

UNCLASSIFIED

AD NUMBER

AD865534

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors; Critical Technology; JAN 1970. Other requests shall be referred to Air Force Technical Application Center, VELA Seismological Center, Washington, DC 20333. This document contains export-controlled technical data.

AUTHORITY

usaf ltr, 25 jan 1972

THIS PAGE IS UNCLASSIFIED



AFTAC Project No. VELA/T/0701/B/ASD

This document is subject to special export controls and each transmission to foreign governments or foreign nationals may be made only with prior approval of Chief, AFTAC

VSC.

also via 22313

SUBOPTIMAL MULTICHANNEL DIGITAL FILTERS
Technical Report No. 2
SEISMIC ARRAY PROCESSING TECHNIQUES

Prepared by

Hal H. Bybee

Stanley J. Laster, Project Scientist

Frank H. Binder, Program Manager
Area Code 214, 358-5181, Ext. 6521

TEXAS INSTRUMENTS INCORPORATED
Services Group
P. O. Box 5621
Dallas, Texas 75222

Contract No. F33657-70-C-0100
Amount of Contract: \$339,052
Beginning 15 July 1969
Ending 14 July 1970

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D. C. 20333

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Order No. 624
ARPA Program Code No. 9F10

28 January 1970

DDC
RECEIVED
MAR 4 1970
RECEIVED
C

Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract No. F33657-70-C-0100.

16

AD 865534



ADDITIONAL FOR	
APPROPRIATE	<input type="checkbox"/>
APPROPRIATE	<input type="checkbox"/>
APPROPRIATE	
SPECIAL	
2	

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of Chief, AFTAC.

Qualified users may request copies of this document from:

Defense Documentation Center
Cameron Station
Alexandria, Virginia 22314



This document is subject to special export controls and may be transmitted to foreign governments or foreign nationals only with prior approval of the AF TAC.

AFTAC Project No. VELA/T/0701/B/ASD

SUBOPTIMAL MULTICHANNEL DIGITAL FILTERS
Technical Report No. 2
SEISMIC ARRAY PROCESSING TECHNIQUES

Prepared by
Hal H. Bybee

Stanley J. Laster, Project Scientist
Frank H. Binder, Program Manager
Area Code 214, 358-5181, Ext. 6521

TEXAS INSTRUMENTS INCORPORATED
Services Group
P. O. Box 5621
Dallas, Texas 75222

Contract No. F33657-70-C-0100
Amount of Contract: \$339,052
Beginning 15 July 1969
Ending 14 July 1970

Prepared for
AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D. C. 20333

Sponsored by
ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Order No. 624
ARPA Program Code No. 9F10

28 January 1970

Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract No. F33657-70-C-0100.



ABSTRACT

This report discusses a new method of generating time-domain filters to extract a signal from digitized multichannel noise. The new filter generation technique is based on the frequency-domain Wiener filter response and uses the mean-square-error of the whole filter set in the transformation back into the time domain. This new, computationally efficient technique was evaluated against a previously used technique, also based on the frequency-domain Wiener filter response. Two different sets of experimental noise data with power spectra specified at 65 frequencies were used in the filter evaluation, and the signal to be detected was assumed to have the same power spectrum as the noise to prevent frequency filtering. For the first noise sample and 37-point long filters, the previous technique gave 0.8 db more error than the optimum frequency domain filter and the new filtering technique gave 0.5 db more error. For the second noise sample, these respective filters had 1.2 db and 0.9 db more error than the optimum filter.



TABLE OF CONTENTS

Section	Title	Page
	ABSTRACT	iii
I	INTRODUCTION	I-1
II	FILTER GENERATION TECHNIQUE	II-1
III	EVALUATION OF TECHNIQUE	III-1
IV	RESULTS AND CONCLUSIONS	IV-1



SECTION I

INTRODUCTION

A new method of generating time-domain filters to extract a signal from digitized multichannel noise has been developed. These filters approximate optimal Wiener filters and for computational efficiency are designed in the frequency domain.

Two other techniques have been used to design multichannel filters. In all three techniques a noise sample, typical of the anticipated noise, is used to design a filter that will extract signals that match some predetermined signal model. The first previous technique was the optimal time-domain filter-design technique. In it the filter's mean-square-error was expressed in terms of the crosscorrelation matrix of the noise. Setting the partial derivatives of the error with respect to each filter-weighting coefficient equal to zero yields a set of simultaneous equations to be solved for these coefficients. If there were C channels of data and a P -point filter were desired, a set of $C \times P$ simultaneous equations would have to be solved.

In the second technique the filter was designed in the frequency domain. Even though this filter does not give as small a mean-square-error as the first filter, it is computationally much more efficient to obtain. In the frequency domain, the optimal Wiener filter frequency response was computed from the crosspower matrix of the noise sample. This step involved only solving one set of C simultaneous equations. In the second step of this technique, each channel of the desired frequency response, which was specified at many frequencies, was individually approximated in the least mean-square-error sense by a time-domain filter with only P points. This was accomplished by taking the inverse Fourier transform and evaluating it at the P desired time points. This system produces more than the optimal



Wiener error because the finite length impulse responses do not exactly fit the desired frequency responses required by the optimum filters. The new method uses this method as a standard for comparison.

The new filter generation technique is based on the frequency-domain Wiener filter response, as was the second filter. It is better than the first technique in that it uses much less computation than the first technique, and it is better than the second technique in that its mean-square-error for the data investigated is smaller. This experiment evaluated the new technique against the second technique since they both were relatively computationally efficient.

In the new technique, the mean-square-error of the whole filter set is used in the transformation back into the time domain, instead of just the error of one channel at a time (as in the second type of filter). The mean-square-error of the filter is expressed in terms of the crosspower matrix of the signal and the weighting coefficients of the filter. By factoring the crosspower matrix into the product of a lower triangular, a diagonal, and an upper triangular matrix before the error is computed, the error can be separated into one part that involves the coefficients and the coefficients of a second channel, one part that involves those two sets of coefficients and the coefficients of a third channel, and so on. A better filter set can then be generated by choosing the weighting coefficients of the first channel of the filter to minimize the first part of the mean-square-error, and then choosing the weighting coefficients of the second channel to minimize the second part of the error with the coefficients of the first channel substituted into this second part of the error, and so on.

This filter generation technique was programmed to evaluate its improvement in mean-square-error over that of the second filter technique discussed above. Two different sets of experimental noise data with



power spectra specified at 65 frequencies were used in the filter evaluation, and the signal to be detected was assumed to have the same power spectrum as the noise to prevent frequency filtering. For the first noise sample, a 37-point one-channel-at-a-time filter had 0.8 db more error than the optimum frequency domain filter and the new triangular-method filter using 37 points had 0.5 db more error. For the second noise sample these respective filters had 1.2 db and 0.9 db additional error. When the filters were restricted to 11 points, on the second noise sample the respective additional errors were 3.5 db and 2.1 db. For these two samples of seismic noise data this new technique which considers the interaction between channels gives a filter better fitting the frequency response needed to minimize the output mean-square-error over all channels.

BLANK PAGE



SECTION II

FILTER GENERATION TECHNIQUE

To illustrate the new triangular method filter technique, a 3-point filter will be analytically derived for a 2-channel signal. Given two digitized noise channels, the fast Fourier transform can be used to obtain an estimate of the spectral matrix of the noise

$$\begin{bmatrix} N_{11}(f) & N_{12}(f) \\ N_{21}(f) & N_{22}(f) \end{bmatrix} = N(f) \quad (2-1)$$

$N(f)$ will be positive definite and Hermitian. A model for the signal spectral matrix which will be Hermitian can be generated

$$\begin{bmatrix} S_{00}(f) & S_{01}(f) & S_{02}(f) \\ S_{10}(f) & S_{11}(f) & S_{12}(f) \\ S_{20}(f) & S_{21}(f) & S_{22}(f) \end{bmatrix} = S(f) \quad (2-2)$$

In Equations 2-1 and 2-2, the subscripts 1 and 2 refer to the two channels and the subscript 0 refers to an origin point at which the signal estimate is desired.



The Wiener optimal filter $H(f)$ for this signal and noise is given by[†]

$$\begin{bmatrix} S_{11}(f) + N_{11}(f) & S_{12}(f) + N_{12}(f) \\ S_{21}(f) + N_{21}(f) & S_{22}(f) + N_{22}(f) \end{bmatrix} \begin{bmatrix} H_1^*(f) \\ H_2^*(f) \end{bmatrix} = \begin{bmatrix} S_{10}(f) \\ S_{20}(f) \end{bmatrix} \quad (2-3)$$

where $*$ denotes the complex conjugate.

The one-channel-at-a-time filter would have been realized from the Wiener response by minimizing the mean-square-error

$$\int_{-w}^w (F_j(f) - H_j(f)) (F_j^*(f) - H_j^*(f)) df \quad j=1,2 \quad (2-4)$$

between the filter response and the Wiener response with respect to the filter coefficients. The filter response given in terms of its coefficients a_{jk} would be

$$F_j(f) = \sum_{k=1}^3 a_{jk} e^{-i2\pi(k+c)\tau f} \quad j=1,2 \quad (2-5)$$

In the above expressions, w and $-w$ are the upper and lower limits of the signal bandwidth of data digitized with period τ . The zero point of the impulse response (Equation 2-5) of the filter is determined by the delay term c . The integrand of the mean-square-error (expression 2-4) can be multiplied by a weighting factor $R(f)$ to modify the impulse response of the filter, if desired.

The new triangular method filter is such a weighted form of expression 2-4. As derived by Burg,[†] the Wiener filter from Equation 2-3 has a mean-square-error between signal and output of

[†]Burg, John P., 1964; Three-Dimensional Filtering with an array of Seismometers, *Geophys.*, v. XXIX, n. 5, Oct. p. 693-713.

[†]Ibid.



$$\int_{-w}^w \left\{ S_{00}(f) - \begin{bmatrix} H_1(f) & H_2(f) \end{bmatrix} \begin{bmatrix} S_{10}(f) \\ S_{20}(f) \end{bmatrix} - \begin{bmatrix} S_{01}(f) & S_{02}(f) \end{bmatrix} \begin{bmatrix} H_1^*(f) \\ H_2^*(f) \end{bmatrix} \right. \\
 \left. + \begin{bmatrix} H_1(f) & H_2(f) \end{bmatrix} \begin{bmatrix} S_{11}(f) + N_{11}(f) & S_{12}(f) + N_{12}(f) \\ S_{21}(f) + N_{21}(f) & S_{22}(f) + N_{22}(f) \end{bmatrix} \begin{bmatrix} H_1^*(f) \\ H_2^*(f) \end{bmatrix} \right\} df \quad (2-6)$$

The error for any other filter G is the same as expression 2-6 with H replaced by G. Since the Wiener error is fixed, it can be subtracted from the error for G to give a simplified form of the error related to G. This excess mean-square-error derived from Equation 2-3 and expression 2-6 is

$$\int_{-w}^w \left\{ \begin{bmatrix} G_1(f) - H_1(f) & G_2(f) - H_2(f) \end{bmatrix} \begin{bmatrix} S_{11}(f) + N_{11}(f) & S_{12}(f) + N_{12}(f) \\ S_{21}(f) + N_{21}(f) & S_{22}(f) + N_{22}(f) \end{bmatrix} \begin{bmatrix} G_1^*(f) - H_1^*(f) \\ G_2^*(f) - H_2^*(f) \end{bmatrix} \right\} df \quad (2-7)$$

This error is approximately minimized to give the new triangular filter by first factoring S+N into the product of a lower triangular, a diagonal, and an upper triangular matrix.

$$\begin{bmatrix} 1 & 0 \\ T_{12}^*(f) & 1 \end{bmatrix} \begin{bmatrix} D_{11}(f) & 0 \\ 0 & D_{22}(f) \end{bmatrix} \begin{bmatrix} 1 & T_{12}(f) \\ 0 & 1 \end{bmatrix} \quad (2-8)$$

By substituting this expression for S+N, the error (expression 2-7) reduces to



$$\int_{-w}^w \left(G_1(f) - H_1(f) + T_{12}^*(f) (G_2(f) - H_2(f)) \right) D_{11}(f) \left(G_1^*(f) - H_1^*(f) + T_{12}(f) (G_2^*(f) - H_2^*(f)) \right) df \quad (2-9a)$$

$$+ \int_{-w}^w (G_2(f) - H_2(f)) D_{22}(f) (G_2^*(f) - H_2^*(f)) df \quad (2-9b)$$

G must be restricted to the form of Equation 2-5 to obtain a digital filter. It should be noted that expression 2-9b is independent of G_1 and is of the form of expression 2-4 with a weighting function of $D_{22}(f)$. For the triangular method filter, expression 2-9b is minimized with respect to the filter coefficients of G_2 . This G_2 is then substituted into expression 2-9a which then takes on the form of expression 2-4 with a weighting function $D_{11}(f)$ and a modified Wiener response. Expression 2-9a is then minimized to evaluate G_1 .

The only computational difference between this triangular method and the old weighted one-channel-at-a-time method is the computation of the modified Wiener response since the normal method of obtaining H from Equation 2-3 requires the triangular factorization of the S+N matrix. The computation of the modified Wiener response involves only the multiplication of a triangular matrix by a filter vector one row at a time as the filter vector is evaluated. This additional computation is very short in comparison to the amount of computation involved in the rest of the filter evaluation.



SECTION III

EVALUATION OF TECHNIQUE

The noise samples used in the computer evaluation of the triangular technique were both derived from actual seismic field measurements. For the measurements an array of 13 seismometers was used. The data coming from the seismometers were sampled, digitized, and recorded in the field. This set of data was returned to the lab where the power spectral matrix was computed for 65 equally-spaced frequencies from DC to the upper limit passed by the sampling unit.

The signal power spectra were assumed to have the same shape as the noise spectra, but the signal was given an infinite propagation velocity to model plane waves arriving normal or nearly normal to the earth's surface. A signal-to-noise ratio of 10 was used in the filter design.



SECTION IV

RESULTS AND CONCLUSIONS

Using the data discussed in Section III, three different length filters were designed and evaluated. Using the first noise sample, 37-point digital filters were designed with the output generated for the central point of the filter (i. e. , the output was delayed half of the length of the filter). The increased mean-square-error relative to the optimum frequency-domain filter was evaluated according to expression 2-7. The one-channel-at-a-time filter had 0.84 db more error than the optimum Wiener filter, and the new triangular method filter had 0.53 db more error than the optimum filter. For the second noise sample for which the difference between spectra for all channels at each frequency was smaller than the first sample, the old-method filter had only 0.88 db excess error. Using the second noise sample, two more sets of shorter filters were evaluated. For the 21-point filter with output in the center, the old method yielded 2.40 db excess error and the new method yielded 1.24 db excess error. When an 11-point center output filter was designed, the excess errors were 3.50 db and 2.12 db, respectively.

The excess errors of both types of filters are, however, functions of the signal and noise power spectra so that for different noise environments and signal characteristics, little can be said about the relative errors. If analytic expressions for noise and signal spectral matrices were known, analytic bounds on the errors could be derived from Equation 2-3 and expressions 2-6 and 2-7. For seismic applications, the results obtained from the noise samples of this report indicate that it should be well worth the investigator's effort to test the triangular method on noise and signals that he considers typical of his problem.

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing are station must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Texas Instruments Incorporated
Services Group
P. O. Box 5621, Dallas, Texas 75222

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

SUBOPTIMAL MULTICHANNEL DIGITAL FILTERS
SEISMIC ARRAY PROCESSING TECHNIQUES TECH. RPT. NO. 2

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical

5. AUTHOR(S)

Hal H. Bybee

6. REPORT DATE

28 January 1970

7a. TOTAL NO. OF PAGES

12

7b. NO. OF REFS

1

8a. CONTRACT OR GRANT NO.

F33657-70-C-0100

b. PROJECT NO.

VELA/T/0701/B/ASD

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of Chief, AFTAC.

11. SUPPLEMENTARY NOTES

ARPA Order No. 624

12. SPONSORING MILITARY ACTIVITY

Advanced Research Projects Agency
Department of Defense
The Pentagon, Washington, D. C. 20301

13. ABSTRACT

This report discusses a new method of generating time-domain filters to extract a signal from digitized multichannel noise. The new filter generation technique is based on the frequency-domain Wiener filter response and uses the mean-square-error of the whole filter set in the transformation back into the time domain. This new, computationally efficient technique was evaluated against a previously used technique, also based on the frequency-domain Wiener filter response. Two different sets of experimental noise data with power spectra specified at 65 frequencies were used in the filter evaluation, and the signal to be detected was assumed to have the same power spectrum as the noise to prevent frequency filtering. For the first noise sample and 37-point long filters, the previous technique gave 0.8 db more error than the optimum frequency-domain filter and the new filtering technique gave 0.5 db more error. For the second noise sample, these respective filters had 1.2 db and 0.9 db more error than the optimum technique.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Suboptimal Multichannel Digital Filters Seismic Array Processing Techniques						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.
- 2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.
- 2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.
3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.
4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.
5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.
6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.
- 7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.
- 8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.
- 8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.
- 9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.
- 9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).
10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.
12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.
13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.