first three equations of the system for $\beta_i^{(t)}$ i = 1, 2, 3. If the crystal is hexagonal one of the α 's naturally leads to an ill-conditioned system if the first three equations are solved for $\beta_1^{(t)}$, $\beta_2^{(t)}$, and $\beta_3^{(t)}$ in terms of $\beta_4^{(t)}$. Thus $\beta_1^{(t)}$ is set equal to 10^{-10} and the system composed of the second, third, and fourth equation are solved for $\beta_2^{(t)}$, $\beta_3^{(t)}$, $\beta_4^{(t)}$. If the case is degenerate the $\beta_i^{(t)}$'s are calculated in the fashion indicated in the analysis.

If $K < 4$ the procedure for calculating the $\beta_i^{(t)}$'s is dependent upon the value of K. If $K \leq 1$ the case terminates since no solution is possible with only one available value of α . If $K > 1$, $\hat{A}_{12}$ and $\hat{A}_{23}$ are investigated to see if they are identically zero.

^{*}See Appendix IV.

If either $\hat{A}_{12}$ or $\hat{A}_{23}$ is not identically zero the case cannot be degenerate. The program proceeds in one of two possible ways. If the crystal is non-piezoelectric and $K = 3$ the $\beta_i^{(t)}$'s are calculated as indicated in the analysis (i.e. $\beta_4^{(t)} = 0$, $\beta_3^{(t)} = 1$ and the first two equations of the system are solved for $\beta_1^{(t)}$ and $\beta_2^{(t)}$). If either $K \neq 3$ or the crystal is piezoelectric the case terminates. This is due to the fact that if the crystal is piezoelectric and non-degenerate, four α 's are necessary for a solution in the general case.

If both $\hat{A}_{12}$ and $\hat{A}_{23}$ are equal to zero, the non-piezoelectric case is degenerate and is treated as follows. $|\hat{A}_{22}|$ is calculated for each value of α . For $K = 2$ it is necessary that $|\hat{A}_{22}|$ be non-zero for both values of α (due to the large magnitude of the individual terms in $\hat{A}_{22}$ it is sufficient to compare $|\hat{A}_{22}|$ to 10^7). If $|\hat{A}_{22}| > 10^7$ for both values of α then we may set

$$\beta_2^{(t)} = 0, \beta_3^{(t)} = 10^{-10}, \text{ and } \beta_1^{(t)} = -\frac{\hat{A}_{13}^{(t)}}{\hat{A}_{11}^{(t)}} \cdot 10^{-10}.$$

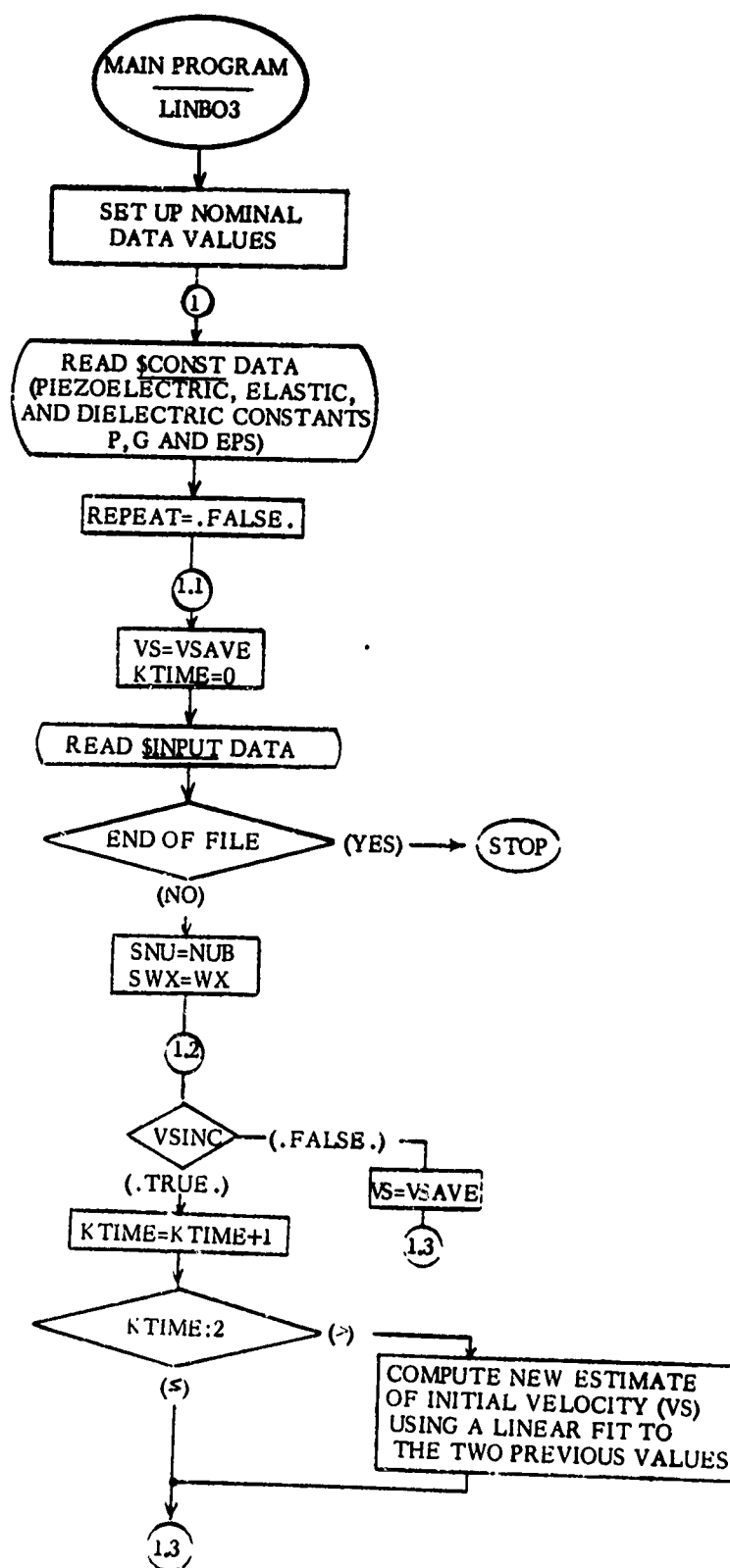
Otherwise the case is terminated. If $K = 3$ the minimum value of $|\hat{A}_{22}|$ is calculated and the corresponding α is discarded. The β 's are then calculated for the other two α 's from the above formulas.

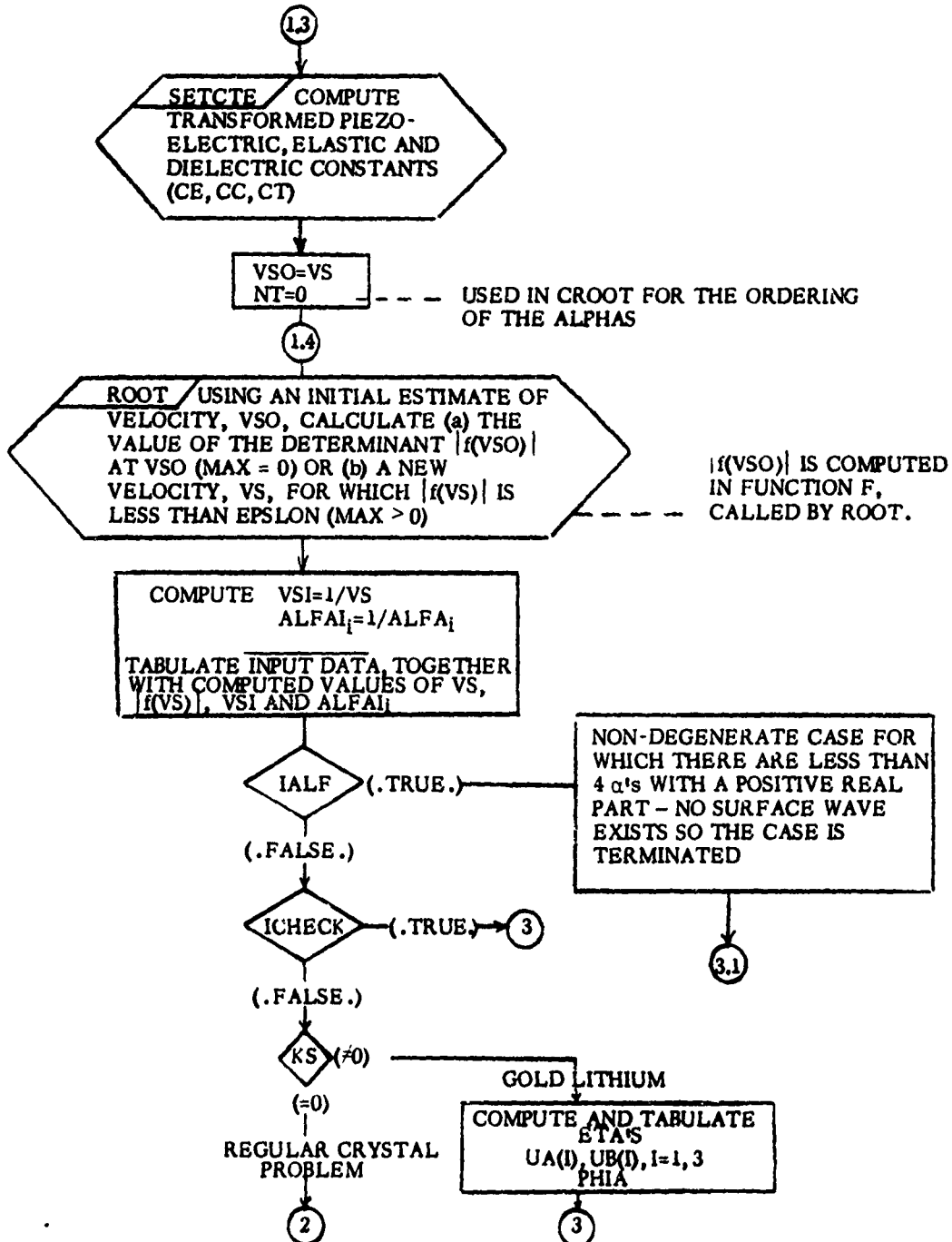
If $\hat{A}_{12}$ and $\hat{A}_{23}$ are identically equal to zero and the crystal is piezoelectric the program proceeds as follows. $\hat{A}_{24}$ is tested and if equal to zero the first degenerate case of the analysis section must be considered. The case is terminated if $K = 2$ but if $K = 3$ a check is made of $|\hat{A}_{22}|$. If $|\hat{A}_{22}| > 10^7$ for all three α 's, the β 's are calculated as indicated in the analysis. If $|\hat{A}_{22}| < 10^7$ for any of the α 's the case is terminated.

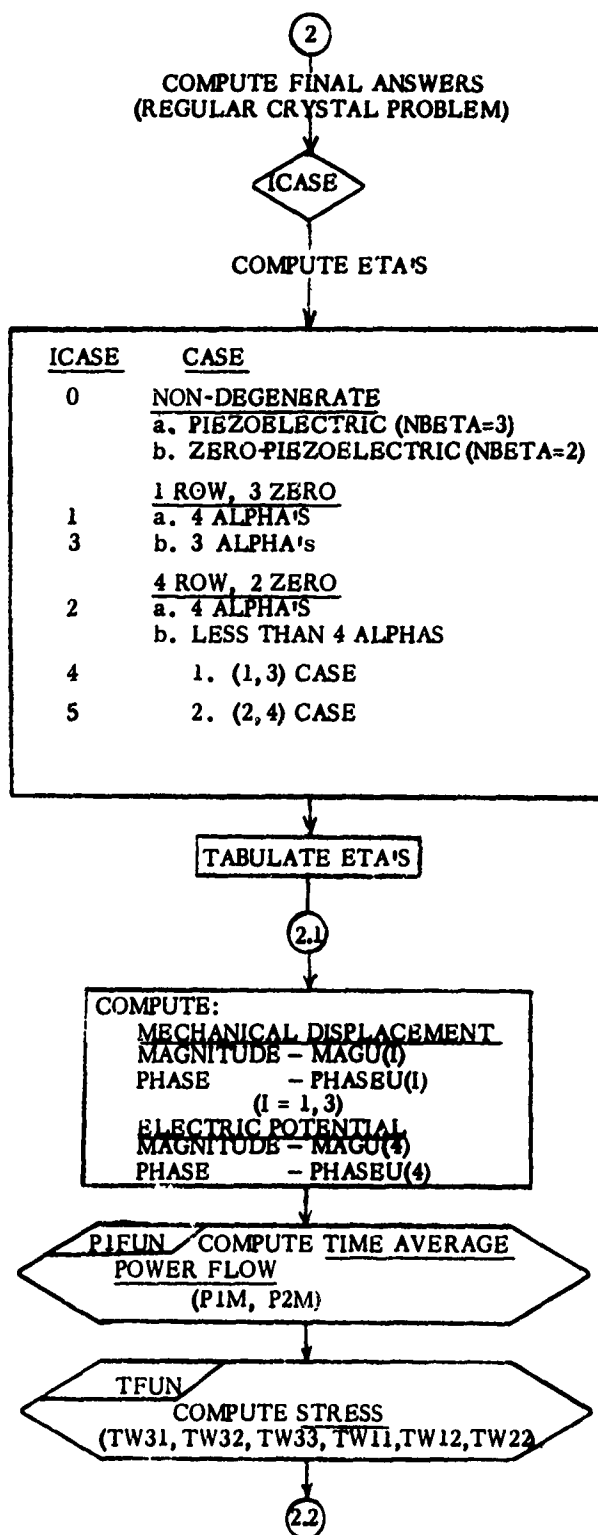
If $\hat{A}_{22} \neq 0$ a check of $\hat{A}_{14}$ and $\hat{A}_{34}$ is made. If they are not both identically equal to zero the case is terminated. If both are equal to zero the second degenerate case of the analysis is considered. In this case $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ (TERMJ in the program) is calculated for each α . If $\text{TERMJ} \geq 10^{-5}$ for two of the α 's the (β_1, β_3) split of the analysis arises and the β 's are appropriately calculated. If $\text{TERMJ} < 10^{-5}$ for two of the α 's the (β_2, β_4) split arises and the β 's are appropriately calculated. Under any other conditions the case is terminated.

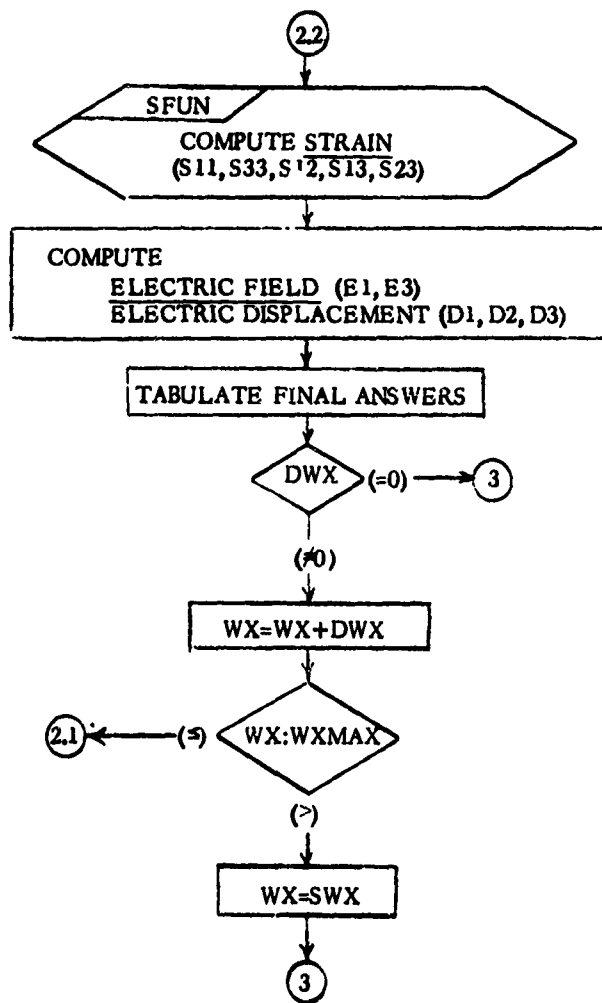
Now that the α 's and β 's are known the boundary condition matrix ($\hat{L}$) is set up and its determinant evaluated. If the problem of a conducting elastic medium in contact with a piezoelectric or elastic medium is being considered

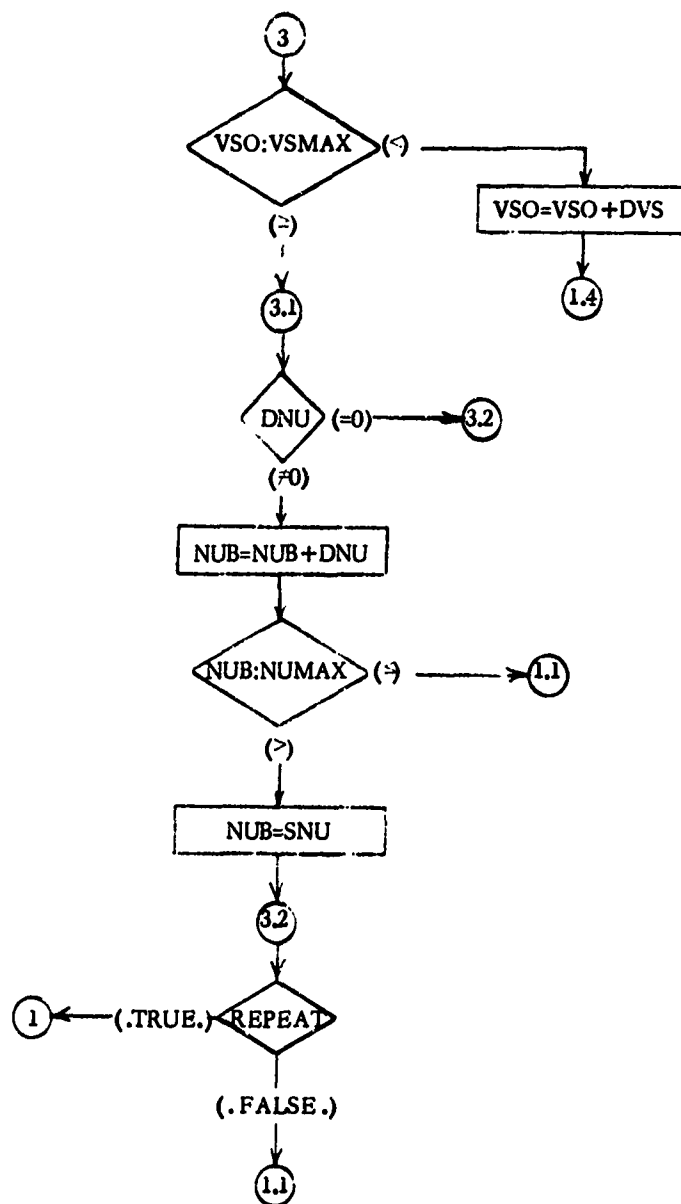
the α 's and β 's appropriate to the conductor are first evaluated then the appropriate boundary condition matrix is set up and its determinant evaluated. This completes the computation of $F(VS)$ whereupon the subroutine is exited back to the main program.







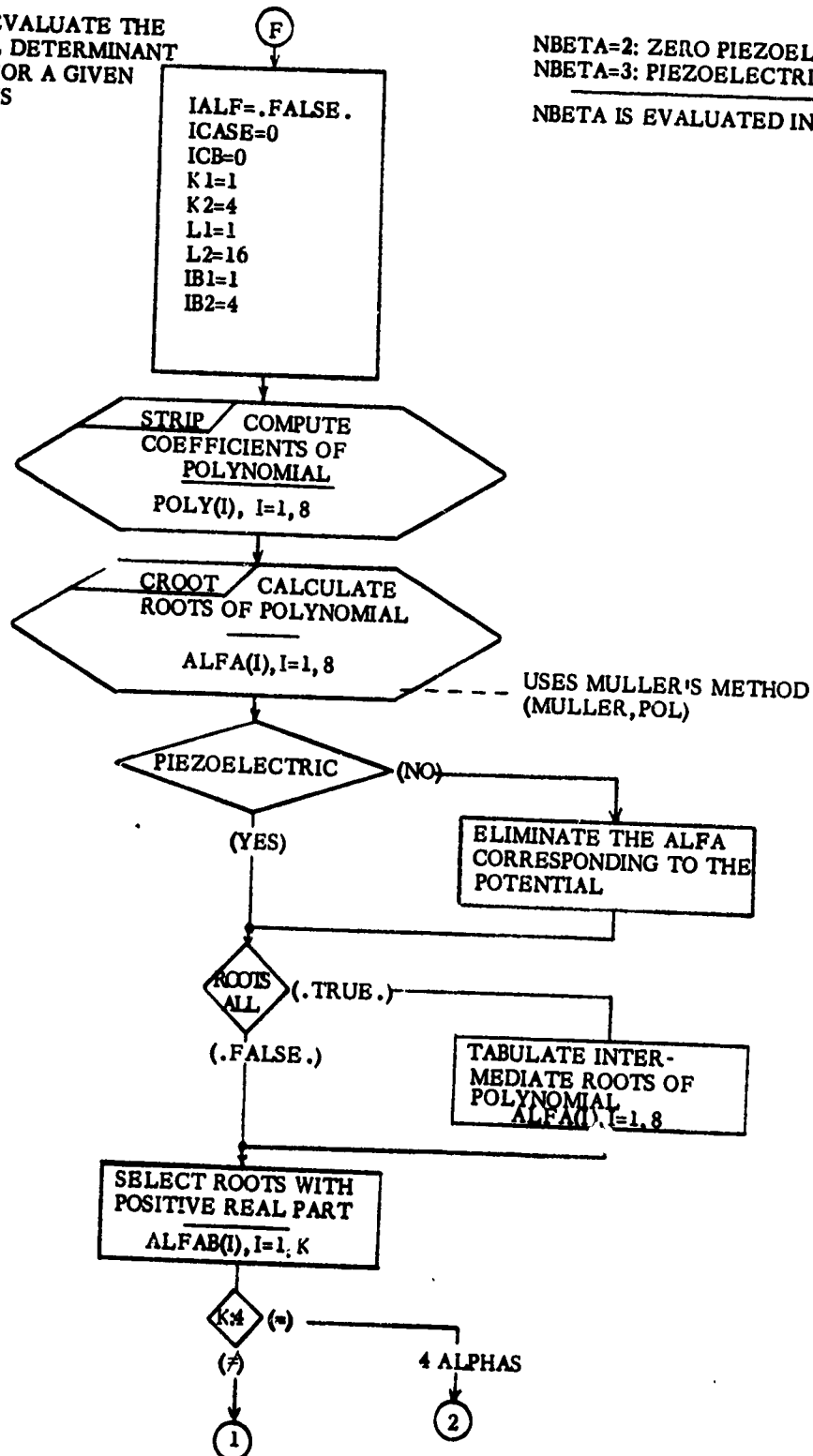


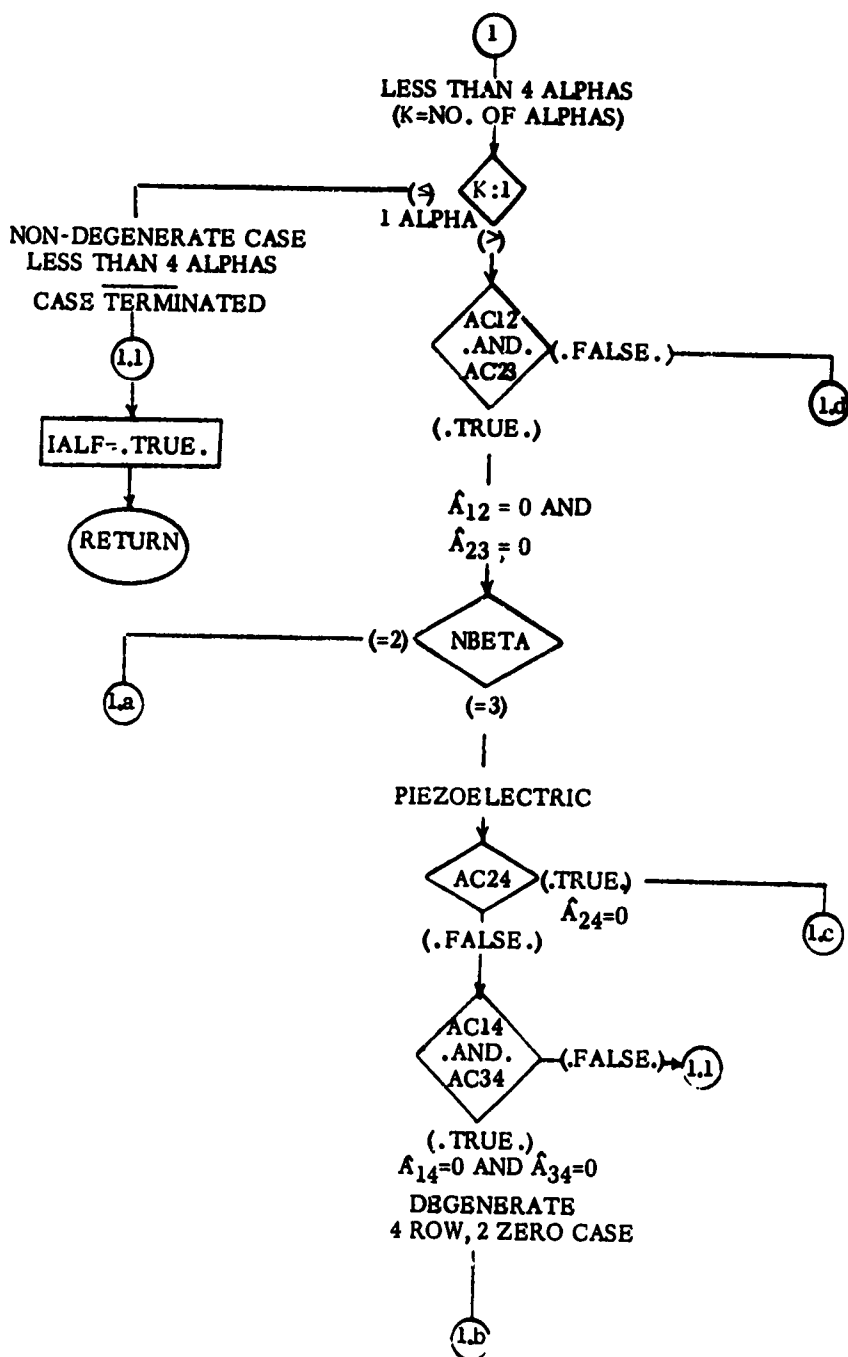


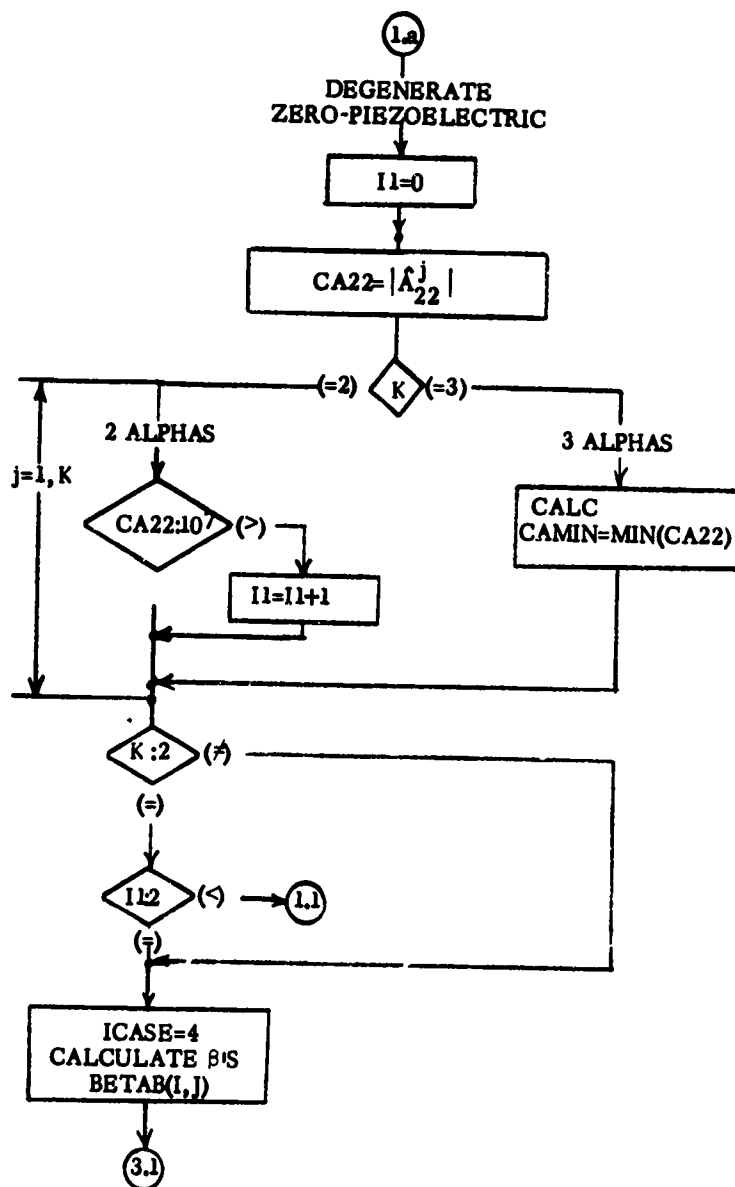
EVALUATE THE
L DETERMINANT
FOR A GIVEN
VS

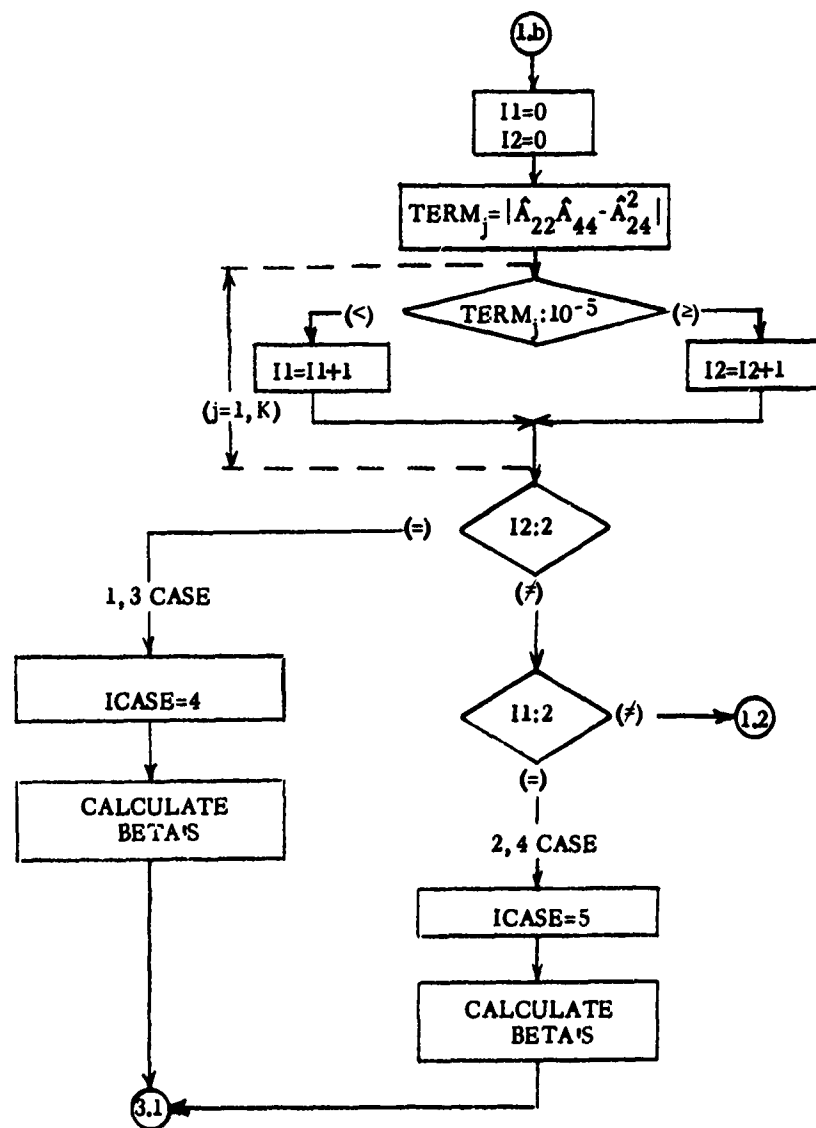
NBETA=2: ZERO PIEZOELECTRIC
NBETA=3: PIEZOELECTRIC

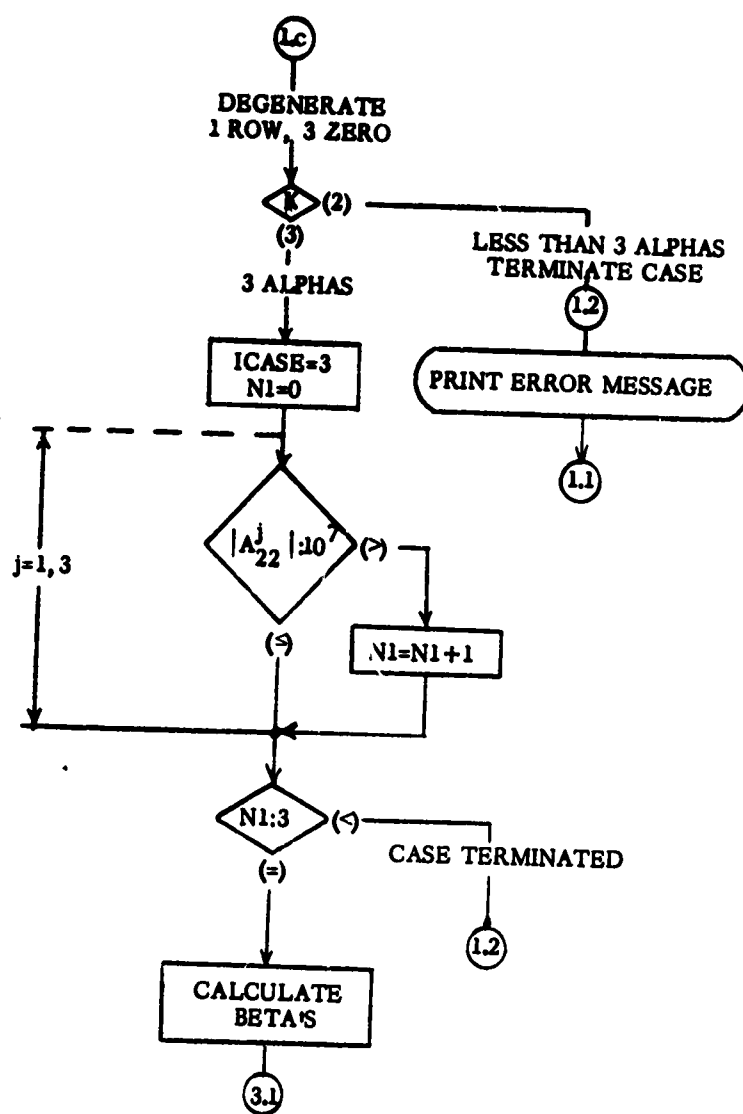
NBETA IS EVALUATED IN SETCTE

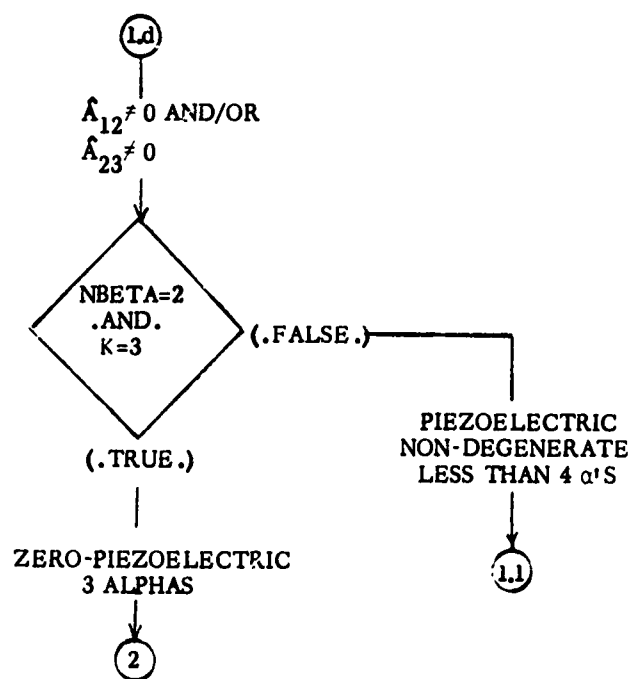


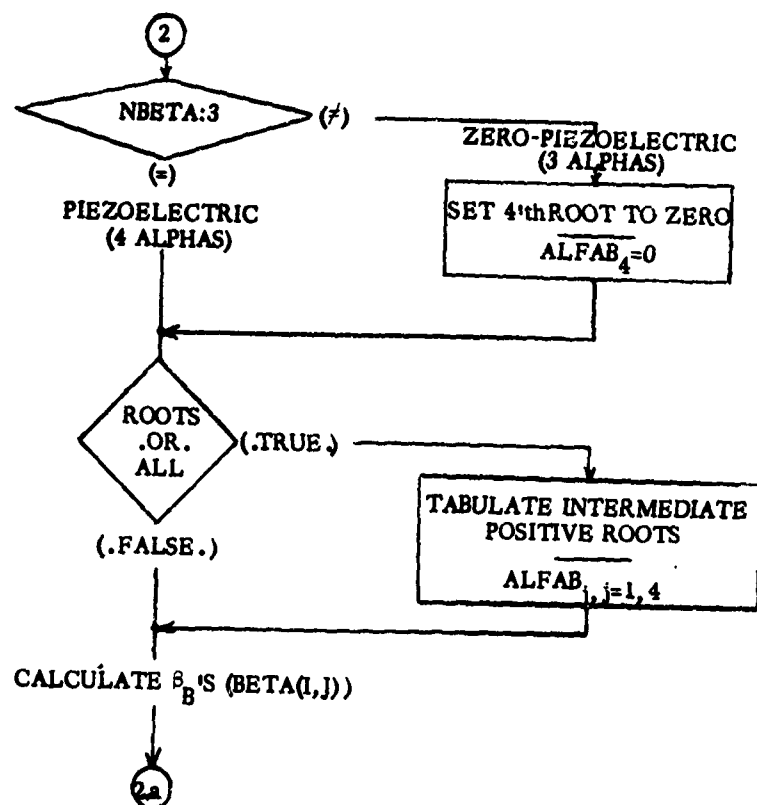


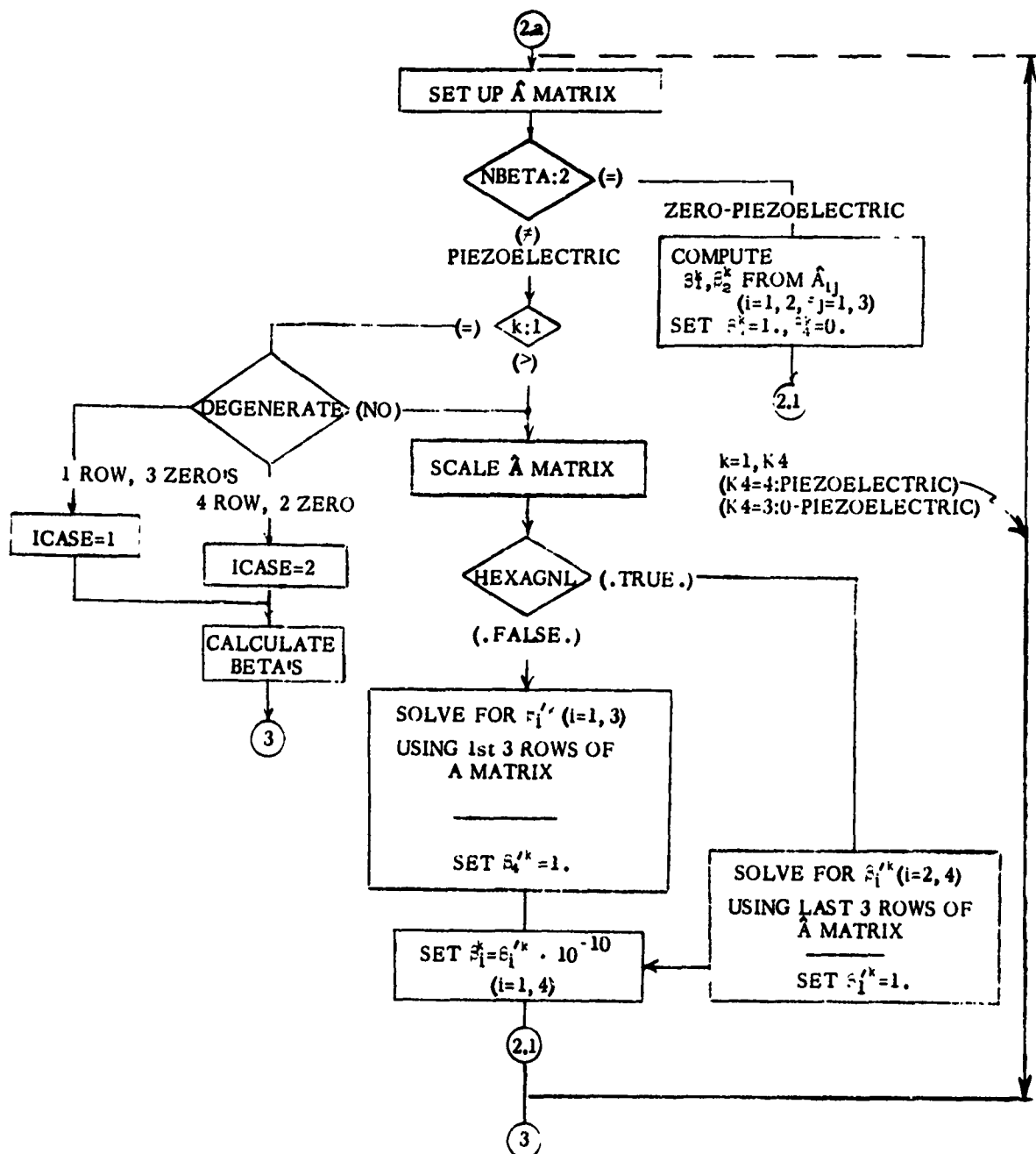


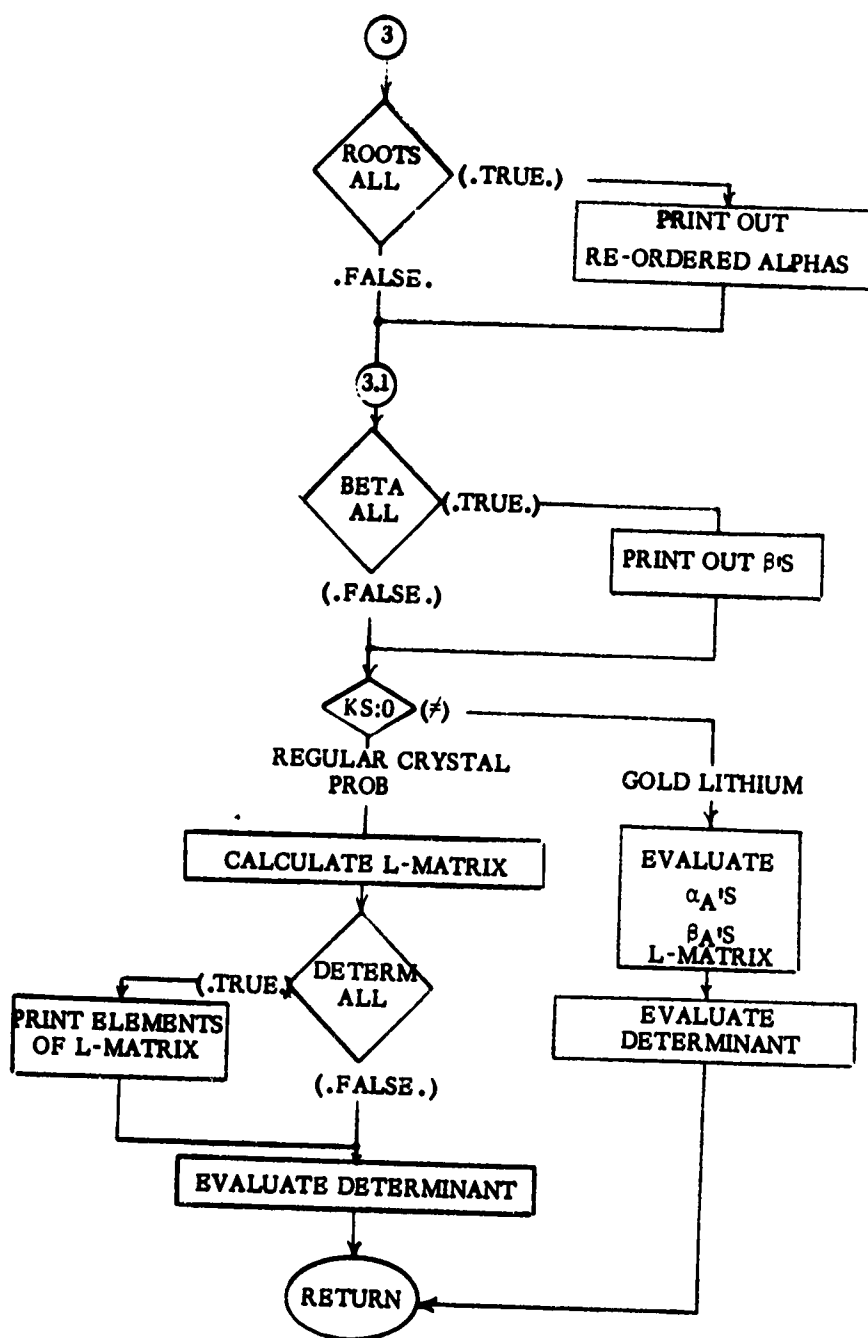












2. Surface Waves at the Boundary Between a Fluid Medium and Piezoelectric Crystal – Program Description

The purpose of this program is to determine the complex velocity of propagation of surface waves at the interface between a semi-infinite fluid and a piezoelectric substrate. Input parameters which define the fluid, the piezoelectric medium, and control the use of the program are described on the following pages.

The program is set up to run on the IBM 7094, using FORTRAN IV and Namelist input and the deck set up is identical with that given for the preceding program.

As in the preceding program, two input sections are required: the first describes the material constants of the piezoelectric crystal and the second describes the orientation of the crystal as well as other information pertinent to the execution of the program. The first data set is called CONST and is identical with that presented in Section IV.1. The following is a definition of each input parameter in the "INPUT" data set. Medium A refers to the dielectric (elastic) layer and medium B, to the piezoelectric substrate.

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
LAMDAB	λ_B	real	Euler angles for Medium B.
MUB	μ_B	real	
NUB	ν_B	real	
DNU	$\Delta\nu$	real	If the user wishes to vary ν (NUB) from some initial value, ν , to some final value, $\nu_{\max}$, in steps of $\Delta\nu$, then set DNU equal to the steps desired; also, see NUMAX. (See VSINC)
NUMAX	$\nu_{\max}$	real	The maximum value of ν (see DNU). $\nu_{\max}$ is only used when DNU $\neq 0$.
VS	v_s	real	Initial estimate of velocity. This initial value will be used to find a final velocity, v_s , such that $ f(v_s) < \epsilon$, where ϵ is input.
DVS	Δv_s	real	If the user does not care to use the root-finding scheme in determining a final value for v_s , but wishes, instead, to evaluate the determinant $ f(v_s) $ for particular values of v_s in the range from v_s to $v_{s\max}$ in steps of Δv_s , then set DVS equal to the step size desired. (For use when MAX = 0.)
VSMAX	$v_{s\max}$	real	Maximum value of v_s to be used when DVS $\neq 0$.
LAMDAA	λ_A	real	Lame constants for Medium A.
MUA	μ_A	real	
RHOA	ρ_A	real	Mass density of Medium A.
RHOB	ρ_B	real	Mass density of Medium B.
EPSLON	ϵ	real	A positive number used as a convergence criterion by the root-finding scheme (MAX > 0). If $ f(v_s) < \epsilon$, then v_s is assumed to be the root required.
EPSO	ϵ_0	real	Permittivity of free space.

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
EPSA	ϵ_A	real	Dielectric constant for Medium A.
WH	ωh	real	Frequency thickness product.
WXA	ωx_A	real	Normalized distance into Medium A.
DWXA	$\Delta \omega x_A$	real	In order to vary ωx_A (WXA) from an initial value, ωx_A , to a final value $\omega x_{A_{max}}$, DWXA must be set equal to the desired step size. See WXAMAX.
WXAMAX	$\omega x_{A_{max}}$	real	The maximum value of ωx_A to be used when DWXA $\neq$ 0.
WXB	ωx_B	real	Normalized distance into Medium B.
DWXB	$\Delta \omega x_B$	real	In order to vary ωx_B from an initial value, ωx_B , to a final value, $\omega x_{B_{max}}$, DWXB must be set equal to the step size desired. See WXBMAX.
WXBMAX	$\omega x_{B_{max}}$	real	The maximum value of ωx_B to be used when DWXB $\neq$ 0.
ICHECK	----	logical	ICHECK = .TRUE. - All FINAL ANSWERS* are computed in addition to the evaluation of the determinant $ f(\nabla_s) $. ICHECK = .FALSE. - FINAL ANSWERS are <u>not</u> computed; evaluate determinant only.
MAX	----	integer	Since an iteration scheme is used for convergence for a final root v_s , there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 15). If MAX is set to zero (MAX = 0) the determinant $ f(\nabla_s) $ will be evaluated for the particular v_s value input - the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which v_s lies.

*The FINAL RESULTS, which are computed for all values of WXA (dielectric layer) and WXB (piezoelectric layer), include the following:

Stress Components
Strain Components
Time Average Power Flow
Electric Displacement

Mechanical Displacement
Electric Potential Magnitude
Electric Field

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
TITLE	----	BCD	An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example TITLE = 6HQWARTZ
HXAGNL	----	logical	Parameter which controls the calculation of betas (β's) for a hexagonal crystal (such as zinc oxide) .TRUE. hexagonal crystal (use special technique) .FALSE. non-hexagonal crystal (use normal procedure)
VSINC	----	logical	VSINC = .TRUE. – New estimates of initial velocity (v_s) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ... , NUMAX) VSINC = .FALSE. – The same initial estimate of velocity is used for all values over the specified range of NUB.
IOP	----	integer	Degenerate case options (used when exactly four α's with positive real part occur). IOP = 1 – seek modes of propagation of the Quasi-Rayleigh or Sesawa type. IOP = 2 – seek modes of propagation of Love type.
REPEAT	----	logical	REPEAT is a logical variable and in its usage, can take only one value: .TRUE. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
			as .TRUE. This means that the \$CONST data will have to be input again (in the other cases above, \$CONST would not have to be input again).

The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

TABCTE	Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.
ROOTS	Print the roots of the polynomial each time they are calculated.
BETA	Print the values of β_{ij} .
DETERM	Print the value of the determinant.
COEFF	Print the coefficients of the 8 th order polynomial.
TABL	Print the L matrix (or $\hat{P}$, $\hat{Q}$, $\hat{R}$, etc., when used).
ALPHA	Print the roots of the polynomial (σ_B^j 's) and the re-ordered roots for degenerate cases.
ALL	Print all of the above.

Data items may be excluded from the input stream at the discretion of the user. Items omitted from the first data set will take on nominal values (i.e.: values assigned within the program). Items omitted from succeeding data sets will take on previously assigned values. The following is a complete list of nominal values:*

*All logical parameters have a nominal value of .FALSE.

<u>Parameter</u>	<u>Nominal Value</u>
LAMDAB	0.
MUB	0.
NUB	0.
DNU	0.
NUMAX	0.
VS	3000.
DVS	0.
VSMAX	0.
LAMDAA	1.5×10^{11}
MUA	2.85×10^{10}
RHOA	1.888×10^4
RHOB	4700.
EPSLON	$1. \times 10^{-11}$
EPSO	8.85×10^{-12}
EPSA	44.25×10^{-12}
WH	0.
WXA	0.
DWXA	0.
WXAMAX	0.
WXB	0.
DWXB	0.
WXBMAX	0.
MAX	15.
TITLE	LITHIUM NIOBATE
IOP	1

The computer program flow described below shares a good many features of the program described in Section IV.1. As in the preceding programs the nominal data values are set up first. Next the piezoelectric (P), elastic (G), and dielectric (EPS) constants are read in (CONST DATA). Following this the rest of the input data is entered (INPUT DATA).

At this point subroutine SETCTE is called to compute the transformed piezoelectric (CE), elastic (CC), and dielectric (CT) constants. Next subroutine ROOT is called. ROOT performs the calculations and calls the subroutines necessary to perform the following tasks:

- (a) Compute $F(VS)$, the boundary condition determinant,
- (b) implement a complex root finding scheme to minimize $|F(VS)|$ as a function of complex velocity (VS),
- (c) perform the perturbation analysis.

ROOT calls subroutine F to perform the manipulations necessary to compute the boundary condition determinant. F will be discussed in some detail below.

After exiting from ROOT and returning to the main program the trial velocity (VS) can be incremented if it is desired only to compute $F(VS)$ at specified velocities rather than implement the root finding scheme or the perturbation scheme. When this has been completed the third Euler angle (NU) can be incremented and all of the steps, from the point where SETCTE is called to calculate the transformed piezoelectric, elastic, and dielectric constants, are repeated for each value of NU.

Following this the program returns to read in new data in either of the following fashions:

- (a) If the crystal is to remain the same but the orientation of the crystal face is changed (new Euler angles) the new data comes from INPUT DATA.
- (b) If the crystal itself is changed as well as the Euler angles the data comes from CONST DATA and INPUT DATA in that order.

When all the data has been exhausted the program stops.

Subroutine F

Upon entering subroutine F a check is made to determine whether the perturbation scheme is to be used. If it is not to be used, VS is set equal to VSO (the input value). If the perturbation scheme is to be used a loop is begun which will allow the numerical derivatives of the various determinants to be taken by setting $VS = VSO + DEL$, $VS = VSO - DEL$ and $VS = VSO$, in that order. For each of these velocities the necessary determinants needed to evaluate Δv_s (the perturbed velocity) are computed. The convergence of the numerical derivatives is checked by noting the differences in the computed determinants and Δv_s as DEL is allowed to take on subsequently smaller values.

The loop is begun by setting counters INIT and IDX both equal to 1. $DEL = EPS(INIT) \cdot VSO$ is calculated and VS is set equal to $VSO + DEL$. $EPS(INIT)$ is a small number depending upon the value of INIT. Three values $EPS(1)$, $EPS(2)$, and $EPS(3)$ will be used eventually as a convergence test on the numerical derivatives. The program now proceeds to set up the M, N, and P matrices as a function of VS (the P matrix is the cofactor of M_{46} and its determinant has been referred to as K in the analysis section, $K = \det(P)$).

When the determinant of the P matrix ($DP(IDX)$) has been evaluated for the first time ($IDX = 1$) a logical check notes that $IDX \neq 3$ and proceeds to evaluate the determinant of the N matrix ($DN(IDX)$). At this point another logical check notes that $IDX = 1$ and proceeds to set $VS = VSO - DEL$ and $IDX = 2$. The program then returns to set up the new M, N, and P matrixes and evaluate $DP(IDX) = DP(2)$. The logical check following the evaluation of $DP(IDX)$ again notes that $IDX \neq 3$ and therefore evaluates $DN(IDX) = DN(2)$. The logical check following evaluation of $DN(IDX)$ now notes that $IDX \neq 1$ as it was before. The program therefore proceeds to calculate the numerical derivatives ($(\det N)'$ and $(\det P)'$) by the approximate formulas

$$\underbrace{DNP(INIT)}_{(\det N)'} = \frac{DN(1) - DN(2)}{2 \cdot DEL} \quad \text{and} \quad \underbrace{DPP(INIT)}_{(\det P)'} = \frac{DP(1) - DP(2)}{2 \cdot DEL} .$$

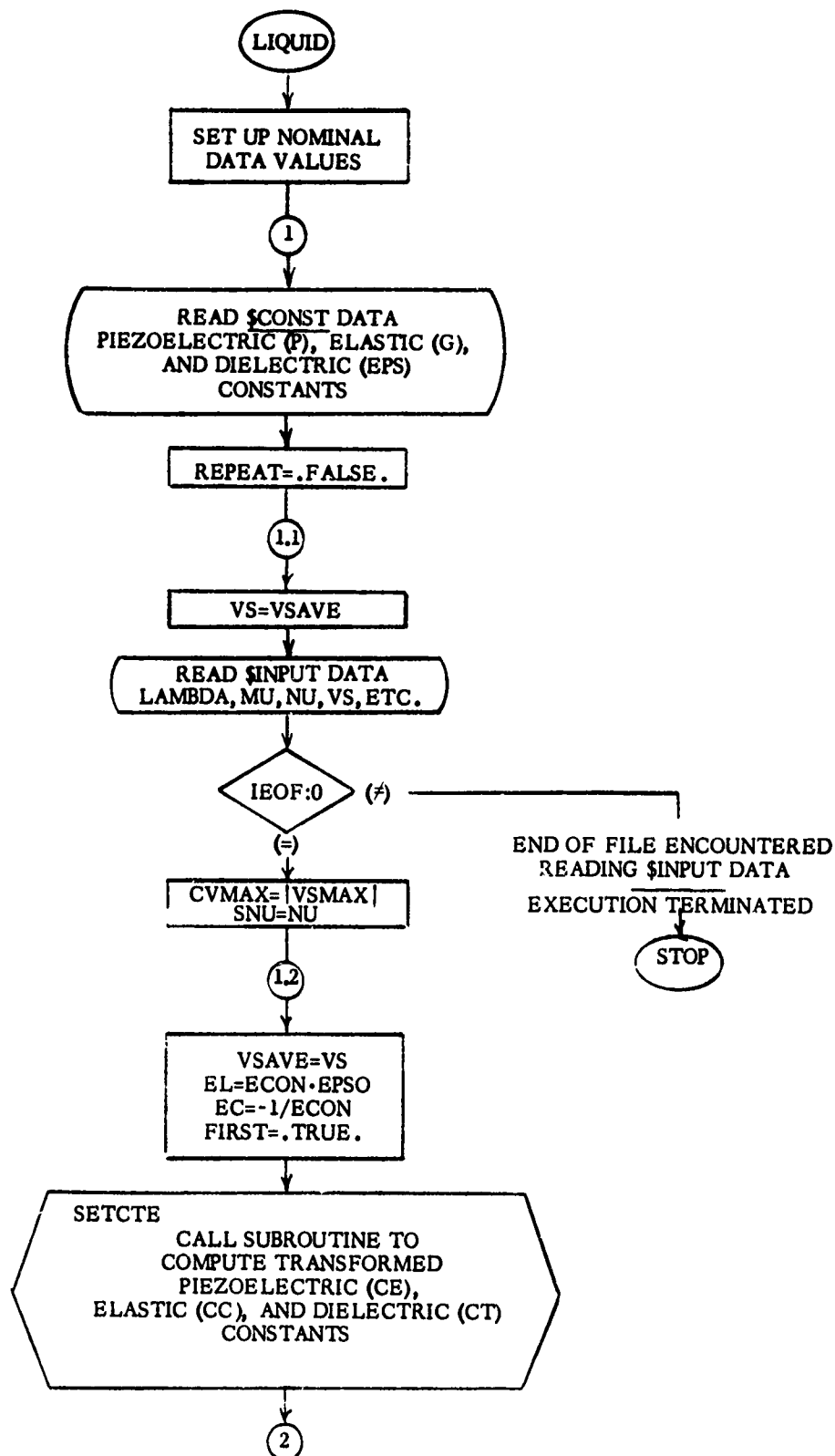
Next a new logical check notes that $INIT \neq 3$ (it is still 1) and returns to the point where INIT and IDX were originally initialized. It now increments INIT by one (i.e. $INIT = 2$ now) and resets $IDX = 1$. $DEL = EPS(2) \cdot VSO$ and $VS = VSO + DEL$ are evaluated and all the steps from this point are repeated until finally the numerical derivatives are again taken with the new value of DEL.

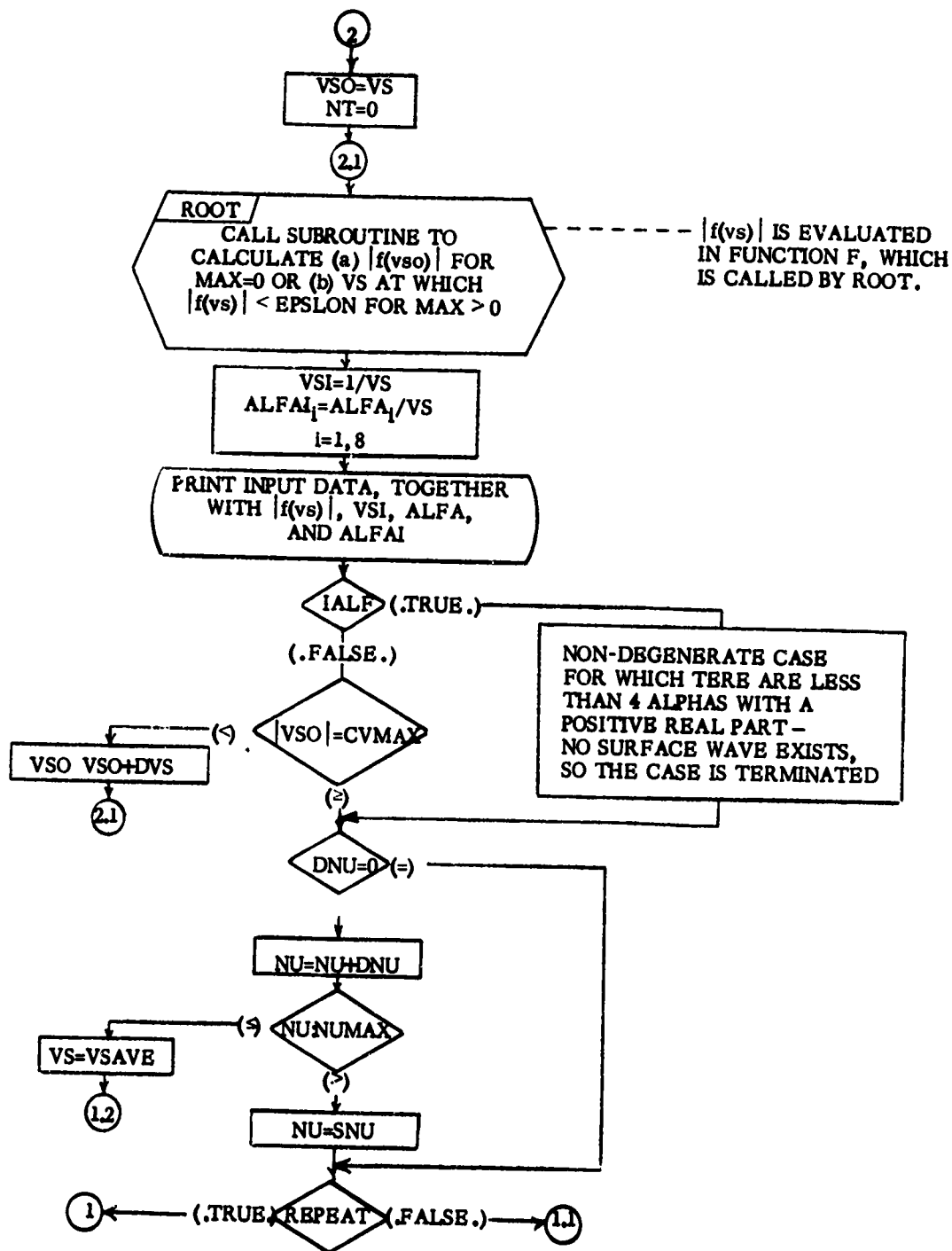
Again the logical check at this point notes $INIT \neq 3$ ($INIT = 2$) and again the point of initialization of $INIT$ and IDX is entered. $INIT$ is again incremented by one ($INIT = 3$ now) and IDX is reset to 1. $DEL = EPS(3) \cdot VSO$ and $VS = VSO + DEL$ are evaluated again and the subsequent steps again taken until the numerical derivatives are taken again at this third value of DEL . The logical check following the evaluation of the numerical derivatives now notes that $INIT = 3$ and thus sets $IDX = 3$ and returns to the point where VS was first set equal to VSO . The perturbation loop has been bypassed and $VS = VSO$ is now used to evaluate M , N , P matrix elements. When the determinant of the P matrix ($DP(IDX)$) has been evaluated the subsequent logical check notes that $IDX = 3$ and all the quantities needed have been evaluated. It therefore proceeds to print out the results. After this the main program is re-entered to look for new cases.

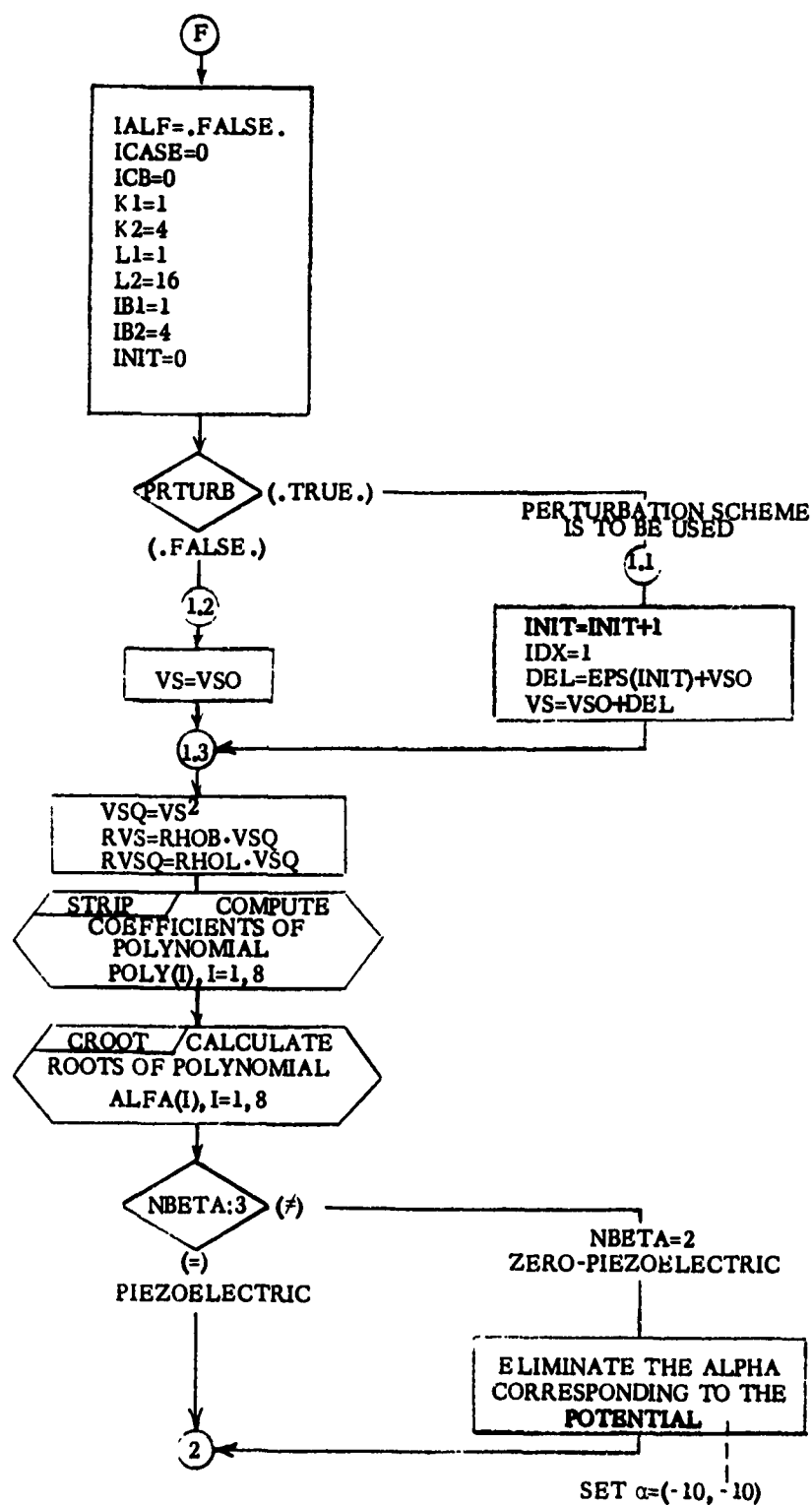
The steps for any particular velocity (VS) taken to evaluate the elements of the M , N , or P matrices will now be discussed. First quantities $VSQ = (VS)^2$ and $\begin{cases} RVS = RHOB \cdot VSQ \\ RVSQ = RHOL \cdot VSQ \end{cases}$ are set up. $RHOL$ is the mass density of the liquid while $RHOB$ is the mass density of the crystal. Next subroutine $STRIP$ is called to compute the coefficients of the eighth order polynomial in α just as was done in the first program. Subroutine $CROOT$ is now called to calculate the roots (α) of the polynomial. If a non-piezoelectric case is being considered the two extraneous roots are eliminated as in the first program. The roots with positive real part are now selected ($ALFAB(I) \ I=1, K$). If $K \leq 1$ the case terminates. If $K = 2$ or 3 checks are made of various elements of the matrix of coefficients ($\hat{A}$) of the relative field amplitudes ($\beta_i^{(l)}$). $\hat{A}_{12}$ and $\hat{A}_{23}$ are tested to see if they are identically zero. If they are not both identically zero a degenerate case cannot exist. If the crystal is piezoelectric, then, there are insufficient α 's and the case terminates. If the crystal is non-piezoelectric and $K = 2$ the case terminates also (insufficient α 's). If the crystal is non-piezoelectric and $K = 3$ the appropriate β 's are calculated as indicated in the analysis of the first problem. If, however, $\hat{A}_{12}$ and $\hat{A}_{23}$ are both identically zero and a non-piezoelectric case is being considered, it is a degenerate case and is so treated. If the crystal is piezoelectric a check of $\hat{A}_{24}$ is made. If $\hat{A}_{24}$ is zero and $K = 2$ the program terminates but if $K = 3$ the first degenerate case of the analysis section of the first problem has arisen ($\beta_1, \beta_3, \beta_4 \neq 0, \beta_2 = 0$) and is treated appropriately.

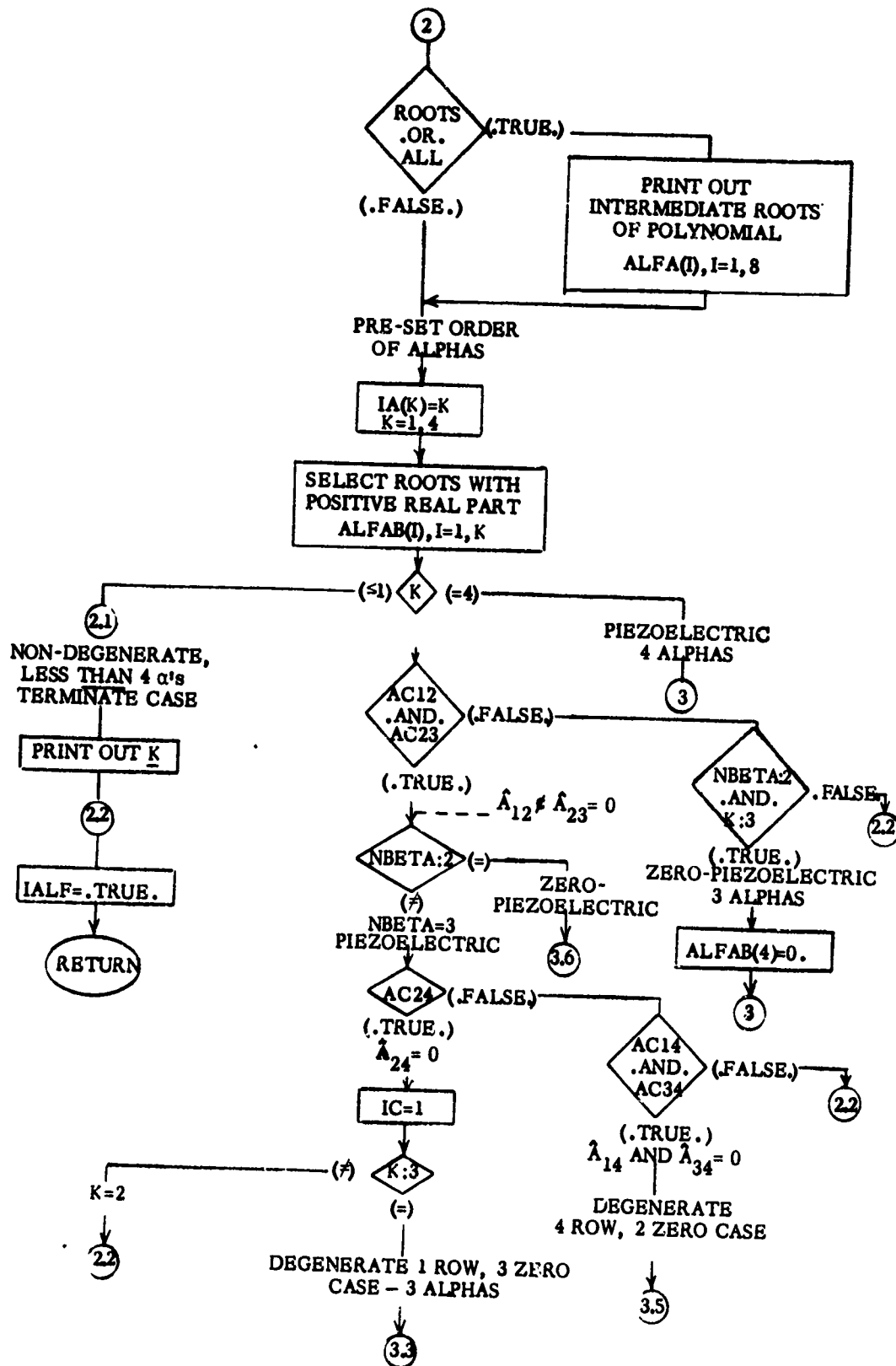
If $\hat{A}_{24}$ is not identically zero a check of $\hat{A}_{14}$ and $\hat{A}_{34}$ is made. If they are not both equal to zero the case terminates since no degenerate case has arisen. If both are identically zero the second degenerate case of the analysis section of the first problem has arisen and is treated accordingly.

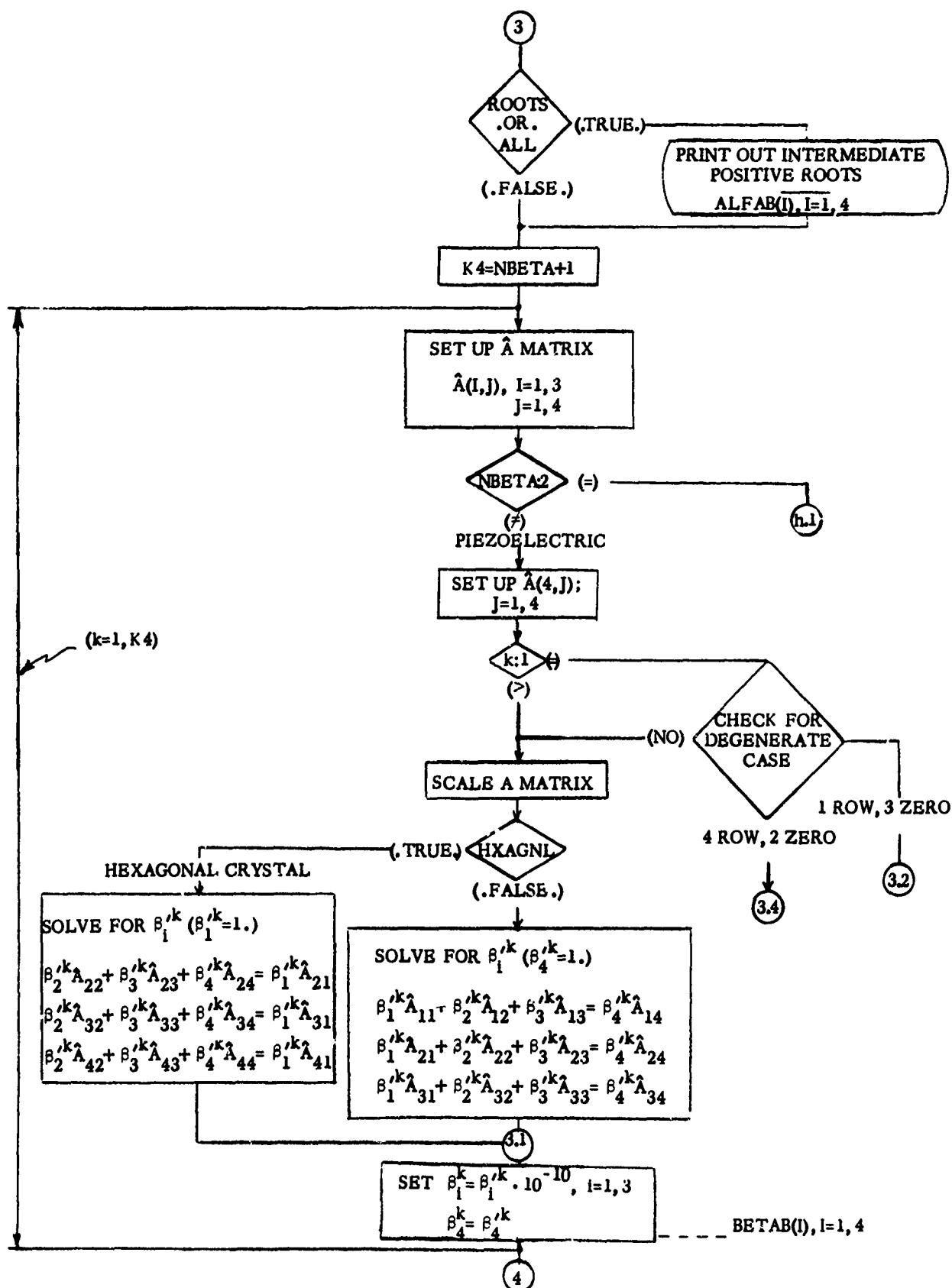
If $K = 4$ and a piezoelectric case is being considered or $K = 3$ and a non-piezoelectric case is being considered the β 's are derived in the fashion indicated in the first program. After the β 's have been computed the quantity $ARAD = 1 - \frac{RVSQ}{LAMDAL}$ is computed where $LAMDAL (\lambda_l)$ is the modulus of compression of the fluid. If no perturbation scheme is to be used the program computes $ALFAL = \sqrt{ARAD}$ and tests to see if the imaginary part of $ALFAL$ ($Im(ALFAL)$) is equal to zero. If $Im(ALFAL) = 0$ the negative square root is taken; otherwise the root is taken so that $Im(ALFAL) < 0$ (this was necessary in order that a velocity with positive imaginary part result as a solution). The elements of the M matrix are set up next and the determinant evaluated ($\det(M) = F(VS)$). If the perturbation scheme is to be used the various matrices indicated are set up as indicated earlier and the velocity perturbation Δv_s is calculated.

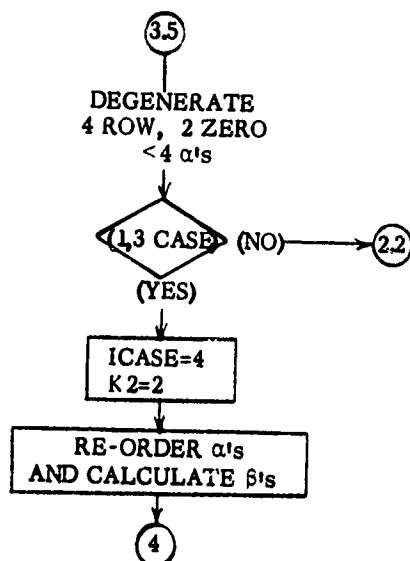
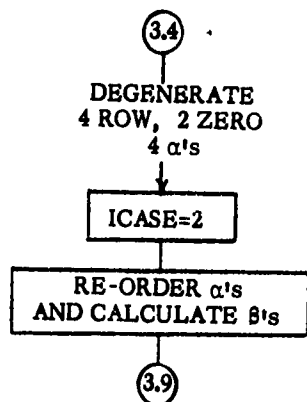
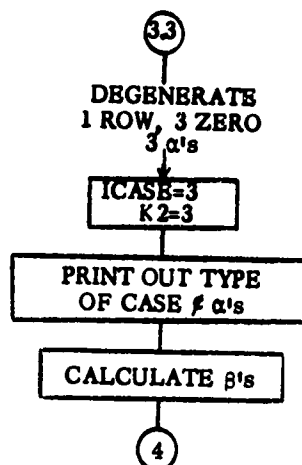
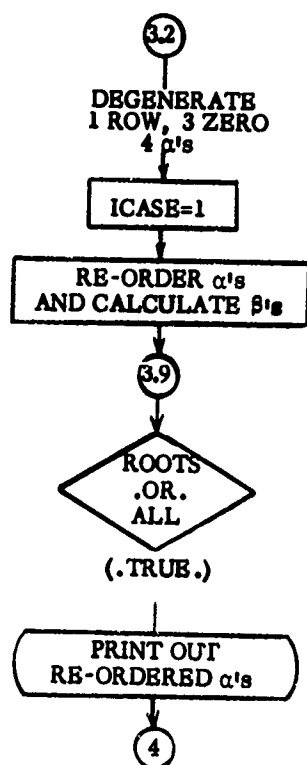


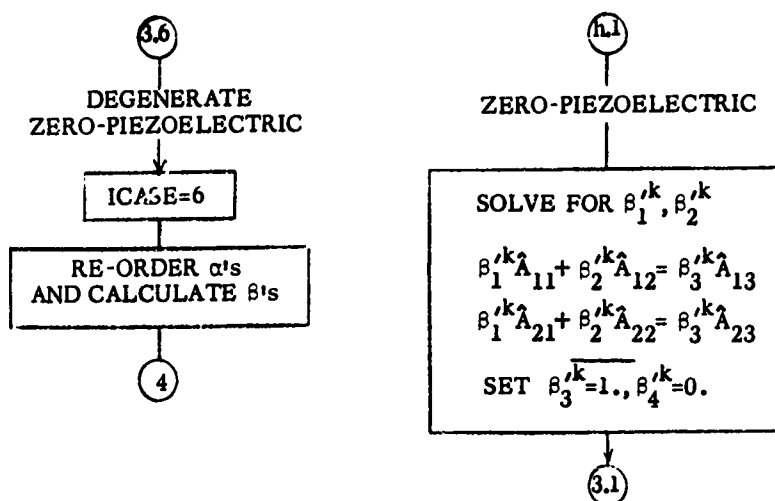


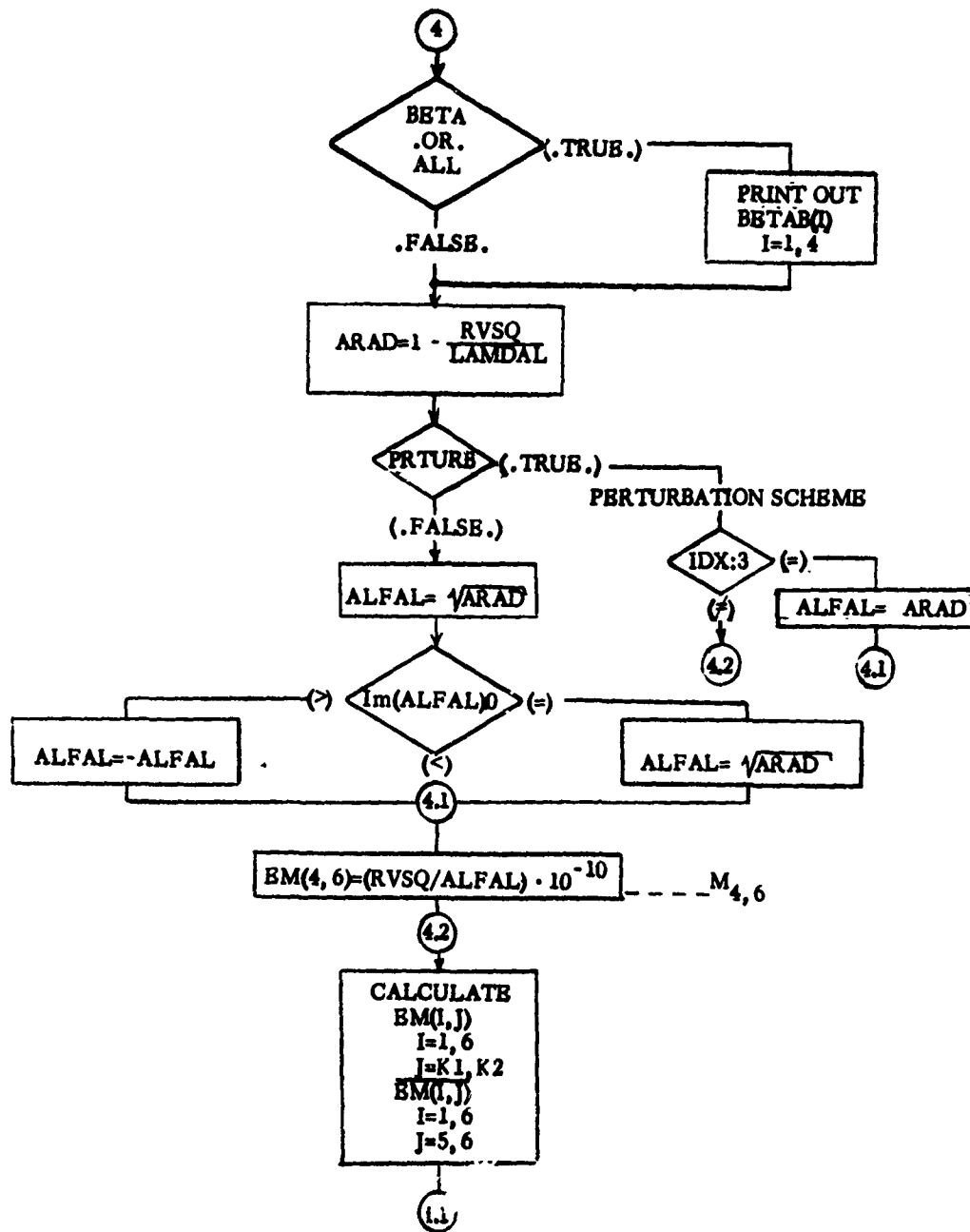


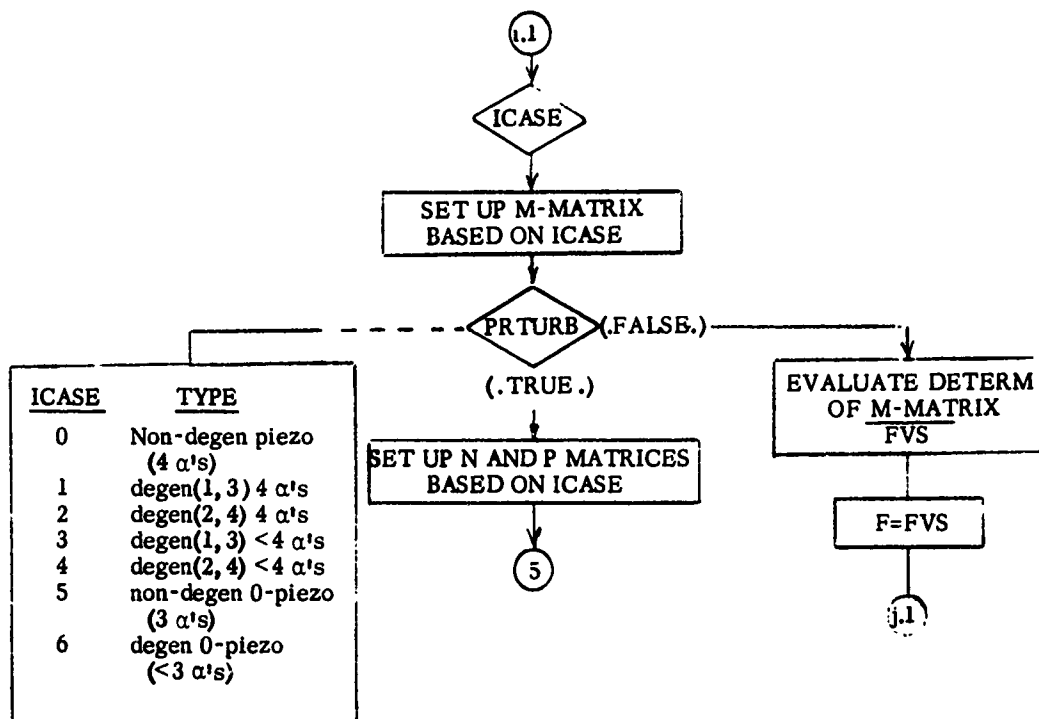


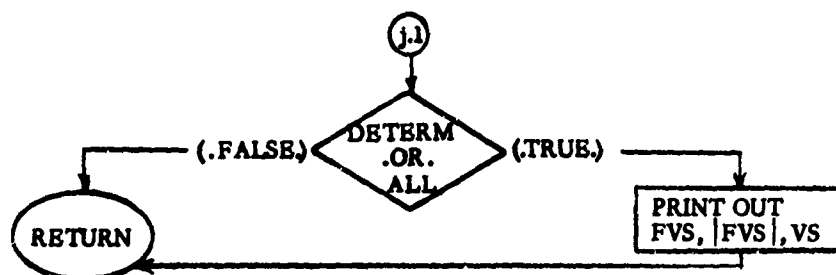


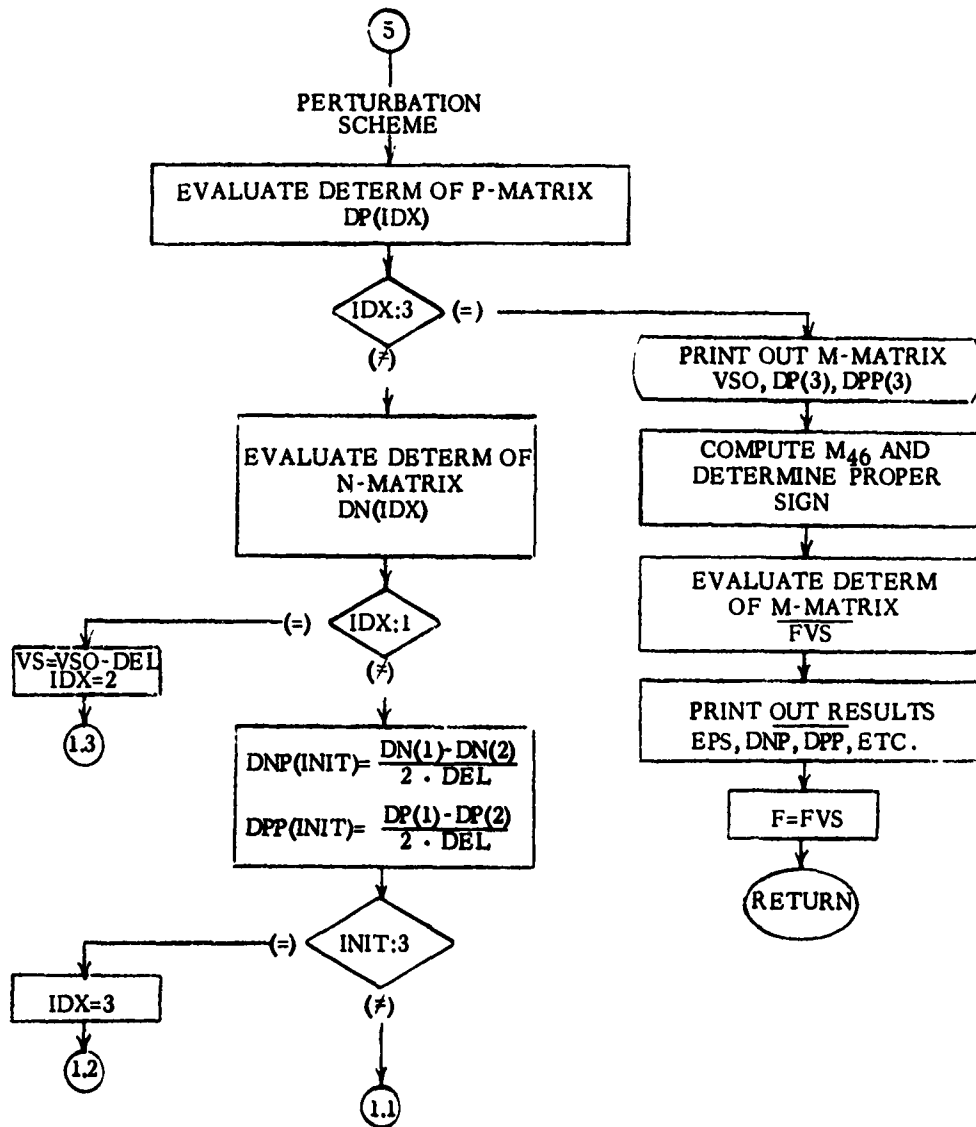












3. Isotropic, Elastic, Dielectric Layer on Piezoelectric Substrate – Program Description

The purpose of this program is to determine the complex velocity of propagation of surface waves at the interface between a semi-infinite fluid and a piezoelectric substrate. The necessary input and control parameters are described on the following pages.

As with the proceeding program, this program is set up to run on the IBM 7094, using FORTRAN IV and Namelist input and the deck set up is again identical with that described in Section IV.1. Again, two input sections are required: the first describes the material constants of the piezoelectric crystal and the second describes the orientation of the crystal as well as other information pertinent to the execution of the program. The first data set is called CONST and is identical with that described in Section IV.1. The second data set is called "INPUT," and the following is a definition of each input parameter. Medium A refers to the dielectric (elastic) layer and medium B, to the piezoelectric substrate.

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
LAMDAB	λ_B	real	Euler angles for Medium B.
MUB	μ_B	real	
NUB	ν_B	real	
DNU	$\Delta\nu$	real	If the user wishes to vary ν (NUB) from some initial value, ν_i , to some final value, $\nu_{\max}$, in steps of $\Delta\nu$, then set DNU equal to the steps desired; also, see NUMAX. (See VSINC)
NUMAX	$\nu_{\max}$	real	The maximum value of ν (see DNU). $\nu_{\max}$ is only used when DNU $\neq 0$.
VS	v_s	real	Initial estimate of velocity. This initial value will be used to find a final velocity, $\bar{v}_s$, such that $ f(v_s) < \epsilon$, where ϵ is input.
DVS	Δv_s	real	If the user does not care to use the root-finding scheme in determining a final value for v_s , but wishes, instead, to evaluate the determinant $ f(v_s) $ for particular values of v_s in the range from v_s to $v_{s\max}$ in steps of Δv_s , then set DVS equal to the step size desired. (For use when MAX = 0.)
VSMAX	$v_{s\max}$	real	Maximum value of v_s to be used when DVS $\neq 0$.
LAMDAA	λ_A	real	Lame constants for Medium A.
MUA	μ_A	real	
RHOA	ρ_A	real	Mass density of Medium A.
RHOB	ρ_B	real	Mass density of Medium B.
EPSLON	ϵ	real	A positive number used as a convergence criterion by the root-finding scheme (MAX > 0). If $ f(v_s) < \epsilon$, then v_s is assumed to be the root required.
EPSO	ϵ_0	real	Permittivity of free space.

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
EPSA	ϵ_A	real	Dielectric constant for Medium A.
WH	ωh	real	Frequency thickness product.
WXA	ωx_A	real	Normalized distance into Medium A.
DWXA	$\Delta \omega x_A$	real	In order to vary ωx_A (WXA) from an initial value, ωx_A , to a final value $\omega x_{A_{\max}}$, DWXA must be set equal to the desired step size. See WXAMAX.
WXAMAX	$\omega x_{A_{\max}}$	real	The maximum value of ωx_A to be used when DWXA $\neq 0$.
WXB	ωx_B	real	Normalized distance into Medium B.
DWXB	$\Delta \omega x_B$	real	In order to vary ωx_B from an initial value, ωx_B , to a final value, $\omega x_{B_{\max}}$, DWXB must be set equal to the step size desired. See WXBMAX.
WXBMAX	$\omega x_{B_{\max}}$	real	The maximum value of ωx_B to be used when DWXB $\neq 0$.
ICHECK	----	logical	ICHECK = .TRUE. - All FINAL ANSWERS* are computed in addition to the evaluation of the determinant $ f(\nabla_s) $. ICHECK = .FALSE. - FINAL ANSWERS are <u>not</u> computed; evaluate determinant only.
MAX	----	integer	Since an iteration scheme is used for convergence for a final root v_s , there must be an indication of how many iterations are to be executed before divergence is assumed. Hence, MAX should be the maximum number of iterations the user wishes the program to make (usually 15). If MAX is set to zero (MAX = 0) the determinant $ f(\nabla_s) $ will be evaluated for the particular v_s value input - the iteration scheme will not be used. This option may be useful if there is difficulty in determining the range in which v_s lies.

*The FINAL RESULTS, which are computed for all values of WXA (dielectric layer) and WXB (piezoelectric layer), include the following:

Stress Components
Strain Components
Time Average Power Flow
Electric Displacement

Mechanical Displacement
Electric Potential Magnitude
Electric Field

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
TITLE	----	BCD	An alphanumeric array of 24 characters or less used to describe the type of crystal, such as lithium niobate. This is input in the following manner: TITLE = nH name of crystal, where n is the number of characters following the H (including blanks). For example TITLE = 6HQWARTZ
HXAGNL	----	logical	Parameter which controls the calculation of betas (β's) for a hexagonal crystal (such as zinc oxide) .TRUE. hexagonal crystal (use special technique) .FALSE. non-hexagonal crystal (use normal procedure)
VSINC	----	logical	VSINC = .TRUE. - New estimates of initial velocity (v_s) are computed using a linear fit to the two previous values. (Used when NUB varies over a range NUB, NUB + DNU, ... , NUMAX) VSINC = .FALSE. - The same initial estimate of velocity is used for all values over the specified range of NUB.
IOP	----	integer	Degenerate case options (used when exactly four α's with positive real part occur). IOP = 1 - seek modes of propagation of the Quasi-Rayleigh or Sesawa type. IOP = 2 - seek modes of propagation of Love type.
REPEAT	----	logical	REPEAT is a logical variable and in its usage, can take only one value: .TRUE. If there are no more cases to run after the current case, REPEAT does not need to be input. If there will be another case to follow, but the crystal coefficients remain the same, then, again, REPEAT does not need to be input. However, if another case is to be run and the coefficients are different, then REPEAT needs to be input

<u>Input Name</u>	<u>Equation Name</u>	<u>Type</u>	<u>Definition</u>
			as .TRUE. This means that the \$CONST data will have to be input again (in the other cases above, \$CONST would not have to be input again).

The following input parameters are all logical variables which are assumed to be false (.FALSE.) in the program. They are used as switches indicating whether or not intermediate calculations are to be printed. If any one, or any combination of these parameters are input as true (.TRUE.), then certain intermediate data will print, according to the following:

TABCTE	Print the constants E, C, and T (the transformed piezoelectric, elastic, and dielectric constants) calculated from the constants P, G, and EPS.
ROOTS	Print the roots of the polynomial each time they are calculated.
BETA	Print the values of β_{ij} .
DETERM	Print the value of the determinant.
COEFF	Print the coefficients of the 8'th order polynomial.
TABL	Print the L matrix (or $\hat{P}$, $\hat{Q}$, $\hat{R}$, etc., when used).
ALPHA	Print the roots of the polynomial (α_B^j 's) and the re-ordered roots for degenerate cases.
ALL	Print all of the above.

Data items may be excluded from the input stream at the discretion of the user. Items omitted from the first data set will take on nominal values (i.e.: values assigned within the program). Items omitted from succeeding data sets will take on previously assigned values. The following is a complete list of nominal values: *

*All logical parameters have a nominal value of .FALSE.

<u>Parameter</u>	<u>Nominal Value</u>
LAMDAB	0.
MUB	0.
NUB	0.
DNU	0.
NUMAX	0.
VS	3000.
DVS	0.
VSMAX	0.
LAMDAA	1.5×10^{11}
MUA	2.85×10^{10}
RHOA	1.888×10^4
RHOB	4700.
EPSLON	$1. \times 10^{-11}$
EPSO	8.85×10^{-12}
EPSA	44.25×10^{-12}
WH	0.
WXA	0.
DWXA	0.
WXAMAX	0.
WXB	0.
DWXB	0.
WXBMAX	0.
MAX	15.
TITLE	LITHIUM NIOBATE
IOP	1

The following is a description of the computer program flow diagram provided at the end of this section.

The computer program for this problem shares many common features with the programs described in the preceding sections. Again the nominal data values are set up first. The piezoelectric (P), elastic (G), and dielectric (EPS) constants are read in next (CONST DATA). Following this the rest of the input data is entered (INPUT DATA).

Subroutine SETCTE computes the transformed piezoelectric (CE), elastic (CC), and dielectric (CT) constants. Next, subroutine ROOT performs the calculations and calls the subroutines necessary to compute F(VS), the boundary condition determinant. ROOT will either minimize F(VS) (root finding scheme) or simply compute it at velocity VS depending on the setting of the counter MAX. Upon returning to the main program an option to increment VS followed by an option to increment the third Euler angle (NU) is available as in the first two programs. As NU is incremented it is also possible to update the initial velocity (VS) if the root finding scheme is being employed so that a closer initial estimate will be had as NU varies. The setting of a logical variable (VSINC) dictates whether this updating scheme is to be used or not.

When the correct velocity of propagation has been found (either from the root finding scheme or from plotting F(VS) as a function of velocity) the relative amplitudes of the partial surface wave fields are calculated (ETA(1), ETA(2), etc.). Next the various quantities of interest are calculated in medium A (the dielectric) as a function of normalized distance (WX_A) into the medium. Following this the same parameters are calculated in medium B (crystal) as a function of normalized distance (WX_B) into the crystal. For both media an incrementation scheme may be used to increase WX_A or WX_B in equal increments (DWXA or DWXB) from some initial value to some final value.

The quantities of interest mentioned above are as follows:

- a) Mechanical Displacement (magnitude and phase) referred to as MAGU(I) and PHASEU(I), I=1 to 3 in the program.
- b) The electric potential (magnitude and phase) referred to as MAGU(4) and PHASEU(4).
- c) The time average power flow computed in the subroutine PIFUN.

- d) The stresses computed in the subroutine TFUN.
- e) The strains computed in subroutine SFUN.
- f) The electric fields (E1, E3) and electric displacement (D1, D2, and D3).

Calculation of F(VS)

Subroutine ROOT calls subroutine F to evaluate the determinant of the boundary condition matrix. F first calls subroutine STRIP to calculate the coefficients of the eighth order equation in α . Next subroutine CROOT computes the roots of the polynomial (ALFA(I), I=1 to 8). The roots with positive real parts (ALFAB(I), I=1 to K) are selected as in the other programs and the extraneous roots in the non-piezoelectric case are eliminated.

If $K = 0$ the case terminates since no solution is possible. If $K \neq 0$ a search for degeneracies follows. $K = 1$ presents a possibility now which was not present in the previous problems.* First $\hat{A}_{12}$ and $\hat{A}_{23}$ are checked ($\hat{A}$ is the matrix of the coefficients of the unknown amplitudes $\beta_i^{(t)}$ as in the previous problems). If $\hat{A}_{12}$ and $\hat{A}_{23}$ are not both equal to zero the case cannot be degenerate. A check is made to see if the following two conditions both hold:

- a) The case is non-piezoelectric
- b) $K \neq 3$

If both of these conditions hold the case terminates. If it is not true that both hold then a test is made to see if the following two conditions hold:

- a) The case is piezoelectric
- b) $K \neq 4$

If both of these conditions hold the case terminates. Otherwise the program continued for now we must have either a non-piezoelectric case with $K = 3$ or a piezoelectric case with $K = 4$, both of which are proper non-degenerate cases.

If $\hat{A}_{12}$ and $\hat{A}_{23}$ are both identically zero and the case is non-piezoelectric, the program proceeds to the section where degenerate, non-piezoelectric cases

*Page 37 of analysis.

are handled. If, however, the case is piezoelectric $\hat{A}_{24}$ is checked. If $\hat{A}_{24}$ is zero the first type of degenerate case of the analysis section arises and the program proceeds to that section which treats such cases. If $\hat{A}_{24}$ is not identically zero then $\hat{A}_{14}$ and $\hat{A}_{34}$ are checked. If they are identically zero, the second degenerate case of the analysis section arises and the program proceeds to the section that treats these cases. If they are not both zero the program goes through the test mentioned above (i.e. is it simultaneously true that (a) the case is piezoelectric and (b) $K \neq 4$).

If (a) and (b) are not simultaneously true then we are dealing with a non-degenerate, piezoelectric case and the program proceeds accordingly.

Non-degenerate Cases

The $\hat{A}$ matrix is set up for each value of α ($k = 1, K4$) where $K4 = 4$ for piezoelectric cases and $K4 = 3$ for non-piezoelectric cases. If the case is non-piezoelectric the program sets $\beta_4 = 0$, $\beta_3 = 10^{-10}$ and solves the first two equations for β_1 and β_2 . If the crystal is piezoelectric and not hexagonal β_4 is set equal to 1 and the first three equations of the set are solved for β_1 , β_2 , and β_3 . If the crystal is piezoelectric and hexagonal β_1 is set equal to 10^{-10} and the second, third, and fourth equations of the set are solved for β_2 , β_3 , and β_4 .

Degenerate Case 1

This case is characterized by a decoupling of the equations for $\beta_1^{(t)}$ so that three of the equations involve β_1 , β_3 , and β_4 only and one involves β_2 only as discussed in the analysis. If there are four α 's with positive real part ($K = 4$) the program calculates $|\hat{A}_{22}|$ for each α and determines which α leads to a minimum of this function. This now becomes $\alpha^{(1)}$ while the other α 's become $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. The program proceeds to work with the relabeled α 's and computes the β 's as follows:

$$\beta_2^{(1)} = 10^{-10}, \beta_i^{(1)} = 0 \quad i = 1, 3, 4; \quad \beta_2^{(t)} = 0, \beta_4^{(t)} = 1$$

$$\beta_1^{(t)} = \frac{\hat{A}_{13}^{(t)} \hat{A}_{34}^{(t)} - \hat{A}_{14}^{(t)} \hat{A}_{33}^{(t)}}{\hat{A}_{11}^{(t)} \hat{A}_{33}^{(t)} - \hat{A}_{13}^{(t)2}}, \quad \text{and} \quad \beta_3^{(t)} = \frac{\hat{A}_{13}^{(t)} \hat{A}_{14}^{(t)} - \hat{A}_{11}^{(t)} \hat{A}_{34}^{(t)}}{\hat{A}_{11}^{(t)} \hat{A}_{33}^{(t)} - \hat{A}_{13}^{(t)2}}$$

$t = 2, 3, 4$. The program now proceeds to set up either the $M(10 \times 10)$ or

N(3 x 3) matrix discussed in the analysis and evaluates its determinant.

If there are less than four α 's with positive real part ($K < 4$) the program calculates $|\hat{A}_{22}|$ for each α and counts the number (N_1) of α 's for which $|\hat{A}_{22}| < 10^7$ (this is close enough to zero considering the magnitude of the individual terms in $\hat{A}_{22}$). If $K = 1$ and $N_1 = 0$ the case terminates (case 1c2). If $K = 1$ and $N_1 = 1$ the program sets $\beta_2^{(1)} = 10^{-10}$ $\beta_i^{(1)} = 0$ $i = 1, 3, 4$ and then proceeds to set up the N matrix and evaluate its determinant (case 1c1).

If $K = 2$ and $N_1 = 0$ the case terminates (case 1b3). If $K = 2$ and $N_1 = 1$ the α that yielded $|\hat{A}_{22}| < 10^7$ becomes $\alpha^{(1)}$. The program now sets $\beta_2^{(1)} = 10^{-10}$ and $\beta_i^{(1)} = 0$ $i = 1, 3, 4$ and then proceeds to set up the N matrix and evaluate its determinant (case 1b1). If $K = 2$ and $N_1 = 2$ this represents an impossible case and the case terminates (1b2).

If $K = 3$ and $N_1 = 0$ then all three roots correspond to the $(\beta_1, \beta_3, \beta_4)$ split. The α 's become $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. The β 's are calculated as they are in the four α case for $\alpha^{(2)}$, $\alpha^{(3)}$, and $\alpha^{(4)}$. Only the M matrix can be set up and evaluated for this case (case 1a1). If $K = 3$ and $N_1 = 1$ the corresponding α becomes $\alpha^{(1)}$ and $\beta_2^{(1)} = 10^{-10}$ while $\beta_i^{(1)} = 0$ $i = 1, 3, 4$. Only the N matrix is set up and evaluated (case 1a2). If $K = 3$ and $N_1 = 2$ or more the case terminates since this is physically impossible.

Degenerate Case 2

This case is characterized by a decoupling of the equations for $\beta_i^{(l)}$ so that two of the equations involve β_1 and β_3 only while the other two involve β_2 and β_4 only. If there are four α 's with positive real part ($K = 4$) the program calculates $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ for each α . The (2) values of $\alpha^{(l)}$ which lead to minimum values of $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ are selected and become $\alpha^{(3)}$ and $\alpha^{(4)}$. The other (2) values of α become $\alpha^{(1)}$ and $\alpha^{(2)}$. The program then computes the β 's as follows:

$$\beta_3^{(1)} = \beta_3^{(2)} = 10^{-10}; \quad \beta_1^{(1)} = \frac{-\hat{A}_{13}^{(1)}}{\hat{A}_{11}^{(1)}} \cdot 10^{-10}, \quad \beta_1^{(2)} = \frac{-\hat{A}_{13}^{(2)}}{\hat{A}_{11}^{(2)}} \cdot 10^{-10};$$

$$\beta_2^{(1)} = \beta_2^{(2)} = 0;$$

$$\beta_4^{(1)} = \beta_4^{(2)} = 0;$$

$$\beta_4^{(3)} = \beta_4^{(4)} = 1; \quad \beta_2^{(3)} = \frac{-\hat{A}_{24}^{(3)}}{\hat{A}_{22}^{(3)}}, \quad \beta_2^{(4)} = \frac{-\hat{A}_{24}^{(4)}}{\hat{A}_{22}^{(4)}};$$

$$\beta_1^{(3)} = \beta_1^{(4)} = 0; \quad \beta_3^{(3)} = \beta_3^{(4)} = 0.$$

The program now proceeds to set up either the P or the Q matrix and evaluates its determinant.

If there are less than four α 's with positive real part ($K < 4$) the program proceeds as follows:

If $K = 1$ the case terminates (case 2c). If $K = 2$ or 3 the program computes the quantity $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ for each α and counts the number (I1) of α 's for which this quantity $< 10^{-5}$ and the number (I2) of α 's for which this quantity $\geq 10^{-5}$. 10^{-5} is close enough to zero due to the magnitudes of the individual terms in the quantity. If $K = 3$ and $I1 = 2$ the α 's become $\alpha^{(3)}$ and $\alpha^{(4)}$ and the β 's are calculated as they were above for $\alpha^{(3)}$ and $\alpha^{(4)}$. Only the Q matrix is set up and evaluated (case 2a1). If $K = 3$ and $I2 = 2$ the α 's become $\alpha^{(1)}$ and $\alpha^{(2)}$ and the β 's are calculated as above for $\alpha^{(1)}$ and $\alpha^{(2)}$. Only the P matrix is set up and evaluated (case 2a2). If $K = 3$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2a3). If $K = 2$ and $I1 = 2$ the β 's are handled as above (case 2b1). If $K = 2$ and $I2 = 2$ the β 's are likewise handled as above (case 2b2). If $K = 2$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2b3).

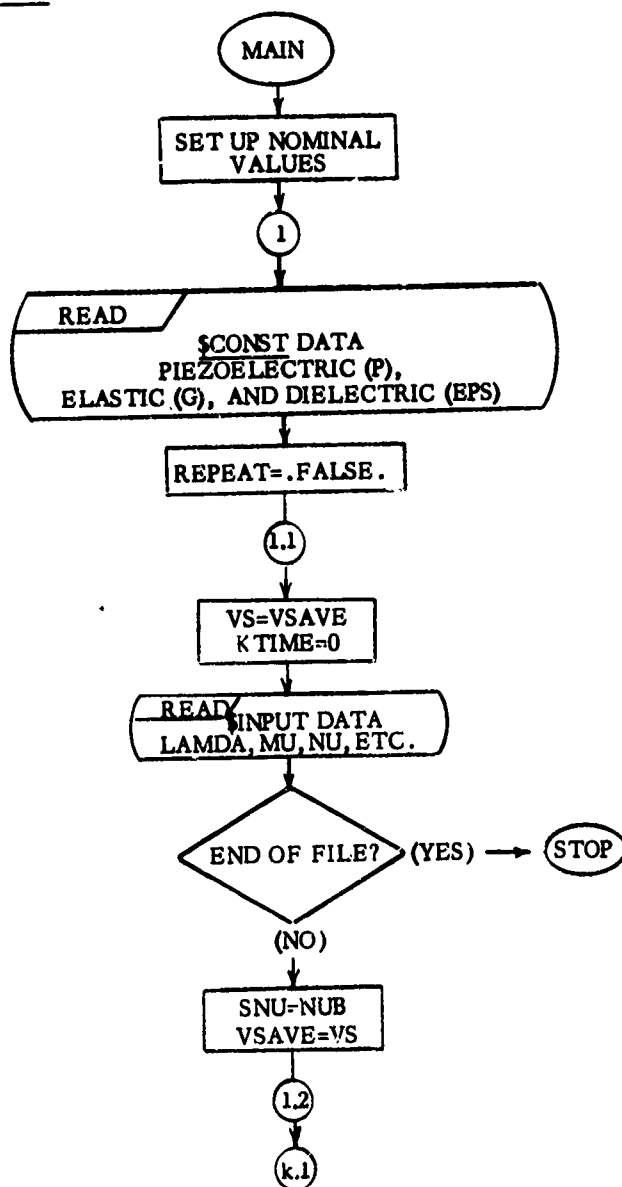
Degenerate Non-piezoelectric Case

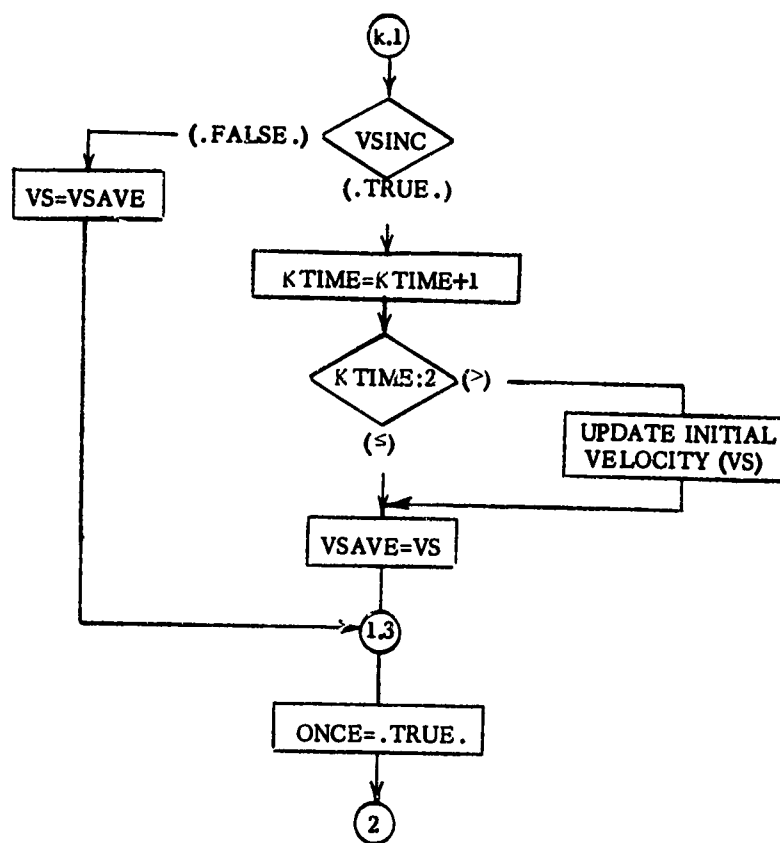
This case is characterized by a decoupling of the equations for $\beta_i^{(l)}$ such that two of the equations involve β_1 and β_3 only and one of the equations involves

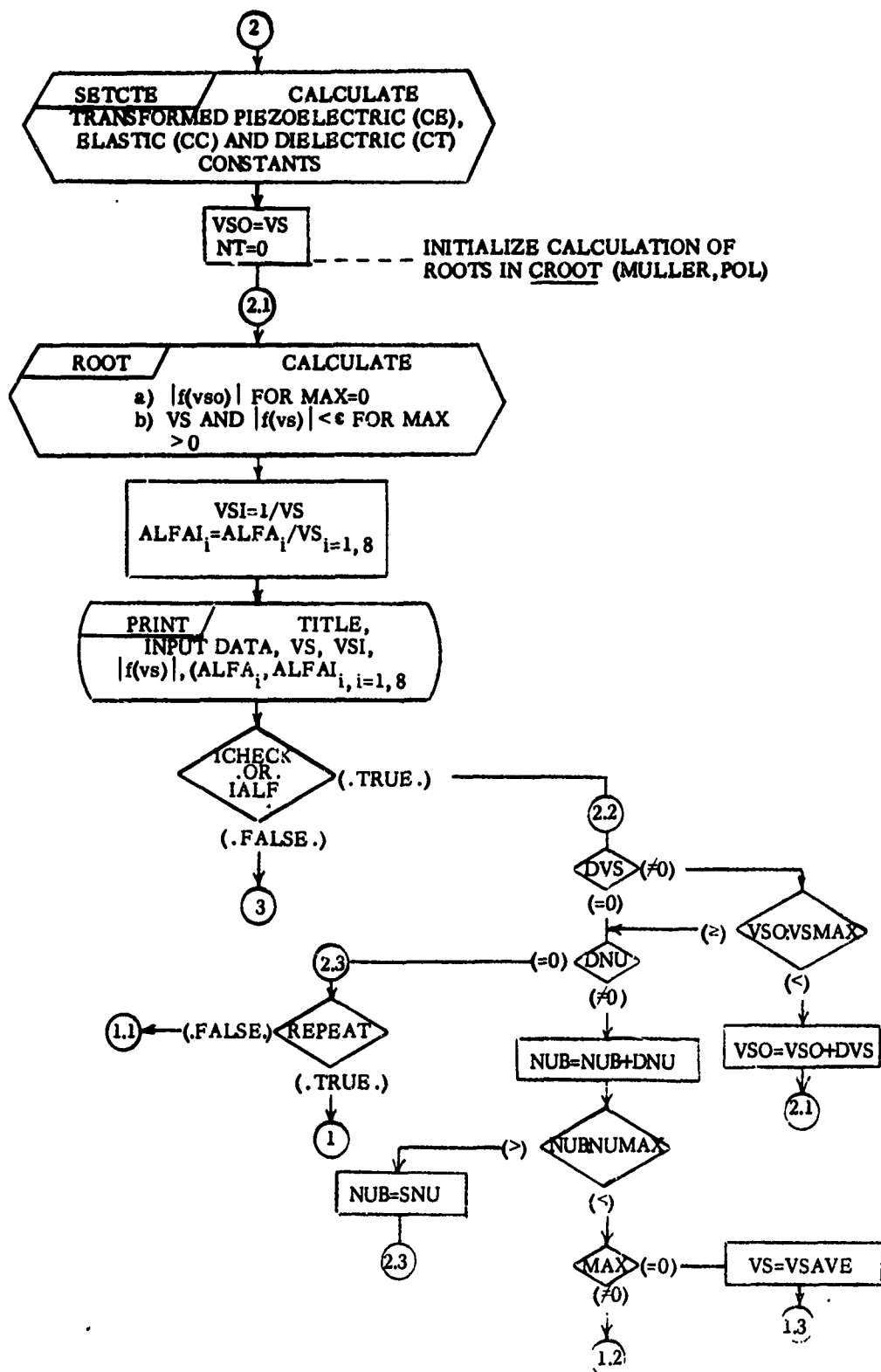
β_2 only. If there are three α 's with positive real part $|\hat{A}_{22}|$ is evaluated for each α . The α leading to the minimum value of $|\hat{A}_{22}|$ becomes $\alpha^{(1)}$ while the others become $\alpha^{(2)}$ and $\alpha^{(3)}$. The program sets $\beta_2^{(1)} = 10^{-10}$, $\beta_i^{(1)} = 0$ $i = 1$ and 3 ; $\beta_2^{(t)} = 0$, $\beta_3^{(t)} = 10^{-10}$, $\beta_1^{(t)} = -\hat{A}_{13}^{(t)} / \hat{A}_{11}^{(t)} \cdot 10^{-10}$ $t = 2$ and 3 . Either the R or N matrix can be set up and evaluated depending on the type of wave sought.

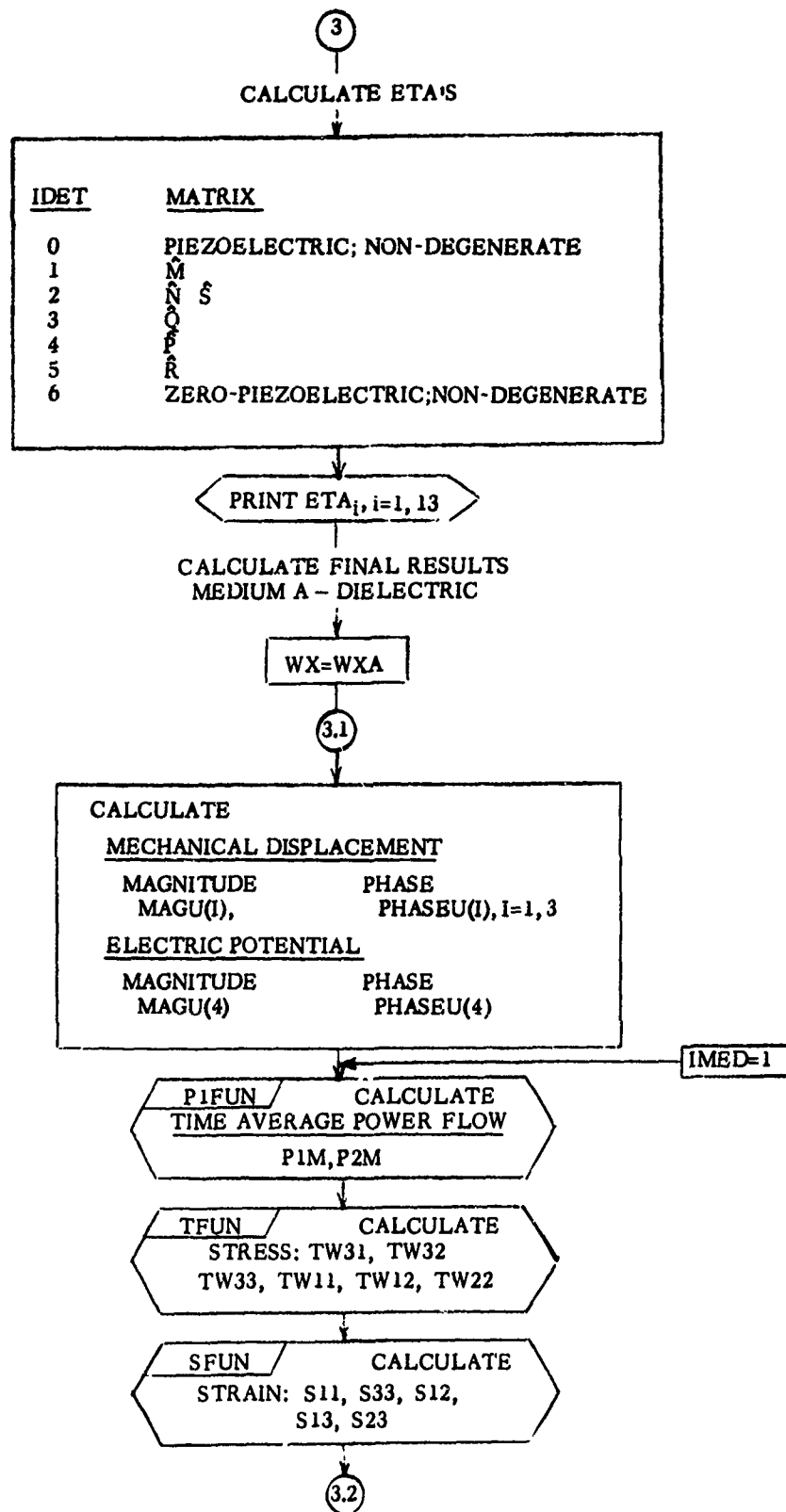
If there are two α 's with positive real part $|\hat{A}_{22}|$ is evaluated for both α 's and compared with 10^7 . (This is close enough to zero considering the magnitudes of the individual terms of $\hat{A}_{22}$). If $|\hat{A}_{22}| > 10^7$ for both α 's they become $\alpha^{(2)}$ and $\alpha^{(3)}$ and the β 's are calculated as above. Only the R matrix is set up and its determinant evaluated (case xa1). If $|\hat{A}_{22}| \leq 10^7$ for one of the α 's this becomes $\alpha^{(1)}$ and the β 's are the same as above. Only the N matrix is set up and its determinant evaluated (case xa2). If $|\hat{A}_{22}| \leq 10^7$ for both α 's the case is terminated (case xa3).

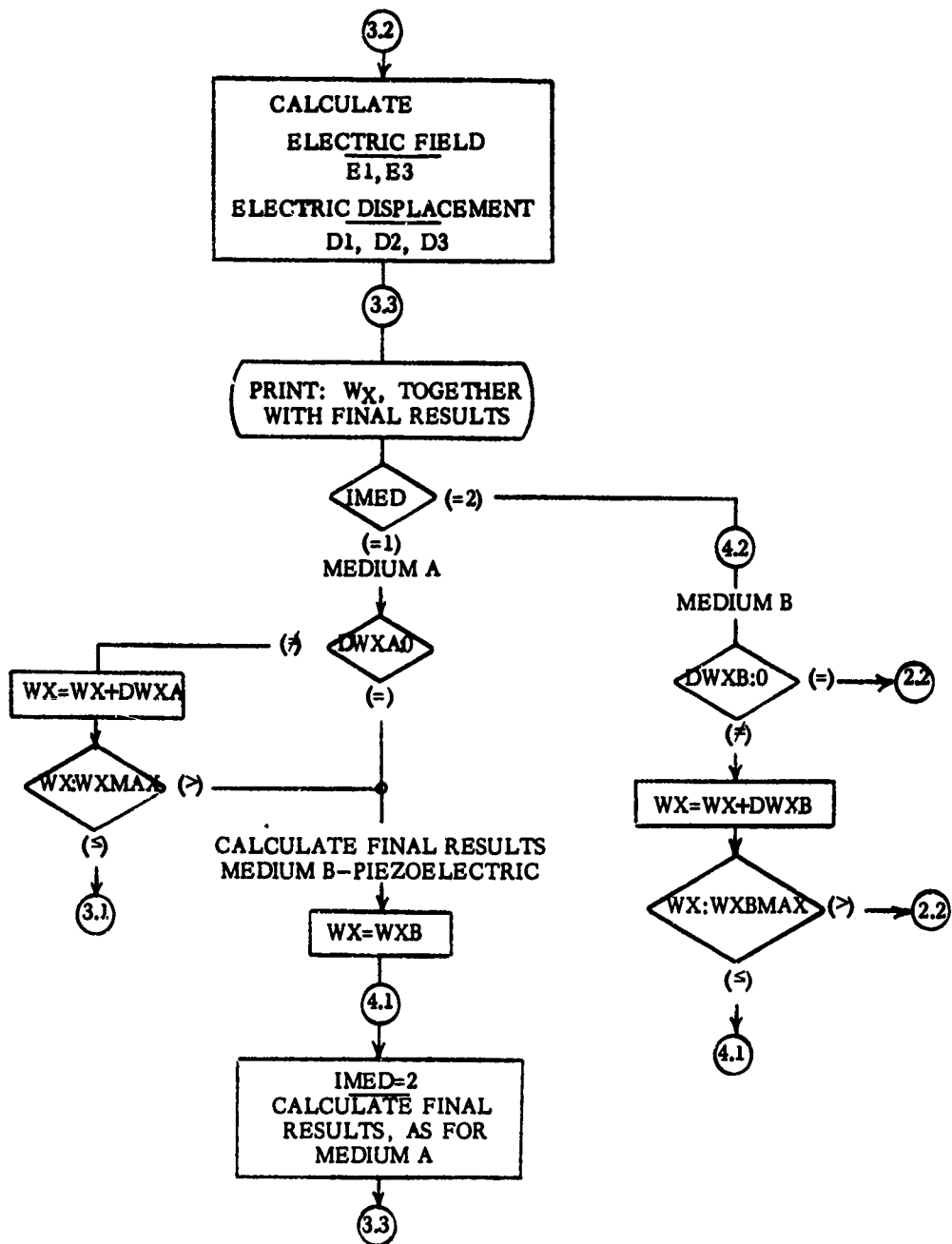
If there is one α with positive real part $|\hat{A}_{22}|$ is evaluated and compared to 10^7 . If $|\hat{A}_{22}| \leq 10^7$ the program sets $\beta_2^{(1)} = 10^{-10}$ $\beta_i^{(1)} = 0$ $i = 1$ and 3 . Only the N matrix is set up and its determinant evaluated (case ya1). If $|\hat{A}_{22}| > 10^7$ the case is terminated (case ya2).

DIELECTRIC

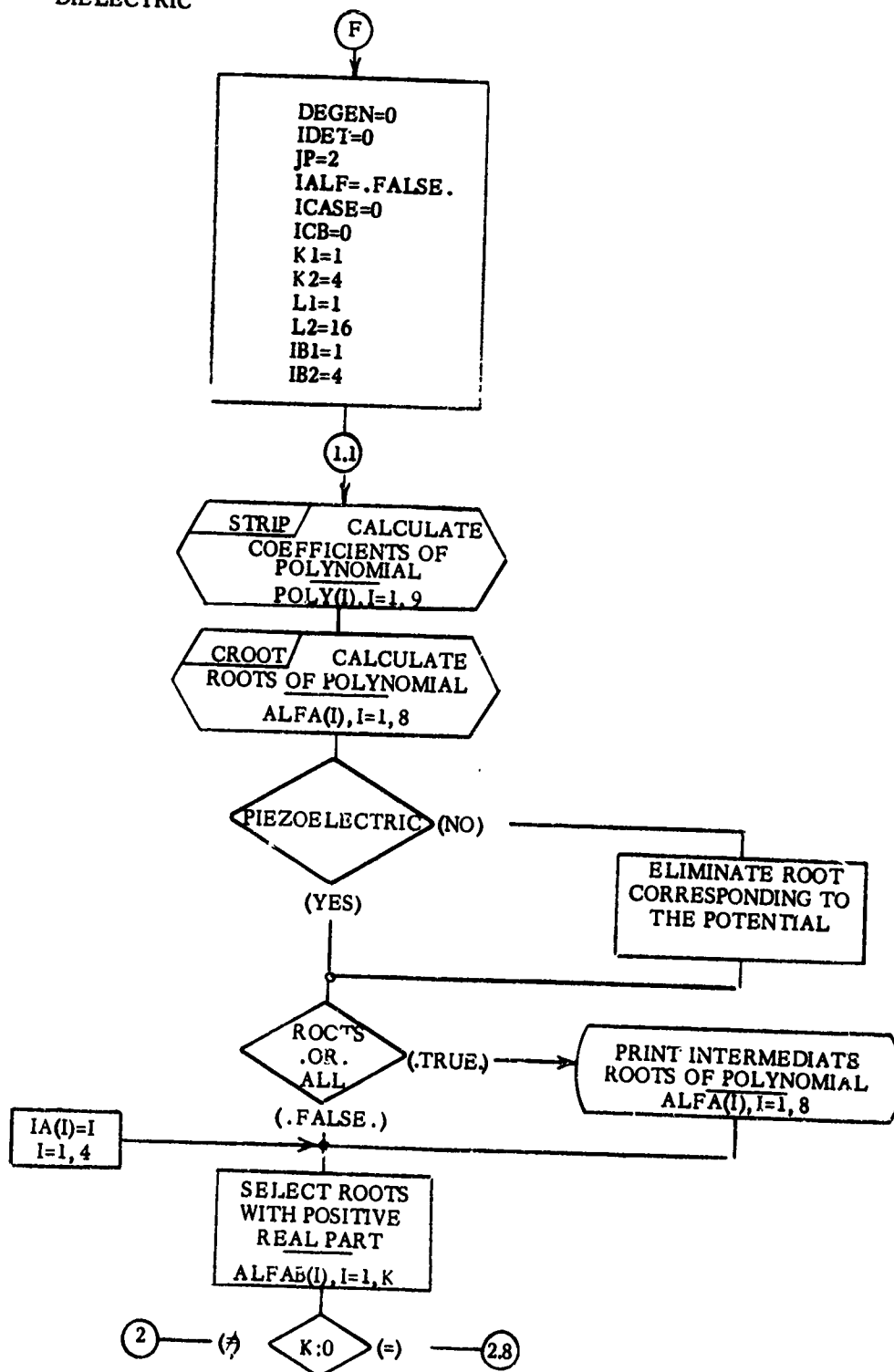








DIELECTRIC



$$\beta_3^{(1)} = \beta_3^{(2)} = 10^{-10}; \quad \beta_1^{(1)} = \frac{-\hat{A}_{13}^{(1)}}{\hat{A}_{11}^{(1)}} \cdot 10^{-10}, \quad \beta_1^{(2)} = \frac{-\hat{A}_{13}^{(2)}}{\hat{A}_{11}^{(2)}} \cdot 10^{-10};$$

$$\beta_2^{(1)} = \beta_2^{(2)} = 0;$$

$$\beta_4^{(1)} = \beta_4^{(2)} = 0;$$

$$\beta_4^{(3)} = \beta_4^{(4)} = 1; \quad \beta_2^{(3)} = \frac{-\hat{A}_{24}^{(3)}}{\hat{A}_{22}^{(3)}}, \quad \beta_2^{(4)} = \frac{-\hat{A}_{24}^{(4)}}{\hat{A}_{22}^{(4)}};$$

$$\beta_1^{(3)} = \beta_1^{(4)} = 0; \quad \beta_3^{(3)} = \beta_3^{(4)} = 0.$$

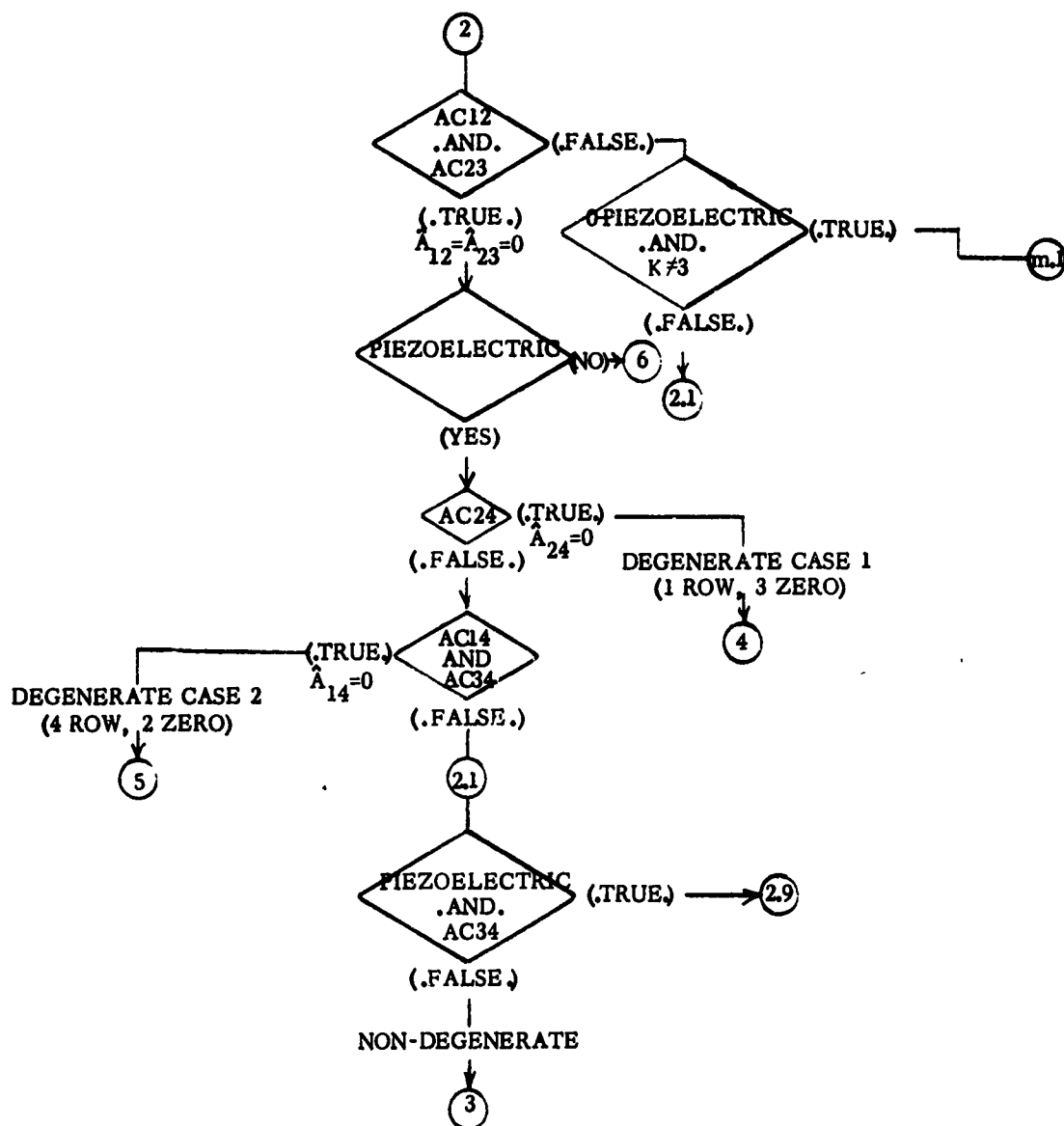
The program now proceeds to set up either the P or the Q matrix and evaluates its determinant.

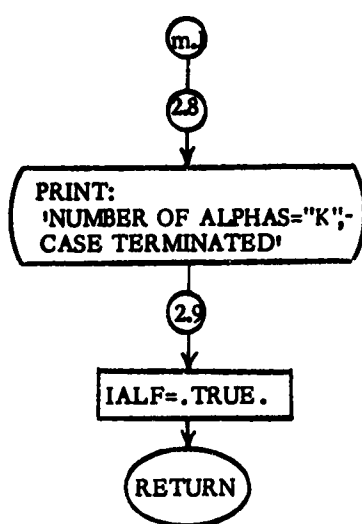
If there are less than four α 's with positive real part ($K < 4$) the program proceeds as follows:

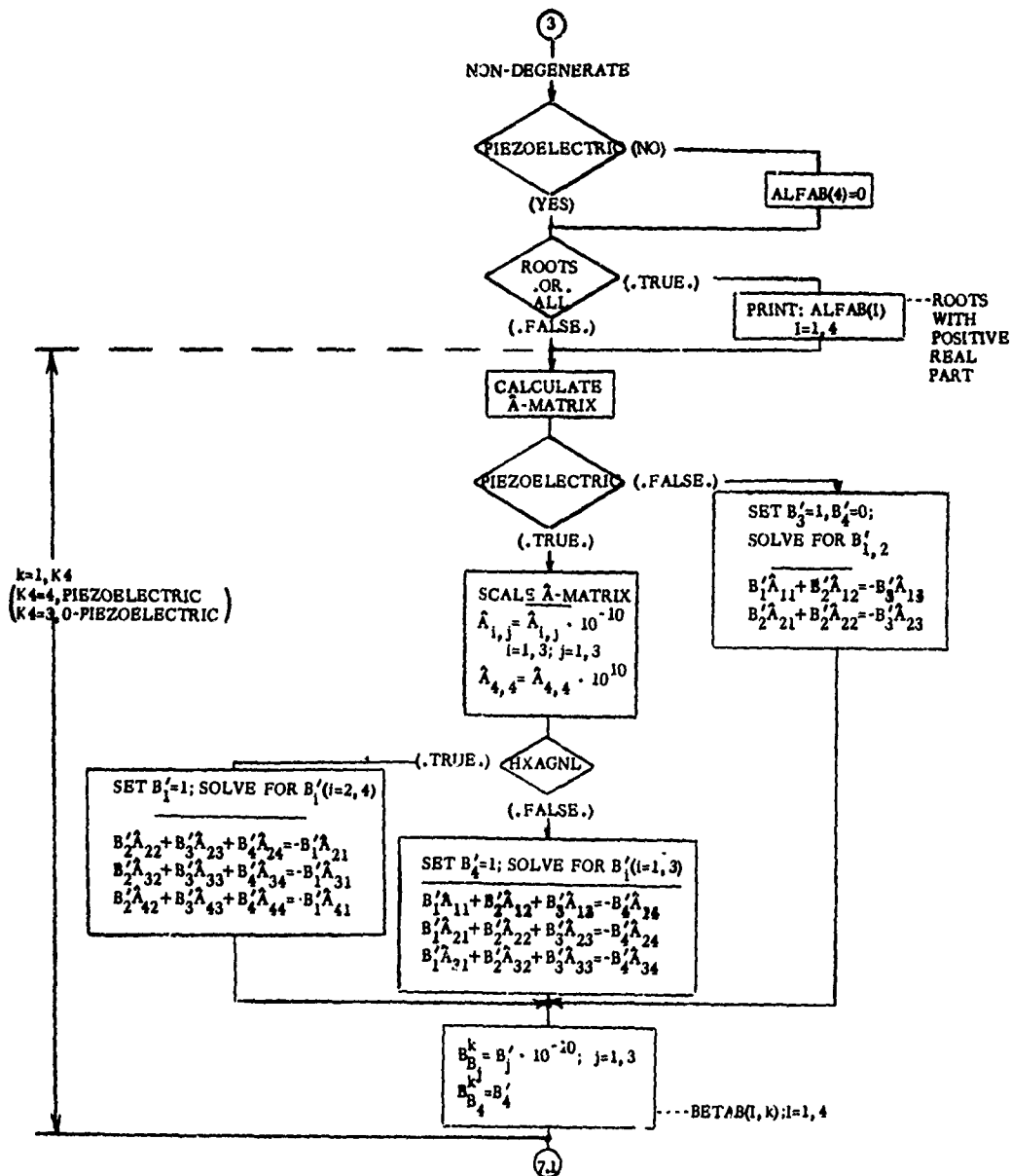
If $K = 1$ the case terminates (case 2c). If $K = 2$ or 3 the program computes the quantity $|\hat{A}_{22}\hat{A}_{44} - \hat{A}_{24}^2|$ for each α and counts the number (I1) of α 's for which this quantity $< 10^{-5}$ and the number (I2) of α 's for which this quantity $\geq 10^{-5}$. 10^{-5} is close enough to zero due to the magnitudes of the individual terms in the quantity. If $K = 3$ and $I1 = 2$ the α 's become $\alpha^{(3)}$ and $\alpha^{(4)}$ and the β 's are calculated as they were above for $\alpha^{(3)}$ and $\alpha^{(4)}$. Only the Q matrix is set up and evaluated (case 2a1). If $K = 3$ and $I2 = 2$ the α 's become $\alpha^{(1)}$ and $\alpha^{(2)}$ and the β 's are calculated as above for $\alpha^{(1)}$ and $\alpha^{(2)}$. Only the P matrix is set up and evaluated (case 2a2). If $K = 3$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2a3). If $K = 2$ and $I1 = 2$ the β 's are handled as above (case 2b1). If $K = 2$ and $I2 = 2$ the β 's are likewise handled as above (case 2b2). If $K = 2$ while $I1 \neq 2$ and $I2 \neq 2$ the case terminates (case 2b3).

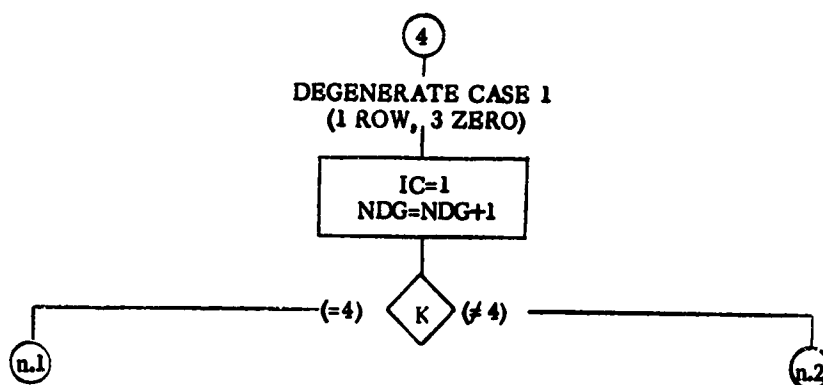
Degenerate Non-piezoelectric Case

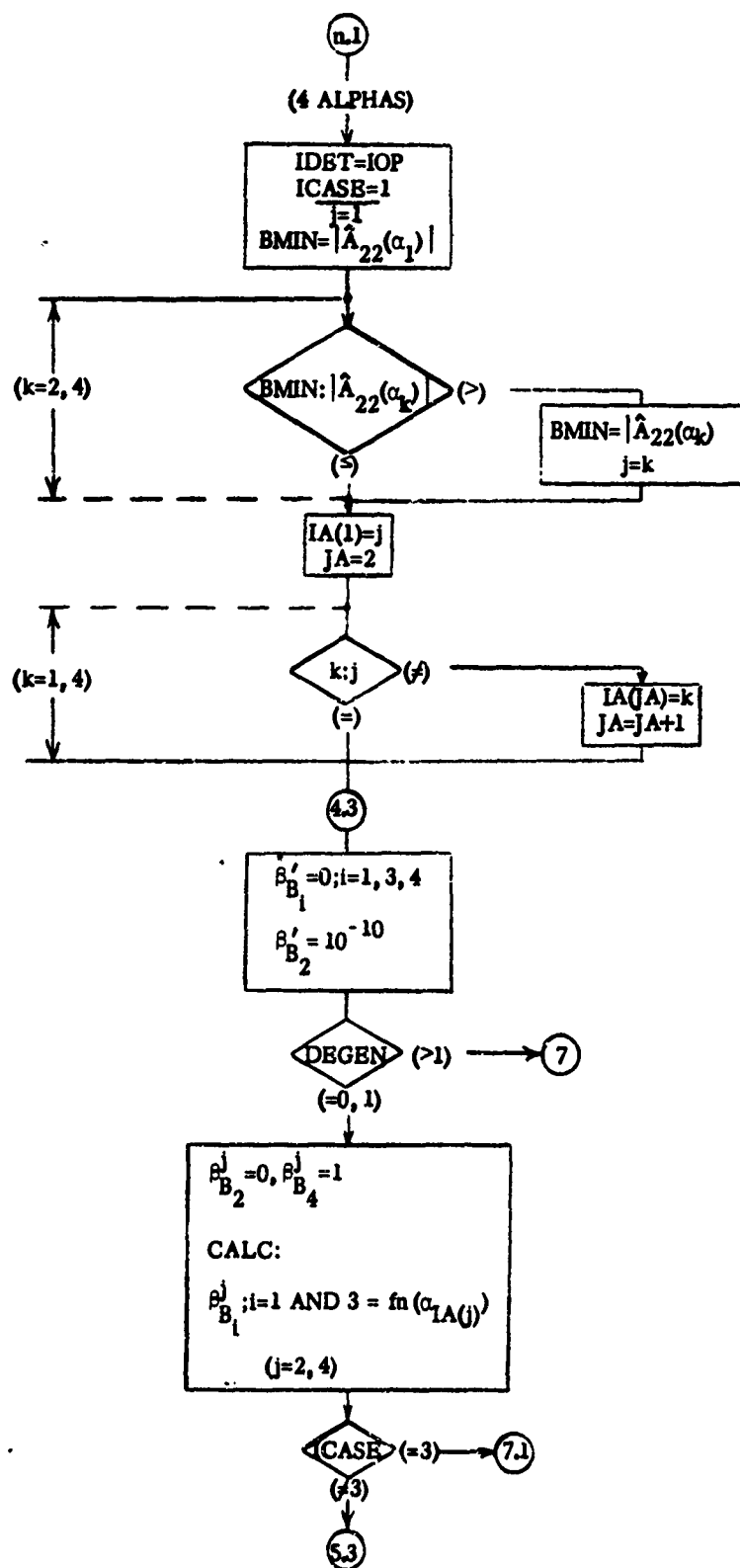
This case is characterized by a decoupling of the equations for $\beta_i^{(l)}$ such that two of the equations involve β_1 and β_3 only and one of the equations involves

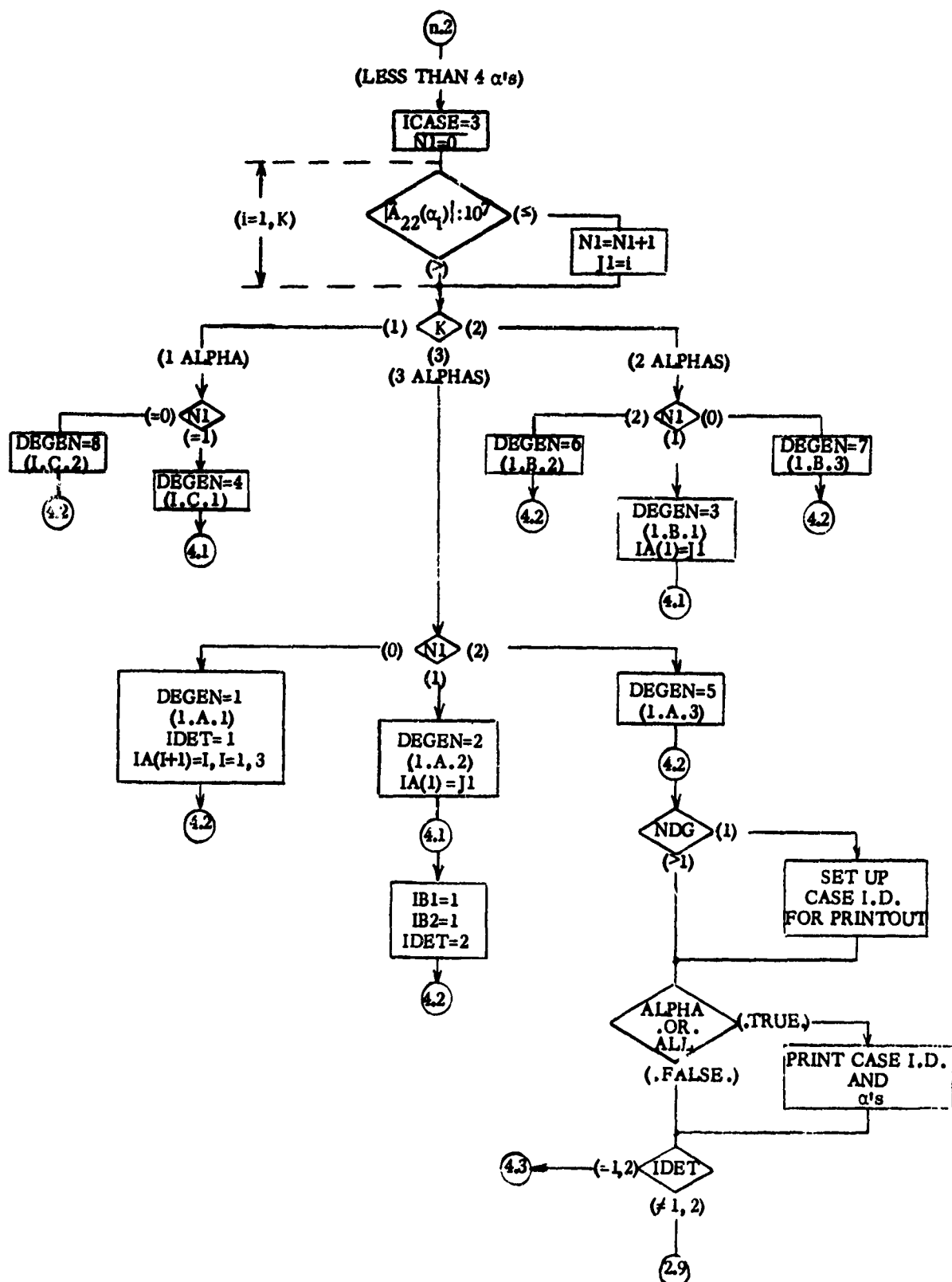


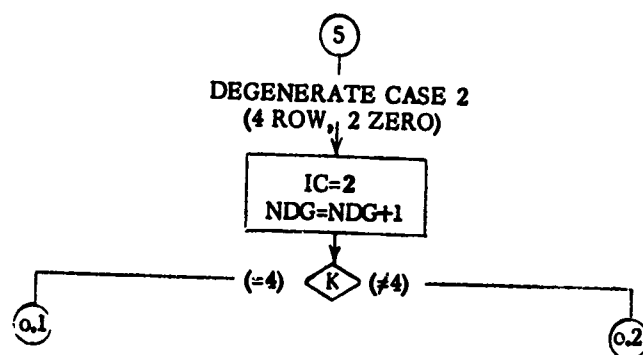


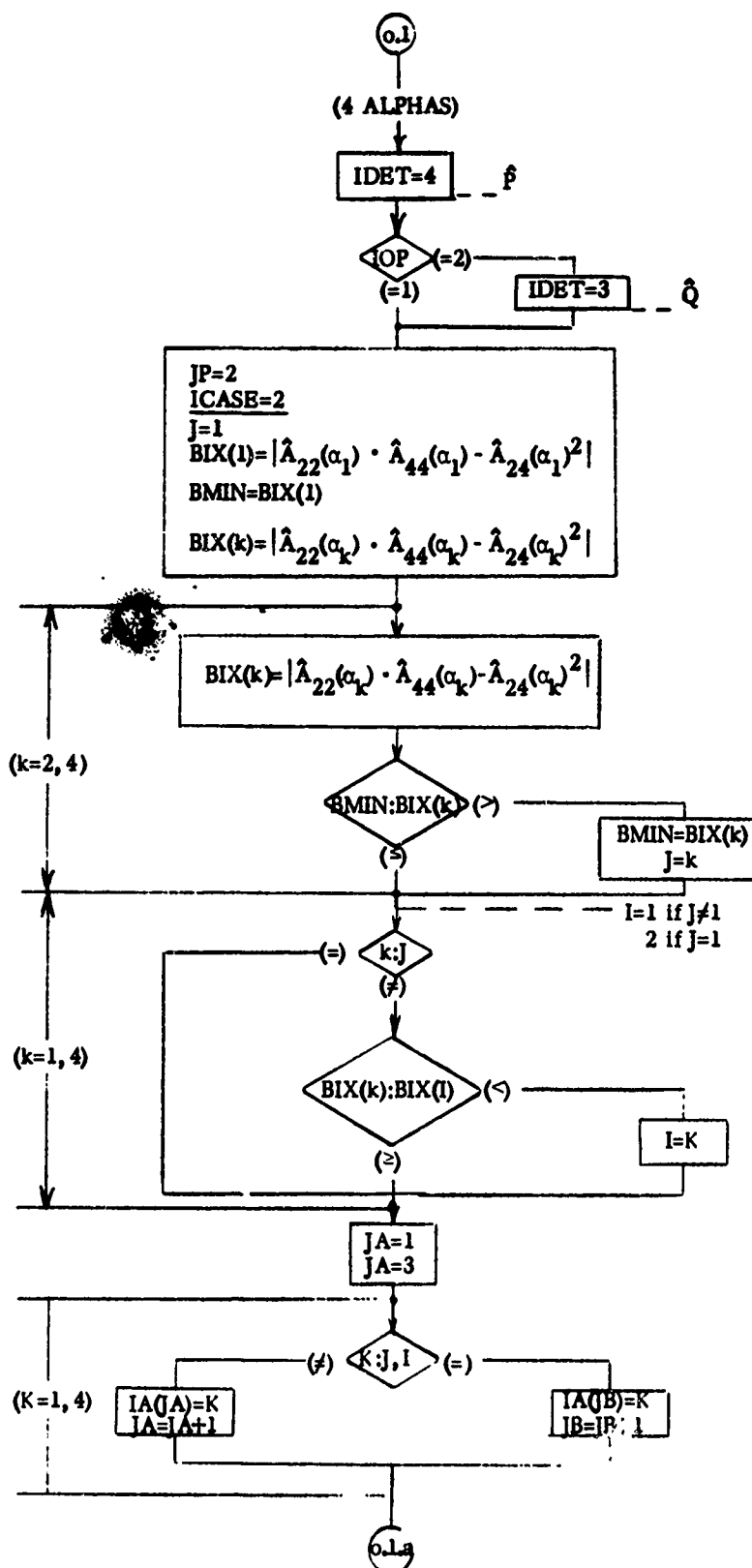


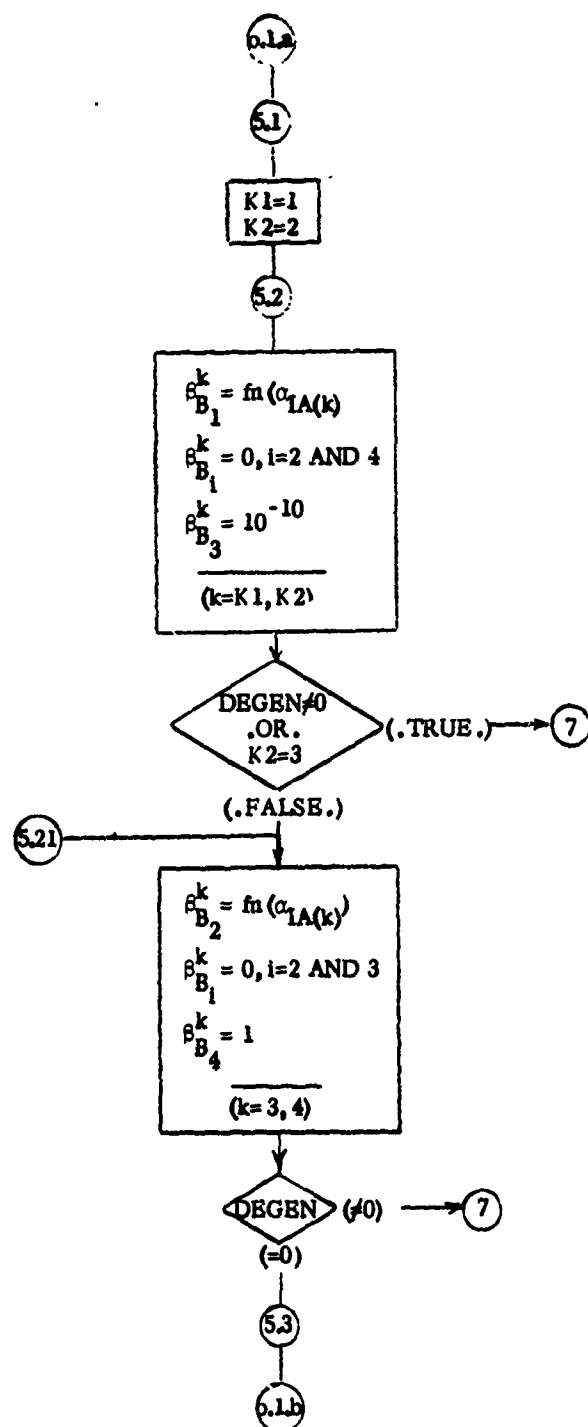


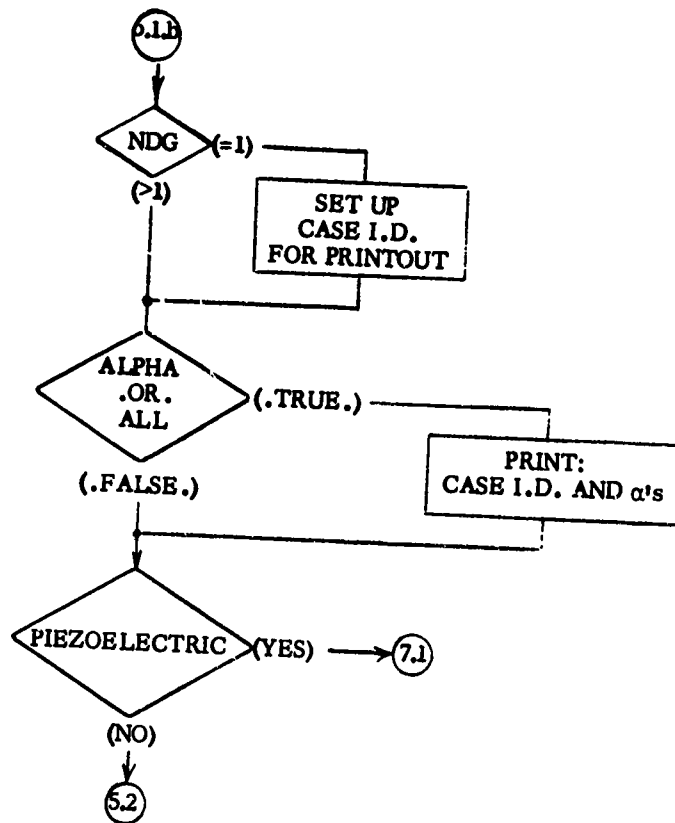


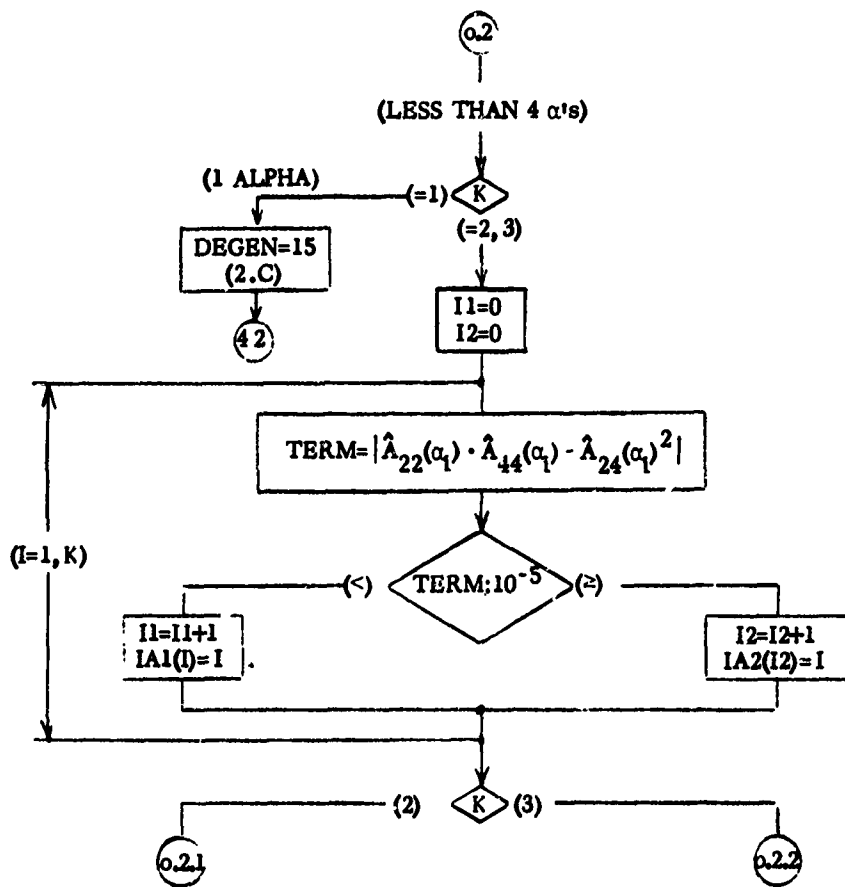


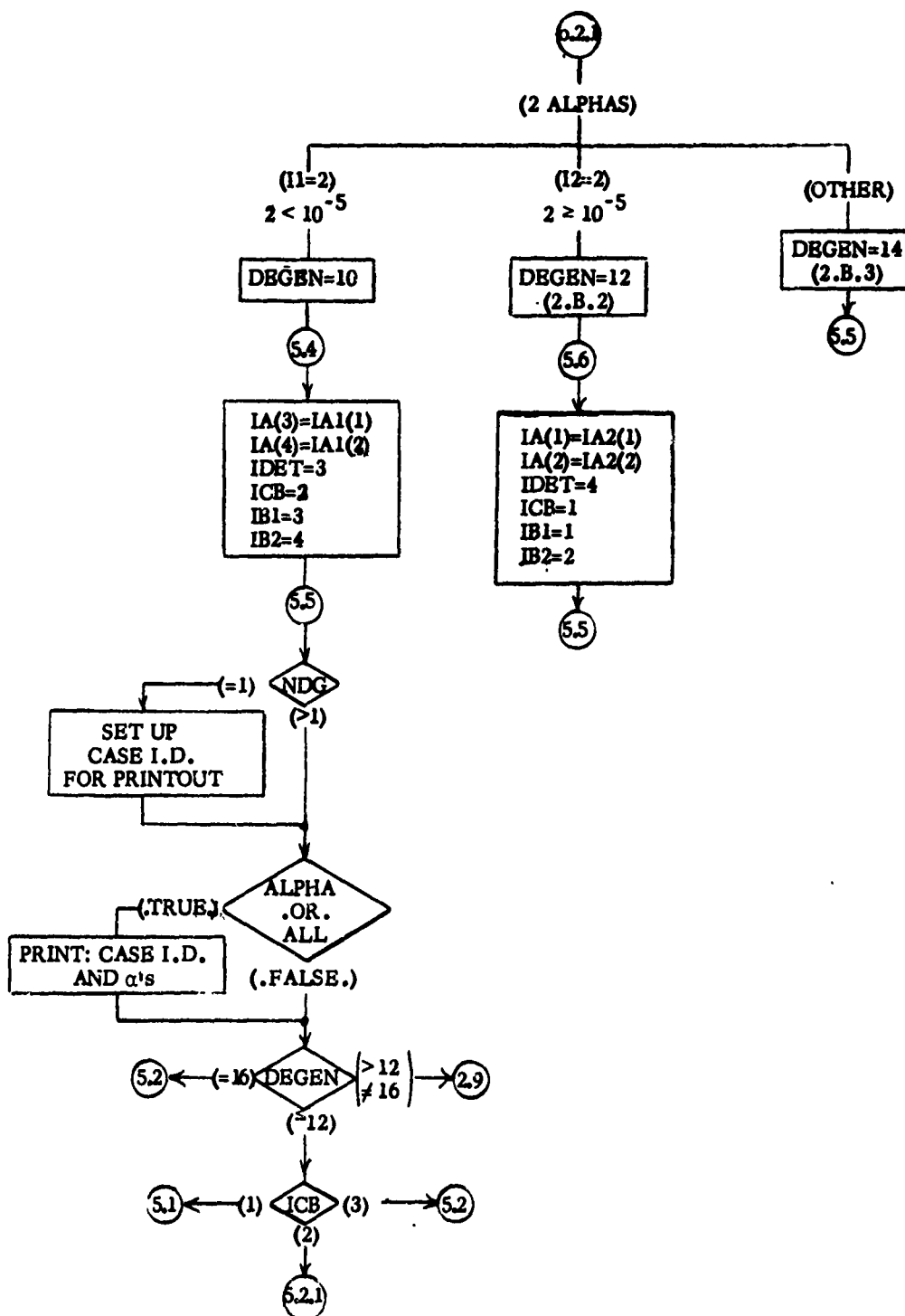


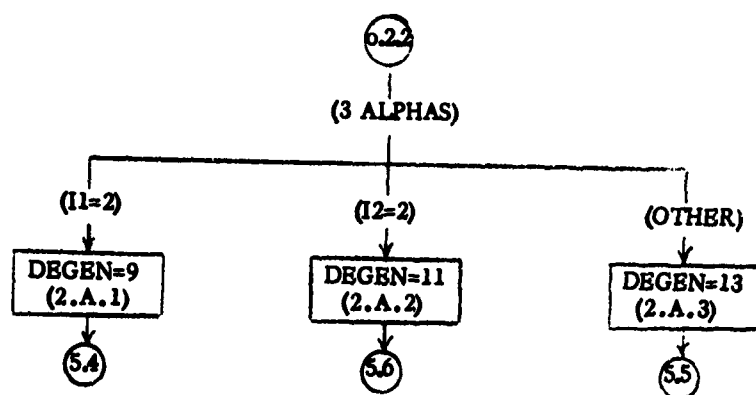


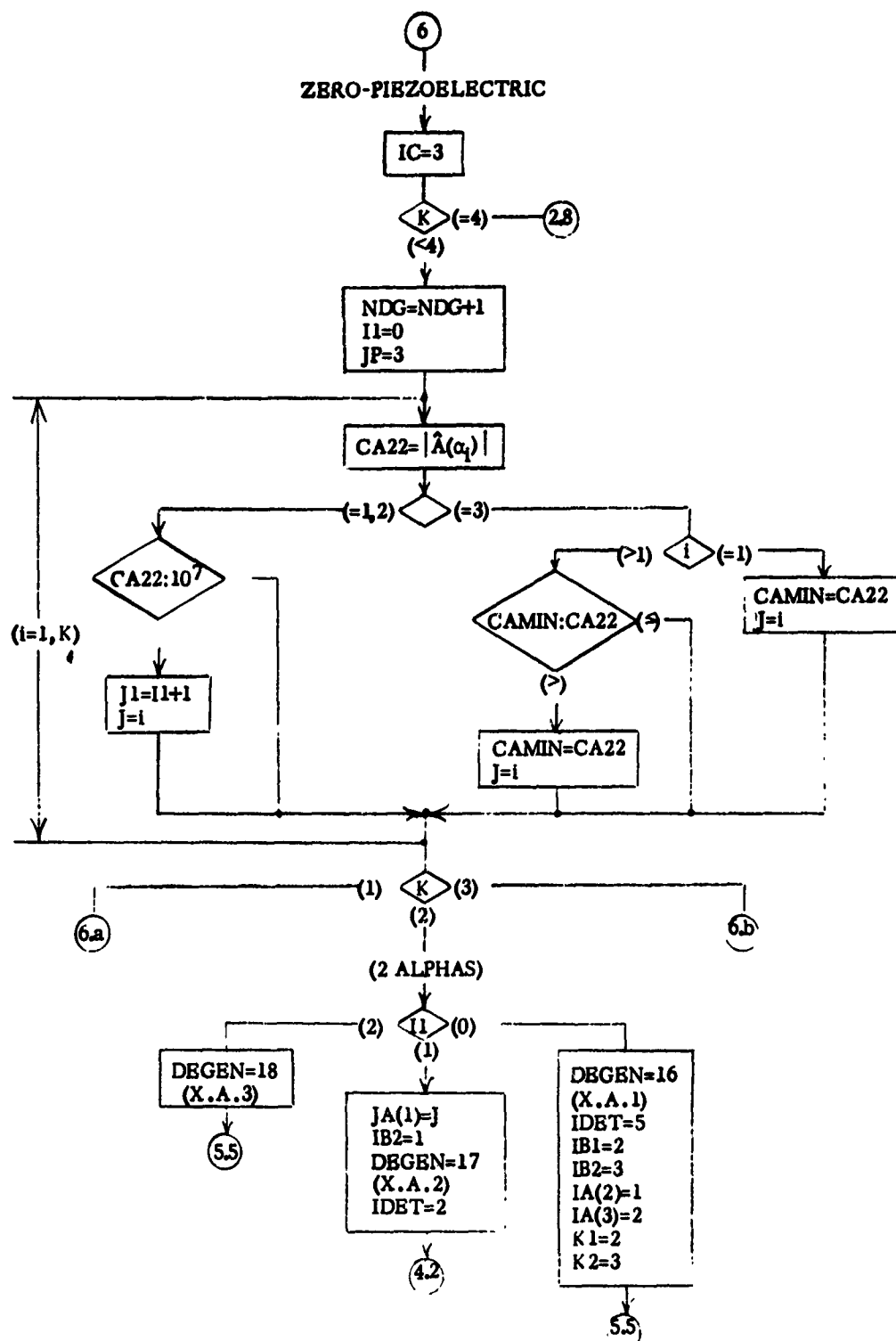


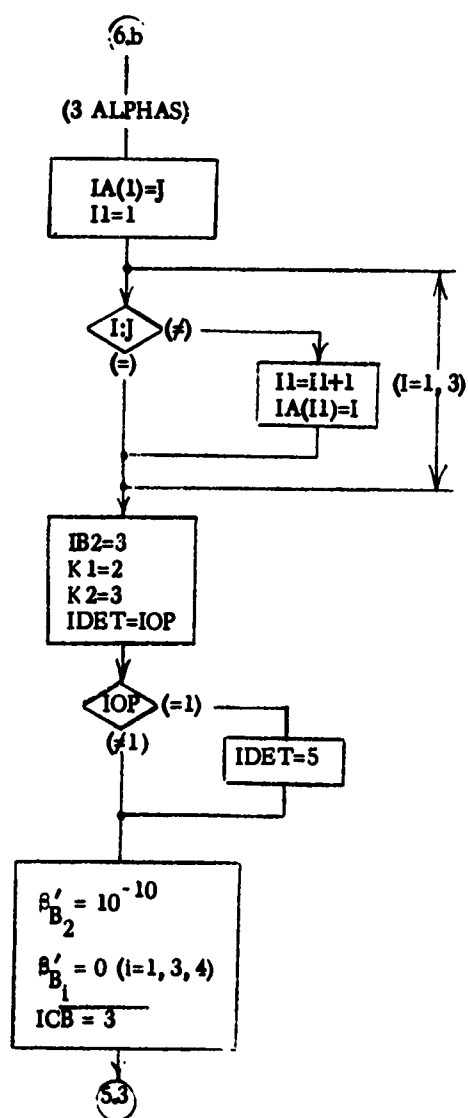
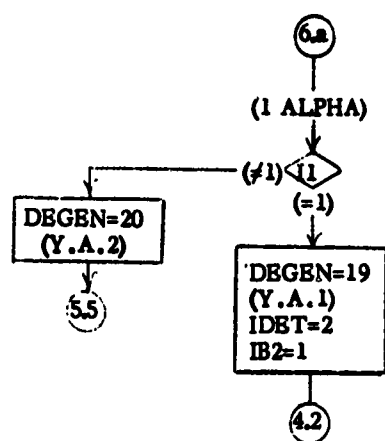


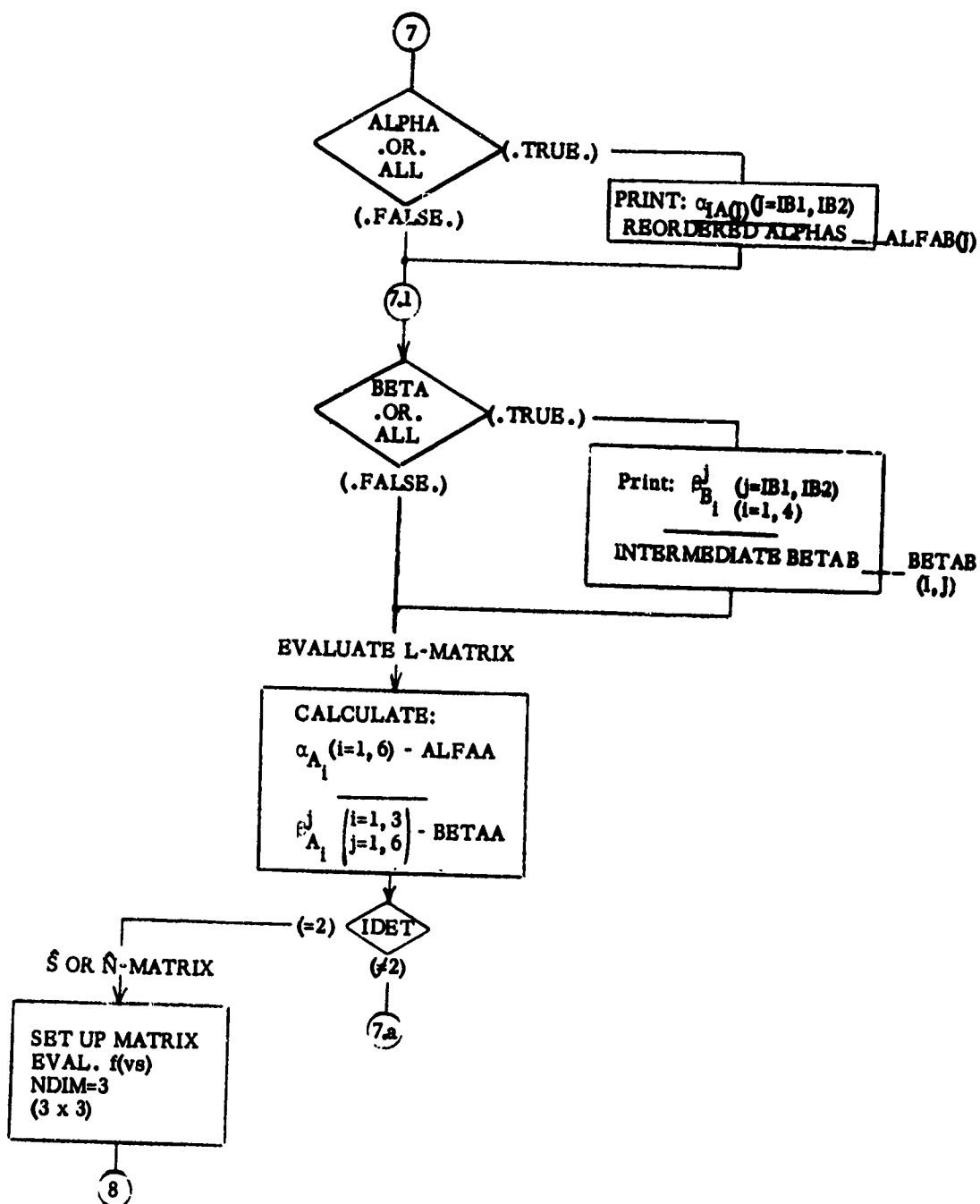


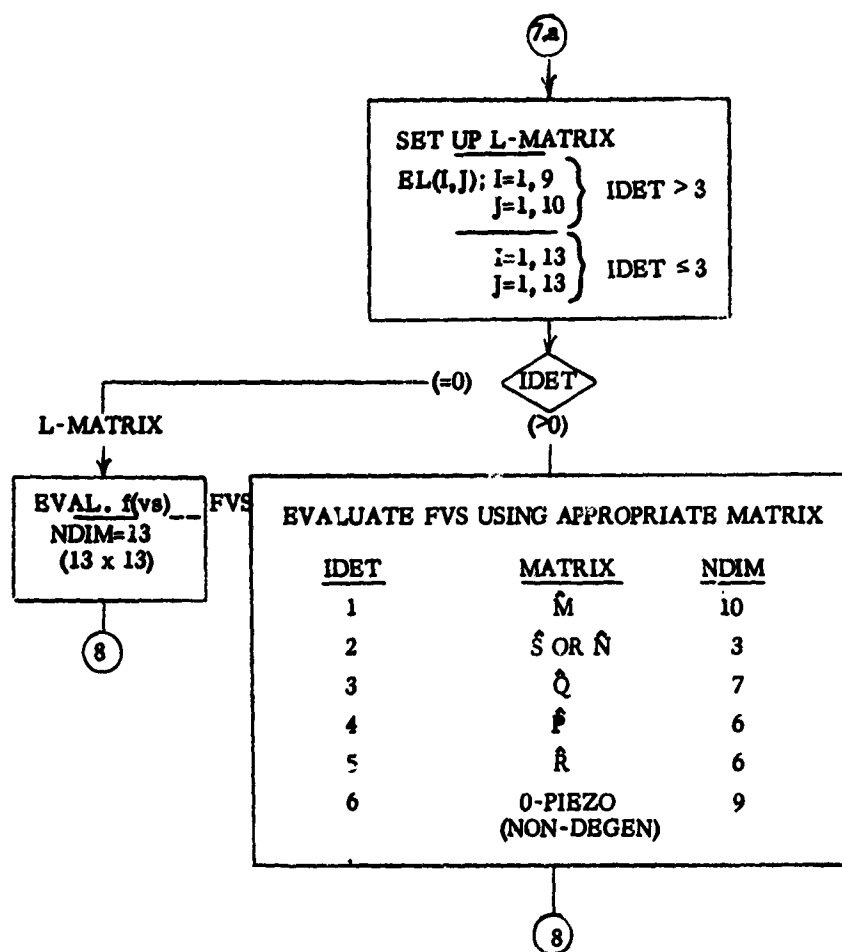


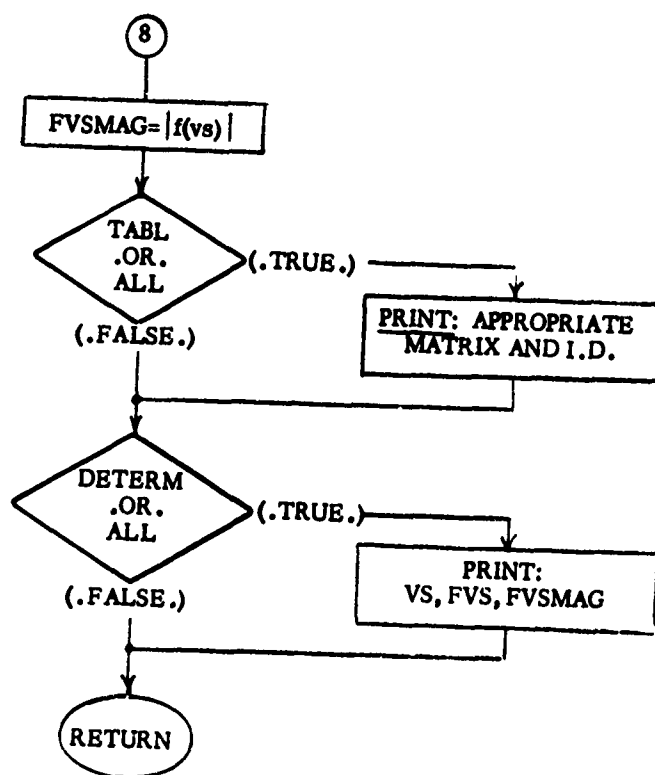












APPENDIX I. ELEMENTS OF THE $\hat{L}$ MATRIX FOR THE ELASTIC-CONDUCTOR PIEZOELECTRIC SUBSTRATE PROBLEM

The subscript p corresponds to the piezoelectric medium while the subscript c corresponds to the conducting elastic medium. The C_{ij} 's and e_{ij} 's correspond to the piezoelectric medium while λ , μ correspond to the conductor (Lame's constants). The expressions for $\alpha_c^{(j)}$ are easily obtainable from equation (20) and are as follows

$$\alpha_c^{(1,2)} = \pm \sqrt{\frac{\mu - \rho_c v_s^2}{\mu}} = \alpha_c^{(5,6)}$$

$$\alpha_c^{(3,4)} = \pm \sqrt{\frac{2\mu + \lambda - \rho_c v_s^2}{2\mu + \lambda}}$$

The elements of the 10 x 10 boundary value determinant are as follows:

$$L_{i\ell} = \beta_{ci}^{(\ell)} [i = 1, 2, 3 \quad \ell = 1, 2, \dots, 6]$$

$$L_{i\ell} = -\beta_{pi}^{(\ell-6)} [i = 1, 2, 3 \quad \ell = 7, 8, 9, 10]$$

$$L_{4\ell} = \beta_{c1}^{(\ell)} \alpha_c^{(\ell)} \mu + j \beta_{c3}^{(\ell)} \mu \quad [\ell = 1, 2, \dots, 6]$$

$$L_{4\ell} = -\beta_{p1}^{(\ell-6)} [j C_{15} + \alpha_p^{(\ell-6)} C_{55}] - \beta_{p2}^{(\ell-6)} [j C_{56} + \alpha_p^{(\ell-6)} C_{45}] \\ - \beta_{p3}^{(\ell-6)} [j C_{55} + \alpha_p^{(\ell-6)} C_{35}] - \beta_{p4}^{(\ell-6)} [j e_{15} + \alpha_p^{(\ell-6)} e_{35}] \quad [\ell = 7, 8, 9, 10]$$

$$L_{5\ell} = \beta_{c2}^{(\ell)} \alpha_c^{(\ell)} \mu \quad [\ell = 1, 2, \dots, 6]$$

$$L_{5\ell} = -\beta_{p1}^{(\ell-6)} [j C_{14} + \alpha_p^{(\ell-6)} C_{45}] - \beta_{p2}^{(\ell-6)} [j C_{46} + \alpha_p^{(\ell-6)} C_{44}] \\ - \beta_{p3}^{(\ell-6)} [j C_{45} + \alpha_p^{(\ell-6)} C_{34}] - \beta_{p4}^{(\ell-6)} [j e_{14} + \alpha_p^{(\ell-6)} e_{34}] \quad [\ell = 7, 8, 9, 10]$$

$$L_{6l} = j \beta_{c1}^{(l)} \lambda + \beta_{c3}^{(l)} \alpha_c^{(l)} (2\mu + \lambda) \quad [l = 1, 2, \dots, 6]$$

$$L_{6l} = -\beta_{p1}^{(l-6)} [j C_{13} + \alpha_p^{(l-6)} C_{35}] - \beta_{p2}^{(l-6)} [j C_{36} + \alpha_p^{(l-6)} C_{34}] \\ - \beta_{p3}^{(l-6)} [j C_{35} + \alpha_p^{(l-6)} C_{33}] - \beta_{p4}^{(l)} [j e_{13} + \alpha_p^{(l-6)} e_{33}] \quad [l = 7, 8, 9, 10]$$

$$L_{7l} = L_{4l} e^{\alpha_c^{(l)} \omega h / v_s} \quad [l = 1, 2, \dots, 6]$$

$$L_{7l} = 0 \quad [l = 7, 8, 9, 10]$$

$$L_{8l} = L_{5l} e^{\alpha_c^{(l)} \omega h / v_s} \quad [l = 1, 2, \dots, 6]$$

$$L_{8l} = 0 \quad [l = 7, 8, 9, 10]$$

$$L_{9l} = L_{6l} e^{\alpha_c^{(l)} \omega h / v_s} \quad [l = 1, 2, \dots, 6]$$

$$L_{9l} = 0 \quad [l = 7, 8, 9, 10]$$

$$L_{10l} = 0 \quad [l = 1, 2, \dots, 6]$$

$$L_{10l} = \beta_{p4}^{(l-6)} \quad [l = 7, 8, 9, 10]$$

APPENDIX II. EXPLICIT FORMS OF THE ELEMENTS OF THE MATRIX M
ASSOCIATED WITH THE FLUID MEDIUM PIEZOELECTRIC
SUBSTRATE PROBLEM

$$M_{1l} = \beta_3^{(l)} \cdot 10^{10}, \quad l = 1, 2, 3, 4,$$

$$M_{15} = 0,$$

$$M_{16} = -1;$$

$$M_{2l} = \beta_4^{(l)}, \quad l = 1, 2, 3, 4$$

$$M_{25} = -1,$$

$$M_{26} = 0;$$

$$M_{3l} = [\beta_1^{(l)}(je_{31} + \alpha_c^{(l)} e_{35}) + \beta_2^{(l)}(je_{36} + \alpha_c^{(l)} e_{34}) \\ + \beta_3^{(l)}(je_{35} + \alpha_c^{(l)} e_{33}) - \beta_4^{(l)}(je_{13} + \alpha_c^{(l)} e_{33})] \cdot 10^{10}, \\ l = 1, 2, 3, 4$$

$$M_{35} = -e_l \cdot 10^{10},$$

$$M_{36} = 0;$$

$$M_{4l} = \beta_1^{(l)}(jC_{13} + \alpha_c^{(l)} C_{35}) + \beta_2^{(l)}(jC_{36} + \alpha_c^{(l)} C_{34}) \\ + \beta_3^{(l)}(jC_{35} + \alpha_c^{(l)} C_{33}) + \beta_4^{(l)}(je_{13} + \alpha_c^{(l)} e_{33}), \\ l = 1, 2, 3, 4,$$

$$M_{45} = 0,$$

$$M_{46} = \frac{\rho_l v_s^2}{\alpha_l} \cdot 10^{-10};$$

$$M_{5l} = \beta_1^{(l)} (jC_{15} + \alpha_c^{(l)} C_{55}) + \beta_2^{(l)} (jC_{56} + \alpha_c^{(l)} C_{45}) \\ + \beta_3^{(l)} (jC_{55} + \alpha_c^{(l)} C_{35}) + \beta_4^{(l)} (je_{15} + \alpha_c^{(l)} e_{35}) ,$$

$$l = 1, 2, 3, 4 ,$$

$$M_{55} = M_{56} = 0 ;$$

$$M_{6l} = \beta_1^{(l)} (jC_{14} + \alpha_c^{(l)} C_{45}) + \beta_2^{(l)} (jC_{46} + \alpha_c^{(l)} C_{44}) \\ + \beta_3^{(l)} (jC_{45} + \alpha_c^{(l)} C_{34}) + \beta_4^{(l)} (je_{14} + \alpha_c^{(l)} e_{34}) ,$$

$$l = 1, 2, 3, 4 ,$$

$$M_{65} = M_{66} = 0 .$$

The factors 10^{10} and 10^{-10} are introduced to make the real and imaginary parts of all elements of the matrix on the order of unity.

APPENDIX III. ELEMENTS OF THE $\hat{L}$ MATRIX FOR THE ELASTIC LAYER
PIEZOELECTRIC SUBSTRATE PROBLEM

$$L_{i\ell} = \beta_{di}^{(\ell)} \cdot 10^{10} \quad [i=1, 2, 3 \quad \ell = 1, 2, \dots, 6]$$

$$L_{i\ell} = -\beta_{ci}^{(\ell-6)} \cdot 10^{10} \quad [i=1, 2, 3 \quad \ell=7, 8, 9, 10]$$

$$L_{i\ell} = 0 \quad [i=1, 2, 3 \quad \ell=11, 12, 13]$$

$$L_{4\ell} = \mu_d \alpha_d^{(\ell)} \beta_{d1}^{(\ell)} + j \mu_d \beta_{d3}^{(\ell)} \quad [\ell=1, 2, \dots, 6]$$

$$\begin{aligned} L_{4\ell} = & -\beta_{c1}^{(\ell-6)} [j c_{15} + \alpha_c^{(\ell-6)} c_{55}] - \beta_{c2}^{(\ell-6)} [j c_{56} + \alpha_c^{(\ell-6)} c_{45}] \\ & -\beta_{c3}^{(\ell-6)} [j c_{55} + \alpha_c^{(\ell-6)} c_{35}] - \beta_{c4}^{(\ell-6)} [j e_{15} + \alpha_c^{(\ell-6)} e_{35}] \\ & [\ell = 7, 8, 9, 10] \end{aligned}$$

$$L_{4\ell} = 0 \quad [\ell = 11, 12, 13]$$

$$L_{5\ell} = \mu_d \alpha_d^{(\ell)} \beta_{d2}^{(\ell)} \quad [\ell = 1, 2, \dots, 6]$$

$$\begin{aligned} L_{5\ell} = & -\beta_{c1}^{(\ell-6)} [j c_{14} + \alpha_c^{(\ell-6)} c_{45}] - \beta_{c2}^{(\ell-6)} [j c_{46} + \alpha_c^{(\ell-6)} c_{44}] \\ & -\beta_{c3}^{(\ell-6)} [j c_{45} + \alpha_c^{(\ell-6)} c_{34}] - \beta_{c4}^{(\ell-6)} [j e_{14} + \alpha_c^{(\ell-6)} e_{34}] \\ & [\ell = 7, 8, 9, 10] \end{aligned}$$

$$L_{5\ell} = 0 \quad [\ell = 11, 12, 13]$$

$$L_{6\ell} = j \lambda_d \beta_{d1}^{(\ell)} + (\lambda_d + 2 \mu_d) \alpha_d^{(\ell)} \beta_{d3}^{(\ell)} \quad [\ell = 1, 2, \dots, 6]$$

$$\begin{aligned} L_{6\ell} = & -\beta_{c1}^{(\ell-6)} [j c_{13} + \alpha_c^{(\ell-6)} c_{35}] - \beta_{c2}^{(\ell-6)} [j c_{36} + \alpha_c^{(\ell-6)} c_{34}] \\ & -\beta_{c3}^{(\ell-6)} [j c_{35} + \alpha_c^{(\ell-6)} c_{33}] - \beta_{c4}^{(\ell-6)} [j e_{13} + \alpha_c^{(\ell-6)} e_{33}] \\ & [\ell = 7, 8, 9, 10] \end{aligned}$$

$$L_{6l} = 0 \quad [l = 11, 12, 13]$$

$$L_{7l} = L_{4l} e^{\alpha_d^{(l)} \omega h / v_s} \quad [l = 1, 2, \dots, 6]$$

$$L_{7l} = 0 \quad [l = 7, 8, \dots, 13]$$

$$L_{8l} = L_{5l} e^{\alpha_d^{(l)} \omega h / v_s} \quad [l = 1, 2, \dots, 6]$$

$$L_{8l} = 0 \quad [l = 7, 8, \dots, 13]$$

$$L_{9l} = L_{6l} e^{\alpha_d^{(l)} \omega h / v_s} \quad [l = 1, 2, \dots, 6]$$

$$L_{9l} = 0 \quad [l = 7, 8, \dots, 13]$$

$$L_{10l} = 0 \quad [l = 1, 2, \dots, 6]$$

$$L_{10l} = \beta_{c4}^{(l-6)} \quad [l = 7, 8, 9, 10]$$

$$L_{10l} = -1 \quad [l = 11, 12]$$

$$L_{10l} = 0 \quad [l = 13]$$

$$L_{11l} = 0 \quad [l = 1, 2, \dots, 10]$$

$$L_{11,11} = e^{-\omega h / v_s}$$

$$L_{11,12} = e^{\omega h / v_s}$$

$$L_{11,13} = -e^{-\omega h / v_s}$$

$$L_{12\ell} = 0 \quad [\ell = 1, 2, \dots, 6]$$

$$L_{12\ell} = \left[\beta_{c1}^{(\ell-6)} [j e_{13} + \alpha_c^{(\ell-6)} e_{35}] + \beta_{c2}^{(\ell-6)} [j e_{36} + \alpha_c^{(\ell-6)} e_{34}] \right. \\ \left. + \beta_{c3}^{(\ell-6)} [j e_{35} + \alpha_c^{(\ell-6)} e_{33}] - \beta_{c4}^{(\ell-6)} [j e_{13} + \alpha_c^{(\ell-6)} e_{33}] \right] \cdot 10^{10}$$

$$\ell = 7, 8, 9, 10$$

$$L_{12,11} = -\epsilon_d \cdot 10^{10}$$

$$L_{12,12} = \epsilon_d \cdot 10^{10}$$

$$L_{12,13} = 0$$

$$L_{13,\ell} = 0 \quad [\ell = 1, 2, \dots, 10]$$

$$L_{13,11} = e^{-j\hbar/v_s}$$

$$L_{13,12} = -e^{j\hbar/v_s}$$

$$L_{13,13} = -\frac{\epsilon_o}{\epsilon_d} e^{-j\hbar/v_s}$$

APPENDIX IV. EXPLICIT FORMS OF THE POLYNOMIAL COEFFICIENTS A_k

The elements of the matrix $\hat{A}$ have the general form $\hat{A}_{ik} = a_{ik}\alpha^2 + j b_{ik}\alpha + d_{ik}$ where a_{ik} , b_{ik} , and d_{ik} are easily deduced from equation (6). Therefore, the determinant of $\hat{A}$ can be expressed as the polynomial

$$A_1\alpha^8 + jA_2\alpha^7 + A_3\alpha^6 + jA_4\alpha^5 + A_5\alpha^4 + jA_6\alpha^3 + A_7\alpha^2 + jA_8\alpha + A_9$$

with coefficients

$$A_1 = \sum_{\{j, k, l, m\}} (-1)^h H_{jkl} S_m$$

$$A_2 = \sum_{\{j, k, l, m\}} (-1)^h [H_{jkl} \bar{U}_m + I_{jkl} S_m]$$

$$A_3 = \sum_{\{j, k, l, m\}} (-1)^h [H_{jkl} V_m - I_{jkl} \bar{U}_m + J_{jkl} S_m]$$

$$A_4 = \sum_{\{j, k, l, m\}} (-1)^h [I_{jkl} V_m + J_{jkl} \bar{U}_m + K_{jkl} S_m]$$

$$A_5 = \sum_{\{j, k, l, m\}} (-1)^h [J_{jkl} V_m - K_{jkl} \bar{U}_m + L_{jkl} S_m]$$

$$A_6 = \sum_{\{j, k, l, m\}} (-1)^h [K_{jkl} V_m + L_{jkl} \bar{U}_m + M_{jkl} S_m]$$

$$A_7 = \sum_{\{j,k,\ell,m\}} (-1)^h [L_{jkl} V_m - M_{jkl} \bar{U}_m + N_{jkl} S_m]$$

$$A_8 = \sum_{\{j,k,\ell,m\}} (-1)^h [M_{jkl} V_m + N_{jkl} \bar{U}_m]$$

$$A_9 = \sum_{\{j,k,\ell,m\}} (-1)^h N_{jkl} V_m$$

$\sum_{\{j,k,\ell,m\}}$ refers to a sum over all permutations of 1, 2, 3, 4. There are 24 terms in each sum. h is the number of interchanges for each term necessary to return the indices to the order 1, 2, 3, 4; and

$$H_{jkl} = a_{1j} a_{2k} a_{3\ell}$$

$$I_{jkl} = a_{1j} a_{2k} b_{3\ell} + (a_{1j} b_{2k} + b_{1j} a_{2k}) a_{3\ell}$$

$$J_{jkl} = a_{1j} a_{2k} d_{3\ell} - (a_{1j} b_{2k} + b_{1j} a_{2k}) b_{3\ell} \\ + (a_{1j} d_{2k} - b_{1j} b_{2k} + d_{1j} a_{2k}) a_{3\ell}$$

$$K_{jkl} = (a_{1j} b_{2k} + b_{1j} a_{2k}) d_{3\ell} - (a_{1j} b_{2k} + b_{1j} a_{2k}) b_{3\ell} \\ + (b_{1j} d_{2k} + d_{1j} b_{2k}) a_{3\ell}$$

$$L_{jkl} = (a_{1j} d_{2k} - b_{1j} b_{2k} + d_{1j} a_{2k}) d_{3\ell} - (b_{1j} d_{2k} + d_{1j} b_{2k}) b_{3\ell} \\ + d_{1j} d_{2k} a_{3\ell}$$

$$M_{jkl} = (b_{1j} d_{2k} + d_{1j} b_{2k}) d_{3\ell} + d_{1j} d_{2k} b_{3\ell}$$

$$N_{jkl} = d_{1j} d_{2k} d_{3\ell}$$

$$S_m = a_{4m}$$

$$\bar{U}_m = b_{4m}$$

$$V_n = d_{4m}$$

APPENDIX V. COMPUTER PROGRAMS

	CONWAY	PHASE	N4.		12/31/69	000109	PAGE 1
SID * 1614 MCGONWAY	PHASE	A4	000109	112244	12/31/69	000109	
STCP	TIME=3,PAGES=20,DUMP						
SSETUP L84	CRTPLT						
SALL	CONTINUE						
SIBSYS							
RETURNING TO IBSYS.							
SIBJOB	DEBUG						
SIBFTC LIN803	DECK.DEBUG						
CLINBC3							MAIN0020
C*	GOLD LITHIUM AND LITHIUM NIOBATE						MAIN0030
C*							MAIN0040
C*****							MAIN0050
							MAIN0060
DIMENSION	PANGLE(181)						
DIMENSION	XX(2),YY(2)						
DIMENSION	DELTA(181)						
DIMENSION	RT1(181),RT11(181),RUI(181),RIU1(181),RE1(181)						
DIMENSION	RE11(181)						
DIMENSION	CETRAY(100),DETIAY(100),VSARAY(100),AAAAA(200)						
COMPLEX	D(3+I4)						
INTEGER	BLIMIT,ELIMIT						
LOGICAL	ROTATE						
LOGICAL	GETOUT						
LOGICAL	NEGAT						
LOGICAL	MULT						
LOGICAL	ICHECK						
COMMON	/ROTAT/ ROTATE						
COMMON	/GET/GETOUT						
COMMON	/PLOTS/ICHECK						
DATA	XX/0.,10./,YY/5.,5./						
DIMENSION	DEG(181), TITLES(4)						
DIMENSION	VELOC(181), VELOC1(181), VEL(362)						
COMMON	/OVR/KO,N1						
COMMON	/ZZTZ/ CC(20),CE(17),CT(5)						
COMMON	... /LINK/ ALFA(8), ALFAI(8), EL(100), ALFAA(6),						MAIN0080
	ALFAB(4), BETAA(3,6), BETAB(4,4), EPSO,						MAIN0090
	MUA, LAMDA, RHOA, MUB, LAMCAB, MUB, RHOB,						MAIN0100
	VS, KS, EPSLON, DIGIT, WH, WXA, WXB, KL,						MAIN0110
	KM, ALL, ROOTS, ITER, COEFF, DETERM, POLY,						MAIN0120
	ALPHA, BETA, MAX						MAIN0130
COMMON	/GPEP5/ G(21), P(18), EPS(9)						MAIN0140
COMMON	/FLAG/ ONCE /BETAN/ NBETA						MAIN0150
	/CSET/ CLIM, ELIM, TLM						MAIN0160
COMMON	/FROOT/ FVSMAG, NY, ICASE						MAIN0170
	/COM/ ACAP, EPSR						MAIN0180
	/CIA/ IA(4)						MAIN0190
	/ALESS/ IMF						MAIN0195
	/CIFL/ WXAGNL						
INTEGER	PLOTIT(6)						
COMPLEX	XEL(3,3), FXL, XL(2,3), XE1(2)						MAIN0200
COMPLEX	ALFA, FVS, EL, EA(6), EB(4), UA(3), UB(3), ALFAI,						MAIN0210
	ALFAA, ALFAB, BETAA, BETAB, ETA(10), PHIO						MAIN0220
COMPLEX	CKO, CX1						MAIN0230
COMPLEX	TFUN, PIFUN, U, EX(4), PIM, P2M,						MAIN0240
	TM31, TM32, TM33, TM11, TM12, TM22, S11, S22, S33,						MAIN0250
	S12, S13, S23, C1, D2, D3, JIMAG, E1, E3						MAIN0260
REAL	MUA, LAMDA, MUB, LAMCAB, MUA, MUB,						MAIN0270
	MAGU(4), PHASEU(4), NUMAX, TITLE(4)						MAIN0280
LOGICAL	ALL, ROOTS, COEFF, DETERM, POLY, ALPHA, BETA,						MAIN0290

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      ICHECK, ACAP, IALF,                                MAIN0300
      *XAGAL.
      REPEAT, ONCE                                         MAIN0310
      LINBO3 CONWAY PHASE M4
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EQUIVALENCE ( VEL(1), VELOC(1) ), ( VELOC(1), VEL(102) )
EQUIVALENCE ( TITLE, TITLES(1) )
EQUIVALENCE (CC(1), C11), (CC(2), C13), (CC(3), C14), (CC(4), C15), MAIN0320
      (CC(5), C33), (CC(6), C34), (CC(7), C35), (CC(8), C36), MAIN0330
      (CC(9), C44), (CC(10), C45), (CC(11), C46), MAIN0340
      (CC(12), C55), (CC(13), C56), (CC(14), C66), MAIN0350
      (CC(15), C16), (CE(1), E11), (CE(2), E13), MAIN0360
      (CE(3), E14), (CE(4), E15), (CE(5), E16), MAIN0370
      (CE(6), E31), (CE(7), E33), (CE(8), E34), MAIN0380
      (CE(9), E35), (CE(10), E36), (CT(1), T11), MAIN0390
      (CT(2), T13), (CT(3), T33) MAIN0400
EQUIVALENCE (CC(16), C12), (CC(17), C25), (CC(18), C26), MAIN0410
      (CC(19), C24), (CC(20), C23), (CE(11), E12), MAIN0420
      (CE(12), E32), (CE(13), E21), (CE(14), E23), MAIN0430
      (CE(15), E24), (CE(16), E25), (CE(17), E26), MAIN0440
      (CT(4), T21), (CT(5), T23) MAIN0450
      MAIN0460
PANELIST /INPUT/ MUA, LAMDA, RMOA, MLB, LAMCAB, MUB, RHOB, MAIN0470
      VS, KS, EPSLON, MH, MKA, MKB, EPSR, MAIN0480
      *NEGAT,
      *XAGNL,
      KL, KM, ALL, ROOTS, COEFF, DETERM, MAIN0490
      POLY, ALPHA, BETA, MAX, EPSO, MX, REPEAT, MAIN0500
      ICHECK, DVS, VSMAX, ACAP, CLIM, ELIM, TLIM, MAIN0510
      DMU, NUMAX, DMX, MXMAX, TITLE MAIN0520
      *MLIT, ROTATE
      NAMELIST /CONST/ G, P, EPS MAIN0530
      MAIN0540
      DATA CX0, CX1/(0.,0.), (1.,0.)/ MAIN0550
      DATA TT31, TT32, TT33, TT11, TT12, TT22 / MAIN0560
      3MT31, 3MT32, 3MT33, 3MT11, 3MT12, 3MT22 / MAIN0570
      DATA SS11, SS22, SS33, SS12, SS13, SS23 / MAIN0580
      3MS11, 3MS22, 3MS33, 3MS12, 3MS13, 3MS23 / MAIN0590
      DATA DD1, DD2, DD3, UU1, UU2, UU3 / MAIN0600
      2MD1, 2MD2, 2MD3, 2MU1, 2MU2, 2MU3 / MAIN0610
      DATA PP1M, PP2M / 3MP1M, 3MP2M / MAIN0620
      DATA EE1, EE3, JIMAG / 2ME1, 2ME3, (0.,1.) / MAIN0630
      DATA TITLE(1) / 24MLITHIUM NIOBATE / MAIN0640
      MAIN0650
      MAIN0660
      MAIN0670
      REAC(5,7773) JJKK
      7773 FCRPAT(12)
      IF(JJKK.EQ.0) GO TO 7774
      CALL SKFILE(2,JJKK)
      7774 CCNTINUE
      CC 20 I=1,101
      20 CEG(I) = 1-1
      CALL CRTPLY(1,0)
      CALL PLOT(0.,-.25,-3)
      CLIM = 1.E5
      ELIM = 1.E-5
      TLIM = 1.E-15
      EPSLON = 1.E-11
      EPSO = 0.05E-12
      EPSR = 1.
      MAIN0680
      MAIN0690
      MAIN0700
      MAIN0710
      MAIN0720
      MAIN0730

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LINB03	- EPN SOURCE STATEMENT -	(FN(S) -		
RMDA = 1.888E4				MAIN0730
MUA = 2.85E10				MAIN0740
LAMDAA = 1.5E11				MAIN0750
VSAVE = 3000.				MAIN0760
DVS = 0.				MAIN0770
VSNAX = 0.				MAIN0780
DIGIT = 3.0				MAIN0790
NUMAX = 0.				MAIN0800
CMJ = 0.				MAIN0810
MXMAX = 0.				MAIN0820
DMS = 0.				MAIN0830
				MAIN0840
				MAIN0850
KL = 0				MAIN0860
KM = 0				MAIN0870
MAX=25				
ALL = .FALSE.				MAIN0890
ROOTS = .FALSE.				MAIN0900
COEFF = .FALSE.				MAIN0920
DETERM = .FALSE.				MAIN0930
NEGAT=.FALSE.				
POLY = .FALSE.				MAIN0940
ALPHA = .FALSE.				MAIN0950
BETA = .FALSE.				MAIN0960
ICHECK = .FALSE.				MAIN0970
ACAP = .FALSE.				MAIN0980
MXAGNL=.FALSE.				
GETCUT=.FALSE.				
MULT=.FALSE.				
ROTATE=.FALSE.				
C.....I N P U T D A T A.....				MAIN0990
510 READ (5, CONST)				MAIN1000
1 FCRMAT(4A6)			22	MAIN1020
READ(5,1) (TITLES(I), I=1,4)			23	
KCUNT = 0				
KONT=0				
KNT1=0				
KNT2=0				
KNT3=0				
REPEAT = .FALSE.				MAIN1030
520 VS = VSAVE				MAIN1040
KTIME = 0				MAIN1050
READ (5, INPUT)				MAIN1060
SMU = MUB			36	MAIN1080
SMX = MX				MAIN1090
READ(5,7) (PLOTIT(I),I=1,6)			38	
7 FCRMAT(6I2)				
530 KTIME = KTIME + 1				MAIN1100
IF (KTIME .LE. 2) GO TO 535				MAIN1110
COA = (VS1 - VS2)/(AN1 - AN2)				MAIN1120
COB = VS1 - COA*AN1				MAIN1130
VS = COA*MUB + COB				MAIN1140
535 VSAVE = VS				MAIN1150
CMCE = .TRUE.				MAIN1160
				MAIN1170

LINBO3 CONWAY PHASE M4
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C.....CALCULATE COEFFICIENTS.....
 CALL SETCTE (MUB, MUB, LANDAB, COEFF .OR. ALL)
 IF (ICHECK) GO TO 2

C.....CALCULATE ROOT AND OUTPUT RESULTS.....
 VSO = VS
 AT = 0
 550 CONTINUE
 CALL ROOT(VSO,VS,N,FVS,EPSLON,MAX,8550)
 IF (GETOUT) GO TO 17
 IF (PLOTIT(1) .EQ. 0) GO TO 32
 KCUNT = KCUNT + 1
 VEL(KCUNT) = VS
 32 CONTINUE
 VS1 = VS2
 VS2 = VS
 AN1 = AN2
 AN2 = MUB
 VS1 = 1./VS
 DO 615 I = 1, 8
 615 ALFA(I) = ALFA(I)/VS
 WRITE (6, 650) KS, TITLE, EPSLON,
 KL, KM, MAX, N, MUB, LANDAB, MUB, LANDAA, MUA,
 RHOA, RHOB, WM, VSO, VS, VS1, FVS,
 (ALFA(I), ALFA(I), I = 1, 8)
 PUNCH 1500, LANDAB, MUB, NLB
 1500 FCRMAT(3F5.1)
 PUNCH 1502, VS, VS1, (ALFA(I), I=1, 8)
 WRITE (2, 650) KS, TITLE, EPSLON,
 KL, KM, MAX, N, MUB, LANDAB, MUB, LANDAA, MUA,
 RHOA, RHOB, WM, VSO, VS, VS1, FVS,
 (ALFA(I), ALFA(I), I = 1, 8)
 650 FCRMAT (1H1, 20X, 4HKS =, 12, 5X, 4A6 /1H0, 6X,
 . 8HEPSLON =, E15.7, 3X, 32PCLOSENESS OF DETERMINANT TO ZERO/1H , 6X, MAIN1330
 . 2HKL, 5X, 1H=, I3, 15X, 34H0 = COMPUTE FOURTH ROW OF L MATRIX/1H , MAIN1340
 . 32X, 22M1 = SET FOURTH ROW = 1/1H , 6X, 2HKL, 5X, 1H=, I3, 15X, MAIN1350
 . 25H0 = ELECTRIC FIELD (COH)/1H , 32X, 25H1 = MAGNETIC FIELD (TANH) MAIN1360
 /1H , 6X, 3HMAX, 4X, 1H=, I3, 15X, 20HMAXIMUM NUMBER OF ITERATIONS/ MAIN1370
 . 1H , 6X, 1HMA, 6X, 1H=, I3, 15X, 34HNUMBER OF ITERATIONS ACTUALLY USED/ MAIN1380
 . 1H , 6X, 8HMMU B =, E15.7, 7X, 9HLMADA B =, E15.7/1H , 6X, 8HMMU B =, MAIN1390
 . E15.7, 7X, 9HLMADA A =, E15.7/1H , 6X, 8HMMU A =, E15.7/1H , 6X, MAIN1400
 . 8HMMO A =, E15.7, 7X, 9HMMO B =, E15.7/1H , 6X, 2HMM, 5X, 1H=, E15.7/ MAIN1410
 . 1H , 6X, MAIN1420
 . 8HVS 0 =, E15.7, 3X, 16HINITIAL VELOCITY/1H0, 6X, 2HVS, 5X, 1H=, E15.7, MAIN1430
 . 3X, 42HFINAL VELOCITY SUCH THAT F(VS) .LT. EPSLON/1H , 6X, MAIN1440
 . 8H1/V5 =, MAIN1450
 . E15.7, 3X, 13HINVERSE OF VS/1H0, 9X, 21H..... DETERMINANT = (E14.7, MAIN1460
 . 1H,, E14.7, 7H) /1H0, 5X, 25HFINAL ROOTS OF POLYNOMIAL, 16X, MAIN1470
 . 13H DIVIDED BY VS/14H (E14.7, 1H,, E14.7, 1H), 7X, 1H, (E14.7, MAIN1480
 . 1H,, E14.7, 1H)) MAIN1490
 MAIN1500
 MAIN1510
 MAIN1520
 MAIN1530
 MAIN1540
 MAIN1550
 MAIN1560
 MAIN1570
 MAIN1580
 MAIN1590
 IF (ALF) GO TO 1707
 IF .NOT. ICHECK) GO TO 660
 IF VSO .GE. VSMAX) GO TO 1708
 WRITE(6, 1705)
 VSO = VSO + DVS

53
 59
 78
 87
 88
 95
 112

IF (ALF) .EQ. XCD)

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GG TO 550                                MAIN1600
660 CONTINUE                             MAIN1610
IF (KS .NE. 0) GO TO 1240                MAIN1630
                                           MAIN1640
C.....CALCULATE FINAL RESULTS FOR LITHIUM NIOBATE AND OUTPUT RESULTS.....MAIN1650
IF( ICASE .EQ. 0 ) GO TO 1000            MAIN1660
GO TO ( 50, 100, 60, 110, 105 ) , ICASE MAIN1670
                                           MAIN1680
                                           MAIN1690
C.....1 ROW, 3 ZERO CASE.....          MAIN1700
                                           MAIN1710
C.....( 4 ALPHAS ).....                 MAIN1720
50 ETA(1) = CX0                          MAIN1730
   ETA(4) = CX1                          MAIN1740
   XL(1,1) = EL(5)                       MAIN1750
   XL(2,1) = EL(7)                       MAIN1760
   XL(1,2) = EL(5)                       MAIN1770
   XL(2,2) = EL(11)                     MAIN1780
   XL(1,3) = - EL(13)                   MAIN1790
   XL(2,3) = - EL(15)                   MAIN1800
   CALL CMATS( XL, XET, 2, 1, $1740 )    MAIN1810
   ETA(2) = XET(1)                      MAIN1820
   ETA(3) = XET(2)                      MAIN1830
   GC TO 1100                           MAIN1840
                                           MAIN1850
C.....( 3 ALPHAS ).....                 MAIN1860
60 ETA(3) = CX1                          MAIN1870
   ETA(4) = CX0                          MAIN1880
   XL(1,1) = EL(1)                      MAIN1890
   XL(2,1) = EL(2)                      MAIN1900
   XL(1,2) = EL(5)                      MAIN1910
   XL(2,2) = EL(7)                      MAIN1920
   XL(1,3) = -EL(9)                    MAIN1930
   XL(2,3) = -EL(11)                   MAIN1940
   CALL CMATS( XL, ETA, 2, 1, $1740 )    MAIN1950
   GC TO 1100                           MAIN1960
                                           MAIN1970
C.....4 ROWS, 2 ZERO CASE.....          MAIN1980
100 EL1 = CABS( EL(1)*EL(7) - EL(5)*EL(3) ) MAIN1990
    EL2 = CABS( EL(10)*EL(16) - EL(14)*EL(12) ) MAIN2000
    IF( EL1 .LE. EL2 ) GC TO 110         MAIN2010
105 ETA(1) = CX0                         MAIN2020
    ETA(2) = CX0                         MAIN2030
    ETA(3) = - EL(14) / EL(10)           MAIN2040
    ETA(4) = CX1                         MAIN2050
    GC TO 1100                           MAIN2060
110 ETA(1) = - EL(5) / EL(1)             MAIN2070
    ETA(2) = CX1                         MAIN2080
    ETA(3) = CX0                         MAIN2090
    ETA(4) = CX0                         MAIN2100
    GC TO 1100                           MAIN2110
                                           MAIN2120
1000 IF (NBETA .EQ. 3) GO TO 1010        MAIN2130
                                           MAIN2140
C.....ZERC - PIEZOELECTRIC CASE.....    MAIN2150
                                           MAIN2160

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LINBO3 CONWAY PHASE N4
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EL(3) = EL(5)
EL(4) = EL(6)
EL(5) = -EL(9)
EL(6) = -EL(10)
GO TO 1095
MAIN2170
MAIN2180
MAIN2190
MAIN2200
MAIN2210
MAIN2220

1010 EL(4) = EL(5)
EL(5) = EL(6)
EL(6) = EL(7)
EL(7) = EL(9)
EL(8) = EL(10)
EL(9) = EL(11)
EL(10) = -EL(13)
EL(11) = -EL(14)
EL(12) = -EL(15)
MAIN2230
MAIN2240
MAIN2250
MAIN2260
MAIN2270
MAIN2280
MAIN2290
MAIN2300
MAIN2310
MAIN2320

1095 KGO = 12
CALL CMATS (EL, ETA, NBETA, 1, $1740)
ETA(4) = (0.,0.)
ETA(NBETA+1) = (1.,0.)
MAIN2330
MAIN2340
MAIN2350
MAIN2360
MAIN2370
MAIN2380

1100 CONTINUE
NBETA1 = NBETA + 1
WRITE (6, 1120) (1, ETA(I), I = 1, NBETA1)
WRITE (2, 1120) (1, ETA(I), I = 1, NBETA1)
MAIN2390
MAIN2390
MAIN2390

1120 FORMAT (1H1/1H , 17X, 36H*** F I N A L A N S W E R S ***/
. 1H0. 30X,13HPARTIAL FIELD/1H ,27X,19HRELATIVE AMPLITUDES/
. /1H , 17X, 12, 3H (, E14.7, 1H,,
. E14.7, 1H)))
PUNCH 1502, (ETA(I),I=1,4)
MAIN2400
MAIN2410
MAIN2420
MAIN2430
MAIN2440
MAIN2450

.....CALCULATE FINAL RESULTS FOR GOLD LITHIUM AND OUTPUT RESULTS.....
1240 M1 = 1
K = 0
DO 1310 I = 1, 9
M2 = M1 + 8
DO 1300 J = M1, M2
K = K + 1
MAIN2460
MAIN2470
MAIN2480
MAIN2490
MAIN2500
MAIN2510
MAIN2520
MAIN2530
MAIN2540
MAIN2550
MAIN2560
MAIN2570
MAIN2580
MAIN2590
MAIN2600
MAIN2610
MAIN2620
MAIN2630
MAIN2640
MAIN2650
MAIN2660
MAIN2670
MAIN2680
MAIN2690
MAIN2700

1300 EL(K) = EL(J)
1310 M1 = M1 + 10
DO 1340 I = 51, 99
K = K + 1
1340 EL(K) = -EL(I)
KGC = 90
CALL CMATS (EL, ETA, 9, 1, $1740)
ETA(10) = (1.,0.)
DO 1490 I = 1, 3
UA(I) = (0.,0.)
UB(I) = (0.,0.)
DO 1480 J = 1, 6
IF (1 .GT. 1) GO TO 1460
EA(I) = CEXP(-ALFAA(I)*MXA/VS)
IF (J .GT. 4) GO TO 1460
EB(I) = CEXP(-ALFAB(I)*MXB/VS)
1460 UA(I) = UA(I) + ETA(J)*BETAA(I,J)*EA(I)
IF (J .GT. 4) GO TO 1480
MAIN2675
MAIN2680
MAIN2690
MAIN2700

```

W = IA(J)

EB(J) = CEXP(-ALFAB(I)*WXB/VS) MAIN2675
MAIN2680

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UB(I) = UB(I) + ETA(J+6)*BETAA(I,J)*EB(J)          MAIN2710
1480 CCNTINUE                                         MAIN2720
1490 CCNTINUE                                         MAIN2730
PHIB = (0.,0.)                                       MAIN2740
CC 1520 J = 1, 4                                     MAIN2750
1520 PHIB = PHIB + ETA(J+6)*EB(J)                   MAIN2760
WRITE (6, 1550) ((I, ETA(I), I = 1, 10), PHIB      MAIN2770
                  (I, UA(I), UB(I), I = 1, 3), PHIB  MAIN2780
                  /10(IH, 17X, 12, 3H (, E14.7, 1H,, 249
1550 FORMAT (1H0/1H, 17X, 36H*** F I N A L   A N S W E R S ***/
          1H0, 30X, 13HPARTIAL FIELD/1H, 27X, 19HRELATIVE AMPLITUDES/
          /10(1H, 17X, 12, 3H (, E14.7, 1H,,
          E14.7, 1H)/1H, 21X, 2HUA, 32X, 2HUB//
          3(1H, 16, 3H (, E14.7, 1H,, E14.7, 4H) (, E14.7, 1H,,
          E14.7, 1H)/1H, 24X, 9HPHI B = (, E14.7, 1H,,
          E14.7, 1H))                                MAIN2850
1708 WRITE (6, 1705)                                MAIN2860
1705 FCNRMAT (1H1)                                   MAIN2870
      IF (CETOUT) GO TO 17
      IF (NEGAT) NUB = -NUB
      IF (NEGAT) NUMAX = -NUMAX
      IF (NEGAT) NUB = -NUB
      IF (VSO .GE. VSMAX) GO TO 1706
      VSO = VSO + DVS
      GC TO 530
1707 WRITE (6, 1705)                                MAIN2890
1706 IF (ONU .EQ. 0.) GO TO 1710                      MAIN2900
      IF (NUT) GO TO 38                               MAIN2905
      NUB = NUB + ONU                                 MAIN2910
      IF ( NUB .GT. NUMAX ) GO TO 1709
      GC TO 39
38 NUB = NUB + ONU
      IF (NUB.GT.NUMAX) GO TO 1709
39 CCNTINUE
      CALL OVERCK
      IF (NEGAT) NUB = -NUB
      IF (NEGAT) NUMAX = -NUMAX
      IF (NEGAT) NUB = -NUB
      IF ( MAX .NE. 0 ) GO TO 530
      VS = VSAVE
      GO TO 535
1709 NUB = SNU
1710 CCNTINUE
C.....INCLUDE PLOT ROUTINES HERE.....
      IF (PLOTIT(1).EQ.2) GC TO 34
      IF (PLOTIT(1).EQ.0) GO TO 34
      CALL SCALE(VEL,10.,KOUNT,1,10.,YMIN,DY)
      IF (.NOT. NUT)
1CALL AXIS(0.,0.,24HDIRECTION OF PROPAGATION,24,10.0,0.0,0.,18.0,
1 10.0)
      IF (NUT) CALL AXIS(0.,0.,25HDIRECTION OF PLATE NORMAL,25,10.,
1 0.0,18.0,10.0)
      CALL AXIS(C,0.0,0.21MSURFACE WAVE VELOCITY,21,10.,90.0,YMIN,DY,
1 10.0)
      CALL AXIS(0.,10.,1H,1,10.0,0.,18.,10.)

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LIMBOS	CONWAY	PHASE	N4	12/31/69	000109	PAGE 7
LIMBOS	EFN	SOURCE STATEMENT	IFN(S)			
		CALL AXIS(10.0.0.1M ,1.10.90.0.YMIN,DY,10.)		325		
		CALL LINE(DEG.VELOC,181.1.0.0.0.18.0.YMIN,DY)		327		
		IF(KCUNT.EQ.181) GO TO 34				
		CALL LINE(DEG.VELOC,181.1.0.0.0.18.0.YMIN,DY)		332		
36		CALL PLOT(17.0.0.-3)		334		
		KCUNT=0				
34		CONTINUE				
		IF(PLOTIT(4).NE.3) GO TO 87				
		CC 86 JK=1.181				
86		DELTA(JK)=(VEL(JK+181)-VEL(JK))/VEL(JK+181)				
		CALL AXIS(0.0.0.24HDIRECTION OF PROPAGATION,24.10.0.0.0.18.10.)		350		
		CALL SCALE(DELTA,10.181.1.10.0.MIN,DY)		352		
		CALL AXIS(0.0.0.11HDELTA V / V,11.10.90.0.0.MIN,DY,10.)		354		
		CALL AXIS(0.0.0.1M ,1.10.0.0.0.18.10.)		356		
		CALL AXIS(10.0.0.1M ,1.10.90.0.0.18.10.)		358		
		CALL LINE(DEG.DELTA,181.1.0.0.0.18.0.MIN,DY)		360		
		CALL PLOT(17.0.0.-3)		362		
87		CONTINUE				
		IF(PLOTIT(2).EQ.0) GO TO 37				
		CALL AXIS(0.0.0.24HDIRECTION OF PROPAGATION,24.10.0.0.0.0.18.0.				
		1 10.0)		367		
		CALL AXIS(0.0.0.33HTIME AVERAGE POWER FLOW DIRECTION,33.10.90.0.				
		1 -25.5.10.)		369		
		CALL AXIS(0.0.0.1M ,1.10.0.0.0.18.10.)		371		
		CALL AXIS(10.0.0.1M ,1.10.90.0.-25.5.10.)		373		
		CALL LINE(XX.YY,2.1.0.0.0.1.0.0.1.)		375		
		CALL LINE(DEG.PANGLE,181.1.0.0.0.18.0.-25.5.)		377		
		CALL PLOT(17.0.0.-3)		379		
		KNT=0				
37		CONTINUE				
		IF(PLOTIT(3).EQ.0) GO TO 75				
		CALL SCALE(RT11,10.0.KNT1.1.10.0.RTMIN,DR)		385		
		CALL AXIS(0.0.0.24HDIRECTION OF PROPAGATION,24.10.0.0.0.0.18.0.				
		1 10.0)		387		
		CALL AXIS(0.0.0.8HMAGN T11,8.10.90.0.RTPIN,DR,10.)		389		
		CALL LINE(DEG.RT11,181.1.0.0.0.18.0.RTMIN,DR)		391		
		CALL PLOT(17.0.0.-3)		393		
		KNT1=0				
75		CONTINUE				
		IF(PLOTIT(4).EQ.0) GO TO 76				
		CALL SCALE(RU1,10.0.KNT2.1.10.0.RTMIN,DR)		399		
		CALL AXIS(0.0.0.24HDIRECTION OF PROPAGATION,24.10.0.0.0.0.18.0.				
		1 10.0)		401		
		CALL AXIS(0.0.0.6HMAG U3,6.10.90.0.RTMIN,DR,10.)		403		
		CALL LINE(DEG.RU1,181.1.0.0.0.18.0.RTMIN,DR)		405		
		CALL PLOT(17.0.0.-3)		407		
		CALL SCALE(RI1,10.0.KNT2.1.10.0.RTMIN,DR)		409		
		CALL AXIS(0.0.0.24HDIRECTION OF PROPAGATION,24.10.0.0.0.0.18.0.				
		1 10.0)		411		
		CALL AXIS(0.0.0.6HPHASES,6.10.90.0.RTMIN,DR,10.)		413		
		CALL LINE(DEG.RI1,181.1.0.0.0.18.0.RTMIN,DR)		415		
		CALL PLOT(17.0.0.-3)		417		
		KNT2=0				
76		CONTINUE				
		IF(PLOTIT(5).EQ.0) GO TO 74				
		CALL SCALE(RE1,10.0.KNT3.1.10.0.RTMIN,DR)		423		

LINBO3 CONWAY PHASE M4		12/31/65	000109	PAGE 8
LINBO3 - EFN SOURCE STATEMENT - IFN(S) -				
CALL AXIS(0.0,24,DIRECTION OF PROPAGATION,24,10.0,0.0,0.0,10.0,				
1 10.0)				425
CALL AXIS(0.0,7,MAGN D1,8,10.0,90.0,RTMIN,DR,1C.)				427
CALL LINE(DEG,REL,181,1.0,0.0,18.0,RTMIN,DR)				429
CALL PLOT(17.0,-3)				431
KNT3=0				
74 CCNTINUE				
END FILE 2				433
17 CCNTINUE				
IF(CEOUT) END FILE 2				435
IF(CEOUT) KOUNT=0				
IF(CEOUT) KONT=0				
IF(CEOUT) KNT1=0				
IF(CEOUT) KNT2=0				
IF(CEOUT) KNT3=0				
GETCUT =.FALSE.				
IF (REPEAT) GO TO 510				
GO TO 520				
C.....ERRCR - L MATRIX SINGULAR.....				
1740 WRITE (6,1750) (EL(I), I = 1, KGO)				
1750 FCPRAT (42H1*** L MATRIX SINGULAR (OUTPUT BY COLUMNS) //				451
(1H , 6E18.7))				
GO TO 1700				
C.....CALCULATE ADDITIONAL PARAMETERS FOR LITHIUM NIOBATE.....				
1790 WRITE(6,1705)				
1800 DC 1810 I = 1, 4				459
J = 1A(I)				
1810 EX(I) = CEXP(-ALFAB(J)*WX/VS)				
CG 1890 I = 1, 4				467
L = (0.0,0.0)				
CC 1850 K = 1, 4				
L = U + ETAK(K)*BETAB(I,K)*EX(K)				
MAGU(I) = CABS(U)				
PHASEU(I) = 0.				481
IF (MAGU(I) .NE. 0.) PHASEU(I) = ATAN2(AIMAG(U),REAL(U))*57.295779				486
1890 CCNTINUE				
E1 = U				
C.....CCMPUTE TIME AVERAGE POWER FLOW.....				
P1M = PIFUN(ETA, C11, C15, C16, C14, C15, C13, E11, E31,				
C16, C56, C66, C46, C56, C36, E16, E36,				
C15, C55, C56, C45, C55, C35, E15, E35)				492
SPECL= SORT(SORT(REAL(P1M)**2+AIMAG(P1M)**2))				493
F2M = PIFUN (ETA, C16, C56, C66, C46, C56, C36, E16, E36,				
C12, C25, C26, C24, C25, C23, E12, E32,				
C14, C45, C46, C44, C45, C34, E14, E34)				495
IF (PLOTIT(2).EQ.0) GC TO 57				
KNT1=KONT+1				
FANGLE(KONT)=ATAN(REAL(P2M)/REAL(P1M))*180./3.1415				500
57 CCNTINUE				
C.....CALCULATE T S.....				
TH31 = TFUN (ETA, WX, C15, C55, C56, C45, C55, C35, E15, E35)				503

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      LINB03 CONWAY PHASE N4
      LINB03 - EFM SOURCE STATEMENT - IFM(S) -
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      TM02 = TFUN(ETA, MX, C14, C45, C46, C44, C45, C34, E14, E34) MAIN3340 504
      TM03 = TFUN(ETA, MX, C13, C35, C36, C34, C35, C33, E13, E33) MAIN3350 505
      TM11 = TFUN(ETA, MX, C11, C15, C16, C14, C15, C13, E11, E31) MAIN3360 506
      TM12 = TFUN(ETA, MX, C16, C56, C46, C56, C36, E16, E36) MAIN3370 507
      TM22 = TFUN(ETA, MX, C12, C25, C26, C24, C25, C23, E12, E32) MAIN3380 508
      IF(PLOTIT(3).EQ.0) GO TO 71
      KNT1=KNT1+1
      RT11(KNT1)=SQRT(REAL(TM11)**2+AIMAG(TM11)**2)/SPECL 513
      RT11(KNT1)=AIMAG(TM11)/SPECL
71 CONTINUE
      S11 = (0.,0.) MAIN3390
      S22 = (0.,0.) MAIN3400
      S33 = (0.,0.) MAIN3410
      S12 = (0.,0.) MAIN3420
      S13 = (0.,0.) MAIN3430
      S23 = (0.,0.) MAIN3440
      DO 2190 I = 1, 4
      J = IA(I) MAIN3450
      S11 = S11 - ETA(I)*BETAB(1,I)*EX(I) MAIN3460
      S33 = S33 - ALFAB(J)*ETA(I)*BETAB(3,I)*EX(I) MAIN3470
      S12 = S12 - ETA(I)*BETAB(2,I)*EX(I) MAIN3480
      S13 = S13 - ETA(I)*BETAB(1,I)*ALFAB(J) + BETAB(3,I)*JIMAG*EX(I) MAIN3490
2190 S23 = S23 - ETA(I)*BETAB(2,I)*ALFAB(J)*EX(I) MAIN3500
      S11 = S11+JIMAG/V5 MAIN3510
      S33 = S33/V5 MAIN3520
      S12 = S12+0.5*JIMAG/V5 MAIN3530
      S13 = 0.5*S13/V5 MAIN3540
      S23 = 0.5*S23/V5 MAIN3550
      C1 = TFUN(ETA, MX, E11, E15, E16, E14, E15, E13,-T11,-T13) MAIN3560
      D2 = TFUN(ETA, MX, E21, E25, E26, E24, E25, E23,-T21,-T23) MAIN3570 544
      C3 = TFUN(ETA, MX, E31, E35, E36, E34, E35, E33,-T13,-T33) MAIN3580 545
      E1 = JIMAG*E1/V5 MAIN3590 546
      E3 = (0.,0.) MAIN3600
      DO 2310 I = 1, 4
      J = IA(I) MAIN3610
      E3 = E3 + ALFAB(J)*ETA(I)*EX(I)*BETAB(4,I) MAIN3620
      E3 = E3/V5 MAIN3630
      IF(PLOTIT(5).EQ.0) GO TO 73
      KNT3=KNT3+1
      RE1(KNT3)=SQRT(REAL(D1)**2+AIMAG(D1)**2)/SPECL 562
73 CONTINUE
C.....OUTPUT RESULTS.....
      WRITE (6, 2340) MX MAIN3670
      WRITE(2,2340) MX MAIN3680 564
2340 FORMAT (10F0.....MX =, 1PE14,7) MAIN3690
      WRITE (6, 2440) TT31, TM31, TT32, TM32, TT33, TM33, MAIN3700
      TT11, TM11, TT12, TM12, TT22, TM22, MAIN3710
      SS11, S11, SS22, S22, SS33, S33, MAIN3720
      SS12, S12, SS13, S13, SS23, S23, MAIN3730
      PP1M, P1M, PP2M, P2M, MAIN3740
      DD1, D1, DD2, D2, DD3, D3, MAIN3750
      UU1, MAGU(1), PHASEU(1), MAIN3760
      UU2, MAGU(2), PHASEU(2), MAIN3770
      UU3, MAGU(3), PHASEU(3), MAIN3780
      MAGU(4), PHASEU(4), MAIN3790

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L[ND03 CONWAY PHASE N4 LIND03 - EFN SOURCE STATEMENT - IFN(S) -		12/31/69 000109 PAGE 10
WRITE (2, 2440)	EE1, E1, EE3, E3 TT31, TW31, TT32, TW32, TT33, TW33, TT11, TW11, TT12, TW12, TT22, TW22, SS11, S11, SS22, S22, SS33, S33, SS12, S12, SS13, S13, SS23, S23, PP1H, P1H, PP2H, P2H, DO1, D1, DO2, D2, DO3, D3, UU1, MAGU(1), PHASEU(1), UU2, MAGU(2), PHASEU(2), UU3, MAGU(3), PHASEU(3), MAGU(4), PHASEU(4), EE1, E1, EE3, E3	MAIN3800 566 MAIN3700 MAIN3710 MAIN3720 MAIN3730 MAIN3740 MAIN3750 MAIN3760 MAIN3770 MAIN3780 MAIN3790 MAIN3800 567
2440 FORMAT (1H0, 20X, 17HSTRESS COMPONENTS//6(1H, 10X, A3, 4H = (,	1PE14.7, 1H,, 1PE14.7, 1H)/1H, 20X, 17HSTRAIN COMPONENTS//6(1H, 10X, A3, 4H = (,	MAIN3810 MAIN3820 MAIN3830 MAIN3840 MAIN3850 MAIN3860 MAIN3870 MAIN3880 MAIN3890 MAIN3900 MAIN3910 MAIN3920 MAIN3930
1PE14.7, 1H,, 1PE14.7, 1H)/1H, 25X, 23HTIME AVERAGE POWER FLOW//2(1H, 19X, A3, 4H = (,	1PE14.7, 1H,, 1PE14.7, 1H)/1H, 26X, 21HELECTRIC DISPLACEMENT//3(1H, 19X, A2, 4H = (,	
1PE14.7, 1H,, 1PE14.7, 1H)/1H, 25X, 23HMECHANICAL DISPLACEMENT//1H, 24X, 9HMAGNITUDE, 7X, 5MPHASE/ 3(1H, 19X, A2, 3H = , 1PE14.7, OPF10.3)/1	1H, 6X, 30HELECTRIC POTENTIAL MAGNITUDE = , 1PE15.7, 9H PHASE = , OPF9.3/1H0, 20X, 14HELECTRIC FIELD// 2(1H, 19X, A2, 4H = (, 1PE14.7, 1H,, 1PE14.7, 1H)/1)	
1502 FCNMT(2E15.8)	PUNCH 1502, P1H, P2H	568
	PUNCH 1502, TW31, TW32, TW33, TW11, TW12, TW22	569
	PUNCH 1502, S11, S22, S33, S12, S13, S23	570
	PUNCH 1506, (MAGU(1), PHASEU(1), 1=1, 4)	571
1506 FCNMT(1E15.8, F10.3)	PUNCH 1502, E1, E3	579
	PUNCH 1502, D1, D2, D3	580
	CC 10 I=1.4	
	RRRR=COS(PHASEU(1)*3.1415/180.)*MAGU(1)	585
	RII=SIN(PHASEU(1)*3.1415/180.)*MAGU(1)	588
	DISP(1)=CMPLX(RRRR,RII)	
10 CCNTINUE	IF(PLOTIT(4).EQ.0) GO TO 72	
	KAT2=KNT2+1	
	RUI(KNT2)=SORT(REAL(DISP(3))*2+AIMAG(DISP(3))*2)/SPECL	590
	RIU1(KNT2)=PHASEU(1)-PHASEU(3)	
72 CCNTINUE	WRITE(6,2441) (DISP(1), I=1.4)	601
	WRITE(2,2441) (DISP(1), I=1.4)	608
2441 FORMAT(25X, 23HMECHANICAL DISPLACEMENT, 3(1/23X, 2E10.8), /, 25X, 1 20HELEC. POT. MAGNITUDE, /, 23X, 2E10.8)	IF (DWH .EQ. 0.) GO TO 1708	
	DW = DW + DWH	MAIN3940
	IF (DW .LE. DWMAX) GO TO 1790	MAIN3950
	DW = SWX	MAIN3960
	GO TO 1708	MAIN3970
2 CCNTINUE		MAIN3980
REAC(5,3) BLIMIT, ELIMIT, INCR		622
3 FCNMT(1315)		

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      LINBO3 CONWAY PHASE M4
      LINBO3 - EFN SOURCE STATEMENT - IFN(S) -
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CC 4 I=BLIMIT,ELIMIT,INCR
K=(I-BLIMIT)/INCR+1
VSARAY(K)=I
VS=I
VSO=VS
CALL ROOT(VSO,VS,N,FVS,EPSLON,MAX)
AAAAA(2*K-1)=AIMAG(FVS)
AAAAA(2*K)=REAL(FVS)
DETRAY(K)=REAL(FVS)
DETIAY(K)=AIMAG(FVS)
CEBUG K,DETRAY(K),DETIAY(K)
4 CCNTINUE
CC 5 JJ=1,K
AAAAA(2*JJ-1)=AAAAA(2*JJ-1)*1.E10
AAAAA(2*JJ)=AAAAA(2*JJ)*1.E10
DETRAY(JJ)=DETRAY(JJ)*1.E10
DETIAY(JJ)=DETIAY(JJ)*1.E10
JJ=2*JJ-1
IF(AAAAA(JJ).GT.1.) AAAAA(JJ)=1.
IF(AAAAA(JJ).LT.-1.) AAAAA(JJ)=-1.
IF(AAAAA(2*JJ).GT.1.) AAAAA(2*JJ)=1.
IF(AAAAA(2*JJ).LT.-1.) AAAAA(2*JJ)=-1.
IF(DETRAY(JJ).GT.1.) DETRAY(JJ)=1.
IF(DETRAY(JJ).LT.-1.) DETRAY(JJ)=-1.
IF(DETIAY(JJ).GT.1.) DETIAY(JJ)=1.
IF(DETIAY(JJ).LT.-1.) DETIAY(JJ)=-1.
5 CCNTINUE
YMIN=-1.
CY=.2
NANAN=2*K
CALL SCALE(VSARAY,15,0,K,1,10.,XMIN,DX)
CALL AXIS(0.,0.,2*VS,2,15,0,0,0,XMIN,DX,10,0)
CALL AXIS(0.,0.,6*DETERM,6,10.,90,0,YMIN,DY,10.)
CALL LINE(VSARAY,DETRAY,K,1,1,4,XMIN,DX,YMIN,DY)
CALL LINE(VSARAY,DETIAY,K,1,1,4,XMIN,DX,YMIN,DY)
CALL PLOT(17,0.,-3)
ICHECK=.FALSE.
GC TO 17
CALL ENDPLT
691
693
695
697
699
701
704

ERROR MESSAGE NUMBER 1
END
MAIN3990

ERROR MESSAGE NUMBER 2

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LINBO3 CONWAY PHASE

N4

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SIBFTC ROCT.. DECK,DEBUG

CROOT SLBRCTINE RCOT (VSO, VS, N, FVS, EPS, MAX)

ROOT0030
ROOT0020
ROOT0040DIMENSION SUB(24)
DATA SUB/1.,-2.,3.,-4.,5.,-6.,7.,-8.,9.,-10.,11.,1.,1.,1.,1.,1.,
U 2.,2.,3.,5.,10.,50.,100.,100./

DATA KKKKK/0/

DATA KICK/1/

COMMON /GET/GETOUT

COMMON/PLOTS/ICHECK

LOGICAL GETCUT

LOGICAL PREV

DATA PREV/.FALSE./

LOGICAL ICHECK

COMMON /FROOT/ FVSMAG, NT, ICASE

/ALESS/ IALF

ROOT0050
ROOT0060
ROOT0070
ROOT0080
ROOT0090
ROOT0100
ROOT0110
ROOT0120
ROOT0130
ROOT0140

COMPLEX F, FVS, FX0, FX1, FX2, FX3, G2, LAMCA3, ROUND,

Y, T1, T2, EL

REAL LAMDA2

LOGICAL IALF

DATA TEN10/1.E9/

KKKKK=KKKKK+1

FACT1=1.02

FACT2=1.005

IF(KKKKK/2*2.NE.KKKKK) FACT1=.99

IF(KKKKK/2*2.NE.KKKKK) FACT2=.995

N = 0

VS = VSO

FVS = RCLND(F(VS))

IF(IALF) GO TO 600

IF(ICHECK) GO TO 530

IF(MAX .EC. 0) GO TO 530

IF(FVSMAG .LT. EPS) GO TO 530

FX0 = FVS

VS=FACT1*VSC

FVS = ROUND(F(VS))

IF(IALF) GO TO 600

IF(FVSMAG .LT. EPS) GO TO 530

FX1 = FVS

X2=FACT2*VSC

VS = X2

FVS = ROUND(F(VS))

IF(IALF) GO TO 600

IF(FVSMAG .LT. EPS) GO TO 530

FX2 = FVS

H2=(-1.)**(KKKKK-1)*.005*VSO

LAMCA2 = -0.5

ROOT0150
ROOT0160
ROOT0170
ROOT0180

8 9

ROOT0190
ROOT0200
ROOT0210ROOT0230
ROOT0240
ROOT0250
ROOT0260

22 23

ROOT0280
ROOT0290
ROOT0300
ROOT0310
ROOT0320

30 31

ROOT0340

30

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      LINDB3  CONWAY  PHASE  N4
      ROOT.. - EFN SOURCE STATEMENT - IFN(S) -
      DELTA2 = 0.5
      C.....B E G I N   I T E R A T I O N.....
      230 G2 = FX0*LANCA2*LANDA2 - FX1*DELTA2*DELTA2 + FX2*(LANDA2 + DELTA2)
      T = CSORT(G2*G2 - 4.*FX2*DELTA2*LANDA2*(FX0*LANDA2
      - FX1*DELTA2 + FX2))
      T1 = G2 + T
      T2 = G2 - T
      T = T1
      IF (CABS(T2) .GT. CABS(T1)) T = T2
      IF (CABS(T) .EQ. 0.) GO TO 530
      300 LANCA3 = -2.*FX2*DELTA2/T
      VS = X2 + REAL(LANCA3*H2)
      FVS = ROUND(FVS)
      IF( IALF ) GO TO 400
      IF( FVSMAG .LT. EPS ) GO TO 530
      N = N + 1
      IF(N.GT.MAX) GO TO 600
      H3 = VS - X2
      LANCA2 = H3/H2
      DELTA2 = 1. + LANCA2
      FX0 = FX1
      FX1 = T1
      FX2 = FVS
      X2 = VS
      H2 = H3
      IF (H2 .NE. 0.) GO TO 230
      C.....E N D   O F   I T E R A T I O N.....
      530 CONTINUE
      IF(PREV) PREV=.FALSE.
      KKKK=0
      KICK=1
      RETRN
      600 WRITE(6,1000) VS
      1000 FORMAT( /// 51H .....LESS THAN 4 ALPHAS WITH A POSITIVE REAL PART
      * / 32H .....CASE TERMINATED ( VS = , 1PE14.7, 2H ) // )
      IF(CHECK) GO TO 530
      IF(KKKK/2*2.NE.KKKK) GO TO 998
      VSO=VSO-SUB(KICK)
      KICK=KICK+1
      IF(KICK.EC.25) GO TO 999
      998 RETURN 1
      999 CONTINUE
      IF(PREV) GETOLT=.TRUE.
      PREV=.TRUE.
      KICK=1
      RETRN
      GC TC 530
      ERROR MESSAGE NUMBER 1
      EAD

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      ROOT0350
      ROOT0360
      ROOT0370
      ROOT0380
      ROOT0390
      ROOT0400
      ROOT0410
      ROOT0420
      ROOT0430
      ROOT0440
      ROOT0450
      ROOT0460
      ROOT0470
      ROOT0480
      ROOT0490
      ROOT0500
      ROOT0510
      ROOT0520
      ROOT0540
      ROOT0550
      ROOT0560
      ROOT0570
      ROOT0580
      ROOT0590
      ROOT0600
      ROOT0610
      ROOT0620
      ROOT0630
      ROOT0640
      ROOT0660
      ROOT0670
      ROOT0680
      ROOT0690
      ROOT0700

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LINBO3 CONWAY PHASE

N4

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000109

PAGE 14

810FTC SETCT. DECK

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CSEYCTE *** SETCTE COMPUTES THE C, T, E COEFFICIENTS FROM CTE00020
C* THE INPUT PARAMETERS G, P, EPS, MU, LAMBDA, NU CTE00030
SUBROUTINE SETCTE (NU, MU, LAMBDA, COEFF) CTE00040
CTE00050

CCMPCN /ROTAT/ROTATE
LCGICAL ROTATE
DIMENSION CPR(4,6),EPR(3,6),EPSPR(3,3)
LCGICAL COEFF CTE00060
LCGICAL AC12, AC23, AC24, AC14, AC34 CTE00070
REAL MU, NU, LAMBDA CTE00080
DOUBLE PRECISION GAMMA(3,3), D, Q, DR, RN, RL, CM, SM, CN, SN, CTE00090
CL, SL, FF, R, TIJ CTE00100
CTE00110
DIMENSION LABE(6), LABC(5), LABT(6) CTE00120
CTE00130

CCMPCN /ZZTZ/ C(20),E(17),T(5)
CCMPCN /FRT/ GAMMA, D(3,3,3,3), Q(3,3,3) CTE00150
CCMPCN /GPEPS/ G(21), P(18), EPSLON(3,3) CTE00160
. /CSET/ CLIM, ELIM, TLIM CTE00170
. /CSET1/ AC12, AC23, AC24, AC14, AC34 CTE00180
. /BETAN/ NBETA CTE00190
CTE00200
DATA DR / 57.29577951308232 / CTE00210
DATA LABE(11)/36HTRANSFORMED PIEZOELECTRIC CONSTANTS /, CTE00220
. LABC(11)/36HTRANSFORMED ELASTIC CONSTANTS /, CTE00230
. LABT(11)/36HTRANSFORMED DIELECTRIC CONSTANTS / CTE00240
CTE00250
CTE00260
AC12 = .FALSE. CTE00270
AC23 = .FALSE. CTE00280
AC24 = .FALSE. CTE00290
AC14 = .FALSE. CTE00300
AC34 = .FALSE. CTE00310
RN = MU/DR CTE00320
RN = MU/DR CTE00330
RL = LAMBDA/DR CTE00340
CALL CSFUN (MU, RN, SM, CM) CTE00350
CALL CSFUN (MU, RN, SN, CN) CTE00360
CALL CSFUN (LAMBDA, RL, SL, CL) CTE00370
GAMMA(1,1) = CL*CN - SL*CM*SN CTE00380
GAMMA(1,2) = SL*CN + CL*CM*SN CTE00390
GAMMA(1,3) = SM*SN CTE00400
GAMMA(2,1) = -CL*SN - SL*CM*CN CTE00410
GAMMA(2,2) = -SL*SN + CL*CM*CN CTE00420
GAMMA(2,3) = SM*CN CTE00430
GAMMA(3,1) = SL*SM CTE00440
GAMMA(3,2) = -CL*SM CTE00450
GAMMA(3,3) = CM CTE00460
CTE00470
CTE00480
C.....SET UP D AND Q MATRICES.....

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2
4
6

LINB03 CONWAY PHASE N4
 SETCT. - EFN SOURCE STATEMENT - (FNIS) -

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C(1.1.1.1) = G(1)
 C(2.2.2.2) = G(2)
 C(3.3.3.3) = G(3)
 C(1.1.2.2) = G(4)
 C(2.2.1.1) = G(4)
 C(1.1.3.3) = G(5)
 C(3.3.1.1) = G(5)
 C(1.1.2.3) = G(6)
 C(1.1.3.2) = G(6)
 C(2.3.1.1) = G(6)
 C(3.2.1.1) = G(6)
 C(1.1.1.3) = G(7)
 C(1.1.3.1) = G(7)
 C(1.3.1.1) = G(7)
 C(3.1.1.1) = G(7)
 C(1.1.1.2) = G(8)
 C(1.1.2.1) = G(8)
 C(1.2.1.1) = G(8)
 C(2.1.1.1) = G(8)
 C(2.2.3.3) = G(9)
 C(3.3.2.2) = G(9)
 C(2.2.2.3) = G(10)
 C(2.2.3.2) = G(10)
 C(2.3.2.2) = G(10)
 C(3.2.2.2) = G(10)
 C(2.2.1.3) = G(11)
 C(2.2.3.1) = G(11)
 C(1.3.2.2) = G(11)
 C(3.1.2.2) = G(11)
 C(2.2.1.2) = G(12)
 C(2.2.2.1) = G(12)
 C(1.2.2.2) = G(12)
 C(2.1.2.2) = G(12)
 C(3.3.2.3) = G(13)
 C(3.3.3.2) = G(13)
 C(2.3.3.3) = G(13)
 C(3.2.3.3) = G(13)
 C(3.3.1.3) = G(14)
 C(3.3.3.1) = G(14)
 C(1.3.3.3) = G(14)
 C(3.1.3.3) = G(14)
 C(3.3.1.2) = G(15)
 C(3.3.2.1) = G(15)
 C(1.2.3.3) = G(15)
 C(2.1.3.3) = G(15)
 C(2.3.2.3) = G(16)
 C(2.3.3.2) = G(16)
 C(3.2.2.3) = G(16)
 C(3.2.3.2) = G(16)
 C(2.3.1.3) = G(17)
 C(2.3.3.1) = G(17)
 C(3.2.1.3) = G(17)
 C(3.2.3.1) = G(17)
 C(1.3.2.3) = G(17)
 C(3.1.2.3) = G(17)
 C(1.3.3.2) = G(17)

CTE00490
 CTE00500
 CTE00510
 CTE00520
 CTE00530
 CTE00540
 CTE00550
 CTE00560
 CTE00570
 CTE00580
 CTE00590
 CTE00600
 CTE00610
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 CTE00690
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 CTE00940
 CTE00950
 CTE00960
 CTE00970
 CTE00980
 CTE00990
 CTE01000
 CTE01010
 CTE01020
 CTE01030
 CTE01040

SETCT.	LINB03 - EFN	CONWAY SOURCE STATEMENT	PHASE - IFN(S)	N4 -	12/31/69	000109	PAGE 16
	C(3,1,3,2)	= G(17)		CTE01050			
	C(2,3,1,2)	= G(18)		CTE01060			
	C(2,3,2,1)	= G(18)		CTE01070			
	D(3,2,1,2)	= G(18)		CTE01080			
	D(3,2,2,1)	= G(18)		CTE01090			
	C(1,2,2,3)	= G(18)		CTE01100			
	C(2,1,2,3)	= G(18)		CTE01110			
	C(1,2,3,2)	= G(18)		CTE01120			
	C(2,1,3,2)	= G(18)		CTE01130			
	D(1,3,1,3)	= G(19)		CTE01140			
	C(1,3,3,1)	= G(19)		CTE01150			
	D(3,1,1,3)	= G(19)		CTE01160			
	C(3,1,3,1)	= G(19)		CTE01170			
	D(1,3,1,2)	= G(20)		CTE01180			
	C(1,3,2,1)	= G(20)		CTE01190			
	C(3,1,1,2)	= G(20)		CTE01200			
	C(3,1,2,1)	= G(20)		CTE01210			
	C(1,2,1,3)	= G(20)		CTE01220			
	C(2,1,1,3)	= G(20)		CTE01230			
	C(1,2,3,1)	= G(20)		CTE01240			
	D(2,1,3,1)	= G(20)		CTE01250			
	C(1,2,1,2)	= G(21)		CTE01260			
	C(1,2,2,1)	= G(21)		CTE01270			
	C(2,1,1,2)	= G(21)		CTE01280			
	C(2,1,2,1)	= G(21)		CTE01290			
	C(1,1,1)	= P(1)		CTE01300			
	C(1,2,2)	= P(2)		CTE01310			
	C(1,3,3)	= P(2)		CTE01320			
	C(1,2,3)	= P(4)		CTE01330			
	C(1,3,2)	= P(4)		CTE01340			
	C(1,1,3)	= P(5)		CTE01350			
	C(1,3,1)	= P(5)		CTE01360			
	C(1,1,2)	= P(6)		CTE01370			
	C(1,2,1)	= P(6)		CTE01380			
	C(2,1,1)	= P(7)		CTE01390			
	C(2,2,2)	= P(8)		CTE01400			
	C(2,3,3)	= P(9)		CTE01410			
	C(2,2,3)	= P(10)		CTE01420			
	C(2,3,2)	= P(10)		CTE01430			
	C(2,1,3)	= P(11)		CTE01440			
	C(2,3,1)	= P(11)		CTE01450			
	C(2,1,2)	= P(12)		CTE01460			
	C(2,2,1)	= P(12)		CTE01470			
	C(3,1,1)	= P(13)		CTE01480			
	C(3,2,2)	= P(14)		CTE01490			
	C(3,3,3)	= P(15)		CTE01500			
	C(3,2,3)	= P(16)		CTE01510			
	C(3,3,2)	= P(16)		CTE01520			
	C(3,1,3)	= P(17)		CTE01530			
	C(3,3,1)	= P(17)		CTE01540			
	C(3,1,2)	= P(18)		CTE01550			
	C(3,2,1)	= P(18)		CTE01560			
	C(1)	= FF(1,1,1,1)		CTE01570	8		
	C(2)	= FF(1,1,3,3)		CTE01580	9		
	C(3)	= FF(1,1,2,3)		CTE01590	10		
	C(4)	= FF(1,1,1,3)		CTE01600	11		

LINE03	CONWAY	PHASE	N4	12/31/69	000109	PAGE 17
SETCT.	- EFN	SOURCE STATEMENT	- IFN(S)			
C(5) =FF(3,3,3)				CTE01610	12	
C(6) =FF(3,3,2)				CTE01620	13	
C(7) =FF(3,3,1)				CTE01630	14	
C(8) =FF(3,3,1,2)				CTE01640	15	
C(9) =FF(2,3,2,3)				CTE01650	16	
C(10) =FF(2,3,1,3)				CTE01660	17	
C(11) =FF(2,3,1,2)				CTE01670	18	
C(12) =FF(1,3,1,3)				CTE01680	19	
C(13) =FF(1,3,1,2)				CTE01690	20	
C(14) =FF(1,2,1,2)				CTE01700	21	
C(15) =FF(1,1,1,2)				CTE01710	22	
C(16) =FF(1,1,2,2)				CTE01720	23	
C(17) =FF(2,2,1,3)				CTE01730	24	
C(18) =FF(2,2,1,2)				CTE01740	25	
C(19) =FF(2,2,2,3)				CTE01750	26	
C(20) =FF(2,2,3,3)				CTE01760	27	
IF(,NOT. RCTATE) GO TO 1601						
CPR(1,1)=FF(1,1,1)					31	
CPR(1,2)=FF(1,1,2)					32	
CPR(1,3)=FF(1,1,3)					33	
CPR(1,4)=FF(1,1,2,3)					34	
CPR(1,5)=FF(1,1,1,3)					35	
CPR(1,6)=FF(1,1,1,2)					36	
CPR(2,2)=FF(2,2,2,2)					37	
CPR(2,3)=FF(2,2,3,3)					38	
CPR(2,4)=FF(2,2,2,3)					39	
CPR(2,5)=FF(2,2,1,3)					40	
CPR(2,6)=FF(2,2,1,2)					41	
CPR(3,3)=FF(3,3,3,3)					42	
CPR(3,4)=FF(3,3,2,3)					43	
CPR(3,5)=FF(3,3,1,3)					44	
CPR(3,6)=FF(3,3,1,2)					45	
CPR(4,4)=FF(2,3,2,3)					46	
CPR(4,5)=FF(2,3,1,3)					47	
CPR(4,6)=FF(2,3,1,2)					48	
CPR(5,5)=FF(1,3,1,3)					49	
CPR(5,6)=FF(1,3,1,2)					50	
CPR(6,6)=FF(1,2,1,2)					51	
1601 CONTINUE						
DO 10 I=1,20				CTE01770		
IF(ABS(C(I)) .LT. CLIM) C(I) = 0.				CTE01780		
10 CONTINUE				CTE01790		
E(1) = R(1,1,1)				CTE01800	63	
E(2) = R(1,3,3)				CTE01810	64	
E(3) = R(1,2,3)				CTE01820	65	
E(4) = R(1,1,3)				CTE01830	66	
E(5) = R(1,1,2)				CTE01840	67	
E(6) = R(3,1,1)				CTE01850	68	
E(7) = R(3,3,3)				CTE01860	69	
E(8) = R(3,2,3)				CTE01870	70	
E(9) = R(3,1,3)				CTE01880	71	
E(10) = R(3,1,2)				CTE01890	72	
E(11) = R(1,2,2)				CTE01900	73	
E(12) = R(3,2,2)				CTE01910	74	
E(13) = R(2,1,1)				CTE01920	75	
E(14) = R(2,3,3)				CTE01930	76	

SECT.	LINBO3	CONWAY	PHASE	N4	12/31/69	000109	PAGE 18
	-	EFN	SOURCE STATEMENT	- IFN(S)			
	E(15) = R(2,2,3)				CTE01940	77	
	E(16) = R(2,1,3)				CTE01950	78	
	E(17) = R(2,1,2)				CTE01960	79	
	IF(.NOT. ROTATE) GO TO 1602						
	EPR(1,1)=R(1,1,1)					83	
	EPR(1,2)=R(1,2,2)					84	
	EPR(1,3)=R(1,3,3)					85	
	EPR(1,4)=R(1,2,3)					86	
	EPR(1,5)=R(1,1,3)					87	
	EPR(1,6)=R(1,1,2)					88	
	EPR(2,1)=R(2,1,1)					89	
	EPR(2,2)=R(2,2,2)					90	
	EPR(2,3)=R(2,3,3)					91	
	EPR(2,4)=R(2,2,3)					92	
	EPR(2,5)=R(2,1,3)					93	
	EPR(2,6)=R(2,1,2)					94	
	EPR(3,1)=R(3,1,1)					95	
	EPR(3,2)=R(3,2,2)					96	
	EPR(3,3)=R(3,3,3)					97	
	EPR(3,4)=R(3,2,3)					98	
	EPR(3,5)=R(3,1,3)					99	
	EPR(3,6)=R(3,1,2)					100	
1602	CCNTINUE				CTE01970		
	CC 20 I=1,17				CTE01980		
	IF(ABS(E(I)) .LT. ELIM) E(I) = 0.				CTE01990		
20	CCNTINUE				CTE02000		
	EPSLCN(3,2) = EPSLON(2,3)				CTE02010		
	EPSLCN(2,1) = EPSLON(1,2)				CTE02020		
	EPSLON(3,1) = EPSLCN(1,3)				CTE02030	112	
	T(1) = TIJ(1,1)				CTE02040	113	
	T(2) = TIJ(1,3)				CTE02050	114	
	T(3) = TIJ(3,3)				CTE02060	115	
	T(4) = TIJ(2,1)				CTE02070	116	
	T(5) = TIJ(2,3)						
	IF(.NOT. ROTATE) GO TO 1603						
	EPSPR(1,1)=TIJ(1,1)					120	
	EPSPR(1,2)=TIJ(1,2)					121	
	EPSPR(1,3)=TIJ(1,3)					122	
	EPSPR(2,1)=TIJ(2,1)					123	
	EPSPR(2,2)=TIJ(2,2)					124	
	EPSPR(2,3)=TIJ(2,3)					125	
	EPSPR(3,1)=TIJ(3,1)					126	
	EPSPR(3,2)=TIJ(3,2)					127	
	EPSPR(3,3)=TIJ(3,3)					128	
1603	CCNTINUE				CTE02080		
	CC 30 I=1,5				CTE02090		
	IF(ABS(T(I)) .LT. TLIM) T(I) = 0.				CTE02100		
30	CCNTINUE				CTE02110		
C.....	SEE IF PIEZOELECTRIC COEFFICIENTS ARE ALL ZERO.....				CTE02120		
	NBETA = 3				CTE02130		
	CC 470 K = 1, 18				CTE02140		
	IF (PIK) .NE. 0.) GC TO 490				CTE02150		
470	CCNTINUE				CTE02160		
	NBETA = 2				CTE02170		
490	CCNTINUE				CTE02180		

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      LIN03  CONWAY  PHASE  N4
SETCT.  -  EFN  SOURCE STATEMENT - IFN(S) -

      IF( C(10).EQ.0. .AND. C(3).EQ.0. .AND. C(13).EQ.0. .AND. CTE02190
      C(15).EQ.0. ) AC12 = .TRUE. CTE02200
      IF( C(6).EQ.0. .AND. C(8).EQ.0. .AND. C(10).EQ.0. .AND. CTE02210
      C(13).EQ.0. ) AC23 = .TRUE. CTE02220
      IF( E(8).EQ.0. .AND. E(3).EQ.0. .AND. E(10).EQ.0. .AND. CTE02230
      E(5).EQ.0. ) AC24 = .TRUE. CTE02240
      IF( E(9).EQ.0. .AND. E(4).EQ.0. .AND. E(6).EQ.0. .AND. CTE02250
      E(1).EQ.0. ) AC14 = .TRUE. CTE02260
      IF( E(7).EQ.0. .AND. E(2).EQ.0. .AND. E(5).EQ.0. .AND. CTE02270
      E(4).EQ.0. ) AC34 = .TRUE. CTE02280
      IF( COEFF ) WRITE(6,1595) LABC, ( C(I), I=1,20 ), CTE02290
      LABE, ( E(I), I=1,17 ), LABT, ( T(I), I=1,5 ) CTE02300
      IF(.NOT. ROTATE) GO TO 1604 162
      WRITE(6,1600) CPR(1,1),CPR(2,2),CPR(3,3),CPR(1,2),
      1 ((CPR(I,J),J=3,6),I=1,2),
      U ((CPR(I,J),J=4,6),I=3,4),CPR(5,5),CPR(5,6),CPR(6,6),
      1 ((EPR(I,J),J=1,6),I=1,3),((EPSPR(I,J),J=1,3),I=1,3) 187
1600 FCPRPAT(5H C 21(/E18.7)/,5H E 18(/E18.7)/,5H EPS,9(/E18.7))
      1 )
1604 CONTINUE
1599 FORMAT( 1H0.4X,5A6, 18H ( C(I), I=1,20 ) // 20( 1H ,1PE18.7/ ) / CTE02310
      1H0.4X,6A6, 18H ( E(I), I=1,17 ) // 17( 1H ,1PE18.7/ ) / CTE02320
      1H0.4X,6A6, 17H ( T(I), I=1,5 ) // 5( 1H ,1PE18.7/ ) ) CTE02330
      RETURN CTE02340
      END CTE02350

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LINBO3 CONWAY PHASE N4

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SIBFTC ACUND. DECK

CRCUND

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      CMPLX FUNCTION ROUND (F)
      REAL I, F(2)
      R = F(1)
      I = F(2)
      IF (R .EQ. 0. .OR. I .EQ. 0.) GO TO 100
      IF (ABS(I/R) .LT. 1.E5) GO TO 50
      R = 0.
      GO TO 100
50 IF (ABS(R/I) .GE. 1.E5) I = 0.
100 ACUND = CMPLX(R, I)
      RETURN
      END

```

```

ROUND020
ROUND030
ROUND040
ROUND050
ROUND060
ROUND070
ROUND080
ROUND090
ROUND100
ROUND110
ROUND120
ROUND130
ROUND140

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LINBO3 CONWAY PHASE

N4

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910FTC CSFUN. DECK

CCSFUA

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SUBROUTINE CSFUN (Y, RX, SX, CX)
DOUBLE PRECISION RX, SX, CX
X = Y
IF (X .LT. 0.) X = X + 360.
IF (X .EQ. 0. .OR. X .EQ. 180.) GO TO 150
IF (X .EQ. 90. .OR. X .EQ. 270.) GO TO 200
SX = DSIN(RX)
CX = DCOS(RX)
100 RETURN
150 CX = 1.
SX = 0.
IF (X .EQ. 180.) CX = -1.
GO TO 100
200 SX = 1.
CX = 0.
IF (X .EQ. 270.) SX = -1.
GO TO 100
END

```

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CSFUN020
CSFUN030
CSFUN040
CSFUN050
CSFUN060
CSFUN070
CSFUN080
CSFUN090 10
CSFUN100 11
CSFUN110
CSFUN120
CSFUN130
CSFUN140
CSFUN150
CSFUN160
CSFUN170
CSFUN180
CSFUN190
CSFUN200

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LIN803 CONNAV PHASE

M4

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SIBFTC FF.... DECK

CFF

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DOUBLE PRECISION FUNCTION FF (I, J, K, L)
  COMMON /FRT/ GAMMA(3,3), D(3,3,3,3), Q(3,3,3)
  DOUBLE PRECISION GAMMA, D, Q
  INTEGER R, S, T, U
  FF= 0.
  CC 50 R = 1, 3
  CC 50 S = 1, 3
  CC 50 T = 1, 3
  CC 50 U = 1, 3
50 FF=FF + GAMMA(I,R)*GAMMA(J,S)*GAMMA(K,T)*GAMMA(L,U)*D(R,S,T,U)
  RETURN
  END

```

```

FF000020
FF000030
FF000040
FF000050
FF000060
FF000070
FF000080
FF000090
FF000100
FF000110
FF000120
FF000130
FF000140

```

LINBO3 CONWAY PHASE N4

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SIOFTC R. DECK

CR

```

CCURLE PRECISION FUNCTION R (I, J, K)
CCMCM /FRT/ GAMMA(3,3), D(3,3,3,3), Q(3,3,3)
CCURLE PRECISION GAMMA, D, Q
INTEGER S, T, U
R = 0.
DO 50 S = 1, 3
CC 50 T = 1, 3
CC 50 U = 1, 3
50 R = R + GAMMA(I,S)*GAMMA(J,T)*GAMMA(K,U)*Q(S,T,U)
RETURN
EAD

```

```

R0000020
R0000030
R0000040
R0000050
R0000060
R0000070
R0000080
R0000090
R0000100
R0000110
R0000120
R0000130

```

LINBO3 CONWAY PHASE

N4

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SIBFTC TIJ. DECK

CTIJ

```

DOUBLE PRECISION FUNCTION TIJ (I, J)
COMMON /FRT/ GAMMA(3,3), JUNK(108)
COMMON /GPEPS/ G(21), P(10), EPSLON(3,3)
DOUBLE PRECISION GAMPA, JUNK
INTEGER R, S
TIJ = 0.
DO 50 R = 1, 3
DO 50 S = 1, 3
50 TIJ = TIJ + GAMMA(I,R)*GAMMA(J,S)*EPSLON(R,S)
RETURN
END

```

```

TIJ00020
TIJ00030
TIJ00040
TIJ00050
TIJ00060
TIJ00070
TIJ00080
TIJ00090
TIJ00100
TIJ00110
TIJ00120
TIJ00130

```

LINBO3 CCNWAY PHASE

N4

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SIOFTC F..... DECK

```

CF      THIS FUNCTION EVALUATES THE L DETERMINANT FOR A VALUE OF VS      F0000020
C*      FOR EITHER THE LITHIUM OR GOLD LITHIUM CASE                      F0000030
C*                                           F0000040
C*****                                F0000050
                                           F0000060
                                           F0000070
                                           F0000080

      COMPLEX FUNCTION F (VS)

      CCMCN /ZZTZ/ CC,CE,CT
      CCMCN /FROOT/ FVSMAG, NT, ICASE      F0000100
      CCMCN /BETAN/ NBETA                  F0000110
      CCMCN /CSET1/ AC12, AC23, AC24, AC14, AC34      F0000120
      CCMCN /LINK / ALFA1(8), ALFA1(8), EL(100), ALFAA(6),      F0000130
      ALFAB(4), BETAA(3,6), BETAB(4,4), EPSO,      F0000140
      MUA, LAMDA, RHOA, MUB, LAMCAB, NUB, RHOB,      F0000150
      VSX, KS, EPSLON, DIGIT, WH, WXA, WXB, KL,      F0000160
      KM, ALL, ROOTS, ITER, COEFF, DETERM, POLYN,      F0000170
      ALPHA, BETA, MAX                      F0000180
      CCMCN /CIA/ IA(4)                    F0000190
      CCMCN /CCM / ACAP, EPSR              F0000200
      CCMCN /GPEPS/ XG(21), XP(18), XEPS(9)      F0000210
      CCMCN /ALESS/ IALF                   F0000220
      CCMCN /CIFE/IXAGNL

      COMPLEX ALFA, FVS, EL, EA(6), EB(4), UA(3), UB(3), ALFA1,      F0000230
      ALFAA, ALFAB, BETAA, BETAB, POLY(9), JIMAG,      F0000240
      B1(4,4), CA, DB, DC, ELL(10,10), ALF(8), AOLD(9)      F0000250
      COMPLEX ALF1, ALF2, XEL(100), XXEL(3,3)      F0000260
      COMPLEX B11, B12, B13, B14, B22, B23, B24, B33, B34, B44, AK,      F0000270
      BB(3,4), BB1(3,3), BETAB1(4),      F0000280
      B1A(2,3), BETAX(2), CXO, CX1, B1B(4,4)
      REAL MUA, LAMCAA, MUB, LAMCAB, NUB, CA(4,4), CB(4,4),      F0000300
      CO(4,4), CC(20), CE(17), CT(5), BID(2,4,4)      F0000310
      LOGICAL ALL, ROOTS, ITER, COEFF, DETERM, POLYN, ALPHA, BETA,      F0000320
      IXAGNL.
      CCMCN ACAP                          F0000330
      LOGICAL AC12, AC23, AC24, AC14, AC34, IALF      F0000340
      F0000350
      DIMENSION ALAB(3,2), IAI(3), IAI(3), LAB(3)      F0000360
      DIMENSION BIX(4), NB(4)              F0000370
      EQUIVALENCE (EL, ELL), (B1, BID),      F0000380
      (CC(1), C11), (CC(2), C13), (CC(3), C14), (CC(4), C15),      F0000390
      (CC(5), C33), (CC(6), C34), (CC(7), C35), (CC(8), C36),      F0000400
      (CC(9), C44), (CC(10), C45), (CC(11), C46),      F0000410
      (CC(12), C55), (CC(13), C56), (CC(14), C66),      F0000420
      (CC(15), C16), (CE(1), E11), (CE(2), E13),      F0000430
      (CE(3), E14), (CE(4), E15), (CE(5), E16),      F0000440
      (CE(6), E31), (CE(7), E33), (CE(8), E34),      F0000450
      (CE(9), E35), (CE(10), E36), (CT(1), T11),      F0000460
      (CT(2), T13), (CT(3), T33)            F0000470
      F0000480

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LINBO3 CONWAY PHASE N4
F..... - EFN SOURCE STATEMENT - IFN(S) -
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CATA LAB(1)/18M (1,3) (2,4) CASE / F0000490
CATA JIMAG / (0.,1.) / . CX0,CX1/(0.,0.), (1.,0.)/ F0000500
CATA ALAB(1,1)/54M1 ROW, 3 ZERO CASE4 ROW, 2 ZERO CASEZEROF0000510
-PIEZUELECTRIC/ TEN,TEN10/1.E-10.1.E1C/

011( AK ) = AK * ( C55*AK + CMPLX10.,(C15+C15)) ) - C11 + RVS F0000530
012( AK ) = AK * ( C45*AK + CMPLX10.,(C14+C56)) ) - C16 F0000540
013( AK ) = AK * ( C35*AK + CMPLX10.,(C13+C55)) ) - C15 F0000550
014( AK ) = -AK * ( E35*AK + CMPLX10.,(E15+E31)) ) + E11 F0000560
022( AK ) = AK * ( C44*AK + CMPLX10.,(C46+C46)) ) - C66 + RVS F0000570
023( AK ) = AK * ( C34*AK + CMPLX10.,(C36+C45)) ) - C56 F0000580
024( AK ) = -AK * ( E34*AK + CMPLX10.,(E14+E36)) ) + E16 F0000590
033( AK ) = AK * ( C33*AK + CMPLX10.,(C35+C35)) ) - C55 + RVS F0000600
034( AK ) = -AK * ( E33*AK + CMPLX10.,(E13+E35)) ) + E15 F0000610
044( AK ) = +AK * ( T33*AK + CMPLX10.,(T13+T13)) ) - T11 F0000620
F0000630
IALF = .FALSE. F0000640
ICASE = 0 F0000650
ICB = 0 F0000660
K1 = 1 F0000670
K2 = 4 F0000680
L1 = 1 F0000690
L2 = 16 F0000700
IB1 = 1 F0000710
IB2 = 4 F0000720
F0000730
F0000740
C.....CALCULATE COEFFICIENTS OF POLYNOMIAL..... F0000750
490 RVS = RHOB*VS*VS F0000760
CALL STRIP (POLY, VS, RHOB, ALL, POLYN) F0000770 30
CALL CROOT (POLY, AOLD, NT, ALFA) F0000780 32
IF (NBETA .EQ. 3) GO TO 520 F0000790
CC = T33 + T33 F0000800
DA = CMPLX10., -T13 - T13 F0000810
CB = CSORT(CMPLX10., -4.*T13 + 4.*T11*T33, 0.) F0000820 37
ALF1 = (DA + CB)/DC F0000830
ALF2 = (DA - CB)/DC F0000840
CC 510 I = 1, 8 F0000850
IF (CABS(ALFA(I) - ALF1) .LE. 1.E-5) ALFA(I) = (-10.,-10.) F0000860 42
IF (CABS(ALFA(I) - ALF2) .LE. 1.E-5) ALFA(I) = (-10.,-10.) F0000870 47
510 CCNTINUE F0000880
520 CC 525 I = 1, 8 F0000890
525 ALF(I) = ALFA(I) F0000900
IF( ROOTS .OR. ALL ) WRITE(6,528) ( ALF(I), I=1,8 ) F0000910 63
528 FORMAT (33P0INTERMEDIATE ROOTS OF POLYNOMIAL/(1H , 1P8E13.5)) F0000920
F0000930
C.....SELECT POSITIVE REAL ROOTS..... F0000940
CC 789 K=1,4 F0000950
789 IA(K) = K F0000960
K = 0 F0000970
CC 630 I = 1, 8 F0000980
RA = REAL(ALFA(I)) F0000990
IF( RA .GT. 0. ) GO TO 610 F0001000
GO TO 630 F0001010
610 K = K + 1 F0001020
ALFAB(K) = ALFA(I) F0001030
IF( K .EQ. 4 ) GO TO 640 F0001040

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      LIN003  CONWAY  PHASE  N4
      F..... - EFN  SOURCE STATEMENT - IFN(S) -

630 CONTINUE
      IF( K .LE. 1 ) GO TO 637
      IF( AC12 .AND. AC23 ) GO TO 631
      IF( NBETA.EC.2 .AND. K.EQ.3 ) GO TO 640
      GO TO 634
631 IF( NBETA .EQ. 2 ) GO TO 400
      IF( .NOT. AC24 ) GO TO 636
      IC = 1
      IF( K .EQ. 3 ) GO TO 140
632 WRITE(6,633) ( ALAB(I,IC), I=1,3 ), K
633 FORMAT( /// 16H *** DEGENERATE ,3A6,3H (,12,9H ALPHAS ) )
634 IALF = .TRUE.
      GO TO 1357
636 IF( .NOT. ( AC14.AND.AC34 ) ) GO TO 634
      GO TO 280
637 WRITE(6,638) K
638 FORMAT( /// 23H *** NUMBER OF ALPHAS =,12,
      .      20H - CASE TERMINATED /
      .      CC TO 634

640 IF( NBETA .EQ. 2 ) ALFAH(4) = (C.,0.)
      IF( ROOTS .OR. ALL ) WRITE(6,788) ( ALFAB(K), K=1,4 )
788 FORMAT( 28H0INTERMEDIATE POSITIVE ROOTS//11H , 1PBE13.5)

C.....CALCULATE BETA B.....
      K4 = NBETA + 1
810 CC 580 K = 1, K4
      B1(1,1) = B11( ALFAB(K) )
      B1(1,2) = B12( ALFAB(K) )
      B1(1,3) = B13( ALFAB(K) )
      B1(1,4) = B14( ALFAB(K) )
      B1(2,2) = B22( ALFAB(K) )
      B1(2,3) = B23( ALFAB(K) )
      B1(2,4) = B24( ALFAB(K) )
      B1(3,3) = B33( ALFAB(K) )
      B1(3,4) = B34( ALFAB(K) )
      B1(2,1) = B1(1,2)
      B1(3,1) = B1(1,3)
      B1(3,2) = B1(2,3)
      IF( NBETA.EC.2 ) GO TO 540
      B1(4,1) = -B1(1,4)
      B1(4,2) = -B1(2,4)
      B1(4,3) = -B1(3,4)
      B1(4,4) = B44( ALFAB(K) )
      IF( K.GT.1 ) GO TO 830

C.....CHECK FOR DEGENERATE CASES.....
      CC 820 I=1,4
      NB(I) = 0
      CC 820 J=1,4
      IF( B10(I,1,J).EQ.0. .AND. B10(2,I,J).EQ.0. ) NB(I) = NB(I) + 1
820 CONTINUE
      IF( NB(2) .EQ. 3 ) GO TO 100
      IF( NB(1).NE.2 .OR. NB(2).NE.2 .OR. NB(3).NE.2 .OR. NB(4).NE.2 )
      .      GO TO 830
      GO TO 200

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F0001050
F0001060
F0001070
F0001072
F0001074
F0001080
F0001090
F0001100
F0001110
F0001120
F0001130
F0001140
F0001150
F0001160
F0001170
F0001180
F0001190
F0001200
F0001210
F0001220
F0001230
F0001240
F0001250
F0001260
F0001270
F0001280
F0001290
F0001300
F0001310
F0001320
F0001330
F0001340
F0001350
F0001360
F0001370
F0001380
F0001390
F0001400
F0001410
F0001460
F0001470
F0001480
F0001490
F0001440
F0001450
F0001500
F0001510
F0001520
F0001530
F0001540
F0001550
F0001560
F0001580

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LIAB03 CONWAY PHASE N4
F..... - EFN SOURCE STATEMENT - IFN(5) - 12/31/69 000109 PAGE 28

C.....PIEZOELECTRIC.....
830 IF(ACAP) WRITE(6,975) K, ((B1(I,J), J=1,4), I=1,4) 185
CC 850 I=1,3
CC 840 J=1,3
840 B1B(J,I)=B1(J,I)*TEN
B1B(4,I)=B1(4,I)
850 B1B(I,4)=B1(I,4)
B1B(4,4)=B1(4,4)*TEN10
KK=4
IF(IXAGNL) GC TO 900
860 CC 870 J=1,4
CC 870 I=1,3
870 BB(I,J)=B1B(I,J)
IF(ACAP) WRITE(6,875) K, ((BB(I,J), J=1,4), I=1,3) 222
875 FORMAT(4H0BB(,11,1H),/(1H,1P8E13.5))
CALL CMATS(BB,BETAB1(1),3,1,81960) 231
880 BETAB1(KK)=CX1
BETAB(4,K)=BETAB1(4)
CC 890 J=1,3
890 BETAB(J,K)=BETAB1(J)*TEN
GC TO 980
C.....HEXAGONAL CRYSTAL.....
900 CC 910 J=1,3
CC 910 I=1,3
910 BB(I,J)=B1B(I,J)
CALL COET(BB1,3,FVS,KEXP) 253
FVS MAG=CABS(FVS) 255
IF(FVS MAG.GT.1.E-5) CO TO 860
KK=1
CC 930 I=1,3
CC 920 J=1,3
920 BB(I,J)=B1B(I+1,J+1)
BB(I,3)=BB(I,3)
930 BB(I,4)=B1B(I+1,1)
IF(ACAP) WRITE(6,875) K, ((BB(I,J), J=1,4), I=1,3) 274
CALL CMATS(BB,BETAB1(2),3,1,81960) 283
GC TO 880
C.....ZERC-PIEZOELECTRIC CASE.....
940 IF(ACAP) WRITE(6,974) K, ((B1(I,J), J=1,3), I=1,2) 287
974 FORMAT(6HOACAP(,11,1H) // (1H,1P6E13.5))
975 FORMAT(6HOACAP(,11,1H) // (1H,1P8E13.5))
B1(3,1)=B1(1,2)
B1(4,1)=B1(2,2)
B1(1,2)=B1(1,3)
B1(2,2)=B1(2,3)
K2=3
CALL CMATS(B1,BETAB(1,K),NBETA,1,81960) 298
BETAB(4,K)=CX0
BETAB(3,K)=CX1
980 CONTINUE
IF (NBETA .EQ. 3) GC TO 983
F0001750
F0001760
F0001770
F0001780
F0001790
F0001800
C.....ZERC - PIEZOELECTRIC CASE.....
CC 982 I = 1, 4
BETAB(1,4) = (0.,0.)
CC 982 J = 1, 3

LINBO3 CONWAY PHASE N4
F..... - EFN SOURCE STATEMENT - IFN(S) -

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982 BETAB(I,J) = BETAB(I,J)*1.E-10
GO TO 983

C.....1 ROW, 3 ZERO CASE.....

C.....(4 ALPHAS).....

100 CONTINUE

ICASE = 1

J = 1

BMIN = CABS(B1(2,2))

CC 110 X=2.4

B1X = CABS(B22(ALFAB(K)))

IF(B1X .GE. BMIN) GO TO 110

J = K

BMIN = B1X

110 CONTINUE

IA(1) = J

JA = 2

CC 120 K=1.4

IF(K .EQ. J) GO TO 120

IA(JA) = K

JA = JA + 1

120 CONTINUE

BETAB(2,1) = 1.E-10

BETAB(1,1) = CXO

BETAB(3,1) = CXO

BETAB(4,1) = CXO

J1 = 2

J2 = 4

125 CC 130 J=J1,J2

K = IA(J)

B1A(1,1) = B11(ALFAB(K))

B1A(2,1) = B13(ALFAB(K))

B1A(3,1) = B33(ALFAB(K))

B1A(1,2) = -B14(ALFAB(K))

B1A(2,2) = -B34(ALFAB(K))

BETAX = B1A(1,1)*B1A(2,2) - B1A(1,2)*B1A(2,1)

BETAB(1,J) = (B1A(2,1)*B1A(2,3) - B1A(1,3)*B1A(2,2)) / BETAX

BETAB(2,J) = CXO

BETAB(3,J) = (B1A(2,1)*B1A(1,3) - B1A(1,1)*B1A(2,3)) / BETAX

130 BETAB(4,J) = CXO

IF(ICASE .EQ. 3) GO TO 983

GO TO 270

C.....(3 ALPHAS).....

140 ICASE = 3

N1 = 0

GO 150 I=1,K

IF(CABS(B22(ALFAB(I))) .GT. 1.E7) N1 = N1 + 1

150 CONTINUE

IF(N1 .LT. 3) GO TO 632

J1 = 1

J2 = 3

K2 = 3

I02 = 3

F0001810
F0001820
F0001830
F0001840
F0001850
F0001860
F0001870
F0001880
F0001890
F0001900
F0001910 325
F0001920
F0001930 330
F0001940
F0001950
F0001960
F0001970
F0001980
F0001990
F0002000
F0002010
F0002020
F0002030
F0002040
F0002050
F0002060
F0002070
F0002080
F0002090
F0002100
F0002110
F0002120
F0002130
F0002140
F0002150
F0002160
F0002170
F0002180
F0002190 361
F0002200
F0002210
F0002220
F0002230
F0002240
F0002250
F0002260
F0002270
F0002280
F0002290
F0002300
F0002310 380
F0002320
F0002330
F0002340
F0002350
F0002360
F0002370

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      LIN003  CONWAY  PHASE  N4
      F..... - EFN  SOURCE STATEMENT - IFN(S) -

      L2 = 12
      ICB = 3
      WRITE(16,160) ( ALAB(I,1), I=1,3 ), ( 1, ALFAB(I), I=1,K )
160  FORMAT( /// 16H *** DEGENERATE ,3A6.14H ( 3 ALPHAS ) /
      .      ( 11H ALPHAB(1,3H) = , 1PZE13.5 / ) )
      GO TO 125

C.....4 ROW, 2 ZERO CASE.....

C.....( 4 ALPHAS ).....
200  CCNTINUE
      ICASE = 2
      BIX(1) = CABS( B1(1,1)*B1(3,3) - B1(1,3)**2 )
      J = 1
      EPIN = BIX(1)
      CC 210 K=2,4
      BIX(K) = CABS( B11(ALFAB(K))*B33(ALFAB(K))-B13(ALFAB(K))**2 )
      IF( BIX(K) .GE. BMIN ) GO TO 210
      J = K
      BMIN = BIX(K)
210  CCNTINUE
      I = 1
      IF( J .EQ. 1 ) I = 2
      CC 220 K=1,4
      IF( K .EQ. J ) GO TO 220
      IF( BIX(K) .GE. BIX(I) ) GO TO 220
      I = K
220  CCNTINUE
230  CCNTINUE
      JA = 3
      JB = 1
      DO 240 K=1,4
      IF( K.EQ.J .OR. K.EQ.I ) GO TO 235
      IA(JA) = K
      JA = JA + 1
      GO TO 240
235  IA(JB) = K
      JB = JB + 1
240  CCNTINUE
245  CC 250 K=1,2
      I = IA(K)
      BETAB(1,K) = - B13(ALFAB(I)) / B11(ALFAB(I)) * 1.E-10
      BETAB(2,K) = CX0
      BETAB(3,K) = 1.E-10
250  BETAB(4,K) = CX0
      IF( ICB .EQ. 1 ) GO TO 263
255  DO 260 K=3,4
      I = IA(K)
      BETAB(1,K) = CX0
      BETAB(2,K) = B24(ALFAB(I)) / B22(ALFAB(I))
      BETAB(3,K) = CX0
      BETAB(4,K) = CX1
260  IF( ICB .EQ. 2 ) GO TO 263
      GO TO 270

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F0002380
F0002390
F0002400 394
F0002410
F0002420
F0002430
F0002440
F0002450
F0002460
F0002470
F0002480
F0002490
F0002500
F0002510 410 411
F0002520
F0002530
F0002540
F0002550 419 420
F0002560
F0002570
F0002580
F0002590
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F0002732
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F0002736
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F0002750
F0002760
F0002770
F0002780
F0002790
F0002800
F0002810
F0002820
F0002830
F0002840
F0002850
F0002860
F0002870
F0002880
F0002890
F0002900

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      LINBO3  CONWAY  PHASE  M4
      F..... - EFN  SOURCE STATEMENT - IFN(S) -
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C.....( LESS THAN 4 ALPHAS ).....
280 IC = 2
   I1 = 0
   I2 = 0
   CC 310 I=1,K
   TERM = CABS( - 822(ALFAB(I))*844(ALFAB(I)) - 824(ALFAB(I))*2 )
   IF( TERM .GE. 1.E-5 ) GO TO 300
   I1 = I1 + 1
   IA1(I1) = 1
   GC TO 310
300 I2 = I2 + 1
   IA2(I2) = 1
310 CONTINUE
   IF( I2 .EQ. 2 ) GO TO 340
   IF( I1 .EQ. 2 ) GO TO 320
   GO TO 632

C.....( 2,4 CASE ).....
320 IA(3) = IA1(I1)
   IA(4) = IA1(I2)
   ICASE = 5
   IB1 = 3
   IC9 = 2
   LI = 9
   KI = 3
   GO TO 360

C.....( 1,3 CASE ).....
340 IA(1) = IA2(I1)
   IA(2) = IA2(I2)
   ICB = 1
350 ICASE = 4
   IB2 = 2
   L2 = 8
   K2 = 2
360 J = IA(IB1)
   K = IA(IB2)
   WRITE(6,370) ( ALAB(I,IC),I=1,3 ), LAB(ICB), IB1,IB2,IB1,ALFAB(J),
   . IB2, ALFAB(K)
370 FCNMT( /// 16H *** DEGENERATE ,4A6 /
   . 43H CALCULATE BETA(I,K) AND L(I,K) FOR K =,12,
   . 4H AND,12,8H (I=1,4) //
   . 2(I1H ALPHAT,I1,3H) =,1P2E13.5 / ) /
   IF( ICB .EQ. 3 ) ICB = 1
   GC TO ( 245, 255 ), ICB

C.....(ERC - PIEZOELECTRIC CASE).....
400 IC = 3
   I1 = 0
   CO 430 I=1,K
   CA22 = CABS( 822( ALFAB(I) ) )
   IF( K .EQ. 3 ) GO TO 410
   IF( CA22 .GT. 1.E7 ) I1 = I1 + 1
   GC TO 430
410 IF( I .GT. 1 ) GO TO 420

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F0002910
 F0002920
 F0002930
 F0002940
 F0002950
 F0002960
 F0002970
 F0002980
 F0002990
 F0003000
 F0003010
 F0003020
 F0003030
 F0003040
 F0003050
 F0003060
 F0003070
 F0003080
 F0003090
 F0003100
 F0003110
 F0003120
 F0003130
 F0003140
 F0003150
 F0003160
 F0003170
 F0003180
 F0003190
 F0003200
 F0003210
 F0003220
 F0003230
 F0003240
 F0003250
 F0003260
 F0003270
 F0003280
 F0003290
 F0003300
 F0003310
 F0003320
 F0003330
 F0003340
 F0003350
 F0003360
 F0003370
 F0003380
 F0003390
 F0003400
 F0003410
 F0003420
 F0003430
 F0003440
 F0003450
 F0003460

511 512

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LINB03 CONWAY PHASE N4
F..... - EFN SOURCE STATEMENT - IFN(S) -

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```

CAMIN = CA22
J = 1
GC TC 430
420 IF( CAMIN .LE. CA22 ) GO TO 430
CAMIN = CA22
J = 1
430 CONTINUE
IF( K .EQ. 2 ) GO TO 450
I1 = 0
CC 440 I=1,3
IF( I .EQ. J ) GO TO 440
I1 = I1 + 1
IA(I1) = I
440 CONTINUE
GO TO 460
450 IF( I1 .NE. 2 ) GO TO 632
460 ICB = 3
GC TC 350

270 IF( .NOT. ( NCOTS .OR. ALL ) ) GO TO 583
I11 = IA(1)
I12 = IA(2)
I13 = IA(3)
I14 = IA(4)
WRITE(6,984) ( ALAB(I,ICASE), I=1,3 ), ALFAB(I11), ALFAB(I12),
ALFAB(I13), ALFAB(I14)
984 FORMAT( 21MORE-ORDERED ALPHAS ( , 3A6, 2H ) // ( 1H ,1P8E13.5 ) )
983 IF( BETA .OR. ALL ) WRITE(6,985) ( ( BETAB(I,J),
J=1,102 ), I=1,4 )
985 FORMAT (20+INTERMEDIATE BETA B//11H , 1P8E13.5)
IF (KS .NE. 0) GO TO 1390

C.....EVALUATE LITHIUM NIOBATE EQUATIONS.....
L = L1
CO 1330 K=K1,K2
J = IA(K)
EL(L) = BETAB(1,K)*(CMPLX(O., C15) + ALFAB(J)*C55)
. + BETAB(2,K)*(CMPLX(O., C56) + ALFAB(J)*C45)
. + BETAB(3,K)*(CMPLX(O., C55) + ALFAB(J)*C35)
. + BETAB(4,K)*(CMPLX(O., E15) + ALFAB(J)*E35)
EL(L+1) = BETAB(1,K)*(CMPLX(O., C14) + ALFAB(J)*C45)
. + BETAB(2,K)*(CMPLX(O., C46) + ALFAB(J)*C44)
. + BETAB(3,K)*(CMPLX(O., C45) + ALFAB(J)*C34)
. + BETAB(4,K)*(CMPLX(O., E14) + ALFAB(J)*E34)
EL(L+2) = BETAB(1,K)*(CMPLX(O., C13) + ALFAB(J)*C35)
. + BETAB(2,K)*(CMPLX(O., C36) + ALFAB(J)*C34)
. + BETAB(3,K)*(CMPLX(O., C35) + ALFAB(J)*C33)
. + BETAB(4,K)*(CMPLX(O., E13) + ALFAB(J)*E33)
IF (KL .EQ. 0) GO TO 1190
1170 EL(L+3) = BETAB(4,K)
GO TO 1330
1190 IF (WM .EQ. 0. .AND. KM .EQ. 0) GO TO 1170
IF (WM .LE. 1.E10) GO TO 1220
ST = EPSR
GC TO 1290

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F0003470
F0003480
F0003490
F0003500
F0003510
F0003520
F0003530
F0003540
F0003550
F0003560
F0003570
F0003580
F0003590
F0003600
F0003610
F0003620
F0003630
F0003640
F0003650
F0003660
F0003670
F0003680
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F0003900
F0003910
F0003920
F0003930
F0003940
F0003950
F0003960
F0003970
F0003980
F0003990
F0004000
F0004010
F0004020

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LINBU3 CONWAY PHASE N4
 F..... - EFM SOURCE STATEMENT - IFN(S) -

1220 IF (WH.NE. 0. .OR. KM.NE. 1) GO TO 1290 F0004030
 ST = 0. F0004040
 GC TO 1290 F0004050
 1250 IF (KM.NE. 0 .OR. WH.EQ. 0. .OR. WH.GE. 1.E10) GO TO 1280 F0004060
 ST=COSH(WH/V5)/SINH(WH/V5)*EPSR
 CO TO 1290
 1280 ST = TANH(WH/V5) * EPSR F0004090
 1290 EL(L+3) = BETAB(1,K)*(CMPLX(0., E31) + ALFAB(J)*E35) F0004100
 + BETAB(2,K)*(CMPLX(0., E36) + ALFAB(J)*E34) F0004110
 + BETAB(3,K)*(CMPLX(0., E35) + ALFAB(J)*E33) F0004120
 -(CMPLX(0., T13) + ALFAB(J)*T33 + EPS0*ST)*BETAB(4,K) F0004130
 EL(L+3) = EL(L+3) * 1.E+10 F0004140
 1330 L = L + 4 F0004150
 IF (NBETA.EQ. 3) GO TO 1340 F0004160
 F0004170
 C.....ZERC - PIEZOELECTRIC CASE..... F0004180
 EL(4) = (0.,0.) F0004190
 EL(8) = (0.,0.) F0004200
 EL(12) = (0.,0.) F0004210
 EL(13) = (0.,0.) F0004220
 EL(14) = (0.,0.) F0004230
 EL(15) = (0.,0.) F0004240
 EL(16) = CX1 F0004250
 F0004260
 1340 IF (DETERM .OR. ALL) WRITE(6,1335) (EL(I), I=L1,L2) F0004270
 1335 FORMAT (33HINTERMEDIATE L MATRIX BY COLUMNS// F0004280
 (1H,1P8E13.5)) F0004290
 ICB1 = ICB + 1 F0004300
 GC TO (1343, 1360, 137C, 1380), ICB1 F0004310
 1343 CC 1345 I=1,16 F0004320
 1345 XEL(I) = EL(I) F0004330
 CALL CDET (XEL, 4, FVS, KEXP) F0004340
 1350 F = FVS F0004350
 FVSMAG = CABS(FVS) F0004360
 IF (DETERM .OR. ALL) WRITE (6, 1355) VS, FVS, FVSMAG F0004370
 1355 FCRMAT (5MOV5 =, E15.7, 5X, 7HFVS) =, 2E15.7, 5X, 5HMG = E15.7) F0004380
 1357 RETURN F0004390
 1360 FVS = EL(1)*EL(7) - EL(5)*EL(13) F0004400
 GC TO 1350 F0004410
 1370 FVS = EL(10)*EL(16) - EL(14)*EL(12) F0004420
 GC TO 1350 F0004430
 1380 J = 1 F0004440
 CC 1385 K=1,3 F0004450
 XXEL(1,K) = EL(J) F0004460
 XXEL(2,K) = EL(J+2) F0004470
 XXEL(3,K) = EL(J+3) F0004480
 1385 J = J + 4 F0004490
 CALL CDET (XXEL, 3, FVS, KEXP) F0004500
 GC TO 1350 F0004510
 F0004520
 C.....EVALUATE GOLD LITHIUM NIOBATE EQUATIONS..... F0004530
 1390 CD = MUA + MUA + LAMCA F0004540
 CA = MUA*DC/1.E20 F0004550
 RVS=RHOA*VS*VS F0004560
 CB = (RVS*(MUA + DD) - 2.*MUA*DC)/1.E20 F0004570
 CC = (RVS - DD)*(RVS - MUA)/1.E20 F0004580

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691 692

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LINBO3 CONWAY PHASE N4
 F..... - EFM SOURCE STATEMENT - IFN(5) -

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FVS = CSQRT(CB*DB - 4.*CA*DC)
ALFAA(1) = CSQRT((-DB + FVS)/(DA + CA))
ALFAA(2) = -ALFAA(1)
ALFAA(3) = CSQRT((-DB - FVS)/(DA + CA))
ALFAA(4) = -ALFAA(3)
ALFAA(5) = CSQRT(CMPLX((MUA - RVS)/MUA, 0.))
ALFAA(6) = -ALFAA(5)
IF( ALPHA .GT. ALL ) WRITE(6,1505) ( ALFAA(I), I=1,6 )
1505 FCRMAT (21)HOINTERMEDIATE ALPHA A//((IM , 1P6E13.5))
CC 1550 K = 1, 4
BETAA(1,K) = CMPLX(0., -LAMBDAA - MUA)*ALFAA(K)/
      ((MUA*ALFAA(K)*ALFAA(K) - DD + RVS)*1.E10)
      BETAA(2,K) = (0.,0.)
1550 BETAA(3,K) = (1.E-10,0.)
      BETAA(2,5) = (1.E-10,0.)
      BETAA(2,6) = (1.E-10,0.)
      BETAA(3,5) = (0.,0.)
      BETAA(3,6) = (0.,0.)
IF( BETA .OR. ALL ) WRITE(6,1595) ( BETAA(I,J), I=1,3 , J=1,6 )
      ((BETAA(I,J), I = 1, 3), J = 1, 6)
1595 FCRMAT (31)HOINTERMEDIATE BETA A BY COLUMNS//((IM , 1P6E13.5))
CC 1620 J = 1, 6
CO 1620 I = 1, 6
1620 ELL(1,J) = BETAA(1,J)
CO 1650 J = 7, 10
CO 1650 I = 1, 3
1650 ELL(1,J) = -BETAB(1,J-6)
CC 1740 J = 1, 6
ELL(4,J) = BETAA(1,J)*ALFAA(1)*MUA + CMPLX(0.,MUA)*BETAA(3,J)
ELL(5,J) = BETAA(2,J)*ALFAA(1)*MUA
ELL(6,J) = CMPLX(0.,LAMBDAA)*BETAA(1,J) + BETAA(3,J)*ALFAA(1)*DD
FVS = CEXP(IALFAA(J)*h/VS)
ELL(7,J) = ELL(4,J)*FVS
ELL(8,J) = ELL(5,J)*FVS
ELL(9,J) = ELL(6,J)*FVS
1740 ELL(10,J) = (0.,0.)
CO 1910 J = 7, 10
I = 1A(J-6)
ELL(4,J) = -BETAB(1,J-6)*(CMPLX(0., C15) + ALFAB(1)*C55)
      - BETAB(2,J-6)*(CMPLX(0., C56) + ALFAB(1)*C45)
      - BETAB(3,J-6)*(CMPLX(0., C55) + ALFAB(1)*C35)
      - BETAB(4,J-6)*(CMPLX(0., E15) + ALFAB(1)*E35)
ELL(5,J) = -BETAB(1,J-6)*(CMPLX(0., C14) + ALFAB(1)*C45)
      - BETAB(2,J-6)*(CMPLX(0., C46) + ALFAB(1)*C44)
      - BETAB(3,J-6)*(CMPLX(0., C45) + ALFAB(1)*C34)
      - BETAB(4,J-6)*(CMPLX(0., E14) + ALFAB(1)*E34)
ELL(6,J) = -BETAB(1,J-6)*(CMPLX(0., C13) + ALFAB(1)*C35)
      - BETAB(2,J-6)*(CMPLX(0., C36) + ALFAB(1)*C34)
      - BETAB(3,J-6)*(CMPLX(0., C35) + ALFAB(1)*C33)
      - BETAB(4,J-6)*(CMPLX(0., E13) + ALFAB(1)*E33)
ELL(7,J) = (0.,0.)
ELL(8,J) = (0.,0.)
ELL(9,J) = (0.,0.)
1910 ELL(10,J) = BETAB(4,J-6)
CO 1915 I = 1, 3
DO 1915 J = 1, 10

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F0004590 762
 F0004600 763
 F0004610
 F0004620 764
 F0004630
 F0004640 765
 F0004650
 F0004660 766
 F0004670
 F0004680
 F0004690
 F0004700
 F0004710
 F0004720
 F0004730
 F0004740
 F0004750
 F0004760
 F0004770
 F0004780 786
 F0004790
 F0004800
 F0004810
 F0004820
 F0004830
 F0004840
 F0004850
 F0004860
 F0004870
 F0004880
 F0004890
 F0004900 843
 F0004910
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 F0004960
 F0004970
 F0004980
 F0004990
 F0005000
 F0005010
 F0005020
 F0005030
 F0005040
 F0005050
 F0005070
 F0005080
 F0005090
 F0005100
 F0005110
 F0005120
 F0005130
 F0005140

LINB03 CONWAY PHASE N4		12/31/69	000109	PAGE 35
F..... - EFN SOURCE STATEMENT - IFAIS) -				
1915	ELL(I,J) = ELL(I,J)*1.E10	F0005150		
	IF(DETERM .OR. ALL) WRITE(6,1930) ((ELL(I,J), J=1,10), I=1,10)	F0005160		
	((ELL(I,J), J = 1, 10), I = 1, 10)	F0005170	905	
1930	FORMAT (17H1*** L MATRIX ***// (1H , 1P8E15.6))	F0005180		
	DO 1940 I=1,100	F0005190		
1940	XEL(I) = ELL(I,1)	F0005200		
	CALL CDET (XEL, 10, FVS, KEXP)	F0005210	933	
	GO TO 1350	F0005220		
		F0005230		
C.....	ERROR IN MATRIX INVERSION.....	F0005240		
1960	WRITE (6, 1970)	F0005250	925	
1970	FORMAT (30H0***SINGULAR MATRIX IN BETA**)	F0005260		
	GO TO 1350	F0005270		
	END	F0005280		

LINB03 CONWAY

PHASE

N4

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SIBFTC STRIP. DECK

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CSTRIP      *** ARRAY OF CONDUCTING STRIPS OVER PIEZOELECTRIC MEDIUM *** STRP0020
C*GV012                                           STRP0030
C*                                           G.V. ROBERTS STRP0040
C*                                           03.15.68 STRP0050
C***** STRP0060
SUBROUTINE STRIP (A, VS, RHO, ALL, COEFF) STRP0070
                                           STRP0080

COMMON /ZZTZ/ CC(20),CE(17),CT(5)
COMMON /FLAG/ ONCE STRP0100
CCOMPLEX A(9), ZERC, CNE, JIMAG STRP0110
REAL ONES(5), CA(4,4), CB(4,4), CC(4,4) STRP0120
INTEGER P STRP0130
LOGICAL ALL, COEFF, ONCE STRP0140
EQUIVALENCE (CC(1), C11), (CC(2), C13), (CC(3), C14), (CC(4), C15), STRP0150
              (CC(5), C33), (CC(6), C34), (CC(7), C35), (CC(8), C36), STRP0160
              (CC(9), C44), (CC(10), C45), (CC(11), C46), STRP0170
              (CC(12), C55), (CC(13), C56), (CC(14), C66), STRP0180
              (CC(15), C16), (CE(1), E11), (CE(2), E13), STRP0190
              (CE(3), E14), (CE(4), E15), (CE(5), E16), STRP0200
              (CE(6), E31), (CE(7), E33), (CE(8), E34), STRP0210
              (CE(9), E35), (CE(10), E36), (CT(1), T11), STRP0220
              (CT(2), T13), (CT(3), T33) STRP0230
              STRP0240

DATA TP1, ZER0, CNE, ONES, JIMAG / 6.2831853, (0.,0.), (1.,0.), STRP0250
              1., -1., -1., 1., 1., (0.,1.) / STRP0260
C.....SET UP MATRICES..... STRP0270
IF (.NOT. ONCE) GO TO 670 STRP0280
ONCE = .FALSE. STRP0290
CA(1,1) = C55 STRP0300
CA(1,2) = C45 STRP0310
CA(2,1) = C45 STRP0320
CA(2,2) = C44 STRP0330
CA(1,3) = C35 STRP0340
CA(3,1) = C35 STRP0350
CA(1,4) = E35 STRP0360
CA(4,1) = E35 STRP0370
CA(2,3) = C34 STRP0380
CA(3,2) = C34 STRP0390
CA(2,4) = E34 STRP0400
CA(3,3) = C33 STRP0410
CA(3,4) = E33 STRP0420
CA(4,3) = E33 STRP0430
CA(4,4) = -T33 STRP0440
CB(1,1) = C15 + C15 STRP0450
CB(1,2) = C14 + C56 STRP0460
CB(2,1) = CB(1,2) STRP0470
CB(1,3) = C13 + C55 STRP0480
CB(3,1) = CB(1,3) STRP0490
              STRP0500
              STRP0510

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LINBO3 CONWAY PHASE M4
STRIP. - EFN SOURCE STATEMENT - IFN(S) -

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CB(1,4) = E15 + E31
CB(4,1) = CB(1,4)
CB(2,2) = C46 + C46
CB(2,3) = C36 + C45
CB(3,2) = CB(2,3)
CB(2,4) = E14 + E36
CB(4,2) = CB(2,4)
CB(3,3) = C35 + C35
CB(3,4) = E12 + E35
CB(4,3) = CB(3,4)
CB(4,4) = -T13 - T13
CD(1,2) = -C16
CD(2,1) = -C16
CD(1,3) = -C15
CD(3,1) = -C15
CD(1,4) = -E11
CD(4,1) = -E11
CD(2,3) = -C56
CD(3,2) = -C56
CD(2,4) = -E16
CD(4,2) = -E16
CD(3,4) = -E15
CD(4,3) = -E15
CD(4,4) = T11

C.....P T E W.....
670 DD 680 I = 1, 9
680 A(I) = ZERO

C.....PREPARE FOR GAMMA(N) LOOP.....
RGAM = RMD*VS*VS
CD(1,1) = RGAM - C11
CD(2,2) = RGAM - C66
CD(3,3) = RGAM - C55

C.....CCOMPUTE COEFFICIENTS OF EIGHT DEGREE PCLYNOMIAL.....
P = 0
CC 1030 J = 1, 4
CC 1020 K = 1, 4
IF (K .EQ. J) GO TO 1020
CC 1010 L = 1, 4
IF (L .EQ. K .OR. L .EQ. J) GO TO 1010
M = 10 - J - K - L
P = P + 1
IF (P .GT. 5) P = 2
T1 = CA(1,J)*CB(2,K) + CB(1,J)*CA(2,K)
T2 = CA(1,J)*CD(2,K) - CB(1,J)*CB(2,K) + CD(1,J)*CA(2,K)
T3 = CB(1,J)*CD(2,K) + CD(1,J)*CB(2,K)
ZH = CA(1,J)*CA(2,K)*CA(3,L)
ZI = CA(1,J)*CA(2,K)*CB(3,L) + T1*CA(3,L)
ZJ = CA(1,J)*CA(2,K)*CD(3,L) - T1*CB(3,L) + T2*CA(3,L)
ZK = CD(3,L)*T1 + CB(3,L)*T2 + CA(3,L)*T3
ZL = T2*CD(3,L) - T3*CB(3,L) + CD(1,J)*CD(2,K)*CA(3,L)
ZM = T3*CD(3,L) + CD(1,J)*CD(2,K)*CB(3,L)
ZN = CD(1,J)*CD(2,K)*CD(3,L)
ZS = CA(4,M)

STRP0520
STRP0530
STRP0540
STRP0550
STRP0560
STRP0570
STRP0580
STRP0590
STRP0600
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STRP0670
STRP0680
STRP0690
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STRP0970
STRP0980
STRP0990
STRP1000
STRP1010
STRP1020
STRP1030
STRP1040
STRP1050
STRP1060
STRP1070

LINBO3 CONWAY PHASE M4
STRIP. - EFN SOURCE STATEMENT - IFN(S) -

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ZU = CB(4,M)
ZV = CD(4,P)
A(1) = A(1) + ONES(P)*ZH*ZS
A(2) = A(2) + ONES(P)*(ZH*ZU + ZI*ZS)
A(3) = A(3) + ONES(P)*(ZH*ZV - ZI*ZU + ZJ*ZS)
A(4) = A(4) + ONES(P)*(ZI*ZV + ZJ*ZU + ZK*ZS)
A(5) = A(5) + ONES(P)*(ZJ*ZV - ZK*ZU + ZL*ZS)
A(6) = A(6) + ONES(P)*(ZK*ZV + ZL*ZU + ZM*ZS)
A(7) = A(7) + ONES(P)*(ZL*ZV - ZM*ZU + ZN*ZS)
A(8) = A(8) + ONES(P)*(ZM*ZV + ZN*ZU)
A(9) = A(9) + ONES(P)*ZN*ZV
1010 CCNTINUE
1020 CCNTINUE
1030 CCNTINUE
DC 1035 I = 2, 8, 2
1035 A(I) = JIMAG*A(I)
IF (ALL .OR. COEFF) WRITE (6, 1050) (A(I), I = 1, 9)
1050 FORMAT (27H0COEFFICIENTS OF POLYNOMIAL/(1M , 2E18.7))

C.....NORMALIZE COEFFICIENTS TO LARGEST VALUE.....
ANORM = CABS(A)
DC 1155 I = 2, 9
1155 ANORM = AMAX1 (ANORM,CABS(A(I)))
DC 1157 I = 1, 9
A(I) = A(I)/ANORM
IF (CABS(A(I)) .LT. 1.E-6) A(I) = ZERO
1157 CCNTINUE
IF (ALL .OR. COEFF) WRITE (6, 1050) (A(I), I = 1, 9)
RETURN
END

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STRP1080
STRP1090
STRP1100
STRP1110
STRP1120
STRP1130
STRP1140
STRP1150
STRP1160
STRP1170
STRP1180
STRP1190
STRP1200
STRP1210
STRP1220
STRP1230
STRP1240
STRP1250
STRP1260
STRP1270
STRP1280
STRP1290
STRP1300
STRP1310
STRP1320
STRP1330
STRP1340
STRP1350
STRP1360
STRP1370

99

1C7

113

122

129

SIBFTC CROOT. DECK

CROOT

C* CROOT CALCULATES THE ROOTS OF AN EIGHTH DEGREE POLYNOMIAL

SUBROUTINE CROOT (A, XO, KODE, X)

COMMON /CROOT/ C, K, M

COMPLEX A(9), C(9), X(8), X(8), X(8), X(8), X(8), X(8), X(8)

EXTERNAL POL

DATA CXO/(0.0,0.0)/

K = 1

DO 100 I = 1, 9

100 C(I) = A(I)

110 IF (CABS(C(1)).NE. 0.) GO TO 130

X(K) = (0.0,0.0)

K = K + 1

IF (K.GT. 8) GO TO 280

M = 10 - K

DO 120 I = 1, M

120 C(I) = C(I+1)

GO TO 110

C.....COMPUTE K-TH ROOT.....

130 X1 = (.5,.5)

IF (KODE.NE. 0) X1 = X(K)

C.....X1 = INITIAL GUESS, USE MULLERS METHOD TO REFINE IT.....

M = 10 - K

CALL MULLER (POL, X1, 20, 1.E-8, JUNK, X(K), N)

X1 = (.5, -.5)

IF (N.GE. 20) CALL MULLER (POL, X1, 20, 1.E-8, JUNK, X(K), N)

X(K) = ROUND(X(K))

K = K + 1

IF (K.GT. 8) GO TO 280

C.....REDUCE COEFFICIENTS.....

DO 240 I = 2, M

240 C(I) = C(I) + X(K-1)*C(I-1)

IF (K.NE. 8) GO TO 130

IF (CABS(C(1)).NE. 0.) GO TO 250

X(8) = CXO

GO TO 280

250 X(8) = -C(2)/C(1)

X(8) = ROUND(X(8))

C.....EXIT.....

280 DO 290 I = 1, 8

290 X(1) = X(I)

KCODE = 1

RETURN

END

CROOT020

CROOT030

CROOT040

CROOT050

CROOT060

CROOT070

CROOT075

CROOT080

CROOT090

CROOT100

CROOT110

CROOT120

CROOT130

CROOT140

CROOT150

CROOT160

CROOT170

CROOT180

CROOT190

CROOT200

CROOT210

CROOT220

CROOT230

CROOT240

CROOT250

CROOT260

CROOT270

CROOT280

CROOT290

CROOT300

CROOT310

CROOT320

CROOT330

CROOT340

CROOT350

CROOT360

CROOT370

CROOT380

CROOT390

CROOT400

CROOT410

CROOT415

CROOT420

CROOT430

CROOT440

CROOT450

CROOT460

CROOT470

CROOT480

12

36

40

43

62

68

LINBO3 CONWAY PHASE

N4

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SIBFTC MULR. DECK

```

CMULLER ***** MULLERS METHOD FOR RCOT FINDING ****
C#
C# G.V. ROBERTS
C# 04.24.68
C*****
SUBROUTINE MULLER (F, XC, MAX, EPSLO%, KODE, X, N)
    COMPLEX F, X, X0, X1, X2, X3, H2, L2, D2, G, FX, FX1, FX2, D,
    TWD, ONE, GPD, GMD, L3, FX3, H1
    DATA TWC, ONE / (0.02,0.), (0.01,0.) /

    FX = F(X0)
    X2 = X0
    KODE = 0
    IF (CABS(X) .EQ. 0.) GO TO 190
    FX1 = F(1.C2*X0)
    FX2 = F(1.01*X0)
    H2 = -0.01*X0
    GO TO 220
190 FX1 = F(TWD)
    FX2 = F(ONE)
    T2 = -0.01
    220 L2 = -0.50
    D2 = 0.50

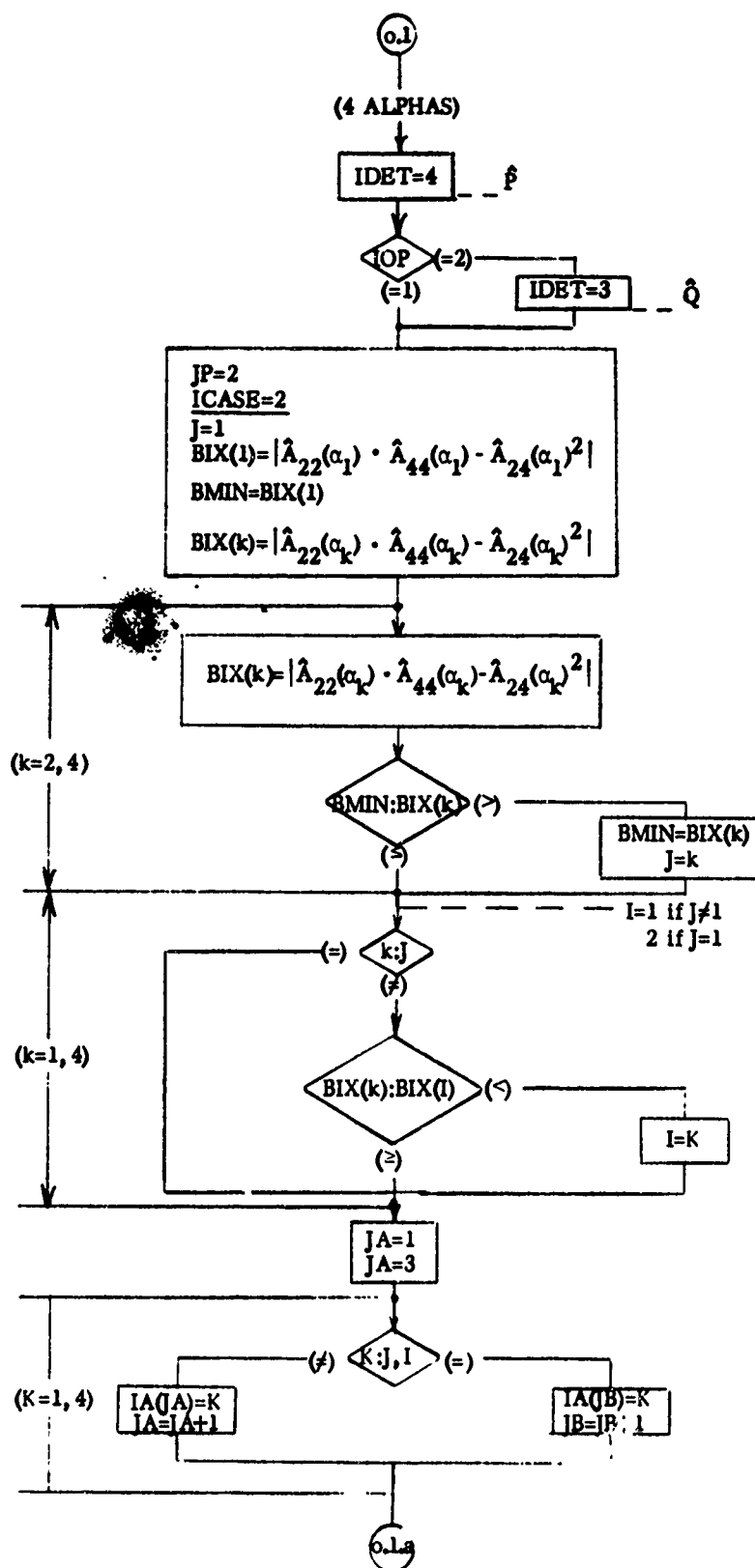
C.....BEGIN ITERATIONS.....
DC 540 K = 1, MAX
N = K
G = FX*L2*L2 - FX1*D2*D2 + FX2*(L2 + D2)
CALL OVERCK
C = CSQRT (G*G - 4.*FX2*D2*L2*(FX*L2 - FX1*D2 + FX2))
GPD = G + C
GMD = G - C
CGPD = CABS(GPD)
CGMD = CABS(GMD)
IF (CGPD .GT. CGMD) GO TO 360
IF (CGMD .EQ. 0.) GO TO 580
L3 = -2.*FX2*D2/GMD
GO TO 370
360 IF (CGPD .EQ. 0.) GO TO 580
L3 = -2.*FX2*D2/GPD
370 X3 = X2 + L3*T2
FX3 = F(X3)
IF (CABS(X3) .EQ. 0.) GO TO 420
IF (CABS((X3 - X2)/X2) .GT. EPSLO%) GO TO 450
CALL OVERCK
390 X = X3
GO TO 570
420 IF (CABS(FX3) .LE. EPSLO%) GO TO 390

```

```

MULR020
MULR030
MULR040
MULR050
MULR060
MULR070
MULR080
MULR090
MULR100
MULR110
MULR120
MULR130
MULR140
MULR150
MULR160
MULR170
MULR180
MULR190
MULR200
MULR210
MULR220
MULR230
MULR240
MULR250
MULR260
MULR270
MULR280
MULR290
MULR300
MULR310
MULR320
MULR330
MULR340
MULR350
MULR360
MULR370
MULR380
MULR390
MULR400
MULR410
MULR420
MULR430
MULR440
MULR450
MULR460
MULR470
MULR480
MULR490

```



LINBO3 CONWAY PHASE N4
MULER. - EFN SOURCE STATEMENT - IFN(S) -

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C.....ACT CONVERGENT YET.....

450 FX = FX1
FX1 = FX2
FX2 = FX3
X = X1
X1 = X2
X2 = X3
F1 = H2
F2 = X2 - X1
L2 = F2/H1
540 D2 = 1. + L2
MCDE= 2
X = X2
570 RETURN
580 X = X2
GC TO 570
END

MULER500 48
MULER510
MULER520
MULER530
MULER540
MULER550
MULER560
MULER570
MULER580
MULER590
MULER600
MULER610
MULER620
MULER630
MULER640
MULER650
MULER660

LINBO3 CCMWAY PHASE

N4

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SIBFTC PCL... DECK

CPOL

```

      COMPLEX FUNCTION POL (X)
      CCMPCN /ORDER/ C, K, M
      COMPLEX T, C(9), X
      PCL = (0.,0.)
      T = (1.,0.)
      J = M
      DO 100 I = 1, M
      PCL = POL + C(I)*T
      CALL CVERCK
      T = T*X
100  J = J - 1
      RETURN
      END

```

```

POL00020
POL00030
POL00040
POL00050
POL00060
POL00070
POL00080
POL00090
POL00100
POL00110
POL00120
POL00130
POL00140

```

7

LINDO3 CONWAY PHASE

N4

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000109

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SIBFTC TFUN.. DECK

```

CTFUN
  CCMPLX FUNCTION TFUN (ETA, MX, C1, C2, C3, C4, C5, C6, C7, C8)
  COMMON /LINK/ JUNK1(244), ALFAB(4), JUNK2(36), BETAB(4,4),
    JUNK3(8), VS, JUNK4(17)
    /CIA / IA(4)
  CCMPLX ALFAB, BETAB, ETA(4)

  CDEBUG ETA, MX, C1, C2, C3, C4, C5, C6, C7, C8
  TFUN = (0., 0.)
  DO 80 K = 1, 4
    I = IA(K)
    CDEBUG I
    CDEBUG TFUN
    80 TFUN = TFUN + ETA(I)*I
      - BETAB(1,K)*(CMPLX(0., C1) + ALFAB(I)*C2)/VS
      - BETAB(2,K)*(CMPLX(0., C3) + ALFAB(I)*C4)/VS
      - BETAB(3,K)*(CMPLX(0., C5) + ALFAB(I)*C6)/VS
      - BETAB(4,K)*(CMPLX(0., C7) + ALFAB(I)*C8)/VS
      + CEXP(-ALFAB(I)*MX)/VS
  CDEBUG TFUN
  RETURN
END

```

TFUN0020
TFUN0030
TFUN0040
TFUN0050
TFUN0060
TFUN0070
TFUN0080

TFUN0090
TFUN0100
TFUN0110

TFUN0120
TFUN0130
TFUN0140
TFUN0150
TFUN0160
TFUN0170

17

TFUN0180
TFUN0190

LINB03 CONWAY

PHASE

N4

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SIBFTC PIFUN. DECK

```

CPIFUN
  CCMPLX FUNCTION PIFUN (ETA, C1, C2, C3, C4, C5, C6, C7,
    .               C8, C9, C10, C11, C12, C13, C14, C15,
    .               C16, C17, C18, C19, C20, C21, C22, C23,
    .               C24)
  CCMCN /LINK/ JUNK1(24), ALFAB(4), JUNK2(36), BETAB(4,4),
    .       JUNK3(26) /BETAN/ NBETA
  .       /CIA / IA(4)
  CCMPLX ALFAB, BETAB, ETA(4), J
  DATA J / (0.,1.) /

  NK4 = NBETA + 1
  PIFUN = (0.,0.)
  DC 230 I = 1, 4
  I = IA(I)
  CC 230 K = 1, NK4
  M = IA(K)
  PIFUN=PIFUN + (ETA(I)*CONJG(ETA(K))/(ALFAB(I) + CONJG(ALFAB(M))))
  .   + (CONJG(BETAB(1,K))*(BETAB(1,I)*(C1 - J*C2*ALFAB(I))
  .   + BETAB(2,I)*(C3 - J*ALFAB(I)*C4)
  .   + BETAB(3,I)*(C5 - J*ALFAB(I)*C6)
  .   + BETAB(4,I)*(C7 - J*ALFAB(I)*C8))
  .   + CONJG(BETAB(2,K))*(BETAB(1,I)*(C9 - J*ALFAB(I)*C10)
  .   + BETAB(2,I)*(C11 - J*ALFAB(I)*C12)
  .   + BETAB(3,I)*(C13 - J*ALFAB(I)*C14)
  .   + BETAB(4,I)*(C15 - J*ALFAB(I)*C16))
  .   + CONJG(BETAB(3,K))*(BETAB(1,I)*(C17 - J*ALFAB(I)*C18)
  .   + BETAB(2,I)*(C19 - J*ALFAB(I)*C20)
  .   + BETAB(3,I)*(C21 - J*ALFAB(I)*C22)
  .   + BETAB(4,I)*(C23 - J*ALFAB(I)*C24)))
230 CCNTINUE
  PIFUN = PIFUN/2.
  RETURN
  END

```

```

PIFUN020
PIFUN030
PIFUN040
PIFUN050
PIFUN060
PIFUN070
PIFUN080
PIFUN090
PIFUN100
PIFUN110
PIFUN120
PIFUN130
PIFUN140
PIFUN150
PIFUN160
PIFUN170
PIFUN180
PIFUN190
PIFUN200
PIFUN210
PIFUN220
PIFUN230
PIFUN240
PIFUN250
PIFUN260
PIFUN270
PIFUN280
PIFUN290
PIFUN300
PIFUN310
PIFUN320
PIFUN330
PIFUN340
PIFUN350

```

LINBO3 CONWAY PHASE

N4

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SIBFTC CDET.. DECK

```

CCDET      SUBROUTINE CDET      6-12-68      CDET0000
C          DETERMINANT OF COMPLEX MATRIX.  DET(A) = F * 10**M      CDET0001
          SUBROUTINE CDET(A,N,F,M)      0001
          DATA PI/3.14159265/      0002
          DIMENSION A(N,M),S(2)      0003
          COMPLEX A,F,B      0004
          EQUIVALENCE (S,PHI,B),S(2),SUM      0005
          ALOGB2(X)=ALCG(X)/ALCG(2.)      2    3
          SGN = 1.      0006
          CC 15 I=2,N      0007
          II=I-1      0008
          CC 15 J=1,II      0009
          IF (REAL(A(I,J)).EQ.0..AND.AIMAG(A(I,J)).EQ.0.) GO TO 15      0010
          IF (REAL(CABS(A(I,J))) - REAL(CABS(A(I,J)))) 11,10,10      20    22
          CC 12 K=J,N      0012
          B = A(J,K)      0013
          A(J,K)=A(I,K)      0014
          A(I,K) = B      0015
          SGN = - SGN      0016
          IF (REAL(A(I,J)).EQ.0..AND.AIMAG(A(I,J)).EQ.0.) GO TO 15      0017
          B = A(I,J)/A(I,J)      0018
          JJ=J+1      0019
          CC 14 L=JJ,N      0020
          A(I,L) = A(I,L) - B*A(J,L)      0021
          15 CONTINUE      0022
          16 SUM=0.      0023
          PHI=0.      0024
          CC 20 I=1,N      0025
          IF (REAL(A(I,I)).EQ.0..AND.AIMAG(A(I,I)).EQ.0.) GO TO 18      0026
          SUM = SUM + ALOGB2(REAL(CABS(A(I,I))))      0027    63
          PHI=PHI+ATAN2(AIMAG(A(I,I)), REAL(A(I,I)))/PI      0028    67
          SUM = SUM*.301029996      0029
          PHI = PI*PHI      0030
          F=CMPLX(COS(PHI),SIN(PHI))      0031    70    71
          S = ABS(SUM) - 37.      0032
          IF (S.LE.0.) GO TO 100      0033
          M = SIGN(S,SUM)      0034
          F = F*10.**((SUM-FLOAT(M)))      0035    76
          GO TO 200      0036
          100 M = 0      0037
          F = F*10.**SUM      0038    80
          200 IF (SUM.GT.0.) RETURN      0039
          F = - F      0040
          RETURN      0041
          M = 0      0042
          F=10.**0.      0043
          RETURN      0044
          END      0045

```

SIBFTE CMATS. DECK

CCMATS	SUBROUTINE CMATS	6-12-68	CMATS020
C	SOLUTION OF COMPLEX LINEAR EQUATIONS		CMATS030
	SUBROUTINE CMATS(A,X,N1,M1,P)		CMATS040
	DIMENSION A(N1,50),X(N1,M1)		CMATS050
	COMPLEX A,X,R		CMATS060
	M=P		CMATS070
	A=N1		CMATS080
	IF (M) 109,199,5		CMATS090
5	NP1 = N - 1		CMATS100
	IF (NM1) 199,160,6		CMATS110
6	PP=A+M		CMATS120
	II = 1		CMATS130
	DO 155 I=2,N		CMATS140
	CC15 J=1,II		CMATS150
	IF (REAL(A(I,J)) .EQ. 0. .AND. AIMAG(A(I,J)) .EQ. 0.) GO TO 15		CMATS160
	IF (CABS(A(J,J)) - CABS(A(I,J))) 11,10,10	22 24	CMATS170
11	DO 12 K=J,PP		CMATS180
	R = A(J,K)		CMATS190
	A(J,K)=A(I,K)		CMATS200
12	A(I,K) = R		CMATS210
	IF (REAL(A(I,J)) .EQ. 0. .AND. AIMAG(A(I,J)) .EQ. 0.) GO TO 15		CMATS220
10	R = -A(I,J)/A(J,J)		CMATS230
	JJ = J + 1		CMATS240
	CC14 K=JJ,PP		CMATS250
14	A(I,K) = A(I,K) + R*A(J,K)		CMATS260
	A(I,J) = (0.,0.)		CMATS270
15	CCNTINUE		CMATS280
155	II = I		CMATS290
160	DO 166 I=1,N		CMATS300
166	IF (REAL(A(I,1)) .EQ. 0. .AND. AIMAG(A(I,1)) .EQ. 0.) RETURN 1		CMATS310
	DO 28 JJ=1,M		CMATS320
	KK=A+J		CMATS330
	X(N,J)=A(N,KK)/A(N,N)		CMATS340
	IF (NM1) 287,287,287		CMATS350
287	JJ = N		CMATS360
	DO 289 I=2,N		CMATS370
	II = JJ		CMATS380
	JJ = JJ - 1		CMATS390
	R = (0.,0.)		CMATS400
	DO 25 K=II,N		CMATS410
25	R = R + A(JJ,K)*X(K,J)		CMATS420
289	X(JJ,J) = (A(JJ,KK) - R)/A(JJ,JJ)		CMATS430
28	CCNTINUE		CMATS440
	RETURN		CMATS450
199	RETURN 1		CMATS460
	END		CMATS470

LIN803 CCNDAY PHASE

N4

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SIBFTC CVADK DECK

```
SUBROUTINE OVERCK  
COMMON/OVR/KO,N1  
EQUIVALENCE (IAND,XAND)  
COMMON KOUNT(1)  
DATA MASK/C000000077777/  
KC=0  
XAND=AND(MASK,KOUNT(1))  
KNTFPT=IAND*239  
KNTSAV=KOUNT(KNTFPT)  
KNT1=KNTSAV/32768  
KNT2=KNTSAV-32768*KNT1  
IF(KNT2.LE.1) RETURN  
KOUNT(KNTFPT)=32768*KNT1+1  
KC=1  
RETURN  
END
```


SYMBOL	LINB03	LINB03	CONWAY	PHASE	H4		TYPE	LOCATION	SYMBOL	TYPE	LOCATION	TYPE
					STORAGE	MAP						
MBETA	00000		I									
	00000	COMMON	BLOCK	CSET	ELIP		00074	ORIGIN		R	00003	
CLIM	00000		R					00001	LENGTH		00002	R
	00000	COMMON	BLOCK	FROOT	NT		00077	ORIGIN	LENGTH		00003	I
FVSMAG	00000		R					00001	ICASE		00002	
	00000	COMMON	BLOCK	CGM	EPSR		00702	ORIGIN	LENGTH		00002	
ACAP	00000		L					00001				
	00000	COMMON	BLOCK	CIA			00704	ORIGIN	LENGTH		00004	
IA	00000		I									
	00000	COMMON	BLOCK	ALESS			00710	ORIGIN	LENGTH		00001	
IALF	00000		L									
	00000	COMMON	BLOCK	CIFL			00711	ORIGIN	LENGTH		00001	
HXAGNL	00000		L									
	00000	COMMON	BLOCK									
DIMENSIONED PROGRAM VARIABLES												
SYMBOL	LOCATION		TYPE	SYMBOL			LOCATION		SYMBOL		LOCATION	TYPE
PANGLE	01470		R	XX			01755		YY		01757	
DELTA	01761		R	R11			02246		RI11		02533	R
	03020		R	RIU1			03305		RE1		03572	R
REI1	04057		R	DETRAY			04344		DETRAY		04510	R
VSARAY	04654		R	AAAAA			05020		DISP		05330	R
DEC	05340		R	TITLES			01464		VELOC		00712	C
VELDCI	01177		R	VEL			00712		PLOTIT		05625	I
XEL	05633		C	XL			05655		XET		05671	C
EA	05675		C	EB			05711		UA		05721	C
UB	05727		C	ETA			05735		EX		05761	C
MAGU	05771		R	PHASEU			05775		TITLE		01464	R
UNDIMENSIONED PROGRAM VARIABLES												
SYMBOL	LOCATION		TYPE	SYMBOL			LOCATION		SYMBOL		LOCATION	TYPE
BLIMIT	06001		I	ELIPIT			06002		NEGAT		06003	L
RUIT	06004		L	FXL			06005		FVS		06007	C
PHIB	06011		C	FXU			06013		CX1		06015	C
U	06017		C	PIH			06021		P2H		06033	C
	06025		C	TW32			06027		TW3		06031	C
TW11	06033		C	TW12			06035		TW22		06037	C
S11	06041		C	S22			06043		S33		06045	C
S12	06047		C	S13			06051		S23		06053	C
D1	06055		C	D2			06057		D3		06061	C
JIMAG	06063		C	E1			06065		E3		06067	C
NUA	06071		R	NUMAX			06072		REPEAT		06073	L
W	06074		R	DVS			06075		VSMAX		06076	R
DMU	06077		R	DMX			06100		WMAX		06101	R
JJK	06102		I	I			06103		VSAVE		06104	R
KOUNT	06105		I	KCNT			06106		KNT1		06107	I
KNT2	06110		I	KNT3			06111		KTIME		06112	I

LINBO3 LINBO3 CONWAY PHASE STORAGE MAP

SMU	06113	R	SMX	06114	R	COA	06115	R
VS1	06116	R	VS2	06117	R	AM1	06120	R
AM2	06121	R	C08	06122	R	VS0	06126	R
N	06124	I	VS1	06125	R	EL1	06131	I
EL2	06127	K	KGO	06130	I	NBETAL	06134	I
W1	06132	I	K	06133	I	M2	06137	R
J	06135	I	YMIN	06136	R	DY	06142	R
OMIN	06140	R	RTMIN	06141	R	DR	06145	R
SPEC	06143	R	TT31	06144	R	TT32	06150	R
TT33	06146	R	TT11	06147	R	TT12	06153	R
TT22	06151	R	SS11	06152	R	SS22	06156	R
SS33	06154	R	SS12	06155	R	SS13	06161	R
SS23	06157	R	PP1M	06160	R	PP2M	06164	R
DD1	06162	R	DD2	06163	R	DD3	06167	R
DD1	06165	R	UU2	06166	R	UU3	06172	R
EE1	06170	R	EE3	06171	R	RRR	06175	R
RII1	06173	I	INCR	06174	I	VS0	06200	I
JJ	06176	I	JJJ	06177	I	NNNN		
XMIN	06201	R	DX	06202	R			

ENTRY POINTS

..... SECTION 17

SUBROUTINES CALLED

TFUN	SECTION 18	PIFUN	SECTION 19	FRDD	SECTION 20
SKFILE	SECTION 21	CRTPLT	SECTION 22	PLT	SECTION 21
-FRCU	SECTION 24	SETCTE	SECTION 25	ROOT	SECTION 26
-CFCP	SECTION 27	-FMAD	SECTION 28	-PSLO	SECTION 29
-CFUN	SECTION 30	CHATS	SECTION 31	CABS	SECTION 32
-CFMP	SECTION 33	CEXP	SECTION 34	OVERCK	SECTION 35
SCALE	SECTION 36	AXIS	SECTION 37	LINE	SECTION 38
-FEFT	SECTION 39	ATAN2	SECTION 40	SORT	SECTION 41
ATAN	SECTION 42	COS	SECTION 43	SIN	SECTION 44
ENDPLT	SECTION 45	-FXEM	SECTION 46	-UMOS	SECTION 47
-FRM	SECTION 48	-FCNV	SECTION 49	-UMOS	SECTION 50
-FFIL	SECTION 51	-UNO2	SECTION 52	E-1	SECTION 53
E-2	SECTION 54	E-3	SECTION 55	E-4	SECTION 56
CC-1	SECTION 57	CC-2	SECTION 58	CC-3	SECTION 59
CC-4	SECTION 60	SYSLOC	SECTION 61		

EFN IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION
7773	7774	7A	20	11A	07570
510	1	FORMAT	520	35A	07725
2	530	45A	535	50A	10006
32	550	58A	650	434A	12754
1500	615	74A	17C7	FORMAT	06435
640	1502	FORMAT	1705	FORMAT	11430
1240	17C8	263A	122A	FORMAT	07102
	175A	141A			10426

	LINB03	CONMAY	PHASE	N4 STORAGE MAP	12/31/65	000109	PAGE 51
100							
105	131A	10534	60	127A	110	139A	10666
110	137A	10641	174C	451A	1100	152A	11017
115	145A	10740	1095	146A	1120	FORMAT	06732
120	145A	10740	1310	188A	1300	184A	11143
125	145A	10740	149C	239A	1480	237A	11457
130	145A	10740	152C	245A	1550	FORMAT	06777
135	145A	10740	1710	302A	38	288A	11656
140	145A	10740	35	292A	34	336A	12106
145	145A	10740	87	363A	86	343A	12113
150	145A	10740	75	45A	76	419A	12654
155	145A	10740	175C	FORMAT	1790	459A	13032
160	145A	10740	189C	489A	1650	476A	13126
165	145A	10740	71	516A	219C	537A	14016
170	145A	10740	73	564A	2340	FORMAT	07121
175	145A	10740	1506	FORMAT	18	591A	15443
180	145A	10740	2441	FORMAT	3	FORMAT	07370
185	145A	10740	32767	FORMAT	5	687A	16041

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 16216.

LIB03 CCNWAY PHASE 12/31/65 000109 PAGE 52
 MOUT.. STORAGE MAP

SUBROUTINE ROCT

COMMON VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	ORIGIN	TYPE	LENGTH	LOCATION	TYPE
GETOUT	00000	L	GET	00001	00001		00001	
ICHECK	00000	L	PLOTS	00002	00002		00001	
FVSMAG	00000	R	FRCCT NT	00003	00003		00003	I
IALF	00000	L	ALESS	00006	00006		00001	

DIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
SUB	00007	R			

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
PREV	00037	L	FXG	00040	C
FX2	00044	C	FX3	00046	C
LANDA3	00052	C	T	00054	C
T2	00060	C	EL	00062	C
KKKK	00065	I	FACT1	00066	R
X2	00070	R	M2	00071	R
H3	00073	R	KICK	00074	I

ENTRY POINTS

RCOT	SECTION	6	EFN	IFN	LOCATION	EFN	IFN	LOCATION
F	SECTION 7							
.GMP.	SECTION 10							
.CFCP.	SECTION 13							
.FFIL.	SECTION 16							
E.2	SECTION 19							
SYSLOC	SECTION 22							

SUBROUTINES CALLED

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
ROUND	SECTION 8		XP2.	SECTION 9	
CSORT	SECTION 11		CABS	SECTION 12	
.FMRD.	SECTION 14		.UNO6.	SECTION 15	
.FCNV.	SECTION 17		E.1	SECTION 18	
E.3	SECTION 20		E.4	SECTION 21	

ROOT..	LINK03	CONWAY	PHASE	N4 STORAGE	MAP	12/21/69	000109	PAGE	53
600	70A	01016	530	65A	01007	230	39A	00353	
300	49A	01023	100C	FORMAT	00132	998	82A	01033	
999	83A	01040							

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 01167.

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SETCT. LINB03 CONWAY PHASE N4 STORAGE MAP

SUBROUTINE SETCTE

COMMON VARIABLES

SYMBOL	LOCATION	COMMON BLOCK	TYPE	ROTAT	SYMBOL	ORIGIN	LOCATION	TYPE	LENGTH	SYMBOL	LOCATION	TYPE
ROTATE	00000		L									
C	00000	COMMON BLOCK	R	ZTZ	E	ORIGIN	00024	00002			00001	
								R				
GAMMA	00000	COMMON BLOCK	D	FRT	D	ORIGIN	00022	00034				
								D				
G	00000	COMMON BLOCK	R	GPEPS	P	ORIGIN	00025	00426				
								R				
CLIM	00000	COMMON BLOCK	R	CSET	ELIM	ORIGIN	00001	00506				
								R				
AC12	00000	COMMON BLOCK	L	CSET1	AC23	ORIGIN	00001	00511				
AC14	00003		L		AC34		00004	L				
								L				
NBETA	00000	COMMON BLOCK	I	BETAN		ORIGIN		00516				

DIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
CPR	00517	R	EPR	00563	R
LAGE	00616	I	LAGE	00624	I

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
DR	00637	D	RM	00641	D
RL	00645	D	CM	00647	D
CH	00653	D	SN	00655	D
SL	00661	D	I	00663	I

ENTRY POINTS

SETCTE	SECTION	9	SUBROUTINES CALLED	TIJ	SECTION	12
FF	SECTION	10	R		SECTION	15
CSFUN	SECTION	13	.FMRC.	.FSLD.	SECTION	19
.UMC6.	SECTION	16	.FFIL.	.FCNV.	SECTION	

SETCT. LINB03 CONWAY		PHASE	N4 STORAGE MAP	12/31/69	000109	PAGE 55
SYSLCC SECTION 19						
EFN	IFN	LOCATION	EFN	IFN	LOCATION	
1501	52A	02326	1602	101A	02731	
20	109A	02742	30	137A	03112	
470	148A	03125	1595	FORMAT	00715	
1604	224A	03430				
		140C				

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 03464.

ROUND.		LINB03	CONWAY	PHASE	N4 STORAGE MAP	12/31/69	000109	PAGE	50
UNDIMENSIONED PROGRAM VARIABLES									
SYMBOL	LOCATION	TYPE	FUNCTION	ROUND	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
P-000C	000C1	C	I		00003	R	R	00004	R
ENTRY POINTS									
PCUND	SECTION	2							
SUBROUTINES CALLED									
SYSLOC	SECTION	3							
EFM	IFN	LOCATION	EFN	IFN	CORRESPONDENCE	EFN	IFN	LOCATION	
100	12A	0C052	50	9A				00041	
THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00C77.									

FF.... LINB03 CONWAY PHASE NA STORAGE MAP 12/31/69 000109 PAGE 58

FUNCTION		FF	TYPE	D
COMMON VARIABLES				
COMMON BLOCK		FRT		
SYMBOL	LOCATION	TYPE	ORIGIN	00001
GAMMA	0000C	D	SYMBOL	LOCATION
			D	00022
UNDIMENSIONED PROGRAM VARIABLES				
SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION
F-0000	00353	D	R	00355
T	00357	I		
ENTRY POINTS				
FF	SECTION	3	SUBROUTINES CALLED	
SYSLOC	SECTION	4		
EFN	IFN	LOCATION	EFN	IFN
50	11A	00465	EFN	IFN
CORRESPONDENCE				
LOCATION				
THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00546.				

R.	LINE	CONWAY	PHASE	M4 STORAGE MAP	12/31/69	000109	PAGE 59
			FUNCTION R	TYPE D			
			COMMON VARIABLES				
			FRT	ORIGIN 00001	LENGTH	00352	
			COMMON BLOCK	TYPE D			
			LOCATION 00000	LOCATION 00022	SYMBOL C	LOCATION 00264	TYPE D
			UNDIMENSIONED PROGRAM VARIABLES				
			SYMBOL F-0000	TYPE D	SYMBOL I	LOCATION 00356	TYPE I
			LOCATION 00353	LOCATION 00355			
			ENTRY POINTS				
			R	SECTION 3			
			SUBROUTINES CALLED				
			SYSLOC	SECTION 4			
			EFN 50	IFN 9A	LOCATION 00443	EFN IFN	LOCATION
			THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00515.				

TIJ.		LINB03	CONWAY	PHASE	N4 STORAGE MAP		12/31/69	000109	PAGE 60
FUNCTION		TIJ	TYPE	D					
COMMON VARIABLES									
COMMON BLOCK		FRT							
SYMBOL	LOCATION	TYPE	ORIGIN	00001	LENGTH	00352	LOCATION	TYPE	
GAMMA	00000	D	SYMBOL	LOCATION	SYMBOL	00060	00047	R	
			JUNK	00022	EPSLON				
COMMON BLOCK		GPEPS	ORIGIN	00353					
G	00000	R	00025	R					
UNDIMENSIONED PROGRAM VARIABLES									
SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	
F.0000	00433	D	R	00435	I				
ENTRY PCINTS									
TIJ		SECTION	4						
SUBROUTINES CALLED									
SYSLOC		SECTION	5						
EFN	IFN	LOCATION	EFN	IFN	CORRESPONDENCE	EFN	IFN	LOCATION	
50	7A	00500							
THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00545.									

F.....				LINB03 CONWAY		PHASE	N4 STORAGE MAP		12/31/65	000109	PAGE 61	
				FUNCTION F		TYPE C	COMMON VARIABLES		LENGTH	LOCATION	TYPE	
				ZTZ			CRIGIN		SYMBOL			
				COMMON BLOCK			COMMON BLOCK					
				LOCATION		TYPE	SYMBOL					
				CE			CE					
CC	00000	R		00000	R		00000	R	CT	00045	R	
C11	00000	R		00000	R		00000	R	C14	00002	R	
C15	00003	R		00003	R		00003	R	C34	00005	R	
C35	00006	R		00006	R		00006	R	C44	00010	R	
C45	00011	R		00011	R		00011	R	C55	00013	R	
C56	00014	R		00014	R		00014	R	C16	00016	R	
E11	00024	R		00024	R		00024	R	E14	00026	R	
E15	00027	R		00027	R		00027	R	E31	00031	R	
E33	00032	R		00032	R		00032	R	E35	00034	R	
E36	00035	R		00035	R		00035	R	T13	00046	R	
T33	00047	R		00047	R		00047	R				
				COMMON BLOCK			COMMON BLOCK					
FVSMAG	00000	R		00000	R		00000	R	LENGTH	00003	I	
				COMMON BLOCK			COMMON BLOCK					
NPETA	00000	I		00000	I		00000	I	ICASE	00002	I	
				COMMON BLOCK			COMMON BLOCK					
AC12	00000	L		00000	L		00000	L	LENGTH	00005	L	
AC14	00003	L		00003	L		00003	L	AC24	00002	L	
				COMMON BLOCK			COMMON BLOCK					
ALFA	00000	C		00000	C		00000	C	LENGTH	00032	C	
ALFAA	00250	C		00250	C		00250	C	BETAA	00374	C	
BETAB	00440	C		00440	C		00440	C	MUA	00501	C	
LANEAB	00502	R		00502	R		00502	R	MUB	00504	R	
VSK	00505	R		00505	R		00505	R	RHOB	00507	R	
DIGIT	00510	R		00510	R		00510	R	EPSLON	00512	R	
MAB	00513	R		00513	R		00513	R	MXA	00515	R	
ALL	00514	R		00514	R		00514	R	KM	00520	R	
COEFF	00521	L		00521	L		00521	L	ITER	00523	L	
ALPHA	00527	L		00527	L		00527	L	POLYN	00526	L	
ELL	00040	C		00040	C		00040	C	MAX	00531	I	
				COMMON BLOCK			COMMON BLOCK					
IA	00000	I		00000	I		00000	I	LENGTH	00004		
				COMMON BLOCK			COMMON BLOCK					
ACAP	00000	L		00000	L		00000	L	LENGTH	00002		
				COMMON BLOCK			COMMON BLOCK					
XG	00000	R		00000	R		00000	R	LENGTH	00040	R	
				COMMON BLOCK			COMMON BLOCK					
IALF	00000	L		00000	L		00000	L	LENGTH	00047		
				COMMON BLOCK			COMMON BLOCK					
	00000	L		00000	L		00000	L	LENGTH	00001		

F.....				N4		PAGE 62	
LINB03		CCNNAY	PHASE	STORAGE MAP		12/31/65	000109
COMMON BLOCK		L	CIFL	ORIGIN	00705	LENGTH	00001
00000				DIMENSIONED PROGRAM VARIABLES			
SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION
EA	00746	C	EB	00762	C	UA	00772
UB	01000	C	POLY	01006	C	01	00706
ALF	01030	C	AGLD	01050	C	XEL	01072
XEL	01422	C	88	01424	C	881	01454
BETAB1	01476	C	B1A	01506	C	BETAX	01522
B1B	01530	C	CA	01570	R	CB	01610
CD	01630	R	B1D	00706	R	ALAB	01650
IA1	01661	I	IA2	01664	I	LAB	01667
BOX	01672	R	MB	01676	I		
UNDIMENSIONED PROGRAM VARIABLES							
SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION
FLO000	01702	C	FVS	01704	C	JIMAG	01706
DA	01710	C	DB	01712	C	DC	01714
ALF1	01716	C	ALF2	01720	C	CHX	01722
CH1	01724	C	RVS	01726	R	ICB	01727
K1	01730	I	K2	01731	I	L1	01732
L2	01733	I	IB1	01734	I	IB2	01735
I	01736	I	K	01737	I	RA	01740
IC	01741	I	K4	01742	I	J	01743
TEN	01744	R	TEN10	01745	R	KK	01746
KEAP	01747	I	BEN	01750	R	JA	01751
J1	01752	I	J2	01753	I	MI	01754
J2	01755	I	J1	01756	I	I2	01757
JB	01758	R	CA22	01761	R	CAMIN	01762
TEHH	01760	R	L	01764	I	I13	01765
I11	01763	I	L	01767	I	ST	01770
I14	01766	I	DD	01772	R		
ICB1	01771	I					
ENTRY POINTS							
F	SECTION	12	SUBROUTINES CALLED				
.CFMP.	SECTION 13		STRIP	SECTION 14	CRDOT	SECTION 15	
CSORT	SECTION 16		.CFDP.	SECTION 17	CABS	SECTION 18	
.FMRD.	SECTION 19		CHATS	SECTION 20	CODET	SECTION 21	
.CXPL.	SECTION 22		CUSH	SECTION 23	SINH	SECTION 24	
TANH	SECTION 25		CERP	SECTION 26	.FXEM.	SECTION 27	
.UNOG.	SECTION 28		.FFIL.	SECTION 29	.FCNV.	SECTION 30	
E.1	SECTION 31		E.2	SECTION 32	E.3	SECTION 33	
E.4	SECTION 34		SYSLOC	SECTION 35			

F.....

L1NB03		CONWAY	PHASE	N4 STORAGE MAP		12/31/65	000109	PAGE 63
EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION
490	29A	0324	520	54A	0325	510	51A	03522
525	58A	0357	528	FORMAT	02063	789	74A	03563
630	94A	03624	610	87A	03636	640	132A	03732
637	130A	03721	631	107A	03651	634	124A	03710
400	560A	03325	636	126A	03713	140	373A	05340
632	117A	03666	633	FORMAT	02075	1357	739A	10262
280	501A	06042	638	FORMAT	02106	788	FORMAT	02131
810	143A	03766	980	303A	04734	940	286A	04635
830	184A	04170	820	174A	04141	100	322A	05003
200	408A	05451	975	FORMAT	02156	850	204A	04265
840	198A	04246	900	243A	04446	860	212A	04314
870	217A	04333	875	FORMAT	02142	1940	935A	12524
880	233A	04410	890	238A	04430	910	248A	04465
930	270A	04553	920	264A	04537	974	FORMAT	02150
983	622A	04545	982	315A	04762	110	335A	05042
120	346A	05070	125	350A	05107	130	365A	05326
270	605A	04450	150	383A	05370	160	FORMAT	02164
210	428A	05600	220	445A	05633	230	448A	05635
240	461A	05674	235	459A	05664	245	464A	05676
250	475A	05755	255	482A	05766	260	493A	06030
310	522A	06157	300	519A	06147	340	538A	05212
320	532A	06173	360	543A	06230	350	539A	06220
370	FORMAT	02210	430	581A	06407	410	574A	06364
420	578A	04376	450	600A	06441	440	596A	06436
460	603A	06445	584	FORMAT	02263	985	FORMAT	02275
1390	761A	10415	1330	708A	10115	1190	577A	07536
1170	673A	07526	1220	684A	07555	1290	696A	07635
1250	688A	07566	1280	694A	07623	1340	715A	10145
1335	FORMAT	02304	1343	725A	10212	1360	740A	10264
1370	742A	10320	1380	744A	10354	1345	729A	10214
1350	735A	10230	1355	FORMAT	02322	1385	755A	10377
1505	FORMAT	02334	1550	782A	11037	1595	FORMAT	02344
1620	811A	11154	1650	822A	11204	1740	850A	11444
1910	889A	12363	1515	859A	12404	1930	FORMAT	02355
1940	928A	12507	1970	FORMAT	02364			

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 12600.

[illegible]

LINB03		CONWAY	PHASE	NA STORAGE MAP		12/31/65	000109	PAGE 65
STRIP.				EFN	IFN	CORRESPONDENCE		
EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION
670	5A	00375	680	9A	00000	1030	88A	01245
1020	86A	01243	101C	84A	01240	1025	94A	01251
1050	FORMAT	00216	1155	111A	01321	1157	126A	01371

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 01526.

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CRUOT. LINB03 CONWAY PHASE NO. STORAGE MAP

SUBROUTINE CROCT

COMMON VARIABLES

COMMON BLOCK	ORDER	SYMBOL	LOCATION	ORIGIN	TYPE	LENGTH	SYMBOL	LOCATION	TYPE
00000	C	K	00000	00001	I	00024	M	00023	I

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
X1	00025	C	CXC	00027	C
h	00032	I	JUNK	00031	I

ENTRY POINTS

CROCT SECTION 3

SUBROUTINES CALLED

ROUND	SECTION	LOCATION	IFN	IFN	LOCATION	IFN	IFN	LOCATION
PULLER	4	00063	110	11A	00070	130	30A	00132
E.1	7	00337	120	24A	00124	240	52A	00252
E.4	10	00314	290	73A	00341			

IFN	IFN	LOCATION	IFN	IFN	LOCATION
100	6A	00063	110	11A	00070
200	69A	00337	120	24A	00124
250	67A	00314	290	73A	00341

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00422.

MULLER. LIMBO3 CONWAY PHASE 12/31/69 000109 PAGE 67

STORAGE MAP

SUBROUTINE MULLER

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
X1	00001	C	X2	00003	C	X3	00005	C
M2	00007	C	L2	00011	C	D2	00013	C
G	00015	C	FX	00017	C	FX1	00021	C
FX2	00023	C	D	00025	C	TMO	00027	C
CNE	00031	C	GPD	00033	C	GPD	00035	C
L3	00037	C	FX3	00041	C	H1	00043	C
K	00045	I	CGPD	00046	R	CGMC	00047	R

ENTRY POINTS

PULLER SECTION 2

SUBROUTINES CALLED

CABS	SECTION	3	•CFMP•	SECTION	4
CSORT	SECTION	6	•CFDP•	SECTION	7
E-2	SECTION	9	E-3	SECTION	10
SYSLOC	SECTION	12			

EFN IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION
190	11A	00175	220	14A	00210
340	31A	00572	580	57A	01023
420	47A	00730	450	51A	00740
570	56A	01022			

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 01076.

EFN	IFN	LOCATION
540	52A	01001
370	34A	00634
390	45A	00725

OVERCK	SECTION	5
E-1	SECTION	8
E-4	SECTION	11

LINB03 CONWAY PHASE N4 STORAGE MAP PAGE 68
 POL.... 12/31/65 000109

COMMON BLOCK:		ORDER		FUNCTION POL TYPE C		COMMON VARIABLES	
SYMBOL	LOCATION	TYPE	ORIGIN	SYMBOL	LOCATION	TYPE	ORIGIN
C	00000	C	00001	R	00022	I	00024
F.0000	00025	C	00027	I	00023	I	00031

UNDIMENSIONED PROGRAM VARIABLES
 ENTRY PCINTS

SUBROUTINES CALLED		OVERCK		E.1		E.4	
SYMBOL	LOCATION	TYPE	ORIGIN	SYMBOL	LOCATION	TYPE	ORIGIN
COMP.	00000	C	00001	R	00022	I	00024
E.2	00025	C	00027	I	00023	I	00031
SYSLCC	00025	C	00027	I	00023	I	00031

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00150.

TFUN..		LINK	CONWAY	PHASE	N4 STORAGE MAP		12/31/69	000109	PAGE 69
		FUNCTION		TFUN	TYPE	C			
		COMMON		VARIABLES					
		LINK	ORIGIN		00001				
		SYMBOL	LOCATION	TYPE	00001				
JUNK1	0000C	I	ALFAB	00364	C				
BETAB	00440	C	JUNK3	00500	I				
JUNK4	00511	I							
		COMMON BLOCK		CORRESPONDENCE					
IA	00000	I	CIA	00533					
		COMMON BLOCK		CORRESPONDENCE					
		COMMON BLOCK		CORRESPONDENCE					
		COMMON BLOCK		CORRESPONDENCE					
		COMMON BLOCK		CORRESPONDENCE					
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		COMMON BLOCK		CORRESPONDENCE					
		COMMON BLOCK		CORRESPONDENCE					

PIFUN.		LINB03	CONWAY	PHASE	M4 STORAGE MAP		12/31/65	000109	PAGE 73					
		FUNCTION		PIFUN	TYPE	C								
COMMON VARIABLES														
COMMON BLOCK														
SYMBOL	LOCATION	COMMON BLOCK	LINK	SYMBOL	LOCATION	TYPE	LENGTH	00532	TYPE					
JUNK1	00000	I		ALFAB	00364	C	SYMBOL	LOCATION	I					
BETAB	00440	C		JUNK3	00500	I	JUNK2	00374						
COMMON BLOCK														
MBETA	00000	I	BETAB	ORIGIN	00533		LENGTH	00001						
COMMON BLOCK														
IA	00000	I	CIA	ORIGIN	00534		LENGTH	00004						
UNDIMENSIONED PROGRAM VARIABLES														
SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE						
F-0000	00540	C	J	00542	C	NR4	00544	I						
I	00545	I	L	00546	I	M	00547	I						
ENTRY POINTS														
PIFUN	SECTION	5	SUBROUTINES CALLED											
-CFMP-	SECTION	6	-CFDP-	SECTION	7	E-1	SECTION	8						
E-2	SECTION	9	E-3	SECTION	10	E-4	SECTION	11						
SVSLCC	SECTION	12	CORRESPONDENCE											
EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION						
230	33A	02102												

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 02233-

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 02333.

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LIBR03 CONWAY PHASE N4 STORAGE MAP
CDET.. SUBROUTINE CDET

SYMBOL LOCATION TYPE SYMBOL LOCATION TYPE
S 00001 R

SYMBOL LOCATION TYPE SYMBOL LOCATION TYPE
B 00001 C
SGN 00003 R
J 00006 I

ENTRY POINTS

CDET SECTION 2

SUBROUTINES CALLED

ALOG SECTION 3
-CFMP. SECTION 6
SIN SECTION 9
E-2 SECTION 12
CC-1 SECTION 15
CC-4 SECTION 18

EFN IFN LOCATION EFN IFN CORRESPONDENCE
6 6A 00073 15 4A 00335
9 18A 00163 11 24A 00212
12 30A 00231 14 44A 00302
20 64A 00410 18 84A 00563
200 81A 00547
THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00703.

SYMBOL LOCATION TYPE
SUM 00002 R
I 00005 I
PI 00010 R

-CFMP. SECTION 8
CC-1 SECTION 11
E-4 SECTION 14
CC-3 SECTION 17

EFN IFN LOCATION
6 13A 00150
10 38A 00253
14 53A 00343
100 78A 00530

CHATS. LIMB03 CCNWAY PHASE NA STORAGE MAP 12/31/65 000109 PAGE 72

SUBROUTINE CHATS

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE	SYMBOL	LOCATION	TYPE
R	00001	C	N	00003	I
AP1	00005	I	MM	00006	I
I	00010	I	J	00011	I
KK	00013	I			

ENTRY POINTS

CHATS SECTION 2

SUBROUTINES CALLED

CHATS	SECTION	SYMBOL	LOCATION	TYPE	CHATS	SECTION	SYMBOL	LOCATION	TYPE
E.1	3	CFDP	5A	I	E.2	7	CFDP	5A	I
E.4	6	SYLOC	5A	I	E.3	8	CFDP	5A	I

E.1 IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION
199	98A	0C610	5	5A	00020
6	8A	0C56	155	5A	00020
11	26A	0C174	10	40A	00323
14	46A	00266	166	63A	00336
287	78A	0C443	289	90A	00527

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS C0733.

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OVERCK LIN803 CONWAY PHASE M4 STORAGE MAP

SUBROUTINE OVERCK

COMMON VARIABLES

COMMON BLOCK OVR

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK //

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

SYMBOL LOCATION TYPE

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COMMON BLOCK 1

SYMBOL LOCATION TYPE

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SYMBOL LOCATION TYPE

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COMMON BLOCK 1

SYMBOL LOCATION TYPE

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

SYMBOL LOCATION TYPE

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COMMON BLOCK 1

SYMBOL LOCATION TYPE

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SYMBOL LOCATION TYPE

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00000 00000 1

COMMON BLOCK 1

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

OVERCK SECTION 4

SYSLCC SECTION 5

SUBROUTINES CALLED

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

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SYMBOL LOCATION TYPE

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SYMBOL LOCATION TYPE

00000 00000 1

COMMON BLOCK 1

SYMBOL LOCATION TYPE

EFN IFN CORRESPONDENCE

EFN IFN LOCATION

EFN IFN LOCATION

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EFN IFN LOCATION

THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 00103.

COMPILATION LIN803

1 SOURCE ERROR 13 LEVEL 1 - WARNING ONLY

THIS STATEMENT CANNOT BE REACHED.

2 SOURCE ERROR 20 LEVEL 1 - WARNING ONLY

THE PROGRAM SHOULD END WITH A TRANSFER. A RETURN HAS BEEN GENERATED.

COMPILATION ROOT..

1 SOURCE ERROR 13 LEVEL 1 - WARNING ONLY

THIS STATEMENT CANNOT BE REACHED.

PHASE 8 DIAGNOSTIC MESSAGES

EFN IFN LOCATION

DIAGNOSTIC MESSAGES

DIAGNOSTIC MESSAGES

	LINB03	CONWAY	PHASE	N4	COMPILATION	ROOT..	12/31/65	000109	PAGE 74
2	SCURCE ERROR DATA STATEMENT VARIABLE NAME -	290 5 TEN10	LEVEL 1 - WARNING ONLY GROUP 1 APPEARS ONLY IN DATA STATEMENT.		COMPILATION STRIP.			DIAGNOSTIC MESSAGES	
					PHASE 8 DIAGNOSTIC MESSAGES				
1	SCURCE ERROR DATA STATEMENT VARIABLE NAME -	290 1 TPI	LEVEL 1 - WARNING ONLY GROUP 1 APPEARS ONLY IN DATA STATEMENT.						

IBLUR	CONWAY	PHASE	N4	12/31/69	000109	PAGE	76
25. FIC8	53442						
26. F10H	54036						
27. F10S	55006						
28. F8CD	55376						
29. F800	55471						
30. F8RD	55544						
31. F8IN	55634						
32. F8R8	56771						
33. F8T1	56737						
34. F8V10	56760						
35. F8V10	57024						
36. F8V10	57025						
37. F8V10	57072						
38. F8V10	57100						
39. F8V10	57101						
40. F8V10	57102						
41. F8V10	57103						
42. F8V10	57104						
43. F8V10	57105						
44. F8V10	57106						
45. F8V10	57107						
46. F8V10	57110						
47. F8V10	57111						
48. F8V10	57112						
49. F8V10	57113						
50. F8V10	57114						
51. F8V10	57115						
52. F8V10	57116						
53. F8V10	57117						
54. F8V10	57118						
55. F8V10	57119						
56. F8V10	57120						
57. F8V10	57121						
58. F8V10	57122						
59. F8V10	57123						
60. F8V10	57124						
61. F8V10	57125						
62. F8V10	57126						
63. F8V10	57127						
64. F8V10	57128						
65. F8V10	57129						
66. F8V10	57130						
67. F8V10	57131						
68. F8V10	57132						
69. F8V10	57133						
70. F8V10	57134						
71. F8V10	57135						
72. F8V10	57136						
73. F8V10	57137						
74. F8V10	57138						
75. F8V10	57139						
76. F8V10	57140						
77. F8V10	57141						
78. F8V10	57142						
79. F8V10	57143						
80. F8V10	57144						
81. F8V10	57145						
82. F8V10	57146						
83. F8V10	57147						
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247. F8V10	57311						
248. F8V10	57312						
249. F8V10	57313						
250. F8V10	57314						
251. F8V10	57315						
252. F8V10	57316						
253. F8V10	57317						

UNLSEC CORE

BEGIN EXECUTION.

TRANSFORMED ELASTIC CONSTANTS (C11, I=1,20)

2.45C0000E 11
7.5C00000E 1C
0.
C.
2.03C0000E 11
0.
9.5C00000E C9
0.
7.5C00000E 10
0.
5.C0C0000F C5
6.5C00000E 1C

0.
6.600000E 10
0.
7.500000E 10
-9.000000E 09
0.
C.
5.300000E 10

TRANSFERRED PIEZOELECTRIC CONSTANTS (E11, 1-1,17)

1.300000E 00
2.000000E-01
0.
C.
0.
C.
-2.500000E 00
0.
3.700000E 00
0.
2.000000E-01
2.500000E 00
0.
2.500000E 00
C.
3.700000E 00

TRANSFERRED DIELECTRIC CONSTANTS (Y11, 1-1,5)

2.570000E-10
0.
3.900000E-10
C.
0.

COEFFICIENTS OF POLYNOMIAL

-0.002938E 24
0.
0.8562924E 23
0.
C.0513205E 24
-0.3294582E 24

-C.61C7594E 24 0.
 0.6680760E 23 0.
 C.1774096E 23 0.
 0.7146826E 12 0.
 -C.4749200E 21 0.

COEFFICIENTS OF POLYNOMIAL

-C.7522391E 00 0.
 0.1005840E 00 0.
 C.10C0000E 01 -0.
 -0.3869567E 00 0.
 -0.7174261E 00 0.
 0.8C58563E-01 0.
 0.2083934E-C1 -0.
 0.5578627E-C3 0.
 -0.5578627E-C3 0.

INTERMEDIATE ROOTS OF POLYNOMIAL

-1.63345E-01 1.20000E-01 1.63345E-C1 1.20000E-01 7.74633E-01 3.96277E-01 1.28413E-01 -6.48500E-02
 -7.74633E-01 3.96277E-01 -1.28413E-01 -6.48500E-02 1.03875E 00 -3.80417E-01 -1.03875E 00 -3.80417E-01

INTERMEDIATE POSITIVE ROOTS

1.63345E-01 1.20000E-01 7.74633E-01 3.96277E-01 1.28413E-01 -6.48500E-02 1.03875E 00 -3.80417E-01

RE-ORDERED ALPHAS (1 ROW, 3 ZERO CASE)

1.63345E-01 1.20000E-01 7.74633E-01 3.96277E-01 1.28413E-01 -6.48500E-02 1.03875E 00 -3.80417E-01

INTERMEDIATE BETA 8

0. 0. 2.07710E-11 5.01161E-11 -6.54416E-11 4.25594E-12 2.27313E-11 -6.97380E-11
 1.00000E-10 0. 2.16897E-11 -5.39077E-11 -2.24545E-10 5.24802E-10 2.08899E-11 -4.05626E-11
 0. 0. 1.00000E 00 0. 1.00000E 00 0. 1.00000E 00 0.

INTERMEDIATE L MATRIX BY COLUMNS

0. 0. 1.2250E 00 -2.54600E-C8 0. 0. 0. 0.
 0.21789E 00 5.29229E 00 0. 0. 2.50323E 00 -5.34639E 00 -2.20857E 00 1.82708E 00
 -3.14539E 01 -1.26878E 01 -0. -0. -4.38118E 00 1.01127E 01 -2.04377E 01 -9.92681E 00
 5.78751E 00 -7.83386E 00 0. 0. -4.56512E 00 -5.00197E 00 -1.81849E 00 -1.43465E 00

VS = 0.3480000E 04 F(VS) = -0.3933011E 02 -0.166C051E 02 MAG = 0.3937967E 02

COEFFICIENTS OF POLYNOMIAL

-0.6025385E 24 0.
 0.87C8330E 24 0.
 0. 0. 0.8562924E 23 0.
 -0.3321240E 24 0.
 -0.6365181E 24 0.
 0. 0. 0.7365250E 23 0.
 0.3101833E 23 0.
 0. 0. 0.4026611E 14 0.
 -0.8934538E 21 0.

COEFFICIENTS OF POLYNOMIAL

-0.6923692E 00 0.
 0.5833018E-01 0.

```

0.10C0000E C1 -0.
-C.7332261E 00 0.3613862E 00
0.
C.3561912E-01 -0.
0.
-0.1025575E-C2 0.

INTERMEDIATE ROOTS CF POLYNOMIAL
-2.04412E-01 1.20000E-01 2.04412E-01 1.20000E-01 7.77745E-01 3.98383E-01 1.53931E-01 -6.58000E-02
-7.77745E-01 3.98383E-01 -1.53931E-01 -6.58000E-02 1.04255E 00 -3.81573E-01 -1.04255E 00 -3.81573E-01

INTERMEDIATE POSITIVE ROOTS
2.04412E-01 1.20000E-01 7.77745E-01 3.98383E-01 1.53931E-01 -6.58000E-02 1.04255E 00 -3.81573E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )
2.04412E-01 1.20000E-01 7.77745E-01 3.98383E-01 1.53931E-01 -6.58000E-02 1.04255E 00 -3.81573E-01

INTERMEDIATE BETA 8
0.
1.00000E-10 0.
0.
0.
2.11134E-11 4.98897E-11 -6.37559E-11 4.84614E-12 2.30816E-11 -6.97654E-11
0.
2.11134E-11 -5.41177E-11 1.00000E 00 0.
0.
1.00000E 00 0.

INTERMEDIATE L MATRIX BY COLUMNS
0.
0.
1.53309E C0 -2.56C00E-C8 C.
8.25920E 00 5.26568E 00 0.
-2.72634E 01 -4.56332E 00 -0.
0.
5.81854E 00 -7.85857E 00 0.
0.
2.47791E 00 -5.43538E 00 -2.19442E 00 0.
-3.71741E 00 1.05007E 01 -1.79988E 01 -7.32309E 00
-4.57388E 00 -5.67108E 00 -1.80946E 00 -1.42852E 00

VS = 0.3445200E 04 F(VS) = -0.1975967E 03 -0.7503187E 02 MAC = 0.2113628E 03

COEFFICIENTS CF POLYNOMIAL
-0.6025385E 24 0.
C.
C.8611016E 24 0.
0.
-0.3207544E 24
0.
-0.4244250E 24 0.
C.
C.24333569E 23 0.
-0.
-0.668C190E 21 0.

COEFFICIENTS OF POLYNOMIAL
-C.70C1543E C0 0.
0.
0.5544150E-C1
0.
-C.
-C.10C0000E C1 -0.
-C.
-C.7253790E 00 0.
C.
C.2826146E-C1 0.8260365E-01
0.
-0.7757725E-C3 0.

INTERMEDIATE ROOTS CF POLYNOMIAL

```


1.52848E-01 1.20000E-01 7.73553E-01 3.55802E-01 1.22012E-01 -6.46375E-02 1.03791E 00 -3.80157E-01

INTERMEDIATE BETA 8

0. 0. 2.06950E-11 5.01652E-11 -6.58226E-11 4.09002E-12 2.26542E-11 -6.97317E-11
 1.00000E-10 0. 0. 0. 0. 0. 0. 0.
 0. 0. 2.18195E-11 -5.38595E-11 -2.47263E-10 5.44002E-10 -2.05302E-11 -4.05048E-11
 0. 0. 1.00000E 00 0. 1.00000E 00 0. 1.00000E 00 0.

INTERMEDIATE L MATRIX BY COLUMNS

0. 0. 1.14436E 00 0. 0. 0. 0. 0.
 6.20871E 00 5.25723F 00 0. 0. 2.50857E 00 -5.32668E 00 -2.21128E 00 1.82950E 00
 -3.26647E 01 -1.40485E 01 -0. -0. -4.56358E 00 9.96079E 00 -2.11046E 01 -1.07797E 01
 5.75671E 00 -7.82842E 00 0. 0. -4.56311E 00 -4.98678E 00 -1.82049E 00 -1.43625E 00

VS = 0.3487601E 04 F(VS) = -0.4064251E 00 -0.1963597E 00 MAG = 0.4522477E 00

COEFFICIENTS OF POLYNOMIAL

-0.4029385E 24 0.
 0. 0. 0.8562924E 23
 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0.
 -C. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 -0.6046426E 24 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0.1415528E 23 0. 0. 0. 0. 0. 0. 0. 0. 0.
 -0. 0. 0. 0. 0. 0. 0. 0. 0.
 -0.3559609E 21 0. 0. 0. 0. 0. 0. 0. 0. 0.

COEFFICIENTS OF POLYNOMIAL

-0.7118658E 00 0.
 0. 0. 0.100991E 00
 0.100000E 01 -0. 0. 0. 0. 0. 0. 0. 0. 0.
 -J. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 -C. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0.1753904E -01 -0. 0. 0. 0. 0. 0. 0. 0. 0.
 -0.4722181E -03 0. 0. 0. 0. 0. 0. 0. 0. 0.

INTERMEDIATE ROOTS CF POLYNOMIAL

-1.52725E-01 1.20000E-01 1.52725E-01 1.20000E-01 7.73945E-01 3.55802E-01 1.21937E-01 1.03790E 00 -3.80157E-01
 -7.73545E-01 3.95802E-01 -1.21937E-01 -6.46348E-02 1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

INTERMEDIATE POSITIVE ROOTS

1.52725E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21937E-01 -6.46348E-02 1.03790E 00 -3.80157E-01

RE-ORDERED ALPHAS (1 ROW, 3 ZERO CASE)

1.52725E-01 1.20000E-01 7.73545E-01 3.55802E-01 1.21937E-01 -6.46348E-02 1.03790E 00 -3.80157E-01

INTERMEDIATE BETA 8

0. 0. 2.06942E-11 5.01658E-11 -6.58268E-11 4.08803E-12 2.26534E-11 -6.97316E-11
 1.00000E-10 0. 0. 0. 0. 0. 0. 0.
 0. 0. 2.18210E-11 -5.38589E-11 -2.47544E-10 5.44229E-10 -2.09306E-11 -4.05041E-11
 0. 0. 1.00000E 00 0. 1.00000E 00 0. 1.00000E 00 0.

```

0.1753852E-C1      -0.
0.
-0.4722051E-C3      0.

INTERMEDIATE ROOTS OF POLYNOMIAL
-1.52723E-01  1.20000E-01  1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21934E-01  1.21934E-01  -6.46346E-02
-7.73945E-01  3.95802E-01  -1.21934E-01  -6.46346E-02  1.03790E 00  -3.80157E-01  -1.03790E 00  -3.80157E-01

INTERMEDIATE POSITIVE ROOTS
1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21934E-01  -6.46346E-02  1.03790E 00  -3.80157E-01

RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )
1.52723E-01  1.20000E-01  7.73945E-01  3.95802E-01  1.21934E-01  -6.46346E-02  1.03790E 00  -3.80157E-01

INTERMEDIATE BETA B
0.
0.
2.06942E-11  5.01459E-11  -6.58269E-11  4.08808E-12  2.26533E-11  -6.97316E-11
1.00000E-10  0.
0.
0.
2.18210E-11  -5.38589E-11  -2.47548E-10  5.44232E-10  -2.09307E-11  -4.05041E-11
0.
0.
1.00000E 00  0.
1.00000E 00  0.
1.00000E 00  0.

INTERMEDIATE L MATRIX BY COLUMNS
0.
0.
1.14542E 00  1.28000E-C8  0.
0.
6.20861E 00  5.25728E 00  0.
2.50863E 00  -5.32645E 00  -2.21131E 00  1.82953E 00
-3.24236E 01  -1.40654E 01  -0.
-4.56617E 00  9.95881E 00  -2.11124E 01  -1.07904E 01
5.75658E 00  -7.82836E 00  0.
-4.56308E 00  -4.98660E 00  -1.82051E 00  -1.43626E 00

VS = 0.3487689E 04  F(VS) = 0.8723089E-C4  0.6554125E-04  MAG = 0.1093503E-03

```

COEFFICIENTS OF POLYNOMIAL

-0.602385E 24 0.
 0.8562924E 23
 0.6469826E 24 0.
 -0.3288656E 24
 -0.6046417E 24 0.
 0.6749291E 23
 0.1465483E 23 0.
 -0.1870369E 14
 -0.3955498E 21 0.

COEFFICIENTS OF POLYNOMIAL

-0.711864E 00 0.
 0.1010992E 00
 0.1000000E 01 -0.
 -0.3882790E 00
 -0.712773E 00 0.
 0.7588629E-01
 0.1753653E-01 0.
 0.4722054E-03 0.

INTERMEDIATE ROOTS OF POLYNOMIAL

-1.52723E-01 1.20000E-01 1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21936E-01 -6.46346E-02
 -7.73945E-01 3.95802E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

INTERMEDIATE POSITIVE ROOTS

1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80157E-01

RE-ORDERED ALPHAS (1 ROW, 3 ZERO CASE)

1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80157E-01

INTERMEDIATE BETA 8

0. 0. 2.06942E-11 5.01459E-11 -4.58268E-11 4.08817E-12 2.26533E-11 -4.97716E-11
 1.00000E-10 0. 0. 2.18210E-11 -5.38589E-11 -2.47547E-10 5.44232E-10 -2.09307E-11 -4.05041E-11
 0. 0. 1.00000E 00 0. 1.00000E 00 0. 1.00000E 00 0.

INTERMEDIATE L MATRIX BY COLUMNS

0. 0. 1.14542E 00 1.28000E-08 0. 0. 0.
 6.20841E 00 5.29728E 00 0. 0. 2.50863E 00 -5.32645E 00 -2.21131E 00 1.82953E 00
 -3.26236E 01 -1.40655E 01 -0. -0. -4.56614E 00 9.95880E 00 -2.11126E 01 -1.07903E 01
 5.75658E 00 -7.82836E 00 0. 0. -4.56308E 00 -4.98660E 00 -1.82051E 00 -1.43626E 00

VS = 0.3487689E 04 FIVS) = 0.2110807E-03 0.1900636E-04 MAG = 0.2119346E-03

COEFFICIENTS OF POLYNOMIAL

-0.602385E 24 0.
 0.8562924E 23
 0.6469826E 24 0.
 -0.3288656E 24
 -0.6046417E 24 0.
 0.6749291E 23
 0.1465483E 23 0.
 -0.1870369E 14
 -0.3955498E 21 0.

```

-C.399545E 21 0.
COEFFICIENTS OF POLYNOMIAL
-C.711864E C0 0.
C. 0.1010992E 00
0.10C0000E C1 -0.
-0. -0.3E82790E 00
-0. -0.7130773E 00
C. 0.7568629E-01
C.1753852E-C1 -0.
0. 0.
-0.4722051E-C3 0.
INTERMEDIATE ROOTS OF POLYNOMIAL
-1.52723E-01 1.20000E-01 1.52723E-01 1.20C00E-01 7.73945E-01 3.95802E-01 1.21936E-01 1.21936E-01 -6.46346E-02
-7.73945E-01 3.95802E-01 -1.21936E-01 -6.46347E-02 1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01
INTERMEDIATE POSITIVE ROOTS
1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80157E-01
RE-ORDERED ALPHAS ( 1 ROW, 3 ZERO CASE )
1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21936E-01 -6.46346E-02 1.03790E 00 -3.80157E-01
INTERMEDIATE BETA B
0. 0. 2.06942E-11 5.01658E-11 -6.58269E-11 4.08808E-12 2.26533E-11 -6.97316E-11
1.0C000E-10 0. 0. 0. 0. 0. 0. 0.
2.18210E-11 -5.38589E-11 -2.47548E-10 5.44232E-10 -2.09307E-11 -4.05041E-11
0. 0. 1.00000E 00 0. 1.00000E 00 0. 1.00000E 00 0.
INTERMEDIATE L MATRIX BY COLUMNS
0. 0. 1.14542E 00 1.28000E-08 0. 0. 0. 0.
6.20861E 00 5.29728E 00 0. 0. 2.50843E 00 -5.32445E 00 -2.21131E 00 1.82953E 00
-3.26236E 01 -1.40658E 01 -0. -0. -4.58617E 00 9.95881E 00 2.11124E 01 -1.07904E 01
5.75638E 00 -7.82836E 00 0. 0. -4.58508E 00 -4.58660E 00 1.82051E 00 -1.43626E 00
VS = 0.3487689E 04 F(VS) = 0.8576001E-04 0.4355446E-04 MAG = 0.9636790E-04
COEFFICIENTS OF POLYNOMIAL
-C.6025385E 24 0.
0. 0.8562924E 23
0.8465826E 24 0.
-0. -0.3288656E 24
-0.6046417E 24 0.
0. 0.6749291E 23
0.148542E 23 0.
-C. 0.
-C.399545E 21 0.
COEFFICIENTS OF POLYNOMIAL
-C.711864E C0 0.
C. 0.1010992E 00
0.10C0000E C1 -0.
-0. -0.3E82790E 00
-0. -0.7130773E 00
C. 0.7568629E-01
0. 0.

```

INTERMEDIATE L MATRIX BY COLUMNS

```

0. 0. 1.14543E 00 1.28000E-08 0. 0. 0. 0. 0. 0.
6.20861E 00 5.29728E 00 0. 2.50803E 00 -5.32645E 00 -2.21131E 00 1.82933E 00
-3.26234E 01 -1.40654E 01 -0. -4.56615E 00 9.95883E 00 -2.11125E 01 -1.07904E 01
5.75658E 00 -7.82836E 00 0. -4.56308E 00 -4.58661E 00 -1.82051E 00 -1.43626E 00

```

VS = 0.3487688E C4 F(VS) = -0.5621333E-C2 -0.2856658E-02 MAG = 0.6305544E-02

COEFFICIENTS OF POLYNOMIAL

```

-C.6025385E 24 0.
0. 0.8562924E 23
C.8465826E 24 0.
-1. -0.3288656E 24
-C.6046417E 24 0.
0. 0.6749291E 23
0. 1.465482E 23 0.
0. -0.3599455E 21 0.
-0.3599455E 21 0.

```

COEFFICIENTS OF POLYNOMIAL

```

-C.711864E 00 0.
0. 0.101092E 00
0. 1.000000E C1 -0.
-0. 0.3882790E 00
-C.7118773E 00 0.
0. 0.7568629E-01
0. 0.1753852E-C1 -0.
0. 0.
-0.4722051E-03 0.

```

INTERMEDIATE L<4 ROOTS OF POLYNOMIAL

```

-1.52723E-01 1.20000E-01 1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21536E-01 -6.46346E-02
-7.73945E-01 3.95802E-01 -1.21536E-01 -6.46346E-02 1.03790E 00 -3.80157E-01 -1.03790E 00 -3.80157E-01

```

INTERMEDIATE POSITIVE ROOTS

1.52723E-01 1.20000E-01 7.73945E-C1 3.95802E-01 1.21536E-01 -6.46346E-02 1.03790F 00 -3.80157E-01

RE-ORDERED ALPHAS (1 ROW, 3 ZERO CASE)

1.52723E-01 1.20000E-01 7.73945E-01 3.95802E-01 1.21536E-01 -6.46346E-02 1.03790E 00 -3.80157E-01

INTERMEDIATE BETA 8

```

0. 0. 2.06942E-11 5.01658E-11 -6.58269E-11 4.08808E-12 2.26533E-11 -6.97316E-11
1.00000E-10 0. 0. 0. 0. 0. 0. 0.
0. 0. 2.18210E-11 -5.38589E-11 -2.47548E-10 5.44232E-10 -2.05307E-11 -4.05041E-11
0. 0. 1.00000E 00 0. 1.00000E 00 0. 1.00000E 00 0.

```

INTERMEDIATE L MATRIX BY COLUMNS

```

0. 0. 1.14542E 00 1.28000E-08 0. 0. 0. 0. 0. 0.
6.20861E 00 5.29728E 00 0. 2.50803E 00 -5.32645E 00 -2.21131E 00 1.82933E 00
-3.26234E 01 -1.40654E 01 -0. -4.56617E 00 9.95881E 00 -2.11126E 01 -1.07904E 01
5.75658E 00 -7.82836E 00 0. -4.56308E 00 -4.58660E 00 -1.82051E 00 -1.43626E 00

```

VS = 0.3487689E C4 F(VS) = 0.1975960E-04 0.4826713E-04 MAG = 0.5226283E-C4

```

KS = 0      LITHIUM NIOBATE

EPSLON = 0.100000E-10
KL      = 0
KM      = 0
MA      = 0
MAX      = 25
N        = 6
MU B    = C.900000E 02
MU B    = C.900000E 02
MU A    = C.285000E 11
RHO A   = C.188000E 05
RHO B   = C.100000E 11
VS 0    = 0.348000E 04

VS      = 0.348768E 04  FINAL VELOCITY SUCH THAT F(VS) .LT. EPSLON
1/VVS   = 0.286722E-03  INVERSE OF VS

***** DETERMINANT = ( 0.872309E-04, 0.659412E-04) *****

FINAL ROOTS OF POLYNOMIAL      DIVIDED BY VS

(-0.152722E 00, C.120000E 00)  (-0.437891E-04, 0.344067E-04)
(-0.152722E 00, C.120000E 00)  (-0.437891E-04, 0.344067E-04)
(-0.775847E 00, 0.395401E 00)  (-0.221907E-03, 0.113485E-03)
(-0.121935E 00, -0.646346E-01)  (-0.349617E-04, -0.185222E-04)
(-0.775847E 00, 0.395401E 00)  (-0.221907E-03, 0.113485E-03)
(-0.121935E 00, -0.646346E-01)  (-0.349617E-04, -0.185222E-04)
(-0.163790E 01, -0.380157E 00)  (-0.257590E-03, -0.108997E-03)
(-0.163790E 01, -0.380157E 00)  (-0.257590E-03, -0.108997E-03)

```

*** F I A L A N S W E R S ***

PARTIAL FIELD
RELATIVE AMPLITUDES

1 (0. , 0.)
2 (-2.7201401E C0, 0.8085244E 00)
3 (-0.1512902E 00,-0.1377944E 00)
4 (0.1000000E 01, 0.)

.....BX = 0.

STRESS COMPONENTS

T31 = (-5.8207661E-11, 0.)
T32 = (-C. , 0.)
T33 = (-C. , -1.4551915E-11)
T11 = (-4.3295741E-03, 1.3513207E-C3)
T12 = (-0. , 0.)
T22 = (-1.3872618E-C3, 3.9637460E-04)

STRAIN COMPONENTS

S11 = (-2.3131124E-14, 6.3502930E-15)
S22 = (-C. , 0.)
S33 = (-7.8239108E-15,-2.1343742E-15)
S12 = (-C. , 0.)
S13 = (-5.8991011E-16,-2.5035702E-16)
S23 = (-C. , 0.)

TIME AVERAGE POWER FLOW

P1M = (2.2182838E-C5,-2.5693692E-11)
P2M = (-0. , 0.)

ELECTRIC DISPLACEMENT

D1 = (-7.7528271E-14, 1.7353723E-14)
D2 = (-C. , 0.)
D3 = (-3.2623770E-16,-1.7019480E-15)

MECHANICAL DISPLACEMENT

U1 = 8.3656159E-11 -105.444
U2 = 0. C.
U3 = 1.2363921E-10 -15.145

ELECTRIC POTENTIAL MAGNITUDE = 6.8294138E-01 PHASE = 79.149

ELECTRIC FIELD

E1 = (-1.9231359E-14, 3.6643880E-05)
E3 = (3.8187657E-C5,-1.3321136E-C5)
MECHANICAL DISPLACEMENT
-C.22282974E-10 -0.80675376E-1C
C.
C.11934538E-C9 -0.32300643E-10
ELEC. POT. MAGNITUDE
C.1285971CE 00 C.67C72476E C0

```
9 112245 C ASSIGN          SYSLB4
9 112451 C $STOP
9 112451 0 PERIPHERAL FILE POSITIONS AT END CF JOB
9 112451 0 SYSPPI          REC. 00942, FILE 00000
9 112451 C SYSOUI          REC. 03745, FILE 00000
9 112451 0 SYSINI          REC. 00002, FILE 00002
9 112451 C END OF JOB
```

REC= 00000 FIL=

01 UNITS. EOF.

4*104318 JOB 105 REEL CRYPT IS REQUIRED
4*110409 JOB 109READY CRYPT ON UNIT C3

001096 1614 McCONWAY PHASE TIME OFF WAS 112451 12/31/69

```
*****  
* 007C64 SERVICE*****  
* 2159 CARDS READ *  
* 979 CARDS PUNCHED *  
* 3716 LINES PRINTED *  
* 007C9411 SERVICE*****  
* 2-C7 MIN. PRE-EXECUTION *  
* 13 MIN. EXECUTION *  
*****
```

REFERENCES

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13. ABSTRACT This report describes the analyses of several piezoelectric and pure elastic surface wave propagation problems and computer programs which implement their numerical study. In addition, the formal analysis of an electric current line source located above a piezoelectric crystal half space is presented in some detail.		

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