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DESIGN OF MAXIMUM THRUST PLUG NOZZLES FOR FIXED INLET GEOMETRY

Robert P. Humphreys, H. Doyle Thompson, and Joe D. Hoffman

Jet Propulsion Center ✓
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**DESIGN OF MAXIMUM THRUST PLUG NOZZLES FOR
FIXED INLET GEOMETRY**

Robert P. Humphreys, H. Doyle Thompson, Joe D. Hoffman

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FOREWORD

The present study is part of the program "An Analytical Study of the Exhaust Expansion System (Scramjet Scientific Technology)" being conducted by the Jet Propulsion Center, Purdue University, under United States Air Force Contract No. F33615-67-C-1068, BPSN 7 (63 301206 6205214). The Air Force program monitor was Lt. Gary J. Jungwirth of the Air Force Aero Propulsion Laboratory. This report presents a formulation, numerical solution technique, and a computer program implementing the technique for the optimization of fixed inlet plug nozzles including boundary layer effects. The problem is formulated for isentropic, rotational and isentropic, irrotational flows.

The authors wish to express their appreciation to the Air Force Frank J. Seiler Research Laboratory for its generous contribution of computer time and to Mr. Gearold R. Johnson for his efforts in making the computer program compatible with the computers at Purdue University and Wright-Patterson AFB.

This report was submitted by the authors on 25 May 1970.

Publication of this report does not constitute Air Force approval of the report's finding or conclusions. It is published only for the exchange and stimulation of ideas.

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ABSTRACT

The techniques of the calculus of variations have been used to determine the configuration of an optimum thrust plug nozzle. The problem is formulated for a fixed thrust injection angle and cowl lip radius, and the resulting plug contour is then an optimum for a given upstream geometry. The optimum values of the injection angle and cowl lip radius are determined by a parametric study. The analysis is carried out for rotational and irrotational flows and includes boundary layer effects. A method is presented for each of the problem formulations to determine if a given contour is an optimum and a relaxation technique is used to obtain a solution to the irrotational flow problem.

A computer program which makes use of the design equations for the irrotational flow problem is developed and described. This program is used to carry out a parametric study to determine the optimum cowl lip radius and injection angle when the plug length is fixed. The resulting optimum nozzle is compared to one designed by Rao's Method. The importance of determining the base pressure accurately is illustrated and an example of scramjet nozzle optimization is presented.

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NOMENCLATURE

English Symbols

a	acoustic speed
C_1, C_2	Lagrange multipliers
E'	Cowl lip location
F	fundamental function, Eq. (9)
g	general isoperimetric constraint
G	boundary requirements, Eqs. (10), (11), and (12)
h_0	throat half-height
I	integral to be maximized, Eq. (8)
m	m'/ϵ
m'	gradient of the nozzle walls at the throat
m''	a mean radius of curvature at the throat
M	Mach number
n	number of generic dependent variables
p_0	total pressure
p	pressure
p_k	z_{k_x} , partial derivative of z_k with respect to x
q_k	z_{k_y} , partial derivative of z_k with respect to y
q	dynamic pressure, $\rho V^2/2$
R	gas constant
T	thrust integral, Eq. (1)
u	x -component of velocity

v	y-component of velocity
V	velocity modulus = $(u^2 + v^2)^{1/2}$
x	spacial coordinate along the axis of symmetry
y	spacial coordinate normal to the axis of symmetry
z_k	typical generic dependent variable

Greek Symbols

α	Mach angle
β	angle of inclination of the general flow direction in the throat to the axis of symmetry
γ	ratio of specific heats
δ^*	a boundary layer thickness
$\delta()$	first variation
δ'	$\delta^* \cos \theta$
ϵ	$R^{-1/2}$, where R is a non-dimensional radius of curvature of a meridian section at the throat
ζ_k	defined by Eq. (B-6)
η	defined by Eq. (B-5), and when used in conjunction with the Moore-Hall analysis η represents the asymmetry of the nozzle profile at the throat
θ	flow angle
λ_1, λ_2	Lagrange multipliers
λ_3, λ_4	
ξ	defined by Eq. (B-4)
ρ	density
ρ_d	downstream radius of curvature of the plug wall at the throat
τ	shear stress

ϕ base pressure contribution to the thrust, Eq. (13)
 ψ $(\gamma - 1)/(\gamma + 1)$

Subscripts

b base
D evaluated at point D
DE evaluated along the line DE
E evaluated at point E
TD evaluated along the boundary TD
w evaluated on the nozzle wall
x partial derivative with respect to x
y partial derivative with respect to y

Other

($\cdot$) total derivative with respect to x
* critical conditions

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SECTION I

INTRODUCTION

Conceptually, nonconventional nozzles such as the plug nozzle or the forced deflection nozzle offer advantages that cannot be achieved with conventional nozzles. The plug nozzle, for example, has the potential advantages of throttleability, thrust vector control, altitude compensation, and a shorter (and presumably lighter) nozzle for the same expansion ratio when compared with a conventional axisymmetric nozzle. This type of nozzle is currently operational on General Electric's TF39 and Pratt and Whitney's JT9D jet engines and shows considerable promise for ramjet, scramjet, and rocket engine applications. Because of its potential importance, the problem of maximizing the thrust of a plug nozzle is of considerable interest and is the subject of this investigation.

The concept of applying optimization techniques to design thrust nozzles was introduced by Guderley and Hantsch (1) in 1955. Subsequently, Rao (2) simplified the analysis and developed a basic design procedure that has gained wide acceptance throughout the industry. Rao also applied his formulation to the plug nozzle design problem (3). His formulations (2,3) are limited to a fixed

nozzle length (or, equivalently, a fixed expansion ratio for a conventional nozzle). More recently Guderley and Armitage (4,5) reformulated the problem for the design of axisymmetric nozzles using a more general approach which provides for a wide selection of the form of the geometric constraint that can be imposed on the design. This additional flexibility is achieved only at the expense of a more complex problem formulation and an order of magnitude increase in the complexity of the numerical solution.

The additional potential of the Guderley-Armitage approach was realized when the method was extended by Hoffman and Thompson (6) to include the design of optimum nozzles for gas-particle flows, by Hoffman (7) to include the design of optimum nozzles for reacting nonequilibrium flows, and by Scofield, Thompson, and Hoffman (8) to include the effects of boundary layer drag in the optimization. The essential difference in applying the method to these various types of flows is in the computation of the basic flow field and not in the method itself.

In addition to the references already cited, it is of interest to note that the Soviets are actively engaged in this field of study (9,10,11,12,13). Krayko (9) has extended the work of Guderley and Armitage (4) to the construction of the rear part of a minimum drag body which is restricted in length. His study included two cases: 1) the pressure in the base region of the body is independent of the upstream body contour, and 2) the pressure in the base

region is dependent upon the upstream body contour.

Pirumov and Rubtsov (10) have developed a computer program to calculate the flow field in axisymmetric plug and expansion-deflection nozzles using a linear sonic line. References 11, 12, and 13 are works of a similar nature.

The objective of this investigation is to determine the configuration of an optimum thrust plug nozzle, including the throat injection angle, cowl lip radius, and the plug contour, by applying the techniques of the calculus of variations. The problem is formulated for a fixed throat injection angle and cowl lip radius, and the resulting plug is then an optimum for the given upstream geometry. The optimum values of the injection angle and cowl lip radius are then determined by a parametric study.

The first two sections which follow contain the problem formulations. The first is for axisymmetric, steady, rotational* flow and the second is for axisymmetric, steady, irrotational** flow. Section IV contains explanations of the solution procedure and the method of plug contour modification. Section V contains: 1) the results of the

* The isentropic, rotational assumption implies that entropy and total enthalpy are constant on a streamline (see Ref. 14).

** The irrotationality assumption implies that entropy and total enthalpy are constant throughout the flow.

parametric study used to determine the optimum injection angle and cowl lip radius and the corresponding optimum plug contour, 2) a comparison of the optimum nozzle to a nozzle designed by Rao's method for the same conditions, 3) an explanation of the effects of changing the base pressure model, and 4) an example which illustrates the optimization of scramjet nozzles. The final section contains the summary and recommendations.

SECTION II

ROTATIONAL FLOW PROBLEM

1. FLOW MODEL

The plug contour to be optimized is that portion between points T' and D', shown in Figure 1 along with the remaining nozzle geometry. The cowl lip radius, y_E , and the injection angle, β , are prescribed, and the nozzle geometry upstream of the characteristic ET is fixed. The resulting optimum contour is then the best for a given upstream geometry. The region (R) is considered to be an inviscid core bounded by the streamline TD, the right characteristic DE, and the left characteristic ET. The streamline TD is separated from the plug surface by a boundary layer thickness δ^* , measured normal to the streamline. Thus, changes in the streamline TD will affect only the flow in region (R).

The axial thrust to be maximized is obtained by summing the integrated pressure and shear forces on the plug T'D' and the pressure forces acting on the base D'C. The thrust expression, developed in Appendix A, is given by:

$$\frac{T}{2\pi} = - \int_{T'}^{D'} [p(y - \delta') + \tau] (y - \delta') dx + (y_D - \delta'_D)^2 p_b / 2 \quad (1)$$

where $\delta' = \delta^* \cos \theta$ and $(\cdot)$ denotes differentiation with respect to x . Equation (1) represents only that portion of the total axial thrust which is to be maximized in the variational problem. The optimization with respect to the injection angle and cowl lip radius is accomplished by a parametric study. Since ambient pressure acts over the area πy_B^2 , the altitude for which the nozzle is designed is specified during the parametric study. Thus, it is desired to find the streamline $y = y(x)$ which maximizes Eq.(1) and from which the wall contour can be obtained. However, it is necessary to introduce certain constraints to assure that the results will be physically realizable.

The governing equations for axisymmetric, steady, isentropic, rotational flow are the following:

$$\rho u_x + \rho v_y + u\rho_x + v\rho_y + \rho v/y = 0 \quad (2)$$

$$\rho u u_x + \rho v u_y + p_x = 0 \quad (3)$$

$$\rho u v_x + \rho v v_y + p_y = 0 \quad (4)$$

$$u p_x + v p_y - a^2 u \rho_x - a^2 v \rho_y = 0 \quad (5)$$

where the subscripts x and y denote partial derivatives. Equation (2) is the continuity equation, Eqs.(3) and (4) are the x and y Euler equations, and Eq.(5) is obtained as a result of the entropy being constant on a streamline.

The boundary TD is to be a streamline, which requires the dependent variables u and v to be related by

$$u\dot{y} - v = 0$$

along TD

This expression is multiplied by $y\rho$ for later convenience in algebraic manipulation

$$y\rho(u\dot{y} - v) = 0$$

along TD (6)

In addition to these constraints most engineering applications require the contour to have either a fixed length, a fixed surface area, or to be restricted in some other way. To place a physical limitation on the flow, a general isoperimetric constraint of the form

$$\int_T^D g(y, \dot{y}, p) dx = \text{constant}$$

along TD (7)

is imposed. A fixed length constraint is obtained by setting $g = 1$ and the condition of a fixed surface area by setting $g = (1 + \dot{y}^2)^{1/2}$.

The constraining relations given by Eqs. (2) through (7) are imposed by utilizing Lagrange multipliers. The functional to be optimized becomes

$$I = \iint_R F dy dx + \int_{TDET} G dx + \phi \quad (8)$$

where

$$\begin{aligned} F = & \lambda_1(\rho u_x + \rho v_y + u\rho_x + v\rho_y + \rho v/y) \\ & + \lambda_2(\rho u u_x + \rho v u_y + p_x) + \lambda_3(\rho u v_x + \rho v v_y + p_y) \\ & + \lambda_4(u p_x + v p_y - a^2 u \rho_x - a^2 v \rho_y) \end{aligned} \quad \text{in } R \quad (9)$$

$$G = -[f + C_1 g + C_2 y \rho(u\dot{y} - v)] \quad \text{along TD} \quad (10)$$

$$G = 0 \quad \text{along DE} \quad (11)$$

$$G = 0 \quad \text{along ET} \quad (12)$$

$$\phi = (y_D - \delta'_D)^2 p_b / 2 \quad \text{at D} \quad (13)$$

$$f = [p(\dot{y} - \dot{\delta}') + \tau](y - \delta') \quad (14)$$

In the above equations λ_1 through λ_4 are functions of x and y , C_2 is a function of x , and C_1 is a constant.

The functional forms assumed for τ and δ' are

$$\tau = \tau(x) \quad , \quad \delta' = \delta'(x)$$

The base pressure is taken as constant over the base of the plug at an effective value which is determined by the flow properties in the region (R). Further, since the base pressure does not affect the flow properties in the region (R), it must be treated in the variational problem as a constant which is not known a priori. As will be explained in more detail later, the optimum contour is approached in an iterative manner in which the value of the base pressure is recalculated in each iteration.

The optimization procedure is independent of the model used to calculate the base pressure. The particular model used in this investigation and other details of base pressure are discussed in Appendix G. In addition it is assumed that the total temperature and pressure are known

for each streamline crossing the characteristic TE.

2. NECESSARY CONDITIONS

In the calculus of variations there are certain necessary conditions arising out of the first variation which have to be met for an extremal solution to exist. These conditions are the Euler Equations, transversality condition, Erdman-Weirstrass corner condition for corner lines, and the corner condition for corner points on a boundary line. The Erdman-Weirstrass condition will not be investigated since flows in which corner lines arise are not to be considered. When the remaining conditions are satisfied it will be assumed on physical grounds that the resulting nozzle surface is indeed the maximizing solution.

The calculus of variations for a functional of the type shown in Eq.(8) is developed in Appendix B. The details of the application of the calculus of variations are given in Appendices C through E and the results are presented in the next few sections.

a. Euler Equations. The general form of the Euler Equations, obtained for arbitrary variations of the generic dependent variables in the region (R), is given by Eq.(B-13). After application of this equation for each of the generic dependent variables u, v, p , and ρ , and some manipulation as shown in Appendix C, Eqs.(C-5), (C-9), (C-13), and (C-20) are obtained. This set of partial differential equations for

determining the Lagrange multipliers in the region (R) is:

$$\begin{aligned}
 & -\lambda_2 u_x - \lambda_3 v_x - (\lambda_4/\rho)(p_x - a^2 \rho_x) + y\lambda_{1x} + u\lambda_{2x} \\
 & + v\lambda_{2y} = \lambda_2 v/y
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 & -\lambda_2 u_y - \lambda_3 v_y - (\lambda_4/\rho)(p_y - a^2 \rho_y) + y\lambda_{1y} + u\lambda_{3x} \\
 & + v\lambda_{3y} = \lambda_3 v/y
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 & \lambda_4 u_x + \lambda_4 v_y + (\lambda_4 a^2/\rho)(u\rho_x + v\rho_y) + \lambda_{2x} + \lambda_{3y} \\
 & + u\lambda_{4x} + v\lambda_{4y} = 0
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 & (\lambda_2/\rho)p_x + (\lambda_3/\rho)p_y + yu\lambda_{1x} + yv\lambda_{1y} + a^2\lambda_{2x} \\
 & + a^2\lambda_{3y} = 0
 \end{aligned} \tag{18}$$

where the subscripts x and y denote partial derivatives.

Equations (2) through (5), together with Eqs. (15) through (18), constitute a system of eight partial differential equations for determining the eight variables $u, v, p, \rho, \lambda_1, \lambda_2, \lambda_3$, and λ_4 . As shown in Appendix F, these eight equations form a system of quasi-linear, nonhomogeneous, first-order partial differential equations of the hyperbolic type. Thus, the system can be replaced by an equivalent system of characteristic and compatibility equations. The characteristic system valid along gas streamlines is defined by Eqs. (F-40), (F-47), (F-48), (F-49), and (F-50). These equations are:

$$dy/dx = v/u \tag{19}$$

$$\rho u du + \rho v dv + dp = 0 \quad (20)$$

$$dp - a^2 d\rho = 0 \quad (21)$$

$$-\lambda_2 du - \lambda_3 dv + y d\lambda_1 + u d\lambda_2 + v d\lambda_3 = (\lambda_2 dx + \lambda_3 dy)(v/y) \quad (22)$$

$$(v\lambda_2 - u\lambda_3)dv + (\lambda_2/\rho)dp - (\lambda_4 u a^2/\rho)(\gamma-1)d\rho + y u d\lambda_1 - a^2 u d\lambda_4 = -(\lambda_4 a^2 v/y)dx \quad (23)$$

The system of equations valid along gas Mach lines is made up of Eqs. (F-42), (F-57), and (F-58). These equations are:

$$dy/dx = \tan(\theta \pm \alpha) \quad (24)$$

$$a^2(vdu - u dv) \pm (a^2/\rho) \cot \alpha dp = (a^2 v/y)(udy - vdx) \quad (25)$$

$$\lambda_2 du + \lambda_3 dv - (\lambda_4/\rho)(dp + a^2 d\rho) - y d\lambda_1 \pm \tan \alpha (v d\lambda_2 - u d\lambda_3) = \mp \tan \alpha (\lambda_3 dx - \lambda_2 dy)(v/y) \quad (26)$$

The upper sign in Eqs. (24), (25), and (26) refers to left-running Mach lines and the lower sign to right-running Mach lines.

b. Transversality Conditions. The general transversality condition is given by Eq. (B-14). A detailed development of the transversality conditions is presented in Appendix D. The following is a summary of the results:

(1) Along ET. There can be no variations in u, v, p, ρ, x , or y along ET (see Figure 1) since the location of this line and the flow properties along it are determined by the

fixed upstream geometry. Therefore, the transversality conditions along ET are satisfied identically.

(2) Along DE. Application of Eq.(B-14) along DE results in the four equations (D-4) through (D-7). These four equations can be combined to yield the equation of a right-running characteristic which can be used to replace any of the four equations. Thus, DE is required to be a right-running characteristic along which Eqs.(D-4), (D-5), and (D-7) must be satisfied in addition to the characteristic equation which is used to replace Eq.(D-6). These equations are:

$$y\dot{y}\lambda_1 + (u\dot{y} - v)\lambda_2 = 0 \quad (27)$$

$$y\lambda_1 - (u\dot{y} - v)\lambda_3 = 0 \quad (28)$$

$$y\lambda_1 - a^2\lambda_4 = 0 \quad (29)$$

$$(u^2 - a^2)\dot{y}^2 - 2uv\dot{y} + (v^2 - a^2) = 0 \quad (30)$$

(3) Along TD. When Eq.(B-14) is applied along TD, Eqs.(D-14), (D-18), and (D-31) result. These equations are as follows:

$$\lambda_1 = C_2 \quad (31)$$

$$u\lambda_3 - v\lambda_2 + uf_p + ug_p C_1 = 0 \quad (32)$$

$$\begin{aligned} dC_2/dx = & (y - \delta')(du/dx + \delta'dv/dx)/y \\ & - [C_1 (\rho ug_p dv/dx - g_y + d(g_y)/dx)/(y\rho u) \\ & + \tau/(y\rho u) \end{aligned} \quad (33)$$

where

$$f_p = (\dot{y} - \dot{\delta}') (y - \delta') \quad (34)$$

c. Corner Conditions. The corner conditions are discussed in detail in Appendix E. Points E and T are considered to be fixed, thus the corner condition is satisfied identically at these two points. The condition which must be satisfied at point D is given by Eq.(B-16). Application of this equation yields Eqs.(E-12) and (E-13) which are:

$$[(y - \delta')(p\dot{\delta}' - \tau) + C_2\rho y\dot{y} - C_1(g - \dot{y}g_y)]_{TD} = 0 \quad (35)$$

$$[p(y - \delta') + C_1g_y + C_2\rho yu]_{TD} = (y_D - \delta'_D)p_b \quad (36)$$

3. METHOD OF SOLUTION

The equations obtained from the variational problem can now be applied in a straightforward manner to determine if a given contour is an optimum. A contour for the plug is assumed and the flow field calculated by the method of characteristics. However, this requires an initial-value line along which the flow properties are known.

In the problem formulation it has been assumed, perhaps naively, that the initial conditions from which the method of characteristic solution can be initiated are provided or can be readily calculated. This is consistent with the fact that the optimization method is independent of the method of obtaining initial conditions, and only requires that the initial conditions represent a physically possible flow that

is compatible with the governing equations for the supersonic flow field. Thus, the same optimization procedure is used for a scramjet, for which the initial conditions at the nozzle entrance are provided either from measured data or from a theoretical analysis of the flow through the combustor, and for a subsonic burning engine for which the initial conditions can be determined from a transonic flow analysis.

Once the flow field has been calculated, values of the Lagrange multipliers λ_1 through λ_4 can be determined at point D from Eqs. (27), (29), (31), (32), (35), and (36). Starting at point D, values of these multipliers can be evaluated along TD using Eqs. (22), (23), (31), (32), and (33). Then, utilizing the characteristic net (see Figure 2) developed in evaluating the flow field, the Lagrange multipliers in the region (R) can be determined.

Starting at point 1 near the base of the plug, the initial data known along TD can be used, along with Eq. (26) which is valid along the two Mach lines intersecting at point 2, and Eqs. (27) and (29) which are valid along DE, to determine the Lagrange multipliers λ_1 through λ_4 at point 2. The values of λ_1 through λ_4 can be determined at point 3 by applying Eq. (26) along the Mach lines intersecting at point 3 and Eqs. (22) and (23) along the gas streamline passing through points 2 and 3. Point 5 can be determined in the same manner as point 2. The Lagrange multipliers can be determined throughout the region (R) by continuing this procedure.

Note that Eq. (28) was not used in the above procedure but

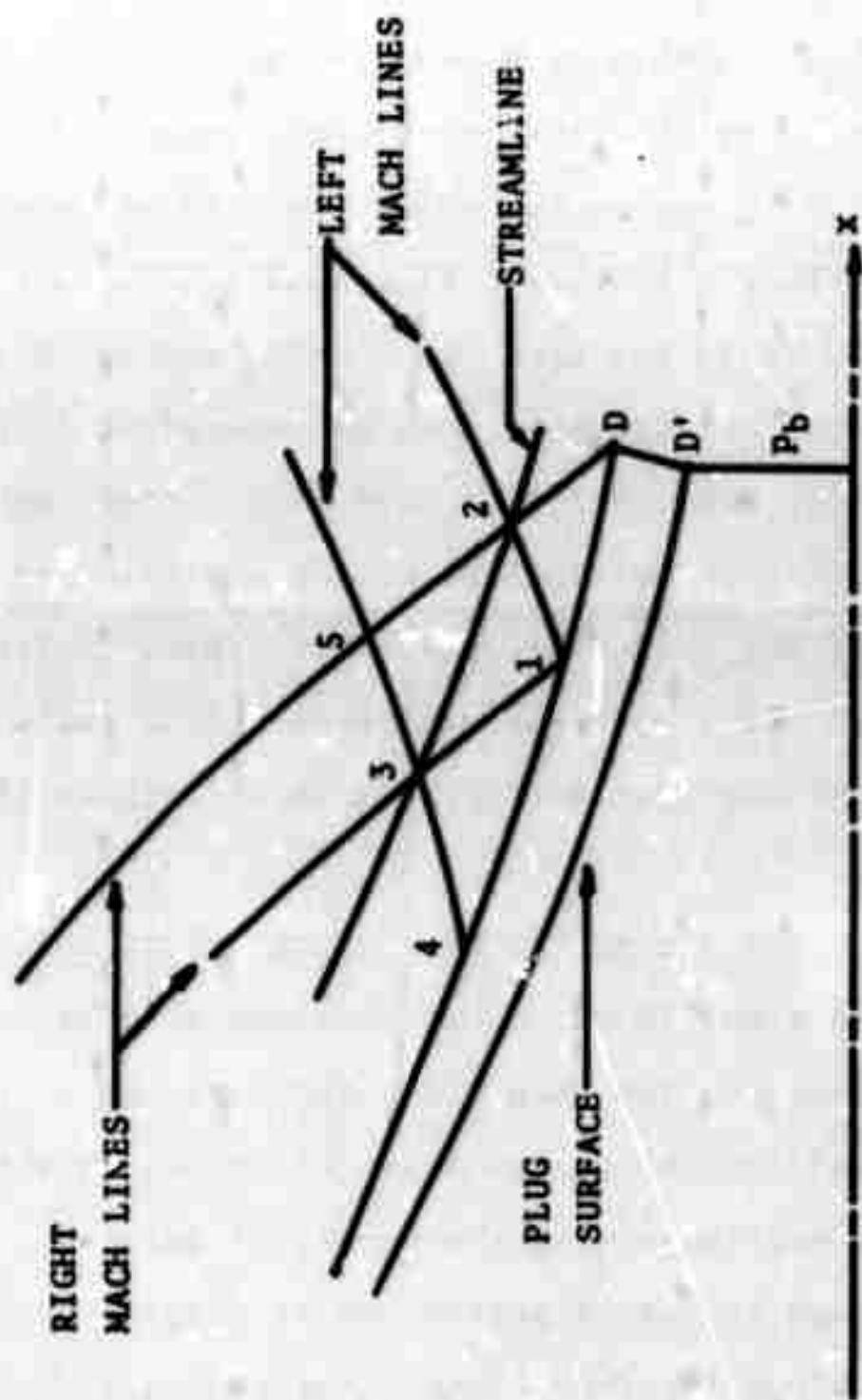


FIGURE 2. CHARACTERISTIC NET

must be satisfied along DE. This relation then serves as a means of checking whether a given contour is an optimum. If the contour is not an optimum, it must be modified to satisfy Eq.(28).

Contour modification techniques have been developed by Guderley and Armitage (5) and Scofield et al.(8) for irrotational flow. Even though the current problem is more complex, it appears that a similar contour modification technique could be developed to obtain an optimum plug contour.

It is possible to significantly simplify the calculation procedure by considering a problem which is slightly more restrictive than the present one and yet retains the significant features of the optimization, thereby providing considerable insight into the design and operation of optimum plug nozzles. This problem is considered in the next section.

SECTION III

IRROTATIONAL FLOW PROBLEM

1. FLOW MODEL

The problem to be considered here is one in which the flow in region (R) (see Figure 1) is irrotational. The thrust to be maximized in the variational problem is given by Eq.(1) which is repeated here for convenience:

$$\frac{T}{2\pi} = - \int_T^D [p(\dot{y} - \dot{\delta}') + \tau](y - \delta')dx + (y_D - \delta_D')^2 p_b / 2 \quad (37)$$

Here again, the optimum values of the injection angle and cowl lip radius are obtained from a parametric study.

The basic difference between this problem formulation and the one presented in Section II is in the governing equations of motion. For the flow field to be isentropic, it must satisfy the irrotationality condition:

$$u_y - v_x = 0 \quad (38)$$

The flow field must also satisfy the continuity equation

$$(y\rho u)_x + (y\rho v)_y = 0 \quad (39)$$

where the subscripts indicate partial derivatives. Along the nozzle wall the dependent variables are related through the

equation for a streamline which is:

$$y\rho(u\dot{y} - v) = 0 \quad (40)$$

Also, a general isoperimetric constraint is imposed along TD.

$$\int_T^D g(y, \dot{y}, p) dx = \text{constant} \quad (41)$$

The constraining relations given by Eqs.(38),(39),(40) and (41) are again imposed by means of Lagrange multipliers such that the functional to be maximized becomes

$$I = \iint_R F dy dx + \oint G dx + \Phi \quad (42)$$

where

$$F = \lambda_1 (u_y - v_x) + \lambda_2 [(y\rho u)_x + (y\rho v)_y] \quad \text{in } R \quad (43)$$

$$G = -[p(\dot{y} - \delta') + \tau](y - \delta') + \lambda_3 y\rho(u\dot{y} - v) + \lambda_4 g \quad \text{along TD} \quad (44)$$

$$G = 0 \quad \text{along DE} \quad (45)$$

$$G = 0 \quad \text{along ET} \quad (46)$$

$$\Phi = (y_D - \delta_D')^2 p_b / 2 \quad \text{at } D \quad (47)$$

In the above equations λ_1 and λ_2 are functions of x and y , λ_3 is a function of x , and λ_4 is a constant.

In addition it is assumed that the total temperature and pressure are known. Since the flow is isentropic the functional relations

$$p = p(u, v) \quad \rho = \rho(u, v) \quad a = a(u, v) \quad \alpha = \alpha(u, v)$$

are valid. Thus, the dependent variables are the velocity components u and v .

2. NECESSARY CONDITIONS

The calculus of variations can be applied to Eq.(42) in a manner similar to the previous case to determine the conditions necessary to an extremal solution. These conditions are as follows:

a. Euler Equations.

$$\lambda_{1y} + \gamma\rho(1 - u^2/a^2)\lambda_{2x} - \gamma\rho(uv/a^2)\lambda_{2y} = 0 \quad (48)$$

$$\lambda_{1x} + \gamma\rho(uv/a^2)\lambda_{2x} - \gamma\rho(1 - v^2/a^2)\lambda_{2y} = 0 \quad (49)$$

In the region of supersonic flow these equations are a system of hyperbolic, partial differential equations with characteristic directions which correspond to the characteristic directions of the basic flow field. The compatibility equations are

$$d\lambda_1 \pm \gamma\rho \cot \alpha d\lambda_2 = 0 \quad (50)$$

Equations (50) are valid along the characteristics of the basic flow field, defined by

$$dy/dx = \tan(\theta \pm \alpha) \quad (51)$$

The upper sign in Eqs.(50) and (51) refers to left-running characteristics and the lower sign to right-running characteristics.

b. Transversality Conditions.

(1) Along TD. Along this line variations in u, v, x , and y can be treated as arbitrary and independent which results in three equations:

$$\lambda_2 = \lambda_3 \quad (52)$$

$$\lambda_1 = \rho u(y - \delta')(\dot{y} - \dot{\delta}') + \lambda_4 \rho u g_p \quad (53)$$

$$\begin{aligned} \dot{\lambda}_3 = & (y - \delta')(du/dx + \delta' dv/dx)/y - (\lambda_4/y)[g_p v \\ & - (g_y - dg_y/dx)/(\rho u)] + \tau/(y \rho u) \end{aligned} \quad (54)$$

(2) Along DE. Along the exit characteristic, DE, variations in u, v, x , and y can be treated as arbitrary and independent. This results in two equations which can be combined to yield the equation of a right-running characteristic. This equation can be used to replace one of the two original equations. Thus, DE is required to be a right-running characteristic along which the following equation must be satisfied:

$$\lambda_1 - \lambda_2 y \rho \cot \alpha = 0 \quad (55)$$

(3) Along ET. Since no variations in the gas properties or in x and y are allowed upstream of the left-running characteristic ET, the transversality condition is satisfied identically along this line.

c. Corner Conditions.

As in the previous case, corner conditions are obtained at point D only. The two conditions which must be satisfied at this point are:

$$[(y - \delta')(\dot{p}\delta' - \tau) - \lambda_4(g - \dot{y}g_y) + \lambda_3 y \rho u \dot{y}]_{TD} = 0 \quad (56)$$

$$[p(y - \delta') + \lambda_3 y \rho u + \lambda_4 g_y]_{TD} = (y_D - \delta'_D)p_b \quad (57)$$

3. METHOD OF SOLUTION

The solution procedure for this problem is similar to that of the previous case, but is somewhat less complicated. First, the flow field is solved by the method of characteristics. The characteristics have directions given by Eq.(51) and the compatibility relations valid along these lines are

$$d\theta \mp \cot \alpha \, dV/V \pm [(\sin \theta \sin \alpha)/(y \sin (\theta \pm \alpha))]dy = 0 \quad (58)$$

The upper sign refers to left-running characteristics and the lower sign to right-running characteristics. Once the flow field is known, Eqs.(52), (56), and (57) can be used to solve for λ_1 and λ_2 at point D.

Starting from point D, Eqs.(52), (53), and (55) can be used to evaluate λ_1 and λ_2 along TD. The plug surface TD then serves as an initial-value line from which to start the method of characteristics solution for λ_1 and λ_2 in the region (R). These two multipliers have the same characteristic directions as the flow field, and their compatibility relations are given by Eqs.(50).

Equation (55) must be satisfied along the exit characteristic, DE, and serves as a check to determine whether or not a given contour is an optimum. If Eq.(55) is not satisfied then the contour must be modified. As mentioned earlier, contour modification techniques have been developed

for the type of flow under consideration, but were applied to converging-diverging nozzles.

4. RAO'S RESULT A SPECIAL CASE

It is of interest to note that the current formulation contains the results of Rao(3) as a special case. If the axial length is held constant, then $g = 1$, and

$$g_p = g_y = g_{\dot{y}} = 0$$

Neglecting the wall shear stress and boundary layer thickness, Eq.(54), valid along TD, becomes

$$d\lambda_3/dx = du/dx$$

or since $\lambda_2 = \lambda_3$ on TD,

$$\lambda_2 - \lambda_{2D} = u - u_D$$

$$\lambda_2 = \lambda_{2D} + V \cos \theta - V_D \cos \theta_D \quad (59)$$

Equation (53), which is also valid on TD, becomes

$$\lambda_1 = \gamma \rho V \sin \theta \quad (60)$$

For any velocity distribution $u(x,y)$, $v(x,y)$ that satisfies the flow equations, the two pairs of functions

$$\lambda_1 = \text{const}$$

$$\lambda_2 = \text{const}$$

$$\lambda_1 = \gamma \rho V \sin \theta$$

$$\lambda_2 = V \cos \theta$$

satisfy the partial differential equations(48) and(49) which

are valid throughout the flow field. Thus, Eqs.(59) and (60) must also be valid along DE where Eq.(55) applies. Substitution of Eqs.(59) and (60) into Eq.(57) yields

$$\gamma \rho V \sin \theta - \gamma \rho \cot \alpha (\lambda_{2D} + V \cos \theta - V_D \cos \theta_D) = 0$$

which reduces directly to

$$\frac{V \cos (\theta + \alpha)}{\cos \alpha} = \text{const} \quad (61)$$

Equations (50) and (55) can be combined to yield

$$\lambda_1 = (C \gamma \rho \cot \alpha)^{1/2}$$

where C is a constant. Substituting this into Eq.(60) yields

$$\gamma \rho V^2 \sin^2 \theta \tan \alpha = \text{const} \quad (62)$$

Equations (61) and (62) must be satisfied along the exit characteristic DE and are identical to the equations obtained by Rao. At point D Rao obtained the corner condition

$$(p - p_b) \cot \alpha / (1/2 \rho V^2) = - \sin 2\theta \quad (63)$$

This equation can also be obtained by neglecting the wall shear and boundary layer thickness in Eqs.(53) and (57), and by substituting Eqs.(52) and (53) into Eq.(55). This results in

$$\lambda_3 = v/\text{ctn } \alpha$$

(64)

which must be satisfied at point D. This result is then substituted into the corner condition, Eq.(57), which reduces directly to Eq.(63).

The problem formulated in Section III, neglecting the boundary layer thickness, has been programmed in Fortran IV. Correction for boundary layer thickness can be made to the optimized contour by using the displacement thickness to adjust the wall coordinates. The numerical techniques, including the method of contour modification, are discussed in the next section.

SECTION IV

NUMERICAL METHODS

1. SOLUTION PROCEDURE

The solution procedure consists of estimating what the optimum contour should be, analyzing the contour to see if it is an optimum, and modifying the contour if it is not an optimum. In order to analyze the contour it is necessary to first solve the flow field by the method of characteristics which requires a start line. The computer program contains two options for this purpose. The start line can either be read in from data cards or generated internally. The internally generated start line is obtained from either a modified Moore-Hall (16) transonic flow analysis or an isentropic flow analysis in which the Mach number is assumed to be constant along a straight line. The details of the Moore-Hall analysis and the necessary modifications are discussed in Appendix H. In any case, the start line will always be from point A to point E' shown in Figure 3.

Before going into the details of how the flow field is calculated, the conditions at point T (see Figure 1) need to be examined. In the problem formulation, point T was considered to be fixed and thus variations in the plug contour are only allowed downstream of point T. As a result, in

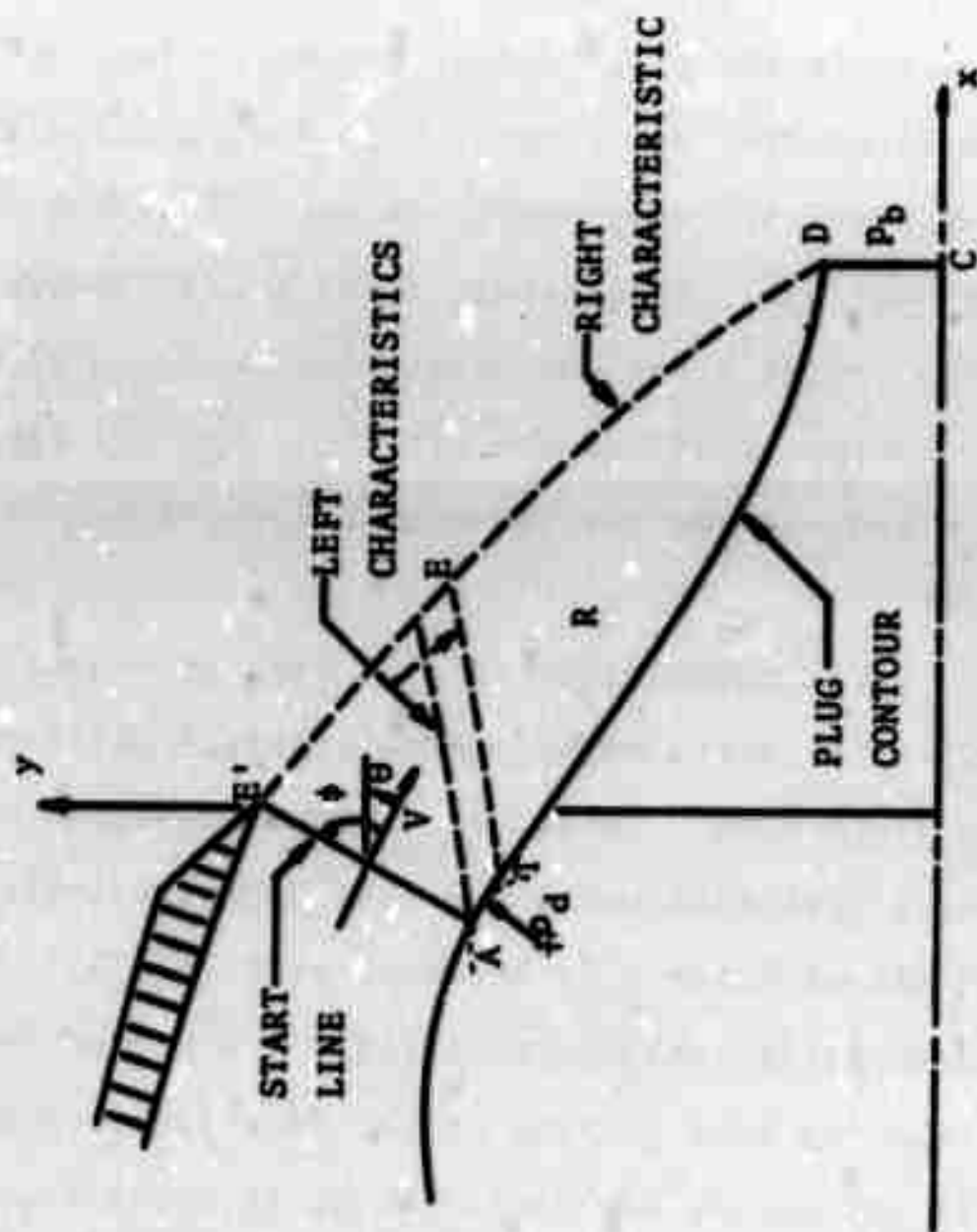


FIGURE 3. NOZZLE GEOMETRY WITH A SPECIFIED THROAT RADIUS

carrying out the optimization process, the contour must be modified, and consequently, a sharp corner can arise at point T. It is thus necessary to specify the plug curvature in this region. The actual nozzle will then follow this contour up to a certain, but not predetermined, point. As a result, point T, shown in Figure 3, is located so that no discontinuity in the plug contour arises as the contour modification is carried out. Point E is then located along the left characteristic originating at point T rather than being on the cowl lip. In general, point T will always be downstream from point A and can never move upstream of point A during the contour modification process. The calculation of the flow field must be carried out in recognition of this situation.

The flow field calculations are started by evaluating the flow properties along right characteristics which originate at the start line. These calculations continue down the right characteristics until the left characteristic which originates at point A is reached(see Figure 3). The properties along this left characteristic are stored to avoid recalculating this portion of the flow field each time the plug contour is adjusted. Thus, as point T moves during the iteration procedure it is necessary to calculate the flow field from the left characteristic originating at point A to the new point T.

Once the flow properties upstream of and along the left characteristic TE are evaluated, TE is divided into (NPTS-1)

equally spaced points and the data are curve fitted. However, an option is provided in the program so that the first (NS-1) points can have closer spacing than the remaining points. Thus, the grid size can be varied in the region (R). The flow calculations in the region (R) proceed from T to E down the right characteristics until the plug contour TD is reached.

Once the flow field is known it is possible to evaluate the Lagrange multiplier field by starting at point D and solving Eqs.(52), (56), and (57) for λ_1 and λ_2 . The transversality conditions (52), (53), and (54) are then used to evaluate λ_1 and λ_2 along TD. Thus, the surface TD can be used as an initial-value line from which to start the method of characteristics solution for λ_1 and λ_2 in (R). Again starting at point T and proceeding towards E, λ_1 and λ_2 are evaluated up the right characteristics starting from the plug surface and stopping when the left characteristic TE is reached. Equation (55) is evaluated as λ_1 and λ_2 are obtained along the exit characteristic. No iteration procedure is required to obtain λ_1 and λ_2 in (R) since Eqs.(50) are valid along the characteristics of the basic flow field which is known. Thus, in finite difference form Eqs.(50) are algebraic equations.

In general Eq.(55) will not be satisfied by the first guess for the optimum surface and it is then necessary to calculate a new wall contour. In calculating the new contour it is convenient to consider Eq.(55) as an error

function such that

$$E = \lambda_1 - \lambda_2 y \rho \cot \alpha \quad (65)$$

Changes in the wall contour are designed to drive E to zero as rapidly as possible and upon this premise a new wall is constructed. This method of contour modification is referred to as a relaxation technique and was developed by Scofield, Thompson, and Hoffman (8). The details of the method are described in the next section.

After the new wall contour is obtained the calculation procedure starts over again, beginning from the stored left characteristic which originates at point A. This iteration procedure continues until the error function E is reduced to an acceptable value near zero.

2. RELAXATION TECHNIQUE

The wall modification procedure or wall relaxation must, above all, produce rapid convergence. The entire procedure is designed with this in mind.

In order to change the wall contour it must first be determined how changes in the contour affect the error function given by Eq.(65). To do this the wall angle θ is chosen as the independent variable and the error function E as the dependent variable. Theoretically the wall angle can be incremented at a point on the wall, the flow field and Lagrange multiplier field recalculated, and the change in the error function along DE evaluated. This procedure

could be repeated until every wall point had been incremented and the corresponding changes in the error function found. This would result in an $n \times n$ matrix (n = number of wall points) relating changes in the wall angle to changes in the error function. This can be written in partial derivative form as

$$\frac{\partial E_i}{\partial \theta_j} = \frac{E_i - E_{i0}}{\theta_j - \theta_{j0}} \quad (i, j = 1, \dots, n) \quad (66)$$

where E_{i0} and θ_{j0} are the initial values of E_i and θ_j .

Once these partial derivatives are known, a truncated Taylor series could be used to relate changes in the wall angle to changes in the error function such that

$$E_i - E_{i0} = \left. \frac{\partial E_i}{\partial \theta_j} \right|_{\theta_{j0}} (\theta_j - \theta_{j0}) \quad (i, j = 1, \dots, n) \quad (67)$$

Since the desired value of the error function is zero, Eqs. (67) could be used to solve for the value of the wall angle that will drive the error function to zero. Thus,

$$\Delta \theta_j = -E_{i0} / \left(\left. \frac{\partial E_i}{\partial \theta_j} \right|_{\theta_{j0}} \right) \quad (i, j = 1, \dots, n) \quad (68)$$

In theory Eqs. (68) could be solved, but in practice this could be difficult when a large number of wall points are involved.

In their investigation of the problem for conventional nozzles, Scofield, Thompson, and Hoffman (8) found that it could be assumed that there is an independent relationship

between changes in the wall angle and changes in the error function which lie on the same right-running characteristic. That is, the main effect on E due to a change in the wall angle propagates down the right-running characteristic which originates at that point. This same type of assumption was found to be valid for the plug nozzle when applied along left-running characteristics originating at the wall. Thus, Eqs.(68) reduce to n simple independent equations which can be solved for $\Delta\theta$ at each of the n wall points.

$$\Delta\theta_i = -E_{i0}/(\partial E_i/\partial\theta_i) \quad (i = 1, \dots, n) \quad (69)$$

Since the problem is nonlinear, the final solution must be approached iteratively. The wall angles for the $(r + 1)$ iteration are given by

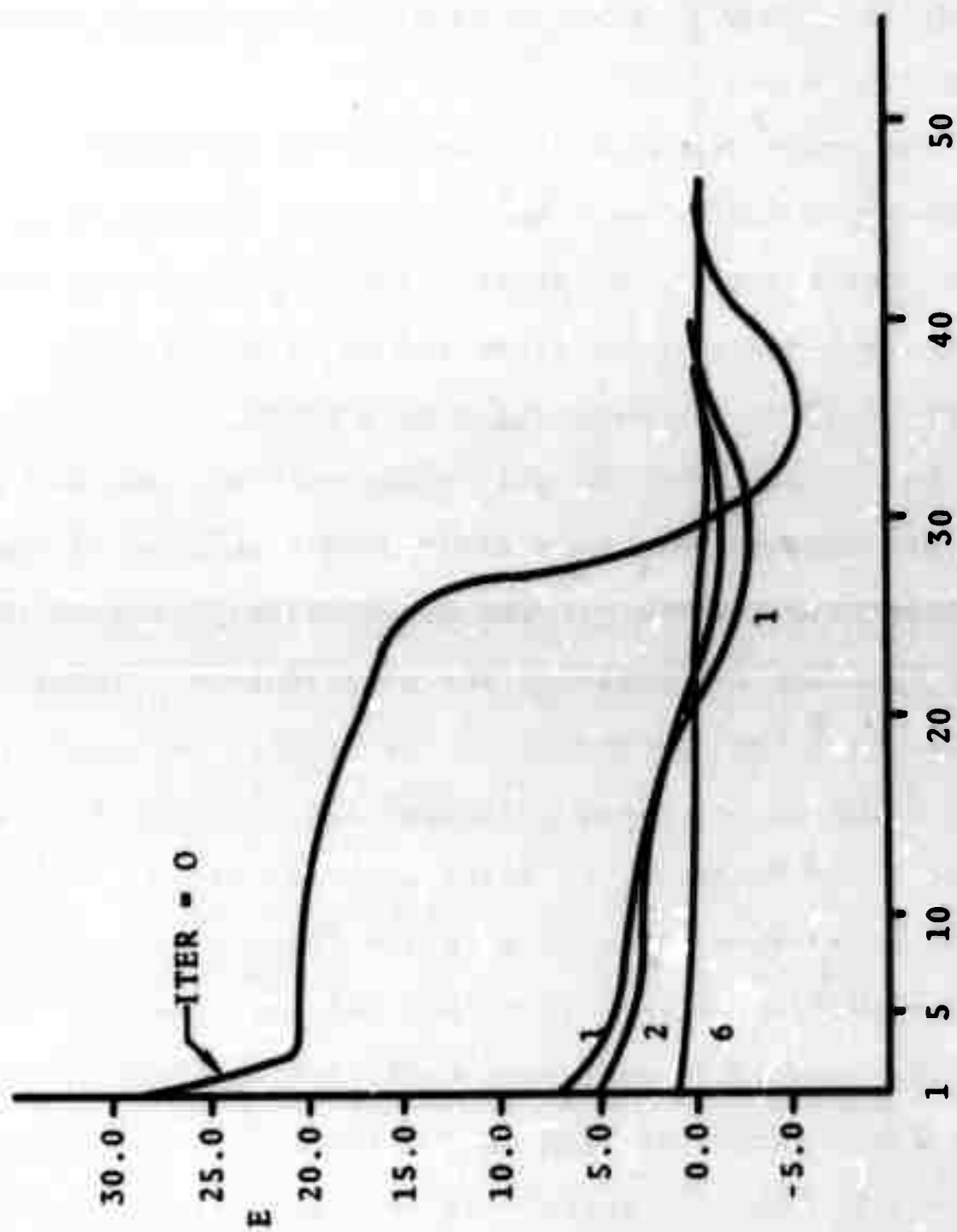
$$\theta_i^{(r+1)} = \theta_i^r + \Delta\theta_i \quad (i = 1, \dots, n) \quad (70)$$

The procedure just described is a simple method of adjusting the wall contour but would require considerable time if carried out in all of its detail. Scofield, Thompson, and Hoffman (8) used two methods to reduce the computer running time and both are used in the current program. First it was noted that a nearly linear relationship exists between the partial derivatives $\partial E_i/\partial\theta_i$ and the corresponding wall axial coordinate x . Because of this linear relationship it is possible to calculate the partial derivatives at a few points (typically ten) and then fit a straight

line through these points by the method of least squares. The remaining partial derivatives can be taken from the fitted straight line and thereby reduce the time required for calculating $\partial E_i / \partial \theta_i$. The second method used to reduce the program running time is by recalculating the partial derivatives only after several iterations rather than after each iteration.

Two other controls are included in the program to assure rapid convergence and to prevent oscillations. These controls are: 1) application of a weighting factor to the wall angle corrections and 2) allowing a maximum change of 5° in the wall angle at a point.

The error function, wall angle corrections, and partial derivatives have been plotted in Figures 4, 5, and 6 to illustrate how the program progressively reduces the error function and modifies the plug contour. These figures have been plotted from the results of Sample Case No. 1 which is discussed in Appendix K. Figure 4 is a plot of the error function, E , after each iteration. As can be seen, the error associated with the first guess for the optimum surface is very large but reduces rapidly. The iteration procedure continues until the absolute value of the error function divided by the local value of λ_1 is less than 0.001. This criterion for an acceptable solution was met after 11 iterations. Figure 5 is a plot of the wall angle corrections, $\Delta\theta$, after each iteration. As might be



POINT ALONG EXIT CHARACTERISTIC

FIGURE 4. REDUCTION OF THE ERROR FUNCTION

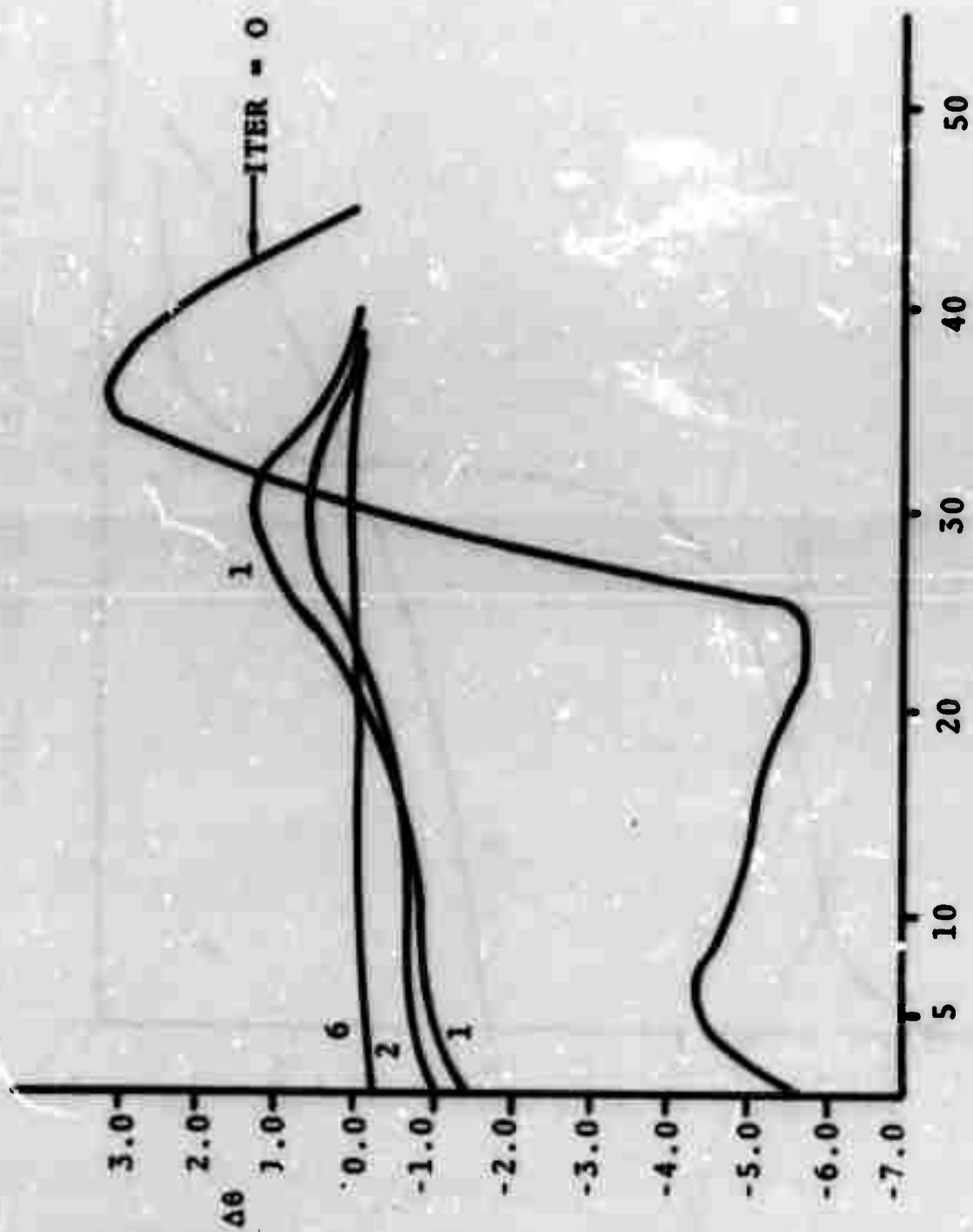


FIGURE 5. BEHAVIOR OF THE WALL ANGLE CORRECTIONS
POINT ALONG THE EXIT CHARACTERISTIC

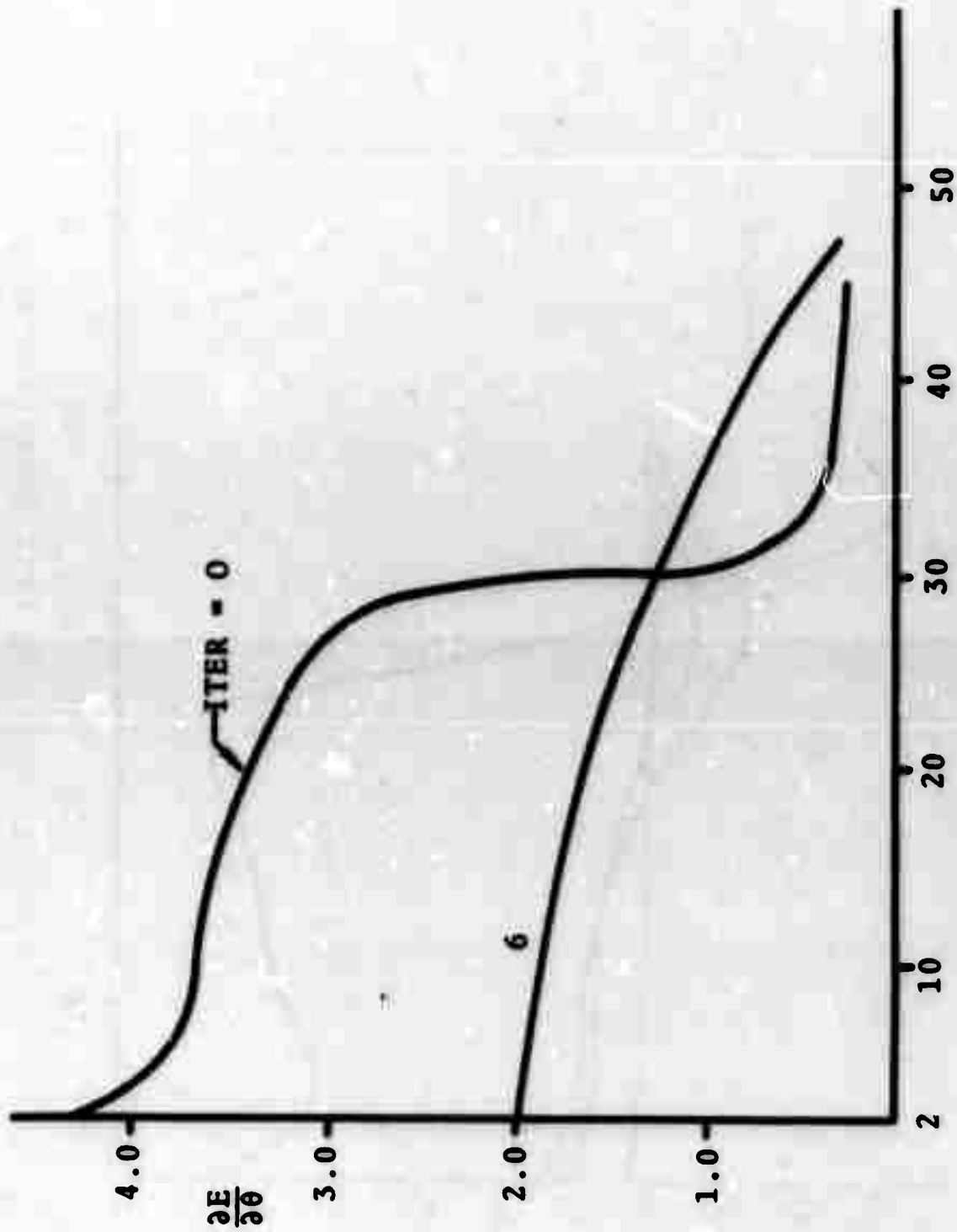


FIGURE 6. BEHAVIOR OF THE ERROR FUNCTION DERIVATIVE
POINT ALONG THE EXIT CHARACTERISTIC

expected, the wall angle corrections are large for the first guess of the optimum contour but decrease rapidly as the error function decreases. The partial derivatives of the error function with respect to the wall angle, $\partial E / \partial \theta$, were calculated for the first guess and again after 5 iterations. These results are shown in Figure 6. It should be noted in each of these figures that the number of wall points on the contour which is to be optimized may change from iteration to iteration. This occurs as the program seeks the specified length and specified number of wall points.

The details concerning the computer program and its operation, including input, output, failure modes, and listings are given in Appendices J, K, and L.

SECTION V

RESULTS

The formulations of Sections II and III assume specified values for the cowl lip radius and initial injection angle. Since both of these parameters have a significant influence upon the total thrust of plug nozzles, it may be desirable, in some cases, to determine the optimum values of these quantities. To illustrate how this can be done, a parametric study was carried out to determine the optimum cowl lip radius and injection angle for a given length nozzle. The purpose of this section is to present the results of the parametric study and to compare the optimum nozzle to one designed by Rao's method (3) for the same mass flow rate and ambient pressure. The effect of changing the base pressure model is illustrated, and an example of the optimization of scramjet nozzles is presented.

1. PARAMETRIC STUDY TO DETERMINE THE OPTIMUM COWL LIP RADIUS AND INJECTION ANGLE

A Fortran IV computer program was written for the irrotational flow problem presented in Section III. The program accounts for the wall shear stress, but does not account for the boundary layer thickness. A correction for the boundary layer thickness can be made to the optimized

contour by displacing the wall contour by the magnitude of the boundary layer thickness.

As mentioned in Section II, the thrust maximized in the variational problem does not represent the total axial thrust. The total thrust is given by the equation

$$\begin{aligned} \frac{T}{2\pi} = & \int_A^E \left[p + \rho V^2 \frac{\sin(\phi - \theta)}{\sin \theta} \cos \phi \right] y dy - y_E^2 p_a / 2 \\ & - \int_A^D [p\dot{y} + \tau] y dx + y_D^2 p_b / 2 \end{aligned} \quad (71)$$

where the angles ϕ and θ are shown in Figure 3. The last two terms in Eq. (71) are obtained by neglecting the boundary layer thickness in Eq. (37). The first term accounts for the pressure and momentum forces across the initial-value line EA, the second term accounts for the ambient pressure, the third term is the pressure thrust generated by the plug contour, and the last term is the plug base thrust. The cowl lip radius and injection angle will influence the first two terms while the contour optimization presented in Section III is used to maximize the last two terms for a specific choice of cowl lip radius and injection angle. However, as explained in Section IV, the radius of curvature of the plug between points A and T is specified a priori, and as a result, this portion of the nozzle wall will not necessarily be an optimum. Thus, the plug contour generated by the present technique yields a thrust maximum for the specified inlet. To determine the best overall nozzle when no constraints are placed on the cowl lip radius and injection angle, these two

quantities can be varied parametrically and the optimum plug contour can be obtained for each set of these quantities. The maximum thrust nozzle is then selected from this group of nozzles. Such a parametric study is presented in this section.

The parametric study was conducted for a nozzle which has a mean radius of curvature at the throat of 0.705 in., a downstream radius of curvature of 0.5 in., and a length from point T to point D (see Figure 3) of 12.0 in. The mass flow rate, ambient pressure, and incompressible skin friction coefficient were selected as 148.08 lbm/sec, 14.7 psia, and 0.002, respectively. The engine was assumed to operate with a chamber pressure of 500.0 psia and a chamber temperature of 6000.0°R. The exhaust products were assumed to have a gas constant of 56.0 (ft-lbf)/(lbm-°R) and a ratio of specific heats of 1.23. The constants for the base pressure model were selected as $AA = 0.846$ and $AB = 1.3$. Each of the above parameters is an input to the computer program which is described in Appendices J, K, and L. These appendices should be consulted for additional details concerning how these parameters are used in the program. The nozzle was designed for a subsonic burning engine, thus the start line described in Appendix H was used. As the cowl lip radius and injection angle are changed, the throat height, $2h_0$, is varied in order to keep the mass flow rate constant. A total of 20 computer runs were made to determine the optimum cowl lip radius and

injection angle. The results of these runs are presented in Table 1.

The data in Table 1, except the last set where $\beta = -34^\circ$ and $y_E = 7.55$ in., are plotted in Figures 7 and 8. Figure 7 is a plot of thrust as a function of cowl lip radius with the injection angle as a parameter, and Figure 8 is a plot of thrust as a function of injection angle with the cowl lip radius as a parameter. From Figure 7, it was determined that the optimum cowl lip radius would be approximately 7.55 in., and the optimum injection angle was determined from Figure 8 to be approximately -34° . One final computer run was made using these values for the cowl lip radius and injection angle. This run resulted in a thrust of 32,881 lbf., which is the maximum of the total of 21 designs. The coordinates and slope of the resulting optimum contour are given in Table 2 and plotted in Figure 9. Two other contours of the same length have also been plotted in Figure 9 for comparison to the optimum. One contour is 0.5 in. above the optimum at point D and the other is 0.5 in. below the optimum at the same point. As expected, both contours produced a lower thrust than the thrust of the optimum contour. The upper contour produced a thrust of 32,556 lbf. and the lower contour produced 32,601 lbf.

Thus, it is possible to determine the optimum values for both the cowl lip radius and injection angle even though these quantities were fixed in the variational problem

TABLE 1. PARAMETRIC STUDY DATA

INJECTION ANGLE (DEGREES)	COWL LIP RADIUS (IN)	THRUST (lbf)
-31.00	7.00	32,803
-31.00	7.50	32,853
-31.00	8.00	32,813
-31.00	8.50	32,699
-33.00	7.00	32,811
-33.00	7.50	32,874
-33.00	8.00	32,842
-33.00	8.50	32,731
-35.00	7.00	32,808
-35.00	7.50	32,877
-35.00	8.00	32,846
-35.00	8.50	32,747
-37.00	7.00	32,791
-37.00	7.50	32,869
-37.00	8.00	32,847
-37.00	8.50	32,753
-39.00	7.00	32,763
-39.00	7.50	32,844
-39.00	8.00	32,829
-39.00	8.50	32,741
-34.00	7.55	32,881

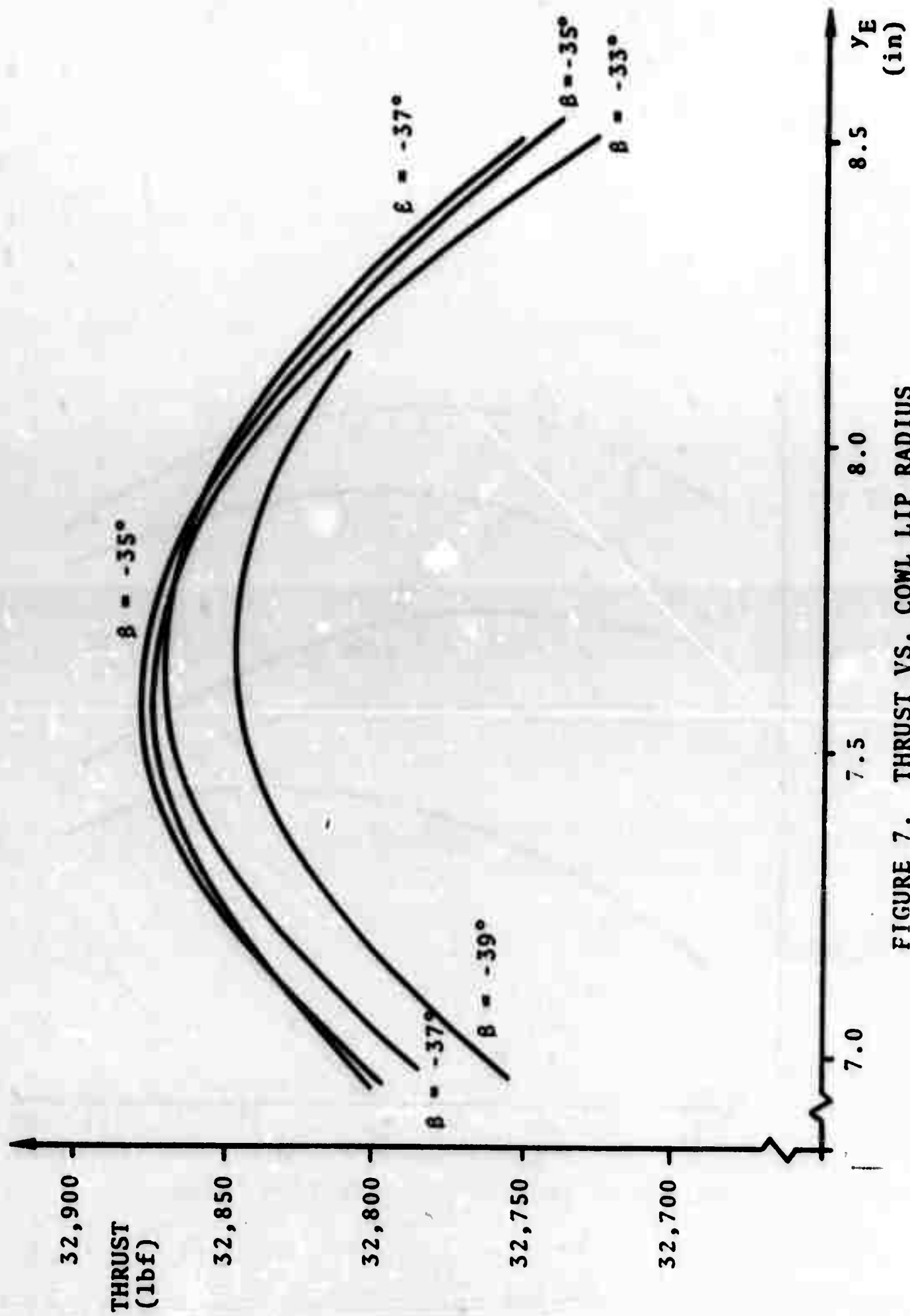


FIGURE 7. THRUST VS. COWL LIP RADIUS

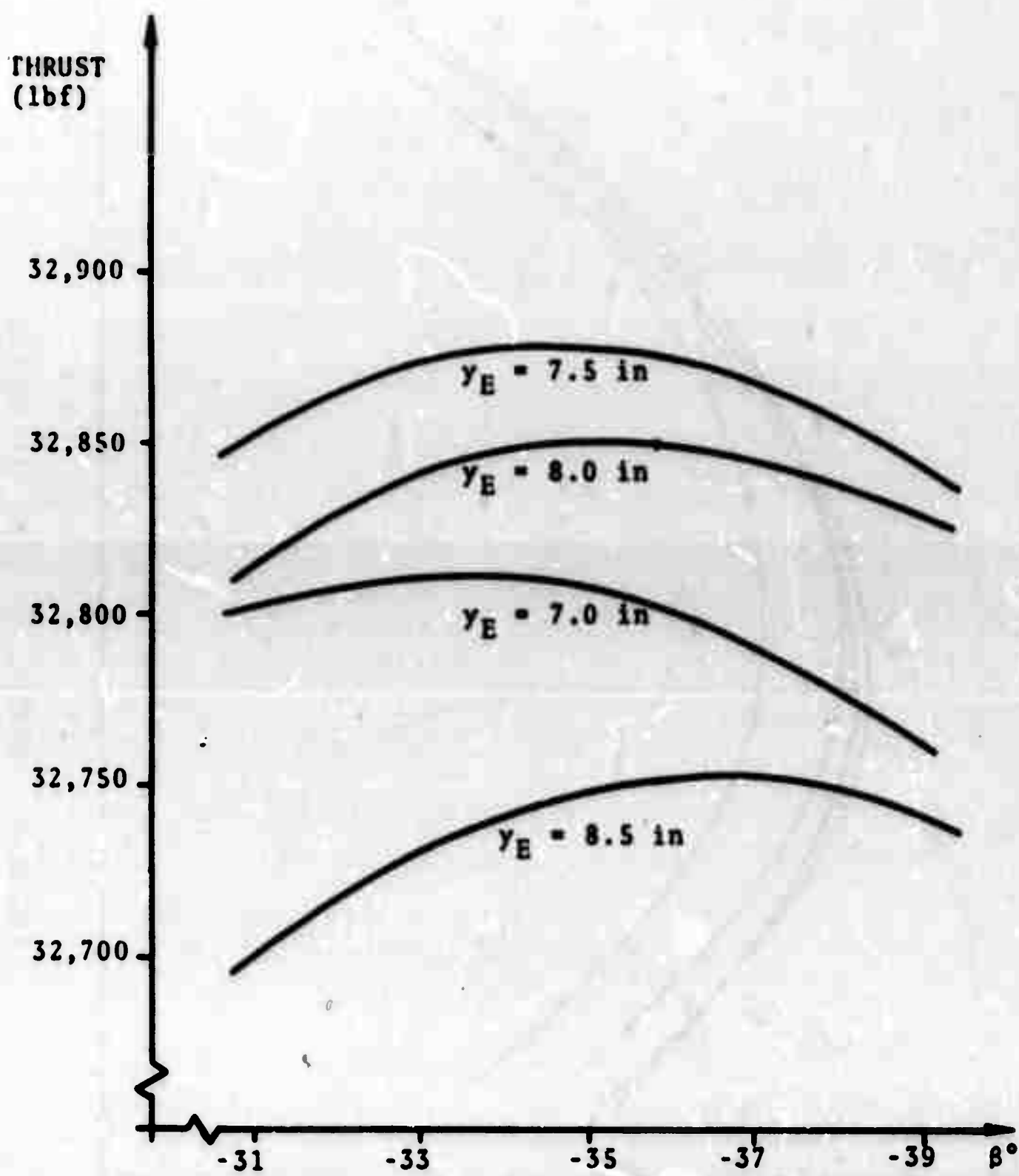


FIGURE 8. THRUST VS. INJECTION ANGLE

TABLE 2. COORDINATES OF THE OPTIMUM PLUG CONTOUR

$\gamma = 1.23$	$p_o = 500.0 \text{ psia}$
$R = 56.0 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot ^\circ\text{R}}$	$T_o = 6000 \text{ }^\circ\text{R}$

$x(\text{in})$	$y(\text{in})$	θ°
-0.56069	6.71874	-36.25027
-0.55023	6.71087	-37.75027
-0.53999	6.70272	-39.25027
-0.52996	6.69430	-40.75027
-0.52016	6.68563	-42.25027
-0.51059	6.67670	-43.75027
-0.50125	6.66753	-45.25027
-0.49216	6.65811	-46.75027
-0.48331	6.64846	-48.25027
-0.40216	6.55777	-48.02999
-0.29091	6.43493	-47.64577
-0.03515	6.15846	-46.81925
0.29695	5.81585	-44.30856
0.99456	5.21035	-37.84839
2.10220	4.45395	-31.24354
3.01742	3.93915	-27.61354
5.08714	2.98654	-22.22618
6.75042	2.35672	-19.37574
8.85315	1.67387	-16.70496
11.51707	0.95441	-13.26458

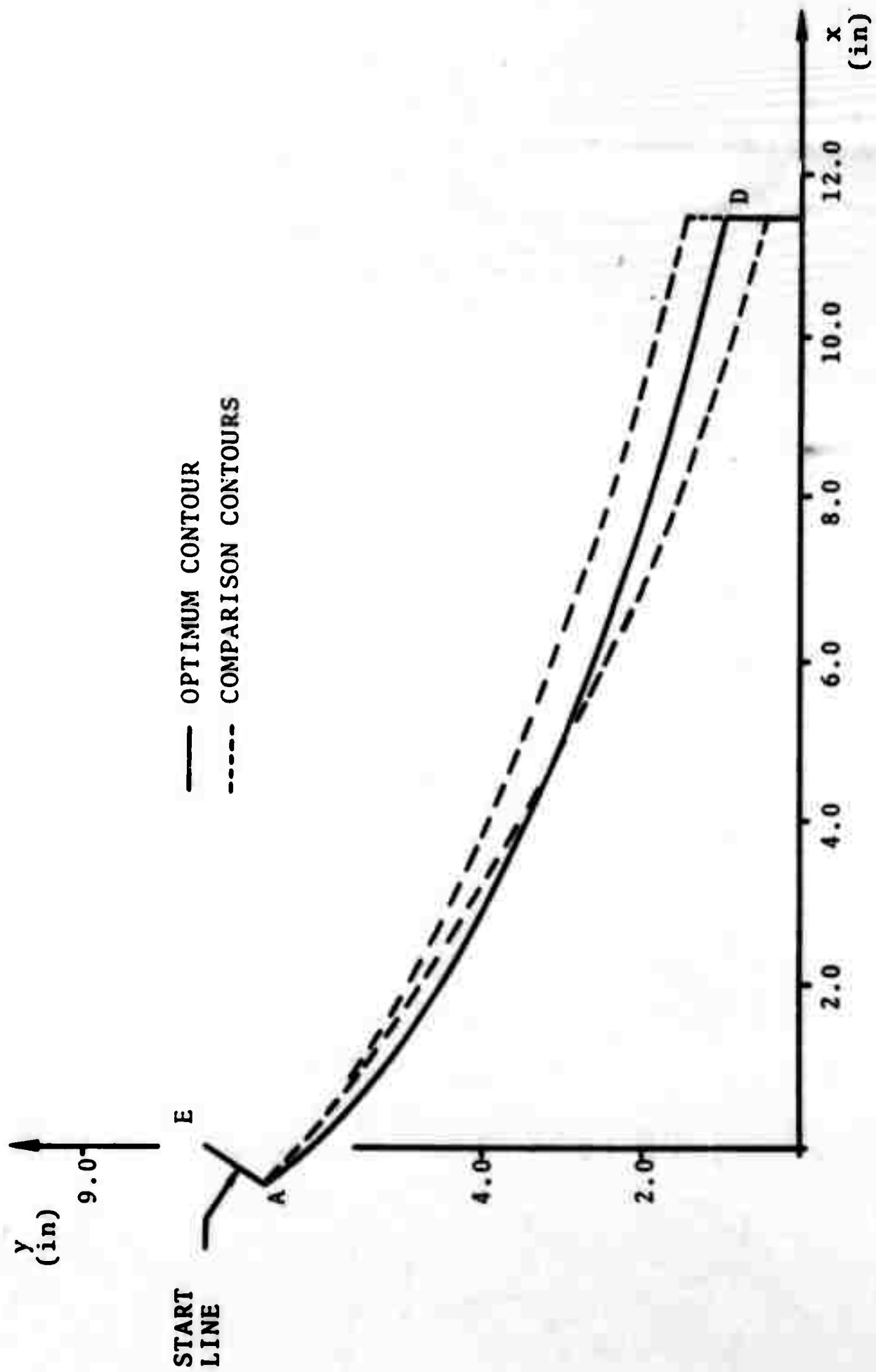


FIGURE 9. OPTIMUM CONTOUR

formulation. This approach to the problem is also useful in cases where either the cowl lip radius or injection angle is dictated by other considerations. This situation could arise, for example, when the cowl lip radius is limited by the vehicle size, but no restrictions are placed on the injection angle. The best injection angle for the given cowl lip radius could be obtained from a plot such as Figure 8.

2. COMPARISON TO RAO NOZZLES

Since the current formulation contains the results of Rao (3) as a special case, it is of interest to compare the nozzles designed by the two methods.

a. Rao Results. The design equations obtained by Rao were programmed in order to compare his technique with the current formulation. A comparison was made with the same mass flow rate, plug length, ambient pressure, thermodynamic properties, and base pressure model as those employed in the parametric study. The Rao method yields an optimum cowl lip radius of 8.33 in. and an optimum injection angle of approximately -58.5° with a resulting thrust of 34,253 lbf. The coordinates and slope of the resulting optimum contour are given in Table 3 and plotted in Figure 10. By comparing the data in Table 3 with that in Table 2, it can be seen that the contour obtained by Rao's method lies above the contour obtained from the parametric study. In order to compare the

TABLE 3. COORDINATES OF THE RAO CONTOUR

$\gamma = 1.23$	$p_o = 500.0 \text{ psia}$	
$R = 56.0 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot ^\circ\text{R}}$	$T_o = 6000^\circ\text{R}$	
$x(\text{in})$	$y(\text{in})$	θ°
-0.63971	7.60953	-60.69463
-0.52553	7.41133	-59.26745
-0.38991	7.19374	-56.76264
-0.28105	7.03417	-54.62384
-0.11511	6.81327	-51.56682
0.08454	6.57580	-48.37210
0.26305	6.38348	-45.92900
0.54982	6.10414	-42.64770
0.80980	5.87499	-40.18842
1.11665	5.62702	-37.74413
1.75877	5.16641	-33.72519
2.24216	4.85831	-31.35732
3.03421	4.40556	-28.40556
4.04195	3.89898	-25.23198
5.33714	3.33063	-12.27169
6.55991	2.85772	-20.07902
8.07079	2.33921	-17.86199
9.28531	1.96623	-16.29294
11.51781	1.37506	-13.25333

shapes of these two contours, the contour which resulted from the parametric study was also plotted in Figure 10. However, 0.42065 in. was added to the y-coordinates of the contour obtained from the parametric study so that the two contours would match at point D. As can be seen from Figure 10, the two contours are almost identical in shape except in the throat region, the most significant difference being in the injection angle and cowl lip radius. Since the start line or sonic line assumptions for these two models are not the same, it was suspected that the differences in the results were caused by the dissimilarities in the start lines. The results of an investigation to determine if this was indeed the case are presented in the next section.

b. Importance of Start Line. Since Rao assumed a linear sonic line along which the flow direction is constant, an attempt was made to duplicate his results by using such a start line along which the Mach number was slightly greater than unity. The cowl lip radius and injection angle were fixed at the values found for the optimum Rao nozzle and the wall shear stress was set equal to zero since Rao did not account for this in his formulation. The general flow condition produced by a parallel uniform flow with the flow direction toward the axis of symmetry is one of compression. Unless the Mach number along the start line is increased to approximately 1.5, the compression results in subsonic Mach numbers in the throat region. However, the assumption of a

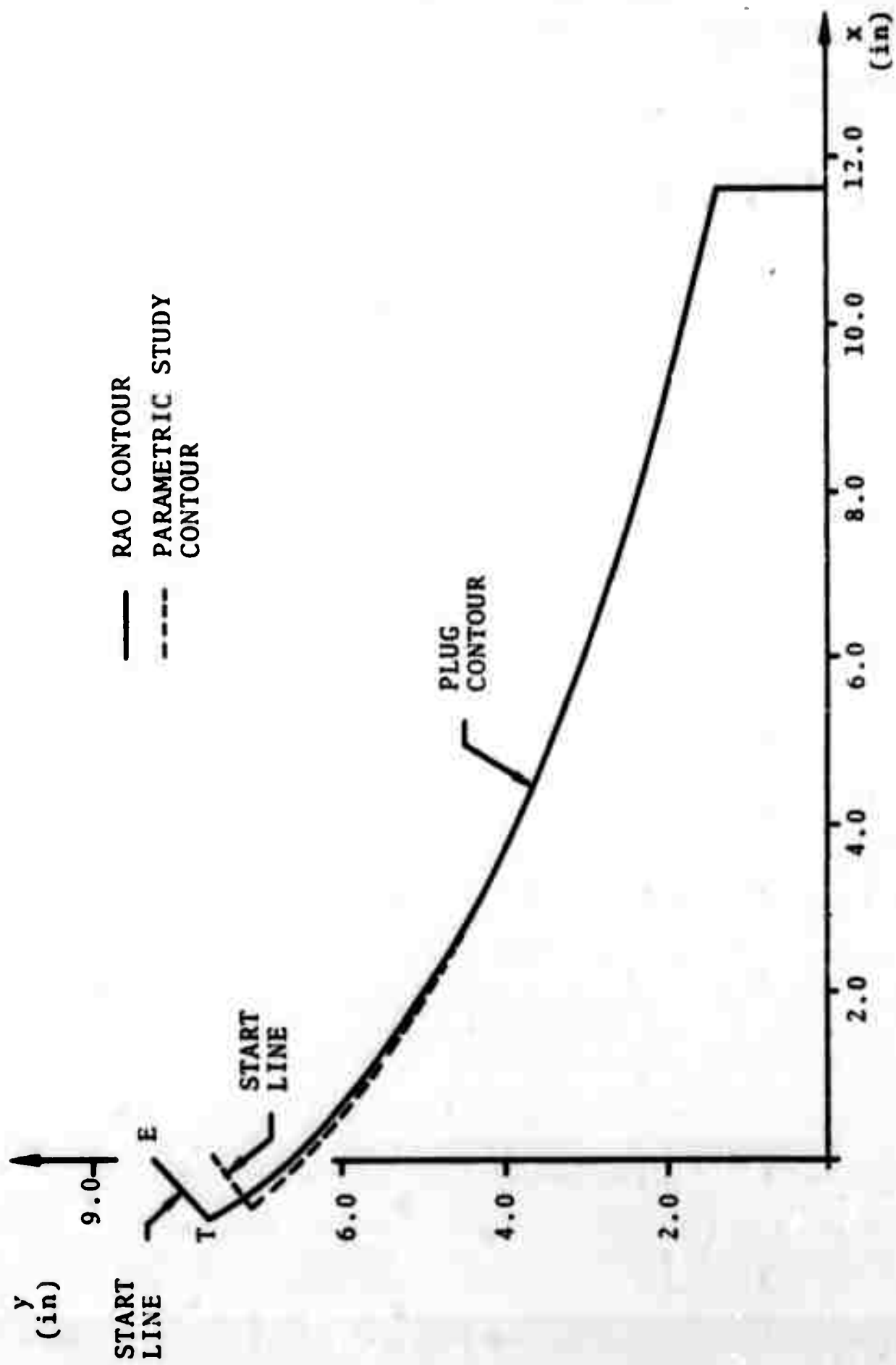


FIGURE 10. CONTOUR COMPARISON

Mach number greater than one along the start line implies that the flow has passed through the minimum area and should be expanding. This requires the flow direction to become more negative along the start line, EA. By decreasing the flow direction from 3 to 6 degrees along the start line, the compression problem was eliminated. Typically, the flow direction at point E was fixed at -56.0° and decreased uniformly to -62.0° at point A. The resulting plug contour matched the Rao contour reasonably well but the thrust was approximately 2700 lbf. lower.

One final attempt was made to duplicate Rao's results. A right-running characteristic near the throat was taken from the Rao nozzle flow field and used as a start line. This start line produced a contour almost identical to Rao's. The y-coordinate at point D was 0.1364 in. lower than Rao's, and the thrust was 34,373 lbf. compared to 34,375 lbf. for Rao's nozzle.

These results indicate that the start line assumptions are very important in the design of optimum plug nozzles, and approximations in this region should be carefully evaluated.

3. EFFECT OF THE BASE PRESSURE MODEL

Another possible source of error in plug nozzle design is the base pressure model. The importance of the base pressure model is illustrated in this section.

After the parametric study was completed and the optimum nozzle configuration determined, the equation used to calculate the base pressure was changed from Eq. (G-1) to Eq. (G-2) to determine the effect of the base pressure model. All other parameters were set at the values used for the parametric study, and the cowl lip radius and injection angle were set at their optimum values of 7.55 in. and -34° . The coordinates and slope of the resulting contour are given in Table 4 and plotted in Figure 11. The contour obtained from the parametric study is also plotted in Figure 11. As can be seen from this figure, the base pressure model has a considerable effect upon the shape of the plug contour. The base height, y_D , increased from 0.954 in. to 2.34 in. and the wall slope at point D increased from -13.26° to -3.08° . In addition, the thrust increased to 32,965 lbf. because of the higher base pressure. Since the thrust has changed, it is expected that the optimum cowl lip radius and injection angle will also be different. Thus, in addition to a direct thrust contribution on the plug base, the value of the base pressure significantly influences the shape of the optimum contour.

4. OPTIMIZATION OF SCRAMJET NOZZLES

The methods and results presented to this point are applicable to supersonic combustion engines as well as to those which burn subsonically. However, the starting conditions for these two applications are not the same. An

TABLE 4. COORDINATES OF AN OPTIMUM CONTOUR
FOR THE ALTERNATE BASE PRESSURE MODEL

$\gamma = 1.23$			$p_o = 500.0 \text{ psia}$		
$R = 56.0 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot ^\circ\text{R}}$			$T_o = 6000^\circ\text{R}$		
x(in)		y(in)		θ°	
-0.56069		6.71874		-36.25027	
-0.55023		6.71087		-37.75027	
-0.53999		6.70272		-39.25027	
-0.52996		6.69430		-40.75027	
-0.52016		6.68563		-42.25027	
-0.51059		6.67670		-43.75027	
-0.49819		6.66442		-45.75027	
-0.44272		6.60622		-46.29203	
-0.33777		6.49721		-45.88483	
-0.19453		6.35086		-45.35040	
-0.00101		6.15730		-44.66440	
0.34386		5.83114		-41.57284	
0.90107		5.38449		-36.00408	
1.72398		4.85222		-30.05107	
2.55635		4.41335		-25.69669	
3.61048		3.95442		-21.49032	
4.92072		3.49325		-17.40459	
6.53289		3.05192		-13.30646	
8.49709		2.66542		- 9.02277	
11.58406		2.34099		- 3.08106	

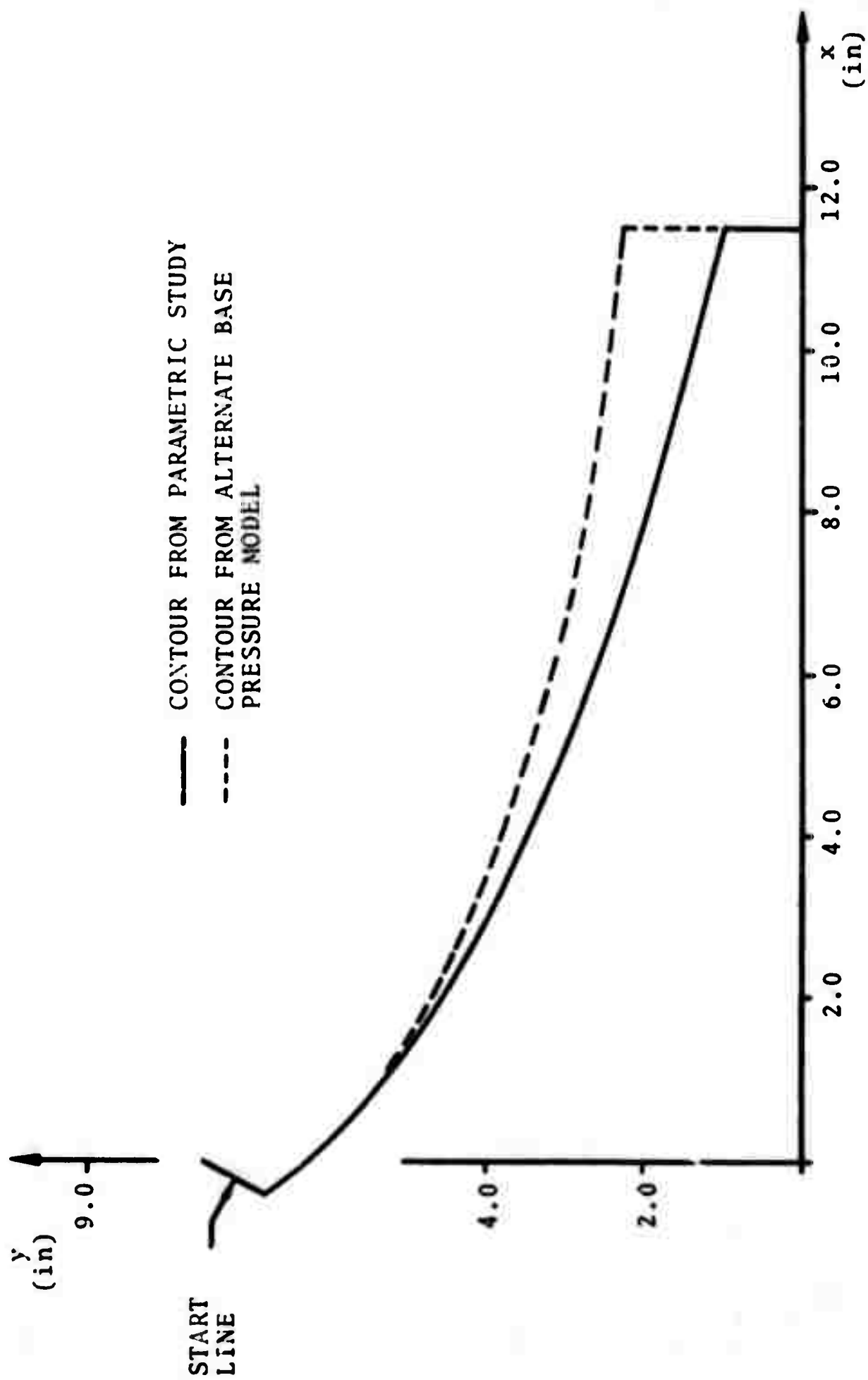


FIGURE 11. OPTIMUM CONTOUR FOR ALTERNATE BASE PRESSURE MODEL

illustration of the optimization of scramjet nozzles is presented in this section.

The flow conditions at the entrance to scramjet nozzles are expected to be nonuniform in nature and well within the supersonic range. The nonuniform nature of the flow is produced by the inlet effects and the combustion process. As was demonstrated in the parametric study, it is advantageous in the case of subsonic burning engines to inject the exhaust gases toward the axis of symmetry. This may not be feasible for scramjets since the flow throughout the engine is supersonic and would require a compression turn. The losses inherent in the resulting shock could outweigh any advantages to be gained. A detailed analysis would have to be carried out before any definite conclusions could be drawn, but for the purpose of illustrating the design of a scramjet nozzle, the injection angle was assumed to be zero. The start line was assumed to be a straight line along which the Mach number had a constant value of 1.5. The downstream radius of curvature was selected as 0.5 in. and the length from point T to point D was chosen as 8.0 in. The mass flow rate, ambient pressure, and incompressible skin friction coefficient were given values of 50.0 lbm/sec, 14.7 psia, and 0.002, respectively. The engine chamber conditions were selected as $p_0 = 500.0$ psia and $T_0 = 6000^\circ\text{R}$. The exhaust products were assumed to have a gas constant of 56.0 (ft-lbf)/(lbm- $^\circ\text{R}$) and a ratio of specific heats of 1.23.

The constants for the base pressure model were selected as $AA = 0.846$ and $AB = 1.3$. Finally, the cowl lip radius was fixed at 6.0 in. The coordinates and slope of the optimum contour which resulted from these data are given in Table 5 and plotted in Figure 12. This contour produced a thrust of 9722 lbf.

As these results indicate, the contour follows the specified radius of curvature at the nozzle inlet until the slope reaches -39.5° . The contour then begins to flatten out and the slope at point D is about the same as obtained in the previous cases. The maximum wall slopes obtained are less than those calculated for the other cases, due to the design constraint which requires the initial injection angle to be zero. Should moderate injection angles be permissible, a parametric study, such as presented at the beginning of this section, could be performed to obtain the best overall design.

TABLE 5. COORDINATES OF AN OPTIMUM SCRAMJET NOZZLE

$\gamma = 1.23$	$p_o = 500.0 \text{ psia}$
$R = 56.0 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot ^\circ\text{R}}$	$T_o = 6000^\circ\text{R}$

$x(\text{in})$	$y(\text{in})$	θ°
0.00000	5.56647	0.00000
0.02181	5.56600	-2.50000
0.04358	5.56457	-5.00000
0.06526	5.56219	-7.50000
0.08682	5.55888	-10.00000
0.10822	5.55462	-12.50000
0.12941	5.54944	-15.00000
0.15035	5.54333	-17.50000
0.17101	5.53632	-20.00000
0.19134	5.52841	-22.50000
0.21131	5.51963	-25.00000
0.23087	5.50998	-27.50000
0.25000	5.49949	-30.00000
0.26865	5.48817	-32.50000
0.28679	5.47605	-35.00000
0.30438	5.46315	-37.50000
0.31804	5.45228	-39.50000
0.32071	5.45007	-39.89710
0.45381	5.34156	-38.42232
0.76198	5.11254	-34.84466
1.51175	4.65257	-28.66838
2.50288	4.16818	-23.76215
3.47465	3.77417	-20.52422
4.92358	3.28370	-17.04437
7.03519	2.71383	-13.31267
8.32071	2.43250	-11.39107

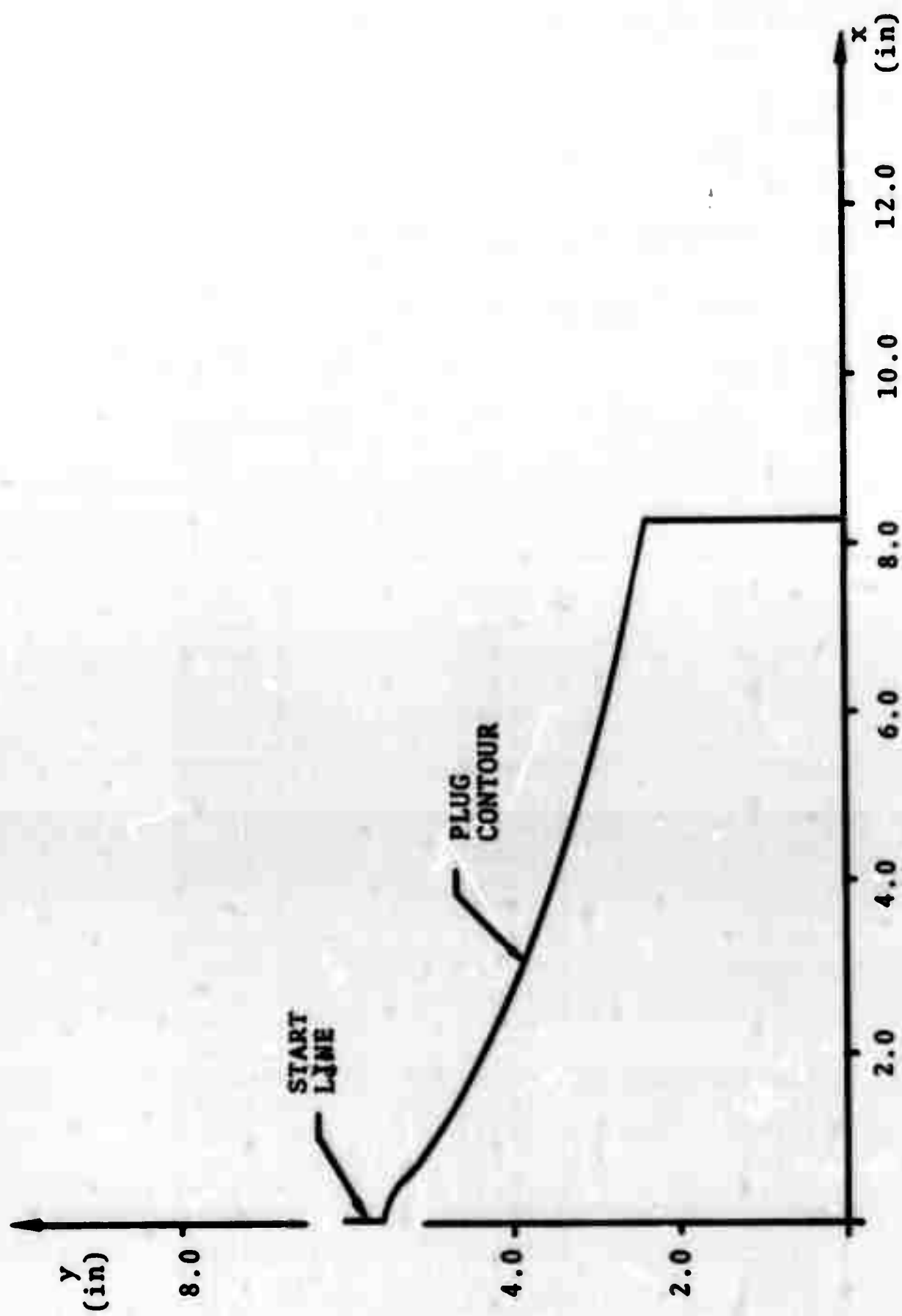


FIGURE 12. OPTIMUM SCRAMJET NOZZLE

SECTION VI

SUMMARY AND RECOMMENDATIONS

An analysis has been presented for the optimization of plug nozzle contours with boundary layer effects accounted for in the optimization. The solution makes no particular assumption about the upstream nozzle geometry but simply requires the flow conditions along a start line to be available in order to initiate the flow field solution. The results can then be applied to supersonic burning engines as well as to those which burn subsonically. The problem was formulated for rotational and irrotational flow. A general isoperimetric constraint was imposed upon the plug contour in the region of supersonic flow. A complete set of partial differential equations with sufficient boundary conditions was obtained for determining the flow properties and Lagrange multipliers. A method was presented for each of the problem formulations to determine if a given contour was an optimum and a relaxation technique was used to obtain a solution to the irrotational flow problem.

The design equations for the irrotational flow problem were programmed in Fortran IV and the computer program was described. This program was used to carry out a parametric

study to determine the optimum cowl lip radius and injection angle when the isoperimetric constraint is one of fixed length. The resulting optimum nozzle was compared to one designed by Rao's method. The importance of determining the base pressure accurately was illustrated and an example of scramjet nozzle optimization was presented.

During this study it has become apparent that several aspects of the plug nozzle optimization problem could benefit from additional work. First, the analytical methods of determining the transonic flow conditions in subsonic burning engines need to be verified experimentally. The flow conditions at the throat determine to a large extent the best injection angle and affect the flow conditions along the exit characteristic. Second, there is a need for a base pressure model which has direct application to plug nozzles. The importance of determining the base pressure accurately has been demonstrated but one finds very little base pressure data in the literature which is directly applicable to plug nozzles. Third, it would be desirable to include the ambient pressure and injection angle in the problem formulation, thereby removing the need for a parametric study to determine the optimum cowl lip radius and injection angle.

REFERENCES

1. Guderley, G. and Hantsch, E., "Beste Formen fur Achsensymmetrische Überschallschubdusen," *Z. Flugwiss.*, Vol. 3, 1955, pp. 305-313.
2. Rao, G. V. R., "Exhaust Nozzle Contour for Optimum Thrust," *Jet Propulsion*, Vol. 28, 1958, pp. 377-382.
3. Rao, G. V. R., "Spike Nozzle Contour for Optimum Thrust," *Ballistic Missile and Space Technology*, Vol. 2, edited by C. W. Morrow, Pergamon Press, New York, 1961, pp. 92-101.
4. Guderley, K. G. and Armitage, John V., "A General Method for the Determination of Best Supersonic Rocket Nozzles," Paper presented at the Symposium on Extremal Problems in Aerodynamics, Boeing Scientific Research Laboratories, Seattle, Washington, December 3-4, 1962.
5. Guderley, K. G. and Armitage, J. V., "General Approach to Optimum Rocket Nozzles," Chap. 11, *Theory of Optimum Aerodynamic Shapes*, edited by Angelo Miele, Academic Press, New York, 1965.
6. Hoffman, Joe D. and Thompson, H. Doyle, "A General Method for Determining Optimum Thrust Nozzle Contours for Gas-Particle Flows," *AIAA Journal*, Vol. 5, No. 10, Oct. 1967, pp. 1886-1887.
7. Hoffman, Joe D., "A General Method for Determining Optimum Thrust Nozzle Contours for Chemically Reacting Gas Flows," *AIAA Journal*, Vol. 5, No. 4, April 1967, pp. 670-676.
8. Scofield, M. Peter, Thompson, H. Doyle and Hoffman, Joe D., "Thrust Nozzle Optimization Including Boundary Layer Effects," Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio, Report No. AFAPL-TR-67-158, Dec. 1967.
9. Krayko, A. N., "Solution of Variational Problems in Supersonic Gas Dynamics," *Prikladnaya Matematika i Mekhanika*, Vol. 30, No. 2, 1966, pp. 312-320.

10. Pirumov, U. C. and Ruhtsov, V. A., "Calculation of Axisymmetric Supersonic Plug Nozzles," AN SSSR, Mechanics and Machines, No. 6, 1961, pp. 17-37.
11. Gajewski, T., "Gas Dynamics of a Variable Supersonic Plug Nozzle," Technika Lotnicza i Astronautyczna, No. 4, 1967, pp. 12-16.
12. Gogish, L. V. and Stephanov, G. Yu., "Classification and Approximate Method for Profiling Plug Nozzles," AN SSSR Mikhanika Zhidkosti i Gaza, No. 4, 1966, pp. 166-171.
13. Pilipko, N. K., "A Method of Shaping a Supersonic Nozzle with an Internal Cone," Izvestiya Kiyevskogo Politekhnicheskogo Instituta, Sbornik Trudov Teplotekhnicheskogo Fakul' teta, Vol. 30, No. 1, pp. 38-52.
14. Von Mises, Richard, Mathematical Theory of Compressible Fluid Flow, Academic Press, New York, 1958, pp. 9-11.
15. Miele, Angelo, "Generalized Approach to the Calculus of Variations in Two Independent Variables," Chap. 4, Theory of Optimum Aerodynamic Shapes, edited by Angelo Miele, Academic Press, New York, 1965.
16. Moore, A. W. and Hall, I. M., "Transonic Flow in the Throat Region of an Annular Nozzle with an Arbitrary Smooth Profile," Aeronautical Research Council Report 26-543, Jan. 4, 1965.
17. Hoffman, Joe D., "Optimum Thrust Nozzle Contours for Chemically Reacting Gas Flows," Jet Propulsion Center, Purdue University, Report No. TM-66-3, April 1966.
18. Shapiro, Ascher M., The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. I, The Ronald Press Company, New York, 1953, pp. 595-608.
19. Courant, R. and Hilbert, D., Methods of Mathematical Physics, Vol. II, Interscience Publishers, 1962, Chapter V, p. 407.
20. Chapman, D. R., "An Analysis of Base Pressure at Supersonic Velocities and Comparison with Experiments," NACA TR-1051, 1951.
21. Korst, H. H., "A Theory for Base Pressure in Transonic and Supersonic Flow," Journal of Applied Mechanics, Vol. 23, No. 1, March 1956, pp. 593-600.

22. Scherberg, M. G. and Smith, H. E., "An Experimental Study of Supersonic Flow Over a Rearward Facing Step," AIAA Journal, Vol. 5, No. 1, June 1967, pp. 51-56.
23. Donaldson, I. S., "On the Separation of a Supersonic Flow at a Sharp Corner," AIAA Journal, Vol. 5, No. 6, June 1967, pp. 1086-1088.
24. Mueller, T. J., "Determination of the Turbulent Base Pressure in Supersonic Axisymmetric Flow," AIAA Paper 67-446 presented at the 3rd Propulsion Joint Specialist Conference, Washington, D. C., July 17-21, 1967.
25. Charwat, A. F. and Schneider, L., "Effect of the Boundary-Layer Profile at Separation on the Evaluation of a Wake," AIAA Journal, Technical Note, Vol. 5, No. 6, June 1967, pp. 1188-1190.
26. Crocco, L. and Lees, L., "A Mixing Theory for the Interaction Between Dissipative Flows and Nearly Isentropic Streams," Journal of the Aeronautical Sciences, Vol. 19, No. 10, Oct. 1952, pp. 649-676.
27. Lykoudis, P. S., "A Review of Hypersonic Wake Studies," AIAA Journal, Vol. 4, No. 4, April 1966, pp. 577-587.
28. Sedney, R., "Review of Base Drag," Ballistic Research Laboratories" Report No. 1337, Oct. 1966.
29. Marik, R. H. and Hosack, G. A., "Theoretical Prediction of Base Pressure in Aerospike Nozzles," AIAA 2nd Propulsion Joint Specialist Conference, Colorado Springs, Colorado, June 13-17, 1966. Confidential.
30. Grahm, A. R., NASA Plug Nozzle Handbook, Developed by General Electric under NASA Contract NAS9-3748.
31. Users Manual for the External Drag and Internal Nozzle Performance Deck. Prepared by Pratt & Whitney Aircraft under USAF Contract No. AF33(615)-3128, 1968.
32. Rom, J., "Analysis of the Near-Wake Pressure in Supersonic Flow Using the Momentum Integral Method," Journal of Spacecraft, Vol. 3, No. 10, October 1966, pp. 1504-1509.

33. Panov, I. A. and Shvets, A.I., "Investigation of the Base Pressure Near the Trailing Edge of Axisymmetric Bodies in Supersonic Flow," *Prikladnaya Mekhanika*, Vol. 2, No. 6, 1966, pp. 105-111.
34. Hopkins, D. F. and Hill, D. E., "Transonic Flow in Unconventional Nozzles," *AIAA Journal*, Vol. 6, No. 5, May 1968, pp. 838-842.
35. Leipman, H. W. and Goddard, "Note on the Mach Number Effect Upon the Skin Friction of Rough Surfaces," *Journal of the Aeronautical Sciences*, Vol. 24, 1957, p. 784.

APPENDIX A

DERIVATION OF THE THRUST EXPRESSION

The axial thrust to be maximized is obtained by summing the integrated pressure and shear forces on the plug T'D' (see Figure 1) and the pressure acting on the base D'C. Consider the forces acting on the plug surface segment shown in Figure A-1. The segment, of length ds , when rotated about

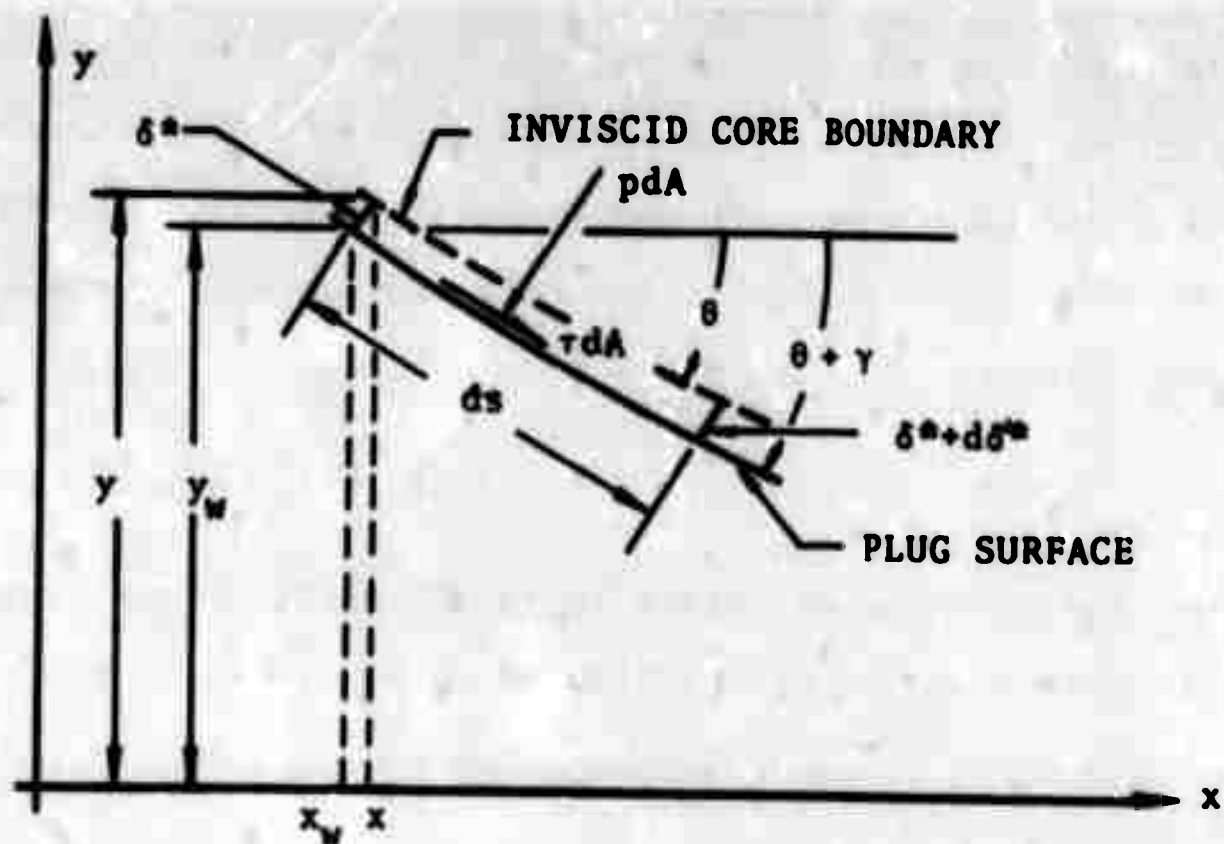


FIGURE A-1. PLUG NOZZLE THRUST SEGMENT
the x -axis sweeps out an area given by

$$dA = 2\pi y_w ds \quad (A-1)$$

The coordinates (x, y) locate a point on the boundary of the inviscid core (Figure A-1) and (x_w, y_w) locate a corresponding point on the plug surface. The relationship between these two sets of coordinates is given by:

$$x_w = x + \delta^* \sin \theta \quad (A-2)$$

$$y_w = y - \delta^* \cos \theta \quad (A-3)$$

where δ^* is a boundary layer thickness measured normal to the boundary of the inviscid core.

It will be assumed that the pressure which exists at a given point on the boundary of the inviscid core is the same at the corresponding wall point. The force due to pressure acts normal to the surface and can be written as $p dA$. The force due to shear must also be considered. This force acts parallel to the wall and is written as τdA . The thrust to be considered is due to the axial components of these two forces. Thus,

$$dT = p dA \sin (\theta + \gamma) + \tau dA \cos (\theta + \gamma) \quad (A-4)$$

where the positive direction is to the right. Equations (A-1) and (A-3) can be substituted in Eq. (A-4) to yield

$$\begin{aligned} dT = & p 2\pi (y - \delta') ds \sin (\theta + \gamma) \\ & + \tau 2\pi (y - \delta') ds \cos (\theta + \gamma) \end{aligned} \quad (A-5)$$

where $\delta' = \delta^* \cos \theta$. It is apparent from Figure A-1 that

$$dy = ds' \sin \theta \quad dx = ds' \cos \theta \quad (A-6)$$

$$dy_w = ds \sin (\theta + \gamma) \quad (A-7)$$

where ds and ds' are related as shown in Figure A-2. The relationship between them can be expressed as:

$$ds' = ds \cos \gamma \quad (A-8)$$

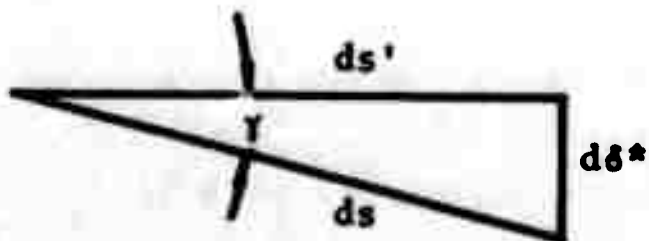


FIGURE A-2. THRUST SEGMENT-INVISCID CORE
BOUNDARY RELATIONSHIP

If the boundary layer thickness, δ^* , is small, then γ will also be small. Thus, for small γ :

$$\cos \gamma = 1$$

$$\sin \gamma = \gamma$$

Therefore, from Eq.(A-8) and the trigonometric identities

$$ds' = ds \quad (A-9)$$

$$\sin (\theta + \gamma) = \sin \theta + \gamma \cos \theta \quad (A-10)$$

$$\cos (\theta + \gamma) = \cos \theta - \gamma \sin \theta \quad (A-11)$$

Substitution of Eq.(A-9) into Eqs.(A-6) yields

$$dy = ds \sin \theta \quad dx = ds \cos \theta \quad (A-12)$$

and substitution of Eqs. (A-3), (A-9), and (A-10) into Eq. (A-7) yields

$$d(y - \delta') = ds \sin \theta + \gamma \cos \theta ds \quad (A-13)$$

Using Eqs. (A-10) and (A-11), Eq. (A-5) can be written as

$$\begin{aligned} \frac{dT}{2\pi} = & p(ds \sin \theta + \gamma \cos \theta ds)(y - \delta') \\ & + \tau(ds \cos \theta - \gamma \sin \theta ds)(y - \delta') \end{aligned} \quad (A-14)$$

which, upon substitution of Eq. (A-10), reduces directly to

$$\frac{dT}{2\pi} = [p(1 + \frac{\gamma}{\tan \theta}) dy + \tau(1 - \gamma \tan \theta) dx](y - \delta') \quad (A-15)$$

Since the effect of wall shear on thrust is being considered, it is desirable to retain those terms which are of the order of τ or lower. Thus, the term $\tau \gamma \tan \theta$ can be dropped since it is of an order higher than τ . By considering Eq. (A-13), Eq. (A-15) can be written as

$$\frac{dT}{2\pi} = [p(\dot{y} - \dot{\delta}') + \tau](y - \delta') dx \quad (A-16)$$

where $(\dot{})$ indicates the total derivative with respect to x . The final axial thrust expression is obtained by integrating Eq. (A-16) from T to D and adding the base pressure contribution. Thus,

$$\frac{T}{2\pi} = \int_T^D [p(\dot{y} - \dot{\delta}') + \tau](y - \delta') dx - (y_D - \delta'_D)^2 p_b / 2 \quad (A-17)$$

Since $\dot{y}$ is negative along TD, Eq.(A-17) will produce a negative value for thrust. Without loss of generality, the signs are changed so that a positive value of thrust is obtained. The thrust expression becomes

$$\frac{T}{2\pi} = - \int_T^D [p(\dot{y} - \delta') + \tau](y - \delta')dx + (y_D - \delta'_D)^2 p_b / 2$$

(A-18)

APPENDIX B

CALCULUS OF VARIATIONS

Miele (15) has presented the calculus of variations for a functional of the form

$$I = \iint_S F(x, y, z_k, p_k, q_k) dx dy + \oint_B G(x, y, z_k, \dot{y}, \dot{z}_k) dx \quad (B-1)$$

The current formulation, however, requires the maximization of a functional which has the form

$$I = \iint_S F(x, y, z_k, p_k, q_k) dx dy + \oint_B G(x, y, z_k, \dot{y}, \dot{z}_k) dx + \phi(y_c, x_c, z_{k_c}) \quad (B-2)$$

Thus, it is necessary to account for the first variation of the term $\phi(y_c, x_c, z_{k_c})$ which is to be evaluated at a corner point c . The starting point for this will be the first variation of the functional (B-1) as given by Miele (15).

For purposes of illustration, Miele assumed the presence of only one corner line in the class of admissible surfaces and the presence of only one corner point in the class of boundary lines examined. His expression for the first variation

is given as

$$\begin{aligned}
 \delta I = & \iint_{S_1} \sum_{k=1}^n [F_{z_k} - (F_{p_k})_x - (F_{q_k})_y] \tilde{\delta z}_k dx dy \\
 & + \iint_{S_2} \sum_{k=1}^n [F_{z_k} - (F_{p_k})_x - (F_{q_k})_y] \tilde{\delta z}_k dx dy \\
 & + \oint_B (\xi \delta x + \eta \delta y + \sum_{k=1}^n \zeta_k \delta z_k) dx \\
 & - \oint_C (\Delta X \delta x + \Delta Y \delta y + \sum_{k=1}^n \Delta z_k \delta z_k) dx \\
 & + [\Delta(G - \dot{y} G_y - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x + \Delta G_y \delta y + \sum_{k=1}^n \Delta G_{z_k} \delta z_k] = 0 \quad (B-3)
 \end{aligned}$$

where p_k and q_k are the partial derivatives of the generic dependent variable, z_k , with respect to x and y respectively, and

$$\xi = X - \dot{y} E(G, y) - \sum_{k=1}^n \dot{z}_k E(G, z_k) \quad (B-4)$$

$$\eta = Y + E(G, y) \quad (B-5)$$

$$\zeta_k = z_k + E(G, z_k), \quad (k = 1, \dots, n) \quad (B-6)$$

$$X = \sum_{k=1}^n p_k F_{q_k} + \dot{y} (F - \sum_{k=1}^n p_k F_{p_k}) \quad (B-7)$$

$$Y = (F - \sum_{k=1}^n q_k F_{q_k}) - \dot{y} \sum_{k=1}^n q_k F_{p_k} \quad (B-8)$$

$$Z_k = -F_{q_k} + \dot{y} F_{p_k}, \quad (k = 1, \dots, n) \quad (B-9)$$

$$E(G, y) = G_y - dG_y/dx \quad (B-10)$$

$$E(G, z_k) = G_{z_k} - dG_{z_k}/dx, \quad (k = 1, \dots, n) \quad (B-11)$$

The $\Delta(\dots)$ under the $\oint$ represents the difference of the quantity (...) evaluated on the outer side of the corner line and the same quantity evaluated on the inner side. The $\Delta(\dots)$ included within the brackets represents the difference of the quantity (...) evaluated before the corner point and the same quantity evaluated after the corner.

The first variation of Eq.(B-2) can be obtained by adding the first variation of ϕ to Eq.(B-3). This variation, given by

$$\delta\phi = \phi_{y_c} \delta y_c + \phi_{x_c} \delta x_c + \sum_{k=1}^n \phi_{z_k} \delta z_{k_c}$$

can be combined with the bracketed terms of Eq.(B-2) to yield

$$\begin{aligned} & \left[(G - \dot{y} G_y - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x + G_y \delta y + \sum_{k=1}^n G_{z_k} \delta z_k \right] \Big|_{x_c^-} + \phi_{x_c} \delta x_c \\ & + \phi_{y_c} \delta y_c + \sum_{k=1}^n \phi_{z_k} \delta z_{k_c} - \left[(G - \dot{y} G_y - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x \right. \\ & \left. + G_y \delta y + \sum_{k=1}^n G_{z_k} \delta z_k \right] \Big|_{x_c^+} \end{aligned} \quad (B-12)$$

where the subscripts c^- and c^+ denote the conditions immediately before and after the corner point of the boundary line, respectively. Since the variation, $\delta\Phi$, enters only the bracketed term from which the corner condition along the boundary line is obtained, the remaining necessary conditions will be exactly those given by Miele (15).

The Euler equation which must be satisfied on the surface S is given by

$$F_{z_k} - (F_{p_k})_x - (F_{q_k})_y = 0, \quad (k = 1, \dots, n) \quad (B-13)$$

When Eq.(B-13) is combined with Eq.(B-3) the transversality condition is obtained,

$$\xi \delta x + \eta \delta y + \sum_{k=1}^n \zeta_k \delta z_k = 0 \quad (B-14)$$

where ξ, η , and δ are defined by Eqs.(B-4) through (B-6). This condition must be satisfied for every set of variations δx , δy and δz_k consistent with the conditions imposed on the boundary line. The Erdmann-Weirstrass corner condition, obtained by combining Eqs.(B-3), (B-13), and (B-14), must be satisfied for every set of variations δx , δy , and δz consistent with the conditions imposed upon the location of the corner line. This condition is given by

$$\Delta X \delta x + \Delta Y \delta y + \sum_{k=1}^n \Delta Z_k \delta z_k = 0 \quad (B-15)$$

The final condition is obtained by combining Eq.(B-3) with

Eqs.(B-13) through (B-15) and the expression (B-12). This corner condition, given by

$$\begin{aligned} & \left[(G - \dot{y}G_y - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x + G_y \delta y + \sum_{k=1}^n G_{z_k} \delta z_k \right] \Big|_{x_{c-}} + \phi_{x_c} \delta x_c \\ & + \phi_{y_c} \delta y_c + \sum_{k=1}^n \phi_{z_k} \delta z_k - \left[(G - \dot{y}G_y - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x + G_y \delta y \right. \\ & \left. + \sum_{k=1}^n G_{z_k} \delta z_k \right] \Big|_{x_{c+}} = 0 \end{aligned} \quad (B-16)$$

must be satisfied for every set of variations δx , δy , and δz_k consistent with the conditions imposed upon the location of the corner point of the boundary line. This last condition is the only one different from those given by Miele. However, in the event that a corner point on the boundary does not have a function ϕ to be evaluated at that location then the corner condition reduces to that given by Miele.

When more than one corner point is present an expression of the type given in Eq.(B-16) arises for each corner and the sum of all such expressions is equated to zero. Only when the variations at the corners are independent can Eq.(B-16) be applied to each corner separately.

APPENDIX C

DERIVATION OF THE EULER EQUATIONS

The general form of the Euler equations is given by Eq.(B-13)

$$F_{z_k} - (F_{p_k})_x - (F_{q_k})_y = 0 \quad (B-13)$$

This equation must be satisfied for each of the generic dependent variables u, v, p and ρ . The fundamental function, F , given by Eq.(9) is

$$\begin{aligned} F = & \lambda_1(\rho u_x + \rho v_y + u\rho_x + v\rho_y + \rho v/y) + \lambda_2(\rho u u_x + \rho v u_y + p_x) \\ & + \lambda_3(\rho u v_x + \rho v v_y + p_y) + \lambda_4(u p_x + v p_y - a^2 u \rho_x \\ & - a^2 v \rho_y) \end{aligned} \quad (9)$$

For $k = 1$: $z_k = u$, $p_k = u_x$, and $q_k = u_y$.

Taking the derivatives indicated in Eq.(B-13) yields

$$F_u = \lambda_1 \rho + \lambda_2 \rho u_x + \lambda_3 \rho v_x + \lambda_4 (p_x - a^2 \rho_x) \quad (C-1)$$

$$F_{u_x} = \lambda_1 \rho_y + \lambda_2 \rho u \quad (C-2)$$

$$F_{u_y} = \lambda_2 \rho v \quad (C-3)$$

When Equations (C-1) through (C-3) are substituted into Eq. (B-13) the following is obtained:

$$\begin{aligned} \lambda_1 y \rho_x + \lambda_2 \rho u_x + \lambda_3 \rho v_x + \lambda_4 (p_x - a^2 \rho_x) - (\lambda_1 \rho y + \lambda_2 \rho u)_x \\ - (\lambda_2 \rho v)_y = 0 \end{aligned} \quad (C-4)$$

Expansion of Eq.(C-4) yields

$$\begin{aligned} -\lambda_2 u_x - \lambda_3 v_x - \frac{1}{\rho} \lambda_4 (p_x - a^2 \rho_x) + y \lambda_{1x} + u \lambda_{2x} + v \lambda_{2y} \\ = \frac{v}{y} \lambda_2 \end{aligned} \quad (C-5)$$

Equation (C-5) is the Euler equation for the dependent variable u . The same procedure is followed to obtain the Euler equation for each of the other dependent variables.

For $k = 2$: $z_k = v$, $p_k = v_x$, and $q_k = v_y$.

$$F_v = \lambda_1 y \rho_y + \lambda_1 \rho + \lambda_2 \rho u_y + \lambda_3 \rho v_y + \lambda_4 (p_y - a^2 \rho_y) \quad (C-6)$$

$$F_{v_x} = \lambda_3 \rho u \quad (C-7)$$

$$F_{v_y} = \lambda_1 \rho y + \lambda_3 \rho v \quad (C-8)$$

Substitution into Eq.(B-13) yields

$$\begin{aligned} \lambda_1 y \rho_y + \lambda_1 \rho + \lambda_2 \rho u_y + \lambda_3 \rho v_y + \lambda_4 (p_y - a^2 \rho_y) - (\lambda_3 \rho u)_x \\ - (\lambda_1 \rho y + \lambda_3 \rho v)_y = 0 \end{aligned}$$

which reduces directly to

$$\begin{aligned}
 & -\lambda_2 u_y - \lambda_3 v_y - \frac{1}{\rho} \lambda_4 (p_y - a^2 \rho_y) + y \lambda_{1y} + u \lambda_{3x} + v \lambda_{3y} \\
 & = \frac{v}{y} \lambda_3
 \end{aligned} \tag{C-9}$$

For $k = 3$: $z_k = p$, $p_k = p_x$, and $q_k = p_y$.

The derivatives with respect to p and ρ are taken using the relation

$$a^2 = \frac{\gamma p}{\rho}$$

such that

$$(a^2)_p = a^2/p, \quad (a^2)_\rho = a^2/\rho$$

Thus,

$$F_p = -\lambda_4 \frac{\gamma}{\rho} (u \rho_x + v \rho_y) \tag{C-10}$$

$$F_{p_x} = \lambda_2 + \lambda_4 u \tag{C-11}$$

$$F_{p_y} = \lambda_3 + \lambda_4 v \tag{C-12}$$

Using the above results, Eq.(B-13) becomes

$$\begin{aligned}
 & \lambda_4 u_x + \lambda_4 v_y + \lambda_4 \frac{a^2}{p} (u \rho_x + v \rho_y) + \lambda_{2x} + \lambda_{3y} + u \lambda_{4x} \\
 & + v \lambda_{4y} = 0
 \end{aligned} \tag{C-13}$$

For $k = 4$: $z_k = \rho$, $p_k = \rho_x$, and $q_k = \rho_y$.

$$F_\rho = \lambda_1 y u_x + \lambda_1 y v_y + \lambda_1 v + \lambda_2 (u u_x + v u_y) + \lambda_3 (u v_x + v v_y) + \lambda_4 (a^2 u \rho_x / \rho + a^2 v \rho_y / \rho) \quad (C-14)$$

$$F_{\rho_x} = \lambda_1 y v - \lambda_4 a^2 v \quad (C-15)$$

$$F_{\rho_y} = \lambda_1 y v - \lambda_4 a^2 v \quad (C-16)$$

Substitution of Eqs.(C-14) through (C-16) into Eq.(B-13) yields

$$\begin{aligned} & \lambda_1 (y u_x + y v_y + v) + \lambda_2 (u u_x + v u_y) + \lambda_3 (u v_x + v v_y) \\ & + \lambda_4 (a^2 u \rho_x / \rho + a^2 v \rho_y / \rho) + (\lambda_1 y u - \lambda_4 a^2 u)_x \\ & - (\lambda_1 y v - \lambda_4 a^2 v)_y = 0 \end{aligned} \quad (C-17)$$

which can be expanded to

$$\begin{aligned} & \lambda_2 (u u_x + v u_y) + \lambda_3 (u v_x + v v_y) - y u \lambda_{1_x} + a^2 (u \lambda_{4_x})_x \\ & + \lambda_4 u (a^2 p_x / \rho - a^2 \rho_x / \rho) + \lambda_4 v (a^2 p_y / \rho - a^2 \rho_y / \rho) - y v \lambda_{1_y} \\ & + a^2 (v \lambda_{4_y})_y - y u \lambda_{1_x} + \lambda_4 a^2 (u \rho_x + v \rho_y) / \rho = 0 \end{aligned} \quad (C-18)$$

Multiplication of Eq.(C-13) by $-a^2$, addition of the result to Eq.(C-18), and use of the two momentum equations

$$u u_x + v u_y = - p_x / \rho$$

$$uv_x + vv_y = -p_y/\rho$$

results in

$$\begin{aligned} & -\lambda_2 p_x/\rho - \lambda_3 p_y/\rho - yu\lambda_{1x} - yv\lambda_{1y} - a^2\lambda_{2x} - a^2\lambda_{3y} \\ & + \lambda_1 a^2 (up_x + vp_y - ua^2\rho_x - va^2\rho_y)/p = 0 \end{aligned} \quad (C-19)$$

The last term in the above equation is identical to Eq.(5).

Thus, Eq.(C-19) reduces to

$$\frac{1}{\rho} \lambda_2 p_x + \frac{1}{\rho} \lambda_3 p_y + yu\lambda_{1x} + yv\lambda_{1y} + a^2\lambda_{2x} + a^2\lambda_{3y} = 0 \quad (C-20)$$

Thus, the Euler equations are Eqs.(C-5), (C-9), (C-13), and (C-20).

APPENDIX D

DERIVATION OF THE TRANSVERSALITY CONDITIONS

Since the function G is not the same on all portions of the boundary of the region (R) and different assumptions apply to the variations, it is necessary to consider each portion of the boundary separately. The general transversality condition which must be satisfied is given by Eq.(B-14)

$$\xi \delta x + \eta \delta y + \sum_{k=1}^n \zeta_k \delta z_k = 0 \quad (\text{B-14})$$

The terms in this equation are defined by Eqs.(B-4) through (B-11) and the function G along each portion of the boundary is given by Eqs.(10) through (12) which are

$$G = -[f + C_1 g + C_2 y \rho(u \dot{y} - v)] \quad \text{along TD} \quad (10)$$

$$G = 0 \quad \text{along DE} \quad (11)$$

$$G = 0 \quad \text{along ET} \quad (12)$$

Since the fundamental function, F , is zero, Eq.(B-14) can be written as

$$\begin{aligned} \sum_{k=1}^4 [(F_{q_k} - \dot{y} F_{p_k})(p_k \delta x + q_k \delta y - \delta z_k) + (\delta z_k - \dot{z}_k \delta x) E(G, z_k)] \\ + E(G, y)(\delta y - \dot{y} \delta x) = 0 \end{aligned} \quad (\text{D-1})$$

1. CONDITIONS ALONG ET

There can be no variations in u , v , p , ρ , x , or y along ET since the flow properties along and the location of this line are determined from the fixed upstream geometry. Therefore, Eq.(B-14) is satisfied identically.

2. CONDITIONS ALONG DE

Since the function $G = 0$ along this portion of the boundary,

$$E(G, z_k) = E(G, y) = 0$$

and the transversality condition, Eq.(D-1), becomes

$$\sum_{k=1}^4 (F_{q_k} - y F_{p_k}) (p_k \delta x + q_k \delta y - \delta z_k) = 0 \quad (D-2)$$

Gas property variations along ED are arbitrary which requires the coefficient of δz_k to be zero. If this is done no requirement need be placed on the variations of δx and δy . Thus, it is required that

$$F_{q_k} - y F_{p_k} = 0 \quad (k = 1, \dots, 4) \quad (D-3)$$

For $k = 1$: $z_k = u$, $p_k = u_x$, and $q_k = u_y$.

The derivations F_{p_k} and F_{q_k} , given by Eqs.(C-2) and (C-3), are substituted into Eq.(D-3) which results in

$$(v - yu)\lambda_2 - \lambda_1 y = 0 \quad (D-4)$$

For $k = 2$: $z_k = v$, $p_k = v_x$, and $q_k = v_y$.

The derivatives F_{p_k} and F_{q_k} , given by Eqs.(C-7) and (C-8), are

also substituted into Eq.(D-3) to yield

$$(v - \dot{y}u)\lambda_3 + \lambda_1 y = 0 \quad (D-5)$$

For $k = 3$: $z_k = p$, $p_k = p_x$, and $q_k = p_y$.

In this case the derivatives F_{p_k} and F_{q_k} are given by Eqs.(C-11) and (C-12) and when substituted into Eq.(D-3) yields

$$\lambda_3 + (v - \dot{y}u) \lambda_4 - \dot{y} \lambda_2 = 0 \quad (D-6)$$

For $k = 4$: $z_k = \rho$, $p_k = \rho_x$, and $q_k = \rho_y$.

The derivatives F_{p_k} and F_{q_k} , given by Eqs.(C-15) and (C-16) are substituted into Eq. (D-3) which results in

$$\lambda_1 y - \lambda_4 a^2 = 0 \quad (D-7)$$

Equations (D-4) through (D-7) must be satisfied along ED. However, these four equations can be combined to yield the equation of a right-running characteristic which can be used to replace one of the four. First, eliminating λ_3 from Eqs. (D-5) and (D-6) results in

$$\lambda_1 y - (v - \dot{y}u)^2 \lambda_4 + \dot{y}(v - \dot{y}u) \lambda_2 = 0 \quad (D-8)$$

An expression for λ_2 can be obtained from Eq.(D-4) and substituted into Eq.(D-8) to yield

$$\lambda_1 y(1 + \dot{y}^2) - (v - \dot{y}u)^2 \lambda_4 = 0 \quad (D-9)$$

Using Eq.(D-7) to eliminate λ_1 from Eq.(D-9) results in

$$\lambda_1 [(u^2 - a^2)\dot{y}^2 - 2uv\dot{y} + (v^2 - a^2)] = 0 \quad (D-10)$$

Since λ_1 will not in general be zero, the bracketed term in Eq.(D-10) must vanish. Thus,

$$[(u^2 - a^2)\dot{y}^2 - 2uv\dot{y} + (\dot{v}^2 - a^2)] = 0 \quad (D-11)$$

As shown in Appendix F, Eq.(D-11) is the equation of a right-running characteristic. For convenience, Eq.(D-11) will be used to replace Eq.(D-6). Thus, the equations which must be satisfied along DE are: (D-4), (D-5), (D-7), and (D-11).

3. CONDITIONS ALONG TD

Along this portion of the boundary variations in the gasdynamic properties are arbitrary, thus, their coefficients must vanish. From Eq.(B-14) it is required that

$$z_k = F_{q_k} - \dot{y}F_{p_k} - E(G, z_k) = 0 \quad (k = 1, \dots, 4) \quad (D-12)$$

For $k = 1$: $z_k = u$, $p_k = u_x$, and $q_k = u_y$.

$$E(G, z_k) = G_u - dG_u/dx = -C_2\rho y\dot{y} \quad (D-13)$$

When equations (C-2), (C-3) and (D-13) are substituted into Eq.(D-12) the following is obtained:

$$-\lambda_1\rho y\dot{y} + C_2\rho y\dot{y} = 0$$

or

$$\lambda_1 = C_2 \quad (D-14)$$

For $k = 2$: $z_k = v$, $p_k = v_x$, and $q_k = v_y$.

$$E(G, z_k) = + C_2 \rho y \quad (D-15)$$

Substitution of Eqs.(C-7), (C-8) and (D-15) into Eq.(D-12) yields

$$\lambda_1 \rho y - C_2 \rho y = 0$$

or

$$\lambda_1 = C_2 \quad (D-16)$$

For $k = 3$: $z_k = p$, $p_k = p_x$, and $q_k = p_y$.

$$E(G, z_k) = -f_p - C_1 g_p \quad (D-17)$$

Substitution of Eqs.(C-11), (C-12) and (D-17) into Eq.(D-12) yields

$$-\dot{y}\lambda_2 + \lambda_1 + f_p + C_1 g_p = 0$$

or

$$u\lambda_1 - v\lambda_2 + uf_p + ug_p C_1 = 0 \quad (D-18)$$

where $f_p = \dot{y}(y - \delta')$

For $k = 4$: $z_k = \rho$, $p_k = \rho_x$, $q_k = \rho_y$.

$$E(G, z_k) = -f_\rho - C_1 g_\rho$$

but $f_p = 0$, therefore

$$E(G, z_k) = -C_1 g_p \quad (D-19)$$

Substitution of Eqs.(C-15), (C-16) and (D-19) into Eq.(D-12) yields only the identity $0 = 0$.

Variations in x and y are also arbitrary along TD. However, it is convenient to rearrange the coefficients of δx and δy in Eq.(B-14) as

$$\xi \delta x + \eta \delta y = (\delta y - \dot{y} \delta x) \left\{ \sum_{k=1}^4 q_k E(G, z_k) \right\} + E(G, y)$$

Thus, for arbitrary variations in x and y it is required that

$$\sum_{k=1}^4 q_k E(G, z_k) + E(G, y) = 0 \quad (D-20)$$

The expressions for $E(G, z_k)$ as k takes on the values 1 to 4, are given by Eqs.(D-13), (D-15), (D-17), and (D-19). The function $E(G, y)$ is written as

$$E(G, y) = G_y - dG_y/dx$$

or

$$E(G, y) = -f_y - C_1 g_y + \frac{d}{dx} (f_y + C_2 \rho y u + C_1 g_y) \quad (D-21)$$

Substitution into Eq.(D-20) yields

$$C_2 [\rho y y u_y - \rho y v_y - \frac{d}{dx} (\rho y u)] - y \rho u \frac{d}{dx} C_2 + C_1 [p_y g_p + g_y - \frac{d}{dx} g_y] + f_y - \frac{d}{dx} (f_y) + f_p p_y = 0 \quad (D-22)$$

Using the continuity equation and the fact that TD is a streamline, it can be shown that

$$-\frac{d}{dx}(\gamma \rho u) = \gamma \rho v_y - \dot{\gamma} \rho y u_y \quad (D-23)$$

Substitution of Eq.(D-23) into Eq.(D-22) shows that the coefficient of C_2 is zero.

Further simplifications to Eq.(D-22) can be obtained by using the following expanded terms

$$f_y - \frac{d}{dx} f_y' = +\tau - (y - \delta') \frac{dp}{dx} \quad (D-24)$$

$$-(y - \delta') \frac{dp}{dx} = -(y - \delta') p_x - (y - \delta') \dot{\gamma} p_y \quad (D-25)$$

$$f_p p_y = \dot{\gamma} (y - \delta') p_y - \delta' (y - \delta') p_y \quad (D-26)$$

Substitution of Eqs.(D-24) through (D-26) into Eq.(D-22) yields

$$\begin{aligned} & -\gamma \rho u \frac{dC_2}{dx} + C_1(p_y g_p + g_y - \frac{d}{dx} g_y) + \tau - (y - \delta') p_x \\ & - \delta' (y - \delta') p_y = 0 \end{aligned} \quad (D-27)$$

From Eqs.(3) and (4)

$$p_x = -\rho u u_x - \rho v u_y = -\rho u \frac{du}{dx} \quad (D-28)$$

$$p_y = -\rho u v_x - \rho v v_y = -\rho u \frac{dv}{dx} \quad (D-29)$$

Substitution of these expressions into Eq.(D-27) gives

$$\begin{aligned}
 & - y \rho u \frac{dC_2}{dx} + C_1 \left(-\rho u g_p \frac{dv}{dx} + g_y - \frac{dg}{dx} \dot{y} \right) + \tau + (y - \delta') \rho u \frac{du}{dx} \\
 & \dot{\delta}' (y - \delta') \rho u \frac{dv}{dx} = 0
 \end{aligned} \tag{D-30}$$

Equation (D-30) can be rewritten as

$$\begin{aligned}
 y \rho u \frac{d}{dx} C_2 & = \rho u (y - \delta') \left(\frac{du}{dx} + \dot{\delta}' \frac{dv}{dx} \right) \\
 & - C_1 \left(\rho u g_p \frac{dv}{dx} - g_y + \frac{dg}{dx} \dot{y} \right) + \tau
 \end{aligned} \tag{D-31}$$

Thus, Eqs. (D-14), (D-16), (D-18), and (D-31) must be satisfied along TD.

APPENDIX E

DERIVATION OF THE CORNER CONDITIONS

Since flows in which corner lines arise are not to be considered, the only corner condition which must be satisfied at points E, T, and D is given by Eq.(B-16)

$$\begin{aligned} & \left[(G - \dot{y}G_y - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x + G_y \delta y + \sum_{k=1}^n G_{z_k} \delta z_k \right] \Big|_{x_c-} \\ & + \phi_{x_c} \delta x_c + \phi_{y_c} \delta y_c + \sum_{k=1}^n \phi_{z_k} \delta z_k - \left[(G - \dot{y}G_y \right. \\ & \left. - \sum_{k=1}^n \dot{z}_k G_{z_k}) \delta x + G_y \delta y + \sum_{k=1}^n G_{z_k} \delta z_k \right] \Big|_{x_c+} = 0 \end{aligned} \quad (B-16)$$

where the subscripts c- and c+ denote the conditions immediately before and after the corner point, respectively.

1. CONDITIONS AT POINTS E AND T

These two points are considered to be fixed which implies that $\delta y = \delta x = \delta z_k = 0$. Hence, the corner condition is satisfied identically at these two points.

2. CONDITIONS AT POINT D

The variations in x, y, u, v, p, and ρ can be treated as arbitrary and independent at this point. Thus, the coefficient

of each of these variables must vanish. Since $G_{z_k} = 0$ on all portions of the boundary of the region (R), it is required that

$$(G - \dot{y}G_y)_{TD} + \phi_{x_D} = (G - \dot{y}G_y)_{DE} \quad (E-1)$$

$$(G_y)_{TD} + \phi_{y_D} = (G_y)_{DE} \quad (E-2)$$

$$(\phi_u)_{TD} = 0 \quad (E-3)$$

$$(\phi_v)_{TD} = 0 \quad (E-4)$$

$$(\phi_p)_{TD} = 0 \quad (E-5)$$

$$(\phi_\rho)_{TD} = 0 \quad (E-6)$$

The function ϕ is independent of u, v, p , and ρ , therefore Eqs.(E-3) through (E-6) are satisfied identically. Also, the function G is 0 along DE which reduces Eqs.(E-1) and (E-2) to

$$(G - \dot{y}G_y)_{TD} + \phi_{x_D} = 0 \quad (E-7)$$

$$(G_y)_{TD} + \phi_{y_D} = 0 \quad (E-8)$$

The derivatives indicated in these two equations are

$$G_y = - [p(y - \delta') + C_1 g_y + C_2 \rho y u] \quad (E-9)$$

$$\phi_{x_D} = 0 \quad (E-10)$$

$$\phi_{y_D} = (y_D - \delta'_D) p_b \quad (E-11)$$

Substitution of Eqs.(E-9) and (E-10) into Eq.(E-7) yields

$$\begin{aligned} & [-p(\dot{y} - \dot{\delta}') (y - \delta') - \tau(y - \delta') - C_1 g + \dot{y} p(y - \delta') \\ & + C_1 \dot{y} g_y + C_2 \rho y \dot{y} u]_{TD} = 0 \end{aligned}$$

or

$$[(y - \delta') (p \dot{\delta}' - \tau) + C_2 \rho u y \dot{y} - C_1 (g - \dot{y} g_y)]_{TD} = 0 \quad (E-12)$$

Substitution of Eqs.(E-9) and (E-11) into Eq.(E-8) yields

$$[p(y - \delta') + C_1 g_y + C_2 \rho u y]_{TD} = (y_D - \delta'_D) p_b \quad (E-13)$$

Thus, Eqs.(E-12) and (E-13) are the corner conditions which must be satisfied at point D.

APPENDIX F

CHARACTERISTIC AND COMPATIBILITY EQUATIONS

In this section the method of characteristics will be used to obtain the characteristic and compatibility equations for the system of partial differential equations consisting of Eqs.(2) through (5) and Eqs.(15) through (18). The method of characteristics as used here is described in such references as (17), (18), and (19).

The system of equations to be considered consists of those mentioned above which are written here for convenience.

$$L_1 = \rho u_x + \rho v_y + u\rho_x + v\rho_y + \frac{\rho v}{y} = 0 \quad (F-1)$$

$$L_2 = \rho u u_x + \rho v u_y + p_x = 0 \quad (F-2)$$

$$L_3 = \rho u v_x + \rho v v_y + p_y = 0 \quad (F-3)$$

$$L_4 = u p_x + v p_y - a^2 u \rho_x - a^2 v \rho_y = 0 \quad (F-4)$$

$$L_5 = -\lambda_2 u_x - \lambda_3 v_x - \frac{1}{\rho} \lambda_4 (p_x - a^2 \rho_x) + y \lambda_{1x} + u \lambda_{2x} \\ + v \lambda_{2y} - \lambda_2 v/y = 0 \quad (F-5)$$

$$\begin{aligned}
L_6 = & -\lambda_2 u_y - \lambda_3 v_y - \frac{1}{\rho} \lambda_4 (p_y - a^2 \rho_y) + y \lambda_{1y} + u \lambda_{3x} \\
& + v \lambda_{3y} - \lambda_3 v/y = 0
\end{aligned} \tag{F-6}$$

$$\begin{aligned}
L_7 = & \lambda_4 u_x + \lambda_4 v_y + \frac{a^2}{p} \lambda_4 (u \rho_x + v \rho_y) + \lambda_{2x} + \lambda_{3y} + u \lambda_{4x} \\
& + v \lambda_{4y} = 0
\end{aligned} \tag{F-7}$$

$$\begin{aligned}
L_8 = & \frac{1}{\rho} \lambda_2 p_x + \frac{1}{\rho} \lambda_3 p_y + y u \lambda_{1x} + y v \lambda_{1y} + a^2 \lambda_{2x} \\
& + a^2 \lambda_{3y} = 0
\end{aligned} \tag{F-8}$$

Equations (F-1) through (F-8) are now multiplied by the arbitrary functions σ_1 through σ_8 to form the differential operator

$$\begin{aligned}
L = & \sigma_1 L_1 + \sigma_2 L_2 + \sigma_3 L_3 + \sigma_4 L_4 + \sigma_5 L_5 + \sigma_6 L_6 \\
& + \sigma_7 L_7 + \sigma_8 L_8 = 0
\end{aligned} \tag{F-9}$$

Equation (F-9) can be rearranged such that

$$\begin{aligned}
L = & A(u_x + \frac{B}{A} u_y) + C(v_x + \frac{D}{C} v_y) + E(p_x + \frac{F}{E} p_y) \\
& + G(\rho_x + \frac{H}{G} \rho_y) + I(\lambda_{1x} + \frac{J}{I} \lambda_{1y}) + K(\lambda_{2x} + \frac{L}{K} \lambda_{2y}) \\
& + M(\lambda_{3x} + \frac{N}{M} \lambda_{3y}) + P(\lambda_{4x} + \frac{Q}{P} \lambda_{4y}) + R = 0
\end{aligned} \tag{F-10}$$

where

$$A = \sigma_1 \rho + \sigma_2 \rho u - \sigma_3 \lambda_2 + \sigma_7 \lambda_4 \quad (F-11)$$

$$B = \sigma_2 \rho v - \sigma_6 \lambda_2 \quad (F-12)$$

$$C = \sigma_3 \rho u - \sigma_5 \lambda_3 \quad (F-13)$$

$$D = \sigma_1 \rho + \sigma_3 \rho v - \sigma_6 \lambda_3 + \sigma_7 \lambda_4 \quad (F-14)$$

$$E = \sigma_2 + \sigma_4 u - \frac{1}{\rho} \sigma_3 \lambda_4 + \frac{1}{\rho} \sigma_8 \lambda_2 \quad (F-15)$$

$$F = \sigma_3 + \sigma_4 v - \frac{1}{\rho} \sigma_6 \lambda_4 + \frac{1}{\rho} \sigma_8 \lambda_3 \quad (F-16)$$

$$G = \sigma_1 u - \sigma_4 a^2 u + \sigma_5 \lambda_4 \frac{a^2}{\rho} + \sigma_7 \lambda_4 \frac{a^2 u}{p} \quad (F-17)$$

$$H = \sigma_1 v - \sigma_4 a^2 v + \sigma_6 \lambda_4 \frac{a^2}{\rho} + \sigma_7 \lambda_4 \frac{a^2 v}{p} \quad (F-18)$$

$$I = \sigma_5 y + \sigma_8 y u \quad (F-19)$$

$$J = \sigma_6 y + \sigma_8 y v \quad (F-20)$$

$$K = \sigma_5 u + \sigma_7 + \sigma_8 a^2 \quad (F-21)$$

$$L = \sigma_5 v \quad (F-22)$$

$$M = \sigma_6 u \quad (F-23)$$

$$N = \sigma_6 v + \sigma_7 + \sigma_8 a^2 \quad (F-24)$$

$$P = \sigma_7 u \quad (F-25)$$

$$Q = \sigma_7 v \quad (F-26)$$

$$R = \sigma_1 \frac{\rho v}{y} - \sigma_5 K_1 - \sigma_6 K_2 \quad (F-27)$$

Under certain conditions Eq.(F-10) can be written as

$$L = A \frac{du}{dx} + C \frac{dv}{dx} + E \frac{dp}{dx} + G \frac{d\rho}{dx} + I \frac{d\lambda_1}{dx} + K \frac{d\lambda_2}{dx} + M \frac{d\lambda_3}{dx} \\ + P \frac{d\lambda_4}{dx} + R = 0$$

or

$$Adu + Cdv + Edp + Gd\rho + Id\lambda_1 + Kd\lambda_2 + Md\lambda_3 + Pd\lambda_4 \\ + Rdx = 0 \quad (F-28)$$

The conditions under which this equation can be written are that the ratios B/A, D/C, F/E, H/G, J/I, L/K, N/M, and Q/P must equal the dy/dx which is designated by λ . If these conditions are to hold then the following eight equations can be written.

$$A\lambda - B = 0 \quad (F-29)$$

$$C\lambda - D = 0 \quad (F-30)$$

$$E\lambda - F = 0 \quad (F-31)$$

$$G\lambda - H = 0 \quad (F-32)$$

$$I\lambda - J = 0 \quad (F-33)$$

$$K\lambda - L = 0 \quad (F-34)$$

$$M\lambda - N = 0 \quad (F-35)$$

$$P\lambda - Q = 0$$

(F-36)

These equations can then be arranged with the multipliers σ_1 through σ_4 as the unknowns. If this system of equations is to have a solution other than the trivial, σ_1 through σ_4 equal to zero, then the coefficient determinant must vanish. This determinant is as follows:

$$\begin{vmatrix} \rho\lambda & \rho(\lambda u-v) & 0 & 0 & -\lambda_2\lambda & \lambda_2 & \lambda_4\lambda & 0 \\ -\rho & 0 & \rho(\lambda u-v) & 0 & -\lambda_3\lambda & \lambda_3 & -\lambda_4 & 0 \\ 0 & \lambda & -1 & (\lambda u-v) & -\lambda_4\frac{\lambda}{\rho} & \frac{1}{\rho}\lambda_4 & 0 & \frac{1}{\rho}(\lambda_2\lambda-\lambda_3) \\ (\lambda u-v) & 0 & 0 & -a^2(\lambda u-v) & \lambda_4\frac{a^2\lambda}{\rho} & \frac{-\lambda_4 a^2}{\rho} & \frac{\lambda_4 a^2(\lambda u-v)}{\rho} & 0 \\ 0 & 0 & 0 & 0 & \lambda y & -y & 0 & y(\lambda u-v) \\ 0 & 0 & 0 & 0 & (\lambda u-v) & 0 & \lambda & a^2\lambda \\ 0 & 0 & 0 & 0 & 0 & (\lambda u-v) & -1 & -a^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & (\lambda u-v) & 0 \end{vmatrix}$$

(F-37)

Since the lower left corner of the above determinant is filled with zeros, the expansion of the determinant (F-37) reduces to the following:

$$|X| \times |Y| = 0 \quad (F-38)$$

where X represents the upper left 4 x 4 submatrix and Y represents the lower right 4 x 4 submatrix. Equation (F-38) can be zero by either the determinant of X or Y being zero. Setting these two determinants to zero results in the identical expression

$$(\lambda u - v)^2 [\lambda^2 (u^2 - a^2) - 2\lambda uv + (v^2 - a^2)] = 0 \quad (F-39)$$

The characteristic curves are found by solving this equation for λ . The resulting expressions are

$$\lambda = \frac{dy}{dx} = \frac{v}{u} \quad (F-40)$$

$$\lambda = \frac{dy}{dx} = \frac{uv \pm a^2 (M^2 - 1)^{1/2}}{u^2 - a^2} \quad (F-41)$$

Equation (F-41) can be rewritten in terms of the Mach angle and flow angle as

$$\lambda = \frac{dy}{dx} = \tan(\theta \pm \alpha) \quad (F-42)$$

Thus, the characteristic curves are the gas streamlines appearing four times and the Mach lines appearing two times each. The system is then totally hyperbolic since there are a total of eight real characteristic curves.

The corresponding compatibility equations are obtained by substituting Eqs.(F-40) and (F-42) into Eqs.(F-29) through (F-36) which are then solved for the multipliers σ_1 through σ_8 . The expressions for σ_1 through σ_8 are then substituted into Eq.(F-28) which is the general compatibility relation.

The compatibility relations along the gas streamlines will be obtained first. Using Eq.(F-40) in Eqs.(F-29) through (F-36) results in four independent relations to determine σ_1 through σ_4 . Thus, four of the σ 's can be arbitrary and these are chosen to be σ_2 , σ_4 , σ_5 , and σ_8 . Expressions for the remaining σ 's are;

$$\sigma_7 = -a^2 \sigma_8 \quad (F-43)$$

$$\sigma_6 = \lambda \sigma_5 \quad (F-44)$$

$$\sigma_3 = \lambda \sigma_2 + \frac{1}{\rho} \sigma_8 (\lambda \lambda_2 - \lambda_3) \quad (F-45)$$

$$\sigma_1 = \frac{a^2}{\rho} \lambda_4 \sigma_8 \quad (F-46)$$

Equations (F-43) through (F-46) are now substituted into Eq.(F-28). Since the multipliers σ_2 , σ_4 , σ_5 , and σ_8 are arbitrary the resulting equation can be zero only if the coefficient of each of the arbitrary multipliers is zero. Equating to zero the coefficient of each of the arbitrary multipliers results in the following ordinary differential equations.

$$\rho u du + \rho v dv + dp = 0 \quad (F-47)$$

$$dp - a^2 d\rho = 0 \quad (F-48)$$

$$-\lambda_2 du - \lambda_3 dv + y d\lambda_1 + u d\lambda_2 + v d\lambda_3 = (v/y)(\lambda_2 dx + \lambda_3 dy) \quad (F-49)$$

$$\begin{aligned} (v\lambda_2 - u\lambda_3)dv + \frac{1}{\rho} \lambda_2 dp - \lambda_4 \frac{ua^2}{\rho}(\gamma - 1)d\rho + yud\lambda_1 \\ - a^2 u d\lambda_4 = -\lambda_4 \frac{a^2 v}{y} dx \end{aligned} \quad (F-50)$$

Proceeding as before, the compatibility relations along the Mach lines can be obtained. Equation (F-41) is used in Eqs.(F-29) through (F-36) which results in six independent relations to determine the eight multipliers σ_1 through σ_8 . Thus, only two of the σ 's are arbitrary in this case and are chosen to be σ_4 and σ_8 . The expressions for the remaining σ 's

are

$$\sigma_7 = 0 \quad (F-51)$$

$$\sigma_6 = \frac{a^2}{\lambda u - v} \sigma_8 \quad (F-52)$$

$$\sigma_5 = \frac{-a^2 \lambda}{\lambda u - v} \sigma_8 \quad (F-53)$$

$$\sigma_3 = \frac{a^2}{\lambda u - v} \sigma_4 + \frac{1}{\rho} \sigma_8 \left[\frac{a^2}{\lambda u - v} \lambda_4 - \lambda_3 \right] \quad (F-54)$$

$$\sigma_2 = \frac{-a^2 \lambda}{\lambda u - v} \sigma_4 - \frac{1}{\rho} \sigma_8 \left[\frac{a^2 \lambda}{\lambda u - v} \lambda_4 + \lambda_2 \right] \quad (F-55)$$

$$\sigma_1 = a^2 \sigma_4 + \frac{a^2}{\rho} \lambda_4 \sigma_8 \quad (F-56)$$

Equations (F-51) through (F-56) are then substituted into Eq.(F-28) and the coefficients of σ_4 and σ_8 are collected. These coefficients are then equated to zero, which results in the following:

$$\begin{aligned} \lambda_2 du + \lambda_3 dv - \frac{1}{\rho} \lambda_4 (dp + a^2 d\rho) - y d\lambda_1 \pm \tan \alpha (vd\lambda_2 - ud\lambda_3) \\ = \mp \tan \alpha (\lambda_1 dx - \lambda_2 dy) (v/y) \end{aligned} \quad (F-57)$$

$$a^2 (vdu - u dv) \pm \frac{a^2}{\rho} \cot \alpha dp = \frac{a^2 v}{y} (udy - vdx) \quad (F-58)$$

The upper signs in Eqs.(F-57) and (F-58) apply to the left-running Mach lines and the lower signs to the right-running Mach lines.

Thus, the system of partial differential equations given by Eqs. (F-1) through (F-8) can be replaced by the equivalent system of characteristic and compatibility equations developed in this section. A total of eight characteristic equations were found, of which three, the two Mach lines and the gas

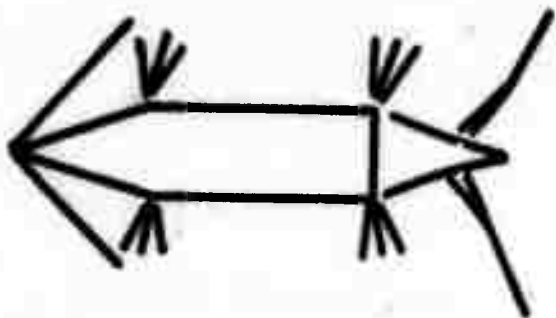
streamline, were distinct. A total of eight compatibility equations, each valid along one of the characteristic curves, were found.

APPENDIX G

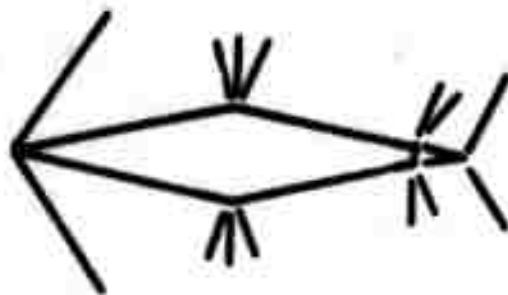
BASE PRESSURE MODEL

The optimization of plug nozzles, as carried out in this report, requires that the base pressure be recalculated each time the plug contour is modified. It is therefore desirable to have a rapid method of calculating the base pressure. The purpose here is to review the fluid mechanics of the base pressure problem in which the flow ahead of the base is supersonic and remains so through the wake. The expression used to calculate the base pressure will be discussed briefly even though the program is not restricted to any particular model. The model can be changed in the program simply by changing subprogram BASE.

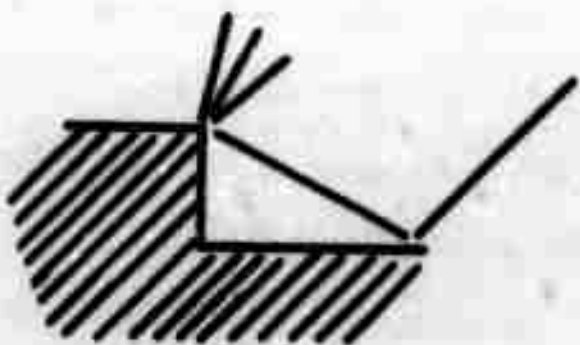
Base pressures of various types have been under study both theoretically and experimentally for several years so that one can find an abundance of information in the literature. The early work was concerned with predicting the base pressure on axisymmetric bodies of revolution traveling at supersonic speeds, the blunt trailing edges of wings, rearward facing steps, and the internal flow situation of an abrupt increase in cross-section. These are shown schematically in Figures G-1a through G-1d.



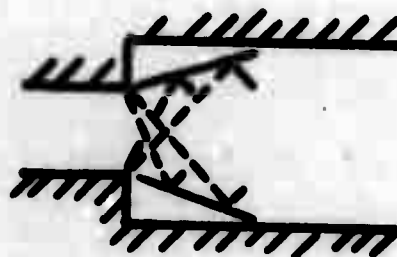
a) Axisymmetric Body of Revolution



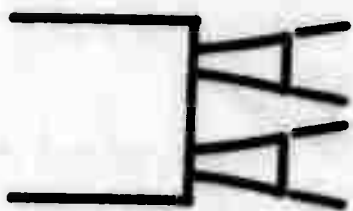
b) Blunt Trailing Edge of a Supersonic Airfoil



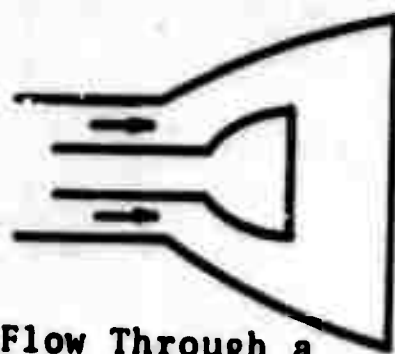
c) Supersonic Flow Over a Rearward Facing Step



d) Supersonic Flow Through an Abrupt Increase in Cross-Section



e) Flow Over a Missile With Engines Operating



f) Flow Through a Forced Deflection Nozzle

FIGURE G-1. TYPICAL FLOW CONFIGURATIONS
FOR BASE PRESSURE PROBLEMS

In more recent years other base pressure problems such as those depicted in Figure G-1e and G-1f have received attention.

The next few paragraphs will be devoted to a discussion of two-dimensional flow. The conceptual model used here can be applied to the axisymmetric case.

1. FLOW DESCRIPTION

a. Two-Dimensional Flow. As in any problem of this nature it is necessary to have a model which accurately represents the actual flow. Figure G-2 depicts the Chapman-Korst model which is frequently used to describe the flow in the base region. The general features of the flow were first described by Chapman(20) and extended

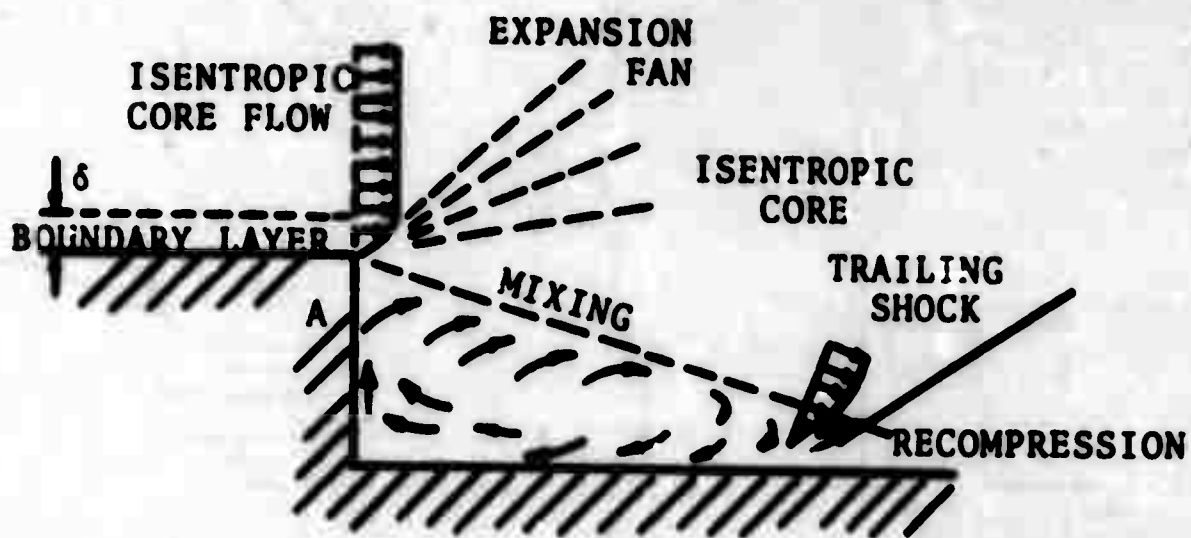


FIGURE G-2. CHAPMAN-KORST MODEL OF SEPARATED FLOW

by Korst(21) in order to formulate an energetic criterion for penetration into the high pressure region of the

wake throat.

The flow, with boundary layer thickness δ , expands around the corner A and separates from the surface at that point. The interaction of the external inviscid flow and the internal dissipative region produces a circular motion of the fluid in the base region and an inherent decrease of base pressure. The flow is deflected towards the surface of the afterbody due to the lower pressure in this region and then must be recompressed as the afterbody surface is reached. The recompression produces waves, as shown in Figure G-2, which coalesce to form the trailing edge shock just downstream of the wake throat. The fluid entrained by the internal flow near the base is returned to the separated flow region near the wake throat. This flow characteristic satisfies the conservation of mass requirement in the base region and helps to define the dividing streamline which is considered to originate at the corner and stagnate at a point near the wake throat. The pressure across the base is considered to be constant. Experimental studies indicate that the pressure downstream of the base region is nearly constant and equal to that on the base for distances up to one half the length of the separated flow region (21).

Each of the characteristics of this model warrant individual attention and discussion in light of the experimental evidence. However, since it is not intended to

solve the base pressure problem here it suffices to say that certain aspects of this model have recently been subject to criticism (22,23).

The flow in plug nozzles is axisymmetric, and there are some significant differences between two-dimensional and axisymmetric flows which should be noted.

b. Axisymmetric Flow. As mentioned earlier the conceptual model used for two-dimensional flow can be applied to the axisymmetric case; i.e., a meridian section of an axisymmetric body would appear exactly as depicted in Figure G-2 for the two-dimensional case. Since plug nozzles do not have an afterbody or sting as shown in Figure G-2, the discussion here will be directed to the no sting situation. Thus, recompression begins to occur as the fluid from opposite sides of the body converge instead of approaching an afterbody surface.

The expansion about the corner at the base of an axisymmetric body is not expected to be the same as for the two-dimensional case. This expansion in two-dimensional flow is frequently taken to be the Prandtl-Meyer expansion-even though there are indications that this is incorrect (22).

The wake flow also has some differences as one might expect. The converging wake in axisymmetric flow is essentially conical and must experience a pressure increase along the cone's surface. This is not the case for

two-dimensional flow where flow along the converging portion of the wake produces no change in pressure. As mentioned by Mueller (24) the pressure increase measured along the axis of the separated flow region is greater behind axisymmetric bodies.

In addition to being axisymmetric, plug nozzle flow is also expected to be turbulent and any adequate theory must give this consideration.

c. Turbulent Flow. It has been shown that the boundary layer profile at separation has a considerable influence upon the wake evolution (25). Thus, a turbulent profile at separation could be expected to influence the base pressure. Crocco and Lees (26) state that one of the major differences between laminar and turbulent flows is the mixing rate. Turbulent mixing rates are from five to ten times larger than laminar mixing rates. The survey article by Lykoudis (27) contains further discussion and references on this topic.

2. BASE PRESSURE DETERMINATION

A review of the literature reveals that methods are available for predicting the base pressure in a turbulent axisymmetric flow. The survey article by Sedney (28) contains a large collection of the references on the base pressure problem. The more recent paper by Mueller (24) enumerates seven methods of calculating the base pressure in this type of flow and adds his own method to bring the

total to eight. None of these methods have direct application to plug nozzles since they assume a uniform axial flow upstream of the plug base. Mueller applies his method to an ideal forced-deflection nozzle by selecting the proper reference conditions but it is unlikely that this could be done in the case of a plug nozzle.

It is also known that methods which are probably more directly applicable have been developed by the industry but have not as yet appeared in the unclassified literature. Work of this nature has been and is being conducted at Rocketdyne (29). Personal conversations with Mr. Grant A. Hosack of that company reveal that all of the details of their methods may not be available even in a classified form since part of the work was done on independent research and development funds. Pratt and Whitney along with General Electric (30) has also been active in this field. A program is in existence at Pratt and Whitney for base pressure calculations in plug nozzles (31).

Even though methods are available for base pressure determination, it is desirable, in the thrust optimization of plug nozzles, to have an analytical expression for base pressure in terms of the upstream flow properties. An expression of this type lends itself to the rapid interaction techniques necessary to this optimization method. Rom (32) has reported empirical expressions for the base pressure in two-dimensional laminar and turbulent flows for backward facing steps and base flows. He points out

that axisymmetric base flows cannot be treated by a simple modification of the two-dimensional analysis because of the difference in the interaction between the external inviscid flow and the viscous mixing region. Rom's attempt to obtain an analytical expression for base pressure is apparently the first to meet with any degree of success and appear in the literature.

3. EQUATION FOR BASE PRESSURE

It is not intended to investigate the validity of the various methods used to compute p_b , but rather to be able to calculate a representative value of base pressure in order that its effect upon the optimization can be demonstrated. It was found that a curve fit of experimental data would serve this purpose. Reference (32) contains a plot which shows the effect of the local Mach number at separation on the base pressure. This experimental data can be represented by the empirical equation

$$p_b = 0.846 p/M^{1.3} \quad (G-1)$$

where p and M are the local values of pressure and Mach number at separation. Equation (G-1) has been incorporated into the computer program for base pressure calculations. In order to determine the importance of accurate base pressure calculations in plug nozzle optimization, an alternate empirical equation for base pressure was obtained from Reference (33). This equation is

$$p_b = p_\infty [1 - .715\gamma(M_\infty^2 \cdot 3 - 0.92M_\infty^2 - 0.03)/M_\infty^2 \cdot 7] \quad (G-2)$$

where p_∞ and M_∞ are free stream values of pressure and Mach number. Since Eq.(G-2) is based upon free stream conditions rather local conditions at separation, it is not expected that base pressures calculated from this equation will accurately represent those occurring in plug nozzles. However, as shown in Section V, it illustrates the importance of calculating the base pressure accurately. Equations (G-1) and (G-2) are plotted as a function of Mach number in Figure G-3.

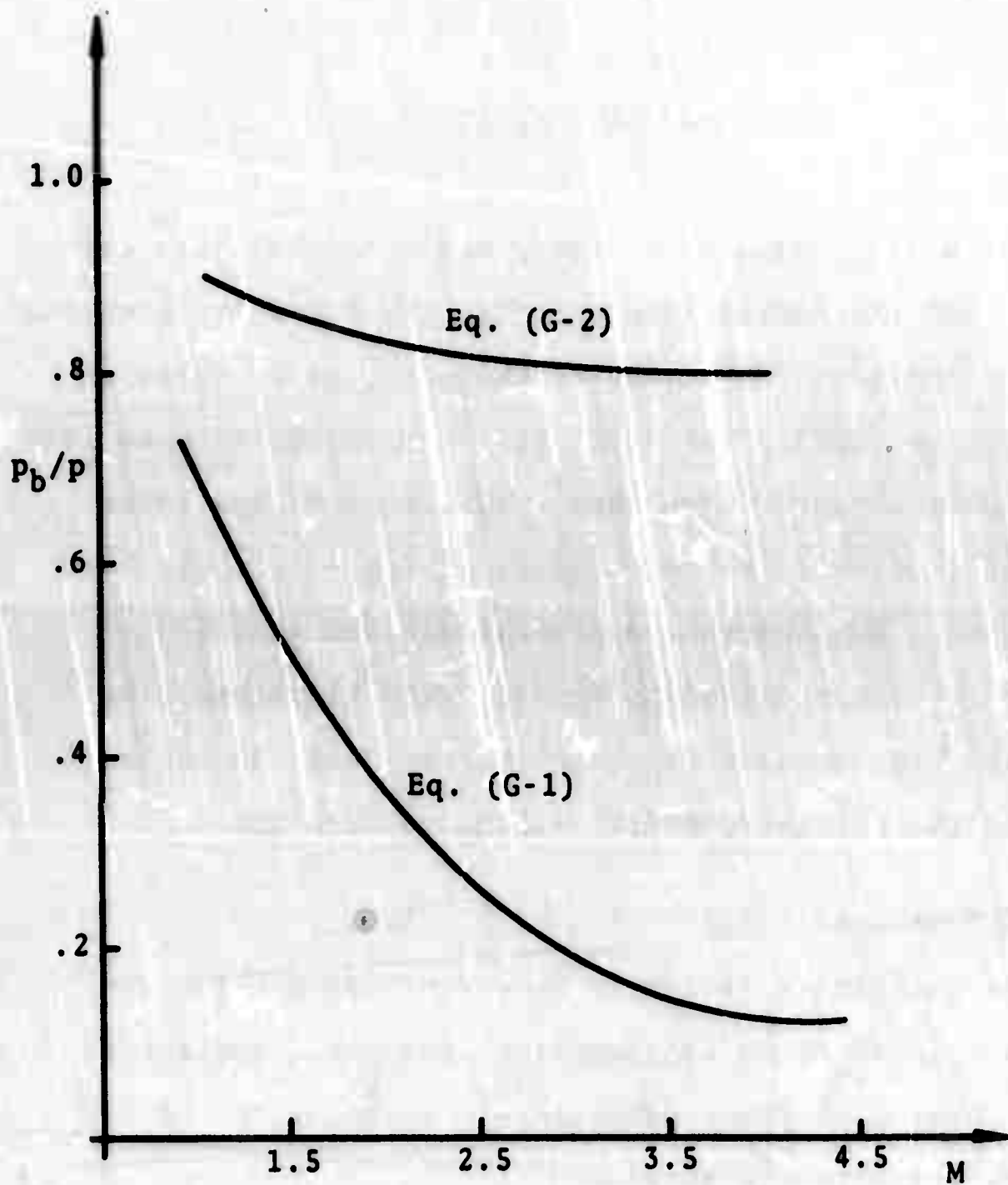


FIGURE G-3. BASE PRESSURE AS A FUNCTION OF MACH NUMBER

APPENDIX H

TRANSONIC FLOW ANALYSIS

An initial-value line, along which the velocity components and thermodynamic properties are known, is required in order to start the method of characteristics solution for the flow field. The start line can either be read into the computer program from data cards or generated internally if the nozzle being analyzed is for a subsonic burning engine. The internally generated start line is obtained from a modified Moore-Hall (16) transonic flow analysis. The results of their analysis will be outlined here and the necessary modifications pointed out.

1. MOORE-HALL ANALYSIS

The coordinate system is shown in Figure H-1. The flow is assumed to be axisymmetric, inviscid, and irrotational. The applicable equations in terms of the (X,Y) coordinate system are given as

$$\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} = 0 \quad (H-1)$$

$$\frac{1}{\rho} \frac{\partial(\rho U)}{\partial x} + \frac{1}{\rho} \frac{\partial(\rho V)}{\partial y} + \frac{V}{y} = 0 \quad (H-2)$$

$$a^2 = \frac{\gamma + 1}{2} a^*{}^2 - \frac{\gamma - 1}{2} (U^2 + V^2) \quad (H-3)$$

where a^* is the critical speed of sound. Since the boundary conditions are complicated in the (X,Y) coordinate system the problem is transformed to the (x,y) system and the solution is then given in terms of the (x,y) system.

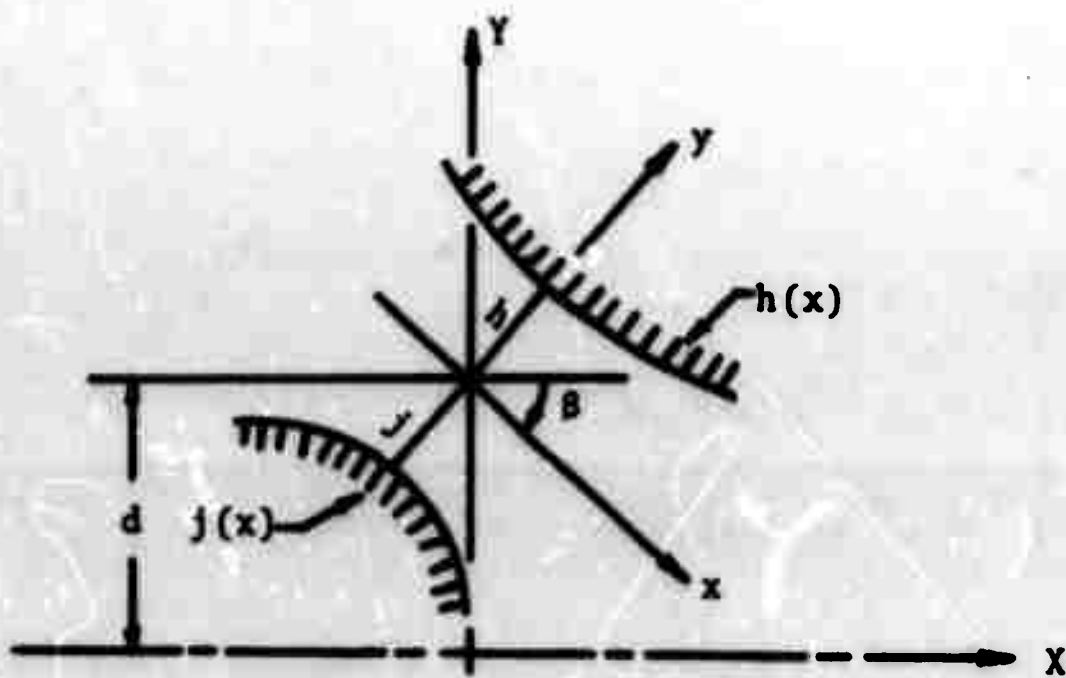


FIGURE H-1. COORDINATE SYSTEM FOR TRANSONIC FLOW ANALYSIS

The velocity components are given as u and v but are written in a non-dimensional form as

$$\frac{u}{a^*} = 1 + u'$$

$$\frac{v}{a^*} = v' \quad (H-4)$$

Without loss of generality the origin is taken to be at the minimum cross-sectional area (throat) and the throat half-height to be unity. The axis is oriented so that the wall slopes at $x = 0$ are equal in magnitude but opposite in sign. The equations which describe the upper and lower

nozzle walls are written in terms of a MacLaurin's series.

These equations are

$$\begin{aligned} h(x) &= 1 - m' x + \frac{1 + \eta}{2} m'' x^2 + \dots \\ j(x) &= 1 + m' x + \frac{1 - \eta}{2} m'' x^2 + \dots \end{aligned} \quad (H-5)$$

where

$$\frac{dh(0)}{dx} = -m'$$

$$\frac{dj(0)}{dx} = m'$$

$$(1 + \eta)m'' = d^2h(0)/dx^2$$

$$(1 - \eta)m'' = d^2j(0)/dx^2$$

with η representing the asymmetry of the nozzle profile and m'' a mean radius of curvature. As a boundary condition, the flow is required to be parallel to the walls. This condition at the outer and inner walls can be written as

$$v'(x, h) = [1 + u'(x, h)]dh/dx$$

$$-v'(x, h) = [1 + u'(x, -j)]dj/dx$$

The quantities x and m' are replaced by ϵz and ϵm where $\epsilon = R^{-1/2}$, m is a quantity of order unity, and R is a non-dimensional radius of curvature of a meridian section at the throat. The velocity components are then sought as series expansions in powers of ϵ . Thus,

$$u' = u_1 \epsilon^2 + u_2 \epsilon^3 + \dots$$

$$v' = m y \epsilon + m^2 \epsilon^2 (y^2 - 1) \cot \beta + v_1 \epsilon^3 + v_2 \epsilon^4 + \dots$$

These equations are substituted into the flow equations and the boundary conditions are required to hold. This results in an infinite sequence of pairs of differential equations, the first of which is solved. The solution can be written in the form

$$\begin{aligned} v_1 &= A'_0(y) + zA'_1(y) \\ u_1 &= A_1(y) + B_0 + B_1 z \end{aligned} \quad (II-6)$$

where

$$\begin{aligned} B_1^2 &= (1 - \psi)(1/2 - m^2) \\ B_0 &= (1 - \psi)m/4B_1 - (1 - \psi)m^3/6B_1 - 1/6 \\ A_1(y) &= (1 - mB_1 - m^2)y^2/2 + \eta y \\ A'_0(y) &= K_0 + K_1 y + K_2 y^2/2 + K_3 y^3/3 \\ K_0 &= m\eta/2 - B_1\eta/(1 - \psi) \\ K_1 &= 2B_0B_1/(1 - \psi) - mB_0 + m^3\text{ctn}^3\beta \\ K_2 &= 2B_1\eta/(1 - \psi) - 3m\eta \\ K_3 &= B_1/(1 - \psi) + 3m^2B_1/2 - 3m + 5m^3/2 - 3m^3\text{ctn}^2\beta \\ A'_1(y) &= \partial A_1(y)/\partial y \end{aligned}$$

The equation for the sonic line is obtained by using the following equation for total velocity

$$q^2 = u^2 + v^2$$

and

$$M^{*2} = q^2/a^{*2} = (1 + u')^2 + (v')^2$$

$$M^{*2} = 1 + 2u' + (u')^2 + (v')^2$$

Since

$$u' = u_1 \epsilon^2 + \dots \quad (H-7)$$

and

$$v' = -m y \epsilon + m^2 \epsilon^2 (y^2 - 1) \operatorname{ctn} \beta + v_1 \epsilon^3 + \dots \quad (H-8)$$

this equation becomes

$$M^{*2} = 1 + (u_1 + m^2 y^2 / 2) \epsilon^2 + O(\epsilon^3) \quad (H-9)$$

When the Mach number is one, $M^* = 1$, and the remaining terms must vanish. Thus

$$Q = u_1 + m^2 y^2 / 2 = 0 \quad (H-10)$$

and the equation for u_1 provides sufficient information to locate the sonic line.

$$z = (Q - A_1 - B_0 - m^2 y^2 / 2) / B_1 + O(\epsilon) \quad (H-11)$$

When $Q = 0$ the above equation locates the sonic line and when Q is greater than zero it locates lines of constant Mach number.

The flow direction relative to the x -axis is θ , where

$$\theta = \tan^{-1} \frac{v'}{1 + u'}$$

2. MODIFICATIONS TO THE MOORE-HALL ANALYSIS

It was found upon programming that the equation given for the sonic line did not yield a local Mach number of unity at all points. This was due to the terms of order ϵ^3 and higher which were dropped from Eq. (H-9). Once

the definitions of u' and v' have been established all of the terms must be retained. Thus, Eq. (H-9) should be written as

$$\begin{aligned} M^{*2} = & 1 + 2(u_1 + m^2 y^2 / 2) \epsilon^2 - 2[m^3 y (y^2 - 1) \text{ctn } \beta] \epsilon^3 \\ & + 2[u_1^2 / 2 - v_1 m y + m^4 (y^2 - 1)^2 \text{ctn}^2 \beta / 2] \epsilon^4 \\ & + 2[v_1 m^2 (y^2 - 1) \text{ctn } \beta] \epsilon^5 + v_1^2 \epsilon^6 \end{aligned} \quad (\text{H-12})$$

and Eq. (H-10) as

$$\begin{aligned} Q = & u_1 + m^2 y^2 / 2 - [m^3 y (y^2 - 1) \text{ctn } \beta] \epsilon \\ & + [u_1^2 / 2 - v_1 m y + m^4 (y^2 - 1)^2 \text{ctn}^2 \beta / 2] \epsilon^2 \\ & + [v_1 m^2 (y^2 - 1) \text{ctn } \beta] \epsilon^3 + v_1^2 \epsilon^4 / 2 \end{aligned} \quad (\text{H-13})$$

Again when $Q = 0$, $M^* = 1$. Thus, when Eqs. (H-6) are substituted into (H-13) the equation of the sonic line results. It was found that this procedure resulted in the local Mach number, M , being unity at every point along the sonic line. Values of Q greater than zero yield the equations of lines of constant Mach number.

A second problem was encountered when an attempt was made to initiate a method of characteristics solution from a line of constant Mach number which passed through the wall points $h(1)$ and $j(-1)$. This particular line was chosen since the flow matches the wall at only these two points. It was found that this line was not a space-like curve, or some of the points on this line lie within the zone of influence of neighboring points. That is, some

points lie within the Mach cone of neighboring points. This is shown schematically in Figure H-2 where point 2 lies in the zone of influence of point 1. Two things were done to minimize this problem. It was found that the

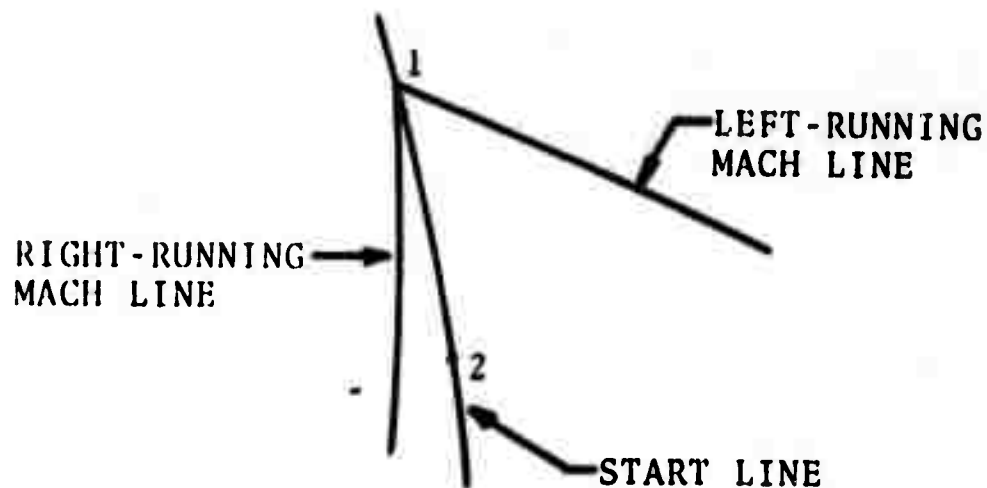


FIGURE H-2. ZONE OF INFLUENCE IN SUPERSONIC FLOW

flow properties could be evaluated along two straight lines passing through $h(1)$ and $j(-1)$ and intersecting at a chosen place on the x -axis. Thus, it was possible to vary the slopes of these two lines and in so doing minimize the number of points lying in the zone of influence of another point. Also, right characteristics were not started from those points which had other points in their zone of influence. This produced very little error since the right characteristics normally pass very close to the neighboring points in this region.

APPENDIX I

WALL SHEAR MODEL

The optimization problem formulated in Section III requires the calculation of the boundary layer thickness and the wall shear. However, the computer program which solves the resulting design equations only accounts for the wall shear. The correction for boundary layer thickness can be made to the optimized contour by using the boundary layer thickness to adjust the wall coordinates. The purpose here is to discuss briefly the wall shear model which has been incorporated into the computer program to evaluate the wall shear.

The shearing stress at the wall is given by Newton's law of friction as

$$\tau = \mu(\partial u / \partial y)_{y=0} \quad (I-1)$$

which requires the evaluation of the velocity gradient normal to the wall. This can be a very difficult task in the case of plug nozzles since 1) the boundary layer is expected to be turbulent, 2) compressibility effects are important, 3) heat transfer to the wall must be considered, and 4) the favorable free stream pressure gradient must be taken into

account. Numerous methods can be found in the literature for evaluating the wall shear stress. Since it was not intended to investigate the validity of the many boundary layer methods used to compute the shear stress, an alternate approach was taken for calculation purposes. The shear stress was written as

$$\tau = c_f \rho V^2 / 2 \quad (I-2)$$

where c_f , ρ , and V are the local values of the skin friction coefficient, density, and velocity. It then remains to find an expression for c_f which can be used in the optimization analysis and which gives reasonably accurate values for the local skin friction coefficient expected in plug nozzles. An expression obtained by Liepman and Goddard (35) for compressible, turbulent flow over a flat plate was selected for this purpose. This expression is

$$c_f = c_{f_i} / [1 + r(\gamma - 1)M^2/2] \quad (I-3)$$

where c_{f_i} is the incompressible skin friction coefficient and r is the recovery factor. A representative value of the recovery factor was chosen as 0.72 and c_{f_i} is an input parameter to the computer program. It should be emphasized that the optimization process is not restricted to this or any other wall shear model. Any desired model can be incorporated into subprogram SKIN which is described in Appendix J.

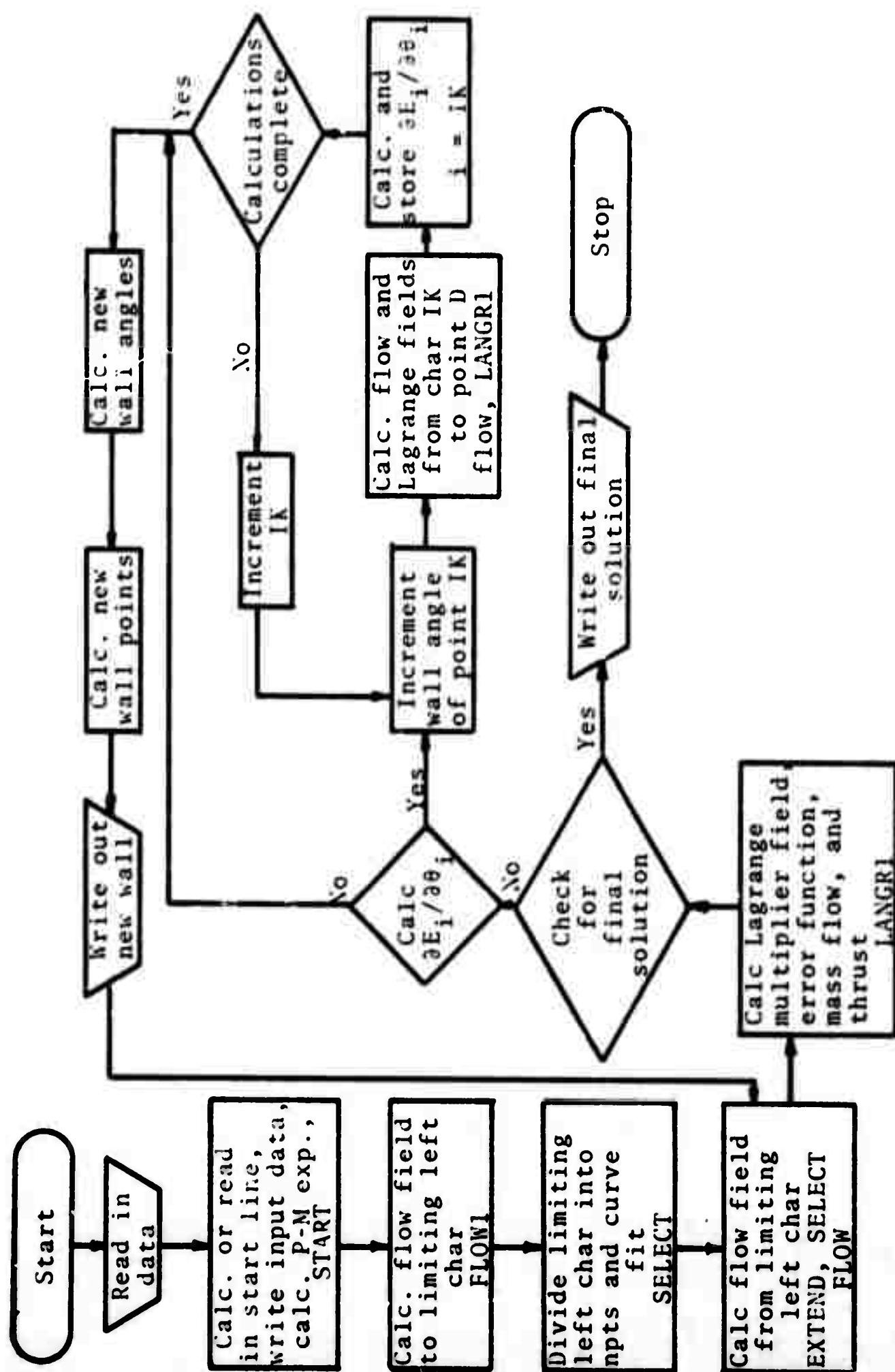
APPENDIX J

COMPUTER PROGRAM DESCRIPTION

The program was written in the Fortran IV language and has been run on the CDC 6500 and IBM 7094 computers at Purdue University and the Burroughs 5500 computer at the United States Air Force Academy. The program fits entirely in the core of these machines without an overlay scheme. It is composed of a main program and fifteen subprograms whose functions are described here. The input and output parameters are discussed in Appendix K with a complete program listing in Appendix L.

1. MAIN PROGRAM

The primary purpose of the main program is to control the program logic even though it contains some calculations. The key functions of the main program are shown in Figure J-1 with the subprograms involved being indicated at the bottom of each block. These subprograms may in turn call upon other subprograms which are not indicated. The main program also reads in all the input data which are discussed in a later section.



2. SUBPROGRAMS

a. Subprogram START. Subprogram START performs several functions. It first calculates the start line by either a modified Moore-Hall analysis or a simple isentropic analysis along a straight line of constant Mach number. At the same time it calculates the thrust and mass flow rate along the initial-value line. If the start line data are read in, the thrust and mass flow rate along this line will also be calculated. If the mass flow rate is not the specified value, FLRTC, and the start line is generated internally, an option can be selected which modifies the throat half-height to obtain the correct mass flow rate. Then the start line solution is written out. Once the start line is complete the Prandtl-Meyer expansion at point E' (Figure 3) is calculated and written out. The expansion is allowed to continue until either the ambient pressure is reached or the last right characteristic generated from E' has a direction parallel to the center line at that point. Finally, the first guess for the optimum contour, which may be read in or generated internally, is written out.

b. Subprogram FLOW1. This subprogram performs one basic function. It calculates the flow field in the region between the start line and exit characteristic DE down to the left characteristic which originates at point A (Figure 3). It makes use of Subprogram QUAD to determine

whether or not points on the start line lie within the zone of influence of each other. Right characteristics are not initiated from those points which have other points in their zone of influence. The output from this subprogram is written out along the right characteristics as the calculations proceed from left to right starting at point A.

c. Subprogram SELECT. Subprogram SELECT divides the left-characteristic which originates at point T (Figure 3) into (NPTS-1) points and curve fits the flow data along this line. This process may involve all or a portion of the data points along the characteristic. Any portion of the (NPTS-1) points designated by (NS-1) can have a reduced spacing determined by dividing the spacing the points would have if all (NPTS-1) points were spaced equally by the factor CS. The remaining (NPTS-NS) points have equal spacing.

d. Subprogram EXTEND. This subprogram extends the flow field calculations from the left characteristic originating at point A down to the left characteristic originating at point T. It also calculates the thrust contribution due to that portion of the plug surface between points A and T. The output from this subprogram is written out along the left characteristics as the calculations proceed downward from point A to point T.

e. Subprogram FLOW. This subprogram calculates the flow field in the region (R). Calculations begin on and are carried out down the right characteristic designated by the subprogram variable NZ. NZ = 1 corresponds to point T and NZ = NPTS corresponds to the exit characteristic DE. The output from this subprogram is written out along the right characteristics as the calculations proceed from left to right starting at the point designated by NZ.

f. Subprogram LANGR1. This subprogram calculates the Lagrange multipliers in the region (R), calculates the thrust developed by the plug surface between points T and D and the total thrust, and evaluates the error function along the right characteristic DE. The Lagrange multipliers calculated by this subprogram are written out along the right characteristics as the calculations proceed from left to right starting from the same point as in FLOW.

g. Subprogram CHAR1. This subprogram is an iteration procedure which solves the flow field characteristic and compatibility equations which, in finite difference form, are

$$y_3 - y_1 = (x_3 - x_1) \tan (\theta_{1,3} - \alpha_{1,3}) \quad (J-1)$$

$$y_3 - y_2 = (x_3 - x_2) \tan (\theta_{2,3} + \alpha_{2,3}) \quad (J-2)$$

$$(\theta_3 - \theta_1) + Q_{1,3}(V_3 - V_1) - G_{1,3}(y_3 - y_1)/y_{1,3} = 0 \quad (J-3)$$

$$(\theta_3 - \theta_2) - Q_{2,3}(V_3 - V_2) + F_{2,3}(y_3 - y_2)/y_{2,3} = 0 \quad (J-4)$$

The double subscript indicates the average value of the

parameter between the two points and

$$Q_{13} = \cot \alpha_{13} / V_{13}$$

$$Q_{23} = \cot \alpha_{23} / V_{23}$$

$$G_{13} = \sin \theta_{13} \sin \alpha_{13} / \sin (\theta_{13} - \alpha_{13})$$

$$F_{23} = \sin \theta_{23} \sin \alpha_{23} / \sin (\theta_{23} + \alpha_{23})$$

The points 1 and 2 are assumed to be known and the procedure solves for the location and flow properties at point 3. This is shown schematically in Figure J-2.

h. Subprogram SURF. This subprogram finds the point where a right characteristic intersects the plug surface TD. Since the wall location and slope are known it is only necessary to satisfy Eqs. (J-1) and (J-3). The location and flow properties at point 1, shown in Figure J-3, are assumed to be known and the location and velocity at point 3 are found.

i. Subprogram LOCAT. This subprogram performs a similar function to that of subprogram SURF. Here it is desired to start from a point on the plug surface where the location and flow angle are known and extend a right characteristic to a known left characteristic. Thus, it is desired to find the velocity at point 1 shown in Figure J-4 and the location and flow properties at point 3. In general, interpolation along the known left characteristic is required.

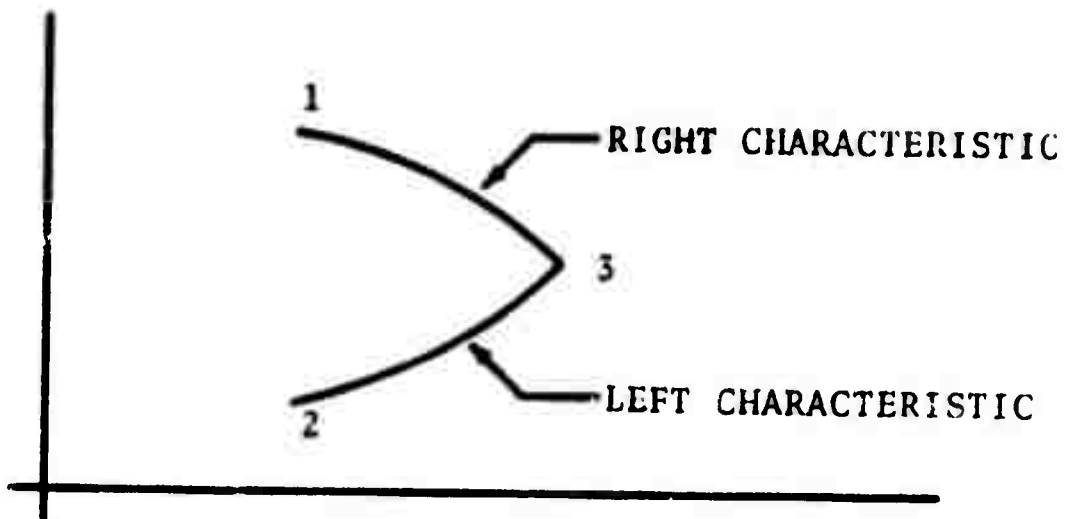


FIGURE J-2. CHARACTERISTIC NET LABELING
FOR SUBPROGRAM CHAR1

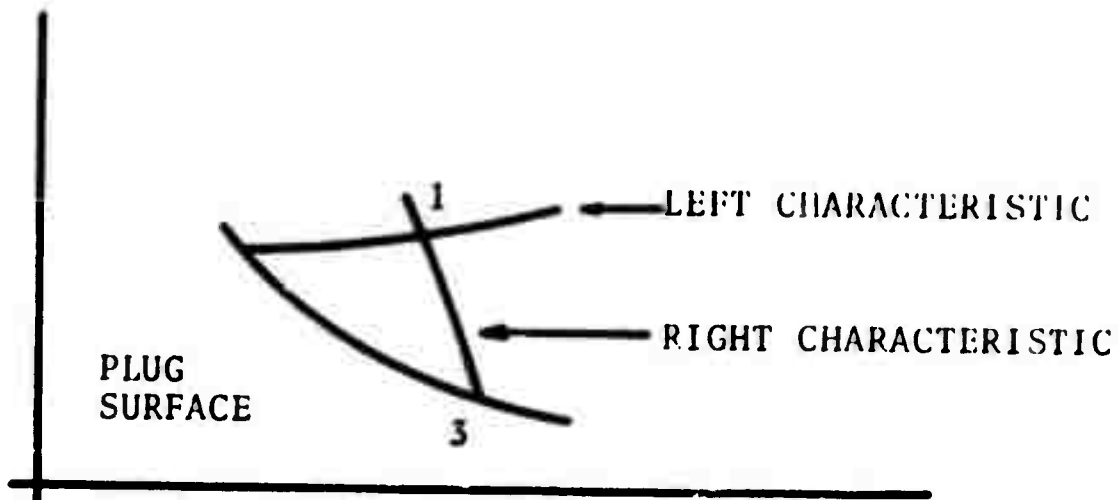


FIGURE J-3. WALL POINT LABELING
FOR SUBPROGRAM SURF

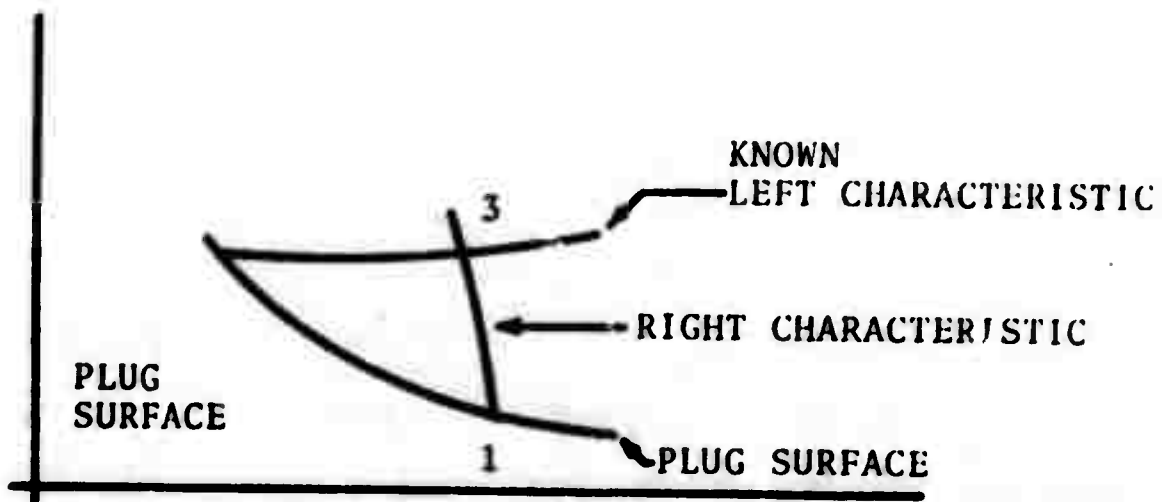


FIGURE J-4. WALL POINT LABELING
FOR SUBPROGRAM LOCAT

j. Subprogram SKIN. This subprogram calculates the wall shear stress at a given point on the plug surface using the equation

$$\tau = \frac{1}{2} C_f \rho V^2 \quad (J-5)$$

where

$$C_f = C_{f_i} / [1 + .72(\gamma-1)M^2/2]^{.578}$$

The incompressible coefficient of friction, C_{f_i} , is constant and is an input parameter to the program. The input arguments to SKIN are the local Mach number and velocity and the incompressible coefficient of friction. The output argument is the value of τ . This subprogram could be easily modified to incorporate any desired wall shear model.

k. Subprogram BASE. Subprogram BASE calculates the base pressure using the empirical equation

$$p_b = \Lambda p / M^B \quad (J-6)$$

where the constants Λ and B are program input parameters. The input arguments to BASE are the x and y coordinates and the local Mach number and velocity. The output arguments are p_b and γp_b . This subprogram could also be modified to incorporate any desired base pressure model.

l. Subprogram REST. This subprogram evaluates the general isoperimetric constraint g and its derivatives g_p , g_y , g_y' , and $d(g_y')/dx$. Since this constraint was taken

to be one which requires the nozzle to have a fixed length, $g = 1$, and all the derivatives are zero. Currently there are no required input arguments to BASE and the output arguments are g , g_p , g_y , $g_{\dot{y}}$, and $dg_{\dot{y}}/dx$. This subprogram could be changed to include other constraints such as constant wetted area by setting

$$g = y (1 + \dot{y}^2) \quad (J-7)$$

$$g_p = 0 \quad (J-8)$$

$$g_y = 1 + \dot{y}^2 \quad (J-9)$$

$$g_{\dot{y}} = 2y\dot{y} \quad (J-10)$$

$$\frac{d}{dx} g_{\dot{y}} = 2(y\ddot{y} + \dot{y}^2) \quad (J-11)$$

m. Subprogram QUAD. This subprogram simply determines the angular location of one point with respect to another. It is used in subprogram FLOW1 to help determine if a point on the start line lies within the zone of influence of another.

n. Subprogram LSQUARE. This subprogram determines the constants A and B for a least squares fit of data to a straight line whose equation is

$$y = Ax + B$$

o. Subprogram AITKEN. This is an interpolating subprogram which uses Aitken's method of interpolation. The maximum degree of the interpolating polynomial is 10.

APPENDIX K

COMPUTER PROGRAM OPERATION

The program input-output parameters are discussed in this Appendix and are illustrated by means of two sample cases. The program failure modes and flags are also discussed.

1. PROGRAM INPUT

The program input parameters are read in from data cards by the main program. All of the input parameters are identified here in the order in which they appear in the data deck and are discussed as necessary.

Card 1 FORMAT (12A6)

This card contains 72 characters of identifying information which is written out on the first page of the computer output.

Card 2 FORMAT (9F8.0)

This card contains the engine operating conditions and information for the first guess of the plug surface.

PA	Atmospheric pressure (lbf/in ²).
PO	Chamber pressure (lbf/in ²).
TO	Chamber temperature (°R).

R Gas constant (ft - lbf/lbm - °R).

G Ratio of specific heats (non-dimensional).

FLRTC Desired mass flow rate (lbm/sec).

XLENG Desired length of the plug surface (in).

YLAST The estimate for y_D (in) when the first guess
of the plug surface is generated internally.

TLAST The estimate for the wall slope (degrees) at
point D when the first guess of the plug surface
is generated internally.

Card 3 FORMAT (7F10.0)

These data are concerned with the specification of the throat geometry and start line. Appendix G contains a detailed discussion of these parameters.

HO Throat half-height (in).

RE Cowl lip radius (in).

BD Mean flow angle at the throat (degrees).

RRE The mean radius of curvature of the nozzle walls
at the throat (in).

EPS Velocity component expansion parameter = $R^{-1/2}$
(nondimensional).

CENTM Desired Mach number along the start line at the
throat center-line or the Mach number along the
linear, internally generated start line.

RHOD The downstream radius of curvature of the plug
surface (in).

Card 4 FORMAT (14I5)

This card contains integer parameters for controlling the numerical algorithm.

- NPTSL Desired number of points on the start line or the number of points on the start line to be read in (maximum = 100). Typically 31.
- NPTS Number of points on the portion of the plug surface to be optimized (maximum = 50). Typically 50.
- NDEGR1 Degree of the interpolating polynomial used for interpolation of the data along the left characteristic originating at point T in subprograms SELECT and EXTEND (maximum = 10). Typically 1.
- NDEGR2 Degree of the interpolating polynomial used in subprogram SURF (maximum = 10). Typically 1.
- NDERMX Number of iterations before recalculating the partial derivatives, $\partial E / \partial \theta$. Typically 5.
- NPTSS One plus the number of wall points to be read in as the first guess for the optimum surface (maximum = 200). Should be at least 15.
- NS Number of points which are to have reduced spacing along the left characteristic originating at point T. NS = 1 if all points are to have equal spacing. Generally less than 10.

NWRITE Controls the output. NWRITE = 1: all calculations are written out; NWRITE = 0: limited output as described in the next section.

NCNT The desired number of points of the internally generated first guess for the optimum surface which are to be on the arc of a circle of radius RHOD. Typically 5.

NREAD1 0: Read in 2 through NPTSS data points as the first guess for the optimum surface. The first point corresponds to the lowest point on the start line and need not be read in.
1: Generate the first guess for the optimum surface internally.

NREAD2 0: If other than a Moore-Hall start line is desired. 1: If a Moore-Hall start line is desired.

OPT1 0: To analyze a given plug contour. 1: To obtain the optimum contour.

OPT2 0: Read in NPTSL data point for the start line.
1: Generate a linear start line internally.

OPT3 0: HO is fixed at the value read in. 1: Change HO to obtain the desired mass flow rate.

Card 5 FORMAT (9F8.0)

This card contains more information for the control of the numerical algorithm.

DELTPM The Mach number increment for the Prandtl-Meyer expansion at point E'. Typically .05.

CFI Incompressible flow coefficient of friction. Typically .003 to .008. CFI = 0.0 if wall shear is to be zero.

ERR The acceptable error along the exit characteristic. Typically 0.001.

WEIGHT The weighting factor applied to changes in the wall angle. Typically 0.85.

DELTAT Wall angle increment from point A to T. Used in generating the first guess for the optimum surface and also in the optimization process. Typically 0.5 degrees.

CS The factor used to reduce the spacing of the points along the left characteristic originating at point T. The discussion of subprogram SELECT contains additional information about CS. CS = 1 if all points are to have equal spacing.

AA Constant coefficient used to calculate the base pressure in subprogram BASE. Typically 0.846.

AB The exponent of the Mach number used to calculate the base pressure in subprogram base. Typically 1.3.

THETAC The decrease in the flow angle (degrees) across the throat. Used when the linear start line is generated internally. Typically 6.0.

Cards 6 FORMAT (3F10.0)

These are (NPTSS-1) cards containing the data for the first estimate of the optimum plug contour. $I = 1$ corresponds to point A shown in Figure 3 and this data point is not read in since it is included in the start line data.

XS(I) Ith value of the x-coordinate (in).
RS(I) Ith value of the y-coordinate (in).
TS(I) Ith value of the wall angle (degrees).

Cards 7 FORMAT (5F10.0)

These are NPTSL cards containing the data for the start line if it is to be read in. $I = 1$ corresponds to point A shown in Figure 3.

XL (I) Ith value of the x-coordinate (in).
RL (I) Ith value of the y-coordinate (in).
ML (I) Ith value of the Mach number.
VL (I) Ith value of the velocity (ft/sec).
TL (I) Ith value of the flow angle (degrees).

2. PROGRAM OUTPUT

The only program output control is the input parameter NWRITE. The value of NWRITE determines whether or not the flow properties and Lagrange multipliers at each point in the flow field will be written out. These are written out as the calculations are made in subprograms FLOW1, EXTEND, FLOW, and LANGR1. The order in which the output appears is explained in the descriptions of the

subprograms in Appendix I.

Portions of the output are printed independent of the value of NWRITE. These data include all input data, the start line, throat half-height, Prandtl-Meyer expansion, first guess for the optimum surface, Lagrange multipliers along the wall, error along the exit characteristic, calculated and adjusted partial derivatives ($\partial E/\partial \theta$), new wall contour, final wall contour, thrust (lbf), mass flow rate (lbm/sec), specific impulse (lbf-sec/lbm), flow injection angle (degrees), and the ambient pressure (lbf/in²).

3. SAMPLE CASES

In this section the input data and selected program output sheets are presented for two sample cases.

a. Sample Case Number One. For this case, it is desired to obtain the optimum plug contour for a nozzle which has a fixed length $XLENG = 12.0$ in., a cowl lip radius $RE = 8.34612$ in., an injection angle $BD = -45.0^\circ$, a downstream radius of curvature $RHOD = 0.5$ in., a mean radius of curvature $RRE = 0.705$ in., and no asymmetry which implies that $ETA = 0.0$. The engine is assumed to operate with a chamber pressure $PO = 500$ psia and a chamber temperature $TO = 6000^\circ R$. The exhaust products are assumed to have a gas constant of $R = 56.0$ (ft - lbf)/(lbm - $^\circ R$) and a ratio of specific heats $G = 1.23$. The nozzle is to be designed for an ambient pressure $PA = 14.7$ psia

and to have a mass flow FLRTC = 148.077 lbm/sec. The throat half-height which will pass this mass flow rate is estimated to be $H_0 = 0.4155$ in. The y coordinate of point D and the wall slope at that point are estimated to be $Y_{LAST} = 1.75$ in. and $T_{LAST} = -13.00^\circ$.

The Moore-Hall start line analysis is to be used and the throat half-height is to be modified to obtain the desired mass flow rate. This requires that $OPT2 = 1$, $NREAD2 = 1$, and $OPT3 = 1$. The Mach number at the throat center line is chosen as $CENTM = 1.1$. A total of 15 data points are to be generated along the start line so that $NPTSL = 15$. It is desired to have the maximum number of points on the plug surface which is to be optimized so that $NPTS = 50$. $NDEGR1$ and $NDEGR2$ are both set equal to 1. Ten points along the left characteristic originating at point T are to have their spacing reduced by a factor of two, thus $NS = 10$ and $CS = 2.0$. It is desired to have the partial derivatives, $\partial E / \partial \theta$, recalculated after every six iterations so that $NDERMX = 6$. The first guess for the optimum surface is to be generated internally with a total of 17 data points. Five of these points are to be located on the circular arc whose radius is $RHOD$. Consequently, $NPTSS = 17$, $NCNT = 5$, and $NREAD1 = 1$. Since the optimization analysis is to be carried out $OPT1 = 1$. $NWRITE$ is set equal to 0 so that the flow field and Lagrange multiplier fields will be written out in limited form.

The Mach number increment for the Prandti-Meyer expansion at point E', the flow angle increment along the circular arc of radius RHOD, and the incompressible skin friction coefficient are selected as DELTPM = 0.05, DELTAT = 0.5, and CFI = 0.002. Values for ERR and WEIGHT are selected as 0.001 and 0.85, respectively. The constants for the base pressure model are AA = 0.846 and AB = 1.3. Finally, the decrease in the flow angle along the linear start line, THETAC, is arbitrarily set equal to zero since it is not used.

The input data sheet for this case is shown in Figure K-1 and typical program output is shown in Figure K-2.

b. Sample Case Number Two. This sample case is similar to the first but will illustrate the use of the linear start line. Only those input parameters which are different or not used will be pointed out.

Since the start line is to be linear, ETA and RRE will not be used but for convenience left at the same values as the previous case. CENTM now becomes the value of the Mach number along the entire start line and is given a value of 1.08. The linear start line option also requires that NREAD2 = 0. The flow angle decrease across the throat THETAC = 6.0°. The nozzle length, cowl lip radius, and injection angle are selected as XLENG = 10.0 in., RE = 9.5 in., and BD = - 50.0°. The nozzle is to be

designed for an ambient pressure $PA = 0.0$. Limited program out is desired so that $NWRITE = 0$.

The input data sheet for this case is shown in Figure K-3 and typical program output is shown in Figure K-4.

4. FAILURE MODES

Only one primary failure mode has been observed in the program. If the first guess for the optimum surface is not smooth and compatible with the upstream geometry, the flow in the throat region may become subsonic and negative square roots will cause the program to stop. The program contains several other error checks and if a specific error is encountered an appropriate message is printed out. Since the program was found to have few error modes, the output control $NWRITE$ was normally set equal to 0.

SAMPLE CASE NO. 1

INPUT DATA

PA = 14.700 PC = 500.000 TC = 600.000 R = 56.000 G = 1.230
 FLRTC = 148.077 XLENG = 12.000 YLAST = 1.750 TLAST = -13.000
 MO = 0.41550 RE = 8.34612 BD = -45.0000 ETA = 0. RRE = 0.7050 CENTM = 1.10000 RMOD = 0.50000
 NWRITE = 0 APTSL = 15 NPTS = 50 NDEGR1 = 1 NDEGR2 = 1 NDERMX = 6 NPTSS = 17 NS = 10
 NCNT = 5 NREAD1 = 1 NREAD2 = 1 CPT1 = 1 CPT2 = 1 OPT3 = 1
 DELTPM = 0.0500 CFI = 0.0020 ERR = 0.0010 HEIGHT = 0.8500 DELTAT = 0.5000 CS = 2.0000
 AA = 0.8460 AB = 1.3000 THETAC = 0.

FIGURE K-2. SELECTED OUTPUT FOR SAMPLE CASE NO. 1

DATA FOR THE START LINE

XC (IN)	RC (IN)	MACH NUMBER	VELOCITY (FPS)	FLOW ANGLE (DEGREES)
-0.00000	8.34612	1.01918	3512.53018	-42.79400
-0.01429	8.27196	1.02682	3536.03900	-42.90326
-0.02859	8.19781	1.03573	3563.37405	-43.07355
-0.04288	8.12365	1.04593	3594.56052	-43.30770
-0.05718	8.04950	1.05745	3629.64050	-43.60805
-0.07147	7.97534	1.07031	3668.67572	-43.97642
-0.08577	7.90119	1.08456	3711.74924	-44.41411
-0.10006	7.82703	1.10027	3758.96758	-44.92189
-0.11422	7.81274	1.08453	3711.66019	-45.43092
-0.12837	7.79844	1.07026	3668.51959	-45.87660
-0.14253	7.78415	1.05738	3629.44690	-46.25991
-0.15668	7.76985	1.04587	3594.36331	-46.58232
-0.17084	7.75556	1.03568	3563.20886	-46.84578
-0.18500	7.74126	1.02679	3535.94055	-47.05271
-0.19915	7.72697	1.01918	3512.53018	-47.20600

MASS FLOW RATE = 148.07654 LBM/SEC

H() = 0.43781 INCHES

FIGURE K-2 (Continued)

DATA FOR PRANDTL-MEYER EXPANSION AT E

XC (IN)	RC (IN)	MACH NUMBER	VELOCITY (FPS)	FLOW ANGLE (DEGREES)
-0.00000	8.34612	1.06918	3665.24677	-42.07338
-0.00000	8.34612	1.11918	3815.49453	-41.05527
-0.00000	8.34612	1.16918	3963.22870	-39.84744
-0.00000	8.34612	1.21918	4108.41064	-38.50356
-0.00000	8.34612	1.26918	4251.00873	-37.05764
-0.00000	8.34612	1.31918	4390.99725	-35.53356
-0.00000	8.34612	1.36918	4528.35681	-33.94911
-0.00000	8.34612	1.41918	4663.07367	-32.31807
-0.00000	8.34612	1.46918	4795.13953	-30.65140
-0.00000	8.34612	1.51918	4924.55133	-28.95795
-0.00000	8.34612	1.56918	5051.31091	-27.24505
-0.00000	8.34612	1.61918	5175.42493	-25.51877
-0.00000	8.34612	1.66918	5296.90393	-23.78424
-0.00000	8.34612	1.71918	5415.76288	-22.04577
-0.00000	8.34612	1.76918	5532.02026	-20.30705
-0.00000	8.34612	1.81918	5645.69812	-18.57123
-0.00000	8.34612	1.86918	5756.82135	-16.84102
-0.00000	8.34612	1.91918	5865.41803	-15.11874
-0.00000	8.34612	1.96918	5971.51855	-13.40639
-0.00000	8.34612	2.01918	6075.15570	-11.70569
-0.00000	8.34612	2.06918	6176.36450	-10.01813
-0.00000	8.34612	2.11918	6275.18146	-8.34497
-0.00000	8.34612	2.16918	6371.34508	-6.68730
-0.00000	8.34612	2.21918	6465.79474	-5.04606
-0.00000	8.34612	2.26918	6557.67153	-3.42205
-0.00000	8.34612	2.31918	6647.31732	-1.81591
-0.00000	8.34612	2.36918	6734.77461	-0.22823
-0.00000	8.34612	2.41918	6820.08636	1.34055
-0.00000	8.34612	2.46918	6903.27657	2.87006
-0.00000	8.34612	2.51917	6984.44952	4.41998
-0.00000	8.34612	2.56917	7063.58917	5.93009
-0.00000	8.34612	2.61917	7140.76007	7.42023
-0.00000	8.34612	2.66917	7216.00623	8.89027
-0.00000	8.34612	2.71917	7289.37213	10.34015
-0.00000	8.34612	2.76917	7360.90157	11.75983
-0.00000	8.34612	2.81917	7430.63837	13.17934
-0.00000	8.34612	2.84953	7472.11627	14.02517

PFINAL = 14.70000 PSIA

FIGURE K-2 (Continued)

FIRST GUESS FOR THE OPTIMUM SURFACE

XS (IN)	RS (IN)	THETA (DEGREES)
-0.61915	7.72697	-47.20600
-0.61620	7.72375	-47.70500
-0.61328	7.72051	-48.20600
-0.61039	7.71725	-48.70600
-0.60752	7.71396	-49.20600
0.39151	6.69466	-42.12827
1.39054	5.87625	-36.66502
2.38957	5.19169	-32.28801
3.38860	4.60454	-28.68724
4.38763	4.09241	-25.66403
5.38666	3.64037	-23.08370
6.38570	3.23786	-20.85124
7.38473	2.87710	-18.89746
8.38376	2.55214	-17.17070
9.38279	2.25834	-15.63155
10.38182	1.99197	-14.24937
11.38085	1.75000	-13.00000

FIGURE K-2 (Continued)

LAM4 = 85.86692

LAGRANGE MULTIPLIERS ALONG WALL

POINT	LAMBDA1	LAMBDA2
1	-217.75	-5186.75
3	-219.63	-5082.05
6	-220.54	-5038.20
10	-220.02	-5031.56
15	-219.48	-5024.78
21	-218.93	-5017.77
28	-218.36	-5010.55
36	-217.79	-5003.14
45	-217.20	-4995.57
55	-216.60	-4987.84
66	-215.24	-4959.99
78	-213.82	-4951.74
91	-212.35	-4933.00
105	-210.84	-4913.83
120	-209.27	-4894.24
136	-207.67	-4874.39
153	-206.02	-4854.18
171	-204.33	-4833.72
190	-202.60	-4813.01
210	-200.83	-4792.08
231	-199.03	-4770.94
253	-197.18	-4749.62
276	-195.31	-4728.13
300	-193.39	-4705.50
325	-187.88	-4609.07
351	-178.31	-4442.70
378	-166.33	-4249.48
406	-152.19	-4032.33
435	-136.35	-3793.86
465	-119.78	-3538.15
496	-103.27	-3271.62
528	-87.51	-2997.71
561	-73.03	-2727.03
595	-60.13	-2455.69
630	-48.91	-2193.37
666	-39.24	-1940.70
703	-31.03	-1703.24
741	-24.07	-1483.67
780	-18.23	-1282.73
820	-13.39	-1100.15
861	-9.40	-934.20
903	-6.20	-780.94
946	-4.27	-641.73

THRUST = 32473.44067 Lbf

FIGURE K-2 (Continued)

ERROR ALONG EXIT CHAR

POINT	ERROR
1	26.75906
2	21.60920
3	17.78381
4	17.76507
5	17.74876
6	17.72935
7	17.70725
8	17.68304
9	17.65718
10	17.63000
11	17.55747
12	17.49206
13	17.42726
14	17.36442
15	17.30502
16	17.25029
17	17.19978
18	17.15779
19	17.12318
20	17.09709
21	17.08020
22	17.07330
23	17.07709
24	17.09224
25	15.62179
26	12.91686
27	9.98359
28	7.10332
29	4.53761
30	2.05996
31	-0.06379
32	-1.48215
33	-2.14844
34	-2.64489
35	-2.70101
36	-2.53269
37	-2.11817
38	-1.55723
39	-0.98753
40	-0.47862
41	-0.07773
42	0.08246
43	0.00073

FIGURE K-2 (Continued)

CALCULATED PARTIAL DERIVATIVES

POINT	DTHETA	DE/DT
43	0.10694	0.32544
39	0.11276	0.74828
35	0.11361	0.89441
31	0.11439	1.53692
27	0.11438	2.80143
23	0.11424	3.55861
19	0.11419	3.60956
15	0.11412	3.67076
11	0.11403	3.73761
7	0.11453	3.77930

FIGURE K-2 (Continued)

ADJUSTED PARTIAL DERIVATIVES

POINT	DE/DT
1	5.88877
2	5.77836
3	5.66796
4	5.55755
5	5.44715
6	5.33674
7	5.22634
8	5.11594
9	5.00553
10	4.89513
11	4.78472
12	4.67432
13	4.56391
14	4.45351
15	4.34311
16	4.23270
17	4.12230
18	4.01189
19	3.90149
20	3.79108
21	3.68068
22	3.57028
23	3.45987
24	3.34947
25	3.23906
26	3.12866
27	3.01825
28	2.90785
29	2.79744
30	2.68704
31	2.57664
32	2.46623
33	2.35583
34	2.24542
35	2.13502
36	2.02461
37	1.91421
38	1.80381
39	1.69340
40	1.58300
41	1.47259
42	1.36219
43	1.25178

FIGURE K-2 (Continued)

NEW WALL COMPUTED, ITERATION = 0

XS (IN)	RS (IN)	THETA (DEGREES)
-0.61915	7.72697	-47.20600
-0.61620	7.72375	-47.10600
-0.61328	7.72051	-48.20600
-0.61039	7.71725	-48.70600
-0.60752	7.71396	-49.20600
-0.60468	7.71064	-49.70600
-0.60188	7.70730	-50.20600
-0.59910	7.70394	-50.70600
-0.59753	7.70201	-50.99053
-0.59129	7.69425	-51.38405
-0.58375	7.68474	-51.80391
-0.57575	7.67458	-51.79632
-0.56741	7.66398	-51.78869
-0.55892	7.65320	-51.78165
-0.55029	7.64224	-51.77543
-0.54151	7.63110	-51.77021
-0.53259	7.61977	-51.76621
-0.52353	7.60827	-51.76362
-0.50314	7.58243	-51.67561
-0.48169	7.55534	-51.58413
-0.45947	7.52736	-51.49025
-0.43648	7.49852	-51.39445
-0.41271	7.46881	-51.29722
-0.38816	7.43822	-51.19906
-0.36281	7.40675	-51.10048
-0.33666	7.37439	-51.00207
-0.30969	7.34115	-50.90446
-0.28190	7.30701	-50.80831
-0.25328	7.27196	-50.71437
-0.22381	7.23600	-50.62350
-0.19349	7.19910	-50.53664
-0.16229	7.16126	-50.45486
-0.10832	7.09661	-49.83930
-0.02802	7.00334	-48.69214
0.07103	6.89330	-47.30678
0.19265	6.76502	-45.72471
0.34327	6.61512	-43.97405
0.53059	6.44005	-42.12433
0.75616	6.24264	-40.23166
1.02693	6.02129	-38.27101
1.36150	5.76696	-36.18232
1.78222	5.47088	-34.06242
2.27106	5.15380	-31.84796
2.87886	4.79266	-29.56016
3.59466	4.40568	-27.20710
4.46347	3.98160	-24.80448
5.49529	3.53049	-22.40295
6.71150	3.05844	-20.00369
8.15566	2.56711	-17.55712
9.87923	2.06312	-15.02504
11.40247	1.68285	-13.00049

FIGURE K-2 (Continued)

NEW WALL COMPUTED,

ITERATION = 12

XS (IN)	RS (IN)	THETA (DEGREES)			
-0.61915	7.12697	-47.20600			
-0.61620	7.12375	-47.10600			
-0.61328	7.12051	-46.20600			
-0.61039	7.11725	-45.70600			
-0.60752	7.11396	-45.20500			
-0.60468	7.11064	-44.70600			
-0.60188	7.10730	-50.20600			
-0.59910	7.10394	-50.70600			
-0.59635	7.10055	-51.20600			
-0.59363	7.69713	-51.70600			
-0.59094	7.69370	-52.20500			
-0.58828	7.69024	-52.70600			
-0.58565	7.68676	-53.20500			
-0.58306	7.68325	-53.70600			
-0.58049	7.67972	-54.20500			
-0.57795	7.67617	-54.70600			
-0.57542	7.67371	-55.05114			
-0.57290	7.67125	-55.05100			
-0.57039	7.66879	-55.84952			
-0.56789	7.66633	-55.74796			
-0.56540	7.66387	-55.64635			
-0.56291	7.66141	-55.54468			
-0.56043	7.65895	-55.44297			
-0.55795	7.65649	-55.34126			
-0.55548	7.65403	-55.23951			
-0.55301	7.65157	-55.13775			
-0.55054	7.64911	-55.03600			
-0.54807	7.64665	-54.93425			
-0.54560	7.64419	-54.83250			
-0.54313	7.64173	-54.73075			
-0.54066	7.63927	-54.62900			
-0.53819	7.63681	-54.52725			
-0.53572	7.63435	-54.42550			
-0.53325	7.63189	-54.32375			
-0.53078	7.62943	-54.22200			
-0.52831	7.62697	-54.12025			
-0.52584	7.62451	-54.01850			
-0.52337	7.62205	-53.91675			
-0.52090	7.61959	-53.81500			
-0.51843	7.61713	-53.71325			
-0.51596	7.61467	-53.61150			
-0.51349	7.61221	-53.50975			
-0.51102	7.60975	-53.40800			
-0.50855	7.60729	-53.30625			
-0.50608	7.60483	-53.20450			
-0.50361	7.60237	-53.10275			
-0.50114	7.60091	-53.00100			
-0.49867	7.59845	-52.89925			
-0.49620	7.59599	-52.79750			
-0.49373	7.59353	-52.69575			
-0.49126	7.59107	-52.59400			
-0.48879	7.58861	-52.49225			
-0.48632	7.58615	-52.39050			
-0.48385	7.58369	-52.28875			
-0.48138	7.58123	-52.18700			
-0.47891	7.57877	-52.08525			
-0.47644	7.57631	-51.98350			
-0.47397	7.57385	-51.88175			
-0.47150	7.57139	-51.78000			
-0.46903	7.56893	-51.67825			
-0.46656	7.56647	-51.57650			
-0.46409	7.56401	-51.47475			
-0.46162	7.56155	-51.37300			
-0.45915	7.55909	-51.27125			
-0.45668	7.55663	-51.16950			
-0.45421	7.55417	-51.06775			
-0.45174	7.55171	-50.96600			
-0.44927	7.54925	-50.86425			
-0.44680	7.54679	-50.76250			
-0.44433	7.54433	-50.66075			
-0.44186	7.54187	-50.55900			
-0.43939	7.53941	-50.45725			
-0.43692	7.53695	-50.35550			
-0.43445	7.53449	-50.25375			
-0.43198	7.53203	-50.15200			
-0.42951	7.52957	-50.05025			
-0.42704	7.52711	-49.94850			
-0.42457	7.52465	-49.84675			
-0.42210	7.52219	-49.74500			
-0.41963	7.51973	-49.64325			
-0.41716	7.51727	-49.54150			
-0.41469	7.51481	-49.43975			
-0.41222	7.51235	-49.33800			
-0.40975	7.50989	-49.23625			
-0.40728	7.50743	-49.13450			
-0.40481	7.50497	-49.03275			
-0.40234	7.50251	-48.93100			
-0.39987	7.50005	-48.82925			
-0.39740	7.49759	-48.72750			
-0.39493	7.49513	-48.62575			
-0.39246	7.49267	-48.52400			
-0.39000	7.49021	-48.42225			
-0.38753	7.48775	-48.32050			
-0.38506	7.48529	-48.21875			
-0.38259	7.48283	-48.11700			
-0.38012	7.48037	-48.01525			
-0.37765	7.47791	-47.91350			
-0.37518	7.47545	-47.81175			
-0.37271	7.47299	-47.71000			
-0.37024	7.47053	-47.60825			
-0.36777	7.46807	-47.50650			
-0.36530	7.46561	-47.40475			
-0.36283	7.46315	-47.30300			
-0.36036	7.46069	-47.20125			
-0.35789	7.45823	-47.09950			
-0.35542	7.45577	-46.99775			
-0.35295	7.45331	-46.89600			
-0.35048	7.45085	-46.79425			
-0.34801	7.44839	-46.69250			
-0.34554	7.44593	-46.59075			
-0.34307	7.44347	-46.48900			
-0.34060	7.44101	-46.38725			
-0.33813	7.43855	-46.28550			
-0.33566	7.43609	-46.18375			
-0.33319	7.43363	-46.08200			
-0.33072	7.43117	-45.98025			
-0.32825	7.42871	-45.87850			
-0.32578	7.42625	-45.77675			
-0.32331	7.42379	-45.67500			
-0.32084	7.42133	-45.57325			
-0.31837	7.41887	-45.47150			
-0.31590	7.41641	-45.36975			
-0.31343	7.41395	-45.26800			
-0.31096	7.41149	-45.16625			
-0.30849	7.40903	-45.06450			
-0.30602	7.40657	-44.96275			
-0.30355	7.40411	-44.86100			
-0.30108	7.40165	-44.75925			
-0.29861	7.39919	-44.65750			
-0.29614	7.39673	-44.55575			
-0.29367	7.39427	-44.45400			
-0.29120	7.39181	-44.35225			
-0.28873	7.38935	-44.25050			
-0.28626	7.38689	-44.14875			
-0.28379	7.38443	-44.04700			
-0.28132	7.38197	-43.94525			
-0.27885	7.37951	-43.84350			
-0.27638	7.37705	-43.74175			
-0.27391	7.37459	-43.64000			
-0.27144	7.37213	-43.53825			
-0.26897	7.36967	-43.43650			
-0.26650	7.36721	-43.33475			
-0.26403	7.36475	-43.23300			
-0.26156	7.36229	-43.13125			
-0.25909	7.35983	-43.02950			
-0.25662	7.35737	-42.92775			
-0.25415	7.35491	-42.82600			
-0.25168	7.35245	-42.72425			
-0.24921	7.35000	-42.62250			
-0.24674	7.34754	-42.52075			
-0.24427	7.34508	-42.41900			
-0.24180	7.34262	-42.31725			
-0.23933	7.34016	-42.21550			
-0.23686	7.33770	-42.11375			
-0.23439	7.33524	-42.01200			
-0.23192	7.33278	-41.91025			
-0.22945	7.33032	-41.80850			
-0.22698	7.32786	-41.70675			
-0.22451	7.32540	-41.60500			
-0.22204	7.32294	-41.50325			
-0.21957	7.32048	-41.40150			
-0.21710	7.31802	-41.29975			
-0.21463	7.31556	-41.19800			
-0.21216	7.31310	-41.09625			
-0.20969	7.31064	-40.99450			
-0.20722	7.30818	-40.89275			
-0.20475	7.30572	-40.79100			
-0.20228	7.30326	-40.68925			
-0.19981	7.30080	-40.58750			
-0.19734	7.29834	-40.48575			
-0.19487	7.29588	-40.38400			
-0.19240	7.29342	-40.28225			
-0.18993	7.29096	-40.18050			
-0.18746	7.28850	-40.07875			
-0.18499	7.28604	-39.97700			
-0.18252	7.28358	-39.87525			
-0.18005	7.28112	-39.77350			
-0.17758	7.27866	-39.67175			
-0.17511	7.27620	-39.57000			
-0.17264	7.27374	-39.46825			
-0.17017	7.27128	-39.36650			
-0.16770	7.26882	-39.26475			
-0.16523	7.26636	-39.16300			
-0.16276	7.26390	-39.06125			
-0.16029	7.26144	-38.95950			
-0.15782	7.25898	-38.85775			
-0.15535	7.25652	-38.75600			
-0.15288	7.25406	-38.65425			
-0.15041	7.25160	-38.55250			
-0.14794	7.24914	-38.45075			
-0.14547	7.24668	-38.34900			
-0.14300	7.24422	-38.24725			
-0.14053	7.24176	-38.14550			
-0.13806	7.23930	-38.04375			
-0.13559	7.23684	-37.94200			
-0.13312	7.23438	-37.84025			
-0.13065	7.23192	-37.73850			
-0.12818	7.22946	-37.63675			
-0.12571	7.22700	-37.53500			
-0.12324	7.22454	-37.43325			
-0.12077	7.22208	-37.33150			
-0.11830	7.21962	-37.22975			
-0.11583	7.21716	-37.12800			
-0.11336	7.21470	-37.02625			
-0.11089	7.21224	-36.92450			
-0.10842	7.20978	-36.82275			
-0.10595	7.20732	-36.72100			
-0.10348	7.20486	-36.61925			
-0.10101	7.20240	-36.51750			
-0.09854	7.20000	-36.41575			
-0.09607	7.19754	-36.31400			
-0.09360	7.19508	-36.21225			
-0.09113	7.19262	-36.11050			
-0.08866	7.19016	-36.00875			
-0.08619	7.18770	-35.90700			
-0.08372	7.18524	-35.80525			
-0.08125	7.18278	-35.70350			
-0.07878	7.18032	-35.60175			
-0.07631	7.17786	-35.50000			
-0.07384	7.17540	-35.39825			
-0.07137	7.17294	-35.29650			
-0.06890	7.17048	-35.19475			
-0.06643	7.16802	-35.09300			
-0.06396	7.16556	-34.99125			
-0.06149	7.16310	-34.88950			
-0.05902	7.16064	-34.78775			
-0.05655					

LAM4 = 85.71503

LAGRANGE MULTIPLIERS ALONG WALL

POINT	LAMBDA1	LAMBDA2
1	-218.99	-4947.75
3	-217.64	-4935.95
6	-216.29	-4924.17
10	-214.94	-4912.28
15	-213.58	-4900.43
21	-212.23	-4888.56
28	-210.87	-4876.69
36	-209.51	-4864.81
45	-208.14	-4852.92
55	-206.77	-4841.02
66	-203.72	-4814.52
78	-200.67	-4787.99
91	-197.60	-4751.44
105	-194.52	-4734.87
120	-191.44	-4708.28
136	-186.41	-4647.48
153	-177.78	-4523.59
171	-168.21	-4382.39
190	-157.93	-4226.53
210	-147.21	-4059.77
231	-136.31	-3885.80
253	-125.45	-3707.02
276	-114.81	-3526.15
300	-104.55	-3345.46
325	-94.79	-3166.79
351	-85.59	-2991.64
378	-77.01	-2821.10
406	-69.05	-2655.97
435	-61.73	-2496.77
465	-55.02	-2343.77
496	-48.89	-2197.10
528	-43.31	-2055.72
561	-38.27	-1923.39
595	-33.73	-1796.86
630	-29.61	-1676.20
666	-25.90	-1561.06
703	-22.56	-1451.72
741	-19.58	-1348.98
780	-16.89	-1250.87
820	-14.46	-1157.06
861	-12.33	-1069.76
903	-10.40	-986.36
946	-8.66	-906.90
990	-7.13	-833.36
1035	-5.75	-763.37
1081	-4.51	-698.76
1128	-4.19	-632.03

THRUST = 32648.37720 LBF

FIGURE K-2 (Continued)

ERROR ALONG EXIT CHAR

POINT	ERROR
1	0.00334
2	0.00321
3	0.00295
4	0.00286
5	0.00274
6	0.00268
7	0.00261
8	0.00256
9	0.00252
10	0.00248
11	0.00223
12	0.00219
13	0.00213
14	0.00215
15	0.00326
16	0.00432
17	0.00222
18	0.00189
19	0.00174
20	0.00151
21	0.00133
22	0.00133
23	0.00129
24	0.00123
25	0.00120
26	0.00116
27	0.00115
28	0.00112
29	0.00106
30	0.00101
31	0.00085
32	0.00065
33	0.00046
34	0.00029
35	0.00009
36	-0.00020
37	-0.00054
38	-0.00072
39	-0.00103
40	-0.00153
41	-0.00170
42	-0.00175
43	-0.00148
44	0.00021
45	0.00304
46	0.00317
47	0.00127

FIGURE K-2 (Continued)

FINAL SOLUTION

XS (IN)	RS (IN)	THETA (DEGREES)
-0.61915	7.72697	-47.20600
-0.61620	7.72375	-47.70600
-0.61328	7.72051	-48.20600
-0.61039	7.71725	-48.70600
-0.60752	7.71396	-49.20600
-0.60468	7.71064	-49.70600
-0.60188	7.70730	-50.20600
-0.59910	7.70394	-50.70600
-0.59635	7.70055	-51.20600
-0.59363	7.69713	-51.70600
-0.59094	7.69370	-52.20600
-0.58828	7.69024	-52.70600
-0.58565	7.68676	-53.20600
-0.58306	7.68325	-53.70600
-0.58049	7.67972	-54.20600
-0.57795	7.67617	-54.70600
-0.57542	7.67261	-55.20600
-0.57290	7.66903	-55.70600
-0.57039	7.66544	-56.20600
-0.56789	7.66183	-56.70600
-0.56540	7.65820	-57.20600
-0.56291	7.65456	-57.70600
-0.56043	7.65091	-58.20600
-0.55795	7.64725	-58.70600
-0.55548	7.64358	-59.20600
-0.55301	7.63989	-59.70600
-0.55054	7.63619	-60.20600
-0.54807	7.63248	-60.70600
-0.54560	7.62876	-61.20600
-0.54313	7.62503	-61.70600
-0.54066	7.62129	-62.20600
-0.53819	7.61754	-62.70600
-0.53572	7.61378	-63.20600
-0.53325	7.60999	-63.70600
-0.53078	7.60619	-64.20600
-0.52831	7.60238	-64.70600
-0.52584	7.59855	-65.20600
-0.52337	7.59471	-65.70600
-0.52090	7.59086	-66.20600
-0.51843	7.58699	-66.70600
-0.51596	7.58311	-67.20600
-0.51349	7.57922	-67.70600
-0.51102	7.57532	-68.20600
-0.50855	7.57141	-68.70600
-0.50608	7.56749	-69.20600
-0.50361	7.56356	-69.70600
-0.50114	7.55962	-70.20600
-0.49867	7.55567	-70.70600
-0.49620	7.55171	-71.20600
-0.49373	7.54774	-71.70600
-0.49126	7.54376	-72.20600
-0.48879	7.53977	-72.70600
-0.48632	7.53577	-73.20600
-0.48385	7.53176	-73.70600
-0.48138	7.52774	-74.20600
-0.47891	7.52371	-74.70600
-0.47644	7.51967	-75.20600
-0.47397	7.51562	-75.70600
-0.47150	7.51156	-76.20600
-0.46903	7.50749	-76.70600
-0.46656	7.50341	-77.20600
-0.46409	7.49932	-77.70600
-0.46162	7.49523	-78.20600
-0.45915	7.49113	-78.70600
-0.45668	7.48702	-79.20600
-0.45421	7.48290	-79.70600
-0.45174	7.47877	-80.20600
-0.44927	7.47463	-80.70600
-0.44680	7.47048	-81.20600
-0.44433	7.46632	-81.70600
-0.44186	7.46215	-82.20600
-0.43939	7.45797	-82.70600
-0.43692	7.45378	-83.20600
-0.43445	7.44958	-83.70600
-0.43198	7.44537	-84.20600
-0.42951	7.44115	-84.70600
-0.42704	7.43692	-85.20600
-0.42457	7.43268	-85.70600
-0.42210	7.42843	-86.20600
-0.41963	7.42417	-86.70600
-0.41716	7.41990	-87.20600
-0.41469	7.41562	-87.70600
-0.41222	7.41133	-88.20600
-0.40975	7.40703	-88.70600
-0.40728	7.40272	-89.20600
-0.40481	7.39840	-89.70600
-0.40234	7.39407	-90.20600
-0.39987	7.38973	-90.70600
-0.39740	7.38538	-91.20600
-0.39493	7.38102	-91.70600
-0.39246	7.37665	-92.20600
-0.39000	7.37227	-92.70600
-0.38753	7.36788	-93.20600
-0.38506	7.36348	-93.70600
-0.38259	7.35907	-94.20600
-0.38012	7.35465	-94.70600
-0.37765	7.35022	-95.20600
-0.37518	7.34578	-95.70600
-0.37271	7.34133	-96.20600
-0.37024	7.33687	-96.70600
-0.36777	7.33240	-97.20600
-0.36530	7.32792	-97.70600
-0.36283	7.32343	-98.20600
-0.36036	7.31893	-98.70600
-0.35789	7.31442	-99.20600
-0.35542	7.30990	-99.70600
-0.35295	7.30537	-100.20600
-0.35048	7.30083	-100.70600
-0.34801	7.29628	-101.20600
-0.34554	7.29172	-101.70600
-0.34307	7.28715	-102.20600
-0.34060	7.28257	-102.70600
-0.33813	7.27798	-103.20600
-0.33566	7.27338	-103.70600
-0.33319	7.26877	-104.20600
-0.33072	7.26415	-104.70600
-0.32825	7.25952	-105.20600
-0.32578	7.25488	-105.70600
-0.32331	7.25023	-106.20600
-0.32084	7.24557	-106.70600
-0.31837	7.24090	-107.20600
-0.31590	7.23622	-107.70600
-0.31343	7.23153	-108.20600
-0.31096	7.22683	-108.70600
-0.30849	7.22212	-109.20600
-0.30602	7.21740	-109.70600
-0.30355	7.21267	-110.20600
-0.30108	7.20793	-110.70600
-0.29861	7.20318	-111.20600
-0.29614	7.19842	-111.70600
-0.29367	7.19365	-112.20600
-0.29120	7.18887	-112.70600
-0.28873	7.18408	-113.20600
-0.28626	7.17928	-113.70600
-0.28379	7.17447	-114.20600
-0.28132	7.16965	-114.70600
-0.27885	7.16482	-115.20600
-0.27638	7.15998	-115.70600
-0.27391	7.15513	-116.20600
-0.27144	7.15027	-116.70600
-0.26897	7.14540	-117.20600
-0.26650	7.14052	-117.70600
-0.26403	7.13563	-118.20600
-0.26156	7.13073	-118.70600
-0.25909	7.12582	-119.20600
-0.25662	7.12089	-119.70600
-0.25415	7.11595	-120.20600
-0.25168	7.11099	-120.70600
-0.24921	7.10602	-121.20600
-0.24674	7.10104	-121.70600
-0.24427	7.09605	-122.20600
-0.24180	7.09105	-122.70600
-0.23933	7.08604	-123.20600
-0.23686	7.08102	-123.70600
-0.23439	7.07599	-124.20600
-0.23192	7.07094	-124.70600
-0.22945	7.06588	-125.20600
-0.22698	7.06080	-125.70600
-0.22451	7.05571	-126.20600
-0.22204	7.05060	-126.70600
-0.21957	7.04548	-127.20600
-0.21710	7.04034	-127.70600
-0.21463	7.03519	-128.20600
-0.21216	7.03003	-128.70600
-0.20969	7.02485	-129.20600
-0.20722	7.01966	-129.70600
-0.20475	7.01445	-130.20600
-0.20228	7.00923	-130.70600
-0.19981	7.00400	-131.20600
-0.19734	6.99875	-131.70600
-0.19487	6.99349	-132.20600
-0.19240	6.98821	-132.70600
-0.18993	6.98292	-133.20600
-0.18746	6.97761	-133.70600
-0.18499	6.97229	-134.20600
-0.18252	6.96695	-134.70600
-0.18005	6.96160	-135.20600
-0.17758	6.95624	-135.70600
-0.17511	6.95087	-136.20600
-0.17264	6.94549	-136.70600
-0.17017	6.94009	-137.20600
-0.16770	6.93468	-137.70600
-0.16523	6.92926	-138.20600
-0.16276	6.92383	-138.70600
-0.16029	6.91838	-139.20600
-0.15782	6.91291	-139.70600
-0.15535	6.90743	-140.20600
-0.15288	6.90193	-140.70600
-0.15041	6.89642	-141.20600
-0.14794	6.89089	-141.70600
-0.14547	6.88534	-142.20600
-0.14300	6.87977	-142.70600
-0.14053	6.87419	-143.20600
-0.13806	6.86859	-143.70600
-0.13559	6.86297	-144.20600
-0.13312	6.85733	-144.70600
-0.13065	6.85167	-145.20600
-0.12818	6.84599	-145.70600
-0.12571	6.84029	-146.20600
-0.12324	6.83457	-146.70600
-0.12077	6.82883	-147.20600
-0.11830	6.82307	-147.70600
-0.11583	6.81729	-148.20600
-0.11336	6.81149	-148.70600
-0.11089	6.80567	-149.20600
-0.10842	6.79983	-149.70600
-0.10595	6.79397	-150.20600
-0.10348	6.78809	-150.70600
-0.10101	6.78219	-151.20600
-0.09854	6.77627	-151.70600
-0.09607	6.77033	-152.20600
-0.09360	6.76437	-152.70600
-0.09113	6.75839	-153.20600
-0.08866	6.75239	-153.70600
-0.08619	6.74637	-154.20600
-0.08372	6.74033	-154.70600
-0.08125	6.73427	-155.20600
-0.07878	6.72819	-155.70600
-0.07631	6.72208	-156.20600
-0.07384	6.71595	-156.70600
-0.07137	6.70979	-157.20600
-0.06890	6.70360	-157.70600
-0.06643	6.69739	-158.20600
-0.06396	6.69115	-158.70600
-0.06149	6.68489	-159.20600
-0.05902	6.67860	-159.70600
-0.05655	6.67229	-160.20600
-0.05408	6.66595	-160.70600
-0.05161	6.65958	-161.20600
-0.04914	6.65319	-161.70600
-0.04667	6.64677	-162.20600
-0.04420	6.64033	-162.70600
-0.04173	6.63387	-163.20600
-0.03926	6.62738	-163.70600
-0.03679	6.62087	-164.20600
-0.03432	6.61433	-164.70600
-0.03185	6.60777	-165.20600
-0.02938	6.60118	-165.70600
-0.02691	6.59457	-166.20600
-0.02444	6.58793	-166.70600
-0.02197	6.58127	-167.20600
-0.01950	6.57458	-167.70600
-0.01703	6.56786	-168.20600
-0.01456	6.56112	-168.70600
-0.01209	6.55436	-169.20600
-0.00962	6.54757	-169.70600
-0.00715	6.54075	-170.20600
-0.00468	6.53390	-170.70600
-0.00221	6.52703	-171.20600
0.00026	6.52013	-171.70600
0.00283	6.51320	-172.20600
0.00540	6.50625	-172.70600
0.00797	6.49927	-173.20600
0.01054	6.49227	-173.70600
0.01311	6.48524	-174.20600
0.01568	6.47818	-174.70600
0.01825	6.47109	-175.20600
0.02082	6.46397	-175.70600
0.02339	6.45682	-176.20600
0.02596	6.44964	-176.70600
0.02853	6.44243	-177.20600
0.03110	6.43519	-177.70600
0.03367	6.42792	-178.20600
0.03624	6.42062	-178.70600
0.03881	6.41329	-179.20600
0.04138	6.40593	-179.70600
0.04395	6.39854	-180.20600
0.04652	6.39112	-180.70600
0.04909	6.38367	-181.20600
0.05166	6.37619	-181.70600
0.05423	6.36868	-182.20600
0.05680	6.36115	-182.70600
0.05937	6.35359	-183.20600
0.06194	6.34599	-183.70600
0.06451	6.33837	-184.20600
0.06708	6.33072	-184.70600
0.06965	6.32304	-185.20600
0.07222	6.31533	-185.70600
0.07479	6.30759	-186.20600
0.07736	6.29982	-186.70600
0.07993	6.29199	-187.20600
0.08250	6.28413	-187.70600
0.08507	6.27624	-188.20600
0.08764	6.26831	-188.70600
0.09021	6.26035	-189.20600
0.09278	6.25236	-189.70600
0.09535	6.24434	-190.20600
0.09792	6.23629	-19

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72																																		
CARD 1	SAMPLE CASE NO. 2																	HEADER																
CARD 2	0.0	PA	500.0	PO	6000.0	T0	R	G	1.23	FLRTC	148.077	XLENG	10.00	YLAST	1.75	TLAST	-13.00																	
CARD 3	4.155	H0	9.5	RE	-50.00	BD	0.0	ETA	.705	RRE	1.08	CENTM	0.5	RH0D																				
CARD 4	13	NPTS L	50	NPTS	17	ONPTS	10	ONWRITE	ONCNT	INREAD 1	ONREAD 2	ONPT 1	ONPT 2	ONPT 3																				
CARD 5	0.05	DELTPM	0.002	CFI	0.001	ERR	0.85	WEIGHT	2.0	CS	0.846	AA	1.3	A3	6.0	THETAC																		

FIGURE K-3. INPUT DATA SHEET FOR SAMPLE CASE NO. 2

SAMPLE CASE NO. 2

INPUT DATA

PA = 0. PD = 500.000 TO = 6000.000 R = 56.000 G = 1.230
 FLRTC = 148.077 XLENG = 10.000 YLAST = 1.750 TLAST = -13.000
 HO = 3.41550 RE = 9.50000 BD = -50.0000 ETA = 0. RRE = 0.7050 CENTM = 1.08000 RMOD = 0.50000
 NWRITE = 0 NPTSL = 15 NPTS = 50 NDEGR1 = 1 NDEGR2 = 1 NDERMX = 6 NPTSS = 17 NS = 10
 NCUT = 5 NREAD1 = 1 NREAD2 = 0 OPT1 = 1 OPT2 = 1 OPT3 = 1
 DELTPY = 0.0500 CFI = 0.0020 ERR = 0.0010 WEIGHT = 0.8500 DELTAT = 0.5000 CS = 2.0000
 AA = 0.8460 AB = 1.3000 THETAC = 6.0000

FIGURE K-4: SELECTED OUTPUT FOR SAMPLE CASE NO. 2

DATA FOR THE START LINE

XC (IN)	RC (IN)	MACH NUMBER	VELOCITY (FPS)	FLOW ANGLE (DEGREES)
0.	9.50000	1.08000	3697.98572	-47.00000
-0.04064	9.46590	1.08000	3697.98572	-47.42857
-0.08128	9.43179	1.08000	3697.98572	-47.85714
-0.12193	9.39769	1.08000	3697.98572	-48.28571
-0.16257	9.36359	1.08000	3697.98572	-48.71428
-0.20321	9.32948	1.08000	3697.98572	-49.14285
-0.24385	9.29538	1.08000	3697.98572	-49.57143
-0.28450	9.26128	1.08000	3697.98572	-50.00000
-0.32514	9.22717	1.08000	3697.98572	-50.42857
-0.36578	9.19307	1.08000	3697.98572	-50.85714
-0.40642	9.15897	1.08000	3697.98572	-51.28571
-0.44707	9.12487	1.08000	3697.98572	-51.71428
-0.48771	9.09076	1.08000	3697.98572	-52.14286
-0.52835	9.05666	1.08000	3697.98572	-52.57143
-0.56899	9.02256	1.08000	3697.98572	-53.00000

MASS FLOW RATE = 148.07702 LBM/SEC

HO = 0.37138 INCHES

FIGURE K-4 (Continued)

DATA FOR PRANDTL-NEYER EXPANSION AT E

XC (IN)	RC (IN)	MACH NUMBER	VELOCITY (FPS)	FLOW ANGLE (DEGREES)
0.	9.50000	1.13000	3847.69293	-45.93457
0.	9.50000	1.18000	3994.87750	-44.69364
0.	9.50000	1.23000	4139.50255	-43.32523
0.	9.50000	1.28000	4281.53760	-41.86061
0.	9.50000	1.33000	4420.95844	-40.32211
0.	9.50000	1.38000	4557.74689	-38.72652
0.	9.50000	1.43000	4691.89032	-37.08692
0.	9.50000	1.48000	4823.38165	-35.41376
0.	9.50000	1.53000	4952.21893	-33.71553
0.	9.50000	1.58000	5078.40509	-31.99925
0.	9.50000	1.63000	5201.94751	-30.27078
0.	9.50000	1.68000	5322.85797	-28.53505
0.	9.50000	1.73000	5441.15198	-26.79623
0.	9.50000	1.78000	5556.84894	-25.05789
0.	9.50000	1.83000	5669.97150	-23.32308
0.	9.50000	1.88000	5780.54523	-21.59440
0.	9.50000	1.93000	5888.59888	-19.87411
0.	9.50000	1.98000	5994.16327	-18.16414
0.	9.50000	2.03000	6097.27167	-16.46617
0.	9.50000	2.08000	6197.95947	-14.78162
0.	9.50000	2.13000	6296.26361	-13.11172
0.	9.50000	2.18000	6392.22290	-11.45754
0.	9.50000	2.23000	6485.87708	-9.81997
0.	9.50000	2.28000	6577.26727	-8.19977
0.	9.50000	2.33000	6666.43549	-6.59759
0.	9.50000	2.38000	6753.42450	-5.01396
0.	9.50000	2.43000	6838.27771	-3.44932
0.	9.50000	2.48000	6921.03887	-1.90404
0.	9.50000	2.53000	7001.75214	-0.37838
0.	9.50000	2.58000	7080.46179	1.12742
0.	9.50000	2.63000	7157.21216	2.61321
0.	9.50000	2.68000	7232.04755	4.07889
0.	9.50000	2.73000	7305.01215	5.52439
0.	9.50000	2.78000	7376.14290	6.94971
0.	9.50000	2.83000	7445.50415	8.35486
0.	9.50000	2.88000	7513.11823	9.73989
0.	9.50000	2.93000	7579.03491	11.10489
0.	9.50000	2.98000	7643.29626	12.44996
0.	9.50000	3.03000	7705.94385	13.77525
0.	9.50000	3.08000	7767.01886	15.08089
0.	9.50000	3.13000	7826.56146	16.36705
0.	9.50000	3.18000	7884.61151	17.63394
0.	9.50000	3.20067	7908.18787	18.19502

PFINAL = 7.78023 PSIA

FIGURE K-4 (Continued)

FIRST GUESS FOR THE OPTIMUM SURFACE

XS (IN)	RS (IN)	THETA (DEGREES)
-0.56899	9.02256	-53.00000
-0.56638	9.01906	-53.50000
-0.56380	9.01554	-54.00000
-0.56125	9.01200	-54.50000
-0.55874	9.00844	-55.00000
0.27374	7.88769	-51.73091
1.10622	6.89303	-48.35806
1.93870	6.01113	-44.89351
2.77118	5.23086	-41.35328
3.60366	4.54280	-37.75689
4.43613	3.93891	-34.12676
5.26861	3.41226	-30.48725
6.10109	2.95680	-26.86361
6.93357	2.56725	-23.28078
7.76605	2.23896	-19.76230
8.59853	1.96777	-16.32934
9.43101	1.75000	-13.00000

FIGURE K-4 (Continued)

LAM4 = 164.97913

LAGRANGE MULTIPLIERS ALCNG WALL

POINT	LAMBDA1	LAMBDA2
1	-275.28	-4752.82
3	-276.40	-4702.85
6	-275.92	-4692.23
10	-275.23	-4685.90
15	-274.53	-4679.76
21	-273.83	-4673.69
28	-273.12	-4667.73
36	-272.42	-4661.88
45	-271.71	-4656.06
55	-271.01	-4650.40
66	-269.43	-4637.97
78	-267.85	-4625.88
91	-266.27	-4614.20
105	-264.68	-4602.81
120	-263.08	-4591.69
136	-261.49	-4580.82
153	-259.89	-4570.22
171	-254.16	-4494.79
190	-244.95	-4380.28
210	-232.11	-4236.02
231	-215.31	-4053.82
253	-194.65	-3865.38
276	-171.29	-3647.21
300	-146.82	-3413.57
325	-123.09	-3166.07
351	-101.24	-2910.39
378	-82.13	-2648.27
406	-66.25	-2371.98
435	-53.21	-2090.78
465	-42.76	-1804.40
496	-34.24	-1521.86
528	-27.11	-1254.22
561	-20.76	-1052.77
595	-14.92	-908.59
630	-9.56	-824.06
666	-6.96	-785.48

THRUST = 35892.77539 LBF

FIGURE K-4 (Continued)

ERROR ALONG EXIT CHAR

POINT	ERROR
1	-6.04158
2	-10.79777
3	-11.86355
4	-11.89563
5	-11.92187
6	-11.94587
7	-11.96653
8	-11.98405
9	-11.99976
10	-12.01137
11	-12.03304
12	-12.03968
13	-12.03462
14	-12.01949
15	-11.99474
16	-11.96166
17	-11.91884
18	-12.94987
19	-14.52193
20	-16.30766
21	-18.03127
22	-19.38413
23	-20.08290
24	-20.03929
25	-19.38645
26	-18.23950
27	-16.80228
28	-15.38812
29	-13.98865
30	-12.65011
31	-11.24749
32	-9.54065
33	-7.20379
34	-4.16381
35	-0.96856
36	0.34578

FIGURE K-4 (Continued)

CALCULATED PARTIAL DERIVATIVES

POINT	DTHETA	DE/DT
36	0.10705	0.51604
32	0.11279	0.02277
28	0.11447	-0.41638
24	0.11475	0.09701
20	0.11463	1.53350
16	0.11440	2.30957
12	0.11437	2.36282
8	0.11450	2.42157
4	0.11451	2.45974
2	0.11923	2.54819

FIGURE K-4 (Continued)

ADJUSTED PARTIAL DERIVATIVES

POINT	DE/DT
1	4.12026
2	4.03236
3	3.94447
4	3.85657
5	3.76867
6	3.68078
7	3.59288
8	3.50499
9	3.41709
10	3.32919
11	3.24130
12	3.15340
13	3.06551
14	2.97761
15	2.88971
16	2.80182
17	2.71392
18	2.62603
19	2.53813
20	2.45023
21	2.36234
22	2.27444
23	2.18655
24	2.09865
25	2.01075
26	1.92286
27	1.83496
28	1.74707
29	1.65917
30	1.57127
31	1.48338
32	1.39548
33	1.30759
34	1.21969
35	1.13179
36	1.04390

FIGURE K-4 (Continued)

NEW WALL COMPUTED,

ITERATION = 0

XS (IN)	RS (IN)	THETA (DEGREES)
-0.56899	9.02256	-53.00000
-0.56077	9.01169	-53.41534
-0.55193	8.99973	-53.64747
-0.54277	8.98730	-53.57762
-0.53341	8.97463	-53.50608
-0.52393	8.96183	-53.43266
-0.51432	8.94889	-53.35740
-0.50459	8.93583	-53.28018
-0.49476	8.92267	-53.20076
-0.48480	8.90938	-53.11926
-0.46248	8.87969	-52.98544
-0.43943	8.84921	-52.84839
-0.41590	8.81822	-52.70842
-0.39189	8.78678	-52.55529
-0.36743	8.75492	-52.41890
-0.34254	8.72266	-52.25911
-0.31722	8.69003	-52.11568
-0.27880	8.64099	-51.72692
-0.22826	8.57755	-51.18012
-0.16449	8.49921	-50.51792
-0.08428	8.40312	-49.77222
0.01726	8.28475	-48.99129
0.14557	8.13927	-48.18552
0.30893	7.95921	-47.37916
0.51752	7.73599	-46.49313
0.77872	7.46548	-45.50348
1.11348	7.13184	-44.29124
1.54871	6.71921	-42.63359
2.08263	6.24491	-40.55438
2.76240	5.68993	-37.84100
3.60098	5.07685	-34.42510
4.59672	4.44342	-30.40973
5.75859	3.82006	-25.92433
7.07806	3.24396	-21.15350
8.56497	2.74170	-16.08883
9.43101	2.51568	-13.14601

FIGURE K-4 (Continued)

FINAL SOLUTION

AS (IN)	RS (IN)	THE TA (DEGREES)
-0.56899	9.02256	-53.00000
-0.56638	9.01906	-53.50000
-0.56380	9.01556	-54.00000
-0.56125	9.01200	-54.50000
-0.55874	9.00844	-55.00000
-0.55625	9.00485	-55.50000
-0.55379	9.00125	-56.00000
-0.55137	8.99762	-56.50000
-0.54898	8.99397	-57.00000
-0.54662	8.99030	-57.50000
-0.54429	8.98661	-58.00000
-0.54199	8.98290	-58.50000
-0.53973	8.97917	-58.85430
-0.53751	8.97549	-58.80886
-0.53532	8.97181	-58.75064
-0.53316	8.96812	-58.69405
-0.53102	8.96441	-58.63828
-0.52891	8.96069	-58.58325
-0.52682	8.95699	-58.53307
-0.52475	8.95328	-58.48229
-0.52270	8.94956	-58.43259
-0.52067	8.94583	-58.38390
-0.51866	8.94211	-58.33029
-0.51667	8.93839	-58.28177
-0.51469	8.93466	-58.23564
-0.51273	8.93093	-58.19164
-0.51079	8.92719	-58.14922
-0.50886	8.92345	-58.10818
-0.50694	8.91971	-58.06809
-0.50503	8.91597	-58.02873
-0.50313	8.91223	-57.99015
-0.50124	8.90849	-57.95235
-0.49936	8.90475	-57.91532
-0.49748	8.90101	-57.87907
-0.49561	8.89727	-57.84359
-0.49375	8.89353	-57.80899
-0.49189	8.88979	-57.77524
-0.49004	8.88605	-57.74235
-0.48819	8.88231	-57.71032
-0.48635	8.87857	-57.67907
-0.48451	8.87483	-57.64859
-0.48267	8.87109	-57.61899
-0.48084	8.86735	-57.59024
-0.47901	8.86361	-57.56235
-0.47718	8.85987	-57.53532
-0.47535	8.85613	-57.50907
-0.47352	8.85239	-57.48359
-0.47169	8.84865	-57.45899
-0.46986	8.84491	-57.43524
-0.46803	8.84117	-57.41235
-0.46620	8.83743	-57.39032
-0.46437	8.83369	-57.36907
-0.46254	8.82995	-57.34859
-0.46071	8.82621	-57.32899
-0.45888	8.82247	-57.31024
-0.45705	8.81873	-57.29235
-0.45522	8.81499	-57.27532
-0.45339	8.81125	-57.25907
-0.45156	8.80751	-57.24359
-0.44973	8.80377	-57.22899
-0.44790	8.79999	-57.21524
-0.44607	8.79625	-57.20235
-0.44424	8.79251	-57.19032
-0.44241	8.78877	-57.17907
-0.44058	8.78503	-57.16859
-0.43875	8.78129	-57.15899
-0.43692	8.77755	-57.15024
-0.43509	8.77381	-57.14235
-0.43326	8.77007	-57.13532
-0.43143	8.76633	-57.12907
-0.42960	8.76259	-57.12359
-0.42777	8.75885	-57.11899
-0.42594	8.75511	-57.11524
-0.42411	8.75137	-57.11235
-0.42228	8.74763	-57.11032
-0.42045	8.74389	-57.10907
-0.41862	8.74015	-57.10859
-0.41679	8.73641	-57.10899
-0.41496	8.73267	-57.11024
-0.41313	8.72893	-57.11235
-0.41130	8.72519	-57.11532
-0.40947	8.72145	-57.11907
-0.40764	8.71771	-57.12359
-0.40581	8.71397	-57.12899
-0.40398	8.71023	-57.13524
-0.40215	8.70649	-57.14235
-0.40032	8.70275	-57.15032
-0.39849	8.69901	-57.15907
-0.39666	8.69527	-57.16859
-0.39483	8.69153	-57.17907
-0.39300	8.68779	-57.19032
-0.39117	8.68405	-57.20235
-0.38934	8.68031	-57.21532
-0.38751	8.67657	-57.22907
-0.38568	8.67283	-57.24359
-0.38385	8.66909	-57.25907
-0.38202	8.66535	-57.27532
-0.38019	8.66161	-57.29235
-0.37836	8.65787	-57.31032
-0.37653	8.65413	-57.32899
-0.37470	8.65039	-57.34859
-0.37287	8.64665	-57.36907
-0.37104	8.64291	-57.39032
-0.36921	8.63917	-57.41235
-0.36738	8.63543	-57.43532
-0.36555	8.63169	-57.45907
-0.36372	8.62795	-57.48359
-0.36189	8.62421	-57.50907
-0.36006	8.62047	-57.53532
-0.35823	8.61673	-57.56235
-0.35640	8.61299	-57.59032
-0.35457	8.60925	-57.61907
-0.35274	8.60551	-57.64859
-0.35091	8.60177	-57.67907
-0.34908	8.59803	-57.71032
-0.34725	8.59429	-57.74235
-0.34542	8.59055	-57.77532
-0.34359	8.58681	-57.80907
-0.34176	8.58307	-57.84359
-0.33993	8.57933	-57.87907
-0.33810	8.57559	-57.91532
-0.33627	8.57185	-57.95235
-0.33444	8.56811	-57.99032
-0.33261	8.56437	-58.02899
-0.33078	8.56063	-58.06859
-0.32895	8.55689	-58.10907
-0.32712	8.55315	-58.15032
-0.32529	8.54941	-58.19235
-0.32346	8.54567	-58.23532
-0.32163	8.54193	-58.27907
-0.31980	8.53819	-58.32359
-0.31797	8.53445	-58.36907
-0.31614	8.53071	-58.41532
-0.31431	8.52697	-58.46235
-0.31248	8.52323	-58.51032
-0.31065	8.51949	-58.55907
-0.30882	8.51575	-58.60907
-0.30699	8.51201	-58.66032
-0.30516	8.50827	-58.71235
-0.30333	8.50453	-58.76532
-0.30150	8.50079	-58.81907
-0.29967	8.49705	-58.87359
-0.29784	8.49331	-58.92907
-0.29601	8.48957	-58.98532
-0.29418	8.48583	-59.04235
-0.29235	8.48209	-59.10032
-0.29052	8.47835	-59.15907
-0.28869	8.47461	-59.21907
-0.28686	8.47087	-59.28032
-0.28503	8.46713	-59.34235
-0.28320	8.46339	-59.40532
-0.28137	8.45965	-59.46907
-0.27954	8.45591	-59.53359
-0.27771	8.45217	-59.59907
-0.27588	8.44843	-59.66532
-0.27405	8.44469	-59.73235
-0.27222	8.44095	-59.80032
-0.27039	8.43721	-59.86907
-0.26856	8.43347	-59.93907
-0.26673	8.42973	-60.01032
-0.26490	8.42600	-60.08235
-0.26307	8.42226	-60.15532
-0.26124	8.41852	-60.22907
-0.25941	8.41478	-60.30359
-0.25758	8.41104	-60.37907
-0.25575	8.40730	-60.45532
-0.25392	8.40356	-60.53235
-0.25209	8.39982	-60.61032
-0.25026	8.39608	-60.68907
-0.24843	8.39234	-60.76907
-0.24660	8.38860	-60.85032
-0.24477	8.38486	-60.93235
-0.24294	8.38112	-61.01532
-0.24111	8.37738	-61.10032
-0.23928	8.37364	-61.18632
-0.23745	8.36990	-61.27332
-0.23562	8.36616	-61.36132
-0.23379	8.36242	-61.45032
-0.23196	8.35868	-61.54032
-0.23013	8.35494	-61.63132
-0.22830	8.35120	-61.72332
-0.22647	8.34746	-61.81632
-0.22464	8.34372	-61.91032
-0.22281	8.33998	-62.00532
-0.22098	8.33624	-62.10132
-0.21915	8.33250	-62.19832
-0.21732	8.32876	-62.29632
-0.21549	8.32502	-62.39532
-0.21366	8.32128	-62.49532
-0.21183	8.31754	-62.59632
-0.20999	8.31380	-62.69832
-0.20816	8.31006	-62.80132
-0.20633	8.30632	-62.90532
-0.20450	8.30258	-63.01032
-0.20267	8.29884	-63.11632
-0.20084	8.29510	-63.22332
-0.19901	8.29136	-63.33132
-0.19718	8.28762	-63.44032
-0.19535	8.28388	-63.55032
-0.19352	8.28014	-63.66132
-0.19169	8.27640	-63.77332
-0.18986	8.27266	-63.88632
-0.18803	8.26892	-63.99932
-0.18620	8.26518	-64.11332
-0.18437	8.26144	-64.22832
-0.18254	8.25770	-64.34432
-0.18071	8.25396	-64.46132
-0.17888	8.25022	-64.57932
-0.17705	8.24648	-64.69832
-0.17522	8.24274	-64.81832
-0.17339	8.23900	-64.93932
-0.17156	8.23526	-65.06132
-0.16973	8.23152	-65.18432
-0.16790	8.22778	-65.30832
-0.16607	8.22404	-65.43332
-0.16424	8.22030	-65.55932
-0.16241	8.21656	-65.68632
-0.16058	8.21282	-65.81432
-0.15875	8.20908	-65.94332
-0.15692	8.20534	-66.07332
-0.15509	8.20160	-66.20432
-0.15326	8.19786	-66.33632
-0.15143	8.19412	-66.46932
-0.14960	8.19038	-66.60332
-0.14777	8.18664	-66.73832
-0.14594	8.18290	-66.87432
-0.14411	8.17916	-67.01132
-0.14228	8.17542	-67.14932
-0.14045	8.17168	-67.28832
-0.13862	8.16794	-67.42832
-0.13679	8.16420	-67.56932
-0.13496	8.16046	-67.71132
-0.13313	8.15672	-67.85432
-0.13130	8.15298	-67.99832
-0.12947	8.14924	-68.14332
-0.12764	8.14550	-68.28932
-0.12581	8.14176	-68.43632
-0.12398	8.13802	-68.58432
-0.12215	8.13428	-68.73332
-0.12032	8.13054	-68.88332
-0.11849	8.12680	-69.03432
-0.11666	8.12306	-69.18632
-0.11483	8.11932	-69.33932
-0.11299	8.11558	-69.49332
-0.11116	8.11184	-69.64832
-0.10933	8.10810	-69.80432
-0.10750	8.10436	-69.96132
-0.10567	8.10062	-70.11932
-0.10384	8.09688	-70.27832
-0.10201	8.09314	-70.43832
-0.10018	8.08940	-70.59932
-0.09835	8.08566	-70.76132
-0.09652	8.08192	-70.92432
-0.09469	8.07818	-71.08832
-0.09286	8.07444	-71.25332
-0.09103	8.07070	-71.41932
-0.08920	8.06696	-71.58632
-0.08737	8.06322	-71.75432
-0.08554	8.05948	-71.92332
-0.08371	8.05574	-72.09332
-0.08188	8.05200	-72.26432
-0.08005	8.04826	-72.43632
-0.07822	8.04452	-72.60932
-0.07639	8.04078	-72.78332
-0.07456	8.03704	-72.95832
-0.07273	8.03330	-73.13432
-0.07090	8.02956	-73.31132
-0.06907	8.02582	-73.48932
-0.06724	8.02208	-73.66832
-0.06541	8.01834	-73.84832
-0.06358	8.01460	-74.02932
-0.06175	8.01086	-74.21132
-0.05992	8.00712	-74.39432
-0.05809	8.00338	-74.57832
-0.05626	7.99964	-74.76332
-0.05443	7.99590	-74.94932
-0.05260	7.99216	-75.13632
-0.05077	7.98842	-75.32432
-0.04894	7.98468	-75.51332
-0.04711	7.98094	-75.70332
-0.04528	7.97720	-75.89432
-0.04345	7.97346	-76.08632
-0.04162	7.96972	-76.27932
-0.03979	7.96598	-76.47332
-0.03796	7.96224	-76.66832
-0.03613	7.95850	-76.86432
-0.03430	7.95476	-77.06132
-0.03247	7.95102	-77.25932
-0.03064	7.94728	-77.45832
-0.02881	7.94354	-77.65832
-0.02698	7.93980	-77.85932
-0.02515	7.93606	-78.06132
-0.02		

APPENDIX L
COMPUTER PROGRAM LISTING

This appendix is a complete listing of the computer program.

618FTC MAIN		
C	PLUG NOZZLE PROGRAM WITH FIXED INLET	MAI 1
C		MAI 2
	DIMENSION REG(1275,7), XL(75), RL(75), ML(75), VL(75), TL(75), XS(MAI	3
	1125), RS(125), TS(125), NPOINT(51), ERROR(50), TX(50), TY(50), HXSMAI	4
	2(50), HRS(50), HTS(50), COEF(50), ED(50), XA(75), YA(75), MA(75), MAI	5
	3VA(75), TA(75), HEADER(12)	MAI 6
	REAL M,ML,MA	MAI 7
	INTEGER OPT1,OPT2,OPT3	MAI 8
	COMMON /B2/ XL,RL,ML,VL,TL	MAI 9
	COMMON /B3/ PA,PO,TO,RHOD,G,R,GO	MAI 10
	COMMON /B4/ HO,RE,BD,ETA,RRE,RHOD,DELTAT,AA,AB,VCNT,THETAC	MAI 11
	COMMON /B5/ REG	MAI 12
	COMMON /B6/ NPOINT	MAI 13
	COMMON /B7/ ERROR	MAI 14
	COMMON /B8/ NWRITE	MAI 15
	COMMON /B9/ RAD,FOOT	MAI 16
	COMMON /B10/ XA,YA,MA,VA,TA	MAI 17
	COMMON /B11/ CF1,CF2,CF3,CFI,FLRT,THRUST,FLRTC	MAI 18
	COMMON /B12/ NREAD1,NREAD2,OPT1,OPT2,OPT3	MAI 19
	COMMON /B13/ XS,RS,TS	MAI 20
C		MAI 21
10	WRITE (6,390)	MAI 22
	READ (5,640) HEADER	MAI 23
	READ (5,450) PA,PO,TO,R,G,FLRTC,XLENG,YLAST,TLAST	MAI 24
	READ (5,460) HO,RE,BD,ETA,RRE,CENTH,RHOD	MAI 25
	READ (5,470) NPTSL,NPTS,NDEGR1,NDEGR2,NDERMX,NPTSS,NS,NWRITE,NCNT,MAI	26
	INHEAD1,NREAD2,OPT1,OPT2,OPT3	MAI 27
	READ (5,480) DELTPH,CFI,ERR,WEIGHT,DELTAT,CS,AA,AB,THETAC	MAI 28
	IF (INHEAD1.EQ.1) GO TO 20	MAI 29
	READ (5,490) (XS(1),RS(1),TS(1),I=2,NPTSS)	MAI 30
20	CONTINUE	MAI 31
	RAD=3.14159/180.	MAI 32
	FOOT=12.0	MAI 33
	GO=32.1739	MAI 34
	IF (OPT2.EQ.1) GO TO 40	MAI 35
	READ (5,500) (XL(1),RL(1),ML(1),VL(1),TL(1),I=1,NPTSL)	MAI 36
	DO 30 I=1,NPTSL	MAI 37
	XL(I)=XL(I)/FOOT	MAI 38
	RL(I)=RL(I)/FOOT	MAI 39
30	CONTINUE	MAI 40
40	CONTINUE	MAI 41
C		MAI 42
C	WRITE THE INPUT DATA	MAI 43
C		MAI 44
	WRITE (6,650) HEADER	MAI 45
	WRITE (6,400) PA,PO,TO,R,G,FLRTC,XLENG,YLAST,TLAST	MAI 46
	WRITE (6,410) HO,RE,BD,ETA,RRE,CENTH,RHOD	MAI 47
	WRITE (6,420) NPTSL,NPTS,NDEGR1,NDEGR2,NDERMX,NPTSS,NS,NWRITE,NCNT,MAI	48
	1,NREAD1,NREAD2,OPT1,OPT2,OPT3	MAI 49
	WRITE (6,430) DELTPH,CFI,ERR,WEIGHT,DELTAT,CS,AA,AB,THETAC	MAI 50
C		MAI 51
	RE=RE/FOOT	MAI 52
	XLENG=XLENG/FOOT	MAI 53
	YLAST=YLAST/FOOT	MAI 54

	HO=HO/FOOT	MAI	55
	DO 50 I=2,NPTSS	MAI	56
	XS(I)=XS(I)/FOOT	MAI	57
	RS(I)=RS(I)/FOOT	MAI	58
50	CONTINUE	MAI	59
	PA=PA*144.0	MAI	60
	PO=PO*144.0	MAI	61
	RHOO=PO/(R*TO)	MAI	62
	RHOD=RHOD/FOOT	MAI	63
	NPTSN=0	MAI	64
	IDERV=0	MAI	65
	ISCIL=0	MAI	66
	ITER=0	MAI	67
	NRECD=0	MAI	68
	NTIP=0	MAI	69
	N1=2	MAI	70
	N2=0	MAI	71
	NZ=2	MAI	72
	PSTOP=PA	MAI	73
C		MAI	74
C	CALCULATE THE START LINE AND THE P-H EXPANSION	MAI	75
C		MAI	76
C	CALL START (CENTH,NPTSL,PSTOP,J,NPTSS,DELTPH,XLENG,YLAST,TLAST)	MAI	77
C		MAI	78
C	CALCULATE THE FLOW IN REGION R1	MAI	79
C		MAI	80
	CALL FLOW1 (NPTSL,J,NTOT)	MAI	81
	XHOLD=XL(1)	MAI	82
	RHOLD=RL(1)	MAI	83
	THOLD=TL(1)/RAD	MAI	84
C		MAI	85
C	DIVIDE THE LIMITING LEFT CHARACTERISTIC INTO NPTS AND CURVE FIT	MAI	86
C		MAI	87
	CALL SELECT (NPTS,NTOT,NDEGR1,NS,CS,XL,RL,ML,VL,TL)	MAI	88
	CF2=0.0	MAI	89
	GO TO 70	MAI	90
C		MAI	91
60	CALL EXTEND (NTOT,NDEGR1,NPTSN,ITOT,NWRITE,NSAVE,XFINAL,ITER)	MAI	92
	NEWS=ITOT	MAI	93
	IF (ITER.GE.2) NEWS=NSAVE	MAI	94
	IF (NTIP.EQ.1) NEWS=NSAVE	MAI	95
	CALL SELECT (NPTS,NEWS,NDEGR1,NS,CS,XA,YA,MA,VA,TA)	MAI	96
70	CONTINUE	MAI	97
C		MAI	98
C	CALCULATE THE MAIN FLOW FIELD	MAI	99
C		MAI	100
	NCHECK=0	MAI	101
	CALL FLOW (NZ,NPTS,NPTSS,NDEGR2,NCHECK,XLENG,NRECD,XFINAL)	MAI	102
C		MAI	103
C	CALCULATE THE LAGRANGIAN MULTIPLIER FIELD	MAI	104
C		MAI	105
	CALL LANGR1 (NRECD,N1,N2,LAN4,OPT1)	MAI	106
	IF (OPT1.EQ.0) GO TO 300	MAI	107
C		MAI	108
C	SUBSTITUTE WALL POINTS	MAI	109
C		MAI	110

DO 80 K=1,NRECRD	MAT 111
J=NPOINT(K)+K	MAT 112
I=NPTSN+K	MAT 113
XS(I)=REG(J,1)	MAT 114
RS(I)=REG(J,2)	MAT 115
TS(I)=REG(J,5)/RAD	MAT 116
EU(K)=ERROR(K)	MAT 117
HXS(K)=XS(I)	MAT 118
HRS(K)=RS(I)	MAT 119
HTS(K)=TS(I)	MAT 120
CONTINUE	MAT 121
C	MAT 122
C CHECK FOR FINAL SOLUTION	MAT 123
C	MAT 124
IF IN=0	MAT 125
IZ=NPOINT(NRECRD)	MAT 126
DO 90 I=1,NRECRD	MAT 127
IZ=IZ+1	MAT 128
IF (ABS(ERROR(I)/REG(IZ,6)).GE.ERR) GO TO 100	MAT 129
CONTINUE	MAT 130
IF IN=1	MAT 131
GO TO 320	MAT 132
C	MAT 133
100 IF (ITER.EQ.0) GO TO 110	MAT 134
IF (IDERIV.EQ.NDERMX) GO TO 110	MAT 135
IF (NRECRD.GT.NRECRDS) GO TO 140	MAT 136
GO TO 210	MAT 137
C	MAT 138
C BEGIN ADJUSTING SLOPES TO DETERMINE PARTIAL DERIVATIVES	MAT 139
C	MAT 140
110 DT=.002	MAT 141
NCAL=10	MAT 142
IF (NCAL.GT.NRECRD) NCAL=NRECRD	MAT 143
IDADJ=(NRECRD+5)/NCAL	MAT 144
WRITE (6,570)	MAT 145
WRITE (6,560)	MAT 146
NADJ=1-IDADJ	MAT 147
NCHECK=1	MAT 148
DO 120 I=1,NCAL	MAT 149
NADJ=NADJ+IDADJ	MAT 150
IK=NRECRD-NADJ+1	MAT 151
IF (IK.LT.2) IK=2	MAT 152
IJ=NPTSN+IK	MAT 153
TS(IJ)=HTS(IK)+DT*180./3.14159	MAT 154
C	MAT 155
C DETERMINE THE FLOW FIELD FROM IK TO NRECRD AND THE LANGR FIELD	MAT 156
C	MAT 157
LMXY=NPTSN+NRECRD	MAT 158
CALL FLOW (IK,NRECRD,LMXY,NDEGR2,NCHECK,XLENG,NRECRD,XFINAL)	MAT 159
N2=1	MAT 160
CALL LANGR1 (NRECRD,IK,N2,LAN4,OPT1)	MAT 161
C	MAT 162
C CALCULATE AND STORE THE PARTIAL DERIVATIVES	MAT 163
C	MAT 164
K=NPOINT(IK)+IK	MAT 165
DS=REG(K,5)*180./3.14159-HTS(IK)	MAT 166

	COEF(1)=(ERROR(1K)-EO(1K))/DS	MAI 167
	WRITE (6,580) 1K,DS,COEF(1)	MAI 168
	TS(1J)=HTS(1K)	MAI 169
120	CONTINUE	MAI 170
	K=1-IDADJ	MAI 171
	DO 130 I=1,NCAL	MAI 172
	K=K-IDADJ	MAI 173
	I1=NPTS-K+1	MAI 174
	IF (I1.LT.2) I1=2	MAI 175
	TX(I)=I1	MAI 176
	TY(I)=COEF(1)	MAI 177
130	CONTINUE	MAI 178
	CALL LSQARE (NCAL, TX, TY, A, B)	MAI 179
	GO TO 150	MAI 180
C		MAI 181
140	CONTINUE	MAI 182
	Y1=COEF(1)*RAD	MAI 183
	Y2=COEF(NRECD)*RAD	MAI 184
	X1=1.0	MAI 185
	X2=NRECD	MAI 186
	A=(Y2-Y1)/(X2-X1)	MAI 187
	B=Y1-A*X1	MAI 188
150	WRITE (6,590)	MAI 189
	WRITE (6,560)	MAI 190
	CSML=0.01*12.*RE	MAI 191
	DO 200 I=1,NRECD	MAI 192
	ZI=I	MAI 193
	COEF(1)=A*ZI+B	MAI 194
	IF (COEF(1)) 160,170,180	MAI 195
160	IF (COEF(1).GT.-CSML) COEF(1)=-CSML	MAI 196
	GO TO 190	MAI 197
C		MAI 198
170	COEF(1)=-CSML	MAI 199
	GO TO 190	MAI 200
C		MAI 201
180	IF (COEF(1).LT.CSML) COEF(1)=CSML	MAI 202
190	WRITE (6,600) 1,COEF(1)	MAI 203
	COEF(1)=COEF(1)*180./3.14159	MAI 204
200	CONTINUE	MAI 205
C		MAI 206
C	CALCULATE CORRECTIONS TO NOZZLE WALL ANGLES	MAI 207
C		MAI 208
210	AMAX2=5.*RAD	MAI 209
	AMAX=5.*RAD	MAI 210
	DO 220 I=1,NRECD	MAI 211
	TS(I)=-EO(I)/COEF(1)	MAI 212
	IF (ABS(TS(I)).LT.AMAX) GO TO 220	MAI 213
	AMAX=ABS(TS(I))	MAI 214
220	CONTINUE	MAI 215
	WF=HEIGHT*AMAX2/AMAX	MAI 216
	DO 230 I=1,NRECD	MAI 217
	HTS(I)=HTS(I)+TS(I)*WF*180./3.14159	MAI 218
C		MAI 219
230	CONTINUE	MAI 220
C		MAI 221
C	CALCULATE NEW WALL POINTS	MAI 222

C	TN=(90.+THOLD)*RAD	MAI 223
	XC=XHOLD-RHOD*COS(TN)	MAI 224
	YC=RHOLD-RHOD*SIN(TN)	MAI 225
	TS(1)=THOLD	MAI 226
	XS(1)=XHOLD	MAI 227
	RS(1)=RHOLD	MAI 228
	TN=(90.+HTS(1))*RAD	MAI 229
	XSTAR=XC+RHOD*COS(TN)	MAI 230
	RSTAR=YC+RHOD*SIN(TN)	MAI 231
	DX=XSTAR-HXS(1)	MAI 232
	DU 240 I=2,200	MAI 233
	TS(I)=TS(I-1)-DELTAT	MAI 234
	TN=(90.+TS(I))*RAD	MAI 235
	XS(I)=RHOD*COS(TN)+XC	MAI 236
	RS(I)=YC+RHOD*SIN(TN)	MAI 237
	IH=1	MAI 238
	IF (TS(I).LE.HTS(1)) GO TO 250	MAI 239
240	CONTINUE	MAI 240
250	CONTINUE	MAI 241
	IF (IH.EQ.2) GO TO 270	MAI 242
260	NIMPTS=IH+NRECRD-1	MAI 243
	NPTSN=IH-1	MAI 244
	TS(I)=HTS(1)	MAI 245
	XS(I)=XSTAR	MAI 246
	RS(I)=RSTAR	MAI 247
	GO TO 300	MAI 248
C		MAI 249
270	CONTINUE	MAI 250
	IF (TS(I).LE.HTS(1).AND.ITER.GT.8) GO TO 290	MAI 251
	IF (TS(I).LE.HTS(1)) GO TO 280	MAI 252
	GO TO 260	MAI 253
C		MAI 254
280	NIMPTS=NRECRD	MAI 255
	IH=1	MAI 256
	NPTSN=IH-1	MAI 257
	DX=0.0	MAI 258
	GO TO 300	MAI 259
C		MAI 260
290	WRITE (6,510)	MAI 261
	GO TO 300	MAI 262
C		MAI 263
300	CONTINUE	MAI 264
	DU 310 I=2,NRECRD	MAI 265
	K=NPTSN+1	MAI 266
	TS(K)=HTS(1)	MAI 267
	XS(K)=HXS(1)+DX	MAI 268
	T1=HTS(1)*3.14159/180.	MAI 269
	T2=HTS(I-1)*3.14159/180.	MAI 270
	TB=(TAN(T1)+TAN(T2))/2.	MAI 271
	HAS(I)=RS(K-1)+(XS(K)-XS(K-1))*TB	MAI 272
	RS(K)=HAS(I)	MAI 273
	IF (RS(K).LT.0.0) MESAG=1	MAI 274
310	CONTINUE	MAI 275
320	CONTINUE	MAI 276
	IF (IFIN.EQ.0) GO TO 340	MAI 277
		MAI 278

	IF (IFIN.EC.1.AND.NEWS.NE.NSAVE) GO TO 330	PAI 279
	IF (IFIN.EC.1) WRITE (6,620)	PAI 280
	GO TO 350	PAI 281
C		PAI 282
330	NTIP=1	PAI 283
	GO TO 400	PAI 284
C		PAI 285
340	WRITE (6,610) ITER	PAI 286
350	WRITE (6,630)	PAI 287
	WRITE (6,560)	PAI 288
	CU 360 I=1,NTWPTS	PAI 289
	XD=XS(1)*FCOT	PAI 290
	RD=RS(1)*FCOT	PAI 291
	WRITE (6,550) XD,RD,TS(1)	PAI 292
360	CONTINUE	PAI 293
	IF (IFIN.EC.1) GO TO 380	PAI 294
	IF (MESAG.EC.1) GO TO 370	PAI 295
	ISCIL=ISCIL+1	PAI 296
	IF (ISCIL.CE.4) ISCIL=0	PAI 297
	ITER=ITER+1	PAI 298
	IF (IDERIV.EQ.NDERMX) IDERIV=0	PAI 299
	IDERIV=IDERIV+1	PAI 300
	IF (ITER.GE.25) NWRITE=1	PAI 301
	N2=0	PAI 302
	APTSS=NTWPTS	PAI 303
	NREAD2=1	PAI 304
	NRECDS=NWFCHR	PAI 305
	GO TO 400	PAI 306
C		PAI 307
370	WRITE (6,660)	PAI 308
380	CONTINUE	PAI 309
	XISP=THRUST/FLRT	PAI 310
	WRITE (6,520) THRUST	PAI 311
	WRITE (6,530) FLRT	PAI 312
	WRITE (6,540) XISP	PAI 313
	PA=PA/144.0	PAI 314
	WRITE (6,440) BU,PA	PAI 315
	GO TO 10	PAI 316
C		PAI 317
C		PAI 318
390	FORMAT (1H1)	PAI 319
400	FORMAT (1H0,55X,10HINPUT DATA//22X,4HPA =,F10.3,2X,4HPI =,F10.3,2X,4HPT	PAI 320
	1,4HTU =,F10.3,2X,3HR =,F10.3,2X,3HG =,F10.3,2X,3HFLTC =,F10.3,2X,3HFLTC	PAI 321
	2X,7HXLNG =,F10.3,2X,7HYLAST =,F10.3,2X,7HFLAST =,F10.3,2X,7HFLAST =,F10.3,2X,7HFLAST	PAI 322
410	FORMAT (1H0,4X,4HMO =,F8.5,2X,4HME =,F8.5,2X,4HMD =,F8.4,2X,5HETA	PAI 323
	1=,F6.4,2X,5HME =,F6.4,2X,7HCENTM =,F10.5,2X,6HMOD =,F10.5,2X,6HMOD	PAI 324
420	FORMAT (1H0,16X,71HPTSL =,14,2X,6HNPST =,14,2X,6HNPST =,14,2X,6HNPST	PAI 325
	INDEGR2 =,14,2X,6HNPST =,14,2X,6HNPST =,14,2X,6HNPST =,14,2X,6HNPST	PAI 326
	2ITE =,14,5X,6HNPST =,15,5X,6HNPST =,15,5X,6HNPST =,15,5X,6HNPST	PAI 327
	31 =,15,5X,6HNPST =,15,5X,6HNPST =,15,5X,6HNPST =,15,5X,6HNPST	PAI 328
430	FORMAT (1H0,12X,8HDELTPM =,F8.4,2X,5HCFI =,F8.4,2X,5HCFI =,F8.4,2X,5HCFI	PAI 329
	1,8HWELOMI =,F8.4,2X,8HDELTPM =,F8.4,2X,8HDELTPM =,F8.4,2X,8HDELTPM	PAI 330
	24,5X,5HAA =,F8.4,5X,9HTEIAC =,F8.4,5X,9HTEIAC =,F8.4,5X,9HTEIAC	PAI 331
440	FORMAT (1H0,46X,7HETA =,F8.3,8HDEGRS,5X,5HPA =,F5.2,5HPSIAIP	PAI 332
450	FORMAT (7F8.0)	PAI 333
460	FORMAT (7F10.0)	PAI 334

470	FORMAT (14I5)	MAI 335
480	FORMAT (9F8.0)	MAI 336
490	FORMAT (3F10.0)	MAI 337
500	FORMAT (5F10.0)	MAI 338
510	FORMAT (1H0,35X,49H WALL ANGLE LARGER THAN SELECTED UPSTREAM GEOMETN	MAI 339
	1RY)	MAI 340
520	FORMAT (1H0,46X,9H THRUST = ,F15.5,4H LBF)	MAI 341
530	FORMAT (1H0,46X,17H MASS FLOW RATE = ,F15.5,8H L3M/SEC)	MAI 342
540	FORMAT (1H0,46X,6H ISP = ,F10.5,12H LBF-SEC/LBM)	MAI 343
550	FORMAT (46X,2F10.5,F15.5)	MAI 344
560	FORMAT (1H0)	MAI 345
570	FORMAT (1H1,50X,30H CALCULATED PARTIAL DERIVATIVES//48X,5H POINT,7X,4H	MAI 346
	16H THETA,10X,5H DE/DT)	MAI 347
580	FORMAT (1H ,48X,13,2F15.5)	MAI 348
590	FORMAT (1H1,50X,20H ADJUSTED PARTIAL DERIVATIVES//54X,5H POINT,10X,5H	MAI 349
	1H DE/DT)	MAI 350
600	FORMAT (1H ,54X,13,2X,F15.5)	MAI 351
610	FORMAT (1H1,45X,12H NEW WALL COMPUTED,5X,12H ITERATION = ,12)	MAI 352
620	FORMAT (1H1,57X,14H FINAL SOLUTION)	MAI 353
630	FORMAT (1H0,51X,2H XS,9X,2H RS,8X,5H THETA/50X,4H (IN),8X,4H (IN),8X,9H	MAI 354
	1 (DEGREES))	MAI 355
640	FORMAT (12A6)	MAI 356
650	FORMAT (1H0,51X,12A6)	MAI 357
660	FORMAT (1H0,43X,34H PLUG SURFACE HAS NEGATIVE Y /ALUES)	MAI 358
	END	MAI 359-

818FTC LANG

SUBROUTINE LANGH1 (N,N1,NX,LAN4,OPTION)	LAN	1
COMMON /H3/ PA,PO,TU,RHOO,G,R,GO	LAN	2
COMMON /H4/ MU,KE,DD,ETA,RRE,RHOD,DELTAT,AA,AB,NCNT,THE TAC	LAN	3
COMMON /H5/ REG	LAN	4
COMMON /H6/ NLUC	LAN	5
COMMON /H7/ ERROR	LAN	6
COMMON /H8/ NWRITE	LAN	7
COMMON /H11/ CF1,CF2,CF3,CFI,FLAT,THRUST,FLRIC	LAN	8
REAL LAN4,MOT	LAN	9
DIMENSION REG(1275,7), NLUC(51), ERROR(50)	LAN	10
INTEGER OPTION	LAN	11
J1=NLUC(N)+N	LAN	12
J2=NLUC(N-1)+N-1	LAN	13
CALL SKIN (REG(J1,4),REG(J1,3),CF1,TAU,TAURD,TAUR,TAURHO,TAUVD,TAULAN	LAN	14
1PD,RTAUIX)	LAN	15
CALL HASE (REG(J1,1),REG(J1,2),REG(J1,4),REG(J1,3),PB,PP4IPY,FPHIPLAN	LAN	16
1X,AA,AR)	LAN	17
FUNCT=1.+(G-1.)*REG(J1,3)**2/2.	LAN	18
TE3A=TO/FUNCT	LAN	19
P3A=PI/FUNCT**((G/(G-1.))	LAN	20
DYDX=TAN(REG(J1,5))	LAN	21
RHOA=RHOD/FUNCT**((1./(G-1.))	LAN	22
U3A=REG(J1,4)*COS(REG(J1,5))	LAN	23
CALL REST (REG(J1,1),H,HP,HR,RHD,DDX)	LAN	24
LAN4=(REG(J1,2)*TAU+P3A*REG(J1,2)*DYDX-DYDX*PPH4PY+PPH4PX)/(-H)	LAN	25
UD=U3A	LAN	26
IF (NX.NE.1.AND.OPTION.NE.0) WRITE (6,120) LAN4	LAN	27
REG(J1,7)=(REG(J1,2)*(TAU-DYDX*TAURD)+LAN4*(H-DYDX*HRD)+PPH4PX)*GOLAN	LAN	28
1/(1+REG(J1,2)*RHOA*U3A*DYDX)	LAN	29
AL2U=REG(J1,7)	LAN	30
REG(J1,6)=+REG(J1,2)*RHOA*REG(J1,4)*SIN(REG(J1,5))-REG(J1,2)*U3A*(LAN	LAN	31
1TAUVD)/REG(J1,4)-RHOA*TAURD-RHOA*TAURHO/(G*GO*R+TE3A))+LAN4*RHOA*U3LAN	LAN	32
2A*HP	LAN	33
G5=0.0	LAN	34
CF1=0.0	LAN	35
JU=N-1	LAN	36
IF (NX.EQ.1) JU=N+1-N1	LAN	37
J4=NLUC(N)+N	LAN	38
DO 1J 1=1,JU	LAN	39
JT=N-1	LAN	40
J1=NLUC(JT+1)+JT+1	LAN	41
J2=NLUC(JT)+JT	LAN	42
DX=REG(J2,1)-REG(J1,1)	LAN	43
DY=REG(J2,2)-REG(J1,2)	LAN	44
DV=REG(J2,4)*SIN(REG(J2,5))-REG(J1,4)*SIN(REG(J1,5))	LAN	45
DV DX=DV/DX	LAN	46
HP1=HP	LAN	47
HR1=HR	LAN	48
HRD1=HRD	LAN	49
TAU1=TAU	LAN	50
TAURD1=TAURD	LAN	51
TAUR1=TAUR	LAN	52
TAURH1=TAURHO	LAN	53
TAUVD1=TAUVD	LAN	54

	TAUPD1=TAUPD	LAN 55
	RTAUX1=RTAUX	LAN 56
	CALL SKIN (REG(J2,4),REG(J2,3),CF1,TAU,TAUMD,TAUM,TAUMH,TAUVD,TAULAN	LAN 57
	1PU,RTAUX)	LAN 58
	CALL RST (REG(J2,1),H,HP,HR,HRD,DDX)	LAN 59
	XE1=(REG(J1,1)+REG(J2,1))/2.	LAN 60
	YE1=(REG(J1,2)+REG(J2,2))/2.	LAN 61
	FE1=(REG(J1,3)+REG(J2,3))/2.	LAN 62
	VE1=(REG(J1,4)+REG(J2,4))/2.	LAN 63
	UE1=(REG(J1,4)*COS(REG(J1,5))+REG(J2,4)*COS(REG(J2,5)))/2.	LAN 64
	TE1=(REG(J1,5)+REG(J2,5))/2.	LAN 65
	FUNCT=1+(G-1)*FF1**2/2.	LAN 66
	RH01=RH0/FUNCT**((1/(G-1)))	LAN 67
	TE1=TE/FUNCT	LAN 68
	PE1=PD/FUNCT**((G/(G-1)))	LAN 69
	DYDX1=(TAN(REG(J1,5))+TAN(REG(J2,5)))/2.	LAN 70
	ASOE1=COS(RH01*TE1)	LAN 71
	HP1=(HP1+HP)/2.	LAN 72
	HR1=(HR1+HR)/2.	LAN 73
	HRD1=(HRD+HRD1)/DX	LAN 74
	TAU1=(TAU1+TAU)/2.	LAN 75
	TAUM1=(TAUM1+TAUM)/2.	LAN 76
	TAUMD1=(TAUMD1+TAUMD)/2.	LAN 77
	TAUVD1=(TAUVD1+TAUVD)/2.	LAN 78
	TAUPD1=(TAUPD1+TAUPD)/2.	LAN 79
	TARHE1=(TARHE1+TARHE)/2.	LAN 80
	RTAXE1=(RTAXE1+RTAXE)/2.	LAN 81
	C1=(HP1+HRD1+HR1+HRD1)*GO/(RHE1+UE1)*LAN4/YE1	LAN 82
	C2=(TAU1+YE1+TAUVD1+RTAXE1)*GO/(VE1+RHE1+UE1)	LAN 83
	C3=C1-C2	LAN 84
	C4=(TAUVD1+DNDX/(VE1+RHE1)-TAUPE1+DNDX-TARHE1+DNDX/ASOE1)	LAN 85
	C5=C1-C4	LAN 86
	UDDT=(REG(J2,4)*COS(REG(J2,5))-REG(J1,4)*COS(REG(J1,5)))/DX	LAN 87
	CF1=CF1+(PE1+DYDX1+TAU1)*YE1+DX	LAN 88
	TE=TE/FUNCT	LAN 89
	FUNCT=1+(G-1)*REG(J2,3)**2/2.	LAN 90
	RH0=RH0/FUNCT**((1/(G-1)))	LAN 91
	ASO=COS(RH0*TE)	LAN 92
	ALP2DX=UDDT-C5	LAN 93
	REG(J2,7)=REG(J1,7)+ALP2DX*DX	LAN 94
	REG(J2,6)=REG(J2,2)+RH0*REG(J2,4)*SIN(REG(J2,5))-REG(J2,2)*REG(J2,7)	LAN 95
	1.4)*COS(REG(J2,5))*(TAUVD/REG(J2,4)-RH0*TAUMD-RH0*TAURH0/ASO)*LAN4	LAN 96
	2)*RH0*REG(J2,4)*COS(REG(J2,5))*HP	LAN 97
10	CONTINUE	LAN 98
	IF (OPTION.FU.0) GO TO 30	LAN 99
	IF (NR.FU.1) GO TO 40	LAN 100
	WRITE (6,140)	LAN 101
	DO 20 I=1,N	LAN 102
	J1=NL0C(I)+1	LAN 103
	WRITE (6,150) J1,REG(J1,6),REG(J1,7)	LAN 104
20	CONTINUE	LAN 105
30	THRUST=2.*1.14159*(CF1+CF2+CF3-R1**2*PA/2.+REG(J4,2)**2*PR/2.)	LAN 106
	IF (OPTION.NF.0) WRITE (6,160) THRUST	LAN 107
	IF (OPTION.FU.0) GO TO 100	LAN 108
C		LAN 109
C	CALCULATE THE LAGRANGIAN MULTIPLIERS IN R	LAN 110

C	IF (NWRITE.NE.0) WRITE (6,110)	LAN 111
	IUM=1	LAN 112
	JUB=1	LAN 113
	IF (NWRITE.NE.0) WRITE (6,170) IUM,JUB,REG(1,6),REG(1,7)	LAN 114
	IF (NWRITE.NE.0) WRITE (6,130)	LAN 115
40	DO 60 I=1,N	LAN 116
	I1=I-1	LAN 117
	I2=I+1	LAN 118
	I5=I+1-N	LAN 119
	JUB=NLUC(I)+1	LAN 120
	IF (NWRITE.NE.0) WRITE (6,170) I,I,REG(JUB,6),REG(JUB,7)	LAN 121
	DO 50 M=1,IS	LAN 122
	M1=I-M	LAN 123
	M2=I2-M	LAN 124
	J1=NLUC(I)+M2	LAN 125
	J2=NLUC(I)+M1	LAN 126
	J3=J1-1	LAN 127
	E13=(REG(J1,3)+REG(J3,3))/2.	LAN 128
	E23=(REG(J2,3)+REG(J3,3))/2.	LAN 129
	Y13=(REG(J1,2)+REG(J3,2))/2.	LAN 130
	Y23=(REG(J2,2)+REG(J3,2))/2.	LAN 131
	RHO13=RHO1/(1.+(G-1.)*E13**2/2.)***((1./(G-1.))	LAN 132
	RHO23=RHO1/(1.+(G-1.)*E23**2/2.)***((1./(G-1.))	LAN 133
	F1=Y13*RHO13*SQRTE13**2-1.)	LAN 134
	F2=Y23*RHO23*SQRTE23**2-1.)	LAN 135
	REG(J3,7)=(REG(J1,6)-REG(J2,6)+REG(J1,7)+F1+REG(J2,7)+F2)/(F1+F2)	LAN 136
	REG(J3,6)=REG(J1,6)-F1*(REG(J3,7)-REG(J1,7))	LAN 137
	JUB=M2-1	LAN 138
	IF (NWRITE.NE.0) WRITE (6,170) I,JUB,REG(J3,6),REG(J3,7)	LAN 139
	IF (I.LT.N) GO TO 50	LAN 140
	ME=N+1-M	LAN 141
	RHO=RHO1/(1.+(G-1.)*REG(J1,3)**2/2.)***((1./(G-1.))	LAN 142
	FRNOM(M)=REG(J1,6)-REG(J1,7)*REG(J1,2)*RHO*SQRTE13**2-1.)	LAN 143
50	CONTINUE	LAN 144
	IF (NWRITE.NE.0) WRITE (6,130)	LAN 145
60	CONTINUE	LAN 146
	IF (ME.NE.2) GO TO 70	LAN 147
	RHO=RHO1/(1.+(G-1.)*REG(J3,3)**2/2.)***((1./(G-1.))	LAN 148
	FRNOM(1)=REG(J3,6)-REG(J3,7)*REG(J3,2)*RHO*SQRTE13**2-1.)	LAN 149
70	CONTINUE	LAN 150
	IF (IX.NE.1) GO TO 80	LAN 151
	GO TO 100	LAN 152
C		LAN 153
80	WRITE (6,140)	LAN 154
	DO 90 I=1,N	LAN 155
	WRITE (6,170) I,ERROR(I)	LAN 156
90	CONTINUE	LAN 157
100	CONTINUE	LAN 158
	RETURN	LAN 159
C		LAN 160
C		LAN 161
C		LAN 162
110	FORMAT (1H1,4X,1H1,4X,1H3,5X,7HLAMBDA1,4X,7HLAMBDA2/1H0)	LAN 163
120	FORMAT (1H1,50X,7HLAMB4 = ,F12.5)	LAN 164
130	FORMAT (11M)	LAN 165
		LAN 166

140	FORMAT (1H0,48X,31HLAGRANGE MULTIPLIERS ALONG WALL//1H0,48X,5HPDIN	LAN 167
	1T,7X,7HLAMDA1,6X,7HLAMDA2)	LAN 168
150	FORMAT (50X,14,4X,F10.2,4X,F10.2)	LAN 169
160	FORMAT (1H0,46X,9HTHRUST = ,F15.5,4H LBF)	LAN 170
170	FORMAT (43X,215,F10.2,4X,F10.2)	LAN 171
180	FORMAT (1H1,50X,21HERROR ALONG EXIT CHAR//53X,54POINT,8X,5HERROR//	LAN 172
	1)	LAN 173
190	FORMAT (54X,13,3X,F12.5)	LAN 174
	END	LAN 175-

618FIC START1

	SUBROUTINE START (F1,N2,PNO,J,NPTSS,DELTPH,XLEN3,YLAST,FLAST)	STA 1
	REAL M,K0,K1,K2,K3,ML,M1,M2,M3,M4,MR,L,NR,KK	STA 2
	INTEGER OPT1,OPT2,OPT3	STA 3
	COMMON /B2/ XL,RL,ML,VL,TL	STA 4
	COMMON /B3/ PA,PU,TU,RH00,G,R,GU	STA 5
	COMMON /B4/ HU,HE,UD,ETA,RHE,RH0D,DELTAT,AA,AB,VCNT,THETAC	STA 6
	COMMON /B8/ WRITE	STA 7
	COMMON /B9/ RAD,FOOT	STA 8
	COMMON /B11/ CF1,CF2,CF3,CFI,FLRT,THRUST,FLRTC	STA 9
	COMMON /B12/ NREAD1,NREAD2,OPT1,OPT2,OPT3	STA 10
	COMMON /B13/ XS,YS,TS	STA 11
	DIPEYSION XL(75), RL(75), ML(75), VL(75), TL(75), XS(125), YS(125)	STA 12
	1,TS(125), A9(4,5)	STA 13
	WRITE (6,496)	STA 14
	THETAC=THETAC+RAD	STA 15
	PN=0	STA 16
10	CF3=0.0	STA 17
	FLRT=0.0	STA 18
	IF (NREAD2.EQ.0) GO TO 140	STA 19
	AS=SQRT((2.0*G*GU*H*TO)/(G+1.0))	STA 20
	MX=RHE/M1	STA 21
	FPS=1.0/SQRT(MX)	STA 22
	F1=(G-1.0)/(G+1.0)	STA 23
	BR=H0*1.14159/180.0	STA 24
	C=RL-H0*GUS(MR)	STA 25
	P=HU*SIN(BR)/(1.0+EPS)	STA 26
	XU=HU*SIN(BR)	STA 27
	P1=SQRT((1.0-F1)*(0.5-M**2))	STA 28
	EU=(1.0-F1)*M/(4.0*B1)-(1.0-F1)*M*M*M/(6.0*B1)-1.0/6.0	STA 29
	K0=M*ETA/2.-B1*ETA/(1.0-F1)	STA 30
	K1=2.0*H0*B1/(1.0-F1)-M*B0*M*M*M*(COS(BR)/SIN(BR))**2	STA 31
	K2=2.0*ETA*B1/(1.0-F1)-3.0*ETA*M	STA 32
	K3=B1/(1.0-F1)+1.5*M**2*B1-3.0*M+2.5*M*M*M-3.0*M*M*M*(COS(BR)/SIN(BR))**2	STA 33
	Y=0.0	STA 34
	Z=0.0	STA 35
	A1=(1.0-M*H1-M**2)*Y**2/2.0+1.0*ETA*Y	STA 36
	A1P=(1.0-M*H1-M**2)*Y+2.0*ETA	STA 37
	A0=K0+K1*Y+0.5*K2*Y**2+K3*Y*Y*Y/3.0	STA 38
20	Z=Z+0.025	STA 39
	U1=A1+H0+B1*Z	STA 40
	V1=A0+7.0*A1P	STA 41
	UP=U1*EPS**2	STA 42
	VP=-M*Y*EPS*(M*EPS)**2*(Y**2-1.0)*COS(BR)/SIN(BR)+V1*EPS**3.0	STA 43
	UY=(1.0+UP)*AS	STA 44
	VY=VP*AS	STA 45
	VEL=SQRT(VY**2+UY**2)	STA 46
	TEM=TU-VFL**2*(G-1.0)/(2.0*G*GU*R)	STA 47
	A=SQRT(G*GU*R*TEM)	STA 48
	EM=VEL/A	STA 49
	IF (EM.LE.E1) GO TO 20	STA 50
	Z0=Z	STA 51
	N=0	STA 52
	N1=N2+1	STA 53
		STA 54

	N3=N2-1	STA 55
	DELY=N3	STA 56
	Y=(DELY+2.)/DELY	STA 57
	DO 90 I=1,N2	STA 58
	N=N+1	STA 59
	N3=N1-1	STA 60
	Y=(Y+DELY-2.)/DELY	STA 61
	IF (Y) 40,40,30	STA 62
30	Z=ZU*(1.0-Y)	STA 63
	GO TO 50	STA 64
C		STA 65
40	Z=ZU*(1.0+Y)	STA 66
50	A1=(1.0-M*RI-M**2)*Y**2/2.0+2.0*ETA*Y	STA 67
	A1P=(1.0-M*B1-M**2)*Y+2.0*ETA	STA 68
	AU=K0+K1*Y+0.5*K2*Y**2+K3*Y*Y*Y/3.0	STA 69
	U1=A1+R0+B1*Z	STA 70
	V1=A0+7*A1P	STA 71
	YP=M0*Y	STA 72
	XP=M0*EPS*Z	STA 73
	UP=U1*EPS**2	STA 74
	VP=-M0*Y*EPS+(M*EPS)**2*(Y**2-1.0)*COS(BR)/SIN(BR)+V1*EPS**3.0	STA 75
	TR=ATAN(VP/(1.0+UP))	STA 76
	TL(N3)=TR+BR	STA 77
	VY=VP*AS	STA 78
	UY=(1.0+UP)*AS	STA 79
	VL(N3)=SQRT(VY**2+UY**2)	STA 80
	XL(N3)=XP*COS(BR)-YP*SIN(BR)+X0	STA 81
	RL(N3)=D+XP*SIN(BR)+YP*COS(BR)	STA 82
	TE=TO-VL(N3)**2*(G-1.)/(2.0*GO*R*G)	STA 83
	A=SQRT(G*GO*R*TE)	STA 84
	ML(N3)=VL(N3)/A	STA 85
	IF (Y.GT.0.U) GO TO 60	STA 86
	PHIN=ATAN(1./(EPS*ZO))+BR	STA 87
	GU TO 70	STA 88
C		STA 89
60	PHIN=3.14159+ATAN(-1./(EPS*ZO))+BR	STA 90
70	CONTINUE	STA 91
	IF (1.TO.1) GO TO 80	STA 92
	T1=(TL(N3)+TL(N3+1))/2.	STA 93
	PHIA=(PHIN+PHI0)/2.	STA 94
	FE1=(ML(N3)+ML(N3+1))/2.	STA 95
	VE1=(VL(N3)+VL(N3+1))/2.	STA 96
	FUNCT=1.+(G-1.)*EE1**2/2.	STA 97
	PE1=PU/FUNCT**G/(G-1.)	STA 98
	RHOE1=RHOO/FUNCT**1./(G-1.)	STA 99
	YE1=(RL(N3)+RL(N3+1))/2.	STA 100
	DY=RL(N3)-RL(N3+1)	STA 101
	FLRT=FLRT-(RHOE1*VE1*SIN(PHIA-T1)*YE1*DY/(SIN(PHIA)))	STA 102
	CF3=CF3-(PE1*RHOE1*VE1**2*SIN(PHIA-T1)*COS(T1)/(GO*SIN(PHIA)))*YE1	STA 103
	1+UY	STA 104
80	PHI0=PHIN	STA 105
90	CONTINUE	STA 106
	FLRT=FLRT*2.*3.14159	STA 107
	IF (OPT3.EQ.0) GO TO 110	STA 108
	IF (MN.GT.0) GO TO 100	STA 109
	IF (ABS(FLRTC-FLRT).LT..001) GO TO 110	STA 110

	FLATS=FLRT	STA 111
	WRITE (6,460) FLRT	STA 112
	HO=HO*FOOT	STA 113
	WRITE (6,470) HO	STA 114
	DHO=HO*0.01	STA 115
	HO=HO+DHO	STA 116
	NN=NN+1	STA 117
	GO TO 10	STA 118
C		STA 119
100	PHDOT=(FLRT-FLATS)/DHO	STA 120
	HO=HO-DHO	STA 121
	DHO=(FLRT-FLATS)/PHDOT	STA 122
	HO=HO+0.75*DHO	STA 123
	NN=0	STA 124
	GO TO 10	STA 125
C		STA 126
110	WRITE (6,500) EPS,ZO	STA 127
120	WRITE (6,510)	STA 128
	WRITE (6,530)	STA 129
	N1=N2+1	STA 130
	DO 130 I=1,N2	STA 131
	N3=N1-1	STA 132
	TD=TL(N3)*180./3.14159	STA 133
	XD=XL(N3)*FOOT	STA 134
	RD=RL(N3)*FOOT	STA 135
	WRITE (6,520) XD,RD,ML(N3),VL(N3),TD	STA 136
130	CONTINUE	STA 137
	WRITE (6,460) FLRT	STA 138
	HO=HO*FOOT	STA 139
	WRITE (6,470) HO	STA 140
	WRITE (6,530)	STA 141
	GO TO 250	STA 142
C		STA 143
140	CONTINUE	STA 144
	IF (OPT2.EQ.1) GO TO 170	STA 145
	WRITE (6,510)	STA 146
	WRITE (6,530)	STA 147
	N1=N2+1	STA 148
	N=0	STA 149
	DO 160 I=1,N2	STA 150
	N3=N1-1	STA 151
	N=N+1	STA 152
	IF (I.EQ.1) GO TO 150	STA 153
	T1=(TL(N3)+TL(N3+1))/2.	STA 154
	PHIA=ATAN((RL(N3+1)-RL(N3))/(XL(N3+1)-XL(N3)))	STA 155
	EE1=(ML(N3)+ML(N3+1))/2.	STA 156
	VE1=(VL(N3)+VL(N3+1))/2.	STA 157
	FUNCT=1.+(G-1.)*EE1**2/2.	STA 158
	PE1=PO/FUNCT**((G/(G-1.))	STA 159
	RHOE1=RHO0/FUNCT**((1./(G-1.))	STA 160
	YE1=(RL(N3)+RL(N3+1))/2.	STA 161
	DY=RL(N3)-RL(N3+1)	STA 162
	FLRT=FLRT-(RHOE1*VE1*SIN(PHIA-T1)*YE1*DY/(SIN(P4IA)))	STA 163
	CF3=CF3-(PE1*RHOE1*VE1**2*SIN(PHIA-T1)*COS(T1)/(GO*SIN(PHIA)))*YE1	STA 164
	1*DY	STA 165
150	TD=TL(N3)/RAD	STA 166

	X0=XL(N3)*FOOT	STA 167
	RO=RL(N3)*FOOT	STA 168
	WRITE (6,520) X0,RO,HL(N3),VL(N3),TO	STA 169
160	CONTINUE	STA 170
	FLRT=FLRT*2.*3.14159	STA 171
	WRITE (6,530)	STA 172
	GO TO 250	STA 173
C		STA 174
170	CONTINUE	STA 175
180	T1=BD*RAD	STA 176
	X2=0.0	STA 177
	CF3=0.0	STA 178
	N=0	STA 179
	FLRT=0.0	STA 180
	Y2=RE	STA 181
	X1=X2+2.*HO*SIN(T1)	STA 182
	Y1=Y2-2.*HO*COS(T1)	STA 183
	IF (T1.EQ.0.0) GO TO 190	STA 184
	B1=(Y2-Y1)/(X2-X1)	STA 185
	B2=Y1-(Y2-Y1)*X1/(X2-X1)	STA 186
	XN=N2-1	STA 187
	DX=(X2-X1)/XN	STA 188
	X=X1-DX	STA 189
	DTHEIA=THETAC/XN	STA 190
	T1=T1-DTHETA-THETAC/2.0	STA 191
	GO TO 200	STA 192
C		STA 193
190	DX=0.0	STA 194
	XN=N2-1	STA 195
	DY=(Y2-Y1)/XN	STA 196
	Y=Y1-DY	STA 197
	DTHEIA=THETAC/XN	STA 198
	T1=T1-DTHETA-THETAC/2.0	STA 199
200	PHIA=90.0*PI/180.0	STA 200
	FUNCT=1.+(G-1.)*E1**2/2.	STA 201
	TE=T0/FUNCT	STA 202
	A1=SQRT(G*GO*R*TE)	STA 203
	VE1=E1*A1	STA 204
	PE1=PU/FUNCT**((G/(G-1.)))	STA 205
	RHUE1=RHO0/FUNCT**((1./(G-1.)))	STA 206
	DO 230 I=1,N2	STA 207
	N=N+1	STA 208
	IF (T1.EQ.0.0) GO TO 210	STA 209
	X=X+DX	STA 210
	XL(I)=X	STA 211
	RL(I)=B1*X+B2	STA 212
	GO TO 220	STA 213
C		STA 214
210	XL(I)=X2	STA 215
	Y=Y+DY	STA 216
	RL(I)=Y	STA 217
220	PL(I)=E1	STA 218
	VL(I)=VE1	STA 219
	T1=T1+DTHETA	STA 220
	TL(I)=T1	STA 221
	IF (I.EQ.1) GO TO 230	STA 222

	N3=I-1	STA 223
	YE1=(RL(I)+RL(N3))/2.	STA 224
	DY=RL(I)-RL(N3)	STA 225
	FLRT=FLRT+(RHOE1*VE1*SIN(PHIA-T1)*YE1*DY/(SIN(PHIA)))	STA 226
	CF3=CF3+(PE1+RHOE1*VE1**2*SIN(PHIA-T1)*COS(T1)/(GO*SIN(PHIA)))*YE1	STA 227
	I*DY	STA 228
230	CONTINUE	STA 229
	FLRT=FLRT+2.*3.14159	STA 230
	IF (OPT3.EQ.0) GO TO 120	STA 231
	IF (MN.GT.0) GO TO 240	STA 232
	IF (ABS(FLATC-FLRT).LT..001) GO TO 120	STA 233
	FLRTS=FLRT	STA 234
	DHO=HO+0.01	STA 235
	HO=HO+DHO	STA 236
	MN=MN+1	STA 237
	GO TO 180	STA 238
C		STA 239
240	PHDOT=(FLRT-FLRTS)/DHO	STA 240
	HO=HO-DHO	STA 241
	DHO=(FLATC-FLRTS)/PHDOT	STA 242
	HO=HO+DHO	STA 243
	MN=0	STA 244
	GO TO 180	STA 245
C		STA 246
C	CALCULATE THE PRANDTL-MEYER EXPANSION AT E	STA 247
C		STA 248
250	DM=DELTPH	STA 249
260	NSTOP=0	STA 250
	J=N	STA 251
	J=J+1	STA 252
	E1=ML(N)	STA 253
	T1=TL(N)	STA 254
	ALFA=ATAN(1.0/SQRT(E1**2-1.0))	STA 255
	TCHK5=T1-ALFA+0.00044	STA 256
	SN2=E1**2/(1.+(G-1.)*E1**2/2.)	STA 257
	SN4=SN2*(1.-G**2)/2.+G	STA 258
	SNA=SN4/SQRT(1.0-SN4**2)	STA 259
	SN6=G-2./SN2	STA 260
	SNE=SN6/SQRT(1.0-SN6**2)	STA 261
	PH1=-SQRT((G+1.0)/(G-1.0))*(ATAN(SNA)-3.14159/2.0)/2.0-(ATAN(SNB)+	STA 262
	13.14159/2.0)/2.0	STA 263
	WRITE (6,540)	STA 264
	WRITE (6,510)	STA 265
	WRITE (6,530)	STA 266
270	E1=E1+DM	STA 267
280	SN2=E1**2/(1.+(G-1.)*E1**2/2.)	STA 268
	SN4=SN2*(1.-G**2)/2.+G	STA 269
	SNA=SN4/SQRT(1.0-SN4**2)	STA 270
	SN6=G-2./SN2	STA 271
	SNE=SN6/SQRT(1.0-SN6**2)	STA 272
	PH2=-SQRT((G+1.0)/(G-1.0))*(ATAN(SNA)-3.14159/2.0)/2.0-(ATAN(SNB)+	STA 273
	13.14159/2.0)/2.0	STA 274
	DT=PH2-PH1	STA 275
	T1=T1+DT	STA 276
	XL(J)=XL(N)	STA 277
	RL(J)=RL(N)	STA 278

	ML(J)=E1	STA 279
	TL(J)=T1	STA 280
	FUNCT=1.0*(G-1.0)*E1**2/2.0	STA 281
	PRES=PO/FUNCT**((G/(G-1.0))	STA 282
	TEA=TO/FUNCT	STA 283
	SPA=SQRT(G*GO*P*TEA)	STA 284
	VL(J)=ML(J)*SPA	STA 285
	PNEW=PHES-PND	STA 286
	IF (NSTOP.EQ.1) GO TO 340	STA 287
	ALFA=ATAN(1.0/SQRT(E1**2-1.0))	STA 288
	TCHK=T1-ALFA+.00044	STA 289
	DTCPM=(TCHK-TCHKS)/DM	STA 290
	IF (TCHK.GT.0.0) GO TO 310	STA 291
	TCHKS=TCHK	STA 292
	IF (PNEW) 330,340,290	STA 293
290	CONTINUE	STA 294
	TD=T1*180./3.14159	STA 295
	XD=XL(J)*FOOT	STA 296
	RD=RL(J)*FOOT	STA 297
	WRITE (6,520) XD,RD,ML(J),VL(J),TD	STA 298
	J=J+1	STA 299
	IF (J.GT.75) GO TO 300	STA 300
	PM1=PM2	STA 301
	GU TO 270	STA 302
C		STA 303
300	DM=DM+.01	STA 304
	GU TO 260	STA 305
C		STA 306
310	CONTINUE	STA 307
	TCHK=TCHKS	STA 308
	NSTOP=1	STA 309
	T1=T1-DT	STA 310
	E1=E1-DM	STA 311
320	DM=-TCHK/DTCPM	STA 312
	DM=DM*0.65	STA 313
	E1=E1+DM	STA 314
	SN2=E1**2/(1.+(G-1.0)*E1**2/2.0)	STA 315
	SN4=SN2*(1.-G**2)/2.+G	STA 316
	SNA=SN4/SQRT(1.0-SN4**2)	STA 317
	SN6=G-2./SN2	STA 318
	SNB=SN6/SQRT(1.0-SN6**2)	STA 319
	PM2=-SQRT((G+1.0)/(G-1.0))*(ATAN(SNA)-3.14159/2.0)/2.0-(ATAN(SNB)+	STA 320
	3.14159/2.0)/2.0	STA 321
	DT=PM2-PM1	STA 322
	T1=T1+DT	STA 323
	ALFA=ATAN(1.0/SQRT(E1**2-1.0))	STA 324
	TCHK=T1-ALFA+.00044	STA 325
	IF (TCHK.GT.-0.00087.AND.TCHK.LT.0.0) GO TO 280	STA 326
	DTCPM=(TCHK-TCHKS)/DM	STA 327
	TCHKS=TCHK	STA 328
	PM1=PM2	STA 329
	GU TO 320	STA 330
C		STA 331
330	CONTINUE	STA 332
	NSTOP=1	STA 333
	T1=T1-DT	STA 334

	E1=E1-DH	STA 335
	E1=SQRT(2.0*((PO/PNO)**((G-1.0)/G)-1.0)/(G-1.0))	STA 336
	GO TO 280	STA 337
C		STA 338
340	CONTINUE	STA 339
	TU=T1*180./3.14159	STA 340
	XD=XL(J)*FOOT	STA 341
	RD=RL(J)*FOOT	STA 342
	WRITE (6,520) XD,RD,ML(J),VL(J),TD	STA 343
	PFINAL=PO/(1.0+(G-1.0)*E1**2/2.0)**((G/(G-1.0))	STA 344
	PFINAL=PFINAL/144.0	STA 345
	WRITE (6,480) PFINAL	STA 346
C		STA 347
C	WRITE OUT THE FIRST GUESS OF THE OPTIMUM SURFACE	STA 348
C		STA 349
	IF (NREAD1.EQ.0) GO TO 440	STA 350
	THOLD=TL(1)/RAD	STA 351
	XHOLD=XL(1)	STA 352
	RHOLD=RL(1)	STA 353
	TN=(90.+THOLD)*RAD	STA 354
	XC=XHOLD-RHOLD*COS(TN)	STA 355
	YC=RHOLD-RHOLD*SIN(TN)	STA 356
	TS(1)=THOLD	STA 357
	XS(1)=XHOLD	STA 358
	RS(1)=RHOLD	STA 359
	DO 350 I=2,NCNT	STA 360
	TS(I)=TS(I-1)-DELTAT	STA 361
	TN=(90.+TS(I))*RAD	STA 362
	XS(I)=RHOLD*COS(TN)+XC	STA 363
	RS(I)=YC+RHOLD*SIN(TN)	STA 364
	IN=I	STA 365
350	CONTINUE	STA 366
	X1=XS(IN)	STA 367
	Y1=RS(IN)	STA 368
	T1=TS(IN)*RAD	STA 369
	X2=XLENG+XL(1)	STA 370
	Y2=YLAST	STA 371
	T2=TLAST+RAD	STA 372
	IF (T1.GT.T2) GO TO 360	STA 373
	E1=-1.	STA 374
	E3=0.0	STA 375
	E4=0.0	STA 376
	E2=-1.	STA 377
	A9(1,1)=X1**2	STA 378
	A9(1,2)=Y1**2	STA 379
	A9(1,3)=X1	STA 380
	A9(1,4)=Y1	STA 381
	A9(2,1)=X2**2	STA 382
	A9(2,2)=Y2**2	STA 383
	A9(2,3)=X2	STA 384
	A9(2,4)=Y2	STA 385
	A9(3,1)=2.0*X1	STA 386
	A9(3,2)=2.0*Y1*TAN(T1)	STA 387
	A9(3,3)=1.0	STA 388
	A9(3,4)=TAN(T1)	STA 389
	A9(4,1)=2.0*X2	STA 390

	23,4)*A9(2,1)*A9(1,2))	STA 447
	ADET3=(A9(1,1)*A9(2,2)*A9(4,3)+A9(2,1)*A9(4,2)*A9(1,3)+A9(4,1)*A9(1,2,3)*A9(1,2))-(A9(1,3)*A9(2,2)*A9(4,1)+A9(2,3)*A9(4,2)*A9(1,1)+A9(1,1)*A9(2,4,3)*A9(2,1)*A9(1,2))	STA 448
	ADET4=(A9(1,1)*A9(2,2)*A9(4,4)+A9(2,1)*A9(4,2)*A9(1,4)+A9(4,1)*A9(1,2,4)*A9(1,2))-(A9(1,4)*A9(2,2)*A9(4,1)+A9(2,4)*A9(4,2)*A9(1,1)+A9(1,1)*A9(2,4,4)*A9(2,1)*A9(1,2))	STA 449
	ADETS=A9(1,1)*A9(2,2)-A9(1,2)*A9(2,1)	STA 450
	ADET=(ADET1+ADET4)-(ADET2+ADET3)	STA 451
	ADET=ADET/ADETS	STA 452
	A9(1,5)=ADET/ADETS	STA 453
	A9(1,1)=A15	STA 454
	A9(2,1)=A25	STA 455
	A9(3,1)=A35	STA 456
	A9(4,1)=A45	STA 457
380	CONTINUE	STA 458
	IF (T1.GT.T2) GO TO 390	STA 459
	M3=(Y2-Y1)/(X2-X1)	STA 460
	C3=Y1-M3*X1	STA 461
	KK=-A9(1,5)*A9(2,5)/(A9(2,5)*M3**2+A9(1,5))	STA 462
	A=A9(1,5)+KK*M3**2	STA 463
	B=-2.*KK*M3	STA 464
	C=A9(2,5)+KK	STA 465
	D=A9(3,5)+2.*KK*C3*M3	STA 466
	E=A9(4,5)-2.*KK*C3	STA 467
	F=1.+KK*C3**2	STA 468
	GU TO 400	STA 469
C		STA 470
390	A=A9(1,5)	STA 471
	B=A9(2,5)	STA 472
	C=A9(3,5)	STA 473
	D=A9(4,5)	STA 474
400	CONTINUE	STA 475
	CNPTS=NPTSS-NCNT	STA 476
	DX=(X2-X1)/CNPTS	STA 477
	X=X1	STA 478
	NNN=NCNT+1	STA 479
	DO 430 I=NNN,NPTSS	STA 480
	X=X+DX	STA 481
	V=B*X+E	STA 482
	W=A*X**2+D*X+F	STA 483
	IF (T1.GT.T2) GO TO 410	STA 484
	RS(1)=(-V-SQRT(V**2-4.*C*W))/(2.*C)	STA 485
	TS(1)=ATAN((-2.*A*X-B*RS(1)-D)/(B*X+2.*C*RS(1)+E))/RAD	STA 486
	GO TO 420	STA 487
C		STA 488
410	RS(1)=A*X**3.+B*X**2+C*X+D	STA 489
	TS(1)=ATAN(3.*A*X**2+2.*B*X+C)/RAD	STA 490
420	XS(1)=X	STA 491
430	CONTINUE	STA 492
440	XS(1)=XL(1)	STA 493
	RS(1)=RL(1)	STA 494
	TS(1)=TL(1)/RAD	STA 495
	WRITE (6,550)	STA 496
	WRITE (6,530)	STA 497
	WRITE (6,560)	STA 498
		STA 499
		STA 500
		STA 501
		STA 502

	WRITE (6,530)	STA 503
	CO 450 1-1,NPTSS	STA 504
	XO=XS(1)*FCOT	STA 505
	MO=MS(1)*FCOT	STA 506
	WRITE (6,570) XO,MO,TS(1)	STA 507
450	CONTINUE	STA 508
	RETURN	STA 509
C		STA 510
C		STA 511
C		STA 512
460	FORMAT (1H0,40X,17HMASS FLOW RATE = ,F15.5,0H LBM/SEC)	STA 513
470	FORMAT (1H0,40X,5HIN = ,F15.5,7H INCHES)	STA 514
480	FORMAT (1H0,42X,9HFINAL = ,F10.5,5H PSIA)	STA 515
490	FORMAT (1H1,40X,23HDATA FOR THE START LINE///)	STA 516
500	FORMAT (44X,7H LPS = ,F6.3,5X,6H ZI = ,F7.5)	STA 517
510	FORMAT (1H0,54X,4HMACH,22X,4HFLOW/35X,2HXC,0X,24HC,5X,6HNUMBER,7X,STA 518	
	1HVELOCITY,7X,5HANGLE/34X,4H(IN),6X,4H(IN),10X,5H(FPS),7X,9HDEGREESTA 519	
	2F5.1)	STA 520
520	FORMAT (31X,10.5,F15.5,F12.5)	STA 521
530	FORMAT (1H0)	STA 522
540	FORMAT (1H1,42X,37HDATA FOR PRANDTL-MEYER EXPANSION AT E)	STA 523
550	FORMAT (1H1,42X,35H1ST GUESS FOR THE OPTIMUM SURFACE)	STA 524
560	FORMAT (1H0,40X,2HXS,0X,2HXS,9X,5HTheta/45X,4H(IN),0X,4H(IN),6X,5HSTA 525	
	1(DEGREE S))	STA 526
570	FORMAT (42X,20.5,F15.5)	STA 527
	END	STA 528-

SUBFTC EXT

	SUBROUTINE EXTEND (NTOT,NDEGR1,NPTSN,ITOT,NWRITE,NSAVE,XFINAL,ITEREXT	1
	1)	EXT 2
	REAL ML,MA,MB	EXT 3
	COMMON /B3/ PA,PO,TO,RHOO,G,R,GO	EXT 4
	COMMON /B2/ XL,RL,ML,VL,TL	EXT 5
	COMMON /B5/ REG	EXT 6
	COMMON /B6/ NPOINT	EXT 7
	COMMON /B9/ RAD,FOOT	EXT 8
	COMMON /B10/ XA,YA,MA,VA,TA	EXT 9
	COMMON /B11/ CF1,CF2,CF3,CFI,FLRT,THRUST,FLRTC	EXT 10
	COMMON /B13/ XS,RS,TS	EXT 11
	DIMENSION REG(1275,7), XL(75), RL(75), ML(75), VL(75), TL(75), NPOEXT	EXT 12
	1INT(51), XS(125), RS(125), TS(125), XA(75), YA(75), MA(75), VA(75)EXT	EXT 13
	2, TA(75), XB(100), YB(100), MB(100), VB(100), TS(100)	EXT 14
	IF (NWRITE.NE.0) WRITE (6,70)	EXT 15
	IF (NWRITE.NE.0) WRITE (6,80)	EXT 16
	DO 10 I=1,NTOT	EXT 17
	XA(I)=XL(I)	EXT 18
	YA(I)=RL(I)	EXT 19
	MA(I)=ML(I)	EXT 20
10	VA(I)=VL(I)	EXT 21
	TA(I)=TL(I)	EXT 22
	ITOT=NTOT	EXT 23
	CF2=0.0	EXT 24
	CALL SKIN (VA(1),MA(1),CF1,TAU,TAURD,TAUR,TAURH),TAUVD,TAUPD,RTAUDEXT	EXT 25
	1X)	EXT 26
	TAU1=TAU	EXT 27
	IF (NPTSN.EQ.0) GO TO 40	EXT 28
	NJOIN=NPTSN+1	EXT 29
	DO 30 J=2,NJOIN	EXT 30
	XB(J)=XS(J)	EXT 31
	YB(J)=RS(J)	EXT 32
	TB(J)=TS(J)*RAD	EXT 33
	CALL LOCAT (ITOT,XB(1),YB(1),TB(1),NDEGR1,V1,E1,X3A,Y3A,E3A,V3A,TEXT	EXT 34
	13A,NSTRT,J)	EXT 35
	MB(1)=E1A	EXT 36
	VB(1)=V1	EXT 37
	KOB=0	EXT 38
	IOB=1	EXT 39
	JOB=J-1	EXT 40
	TD=TB(1)/RAD	EXT 41
	XD=XB(1)*FOOT	EXT 42
	RD=YB(1)*FOOT	EXT 43
	IF (NWRITE.NE.0) WRITE (6,90) IOB,JOB,XD,RD,MB(1),VB(1),TD,KOB	EXT 44
	EE1=(MA(1)+MB(1))/2.	EXT 45
	VE1=(YA(1)+YB(1))/2.	EXT 46
	CV1=YB(1)-YA(1)	EXT 47
	CALL SKIN (VB(1),MB(1),CF1,TAU,TAURD,TAUR,TAURH),TAUVD,TAUPD,RAUDXEXT	EXT 48
	1)	EXT 49
	TAUE1=(TAU1+TAU)/2.	EXT 50
	DYDX=(TAN(TA(1))+TAN(TB(1)))/2.	EXT 51
	PE1=PO/(1.+(G-1.)*EE1**2/2.)**(G/(G-1.))	EXT 52
	DX=XB(1)-XA(1)	EXT 53
	CF2=CF2-(PE1*DYDX+TAUE1)*VE1*DX	EXT 54

TAU1=TAU	EXT 55
L=0	EXT 56
NSTOP=ITOT	EXT 57
DO 20 K=NSTRT,NSTOP	EXT 58
L=L+1	EXT 59
CALL CHAR1 (XA(K),XB(L),YA(K),YB(L),VA(K),VB(L),TA(K),TB(L),MA(K),	EXT 60
MB(L),TD,G,GO,R,X3A,Y3A,V3A,T3A,E3A,I)	EXT 61
XB(L+1)=X3A	EXT 62
YB(L+1)=Y3A	EXT 63
MB(L+1)=E3A	EXT 64
VB(L+1)=V3A	EXT 65
TD=T3A/RAD	EXT 66
IOB=IOB+1	EXT 67
XD=X3A*FOOT	EXT 68
RD=Y3A*FOOT	EXT 69
IF (NWRITE.NE.0) WRITE (6,90) IOB,JOB,XD,RD,E3A,V3A,TD,I	EXT 70
TB(L+1)=T3A	EXT 71
XA(L)=XB(L)	EXT 72
YA(L)=YB(L)	EXT 73
MA(L)=MB(L)	EXT 74
VA(L)=VB(L)	EXT 75
TA(L)=TB(L)	EXT 76
20 ITOT=L+1	EXT 77
XA(ITOT)=XB(ITOT)	EXT 78
YA(ITOT)=YB(ITOT)	EXT 79
IF (NWRITE.NE.0) WRITE (6,100)	EXT 80
MA(ITOT)=MB(ITOT)	EXT 81
VA(ITOT)=VB(ITOT)	EXT 82
30 TA(ITOT)=TB(ITOT)	EXT 83
40 CONTINUE	EXT 84
DO 50 I=1,NTOT	EXT 85
ISV=1	EXT 86
XCK=XFINAL-XA(I)	EXT 87
IF (XCK) 60,60,50	EXT 88
50 CONTINUE	EXT 89
60 NSUB=ISV+2	EXT 90
NSAVE=ISV+12-ITER	EXT 91
IF (NSAVE.LT.NSUB) NSAVE=NSUB	EXT 92
IF (NSAVE.GT.ITOT) NSAVE=ITOT	EXT 93
RETURN	EXT 94
C	EXT 95
C	EXT 96
C	EXT 97
70 FORMAT (1H1,39X,32HFLOW FIELD EXTENSION FROM A TO T)	EXT 98
80 FORMAT (1H0,54X,4HMACH,22X,4HFLOW/25X,1H1,4X,1HJ,4X,2HXC,8X,2HRC,6EXT	EXT 99
1X,6HNUMBER,6X,8HVELOCITY,7X,5HANGLE,6X,4HITER/34X,4H(IN),6X,4H(IN)EXT	EXT 100
2,18X,5H(FPS),7X,9H(DEGREES))	EXT 101
90 FORMAT (21X,2I5,3F10.5,F15.5,F12.5,I7)	EXT 102
100 FORMAT (1H0)	EXT 103
END	EXT 104-

SUBROUTINE LOCAT (ITOT,X1,Y1,T1,ND,V1,E1A,X3A,Y3A,E3A,V3A,T3A,NSTH		LUC 1
IT,J)		LUC 2
REAL M4		LUC 3
COMMON /R13/ XS,MS,TS		LUC 4
COMMON /R10/ XA,YA,MA,VA,TA		LUC 5
COMMON /R3/ PA,PU,TO,RHOC,C,K,GO		LUC 6
DIMENSION XA(75), YA(75), MA(75), VA(75), TA(75), XS(125), MS(125)		LUC 7
1), TS(125)		LUC 8
KP=J-1		LUC 9
I=0		LUC 10
X3A=(X1+XS(KP))/2.0		LUC 11
GO TO 20		LUC 12
C		LUC 13
10	ADIV=1	LUC 14
	X3A=XS(KP)+(X1-XS(KP))/ADIV	LUC 15
20	I=I+1	LUC 16
	CALL AITKEN (XA,YA,ITOT,ND,X3A,YB)	LUC 17
	Y3A=YB	LUC 18
	CALL AITKEN (XA,MA,ITOT,ND,X3A,YB)	LUC 19
	F3A=YB	LUC 20
	CALL AITKEN (XA,VA,ITOT,ND,X3A,YB)	LUC 21
	V3A=YB	LUC 22
	CALL AITKEN (XA,TA,ITOT,ND,X3A,YB)	LUC 23
	T3A=YB	LUC 24
	A3A=ATAN(1./SQRT(E3A**2-1.))	LUC 25
	Y13=(Y1+Y3A)/2.	LUC 26
	T13=(T1+T3A)/2.	LUC 27
	IF (I.NE.1) GO TO 30	LUC 28
	G3=COS(A3A)/(SIN(A3A)*V3A)	LUC 29
	G3=SIN(T3A)*SIN(A A)/SIN(T3A-A3A)	LUC 30
	A13=A3A	LUC 31
	C13=G3	LUC 32
	G13=G3	LUC 33
	GO TO 40	LUC 34
C		LUC 35
30	A13=(A1+A3A)/2.	LUC 36
	V13=(V1+V3A)/2.	LUC 37
	C13=COS(A13)/(SIN(A13)*V13)	LUC 38
	G13=SIN(T13)*SIN(A13)/SIN(T13-A13)	LUC 39
40	C=TAN(T13-A13)	LUC 40
	E=Y1-C*X1	LUC 41
	KP=1	LUC 42
	KX=2	LUC 43
	IF (X3A.GT.XA(KX)) GO TO 70	LUC 44
	IF (I.GE.50) GO TO 140	LUC 45
50	XX1=XA(KP)	LUC 46
	YY1=YA(KP)	LUC 47
	XX2=X3A	LUC 48
	YY2=Y3A	LUC 49
	KY=KP+1	LUC 50
	IF (XX1.EQ.XX2) GO TO 60	LUC 51
	GO TO 40	LUC 52
C		LUC 53
60	XX1=XA(KY)	LUC 54

	YY1=YA(KY)	LOC 55
	GO TO 90	LOC 56
C		LOC 57
70	ETJ 80 KQ=3, ITOT	LOC 58
	KX=KQ	LOC 59
	KP=KX-1	LOC 60
	IF (X3A.LE.XA(KQ)) GO TO 90	LOC 61
80	CONTINUE	LOC 62
	KP=1	LOC 63
	GO TO 90	LOC 64
C		LOC 65
90	A=(YY2-YY1)/(XX2-XX1)	LOC 66
	P=YY2-A*XX2	LOC 67
	X3A=(D-B)/(A-C)	LOC 68
	IF (X3A.LE.XS(1)) GO TO 10	LOC 69
	V1=Y3A*(T3A-T1-G13*(Y3A-Y1)/Y13)/Q13	LOC 70
	TE1=TU-V1**2*(G-1.)/(2.*C*GO*R)	LOC 71
	SPI=SQR(G*GO*R*TE1)	LOC 72
	E1A=V1/SPI	LOC 73
	A1=ATAN(1./SQR(E1A**2-1.))	LOC 74
	IF (1.EQ.1) GO TO 100	LOC 75
	X3P=(X3A-X3)/X3	LOC 76
	Y3P=(Y3A-Y3)/Y3	LOC 77
	E1P=(E1A-E1)/E1	LOC 78
	T3P=T3A-T3	LOC 79
	IF (ABS(X3P).LT..0001.AND.ABS(Y3P).LT..0001.AND.ABS(E1P).LT..0001.	LOC 80
100	AND.ABS(T3P).LT..0001) GO TO 110	LOC 81
	X3=X3A	LOC 82
	Y3=Y3A	LOC 83
	E1=E1A	LOC 84
	T3=T3A	LOC 85
	GO TO 20	LOC 86
C		LOC 87
110	CONTINUE	LOC 88
	DO 120 I=1, ITOT	LOC 89
	NSIRI=1	LOC 90
	XCK=X3A-XA(1)	LOC 91
	IF (XCK) 130, 130, 120	LOC 92
120	CONTINUE	LOC 93
130	CONTINUE	LOC 94
	RETURN	LOC 95
140	WRITE(6, 150)	LOC 96
	STOP	LOC 97
150	FORMAT (1H0, 28H TOO MANY ITERATIONS IN LUCAT)	LOC 98
	END	LOC 99-

SIBFTC FLOW1

SUBROUTINE FLOW1 (N,J,NTOT)

REAL ML

COMMON /B2/ XL,RL,ML,VL,TL

COMMON /B3/ PA,PO,TO,RHOO,G,R,GO

COMMON /B5/ REG

COMMON /B8/ NWRITE

COMMON /B9/ RAD,FOOT

DIMENSION REG(1275,7), XL(75), RL(75), ML(75), VL(75), TL(75)

IF (NWRITE.EQ.0) GO TO 10

WRITE (6,150)

WRITE (6,160)

WRITE (6,170)

10

NJUMP=0

REG(1,1)=XL(1)

REG(1,2)=RL(1)

REG(1,3)=ML(1)

REG(1,4)=VL(1)

REG(1,5)=TL(1)

KOB=0

IOB=1

JOB=1

TD=TL(1)*180.0/3.14159

XD=XL(1)*FOOT

RD=RL(1)*FOOT

IF (NWRITE.NE.0) WRITE (6,180) IOB,JOB,XD,RD,ML(1),VL(1),TD,KOB

IF (NWRITE.NE.0) WRITE (6,170)

DO 140 I1=2,J,1

IF (I1.GT.N) GO TO 20

X1=XL(I1)

X2=REG(1,1)

Y1=RL(I1)

Y2=REG(1,2)

T1=TL(I1)

T2=REG(1,5)

V1=VL(I1)

V2=REG(1,4)

E1=ML(I1)

E2=REG(1,3)

GO TO 30

C

20

X1=XL(I1)

X2=REG(2,1)

Y1=RL(I1)

Y2=REG(2,2)

E1=ML(I1)

E2=REG(2,3)

V1=VL(I1)

V2=REG(2,4)

T1=TL(I1)

T2=REG(2,5)

30

XX=REG(1,1)

RX=REG(1,2)

EX=REG(1,3)

VX=REG(1,4)

FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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FL1

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TX=REG(1,5)	FL1	55
REG(1,1)=XL(11)	FL1	56
REG(1,2)=RL(11)	FL1	57
REG(1,3)=ML(11)	FL1	58
REG(1,4)=VL(11)	FL1	59
REG(1,5)=TL(11)	FL1	60
CALL CHAR1 (X1,X2,Y1,Y2,V1,V2,T1,T2,E1,E2,TO,G,DO,R,X3A,Y3A,V3A,T3	FL1	61
IA,E3A,I)	FL1	62
CALL QUAD (X1,X2,Y1,Y2,THETA)	FL1	63
T1=THETA	FL1	64
CALL QUAD (X1,X3A,Y1,Y3A,THETA)	FL1	65
T2=THETA	FL1	66
IF (I2.GE.0.0.AND.T2.LT.3.14159.AND.T1.GT.3.*3.14159/2..AND.T1.LT.	FL1	67
16.28318) T1=T1-6.28318	FL1	68
IF (I2-T1) 90,90,40	FL1	69
40 CONTINUE	FL1	70
KOB=0	FL1	71
JOB=1	FL1	72
IOB=11-NJUMP	FL1	73
TD=TL(11)*180.0/3.14159	FL1	74
XD=X1*FOOT	FL1	75
RD=Y1*FOOT	FL1	76
IF (INWRITE.NE.0) WRITE (6,180) IOB,JOB,XD,RD,E1,V1,TD,KOB	FL1	77
TD=T3A*180./3.14159	FL1	78
JOB=2	FL1	79
XD=X3A*FOOT	FL1	80
RD=Y3A*FOOT	FL1	81
IF (INWRITE.NE.0) WRITE (6,180) IOB,JOB,XD,RD,E3A,V3A,TD,I	FL1	82
IF (I1.GT.N) GO TO 70	FL1	83
IF (I1.EQ.2) GO TO 60	FL1	84
I3=I1-1	FL1	85
DO 50 I2=2,I3	FL1	86
X2=REG(I2,1)	FL1	87
Y2=REG(I2,2)	FL1	88
E2=REG(I2,3)	FL1	89
V2=REG(I2,4)	FL1	90
T2=REG(I2,5)	FL1	91
REG(I2,1)=X3A	FL1	92
REG(I2,2)=Y3A	FL1	93
REG(I2,3)=E3A	FL1	94
REG(I2,4)=V3A	FL1	95
REG(I2,5)=T3A	FL1	96
X1=X3A	FL1	97
Y1=Y3A	FL1	98
E1=E3A	FL1	99
V1=V3A	FL1	100
T1=T3A	FL1	101
CALL CHAR1 (X1,X2,Y1,Y2,V1,V2,T1,T2,E1,E2,TO,G,DO,R,X3A,Y3A,V3A,T3	FL1	102
IA,E3A,I)	FL1	103
JOB=I2+1	FL1	104
TD=T3A*180./3.14159	FL1	105
XD=X3A*FOOT	FL1	106
RD=Y3A*FOOT	FL1	107
IF (INWRITE.NE.0) WRITE (6,180) IOB,JOB,XD,RD,E3A,V3A,TD,I	FL1	108
50 CONTINUE	FL1	109
XL(IOB)=X3A	FL1	110

	RL(108)=Y3A	FL1 111
	ML(108)=E3A	FL1 112
	VL(108)=V3A	FL1 113
	TL(108)=T3A	FL1 114
	REG(11,1)=X3A	FL1 115
	REG(11,2)=Y3A	FL1 116
	REG(11,3)=E3A	FL1 117
	REG(11,4)=V3A	FL1 118
	REG(11,5)=T3A	FL1 119
	GO TO 130	FL1 120
C		
60	XL(108)=X3A	FL1 121
	RL(108)=Y3A	FL1 122
	ML(108)=E3A	FL1 123
	VL(108)=V3A	FL1 124
	TL(108)=T3A	FL1 125
	REG(2,1)=X3A	FL1 126
	REG(2,2)=Y3A	FL1 127
	REG(2,3)=E3A	FL1 128
	REG(2,4)=V3A	FL1 129
	REG(2,5)=T3A	FL1 130
C		FL1 131
	GO TO 130	FL1 132
C		FL1 133
70	DO 80 12=3,N	FL1 134
	REG(12-1,1)=X3A	FL1 135
	REG(12-1,2)=Y3A	FL1 136
	REG(12-1,3)=E3A	FL1 137
	REG(12-1,4)=V3A	FL1 138
	REG(12-1,5)=T3A	FL1 139
	X1=X3A	FL1 140
	Y1=Y3A	FL1 141
	E1=E3A	FL1 142
	V1=V3A	FL1 143
	T1=T3A	FL1 144
	CALL CHAR1 (X1,REG(12,1),Y1,REG(12,2),V1,REG(12,4),T1,REG(12,5),E1	FL1 145
	1,REG(12,3),TO,G,GO,R,X3A,Y3A,V3A,T3A,E3A,1)	FL1 146
	TD=T3A*100./3.14159	FL1 147
	X0=X3A*FOOT	FL1 148
	RD=Y3A*FOOT	FL1 149
80	IF (NWRITE,NE,0) WRITE (6,100) 108,12,X0,RD,E3A,V3A,TD,1	FL1 150
	REG(N,1)=X3A	FL1 151
	REG(N,2)=Y3A	FL1 152
	REG(N,3)=E3A	FL1 153
	REG(N,4)=V3A	FL1 154
	REG(N,5)=T3A	FL1 155
	XL(108)=X3A	FL1 156
	RL(108)=Y3A	FL1 157
	ML(108)=E3A	FL1 158
	VL(108)=V3A	FL1 159
	TL(108)=T3A	FL1 160
	GO TO 130	FL1 161
C		FL1 162
90	IF (11.GT.N) GO TO 100	FL1 163
	GO TO 110	FL1 164
C		FL1 165
		FL1 166

100	IF (NWRITE.NE.0) WRITE (6,190)	FL1 167
	NJUMP=NJUMP+1	FL1 168
	GO TO 140	FL1 169
C		
110	IF (NWRITE.NE.0) WRITE (6,190)	FL1 170
	NJUMP=NJUMP+1	FL1 171
	JOB=1	FL1 172
	KOB=0	FL1 173
	IUB=11-NJUMP	FL1 174
	TD=TL(11)*180.0/3.14159	FL1 175
	XD=X1*FOOT	FL1 176
	RD=Y1*FOOT	FL1 177
	IF (NWRITE.NE.0) WRITE (6,180) IOB,JOB,XD,RD,E1,V1,TD,KOB	FL1 178
	X3A=REG(2,1)	FL1 179
	Y3A=REG(2,2)	FL1 180
	E3A=REG(2,3)	FL1 181
	V3A=REG(2,4)	FL1 182
	T3A=REG(2,5)	FL1 183
	REG(2,1)=XX	FL1 184
	REG(2,2)=RX	FL1 185
	REG(2,3)=EX	FL1 186
	REG(2,4)=VX	FL1 187
	REG(2,5)=TX	FL1 188
	JOB=2	FL1 189
	TD=REG(2,5)*180./3.14159	FL1 190
	XD=REG(2,1)*FOOT	FL1 191
	RD=REG(2,2)*FOOT	FL1 192
	IF (NWRITE.NE.0) WRITE (6,180) IOB,JOB,XD,RD,REG(2,3),REG(2,4),TD,	FL1 193
	IKOB	FL1 194
	DO 120 I2=3,11	FL1 195
	XX=REG(12,1)	FL1 196
	RX=REG(12,2)	FL1 197
	EX=REG(12,3)	FL1 198
	VX=REG(12,4)	FL1 199
	TX=REG(12,5)	FL1 200
	REG(12,1)=X3A	FL1 201
	REG(12,2)=Y3A	FL1 202
	REG(12,3)=E3A	FL1 203
	REG(12,4)=V3A	FL1 204
	REG(12,5)=T3A	FL1 205
	TD=REG(12,5)*180./3.14159	FL1 206
	XD=REG(12,1)*FOOT	FL1 207
	RD=REG(12,2)*FOOT	FL1 208
	IF (NWRITE.NE.0) WRITE (6,180) IOB,I2,XD,RD,REG(12,3),REG(12,4),TD,	FL1 209
	IKOB	FL1 210
	X3A=XX	FL1 211
	Y3A=RX	FL1 212
	E3A=EX	FL1 213
	V3A=VX	FL1 214
	T3A=TX	FL1 215
120	CONTINUE	FL1 216
130	IF (NWRITE.NE.0) WRITE (6,170)	FL1 217
140	CONTINUE	FL1 218
	NTOT=J-NJUMP	FL1 219
	RETURN	FL1 220
C		FL1 221
		FL1 222

C		FL1 223
C		FL1 224
150	FORMAT (1H1,37X,46HDATA FOR THE FLOW FIELD IN THE SMALL REGION R1)	FL1 225
160	FORMAT (1H0,54X,4HMACH,22X,4HFLOW/25X,1H1,4X,1HJ,4X,2HXC,8X,2HRC,7FL1 226	
	1X,6HNUMBER,5X,8HVELOCITY,7X,5HANGLE,6X,4HITER/36X,4H(IN),6X,4H(IN)FL1 227	
	2,18X,5H(FPS),7X,9H(DEGREES))	
170	FORMAT (1H0)	FL1 228
180	FORMAT (21X,215,3F10.5,F15.5,F12.5,17)	FL1 229
190	FORMAT (43X,34H RIGHT CHARACTERISTIC SKIPPED HERE)	FL1 230
	END	FL1 231
		FL1 232-

518F10.511

	SUBROUTINE SELECT (NPTS,K,KD,NS,CS,X,R,M,V,T)	SEL	1
	REAL A	SEL	2
	COMMON /NS/ REG	SEL	3
	COMMON /NS/ NPOINT	SEL	4
	DIMENSION REG(1275,7), X(75), R(75), M(75), V(75), T(75), NPOINT(5	SEL	5
	11)	SEL	6
C	DZ=X(K)-X(1)	SEL	7
	N=NPTS-1	SEL	8
	DN=N	SEL	9
	DELTAZ=DZ/DN	SEL	10
	DELTAZ1=DELTAZ/CS	SEL	11
	NQ=NS-1	SEL	12
	SN=1.0	SEL	13
	PN=NPTS-NS	SEL	14
	X5=X(1)+SN*DELTAZ1	SEL	15
	DELTAZ=(X(K)-X5)/DN	SEL	16
	Z=X(1)	SEL	17
	REG(1,1)=X(1)	SEL	18
	REG(1,2)=R(1)	SEL	19
	REG(1,3)=M(1)	SEL	20
	REG(1,4)=V(1)	SEL	21
	REG(1,5)=T(1)	SEL	22
	NPOINT(1)=0	SEL	23
	DO 10 I=2,51	SEL	24
10	NPOINT(I)=NPOINT(I-1)+1-1	SEL	25
	DO 20 I=2,NPTS	SEL	26
	IF (1.0-NS) DELTAZ1=DELTAZ	SEL	27
	Z=Z+DELTAZ1	SEL	28
	NJ=NPOINT(I)+1	SEL	29
	REG(NJ,1)=Z	SEL	30
	CALL ATIKEN (X,R,K,KD,Z,YB)	SEL	31
	REG(NJ,2)=YB	SEL	32
	CALL ATIKEN (X,M,K,KD,Z,YB)	SEL	33
	REG(NJ,3)=YB	SEL	34
	CALL ATIKEN (X,V,K,KD,Z,YB)	SEL	35
	REG(NJ,4)=YB	SEL	36
	CALL ATIKEN (X,T,K,KD,Z,YB)	SEL	37
	REG(NJ,5)=YB	SEL	38
20	CONTINUE	SEL	39
	RETURN	SEL	40
	END	SEL	41
		SEL	42-

41BFTC FLOW

SUBROUTINE FLOW (NZ,NY,NJ,NK,NCHECK,XLENG,NRECRD,XFINAL)	FLO 1
COMMON /B3/ PA,PD,TD,RHDD,G,R,GO	FLO 2
COMMON /B5/ REG	FLO 3
COMMON /B6/ NP	FLO 4
COMMON /B8/ INWRITE	FLO 5
COMMON /B9/ RAD,FOOT	FLO 6
COMMON /B13/ XS,RS,TS	FLO 7
DIMENSION REG(1275,7), NP(51), XS(125), RS(125), TS(125)	FLO 8
IF (INWRITE.EQ.0) GO TO 10	FLO 9
WRITE (6,110)	FLO 10
WRITE (6,130)	FLO 11
WRITE (6,120)	FLO 12
IOB=1	FLO 13
JOB=1	FLO 14
KOB=0	FLO 15
TD=REG(1,5)/RAD	FLO 16
XD=REG(1,1)*FOOT	FLO 17
RD=REG(1,2)*FOOT	FLO 18
WRITE (6,140) IOB,JOB,XD,RD,REG(1,3),REG(1,4),TD,KOB	FLO 19
WRITE (6,120)	FLO 20
IOB=2	FLO 21
TD=REG(2,5)/RAD	FLO 22
XD=REG(2,1)*FOOT	FLO 23
RD=REG(2,2)*FOOT	FLO 24
WRITE (6,140) IOB,JOB,XD,RD,REG(2,3),REG(2,4),TD,KOB	FLO 25
IOB=2	FLO 26
DO 70 N=NZ,NY	FLO 27
IF (NCHECK.NE.1) NRECRD=N	FLO 28
NS=N-2	FLO 29
IF (NS.LE.0) GO TO 30	FLO 30
MX=1	FLO 31
IF (NZ.NE.2) MX=MY	FLO 32
NW=NP(N)+1	FLO 33
TD=REG(NW,5)/RAD	FLO 34
IOB=1	FLO 35
XD=REG(NW,1)*FOOT	FLO 36
RD=REG(NW,2)*FOOT	FLO 37
IF (INWRITE.NE.0) WRITE (6,140) N,JOB,XD,RD,REG(NW,3),REG(NW,4),TD,	FLO 38
KOB	FLO 39
DO 20 M=MX,NS	FLO 40
N1=NP(N)+M	FLO 41
N2=NP(N-1)+M+1	FLO 42
CALL CHAR1 (REG(N1,1),REG(N2,1),REG(N1,2),REG(N2,2),REG(N1,4),REG(N2,4),	FLO 43
REG(N1,5),REG(N2,5),REG(N1,3),REG(N2,3),TD,G,GO,R,X3A,Y3A,V3A,	FLO 44
2A,T3A,E3A,1)	FLO 45
N3=NP(N)+M+1	FLO 46
REG(N3,1)=X3A	FLO 47
REG(N3,2)=Y3A	FLO 48
REG(N3,3)=E3A	FLO 49
REG(N3,4)=V3A	FLO 50
REG(N3,5)=T3A	FLO 51
IOB=M+1	FLO 52
TD=T3A*180./3.14159	FLO 53
XD=X3A*FOOT	FLO 54

	RD=Y3A*FOOT	FLO 55
	IF (NWRITE.NE.0) WRITE (6,140) N,JOB,XD,RD,E3A,V3A,TD,I	FLO 56
20	CONTINUE	FLO 57
	GO TO 40	FLO 58
C		FLO 59
30	X3A=REG(2,1)	FLO 60
	Y3A=REG(2,2)	FLO 61
	E3A=REG(2,3)	FLO 62
	V3A=REG(2,4)	FLO 63
	T3A=REG(2,5)	FLO 64
40	X1=X3A	FLO 65
	Y1=Y3A	FLO 66
	T1=T3A	FLO 67
	E1=E3A	FLO 68
	V1=V3A	FLO 69
	N4=NP(N-1)+N-1	FLO 70
	X2=REG(N4,1)	FLO 71
	Y2=REG(N4,2)	FLO 72
	T2=REG(N4,5)	FLO 73
	CALL SURF (X1,Y1,X2,Y2,T2,T1,E1,V1,NJ,NX,X3A,Y3A,V3A,T3A,E3A,I)	FLO 74
	N3=NP(N)+N	FLO 75
	REG(N3,1)=X3A	FLO 76
	REG(N3,2)=Y3A	FLO 77
	REG(N3,3)=E3A	FLO 78
	REG(N3,4)=V3A	FLO 79
	REG(N3,5)=T3A	FLO 80
	TD=Y3A*180./3.14159	FLO 81
	XD=X3A*FOOT	FLO 82
	RD=Y3A*FOOT	FLO 83
	IF (NWRITE.NE.0) WRITE (6,140) N,N,XD,RD,E3A,V3A,TD,I	FLO 84
	IF (NWRITE.NE.0) WRITE (6,120)	FLO 85
	IF (NCHECK.EQ.1) GO TO 70	FLO 86
	XCK=XLENG-(X3A-REG(1,1))	FLO 87
	IF (XCK.GT.0.0.AND.N.EQ.NY) WRITE (6,90)	FLO 88
	NYQ=NP(N)+1	FLO 89
	IF (XCK.GT.0.0.AND.N.EQ.NY) XFINAL=REG(NYQ,1)	FLO 90
	IF (XCK) 50,80,70	FLO 91
50	NE=N-1	FLO 92
	NF=NP(NE)+NE	FLO 93
	DX=XLENG-(REG(NF,1)-REG(1,1))	FLO 94
	DX1=REG(N3,1)-REG(NF,1)	FLO 95
	ZQ=XLENG+REG(1,1)	FLO 96
	REG(N3,1)=ZQ	FLO 97
	CALL AITKEN (XS,RS,NJ,NX,ZQ,YB)	FLO 98
	REG(N3,2)=YB	FLO 99
	CALL AITKEN (XS,TS,NJ,NX,ZQ,YB)	FLO 100
	REG(N3,5)=YB*3.14159/180.	FLO 101
	REG(N3,3)=REG(NF,3)+DX*(REG(N3,3)-REG(NF,3))/DX1	FLO 102
	REG(N3,4)=REG(NF,4)+DX*(REG(N3,4)-REG(NF,4))/DX1	FLO 103
	IF (NWRITE.NE.0) WRITE (6,100)	FLO 104
	IC=N	FLO 105
	N1=NP(N)+N	FLO 106
	N2=NP(NE)+NE	FLO 107
	IF (NWRITE.NE.0) WRITE (6,120)	FLO 108
	XD=REG(N3,1)*FOOT	FLO 109
	RD=REG(N3,2)*FOOT	FLO 110

	TD=REG(N3,5)/RAD	FLO 111
	IY=0	FLO 112
	IF (NWRITE.NE.0) WRITE (6,140) N,Y,XD,RD,REG(N1,3),REG(N1,4),TD,IY	FLO 113
	DO 60 IX=1,NE	FLO 114
	IC=IC-1	FLO 115
	CALL CHAR1 (REG(N1,1),REG(N2,1),REG(N1,2),REG(N2,2),REG(N1,4),REG(N2,4),REG(N1,5),REG(N2,5),REG(N1,3),REG(N2,3),TJ,G,GO,R,X3A,Y3A,V3	FLO 116
	2A,T3A,E3A,I)	FLO 117
	N3=N1-1	FLO 118
	IF (IX.EQ.NE) XFINAL=X3A	FLO 119
	REG(N3,1)=X3A	FLO 120
	REG(N3,2)=Y3A	FLO 121
	REG(N3,3)=E3A	FLO 122
	REG(N3,4)=V3A	FLO 123
	REG(N3,5)=T3A	FLO 124
	TD=T3A/RAD	FLO 125
	XD=X3A*FOOT	FLO 126
	RD=Y3A*FOOT	FLO 127
	IF (NWRITE.NE.0) WRITE (6,140) N,IC,XD,RD,REG(N3,3),REG(N3,4),TD,IY	FLO 128
	N1=N1-1	FLO 129
60	N2=N2-1	FLO 130
	GO TO 80	FLO 131
C		FLO 132
70	CONTINUE	FLO 133
80	CONTINUE	FLO 134
	RETURN	FLO 135
C		FLO 136
C		FLO 137
C		FLO 138
90	FORMAT (1H0,43X,34HTHE SELECTED UPSTREAM GEOMETRY AND/41X,36HAMBIE	FLO 139
	1NT PRESSURE PREVENT THE DESIRED/47X,26HLENGTH FROM BEING OBTAINED)	FLO 140
100	FORMAT (1H0,36X,47HTHE PREVIOUS RIGHT CHARACTERISTIC IS BEYOND THE	FLO 141
	1/40X,41HREGION R AND IS REPLACED BY THE FOLLOWING)	FLO 142
110	FORMAT (1H1,46X,28HDATA FOR THE MAIN FLOW FIELD)	FLO 143
120	FORMAT (1H0)	FLO 144
130	FORMAT (1H0,54X,4HMACH,22X,4HFLOW/25X,1H1,4X,1HJ,4X,2HXC,8X,2HRC,7	FLO 145
	1X,6HNUMBER,5X,8HVELOCITY,7X,5HANGLE,6X,4HITER/34X,4H(IN),6X,4H(IN)	FLO 146
	2,14X,5H(FPS),7X,9H(DEGREES))	FLO 147
140	FORMAT (21X,215,3F10.5,F15.5,F12.5,17)	FLO 148
	END	FLO 149
		FLO 150-

SIBFTC SKIN

SUBROUTINE SKIN (VD,ED,CFI,TAU,TAURD,TAUR,TAURHJ,TAUVD,TAUPD,RTAUDSKI	SKI	1
IX)	SKI	2
COMMON /B3/ PA,PO,TO,RHOO,G,R,GO	SKI	3
FUNCTION=1.+(G-1.)*ED**2/2.	SKI	4
RHO=RHOO/FUNCTION**((1./(G-1.))	SKI	5
CF=CFI/(1.0+.72*(G-1.0)*ED**2/2.0)**0.578	SKI	6
TAU=0.5*CF*RHO*VD**2/GO	SKI	7
TAUVD=0.0	SKI	8
TAURD=0.0	SKI	9
TAUR=0.0	SKI	10
TAURHO=0.0	SKI	11
TAUPD=0.0	SKI	12
RTAUDX=0.0	SKI	13
RETURN	SKI	14
END	SKI	15-

SIBFTC BASE1

SUBROUTINE BASE (XD, RD, VD, M1, PB, PPHIPY, PPHIPX, AL, AB)

REAL M1

COMMON /B3/ PA, PD, TO, RHOO, G, R, GO

P=PD/(1.0+(G-1.0)*M1**2/2.0)**(G/(G-1.0))

PPHIPX=0.0

PB=AA*P/M1**AB

PPHIPY=RD*PB

RETURN

END

BAS	1
BAS	2
BAS	3
BAS	4
BAS	5
BAS	6
JAS	7
BAS	8
BAS	9-

SUBROUTINE REST (X,H,HP,HR,HRD,DDX)
H=1.
HP=0.0
HR=0.0
HRD=0.0
DDX=0.0
RETURN
END

RES 1
RES 2
RES 3
RES 4
RES 5
RES 6
RES 7
RES 8-

818FTC QUAD1

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SUBROUTINE QUAD (X1,X2,Y1,Y2,THETA)
DX=X2-X1
DY=Y2-Y1
D=SQRT(DX**2+DY**2)
ST=DY/D
CT=DX/D
IF (ST.GT.0.0.AND.CT.GT.0.0) GO TO 10
IF (ST.GT.0.0.AND.CT.LT.0.0) GO TO 20
IF (ST.LT.0.0.AND.CT.LT.0.0) GO TO 20
IF (ST.LT.0.0.AND.CT.GT.0.0) GO TO 30
IF (ST.EQ.0.0.AND.CT.EQ.1.0) GO TO 40
IF (ST.EQ.0.0.AND.CT.EQ.-1.0) GO TO 50
IF (ST.EQ.1.0.AND.CT.EQ.0.0) GO TO 60
IF (ST.EQ.-1.0.AND.CT.EQ.0.0) GO TO 70
10 THETA=ATAN(DY/DX)
GO TO 80
C
20 THETA=3.14159+ATAN(DY/DX)
GO TO 80
C
30 THETA=6.28318+ATAN(DY/DX)
GO TO 80
C
40 THETA=0.0
GO TO 80
C
50 THETA=3.14159
GO TO 80
C
60 THETA=3.14159/2.0
GO TO 80
C
70 THETA=3.*3.14159/2.
80 CONTINUE
RETURN
END

```

```

QUA 1
QUA 2
QUA 3
QUA 4
QUA 5
QUA 6
QUA 7
QUA 8
QUA 9
QUA 10
QUA 11
QUA 12
QUA 13
QUA 14
QUA 15
QUA 16
QUA 17
QUA 18
QUA 19
QUA 20
QUA 21
QUA 22
QUA 23
QUA 24
QUA 25
QUA 26
QUA 27
QUA 28
QUA 29
QUA 30
QUA 31
QUA 32
QUA 33
QUA 34
QUA 35
QUA 36-

```

SIBFIC CHAR1		
	SUBROUTINE CHAR1 (X1,X2,Y1,Y2,V1,V2,T1,T2,E1,E2,TO,G,GO,R,X3A,Y3A,CHA	1
	1V3A,T3A,E3A,I)	CHA 2
C		CHA 3
	D1=1.0/SQRT(E1**2-1.0)	CHA 4
	D2=1.0/SQRT(E2**2-1.0)	CHA 5
	A1=ATAN(D1)	CHA 6
	A2=ATAN(D2)	CHA 7
	Q1=COS(A1)/(SIN(A1)*V1)	CHA 8
	Q2=COS(A2)/(SIN(A2)*V2)	CHA 9
	F2=SIN(T2)*SIN(A2)/SIN(T2+A2)	CHA 10
	G1=SIN(T1)*SIN(A1)/SIN(T1-A1)	CHA 11
	A13=A1	CHA 12
	A23=A2	CHA 13
	T13=T1	CHA 14
	T23=T2	CHA 15
	Q13=Q1	CHA 16
	Q23=Q2	CHA 17
	F23=F2	CHA 18
	G13=G1	CHA 19
	I=0	CHA 20
20	I=I+1	CHA 21
	X3A=(Y2-Y1*X1*SIN(T13-A13)/COS(T13-A13)-X2*SIN(T23+A23)/COS(T23+A23	CHA 22
	13))/SIN(T13-A13)/COS(T13-A13)-SIN(T23+A23)/COS(T23+A23))	CHA 23
	Y3A=Y1+(X3A-X1)*SIN(T13-A13)/COS(T13-A13)	CHA 24
	Y13=(Y1+Y3A)/2.0	CHA 25
	Y23=(Y2+Y3A)/2.0	CHA 26
	IF (ABS(T13-A13).LT..314159) GO TO 20	CHA 27
	IF (ABS(T23+A23).LT..314159) GO TO 30	CHA 28
	V3A=(T1-T2+Q13*V1+Q23*V2+G13*(Y3A-Y1)/Y13+F23*(Y3A-Y2)/Y23)/(Q13+Q	CHA 29
	123)	CHA 30
	T3A=T1-Q13*(V3A-V1)+G13*(Y3A-Y1)/Y13	CHA 31
	GO TO 40	CHA 32
C		CHA 33
20	G13=SIN(T13)*SIN(A13)/COS(T13-A13)	CHA 34
	V3A=(T1-T2+Q13*V1+Q23*V2+G13*(X3A-X1)/Y13+F23*(Y3A-Y2)/Y23)/(Q13+Q	CHA 35
	123)	CHA 36
	T3A=T1-Q13*(V3A-V1)+G13*(X3A-X1)/Y13	CHA 37
	GO TO 40	CHA 38
C		CHA 39
30	F23=SIN(T23)*SIN(A23)/COS(T23+A23)	CHA 40
	V3A=(T1-T2+Q13*V1+Q23*V2+G13*(Y3A-Y1)/Y13+F23*(X3A-X2)/Y23)/(Q13+Q	CHA 41
	123)	CHA 42
	T3A=T1-Q13*(V3A-V1)+G13*(Y3A-Y1)/Y13	CHA 43
40	TE3A=TO-V3A**2*(G-1.0)/(2.0*G+GO**2)	CHA 44
	SP3A=SQRT(G*GO**2+TE3A)	CHA 45
	E3A=V3A/SP3A	CHA 46
	IF (E3A.LE.1.0.AND.I.LE.10) E3A=1.09	CHA 47
	C3A=1.0/SQRT(E3A**2-1.0)	CHA 48
	A3A=ATAN(D3A)	CHA 49
	IF (I.EQ.1) GO TO 50	CHA 50
	X3P=(X3A-X3)/X3	CHA 51
	Y3P=(Y3A-Y3)/Y3	CHA 52
	E3P=(E3A-E3)/E3	CHA 53

	T3P=T3A-T3	CHA	54
	IF (ABS(K3P).LT..0001.AND.ABS(Y3P).LT..0001.AND.ABS(E3P).LT..0001.	CHA	55
	1AND.ABS(T3P).LT..0001) GO TO 60	CHA	56
50	T13=(T1+T3A)/2.0	CHA	57
	T23=(T2+T3A)/2.0	CHA	58
	A13=(A1+A3A)/2.0	CHA	59
	A23=(A2+A3A)/2.0	CHA	60
	V13=(V1+V3A)/2.0	CHA	61
	V23=(V2+V3A)/2.0	CHA	62
	Q13=COS(A13)/(SIN(A13)*V13)	CHA	63
	Q23=COS(A23)/(SIN(A23)*V23)	CHA	64
	F23=SIN(T23)*SIN(A23)/SIN(T23+A23)	CHA	65
	G13=SIN(T13)*SIN(A13)/SIN(T13-A13)	CHA	66
	X3=X3A	CHA	67
	Y3=Y3A	CHA	68
	T3=T3A	CHA	69
	E3=E3A	CHA	70
	GO TO 10	CHA	71
C		CHA	72
60	CONTINUE	CHA	73
	RETURN	CHA	74
	END	CHA	75-

SUBROUTINE SURF (X1,Y1,X2,Y2,T1,E1,V1,N,M,X3A,Y3A,V3A,T3A,E3A,ISUR		1
1)		SUR 2
	DIMENSION XS(125), RS(125), TS(125)	SUR 3
	COMMON /B3/ PA,PD,TD,RH00,G,R,GD	SUR 4
	COMMON /B13/ XS,RS,TS	SUR 5
	D1=1.0/SQRT(E1**2-1.0)	SUR 6
C		SUR 7
	A1=ATAN(D1)	SUR 8
	Q1=COS(A1)/(SIN(A1)*V1)	SUR 9
	G1=SIN(T1)*SIN(A1)/SIN(T1-A1)	SUR 10
	A13=A1	SUR 11
	G13=Q1	SUR 12
	G13=G1	SUR 13
	I=0	SUR 14
	I1=0	SUR 15
	X3A=X1	SUR 16
	T13=T1	SUR 17
	A13=A1	SUR 18
10	I=I+1	SUR 19
	C=TAN(T13-A13)	SUR 20
	D=Y1-C*X1	SUR 21
20	I1=I1+1	SUR 22
	CALL AITKEN (XS,RS,N,M,X3A,YB)	SUR 23
	Y3A=YB	SUR 24
	CALL AITKEN (XS,TS,N,M,X3A,YB)	SUR 25
	T3A=YB*3.14159/180.	SUR 26
	A=(TAN(T2)+TAN(T3A))/2.	SUR 27
	B=Y2-A*X2	SUR 28
	IF (I1.EQ.1) GO TO 50	SUR 29
	X3A=(D-B)/(A-C)	SUR 30
	IF (X3.EQ.0.0.OR.Y3.EQ.0.0) GO TO 30	SUR 31
	X3P=(X3A-X3)/X3	SUR 32
	Y3P=(Y3A-Y3)/Y3	SUR 33
	GO TO 40	SUR 34
C		SUR 35
30	X3P=X3A-X3	SUR 36
	Y3P=Y3A-Y3	SUR 37
40	CONTINUE	SUR 38
	IF (ABS(X3P).LT..0001.AND.ABS(Y3P).LT..0001) GO TO 60	SUR 39
50	X3=X3A	SUR 40
	Y3=Y3A	SUR 41
	IF (I1.GE.100) GO TO 60	SUR 42
	GO TO 20	SUR 43
C		SUR 44
60	CONTINUE	SUR 45
	T13=(T1+T3A)/2.0	SUR 46
	Y13=(Y1+Y3A)/2.0	SUR 47
	V3A=V1+(G13*(Y3A-Y1)/Y13-(T3A-T1))/Q13	SUR 48
	TE3A=TD-V3A**2*(G-1.0)/(2.0*G*GD*R)	SUR 49
	SP3A=SQRT(G*GD*R*TE3A)	SUR 50
	E3A=V3A/SP3A	SUR 51
	IF (E3A.LE.1.01.AND.1.LE.20) E3A=1.05	SUR 52
	D3A=1.0/SQRT(E3A**2-1.0)	SUR 53
	A3A=ATAN(D3A)	SUR 54

```

IF (I.EQ.1) GO TO 70
E3P=(E3A-E3)/E3
T3P=T3A-T3
IF (I1.GE.100.OR.I.GE.100) GO TO 80
IF (ABS(E3P).LT..0001.AND.ABS(T3P).LT..0001) GO TO 90
70  A13=(A1+A3A)/2.
    V13=(V1+V3A)/2.0
    Q13=COS(A13)/(SIN(A13)+V13)
    G13=SIN(T13)+SIN(A13)/SIN(T13-A13)
    X3=X3A
    Y3=Y3A
    T3=T3A
    E3=E3A
    GO TO 10
C
80  CONTINUE
    WRITE (6,100)
    STOP
C
90  CONTINUE
    RETURN
C
C
C
100 FORMAT (1H0,27H100 MANY ITERATIONS IN SURF)
    END

```

```

SUR 55
SUR 56
SUR 57
SUR 58
SUR 59
SUR 60
SUR 61
SUR 62
SUR 63
SUR 64
SUR 65
SUR 66
SUR 67
SUR 68
SUR 69
SUR 70
SUR 71
SUR 72
SUR 73
SUR 74
SUR 75
SUR 76
SUR 77
SUR 78
SUR 79
SUR 80-

```

11071C LSO

SUBROUTINE LSOARE (N,K,Y,A,B)
 DIMENSION X(N), Y(N)

C

LEAST SQUARE FIT TO A STRAGHT LINE $Y=A \cdot X + B$

C

FORM SUMS

C

SK=0.0

SY=0.0

SKY=0.0

SK2=0.0

DO 10 I=1,N

SK=SK+X(I)

SY=SY+Y(I)

SKY=SKY+X(I)*Y(I)

SK2=SK2+X(I)**2

10

CONTINUE

SKQ2=SK**2

AN=N

C

CALCULATE CONSTANTS

C

A=(AN*SKY-SY*SK)/(AN*SK2-SKQ2)

B=(SY-A*SK)/AN

RETURN

END

LSQ 1
 LSQ 2
 LSQ 3
 LSQ 4
 LSQ 5
 LSQ 6
 LSQ 7
 LSQ 8
 LSQ 9
 LSQ 10
 LSQ 11
 LSQ 12
 LSQ 13
 LSQ 14
 LSQ 15
 LSQ 16
 LSQ 17
 LSQ 18
 LSQ 19
 LSQ 20
 LSQ 21
 LSQ 22
 LSQ 23
 LSQ 24
 LSQ 25
 LSQ 26-

*1BFTC AITKEN	
SUBROUTINE AITKEN (X,Y,N,K,XB,YB)	
C	AIT 1
C AITKEN INTERPOLATION SUBROUTINE	AIT 2
C CALLING SEQUENCE...	AIT 3
C CALL AITKEN(X,Y,N,K,XB,YB)	AIT 4
C X IS A ONE DIMENSIONAL ARRAY OF INDEPENDENT	AIT 5
C VARIABLE(INCREASING OR DECREASING)	AIT 6
C Y IS A ONE DIMENSIONAL ARRAY OF DEPENDENT	AIT 7
C VARIABLE	AIT 8
C N IS NO. OF X,Y PAIRS	AIT 9
C K IS DEGREE OF INTERPOLATING POLYNOMIAL (MAX = 10)	AIT 10
C YB IS INTERPOLATED RESULT	AIT 11
C XB IS INDEP. VARIABLE ARGUMENT	AIT 12
C	AIT 13
DIMENSION X(N), Y(N), XX(11), YY(11)	AIT 14
K1=K+1	AIT 15
10 IF (X(N)-X(1)) 100,10,10	AIT 16
20 IF (XB-X(1)) 20,20,30	AIT 17
LL=0	AIT 18
GO TO 190	AIT 19
C	AIT 20
30 IF (X(N)-XB) 40,40,50	AIT 21
40 LL=N-K1	AIT 22
GO TO 190	AIT 23
C	AIT 24
50 LL=1	AIT 25
LU=N	AIT 26
60 IF (LU-LL-1) 170,170,70	AIT 27
70 LL=(LL+LU)/2	AIT 28
IF (X(LL)-XB) 80,80,90	AIT 29
80 LL=LL	AIT 30
GO TO 60	AIT 31
C	AIT 32
90 LU=LL	AIT 33
GO TO 60	AIT 34
C	AIT 35
100 IF (XB-X(1)) 110,20,20	AIT 36
110 IF (X(N)-XB) 120,40,40	AIT 37
120 LL=1	AIT 38
LU=N	AIT 39
130 IF (LU-LL-1) 170,170,140	AIT 40
140 LL=(LL+LU)/2	AIT 41
IF (X(LL)-XB) 150,150,160	AIT 42
150 LU=LL	AIT 43
GO TO 130	AIT 44
C	AIT 45
160 LL=LL	AIT 46
GO TO 130	AIT 47
C	AIT 48
170 LL=LL-(K1+1)/2	AIT 49
IF (LL) 20,190,180	AIT 50
180 IF (LL+K1-N) 190,190,40	AIT 51
190 DO 200 I=1,K1	AIT 52
LL=LL+1	AIT 53
	AIT 54

```

200  XX(I)=X(I)-XB
      YY(I)=Y(I)
      DO 210 I=1,K
      DU 210 J=1,K
210  YY(J+1)=(1./(XX(J+1)-XX(I)))*(YY(I)*XX(J+1)-YY(J+1)*XX(I))
      YB=YY(K)
      RETURN
      END
$DATA

```

```

AIT 55
AIT 56
AIT 57
AIT 58
AIT 59
AIT 60
AIT 61
AIT 62-

```

UNCLASSIFIED

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13. ABSTRACT → The techniques of the calculus of variations have been used to determine the configuration of an optimum thrust plug nozzle. The problem is formulated for a fixed thrust injection angle and cowl lip radius, and the resulting plug contour is then an optimum for a given upstream geometry. The optimum values of the injection angle and cowl lip radius are determined by a parametric study. The analysis is carried out for rotational and irrotational flows and includes boundary layer effects. A method is presented for each of the problem formulations to determine if a given contour is an optimum and a relaxation technique is used to obtain a solution to the irrotational flow problem. A computer program which makes use of the design equations for the irrotational flow problem is developed and described. This program is used to carry out a parametric study to determine the optimum cowl lip radius and injection angle when the plug length is fixed. The resulting optimum nozzle is compared to one designed by Rao's Method. The importance of determining the base pressure accurately is illustrated and an example of scramjet nozzle optimization is presented.			

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