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STRESS DISTRIBUTION AND MATERIAL CHARACTERISTICS OF  
COMPOSITE MATERIALS UNDER OBLIQUE LOADINGS

By

Juan Haener  
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July 1970

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FORT EUSTIS, VIRGINIA

CONTRACT DAAJ02-69-C-0029  
WHITTAKER CORPORATION  
RESEARCH AND DEVELOPMENT DIVISION  
SAN DIEGO, CALIFORNIA

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This program was performed under Contract DAAJ02-69-C-0029 with Whittaker Corporation, San Diego, California.

The data contained in this report are the result of research concerned with the load transfer through a composite with unidirectional fibers embedded in a matrix under consideration of oblique loading.

The report has been reviewed by this command and is considered to be technically sound; however, experimental verification has not been completed, and the information should be used accordingly. It is published for the exchange of information and the stimulation of future research.

Task 1F162204A17002  
Contract DAAJ02-69-C-0029  
USAAVLABS Technical Report 70-15  
July 1970

STRESS DISTRIBUTION AND MATERIAL CHARACTERISTICS OF  
COMPOSITE MATERIALS UNDER OBLIQUE LOADINGS

Final Report

By

Juan Haener  
and  
Ming-yuan Feng

Prepared by

Whittaker Corporation  
Research & Development Division  
San Diego, California

for

U. S. ARMY AVIATION MATERIEL LABORATORIES  
FORT EUSTIS, VIRGINIA

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## ABSTRACT

The critical loadings at failure of a unidirectional fibrous composite under any oblique loading are the most important results obtained in this work. For this purpose the von Mises energy criterion was applied to the components of the composite. Debonding failures at the interfaces were also considered.

Other results of this research include composite elastic engineering constants in any angular direction as a function of fiber density and component properties.

Diagrams are presented which exhibit the critical loading and the elastic coefficients of a series of composites as a function of the loading directions, component material constants, and geometry.

The fundamentals to this work are based on the micromechanical stress fields in the fibers and in the matrix.

## FOREWORD

This final report was prepared by Whittaker Corporation, Research and Development Division, under U. S. Army Contract DAAJ02-69-C-0029 (Task 1F162204A17002) for the U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia. Mr. A. Gustafson, Jr., was the Army Project Officer for this program.

This report covers the work accomplished during the period from 31 October 1968 through 31 October 1969.

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### LIST OF SYMBOLS

a	Radius of fiber in inches
{a}	Column vector of the coefficients relating displacement components to local coordinates of the nodal points
[A]	Coordinates matrix
$A_{x,i}$	Denoting the area of the plane of triangular prism i along the boundary $x = c$ in the basic representative element
$A_{y,j}$	Denoting the area of the plane of triangular prism j along the boundary $y = b/2$ in the basic representative element
$A_{z,k}$	Denoting the area of the plane of triangular prism k along the boundary $z = h$ in the basic representative element
$A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$	Constants
b	Half the distance between the axes of two neighboring fibers in inches; also the dimension of a basic representative element in y-direction
c	Half of the dimension in inches of a basic representative element in x-direction
[d]	Direction cosine column vector of the direction normal d at the interface
[D]	Coefficients matrix relating strain components of coefficients in the expression of displacements in psi
E	Major Young's modulus of the composite in the direction of loading p in psi
$E_{11}, E_r$	Young's modulus of the composite in the direction perpendicular to the fiber axis in psi
$E_{33}, E_z$	Young's modulus of the composite in the direction of fiber axis in psi

LIST OF SYMBOLS (Continued)

$E_f$	Young's modulus of fiber in psi
$E_m$	Young's modulus of matrix in psi
$G$	Shear modulus of the composite in the plane parallel or perpendicular to the direction of the loading $p$ in psi
$G_{31}$	Shear modulus of the composite in relation to plane normal to plane 3 in direction 1 or $z = x$ axes in psi
$G_f$	Shear modulus of fiber in psi
$G_m$	Shear modulus of matrix in psi
$h$	Height of a basic representative element in $z$ -direction in inches
$i, j, k$	Representing natural sequential integers initialized to 1
$I, J, K$	Denoting the total numbers of triangular elements along the boundaries $x = c$ , $y = b/2$ and $z = h$ respectively (in the present analysis $I = 6$ , $J = 11$ , $K = 158$ )
$m$	$= \cos\theta$
$n$	$= \sin\theta$
$[N]$	Macrostress column vector due to a generalized external loading(s)
$N_x, N_y, N_z, T_{yz}, T_{zx}, T_{xy}$	Macrostress components due to a generalized external loading(s)
$\bar{N}_x, \bar{N}_y, \bar{N}_z, \bar{T}_{yz}, \bar{T}_{zx}, \bar{T}_{xy}$	Macrostress components (fundamental loadings) actually used as the inputs of a basic representative element for six fundamental cases
$P$	External oblique loading on $x'-z'$ (or $4'-3'$ ) plane in psi

LIST OF SYMBOLS (Continued)

$(p_{cr})_c$	Critical loading calculated by von Mises'-Hencky Distortion Energy Theory applied at the triangular prisms of the constituents of the composite in psi
$(p_{cr})_{is}$	Critical loading based on the interfacial shear bonding strength between fiber and matrix in psi
$(p_{cr})_{in}$	Critical loading based on the normal bonding strength at the interface between fiber and matrix in psi
$[S]$	6 x 6 matrix of stress components of six fundamental loadings
$S_{11}, S_{21}, S_{31},$ $S_{41}, S_{51}, S_{61}$	Stress components produced in the composite due to unity transverse loading $N_x$ ( $S_{41}=S_{51}=0$ ) (Revised Fundamental Case $N_y$ )
$S_{12}, S_{22}, S_{32},$ $S_{42}, S_{52}, S_{62}$	Stress components produced in the composite due to unity transverse loading $N_y$ ( $S_{42}=S_{52}=0$ ) (Revised Fundamental Case $N_z$ )
$S_{13}, S_{23}, S_{33},$ $S_{43}, S_{53}, S_{63}$	Stress components produced in the composite due to axial loading $N_z$ ( $S_{43}=S_{53}=0$ ) (Revised Fundamental Case $N_z$ )
$S_{14}, S_{24}, S_{34},$ $S_{44}, S_{54}, S_{64}$	Stress components produced in the composite due to unity longitudinal shear $T_{yz}$ in Revised Fundamental Case $T_{yz}$ ( $S_{14}=S_{24}=S_{34}=S_{64}=0$ )
$S_{15}, S_{25}, S_{35},$ $S_{45}, S_{55}, S_{65}$	Stress components produced in the composite due to unity longitudinal shear $T_{zx}$ in Revised Fundamental Case $T_{zx}$ ( $S_{15}=S_{25}=S_{35}=S_{65}=0$ )
$S_{16}, S_{26}, S_{36},$ $S_{46}, S_{56}, S_{66}$	Stress components produced in the composite due to unity transverse shear $T_{xy}$ in Revised Fundamental Case $T_{xy}$ ( $S_{46}=S_{56}=0$ )

LIST OF SYMBOLS (Continued)

$\bar{S}_{11}, \bar{S}_{21}, \bar{S}_{31},$ $\bar{S}_{41}, \bar{S}_{51}, \bar{S}_{61}$	Stress components produced in the composite due to unity transverse normal loading $\bar{N}_x$ in Fundamental Case $N_x$ ( $\bar{S}_{41} = \bar{S}_{51} = 0$ )
$\bar{S}_{12}, \bar{S}_{22}, \bar{S}_{32},$ $\bar{S}_{42}, \bar{S}_{52}, \bar{S}_{62}$	Stress components produced in the composite due to unity transverse normal loading $\bar{N}_y$ in Fundamental Case $N_y$ ( $\bar{S}_{42} = \bar{S}_{52} = 0$ )
$\bar{S}_{13}, \bar{S}_{23}, \bar{S}_{33},$ $\bar{S}_{43}, \bar{S}_{53}, \bar{S}_{63}$	Stress components produced in the composite due to unity axial loading $\bar{N}_z$ in Fundamental Case $N_z$ ( $\bar{S}_{43} = \bar{S}_{53} = 0$ )
$\bar{S}_{14}, \bar{S}_{24}, \bar{S}_{34},$ $\bar{S}_{44}, \bar{S}_{54}, \bar{S}_{64}$	Stress components produced in the composite due to unity longitudinal shear $\bar{T}_{yz}$ in Fundamental Case $T_{yz}$ ( $\bar{S}_{14} = \bar{S}_{24} = \bar{S}_{34} = \bar{S}_{64} = 0$ )
$\bar{S}_{15}, \bar{S}_{25}, \bar{S}_{35},$ $\bar{S}_{45}, \bar{S}_{55}, \bar{S}_{65}$	Stress components produced in the composite due to unity longitudinal shear $\bar{T}_{xy}$ in Fundamental Case $T_{yz}$ ( $\bar{S}_{15} = \bar{S}_{25} = \bar{S}_{35} = \bar{S}_{65} = 0$ )
$\bar{S}_{16}, \bar{S}_{26}, \bar{S}_{36},$ $\bar{S}_{46}, \bar{S}_{56}, \bar{S}_{66}$	Stress components produced in the composite due to unity transverse shear $\bar{T}_{xy}$ in Fundamental Case $T_{xy}$ ( $\bar{S}_{16} = \bar{S}_{26} = 0$ )
[T]	Matrix of stress components
[ ] <sup>T</sup>	Transposed matrix
[T <sub>d</sub> ]	Stress column vector associated with the direction normal at the interface
{u}	Displacement column vector at nodal points
$U_d$	Denoting distortion energy at a point in the matrix due to actual combined loadings
$U_{df}$	Denoting distortion energy at failure due to a simple tension test
u, v, w	Displacement components in x, y, z-directions respectively

LIST OF SYMBOLS (Continued)

$V_f$	Percentage of volumetric content of fiber(s) in a composite
$V$	Volume of the triangular prism
$x, y, z$	Cartesian coordinates in right-handed coordinate system, with $z$ (or 3) axis parallel to fiber axis
$x', y', z'$ or $1', 2', 3'$	Cartesian coordinates in right-handed coordinate system, with $z'$ (or 3') axis parallel to the direction of loading $p$
$\alpha$	Arc tangent of the ratio between $\tau_{yz}$ and $\tau_{xz}$
$\beta$	Ratio of Poisson's ratios between fiber and matrix
$\gamma$	Ratio of Young's modulus between fiber and matrix
$\{\epsilon\}$	Strain components
$\{\epsilon_1\}$	Strain column vector defined by equation (5)
$\{\epsilon_2\}$	Strain column vector defined by equation (6)
$\eta$	Coupling factor
$\theta$	Angle between directions of fiber axis and loading $p$ in $x = z$ (or 4-3, $x'-z'$ or $1'-3'$ ) plane
$\nu$	Major Poisson's ratio of a unidirectional fiber-reinforced composite due to an oblique loading $p$
$\nu_{31}$	Poisson's ratio due to loading in the direction of fiber axis
$\nu_{13}$	Poisson's ratio due to loading in the direction perpendicular to fiber axis
$\nu_f$	Poisson's ratio of fiber
$\nu_m$	Poisson's ratio of matrix
$\{\sigma\}$	Stress column vector

LIST OF SYMBOLS (Continued)

$\sigma_x, \sigma_y, \sigma_z,$ $\tau_{yz}, \tau_{zx}, \tau_{xy}$	Microstress components in a composite due to a generalized loading $p$ in psi
$\sigma_1, \sigma_2, \sigma_3,$ $\sigma_4, \sigma_5, \sigma_6$	Microstress components in contracted notation, corresponding to $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{yz}, \tau_{zx}, \tau_{xy}$
$\sigma_x', \sigma_y', \sigma_z',$ $\tau_{xy}', \tau_{xz}', \tau_{yz}'$	Externally applied stress components (in the present analysis, $\sigma_z' = p$ , others equal to 0)
$\sigma_{x,i}$	Denoting the normal stress in x-direction of triangular prism $i$ along the boundary $x = c$ , due to the boundary conditions used in the computation program in fundamental case $N_x$
$\sigma_{y,j}$	Denoting the normal stress in y-direction of triangular prism $j$ along the boundary $y = b/2$ due to the boundary condition used in the computation program in fundamental case $N_y$
$\sigma_{z,k}$	Denoting the normal stress in z-direction of triangular prism $k$ along the boundary $z = h$ , due to the boundary conditions used in the computation program in fundamental case $N_z$
$\sigma_f$	Failure strength of the material due to a simple tension or compression test in psi
$\sigma_{fc}$	Yield strength in compression of the material due to a simple compression test in psi
$\sigma_{ft}$	Yield strength in tension of the material due to a simple tension test in psi
$\sigma_n$	Normal stress at the interface in psi
$\sigma_{nf}$	Normal failure debonding strength at the interface between fiber and matrix
$\tau_t$	Tangential stress at the interface in psi
$\tau_{tf}$	Interfacial tangential fibers debonding strength between fiber and matrix in psi

LIST OF SYMBOLS (Continued)

- $\tau_{yz,j}$  Denoting the shear stress of triangular prism  $j$  along the boundary  $y=b/2$ , due to the boundary conditions used in the computation program in fundamental case  $T_{yz}$
- $\tau_{zx,i}$  Denoting the shear stress of triangular prism  $i$  along the boundary  $x=c$ , due to the boundary conditions used in the computation program in fundamental case  $T_{zx}$
- $\tau_{xy,i}$  Denoting the shear stress of triangular prism  $i$  along the boundary  $x=c$ , due to the boundary conditions used in the computation program in fundamental case  $T_{xy}$
- $\Omega$  Area of the triangle of each triangular prism

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## INTRODUCTION

In ideal cases only, the loading of a composite material is in the direction of the reinforcements. Normally, oblique loading is felt by the composite of which a structural component is made. Examples are materials consisting of laminates in which each laminate has a definite fiber orientation. Other examples where oblique loading is involved are the different kinds of mechanical and other joints.

Therefore, the problem to be investigated is a unidirectional fiber reinforced composite under generalized oblique loading.

To obtain the effect of an oblique loading in any direction to the fiber axis, six fundamental loadings have to be combined. Based on micro-mechanical considerations and by linear superposition (References 1 and 2) of stresses due to the six fundamental loadings, the combined micro-stresses are then computed. References 3, 4, and 5 give the analyses of the following six fundamental loading cases: axial loading, transverse normal (two cases), longitudinal shear, and transverse shear (two cases)\*. Then, having the combined stresses which are acting on each particle of the reinforcement and the matrix, a failure criterion can be adopted. Here the von Mises distortion energy theory was used to calculate the critical loading in each element. For the points at the interfaces between fibers and matrix, debonding failures have to be evaluated in normal as well as tangential directions of the curved surfaces. It is self-evident that only the smallest critical loading of a composite has to be determined by the above outlined procedure. Which of the particles under stress will be first subjected to failure depends on geometry and material combinations and will be automatically considered. Failure may occur anywhere at the interface, in the fiber or in the resin.

The elastic engineering constants computed for each fundamental loading case will then be used to obtain, through transformation laws for fourth-order tensors, the composite elastic constants in any direction of the composite.

---

\* Later it will be explained why two cases are necessary in the transverse direction.

### TECHNICAL DISCUSSION

In the present problem the internal microstresses produced by external oblique loading can be envisioned as linear superimposed stresses produced by six independent fundamental loadings. It was assumed that the fiber packing is hexagonal since a model of such an array provides, as explained in Reference 6, better agreement with experiments than any other model. Therefore, the oblique loading can be produced by the following fundamental loadings:

Transverse normal loadings	$N_x, N_y$
Axial loading	$N_z$
Longitudinal shear loadings	$N_{yz}, T_{zx}$
Transverse shear loading	$T_{xy}$

Figure 1 shows these fundamental loadings acting on the surface of the representative element.

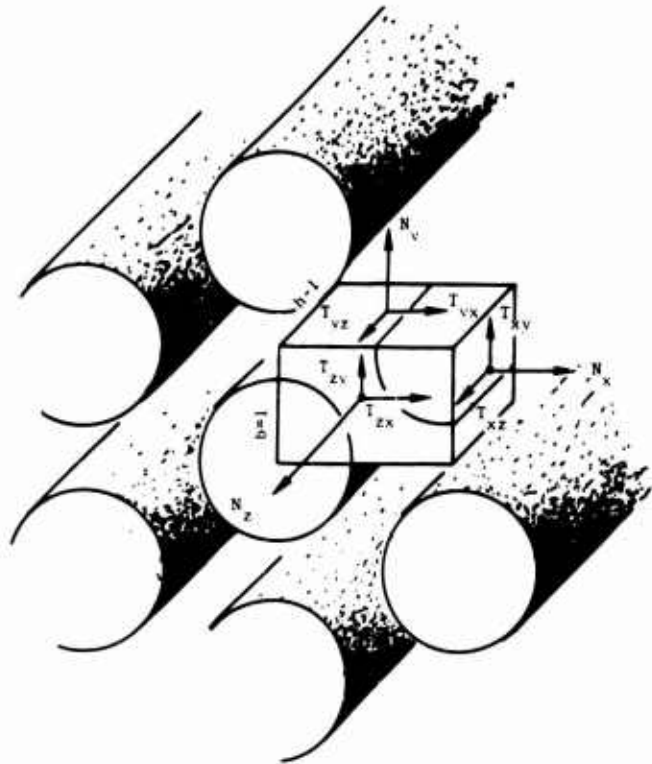


Figure 1. Basic Representative Element and the Loadings on the Surfaces of the Element.

The stress tensor is assumed to be symmetric,  $T_{ij} = T_{ji}$ , so that only six of the nine loads shown in Figure 1 have to be considered. Since external loadings are usually inclined at an angle with the fiber axis, the loadings as shown are the outputs of external loadings actually applied to the surface of the composite. For this purpose the transformation law for rank two tensors (References 7 and 8) has been applied. At this state of manipulation, the transformation law is valid.

These so-obtained loadings are the load boundary conditions for the combined internal microstresses which they produce. In other words, these loadings are the inputs of the analysis of the fundamental cases for which the microstresses will be deduced.

The microstresses are then added linearly in each particle of the reinforcement and matrix and introduced into the von Mises criterion so that the critical load in the constituents due to combined loading of a component is obtained.

Instead of beginning with the transformation of the oblique load into fundamental loads, we present first the latter ones. The finite element method itself will not be repeated here since this technique was already explained in detail in Reference 3.

#### TRANSVERSE NORMAL LOADINGS, $N_x$ AND $N_y$

The fundamental load case  $N_x$  was solved by finite element methods as a three-dimensional as well as a two-dimensional problem in Reference 3. It was found that accuracy of a two-dimensional analysis is sufficient for continuous fiber composites. In the present work, additional two-dimensional plane-strain solutions have been obtained separately for the  $N_x$  and  $N_y$  loadings.

The boundary conditions used in the computation program are for transverse normal loading  $N_x$ :

- 1) Displacements in x-direction along the boundary planes perpendicular to x-axis are 1 and -1 respectively; i.e.,  $u = 1$  at  $x = \frac{1}{2}\sqrt{3}$  and  $u = -1$  at  $x = -\frac{1}{2}\sqrt{3}$ .
- 2) Displacements in y-direction along the boundary planes perpendicular to y-axis are zeroes; i.e.,  $v = 0$  at  $y = \pm\frac{1}{2}$ .
- 3) Transverse shear stresses are zeroes along all boundaries.

For transverse normal loading case  $N_y$ :

- 1) Displacements in x-direction along the boundary planes perpendicular to x-axis are zeroes; i.e.,  $u = 0$  at  $x = \pm \frac{1}{2}\sqrt{3}$ .
- 2) Displacements in y-direction along the boundary planes perpendicular to y-axis are 1 and -1 respectively; i.e.,  $u = 1$  at  $y = \frac{1}{2}$  and  $v = -1$  at  $y = -\frac{1}{2}$ .
- 3) Transverse shear stresses are zeroes along all boundaries.

In order to convert the stresses caused by the above boundary conditions into the stresses corresponding to unity loadings in each case ( $\bar{N}_x = \bar{N}_y = 1$ ), the normalized factors  $1/\sqrt{3} \sum_i \bar{\tau}_{xi} A_{xi}$  and  $\sqrt{3}/\sum_j \bar{\tau}_{yj} A_{yj}$  were used in fundamental cases  $N_x$  and  $N_y$  respectively. The conversions for these and the subsequent fundamental cases are deemed to be necessary for the convenience of linear superposition.

The resulting stresses produced for case  $N_x$  are  $\bar{S}_{11}, \bar{S}_{21}, \bar{S}_{31}, \bar{S}_{41}, \bar{S}_{51}$  and  $\bar{S}_{61}$  ( $\bar{S}_{41} = \bar{S}_{51} = 0$ ), and for case  $N_y$ ,  $\bar{S}_{12}, \bar{S}_{22}, \bar{S}_{32}, \bar{S}_{42}, \bar{S}_{52}$ , and  $\bar{S}_{62}$  ( $\bar{S}_{42} = \bar{S}_{52} = 0$ ).\*

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\* All stress components indicated here and hereafter are in their orders corresponding to final microstresses:  $\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_z, \bar{\tau}_{yz}, \bar{\tau}_{zx}$ , and  $\bar{\tau}_{xy}$ .

## AXIAL LOADING, $N_z$

The problem of an axially loaded composite was solved in Reference 4 by an analytical method with the use of different coordinate axes of reference. Therefore, in this analysis, it must be solved numerically by applying the same type of finite elements used in the previous cases and using the coordinate axis of reference as shown in Figure 1.

The present problem consists of two parts: (1) a two-dimensional plane-strain problem which is the same as the previous cases except, of course, for boundary conditions; and (2) a two-dimensional problem which produces a unit strain in the direction of fiber axis and is slightly different from the so-called generalized-plane-strain problem in that it does not produce stresses in x- and y-directions.

The equations used for calculation are presented as follows (most of the notations are the same as those used in Reference 3):

The displacements at the nodes are expressed as

$$\{u\} = [A]\{a\} \quad (1)$$

or

$$\{a\} = [A]^{-1}\{u\} \quad (2)$$

The strain components are

$$\{\epsilon\} = [D]\{a\} \quad (3)$$

However, the strain column vector is a combination of strain components for a two-dimensional plane-strain solution and of strain components producing unit strain in fiber axis direction; i.e.,

$$\{\epsilon\} = \{\epsilon_1\} + \{\epsilon_2\} \quad (4)$$

where

$$\{\epsilon_1\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (5)$$

and

$$\{\epsilon_2\} = \begin{Bmatrix} -v \\ -v \\ 0 \end{Bmatrix} \quad (6)$$

The stresses produced are then

$$\{\sigma\} = [C]\{\epsilon_1\} = [C](\{\epsilon\} - \{\epsilon_2\}) \quad (7)$$

or

$$\{\sigma\} = [C]([D][A]^{-1}\{u\} - \{\epsilon_2\}) \quad (8)$$

The strain energy is

$$W = \iiint \{\epsilon\}^T \{\sigma\} t \, dx \, dy \, d\epsilon$$

Since  $\{\epsilon\}^T \{\sigma\}$  represents energy density per unit strain, integration over strain  $\epsilon$  must be carried out so that the strain energy is

$$W = \frac{1}{2} \iint (\{u\}^T ([A]^{-1})^T [D]^T [C] [D] [A]^{-1} \{u\} - 2\{u\}^T ([A]^{-1}) [D]^T [C] \{\epsilon_2\}) t \, dx \, dy \quad (9)$$

Then the force components at nodal points are

$$\{P_1\} = [K]\{u\} - \{P_2\} \quad (10)$$

or

$$\{P_1\} + \{P_2\} = [K]\{u\} \quad (11)$$

where

$$[K] = ([A]^{-1})^T [D]^T [C] [D] [A]^{-1} V \quad (12)$$

and

$$\{P_2\} = \{u\}^T ([A]^{-1}) [D]^T [C] \{\epsilon_2\} V \quad (13)$$

The rest of the equations will be as in Reference 5 and will not be repeated here.

The boundary conditions used in the computation program were:

- 1) All displacement components are zeroes along the boundaries; i.e.,  $u = 0$  at  $x = \pm \frac{1}{2} \sqrt{3}$  and  $v = 0$  at  $y = \pm \frac{1}{2}$ .
- 2) Transverse shear stresses along the boundaries are zeroes.

The normalized factor used in the present case was  $\sqrt{3} / (\sum_k^K \bar{\sigma}_{z,k} A_{z,k})$ .

The resulting stresses produced are  $\bar{S}_{13}, \bar{S}_{23}, \bar{S}_{33}, \bar{S}_{43}, \bar{S}_{53}$ , and  $\bar{S}_{63}$  ( $\bar{S}_{43} = \bar{S}_{53} = 0$ ).

#### LONGITUDINAL SHEAR LOADINGS, $T_{yz}$ and $T_{zx}$

The boundary value problem of a longitudinally shear-loaded composite is quite different from the previous cases and the subsequent case, although they are all two-dimensional. For the latter cases the governing differential equation for stress function is biharmonic, while for the former it is harmonic (as a torsional problem).

The displacement components at the nodal points are expressed as

$$\{w\} = [A]\{a\} \quad (14)$$

In matrix form, the displacements in the z-direction are

$$\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & \xi_2 & \eta_2 \\ 1 & \xi_3 & \eta_3 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \quad (15)$$

$$\{a\} = [A]^{-1}\{w\} \quad (16)$$

where

$$[A]^{-1} = \begin{bmatrix} \frac{\xi_2 \eta_3 - \xi_3 \eta_2}{\Delta} & 0 & 0 \\ \frac{-\eta_3 - \eta_2}{\Delta} & \frac{\eta_3}{\Delta} & \frac{-\eta_2}{\Delta} \\ \frac{\xi_3 - \xi_2}{\Delta} & \frac{-\xi_3}{\Delta} & \frac{\xi_2}{\Delta} \end{bmatrix} \quad (17)$$

$$\text{and } \Delta = \xi_2 \eta_3 - \xi_3 \eta_2 \quad (18)$$

The strain components are

$$\{\gamma\} = [D]\{a\} = [D][A]^{-1}\{w\} \quad (19)$$

or

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \quad (20)$$

and the stress components are

$$\{\tau\} = [C]\{\gamma\} \quad (21)$$

or

$$\{\tau\} = [C][D][A]^{-1}\{w\} \quad (22)$$

Therefore, the strain energy is given by

$$\begin{aligned} W &= \frac{1}{2} \iint \{\gamma\}^T \{\tau\} dx dy \\ &= \frac{1}{2} \iint \{w\}^T ([A]^{-1})^T [D]^T [C] [D] [A]^{-1} \{w\} dx dy \end{aligned} \quad (23)$$

By Castigliano's second principle, we have

$$\{P\} = [K]\{w\} \quad (24)$$

where

$$[K] = ([A]^{-1})^T [D]^T [C] [D] [A]^{-1} \Omega \quad (25)$$

The rest of the equations will be similar to those in Reference 3 and will not be repeated here.

In the computation program, the boundary conditions used are:  
Fundamental Longitudinal Shear Loading Case  $T_{yz}$ :

- 1) Longitudinal displacement of the boundary plane perpendicular to the y-axis in the positive direction is unity; i.e.,  $w = 1$  at  $y = \frac{1}{2}$ .
- 2) Longitudinal displacement of the boundary plane perpendicular to the y-axis in the negative direction is negative unity; i.e.,  $w = -1$  at  $y = -\frac{1}{2}$ .

Fundamental Longitudinal Shear Loading Case  $T_{zx}$ :

- 1) Longitudinal displacement of the boundary plane perpendicular to the x-axis in the positive direction is unity; i.e.,  $w = 1$  at  $x = \frac{1}{2}\sqrt{3}$ .
- 2) Longitudinal displacement of the boundary plane perpendicular to the x-axis in the negative direction is negative unity; i.e.,  $w = -1$  at  $x = -\frac{1}{2}\sqrt{3}$ .

The normalized converting factors used in fundamental cases  $T_{yz}$  and  $T_{zx}$

were  $\sqrt{3}/(\sum_j \tau_{yz,j} A_{y,j})$  and  $1/(\sum_i \tau_{zx,i} A_{x,i})$  respectively.

The stresses produced for case  $T_{yz}$  are  $\bar{S}_{14}, \bar{S}_{24}, \bar{S}_{34}, \bar{S}_{44}, \bar{S}_{54}$ , and  $\bar{S}_{64}$  ( $\bar{S}_{14} = \bar{S}_{24} = \bar{S}_{34} = \bar{S}_{64} = 0$ ), and for case  $T_{zx}$ ,  $\bar{S}_{15}, \bar{S}_{25}, \bar{S}_{35}, \bar{S}_{45}, \bar{S}_{55}$ , and  $\bar{S}_{65}$  ( $\bar{S}_{15} = \bar{S}_{25} = \bar{S}_{35} = \bar{S}_{65} = 0$ ).

#### TRANSVERSE SHEAR LOADING, $T_{xy}$

The problem of a unidirectional fiber-reinforced composite subjected to transverse shear loading is the same as fundamental cases 1 and 2, except the boundary conditions. It was found that the boundary conditions in the present case (see Appendix I) are complementary to those of fundamental transverse loading case  $N_x$ .

In addition to the study made in Appendix I on the boundary conditions, an analytical study (see Appendix II) as well as a numerical analysis (see Appendix III) by finite-element method on the symmetry relations in perforated plates has been performed. The analytical study and the numerical analysis justify the conclusions made in Appendix I.

The boundary conditions used in the computation program are:

- 1) Displacements in the y-direction along the boundary planes perpendicular to the x-axis are 1 and -1 respectively; i.e.,  $v = 1$  at  $x = \frac{1}{2}\sqrt{3}$  and  $v = -1$  at  $x = -\frac{1}{2}\sqrt{3}$ .
- 2) Displacements in the x-direction along the boundary planes perpendicular to the y-axis are zeroes; i.e.,  $u = 0$  at  $y = \pm\frac{1}{2}$ .
- 3) Normal stresses along the boundary planes are zeroes.

The normalized factor for converting the loading condition of the above

boundary conditions into the unity loading condition was  $1/(\sum_{i=1}^I \tau_{xy,i} A_{x,i})$ .

The resulting stresses in this case are  $\bar{S}_{16}, \bar{S}_{26}, \bar{S}_{36}, \bar{S}_{46}, \bar{S}_{56}$ , and  $\bar{S}_{66}$  ( $\bar{S}_{46} = \bar{S}_{56} = 0$ ).

#### PARAMETRIC STUDIES AND FINAL STRESSES IN A COMPOSITE DUE TO A GENERALIZED PLANE OBLIQUE LOADING

The coordinate system of oblique loading, which in most cases may be parallel to the surface of a unidirectional composite, will be called the loading or structural geometry system. The system with one coordinate axis parallel to the fiber axis will be called the material property system. In the previous studies of fundamental loading cases, the material property system was adopted for simplicity in description of the boundary conditions. Both systems are inclined by angle  $\psi$  with their coordinate axis (Figure 2); therefore, the loadings applied on such a composite must be transformed into the loading corresponding to the material property system before the linear superposition of any desired quantities can take place. Based on the equilibrium conditions of loading and the transformation law for rank two tensors, the three-dimensional equations of transformation have the following form:

$$N_{ij} = l_{ik} l_{jl} \sigma'_{kl} \quad (26)$$

where  $\sigma'_{kl}$  are applied loads in psi, and  $N_{ij}$  are the transformed loadings in psi. Further,  $l_{ik}$  and  $l_{jl}$ , the direction cosines between the new coordinate axis x y z (material property axis) (see Figure 2) and the original coordinate axis x'y'z' (loading coordinate axis), are defined in the following table.

	x'	y'	z'
x	$l_{11}$	$l_{12}$	$l_{13}$
y	$l_{21}$	$l_{22}$	$l_{23}$
z	$l_{31}$	$l_{32}$	$l_{33}$

Equation (26) becomes after summing over k and l

$$N_x = N_{11}$$

$$N_{11} = l_{11}l_{11}\sigma_{11} + l_{11}l_{12}\sigma_{12} + l_{11}l_{13}\sigma_{13} + l_{12}l_{11}\sigma_{21} + l_{12}l_{12}\sigma_{22} + l_{12}l_{13}\sigma_{23} + l_{13}l_{11}\sigma_{31} + l_{13}l_{12}\sigma_{32} + l_{13}l_{13}\sigma_{33}$$

$$T_{xy} = N_{12}$$

$$N_{12} = l_{11}l_{21}\sigma_{11} + l_{11}l_{22}\sigma_{12} + l_{11}l_{23}\sigma_{13} + l_{12}l_{21}\sigma_{21} + l_{12}l_{22}\sigma_{22} + l_{12}l_{23}\sigma_{23} + l_{13}l_{21}\sigma_{31} + l_{13}l_{22}\sigma_{32} + l_{13}l_{23}\sigma_{33}$$

$$T_{xz} = N_{13}$$

$$N_{13} = l_{11}l_{31}\sigma_{11} + l_{11}l_{32}\sigma_{12} + l_{11}l_{33}\sigma_{13} + l_{12}l_{31}\sigma_{21} + l_{12}l_{32}\sigma_{22} + l_{12}l_{33}\sigma_{23} + l_{13}l_{31}\sigma_{31} + l_{13}l_{32}\sigma_{32} + l_{13}l_{33}\sigma_{33} \quad (27)$$

$$N_y = N_{22}$$

$$N_{22} = l_{21}l_{21}\sigma_{11} + l_{21}l_{22}\sigma_{12} + l_{21}l_{23}\sigma_{13} + l_{22}l_{21}\sigma_{21} + l_{22}l_{22}\sigma_{22} + l_{22}l_{23}\sigma_{23} + l_{23}l_{21}\sigma_{31} + l_{23}l_{22}\sigma_{32} + l_{23}l_{23}\sigma_{33}$$

$$T_{yz} = N_{23}$$

$$N_{23} = l_{21}l_{31}\sigma_{11} + l_{21}l_{32}\sigma_{12} + l_{21}l_{33}\sigma_{13} + l_{22}l_{31}\sigma_{21} + l_{22}l_{32}\sigma_{22} + l_{22}l_{33}\sigma_{23} + l_{23}l_{31}\sigma_{31} + l_{23}l_{32}\sigma_{32} + l_{23}l_{33}\sigma_{33}$$

$$N_z = N_{33}$$

$$N_{33} = l_{31}l_{31}\sigma_{11} + l_{31}l_{32}\sigma_{12} + l_{31}l_{33}\sigma_{13} + l_{32}l_{31}\sigma_{21} + l_{32}l_{32}\sigma_{22} + l_{32}l_{33}\sigma_{23} + l_{33}l_{31}\sigma_{31} + l_{33}l_{32}\sigma_{32} + l_{33}l_{33}\sigma_{33}$$

In the present analysis, only a plane oblique loading  $p$  is applied on the surface of the composite. Therefore, the transformation is two-dimensional and is simplified as (see Figure 2):

$$N_x = p \sin^2 \theta \quad (28)$$

$$N_z = p \cos^2 \theta \quad (29)$$

$$T_{zx} = \frac{1}{2} p \sin(2\theta) \quad (30)$$

$$N_y = T_{yz} = T_{xy} = 0 \quad (31)$$

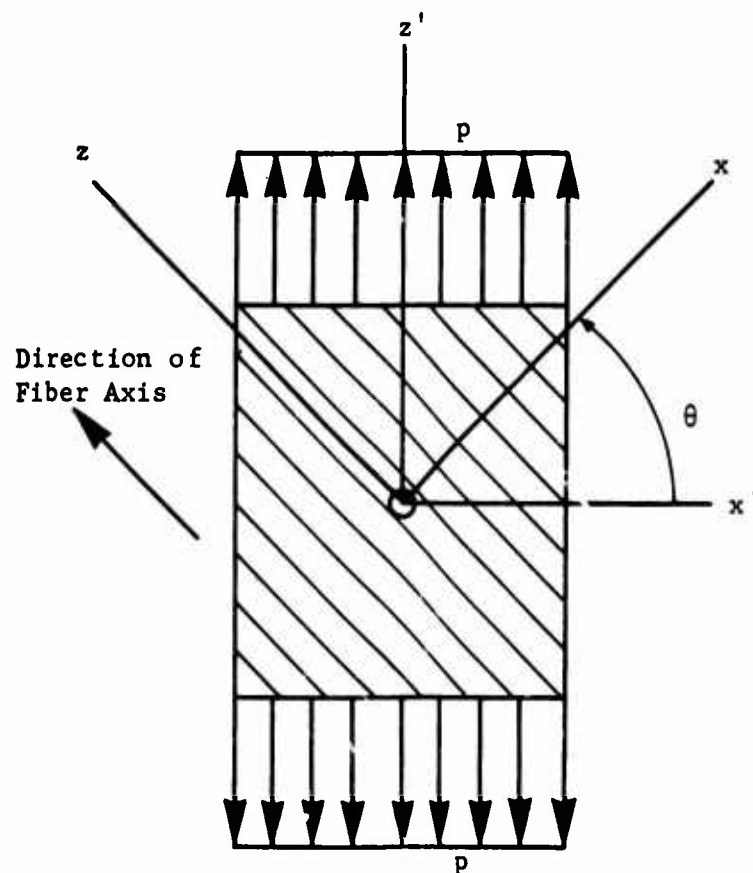


Figure 2. Loading (Geometry) Coordinate Axis and Material Property Coordinate Axes.

However, in the fundamental cases the inputs (loadings) used were  $\bar{N}_x$ ,  $\bar{N}_y$ ,  $\bar{N}_z$ ,  $\bar{T}_{yz}$ ,  $\bar{T}_{zx}$ , and  $\bar{T}_{xy}$  respectively. These loadings are not completely the same as those of the left-hand side of equations (27). From the observation of the boundary conditions of fundamental cases  $N_x N_y N_z$ , it can be found that the internal applied loading (macrostress) on the characteristic element in each case is not one simple loading but a combination of three normal loadings (although the other two are comparatively small). In order to convert the stress components generated in these cases into those really produced by  $N_x$ ,  $N_y$ , and  $N_z$ , the following manipulation must be taken to obtain the revised cases.

For revised case  $N_x$ , the simultaneous equations used to determine the constants in order to calculate stress  $S_{11}, S_{21}, S_{31}, S_{41}, S_{51}$ , and  $S_{61}$  due to loading  $N_x$  are:

$$1 = A_1 \sum_{i=1}^6 (\bar{S}_{11})_i A_{x,i} + A_2 \sum_{i=1}^6 (\bar{S}_{12})_i A_{x,i} + A_3 \sum_{i=1}^6 (\bar{S}_{13})_i A_{x,i} \quad (32)$$

$$0 = A_1 \sum_{j=1}^{11} (\bar{S}_{21})_j A_{y,j} + A_2 \sum_{j=1}^{11} (\bar{S}_{22})_j A_{y,j} + A_3 \sum_{j=1}^{11} (\bar{S}_{23})_j A_{y,j} \quad (33)$$

$$0 = A_1 \sum_{k=1}^{158} (\bar{S}_{31})_k A_{z,k} + A_2 \sum_{k=1}^{158} (\bar{S}_{32})_k A_{z,k} + A_3 \sum_{k=1}^{158} (\bar{S}_{33})_k A_{z,k} \quad (34)$$

Solving equations (32) to (34), we get constants  $A_1, A_2$ , and  $A_3$ .

Then we can calculate the stresses produced in the revised fundamental case  $N_x$  as follows:

$$S_{11} = A_1 \bar{S}_{11} + A_2 \bar{S}_{12} + A_3 \bar{S}_{13} \quad (35)$$

$$S_{21} = A_1 \bar{S}_{21} + A_2 \bar{S}_{22} + A_3 \bar{S}_{23} \quad (36)$$

$$S_{31} = A_1 \bar{S}_{31} + A_2 \bar{S}_{32} + A_3 \bar{S}_{33} \quad (37)$$

$$S_{61} = A_1 \bar{S}_{61} + A_2 \bar{S}_{62} + A_3 \bar{S}_{63} \quad (38)$$

$$\text{and } S_{41} = S_{51} = 0. \quad (39)$$

For revised fundamental case  $N_y$ , we have the simultaneous equations

$$0 = B_1 \sum_{i=1}^6 [(\bar{S}_{11})_i A_{x,i}] + B_2 \sum_{i=1}^6 [(\bar{S}_{12})_i A_{x,i}] + B_3 \sum_{i=1}^6 [(\bar{S}_{13})_i A_{x,i}] \quad (40)$$

$$\sqrt{3} = B_1 \sum_{j=1}^{11} [(\bar{S}_{21})_j A_{y,j}] + B_2 \sum_{j=1}^{11} [(\bar{S}_{22})_j A_{y,j}] + B_3 \sum_{j=1}^{11} [(\bar{S}_{23})_j A_{y,j}] \quad (41)$$

$$0 = B_1 \sum_{k=1}^{156} [(\bar{S}_{31})_k A_{z,k}] + B_2 \sum_{k=1}^{156} [(\bar{S}_{32})_k A_{z,k}] + B_3 \sum_{k=1}^{156} [(\bar{S}_{33})_k A_{z,k}] \quad (42)$$

Solving for  $B_1, B_2$ , and  $B_3$ , we can find  $S_{12}, S_{22}, S_{32}$ , and  $S_{32}$  as follows:

$$S_{12} = B_1 \bar{S}_{11} + B_2 \bar{S}_{12} + B_3 \bar{S}_{13} \quad (43)$$

$$S_{22} = B_1 \bar{S}_{21} + B_2 \bar{S}_{22} + B_3 \bar{S}_{23} \quad (44)$$

$$S_{32} = B_1 \bar{S}_{31} + B_2 \bar{S}_{32} + B_3 \bar{S}_{33} \quad (45)$$

$$S_{32} = B_1 \bar{S}_{31} + B_2 \bar{S}_{32} + B_3 \bar{S}_{33} \quad (46)$$

and

$$S_{42} = S_{52} = 0 \quad (47)$$

For revised fundamental case  $N_z$ , the simultaneous equations are:

$$0 = C_1 \sum_{i=1}^6 [(\bar{S}_{11})_i A_{x,i}] + C_2 \sum_{i=1}^6 [(\bar{S}_{12})_i A_{x,i}] + C_3 \sum_{i=1}^6 [(\bar{S}_{13})_i A_{x,i}] \quad (48)$$

$$0 = C_1 \sum_{j=1}^6 [(\bar{S}_{21})_j A_{y,j}] + C_2 \sum_{j=1}^{11} [(\bar{S}_{22})_j A_{y,j}] + C_3 \sum_{j=1}^{11} [(\bar{S}_{23})_j A_{y,j}] \quad (49)$$

$$\sqrt{3} = C_1 \sum_{k=1}^{156} [(\bar{S}_{31})_k A_{z,k}] + C_2 \sum_{k=1}^{156} [(\bar{S}_{32})_k A_{z,k}] + C_3 \sum_{k=1}^{156} [(\bar{S}_{33})_k A_{z,k}] \quad (50)$$

After solving for  $C_1, C_2$ , and  $C_3$ , we can get the stress components in the revised case  $N_z$  as follows:

$$S_{13} = C_1 \bar{S}_{11} + C_2 \bar{S}_{12} + C_3 \bar{S}_{13} \quad (51)$$

$$S_{23} = C_1 \bar{S}_{21} + C_2 \bar{S}_{22} + C_3 \bar{S}_{23} \quad (52)$$

$$S_{33} = C_1 \bar{S}_{31} + C_2 \bar{S}_{32} + C_3 \bar{S}_{33} \quad (53)$$

$$S_{33} = C_1 \bar{S}_{31} + C_2 \bar{S}_{32} + C_3 \bar{S}_{33} \quad (54)$$

$$\text{and} \quad S_{43} = S_{53} = 0 \quad (55)$$

The stresses produced in the composite due to shears  $T_{yz}$ ,  $T_{zx}$  and  $T_{xy}$  will be the same as those generated in the corresponding fundamental cases. Therefore, no revision for the shear cases is necessary. For easy identification, these stresses are reassigned as follows:

$$\begin{aligned} &S_{14}, S_{24}, S_{34}, S_{44}, S_{54} \text{ and } S_{64} \text{ due to shear } T_{yz}, \\ &S_{15}, S_{25}, S_{35}, S_{45}, S_{55} \text{ and } S_{65} \text{ due to shear } T_{zx}, \\ &\text{and } S_{16}, S_{26}, S_{36}, S_{46}, S_{56} \text{ and } S_{66} \text{ due to shear } T_{xy}. \end{aligned}$$

$$(S_{14} = S_{24} = S_{34} = S_{64} = S_{15} = S_{25} = S_{35} = S_{65} = S_{46} = S_{56} = 0)$$

After obtaining the stresses in the revised fundamental cases, we then can get the final stresses by the use of the principle of linear superposition, as follows:

$$\{\sigma\} = [S]\{N\} \quad (56)$$

Written out in detail, one gets

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & 0 \\ S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66} \end{bmatrix} \begin{pmatrix} N_x \\ N_y \\ N_z \\ T_{yz} \\ T_{zx} \\ T_{xy} \end{pmatrix} \quad (57)$$

#### FAILURE CRITERIA OF THE COMPOSITE

In the present analysis, the failure (yield) criteria were determined by examining the constituents themselves as well as their interfaces

##### Failure in the Constituents

The von Mises Theory of Distortion Energy was adopted as the failure criterion of the composite constituents. This theory states that the load condition  $(p_{cr})_c$  is critical when the distortional energy due to

this load condition is equal to the distortional energy at failure under simple tension or compression. In mathematical form:

$$(p_{cr})_c \text{ occurs when } \frac{U_{df}}{U_d(p_{cr})} = 1$$

or

$$1 = \sqrt{2} \sigma_f / [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2)]^{1/2} \quad (59)$$

where  $\sigma_f$  is the yield strength of the material due to a simple tension or compression test and is equal to  $\sigma_{ft}$  when  $\sigma_x + \sigma_y + \sigma_z \geq 0$  or  $\sigma_{fc}$  when  $\sigma_x + \sigma_y + \sigma_z < 0$ .

After obtaining the microstresses of each triangular prism in both constituents from the previous section, we can then find the critical loadings for all finite elements by the above equation. The smallest one will be the critical loading of the composite as far as the composite materials are concerned. However, failure may occur at the interface.

#### Failures at the Interface

Debonding failure at the interface between fiber and matrix is actually a very difficult "micro-micro" problem. Many investigators have attacked it on different micro-levels. Based on our survey of the literature, we found that none of them have solved it successfully in reality. Here, in our study, we have used two criteria to determine the bonding failure conditions. They are normal bonding strength and interfacial shear bonding strength between two materials.

The stress vector associated with the direction normal  $d$  at the interface is

$$[T_d] = [T][d] \quad (60)$$

or

$$[T_d] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} \cos t \\ \sin t \\ 0 \end{bmatrix} \quad (61)$$

where  $\phi$  is the angle between direction normal at the interface and x-axis of the basic representative element.

Therefore, the normal stress of the finite elements at the interface is

$$\begin{aligned}\sigma_n &= [T_d]^T [d] \\ &= \sigma_x \cos^2 \phi + \sigma_y \sin^2 \phi + 2\tau_{xy} \sin \phi \cos \phi\end{aligned}\quad (62)$$

and the tangential stress of finite elements at the interface is then

$$\begin{aligned}\tau_t &= \left\{ \left[ \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\phi - \tau_{xy} \cos 2\phi \right]^2 \right. \\ &\quad \left. + (\tau_{xz} \cos \phi + \tau_{yz} \sin \phi)^2 \right\}^{\frac{1}{2}}\end{aligned}\quad (63)$$

One can then define each critical load condition  $(p_{cr})_{in}$  and  $(p_{cr})_{is}$  as that for which a developed stress is equal to a critical stress; i.e.,

$$(p_{cr})_{in} \text{ occurs when } \frac{\sigma_{nf}}{[\sigma_n(p_{cr})]_{av}} = \frac{\sigma_{nf}}{(\sigma_n)_{av}} = 1 \quad (64)$$

$$(p_{cr})_{is} \text{ occurs when } \frac{\tau_{tf}}{[\tau_t(p_{cr})]_{av}} = \frac{\tau_{tf}}{(\tau_t)_{av}} = 1 \quad (65)$$

Here  $(\sigma_n)_{av}$  and  $(\tau_t)_{av}$  are average values of  $\sigma_n$  and  $\tau_t$  of neighboring triangular prisms at the interface respectively.

From equations (59), (64), and (65), we can calculate different critical loads based on the criteria of the constituents strength and the normal and tangential bonding strength of the interface. These values will be automatically compared with other stresses in the material. The smallest of these values will be the critical loading of the composite as a whole, if there is no smaller value present elsewhere in the material.

#### ELASTIC CONSTANTS E, G, $\nu$ , and $\eta$ OF A UNIDIRECTIONAL COMPOSITE IN THE DIRECTION OF THE LOADING

For each fundamental loading case,  $N_x, N_y, N_z, N_{xy}, N_{xz}, N_{yz}$ , the corresponding engineering elastic constants and Poisson's ratios  $\nu$  have been computed from the numerical solutions. As mentioned before, the loads acting on structures are seldom in the direction of the fibers, and they are not aligned in the direction of a Cartesian coordinate system as the fundamental loading cases are. Therefore, the elastic constants have to be calculated in the direction of the load of interest. They can be found by the following formulas<sup>9</sup> derived through the use of the transformation law for fourth-order tensors.

Modulus of elasticity of a composite in direction  $\theta$ :

$$E_{\theta} = \left[ \frac{m^4}{E_z} + \frac{n^4}{E_r} + \left( \frac{1}{G_{zr}} - \frac{\nu_{zr}}{E_z} - \frac{\nu_{rz}}{E_r} \right) m^2 n^2 \right]^{-1} \quad (66)$$

Shear modulus of a composite in direction  $\theta$ :

$$G_{\theta} = \left[ \frac{1}{G_{zr}} + 4m^2 n^2 \left( \frac{1+\nu_{zr}}{E_z} + \frac{1+\nu_{rz}}{E_r} - \frac{1}{G_{zr}} \right) \right]^{-1} \quad (67)$$

Poisson's ratio connected with direction  $\theta$ :

$$\nu_{\theta} = E_{\theta} \left[ \frac{\nu_{zr}}{E_z} - m^2 n^2 \left( \frac{1+\nu_{zr}}{E_z} + \frac{1+\nu_{rz}}{E_r} - \frac{1}{G_{zr}} \right) \right] \quad (68)$$

Shear coupling factor connected with direction  $\theta$ :

$$\eta_{\theta} = E_{\theta} mn \left[ \frac{2m^2}{E_z} - \frac{2n^2}{E_r} + (m^2 - n^2) \left( \frac{\nu_{zr}}{E_z} + \frac{\nu_{rz}}{E_r} - \frac{1}{G_{zr}} \right) \right] \quad (69)$$

### NUMERICAL RESULTS

Once the details of the stress-strain fields are known for the fundamental loading cases, the final stresses due to oblique loading can be computed by using equation (57). In this phase, the obtained stresses have been numerically introduced into the von Mises criterion equations [(59), (64), and (65)] for the finite elements shown in Figure 3.

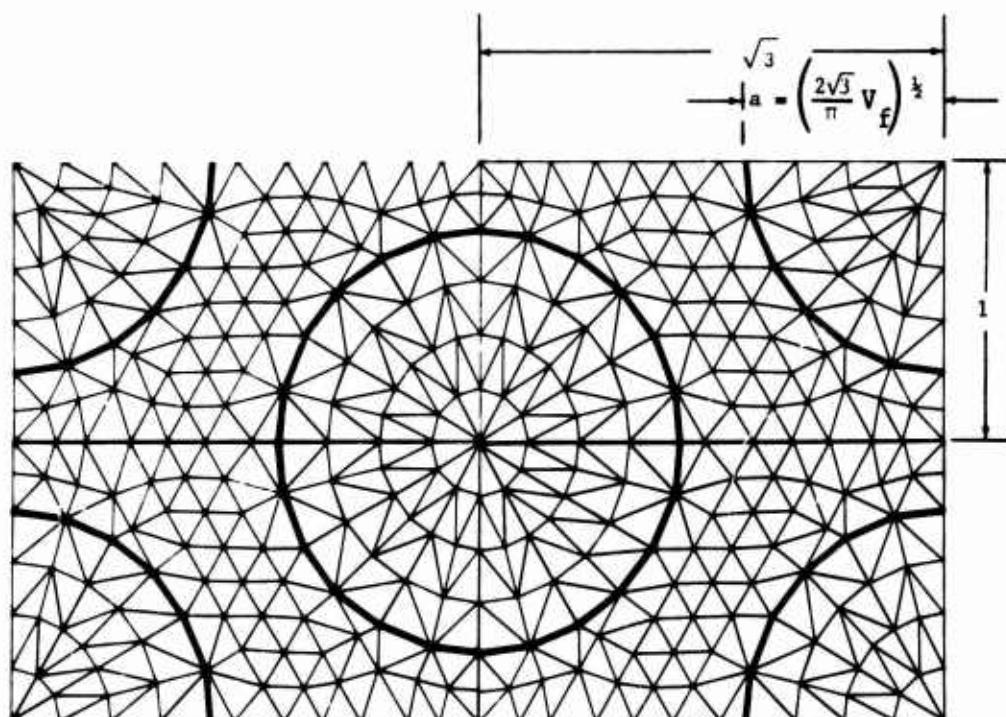


Figure 3. The Finite Elements for Which the Stresses Were Computed and the von Mises Failure Criterion Was Applied.

In this manner, the smallest critical load  $p_{cr}$ , which is the critical load of the composite, is obtained (Figure 4 through Figure 8). Equations (66) to (69) have been used to calculate the elastic constants (Figures 9 through 18), Poisson's ratio (Figure 19 through Figure 24), and shear coupling factors (Figures 25 and 26).

The different combinations of computer inputs that were used in the fundamental loading cases are for  $v_f = 0.2$  and  $v_f/v_m = 0.5714$  (see table below).

COMBINATIONS OF COMPONENTS, MATERIAL CONSTANTS, AND VOLUME PERCENTAGES FOR WHICH THE COMPOSITE MATERIAL CONSTANTS AND CRITICAL LOAD HAVE BEEN COMPUTED							
	$V_f$	$E_f/E_m$	$E_f \cdot 10^{-6}$		$V_f$	$E_f/E_m$	$E_f \cdot 10^{-6}$
1	0	1	0.38	22	0.5	30	11.40
2	0.5	2	0.76	23	0.5	60	22.80
3	0.5	4	1.52	24	0.5	90	34.20
4	0.5	6	2.28	25	0.5	120	45.60
5	0.5	10	3.80	26	0.5	160	60.80
6	0.5	20	7.60	27	0.6	30	11.40
7	0.6	2	0.76	28	0.6	60	22.80
8	0.6	4	1.52	29	0.6	90	34.20
9	0.6	6	2.28	30	0.6	120	45.60
10	0.6	10	3.80	31	0.6	160	60.80
11	0.6	20	7.60	32	0.7	30	11.40
12	0.7	2	0.76	33	0.7	60	22.80
13	0.7	4	1.52	34	0.7	90	34.20
14	0.7	6	2.28	35	0.7	120	45.60
15	0.7	10	3.80	36	0.7	160	60.80
16	0.7	20	7.60	37	0.8	30	11.40
17	0.8	2	0.76	38	0.8	60	22.80
18	0.8	4	1.52	39	0.8	90	34.20
19	0.8	6	2.28	40	0.8	120	45.60
20	0.8	10	3.80	41	0.8	160	60.80
21	0.8	20	7.60				

In Figure 4 the composite critical strength  $p_{cr}$  is given as a function of fiber yield strength  $\sigma_f$ , matrix strength  $\sigma_m$ , component moduli  $E_f$  and  $E_m$ , and the angle  $\theta$  between fiber axis and external load. A composite with 50 percent fiber volume and Poisson's ratios  $\nu_f = 0.2$ ,  $\nu_m = .35$  is assumed. The results shown are valid for both tension and compression, provided the correct ratio  $\sigma_f/\sigma_m$  is used (in some materials, this ratio may be different in tension and compression). Consider, for example, a case where  $\sigma_f/\sigma_m = 25$  in tension and  $\sigma_f/\sigma_m = 12.5$  in compression. Assume a load acting at an angle  $\theta = 50^\circ$  to the fiber axis. The tensile strength in this case is  $p_{cr} = 0.04 \sigma_f$ . The compressive strength, on the other hand, would be  $p_{cr} = 0.08 \sigma_f$ .

The dotted curves in Figure 4 indicate that for this part the critical strength has not been computed; values are plotted only for composites where  $E_f/E_m > \sigma_f/\sigma_m$ .

In Figure 5 the composite critical strength  $\sigma_f$  is presented, like in Figure 4, as function of the components properties  $\sigma_f, \sigma_m, E_f, E_m$ , and  $\theta$  for 60 percent of fiber content by volume. Strength values are plotted for  $E_f/E_m > \sigma_f/\sigma_m$  and for  $E_f/E_m < \sigma_f/\sigma_m$ . At the angle  $\theta = 0$  or close to zero, the strength curves for  $E_f/E_m \leq 90$  separate from the strength curves for values  $E_f/E_m \geq 90$ . This fact is indicated by an arrow for  $E_f/E_m = 30$  and  $E_f/E_m = 90$ .

Figure 6 shows the strength of a composite, like in Figures 4 and 5, but for 70 percent of fiber content by volume. The enlarged section at the top of the figure represents the curves on an extended scale from  $\theta = 0^\circ$  to  $\theta = 10^\circ$ .

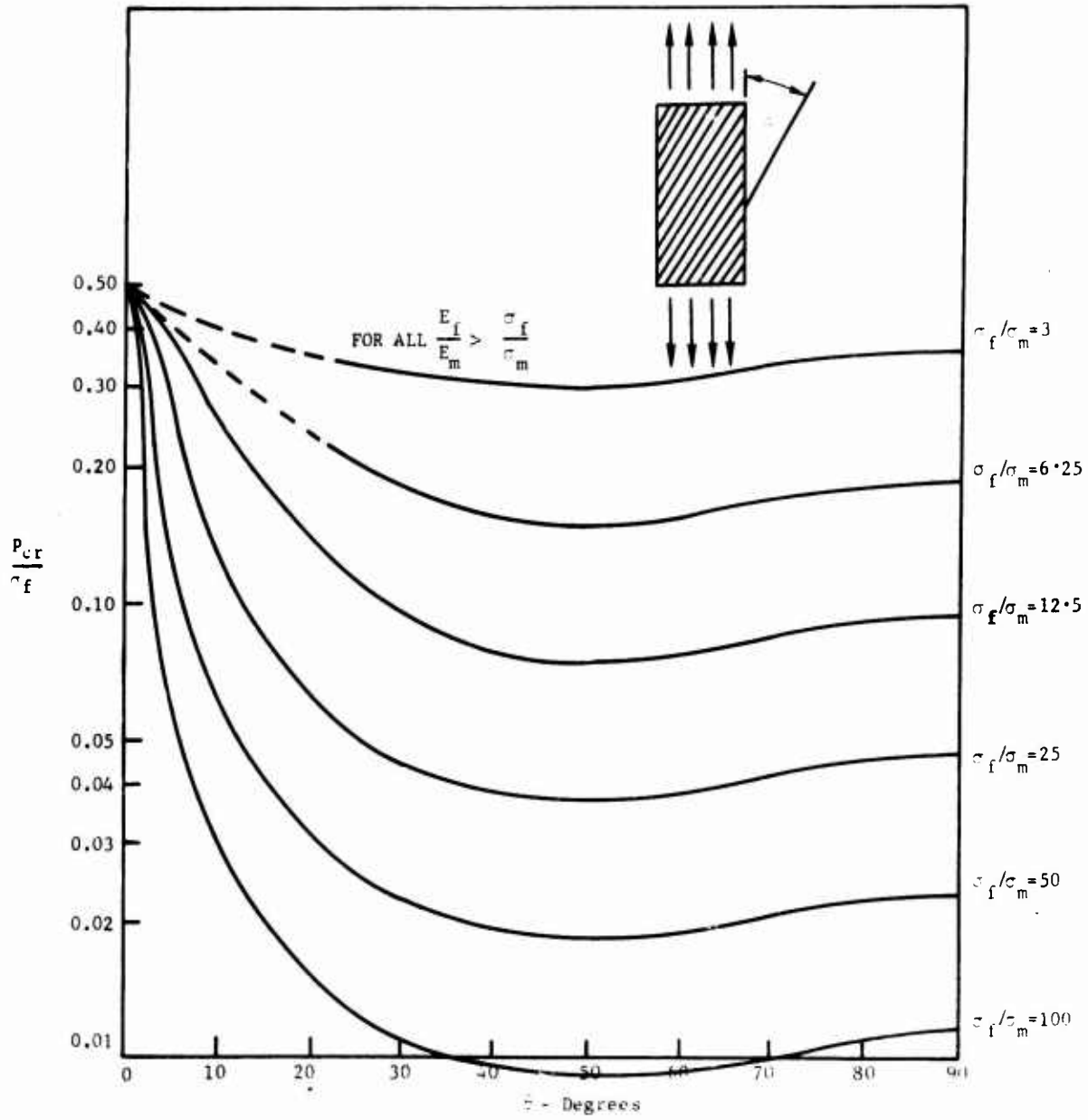


Figure 4. Critical Load ( $p_{cr}$ ) of a Composite ( $V_f=0.5$ ) as a Function of Loading Angle ( $\alpha$ ), Fiber and Matrix Yield Strength, and Moduli of Elasticity ( $\sigma_f$ ,  $\sigma_m$ , and  $E_f$ ,  $E_m$ ).

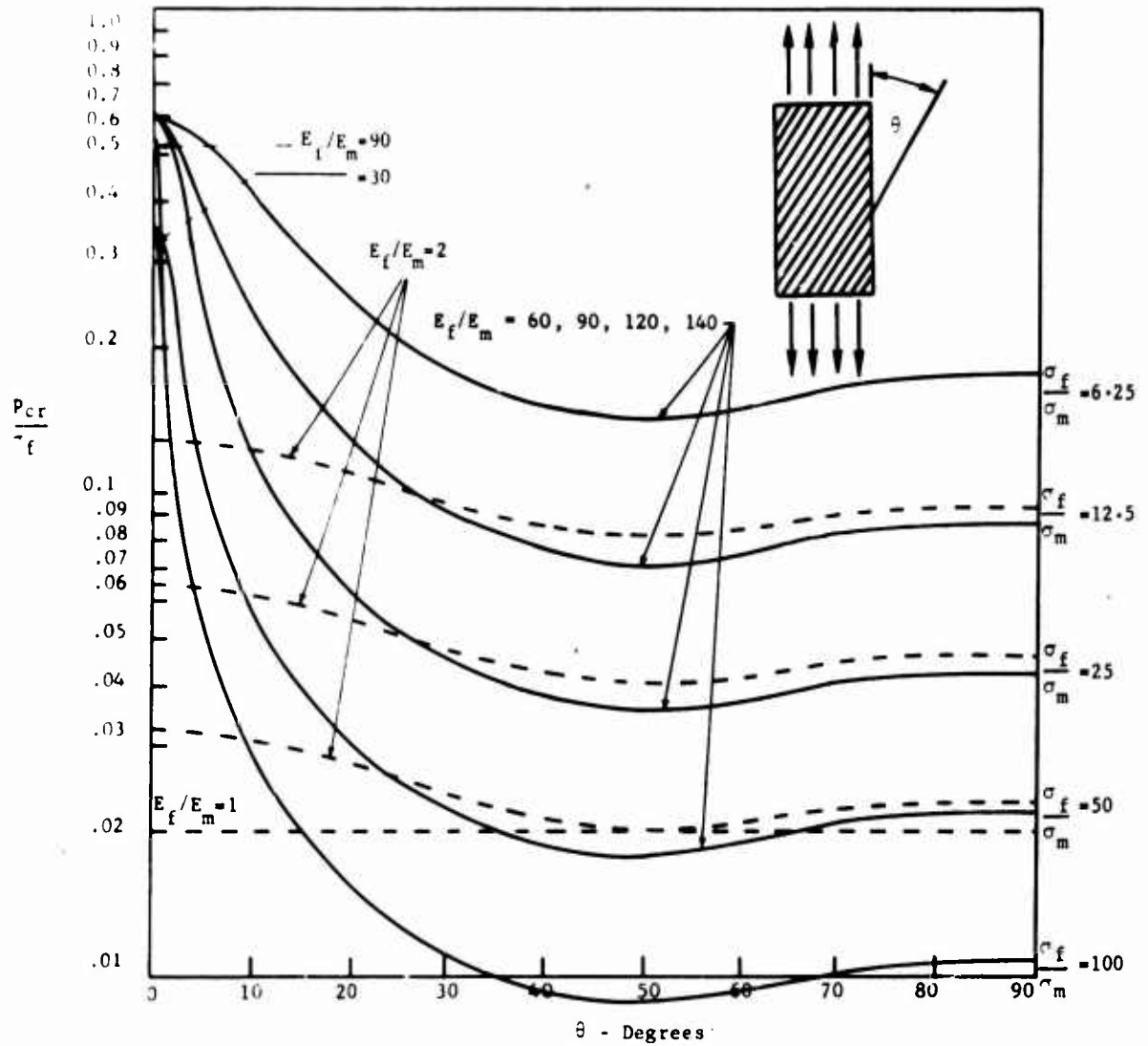


Figure 5. Critical Loading ( $p_{cr}$ ) of a Composite ( $V_f=0.6$ ) Under Oblique Loading as a Function of Loading Angle ( $\theta$ ), Fiber and Matrix Yield Strength, and Moduli of Elasticity ( $\theta$ ,  $\sigma_f$ ,  $\sigma_m$ , and  $E_f$ ,  $E_m$ ).

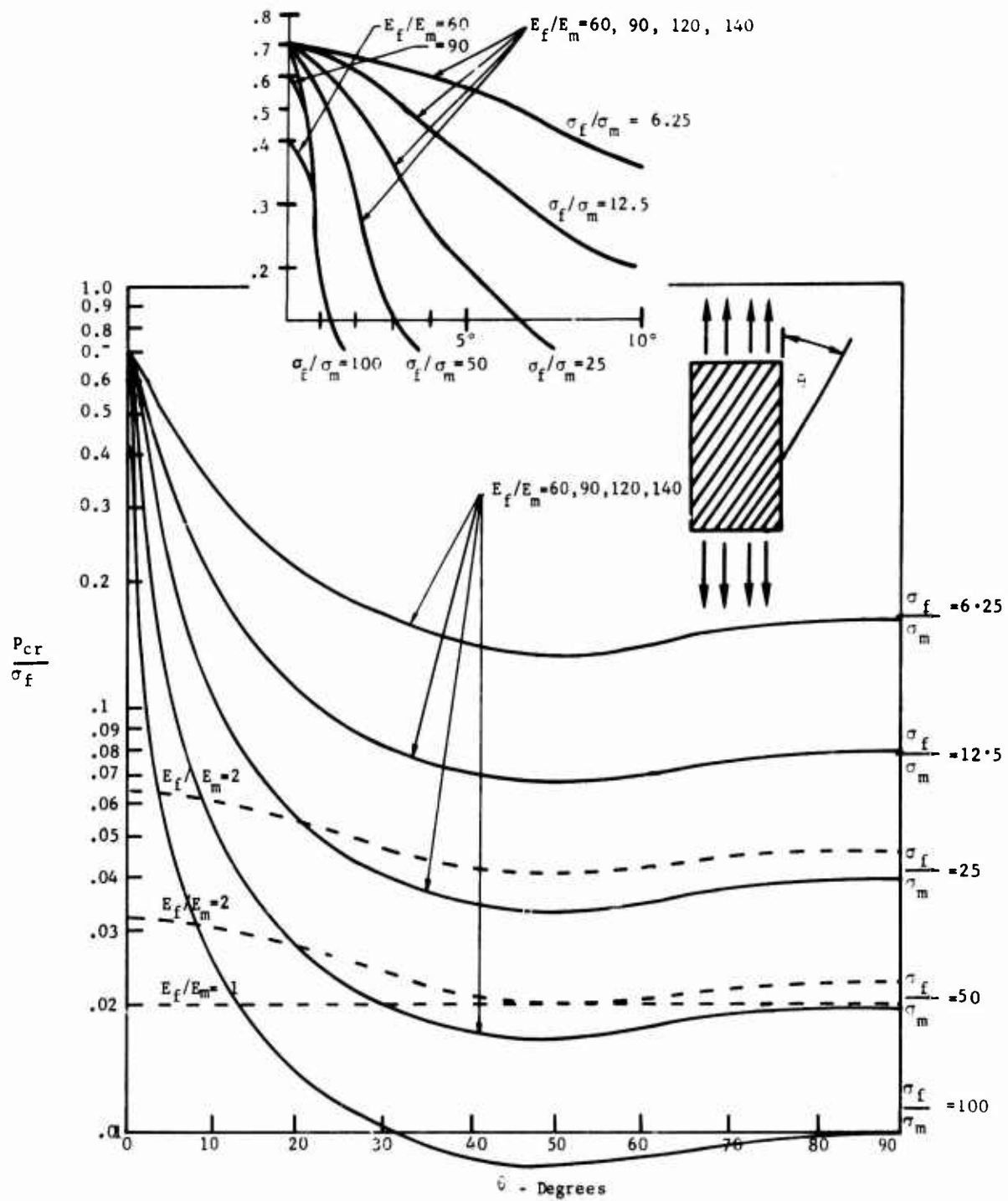


Figure 6. Critical Loading ( $p_{cr}$ ) of a Composite ( $V_f=0.7$ ) Under Oblique Loading as a Function of Loading Angle ( $\theta$ ), Fiber and Matrix Yield Strength, and Moduli of Elasticity ( $\theta$ ,  $\sigma_f$ ,  $\sigma_m$ , and  $E_f$ ,  $E_m$ ).

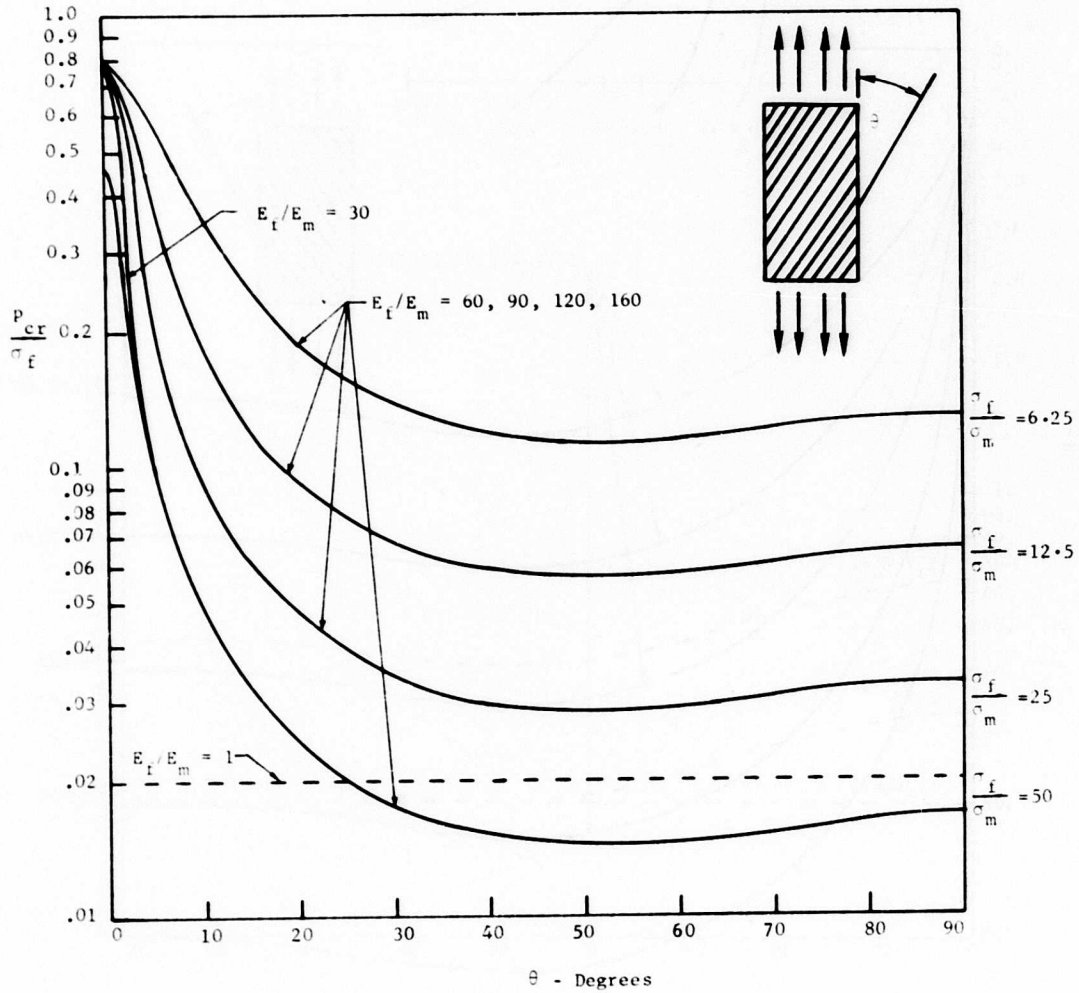


Figure 7. Critical Loading ( $p_{cr}$ ) of a Composite ( $V_f=0.8$ ) Under Oblique Loading as a Function of Loading Angle ( $\theta$ ), Fiber and Matrix Yield Strength, and Moduli of Elasticity ( $\theta$ ,  $\sigma_f$ ,  $\sigma_m$ , and  $E_f$ ,  $E_m$ ).

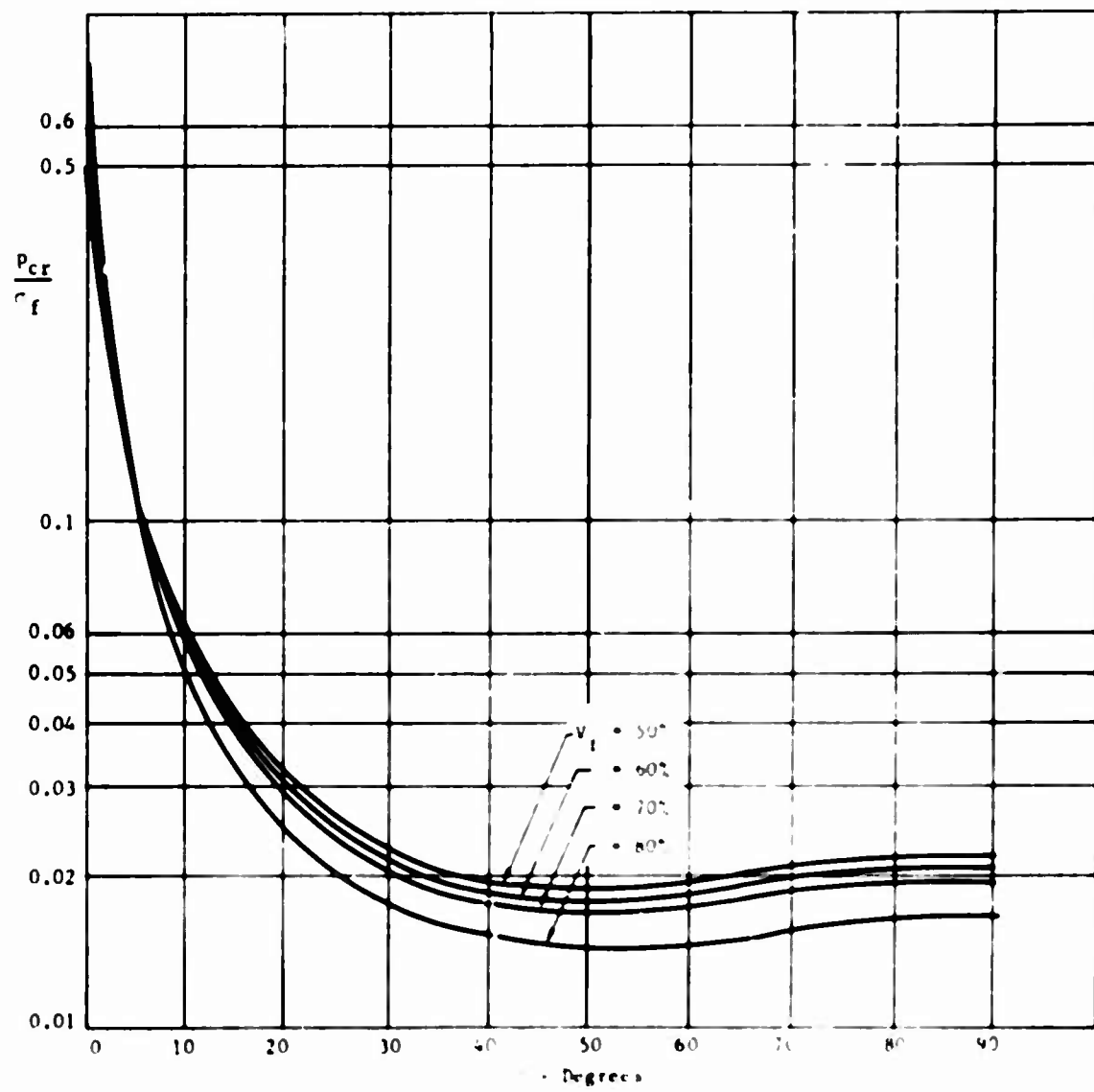


Figure 8. Critical Loading of a Composite Under Oblique Loading.

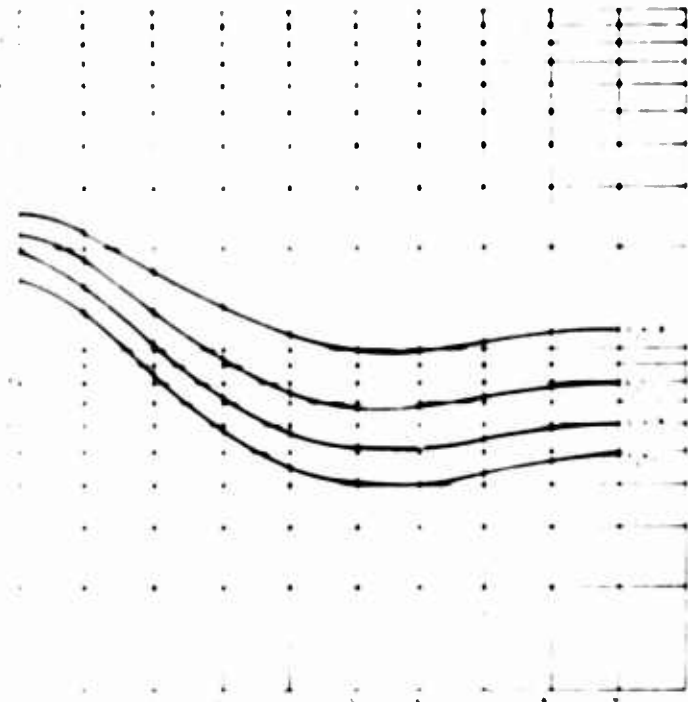


Figure 9. Major Composite - Young's Modulus vs. Fiber Orientation ( $E_f/E_m = 30$ ).

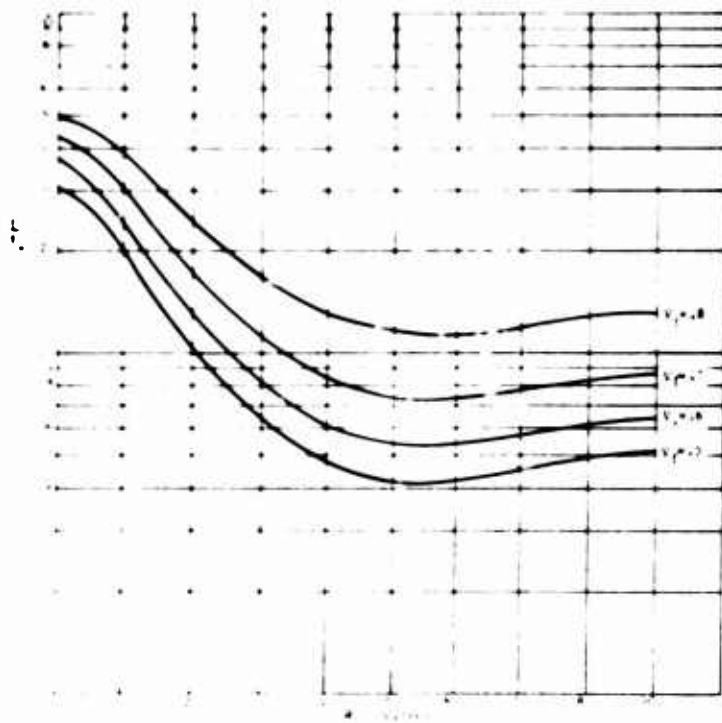


Figure 10. Major Composite - Young's Modulus vs. Fiber Orientation ( $E_f/E_m = 60$ ).

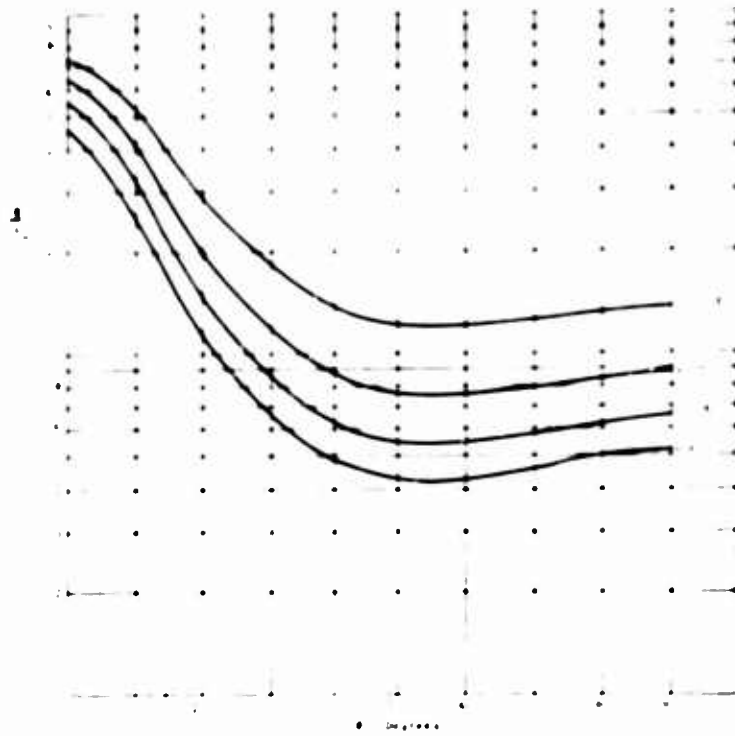


Figure 11. Major Composite - Young's Modulus vs. Fiber Orientation ( $E_f/E_m = 90$ ).

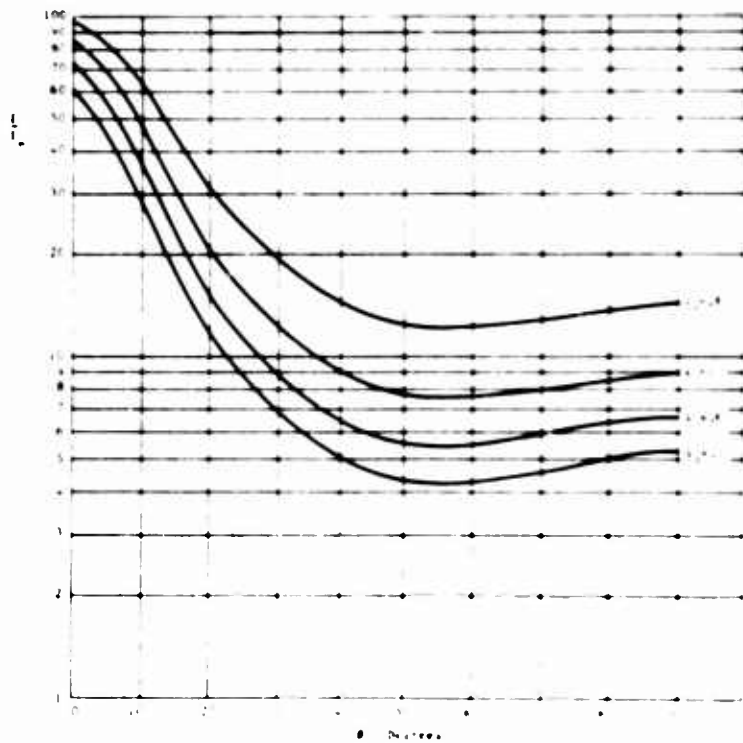


Figure 12. Major Composite - Young's Modulus vs. Fiber Orientation ( $E_f/E_m = 120$ ).

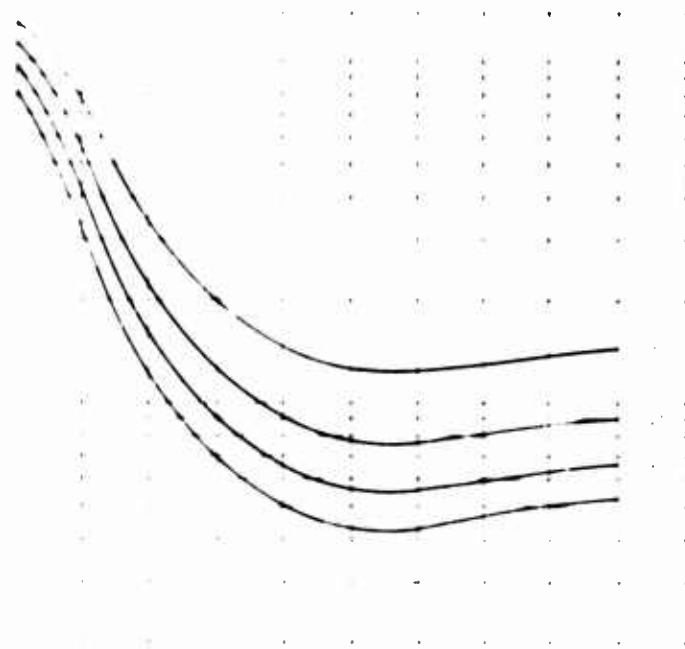


Figure 13. Major Composite - Young's Modulus vs. Fiber Orientation ( $E_f/E_m = 160$ ).

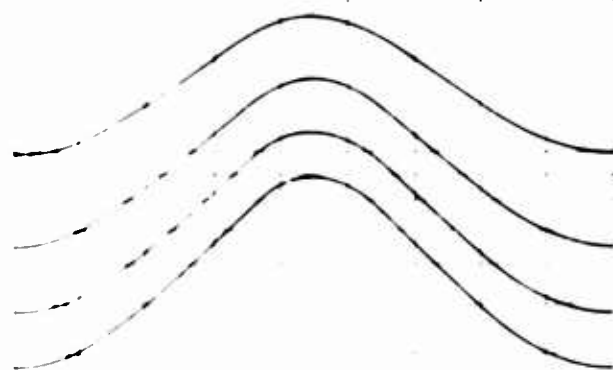


Figure 14. Composite Shear Modulus vs. Fiber Orientation ( $E_f/E_m = 30$ ).

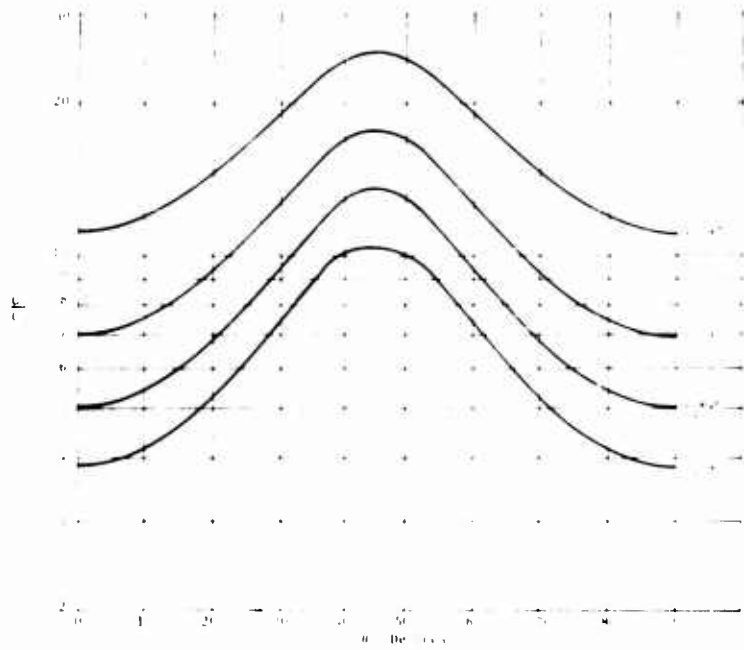


Figure 15. Composite Shear Modulus vs. Fiber Orientation ( $E_f/E_m = 60$ ).

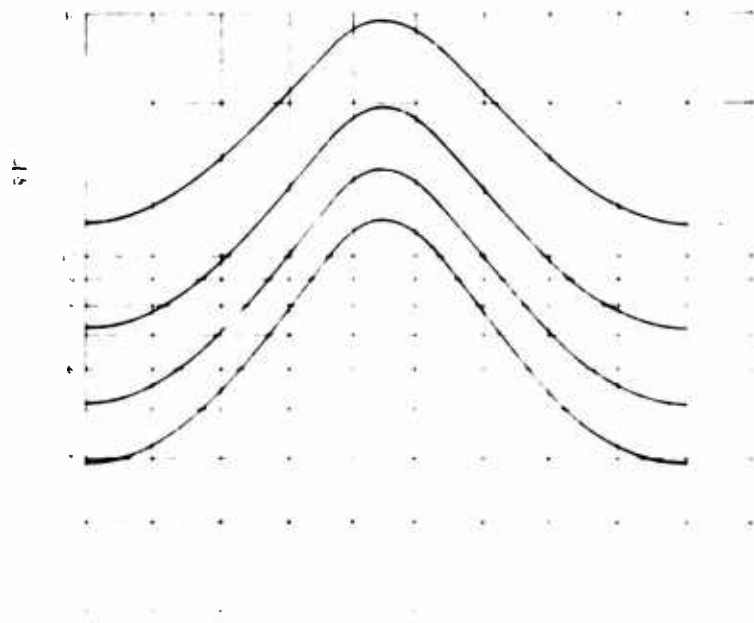


Figure 16. Composite Shear Modulus vs. Fiber Orientation ( $E_f/E_m = 90$ ).

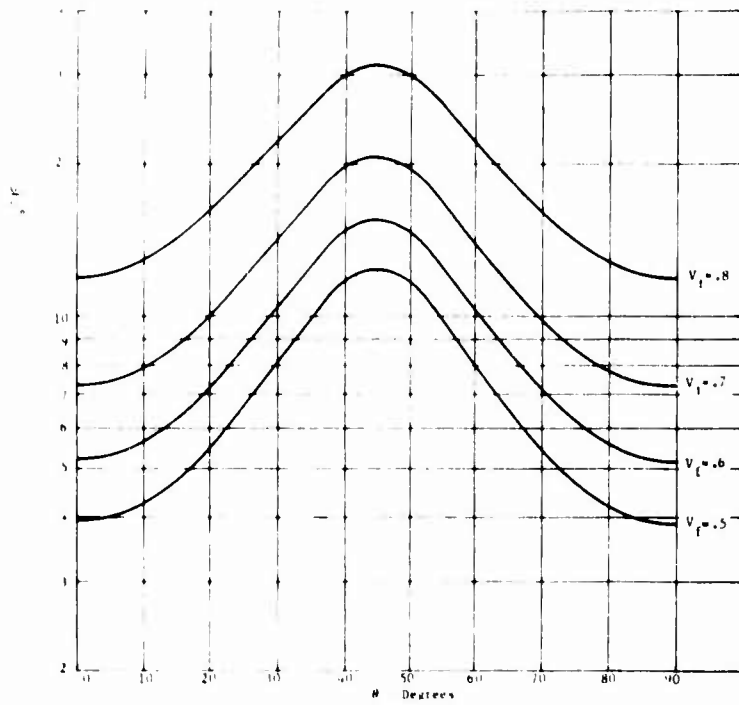


Figure 17. Composite Shear Modulus vs. Fiber Orientation ( $E_f/E_m = 120$ ).

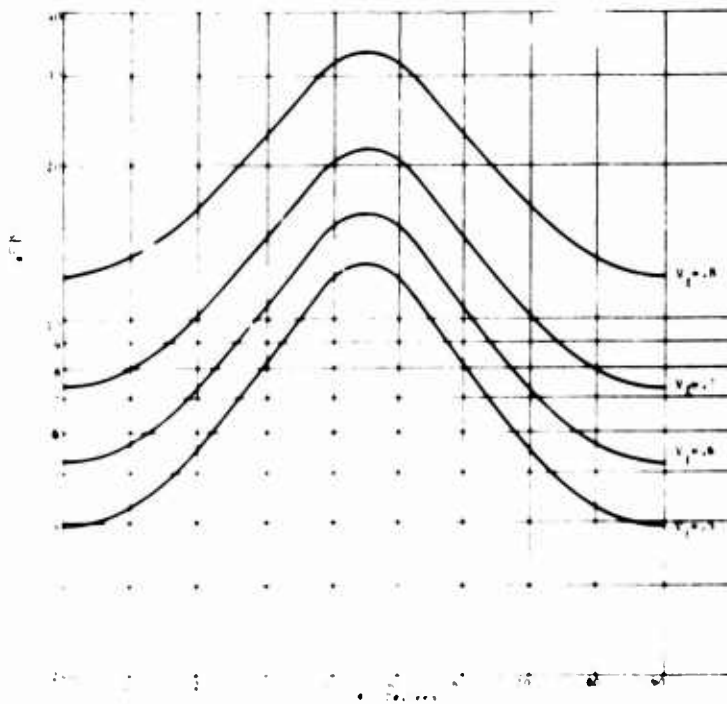


Figure 18. Composite Shear Modulus vs. Fiber Orientation ( $E_f/E_m = 160$ ).

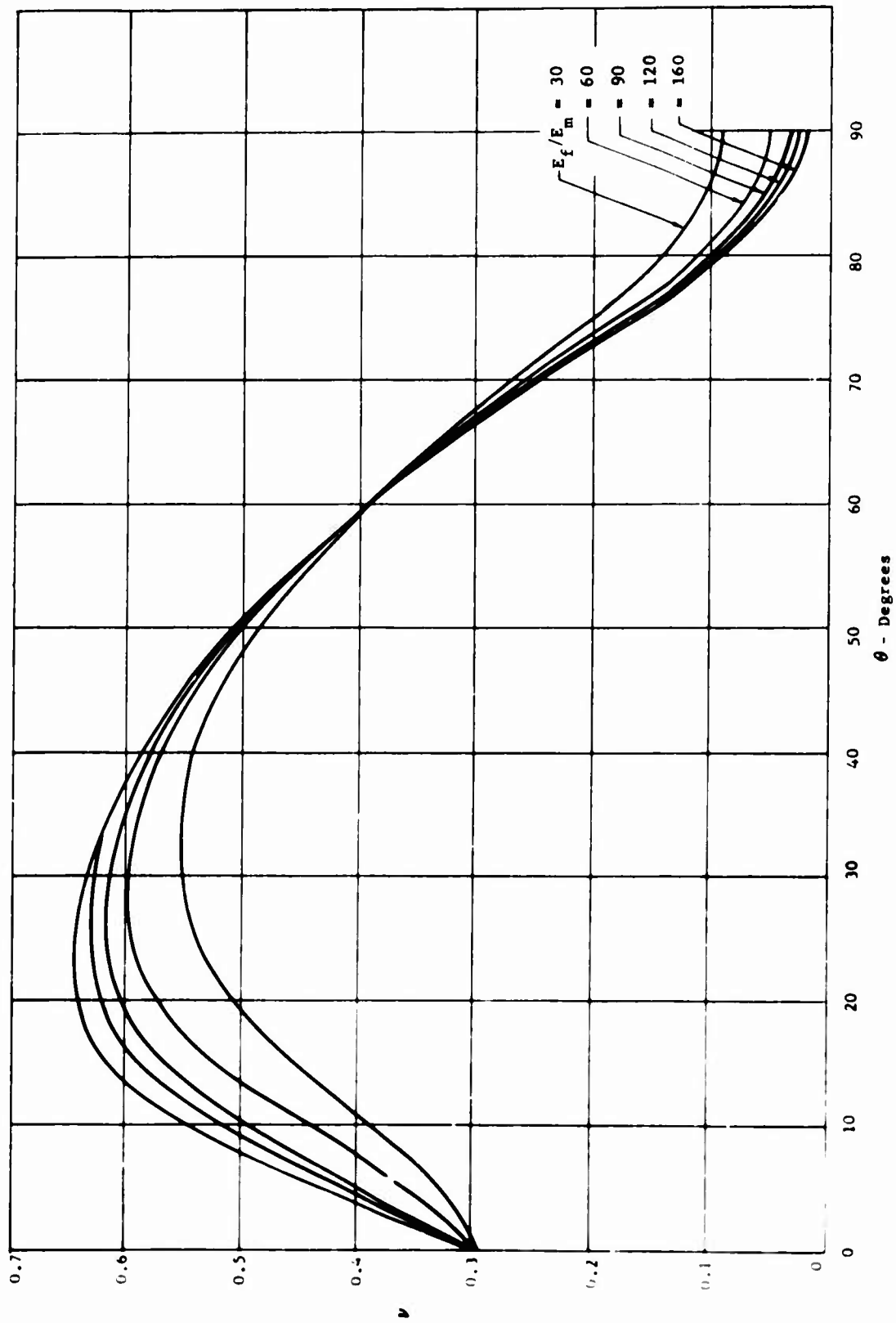


Figure 19. Major Composite - Poisson's Ratio vs. Fiber Orientation ( $V_f = .5$ ).

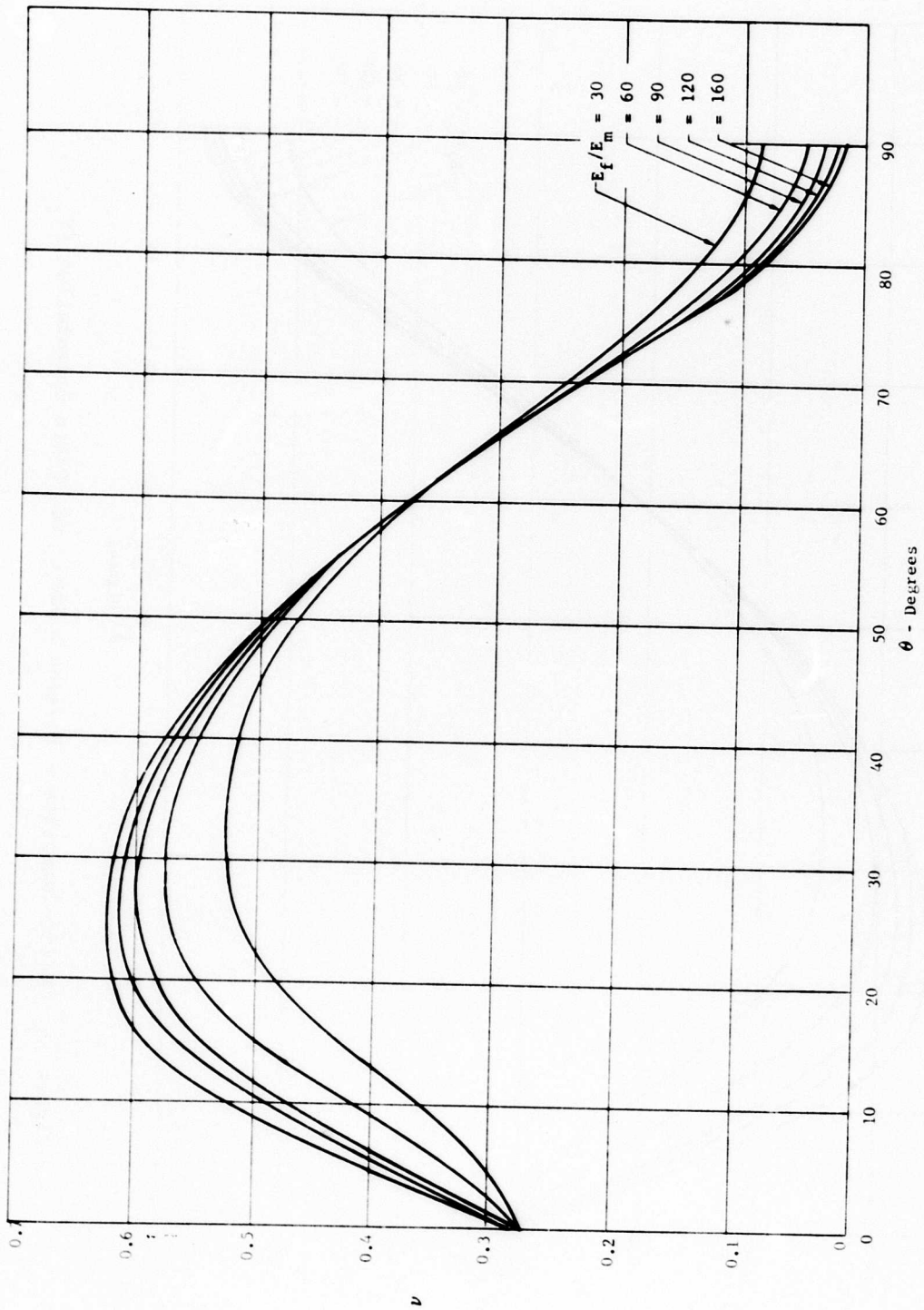


Figure 20. Major Composite - Poisson's Ratio vs. Fiber Orientation ( $V_f = .6$ ).

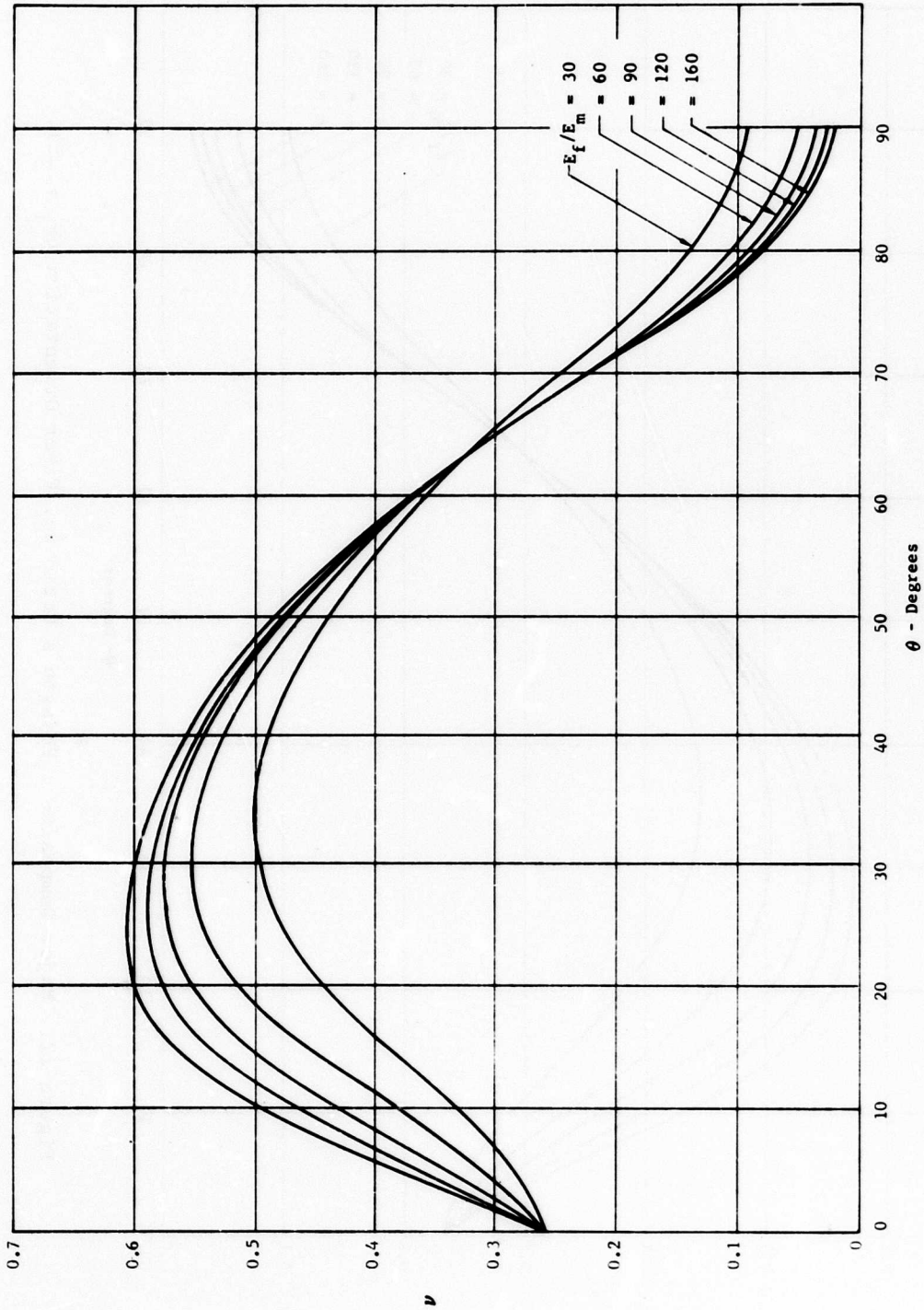


Figure 21. Major Composite - Poisson's Ratio vs. Fiber Orientation ( $V_f = .7$ ).

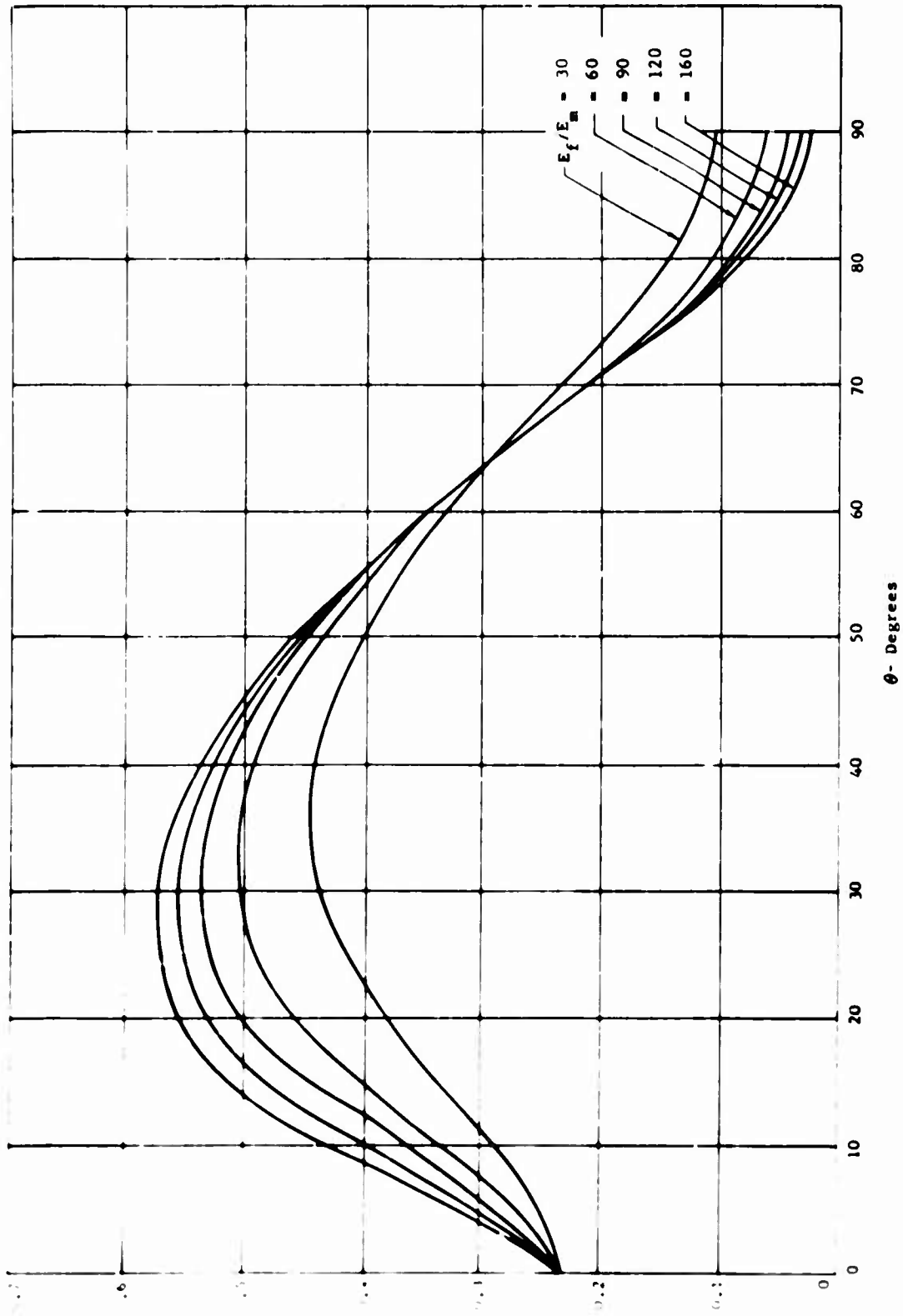


Figure 22. Major Composite - Poisson's Ratio vs. Fiber Orientation ( $V_f = .8$ ).

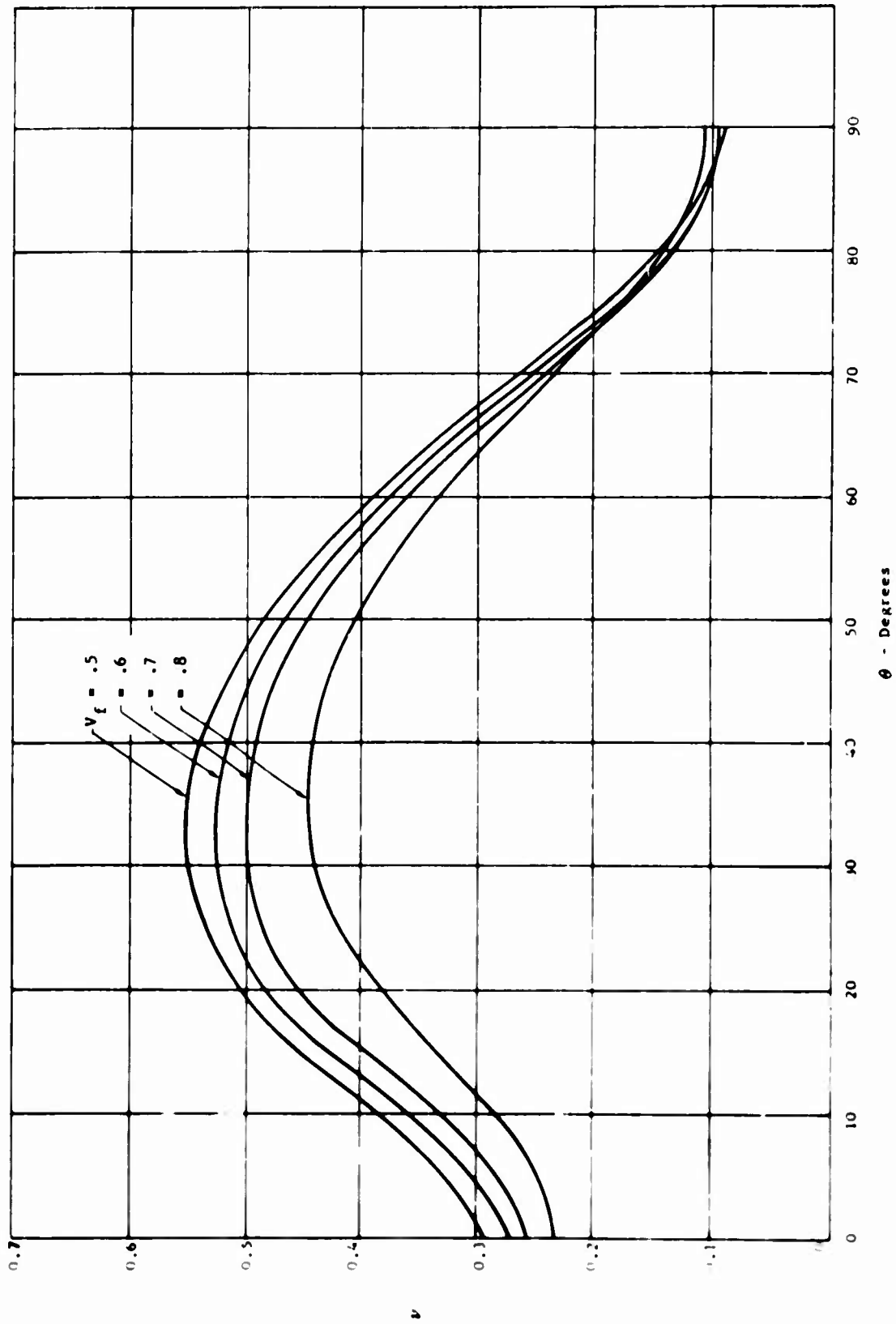


Figure 23. Major Composite - Poisson's Ratio vs. Fiber Orientation for  $E_f/E_m = 30$ .

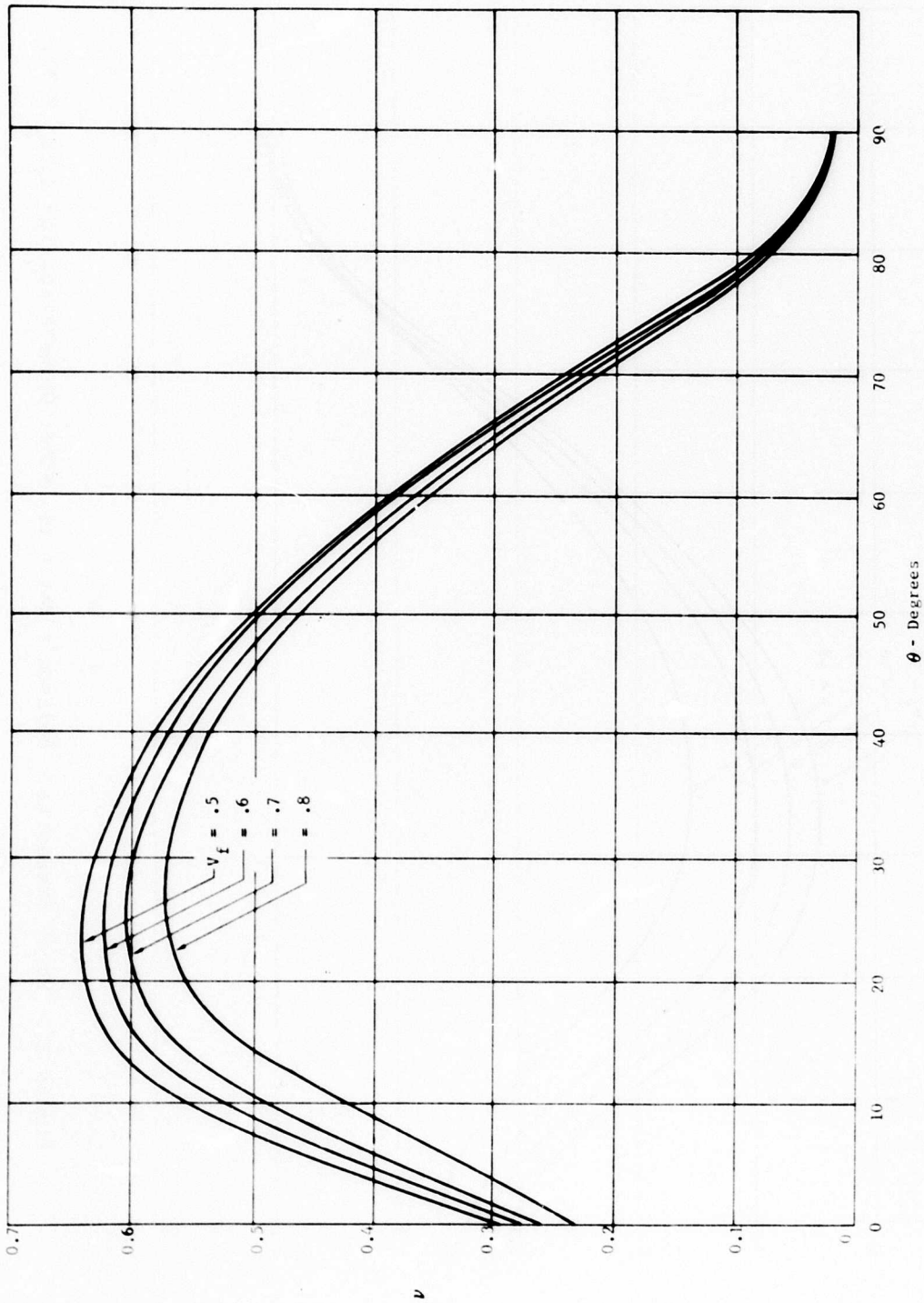


Figure 24. Major Composite - Poisson's Ratio vs. Fiber Orientation for  $E_f/E_m = 160$ .

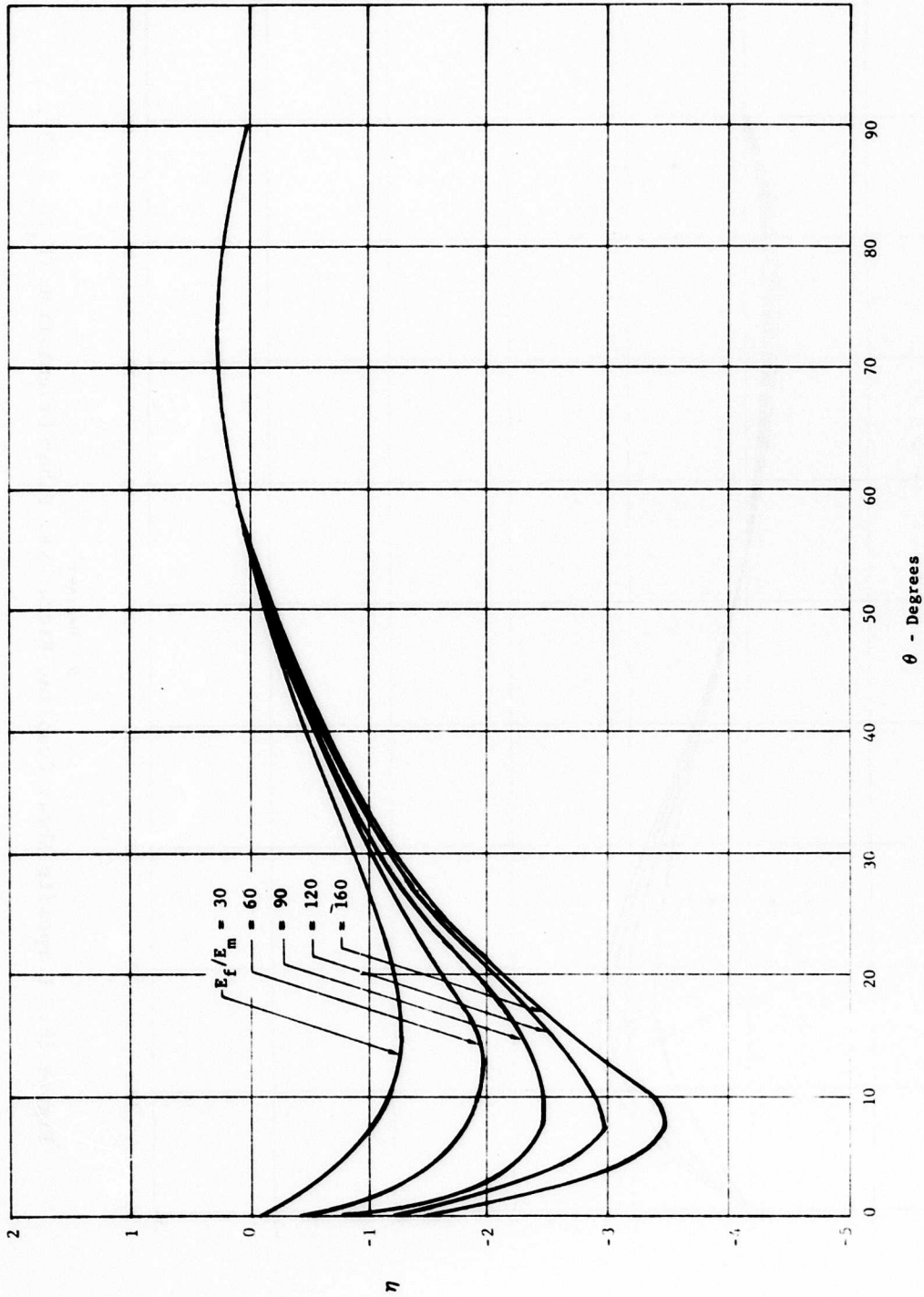


Figure 25. Composite Shear Coupling Factor vs. Fiber Orientation ( $V_f = .5$ ).

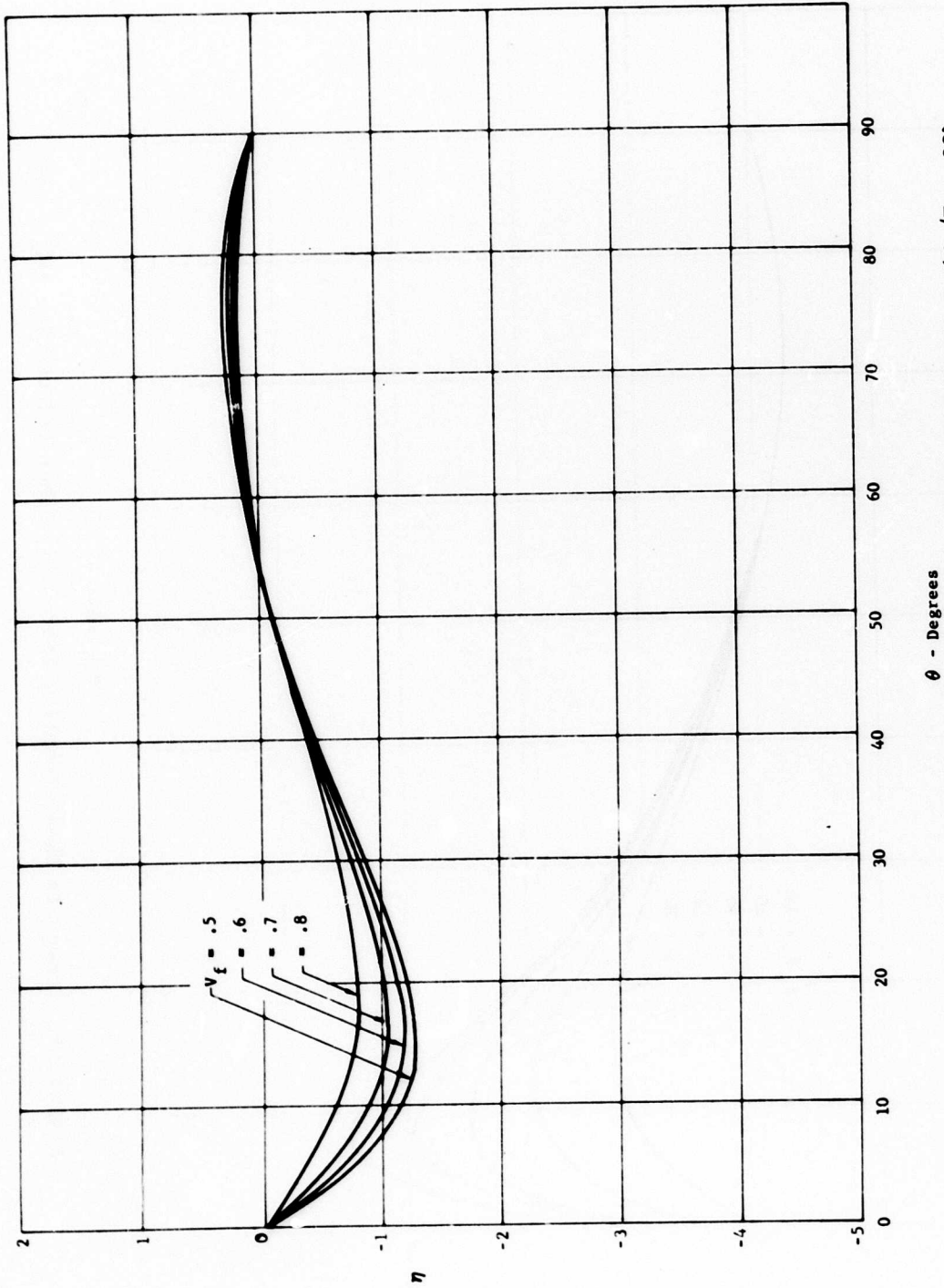


Figure 26. Composite Shear Coupling Factor vs. Fiber Orientation ( $E_f/E_m = 30$ ).

### CONCLUSIONS AND RECOMMENDATIONS

From the data presented herein and from comparisons with tests and analyses performed by other investigators (References 10, 11, and 12), it can be concluded that this kind of analysis results in valid solutions for the strength criterion of a composite. The authors of the cited papers critically compared their results with test data stemming from a variety of sources. Furthermore, results obtained by Tsai (References 13 and 14) also contribute to the confidence of the values published in this report. The comparison could be made only for axial and transverse loading because the analysis and the tests published in the literature are concerned only with these two loading conditions while the present work contains all possible analysis of loadings.

The combined stresses in each particle of the reinforcement and the matrix introduced into a strength criterion give a real picture of the strength of a composite. As can be seen from the curves, a composite loses much of its strength when the loading is not in the direction of the reinforcement. When loading in the fiber direction is present, then the failure occurs in the fiber as long as  $\frac{E_f}{E_m} > \frac{\sigma_{fer}}{\sigma_{mer}}$ ; otherwise, failure occurs in the

matrix. In cases where the loading is inclined to the fiber direction, even at very small angles, failure occurs in the resin, at the interface, and at those points where the surfaces of two adjacent fibers are closest together. The strength of a composite obtained with this analysis is on the conservative side. In reality, a composite loaded in the transverse direction, for instance, fails initially in the resin but is then capable of taking twice the load of initial failure before catastrophic structural failure occurs. The reason for this is that the matrix of most materials becomes nonlinear, and instead of failing, it smooths out the stress peaks.

An analysis which takes into account a nonlinear stress/strain relation would be the natural continuation of this work. It would reveal higher strength of a composite in the transverse direction. A realistic nonlinear analysis of a composite can be based only on micromechanics where the stress distribution in the components and the dislocations of the fibers due to loads are obtainable in detail. The present analysis possesses the above-mentioned capabilities.

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## APPENDIX I

### BOUNDARY CONDITIONS FOR THE TWO-DIMENSIONAL TRANSVERSE SHEAR PROBLEM

#### ANALYTICAL PART

To find the stress distribution within a typical micromechanical element of a fiber-matrix configuration, proper conditions at its boundaries must be established first. The objective of this part of the analysis is to derive the boundary conditions from the geometry aspects inherent to the load configuration. In order to define the problem we refer to Figure 27, showing the assembly of micromechanical elements as they would appear within a fiber-matrix material far away from its edges and loads. Under this assumption we can consider that the stress-strain distribution around each fiber center must be identical for any arbitrarily chosen fiber center. The external loads causing the so-called "transverse shear deformation" are applied force couples, one acting in the  $\pm$  x-direction and the other in the  $\mp$  y-direction. In Figure 27 the forces are denoted by  $P_1, -P_1$  and  $P_2, -P_2$ , which are parallel to the x- and y-axes, respectively. In general, magnitudes of the force couples are independent of each other, but are usually the results of some equilibrium conditions.

In order to find the boundary conditions for the rectangle ABCD, we must also consider the two adjacent rectangles A'DCB' and CBA'D'' and investigate how the displacement vectors in these rectangles can be related to each other. For this purpose, we introduce three local Cartesian coordinate systems with their origins at the rectangle centroids  $M_0, M_0'$  and  $M_0''$ . The above-defined coordinate systems are the x, y - system, the x'y' - system, and the x'', y'' - system. The two latter coordinate axes are generated from the former by translation; i.e.,

$$x' = x \quad (70)$$

$$y' = y - b$$

$$x'' = x - 2c \quad (71)$$

$$y'' = y$$

Let us now consider a point P within the boundaries of rectangle ABCD which has coordinates (x, y) with respect to the non-primed coordinate system. We denote the displacement vector at P by  $\bar{u}(x,y)$ , which can be expressed by its components as follows:

$$\bar{u}(x,y) = \bar{i} u_x(x,y) + \bar{j} u_y(x,y) \quad (72)$$

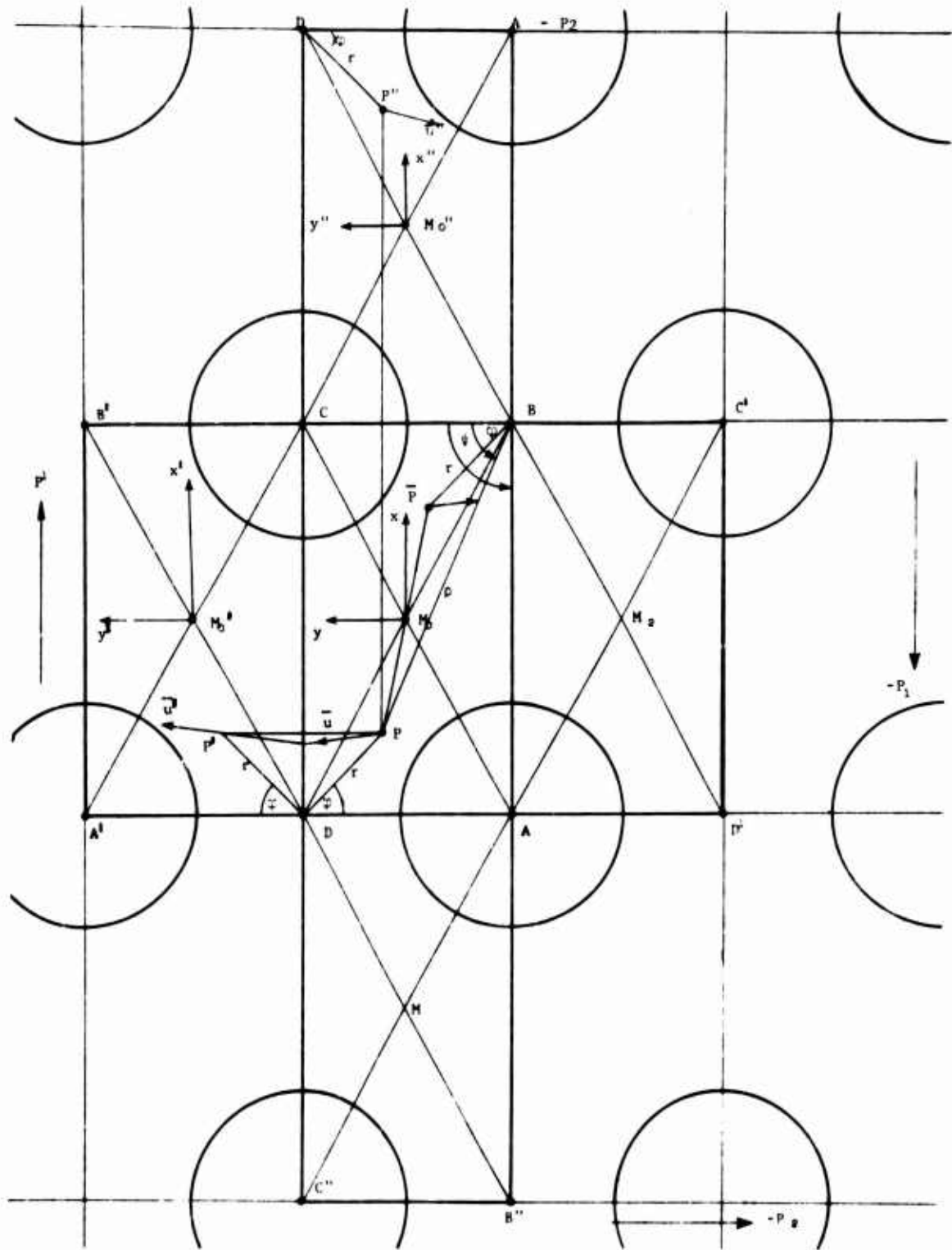


Figure 27. Geometry of the Basic Representative Element.

The displacement  $\bar{u}(x,y)$  is in reference to point  $M_0$ , and we note that

$$\bar{u}(0,0) = 0$$

and therefore

$$\begin{aligned} u_x(0,0) &= 0 \\ u_y(0,0) &= 0 \end{aligned} \tag{73}$$

If we now look at the rectangle upside down, then we notice that the configuration as well as external force geometry remains unchanged. That means that the displacement vector at the point  $\bar{P}(-x,-y)$  in the  $180^\circ$  rotated coordinate system must be identical to the displacement vector at  $P(x,y)$ . Therefore,

$$\bar{i} u_x(x,y) + \bar{j} u_y(x,y) = -\bar{i} u_x(-x,-y) - \bar{j} u_y(-x,-y)$$

or

$$\begin{aligned} u_x(x,y) &= -u_x(-x,-y) \\ u_y(x,y) &= -u_y(-x,-y) \end{aligned} \tag{74}$$

Equation (74) shows the fact that within each rectangle there exists a pair of points which are located centrally symmetrical with respect to the centroid and for which the relative displacements are also centrally symmetrical with respect to each other.

Next we have to consider rectangle  $A'DCB'$  and investigate the relative displacement vector with respect to centroid  $M'_0$  at a point  $\bar{P}'(x',y')$  within that rectangle. The relative displacement at  $P'$  is

$$\bar{u}'(x',y') = \bar{i} u'_x(x',y') + \bar{j} u'_y(x',y') \tag{75}$$

At the point  $M'_0(x'=0,y'=0)$ , we have

$$\begin{aligned} u'_x(0,0) &= 0 \\ u'_y(0,0) &= 0 \end{aligned} \tag{76}$$

The fiber configuration within rectangle  $A'DCB'$  is the mirror image of the fiber configuration within rectangle  $ABCD$  with the mirror line  $y' = -\frac{b}{2}$  (or  $y = +\frac{b}{2}$ ). However, the mirror images of the external force configuration become antidirectional but collinear. This means that by changing

the sign of all external forces, the relative displacement vector in rectangle A'DCB' will be the mirror image of the relative displacement vector within rectangle ABCD. The mirror line is again  $y' = -\frac{b}{2}$ . Changing the sign of external force for rectangle A'DCB' is equivalent to changing the sign of the displacement vector. Let us consider point  $P'(x_M, y_M)$  being the mirror image of point  $P(x, y)$  with respect to  $y = +\frac{b}{2}$ ; then

$$\begin{aligned}x_M &= x \\y_M &= y + 2\left(\frac{b}{2} - y\right) = b - y\end{aligned}$$

or with (70),

$$\begin{aligned}x'_M &= x \\y'_M &= y_M - b = b - y - b = -y\end{aligned}\tag{77}$$

The negative mirror image of vector  $\bar{u}(x, y)$  will be  $\bar{u}'(x'_M, y'_M)$  as follows:

$$\bar{u}'(x'_M, y'_M) = - \left\{ \bar{i} u_x(x, y) - \bar{j} u_y(x, y) \right\}\tag{78}$$

Because of (77) and  $\bar{i} = \bar{i}'$ ,  $\bar{j} = \bar{j}'$ ,

$$u'(x'_M, y'_M) = \bar{i} u'_x(x, -y) + \bar{j} u'_y(x, -y)\tag{79}$$

Combination of (78) and (79) gives

$$\bar{i} \left[ u'_x(x, -y) + u_x(x, y) \right] + \bar{j} \left[ u'_y(x, -y) - u_y(x, y) \right] = 0\tag{80}$$

This vector equation can only be satisfied if each component becomes zero. Therefore, the following two equations are obtained:

$$u'_x(x, -y) = -u_x(x, y)\tag{81}$$

$$u'_y(x, -y) = u_y(x, y)\tag{82}$$

Equations (81) and (82) show the relative displacements in rectangles A'DCB' and ABCD to each other.

In a similar fashion we can get a relation between the relative displacements in rectangle BA'D'C and the corresponding relative displacements in rectangle ABCD. We make use of the mirror symmetry about the line  $x = c$  and apply the same line of reasoning as before (to distinguish from other symmetries, the sub-index M is primed).

The relative displacement at a point P'' ( $x'', y''$ ) within rectangle BA'D'C is

$$\bar{u}''(x'', y'') = \bar{i}'' u_x''(x'', y'') + \bar{j}'' u_y''(x'', y'') \quad (83)$$

Since  $\bar{u}''(x'', y'')$  is in reference to  $M''_0$ , it is again

$$u''(0,0) = 0 \quad (84)$$

or

$$u_x''(0,0) = 0 \text{ and } u_y''(0,0) = 0$$

The mirror symmetric point to P with respect to line  $x = c$  has the coordinates  $x_M', y_M'$ , as follows:

$$x_{M'} = x + 2(c - x) = 2c - x$$

$$y_{M'} = y$$

or with (71),

$$x_{M'}'' = -x; y_{M'}'' = y \quad (85)$$

We consider the negative mirror image of vector  $\bar{u}(x,y)$  with respect to line  $x = c$  and obtain

$$\bar{u}''(x_{M'}'', y_{M'}'') = - \left\{ -\bar{i} u_x(x,y) + \bar{j} u_y(x,y) \right\} \quad (86)$$

Because of (83) and  $\bar{i} = \bar{i}'', \bar{j} = \bar{j}'',$  we get

$$\bar{u}''(x_{M'}'', y_{M'}'') = \bar{i} u_x''(-x,y) + \bar{j} u_y''(-x,y) \quad (87)$$

and therefore

$$\bar{i} \left[ u_x''(-x,y) - u_x(x,y) \right] + \bar{j} \left[ u_y''(-x,y) + u_y(x,y) \right] = 0$$

Hence, we get the desired relations:

$$u_x''(-x,y) = u_x(x,y) \quad (88)$$

$$u_y''(-x,y) = -u_y(x,y) \quad (89)$$

The displacement vectors  $\bar{u}'$  and  $\bar{u}''$  at points  $P'$  and  $P''$  are in reference to  $M'_0$  and  $M''_0$  respectively. In order to obtain the corresponding displacement vectors with respect to  $M_0$ , we must introduce the relative displacement vectors of  $M'_0$  and  $M''_0$  with respect to  $M_0$ , which are denoted by  $\bar{U}_0, \bar{V}_0$ . In general the relative displacement vectors for points along the line  $x = 0$  and  $y = 0$  are as follows:

$$\bar{U}(x,y) = \bar{i} U_x(y) + \bar{j} U_y(y) \quad (90)$$

$$\bar{V}(x,y) = \bar{i} V_x(x) + \bar{j} V_y(x) \quad (91)$$

Because  $U_y(x,y) = \text{const}$  and  $V_x(x,y) = \text{const}$  and the symmetry relations are as follows, we have

$$\bar{U}(y) = - \bar{U}(x,-y)$$

and

$$\bar{V}(x) = - \bar{V}(-x)$$

Therefore,

$$U_y(y) = 0 \quad (92)$$

$$V_x(x) = 0 \quad (93)$$

and

$$U_x(y) = - U_x(-y) \quad (94)$$

$$V_y(x) = - V_y(-x) \quad (95)$$

Also

$$\bar{U}_0 = \bar{i} U_x(0,b) \quad (96)$$

and

$$\bar{V}_0 = \bar{j} V_y(2c,0) \quad (97)$$

The displacement vector at point P' relative to  $M_0$  is therefore

$$\bar{u}'_{TOT}(x, -y) = \bar{u}'(x, -y) + \bar{U}_0 \quad (98)$$

and the displacement vector at point P'' relative to  $M_0$  is similar; i.e.,

$$\bar{u}''_{TOT}(-x, y) = \bar{u}''(-x, y) + \bar{V}_0 \quad (99)$$

With (77), (83), (96), and (97), we get

$$\bar{u}'_{TOT}(x, -y) = \bar{i} \left[ u'_x(x, -y) + U_x(b) \right] + \bar{j} u'_y(x, -y) \quad (100)$$

$$\bar{u}''_{TOT}(-x, y) = \bar{i} u''_x(-x, y) + \bar{j} \left[ u''_y(-x, y) + V_y(2c) \right] \quad (101)$$

These two relations are meaningful only when there are continuities of the displacement vector at the lines  $y = \frac{b}{2}$  and  $x = c$ .

At these lines, we must have

$$\bar{u}'_{TOT}\left(x, +\frac{b}{2}\right) = \bar{u}\left(x, \frac{b}{2}\right) \quad (102)$$

and

$$\frac{\partial^n}{\partial y^n} \bar{u}''_{TOT}\left(x, -\frac{b}{2}\right) = \frac{\partial^n}{\partial x^n} \bar{u}\left(x, \frac{b}{2}\right) \quad (103)$$

$$(n = 1, 2, 3, \dots)$$

also,

$$\bar{u}''_{TOT}(-c, y) = \bar{u}(c, y) \quad (104)$$

and

$$\frac{\partial^n}{\partial y^n} \bar{u}''_{TOT}(-c, y) = \frac{\partial^n}{\partial y^n} \bar{u}(c, y) \quad (105)$$

From (102), (100), and (72), we get

$$\bar{i} \left[ u'_x\left(x, -\frac{b}{2}\right) - u\left(x, \frac{b}{2}\right) + U_x(b) \right] + \bar{j} \left[ u'_y\left(x, -\frac{b}{2}\right) - u_y\left(x, \frac{b}{2}\right) \right] = 0 \quad (106)$$

Because of (81) and (82) and the fact that each component of the vector equation above must vanish, it follows that

$$u(x, \frac{b}{2}) = \frac{1}{2} U_x(b) \quad (107)$$

(107) represents one boundary condition along line  $y = b$  (in Figure 28 line CD).

From (103) ( $n = 1$ ), we have

$$i \frac{\partial u'}{\partial x}(x, -\frac{b}{2}) - \frac{\partial u}{\partial x}(x, \frac{b}{2}) + j \frac{\partial u'}{\partial x}(x, -\frac{b}{2}) - \frac{\partial u}{\partial x}(x, +\frac{b}{2}) = 0 \quad (108)$$

From (9), (10) and their partial differentiation with respect to  $x$  at  $y = \frac{b}{2}$ , we get

$$\frac{\partial u'}{\partial x}(x, -\frac{b}{2}) = -\frac{\partial u}{\partial x}(x, \frac{b}{2}) \quad (109)$$

and

$$\frac{\partial u'}{\partial y}(x, -\frac{b}{2}) = \frac{\partial u}{\partial y}(x, \frac{b}{2}) \quad (110)$$

Equations (108), (109), and (110) give

$$\frac{\partial u}{\partial x}(x, \frac{b}{2}) = 0 \quad (111)$$

Differentiation of (82) with respect to  $y$  at  $y = \frac{b}{2}$  gives

$$-\frac{\partial u'}{\partial y}(x, -\frac{b}{2}) = \frac{\partial u}{\partial y}(x, \frac{b}{2}) \quad (112)$$

and with (82) follows

$$-\frac{\partial u}{\partial y}(x, \frac{b}{2}) = \frac{\partial u}{\partial y}(x, \frac{b}{2}) \quad (113)$$

or

$$\frac{\partial u}{\partial y}(x, \frac{b}{2}) = 0 \quad (114)$$

(111) and (114) issue the second boundary conditions along line  $y = b$ . For any two-dimensional problem, the normal stress is a homogeneous linear function of  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$ . It therefore follows that

$$\sigma_x(x, \frac{b}{2}) = 0 \quad (115)$$

$$\sigma_y(x, \frac{b}{2}) = 0 \quad (116)$$

where (116) represents the second boundary condition along line  $y = b$ .

Similar boundary conditions follow from (104) and (105). First we obtain from (104), (99), (87), (72), and (97)

$$\bar{i} \left[ u_x''(-c, y) - u_x''(c, y) \right] + \bar{j} \left[ u_y''(-c, y) - u_y''(c, y) + v_y(2c) \right] = 0 \quad (117)$$

and because of (88) and (89),

$$u_y''(c, y) = \frac{1}{2} v_y(2c) \quad (118)$$

Equation (118) is the first boundary condition at line  $x = c$ .

Setting  $n = 1$  in equation (105) and combining with (99) and (87), we get

$$\bar{i} \left[ \frac{\partial u_x''}{\partial y}(-c, y) - \frac{\partial u_x''}{\partial y}(c, y) \right] + \bar{j} \left[ \frac{\partial u_y''}{\partial y}(-c, y) - \frac{\partial u_y''}{\partial y}(c, y) \right] = 0 \quad (119)$$

The partial differentiation of (89) with respect to  $y$  at  $x = c$  gives

$$\frac{\partial u_y''}{\partial y}(-c, y) = - \frac{\partial u_y''}{\partial y}(c, y) \quad (120)$$

From the vector equation (119), we obtain

$$\frac{\partial u_y''}{\partial y}(-c, y) = \frac{\partial u_y''}{\partial y}(c, y) \quad (121)$$

**CONCLUSION**

The problem of the elastic medium under transverse shear load can be completely attacked by solving a special problem of the basic representative element ABCD of Figure 27. This rectangle has been reproduced below for convenience.

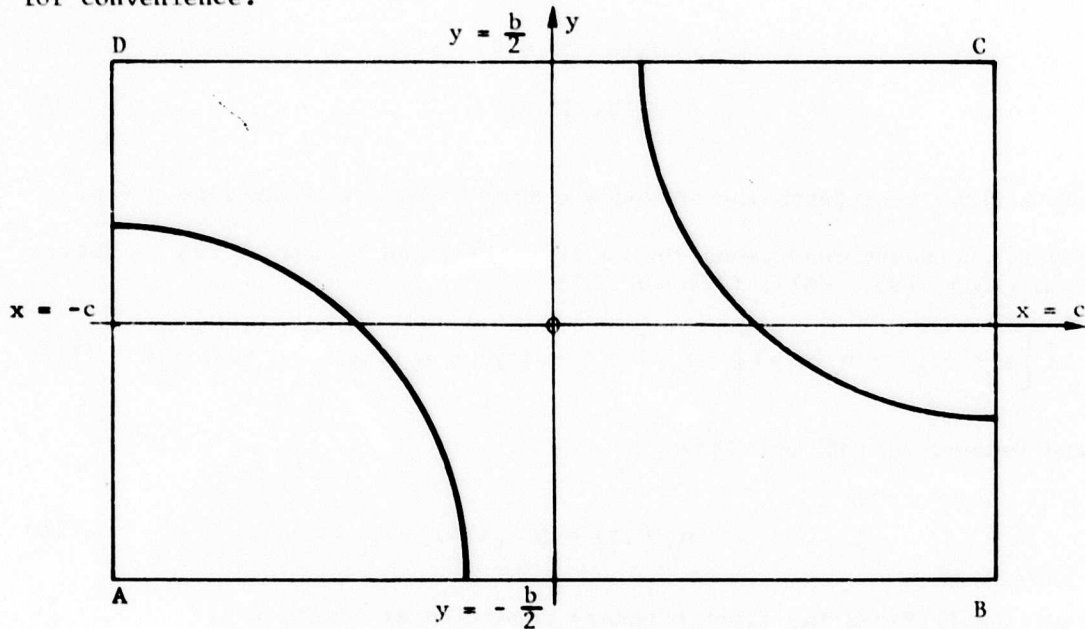


Figure 28. Rectangle ABCD.

From the analysis, the boundary conditions have been established for all boundary lines of Figure 28. For line CD, from equations (107) and (116), we have

$$u_x(x, \frac{b}{2}) = k_1 \tag{129}$$

where  $k_1$  is an arbitrary constant,

and

$$\sigma_y(x, \frac{b}{2}) = 0 \tag{130}$$

$$-c \leq x \leq +c \tag{131}$$

Then

$$\frac{\partial u}{\partial y}(c, y) = - \frac{\partial u}{\partial y}(c, y) \quad (122)$$

or

$$\frac{\partial u}{\partial y}(c, y) = 0 \quad (123)$$

Partial differentiation of (88) with respect to  $x$  at  $x = c$  gives

$$- \frac{\partial u''}{\partial x}(c, y) = \frac{\partial u}{\partial x}(c, y) \quad (124)$$

According to (88), the left-hand side function of (124) can be replaced; thereby

$$- \frac{\partial u}{\partial x}(c, y) = \frac{\partial u}{\partial x}(c, y) \quad (125)$$

is obtained. This means that

$$\frac{\partial u}{\partial x}(c, y) = 0 \quad (126)$$

For the same reason as pointed out above, and because of (123) and (126), it follows that

$$\sigma_x(c, y) = 0 \quad (127)$$

$$\sigma_y(c, y) = 0 \quad (128)$$

The first relation, equation (127), represents the second boundary condition for the line  $x = c$  (in Figure 27 it is the line CB). (118) and (127) are the set of boundary conditions along line  $x = c$ .

Along line CB we obtain the two boundary conditions from (118) and (127)

$$u_y(c, y) = k_2 \quad (132)$$

where  $k_2$  is another arbitrary constant,

and

$$\sigma_x(c, y) = 0 \quad (133)$$

for all values of  $y$  defined by the inequality

$$-\frac{b}{2} \leq y \leq \frac{b}{2} \quad (134)$$

In order to find the boundary conditions along the lines  $\overline{AD}$  and  $\overline{AB}$ , we make use of the central symmetrical property of displacement with respect to the origin of Figure 28. Equation (74) expresses this property in analytic form and we obtain therefore from (129), (130), (132) and (133) with (74):

for line AB,

$$u_x(-x, -\frac{b}{2}) = -k_1 \quad (135)$$

$$\sigma_y(-x, -\frac{b}{2}) = 0 \quad (136)$$

where

$$-c \leq x \leq +c \quad (137)$$

and for line  $\overline{AD}$ ,

$$u_y(-c, -y) = -k_2 \quad (138)$$

$$\sigma_x(-c, -y) = 0 \quad (139)$$

where

$$-\frac{b}{2} \leq y \leq \frac{b}{2} \quad (140)$$

With the boundary conditions (129) to (140), the problem of elasticity is defined and therefore unique solutions must exist. The boundary conditions are especially well suited for the method of finite elements and, therefore, directly applicable to the existing standard two-dimensional numerical program.

In our Fundamental Case  $T_{xy}$  of transverse shear, we take  $k_1 = 0$  and  $k_2 = 1$ .

APPENDIX II

SYMMETRY RELATIONS IN PERFORATED PLATES

In the analysis of composite materials, it is necessary to consider a nonhomogenous medium consisting of a matrix and an array of fibers (or flakes). When the fibers are collimated, equally sized, and regularly arrayed, the symmetry of displacements in the composite can sometimes be deduced without recourse to complete solutions.

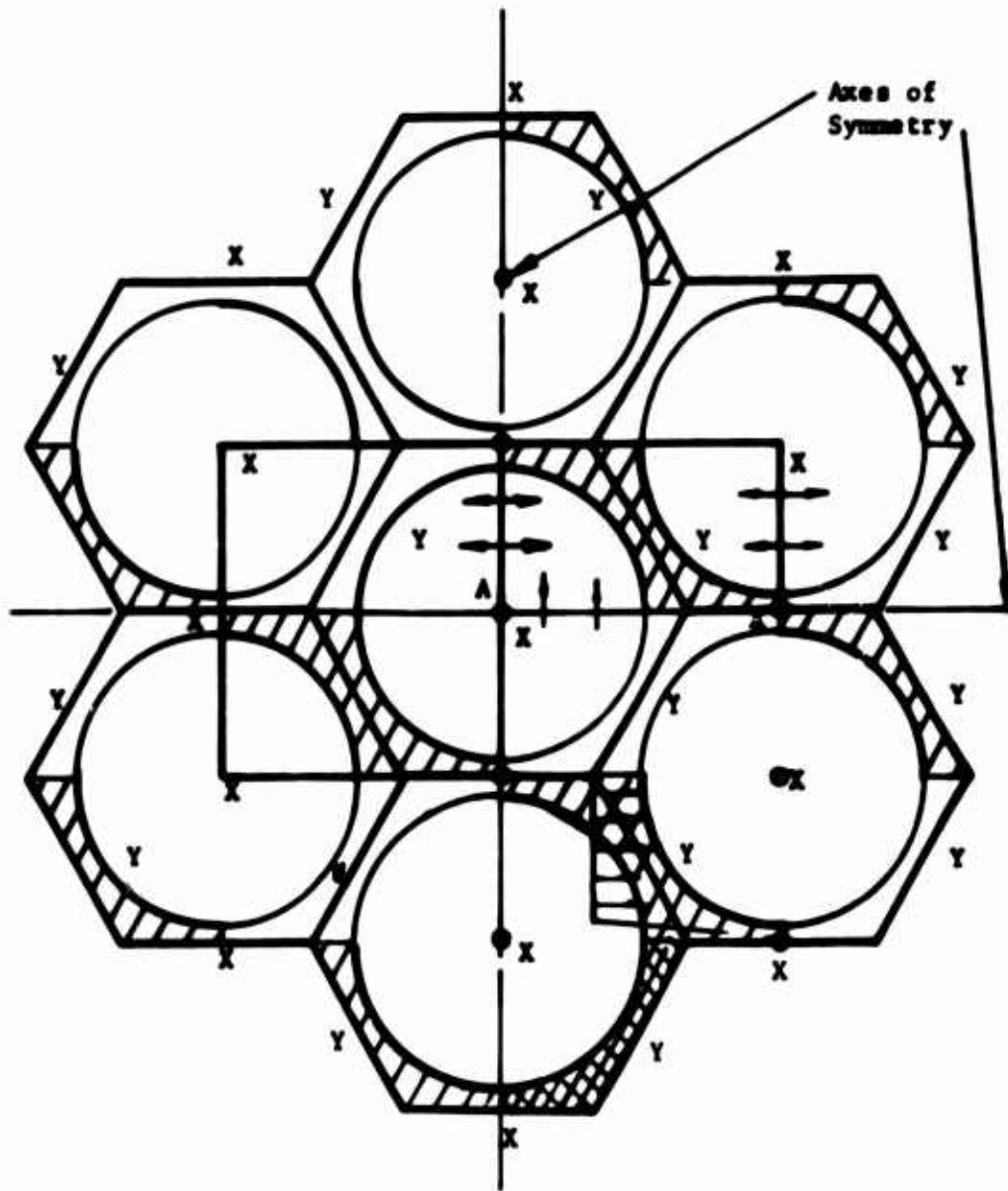


Figure 29. Basic Pattern for Fiber Spacing in Infinitely Large Cross Section (X, Y mark poles).

Let us consider a composite whose fibers are spaced in the hexagonal pattern shown in Figure 29. This pattern is repeated throughout the rectangular cross-section shown in Figure 30. The axes of geometry and physical symmetry pass through point A.

In many typical experiments performed on this model, the resultant loading is inevitably radially symmetric with respect to point A; that is, the applied force vector  $\vec{F}(\vec{r})$  at any point  $\vec{r}$  is the negative of the force vector at  $-\vec{r}$  (see Figure 30).

$$\vec{F}(\vec{r}) = -\vec{F}(-\vec{r}) \quad (141)$$

In view of equation (141), the displacement response  $\vec{u}(\vec{r}, B)$  due to radially symmetric load system B is also radially symmetric; that is,

$$\vec{u}(\vec{r}, B) = -\vec{u}(-\vec{r}, B) \quad (142)$$

In Figure 30, therefore, only elements I and II need be considered further. Responses in elements III and IV can be described using equation (142).

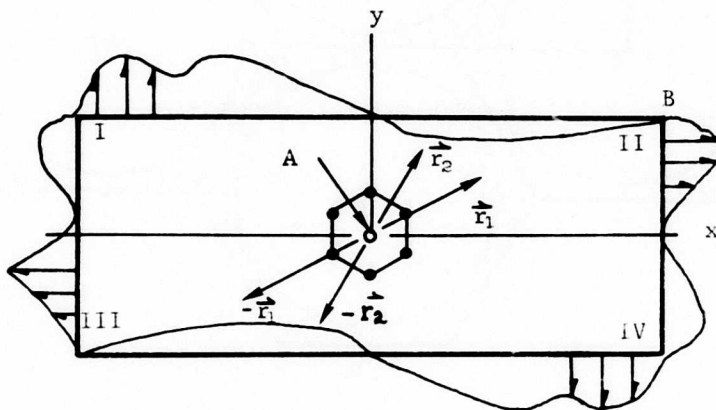


Figure 30. Cross Section Under Radially Symmetric Loading.

Another useful relation is obtained if the observer moves "behind" the section of Figure 30. Equivalently, the plate with its attached loads could be rotated  $180^\circ$  about the y-axis.\* Since the ordinate is an axis of symmetry, the response of the rotated plate is the same as that of the original plate under a rotated load system B. See Figure 31 (a) and (b).

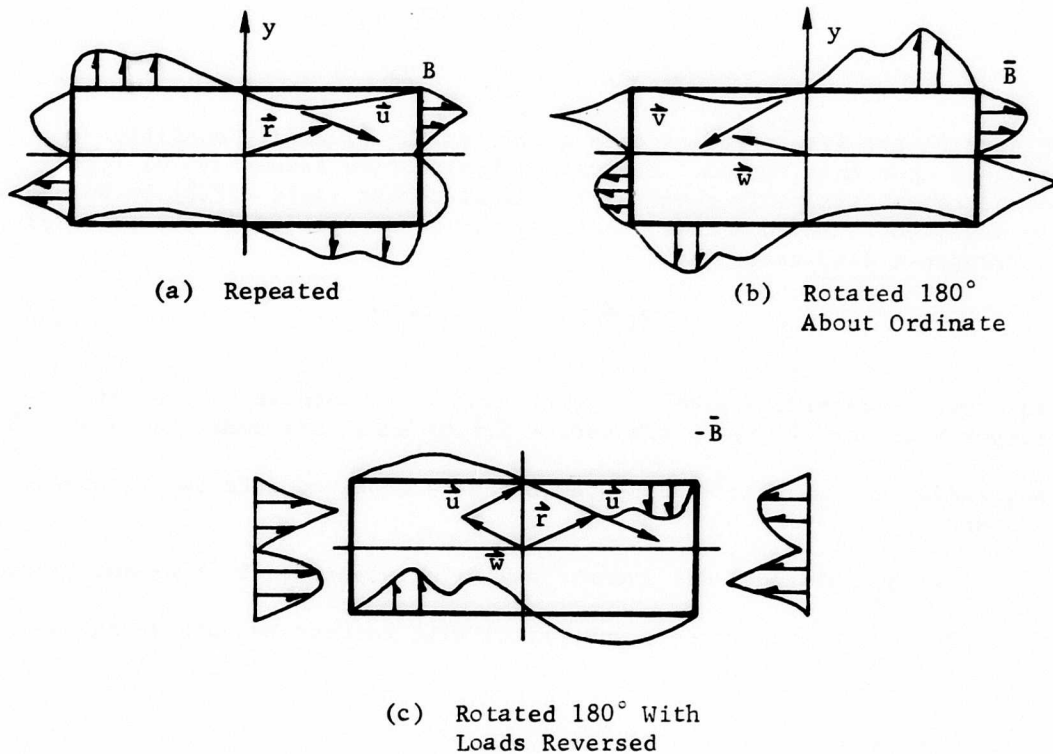


Figure 31. Illustration of Rotated System.

\* Due to the symmetries, rotation about the x-axis would yield the same result.

This can be expressed symbolically if we define the vector operator  $\text{Rot}_y[\vec{r}]$  as the mirror-imaging process just described. In this case, referring to Figure 31,

$$\vec{w} = \text{Rot}_y [\vec{r}] \quad (143)$$

$$\vec{v}(\vec{w}, \bar{B}) = \text{Rot}_y [\vec{u}(\vec{r}, B)] \quad (144)$$

$$\bar{B}(\vec{w}) = \text{Rot}_y B(\vec{r}) \quad (145)$$

Figure 31 (c) illustrates a consequence of the linearity condition imposed upon this system. The rather restrictive assumption of force-displacement linearity requires the displacement field  $\vec{u}(\vec{r}, B)$  to be small in magnitude. In this case, reversal of forces results in the reversal of response displacements.

$$\vec{v}(\vec{w}, \bar{B}) = -\vec{v}(\vec{w}, -\bar{B}) \quad (146)$$

In order to determine symmetry relations, it is necessary to relate the response vector  $\vec{v}(\vec{w}, B)$  to the vector  $\vec{u}(\vec{r}, B)$  under the same load system B.

Any radially symmetric load system B can be resolved into two component systems:

1.  $B_s$ , having loads symmetrical with respect to both x- and y-axes
2.  $B_a$ , having loads asymmetrical with respect to both x- and y-axes

These systems are illustrated in Figure 32.

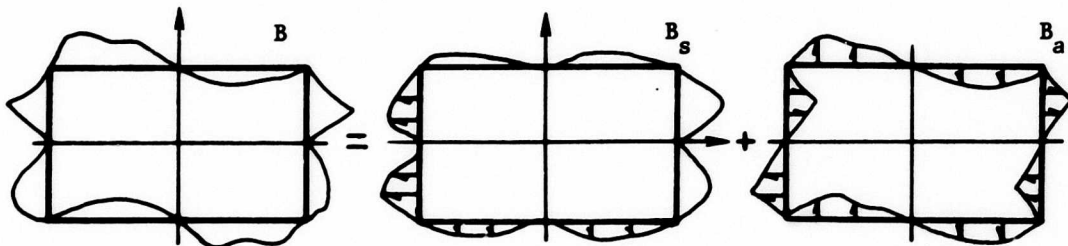


Figure 32. Resolution of Load System B.

If the rotation operations are performed on each component system, we find that symmetry or asymmetry of loading can be defined by equations (147) and (148).

$$\text{Symmetry: } \bar{B}_s(\vec{w}) = \text{Rot}_y [B_s(\vec{r})] = B_s(\vec{w}) \quad (147)$$

$$\text{Asymmetry: } \bar{B}_a(\vec{w}) = \text{Rot}_y [B_a(\vec{r})] = -[B_a(\vec{w})] \quad (148)$$

Upon application to equation (144), the following relationships are obtained

$$\vec{v}(\vec{w}, \bar{B}_s) = \vec{v}(\vec{w}, B_s) = \text{Rot}_y [\vec{u}(\vec{r}, B_s)] \quad (149)$$

That is,

$$\vec{v}(\vec{w}, B_s) = \text{Rot}_y [\vec{u}(\vec{r}, B_s)] \quad (150)$$

$$\vec{v}(\vec{w}, \bar{B}_a) = \vec{v}(\vec{w}, -B_a) = -\vec{v}(\vec{w}, B_a) = \text{Rot}_y [\vec{u}(\vec{r}, B_a)] \quad (151)$$

and

$$\vec{v}(\vec{w}, B_a) = -\text{Rot}_y [\vec{u}(\vec{r}, B_a)] \quad (152)$$

The response  $\vec{v}$  at point  $\vec{w}$  in element I is determined from the response  $\vec{u}$  at  $\vec{r}$  in element II by rotating  $\vec{u}(\vec{r})$  and then multiplying by  $\pm 1$ . It is important to note that only one element need be considered under either load system.

Some boundary conditions can be obtained by considering conditions along the y axis. In this case, equations (150) and (152) yield

$$\vec{r}_y = \vec{w}_y, \vec{u}(\vec{r}_y, B) = \vec{v}(\vec{r}_y, B) \quad (153)$$

$$\vec{v}(\vec{r}_y, B_s) = \text{Rot}_y [\vec{u}(\vec{r}_y, B_s)] = \vec{u}(\vec{r}_y, B_s) \quad (154)$$

$$\vec{v}(\vec{r}_y, B_a) = -\text{Rot}_y [\vec{u}(\vec{r}_y, B_a)] = -\vec{u}(\vec{r}_y, B_a)$$

Equation (153) implies that  $\vec{u}(\vec{r}_y, B_s)$  is parallel to the y-axis, while equation (154) implies that  $\vec{u}(\vec{r}_y, B_a)$  is perpendicular to the y-axis. Typical boundary conditions are illustrated in Figure 33.

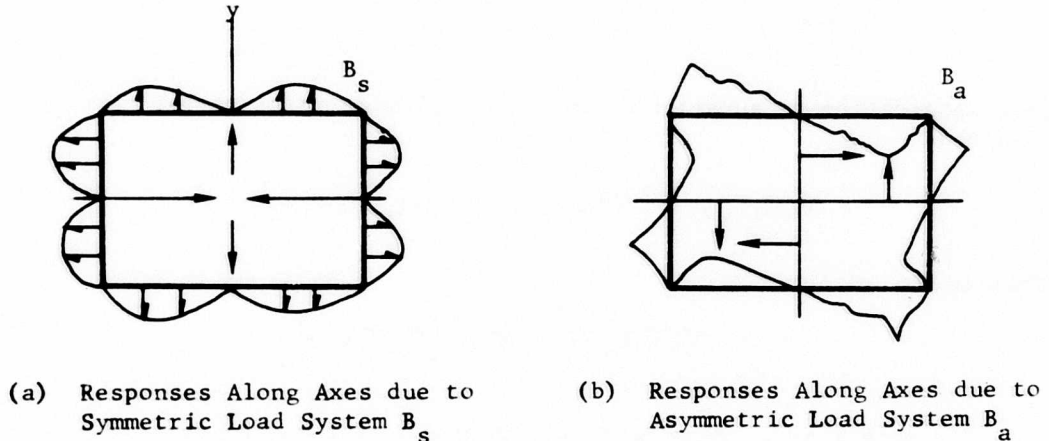


Figure 33. Responses of Points on Axes of Symmetry.

### DISCUSSION

The results of equations (149) through (154) hold true only if point A is (1) a pole of lead radial symmetry and (2) an origin of plate symmetry.

In an infinite plate under uniform shear loading at the boundaries, an infinite number of poles of type A exist. In Figure 29, these poles are indicated by X and Y. The coordinate axes (axes of symmetry) for points X are vertical and horizontal in this figure, while for points Y the coordinate systems are rotated. For the infinite plate only one sub-element, such as the one shaded in Figure 29, has to be analyzed to obtain a complete response description. For the finite plate, on the other hand, an entire quadrant must be analyzed. In a practical experiment, such a point A must exist, and the symmetries of the loading must be known if adequate verification is to be performed.

APPENDIX III

COMPUTER PROGRAM

INPUT DECK SETUP

In this program the six fundamental load cases are referred to as cases 1 through 6 as indicated in the following table:

	$N_x$	$N_y$	$N_z$	$T_{yz}$	$T_{zx}$	$T_{xy}$
case	1	2	3	4	5	6

FUNDAMENTAL CASES ONE THROUGH FOUR

<u>CARD</u>	<u>COLUMNS</u>	<u>FORMAT</u>	<u>CONTAINING</u>
1.	1-80	10A8	Run Identification
2.	1-10	E10.3	$V_f$
	11-20	E10.3	$E_I/E_{II}$
	21-30	E10.3	$E_I$
	31-40	E10.3	$v_I$
	41-50	E10.3	$v_I/v_{II}$

Repeat cards 1 and 2 up to 10 runs

3. Two blank cards end data

FUNDAMENTAL CASES FOUR AND FIVE

<u>CARD</u>	<u>COLUMNS</u>	<u>FORMAT</u>	<u>CONTAINING</u>
1.	1-80	10A8	Run Identification
2.	1-10	E10.3	$G_I$
	11-20	E10.3	$G_{II}$
	21-30	E10.3	$V_f$

Repeat cards 1a and 2 for same runs made by cases one through four

3. Two blank cards end data

PROGRAM FENG

<u>CARD</u>	<u>COLUMNS</u>	<u>FORMAT</u>	<u>CONTAINING</u>
1.	1	I1	1 for 1st 6 files 2 for 2nd 6 files
2.	1-10	E10.3	$V_f$
	11-20	E10.3	$E_I/E_{II}$
	21-30	E10.3	$E_I$
	31-40	E10.3	$v_I$
	41-50	E10.3	$v_I/v_{II}$

Repeat card 2 NS times, in the order the runs were made in Phase I

3. One blank card ends the run

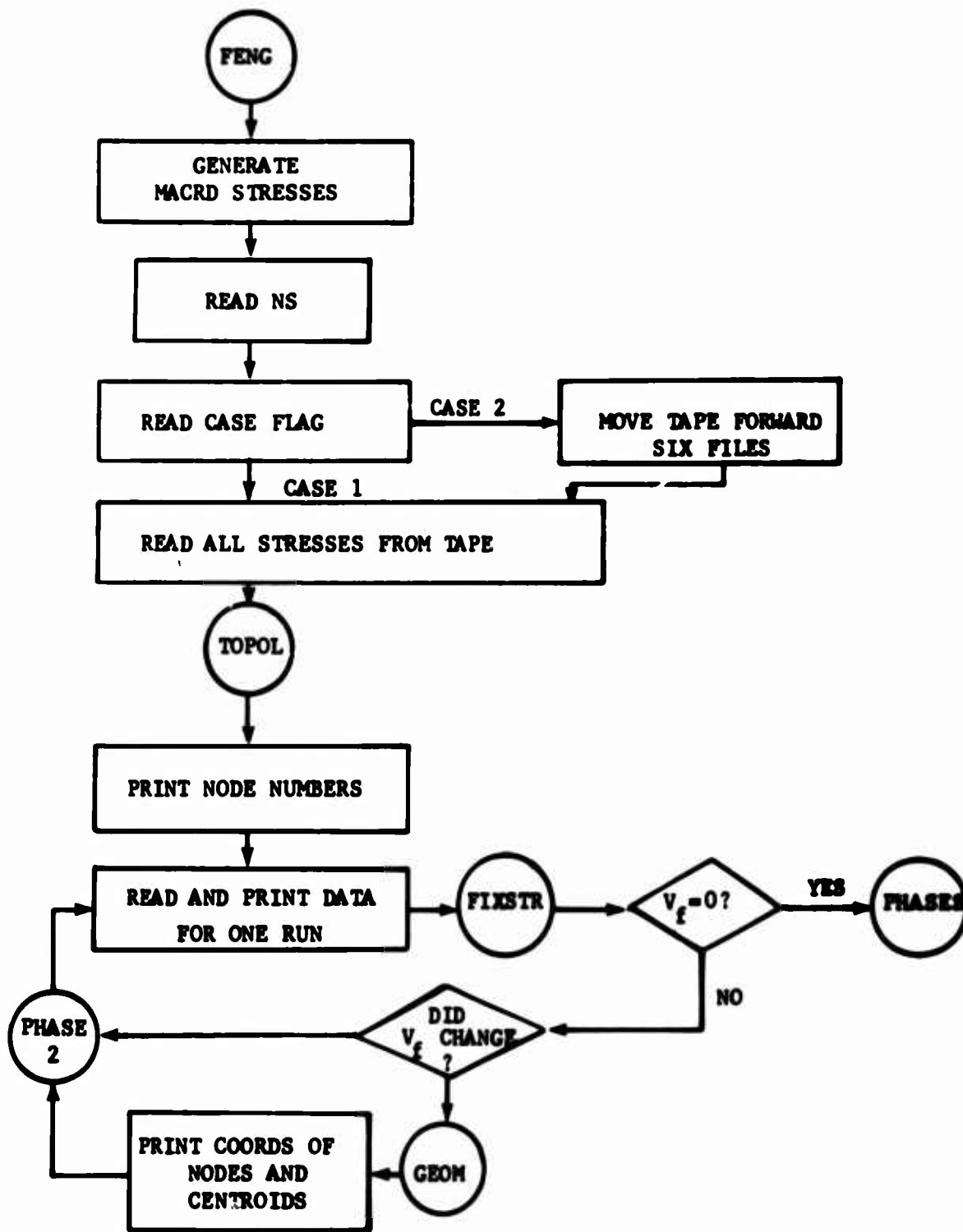
Because of core size limitations, a limit of 10 runs of six fundamental cases can be run at one time. Also, because only 20 cases were needed for this study, Program FENG was written to accept only two sets of six files, these to be run separately. If more files are to be desired on one tape, appropriate tape spacing logic must be added to the program.

PROCEDURE FOR WRITING DATA TAPE 20

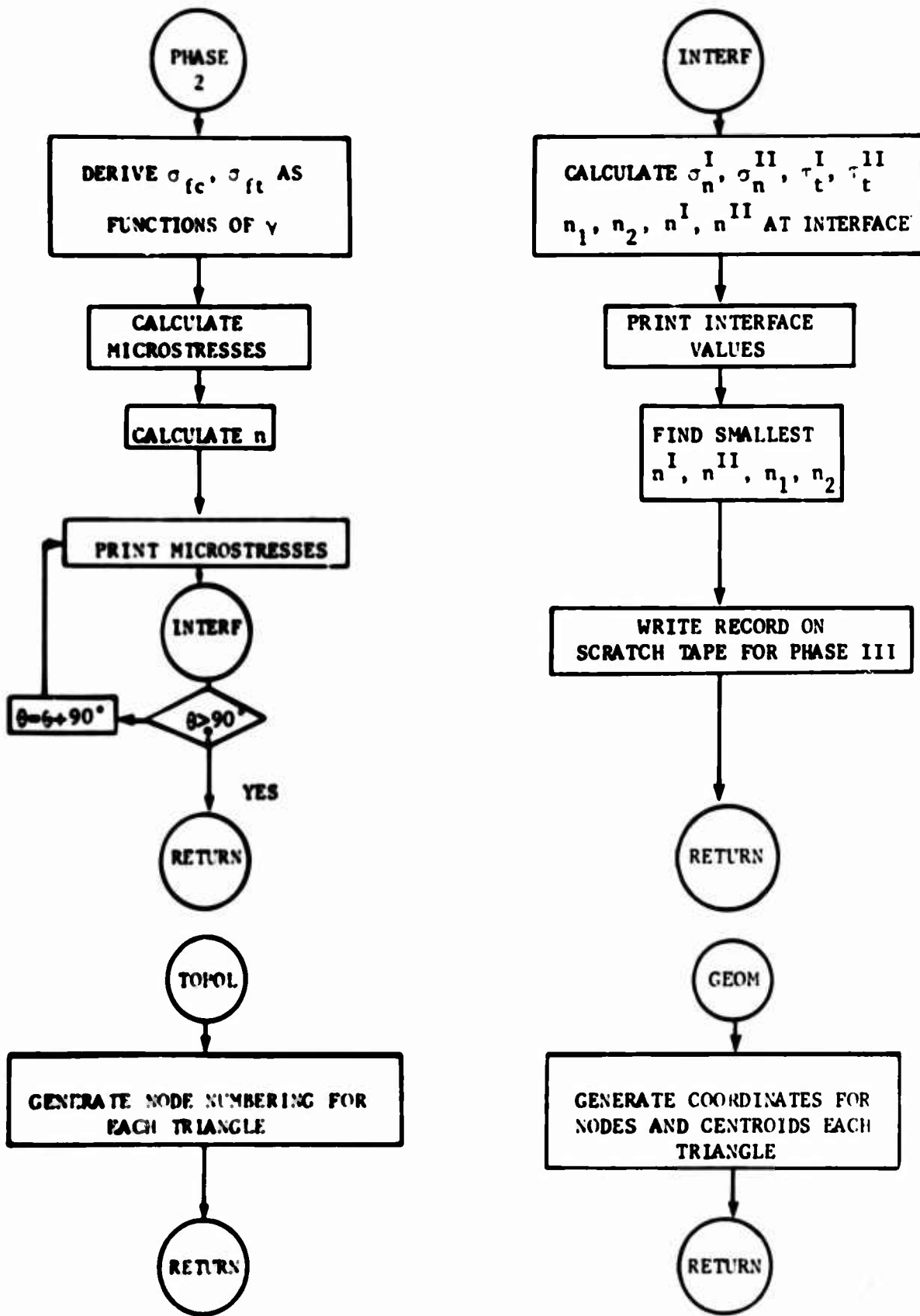
Before running Program FENG, it is necessary to write a data tape containing stress values for each of 79 triangular segments. This is accomplished by running sequentially Fundamental Case 1 through Fundamental Case 6, one or more times. Each fundamental case generates one file on Tape 20 which is an input to Program FENG.

For each set of six files, the Program MAIN must be adjusted for cases 1 through 4, and Program LONTUD must be modified for cases 5 and 6, as follows:

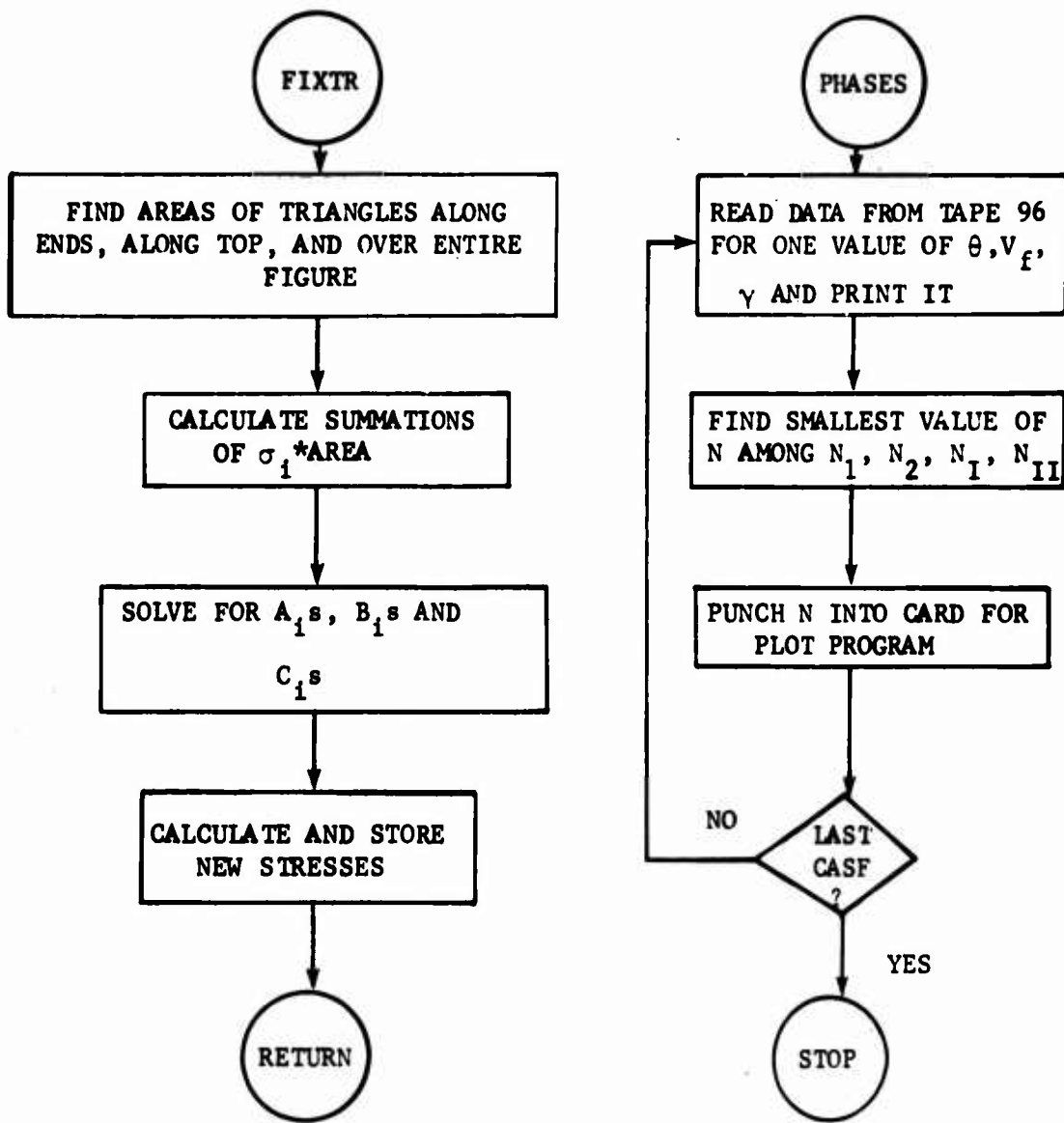
- The first record of the first file on the tape must contain the integer  $N_s$  = number of runs per file.
- Each Fundamental Case must space Tape 20 an appropriate number of files so as to leave room for files already written.



PROGRAM FENG



SUBROUTINES PHASE 2, TOPOL, GEOM, INTERF



SUBROUTINES FIXSTR (KASE), PHASES

PHASE ONE

FUNDAMENTAL CASES ONE, TWO, AND SIX  
(TRANSVERSE LOADING)

FUNDAMENTAL CASE 1

```
PROGRAM MAIN
C
C THIS IS THE EXECUTIVE PROGRAM USED WITH SUBROUTINE BIGMX AND SUBROUTINE
C CHLSKY TO GENERATE AND SOLVE LARGE SYSTEMS OF LINEAR EQUATIONS
C
C REWIND 20
CALL TAPESKIP(20,6,0)
1 CONTINUE
CALL INPUT
C
C NOW HAVE ALL K-PRIME AND CDA MATRICES
C
C CALL CHLSKY
C
C NOW HAVE SOLUTION U IN SP
C
C CALL STRESS
C
C ALL STRESSES NOW PRINTED OUT
C
C GO TO 1
END
```

CASE 1

```

SUBROUTINE INPUT
C
C   TRANSVERSE NORMAL CASE 1
C
C   THIS SUBROUTINE READS AND PRINTS THE INPUTS FOR THE PLANAR FINITE
C   ELEMENT PROGRAM. ALL INPUTS NOT READ ARE GENERATED HERE.
C
COMMON /STRSS/ ALF(155), MSIZE, R, GAMMA, EI, GII, XNU(2)
COMMON /1/ S(32,32,5), L(300,3), G(32), SPACE(20)
1      , NUF(3,314)      , X(155), Y(155), CDA(3,6,250)
2      , KPR(6,6,250), MSK(310), JTOTAL, N, KREM, M , IMR, IPL
3      , P(155,2) , D(155,2)
COMMON /LIM/ LIM1(10), LIM2(10)
DIMENSION SP(32,10)
DIMENSION NUYX(250), NUXY(250), GXY(250), EX(250), EY(250)
DIMENSION BUMP(10) , BUMP1(5)
EQUIVALENCE (NUF, SP)
EQUIVALENCE (C, CP)
DIMENSION JL(79,3)
DIMENSION COMENT(10)
C
C   NUYX = NUXY, THIS MODIFICATION
EQUIVALENCE (NUXY, NUYX)
DIMENSION TH(155) , CP(3,3) , T1(3,3,2), T2(3,3), DTD(18),
1 T(6,6), KMX(6,6), C(3,3), A(6,6,2), DO(3,6), DD(18), DT0(6,3)
EQUIVALENCE (S, F) , (S(901), VF) , (S(1801), PHI) , (S(2701), ER)
1      , (S(3001), EF) , (S(3901), NUR)
2      , (DO, DD) , (DT0, DTD) , (S(4356), TH)
3      , (S(4511), CP) , (S(4521), T1) , (S(4541), T2)
4      , (S(4551), CABC) , (S(4581), T) , (S(4621), KMX)
5      , (S(4671), A)
TYPE REAL NUXY
TYPE INTEGER BUMP, BUMP1
TYPE REAL NUYX, NUR, NUF, KPR, KMX
DATA (KTOTAL = 97)
DATA (BUMP= 2,4,7,7,15,1 ,9 ,9 ,12,-1)
DATA (BUMP1= 5,7,7,7,5 ) , (PI= 3.1415927)
DATA (RAD = 57.29578)
DATA (XLIM=1.E-8)
DATA (PK=999.)
DATA ((DD(JJ),JJ=1,18) = 0., 0., 0., 1., 0., 0., 0., 0., 1.,
1      0., 0., 0., 0., 0., 1., 0., 1., 0.)
DATA ((DTD(JJ),JJ=1,18) = 0., 1., 0., 0., 0., 0., 0., 0., 0.,
1      0., 0., 1., 0., 0., 0., 1., 0.)
DATA(((JL(I,J),I=1,79),J=1,3) =
1 1,1,1,2,2,3,3,3,4,4,4,5,6,7,7,8,8,8,9,10,10,11,11,12,13,13,
2 14,15,15,16,16,17,17,18,18,19,20,23,24,24,25,25,25,26,28,29,
3 29,30,30,35,21,21,21,21,22,22,23,23,27,27,28,28,36,46,37,
4 47,38,48,39,39,40,41,41,41,42,42,43,43,
5 2,3,4,6,7,7,8,9,9,10,11,11,13,6,14,14,15,16,16,16,17,10,
6 18,18,20,21,21,21,22,15,23,23,24,17,25,25,35,27,27,28,28,29,
7 30,30,32,32,33,33,34,36,35,37,38,39,39,40,40,41,41,42,43,43,44,
8 45,37,46,38,47,39,48,49,49,40,50,51,51,52,52,53,
9 3,4,5,7,3,8,9,4,10,11,5,12,14,14,8,15,16,9,10,17,18,18,12,
1 19,21,14,15,22,23,23,17,24,25,25,19,26,21,24,28,25,29,30,26,
2 31,29,33,30,34,31,37,37,38,39,22,40,23,41,27,42,43,28,44,32,
3 46,36,47,37,48,38,49,40,50,50,51,42,52,43,53,44 )
C
READ 1000,(COMENT(I),I=1,10)
1000 FORMAT(10A8)
PRINT 1001,(COMENT(I),I=1,10)

```

```

1001 FORMAT(1H1,10A8)
C
C   KTOTAL - TOTAL NUMBER OF NODES CONSIDERED
C
C   READ AND PRINT
C
      JTOTAL = 158
      READ 1003, VF, GAMMA, EI, XNUI, BETA
1003 FORMAT(5E10.4)
      IF (VF.EQ.0.) GO TO 950
      PRINT 1018
      PRINT 1016
      PRINT 1017, VF, GAMMA, EI, XNUI, BETA
1016 FORMAT(1H ,13X,2HV, 10X,5HGAMMA, 13X,2HEI, 11X,4HXNUI, 11X4HBETA)
1017 FORMAT(1H ,5(E15.8))
1018 FORMAT(////)

      S3 = SQRT(3.)
      S302 = S3/2.
      EII = EI / GAMMA
      XNUII = XNUI/BETA
      GII = EII/(2.*(1.+XNUII))
      XNU(1) = XNUI
      XNU(2) = XNUII
C
      EPI = EI
      EPII = EII
      XNUPI = XNUI
      XNUPII = XNUII
      EPI = EI/(1.-XNUI**2)
      EPII = EII/(1.-XNUII**2)
C
      XNUPI = XNUI/(1.-XNUI)
      XNUPII = XNUII/(1.-XNUII)
      DO 200 I = 1,97
      TH(I) = 1.
      ALF(I) = 0.
      D(I,1) = 1000.
      D(I,2) = 1000.
      P(I,1) = 0.
      P(I,2) = 0.
200 CONTINUE
      I = 0
      DO 201 J = 1,10
      I = I + BUMP(J)
      P(I,2) = 1000.
      P(98-I,2) = 1000.
      D(I,2) = 0.
      D(98-I,2) = 0.
201 CONTINUE
      I = 0
      DO 202 J = 1,5
      I = I + BUMP(J)
      P(I,1) = 1000.
      P(98-I,1) = 1000.
      D(I,1) = 1.
      D(98-I,1) = -1.
202 CONTINUE
      P(1,1) = 1000.
      P(1,2) = 1000.
      D(1,1) = 1.

```

```

D(1,2) = 0.
P(34,1) = 1000.
P(34,2) = 1000.
D(34,1) = 1.
D(34,2) = 0.
P(64,1) = 1000.
P(64,2) = 1000.
D(64,1) = -1.
D(64,2) = 0.
P(97,1) = 1000.
P(97,2) = 1000.
D(97,1) = -1.
D(97,2) = 0.
R = SORT(2.0*S3*VF/PI)
X(1) = S302
Y(1) = .5
DO 210 I=1,4
X(I+1) = S302 - R/4.*COS(PI*(I-1)/6.)
Y(I+1) = .5 - R/4.*SIN(PI*(I-1)/6.)
210 CONTINUE
C
DO 220 I=1,7
X(I+5) = S302 - R/2.*COS(PI*(I-1)/12.)
Y(I+5) = .5 - R/2.*SIN(PI*(I-1)/12.)
C
X(I+12) = S302 - 3.*R/4.*COS(PI*(I-1)/12.)
Y(I+12) = .5 - 3.*R/4.*SIN(PI*(I-1)/12.)
C
X(I+19) = S302 - R * COS(PI*(I-1)/12.)
Y(I+19) = .5 - R * SIN(PI*(I-1)/12.)
220 CONTINUE
X(34) = S302
Y(34) = -.5
X(31) = S302
Y(31) = (Y(26)+Y(34)) / 2.
X(45) = -.5 * TAN(PI/6.)
Y(45) = .5
DX = (1.-R)/(2.*COS(PI/6.))
X(36) = X(45) + DX
Y(36) = .5
X(35) = (X(20)+X(36))/2.
Y(35) = .5
C
DO 230 I = 1,4
X(I+45) = (4 - I) * X(45) / 4.
Y(I+45) = (4 - I) * Y(45) / 4.
230 CONTINUE
C
DELX = X(46) - X(45)
DELY = Y(46) - Y(45)
DO 240 I = 1,8
X(I+36) = X(I+35) + DELX
Y(I+36) = Y(I+35) + DELY
240 CONTINUE
X(32) = X(44) + (X(34) - X(44))/3.
Y(32) = -.5
X(33) = 2.*X(32) - X(44)
Y(33) = -.5
X(27) = X(23) + (X(32) - X(23))/3.
Y(27) = Y(23) + (Y(32) - Y(23))/3.
X(28) = 2.*X(27) - X(23)
Y(28) = 2.*Y(27) - Y(23)

```

```

X(29) = X(28) + (X(31) - X(28)) / 3.
Y(29) = Y(28) + (Y(31) - Y(28)) / 3.
X(30) = 2. * X(29) - X(28)
Y(30) = 2. * Y(29) - Y(28)
C
C
DO 250 I=50,97
X(I) = -X(98-I)
Y(I) = -Y(98-I)
250 CONTINUE
C
C
C PRINT OUT NODAL DATA
I = 1
12 CONTINUE
LINE = 4
PRINT 1010
13 CONTINUE
PRINT 1011, I, X(I), Y(I), P(I,1), P(I,2), D(I,1), D(I,2),
1 TH(I), ALF(I)
I = I+1
IF(I.GT.KTOTAL) GO TO 14
LINE = LINE + 2
IF(LINE.GT.56) GO TO 12
GO TO 13
14 CONTINUE
1011 FORMAT(1H I4,7(3XE11.4),5X,F11.2/)
1010 FORMAT(5H1NODE,7X,1HX,13X,1HY,13X,2HP1,12X,2HP2,12X,2HD1,12X,2HD2,
1 8X,9HTHICKNESS,9X,5HALPHA /)
C
C I - NUMBER OF NODE
C X - X- COORDINATE OF ITH NODE
C Y - Y- COORDINATE OF ITH NODE
C P(I,1) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 1 DIRECTION
C P(I,2) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 2 DIRECTION
C D(I,1) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 1 DIRECTION
C D(I,2) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 2 DIRECTION
C ALF(I) - ANGLE BETWEEN X DIRECTION AND 1 DIRECTION (POSITIVE WHEN
C COUNTER-CLOCKWISE)
C TH(I) - PLATE THICKNESS AT ITH NODE
C
C GET MSK MATRIX
JP = 0
DO 27 J=1,KTOTAL
DO 27 I=1, 2
IF (P(J,I).GT.PK) GO TO 27
JP = JP+1
MSK(JP) = 2 * (J-1) + I
27 CONTINUE
C MSK IS MATRIX OF INDICES OF KNOWN FORCES
C IF FORCE P IS UNKNOWN, IT IS INPUT AS 1000.
C NOW READ IN TRIANGLE DATA
C
C JTOTAL - TOTAL NUMBER OF TRIANGLES
DO 19 I=1,79
DO 19 J=1,3
19 L(I,J) = JL(I,J)
DO 260 I= 80,158
DO 260 J= 1,3
L(I,J) = 98 - L(159-I,J)
260 CONTINUE
C L(J,1) - INDEX OF THE FIRST NODE OF THE JTH TRIANGLE

```

```

C L(J,2) - INDEX OF THE SECOND NODE OF THE JTH TRIANGLE
C L(J,3) - INDEX OF THE THIRD NODE OF THE JTH TRIANGLE
C
DO 20 I=1,JTOTAL
IF(I.LE.36.OR.I.GE.123) GO TO 300
EX(I) = EPII
EY(I) = EPII
NUXY(I) =XNUPII
GXY(I) = EPII/(2.*(1.+XNUPII))
GO TO 310
300 EX(I) = EPI
EY(I) = EPI
NUXY(I) = XNUPI
GXY(I) = EPI / (2.*(1.+XNUPI))
310 CONTINUE
20 CONTINUE
C PRINT OUT TRIANGLE DATA
LINE = 4
PRINT 1012
DO 24 I=1,JTOTAL
IF(LINE.LT.54) GO TO 22
LINE = 4
PRINT 1012
1012 FORMAT(1H1,8HTRIANGLE,4X6HNODE 1,3X,6HNODE 2,3X,6HNODE 3,10X,2HEX,
1 18X 2HEY,16X,4HNUIX,17X,3HGXY//)
22 LINE = LINE+2
24 PRINT 1023, I, (L(I,J),J=1,3),EX(I),EY(I),NUYX(I),GXY(I)
1023 FORMAT(1H ,5X,I3,7X,I3,2(6X,I3),4(5X,E15.8)//)
C
C
MSIZE = 156
KREM = 6
N = 25
M = MSIZE / N
IF(KREM.NE.N) M = M+1
C
LIM1(1) = 1
LIM1(2) = 1
LIM1(3) = 1
LIM1(4) = 1
LIM1(5) = 1
LIM1(6) = 1
LIM1(7) = 1
LIM2(1) = 158
LIM2(2) = 158
LIM2(3) = 158
LIM2(4) = 158
LIM2(5) = 158
LIM2(6) = 158
LIM2(7) = 158
C
C PRINT OUT PARTITION INFORMATION
PRINT 1008
1008 FORMAT(1H1,12X,20HTRIANGLES CONSIDERED)
DO 31 I=1,M
ISIZE = N
IF(I.EQ.1) ISIZE = KREM
PRINT 1009, I,LIM1(I),LIM2(I), ISIZE
31 CONTINUE
1009 FORMAT(10H PARTITION,6X 5HFIRST,2X,2HTO,2X,4HLAST, 6X,9HDIMENSION/
1 4X,I3,11X,I3,6X,I3,10X,I3)

```

```

DO 28 I = 1,KTOTAL
ALF(I) = ALF(I)/RAD
28 CONTINUE
C ALL ANGLES NOW IN RADIANS
DO 400 I=1, JTOTAL
C I IS A TRIANGLE COUNTER
B11 = 1./EX(I)
B12 = -NUYX(I) / EY(I)
B22 = 1. / EY(I)
B33 = 1. / GXY(I)
DELTA = B11*B22 -B12**2
CP(2,1) = - B12 / DELTA
CP(1,1) = B22 / DELTA
CP(3,1) = 0.
CP(1,2) = CP(2,1)
CP(2,2) = B11/ DELTA
CP(3,2) = 0.
CP(1,3) = 0.
CP(2,3) = 0.
CP(3,3) = 1./B33
C
C C NOW IN C(3,3) . MATRIX I
C
30 CONTINUE
THOMEG = X(L(I,2))*Y(L(I,3)) +X(L(I,1))*Y(L(I,2))
1 + Y(L(I,1))*X(L(I,3)) -X(L(I,3))*Y(L(I,2))
2 - X(L(I,1))*Y(L(I,3)) -Y(L(I,1))*X(L(I,2))
THOMEG=(TH(L(I,1))+TH(L(I,2))+TH(L(I,3)))/ 6. * THOMEG
X12 = X(L(I,2))- X(L(I,1))
X13 = X(L(I,3))- X(L(I,1))
ETA2= Y(L(I,2))- Y(L(I,1))
ETA3= Y(L(I,3))- Y(L(I,1))
DELTA = X12*ETA3 - X13*ETA2
DO 38 II = 1,3
DO 38 JJ = 1,3
A(II+3, JJ,1) = 0.
A(II, JJ+3,1) = 0.
A(II+3, JJ,2) = 0.
A(II, JJ+3,2) = 0.
38 CONTINUE
A(1,1,1) = 1.
A(2,1,1) = -(ETA3-ETA2) / DELTA
A(3,1,1) = (X13 - X12) / DELTA
A(1,2,1) = 0.
A(2,2,1) = ETA3 / DELTA
A(3,2,1) = -X13 / DELTA
A(1,3,1) = 0.
A(2,3,1) = -ETA2 / DELTA
A(3,3,1) = X12 / DELTA
DO 39 II = 1,3
DO 39 JJ = 1,3
A(II+3, JJ+3,1) = A(II, JJ,1)
A(II, JJ,2) = A(JJ, II,1)
A(II+3, JJ+3,2) = A(JJ, II,1)
39 CONTINUE
C
C TRANSPOSE OF INVERSE OF A NOW IN A(1,1,2) . A INVERSE STILL IN A
C
CALL MXMULT(DO,A,KMX(1,1),3,6,6)
CALL MXMULT(C,KMX(1,1) ,CDA(1,1,I),3,3,6)
C PRODUCT C*D*(A**-1) NOW IN CDA(1,1,I). ITH TRIANGLE
C

```

```

CALL MXMULT(A(1,1,2),DT(,A,6,6,3)
CALL MXCON(A,KPR(1,1,1),THOMEG,6,3)
CALL MXMULT(KPR(1,1,1),CDA(1,1,1),KMX,6,3,6)
C
C MATRIX K(I) NOW IN KMX, TRIANGLE I
DO 40 II =1,6
DO 40 JJ =1,6
T(II,JJ) = 0.
40 CONTINUE
T(1,1) = COS(ALF(L(1,1)))
T(4,1) = SIN(ALF(L(1,1)))
T(2,2) = COS(ALF(L(1,2)))
T(5,2) = SIN(ALF(L(1,2)))
T(3,3) = COS(ALF(L(1,3)))
T(6,3) = SIN(ALF(L(1,3)))
T(1,4) = -T(4,1)
T(4,4) = T(1,1)
T(2,5) = -T(5,2)
T(5,5) = T(2,2)
T(3,6) = -T(6,3)
T(6,6) = T(3,3)
CALL MXMULT (KMX,T,A,6,6,6)
T(4,1) = - T(4,1)
T(5,2) = - T(5,2)
T(6,3) = - T(6,3)
T(1,4) = - T(1,4)
T(2,5) = - T(2,5)
T(3,6) = - T(3,6)
C
C INVERSE OF T NOW IN T
C
CALL MXMULT (T,A,KPR(1,1,1),6,6,6)
C
C K-PRIME NOW IN KPR . A HAS BEEN CLOBBED.
400 CONTINUE
RETURN
800 PRINT 1051,II, JJ,II
1051 FORMAT(1M1, 5M EF(,13,1M,13,7M) = ER(,13,6M) = 0.)
STOP
900 CONTINUE
PRINT 1056 , J
1056 FORMAT(1M1 35HCOULD NOT INVERT MATRIX T1,TRIANGLE,13)
950 CONTINUE
REWIND 20
STOP
END

```

CASE 1

```

SUBROUTINE STRESS
C
C STRESS SUBROUTINE CASE 1
C
C THIS SUBROUTINE DERIVES AND PRINTS STRESSES
C
COMMON /STRSS/ ALF(155), MSIZE, R, GAMMA, EI, GII, XNU(2)
COMMON /1/ S(32,32,5), L(300,3), G(32), SPACE(20)
1      , NUF(3,314)      , X(155), Y(155), CJA(3,6,250)
2      , KPR(6,6,250), MSK(310), JTOTAL, N, KREM, M, IMPR, IPL
3      , P(155,2) , D(155,2)
COMMON/LIM/ LIM1(10), LIM2(10)
DIMENSION DVX(6), SIGOUT(632)
DIMENSION SP(32,10)
EQUIVALENCE(S,SIGOUT) ,(NUF,SP)
TYPE REAL KPR
EQUIVALENCE (S,SIG) , (S(931),PSTR) , (S(1861),X0), (S(2161),Y0)
EQUIVALENCE (S(2461),DEL) , (S(2761),DX)
DIMENSION ERR(310)
DIMENSION DX(6) , SIG(6,168) , PSTR(3,310)
DIMENSION X0(300) , Y0(300) , DEL(310)
DIMENSION KS22(32,96)
EQUIVALENCE (KPR,KS22)
TYPE REAL KS22
DATA (PK=999)
C REMOVE GAPS FROM SP(32,10) = DEL(310)
DO 5 J=1,KREM
DEL(J) = SP(J,1)
5 CONTINUE
KLOC = KREM -N
DO 10 I = 2,M
KLOC = KLOC + N
DO 10 J = 1,N
DEL(KLOC+J) = SP(J,1)
10 CONTINUE
PRINT 1010
1010 FORMAT(1H1,50X,13HDISPLACEMENTS,/)
NDEL = MSIZE/7
JCNT = 0
DO 15 J=1,NDEL
JCNT = JCNT + 1
IF(JCNT.LE.18) GO TO 14
PRINT 1010
JCNT = 0
14 JFIR = 7*(J-1) + 1
JLAST = JFIR + 6
PRINT 1011 , (K,K=JFIR,JLAST)
1011 FORMAT(1H ,7(8X,4HDEL(,13,1H) )
PRINT 1012 , (DEL(K),K=JFIR,JLAST)
1012 FORMAT(1H ,7(2X,E14.7)/)
15 CONTINUE
LOC1 = 7*NDEL+1
LOC2 = MSIZE
IF(LOC1.GT.LOC2) GO TO 20
PRINT 1011,(K,K=LOC1,LOC2)
PRINT 1012 , (DEL(K),K=LOC1,LOC2)
20 CONTINUE
DO 855 I=1,JTOTAL
J = I
KZ = 0
DO 831 KK =1,3

```

```

DO 832 KJ = 1,2
KZ = KZ + 1
IF( P(L(J,KK),KJ).GT.PK) GO TO 828
IF(2*(L(J,KK)-1)+KJ-MSK(IPL)) 801, 802, 803
802 I12 = IPL
GO TO 827
801 IMR = IPL - 1
806 IF(2*(L(J,KK)-1) +KJ -MSK(IMR)) 804, 805, 805
805 I12 = IMR
IPL = IMR
GO TO 827
804 IMR = IMR - 1
GO TO 806
803 IMR = IPL + 1
807 IF(2*(L(J,KK)-1) +KJ -MSK(IMR)) 805, 805, 810
810 IMR = IMR + 1
GO TO 807
827 DVX(KZ) = DEL(I12)
GO TO 832
828 DVX(KZ) = D(L(J,KK),KJ)
832 CONTINUE
DX(KZ-1) = DVX(KZ-1)*COS(ALF(L(J,KK))) - DVX(KZ)*SIN(ALF(L(J,KK)))
DX(KZ) = DVX(KZ-1)*SIN(ALF(L(J,KK))) + DVX(KZ)*COS(ALF(L(J,KK)))
831 CONTINUE
DX2 = DX(2)
DX(2) = DX(3)
DX(3) = DX(5)
DX(5) = DX(4)
DX(4) = DX2
CALL MXMULT(CDA(1,1,1),DX, SIG(1,1),3,6,1)
KK3 = 1
IF(1.GE.37.AND.1.LE.122) KK3 = 2
SIG(4,1) = SIG(3,1)
SIG(3,1) = XNU(KK3)*(SIG(1,1)+SIG(2,1))
C SIGMA NOW IN SIG(1,1) , TRIANGLE I
X0(1) = 0.
Y0(1) = 0.
DO 840 K = 1,3
X0(1) = X0(1) + X(L(I,K))
Y0(1) = Y0(1) + Y(L(I,K))
840 CONTINUE
X0(1) = X0(1) / 3.
Y0(1) = Y0(1) / 3.
850 CONTINUE
855 CONTINUE
ENBARX = R/4.*(SIG(1,3)+SIG(1,12)+SIG(1,24)+SIG(1,36) )
1 + (1.-R)/2. *(SIG(1,44)+SIG(1,49) )
ENBARX = 1./ENBARX
DO 860 I=1,632
860 SIGOUT(I) = SIGOUT(I)*ENBARX
I = 1
870 CONTINUE
PRINT 1000
LINE = 3
1000 FORMAT(1H1,2(8HTRIANGLE,6X,8HCENTROID,9X,12HVECTOR SIGMA,10X)/)
871 CONTINUE
II = I+1
DO 880 J=1,4
GO TO (873,874,875,875), J
873 PRINT 1001, I, X0(I), SIG(J,I), II, X0(II), SIG(J,II)
GO TO 878
874 PRINT 1002, Y0(I), SIG(J,I), Y0(II), SIG(J,II)

```

```

GO TO 878
875 PRINT 1003, SIG(J,I), SIG(J,II)
878 CONTINUE
880 CONTINUE
    LINE = LINE + 6
    I = I + 2
    PRINT 1004
1001 FORMAT(1H 2(5X,I3,F14.6, 7X,E15.8,10X))
1002 FORMAT(1H 2(8X,F14.6, 7X,E15.8,10X) )
1003 FORMAT(1H 2(29X,E15.8,10X))
1004 FORMAT(/)
1069 FORMAT(1H ,30X,E15.8)
    IF(I.GT.JTOTAL) GO TO 890
    IF (LINE.GT.54) GO TO 870
    GO TO 871
890 CONTINUE
    WRITE (20) (SIGOUT(I),I=1,316)
    PRINT 8787, ENBARX
8787 FORMAT(/23H NORMALIZATION FACTOR =,2X,E15.7)
    RETURN
    END

```

CASE 1

2026 CARDS

PROGRAM MAIN

```

C
C THIS IS THE EXECUTIVE PROGRAM USED WITH SUBROUTINE BIGMX AND SUBROUTINE
C CHLSKY TO GENERATE AND SOLVE LARGE SYSTEMS OF LINEAR EQUATIONS
C

```

```

    REWIND 20
    CALL TAPESKIP(20,6,0)
    CALL TAPESKIP(20,1,0)
1 CONTINUE
    CALL INPUT

```

CASE 2

```

C
C NOW HAVE ALL K-PRIME AND CDA MATRICES
C
    CALL CHLSKY
C
C NOW HAVE SOLUTION U IN SP
C
    CALL STRESS
C
C ALL STRESSES NOW PRINTED OUT
C
    GO TO 1
    END

```

23 CARDS

FUNDAMENTAL CASE 2

```

SUBROUTINE INPUT
C
C   TRANSVERSE NORMAL- CASE 2
C
C   THIS SUBROUTINE READS AND PRINTS THE INPUTS FOR THE PLANAR FINITE
C   ELEMENT PROGRAM. ALL INPUTS NOT READ ARE GENERATED HERE.
C
COMMON /STRSS/ ALF(155), MSIZE, R, GAMMA, EI, GII, XNU(2)
COMMON /I/ S(32,32,5), L(300,3), G(32), SPACE(20)
      , NUF(3,3,4)      , X(155), Y(155), CDA(3,6,250)
2      , KPR(6,6,250), MSK(310), JTOTAL, N, KREM, M , IMR, IPL
3      , P(155,2) , DI(155,2)
COMMON/LIM/ LIM1(10), LIM2(10)
DIMENSION SP(32,10)
DIMENSION NUYX(250), NUXY(250), GXY(250), EX(250), EY(250)
DIMENSION BUMP(10) , BUMP1(5)
EQUIVALENCE (NUF, SP)
EQUIVALENCE (C, CP)
DIMENSION JL(79,3)
DIMENSION COMENT(10)
C
C   NUYX = NUXY, THIS MODIFICATION
EQUIVALENCE (NUXY, NUYX)
DIMENSION TH(155) , CP(3,3) , T1(3,3,2), T2(3,3), DTD(18),
1 T(6,6), KMX(6,6), C(3,3), A(6,6,2), D0(3,6), DD(18), DT0(6,3)
EQUIVALENCE (S, F) , (S(90), VF), (S(180), PHI), (S(270), ER)
1      , (S(300), EF), (S(390), NUR)
2      , (DO, DD) , (DT0, DTD) , (S(4356), TH)
3      , (S(4511), CP) , (S(4521), T1) , (S(4541), T2)
4      , (S(4551), CABC) , (S(4581), T) , (S(4621), KMX)
5      , (S(4671), A)
TYPE REAL NUXY
TYPE INTEGER BUMP, BUMP1
TYPE REAL NUYX, NUR, NUF, KPR, KMX
DATA (KTOTAL = 97)
DATA (BUMP = 2, 4, 7, 7, 15, 1, 9, 9, 12, -1)
DATA (BUMP1 = 5, 7, 7, 7, 5) , (PI = 3.1415927)
DATA (RAD = 57.29578)
DATA (XLIM = 1.E-8)
DATA (PK = 999.)
DATA ((DD(JJ), JJ=1, 18) = 0., 0., 0., 1., 0., 0., 0., 0., 0., 1.,
1      0., 0., 0., 0., 0., 0., 1., 0., 1., 0.)
DATA ((DTD(JJ), JJ=1, 18) = 0., 1., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
1      0., 0., 1., 0., 0., 1., 0., 0.)
DATA(((JL(I, J), I=1, 79), J=1, 3) =
1 1, 1, 1, 2, 2, 3, 3, 3, 4, 4, 4, 5, 6, 7, 7, 8, 8, 8, 9, 10, 10, 11, 11, 12, 13, 13,
2 14, 15, 15, 16, 16, 17, 17, 18, 18, 19, 20, 23, 24, 24, 25, 25, 25, 26, 28, 29,
3 29, 30, 30, 35, 21, 21, 21, 21, 22, 22, 23, 23, 27, 27, 27, 28, 28, 36, 46, 37,
4 47, 38, 48, 39, 39, 40, 41, 41, 41, 42, 42, 43, 43,
5 2, 3, 4, 6, 7, 7, 8, 9, 9, 10, 11, 11, 13, 6, 14, 14, 15, 16, 16, 16, 17, 10,
6 18, 18, 20, 21, 21, 21, 22, 15, 23, 23, 24, 17, 25, 25, 35, 27, 27, 28, 28, 29,
7 30, 30, 32, 32, 33, 33, 34, 36, 35, 37, 38, 39, 39, 40, 40, 41, 41, 42, 43, 43, 44,
8 45, 37, 46, 38, 47, 39, 48, 49, 49, 40, 50, 51, 51, 52, 52, 53,
9 3, 4, 5, 7, 3, 8, 9, 4, 10, 11, 5, 12, 14, 14, 8, 15, 16, 9, 10, 17, 18, 18, 12,
1 19, 21, 14, 15, 22, 23, 23, 17, 24, 25, 25, 19, 26, 21, 24, 28, 25, 29, 30, 26,
2 31, 29, 33, 30, 34, 31, 37, 37, 38, 39, 22, 40, 23, 41, 27, 42, 43, 28, 44, 32,
3 46, 36, 47, 37, 48, 38, 49, 40, 50, 50, 51, 42, 52, 43, 53, 44 )
C
READ 1000, (COMENT(I), I=1, 10)
1000 FORMAT(10A8)
PRINT 1001, (COMENT(I), I=1, 10)

```

```

1001 FORMAT(1H1,10A8)
C
C KTOTAL - TOTAL NUMBER OF NODES CONSIDFRED
C
C
C READ AND PRINT
C
      JTOTAL = 158
      READ 1003, VF, GAMMA, EI, XNUI, BETA
1003 FORMAT(5E10.4)
      IF (VF.EQ.0.) GO TO 950
      PRINT 1018
      PRINT 1016
      PRINT 1017, VF, GAMMA, EI, XNUI, BETA
1016 FORMAT(1H ,13X,2HVVF, 10X,5HGAMMA, 13X,2HEI, 11X,4HXNUI, 11X4HBETA)
1017 FORMAT(1H ,5(E15.8))
1018 FORMAT(///// )

      S3 = SQRT(3.)
      S302 = S3/2.
      EII = EI / GAMMA
      XNUII = XNUI/BETA
      GII = EII/(2.*(1.+XNUII))
      XNU(1) = XNUI
      XNU(2) = XNUII
C
      EPI = EI
      EPII = EII
      XNUPI = XNUI
      XNUPII = XNUII
      EPI = EI/(1.-XNUI**2)
      EPII = EII/(1.-XNUII**2)
C
      XNUPI = XNUI/(1.-XNUI)
      XNUPII = XNUII/(1.-XNUII)
      DO 200 I = 1,97
      TH(I) = 1.
      ALF(I) = 0.
      D(I,1) = 1000.
      D(I,2) = 1000.
      P(I,1) = 0.
      P(I,2) = 0.
200 CONTINUE
      I = 0
      DO 201 J = 1,10
      I = I + BUMP(J)
      P(I,2) = 1000.
      P(98-I,2) = 1000.
      D(I,2) = 1.
      D(98-I,2) = -1.
201 CONTINUE
      I = 0
      DO 202 J = 1,5
      I = I + BUMP1(J)
      P(I,1) = 1000.
      P(98-I,1) = 1000.
      D(I,1) = 0.
      D(98-I,1) = 0.
202 CONTINUE
      P(1,1) = 1000.
      P(1,2) = 1000.
      D(1,1) = 0.

```

```

D(1.2) = 1.
P(34.1) = 1000.
P(34.2) = 1000.
D(34.1) = 0.
D(34.2) = -1.
P(64.1) = 1000.
P(64.2) = 1000.
D(64.1) = 0.
D(64.2) = 1.
P(97.1) = 1000.
P(97.2) = 1000.
D(97.1) = 0.
D(97.2) = -1.
R = SQRT(2.0530VF/PI)
X(1) = S302
Y(1) = .5
DO 210 I=1,4
X(I+1) = S302 - R/4.0 * COS(PI*(I-1)/6.0)
Y(I+1) = .5 - R/4.0 * SIN(PI*(I-1)/6.0)
210 CONTINUE
C
DO 220 I=1,7
X(I+5) = S302 - R/2.0 * COS(PI*(I-1)/12.0)
Y(I+5) = .5 - R/2.0 * SIN(PI*(I-1)/12.0)
C
X(I+12) = S302 - 3.0*R/4.0 * COS(PI*(I-1)/12.0)
Y(I+12) = .5 - 3.0*R/4.0 * SIN(PI*(I-1)/12.0)
C
X(I+19) = S302 - R * COS(PI*(I-1)/12.0)
Y(I+19) = .5 - R * SIN(PI*(I-1)/12.0)
220 CONTINUE
X(34) = S302
Y(34) = -.5
X(35) = S302
Y(35) = (Y(26)+Y(34)) / 2.
X(45) = -.5 * TAN(PI/6.0)
Y(45) = .5
DX = (1.0-R)/(2.0 * COS(PI/6.0))
X(36) = X(45) + DX
Y(36) = .5
X(39) = (X(20)+X(36))/2.
C
Y(39) = .5
DO 230 I = 1,4
X(I+45) = (4 - I) * X(45) / 4.
Y(I+45) = (4 - I) * Y(45) / 4.
230 CONTINUE
C
DELX = X(46) - X(45)
DELY = Y(46) - Y(45)
DO 240 I = 1,8
X(I+36) = X(I+35) + DELX
Y(I+36) = Y(I+35) + DELY
240 CONTINUE
X(32) = X(44) + (X(34) - X(44))/3.
Y(32) = -.5
X(33) = 2.0 * X(32) - X(44)
Y(33) = -.5
X(27) = X(29) + (X(32) - X(29))/3.
Y(27) = Y(29) + (Y(32) - Y(29))/3.
X(28) = 2.0 * X(27) - X(29)
Y(28) = 2.0 * Y(27) - Y(29)

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X(29) = X(28) + (X(31) - X(28)) / 3.
Y(29) = Y(28) + (Y(31) - Y(28)) / 3.
X(30) = 2. * X(29) - X(28)
Y(30) = 2. * Y(29) - Y(28)
C
C
DO 250 I=50,97
X(I) = -X(98-I)
Y(I) = -Y(98-I)
250 CONTINUE
C
C
C PRINT OUT NODAL DATA
I = 1
12 CONTINUE
LINE = 4
PRINT 1010
13 CONTINUE
PRINT 1011, I, X(I), Y(I), P(I,1), P(I,2), D(I,1), D(I,2),
1 TH(I), ALF(I)
I = I+1
IF(I.GT.KTOTAL) GO TO 14
LINE = LINE + 2
IF(LINE.GT.56) GO TO 12
GO TO 13
14 CONTINUE
1011 FORMAT(1H 14,7(3XE11.4),5X,F11.2/)
1010 FORMAT(5H1NODE,7X,1HX,13X,1HY,13X,2HP1,12X,2HP2,12X,2HD1,12X,2HD2,
1 8X,9HTHICKNESS,9X,5HALPHA /)
C
C I - NUMBER OF NODE
C X - X- COORDINATE OF ITH NODE
C Y - Y- COORDINATE OF ITH NODE
C P(I,1) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 1 DIRECTION
C P(I,2) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 2 DIRECTION
C D(I,1) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 1 DIRECTION
C D(I,2) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 2 DIRECTION
C ALF(I) - ANGLE BETWEEN X DIRECTION AND 1 DIRECTION (POSITIVE WHEN
C COUNTER-CLOCKWISE)
C TH(I) - PLATE THICKNESS AT ITH NODE
C
C GET MSK MATRIX
JP = 0
DO 27 J=1,KTOTAL
DO 27 I=1,2
IF (P(J,I).GT.PK) GO TO 27
JP = JP+1
MSK(JP) = 2 * (J-1) + I
27 CONTINUE
C MSK IS MATRIX OF INDICES OF KNOWN FORCES
C IF FORCE P IS UNKNOWN, IT IS INPUT AS 1000.
C NOW READ IN TRIANGLE DATA
C
C JTOTAL - TOTAL NUMBER OF TRIANGLES
DO 19 I=1,79
DO 19 J=1,3
19 L(I,J) = JL(I,J)
DO 260 I= 80,158
DO 260 J= 1,3
L(I,J) = 98 - L(159-I,J)
260 CONTINUE
C L(I,J) - INDEX OF THE FIRST NODE OF THE JTH TRIANGLE

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C L(J,2) - INDEX OF THE SECOND NODE OF THE JTH TRIANGLE
C L(J,3) - INDEX OF THE THIRD NODE OF THE JTH TRIANGLE
C
DO 20 I=1,JTOTAL
  IF(I.LE.36.OR.I.GE.123) GO TO 300
  EX(I) = EPII
  EY(I) = EPII
  NUXY(I) = XNUPII
  GXY(I) = EPII/(2.*(1.+XNUPII))
  GO TO 310
300 EX(I) = EPI
  EY(I) = EPI
  NUXY(I) = XNUPI
  GXY(I) = EPI / (2.*(1.+XNUPI))
310 CONTINUE
20 CONTINUE
C PRINT OUT TRIANGLE DATA
  LINE = 4
  PRINT 1012
  DO 24 I=1,JTOTAL
    IF(LINE.LT.54) GO TO 22
    LINE = 4
    PRINT 1012
1012 FORMAT(1H1,8HTRIANGLE,4X6HNODE 1,3X,6HNODE 2,3X,6HNODE 3,18X,2HEX,
1 18X 2HEY,16X,4HNXYX,17X,3HGXY//)
  22 LINE = LINE+2
  24 PRINT 1023, I, (L(I,J),J=1,3),EX(I),EY(I),NUXY(I),GXY(I)
1023 FORMAT(1H ,5X,I3,7X,I3,2I6X,I3),4(5X,E15.8I//)
C
C
MSIZE = 156
KREM = 6
N = 25
M = MSIZE / N
IF(KREM.NE.N) M = M+1
C
LIM1(1) = 1
LIM1(2) = 1
LIM1(3) = 1
LIM1(4) = 1
LIM1(5) = 1
LIM1(6) = 1
LIM1(7) = 1
LIM2(1) = 158
LIM2(2) = 158
LIM2(3) = 158
LIM2(4) = 158
LIM2(5) = 158
LIM2(6) = 158
LIM2(7) = 158
C
C PRINT OUT PARTITION INFORMATION
PRINT 1008
1008 FORMAT(1H1,12X,20HTRIANGLES CONSIDERED)
DO 31 I=1,M
  ISIZE = N
  IF(I.EQ.1) ISIZE = KREM
  PRINT 1009, I,LIM1(I),LIM2(I), ISIZE
31 CONTINUE
1009 FORMAT(10H PARTITION,6X 5MFIRST,2X,2MTO,2X,4HLAST, 6X,9MDIMENSION/
1 4X,I3,11X,I3,6X,I3,10X,I3)

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DO 28 I = 1,KTOTAL
ALF(I) = ALF(I)/RAD
28 CONTINUE
C ALL ANGLES NOW IN RADIAN
DO 400 I=1, JTOTAL
C I IS A TRIANGLE COUNTER
B11 = 1./EX(I)
B12 = -NUYX(I) / EY(I)
B22 = 1. / EY(I)
B33 = 1. / GXY(I)
DELTA = B11*B22 -B12**2
CP(2,1) = - B12 / DELTA
CP(1,1) = B22 / DELTA
CP(3,1) = 0.
CP(1,2) = CP(2,1)
CP(2,2) = B11/ DELTA
CP(3,2) = 0.
CP(1,3) = 0.
CP(2,3) = 0.
CP(3,3) = 1./B33
C
C C NOW IN C(3,3) , MATRIX I
C
30 CONTINUE
THOMEG = X(L(I,2))*Y(L(I,3)) +X(L(I,1))*Y(L(I,2))
1 + Y(L(I,1))*X(L(I,3)) -X(L(I,3))*Y(L(I,2))
2 - X(L(I,1))*Y(L(I,3)) -Y(L(I,1))*X(L(I,2))
THOMEG=(TH(L(I,1))+TH(L(I,2))+TH(L(I,3)))/ 6. * THOMEG
X12 = X(L(I,2))- X(L(I,1))
X13 = X(L(I,3))- X(L(I,1))
ETA2= Y(L(I,2))- Y(L(I,1))
ETA3= Y(L(I,3))- Y(L(I,1))
DELTA = X12*ETA3 - X13*ETA2
DO 38 II = 1,3
DO 38 JJ = 1,3
A(II+3, JJ,1) = 0.
A(II, JJ+3,1) = 0.
A(II+3, JJ,2) = 0.
A(II, JJ+3,2) = 0.
38 CONTINUE
A(1,1,1) = 1.
A(2,1,1) = -(ETA3-ETA2) / DELTA
A(3,1,1) = (X13 - X12) / DELTA
A(1,2,1) = 0.
A(2,2,1) = ETA3 / DELTA
A(3,2,1) = -X13 / DELTA
A(1,3,1) = 0.
A(2,3,1) = -ETA2 / DELTA
A(3,3,1) = X12 / DELTA
DO 39 II = 1,3
DO 39 JJ = 1,3
A(II+3, JJ+3,1) = A(II, JJ,1)
A(II, JJ,2) = A(JJ, II,1)
A(II+3, JJ+3,2) = A(JJ, II,1)
39 CONTINUE
C
C TRANSPOSE OF INVERSE OF A NOW IN A(1,1,2) , A INVERSE STILL IN A
C
CALL MXMULT(DO,A,KMX(1,1),3,6,6)
CALL MXMULT(C,KMX(1,1) ,CDA(1,1,1),3,3,6)
C PRODUCT C*D*(A**-1) NOW IN CDA(1,1,1), ITH TRIANGLE
C

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```

CALL MXMULT(A(1,1,2),DT0,A,6,6,3)
CALL MXCON(A,KPR(1,1,1),THOMEG,6,3)
CALL MXMULT(KPR(1,1,1),CDA(1,1,1),KMX,6,3,6)
C
C MATRIX K(I) NOW IN KMX, TRIANGLE I
DO 40 II =1,6
DO 40 JJ =1,6
T(II,JJ) = 0.
40 CONTINUE
T(1,1) = COS(ALF(L(I,1)))
T(4,1) = SIN(ALF(L(I,1)))
T(2,2) = COS(ALF(L(I,2)))
T(5,2) = SIN(ALF(L(I,2)))
T(3,3) = COS(ALF(L(I,3)))
T(6,3) = SIN(ALF(L(I,3)))
T(1,4) = -T(4,1)
T(4,4) = T(1,1)
T(2,5) = -T(5,2)
T(5,5) = T(2,2)
T(3,6) = -T(6,3)
T(6,6) = T(3,3)
CALL MXMULT(KMX,T,A,6,6,6)
T(4,1) = - T(4,1)
T(5,2) = - T(5,2)
T(6,3) = - T(6,3)
T(1,4) = - T(1,4)
T(2,5) = - T(2,5)
T(3,6) = - T(3,6)
C
C INVERSE OF T NOW IN T
C
CALL MXMULT (T,A,KPR(1,1,1),6,6,6)
C
C K-PRIME NOW IN KPR . A HAS BEEN CLOBBERED.
400 CONTINUE
RETURN
800 PRINT 1051,II,JJ,II
1051 FORMAT(1H1, 5H EF(,I3,1H,I3,7H) = ER(,I3,6H) = 0.)
STOP
900 CONTINUE
PRINT 1050 , J
1050 FORMAT(1H1 35HCOULD NOT INVERT MATRIX T1,TRIANGLE,I3)
950 CONTINUE
REWIND 20
STOP
END

```

CASE 2

```

SUBROUTINE STRESS
C
C   STRESS SUBROUTINE CASE 2
C
C   THIS SUBROUTINE DERIVES AND PRINTS STRESSES
C
COMMON /STRESS/ ALF(155), MSIZE, R, GAMMA, EI, GII,XNU(2)
COMMON /1/ S(32,32,5),L(300,3),G(32),SPACE(20)
1   . NUF(3,314) . X(155),Y(155),CDA(3,6,250)
2   . KPR(6,6,250),MSK(310),JTOTAL,N,KREM,M . IMR, IPL
3   . P(155,2) . D(155,2)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION DVX(6), SIGOUT(632)
DIMENSION SP(32,10)
EQUIVALENCE(S,SIGOUT) ,(NUF,SP)
TYPE REAL KPR
EQUIVALENCE (S,SIG) , (S(931),PSTR) , (S(1861),X0),(S(2161),Y0)
EQUIVALENCE (S(2461),DEL) , (S(2761),DX)
DIMENSION ERR(310)
DIMENSION DX(6) , SIG(4,168) , PSTR(3,310)
DIMENSION X0(300) , Y0(300) , DEL(310)
DIMENSION KS22(32,96)
EQUIVALENCE (KPR,KS22)
TYPE REAL KS22
DATA (PK=999.)
C REMOVE GAPS FROM SP(32,10) = DEL(310)
DO 5 J=1,KREM
DEL(J) = SP(J,1)
5 CONTINUE
KLOC = KREM -N
DO 10 I = 2,M
KLOC = KLOC + N
DO 10 J = 1,N
DEL(KLOC+J) = SP(J,I)
10 CONTINUE
PRINT 1010
1010 FORMAT(1H1,50X,13HDISPLACEMENTS,/)
NDEL = MSIZE/7
JCNT = 0
DO 15 J=1,NDEL
JCNT = JCNT + 1
IF(JCNT.LE.18) GO TO 14
PRINT 1010
JCNT = 0
14 JFIR = 7*(J-1) + 1
JLAST = JFIR + 6
PRINT 1011 , (K,K=JFIR,JLAST)
1011 FORMAT(1H ,7(8X,4HDEL(,I3,1H) ))
PRINT 1012 , (DEL(K),K=JFIR,JLAST)
1012 FORMAT(1H ,7(2X,E14.7)/)
15 CONTINUE
LOC1 = 7*NDEL+1
LOC2 = MSIZE
IF(LOC1.GT.LOC2) GO TO 20
PRINT 1011,(K,K=LOC1,LOC2)
PRINT 1012 , (DEL(K),K=LOC1,LOC2)
20 CONTINUE
DO 855 I=1,JTOTAL
J = I
KZ = 0
DO 831 KK =1,3

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```

DO 832 KJ = 1,2
KZ = K + 1
IF( PI (J, KK), KJ), GT, PK) GO TO 828
IF(2*(L(J, KK)-1)+KJ-MSK(IPL)) 801, 802, 803
802 I12 = IPL
GO TO 827
801 IMR = IPL - 1
806 IF(2*(L(J, KK)-1) +KJ -MSK(IMR)) 804, 805, 805
805 I12 = IMR
IPL = IMR
GO TO 827
804 IMR = IMR - 1
GO TO 806
803 IMR = IPL + 1
807 IF(2*(L(J, KK)-1) +KJ -MSK(IMR)) 805, 805, 810
810 IMR = IMR + 1
GO TO 807
827 DVX(KZ) = DEL(I12)
GO TO 832
828 DVX(KZ) = D(L(J, KK), KJ)
832 CONTINUE
DX(KZ-1) = DVX(KZ-1)*COS(ALF(L(J, KK))) - DVX(KZ)*SIN(ALF(L(J, KK)))
DX(KZ) = DVX(KZ-1)*SIN(ALF(L(J, KK))) + DVX(KZ)*COS(ALF(L(J, KK)))
831 CONTINUE
DX2 = DX(2)
DX(2) = DX(3)
DX(3) = DX(5)
DX(5) = DX(4)
DX(4) = DX2
CALL MXMULT(CDA(1,1,1), DX, SIG(1,1), 3, 6, 1)
KK3 = 1
IF(1, GE, 37, AND, 1, LE, 122) KK3 = 2
SIG(4,1) = SIG(3,1)
SIG(3,1) = XNU(KK3)*(SIG(1,1)+SIG(2,1))
C SIGMA NOW IN SIG(1,1) , TRIANGLE 1
X0(1) = 0.
Y0(1) = 0.
DO 840 K = 1,3
X0(1) = X0(1) + X(L(1, K))
Y0(1) = Y0(1) + Y(L(1, K))
840 CONTINUE
X0(1) = X0(1) / 3.
Y0(1) = Y0(1) / 3.
850 CONTINUE
855 CONTINUE
ENBARY = R/4.*(SIG(2,1)+SIG(2,4)+SIG(2,13)+SIG(2,25))
1 +(X(20)-X(35))*(SIG(2,37)+SIG(2,50))
2 +(X(36)-X(45))*(SIG(2,64)+SIG(2,80))
3 +(X(54)-X(64))/3.*(SIG(2,96)+SIG(2,111)+SIG(2,113))
ENBARY = SQRT(3.)/ENBARY
DO 860 I=1,632
860 SIGOUT(I) = SIGOUT(I)*ENBARY
I = 1
870 CONTINUE
PRINT 1000
LINE = 3
1000 FORMAT(1H1,2(8HTRIANGLE,6X,8HCENTROID,9X,12HVECTOR SIGMA,10X)/)
871 CONTINUE
II = I+1
DO 880 J=1,4
GO TO (873,874,875,875), J
873 PRINT 1001, I, X0(1), SIG(J,1), II, X0(II), SIG(J,II)

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      GO TO 878
874 PRINT 1002,  Y0(I),SIG(J,I),      Y0(I), SIG(J,I)
      GO TO 878
875 PRINT 1003, SIG(J,I), SIG(J,I)
878 CONTINUE
880 CONTINUE
      LINE = LINE + 6
      I = I + 2
      PRINT 1004
1001 FORMAT(1H 2(5X,I3,F14.6, 7X,E15.8,10X))
1002 FORMAT(1H 2(8X,F14.6, 7X,E15.8,10X) )
1003 FORMAT(1H 2(29X,E15.8,10X))
1004 FORMAT(/)
1069 FORMAT(1H ,30X,E15.8)
      IF(I.GT.JTOTAL) GO TO 890
      IF (LINE.GT.54) GO TO 870
      GO TO 871
890 CONTINUE
      WRITE (20) (SIGOUT(I),I=1,316)
      PRINT 8787, ENBARY
8787 FORMAT(/23H NORMALIZATION FACTOR =,2X,E15.7)
      RETURN
      END

```

CASE 2

PROGRAM MAIN

```

C
C THIS IS THE EXECUTIVE PROGRAM USED WITH SUBROUTINE BIGMX AND SUBROUTINE
C CHLSKY TO GENERATE AND SOLVE LARGE SYSTEMS OF LINLAR EQUATIONS
C

```

```

      REWIND 20
      CALL TAPESKIP(20,6,0)
      CALL TAPESKIP(20,3,0)
1 CONTINUE
      CALL INPUT

```

CASE 6

```

C
C NOW HAVE ALL K-PRIME AND CDA MATRICES
C

```

```

      CALL CHLSKY

```

```

C
C NOW HAVE SOLUTION U IN SP
C

```

```

      CALL STRESS

```

```

C
C ALL STRESSES NOW PRINTED OUT
C

```

```

      GO TO 1

```

22 CARDS

SUBROUTINE INPUT

TRANSVERSE SHEAR CASE 6

THIS SUBROUTINE READS AND PRINTS THE INPUTS FOR THE PLANAR FINITE ELEMENT PROGRAM. ALL INPUTS NOT READ ARE GENERATED HERE.

```
COMMON /STRSS/ ALF(155), MSIZE, R, GAMMA, EI, GII, XNU(2)
COMMON /I/ S(32,32,5), L(300,3), G(32), SPACE(20)
1   , NUF(3,314) , X(155), Y(155), CDA(3,6,250)
2   , KPR(6,6,250), MSK(310), JTOTAL, N, KREM, M , IMR, IPL
3   , P(155,2) , D(155,2)
COMMON/LIM/ LIM1(10), LIM2(10)
DIMENSION SP(32,10)
DIMENSION NUYX(250), NUXY(250), GXY(250), EX(250), EY(250)
DIMENSION BUMP(10), BUMP1(5)
EQUIVALENCE (NUF, SP)
EQUIVALENCE (C, CP)
DIMENSION JL(79,3)
DIMENSION COMENT(10)
```

NUYX = NUXY, THIS MODIFICATION

```
EQUIVALENCE (NUXY, NUYX)
DIMENSION TH(155), CP(3,3), T1(3,3,2), T2(3,3), DTD(18),
1 T(6,6), KMX(6,6), C(3,3), A(6,6,2), DO(3,6), DD(18), DTO(6,3)
EQUIVALENCE (S, F), (S(901), VF), (S(1801), PHI), (S(2701), ER)
1   , (S(3001), EF), (S(3901), NUR)
2   , (DO, DD), (DT, DTD), (S(4356), TH)
3   , (S(4511), CP), (S(4521), T1), (S(4541), T2)
4   , (S(4551), CAB), (S(4581), T), (S(4621), KMX)
5   , (S(4671), A)
```

TYPE REAL NUYX

TYPE INTEGER BUMP, BUMP1

TYPE REAL NUYX, NUR, NUF, KPR, KMX

DATA (KTOTAL = 97)

DATA(BUMP= 2,4,7,7,15,1,9,9,12,-1)

DATA(BUMP1= 5,7,7,7,5), (PI= 3.1415927)

DATA (RAD = 57.29578)

DATA (XLIM=1.E-8)

DATA (PK=999.)

DATA ((DD(JJ), JJ=1,18) = 0., 0., 0., 1., 0., 0., 0., 0., 1.,

0., 0., 0., 0., 0., 0., 1., 0., 1., 0., )

DATA ((DTD(JJ), JJ=1,18) = 0., 1., 0., 0., 0., 0., 0., 0., 0., 0.,

0., 0., 1., 0., 0., 0., 1., 0., 0., )

DATA(((JL(I, J), I=1,79), J=1,3) =

1 1,1,1,2,2,3,3,3,4,4,4,5,6,7,7,8,8,8,9,10,10,11,11,12,13,13,

2 14,15,15,16,16,17,17,18,18,19,20,23,24,24,25,25,26,28,29,

3 29,30,30,35,21,21,21,21,22,22,23,23,27,27,27,28,28,36,46,37,

4 47,38,48,39,39,40,41,41,41,42,42,43,43,

5 2,3,4,6,7,7,8,9,9,10,11,11,13,6,14,14,15,16,16,17,10,

6 18,18,20,21,21,21,22,15,23,23,24,17,25,25,35,27,27,28,28,29,

7 30,30,32,32,33,33,34,36,35,37,38,39,39,40,40,41,41,42,43,43,44,

8 45,37,46,38,47,39,48,49,49,40,50,51,51,52,52,53,

9 3,4,5,7,3,8,9,4,10,11,5,12,14,14,8,15,16,9,10,17,18,18,12,

1 19,21,14,15,22,23,23,17,24,25,25,19,26,21,24,28,25,29,30,26,

2 31,29,33,30,34,31,37,37,38,39,22,40,23,41,27,42,43,28,44,32,

3 46,36,47,37,48,38,49,40,50,50,51,42,52,43,53,44 )

READ 1000, (COMENT(I), I=1,10)

1000 FORMAT(10A8)

PRINT 1001, (COMENT(I), I=1,10)

```

1001 FORMAT(1H1,10A8)
C
C   KTOTAL - TOTAL NUMBER OF NODES CONSIDERED
C
C   READ AND PRINT
C
      JTOTAL = 158
      READ 1003, VF, GAMMA, EI, XNUI, BETA
1003 FORMAT(5E10.4)
      IF (VF.EQ.0.) GO TO 950
      PRINT 1018
      PRINT 1016
      PRINT 1017, VF, GAMMA, EI, XNUI, BETA
1016 FORMAT(1H ,13X,2HV, 10X,5HGAMMA, 13X,2HEI, 11X,4HXNUI, 11X4HBETA)
1017 FORMAT(1H ,5(E15.8))
1018 FORMAT(////)

      S3 = SQRT(3.)
      S302 = S3/2.
      EII = EI / GAMMA
      XNUII = XNUI/BETA
      GII = EII/(2.*(1.+XNUII))
      XNU(1) = XNUI
      XNU(2) = XNUII
C
      EPI = EI
      EPII = EII
      XNUPI = XNUI
      XNUPII = XNUII
      EPI = EI/(1.-XNUI**2)
      EPII = EII/(1.-XNUII**2)
C
      XNUPI = XNUI/(1.-XNUI)
      XNUPII = XNUII/(1.-XNUII)
      DO 200 I = 1,97
      TH(I) = 1.
      ALF(I) = 0.
      D(I,1) = 1000.
      D(I,2) = 1000.
      P(I,1) = 0.
      P(I,2) = 0.
200 CONTINUE
      I = 0
      DO 201 J = 1,10
      I = I + BUMP(J)
      D(I,1) = 0.
      D(98-I,1) = 0.
      P(I,1) = 1000.
      P(98-I,1) = 1000.
201 CONTINUE
      I = 0
      DO 202 J = 1,5
      I = I + BUMP1(J)
      D(I,2) = 1.
      D(98-I,2) = -1.
      P(I,2) = 1000.
      P(98-I,2) = 1000.
202 CONTINUE
      D(1,1) = 0.
      D(1,2) = 1.
      P(1,1) = 1000.

```

```

P(1,2) = 1000.
D(34,1) = 0.
D(34,2) = 1.
P(34,1) = 1000.
P(34,2) = 1000.
D(64,1) = 0.
D(64,2) = -1.
P(64,1) = 1000.
P(64,2) = 1000.
D(97,1) = 0.
D(97,2) = -1.
P(97,1) = 1000.
P(97,2) = 1000.
P(64,2) = 1000.
R = SQRT(2.*S3*VF/P1)
X(1) = S302
Y(1) = .5
DO 210 I=1,4
X(I+1) = S302 - R/4.*COS(PI*(I-1)/6.)
Y(I+1) = .5 - R/4.*SIN(PI*(I-1)/6.)
210 CONTINUE
C
DO 220 I=1,7
X(I+5) = S302 - R/2.*COS(PI*(I-1)/12.)
Y(I+5) = .5 - R/2.*SIN(PI*(I-1)/12.)
C
X(I+12) = S302 - 3.*R/4.*COS(PI*(I-1)/12.)
Y(I+12) = .5 - 3.*R/4.*SIN(PI*(I-1)/12.)
C
X(I+19) = S302 - R * COS(PI*(I-1)/12.)
Y(I+19) = .5 - R * SIN(PI*(I-1)/12.)
220 CONTINUE
X(34) = S302
Y(34) = -.5
X(31) = S302
Y(31) = (Y(26)+Y(34)) / 2.
X(45) = -.5 * TAN(PI/6.)
Y(45) = .5
DX = (1.-R)/(2.*COS(PI/6.))
X(36) = X(45) + DX
Y(36) = .5
X(35) = (X(21)+X(36))/2.
Y(35) = .5
C
DO 230 I = 1,4
X(I+45) = (4 - I) * X(45) / 4.
Y(I+45) = (4 - I) * Y(45) / 4.
230 CONTINUE
C
DFLX = X(46) - X(45)
DELY = Y(46) - Y(45)
DO 240 I = 1,8
X(I+36) = X(I+35) + DELX
Y(I+36) = Y(I+35) + DELY
240 CONTINUE
X(32) = X(46) + (X(34) - X(46))/3.
Y(32) = -.5
X(33) = 2.*X(32) - X(46)
Y(33) = -.5
X(27) = X(23) + (X(32) - X(23))/3.
Y(27) = Y(23) + (Y(32) - Y(23))/3.
X(28) = 2.*X(27) - X(23)

```

```

Y(28) = 2.*Y(27)-Y(23)
X(29) = X(28) +(X(31)-X(28))/3.
Y(29) = Y(28) +(Y(31)-Y(28))/3.
X(30) = 2.*X(29)-X(28)
Y(30) = 2.*Y(29)-Y(28)
C
C
DO 250 I=50,97
X(I) = -X(98-I)
Y(I) = -Y(98-I)
250 CONTINUE
C
C
C PRINT OUT NODAL DATA
I = 1
12 CONTINUE
LINE = 4
PRINT 1010
13 CONTINUE
PRINT 1011, I, X(I), Y(I), P(1,1), P(1,2), D(1,1), D(1,2),
1 TH(I), ALF(I)
I = I+1
IF(I.GT.KTOTAL) GO TO 14
LINE = LINE + 2
IF(LINE.GT.56) GO TO 12
GO TO 13
14 CONTINUE
1011 FORMAT(1H 14,7(3XE11.4),5X,F11.2/)
1010 FORMAT(5H1NODE,7X,1HX,13X,1HY,13X,2HP1,12X,2HP2,12X,2HD1,12X,2HD2,
1 8X,9HTHICKNESS,9X,5HALPHA /)
C
C I - NUMBER OF NODE
C X - X- COORDINATE OF ITH NODE
C Y - Y- COORDINATE OF ITH NODE
C P(1,1) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 1 DIRECTION
C P(1,2) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 2 DIRECTION
C D(1,1) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 1 DIRECTION
C D(1,2) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 2 DIRECTION
C ALF(I) - ANGLE BETWEEN X DIRECTION AND 1 DIRECTION (POSITIVE WHEN
C COUNTER-CLOCKWISE)
C TH(I) - PLATE THICKNESS AT ITH NODE
C
C GET MSK MATRIX
JP = 0
DO 27 J=1,KTOTAL
DO 27 I=1, 2
IF (P(J,I).GT.PK) GO TO 27
JP = JP+1
MSK(JP) = 2 * (J-1) + I
27 CONTINUE
C MSK IS MATRIX OF INDICES OF KNOWN FORCES
C IF FORCE P IS UNKNOWN, IT IS INPUT AS 1000.
C NOW READ IN TRIANGLE DATA
C
C JTOTAL - TOTAL NUMBER OF TRIANGLES
DO 19 I=1,79
DO 19 J=1,3
19 L(I,J) = JL(I,J)
DO 260 I= 80,158
DO 260 J= 1,3
L(I,J) = 98 - L(159-I,J)
260 CONTINUE

```

```

C L(J,1) - INDEX OF THE FIRST NODE OF THE JTH TRIANGLE
C L(J,2) - INDEX OF THE SECOND NODE OF THE JTH TRIANGLE
C L(J,3) - INDEX OF THE THIRD NODE OF THE JTH TRIANGLE
C
  DO 20 I=1,JTOTAL
  IF(I.LE.36.OR.I.GE.123) GO TO 300
  EX(I) = EPII
  EY(I) = EPII
  NUXY(I) = XNUPII
  GXY(I) = EPII/(2.*(1.+XNUPII))
  GO TO 310
300 EX(I) = EPI
  EY(I) = EPI
  NUXY(I) = XNUPI
  GXY(I) = EPI / (2.*(1.+XNUPI))
310 CONTINUE
20 CONTINUE
C PRINT OUT TRIANGLE DATA
  LINE = 4
  PRINT 1012
  DO 24 I=1,JTOTAL
  IF(LINE.LT.54) GO TO 22
  LINE = 4
  PRINT 1012
1012 FORMAT(1H1,8HTRIANGLE,4X6HNODE 1,3X,6HNODE 2,3X,6HNODE 3,18X,2HEX,
1 18X 2HEY,16X,4HNUYX,17X,3HGXY//)
  22 LINE = LINE+2
  24 PRINT 1023, I, (L(I,J),J=1,3),EX(I),EY(I),NUYX(I),GXY(I)
1023 FORMAT(1H ,5X,I3,7X,I3,2(6X,I3),4(5X,E15.8)/)
C
C
  MSIZE = 156
  KREM = 6
  N = 25
  M = MSIZE / N
  IF(KREM.NE.N) M = M+1
C
  LIM1(1) = 1
  LIM1(2) = 1
  LIM1(3) = 1
  LIM1(4) = 1
  LIM1(5) = 1
  LIM1(6) = 1
  LIM1(7) = 1
  LIM2(1) = 158
  LIM2(2) = 158
  LIM2(3) = 158
  LIM2(4) = 158
  LIM2(5) = 158
  LIM2(6) = 158
  LIM2(7) = 158
C
C PRINT OUT PARTITION INFORMATION
  PRINT 1008
1008 FORMAT(1H1,12X,20HTRIANGLES CONSIDERED)
  DO 31 I=1,M
  ISIZE = N
  IF(I.EQ.1) ISIZE = KREM
  PRINT 1009, I,LIM1(I),LIM2(I), ISIZE
  31 CONTINUE
1009 FORMAT(10H PARTITION,6X 5HFIRST,2X,2HTO,2X,4HLAST, 6X,9HDIMENSION/

```

```

1  4X.13.11X.13.6X.13.10X.13)
DO 28 I = 1,KTOTAL
ALF(I) = ALF(I)/RAD
28 CONTINUE
C ALL ANGLES NOW IN RADIANS
DO 400 I=1, JTOTAL
C I IS A TRIANGLE COUNTER
B11 = 1./EX(I)
B12 = -NUYX(I) / EY(I)
B22 = 1. / EY(I)
B33 = 1. / GXY(I)
DELTA = B11*B22 - B12**2
CP(2,1) = - B12 / DELTA
CP(1,1) = B22 / DELTA
CP(3,1) = 0.
CP(1,2) = CP(2,1)
CP(2,2) = B11/ DELTA
CP(3,2) = 0.
CP(1,3) = 0.
CP(2,3) = 0.
CP(3,3) = 1./B33
C
C NOW IN C(3,3) . MATRIX I
C
30 CONTINUE
THOMEG = X(L(I,2))*Y(L(I,3)) + X(L(I,1))*Y(L(I,2))
1 + Y(L(I,1))*X(L(I,3)) - X(L(I,3))*Y(L(I,2))
2 - X(L(I,1))*Y(L(I,3)) - Y(L(I,1))*X(L(I,2))
THOMEG=(TH(L(I,1))+TH(L(I,2))+TH(L(I,3)))/ 6. * THOMEG
X12 = X(L(I,2))- X(L(I,1))
X13 = X(L(I,3))- X(L(I,1))
ETA2= Y(L(I,2))- Y(L(I,1))
ETA3= Y(L(I,3))- Y(L(I,1))
DELTA = X12*ETA3 - X13*ETA2
DO 38 II = 1,3
DO 38 JJ = 1,3
A(II+3, JJ+3) = 0.
A(II, JJ+3) = 0.
A(II+3, JJ+2) = 0.
A(II, JJ+3+2) = 0.
38 CONTINUE
A(1,1,1) = 1.
A(2,1,1) = -(ETA3-ETA2) / DELTA
A(3,1,1) = (X13 - X12) / DELTA
A(1,2,1) = 0.
A(2,2,1) = ETA3 / DELTA
A(3,2,1) = -X13 / DELTA
A(1,3,1) = 0.
A(2,3,1) = -ETA2 / DELTA
A(3,3,1) = X12 / DELTA
DO 39 II = 1,3
DO 39 JJ = 1,3
A(II+3, JJ+3,1) = A(II, JJ,1)
A(II, JJ,2) = A(JJ, II,1)
A(II+3, JJ+3,2) = A(JJ, II,1)
39 CONTINUE
C
C TRANSPOSE OF INVERSE OF A NOW IN A(1,1,2) . A INVERSE STILL IN A
C
CALL MXMULT(DO,A,KMX(1,1),3,6,6)
CALL MXMULT(C,KMX(1,1) ,CDA(1,1,1),3,3,6)
C PRODUCT C*D*(A** -1) NOW IN CDA(1,1,1), ITM TRIANGLE

```

```

C
CALL MXMULT(A(1:1,2),DT(A,6,6,3)
CALL MXCON(A,KPR(1:1,1),WOMFG,6,3)
CALL MXMULT(KPR(1:1,1),CUA(1:1,1),KMX,6,1,6)
C
C MATRIX K(1) NOW IN KMX. TRIANGLE 1
DO 40 II =1,6
DO 40 JJ =1,6
T(II,JJ) = 0.
40 CONTINUE
T(1,1) = COS(ALFIL(1,1))
T(4,1) = SIN(ALFIL(1,1))
T(2,2) = COS(ALFIL(1,2))
T(5,2) = SIN(ALFIL(1,2))
T(3,3) = COS(ALFIL(1,3))
T(6,3) = SIN(ALFIL(1,3))
T(1,4) = -T(4,1)
T(4,4) = T(1,1)
T(2,5) = -T(5,2)
T(5,5) = T(2,2)
T(3,6) = -T(6,3)
T(6,6) = T(3,3)
CALL MXMULT(KMX,T,A,6,6,6)
T(4,1) = -T(4,1)
T(5,2) = -T(5,2)
T(6,3) = -T(6,3)
T(1,4) = -T(1,4)
T(2,5) = -T(2,5)
T(3,6) = -T(3,6)
C
C INVERSE OF T NOW IN T
C
CALL MXMULT(T,A,KPR(1:1,1),6,6,6)
C
C K-PRIME NOW IN KPR. A HAS BEEN CLOBBERED.
400 CONTINUE
RETURN
800 PRINT 1051,11,11,11
1051 FORMAT(1M) 5M (F1.13,1M,13.7M) = (R1.13,6M) = 0.1
STOP
900 CONTINUE
PRINT 1050 , J
1050 FORMAT(1M) 15M COULD NOT INVERT MATRIX 11,TRIANGLE,13)
950 CONTINUE
REWIND 20
STOP
END
CASE 6

```

## FUNDAMENTAL CASE 6

```

SUBROUTINE STRESS
C
C STRESS SUBROUTINE CASE 6
C
C THIS SUBROUTINE DERIVES AND PRINTS STRESSES
C
COMMON /STRSS/ ALF(155), MSIZE, R, GAMMA, EI, GII,XNU(2)
COMMON /I/ S(32,32,5),L(300,3),G(32),SPACE(20)
1   , NUF(3,314) , X(155),Y(155),CDA(3,6,250)
2   , KPR(6,6,250),MSK(310),JTOTAL,N,KREM,M , IMR, IPL
3   , P(155,2) , D(155,2)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION DVX(6), SIGOUT(632)
DIMENSION SP(32,10)
EQUIVALENCE(S,SIGOUT) ,(NUF,SP)
TYPE REAL KPR
EQUIVALENCE (S,SIG) , (S(931),PSTR) , (S(1861),X0),(S(2161),Y0)
EQUIVALENCE (S(2461),DEL) , (S(2761),DX)
DIMENSION ERR(310)
DIMENSION DX(6) , SIG(4,168) , PSTR(3,310)
DIMENSION X0(300) , Y0(300), DEL(310)
DIMENSION KS22(32,96)
EQUIVALENCE (KPR,KS22)
TYPE REAL KS22
DATA (PK=999,.)
C REMOVE GAPS FROM SP(32,10) = DEL(310)
DO 5 J=1,KREM
DEL(J) = SP(J,1)
5 CONTINUE
KLOC = KREM -N
DO 10 I = 2,M
KLOC = KLOC + N
DO 10 J = 1,N
DEL(KLOC+J) = SP(J,I)
10 CONTINUE
PRINT 1010
1010 FORMAT(1H1,5X,13HDISPLACEMENTS,/)
NDEL = MSIZE/7
JCNT = 0
DO 15 J=1,NDEL
JCNT = JCNT + 1
IF(JCNT.LE.18) GO TO 14
PRINT 1010
JCNT = 0
14 JFIR = 7*(J-1) + 1
JLAST = JFIR + 6
PRINT 1011 , (K,K=JFIR,JLAST)
1011 FORMAT(1H ,7(8X,4HDEL(,I3,1H) ))
PRINT 1012, (DEL(K),K=JFIR,JLAST)
1012 FORMAT(1H ,7(2X,E14.7)/)
15 CONTINUE
LOC1 = 7*NDEL+1
LOC2 = MSIZE
IF(LOC1.GT.LOC2) GO TO 20
PRINT 1011,(K,K=LOC1,LOC2)
PRINT 1012, (DEL(K),K=LOC1,LOC2)
20 CONTINUE
DO 855 I=1,JTOTAL
J = I
KZ = 0
DO 831 KK = 1,3

```

```

      DO 832 KJ = 1,2
      KZ = KZ + 1
      IF( P(L(J, KK), KJ), GT, PK) GO TO 828
      IF(2*(L(J, KK)-1)+KJ-MSK(IPL)) 802, 803
802  I12 = IPL
      GO TO 827
801  IMR = IPL - 1
806  IF(2*(L(J, KK)-1) +KJ -MSK(IMR)) 804, 805, 805
805  I12 = IMR
      IPL = IMR
      GO TO 827
804  IMR = IMR - 1
      GO TO 806
803  IMR = IPL + 1
807  IF(2*(L(J, KK)-1) +KJ -MSK(IMR)) 805, 805, 810
810  IMR = IMR + 1
      GO TO 807
827  DVX(KZ) = DEL(I12)
      GO TO 832
828  DVX(KZ) = D(L(J, KK), KJ)
832  CONTINUE
      DX(KZ-1) = DVX(KZ-1)*COS(ALF(L(J, KK))) - DVX(KZ)*SIN(ALF(L(J, KK)))
      DX(KZ) = DVX(KZ-1)*SIN(ALF(L(J, KK))) + DVX(KZ)*COS(ALF(L(J, KK)))
831  CONTINUE
      DX2 = DX(2)
      DX(2) = DX(3)
      DX(3) = DX(5)
      DX(5) = DX(4)
      DX(4) = DX2
      CALL MXMULT(CDA(1,1,1), DX, SIG(1,1), 3, 6, 1)
      KK3 = 1
      IF(I, GE, 37, AND, J, LE, 122) KK3 = 2
      SIG(4,1) = SIG(3,1)
      SIG(3,1) = XNU(KK3)*(SIG(1,1)+SIG(2,1))
C  SIGMA NOW IN SIG(1,1) . TRIANGLE I
      XO(I) = 0.
      YO(I) = 0.
      DO 840 K = 1,3
      XU(I) = XU(I) + X(L(I, K))
      YO(I) = YO(I) + Y(L(I, K))
840  CONTINUE
      XO(I) = XO(I) / 3.
      YO(I) = YO(I) / 3.
850  CONTINUE
855  CONTINUE
      TAUBAR = R/4.*(SIG(4,3)+SIG(4,12)+SIG(4,24)+SIG(4,36))
      + (1.-R)/2.*(SIG(4,44)+SIG(4,49))
      TAUBAR = 1./TAUBAR
      DO 860 I = 1, 632
860  SIGOUT(I) = SIGOUT(I) * TAUBAR
      I = I + 1
870  CONTINUE
      PRINT 1000
      LINE = 3
1000  FORMAT(1H1, 2(8HTRIANGLE, 6X, 8HCENTROID, 9X, 12HVECTOR SIGMA, 10X)/)
871  CONTINUE
      II = I + 1
      DO 880 J = 1, 4
      GO TO (873, 874, 875, 875), J
873  PRINT 1001, I, XO(I), SIG(J, I), II, XO(II), SIG(J, II)
      GO TO 878
874  PRINT 1002, YO(I), SIG(J, I), YO(II), SIG(J, II)

```

```

GO TO 878
875 PRINT 1003, SIG(J,1), SIG(J,11)
878 CONTINUE
880 CONTINUE
LINE = LINE + 6
I = I + 2
PRINT 1004
1001 FORMAT(1H 2(5X,F14.6, 7X,E15.8,10X))
1002 FORMAT(1H 2(8X,F14.6, 7X,E15.8,10X) )
1003 FORMAT(1H 2(29X,E15.8,10X))
1004 FORMAT(/)
1069 FORMAT(1H ,30X,E15.8)
IF(I.GT.JTOTAL) GO TO 890
IF (LINE.GT.54) GO TO 870
GO TO 871
890 CONTINUE
WRITE (20) (SIGOUT(I),I=1,316)
PRINT 8787, TAUBAR
8787 FORMAT(//23H NORMALIZATION FACTOR =,2X,E15.7)
RETURN
END

```

CASE 6

## AUXILIARIES

```

SUBROUTINE BIGMX(ICYCLE)
C
C THIS SUBROUTINE SEARCHES THE K-PRIME MATRICES BETWEEN LIM1(ICYCLE)
C AND LIM2(ICYCLE) TO SET UP THE KREM OR N ROWS OF S=K*22 REQUIRED
C BY THE EQUATION SOLVING ROUTINE CHLSKY
C
COMMON /1/ S(32,32,5),L(300,3),G(32),SPACE(20)
1      , NUF(3,314)      ,X(155),Y(155),CDA(3,6,250)
2      , KPR(6,6,250),MSK(317),JTOTAL,N,KRFM,M , IMR, IPL
3      , P(155,2) ,D(155,2)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION SP(32,10)
EQUIVALENCE (NUF,SP)
DIMENSION KS22(32,96) , KS21(32)
DIMENSION Q(5120)
EQUIVALENCE (S,KS22,Q) , (G,KS21)
TYPE REAL KPR
TYPE REAL KS22 , KS21
DATA (PK = 999.)
IF(ICYCLE.GT.1) GO TO 1
ISHIFT = 0
JSHIFT = 0
K1 = KREM
K2 = KREM + N
GO TO 109
1 CONTINUE
IF (ICYCLE.EQ.M) GO TO 3
K1 = N
K2 = 3*N
GO TO 2
3 CONTINUE
K1 = N
K2 = 2*N
2 CONTINUE
IF(ICYCLE.GT.2) GO TO 110
JSHIFT = 0
GO TO 111
110 CONTINUE
JSHIFT = KREM + (ICYCLE-3)*N
111 CONTINUE
ISHIFT = KREM + (ICYCLE-2)*N
C
C ISHIFT IS A BIAS TO FIT THE EQUATIONS INTO THE CORRECT ROWS
C JSHIFT IS A BIAS TO FIT THE EQUATIONS INTO THE CORRECT COLUMNS
C
109 CONTINUE
C ZERO OUT KS22, KS21, AS REQUIRED
DO 8 I=1,K1
G(I) = 0.
DO 8 J=1,K2
KS22(I,J) = 0.
8 CONTINUE
IPL = 1
NLO = LIM1(ICYCLE)
NHI = LIM2(ICYCLE)
DO 100 J=NLO,NHI
DO 100 I=1,6
I2 = I
I1 = I
15 IF(I1-3) 17, 18, 19
19 I1 = I1-3
I2 = I2+1

```

```

      GO TO 15
18  IS = 3
      IT = 12
      IF ((P(L(J,3),I2)-PK).GT.0.) GO TO 99
C  HERE IF FORCE IS KNOWN
      P1 = P(L(J,3),I2)
      GO TO 25
17  IF(I1-2) 20,21,21
20  IS = 1
      IT = 12
      IF ((P(L(J,1),I2)-PK).GT.0.) GO TO 99
      P1 = P(L(J,1),I2)
      GO TO 25
21  IS = 2
      IT = 12
      IF ((P(L(J,2),I2)-PK).GT.0.) GO TO 99
      P1 = P(L(J,2),I2)
24  IF (2*(L(J,IS)-1)+IT-MSK(IPL)) 301,302,303
302  I11 = IPL
      GO TO 327
301  IMR = IPL - 1
306  IF(2*(L(J,IS)-1) + IT-MSK(IMR)) 304, 305, 305
305  I11 = IMR
      IPL = IMR
      GO TO 327
304  IMR = IMR-1
      GO TO 306
303  IMR = IPL + 1
307  IF (2*(L(J,IS)-1)+IT-MSK(IMR)) 309, 309, 310
309  I11 = IMR
      IPL = IMR
      GO TO 327
310  IMR = IMR+1
      GO TO 307
327  IF(I11-ISHIFT.GT.K1) GO TO 328
      IF(I11-ISHIFT.LT.1) GO TO 328
      IF(KS21(I11-ISHIFT).NE.0.) GO TO 328
      KS21(I11-ISHIFT) =KS21(I11-ISHIFT) + P1
328  CONTINUE
      DO 100  KJ = 1,2
      DO 100  KK = 1,3
      IF ((P(L(J,KK),KJ)-PK).GT.0.) GO TO 28
      IF (2*(L(J,KK)-1)+KJ-MSK(IPL)) 11, 12, 13
12  I12 = IPL
      GO TO 27
11  IMR = IPL-1
6  IF(2*(L(J,KK)-1) + KJ - MSK(IMR)) 4,5,5
5  I12 = IMR
      IPL = IMR
      GO TO 27
4  IMR = IMR-1
      GO TO 6
13  IMR = IPL + 1
7  IF (2*(L(J,KK)-1) + KJ - MSK(IMR)) 5, 5, 10
10  IMR = IMR + 1
      GO TO 7
27  IF(I11-ISHIFT.GT.K1) GO TO 99
      IF(I11-ISHIFT.LT.1) GO TO 99
      IF(I12-JSHIFT.GT.K2) GO TO 99
      KS22(I11-ISHIFT,I12-JSHIFT) = KS22(I11-ISHIFT,I12-JSHIFT)
      + KPR(1,3*(KJ-1)+KK,J)
      GO TO 99

```

```

28 IF(DIL(J,KK),KJ).EQ.0.) GO TO 99
   IF(III)-ISHIFT.GT.K1) GO TO 99
   IF(III)-ISHIFT.LT.1) GO TO 99
   KS2(III)-ISHIFT) = KS2(III)-ISHIFT) -KPR(1,3*(KJ-1)+KK,J)
   1 * DIL(J,KK),KJ)
99 CONTINUE
100 CONTINUE
C HAVE EQUATIONS, NOW SHIFT THE SUBMATRICES TO THE PROPER POSITION FOR
C SUBROUTINE CHLSKY
   IF(ICYCLE.GT.1) GO TO 430
C S(1,1) IS (KREM X KREM), S(1,2) IS (KREM X N)
   DO 410 J = 1, KREM
   DO 410 I = 1, N
   LOCL = J + KREM*(I-1)
   LOCR1 = J + 32*(I-1)+KREM)
C SET UP S(1,2) IN S(1,1,4)
   Q(LOCL+3072) = Q(LOCR1)
410 CONTINUE
   DO 420 J = 1, KREM
   DO 420 I = 1, KREM
   LOCL = J + KREM*(I-1)
   LOCR2 = J + 32*(I-1)
C SET UP S(1,1) IN S(1,1,2)
   Q(LOCL+1024) = Q(LOCR2)
420 CONTINUE
   GO TO 480
430 CONTINUE
   IF(ICYCLE.GT.2) GO TO 440
C S(2,1) IS (N X KREM), S(2,2) AND S(2,3) ARE (N X N)
   K1 = KREM
   GO TO 450
440 CONTINUE
C S(1,I-1), S(1,I), S(1,I+1) ARE (N X N)
   K1 = N
450 CONTINUE
C SET UP S(ICYCLE,ICYCLE+1) IN S(1,1,4)
   IF(ICYCLE.FQ.M) GO TO 461
   DO 460 I = 1, N
   DO 460 J = 1, N
   LOCL = J+N*(I-1)
   LOCR1 = J + 32*(I-1)+K1+N)
   Q(LOCL+3072) = Q(LOCR1)
460 CONTINUE
461 CONTINUE
C SET UP S(ICYCLE,ICYCLE) IN S(1,1,3)
   DO 468 I=1,N
   DO 468 J=1,N
   LOCR2 = J+32*(I-1)+K1)
   LOCL = J + N * (I-1)
   Q(LOCL+2048) = Q(LOCR2)
468 CONTINUE
C MOVE S(ICYCLE,ICYCLE) TO S(1,1,2)
   NSQ = N**2
   DO 470 II=1,NSQ
   Q(II+1024) = Q(II+2048)
470 CONTINUE
C REARRANGE S(ICYCLE,ICYCLE-1) WITHIN S(1,1,1)
   KLOC = 0
   DO 480 I=2,K1
   KLOC = KLOC+N
   DO 480 J = 1,N
   Q(KLOC+J) = S(J,I,1)

```

```

480 CONTINUE
RETURN
999 CONTINUE
PRINT 1000, ICYCLE
1000 FORMAT (H1,35H BANDWIDTH EXCEEDFD, PARTITION ROW, I3)
IROW1 = I11 + ISHIFT
JROW1 = I12 + JSHIFT
PRINT 1001, IROW1, JROW1
1001 FORMAT (1H ,4HROW=, I3, 6X, 7HCOLUMN=, I3)
PRINT 9120, I11, I12, ISHIFT, JSHIFT
9120 FORMAT (1H ,4HI11=, I3, 4HI12=, I3, 7HISHIFT=, I3, 7HJSHIFT=, I3)
PRINT 9121, K1, K2, KREM, N, M
9121 FORMAT (1H ,3HK1=, I3, 3HK2=, I3, 5HKREM=, I3, 2HN=, I3, 2HM=, I3)
PRINT 9122, I, J, IPL, IMR, IS, IT
9122 FORMAT (1H ,2HI=, I3, 2HJ=, I3, 4HIPL=, I3, 4HIMR=, I3, 3HIS=, I3, 3HIT=, I3)
STOP
END

```

```

SUBROUTINE CHLSKY
C
C THIS ROUTINE SOLVES  $SUUG$ , WHERE  $S$  IS A TRI-DIAGONAL MATRIX IN
C SUBMATRICES, WITH ELEMENTS OF ORDER  $N$ .
C  $S$  IS KNOWN
C  $SP$  IS A VECTOR OF DIMENSION  $(NXM)$  WHERE  $M$  IS THE NUMBER OF DIVISIONS OF  $S$ 
C  $C$  IS WRITTEN ONTO TAPE AFTER DERIVATION ON THE FORWARD PASS.
C AND READ BACK IN ON THE BACKSWEEP
C  $S(1,1,1)$  INITIALLY CONTAINS  $S(1,1-1)$ 
C  $S(1,1,2)$  INITIALLY CONTAINS  $S(1,1)$ 
C  $S(1,1,4)$  INITIALLY CONTAINS  $S(1,1+1)$ 
C  $SP$  CORRESPONDS TO  $P$  IN THE WRITEUP BY GATEWOOD ON THE FORWARD PASS.
C ON THE BACKSWEEP, IT CORRESPONDS TO  $U$ .
COMMON /1/ S(32,32,5),L(300,3),G(32),SPACE(20)
1      .NUF(3,314)      .X(155),Y(155),CDA(3,6,250)
2      .KPR(6,6,250),MSK(310),JTOTAL,N,KREM,M,IMR,IPL
3      .P(155,2)      .D(155,2)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION SP(32,10)
EQUIVALENCE (NUF,SP)
TYPE REAL KPR
DIMENSION C(1024)
EQUIVALENCE (S(4097),C)
DATA (XLIMIT = 1.E-8)
REWIND 96
N2 = 2*N
KREM2 = 2*KREM
DO 30 ICYCLE = 1,M
CALL BIGNX(ICYCLE)
IF(ICYCLE-2) 1,2,3
1 K1 = KREM
K3 = KREM2
GO TO 10
2 K1 = N
K2 = KREM
K3 = N2
GO TO 4
3 K1 = N
K2 = N
K3 = N2
4 CONTINUE
5 IF (UNIT,96) 6,7,600,600
6 GO TO 5
7 CONTINUE
CALL MXMULT(S(1,1,1) , S(1,1,5) , S(1,1,3) ,K1,K2,K1)
C
C S(1,1,5) CONTAINS C FROM LAST CYCLE
C
CALL MXSUB(S(1,1,2) , S(1,1,3) , S(1,1,2) ,K1,K1)
10 CONTINUE
C
C B(1,1) NOW IN S(1,1,2)
C
CALL INVERT(S(1,1,2) , K1 , K3, XLIMIT , FLAG)
IF (FLAG.NE.0.) GO TO 500
C
C INVERSE OF B(1,1) NOW IN S(1,1,2)
C
IF (ICYCLE.EQ.1) GO TO 20
CALL MXMULT(S(1,1,1) ,SP(1,ICYCLE-1) , S(1,1,3) , N, K2 , 1)
CALL MXSUB (G , S(1,1,3) , G, N, 1)
20 CONTINUE

```

```

CALL MMULT(S(1,1,2), G, SP(1, ICYCLE, 1), K1, K1, 1)
IF (ICYCLE.GF.M) GO TO 35
CALL MMULT(S(1,1,2), S(1,1,4), S(1,1,5), K1, K1, M)
NSQ = K1*M
BUFFER OUT (96,1) IC(1), C(NSQ)
32 CONTINUE
34 CONTINUE
C
C NOW IN BACKSWEEP, SOLVING FOR U
C
DO AD I = 2,M
JCYCLE = M-I+1
IF(JCYCLE.GT.1) GO TO 36
K1 = KREM
GO TO 37
36 K1 = M
37 CONTINUE
NSQ = K1*M
IF (JCYCLE.EQ.M-1) GO TO 41
BACKSPACE 96
41 CONTINUE
BACKSPACE 96
BUFFER IN (96,1) IC(1),C(NSQ)
42 IF(UNIT,96) 43, 44,700,700
43 GO TO 42
44 CONTINUE
C
C U(M)=SP(M), CONSIDER FIRST (M-1)TH CYCLE
C
CALL MMULT(S(1,1,5), SP(1,JCYCLE+1), S(1,1,1), K1, M, 1)
CALL MSUB(SP(1,JCYCLE), S(1,1,1), SP(1,JCYCLE), K1, 1)
60 CONTINUE
C
C U(NS,1) NOW STORED IN SP(N,1), I=1,M
C
RETURN
500 CONTINUE
PRINT 1000, ICYCLE
1000 FORMAT(13)MICOULD NOT INVERT MATRIX IN ROW,12)
STOP
600 CONTINUE
PRINT 1001, ICYCLE
1001 FORMAT(13)M1 ERROR READING C INTO CORE ON 12, 7TH ROW.)
STOP
700 CONTINUE
PRINT 1002, JCYCLE
1002 FORMAT(13)M1 ERROR WRITING C ONTO TAPE ON 12, 7TH ROW.)
STOP
END

```

```

SUBROUTINE MXMULT(A,M,C,M,N,K)
C
C THIS SUBROUTINE MULTIPLIES MATRIX A BY MATRIX B AND STORES THE
C PRODUCT IN C. (C CANNOT BE THE SAME AS A OR B.)
C
C A IS (M X N)
C B IS (N X K)
C C IS (M X K)
C
C DIMENSION A(M,N) , B(N,K) , C(M,K)
C
C DO 1 I=1,M
C DO 1 L=1,K
C C(I,L) = 0.
C DO 1 J=1,N
C C(I,L) = C(I,L) + A(I,J) * B(J,L)
1 CONTINUE
RETURN
END

```

```

SUBROUTINE MXSUB(A,B,C,M,N)
C
C THIS SUBROUTINE SUBTRACTS MATRIX B FROM MATRIX A, STORES RESULT IN C
C
C A, B, AND C ARE (M X N) (C CAN BE THE SAME AS A OR B)
C
C DIMENSION A(M,N) , B(M,N) , C(M,N)
C
C DO 1 I=1,M
C DO 1 J=1,N
C C(I,J) = A(I,J) - B(I,J)
1 CONTINUE
RETURN
END

```

```

SUBROUTINE MXCON(A,B,X,M,N)
C
C THIS SUBROUTINE MULTIPLIES MATRIX A (MXN) BY CONSTANT X, RESULT IN B
C A MAY BE SAME AS B.
C THIS SUBROUTINE MULTIPLIES MATRIX A (MXN) BY CONSTANT X, RESULT IN B
C A MAY BE SAME AS B.
C DIMENSION A(M,N) , B(M,N)
C DO 1 I=1,M
C DO 1 J=1,N
C B(I,J) = X*A(I,J)
1 CONTINUE
RETURN
END

```

SUBROUTINE INVERT(B,K2,XMIN,FLAG)

C THIS SUBROUTINE SETS UP A UNIT MATRIX ADJACENT TO B(K,K)  
C ELEMENTARY ROW OPERATIONS ARE THEN PERFORMED ON THE NEW K X 2K MATRIX  
C TO REDUCE B(K,K) TO A UNIT MATRIX. THIS WILL PLACE THE INVERSE OF  
C THE MATRIX B(K,K) IN THE RIGHT HALF OF B(K,2K)  
C ON EXIT, THE INVERSE OF B REPLACES B  
C B(9) IS AN ARRAY OF 2\*K\*\*2 LOCATIONS CONTAINING THE MATRIX  
C K IS THE DIMENSION OF THE SQUARE MATRIX B  
C K2 IS 2\*K  
C XMIN IS THE SMALLEST ALLOWABLE MAGNITUDE OF THE PIVOT  
C FLAG WILL BE RETURNED AS 0. IF THE INVERSION WENT OFF OK  
C FLAG WILL BE RETURNED AS 1. IF A PIVOT ELEMENT WAS TOO SMALL  
C FLAG SHOULD BE TESTED AFTER EACH CALL TO THIS ROUTINE

C DIMENSION B(K,K2)

C FLAG = 0.

C SET UP UNIT MATRIX

C IF(K.GT.1) GO TO 20  
C IF(ABS(B(1,1)).LT.XMIN) GO TO 10  
C B(1,1) = 1./B(1,1)  
C RETURN

20 CONTINUE

DO 1 I=1,K  
DO 1 J=1,K  
B(I,K+J) = 0.  
IF(I.EQ.J) B(I,K+J) = 1.  
1 CONTINUE

C FIND LEADING ELEMENT WITH GREATEST MAGNITUDE

DO 6 J=1,K  
M = J  
N = J+1  
IF(J.EQ.K) GO TO 21  
DO 2 L=N,K  
IF(ABS(B(M,J)).LT.ABS(B(L,J))) M=L  
2 CONTINUE  
21 CONTINUE  
IF(ABS(B(M,J)).LT.XMIN) GO TO 10  
IF(J.EQ.K) GO TO 31

C INTERCHANGE JTH AND MTH ROWS

DO 3 L=J,K2  
D = B(J,L)  
B(J,L) = B(M,L)  
B(M,L) = D  
3 CONTINUE  
31 CONTINUE

C ZERO OUT PIVOTAL JTH COLUMN, SKIPPING PIVOTAL JTH ELEMENT

C DIVIDE JTH ROW BY PIVOT

DO 4 M=N,K2  
B(J,M) = B(J,M) / B(J,J)  
4 CONTINUE  
DO 6 M=1,K

```

C
C M DETERMINES ROW BEING MODIFIED. ONE WHOLE ROW AT A TIME
C
C   IF ( M.EQ.J ) GO TO 6
C   DO 5 L=N.K2
C
C L DETERMINES ELEMENT IN THE MTH ROW
C
C   B(M,L) = B(M,L) - B(M,J) * H(J,L)
C   5 CONTINUE
C   6 CONTINUE
C
C INVERSE OF B IS NOW IN RIGHT HALF OF B(K,K2)
C NOW MOVE B INVERSE TO WHERE H WAS
C   DO 7 I=1.K
C   DO 7 J=1.K
C   B(I,J) = B(I,J+K)
C   7 CONTINUE
C   RETURN
10 FLAG = 10.
C   RETURN
C   END

```

FUNDAMENTAL CASE THREE  
(AXIAL LOADING)

```
PROGRAM MAINA
C
C THIS IS THE EXECUTIVE PROGRAM USED WITH SUBROUTINE RIGMX AND SUBROUTINE
C CHLSKY TO GENERATE AND SOLVE LARGE SYSTEMS OF LINEAR EQUATIONS
C
C     REWIND 20
C     CALL TAPESKIP(20*6.0)
C     CALL TAPESKIP(20*2.0)
C     1 CONTINUE
C     CALL INPUTA
C
C NOW HAVE ALL K-PRIME AND CDA MATRICES
C
C     CALL CHOLA
C
C NOW HAVE SOLUTION U IN SP
C
C     CALL STRESA
C
C ALL STRESSES NOW PRINTED OUT
C
C     GO TO 1
C     END
```

CASE 3

23 CARDS

SUBROUTINE INPUTA

```

C
C THIS SUBROUTINE READS AND PRINTS THE INPUTS FOR THE PLANAR FINITE
C ELEMENT PROGRAM. ALL INPUTS NOT READ ARE GENERATED HERE.
C
COMMON /1/ S(32,32,5),L(160,3),G(32),SPACE(20),XNU(2),E(2)
1      , NUF(3,160) , X(155),Y(155),CDA(3,6,160)
2      , KPR(6,6,160),MSK(310),JTOTAL,N,KREM,M , IMR, IPL
3      , P(155,2) , D(155,2) , EPT(3), PT(6,160), SIT(3,160)
COMMON /STRESS/ ALF(155) , MSIZE , R , GAMMA
EQUIVALENCE(XNU(1),XNU1), (E(1),E1)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION SP(32,10)
DIMENSION NUYX(250),NUXY(250), GXY(250), EX(250), EY(250)
DIMENSION COMENT(10) , PTM(6)
DIMENSION BUMP(10) , BUMP1(5)
EQUIVALENCE (NUF,SP)
EQUIVALENCE (C,CP)
DIMENSION JL(79,3)
C
C NUYX = NUXY, THIS MODIFICATION
EQUIVALENCE (NUXY,NUYX)
DIMENSION TH(155) , CP(3,3) , T1(3,3,2),T2(3,3),DTD(18),
1 T(6,6),KMX(6,6), C(3,3),A(6,6,2),DO(3,6),DD(18),DTC(6,3)
EQUIVALENCE (S,F) , (S(901),VF), (S(1801),PHI), (S(2701),ER)
1      , (S(3001),EF), (S(3901),NUR)
2      , (DO,DD) , (DTU,DTD) , (S(4356),TH)
3      , (S(4511),CP) , (S(4521),T1) , (S(4541),T2)
4      , (S(4551),CABC) , (S(4581),T) , (S(4621),KMX)
5      , (S(4671),A)
DIMENSION EPT1(3,2)
TYPE REAL NUXY
TYPE INTEGER BUMP, BUMP1
TYPE REAL NUYX, NUR, NUF, KPR, KMX
DATA (KTOTAL = 97)
DATA (BUMP = 2,4,7,7,15,1 ,9 ,9 ,12,-1)
DATA (BUMP1 = 5,7,7,7,5 ) , (PI = 3.1415927)
DATA (RAD = 57.29578)
DATA (XLIM = 1.E-8)
DATA (PK = 999.)
DATA ((DD(JJ),JJ=1,18) = 0., 0., 0., 1., 0., 0., 0., 0., 1.,
1      0., 0., 0., 0., 0., 0., 1., 0., 0., 1., 0.)
DATA ((DTD(JJ),JJ=1,18) = 0., 1., 0., 0., 0., 0., 0., 0., 0., 0.,
1      0., 0., 1., 0., 0., 0., 1., 0., 0., 1., 0.)
DATA(((JL(I,J),I=1,9),J=1,3) =
1 1,1,1,2,2,3,3,3,4,4,4,5,6,7,7,8,8,8,9,10,10,11,11,12,13,13,
2 14,15,15,16,16,17,17,18,18,19,20,23,24,24,25,25,25,26,28,29,
3 29,30,30,35,21,21,21,21,22,22,23,23,27,27,27,28,28,36,46,37,
4 47,38,48,39,39,40,41,41,41,42,42,43,43,
5 2,3,4,6,7,7,8,9,9,10,11,11,13,6,14,14,15,16,16,16,17,10,
6 18,18,20,21,21,21,22,15,23,23,24,17,25,25,35,27,27,28,28,29,
7 30,30,32,32,33,33,34,36,35,37,38,39,39,40,40,41,41,42,43,43,44,
8 45,37,46,38,47,39,48,49,49,40,50,51,51,52,52,53,
9 3,4,5,7,3,8,9,4,10,11,5,12,14,14,8,15,16,9,10,17,18,18,12,
1 19,21,14,15,22,23,23,17,24,25,25,19,26,21,24,28,25,29,30,26,
2 31,29,33,30,34,31,37,37,38,39,22,40,23,41,27,42,43,28,44,32,
3 46,36,47,37,48,38,49,40,50,50,51,42,52,43,53,44 )
C
READ 1000,(COMENT(I),I=1,10)
1000 FORMAT(10A8)
PRINT 1001,(COMENT(I),I=1,10)
1001 FORMAT(1H1,10A8)

```

```

C
C XTOTAL = TOTAL NUMBER OF NODES CONSIDERED
C
C
C READ AND PRINT
C
      JTOTAL = 158
      READ 1003,VF,GAMMA,E(1),XNU(1),BETA
      IF (VF.EQ.0.) GO TO 95
      XNU(2) = XNU(1)/BETA
      E(2) = E(1)/GAMMA
      EPT1(1,1) = -XNU(1)
      EPT1(1,2) = -XNU(2)
      EPT1(2,1) = EPT1(1,1)
      EPT1(2,2) = EPT1(1,2)
      EPT1(3,1) = 0.
      EPT1(3,2) = 0.
      E(1) = E(1)/(1.-XNU(1)**2)
      E(2) = E(2)/(1.-XNU(2)**2)
      XNU(1) = XNU(1)/(1.-XNU(1))
      XNU(2) = XNU(2)/(1.-XNU(2))
1003 FORMAT(5F10.4)
      PRINT 1018
      PRINT 1016
      PRINT 1017, VF, GAMMA, E1, XNU1, BETA
1016 FORMAT(1H,13X,2MVF,10X,5HGAMMA,13X,2HE1,11X,4HXNU1,11X,4HBETA)
1017 FORMAT(1H,5F15.8)
1018 FORMAT(7F7.3)

      S3 = SORT(3.)
      S302 = S3/2.
C
      EPI = E(1)
      EPII = E(2)
C
      XNUPI = XNU(1)
      XNUPII = XNU(2)
      DO 200 I = 1,97
      TH(I) = 1.
      ALF(I) = 0.
      D(I,1) = 1000.
      D(I,2) = 1000.
      P(I,1) = 0.
      P(I,2) = 0.
200 CONTINUE
      P(64,1) = 1000.
      P(64,2) = 1000.
      D(64,1) = 0.
      D(64,2) = 0.
      P(97,1) = 1000.
      P(97,2) = 1000.
      D(97,1) = 0.
      D(97,2) = 0.
      P(34,1) = 1000.
      P(34,2) = 1000.
      D(34,1) = 0.
      D(34,2) = 0.
      P(1,1) = 1000.
      P(1,2) = 1000.
      D(1,1) = 0.
      D(1,2) = 0.
      I = 0

```

```

DO 201 J= 1,10
I = I +BUMP(J)
D(I,2) =0.
P(I,2) =1000.
D(98-I,2) =0.
P(98-I,2) =1000.
201 CONTINUE
I = 0
DO 202 J = 1,5
I = I+ BUMP(J)
D(I,1) =0.
D(98-I,1) =0.
P(I,1) = 1000.
P(98-I,1) = 1000.
202 CONTINUE
R= SQRT(2.*S3*VF/PI)
X(1) = S302
Y(1) = .5
DO 210 I=1,4
X(I+1) = S302 - R/4.*COS(PI*(I-1)/6.)
Y(I+1) = .5 - R/4. *SIN(PI*(I-1)/6.)
210 CONTINUE
C
DO 220 I=1,7
X(I+5) = S302 - R/2.* COS(PI*(I-1)/12.)
Y(I+5) = .5 - R/2.* SIN(PI*(I-1)/12.)
C
X(I+12) = S302 - 3.*R/4.*COS(PI*(I-1)/12.)
Y(I+12) = .5 - 3.*R/4.*SIN(PI*(I-1)/12.)
C
X(I+19) = S302 - R * COS(PI*(I-1)/12.)
Y(I+19) = .5 - R * SIN(PI*(I-1)/12.)
220 CONTINUE
X(34) = S302
Y(34) = - .5
X(31) = S302
Y(31) = (Y(26)+Y(34)) / 2.
X(45) = -.5* TAN(PI/6.)
Y(45) = .5
DX = (1.-R)/(2.*COS(PI/6.))
X(36) = X(45) + DX
Y(36) = .5
X(35) = (X(20)+X(36))/2.
Y(35) = .5
C
DO 230 I = 1,4
X(I+45) = (4 -I)* X(45) /4.
Y(I+45) = (4 -I)* Y(45) /4.
230 CONTINUE
C
DELX = X(46) -X(45)
DELY = Y(46)-Y(45)
DO 240 I = 1,8
X(I+36) = X(I+35) +DELX
Y(I+36) = Y(I+35) +DELY
240 CONTINUE
X(32) = X(44) +(X(34)-X(44))/3.
Y(32) = -.5
X(33) = 2.*X(32) - X(44)
Y(33) = -.5
X(27) = X(23) + (X(32)-X(23))/3.
Y(27) = Y(23) + (Y(32)-Y(23))/3.

```

```

X(28) = 2.*X(27)-X(23)
Y(28) = 2.*Y(27)-Y(23)
X(29) = X(28) +(X(31)-X(28))/3.
Y(29) = Y(28) +(Y(31)-Y(28))/3.
X(30) = 2.*X(29)-X(28)
Y(30) = 2.*Y(29)-Y(28)
C
C
DO 250 I=50,97
X(I) = -X(98-I)
Y(I) = -Y(98-I)
250 CONTINUE
C
C
C PRINT OUT NODAL DATA
I = 1
12 CONTINUE
LINE = 4
PRINT 1010
13 CONTINUE
PRINT 1011, I, X(I), Y(I), P(I,1), P(I,2), D(I,1), D(I,2),
1 TH(I), ALF(I)
I = I+1
IF(I.GT.KTOTAL) GO TO 14
LINE = LINE + 2
IF(LINE.GT.56) GO TO 12
GO TO 13
14 CONTINUE
1011 FORMAT(1H 14,7(3X E11.4),5X,F11.2/)
1010 FORMAT(5HINODE,7X,1HX,13X,1HY,13X,2HP1,12X,2HP2,12X,2HD1,12X,2HD2,
1 8X,9HTHICKNESS,9X,5HALPHA /)
C
C I - NUMBER OF NODE
C X - X- COORDINATE OF ITH NODE
C Y - Y- COORDINATE OF ITH NODE
C P(I,1) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 1 DIRECTION
C P(I,2) - KNOWN AND UNKNOWN FORCE COMPONENTS ALONG 2 DIRECTION
C D(I,1) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 1 DIRECTION
C D(I,2) - KNOWN AND UNKNOWN DISPLACEMENTS ALONG 2 DIRECTION
C ALF(I) - ANGLE BETWEEN X DIRECTION AND 1 DIRECTION (POSITIVE WHEN
C COUNTER-CLOCKWISE)
C TH(I) - PLATE THICKNESS AT ITH NODE
C
C GFT MSK MATRIX
JP = 0
DO 27 J=1,KTOTAL
DO 27 I=1, 2
IF (P(J,I).GT.PK) GO TO 27
JP = JP+1
MSK(JP) = 2 * (J-1) + I
27 CONTINUE
C MSK IS MATRIX OF INDICES OF KNOWN FORCES
C IF FORCE P IS UNKNOWN, IT IS INPUT AS 1000.
C NOW READ IN TRIANGLE DATA
C
C JTOTAL - TOTAL NUMBER OF TRIANGLES
DO 19 I=1,79
DO 19 J=1,3
19 L(I,J) = JL(I,J)
DO 260 I= 80,158
DO 260 J= 1,3
L(I,J) = 98 - L(159-I,J)

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```

260 CONTINUE
C L(J,1) - INDEX OF THE FIRST NODE OF THE JTH TRIANGLE
C L(J,2) - INDEX OF THE SECOND NODE OF THE JTH TRIANGLE
C L(J,3) - INDEX OF THE THIRD NODE OF THE JTH TRIANGLE
C
DO 20 I=1,JTOTAL
IF(I.LE.36.OR.I.GE.123) GO TO 300
EX(I) = EPII
EY(I) = EPII
NUXY(I) = XNUPII
GXY(I) = EPII/(2.*(1.+XNUPII))
GO TO 310
300 FX(I) = EPI
FY(I) = FPI
NUXY(I) = XNUPI
GXY(I) = EPI / (2.*(1.+XNUPII))
310 CONTINUE
20 CONTINUE
C PRINT OUT TRIANGLE DATA
LINE = 4
PRINT 1012
DO 24 I=1,JTOTAL
IF(LINE.LT.54) GO TO 22
LINE = 4
PRINT 1012
1012 FORMAT(1H1,8HTRIANGLE,4X6HNODE 1,3X,6HNODE 2,3X,6HNODE 3,10X,2HEX,
1 18X 2HEY,16X,4HNUYX,17X,3HGXY//)
22 LINE = LINE+2
24 PRINT 1023, I, (L(I,J),J=1,3),EX(I),EY(I),NUYX(I),GXY(I)
1023 FORMAT(1H ,5X,I3,7X,I3,2(6X,I3),4(5X,E15.8)//)
C
C
MSIZE = 156
KREM = 6
N = 25
M = MSIZE / N
IF(KREM.NE.N) M = M+1
C
LIM1(1) = 1
LIM1(2) = 4
LIM1(3) = 25
LIM1(4) = 50
LIM1(5) = 74
LIM1(6) = 96
LIM1(7) = 128
LIM2(7) = 158
LIM2(6) = 139
LIM2(5) = 122
LIM2(4) = 91
LIM2(3) = 67
LIM2(2) = 37
LIM2(1) = 12
C
C PRINT OUT PARTITION INFORMATION
PRINT 1008
1008 FORMAT(1H1,12X,20HTRIANGLES CONSIDERED)
DO 31 I=1,M
ISIZE = N
IF(I.EQ.1) ISIZE = KREM
PRINT 1009, I,LIM1(I),LIM2(I), ISIZE
31 CONTINUE

```

```

1009 FORMAT(10H PARTITION,6X 5HFIRST,2X,2HTO,2X,4HLAST, 6X,9HDIMENSION/
1 4X,I3,11X,I3,6X,I3,10X,I3)
DO 28 I = 1,KTOTAL
ALF(I) = ALF(I)/RAD
28 CONTINUE
C ALL ANGLES NOW IN RADIANS
DO 400 I=1, JTOTAL
C I IS A TRIANGLE COUNTER
KK3 = 2
IF(I.LE.36.OR.I.GE.123) KK3 = 1
B11 = 1./EX(I)
B12 = -NUYX(I) / EY(I)
B22 = 1. / EY(I)
B33 = 1. / GXY(I)
DELTA = B11*B22 -B12**2
CP(1,1) = B22 / DELTA
CP(2,1) = - B12 / DELTA
CP(3,1) = 0.
CP(1,2) = CP(2,1)
CP(2,2) = B11/ DELTA
CP(3,2) = 0.
CP(1,3) = 0.
CP(2,3) = 0.
CP(3,3) = 1./B33
C
C C NOW IN C(3,3) , MATRIX I
C
30 CONTINUE
THOMEG = X(L(I,2))*Y(L(I,3)) +X(L(I,1))*Y(L(I,2))
1 + Y(L(I,1))*X(L(I,3)) -X(L(I,3))*Y(L(I,2))
2 - X(L(I,1))*Y(L(I,3)) -Y(L(I,1))*X(L(I,2))
THOMEG=(TH(L(I,1))+TH(L(I,2))+TH(L(I,3)))/ 6. * THOMEG
X12 = X(L(I,2))- X(L(I,1))
X13 = X(L(I,3))- X(L(I,1))
ETA2= Y(L(I,2))- Y(L(I,1))
ETA3= Y(L(I,3))- Y(L(I,1))
DELTA = X12*ETA3 - X13*ETA2
DO 38 II = 1,3
DO 38 JJ = 1,3
A(II+3, JJ,1) = 0.
A(II, JJ+3,1) = 0.
A(II+3, JJ,2) = 0.
A(II, JJ+3,2) = 0.
38 CONTINUE
A(1,1,1) = 1.
A(2,1,1) = -(ETA3-ETA2) / DELTA
A(3,1,1) = (X13 - X12) / DELTA
A(1,2,1) = 0.
A(2,2,1) = ETA3 / DELTA
A(3,2,1) = -X13 / DELTA
A(1,3,1) = 0.
A(2,3,1) = -ETA2 / DELTA
A(3,3,1) = X12 / DELTA
DO 39 II = 1,3
DO 39 JJ = 1,3
A(II+3, JJ+3,1) = A(II, JJ,1)
A(II, JJ,2) = A(JJ, II,1)
A(II+3, JJ+3,2) = A(JJ, II,1)
39 CONTINUE
C
C TRANSPOSE OF INVERSE OF A NOW IN A(1,1,2) , A INVERSE STILL IN A
C

```

```

      CALL MXMULT(DO,A,KMX(1,1),3,6,6)
      CALL MXMULT(C,FPT1(1,KK3),SIT(1,1),3,3,1)
      CALL MXMULT(C,KMX(1,1),CDA(1,1,1),3,3,6)
C   PRODUCT C*D*(A**-1) NOW IN CDA(1,1,1), IT- TRIANGLE
C
      CALL MXMULT(A(1,1,2),DT,A,6,6,3)
      CALL MXMULT(EPT1(1,KK3),CDA(1,1,1),PTM,1,3,6)
      CALL MXCON(PTM,PTM,THCMFG,1,6)
      CALL MXCON(A,KPR(1,1,1),THOMFG,6,3)
      CALL MXMULT(KPR(1,1,1),CDA(1,1,1),KMX,6,3,6)
C
C   MATRIX K/I) NOW IN KMX, TRIANGLE I
      DO 40 II =1,6
      DO 40 JJ =1,6
      T(II,JJ) = 0.
40  CONTINUE
      T(1,1) = COS(ALF(L(1,1)))
      T(4,1) = SIN(ALF(L(1,1)))
      T(2,2) = COS(ALF(L(1,2)))
      T(5,2) = SIN(ALF(L(1,2)))
      T(3,3) = COS(ALF(L(1,3)))
      T(6,3) = SIN(ALF(L(1,3)))
      T(1,4) = -T(4,1)
      T(4,4) = T(1,1)
      T(2,5) = -T(5,2)
      T(5,5) = T(2,2)
      T(3,6) = -T(6,3)
      T(6,6) = T(3,3)
      CALL MXMULT(KMX,T,A,6,6,6)
      CALL MXMULT(T,PTM,PT(1,1),6,6,1)
      T(4,1) = - T(4,1)
      T(5,2) = - T(5,2)
      T(6,3) = - T(6,3)
      T(1,4) = - T(1,4)
      T(2,5) = - T(2,5)
      T(3,6) = - T(3,6)
C
C   INVERSE OF T NOW IN T
C
      CALL MXMULT(T,A,KPR(1,1,1),6,6,6)
C
C   K-PRIME NOW IN KPR , A HAS BEEN CLOBBERED.
400 CONTINUE
      RETURN
800 PRINT 1051,II,JJ,II
1051 FORMAT(1H1, 5H EF(,13,1H,13,7H) = ER(,13,6H) = 0.)
      STOP
900 CONTINUE
      PRINT 1050 , J
1050 FOPMAT(1H1 35HCOULD NOT INVERT MATRIX T1,TRIANGLE,13)
950 CONTINUE
      REWIND 20
      STOP
      END

```

CASE 3

1406 CARDS

```

SUBROUTINE STRESA
C
C THIS SUBROUTINE DERIVES AND PRINTS STRESSES
C
COMMON /1/ S(32,32,5),L(160,3),G(32),SPACE(20),XNU(2),E(2)
1      ,NUF(3,160)      ,X(155),Y(155),CDA(3,6,160)
2      ,KPR(6,6,160),MSK(310),JTOTAL,N,KREM,M ,IMR, IPL
3      ,P(155,2) ,D(155,2) ,EPT(3),PT(6,160),SIT(3,160)
COMMON /STRSS/ ALF(155) ,MSIZE ,P ,GAMMA
EQUIVALENCE(XNU(1),XNU1), (E(1),E1)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION DVX(6),SIGOUT(632)
EQUIVALENCE (S,SIGOUT)
DIMENSION SP(32,10)
EQUIVALENCE (NUF,SP)
TYPE REAL KPR
EQUIVALENCE (S,SIG) , (S(931),PSTR) , (S(1861),X0),(S(2161),Y0)
EQUIVALENCE (S(2461),DEL) , (S(2761),DX)
DIMENSION ERR(310)
DIMENSION DX(6) , SIG(4,310) , PSTR(3,310)
DIMENSION XU(300) , YO(300), DEL(310)
DIMENSION KS22(32,96) ,PZ(160)
EQUIVALENCE (KPR,KS22)
TYPE REAL KS22
DATA (PK=999.)
E(1) = E(1)*(1.-(XNU(1)/(1.+XNU(1)))**2)
E(2) = E(2)*(1.-(XNU(2)/(1.+XNU(2)))**2)
C REMOVE GAPS FROM SP(32,10) = DEL(310)
DO 5 J=1,KREM
DEL(J) = SP(J,1)
5 CONTINUE
KLOC = KREM -N
DO 10 I = 2,M
KLOC = KLOC + N
DO 10 J = 1,N
DEL(KLOC+J) = SP(J,I)
10 CONTINUE
PRINT 1010
1010 FORMAT(1H,50X,13HDISPLACEMENTS,/)
NDEL = MSIZE/7
JCNT = 0
DO 15 J=1,NDEL
JCNT = JCNT + 1
IF(JCNT.GT.18) GO TO 14
JFIR = 7*(J-1) + 1
JLAST = JFIR + 6
PRINT 1011 , (K,K=JFIR,JLAST)
1011 FORMAT(1H ,7(8X,4HDEL(,13,1H) ))
PRINT 1012 , (DEL(K),K=JFIR,JLAST)
1012 FORMAT(1H ,7(2X,E14.7)/)
GO TO 15
14 PRINT 1010
JCNT = 0
15 CONTINUE
LOC1 = 7*NDEL+1
LOC2 = MSIZE
IF(LOC1.GT.LOC2) GO TO 20
PRINT 1011,(K,K=LOC1,LOC2)
PRINT 1012,(DEL(K),K=LOC1,LOC2)
20 CONTINUE
DO 855 I=1,JTOTAL
J = I

```

```

KK3 = 2
IF(1.LE.36.OR.1.GE.123) KK3 = 1
KZ = 0
DO 831 KK = 1,3
DO 832 KJ = 1,2
KZ = KZ + 1
IF( P(L(J, KK), KJ).GT.PK) GO TO 828
IF(2*(L(J, KK)-1)+KJ-MSK(IPL)) 801, 802, 803
802 II2 = IPL
GO TO 827
801 IMR = IPL - 1
806 IF(2*(L(J, KK)-1) +KJ -MSK(IMR)) 804, 805, 805
805 II2 = IMR
IPL = IMR
GO TO 827
804 IMR = IMR - 1
GO TO 806
803 IMR = IPL + 1
807 IF(2*(L(J, KK)-1) +KJ -MSK(IMR)) 805, 805, 810
810 IMR = IMR + 1
GO TO 807
827 DVX(KZ) = DEL(II2)
GO TO 832
828 DVX(KZ) = D(L(J, KK), KJ)
832 CONTINUE
DX(KZ-1) = DVX(KZ-1)*COS(ALF(L(J, KK))) - DVX(KZ)*SIN(ALF(L(J, KK)))
DX(KZ) = DVX(KZ-1)*SIN(ALF(L(J, KK))) + DVX(KZ)*COS(ALF(L(J, KK)))
831 CONTINUE
DX2 = DX(2)
DX(2) = DX(3)
DX(3) = DX(5)
DX(5) = DX(4)
DX(4) = DX2
CALL MXMULT(CDA(1,1,1), DX, SIG(1,1), 3, 6, 1)
DO 838 J=1,3
838 SIG(J,1) = SIG(J,1) - SIT(J,1)
SIG(4,1) = SIG(3,1)
Y1 = Y(L(I,1))
Y2 = Y(L(I,2))
Y3 = Y(L(I,3))
AREA = (X(L(I,1))*(Y2-Y3)+X(L(I,2))*(Y3-Y1)+X(L(I,3))*(Y1-Y2))/2.
SIGZ = (SIG(1,1)+SIG(2,1))*XNU(KK3)/(1.+XNU(KK3))+E(KK3)
SIG(3,1) = SIGZ
PZ(1) = SIGZ*AREA
C SIGMA NOW IN SIG(1,1) , TRIANGLE I
XO(1) = 0.
YO(1) = 0.
DO 840 K = 1,3
XO(1) = XO(1) + X(L(I,K))
YO(1) = YO(1) + Y(L(I,K))
840 CONTINUE
XO(1) = XO(1) / 3.
YO(1) = YO(1) / 3.
850 CONTINUE
855 CONTINUE
ENBARZ = 0.
DO 860 I=1,158
860 ENBARZ = ENBARZ + PZ(I)
ENBARZ = SQRT(3.)/ENBARZ
DO 865 I=1,632
865 SIGOUT(I) = SIGOUT(I)*ENBARZ
I = 1

```

```

870 CONTINUE
PRINT 1000
LINE = 3
1000 FORMAT(1H1,2(8HTRIANGLE,6X,8HCFNTROID,9X,12HVECTOR SIGMA,10X)/)
871 CONTINUE
II = I+1
DO 880 J=1,4
GO TO (873,874,875,875), J
873 PRINT 1001, I, XU(II), SIG(J,I), II, X0(II), SIG(J,II)
GO TO 878
874 PRINT 1002, Y0(II), SIG(J,I), Y0(II), SIG(J,II)
GO TO 878
875 PRINT 1003, SIG(J,I), SIG(J,II)
878 CONTINUE
880 CONTINUE
LINE = LINE + 6
I = I + 2
PRINT 1004
1001 FORMAT(1H 2(5X,13,F14.6, 7X,E15.8,10X))
1002 FORMAT(1H 2(8X,F14.6, 7X,E15.8,10X) )
1003 FORMAT(1H 2(29X,E15.8,10X))
1004 FORMAT(/)
IF (I.GT.JTOTAL) GO TO 890
IF (LINE.GT.54) GO TO 870
GO TO 871
890 CONTINUE
WRITE (20) (SIGOUT(I),I=1,316)
SIGMA = 0.
DO 900 I=1,JTOTAL
SIGMA = SIGMA+PZ(I)
900 CONTINUE
EC = SIGMA/SQRT(3.)
CHI = EC/E(1)
PRINT 1017 , CHI,EC
1017 FORMAT( 7H1 CHI=, E13.6,4X,6H EC=,E13.6)
PRINT 8787, ENBARZ
8787 FORMAT(/23H NORMALIZATION FACTOR =,2X,E15.7)
RETURN
END

```

CASE 3

SUBROUTINE CHOLA

```

C
C THIS ROUTINE SOLVES  $SU=G$ , WHERE S IS A TRI-DIAGONAL MATRIX IN
C SUBMATRICES, WITH ELEMENTS OF ORDER N.
C S IS KNOWN
C SP IS A VECTOR OF DIMENSION (NXM) WHERE M IS THE NUMBER OF DIVISIONS OF S
C C IS WRITTEN ONTO TAPE AFTER DERIVATION ON THE FORWARD PASS.
C AND READ BACK IN ON THE BACKSWEEP
C S(1,1,1) INITIALLY CONTAINS S(I,I-1)
C S(1,1,2) INITIALLY CONTAINS S(I,I )
C S(1,1,4) INITIALLY CONTAINS S(I,I+1)
C SP CORRESPONDS TO P IN THE WRITEUP BY GATEWOOD ON THE FORWARD PASS.
C ON THE BACKSWEEP, IT CORRESPONDS TO U.
COMMON /1/ S(32,32,5),L(160,2),G(32),SPACE(20),XNU(2),E(2)
1      .NUF(3,160)      .X(155),Y(155),CDA(3,6,160)
2      .KPR(6,6,160),MSK(310),JTOTAL,N,KREM,M,IMR,IPL
3      .P(155,2) .D(155,2) .EPT(3) .PT(6,160) .SIT(3,160)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION SP(32,10)
EQUIVALENCE (NUF,SP)
TYPE REAL KPR
DIMENSION C(1024)
EQUIVALENCE (S(4097),C)
DATA (XLIMIT =1.E-8)
REWIND 96
N2 = 2*N
KREM2 = 2*KREM
DO 30 ICYCLE =1,M
CALL BIGNXA(ICYCLE)
IF(ICYCLE-2) 1,2,3
1 K1 = KREM
K3 = KREM2
GO TO 10
2 K1 = N
K2 = KREM
K3 = N2
GO TO 4
3 K1 = N
K2 = N
K3 = N2
4 CONTINUE
5 IF (UNIT,96) 6,7 ,600, 600
6 GO TO 5
7 CONTINUE
CALL MXMULT(S(1,1,1) , S(1,1,5) , S(1,1,3) ,K1,K2,K1)
C
C S(1,1,5) CONTAINS C FROM LAST CYCLE
C
CALL MXSUB(S(1,1,2) , S(1,1,3) , S(1,1,2) ,K1,K1)
10 CONTINUE
C
C B(1,1) NOW IN S(1,1,2)
C
CALL INVERT(S(1,1,2) , K1 , K3, XLIMIT , FLAG)
IF (FLAG,NE,0.) GO TO 500
C
C INVERSE OF B(1,1) NOW IN S(1,1,2)
C
IF (ICYCLE,EQ,1) GO TO 20
CALL MXMULT(S(1,1,1) ,SP(1,ICYCLE-1) , S(1,1,3) , N, K2 , 1)
CALL MXSUB (G , S(1,1,3), G, N, 1)
20 CONTINUE

```

```

      CALL MXMULT(S(1,1,2) , G , SP(1, ICYCLE , 1) , K1 , K1 , 1)
      IF (ICYCLE.GE.M) GO TO 35
      CALL MXMULT (S(1,1,2) , S(1,1,4) , S(1,1,5) , K1 , K1 , N)
      NSQ = K1*N
      BUFFER OUT (96,1) (C(1), C(NSQ))
30 CONTINUE
35 CONTINUE
C
C NOW IN BACKSWEEP, SOLVING FOR U
C
      DO 60 I = 2,M
      JCYCLE = M-I+1
      IF (JCYCLE.GT.1) GO TO 36
      K1 = KREM
      GO TO 37
36 K1 = N
37 CONTINUE
      NSQ = K1*N
      IF (JCYCLE.EQ.M-1) GO TO 41
      BACKSPACE 96
41 CONTINUE
      BACKSPACE 96
      BUFFER IN (96,1) (C(1),C(NSQ))
42 IF (UNIT,96) 43, 44,700,700
43 GO TO 42
44 CONTINUE
C
C U(M)=SP(M) , CONSIDER FIRST (M-1)TH CYCLE
C
      CALL MXMULT(S(1,1,5) , SP(1,JCYCLE+1),S(1,1,1),K1,N, 1)
      CALL MXSUB(SP(1,JCYCLE), S(1,1,1) ,SP(1,JCYCLE), K1 , 1)
60 CONTINUE
C
C U(NS,1) NOW STORED IN SP(N,1) , I=1..M
C
      RETURN
500 CONTINUE
      PRINT 1000 , ICYCLE
1000 FORMAT (31)C(1)COULD NOT INVERT MATRIX IN ROW,12)
      STOP
600 CONTINUE
      PRINT 1001,ICYCLE
1001 FORMAT(37)1 ERROR READING C INTO CORE ON 12, 7TH ROW.)
      STOP
700 CONTINUE
      PRINT 1002, JCYCLE
1002 FORMAT(37)1 ERROR WRITING C ONTO TAPE ON 12, 7TH ROW.)
      STOP
      END

```

```

SUBROUTINE BIGMXA(ICYCLE)
C
C THIS SUBROUTINE SEARCHES THE K-PRIME MATRICES BETWEEN LIM1(ICYCLE)
C AND LIM2(ICYCLE) TO SET UP THE KREM OR N ROWS OF S=K*22 REQUIRED
C BY THE EQUATION SOLVING ROUTINE CHLSKY
C
COMMON /1/ S(32,32,6),L(160,3),G(32),SPACE(20),XNU(2),E(2)
1      , NUF(3,160)      ,X(155),Y(155),CDA(3,6,160)
2      , KPR(6,6,160),MSK(310),JTOTAL,N,KREM,M , IPR, IPL
3      , P(155,2) ,D(155,2) , EPT(3), PT(6,160), SI(3,160)
COMMON/LIM/ LIM1(10),LIM2(10)
DIMENSION SP(32,10)
EQUIVALENCE (NUF,SP)
DIMENSION KS22(32,96) , KS21(32)
DIMENSION Q(5120)
EQUIVALENCE (S,KS22,Q) , (G,KS21)
TYPE REAL KPR
TYPE REAL KS22 , KS21
DATA (PK = 999.)
IF(ICYCLE.GT.1) GO TO 1
ISHIFT = 0
JSHIFT = 0
K1 = KREM
K2 = KREM + N
GO TO 109
1 CONTINUE
IF (ICYCLE.EQ.M) GO TO 3
K1 = N
K2 = 3*N
GO TO 2
3 CONTINUE
K1 = N
K2 = 2*N
2 CONTINUE
IF(ICYCLE.GT.2) GO TO 110
JSHIFT = 0
GO TO 111
110 CONTINUE
JSHIFT = KREM + (ICYCLE-3)*N
111 CONTINUE
ISHIFT = KREM + (ICYCLE-2)*N
C
C ISHIFT IS A BIAS TO FIT THE EQUATIONS INTO THE CORRECT ROWS
C JSHIFT IS A BIAS TO FIT THE EQUATIONS INTO THE CORRECT COLUMNS
C
109 CONTINUE
C ZERO OUT KS22, KS21, AS REQUIRED
DO 8 I=1,K1
G(I) = 0.
DO 8 J=1,K2
KS22(I,J) = 0.
8 CONTINUE
IPR = 1
NLO = LIM1(ICYCLE)
NHI = LIM2(ICYCLE)
DO 100 J=NLO,NHI
DO 100 I=1,6
I2 = 1
I1 = I
15 IF(I1-3) 17, 18, 19
19 I1 = I1-3
I2 = I2+1

```

```

      GO TO 15
18  IS = 3
      IT = 12
      IF ((P(L(J,3),I2)-PK).GT.0.) GO TO 99
C  HERE IF FORCE IS KNOWN
      PI = P(L(J,3),I2)
      GO TO 25
17  IF(I1-2) 20,21,21
20  IS = 1
      IT = 12
      IF ((P(L(J,1),I2)-PK).GT.0.) GO TO 99
      PI = P(L(J,1),I2)
      GO TO 25
21  IS = 2
      IT = 12
      IF ((P(L(J,2),I2)-PK).GT.0.) GO TO 99
      PI = P(L(J,2),I2)
25  CONTINUE
      IF (2*(L(J,IS)-1)+IT-MSK(IPL)) 301,302,303
302  I11 = IPL
      GO TO 327
301  IMR = IPL - 1
306  IF(2*(L(I,IS)-1) + IT-MSK(IMR)) 304, 305, 305
305  I11 = IMR
      IPL = IMR
      GO TO 327
304  IMR = IMR-1
      GO TO 306
303  IMR = IPL + 1
307  IF (2*(L(J,IS)-1)+IT-MSK(IMR)) 309, 309, 310
309  I11 = IMR
      IPL = IMR
      GO TO 327
310  IMR = IMR+1
      GO TO 307
327  IF(I11-ISHIFT.GT.K1) GO TO 328
      IF(I11-ISHIFT.LT.1) GO TO 328
      KS21(I11-ISHIFT) =KS21(I11-ISHIFT) + P(I1,J)
328  CONTINUE
      DO 100  KJ = 1,2
      DO 100  KK = 1,3
      IF ((P(L(J,KK),KJ)-PK).GT.0.) GO TO 28
      IF (2*(L(J,KK)-1)+KJ-MSK(IPL)) 11, 12, 13
12  I12 = IPL
      GO TO 27
11  IMR = IPL-1
      6  IF(2*(L(J,KK)-1) + KJ - MSK(IMR)) 4,5,5
      5  I12 = IMR
      IPL = IMR
      GO TO 27
      4  IMR = IMR-1
      GO TO 6
13  IMR = IPL + 1
      7  IF (2*(L(J,KK)-1) + KJ - MSK(IMR)) 5, 5, 10
10  IMR = IMR + 1
      GO TO 7
27  IF(I11-ISHIFT.GT.K1) GO TO 99
      IF(I11-ISHIFT.LT.1) GO TO 99
      IF(I12-JSHIFT.GT.K2) GO TO 999
      KS22(I11-ISHIFT,I12-JSHIFT) = KS22(I11-ISHIFT,I12-JSHIFT)
      1  + KPR(1,3*(KJ-1)+KK,J)
      GO TO 99

```

```

28 IF(D(L(J,KK),KJ).EQ.0.) GO TO 99
   IF(III-ISHIFT.GT.K1) GO TO 99
   IF(III-ISHIFT.LT.1) GO TO 99
   KS21(III-ISHIFT) = KS21(III-ISHIFT) -KPR(1,3*(KJ-1)+KK,J )
   1 * D(L(J,KK),KJ)
99 CONTINUE
100 CONTINUE
C HAVE EQUATIONS. NOW SHIFT THE SUBMATRICES TO THE PROPER POSITION FOR
C SUBROUTINE CHLSKY
   IF(ICYCLE.GT.1) GO TO 430
C S(1,1) IS (KREM X KREM), S(1,2) IS (KREM X N)
   DO 410 J = 1, KREM
   DO 410 I = 1,N
   LOCL = J + KREM*(I-1)
   LOCR1 = J + 32*(I-1+KREM)
C SET UP S(1,2) IN S(1,1,4)
   Q(LOCL+3072) = Q(LOCR1)
410 CONTINUE
   DO 420 J = 1, KREM
   DO 420 I = 1,KREM
   LOCL = J + KREM*(I-1)
   LOCR2 = J + 32*(I-1)
C SET UP S(1,1) IN S(1,1,2)
   Q(LOCL+1024) = Q(LOCR2)
420 CONTINUE
   GO TO 480
430 CONTINUE
   IF (ICYCLE.GT.2) GO TO 440
C S(2,1) IS (N X KREM) , S(2,2) AND S(2,3) ARE (N X N)
   K1 = KREM
   GO TO 450
440 CONTINUE
C S(1,1-1) , S(1,1) , S(1,1+1) ARE (N X N)
   K1 = N
450 CONTINUE
C SET UP S(ICYCLE,ICYCLE+1) IN S(1,1,4)
   IF (ICYCLE.EQ.M) GO TO 461
   DO 460 I = 1,N
   DO 460 J = 1,N
   LOCL = J+N*(I-1)
   LOCR1 = J + 32*(I-1+K1+N)
   Q(LOCL+3072) = Q(LOCR1)
460 CONTINUE
461 CONTINUE
C SET UP S(ICYCLE,ICYCLE) IN S(1,1,3)
   DO 468 I=1,N
   DO 468 J=1,N
   LOCR2 = J+32*(I-1+K1)
   LOCL = J + N * (I-1)
   Q(LOCL+2048) = Q(LOCR2)
468 CONTINUE
C MOVE S(ICYCLE,ICYCLE) TO S(1,1,2)
   NSQ = N**2
   DO 470 II=1,NSQ
   Q(II+1024) = Q(II+2048)
470 CONTINUE
C REARRANGE S(ICYCLE,ICYCLE-1) WITHIN S(1,1,1)
   KLOC = 0
   DO 480 I=2,K1
   KLOC = KLOC+N
   DO 480 J = 1,N
   Q(KLOC+J) = S(J,I,1)

```

```

480 CONTINUE
RETURN
999 CONTINUE
PRINT 1000, ICYCLE
1000 FORMAT (M1, 35H BANDWIDTH EXCEEDED, PARTITION ROW, I3)
IROW1 = I11 + ISHIFT
JROW1 = I12 + JSHIFT
PRINT 1001, IROW1, JROW1
1001 FORMAT (1H, 4HROW=, I3, 6X, 7HCOLUMN=, I3)
PRINT 9120, I11, I12, ISHIFT, JSHIFT
9120 FORMAT (1H, 4HI11=, I3, 4HI12=, I3, 7HISHIFT=, I3, 7HJSHIFT=, I3)
PRINT 9121, K1, K2, KREM, N, M
9121 FORMAT (1H, 3HK1=, I3, 3HK2=, I3, 5HKREM=, I3, 2HN=, I3, 2HM=, I3)
PRINT 9122, I, J, IPL, IMR, IS, IT
9122 FORMAT (1H, 2HI=, I3, 2HJ=, I3, 4HIPL=, I3, 4HIMR=, I3, 3HIS=, I3, 3HIT=, I3)
STOP
END

```

```

SUBROUTINE MXMULT(A,B,C,M,N,K)

```

```

C
C THIS SUBROUTINE MULTIPLIES MATRIX A BY MATRIX B AND STORES THE
C PRODUCT IN C. (C CANNOT BE THE SAME AS A OR B.)
C
C A IS (M X N)
C B IS (N X K)
C C IS (M X K)
C
C DIMENSION A(M,N) , B(N,K) , C(M,K)
C
DO 1 I=1,M
DO 1 L=1,K
C(I,L) = 0
DO 1 J=1,N
C(I,L) = C(I,L) + A(I,J) * B(J,L)
1 CONTINUE
RETURN
END

```

```

SUBROUTINE MXSUB(A,B,C,M,N)
C
C THIS SUBROUTINE SUBTRACTS MATRIX B FROM MATRIX A, STORES RESULT IN C
C
C A, B, AND C ARE (M X N)      (C CAN BE THE SAME AS A OR B)
C
C DIMENSION A(M,N), B(M,N), C(M,N)
C
DO 1 I=1,M
DO 1 J=1,N
C(I,J) = A(I,J) - B(I,J)
1 CONTINUE
RETURN
END

```

```

SUBROUTINE MXCON(A,B,X,M,N)
C
C THIS SUBROUTINE MULTIPLIES MATRIX A (MXN) BY CONSTANT X, RESULT IN B
C A MAY BE SAME AS B.
C THIS SUBROUTINE MULTIPLIES MATRIX A (MXN) BY CONSTANT X, RESULT IN B
C A MAY BE SAME AS B.
C DIMENSION A(M,N), B(M,N)
DO 1 I=1,M
DO 1 J=1,N
B(I,J) = X*A(I,J)
1 CONTINUE
RETURN
END

```

SUBROUTINE INVERT(B,K,K2,XMIN,FLAG)

C THIS SUBROUTINE SETS UP A UNIT MATRIX ADJACENT TO B(K,K)  
 C ELEMENTARY ROW OPERATIONS ARE THEN PERFORMED ON THE NEW K X 2K MATRIX  
 C TO REDUCE B(K,K) TO A UNIT MATRIX. THIS WILL PLACE THE INVERSE OF  
 C THE MATRIX B(K,K) IN THE RIGHT HALF OF B(K,2K)  
 C ON EXIT, THE INVERSE OF B REPLACES H  
 C B9+S N ARRAY OF 2\*K\*\*2 LOCATIONS CONTAINING THE MATRIX  
 C K IS THE DIMENSION OF THE SQUARE MATRIX B  
 C K2 IS 2\*K  
 C XMIN IS THE SMALLEST ALLOWABLE MAGNITUDE OF THE PIVOT  
 C FLAG WILL BE RETURNED AS 0. IF THE INVERSION WENT OFF OK  
 C FLAG WILL BE RETURNED AS 1. IF A PIVOT ELEMENT WAS TOO SMALL  
 C FLAG SHOULD BE TESTED AFTER EACH CALL TO THIS ROUTINE

C DIMENSION B(K,K2)

C FLAG = 0.

C SET UP UNIT MATRIX

C IF(K.GT.1) GO TO 2J  
 C IF(ABS(B(1,1)).LT.XMIN) GO TO 10  
 C B(1,1) = 1./B(1,1)  
 C RETURN  
 C 20 CONTINUE  
 C DO 1 I=1,K  
 C DO 1 J=1,K  
 C B(I,K+J) = 0.  
 C IF(I.EQ.J) B(I,K+J) = 1.  
 C 1 CONTINUE

C FIND LEADING ELEMENT WITH GREATEST MAGNITUDE

C DO 6 J=1,K  
 C M = J  
 C N = J+1  
 C IF(J.EQ.K) GO TO 21  
 C DO 2 L=M,K  
 C IF (ABS(B(M,J)).LT.ABS(B(L,J))) M=L  
 C 2 CONTINUE  
 C 21 CONTINUE  
 C IF (ABS(B(M,J)).LT.XMIN) GO TO 10  
 C IF(J.EQ.K) GO TO 31

C INTERCHANGE JTH AND MTH ROWS

C DO 3 L=J,K2  
 C D = B(J,L)  
 C B(J,L) = B(M,L)  
 C B(M,L) = D  
 C 3 CONTINUE  
 C 31 CONTINUE

C ZERO OUT PIVOTAL JTH COLUMN, SKIPPING PIVOTAL JTH ELEMENT

C DIVIDE JTH ROW BY PIVOT

C DO 4 M=N,K2  
 C B(J,M) = B(J,M) / B(J,J)  
 C 4 CONTINUE  
 C DO 6 M=1,K

```

C
C M DETERMINES ROW BEING MODIFIED, ONE WHOLE ROW AT A TIME
C
  IF ( M.EQ.J ) GO TO 6
  DO 5 L=N,K2
C
C L DETERMINES ELEMENT IN THE MTH ROW
C
  B(M,L) = B(M,L) - B(M,J) * B(J,L)
  5 CONTINUE
  6 CONTINUE
C
C INVERSE OF B IS NOW IN RIGHT HALF OF B(K,K2)
C NOW MOVE B INVERSE TO WHERE B WAS
  DO 7 I=1,K
  DO 7 J=1,K
  B(I,J) = B(I,J+K)
  7 CONTINUE
  RETURN
10 FLAG = 10.
  RETURN
  END

```

FUNDAMENTAL CASES FIVE AND FOUR (LONGITUDINAL LOADING)

FUNDAMENTAL CASE 5

```

PROGRAM LONTUD
C
C   LONTUD   CASE 5
C
C
C   THIS PROGRAM SOLVES THE LONGITUDINAL SHEAR PROBLEM
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1          CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2          NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
COMMON/2/ VF
DIMENSION DUM(10) , DMXT(6), DMX(6)
DATA (DMX = 0.,0.,1.,0.,0.,1.) ,(DMXT = 0.,1.,0.,0.,0.,1.)
C   NDIR(I,4) - MATERIAL, 1 OR 2
C   NDIR(I,J) - JTH DIRECTION/NODE IN ITH TRIANGLE ,(J=1,3)
C   NT - NUMBER TRIANGLES
C   NN - NUMBER NODES/DIRECTIONS
C
C
C
C   NUD - NUMBER UNKNOWN DISPLACEMENTS
C   NKD - NUMBER KNOWN DISPLACEMENTS
C
C   REWIND 20
C   CALL TAPESKIP(20,6,0)
C   CALL TAPESKIP(20,4,0)
30 READ 1000,DUM
PRINT 1001, DUM
READ 1002,AG(1),AG(2),VF
IF(VF.EQ.0.) GO TO 950
PRINT 1003,AG(1),AG(2),VF
M = 5
N = 17
KREM = 15
NT = 158
NN = 97
NUD = 83
NKD = 14
CALL CONFIG
PRINT 1009
DO 100 I=1,14
J = 83+I
PRINT 1010,J,FD(I)
100 CONTINUE
LINE = 4
PRINT 1004,(J,J=1,3),(J,J=1,9)
DO 130 I=1,157,2
IF(LINE.LT.54) GO TO 120
LINE = 4
PRINT 1004,(J,J=1,3),(J,J=1,9)
120 LINE = LINE + 2
II=I+1
130 PRINT 1005,I, (NDIR(I,J),J=1,4),II,(NDIR(II,J),J=1,4)
C
C   PRINT OUT NODE INFORMATION
C
C
I = 1
190 CONTINUE
LINE = 4
PRINT 1007
200 CONTINUE
IF(I.NE.97) GO TO 210
PRINT 1008, I, X(I), Y(I)
GO TO 215

```

CASE 5

CASE 5

```

210 IP2 = I+2
    PRINT 1018,((J,X(J),Y(J)),J=1,IP2)
215 I=I+3
    IF(I.GT.NN) GO TO 220
    LINE = LINE + 2
    IF(LINE.GT.56) GO TO 190
    GO TO 200
220 CONTINUE
C
C   DERIVE ALL MATRICES SMALL K
C
    DO 300 J=1,NT
    CALL KSMALL(J)
300 CONTINUE
    DO 400 I=1,NN
    NTRI(I,10)= 0
400 CONTINUE
    DO 500 I = 1,NT
    DO 500 J = 1,3
    K = NDIR(I,J)
    NTRI(K,10) = NTRI(K,10) + 1
    L = NTRI(K,10)
    NTRI(K,L) = I
500 CONTINUE
C
    CALL PUCHOL
    GO TO 30
1000 FORMAT(10A8)
1001 FORMAT(1H1,10A8)
1002 FORMAT(3E15.4)
1003 FORMAT(30X,3HG 1,20X,3HG 2,20X,3HV F/10X,3(8X,E15.5))
1004 FORMAT(1H1, 8HTRIANGLE,3(2X,5HNODE ,I1),4X,8HMATERIAL,10X,
1      8HTRIANGLE,3(2X,5HNODE ,I1),4X,8HMATERIAL/)
1005 FORMAT(1X,2(5X,I3,5X,I3,5X,I3,5X,I3 ,9X,I3,10X)/)
1007 FORMAT(1H1,3(4HNODE,12X,1HX,12X,1MY,8X)/)
1008 FORMAT(2X,I3,2(3X,F10.5)/)
1009 FORMAT(/////,4X,15HNON-ZERO KNOWNS./)
1010 FORMAT(5H ROW(,I2,2H)=,F11.4)
1018 FORMAT(1X,3(1X,I3,3X,F10.5,3X,F10.5,8X)/)
950 CONTINUE
    REWIND 20
    STOP
    END
CASE 5
CASE 5

```

```

SUBROUTINE CONFIG
C
C
C CONFIG CASE 5
C
C THIS SUBROUTINE GENERATES THE COORDINATE AND FORCE CONFIGURATION
C FOR THE PROGRAM LONTUD
C
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1 CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2 NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
COMMON/2/ VF
DIMENSION JNDIR(158),KNDIR(158),LNDIR(158)
DATA (RAD = 57.29578)
DATA (JNDIR =
1 1. 2. 3. 1. 1. 2. 2. 2. 3. 3. 3. 9. 4. 4. 5. 6. 6. 6. 7. 8.
2 8. 8. 9.15.10.10.11.12.12.12.13.14.14.15.15.21.16.20.20.20.
3 21.21.21.25.23.24.24.25.25.28.28.17.17.17.18.18.19.19.22.22.
4 22.23.23.29.29.30.30.31.31.32.32.33.33.34.34.35.35.36.36.38.
5 38.39.39.40.40.41.41.42.42.43.43.44.44.45.45.47.47.48.48.49.
6 50.50.51.51.52.52.53.54.54.59.57.57.57.58.59.59.59.60.61.61.
7 62.56.63.63.63.64.65.65.65.66.67.67.67.69.69.69.69.70.71.
8 71.71.72.73.73.73.75.75.75.76.77.77.78.79.79.81.81.82)
DATA (KNDIR =
1 2. 3.85. 4. 5. 5. 6. 7. 7. 8. 9.86.10.11.11.11.12.13.13.13.
2 14.15.15.87.16.17.17.17.18.19.19.19.20.14.21.88.28.19.22.23.
3 23.24.25.89.26.26.27.27.90.29.30.30.31.32.32.33.33.34.34.35.
4 36.36.37.38.39.39.40.40.41.41.42.42.43.43.44.44.45.45.46.47.
5 48.48.49.49.50.50.51.51.52.52.53.53.54.54.55.58.61.61.62.62.
6 62.65.65.66.66.67.67.67.56.97.97.59.60.60.96.95.63.63.63.64.
7 64.67.95.94.69.70.70.70.71.72.72.72.73.74.94.93.75.76.76.76.
8 77.78.78.78.79.80.93.92.81.81.81.82.82.82.83.92.91.91)
DATA (LNDIR =
1 84.84.84. 5. 2. 6. 7. 3. 8. 9.85.85.11. 5. 6.12.13. 7. 8.14.
2 15. 9.86.86.17.11.12.18.19.13.14.20.21.21.87.87.17.22.23.21.
3 24.25.88.88.24.27.25.90.89.30.17.31.32.18.33.19.34.22.35.36.
4 23.37.26.39.30.40.31.41.32.42.33.43.34.44.35.45.36.46.37.48.
5 39.49.40.50.41.51.42.52.43.53.44.54.45.55.46.61.48.62.49.50.
6 65.51.66.52.67.53.54.56.55.96.59.60.58.61.95.63.60.61.64.62.
7 65.68.94.69.70.64.65.71.72.66.67.73.74.68.93.75.76.70.71.77.
8 78.72.73.79.80.74.92.81.76.77.82.78.79.83.80.91.82.83)
DO 10 I=1,36
NDIR(I,4) = 1
10 NDIR(159-I,4) = 1
DO 20 I = 1,86
NDIR(36+I,4) = 2
20 CONTINUE
DO 50 I = 1,7
FD(I) = 1
50 FD(I+7) = -1
DO 100 I = 1, 158
NDIR(I,1) = JNDIR(I)
NDIR(I,2) = KNDIR(I)
NDIR(I,3) = LNDIR(I)
100 CONTINUE
S3 = SQRT(3.)
S302 = S3/2.
PI = 3.1415927
R = SQRT(2.*S3*VF/PI)
X(84) = S302
Y(84) = .5
DO 300 I=1,3

```

```

X(I) = S302-R/4.*COS(PI*(I-1)/6.)
300 Y(I)= .5 -R/4.*SIN(PI*(I-1)/6.)
C
DO 310 I=1,6
X(I+3) = S302 - R/2.* COS(PI*(I-1)/12.)
Y(I+3) = .5 - R/2.* SIN(PI*(I-1)/12.)
C
X(I+9) = S302 - 3.*R/4.*COS(PI*(I-1)/12.)
Y(I+9) = .5 - 3.*R/4.*SIN(PI*(I-1)/12.)
C
X(I+15) = S302 - R * COS(PI*(I-1)/12.)
310 Y(I+15) = .5 - R * SIN(PI*(I-1)/12.)
DO 320 I =1,7
X(83+I) = S302
320 CONTINUE
DO 330 I =1,5
Y(83+I)= .5- (I-1)/4.*R
330 CONTINUE
Y(90) = -.5
Y(89) = (Y(88)+Y(90))/2.
X(38) = -.5 *TAN(PI/6.)
Y(38) = .5
DX = (1.-R)/(2.*COS(PI/6.))
X(29) = X(38) + DX
Y(29) = .5
X(28) = (X(29)+X(16))/2.
Y(28) = .5
DO 340 I =1,4
X(I+38) = (4-I) * X(38)/4.
340 Y(I+38) = (4-I) * Y(38)/4.
DELX = X(39) - X(38)
DELY = Y(39) - Y(38)
DO 350 I = 1,8
X(I+29) = X(I+28) + DELX
350 Y(I+29) = Y(I+28) + DELY
X(26) = X(37)+(X(90)-X(37))/3.
Y(26) = -.5
X(27) = 2.*X(26) - X(37)
Y(27) = -.5
X(22) = X(19) + (X(26)-X(19)) /3.
Y(22) = Y(19) + (Y(26)-Y(19)) /3.
X(23) = 2.*X(22)-X(19)
Y(23) = 2.*Y(22)-Y(19)
X(24) = X(23)+(X(89)-X(23))/3.
Y(24) = Y(23)+(Y(89)-Y(23))/3.
X(25) = 2.*X(24)-X(23)
Y(25) = 2.*Y(24)-Y(23)
C
DO 360 I= 43,83
X(I) = -X(84-I)
360 Y(I) = -Y(84-I)
C
DO 370 I= 1,7
X(90+i) = -X(83+I)
370 Y(90+i) = -Y(83+I)
C
RETURN
END

```

```

SUBROUTINE FINAL
C
C   FINAL   CASE 5
C
C   THIS SUBROUTINE CALCULATES STRESSES FOR PROGRAM LONTUD
C
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1          CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2          NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
COMMON/2/ VF
DIMENSION SIG(4,158)
EQUIVALENCE (SIG,STR)
DIMENSION SIGOUT(632)
DIMENSION DEL(200), DX(3), STR(4,200)
DATA (PI=3.1415927)
R=SQRT(3.)
R=SQRT(2.*R*VF/PI)
DEG = 57.29578
DO 10 J = 1,KRFM
DEL(J) = SP(J,1)
10 CONTINUE
KLOC = KREM - N
DO 20 I = 2,M
KLOC = KLOC +N
DO 20 J =1,N
DEL(KLOC+J) = SP(J,1)
20 CONTINUE
DO 25 I = 1, NKD
25 DEL(NUD+I) = FD(I)
PRINT 1010
NDEL = NN / 7
JCNT = 0
DO 35 J = 1,NDEL
JCNT = JCNT + 1
IF(JCNT.LE.10) GO TO 30
PRINT 1010
JCNT = 0
30 JFIR = 7*(J-1)+1
JLAST = JFIR + 6
PRINT 1011, (K,K=JFIR,JLAST)
PRINT 1012, (DEL(K),K=JFIR,JLAST)
35 CONTINUE
LOC1 = 7*NDEL+1
IF(LOC1.GT.NN) GO TO 40
PRINT 1011, (K,K=LOC1,NN)
PRINT 1012, (DEL(K),K=LOC1,NN)
40 CONTINUE
C
C   FIND STRESSES
C
DO 100 I=1,NT
I1 = NDIR(I,1)
I2 = NDIR(I,2)
I3 = NDIR(I,3)
DX(I) = DEL(I1)
DY(I) = DEL(I2)
DZ(I) = DEL(I3)
CALL MMULT(CDA(I,1,1),DX,STR(1,1),2,3,1)
STR(9,1) = SORT( STR(1,1)*2 + STR(2,1)*2)
100 STR(4,1) = DEG * ATAN(STR(2,1) / STR(1,1) )

```

```

    TAUXZ = R/4.*(SIG(1,3)+SIG(1,12)+SIG(1,24)+SIG(1,36))
1    + (1.-R)/2.*(SIG(1,44)+SIG(1,49))
    TAUXZ = 1./TAUXZ
    DO 110 J=1,158
    STR(3,J) = STR(3,J)*TAUXZ
    DO 110 I=1,2
    K = (J-1)*2+I
    SIGOUT(K) = STR(I,J)*TAUXZ
110 STR(I,J) = SIGOUT(K)
C
    LINE = 4
    PRINT 1005
    DO 150 I = 1,NT
    IF(LINE.LT.54) GO TO 140
    LINE = 4
    PRINT 1005
140 LINE = LINE + 2
150 PRINT 1006,I,(CENT(I,J),J=1,2),(STR(J,I),J=1,4)
    PRINT 8787,TAUXZ
8787 FORMAT(//23H NORMALIZATION FACTOR =,2X,E15.7)
    WRITE (20) (SIGOUT(I),I=1,158)
1005 FORMAT(9HTRIANGLE,4X10HCENTROID X,4X,10HCENTROID Y,6X,8MSIGMA XZ,
1    6X,8MSIGMA YZ,6X,8MGRADIENT,9X,5HALPHA/)
1006 FORMAT(6X,I3,2(4X,F10.4),4(2X,E12.5)/)
1010 FORMAT(1H1,50X,13HDISPLACEMENTS,/)
1011 FORMAT(1H .7(8X,4MDEL(.I3,1H) ) )
1012 FORMAT(1H .7(2X,E14.7)/)
    RETURN
    END

```

FUNDAMENTAL CASE 4

```

PROGRAM LONTUD
C
C LONTUD CASE 4
C
C THIS PROGRAM SOLVES THE LONGTUDINAL SHEAR PROBLEM
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1 CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2 NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
COMMON/2/ VF
DIMENSION DUM(10) , DMXT(6), DMX(6)
DATA (DMX = 0.,0.,1.,0.,0.,1.) ,(DMXT = 0.,1.,0.,0.,0.,1.)
C NDIR(I,4) - MATERIAL, 1 OR 2
C NDIR(I,J) - JTH DIRECTION/NODE IN ITH TRIANGLE ,(J=1,3)
C NT - NUMBER TRIANGLES
C NN - NUMBER NODES/DIRECTIONS
C
C
C
C NUD - NUMBER UNKNOWN DISPLACEMENTS
C NKD - NUMBER KNOWN DISPLACEMENTS
REWIND 20
CALL TAPESKIP(20,6,0)
CALL TAPESKIP(20,5,0)
10 READ 1000,DUM
PRINT 1001, DUM
READ 1002,AG(1),AG(2),VF
IF(VF.EQ.0.) GO TO 950
PRINT 1003,AG(1),AG(2),VF
M = 5
N=15
KREM = 13
NUD = 73
NT = 158
NN = 97
NKD = 24
CALL CONFIG
PRINT 1009
DO 100 I=1,24
J = 73+I
PRINT 1010,J,FD(I)
100 CONTINUE
LINE = 4
PRINT 1004,(J,J=1,3),(J,J=1,3)
DO 130 I=1,157,2
IF LINE.LY.54) GO TO 120
LINE = 4
PRINT 1004,(J,J=1,3),(J,J=1,3)
120 LINE = LINE + 2
II=I+1
130 PRINT 1005,I, (NDIR(I,J),J=1,4),II,(NDIR(II,J),J=1,4)
C
C PRINT OUT NODE INFORMATION
C
I = 1
190 CONTINUE
LINE = 4
PRINT 1007
200 CONTINUE
IF(I.NE.97) GO TO 210
PRINT 1008, I, X(I), Y(I)
GO TO 215

```

CASE 4

CASE 4



SUBROUTINE CONFIG

CONFIG CASE 6

THIS SUBROUTINE GENERATES THE COORDINATE AND FORCE CONFIGURATION FOR THE PROGRAM LONTUD

```

COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1          CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2          NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
COMMON/2/ VF
DIMENSION JNDIR(158),KNDIR(158),LNDIR(158)
DATA (RAD = 57.29578)
DATA (JNDIR =
1   1, 1, 2, 4, 1, 1, 1, 1, 2, 2, 2, 3,10, 4, 4, 5, 5, 5, 6, 7,
2   7, 7, 8, 9,16,10,10,11,11,11,12,13,13,13,14,15,16,18,19,19,
3   20,20,20,21,23,24,24,25,25,27,16,16,16,16,17,17,18,22,22,22,
4   22,23,23,34,27,27,27,28,28,29,29,30,30,31,31,32,32,33,33,41,
5   34,34,34,35,35,36,36,37,37,38,38,39,39,40,40,51,41,41,41,42,
6   43,43,44,44,45,45,46,47,47,48,49,49,50,50,48,49,49,50,51,51,
7   52,58,53,54,54,54,55,56,56,56,57,58,58,59,60,60,60,61,62,
8   62,62,63,64,64,64,65,66,66,67,68,68,69,70,70,71,72,73 )
DATA (KNDIR =
1   74, 2, 3,75,75, 4, 5, 6, 6, 7, 8, 8,76,76,10,10,11,12,12,12,
2   13,14,14,14,77,77,16,16,17,18,18,18,19,20,20,20,78,22,22,23,
3   23,24,25,25,88,88,87,87,86,79,79,27,28,29,29,30,30,18,31,32,
4   33,33,89,80,80,34,35,35,36,36,37,37,38,38,39,39,40,40,90,81,
5   81,41,42,42,43,43,44,44,45,45,46,46,47,47,91,82,82,51,52,52,
6   52,56,56,57,57,58,58,58,92,49,84,50,83,51,53,53,54,54,54,55,
7   55,93,59,59,60,61,61,61,62,63,63,63,64,94,65,65,66,67,67,67,
8   68,69,69,69,70,95,71,71,72,72,72,73,73,73,96,97,97,97 )
DATA (LNDIR =
1   75,74,74,76, 4, 5, 6, 2, 7, 8, 3, 9,77,10, 5,11,12, 6, 7,13,
2   14, 8, 9,15,78,16,11,17,18,12,13,19,20,14,15,21,79,19,23,20,
3   24,25,21,26,24,87,25,86,26,80,27,28,29,17,30,18,31,31,32,33,
4   23,89,88,81,34,35,28,36,29,37,30,38,31,39,32,40,33,90,89,82,
5   41,42,35,43,36,44,37,45,38,46,39,47,40,91,90,83,51,52,42,43,
6   56,44,57,45,58,46,47,92,91,85,85,84,84,83,49,54,50,51,55,52,
7   56,92,54,60,61,55,56,62,63,57,58,64,94,93,60,66,67,61,62,68,
8   69,63,64,70,95,94,66,72,67,68,73,69,70,96,95,72,73,96 )
DO 10 I=1,36
  NDIR(I,4) = 1
10  NDIR(159-I,4) = 1
  DO 20 I = 1,86
    NDIR(36+I,4) = 2
20  CONTINUE
  DO 50 I = 1, 12
    FD(I) = 1.
50  FD(I+12) = -1.
  DO 100 I = 1, 158
    NDIR(I,1) = JNDIR(I)
    NDIR(I,2) = KNDIR(I)
    NDIR(I,3) = LNDIR(I)
100 CONTINUE
  S3 = SORT(3.)
  S302 = S3/2.
  PI = 3.1415927
  R = SORT(2.*S3*VF/PI)
  DO 300 I=1,3
    X(I) = S302-R/4.*COS(PI*I/6.)
300 Y(I) = .5 - R/4.*SIN(PI*I/6.)

```

```

C
DO 310 I=1,6
X(I+3) = S302 - R/2.*COS(PI*I/12.)
Y(I+3) = .5 - R/2.*SIN(PI*I/12.)
C
X(I+9) = S302 - 3.*R/4.*COS(PI*I/12.)
Y(I+9) = .5 - 3.*R/4.*SIN(PI*I/12.)
C
X(I+15) = S302 - R*CCS(PI*I/12.)
310 Y(I+15) = .5 - R* SIN(PI*I/12.)
DO 320 I = 1,5
X(I+73) = S302 - R*(I-1)/4.
320 Y(I+73) = .5
DO 330 I=1,4
330 Y(85+I) = -.5
X(86) = S302
X(26) = S302
Y(26) = (Y(21)+ Y(86)) /2.
Y(81) = .5
Y(80) = .5
Y(79) = .5
X(81) = -.5 * TAN(PI/6.)
DX = (1.-R) / (2.*COS(PI/6.))
X(80) = X(81) + DX
X(79) = (X(80) + X(78))/2.
DO 340 I=1,4
X(I+33) = (4-I)* X(81)/4.
340 Y(I+33) = (4-I)* Y(81)/4.
DELX = X(34)-X(81)
DELY = Y(34)-Y(81)
DO 350 I = 1,7
X(I+26) = X(80)+DELX*I
350 Y(I+26) = Y(80)+DELY*I
X(89) = X(33)+DELX
X(88) = X(89)+ (X(86)-X(89))/3.
X(87) = 2.*X(88)-X(89)
X(22) = X(18) + (X(88)-X(18)) /3.
Y(22) = Y(18) + (Y(88)-Y(18)) /3.
X(23) = 2.*X(22) - X(18)
Y(23) = 2.*Y(22) - Y(18)
X(24) = X(23)+(X(26)-X(23))/3.
Y(24) = Y(23)+(Y(26)-Y(23))/3.
X(25) = 2.*X(24) -X(23)
Y(25) = 2.*Y(24) -Y(23)
DO 360 I= 38,73
X(I) = -X(74-I)
360 Y(I) = -Y(74-I)
DO 370 I= 1, 4
X(89+I) = -X(82-I)
Y(89+I) = -.5
X(93+I) = -X(78-I)
Y(93+I) = -.5
X(81+I) = - X(90-I)
370 Y(81+I) = +.5
C
RETURN
END

```

```

SUBROUTINE FINAL
C
C FINAL CASE 6
C
C THIS SUBROUTINE CALCULATES STRESSES FOR PROGRAM LONTUD
C
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1 CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2 NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
COMMON/2/ VF
DIMENSION SIG(4,158)
EQUIVALENCE (SIG,STR)
DIMENSION SIGOUT(632)
DIMENSION DEL(200), DX(3), STR(4,200)
DATA (PI=3.1415927)
R=SQRT(3.)
R=SQRT(2.*R*VF/PI)
DEG = 57.29578
DO 10 J = 1,KREM
DEL(J) = SP(J,1)
10 CONTINUE
KLOC = KREM - N
DO 20 I = 2,M
KLOC = KLOC +N
DO 20 J =1,N
DEL(KLOC+J) = SP(J,I)
20 CONTINUE
DO 25 I = 1, NKD
25 DEL(NUD+I) = FD(I)
PRINT 1010
NDEL = NN / 7
JCNT = 0
DO 35 J = 1,NDEL
JCNT = JCNT + 1
IF(JCNT.LE.18) GO TO 30
PRINT 1010
JCNT = 0
30 JFIR = 7*(J-1)+1
JLAST = JFIR + 6
PRINT 1011, (K,K=JFIR,JLAST)
PRINT 1012, (DEL(K),K=JFIR,JLAST)
35 CONTINUE
LOC1 = 7*NDEL+1
IF(LOC1.GT.NN) GO TO 40
PRINT 1011, (K,K=LOC1,NN)
PRINT 1012, (DEL(K),K=LOC1,NN)
40 CONTINUE
C
C FIND STRESSES
C
DO 100 I=1,NT
I1 = NDIR(I,1)
I2 = NDIR(I,2)
I3 = NDIR(I,3)
DX(1) = DEL(I1)
DX(2) = DEL(I2)
DX(3) = DEL(I3)
CALL MXMULT(CDA(1,1,I),DX,STR(1,I),2,3,1)
100 STR(3,I) = SQRT( STR(1,I)**2 + STR(2,I)**2)
TAUZY = R/4.*(SIG(2,1)+SIG(2,4)+SIG(2,13)+SIG(2,25))

```

```

1      + (X(78)-X(79))*(SIG(2,37)+S.G(2,50))
2      + (X(80)-X(81))*(SIG(2,64)+SIG(2,80))
3      + (X(82)-X(85))/3.*(SIG(2,96)+SIG(2,111)+SIG(2,113))
TAUYZ=SQRT(3.)/TAUYZ
DO 110 J=1,158
IF(1.E10*ABS(STR(1,J)).LT.ABS(STR(2,J))) GO TO 80
STR(4,J)=DEG*ATAN(STR(2,J)/STR(1,J))
GO TO 90
80 STR(4,J) = 90.
90 CONTINUE
STR(3,J) = STR(3,J)*TAUYZ
DO 110 I=1,2
K= (J-1)*2+I
SIGOUT(K) = STR(I,J)*TAUYZ
110 STR(I,J) = SIGOUT(K)
C
LINE = 4
PRINT 1005
DO 150 I =1,NT
IF(LINE.LT.54) GO TO 140
LINE = 4
PRINT 1005
140 LINE = LINE + 2
150 PRINT 1006,I,(CENT(I,J),J=1,2),(STR(J,I),J=1,4)
PRINT 8787,TAUYZ
8787 FORMAT(//23H NORMALIZATION FACTOR =,2X,E15.7)
WRITE (20) (SIGOUT(I),I=1,158)
1005 FORMAT(9H1TRIANGLE,4X10HCENTROID X,4X,10HCENTROID Y,6X,8HSIGMA XZ,
1      6X,8HSIGMA YZ,6X,8HGRADIENT,9X,5HALPHA/)
1006 FORMAT(6X,I3,2(4X,F10.4),4(2X,E12.5)/)
1010 FORMAT(1H1,5UX,13HDISPLACEMENTS,/)
1011 FORMAT(1H ,7(8X,4HDEL(.I3,1H) ) )
1012 FORMAT(1H ,7(2X,E14.7)/)
RETURN
END
CASE 6

```

## AUXILIARIES

```

SUBROUTINE PUCHOL
C
C THIS ROUTINE SOLVES  $SU=G$ , WHERE S IS A TRI-DIAGONAL MATRIX IN
C SUBMATRICES, WITH ELEMENTS OF ORDER N.
C S IS KNOWN
C SP IS A VECTOR OF DIMENSION (NXM) WHERE M IS THE NUMBER OF DIVISIONS OF S
C C IS WRITTEN ONTO TAPE AFTER DERIVATION ON THE FORWARD PASS.
C AND READ BACK IN ON THE BACKSWEEP
C S(1,1,1) INITIALLY CONTAINS S(1,1-1)
C S(1,1,2) INITIALLY CONTAINS S(1,1 )
C S(1,1,4) INITIALLY CONTAINS S(1,1+1)
C SP CORRESPONDS TO P IN THE WRITEUP BY GATEWOOD ON THE FORWARD PASS.
C ON THE BACKSWEEP, IT CORRESPONDS TO U.
C
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,MN,NT,G(30),
1          CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2          NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
DIMENSION C(900)
EQUIVALENCE (S(3601),C)
DATA (XLIMIT =1.E-8)
REWIND 96
N2 = 2*N
KREM2 = 2*KREM
DO 30 ICYCLE =1,M
IF(ICYCLE-2) 1,2,3
1 K1 = KREM
K2 = KREM
K3 = KREM2
GO TO 4
2 K1 = N
K2 = KREM
K3 = N2
GO TO 4
3 K1 = N
K2 = N
K3 = N2
4 CONTINUE
K4 = N
IF(ICYCLE.EQ.M) K4 = NKD
CALL KLARGE(ICYCLE,S,S(1,1,2),S(1,1,4),S(1,1,3),K1,K2,K4,NKD)
IF(ICYCLE.EQ.1) GO TO 10
5 IF (UNIT,96) 6,7 ,600, 600
6 GO TO 5
7 CONTINUE
CALL MXMULT(S(1,1,1) , S(1,1,5) , S(1,1,3) ,K1,K2,K1)
C
C S(1,1,5) CONTAINS C FROM LAST CYCLE
C
CALL MXSUB(S(1,1,2) , S(1,1,3) , S(1,1,2) ,K1,K1)
10 CONTINUE
C
C B(I,I) NOW IN S(1,1,2)
C
CALL INVERT(S(1,1,2) , K1 , K3, XLIMIT , FLAG)
IF (FLAG.NE.0.) GO TO 500
C
C INVERSE OF B(I,I) NOW IN S(1,1,2)
C
IF (ICYCLE.EQ.1) GO TO 20
CALL MXMULT(S(1,1,1) ,SP(1,ICYCLE-1) , S(1,1,3) , N, K2 , 1)
CALL MXSUB (G , S(1,1,3), G, N, 1)
20 CONTINUE

```

```

CALL MXMULT (1, 2, 3, SP(1, JCYCLE), K1, K1, 1)
IF (JCYCLE.GE.M) GO TO 35
CALL MXMULT (S(1,1,2), S(1,1,4), S(1,1,5), K1, K1, N)
NSQ = K1*N
BUFFER OUT (96,1) (C(1), C(NSQ))
30 CONTINUE
35 CONTINUE
C
C NOW IN BACKSWEEP, SOLVING FOR U
C
DO 60 I = 2,M
JCYCLE = M-I+1
IF(JCYCLE.GT.1) GO TO 36
K1 = KREM
GO TO 37
36 K1 = N
37 CONTINUE
NSQ = K1*N
IF (JCYCLE.EQ.M-1) GO TO 41
BACKSPACE 96
41 CONTINUE
BACKSPACE 96
BUFFER IN (96,1) (C(1),C(NSQ))
42 IF(UNIT,96) 43, 44,700,700
43 GO TO 42
44 CONTINUE
C
C U(M)=SP(M), CONSIDER FIRST (M-1)TH CYCLE
C
CALL MXMULT(S(1,1,5), SP(1,JCYCLE+1),S(1,1,1),K1,N, 1)
CALL MXSUB(SP(1,JCYCLE), S(1,1,1), SP(1,JCYCLE), K1, 1)
60 CONTINUE
C
C U(NS,1) NOW STORED IN SP(N,1), I=1,M
C
CALL FINAL
RETURN
500 CONTINUE
PRINT 1000, JCYCLE
STOP
600 CONTINUE
PRINT 1001,JCYCLE
STOP
700 CONTINUE
PRINT 1002, JCYCLE
1000 FORMAT (31M|COULD NOT INVERT MATRIX IN ROW,12)
1001 FORMAT(37M1 ERROR READING C INTO CORE ON 12, 7THM ROW.)
1002 FORMAT(37M1 ERROR WRITING C ONTO TAPE ON 12, 7THM ROW.)
STOP
END

```

```

SUBROUTINE KSMALL(I)
C
C THIS SUBROUTINE DERIVES THE MATRIX SMALL K AND THE MATRIX
C C=D* A**(-1) FOR THE TRIANGLE I.
C
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1 CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2 NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
DIMENSION D(6), AINV(3,3), C(4) ,DMXT(6)
DATA (DMXT = 0.,1.,0.,0.,0.,1.)
DATA (D=0.,0.,1.,0.,0.,1.) , (C = 0.,0.,0.,0.)
AINV(1,1) = 1.
AINV(1,2) = 0.
AINV(1,3) = 0.
C
IN1 = NDIR(I,1)
IN2 = NDIR(I,2)
IN3 = NDIR(I,3)
X12 = X(IN2) - X(IN1)
X13 = X(IN3) - X(IN1)
ETA2= Y(IN2) - Y(IN1)
ETA3= Y(IN3) - Y(IN1)
C
CENT(I,1) = (X(IN1) +X(IN2) + X(IN3) ) / 3.
CENT(I,2) = (Y(IN1) +Y(IN2) +Y(IN3) ) / 3.
C
DELTA = X12*ETA3 - X13*ETA2
IF(ABS(DELTA).GT.1.E-10) GO TO 77777
PRINT 88888,I
88888 FORMAT(1H1,3HI =,I3)
77777 CONTINUE
C
AINV(2,2) = ETA3/DELTA
AINV(2,3) = -ETA2/DELTA
AINV(2,1) = -(AINV(2,2) + AINV(2,3) )
AINV(3,2) = -X13/DELTA
AINV(3,3) = X12/DELTA
AINV(3,1) = -(AINV(3,2) + AINV(3,3) )
C
IMAT = NDIR(I,4)
C(1) = AG(IMAT)
C(4) = C(1)
CALL MXMULT(D,AINV,CAY(1,1,I),2,3,3)
CALL MXMULT(C,CAY(1,1,I),CDA(1,1,I),2,2,3)
DO 100 J=2,3
JJ=J-1
DO 100 K=1,JJ
TEMP = AINV(J,K)
AINV(J,K) = AINV(K,J)
100 AINV(K,J) = TEMP
OMEGA = DELTA / 2.
CALL MXMULT(DMXT,CDA(1,1,I),CAY(1,1,I+1),3,2,3)
CALL MXMULT(AINV,CAY(1,1,I+1),CAY(1,1,I),3,3,3)
DO 110 J = 1,3
DO 110 K = 1,3
CAY(J,K,I) = CAY(J,K,I)* OMEGA
110 CONTINUE
RETURN
END

```

```

SUBROUTINE KLARGE(I,SIM1,S1,SIP1,CAY12, I1,I2,I3,I4)
C
C THIS SUBROUTINE IS CALLED BY SUBROUTINE PUCHOL TO GENERATE THE I-TH
C ROW OF SUBMATRICES S(I,I-1), S(I,I), S(I,I+1) AND THE PART OF THE
C INDEPENDENT VECTOR REQUIRED BY THE CHOLESKI PROCESS ON THE I-TH PASS
C
COMMON /1/ X(100), Y(100), NDIR(200,4),M,N,KREM,NN,NT,G(30),
1 CAY(3,3,201),CENT(200,2),AG(2), CDA(2,3,200),
2 NTRI(100,10),NUD, NKD, S(30,30,5),FD(30) , SP(30,15)
DIMENSION SIM1(I1,I2), S1(I1,I1), SIP1(I1,I3) , CAY12(I1,I4)
KS2= 4
IF(I.GT.1) GO TO 10
KS1= 2
IS = 0
JS = - I2
GO TO 40
10 IF(I.GT.2) GO TO 20
JS = 0
GO TO 30
20 JS = KREM + (I-3)*N
30 IS = KREM + (I-2)*N
C
C IS - ROW BIAS
C JS - COLUMN BIAS
C
KS1 = 1
40 JS1 = JS
JS2 = JS1 + I2
JS3 = JS2 + I1
JS4 = JS3 + I3
IROW1 = 1 + IS
IROW2 = I1 + IROW1 - 1
DO 300 KSS =KS1,KS2
C
GO TO (100,110,120,130),KSS
100 ICOL1 = JS1 +1
ICOL2 = JS2
GO TO 150
110 ICOL1 = JS2+1
ICOL2 = JS3
GO TO 150
120 IF(I.EQ.M) GO TO 300
ICOL1 = JS3+1
ICOL2 = JS4
GO TO 150
130 ICOL1 = NUD + 1
ICOL2 = NN
150 CONTINUE
DO 300 II =IROW1,IROW2
DO 300 JJ =ICOL1,ICOL2
TEMP = 0.
NTRI9 = NTRI(II,10)
NTRJ9 = NTRI(JJ,10)
C
C NTRI9 - NUMBER TRIANGLES TOUCHING NODE II
C NTRJ9 - NUMBER TRIANGLES TOUCHING NODE JJ
C
DO 200 K = 1,NTRI9
NTRII = NTRI(II,K)
DO 200 KK = 1,NTRJ9
IF(NTRII.NE.NTRI(JJ,KK))GO TO 200
DO 180 L = 1,3

```

```

      IF(II.EQ.NDIR(NTRII,L))IK = L
      IF(JJ.EQ.NDIR(NTRII,L))JK = L
180 CONTINUE
      TEMP = TEMP + CAY(IK,JK,NTRII)
200 CONTINUE
      GO TO(210,220,230,240), KSS
210 SIM1(II-IS,JJ-JS1) = TEMP
      GO TO 300
220 SI(II-IS,JJ-JS2) = TEMP
      GO TO 300
230 SIP1(II-IS,JJ-JS3) = TEMP
      GO TO 300
240 CAY12(II-IS,JJ-NUD) =-TEMP
300 CONTINUE
C
C NOW CALCULATE INDEPENDENT TERM G = K12 * (KNOWN DISPLACEMENTS)
C
      CALL MXMULT(CAY12,FD, G,11,14,1)
      RETURN
      END

```

```

      SUBROUTINE TMMUL(A,B,C,N,M,K)
C
C THIS SUBROUTINE MULTIPLIES MATRIX B BY THE TRANSPOSE OF MATRIX A
C THE PRODUCT IS ADDED TO C
C
C A IS (N X M)
C B IS (N X K)
C C IS (M X K)
C
      DIMENSION A(N,M) , B(N,K) , C(M,K)
      DO 1 I=1,M
      DO 1 L=1,K
C
C C(I,L) = 0.
      DO 1 J=1,N
      C(I,L) = C(I,L) + A(J,I) * B(J,L)
1 CONTINUE
      RETURN
      END

```

148 CARDS

```

      SUBROUTINE MXMULT(A,B,C,M,N,K)
C
C THIS SUBROUTINE MULTIPLIES MATRIX A BY MATRIX B AND STORES THE
C PRODUCT IN C. (C CANNOT BE THE SAME AS A OR B.)
C
C A IS (M X N)
C B IS (N X K)
C C IS (M X K)
C
      DIMENSION A(M,N) , B(N,K) , C(M,K)
C
      DO 1 I=1,M
      DO 1 L=1,K
      C(I,L) = 0.
      DO 1 J=1,N
      C(I,L) = C(I,L) + A(I,J) * B(J,L)
1 CONTINUE
      RETURN
      END

```

```

SUBROUTINE MXCON(A,B,X,M,N)
C
C THIS SUBROUTINE MULTIPLIES MATRIX A (MXN) BY CONSTANT X. RESULT IN B
C A MAY BE SAME AS B.
C THIS SUBROUTINE MULTIPLIES MATRIX A (MXN) BY CONSTANT X. RESULT IN B
C A MAY BE SAME AS B.
  DIMENSION A(M,N) , B(M,N)
  DO 1 I=1,M
  DO 1 J=1,N
  B(I,J) = X*A(I,J)
1 CONTINUE
  RETURN
  END

```

```

SUBROUTINE MXSUB(A,B,C,M,N)
C
C THIS SUBROUTINE SUBTRACTS MATRIX B FROM MATRIX A. STORES RESULT IN C
C
C A, B, AND C ARE (M X N)      (C CAN BE THE SAME AS A OR B)
C
  DIMENSION A(M,N) , B(M,N) , C(M,N)
  DO 1 I=1,M
  DO 1 J=1,N
  C(I,J) = A(I,J) - B(I,J)
1 CONTINUE
  RETURN
  END

```

```

SUBROUTINE INVERT(B,K,K2,XMIN,FLAG)
C THIS SUBROUTINE SETS UP A UNIT MATRIX ADJACENT TO B(K,K)
C ELEMENTARY ROW OPERATIONS ARE THEN PERFORMED ON THE NEW K X 2K MATRIX
C TO REDUCE B(K,K) TO A UNIT MATRIX. THIS WILL PLACE THE INVERSE OF
C THE MATRIX B(K,K) IN THE RIGHT HALF OF B(K,2K)
C ON EXIT, THE INVERSE OF B REPLACES B
C B IS AN ARRAY OF 2*K**2 LOCATIONS CONTAINING THE MATRIX
C K IS THE DIMENSION OF THE SQUARE MATRIX B
C K2 IS 2*K
C XMIN IS THE SMALLEST ALLOWABLE MAGNITUDE OF THE PIVOT
C FLAG WILL BE RETURNED AS 0. IF THE INVERSION WENT OFF OK
C FLAG WILL BE RETURNED AS 1. IF A PIVOT ELEMENT WAS TOO SMALL
C FLAG SHOULD BE TESTED AFTER EACH CALL TO THIS ROUTINE
C
C   DIMENSION B(K,K2)
C
C   FLAG = 0.
C
C SET UP UNIT MATRIX
C
C   IF(K.GT.1) GO TO 20
C   IF(ABS(B(1,1)).LT.XMIN) GO TO 10
C   B(1,1) = 1./B(1,1)
C   RETURN
20 CONTINUE
C   DO 1 I=1,K
C   DO 1 J=1,K
C   B(I,K+J) = 0.
C   IF(I.EQ.J) B(I,K+J) = 1.
1 CONTINUE
C
C FIND LEADING ELEMENT WITH GREATEST MAGNITUDE
C
C   DO 6 J=1,K
C   M = J
C   N = J+1
C   IF(J.EQ.K) GO TO 21
C   DO 2 L=N,K
C   IF (ABS(B(M,J)).LT.ABS(B(L,J))) M=L
2 CONTINUE
21 CONTINUE
C   IF (ABS(B(M,J)).LT.XMIN) GO TO 10
C   IF(J.EQ.K) GO TO 31
C
C INTERCHANGE JTH AND MTH ROWS
C
C   DO 3 L=J,K2
C   D = B(J,L)
C   B(J,L) = B(M,L)
C   B(M,L) = D
3 CONTINUE
31 CONTINUE
C
C ZERO OUT PIVOTAL JTH COLUMN, SKIPPING PIVOTAL JTH ELEMENT
C
C DIVIDE JTH ROW BY PIVOT
C
C   DO 4 M=N,K2
C   B(J,M) = B(J,M) / B(J,J)
4 CONTINUE
C   DO 6 V=J,K

```

```

C
C M DETERMINES ROW BEING MODIFIED. ONE WHOLE ROW AT A TIME
C
C   IF ( M.EQ.0 ) GO TO 6
C   DO 5 L=N+K2
C
C L DETERMINES ELEMENT IN THE MTH ROW
C
C   R(M,L) = B(M,L) - R(M,J) * R(J,L)
C   5 CONTINUE
C   6 CONTINUE
C
C INVERSE OF B IS NOW IN RIGHT HALF OF B(K,K2)
C NOW MOVE A INVERSE TO WHERE A WAS
C   DO 7 I=1,K
C   DO 7 J=1,K
C   R(I,J) = R(I,J+K)
C   7 CONTINUE
C   RETURN
C 10 FLAG = 10.
C   RETURN
C   END

```

DECK SET UP FOR TEN RUNS OF FUNDAMENTAL  
CASES ONE THROUGH THREE AND SIX

RUN	1					
.5	30.	1.14F+7	.2	.5714285		1
RUN	2					
.5	60.	2.28F+7	.2	.5714285		2
RUN	3					
.5	90.	3.42F+7	.2	.5714285		3
RUN	4					
.5	120.	4.56E+7	.2	.5714285		4
RUN	5					
.5	160.	6.06E+7	.2	.5714285		5
RUN	6					
.6	30.	1.14E+7	.2	.5714285		6
RUN	7					
.6	60.	2.28F+7	.2	.5714285		7
RUN	8					
.6	90.	3.42F+7.	.2	.5714285		8
RUN	9					
.6	120.	4.56E+7	.2	.5714285		9
RUN	10					
.6	160.	6.06F+7	.2	.5714285		10
						(BLANK)
						(BLANK)

DECK SET UP FOR TEN RUNS OF FUNDAMENTAL  
CASES FIVE AND FOUR

RUN	1				1
	4.75E+6	1.4074E+5	.5		
RUN	2				2
	9.50E+6	1.4074E+5	.5		
RUN	3				3
	14.25E+6	1.4074E+5	.5		
RUN	4				4
	19.00E+6	1.4074E+5	.5		
RUN	5				5
	25.33333E+6	1.4074E+5	.5		
RUN	6				6
	4.75E+6	1.4074E+5	.5		
RUN	7				7
	9.50E+6	1.4074E+5	.6		
RUN	8				8
	14.25E+6	1.4074E+5	.6		
RUN	9				9
	19.00E+6	1.4074E+5	.6		
RUN	10				10
	25.33333E+6	1.4074E+5	.6		
					(BLANK)
					(BLANK)

DECK SET UP FOR RUNNING PHASE TWO AND  
 PHASE THREE PROGRAMS. THIS DECK ASSUMES THE  
 SAME TEN RUNS AS THE SAMPLE DATA FOR THE  
 FUNDAMENTAL CASES.

.5	20.	1.14E+7	.2	.5714285	1
.5	60.	2.28E+7	.2	.5714285	2
.5	90.	3.42E+7	.2	.5714285	3
.5	120.	4.56E+7	.2	.5714285	4
.5	150.	6.00E+7	.2	.5714285	5
.6	30.	1.14E+7	.2	.5714285	6
.6	60.	2.28E+7	.2	.5714285	7
.6	90.	3.42E+7	.2	.5714285	8
.6	120.	4.56E+7	.2	.5714285	9
.6	160.	6.00E+7	.2	.5714285	10

20 CARDS

PHASES TWO AND THREE

```

PROGRAM FENG
COMMON /1/
1   S(316,10,6),X(53),Y(53), NDIR(79,4),
2   M,N,CENT(79,2), VF,E(2),G(2),XNU(2) ,
3   SMAC(6,13), SMIC(6,79),EN(79) ,GAMMA ,NS
DATA (PI=3.14159)

C
C THIS PROGRAM HEADS STRESSES FROM TAPE 20 ASSUMED WRITTEN AS FOLLOWS
C
C SIX FUNDAMENTAL CASES, EACH OF WHICH WAS RUN CONSECUTIVELY
C NS TIMES, EACH RUN PRODUCING STRESSES FOR
C 79 TRIANGLES.
C
C NS IS THE FIRST RECORD ON THE TAPE (NS.LE.10)
C FOLLOWING ARE NS RECORDS OF 316 STRESSES FOR EACH OF THE
C FUNDAMENTAL CASES ONE THRU FOUR.
C FOLLOWING ARE NS RECORDS OF 158 STRESSES FOR EACH OF THE
C FUNDAMENTAL CASES FIVE THROUGH SIX
REWIND 20

C
DO 10 I=2,6,2
DO 10 J=1,13
10 SMAC(I,J) = 0.
READ (20) NS
PIQ24=PI/24.
DO 15 I=1,13
ALP = (I-1)*PIQ24
SMAC(5,I) = COS(ALP)*2
SMAC(1,I) = SIN(ALP)*2
15 SMAC(9,I) = SIN(2.*ALP)/2.
READ 1020,IJUMP

C
C IJUMP = 1, FIRST TEN CASES
C IJUMP = 2, SECOND TEN CASES
C
GO TO (18,16),IJUMP
16 REWIND 20
CALL TAPESKIP(20,6,J)
18 CONTINUE
DO 30 K=1,4
DO 20 J=1,NS
20 READ (20) (S(I,J,K),I=1,316)

C
C CROSS OVER THE EOF GAP
C
READ (20) EOF
30 CONTINUE
DO 50 K=5,6
DO 40 J=1,NS
40 READ (20) (S(I,J,K),I=1,158)
READ (20) EOF
50 CONTINUE

C
C NOW HAVE ALL RAW STRESS DATA FOR NS RUNS OF 6 FUNDAMENTAL CASES
CASE = 0

```

```

C
  CALL TOPOL
  LINE = 4
  PRINT 1004,(J,J=1,3),(J,J=1,3)
  DO 130 I=1,57,2
  IF(LINE.LT.54) GO TO 120
  LINE = 4

  PRINT 1004,(J,J=1,3),(J,J=1,3)
120 LINE = LINE + 2
  II=I+1
130 PRINT 1005,I,(NDIR(I,J),J=1,4),II,(NDIR(II,J),J=1,4)
  I=79
  PRINT 1015,I,(NDIR(I,J),J=1,4)

C
  VF1 = 0.
150 READ 1000, VF, GAMMA, E(1),XNU(1),BETA
  KASE = KASE+1
  IF(VF.EQ.0.) GO TO 950
  E(2) = E(1)/GAMMA
  XNU(2) = XNU(1)/BETA
  G(1) = E(1)/(2.*(1.+XNU(1)))
  G(2) = E(2)/(2.*(1.+XNU(2)))
  PRINT 1001, VF,E(1),E(2),G(1),G(2),XNU(1),XNU(2)
  IF(ABS(VF-VF1).LT.1.E-6) GO TO 350
  VF1 = VF
  CALL GEOM
C PRINT OUT NODE INFORMATION
C
  I = 1
190 CONTINUE
  LINE = 4
  PRINT 1007
200 CONTINUE
  IF(I.NE.52) GO TO 210
  IP1 = I+1
  PRINT 1018, I, X(I), Y(I), IP1, X(IP1), Y(IP1)
  GO TO 220
210 IP2 = I+2
  PRINT 1018,((J,X(J),Y(J)),J=I,IP2)
215 I=I+3
  LINE = LINE + 2
  IF(LINE.GT.56) GO TO 190
  GO TO 200
220 CONTINUE

C
250 CONTINUE
  PRINT 1008
  DO 300 I=1,77,2
  II = I+1
  PRINT 1009,I,CENT(I,1),CENT(I,2),II,CENT(II,1),CENT(II,2)
300 CONTINUE
  I = 79
  PRINT 1009,I,CENT(I,1),CENT(I,2)
350 CALL FIXSTR(KASE)
  CALL PHASE2(KASE)
  GO TO 150
950 REWIND 20
  CALL PHASE3
  STOP

```

```

1000 FORMAT(5E10.3)
1001 FORMAT(1M1,15X,2MVF,13X,4HE(1),13X,4HE(2),13X,4MG(1),13X,4MG(2),
1 12X,5HNU(1),12X,5HNU(2)/1X,7(4X,E13.6) ///)
1004 FORMAT(1M1, 8MTRIANGLE,3(2X,5HNODE ,I1),4X,8MMATERIAL,10X,
1 8MTRIANGLE,3(2X,5HNODE ,I1),4X,8MMATERIAL/)
1005 FORMAT(1X,2I5X,I3,5X,I3,5X,I3,5X,I3 ,9X,I3,10X)/)
1007 FORMAT(1M1,3(4HNODE,12X,1HX,12X,1HY,8X)/)
1008 FORMAT(1M1,2(8MTRIANGLE,5X,10HCENTROID X,5X,10HCENTROID Y)/)
1009 FORMAT(1M ,2(5X,I3,5X,E10.3,5X,E10.3))
1015 FORMAT(1X,4(5X,I3),9X,I3/)

1018 FORMAT(1X,3(1X,I3,3X,F10.5,3X,F10.5,8X)/)
1020 FORMAT(I1)
END

```

```

SUBROUTINE MXSOL(B,K,K2,XMIN,FLAG)
C THIS SUBROUTINE SOLVES (K X K) SYSTEM OF EQUATIONS AND PLACES
C THE RESULT IN THE (K+1ST) COLUMN OF B. B IS A (K X K2) MATRIX.
C WHERE K2 = K+1.
C XMIN IS THE SMALLEST ALLOWABLE MAGNITUDE OF THE PIVOT
C FLAG WILL BE RETURNED AS 0. IF THE INVERSION WENT OFF OK
C FLAG WILL BE RETURNED AS 10. IF A PIVOT ELEMENT WAS TOO SMALL
C FLAG SHOULD BE TESTED AFTER EACH CALL TO THIS ROUTINE
C
C   DIMENSION B(K,K2)
C
C   FLAG = 0.
C
C   FIND LEADING ELEMENT WITH GREATEST MAGNITUDE
C
C     DO 6 J=1,K
C     M = J
C     N = J+1
C     IF(N.GT.K) GO TO 21
C     DO 2 L=N,K
C     IF (ABS(B(M,J)).LT.ABS(B(L,J))) M=L
C 2 CONTINUE
C 21 CONTINUE
C     IF (ABS(B(M,J)).LT.XMIN) GO TO 10
C
C   INTERCHANGE JTH AND MTH ROWS
C
C     DO 3 L=J,K2
C     D = B(J,L)
C     B(J,L) = B(M,L)
C     B(M,L) = D
C 3 CONTINUE
C
C   ZERO OUT PIVOTAL JTH COLUMN. SKIPPING PIVOTAL JTH ELEMENT
C
C   DIVIDE JTH ROW BY PIVOT
C
C     DO 4 M=N,K2
C     B(J,M) = B(J,M) / B(J,J)
C 4 CONTINUE
C     DO 6 M=1,K
C
C   M DETERMINES ROW BEING MODIFIED. ONE WHOLE ROW AT A TIME
C
C     IF ( M.EQ.J ) GO TO 6
C     DO 5 L=N,K2
C
C   L DETERMINES ELEMENT IN THE MTH ROW
C
C     B(M,L) = B(M,L) - B(M,J) * B(J,L)
C 5 CONTINUE
C 6 CONTINUE
C
C   RETURN
C 10 FLAG = 10.
C   RETURN
C   END

```

SUBROUTINE TOPOL

C  
 C THIS SUBROUTINE SETS UP THE TOPOLOGICAL RELATIONSHIPS FOR PROGRAM  
 C FENG. THESE REMAIN INVARIANT WHEN THE GEOMETRY CHANGES WITH VF.  
 C  
 C  
 C

COMMON /1/

```

1  S(316,10,6),X(53),Y(53), NDIR(79,4),
2  M,N,CENT(79,2), VF,E(2),G(2),XNU(2) ,
3  SMAC(6,13), SMIC(6,79),EN(79) ,GAMMA ,NS
  DIMENSION JNDIR(79) , KNDIR(79) , LNDIR(79)
  DATA (JNDIR =
1  1,1,1,2,2,3,3,3,4,4,4,5,6,7,7,8,8,8,9,10,10,11,11,12,13,13,
2  14,15,15,16,16,17,17,18,18,19,20,23,24,24,25,25,26,28,29,
3  29,30,30,35,21,21,21,21,22,22,23,23,27,27,27,28,28,36,46,37,
4  47,38,48,39,39,40,41,41 41,42,42,43,43)
  DATA (KNDIR =
5  2,3,4,6,7,7,8,9,9,10,11,11,13,6,14,14,15,16,16,16,17,10,
6  18,18,20,21,21,21,22,15,23,23,24,17,25,25,35,27,27,28,28,29,
7  30,30,32,32,33,33,34,36,35,37,38,39,39,40,40,41,41,42,43,43,44,
8  45,37,46,38,47,39,48,49,49,40,50,51,51,52,52,53)
  DATA (LNDIR =
9  3,4,5,7,3,8,9,4,10,11,5,12,14,14,8,15,16,9,10,17,18,18,12,
1  19,21,14,15,22,23,23,17,24,25,25,19,26,21,24,28,25,29,30,26,
2  31,29,33,30,34,31,37,37,38,39,22,40,23,41,27,42,43,28,44,32,
3  46,36,47,37,48,38,49,40,50,50,51,42,52,43,53,44 )
  DO 10 I=1,36
10  NDIR(I,4) = 1
  DO 20 I=37,79
  NDIR(I,4) = 2
20  CONTINUE
  DO 100 I = 1, 79
  NDIR(I,1) = JNDIR(I)
  NDIR(I,2) = KNDIR(I)
  NDIR(I,3) = LNDIR(I)
100 CONTINUE
  RETURN
  END
  
```

SUBROUTINE GEOM  
 C THIS SUBROUTINE IS CALLED TO GENERATE NEW COORDINATES AND CENTROIDS  
 C WHENEVER A NEW VALUE OF VF IS CONSIDERED IN PROGRAM FENG.  
 C

```

COMMON /1/
1  S(316,10,6),X(53),Y(53), NDIR(79,4),
2  M,N,CENT(79,2), VF,E(2),G(2),XNU(2) ,
3  SMAC(6,13), SMIC(6,79),EN(79) ,GAMMA ,NS
COMMON /2/ X1(6),Y1(6)
DIMENSION NOTRI(6,2)
DATA (NOTRI = 25, 28, 29, 32, 33, 36,
1  37, 54, 56, 38, 40, 43)
DATA (RAD = 57.29578)
S3 = SQRT(3.)
S302 = S3/2.
PI = 3.1415927
R = SQRT(2.*S3*VF/PI)
X(1) = S302
Y(1) = .5
DO 210 I=1,4
X(I+1) = S302 - R/4.*COS(PI*(I-1)/6.)
Y(I+1) = .5 - R/4. *SIN(PI*(I-1)/6.)
210 CONTINUE
C
DO 220 I=1,7
X(I+5) = S302 - R/2.* COS(PI*(I-1)/12.)
Y(I+5) = .5 - R/2.* SIN(PI*(I-1)/12.)
C
X(I+12) = S302 - 3.*R/4.*COS(PI*(I-1)/12.)
Y(I+12) = .5 - 3.*R/4.*SIN(PI*(I-1)/12.)
C
X(I+19) = S302 - R * COS(PI*(I-1)/12.)
Y(I+19) = .5 - R * SIN(PI*(I-1)/12.)
220 CONTINUE
X(34) = S302
Y(34) = -.5
X(31) = S302
Y(31) = (Y(26)+Y(34)) / 2.
X(45) = -.5 * TAN(PI/6.)
Y(45) = .5
DX = (1.-R)/(12.*COS(PI/6.))
X(36) = X(45) + DX
Y(36) = .5
X(35) = (X(20)+X(36))/2.
Y(35) = .5
C
DO 230 I = 1,8
X(I+45) = (4 - I) * X(45) / 4.
Y(I+45) = (4 - I) * Y(45) / 4.
230 CONTINUE
C
DELX = X(46) - X(45)
DELY = Y(46) - Y(45)
DO 240 I = 1,8
X(I+36) = X(I+35) + DELX
Y(I+36) = Y(I+35) + DELY
240 CONTINUE
X(32) = X(44) + (X(34) - X(44))/3.
Y(32) = -.5
X(33) = 2.*X(32) - X(44)
Y(33) = -.5
X(27) = X(23) + (X(32) - X(23))/3.

```

```

Y(27) = Y(23) + (Y(32)-Y(23))/3.
X(28) = 2.*X(27)-X(23)
Y(28) = 2.*Y(27)-Y(23)
X(29) = X(28) + (X(31)-X(28))/3.
Y(29) = Y(28) + (Y(31)-Y(28))/3.
X(30) = 2.*X(29)-X(28)
Y(30) = 2.*Y(29)-Y(28)

```

C  
C

```

DO 200 I=1,79
  NDIR1 = NDIR(I,1)
  NDIR2 = NDIR(I,2)
  NDIR3 = NDIR(I,3)
  CENT(I,1) = (X(NDIR1)+X(NDIR2)+X(NDIR3))/3.
200 CENT(I,2) = (Y(NDIR1)+Y(NDIR2)+Y(NDIR3))/3.
  DO 300 I=1,6
    X1(I) = (X(I+19)+X(I+20))/2.
300 Y1(I) = (Y(I+19)+Y(I+20))/2.
  RETURN
  END

```

```

SUBROUTINE PHASE2(J)
C
C THIS SUBROUTINE CALCULATES AND PRINTS THE PARAMETERS REQUIRED
C IN PHASE II. ONE CALL TO THIS SUBROUTINE PRODUCES 12 BLOCKS OF 79
C SETS OF DATA, ONE FOR EVERY TRIANGLE FOR EACH OF 12 VALUES OF
C ALPHA.
C
C K COUNTS ALPHA
C I COUNTS TRIANGLES
C M COUNTS FUNDAMENTAL CASES
C L COUNTS POSITIONS WITHIN MICROSTRESS VECTOR
C J RUN NUMBER
C
COMMON /1/
1 S(316,10,6),X(53),Y(53), NDIR(79,4),
2 M,N,CENT(79,2), VF,E(2),G(2),XNU(2) ,
3 SMAC(6,13), SMIC(6,79),EN(79) ,GAMMA ,NS
DATA (SIGFT2=7000.) , (SIGFC2=17000.)
IF(ABS(GAMMA- 1.)>.LE.1.) GO TO 100
IF(ABS(GAMMA- 30.)>.LE.1.) GO TO 120
IF(ABS(GAMMA- 60.)>.LE.1.) GO TO 140
IF(ABS(GAMMA- 90.)>.LE.1.) GO TO 160
IF(ABS(GAMMA-120.)>.LE.1.) GO TO 180
IF(ABS(GAMMA-160.)>.LE.1.) GO TO 200
100 SIGFT1 = 7000.
SIGFC1 = 17000.
GO TO 250
120 SIGFT1 = 160000.
SIGFC1 = 160000.
GO TO 250
140 SIGFT1 = 200000.
SIGFC1 = 200000.
GO TO 250
160 SIGFT1 = 250000.
SIGFC1 = 250000.
GO TO 250
180 SIGFT1 = 300000.
SIGFC1 = 300000.
GO TO 250
200 SIGFT1 = 350000.
SIGFC1 = 350000.
250 CONTINUE
DO 500 K=1,13
ALP = 7.5*(K-1)
DO 350 I=1,79
I4 = 4*(I-1)
I2=2*(I-1)
DO 260 L=1,4
SMIC(L,I) = 0.
DO 260 M=1,4
260 SMIC(L,I) = SMIC(L,I)+S(I4+L,J,M)*SMAC(M,K)
DO 270 L=1,2
SMIC(L+4,I) = 0.
DO 270 M=5,6
270 SMIC(L+4,I) = SMIC(L+4,I)+S(I2+L,J,M)*SMAC(M,K)
IF(I>.LE.76) GO TO 300
XYZ = SMIC(1,I)+SMIC(2,I)+SMIC(3,I)
IF(XYZ>.GE.7.) GO TO 280
SIGF = SIGFC2
GO TO 290
280 SIGF = SIGFT2
290 GO TO 320

```

```

300 SXYZ = SMIC(1,1)+SMIC(2,1)+SMIC(3,1)
    IF(SXYZ.GE.0.) GO TO 310
    SIGF = SIGFC1
    GO TO 320
310 SIGF = SIGFT1
320 EN(1)=SIGF*SQRT(2./((SMIC(1,1)-SMIC(2,1))**2
1      +(SMIC(2,1)-SMIC(3,1))**2+(SMIC(3,1)-SMIC(1,1))**2
2      +6.*(SMIC(4,1)**2+SMIC(5,1)**2+SMIC(6,1)**2)))
350 CONTINUE
    PRINT 1002,ALP
    PRINT 1000
    DO 400 N=1,79
    IF(N.NE.54) GO TO 400
    PRINT 1003
    PRINT 1000
400 PRINT 1001, N, (SMIC(JJ,N),JJ=1,6),EN(N)
    CALL INTERF(K,J)
500 CONTINUE
1000 FORMAT(9H TRIANGLE,8X,7HSIGMA X,8X,7HSIGMA Y,8X,7HSIGMA Z,
1      9X,6HTAU XY,9X,6HTAU XZ,9X,6HTAU YZ,14X,1HM/)
1001 FORMAT(6X,13,7(5XE10,3))
1002 FORMAT(8H1THETA =,F10,3/)
1003 FORMAT(1H1)
    RETURN
    END

```

SUBROUTINE PHASE3

C THIS SUBROUTINE READS AND PRINTS ALL PHASE III DATA FROM TAPE 96  
C

```
COMMON /1/
1   S(316,10,6),X(53),Y(53), NDIR(79,4),
2   M,N,CENT(79,2), VF,E(2),G(2),XNU(2) ,
3   SMAC(6,13), SMIC(6,79),EN(79) ,GAMMA ,NS
DIMENSION ENS(4,5,13)
DIMENSION SM(6,4)
REWIND 96
IVF = 0
VF1 = 0.
GAMMA1 = 1000.
PRINT 1000
NPAGE =(NS*13)/2
```

C  
C NS ASSUMED EVEN  
C

```
DO 300 IP=1,NPAGE
PRINT 1001
DO 300 J=1,2
READ (96) VF,EI,EII,GI,GII,XNUI,XNUII,N,ENI,(SM(K,1),K=1,6),
1   M,ENII,(SM(K,2),K=1,6), X1,Y1,EN1,SNAV,
2   X2,Y2,EN2,TAUTAV,I
GAMMA = EI/EII
IF(I.NE.1) GO TO 100
IF(GAMMA.LE.GAMMA1+.5) GO TO 60
IGAM = IGAM+1
GO TO 100
60 IVF = IVF+1
IGAM = 2
GAMMA1 = GAMMA
100 ENST = ENI
IF(ENST.GT.ENII) ENST = ENII
IF(EN1.LE.-1.E+8) GO TO 110
IF(ENST.GT.EN1) ENST = EN1
110 IF(ENST.GT.EN2) ENST = EN2
ENS(IVF,IGAM,I) = ENST
ALP = 7.5*(I-1)
PRINT 1002
PRINT 1003, VF,EI,EII,GI,GII,XNUI,XNUII,ALP
PRINT 1004
PRINT 1005,(SMAC(K,1),K=1,6)
PRINT 1006
PRINT 1007, N,ENI,(SM(K,1),K=1,6)
PRINT 1008
PRINT 1007, M,ENII,(SM(K,2),K=1,6)
PRINT 1009
IF(EN1.LT.-1.E+8) GO TO 180
150 CONTINUE
PRINT 1010
PRINT 1017, X1,Y1,EN1,SNAV
180 CONTINUE
PRINT 1011
PRINT 1017, X2,Y2,EN2,TAUTAV
PRINT 1013
300 CONTINUE
DO 400 I=1,2
DO 400 J=2,IGAM
DO 400 K=1,13
```

C

```

C   INDEX 1 FOR GAMMA IS SAVED FOR HOMOGENEOUS CASE, RUN SEPARATELY
C
      PUNCH 2000,I,J,K,ENS(I,J,K)
400  CONTINUE
1000 FORMAT(1H1,50X,9HPPHASE III)
1001 FORMAT(1H1)
1002 FORMAT(13X,2HVF,12X,3HE I,11X,4HE II,12X,3HG I,11X,4HG II,
1     11X,4HNU I,10X,5HNU II,10X,5HMET A)
1003 FORMAT(8(5X,E10.3)/)
1004 FORMAT(1X,13X,2HMX,13X,2HNY,13X,2HNZ,12X,3HTXY,12X,3HTXZ,
1     12X,3HTXZ)
1005 FORMAT(1X,6(5X,E10.3)/)
1006 FORMAT(1X,8HTRIANGLE,4X,12HSMALLEST N1,8X,7HSIGMA X,8X,7HSIGMA Y,
1     8X,7HSIGMA Z,9X,6HTAU XY,9X,6HTAU XZ,9X,6HTAU YZ)
1007 FORMAT(1X,6X,12,6X,E10.3,6(5X,E10.3)/)
1008 FORMAT(1X,8HTRIANGLE,4X,12HSMALLEST NII,8X,7HSIGMA X,8X,7HSIGMA Y,
1     8X,7HSIGMA Z,9X,6HTAU XY,9X,6HTAU XZ,9X,6HTAU YZ)
1009 FORMAT(50X,9HINTERFACE)
1010 FORMAT(8X,1HX,11X,1HY,4X,12H SMALLEST N1,3X,15HAVERAGE SIGMA N)
1011 FORMAT(8X,1HX,11X,1HY,4X,12H SMALLEST N2,3X,15H AVERAGE TAU T)
1013 FORMAT(//)
1017 FORMAT(1X,F8.3,4X,F8.3,2E16.3/)
1020 FORMAT(8X,1HX,11X,1HY,1X,15HAVERAGE SIGMA N)
2000 FORMAT(3I3,F15.8)
      STOP
      END

```

```

SUBROUTINE FIXSTR(KASE)
C
C THIS SUBROUTINE MODIFIES THE STRESSES S(I,J,K), I=1,316, K=1,4 .
C RUN= KASE , PRODUCED BY THE NS RUNS OF FUNDAMENTAL CASES ONE
C THRU FOUR.
C
COMMON /1/
1 S(316,1,6),X(53),Y(53), NDIR(79,4),
2 M,N,CENT(79,2), VF,E(2),G(2),XNU(2) ,
3 SMAC(6,13), SMIC(6,79),EN(79),GAMMA,NS
DIMENSION A(3,4,3),ARC(3,3),AREA(79), Z(3,4), ARTOP(11),AREND(6)
DIMENSION PTEND(7), PTTOP(13),TRITOP(11),TRIEND(6), TEMP(316,4)
TYPE INTEGER PTEND,TRIEND,PTTOP,TRITOP
DATA (TRITOP=1,4,13,25,37,50,64,79,63,46,48)
DATA (TRIEND=3,12,24,36,44,49)
DATA (PTEND=1,5,12,19,26,31,34)
DATA (PTTOP=1,2,6,13,20,35,36,45,53,44,32,33,34)
DO 50 I=1,7
J1 = PTTOP(I)
J2 = PTTOP(I+1)
50 ARTOP(I) = X(J1)-X(J2)
DO 100 J=9,12
J1=PTTOP(J)
J2=PTTOP(J+1)
100 ARTOP(J-1) = X(J2)-X(J1)
DO 110 I= 1,6
J1 = PTEND(I)
J2 = PTEND(I+1)
110 AREND(I)=Y(J1)-Y(J2)
C
DO 120 I= 1,79
J1 = NDIR(I,1)
J2 = NDIR(I,2)
J3 = NDIR(I,3)
120 AREA(I) = X(J1)*(Y(J2)-Y(J3))+X(J2)*(Y(J3)-Y(J1))
+X(J3)*(Y(J1)-Y(J2))
DO 130 I=1,3
DO 130 J=1,4
130 Z(I,J) = 0.
DO 150 I= 1,6
I1 = TRIEND(I)
I2 = (I1-1)*4 +1
DO 150 J= 1,3
150 Z(I,J)= Z(I,J)+AREND(I1)*S(I2,KASE,J)
C
DO 160 I=1,11
I1 = TRITOP(I)
I2 = (I1-1)*4 +2
DO 160 J=1,3
160 Z(2,J) = Z(2,J)+ARTOP(I1)*S(I2,KASE,J)
C
DO 180 I = 1,79
I2 = (I-1)*4+3
DO 180 J = 1,3
180 Z(3,J) = Z(3,J)+AREA(I)*S(I2,KASE,J)
C
C NOTE THAT AREA ALREADY CONTAINS A FACTOR OF TWO FROM ITS DERIVATION
C
DO 300 I=1,3
DO 200 J=1,3
DO 200 K=1,4
200 A(J,K,I) = Z(J,K)

```

```

      GO TO(210,220,230),I
210 A(I,4,I) = 1.
      GO TO 240
220 A(2,4,2) = SORT(3.)
      GO TO 240
230 A(3,4,3) = SORT(3.)
240 CALL MRSOL(A(1,1,1),3,4,1.E-7,FLAG)
      IF(FLAG.NE.0.) GO TO 600
      DO 250 J=1,3
250 ABC(J,I) = A(J,4,I)
C
C ABC(I,1) CONTAINS A(I) , I=1,2,3
C ABC(I,2) CONTAINS B(I) , I=1,2,3
C ABC(I,3) CONTAINS C(I) , I=1,2,3
C
      DO 280 J=1,316
280 TEMP(I,J) = ABC(1,I)*S(J,KASE,1)+ABC(2,I)*S(J,KASE,2)
      I
      + ABC(3,I)*S(J,KASE,3)
300 CONTINUE
      DO 400 I=1,316
      DO 400 J=1,3
400 S(I,KASE,J) = TEMP(I,J)
      RETURN
600 PRINT 1000, KASE, I
      STOP
1000 FORMAT(14H1BLEW UP, RUN,13,6X,16HFUNDAMENTAL CASE,13)
      END

```

```

SUBROUTINE INTERF(I,L)
C
C I -- INDEX OF ALPHA, I=1,13
C L -- R-IN NUMBER (DETERMINED BY VF,EI COMBINATION)
C
COMMON /1/
1 S(316.1),X(53),Y(53),NDIR(79,4),
2 M,N,CFNT(79,2),VF,E(2),G(2),XNU(2),
3 SMAC(6,13),SMIC(6,79),EN(79),GAMMA,NS
COMMON /2/ X1(6),Y1(6)
DIMENSION SNI(6),SNI1(6),TAUTI(6),TAUTI1(6),SNAV(6),TAUTAV(6)
DIMENSION EN1(6),EN2(6),K1(6),K2(6)
DIMENSION NOTRI(6,2)
C
C NOTRI(M,N) -- NTH INTERFACE TRIANGLE, MATERIAL M
C
DATA (SNF=10000.), (TAUTF = 10000.)
DATA (NOTRI = 25, 28, 29, 32, 33, 36,
1 37, 54, 56, 38, 40, 43)
DATA (PI=3.14159 )
C
DO 100 K=1,6
K1(K) = NOTRI(K,1)
K2(K) = NOTRI(K,2)
L1 = K1(K)
L2 = K2(K)
PHI = PI*(K-.5)/12.
CPHI = COS(PHI)
SPHI = SIN(PHI)
S2PHI = SIN(2.*PHI)
C2PHI = COS(2.*PHI)
SNI(K) = CPHI**2*SMIC(1,L1) + SPHI**2*SMIC(2,L1)
1 + 2.*SMIC(4,L1)*SPHI*CPHI
SNI1(K) = CPHI**2*SMIC(1,L2) + SPHI**2*SMIC(2,L2)
1 + 2.*SMIC(4,L2)*SPHI*CPHI
TAUTI(K) = SQRT(((SMIC(1,L1)-SMIC(2,L1))*S2PHI/2.-SMIC(4,L1)*C2PHI)
1 **2 + (CPHI*SMIC(5,L1)+SPHI*SMIC(6,L1))**2)
TAUTI1(K) = SQRT(((SMIC(1,L2)-SMIC(2,L2))*S2PHI/2.-SMIC(4,L2)*C2PHI)
1 **2 + (CPHI*SMIC(5,L2)+SPHI*SMIC(6,L2))**2)
SNAV(K) = (SNI(K)+SNI1(K))/2.
TAUTAV(K) = (TAUTI(K)+TAUTI1(K))/2.
EN1(K) = SNF / SNAV(K)
EN2(K) = TAUTF / TAUTAV(K)
100 CONTINUE
PRINT 1000
PRINT 1001
DO 150 K=1,6
150 PRINT 1002,K1(K),SNI(K),TAUTI(K),K2(K),SNI1(K),TAUTI1(K)
PRINT 1003
IF(SNI(1))210,190,190
190 PRINT 1004
DO 200 K=1,6
200 PRINT 1005,K,SNAV(K),TAUTAV(K),EN1(K),EN2(K)
GO TO 230
210 PRINT 1014
DO 220 K=1,6
220 PRINT 1005,K,SNAV(K),TAUTAV(K),EN2(K)
230 CONTINUE
C
C CALCULATE AND WRITE ON TAPE THE NECESSARY PHASE III VALUES
C
N = 1

```

```

DO 250 J=2,36
IF(EN(J).GE.EN(N)) GO TO 250
N=J
250 CONTINUE
M = 37
DO 260 J=38,79
IF(EN(J).GE.EN(M)) GO TO 260
M=J
260 CONTINUE
L1 = 10
ENWUN = 10.E+10
DO 265 J=1,6
IF(EN1(J).GT.ENWUN.OR.EN1(J).LT.0.) GO TO 265
L1 = J
ENWUN = EN1(J)
265 CONTINUE
IF(L1.LT.7) GO TO 270
L1 = 1
EN1(L1) = -10.E+8
270 CONTINUE
L2 = 1
DO 280 J=2,6
IF(EN2(J).GE.EN2(L2)) GO TO 280
L2 = J
280 CONTINUE
WRITE (96) VF,E(1),E(2),G(1),G(2),XNU(1),XNU(2),
1 N, EN(N),(SMIC(J,N),J=1,6), M, EN(M),(SMIC(J,M),J=1,6),
2 X1(L1),Y1(L1),EN1(L1),SNAV(L1),
3 X1(L2),Y1(L2),EN2(L2),TAUTAV(L2),I
1000 FORMAT(///50X,9MINTERFACE)
1001 FORMAT(27X,5MFIBER,54X,5MRESIN/1X,2(8MTRIANGLE,8X,7MSIGMA N,
1 10X,5MTAU T,20X)/)
1002 FORMAT(1X,2(5X,13.5X,E10.3,5X,E10.3,20X) )
1003 FORMAT(/5CX,7MAVERAGE/)
1004 FORMAT(1X,8MPOSITION,13X,7MSIGMA N,15X,5MTAU T,18X,2MM1,18X,2MM2/)
1005 FORMAT(1X,7X,11.4(10X,E10.3))
1014 FORMAT(1X,8MPOSITION,13X,7MSIGMA N,15X,5MTAU T,18X,2MM2/)
RETURN
END

```

## ENGINEERING CONSTANTS

```

100 DIMENSION E33(13<20),E11(13,20),G31(13,20),ANU31(13,20),
110 & ANU13(13,20),EP(13,20),GP(13,20),ANUP(13,20),ETA(13,20),
120 & CC(13,20),SS1(13,20),SC(13,20),SS(13,20),CC(13,20),OS(13,20)
130 DATA E33/13*6.110E6,13*1.176E7,13*1.739E7,13*2.303E7,13*3.055E7,
140 & 13*7.229E6,13*1.401E7,13*2.072E7,13*2.754E7,13*3.656E7,
150 & 13*6.366E6,13*1.629E7,13*2.419E7,13*3.208E7,13*4.260E7,
160 & 13*9.548E6,13*1.635E7,13*2.767E7,13*3.670E7,13*4.874E7/
170 & E11/13*1.362E6,13*1.945E6,13*1.971E6,13*1.985E6,13*1.995E6,
180 & 13*2.307E6,13*2.439E6,13*2.487E6,13*2.511E6,13*2.530E6,
190 & 13*3.11E6,13*3.270E6,13*3.367E6,13*3.418E6,13*3.458E6,
200 & 13*4.334E6,13*4.925E6,13*5.237E6,13*5.380E6,13*5.492E6/
210 & G31/13*5.250E5,13*5.449E5,13*5.519E5,13*5.555E5,13*5.582E5,
220 & 13*6.749E5,13*7.103E5,13*7.230E5,13*7.295E5,13*7.345E5,
230 & 13*9.167E5,13*9.868E5,13*1.013E6,13*1.026E6,13*1.037E6,
240 & 13*1.392E6,13*1.571E6,13*1.641E6,13*1.679E6,13*1.709E6/
250 & ANU31/65*.293,65*.272,65*.256,65*.232/
300 DO 10 I=1,20
310 DO 20 J=1,13
320 BJ=J-1
330 THETA=BJ*3.141593/24.
340 CC(J,I)=COS(THETA)
350 SS1(J,I)=SIN(THETA)
360 SC(J,I)=(COS(THETA))**2
370 SS(J,I)=(SIN(THETA))**2
380 CC(J,I)=(COS(THETA))**4
390 OS(J,I)=(SIN(THETA))**4
400 ANU13(J,I)=ANU31(J,I)*E11(J,I)/E33(J,I)
500 EP(J,I)=1./CC(J,I)/E33(J,I)+OS(J,I)/E11(J,I)+(1./G31(J,I)
510 & -ANU31(J,I)/E33(J,I)-ANU13(J,I)/E11(J,I))*SC(J,I)*SS(J,I)
600 GP(J,I)=1./(1./G31(J,I)+4.*SC(J,I)*SS(J,I))*((1.+ANU31(J,I))/
610 & E33(J,I)+(1.+ANU13(J,I))/E11(J,I)-1./G31(J,I))
700 ANUP(J,I)=EP(J,I)*(ANU31(J,I)/E33(J,I)-SC(J,I)*SS(J,I)*
710 & ((1.+ANU31(J,I))/E33(J,I)+(1.+ANU13(J,I))/E11(J,I)-1./G31(J,I)))
800 ETA(J,I)=EP(J,I)*CC(J,I)*SS1(J,I)*(2.*SC(J,I)/E33(J,I)
810 & -2.*SS(J,I)/E11(J,I)+(SC(J,I)-SS(J,I))*(ANU31(J,I)/E33(J,I)
820 & /ANU13(J,I)/E11(J,I)-1./G31(J,I)))
900 20 CONTINUE
910 PRINT,(EP(J,I)/3.2E5,GP(J,I)/1.407E5,ANUP(J,I),ETA(J,I),J=1,13)
920 10 CONTINUE
1000 30 STOP;END

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13. ABSTRACT The critical loadings at failure of a unidirectional fibrous composite under any oblique loading are the most important results obtained in this work. For this purpose the von Mises energy criterion was applied to the components of the composite. Debonding failures at the interfaces were also considered.  Other results of this research are the composite elastic engineering constants in any angular direction as a function of fiber density and component properties.  Diagrams are presented which exhibit the critical loading and the elastic coefficients of a series of composites as a function of the loading direction, component material constants, and geometry.  The fundamentals to this work are based on the micromechanical stress fields in the fibers and in the matrix.		

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