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RPL-TDR-64-25

AEROSPACE FLUID COMPONENT DESIGNERS' HANDBOOK

VOLUME II

Revision D

TECHNICAL DOCUMENTARY REPORT NO. RPL-TDR-64-25

FEBRUARY 1970

AIR FORCE ROCKET PROPULSION LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
Edwards, California 93523

PROJECT NO. 3058

Prepared Under

Contracts AF04(611)-8385,
AF04(611)-11316,
F04611-67-C-0062, and
F04611-68-C-0065
by

TRW
SYSTEMS GROUP

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AEROSPACE FLUID COMPONENT DESIGNERS' HANDBOOK

VOLUME II
Revision D

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Superseded AD-865277

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AEROSPACE FLUID COMPONENT
DESIGNERS' HANDBOOK

First Issue: MAY 1964
Revised: OCTOBER 1965
Revised: MARCH 1967
Revised: NOVEMBER 1968
Revised: FEBRUARY 1970

Qualified requesters may obtain copies of this report from the
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7.1 INTRODUCTION

Complex fluid components, such as hydraulic servo-actuators and propellant tank pressure regulators, are small-scale control systems. Thus, they are dynamic systems in which a fast response to a change in input or demand is required. In addition, the units must be inherently stable in operation. The dynamic problems which are experienced in a complex fluid component can be divided into two groups:

1) Dynamic performance—the problems involved in obtaining the required response rate and stability in the unit.

2) The effects of vibration and shock on the dynamic performance and structural integrity of the unit.

The following sub-sections of this handbook deal with the dynamic problems in fluid components. Sub-Section 7.2 gives a general outline of control system theory; Sub-Section 7.3 gives a similar outline of vibration theory; and Sub-Section 7.4; covers dynamic performance analysis, which illustrates how the theory of Sub-Section 7.2 is applied to the design and performance analysis of fluid components.

7.2 CONTROL SYSTEM THEORY

Sub-Section 7.2 consists of seven sub-topics reprinted from *Machine Design* (copyrighted by Penton Publishing Company) with permission of the publisher. These sub-topics originally appeared in *Machine Design* as seven articles (References 1-272, 1-273, 1-274, 1-275, 1-276, 1-277, 1-278) in an eighteen-part series written by J. M. Nightingale. The series was subsequently reprinted by the Penton Publishing Company in three volumes, entitled "Hydraulic Servo Fundamentals" (References 1-128, 1-129, 1-131). In adapting the material for use in this handbook, the first example in Sub-Topic 7.2.1 was modified from that appearing in the original reference (Reference 1-272) to conform with the handbook content, and an example which appeared in the original *Machine Design* series at the end of article number five (Reference 1-276) was deleted in the handbook adaptation. Any other changes in the original articles consist of minor additions or deletions in order to conform to the typographical style of the handbook and do not affect content.

7.2.1 An Introduction to Automatic Control Systems

Accuracy, sensitivity, speed, and muscle needed for control of many modern machines are often beyond human capabilities. Environmental conditions and fatigue are but two of the factors which make human control unsatisfactory in many instances. Not only do these facts establish a need for automatic control systems, they lead to a broad definition. Automatic control is the regulation of some variable—called the controlled variable—in accordance with a sequence of desired conditions without human aid. Since control problems occur in many fields, the controlled variable may be of any physical nature. Displacement, speed, pressure, temperature, or voltage are but a few possibilities.

Closed-Loop Concept. Control mechanisms, too, may be susceptible to certain almost human weaknesses, these must, of course, be eliminated from a successful control system by proper design. To illustrate some of the weaknesses and how they may be overcome, the problem of regulating the rotational speed of a turbine-driven pump will be considered. Turbopumps are used in many liquid rocket engines to deliver propellants to the engine thrust chamber. Figure 1(a) illustrates a layout of a turbopump for a monopropellant engine. The centrifugal pump draws the liquid monopropellant from a tank at low pressure and pumps it to the thrust chamber at high pressure. The pump is driven through a gear box by a turbine. The operating fluid in the turbine is a gas which is supplied at high pressure by a gas generator. The flow rate from the gas generator is controlled by a valve. The gas flow rate sets the turbopump speed, which sets the flow rate of the propellant being pumped.

In constant thrust rocket engines, it is necessary to maintain the propellant flow rate, and thus the pump speed, at a constant level. The simplest method of achieving this is to use a valve with a fixed setting in the location shown in Figure 1(a). The valve setting is calibrated to provide a gas flow rate which gives the required turbopump speed. This system will give a fairly constant speed only if propellant temperature and pressure, gas generator efficiency, and other possible variables remain within tolerable limits, since the valve is pre-set for only one set of operating conditions.

Greater accuracy in speed regulation can be obtained by adding a governor or speed regulator to the system, as shown in Figure 1(b). In this case, the valve referred to above becomes a controllable throttle valve. The regulator senses the pump speed, compares it with the required value, and makes a correction to the throttle valve setting if necessary. If the speed sensed is too low, for example, the regulator opens the valve to increase speed and vice versa. With this

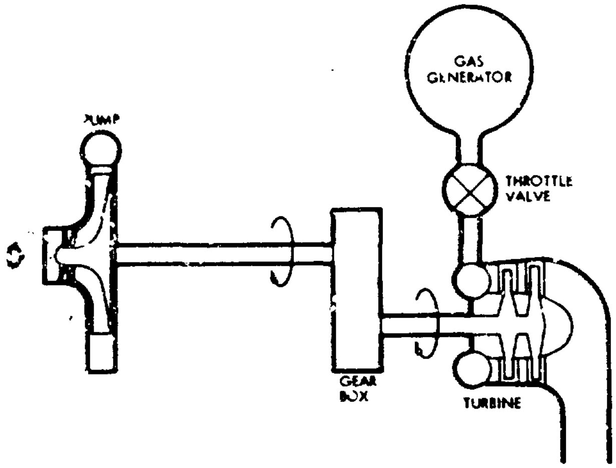


Figure 1a. Turbopump with Passive Speed Regulation

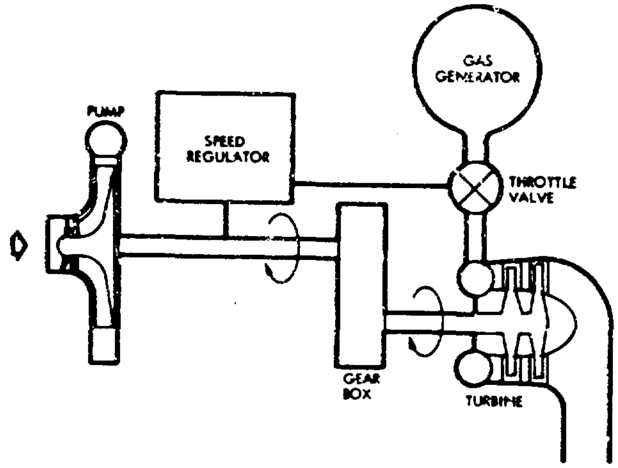


Figure 1b. Turbopump with Closed-Loop Regulation

method, changes in speed due to changes in gas and propellant conditions are readily corrected for, so that a virtually constant speed is maintained.

Figure 1(b) is an example of a closed-loop control system, which may be formally defined as a system in which the true state of the controlled variable (the output) is continuously compared with the desired state (the input), and a signal depending on the difference between the two (the error) operates a controlling element which then acts on the rest of the system to reduce the error to zero. Almost all practical, sensitive, control systems are based on this closed-loop principle.

Mention should be made of the difference between an automatic control system and a manual system. Figure 1(b), in which speed regulation is obtained by having an automatic governor or regulator in the control loop, is an automatic system. A human operator, however, could theoretically take the place of the regulator. He would read the pump speed on a tachometer and would then adjust the gas valve setting by hand to obtain the correct speed. Such a system would be a manual control system. This system, like the automatic, is a closed-loop system. The manual system, however, would have limitations due to the response and fatigue characteristics of the human operator.

Automatic control systems of the closed-loop type are usually classified as either servo systems or regulators. The difference between the two is primarily a matter of application. In servo systems, input varies continuously and often arbitrarily, and the purpose of the system is to follow the input closely, as illustrated in Figure 2. In a regulator, the input is constant for relatively long periods of time, and the purpose of the system is to maintain constant output despite fluctuations in power supply or external load.

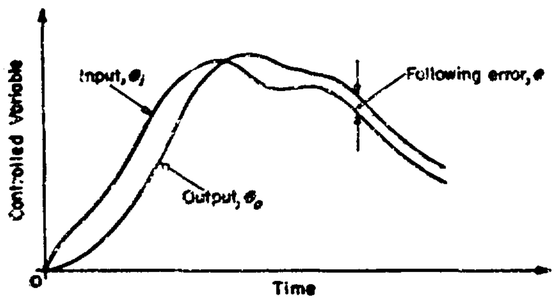


Figure 2. One of the primary purposes of a servo system is to vary output to closely follow a varying input to the system.

Servo Systems. All closed-loop control systems with power amplification around the loop are usually referred to as servo systems or servos. The term servomechanism is reserved for those servo systems having a mechanical output. Further subclassification of servomechanisms is based upon the classification of the output means. For example, a hydraulic servomechanism uses a rotary hydraulic motor or a hydraulic cylinder as the output device. However, certain electrical or electronic devices might be used in a hydraulic servomechanism.

Some examples of servomechanism applications are power steering of vehicles, auto-pilots for aircraft and missiles (including power controls for operating surfaces), machine tracing tools, automatic tracking radar, and remote gun control systems.

A hydraulic servomechanism for controlling angular displacement of a shaft (Figure 3) illustrates the components of a typical servomechanism. Additionally, the circuit illustrates two important functions of a servomechanism: (1) remote control (usually of position), and (2) power amplification. Either may be the predominant requirement of a particular system, but often both are required to some extent. Here the error signal ultimately controls the output displacement by varying the speed of a final drive motor or servomotor. In this case, power supply for the servomotor is a rotary hydraulic pump. Power flow is metered by a controlling element, such as hydraulic slide valve or servo valve. Since the power needed to operate the valve is negligibly small compared with that metered to the servomotor, the slide valve acts as a power amplifier.

Although the input is defined as the desired state of the controlled variable, which is an angular displacement in this example, the commanding signal is a voltage. The device supplying this information is called the input element. Thus, a measure of the output displacement has to be obtained as a voltage for comparison with the command signal. This is achieved by a potentiometer measuring device, and it is the constant of this measuring device which relates the input to the command signal. In this system, a measure of the error can be obtained as a voltage by simply subtracting the output voltage from the command signal in the electronic amplifier. In some systems, however, some form of comparator or differential must be used (Figure 4).

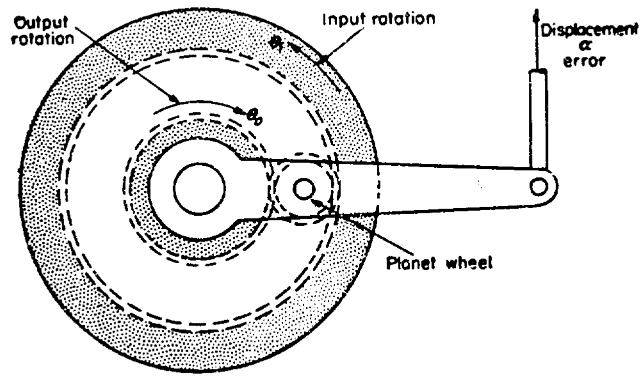


Figure 4. In some servomechanisms a comparator or mechanical differential must be used to add or subtract the feedback signal from the input or command signal.

Since the output signal can be transmitted by wires, the input and output stations can be quite remote in a mechanical sense, provided the output signal voltage does not deteriorate during transmission. Systems for transmitting signals from one place to another are called data transmission systems. Often in mechanical systems, rods and cables must be used to transmit the feedback signal. In this case, even with gears and levers the remoteness of the output station is limited.

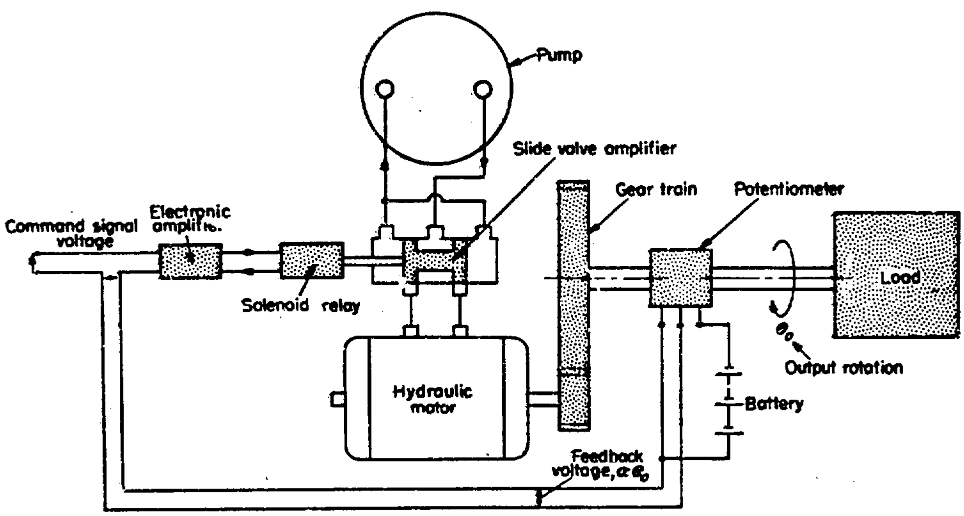


Figure 3. A hydraulic servomechanism for position control illustrates the components of a typical servomechanism.

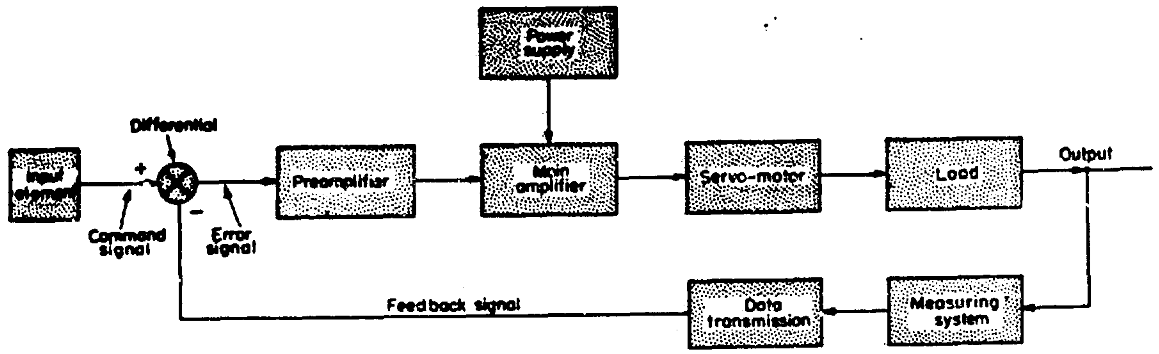


Figure 5. A block diagram of the servomechanism shown in Figure 3 illustrates the general terminology used for servo system components.

Devices concerned with the measurement of the output and the transmission of a signal back to the differential are generally called feedback elements. Since the accuracy of the whole system depends upon accuracy of the signal arriving at the differential, feedback elements must be linear, accurate, and lightly loaded.

Components of the typical system not yet discussed are the electronic amplifier and the solenoid. The purpose of the amplifier is to raise the power level of the error signal. The solenoid operates the slide valve. Such elements are called preamplifiers or signal amplifiers, and transducers, respectively.

Using the general terms established for the specific elements of this typical servomechanism, a block diagram (Figure 5) showing at least the basic elements of nearly all servomechanisms can be constructed. In specific servomechanisms, some of the elements shown may not be present, while other subsidiary elements might be included. Sometimes two or more elements perform one of the functions described, and sometimes two or more functions are performed by a single element. Frequently the feedback path is purely virtual; that is, the input and output are directly compared, no feedback elements or differential being necessary (Figure 6).

Regulators. Typical aerospace applications of automatic control systems as regulators are found in the regulation of pressure in a missile propellant tank and in the control of thrust level in a constant-thrust rocket engine. In these cases, the objective is to maintain the controlled variable at a steady value over a period of time. Pressure regulators are described in detail in Sub-Section 5.4 of this handbook. An analysis of the dynamic performance of a pressure regulator or reducer is given in Sub-Topic 7.4.8. The

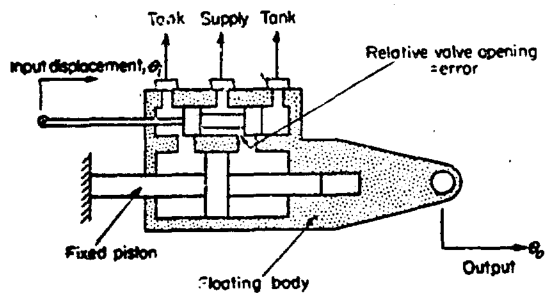


Figure 6. A feedback path may be purely virtual. In this hydraulic servomechanism, for example, the floating valve directly senses and corrects the error.

regulation of thrust in a liquid rocket engine was discussed previously in the present section. Figure 7 gives the block diagram of the pump speed regulation system of Figure 1(b). This diagram is similar in principle to Figure 5, the block diagram of a servomechanism.

Control Theory: All types of servos can be treated by control theory, subject to certain mathematical limitations. At first, however, only a simple system in which an input θ_i causes an output θ_o will be considered.

First the effect of closed-loop operation on the static accuracy of control will be demonstrated by comparing it with open-loop control. In the simple open-loop system, *Fig. 8*, it is assumed that the application of a constant input θ_i will lead ultimately to a steady output θ_o, or

$$\theta_o = A\,\theta_i \tag{1}$$

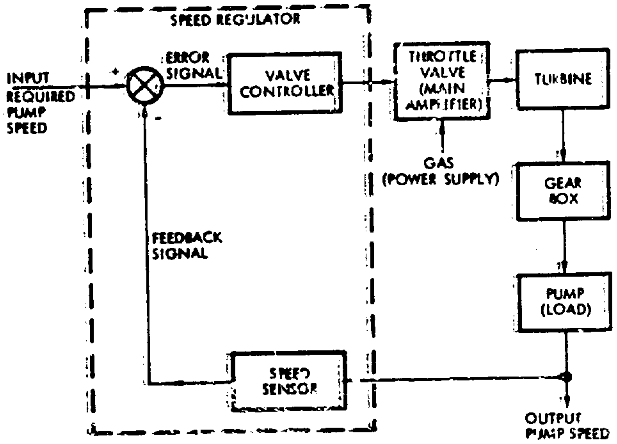


Figure 7. Block Diagram of Turbopump Speed Regulating System

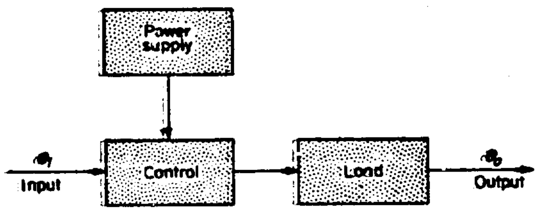


Figure 8. The simple open-loop control system represented by this block diagram is too sensitive to changes in load and power supply to serve satisfactorily in many industrial control applications.

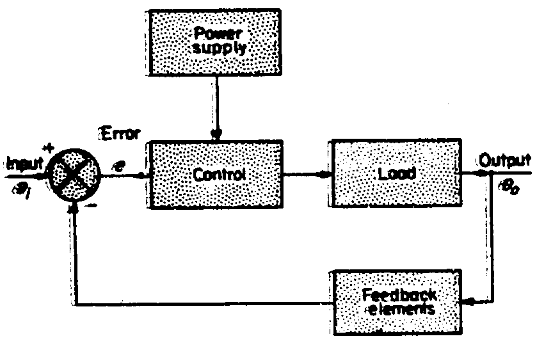


Figure 9. A closed-loop control system of the type represented by this block diagram offers much more accurate control than an open-loop system. Analysis of this system shows that its accuracy depends upon sensitivity of the controller as well as the accuracy of the feedback elements.

where A depends on the system components. Although it would be convenient for A to be constant, this is impossible because of fluctuations in the power supply and load. As a result of such fluctuation assume that A increases by some small amount a.

If the new steady-state is θ_o then

$$\theta_o' = (A + a)\,\theta_i \tag{2}$$

and the fractional change in output is

$$\delta = \frac{\theta_o' - \theta_o}{\theta_o} = \frac{a}{A} \tag{3}$$

Thus δ gives a simple measure of the inaccuracy of the system. If $a/A = 0.1$, the output has the same fractional error. Such an error would be quite unsuitable in industrial controls.

If a closed-loop system, *Fig. 9* , were sensitive to the same error, then

$$\theta_o = Ae \tag{4}$$

where

$$e = \theta_i - \theta_o \tag{5}$$

By eliminating e from Equations 4 and 5

$$\theta_o = \frac{A\theta_i}{(1 + A)} = \frac{\theta_i}{\left(1 + \frac{1}{A}\right)} \tag{6}$$

If A is very large, say $A = 100$, the output will very nearly equal the input. When A increases by an amount a as before, the fractional change in output is

$$\delta = \frac{a}{A}\left(\frac{1}{1 + A + a}\right) \tag{7}$$

Substitution of $a/A = 0.1$ as before, and $A = 100$ in Equation 7, shows that the fractional change in the output is now only 0.001. This is a marked improvement upon the open-loop system. Making A large implies using a very sensitive controller.

Now the effect of feedback elements on closed-loop systems will be considered. Suppose the signal fed back to the differential is $B\theta_o$, where B is ideally 1.0. Then Equation 5 becomes

$$e = \theta_i - B\theta_o \tag{8}$$

and from Equations 4 and 8

$$\theta_o = \left(\frac{A}{1 + AB}\right)\theta_i \tag{9}$$

If B now changes by some small amount b, then the fractional change in the output is

$$\delta\left[\frac{Ab}{1 + A(B + b)}\right] \tag{10}$$

If $A = 100$ as before, and $b/B = 0.1$, then $\delta = 0.1$; furthermore this inaccuracy increases if A is increased. In other words the accuracy of a closed-loop control system is of the same order as the accuracy of the feedback elements, no matter how sensitive the controller. This is a very important point.

This analysis has been qualitative rather than quantitative. In practice the characteristics of system components can rarely be represented by constants such as A and B. One reason is that power amplification is always accompanied by time lags, and so a detailed analysis of servos must be based on the differential equations of motion which relate their input and output. From this analysis stem the standard techniques which make up control theory.

Basic theoretical techniques apply to those servos which are both continuous and linear, that is, systems in which the error is measured continuously and acts on the controlling element in a proportional manner.

There are, however, two widely used types of discontinuous servos: on-off and sampling servos. On-off servos are also known as *relay or bang-bang* servos. Here the error must reach a certain magnitude before it acts on the controller. Then full power is applied to the servomotor through a switch or relay. There is a dead spot in the control for small errors, within which the system can wander. The magnitude of the dead spot is usually critical to the stability. Sampling servos *are also called pulsed data* and *definite correction* servos. Here a measure of the error is obtained at discrete intervals of time and the control acts in a series of finite steps.

All servos are nonlinear to some extent, but very often a good approximation can be obtained by assuming linearity. The justification for the assumption lies in the accuracy of the predicted results.

7.2.2 A Basic Outline of Servo Mathematics

A comprehensive investigation of control system performance requires a knowledge of certain mathematical techniques, based on differential equation analysis. These techniques are summarized in the present article. Space limitations prevent a rigorous treatment.

Input-Output Relationships: A servo system can be represented as a sequence of elements in a block diagram. Each element has an input and an output. Thus, in Fig. 1 $x(t)$ is the input and $y(t)$ is the output. If the relationship between them is of the form $y = kx$ then at any time the relationship between x and y can be represented as a straight line, Fig. 2*a*. This is called a linear relationship, whereas $y = kx^3$ is a nonlinear relationship. Here the relationship gives a curved graph, Fig. 2*b*.

Linear or proportional relations lead to differential equations which can be handled in a methodical and often simple manner. On the other hand nonlinear relationships lead to equations which are difficult, if not impossible to solve. The general theory of control deals with linear systems. No general method of approach exists for nonlinear servos, although considerable attention is being given to certain types of nonlinear systems.

In general an element having an input $x(t)$ and an output $y(t)$, both varying with time t,

Figure 1. Any servo element may be represented by a box having an input, x(t), and output y(t).

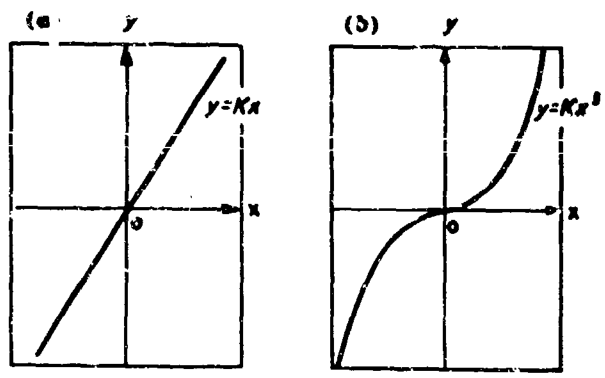


Figure 2. Relationship of input and output of a servo element may be linear, a, or non-linear, b. General servo theory deals with linear systems.

will be related by an equation involving their derivatives as well as x and y themselves. Once again linearity implies proportionality between effects. Thus for a simple mechanical network, Fig. 3,

$$m \frac{d^2y}{dt^2} + f \frac{dy}{dt} + ky = f \frac{dx}{dt} + kx \qquad (1)$$

This is a linear differential equation with constant coefficients.

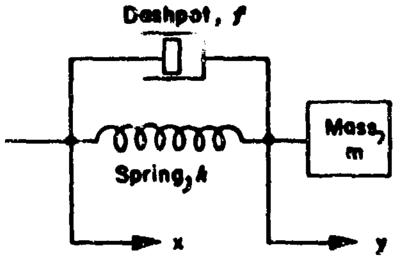


Figure 3. In this simple mechanical network the output for a given input depends upon mass m, damping f, and spring constant k.

For any given input, the output will depend only on the coefficients, such as m, f and k in Equation 1. Thus the element can be thought of as operating on the input to give the output. Servo elements are therefore similar to the filters of the communications engineer, and are sometimes given the same name. They are also called *transfer elements*.

The general relation between the input and output of a linear element can be written in the form

$$(a_n D^n + a_{n-1} D^{n-1} + \ldots + a_1 D + a_0)\, y = (b_m D^m + \ldots + b_1 D + b_0)\, x \qquad (2)$$

where D is a shorthand notation for d/dt and where the a and b factors are all constant.

Any element governed by such an equation is said to be linear. One important property of such elements is that if an input x_1 causes an output y_1, and input x_2 causes an output y_2, then an input $(c_1x_1 + c_2x_2)$ causes an output $(c_1y_1 + c_2y_2)$, where c_1 and c_2 are constants.

This is known as *linear superposition*. It is sometimes given as the definition of a linear system, but since it holds good even if the constants are functions of time, it is not sufficiently precise in this instance.

A satisfactory definition of a linear system is that it is one which under steady conditions gives a sinusoidal output for a sinusoidal input of the same period. Although this is not a mathematically precise definition, it permits treating certain nonlinear elements as linear ones when a sinusoidal input causes an output which, although not sinusoidal, is periodic and of the same frequency as the input. Then only the first harmonic of the output is considered. The justification for this lies only in the accuracy of the results it yields.

If the input $x(t)$ is known, then the right-hand side of Equation 2 is a known function of time, say $f(t)$. Then the output can be obtained by solving

$$(a_n D^n + \ldots + a_1 D + a_0)\, y = f(t) \qquad (3)$$

To do this either the so-called classical or operational methods of differential equation analysis may be used. Of these the latter is quicker and far more suited to servo work.

Nomenclature

A_n =	Residues of partial fraction expansion of $Y_c(s)$
a, b, c =	Constants
D =	Differential operator, d/dt
e_s =	Steady-state position error
f =	Damping constant of mechanical system
$\bar{f}(s)$ =	Laplace transformation of $f(t)$
$f(t)$ =	Arbitrary function of time
h_n =	Roots of characteristic equation
j =	Square root of -1
K =	Scalar gain constant
k =	Spring constant of mechanical system
m =	Mass constant of mechanical system
r =	Order of servo
s =	Laplace operator
T =	Time constant
T_b =	Buildup time
T_d =	Decay time
t =	Time variable
$U(t)$ =	Unit step function
$W(t)$ =	Weighting function of servo
x =	Input to transfer element
$Y_c(j\omega)$ =	Overall harmonic response function $= Me^{j\phi}$
$Y_c(s)$ =	Overall transfer function of servo
$Y_o(j\omega)$ =	Loop harmonic response function $= Ne^{j\theta}$
$Y(s)$ =	Transfer function of element
$Y_o(s)$ =	Loop transfer function of servo
y =	Output of transfer element
$\delta(t)$ =	Unit impulse function
Ω =	$0.01\,\omega$
ω =	Angular frequency, rad per sec

Laplace Transformation: Probably the best known and most useful form of operational calculus is Laplace transformation. Even in moderately experienced hands Laplace transforms are powerful tools for solving differential equations. Briefly, Laplace transformation turns a differential equation in which the variables are functions of time, t, into an algebraic equation in which the variables are functions of a new variable, s, called the Laplace Operator.

Before a Laplace transform is defined, two functions which will be of interest might first be considered:

1. *Unit Step Function:* This represents a sudden change from zero to one at time $t = 0$, Fig. 4*a*. In order for this function to be amenable to the mathematical rules of differentiation and integration, it is defined as the limit of a continuous function, such as that shown dotted in Fig. 4*a*, as the build-up time τ tends to zero. When defined in this way the function is called the *Heaviside unit step function* $U(t)$.
2. *Unit Impulse Function:* This is defined as the limit as $\tau \to 0$ of the continuous function shown dotted in Fig. 4*b*. The function is continuous, equally spaced about the origin and its area remains unity as $\tau \to 0$. Defined in this way, the function is called the *Dirac unit impulse* $\delta(t)$; it is the derivative of $U(t)$. Terms $U(t - t_o)$ and $\delta(t - t_o)$ are respectively unit step and unit impulse functions at time t_o

The Laplace transform $\bar{f}(s)$ of a function $f(t)$ is defined as

$$\bar{f}(s) = \lim_{h \to 0} \int_{-h}^{\infty} f(t)\, e^{-st}\, dt \qquad (4)$$

or as it is normally written

$$\bar{f}(s) = \int_{0-}^{\infty} f(t)\, e^{-st}\, dt \qquad (4a)$$

Making the lower limit of integration 0-, instead of simply 0, insures that the full contribution of any impulse function at the origin is included.

Many textbooks give comprehensive tables of Laplace transforms. The more important ones are listed in Table 1.

In servo work, only functions which are zero for negative time are involved. The time origin, $t = 0$ is the time when an input is applied to the system. Some extremely useful theorems for such functions are given in Table 2. In connection with these theorems the following notation is used:

$$\bar{f}(s) = \mathcal{L} f(t),\ f(t) = \mathcal{L}^{-1} \bar{f}(s)$$

To illustrate the application of the theorems in Table 2 to the solution of differential equations, Theorems 1 and 2 are first applied to Equation 1 to give

$$(ms^2 + fs + k)\, \bar{y}(s) = (fs + k)\, \bar{x}(s)$$

and since this equation can now be handled algebraically,

$$\bar{y}(s) = \frac{(Fs + k)\, \bar{x}(s)}{ms^2 + Fs + k} \qquad (5)$$

Table 1—Laplace Transforms For Servo Analysis

$f(t)$	$\bar{f}(s)$
$U(t)$	$1/s$
$\delta(t)$	1
t^n	$n!/s^{n+1}$
e^{at}	$1/(s - a)$
$\sin \omega t$	$\omega/(s^2 + \omega^2)$
$\cos \omega t$	$s/(s^2 + \omega^2)$

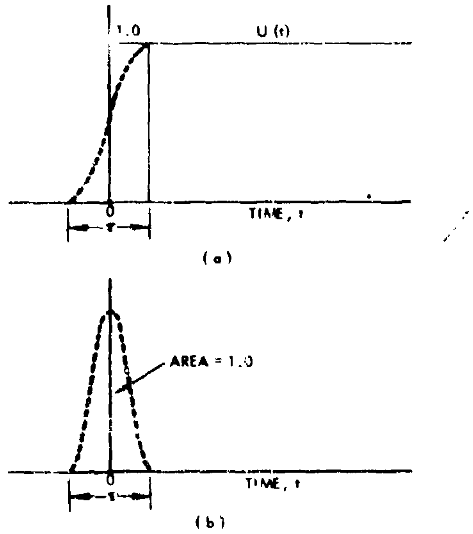


Figure 4. The unit step function is defined as a sudden change from zero to one at time $t = o$, *a*. Mathematically it is defined as the limit of a continuous function such as that shown dotted at *a*. The unit impulse function is the limit, as τ approaches zero, of the continuous function shown dotted at *b*.

Then if $x(t)$ is specified, $\bar{x}(s)$ can be obtained and substituted in Equation 5. The output can then be obtained as a function of time, t, by inverse transforming the right hand side of Equation 5. To do this a comprehensive table of transforms is very useful. However, the short list given in Table 1 may be expanded by using the theorems in Table 2. As a simple example

$$\mathcal{L}\,(e^{-at}\sin \omega t) = \frac{\omega}{(s+a)^2+\omega^2}$$

For any general element, by transforming Equation 2,

$$\frac{y}{x}\,(s) = \left(\frac{b_m s^m + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0}\right) \equiv Y(s) \qquad (6)$$

where $Y(s)$ is a property of the element only and is called its *transfer function*. Obviously the transfer function of an element governed by a linear differential equation is a rational function of s, as shown in Equation 6.

As long as it is realized that transformed quantities are being considered the $\bar{x}(s)$ notation can be discarded and $x(s)$ or simply x can be used.

It is possible to represent each element by a simple block diagram. If two such elements are in series, the output of the first being the input to the second, Fig. 5*a*, and if they are governed by the respective equations,

$$\left.\begin{aligned} y &= Y_1\,x \\ z &= Y_2\,y \end{aligned}\right\} \qquad (7)$$

Then, since the equations can be handled algebraically

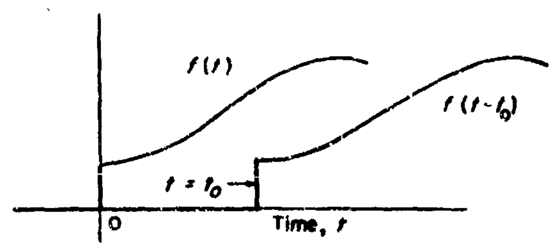
$$z = Y_1 Y_2\, x \qquad (8)$$

This shows that the two boxes in series can be replaced by a single box containing the operator Y_1Y_2 Fig. 5*b*. This can be extended to any number of elements in series. However, this is true only when the elements do not interact, that is, provided the output of any element depends only on its input and not upon the output of the succeeding elements. This is only approximately true in practice. Serious interaction results where the succeeding elements seriously overload the power source of the system.

This technique can be extended to a servo system comprising a sequence of noninteracting elements and a feedback loop, Fig. 5*c*. As shown here the system is a single-loop system. More

Table 2—Useful Theorems for Laplace Transforms

Theorem	No.
$\mathcal{L}\,[af(t) + bg(t)] = a\bar{f}(s) + b\bar{g}(s)$	1
where a, b are constants.	
$\mathcal{L}\left[\frac{df}{dt}\right] = s\bar{f}(s)$	2
$\mathcal{L}\left[\int_{0-}^{t} f(t)\,dt\right] = \frac{1}{s}\,\bar{f}(s)$	3
$\mathcal{L}\,[e^{-at} f(t)] = \bar{f}(s+a)$	4
$\mathcal{L}\,[F(t-t_0),\, U(t-t_0)] = e^{-st_0} F(s)$	5
where $f(t) - t_0)$ is $f(t)$ shifted forward by t_0.	



If

$$C(t) \equiv \int_{0-}^{\infty} f(\tau)\,g(t-\tau)\,d \equiv \int_{0-}^{\infty} g(\tau)\,f(t-\tau)\,d \qquad 6$$

then

$$\bar{C}(s) = \bar{f}(s)\,\bar{g}(s)$$

$$\mathcal{L}\left[\,tf(t)\,\right] = -\left[\frac{d}{ds}\,\bar{f}(s)\right] \qquad 7$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0}\,[s\bar{f}(s)] \qquad 8$$

provided this limit exists.

$$\lim_{t\to 0+} f(t) = \lim_{s\to\infty}\,[s\bar{f}(s)] \qquad 9$$

provided this limit exists.

If $f(t)$ contains a term $A\,\delta\,(t)$,

then

$$A = \lim_{s\to 0} \bar{f}(s) \text{ and}$$

$$\lim_{t\to 0+} f(t) = \lim_{s\to\infty}\,[s\{\bar{f}(s) - A\}]$$

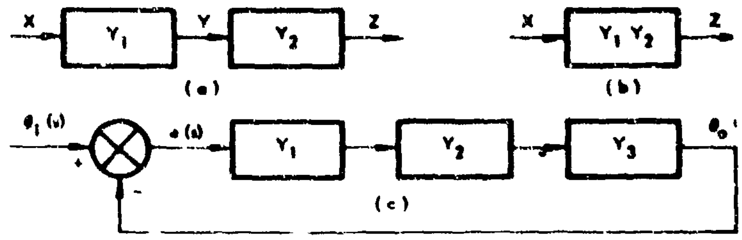


Figure 5. Two servo elements in series, a, may be considered as a single element, b, for purposes of analysis. This principle can be extended to any number of elements as well as to servo systems comprised of several elements and a feedback loop, c.

complicated multiloop systems differ only in detail rather than principle.

The transfer function relating the output, θ_o, to the error, e, of this circuit, Fig. 5c, is called the *open-loop* or the *loop* transfer function. This is

$$Y_o(s) \equiv \frac{\theta_o}{e}(s) = Y_1 Y_2 Y_3 \tag{9}$$

At the differential we have the subtraction

$$e = \theta_i - \theta_o \tag{10}$$

where θ_i is the input. Then by eliminating e from Equations 9 and 10

$$Y_c(s) \equiv \frac{\theta_o}{\theta_i}(s) = \left(\frac{Y_o}{1 + Y_o}\right) \tag{11}$$

where $Y_c(s)$ which relates the transformed output and input of the servo is called the *closed-loop* or *overall transfer function*.

Although primary concern is with output-input relations, it is very convenient to work with the loop transfer function, Y_o as will be shown. Individual transfer functions of servo elements are of the form

$$K, \frac{K}{s}, \frac{K}{1+Ts}, \frac{K}{s(1+Ts)}, \frac{K(1+Ts)}{1+T_1 s},$$
$$\frac{K}{1 + T_1 s + T_2^2 s^2},$$

and so on. If several of these are compounded in the loop of a servo, as in Fig. 5c, the loop transfer function will be of the form

$$Y_o(s) = \frac{K f(s)}{s^r g(s)} \tag{12}$$

where f and g are finite polynomials in s which tend to 1 as $s \to 0$. Thus f and g are of the form

$$\left.\begin{aligned} f(s) &= 1 + T_1 s + T_2^2 s^2 + T_3^3 s^3 + \ldots \\ g(s) &= 1 + (T_1')s + (T_2')^2 s^2 + \ldots \end{aligned}\right\} \tag{13}$$

where K is a constant called the *scalar gain constant* of the system. It is sometimes called simply the gain, but this may lead to confusion with a similarly named term.

From Equation 11 it follows that

$$Y_c(s) = \frac{Kf(s)}{Kf(s) + s^r g(s)} \tag{14}$$

This can be written in the more general form

$$Y_c(s) = \frac{F(s)}{G(s)} = \left(\frac{b_m s^m + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0}\right) \tag{15}$$

Comparing this with Equation 6 shows that the servo is itself a linear filter, operating on the input to give the output. If $\theta_i(t)$ and hence $\theta_i(s)$ are known, then the output $\theta_o(t)$ can be found from

$$\theta_o(t) = \mathcal{L}^{-1}[Y_c(s)\,\theta_i(s)] \tag{16}$$

Servo Input Functions: It is not possible to generalize on the type of input likely to be encountered in servo work. Indeed the kinds of inputs normally encountered do not yield themselves to analytical expression. Instead, three idealized input functions upon which to base an analytical approach are chosen. They are:

1. *Unit Impulse Function*, $\delta(t)$: Here $\theta_i(s) = 1$. The output in this case is called the *Weighting Function*, $W(t)$, of the system. From Equation 16

$$W(t) = \mathcal{L}^{-1}[Y_c(s)] \tag{17}$$

or

$$Y_c(s) = \mathcal{L}\, W(t) \tag{18}$$

This shows that the weighting function is an important property of the servo. From Equations 16 and 18 and Theorem 6, it can be seen that if $W(t)$ is known, the response to any input $\theta_i(t)$ can be found from

$$\theta_o(t) = \int_{0-}^{\infty} \theta_i(\tau)\, W(t-\tau)\, d\tau \qquad (19)$$

Thus the response to a complicated input can be obtained.

If the input can be thought of as a series of impulses of duration, $\Delta\tau$, Equation 19 means that as $\Delta\tau$ becomes very small the system is unable to distinguish between the series of impulses and continuous input, Fig. 6. This concept is very helpful in assessing and improving the performance of existing systems, for if $W(t)$ can be determined experimentally, it is possible to calculate how the system will respond to any input. The difficulty is to generate an impulse of sufficiently short duration to approximate a δ-function. Generally, if the duration of the pulse is much smaller than any natural period of the system, very good results are obtained.

If the denominator of $Y_c(s)$ is expressed in the form

$$G(s) \equiv (s-h_1)(s-h_2)\ldots(s-h_n) \qquad (20)$$

where $h_1, h_2, \ldots, h_n$ are the roots of $G(s) = 0$, called the *characteristic equation*, $Y_c(s)$ can be split into partial fractions, thus

$$Y_c(s) = \frac{A_1}{s-h_1} + \frac{A_2}{s-h_2} + \ldots + \frac{A_n}{s-h_n} \qquad (21)$$

where $A_1, A_2, \ldots$ are the normal partial fraction constants.

Then using Table 1

$$W(t) = A_1 e^{h_1 t} + A_2 e^{h_2 t} + \ldots + A_n e^{h_n t} \qquad (22)$$

It has been assumed that the roots of the characteristic equation are all distinct. If, on the other hand, there are repeated roots such as $(s-h)^2$, the weighting function will contain terms such as Bte^{ht}. The most general form of the weighting function is, therefore, written

$$W(t) = \sum (A + Bt + Ct^2 + Dt^3 + \ldots)\, e^{ht} \qquad (23)$$

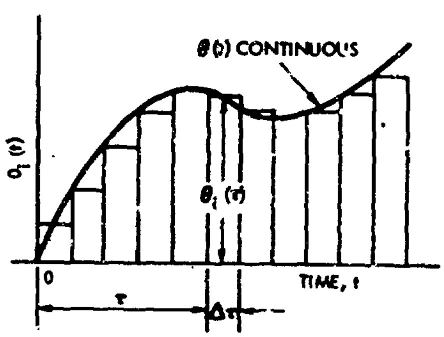


Figure 6. When servo input is a series of pulses of short duration, the servo is unable to distinguish the pulses from a continuous function. This concept is helpful in assessing and improving system performance.

A typical weighting function of a linear servo is shown in Fig. 7.

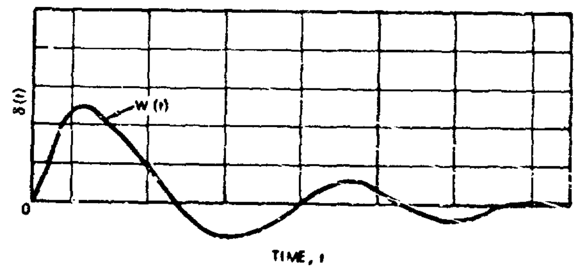


Figure 7. Typical weighting function of a linear servo.

2. *Unit Step Function* $U(t)$: Here $\theta_i(s) = 1/s$ and from Equation 11

$$\theta_o(s) = \frac{Y_c(s)}{s} \qquad (24)$$

From Theorem 3, Table 2, it follows that

$$\theta_o(t) = \int_{0-}^{t} W(t)\, dt \qquad (25)$$

It is more likely that the response to a step function would be obtained directly from Equation 24. Thus expanding by partial fractions

$$\theta_o(s) = \frac{A_0}{s} + \frac{A'_1}{s-h_1} + \ldots + \frac{A'_n}{s-h_n} \qquad (26)$$

where, in general,

$$A_o = Y_c(O),\ A'_r = \lim_{s \to h_r} \left[\frac{(s-h_r)\, F(s)}{sG(s)} \right]$$

The general form of the output response, if there are repeated roots in the characteristic equation is, therefore,

$$\theta_o(t) = A_o U(t) + \sum [(A' + B' + C' t^2 + \ldots +)\, e^{ht}] \qquad (27)$$

A typical response is shown in Fig. 8. Such a curve is very informative because it gives a simple pictorial representation of the response to a sudden jump in the input. Thus, in the diagram, T_3 gives a measure of the sensitivity, e_s gives a measure of the steady-state accuracy, and M and T_4 give measures of the stability.

3. *Sinusoidal Input Function*: Output here is called the *frequency response*. Instead of a real sinusoidal input, eg. sin ωt, the complex form of a harmonic quantity will be considered, that is

$$\theta_i = e^{j\omega t} \equiv (\cos \omega t + j \sin \omega t) \qquad (38)$$

where ω is the frequency in radians per second,

Manipulation of Complex Quantities

Addition: Two response functions such as $N_1 e^{j\psi_1}$ and $N_2 e^{j\psi_2}$ must be added according to the parallelogram law of vectors, Sketch 1.

Multiplication: If $Ne^{j\psi} = N_1 e^{j\psi_1} \times N_2 e^{j\psi_2}$, then $N = N_1 N_2$; that is, moduli are multiplied, and $\psi = \psi_1 + \psi_2$, the phase angles are added, Sketch 2.

Division: If $Ne^{j\psi} = N_1 e^{j\psi_1}/N_2 e^{j\psi_2}$, then $N = N_1/N_2$ and $\psi = \psi_1 - \psi_2$.

To illustrate the application of these methods, suppose $Y_o(j\omega)$ is known for a particular frequency. Then $1 + Y_o(j\omega)$ can be obtained by addition, Sketch 3*a*, and $Y_c(j\omega)$ can then be obtained by division from Equation 36, as shown in Sketch 3*b*.

Numerical Example: Suppose a servo has the loop transfer function

$$Y_o(s) = \frac{51.3\,(1 + 0.0225\,s)(1 + 0.2\,s)}{s\,(1 + 0.00435\,s + 0.00045\,s^2)}$$

Then the loop response function is

$$Y_o(j\omega) = \frac{51.3\,(1 + 0.0225\,j\omega)(1 + 0.2\,j\omega)}{j\omega\,(1 - 0.00045\,\omega^2 + 0.00435\,j\omega)}$$

Thus the modulus and phase are given. To change to a more convenient frequency scale,

$$N \equiv |Y_o(\Omega)| = \frac{0.513}{\Omega}\sqrt{\frac{(1 + 5.06\Omega^2)(1 + 400\Omega^2)}{(1 - 4.5\Omega^2)^2 + 0.19\Omega^2}}$$

$$\psi \equiv \arg\ Y_o(\Omega) = \tan^{-1}(2.25\,\Omega) + \tan^{-1}(20\,\Omega) - \tan^{-1}\left(\frac{0.435\,\Omega}{1 - 4.5\,\Omega^2}\right) - \frac{\pi}{2}$$

where $\Omega = 0.01\omega$. The loop response vector has been plotted for these values in Fig. 16.

The overall transfer function is given by

$$Y_c(s) = \frac{50.4\,s^2 + 2490\,s + 113{,}000}{s^3 + 59.6\,s^2 + 4690\,s + 113{,}300}$$

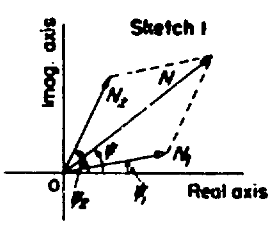
The denominator can be factored into $(s + 29.6)\,(s^2 + 30s + 3800)$. Then for a unit step input

$$\theta_o(s) = \left[\frac{Y_c(s)}{s}\right] = \left[\frac{1}{s} - \frac{0.71}{s + 29.6} + \frac{20.7 - 0.29\,s}{s^2 + 30s + 3800}\right]$$

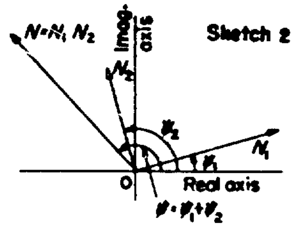
Inverse transforming gives the transient response as

$$\theta_o(t) = U(t) - 0.71\,e^{-29.6t} + 0.4_e^{-15t}(\sin 60t - 0.725 \cos 60t)$$

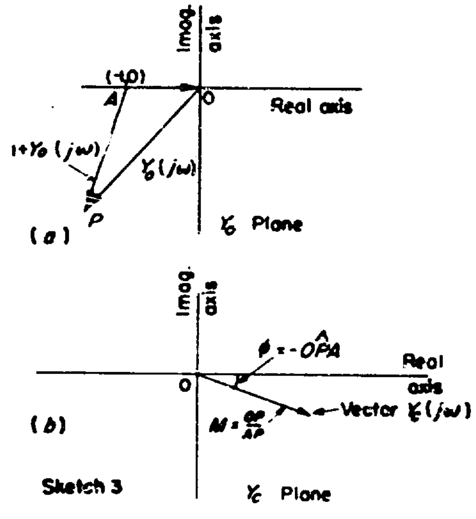
This is plotted in Sketch 4.



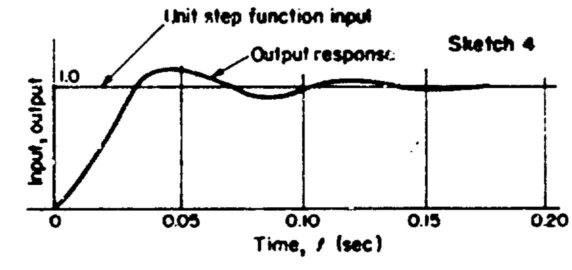
Sketch 1



Sketch 2



Sketch 3



Sketch 4

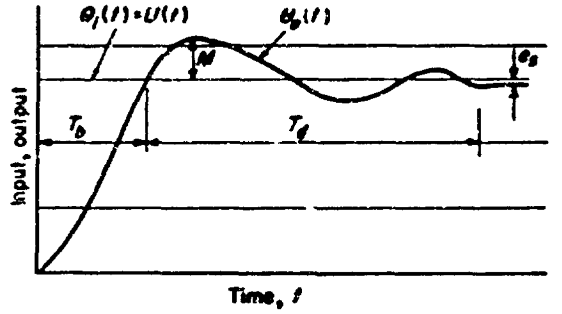


Figure 8. Typical output response to a step function input shows pictorially the change in output corresponding to any sudden change in input.

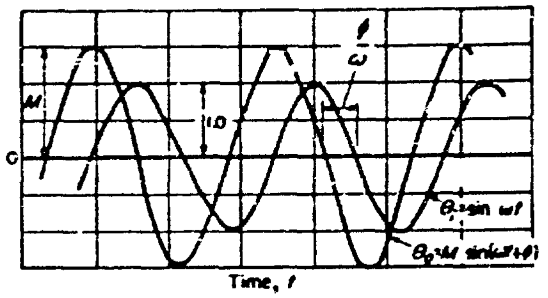


Figure 9. Servo response to pure harmonic input is sinusoidal and of the same frequency. The amplitude, however, is increased in the ratio M:1 and phase is shifted by an angle, ϕ.

and j is the symbolic quantity for $\sqrt{-1}$. This is a dodge which greatly simplifies the mathematics. It is justified because of the principle of linear superposition, since the real part of the output can be considered as the response of $\cos \omega t$ and the imaginary part as the response to $\sin \omega t$.

From Table 1 $\theta_i(s) = 1/(s - j\omega)$ and, therefore,

$$\theta_o(s) = \left[\frac{Y_c(s)}{s - j\omega}\right] = \frac{F(s)}{(s - j\omega)\,G(s)} \tag{29}$$

This may be expanded in partial fractions, giving

$$\theta_o(s) = \sum \left[\frac{A}{s - h}\right] + \frac{B}{s - j\omega} \tag{30}$$

The time variation of the output is, therefore,

$$\theta_o(t) = \sum Ae^{ht} + Be^{j\omega t} \tag{31}$$

The first term, ΣAe^{ht}, represents a transient which ultimately disappears if the servo is stable. The remainder $Be^{j\omega t}$ is the *steady-state frequency response*. The value of B is

$$B = \lim_{s \to j\omega} [(s - j\omega)\,\theta_o(s)] = Y_c(j\omega) \tag{32}$$

$Y_c(j\omega)$ is the *overall harmonic response function*. It is obtained simply by substituting $j\omega$ for s in $Y_c(s)$. Thus the steady-state response to a complex harmonic input of frequency ω is

$$\theta_o(t) = Y_c(j\omega)\,e^{j\omega t} \tag{33}$$

Term $Y_c(j\omega)$ is in general a complex quantity which can be written

$$Y_c(j\omega) = M(\omega)\,e^{j\phi(\omega)} \tag{34}$$

where $M(\omega) = |Y_c(j\omega)|$, sometimes written as $|(\theta_o/\theta_i)(j\omega)|$, and called the *overall amplitude ratio;* $\phi(\omega) = \arg[Y_c(j\omega)]$, sometimes written $\arg[(\theta_o/\theta_i)(j\omega)]$, and called the *overall phase angle*. Thus Equation 33 can be written

$$\theta_o(t) = Me^{j(\omega t + \phi)} = M[\cos(\omega t + \phi) + j\sin(\omega t + \phi)] \tag{35}$$

Separating the real and imaginary parts shows that the response to the real inputs $\cos \omega t$ and $\sin \omega t$ are respectively $M\cos(\omega t + \phi)$ and $M\sin(\omega t + \phi)$. That is, the response to any pure harmonic input is also sinusoidal and of the same frequency, but the amplitude is increased in the ratio $M:1$, and the phase is shifted by an angle ϕ with respect to the input, Fig. 9. In practical systems the output will lag the input; that is, ϕ will be negative.

It is possible to draw $Y_c(j\omega)$ as a vector in the complex plane. If this vector is drawn for all frequencies between 0 and ∞, then its end point will trace out a continuous curve in the Y_c-plane, as shown dotted in Fig. 10*a*. In practice, however, it is more usual to plot the overall frequency response characteristics as separate curves of M and ϕ plotted against ω, Fig. 10*b*.

Just as it is possible to work with the loop transfer function, the loop harmonic response function, $Y_o(j\omega)$, can also be used. This is obtained simply by putting $s = j\omega$ in $Y_o(s)$. Then

$$Y_c(j\omega) = \frac{Y_o(j\omega)}{1 + Y_o(j\omega)} \tag{36}$$

It is usual to plot $Y_o(j\omega)$ as a vector in the complex-plane. To do this $Y_o(j\omega)$ must be expressed in the form; $Y_o(j\omega) = u(\omega) + jv(\omega)$ where $u(\omega)$ is the real part and is plotted along the horizontal axis and $v(\omega)$ is the imaginary part and is plotted vertically, Fig. 11.

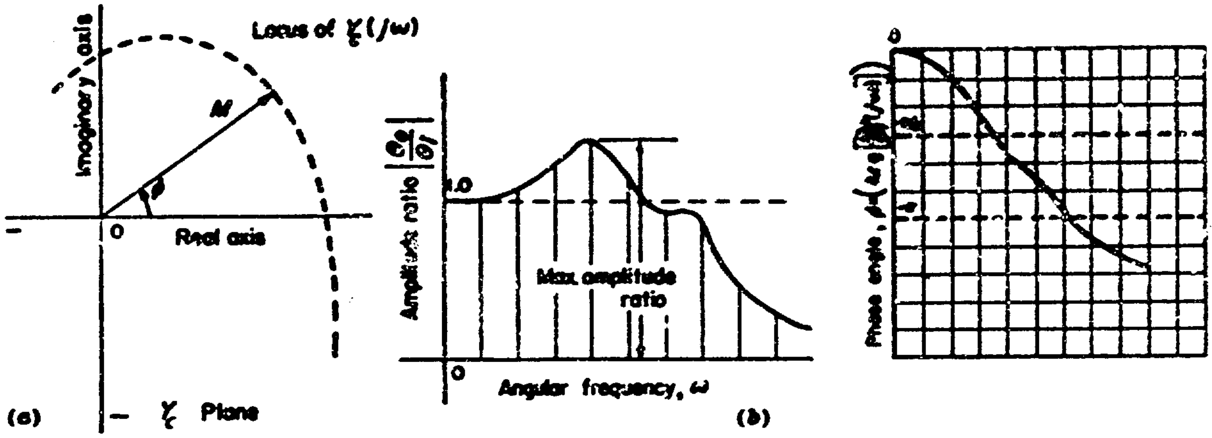


Figure 10. Overall harmonic response function $Y_o(j\omega)$, may be represented by a vector in the complex plane. If the vectors corresponding to all frequencies are drawn, a continuous curve will be traced as shown by the dotted line, a. It is more usual, however, to plot overall frequency response as separate curves, b, of M and θ against ω.

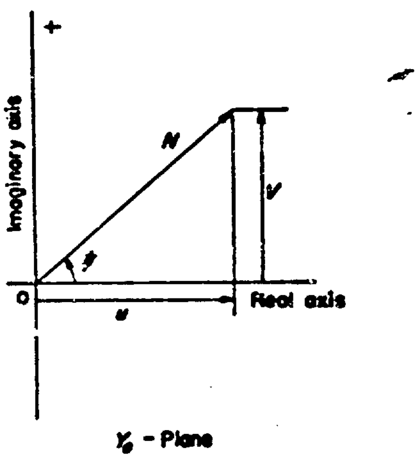


Figure 11. The loop harmonic function $Y_o(j\omega)$ is usually plotted as a vector in the complex plane. The real part, $u(\omega)$, is plotted on the horizontal axis while the imaginary part $v(\omega)$ is plotted vertically.

Alternatively, the loop harmonic response function may be expressed as

$$Y_o(j\omega) \equiv \left[\frac{\theta_o}{e}(j\omega)\right] = N(\omega)\, e^{j\phi(\omega)} \qquad (37)$$

where $N(\omega) = |Y_o(j\omega)| = \sqrt{u^2+v^2}$, and is called the *loop gain* or *loop amplitude ratio* and $\phi(\omega) = \arg[Y_o(j\omega)] = \tan^{-1}(v/u)$, and is the loop phase angle.

As ω is varied from 0 to ∞, the tip of the $Y_o(j\omega)$ vector will trace out a continuous curve in the Y_o-plane, Fig. 12. This curve is called the *loop vector locus* or the *Nyquist plot*. Its very great value in servo analysis will be discussed in the next article, which will deal with performance criteria.

7.2.3 Criteria for Evaluating Servo System Performance

Performance can be described generally in terms of two qualities: (1) stability and (2) response. Stability describes the ability of a servo to settle down after a disturbance has been removed. It is closely related to the response of the system. Response is the term used to describe the accuracy and sensitivity of the system when responding to some input or command signal.

7.2.3.1 STABILITY

The formal definition of a stable servo is very clear-cut. It is a system in which the output is always finite, or limited, for any finite input. An unstable servo is one in which the output drifts away from the input without limit. This does not necessarily happen for all inputs, but if it will

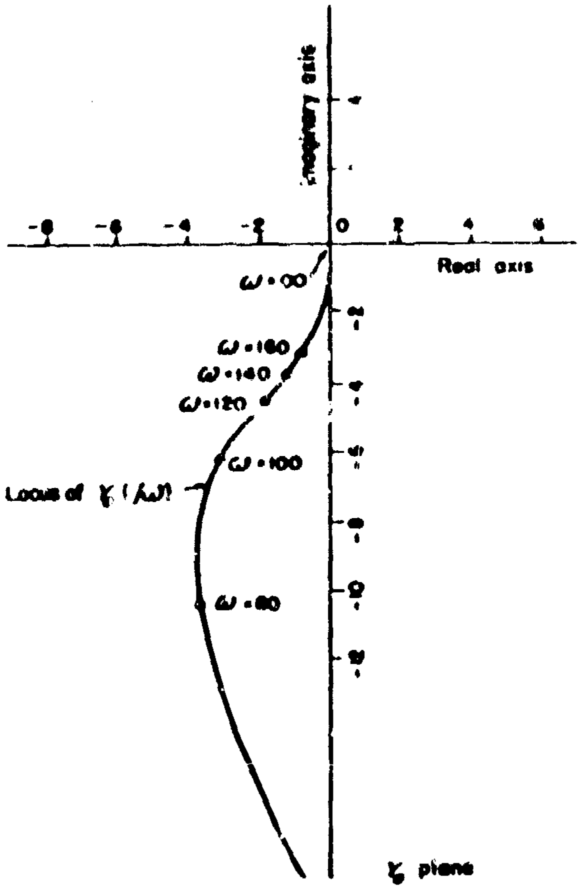


Figure 12. The servo loop vector locus or Nyquist plot is of great value in servo analysis. It is obtained by plotting the loop harmonic response function, $Y_o(j\omega)$, for values of ω from zero to infinity.

occur for any input then the system is obviously unsatisfactory. The idea of output increasing without limit is only a mathematical concept. What happens in practice is that output will only increase until some component in the system breaks down, or until some nonlinearity intervenes to constrain the output.

Although this definition gives a definite division between stable and unstable servos, the term stability is generally used in a relative sense. A system with good relative stability characteristics, Fig. 1a, might have a maximum overshoot of 0.3 and its oscillations would decay in a comparatively short time such as four times the buildup time. On the other hand, a system having a maximum overshoot of 0.8 and a decay time equal to ten times buildup time, Fig. 1b, although stable in an absolute sense, would be said to have poor relative stability characteristics.

The mathematical definition of stability is that

$$\int_0^{\infty} |W(t)|\, dt$$

must exist and be finite, where $W(t)$ is the weighting function.[*] In practical servos a sufficient condition is that $W(t) \to 0$ as $t \to \infty$. Physically this means that the output must return to its initial position if the system is given a sudden impulsive kick at the input.

The most general expression for $W(t)$, the weighting function is

$$W(t) = (A_1 + B_1 t + C_1 t^2 + \dots) e^{b_1 t} + (A_2 + B_2 t + \dots) e^{b_2 t} \quad (1)$$

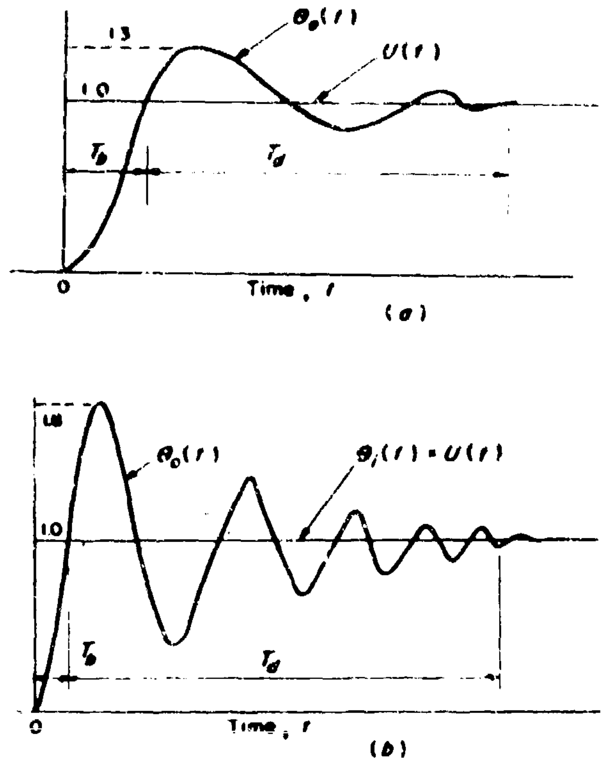


Figure 1. A servo system with good relative stability characteristics, a, may be defined as one having a maximum overshoot of 0.3 and comparatively short decay time, T_d, equal to about four times the buildup time, 1b. A relative unstable system, b, has overshoot of 0.8 and decay time equal to 10 times the buildup time. The relatively unstable system does, however, possess absolute stability since oscillations dieout.

where h_1, h_2, etc are all the values of s which make $G(s)$, the denominator of the overall transfer function, $Y_c(s)$, zero. Each h may be either real, imaginary, or in the most general case complex. Any complex root can be written in the form $h = \alpha + j\Omega$. Presence of such a root indicates a damped sinusoid in the weighting function. Only if α is negative will this oscillation decay as time t increases. Thus, a necessary condition for stability is that *all the roots of $G(s) = 0$ must possess a negative real part.*

Note that in practical systems the coefficients of powers of s in $G(s)$ are positive and real. This implies that complex roots occur in conjugate pairs. That is if $\alpha + j\Omega$ is a root, then $\alpha - j\Omega$ is also a root.

The presence of a purely imaginary root, say $s = j\Omega_o$, is to be deplored. It does not satisfy the above condition for stability and means that there is an undamped oscillation in the weighting function. With a periodic function input of frequency Ω_o, the output can increase without limit, at least in theory.

It is therefore possible to investigate the stability of a servo by finding the roots of the characteristic equation

$$G(s) \equiv a_n s^n + a_{n-1} s^{n-1} + a_1 s + a_0 = 0 \qquad (2)$$

This can be very tedious if $n>3$, as it probably will be in most servos. Further on rapid methods for investigating the absolute and the relative stability of systems will be discussed.

There are certain helpful rules regarding stability based upon the transfer function,

$$\frac{\theta_o}{\theta_i}(s) = Y_c(s) = \frac{b_m s^m + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0} \qquad (3)$$

of a linear servo. These rules are:

1. If $m > n$, the system is physically unrealizable.
2. If any of the a coefficients in the denominator is negative, then the system is in general unstable.
3. If a_n exists and any of other coefficients $a_{n-1}, \ldots, a_0$ is zero, then the system is unstable.

It must be realized that although these rules can reveal an unstable servo, they cannot prove that a system is stable. In other words they are not sufficient tests for stability.

Frequency Response and Stability: Suppose $s = j\Omega$ is an imaginary root of $G(s) = 0$. Then for a complex sinusoidal input of frequency ω, the transformed output is given by

$$\theta_o(s) = \frac{Y_c(s)}{s - j\omega} = \frac{F(s)}{(s-h_1)(s-h_2)\ldots(s-j\Omega)(s-j\omega)} \qquad (4)$$

So that when the input frequency ω is equal to Ω, the partial fraction expansion for $\theta_o(s)$ will contain the term, $C/(s - j\Omega)^2$. This results in the term $Cte^{j\Omega t}$ in the weighting function. This component of the response is an oscillation whose successive amplitudes increase linearly without limit. The system is therefore unstable. This phenomenon is known as resonance.

In practice, as previously stated, the output amplitude can only increase until the system fails or until some nonlinearity, such as saturation of the power source, intervenes to limit the amplitude. A self-maintained oscillation is then set up. This phenomenon, called hunting or limit cycling, will only occur in practice where a closed-loop sequence monitors a power source. Self-maintained oscillation in other spheres (for example, aircraft flutter vibrations) can be traced to the same cause.

It is possible to plot $Y_c(j\omega)$ against frequency, ω. Thus,

$$|Y_c(j\omega)| = M(\omega) = \frac{|F(j\omega)|}{\sqrt{(\omega^2 + h_1^2)(\omega^2 + h_2^2)\ldots(\omega^2 + h_n^2)}} \qquad (5)$$

Therefore, if $h = j\Omega$ is an imaginary root of $G(s) = 0$, $M(\omega)$ will become infinite when $\omega = \Omega$, Fig. 2. Thus, if the overall amplitude response curve becomes infinite at any frequency, it indicates the presence of an undamped oscillation in the weighting function, and therefore instability.

A servo will also be unstable if there is a root of the form $\alpha' + j\Omega'$, where α' is positive. In this case the amplitude plot would be the same if we replaced the unstable root by $-\alpha' + j\Omega'$. This method does not give conclusive proof of stability, although as will be shown later, once absolute stability has been established, $M(\omega)$ and $\phi(\omega)$ give useful information on relative stability.

Obviously, some simple and conclusive tests for stability would be very helpful. Two approaches to this problem will be outlined. They are: (1) The Nyquist criterion and (2) algebraic criteria.

Nyquist Criterion: This utilizes the open-loop harmonic response function $Y_o(j\omega)$, and is based

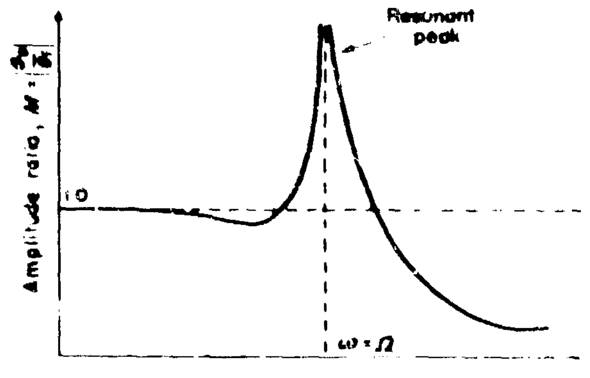


Figure 2. Resonance in a servo system is one of the possible types of instability. If h jΩ is an imaginary root of the denominator of the overall servo transfer function, it indicates that at some frequency, Ω, there is a resonant peak. Some component of the servo system would be overloaded and fail at this frequency as the amplitude ratio tended to infinity.

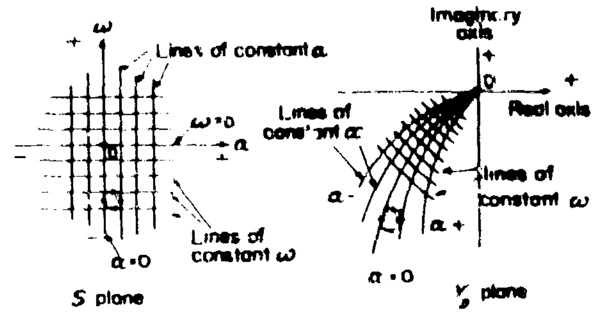


Figure 3. Lines of constant α and ω in the s plane correspond to similar contours in the Y_e plane which depend on the function $Y_e(s)$. This is known as *conformal mapping*. The small shaded square in the s plane corresponds in the limit to the small shaded area in the Y_e plane.

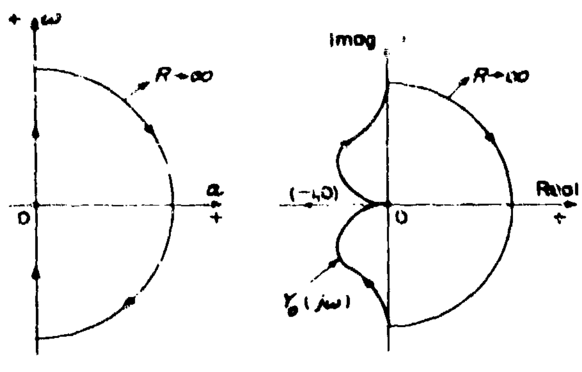


Figure 4. The Nyquist criterion for stability is that the point (−1,0) shall not fall within the shaded region in the Y_e plane obtained by conformal transformation of and corresponding to the shaded region in the s plane. The criterion holds true provided all system elements are themselves stable.

upon the properties of functions of a complex variable. Consider first the loop transfer function $Y_e(s)$, where in general s is a complex number of the form $s = \alpha + j\omega$. Corresponding to each value of s there is particular value of $Y_e(s)$. This can be shown by showing the value of s as a point in a complex plane called the s plane, and the corresponding value of $Y_e(s)$ as a point on another complex plane, called the Y_e plane. Corresponding to a contour in the s plane there is a contour in the Y_e plane. The shape of the latter depends on the function $Y_e(s)$, and hence on the parameters of the servo it represents.

Thus, if the s plane is divided into a net of lines of constant α and constant ω, parallel to the axes, Fig. 3, there is a corresponding pattern of lines in the Y_e plane. This is called *conformal mapping*. If $Y_e(s)$ is what is known as an analytic function, and it certainly is for the linear servos being considered, then small squares in the s plane correspond in the limit to small squares in the Y_e plane, Fig. 3 This is called a *conformal transformation*. The important point is that the squares are traversed in the same sense, as will be shown.

The point $(-1 + j0)$, written $(-1, 0)$, in the Y_e plane corresponds to a point $(\alpha_1 + j\omega_1)$ in the s plane. That is,

$$Y_e\ (\alpha_1 + j\omega_1) = -1 \tag{6}$$

In other words $(\alpha_1 + j\omega_1)$ is a root of the characteristic equation $G(s) = 1 + Y_e(s) = 0$. For stability α_1 must be negative, or $(\alpha_1 + j\omega_1)$ must not lie in the region shown shaded in Fig. 4a. Corresponding to this region there is a shaded region in the Y_e plane as shown in Fig. 4b. Because of the previously mentioned conformal transformation, this region is bounded by the contour $Y_e(j\omega)$ and lies to the right of it as the contour is traversed from $\omega = -\infty$ through $\omega = 0$ to $\omega = +\infty$. The condition for stability is therefore that the point $(-1, 0)$ shall not lie in this shaded region of Y_o plane.

The condition stated holds if all the elements in the system are themselves stable. Very occasionally systems do contain unstable components, usually due to some local positive feedback loop around a component. This does not necessarily mean that the overall system is unstable, but in this case the condition for stability depends on how many times the contour $Y_o(j\omega)$ encircles the point $(-1, 0)$. In determining the stability of these so-called *nonminimum-phase* systems the exact form of the loop transfer function must first be obtained. However, they are sufficiently rare in mechanical servos to be neglected in this discussion. They will be discussed in a later article.

In the condition for stability just stated it would be necessary to draw the whole of the $Y_o(j\omega)$ contour, including a large circular arc. The sweep of this arc depends on the power r in the denominator of the loop transfer function (Equation 12, Ref. 2). But in practical servos it is unnecessary to go to all this complication. If the $Y_o(j\omega)$ contour from $\omega = 0$ to $\omega = +\infty$ is plotted, then the condition for stability is: *The point $(-1, 0)$ must always lie to the left of the contour when it is traversed in the direction of increasing ω,* Fig. 5. A contour passing through the point $(-1, 0)$ represents the critical stability boundary.

The Nyquist criterion can be given a simple physical explanation. Where $Y_o(j\omega)$ crosses the negative real axis, the output lags the error by 180 degrees. Thus any sinusoidal pulse introduced as an error passes through the loop to the output and is reintroduced as an error 180 degrees behind the initial pulse, as shown in Fig. 6*a*. The amplitude of this pulse will be $|Y_o|$ times the amplitude of the initial pulse. Thus, if $|Y_o| = 1$ at this frequency, a continuous oscillation can be maintained, since this second pulse will cause an equal and opposite one to be introduced, and so on. If $|Y_o| > 1$ at 180-degree phase lag, the oscillation will increase in amplitude, Fig. 6*b*. Obviously the desired condition for stability is $|Y_o| < 1$ at the given frequency.

In most servos stability depends on the value of the scalar gain K, where

$$Y_o(s) = \frac{K f(s)}{s^r g(s)} \tag{7}$$

and $f(s)/g(s) = 1$, when $s = 0$.

That is to say there is a critical value for K

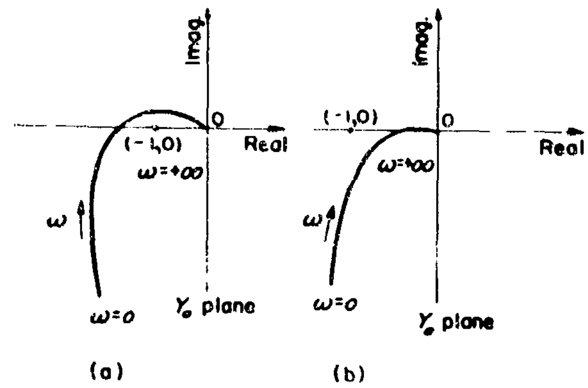


Figure 5. Application of the Nyquist criterion for stability can be simplified in practice by plotting the $Y_o(j\omega)$ contour only from $\omega = 0$ to $\omega = \infty$. The condition for stability then becomes that the point $(-1,0)$ must always be to the left of the contour when it is traversed in the direction of increasing ω. The plot for an unstable system is shown at *a*. The contour at *b* fulfills the conditions for stability.

above which the servo becomes unstable. In practice, for good relative stability, K must be set somewhat less than this critical value, as will be shown. From Equation 7 it can be seen that changing K merely alters the scale of the $Y_o(j\omega)$ contour, or *Nyquist plot* as it is frequently called.

Algebraic Criteria: These are expressed in terms of relations between the coefficients of the powers of s in the characteristic equation.

$$G(s) \equiv a_n s^n + \dots + a_1 s + a_0 \tag{8}$$

One of these criteria is due to Hurwitz. This is as follows: Write down the determinant of order $n - 1$.

$$\Delta \equiv \begin{vmatrix} a_1 & a_0 & 0 & 0 & 0 & 0 & \cdots \\ a_3 & a_2 & a_1 & a_0 & 0 & 0 & \cdots \\ a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & \cdots \\ a_7 & a_6 & \cdots & & & & \cdots \\ \cdots & & & & & & \cdots \end{vmatrix} \tag{9}$$

Then for stability all the a's must be of the same sign, and Δ must be positive when evaluated. For example, if

$$G(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 \tag{10}$$

then

$$\Delta = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} \tag{11}$$

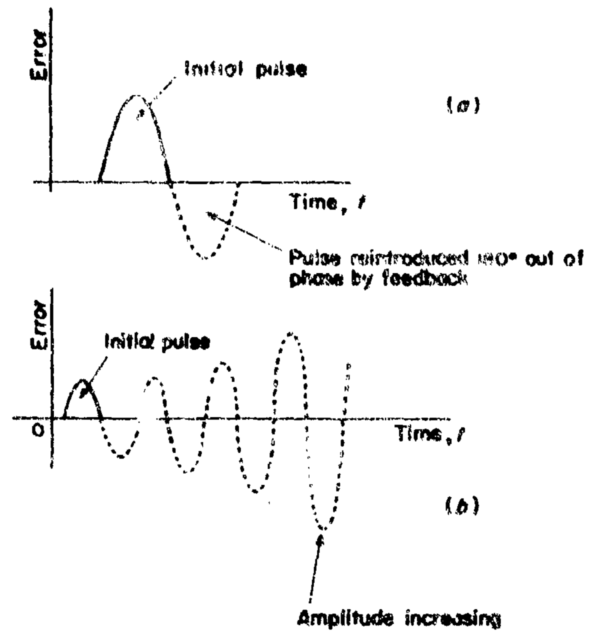


Figure 6. If the $Y_o(j\omega)$ contour crosses the negative real axis, servo output lags the error by 180 deg. Any sinusoidal pulse introduced as an error passes through the servo loop to the output and is reintroduced as an error 180 deg behind the initial pulse, *a*. Amplitude of this pulse is $|Y_o|$ times the amplitude of the initial pulse. Therefore, if $|Y_o| = 1$, continuous oscillation will be maintained. If $|Y_o| > 1$, the oscillation will increase in amplitude, *b*. Obviously for a stable system $|Y_o|$ must be less than one. It is this condition which the Nyquist plot serves to establish.

and the condition for stability is

$$a_1 \, a_2 > a_0 \, a_3 \tag{12}$$

As a further illustration, if

$$G(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \tag{13}$$

then

$$\Delta = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix} \tag{14}$$

and the condition for stability is

$$a_1 \, (a_2 \, a_3 - a_1 \, a_4) > a_3^2 \, a_0 \tag{15}$$

There are other similar algebraic criteria—for example, Routh's criterion. Although they differ in method they give the same results.

The advantage of algebraic methods is that they are simple to apply and give clear-cut decisions. However, they only give the conditions for absolute stability, and do not give any data on the relative stability of the system.

On the other hand, the Nyquist criterion is sometimes difficult to use when determining absolute stability, although if correctly used it always gives the right results. The great advantage in drawing a Nyquist plot is that it can also be used to determine the relative stability and response characteristics. In practice it is a good idea to use both Nyquist and algebraic methods.

7.2.3.2 RESPONSE

With the necessary conditions for absolute stability discovered response characteristics can be evaluated. No clear-cut response criteria can be laid down since they depend on the field of application and on the types of inputs likely to be encountered. In servomechanisms (for example, remote-position-controllers), the input is likely to change continuously and rapidly, with perhaps many changes of direction per second. In general the output must have small following errors, and this means high sensitivity as well as static accuracy.

In automatic regulators (for example speed-governors), the input is likely to remain constant over long intervals of time. The output response to change in input setting must usually be accurate rather than sensitive. In fact, the control must sometimes react slowly to input change so as not to overload the system. Continuous excitation may come from some unwanted external disturbance, and it is desirable that the system does not respond very much to this disturbance. In process controls, which are special forms of regulators, the time scale may be very different from that of servomechanisms. Here there may be very large time lags, especially in the plant itself.

Since inputs are so variable, the analysis presented here will be performed by considering the response to certain idealized input functions.

The choice of which method to use for design purposes is purely optional and depends ultimately on the preferences of the designer. Each method has certain advantages and disadvantages which will be briefly outlined.

The transient response method is usually based on response to the Heaviside unit step function, $U(t)$. Results are easy to interpret when plotted graphically, but they are difficult and tedious to obtain because the characteristic equation has to be solved, and then the final expression plotted in graphical form. Another big disadvantage is that if any parameter is changed or if additional elements are put into the loop, the whole process has to be reworked. It is also very difficult to associate any characteristic in the response with particular elements in the loop.

Thus, while transient response can be used to identify a good or bad system, it does not often suggest how to modify the system so as to improve its response. These faults become very much worse when the degree, n, of the characteristic equation is greater than three.

With frequency response methods, mathematical labor is shorter and simpler. Also in this direction some simple aids exist. These will be discussed in a later section. The great advantage of frequency response methods is that the effect of modifying the elements in the system, or adding new components, can be easily accounted for. The disadvantage is that the response vector curves do not give a physical picture of system behavior. That means that a set of rules must be available to correlate frequency response curves with the transient behavior of the system. No concrete set of such rules exists, unfortunately, but there are some approximate rules which will shortly be given.

Successful use of either of these design techniques, therefore, depends largely on the skill of the engineer. Only with experience can he weigh the value of any design criteria.

In practice it is convenient to do the initial design work using frequency response methods. Once the design has been more or less finalized in this way, then a check can be made by plotting its transient response.

Response Criteria: Based on transient response to the unit step function, $U(t)$, response of a stable system will in general involve an overshoot, followed by a decaying oscillation. The response is generally considered satisfactory if the maximum overshoot is about 30 per cent of the step, with only two or three large overswings following it. Fig. 1*a*. Less than 10 per cent overshoot is sometimes necessary.

Decay of the oscillations depends on the values of the roots, $(-\alpha + j\Omega)$, of the characteristic equation. All oscillations will have substantially disappeared at time $t_d = 4/\alpha_m$, where α_m is the magnitude of the smallest real component of all the roots. The number of oscillations depends on the ratio α/Ω for each of the roots. A value of about 0.5 is usually quoted as satisfactory for this ratio.

A measure of sensitivity is given by the build-up time T_b. This has been variously defined as:

1. Time to pass through 1.0 for first time.
2. Time to get within a steady 2 per cent of 1.0.
3. Time to swing through 1.0 at maximum rate of response.

Based on the overall response function $Y_c(j\omega)$, the requirement for no steady-state positional error is that $M = 1$ when $\omega = 0$, or that $a_0 = b_0$, where

$$Y_c\,(j\omega) \equiv Me^{j\phi} = \frac{b_0 + b_1\,j\omega + \ldots + b_m\,(j\omega)^m}{a_0 + a_1\,j\omega + \ldots + a_n\,(j\omega)^n} \tag{16}$$

In practical servos $n > m$, so that $M \to 0$ and ϕ is negative as $\omega \to \infty$. Thus a typical response is of the form shown in Fig. 7.

The amplitude or $M(\omega)$ curve is very informative. High resonant peaks correspond to lightly damped roots in the characteristic equation; that is, α/Ω is about 0.2 or less. An ideal type of characteristic is shown in Fig. 7. If the maximum value of M is limited to 1.3 or 1.5, then in general a good transient response is obtained without too many overshoots.

Sensitivity is determined by the bandwidth ω_b. This is variously defined as:

1. $M(\omega_b) = 1.0$ beyond resonant peak, if response is of type shown in Fig. 7.
2. $\displaystyle\int_0^\infty \frac{M^2\,d\omega}{\omega_b} = 1.0$ holds for curve with no resonant peaks.

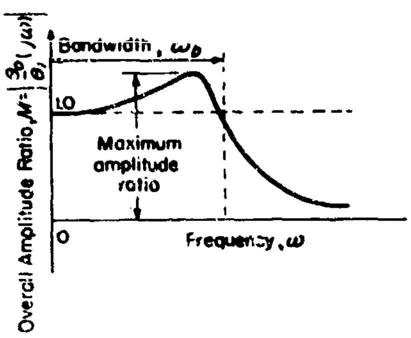


Figure 7. Limiting the maximum value of M to 1.3 to 1.5 results in an ideal transient response characteristic without too many overshoots.

3. $M(\omega_b) = 1/2$ beyond any resonant peak.

Since both relate to sensitivity a relationship between the bandwidth ω_b and the build-up time T_b might be expected. There is an approximate relationship between the two, but generally nothing more can be said except that increasing the bandwidth reduces the build-up time and hence improves the sensitivity of the servomechanism. An approximate relationship between the two can be established if an idealized frequency response, Fig. 8, is considered. Here $M = 1$ up to the bandwidth frequency ω_b, and is zero for all higher frequencies, while the phase angle is linear in bandwidth. The response of a system, having such a characteristic, to a step function is shown in Fig. 9. This response has a small value when $t = 0$, so the system is not physically realizable. Apart from this, its response is very much as desired.

Using the third of the definitions of T_b previously given, it can be shown that

$$T_b = \frac{\pi}{\omega_b} \tag{17}$$

This supports the previous remark on increasing the bandwidth. Generally to increase ω_b to achieve a more rapid response *the scalar gain constant K must be as large as possible.* However, as is shown by the Nyquist criterion, this can lead to instability, and almost invariably means a more oscillatory response. Therefore, a compromise value for K must be achieved. One of the fundamental problems of servo design is to get the maximum possible bandwidth for a given scalar gain K.

Nomenclature

a, b = Constants
j = Square root of minus one (symbolic)
K = Scalar gain constant
r = Order of servo
s = Laplace operator
T_b = Buildup time
T_d = Decay time
t = Time variable
$U(t)$ = Unit step function
$W(t)$ = Weighting function
$Y_c(s)$ = Overall servo transfer function
= $F(s)/G(s)$
$Y_o(s)$ = Loop transfer function
ω = Angular frequency, rad per sec
ω_b = Bandwidth of amplitude response

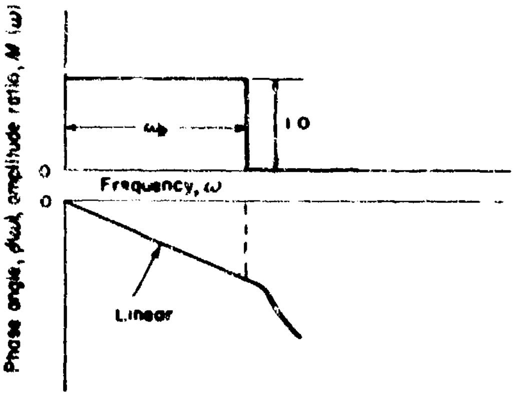


Figure 8. With the idealized frequency response shown here, $M = 1$ over the entire bandwidth and zero at higher frequencies. The phase angle is linear throughout the bandwidth.

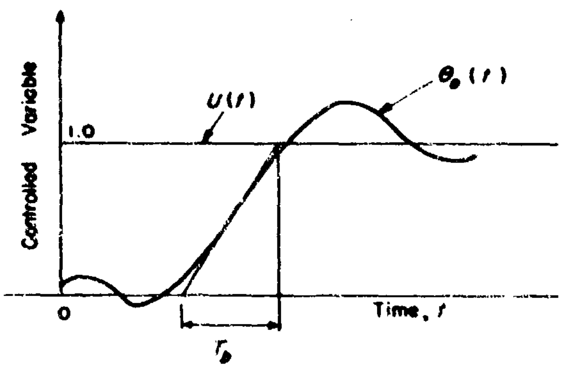


Figure 9. The response of a system with the characteristics shown in Figure 8 to a step function, $U(t)$, illustrates the relationship between bandwidth and buildup time. Response of such a system is much as desired; however, the system is physically unrealizable since response is so small at $t = 0$.

Steady-State Errors: Apart from sensitivity and stability another important factor in assessing performance is accuracy. Obviously, in any servo, high static accuracy is essential. A measure of static accuracy is given by the steady-state error in the response to a unit step function input $U(t)$, Fig. 10a.

In some systems dynamic accuracy is also very important. That is, following errors to continuously varying inputs must be very small. To assess the dynamic accuracy, the steady-state error when following an input which is increasing at unit rate

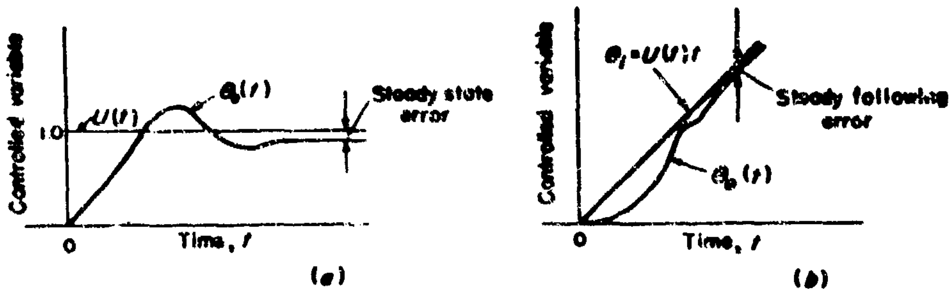


Figure 10. Static accuracy of a servo system is indicated by the steady-state error in response to a unit step-function input, a. Dynamic accuracy can be assessed by finding the steady-state error in response to a unit step velocity input, b.

can be easily found. In the case of position servos this input $U(t)t$ is called a *unit step velocity*, Fig. 10b.

Occasionally, in some position control servos, the output must be able to follow, with a small steady error, a constant acceleration input. Such an input is the *unit step acceleration* $U(t)t^2/2$. The difficulties involved here will shortly be discussed.

The relationship between the error and input is

$$\frac{e}{\theta_i}(s) = \frac{1}{1+Y_o(s)} \tag{18}$$

Substituting from Equation 7 gives

$$e(s) = \frac{\theta_i(s)}{\left[1+\frac{Kf(s)}{s^r g(s)}\right]} \tag{19}$$

Now Theorem 8, Sub-Topic 7.2.2, is used to obtain the steady-state error. This is

$$e_s = \lim_{t\to\infty}[e(t)] = \lim_{s\to 0}[se(s)] =$$

$$\lim_{s\to 0}\left[\frac{s^{r+1}\theta_i(s)}{s^r+K}\right] \tag{20}$$

since $f/g \to 1$ as $s \to 0$.

In the case of a unit step function $U(t)$, $\theta_i(s) = 1/s$, so that

$$e_s = \lim_{s\to 0}\left[\frac{s^r}{s^r+K}\right] \tag{21}$$

Obviously, for zero steady-state error, the requirement is that $r \geqq 1$. If $r = 0$, there is a static error of $e_s = 1/(1+K)$. For a unit velocity step input $\theta_i(t) = U(t)t$, $\theta_i(s) = 1/s^2$. Therefore,

$$e_s = \lim_{s\to 0}\left[\frac{s^{r-1}}{s^r+K}\right] \tag{22}$$

where the integer, r, is called the *order* of the servo.

Thus for zero steady following error $r \geqq 2$ is required. If $r = 1$, there is a steady following error $e_s = 1/K$. If $r = 0$, then the following error increases without limit. In other words, the servo is incapable of following the input.

This approach leads to Table 2 which can be extended at will and is symmetrical apart from the first term. As the table shows, a first order servo ($r = 1$) has a zero static error, but a finite steady following error to a step velocity input. A second-order servo has a zero steady following error for a step velocity input. For this reason second-order servos are frequently called *zero-velocity-error servos*, particularly in the case of displacement controllers.

Probably most mechanical servos are of the first-order kind. Where high dynamic accuracy is required, for example in gun-control systems, second and even third order systems are sometimes used. Here there are inherent stability problems to be solved. This will be illustrated with a very simple example.

Table 3—Steady Following Errors of Servos

Input $\theta_i(s)$	Order r	Steady Error e_s
$1/s$	0	$1/(1+K)$
	1	0
	2	0
	3	0
	4	0
$1/s^2$	0	∞
	1	$1/K$
	2	0
	3	0
	4	0
$1/s^3$	0	∞
	1	∞
	2	$1/K$
	3	0
	4	0

Suppose a second-order servo has the loop transfer function,

$$Y_o(s) = \frac{K}{s^2} \tag{23}$$

where the effects of time lags have been neglected for simplicity. The characteristic equation is therefore

$$s^2 + K = 0 \tag{24}$$

This has two imaginary roots $\pm j\sqrt{K}$, and so the system is unstable.

Now suppose the system loop transfer function is modified to

$$Y_o'(s) = \frac{K(1+Ts)}{s^2} \tag{25}$$

The characteristic equation is now

$$s^2 + KTs + K = 0 \tag{26}$$

The system is now stable, and still retains its zero-velocity-error characteristics. This type of problem will be investigated more thoroughly in a later article.

Performance Characteristics from Nyquist Plot: In a typical Nyquist plot, Fig. 11, the vector $\overline{OP}$ represents $Y_o(j\omega)$ for some particular frequency ω. Then to the same scale the vector $\overline{AP}$ represents $1 + Y_o(j\omega)$. Then the overall response function can be found by division.

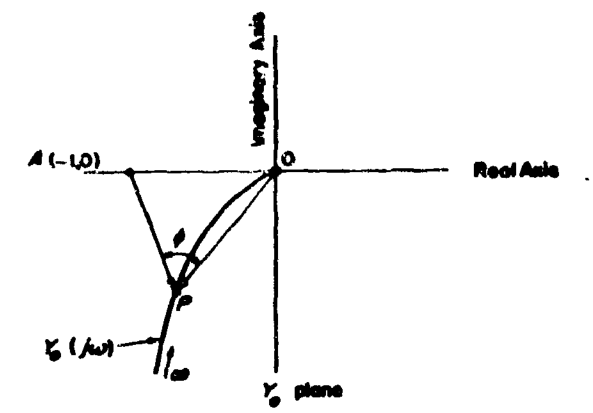


Figure 11. The vector OP in this typical Nyquist plot represents the value of $Y_o(j\omega)$ for some frequency, ω. The vector AP represents $1 + Y_o(j\omega)$. The overall response function is found by dividing OP by AP.

Thus

$$Me^{j\phi} \equiv Y_o(j\omega) = \frac{\overline{OP}}{\overline{AP}}$$

Results of this division are $M(\omega) = (OP)/(AP)$ (ratio of lengths) and $\phi(\omega) = \psi - \gamma = OPA$ (negative as shown).

Performing this process for a number of frequencies permits plotting of $M(\omega)$ and $\phi(\omega)$. This is a rather tedious task. It can however be avoided by superposing curves of constant M and ϕ on the Y_o plane. These contours are orthogonal circles, Fig. 12. M contours have their centers at $[-M^2/(M^2-1) + j0]$ and radii of $|M/(M^2-1)|$. The ϕ contours have their centers at $[-\frac{1}{2} - \frac{1}{2}j \cot \phi]$ and radii of $|\frac{1}{2} \csc \phi|$.

If as previously suggested the maximum amplitude ratio is limited to 1.3, then the region shown shaded in Fig. 12 is prohibited. The servo having the loop response shown plotted in Fig. 12 obviously has a maximum overall amplitude ratio of 1.3.

Since changing the scalar gain constant K changes the scale of the Nyquist plot it obviously must be set so that the curve does not enter the prohibited region. The best way to do this is first to plot a curve of

$$\frac{f(j\omega)}{(j\omega)^r g(j\omega)}$$

which is just $Y_o(j\omega)$ with K omitted. Then the critical stability point is $(-1/K, 0)$ instead of $(-1, 0)$, Fig. 13. Thus instead of altering the contour when K is changed, the critical stability point is moved until the contour is in the right position relative to it. Unfortunately, changing the critical stability point involves changing the scale and location of the M and ϕ constant contours. There are constructions for insuring that the critical stability point $(-1/K, 0)$ is positioned so that the $Y_o(j\omega)$ locus just touches the required M contour, thus fixing the optimum value of K.

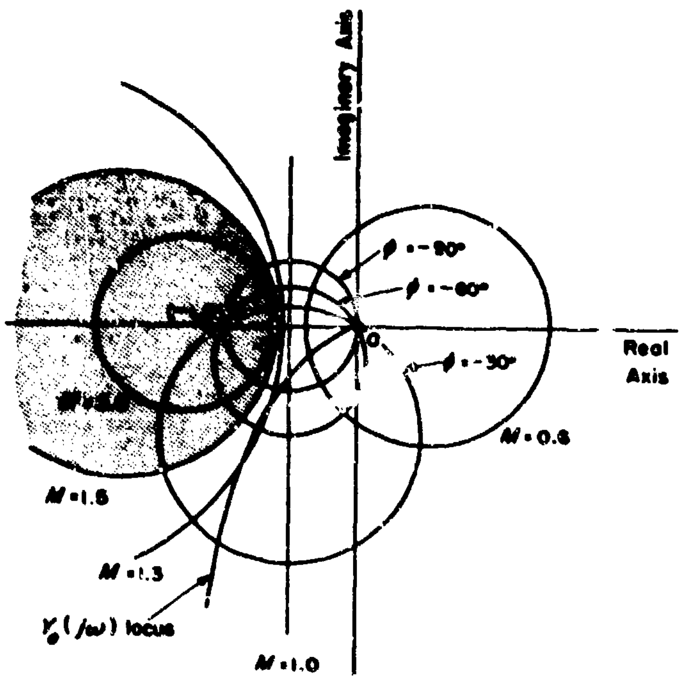


Figure 12. Plotting a series of orthogonal circles representing constant M and ϕ in the same plane as the Nyquist plot simplifies determination of the maximum amplitude ratio M. The servo having the loop response plo here has a maximum amplitude ratio of

These constructions are somewhat complicated and some prefer a simpler method involving two figures of merit known as the *gain margin*, G, and *phase margin* β, Fig. 13. Desired values are: G from 0.5 to 0.8, and β from 35 to 45 degrees. Thus once the point A and hence the value K have been fixed to agree with these figures, it is possible to plot $Y_o(j\omega)$ to the correct scale on a graph containing contours of constant M and ϕ.

Value of the gain margin and phase margin is purely their use in obtaining very simply an approximate best gain constant K. They are not reliable figures of merit to assess performance, although some have used them as such. The danger of doing this is demonstrated by the dotted re-

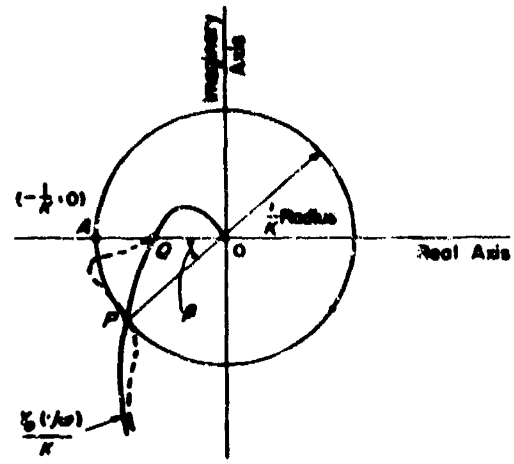


Figure 13. One way of insuring satisfaction of the Nyquist criterion is to move the critical stability point by changing the value of K, the scalar gain constant. Construction insures that the $Y_o(j\omega)$ locus just touches the required M value to fix the optimum value of K.

sponse shown in Fig. 13. Although this satisfies the optimum values of G and β, the curve comes very close to the critical point and has a high maximum M. Thus the value of K would have to be much less than that predicted by the above method, unless the locus is modified to give better characteristics in the neighborhood of the critical point. That is probably what would happen.

The order of the servo and therefore its steady-state errors are also revealed by the Nyquist plot. This is because $Y_o(j\omega)$ behaves like $K/(j\omega)^r$ at low frequencies. Thus for $r = 1$, the curve approaches the negative imaginary axis asymptotically. Fig. 14. While for $r = 2$ the loop response locus approaches the negative real axis asymptotically, and so on. This is particularly useful if only an experimental Nyquist plot is available. Then if the order can be found and K is known, the steady-state errors can be obtained from Table 3.

As previously stated, second and higher-order

Nomenclature

- $e(s)$ = Transformed error
- G = Gain margin
- K = Scalar gain constant
- $M(\_)$ = Modulus of $Y_c(j\omega)$
- r = Order of servo
- T = Time constant
- $U(t)$ = Unit step function
- $Y_c(j\omega)$ = Overall harmonic response function
- $Y_o(s)$ = Loop transfer function
- β = Phase margin
- $\theta_i(s)$ = Transformed input
- $\theta_o(s)$ = Transformed output
- $\phi(\omega)$ = Phase or argument of $Y_c(j\omega)$

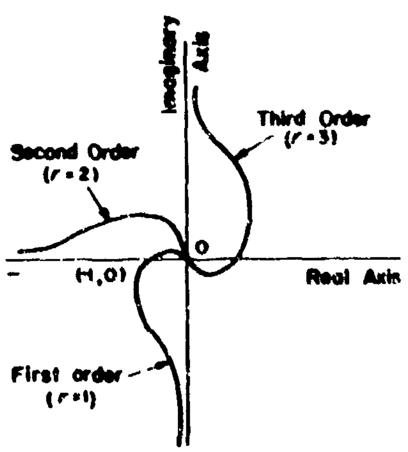


Figure 14. Nyquist plots may be used to determine the order, r, of a servo because $Y_o(j\omega)$ behaves like $K/(j\omega)^r$ at low frequencies. Generalized Nyquist plots for first, second and third-order servos are shown.

servos are inherently unstable. Curve *a*, Fig. 15 shows a second-order servo with one time lag in the loop, which can be represented by the response function,

$$Y_o(j\omega) = \frac{K}{(j\omega)^2 (1 + j\omega T_1)} \qquad (27)$$

This is seen to be unstable. By modifying the response function to

$$Y_o'(j\omega) = \frac{K(1 + j\omega T)}{(j\omega)^2 (1 + j\omega T_1)} \qquad (28)$$

the system can be made conditionally stable if T is large enough.

Transient Response Criteria from Transfer Function: As stated previously, it is not practical to attempt to express the transient response in terms of the system parameters. Indeed if $n > 4$, then this is not possible. However, an attempt has been made to give relations between the parameters for certain optimum types of response.

Whiteley's figures, Table 4, are normally for a slightly overdamped response (all roots of characteristic equation real and negative). The figures are given for systems according to their order r and the degree n of the characteristic equation. Whiteley considers a system with a loop transfer function of the form,

$$Y_o(s) = \frac{C_{r-1}\,\Omega^{n-r+1} s^{r-1} + C_{r-2}\,\Omega^{n-r+2} s^{r-2} + \ldots + \Omega^n}{C_n s^n + C_{n-1}\,\Omega s^{n-1} + \ldots + C_r\,\Omega^{n-r} s^r} \qquad (29)$$

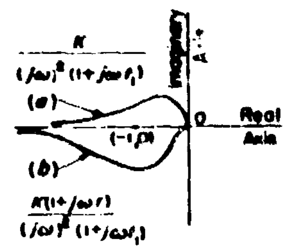


Figure 15. A second-order servo with one time lag in the loop is seen to be unstable, *a*. The system can be made conditionally stable, *b*, by modifying the response function and making t sufficiently large.

and Table 4 quotes his figures for the coefficients C, the maximum overshoot and the build-up time. To illustrate the use of the table, a second-order servo with a fifth-degree characteristic equation will be considered. By substituting values from Table 4 in Equation 29

$$Y_o(s) = \frac{18\,\Omega^4 s + \Omega^5}{s^2 (s^3 + 9\,\Omega s^2 + 29\,\Omega^2 s + 38\,\Omega^3)} \qquad (30)$$

The scalar gain factor, $K = \Omega^2/38$. Hence the build-up time $T_b = 3.85/\Omega = 0.625/\sqrt{K}$.

Table 4—Whitely's Optimum Parameters

Servo Order		Coefficients					Buildup Time	Overshoot, max.
r	n	C_1	C_2	C_3	C_4	C_5	ΩT_b	
1	2	1.4	1	—	—	—	3.3	0.045
1	3	2	2	1	—	—	3.8	0.06
1	4	2.6	3.4	2.6	1	—	4.2	0.10
2	2	2.5	1	—	—	—	0.85	0.10
2	3	6.3	5.1	1	—	—	1.65	0.10
2	4	12	16	7.2	1	—	2.65	0.10
2	5	18	38	29	9	1	3.85	0.10

Based on response to a unit step input.

Relative Damping Criterion: Transient response to a step function is largely determined by the roots of the characteristic equation $G(s) = 0$. A root of the form $s = -\alpha + j\Omega$ gives rise to a term of the form

$$Ae^{-\alpha t} (\sin \Omega t + \cos \Omega t) \qquad (31)$$

As previously stated, the magnitude of α fixes the time taken for this particular component oscillation to die away. The ratio α/Ω determines the decay of amplitude per cycle of oscillation. Some fix the minimum value of this ratio as $\alpha/\Omega = 0.5$. This gives a decay in the ratio of 0.206 per half-cycle. That is $A_2/A_1 = 0.206$ in Fig. 16.

If 0.5 is used as the minimum ratio of α/Ω for each root of the characteristic equation, all

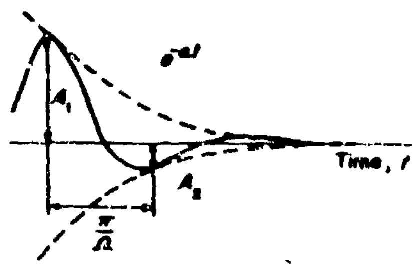


Figure 16. Time required for oscillation caused by a transient to die away is determined by the value of α. The ratio, α/Ω determines the decay in amplitude per cycle of oscillation. An accepted minimum value of α/Ω for a suitably stable servo is 0.5.

the roots must lie in the region of the s plane shown shaded in Fig. 17a. Making this restraint permits the Nyquist and Algebraic stability criteria to be modified so that they become relative as well as absolute criteria. This can be most easily done with the algebraic criteria. The modified characteristic equation is

$$G'(s) \equiv G_1(s)\ G_2(s) = 0 \qquad (32)$$

where

$$G_1(s) = a_n s^n + a_{n-1} e^{-j\lambda} s^{n-1} + \ldots + a_1 e^{-(n-1)j\lambda} s + a_0 e^{-nj\lambda}$$

$$G_2(s) = a_n s^n + a_{n-1} e^{j\lambda} s^{n-1} + \ldots + a_1 e^{(n-1)j\lambda} s + a_0 e^{nj\lambda}$$

and

$$\tan \lambda = \frac{\alpha}{\Omega} \qquad (33)$$

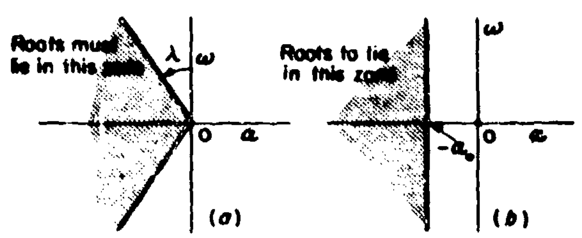


Figure 17. If 0.5 is accepted as a minimum for α/Ω, all roots of the characteristic equation must fall within the shaded zone at a. If all roots fall within the shaded zone at b, total oscillations will decay in time determined by α_0.

Equation 32 is a polynomial in s with real coefficients, so Hurwitz' criterion can be applied. This yields desired relationships between the system coefficients. The method may involve some tedious numerical work since the degree of the characteristic equation is doubled. Some simplifying techniques have been developed, Reference 436-1 but these are too lengthy to discuss here.

Others have developed similar criteria locating the roots in other restricted regions of the s plane. For instance, to insure that all roots are in the shaded region in Fig. 17b, magnitude of all the real parts of the roots is made greater than a certain value α_0. This means the total oscillations will decay within a time determined by α_0.

7.2.4 Analyzing a Servo System

Transient response criteria provide methods which are particularly suited to the analysis of relatively simple servo systems. Correspondingly, the equations for the system must be relatively manageable. A simple position control servomechanism will be analyzed in this Sub-Topic. Sub-Topics 7.2.1 through 7.2.3 have outlined the fundamental concepts of closed-loop control, briefly discussed the mathematics of control systems, and outlined performance criteria. This Sub-Topic illustrates the application of this material.

Position Control Servo: Function of the position control system, Fig. 1, is to insure alignment between two shafts. Potentiometers attached to the input and output shafts give voltages which are proportional to the input and output displacements, respectively. These voltages are subtracted, and the difference between them gives a measure of error, or

$$V_e = V_i - V_o = \beta(\theta_i - \theta_o) = \beta e \qquad (1)$$

where the voltage-displacement ratios of the two potentiometers are taken to be equal and constant. This error voltage is then fed to an electronic amplifier.

The amplified voltage, $V = K_a V_e$, is then applied to a dc motor which gives a roughly proportional torque, thus

$$T = K_m V \qquad (2)$$

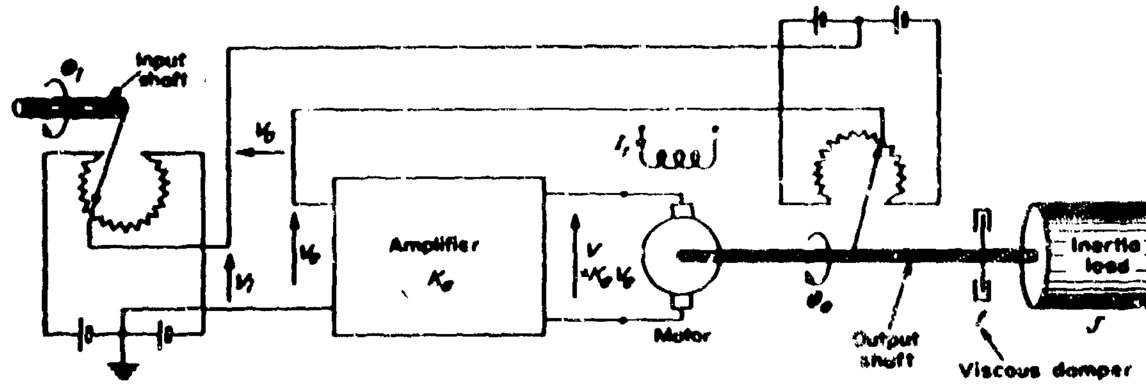


Figure 1. This simple position-control servomechanism is used to illustrate the application of servo theory to an actual system. Function of the system is simply to rotate the output shaft to the same position as that of the input shaft.

This torque in driving the output shaft is opposed by a load, which in this case is the result of an inertia J and a damper f. This load is inclusive of the inertia and mechanical resistance of the motor itself. Relationship between the output displacement, θ_o, and torque, T, is, therefore,

$$J \frac{d^2\theta_o}{dt^2} + f \frac{d\theta_o}{dt} = T \quad (3)$$

Laplace transformation of Equation 3, taking zero initial conditions, results in the transfer function,

$$\frac{\theta_o}{T}(s) = \frac{1}{Js^2 + fs} \quad (4)$$

It is now possible to construct a block diagram for the complete system, Fig. 2a. This differs from the conventional block diagram of a closed-loop system in that quantities proportional to the output and input are subtracted at the differential, rather than the quantities themselves. If, however, the constants of the potentiometers are equal, then it is possible to redraw the diagram in the conventional manner, Fig. 2b. Here the potentiometer constant is included in the loop transfer function. This change is made purely to conform with normal practice in representing servos by block diagrams.

It can be seen that the loop transfer function is given by

$$\frac{\theta_o}{e}(s) = Y_o(s) = \frac{K}{s\left(1 + \frac{Js}{f}\right)} \quad (5)$$

where $K = K_a K_m \beta / f$. Then the overall transfer function is given by

$$\frac{\theta_o}{\theta_i}(s) = Y_c(s) = \frac{Kf}{Js^2 + fs + Kf} \quad (6)$$

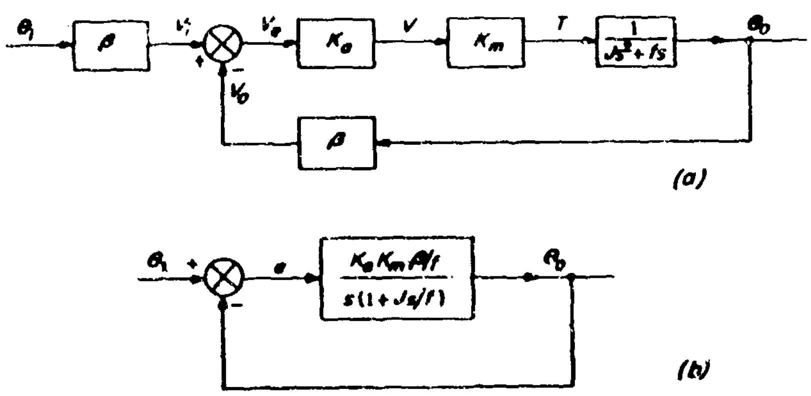


Figure 2. Block diagrams of the simple position-control system may be constructed, a, to simulate actual system layout or b, in line with conventional servo practice.

In dealing with quadratic factors such as the denominator of the transfer function in Equation 6, it is very helpful to adopt a well-known notation. Then Equation 6 can be written as

$$\frac{\theta_o}{\theta_i}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (7)$$

where

$$\omega_n = \sqrt{\frac{Kf}{J}} = \sqrt{\frac{K_a K_m \beta}{J}} \qquad (8)$$

$$\zeta = \frac{f}{\sqrt{4 K_a K_m \beta J}} \qquad (8a)$$

The servo system is absolutely stable provided f is positive. In order to determine its relative stability by transient response methods it is necessary to determine the roots of the characteristic equation. In this case the roots are located in the left half of the s plane as shown in Fig. 3.

Once the roots have been found, the response of the system to a unit-step function input can be found. The type of response depends on the value of ζ. If $\zeta > 1$ the response is purely exponential, but if $\zeta < 1$, the response also contains oscillatory components. Demarcation between the two types of responsible exists when $\zeta = 1$ and is called the *critically damped* case. Expressions for the response to unit step function input for various values of ζ are given in Table 1, while Fig. 4 plots these responses for numerical values of ζ.

In this application, K_a, the gain of the amplifier,

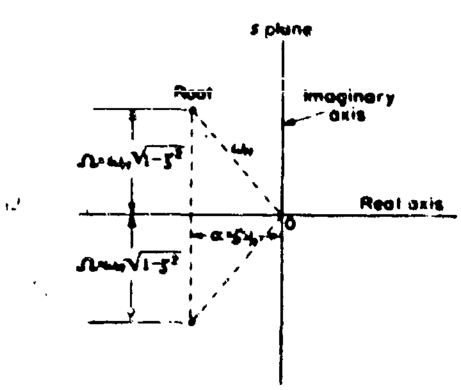


Figure 3. Roots of the characteristic equation are located in the left half of the s plane. Relative stability of the servo is determined by these roots.

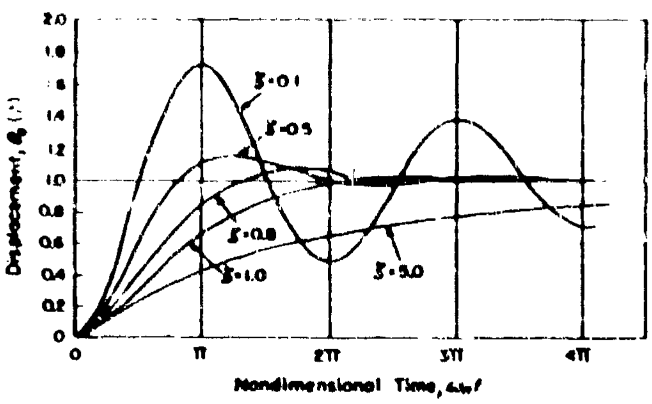


Figure 4. Response of a servomechanism with a quadratic characteristic equation to a unit step-function input for various values of ζ.

is an easily adjusted parameter, and may be set to give optimum response. Often, $\zeta = 0.5$ is taken as the most desirable case. If this value is substituted in Equation 8a, K_a becomes

$$K_a = \frac{f^2}{K_m J \beta} \qquad (9)$$

In some applications, however, it may be necessary to have a more heavily damped response. For instance by choosing $\zeta = 0.8$, very little overshoot or oscillation is obtained, Fig. 4. This increased damping unfortunately results in a more sluggish response with a longer build-up time.

This servo is of the first order as shown by Equation 5. Therefore, it has zero steady-state positional error. Here, however, positional accuracy really depends on the accuracy of the potentiometers. In response to a unit-velocity input, $U(t)t$, there is a steady following error of $1/K$. Thus the so-called velocity error can be reduced by increasing K_a, but here again improvement is

Table 1—Response to Unit Step Function Input

Values of ζ	Response Equation
>1	$\theta_o(t) = 1 - e^{-\zeta\omega_n t}\left[\cosh\gamma t + \frac{\zeta}{\sqrt{\zeta^2 - 1}}\sinh\gamma t\right]$
$=1$	$\theta_o(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$
<1	$\theta_o(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\Omega t + \frac{\zeta}{\sqrt{1 - \zeta^2}}\sin\Omega t\right]$

achieved at the cost of reducing stability.

In most practical servos the equations will be complicated by time lags in the control equipment. Suppose that in the present case there is a time lag τ in the motor between the application of voltage V and the development of torque T. Mathematically this can be written

$$\frac{T}{V}(s) = \frac{K_m}{1 + \tau s} \qquad (10)$$

The loop and overall transfer functions of the complete system now become, respectively,

$$\frac{\theta_o}{e}(s) = \frac{K}{s\left(1 + \dfrac{Js}{f}\right)(1 + \tau s)} \qquad (11)$$

$$\frac{\theta_o}{\theta_i}(s) = \frac{Kf}{[J\tau s^3 + (J + f\tau)s^2 + fs + Kf]} \qquad (12)$$

The stability of the system is now dependent upon the magnitude of K. Also, the increased complexity of the transfer functions makes analysis by transient response methods very tedious. In the earlier case, in which there were no time lags in the control equipment, it was possible to analyze the system by transient methods, because the assumptions made kept the system equations fairly simple. As more complex systems are encountered, it will be found that their analysis by transient methods becomes extremely difficult, and frequency response techniques will have to be used.

Nomenclature

- e = Error
- f = Viscous damping coefficient
- J = Moment of inertia
- K = Overall gain constant
- K_a = Amplifier gain constant
- K_m = Motor gain constant
- s = Laplace operator
- T = Torque
- t = Time, variable
- V = Voltage
- $Y_o(s)$ = Overall transfer function
- $Y_e(s)$ = Loop transfer function
- α = Real part of complex conjugate root $= \omega_n \zeta$
- β = Voltage displacement constant of potentiometer
- $\gamma = \omega_n \sqrt{\zeta^2 - 1}$ for $\zeta > 1$.
- ζ = Damping ratio
- θ_i = Input rotation
- θ_o = Output rotation
- τ = Time constant
- Ω = Imaginary part of complex root $= \omega_n \sqrt{1 - \zeta^2}$ for $\zeta < 1$
- ω_n = Modulus of complex conjugate root

7.2.5 Methods for Determining Transient Response of Servo Systems

7.2.5.1 RELATION BETWEEN TRANSIENT RESPONSE AND FREQUENCY RESPONSE

The usefulness of frequency-response techniques in the design of closed-loop systems is that the characteristics of the component elements of the loop can be combined by simple arithmetical manipulations of addition and multiplication. Also, since the relationship between open-loop and closed-loop characteristics is clear-cut in the frequency domain, it is possible to use open-loop curves for system design.

Pure sinusoidal input functions are unlikely to be encountered in practice, and the response to more realistic inputs should be considered. In an attempt to represent severe demands on the system a designer usually considers impulse, step, and constant-velocity input functions. Although these are rather idealized inputs it is possible to assess response to them in terms of a few simple criteria.

The task of determining response to these or more general inputs is very difficult for other than simple systems. To attempt to design complex servos in terms of transient response would be tiresome unless special techniques were available. In earlier sections some simple empirical relations between frequency and transient response were given. But these are not rigorous. Hence, even if the frequency response is satisfactory according to gain and phase margins, etc., it is not certain that the transient response will be at all satisfactory. Therefore, as a final check on design values, it is highly desirable to plot the transient response, usually to a step-function input. This may be approached either from knowledge of the harmonic-response function or directly from the transfer function. Of these the latter is perhaps more general, although the former is very convenient.

As a background for the techniques to be outlined, the simple concept of frequency response will be a starting point and from it the idea of Fourier and Laplace transforms will be developed.

Fourier's Theorem: More general types of functions than sinusoids are general periodic functions, Fig. 1. Here the repetition period is T. Fourier's theorem is a mathematical way of saying that the periodic function can be broken down into a constant, or "dc," component, plus a fundamental sine wave of period T, plus second, third and higher harmonic components. Mathematically

$$\theta_i(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \tag{1}$$

where $\omega_0 = 2\pi/T$ = fundamental frequency.

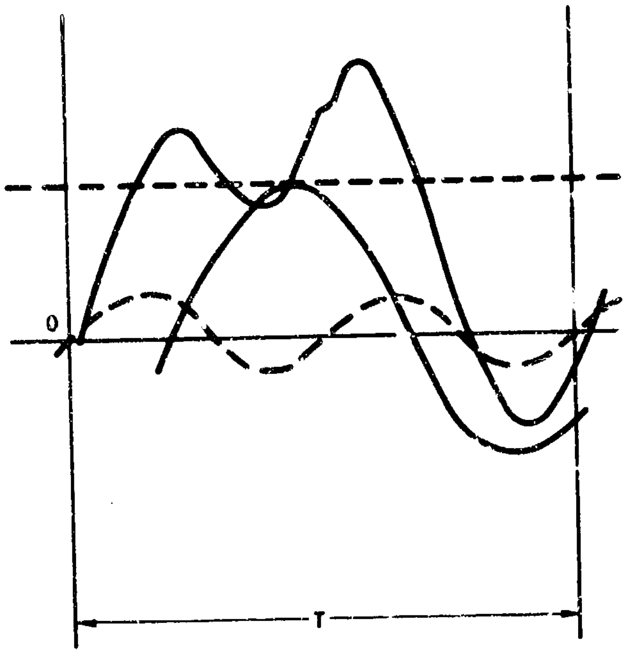
For convenience, the exponential form has been used for harmonic components. Hence the c_n coefficients, denoting the relative amplitude and phase of each component, are in general complex. They are determined by

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} \theta_i(t) e^{-jn\omega_0 t}\, dt \tag{2}$$

An important feature of linear systems is that the response to an input containing several components is the sum of the responses to the separate components. Thus, in this case, the response is the sum of responses to the dc term and fundamental and higher harmonics. Thus if $Y_c(j\omega)$ is the overall harmonic-response function for frequency ω, the system output is

$$\theta_o(t) = \sum_{n=-\infty}^{\infty} c_n Y_c(jn\omega_0) e^{jn\omega_0 t} \tag{3}$$

Thus each component is amplified and phase-shifted according to the value of $Y_c(j\omega)$ at its particular frequency. Diagrammatically this can be illustrated by means of a frequency spectrum for $\theta_i(t)$. This is shown in Fig. 2a; Fig. 2b shows a typical response function. These may be combined to give the spectrum of the output as shown in Fig. 2c.



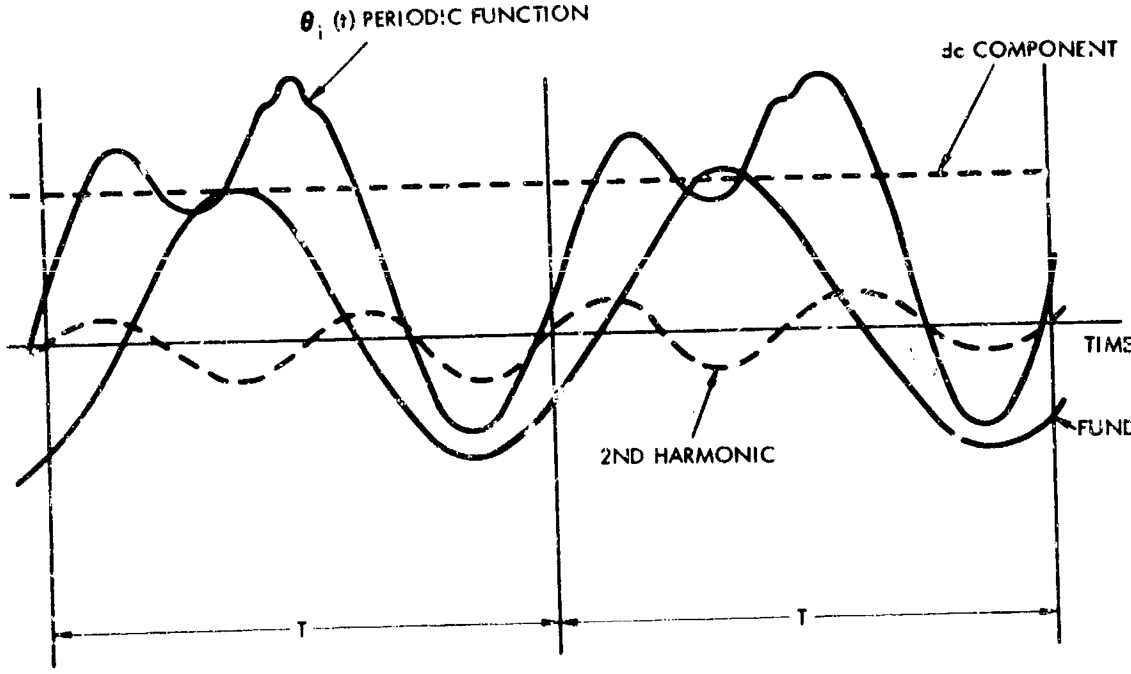


Figure 1. Periodic input function. By means of Fourier's theorem it is possible to evaluate dc component, and fundamental and higher harmonics.

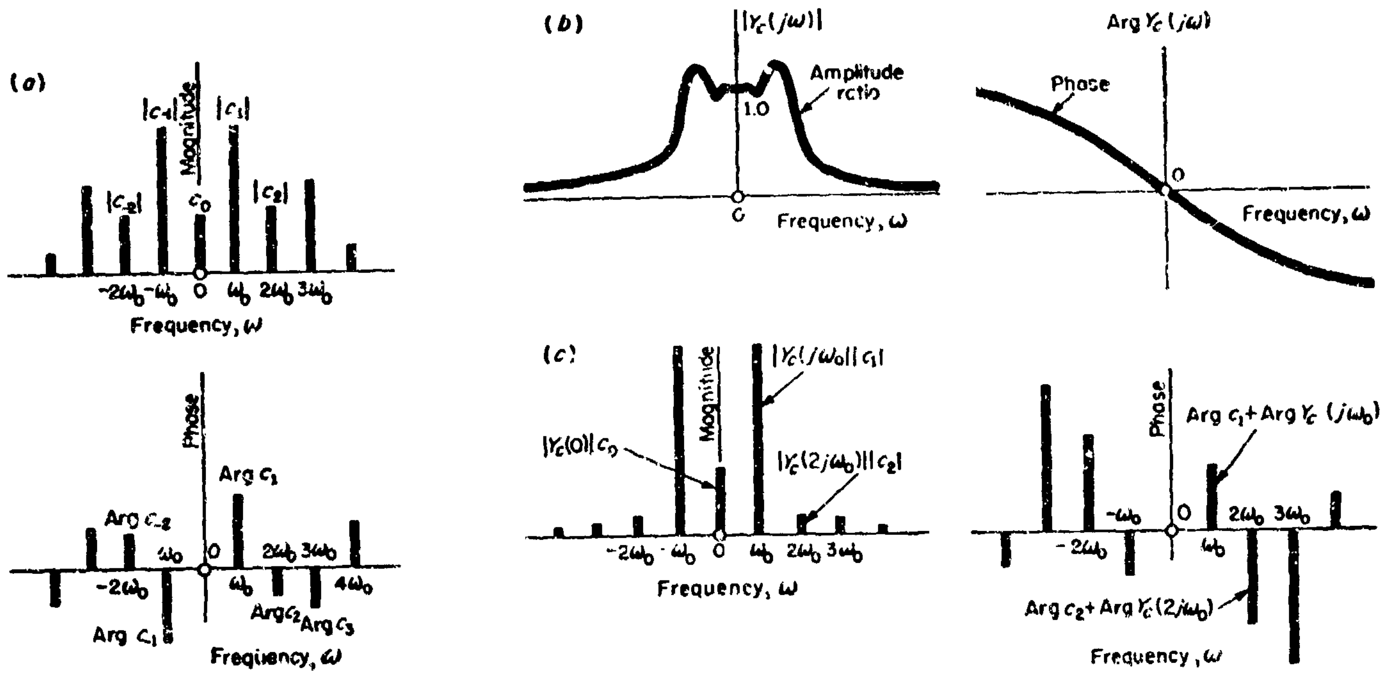


Figure 2. The spectrum of a periodic input, shown by magnitude and phase curves at *a*. The spectrum occurs at discrete multiples of the fundamental frequency ω_0. The amplitude and phase curves of a typical system are shown at *b*. These act on the input spectrum to produce the discrete output spectrum at *c*.

Although more general than sinusoidal functions, periodic functions are still too restrictive to be classed as general inputs. Therefore, can the spectrum ideas be extended to an aperiodic function, Fig. 3*a*? This can be done by the Fourier integral theorem, which states mathematically

$$\theta_i(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Theta_i(j\omega)\, e^{j\omega t}\, d\omega \qquad (4)$$

Equation 4 is developed from Equation 1 by first assuming function $\theta_i(t)$ to be part of a periodic wave of very large period T. Such a periodic wave would have a discrete frequency spectrum of spacing $2\pi/T$. Then if T is considered to become infinite, the segment of the periodic wave becomes the aperiodic function. Also the spectrum closes up and becomes ultimately a continuous curve, Fig. 3*b*. Then instead of definite components at 0, ω_0, $2\omega_0$, . . . , the harmonic components become continuously distributed throughout all frequencies. An amount $\Theta_i(j\omega)d\omega$ can be thought of to lie in the range ω to $\omega + d\omega$. The spectral function is given by

$$\Theta_i(j\omega) = \int_{-\infty}^{\infty} \theta_i(t)\, e^{-j\omega t}\, dt \qquad (5)$$

where $\theta_i(t)$ and $\Theta_i(j\omega)$ are said to be a Fourier transform pair.

Once again the system responds separately to each component $\Theta_i(j\omega)d\omega\; e^{j\omega t}$ so that the output is given by

$$\theta_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Theta_i(j\omega)\, Y_c(j\omega)\, e^{j\omega t}\, d\omega \qquad (6)$$

If the input spectral function can be found, the output response can be evaluated from Equation 6. Most of this part of this article will be devoted to approximate methods of achieving this. These approximations have to be used because it is often impossible to obtain $\Theta_i(j\omega)$ explicitly. The necessary condition for doing so is that

Nomenclature

$A(t)$ = Response to unit-step function
s = Complex frequency variable
T = Periodic time
t = Time variable
$U(\omega)$, $V(\omega)$ = Real and imaginary parts of $Y_c(j\omega)$
$u(t)$ = Unit-step function
$W(t)$ = Weighting function
$Y_c(s)$ = Overall complex frequency-response function
$\delta(t)$ = Unit-impulse function
$\Theta_i(s)$, $\Theta_o(s)$ = Transformed input and output
$\theta_i(t)$, $\theta_o(t)$ = Input and output
ω = Frequency variable

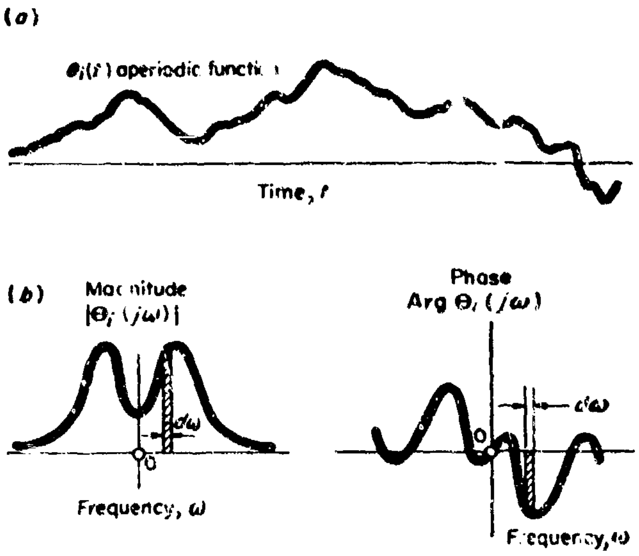


Figure 3. For a general aperiodic input function frequency components do not occur at discrete values but are distributed continuously. Fourier integral enables one to obtain for a general function at a the continuous spectrum at b.

$$\int_{-\infty}^{\infty} |\theta_i(t)|\,dt$$

exists as a finite value. This is not easily satisfied; for example, step-function input violates this condition. For this reason the Laplace transform method is much more powerful.

Laplace Transform Method: Briefly this method splits $\theta_i(t)$ into damped sinusoidal components, e^{st}, where $s = \alpha + j\omega$, each component being $\Theta_i(s)$ $e^{st}ds$. Since s is complex, $\Theta_i(s)$ is a function of a complex variable. Analogous to Equation 5, $\Theta_i(s)$, called the Laplace transform of $\theta_i(t)$, is given by

$$\Theta_i(s) = \int_0^{\infty} \theta_i(t)\, e^{-st}\, dt \tag{7}$$

A lower limit of 0 is taken since practical input functions must have a time origin.

The task of recovering $\theta_i(t)$ from $\Theta_i(s)$ involves a difficult contour integration. Thus

$$\theta_i(t) = \frac{1}{2\pi j}\int_C \Theta_i(s)\, e^{st}\, ds \tag{8}$$

The contour chosen is a line parallel to the ω axis to the right of all singularities of $\Theta_i(s)$. Evaluation of Equation 8 is normally a specialist's task, but standard tables exist for many functions.

Again by linear superposition, the output is the sum of the responses to the component damped sinusoids. Thus the output complex frequency spectrum is given by $\Theta_o(s) = Y_c(s)\ \Theta_i(s)$. As a time function,

$$\theta_o(t) = \frac{1}{2\pi j}\int_C Y_c(s)\,\Theta_i(s)\, e^{st}\, ds \tag{9}$$

Methods of overcoming this formidable integral will be discussed in a later part of this article. Here the calculation of output response from Equation 6 is resumed.

Output Response: The response to a δ-type impulse is called the weighting function $W(t)$ of the servo. The response to unit step function is denoted by $A(t)$.

The spectral function of a δ function is unity. That is, it contains equal amounts of all frequency components. Substituting in Equation 6 gives

$$W(t) = \int_{-\infty}^{\infty} Y_c(j\omega)\, e^{j\omega t}\, d\omega \tag{10}$$

Since $W(t)$ must be zero for all negative time, this can be simplified to

$$W(t) = \frac{2}{\pi}\int_0^{\infty} U(\omega)\cos\omega t\, d\omega \tag{11a}$$

or

$$W(t) = \frac{-2}{\pi}\int_0^{\infty} V(\omega)\sin\omega t\, d\omega \tag{11b}$$

Exact evaluation of these integrals is usually very difficult, but a number of approximate ways have been developed, as well as mechanical computation aids. Better approximations can be found for $A(t)$ rather than for $W(t)$.

Since

$$A(t) = \int_0^t W(t)\,dt$$

one gets

$$A(t) = U(0) + \frac{2}{\pi}\int_0^{\infty} \frac{V(\omega)}{\omega}\cos\omega t\, d\omega \tag{12a}$$

or

$$A(t) = -\frac{2}{\pi}\int_0^{\infty} \frac{U(\omega)}{\omega}\sin\omega t\, d\omega \tag{12b}$$

Either Equation 12a or Equation 12b can be used to find $A(t)$ but one integrand will usually converge to zero more rapidly than the other, making it more suitable for approximation purposes.

Equation 12 can be obtained in a more illustrative way. The Fourier integral expression for unit step function is

$$u(t) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\sin \omega t}{\omega}\, d\omega \qquad (13)$$

Then, considering system response to separate components,

$$A(t) = \frac{Y_c(0)}{2} + \frac{1}{\pi}\int_0^\infty \frac{M(\omega)\sin(\omega t + \phi)}{\omega}\, d\omega \qquad (14)$$

A little manipulation then leads to the two forms of Equation 12.

Approximate Evaluation of Integrals: Fig. 4a shows the variation of a typical function $U(\omega)/\omega$ against ω, and in Fig. 4b $\sin \omega t$ is plotted against ω. The product of these two functions is the integrand of Equation 12b, Fig. 4c. For computation the integrand must be finite at $\omega = 0$ and should converge rapidly. If not, the other component must be used or else some other artifice used.[a] Computational integration must stop at a finite frequency Ω. Errors involved will be small if Ω is chosen sufficiently large.

The order of error can be estimated if U/ω or V/ω (whichever is used) is approximated by c/ω^2 for $\omega > \Omega$, where $c = \Omega U(\Omega)$ or $\Omega V(\Omega)$. Then a pessimistic estimate of the error is $2U(\Omega)/\pi$ or $2V(\Omega)/\pi$.

A series of aids for carrying out this computation is based on approximating the form of U/ω or V/ω up to $\omega = \Omega$. One method[b] approximates the curve as the sum of a number of trapezoidal components, chosen by cut-and-try, Fig. 5a. Then the response is the sum of the contributions due to the separate trapezoids. One such trapezoid is shown in Fig. 5b, suitably labelled. The response component due to this trapezoid is

$$\theta_o(t) = \frac{2A_t}{\pi}\left[\frac{\sin\left(\frac{\omega_b - \omega_a}{2}\right)t}{\left(\frac{\omega_b - \omega_a}{2}\right)t} \times \frac{\sin\left(\frac{\omega_a + \omega_b}{2}\right)t}{\left(\frac{\omega_b + \omega_a}{2}\right)t}\right] \qquad (15)$$

where A_t is the area of the trapezoid. All such components must be added. Both terms in square brackets in Equation 15 are of the form $(\sin x)/x$ and this function has been extensively tabulated in Reference 18. By the use of these tables the computation becomes extremely simple.

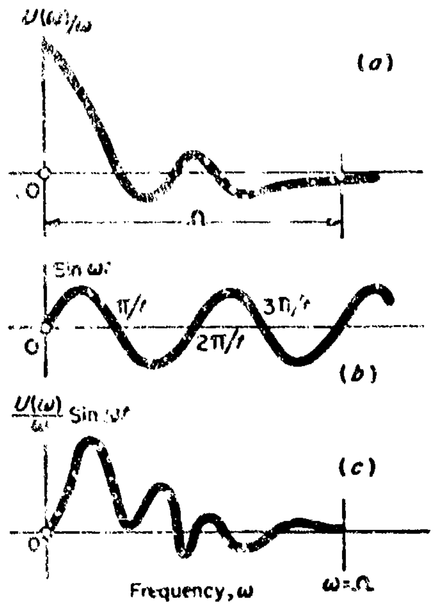


Figure 4. Typical functions involved in Fourier integral calculation of $A(t)$. $U(\omega)/\omega$ or $V(\omega)/\omega$ is plotted at a, while b shows $\sin \omega t$ for a particular time instant t. The product of these functions at t is the required integrand.

In another method[c] for performing the integration, U/ω or V/ω is approximated by straight-line segments, Fig. 6. Once again components of response due to the separate segments must be added. The component due to the segment in the interval ω_a to ω_b, if Equation 12b is used, is

$$\theta_o(t) = \frac{2}{\pi}\left[\frac{\cos \omega_a t}{t}(a - b) - \frac{\cos \omega_b t}{t}(a + b) + \frac{2b}{(\omega_b - \omega_a)t^2}(\sin \omega_b t - \sin \omega_a t)\right] \qquad (16)$$

If Equation 12a is chosen, a slightly different equation[c] must be used instead of Equation 16.

A third method involves the expression of U/ω (or V/ω) as a series of frequency impulses. The curve is divided into strips of width $\Delta\omega$, Fig. 7, and height $a_1, a_2, a_3, \ldots$. Then each impulse is taken at the center of a strip with weight equal to the area of the particular strip. Thus,

$$\frac{U(\omega)}{\omega} = \Delta\omega \sum_n a_n \delta(\omega - \omega_n) \qquad (17)$$

where $a_n = U(\omega_n)/\omega_n$.

Note, a δ function of frequency is defined exactly

a. *Reference 439-1*
b. *Reference 426-1*
c. *Reference 439-1*

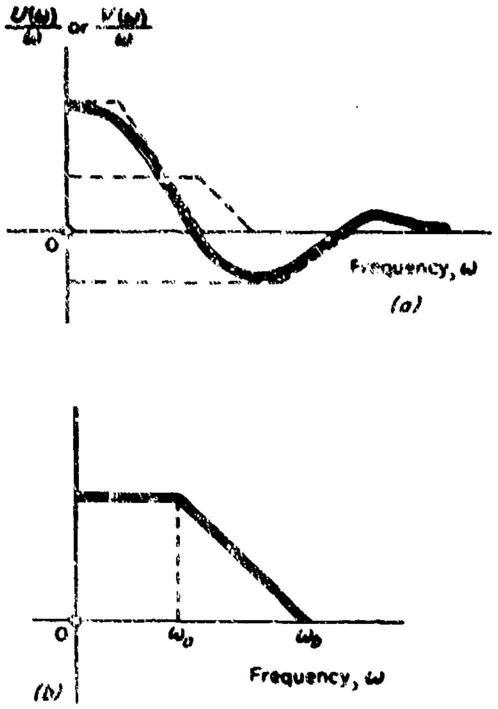


Figure 5. Approximation to $U(\omega)/\omega$ or $V(\omega)/\omega$ by trapezoidal segments is shown at a. A particular trapezoid is shown at b.

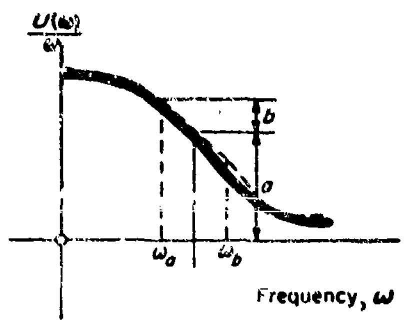


Figure 6. Approximation by straight-line segments.

as a δ function of time[2], except that ω is now the variable.

Substituting in Equation 12b gives

$$A(t) = \frac{2\Delta\omega}{\pi} \sum_{n} a_n \sin \omega_n t \tag{18}$$

Summation should stop at $\omega = \Omega$ as before. A similar method can easily be applied to Equation 12a.

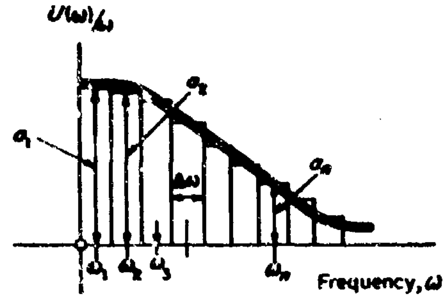


Figure 7. Approximation by a series of weighted impulses.

Guillemin[a] has developed a method combining straight-line and impulse approximations. Briefly, the first or a higher derivative of U/ω is approximated by straight-line segments. This approximation is then differentiated twice to give a series of impulses as before.

Yet another method is to expand U/ω as a Fourier series[d] with 2Ω as the repetition interval. For example,

$$\frac{U(\omega)}{\omega} = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi\omega}{\Omega}\right) \tag{19}$$

for $-\Omega < \omega < \Omega$. The coefficients can be calculated from

$$a_n = \frac{2}{\Omega}\int_0^{\Omega} \frac{U(\omega)}{\omega} \sin\left(\frac{n\pi\omega}{\Omega}\right) d\omega \tag{20}$$

Substitution from Equation 19 in Equation 12b leads to

$$A(t) = 2\Omega \sin \Omega t \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n a_n}{(n^2\pi^2 - \Omega^2 t^2)} \tag{21}$$

Usually this expression converges rapidly so that only the first few terms of the series need be evaluated.

Instead of approximating the characteristic of the system it is alternatively possible to approximate the input function. For example, if unit step function input $u(t)$ is replaced by a square wave of duration T, Fig. 8, then the response to the front step of the square will differ little from $A(t)$, provided T is much greater than the settling time of the servo. To tie in with system approximation accuracy, T should be numerically comparable with π/Ω. The simplification to calculation occurs if the square pulse is considered to be part

a. *Reference 440-1*

d. *Reference 127-40*

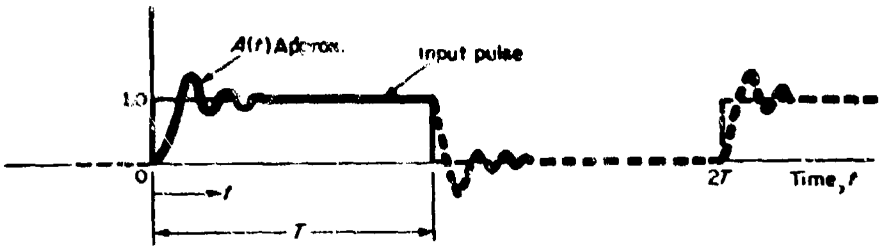


Figure 8. An approximate expression for transient response can be obtained if input is a repetitive square wave. Response differs little from step-function response if T is much longer than any time constant associated with the system.

of a repetitive train of period $2T$ (dotted curve, Fig. 8). It is then possible, by Fourier series, to approximate the input by

$$\theta_i(t) = \frac{1}{2} + \frac{2}{\pi}\left(\sin \omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \ldots\right) \qquad (22)$$

where $\omega_0 = \pi/T$. Considering the response to each component leads to

$$A(t) \approx \frac{4}{\pi}\left[U(\omega_0)\sin \omega_0 t + \frac{U(3\omega_0)}{3}\sin 3\omega_0 t + \frac{U(5\omega_0)}{5}\sin 5\omega_0 t\right] \qquad (23)$$

Similar results can be achieved by other approximations to a step function which have finite duration.

7.2.5.2 TRANSIENT RESPONSE FROM TRANSFER FUNCTIONS

If $\theta_i(t)$ can be expressed in terms of its complex frequency spectrum $\Theta_i(s)$ [mathematically $\Theta_i(s)$ is the Laplace transform of $\theta_i(t)$] then the output spectrum is given by

$$\Theta_o(s) = Y_c(s)\Theta_i(s) \qquad (24)$$

Recovery of the output as a function of time is simplified if a physical linear system with constant parameters is being considered since, in this case, $Y_c(s)$ can be factored. Thus,

$$Y_c(s) = \frac{H(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)} \qquad (25)$$

The poles p_1, p_2, . . . determine the natural or "free-running" modes of the system. Generally the poles are complex so that the natural modes are damped sinusoidal time functions.

Unit-step function is most generally chosen as a representative input. In this case $\Theta_i(s) = 1/s$ (Sub-Topic 7.2.1). This relationship may be substituted into Equation 25 and the resulting expression expanded into a partial fraction. Thus

$$\Theta_o(s) = \frac{A}{s} + \frac{B_1}{s - p_1} + \frac{B_2}{s - p_2} + \ldots + \frac{B_n}{s - p_n} \qquad (26)$$

From a table of inverse transforms, it follows that

$$\theta_o(t) = Au(t) + (B_1 e^{p_1 t} + B_2 e^{p_2 t} + \ldots + B_n e^{p_n t})u(t) \qquad (27)$$

The coefficients are given by

$$A = (-1)^{m-n}\left[\frac{z_1 \ldots z_m}{p_1 \ldots p_n}\right]$$

$$B_i = \left[(s - p_i)\frac{Y_c(s)}{s}\right]_{s=p_i}$$

Normally A is unity (for systems of order higher than one). Poles and zeros can be located on the complex s plane, Fig. 9. Then if all poles are distinct the coefficients can be found by vector multiplication. Thus in Fig. 9, where $m = 2$ and $n = 3$, for example,

$$B_1 = \frac{V_1 V_2}{V_3 V_4} \qquad (28)$$

Vectors V_1, etc., represent complex numbers and must be manipulated accordingly. In the case where more than one pole occurs at some point, such as a term of the form $(s - p_j)^2$ occurring in denominator of $Y_c(s)$, the procedure is slightly different.

Obviously the first step in determining response is to find the poles p_1, ... These are the roots of the characteristic equation $1 + Y_c(s) = 0$. The difficulty is that this is usually something worse than a cubic in s and any direct approach to its solution will likely lead to considerable toil. However, ingenious methods have been derived to

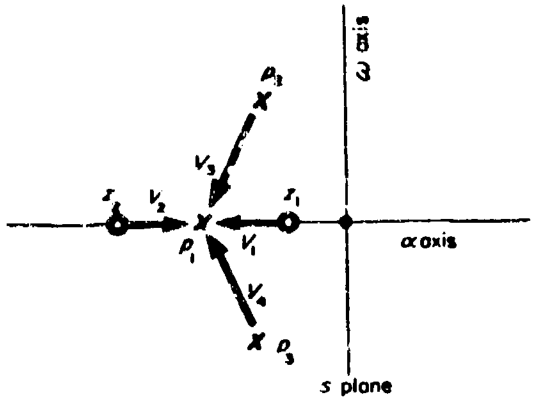


Figure 9. Response coefficients can be obtained from diagram showing location of closed-loop poles and zeros.

evade direct solution and the rest of this article is devoted to a brief summary of some of these. More detailed discussion is contained in the texts mentioned at the end of this article.

Most methods start with knowledge of the loop transfer function $Y_o(s)$. A typical example might be

$$Y_o(s) = \frac{K(1 + T_1 s)}{s(1 + T_2 s)(1 + T_3 s)} \tag{29}$$

The problem now is to relate the closed-loop poles to the open-loop poles and zeros, in this case 0, $-1/T_1$, $-1/T_2$, $-1/T_3$.

Root-Locus Method: Particularly useful, the root-locus method of Evans[a] traces out how the closed-loop poles move in the s plane as the gain constant K, or any other parameter in $Y_o(s)$, is varied. A very short account of this technique will now be given. It will be convenient to proceed with the example chosen in Equation 29. It follows that

$$Y_c(s) = \frac{K(1 + T_1 s)}{s(1 + T_2 s)(1 + T_3 s) + K(1 + T_1 s)} \tag{30}$$

Obviously the zeros of $Y_c(s)$ are the same as those of $Y_o(s)$, but the poles must satisfy:

$$\frac{K(1 + T_1 s)}{s(1 + T_2 s)(1 + T_3 s)} = -1 \tag{31}$$

Since s is in general a complex quantity, Equation 31 contains two conditions:

$$\frac{K\,|1 + T_1 s|}{|s|\,|1 + T_2 s|\,|1 + T_3 s|} = 1 \tag{32}$$

a. *Reference 437-1*

$$\arg(1 + T_1 s) - \arg s - \arg(1 + T_2 s) - \arg(1 + T_3 s) = 180 \text{ deg} + k\,360 \text{ deg} \tag{33}$$

where k is any integer, 0, ± 1, ± 2, . . .

For some point s satisfying these conditions, Fig. 10.

$$\left[\frac{KT_1}{T_2 T_3}\right]\left[\frac{l_1}{l_0\, l_2\, l_3}\right] = 1 \tag{34}$$

$$\phi_1 - \phi_0 - \phi_2 - \phi_3 = 180 \text{ deg} + k\,360 \text{ deg} \tag{35}$$

Of these, the second is the one fundamental to the root-locus concept. If a value of s can be found to satisfy Equation 35, then the value of K in Equation 34 can be adjusted to satisfy Equation 34. It is found that values of s satisfying Equation

Nomenclature

$A, B_1, \ldots$	Constants
$A(t)$ =	Response to unit-step function
a_i =	Ordinates of $\theta_o(t)$
$l_1, D_1, \ldots$ =	Corrections to approximate closed-loop poles
$e_0, \ldots$ =	Ordinates of $e(t)$
$h(t)$ =	Open-loop response to unit impulse
$h_0, h_1, \ldots$ =	Ordinates of $h(t)$
K =	Scalar gain constant
k =	Any integer 0, ± 1, ± 2, . . .
$l_1, \ldots$ =	Lengths
m, m' =	Number of closed-loop and loop zeros
n, n' =	Number of closed-loop and loop poles
P, Q =	Slope of gain and phase curves evaluated at closed-loop pole
$p_1, \ldots$ =	Poles
R, θ =	Polar co-ordinates of s
s =	Complex frequency variable (Laplace operator)
$T_1, \ldots$ =	Time constants
t =	Time variable
$u(t)$ =	Unit step function
$Y_o(s), Y_c(s)$ =	Loop and overall transfer functions
z =	Shift operator
$z_1, \ldots$ =	Zeros
α, ω =	Real and imaginary parts of s
$\Delta\alpha, \Delta\omega, \Delta X$ =	Small finite increments
$\delta(t)$ =	Unit-impulse function
β, γ, δ =	Angles
ϵ =	Small number
ζ =	$-\cos\theta$
$\Theta_i(s), \Theta_o(s), E(s)$ =	Transformed input, output, and error
$\theta_i(t), \theta_o(t), e(t)$ =	Input, output, and error
$\phi_1, \ldots$ =	Angles
τ =	Time interval

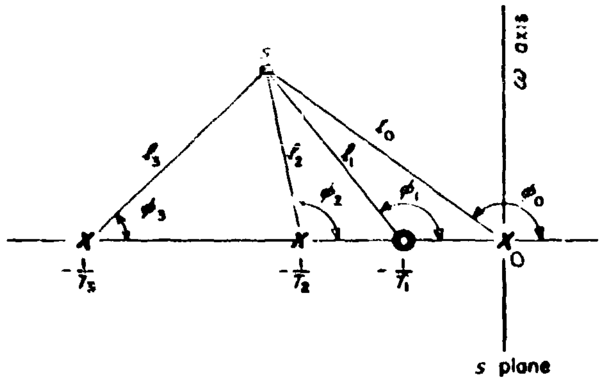


Figure 10. Any point s on a root-locus must satisfy the gain and phase relations implied by the characteristic equation.

35 lie on curves in the s plane, Fig. 11. To each point on a curve there corresponds a value of K necessary to satisfy Equation 34.

The formal definition of a root-locus is a contour in the s plane so that if the value of s at any point on the contour is substituted in $Y_o(s)$, the argument of $Y_o(s)$ is 180 deg + k360 deg.

As described so far, each point on the locus corresponds to a particular value of K so that effectively root-loci tell how the closed-loop poles move when K is adjusted, other parameters being fixed. It is, however, possible to investigate changes in poles as other parameters are varied.[a]

Root-Loci Rules: Some simple rules for the construction of root-loci will now be given without rigorous justification. For the latter, one of the other references listed may be consulted.

1. Loci start on open-loop poles.
2. Loci terminate on open-loop zeros. Here the sense of traversing the loci is in the direction of increasing K. That is, rule 1 corresponds to $K = 0$, while rule 2 corresponds to $K = \infty$.
3. Loci appear in distinct segments. Number of segments equals the greater m' or n' (see Nomenclature).
4. Loci occur in conjugate pairs. That is, diagram is symmetrical about real axis, Fig. 11.
5. For large s, loop function $Y_o(s)$ behaves like C/s^q where $q = n' - m'$, so that the asymptotes to the loci make angles $(180 + k360)/q$ deg with the positive real axis. In *Fig. 11*, the loci belong to a system having $q = 3$. For the system of Equation 29, for large s, $Y_o(s) \approx KT_1/T_2T_3s^2$ so that $q = 2$ and the asymptote angles are ± 90 deg, Fig. 12.

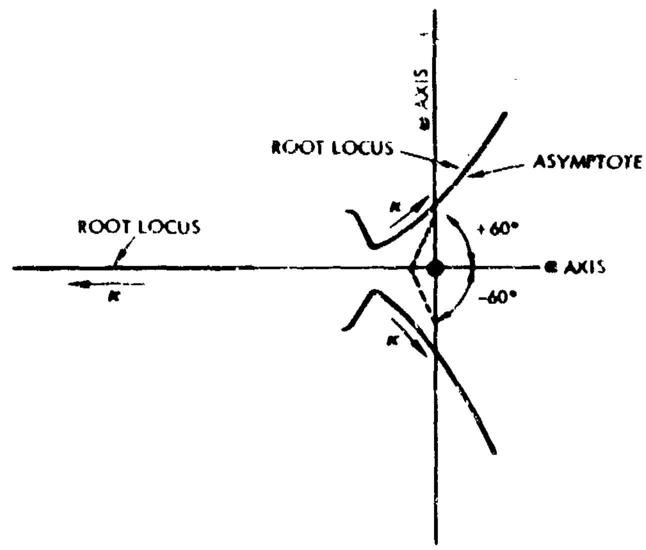


Figure 11. Values of s satisfying characteristic equation trace out continuous curves in the s plane. These are called root loci. At each point K must be adjusted to satisfy magnitude condition. Hence, curves may be shown with K as parameter varying along the contour. Root loci can be drawn for variation of other parameters in $Y_o(s)$.

6. Asymptotes do not radiate from the origin but intersect at a point on the real axis s_1 given by

$$s_1 = \frac{\Sigma \text{ open-loop poles} - \Sigma \text{ open-loop zeros}}{q} \qquad (36)$$

Thus for the system of Equation 29, Fig. 12,

$$s_1 = \frac{1}{2}\left[\frac{1}{T_1} - \frac{1}{T_2} - \frac{1}{T_3}\right] \qquad (37)$$

7. On real axis, loci lie only in sections to the left of an odd number of open-loop poles and zeros. This condition is also illustrated in Fig. 12 which shows the root-locus for the system of Equation 29.
8. Point of intersection of loci with imaginary axis can often be easily determined directly by substituting $s = j\omega$ in characteristic equation and solving directly. The corresponding value of K is simply determined by Routh's criterion.
9. Angle at which locus leaves an open-loop pole is indicative—a point that can best be illustrated by an example. A system has loop poles at $s = p_1, p_2, p_3, p_4$ and a loop zero at $s = z_1$ as shown in Fig. 13. Suppose it is required o find

a. *Reference 436-1*

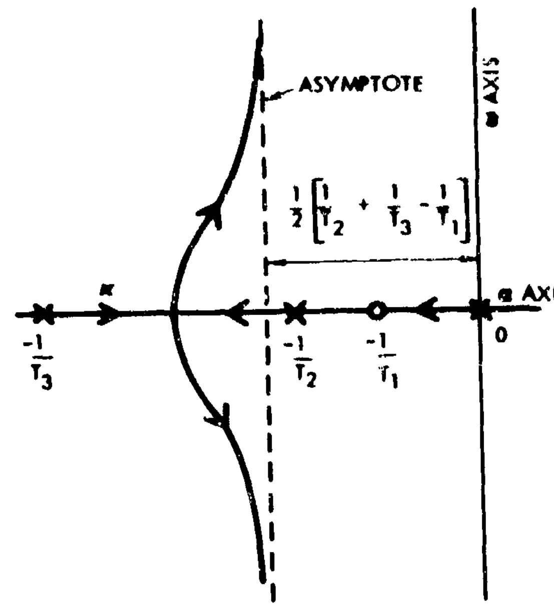


Figure 12. Root locus for system of Equation 29, showing location of branches of locus, asymptotes, and break-away from real axis.

the inclination γ of the locus leaving p_2. At point on locus near p_2 arguments of vectors from p_1, . . . , p_4, z_1 should add to give an angle of 180 deg + k360 deg provided contributions from zeros are added and contributions from poles are subtracted. Thus the inclination γ can be found from

$$\gamma + (\phi_2 - \phi_1 - \phi_3 - \phi_4) = 180 \text{ deg} + k\,360 \text{ deg} \quad (38)$$

Angles ϕ_1, ϕ_2, ϕ_3, ϕ_4, can be measured directly, being the arguments of vectors drawn to p_2 from the other poles and zeros.

An exactly similar argument can be used to find the angle at which loci enter open-loop zeros.

10. Where loci exist on segments of the real axis between two loop poles, the contour must split away in two branches from the real axis to satisfy rule 2, *Fig. 13b*. If $-\alpha$ is the abscissa of the break-away point, then α can be estimated by considering a point on the branch very close to break-away with co-ordinates $(-\alpha, \epsilon)$, ϵ being very small. Once again the sum-of-argument condition must be satisfied, and in this case all arguments can be expressed to first-order approximation as proportional to ϵ. Then ϵ can be cancelled from the equation, leaving an expression for α.

In the example shown in *Fig. 13*

$$\frac{\epsilon}{\alpha_2 - \alpha} + \frac{\epsilon}{\alpha_3 - \alpha} + \left(\pi - \frac{\epsilon}{\alpha}\right) = \pi$$

giving

$$\frac{1}{\alpha} = \frac{1}{\alpha_2 - \alpha} + \frac{1}{\alpha_3 - \alpha} \quad (39)$$

Equation 39 can conveniently be solved by trial-and-error methods.

Other rules exist, and are extensively treated in *References 359-1 and 437-1.*

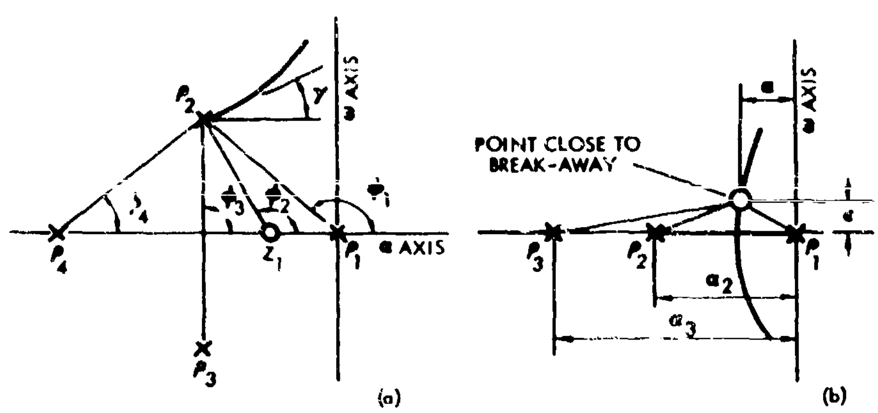


Figure 13. (above) At *a*, diagram illustrating determination of angle at which locus leave an open-loop pole. At *b*, diagram illustrating determination of point at which branches of locus break away from real axis.

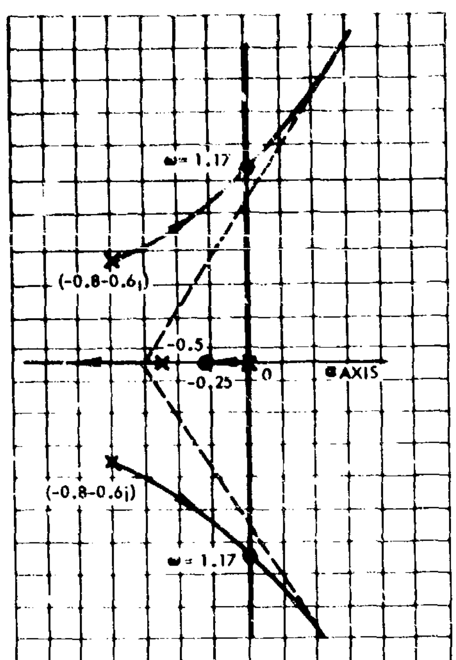


Figure 14. (right) Root locus for system having $Y_o(s) = K(1 + 4s)/s(1 + 2s)(1 + 1.6s + s^2)$.

Example: A simple example will illustrate the application of the above ten rules to the construction of root-loci. Consider

$$Y_o(s) = \frac{K(1 + 4s)}{s(1 + 2s)(1 + 1.6s + s^2)} \qquad (40)$$

Loop poles are at 0, -0.5, $(-0.8 \pm 0.6j)$. Loop zero is at -0.25. Here $q = 3$ so that the asymptotes are equally spaced at 120 deg, Fig. 14. Equation 36 shows that the asymptotes meet at point $-0.616, 0$. The characteristic equation is

$$2s^4 + 4.2s^3 + 3.6s^2 + (1 + 4K)s + K = 0 \qquad (41)$$

Putting $s = j\omega$ and separating real and imaginary parts lead to

$$\left.\begin{array}{l} 2\,\omega^4 - 3.6\,\omega^2 + K = 0 \\ 1 + 4K - 4.2\,\omega^2 = 0 \end{array}\right\} \qquad (42)$$

Eliminating K and solving give $\omega = \pm 1.17$ as the points where the loci cut the ω axis.

The loci leave the complex poles at an angle of 35 deg (rule 9).

Now from rules 1, 2, 3, 4, and 7, it is possible to sketch the full locus as shown in Fig. 14. Despite the complexity of $Y_o(s)$, the procedure can be carried out in a very short time. It is advisable to check the accuracy of the sketched parts by checking that a few points on the locus satisfy the required conditions.

Phase-Angle Locus Method: Another approach to the task of loci construction is to map the s plane with lines of constant phase angles for each pole or zero in $Y_o(s)$. Points can then be found at which the total phase angle adds up to 180 deg $+ k360$ deg. This method, called phase-angle locus method, is again quite easy to use.[a]

With the locus sketched it is now possible to indicate the variation of closed-loop poles with K. A simple way is to select a number of points on the locus, and then to apply the modulus condition—Equation 32, for example—adjusting K so that the two sides of the equation balance. Other quantities in the equation can be measured directly from the diagram.

The root-locus method is invaluable as a design tool. Previous articles have shown that it is necessary to restrict closed-loop poles to certain regions of the s plane. This restriction sets a limiting value of K and, for this value, the complete closed-loop pole-zero configuration is known. If this is so, it is an easy matter to find the transient response by the semigraphical methods outlined by Equations 25 to 28. Thus the root-locus method permits design with transient requirements in mind, a hitherto difficult task. Although only parameter adjustment has been discussed here, the method is particularly useful in that suitable modifying networks can also be specified to improve performance or stability, contrasting favorably with frequency-response methods in that transient response can be controlled directly. Space shortage prevents a more complete discussion on the use of root-loci in design, but more detail will be found in *References 18-22, 359-1, 437-1, 127-41, 436-1, and 127-2.*

Other Methods: To find the roots of the characteristic equation, it is necessary to find complex values $s = \alpha + j\omega$ which satisfy

$$Y_o(s) = -1 \qquad (43)$$

Nyquist's criterion shows that, if $s = j\omega$ satisfies Equation 43, for some particular value of ω, a plot of $Y_o\,(j\omega)$ passes through $-1, 0$.

Therefore, it is to be expected that, if $\alpha + j\omega$ is a solution, a plot of $Y_o(\alpha + j\omega)$ would also pass through $-1, 0$. In Reference 3 it was shown that all points on the s plane could be transformed into corresponding points on the Y_o plane and use can be made of this fact to solve Equation 43. Since the method is approximate only, the s plane must be divided into a finite number of points. One way of doing this is with a grid of lines parallel to the axes, forming small squares, Fig. 15*a*. The transformation of this grid in the Y_o plane is also a grid of squares, although they are "curvilinear squares" in this case. Another way of saying this is that lines of constant α and constant ω intersect at 90 deg in the Y_o plane.

The small-square construction can now be used to map the grid on the Y_o plane. First, the locus of $Y_o(j\omega)$ (Nyquist plot) is drawn, divided by equal increments $\Delta\omega$ in frequency corresponding to vertical divisions of the s plane. Then squares can be drawn corresponding to the small squares produced in moving by distance $\Delta\alpha(= \Delta\omega)$ to the left in the s plane. Smoothing off the squares, Fig. 15*b*, gives an approximation to $Y_o(-\Delta\alpha + j\omega)$. The method can then be continued to obtain $Y_o(-2\Delta\alpha + j\omega)$, $Y_o(-3\Delta\alpha + j\omega)$, . . . and so on. Ultimately a value of α is found for which the curve passes through $-1, 0$ and the corresponding value of ω can be read off. Thus in Fig. 15*b*, $\alpha = -3\Delta\alpha = -3\Delta\omega$; $\omega = 5\Delta\omega$ gives one value of s satisfying characteristic Equation 43.

This method is often not successful for finding all the roots. However, in many practical systems it is found that one pair of complex roots, the

a. *Reference 127-41*

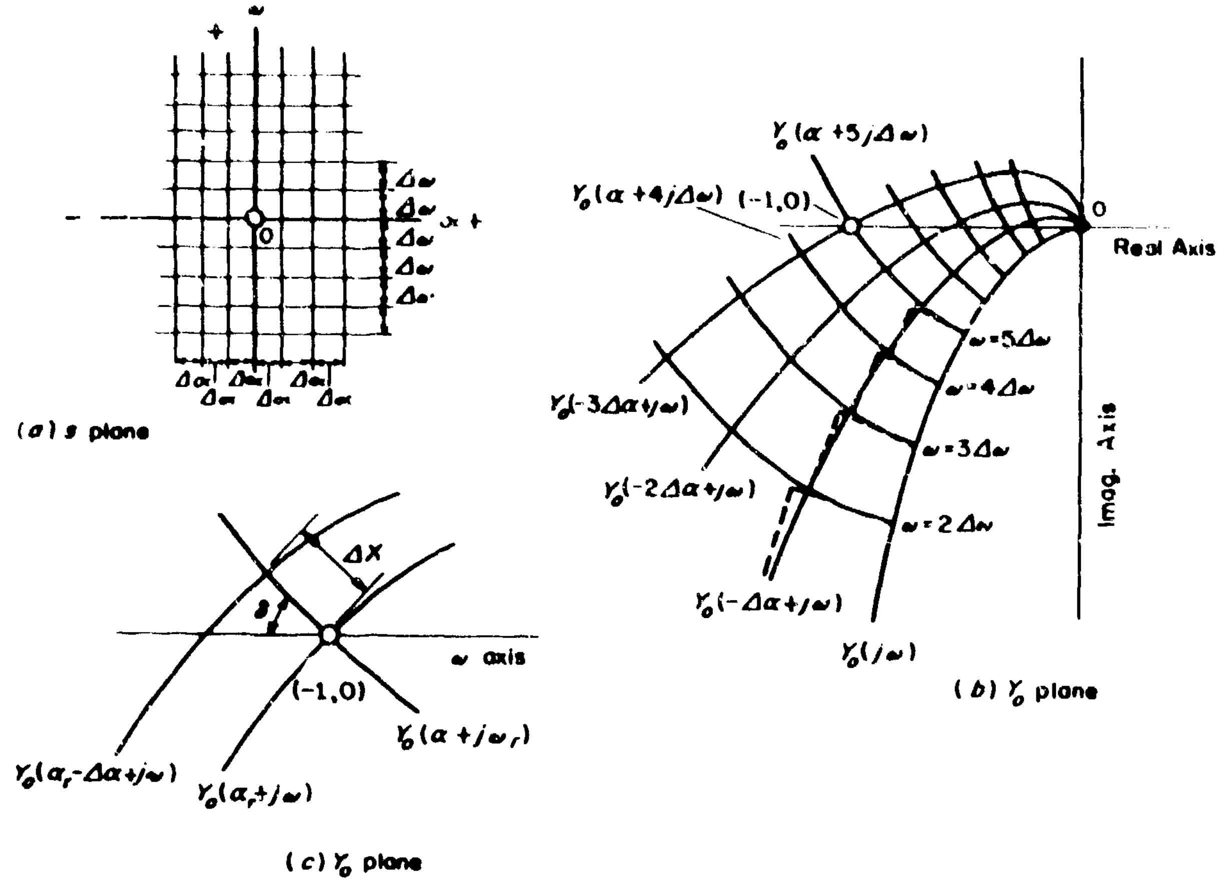


Figure 15. At a, s plane divided into grid of small squares. At b, transformation of square grid into $Y_o(s)$ plane. Small square construction can be used to construct this mesh starting with $Y_o(j\omega)$ plot. At c, quantities which may be used to find the derivative of $Y_o(s)$, approximately, at point $-1, 0$.

most lightly damped, dominate the oscillatory part of the transient response.

If the roots are found by the method just discussed a simple artifice can be used to find the transient response. Suppose the root that has been found is $p_r = \alpha_r + j\omega_r$. Then there is a term in the transient response given by $B_r e^{p_r t}$. For unit-step input the coefficient is given by

$$B_r = \left[\frac{(s - p_r)\, Y_c(s)}{s} \right]_{s=p_r} \qquad (44)$$

Putting $s = p_r + \Delta s$,

$$B_r = \lim_{\Delta s \to 0} \left[\frac{\Delta s}{p_r} Y_c(p_r + \Delta s) \right]$$

$$= \lim_{\Delta s \to 0} \left[\frac{\Delta s}{p_r} \frac{Y_o(p_r + \Delta s)}{[1 + Y_o(p_r + \Delta s)]} \right]$$

Expanding $Y_o(p_r + \Delta s)$ about p_r, remembering that $Y_o(p_r) = -1$, gives finally

$$B_r = \frac{\dfrac{-1}{p_r}}{\left(\dfrac{d Y_o}{ds}\right)_{s=p_r}} \qquad (45)$$

An approximation to the derivative in Equation 45 can be obtained by measurement from the plot of $Y_o(s)$, Fig. 15c:

$$\left(\frac{d Y_o}{ds}\right)_{s=p_r} \approx \frac{\Delta X}{\Delta\alpha e^{j\theta}} \qquad (46)$$

Thus, for the component of response due to root p_r,

$$\frac{-\Delta\alpha}{\Delta X} \frac{e^{p_r t + j\theta}}{p_r}$$

The conjugate of this term must also be present in the response since $\alpha_r - j\omega_r$ is also a root. If the combined component of these roots dominates the response, there results, provided the system is at least of first order,

$$A(t) = u(t) \left\{ 1 - \frac{2\Delta a}{\Delta X \sqrt{a_r^2 + \omega_r^2}} e^{a_r t} \times \right.$$

$$\left. \cos (\omega_r t + \delta - \beta) \right\} \tag{47}$$

where $\beta = \tan^{-1}(a_r/\omega_r)$.

A typical plot, Fig. 16, shows that in most cases there is some error for small values of t. The error is due to the absence of terms due to less important roots.

It is often profitable to divide the s plane by radial lines as shown in Fig. 17. The technique here is to plot the Y_e transformation corresponding to radial lines. Any point on a line is given by $s = Re^{j\theta}$, but Kusters and Moore[a] introduce notation

$$s = (-\zeta + j\sqrt{1 - \zeta^2})R \tag{48}$$

where $\zeta = -\cos \theta$. They also introduce the idea of plotting $Y_e(s)$ on a logarithmic basis, similar to logarithmic frequency-response curves.[14, 15] Reference 21 gives plots of magnitude and phase curves versus log R for given values of ζ for simple lag and quadratic lag terms. These may be added to give loop plots, for example, Fig. 18. For certain values of ζ, 0 db gain and -180 deg phase occur at the same value of R, such as ζ_0 and R_0 in Fig. 18, and these values substituted in Equation 48 give the roots of the characteristic equation.

From the gain and phase curves it is also possible to calculate the coefficient B_r of the component of response to unit-step input due to a root at $s = p_r$. Real roots occur for $\zeta = 1$ and, in this case, at $s = p_r$

$$B_r = \frac{20}{P} \tag{49}$$

where P = slope of gain curve, db per decade. For complex roots

$$B_r = \frac{1}{\dfrac{P}{20} + j\dfrac{Q}{2.3}} \tag{50}$$

where P = slope of gain curve, db per decade, for particular ζ, R at $s = p_r$, and Q = slope of phase curve, rad per decade, for same ζ, R at $s = p_r$.

a. *Reference 436-1*

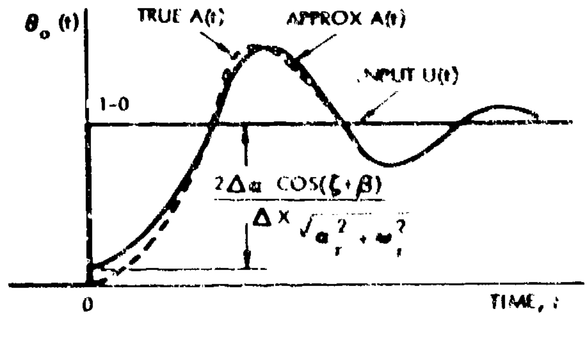


Figure 16. Plot of approximate response obtained from dominant roots of characteristic equation. Often there is some error for small values of time, but the approximation gives accurate indication of overshoot and oscillation.

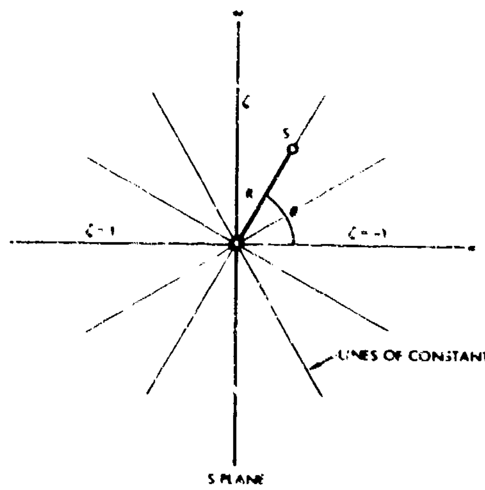


Figure 17. The s plane divided by radial lines gives another approach to solution of characteristic equation.

For each R there is a given ζ for which arg $Y_e(s) = 180$ deg $+ k360$ deg. For these ζ, R values the gain curve can be lifted by adjusting K to cut the 0 db line. Thus the root-locus conditions have been satisfied. Since the given values of ζ, R define a curve in the s plane, satisfying these conditions, yet another way of constructing root-loci, with K as a parameter, is available.

Locating closed-loop poles by the method just outlined can be tedious since a wide range of values of ζ and R must be covered to locate all the poles. In order to reduce the amount of work it would be convenient to locate the poles approximately as a first step. Biernson[a] suggests a good approach to the task. First step is a crude approximation giving three locations for poles:

a. *Reference 127-2*

1. A pole at $s = -\omega_c$, where ω_c is frequency at which Bode gain plot, $20 \log_{10}|Y_o(j\omega)|$, cuts the 0-db line.

2. Poles at $z_1, z_2, \ldots$ = zeros of $Y_o(s)$ for which $|z_1| < \omega_c, |z_2| < \omega_c, \ldots$

3. Poles at $p_1, p_2, \ldots$ = poles of $Y_o(s)$ for which $|p_1| > \omega_c, |p_2| > \omega_c, \ldots$

It is found that poles distant from $-\omega_c$ are quite accurate, but those nearer need refining. Next step is to make a correction to the rough estimates. For example, a better approximation to the closed-loop pole near the loop zero z_1 is taken to be $z_1 + d_1$. Then, to a first order, d_1 is given by

$$d_1 = \left[\frac{(s - z_1)}{Y_o(s)} \right]_{s \to z_1} \tag{51}$$

A better approximation to the pole near open-loop pole p_1 is $p_1 + D_1$, where

$$D_1 = \Big[(s - p_1) Y_o(s) \Big]_{s \to p_1} \tag{52}$$

Application of this method successively leads to rapid convergence toward true positions for distant poles, but for poles near $-\omega_c$ final adjustment must be made by the graphical method outlined in the last section. However, the advantage is that a good idea is obtained of where the poles will lie and the graphical plots can be localized.

A number of methods have been devised for determining closed-loop transient response from the transient response of the system with the loop opened. For example, if

$$Y_o(s) = \frac{K(1 + T_1 s)}{s(1 + T_2 s)(1 + T_3 s)} \tag{53}$$

then the response of the system to unit impulse when the loop is opened, say $h(t)$, is the inverse Laplace transform of $Y_o(s)$ and can easily be determined since the location of the loop poles is known to be $0, -1/T_2, -1/T_3$. Now the objective is to find closed-loop response when input is unit-step. It is convenient to find how the error $e(t)$ varies with time. In this case, the transformed error $E(s)$ is given by

$$[1 + Y_o(s)]E(s) = \theta_i(s) \tag{54}$$

Next step is to approximate $h(t)$, $e(t)$ and $\theta_i(t)$ as a sum of impulses, Fig. 19. Thus,

$$\begin{aligned} h(t) &\simeq \tau[h_0\, \delta(t) + h_1\, \delta(t - \tau) + h_2\, \delta(t - 2\tau) + \ldots] \\ e(t) &\simeq \tau[e_0\, \delta(t) + e_1\, \delta(t - \tau) + e_2\, \delta(t - 2\tau) + \ldots] \\ \theta_i(t) &= u(t) \simeq \tau[\delta(t) + \delta(t - \tau) + \delta(t - 2\tau) + \ldots] \end{aligned} \tag{55}$$

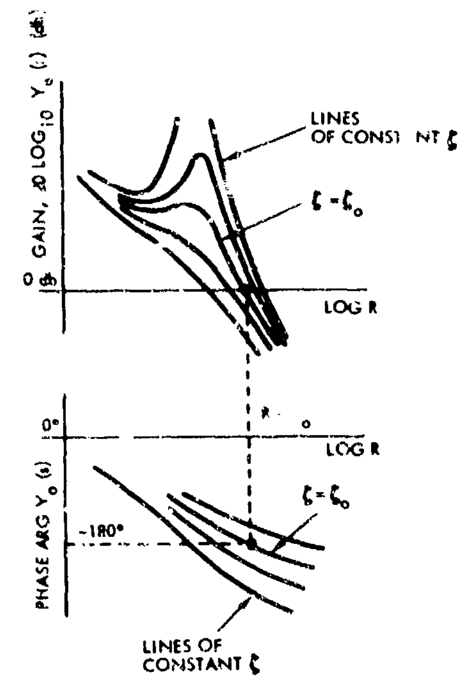


Figure 18. Values of R and ζ satisfying characteristic equation must be such that gain is 0 db when phase is 180 deg + K360 degrees.

Taking Laplace transforms of these three quantities,

$$\begin{aligned} Y_o(s) &= \tau[h_0 + h_1 z + h_2 z^2 + \ldots] \\ E(s) &= \tau[e_0 + e_1 z + e_2 z^2 + \ldots] \\ \theta_i(s) &= \tau[1 + z + z^2 + \ldots] \end{aligned} \tag{56}$$

where $z = e^{-s\tau}$ (Sub-Topic 7.2.1). Substituting from Equation 56 in Equation 54 and collecting like terms in z,

$$\begin{aligned} (1 + h_0)e_0 + [(1 + h_0)e_1 + h_1 e_0]\, z &+ \\ [(1 + h_0)e_2 + h_1 e_1 + h_2 e_0]\, z^2 &+ \\ \ldots = 1 + z + z^2 + \ldots \end{aligned} \tag{57}$$

In this equation, $h_0, h_1, \ldots$ are known; $e_0, e_1, \ldots$ are unknown. By equating like powers of z it is possible to obtain expressions for the error coordinates, thus

$$\begin{aligned} e_0 &= \frac{1}{1 + h_0} \\ e_1 &= \frac{1 + h_0 - h_1}{(1 + h_0)^2} \end{aligned} \tag{58}$$

and so on

A smooth $e(t)$ curve can be sketched through the ordinates so calculated. Finally $\theta_o(t) = 1 - e(t)$ is obtained. The success of this method depends

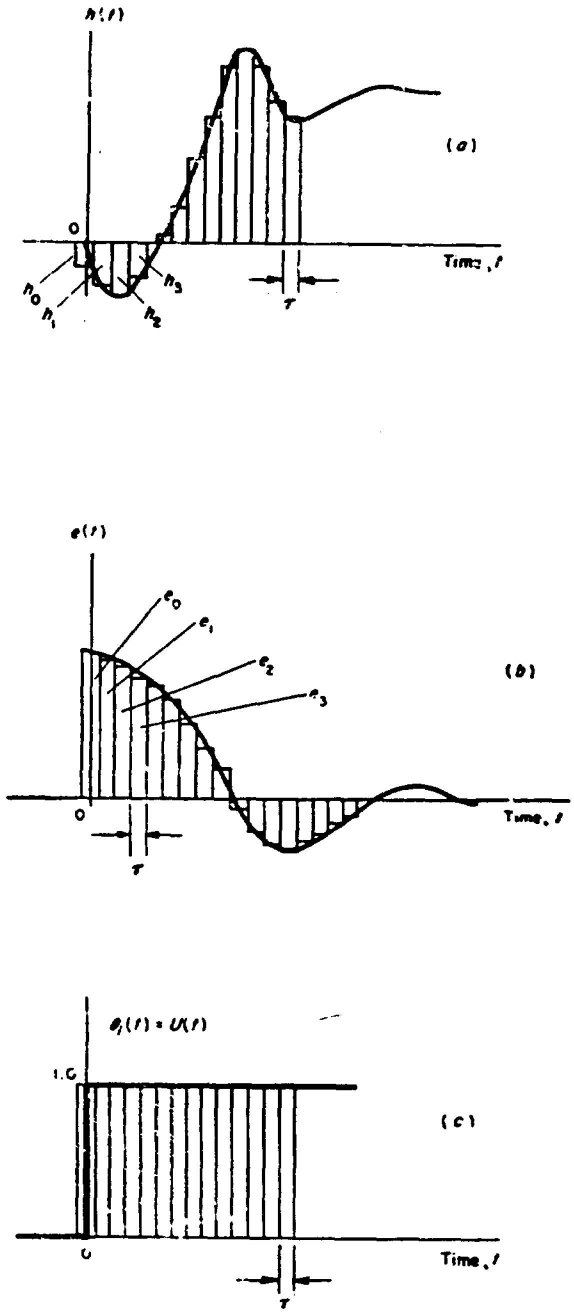


Figure 19. Approximate representations of $h(t)$, $e(t)$ and $\theta_i(t)$, in this case a step function, by series of impulses. Values h_0, h_1, . . ., e_0, e_1, . . . are successive ordinates of $h(t)$ and $e(t)$ at intervals of τ.

largely on selecting τ sufficiently small, about 1/20 of any oscillation period expected in the response.

A similar method is to attempt to express the transform of the output directly as a power series in z, thus

$$\Theta_o(s) = \frac{Y_o(s)}{s[1 + Y_o(s)]} = \tau(a_0 + a_1 z + a_2 z^2 + \ldots) \quad (59)$$

Transforming gives

$$\theta_o(t) = \tau[a_0\,\delta(t) + a_1\,\delta(t-\tau) + a_2\,\delta(t-2\tau) + \ldots] \quad (60)$$

Here a_0, a_1, a_2, . . . give output ordinates at times $t = 0, \tau, 2\tau, \ldots$. The difficulty lies in making the expansion, Equation 59. In the left-hand side s must be replaced by $1/\tau(\log z)$, but the result cannot be expanded as a power series in z. However, by replacing differentiation by difference of ordinates, it is possible to obtain suitable approximations. The simplest of these is

$$s = \frac{2}{\tau}\left(\frac{z-1}{z+1}\right) \quad (61)$$

Substituting Equation 61 for s in the left-hand side of Equation 59 permits a suitable expansion in powers of z. Effectively the system has been described by a linear-difference equation rather than by a differential equation.

7.3 VIBRATION AND SHOCK ANALYSIS

7.3.1 General

Vibration is a periodic or random displacement of a body from its equilibrium position. All bodies possessing mass and elasticity are subject to vibration along, or transverse to, any axis of the body.

Vibrations may be free or forced. Free vibration in an elastic system refers to a system free of impressed forces but under the action of forces inherent in the system itself. A freely vibrating system will vibrate at its natural frequency or frequencies. Forced vibration refers to a vibrating system under the excitation of an external force (forcing function). The frequency of the exciting force is independent of the natural frequency of the system. When the frequency of the exciting force coincides with one of the natural frequencies, resonance may occur.

The simplest form of periodic motion is simple harmonic motion, which can be represented by the sine or cosine functions. A periodic motion which is not harmonic can be represented by a series of harmonic motions (Fourier series) which have frequencies that are multiples of the given frequencies. The first term in the series is called the fundamental, and has the same frequency as that of the periodic motion. The second term has a frequency equal to twice the fundamental, and is known as the second harmonic, etc.

7.3.2 Harmonic Motion

Harmonic motion may be represented by the following equations:

Displacement

$$x = X \sin \omega t \qquad \textbf{(Eq 7.3.2a)}$$

Velocity

$$\dot{x} = X\omega \cos \omega t = X\omega \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \textbf{(Eq 7.3.2b)}$$

Acceleration

$$\ddot{x} = -X\omega^2 \sin \omega t = X\omega^2 \sin(\omega t + \pi) \qquad \textbf{(Eq 7.3.2c)}$$

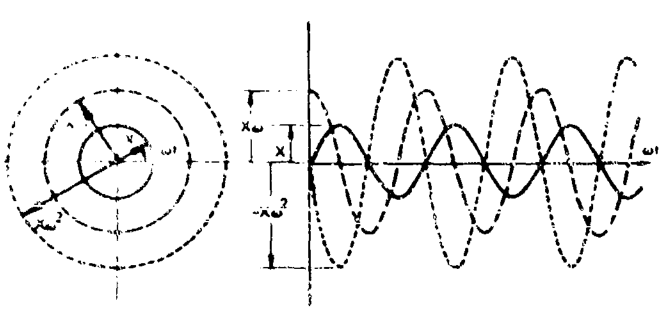
where $\omega = 2\pi f$ = angular frequency of the motion, rad/sec

f = frequency of motion, cycles/sec (cps)

X = amplitude of displacement, in., ft.

These equations can be represented by vectors rotating with velocity, ωt, as shown in Figure 7.3.2.

The vector with magnitude $X\omega$ represents the velocity, and is 90° ahead of the displacement. The acceleration vector, $X\omega^2$, is 180° ahead of the displacement. These angles are called phase angles.



7.3.3 Natural Frequencies of Spring-Mass Systems

The natural frequency is the free vibration frequency of a system. The natural frequencies of a multiple degree of freedom system are the frequencies of the normal modes of vibration. The equations for calculating the natural frequencies of some common systems are given in Table 7.3.3.

7.3.4 Elements of a Vibratory System

The elements of a vibratory system include a mass, a spring, and a damper. A mass is a rigid body which in a vibratory system stores kinetic energy and has an acceleration, $\ddot{x}$, proportional to the force, F, acting on the mass

$$F = m\ddot{x} \qquad \textbf{(Eq 7.3.4a)}$$

A spring provides a means for storing potential energy. The ideal spring is linear and is assumed to have no mass. The change in length, x, of a linear spring is proportional to the force, F, acting along its length

$$F = kx \qquad \textbf{(Eq 7.3.4b)}$$

where k is the spring constant or stiffness factor. Stiffness factors for some springs and shafts are presented in Figure 7.3.4a.

A damper provides a means of absorbing energy. Two commonly used types of dampers are presented in Figure 7.3.4b. In the viscous damper, the absorbed energy is due to viscous friction. The applied force is proportional to the velocity and can be expressed by the equation

$$F_d = c\dot{x} \qquad \textbf{(Eq 7.3.4c)}$$

where F_d = viscous damping force
c = coefficient of viscous damping
$\dot{x}$ = velocity

In coulomb damping, the energy absorbed is due to the friction between solid members. The coulomb frictional force is independent of the displacement and frequency of the vibrating mass and is expressed by the equation

$$F = \mu N \qquad \textbf{(Eq 7.3.4d)}$$

where μ is the coefficient of friction and N is the normal load.

7.3.5 Systems with One Degree of Motion

A mechanical system capable of vibration is shown in Figure 7.3.5a. The different cases of vibratory motion are discussed as follows:

CASE I Free Vibration Without Damping $F(t) = c = 0$

If the mass is displaced from its equilibrium position and released, the system will undergo harmonic oscillations. The sum of the forces acting on the mass must equal zero

$$\begin{matrix}\text{Acceleration} \\ \text{Force}\end{matrix} + \begin{matrix}\text{Spring} \\ \text{Force}\end{matrix} = 0$$

The equation of motion is

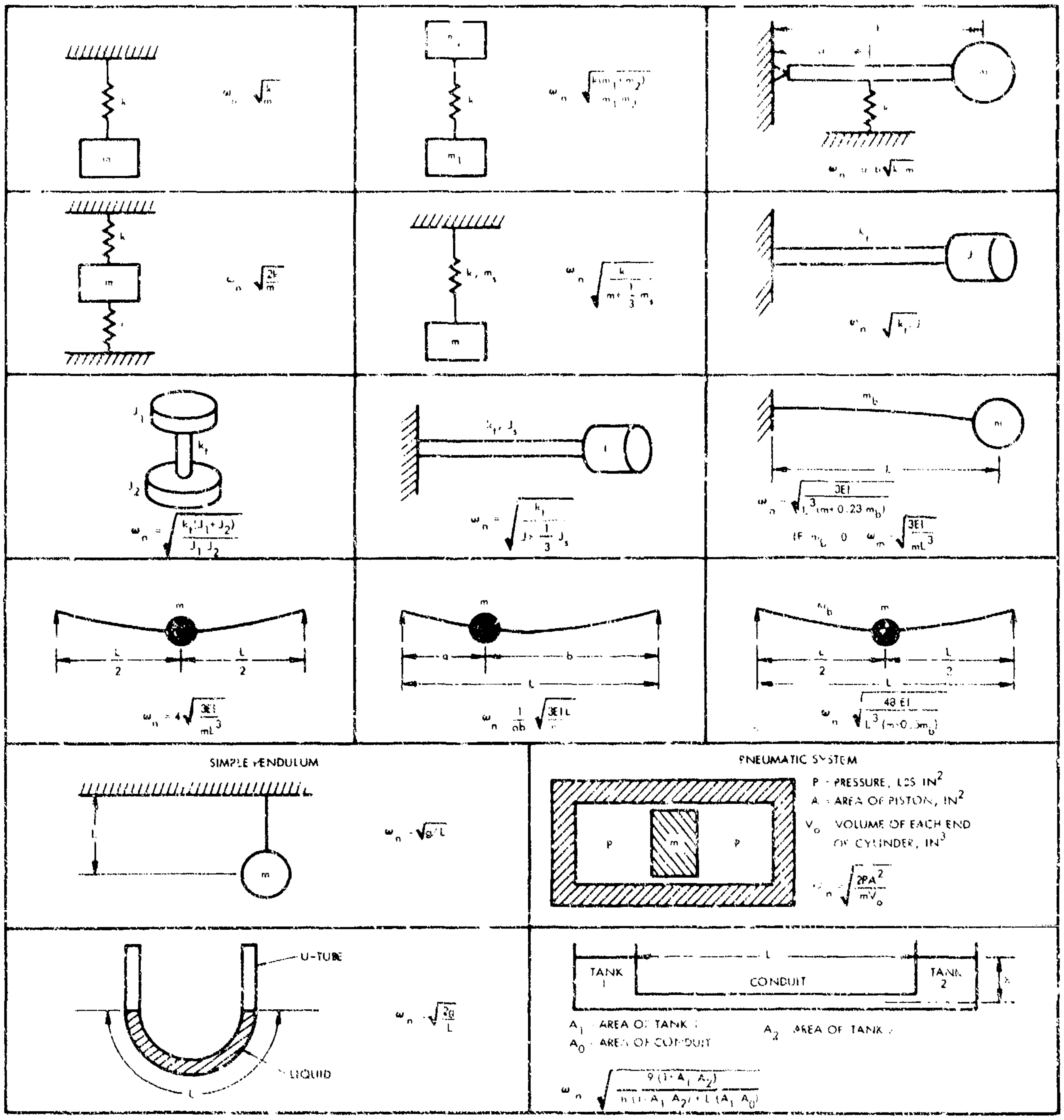
$$m\ddot{x} + kx = 0 \qquad \textbf{(Eq 7.3.5a)}$$

and the general solution of Equation (7.3.5a) is

$$x = A \cos \omega_n t + B \sin \omega_n t \qquad \textbf{(Eq 7.3.5b)}$$

A and B are constants which must be determined from initial conditions. If the mass is displaced a distance, x_o,

Table 7.3.3. Equations to Calculate Natural Frequencies of Some Common Systems



ω_n = ANGULAR NATURAL FREQUENCY, RADIANS/SEC

k = SPRING STIFFNESS, LB/IN

k_t = TORSIONAL STIFFNESS, LB-IN/RADIAN

J = MASS MOMENT OF INERTIA OF ROTOR, LB-IN-SEC^2

I = AREA MOMENT OF INERTIA, IN^4

m = MASS OF LOAD, LB-SEC^2/IN

m_s = MASS OF SPRING, LB-SEC^2/IN

m_b = MASS OF BEAM, LB-SEC^2/IN

E = YOUNGS MODULUS, LB/IN^2

g = ACCELERATION OF GRAVITY, IN/SEC^2

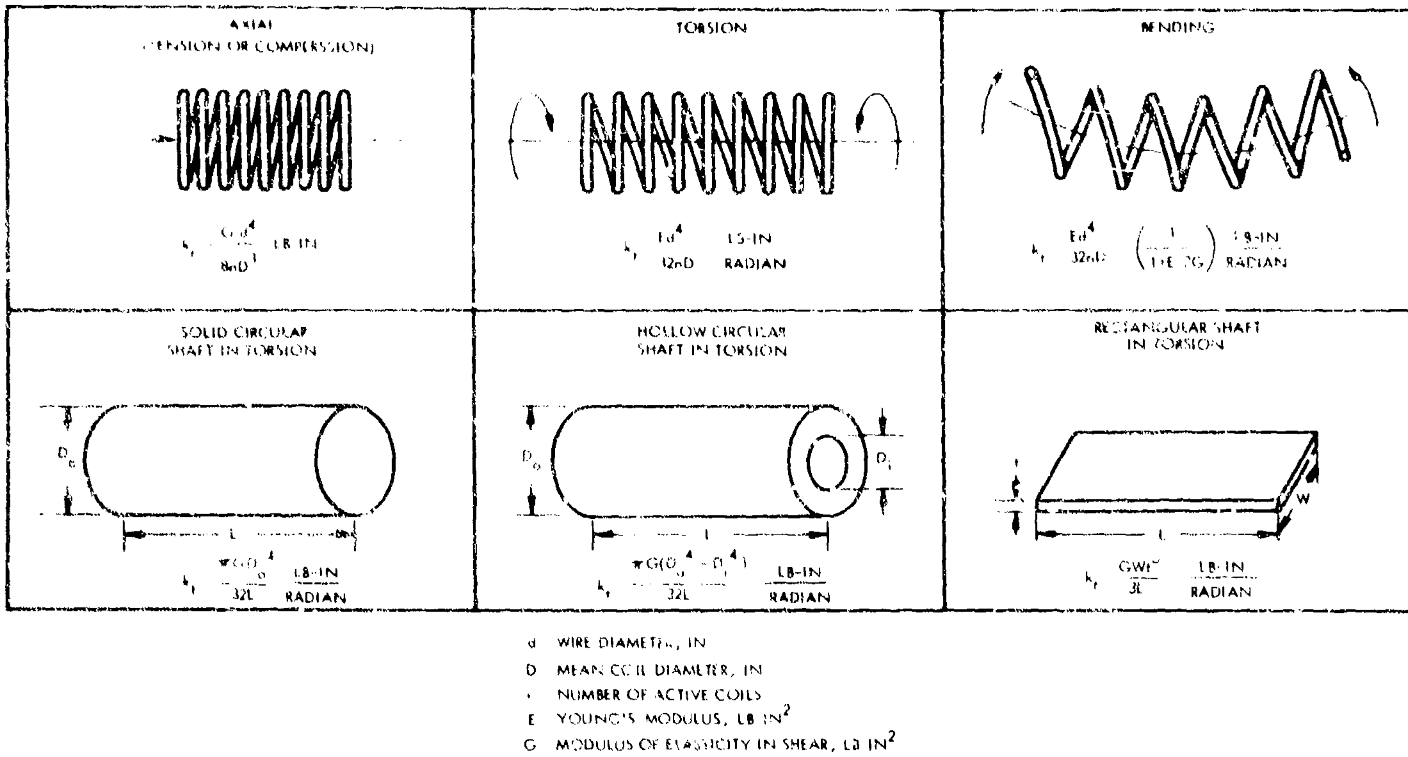


Figure 7.3.4a. Stiffness Factors

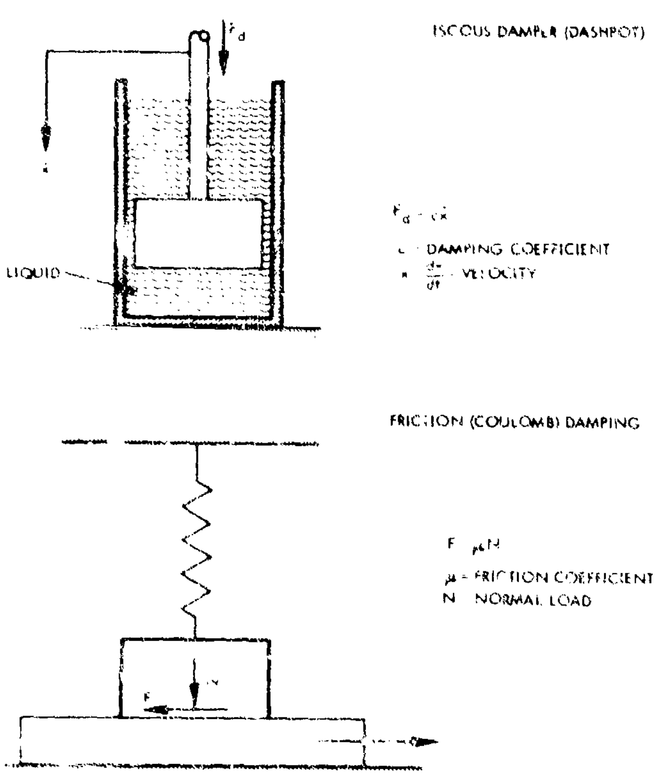


Figure 7.3.4b. Common Damping Devices

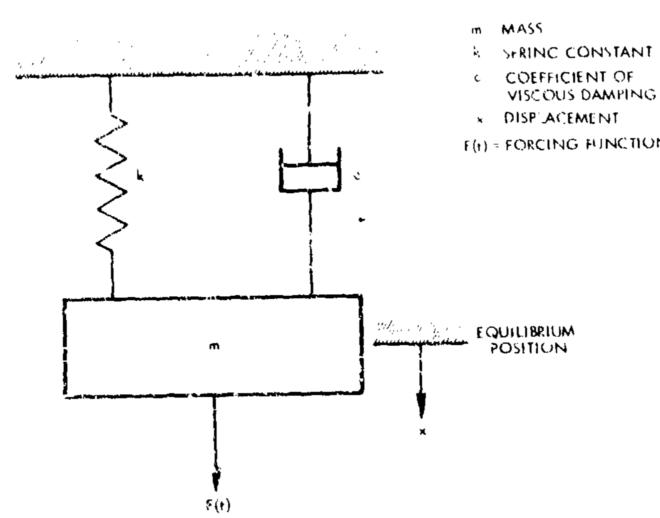


Figure 7.3.5a. Mass, Spring, Dashpot System

and released with an initial velocity, $\dot{x}_o$, the initial conditions at $t = 0$ are

$$x = x_o, \dot{x} = \dot{x}_o \qquad \textbf{(Eq 7.3.5c)}$$

and the specific solution of Equation (7.3.5a) is

$$x = x_o \cos \omega_n t + \frac{\dot{x}_o}{\omega_n} \sin \omega_n t \qquad \text{(Eq 7.3.5d)}$$

where the undamped natural angular frequency of vibration in radians/sec is

$$\omega_n = \sqrt{\frac{k}{m}}$$

The period of vibration in seconds is

$$T = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m}{k}} \qquad \text{(Eq 7.3.5e)}$$

The natural frequency in cycles/sec (cps) is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{(Eq 7.3.5f)}$$

CASE II Free Vibration With Viscous Damping $F(t) = 0$; c = constant

The equation of motion is

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \text{(Eq 7.3.5g)}$$

and its general solution is

$$x = Ae^{S_1 t} + Be^{S_2 t} \qquad \text{(Eq 7.3.5h)}$$

where

$$S_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$= \left[-\zeta \pm \sqrt{\zeta^2 - 1}\right]\omega_n \qquad \text{(Eq 7.3.5i)}$$

$$\zeta = \frac{c}{c_c} \qquad \text{(Eq 7.3.5j)}$$

$$c_c = 2m\,\omega_n \qquad \text{(Eq 7.3.5k)}$$

ζ is called the damping factor and c_c is denoted the critical damping coefficient.

If $\zeta > 1$ (overdamped), the motion is not periodic and no vibration takes place (aperiodic motion); thus

$$x = Ae^{[-\zeta + \sqrt{\zeta^2 - 1}]\omega_n t} + Be^{[-\zeta - \sqrt{\zeta^2 - 1}]\omega_n t} \qquad \text{(Eq 7.3.5l)}$$

If $\zeta < 1$ (light damping), the radical of $S_{1,2}$ is imaginary and motion is oscillatory; thus

$$x = e^{-\zeta\omega_n t}\left(Ce^{i\sqrt{1-\zeta^2}\,\omega_n t} + De^{-i\sqrt{1-\zeta^2}\,\omega_n t}\right) \qquad \text{(Eq 7.3.5m)}$$

If $\zeta = 1$ (critical damping), the body returns to the equilibrium position in the shortest time without oscillation; thus

$$x = (E + Ft)e^{-\omega_n t} \qquad \text{(Eq 7.3.5n)}$$

CASE III Forced Vibration With Viscous Damping $F(t) = F_o \sin \omega_d t$, c = constant

If the harmonic driving force is

$$F(t) = F_o \sin \omega_d t \qquad \text{(Eq 7.3.5o)}$$

where F_o = the maximum value of the force and ω_d = the angular frequency of the driving force, the equation of motion may be written as

$$m\ddot{x} + c\dot{x} + kx = F_o \sin \omega_d t \qquad \text{(Eq 7.3.5p)}$$

The resulting solution consists of two parts, (1) free damped vibration as represented by the three types for free vibration with damping, Equation (7.3.5h), and (2) a particular solution expressed by the steady-state oscillation which remains after the damped motion of the transient solution dies out.

The steady-state oscillation is represented by

$$x = X \sin(\omega_d t - \phi) \qquad \text{(Eq 7.3.5q)}$$

where ϕ is the phase angle by which the motion lags the impressed force, and X is the amplitude of steady oscillation.

If $\zeta > 1$, the complete solution for an overdamped system is

$$x = Ae^{[-\zeta + \sqrt{\zeta^2 - 1}]\omega_n t} + Be^{[-\zeta - \sqrt{\zeta^2 - 1}]\omega_n t} + X \sin(\omega_d t - \phi) \qquad \text{(Eq 7.3.5r)}$$

If $\zeta < 1$ (light damping), the displacement is

$$x = e^{-\zeta\omega_n t}\left[Ce^{i\sqrt{1-\zeta^2}\,\omega_n t} + De^{-i\sqrt{1-\zeta^2}\,\omega_n t}\right] + X \sin(\omega_d t - \phi) \qquad \text{(Eq 7.3.5s)}$$

If $\zeta = 1$ (critical damping), the displacement is

$$x = (E + Ft)e^{-\omega_n t} + X \sin(\omega_d t - \phi) \qquad \text{(Eq 7.3.5t)}$$

where $\omega_d = 2\pi f_d$ = angular frequency of driving force

f_d = frequency of driving force

Differentiating Equation (7.3.5q) for $\dot{x}$ and $\ddot{x}$, and substituting into Equation (7.3.5r)

$$-m\omega_d^2 X \sin(\omega_d t - \phi) + c\omega_d X \sin\left(\omega_d t - \phi + \frac{\pi}{2}\right) + kX\sin(\omega_d t - \phi) - F_o \sin \omega_d t = 0 \qquad \text{(Eq 7.3.5u)}$$

Then

$$X = \frac{F_o}{\sqrt{(k - m\omega_d^2)^2 + (c\omega_d)^2}} \qquad \text{(Eq 7.3.5v)}$$

and

$$\tan \phi = \frac{c\omega_d}{k - m\omega_d^2} \qquad \text{(Eq 7.3.5w)}$$

Let

$$X_o = \frac{F_o}{k} \quad \text{zero frequency deflection of mass under impressed force, } F_o$$

Then the magnification factor (steady-state) is

(Eq 7.3.5x)

$$\frac{X}{X_o} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_d}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega_d}{\omega_n}\right)^2}}$$

$$\tan\phi = \frac{2\zeta\frac{\omega_d}{\omega_n}}{1 - \left(\frac{\omega_d}{\omega_n}\right)^2} \qquad \text{(Eq 7.3.5y)}$$

The magnification factor is the ratio of the amplitude of steady oscillation to the deflection of mass under the static force, F_o.

The equation for the magnification factor is plotted in Figure 7.3.5b. For relatively small values of ζ, resonance occurs when the driving frequency is near the undamped natural frequency of the system. For large values of ζ, resonance occurs at ratios of driving frequency to undamped natural frequency which approach zero as ζ approaches $0.707\left(\frac{1}{\sqrt{2}}\right)$

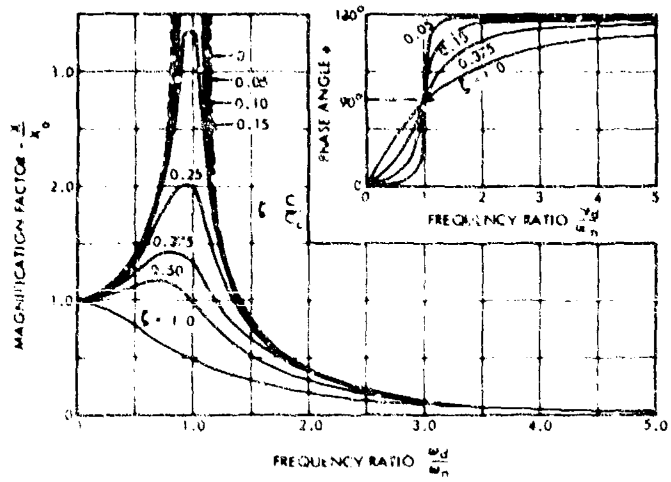


Figure 7.3.5b. Plot of Equations 7.3.5x and 7.3.5y for the Vibration of a Viscously Damped System

CASE IV Free Vibration With Coulomb Damping $F(t) = 0$, μ = constant

Coulomb (friction) damping is due to frictional forces and is considered independent of displacement, velocity, and acceleration. The sign of the force cannot be taken into account for a complete cycle. However, energy methods may be used for conducting the analysis.

Referring to Figure 7.3.5c and equating the work done by the spring, the work of friction to the kinetic energy per half cycle is

(Eq 7.3.5z)

$$\frac{1}{2}k\,x_o^2 - \frac{1}{2}k\,(x_o - b)^2 - F\,(2x_o - b) = \frac{1}{2}m\,(v_i^2 - v_f^2) = 0$$

where $v_i = v_f = 0$

x_o = initial spring displacement

b = decrease in amplitude per half cycle = $2F/k$

The amplitude decrease is constant for each half cycle, and the decay per cycle is thus

$$2b = \frac{4F}{k} \qquad \text{(Eq 7.3.5a')}$$

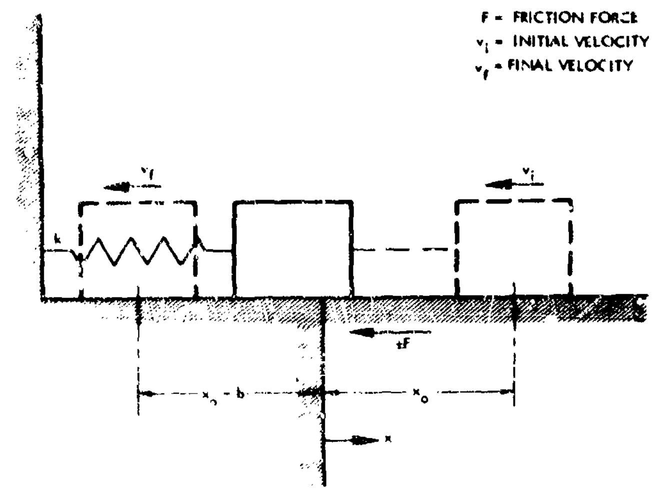


Figure 7.3.5c. Vibrating System with Coulomb Damping

The rate of decay is shown in Figure 7.3.5d. The motion will cease when the spring force is insufficient to overcome the static friction force.

7.3.6 Vibration Isolation and Transmissibility

An element rigidly attached to a foundation or supporting structure will transmit to that support any vibration originating from it. Conversely, any vibration of the supporting structure is transmitted to the element. Vibration isolators are a means of minimizing the transmitted vibration These isolators may take the form of rubber mounts, springs, padding, dashpot dampers, etc. Assuming that isolators can be represented by the spring and dashpot shown in Figure 7.3.5a, the magnitude of the transmitted force is given by

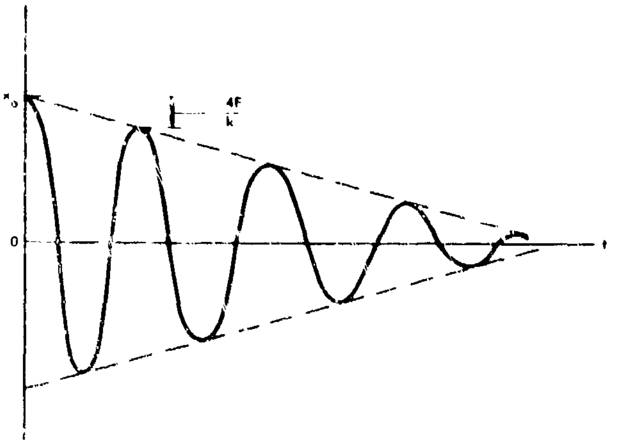


Figure 7.3.5d. Rate of Decay of Amplitude with Coulomb Damping is Linear with Time
Figure 7.3.6. Transmissibility

(Eq 7.3.6a)

$$F_{TR} = \sqrt{(kX)^2 + (c\omega_d X)^2}$$

$$= kX\sqrt{1 + \left(2\zeta\frac{\omega_d}{\omega_n}\right)^2}$$

where X is the amplitude of steady oscillation, given in Sub-Topic 7.3.5, Case III. Then

(Eq 7.3.6b)

$$F_{TR} = \frac{F_o\sqrt{1 + \left(2\zeta\frac{\omega_d}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega_d}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega_d}{\omega_n}\right)^2}}$$

The transmissibility, TR, of the system is the ratio of the force transmitted through the springs, plus damper to the force transmitted when the mass is mounted rigidly to the foundation.

(Eq 7.3.6c)

$$TR = \frac{F_{TR}}{F_o} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega_d}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega_d}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega_d}{\omega_n}\right)^2}}$$

The TR equation is plotted in Figure 7.3.6. When TR = 1, all the curves pass through the point where $\frac{\omega_d}{\omega_n} = \sqrt{2}$. Figure 7.3.6 also shows that for values of $\frac{\omega_d}{\omega_n} < \sqrt{2}$, the transmitted force is greater than the value for rigid mounting. Vibration isolation then is possible only when $\frac{\omega_d}{\omega_n} > \sqrt{2}$. If damping is negligible, $\zeta = 0$, then

$$TR = \frac{1}{\left(\frac{\omega_d}{\omega_n}\right)^2 - 1} \qquad \text{(Eq 7.3.6d)}$$

where $\frac{\omega_d}{\omega_n} > \sqrt{2}$

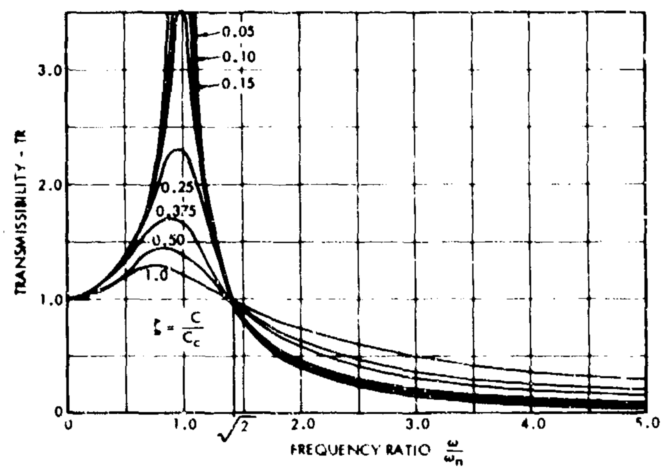


Figure 7.3.6. Transmissibility Versus Frequency Ratio

7.3.7 Self-Excited Vibrations

A self-excited vibration may occur when the exciting force is a function of the displacement, velocity, or acceleration. If a system is excited by a force proportional to the velocity of the mass, the equation of motion for a single degree of freedom system is

$$m\ddot{x} + c\dot{x} + kx = a\dot{x} \qquad \text{(Eq 7.3.7a)}$$

The term $\dot{x}$ is the forcing function. Equation (7.3.7a) transformed becomes

(Eq 7.3.7b)

$$\ddot{x} + \frac{(c - a)\dot{x}}{m} + \frac{k}{m}x = 0$$

The general solution of Equation (7.3.7b) is

$$x = Ae^{S_1 t} + Be^{S_2 t} \qquad \text{(Eq 7.3.7c)}$$

(Eq 7.3.7d)

where $S_{1,2} = -\frac{(c-a)}{2m} \pm \sqrt{\left(\frac{c-a}{2m}\right)^2 - \frac{k}{m}}$

If $a > c$, this system is negatively damped, causing the amplitude to increase exponentially. The system is then referred to as dynamically unstable. The equation of motion of this system is similar to the equation of motion for free vibration with viscous damping given in Sub-Topic 7.3.5, except that the sign of c is negative. In a physical system, nonlinear effects enter eventually, and Equation (7.3.7b) fails to represent the system.

7.3.8 Random Vibration

The vibration environment to which a component or part is exposed when functioning as part of a system is very seldom in the form of a single frequency sinusoidal environment, but rather is a combination of frequencies occurring simultaneously in a random manner.

Random vibrations are the result of a number of events occurring by chance and may be characterized by any frequency spectrum. An acceleration-time curve describing random motion is illustrated in Figure 7.3.8a. The magnitude of the acceleration and the period between zero accelerations varies erratically. The frequency of a random vibrating system cannot be specified because several frequencies are present simultaneously. In order to deal with random vibration, it is necessary to deal with the total energy of some specified band of frequencies. Random motion can be conveniently represented as a single component frequency by a concept known as acceleration density. A plot of the acceleration density at each frequency gives a curve of g^2/cps versus frequency over the frequency spectrum of interest and is known as the *power spectral density* (PSD) curve. The PSD curve is used to give a complete description of a random vibration test requirement. When the PSD curve is flat, the random motion is referred to as *white noise*. Figure 7.3.8b represents typical PSD curves.

The equation for the acceleration density is

$$G = \lim_{B \to 0} \frac{a^2}{B} \qquad \text{(Eq 7.3.8a)}$$

where G = acceleration density, g^2/cps

a = root mean squared (rms) average of the random accelerations

B = range of frequency under consideration, referred to as the bandwidth

The value of a may be calculated by squaring the instantaneous accelerations, computing the average or mean of the squared values, and then taking the square root of the average. For example, given the instantaneous accelerations of 1 g and 7 g's, the rms average of random acceleration is calculated

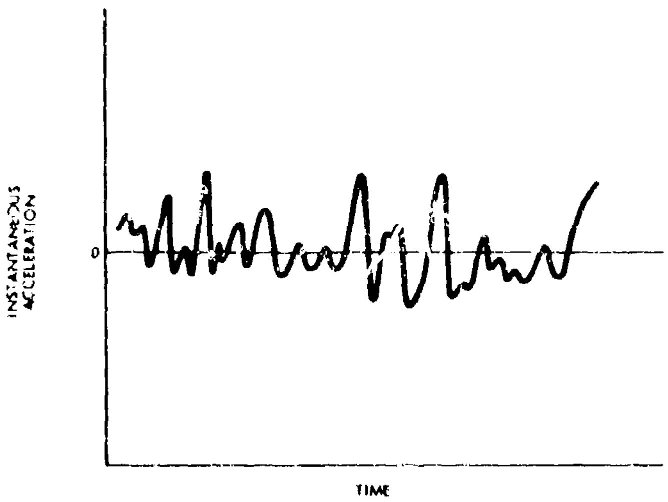


Figure 7.3.8a. Acceleration Time Curve Describing Random Motion

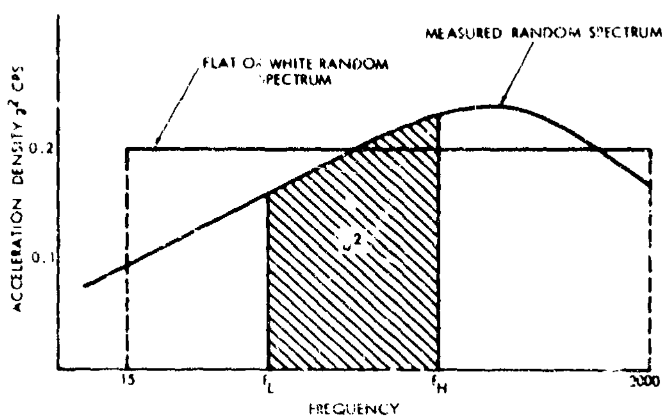


Figure 7.3.8b. Power Spectral Density Curves

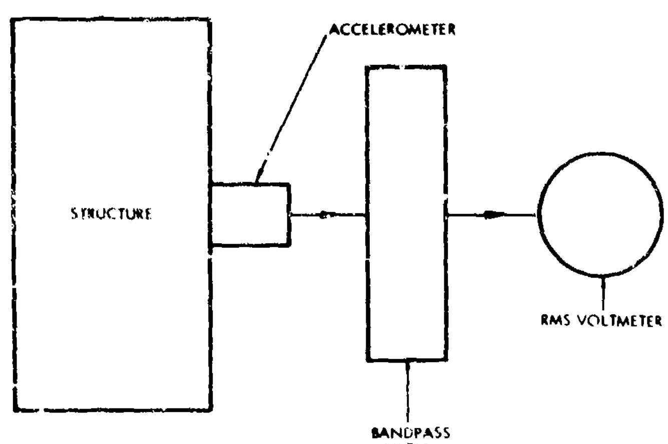


Figure 7.3.8c. Basic Instruments Necessary to Measure the Acceleration Density of a Structure in Random Vibration

$$1^2 + 7^2 = 1 + 49 = 50$$

$$50/2 = 25$$

$$\sqrt{25} = 5 \text{ g rms}$$

For a sinusoidal wave, the rms value is

(Eq 7.3.8b)

$$a\ (\text{rms}) = \sqrt{\frac{1}{T}\int_o^T a_m{}^2 \sin^2 \frac{2\pi t}{T}\, dt} = \frac{a_m}{\sqrt{2}}$$

where a_m is the peak or maximum value.

Acceleration, a, can also be measured directly from the random vibrating structure. An instrumented structure capable of random vibrations is shown in Figure 7.3.8c and is described as follows:

1) The accelerometer attached to the structure produces a voltage proportional to the instantaneous acceleration of the structure.
2) The voltage is then filtered through a bandpass filter which permits only the frequencies within the bandwidth B to pass through.
3) The rms voltmeter is used to read the output voltage of the bandpass filter. The voltage reading is the root mean square value of the voltage waveform and, therefore, represents the rms random acceleration.

The system may be calibrated by exciting the structure sinusoidally at one g rms and the rms voltmeter set to read 1. The one g rms is equivalent to a peak acceleration of 1.41g. As the frequency of the sinusoidal motion is varied, the meter will read the value 1 if the frequency is within the bandwidth B. The bandwidth B is defined by the equation

$$B = f_H - f_L \qquad \text{(Eq 7.3.8c)}$$

where f_H = high cutoff frequency

f_L = low cutoff frequency

At any frequency higher than f_H or lower than f_L, the voltmeter will read zero.

After the system is calibrated and is excited randomly, only the frequency band of the random voltage within the bandwidth is supplied to the voltmeter. The random acceleration, a, in rms g for the frequency band is read directly off the voltmeter.

If the bandwidth is reduced to smaller values, the voltage indicator will fluctuate about a mean value, with the frequency of fluctuations approximately the bandwidth of the filter in cps. If the bandwidth is reduced by successive factors of four, the rms random acceleration will drop by factors of approximately two. The square of the rms random acceleration is proportional to the bandwidth, $a^2 \propto B$. As B decreases, the quantity a^2/B approaches a constant value which is the acceleration density, G, given in Equation (7.3.8a). An acceleration density measured in a typical random environment is shown in Figure 7.3.8b.

The measured acceleration density is not constant but is a function of frequency. An environment with a constant acceleration density as a function of frequency is called a *white* or *flat* random motion spectrum and is designated G_o, as shown in Figure 7.3.8b. For a white spectrum, the acceleration density is independent of bandwidth and the equation is

$$G_o = \frac{a^2}{B} \qquad \text{(Eq 7.3.8d)}$$

An example of a random motion test specification is a constant acceleration density of 0.2 g^2/cps over the bandwidth from 15 cps to 2015 cps. From the equation for the acceleration density for the white spectrum

$$a = \sqrt{(2015\text{-}15)\ (0.2)} = 20 \text{ g rms}$$

7.3.9 Shock and Resulting Stresses

Mechanical shock is a sudden, non-periodic disturbance occurring when environmental accelerations are applied for short but definite periods of time. One concept is the velocity shock, or the rapid variation of velocity causing large accelerations. The resulting accelerations, called pulses, are specified by acceleration amplitude, time duration, and pulse shape. Actual pulse shapes are usually complex; they do not readily lend themselves to mathematical description, but are approximated by comparing them with simple pulse shapes such as those given in Figure 7.3.9a. For detailed analysis of shock loading, see References 388-1, 388-2, and 388-3.

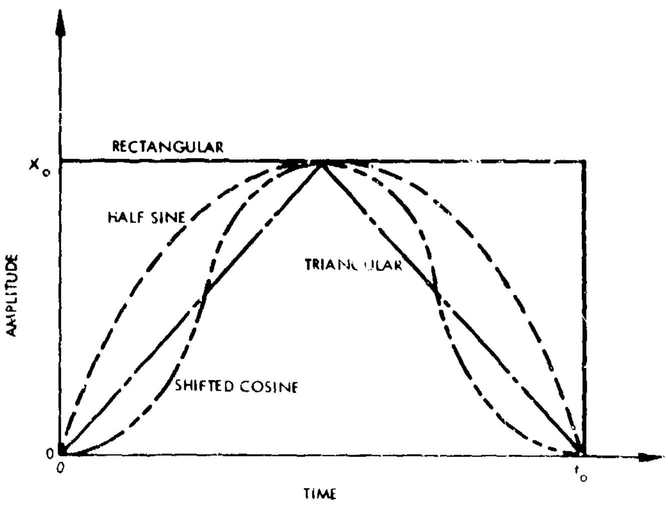


Figure 7.3.9a Pulse Load Shapes

In general, the characteristic of shock which makes it different from static loading is the time required for the acceleration to rise from zero to a maximum. If the time of acceleration rise is less than one half the natural period of the structure, shock conditions are said to exist; and, if the time of acceleration rise is greater than three times the natural period, the static conditions, or loads, are said to exist.

To protect against shock, it is common design practice to multiply the calculated load under static conditions by a shock or load factor and then design the part to resist the corrected static load. A load factor of two is usually recommended for shock loads. However, the factor of two applies only when the system is under a rectangular pulse shape loading. For other pulse shapes, the load factor may be somewhat less, as illustrated in Table 7.3.9. It should be pointed out that these load factors are maximum and only apply when structural deflection is linear and the period of the applied load and the natural period of the structure have a defined relationship.

To illustrate the use of shock loading factors, consider the following example:

A body is subjected to a suddenly-applied acceleration rise of 10g's, as shown in Figure 7.3.9b. The body weighs one pound and the connecting rod has a cross sectional area of one square inch. Find the stress in the rod.

The force applied to the rod is

$$F = ma = \frac{1\ \mathrm{lb}}{g} \times 10g = 10\ \mathrm{lb_f}$$

The equivalent static tensile stress

$$\sigma_{static} = \frac{F}{A} = \frac{10}{1}\ \mathrm{lb_f/in^2}$$

The dynamic stress for shock loading with a rectangular pulse shape is

$$\sigma_{dynamic} = 2\,\sigma_{static} = 20\ \mathrm{lb_f/in^2}$$

The factor of two could have been applied to the acceleration environment and the actual stress computed directly.

If the body were suspended vertically, the acceleration would be 11g's, since the body is initially under a load of 1g. The total stresses then are determined as follows:

Table 7.3.9. Load Factors for Several Pulse Shapes

(Reference 1 268)

PULSE SHAPE	LOAD FACTOR
Rectangular	2
Half sine	1.8
Shifted cosine	1.7
Triangular (equilateral)	1.5

$$F = \frac{W}{g}a + W = W\left(\frac{a}{g} + 1\right) = W(10 + 1) = 11W\ \mathrm{lb_f}$$

The equivalent static tensile stress is

$$\sigma_{static} = 11\ \mathrm{lb_f/in^2}$$

The dynamic stress is

$$\sigma_{dynamic} = 22\ \mathrm{lb_f/in^2}$$

Nomenclature

SYMBOL	QUANTITY	DIMENSION
a	rms average of random acceleration	L^2/t^4
A	Constant	L
B	Constant	L
B	Frequency range ($f_u - f_l$)	$1/t$
c_c	Critical damping coefficient	M/t
C	Damping coefficient	Ft/L
C	Constant	L
D	Constant	L
E	Constant	L
f	Frequency	$1/t$
F	Force	F
F	Constant	L
G	Acceleration density	L^2/t^3
k	Spring constant	F/L
m	Mass	M
N	Normal force	F
t	Time	t
T	Period	t
TR	Transmissibility	—
W	Weight	F
x	Displacement	L
X	Amplitude of displacement	L
ζ	Damping factor	—
μ	Coefficient of friction	—
σ	Stress	F/L^2
ϕ	Phase angle	radians
ω	Angular frequency	$1/t$
ω_d	Angular frequency of driving force	$1/t$
ω_n	Natural frequency	$1/t$

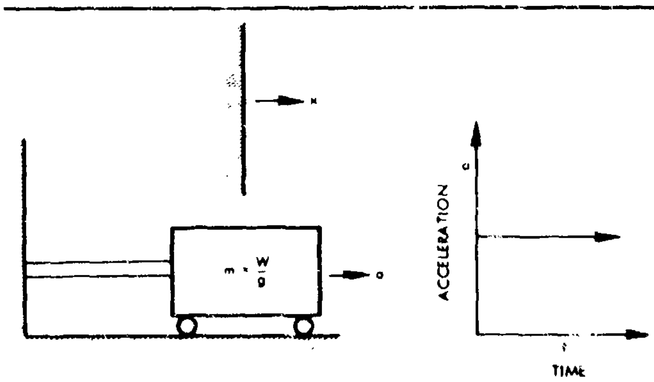


Figure 7.3.9b. A System Under the Influence of a Suddenly Applied Acceleration

7.4 DYNAMIC PERFORMANCE ANALYSIS

7.4.1 Introduction

Many fluid components used in aerospace applications have a control or regulation function. Dynamic characteristics such as response rate and stability are thus important performance parameters in these components. In designing the units, liberal use is made of control system theory, which deals with the design and performance analysis of control systems and which is reviewed in Sub-Section 7.2. The purpose of the following paragraphs is to illustrate how control system theory is applied to fluid component design.

7.4.2 Methods of Component Design

Fluid components, such as hydraulic servo-actuators and propellant tank pressure regulators, have control or regulation functions. There are two basic design approaches for components in this class, synthesis and analysis. Using synthesis, the designers are given a set of requirements concerning component performance, weight, size, and other factors. They are asked to synthesize directly from available design data a component which fulfills the requirements. The synthesis includes the design or selection of suitable elements. Using analysis, the designers or analysts are given an existing component (one which exists on paper or in hardware) and are asked to determine how closely the unit meets the required performance and other specifications. If the component does not fulfill the requirements, it is either modified or a new component is laid out, and the procedure is repeated.

A comparison of synthesis and analysis shows that the former is the ideal design approach. Where synthesis can be practiced, the cut-and-try approach of analysis is eliminated, and a final design is arrived at in the shortest possible time.

Pure synthesis, however, requires specialized mathematical techniques for the type of component being designed, along with complete design data. The necessary mathematical techniques and data are available in well-established fields of component design, but are generally lacking in the newer fields such as aerospace components. As a result, the pure synthesis approach is not employed in designing the latter units. The present design procedure is a combination of analysis and synthesis, which is a less demanding but likewise less direct approach than synthesis alone. It is described in more detail in the following Sub-Topic.

7.4.3 Synthesis by Analysis

Aerospace components, as noted, are designed by a procedure combining analysis and synthesis. C. J. Savant, Jr., in the preface of Reference 403-1, discusses this kind of procedure in connection with feedback control systems, employing the term *synthesis by analysis* to describe the process. The following excerpt from the preface of Reference 403-1 illustrates the use of the term:

"From my own experience the following philosophy is presented as being basic in feedback-control-system design: feedback control systems are designed by trial and error. Each trial is analyzed, and the results of the analysis are then examined to determine the next. The first guess might possibly be the type of equalizer (series, parallel, etc.) necessary to improve system performance. The next may be the component values of a resistance-capacitance network or the amplifier gain. The engineer continues varying system quantities until satisfactory performance is obtained. This design technique, which I call 'synthesis by analysis', requires that the engineer be able to analyze rapidly each subsequent trial."

The description *synthesis by analysis* applies equally to the design and development of a typical aerospace component with a control function. The principal steps in the design procedure for this class of component are given below. In this procedure it is assumed that complete design specifications have been formulated and are available.

1) Configuration studies are made, and a basic configuration for the component, including elements, is selected. From the specifications, initial estimates are made of the following design parameters: power requirements, fluid pressures, flow rates, forces, structural stresses, size and weight of elements, and the complete component.

These estimates are preliminary and may be revised later.

2) A dynamic performance analysis is made of the component design laid out in Step 1. The purpose of this analysis is to establish the response and stability characteristics of the unit. The form of the analysis may be one or a combination of the following: mathematical-graphical (frequency response, root-locus, etc.), computer simulation, or testing of breadboard model or prototype.

The dynamic characteristics of the component as determined by this analysis are compared with the characteristics required in the specifications. Revisions in the design are then made where necessary, and a re-analysis is carried out. This procedure is followed until satisfactory dynamic performance is attained.

3) Testing of the actual component under extreme environmental conditions is performed. Further revisions in the design are made where required, until the unit meets the specifications with respect to environmental effects.

7.4.4 Performance Specifications for Closed-Loop Systems

The dynamic performance analysis referred to in Step 2 of Sub-Topic 7.4.3 is a key phase in the development of a new control component. The purpose of this analysis, as noted, is to determine the characteristics of a trial design and to correct the design, if necessary, to meet the required performance specifications. Concerning the latter specifications, components such as servo-actuators and pressure regulators are small-scale, closed-loop control systems, and on this account they are defined by the performance specifications for closed-loop systems. These specifications will be reviewed in this section.

In a typical control system, the important performance characteristics are speed of response, accuracy, and stability. In a new system design, it is necessary to specify the requirements in these three areas as accurately as possible. Various specifications based on control theory have been developed for this purpose. It has not been possible to derive a single simple and usable specification for each of the characteristics of response, accuracy, and stability. As a result, numerous specifications have been derived, some of which define a single characteristic while others define the over-all system performance. Table 7.4.4 describes eleven of the most common specifications. Each of the criteria in this table has certain advantages and limitations when applied to a given system. The columns in the table list the name of the specification, its type, a definition of the specification, how it is calculated, and the advantages and limitations in its use.

7.4.5 Methods of Dynamic Performance Analysis

In the development of a new control component the dynamic perfromance analysis of several successive trial designs or configurations may be required, as noted. This analysis establishes the dynamic characteristics of the trial units. From these characteristics, design improvements are derived which lead eventually to the final configuration. To complete the development program within a reasonable length of time and cost, rapid and accurate methods of dynamic performance analysis are required. The methods employed today include mathematical and graphical analysis, computer simulation, and the testing of breadboard models or simplified prototypes. One or a combination of these techniques may be used, depending on the type of component being developed. All of the methods have limitations and approximations, and in certain cases will not give useful results. In general, however, the methods are a major help in studying the performance of a new unit, and in deducing improvements in the design. While several methods of dynamic analysis may be used, there are common steps in the procedure of applying them. These steps are outlined below.

7.4.5.1 GENERAL PROCEDURE

1) *Schematic Diagram.* The original step in the analysis of a control component is the layout of the schematic diagram. This diagram provides a physical picture of the configuration, and illustrates the operation of the individual elements and the system. Figure 7.4.7.1a is a representative schematic diagram of a hydraulic actuator. The function of this valve-cylinder servo-mechanism is to move a load a displacement, θ_o, in response to an input signal θ_i. In more advanced servos of this type, the load could be a rocket engine nozzle or a complete engine which is gimballed for thrust vector control. In the system of Figure 7.4.7.1a, the input signal or displacement is converted to voltage, $K_1\theta_i$, by potentiometer A. The difference, V, between the input and feedback voltages is electronically amplified and used to control a solenoid valve. The latter controls the main valve, which controls the hydraulic cylinder. Feedback is provided by potentiometer B which is connected to the cylinder and converts the output displacement, θ_o, to voltages, $K_1\theta_o$. A dynamic analysis of this actuator is given in Sub-Topic 7.4.7.

2) *Differential Equations.* The differential equations of the component elements and the complete unit are derived from the fundamental laws of energy, momentum, and continuity, as applied to the system.

3) *Transfer Functions.* The transfer functions of the elements are obtained from the corresponding differential equations by the methods described in Sub-Topic 7.2.2. From these transfer functions, the open-loop and closed-loop transfer functions of the complete unit are obtained.

4) *Block Diagram.* This type of diagram has been described in Sub-Section 7.2. It is a figure showing only the functions of the system elements and the interconnections between elements, without showing how the functions are accomplished. The block diagram method of representation can take many forms, depending on its purpose. Each block may represent a single element or a combination of elements. The functions may be shown descriptively, as mathematical expressions, or as transfer functions. Figure 7.4.7.1b is a block diagram for the hydraulic actuator of Figure 7.4.7.1a. The elements in this case are represented by their transfer functions. A detailed explanation of how this diagram was derived and how it is used is given in Sub-Topic 7.4.7.

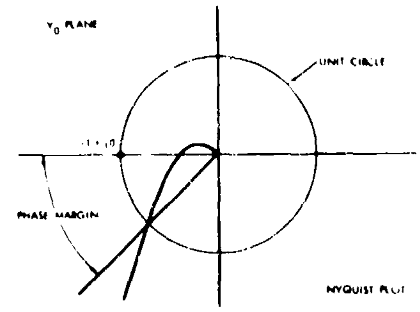
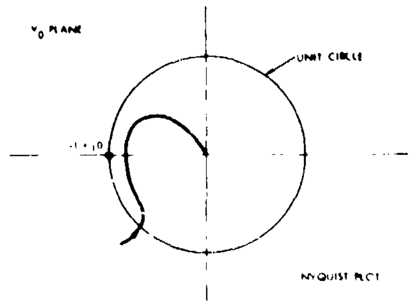
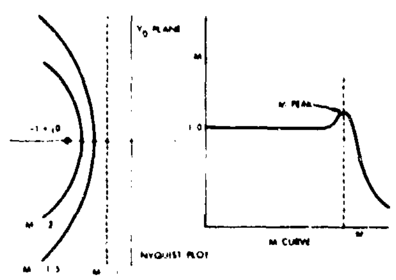
5) *Synthesis by Analysis.* Following the derivation of the schematic diagram, differential equations, transfer functions, and block diagram for a new component design, the *synthesis by analysis* process previously described is then carried out. This process, as noted, consists of the following basic steps which are repeated until an effective over-all configuration is synthesized: (a) dynamic performance analysis of a trial design, and (b) correction of the design based on the results of the analysis.

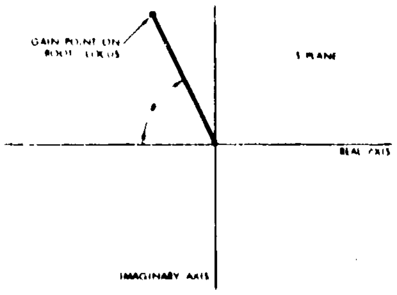
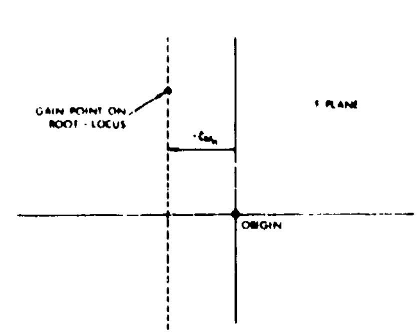
Notations for Table 7.4.4

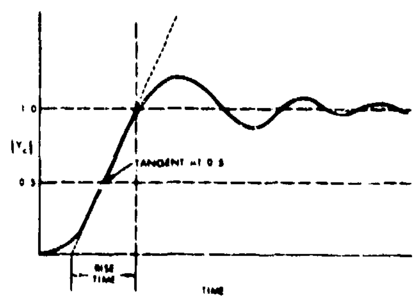
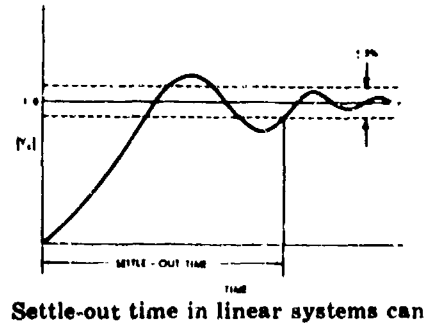
db	Decibel
F(s)	Transform of f(t)
f(t)	Function of time
$\mathcal{L}$	Laplace transform
M	$= Y_c(s) =$ Amplitude ratio
s	Laplace variable
Y_c	Closed-loop transfer function
Y_o	Open-loop transfer function
ζ	Damping ratio
θ	$= \cos^{-1} \zeta$
ω	Angular frequency (radians/sec)
ω_n	Natural frequency (radians/sec)

ISSUED: OCTOBER 1965
SUPERSEDES: MAY 1964

Table 7.4.4. The Eleven Most Common Performance Specifications
(Reference 47-1)

NAME	TYPE OF SPECIFICATION	DEFINITION AND METHOD OF COMPUTATION	GENERAL REMARKS
1) Gain Margin	Stability	Ratio of maximum stable gain to actual gain at a phase angle of 180°. Can be calculated by Routh's criterion, or from Nyquist plot or root-locus diagram.	Simplest stability specification, but gives little information on relative stability and thus is seldom used. However, if gain margin is less than 10, system may be "rubbery" and overshoot excessively.
2) Phase Margin	Frequency, domain stability	On Nyquist diagram, the angle between the negative real axis and the Nyquist curve at the unit circle.  On Bode plot, read frequency at unity gain, determine phase shift at this frequency, and subtract from 180°.	Indicates absolute and relative stability. For well-behaved systems, a phase margin of 45° means one overshoot in response to a step input. Phase margin is an easy specification to use, but there is the possibility that a system with supposedly adequate phase margin may overshoot several times. For example, a system with under-damped quadratic roots at a frequency just above crossover frequency (frequency at which Nyquist plot crosses unit circle) can have a Nyquist plot as follows:  The system is under-damped and oscillates as shown by its high M peak ($M_{max} > 2.0$) and low damping ratio ($\zeta < 0.2$).
3) M Peak  	Frequency, domain stability	M is the magnitude of the closed-loop transfer function for a system with unity feedback $$M = \left\| \frac{y_o}{1 + y_o} \right\|$$ M peak is the maximum value of M. It is obtained by plotting circles of constant M on the Nyquist plot. M peak is the value corresponding to the circle tangent to the Nyquist curve.	The M peak criterion is more difficult to check than phase margin, but is superior in that a given M peak assures that the Nyquist plot never approaches the $(-1 + j0)$ point for any frequency. An M peak of 1.4 usually means one overshoot. However, a rapid phase shift near unity gain, such as is associated with underdamped quadratic terms, can result in several small-amplitude oscillations in a step-function response, even for M peaks as low as unity.

NAME	TYPE OF SPECIFICATION	DEFINITION AND METHOD OF COMPUTATION	GENERAL REMARKS
4) Damping Ratio	Stability	Defined as ζ in the quadratic term $s^2 + 2\zeta\omega_n s + \omega_n^2$. This indicates the decay per cycle of the natural frequency. On the root-locus diagram, ζ equals $\cos\theta$ where θ is defined: 	For a simple quadratic transfer function, $\zeta = 1$ for a critically damped response, while $\zeta = 0.7$ means one overshoot. Damping ratio is extended to a system with more than two closed-loop poles by analogy. Thus its meaning differs with different systems. The advantage of damping ratio over the previous specifications is that it is pessimistic in at least one respect. A damping ratio of 0.7 could mean an overdamped response for certain systems, but the response could never be more oscillatory than the simple quadratic response for the same ζ. However, the damping ratio gives no indication of the first overshoot. It is possible for a system with ζ as large as unity to exhibit an extremely large overshoot in response to a step-function input. This possibility must be suspected if one of the closed-loop roots is close to the origin for the gain constant used. A damping ratio of 0.6 to 0.8 is usually specified.
5) Damping Factor, $\zeta\omega_n$, or Decrement Factor 	Stability	Defined by the factors in a quadratic system, as was damping ratio. It is $\zeta\omega_n$ in the roots $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ which give a characteristic equation of the form $e^{-\zeta\omega_n t}(c_1 \cos \omega_n t\sqrt{1-\zeta^2} + c_2 \sin \omega_n t\sqrt{1-\zeta^2})$ It thus determines the rate of decay of the transient. $\zeta\omega_n$ may be found directly from the root-locus diagram.	This has had extensive use in systems demanding prescribed transient performance, particularly when the time decay is important, e.g., in autopilots, and is a convenient method of interpreting more complex systems in terms of quadratic systems. The damping factor of the roots closest to the imaginary axis is presumed to have the greatest influence on system response. If damping factor is specified along with damping ratio, the possibility of a large initial overshoot may be eliminated.
6) Percent Overshoot	Direct evaluation of system relative stability	Ratio of peak of transient to final value, in response to a step input. Computation of this characteristic in a linear system involves solving the inverse Laplace transform either analytically or by picking values off root-locus plot. Direct determination in linear and non-linear systems is easiest with an analog computer.	Useful with non-linear systems. Used for regulators, meters, and position servomechanisms which are normally excited by step inputs and are underdamped. Usually a 20 to 30 percent overshoot is not considered deleterious if accompanied by a fast settle-out; 60 percent is large.

NAME	TYPE OF SPECIFICATION	DEFINITION AND METHOD OF COMPUTATION	GENERAL REMARKS
7) System Final Value	Steady-state accuracy	Final value of output. Calculated from system static characteristics, or by using final value theorem on Laplace transform of output for given input. For $\mathcal{L}\,[f(t)] = F(s)$ the final value theorem is $\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$	System final value minus desired output gives steady-state error or measure of steady-state accuracy. Steady-state error depends on gain constant and number of integrations in loop; it is independent of the system time constants.
8) Bandwidth	Frequency domain, speed of response specification	Usually defined as the frequency at which the closed-loop frequency response falls to 0.707 or -3 db of its low frequency value. Bandwidth is available from the curve of M versus frequency, or can be calculated from the Nyquist plot. A different definition of bandwidth is the cross-over frequency.	Bandwidth is directly related to speed of response and to system accuracy for rapidly changing input. The exact response of a system with a given bandwidth, however, depends somewhat on the particular system under consideration. Bandwidth is an indication of the highest input frequency that can be handled by a system.
9) Rise Time 	Speed of response	The following are definitions of rise time: a) 1/Bandwidth (see above) b) $1/\omega_{M\ peak}$ c) Time to first zero error, in response to step input d) See figure at left	Direct speed of response specification. Rise time defined as 1/bandwidth gives as good a measure of system performance as any of the remaining definitions. Time to first zero error is convenient if an analog computer is available.
10) Settle-out Time, or Synchronization Time	Over-all performance	The time required for the system response to a step input to reach and remain within a given tolerance to the final value. The usual tolerance is ± 5 percent.  Settle-out time in linear systems can be calculated analytically by the inverse Laplace transform, or by picking values off the root-locus plot. Determination in linear and nonlinear systems is easiest with an analog computer.	Used with systems requiring rapid synchronization. Settle-out time is the simplest of the over-all performance specifications. However, it is usually necessary to specify maximum allowable overshoot and steady-state error in addition.

NAME	TYPE OF SPECIFICATION	DEFINITION AND METHOD OF COMPUTATION	GENERAL REMARKS
11) ITAE, or Integrated Value of the Product of Time and Absolute Value of Error	Over-all performance	The ITAE specification is defined by the integral $\int_0^\infty t\|e\|dt$ where t is time and $\|e\|$ is the magnitude of the error. Minimize the value of this integral for optimum performance. This specification yields a number which depends on the particular system being considered. It would probably be an improvement to generalize the specification by normalizing to the rated output quantity. The dimensions would then be sec². Another problem with the given definition is that for a class of inputs causing the system to have a constant final error (which may actually be insignificant), the ITAE specification goes to infinity. This could be avoided by integrating to some arbitrary large time, such as 10 times constants, rather than to infinity.	ITAE is one of several attempts to define an over-all figure of merit for system operation. Rather than simply summing the error of a system, the error is progressively weighted more heavily as time goes on. This puts a premium on rapid, accurate settle-out, and allows for an unavoidable initial large error. While a generalized specification of system performance such as ITAE is desirable, any broad figure of merit must be used with caution lest it obscure an important specific advantage or disadvantage of a particular system.

7.4.5.2 SPECIFIC METHODS OF ANALYSIS

1) *Mathematical-Graphical Techniques*. Table 7.4.5.2 summarizes the six major mathematical or mathematical-graphical techniques of dynamic performance analysis. The first four methods are reviewed in Sub-Section 7.2. Reference 371-1 gives a detailed treatment of all of the techniques, plus numerous additional references on each.

2) *Computer Analysis*. The use of computers is described as follows:

Analog Computer Simulation. The mathematical and mathematical-graphical techniques listed above are extremely useful in their areas of application. With many complex and non-linear components, however, these methods are either inadequate or cumbersome to apply. In these cases, analog computers provide one of the best means of performing the required dynamic performance analysis. Simulation of a component on an analog computer allows rapid and complete (often visual) evaluation of the dynamic performance. This approach is discussed in Sub-Section 8.2.

Analysis Using Digital Computers. In dynamic performance analysis, the primary function of the digital computer is the high-speed performance of the calculations required in the mathematical methods referred to in (1) above. The rapid computing rate of these machines makes possible the analysis of complex control and regulation components. Digital computers and their applications are discussed in Sub-Section 8.3.

3) *Testing of Breadboard Model or Prototype*. Certain components cannot be adequately analyzed either by mathematical methods or by computer simulation. With these units, unrealistic assumptions or approximations may be required in order to carry through the analysis. Alternately, a great deal of manual computing effort or a long and expensive series of computer runs may be required. For these components, the construction and testing of breadboard models or simplified prototypes may represent the best method of establishing and improving the dynamic performance. With some components, a combination of mathematical and computer analysis and prototype testing is used to advantage in the development process. Mathematical analysis establishes the basic component configuration. Next, the component is simulated on an analog computer to determine its dynamic performance more accurately and to study the effect of varying critical parameters. Finally, a breadboard model or prototype is built and tested.

7.4.6 Advantages and Limitations in the Methods of Analysis

The methods of dynamic performance analysis used in developing control-type components have been listed as mathematical-graphical techniques, computer analysis, and the testing of breadboard models or simplified prototypes. While all of these methods are employed in component analysis, the first two are referred to as *analytical* whereas the third is *experimental*. This distinction will be used in the following paragraphs.

There is a question as to which type of approach, analytical or experimental, is the most useful in developing and analyzing fluid components. This question is being raised more frequently as regulators, relief valves, servo valves, etc., become more complex. Today, as in the past, fluid components are generally developed by the experimental or trial-and-error approach. After the basic configuration of a new component has been selected and the steady-state design calculations made, a breadboard model or prototype is built as rapidly as possible and tested to determine the dynamic

Table 7.4.5.2. Summary of Major Analytical Techniques
(Reference 371 1)

TYPE	USEFULNESS
1) Differential equations	Classical solutions of differential equations are generally too involved for practical use in synthesis. Non dimensional performance charts help on second order systems. Significance of individual system component values difficult to ascertain.
2) Routh-Hurwitz criterion	Used to determine the limiting stability conditions. Can be extended to include damping factor only with difficulty. Limited usefulness.
3) Root locus	The best solution to the problem of directly synthesizing the time response. Particularly useful when the performance specifications are in terms of the time response. Construction of the diagrams can be time-consuming and the performance can be sensitive to small relative changes of locus in low-frequency region.
4) Frequency response	The most used approach presently available. The locus can be plotted in the form of a Nyquist, log magnitude-angle diagram, or the log magnitude and phase diagram. The latter has the advantages of easy construction by templates and of easy introduction of compensating characteristics. Easy to include experimental data in frequency response analysis. The difficulty of relating transient and frequency response is a limitation.
5) Describing functions	An extension of the frequency response techniques to non-linear systems. Good performance criterion not available. Method can treat higher order systems.
6) Closed loop, pole-zero location	Requires determining realizable and practical components after the definition of the system response. Not in wide use as yet, but possesses the good feature of working directly from the desired closed loop response

performance of the unit. The design is then modified, if necessary, to improve the performance. Eventually a unit is obtained which meets the requirements. This approach has the advantage of dealing with actual hardware, rather than with mathematical or computer models which may incorporate unrealistic assumptions or approximations. On the other hand, the experimental approach can be extremely lengthy and expensive, particularly when it is applied to complex components, because of the cut-and-try aspect of the process. Physical testing of a component frequently does not show or give an understanding of what is taking place internally in the unit. As a result, design changes to improve the performance have to be guessed at in a hit-or miss manner, rather than arrived at logically. For this reason, improvements come slowly.

Because of the disadvantages in the experimental approach, the application of analytical techniques to fluid component development is being actively investigated today. In this approach, the paper or hypothetical design of a new component is dynamically analyzed by mathematical and graphical methods or by computer simulation. This analysis serves two useful purposes. First, it gives an early understanding of the operation of the component. Second, in applications where the analysis is known to be highly accurate, it establishes the performance of the unit and allows corrections in the design to be made before any metal is cut. Thus, a near-final design is obtained before the expensive fabrication of the prototype component is initiated. Even in cases where mathematical analysis or computer simulation do not give exact numerical results, the methods give an insight into the operation of the unit, an insight which may lead quickly to design improvements and a [illegible]er development period. Examples of component dynamic analysis which illustrate the usefulness of this approach are given in Sub-Topic 7.4.7, an analysis of a hydraulic servo-actuator by J. M. Nightingale. Sub-Topic 7.4.8 reproduces a paper by D. H. Tsai and E. C. Cassidy in which the dynamic behavior of a simple pneumatic pressure regulator or reducer is studied. Sub-Topic 7.4.9 is an analysis of a pneumatic dashpot in a regulator control element.

At the present stage of development, the analytical approach as yet does not deal adequately with certain types of components, for example, complex gas pressure regulators. In these cases, questionable approximations are often required in order to carry out the analysis. In addition, laborious hand calculations, or long and expensive computer runs may be required. References 33-1 and 35-1 describe the design and development of two pressure regulators of advanced configuration. In developing these units, it was intended to apply analytical techniques wherever possible. The operation of one regulator (Reference 33-1) was extensively studied on an analog computer. The second unit was analyzed in detail with the aid of a digital computer. These studies gave some insight into the operation of the regulators, but contributed few useful results to the development program.

The above remarks indicate that the advantages in the analytical approach are not always clear cut. Nevertheless, as improvement of the method continues, it is becoming an indispensable part of fluid component development. The ideal development cycle is one which combines the analytical and experimental approaches. The process is initiated with mathematical studies of the theoretical design and completed with operational testing of the hardware, with intervening analytical and experimental phases. Table 7.4.6 summarizes the advantages and disadvantages in the two approaches to component development.

Table 7.4.6. Summary of Design Approaches

ADVANTAGES	DISADVANTAGES
Analytical Approach: Where applicable, this approach gives an early insight into the operation of a component. It permits problems in dynamic performance to be solved and a near final design to be obtained before metal is cut. In these cases, it saves development time and cost. Furthermore, if good agreement between test and analysis has been achieved for one basic unit, analysis will help establish the design parameters of similar units prior to building and testing hardware.	With complex components, the analytical approach frequently requires questionable approximations, and either a laborious manual computing effort or long and expensive computer runs. Latter may exceed the time and cost of building and testing prototype unit. The theoretical design may not demonstrate predicted performance when converted into hardware.
Experimental Approach: This approach deals with actual component hardware, thus gives realistic test results. When a satisfactory unit is obtained, the design can be quickly put into production.	It gives little understanding of the internal operation of a component. Thus, design improvements have to be obtained by a hit-or-miss procedure. On this account, an experimental development program can also be long and expensive.

7.4.7 Analysis of a Hydraulic Servo-Actuator

7.4.7.1 INTRODUCTION. An example of the dynamic performance analysis of a fluid component is the valve-cylinder servomechanism shown in Figures 7.4.7.1a and 7.4.7.1b. The following example is based on an analysis given by J. M. Nightingale in Reference 1-128. In this example, the frequency response method is used.

The purpose of the servo-actuator of Figure 7.4.7.1a is to provide power amplification with positional accuracy. The servo, in response to a weak input signal or displacement, θ_i, exerts a strong output force over a displacement, θ_o. The output force may be used to move an aerodynamic control surface, for example, or a gimballed rocket engine for thrust vector control. The internal operation of the servo was briefly explained in Sub-Topic 7.4.5, in the paragraph titled "Schematic Diagram." The principal components of the system are the main valve and hydraulic cylinder, which in combination are referred to as the *valve-cylinder relay*. This relay constitutes the muscles of the system, exerting the required output force over the required displacement. Because the force is usually large, and the operation of the main valve and cylinder involves time lags, the performance of the relay usually determines the performance of the servo system as a whole. An analysis of the valve-cylinder relay alone will be given first.

7.4.7.2 DESIGN PARAMETERS OF VALVE-CYLINDER RELAY. The first phase in the development of a new system is the selection or calculation of the design parameters such as, in the case of a hydraulic servo, the supply pressure, valve travel, etc. The system design is then dynamically analyzed. Figure 7.4.7.2a shows a simplified version of the valve-cylinder relay of Figure 7.4.7.1a, in which the solenoid and main valves have been replaced by a single lever-operated valve. The design parameters for this simplified relay will first be obtained. For a specific application, the following parameters will usually be known or selected in advance: constant supply pressure, P_s; cylinder stroke, L; maximum cylinder load, F_m; certain required operating times of the valve-cylinder combination; and bandwidth, ω_b. The following design parameters will then be calculated: cylinder piston area, A; fully-open valve port area, a_m; differential pressure across the cylinder piston, P_c; no-load volume flow, Q_o; and valve travel, X_m; from zero port area to area, a_m. In order to obtain explicit expressions for these quantities, various approximations will necessarily be made.

Flow from the supply line through the right-hand valve port in Figure 7.4.7.2a is given by

$$Q = B\,a(X)\sqrt{P_s - P_1} \qquad \text{(Eq 7.4.7.2a)}$$

where B is a constant and a(X) is the valve port area as a function of the valve travel, X. Similarly, flow from the cylinder through the left-hand valve port is

$$Q = B\,a(X)\sqrt{P_2} \qquad \text{(Eq 7.4.7.2b)}$$

Assuming that leakage is small and both port areas are equal, these two flow rates will be approximately equal. Equations (7.4.7.2a) and (7.4.7.2b) give

$$P_1 + P_2 = P_s \qquad \text{(Eq 7.4.7.2c)}$$

The differential pressure acting on the cylinder piston is

$$P_c = P_1 - P_2 \qquad \text{(Eq 7.4.7.2d)}$$

Combining Equations (7.4.7.2c) and (7.4.7.2d)

$$P_s - P_1 = P_2 = \frac{P_s - P_c}{2} \qquad \text{(Eq 7.4.7.2e)}$$

Substituting for P_2 in Equation (7.4.7.2b)

$$Q = B\,a(X)\sqrt{\frac{P_s - P_c}{2}} \qquad \text{(Eq 7.4.7.2f)}$$

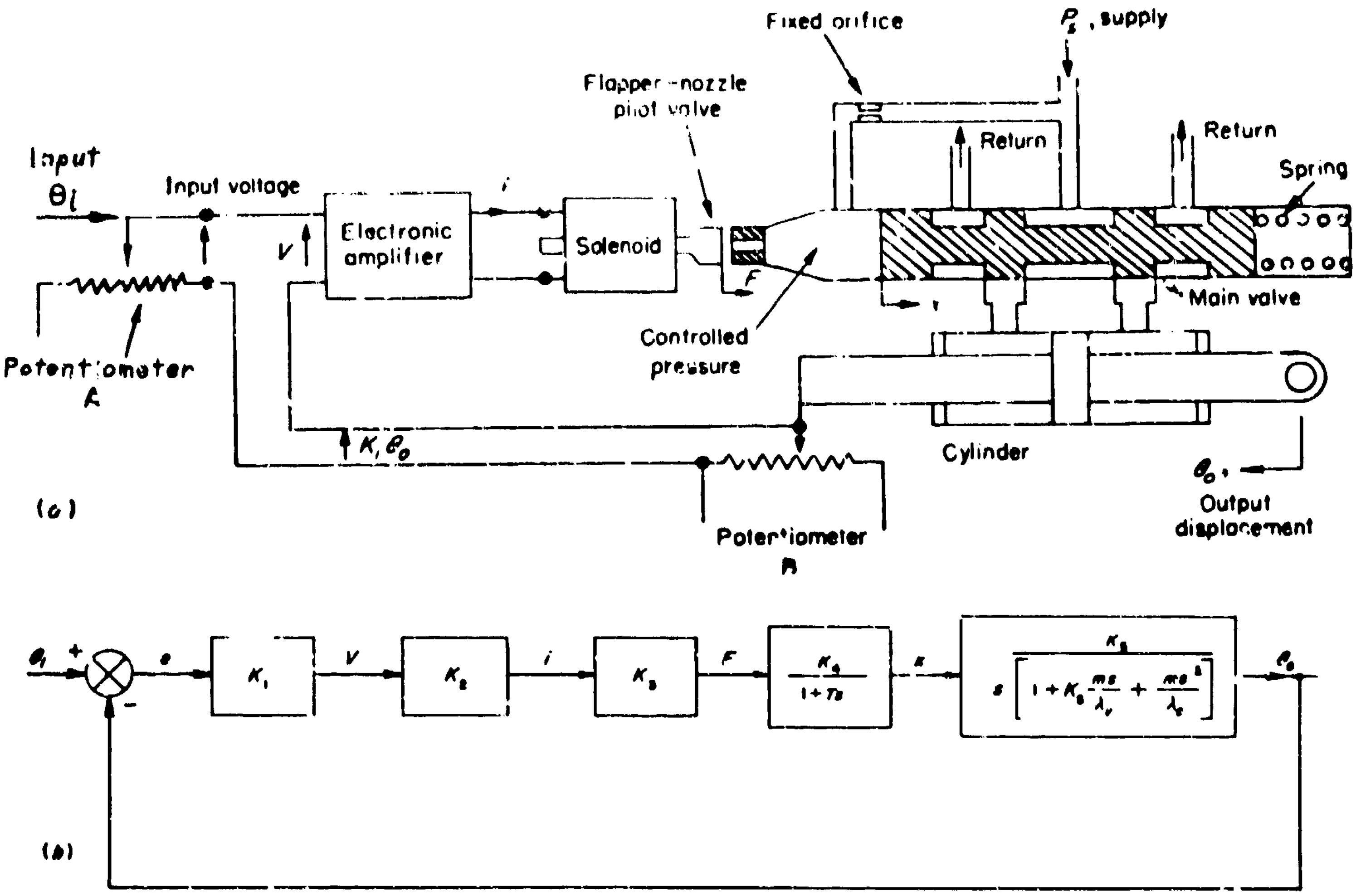


Figure 7.4.7.1a, b. In this valve-cylinder servomechanism, input is a voltage which is electronically amplified and used to control a solenoid valve. Feedback quantity is also a voltage supplied by a potentiometer connected to the output device, a hydraulic cylinder. A schematic diagram is seen in (a), and a block diagram for the system in (b).

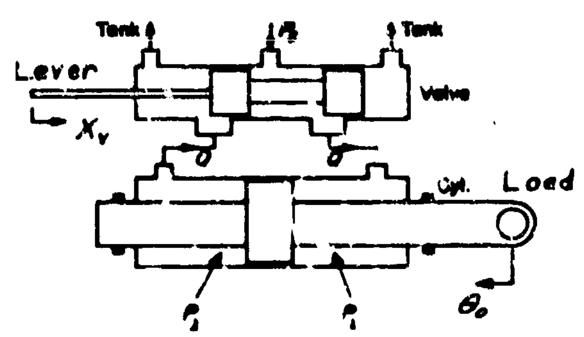


Figure 7.4.7.2a. Basic components of one type of power drive often used in hydraulic servo systems are a control valve and hydraulic cylinder. This section includes a mathematical analysis of the performance of such a system.

Any desired port area function a(X) can be approximately reproduced in the valve by making each port a row of spaced holes. For the moment, however, it will be assumed that a(X) is linear. Then

$$Q = \frac{B\,X\,a_m}{X_m}\sqrt{\frac{P_s - P_c}{2}} \qquad \text{(Eq 7.4.7.2g)}$$

Equation (7.4.7.2g) shows that flow varies not only with valve travel, X, but also with pressure, P_c, and hence with the cylinder load, since load = $P_c A$. From the form of Equation (7.4.7.2g), moreover, a varying load introduces a nonlinearity into the system equations which must be taken into consideration. However, for the present the cylinder load will be considered constant. Flow is then proportional to valve displacement only. Neglecting the compressibility of the fluid within the cylinder and the leakage across the piston, the flow into the cylinder is given by

$$Q = A\frac{d\theta_o}{dt} \qquad \text{(Eq 7.4.7.2h)}$$

From Equations (7.4.7.2g) and (7.4.7.2h), the transfer function relating cylinder displacement to valve displacement is

$$\frac{\theta_o}{X}(s) = \frac{C}{s} \qquad \text{(Eq 7.4.7.2i)}$$

$$\text{where} \quad C = \frac{B\,a_m}{A\,X_m}\sqrt{\frac{P_s - P_L}{2}} \qquad \text{(Eq 7.4.7.2j)}$$

In order to complete the servo the loop must be closed. The loop will often include other components such as preamplifiers, transducers, etc. In many cases, however, the loop contains only the valve and cylinder, and is closed by virtual feedback, as in Figure 7.4.7.2b. Feedback is virtual in this configuration due to the fact that the valve casing and the cylinder are integral and floating. The block diagram of this servo is given in Figure 7.4.7.2c. The open-loop transfer function, from Equation (7.4.7.2i) is

$$Y_o(s) = \frac{C}{s} \qquad \text{(Eq 7.4.7.2k)}$$

The closed-loop transfer function is

$$Y_c(s) = \frac{1}{1 + \tau s} \qquad \text{(Eq 7.4.7.2l)}$$

where $\tau = 1/C$.

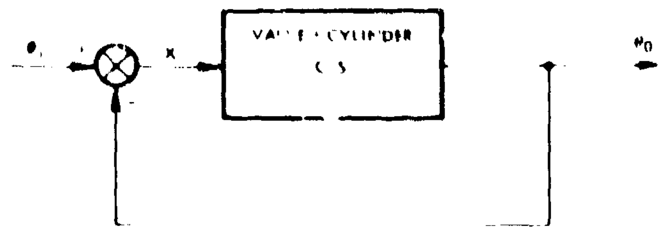


Figure 7.4.7.2c. Block Diagram of Valve and Cylinder.

Power delivered to the cylinder by the fluid is proportional to $P_L Q$ or $P_L\sqrt{P_s - P_L}$, which can be shown to be a maximum when $P_L = 2/3\,P_s$. The maximum value of P_L, therefore, should be two-thirds of the supply pressure, since maximum power is needed at maximum pressure differential in the cylinder to obtain the best possible performance.

This fixes the effective piston area, which is obtained from

$$A = \frac{F_m}{(P_L)_{max}} = \frac{3F_m}{2P_s} \qquad \text{(Eq 7.4.7.2m)}$$

An expression will next be derived for the operating time of the hydraulic cylinder in moving the full stroke distance, L. This expression will be useful in obtaining equations for the remaining design parameters a_m, Q_m, and X_m. Assume first that the cylinder in Figure 7.4.7.2b is fully to the left ($\theta_o = 0$), and that the valve is in the closed position. Assume next that the valve lever is moved to the right a distance, L. That is, an input $\theta_i = L$ is made. In servomechanisms of the type being studied, L (the cylinder stroke distance) is considerably larger than X_m (the valve travel from the

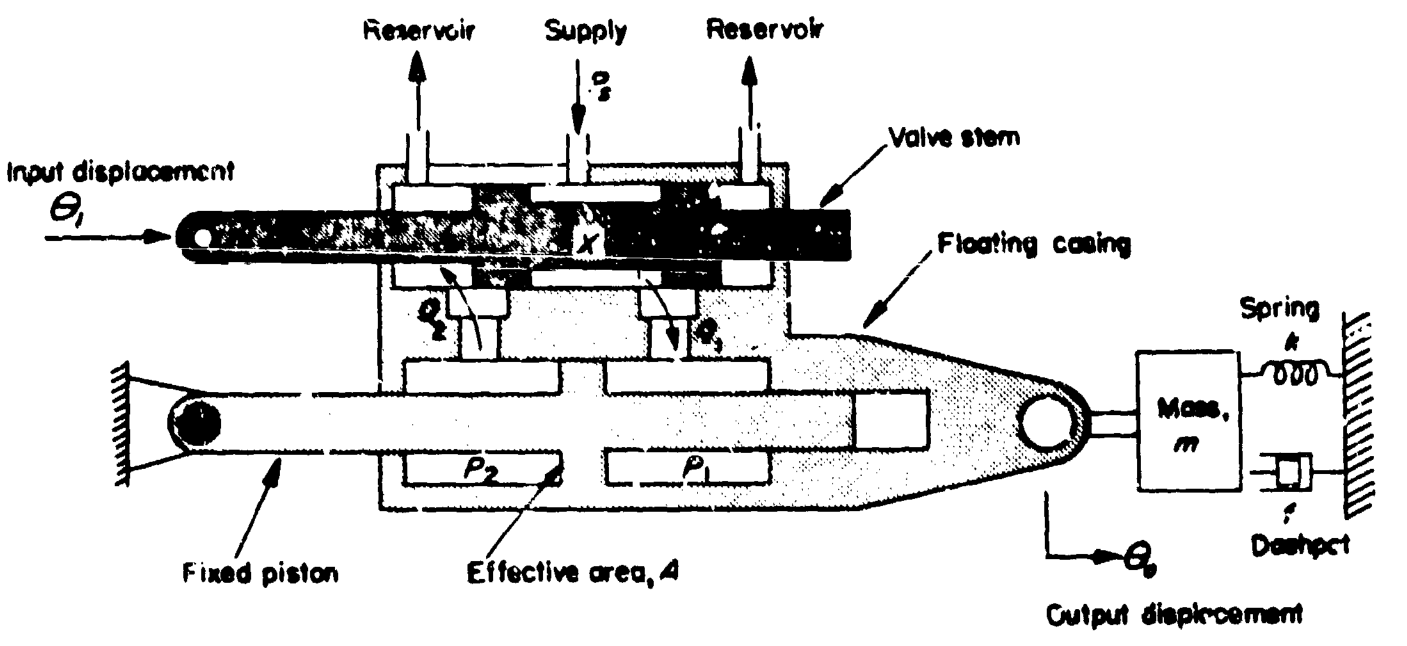


Figure 7.4.7.2b. Section through a typical valve-cylinder relay. Feedback is virtual since cylinder and valve bodies are integral and floating. Relative valve travel gives direct measure of error. Analysis of this system is complicated by high output loading.

closed position to fully open). Thus when an input, $\theta_i = L$, to the valve lever is made, the valve spool moves past the fully open position. Flow through the valve now occurs, giving the cylinder the time response shown in Figure 7.4.7.2d.

Note that this time response can be divided into two phases: (a) during which the valve is fully uncovered and thus the cylinder velocity, $\frac{d\theta_o}{dt}$, is constant, making the displacement curve a straight line with time (the distance the cylinder travels during this phase is $L - X_m$, as shown); and (b) during which the continuing movement of the cylinder causes the valve to close, thus reducing the cylinder velocity with time

During phase (a), the valve is said to be "saturated," that is, the valve is at or beyond the fully-open position. The time of cylinder travel in this phase is

$$T_s = \frac{A(L - X_m)}{Q_s} \qquad \text{(Eq 7.4.7.2n)}$$

where Q_s is the constant saturated valve flow. From Equation (7.4.7.2g)

$$Q_s = B a_m \sqrt{\frac{P_s - P_\ell}{2}} \qquad \text{(Eq 7.4.7.2o)}$$

During phase (b), the equation governing the cylinder motion is Equation (7.4.7.2l), which applies only when the valve is opening or closing. The time in phase (b) is approximately equal to 2τ, where $\tau = 1/C$. Thus from Equations (7.4.7.2j), (7.4.7.2l), and (7.4.7.2o), the total time for the cylinder stroke is approximately:

$$\text{(Eq 7.4.7.2p)}$$

$$T_t = T_s + 2\tau = \frac{AL}{Q_s}\left(1 + \frac{X_m}{L}\right)$$

Total time, T_t, varies with the output load. It is generally specified for the no-load condition as one of the design requirements. For this condition, $P_\ell = 0$ and from Equation (7.4.7.2o)

$$Q_n = B a_m \sqrt{\frac{P_s}{2}} \qquad \text{(Eq 7.4.7.2q)}$$

In Equation (7.4.7.2p), if substitutions are made for Q_s from Equation (7.4.7.2q) and A from Equation (7.4.7.2m), and if the small term X_m/L is neglected

$$T_n = \frac{3 F_m L}{\sqrt{2} B a_m P_s^{3/2}} \qquad \text{(Eq 7.4.7.2r)}$$

where T_n is the no-load time. In the full-load case, operating time is a minimum when $P_\ell = 2/3\ P_s$. For this case, from Equations (7.4.7.2m), (7.4.7.2o), (7.4.7.2p), and (7.4.7.2r), neglecting X_m/L in Equation (7.4.7.2p):

$$T_t = \sqrt{3}\, T_n \qquad \text{(Eq 7.4.7.2s)}$$

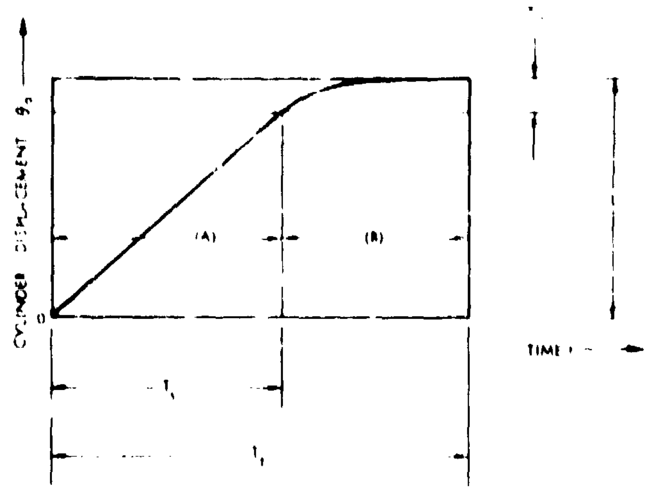


Figure 7.4.7.2d. Time Response of Cylinder.

From Equation (7.4.7.2r):

$$a_m = \frac{3 F_m L^2}{\sqrt{2} B T_n P_s^{3/2}} \qquad \text{(Eq 7.4.7.2t)}$$

Maximum flow demanded from the supply occurs under no load and, from Equations (7.4.7.2q) and (7.4.7.2t), it is

$$Q_n = \frac{3 F_m L}{2 P_s T_n} \qquad \text{(Eq 7.4.7.2u)}$$

All of the required parameters have now been determined except the valve travel, X_m. In the region $0 < X < X_m$, as noted, Equation (7.4.7.2l) governs the cylinder motion. The frequency response bandwidth within this zone is thus

$$\omega_b = 1/\tau = C \qquad \text{(Eq 7.4.7.2v)}$$

Substituting from Equations (7.4.7.2j), (7.4.7.2m), (7.4.7.2o), and (7.4.7.2r) in (7.4.7.2v), taking $Q_s = Q_n$

$$\omega_b = \frac{L}{X_m T_n} \qquad \text{(Eq 7.4.7.2w)}$$

Thus for inputs of approximate amplitude, X_m, and for frequencies up to ω_b, the output is a fairly faithful reproduction of the input. For any greater input signal of magnitude, $|\theta_i|$, frequency response is limited by flow saturation of the valve. In this case, as can be shown, accurate response can be obtained only up to a frequency

$$\omega_c = \frac{L}{|\theta_i|\, T_n} \qquad \text{(Eq 7.4.7.2x)}$$

7.4.7.2 ANALYSIS OF VALVE-CYLINDER RELAY. In the previous paragraph, several assumptions were made in order to determine the design parameters of the valve-cylinder servomechanism shown in Figure 7.4.7.2b. One of the assumptions, which is true only for small output loads, was that the hydraulic fluid in the cylinder is incompressible. This assumption was used in writing Equation (7.4.7.2h). In the following dynamic analysis, compressibility of the fluid is taken into account in the corresponding equation. The valve-cylinder relay of Figure 7.4.7.2b is a self-contained, closed-loop servo system. It will be analysed by the frequency response method.

The flow through the valve ports is given, with suitable accuracy, by Equation (7.4.7.2f). In this equation, $a(X)$ represents the valve port area as a function of the valve travel X. In order to use linear analysis, the relation between Q and X in Equation (7.4.7.2i) should be linearly proportional, but in practice this is hardly likely either by purpose or accident. If, then, some general, non-linear function $a(X)$ is assumed, the problem becomes one in non-linear mechanics and requires more advanced methods of analysis. Linear theory, however, provides such a clear picture of system behavior and is so convenient for investigating the effect of changing parameters that it is worthwhile trying to employ it in the present case, although it is at best only an approximation.

Various techniques exist for linearizing non-linear equations such as Equation (7.4.7.2f). The simplest of these, known as the small perturbation method, will be adopted here. The method consists of considering only small disturbances which are superimposed on a relatively gradual or steady-state motion. The variables in the steady-state motion are considered to change so slowly that they may be treated as constants relative to the disturbances. Although only an artifice, this quasi-steady-state method gives useful results in servo system analysis. The closeness between theory and practice is influenced by the fact that proportional feedback tends to reduce nonlinear distortion and also because higher harmonics introduced by nonlinearity are severely attenuated or cut off above the bandwidth of the servo.

A word of caution must be given here since both these mitigating influences will not hold for very large disturbances. These disturbances can only be investigated by nonlinear mechanics through the use of an analog computer.

The small perturbation method is based upon an approximation obtained from a Taylor series expansion of a function. Thus, if X_o is a constant value of X, and x is a small increment or perturbation in X around X_o

(Eq 7.4.7.3a)

$$f(X) = f(X_o + x) = f(X_o) + x\,\frac{d\,f(X_o)}{dX}$$

Higher terms of the expansion are neglected. The artifice adopted in linearization is to say that while X_o is not actually constant, it represents a temporary mean value which changes so slowly compared to x that it can be regarded as constant. Then the right-hand side of Equation (7.4.7.3a) may be separated into two components, a steady-state quantity, $f(X_o)$, and a small perturbation term, $x\,\frac{d\,f(X_o)}{dX}$. In the latter term, $\frac{d\,f(X_o)}{dX}$ is also considered constant and so the perturbation term is a linear function of the perturbation, x. By considering only small perturbations in all the system variables, a set of linear equations is obtained which describes the disturbed state of the system. It is then possible to investigate the stability of the system.

Since P_c in Equation (7.4.7.2f) depends on the output load and thus cannot be regarded as constant, it is necessary to consider a small perturbation p_c about a steady quantity P_{co}. Disturbed flow from the valve is:

(Eq 7.4.7.3b)

$$Q_d = Q_o + q = B\,a(X_o)\sqrt{\frac{P_s - P_{co}}{2}} + x\left(\frac{\partial Q}{\partial X}\right)_{P_c = P_{co}} + p_c\left(\frac{\partial Q}{\partial P_c}\right)_{x = x_o}$$

where $(\partial Q/\partial X)_{P_c = P_{co}}$ and $(\partial Q/\partial P_c)_{x=x_o}$ are both evaluated for $X = X_o$ and $P_c = P_{co}$.

By separating out the perturbed components of Equation (7.4.7.3b) and introducing convenient parameters K and λ_v

$$q = A\,K\left(x - \frac{A\,p_c}{\lambda_v}\right) \qquad \textbf{(Eq 7.4.7.3c)}$$

where

$$K = \left(\frac{1}{A}\right)\left(\frac{\partial Q}{\partial X}\right)\Bigg|_{\substack{P_c = P_{co}\\ x = x_o}} \qquad \textbf{(Eq 7.4.7.3d)}$$

(Eq 7.4.7.3e)

$$\lambda_v = -A\left(\frac{\dfrac{\partial Q}{\partial X}}{\dfrac{\partial Q}{\partial P_c}}\right) = A\left(\frac{d\,P_c}{d\,X}\right)\Bigg|_{Q = Q_o}$$

Factor K is the slope gain constant, while λ_v is called the stiffness of the valve and gives the gradient of cylinder effort against valve displacement for constant flow Q_o.

In addition to the component of flow tending to displace the cylinder, a component due to the compressibility of the fluid in the cylinder must be considered. This depends on the rate of change of P_1 and P_2. If the cylinder is in its mid-position, balanced flow exists for each side of the cylinder piston. Then for the right side in Figure 7.4.7.2b, the continuity equation is

(Eq 7.4.7.3f)

$$\rho_1 Q = \frac{d}{dt}(\rho_1 V_r) = \rho_1\frac{dV_r}{dt} + V_r\frac{d\rho_1}{dt}$$

where ρ_r = density of fluid in right side of cylinder
V_a = volume of fluid in one half of cylinder

From Equation (7.4.7.3f):

(Eq 7.4.7.3g)

$$Q = A \frac{d\theta_o}{dt} + \frac{V_a}{E}\frac{dP_1}{dt}$$

where E = bulk modulus of fluid.

From Equations (7.4.7.2c) and (7.4.7.2d),

(Eq 7.4.7.3h)

$$\frac{dP_1}{dt} = \frac{1}{2}\frac{dP_c}{dt}$$

and thus

$$Q = A \frac{d\theta_o}{dt} + \frac{V_t}{4E}\frac{dP_c}{dt} \quad \text{(Eq 7.4.7.3i)}$$

where V_t = total volume of fluid in cylinder.

The same equation can be obtained for the left side of the cylinder. By considering only small perturbations in Q, θ_o, and P_c, and introducing Laplace transforms

$$q = A s \left(\theta_o + \frac{A p_c}{\lambda_c}\right) \quad \text{(Eq 7.4.7.3j)}$$

where λ_c = cylinder stiffness
$= 4 A^2 E/V_t$

Finally, the load on the cylinder must be considered. In the general case, the load includes inertia, damping, and spring loads. Thus

(Eq 7.4.7.3k)

$$A P_c = m \frac{d^2\theta_o}{dt^2} + f \frac{d\theta_o}{dt} + k\theta_o$$

Considering perturbations and transforming

$$A p_c = \zeta(s)\,\theta_o \quad \text{(Eq 7.4.7.3l)}$$

where $\zeta(s) = ms^2 + fs + k$. (Eq 7.4.7.3m)

By eliminating between Equations (7.4.7.3c), (7.4.7.3j), and (7.4.7.3l), the open-loop transfer function is obtained

(Eq 7.4.7.3n)

$$Y_o(s) = \frac{\theta_o}{x}(s) = \frac{K}{s\left(1 + \frac{\zeta}{\lambda_c}\right) + \frac{K\zeta}{\lambda_v}}$$

The closed-loop transfer function is therefore

(Eq 7.4.7.3o)

$$Y_c(s) = \frac{\theta_o}{\theta_i}(s) = \frac{K}{s\left(1 + \frac{\zeta}{\lambda_c}\right) + K\left(1 + \frac{\zeta}{\lambda_v}\right)}$$

Equations (7.4.7.3n) and (7.4.7.3o) describe the behavior of the servo when subjected to small to medium disturbances. The equations will be used to investigate system stability and response.

Stability. As always, stability must be the first consideration. It can be shown that the worst conditions for absolute stability exist when the cylinder piston is in its midposition and when the output load is simply an inertia force. In this case, from Equation (7.4.7.3m):

$$\zeta(s) = ms^2 \quad \text{(Eq 7.4.7.3p)}$$

The open and closed-loop transfer functions for this condition are

(Eq 7.4.7.3q)

$$Y_o(s) = \frac{K}{s\left(1 + \frac{K\,ms}{\lambda_v} + \frac{ms^2}{\lambda_c}\right)}$$

(Eq 7.4.7.3r)

$$Y_c(s) = \frac{K}{\left(K + s + \frac{K\,ms^2}{\lambda_v} + \frac{ms^3}{\lambda_c}\right)}$$

From these expressions absolute stability can easily be investigated by the Hurwitz criterion. This gives the necessary condition for stability as $K\,m/\lambda_v > K\,m/\lambda_c$ or

$$\lambda_c > \lambda_v \quad \text{(Eq 7.4.7.3s)}$$

This criterion can be interpreted physically as follows. If the output inertia load increases, then the fluid column in the cylinder will shorten because of compressibility, causing relative movement between the valve casing and spool. The resulting valve opening causes a change in the differential cylinder pressure. The change in effort produced by this must not exceed the change in the output load. If the valve does over-compensate for the change in load, then the cylinder will move back and a continuous cycle will be set up.

To reduce valve stiffness, λ_v, sufficiently to satisfy equation (7.4.7.3s), there must be some constant leakage past the valve lands. Two possible ways of achieving this are by means of underlap or overlap at the valve ports, as in Figure 7.4.7.3a. Within the region of valve lap the ports act as a variable resistance bridge. Differential pressure depends on the resistance ratios and hence on the relative valve displacement. If no lap is provided, a very small valve displacement applies full system pressure across the cylinder piston, with resulting over-correction.

Even if stability is not satisfied, it does not necessarily mean that an oscillation can grow indefinitely. The equations have been linearized and deal only with relatively small movements. As the amplitude increases, however, parameters K and λ_v will change sufficiently to restore stability or limit the amplitude, although this may not happen until the valve ports are fully exposed and the system saturates. This means that a steady oscillation will

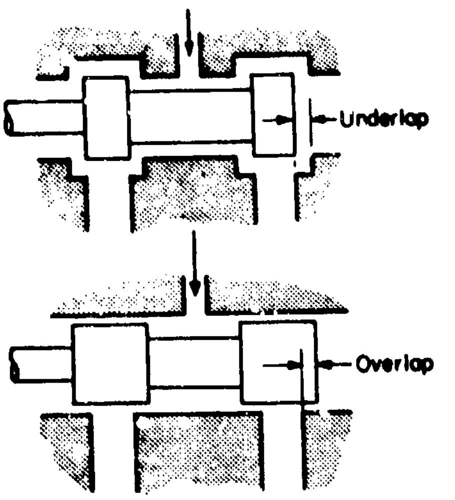


Figure 7.4.7.3a. Valve stiffness in the neutral position may be reduced by either underlap or overlap. Underlap signifies that the valve spool does not completely close the ports when the spool is centered; overlap means that the valve spool lands exceed the width of the ports.

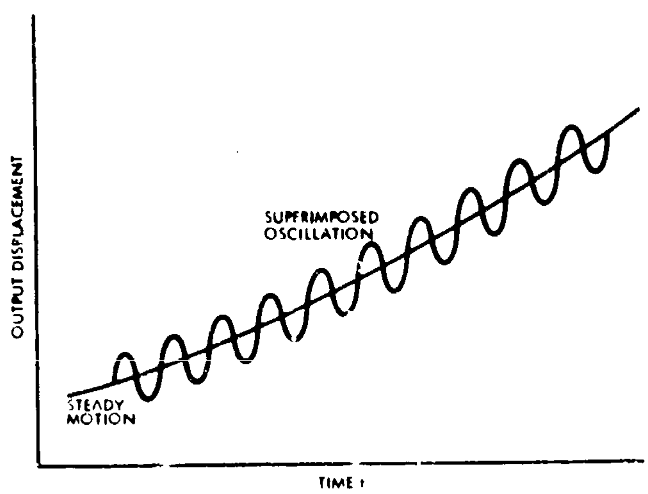


Figure 7.4.7.3b. Oscillation does not necessarily increase indefinitely. It may be a small, steady oscillation superimposed on the steady output motion of the servo.

be superimposed on the steady-state motion, as in Figure 7.4.7.3b. In practice it has been found satisfactory from the stability viewpoint to satisfy Equation (7.4.7.3s) for all possible values of X within the range of maximum travel in either direction and for all possible values of P_r within the range of allowable output loading.

Response. The valve travel, X, as noted earlier, can be divided into two zones: $0 \leq X \leq X_m$, where the valve is not saturated, and $X > X_m$ where it is. The $0 \leq X \leq X_m$ zone is also referred to as the *linear zone* of servo operation, although the valve port area function, a(X), in this zone is not necessarily purely linear with X.

Along with stability, satisfactory response within the linear zone of the servo is a primary performance requirement. As previously noted, response characteristics are seriously impaired by flow-saturation outside the linear zone. Provided small motions only are considered, Equation (7.4.7.8r) can be used to obtain the response characteristics. If a sudden step from one steady demand to another which is quite close to the first occurs, the output should respond quickly and without too much oscillation, as in Figure 7.4.7.3c. It must be stated that for large disturbances the problem actually belongs to the field of nonlinear mechanics. Yet it is surprising how close the small perturbation methods are, even for quite large disturbances.

Linear analysis methods can be used to investigate response from Equations (7.4.7.3q) or (7.4.7.3r). Since the denominator of Equation (7.4.7.8r) is in cubic inches, it is possible to use transient response methods fairly simply. However, parameters K and λ_v cannot be easily adjusted independently and so it is difficult to attempt to optimize design parameters by this technique. Therefore, frequency response techniques will be used.

Substituting $j\omega$ for s in Equation (7.4.7.3q) gives the open-loop harmonic response function

(Eq 7.4.7.3t)

$$Y_o(j\omega) = \frac{K}{j\omega\left(1 + \frac{jKm\omega}{\lambda_v} - \frac{m\omega^2}{\lambda_o}\right)}$$

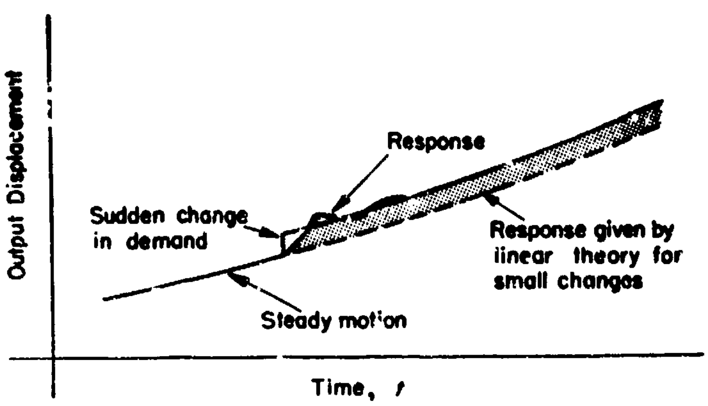


Figure 7.4.7.3c. If established response requirements are met, servo response will be quick and well damped. Although a small change in required output is considered, response to large changes in output requirement may be similar.

From this a Nyquist plot can be made, as shown in Figure 7.4.7.3d. As Equation (7.4.7.3t) shows, a change in K not only alters the scale of the diagram, but also the shape of the $Y_o(j\omega)$ curve. This is often a feature of systems in which there is interaction, as between output load and valve flow. This interaction is illustrated by the subsidiary feedback loops in the servo block diagram, Figure 7.4.7.3e, which represents Equations (7.4.7.3c), (7.4.7.3j), and (7.4.7.3l)

The phase angle of $Y_o(j\omega)$ is $-180°$ when $\omega = \sqrt{\lambda_c/m}$. Thus gain margin is

$$G = \frac{1}{|Y_o|_{-180°}} = \frac{\lambda_c}{\lambda_v} \qquad \text{(Eq 7.4.7.3u)}$$

From Equation (7.4.7.3s), the critical condition for stability is reached when the gain margin equals 1.0. Because of the simple shape of the Nyquist curve for $Y_o(j\omega)$, Figure 7.4.7.3d, a satisfactory value of gain margin in this case is 4.0. More complex shapes will require considerably higher gain margins.

By now adjusting the value of K it is possible to achieve a suitable phase margin. By making the phase margin 45° and taking $|Y_o| = 1.0$ in Equation (7.4.7.3t), the optimum gain constant is

(Eq 7.4.7.3v)

$$K = 1.2\left(\frac{\lambda_v}{m}\right)^{1/2}\left(1 - \frac{\lambda_v}{\sqrt{2}\,\lambda_c}\right)$$

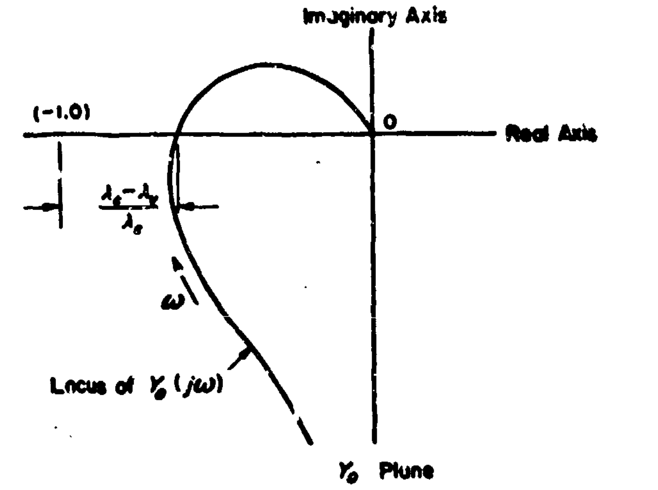


Figure 7.4.7.3d. Typical Nyquist plot for a hydraulic relay. A change in gain constant, K, can alter the shape as well as the scale of the plot. This often happens in systems where interaction occurs.

This is an easy method of determining K when numerical data are available, but when dealing with algebraic expressions it can become tedious. In the present case, when certain of the parameters such as K and λ_v can assume widely varying values, depending on the particular steady-state condition chosen, it is very helpful from the standpoint of understanding system behavior to work with algebraic expressions. In this connection, some useful approximations can often be made.

From the closed-loop harmonic response function

(Eq 7.4.7.3w)

$$Y_c(j\omega) = \frac{K}{K\left(1 - \frac{m\omega^2}{\lambda_v}\right) + j\omega\left(1 - \frac{m\omega^2}{\lambda_c}\right)}$$

the amplitude response is

(Eq 7.4.7.3x)

$$|Y_c(j\omega)| = \left|\frac{\theta_o}{\theta_i}(j\omega)\right|$$

$$= \frac{K}{\sqrt{K^2\left(1 - \frac{m\omega^2}{\lambda_v}\right)^2 + w^2\left(1 - \frac{m\omega^2}{\lambda_c}\right)^2}}$$

This may be plotted against frequency, as in Figure 7.4.7.3f. The response tends to two resonant peaks at frequencies

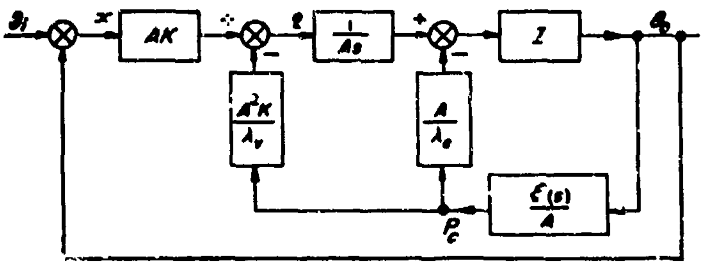


Figure 7.4.7.3e Subsidiary feedback loops in this block diagram of a hydraulic relay represent interaction between components.

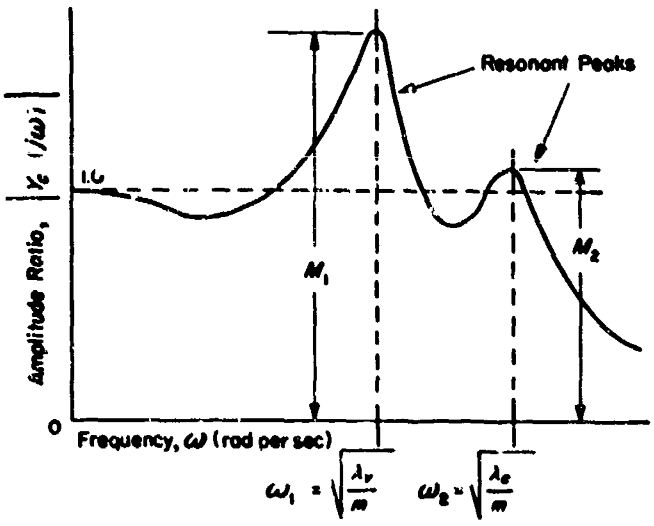


Figure 7.4.7.3f. A plot amplitude ratio versus frequency shows that there are generally two resonant frequencies. The two peaks must not coincide or the system will be unstable. Ordinarily, ω_2 should be larger than ω_1.

approximately given by $\omega_1 = \sqrt{\lambda_v/m}$ and $\omega_2 = \sqrt{\lambda_c/m}$. Thus the stability criterion, Equation (7.4.7.3s) can be written as $\omega_1 < \omega_2$. If $\omega_1 = \omega_2$, the two peaks will merge into a single peak of infinite height, indicating that the output will increase indefinitely at this frequency of excitation.

For a recommended gain margin λ_c/λ_v of 4.0 or more, $\omega_2 \geq 2\,\omega_1$. For this condition, the peaks are definitely separated and compressibility in the cylinder, as represented by $1/\lambda_c$, will have little influence on the maximum amplitude ratio M_1 at ω_1. This ratio is obtained from Equation (7.4.7.3x) by making the substitutions $\omega = \omega_1$, $\sqrt{\frac{\lambda_v}{m}} = \omega_1$, and $\sqrt{\frac{\lambda_c}{m}} = \omega_2$

$$M_1 = \frac{K\,\omega_2/\omega_1}{\sqrt{\omega_2^2 - \omega_1^2}} \qquad \text{(Eq 7.4.7.3y)}$$

If ω_2 is very much larger than ω_1, $K \approx M_1\,\omega_1$. Since ω_1 is some measure of the bandwidth, this shows that K influences both maximum amplitude ratio and bandwidth. If a maximum amplitude ratio $M_1 = 1.5$ is assumed, Equations (7.4.7.3x) and (7.4.7.3y) give the optimum gain as

$$K = 1.5\sqrt{\frac{\lambda_v}{m}\left(1 - \frac{\lambda_v}{\lambda_c}\right)} \qquad \text{(Eq 7.4.7.3z)}$$

This K is slightly higher than that given by the phase margin criterion, as in Equation (7.4.7.3v).

In practical cases, it may not be possible to isolate ω_1 and ω_2 sufficiently to make these approximations, and exact and tedious algebraic expressions must be used. Great simplification can be achieved if the maximum amplitude ratio is limited to $M_1 = 1.0$. Although in theory this tends to give an overdamped, sluggish response, practical results indicate that the choice is satisfactory in the present case. If the conditions $M_1 = |Y_c| = 1.0$ and $\frac{d\,|Y_c|}{d\omega} = 0$ are applied to Equation (7.4.7.3x), the following values are obtained for the optimum gain constant, and for the frequency at the peak where $M_1 = 1$

$$K = \frac{2\,\lambda_v}{\lambda_c}\sqrt{\frac{\lambda_c - 2\,\lambda_v}{m}} \qquad \text{(Eq 7.4.7.3a')}$$

$$\omega_1 = \sqrt{\frac{4\,\lambda_v - \lambda_c}{m}} \qquad \text{(Eq 7.4.7.3b')}$$

The amplitude ratio plot for this case is given by Figure 7.4.7.3g As shown, the unit is so heavily damped that no second peak occurs. If $\lambda_c/\lambda_v < 4.0$, there will be no resonant peak, as seen in Figure 7.4.7.3h, but the actual transient response will be far too sluggish.

The preceding analysis shows that the relationship between λ_c and λ_v is critical for stability. In practice it may be difficult to increase λ_c so as to isolate compressibility effects from the operating range. Stiffness of the fluid column in the cylinder, from before, is

(a)

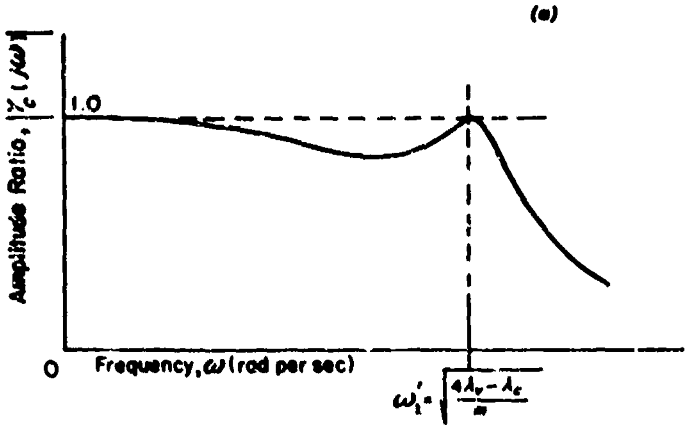


Figure 7.4.7.3g. By limiting maximum amplitude ratio to 1.0, a satisfactory, practical system is achieved, although theoretically the system is overdamped.

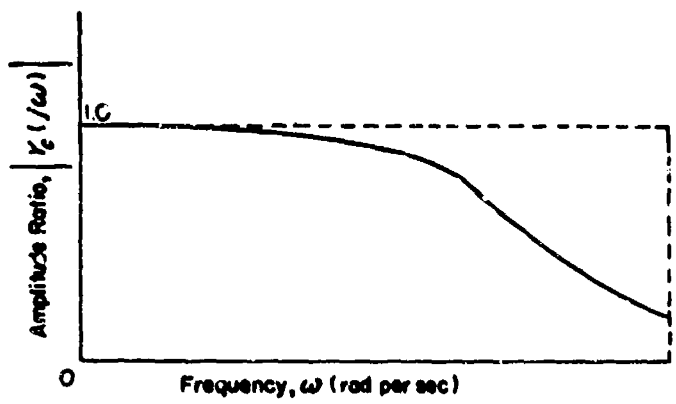


Figure 7.4.7.3h. Damping can be incorporated to eliminate all resonant points. In this case, transient response would be far too sluggish for a practical system.

$$\lambda_c = 4A^2E/V_t = 4AE/L \qquad \text{(Eq 7.4.7.3c')}$$

Thus to increase λ_c, the jack stroke, L, should be kept as short as possible and the area, A, as large as possible. Also, E should be as high as possible. Under actual operating conditions, the hydraulic fluid will absorb air bubbles which will be suspended in the fluid. This tends to lower the effective E according to the volumetric fraction of air contained, and suggests the possibility of maintaining the fluid in the cylinder at a base pressure. Since minimum pressure on one side of the cylinder occurs under maximum output load, and from Equations (7.4.7.2c) and (7.4.7.2d) is $(P_s - P_{r\,max})/2$, it is desirable to limit $P_{r\,max}$. It was previously suggested that $P_{r\,max}$ be limited to 2/3 P_s by power

ISSUED: OCTOBER 1965
SUPERSEDES: MAY 1964

considerations. This gives a minimum pressure of $P_s/6$ in the cylinder—normally a satisfactory value. Such a base pressure can be maintained by neutral-lap leakage past the valve lands.

Physical Parameters. Results thus far have been obtained in terms of two rather unreal parameters, K and λ_v. Now these results must be related to physical design parameters. Response and stability must be investigated for all possible combinations of these two parameters. To do this work, the concept of a K, λ_v plane containing all possible values is necessary. On this plane it is possible to plot contours representing the stability limits and also lines of constant, M_1, and constant bandwidth, ω_b. Figure 7.4.7.3i is such a plot.

By superimposing on this diagram characteristics representing the limits of linear operation (that is, operation in the $0 \leq X \leq X_m$ zone of valve travel), it is possible to inspect performance at all steady operating points. Equation (7.4.7.2f) gives the valve port flow for a general port area function, a(X). If the valve port area is made to vary linearly with X, the flow is given by Equation (7.4.7.2g). From Equations (7.4.7.2g), (7.4.7.3d), and (7.4.7.3e):

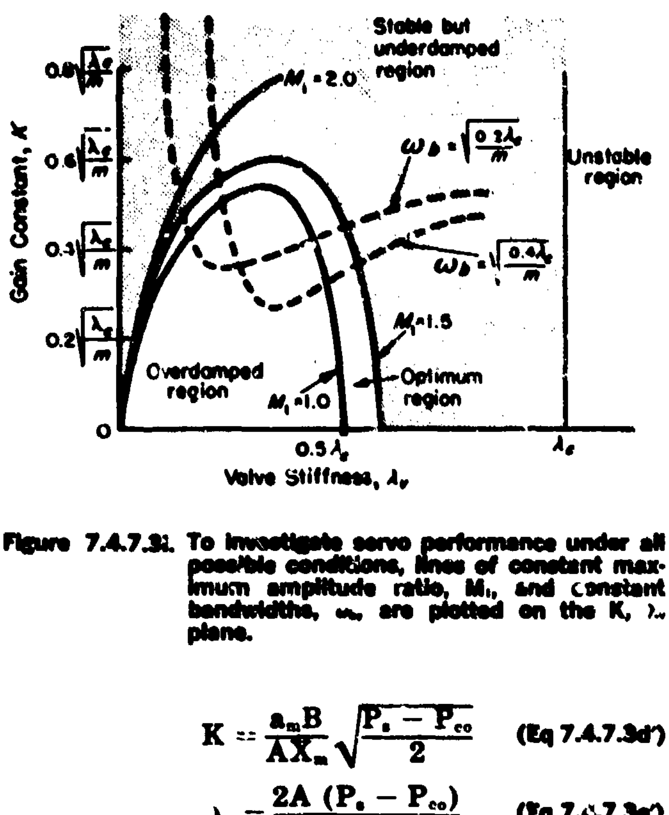


Figure 7.4.7.3i. To investigate servo performance under all possible conditions, lines of constant maximum amplitude ratio, M_1, and constant bandwidths, ω_b, are plotted on the K, λ_v plane.

$$K = \frac{a_m B}{A X_m}\sqrt{\frac{P_s - P_{co}}{2}} \qquad \text{(Eq 7.4.7.3d')}$$

$$\lambda_v = \frac{2A\,(P_s - P_{co})}{X_o} \qquad \text{(Eq 7.4.7.3e')}$$

The range of possible operating conditions is given by $0 \leq X_o \leq X_m$, $0 \leq P_{co} \leq P_{c\,max}$, where the subscript (o) in X_o and P_{co} signifies steady values of X and P_c. From Equation (7.4.7.3d'), lines of constant P_{co} are parallel to the λ_v axis, as in Figure 7.4.7.3j. From Equations (7.4.7.3d') and (7.4.7.3e'), lines of constant, X_o, are given by

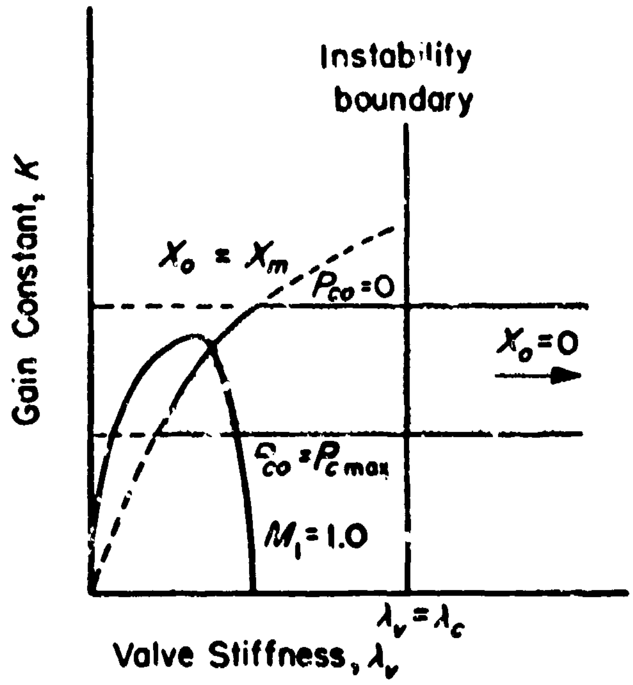


Figure 7.4.7.3j. Superimposition of actual valve operation conditions on the K, λ_v plane shows how a system using a valve with an overlap or underlap becomes unstable when the spool is in neutral position.

$$K = \frac{a_m B}{2X}\sqrt{\frac{\lambda_v X_o}{A^3}} \qquad \text{(Eq 7.4.7.3f')}$$

The shaded region in Figure 7.4.7.3i is obtained by substituting the extreme values of P_{co} and X_o in Equations (7.4.7.3d') and (7.4.7.3f'). This region thus represents all possible operating conditions of the servo. Combining Figures 7.4.7.3i and 7.4.7.3j, it is easy to study the performance variation over the operating range.

From Equation (7.4.7.3e') it can be seen that at the neutral valve position, $X_o = 0$, the valve stiffness, λ_v, becomes infinite. Hence, in theory the stability criterion, Equation (7.4.7.3a), is not satisfied. This is confirmed in practice by the continuous hunting of many valve-cylinder servos in the neutral region. Stability in this region can be achieved by two methods.

a) Providing lap at the valve lands, as in Figure 7.4.7.3a. For a suitable lap, h, the operating range in the K, λ_v plane becomes the shaded area in Figure 7.4.7.3k, which is completely to the left of the unstable region.

b) A small leak across the cylinder piston, together with a region of reduced gain near the neutral valve position.

Method (b) is the more effective and economic technique.

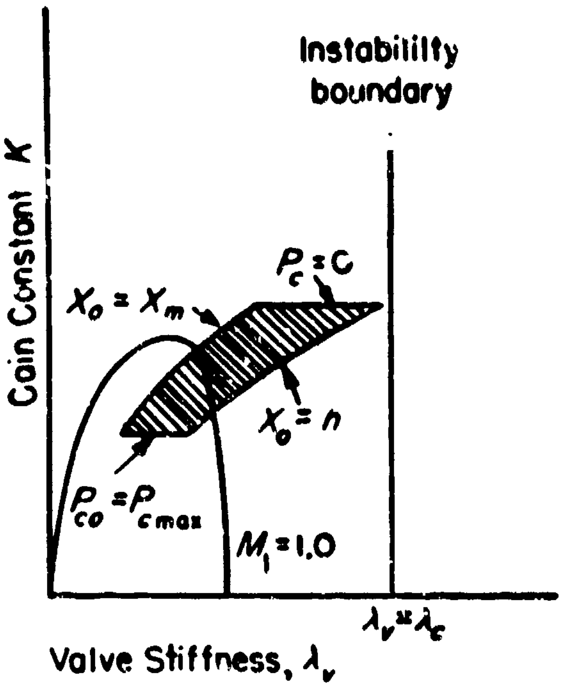


Figure 7.4.7.3k Use of lap on valve lands permits the system to be kept out of the unstable region.

7.4.7.4 ANALYSIS OF COMPLETE SYSTEM. The main valve-cylinder relay of the servo system shown in Figure 7.4.7.1a, analyzed in the previous section, treated the relay as a self-contained closed loop servo system. The complete system of Figure 7.4.7.1a will be analyzed next. In this system, the valve-cylinder relay is an open-loop device, and feedback is provided by electrical components as shown. The open-loop transfer function of the relay, which was derived in the last section, will be used in the following analysis.

System Equations. The system input, θ_i, is converted to a voltage, $K_1\theta_i$, by potentiometer A (Figure 7.4.7.1a). The system output, θ_o, is converted to the feedback voltage, $K_1\theta_o$, by potentiometer B, which has the same gain constant as A. V is the difference between the two voltages

$$V = K_1 (\theta_i - \theta_o) \qquad \text{(Eq 7.4.7.4a)}$$

V is amplified and used to drive the magnetic solenoid relay. Current delivered by the amplifier is

$$i = K_2 V \qquad \text{(Eq 7.4.7.4b)}$$

The relay develops a force F proportional to this current

$$F = K_3 i \qquad \text{(Eq 7.4.7.4c)}$$

This force is developed at the flapper of a flapper-nozzle-type hydraulic pilot valve. A roughly proportional pressure is developed in the chamber of the pilot valve and this in turn produces a proportional displacement of the main spool valve. The transfer function relating the main spool displacement with the flapper force includes a simple time lag term and is therefore

$$\frac{x}{F}(s) = \frac{K_4}{1 + \tau_4 s} \qquad \text{(Eq 7.4.7.4d)}$$

Finally for the main valve-cylinder relay, the open-loop transfer function is given by Equation (7.4.7.3q), assuming that the output load is simply an inertia

$$\frac{\theta_o}{x}(s) = \frac{K_5}{s\left(1 + \frac{K_5 ms}{\lambda_v} + \frac{ms^2}{\lambda_c}\right)} \qquad \text{(Eq 7.4.7.4e)}$$

Here K_5 is the slope gain constant. Based upon the above equations, the block diagram for the complete system, Figure 7.4.7.1b, can be constructed. In this diagram, the blocks for potentiometers A and B in Figure 7.4.7.1a are represented by the single equivalent block, K_1, between e and V, where

$$e = \theta_i - \theta_o \qquad \text{(Eq 7.4.7.4f)}$$

The open-loop transfer function of the system can now be obtained simply by multiplying the transfer functions of each box in the forward path

$$Y_o(s) = \frac{\theta_o}{e}(s) = \frac{K}{s(1 + \tau_4 s)\left(1 + \frac{K_5 ms}{\lambda_v} + \frac{ms^2}{\lambda_c}\right)} \qquad \text{(Eq 7.4.7.4g)}$$

where $K = K_1 K_2 K_3 K_4 K_5$ = net open-loop gain constant. The closed-loop transfer function is

$$Y_c(s) = \frac{\theta_o}{\theta_i}(s) = \frac{K}{b\tau_4 s^4 + (a\tau_4 + b)s^3 + (a + \tau_4)s^2 + s + K} \qquad \text{(Eq 7.4.7.4h)}$$

where $a = K_5 m/\lambda_v$ and $b = m/\lambda_c$.

Performance. The first step in analyzing this system is to investigate the condition for absolute stability. The condition to be achieved (Reference 1-131, page 20, Equation 15) is

$$K \leq \frac{a(b + a\tau_4 + \tau_4^2)}{(b + a\tau_4)^2} \qquad \text{(Eq 7.4.7.4i)}$$

It should be noted that the value of K can be adjusted independently of K_5, the gain constant of the valve-cylinder relay. In practice, the gain constant, K_2, of the electronic amplifier is usually readily adjusted to change the gain setting of the system. Thus K_5 can be fixed at a value suitable for the design of the valve-cylinder relay. From gain margin considerations, K should be about one-fourth of the right hand side of Equation (7.4.7.4i) or less.

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SUPERSEDES: MAY 1964

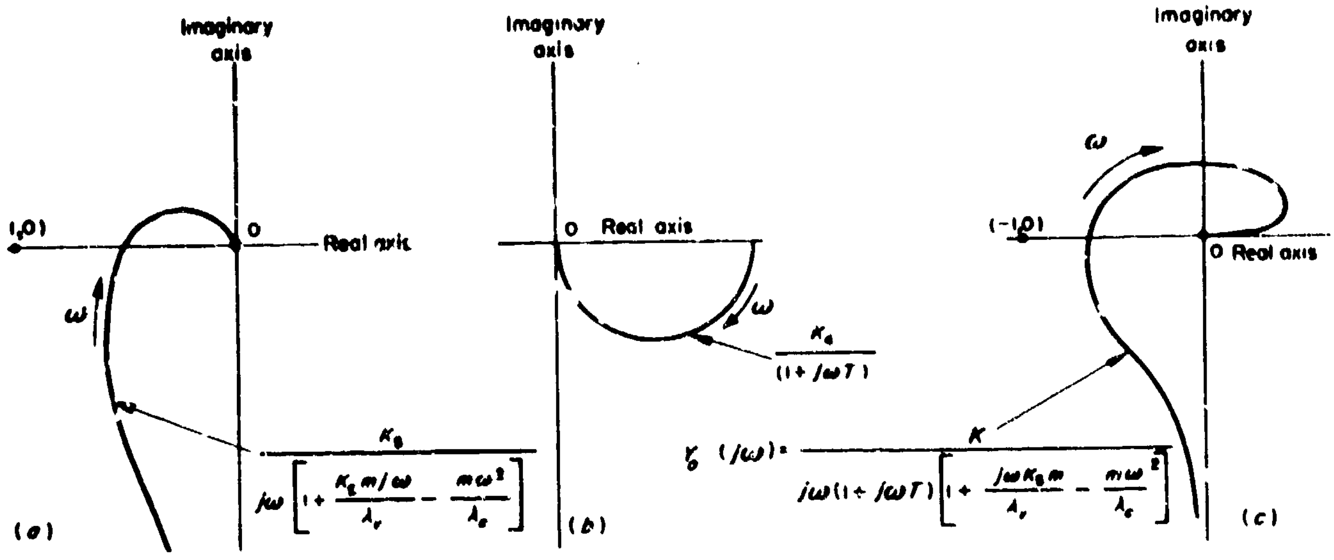

Figure 7.4.7.4a, b, c. Nyquist plots for (a) the main valve and cylinder; (b) the flapper-nozzle pilot valve and main valve spool; and (c) the open-loop plot for the entire system, shown in Figure 7.4.7.1a, b. Note that this open-loop plot approaches the critical point more closely than the plot for the hydraulic relay alone.

Now it is necessary to optimize the parameters of the system to obtain suitable performance. Either frequency or transient response methods can be used. In view of the complexity of the transfer functions, frequency response methods are preferred. Characteristics of those components which are dependent on frequency are plotted in Figures 7.4.7.4a and b. With suitable change of scale to include the effect of other scalar gain constants in the loop, they may be combined to give the system $Y_o(j\omega)$ locus or Nyquist plot, Figure 7.4.7.4c. The distance of this locus from the (-1,0) point is, as noted, a measure of system stability. The effect of the time lag τ_i in the pilot valve is to make this distance less in the system Nyquist plot than in the plot for the valve-cylinder relay alone (Figure 7.4.7.4a), and thus to bring the complete system closer to instability. If the locus of the relay alone is not well away from the (-1, 0) point, the signal (c) in Figure 7.4.7.1b must be partly attenuated before reaching the main valve. That is, the product of the gain constants, $K_1 K_2 K_3 K_4$, must be less than unity. A suitable gain setting may be obtained by limiting the maximum amplitude ratio to 1.5. Thus the $Y_o(j\omega)$ locus must not cut the $M = 1.5$ circle, as shown in Figure 7.4.7.4d.

A numerical example will illustrate the method. Assume $\tau_i = 0.003$ sec, a = 0.019 sec and b = 6.25×10^{-4} sec^2. Then the open-loop harmonic response function is

(Eq 7.4.7.4j)

$$\frac{\theta_o}{e}(ju) = \frac{K'}{ju\,(1 + 3ju)\,(1 + 9.3ju)\,(1 + 0.67ju)}$$

where for convenience $u = 0.001\,\omega$ and $K' = 0.001$ K. The locus of this function for varying ω is plotted in Figure 7.4.7.4d. If the maximum amplitude ratio is to be 1.5, then the scalar gain constant, K, must be set at 100. The bandwidth is approximately 16 cps. From Equation (7.4.7.4g) it can be seen that the servo is of the first order, where order is defined in Reference 1-131, page 24. The steady following error for a constant velocity input of 1.0 inches per second, given by 1/K, is therefore 0.010 inches.

In this example, an expression has been obtained only for the optimum gain constant; the other parameters have been determined by some prior considerations. It is likely, however, that at an initial design stage there will be quite a degree of freedom of choice among the various parameters of the complete system. It seems desirable, therefore, to obtain some relations among the parameters in order to achieve a certain performance. One way this might be attempted is to obtain an algebraic expression by limiting maximum, M, to 1.5 in the expression for the closed-loop harmonic response function. This method was employed quite successfully in the case of the closed-loop valve-cylinder relay; but now the system is more complex and the algebraic expressions are almost unmanageable. It is possible, however, to obtain a set of relations by transient response considerations provided a slightly overdamped response can be tolerated. The method involves the use of Whiteley's coefficients (Reference 1-131, page 26). In the present case, the order of the servo is 1, and the degree of the denominator of $Y_o(s)$ is n = 4. Whiteley's table therefore gives an optimum open-loop transfer function of the form

(Eq 7.4.7.4k)

$$Y_o(s) = \frac{\Omega^4}{s(s^3 + 2.6\,\Omega\, s^2 + 3.4\,\Omega^2 s + 2.6\,\Omega^3)}$$

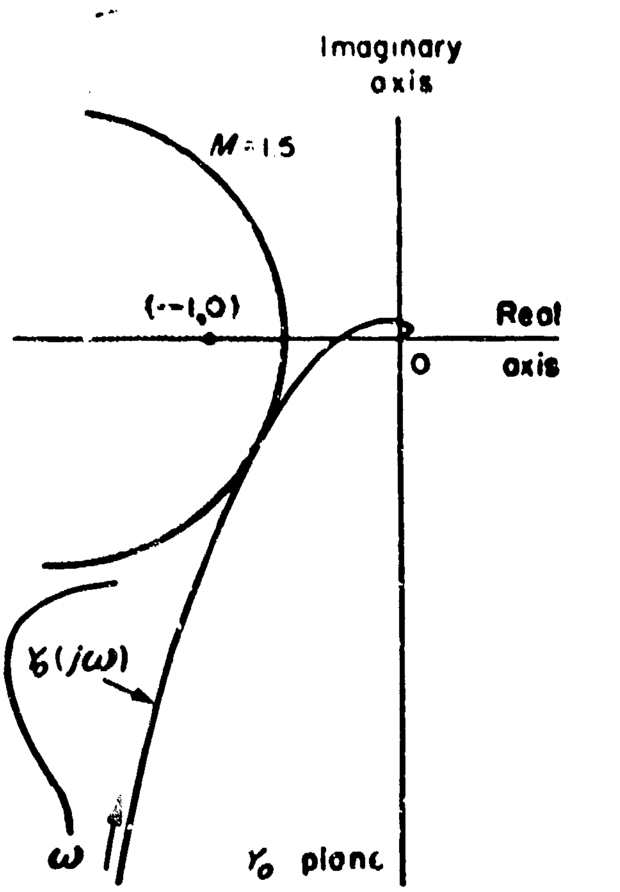

Figure 7.4.7.4d. In addition to the requirement that the Nyquist plot should not approach the −1, 0 point too closely, the loop locus must not intersect the M=1.5 circle for stability. This prevents instability of the hydraulic relay.

Equation (7.4.7.4f) can be written in the same form

$$Y_o(s) = \frac{K/b\tau_4}{s\left[s^3 + \left(\frac{a\tau_4 + b}{b\tau_4}\right)s^2 + \left(\frac{a + \tau_4}{b\tau_4}\right)s + \frac{1}{b\tau_4}\right]} \qquad \text{(Eq 7.4.7.4l)}$$

By equating similar coefficients of Equation (7.4.7.4k) and Equation (7.4.7.4l) : $K/b\tau_4 = \Omega^4$, $1/b\tau_4 = 2.6\ \Omega^3$, $(a + \tau_4)/b\tau_4 = 3.4\ \Omega^2$, and $(a\tau_4 + b)/b\tau_4 = 2.6\ \Omega$. Now the transient response is determined by the parameter, Ω. The relationship among the coefficients in Equation (7.4.7.4k) insures that the maximum overshoot for a step function is limited to 10 percent of the step, while the build-up time is given by $T_b = 4.2/\Omega$. Therefore, by fixing a suitable build-up time and hence, Ω, four relationships among the parameters a, b, τ_4, and K are obtained. It is possible to solve these, but this is a difficult task since it involves the explicit solution of a cubic equation. It is possible, however, to obtain a graphical solution quite easily. Suppose the desired build-up time is 0.016 seconds. This gives $\Omega = 260$. Direct substitution in the first two equations obtained above by equating coefficients of Equations (7.4.7.4k) and 7.4.7.4l) gives K = 100. Substituting for Ω in the other two yields the expressions

$$a + \tau_4 = 0.005 \qquad \text{(Eq 7.4.7.4m)}$$

$$a = \frac{14.8\,\tau_4 - 0.022}{10^6\,\tau_4^2} \qquad \text{(Eq 7.4.7.4n)}$$

These equations are solved by plotting a versus τ_4, as given by both Equation (7.4.7.4m) and (7.4.7.4n). The point of intersection of the two curves, which is the solution, gives

$$a = 0.0023 \text{ seconds}, \quad \tau_4 = 0.0027 \text{ seconds}$$

Substitution of the value of τ_4 in $1/b\tau_4 = 2.6\ \Omega^3$ (from above) gives $b = 6.1 \times 10^{-6}\ \text{sec}^2$. In this case, the following error for a constant-velocity input of 1.0 inches per second is 0.010 inch.

The sort of delays in the hydraulic relay implied by the values of (a) and (b) obtained are quite small, and in practice may be difficult to achieve. With the values likely to be encountered, the value of gain tolerated would not be so great. There are ways, however, of compensating for lags in power amplifiers, such as the present servo system, so that high gains can be used to improve response and accuracy without causing instability problems.

NOMENCLATURE

Symbol	Term	Dimension
A	Cylinder piston area	L^2
a	$= K_v \frac{m}{\lambda_v}$	
a_m	Fully open valve port area	L^2
a(X)	Valve port area as a function of X	L^2
B	Constant	
b	$\frac{m}{\lambda_v}$	
C	Gain constant	
db	Decibel	
E	Hydraulic fluid bulk modulus	F/L^2
e	$= e_i - e_o$	L
F	Solenoid force	F
F_m	Maximum cylinder load	F
F(s)	Transform of $f(t)$	
f	Damping force coefficient	Ft/L
$f(t)$	Function of time	
$f(X)$	Function of X	
G	Gain margin	

NOMENCLATURE

Symbol	Term	Dimension
h	Valve lap	L
i	Current	*amperes*
K	(1) Slope gain constant (2) $K = K_1 K_2 K_3 K_4 K_5$	
K'	$= 0.001\ K$	
K_1	Potentiometer gain	
K_2	Amplifier gain	
K_3	Solenoid gain	
K_4	Pilot valve gain constant	
K_5	Valve-cylinder relay slope gain constant	
k	Spring constant	F/L
L	Cylinder stroke	L
$\mathcal{L}$	Laplace transform	
M	$= \lvert Y_c(s) \rvert$ = Amplitude ratio	
M_r	Maximum amplitude ratio at ω_r	
m	Mass	M
P_c	Differential pressure across cylinder piston	F/L^2
P_{co}	Steady value of P_c	F/L^2
P_s	Supply pressure	F/L^2
P_1, P_2	Cylinder pressures	F/L^2
p_c	Small perturbation in P_c	F/L^2
Q	Volume flow rate	L^3/t
Q_d	Disturbed flow rate	L^3/t
Q_n	Flow rate at no-load	L^3/t
Q_o	Steady value of Q	L^3/t
Q_s	Saturated valve flow rate	L^3/t
q	Small perturbation in Q	L^3/t
s	Laplace variable	$1/t$
T_b	Build-up time	t
T_n	Time of cylinder travel over-stroke L with no-load	t
T_s	Time of cylinder travel with valve saturated	t
T_t	Total time of cylinder travel	t
t	Time	t
u	$= 0.001\ \omega$	$1/t$
V	$= K_1 (\theta_i - \theta_o)$ = voltage difference	*volts*
V_o	Volume of fluid in one half of cylinder	L^3
V_t	Total volume of fluid in cylinder	L^3
X	Valve displacement	L
X_m	Valve travel from zero to fully-open port area	L
X_o	Steady value of X	L
x	Small perturbation in X	L
$Y_c(s)$	Closed-loop transfer function	
$Y_o(s)$	Open-loop transfer function	
λ_c	Cylinder stiffness	
λ_v	Valve stiffness	
$\zeta(s)$	Dynamic stiffness	F/L
ϕ	$= \cos^{-1} \rho$	
θ_i	Input displacement	L
θ_o	Output displacement	L
ρ	Damping ratio	
σ	Density	M/L^3
τ	Time constant	t
τ_4	Time constant of pilot valve	t
Ω	$= 0.01\ \omega$	$1/t$
ω	Angular frequency (radians/sec)	$1/t$
ω_b	Bandwidth (radians/sec)	$1/t$
ω_c	Frequency limit for accurate response	$1/t$
ω_n	Natural frequency (radians/sec)	$1/t$

7.4.8 Dynamic Behavior of a Simple Pneumatic Pressure Regulator

This section is a reprint of an article entitled "Dynamic Behavior of a Simple Pneumatic Pressure Reducer," by D. H. Tsai and E. C. Cassidy, published in the Journal of Basic Engineering, June 1961, Copyrighted by the American Society of Mechanical Engineers (Reference 26-105).

Throughout the article, reference to either the *pressure reducer* or *reducer* connotes the handbook's term, *regulator*.

A PRESSURE reducer is an automatic fluid-mechanical device for reducing the pressure of the working fluid, either hydraulic or pneumatic, from a higher level to a preselected lower level over a wide range of flow rates and upstream pressures. This type of device is used in many fluid-control and fluid-power systems, ranging from the simple domestic water line to the highly sophisticated servomotor control systems in modern aircraft and missiles.

The control of pressure is usually accomplished by a sensing element which senses the change in the regulated pressure, and automatically adjusts the flow rate so as to maintain the desired pressure. In its simple form, a reducer may contain a single sensing element acting directly on a flowmetering valve. In the more elaborate designs, a reducer may contain two or three sensing elements and metering valves, cascaded into two or three stages, so as to achieve the desired characteristics in pressure regulation. Whether the design is simple or complex, the operation of these reducers under static or steady-state conditions is

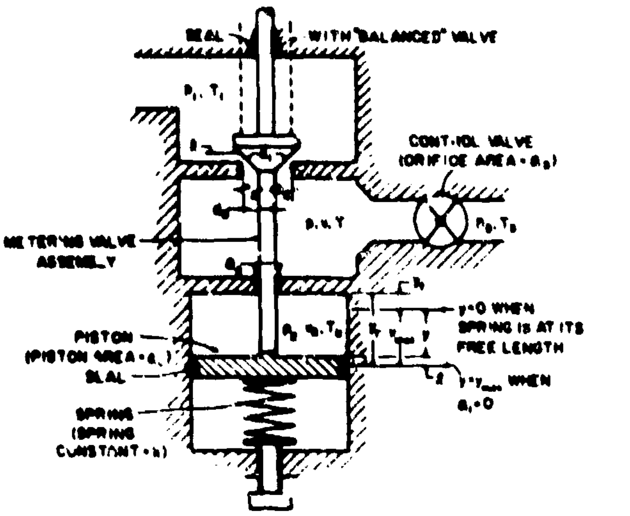

Fig. 1 Schematic diagram of a simple pressure reducer

fairly simple in principle. If information is available on the flow-rate and flow-force characteristics of the metering valve, the analysis of the pressure-flow characteristics of the reducer under steady-state conditions is straightforward [1].[2]

In actual service, however, a reducer seldom operates under purely steady-state conditions. Also, the vibrations of the mounting structure may induce undesirable oscillations in the reducer. To obtain a complete picture of its performance, one must therefore investigate the dynamic characteristics of the reducer and analyse the problems of its natural frequency and stability. This is difficult to do, even in the case of a simple reducer, because of the rather complex interaction between the fluid and the mechanical parts. If the fluid is compressible, additional difficulty is introduced. Perhaps because of these difficulties there is very little analytical work on this subject in the open literature. In the few published articles (see, for example, [2, 3]) the analyses, for the most part, have been handicapped by the rather drastic simplifying assumptions, and the results have been incomplete, especially in the study of the various nonlinear effects in the reducer system. In so far as the authors are aware, there has been no systematic study of the reducer-stability problem as affected by the various design and operating parameters. The design and manufacture of pressure reducers, therefore, have been carried out largely on a trial-and-error basis. This is expensive, and promises only uncertain results.

This paper presents an analysis of the dynamic behavior of a simple pneumatic pressure reducer with a view to clarifying some of the problems brought out in the foregoing discussion. Both the nonlinear and the linearized problems were studied. Some experimental results were also obtained on a working reducer model to check the validity of the analysis. The nonlinear and the linearized solutions were compared in detail so as to bring out the essential features of the dynamic behavior in both cases. The stability problem was also studied in the linearized case, and a set of stability criteria was formulated in terms of the design and operating parameters of the reducer. These results were also compared with experimental results.

In the analytical work, the upstream pressure was assumed to be steady, and the regulated pressure downstream of the flow-metering valve was assumed to be uniform at each instant of time. Thus the dynamic effects of the conduits upstream and downstream of the reducer were neglected. Except for these effects, an effort was made to keep the simplifying assumptions to a minimum. For example, proper account was taken of the flow forces on the metering valve. To do this, it was necessary to obtain first-hand experimental measurements, because of the lack of information in the literature. The results of the flow-force measurements on several typical valves are summarized in Appendix 2.

Despite these efforts, the analysis, of course, remains limited by the very simplicity of the reducer model. To gain a more complete concept, similar analyses should be carried out to study the effects of conduit dynamics mentioned earlier, the interaction between stages in a multistage pressure reducer, the transmission and attenuation of large-amplitude pressure waves, and other related problems. Clearly, the effective use of pressure reducers, and indeed of all fluid-control and fluid-power systems, depends on an understanding of all these various problems.

Analysis

The analysis was made for the case of a simple pressure reducer shown in Fig. 1.[3] In steady-state operation, the mass rate

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[1] The work reported in this paper was carried out under the sponsorship of the Airborne Equipment Division of the Bureau of Aeronautics (now Bureau of Naval Weapons) Department of the Navy—Reference NAer 01826, NAer 01880, NAer 01881.

Contributed by the Instruments and Regulators Division and presented at the Winter Annual Meeting, New York, N. Y., November 27-December 2, 1960, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters, August 11, 1960. Paper No. 60—WA-186.

[2] Numbers in brackets designate References at end of paper.

[3] A common configuration of the poppet valve is the "balanced" valve shown by the dashed outline in Fig. 1. The flow force on the balanced valve is different from that on an unbalanced valve, but there is no basic difference in the method of analysis.

Nomenclature

Dimensional Terms:

- a_1, a_2, a_3 = orifice areas, sq in., Fig. 1
- a = seating area of metering valve, sq in.
- a_p = piston area, sq in., Fig. 1
- a, a_1, a_2 = sonic velocities corresponding to T, T_1, and T_2, respectively, ips
- c_d = viscous damping constant, lb$_F$-sec/in. (lb$_F$ = pound force)
- c_p = specific heat of gas at constant pressure, Btu/lb$_M$-deg R (lb$_M$ = pound mass)
- c_v = specific heat of gas at constant volume, Btu/lb$_M$-deg R
- f = flow force on metering valve, lb$_F$
- g = gravitational constant conversion factor = 384 lb$_M$-in/lb$_F$-sec^2
- J = mechanical equivalent of heat = 778 ft-lb$_F$/Btu
- k = spring constant, lb$_F$/in.
- l = lift of metering valve from seat, in.
- m = mass of gas contained in volume v, lb$_M$
- m_t = mass of metering valve assembly (metering valve, valve rod, piston, and so on), lb$_M$
- $\dot{m}_1, \dot{m}_2, \dot{m}_3$ = mass rate of flow of gas through a_1, a_2, a_3, respectively, lb$_M$/sec
- $\dot{m}_{i1}, \dot{m}_{i2}, \dot{m}_{i3}$ = isentropic mass rate of flow through a_1, a_2, a_3, respectively, lb$_M$/sec
- p, p_1, p_2, p_3 = pressures referred to in Fig. 1, psia
- p_k = pressure in pneumatic spring chamber, psia
- $\mathfrak{R}$ = gas constant = 53.3 lb$_F$-ft/lb$_M$-deg R for air
- t = time, sec
- T, T_1, T_2, T_3 = temperatures corresponding to p, p_1, p_2, p_3, respectively, deg R, Fig. 1
- v, v_2 = volumes, cu in., Fig. 1
- v_k = volume of pneumatic spring chamber, cu in.
- y, y_f, y_{max} = positions of piston in v_2, in., Fig. 1

Reference Terms

- a_0 = reference area, sq in., Fig. 1
- a_0 = reference sonic velocity in gas in v when reducer is in *reference* steady-state operation, ips
- d_0 = reference diameter, in., Fig. 1
- f_0 = flow force on metering valve at zero lift, lb$_F$
- m_0 = reference mass of gas in v at p_0, T_0, lb$_M$
- p_0 = reference pressure in v when reducer is in *reference* steady-state operation, psia
- T_0 = reference temperature in v corresponding to a_0, deg R
- v_{20} = reference volume of v_2, when $a_1 = 0$, cu in.
- y_0 = position of piston during *reference* steady-state operation, in.
- y_r = $y_f + y_{max}$, in., Fig. 1
- ω_0 = reference value of ω, radians/Z

Dimensionless Terms

- A_1, A_2, A_3 = a_1/a_0, a_2/a_0, a_3/a_0, respectively
- $\mathfrak{A}, \mathfrak{A}_1, \mathfrak{A}_2$ = a/a_0, a_1/a_0, a_2/a_0, respectively
- C_1, C_2, C_3 = discharge coefficients of a_1, a_2, a_3, respectively; $C_1 = \dot{m}_1/\dot{m}_{i1}$, and so on
- F_r = $f_0/a_0(p_1 - p) = a/a_0$, Appendix 2
- F_y = f/f_0, Appendix 2
- L = $l/d_0 = (y_r/d_0)(Y_{max} - Y)$
- P, P_1, P_2, P_3 = p/p_0, p_1/p_0, p_2/p_0, p_3/p_0, respectively
- R_1, R_{2c}, R_{21}, R_3 = p/p_1, p_2/p, p/p_2, p_3/p, respectively
- V = v/v_{20}
- V_2 = v_2/v_{20}
- Y, Y_f, Y_{max}, Y_0 = y/y_r, y_f/y_r, y_{max}/y_r, y_0/y_r, respectively
- Z = dimensionless time = $a_0 a_0 t/v$
- γ = ratio of specific heats = c_p/c_v = 1.4 for air
- ζ = damping coefficient = $c_d g v/m_t a_0 a_0$
- ζ_1, ζ_2 = damping ratios of pressure reducer in linearized solution (associated with natural frequencies ω_{n1} and ω_{n2}, respectively)
- η = force coefficient for a_p = $(g/m_t y_r)(v/a_0 a_0)^2 a_p p_0$
- ξ = force coefficient for a = $(g/m_t y_r)(v/a_0 a_0)^2 a_0 p_0$
- ω = natural frequency (radians/Z) of spring-mass system in reducer = $(kg/m_t)^{1/2}(v/a_0 a_0)$
- ω_{n1}, ω_{n2} = natural frequencies (radians/Z) of pressure reducer in linearized solution (associated with damping ratios ζ_1 and ζ_2, respectively)

of flow through the metering valve is equal to that through the control valve, so that p = const = p_0, and the metering-valve assembly is stationary. This equilibrium position is maintained by the balance of forces on the metering-valve assembly. On one side (the bottom side in Fig. 1) the piston is acted on by a reference-spring force obtained from compression of the spring. On the other side the piston is acted on by pressure p_2. The metering valve is also acted on by a flow force due to the pressure forces (and viscous forces) integrated over the face of the valve. There also may be some friction forces between the metering-valve assembly and the casing. But under steady-state or slowly varying conditions, the inertia force is either absent or negligible.

The regulation of the pressure in v is then effected by the feedback of the pressure signal from v to v_2, in response to which the piston in v_2 adjusts the position of the metering valve, and hence the mass rate of flow, so that p is held nearly constant. The regulating quality of a reducer may be expressed in terms of the change in p in the range of flow rates and upstream pressures the reducer is designed to handle. For perfect regulation, the change in p would have to be zero. This is difficult to achieve in the simple reducer under study. But there is no problem of stability under the steady-state condition, because the feedback arrangement always produces a restoring force to balance the change of forces caused by a disturbance in the flow. To satisfy the steady-state condition (no inertia force), the disturbance, of course, either must be very small or must occur very slowly.

In unsteady or dynamic operation, the inertia force may become quite large. Also p_2 may differ appreciably from p, and both may differ appreciably from the reference steady-state operating pressure p_0. Under these conditions, self-sustained oscillations may be induced in the reducer, so that its function as a pressure regulator is impaired. In more serious cases, such oscillations may even damage some mechanical parts, or materially shorten the service life of the reducer. It is therefore of interest to study in some detail the interactions between the fluid and the mechanical parts, in order to gain an understanding of the conditions under which the operation of the pressure reducer is dynamically stable.

The Governing Equations. This section gives a brief discussion of the governing equations used in the present analysis. The basic assumptions and the derivation of these equations are given in Appendix 1.

The dynamical equation was obtained by equating to zero the algebraic sum of the inertia, damping, spring, pressure, and flow forces, acting on the piston and metering valve. In nondimensional form, this equation is

$$D^2Y + \zeta DY + \omega^2 Y - \eta P_2 - \xi F_r F_y (P_1 - P) = 0. \qquad (1)$$

Here $D = d/dZ$, and $D^2 = d^2/dZ^2$. Y is the piston position, and P_2, P_1, P are pressures.[4] The coefficients ζ, ω^2, η, and ξ are mainly design parameters and remain constant for a given reducer. F_r is a function which describes the variation of the flow force with the pressure difference $(P_1 - P)$; F_y is a function which describes the variation of the flow force with the valve position Y. These functions were determined experimentally as discussed in Appendix 2.

The gas pressures P, P_2 and sonic velocities $\mathfrak{A}$, $\mathfrak{A}_2$ were obtained by considering the perfect gas, continuity and energy relationships for the gas in volumes V and V_2. Appendix 1 shows that there were two cases to be considered:

Case I. Flow through A_2 was from V to V_2:

$$\left\{\begin{aligned} &DP - 2PD\mathfrak{A}/\mathfrak{A} = \mathfrak{A}^2[(C_1A_1P_1\varphi_1/\mathfrak{A}_1) \\ &\qquad\qquad - (C_2A_2P\varphi_{2e}/\mathfrak{A}) - (C_3A_3P\varphi_3/\mathfrak{A})] &(2)\\ &(Y + Y_f)DP_2 + \gamma P_2DY = \gamma VC_2A_2\mathfrak{A}P\varphi_{2e} &(3)\\ &DP = \gamma[C_1A_1\mathfrak{A}_1P_1\varphi_1 - C_2A_2\mathfrak{A}P\varphi_{2e} - C_3A_3\mathfrak{A}P\varphi_3] &(4)\\ &2(Y + Y_f)D\mathfrak{A}_2/\mathfrak{A}_2 = (VC_2A_2\mathfrak{A}P\varphi_{2e}/P_2)[\gamma - (\mathfrak{A}_2/\mathfrak{A})^2] \\ &\qquad\qquad - (\gamma - 1)DY. &(5)\end{aligned}\right.$$

Case II. Flow through A_2 was from V_2 to V:

$$\left\{\begin{aligned} &DP - 2PD\mathfrak{A}/\mathfrak{A} = \mathfrak{A}^2[(C_1A_1P_1\varphi_1/\mathfrak{A}_1) \\ &\qquad\qquad + (C_2A_2P_2\varphi_{2i}/\mathfrak{A}_2) - (C_3A_3P\varphi_3/\mathfrak{A})] &(6)\\ &(Y + Y_f)DP_2 + \gamma P_2DY = -\gamma VC_2A_2\mathfrak{A}_2P_2\varphi_{2i} &(7)\\ &DP = \gamma[C_1A_1\mathfrak{A}_1P_1\varphi_1 + C_2A_2\mathfrak{A}_2P_2\varphi_{2i} - C_3A_3\mathfrak{A}P\varphi_3] &(8)\\ &P_2 = \mathfrak{A}_2^{2\gamma/(\gamma-1)}. &(9)\end{aligned}\right.$$

In each case the first equation is the continuity equation for the gas in V, the second is the energy equation applied to the gas in V_2, and the third is the energy equation applied to the gas in V. The fourth equation expresses the relationship between $\mathfrak{A}_2$ and P_2 (and other quantities) for determining the state of the gas in V_2.

The equations for these two cases, together with equation (1), make up two sets of five simultaneous equations, with the independent variable Z and the dependent variables P, $\mathfrak{A}$, P_2, $\mathfrak{A}_2$, and Y. Given suitable initial and boundary conditions, these equations may be solved simultaneously for the five dependent variables [4].

The following functions were assumed:

$$A_1 = -A_1'(Y_{max} - Y), \quad A_1' = dA_1/dY;$$

$$\varphi_n = \varphi(R_n), \text{ where } n = 1, 2e, 2i, 3;$$

and

$$\varphi(R_n) = [2(R_n^{2/\gamma} - R_n^{(\gamma+1)/\gamma})/(\gamma - 1)]^{1/2} \quad \text{for } 0.528 \le R_n \le 1,$$

and

$$\varphi(R_n) = 0.579 \quad \text{for} \quad R_n < 0.528.$$

F_y and F_r were taken from experimental values (Appendix 2). For simplicity, P_1, P_3, A_1' and the discharge coefficients C_1, C_2, and C_3 were assumed to remain constant. Specifically, C_2 was assumed to remain the same for flow in either direction through A_2.

The boundary conditions for this problem were taken to be the following: When $P_2 < P$, the equations for Case I were used. When $P < P_2$, the equations for Case II were used. Also, when $Y = Y_{max}$, the valve A_1 was closed, and the metering-valve assembly was constrained from further closing. Therefore, $Y \ngtr Y_{max}$, and at $Y = Y_{max}$, DY was assumed to be zero (no rebound). Finally, since the equations for reverse flow through A_1 and A_3 were not formulated, P could not be allowed to be greater than P_1 or smaller than P_3. These last conditions were not too restrictive and could be satisfied in most cases of interest.

Method of Solution. The two sets of equations discussed in the foregoing section are nonlinear, and do not admit of a general solution. In order to gain some understanding of the dynamical behavior of the reducer, these equations were therefore solved by an approximate numerical method, and several solutions were obtained with the aid of a digital computer. In the numerical integration, the Runge-Kutta recurrence formula [5] was used to evaluate the increments ΔP, $\Delta\mathfrak{A}$, and so on, for each step of ΔZ. The optimum step size was not investigated. However, an effort was made to keep the step size small so as to limit the truncation error, but not so small as to lengthen the computing time too much. In one problem it was found that an increase in ΔZ by a factor of 10 changed the frequency of oscillation of P, P_2, and Y by approximately 5 per cent and the amplitude of oscillation by about 2 or 3 per cent. These changes were considered not too serious, and the larger ΔZ was taken. In other problems, this example was used as a guide to the correct choice of step size.

Linearization of Governing Equations. Because of the lack of a general solution to the nonlinear governing equations, it was not possible to formulate a general set of stability criteria for this problem. In order to progress further, it was necessary to limit the original objective, and to restrict attention to the linearized equations. Unfortunately, as later discussion shows, even with the linearized equations, it was difficult to establish quantitative stability criteria because of the large number of design and operating parameters involved. Nevertheless, the linearized problem served to provide qualitative information on the effect of the various parameters and, in so doing, led to a better under-

[4] Y, P_2, and so on, are dimensionless ratios (see Nomenclature). However, the terms "dimensionless" and/or "ratio" are omitted here for brevity. This rule will be applied to all other dimensionless terms in the text.

standing of the nonlinear stability problem as well.

In the linearised problem, $\mathcal{Q}$ and $\mathcal{Q}_2$ were assumed to remain constant (and equal to $\mathcal{Q}_1$). This could be justified by the numerical solutions to the nonlinear equations obtained for a few cases, wherein it was found that the changes in $\mathcal{Q}$ and $\mathcal{Q}_2$ were small compared to those in P and P_2, respectively, if P and P_2 did not themselves vary too much from the reference steady-state value of unity. With the variables $\mathcal{Q}$ and $\mathcal{Q}_2$ removed, only three equations were now needed for the three remaining variables P, P_2, and Y. Equations (2), (5), (6), and (9), were therefore abandoned. Actually, with $d\mathcal{Q} = 0$, the continuity equations (2) and (6) became quite similar to the two energy equations (4) and (8). The former were discarded in preference to the latter because it was thought that the continuity equations were less stringent, inasmuch as the derivation of the energy equations actually involved a consideration of mass continuity.

To simplify the remaining equations further, it was assumed that $P \approx P_2$, and that

$$\varphi_2 = \nu(P - P_2)/P \approx \nu(P - P_2)/P_2,$$

where ν is a proportionality factor. When $P_2 < P$, $\varphi_2 \approx \varphi_{2a} > 0$, and when $P_2 > P$, $\varphi_2 \approx \varphi_{2t} < 0$. So both φ_{2a} and φ_{2t} could be represented by a single φ_2, and the two sets of equations (1), (3), (4) and (1), (7), (8) were reduced to one set:

$$\begin{cases} D^2Y + \zeta DY + \omega^2 Y - \eta P_2 - \xi F_y F_v (P_1 - P) = 0 & (10) \\ DP = \gamma[\mathcal{Q}_1 C_1 A_1 P_1 \varphi_1 - C_2 A_2 \nu (P - P_2) - C_3 A_3 P \varphi_3] & (11) \\ (Y + Y_f) DP_2 = \gamma[V C_2 A_2 \mathcal{Q} \nu (P - P_2) - P_2 DY]. & (12) \end{cases}$$

The use of the approximate φ_2 undoubtedly involves some error, especially if the value chosen for ν should be inaccurate. (The slopes of φ_{2a} and φ_{2t} approach infinity as R_{2a} and R_{2t} approach unity.) However, ν always could be combined with the quantity C_2A_2, so that its effect would be the same as that of C_2A_2.

To linearize equations (10), (11), and (12) let

$$\begin{cases} Y = Y_0 + Y' \\ P = 1 + P' \\ P_2 = 1 + P_2', \end{cases}$$

and

$$\begin{cases} Y' \ll Y_0 \\ P' \ll 1 \\ P_2' \ll 1. \end{cases}$$

Also

$$F_v,\ \varphi_1,\ \varphi_3 = \text{const};$$
$$F_y = 1 - F_y'(Y_{max} - Y_0 - Y'), \quad F_y' = dF_y/dY = \text{const};$$
$$A_1 = -A_1'(Y_{max} - Y_0 - Y').$$

Substituting all these conditions into equations (10), (11), and (12), and neglecting higher-order terms, one obtains, after some manipulation, the linearised equations as follows:

$$\begin{cases} [D^2 + \zeta D + (\omega^2 - K_1)]Y' + K_2P' - \eta P_2' \\ \qquad = K_3 + \eta - \omega^2 Y_0 & (13) \\ K_4Y' + (D + K_5 + K_6)P' - K_5P_2' = K_7 - K_6 & (14) \\ K_8DY' - K_9P' + (D + K_9)P_2' = 0, & (15) \end{cases}$$

with

$$\begin{aligned} K_1 &= \xi F_v(P_1 - 1)F_y' \\ K_2 &= \xi F_v[1 - F_y'(Y_{max} - Y_0)] \\ K_3 &= (P_1 - 1)K_2 \\ K_4 &= -\gamma\mathcal{Q}P_1\varphi_1C_1A_1' \\ K_5 &= \gamma\mathcal{Q}C_2A_2\nu \\ K_6 &= \gamma\mathcal{Q}C_3A_3\varphi_3 \\ K_7 &= K_4(Y_{max} - Y_0) \\ K_8 &= \gamma/(Y_0 + Y_f) \\ K_9 &= VK_5K_8/\gamma. \end{aligned}$$

Stability of the Linearized Equations. From equations (13), (14), and (15), the characteristic equation was obtained next by setting the determinant formed by the coefficients of Y', P', and P_2' equal to zero, in the conventional manner [4]:

$$\begin{vmatrix} D^2 + \zeta D + (\omega^2 - K_1) & K_2 & -\eta \\ K_4 & D + K_5 + K_6 & -K_5 \\ K_8D & -K_9 & D + K_9 \end{vmatrix} = 0.$$

Therefore the characteristic equation is

$$\lambda^4 + \alpha_3\lambda^3 + \alpha_2\lambda^2 + \alpha_1\lambda + \alpha_0 = 0, \qquad (16)$$

with

$$\begin{aligned} \alpha_3 &= \zeta + K_5 + K_6 + K_9 \\ \alpha_2 &= \omega^2 - K_1 + \zeta(K_5 + K_6 + K_9) + K_5K_9 + \eta K_8 \\ \alpha_1 &= (\omega^2 - K_1)(K_5 + K_6 + K_9) \\ &\qquad + \zeta K_6K_9 - K_2K_4K_9 + \eta K_8(K_5 + K_6) - K_2K_4 \\ \alpha_0 &= (\omega^2 - K_1)K_6K_9 + K_4K_9(\eta - K_2) \end{aligned}$$

The stability problem associated with the quartic equation (16) was studied by Routh [7] and also independently by Hurwitz [8]. To insure stability, the real parts of all the roots of equation (16) must be negative, and to satisfy this condition, the following criteria, known as the Routh-Hurwitz criteria, must be satisfied [6]:

1. The coefficient α's must all be positive,
2. $\alpha_1\alpha_2\alpha_3 > \alpha_0\alpha_3^2 + \alpha_1^2$,
3. $\alpha_2^2 > 4\alpha_0$.

The problem of stability of the pressure reducer is therefore one of obtaining the values of the K's and α's from the design and operating conditions, and then testing the α's according to the stability criteria.

In a few simpler cases, the qualitative effect of a parameter on stability is evident by inspection. For example, an increase in ω (the natural frequency of the spring-mass system in the reducer) makes α_2, α_1, and α_0 more positive, and hence the system would be more stable, according to the first criterion. In contrast, an increase in K_1 (by increasing P_1, say) would have the opposite effect. Also, an increase in ζ (damping coefficient) would not affect α_0, but would tend to make α_3, α_2, and α_1 more positive and the product $\alpha_3\alpha_2\alpha_1$ larger, and hence the system would be more stable according to all three criteria. These deductions are in general agreement with experience.

To obtain quantitative information on a particular parameter, equation (16) would have to be solved with the parameter in question varied systematically over the range of interest. A few parameters were studied in this manner. However, it was not possible to study all the parameters in their various combinations, because of the large number involved. The solution to equation (16) was obtained by the conventional method of algebra [6]. The computation was again performed with the aid of a digital computer. The results were obtained in terms of the damping ratios ζ_1 and ζ_2 and frequency ratios ω_{n1}/ω and ω_{n2}/ω. These ratios are more descriptive of the dynamic behavior than the four quartic roots λ_1, λ_2, λ_3, and λ_4 of equation (16). They are related to the latter through the following equations:

$$\lambda_1 = -[\zeta_1\omega_{n1} - \omega_{n1}(\zeta_1^2 - 1)^{1/2}]$$
$$\lambda_2 = -[\zeta_1\omega_{n1} + \omega_{n1}(\zeta_1^2 - 1)^{1/2}]$$
$$\lambda_3 = -[\zeta_2\omega_{n2} - \omega_{n2}(\zeta_2^2 - 1)^{1/2}]$$
$$\lambda_4 = -[\zeta_2\omega_{n2} + \omega_{n2}(\zeta_2^2 - 1)^{1/2}].$$

Thus, if $|\zeta_1| < 1$ and $|\zeta_2| < 1$, the four λ's form two pairs of complex-conjugate roots, and ω_{n1} and ω_{n2} are the two natural frequencies of the reducer system, and ζ_1 and ζ_2 are the damping ratios associated, respectively, with the two oscillatory components. If $|\zeta_1| < 1$, and $|\zeta_2| \geq 1$, only the first component is oscillatory with a natural frequency of ω_{n1}, and ω_{n2} has no physical meaning. If both $|\zeta_1| \geq 1$ and $|\zeta_2| \geq 1$, there is no oscillatory component; both ω_{n1} and ω_{n2} have no physical meaning. The system is stable (to a small disturbance) if both ζ_1 and ζ_2 are positive, and unstable if either one (or both) becomes negative. On the boundary of stability, either ζ_1 or ζ_2 is zero while the other remains positive, or both ζ_1 and ζ_2 are zero.

Experiment

Some experimental work was carried out on a working model of a simple pressure reducer to check the validity of the governing equations and the method of solution. The model was similar to that shown in Fig. 1. The physical dimensions are given in the caption of Fig. 2. The metering valve was a 45-deg poppet valve. In the model, the control valve was replaced by a simple orifice to facilitate sudden opening and closing of a_0. The mechanical spring was replaced by a pneumatic spring. This was accomplished by charging the spring chamber to a pressure p_b. The spring constant was determined from the simple isentropic relationship for the gas in the spring chamber:

$$k = \gamma p_b a_p^2/v_b$$

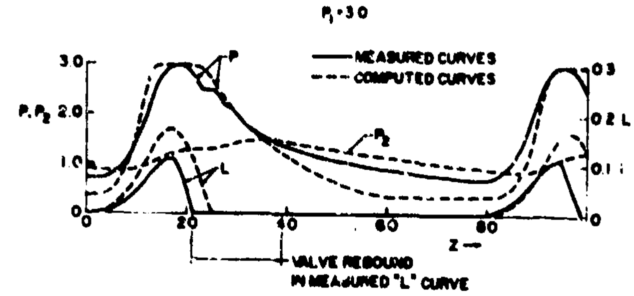
where v_b was the volume of the spring chamber. For small piston displacements, p_b and v_b were approximately constant, and the pneumatic spring therefore was approximately linear. In operation p_b was taken as the pressure in the spring chamber when the reducer was in steady-state operation. This pressure was measured by means of a bourdon-type pressure gage to an accuracy of about 1 psi.

The upstream section of the model was supplied with compressed air at pressure p_1, maintained at the desired level by the use of an auxiliary pressure reducer connected to a high-pressure (3000-psi) source. Fluctuations in p_1 were minimized by the use of a large (3-cu ft) surge tank between the auxiliary reducer and the upstream section of the model. Pressure p_1 was measured by means of a calibrated bourdon gage. The accuracy of the measurement was 1 psi. The regulated downstream pressure p was measured by means of a calibrated strain-gage-type pressure transducer. Since the volume v was small (2.88 cu in.) and compact, p was nearly uniform throughout v at any instant of time. The valve lift l was measured by means of a calibrated linear differential transformer driven at a frequency of 1000 cps. The voltage outputs from the pressure transducer and the linear differential transformer were displayed on a dual-beam cathode-ray oscilloscope. The accuracy of the p and l measurements was within 3 psi and 0.002 in., respectively.

In operation, the auxiliary reducer was adjusted to give the desired level of p_1. The pressure p_b was then adjusted to give the desired downstream pressure p under steady-state conditions. The disturbance was introduced by first closing the orifice a_0 and then suddenly opening it. The subsequent oscillation of p and l was photographed from the oscilloscope screen.

Results

Nonlinear Solutions and Comparison With Experiment. Fig. 2 shows the steady-state oscillations of the measured and the computed pressure P and valve lift L versus time Z obtained under the conditions given in the figure. The design parameters were



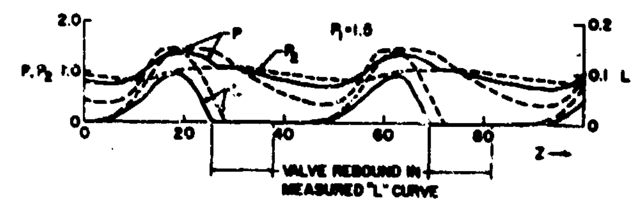


Fig. 2 Comparison of nonlinear solutions with experimental results for P_1 = 3.0 and 1.5

Initial Conditions:

$P = P_2 = 1.0$, $\mathcal{Q} = \mathcal{Q}_2 = 1.0$, $DY = DP = DP_2 = D\mathcal{Q} = D\mathcal{Q}_2 = 0$

Step Disturbance:

$A_0 = 0$ to $A_0 = 0.075$

Constants:

a_p = 3.14 sq in.
a = 0.279 sq in.
a_0 = 13200 in/sec
c_d = 0.713 lb$_F$-sec/in.
d_0 = 0.625 in.
k = 171 lb$_F$/in. for P_1 = 3.0,
= 147 lb$_F$/in. for P_1 = 1.5
m_t = 1.07 lb$_M$
p_0 = 40 psia
v = 2.88 in.^{3}
y_r = 1.12 in.
A_1' = −5.39

A_2 = 0.021
$\mathcal{Q}_1$ = 1.0
$C_t = C_2 = C_3$ = 0.9
F_r = 1.07
F_y' = 5.53
P_3 = 0.375
V = 0.855
ζ = 0.20
η = 0.025
ξ = 0.0022
ω = 0.037 for P_1 = 3.0,
= 0.032 for P_1 = 1.5

fixed by the physical dimensions of the model. The operating conditions were chosen mainly for convenience but otherwise the choice was arbitrary. The computed curves of P_2 are also shown. The results were obtained by disturbing the reducer by a sudden opening of A_2. In the computed case, the steady-state solutions were obtained in two or three cycles of oscillations after the initial disturbance. For comparison, the Z-axis of the computed curves was shifted so that the initial valve lift was the same as in the experiment. The value of Z at this point was arbitrarily taken as zero.

The conditions for the computed solutions listed in the caption did not agree exactly with the conditions of the experiment. The more questionable conditions used for computation were the following. C_1 was assumed constant and equal to 0.9, although experiment showed that it varied with the lift and the pressure ratio across the valve (see Fig. 7(d), Appendix 2). C_2 and C_3 were also assumed equal to 0.9 for the sake of simplicity. The friction at the O-ring seal was difficult to estimate and was arbitrarily assumed to remain constant and to be viscous in nature. The value of ζ was taken as 0.2. The flow force and the pressure force (due to p_2) acting on the metering-valve assembly were somewhat in error because the pressure force acting on the cross-sectional area of the valve rod was neglected. Some error was also involved in the assumption of a linear pneumatic spring. Finally, the coefficient of restitution between the valve and the seat was assumed to be zero (no rebound), and hence the effects of the rebound on P and P_2 were not present in the computation. In the experiment, the details of the rebound were obscured by a high-frequency chatter in the oscillograph record of the L-curve. For this reason, only the duration of the rebound was indicated. However, the small fluctuations in the P-curve, caused by the rebound, were clearly visible.

Because of these assumed conditions, the results should be compared only in a qualitative way. As Fig. 2 shows, the qualitative agreement was satisfactory. The general shape of the P and L-curves was the same. In fact, the computed period of P and L and the maximum amplitude of P agreed quantitatively with the measured values. Moreover, when P_1 was changed from 3.0 to 1.5, these points of agreement remained satisfactory. Finally, the analytical solutions were also correct in showing the self-sustained oscillations observed in the experiment. These results, therefore, show that the assumptions, the governing equations, and the method of solution were reasonably valid.

Linearized Solutions. The nonlinear solutions discussed in the foregoing section reveal many details of the dynamic behavior of the reducer. However, these solutions are cumbersome, and they give very little information on the degree of stability. Moreover, since there is no general solution to the nonlinear problem, it is difficult to evaluate the effect of a change in a design or operating parameter on stability. It is therefore of interest to study the linearized problem outlined in the "Analysis," in order to gain some qualitative understanding of the stability problem.

Fig. 3 shows a series of solutions to the linearized characteristic equation (16) with the parameters A_2 and $(\omega/\omega_b)^2$ varied and with the other parameters held constant at the values given in the caption. The selection of A_2 and $(\omega/\omega_b)^2$ as the variable parameters was again arbitrary, and was intended as a further illustration of the effects of these parameters on the stability problem. The results show the effect of A_2 and $(\omega/\omega_b)^2$ on the damping ratios (the ordinate) and the frequency ratios (the abscissa) of the reducer. Along each dashed curve, A_2 is constant, and $(\omega/\omega_b)^2$ is variable. Along each solid curve, $(\omega/\omega_b)^2$ is constant, and A_2 is variable. The solution for each pair of values of A_2 and $(\omega/\omega_b)^2$, therefore, is shown as a pair of points, representing the two components of the solution, with co-ordinates $(\zeta_1, \omega_{n1}/\omega)$ and $(\zeta_2, \omega_{n2}/\omega)$, as described in the Analysis. Solutions in the unshaded areas have two oscillatory components. Solutions in the shaded areas have one oscillatory component and one nonoscillatory component.

In the figure, A_2 was varied from the very small value of 0.001 to a value of 3.0 (equal to $a_p/3$). Further increase in A_2 resulted in very little change in the computed damping and frequency ratios, showing that at this value, A_2 offered very little resistance to flow between V and V_2. $(\omega/\omega_b)^2$ was varied over a smaller range, but it can be shown that, if ω approached infinity, one component would approach $\zeta_1 = 0$, $\omega_{n1}/\omega = 1$, and the other component would approach the line $\omega_{n2}/\omega = 0$, ζ_2 being indeterminate.

The heavy dashed curves, $A_2 = 0.0496$, and the heavy solid curves $(\omega/\omega_b)^2 = 1.566$ are of some special interest because at their intersection (point X), $\zeta_1 = \zeta_2$ and $\omega_{n1} = \omega_{n2}$. These curves and the curves for the extreme values of A_2 and $(\omega/\omega_b)^2$ are helpful in that they outline regions in each of which the trend of variation of the intermediate curves may be readily traced.

Note that, in these different regions, A_2 and $(\omega/\omega_b)^2$ affected (ζ_1, ω_{n1}) and (ζ_2, ω_{n2}) in different ways. It may be expected that this also would be the case with the other design and operating parameters, inasmuch as the coefficients α_2, α_3, α_1, and α_0 contain all the parameters in different combinations, equation (16). For example, Fig. 3 shows that the computed solutions were stable for many combinations of A_2 and $(\omega/\omega_b)^2$ even though the damping coefficient ζ was assumed to be zero. When ζ was increased (solutions not shown here) it was found that the curves of constant A_2 and $(\omega/\omega_b)^2$ became "distorted" from those which appear in Fig. 3: The point X now moved up vertically, showing an increase in ζ_1 and ζ_2 without affecting ω_{n1} and ω_{n2}; but the curves for small A_2 or large $(\omega/\omega_b)^2$ were not very much affected, showing that, in these regions, ζ was not very effective in changing the damping of the reducer system. Thus it is not easy to classify the design and operating parameters according to their effect on the ζ's and ω_n's. To get a complete picture of the dynamic behavior of the reducer, one must therefore study the effects of all the parameters in all combinations over a wide range.

Comparison of Nonlinear and Linearized Solutions. Fig. 4 shows a representative series of nonlinear solutions obtained with P_1 varied from 4.0 to 1.05, and with the other conditions held constant at the values listed in the caption. These solutions also turned out to be unstable, and the figure shows the self-sustained oscillations in P, P_2, and L under steady-state or near steady-state conditions. The time scale was shifted, as in Fig. 2, so that the comparison of the solutions may be made more readily.

The oscillations in P, P_2, and L were the smallest in the case of $P_1 = 1.05$. Also, in this case, the lift curve shows that the metering valve barely touched the valve seat with each cycle of oscillation, so that the nonlinearity introduced by the valve hitting a stiff valve seat was almost absent. Since these conditions agreed well with the assumptions in the linearized problem, the linearized results for $P_1 = 1.05$ may be expected to agree with the nonlinear results.

Table 1 shows that this was indeed the case. This table lists first the damping ratio ζ_1 and the natural period, $2\pi/\omega_{n1}$, of the unstable component of the linearized solution. For the nonlinear solutions, Fig. 4, Table 1 lists two periods; the over-all period of the cyclic phenomena in P, P_2, and L, and the period of the metering valve during the "free" part of its travel. This latter period was taken as the time duration in which the valve was lifted from the valve seat. In this period, the valve motion (and P, P_2,

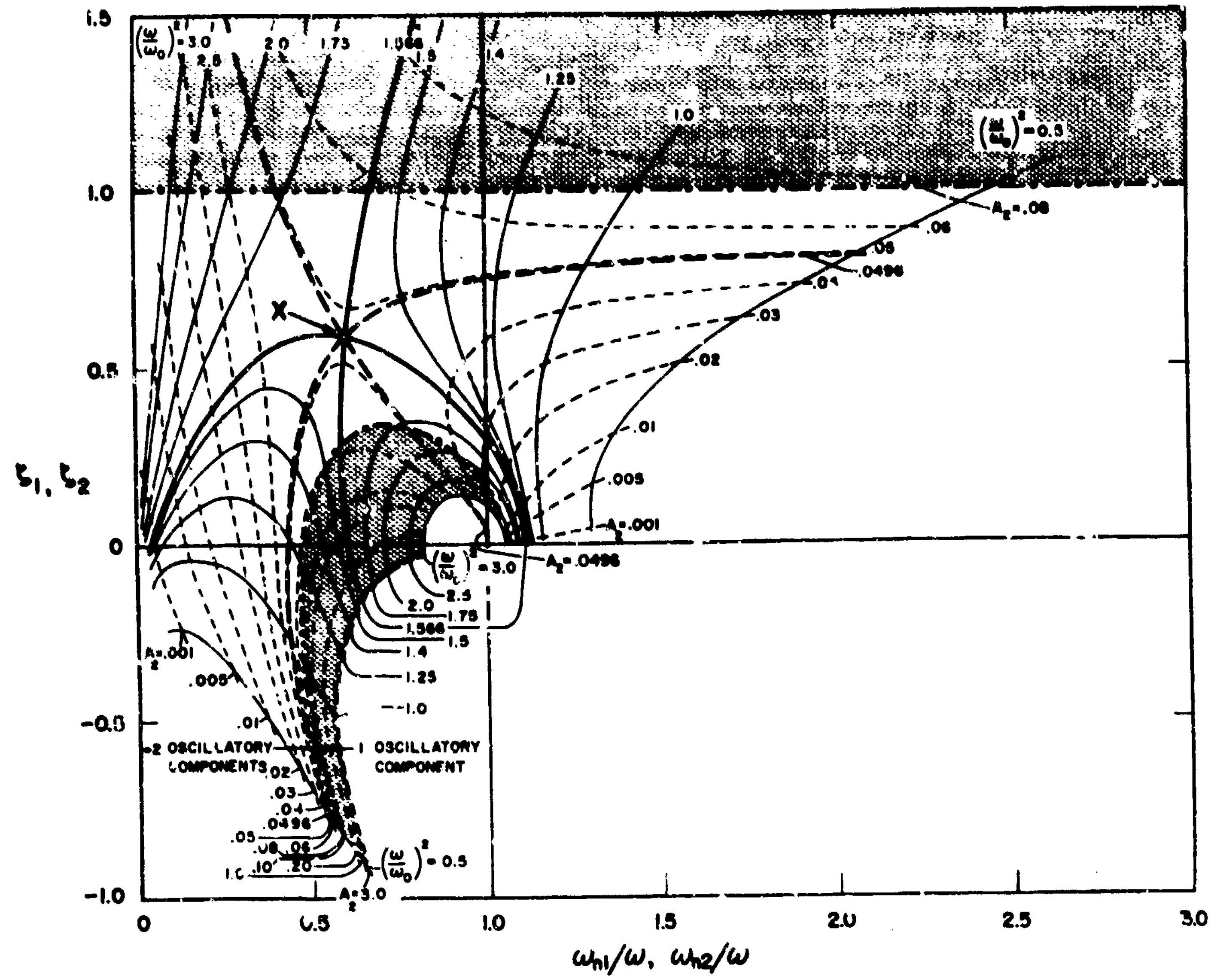


Fig. 3 Effect of A_2 and $(\omega/\omega_0)^2$ on damping ratio ζ_1, ζ_2 and frequency ratios ω_{n1}/ω, ω_{n2}/ω of the linearized solution

Conditions for Computation:

a_p = 3.0 sq in.	y_r = 1.0 in.	P_1 = 3.0
a_0 = 0.30 sq in.	A_1' = −4.82	P_3 = 0.5
ω_0 = 13200 in/sec	A_3 = 0.20	V = 120
c_d = 0.086 lbf-sec/in.	α = 1.0	ζ = 0
d_0 = 0.625 in.	C_1 = 0.80	η = 1904
k = 316 lbf/in.	C_2 = C_3 = 1.0	ξ = 190.4
m_t = 0.5 lbf	F_r = 1.07	ν = 9.3
p_0 = 100 psia	F_y' = 4.93	ω_0^2 = 2007
v = 360 in.^{3}	P = P_3 = 1.0	

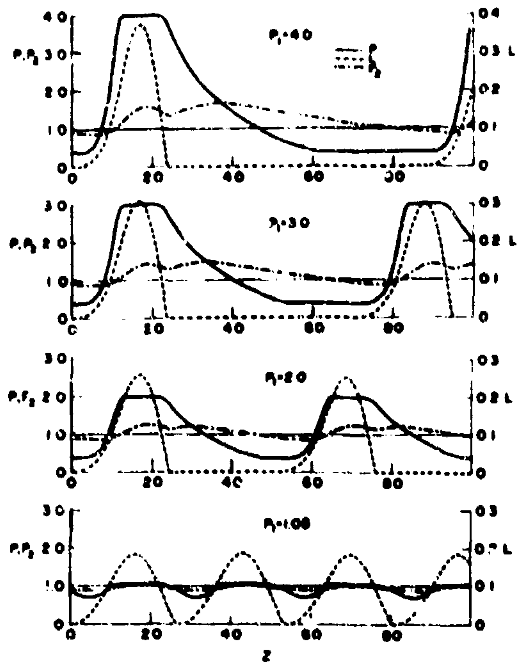


Fig. 4 Computed nonlinear solutions for P_1 = 4.0, 3.0, 2.0, and 1.06

Initial Conditions:

Same as in Fig. 2

Step Disturbance:

$$A_0 = 0 \text{ to } A_0 = 0.077$$

Constants:

Same as in Fig. 2 except:

a_0 = 0.30 sq in.	C_2 = 1.0	ν = 0.021
c_d = 0.086 lb_f-sec/in.	$\bar{\gamma}_b'$ = 9.90	ξ = 0.0020
k = 139 lb_f/in.	ζ = 0.022	ω^3 = 0.026
C_1 = 0.8		

Table 1 Comparison of nonlinear and linearized solutions

	Linearized solutions* (unstable component)		Nonlinear solutions, Fig. 4	
P_1	ζ_1	Period ($2\pi/\omega_{n1}$), Z/cycle	Over-all period, Z/cycle	Period of poppet valve during free travel, Z/cycle (see text)
4.0	−0.64	16.1	97	25
3.0	−0.60	17.5	72	25
2.0	−0.54	19.0	53	26
1.05	−0.31	25.9	27	27

* ν = 9.8 for this series of solutions; other conditions same as those shown in the caption of Fig. 4.

and so on) was governed by the complete set of equations (1) through (9), with the attendant nonlinear effects. In a very approximate way, this period may be considered as comparable to the natural period in the linearized calculation. When the solution was nearly linear (P_1 = 1.06), the over-all period was the same as the period of the metering valve in free oscillation, and Table 1 shows that both were very close to the natural period of the linearized solution. For this case, Fig. 4 shows that the nonlinear effect was to distort the wave form of the P, P_3, and L-curves which otherwise would have been sinusoidal in the linearized case.

With increasing P_1, the nonlinear solutions became more unstable in the sense that the amplitudes of P, P_3, and L became larger, and that the average velocity of the valve in its free travel was higher. This was in qualitative agreement with the linearized solutions in Table 1, which shows that ζ_1 became more negative with increasing P_1. The period of the free part of the valve motion decreased slightly and this also was in qualitative agreement with the linearized solutions, although the latter showed a much larger decrease in $2\pi/\omega_{n1}$ in the same range of P_1. However, this trend of variation was completely opposite to that of the over-all period of the nonlinear solution. Thus the linearized solution proved to be inadequate for predicting the correct natural frequency of the over-all reducer system, except in the case of small-amplitude oscillations.

In the remaining part of the over-all period, the valve was stopped at the valve seat, and equation (1) was not applicable. Therefore, the nonlinear effects were associated primarily with the flow equations, (2) through (9), the effect of valve rebound being ignored. This portion of the over-all period was therefore controlled primarily by the time required for P and P_3 to fall to the proper levels so that the valve could be lifted again by the spring force. This appears to be the principal mechanism responsible for the dependence of the over-all period on the amplitude of oscillation in unstable operation. The amplitude of P was limited by P_1 and P_3. P_3 was controlled primarily by P and also, to a much lesser extent, by the motion of the piston in V_3. The valve travel was limited on the one side by the valve seat, and on the other side by the dynamic behavior of the valve mechanism, and therefore by P_3 and P. Thus, in unstable operation, the oscillations in P, P_3, and L were amplitude limited. This, in turn, controlled the over-all period of the oscillation.

An additional point of interest is the effect of the size of a disturbance on the stability of the pressure reducer. In the linearized case, the disturbance was assumed to be small. Then, if the solution was stable, the small disturbance would be damped out. The question therefore is this: If the disturbance were not small, would a linearly stable solution remain stable? To answer this question, the transient response of the reducer to two different step disturbances was computed from the nonlinear equations. These solutions are shown in Fig. 5 (note the difference in the P and L-scales in the two solutions), and the applicable conditions, as well as the linearized results, are given in the caption. With ζ_1 and ζ_2 both greater than zero, the linearized solution should be stable. However, the nonlinear solutions show that the system was stable only to a small disturbance, but was unstable to a large disturbance.

In the case of the large disturbance, Fig. 5 also shows that the period of the P and L-curves in the first cycle of oscillation was about 80 Z/cycle. This was in fair agreement with the value of 77.0 Z/cycle for the period of the linearized solution. In subsequent cycles, the period became longer, due to the nonlinear effects noted earlier. Now, in the case of the small disturbance, the period of the P and L-curves in the first cycle of oscillation was only about 50 Z/cycle, and in subsequent cycles the period decreased to about 36 Z/cycle. Thus the agreement between the nonlinear and the linearized solutions was actually poorer when the changes in P and L were smaller. This phenomenon was due to the fact that the linearization (see Analysis) did not take into account the degenerate case in which P and P_3 remained almost constant. In this case, the reducer would be operating almost in a steady-state condition, so that only equation (1) need be considered. Then the natural period would be simply $2\pi/\omega$, or 38 Z/cycle, which is the same as the period of the last cycle of the nonlinear solution. Thus the natural period of the reducer could change from $2\pi/\omega$, when P and P_1 were nearly constant; to

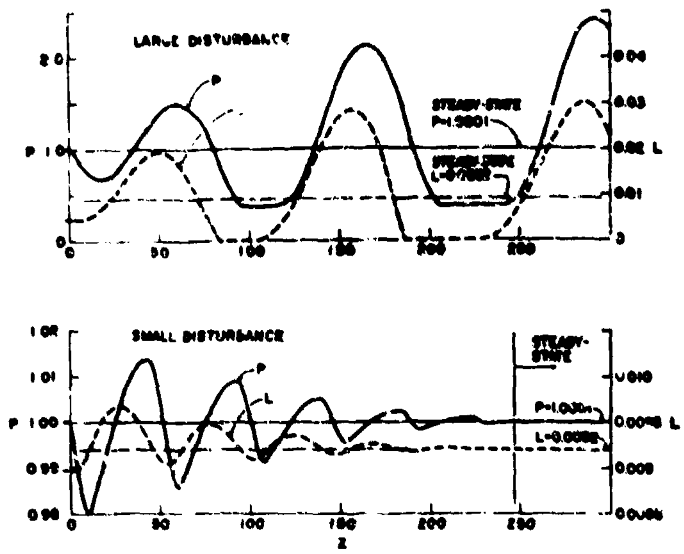


Fig. 5 Effect of size of disturbance on stability of reducer

Initial Conditions:

Same as in Fig. 2

Step Disturbance:

Large disturbance $A_6 = 0.0332$ to $A_6 = 0.0665$
Small disturbance $A_6 = 0.0648$ to $A_6 = 0.0665$

Constants:

Same as in Fig. 2 except

$c_d = 5$ sec/in. $C_1 = C_2 = C_3 = 0.8$ $\zeta = 15$
$k = 1$ in. $F_s' = 9.90$ $\omega^2 = 0.0305$
$A_6 = 0.01$ $P_1 = 3.0$

Linearized Sol. With $\nu = 9.3$:

$2\pi/\omega_{n1} = 77.9\ Z/\text{cycle},\ \zeta_1 = 0.039$
$2\pi/\omega_{n2} = \text{nonoscillatory},\ \zeta_2 = 2.81$

$2\pi/\omega_{n1}$ and $2\pi/\ldots$ when P and P_1 were in small oscillation; and finally to still some other natural period dependent on the amplitudes of P, P_1, L, and so on, when these were in large oscillation.

This point is further clarified by an examination of the damping characteristics of the nonlinear solution. The damping ratio as determined by the logarithmic decrement of the first two cycles of the P and L-curves (Fig. 5, small disturbance) was somewhat greater than the damping ratio $\zeta_1 = 0.039$ in the linearized case. In subsequent cycles, the damping ratio increased even further. Toward the last cycle of oscillation, the reducer behaved as the spring-mass system of equation (1), and the damping was therefore determined by ζ, which in this case was as much as 43 times greater than the critical damping for the spring-mass system. Thus the stability of the reducer was controlled by ζ when P and P_1 were nearly constant, by ζ_1 and ζ_2 when P and P_1 were in small oscillation, and by still other nonlinear effects when P, P_1, L, and so on, were in large oscillation. Incidentally, the large decrease in damping in Fig. 5, from $\zeta = 15$ to $\zeta_1 = 0.039$ and to negative damping, shows that in this instance the viscous damping on the mechanical parts was rather ineffective for stabilizing the reducer system. This, of course, is the same point noted earlier in the discussion of the linearized solution.

Summary

The agreement between the experimental and the analytical results showed that the assumptions used in the analysis were reasonable, and that the governing equations were essentially correct in describing the dynamic behavior of a simple pressure reducer. It was found that the governing equations were rather highly nonlinear, and the effect of the nonlinearity was to distort the wave form of the oscillations of the dependent variables (P, P_1, L, and so on), and to make the natural frequency and the damping associated with the oscillations amplitude-dependent. A set of stability criteria was obtained for the linearized governing equations, under the assumption of small-amplitude oscillations. With these criteria, it was possible to evaluate the qualitative effects of the various design and operating parameters on the stability of the reducer system. The quantitative problem of stability was complicated by the dependence of damping on the amplitude of oscillation. For example, it was found that the stability of the reducer was affected by the size of the disturbance. Further study of the nonlinear properties of the governing equations is required to clarify this and other related problems.

Acknowledgments

The authors wish to thank Mr. Alfred E. Beam and Dr. Walter Gautschi for their help in the numerical analysis and in programming the problem for digital computation. They also wish to thank Mrs. Ruth R. Capuano for her help in preparing the input data for the computer.

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APPENDIX 1

Derivation of Governing Equations

The fluid medium is assumed to be a perfect gas with constant specific heats, so that

$$pv = m\mathcal{R}T, \tag{17}$$

$$\gamma = c_p/c_v = \text{const } (= 1.4 \text{ for this problem}). \tag{18}$$

The symbols in these equations and in those to follow below are explained in the Nomenclature.

The gasdynamic process may be assumed to be adiabatic and quasi-steady (no pressure waves). Then the mass rate of flow through a_1 is

$$\dot{m}_1 = C_1 a_1 p_1 (g\gamma/\mathcal{R})^{1/2} \varphi(R_1)/(T_1)^{1/2}, \tag{19}$$

where

$$C_1 = \text{discharge coefficient of } a_1,$$

$$\varphi(R_1) = [2(R_1^{2/\gamma} - R_1^{(\gamma+1)/\gamma})/(\gamma - 1)]^{1/2}, \tag{20}$$

$$R_1 = p/p_1. \tag{21}$$

At $R_1 = 0.528$, $\varphi(R_1)$ has a maximum value of 0.579. At this point sonic velocity is reached in a_1, and further decrease of R_1 does not make $\dot{m}_1$ larger. Hence $\varphi(R_1)$ remains at 0.579 for $R_1 < 0.528$.[8]

Similar equations may be written for $\dot{m}_{2o}$ or $\dot{m}_{2i}$, and for $\dot{m}_3$, for flows through a_2 and a_3, respectively, with subscript $2o$ referring to flow from v to v_2, and $2i$ referring to flow in the opposite direction.

Denoting the gas mass in v by m, and using subscript o to indicate the reference condition of steady-state operation for the pressure reducer, one may write,

$$m = pv/\mathcal{R}T, \quad m_o = p_o v/\mathcal{R}T_o, \quad a^2 = \gamma g\mathcal{R}T, \tag{22a, b, c}$$

$$dm/dm_o = (a_o/a)^2[(dp/p_o) - 2(p/p_o)(da/a)] - (1/m_o)(\dot{m}_1 + \dot{m}_2 + \dot{m}_3)dt \tag{23}$$

By substitution of equations (22) into (19), one has

$$\dot{m}_1 dt/m_o = (C_1 a_1/a_o)(a_o/a_1)(p_1/p_o)\varphi(R_1)dZ \tag{24}$$

where a_o is a reference area, and $dZ = a_o a_o dt/v$. Similar expressions may be written for $\dot{m}_{2o}dt/m_o$, $\dot{m}_{2i}dt/m_o$, and $\dot{m}_3 dt/m_o$. When these are substituted into equation (23), with the simplified dimensionless notations $a_1/a_o = A_1$, $a_1/a_o = \mathcal{C}_1$, $p_1/p_o = P_1$, $\varphi(R_1) = \varphi_1$, and so on, one obtains the following continuity equation for flow from v to v_2 through A_2:

$$DP - 2PD\mathcal{C}/\mathcal{C} = \mathcal{C}^2[(C_1A_1P_1\varphi_1/\mathcal{C}_1) - (C_2A_2P\varphi_{2o}/\mathcal{C}) - (C_3A_3P\varphi_3/\mathcal{C})], \tag{25}$$

here $D = d/dZ$.

For flow in the opposite direction (v_2 to v), the continuity equation is similar except that the term $-C_2A_2P\varphi_{2o}/\mathcal{C}$ is replaced by $+C_2A_2P_2\varphi_{2i}/\mathcal{C}_2$. The discharge coefficient C_2 is assumed to remain the same for flow in either direction.

The energy equation for the gas in v_2 is obtained by considering the change in the internal energy in v_2 due to the work done on the piston and the energy brought into v_2 by the flow process through a_2. The heat energy transferred to v_2 is assumed to be zero. For flow into v_2 (from v), this energy equation is

$$(c_v/\mathcal{R})d(p_2v_2)/dt = c_pT\dot{m}_{2o} - (1/J)p_2dv_2/dt. \tag{26}$$

In dimensionless terms, with $v_2/v_{2o} = V_2$ and $v/v_{2o} = V$, where v_{2o} is the reference volume of v_2 (the valve A_1 is in the closed position), and also with V_2 expressed in terms of the dimensionless piston position Y, the energy equation for the gas in V_2 becomes

$$(Y + Y_f)DP_2 + \gamma P_2DY = \gamma VC_2A_2\mathcal{C}P\varphi_{2o} \tag{27}$$

for flow from V into V_2. The corresponding energy equation for flow in the reverse direction is similar, except the right-hand side is replaced by $-\gamma VC_2A_2\mathcal{C}_2P_2\varphi_{2i}$.

The energy equation for the gas in V may be written in a similar fashion by equating the change in the internal energy in V to the energies brought into V and/or removed from V by the flow processes through A_1, A_2, and A_3. Actually the boundary of V is somewhat indefinite in the neighborhood of the metering valve because it is not stationary. In fact, there must be some work done on V by the motion of the metering valve. But these effects are assumed to be negligible. Also, the effect of heat transfer is neglected. Then, omitting the intermediate steps and writing the energy equation directly in dimensionless terms, one has,

$$DP = \gamma[C_1A_1\mathcal{C}_1P_1\varphi_1 - C_2A_2\mathcal{C}P\varphi_{2o} - C_3A_3\mathcal{C}P\varphi_3] \tag{28}$$

for flow from V to V_2. Again, the corresponding energy equation for flow in the opposite direction is similar, except that the term $-C_2A_2\mathcal{C}P\varphi_{2o}$ is replaced by $+C_2A_2\mathcal{C}_2P_2\varphi_{2i}$.

An equation is also needed for determining an additional property of the gas in V_2. The equation for $\mathcal{C}_2$ for flow from V to V_2 is obtained by combining equations (27), (17), and (22c):

$$2(Y + Y_f)D\mathcal{C}_2/\mathcal{C}_2 = (VC_2A_2\mathcal{C}P\varphi_{2o}/P_2)[\gamma - (\mathcal{C}_2/\mathcal{C})^2] - (\gamma - 1)DY. \tag{29}$$

The equation for $\mathcal{C}_2$ for flow from V_2 to V is obtained by assuming the process in V_2 to be isentropic:

$$P_2 = \mathcal{C}_2^{2\gamma/(\gamma-1)}. \tag{30}$$

The dynamical equation is obtained by equating the inertia force to the sum of the damping, spring, pressure, and flow forces acting on the metering-valve assembly. Here the pressure force refers to the force on the piston due to pressure p_2, and the flow force refers to the force on the metering valve due to the presence of flow over the surface of the valve. The flow force is discussed in greater detail in Appendix 2. Generally speaking, this force depends on the geometry of the valve and of the flow passage, the pressure (or density) and also the pressure difference across the valve. Because of the complexity of the flow pattern, it is generally very difficult to calculate theoretically the flow force involved. However, in Appendix 2, it is shown that for a valve

[8] For a more detailed discussion of equations (19) to (21) and of this method of defining the discharge coefficient C_1, see D. H. Tsai and M. M. Slawsky, "Determination and Correlation of Flow Capacities of Pneumatic Components," NBS Circular 588, Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., October 15, 1960.

of given geometry the dimensionless flow force f/a_0p_0 is given by the expression

$$f/a_0p_0 = F_rF_y(P_1 - P), \quad (31)$$

and F_r and F_y are experimentally obtained functions as discussed in Appendix 2. In terms of dimensionless Y and Z, the dynamical equation then becomes

$$D^2Y + \zeta DY + \omega^2 Y - \eta P_1 - \xi F_rF_y(P_1 - P) = 0. \quad (32)$$

The coefficients ζ, ω^2, η, ξ are defined in the Nomenclature.

To summarize, equations (26), (27), (28), (29), and (32) describe the dynamic behavior of the pressure reducer when the flow through a_1 is from v to v_1. A similar set of five equations applies when the flow through a_1 is from v_1 to v. These equations contain five unknowns: P, $\mathfrak{A}$, P_1, $\mathfrak{A}_1$, and Y. With a given set of initial conditions, these equations, therefore, may be solved simultaneously, and the solutions may be obtained as functions of Z. The method of solution is discussed in the text.

APPENDIX 2

Determination of Flow Forces and Discharge Coefficients for Various Poppet Valves

Experimental Setup. The metering-valve assembly, shown schematically in Fig. 1, was modified for the flow force and flow-rate measurements. The orifice a_2 was sealed, and the spring and the piston were removed. Sealing gaskets were installed to prevent leakage between the valve rod and the casing, and the rod was rotated (by means of an electric motor) to reduce friction in the axial direction of the rod. The flow force on the poppet valve was measured by means of a hydraulic scale [9] attached to the valve rod. The accuracy of the measurement was about 0.03 lb. The valve lift was measured by means of a precision dial gage which read to 0.001 in. The valve-opening area a_1 was computed from the lift and the geometry of the setup. The pressures p_1 and p were measured by means of calibrated bourdon gages with an accuracy of 1 psi. The mass rate of air flow $\dot{m}_1$ through the modified reducer was controlled by the valve a_3 and measured by means of a nozzle-type flowmeter built to the specifications given in references [10, 11]. According to reference [11], measurements made with this flowmeter were accurate to somewhat better than 1 per cent.

Experimental Procedure. The flow force and flow-rate measurements were made first on the 45-deg poppet valve. The valve was first installed on the upstream side of the valve seat. The flow was then in the direction I as shown in Fig. 7. The upstream pressure was held constant at p_1 = 115 psia and measurements were made at pressure ratios ($R_1 = p/p_1$) ranging from 0.390 to 0.913 and at valve lifts ranging from 0 to 0.080 in. This procedure was repeated with the valve installed on the downstream side of the seat. In this case the flow was in direction II as shown in Fig. 7. The same measurements were then repeated for the case of the ball valve and the flapper valve, at p_1 = 55, 115, and 165 psia.

Results. The flow force considered here was the net pressure forces integrated over the surface of the valve in the axial direction. The measured flow force f on the 45-deg valve is plotted against the lift l in Fig. 6 for various downstream pressures. These curves show that for a given l, f decreased with increasing downstream pressure and that for a given R_1 (or downstream pressure) f decreased with increasing l. These results were as expected because with increasing downstream pressure, the difference between the upstream and downstream pressures decreased, and, with increasing l, less of the downstream area of the valve was acted on by the downstream pressure. In fact, if l were very large, the entire valve would be under pressure p_1, and f would be zero. As l decreased, the valve would move into the stream and f would increase. At zero lift, the flow force f_0 would be equal to $\bar{a}(p_1 - p)$ where $\bar{a}$ is the seating area. This area, in general, would not be equal to a_0, Fig. 1. With the present setup, f_0 could not be measured easily. This force was therefore obtained by extrapolating the flow-force curves to zero lift.

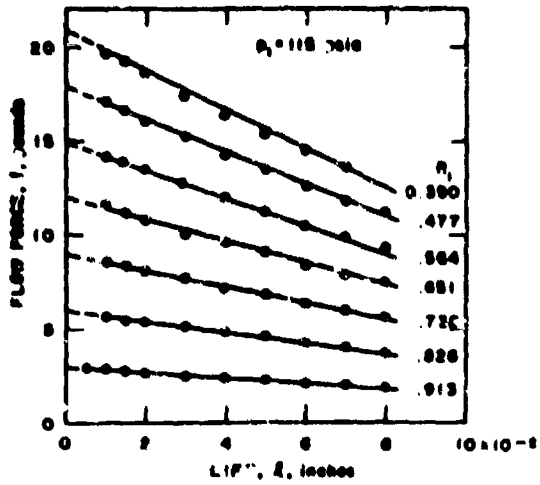


Fig. 6 Measured flow forces on a 45-deg poppet valve at various lifts and pressure ratios. Valve installed as in Fig. 1.

It was found that the data in Fig. 6 could be correlated by dividing the ordinates of curves of constant R_1 by their respective values of f_0. This correlation reduced the flow-force curves to the solid F_y versus A_1-curve (flow in direction I) in Fig. 7(a). Here the ordinate is the dimensionless flow force $F_y = f/f_0$; the abscissa is the dimensionless valve opening $A_1 = a_1/a_0$. This curve indicates that the change in the pressure distribution over the surface of the valve due to a change in the lift was similar at different pressure ratios across the valve.

The area $\bar{a}$ in the expression $f_0 = \bar{a}(p_1 - p)$ may be expected to vary with the seating condition of the valve and with the pressure distribution over the valve seat and hence with R_1. Since it is generally difficult to determine the conditions at the seat, the variation of $\bar{a}$ is best obtained experimentally. The solid curve of $F_r = f_0/a_0(p_1 - p) = \bar{a}/a_0$ against R_1 in Fig. 7(a) shows that F_r = 1.07 and is nearly constant over the range of R_1 tested.

The flow force may now be expressed as

$$f = a_0F_rF_y(p_1 - p).$$

From a dimensional consideration, the flow pattern and the pressure distribution over a given valve at a given lift should be independent of the pressure level if R_1 is held constant. Therefore, the foregoing expression may be extended to apply to other cases with different p_1 and/or p by making f dimensionless. The reference force is most conveniently taken as a_0p_0. Thus

$$f/a_0p_0 = F_rF_y(p_1 - p)/p_0.$$

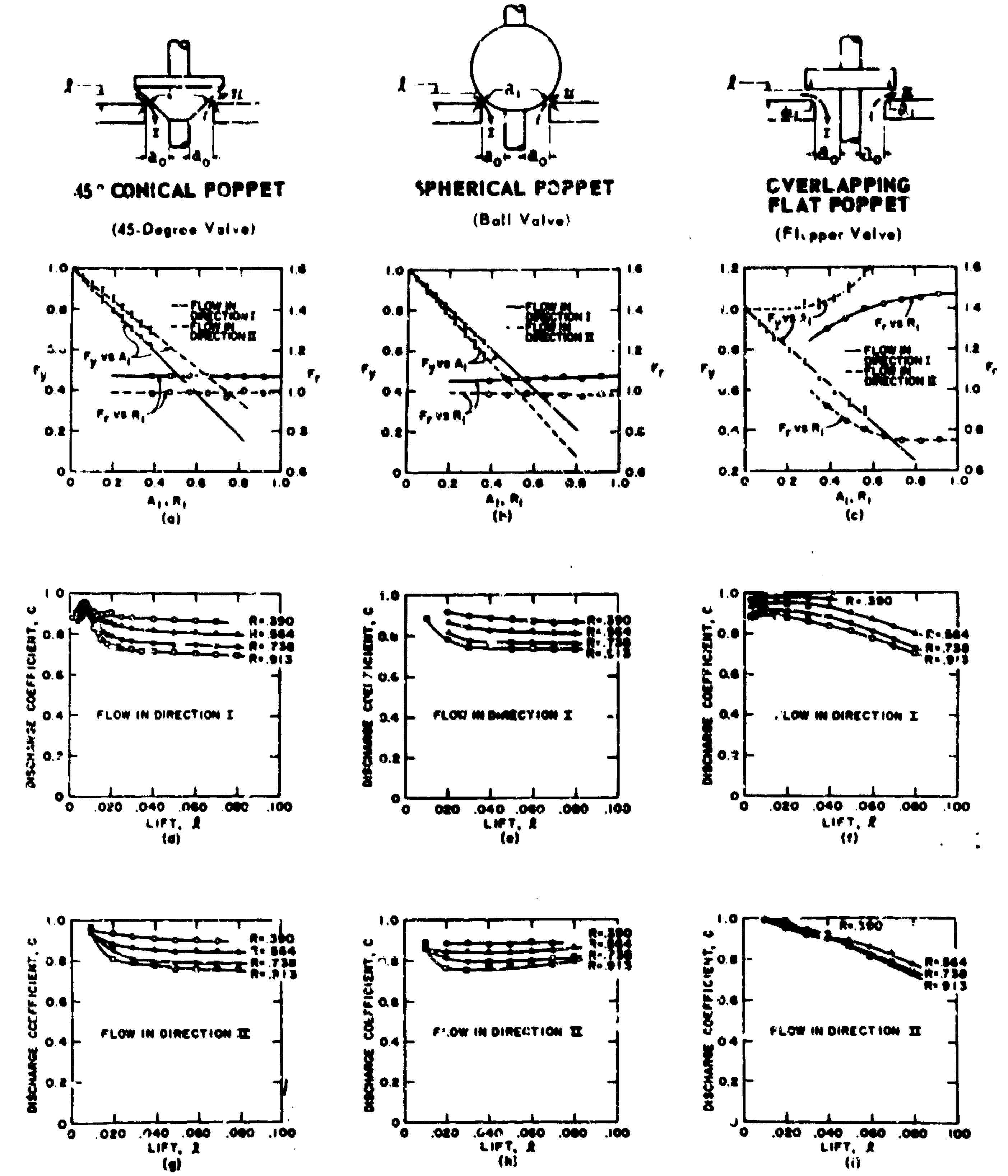


Fig. 7 F_y, F_r, and discharge coefficients for three typical valves

(Height of symbol "I" in F_y curves indicates spread of experimental data.)

This expression indeed was found to apply to the other valves tested at p_1 = 65, 115, and 165 psia, Figs. 7 (a, b, and c)).

The discharge coefficient [equation (10), Appendix 1] of the poppet valve was computed from the measured flow-rate data. Figs. 7(d through i) show the effect of t, R_1, and the flow direction on the discharge coefficient for the three valves at the top of the figure. These results were obtained at p_1 = 115 psia. The results obtained at p_1 = 65 and 165 psia were nearly identical to those obtained at 115 psia.

7.4.9 Dynamic Analysis of Pneumatic Dashpot for a Regulator Control Element

7.4.9.1 INTRODUCTION. A fluid component mounted in a missile or space vehicle will experience vibration of the mounting structure during part or all of the operating period. Vibration will occur over a range of frequencies and g-levels. If an element in the component is resonant with any of the vibration frequencies, the dynamic performance of the component may be seriously impaired, or the unit may fail structurally.

These effects can be prevented by either modifying the design of the resonant element or by other methods. The methods available include changing the resonant frequency of the vibrating part, damping or balancing the part, or the use of vibration isolators.

Reference 35-1 describes the design and development of a helium gas pressure regulator which was intended for use in a missile booster or space propulsion module. The purpose of the unit was to maintain the pressure in a propellant tank at the required level. The specifications for the regulator called for satisfactory operation of the unit in the following environmental conditions:

Vibration: 5 to 2000 cps, at 25 g
Sustained Acceleration: up to 15 g
Ambient Temperature: 300 to +165°F

When the design of the regulator was initiated, it was seen that the sensor element would have a resonant frequency in the above range of vibration frequencies. It was decided to solve this problem by using a pneumatic dashpot to critically damp the sensor. The sensor and the dashpot are part of the regulator control element or "controller." A dynamic analysis of the controller dashpot was carried out to determine the optimum design parameters of the unit. This analysis is given in Reference 35-1 and will be repeated in Detailed Topic 7.4.9.2. As an introduction to this section, the reader should refer to Sub-Topic 5.4.5 for a description of the regulator and its controller.

Figure 5.4.5 is a schematic diagram of the regulator. The pressure at the outlet port and at the sensing port is the tank pressure, which is the pressure being regulated. The regulator is composed of three sections, the bleed regulator, the actuator, and the controller. The operation of the complete regulator is relatively complicated and will not be given here; it is explained in Sub-Topic 5.4.5.

The controller shown in Figure 5.4.5 consists of the following: a sensor (68) which is a bellows; a chamber at the tank pressure and connected to the tank pressure sensing port shown in Figure 5.4.5; a diaphragm (61); a connecting link (64) between the sensor and the diaphragm; a pilot valve (62); a chamber at the controller damping pressure; a small passage around the connecting link (64) between the tank pressure chamber and the damping pressure chamber.

The sensor (68), connecting link (64), and diaphragm (61) constitute a spring-mass system with a resonant frequency. The volume of gas in the chamber at the controller damping pressure, the diaphragm (61), and the passage around the connecting link (64), constitute a pneumatic dashpot which dampens the motion of the spring-mass system. Movement of the diaphragm results in gas flow between the chamber at tank pressure and the controller damping pressure via the passage around the connecting link (64), which is small and acts as a restriction, thus producing the desired damping effect. The following Detailed Topic, which is taken from Reference 35-1, pages 93-103, is a dynamic analysis of this pneumatic dashpot.

NOMENCLATURE

Symbol	Term	Units
A_d	Effective area of diaphragm	in²
A_p	Annular area of passage around connecting pin (64) (Figure 5.4.5)	in²
C	Flow coefficient of area, A_p	---
$C_1 =$	$\dfrac{\theta_o}{P_o^{\frac{\gamma-1}{\gamma}}}$	---
K	Spring rate	lb/in.
M	Effective mass of moving parts	$\frac{\text{lb/sec.}^2}{\text{in.}}$
P_o	Tank pressure, the pressure being regulated	lb/in²
P_1	Controller pressure damping (Figure 5.4.5)	lb/in²
R	Gas constant	in/°R
V_o	Initial volume of chamber at controller damping pressure (Figure 5.4.5)	in³
V_1	Volume of chamber at controller damping pressure (Figure 5.4.5)	in³
W	Weight of gas in volume V_1	lb
X	Position of diaphragm	in.
X_o	Initial position of diaphragm	in.
γ	Ratio of specific heats	—
θ_o	Initial temperature in chamber at controller damping pressure (Figure 5.4.5)	°R
θ_1	Temperature in chamber at controller damping pressure (Figure 5.4.5)	°R

7.4.9.2 ANALYSIS. It was obvious from past experience that an accurate low-friction sensor operating a conventional pilot valve would exhibit an excessive error in regulated pressure under the specified environmental vibration levels. An important design objective was to devise some means of reducing this error. Methods considered were:

a) Damping

b) Balancing (acceleration compensation)

c) Vibration isolators

d) Use of a sensor that did not resonate in the specified frequency range.

Method 3 had been tried in the past. Though reasonably effective, it was not selected due to bulkiness, and the nonavailability of an isolator that would function over the −300 to +165°F temperature range, with 15 g sustained acceleration in any direction.

Method 4 would have precluded the use of a simple bellows-type sensor. To obtain a resonant frequency above 2000 cps, the sensor spring rate would have to be excessively high. Such a sensor would not have a large enough deflection per psi to be usable without the complexities of additional amplification by mechanical linkages, or pneumatic circuitry. Therefore, this method was not selected.

In theoretically evaluating the effect of Method 1 and Method 2, the structural dynamics of the sensor are of decisive importance. The sensors to be used were bellows. Bellows behave under vibration in a manner analogous to helical springs. There are two elementary modes of resonance: (1) one end fixed and one end free, with the free end resonating with respect to the fixed end; and (2) both ends fixed with the center of the spring resonating with respect to the ends. Because the lower end of the bellows was not attached to, nor normally in contact with, any positive stop, it was assumed that the bellows would resonate in mode 1. It was also assumed that the bellows was mounted in a rigid frame, so that the vibration input to the regulator was always equal to the vibration input to the bellows.

If these assumptions are fulfilled, it is obvious that either damping or perfect static balancing would eliminate high amplitude oscillation of the sensor due to vibration.

An analog computer study of another Rocketdyne pressure regulator (using the same assumptions as above) had predicted that if the sensor (68) of Figure 5.4.5, were critically damped, the error in regulated pressure due to vibration would be greatly reduced. The results of this study are in Figure 7.4.9.2a.

Based on this result, it was desired to design a device that would critically damp the sensor. A pneumatic dashpot was selected for trial because it appeared that it could more readily be made to operate over the −300°F to +165°F temperature range than other damping devices.

A pneumatic dashpot does not give true viscous damping. Because of the compressibility of the gas in the damping chamber, the dashpot can act more like a spring than a damper if too small a damping restriction is used. In addition, the low viscosity of helium (the gas being used) causes the damping restriction to behave as an orifice and have a pressure drop proportional to the square of the flow rate, introducing a non-linear damping characteristic.

The non-linear differential equations of the dashpot and spring-mass system are shown in Table 7.4.9.2. A digital computer was used to obtain numerical solutions to the equations for a variety of operating conditions and choices of design parameters.

Figure 7.4.9.2b shows the time response of the spring-mass dashpot system. The system is deflected 0.015 inch upward from its equilibrium position, and released at time t = 0. As originally designed, the response shown in curve A was obtained, a highly underdamped oscillation superimposed on a slowly decaying exponential. The oscillation was apparently due to the pneumatic spring action of the dashpot internal volume. When the damping restriction area was increased by a factor of four, the oscillations were greatly reduced, as shown by curve B. On the other hand, the slowly decaying exponential component of the response could be speeded up by reducing the internal volume by a factor of three, as shown in curve C. When both modifications were incorporated simultaneously, a nearly optimum, critically damped response was obtained, as shown in curve D.

The schematic diagram at the head of Table 7.4.9.2 shows that the nominal pressure in the dashpot is the regulated pressure. This may vary from 10 to 75 psia, depending on the set point selected and the altitude. Figure 7.4.9.2c shows that if the dashpot is designed for optimum response at 75 psia, the response will at 10 psia become more rapid and underdamped.

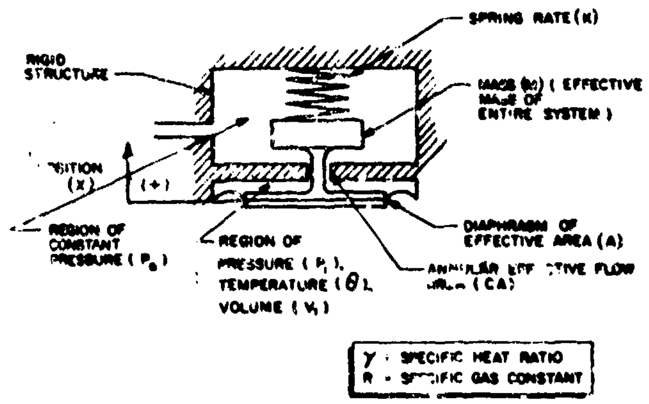
The temperature of the gas also affects response in nearly the same way as does pressure. The effects of both are taken into account by considering gas density in Figures 7.4.9.2b and 7.4.9.2c; gas temperature is 70°F. Figure 7.4.9.2e shows the response at maximum gas density (75 psia and 160°R), and at minimum gas density (10 psia and 960°R). Considering that the density variation is 45 to 1, the response varies remarkably little. It was found that to obtain best average response over this density range, the damping restriction area had to be decreased by a factor of two from the optimum value of Figure 7.4.9.2b. Thus at the highest density, response is somewhat overdamped, and at the lowest density it is slightly underdamped.

It was felt that the response of Figure 7.4.9.2d was satisfactory, and the values of design parameters chosen for this figure were selected for the final design.

It was learned during development of the regulator that the sensor was resonating in a more complex manner than that which has been assumed in this analysis. Based on the observations discussed in the development section, it is suggested that in future analyses, the equivalent spring-mass system for the sensor be taken as a two-degree of freedom system with limit stops for the lower mass, as shown in Figure 7.4.9.2e. A damping term should be included for the "upper mass" based on the structural damping coefficient for the bellows material.

It is clear that neither static bu ing nor critical damping of the bellows (for one-end-fixed and one-end-free resonance) can prevent resonance of the above system.

Table 7.4.9.2. Equations for Dynamics of Pneumatic Dashpot



EQUATIONS OF SYSTEM*

$$X = \frac{1}{M}\iint\left[A(P_0 - P_1) - KX\right] dt\, dt$$ Newtons 2nd law

$$W = \int \frac{C_d A}{\sqrt{R}} Y$$ Compressible flow through orifice

where

$$Y = \begin{cases} \dfrac{P_0}{\sqrt{\theta_0}} \sqrt{\dfrac{2g\gamma}{\gamma - 1}\left[\left(\dfrac{P_1}{P_0}\right)^{2/\gamma} - \left(\dfrac{P_1}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right]} & \text{if } P_1 \leq P_0 \\ -\dfrac{P_0}{\sqrt{\theta_1}} \sqrt{\dfrac{2g\gamma}{\gamma - 1}\left[\left(\dfrac{P_0}{P_1}\right)^{2/\gamma} - \left(\dfrac{P_0}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]} & \text{if } P_0 < P_1 \end{cases}$$

$$V_1 = V_0 - AX$$ From geometrical considerations

$$\left.\begin{aligned} P_1 &= \left(\frac{WRC_2}{V_1}\right)^{\gamma} \\ \theta_1 &= C_2 P_1^{\frac{\gamma - 1}{\gamma}} \end{aligned}\right\} \text{ where } C_2 = \frac{\theta_0}{P_0^{\frac{\gamma-1}{\gamma}}}$$ Perfect gas law plus adiabatic process

*All equations: in-lb-sec units

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7.2 Control System Theory, 1-128, 1-129, 1-130, 1-272, 1-273, 1-274, 1-275, 1-276, 1-277, 1-278, 127-2, 127-40, 127-41, 259-1, 259-2, 424-1, 425-1, 426-1, 427-1, 428-1, 429-1, 429-2, 430-1, 431-1, 432-1, 433-1, 434-1. 435-1, 436-1, 437-1, 439-1, 440-1.

7.3 Vibration and Shock Analysis, 1-268, 332-1, 388-1, 388-2, 388-3, 421-1.

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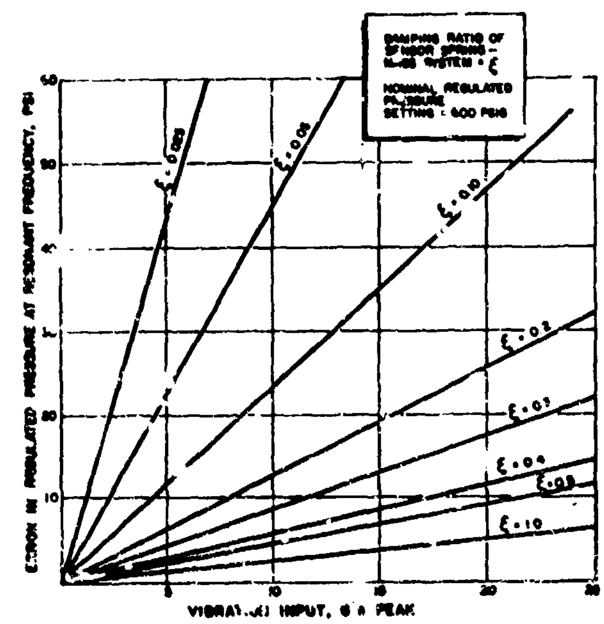

Figure 7.4.9.2a. Error in Regulated Pressure Versus Vibration Input

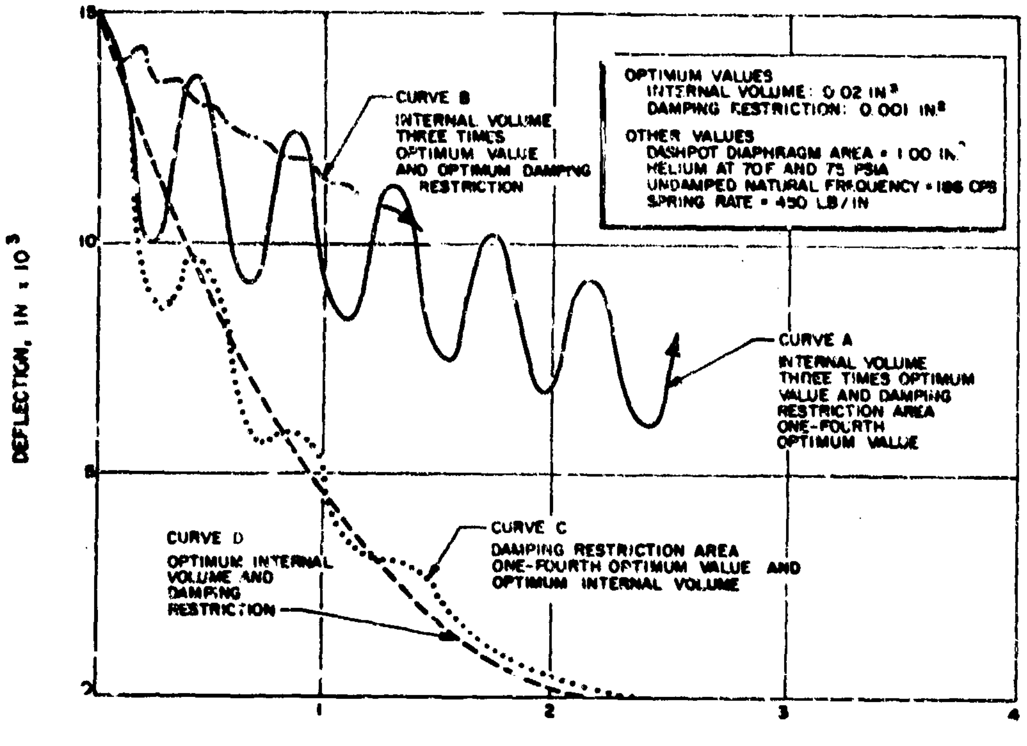

Figure 7.4.9.2b. Response of Spring and Mass with Pneumatic Dashpot — Effect of Changes in Design Parameters

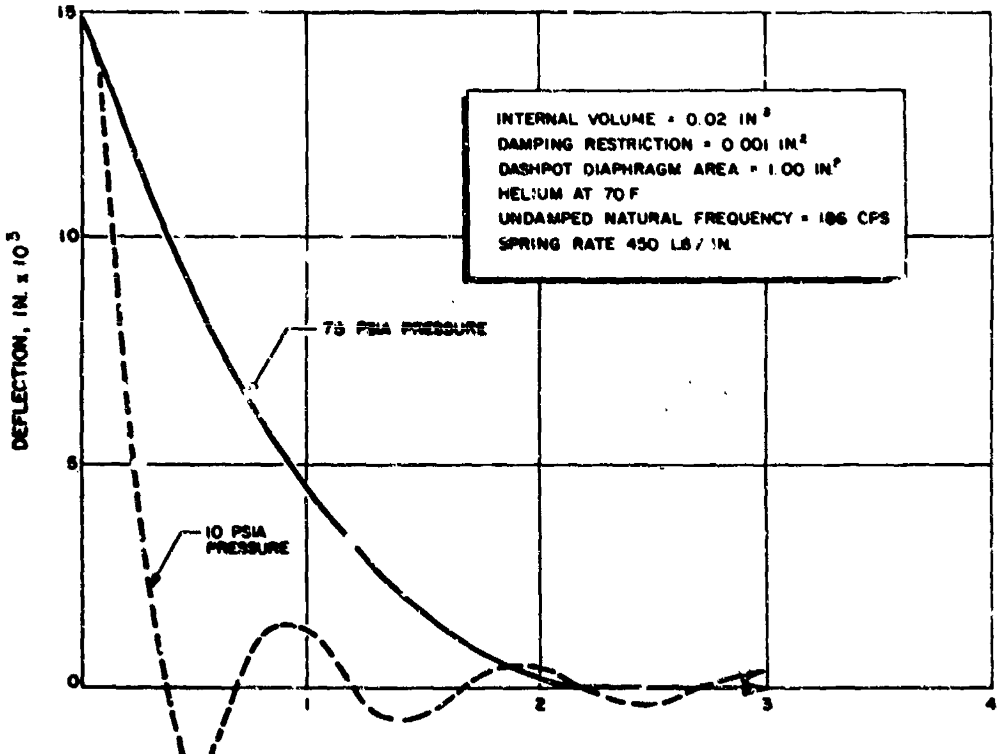

Figure 7.4.9.2c. Response of Spring and Mass with Pneumatic Dashpot — Effect of Changes in Design Parameters

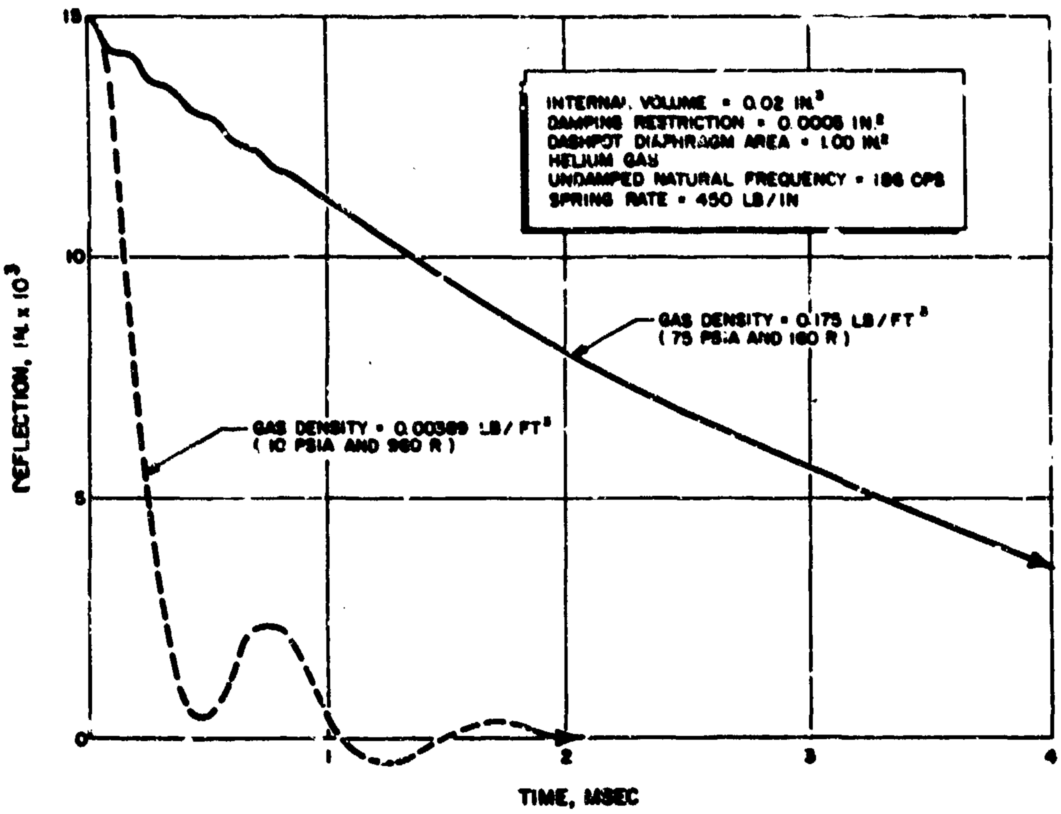

Figure 7.4.8.2d. Response of Spring and Mass with Pneumatic Dashpot — Effect of Changes in Gas Density

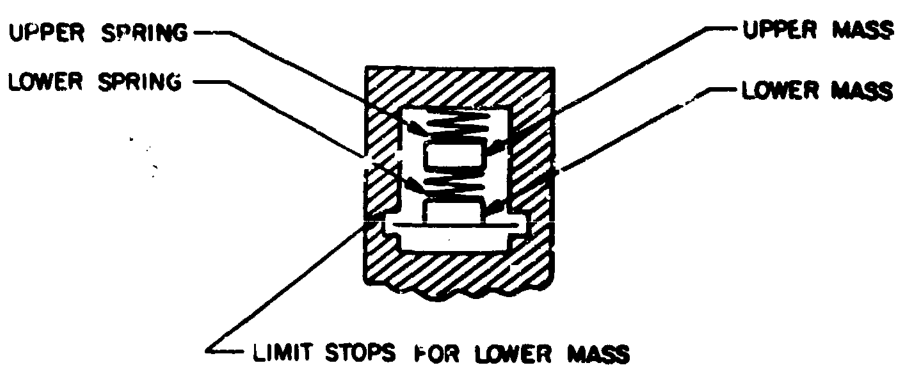

Figure 7.4.9.2e. Two-Degree of Freedom Mass System with Limit Stops for the Lower Mass

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ISSUED: NOVEMBER 1966
SUPERSEDES: MAY 1964

8.1 INTRODUCTION

A variety of computing devices have been developed over the years to aid in the solution of complex engineering problems. Of these, the electronic differential analyzer and the stored program, general purpose, digital computer — generally referred to as analog and digital computers, respectively — have emerged as the most powerful and widely accepted. The purpose of this section is to explain the basic characteristics of analog and digital computers and to indicate the techniques involved in handling various types of engineering problems.

Analog and digital computers differ in almost all respects. The analog computer provides a means of simulating the mathematical model of a system by interconnecting electronic components which are capable of performing basic mathematical operations in accordance with the equations that describe the system of interest. When the analog computer is excited by the application of appropriate initial conditions and forcing functions, all portions of the computer *simultaneously* and *continuously* react in a manner analogous to the system being modeled on it. (Thus the term *analog* computer.) Means are provided for recording the variables of interest, and the user is provided with an immediate display of the activity within the system as it reacts to the forcing functions. The digital computer is a device capable of performing arithmetic and elementary decision operations at high rates of speed. When it is given a sequence of instructions, it can solve a problem in a manner similar to that used in solving a problem with a desk calculator. The instructions and data are stored in a memory unit and executed at rates of several thousand per second. The results of these operations are usually displayed as listings of numerical values.

Several interacting factors to be considered when comparing the suitability of analog and digital computers for application to engineering problems are:

a) *Versatility.* The digital computer is capable of solving a wider range of problems than the analog computer, including any problem that can be solved on an analog computer. Problems that can be reduced to a sequence of arithmetic operations and a combination of simple yes-no decisions can be solved on a digital computer, while the analog computer is limited to the solution of problems associated with differential equations, and is most often used in the design of dynamic systems.

b) *Accuracy.* The accuracy of a digital computer is determined chiefly by the number of significant figures that its memory is designed to handle. This varies widely, depending upon the model of the computer, but is usually between six and twelve decimal places. An analog computer, however, is limited to four place accuracy (0.01 percent), and complex problems often yield results accurate to only two or three places. Although this appears to make the analog computer relatively useless, it should be remembered that the accuracy of the data involved in many engineering problems, particularly those involving preliminary design of dynamic systems, is often limited to two or three places, thus the solution of any equations involving such data is only accurate to the same number of places.

c) *Speed.* The speed at which a digital computer solution is found is determined by the rate at which the computer can perform arithmetic operations, and by the complexity of the problem. In the analog computer, all computing elements operate simultaneously, therefore the speed of the solution is independent of the complexity of the problem. This characteristic makes it possible to program the analog computer so that the time constants and frequencies of the computer solution are equal to those of the physical system which the computer is simulating.

d) *Economy.* Digital computers vary in operating costs, from a low of approximately \$10 per hour to as high as \$600 per hour. This cost range represents a considerable variation in computer speed and size. Digital computer speeds vary from several hundred to several hundred thousand arithmetic operations per second, and memory capacities vary from approximately 2,000 to over 100,000 words. (A word represents one data value or instruction.) The cost of analog computers varies from approximately \$5 to \$60 per hour, depending upon size. The size of an analog computer is measured by the number of its independent computing elements, and varies from 50 to over 500 such elements.

To summarize, analog computers are low cost, high speed, low accuracy machines used primarily to study problems arising from the design and analysis of dynamic systems. The results of analog computer operations are displayed in graphical form, providing the operator with an immediate picture of the system activity. Digital computers provide high accuracy, and are more versatile. They are more suited to the solution of problems that are algebraic or numerical in nature, and they usually display results in the form of a numerical printout.

8.2 ANALOG COMPUTERS

8.2.1 The Nature of Analog Computation

8.2.1.1 THE PRINCIPLE OF ANALOG COMPUTATION. The analog computer is an engineering tool used in the laboratory to study physical systems which are too complicated to analyze with conventional mathematical techniques and for which the "build and try" process of design and test is prohibitively expensive and time consuming. It contains a number of electronic components that can be interconnected to simulate the mathematical description of a system or component. The computer thus becomes an electronic analog of the object system. It is easily manipulated in order to determine optimum design criteria, and readily subjected to a variety of engineering tests ranging from frequency response tests to the application of "worst case" forcing functions.

The basis for using electronic analogs to simulate a deverse class of physical systems lies in the mathematical equivalence of the equations that describe those systems. Consider, for example, the models shown in Figure 8.2.1.1. The velocity of the spring-mass system and the instantaneous current in the electrical circuit are each described by an integro-differential equation of the form

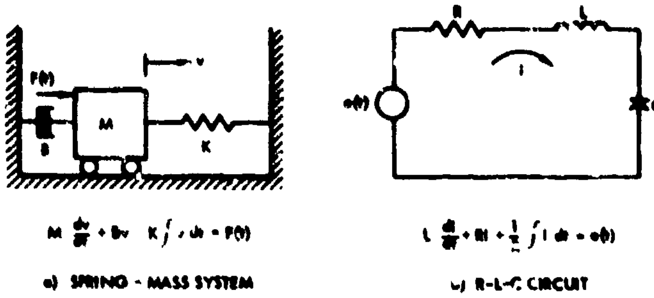


Figure 8.2.1.1. Different Physical Systems with Equivalent Mathematical Models

(Eq 8.2.1.1)

$$a\frac{dy}{dt} + b\,y + c\int y\,dt = g(t)$$

The solution of Equation (8.2.1.1) for y as a function of t is a mathematical process completely independent of the physical significance of the parameters in the equations. It follows, therefore, that a solution for *y* can represent a solution for any system described by the same mathematical form provided that the parameters and initial conditions of Equation (8.2.1.1) are proportional to those of the system.

Conversely, if an arbitrary system can be made to perform in accordance with specified equations, it follows that the activity of such a system will be analogous to that of any other system defined by the same equations. The electronic analog computer represents such an arbitrary system.

8.2.1.2 FUNCTIONAL CHARACTERISTICS. Electronic analog computers contain electronic components that accurately simulate the mathematical operations of addition, integration with respect to time, multiplication by a constant, and the multiplication of variables.

Additional components and techniques provide means of simulating a variety of non-linearities as well as the capability of generating arbitrary functions of variables. The computer components perform these operations on voltages. A multiplier, for example, produces at its output a voltage variation proportional to the product of the voltage variations applied to its input terminals. The voltage variations at the outputs of the various components are related to the variables of the system under study through constants of proportionality known as scale factors. For example, in the process of solving differential equations two integrators might be connected in a tandem arrangement such that the output of the first integrator serves as the input to the second. Typically, the output of the first integrator represents the velocity of a variable, while the output of the second integrator represents the displacement of that variable. Both outputs are in reality voltages, and each voltage is related to its corresponding variable in the physical system through scale factors. Ten volts at the output of the first integrator might correspond to five feet/second velocity, while ten volts at the output of the second integrator might correspond to a twenty foot displacement. The scale factor in this example would be "2 volts per feet/second" and "0.5 volts per foot," respectively.

The inputs and outputs of all components are terminated in a central location where they are interconnected in accordance with the equations that describe a system. The computer then becomes an electronic model of the system. When it is excited by the appropriate application of initial conditions and forcing functions, all elements of the computer *simultaneously* and *continuously* react in a manner analogous to that of the system. The variables of interest can be plotted either as functions of time or as functions of each other. Thus, the user is provided with an immediate display of the system activity and is aided immeasurably in developing a feel for the system operation.

When programming the analog computer, one has the option of speeding up, slowing down, or equating the speed of the computer solution with respect to the time response of the physical system. This is known as time scaling the problem. In principle, the choice of a time scale is arbitrary. In practice, however, it is governed by a number of considerations such as the natural frequencies of the system compared with the frequency limitations of the computer components and recording equipment. Once the time scale is chosen, the speed of solution is independent of the system complexity. Thus, the system equations can be modified at will without affecting the time required to obtain a solution.

8.2.1.3 ACCURACY. The accuracy of the analog computer represents its most significant limitations; within the current state-of-the-art it is limited to 0.01 percent of the full scale voltage range of the computer. (In this section, accuracy will always be expressed as a percentage of full scale voltage range.) Most computers have a voltage range of ± 100 volts, which is more than adequate for a majority of engineering applications; but experience has shown that the accuracy that can be obtained realistically ranges from 0.1 to 10 percent, depending upon the complexity of the problem. In reference to accuracy it has been said that analog computers are to differential equations what the slide rule is to arithmetic. And, as the slide rule is replaced by a desk calculator when more accuracy is required, the analog computer is replaced by a digital computer.

It should be understood, however, that extreme accuracy is not the primary objective in the use of analog computers, nor is it always necessary. Frequently these computers are used in the analysis of problems in which parameter data are not accurate to more than a few percent. If the accuracy obtainable is not sufficient for a particular problem, analog computers are useful for obtaining fast qualitative results, or for determining approximate parameter values, the problem then can be programmed on a digital computer to obtain the required accuracy.

8.2.2 Analog Computer Components

8.2.2.1 Linear Components. Four linear components — the operational amplifier, summer, integrator, and coefficient potentiometer are discussed as follows:

1) *The Operational Amplifier.* The heart of the analog computer is the operational amplifier. It is a high gain, direct-coupled amplifier with high input and low output impedance characteristics. When connected with passive input and feedback impedance elements as shown in Figure 8.2.2.1a, the input-output relationship is given by*

(Eq 8.2.2.1)

$$e_o = -Z_f\left(\frac{e_1}{Z_1} + \frac{e_2}{Z_2} + \cdots + \frac{e_n}{Z_n}\right)$$

where e_o = output voltage
$e_1, e_2, \ldots$ = input voltages
Z_f = feedback impedance
$Z_1, Z_2, \ldots$ = input impedances

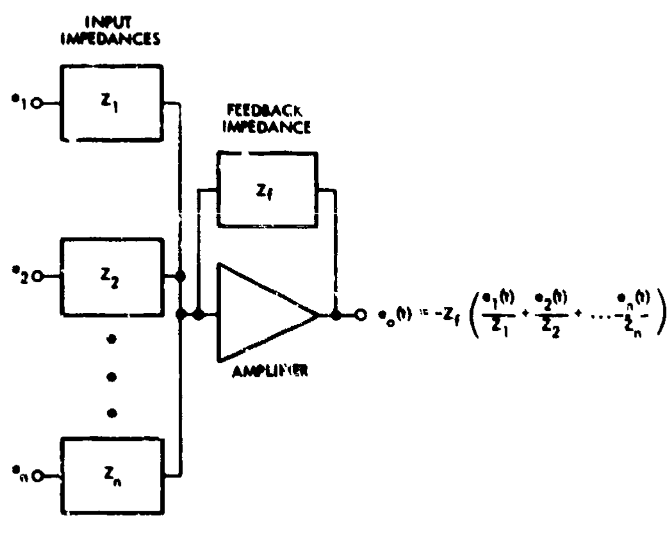


Figure 8.2.2.1a. Operational Amplifier with Passive Input and Feedback Impedances

*Equation (8.2.2.1) is a derived relationship for which approximating assumptions are made. Errors associated with these assumptions, however, are negligible when compared with the accuracy limitations of the impedances.

The significant features expressed in Equation (8.2.2.1) are:

a) The output is a *negative* function of the *sum* of the input terms.

b) The mathematical relationship of the output with respect to the inputs is determined by the nature of the input and feedback impedances.

c) The accuracy of the amplifier is determined by the accuracy of the impedances.

The accuracy of the components is typically 0.01 percent, establishing the highest possible accuracy of the computer.

2) *The Summer.* When resistors are used as both input and feedback elements, as shown in Figure 8.2.2.1b, the operational amplifier becomes a summer. The feedback resistor, R_f, is common to all inputs, and the gain of each input is determined by the value of the resistor associated with that input. The number of inputs and variety of gains available in a summer amplifier are usually fixed for a given computer. A typical computer might provide four unity-gain inputs and three ten-gain inputs per amplifier, with means provided for adding additional input resistors should they be required. The notation used for indicating summer amplifiers on computer diagrams is shown in Figure 8.2.2.1c.

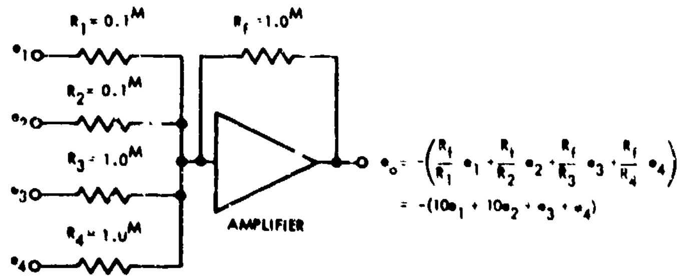


Figure 8.2.2.1b. Simplified Diagram of a Summer Amplifier

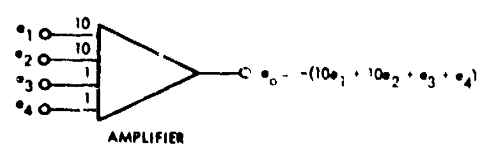


Figure 8.2.2.1c. Computer Diagram Notation for a Summer Amplifier

3) *The Integrator.* When a capacitor is used as a feedback element, the output is the integral with respect to time of the sum of the inputs, as shown in Figure 8.2.2.1d.

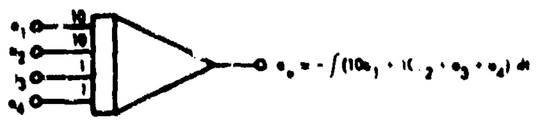


Figure 8.2.2.1d. Simplified Diagram of an Integrating Amplifier

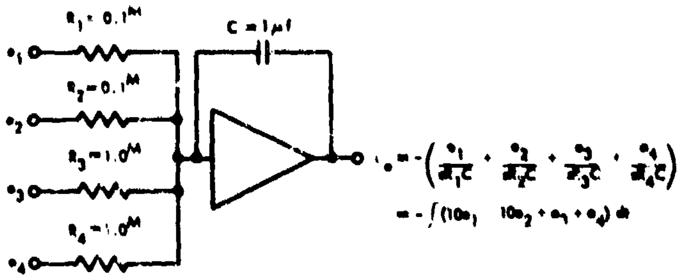


Figure 8.2.2.1e. Computer Diagram Notation for an Integrating Amplifier

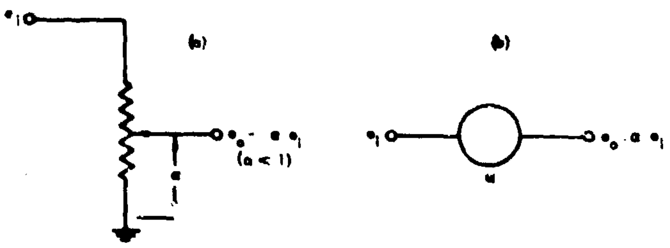


Figure 8.2.2.1f. Schematic of a Coefficient Potentiometer

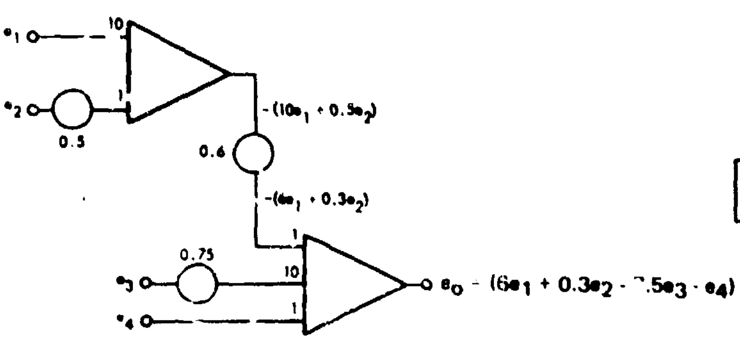


Figure 8.2.2.1g. How Coefficient Potentiometers and Amplifier Gains Achieve Multiplication by Arbitrary Constants

Thus the processes of integration and addition are combined in one unit. A given computer will generally have the same number of inputs and gains for the integrator amplifiers as provided for the summer amplifiers. In addition to the function inputs, provisions are made for applying initial condition voltages to each integrator. The diagrammatic notation for an integrator is shown in Figure 8.2.2.1e.

4) *The Coefficient Potentiometer.* The multiplication of voltages by a constant less than one is obtained through the use of a high resolution voltage dividing potentiometer (commonly referred to as a coefficient potentiometer), as shown in Figure 8.2.2.1f. These potentiometers are usually ten-turn devices capable of a resolution of about one part in ten thousand.

Figure 8.2.2.1g illustrates how coefficient potentiometers can be combined with amplifier gains to achieve multiplication by arbitrary constants.

8.2.2.2 NON-LINEAR DEVICES. Four non-linear devices — multipliers, resolvers, function generators, and relay amplifiers are discussed as follows:

1) *Multipliers.* There are three types of multipliers in common use today: the servomultiplier, the time-division multiplier, and the quarter-square multiplier.

The servomultiplier is an electromechanical device illustrated schematically in Figure 8.2.2.2a. The wipers of several potentiometers are fixed to a common shaft so that their mechanical positions are always aligned. The shaft is positioned by a servomechanism to correspond to one of the variables, x, and if voltages y_i are applied across the multiplying potentiometers, the outputs will be proportional to xy_i. One of the potentiometers is used as a feedback element to convert shaft position into a voltage for

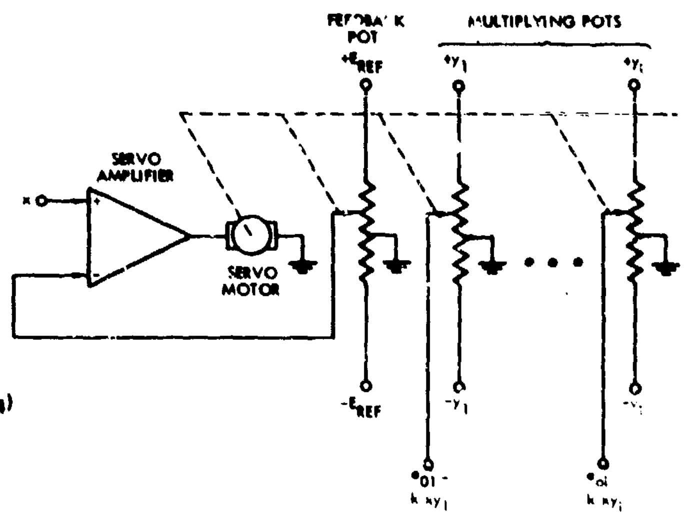


Figure 8.2.2.2a. Schematic Diagram of a Servomultiplier

comparison with the input voltage x. Any difference between the x input and the voltage at the wiper of the feedback potentiometer is amplified and fed to the servo motor to drive the servomechanism to a null. The servomultiplier has severe frequency and rate limitations associated with the function applied to the servo amplifier input. Multipliers that use 60 cps motors are usually limited to a frequency response of less than one cycle per second, and those that use 400 cps motors are good to frequencies that approach 30 cycles per second. The chief advantages of the servomultiplier are (1) several products can be obtained with one variable with a single component, and (2) high accuracy types are capable of up to 0.02 percent accuracy and resolution when the frequency and rate limitations of the x input are maintained.

Both the time division and quarter-square multipliers are all electronic devices, and are useful at problem frequencies ranging from d.c. to an excess of 200 cycles per second. The accuracy of electronic multipliers depends upon a number of considerations that are beyond the scope of this section. In general, the accuracy available varies from 0.05 to about 2 percent, depending primarily upon the frequency characteristics of the input variables. A significant advantage of electronic multipliers is that they can usually be converted to function dividers through the operation of a control switch. The diagrammatic notation for electronic multipliers is shown in Figure 8.2.2.2b.

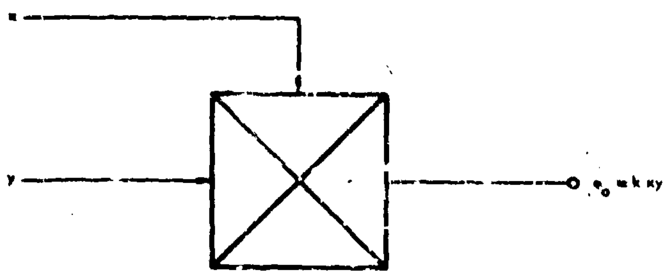


Figure 8.2.2.2b. Diagrammatic Notation for Electronic Multipliers

2) *Resolvers.* The resolver is a device used for coordinate transformations and the generation of the sine and cosine of angles of dependent variables. It can perform polar-to-rectangular transformations or rectangular-to-polar transformations depending upon the setting of a control switch. When resolving vectors into rectangular coordinates, the inputs are the vector magnitude, R, and angle, θ, the outputs are R sin θ and R cos θ. When performing rectangular-to-polar transformations, the inputs are the rectangular coordinates x and y, and the outputs are the vector magnitude R and angle θ.

Resolvers, like multipliers, are either electromechanical or all electronic and have performance characteristics similar to their multiplier counterparts.

3) *Function Generators.* It is frequently necessary to be able to generate functions, based upon empirical data which cannot be generated by conventional analytical techniques. Also, analytical functions occasionally arise that require simulation by an excessive number of computer components. Function generators are devices that have been developed to handle these situations, and thus extend the versatility of the analog computer. Like multipliers and resolvers, there are two types of function generators in common use today, electromechanical and electronic.

The electromechanical function generator is simply a servomultiplier unit with one or more tapped potentiometers in place of ordinary multiplying potentiometers. The principle of the tapped servofunction generator is illustrated in Figure 8.2.2.2c. Up to twenty or more equally spaced taps are provided on a multiturn potentiometer that is fixed to the same shaft as the feedback potentiometer. By applying arbitrary voltages to these taps, a sequence of straight line segments can be made to approximate a desired function. As the servomultiplier unit is positioned by the input function, x, the output tracks the programmed function, f(x).

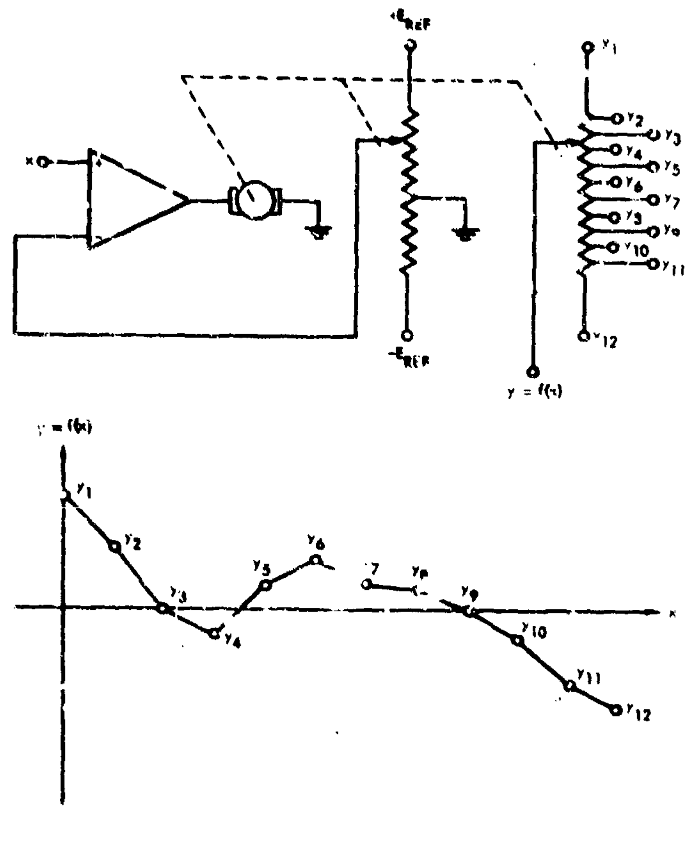


Figure 8.2.2.2c. Schematic of a Tapped Servo Function Generator

(Voltages proportional to y_1, y_2 . . . are applied to the taps on the function generating potentiometer. As the arm of the function potentiometer is positioned in proportion to x, the function y = f(x) is approximated by a sequence of straight line segments, as shown at the right.)

The all electronic diode function generator, DFG, allows an arbitrary function to be represented by a series of straight-line segments. It employs diode networks to change the slope from one segment to the next as the input voltage proportional to the independent variable is increased. Hence, the same general technique of straight line segment approximations to the actual function is used in DFG's as in tapped servofunction generators. The frequency characteristics of diode function generators and tapped servofunction generators are similar to those of electronic multipliers and servomultipliers, respectively.

4) *Relay Amplifiers*. Relay amplifiers, also known as comparator amplifiers or differential relays, are high speed relays driven by high sensitivity difference amplifiers that make it possible to perform switching operations based upon the accurate comparison of voltages. These units usually have relay throw times of less than one millisecond and are capable of sensing the difference of two voltages to within ten or twenty millivolts. The relay contacts are usually double-pole, double-throw, as shown in Figure 8.2.2.2d.

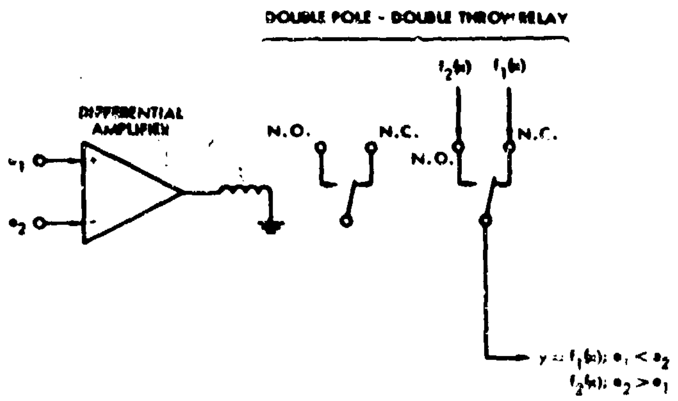


Figure 8.2.2.2d. Relay Amplifier

The principle of operation is quite simple. As long as the input voltages e_1 and e_2 are such that (e_1-e_2) is algebraically less than zero, the relay is de-energized, and the relay contacts are in the normally closed, N.C., position. When (e_1-e_2) becomes greater than zero, the relay is energized, and the contacts are switched to the normally open, N.O., position.

Relay amplifiers make it possible to perform a number of logical operations on the analog compute For example, if it is necessary to satisfy the relationships

$$y \quad f_1(x) : e_1 < e_2$$

$$y \quad f_2(x) : e_1 > e_2$$

this is readily accomplished by comparing e_1 with e_2, as shown in Figure 8.2.2.2d. The function $f_1(x)$ is applied to a normally closed contact of the relay, and $f_2(x)$ to a normally open contact. As long as e_1 is algebraically less than e_2, the relay is de-energized and $f_1(x)$ is coupled to the relay arm. When e_1 exceeds e_2, the relay is energized and $f_2(x)$ is coupled to the relay arm.

8.2.2.3 OUTPUT DEVICES. Three output devices — voltmeters, recorders, and plotters — are described as follows:

1) *Voltmeters*. Voltmeters serve the purpose of monitoring problem variables throughout the computer. They are used for setting coefficient potentiometers, setting initial condition voltages on integrators, reading final values, etc. Four place digital voltmeters are widely used in order to meet resolution and precision requirements commensurate with the computer accuracy, although some computers use conventional d'Arsonval movements in conjunction with a four place reference nulling device. The outputs of the computer components are connected to the voltmeter through an address selector system that consists of pushbutton or rotary selector switches.

2) *Recorders*. A paper strip-chart recorder plots the problem variables against time. In the recorder, paper is drawn at constant speed under pens which are deflected in proportion to the variables being recorded. Normally six or eight channels are available, depending upon the model, allowing a number of voltages to be recorded side-by-side, simultaneously. Each channel has many sensitivity ranges permitting both large and small voltage variations to be accommodated with the same relative accuracy. The frequency response of recorders is usually flat out to 30 to 60 cycles per second. Resolution limitations allow interpretation to no better than two percent of full scale, the recorder is used primarily to obtain qualitative results.

3) *Plotters*. When higher resolution and accuracy than can be obtained with recorders are required, an XY plotting table is used. It allows any two problem variables to be plotted against each other, usually on 11 x 17-inch graph paper. Plotters employ a dual servo system to drive a pen along an arm in the Y direction, and the arm in the X direction. A number of sensitivities are available, allowing large and small voltage variations to be recorded with equal accuracy. The static accuracy of plotters is approximately 0.1 percent, but they are limited to deflection rates of ten to fifteen inches/second.

8.2.3 Applications

8.2.3.1 SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS. Analog computers are frequently used in applications that involve the study of dynamic systems described by linear or non-linear ordinary differential equations with constant or time varying coefficients. They are naturally suited for this application because of the integrator.

Linear Differential Equations. To illustrate the technique employed in solving linear differential equations, consider the second-order differential equation

(Eq 8.2.3.1a)

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = g(t)$$

The first step is to solve the equation for the highest order derivative, thus

(Eq 8.2.3.1b)

$$\frac{d^2y}{dt^2} = \frac{1}{a}g(t) - \frac{b}{a}\frac{dy}{dt} - \frac{c}{a}y$$

Equation (8.2.3.1b) is then integrated directly to obtain the first derivative in the implicit form

(Eq 8.2.3.1c)

$$\frac{dy}{dt} = \int_0^t \left[\frac{1}{a}g(t) - \frac{b}{a}\frac{dy}{dt} - \frac{c}{a}y\right]dt + \frac{dy}{dt}(0)$$

The computer diagram for the solution of Equation (8.2.3.1c) is shown in Figure 8.2.3.1a. The first derivative is formed by integrating the sum of the terms indicated in Equation (8.2.3.1b), and the function y is formed by integrating the first derivative. The variable y and its first derivative are then multiplied by the appropriate coefficients and added with the forcing function at the input to the first integrator, resulting in a closed loop system that simulates the original equation.

Four fundamental points worth noting are:

1) The solution process is based upon the repeated integration of derivatives to obtain the dependent variable, rather than the more straightforward process of assuming the variable and repeatedly differentiating. From a mathematical point of view, either technique is valid. However, from an engineering point of view the process of differentiation has a serious drawback. Differentiation is a noise amplifying process and, since all electronic equipment unavoidably generates random noise, the noise, however slight, would be amplified by the differentiation process. Integration, on the other hand, is a smoothing and averaging process, and minor noise effects are minimized.

2) The second derivative does not appear explicitly in the solution shown in Figure 8.2.3.1a, but can be formed explicitly as indicated in Equation (8.2.3.1b) and shown diagrammatically in Figure 8.2.3.1b. In doing so two extra amplifiers are required. It is general practice to attempt to minimize the number of amplifiers in a computer setup in order to conserve equipment and minimize sources of errors in the programming and solution of a problem. As a result, the highest order derivative is formed implicitly, as indicated in Figure 8.2.3.1a, unless it is required elsewhere in the solution or is to be recorded.

3) The sign inverting characteristics of the amplifiers must always be kept in mind when preparing the computer diagram.

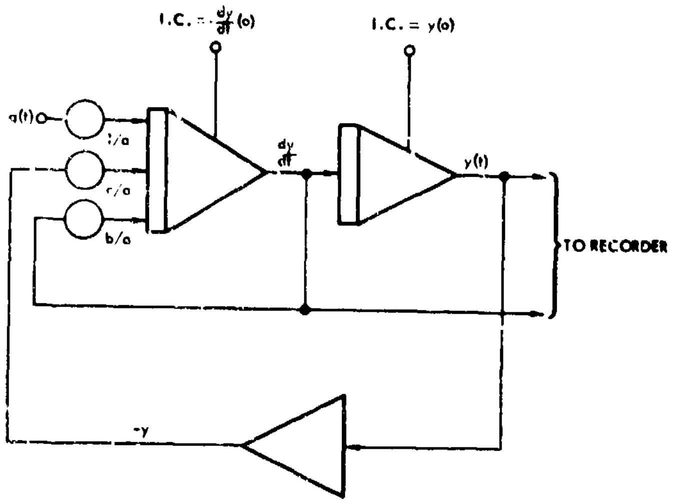


Figure 8.2.3.1a. Analog Computer Schematic for Solving Equation (8.2.3.1c) by Solving for the First Derivative Implicitly

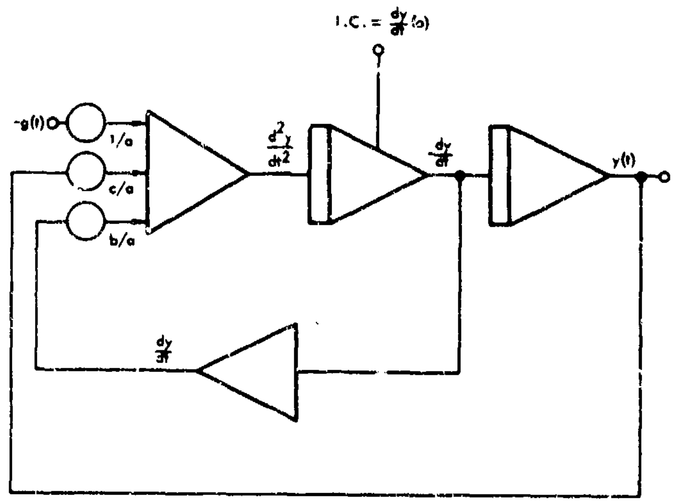


Figure 8.2.3.1b. Analog Computer Schematic for Solving Equation (8.2.3.1b) by Solving for the Second Derivative Explicitly

4) The analog computer should be regarded as a readily manipulated model of the physical system being studied. The system coefficients and parameters generally occur as settings of coefficient potentiometers that are easily changed between solutions. An important advantage of using analog computers is the instantaneous communication that exists between the user and the computer. The response of the important system variables is immediately and simultaneously displayed on the recorder. For example, if the problem solution indicates that the damping is incorrect, the potentiometer representing damping is readily changed and a new solution started. It is possible to optimize, modify, or test physical systems rapidly and economically.

Non-Linear Differential Equations. Non-linear differential equations arise in nearly every phase of engineering design. Because of the severe difficulty of obtaining solutions to most non-linear problems, methods of analysis and synthesis generally emphasize the use of linear approximations that can be solved by conventional means. Unfortunately this leads most engineers to distrust non-linearities, when actually the deliberate incorporation of non-linearities can often result in significant improvements in system performance, or in a reduction in hardware complexity. Although the analytical treatment of a particular non-linear system may only be approximated, the analog computer can readily be programmed to simulate it directly; thus it can often be used to investigate the possible advantages to be gained by deliberately introducing non-linear phenomena into systems.

To illustrate the ease with which non-linearities can be handled, consider a spring-mass system with a non-linear spring that has a restoring force given by

$$F = Ax + Bx^3 \qquad \textbf{(Eq 8.2.3.1d)}$$

The system equation is described by

$$M\ddot{x} + b\dot{x} + Ax + Bx^3 = f(t) \qquad \textbf{(Eq 8.2.3.1e)}$$

The computer diagram for this solution is shown in Figure 8.2.3.1c; the only requirements being two multipliers to generate the x^3 term for the solution.

While in principle it is a simple matter to include non-linearities in the computer setup of a problem, thought and care must be exercised in order to arrive at valid results. Complex problems often require an evolution of setups in order to minimize sources of error, because the computer components are not perfect. For instance, multipliers do not always yield the true product of two voltages, and operational amplifiers (hence, summers and integrators) have finite bandwidth limitations and phase shift characteristics that vary with frequency, often causing computer instabilities.

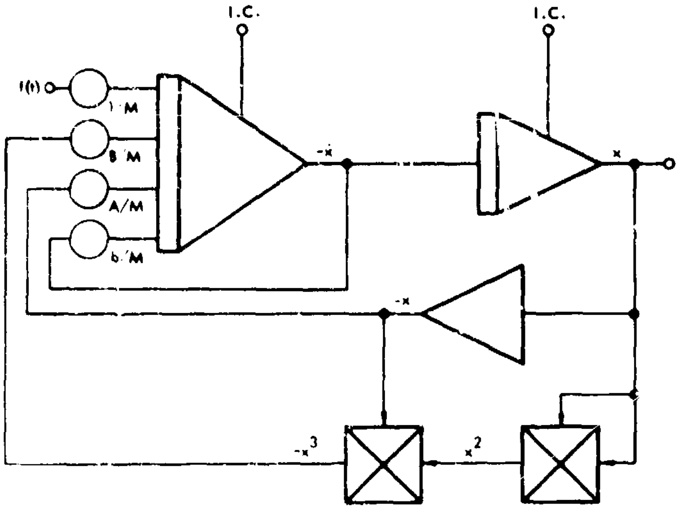


Figure 8.2.3.1c. Computer Solution for the Non-Linear Differential Equation $M\ddot{x} + b\dot{x} + Ax + Bx^3 = f(t)$

8.2.3.2 ANALYSIS OF FEEDBACK CONTROL SYSTEMS. Analog computers are used extensively in the analysis and design of feedback control systems. The general technique employed is to simulate the block diagram of the control system directly on the computer, using special impedance networks in conjunction with operational amplifiers to simulate the individual transfer functions.

The control system diagram shown in Figure 8.2.3.2a in Laplace transform notation illustrates the use of computers in control system analysis. The difference device at the input can be obtained by using an operational amplifier as an adder. The transfer function $G_1(s)$ is simulated by using a resistor and capacitor in parallel in the feedback path of an operational amplifier, as shown in Figure 8.2.3.2b. (Recall that the transfer function of an operational amplifier is determined by the ratio of the feedback impedance to the input impedance.) The Laplace notation for the feedback impedance in Figure 8.2.3.2a is

$$Z_f(s) = \frac{R_f}{sR_fC + 1} \qquad \textbf{(Eq 8.2.3.2a)}$$

and the input impedance is simply R_i. Hence the amplifier transfer function, including the effect of the input potentiometer is

$$\frac{e_o}{e_{in}} = -\frac{\alpha R_f}{R_i}\left(\frac{1}{sR_fC + 1}\right) \qquad \textbf{(Eq 8.2.3.2b)}$$

By making $\frac{\alpha R_f}{R_i}$ proportional to K_1, and R_fC proportional to T_1, the amplifier can be made to simulate $G_1(s)$.

The computer diagram for the simulation of the feedback control system shown in Figure 8.2.3.1c is given in Figure 8.2.3.2c. The system time constants and gains occur as potentiometer settings, making it a simple matter to adjust the important parameters in order to optimize the system performance. The transfer function $G_2(s)$ is divided into two circuits, in order to obtain the rate feedback term, $\dot{C}$, explicitly rather than have to differentiate the output, C. As noted, differentiation is to be avoided whenever possible because of its noise amplifying characteristics.

A number of non-linear characteristics encountered in control systems, such as saturation and deadzone effects, are dependent upon signal amplitude. Biased or zener diodes

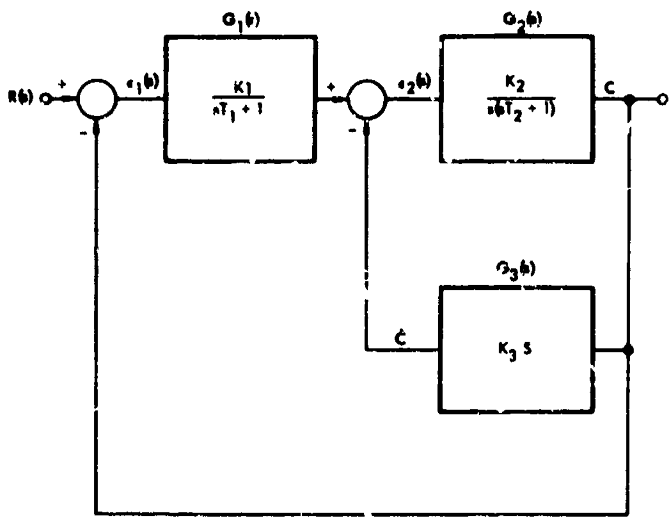


Figure 8.2.3.2a. Feedback Control System in Laplace Transform Notation

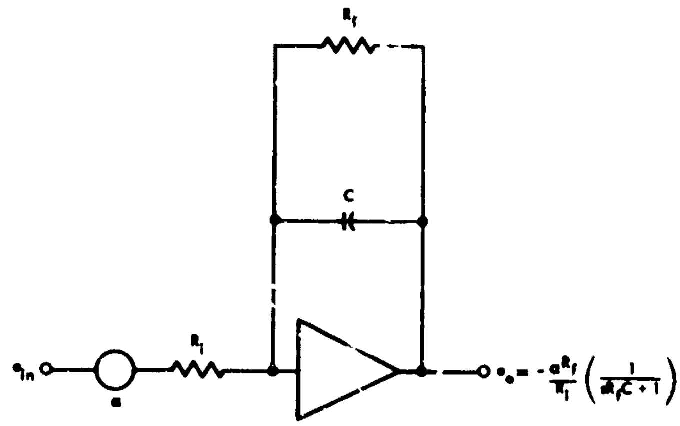
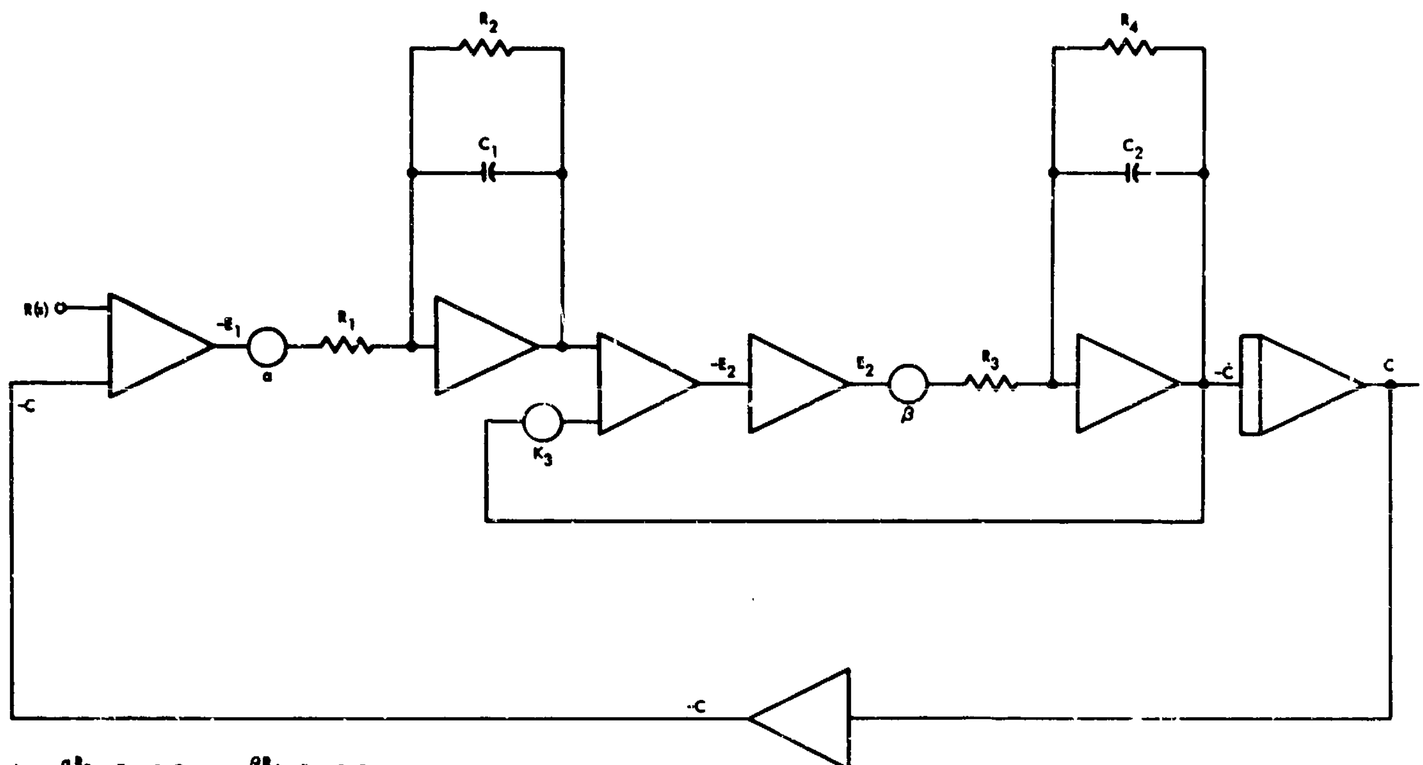


Figure 8.2.3.2b. Operational Amplifier Schematic for Simulating a First Order Lag Function



$k_1 = \frac{aR_2}{R_1}$; $T_1 = R_2C_1$; $K_2 = \frac{\beta R_4}{R_3}$; $T_2 = R_4C_2$

Figure 8.2.3.2c. Computer Diagram for Simulation of the Control System Shown in Figure 8.2.3.2a are commonly used to simulate these effects. Figure 8.2.3.2d shows how zener diodes can be used in the input impedance of an amplifier to simulate deadzone effects. For input voltages with magnitudes less than the breakdown voltage of the diodes, the diodes act as open circuits, and the output remains at zero. When the breakdown voltage of the diode is exceeded, it acts as a short circuit and the amplifier acts as an inverter.

Figure 8.2.3.2e shows how zener diodes can be used in the feedback circuit to simulate saturation effects. As long as the output voltage is less than the breakdown voltage of the diodes, they act as an open circuit and the output follows the input in a linear manner. When the breakdown voltage is exceeded, the output is clamped at a potential equal to the breakdown voltage.

The above examples illustrate only the principles involved in the simulation of discontinuous non-linearities. The actual circuits employed and the errors that might be introduced are discussed at length in the references listed at the end of Section 8.0.

8.2.3.3 SOLUTION OF ALGEBRAIC EQUATIONS.** The solution of systems of linear algebraic equations, and related problems such as the determination of eigenvalues, matrix inversion, and matrix multiplication, are most naturally handled by digital techniques. The analog computer can be used successfully if the number of equations is not too large and if the precision requirements do not exceed 1 percent. If these conditions exist, analog computers are faster to program, and produce results more rapidly, (usually in less than one second) than digital computers. Analog computers are particularly useful if the matrix coefficients are to be varied because, as is generally the case in analog computation, the coefficients occur as potentiometer settings, and can be rapidly changed between solutions.

When programming algebraic problems for solution on the analog computer, it is necessary to exercise considerable care in order to prevent instability — not because of the mathematical nature of an algebraic problem, but because of the frequency and feedback characteristics of the analog computer amplifiers. A general technique has been developed which will circumvent these difficulties and assure the stability of the solution; however, it requires an excessive number of components.

To illustrate the use of this technique, consider the following system of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad \text{(Eq 8.2.3.3a)}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \text{(Eq 8.2.3.3b)}$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad \text{(Eq 8.2.3.3c)}$$

**Rogers and Connolly, *Analog Computation in Engineering Design*, McGraw-Hill Book Company, 1960, Chap 6.

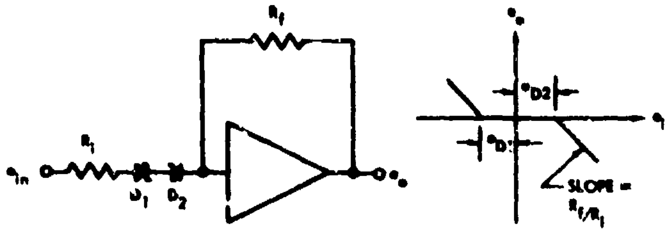


Figure 8.2.3.2d. Operational Amplifier Circuits Illustrating Techniques Used for Simulating Deadzone Effects

(The circuits shown here demonstrate the principals involved, and are not realistic. See text.)

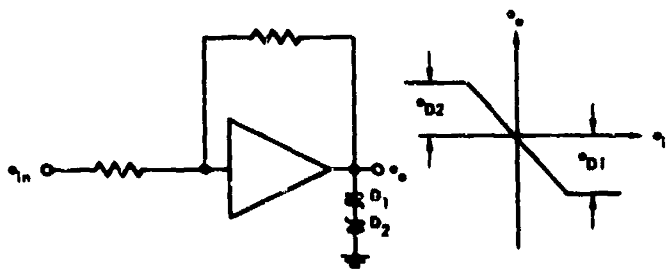


Figure 8.2.3.2e. Operational Amplifier Circuits Illustrating Techniques Used for Simulating Saturation Effects

(The circuits shown here demonstrate the principals involved, and are not realistic. See text.)

which can be examined in two steps. The first step is to augment the system of equations by adding a first derivative column vector. This results in the set of differential equations

$$\dot{x}_1 + a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad \text{(Eq 8.2.3.3d)}$$

$$\dot{x}_2 + a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \text{(Eq 8.2.3.3e)}$$

$$\dot{x}_3 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad \text{(Eq 8.2.3.3f)}$$

The system of differential equations is now solved and, assuming that a steady-state solution exists, the values of x_i at steady-state will represent solutions to the original system of equations because the derivative terms will have vanished.

In order to assure a steady-state solution, the matrix of coefficients in the original system of equations must exhibit the positive-definite property. In general, one cannot readily establish whether or not a matrix is positive-definite, but this property is assured if the original matrix is premultiplied by the transpose of the original coefficient matrix prior to the addition of the first derivative column vector. In matrix notation, the equations to be solved on the computer are

(Eq 8.2.3.3g)

$$(\dot{x}) + A^T A\,(x) = A^T\,(b)$$

where A^T is the transpose of the coefficient matrix A.

The differential equations given above are readily simulated on the computer. The equipment required for a set of n equations consists of $(2n^2 + n)$ coefficient potentiometers, (n) integrators, and $(2n + p)$ summers, where p is the number of inverters required to effect negative coefficients a_{ij}. The amount of equipment required is such that most general purpose analog computers would be restricted to a system of approximately eight linear equations. Two or more computers can be interconnected through trunk lines in order to handle larger systems of equations. Special purpose electronic and analog computers have recently been designed and built for the solution of matrix problems. Such a computer was completed in 1957 by Electronic Associates, Inc. and will handle matrices up to 14 by 14. Solutions can often be obtained with an accuracy of 0.2 percent and a precision of three significant figures. The computation time required for the solution of simultaneous equations or for obtaining each column of the inverse of a matrix is approximately 0.1 seconds.

8.2.3.4 **REAL TIME SIMULATION.** An important characteristic of analog computation is that the computer can be programmed to solve the equations describing a physical system in real time; i.e., the computer can be programmed so that a one-to-one correspondence exists between the time history of the computer variables and that of the physical system. This characteristic has been exploited for a number of applications.

There are occasions when a system to be studied on the analog computer contains a non-linear component that cannot be described with sufficient accuracy by mathematical means. For example, such a component might have a hysteresis characteristic that varies with frequency and amplitude. It might be possible to connect the component to the computer through suitable transducers, with the remainder of the system equations programmed on the computer. This procedure would eliminate the errors that would be introduced due to an inadequate mathematical description of the component's behavior.

A variation of this example is when a component or system is interconnected with the computer for purposes of testing and evaluation. The operation of these system components may then be observed in the laboratory. Thus, a computer can be used to simulate the load of an automatic control system under test, or the computer may serve as a simulated controller operating an actual motor and load. The computer as a simulator permits dynamic as well as operational analysis of a system and enables the prediction and optimization of the performance of the system in the laboratory.

8.2.4 Analog Computer Systems

A comparison of several general purpose analog computers with regard to their components and costs is presented in Table 8.2.4. The computers were selected at random; selection was *not* based on superiority over other computers in their respective price ranges.

Table 8.2.4 Analog Computers: Components and Cost Comparison

COMPONENTS	ELECTRONIC ASSOCIATES (EAI)			BECKMAN INSTRUMENTS		APPLIED DYNAMICS		
	TR10-3	TR48-3	231R-5	2132	2133	32	34-C	256
Amplifiers								
Total	20	48	80	72	120	32	64	256
Integrators	8	11	30	48	72	12	24	64
Potentiometers								
Handset	24	60	20	12	24	40	80	40
Servo	—	—	150	180	240	—	—	160
Multipliers	2	5	45	24	48	10	16	24
Resolvers	—	—	5	4	6	—	2	12
DFG	1	5	20	12	24	—	8	48
Comparators	2	4	10	8	16	2	6	32
Function Switches	2	5	20	24	32	4	2	80
Maximum accuracy	0.1%	0.01%	0.01%	0.01%	0.01%	0.1%	0.01%	0.01%
Approximate Cost, $	11,000	30,000	200,000	200,000	250,000	16,000	47,000	250,000
Transistorized, ±10 volts	X	X						
Vacuum Tubed, ±100 volts			X	X	X	X	X	X

8.3 DIGITAL COMPUTERS

Sub-Topics 8.3.1 through 8.3.5 were adapted to the handbook format from a series of articles by Dewitt W. Cooper, in *Machine Design*, from October 11 to December 6, 1962 (References 1-227, 1-230, 1-232, 1-235, and 1-243), published and copyrighted by the Penton Publishing Company, Cleveland, Ohio.

8.3.1 Introduction to the Digital Computer

The digital computer is a discrete-action device which operates directly on numbers according to a set of instructions introduced into the computer. This computer offers extremely high speed and a high level of accuracy. Digital computers perform only the four basic arithmetic operations; however, methods of numerical analysis may be used to obtain solutions to a wide variety of complex problems.

Computers are generally classified according to their function. General purpose machines are designed to solve problems requiring a large number of mathematical operations at high speed and with great accuracy, and are most often used in engineering departments. Special purpose computers are used to carry out solutions to a specific problem.

8.3.1.1 BASIC COMPONENTS. A digital computer system can be divided into four distinct sections, (Figure 8.3.1): 1) storage, consisting of one or more units in which data are stored, 2) control unit, which synchronizes mathematical operations and data transfer, 3) arithmetic unit, in which mathematical operations are performed, 4) input and output equipment.

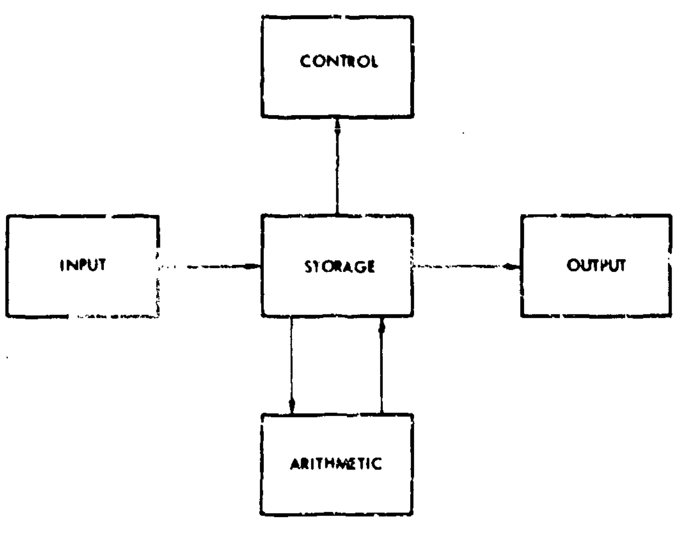


Figure 8.3.1.1. Main Steps Carried Out by a Digital Computer in Processing Information

Storage. That part of a computer which most differentiates it from a calculator (desk-type, slide rule, adding machine) is its storage, or "memory." A computer can store numbers, alphabet characters, and some special symbols. In any calculation it is necessary to store the numbers which begin the calculations, intermediate results, and — at least temporarily — final results. A desk calculator has a very limited ability to store data; the operator usually must enter each number as it is needed. On the other hand, a small computer (such as the IBM 1620) can store 20,000 digits and call them out of storage whenever they are needed.

Stored Programs. The ability to store large amounts of data that are to be used in or have been developed by calculations is only part of the function of storage. Equally important, the computer contains within storage the program for the calculations to be performed.

A program is made up of instructions which cause the computer to go through the sequence of operations necessary to arrive at a meaningful result. A single instruction may:

- Cause data to be brought into storage from some external source such as a card reader
- Cause a specified arithmetic operation to be performed on selected numbers
- Constitute a logical test to determine what part of the program should be performed next
- Cause results to be sent from storage to a recording device such as a typewriter.

After both the data to be operated upon and the program which describes the operations are in storage, the computer is free to proceed with a series of calculations at a speed ranging from 50 to 500,000 additions per second.

A computer can change or modify its own program. Since the program is stored in much the same form as data, one portion of a program may be a sequence of operations which will examine another portion of the same program and change it by arithmetic or logical manipulations. This means that one set of instructions can be used to operate on a number of sets of data stored in different locations in storage. As each set of data is processed, the program is changed to refer to the next set of data. In addition, this feature allows the program to be changed on the basis of conditions which arise during the calculations. For example, suppose a calculation involves the evaluation of

$$y = x^2 \text{ for } x \leq 1$$

$$y = -x^2 + 4x - 2 \text{ for } x > 1$$

At the point in the calculations where the value of x becomes greater than 1, that portion of the program involved in the evaluation of y can be changed to use the second of the two equations. This change might be accomplished by replacing one set of instructions with a new set stored elsewhere for that purpose, or it could be done by causing the computer to select one of two instruction sequences on the basis of whether x is greater than 1 or not greater than 1.

Addresses. To be able to select the item of data or the instruction to be operated upon next, the computer must be provided with a means of locating the desired information in storage. For this reason, storage is divided into units, with each unit identified by an address. Different computers are designed with different size units of storage. The basic unit may vary from one decimal digit (as in the IBM 1620) to 36 binary digits (as in the IBM 7090). In other computers, units of one alphameric character (that is, a decimal digit, letter of the alphabet, or special character) or ten decimal digits are used.

Whatever the size of the basic unit, each is assigned a numerical address which identifies the location. Manipulation of the data stored in a location is accomplished through the use of the address corresponding to the location.

Control Unit. This unit causes the desired operations to be performed in the sequence specified. It reads an instruction from storage, examines it, and sets up the circuit conditions to perform the operation. When these operations are completed, the whole process is repeated.

Generally, the first operation to be performed is manually entered into the control unit by the machine operator. Thereafter the action of the computer is completely controlled by the program in storage as interpreted by the control unit.

Arithmetic Unit. This component contains the circuitry which performs arithmetic on numbers taken from storage. It usually includes a limited amount of storage in which to hold the operands involved in the arthmetic.

Present day computers use the binary numbering system rather than the more familiar decimal system. To store and manipulate decimal digits requires a device capable of assuming ten stable and unique states — one state to represent each of the digits 0 to 9. While such devices are available (the notched wheels in a desk calculator are an example), they lack the speed of operation and small size necessary for a computer.

Many electronic devices are available which can assume two stable and unique states. A tube, for example, can be conducting or non-conducting; a magnetic field can have clockwise or counterclockwise rotation; a pulse can be transmitted or blocked.

The computer designer has at his disposal a number of devices and techniques for operating a number system with the base 2 — the binary number system. The rules for arithmetic are also much simpler in the binary number system than in the decimal system (base 10). As a result, there are two basic types of digital computers: one which operates entirely in the binary number system, and another which codes decimal digits as binary numbers. For the latter, the decimal digits appear as their binary equivalents:

Decimal	Binary	Decimal	Binary
0	0	5	101
1	1	6	110
2	10	7	111
3	11	8	1000
4	100	9	1001

The user need not be familiar with the binary nature of the computer. The computer is capable of translating the engineer's language to its own before starting a program, and performs the reverse operation when communicating its results to the engineer.

Input and Output. Every computer must have a means for communicating with the user. Typical input devices include punched-card readers, papertape readers, and manual keyboards. Card and papertape punches, typewriters, and printers are typical output devices. Magnetic tapes and magnetic discs provide a means of storing data externally from the computer in a form which allows rapid re-entry.

Input-output devices are controlled by the stored program. For example, an instruction to read a card will cause the card reader to start up, feed and read one card, and transmit what has been read into storage.

8.3.1.2 COMPUTER LANGUAGES. A set of instructions must be coded before it can be fed into the computer. Coding is the process of writing a computer program in language that the machine will understand.

A computer language generally consists of rules for writing or coding a problem — solution procedure and a standard or general purpose set of computer instructions for translating or correcting the resultant code into a special purpose set of computer instructions.

Types of Languages. Historically, the general development of languages may be listed in the order of their impact and degree of usage:

- Machine language
- Symbolic language
- Interpretive languages
- Compiler language
- Problem-oriented languages

Machine Language. This is the least practical of computer languages used in coding problems for a computer. Its only major advantage is that a machine-language program, once written, can immediately be executed by the computer. The use of machine-language coding is today generally restricted to programming of higher-level languages.

Symbolic Language. This language greatly reduces clerical effort in programing. The assembler (the general purpose program which must first treat the written symbolic program as data, then produce a machine-language program) makes the actual storage assignment. In addition, the programmer codes in symbols which have some mnemonic value. However, symbolic programing usually requires a few more instructions in coding than does machine language.

Interpretive Languages. These languages reduce the number and complexity of instructions required to write a program, yet eliminate the need for a translation or assembly run on the computer in producing a machine-language program. The language is still relatively unintelligible to a human reader. A more serious disadvantage is the necessary presence of a general purpose program in storage with the specific program reduces the effective size of the computer storage.

Compiler Language. Today the compiler language is accepted as the standard method of programing engineering and scientific problems for a computer. Some of its advantages are simple mathematical-like language, shorter programs, and improved readability.

The greatest advantage of compilers is that a program written in a compiler language can be run on any machine for which there exists the necessary general purpose processor for conversion from compiler language to machine language. This is not true of any of the previously mentioned languages.

The FORTRAN compiler language, introduced in 1955, first demonstrated the feasibility of compilers. Processors are now available for most scientific computers to translate from FORTRAN to machine language. FORTRAN will be used throughout this Sub-Section.

Problem-Oriented Languages. Attention has been given to general purpose programs which will solve any problem within a given technical category. Again, the specific problem is presented to the computer in a simple descriptive language. The general purpose program then develops the mathematics needed to solve the problem, and finally does the calculations to give the solution. This approach has been successful in vibration studies of spring, mass, and damper systems.

Subroutines. In the engineering and scientific fields, many arithmetic evaluations of mathematical statements can be standardized. Therefore, once a computer program is coded for solving a given problem, the same program can be used for any problem of the same type. This is done with subroutines. Compilers can supply these subroutines, called "closed" because they appear only once in each program.

Floating-Point Subroutines. Engineering calculations often involve handling of very large or very small numbers in a computer. To avoid carrying many digits and to eliminate the effort of keeping track of the location of the decimal point, a floating-point notation is used. A common procedure is to maintain perhaps 6 to 15 most significant digits (mantissa) of a number plus a two-digit characteristic to indicate the proper position of the decimal point. The characteristic is developed from the exponent of 10 (assuming use of the decimal rather than binary system).

The internal representation of such a number in a computer may take several forms, depending on the logic of the computer hardware (decimal versus binary, fixed versus variable word length alphanumeric versus numeric characters, etc.). Two possible internal representations for the number 6 1 9 5 7 5 . 3 3 are 6 1 9 5 7 5 3 3 0 6 (variable, decimal, alphanumeric computer) and 5 6 6 1 9 5 7 5 3 3 (fixed, decimal, numeric computer).

Many computers, particularly large scale scientific systems, can automatically handle arithmetic with numbers in floating-point form. In the others, this function is simulated by programing. The programs for arithmetic operations, once programed for a specific machine, become subroutines.

Mathematical-Function Subroutines. Some of the common mathematical functions used repeatedly in engineering work, including the trigonometric functions, the hyperbolic functions, and the exponential functions, are evaluated through the use of subroutines.

Input and Output Subroutines. The entry and exit of information to a computer usually requires complex programing. Subroutines to perform these complex tasks can greatly reduce the effort of programing a specific problem.

Open Subroutines. The bulk of a machine program generated by a compiler processor is made up of another type of subroutine, called an "open" subroutine. These subroutines handle such operations as data transfers in storage, comparison of data, logical branches from one part of a program to another, and the actual arithmetic calculations for those machines not needing the closed subroutines for floating-point arithmetic. The decision to make a particular function an open rather than a closed subroutine depends generally on whether the length of the open subroutine is the same or less than the linkage instructions required for the use of a closed subroutine.

Subprograms. An important development in the programing of large scale problems for large computer systems is the use of subprograms with compiler languages. Previously, an individual program was a one-man job. With subprograming, a major job with distinct logical phases of calculation can be assigned to several people. Each person needs to know only the meaning, order, and form of the data to be accepted by his phase and the meaning, order, and form of the calculated results he is to pass on to the next phase of the program.

Systems Monitors. Most of the large scale scientific computing installations are presently using another type of program, the monitor or executive system, to actually operate the computer system. Jobs to be handled by the computer are stacked on magnetic tape. The monitor system calls on one job after another, performs necessary translation (several languages may be available), and handles error situations. Operator intervention is kept to a minimum, thus reducing the idle time for the computer system.

The FORTRAN Language. FORTRAN is a typical computer language. It uses symbols that the computer can understand and requires that the rules for their use be closely followed. It also eliminates many of the detailed computer-control operations from the programs and uses a problem-statement format close to that of the mathematical equation.

The engineer describes his problem in FORTRAN, which is then translated into machine language by the computer itself with the aid of a program called the FORTRAN Processor. The resulting machine-language program is then ready for use, (Figure 8.3.1.2a).

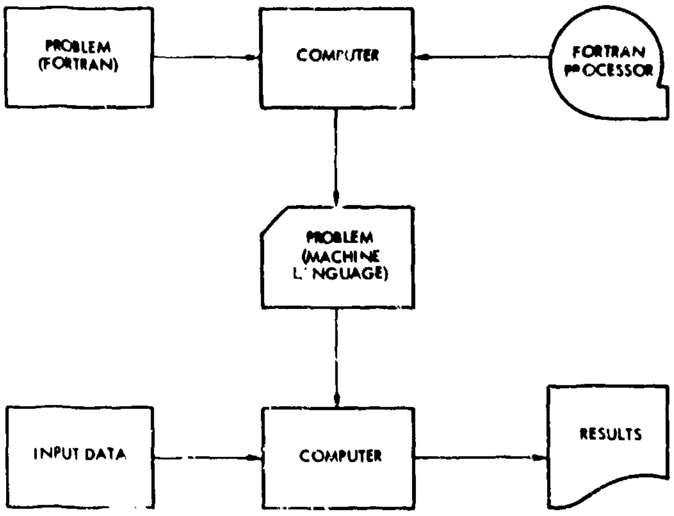


Figure 8.3.1.2a. Procedure Followed by Computer and Peripheral Equipment When Program is Written in FORTRAN

(Program is translated from FORTRAN into machine language by a FORTRAN Processor, and then operates on input data.)

Statements are the sentences of FORTRAN. They may:

1) Define the arithmetic steps to be accomplished by the computer

2) Provide information for control of the computer during execution of the program

3) Describe input and output operations necessary to bring in data and write the results

4) Specify certain additional facts such as the dimensions of a variable which appears in the program with subscripts.

A program for evaluating the roots of $ax^2 + bx + c = 0$ is shown in Figure 8.3.1.2b.

```
1    READ(5,2) A,B,C
2    FORMAT (3F10.0)
     D=B**2-4.0*A*C
     IF(D) 5,3,3
3    ROOT1=(-B+SQRT(D))/(2.0*A)
     ROOT2=(-B-SQRT(D))/(2.0*A)
     WRITE(6,4) A,B,C,ROOT1,ROOT2
4    FORMAT (3F10.0,2F10.4)
     GO TO 1
5    WRITE(6,2) A,B,C
     GO TO 1
     END
```

Figure 8.3.1.2b. Typical FORTRAN Program for Evaluating a Quadratic Equation

(Program is set up to be repeated until the input data, in this case punched cards, are exhausted.)

An example of an arithmetic statement as it would appear on a FORTRAN coding sheet is:

RØØT = (− B + SQRT (B** 2 − 4.*A*C))/(2.*A)

which means: the quantity to be known as RØØT is equal to — that is, can be determined by — evaluating

$$\frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

where A, B, C are given values stored within the computer.

Arithmetic statements. These look like simple statements of equality. The right side of all arithmetic statements is an expression which may involve parentheses, operation symbols, constants, variables, and functions combined in accordance with a set of rules much like that of ordinary algebra. The symbols + and − are employed in the usual way for addition and subtraction. The symbol * is used for multiplication, and the symbol / is used for division. The fifth basic operation, exponentiation, is represented by the symbol **. A**3 is used to represent A^3.

The FORTRAN arithmetic expression A**B*C + D**E/F − G means $A^B C + (D^E/F) - G$. Thus, if parentheses are not used to specify the order of operations, the order is assumed to be: 1) exponentiation 2) multiplication and division 3) addition and subtraction. Parentheses are employed in the usual way to specify order. For example, $(A(B + C))^D$ is written in FORTRAN as (A*(B + C)) **D.

ISSUED NOVEMBER 1968
SUPERSEDES MAY 1964

There are three exceptions to the ordinary rules of mathematical notation. These are:

1) In ordinary notation AB means A · B or A times B. However, AB never means A*B in FORTRAN. The multiplication symbol cannot be omitted.

2) In ordinary usage, expressions like A/B · C and A/B/C are considered ambiguous. However, such expressions are allowed in FORTRAN and are interpreted as follows:

A/B*C means (A/B)*C
A*B/C means (A*B)/C
A/B/C means (A/B)/C

Thus, for example, A/B/C*D*E/F means ((((A/B)/C)*D)*E)/F. That is, the order of operations is simply taken from left to right, in the same way that A + B − C + D − E means (((A + B) − C) + D) − E.

3) The expression A^{B^C} is often encountered. However, the corresponding expression using FORTRAN notation, A**B**C, is not allowed in the FORTRAN language. It should be written as (A**B)**C if $(A^B)^C$ is meant, or as A**(B**C) if $A^{(B^C)}$ is meant.

In addition to constants, simple variables and operations and functions may also be expressed. For example, SQRT (   ) indicates the square root of the expression in parentheses. Typical functions are:

ABS(X)	$\lvert X \rvert$
SQRT(X)	$\sqrt{X}$
SIN(X)	sin X
CØS(X)	cos X
ATAN(X)	arctan X
EXP(X)	e^x
ALØG(X)	$\log_e X$

Input/Output Statements. These are used to bring data into the computer to be stored for processing, and to send out results. Typical examples are:

READ 1, A, B, C — This statement would cause the next card in the card reader to be read and the three numbers on it stored in locations assigned to the values A, B, and C.

PRINT 2, RØØT — This would cause the number in storage identified as the variable RØØT to be printed.

PUNCH 4, SUM A, SUM B — This statement would cause the two values SUM A and SUM B to be punched on a single card.

Similar input-output statements are included in the program for reading and writing on magnetic tapes and magnetic drums, and for such operations as rewinding or backspacing tapes. The numbers which follow the statements in these examples specify the format in which the input or output should appear.

Control Statements. In a FORTRAN program, any statement which is referred to by another statement must be given an identifying number. The control statements refer to these identifying numbers for the purpose of branching from one part of the program to another.

Important statements in this category are illustrated by the following examples:

GO TO 4 — This statement indicates that the next statement to be executed (after having been converted to a machine-language program, of course) is the one numbered 4.

GO TO (4, 18, 20, 40), K — This statement is referred to as a computed GO TO since the value of K is computed in a previous statement. If at the time this GO TO is executed, K = 3, then the third alternate (statement 20) would be the one chosen.

8.3.1.3 METHODS AND TECHNIQUES. This section explains some of the procedures or techniques used in problem definition to simplify communication with the computer.

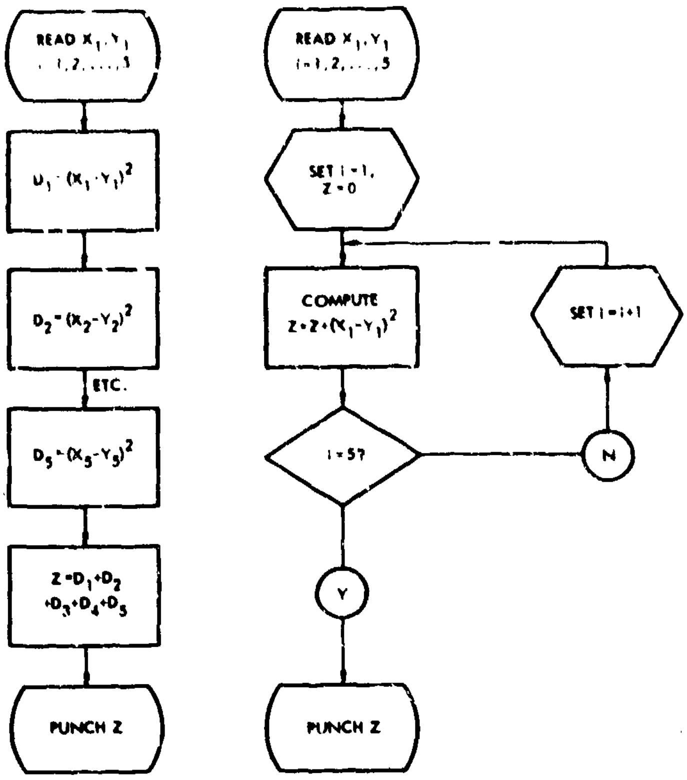
Block Diagramming. A block diagram is a picture of the steps which must be performed to accomplish a particular job. The major function of the diagram is to clarify what must be done as a result of each decision. Since the computer has no way of anticipating the requirements of a program, it must be provided with all the information needed to reach a solution. The amount of information put into the block diagram depends upon personal preference, programing techniques, and type of computer to be used. To illustrate, consider a problem which involves repetition of the same operation a number of times. Let the problem be to evaluate

$$Z = \sum_{i=1}^{5} (X_i - Y_i)^2$$

This might be block diagramed in either of the two ways illustrated in Figure 8.3.1.3.

The two diagrams in Figure 8.3.1.3 illustrate an important programing concept — that of looping. Diagram *a* corresponds to a program in which each $(X_i - Y_i)^2$ is computed separately and then all are summed to obtain Z. With only five values of i, this is not too unlikely a method of approaching the problem. But, consider a situation in which i = 100 or 1000 or may vary according to some other characteristic of the problem of which this computation is a part. In such a case, both the diagram and the program would be large and time consuming.

In diagram *b*, advantage has been taken of the computer's ability to make logical decisions and to modify its own program. Since the number of times the computation is repeated depends only on the value of i (and the presence of successive values X_i and Y_i), this one diagram — and the program which would be based on it — applies for any value of i.



(a) (b)

Figure 8.3.1.3. Block Diagrams for Use in Evaluating $Z = \sum_{i=1}^{5} (X_i - Y_i)^2$

(Diagram (a) corresponds to a program in which each $(X_i - Y_i)^2$ is computed separately; the answers are then summed up. This approach is adequate when only a few values are used. For larger numbers of operations, "looping" is used, as shown in (b). This method is based on the ability of the computer to make logical decisions and to modify its own program.)

Subscripting. In computer work, subscripts are used in two ways:

1) To specify elements of arrays such as:

$$\begin{array}{llllll} A_1 & \text{or} & A_{11} & A_{12} & \cdots & A_{1N} \\ A_2 & & A_{21} & A_{22} & \cdots & A_{2N} \\ A_3 & & A_{31} & A_{32} & \cdots & A_{3N} \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ A_M & & A_{M1} & A_{M2} & \cdots & A_{MN} \end{array}$$

This allows reference to elements of an array through simple manipulation of the subscripts.

2) To specify the chronological order in which a procedure occurs. For example, in Figure 8.3.1.3, diagram *b*, subscript i is used not only to denote which member of the X and Y array is being operated on, but also to indicate how many times the computational step $Z = Z + (X_i - Y_i)^2$ has been performed. This use is not generally familiar to the engineer, but is important in iteration.

Absolute Values. This concept is important because of the way in which a computer makes logical decisions. Computer decisions are based on whether a number is positive, zero, or negative. For example, assume that two possibilities exist, depending on whether A < 500 or A ≥ 500. First, subtract 500 from A, so that A − 500 = E. Then, the decision is based upon the size of E. If E is negative, procedure 1 is carried out; if E is positive or zero, procedure 2 is used.

If the difference represents the error in a procedure (that is, 500 is the true value and A is the estimate), the absolute value of the error must be less than a prescribed amount e and the test is: If |E| − e ≥ 0, use procedure 1; if |E| − e < 0, use procedure 2. The absolute value must be used, for in general there is no prior knowledge of whether the difference A − 500 will be positive or negative. All the alternatives must be spelled out to the computer in the program.

In many practical cases the relative error is a better measure of the error than the absolute error of a result. The relative error test for the preceding situation would be stated: if |E/500| − e ≥ 0, use procedure 1; if |E/500| − e < 0, use procedure 2.

Errors. There are several sources of errors in computation which are important in computer work.

Initial Error. If x is the true value of a data reading and x^* is the reading used in computation (reflecting an error in measurement, perhaps), the initial error is $x - x^*$.

Rounding Error. This type of error results when the less significant digits of a quantity are deleted and a rule of correction is applied to the remaining part.

Truncation Error. To simply chop off at four decimal places for pi, giving 3.1415, would result in a truncation error. Another common source of truncation error is in chopping off all terms in an infinite series expansion after a particular term. For example, cutting the series for e^x at

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

gives a truncation error, sometimes called residual error for series approximations.

Propagated Error. If x is the true value of a variable and x^* the value used in computation, then $f(x) - f(x^*)$ is the propagated error.

Algorithms. An algorithm is a theorem which may state that a solution to a problem, and or a procedure for obtaining the solution, exists. The term is frequently encountered in computer literature because the form of an equation is often all important in programing efficiently. The following example illustrates the use of algorithms.

Example. The square root of a positive real number, A, can be computed by using the algorithm

$$x_{i+1} = \frac{1}{2}\left(x_i + \frac{A}{x_i}\right)$$

This equation also introduces the idea of iteration. For example, to obtain an estimate of $A^{1/2}$, start with a first guess x_0. Substitute this into the right side of the equation to obtain the next estimate of $A^{1/2}$. A few of the steps are:

$$x_1 = \frac{1}{2}\left(x_0 + \frac{A}{x_0}\right)$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{A}{x_1}\right)$$

$$x_3 = \frac{1}{2}\left(x_2 + \frac{A}{x_2}\right)$$

$$x_{i+1} = \frac{1}{2}\left(x_i + \frac{A}{x_i}\right)$$

Note that the algorithm states a procedure for solution of the problem, not just one formula evaluation. Also, note the use of the subscript i; that is, i + 1 indicates a result dependent upon the previous result subscripted by i.

Taking a value of A (say 25) for which the square root is known, and performing the indicated operations will make this algorithm clear. If 2 is used as a starting estimate, then

$$x_0 = 2$$

$$x_1 = \frac{1}{2}\left(2 + \frac{25}{2}\right) = 7.25$$

$$x_2 = \frac{1}{2}\left(7.25 + \frac{25}{7.25}\right) = 5.35$$

$$x_3 = \frac{1}{2}\left(5.35 + \frac{25}{5.35}\right) = 5.01$$

8.3.2 Principles of Iteration

Certain classes of problems can be solved by standard mathematical methods, such as the evaluation of a general formula. In many cases, however, the formulas are too complex for easy solution, or — as in the case of equations of a higher order than quartic — no general formulas can be developed. In such cases, some method of approximation or iteration must be used to arrive at a solution.

Any problem requiring simple mathematical analysis can be handled by a digital computer. But because of its great speed, the computer can go far beyond such methods. Through successive approximation, or iteration, it can arrive quickly at answers of any desired accuracy. Thus, iteration is one of the most powerful tools available to the engineer working with a digital computer. This article shows some of the types of problems that can be handled by a computer through iteration.

There are three cases in which engineering problems are well suited to iteration procedures:

1) The mathematical statement of the problem requires an iterative approach for evaluating one or more of the variables.

2) Many possibilities are to be evaluated to find the best design. Often this problem can be reduced to the preceding situation by selection of an appropriately expressed mathematical criterion of the optimum solution.

3) The mathematical expression for the physical problem is to be evaluated for many values of one or more known parameters, such as time in a motion problem or degree of rotation in a geometry problem.

8.3.2.1 GENERAL PROCEDURE. An equation which arises frequently in absorption problems in optics, electronics, and nuclear engineering illustrates the iterative approach to problem solving:

$$x = a e^{bx} \qquad \textbf{(Eq 8.3.2.1a)}$$

where a and b are constants.

This equation cannot be solved explicitly for x, so the following iteration procedure is used:

1. Guess a value for x.
2. Use this guess with Equation (8.3.2.1a) to give a new value for x.
3. Consider this new value of x the next guess.
4. Repeat steps 2 and 3 until two successive guesses either agree, or differ by an amount less than the allowable error.

A FORTRAN program to solve this problem, where $x = 0.2e^{[illegible]}$, is shown in Figure 8.3.2.1. If this program were translated into machine language and the resulting program run on a computer the successive estimates of x would be:

1.000000
0.329744
0.235848
0.225032
0.223818
0.223682
0.223667

The last value in this list satisfies the requirement that the difference between it and the previous estimate be less than 0.00065 absolute value.

This simple direct iteration procedure is limited in the number of problems it will solve. Direct iteration is based on the formula $x_{n+1} = f(x_n)$. This procedure will converge only for $-1 < f'(x_n) < 1$, where $f(x_n) = Ae^{bx_n}$ for this case, and f′ indicates the first derivative.

When simple iteration fails to produce convergence, the Newton Raphson method is used to obtain an estimate. The Newton-Raphson iterative equation is

```
C READ VALUES FOR A,B, AND AN INITIAL X
      READ(5,1) A,B,X
  1   FORMAT (3F10.6)
C WRITE OUT ESTIMATE OF X
  2   WRITE(6,2) X
  3   FORMAT (F10.6)
C FORTRAN ARITHMETIC STATEMENT
C OF EQUATION TO BE SOLVED
      XNEW=A*EXP(B*X)
C FIND ABSOLUTE VALUE OF DIFFERENCE
C IN LAST TWO ESTIMATES
      TEST=ABS(X-XNEW)
C IF DIFFERENCE IS LESS OR EQUAL TO
C .00005 GO TO 5,OTHERWISE 4
      IF (TEST-.00005) 5,5,4
C STORE NEW ESTIMATE OF X
  4   X=XNEW
C RETURN TO STEP 2 AND REPEAT
      GO TO 2
C WRITE OUT LAST ESTIMATE OF X
  5   WRITE(6,3) XNEW
C END OF PROGRAM
      STOP
      END
```

Figure 8.3.2.1. FORTRAN Program for Evaluating a Typical Iterative Problem, in this case, $x = 0.2e^{x}$

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \qquad \textbf{(Eq 8.3.2.1b)}$$

Applying this method to Equation (8.3.2.1a) gives $F(x_n) = x_n - Ae^{Bx_n}$. From the Newton-Raphson formula, Equation (8.3.2.1b)

$$x_{n+1} = x_n - \frac{x_n - A\,e^{Bx_n}}{1 - B\,A\,e^{Bx_n}} \qquad \textbf{(Eq 8.3.2.1c)}$$

To solve this problem in FORTRAN, replace the statement for XNEW in Figure 8.3.2.1 with the arithmetic statement : 2 XNEW = X − (X − A* EXP (B*X)/(1 − A*B*EXP (B*X). This program will produce the following successive values of x:

1.00000
0.19742
0.22364
0.22366

Note the increased rate of convergence obtained with this method. Only three new estimates are computed, compared to seven in the previous example.

8.3.2.2 OPTIMIZATION. The following problem illustrates one origin of the type of equation just discussed. Analysis of the problem is based on the classical optimization principle of equating the derivative of a function of one variable to zero. The values of the variable which satisfy the resulting equation are those for which the original function is either a maximum or a minimum.

For high-potential conduction through walls, tubular insulators are used, (see Figure 8.3.2.2). These are covered with metal on the inner and outer surfaces. What must be the ratio of the external diameter, 2R, to the bore, 2r, for the cross section Q to be a minimum?

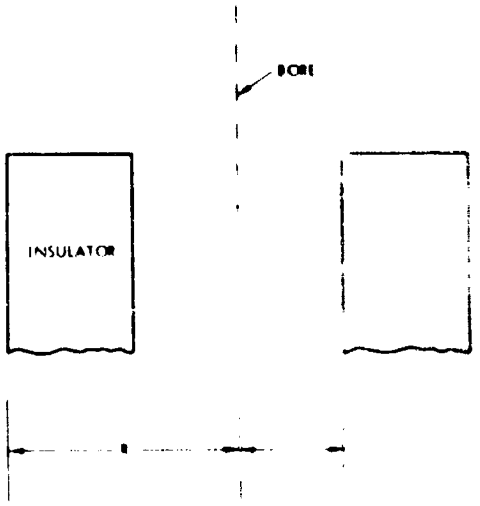


Figure 8.3.2.2. Tubular Insulator
(Discussed in example problem.)

Ratio q of the line voltage to the maximum admissible field strength is

$$q = r \ln \frac{R}{r} \qquad \textbf{(Eq 8.3.2.2a)}$$

Cross section Q is

$$Q = \pi (R^2 - r^2) \qquad \textbf{(Eq 8.3.2.2b)}$$

With $x = R/r$, then $r = q/\ln x$. Thus,

$$Q = \frac{\pi q^2 (x^2 - 1)}{(\ln x)^2} \qquad \textbf{(Eq 8.3.2.2c)}$$

Since Q is to be a minimum, it is necessary to find a point on the curve defined by Equation (8.3.2.2c) where the slope is zero. To do this, the first derivative is set equal to zero. First,

(Eq 8.3.2.2d)

$$\frac{dQ}{dx} = \pi q^2 \left[\frac{2x}{(\ln x)^2} - \frac{2(x^2 - 1)}{x(\ln x)^3} \right]$$

From the bracketed expression, $x - (x^2 - 1)/x \ln x = 0$, or $\ln x = 1 - 1/x^2$. This in turn gives $x - e^{1-1/x^2} = 0$.

The expression for x is easily recognizable as the type of problem discussed in Detailed Topic 8.3.2.1.

8.3.2.3 GEOMETRIC RELATIONSHIPS. Typical examples of problems which can be solved by an interactive solution are presented as follows:

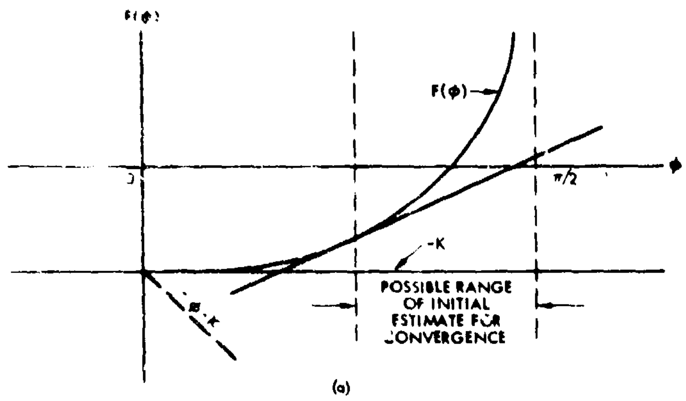
Helical Gears. An example which lends itself to an iterative solution is an equation which occurs in the design of helical gears:

(Eq 8.3.2.3a)

$$K = \text{involute of } \phi = \tan\phi - \phi$$

where K is obtained from standard tables. Find an angle ϕ which satisfies the equation.

The Newton-Raphson method can again be used. However, it is worthwhile to consider the best way to get a first estimate of ϕ, since this estimate will affect the rate of convergence.



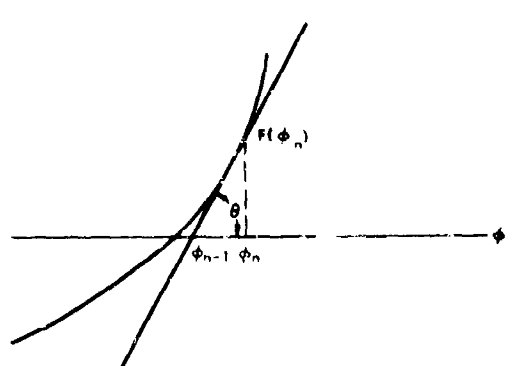


Figure 8.3.2.3a, b. **Graph of Newton's Function, F(φ) = tan φ-φ-K**

(Solved in example problem. Graph in (b) shows enlargement of area of intersection of F (φ) with the φ axis and illustrates the basis for Newton's procedure.)

A graph of Newton's function, $F(\phi) = \tan\phi - \phi - K$, is shown in Figure 8.3.2.3a. An enlargement in the area of intersection of $F(\phi)$ with the ϕ axis shows the basis for Newton's procedure, Figure 8.3.2.3b. If ϕ_n is the first estimate of the root, Figure 8.3.2.3b indicates that the next good estimate would be the intersection of the projected slope of $F(\phi)$ evaluated for ϕ_n. The following use of the construction angle θ shows this to be Newton's procedure

(Eq 8.3.2.3b)

$$\tan\theta = \frac{F(\phi_n)}{\phi_n - \phi_{n+1}} = F'(\phi_n)$$

or

$$\phi_{n+1} = \phi_n - \frac{F(\phi_n)}{F'(\phi_n)} \qquad \textbf{(Eq 8.3.2.3c)}$$

The dotted lines in Figure 8.3.2.3a show the limits on the range of the initial estimate for which Newton's technique will converge to the proper value.

A good first estimate can often be determined by a careful examination of the problem to be solved. In this case the first two terms of the trigonometric series for $\tan\phi$ give

$$\tan\phi \cong \phi + \frac{\phi^3}{3} \qquad \textbf{(Eq 8.3.2.3d)}$$

Substituting this in Equation (8.3.2.3a) and solving for θ gives $\theta \cong (3K)^{1/3}$. Thus, an initial guess which should be reasonably close to the solution can be computed from the given value of K. This can be an important advantage where convergence is likely to be slow.

To illustrate, Table 8.3.2.3 shows a solution for two values of K (0.001 and 0.01). In each case, the initial estimate of $\theta = 0.52359$ radians (30 degrees) is used rather than a value computed as described above.

Table 8.3.2.3 Values of Angle φ (rad)

K = 0.001	K = 0.01
0.52359	0.52359
0.36535	0.39235
0.25482	0.32546
0.18624	0.30818
0.15296	0.30702
0.14440	0.30701
0.14385	
0.14372	

Note that for $K = 0.01$, fewer iterations were required because the initial estimate was closer to the correct result. Had the approximation formula developed earlier been used, the initial estimates would have been 0.14423 and 0.31072, and no more than two iterations would have been required in either case.

Circular Segment. A simple geometry problem which lends itself to iterative solution is that of finding the angle for which the arc and chord in a circle of given radius will enclose a stated area, (Figure 8.3.2.3c).

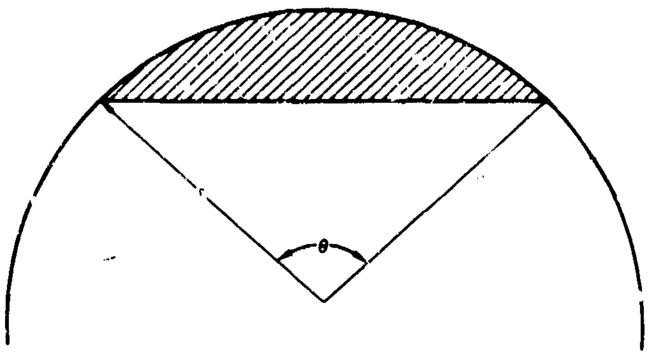


Figure 8.3.2.3c. Geometry Problem Discussed In Text
(Problem is to find the angle for which the arc and chord in a circle of given radius will enclose a stated area.)

Area A is

$$A = \frac{\pi r^2 \theta}{360} - \frac{r^2 \sin\theta}{2} \quad \text{(Eq 8.3.2.3e)}$$

Since θ cannot be found directly, the Newton-Raphson technique is used to find a solution. Converting θ to radians and applying the Newton-Raphson formula gives

$$\theta_{n+1} = \theta_n - \frac{F(\theta_n)}{F'(\theta_n)} \quad \text{(Eq 8.3.2.3f)}$$

where

$$F(\theta_n) = \frac{r^2 \theta_n}{2} - \frac{r^2 \sin\theta_n}{2} - A \quad \text{(Eq 8.3.2.3g)}$$

and

$$F'(\theta_n) = \frac{r^2}{2} - \frac{r^2 \cos\theta_n}{2} \quad \text{(Eq 8.3.2.3h)}$$

8.3.2.4 POLYNOMIALS. Iteration can be used to find a root of a cubic such as $ax^3 + bx^2 + cx + d = 0$. Newton's technique generally suffices for this case.

For polynomials of degree greater than 4, some type of iterative procedure must be used. There are many techniques available to handle the higher degree polynomials, but they will not be discussed here. Standard programs for evaluating such equations have been developed.

The following problem shows how a simple circuit can give rise to a cubic equation. Generally it is simpler to make successive approximations than to solve the cubic.

In the circuit shown in Figure 8.3.2.4, find the potential difference V from (1) to (2) and the current I flowing in the circuit. Equation for current is: $I = kV^{3/2}$; given values are: $k = 10^{-6}$ amps/volts$^{3/2}$; $E = 100$ volts; $R = 5000$ ohms.

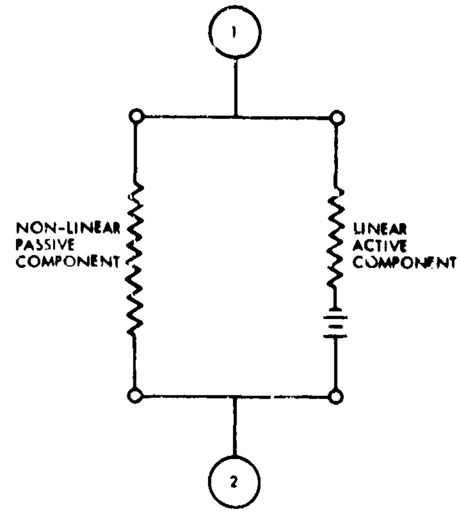


Figure 8.3.2.4. Electrical Circuit for Problem Illustrating Computer Solution of Polynomials
(Solution of this circuit involves a cubic equation)

From the basic relationship for potential drop, $V = E - IR$

$$V = 100 - 5000\,kV^{3/2} \quad \text{(Eq 8.3.2.4a)}$$

or

$$V = 100 - 0.05\,V^{3/2} \quad \text{(Eq 8.3.2.4b)}$$

This cubic equation is easily solved with Newton's technique.

8.3.2.5 DIFFERENTIAL EQUATIONS. Iteration can be used to obtain numerical solutions to differential equations. To illustrate, consider a problem for which the exact integral is known

$$\frac{dy}{dx} = xy \quad \text{(Eq 8.3.2.5a)}$$

Find y as x varies from 0 to 1.0. Initial conditions are $x = 0$, $y = 1.0$.

Euler's method for evaluating the equation numerically for $y = f(x)$ over the range in x is illustrated in Figure 8.3.2.5a.

Taylor's series may be used to estimate the value of a function $y = f(x)$ in the vicinity of a given point y_n

$$y_{n+1} = y_n + \left(\frac{dy}{dx}\right)_n \frac{\Delta x}{1\,!} + \left(\frac{d^2y}{dx^2}\right)_n \frac{\Delta x}{2\,!} + \cdots \quad \text{(Eq 8.3.2.5b)}$$

Consideration of the first two terms only gives

$$y_{n+1} = y_n + \left(\frac{dy}{dx}\right)_n \Delta x \quad \text{(Eq 8.3.2.5c)}$$

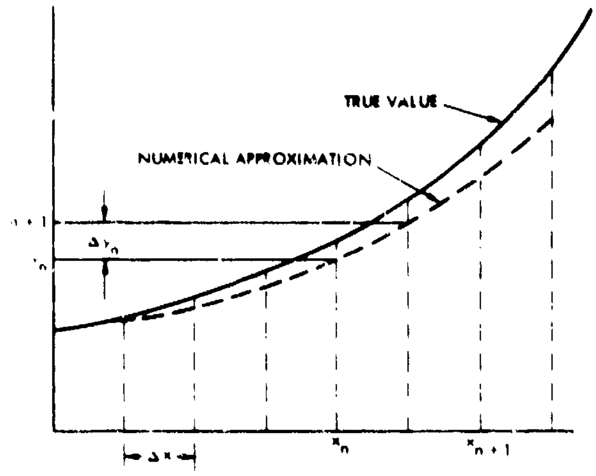


Figure 8.3.2.5a. Euler's Method for Numerically Evaluating Example Equation, Illustrating Computer Solution of a Differential Equation

where, from the given differential equation

$$\left(\frac{dy}{dx}\right)_n = x_n y_n \qquad \text{(Eq 8.3.2.5d)}$$

Figure 8.3.2.5a shows that

$$x_{n+1} = x_n + \Delta x \qquad \text{(Eq 8.3.2.5e)}$$

Successive evaluation of Equations (8.3.2.5c), (8.3.2.5d) and (8.3.2.5e) form the iterative scheme necessary to calculate repeatedly the numerical approximation for y at regular intervals in x over the required range in x.

First step in using these iterative equations is to apply the initial conditions as follows:

$$\left(\frac{dy}{dx}\right)_0 = x_0 y_0$$

$$y_1 = y_0 + \left(\frac{dy}{dx}\right)_0 \Delta x$$

$$x_1 = x_0 + \Delta x$$

The FORTRAN program to evaluate the stated problem is shown in Figure 8.3.2.5b. Table 8.3.2.5 shows results for $\Delta x = 0.05$.

The analytical solution to Equation (8.3.2.5a) for the stated boundary conditions is $y = e^{x^2/2}$; and for $x = 1.0$, $y = 1.64872$. The error in the numerical solution is 1.59594 − 1.64872 = 0.05278 or about 3 percent. Considering the size of the interval, this is not out of line.

Increased accuracy could be obtained by decreasing Δx or by using a more sophisticated technique. In the first case, decreasing the value of Δx may increase the rounding error and hence make it necessary to carry along a greater number of significant digits. The second alternative must be viewed in the light of the accuracy required. The more sophisticated techniques require greater programing effort and more computer time. It is illogical to spend this time and effort to obtain accuracy of 0.01 percent if the required accuracy is only 2 percent — particularly in cases where the input data is accurate to, for instance, 5 percent.

The more sophisticated techniques usually make use of a weighting of several previous estimates for the derivative in an equation similar to

$$y_{n+1} = y_n + w_1(w_2 y_n' + w_2 y_{n-1}' + \cdots)\Delta x \qquad \text{(Eq 8.3.2.5f)}$$

```
      DELX=.05
      X=0.0
      Y=1.0
1     WRITE(6,2) X,Y
2     FORMAT (2F7.5)
      DELY=X*Y*DELX
      X=X+DELX
      Y=Y+DELY
      IF(X-1.0) 1,1,3
3     STOP
      END
```

Figure 8.3.2.5b. Program for Evaluating Differential Equation

Table 8.3.2.5 Printout Values for Program in Figure 8.3.2.5b

x	y
0.00000	1.00000
0.05000	1.00000
0.10000	1.00250
0.15000	1.00751
0.20000	0.01507
0.25000	1.02522
0.30000	1.03803
0.35000	1.05361
0.40000	1.07204
0.45000	1.09348
0.50000	1.11809
0.55000	1 14604
0.60000	1.17756
0.65000	1.21288
0.70000	1.25230
0.75000	1.29613
0.80000	1.34474
0.85000	1.39852
0.90000	1.45796
0.95000	1.52357
1.00000	1.59594

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SUPERSEDES: MAY 1964

where the prime indicates a derivative and the w_i are selected weighting constants. However, the logic of predicting the next value for y in terms of the last calculated point and the derivative evaluated according to the given differential equation, remain the same.

8.3.2.6 HIGHER ORDER DIFFERENTIALS. It is easy to extend the simple numerical integration procedure just discussed to higher order differential equations. For example, suppose the following is to be solved over the interval 0 to 1.0:

(Eq 8.3.2.6a)

$$\frac{d^2y}{dx^2} + A\frac{dy}{dx} + Bxy = 0$$

where $x_0 = 0$, $y_0 = 1.0$, $(dy/dx)_0 = 1$, $A = 2$, and $B = 3$.

The analytic solution must be in terms of an infinite series, therefore, whether a series is determined or numerical integration is performed, the solution involves considerable computation. The series representation allows control of the error, while the stepwise integration procedure to be described here may not.

One approach to this problem is to start with

(Eq 8.3.2.6b)

$$\left(\frac{d^2y}{dx^2}\right)_0 = -A\left(\frac{dy}{dx}\right)_0 - Bx_0y_0$$

and then obtain successive values of each of the variables from

$$y_{n+1} = y_n + \left(\frac{dy}{dx}\right)_n \Delta x \qquad \textbf{(Eq 8.3.2.6c)}$$

$$x_{n+1} = x_n + \Delta x \qquad \textbf{(Eq 8.3.2.6d)}$$

(Eq 8.3.2.6e)

$$\left(\frac{dy}{dx}\right)'_{n+1} = \left(\frac{dy}{dx}\right)_n + \left(\frac{d^2y}{dx^2}\right)_n \Delta x$$

(Eq 8.3.2.6f)

$$\left(\frac{d^2y}{dx^2}\right)'_{n+1} = -A\left(\frac{dy}{dx}\right)_{n+1} - Bx_{n+1}y_{n+1}$$

A computer solution to this problem is shown in Figure 8.3.2.6.

In the first few problems shown, iteration was used to improve the estimate of a single solution. In the case of differential equations, iteration provided successive values of y for finite changes in x. These are two entirely different concepts of iteration.

```
      A=2.0
      B=3.0
      XN=0.0
      YN=1.0
      DYDX=1.0
1     D2YDX=-A*DYDX-B*XN*YN
      WRITE(3,2) XN,YN,DYDX,D2YDX
2     FORMAT (4F11.8)
      IF(XN-1.0) 3,4,4
3     YN=YN+DYDX*.05
      XN=XN+.05
      DYDX=DYDX+D2YDX*.05
      GO TO 1
4     STOP
      END
```

XN	YN	DYDX	D2YDX
.00000000	1.0000000	1.00000000	—2.0000000
.05000000	1.0500000	.90000000	—1.9575000
.10000000	1.0950000	.80212500	—1.9327500
.15000000	1.1351063	.70548750	—1.9217729
.20000000	1.1703807	.60939886	—1.9210261
.25000000	1.2008506	.51334756	—1.9273331
.30000000	1.2265180	.41698091	—1.9378280
.35000000	1.2473670	.32008951	—1.9499144
.40000000	1.2633715	.22259379	—1.9612334
.45000000	1.2745012	.12453212	—1.9696408
.50000000	1.2807278	.02605008	—1.9731919
.55000000	1.2820303	—.07260961	—1.9701310
.60000000	1.2783998	—.17111607	—1.9588875
.65000000	1.2698440	—.26906945	—1.9380749
.70000000	1.2563910	—.36596420	—1.9064927
.75000000	1.2380928	—.46128884	—1.8631311
.80000000	1.2150284	—.55444540	—1.8071774
.85000000	1.1873061	—.64480427	—1.7380221
.90000000	1.1550659	—.73170538	—1.6552671
.95000000	1.1184806	—.81446874	—1.5587323
1.00000000	1.0777572	—.89240536	—1.4484609

Figure 8.3.2.6. FORTRAN Program and Printout for Higher Order Differential Equations

(Solution of these equations involves an extension of simple numerical-integration procedures used for ordinary differential equations.)

8.3.3 Mathematical Models

As its name implies, a mathematical model is simply the representation of a part, system, or process by suitable mathematical relationships. The model may be used to simulate actual performance, much as with a physical prototype. Size and complexity of the system represented may range from a simple gear train to an entire automobile.

Although mathematical models serve much the same function as a physical model, (see Figure 8.3.3), they have their own characteristics, limitations, and advantages.

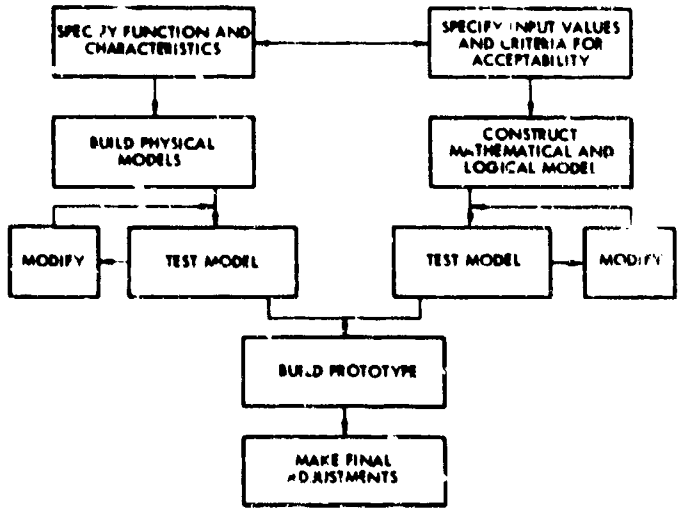

Figure 8.3.3. Two Approaches to Designing a Part or System
(The "build and try" approach shown at left is analogous to the computer approach at right, but is more time consuming.)

In the build-and-try approach, the specification of parameters (size, shape, material) may often be presented in qualitative terms. In a mathematical model, these qualitative values must be replaced by specific quantitative values. Criteria for a better design must be expressed in precise mathematical terms.

Although mathematics may suffice to describe completely a component of the system, rarely will known relationships be adequate to describe completely the effects of components working together. Considerable logic, often taken from engineering experience, must be used to create a realistic, accurate representation of the system to be studied.

Finally, possible modifications may be part of the mathematical model and programed so that automatic modifications take place as a result of decisions based on steps in the calculation. Naturally, these modifications must have been considered during construction of the model. They often represent a search for a better design according to the defined criteria.

The examples which are given in this Sub-Topic illustrate various aspects of mathematical model building for computer analysis. Most of the descriptions are concerned only with that facet of the physical problem which requires something more than conventional analytic techniques. In a practical situation, many more calculations are performed than indicated here.

8.3.3.1 SOLENOID DESIGN. The following problem is most typical of design problems where several variables are involved and there is no obvious procedure for arriving at the best design. Choose a wire for a solenoid such that the power consumed will be less than some fixed amount and the solenoid will give a fixed pull, (see Figure 8.3.3.1).

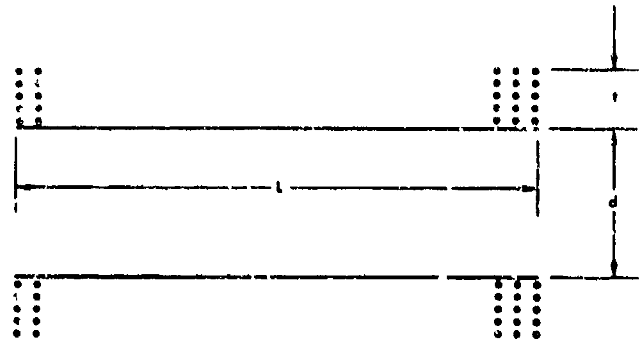

Figure 8.3.3.1. Solenoid in Example Problem

Engineering handbooks provide the following relationships. Pull $F = IN/l$; current $I = V/R$; power $P = V^2/R$; resistance $R = 4\alpha^2\rho L/\pi$; wire length $L = \pi DN$; solenoid diameter $D = d + t$; coil thickness $t = n\alpha$; number of layers $n = N/\alpha l$. In these equations, N = total number of turns; V = applied voltage; ρ = resistivity of wire; α = reciprocal of wire diameter; l = length of solenoid, and d = inside diameter of coil, (Figure 8.3.3.1).

The usual rules of analysis apply. That is, it is best to do some algebraic manipulation before substituting numbers, to give

$$P = \frac{FV}{\left(\frac{V}{4F\rho l} - \alpha^2 d\right)} \qquad \textbf{(Eq 8.3.3.1)}$$

which gives power in terms of wire size.

The important aspect of this problem is not that one can now compute the power output for a given size and thus choose an acceptable wire size, but rather that a computer program can be generalized such that, given any requirements for a solenoid, many different combinations may be tried quickly to find the suitable combination for the job at hand.

Thus, to design a solenoid for a given pull F, it might be desirable to vary l, d, and V or, perhaps, to limit t. With the given equations, a variety of optimizing programs can be written for any solenoid design.

8.3.3.2 HEAT TRANSFER PROBLEM. An insulating wall, (Figure 8.3.3.2a), is made of three parallel layers of different insulating materials. The outside temperatures, t_1 and t_4, are known. Conductivity, k_i, of each layer, (a straight line function of the mean temperature t_i'), is known. Find the quality of heat, Q, passing through a unit area of the wall per unit of time.

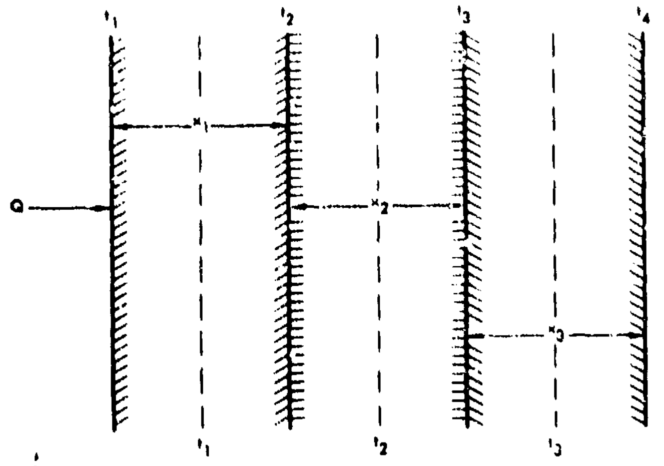

Figure 8.3.3.2a. Insulating Wall Discussed in Heat Transfer Problem

Heat Q is

(Eq 8.3.3.2a)

$$Q = \frac{t_1 - t_4}{\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}}$$

and values of k_i are

$$k_1 = a_1 t_1 + b_1 \qquad \text{(Eq 8.3.3.2b)}$$

$$k_2 = a_2 t_2 + b_2 \qquad \text{(Eq 8.3.3.2c)}$$

$$k_3 = a_3 t_3 + b_3 \qquad \text{(Eq 8.3.3.2d)}$$

where values for a_i and b_i are given constants for the particular intervals. Also

(Eq 8.3.3.2e)

$$t_1' = t_1 - \frac{(t_1 - t_2)}{2} = \frac{1}{2}(t_1 + t_2)$$

(Eq 8.3.3.2f)

$$t_2' = t_2 - \frac{(t_2 - t_3)}{2} = \frac{1}{2}(t_2 + t_3)$$

(Eq 8.3.3.2g)

$$t_3' = t_3 - \frac{(t_3 - t_4)}{2} = \frac{1}{2}(t_3 + t_4)$$

Temperatures t_2 and t_3 are not known.

If the k_i values are known, t_2 and t_3 may be determined from the steady-state requirement that the quantity of heat passing through each layer per unit of time must be the same. Thus

(Eq 8.3.3.2h)

$$\frac{k_1}{x_1}(t_1 - t_2) = \frac{k_2}{x_2}(t_2 - t_3) = \frac{k_3}{x_3}(t_3 - t_4)$$

which gives

(Eq 8.3.3.2i)

$$t_3 = t_4 + \frac{\frac{x_3}{k_3}(t_1 - t_4)}{\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}}$$

and

(Eq 8.3.3.2j)

$$t_2 = t_1 - \frac{\frac{x_1}{k_1}(t_1 - t_4)}{\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}}$$

But this produces a vicious circle; values of k_i are needed to determine t_2 and t_3, and vice versa. Therefore, iteration is required.

The procedure for an iterative solution is:

1. Make a reasonable guess at t_2 and t_3.
2. Use Equations (8.3.3.2e), (8.3.3.2f), and (8.3.3.2g) to determine the t_i' values.
3. Use Equations (8.3.3.2b), (8.3.3.2c), and (8.3.3.2d) to determine the k_i values.
4. From Equation (8.3.3.2a), determine Q.
5. Since values are known for k_i, once again compute t_2 and t_3 using Equations (8.3.3.2i) and (8.3.3.2j).
6. Repeat steps 2 through 5 until two successive values for Q agree.

This problem converges easily to a solution with the simplest iterative scheme, $x = f(x, y, z)$. Experience has shown that the initial estimates of the internal temperatures are not critical, except that t_1 must be greater than t_2, which must be greater than t_3, etc.

Data used in this example are: for layer 1, $b_1 = 0.0623$, $a_1 = 0.00010$; for layer 2, $b_2 = 0.0255$, $a_2 = 0.00005$; for layer 3, $b_3 = 2.4395$, $a_3 = 0.00060$; $t_1 = 1400$ F; $t_4 = 200$ F; $x_1 = 2.15$ in.; $x_2 = 2.0$ in.; and $x_3 = 6.0$ in.

Figure 8.3.3.2b shows the iterated solution to this problem with $Q = 25.38$ Btu/min with the corresponding mean temperature t_1', t_2' and t_3'.

Although convergence is possible in this problem, this is not true for all situations. A mathematical procedure of this type is valid only within a certain range of selected data.

```
      PQ=0.0
      READ(5,2) B1,B2,B3,T1,T4,X1,X2,X3
 1    READ(5,2) A1,A2,A3
 2    FORMAT (11F10.6)
      T2=2.0*(T1-T4)/3.0+T4
      T3=(T1-T4)/3.0+T4
 3    T1B=.5*(T1+T2)
      T2B=.5*(T2+T3)
      T3B=.5*(T3+T4)
      Z1=A1*T1B+B1
      Z2=A2*T2B+B2
      Z3=A3*T3B+B3
      Q=(T1-T4)/(X1/Z1+X2/Z2+X3/Z3)
      WRITE(6,2) Q,T1B,T2B,T3B
      IF(ABS(PQ-Q)-.00005) 1,1,4
 4    PQ=Q
      T3=T4+X3/Z3*Q
      T2=T1-X1/Z1*Q
      GO TO 3
      END
```

Iteration	Q	t_1'	t_2'	t_3'
1	26.925558	1200.0000	800.00000	400.00000
2	25.146838	1241.2234	671.36960	230.14618
3	25.385972	1254.9914	684.25925	229.26788
4	25.384732	1254.6856	684.23785	229.55224
5	25.384376	1254.6691	684.21790	229.54884
6	25.384383	1254.6698	684.21825	229.54845

Figure 8.3.3.2b. Program and Printout for Heat Transfer Problem

8.3.3.3 ROCKER ARM CAM PROBLEM. Figure 8.3.3.3 shows a rocker arm and cam to be used in a fuel pump. Specified are:

1) Axis of rotation of the off-center circular cam (X_2, Y_2)

2) Eccentricity, E, and radius, R, of cam

3) Axis of rotation of rocker arm (X_1, Y_1)

4) Required angular rotation, α, of rocker arm to give desired travel of pull rod.

To be determined are the necessary angle, θ, between rocker arm and vertical axis; and the necessary offset, L, for rocker arm.

First, boundary conditions are considered to obtain ex-

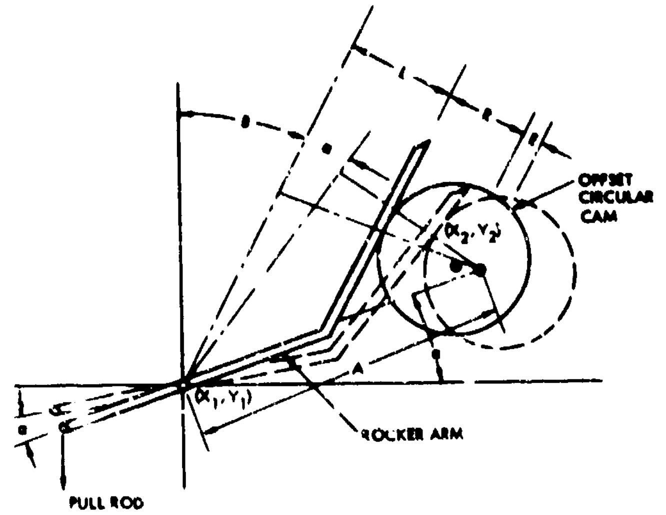

Figure 8.3.3.3. Fuel Pump Rocker Arm and Cam Discussed in Example Problem

pressions for B and L. For the high position of the rocker arm

(Eq 8.3.3.3a)

$$B = 90 \text{ deg} - \gamma - \text{arc sin}\left(\frac{L + R + E}{A}\right)$$

For the low position of the rocker arm

(Eq 8.3.3.3b)

$$L = A \sin (90 \text{ deg} - B - \gamma - \alpha) - (R - E)$$

Because of the transcendental functions these equations may not be solved explicitly for both L and B.

The direct iterative procedure for solution is:

1. Make a reasonable guess at L
2. Determine B from Equation (8.3.3.3a)
3. Use this value of B in Equation (8.3.3.3b) to obtain a value for L
4. Repeat steps 2 and 3 until two successive values for L agree.

This model requires iteration for the basic variables B and L. In practice, many other parameters are to be specified and can be computed directly once B and L have been determined.

This case is interesting because the straightforward iteration procedure diverges. If Equations (8.3.3.3a) and

(8.3.3.3b) are stated as $B = f_1(L)$, $L = f_2(B)$, then a successful iterative equation to replace Equation (8.3.3.3b) in step 3 is

(Eq 8.3.3.3c)

$$L_{i+1} = L_i - \left(\frac{f_2(B_{i+1}) - L_i}{2}\right)$$

where $B_{i+1} = f_1(L_i)$.

With this iterative procedure, convergence is relatively slow. An attempt to improve the convergence using the Newton-Raphson technique gives a divergent iterative scheme. However, a modification of the procedure shown does improve convergence. The improvement is made simply by changing the equation to read

(Eq 8.3.3.3d)

$$L_{i+1} = L_i - [f_2(B_{i+1}) - L_i]$$

Although this approach is obviously dangerous because arbitrary changes in the iterative equation are not easily justified, for this problem it does work. This manipulation also shows that experimentation with techniques is necessary in some instances when standard approaches fail; however, it is important to have some way of judging correctness of the results.

The direct and Newton-Raphson iterative techniques are not the only procedures available. For example, wide use has been made of a procedure credited to J. H. Wegstein of the National Bureau of Standards. The Wegstein method does not require evaluation of the first derivative and therefore is a powerful tool in cases where the first derivative is difficult or impossible to evaluate.

Both the Newton-Raphson and Wegstein methods may be extended to solve simultaneous equations

8.3.3.4 SPRING DESIGN. The design of springs is an excellent computer application. The following example concerns the design of a particular type of spring, but the general method of solution by computer applies to almost any spring problem.

A double helical spring is to support a given torque winding. Mean diameter and length of the spring must be within set limits, and length must be as small as possible. The variables specified (subscript 1 refers to inner spring, subscript 2 to outer spring) are: winding torque, M_1, M_2; maximum selected stress, or ultimate stress, of material, S_1, S_2; maximum torsion angle, T_1, T_2; required spacing between turn, K_1, K_2; required spacing between coils, C; and Young's modulus, E. Find: h_1, h_2 (Figure 8.3.3.4a); b_1, b_2 (Figure 8.3.3.4a); D_1, D_2 (spring diameter); L_1, L_2 (spring length); N_1, N_2 (number of turns in the springs).

Winding torque for a given torsion angle is

$$M = \frac{E\,b\,h^3\,T}{3.6\,DN} \qquad \text{(Eq 8.3.3.4a)}$$

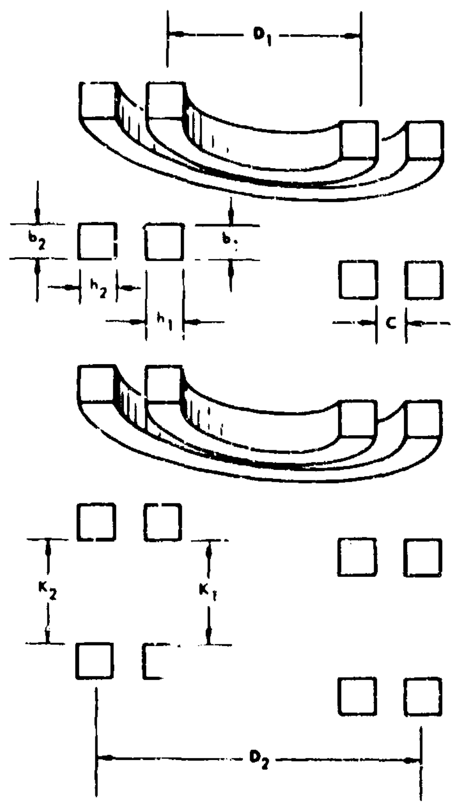

Figure 8.3.3.4a. Double Helical Spring Analyzed in Example Problem

Winding torque for ultimate stress of the material is

$$M = \frac{S\,b\,h^2}{6} \qquad \text{(Eq 8.3.3.4b)}$$

Outer length is

(Eq 8.3.3.4c)

$$L_2 = (N_2 + 1)(b_2 + K_2)$$

Inner diameter is

(Eq 8.3.3.4d)

$$D_1 = D_2 - h_1 - h_2 - 2C$$

For the outer spring, five variables (h_2, b_2, D_2, L_2, N_2) must be determined, but there are only three equations which apply. One approach to obtaining a solution is as follows:

1. Set D_2 (because limit exists), and N_2 (because this must be either an integer or an integer plus 0.5 — successive guesses will be simpler)

2. With Equations (8.3.3.4a) and (8.3.3.4b), solve for h:

$$h_2 = \frac{1.1\,D_2S_2N_2}{ET_2} \qquad \text{(Eq 8.3.3.4e)}$$

3. From Equation (8.3.3.4b),

$$b_2 = \frac{6M_2}{S_2 h_2^2} \qquad \text{(Eq 8.3.3.4f)}$$

4. Find spring length from

$$L_2 = (N_2 + 1)(b_2 + K_2) \qquad \text{(Eq 8.3.3.4g)}$$

This provides the parameters for the outer spring. At this point it is necessary to make an additional guess for the inner spring.

5. Set N_1, then solve $D_1 = D_2 - h_1 - h_2 - 2C$ and $h_1 = 1.1\, D_1 S_1 N_1 / (ET_1)$ simultaneously for h_1 to obtain

$$h_1 = \frac{1.1\, S_1 N_1 (D_2 - h_2 - 2C)}{ET_1 + 1.1\, S_1 N_1} \qquad \text{(Eq 8.3.3.4h)}$$

6. From Equation (8.3.3.4b)

$$b_1 = \frac{6M_1}{S_1 h_1^2} \qquad \text{(Eq 8.3.3.4i)}$$

7. Finally

$$L_1 = (N_2 + 1)(b_2 + K_2) \qquad \text{(Eq 8.3.3.4j)}$$

L_2 must be equal to L_1.

A decision must now be made to accept or reject this design. In case of rejection, the selection for D_2, N_2, and N_1 can be modified and the indicated calculations repeated. A logic diagram of this procedure is shown in Figure 8.3.3.4b.

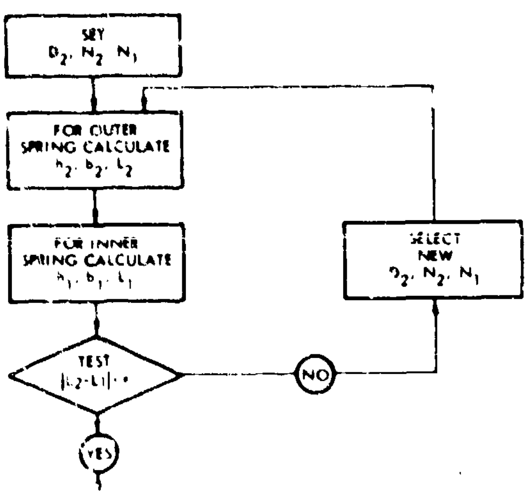

Figure 8.3.3.4b. Logic Diagram of Procedure for Designing Double Helical Spring

The question of how to modify the selection of D_2, N_2, and N_1 must be considered. The mathematical nature of the problem (non-linear equations with an infinite number of solutions) does not permit use of the iterative techniques mentioned thus far. Two possible procedures for varying the selection are *scattering* and *search*.

Both of these procedures assume the existence of a maximum or minimum for the function G which is used to denote acceptability. For the preceding problem this function is $G = |L_2 - L_1| = G(D_2, N_2, N_1)$.

Scattering. A set of selected values for D_2 and N_2 (in three dimensions: D_2, N_2, and N_1), which create a grid covering the possible range in both D_2 and N_2 is used to compute corresponding values for G. The grid is shown in Figure 8.3.3.4c as a set of O's. A smaller area is selected around the minimum G and the process is repeated, as indicated by the Δ's.

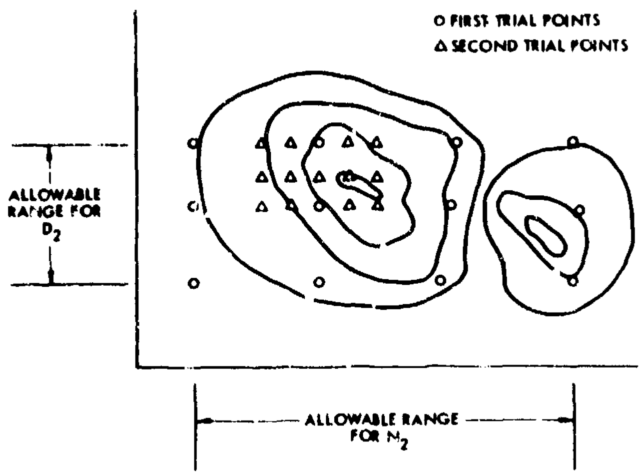

Figure 8.3.3.4c. Example of "Scattering" Method of Modifying D_2, N_2, and N_1 in Helical Spring Problem

This corresponds to the familiar trial and error approach, except that many tries may be run at one time on a computer. Each solution may be tried to determine if it meets secondary design criteria. In the spring problem, a minimum $|L_2 - L_1|$ is not sufficient for a good design; the ratios h_1/b_1, h_2/b_2, and h_1/h_2 are also important. It is quite probable that the spring finally selected will have a small but not the smallest $|L_2 - L_1|$.

Search. One starting point in n dimensions is selected (shown in Figure 8.3.3.4d for the two-dimensional case).

For each variable in turn

1. The variable is changed by some small amount and G is recomputed.

2. If G decreases, step 1 is repeated until G begins to increase, and then the variable is set to correspond to the smallest G.

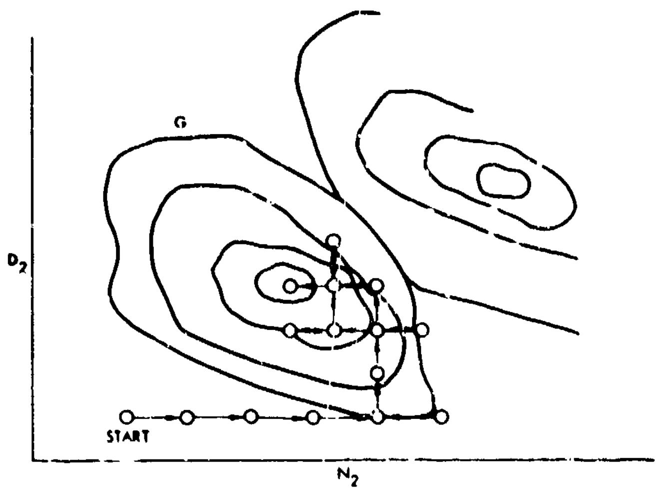

Figure 8.3.3.4d. "Search" Method of Modifying D_2, N_2, and N_1 in Helical Spring Problem

3. If G increases for step 2, an equal change is made to move in the opposite direction in a manner similar to step 2.

Steps 1, 2, and 3 are repeated for each variable until new changes within the practical bounds fail to decrease G significantly.

This method moves more quickly to the solution than scattering and represents a more fully automated procedure. However, the behavior of the function (in this case, G) must be well understood. Discontinuities or lesser *minimums* can destroy the effectiveness of the method.

It is possible to add other selection criteria in the search technique. For example, the material cost might be used, so that the selected spring has the lowest material cost of those designs with a value of G within a specified range.

8.3.4 More Advanced Techniques

Because of their high operating speeds, digital computers are useful where a great number of repetitive calculations are necessary. This capability is especially valuable for handling such complex mathematical techniques as matrix calculations, eigenvalue problems, partial differentiation, and relaxation.

8.3.4.1 MATRICES. A matrix is simply an array of numbers. There are two basic forms which are of interest to the computer user:

1. The rectangular matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix} \quad \text{(Eq 8.3.4.1a)}$$

where m is the row number and n the column number.

2. The column (column-vector) matrix

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_m \end{bmatrix} \quad \text{(Eq 8.3.4.1b)}$$

Addition and Multiplication. The most common application of matrix notation is in transformations, which are very useful for motion problems. A linear transformation can be expressed as the addition of two column matrices. Figure 8.3.4.1a illustrates a linear transformation of the coordinates for point P from the x, y, and z coordinates to the x′, y′, and z′ coordinates. If the coordinates of P in the unprimed coordinate system are represented by the column matrix

$$A = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{(Eq 8.3.4.1c)}$$

the coordinates of O′ are represented by

$$B = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad \text{(Eq 8.3.4.1d)}$$

and the coordinates of P in the primed system by

$$C = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad \text{(Eq 8.3.4.1e)}$$

Thus, transformation (shift in space) from an unprimed to a primed system can be expressed as $C = A - B$ or

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} \quad \text{(Eq 8.3.4.1f)}$$

The inverse transformation is $A = C + B$ or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' + x_0 \\ y' + y_0 \\ z' + z_0 \end{bmatrix}. \quad \text{(Eq 8.3.4.1g)}$$

Another common type of point translation is rotation about an axis, (see Figure 8.3.4.1b). The illustrated rotation of point P about the z axis through angle θ can be expressed

$$\begin{aligned} x' &= \cos\theta\, x + \sin\theta\, y \\ y' &= -\sin\theta\, x + \cos\theta\, y \\ z' &= z \end{aligned} \quad \text{(Eq 8.3.4.1h)}$$

or in matrix notation: $A = BC$

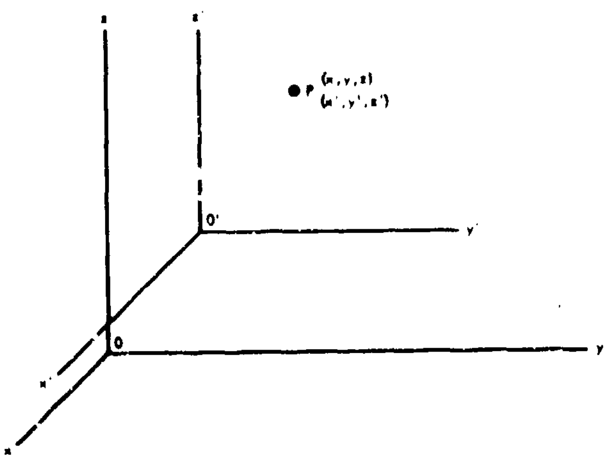

Figure 8.3.4.1a. Linear Transformation of Coordinates of Point P from x, y, and z Coordinates to x', y', and z' Coordinates
(A transformation of this type can be expressed as the addition of two column matrices.)

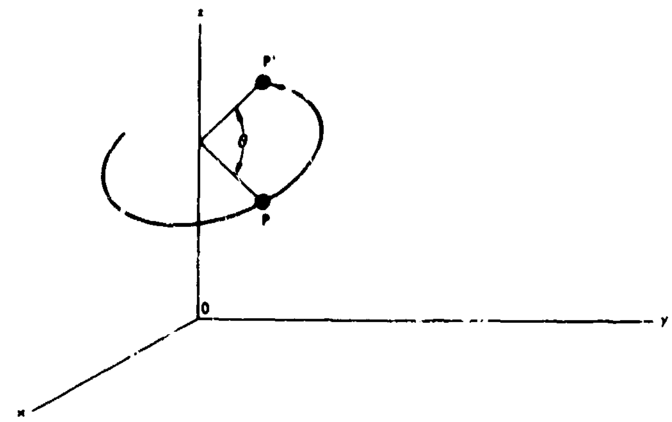

Figure 8.3.4.1b. Rotation of Point About an Axis
(This can be expressed as matrix multiplication.)

where

(Eq 8.3.4.1i)

$$A = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad B = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Frequently it is necessary to transform from an unprimed coordinate system to a primed coordinate system which is both linearly and rotationally different (see Figure 8.3.4.1c). This requires both matrix addition and multiplication. With known coordinates x, y, and z of P, the transformation to primed coordinates is

$$\begin{aligned} x' &= \alpha_1 x + \beta_1 y + \gamma_1 z + x_0 \\ y' &= \alpha_2 x + \beta_2 y + \gamma_2 z + y_0 \\ z' &= \alpha_3 x + \beta_3 y + \gamma_3 z + z_0 \end{aligned} \quad \textbf{(Eq 8.3.4.1j)}$$

where $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$, and $(\alpha_3, \beta_3, \gamma_3)$ are the direction cosines of the x', y', and z' axes in the x, y, z coordinate system. In matrix notation this is: $A = BC + D$.

Transposition. The transpose of a matrix is obtained by interchanging rows and columns in the matrix. For example, the transpose of

$$B = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix} \quad \textbf{(Eq 8.3.4.1k)}$$

is

$$B' = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \quad \textbf{(Eq 8.3.4.1l)}$$

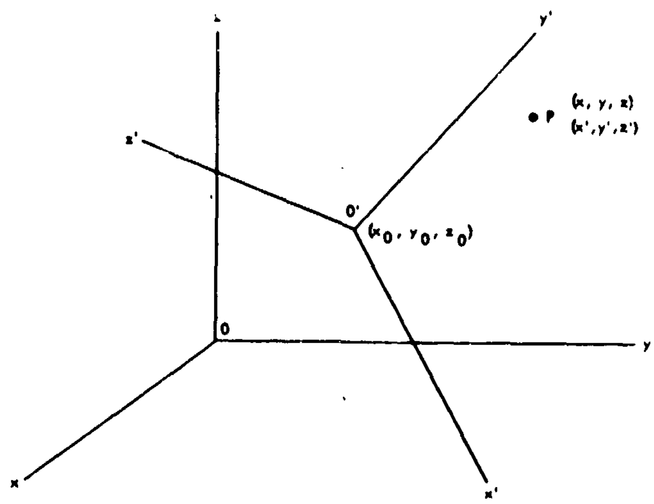

Figure 8.3.4.1c. Transformation of Point P from One Coordinate System to Another Both Linearly and Rotationally Different

Then, the inverse transformation from prime to unprimed coordinate system *for the case under discussion* can be expressed $C = B'(A - D)$.

8.3.4.2 SIMULTANEOUS EQUATIONS. A major application of computers is handling the solution of large sets of simultaneous equations which may occur in such engineering areas as stress analysis, statistical least squares, and circuit analysis. One example is the circuit shown in Figure 8.3.4.2. Values of the resistances are known, and the currents are to be determined. For this circuit, Kirchhoff's law can be used to establish the set of linear equations shown in Figure 8.3.4.2.

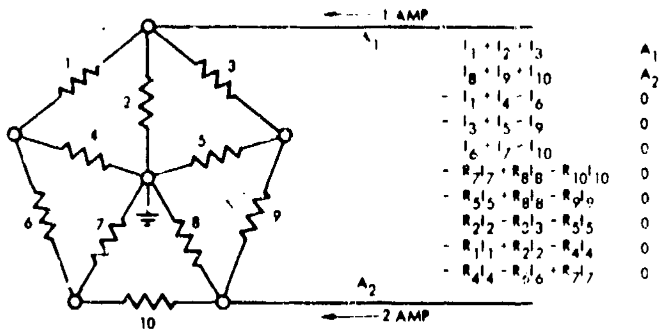

Figure 8.3.4.2. Circuit Yielding Numerous Simultaneous Equations
(Set of linear equations shown is based on Kirchhoff's Law.)

Matrix notation is particularly useful when dealing with large sets of linear equations. For example, the preceding set of equations may be represented by $AX = B$. The solution of such a set of equations can be obtained by one of several exact methods or by one of many iterative methods. Both methods are used in computer programming, but the exact method is preferred for its accuracy and reliability. However, the size of the equation set may prevent the use of this method. The iterative method requires considerably less computer memory for the handling of simultaneous equations.

In matrix notation, the solution of the equations $AX = B$ is equivalent to finding the inverse A^{-1} of the matrix A. One of the properties of A^{-1} is $A^{-1}A = I$, where I is the identity matrix with the form

$$I = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 1 \end{bmatrix} \qquad \textbf{(Eq 8.3.4.2a)}$$

Diagonal elements have a value of one, and all other elements are zero. If a matrix is multiplied by the I matrix, its value is unchanged, and $IX = X$. Therefore, if both sides of $AX = B$ are multiplied by A^{-1}, the result is $X = A^{-1}B$. Thus, a simultaneous equation can be solved by calculating the inverse of the coefficient matrix.

Matrix inversion can be done by elimination. This is an exact method, illustrated by the following example:

Given the coefficient matrix

(Eq 8.3.4.2b)

$$\begin{array}{cccccc|c|} a_{11} & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{1n} & 1 \\ a_{21} & a_{22} & a_{23} & \cdot & \cdot & \cdot & a_{2n} & 0 \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \cdot & \cdot & \cdot & a_{nn} & 0 \\ \square & \square & \square & \cdot & \cdot & \cdot & \square & \end{array}$$

add a column—a unit vector—which contains a 1 in the first row and zeros elsewhere. At the same time, add a row—called the pivot row—denoted by [ ]. Then perform the following computations to arrive at a new array:

1. For the pivot-row elements

$$a_{p,j} = \frac{a_{1,j+1}}{a_{1,1}} \qquad \textbf{(Eq 8.3.4.2c)}$$

where $j = 1, 2, \cdots, n$.

2. For all other elements, compute a new value

(Eq 8.3.4.2d)

$$a_{i,j}' = a_{i,j+1} - (a_{i,1})(a_{p,j})$$

where $i = 1, 2, \cdots, n$.

3. As a result of step 2, all the new elements of row 1 are zero. This row is dropped, and the remaining n rows renumbered 1 through n. Thus, for the last row

$$a_{n,j}' = a_{p,j} \qquad \textbf{(Eq 8.3.4.2e)}$$

4. Add a new unit vector and pivot row and repeat steps 1, 2, and 3 a total of n times. The resulting array is the inverse of the original matrix.

For a set of simultaneous equation such as

(Eq 8.3.4.2f)

$$\begin{array}{ccccc} a_{11}x_1 + a_{12}x_2 + & \cdot \; \cdot \; \cdot & + a_{1n}x_n & = & b_1 \\ \cdot \qquad \cdot & & \cdot & & \cdot \\ \cdot \qquad \cdot & & \cdot & & \cdot \\ a_{n1}x_1 + a_{n2}x_2 + & \cdot \; \cdot \; \cdot & + a_{nn}x_n & = & b_n \end{array}$$

the solution can be obtained directly by starting with an $n + 1$ by n array in which the original matrix is augmented by the b vector. If values of x are required for more than one set of b values, the additional b vectors can be incorporated in the original array, thus

(Eq 8.3.4.2g)

$$\begin{array}{cccccccc} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} & b_{11} & b_{12} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} & b_{21} & b_{22} \\ \cdot & \cdot & & & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & & & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & \cdot & a_{nn} & b_{n1} & b_{n2} \end{array}$$

Using the preceding four-step procedure on this m by n matrix a total of m times gives the array

(Eq 8.3.4.2h)

$$\begin{array}{cccccccc} x_{11} & x_{12} & a_{11}' & a_{12}' & \cdot & \cdot & \cdot & a_{1n}' \\ x_{21} & x_{22} & a_{21}' & a_{22}' & \cdot & \cdot & \cdot & a_{2n}' \\ \cdot & \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & & \cdot \\ x_{n1} & x_{n2} & a_{n1}' & a_{n2}' & \cdot & \cdot & \cdot & a_{nn}' \end{array}$$

where the a_{ij}' are the elements of the inverse of the original coefficient matrix, and the x_{ij} are the solutions for each of the two b vectors of the matrix equation AX = B.

8.3.4.3 DIFFERENTIAL EQUATIONS. In addition to their usefulness for problems presented directly in matrix form, matrix methods have also been used extensively for solving differential equations and eigenvalue problems associated with differential equations. The example shown here illustrates the use of a computer in handling sets of differential equations. It illustrates the meaning of eigenvalue for a set of differential equations which describe the motion of a mechanical system.

The problem is to find the normal modes of oscillation of the system shown in Figure 8.3.4.3a.

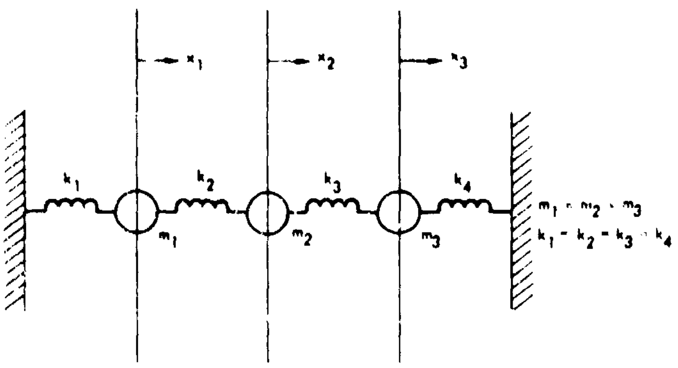

Figure 8.3.4.3a. Spring and Mass System
(Differential equations of motion associated with this system provide an excellent application for computer handling.)

The differential equations of motion are

(Eq 8.3.4.3a)

$$m\ddot{x}_1 = -kx_1 + k(x_2 - x_1) = k(x_2 - 2x_1)$$

$$\begin{aligned} m\ddot{x}_2 &= -k(x_2 - x_1) - k(x_2 - x_3) \\ &= -k(2x_2 - x_1 - x_3) \end{aligned}$$

$$m\ddot{x}_3 = -kx_3 + k(x_2 - x_3) = k(x_2 - 2x_3)$$

One procedure for solving these differential equations is to assume solutions of the form

(Eq 8.3.4.3b)

$$x_1 = Ae^{i\omega t}\ ;\ x_2 = Be^{i\omega t}\ ;\ x_3 = Ce^{i\omega t}$$

Substituting these into Equation (8.3.4.3a), performing the indicated differentiations, and rearranging terms gives

$$GA - \frac{k}{m}B = 0 \qquad \textbf{(Eq 8.3.4.3c)}$$

$$-\frac{k}{m}A + GB - \frac{k}{m}C = 0$$

$$-\frac{k}{m}B + GC = 0$$

where $G = (2k/m) - \omega^2$. This can be written in matrix form as

(Eq 8.3.4.3d)

$$\left[\begin{bmatrix} 2\frac{k}{m} & -\frac{k}{m} & +0 \\ \frac{-k}{m} & +\frac{2k}{m} & -\frac{k}{m} \\ 0 & -\frac{k}{m} & +\frac{2k}{m} \end{bmatrix} - \begin{bmatrix} \omega^2 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{bmatrix}\right] \begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0$$

or more simply,

$$(D - \omega^2 I)\ X = 0 \qquad \textbf{(Eq 8.3.4.3e)}$$

For A, B, and C to satisfy Equation (8.3.4.3c), the determinant of the coefficient matrix must vanish. In other words

$$\det|(D - \omega^2 I)| = 0 \qquad \textbf{(Eq 8.3.4.3f)}$$

Evaluating the determinant for Equation (8.3.4.3c) and equating it to zero gives a polynomial—called the characteristic equation—in ω^2

(Eq 8.3.4.3g)

$$\left(\frac{2k}{m} - \omega^2\right)^3 - 2\frac{k^2}{m}\left(2\frac{k}{m} - \omega^2\right) = 0$$

Roots are

$$\omega_1^2 = \frac{2k}{m} \qquad \textbf{(Eq 8.3.4.3h)}$$

$$\omega_2^2 = \frac{2k}{m} \pm \sqrt{2}\frac{k}{m}$$

The values of ω which satisfy these equations are called eigenvalues. In general, values of ω which satisfy Equation (3.3.4.3e) are eigenvalues. Vector X is called the eigenvector. For this problem the eigenvalues give the natural modes of vibration for the mass-spring system. This calculation of eigenvalues for differential equations is termed frequency analysis.

ISSUED: NOVEMBER 1968
SUPERSEDES: MAY 1964

Solving differential equations in motion problems amounts to determining the displacements as a function of time. This is called amplitude analysis. Both frequency analysis and amplitude analysis are important computer applications.

The spring and mass system shown in Figure 8.3.4.3b can be used to illustrate a computer solution to a frequency analysis. The differential equations which describe this system are

(Eq 8.3.4.3i)

$$\frac{d^2x_1}{dt^2} + a_{11}x_1 + a_{12}x_2 + a_{13}\theta_1 + a_{14}\theta_2 = 0$$

$$\frac{d^2x_2}{dt^2} + a_{21}x_1 + a_{22}x_2 + a_{23}\theta_1 + a_{24}\theta_2 = 0$$

$$\frac{d^2\theta_1}{dt^2} + a_{31}x_1 + a_{32}x_2 + a_{33}\theta_1 + a_{34}\theta_2 = 0$$

$$\frac{d^2\theta_2}{dt^2} + a_{41}x_1 + a_{42}x_2 + a_{43}\theta_1 + a_{44}\theta_2 = 0$$

where the $a_{i,j}$ values depend on the spring constants and the masses.

Assume solutions of the form

(Eq 8.3.4.3j)

$$x_1 = x_{10} \cos \omega t$$
$$\theta_1 = \theta_{10} \cos \omega t$$
$$x_2 = x_{20} \cos \omega t$$
$$\theta_2 = \theta_{20} \cos \omega t$$

where x_{10}, x_{20}, θ_{10}, and θ_{20} are the initial displacements. The appropriate differentiations and substitutions give a homogeneous set of linear equations of the form

(Eq 8.3.4.3k)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \\ \theta_{10} \\ \theta_{20} \end{bmatrix} = \omega^2 \begin{bmatrix} x_{10} \\ x_{20} \\ \theta_{10} \\ \theta_{20} \end{bmatrix}$$

The iterative procedure for the solution of these equations involves the following steps:

1. With $x_{10} = x_{20} = \theta_{10} = \theta_{20} = 1$, evaluate the left-hand side for new values of $\omega^2 x_{10}$, $\omega^2 x_{20}$, $\omega^2\theta_{10}$, and $\omega^2\theta_{20}$; that is, $\omega^2 x_{10} = a_{11}x_{10} + a_{12}x_{20} + a_{13}\theta_{10} + a_{14}\theta_{20}$, and so on.

2. "Normalize" for new guesses at x_{10}, x_{20}, θ_{10}, and θ_{20} by setting $x_{10} = 1$; $\theta_{10} = \omega^2\theta_{10}/(\omega^2 x_{10})$; $x_{20} = \omega^2 x_{20}/(\omega^2 x_{10})$; $\theta_{20} = \omega^2\theta_{20}/(\omega^2 x_{10})$.

3. Repeat steps 1 and 2 until successive values of x_{10}, x_{20}, θ_{10}, and θ_{20} are very close. At this time, convergence has occurred and ω^2 can be computed.

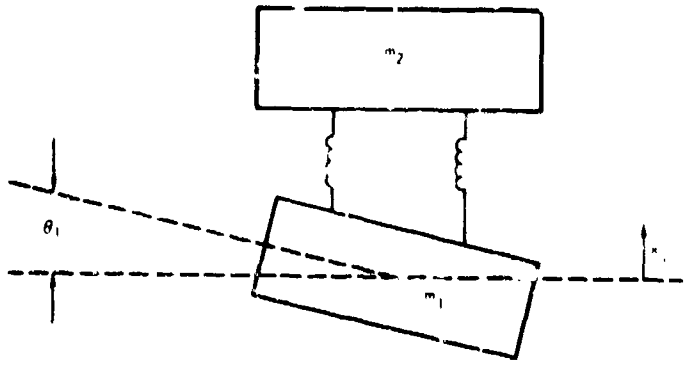

Figure 8.3.4.3b. Spring and Mass System
(Solution of this problem illustrates frequency analysis.)

4. Upon convergence, ω^2 can be evaluated from the given value of $\omega^2 x_{10}$, or $\omega^2 x_{10} = K$.

The iterative solution for ω^2 of the sample set of coefficients:

(Eq 8.3.4.3l)

$$\begin{bmatrix} +929 & 332 & +3590 & -3590 \\ -748 & +748 & -8090 & +8090 \\ +4.17 & -4.17 & +36,400 & 11,760 \\ -14.31 & +14.31 & -40,300 & +40,300 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \omega^2 \begin{bmatrix} x_1 \\ x_2 \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

is

(Eq 8.3.4.3m)

	x_1	x_2	θ_1	θ_2
1	1	1	1	1
2	597	0	23,640	0
3	143,085	321,095	1,401,773	1,595,813
4	76,883	171,908	477,975	844,314
5	63,414	141,558	349,233	693,154
6	60,681	135,398	323,511	662,485
7	60,002	133,869	317,130	654,870
8	59,825	133,468	315,460	652,876
9	59,777	133,362	315,017	652,347

From this, $\omega^2 = 59,777$.

Since x_{10} is to be set equal to 1, then $\omega^2 = K$. In clarification of the preceding solution, it should be remembered that the set of equations has no constant term — it is homogeneous. Essentially this means that there are an infinite number of solutions which satisfy the equations. This is reasonable when the physical system under consideration is examined. In a vibration problem of this kind the initial displacements

x_1, x_2, θ_1, and θ_2 must be expected to take on different values. In the above procedure a value of x_{10} was selected, which then fixed the values of the variables x_{20}, θ_{10}, and θ_{20} and allowed ω to be determined.

A similar iterative scheme, without the normalization step, can be used to solve sets of non-homogeneous linear equations.

8.3.4.4 PARTIAL DIFFERENTIAL EQUATIONS. Many engineering problems involve the handling of partial differential equations. These equations fall into three classes:

1. Elliptical equations (describing potential fields)

$$\nabla^2\phi = g(x, y, z) \qquad \textbf{(Eq 8.3.4.4a)}$$

2. Parabolic equations (describing heat flow and diffusion)

$$\nabla^2 = k\frac{\partial\phi}{\partial t} \qquad \textbf{(Eq 8.3.4.4b)}$$

3. Hyperbolic equations (describing wave action)

$$\nabla^2\phi = \frac{1}{C^2}\frac{\partial^2\phi}{\partial t^2} \qquad \textbf{(Eq 8.3.4.4c)}$$

In these equations ∇^2 is the Laplacian operator in rectilinear coordinates

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} \qquad \textbf{(Eq 8.3.4.4d)}$$

A basic approach to handling partial differential equations when describing a particular material or space is to create a grid of points covering the space, (see Figure 8.3.4.4a).

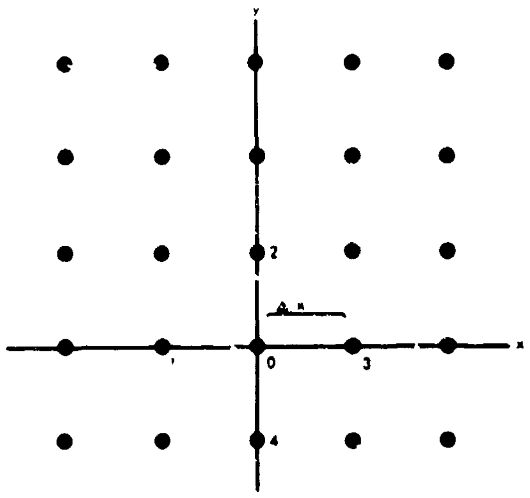

Figure 8.3.4.4a. Point Grid Used in Handling Partial Differential Equations Describing a Material or Space

Then, at any point 0 the first derivative with respect to x can be approximated in one of two ways:

$$\left(\frac{\partial\phi}{\partial x}\right)_1 \cong \frac{\phi_3 - \phi_0}{\Delta x} \qquad \textbf{(Eq 8.3.4.4e)}$$

or

$$\left(\frac{\partial\phi}{\partial x}\right)_2 \cong \frac{\phi_0 - \phi_1}{\Delta x} \qquad \textbf{(Eq 8.3.4.4f)}$$

The second derivative can be approximated as

$$\frac{\partial^2\phi}{\partial x^2} \cong \frac{\left(\frac{\partial\phi}{\partial x}\right)_1 - \left(\frac{\partial\phi}{\partial x}\right)_2}{\Delta x} \qquad \textbf{(Eq 8.3.4.4g)}$$

$$= \frac{\phi_3 + \phi_1 - 2\phi_0}{(\Delta x)^2}$$

Derivatives in the y direction can be obtained in the same way. With this procedure, any partial differential equation can be reduced to a difference equation which can be solved on a computer.

The following problem illustrates the use of the relaxation technique to the solution of a partial differential equation. Find the potential distribution in a square whose sides are maintained at voltages $(V_1)_B$, $(V_2)_B$, $(V_3)_B$, and $(V_4)_B$, (see Figure 8.3.4.4b).

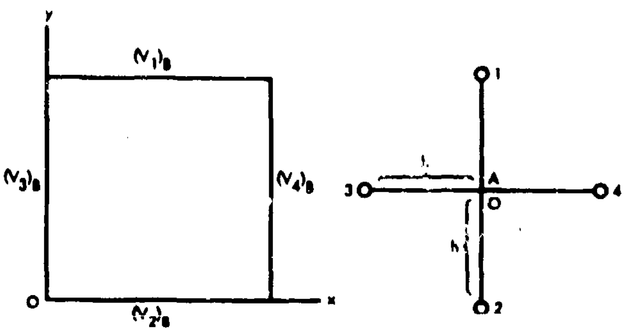

Figure 8.3.4.4b. Square with Sides Maintained at Given Voltages
(Discussed in example problem illustrating relaxation techniques.)

If there is no charge within the square, the potential distribution is defined by the Laplace equation,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \qquad \textbf{(Eq 8.3.4.4h)}$$

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Setting up a square grid system to cover the square for the general point A gives the following approximations for the partial derivatives

(Eq 8.3.4.4i)

$$\left(\frac{\partial V}{\partial x}\right) = \frac{V_4 - V_0}{h}$$

$$\left(\frac{\partial V}{\partial x}\right) = \frac{V_0 - V_3}{h}$$

$$\frac{\partial^2 V}{\partial x^2} \cong \frac{V_4 + V_3 - 2V_0}{h^2}$$

Similarly, for the y dimension

(Eq 8.3.4.4j)

$$\frac{\partial^2 V}{\partial y^2} \cong \frac{V_2 + V_1 - 2V_0}{h^2}$$

Then, Equation (8.3.4.4h) becomes

(Eq 8.3.4.4k)

$$V_1 + V_2 + V_3 + V_4 - 4V_0 = 0$$

This is the basic relaxation equation. It is applied in the following way:

1. A first guess at the potential of each point on the grid is made on the basis of the known boundary conditions.

2. Moving systematically through the points of the grid, compute the quantity called the residual for each point, and store this value. The residual is given by $R_n = V_1 + V_2 + V_3 + V_4 - 4V_n$. Initially Equation (8.3.4.4k) will not be satisfied, since the potentials are only guesses.

3. Again moving systematically and considering each point not on the boundary, adjust the potential to make the residual for the point equal to zero by applying the following equation: $V_n' = V_n + R_n/4$, where V_n' is the new V_n.

4. Since Step 3 affects the residuals of the surrounding points, they are adjusted by: $R_i' = R_i + R_n/4$, where R_i' is the new R_i.

5. Steps 3 and 4 are repeated until no residual is found whose absolute value is greater than some predetermined limit of accuracy. At this time the relaxation equation is satisfied and the potential distribution is known.

It is possible to write a FORTRAN program quickly to do the necessary computation. For this problem, let M = number of points in the grid on the x axis (200 max); N = number of points in the grid on the y axis (200 max); V (I, J) = potential at points on grid (initial guesses) plus boundary values; R (I, J) = associated residual, and DEL = limit of accuracy desired. The resulting FORTRAN program is shown in Figure 8.3.4.4c.

```
      DIMENSION V(200),R(200,200)
      READ(5,2) V,M,N,DEL
 2    FORMAT(20F10.2,2I3,E20.7)
      L=M-1
      L1=N-1
      DO 3 I=2,L
      DO 3 J=2,L1
      K=L*(J-1)+I
 3    R(I,J)=V(I-1)+V(I+1)+V(J-1)+V(J+1)-4.*V(K
 4    K=2
      DO 9 I=2,L
 5    DO 9 J=2,L1
 6    RAB=ABS(R(I,J))
 7    IF(RAB-DEL) 9,9,8
 8    K=1
      RDEL=R(I,J)/4.0
      KL=L*(J-1)+I
      V(KL)=V(KL)+RDEL
      R(I,J)=0.
      R(I-1,J)=R(I-1,J)+RDEL
      R(I+1,J)=R(I+1,J)+RDEL
      R(I,J-1)=R(I,J-1)+RDEL
      R(I,J+1)=R(I,J+1)+RDEL
 9    CONTINUE
      GO TO (4,11),K
 11   CONTINUE
      WRITE(6,12) V
 12   FORMAT (F10.2)
      STOP
      END
```

Figure 8.3.4.4c. FORTRAN Program Illustrating Relaxation Technique of Figure 8.3.4.4b

8.3.5 Empirical Relationships

Empirical data drawn from experiments or tests can be used in two ways: conclusions can be drawn from tables of data, or empirical relationships can be derived to fit the data. A problem often consists of a mixture of theoretical equations, tabular data, and empirically derived equations.

Various methods are available for computer handling of tables of data based on functions of a single variable or functions of multiple variables.

8.3.5.1 FUNCTIONS OF A SINGLE VARIABLE. As an example of the use of a computer in manipulating tabular data consider the following:

x	x_i	y	y_i
0.0	x_1	0.912	y_1
4.0	x_2	0.930	y_2
8.0	x_3	0.941	y_3
12.0	x_4	0.946	y_4
16.0	x_5	0.948	y_5
20.0	x_6	0.950	y_6
24.0	x_7	0.951	y_7
28.0	x_8	0.948	y_8
32.0	x_9	0.944	y_9
36.0	x_{10}	0.938	y_{10}
40.0	x_{11}	0.928	y_{11}
44.0	x_{12}	0.914	y_{12}

The problem is to determine the proper value of y for a given value of x. This can be accomplished either by ordinary table look-up or by data fitting, depending on the number of values to be found.

Table Look-up. This method can be carried out by loading the entire table into computer storage, then searching the table for the value of y that corresponds to a given value of x.

A FORTRAN program for loading the table into storage and for table look-up for several values of x is shown in Figure 8.3.5.1a. Note that the search is accomplished with an "IF" statement within a "DO" loop. Where the argument x is equal to a table entry value of x, the corresponding value of y is simply selected and printed. When the argument x falls between two table entries, linear interpolation is performed, based on the linear-interpolation equation

(Eq 8.3.5.1a)

$$y = y_i + \frac{(y_{i+1} - y_i)(x - x_i)}{x_{i+1} - x_i}$$

which is the equation of the straight line joining points (x_i, y_i) and (x_{i+1}, y_{i+1}). Evaluation of the right hand side of the equation for a particular value of x gives the corresponding value of y. This equation is used in programing statement 4 in Figure 8.3.5.1a.

If the approximation of the function by a straight line in the interval $(y_{i+1} - y_i)$ is not sufficiently accurate, the function may then be approximated by a parabola or higher degree polynomial by a method such as the Lagrange interpolation formula.

Data Fitting. If the problem involving use of the table is to be run many times on a computer, consideration might be given to finding an equation which will pass either through, or within tolerance of, all the points in the table. In most engineering problems it is sufficient to find an equation which passes within a specified tolerance of all the points in a table of data.

Model Selection. First step in fitting an equation to tabular data is to select the equation form A few of the possible selections are:

Polynomial	From $y = a_0 + a_1x$ to $y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_nx_n$. Generally restricted to this range.
Logarithmic	$y = a_0 + a_1 \log x$
Exponential	$y = a_0a_1^x$
Power	$y = a_0x^{a_1}$
Fourier series	$y = a_0 + \sum_{n=1}^{m} (a_n \cos nx + b_n \sin nx)$

The choice may be based on theory, preliminary plotting, past experience, or on trial and error.

Model Fitting. After the equation form has been determined, the next step is to select a method for fitting the equation form (finding values for the values of a_i). There are three widely used methods, selected points, harmonic analysis, and least squares.

1. *Selected Points.* As many sets of observed data as there are values of a_i to be determined are substituted into the selected equation, and the resulting system of equations is solved for a Although this method is very crude, it may be of value in situations where available data are limited.

```
C       TABLE LOOKUP PROGRAM
        DIMENSION X(12),Y(12)
        READ(5,2) X,Y
  1     READ(5,2) TX
  2     FORMAT (F6.3)
        DO 6 I=1,12
        IF(TX-X(I)) 4,5,6
  6     CONTINUE
        GO TO 1
  4     CORY=Y(I-1)+(Y(I)-(Y(I-1))*
       1 (TX-X(I-1))/(X(I)-X(I-1))
        GO TO 8
  5     CORY=Y(I)
  8     WRITE(6,9) TX,CORY
  9     FORMAT (2F6.3)
        GO TO 1
        END
```

Figure 8.3.5.1a. FORTRAN Program Used for Loading Tabular Data into a Computer and for Finding Values of y for Given Values of x

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2. *Harmonic Analysis.* This widely used computer application is useful in fitting a Fourier series to a set of periodic data.

3. *Least Squares.* This is the most commonly used procedure for calculating parameters a_i for the selected model.

The five types of equations, or models, already mentioned (except the Fourier series) may be fitted to a set of data by the least squares method. The principle can be understood by considering the simple linear model

$$y = a_0 + a_1 x \qquad \text{(Eq 8.3.5.1b)}$$

Given a table of n sets of data, determine a_0 and a_1.

The least squares approach, Figure 8.3.5.1b, consists of determining a_0 and a_1 so that the sum of the squares of the vertical distance between the data points and the straight line is a minimum. From Figure 8.3.5.1b this may be stated mathematically as

(Eq 8.3.5.1c)

$$\sum_{i=1}^{n} \epsilon_i^2 \quad \text{minimum}$$

This is true only if

(Eq 8.3.5.1d)

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial a_0} = 0 \text{ and } \frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial a_1} = 0$$

The sum may be expressed in terms of the equation to be fitted and the original data points, $\epsilon_i = y_i - (a_0 + a_1 x_i)$. Then

(Eq 8.3.5.1e)

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial a_0} = \frac{\partial \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)]^2}{\partial a_0} = 0$$

(Eq 8.3.5.1f)

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial a_1} = \frac{\partial \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)]^2}{\partial a_1} = 0$$

Differentiating and simplifying gives

(Eq 8.3.5.1g)

$$\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} a_1 x_i - n a_0 = 0$$

(Eq 8.3.5.1h)

$$\sum_{i=1}^{n} x_i y_i - a_1 \left(\sum_{i=1}^{n} x_i\right)^2 - a_0 \sum_{i=1}^{n} x_i = 0$$

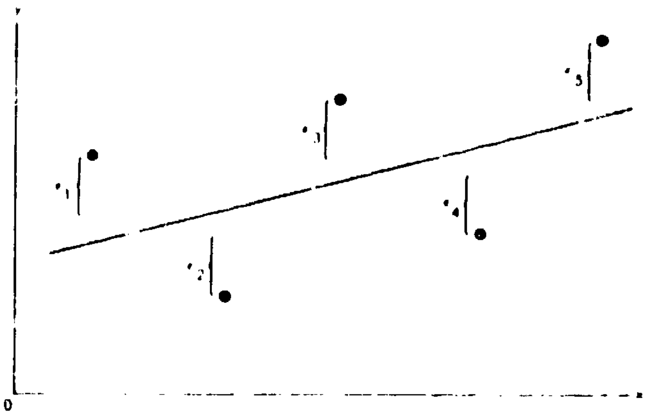

Figure 8.3.5.1b. Least Square Method of Fitting a Line to a Set of Points

These two linear non-homogeneous equations may be solved for a_0 and a_1. Parameters a_0 and a_1, therefore, are computed in terms of sums, and sums of cross products of the raw data.

8.3.5.2 Functions of Multiple Variables. Tabular data involving functions of multiple variables are used in basically the same way as those for a single variable; however, the methods are correspondingly more complex.

Table Look-up. Given the following data, assume that x is to be computed for various sets of values of y and z:

x_1	y_1	
x_2	y_2	z_1
x_3	y_3	
x_4	y_4	
x_5	y_5	
x_6	y_6	z_2
x_7	y_7	
x_8	y_8	
x_9	y_9	
x_{10}	y_{10}	
x_{11}	y_{11}	z_3
.	.	
.	.	
.	.	

To solve this problem, table look-up and interpolation (more complex logically than for a single variable) may again be used. Use the following procedure:

1. For z_1, interpolate for x as a function of y alone.

2. Store the resultant value of x along with z_1.

3. Repeat steps 1 and 2 for all values of z to obtain a complete table of x as a function of z alone.

4. Interpolate in this resultant table for the final value of x.

Data Fitting. Quite often table look up is impractical. Either the problem demands an equation to provide a meaningful solution, or the data cannot be obtained in a form similar to that shown in the previous table. When this is true, curve fitting may be used.

For example, an equation in the description of a vehicle simulation model can be presented as an empirical relationship, $R = aM + bAV^2$. In this case R is a function of three variables — M, A, and V.

The least squares method may again be used in conjunction with experimental data to determine values for a and b which best fit the equation to the data. To apply least squares relationships in this case would require considering AV^2 as a new variable, $x_2 = AV^2$, to give a linear relationship of the form $y = ax_1 + bx_2$.

In general, data fitting of linear equations is called linear regression. If, as in this case, there is more than one independent variable, the procedure is called multiple linear regression.

The method of first assuming the form of a relationship and then using mathematical criteria to fit the relationship to experimental data can be thought of as a search for a useful prediction equation.

In this discussion, the concept of "best fitting" predictive equations to data has been used. The assumption has been made that a useful equation need not fit the complete data set exactly. This assumption is based on statistical principles. Statistics indicate also that predictions, or useful engineering conclusions can be drawn from data without necessarily performing data fitting to arrive at an equation.

8.3.5.3 STATISTICS. The most important statistical procedures have been programed for many computers. Suitably complete statistical reduction of information requires only that the data be prepared in a form acceptable to one of the available programs.

The form of an experiment or test should depend upon the statistical procedures to be applied to the resulting data. Before any data are taken, the engineering hypotheses which are being tested should be clearly defined. Statistical procedures available to resolve the test should be determined. Techniques have been programed to assist in determining the number and order of data required in an experimental design.

Many statistical methods may be applied to a set of data. At the same time, data may arise in an infinite number of forms. The more common statistical methods and the desired results fall into a pattern which requires a progression from simple to complex calculation procedures. There are four levels in this progression:

1. Probability analysis of a random variable is useful in quality control, testing of vendor products, and field performance of products. No cause and effect is considered.

2. Analysis of variance is used to determine the significance of differences between classed or grouped data, such as the differences caused by variation in the process for preparing a product. This represents a test for the existence of cause and effect.

3. Correlation analysis gives a measure of the linear relationship, dependence, or association between two variables. It represents an attempt to place a measure on the cause and effect relationship.

4. Regression analysis is a computational method for determining parameters in an assumed equation form which expresses the dependence of one variable (the dependent variable) on one or more other variables (independent variables) when data on all variables are available. This is a method for defining the cause and effect relationship to the extent that useful predictions can be made for the behavior of the dependent variable.

Probability Analysis. For many problems in which probabilities arise, the behavior of events in the system is known beforehand, so that the events are ruled by well defined laws of probability. But in engineering, probabilities for an event are determined on the basis of data obtained by experiment or testing. This method represents a useful estimate of the true probability.

If every possible trial is made (each *event* tested), the *population* has been tested. However, it is generally practical to test only a sample of the population. From this testing it is possible to obtain an inventory of all possible values for the event, and to determine the probabilities of the event taking on each value. This inventory is called a probability distribution.

Discrete distributions are used to describe probabilities for which events can take on only discrete values. Most engineering problems, however, involve continuous distributions. This discussion will be restricted to the normal distribution for continuous distributions of probabilities. Statistical tests can be made to determine the suitability of application of the normal distribution to a particular set of data.

As an illustration of the use of probabilities and the normal probability distribution, consider Table 8.3.5.3, which shows results of life tests for a brake shoe. A common method for displaying this information is to construct one bar graph to show the frequency distribution and another to show the accumulative distribution, (see Figure 8.3.5.3a). It can be assumed that for successive decreases in interval size and increases in sample size, the graphs in Figure 8.3.5.3a will approach the continuous curves shown in Figure 8.3.5.3b. The information shown in Figure 8.3.5.3a and b and in Table 8.3.5.3 can be summed up in the two following statistics, assuming a normal distribution:

1. The mean of the sample is

$$x' = \left(\sum_{i=1}^{N} x_i\right) \Big/ N \qquad \text{(Eq 8.3.5.3a)}$$

Table 8.3.5.3. Results of Tests on Brake Shoe

LIFE, x_i*	FREQUENCY, F_i**	NORMALIZED FREQUENCY, f_i	$F_i \times x_i$
10	50	0.050	500
20	75	0.075	1500
30	100	0.100	3000
40	200	0.200	8000
50	250	0.250	12,500
60	150	0.150	9000
70	75	0.075	5250
80	75	0.075	6000
90	25	0.025	2250

*Midpoint of interval.
**Number of failures in interval.

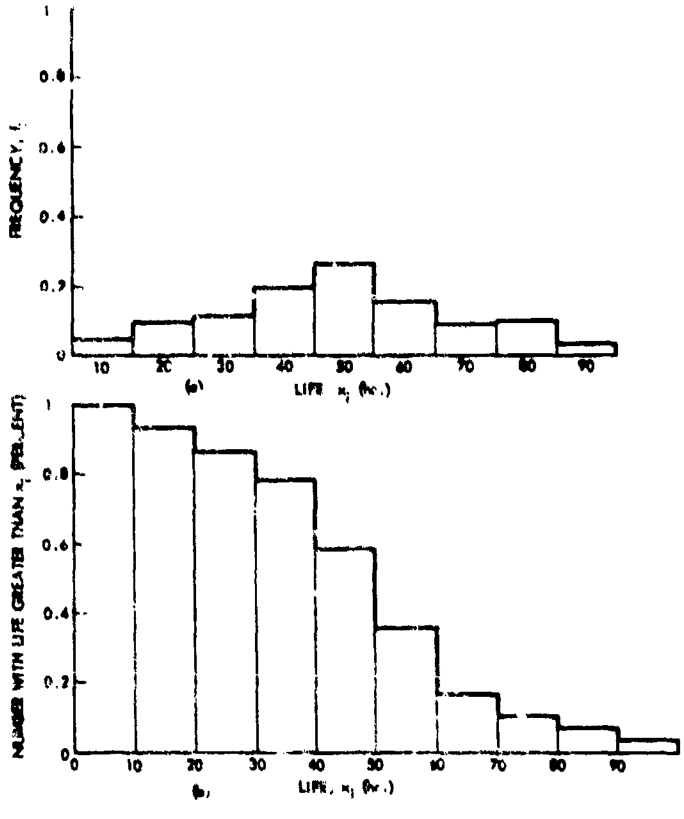

Figure 8.3.5.3a. Bar Graph Method of Displaying Information
(Graph in (a) shows frequency distribution; (b) shows accumulative distribution.)

where N = number of observations for ungrouped data; or

(Eq 8.3.5.3b)

$$x' = \left(\sum_{i=1}^{\eta} F_i x_i\right) \Big/ \sum_{i=1}^{\eta} F_i$$

for grouped data, as in Table 8.3.5.3, where η = number of

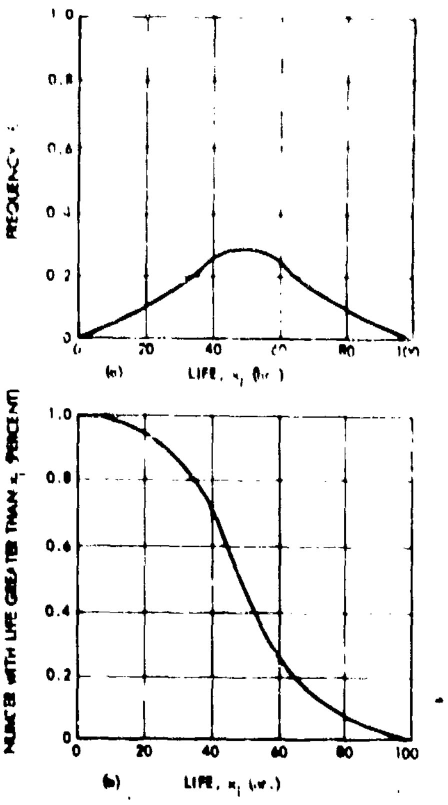

Figure 8.3.5.3b. Continuous Curves Representing (Approximately) the Bar Graphs in Figure 8.3.5.3a
(Such graphs are useful for obtaining statistical information.)

intervals. Mean, x', is an estimate for μ, the mean of the population. In computer handling of data, the grouping of data — a device for simplifying calculations — is seldom used.

2. The standard deviation of the sample is

(Eq 8.3.5.3c)

$$s = \left[\frac{\sum_{i=1}^{N} (x_i - x')^2}{N - 1}\right]^{1/2}$$

where s is an estimate of σ, the standard deviation of the population. Value s^2 is called the variance. For a normal distribution, the probability of an individual reading lying within $x' \pm \sigma$ is approximately 0.68.

With x' and s, the graph in (a) of Figure 8.3.5.3b can be constructed from the normal distribution function.

(Eq 8.3.5.3d)

$$f(x) = \frac{1}{s\sqrt{2\pi}} e^{-1/2[(x-x')/s]^2}$$

This function has been tabulated for the standard normal distribution where the transformation $t = (x - x')/s$ gives the following distribution function:

$$f(t) = (1/\sqrt{2\pi})e^{-t^2/2} \qquad \text{(Eq 8.3.5.3e)}$$

The graph or information shown in (b) of Figure 8.3.3.3 can be reconstructed from the integral function

$$\phi(x) = 1 - \int_{-\infty}^{x} f(x)\,dx \qquad \text{(Eq 8.3.5.3f)}$$

to give the probability of an individual reading being greater than x. Again, this function has been tabulated for the variable $t = (x - x')/s$ or

$$\phi(t) = 1 - \int_{-\infty}^{t} f(t)\,dt \qquad \text{(Eq 8.3.5.3g)}$$

For hand calculation, a listing of the general form

t	$f(t)$	ϕ
-2	0.05399	0.0228
-1	0.24197	0.1587
0	0.31594	0.5000
1	0.24197	0.6587
2	0.05399	0.9772

can be used, along with the transformation equation $t = (x - x')/s$ to provide answers to frequency and probability questions.

For example, with a mean $x' = 5$ and standard deviation $s = 2$, the probability of an individual reading lying between 1 and 9 can be calculated as

$$P_x = \int_{-\infty}^{t_2} f(t)\,dt - \int_{-\infty}^{t_1} f(t)\,dt = \phi(t_2) - \phi(t_1) \qquad \text{(Eq 8.3.5.3h)}$$

Since $t_1 = (1 - 5)/2 = -2$, and $t_2 = (9 - 5)/2 = 2$, then the probability is $\phi(t_2) - \phi(t_1) = 0.9772 - 0.0228 = 0.9544$. In computer handling of probability problems it is simpler to calculate values for $f(t)$ and $\phi(t)$ than to store the large table in the computer. The integral evaluation for $\phi(t)$ is difficult because an exact integration cannot be performed. One of Hastings' approximations

$$\frac{2}{\sqrt{\pi}}\int_0^{y} e^{-y^2} \cong 1 - \frac{1}{1 + a_1y + a_2y^2 + a_3y^3 + a_4y^4} \qquad \text{(Eq 8.3.5.3i)}$$

where $a_1 = 0.278393$, $a_2 = 0.230389$, $a_3 = 0.000972$, $a_4 = 0.078108$, is often used with appropriate transformations to evaluate the integral in a computer program.

The confidence which may be placed in the calculated mean and in the chosen sample size can be illustrated by consideration of the following statistics:

$$t_c = \frac{x' - \mu}{s/\sqrt{N}} \qquad \text{(Eq 8.3.5.3j)}$$

which is the Students t distribution. The Students t distribution approaches the normal distribution as the sample size increases, and in this discussion a normal distribution will be assumed.

For computing the range of the population mean for a given confidence, t_c depends on the confidence (from preceding table, for $t_c = 2$ the confidence that the population mean will be such that the calculated $t_c \leq 2$ is approximately 0.97). If $\mu = x' \pm \Delta$, then

$$\Delta = \frac{t_c s}{\sqrt{N}} \qquad \text{(Eq 8.3.5.3k)}$$

and it can be said with a 97 percent probability of being correct that, from the sample data, the mean of the population lies within $\mu = x' \pm \Delta$.

Equation (8.3.5.3j) may also be used to determine whether the sample size is sufficient to give an adequate confidence in the population mean lying within an acceptable percentage variation, K, of the calculated sample mean. If $\mu = x' \pm Kx'$ for $|\Delta| \geq Kx$, then from Equation (8.3.5.3k), $Kx' = t_c s/\sqrt{N_c}$ and

$$N_c = \frac{t_c^2 s^2}{(Kx')^2} \qquad \text{(Eq 8.3.5.3l)}$$

where N_c is to be compared with the actual sample size N. If $N_c > N$, a larger sample will be required for the necessary confidence. If $N_c \leq N$, a sufficient sample size has been used.

It should be noted that, because of approximations made and arbitrary choice of initial sample size, the value of N indicates only the direction in which the sample size should be changed, and not the actual size of change required. Several iterations might be necessary to determine a best value for N.

Analysis of Variance. The statistic $F = s_1^2/s_2^2$, where s_1^2 and s_2^2 are variances of samples from populations with normal distributions whose true variances are equal, has a distribution of the shape shown in Figure 8.3.5.3c.

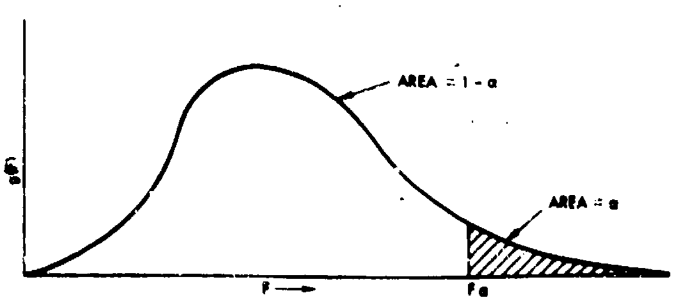

Figure 8.3.5.3c. Distribution for Statistic $F = s_1^2/s_2^2$ As Discussed in Text

ISSUED: NOVEMBER 1969
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The F test is used to test whether there is a significant difference in the two sample variances. The test consists of setting a confidence level (as in the previous discussion on the mean, using the normal distribution) in terms of percent of area (shaded area = α and total area = 1) which may lie to the right of F_α in the distribution curve. This determines the F_α and means that any given F greater than F_α has only a probability α of occurring due to random chance alone. The confidence in F being less than F_α is $1 - \alpha$.

Distribution function, $g(F)$ is a complex multi-variate function (dependent on F, degree of freedom of s_1^2, and degree of freedom of s_2^2). For this reason, tables are generally tabulated only for $\alpha = 0.01$ and $\alpha = 0.05$. For the same reason, $g(F)$ is not as often calculated as part of a computer program as is $\phi(t)$ for the normal distribution.

Next, the sample variances s_1^2 and s_2^2 are calculated, and observed F is computed. If $F > F_\alpha$, there is confidence $1 - \alpha$ that a significant difference in the sample variances exists. For analysis of variance, s_1^2 is a measure of the variation in test data caused by a difference in the process or treatment, while s_2^2 is a measure of the purely random variation of the test data. If the resultant F is greater than the preset F_α, then the effect on the data results can be attributed to the variation in the treatment.

A sample analysis of variance calculation performed on a computer is shown in Figure 8.3.5.3d. The observed F ratio of 15.22 when compared with tabulated F (degree of freedom for s_1^2 = 8 and degree of freedom s_2^2 = 65, the closest entry to 63) where F_α = 2.08 for α = 0.05 and F_α = 2.79 for α = 0.01, shows that the variation in data due to the variation in treatment is significant at both the 5 percent level and the more stringent 1 percent level. The treatment has a cause and effect relationship with the variable for which the data were recorded.

Correlation and Regression Analysis. Discussion of computer applications in the calculation of correlations and regression equations is simplified by the use of matrix notation. These paragraphs will discuss information to be gained from test or experimental data which is available in the following form:

Observation	Dependent Variable, y	Independent Variables x_1 x_2 x_3 $\cdots$ x_p
1	η_1	x_{11} x_{12} x_{13} $\cdots$ x_{1p}
2	η_2	x_{21} x_{22} x_{23} $\cdots$ x_{2p}
.	.	. . .
.	.	. . .
N	η_N	x_{N1} x_{N2} x_{N3} $\cdots$ x_{Np}

Regression analysis consists of using such data to determine, according to the least squares criterion, the value of b_i which best fit an equation of the form $y = b_1x_1 + b_2x_2 + b_px_p$ to the data. The objective is to obtain a useful prediction equation.

The value of p must be less than N for a correct analysis. If a constant term is desired in the equation — that is, $y = b_1x_1 + b_2x_2 + b_{p-1}x_{p-1} + b_p$ — a column of one's replaces the x_p column in the table.

The least squares analysis for this case can be most simply described by reference to matrix handling rules. Let λ = observed column matrix of observed y values; B = parameters b_i to be determined (column matrix); and D = rectangular matrix of observed values for the independent variables. Then

$$\eta = DB \qquad \textbf{(Eq 8.3.5.3m)}$$

or

$$\begin{aligned}
\eta_1 &= x_{11}b_1 + x_{12}b_2 + \cdots + x_{1p}b_p \\
\eta_2 &= x_{21}b_1 + x_{22}b_2 + \cdots + x_{2p}b_p \\
&\vdots \\
\eta_N &= x_{N1}b_1 + x_{N2}b_2 + \cdots + x_{Np}b_p
\end{aligned}$$

where $N > p$.

These equations cannot be solved for B since D has more rows than columns. However, multiplication of both sides of the above equation by the transpose D′ of the D matrix gives

$$D'\eta = D'DB \qquad \textbf{(Eq 8.3.5.3n)}$$

where D′D is a square matrix. These equations are equivalent to the summation equations which resulted in the least squares analysis, and are called normal equations.

The solution of these equations for B is found by first obtaining the inverse of the D′D matrix and multiplying both sides of Equation (8.3.5.3n) by the inverse $D'D^{-1}$ to give $(D'D)^{-1}D'\eta = (D'D)^{-1}(D'D)B$,

or

$$B = (D'D)^{-1}D'\eta \qquad \textbf{(Eq 8.3.5.3o)}$$

The several matrix manipulations indicated in this solution for the values of b_i require so much computation that for any problem involving four or more independent variables a solution without a computer is virtually impossible. Once the evaluation of b_i by Equation (8.3.5.3o) has been completed, several statistics become available to judge the value of the analysis for prediction purposes. These are:
1. "Goodness of fit," or standard error of the estimate. Let y_i = predicted value for y using the original data: $y_i = b_1x_1 + b_2x_{i2} + \cdots + b_pX_{ip}$; η_i = observed value for y; and $e_i = y_i - \eta_i$. Then the standard error of the estimate is given by

$$s_e^2 = \left(\sum_1^N e_i^2\right) \Big/ (N - p)$$

a statistic analogous to the variance for a single random variable; that is, 68 percent of the predicted values should be in error by less than $\pm s_e$.

Treat-ment No.	No. of Reps	Replications							
1	8	6.60	5.11	7.14	4.89	6.81	4.93	4.33	5.87
2	8	2.67	2.19	2.36	0.31	1.48	0.68	1.19	2.90
3	8	5.90	4.58	2.08	3.07	3.12	3.73	4.00	2.20
4	8	5.80	5.92	4.88	5.32	5.36	4.64	5.68	2.46
5	8	5.00	5.58	6.08	4.41	5.37	5.89	6.34	6.14
6	8	2.95	4.58	2.89	3.93	1.67	4.61	3.28	2.75
7	8	5.86	6.14	5.93	4.23	4.59	5.18	5.60	5.11
8	8	4.80	6.47	5.63	4.68	4.05	6.25	5.32	5.36
9	8	6.98	5.13	6.41	5.42	6.93	3.62	5.14	4.09

Treat-ment No.	Sum Y	Sum Y2	Mean
1	45.68	268.45	5.71
2	13.78	30.07	1.72
3	28.68	114.02	3.58
4	40.06	209.38	5.01
5	44.81	253.98	5.60
6	26.66	95.84	3.33
7	42.64	239.47	5.33
8	42.56	231.00	5.32
9	43.72	249.78	5.46

Analysis of Variance:

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Among Treatments	8	120.84	15.11	15.22
Within Treatments	63	62.54	0.99	
Total	71	183.38		

Figure 8.3.5.3d. Sample Analysis of Variance Calculation Performed on a Computer

2. Simple correlation. The elements of the D'D matrix are made up of the simple correlation coefficients r_{ij} between each possible combination of two variables at a time. Another method for computing simple correlation coefficients is from $r_{ij} = V_{ij}/s_i s_j$, where $V_{ij} = 1/N\ [\Sigma x_i x_j - x_i' \Sigma x_j]$; $s_i = (V_{ii})^{1/2}$; and y can be considered one of the variables x_i or x_j.

The simple correlation coefficient between two variables can be interpreted as follows. The square of r_{ij} is the percentage of the variance of x_i that is accounted for by its relationship with x_j. This applies only if a linear relationship can be assumed and ignores the possibility of intercorrelations with other variables.

3. Partial correlation coefficients. Let a_{ij} be the elements of the inverse matrix $(D'D)^{-1}$. Then, $\gamma_{ij} = a_{ij}/(a_{ii}a_{jj})^{1/2}$ are the partial correlation coefficients.

This statistic gives the true correlation between each pair of two variables (one must be the dependent variable) out of the total investigated, after the effects of the remaining variables have been taken into consideration.

4. Multiple correlation coefficient. This is a measure of the total variation of dependent variable y that has been accounted for by the regression analysis, and is analogous to the simple correlation coefficient for two variables only. This coefficient is given by $R = 1 - (1/a_{ii})^{1/2}$. This gives an

excellent measure of the success of the regression analysis.

5. Alternate calculations of the standard error of the estimate. The calculation of s_e given earlier implied a considerable amount of addition calculation. There are two other ways of calculating standard error of this estimate after the inverse matrix $(D'D)^{-1}$ is available. They are: biased standard error of the estimate, $s_e' = s_i/\sqrt{a_{ii}}$; and unbiased standard error of the estimate, $s_e = [N/(N - p + \ )]^{1/2} s_e'$.

The preceding discussion has stated that the models chosen for multiple regression must be linear models. This means that the partial derivative of the model function with respect to one of the parameters must be independent of that parameter. This mathematical restriction is severe for some desired applications.

8.3.6 Comparison of Digital Computer Characteristics

The majority of digital computers available today are rented rather than sold, with rental rates varying widely from one computer manufacturer to the next. Optional equipment may cause a ± 30 percent variance from the average rental rates depending on the particular configuration desired by the user. To determine the approximate purchase price of a particular computer, multiply its monthly rental rate by fifty.

The first electronic computers available were non-solid state machines using conventional vacuum tubes in their logic systems. Consequently, the machines produced a quantity of heat, necessitating frequent replacement of the tubes. Most computers now being manufactured are units which have logic systems composed almost entirely of solid state magnetic devices, transistors, and diodes. These machines require less power, generate less heat, are more compact and reliable, and have longer life.

There are several types of internal storage, the most common being drum and magnetic core memories. Drum memories respond more slowly than core memories, because the sensor must sometimes scan the entire drum surface before the data is located. However, some computer manufacturers build a rapid access scheme into their drum units to accelerate the internal processing rate. Most of the newer computer models have magnetic core memories where thousands of tiny cores are assembled into a single logic unit. Magnetic core systems are considered superior to their drum counterparts, since they have no moving or wearing components.

Information is retrieved from a stored program computer by testing to see whether certain elements are in a magnetized or non-magnetized state. These computers are considered *binary*, which implies that all information is processed in terms of ones and zeroes. Multiple binary digits (bits) represent a word or decimal figure. In magnetic core memories, a word is determined by the *sequence* of the magnetized and non-magnetized cores. Essentially the same procedure applies to drum memories; however, instead of using a matrix of cores, bands or tracks on the surface of a rotating drum are used. Different computers are capable of handling different word sizes (word size determines the magnitude of the numbers with which one can operate.) A machine with 64 bit capacity could work with whole numbers up to 20 digits. In general, it takes about 3.3 bits to represent the information contained in one decimal digit.

Although all internal storage computers use the binary principle of magnetized and non-magnetized elements, their internal components may be wired and arranged in markedly different schemes and, as a result, are programmed differently. Stored program computers can be divided into three classes: regular binary computers, alphanumeric computers, and decimal computers.

The *binary computer* performs fewer and faster internal operations than other computer types and is well suited to solve complex engineering and scientific problems. However, communication with this kind of computer is inherently difficult, and usually requires the use of special programs for translations to and from binary.

The *alphanumeric computer* is used primarily for business applications on problems such as payroll, inventory, or other areas represented in alphabetic or numeric terms.

The *decimal computer* may be categorized between the binary and alphanumeric machines and are programmed using numeric digits only. Two numeric digits are used to represent an alphabetic character. This type of computer is versatile because it can do both scientific and business processing problems on a fairly large scale, although it is not as efficient on business and alphanumeric problems as the alphanumeric computer, nor as fast on engineering problems as the binary computer.

When a computer is described as being suited to business applications, it does not imply that it cannot be used in the other areas, and vice versa. Any computer can solve various kinds of problems if the programming is adjusted to its special requirements.

A computer's speed may be attributed to a number of factors:

Instruction addresses are separate storage areas in a computer. (Digital machines may have one or several instruction addresses.) The advantage of a three-instruction address system lies in the fact that only one instruction may be needed for certain three-step operations, whereas three separate instructions are needed in a single-address system for the same sequence of operations.

Add time is the time required by a computer to execute an ADD instruction.

Average access time is the time required by a computer to obtain a piece of data or instruction from memory. This is part of the add time.

Magnetic tape speed indicates how quickly data can be brought into or out of a computer from external tape units.

Time sharing describes how many functions a computer can perform simultaneously (reading (R), writing (W), and computing (C)). Some machines can perform all three simultaneously (RWC); others can do multiple reading, writing and computing (MRWC). The latter allows multiple operations to be processed concurrently.

Random access file is a large capacity, auxiliary storage unit which has slightly slower access than internal or "fast" storage, because the disc storage file is external. There is a time-consuming mechanical action involved in choosing the required disc from a stack of discs which are stored externally.

Peripheral equipment relates to a computer's speed in assimilating incoming data (input) and producing final tabulated results (output). Results may be in the form of punched cards, punched paper tape, or printed lines.

Table 8.3.6 compares a selection of available, general purpose, digital computer systems, listed with regard to ascending monthly rental rates. The computers were selected at random: selection was *not* based on superiority over other computers in their respective price ranges.

REFERENCES

Analog Computers	Digital Computers
6-8	1-227
20-14	1-230
26-47	1-232
80-7	1-235
33-6	1-243
158-2	19 219
192-4	386-1
257-3	401-1

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COMPUTER	GENERAL CHARACTERISTICS: AVERAGE MONTHLY RENTAL (RANGE)	SOLID STATE	STORAGE CAPACITY AND TYPE (K = 1000 WORDS)	WORD SIZE	INSTRUCTION ADDRESS	FILE S: ADD TIME (μ = MICRO SECONDS)
IBM 7090	$63,000 (55—69)	*	32K Core 186K drum	36b	1	4.4μ
UNIVAC 1107	$45,000 (32—60)	*	16—65K core 128 film	36b	1	4μ
PHILCO 2000 MOD. 210,211	$40,000 (24—66)	*	8—32K core	48b	1	15μ 0.75μ
CONTROL DATA 1604	$34,000 (19—35)	*	8—32K core	48b	1	4.8μ
UNIVAC II	$28,000 (25—30)	—	2K core	12a	1	200μ
HONEYWELL 800	$22,000 (12—30)	*	4—32K core	12d	3	24μ
BURROUGHS 220	$17,000 (8—35)	—	2—10K core	10d	1	200μ
IBM 1410	$13,500 (6—32)	*	10—80K core	1a	2	88μ
IBM 650	$9,000 (3.7—16)	—	1—4K drum 60 core	10d	1	700μ
CONTROL DATA 160A	$4,000 (2.2—9.5)	*	8—32K core	12b	1	12.8μ
PACKARD BELL PB440	$3,500	*	4—28K core 2—4K biax	24b	0	1μ
AUTONETICS RECOMP II	$2,500 (2.5—4.5)	*	4K disc 16 fast	40b	1	1.08μ
RAMO WOOLRIDGE TRW 230	$2,200 (1.8—6.5)	*	8—32K core	15b	0—1	12μ
SCIENTIFIC DATA SDS 910	$1,700 (1.5—8)	*	2—16K core	24b	1	16μ
CONTROL DATA G15	$1,500 (1.5—4)	—	2K drum 16 fast	29b	1	540μ
PACKARD BELL PB250	$1,200 (1.2—6)	*	2.3—15K delay 16 fast	22b	1	24μ
BURROUGHS E101	$875 (0.8—1.4)	—	220 drum	12d	1	50m
HW 15K	$355 (0.35—6)	*	4K drum	24b	1	700μ

ISSUED: NOVEMBER 1968
SUPERSEDES: MARCH 1967

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Table 8.3.6 Typical Digital Computer Systems
(Reference 386-1 and 401-1)

FILE STORAGE		PERIPHERAL EQUIPMENT			INPUT		OUTPUT		
ADD TIME (μ = MICRO SECONDS)	AVERAGE ACCESS TIME (m = MILLI-SECONDS)	AVERAGE TAPE SPEED (THOUSANDS OF CHARACTERS PER SECOND)	TIME SHARING	RANDOM ACCESS FILE	CARDS PER MINUTE	PAPER TAPE (CHARACTERS PER SECOND)	CARDS PER MINUTE	PAPER TAPE (CHARACTERS PER SECOND)	PRINTER LINES PER MINUTE
4.4μ	2.2μ	15—170	MRWC	•	250	—	100	—	150
4μ	4μ	25—120	MRWC	•	600	400	300	110	600 700
15μ 0.75μ	10μ 1.5μ	90	MRWC	•	2000	1000	100	100	900
4.8μ	6.4μ	30—83	MRWC	—	1300	350	100	110	150 1000
200μ	40μ	25	RWC	—	—	—	—	—	—
24μ	6μ	64—124	MRWC	•	800	1000	250	110	150 900
200μ	10μ	25	—	•	300	1000	100	60	150 1500
88μ	4.5μ	7.2—90	RWC	•	800	500	250	—	600
700μ	4.8m 100μ	15	RC, WC	•	155	60	100	—	150
12.8μ	6.4μ	15—83	RC, WC or RW	—	1300	350	100	110	—
1μ	6μ 1μ	42—62	MRWC	—	800	500	250	110	1000
1.08μ	9m 950μ	1.8	—	—	20	600	15	150	—
12μ	6μ	15—41	—	—	200	300	—	60	150
16μ	8μ	3.5—41	MRWC	—	200	300	—	60	300
540μ	29.5m 1.08m	0.43	RC, WC	—	100	400	100	100	100
24μ	3.07m 12μ	2	—	—	400	300	—	110	500
50m	20m	—	—	—	—	20	—	13	60
700μ	16.7m	—	—	—	200	20	200	20	—

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TABLES

9.1 INTRODUCTION

The importance of adequate component procurement specifications to the success of a hardware development program cannot be overemphasized. Specifications which are too stringent can be as detrimental as specifications which are too lax. Performance specifications, for instance, must not only clearly identify all the component requirements, but must also include sufficient quality assurance provisions so that compliance can be verified.

The purpose of this section of the handbook is to describe specification types, present guidance for the adequate preparation of fluid component specifications, and identify applicable documents commonly referenced in fluid component specifications.

9.2 SPECIFICATION TYPES

Fluid component specifications can be categorized according to one of the following three basic types: performance, manufacturing, and proprietary. Each type is outlined below and discussed in detail in subsequent paragraphs.

1) **Performance Specification.** Identifies the constraining parameters, details the required performance, and specifies the tests needed to verify conformance of the product to performance requirements.

2) **Manufacturing Specification.** Identifies the complete design, including materials, processes, tolerances, dimensions, and configuration, in sufficient detail for any qualified manufacturer to produce the product.

3) **Proprietary Specification.** Identifies the exact make, model, or part number and allows no latitude for deviation from the specified item(s).

9.2.1 Performance Specification

A performance specification is a clear and accurate description of the design, construction, and performance requirements of a product, with provisions for determining compliance of the end product to the description. A performance specification is written as the basis for the procurement of an end product which will completely fulfill all specified requirements.

To accomplish this objective, a performance specification must provide complete and thorough answers to the following basic questions:

What is the product?

What physical, chemical or mechanical constraints are imposed on the product?

What must the product do?

In what environments must it function, and within what limits?

What tests and inspections will prove performance and compliance with requirements?

How is the product to be finished, marked, cleaned, packaged, etc.?

What documentation is required?

What are the life and reliability requirements, and how is compliance to be demonstrated?

What are the maintenance requirements?

A well-written specification will answer each of the above questions clearly. If any question is not answered, it is possible that something has been overlooked, and trouble may be experienced during procurement or application of the product.

Many component problems in aerospace fluid systems can be traced to performance specifications which either lack important information, or are based on unrealistically stringent requirements. Specifications which do not adequately cover component requirements will usually result in components failing to meet their intended function, with an expensive redevelopment and retrofit program then required to correct component deficiencies resulting from the specification errors.

Alternately, unnecessarily stringent or conservative specifications will require excessively long and expensive development programs, resulting in over-designed units.

The added design complexity required to meet unreasonably severe functional requirements such as response time, leakage, regulation bands, and unrealistic environmental requirements, often results in excessive costs, long delivery times, and unreliable systems. A good performance specification, therefore, must carefully consider all the requirements of the component for its intended function, but should also avoid placing on the component severe performance or environmental margins which could seriously compromise the end result.

Many problems in aerospace fluid systems can also be traced to test programs which were either not rigorous enough to evaluate the performance adequately, or were so stringent that time and money were wasted by testing for objectives that were impossible or unnecessary to achieve. Failure to specify and test for (1) dynamic conditions involving component-system interactions, (2) environmental transients, such as thermal shock, and (3) vibratory conditions are common component specification shortcomings which can result in serious setbacks to system development programs.

9.2.2 Manufacturing Specification

A manufacturing specification is a document containing enough detailed information to produce the end product described without requiring any additional design work. This specification contains the necessary design, materials, dimensions, and manufacturing methods information necessary to produce the product.

This type of specification is often used to obtain competitive procurement of a well-seasoned design, and if properly prepared and administered, can produce a competitive procurement of a very complex product at a minimum cost and with short delivery schedules.

Particular care should be taken to prevent inclusion of performance specifications and tests in a manufacturing specification. Such a combination of performance and manufacturing specifications may be unenforceable, because if for some reason the specified design and manufacturing data do not produce a component with the specified performance, the specification is obviously in conflict within its own sections, and the contractor cannot be held responsible.

9.2.3 Proprietary Specification

The proprietary specification specifies the required product by make, model, and manufacturer's part or catalog number. This type of specification is the easiest means of delineating the required item, and assures . the specific component desired will be furnished. The proprietary specification should never include the words *or equal* or similar phraseology, because the burden of proof of equality is on the purchaser. If the words *or equal* or similar phraseology are required by governmental regulations, then a performance specification should be used, with clearly defined tests and inspections included to verify equality. Sub-Topic 9.5.5 discusses the *or equal* clause. When performance and test requirements are included, the specification is no longer a proprietary specification, but becomes a performance specification.

9.3 SPECIFICATION FORMAT

The format of a specification should be as simple as possible, and arranged in such a manner that information of a specific type may be readily located and referenced. AFSCM 375-1 and Defense Standardization Manual M-200 present the general format which is widely used both in governmental and industrial specifications. The major sections of a specification, listed below in the commonly accepted order, are:

- Scope
- Applicable Documents
- Requirements
- Quality Assurance Provisions
- Preparation for Delivery
- Notes

The level of detail under each heading is a function of the complexity of the device or system, and the type of specification, either proprietary, manufacturing, or performance. Since the proprietary specification usually requires only the part number to describe an item, the standard format will contain sections which are not necessary, such as "Scope" and "Applicable Documents." The use of section headings is still suggested, however, in assuring that quality assurance and preparation for delivery are adequately specified.

The manufacturing and performance specifications utilize all sections of the standard format, and contain a high level of detail in each section.

9.4 SPECIFICATION CONTENT

The contents of each of the six standard specification sections are discussed in the following paragraphs. Table 9.4 lists the major topics which should be included in each section.

Table 9.4. Specification Content

Section 1 — Scope

a) Brief statement of coverage
b) Brief description of item
c) Type or class of item

Section 2 — Applicable Documents

a) Referenced specifications
b) Referenced standards
c) Referenced drawings
d) Referenced exhibits
e) Referenced publications

Section 3 — Requirements

a) Performance
b) Qualification
c) Sample or pilot model
d) Materials
e) Design details
f) Construction
g) Operating environmental
h) Lubrication
i) Standard part
j) Interchangeability
k) Weight and dimensional
l) Finish
m) Connection and interface
n) Locking
o) Contamination
p) Reliability
q) Maintainability
r) Workmanship
s) Radio interference
t) Storage
u) Cleanability (drainage, trapped areas, etc.)
v) Minimum design safety factors

Section 4 — Quality Assurance Provisions

a) Test methods and procedures to support requirements stated in Section 3., including criteria for success:
   - Development
   - Design verification
   - Qualification
   - Production
   - Acceptance

b) Sampling requirements and procedures
c) Examinations and inspections

Section 5 — Preparation for Delivery

a) Cleaning
b) Painting
c) Packaging
d) Preserving
e) Marking
f) Identification

Section 6 — Notes

a) Safety
b) Intended use
c) Drawing and data requirements
d) Test reports
e) Ordering data
f) Maintenance data requirements
g) Special tools
h) Symbols
i) Definitions
j) Miscellaneous
k) Failure analysis reports

9.4.1 Scope (Section 1)

The "Scope" section may be a very brief statement indicating the coverage of the specification for a simple device, or it may require a long description of limiting parameters for a more complex device or system having a difficult interface definition.

9.4.2 Applicable Documents (Section 2)

The proper use and application of referenced documents is one of the most difficult aspects of specification writing. The specification writer, usually pressed for time, is often unable to investigate thoroughly the content and applicability of the referenced documents. As a result, specifications may not spell out the extent of applicability of referenced documents, and documents are often listed which are never again referenced or used in the specification. A tabulation of frequently-used applicable documents for fluid component specifications is shown in Subsection 9.6.

Several rules which are commonly followed as an aid in the preparation of an Applicable Documents section are:

a) List only those documents which are actually referenced in the specification text.

b) In the specification text indicate the specific portions of the applicable document which are pertinent.

c) Specify the date of issue or date of applicability of the referenced document. For example, the words "latest issue" are not enforceable and should not be used because a contractor can only bid on a definable set of specifications of a specific date. The date of bid is commonly used as date of applicability. In some procurements, an earlier issue of the referenced document may be desired and thus specified to utilize desirable features of an out-dated document.

d) Review the referenced documents to assure that they are actually applicable.

9.4.3 Requirements (Section 3)

The "Requirements" section should be the basis for the specification, with all other sections supporting this key section. It should contain a complete description of the performance, test, design, construction, and other characteristics required of the product. The performance requirements of a performance specification should be clearly stated in this section, and the remainder of the specification should be tailored to assure that the item is tested, packaged, inspected, and documented to assure this performance. Test requirements should be stated very briefly, and the detailed test procedures to implement the test requirements should be included under "Quality Assurance Provisions."

The inclusion of all critical performance parameters in the "Requirements" section is of utmost importance in the preparation of a performance specification. As an aid in the preparation of performance requirements, Table 9.4.3 lists typical fluid component performance parameters, indicating the types of components to which they normally apply.

The "Requirements" section of a performance specification should define each operating parameter under which the device being specified must perform. This definition should include the operating environment as well as the interaction of the component with the system in which it is installed. Performance requirements should include the number of operating cycles required of the component.

It is important that in addition to steady-state factors the performance requirements should include dynamic or transient conditions. Other factors, such as thermal interaction and contamination to and from the system, should be clearly defined.

The ideal performance specification contains the actual required upper and lower performance limits of a component. In a new field involving research and development, however, the performance margins may not be well defined. Under these circumstances, a safety factor may have to be applied to certain performance parameters to assure a successful piece of hardware. Safety factors should be selected with great care to assure a reliable end product as a minimum requirement, and still stay within the limits of practicability and cost at the other extreme. Another precaution which should be taken when safety factors are being assigned is to assure that the safety factor is only taken once. In large programs involving many persons, groups and agencies, there have been instances where each group takes an additional safety factor, compounding the original and valid requirement. When this happens, cost and weight are almost always excessive and occasionally the limits of practicability are exceeded.

After the performance requirements have been specified, a cross-check should be made with test requirements to assure compatibility of the two sections. Because a performance requirement is meaningful only if a means of testing the performance can be accomplished, a test should be provided for each performance requirement and each test requirement should relate to one or more performance requirements.

9.4.4 Quality Assurance Provisions (Section 4)

This section should include all test methods, test procedures, and inspections necessary to support the "Requirements" section of the specification. (Testing provisions are normally applicable only to a performance specification.) Test requirements should be stated in sufficient detail to establish communication between buyer and seller. Test *plans*, which describe, in general, what testing is to be accomplished, should be included under Quality Assurance Provisions. The supplier produces detailed test procedures from the plans.

9.4.4.1 TYPES OF TESTS. There are three basic reasons for testing a device or system: to determine (1) what the component or system will do, (2) the ability of a device or system to withstand the operating environment, and (3) how long or how reliably the component or system will perform without failure. The tests used to make these determinations are called:

a) Functional tests (performance)
b) Environmental tests
c) Reliability tests (life and limit)
d) Development tests
e) Design verification tests
f) Prequalification tests
g) Qualification tests
h) Preproduction, pilot model, pilot lot tests
i) Production acceptance tests
j) Production monitoring tests
k) System integration tests.

The extent of testing is usually a compromise between (1) testing which is necessary to assure reliability, and (2) the time, money, and facilities available to perform the test. This tradeoff is especially difficult to make in components which are to be utilized in space vacuum and zero gravity, because of the cost associated with environmental simulation.

Functional Tests. Functional tests are performed to determine the operating parameters of a component or system; they determine such characteristics as:

- Flow rate
- Pressure drop
- Strength (proof or burst)
- Internal leakage
- External leakage
- Flow and pressure control
- Response
- Power requirements
- Repeatability
- Contamination tolerance

Environmental Tests. Environmental tests are specified to simulate the most severe standby or operating conditions anticipated for the component or system. Compatibility of a component with its operating environment is normally determined by testing under separate environments, e.g., vibration, low temperature, etc. The effect of combined

Table 9.4.3. Performance Parameters for Typical Fluid Components

	Shutoff Valves	Flow Control Valves	Pressure Regulators	Relief Valves	Servo Valves	Explosive Valves	Multiple Passage Valves	Check Valves
WORKING FLUID(S)	X	X	X	X	X	X	X	X
PRESSURE CONSIDERATIONS:								
Burst Pressure	X	X	X	X	X	X	X	X
Proof Pressure	X	X	X	X	X	X	X	X
Operating Inlet Pressure	X	X	X	X	X	X	X	X
Operating Outlet Pressure	X	X	X	X	X	X	X	X
Differential Pressure at Rated Flow	X	X		X	X	X	X	
Cracking Pressure				X				X
Reseat Pressure				X				X
Outlet Pressure Symmetry (multiple outlet ports)					X		X	
Lockup Pressure			X					
Lockup Differential Pressure			X					
Outlet Pressure Regulation Range		X	X		X			
Pressure (load) Droop			X					
Reference Pressure Sensing Considerations			X	X				
FLOW CONSIDERATIONS:								
Rated Flow (load and no-load as applicable)	X	X	X	X	X	X	X	X
Flow Range (throttleability)		X	X		X			
Flow Coefficient	X	X				X	X	
Flow Characteristics (linear, parabolic, etc.)		X			X			
Load Flow — Pressure Characteristics					X			
Saturation Flow					X			
Outlet Flow Symmetry (multiple outlet ports)					X		X	
LEAKAGE CONSIDERATIONS:								
External Leakage	X	X	X	X	X	X	X	X
Internal Leakage	X	X	X	X	X	X	X	X
Null Leakage					X			
Quiescent Flow (leakage flow versus spool position, max at null)					X			
INPUT POWER OR FORCE CONSIDERATIONS:								
Actuation Force or Torque	X	X					X	
Actuation Power	X	X			X	X	X	
Pull-In Voltage	X						X	
Holding Voltage	X						X	
Drop-Out Voltage	X						X	
Coil Resistance and/or Impedance	X				X	X	X	
Rated Current	X				X	X	X	
Quiescent Current					X			
Null Bias Current					X			
Dither Current					X			
All-Fire Current						X		
No-Fire Current						X		
Phasing and/or Polarity	X				X		X	
OUTPUT VERSUS INPUT CONSIDERATIONS:								
Time Response	X	X				X	X	
Deadband		X	X		X			
Linearity		X			X			
Hysteresis		X	X		X			
Resolution		X			X			
Gain (flow and/or pressure)		X			X			
Null Pressure Gain					X			
Hydraulic or Flow Null					X			
Null Shift (pressure and temperature effects)					X			
Transient (step) Response Time			X		X			
Overshoot Allowable and/or Settling Time From Step Input			X		X			
Frequency Response, Phase Lag, Amplitude Ratio (load and no-load)		X	X		X			
Feedback Considerations					X			
LIFE CONSIDERATIONS:								
Duty Cycle	X	X	X	X	X	X	X	X
Operating Cycle (total time and/or number of cycles)	X	X	X	X	X	X	X	X

environments operating simultaneously is an important consideration, however, and should be considered as part of an environmental test program. As combined environmental testing is very expensive, complete combined environmental simulation is not practical. Tests must be carefully selected to provide the best simulation within the confines of budget and schedule. Typical exposures included in environmental test specifications are:

- Temperature
- Humidity
- Salt spray
- Sand and dust
- Altitude (vacuum)
- Shock
- Vibration
- Acceleration
- Acoustic noise
- Chemical compatibility
- Radiation
- Fungus

Reliability Tests. Reliability tests are performed to determine the probability that a component will fulfill its intended function. Components which are cyclic in operation are usually tested for number of operating cycles until failure, and components which operate continuously are usually tested to determine the mean time to failure. Cyclic tests can usually be repeated with sufficient frequency to simulate the operating cyclic life in a reasonably short test period. On the other hand, continuous life tests may be difficult to simulate, particularly on components designed to operate thousands of hours in normal service.

Limit testing, or performance margin testing, determines the margin of safe operation over and above design conditions. Limit tests are conducted by progressively increasing the severity of a test parameter, such as temperature, until the component fails. The margin of safe operation over the design conditions is a measure of the component's functional reliability.

Tests may also be categorized according to the time or phase of a program during which the tests are performed. Such tests are:

- Development Tests
- Design Verification Tests
- Prequalification Tests
- Qualification Tests
- Preproduction Tests
- Production Acceptance Tests
- Production Monitoring Tests
- System Integration Tests

All of these include functional tests, and may also include environmental and reliability tests.

Development Tests. These tests are performed on initial prototype hardware or sub-assemblies to check out the design parameters during the development process. Development tests should be used to verify such factors as flow areas, pressure drops, sizing of subcomponents for power drain, and functional operation plus all other requirements necessary to produce a complete set of engineering drawings which will describe a component capable of meeting its specification requirements. The model used for such tests is usually a "breadboard," "boiler plate," or "engineering model" which has been produced specifically for these tests. The tests should serve to provide data required to make final a new design or to optimize an existing design to comply with new requirements. Adjustments, rework, repair, and retest are normal functions during a development test. Specifications should require that all activities, adjustments, and repairs be accurately recorded during testing. Reasons for repair as well as details of all repairs and adjustments should be documented for future correlation with the production unit.

Design Verification Tests. These tests should be run on initial prototype hardware prior to proceeding to production drawings and actual fabrication of production hardware. Test requirements, toward which the manufacturer should design, should be spelled out specifically in the component specification. Design verification tests are planned to prove that a component has the capability to meet all of its functional and the most critical of its environmental requirements. Component design verification tests allow system tests to be started with maximum assurance that components have proven the capability for performing their system function prior to performing time-consuming life or reliability tests.

Prequalification Tests. Prequalification tests (also called design approval tests, preliminary flight rating tests, and flight certification tests) are run on production hardware prior to their use for flight testing to determine whether the article fabricated by production tooling and techniques will perform as capably as when fabricated as a prototype. These tests should include all functional and environmental requirements, and some life-cycle tests. The tests must prove at this point that the production hardware is capable of meeting all of the required parameters for at least the length of time required by the flight test program. Special "stress to failure" tests are sometimes included as part of prequalification testing. These tests, which can be destructive, are designed to prove margins of safety over minimum design requirements.

Qualification Tests. Qualification tests are normally formal demonstrations (in contrast to evaluations) with production hardware, and are the final test requirements to be met by the component. A primary difference between formal qualification tests and other tests is that this test is used to demonstrate rather than evaluate the product. They should consist of all the steps taken in prequalification tests, as well as the following:

1) The component tested should be randomly selected, representative production-type hardware and made entirely with the manufacturer's production tooling and processes.

2) The number of samples tested should be adequate to prove that the components are statistically capable of meeting their reliability requirements.

3) The tests should be repeated at various undefined points during the production phase of the program to assure that the last components made meet the same standards as the first.

Preproduction, Pilot Model, Pilot Lot Tests. When an extensive production run of products is anticipated, tests are often performed to check the conformance of the preproduction or pilot units prior to commencing a full scale production run. These tests are called preproduction tests, pilot model tests, or pilot lot tests. The individual tests may consist of any or all of the tests in the categories of functional, environmental, or reliability testing.

Production Acceptance Tests. These are non-destructive tests run on deliverable production-type hardware to assure that they are identical in design and manufacture to those components which have previously completed the formal qualification and/or prequalification test programs. Although these tests are of a quality-control nature, they are an integral part of the step-by-step program to ensure a satisfactory end product. During early hardware production, acceptance tests may include limited environmental testing. Testing of this nature is commonly called Production Environmental Testing (PET). These tests usually start on a 100 percent basis, with the number of parts tested reduced to a sampling basis as confidence in the production is increased, until the PET testing is ultimately dropped with subsequent acceptance testing limited to the normal perfunctory bench-type functional tests.

Production Monitoring Tests. These tests are conducted at prescribed intervals to subject the product to more intensive or extensive conditions than are encountered in the normal production acceptance test. These tests can be either destructive or non-destructive and are performed on a sampling basis.

System Integration Tests. These tests are performed to evaluate the compatibility of the components with system requirements, and serve to evaluate and optimize checkout and operating procedures. Although a component may have been correctly designed to fulfill its own function, its compatibility with related equipment and its workability as part of an integrated system must be demonstrated.

9.4.4.3 CRITERIA FOR SUCCESS. Each test section in a performance specification should contain a clear statement of criteria for successful completion of the test. Unless this is done, enforcement of performance requirements cannot be accomplished.

An example of the importance of success criteria was demonstrated on a pump procurement based on a test specification requiring a 1000-hour life test. However, criteria for successful test completion were not specified. The pump operated 1000 hours successfully, but disassembly after test revealed cracked and broken bearings. Since adequate criteria for success had not been specified, the test was considered to have been successful. The production run of several hundred pumps experienced similar cracked bearings in service, and were later rebuilt at an extremely high cost.

9.4.5 Preparation for Delivery (Section 5)

This section should include all necessary information on the packaging and packing of the component to assure safe delivery to the destination, and should take into consideration the duration and environment of the storage to which the product will be subjected prior to ultimate use. Particular attention should be given to the cleaning portion of a specification for fluid components to assure that the cleanliness requirements are realistic and that cleanliness standards can be achieved at a reasonable cost.

9.4.6 Notes (Section 6)

This section is designed to include any information which does not readily fit into the other sections, and usually includes such information as intended use, ordering data, symbols, and definitions. The information related to intended use is of particular importance to a manufacturer, and inclusion of this information may eliminate many misunderstandings between the procuring agency and the producer.

9.5 SPECIFICATION LANGUAGE

9.5.1 General

The success of a device or system is highly dependent upon the quality of the specification to which the item or system is constructed. The wording of a specification must be clear, concise, and non-conflicting.

9.5.2 Contractual Language

The word "shall" is used for all contractually binding requirements. The use of "will," "should," or "may" indicates recommended, desirable, or preferable, but non-mandatory requirements. When "shall" is used, the requirement is binding on either the contractor or the purchaser. The word "will" is used to express a declaration of purpose on the part of the purchaser.

9.5.3 Measurement Terminology

Dimensions, capacities, sizes, temperatures, accuracies, and tolerances should be specified in accordance with acceptable governmental or industrial practice. The use of percentage tolerances should be avoided when absolute values can be assigned. For instance, 95 to 105 volts would be preferable to 100 volts ± 5 percent. The use of absolute values eliminates the need for unnecessary arithmetic. If there is a strong desire to indicate a nominal value, 100 ± 5 volts would be used. The specification of thickness or diameter by a gage number alone should not be used. If a gage number is indicated, the actual thickness or diameter should also be indicated.

9.5.4 Unenforceable Phraseology

A specification has little value if it contains expressions and phrases which cannot be enforced contractually. A review of specifications will often reveal phrases similar to the following:

a) "In accordance with good commercial practice..."
This phrase is meaningless because good commercial practice may not be satisfactory, and even if commercial practice *is* satisfactory, the phrase does not refer to any industrial standard or code for performance standards, and therefore is unenforceable.

b) "The intent of this specification is to..."
This phrase appears frequently, and is usually ineffective because contracting personnel can only enforce the requirements of the contract, and may experience difficulty enforcing the "intent" of the specifications. A specification should contain the *requirement* rather than the *intent* of the purchaser.

c) "The relays shall be capable of closing when a 28-volt signal is applied..."
Whether the relay is *capable* of closing or not is unimportant. The important fact is whether or not the relay closed. The specification should have read... "The relay contacts shall close when a 28-volt signal is applied." The word "capable" is used only where some necessary condition of performance has not been stated. For example, "The pyrometer shall be capable of indicating ambient temperature with the range of 32 to 100 degrees Fahrenheit."

d) "As a design objective the..."
This phrase implies that it may be impossible to meet some objective or criteria, and that the contractor should at least try to approach the requirement. Phrases of this type cannot be administered or enforced. Wherever possible, a specification should contain firm quantitative requirements which can be evaluated, rather than the unenforceable qualitative words illustrated in this example. If such unenforceable phraseology is used, the specification should include elsewhere definite, required levels of the same parameters discussed under "Design objective."

e) "...consistent with good engineering practice..."
This phrase is of dubious value and implies that there is possibility of receiving bad engineering practice. Phrases of this type should refer to a specific engineering code or standard rather than generalities.

f) "...suitable for the purpose intended..."
This example contains two unenforceable phrases. *Suitable for* is an unforceable generality, and *purpose intended* is a matter of judgement or interpretation. In many cases the manufacturer has little or no knowledge of the detailed system into which a component will be assembled. For this reason, the component manufacturer may not be capable of determining suitability for a specific purpose.

g) "The equipment shall be suitably protected..."
Specifications should be definitive in requirements. The words *suitably protected* in a component specification might mean protection by means of anything from a plastic bag to a steel shipping container. The protection requirements should be detailed in specific terms rather than the vague phraseology used in this example.

h) "Only long life components shall be used..."
Vague performance requirements such as this example are meaningless. The number of operating cycles or mean time to failure should be specified, and tests to prove the life characteristics of the component should be included.

9.5.5 Use of *"or equal"*

There have been many legal cases involving the use of the phrase *or equal* in specifications; likewise, there is often considerable doubt in the minds of specification writers regarding its proper use. Regulations governing the use of *or equal* are contained in Armed Services Procurement Regulations, Section 1-1206.2, "Brand Name or Equal Purchase Descriptions." These regulations have been included by reference only in this subsection because they are in a constant state of flux. The current regulations require, under certain circumstances, that the contractor submit to the contracting officer certain data and information for evaluation.

These regulations assume that the contractor will give the pertinent data for comparison, and assume also that the contracting officer has equivalent data on the brand names specified and is technically capable of evaluating the comparative data. The current regulations appear to make the contracting officer responsible for determination of equality.

If vendors misrepresent the capabilities of their products in their standard published data and contracting officers do not have equivalent data on the specified brand names, the enforcement of an *or equal* clause is virtually impossible.

To evaluate equality properly, a specification should contain the critical performance requirements and tests to evaluate compliance with these requirements. The burden of proof of equality should be the responsibility of the contractor; he should be required to demonstrate compliance by test.

Any contractual arrangement which requires a comparative evaluation on any basis other than tests may result in the delivery of an inferior product.

ISSUED FEBRUARY 1970
SUPERSEDES: OCTOBER 1965

9.6 APPLICABLE DOCUMENTS

Table 9.6 lists applicable documents commonly cited in fluid component specifications.

Military specifications, standards, etc., are catalogued in the Department of Defense "Index of Specifications and Standards." Part I, Alphabetical Listing, and Part II, Numerical Listing. The "Index of Specifications and Standards" can be obtained from:

Commanding Officer
USN Supply Depot (NSD 603)
5801 Tabor Avenue
Philadelphia, Pa. 19120

Military specifications (MIL) and military standards (MS) can be obtained by contractors or other qualified requestors from:

Receiving Officer
Naval Supply Depot
5801 Tabor Avenue
Philadelphia, Pa. 19120

All other requestors can obtain military specifications and standards from:

Superintendent of Documents
U.S. Government Printing Office
Washington, D.C. 20005

Copies of individual Air Force-Navy Aeronautical specifications (AN) and standards (AN, AND) may be obtained from:

Receiving Officer
Naval Supply Depot
5801 Tabor Avenue
Philadelphia, Pennsylvania 19120

Complete sets of Air Force-Navy Aeronautical specifications and standards may be obtained from:

National Standards Association, Inc.
1321 - 14th Street NW
Washington, D.C. 20005

Society of Automotive Engineers (SAE): Aerospace Standards (AS), Aerospace Recommended Practices (ARP), and Aerospace Information Reports (AIR) can be obtained from:

Society of Automotive Engineers, Inc.
Aeronautical Material Specifications
485 Lexington Avenue
New York, N.Y. 10017

9.7 MODEL SPECIFICATIONS

Model specifications have been prepared in some instances for use as guides in writing specifications for individual components.

A particularly applicable military specification covering general components for rocket propulsion systems is:

MIL-C-27410, Component, Rocket Propulsion Fluid System, General Specification for.

A useful military specification covering servo valves is:

MIL-V-27162, Military Specification, Valves, Servo Control, Electro-Hydraulic, General Specification for.

A variety of fluid component specifications written for various Department of Defense programs are published by the Interservice Data Exchange Program (IDEP). Most prime aerospace contractors maintain an IDEP file.

REFERENCES

1-295	65-39	447-6
65-36	65-40	452-2
65-37	65-41	500-1
65-38		

Table 9.6. Applicable Documents

SUBJECT	SPECIFICATIONS: MILITARY	SPECIFICATIONS: NASA	STANDARDS: MILITARY	STANDARDS: INDUSTRY	OTHER PUBLICATIONS
DESIGN PRACTICES	MIL-C-5015 Connectors, Electric, AN Type MIL-P-5514 Packings; Installation and Gland Design, Hydraulic, General Requirements For MIL-S-7742 Screw Threads, Standard Optimum Selected Series; General Specification For MIL-W-8160 Wiring, Guided Missile, Installation of, General Specification For		MS-24385 Fitting End, Flared Tube Connection and Gasket Seal, Precision Type, Standard Dimensions MS-24386 Fitting End; Bulkhead Flared Tube Connection and Gasket Seal, Precision Type, Standard Dimensions MS-33514 Fitting End, Standard Dimensions for Flareless Tube Connection and Gasket Seal MS-33515 Fitting End, Standard Dimensions for Bulkhead Flareless Tube Connections MS-33540 Safety Wiring, General MS-33649 Bos vs. Fluid Connection, Internal Straight Thread MS-33656 Fitting End, Standard Dimensions for Bulkhead Flared Tube Connections MS-3102 Connector, Receptacle, Electric, Box Mounting	ARP 24A Determination of Hydraulic Pressure Drop AS 130A Bending Radius, Tube ARP 584 Coiled Tubing ARP 735 Aerospace Vehicle Cryogenic Ducting ARP 739 Gas Power Servos and Reaction Control Systems ARP 745 General Components Specification for Explosive Actuated Valves One Cycle ARP 777 Gas Actuators (Linear and Vane Rotary Type) AIR 867 Long Term Storage For Missile Hydraulic Control Systems, Design and Operational Practices Guide	USASI B46.1-1962 Surface Texture, Surface Roughness Waviness and Lay
STANDARD PARTS	MIL-S-4040 Solenoid, Electrical, General Specification For MIL-F-5509 Fittings; Flared Tube, Fluid Connection				Bulletin ANA 438 Age Controls of Age-Sensitive Elastomeric Items
INTERCHANGEABILITY	MIL-I-8500 Interchangeability and Replaceability of Component Parts for Aircraft and Missiles				
MAINTAINABILITY			MIL-STD-470 Maintainability, Program Requirements for Systems and Equipment MIL-STD-471 Maintainability Demonstration		
ENVIRONMENT AND TESTING	MIL-H-25475 Hydraulic System, Missile, Design, Installation Tests, and Data Requirements, General Requirements for MIL-E-5272 Environmental Testing, Aeronautical and Associated Equipment, General Specification for MIL T-5522 Test Procedure for Aircraft Hydraulic and Pneumatic Systems, General		MIL-STD-810 Environmental Test Methods	ARP 219 Procedure and Method for Conducting Tests of Hydraulic Components in Contamination Controlled System ARP 849 Method of Evaluating Effectiveness of Inspection Procedures for Adequacy of Filter Element Cleaning	AFSCM 80-6 Handbook of Instructions for Aerospace Ground Equipment Design
RELIABILITY		NASA NPC 250-1 Reliability Program Provisions for Space System Contractors	MIL-STD-785 Requirements For Reliability Program for Systems and Equipments		
IDENTIFICATION AND MARKING			MIL-STD-130 Identification Marking of United States Military Property MIL-STD-1247 Identification of Pipe, Hose, and Tube Lines for Aircraft, Missile and Space Systems	ARP 568 Uniform Dash Numbering System for O-Rings	

Table 9.6. Applicable Documents (Continued)

SUBJECT	SPECIFICATIONS		STANDARDS		OTHER PUBLICATIONS
	MILITARY	NASA	MILITARY	INDUSTRY	
ASSEMBLY				ARP 683 Installation Procedures and Torques for Fluid Connections	
DOCUMENTATION	MIL-D-5480 Data, Engineering and Technical, Reproduction Requirements for MIL-D-1000 Drawings, Engineering and Associated List		MIL-STD 831 Test Reports, Preparation of		
SPECIFICATIONS			MIL-STD-143 Specifications and Standards, Order of Precedence for the Selection of	AIR 737 Aerospace Hydraulic and Pneumatic Specifications and Standards	AFSCM 375-1 Configuration Management During Definition and Acquisition Phases
MATERIALS AND PROCESSES	MIL-P-116 Preservation, Methods of MIL-A-8625 Anodic Coatings, for Aluminum and Aluminum Alloys		MS-33586 Metals, Definition of Dissimilar	AIR 786 Elastomer Compatibility Considerations Relative to O-Ring and Sealant Selection AIR 810 Degradation Limits of MIL-H-5606 Hydraulic Fluid	
CLEANING			MIL-STD-1246 Degree of Cleanliness and Clean Room Requirements	ARP 598 Procedure for the Determination of Particulate Contamination of Hydraulic Fluids by the Particle Count Method ARP 599 A Dynamic Test Method for Determining the Degree of Cleanliness Filter Elements ARP 785 Procedure for the Determination of Particulate Contamination in Hydraulic Fluids by the Control Filter Gravimetric Procedure	Air Force Technical Manual TO 42C-1-11C Cleanliness Standards, Cleaning and Inspection Procedures for Ballistic Missile Systems
ASSEMBLY				ARP 600 Torque Determination, Method of, for Tube or Hose End Fitting Connections, Flared, Flareless, or Miscellaneous Screw Thread Style	
QUALITY ASSURANCE AND QUALITY CONTROL	MIL-Q-9858 Quality Program Requirements	NASA NPC 200-1 Quality Assurance Provisions for Inspection Agencies NASA NPC 200-2 Quality Program Provisions for Space System Contractors NASA NPC 200-3 Inspection System Provisions for Suppliers of Space Materials, Parts Components and Services	MIL-STD-109 Quality Assurance Terms and Definitions		Air Force Technical Manual TO 42B7-2-1-7 Quality Control of Propellant Nitrogen Tetroxide TO 42B7-2-1-4 Quality Control of Propellant Hydrazine UDMH (50/50) Mixture

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10.1 INTRODUCTION

The subject of fluid systems contamination has become progressively more important with the increased evidence that fluid component malfunctions are intimately related with fluid contamination. Contamination not only affects system performance, but is also a significant factor in determining component overhaul and system maintenance costs. This problem reaches its peak of criticality in airborne missile fluid systems — propulsion, pressurization, and hydraulic controls — as space, weight, and reliability requirements have become more exacting, and smaller and more precise devices are developed to meet these demands.

It is the intent of this section to present to the fluid component designer an over-all picture of the subject of contamination and cleaning. To this effect, the basic elements of fluid system contamination and its control are presented in the following sequence: (1) nature of contamination, (2) effects of contaminants, (3) cleanliness requirements, (4) contamination control measures in components, systems, fluids, and environment, and (5) contamination considerations in design.

10.2 NATURE OF CONTAMINATION

The most acceptable definition of contamination as it applies to all airborne fluid systems is given in Reference 89-1 as follows:

> *"Any amount of material which constitutes a chemically reactive hazard or a mechanical impairment to proper function of the equipment and/or the performance of the system."*

Since contamination must be defined in relation to the requirements of a specific system, engineering specifications should enlarge upon the above definition by quantifying and qualifying the contaminants that actually constitute a hazard or impairment.

10.2.1 Types of Contamination

The forms and kinds of contaminants found in fluid systems cover the complete range of material and fluids used in the systems, as well as ambient contaminants found in the environments in which the systems operate. For instance, the following contaminants were found in one sample of hydraulic oil (Reference 6-38):

fibers	paint
fly ash	plaster
glass	rubber
lanolin	rust
lint	sand
mercury	silica
mica	steel

Since general terms such as contaminants, debris, foreign matter, dirt, and "gunk" do not accurately describe the character and condition of the materials found in a fluid system, it is necessary to sort and classify these bulk contaminants according to their most relevant properties. Reference 89-1 classifies fluid system contaminants in seven groups related to their chemical nature and conditions (for the purpose of facilitating their future removal) as follows:

1) Carbon and combustion products
2) Loose particulate material
3) Preservatives and lubricants
4) Weld slag, oxides, and scale
5) Rust and corrosion products
6) Waxes, tar, adhesives, and masking
7) Residual fluids, fuels, and propellants

Because the most common forms of contamination affecting fluid systems are particles, fibers, water, and reactive residues, process specifications and control documents usually group all contaminants into the three following categories (References 89-1, 457-1, 457-3, and 457-4):

1) Non-combustible contaminants (gross matter, and particulate matter such as particles and fibers)
2) Combustible contaminants
3) Water

Non-combustible contaminants consist of solid and insoluble matter which can be carried by the fluid in the system and which may lead to mechanical malfunction of the equipment. Gross matter is considered to be large pieces of material or whole objects which are easily detected by the naked eye, such as metal chips, rocks, tools, scale, screws, corrosion products, and coarse granular materials. Particulate matter consists of finely divided forms of the above listed gross, non-combustible contaminants which usually require optical aids for their examination and detection. They are classified into particles and fibers according to their shape and dimensions. Combustible contaminants are reactive, incompatible substances such as paint, organic matter, solvents, rags, lint, etc., which may produce fire or explosion hazards. Water, whether it is present in free form as entrained moisture or as vapor, may be detrimental to fluid stability and system performance and usually promotes corrosion.

10.2.2 Sources of Contamination

There are four primary ways in which contaminants may be introduced to, or developed within a system: negligence, system wear, fluid, and environment. These sources of contamination can be divided into two groups:

1) Internal contamination (contaminants initially in the system or generated by the system)

2) External contamination (airborne contaminants, and foreign or contaminated fluids)

10.2.2.1 INTERNAL CONTAMINATION. Internal contaminants are the most numerous and difficult to control, since their origin includes the attrition and breakdown processes of all parts of the system after it has been designed, assembled, and tested.

Contaminants Initially in the System. Even before a fluid system is operated for the first time, it may already be contaminated by unclean components, leftover dirt, or poor installation procedures. The most common sources of such built-in contaminants are manufacturing operations, assembly and installation, contaminated test stands, and contaminated fluid.

Contaminants left over from manufacturing operations are among the most hazardous because they usually are hard and abrasive. In the case of lapping compound, they are prone to cause silting (accumulation in stagnant areas) because of their minute size. Test systems and fluids are quite common but necessarily critical sources of contamination, not only because they tend to be overlooked on the assumption that they are clean, but also because system checkout and fluid filling are usually the last operations performed before the activation of a system. It is estimated that some pressure-sensitive hydraulic valve circuits receive much of their contamination during testing operations. (References 1-25, 1-26, 1-107, 6-162, and 281-2.)

System-Generated Contaminants. The instant a fluid system is actuated, a constant source of contamination is activated which will continue for the life of the system. This source is the generation of particles and substances as a result of the wear and deterioration of the fluids and components through mechanical and chemical action. This situation becomes evident when it is recognized that surfaces which to the naked eye and to the touch appear smooth and flat are in reality a mass of jagged asperities and sawtoothed configurations (Figure 10.2.2.1). Under sliding friction from similarly finished surfaces, the surfaces mutually fracture and splinter each other into myriads of micronic and submicronic particles. Mechanically generated particles are the most numerous, resulting from moving mechanisms in the course of normal wear, or the result of improper design features which tend to precipitate wear, promote disintegration, or produce traps for the accumulation of dirt. Most investigators concede that of all components, pumps are the largest source of contamination, followed by other sliding mechanisms, close fitting mechanisms, and filter media migration (Table 10.2.2.1). Chemically generated contaminants are those related to the action of the fluids in the system, or to the action of the system in the degradation of the fluids. (References 1-25, 1-26, 1-107, 6-38, 453-1.)

10.2.2.2 EXTERNAL CONTAMINATION. When the internal surfaces of a fluid system are exposed to the atmosphere or to a new quantity of fluid, contamination is introduced into the system. Airborne contaminants vary in type and magnitude according to the location, degree of atmospheric control exerted, and proficiency of the operating personnel. The most common fluid that can be introduced into a system is water, which can cause detrimental changes in the fluid and promote corrosion and bacterial growth.

Airborne contamination sources are:

- Exposed cylinder rods
- Relief valves
- Breather vents
- Sampling operations
- Air moisture
- Filling ports

Table 10.2.2.1 Sources of Contamination in a Hydraulic System

(Reference 6-38)

COMPONENT	CONTAMINANT: OXIDE SCALE	PLASTICS AND ELASTOMERS	OIL ADDITIVES	METAL PARTICLES	AIRBORNE DIRT	SILICA SAND	LAPPING COMPOUND	PROCESS RESIDUES	FIBERS
Oil	X		X		X	X			X
Tank	X	X		X	XX	X			X
Relief valve		X		XX	X	X	X		X
Accumulator (bladder and piston types)	X	X		X	X			X	X
Filter	X	X		X	XX				X
Piping, fittings, and rubber tubing	X	X		X	XX			X	X
Control valves		X		X	X		X		X
Actuators		X		X	X				X
Pump		X		XXX	X	X			X

X = Noticeable
XX = Medium
XXX = Strong

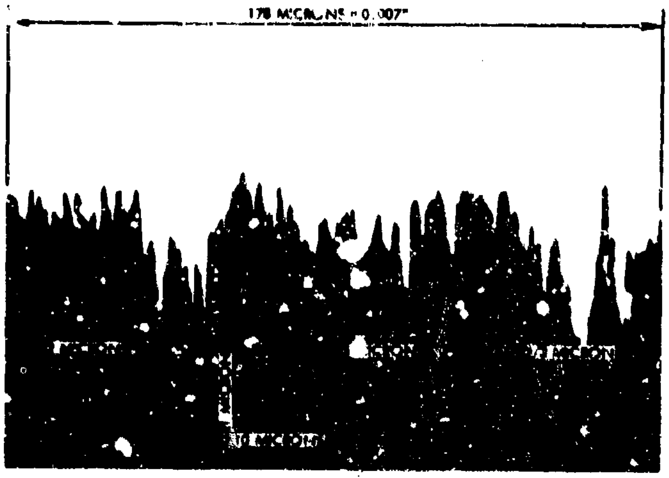

Figure 10.2.2.1. Magnified Profile of an 8 Micro-Inch RMS Surface Finish
(From "Research and Development," A. Lieberman, January 1962, Copyright 1962 by F. D. Thompson Publications, Inc., Chicago, Illinois)

Fluidborne contamination sources are:
- Contaminated fluids
- Improper fluid
- Sampling operation
- Flushing fluids, residues
- Water
- Pressurization gases

10.3 EFFECTS OF CONTAMINANTS

Any amount of contamination will affect a fluid system. How critical this effect may be depends on the nature of the system, the design tolerances of its components, and the nature and extent of the contamination. A true picture of the effect of contaminants on a system can be obtained only through statistical analysis; however, past experience indicates that there are four main areas which are conspicuously affected by particles and other contaminants. These are: (1) interference with moving mechanisms, (2) clogging of filters, (3) erosion of flow surfaces, and (4) breakdown of fluids. In addition, the presence of certain contaminants may present a reaction hazard with some high energy rocket oxidizers.

10.3.1 Effect on Moving Mechanisms

In a fluid system, the components most susceptible to contamination are those with moving parts, and of these, hydraulic servovalves are found to be the most sensitive (References 6-1, 6-22, 6-26, 6-28, 6-56, 6-162, and 6-195). The effect of contaminants on other fluid system components is similar to their effect on those with moving parts, but to a lesser degree.

The most common problems associated with contamination of hydraulic valves are:

a) Sticking of sliding surfaces
b) Plugged orifices
c) Scored surfaces
d) Increased wear and friction
e) Jammed mechanisms
f) Prevention of proper valve seating
g) Upsetting of system pressure balance
h) Alteration of fluid flow direction
i) Interference with alignment.

10.3.1.1 STICKING OF SLIDING MECHANISMS. Close fitting surfaces of the spool and slide variety are very susceptible to contamination, and are the major source of failure in hydraulic servovalves. The sticking action can be caused by dirt lock, stiction, and weldment.

Dirt Lock occurs when stray particles wedge or jam up a mechanism.

Stiction is the most common source of failure in hydraulic valves. It occurs when minute particles carried by fluids across a stationary clearance wedge themselves or build up between the mating surfaces. The process of accumulation and settling is aggravated by inactivity of half an hour or more and is known as silting. This form of sticking action is usually accompanied by hydraulic lock and jamming of misaligned moving parts by system pressure, and results in valve hunting, upset control regulation, or hysteresis, and eventually complete impedance of movement.

Weldment occurs when soft metal particles are wedged between close surfaces and are spread or burnished on the surfaces with the net effect of reducing the clearances.

10.3.1.2 PLUGGED ORIFICES. Orifices in both hydraulic and pneumatic components used for such critical purposes as bleed outlets, balancing pressure connections, or metering orifices are small and require close tolerances which make them very susceptible to plugging. Since they are usually alone, they require individual filter protection.

10.3.1.3 SCORED SURFACES. Particles with a hardness higher than that of the moving parts, such as shafts, rods, and slides, cause scoring and provide leakage paths.

10.3.1.4 WEAR AND FRICTION. Particles increase the rate of wear and abrasion of seals and all moving surfaces. Friction is increased through dirt lock or stiction. Scored seals and seats allow leakage.

10.3.1.5 JAMMING OF MECHANISMS. Very small mechanisms can be jammed by small particles, and if there is no protection, larger mechanisms can be jammed by correspondingly larger particles.

10.3.1.6 VALVE SEATING INTERFERENCE. Particles on a valve seat can allow leakage.

10.3.1.7 UNBALANCED SYSTEM PRESSURE. Pressure balance in a system can be changed by dammed or restricted flow.

10.3.1.8 ALTERED FLOW DIRECTION The flow path of fluids may be changed or stopped altogether by damaged lines or orifices, and precipitate the use of by-pass circuits or relief outlets.

10.3.1.9 MISALIGNMENT. Particles can interfere with the alignment of spools, slide gates, and other moving parts.

10.3.2 Clogging of Filters

Although the purpose of filters is to trap contaminants, the performance and life of the filter can be affected by an excess of the wrong type or size of contaminant. The most common effect is clogging, followed by silting (an overabundance of small particles which reduce filter life and restrict flow). The clogging effect also can be obtained by excess sludge or water emulsions.

10.3.3 Flow Erosion

Hard particles moving at high velocity through restricted orifices or impinging on surfaces can erode metering edges, valve seats, nozzles, and sharp angle turns. This effect is dependent on the type of fluid, rate of flow, and the hardness of the particles. For instance, particles in a helium stream travel much faster than in any other fluid, not only eroding surfaces but also penetrating through the walls of some components such as regulator diaphragms.

10.3.4 Deterioration of Fluids

Contaminants can cause breakdown or alteration of fluids in the fluid systems by direct chemical reaction, particle surface catalysis, heat from friction, formation of sludge, and emulsification with water.

10.4 CLEANLINESS REQUIREMENTS

The establishment of adequate cleanliness criteria is the basis of any contamination control program and determines its success as well as its cost.

Although any or all contaminants can be considered a threat to system performance, to eliminate them totally is difficult and often impossible. Furthermore, since dynamic fluid systems begin generating contaminants the instant they are set in operation, the realistic basis of a contamination control program is the acceptance of the premise that there is no absolute cleanliness; all that can be expected is to be able to control the contaminants to levels acceptable to the performance requirements of a system.

10.4.1 Components Cleanliness Requirements

The cleanliness requirements of fluid components not only reflect on component's design characteristics, but also reflect the characteristics of the system in which they will ultimately perform. Customarily, component requirements are based on the following considerations:

a) Maximum particle size that can be tolerated, usually one-half the minimum orifice or clearance in the system

b) Type of filters in the system

c) Reliability

d) Reactivity of residues with fluids. Limits of reactive residues are particularly important in oxygen and fluorine systems. Current requirements for oxygen components are set at a maximum of 4 mg of hydrocarbons per square foot of component surface (Reference 89-1).

e) Statistical analysis of past performances

f) Use of post-assembly cleaning. If the system is amenable to post-assembly cleaning, individual component requirements may be relaxed, and overall system limits may be met by in-place cleaning or flushing operations (Reference 23-53).

10.4.2 System Cleanliness Requirements

Cleanliness requirements for fluid systems are usually based on considerations involving the fluid as well as the components in the system. Such considerations are:

a) System performance with known amounts and types of contaminants (Table 10.4.2)

b) Practical extent to which the system can be cleaned and maintained in operation

c) Comparison of dirt sensitivity to similar systems

d) Quality of filtration equipment available

e) Type of fluid (liquid or gas).

10.4.3 Fluid Cleanliness Requirements

Fluid cleanliness refers to the condition of the fluid *before* it is placed in service; afterwards, it is only one of the factors determining a system's cleanliness requirements. Cleanliness requirements for unused fluids are based on the following factors:

a) Accumulation of contaminants between the point of manufacture and the point of use. Accumulations are reflected in the scaled requirements presented in Tables 10.4.3a and 10.4.3b.

b) Requirements of the system components. In some instances manufacturers have been misled to design hardware based on probable high levels of fluid cleanliness (Reference 23-33).

c) Cost of cleaning the fluid. "Micronically clean" hydraulic fluid (to levels below 5 microns) costs $2.50 per gallon, while standard oil per MIL-H-5606 costs $1.19 per gallon (References 23-33 and 23-53).

d) Type of fluid or system in which the fluid will be used.

Table 10.4.2. Titan II Equipment Cleanliness Requirements
(Reference 454-1)

PARTICULATE REQUIREMENTS	COMPONENTS	SUBSYSTEMS		PTS PIPING AND SKID UNITS	TANKS: STORAGE (TRANSPORT, HOLDING)	TANKS: MISSILE	ROCKET ENGINE
Total Filterable Solids, mg/ft²	2.0	4.0		4.0	5.0	5000 mg/ Stage I 2000 mg/ Stage II	None
Particles/ft² (micron size)		*Fluid Flush*	*Gas Purge*				
300-500 µ	10	20	*	*	—		
500-1000 µ	2	4	*	*	—	650-1500 µ: 85 per tank 1500-5000 µ: 10 per tank	None
Over 1000 µ	0	0	0	0	No particles over 1500 µ	Over 5000 µ: 3 per tank	
Fibers/ft² (micron size)							
750-2000 µ × 25 µ	20	40		—	—		
2000-6000 µ × 40 µ	2	4		—	—		None
Over 6000 µ × 40 µ	0	0	0	0	No fibers over 9000 µ		

*Particles less than 1000 microns are not counted, but are accounted for on a weight basis as total filterable solids.

Table 10.4.3c shows the variations in the cleanliness requirements for propellants, hydraulic fluids, and aircraft fuels.

e) Reactivity of fluids. Oxygen systems demand strict control on hydrocarbons as well as in particles (References 183-7 and 457-1).

f) Silting characteristics of the fluid. Freedom from large numbers of particles below 10 microns is essential for sensitive servovalves with spool clearances between 1 to 10 microns. It is estimated that the maximum quantity of particles per milliliter in the 1 to 5 micron category which can cause silting is between 250,000 and 500,000. New fluid contains between 160,000 and 700,000 particles per milliliter in the 1 to 5 micron range (Reference 6-38).

10.4.4 Environmental Cleanliness Requirements

To prevent the deposit of airborne contaminants in cleaned parts, it is necessary to clean the air in the clean room to an acceptable level (Table 10.4.4a). Specific control requirements are dictated by the cleanliness requirements of the parts which will be processed in the cleaning facility. The customary criterion for clean room atmospheric control is not to allow particles in the air which exceed the maximum size allowed in the parts being cleaned. More specific requirements, discussed at length in the references are:

a) Maximum air contamination limits should not exceed the maximum allowable particle size in the most critical component processed. Airborne hydrocarbons should not exceed 8 ppm (Reference 33-53).

b) Currently there are two official documents which specify requirements for clean rooms processing missile fluid system components. Reference 454-1 sets a maximum limit of 200 microns for particles and 700 microns for fibers. Reference 454-2 allows various degrees of air cleanliness according to operational requirements, grouping them into four classes (Table 10.4.4b).

c) Typical specifications for a portable "white room" are given as 10,000 particles per cubic foot in the range between 0.5 and 10 microns (Reference 451-1).

d) Two criteria for establishing clean room requirements are given in Reference 448-2 as follows: 1) the contamination level must be less than that of an ordinary air or factory but not so low that it is difficult to achieve or maintain; and 2) particle size lower limit and statistical contamination level must be relevant, reasonable, and compatible with a large majority of components.

10.5 CONTAMINATION CONTROL MEASURES

There are two basic ways of controlling contamination: physical removal of the contaminants through cleaning

Table 10.4.3a. Operational Titan II Oxidizer Requirements
(Reference 457-4)

REQUIREMENTS*	SUPPLIED S-1	STORED S-2	OPERATIONAL S-3	TEST METHOD
Chemical				
N_2O_4 Assay	99.5 Minimum	99.5 Minimum	99.4 Minimum	MIL-P-26539 as referee
Water Equivalent	0.1 Maximum	0.15 Maximum	0.2 Maximum	
Cl as NOCl	0.08 Maximum	—	—	
Total Filterable Solids**	0.001 Maximum or 15 mg/l	0.0014 Maximum or 20 mg/l	0.0017 Maximum or 25 mg/l	Gravimetric
Appearance	Free from undissolved water, sediment, and suspended matter			

*Tabulated values in weight percent.
**Retained on 10 micron plastic membrane filter when sampled downstream of 40 micron system filter.
***There is no color restriction in the present supplied fuel specification, MIL-P-27402.

Note: See also T.O. 42B7-2-1-3 Quality Control of Propellant Nitrogen Tetroxide

Table 10.4.3b. Operational Titan II Fuel Requirements
(Reference 457-4)

REQUIREMENTS*	SUPPLIED S-1	STORED S-2	OPERATIONAL S-3	TEST METHOD
Chemical				
Hydrazine Assay	51.0 ± 0.8	51.0 ± 0.8	51.0 ± 0.9	MIL-P-27402 as referee
UDMH Assay	47.0 Minimum	47.0 Minimum	46.9 Minimum	
Total, N_2H_4 + UDMH + Amines	98.2 Minimum	98.1 Minimum	98.0 Minimum	
Water and Dissolved Impurities	1.8 Maximum	1.9 Maximum	2.0 Maximum	
Total Filterable Solids**	0.001 Maximum or 10.0 mg/l	0.002 Maximum or 20 mg/l	0.0025 Maximum or 25 mg/l	Gravimetric
Appearance***	← Clear, colorless, homogeneous liquid →			

*Tabulated values in weight percent.
**Retained on 10 micron plastic membrane filter when sampled downstream of 40 micron system filter.
***There is no color restriction in the present supplied fuel specification, MIL-P-27402.

Note: See also T.O. 42B7-2-1-4 Quality Control of Propellant Hydrazine-UDMH (50/50) Mixture

operations, and designing it out of the system and components. This Sub-Section will outline the procedures used for cleaning contaminants out of components, fluids, fluid systems, and the working environment. The design aspects of contamination control will be discussed in Sub-Section 10.6.

10.5.1 Cleaning Components

The purpose of this Sub-Topic is to inform the design engineer of the processes, cost, facilities, and methods used in cleaning fluid system components. Detailed cleaning procedures can be found in References 1-270, 65-27, and V-220.

There are two main types of component cleaning during manufacturing — shop cleaning and final cleaning. Shop cleaning involves standard clean-up operations used during manufacturing operations. Regarding fluid components, this phase of component manufacturing becomes increasingly important, for unless some parts and assemblies are cleaned as they are finished, before going into further assembly, some contaminants become hopelessly entrapped. This situation applies particularly to filters (References 1-25, 1-26, and 1-107). To prevent potential contamination and also to reduce the load of final cleaning operations, a "clean-as-you-go" system has been developed (Reference 136-4) in which each part or subassembly is thoroughly cleaned before being assembled into a larger subassembly, until the entire component is completed.

Final cleaning includes all of the decontamination operations performed on parts or components after each of the manufacturing and finishing processes have been completed. Since final cleaning is the last operation prior to installation, or sealing for future use, it is a very critical phase of fluid component manufacture, affecting not only production costs but the ultimate performance of the component and the reliability of the system. It has been estimated that it can cost five times the purchase price of some low-cost components to ensure the absence of particles over 100 microns in size (Reference 19-221). The average cost for cleaning a typical valve is normally between $15 and $30 (Reference V-426).

ISSUED: MARCH 1967
SUPERSEDES: MAY 1964

Table 10.4.3c. Aerospace Fuel Cleanliness Requirements

(Reference V-158)

Hydraulic Fluids

SAE, ASTM, AIA Tentative Hydraulic Contamination Standards Particles per 100 ml by Class of System (Tentative)

SIZE RANGE	0	1	2	3	4	5	6	7-10
2.5-5μ	←			Pending			→	
5-10μ	[illegible]	4600	9700	24,000	32,000	87,000	128,000	Pending
10-25μ	670	1340	2680	5,360	10,700	21,400	42,000	↓
25-50μ	93	210	380	780	1,510	3,130	[illegible]	
50-100μ	16	28	56	110	225	430	1,000	
> 100μ	1	3	5	11	[illegible]	41	92	

Typically and approximately:
Class 0 — rarely attained
Class 1 — MIL-H-5606B
Class 2 — good missile system
Class 3 & 4 — critical system, in general
Class 5 — poor missile system
Class 6 — fluid as received
Class 7 — industrial service

AIRCRAFT FUELS

ACTIVITY	TOTAL SEDIMENT	HYDROPHILIC FRACTION
Commercial Average	0.2 mg/liter	0.1 mg/liter
Int. Air Transp. Assoc.	1.0 mg/USG (maximum)	not established
Military Standards	4.0 mg/USG (maximum)	not established

Aircraft fuel contamination levels are surprisingly low for a bulk fluid because of their low viscosity and the effectiveness, therefore, of settling methods employed in their handling. Typical "in-service" levels (by no means ideal) of contamination are shown in terms of gravimetric analyses. Particle count analyses are seldom used for fuels and other low-viscosity liquids.

ROCKET PROPULSION & SERVICE FLUIDS*

FLUID	AIR FORCE USE LIMITS	AIR FORCE PROCUREMENT LIMITS
LO_2	2.5 mg/liter	1.0 mg/liter
LN_2	2.5 mg/liter	1.0 mg/liter
RP-1	1.5 mg/liter	not applicable
GO_2	0.01 mg/liter	not applicable
GN_2	0.01 mg/liter	not established
He	0.01 mg/liter	not established

*per AFBS 61-3 (revised)

Because of the more generous metering and pumping clearances for missile propellants and service gases, relatively high contamination levels are tolerated. The major (particulate) risk is in clogging of pump inlet screens. Fibers which will initiate clogging and silting, therefore, are specially controlled and held typically to 400μ maximum size.

Table 10.4.4a. Cleanliness Levels of Ambient Air

(Reference V-158)

SIZE RANGE	A	B	C	D	E
	PARTICLES PER CUBIC FOOT				
5-25μ	20	180	300	500	2000+
25-100μ	2	14	150	10	500+
> 100μ	1	6	30	1	50+
Total	23	200	480	510	2550+

A — Superior clean room
B — Ordinary clean room
C — "Dust controlled" assembly area
D — Country air (still day)
E — City air

Table 10.4.4b. Air Force Clean Area Contamination Standards

(Reference 454-2)

FACILITY	CONDITION	MAXIMUM ALLOWABLE PARTICLE CONTENT PER CUBIC FOOT OF AIR: PARTICLE SIZE (μ)	NUMBER
Air Force standard clean room	Operational	Over 0.5	120,000
	At Rest	Over 1.0	20,000
		Over 0.5	20,000
		Over 1.0	4,000
Air Force standard clean work station	Operational	Over 0.5	1,000
	At Rest	Over 0.5	100

The problems of cleaning a particular component are controlled by factors affecting the selection of (1) cleaning materials, (2) cleaning methods, and (3) processing equipment.

10.5.1.1 SELECTION OF CLEANING AGENTS. The factors affecting the selection of cleaning materials are the same as those affecting the successful interaction of cleaning agents, contaminants, and materials in the component.

Nature of the Contaminants. It is important to know the nature of the contaminants to be removed by cleaning, because of the reactivity of some contaminant materials with various cleaning agents (Table 10.5.1.1a). If contaminant and cleaning agent are properly matched, a fast and simple cleaning operation may result, with a minimum amount of damage to the component surfaces.

Conditions of the Contaminants. Contamination may be present in thin films which comprise thick layers. It may be loose, or it may be tightly adhered, needing the action of a penetrating cleaning agent. The contaminants may be bound by grease or oil, or they may consist of minute particles embedded in the component's surface. Each of these conditions calls for specific cleaning actions and cleaning mechanisms which will get the contaminants into suspension so they can be flushed away (Reference 19-221).

Materials Compatibility. Whenever possible, cleaning solutions should be buffered, inhibited, or stabilized to prevent the development of corrosion along with the cleaning action. The cleaning operation should not be detrimental to the materials of the component during or after the cleaning process. Proper consideration of the compatibility of materials will avoid detrimental effects such as (a) reduced tolerances, hydrogen embrittlement, and stress corrosion in metal surfaces; (b) swelling, polymerization, and disintegration in elastomeric materials. Data for matching materials, contaminants, and cleaning agents are presented in Table 10.5.1.1b.

Cleaning Agent Residuals. Some cleaning agents will leave films or residues on a surface which, in some fluid systems, can be hazardous. Therefore, it is necessary to select cleaning agents which will yield surfaces compatible with the levels of cleanliness specified. Unless the cleaning methods are properly controlled, and unless provisions are made for controlling the strength of solutions, and for thorough neu-

Table 10.5.1.1a. Specific Action of Cleaning Agents
(References V-280 and 434-…)

AGENT	CONTAMINANT
Acid cleaner	Scale, rust, oxides, organic and inorganic matter
Alkaline cleaner	Shop dirt, cutting oil, drawing lubricants, organic and inorganic particles
Detergent	Residues left over from alkaline cleaning
Soap	Drawing lubricants, rust preventative compounds, buffing and lapping compounds
Emulsion	Carbonized oils, mixed soils, organic matter, coloring compounds
Solvent	Organic materials, oils, lubricants, and grease
Water	Dust, particles, and residues

Notations for Table 10.5.1.1b

1. **Acid Cleaning:** used to remove contamination not soluble in milder solutions.
   - a) Nitric-hydrofluoric acids
   - b) Nitric acid
   - c) Chromic acid
   - d) Inhibited hydrochloric acid
   - e) Inhibited sulfuric acid
   - f) Inhibited phosphoric acid
   - g) Mixed acid deoxidizers
   - h) Alcohol-phosphoric acid
   - i) Carbon removal systems
2. **Alkaline Cleaning:** used to remove inorganic and organic matter susceptible to solution or emulsification.
   - i) Inhibited alkaline cleaners
   - j) Alkaline rust strippers
   - k) Heavy duty alkaline cleaners
   - l) Carbon removal systems
   - m) Detergents
3. **Solvent Cleaning:** used to remove soluble organic materials.
   - n) Halogenated hydrocarbon solvents
4. **Rinsing and Flushing:** used to rinse solid and liquid residues.
   - n) Halogenated hydrocarbon solvents
   - o) Water
5. **Neutralizing and Passivating:** supplementary treatment to acid and alkaline cleaning to prevent corrosions.
   - b) Nitric acid
   - c) Chromic acid
   - h) Alcoholic phosphoric acid
   - p) Alkali
   - q) Nitrate or phosphate
   - r) Alkali and nitrite or phosphate
6. **Mechanical Cleaning:** used to remove contamination by abrasive action (scrubbing, brushing, etc.).

Table 10.5.1.1b. Selection of Compatible Cleaning Agents
(Reference 89 1)

MATERIAL \ CONTAMINANT	CARBON (TIGHTLY ADHERING COMBUSTION PRODUCTS)	LOOSE PARTICULATE MATERIAL	PRESERVATIVE COMPOUNDS, LUBRICANTS, OILS, PETROLEUM, FUELS	OXIDES, SCALE, WELD SLAG	LIGHT RUST OR OTHER CORROSION PRODUCTS	WAXES, TAPE ADHESIVES, MARKINGS
Aluminum a. Bare alloys b. Coated (anodized, chem film, etc.)	1–l 2–l 6	2–i,m 3–n 4–n,o	2–i,m 3–n 4–n,o*	1–g,h 6	1–g,h 5–h 6	2–i,m 3–n 4–n
Corrosion-resistant steels	1–l 2–l 6	2–i,k,m 3–n 4–n,o	2–i,k,m 3–n 4–n,o*	1–a,b,c,l 2–j,l 5–b,c	1–a,b,c,f,g,h 2–j 5–b,c,h 6	2–i,k,m 3–n 4–n
Nickel and nickel alloys	1–l 2–l 6	2–i,k,m 3–n 4–n,o	2–i,k,m 3–n 4–n,o*	1–d,g 2–j 6	1–d,f,g,h 5–h 6	2–i,k,m 3–n 4–n
Copper and copper alloys	1–l 2–l 6	2–i,k,m 3–n 4–n,o	2–i,k,m 3–n 4–n,o*	1–d,e,g 2–k 6 (electrolytic)	1–d,e,f,g 2–k 6 (electrolytic)	2–i,k,m 3–n 4–n
Titanium	1–l 2–l 6	2–i,k,m 3–n 4–n,o	2–i,k,m 3–n 4–n,o*	1–a**,b**,c** 2–j 5–b**,c** 6	1–a**,b**,c** 2–j 5–b**,c** 6	2–i,k,m 3–n 4–n
Silver and silver alloys (brazes, plating)	1–l 2–l 6	2–i,k,m 3–n 4–n,o	2–i,k,m 3–n 4–n,o*	1–f,g,h 5–h 6	1–f,g,h 5–h 6	2–i,k,m 3–n 4–n
Magnesium	6	2–i,m 3–n 4–n,o	2–i,m 3–n 4–n,o*	2–i,g 6	2–i,g 6	2–i,k,m 3–n 4–n
Chromium plating		2–i,m 3–n 4–n,o	2–i,m 3–n 4–n,o*	N.A.***	N.A.	2–i,m 3–n 4–n
Cadmium plating		2–i,m 3–n 4–n,o	2–i,m 3–n 4–n,o*	N.A.	N.A.	2–i,m 3–n 4–n
Gold plating	1–l 2–l 6	2–i,m 3–n 4–n,o	2–i,m 3–n 4–n,o*	N.A.	N.A.	2–i,m 3–n 4–n
Carbon and low alloy steels	1–l 2–l 6	2–i,k,m 3–n 4–n,o	2–i,k,m 3–n 4–n,o*	1–d,e 2–j 6	1–d,e,f,h 2–j 5–h 6	2–i,k,m 3–n 4–n
Elastomeric materials, rubber, synthetic materials	N.A.	2–i,m 4–o 6	2–i,m	N.A.	N.A.	2–i,m 6
Surface coatings (paints, lacquers, etc.)	2–l 4–o 6	2–m 4–o	2–m 4–o*	N.A.	N.A.	2–m 6
Thermoplastics† Kel-F, Teflon, nylon, etc.	6	2–i,m 3–n 4–n,o	2–i,m 3–n 4–n,o*	N.A.	N.A.	2–i,m 3–n 4–n 6
Reinforced or filled plastics	6	2–i,m 4–o	2–i,m 4–o*	N.A.	N.A.	2–i,m 6
Fibrous materials asbestos, etc.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Carbon seals	N.A.	3–n 4–n	3–n 4–n	N.A.	N.A.	3–n 4–n
Ceramics and cermets	6	3–n 4–n	3–n 4–n	N.A.	N.A.	3–n 4–n 6
Solders (lead-tin, low melting)	6	2–i,m 3–n 4–n,o 6	2–i,m 3–n 4–n,o*	1–f,g,h 5–h 6	1–f,g,h 5–h 6	2–i,m 3–n 4–n

*For water-soluble preservative compounds
**Concentrations are critical
***Not applicable
†No cleaning or drying temperature to exceed 140°F

tralization, passivation and rinsing of surfaces, the following effects will be produced by the cleaning agent on the component (Reference 65-27):

Acid cleaners	Slight etching, residual organic film organic matter and moisture
Alkaline cleaners	Slight etching of light metals, alkaline residue, residual moisture
Detergents	Organic residue, moisture
Soaps	Organic film, soapy film, moisture
Emulsions	Organic films and residues, moisture
Solvents	Residual additives and stabilizers, inorganic residues
Water	Organic film moisture

ULTRASONIC CLEANING. Ultrasonic cleaning is accomplished by immersing a fluid element (most commonly filters) in a solvent solution and applying sonic energy to the system through transducers mounted on or within the tank. The sonic energy will produce cavitation on the surface of elements immersed in the solution which will loosen contaminants. The effectiveness of the cleaning process depends on thorough flushing or rinsing through the element between ultrasonic cycles. Flow while in the ultrasonic environment is relatively ineffective due to decreased cavitation. The successive application of ultrasonic cleaning treatments to new or contaminated filter elements will produce a decreasing downstream particle count until a minimum level is reached. Beyond this point, subsequent application of ultrasonics will not usually produce significant reduction in particle count.

10.5.1.2 Selection of Cleaning Methods. The factors affecting the selection of cleaning methods — whether the component can be flushed, sprayed, soaked, or vapor degreased — are those determined by the design characteristics, ease of disassembly, and nature of the component.

Design Characteristics. The size, shape, and configuration of a component can determine the ease of handling and cleaning; in addition, some design features can make a component susceptible to damage, or can impede the cleaning processes altogether. Components with sharp edges, fine threads, and close tolerances can be easily damaged by etching or corrosion. Pores, crevices, cored holes, and recesses are examples of areas which resist simple cleaning methods (Reference 65-27).

Disassembly. For optimum cleaning operations, components should be fully disassembled. Those components which cannot be taken apart must be cleaned by special procedures or by systems cleaning methods.

Residual Propellants. Residual toxic or corrosive fluids should be thoroughly neutralized and inerted before standard cleaning operations are started. Description of the methods and materials used to remove residual storable propellants (nitrogen tetroxide and hydrazine) are presented in Reference 450-1.

Filters. Filter elements present a special problem in cleaning because they are purposely designed to trap materials. Customarily, filter elements are cleaned by back-flushing, however, this procedure is time consuming and far from satisfactory. An improvement of this method consists of alternate rinsing or back-flushing with ultrasonic cleaning (Reference 51-3). This reduces the contaminant population in the elements to lower levels than those achieved by plain rinsing, vibration, or submitting the element to the conditions of static firing of a rocket engine (Reference 6-2).

Pressure Gages. Pressure sensing instruments with intricate and fine tubing, such as Bourdon tubes, can be cleaned by means of a vacuum injection of the cleaning fluids, followed by vacuum drying (References 450-4 and 23-53).

O-rings. Elastomeric seals may exhibit a powdering condition on the surface known as "O-ring bloom." This built-in contaminant inherited from manufacturing and compounding operations can be removed by a series of washings with hydraulic fluid, naphtha, and isopropyl alcohol (Reference 450-2).

10.5.2 Cleaning the Fluids

To control the introduction of contaminants by any of the fluids coming in contact with the component's surface calls for an integrated control program covering all aspects of fluid usage including new fluids, test fluids, fill-up fluids, and working fluids.

The principal methods of cleaning missile fluids are filtration and separation. Recent developments in missile fluid filtration and separation equipment have widened the range of particle removal capability available to the designer. The choice of filtration equipment to clean up missile fluids constitutes a specialized process predicated upon a thorough understanding of the characteristics of the filters, the fluids, and the fluid system. Filters are discussed in Sub-Section 5.10 of this handbook.

10.5.3 Cleaning Fluid Systems

When fluid systems become contaminated beyond the scope or protection provided by filters it becomes necessary to actually clean the system lines or circuit. Such conditions may arise as a result of complete disintegration of components, heavily encrusted corrosion, introduction of tarry materials, breakdown of fluids, introduction of incompatible fluids, or disintegration of seals and sealants. Depending on the degree and type of contamination, there are two methods that can be used to clean a system:

1) Flushing with a new working fluid
2) Chemical cleaning
   a) Disassembly of the system and recleaning of all individual components
   b) In-place cleaning of the entire system

10.5.3.1 FLUID FLUSHING. In minor stages of contamination, it is possible to recondition a system by draining it, replacing the old filters, and flushing it with large volumes of the working fluid used in the system. From the point of view of compatibility and systems simulation, this is almost an ideal remedy. Since hydraulic systems are flushed with hydraulic fluids, cryogenic systems can be flushed with liquid nitrogen, and pressurization systems can be purged with gaseous nitrogen or helium. This procedure is acceptable for the removal of loose particulate matter, but it is limited by the lack of solubility of most contaminants in the working fluids and, therefore, cannot be used to dissolve and remove adhered and entrapped substances.

10.5.3.2 CHEMICAL CLEANING. Chemical cleaning consists of processing either the entire system, or its individual components, with chemical solutions similar to those used to clean the components before their original assembly. To perform such an operation, it is necessary to either disassemble the system and clean each component individually, or to flow or recirculate the solutions through the entire system. The choice of method depends on:

a) Type of fluid system

b) Materials compatibility with cleaning solutions

c) Flow continuity through the system

d) Ability to activate and operate the system with the cleaning solutions

e) Ease of disassembly.

Disassembly and Recleaning. Complete disassembly of a system and cleaning of each individual component can be a costly and time consuming operation. However, this is the only way to recondition equipment having delicate materials or design features, or systems lacking continuity in their flowpath.

In-Place Cleaning. To clean equipment which cannot easily be dismantled, and most airborne systems cannot, an alternative is in-place cleaning of the entire system. This procedure varies with the type of system and its materials, but usually entails draining all the working fluid, removing any failed parts and spent filters, and filling the system with solutions of chemicals or their vapors. The fluids are then recirculated or allowed to soak for a given period of time at various flow rates and temperatures, followed by rinsing, purging, drying, testing, and finally reassembly and sealing. This procedure has the following advantages and limitations:

Advantages:

Only way to clean systems which are bulky, fixed, or not easily disassembled.

Eliminates cost of disassembly and reassembly.

Flushing follows the path of the working fluids.

Limitations:

Possible incompatibility of seals and delicate surfaces with cleaning fluids.

Danger of hazardous residues and films.

Possible entrapment of cleaning fluids in dead ends and low points.

Deterioration of moving parts when actuated with the cleaning fluids.

A good example of the use of in-place chemical cleaning is presented in Reference 450-1, which describes the development of fluids and processes for decontaminating the TITAN II rocket engines after their final acceptance test by static firing. An excerpt from the reference is as follows:

"The cleaning process consists of two water cycles, three cycles with neutralizing solution, three water cycles, and a final hot nitrogen purge. A single cycle operation consists of filling the engine, holding the liquid under pressure while the engine turbine pump assembly is rotated at 200 rpm, and thorough draining of the cleaning liquid."

10.5.4 Cleaning the Environment

A basic element of a contamination control program is control of the environment in which the components or the equipment are being cleaned. It is very difficult to clean components and maintain their cleanliness when they are exposed to contamination from the surrounding environment.

The need for environmental control arises from the fact that normal manufacturing atmosphere and conditions are inadequate to ensure the attainment of the cleanliness required by precision components. City air is heavily laden with vapors, gases, and particulate matter; shop operations only serve to increase this concentration level (Table 10.5.4). Thus, if a component is to be processed and decontaminated to levels below those found in the air about us, it is necessary to bring it into a special facility where the sources of environmental contamination—people, processes, surfaces, and airborne matter—can be controlled within specified limits. Such special facilities are known as *clean rooms.*

A clean room may be defined as a facility or enclosure in which the air contents and conditions are controlled and maintained at a specific level by means of special construction and facilities, special operating processes, and specially trained personnel. The degree to which air conditioning is controlled is usually dictated by the cleanliness requirements of the parts and components that are to be processed in it. They include four parameters: temperature, humidity, air pressure, and airborne contamination contents.

Variations in the requirements of these four parameters, particularly the concentration and distribution of airborne contaminants, have given origin to many names for a clean room, such as white room (Reference 451-1), dust-controlled area, environmentally controlled area, dust free area, super-

Table 10.5.4. Typical Dust Levels in Rural, Urban, and Shop Air
(Reference 45?-1)

PARTICLE DIAMETERS (μ)	CONCENTRATION OF PARTICLES (PER LITER) RURAL	URBAN	SHOP
0.7 - 1.4	1,250	47,000	75,000
1.4 - 2.8	450	4,700	4,000
2.8 - 5.6	100	1,400	100
5.6 - 11.2	40	120	60
11.2 - 22.4	0	20	15

clean room, cleaning facility, and cleaning establishment. Although some of these names are intended to be synonymous with high degrees of cleanliness and others are obviously misnomers, they are all indicative of the basic role that cleanliness requirements play in the design and operation of a clean room.

The cleanliness requirements and environmental controls of a clean room should satisfy the cleanliness levels of the components processed there throughout operations of cleaning, testing, inspection and sealing. The criteria for cleanliness levels in clean rooms as summarized in Table 10.4.4b indicate the requirements set by the only official document available on this subject.

The design characteristics of clean rooms vary widely, in accordance with the character of the specific cleanliness levels maintained. However, based on the premise that clean rooms must be designed to control the generation of particles within the area and the introduction of particles into the area, they must fulfill the following minimum design requirements (References 89-1, 451-1, 454-1, and 454-2):

1) The area must be entirely enclosed
2) The interiors must hold contaminant generation and entrapment areas to a minimum
3) Illumination should provide a minimum of 100 ft-candles at bench level
4) Entry of personnel and hardware must be made through air locks that maintain the cleanliness levels of the area
5) Work surfaces must not generate particles
6) Equipment in the area must not be a source of contamination

The heart of the clean room is the air conditioning and filtration equipment. Typical requirements of operation are as follows:

1) Air supply should be changed once every three minutes
2) Air temperature should be 72° F ± 5°
3) Relative humidity should be 45 percent ± 5%
4) Positive air pressure on adjacent areas should be 0.10 inch of water minimum
5) Particle content of the air should be maintained within limits outlined in Table 10.4.4b
6) Air filters should remove 99.95 percent of all particles above 0.3 microns
7) Flow of air should be downward from the ceiling

Much of the work involved in the control of airborne contaminants and air conditioning in clean rooms has been greatly simplified recently by the use of the Whitfield Principle of laminar air flow. This innovation consists of introducing air in the room through an overhead bank of filters and moving the entire body of air across to the opposite side. There it is exhausted through a grated floor (Figure 10.5.4 and References 448-3, 455-1, and 456-1). By this method, the incoming filtered air is continuously washing away all of the particles inside the area, instead of just agitating them and partially exhausting them outside.

The Whitfield Principle of laminar air flow also can be used to flow air horizontally in individual work stations, or across entire rooms in which opposite walls are used as the air inlets and outlets. The downward flow of air exhausting through a grated floor is the most efficient arrangement and provides the highest level of contamination control for an entire area (Reference 448-3). However, even horizontal flow, which may be more convenient and easier to construct, can produce levels of cleanliness claimed to be 200 times cleaner than a hospital operating room (Reference 448-1). The two primary requirements for clean rooms using this principle of air contamination control are (Reference 448-3):

1) The space to be kept clean must have walls or sides which help maintain laminar flow.
2) The air inlet and outlet must each have a total area equal to that of the cross section of the confined space.

The control of environmental contaminants does not end with the installation of a well-designed and equipped clean room. To achieve and maintain a cleanliness level commensurate with the component being processed requires three additional indispensable factors: (1) strict and proper operating procedures, (2) continuous maintenance control, and (3) properly trained personnel.

Clean room operating procedures involve a multitude of details, described in References 449-1, 454-1, 454-2, and 458-1. The most important functions to be observed are:

1) All objects, tools, and materials must be cleaned before going into a clean room
2) All personnel must be properly cleaned and attired before going into a clean room.
3) Access to the clean room must be strictly controlled and the number of personnel within it (including workers) maintained at an absolute minimum.

ISSUED: MARCH 196?
SUPERSEDES: MAY 1964

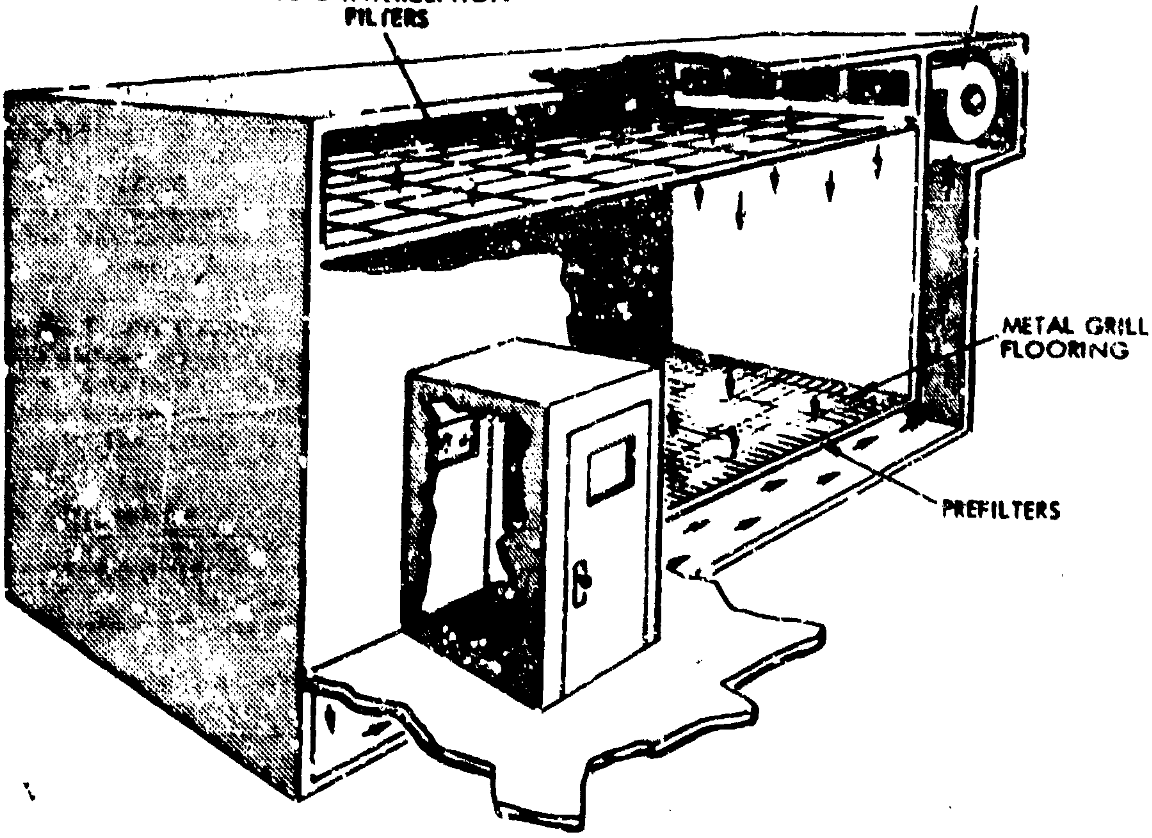

Figure 10.5.4. Clean Room Using Whitfield Laminar Air Flow
(From USAF Technical Order 00-21-203)

The maintenance and control requirements of clean rooms consist of a strict housekeeping program and around-the-clock periodic inspection of the five basic parameters of air conditioning, temperature, humidity, air pressure, and airborne contamination contents (References 449-1, 451-1, and 458-1).

Personnel working in a clean room must be properly trained, equipped, and indoctrinated, since the products evolving from a clean room are only as clean as the personnel working in it. To this effect personnel must be fully cognizant of the nature of contamination and its control, and their operation within the clean area must adhere to approved procedures. The extent and importance of such procedures are indicated by the fact that merely crumpling a piece of paper generates clouds of particles 64 microns and larger (Reference 449-1).

10.6 CONTAMINATION CONSIDERATIONS IN DESIGN

Next to the functional design parameters of a fluid component, contamination control is perhaps the most important design consideration. It is the responsibility of the fluid component designer to develop contaminant-conscious designs which will facilitate the control of contamination. It is within his province to determine the critical factors that bring about the need for such control: dimensional tolerances, materials, fluids, finishes, flow rates, etc. The degree of protection required by these critical factors determines the level of contamination control necessary to assure operational capability and performance requirements. These levels of contamination in turn affect contamination levels of working fluids, clean room specifications, filtration requirements, maintenance schedules, and the over-all cost of cleaning.

10.6.1 Design Criteria

The approaches that the component designer can take to facilitate the task of contamination control fall into the following five categories:

1. Reduce the sources of contaminants in systems and components by selecting materials, fluids, and mechanical designs which will reduce the rate of wear, friction, stress, and fluid decomposition.

2. Increase the tolerance of components to contamination by increasing the dimensional clearances to the maximum values compatible with functional requirements.

3. Protect components and systems from contaminants by means of adequate filtration, sealed modules, clean fluids, and clean environment during assembly and installation.

4. Provide accessibility for the inspection of systems and components and for the removal of contaminants by allowing means of disassembly for cleaning, drainage, post-

ISSUED: MARCH 1967
SUPERSEDES: MAY 1964

assembly cleaning, and maintenance operations.

5. Establish adequate levels of contamination control by relating the cleanliness requirements to the actual needs and nature of the system and components at a given stage of development; all airborne components cannot be treated as if they were hydraulic servovalves.

The following review of current engineering design practices demonstrates how the designer can use these five approaches to overcome the problem of contamination in components, systems, fluids, and the working environment.

10.6.2 Component Design

All design considerations for contamination control should be viewed in the light of the over-all fluid system and the eventual role of the fluid component in an airborne vehicle. Even though contamination control begins with the design of a component, the designer's efforts cannot stop there. Because of his over-all knowledge of the system, the designer must anticipate possible problems and provide design guide lines that will carry through subsequent operations such as materials selection, manufacturing processes, testing, cleaning, packaging, and installation.

10.6.2.1 DESIGN CONFIGURATION. The design features of a component can provide two solutions for keeping contaminants under control: the reduction of generating services and the reduction of the susceptibility of the system and components to the contaminants. Coincidentally, a design that is tolerant of some contamination is less prone to generate contaminants because it has fewer points of friction and wear (Reference 453-1). Examples of these two approaches and other means of control at the design level are listed below.

a) The interior of all fluid components should be smooth (to eliminate flaking), and continuous (to promote flushing action during flow). Pockets, dead ends, crevices, labyrinth areas, and cavities should be eliminated; they collect dirt which is later released during peak flow rates.

b) The component should be able to be disassembled and accessible for cleaning.

c) Avoid threaded joints.

d) Use strong positioning and actuating forces to preclude jamming of mechanisms by particles.

e) Eliminate feather edges and other delicate features susceptible to cracking. Reduce the number of abrading surfaces and friction points. Rubbing surfaces should be carefully balanced to prevent excessive wear.

f) Protect delicate design features by sealing them and providing caps and boots to keep out airborne dirt.

g) Provide the widest possible tolerances in orifices and clearances. Design the components to operate with a fluid contaminated with the largest particles tolerable.

h) Minimize screw-type fasteners and other particle-generating connectors or devices.

i) Pumps, actuators, and dynamic mechanisms with wearable surfaces should be put through a breaking-in period to run-in the friction points and abrading surfaces. The intended working fluid and a return filter should be used. After the operation, the pump or component should be disassembled, inspected for excessive wear, recleaned, and reassembled.

10.6.2.2 MATERIALS SELECTION. Wear and corrosion of components constitute large sources of contamination; to reduce them, proper attention must be given to the process of materials selection and application. The following guide lines are the most outstanding and most often recommended:

a) Select materials to resist wear according to the following steps (Reference 19-220):
   List the materials that fulfill the mechanical requirements of the parts.
   Determine the expected service conditions of the part.
   From the above data, narrow the number by picking only those materials that have shown low wear in similar applications.
   Follow up by actual testing.

b) It is generally advisable to use stainless steel in all components, fittings, and tubing, particularly in components which will stand idle or will be stored and are subject to internal moisture condensation (Reference 51-12).

c) Hoses and flexible connections should be made of Teflon. Pressure hoses should be made of Teflon reinforced with stainless steel braid. Rubber hose is difficult to clean, and filler materials can be washed off (References 1-25, 1-26, 1-107, 6-32, 51-6, 51-12, and V-158).

d) Aluminum castings shed large quantities of very fine particles in the 3 micron range (Reference 6-38). Aluminum alloys need to be anodized to prevent them from adding particles and corrosion products to the fluid stream (Reference 51-12).

e) Avoid any flaky or friable surface finishes. Cadmium, zinc, and tin plating flake or scrape easily. Use electroless nickel, chromium or anodizing (Reference 6-32). Hard chrome plating is susceptible to fatigue unless it is supported on a firm base (Reference 6-32).

f) Do not use ceramics, particularly those with unglazed surfaces (Reference V-158).

g) Soft and stringy packings are gradually deposited in the fluid stream (Reference V-158).

h) Use filter construction materials that are structurally adequate, corrosion and temperature-resistant to the fluids, and will not migrate to the downstream portions of the system (References 1-25, 1-26, 1-107, and 450-2).

i) Careful consideration of material, hardness, and finish will reduce wear and particle generation. Specify that surfaces which rub against each other are smooth and of different hardness. A surface hardness of 60 RC is considered optimum (References 6-1, 19-221, 114-4, and 453-1).

10.6.2.3 MANUFACTURING OPERATIONS. Improvements in manufacturing operations can reduce built-in contamination from this primary source. Anticipate manufacturing or fabrication techniques that will increase the contamination levels. For example, shrink-fitting of mating parts instead of press-fitting or threading prevents shaving off metal slivers which later contaminate the system (References 65-27 and 6-1). Other common recommendations aimed at curtailing the generation of contaminants during manufacture of components and their parts are:

a) Avoid metallic castings, since they generally carry inclusions and molding residues such as core sand. Use forged stock or raw stock. If castings are to be used, the cast surfaces should be machined (References 1-25, 1-26, 1-107, and 6-32).

b) Clean tools and molds during forming and drawing operations in order to avoid embedding hard metallic particles (Reference 6-32).

c) Deburr each part before each forming operation, and degrease it using ultrasonic cleaning and vapor degreasing (Reference 6-32).

d) Weld, braze, and solder before the final machining, or clean immediately afterward. If the weld areas are exposed to the fluid, they should be machined to remove oxides (Reference 6-32).

e) To eliminate weld spatter in filters, use seam welding of overlapping joints, avoid arc welding of open sections, and provide ample openings for final cleaning (References 1-25, 1-26, and 1-107).

f) All functional surfaces should be finished as smooth as possible so they can operate efficiently without generating contaminants. In the design of hydraulic components, a surface finish of 8 to 16 microinches is recommended by some investigations, while others consider a range of 5 to 10 as the optimum finish generating the least amount of contaminants. If the finish is finer, the parts will not retain oil; if it is more coarse, it will abrade contacting surfaces (References 6-32, 19-220, 114-4, and 450-2).

g) Heat treat before machining the surfaces of internal parts, otherwise the oxides formed during heating will be difficult to remove (Reference 6-32).

h) Do not plate springs or other parts subject to high torsional stress in order to avoid flaking the coating (Reference 6-32).

i) Avoid cored passages, particularly in casting; drilled passages are smoother and do not hold core molding materials (Reference 6-32).

10.6.2.4 COMPONENT TESTING. The last and most often overlooked step in the production of fluid components is the performance checkout. This function has been attributed to being a frequent source of contamination in fluid components, and to reduce this source, the following procedures are recommended:

a) Maintain close control over contamination level in all test equipment. All fittings, fluids, and assemblies used to test components must be as clean as or cleaner than the component being tested and should be used only once for each test setup (References 6-32 and 19-220).

b) Clean all test equipment connectors thoroughly before making connections (References 6-1 and 51-6).

c) Use dummy components to prevalidate the cleanliness of the test circuits (Reference 6-32).

10.6.2.5 CLEANING AND PACKAGING. Not all the sources of contamination encountered during manufacturing can be eliminated completely, but they can be reduced substantially by precleaning components to a specified level, and by maintaining them in such a condition during assembly and installation by means of a clean area where contaminants have been reduced to a correspondingly low level (Reference 51-6). Other specific recommendations to the designer concerning component cleanliness are:

a) Clean components and parts immediately after machining before cutting oils set (Reference 6-1).

b) Clean all surfaces and channels of filters thoroughly, using a combination of ultrasonic cleaning and flushing (Reference 6-1).

c) Clean parts with small cored passages, such as castings, carefully before assembly to remove contaminants from these blind passages (Reference 6-32).

d) Take pains to clean at the component level, even when components are not contaminant-sensitive, to avoid contributing later to the system contamination which is harder to eradicate (Reference 6-32).

e) After cleaning, parts and components should be packaged in heat-sealed plastic bags, avoiding the use of preservatives and coatings (References 6-32, 19-221, and 136-4).

f) Cleaned components should have all parts and connections capped. Male fittings should be capped with anodized aluminum caps. Female fittings should be plugged with the fittings used in flight and capped with aluminum caps. Avoid the use of plastic and soft metals for capping (Reference 6-32).

g) A practical and feasible level of filter element cleanliness is that point when approximately the same amount of particles that could be removed by vibration and flow are removed by process cleaning (Reference 51-6).

10.6.2.6 ASSEMBLY AND INSTALLATION. Introduction of foreign matter and airborne dust must be controlled until the last moment when the component is assembled into a system. Recommendations are:

a) Final assembly, cleaning, and inspection of components must be done in a clean environment commensurate with the levels of cleanliness required (Reference 6-32).

b) Thread compounds and lubricants should be avoided. Use of Teflon tape is recommended (Reference 51-12). If lubricants must be used, they should be compatible with the fluids (Reference 28-49).

10.6.2.7 HYDRAULIC SERVOVALVES. From the contamination standpoint, hydraulic servo systems are undoubtedly the most crtical airborne systems. Next to inertial guidance system components, hydraulic servo components demand the mort extreme accuracy, stability, and response. To meet these stringent operation requirements, servovalves usually have minute tolerances, openings, and actuating forces. The critical nature of these dimensions becomes apparent by looking at the critical clearances of a typical servovalve (Table 10.6.2.7a).

Table 10.6.2.7a. Average Critical Dimensions of Hydraulic Servovalves
(Reference 6-28)

AREA	CLEARANCE (μ)
Spool diametral clearance	1-10
Flapper nozzle clearance	25-38
Metering orifice	75-38
Nozzle clearance	254-1016
Pole piece clearance	254-1270
Drain bleed orifice	305-660

Servovalve failures due to contamination are usually of two types—failures caused by plugging of orifices and nozzles, and those caused by sticking of sliding mechanisms. Both of these conditions can be considerably corrected at the design level by keeping the contaminants under control and reducing the susceptibility of the servovalves to the contaminants. The effects of contaminants on servovalves, along with corrective action, are given in Table 10.6.2.7b.

10.6.3 Fluids Compatibility

The internal environment of fluid components is mostly determined by the working fluids. Each fluid system — pneumatic, hydraulic, fuel, pressurization, or propellant — has its own characteristics and requirements of chemical and physical compatibility. If these requirements are not satisfied, the least that can happen is a heavy generation of contaminants. Corrosive propellants can generate corrosion products; cryogenic fluids may crack seals; overheated hydraulic fluids may break down and deposit gums or varnish; particles entrained in high pressure gases will erode surfaces; and, of course, sediment and sludge will plug filters, injector orifices, and small lines. Such problems are heavily compounded in closed recirculating circuits such as hydraulic and lubricating systems where the fluid is continuously subjected to repeated trials and stresses, while the amount of particles due to system wear are continuously increasing.

10.6.4 Systems Design

The component designer can do much at the system level to keep up the contamination control effort originating with the design of fluid components. He can aid in the immuniza-

Table 10.6.2.7b. Characteristics of System Contamination on Servovalves
(Reference 6-42)

PARTICLE PROPERTIES	EFFECT ON SERVOVALVE	CORRECTION
Size	Orifices and nozzles are plugged by particles their same size or larger. Valve spools stick because of trapped particles. Hysteresis increases.	Adjust filters to particle size. Since system null leakage and dead band requirements establish diametral clearance, correction depends on valve design.
Shape	Filters are effective on particle's smallest dimension only. Orifices, nozzles, and passages trap particles because of largest dimension.	Eliminate fibrous contaminants which can pass filters and build up in the valve.
Material	Magnetized particles can form clusters. Sticking, change in leakage, and deadband characteristics are caused by hard, abrasive particles. Flexible materials work through filters, change shape, and build up.	Properly design the valve to eliminate contaminants.
Suspension	Malfunction is caused by suspended particles being carried into the valve. (Light particles will suspend in fluid more than heavy particles; small ones more than large ones of the same density)	Analyze tubing configuration, fluid viscosity, and environmental conditions such as vibration and temperature which affect particle suspension.

tion of the system against contamination by giving proper consideration to the location of components, circuit configuration, assembly methods, selection of fittings and piping, and analysis of the over-all system contamination sources. In essence, the criteria behind these considerations is a corollary of the principles used in the design of contaminant-conscious components: to channel any possible design features towards the control of contamination by reduction and removal of the contaminants or protection and desensitization of the system. A review of current engineering design practices covering these concepts of contamination control is presented below.

10.6.4.1 INCREASE TOLERANCE TO CONTAMINATION. Most contamination problems originate from inadequate, unnecessary, or over-strict cleanliness requirements. Wherever possible, the system should be designed for a maximum of dirt tolerance. The alternative to this is the installation of finer or bigger filters, larger pumps, and the implementation of costly cleaning operations and quality control procedures. The following recommendations are the most commonly suggested to help develop fluid systems with a reasonable contamination tolerance:

a) Design systems to operate with a reasonable amount of dirt, based on the tolerance of the most critical components. If particles of over several hundred microns can be tolerated and dissolved, matter is no problem, and ordinary design and quality control procedures will suffice to maintain system performance (References 19-221, 136-4, and 450-4).

b) Dirt tolerance of a system should not be predicted upon the quality of fluids available, since the initial condition of the fluid is lost when flow begins, particularly in a recirculating system (References 23-33 and 453-1).

c) To determine the dirt tolerance of a system, the following guide lines are given in References 19-221 and 136-4:

   Determine which components are most susceptible to malfunction because of contamination.

   Calculate maximum flow rates and pressure at susceptible points to determine what problems exist.

   Define how long the system must function between maintainance periods. If a system can be cleaned frequently, larger quantities of contaminants can be tolerated.

   Define the types of contaminants expected. Determine the possible effect of soluble contaminants and the size and amount of insoluble materials. Insoluble materials are usually the most critical.

d) Install sampling points at proper locations to allow adequate monitoring of system cleanliness (Reference 136-4).

10.6.4.2 REDUCING CONTAMINATION SOURCES. The best defense against contaminants is prevention. It is easier and cheaper to reduce the sources and avenues of entry than to clean and filter the unwanted particles and substances. A considerable reduction in contaminants can be obtained by care in assembly and installation, by simple design, and by paying attention to common everyday housekeeping and plumbing practices as indicated by the following recommendations:

a) Use lubricants compatible with the fluids to be handled, and only when absolutely necessary. Avoid pipe compounds; use Teflon tape wrapped two threads back from front end (References 1-25, 1-26, 1-107, 28-49, 51-12, 450-2, and V-158).

b) Do not use soft or stringy packings requiring periodic replacement; they are gradually deposited in the fluid stream (Reference V-158).

c) Minimize pipe and tubing runs. The shortest length will have the smallest surface area and, correspondingly, the lowest potential source of contaminants. Minimize the number of tees, crosses, bends, and other fittings that generate and trap particles. Use manifolds wherever possible (References 19-221, 51-12, 453-1, and V-158).

d) Eliminate all possible close fitting dynamic parts, connections, and components susceptible to obstruction (References 136-4 and V-158).

e) Use only clean, bagged, and sealed components to assemble the system. Inspect the bags for the presence of talc or other plastic extrusion lubricants (Reference V-158).

f) Minimize vibration and shock, particularly around filters (Reference V-158).

g) Do not allow flow across threaded connections to go against male fittings to avoid scrubbing particles out of threaded crevices (Reference V-158).

h) Flush assembled systems whenever possible with low viscosity filtered solvents at high velocity (Reference V-158).

i) Place gaskets or seals to permit minimum contact with the bulk of the working fluid (Reference 136-4).

j) Perform assembly and disassembly operations in environmentally-controlled areas commensurate with the degree of cleanliness in the components being used. Avoid exposing components to paints, coatings, and airborne ducts (Reference 136-4).

k) Avoid the need for assembly and installation of fittings on a vessel after it has been fabricated and cleaned (Reference 136-4).

10.6.4.3 PROTECTING THE SYSTEM COMPONENTS. After the components are assembled into a system, the only way to control contamination is through adequate system filtration. Proper filtration demands that close attention be paid to the location of filters and their filtering characteristics. The finer the filtration, the higher the costs in terms of equipment, pressure drops, and maintenance requirements. Therefore, unnecessary protection may be wasteful. The following paragraphs list the most common recommendations for protecting the system from post-assembly contamination:

a) Vent lines should have breather caps with adequate filters, and fill ports should have strainers (References 1-25, 1-26, and 1-107).

b) Dual filters are recommended to provide adequate filtration during service and to provide ease of removal for cleaning and maintenance (Reference 136-4).

c) Provide for proper differential pressure during service and consider pressure fluctuations due to temperatures (Reference 136-4).

d) Consider the addition of a by-pass system for each filter to facilitate service and testing operations (Reference 136-4).

e) Locate filters upstream of servovalves and as close to them as possible (Reference 6-1).

f) Use closed circuit designs such as plug-in modules or integrated assemblies so dirt cannot enter after assembly (References 6-1, 6-205, 23-33, and 453-1).

g) Since the fastest accumulation of contaminants in recirculated circuits occurs during the initial run-in period, when leftover dirt and wear products accumulate, the first replacement of filter elements should be performed at an earlier time than planned in routine maintenance (References 1-25, 1-26, and 1-107).

h) Use filters at strategic locations in the system, and provide access for examination, cleaning, and maintenance (Reference 136-4).

The following specific recommendations for filter locations in open and recirculating systems are adapted from Reference V-158:

a) Open and fluid circuits:
   Place fine filter at last possible point in the system.
   Place a large size depth filter upstream of the fine filter.
   Conduct fluid directly to the destination without rehandling.
   Keep design simple by using a minimum of piping, fittings, and components.

b) Recirculating Systems:
   Place outlet at lowest possible point of cone-shaped reservoir.
   Return all fluids through an in-line fine filter.
   If peak flow rates are too high, use extra reservoir and transfer later through fine filter to clean reservoir.
   Recirculate contaminated fluid in open reservoirs at high flow rates from both top and bottom areas and back to the reservoir through a fine filter.

10.6.4.4 REMOVAL OF CONTAMINANTS. The basis for optimum removal of contaminants lies in planning the fluid circuit and its components so that any contaminants in the fluid stream are continuously being pushed toward the filter elements. To this effect fluid flow surfaces should be as flush as possible and attention should be given to the maintainability of filter elements as well as maintenance schedules. Other pertinent recommendations concerning optimum measures and aids in the removal of contaminants are:

a) Avoid bellows and spirals since they are hard to clean and are dirt entrapment areas (Reference V-158).

b) Design out all possible dead ends and provide bleed drains for those remaining (Reference V-158)

c) Mount accumulators vertically so they will drain down and across a line instead of at a dead end (References 6-32 and V-158).

d) Install components to provide a maximum accessibility to facilitate maintenance and inspection. Wherever possible the design and layout should be aimed at a one-man service operation (Reference 51-12).

e) In airborne tankage, provide free draining structures; baffles and supports should have minimum entrapment areas (Reference 136-4).

f) Vessels and other reservoirs in recirculated systems should have tangential return lines, to keep fluid agitated, and sloping bottoms tapered towards the main outlet (Reference 6-32).

g) Install drains at all low points in the system (Reference 136-4).

h) All systems should disassemble in sections of 20 feet or less so that available cleaning tanks may be used (Reference 136-4).

i) Design systems so that fluids can be recirculated through provisional filters until desired contamination level is achieved (Reference 136-4).

10.6.5 Environmental Factors

The component designer generally has little control over the environment in which the components will be used. However, it is possible to anticipate the approximate operating conditions that will be encountered—such as temperature, pressure, gravity, and quality of the ambient atmosphere—and design with them in mind. Rust and corrosion may be prevented by anticipating conditions of extended storage or inactivity. The only environmental control that the designer is able to specify is having the components assembled and installed in a clean room or a clean area.

10.6.6 Control Methods

Summarizing, there are four methods by which the fluid component designer can control the degrees and effects of contamination at the system level. All four approaches can be specified objectively and are within the exclusive jurisdiction of the component designer. They are:

a) Specify the maximum size, amount, and type of contaminants that can be tolerated by the most susceptible components.

b) Specify the type and size of filter that will protect a given design feature of a critical component.

c) Specify the finishes, clearances, materials, and fluids that will generate the least amount of particles.

d) Specify the maximum cleanliness that must be achieved when contaminants are removed from the components by means of specific cleaning procedures.

Detailed references on how these methods may be developed have been presented in sub-sections of this section, particularly Sub-Section 10.4 "Cleanliness Requirements."

REFERENCES

1-25	19-220	450-1
1-26	19-221	450-2
1-107	23-33	450-4
1-270	23-34	451-1
6-1	23-53	452-1
6-2	28-49	453-1
6-22	51-6	454-1
6-25	51-12	454-2
6-26	65-27	455-1
6-28	81-4	456-1
6-32	89-1	457-1
6-38	114-4	457-2
6-42	136-4	457-3
6-56	183-6	457-4
6-162	183-7	458-1
6-164	281-2	V-158
6-195	448-1	V-270
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6-206	448-3	V-280
6-207	449-1	

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11.1 INTRODUCTION

Reliability is an index of design excellence and product operational integrity. Reliability is defined as the probability that a device will perform a specified function for a given period of time under given environmental conditions. The current trend in design of missile and space vehicle systems is toward higher performance, more complexity, and longer periods of unattended automatic operation, thus placing an ever-increasing importance on high reliability. It is the purpose of this section to define reliability terminology, present some basic reliability mathematical relationships, and give the fluid component designer some practical guidelines for reliable design.

11.2 RELIABILITY DEFINITIONS AND MATHEMATICS

With the increasingly important role reliability considerations are playing in aerospace design, it is important that the fluid component designer have some knowledge of the basic terms and fundamental relationships used by the reliability engineer. In the following paragraphs reliability terms are defined and, where applicable, pertinent mathematical relationships are presented.

11.2.1 Probability

Probability is the percentage of time that an event is predicted to occur, relative to a large number of observations of similar events.

11.2.2 Reliability Index

The reliability index expresses the probability that a part will operate without failure for a specified period of time. Measured on a 0 to 1 scale, the reliability index of an item may have any value from 0 (meaning that it is certain not to operate) to 1 (meaning that the part is certain to operate without failure for the specified time period).

11.2.3 Wear Out Failures

Wear out failures are failures which occur as a result of normal mechanical, chemical, or electrical degradation.

11.2.4 Random Failures

Random failures are failures that occur before wear out and are not predictable or associated with any pattern of similar failures. However, it should not be assumed that the cause of a random failure cannot be found.

11.2.5 Independent Failure

An independent failure is a failure of a device which is not caused by concurrent failure of another device.

11.2.6 Secondary Failure

Secondary failure is the failure of a device resulting directly from the failure of another device.

11.2.7 Mean Time Between Failure

The mean time between failure (MTBF) is the average operating time or number of cycles of a part, determined by adding the individual in-service operating times, number of cycles, and dividing by the total number of times the part is put into service after repair. Where a number of different parts of the same design are used for generating data, the mean time between failure is the average operating time before failure of all parts under consideration. Reliability and mean time between failure values are related by the following equation:

$$R = e^{-\frac{t}{m}} \qquad \text{(Eq 11.2.7)}$$

where
- R = reliability
- m = mean time between failure (mean number of cycles between failure)
- t = individual operating time (number of cycles)
- e = constant = 2.718

If the mean time between the failure for a part has been determined to be one hour, and it is desired to determine the reliability of this part based on this data for one hour, the reliability $R = 2.718^{-1/1} = 0.368$. This means that the part could be expected to operate completely and successfully throughout the one hour only 37 times out of 100 times attempted. To increase reliability, either the individual operating time, t, must be shortened or the mean time between failure must be increased.

11.2.8 Mean Time to Failure

Mean time to failure (MTTF) is an alternate means of expressing MTBF.

11.2.9 Failure Rate

The failure rate is a measure of the probable number of times that a given component will fail during a given time period of operation. Failure rate, λ, is often expressed as failures per million hours.

$$\lambda = \frac{1}{m} \qquad \text{(Eq 11.2.9a)}$$

where
- λ = failure rate, failures/hour
- m = mean time between failure, hours

Reliability expressed as a function of failure rate is

$$R = e^{-\lambda t} \qquad \text{(Eq 11.2.9b)}$$

where
- R = reliability
- e = constant = 2.718
- λ = failure rate, failures/hour
- t = operating time, hours

11.2.10 Confidence Level

Confidence level is the certainty with which conclusions can be drawn from a given group of data. For example, at a 95 percent confidence level the conclusions drawn will be in error 5 percent of the time, or an averaged one in twenty. To demonstrate a given reliability (the conclusion) the higher the confidence level selected, the greater must be the number of tests, as indicated by confidence level tables.

11.2.11 Confidence Limits

Confidence limits are the computed upper and lower limits of the desired value of a physical quantity (e.g., failure rate) for a specified confidence level; that is, the true value of a physical quantity or a parameter which can be stated as falling between upper and lower limits with a certain level of confidence as determined by the sample size. The closer these limits are, the lower the confidence level for a fixed sample size, or number of tests. Conversely, more tests are needed as the specified confidence limits are narrowed to maintain a given confidence level. The upper confidence limit is used to determine a reliability index where the confidence limit is expressed as failure rate in percent, or in number of failures per 100 tests. For example, if actual test data shows 5 failures in 100 tests, the lower and upper confidence limits can statistically be shown to be 3 and 9 percent failure rate, respectively, at a confidence level of 80 percent over the interval. This states that on the average, 80 percent of the time a sample of 100 tests will have no failures equal to or greater than 9, and no failures equal to or less than 3. To express the same information in terms of reliability, only the upper confidence limit is used. This states that there will be no failures equal to or greater than 9 percent, or a reliability of at least 91 percent. When only a single confidence limit (higher limit for reliability) or singular limit is used, the confidence level is higher by one-half the difference between the confidence interval confidence level and 100 percent. Therefore, in the example cited, the reliability is 91 percent with an $80 + \frac{100-80}{2} = 90$ percent confidence level.

11.2.12 Confidence Interval

The confidence interval is the interval determined by the upper and lower confidence limits. In the example cited in Sub-Topic 11.2.11, the confidence interval is 0.03 to 0.09.

11.2.13 Series Configurations

If items are arranged to perform their functions in a configuration such that failure of any one item results in failure of the configuration, then the configuration is said to be a series configuration. The reliability of a series configuration is equal to the product of the separate reliabilities of the components.

(Eq 11.2.13)

$$R_s = R_1 \times R_2 \times R_3 \times \ . \ . \ . \ R_n$$

where R_s = reliability of series configurations
R_n = reliability of nth component

11.2.14 Parallel Configurations (Redundancy)

A parallel or redundant system is one in which multiple devices, structural elements, parts, or mechanisms are employed in combination for the purpose of increasing the reliability of a particular function or operation. In a parallel system, when one item fails all or any one of the remaining items are capable of continuous operation and accomplishing their functions. For a parallel system of n components, the system reliability is

(Eq 11.2.14)

$$R_p = 1 - (1-R_1)(1-R_2) \ . \ . \ . \ (1-R_n)$$

where R_p = reliability of parallel system
R_n = reliability of nth component

Redundancy in fluid systems can be achieved either by redundant components in a common housing or as separate units. A redundant shutoff valve system is illustrated schematically in Figure 11.2.14. This combination series and parallel arrangement, known as a quad valve, provides a system whereby any one of the four valves can fail either open or closed without causing a system failure. See Sub-Topic 11.3.11 for further discussion of redundancy in fluid component design.

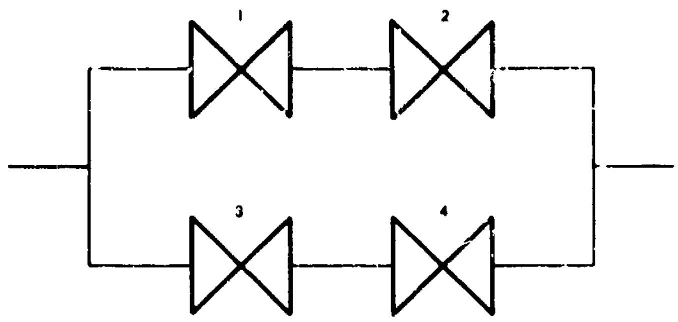

Figure 11.2.14. Quad Shutoff Valve Arrangement

11.2.15 Frequency Distribution

Frequency distribution, also called probability density function, describes the spread of characteristic values for a given set of statistical data. Some of the more common mathematically described frequency distributions include Gaussian (normal), exponential, Weibull, Gamma, and Log-normal.

11.2.16 Normal or Gaussian Frequency Distribution

A normal distribution shows a central tendency of values (measurements) at which point (mean value) the largest frequency of occurrences is observed. The normal curve is characteristically bell shaped (Figure 11.2.16) having equal areas on either side of the center value. Many processes have been discovered by measurement to follow a normal distribution. This is the distribution which is usually assumed by most statisticians when the true distribution of values is unknown. The curve has equal areas on either side of the center or mean value μ. Characteristic values found other than at the center or mean value are said to be scattered or diverse values. Such values are also called deviations from the center value. It is from this word that the phrase "standard deviation," so frequently used in statistical mathematics, is derived. The area under the curve

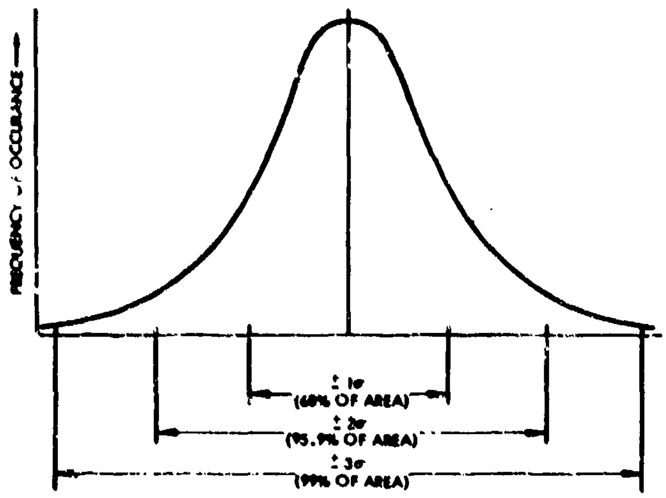

Figure 11.2.16. Normal Frequency Distribution

is a function of the standard deviation, σ, and is indicated in Figure 11.2.16. The area under a normal curve between ± 1 standard deviations is 68 percent of the total area under the curve. For ± 2 standard deviations it is 95.5 percent, and for ± 3 standard deviations it is 99.7 percent. The equation for the normal distribution curve is

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad \text{(Eq 11.2.16)}$$

where
- y = frequency of occurrence
- x = measurement value
- σ = standard deviation
- μ = mean value

11.2.17 Universe (Population) and Sample

In handling statistical data, a distinction is made between the universe—or the entire population of possible measurements—and the limited sample measurements normally available. The larger the sample size, the more closely the sample mean and standard deviations ($\bar{X}$ and S) approximate the corresponding universe values (μ and σ). The following definitions apply to an entire population of data:

a) μ = mean of population. The sample mean $\bar{X}$ is used to denote the best estimate of μ.

b) σ = standard deviation of population. The sample standard deviation, S, denotes an estimate of σ.

c) σ^2 = variance of population. S^2 is the best estimate of σ^2.

11.2.18 Standard Deviation (S)

The standard deviation of a sample provides a measure of the amount of dispersion or scatter about a typical (mean or average) value. The standard deviation indicates the general shape of the distribution curve (Figure 11.2.16) by describing how the area under the curve is distributed about the mean. A small value of standard deviation indicates a tall slender curve, whereas a large value standard deviation indicates a short, spreadout curve. The sample standard deviation is expressed mathematically as follows:

$$S = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N-1}} \qquad \text{(Eq 11.2.18)}$$

where
- S = standard deviation
- X = individual value in the sample
- $\bar{X}$ = mean value of the sample
- N = number of measurements (sample size)

11.2.19 Mean

The mean is the value about which the greatest concentration of data occurs. The mean of the sample is the arithmetic average expressed as follows:

$$\bar{X} = \frac{\Sigma X}{N} \qquad \text{(Eq 11.2.19)}$$

where
- $\bar{X}$ = mean value
- X = individual value
- N = number of measurements (sample size)

11.2.20 Range

The range is the difference between the maximum and minimum measured values.

11.2.21 Variance

The variance of the sample is defined as the square of the standard deviation (S^2).

11.2.22 Example—Determining Range, Mean, Variance, and Standard Deviation

The measurements—mean, range, variance, and standard deviation—used to characterize the frequency distribution of statistical data are shown in the following example:

A life cycle test program on a particular valve design resulted in cycle to failure data on 26 parts as follows:

Cycle to Failure (X)	Number of Parts (f)	fX	X-X̄	(X-X̄)²	f(X-X̄)²
82 (81.6 - 82.5)	1	82	−3	9	9
83 (82.6 - 83.5)	2	166	−2	4	8
84 (83.6 - 84.5)	5	420	−1	1	5
85 (84.6 - 85.5)	10	850	0	0	0
86 (85.6 - 86.5)	5	430	+1	1	5
87 (86.6 - 87.5)	2	174	+2	4	8
88 (87.6 - 88.5)	1	88	+3	9	9
	N (Total Measurements) = 26	ΣfX = 2210			Σf(X-X̄)² = 44

The range, mean, variance, and standard deviation as determined from the data shown above are:

$$\text{Range, } R = 88 - 82 = 6$$

$$\text{Mean, } \bar{X} = \frac{\Sigma f X}{N} = \frac{2210}{26} = 85 \text{ (best estimate of } \mu\text{)}$$

$$\text{Variance, } S^2 = \frac{\Sigma f (X - \bar{X})^2}{N-1} = \frac{44}{26-1} = 1.76$$

Standard Deviation, $S = 1.33$ cycles (best estimate of σ)

The sample distribution is shown graphically by plotting a histogram (Figure 11.2.22).

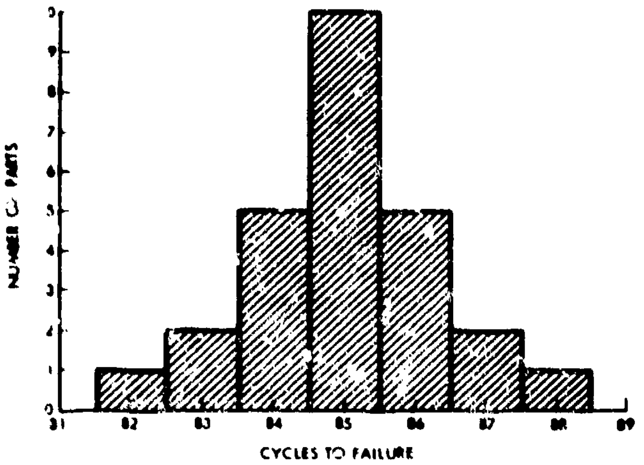

Figure 11.2.22. Histogram of a Sample Distribution

11.3 DESIGNING FOR RELIABILITY

Major considerations in achieving reliability in fluid components are adequate specifications, good design, adequate inspection of materials and components, and adequate testing. Some of the important design considerations influencing reliability are given below.

11.3.1 Select Reliability Goals Appropriate to the System

When reliability requirements are high, the designer must put greater emphasis on these requirements as he considers other design factors such as weight, cost, ease of fabrication, and testing costs.

11.3.2 Design for Simplicity

The designer should attempt to minimize the number of parts, avoid moving parts, avoid delicate mechanisms, and avoid close clearance sliding fits.

11.3.3 Design for Component Assembly and Installation

Common fluid component problems such as over-torquing and reversal of both electrical and fluid connections can be greatly minimized by careful design and clear and distinct markings. One good way to protect a design against human error is to design for one-way assembly and installation. Another way is to design a part, such as a seal, so that it can be installed correctly in more than one way.

11.3.4 Design for Maintenance

Components should be designed for easy maintenance when maintenance requirements are involved. There should be a minimum need for maintenance training and judgment by maintenance personnel. Maintenance procedures should be carefully and properly specified.

11.3.5 Design for Contamination Tolerance

Since a certain amount of fluid contamination is inevitable, a primary consideration should be given to designing a component which will tolerate a reasonable amount of contamination, rather than trying to eliminate all contaminants through excessive filtration.

11.3.6 Design for Minimizing Contamination Generation

The effects of surface finishes and seal and packing materials on the contamination level in a system should be carefully considered. Improper surface finishes combined with shreddable packing in missile system shutoff valves, have resulted in serious contamination problems from packing migration.

11.3.7 Use Proven Designs

Items which have been in quantity production are usually more reliable than new items which have been especially developed for a component or system. Novel design approaches should generally be avoided in favor of proven concepts. This is particularly true in the design of modules such as springs, bearings, etc. It should be pointed out, however, that a proven design may not always adequately meet the requirements and in such cases new designs are perfectly justified.

11.3.8 Design for Safety

The components should be designed so that failure will cause a minimum of impairment to system operation and will minimize personnel hazards. Fail-safe features should be employed such that loss of power will not present an unsafe condition. Safety considerations often dictate the use of valve designs that will automatically close in the event of actuator or power failure.

11.3.9 Design for Environmental Extremes

The designer should consider the worst possible chemical and physical effects that could result from the environmental extremes to which the components must be exposed.

11.3.10 Design Modules with Liberal Stress and Load Margins

The designer should apply adequate safety factors and utilize performance derating to extend service life and increase reliability of component parts (modules) such as springs, bellows, housings, bearings, etc. Vital springs, for instance, should be designed to operate at 80 percent of the normal design stress for the material.

11.3.11 Design Components with Liberal Performance Margins

System reliability can be increased by using components which have liberal performance margins. Thus, successful systems performance can be attained in spite of failure of a component to meet design requirements completely. Some examples indicating how reliability can be improved by designing components with increased performance margins are:

a) Provide actuator force margins such that failure of an actuator to develop its design force still gives adequate force to actuate the valve.

b) Size components large enough so that partial opening of a valve or blockage of a flow passage still provides sufficient flow.

c) Design regulators and relief valves to operate with narrower regulation or crack and reseat bands than required by the system, so that failure to meet component design requirements still fulfills the system objective.

It is important to note that this general approach to increasing system reliability can easily result in unreliability if not used with discretion. Tightening of design requirements cannot be done arbitrarily, as the result may be a component so complex that its inherent reliability is considerably lower than a simpler component which just meets system performance requirements. The use of liberal performance margins should be employed only when apparent system reliability gains are not offset by added component design complexity.

11.3.12 Design for Redundancy

Redundancy, as defined in Sub-Topic 11.2.14, is a common technique for increasing component and system reliability. A good example of redundancy in fluid component design is the use of primary and secondary seals in both static and dynamic applications so that leakage through the primary seal is stopped by the secondary or backup seal. The secondary seal is unnecessary as long as the primary seal is functioning properly, but in the event of primary seal failure, the redundant or secondary seal increases the probability of successful operation of the component and its associated system. Redundancy in design must be used with great care, since it is possible to decrease reliability through improper use of redundant design techniques. The theoretical gain in reliability achieved by redundant design must be carefully weighed against such factors as increased cost, increased weight, and added over-all complexity. Increased complexity alone could potentially offset the theoretical gain in reliability achieved by the use of redundant design.

11.4 RELIABILITY DESIGN REVIEW

The reliability of the final product can be greatly improved by a systematic design review program. Some of the questions that should be asked about a fluid component design are:

11.4.1 Bearings

Are bearings protected from corrosion and galling due to dirt, moisture, and inefficient lubricants? Are bearings protected from brineiling due to vibration, shock, or soft materials? Are bearings adequately protected against the adverse effects of vacuum exposure?

11.4.2 Filters

Are integral filters used to protect the sensitive elements of a component from contamination failure? Do filters have sufficient dirt-holding capability?

11.4.3 Mechanical Linkages

Are actuated surfaces and arms protected from over travel? Have lubrication requirements for linkages been kept to a minimum?

11.4.4 Seals

Are bolt torque requirements specified on the assembly drawings? Are locking devices provided? If lock wire is used, its length should be kept to an absolute minimum. Can O-rings and seals be installed easily without being cut by sharp edges, resulting in seal damage and subsequent leakage?

11.4.5 Flow Passages

Are there flow passages small enough to become clogged with contaminants?

11.4.6 Fasteners

Do all fastened assemblies contain adequate locking devices and/or possess practical, but effective, torquing requirements? Are all fasteners (nuts, bolts, etc.) easily accessible to maintenance personnel?

11.4.7 Corrosion

Are there water or liquid traps formed by brackets, etc.? Is the component splash-proof, water-proof, ice-proof, and salt spray-proof? Are there dissimilar metal shims, fittings, or miscellaneous hardware in intimate contact? Are lock washers of the type that break through protected films? Have all corrosion-prone surfaces been protected?

11.4.8 Maintenance

Are all lines, devices, etc. designed so they cannot be used as handles, steps, or seats? Will all routine maintenance points, drains, etc., be accessible after installation? Have parts been designed so they cannot be assembled incorrectly? Has the number of special tools been kept to a minimum?

11.4.9 Vibration

Are there cantilevered parts, brackets, arms, or linkages which will vibrate? How close are resonant frequencies to the environmental imposed spectrum? Can damping be added if vibration problems are encountered?

11.4.10 Fluid Fittings

Are the number of fittings in external lines kept to a minimum to reduce the number of leakage points?

11.4.11 Materials

In the selection of materials, have the following been investigated: weldability, machineability, formability, fluid compatibility, heat-treat distortion, heat-treat contamination, cost, and availability? Have materials, heat treatments, and stress levels been considered in terms of possible stress corrosion effects? Has the effect of creep been determined? Has material fatigue been determined and provided for? Have the effects of elevated and low temperature service upon the material been determined? Have the effects of thermal gradients been considered?

11.4.12 Manufacturing

Are tolerances so excessively stringent that the shop will not be able to fabricate within these tolerances without excessive cost? Are the capabilities of the manufacturing equipment and facilities within the requirements? Have critical dimensions and properties been designated on the drawings for special attention during the manufacturing and inspection process? Has the proper heat treatment been specified on the drawings for each material heat number? Have parts and subassemblies been adequately identified? Has the cleaning method been specified? Have allowable torques been specified on the drawing? Have distortion and buckling as a result of fabrication processes been considered? Are all fillet radii as large as possible? Have steps been taken to eliminate the possibility of burrs from machining which could break loose during operation and cut seals or cause clogging of the system?

REFERENCES

19-148
28-51
147-5
308-2
390-1
407-1

12.0 MATERIALS

ILLUSTRATIONS

Figure

TABLES

Table

TABLES (Continued)

TABLES (Continued)

12.1 INTRODUCTION

The purpose of the Materials Section is to provide general data on metals, nonmetals, liquids, and gases typical to rocket propulsion systems and components. Since extensive treatment is given to these subjects in readily available literature of extensive volume, most of the data presented reflects general trends and is intended only for the purpose of general design calculations. References are provided on sources of detailed materials data, and it is recommended that the reader make use of those data sources whenever possible in order to insure that he has the best possible understanding of the accuracy for the materials property variable in his engineering calculation. This section is divided into subsections covering properties of fluids, including both liquids and gases; properties of polymers, including plastic and elastomeric materials; properties of metals; the chemical compatibility of materials with rocket propellants; and permeability data and friction coefficients. It is intended that additional data will be included as it becomes available, hence there are blank spaces in the various tables which reflect data that is currently being sought.

Following is an outline of other materials data which have been included elsewhere in the handbook in support of specific subjects:

Section 2.0, Heat Transfer — Thermal conductivity, emissivity, and absorptivity data for use in heat transfer calculations.

Section 3.0, Fluid Mechanics — Density, viscosity, specific heat, bulk modulus, vapor pressure, and sonic velocity data for various fluids for fluid flow calculations.

Section 6.0, Modules — Mechanical properties, compatibility, and friction coefficient data related to the design of various modules.

Section 13.0, Environments — Properties related to environmental analysis such as ozone resistance, oxidation rate, creep strength, thermal expansion, temperature limitation, sublimation rate, vacuum decomposition rate, radiation resistance, and corrosion resistance.

Section 14.0, Stress Analysis — Mechanical properties data.

12.2 PROPERTIES OF FLUIDS

Properties of gases and liquids commonly used in aerospace applications are presented in tabular form. References on more detailed data are listed at the end of each sub-topic.

Unless otherwise noted, all data are given for a pressure of one atmosphere at room temperature.

12.2.1 Storable Rocket Propellants

The most general definition of a storable propellant is a propellant which may be stored in a rocket system unattended for extended periods and ready for instant use under the rocket system storage conditions. Physical property data are presented on the following storable rocket fuels and oxidizers:

Aerozine-50
Ammonia
Chlorine pentafluoride
Chlorine trifluoride
Hydrazine
Hydrogen peroxide (100% and 90%)
Chlorine trifluoride
Hydrazine
Hydrogen peroxide
Monomethylhydrazine
Nitric acid, red fuming
Nitric acid, white fuming
Nitrogen tetroxide
Pentaborane
Perchloryl fluoride
RP-1
Unsymmetrical dimethylhydrazine

12.2.1.1 AEROZINE-50 (A-50) MIL-P-27402 (USAF). Aerozine-50 is a nominal 50:50 mixture by weight of hydrazine and unsymmetrical dimethylhydrazine (UDMH). It is a clear, colorless, hygroscopic (absorbs moisture readily) liquid, with a characteristic ammoniacal odor. When exposed to the air, a distinct fishy odor is evident in addition to the ammonia odor, probably due to air oxidation of UDMH. To prevent degradation of performance due to moisture absorption from the air, Aerozine-50 should be stored and handled in closed dry equipment under a blanket of nitrogen. Aerozine-50 is insensitive to mechanical shock but is flammable in both liquid and vapor states. At room temperature the vapor over Aerozine-50 is greater than 90 percent UDMH. Aerozine-50 is considered to be a hazardous propellant due to its toxicity and flammability.

Table 12.2.1.1. Properties of Aerozine-50 (A-50, 50/50, UDMH/Hydrazine), $(CH_3)_2N_2H_2/N_2H_4$

PROPERTY	VALUES	REF.
Molecular Weight	41.805	81-11
Boiling Point, °F	158	81-11
Freezing Point, °F	22	81-11
Critical Temperature, °F	633	81-11
Critical Pressure, psia	1751	81-11
Density, lb_m/ft^3	1 atm: 20°F [illegible]; 60°F 56.6; 77°F* 56.1; 100°F 55.4; 160°F 53.4 1000 psia: 20°F -; 60°F 57.0; 77°F* -; 100°F 55.7; 160°F 53.9	81-4 81-11*
Vapor Pressure, psia	20°F 0.60; 60°F 1.70; 77°F* 2.68; 100°F 4.60; 120°F 7.00; 160°F 15.0	81-4 81-11*
Heat of Vaporization, Btu/lb_m	425.8 (at NBP)	81-11
Heat of Fusion, Btu/lb_m		
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	1 atm: 20°F 1.505 / 10.11; 60°F 0.944 / 6.35; (77°F 0.809 / 5.43); 100°F 0.665 / 4.47; 160°F 0.453 / 3.04 1000 psia: 20°F - / -; 60°F 0.949 / 4.50; 100°F 0.669 / 4.50; 160°F 0.457 / 3.07	81-4 81-11*
Specific Heat, Btu/lb_m °F	0.707 + 0.00026 (F) | 0.732 at 77°F	81-11
Enthalpy, Btu/lb_m	342 (@ 77°F)	81-11
Surface Tension, lb_f/ft	1.99×10^{-3} (@ 77°F)	81-11
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$K = 0.[illegible]71 - 6.45 \times 10^{-5} (F) - 1.25 \times 10^{-7} (F)^2$ | 0.151 at 200°F	81-11
Electrical Conductivity, mho/cm	$[illegible].67 \times 10^{-3}$ @ 78°F | CAUTION: Value at left may be incorrect. A. E. Sherburne of Trans-Sonice, Inc., Lexington, Mass. suggests a value of 5×10^{-6} to 10^{-6} mho/cm	81-11
Bulk Modulus, psi	$\dfrac{1}{2.324 \times 10^{-6} + 6.922 \times 10^{-9} (F) + 1.047 \times 10^{-11} (F)^2 + 4.310 \times 10^{-14} (F)^3}$	81-11
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	5276 (@ 77°F)	81-11

12.2.1.2 AMMONIA (NH_3) JAN-A-182. Ammonia is colorless in both gas and liquid states and has a strong, irritating characteristic odor. It is toxic and will form flammable and explosive mixtures with air. Ammonia is insensitive to shock and is thermally stable up to 950°F.

Table 12.2.1.2. Properties of Ammonia, NH_3

PROPERTY	VALUES	REF.
Molecular Weight	17.036	81-11
Boiling Point, °F	-28.03	81-11
Freezing Point, °F	-107.95	81-11
Critical Temperature, °F	271	331-1
Critical Pressure, psia	1634.2	81-11
Density, lb_m/ft^3	68°F / 37.6; 160°F / 32.0; 42.5696 (liquid at NBP)*	331-1 81-11*
Vapor Pressure, psia	68°F / 12[illegible]; 77°F* / 145.45; 160°F / 515	331-1 81-11*
Heat of Vaporization, Btu/lb_m	588.16 (at NBP)	81-11
Heat of Fusion, Btu/lb_m	142.75 (at MP of -107.95°F)	81-11
Viscosity, Centipoise Viscosity, lb_m/sec ft	(-28.03°F* / 0.2527 / 1.698×10^{-4}); -28.3°F / 0.266 / 17.9×10^{-5}; 41°F / 0.1618 / 10.83×10^{-5}; 59°F / 0.1479 / 9.88×10^{-5}; 77°F / 0.1350 / 9.07×10^{-5}	486-1 81-11*
Specific Heat, Btu/lb_m °F	160°F / 1.055; -28°F / 1.066; (-28.03°F* / 1.066)	34-19 81-11*
Enthalpy, Btu/lb_m	-40°F / 0; 77°F / 128.5	81-11
Surface Tension, lb_f/ft	-68.8°F / 0.002683; 60.0°F / 0.00155; 138.16°F / 0.000887; 0.00233 (at NBP)*	152-7 81-11*
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$K = 9.9144 + 8.6230 \times 10^{-8}(R) - 2.4353 \times 10^{-10}(R)^2$ 0.912 (at NBP)	81-11
Electrical Conductivity, mho/cm	0.13×10^{-6} (at -110.2°F)	486-1
Bulk Modulus, psi		
Expansivity, $\frac{\Delta V}{V}$ per °F	68°F / 1.3×10^{-3}; 122°F / 1.74×10^{-3}	492-1
Velocity of Sound, ft/sec	615 psia NBP* / 5679; 65°F / 4570; 135°F / 3800; 172°F / 3020	152-7 81-11*

12.2.1.3a CHLORINE PENTAFLUORIDE (Compound A). Chlorine pentafluoride is a halogen fluoride bearing many similarities to the more familiar chlorine trifluoride. It is insensitive to mechanical shock non-flammable in air, and exhibits excellent thermal stability over its entire liquid range. Chlorine pentafluoride is white in the solid state, water-white in the liquid state, and colorless in the gaseous state. Its odor has been described as both sweet and pungent, similar to chlorine, fluorine, or mustard. Chlorine pentafluoride is an extremely hazardous propellant because of its toxicity and reactivity. It reacts with the vast majority of organic and inorganic compounds (including water) and under proper conditions, with most common metals.

Table 12.2.1.3a. Properties of Chlorine Pentafluoride, (CPF) ClF_5

PROPERTY	VALUES	REF.
Molecular Weight	130.443	35-19
Boiling Point, °F	7.3	35-19
Freezing Point, °F	-153.4 ± 7.2	35-19
Critical Temperature, °F	289.4	35-19
Critical Pressure, psia	771	35-19
Density, lb_m/ft^3	$\rho = 221.8 - 48.42 \times 10^{-2}R + 87.96 \times 10^{-5} R^2 - 67.55 \times 10^{-8} R^3$	35-19
Vapor Pressure, psia	$\log P = 5.7701 - 2154.6/R$	35-19
Heat of Vaporization, Btu/lb_m	76.04 at NBP	35-19
Heat of Fusion, Btu/lb_m		
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	$\log \mu_{(cp)} = -1.62875 + 335.636/K$ $\log \mu_{(lb_m/sec\text{-}ft)} = -4.80138 + 604.145/R$	35-19
Specific Heat Btu/lb_m °F		
Enthalpy, Btu/lb_m	-19.89 (at 8°F)	35-19
Surface Tension, lb_f/ft	$\gamma = 3.9708 \times 10^{-3} - 0.5506 \times 10^{-5}R$	35-19
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	0.111	35-19
Electrical Conductivity, mho/cm	0.45×10^{-9} at 1.4°F	35-19
Bulk Modulus, psi	$\dfrac{1}{6.4065\times10^{-6} + 4.0065\times10^{-8}(F) + 1.4103\times10^{-10}(F)^2 + 9.0915\times10^{-12}(F)^3}$	35-19
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	$c = 5758 - 7.426R + 6.011 \times 10^{-4}R^2$	35-19

12.2.1.3b CHLORINE TRIFLUORIDE (CTF). Chlorine trifluoride is a halogen fluoride similar in reactivity to elemental fluorine. It is insensitive to mechanical shock, non-flammable in air, and exhibits excellent thermal stability at ambient temperatures. The propellant is a very pale, greenish-yellow color in the liquid state, and nearly colorless in the gaseous state. Its odor has been described as both sweet and pungent, similar to chlorine or mustard. Chlorine trifluoride is an extremely hazardous propellant due to its toxicity and reactivity, its reactivity being surpassed only by liquid fluorine.

Table 12.2.1.3b. Properties of Chlorine Trifluoride, (CTF), ClF_3

PROPERTY	VALUES	REF.
Molecular Weight	92.45	35-19
Boiling Point, °F	53.2	35-19
Freezing Point, °F	-105.4	35-19
Critical Temperature, °F	355.3	35-19
Critical Pressure, psia	961	35-19
Density, lb_m/ft^3	$\rho = 121.360 - 1.226 \times 10^{-1}(F) + 2.127 \times 10^{-4}(F)^2 - 8.850 \times 10^{-7}(F)^3$; 115 @ NBP	35-19
Vapor Pressure, psia	$\text{Log}_{10} P = 5.65350 - \frac{1974.451}{(F + 386.95)}$; 15 @ NBP	35-19
Heat of Vaporization, Btu/lb_m	128.1 @ 53.2 °F	35-19
Heat of Fusion, Btu/lb_m	35.4 @ F.P.	35-19
Viscosity, lb_m/sec ft	$\text{Log}\,\mu\ (lb/sec\text{-}ft) = -5.00291 + \frac{774.931}{(R)}$	35-19
Specific Heat, Btu/lb_m °R	$C_s = 0.4673 - 1.204 \times 10^{-3}(R) + 2.543 \times 10^{-6}(R)^2 - 1.581 \times 10^{-9}(R)^3$; 0.304 @ NBP	35-19
Enthalpy, Btu/lb_m	-177.6 @ -105.4 °F	35-19
Surface Tension, lb_f/ft	$\sigma = 2.03 \times 10^{-3} - 6.099 \times 10^{-6}$ (F); 1.699 @ NBP	35-19
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-100 °F: 0.148; -50 °F: 0.143; 0 °F: 0.140; 50 °F: 0.137; (77 °F*: 0.1307)	34-19 35-19*
Electrical Conductivity, mho/cm	1.4×10^{-8} @ 32 °F	35-19
Bulk Modulus, psi	$\frac{1}{3.5921 \times 10^{-6} + 1.8837 \times 10^{-8}(F) + 5.7058 \times 10^{-11}(F)^2 + 1.3434 \times 10^{-13}(F)^3}$	35-19
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	C = 6401.8 - 6.8348 (R)	35-19

12.2.1.4 HYDRAZINE MIL-P-26536A (USAF). Hydrazine is a toxic, flammable, caustic liquid and a strong reducing agent. It is a clear, water-white, hygroscopic liquid with an odor similar to ammonia, though less strong. Several of the physical properties of hydrazine are similar to water. Hydrazine is insensitive to mechanical shock. It is considered a hazardous propellant due to its toxicity, reactivity, and flammability. Due to its hygroscopic nature and the fact that it readily forms flammable mixtures in air, nitrogen blanketing of hydrazine containers is required. When exposed to air, hydrazine produces white vapors.

Table 12.2.1.4. Properties of Hydrazine, N_2H_4

PROPERTY	VALUES		REF.
Molecular Weight	32.04		81-11
Boiling Point, °F	236.3		81-11
Freezing Point, °F	35.6		81-11
Critical Temperature, °F	716		81-11
Critical Pressure, psia	2131		81-11
Density, lb_m/ft^3	$\rho = 76.8353 + .021735(R) - 8.7254 \times 10^{-6}(R)^2$	62.93 @ 68°F	35-22
Vapor Pressure, psia	$Log_{10}P = 7.07299 - \frac{4100.29}{R}$	0.267 @ 77°F	35-22
Heat of Vaporization, Btu/lb_m	540 (at 236°F) 602 (at 77°F)*		81-11 331-1*
Heat of Fusion, Btu/lb_m	170 (at 35.6°F)		81-11
Viscosity, Centipoise Viscosity, lb_m/sec ft	$Log_{10}\mu = -1.1280 + \frac{20.176}{R} + \frac{91460}{R^2}$ $Log_{10}\mu = -4.3006 + \frac{36.215}{R} + \frac{296300}{R^2}$		35-21
Specific Heat, Btu/lb_m °F	$C_p = 0.7220 + 1.357 \times 10^{-4}(F) + 6.491 \times 10^{-7}(F)^2$	0.737 @ 77°F	35-21
Enthalpy, Btu/lb_m	$\frac{0°F}{116}$ $\frac{77°F}{158.2}$ $\frac{260.6°F}{337.4}$ $\frac{620.6°F}{426.0}$		287-4
Surface Tension, lb_f/ft	0.004551 @ 77°F 0.004270 @ 95°F		35-21
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$K = 0.2793 + 1.134 \times 10^{-4}(F) - 8.341 \times 10^{-7}(F)^2$		35-21
Electrical Conductivity, mho/cm	2.3 to 2.8×10^{-6} (at 77°F)		81-11
Bulk Modulus, psi	$\frac{1}{1.297 \times 10^{-6} + 3.530 \times 10^{-9}(F) + 5.73 \times 10^{-12}(F)^2}$		35-21
Expansivity, $\frac{\Delta V}{V}$ per °F			
Velocity of Sound, ft/sec	6840 (at 77°F)		81-11

ISSUED: FEBRUARY 1970
SUPERSEDES: MARCH 1967

12.2.1.5a HYDROGEN PEROXIDE MIL-H-16005C. Hydrogen peroxide is a slightly acidic, clear, colorless, odorless liquid. It is miscible with water in all proportions. Hydrogen peroxide is nonflammable and insensitive to mechanical shock under normal conditions. It is stable when pure but will decompose if it becomes contaminated. Heat accelerates decomposition, which may reach explosive violence at 300°F. The decomposition products are oxygen and water vapor. Hydrogen peroxide is nontoxic.

See 12.2.1.5b for 90% hydrogen peroxide.

Table 12.2.1.5a. Properties of 100 Percent Hydrogen Peroxide, H_2O_2

PROPERTY	VALUES			REF.
Molecular Weight	34.02			331-1
Boiling Point, °F	302			331-1
Freezing Point, °F	31			331-1
Critical Temperature, °F	855			331-1
Critical Pressure, psia	3145			331-1
Density, lb_m/ft^3	68°F: 90.2	160°F: 84.7		331-1
Vapor Pressure, psia	68°F: 0.1	160°F: 0.61		331-1
Heat of Vaporization, Btu/lb_m	653 (at 77°F)			331-1
Heat of Fusion, Btu/lb_m	158 (at F.P.)			331-1
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	68°F: 1.26 8.47×10^{-4}	160°F: 0.66 4.44×10^{-4}		331-1
Specific Heat, $Btu/lb_m\ °F$	50°F: 0.629	150°F: 0.659	300°F: 0.705	34-19
Enthalpy, Btu/lb_m	148.8			486-1
Surface Tension, lb_f/ft	32.4°F: 5.395×10^{-3}	51.8°F: 5.311×10^{-3}	64.8°F: 5.204×10^{-3}	486-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	50°F: 0.279	150°F: 0.275	300°F: 0.265	34-19
Electrical Conductivity, mho/cm	4.0×10^{-7} (at 77°F)			486-1
Bulk Modulus, psi				
Expansivity, $\frac{\Delta V}{V}$ per °F				
Velocity of Sound, ft/sec				

12.2.1.5b 90% HYDROGEN PEROXIDE. This common aqueous solution of 90% H_2O_2/10% H_2O by weight is presented because it better represents propellant grade hydrogen peroxide than does the 100% H_2O_2 described in Detailed Topic 12.2.1.5a. Aqueous hydrogen peroxide solutions are more dense, slightly more viscous, and have higher boiling and lower freezing points than water. Because of their strong oxidizing nature and the liberation of oxygen and heat during their decomposition, propellant-grade solutions can initiate the vigorous combustion of many common organic materials such as clothing, wood, wastes, etc.

Table 12.2.1.5b. Properties of 90 Percent Hydrogen Peroxide, H_2O_2/H_2O

PROPERTY	VALUES	REF.
Molecular Weight	31.24	35-18
Boiling Point, °F	286.2	35-18
Freezing Point, °F	11.3	35-18
Critical Temperature, °F	833	35-18
Critical Pressure, psia	3556	35-18
Density, lb_m/ft^3	$\rho_{(lb/cu\ ft)} = 66.166 + 1.577 \times 10^{-1} W + 1.112 \times 10^{-3} W^2 - 2.31 \times 10^{-2} T(F) - 4.7 \times 10^{-6} T(F)^2 - 1.38 \times 10^{-4} WT(F)$ $W = \%\ H_2O_2$	35-18
Vapor Pressure, psia	$\log P = 5.95936 - \frac{2891.65}{(R)} - \frac{510,504}{(R)^2}$	35-18
Heat of Vaporization, Btu/lb_m	700.3	35-18
Heat of Fusion, Btu/lb_m	148	35-18
Viscosity, micropoise	$\mu_{(micropoise)} = 134 + 0.35 \left[T_{(C)} - 100\right] - 14Y$ Y = mole fraction H_2O_2 in vapor	35-18
Specific Heat, $Btu/lb_m\ °F$	0.62	35-18
Enthalpy, Btu/lb_m	15.34 at 100°F	35-18
Surface Tension, lb_f/ft	5.42 at 68°F	35-18
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	0.34	35-18
Electrical Conductivity, mho/cm	11.5 at 77°F	35-18
Bulk Modulus, psi		
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	5745	35-18

12.2.1.6 MONOMETHYLHYDRAZINE (MMH) MIL-P-27404 (AF). Monomethylhydrazine is a clear, water-white, hygroscopic, toxid liquid. It has a sharp ammoniacal or fishy odor detectable in concentrations of 1 to 3 ppm. Liquid MMH is not sensitive to impact and is more stable than hydrazine under conditions of mild heating, however it is similar to hydrazine in sensitivity to catalytic oxidation. The flammability characteristics of MMH with air are close to those of hydrazine and UDMH; consequently, it should be maintained under a nitrogen blanket at all times.

Table 12.2.1.6. Properties of Monomethylhydrazine (MMH), $CH_3NH\ NH_2$

PROPERTY	VALUES		REF.
Molecular Weight	46.072		35-20
Boiling Point, °F	190.3		35-20
Freezing Point °F	-62.27		35-20
Critical Temperature, °F	593.7		35-20
Critical Pressure, psia	1195		35-20
Density, lb_m/ft^3	54.60 (@ 70°F)	$\rho = 56.86 - 3.21 \times 10^{-2}\ (F)$	35-20
Vapor Pressure, psia	0.72 (@ 70°F)	$\text{Log } P = 5.5775 \quad \frac{-2356.5}{°F + 344}$	35-20
Heat of Vaporization, Btu/lb_m	376.9 at NBP		35-20
Heat of Fusion, Btu/lb_m	97.30 at MP		35-20
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	$\text{Log}\ \mu_{(CP)} = -8.1869 + \frac{6297.3}{K} - \frac{1.7969 \times 10^6}{K^2} + \frac{1.900 \times 10^8}{K^3}$ $\text{Log}\ \mu = -11.3266 + \frac{1.1284 \times 10^4}{R} - \frac{5.7963 \times 10^6}{R^2} + \frac{1.10399 \times 10^9}{R^3}$	0.870 at 68°F 0.000555 at 70°F	35-20
Specific Heat, $Btu/lb_m\ °F$	0.698 (@ 70°F) $C_p = 0.6859 + 1.36 \times 10^{-4}(F) + 8.09 \times 10^{-7}(F)^2 - 2.3 \times 10^{-9}(F)^3$		35-20
Enthalpy, Btu/lb_m	370 (@ 70°F)		35-20
Surface Tension, lb_f/ft	2.345 (@ 70°F) $\sigma = 2.607 \times 10^{-3} - 3.76 \times 10^{-6}(F)$		35-20
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	0.1434 (@ 70°F) $k = 0.146 - 1.63 \times 10^{-5}(F) - 3.39 \times 10^{-7}(F)^2$		35-20
Electrical Conductivity, mho/cm	4.1×10^{-5} (@ 73.4°F)		35-20
Bulk Modulus, psi	3.07×10^{-7} (@ 70°F)	$\frac{1}{\text{Bulk Modulus}} = 2.572 \times 10^{-6} +$ [illegible]$.083 \times 10^{-[illegible]}(F)$ $+ 1.266 \times 10^{-11}(F)^2 + 6.17 \times 10^{-[illegible]}(F)^3$	35-20
Expansivity, $\frac{\Delta V}{V}$ per °F	5.90×10^{-4}		35-20
Velocity of Sound, ft/sec	5125 (@ 70°F) $C = 5629.5 - 7.113\ (F)$		35-20

12.2.1.7 NITRIC ACID, RED FUMING (RFNA) MIL-P-7254 (82.4 to 85.4 percent by weight HNO_3, 14 percent by weight NO_2, and 1.5 to 2.5 percent by weight H_2O). Red fuming nitric acid is a highly corrosive, toxic, nonflammable liquid mixture of nitric acid (HNO_3) and dissolved nitrogen dioxide (NO_2). Its color is light orange to orange-red, depending upon the amount of dissolved NO_2, and it has an acrid odor. Addition of 0.7 percent by weight hydrogen fluoride (HF) inhibits corrosion of container materials by RFNA. With the HF additive it is called inhibited red fuming nitric acid (IRFNA).

Table 12.2.1.7. Properties of Red Fuming Nitric Acid (RFNA)

PROPERTY	VALUES					REF.
Molecular Weight	59.7					34-19
Boiling Point, °F	148					34-19
Freezing Point, °F	-56					34-19
Critical Temperature, °F	520					34-19
Critical Pressure, psia	1286					34-19
Density, lb_m/ft^3	-50°F: 160	0°F: 102	50°F: 100	140°F: 95.7	148°F: 91.2	34-19
Vapor Pressure psia	0°F: 0.2	50°F: 1.2	100°F: 5.0	148°F: 15.0		34-19
Heat of Vaporization, Btu/lb_m	247					34-19
Heat of Fusion, Btu/lb_m						
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	-50°F: 8.93 60.0×10^{-4}	0°F: 3.28 22.0×10^{-4}	50°F: 1.56 10.48×10^{-4}	100°F: 0.97 6.52×10^{-4}	148°F: 0.715 4.80×10^{-4}	34-19
Specific Heat, $Btu/lb_m\ °F$	-50°F: 0.410	0°F: 0.414	50°F: 0.417	100°F: 0.422	148°F: 0.425	34-19
Enthalpy, Btu/lb_m						
Surface Tension lb_f/ft						
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-50°F: 0.182	0°F: 0.178	50°F: 0.172	100°F: 0.165	148°F: 0.158	34-19
Electrical Conductivity, mho/cm	32°F: 11.46×10^{-2}		86°F: 15.16×10^{-2}			486-1
Bulk Modulus, psi						
Expansivity, $\frac{\Delta V}{V}$ per °F						
Velocity of Sound, ft/sec	4525					34-19

12.2.1.8 NITRIC ACID, WHITE FUMING (WFNA) MIL-P-7254 (97.5 percent by weight HNO_3, 0.5 percent by weight NO_2, and 2.0 percent by weight H_2O). White fuming nitric acid is a highly corrosive, toxic, nonflammable, oxidizer with a color varying from straw to light green.

Table 12.2.1.8. Properties of White Fuming Nitric Acid (WFNA)

PROPERTY	VALUES	REF.
Molecular Weight		
Boiling Point, °F	191.0 (95.0% HNO_3 + 5.0% H_2O)	486-1
Freezing Point, °F	-43.7 (99.65% HNO_3 + 0.35% H_2O)	486-1
Critical Temperature, °F		
Critical Pressure, psia		
Density, lb_m/ft^3	43°F: 96.45; 77°F: 93.16 (98.0% HNO_3 2.0% H_2O)	486-1
Vapor Pressure, psia	32°F: 0.280; 68°F: 0.924 (99.80% HNO_3)	486-1
Heat of Vaporization, Btu/lb_m		
Heat of Fusion, Btu/lb_m		
Viscosity, Centipoise Viscosity, lb_m/sec ft	40°F: 1.101, 7.405×10^{-4}; 70°F: 0.863, 5.80×10^{-4} (99.51% HNO_3)	486-1
Specific Heat, Btu/lb_m °F	0°F: 0.423; 100°F: 0.423; 200°F: 0.424 (95 - 97% HNO_3)	486-1
Enthalpy, Btu/lb_m		
Surface Tension, lb_f/ft	0.002809	287-4
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-25°F: 0.146; 75°F: 0.162; 200°F: 0.183 (99.01% HNO_3)	486-1
Electrical Conductivity, mho/cm	1.51×10^{-2} (98% HNO_3)	486-1
Bulk Modulus, psi		
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	4067 (97.52% HNO_3 + 0.28% N_2O_4 + 2.11% H_2O)	486-1

12.2.1.9 NITROGEN TETROXIDE MIL-P-26539 (USAF). Nitrogen tetroxide, also known as dinitrogen tetroxide and NTO, is actually an equilibrium mixture of nitrogen tetroxide (N_2O_4) and nitrogen dioxide (NO_2); the percentage of NO_2 increases with increasing temperature. In the solid state, N_2O_4 is colorless; in the liquid state the equilibrium mixture is yellow to red-brown, varying with temperature and pressure; in the gaseous state it is red-brown. Nitrogen tetroxide is a highly reactive, toxic oxidizer which is thermally stable and insensitive to all types of mechanical shock and impact. Although nonflammable, it will support combustion, and upon contact with high-energy fuels such as hydrazine, will react hypergolically. It has an irritating, unpleasant, acid-like odor.

"Green" nitrogen tetroxide has been specified for applications such as NASA Apollo propulsion to minimize stress-corrosion cracking of titanium tanks (see NASA TN D-4289). Green N_2O_4 usually contains 0.45 to 0.85 percent nitric oxide (NO) and is identified by a characteristic green color when frozen. A similar color is also obtained if N_2O_4 contaminated with water is frozen.

Table 12.2.1.9. Properties of Nitrogen Tetroxide, (NTO), N_2O_4

PROPERTY	VALUES		REF.
Molecular Weight	92.016		773-1
Boiling Point, °F	70.2		773-1
Freezing Point, °F	11.8		773-1
Critical Temperature, °F	316.8		773-1
Critical Pressure, psia	1468		773-1
Density, lb_m/ft^3	$\rho = 95.26 - 0.06577(F) - 1.10 \times 10^{-4}(F)^2 + 2.29 \times 10^{-4}(PSIA) + 4.94 \times 10^{-6}(PSIA)(F)$	90.0 @ 70°	35-21
Vapor Pressure, psia	$\log 10\ P = 8.11012 - \frac{4197.55}{R} + \frac{273994}{R^2}$	14.0 @ 70°	35-21
Heat of Vaporization, Btu/lb_m	178.2 at 70°F		773-1
Heat of Fusion, Btu/lb_m	68.5		773-1
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	$\mu_{(CP)} = 0.2005 - \frac{72.003}{°R} + \frac{74314.6}{R^2}$ $\mu\ (lb/ft\ sec) = 1.347 \times 10^{-4} - \frac{8.710 \times 10^{-2}}{R} + \frac{161.8}{R^2}$		35-21
Specific Heat, Btu/lb_m °F	$C_p = 0.340 + 7.87 \times 10^{-4}(F) - 6.09 \times 10^{-6}(F)^2 + 3.03 \times 10^{-8}(F)^3$	0.376 @ 70°	35-21
Enthalpy, Btu/lb_m			
Surface Tension, lb_f/ft	0.00185 @ 68°		773-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$K = 8.405 \times 10^{-2} - 6.219 \times 10^{-5}(F) + 2.121 \times 10^{-6}(PSIA) - 4.081 \times 10^{-9}(F)(PSIA) - 6.986 \times 10^{-7}(F)^2$		35-21
Electrical Conductivity, mho/cm	3.1×10^{-13} @ 77°F		773-1
Bulk Modulus, psi	$\frac{1}{3.394 \times 10^{-6} + 1.260 \times 10^{-8}(F) + 1.486 \times 10^{-10}(F)^2 - 6.329 \times 10^{-13}(F)^3}$	1.18×10^5	773-1
Expansivity, $\frac{\Delta V}{V}$ per °F	0.001		773-1
Velocity of Sound, ft/sec	C = 3825 - 8.065 (F)	3260 @ 70°F	35-21

12.2.1.10 PENTABORANE MIL-P-27403. Pentaborane, a boron hydride, is an extremely hazardous, high-energy rocket propellant. It is considered hazardous due to its toxicity, high reactivity, and erratic pyrophoricity (spontaneous flammability in air) and must be stored under a dry, inert gas blanket. In its pure state the propellant is a clear, water-white liquid at normal atmospheric conditions. It has a characteristic pungent odor which has been described as sickeningly sweet, similar to that of garlic, acetylene, or burnt rubber.

Table 12.2.1.10. Properties of Pentaborane, B_5H_9

PROPERTY	VALUES					REF.
Molecular Weight	63.17					35-7
Boiling Point, °F	140					35-7
Freezing Point, °F	-53					35-7
Critical Temperature, °F	435					35-7
Critical Pressure, psia	557					35-7
Density, lb_m/ft^3	39.14 at 68°F					35-7
Vapor Pressure, psia	-50°F / 0.07	-20°F / 0.225	20°F / 0.89	(68°F / 0.89)*	76°F / 4.0	331-1 35-7*
Heat of Vaporization, Btu/lb_m	219					35-7
Heat of Fusion, Btu/lb_m	92					35-7
Viscosity, Centipoise Viscosity, lb_m/sec ft	-50°F / 0.82 5.51×10^{-4}	0°F / 0.49 3.30×10^{-4}	100°F / 0.27 1.81×10^{-4}	140°F / 0.22 1.48×10^{-4}		34-19
Specific Heat, Btu/lb_m °F	-50°F / 0.4[illegible]	0°F / 0.50	100°F / 0.59	140°F / 0.64		34-19
Enthalpy, Btu/lb_m						
Surface Tension, lb_f/ft	1.465×10^{-3} at 68°F					35-7
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-50°F / 0.089	0°F / 0.096	100°F / 0.111	140°F / 0.118		34-19
Electrical Conductivity, mho/cm						
Bulk Modulus, psi						
Expansivity, $\frac{\Delta V}{V}$ per °F						
Velocity of Sound, ft/sec						

12.2.1.11 PERCHLORYL FLUORIDE. Perchloryl fluoride (ClO_3F) is a colorless gas under normal atmospheric conditions; the liquid is water-white. The propellant is relatively stable at temperatures up to 850°F. Although not shock-sensitive itself, in combination with porous organic or inorganic materials it can produce a potentially shock-sensitive mixture. It is a moderately toxic, strong oxidizing agent, and has a mild, sweetish odor detectable at a concentration of approximately 10 ppm in air.

Table 12.2.1.11. Properties of Perchloryl Fluoride, ClO_3F

PROPERTY	VALUES	REF.
Molecular Weight	102.5	35-19
Boiling Point, °F	-52.2	35-19
Freezing Point, °F	-234	35-19
Critical Temperature, °F	203.4	35-19
Critical Pressure, psia	778.9	35-19
Density, lb_m/ft^3	89.2 at 68°F; $\rho_{(gm/cc)} = 2.226 - 1.603 \times 10^{-3}$	35-19
Vapor Pressure, psia	$\frac{32°F}{86.7}$ $\frac{77°F}{176.1}$ $\frac{122°F}{322}$ $\left(\frac{160°F}{490}\right)^*$	35-19 (331-1)*
Heat of Vaporization, Btu/lb_m	81.1 at -52.2°F	35-19
Heat of Fusion, Btu/lb_m	16.09	35-19
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	$\log_{10}\mu_{(cp)} = \frac{299}{K} - 1.755$	35-19
Specific Heat, Btu/lb_m °F	$\frac{14°F}{0.432}$ $\frac{50°F}{0.228}$	35-19
Enthalpy, Btu/lb_m		
Surface Tension, lb_f/ft	$\frac{68°F}{1.46 \times 10^{-3}}$ $\frac{104°F}{1.65 \times 10^{-3}}$	35-19
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$\frac{-230°F}{0.096}$ $\frac{-150°F}{0.092}$ $\frac{-50°F}{0.087}$ $\left(\frac{75°F}{541.3}\right)^*$	331-1 (35-19)*
Electrical Conductivity, mho/cm		
Bulk Modulus, psi		
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec		

12.2.1.12 RP-1 MIL-R-25576B. RP-1 is a hydrocarbon fuel which can be described as a high-boiling kerosene fraction. The fuel is a clear liquid ranging in color from water-white to a very pale yellow. RP-1 reacts only under strong oxidizing conditions or at extremes of pressure and temperature. The fuel is flammable and its vapors form explosive mixtures with air. It is chemically stable and insensitive to mechanical shock.

Table 12.2.1.12. Properties of RP-1 (Rocket Propellant -1)
(Reference – Military Specification MIL-F-25576B)

PROPERTY	VALUES					REF.
Molecular Weight	172					34-19
Boiling Point, °F	422					34-19
Freezing Point, °F	-50 to -100					34-19
Critical Temperature, °F	758					34-19
Critical Pressure, psia	315					34-19
Density, lb_m/ft^3	0°F: 50.9	100°F: 49.3	300°F: 44.4			34-19
Vapor Pressure, psia	100°F: 0.03	200°F: 0.02	300°F: 2.0	400°F: 10.0	600°F: 100	34-19
Heat of Vaporization, Btu/lb_m	125					34-19
Heat of Fusion, Btu/lb_m						
Viscosity, Centipoise Viscosity, lb_m/sec ft	-30°F: 13.45 90.4×10^{-4}	0°F: 4.77 32.0×10^{-4}	100°F: 1.3 8.73×10^{-4}	300°F: 0.39 2.62×10^{-4}		34-19
Specific Heat, Btu/lb_m °F	-50°F: 0.41	0°F: 0.45	100°F: 0.49	300°F: 0.60	600°F: 0.77	34-19
Enthalpy, Btu/lb_m						
Surface Tension, lb_f/ft						
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-50°F: 0.087	0°F: 0.081	100°F: 0.078	300°F: 0.073	600°F: 0.066	34-19
Electrical Conductivity, mho/cm	Between 10^{-19} and 10^{-12}					287-4
Bulk Modulus, psi	179,000					34-10
Expansivity, $\frac{\Delta V}{V}$ per °F						
Velocity of Sound, ft/sec	2300					34-10

12.2.1.13 UNSYMMETRICAL DIMETHYLHYDRAZINE (UDMH) MIL-D-25604-B. UDMH is a clear, colorless, hygroscopic liquid with a rather sharp ammoniacal or fishy odor characteristic of amines; its vapors are detectable in concentrations of 5 ppm or less. UDMH is moderately toxic and shock insensitive. It exhibits excellent thermal stability and resistance to catalytic breakdown. Due to an extremely wide flammability range in air and the possibility that explosive vapor/air mixtures may be found above the liquid, UDMH should not be exposed to open air. Instead it should be stored in a closed container under a nitrogen blanket. It absorbs both oxygen and carbon dioxide.

Table 12.2.1.13. Properties of Unsymmetrical Dimethylhydrazine (UDMH), $(CH_3)_2N_2H_2$

PROPERTY	VALUES	REF.
Molecular Weight	60.102	81-11
Boiling Point, °F	144.18	81-11
Freezing Point, °F	-70.97	81-11
Critical Temperature, °F	482	81-11
Critical Pressure, psia	867	81-11
Density, lb_m/ft^3	46.55 at 77°F $\rho = 66.1991 - 2.6881 \times 10^{-4}(R) - 9.3735 \times 10^{-6}(R)^2$	3 35-21
Vapor Pressure, psia	3.[illegible]32 at 77°F $\log P = 5.02218 - \frac{1576.61}{R} - \frac{453602}{R^2}$	35-21
Heat of Vaporization, Btu/lb_m	250.55 (at 77°F)	81-11
Heat of Fusion, Btu/lb_m	72.1 (at -71°F)	81-11
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	-65°F: 5.114, $34.[illegible] \times 10^{-4}$ 32°F: 0.783, 5.26×10^{-4} 77°F*: 0.492, 3.309×10^{-4} 100°F: 0.413, 2.78×10^{-4} 140°F: 0.316, 2.12×10^{-4}	34-14 81-11*
Specific Heat, $Btu/lb_m\ °F$	0.6708 at 77°F $C_p = 0.630 + 0.00032\ (F)$	35-21
Enthalpy, Btu/lb_m		
Surface Tension, lb_f/ft	1.651×10^{-3} at 77°F $\gamma = 1.954 \times 10^{-3} - 4.399 \times 10^{-6}(F) + 4.390 \times 10^{-9}(F)^2$	35-21
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	0.0905 at 77°F $K = 0.1014 - 1.368 \times 10^{-4}\ (F)$	35-21
Electrical Conductivity, mho/cm		
Bulk Modulus, psi	$\frac{1}{3.990\times10^{-6} + 1.669\times10^{-8}(F) + 5.145\times10^{-11}(F)^2 + 2.49\times10^{-14}(F)^3}$	35-21
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	4,078 at 77°F $C = 8471.4 - 8.162(R)$	35-21

12.2.2 Cryogenic Fluids

A cryogenic fluid is generally accepted as a liquid whose normal boiling point is below -238°F (-150°C). Physical property data are presented for the following cryogenic fluids used in rocket propellant systems:

- Liquid fluorine
- Liquid helium
- Liquid hydrogen
- Liquid nitrogen
- Liquid oxygen
- Oxygen difluoride
- Diborane.

12.2.2.1 LIQUID FLUORINE. Fluorine is the most powerful chemical oxidizing agent known. It reacts with practically all organic and inorganic substances, a few exceptions being inert gases, some metal fluorides, and a few uncontaminated fluorinated organic compounds. Fluorine exhibits excellent thermal stability and resistance to catalytic breakdown. It is a dense liquid (50 percent heavier than water) at atmospheric pressure in the temperature range -306°F to -363°F. Fluorine is a highly toxic gas with a pungent halogen odor (similar to chlorine bleach), detectable in concentrations of less than 1/10 ppm. Liquid fluorine has a clear yellow color, while fluorine gas at room temperature is a pale greenish-yellow.

Table 12.2.2.1. Properties of Liquid Fluorine (LF_2), F_2

PROPERTY	VALUES	REF.
Molecular Weight	38.0	34-35
Boiling Point, °F	-306.6	34-35
Freezing Point, °F	-363.3	34-35
Critical Temperature, °F	-200.2	34-35
Critical Pressure, psia	808.3	486-1
Density, lb_m/ft^3	94.2 at NBP | $\rho = 110.05 - 7.634 \times 10^{-2}R - 5.6d0 \times 10^{-4}(R)^2$	34-35
Vapor Pressure, psia	0.0456 at -362F, 1.785 at -334F, 14.75 at -306F, 38.1 at -290F, 379.5 at -227.5F $Log_{10} P_{psia} = 5.374 - 643.06/R - 1.8 \times 10^{15}/(R)^8$	34-35
Heat of Vaporization, Btu/lb_m	74.08	34-35
Heat of Fusion, Btu/lb_m	5.77	34-35
Viscosity, Centipoise Viscosity, lb_m/sec ft	$\frac{-335}{.414}$ .00028 $\frac{-319}{.299}$ .00020 $\frac{-309.8}{.257}$ .00017	34-35
Specific Heat, Btu/lb_m °F	$\frac{-310°F}{0.366}$ $\frac{-335°F}{0.357}$ $\frac{-360°F}{0.361}$	81-4
Enthalpy, Btu/lb_m	$\frac{-306.6°F}{-44.376}$ $\frac{-280°F}{-34.18}$ $\frac{-208°F}{-5.97}$ (Sat. Liquid)	106-10
Surface Tension, lb_f/ft	$\frac{-357°F}{100.1 \times 10^{-5}}$ $\frac{-350°F}{97.0 \times 10^{-5}}$ $\frac{-332°F}{83.6 \times 10^{-5}}$ $\frac{-313°F}{71.3 \times 10^{-5}}$	485-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$\frac{-310°F}{0.16}$ $\frac{-360°F}{0.19}$	81-4
Electrical Conductivity, mho/cm		
Bulk Modulus, psi		
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	(Approximately 270 m/sec at 273°K, 1 atm)	27-50

12.2.2.2 LIQUID HELIUM. Liquid helium is an extremely light fluid weighing only 1.04 pounds per gallon. It exists in two stable isotopic forms, He^3 and He^4. Liquid helium is a colorless, odorless fluid having the lowest boiling point of all elements (-452 F). It is nontoxic, nonflammable, and chemically inert. Although helium is relatively scarce, its liquid properties have been more extensively investigated than those of any other fluid with the possible exception of water, due primarily to the unusual properties of normal helium (He^4) below -456.1 F (2.18 K). Below this temperature, known as the lambda transformation temperature, the fluid is designated as helium II. Super fluidity is one of the unusual phenomena exhibited by helium II. The viscosity of helium, which at its normal boiling point is 79 times lower than water, approaches zero below the lambda transformation temperature, making it an almost frictionless fluid. The isotope He^3 does not have a lambda point and at all times behaves as a normal fluid.

The primary source of helium is natural gas, although at a considerably higher cost it is possible to separate helium from air. Although nontoxic, like nitrogen, concentrations of helium gas in confined spaces should be avoided, since replacement of oxygen in the atmosphere can lead to asphyxiation.

Helium is unlike other fluids in that it has no triple point (no condition of temperature and pressure where solid, liquid, and vapor can coexist) and is the only substance which remains liquid down to absolute zero. A minimum of approximately 23 atmospheres of pressure is required to obtain solid helium.

Table 12.2.2.2. Properties of Liquid Helium (LHe), He

PROPERTY	VALUES	REF
Molecular Weight	4.0	476-1
Boiling Point, °F	-452.1	476-1
Freezing Point, °F	does not freeze at 1 atm pressure	
Critical Temperature, °F	-450.23	476-1
Critical Pressure, psia	33.34	476-1
Density, lb_m/ft^3	Sat: -457°F 9.1; -452°F 7.8; -442°F -; -433°F - 4 atm.: -457°F -; -452°F 8.73; -442°F 13.6; -433°F 8.1 10 atm.: -457°F -; -452°F 9.5; -442°F 18.6; -433°F 22.4	82-15
Vapor Pressure, psia	-456°F: 0.87; -452°F: 17.8; -450°F: 33.8	476-1
Heat of Vaporization, Btu/lb_m	9.0 (at B.P.)	476-1
Heat of Fusion, Btu/lb_m	1.8 (at 4.1 [illegible])	476-1
Viscosity, Centipoise Viscosity, lb_m/sec ft	-456°F: 8.4×10^{-4}; 5.65×10^{-7} -455°F: 2.51×10^{-3}; 1.685×10^{-6} -453°F: 2.89×10^{-3}; 1.94×10^{-6}	476-1
Specific Heat, Btu/lb_m °F	-456°F: 1.[illegible]; -455°F: 0.595; -451°F: 2.75 (at Sat. Pressure)	82-15
Enthalpy, Btu/lb_m (See Figs. 12.2.2.4 b, c)	-456°F: 0.3; -451°F: 0.25 (Sat. Liquid)	155-1
Surface Tension, lb_f/ft	-456°F: 2.10×10^{-5}; -453°F: 7.67×10^{-6}; -451°F: 7.54×10^{-7} (under its own vapor pressure)	476-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-456°F: 0.0107; -452°F: 0.0157	476-1
Electrical Conductivity, mho/cm		
Bulk Modulus, psi		
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	Vapor Press: 590; 5 atm: 790; 10 atm: 935; 20 atm: 1132 (at -452°F)	476-

12.2.2.3 LIQUID HYDROGEN. Liquid hydrogen is a transparent, water-white, odorless, nontoxic fluid having about 1/14 the density of water. If spilled, hydrogen evaporates immediately, diffusing into the air faster than any other propellant. Hydrogen gas reacts in air at a very low energy level. The flammability limits of hydrogen in air range from 4.0 to 75 percent. Hydrogen flames are usually colorless, but often assume a yellow cast due to the presence of impurities in the air. The entrance of air in a hydrogen system presents a hazard due to the formation of solid oxygen and/or solid air, the liquid hydrogen-solid oxygen combination being extremely shock-sensitive. Hydrogen systems must be flushed with an inert gas such as helium or nitrogen to exclude air and oxygen. Normally hydrogen does not present an explosive hazard when it evaporates and mixes with air in an unconfined space.

The hydrogen molecule exists in two basic forms, ortho and para, the form depending on the relative direction of nuclear spins. There is no difference in the chemical properties of the two forms, but there is a slight difference in most of the physical properties due to the difference in nuclear spins. Equilibrium liquid hydrogen is essentially para-hydrogen, the normal boiling (-423°F) equilibrium consisting of 99.79 percent para hydrogen and 0.21 percent ortho-hydrogen. "Normal" hydrogen refers to the equilibrium condition of hydrogen gas at high temperatures (room temperature and above) which is a mixture containing 75 percent ortho hydrogen and 25 percent para-hydrogen. The ortho-to-para conversion is an exothermic process, the heat generated being greater than the heat of vaporization. To prevent high boil-off losses during conversion, liquid hydrogen for military and space uses is specified as having a minimum of 95 percent para content. Conversion is accelerated through the use of a catalyst.

Table 12.2.2.3. Properties of Liquid Hydrogen (LH_2) (Para-Hydrogen), H_2

PROPERTY	VALUES	REF
Molecular Weight	2.016	486-1
Boiling Point, °F	-423.19	486-1
Freezing Point, °F	-434.82	486-1
Critical Temperature, °F	-400.31	14-15
Critical Pressure, psia	187.50	14-15
Density, lb_m/ft^3	Sat / 1000 psia: -430°F 4.65 / 4.97; -423°F 4.41 / 4.81; -410°F 3.70 / 4.47	14-15
Vapor Pressure, psia	-434.6°F: 1.044; -420.2°F: 22.99; -417.3°F: 33.90	486-1
Heat of Vaporization, Btu/lb_m	-432.8°F: 197.1; -423.8°F: 194.9	486-1
Heat of Fusion, Btu/lb_m	252 (at 434.6°F, 1.044 psia)	486-1
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft$	-432.6°F: 0.023, 1.54×10^{-5}; -425.9°F: 0.016, 1.075×10^{-5}; -422.5°F: 0.0136, 9.13×10^{-6}	486-1
Specific Heat, Btu/lb_m °F	-423°F: 2.33; -434°F: 1.67 (at Sat. Pressure)	81-4
Enthalpy, Btu/lb_m (See Figs. 12.2.4d, e)	Press., psia / Temp., °F: 14.7 / -423: -110.27; 1000 / -423: -81.57; 14.7 / -404: -114.17; 1000 / -404: -34.05	522-1
Surface Tension, lb_f/ft	-431°F: 2.036×10^{-4}; -424°F: 1.542×10^{-4}; -417°F: 1.023×10^{-4}	486-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-441°F: 0.0627; -423°F: 0.0681; -406°F: 0.0816	522-1
Electrical Conductivity, mho/cm		
Bulk Modulus, psi	14,500 (at B.P.)	14-15
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	3580 (at B.P.)	14-15

12.2.2.4 LIQUID NITROGEN. Liquid nitrogen is a nontoxic, colorless, odorless fluid. Although nontoxic, it does present a safety hazard: if copious quantities are released in a relatively confined space, the reduction in oxygen concentration can present potential asphyxiation to personnel in the area.

Table 12.2.2.4. Properties of Liquid Nitrogen (LN_2), N_2

PROPERTY	VALUES	REF.
Molecular Weight	28.02	81-4
Boiling Point, °F	-320	81-4
Freezing Point, °F	-346	81-4
Critical Temperature, °F	-233	81-4
Critical Pressure, psia	492	81-4
Density, lb_m/ft^3	$\frac{-345°F}{54.4}$ $\frac{-320°F}{50.4}$ (at 1 atm.)	81-4
Vapor Pressure, psia	$\frac{-334°F}{5.56}$ $\frac{-320°F}{14.7}$ $\frac{-280°F}{113}$	155-1
Heat of Vaporization, Btu/lb_m	85.8	81-4
Heat of Fusion, Btu/lb_m	$\frac{-345.7°F}{11.1}$ (at 1 atm)	486-1
Viscosity, Centipoise Viscosity, lb_m/sec ft	$\frac{-320°F}{0.19}$ 1.28×10^{-4} ; $\frac{-340°F}{0.33}$ 2.22×10^{-4}	81-4
Specific Heat, Btu/lb_m °F	$\frac{-320°F}{0.57}$	81-4
Enthalpy, Btu/lb_m (See Fig. 12.2.4g)	Press., psia: 14.7, 45, 140, 300 Temp., °F: $\frac{-320}{0}$ $\frac{-300}{10}$ $\frac{-275}{22}$ $\frac{-250}{34}$	493-1
Surface Tension, lb_f/ft	$\frac{-334°F}{7.2 \times 10^{-4}}$ $\frac{-307°F}{4.94 \times 10^{-4}}$	476-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$\frac{-352°F}{0.094}$ $\frac{-320°F}{0.080}$ $\frac{-262°F}{0.055}$	81-4
Electrical Conductivity, mho/cm		
Bulk Modulus, psi	$\frac{-321.4°F}{91,300}$ $\frac{343°F}{127,200}$	155-1
Expansivity, $\frac{\Delta V}{V}$ per °F	$\frac{-340°F}{0.00233}$ $\frac{-300°F}{0.00112}$ $\frac{-245°F}{0.00072}$	82-15
Velocity of Sound, ft/sec	$\frac{-330°F}{3320}$ 35[illegible] / 2900 $\frac{298°F}{2400}$	34-15

ISSUED: MARCH 1967
SUPERSEDES: OCTOBER 1965

12.2.2.5 LIQUID OXYGEN (LOX) MIL-P-25508 (USAF). Liquid oxygen is a nontoxic, nonflammable, and nonexplosive oxidizing agent having a reactivity much lower than gaseous oxygen. Mixing of liquid oxygen with a hydrocarbon fuel will cause the latter to solidify, forming an extremely shock-sensitive gel. High purity liquid oxygen is a light blue transparent liquid which has no characteristic odor. It does not burn, but will support combustion vigorously

Table 12.2.2.5. Properties of Liquid Oxygen (LOX, LO_2), O_2

PROPERTY	VALUES	REF.
Molecular Weight	32.00	486-1
Boiling Point, °F	-297.4	486-1
Freezing Point, °F	-362.0	486-1
Critical Temperature, °F	-181.8	486-1
Critical Pressure, psia	730.4	486-1
Density, lb_m/ft^3	-362°F: 85.65; -297°F: 71.20 (liquid)	486-1
Vapor Pressure, psia	-361°F: 0.027; -325°F: 2.102; -235°F: 196.692	486-1
Heat of Vaporization, Btu/lb_m	91.62	486-1
Heat of Fusion, Btu/lb_m	5.979 (at 1 atm and 361.79°F)	486-1
Viscosity, Centipoise Viscosity, lb_m/sec ft	-363°F: 0.87, 5.85×10^{-4}; -330°F: 0.32, 2.15×10^{-4}; -297°F: 0.19, 1.28×10^{-4}	486-1
Specific Heat, Btu/lb_m °F	-350°F: 0.110; -320°F: 0.096; -300°F: 0.088	81-4
Enthalpy, Btu/lb_m (See Fig. 12.2.2h)	Press., psia: 14.70, 34.98, 410.02 Temp., °F: -297.4, -261.7, -207.7 8.89, 23.6, 48.4 (liquid)	486-1
Surface Tension, lb_f/ft	-334°F: 1.257×10^{-3}; -318°F: 1.096×10^{-3}; -298°F: 0.907×10^{-3}	486-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	-360°F: 0.110; -320°F: 0.096; -300°F: 0.088	81-4
Electrical Conductivity, mho/cm		
Bulk Modulus, psi	-350°F: 237,000; -320°F: 177,300; -300°F: 142,500 (adiabatic), -297°F: 129,000	34-15 34-10
Expansivity, $\frac{\Delta V}{V}$ per °F	-319°F: 0.00228; -422.7°F: 0.0087	106-10
Velocity of Sound, ft/sec	-351°F: 3703; -325°F: 3353; -297°F: 2900 (liquid)	486-1

12.2.2.6 OXYGEN DIFLUORIDE. Oxygen difluoride is a colorless gas at room temperature and atmospheric pressure, and a yellow liquid at 229.5°F. It has a foul odor detected in air in concentrations of less than 0.5 ppm. Oxygen difluoride is a powerful oxidizing agent similar to fluorine and the halogens of fluoride. It is a relatively stable compound in that it does not detonate by sparking and is found to be insensitive to shock. However, it does decompose thermally at approximately 480°F.

Table 12.2.2.6. Properties of Oxygen Difluoride, OF_2

PROPERTY	VALUES		REF.
Molecular Weight	54.0		476-7
Boiling Point, °F	-229.5		476-7
Freezing Point, °F	-370.8		476-7
Critical Temperature, °F	-75.5		476-7
Critical Pressure, psia	718.8		509-3
Density, lb_m/ft^3	$\frac{-300°F}{107.9}$ $\frac{-250°F}{98.8}$ $\left(\frac{-230°F}{94.95}\right)^*$ $\frac{-200°F}{88.9}$		476-7 (509-3)*
Vapor Pressure, psia	Log P = 5.4258 - 995.02/R	88.67 (at B.P.)*	509-3 (476-7)*
Heat of Vaporization, Btu/lb_m	88.7 (at NBP)		476-7
Heat of Fusion, Btu/lb_m			
Viscosity, Centipoise Viscosity, lb_m/sec ft	Log $\mu_{(cp)}$ = 112.4/K-1.4508 (Valid from 11 to 760 mmHg pressure)	0.2826 cp* 1.90×10^{-4} lb_m/sec ft*	509-3 *106-10
Specific Heat, Btu/lb_m °F	0.35 at -150°F		509-3
Enthalpy, Btu/lb_m	135.6 at -230 °F		509-3
Surface Tension, lb_f/ft			
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	$\frac{-320.4°F}{0.140}$ $\frac{-297.4°F}{0.148}$		476-7
Electrical Conductivity, mho/cm			
Bulk Modulus, psi			
Expansivity, $\frac{\Delta V}{V}$ per °F			
Velocity of Sound, ft/sec			

ISSUED FEBRUARY 1970
SUPERSEDES: OCTOBER 1963

12.2.2.7 DIBORANE. Diborane is a colorless gas at standard conditions. Its odor is described as similar to rotten eggs and sickly sweet. The threshold of detection by odor is between 1 and 10 ppm, however, due to extreme toxicity, these concentrations are higher than the standards set for safe working conditions. Diborane is marginally pyrophoric — apparently moisture or contamination must be present to cause spontaneous combustion with air — and it produces a green flame. It also reacts with certain reducible compounds (some metal oxides, organic materials, etc.). Special precautions must be observed in cleaning hardware for diborane service as diborane may react explosively with residual halogenated solvents such as carbon tetrachloride. Slow leaks often leave a tell-tale sign of boron salts where the emerging diborane reacts with humidity in the air. Because diborane is so toxic, all waste should be incinerated under controlled conditions.

Quantities of diborane are usually stored in the liquid phase. As a liquid, it is water-white and is considered to be a mild cryogen since it is normally kept at temperatures below 0°F. Stored diborane slowly decomposes to higher molecular-weight boron hydrides and hydrogen. When used as a liquid rocket propellant diborane is hypergolic and yields high performance with several of the high energy oxidizers. Its drawbacks are toxicity, low density, low boiling point, formation of solid products of reaction, and high combustion temperatures. It is not a good coolant. Reference 505-1 is a comprehensive report on diborane published by Callery Chemical Company.

Table 12.2.2.7. Properties of Diborane, B_2H_6

PROPERTY	VALUES	REF.
Molecular Weight	27.69	774-1
Boiling Point, °F	-134.6	476-7
Freezing Point, °F	-264.8	476-7
Critical Temperature, °F	62.1	476-7
Critical Pressure, psia	581	476-7
Density, lb_m/ft^3	$\frac{-250^\circ F}{32.1}$ $\frac{-150^\circ F}{27.8}$ $\frac{-50^\circ F}{22.4}$ $\frac{+20^\circ F}{17.6}$	476-7
Vapor Pressure, psia	$\frac{-250^\circ F}{0.044}$ $\frac{-150^\circ F}{9.0}$ $\frac{-50^\circ F}{109}$ $\frac{+20^\circ F}{335}$	476-7
Heat of Vaporization, Btu/lb_m	221.9	476-7
Heat of Fusion, Btu/lb_m	65.5	774-1
Viscosity, Centipoise / Viscosity, $lb_m/sec\ ft$	Liquid $\frac{-200^\circ F}{0.25/1.68 \times 10^{-4}}$ $\frac{-150^\circ F}{0.153/1.03 \times 10^{-4}}$ $\frac{-100^\circ F}{0.10/0.672 \times 10^{-4}}$	476-7
Specific Heat, Btu/lb_m °F	Liquid $\frac{-250^\circ F}{0.65}$ $\frac{-150^\circ F}{0.66}$ $\frac{-50^\circ F}{0.74}$ $\frac{+20^\circ F}{0.98}$	774-1
Enthalpy, Btu/lb_m	As liquid under own vapor pressure H = 0 at 0°R $\frac{-100^\circ F}{226}$ $\frac{-50^\circ F}{263}$ $\frac{+20^\circ F}{323}$	774-1
Surface Tension, lb_f/ft	$\frac{-200^\circ F}{1.36 \times 10^{-3}}$ $\frac{-170^\circ F}{1.15 \times 10^{-3}}$	476-7
Thermal Conductivity $Btu/ft^2/hr/(^\circ F/ft)$		
Electrical Conductivity, mho/cm		
Bulk Modulus, psi	Liquid at 20 atm $\frac{-150^\circ F}{70,000}$ $\frac{-100^\circ F}{47,000}$ $\frac{-50^\circ F}{24,000}$	476-7
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec		

12.2.3 Water and Hydraulic Fluids

The following tables include water and liquids commonly used for the transmission of power.

Table 12.2.3a. Properties of Water

PROPERTY	VALUES	REF.
Molecular Weight	18.02	508-1
Boiling Point, °F	212	409-2
Freezing Point, °F	32	508-1
Critical Temperature, °F	705.47	508-1
Critical Pressure, psia	3206.2	409-2
Density, lb_m/ft^3	32°F: 62.427; 60°F: 62.375; 100°F: 62.003; 150°F: 61.218; 212°F: 59.842	508-1
Vapor Pressure, psia	32°F: 0.0885; 50°F: 0.1781; 75°F: 0.4298; 100°F: 0.9492; 150°F: 3.718; 212°F: 14.696	409-2
Heat of Vaporization, Btu/lb_m	32°F: 1075.8; 50°F: 1065.[illegible]; 75°F: 105[illegible].5; 100°F: 1037.2; 150°F: 1008.2; 212°F: 970.3	409-2
Heat of Fusion, Btu/lb_m	143.3 (at 32°F and 1 atm.)	508-1
Viscosity, Centipoise Viscosity, $lb_m/sec\ ft (x10^4)$	32°F: 1.794 / 12.06; 60°F: 1.131 / 7.61; 75°F: 0.908 / 6.1; 100°F: 0.627 / 4.22; 150°F: 0.434 / 2.92; 212°F: 0.284 / 1.91	508-1
Specific Heat, Btu/lb_m °F	32°F: 1.001; 50°F: 1.002; 100°F: 1.004; 150°F: 1.009; 212°F: 1.021	132-1
Enthalpy, Btu/lb_m	32°F: 0.00; 60°F: 28.05; 100°F: 67.97; 150°F: 117.89; 212°F: 180.07	409-2
Surface Tension, lb_f/ft	32°F: 0.0052; 60°F: 0.0050; 100°F: 0.0048; 150°F: 0.0045; 212°F: 0.0040	461-1
Thermal Conductivity $Btu/ft^2/hr/(°F/ft)$	32°F: 33.030; 50°F: 33.910; 100°F: 36.210; 150°F: 38.240; 175°F: 39.230; 212°F: 39.290	508-1
Electrical Conductivity, mho/cm	32°F: 3.02×10^{-8}; 77°F: 13.97×10^{-8}; 122°F: 43.43×10^{-8}	508-1
Bulk Modulus, psi	32°F: 287,000; 50°F: 313,000; 100°F: 331,000; 150°F: 328,000; 212°F: 300,000 (See Fig. 3.3.7)	464-1
Expansivity, $\frac{\Delta V}{V}$ per °F		
Velocity of Sound, ft/sec	32°F: 4610; 60°F: 4810; 100°F: 4960; 150°F: 4990; 212°F: 4810	

Table 12.2.3b. Properties of MIL-H-5606, Hydraulic Fluid (Red Oil), Mineral Oil Base, Hydrocarbon

PROPERTY	VALUES			REF.
Kinematic Viscosity, Centistokes ft^2/sec	-65°F 2000 2.15×10^{-2}	160°F 7 7.54×10^{-5}	275°F 3.3 3.55×10^{-5}	42-1
Specific Gravity	-65°F 0.90	160°F 0.83	275°F 0.80	42-1
Flash Point, °F	200			V-274
Pour Point, °F	-70			V-274
Specific Heat, Btu/lb_m °F	-65°F 0.41	160°F 0.506	275°F 0.576	42-1
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	-65°F 0.083	160°F 0.077	275°F 0.074	42-1
Vapor Pressure, psi		160°F 0.445	275°F 5.81	42-1
Bulk Modulus, psi	14.7 psia 100°F 21.0×10^4 500°F 4.3×10^4	1000 psia 24.0×10^4 5.2×10^4	5000 psia 29.5×10^4 10.0×10^4 (See Fig. 3.5.7)	42-1

Table 12.2.3c. Properties of MIL-L-7808 Hydraulic Fluid, Synthetic Diester

(Note: MIL-L-7808 is dual-purpose; used as both lubricant and hydraulic fluid.)

PROPERTY	VALUES			REF.
Kinematic Viscosity, Centistokes ft^2/sec	-40°F 2000 2.15×10^{-2}	230°F 3 3.23×10^{-5}	350°F 1.5 1.6×10^{-5}	42-1
Specific Gravity	-40°F 0.96	230°F 0.86	350°F 0.81	42-1
Flash Point, °F	400			42-1
Pour Point, °F	-75			42-1
Specific Heat, Btu/lb_m °F	-40°F 0.32	230°F 0.50	350°F 0.57	42-1
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	-40°F 0.09	230°F 0.082	350°F 0.077	42-1
Vapor Pressure, psi		230°F 0.00387	350°F 0.00581	42-1
Bulk Modulus, psi	-40°F 1.4×10^5	230°F 2.8×10^5	350°F 1.7×10^5	42-1

ISSUED: FEBRUARY 1970
SUPERSEDES: MARCH 1967

Table 12.2.3d. Properties of MIL-H-8446B Hydraulic Fluid (Oronite 8515), Synthetic Silicate Ester and Diester

PROPERTY	VALUES			REF.
Kinematic Viscosity, Centistokes ft^2/sec	-65°F 2500 2.69×10^{-2}	300°F 4.2 4.52×10^{-5}	450°F 2.5 2.69×10^{-5}	42-1
Specific Gravity	-65°F 0.98	300°F 0.83	450°F 0.77	42-1
Flash Point, °F	395			V-274
Pour Point, °F	Below -100			V-274
Specific Heat, Btu/lb_m °F	-65°F 0.38	300°F 0.56	450°F 0.65	42-1
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	-65°F 0.0[illegible]	300°F 0.066	450°F 0.055	42-1
Vapor Pressure, psi		300°F 0.0087	450°F 0.68	42-1
Bulk Modulus, psi	14.7 psia 100°F 20.0×10^4 500°F 4.6×10^4	1000 psia 22.0×10^4 5.5×10^4	5000 psia 27.5×10^4 10.2×10^4	42-1

12.2.4 Gases

Thermophysical property data for fluids commonly used in the gaseous state are presented in this sub-topic in the form of tables and charts.

Table 12.2.4a. Properties of Air

PROPERTY	VALUES	REF.
Molecular Weight	28.96 average	486-1
Critical Temperature, °F	-220.3	132-1
Critical Pressure, psia	546	132-1
C_p, Btu/lb_m °F	0.241	132-1
C_v, Btu/lb_m °F	0.1725	132-1
Ratio of Specific Heats, C_p/C_v	1 atm: -100°F 1.4057; 60°F 1.406; 260°F -; 620°F - 1000 psia: -100°F -; 60°F -; 260°F 1.3961; 620°F 1.3760	132-1, 486-1
Gas Constant, ft-lb_f/lb_m °F	53.30	486-1
Density, lb_m/ft^3	-100°F: 0.1104; 60°F: 0.07658; 260°F: 0.0551	132-1, 486-1
Viscosity, lb_m/ft sec	-280°F: 4.66×10^{-4}; -100°F: 8.93×10^{-4}; 260°F: 1.536×10^{-5}	486-1
Enthalpy, Btu/lb_m See Fig. 12.2.4a	-100°F: 65.9; 260°F: 172.3; 620°F: 261.2	486-1
Mean Free Path, cm		
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	-280°F: 0.00962; -100°F: 0.01882; 260°F: 0.03502	486-1
Compressibility Factor $Z = \frac{PV}{RT}$	1 atm: -100°F 0.99767; 80°F 0.99970; 260°F 1.00019 100 atm: -100°F 0.8105; 80°F 0.9933; 260°F 1.0299	287-4
Velocity of Sound, ft/sec	32°F: 1 atm 1088; 25 atm 1089; 50 atm 1098; 100 atm 1150 212°F: 1 atm 1266 1832°F: 1 atm 2297	248-1

ISSUED: FEBRUARY 1970
SUPERSEDES: MARCH 1967

Table 12.2.4b. Properties of Gaseous Helium (GHe), He

PROPERTY	VALUES					REF.
Molecular Weight	4.003					486-1
Critical Temperature, °F	-450.2					486-1
Critical Pressure, psia	33.2					486-1
C_p, Btu/lb_m °F		-200°F	60°F	200°F		155-1
	1 atm	1.248	1.248	1.248		
	1000 psia	1.259	1.252	1.25[illegible]		
C_v, Btu/lb_m °F		200°F	60°F			
	1 atm	0.752	0.752			
	1000 psia	0.763	0.758			
Ratio of Specific Heats, C_p/C_v		-200°F	60°F			486-1
	1 atm	1.66	1.66			
	1500 psia	1.65	1.655			
Gas Constant, ft-lb_f/lb_m °F	386.3					
Density, lb_m/ft^3	32°F					486-1
	0.0111					
Viscosity, lb_m/ft sec	-280°F	-100°F	80°F	260°F		486-1
	1.54×10^{-5}	1.75×10^{-5}	2.06×10^{-5}	2.30×10^{-5}		
Enthalpy, Btu/lb_m		-100°F	200°F	620°F	1340°F	486-1
	1 atm	447	821	1340	2240	
	1500 psia	463	836			
Mean Free Path, cm	27.45×10^{-6}					248-1
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	-269°F	-10°F	80°F	260°F		486-1
	[illegible]	774.4	866.5	1029.5		
Compressibility Factor $Z = \frac{PV}{RT}$		-200°F	60°F	200°F		155-1
	1 atm	1.002	1.001	1.001		
	1500 psia	1.103	1.051	1.044		
Velocity of Sound, ft/sec		-400°F	100°F	600°F		152-7
	1 atm	1110	3399	4657		
	1500 psia	1341	3480	4739		

Table 12.2.4c. Properties of Gaseous Hydrogen (GH_2) (Normal-Hydrogen), H

PROPERTY	VALUES	REF.
Molecular Weight	2.014	486-1
Critical Temperature, °F	-399.6	486-1
Critical Pressure, psia	188.11	486-1
C_p, Btu/lb_m °F	-200°F, 60°F 1 atm: 3.0, 3.41 1500 psia: 3.25, 3.47	155-1
C_v, Btu/lb_m °F	-200°F, 60°F 1 atm: 2.02, 2.44 1500 psia: 2.02, 2.46	155-1
Ratio of Specific Heats, C_p/C_v	-200°F, 60°F 1 atm: 1.49, 1.40 1500 psia: 1.60, 1.41	155-1
Gas Constant, ft-lb_f/lb_m °F	766.8	155-1
Density, lb_m/ft^3	-100°F, 60°F, 500°F 1 atm: 0.0079, 0.0052, 0.003 100 psia: 0.051, 0.035, 0.019 1000 psia: 0.50, 0.34, 0.20	34-15
Viscosity, lb_m/ft sec	-100°F: 4.57×10^{-6} 60°F: 6.00×10^{-6} 500°F: 9.00×10^{-6}	34-15
Enthalpy, Btu/lb_m	-100°F, 60°F 1 atm: 1214, 1818 1000 psia: 1211, 1803	155-1
Mean Free Path, cm	17.44×10^{-6}	248-1
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	-100°F: 0.0741 60°F: 0.1050	155-1
Compressibility Factor $Z = \frac{PV}{RT}$	-100°F, 60°F 1 atm: 0.995, 1.0 100 psia: 0.995, 1.0 1000 psia: 1.04, 1.035	155-1
Velocity of Sound, ft/sec	-360°F, -260°F, 60°F, 1000°F 1 atm: 2000, 2650, 4250, 7100 1000 psia: 2650, 2820, 4430, 7300 2400 psia: 4050, 3400, 4730, 7400	522-1

Table 12.2.4d. Properties of Gaseous Nitrogen (GN_2), N_2

PROPERTY	VALUES	REF.
Molecular Weight	28.02	486-1
Critical Temperature, °F	-233	486-1
Critical Pressure, psia	491	486-1
C_p, Btu/lb_m °F	0.247	155-1
C_v, Btu/lb_m °F	0.1761	155-1
Ratio of Specific Heats, C_p/C_v	-100°F: 1.404; 620°F: 1.382; 980°F: 1.360	486-1
Gas Constant, ft-lb_f/lb_m °F	55.1	155-1
Density, lb_m/ft^3	-100°F: 0.1068; 80°F: 0.0710; 440°F: 0.0426	486-1
Viscosity, lb_m/ft sec	-280°F: 1.41×10^{-5}; 80°F: 1.91×10^{-5}; 440°F: 2.39×10^{-5}	486-1
Enthalpy, Btu/lb_m	-100°F: 89.0; 620°F: 269.6; 980°F: 364.5	486-1
Mean Free Path, cm	9.29×10^{-6}	248-1
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	1 atm: -100°F: 0.01; 80°F: 0.015; 500°F: 0.023 1000 psia: -100°F: 0.015; 80°F: 0.017; 500°F: 0.024	486-1
Compressibility Factor $Z = \frac{PV}{RT}$	1 atm: -200°F: 0.995; 40°F: 0.997 1000 psia: -200°F: 0.51; 40°F: 0.990	155-1
Velocity of Sound, ft/sec	1 atm: -100°F: 935; 60°F: 1135; 440°F: 1490 100 atm: -100°F: 1040; 60°F: 1237; 440°F: 1588	152-7

Table 12.2.4a. Properties of Gaseous Oxygen (GO$_2$), O$_2$

PROPERTY	VALUES				REF.
Molecular Weight	32.00				34-15
Critical Temperature, °F	-181.8				34-15
Critical Pressure, psia	730.4				34-15
C_p, Btu/lb$_m$ °F		-100°F	60°F	500°F	34-15
	1 atm	0.22	0.22	0.235	
	2000 psia	0.53	0.29	0.25	
C_v, Btu/lb$_m$ °F		-100°F	60°F	500°F	34-15
	1 atm	0.1556	0.157		
	1500 psia		0.1617		
Ratio of Specific Heats, C_p/C_v	-100°F	60°F			34-15
	1.414	1.402			
Gas Constant, ft-lb$_f$/lb$_m$ °F	48.3				34-15
Density, lb$_m$/ft^3		-100°F	60°F	500°F	34-15
	1 atm	0.125	0.081	0.0437	
	100 psia	0.874	0.56	0.312	
	1000 psia	11.87	5.86	3.1	
Viscosity, lb$_m$/ft sec		-100°F	60°F	500°F	34-15
	1 atm	9.80×10^{-6}	1.45×10^{-5}	2.1×10^{-5}	
	1500 psia	1.45×10^{-5}	1.50×10^{-5}	2.1×10^{-5}	
Enthalpy, Btu/lb$_m$	-100°F	60°F	240°F		493-1
	76	97.2	149.5		
Mean Free Path, cm	9.93×10^{-6}				248-1
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)		-100°F	60°F	500°F	34-15
	1 atm	0.0105	0.015	0.025	
	1500 psia	0.02	0.018	0.0265	
Compressibility Factor $Z = \frac{PV}{RT}$		-100°F	80°F	440°F	287-4
	1 atm	0.99201	0.99939	1.00022	
	100 atm	0.6871	0.9541	1.0256	
Velocity of Sound, ft/sec		-100°F	60°F	440°F	152-7
	1 atm	880	1063	1385	
	100 atm		1095	144[illegible]	

ISSUED: MARCH 1967
SUPERSEDES: OCTOBER 1965

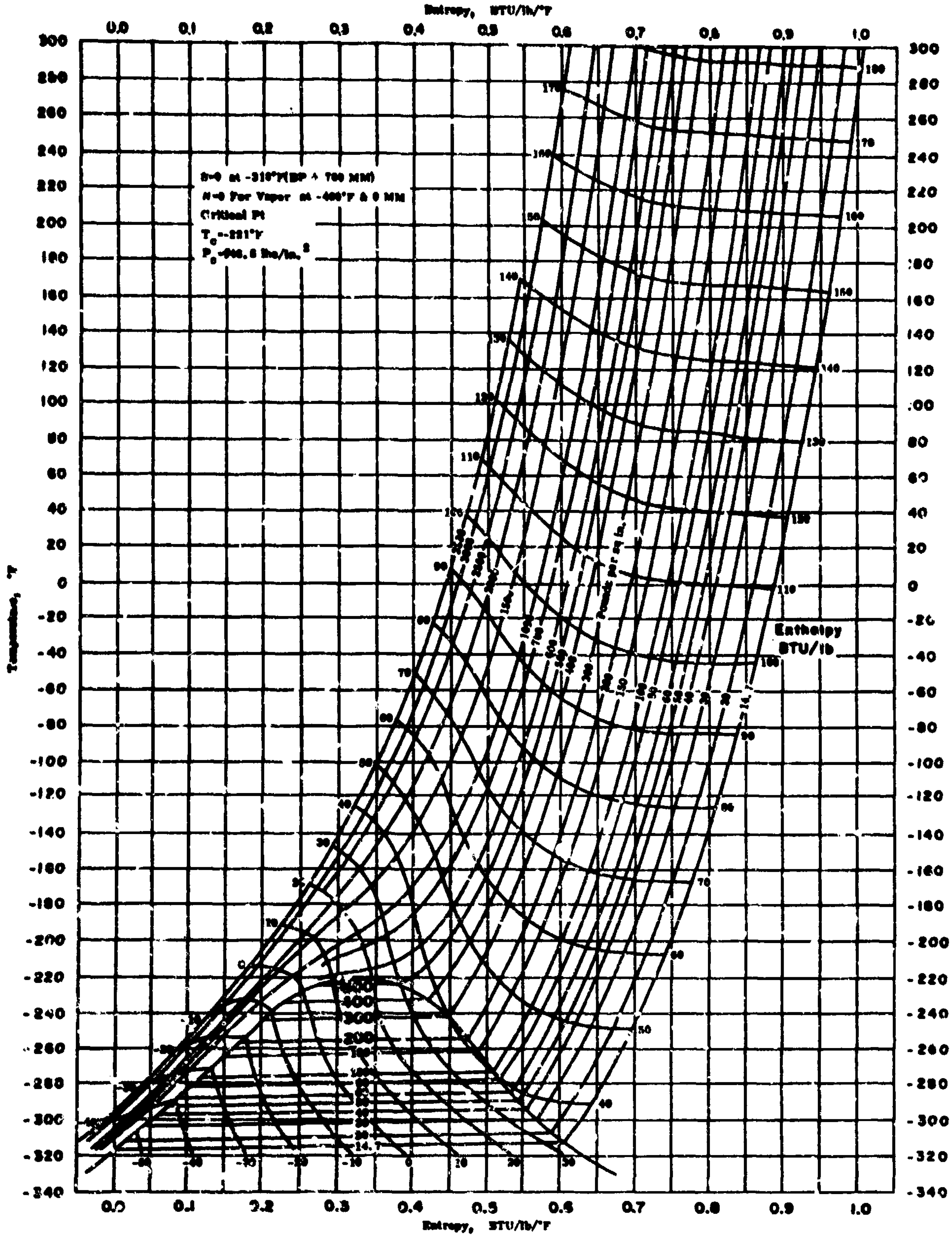

Figure 12.2.4a. Temperature-Entropy Diagram for Air

(Adapted with permission from "Cryogenic Engineering," J. H. Bell, Jr., Copyright 1963, Prentice-Hall, Inc., Englewood Cliffs, New Jersey)

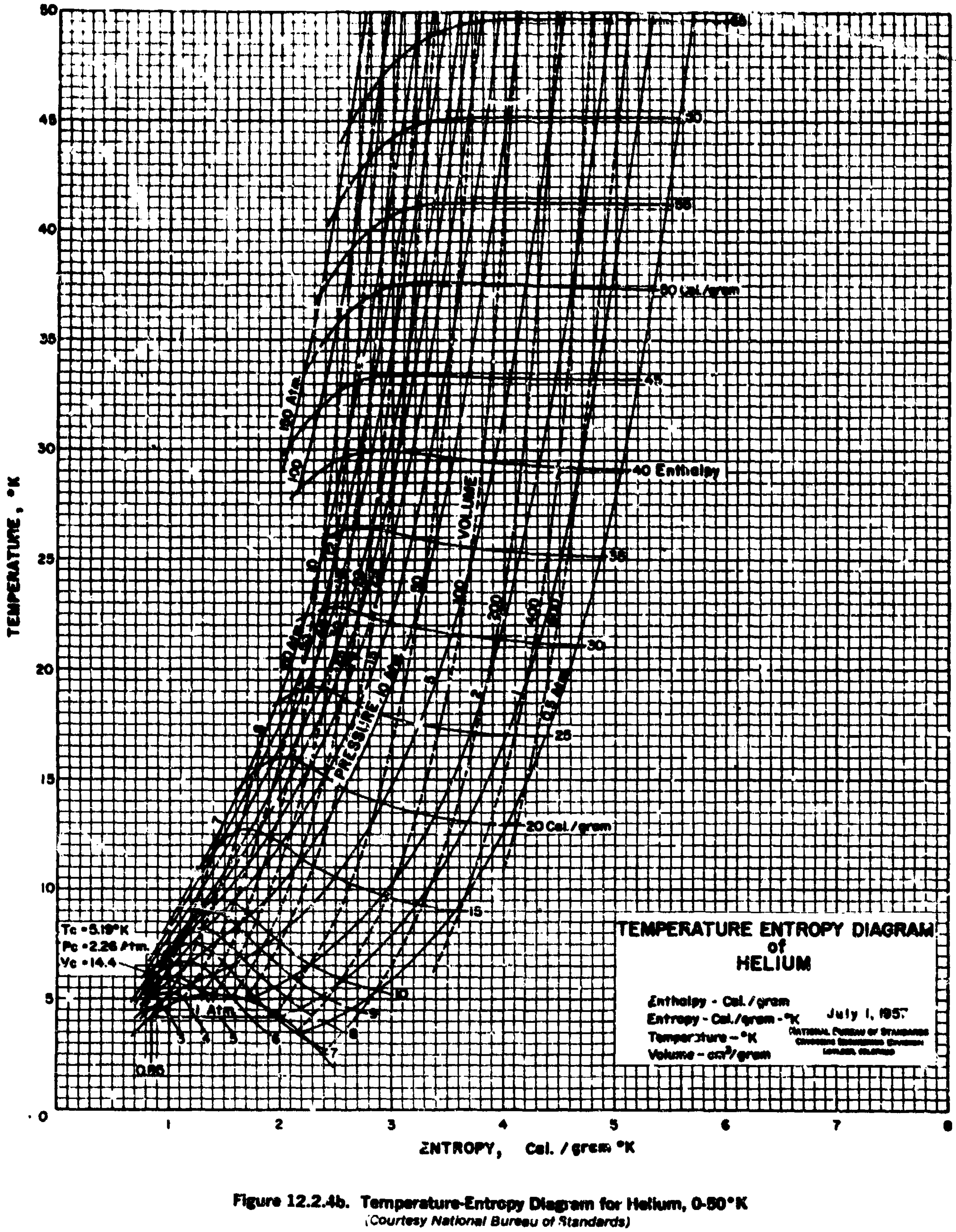

Figure 12.2.4b. Temperature-Entropy Diagram for Helium, 0-50°K
(Courtesy National Bureau of Standards)

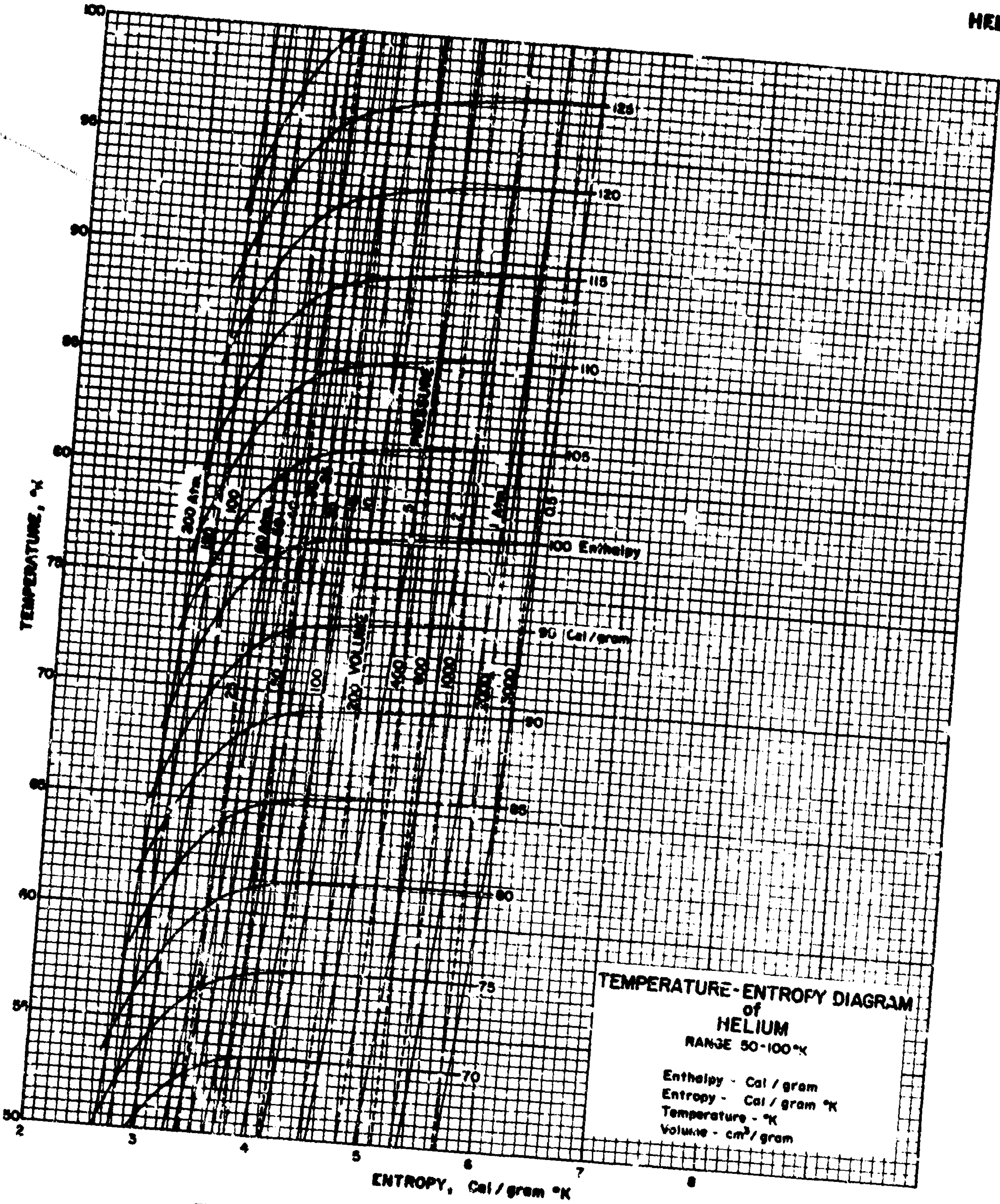

Figure 12.2.4c. Temperature-Entropy Diagram for Helium, 50-100°K
(Courtesy National Bureau of Standards)

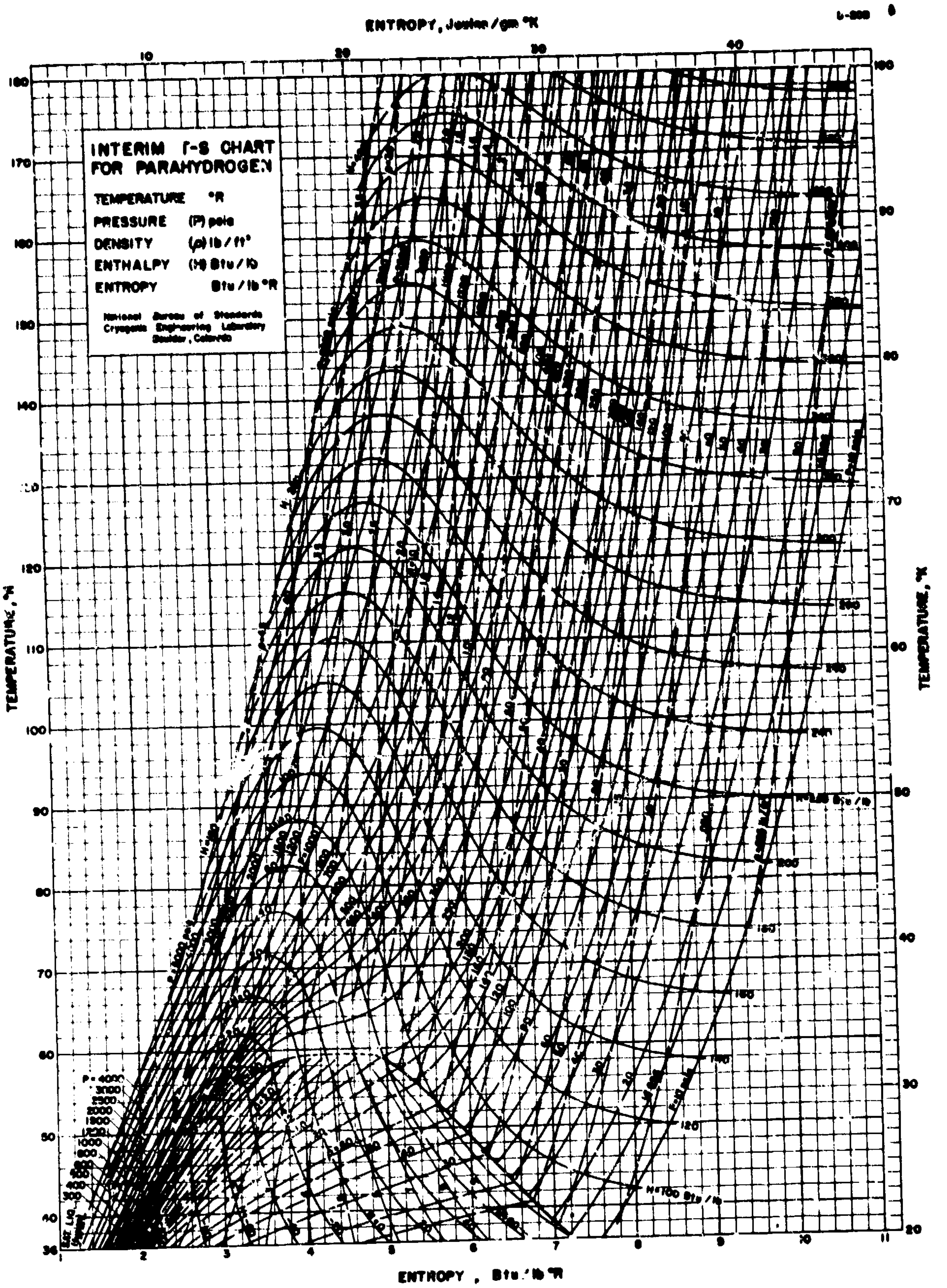

Figure 12.2.4d. Temperature-Entropy Diagram for Para-Hydrogen, 20-100°K

(Courtesy National Bureau of Standards)

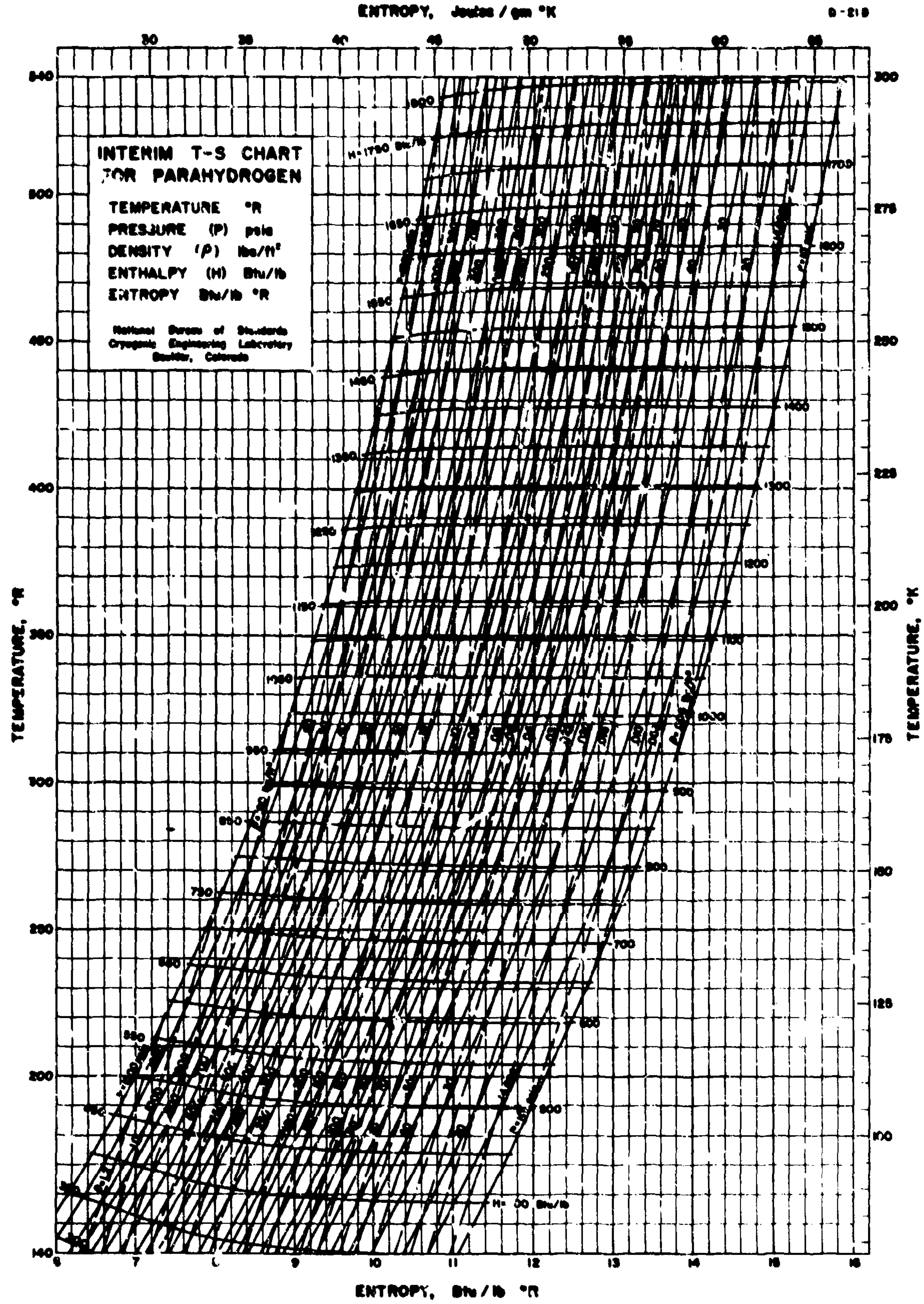

Figure 12.2.4e. Temperature-Entropy Diagram for Para-Hydrogen, 100-300°K
(Courtesy National Bureau of Standards)

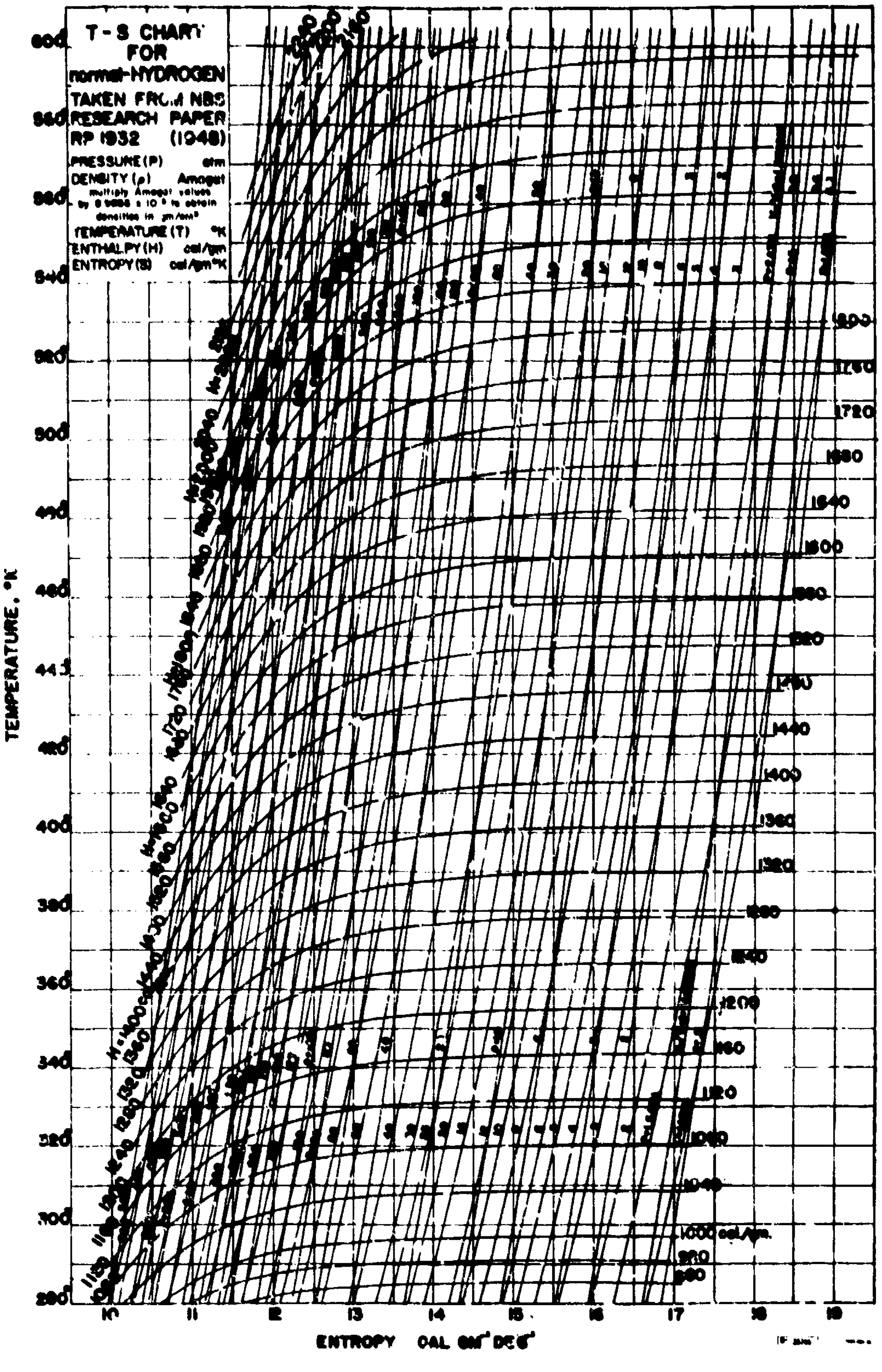

Figure 12.2.4f. Temperature-Entropy Diagram for Normal-Hydrogen, 280-600°K
(Courtesy National Bureau of Standards)

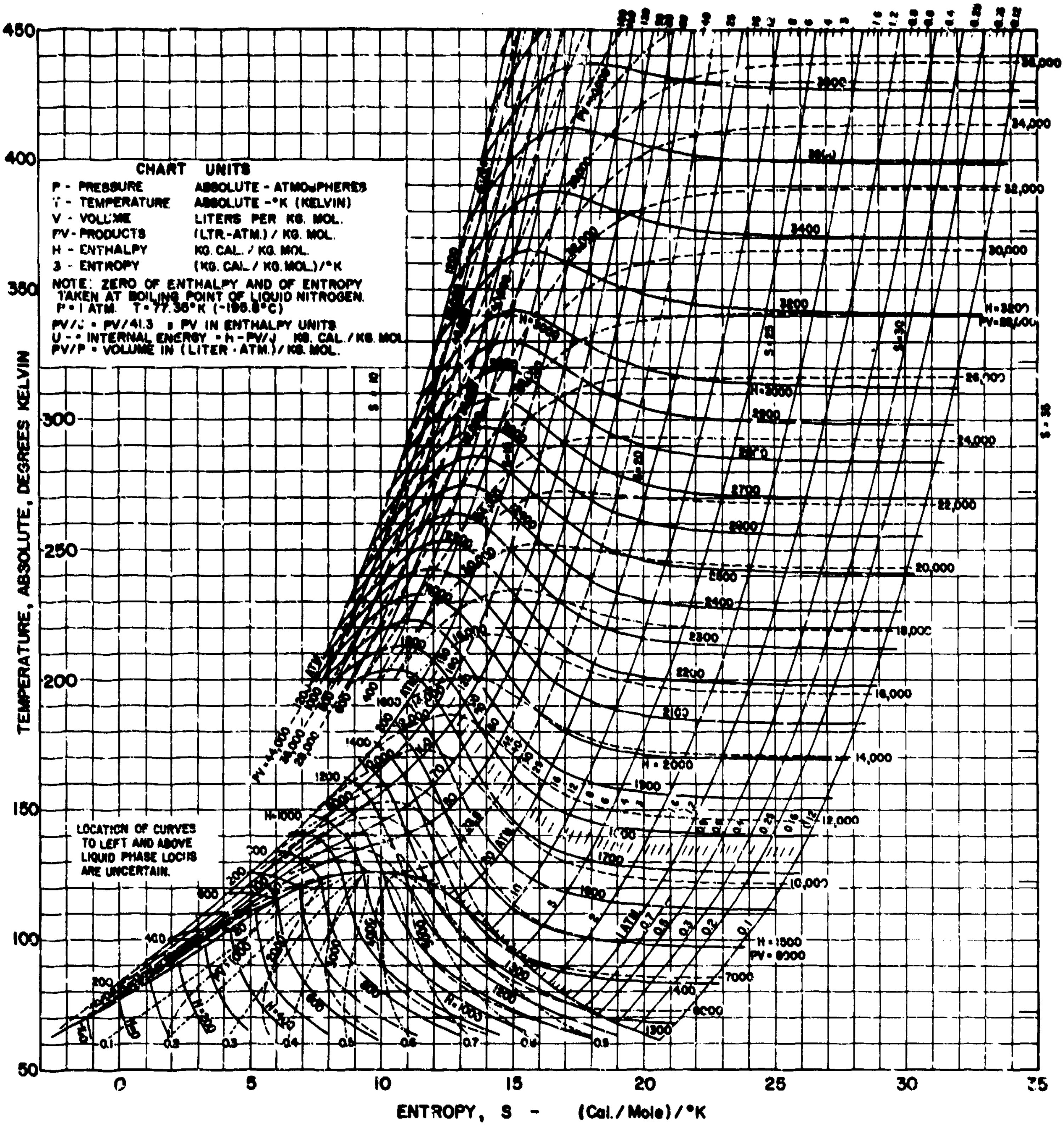

Figure 12.2.4g. Temperature-Entropy Diagram for Nitrogen, 50-450°K
(Courtesy National Bureau of Standards)

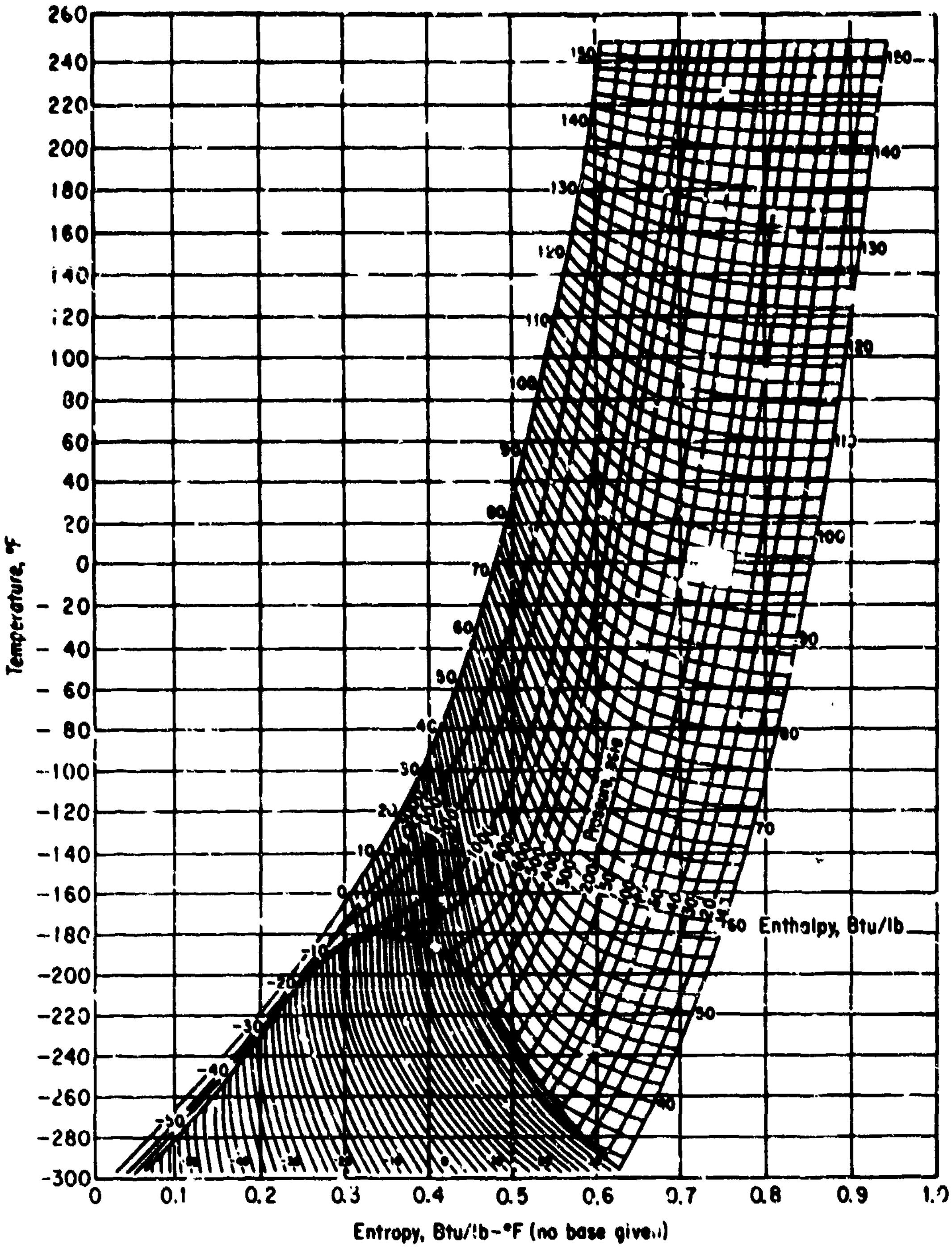

Figure 12.2.4h. Temperature-Entropy Diagram for Oxygen, −300 to 260°F

(Adapted with permission from "Cryogenic Engineering," J. H. Bell, Jr., Copyright 1963, Prentice-Hall, Inc., Englewood Cliffs, New Jersey)

12.3 PROPERTIES OF POLYMERS

In contrast to the ordered and rigid crystalline arrangement of atoms in metals, polymers are composed of long molecular chains of monomers linked end to end. The monomer, which is the basic element in a polymer, is the link in the chain that determines the chemical character. Polymers are used extensively in fluid components for applications such as seals, packings, bushings, bearings, and diaphragms. Polymers of interest to the fluid component designer can be broadly classified as either elastomers or plastics (properties of elastomeric materials and plastics are tabulated respectively in two separate tables). The accepted definition of a *plastic* is "a material having a high molecular weight, which while solid in its finished state, at some stage in its manufacture is soft enough to be formed through application of heat and/or pressure." An *elastomer* is commonly defined by its performance rather than by its composition as "a polymeric material which at room temperature can be stretched to at least twice its original length and upon immediate release of stress will return quickly to approximately its original length." Rubbers are commonly softer than plastics; however, there is a hardness overlap. A representative hardness spectrum for elastomers and plastics is shown in Figure 12.3.

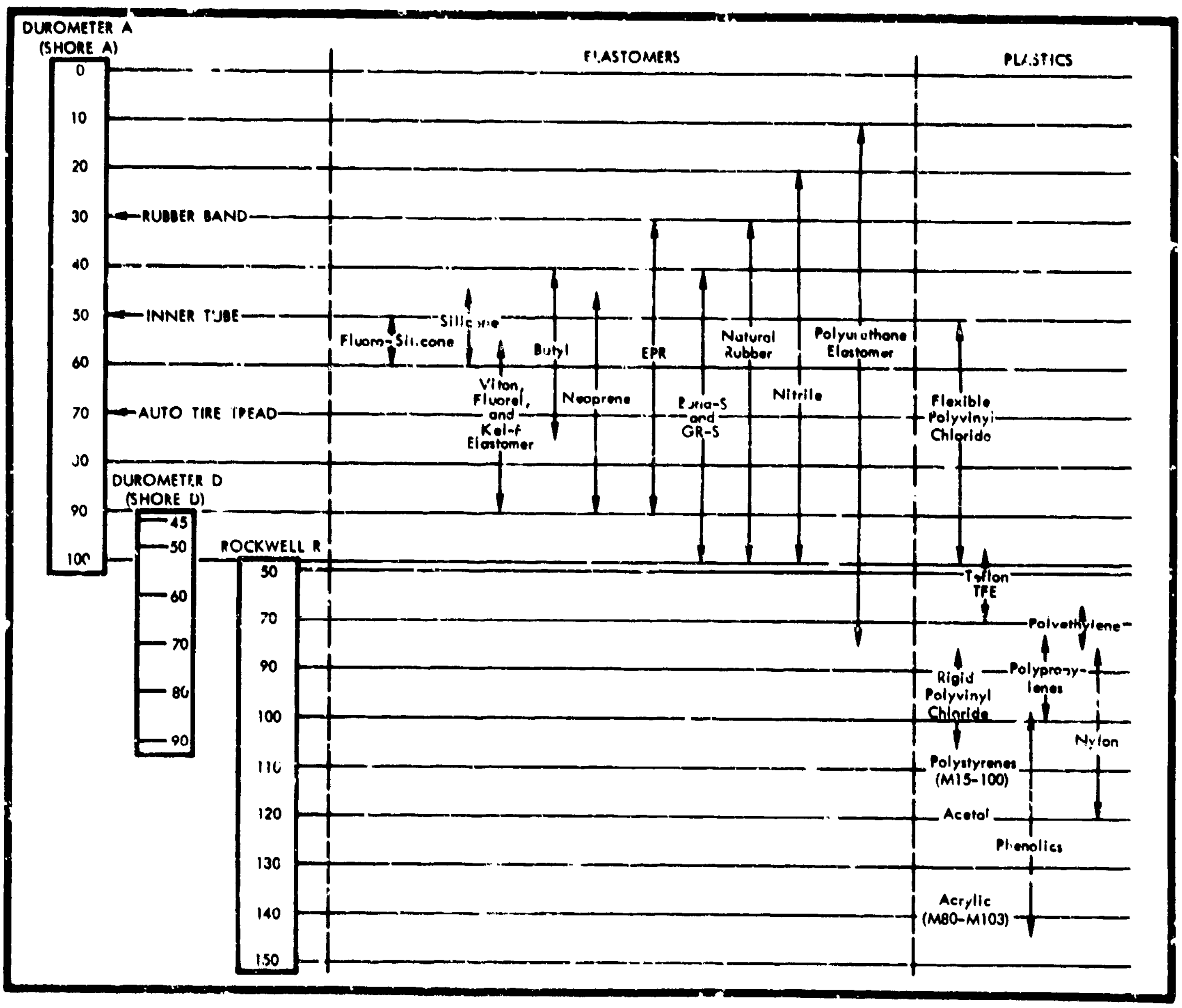

Figure 12.3. Hardness Spectrum for Elastomers and Plastics

12.3.1 Elastomers

At one time, natural rubber was the only elastomer available, while today there are a variety of synthetic rubber compounds available to meet a wide range of requirements. Table 12.3.1 presents representative properties of basic types of elastomers used in fluid component applications. Within each basic elastomer type there can be a great variety of properties achievable depending upon compounding and curing techniques. Ingredients such as fillers, plasticizers, accelerators, and vulcanizing agents along with various curing techniques result in a wide range of possible properties with a given base polymer. Carbon black is a common inert additive which increases the strength and hardness of most elastomers. Since it would be impossible to include data on each compound available within a given elastomer type, the table gives representative data indicating the range of usefulness of each material. Detailed property data on specific compounds can best be obtained from the fabricator or molder. Good sources of data on elastomers are the "Handbook of Design Data on Elastomeric Materials Used in Aerospace Systems, ASD-TR-61-234," and "MIL-HDBK-149A, Reference 547-3."

12.3.1.1 **NATURAL RUBBER.** The chief source of natural rubber is the *hevea brasiliensis* tree grown principally in the Far East. It is a high molecular weight material based on the monomer isoprene (C_5H_8). Natural rubber does not age as well as many synthetics nor is it as chemically inert as some. It is inferior to many synthetics for heat aging and resistance to sunlight, oxygen, ozone, solvents, or oils. Natural rubber, however, has good low temperature flexibility, low compression set, and tear and abrasion resistance superior to almost all synthetics.

12.3.1.2 **NITRILE RUBBER.** Nitrile rubber, commonly referred to as Buna N, is a copolymer of butadiene and acrylonitrile. A number of polymers are commercially available with various monomer ratios. Those containing a high proportion of acrylontrile have excellent resistance to petroleum oils and solvents, good tensile strength, and good abrasion resistance, but relatively poor low temperature flexibility and ozone or sunlight resistance. Decreasing acrylonitrile content yields great improvement in low temperature capability, but at the expense of oil resistance. The most useful compounds are based on medium grades that effect a good balance of properties. Nitrile rubber is the best polymer for most hydraulic applications where petroleum-based fluids are the operating medium.

12.3.1.3 **ISOBUTYLENE ISOPRENE (BUTYL) RUBBER.** Butyl rubber is a high molecular weight copolymer of isobutylene with small amounts of isoprene. It is not resistant to petroleum oil and solvents, but displays excellent resistance to the phosphate ester type fire-resistant hydraulic fluids. Butyl has the lowest permeability rate of any rubber and for this reason is often used for sealing gases and for vacuum seals.

12.3.1.4 **CHLOROPRENE RUBBER.** Chloroprene rubber, more commonly known by the du Pont trade name Neoprene, exhibits excellent resistance to sunlight, ozone, and weathering, in addition to moderate resistance to petroleum oils. Neoprene is frequently used for seals and diaphragms because of its good resistance to a wide variety of fluids and chemicals.

12.3.1.5 **SILICONE RUBBER.** The silicones are a group of elastomeric materials made from silicone, oxygen, hydrogen, and carbon. Although having inferior physical properties such as tensile strength and tear resistance when compared with a number of other elastomers, silicone rubber is primarily useful in applications involving a wide range of temperatures and where compatibility with reactive fluids is required. Silicones are especially useful for low temperature service because they do not employ plasticizers to maintain rubberiness, loss of plasticizer being a problem with organic rubbers. Special compounds are available that remain usefully flexible down to −180°F. The continuous duty upper temperature limit is 500°F; however, certain compounds tolerate much higher temperatures for brief periods.

12.3.1.6 **STYRENE BUTADIENE RUBBER.** Styrene butadiene is a synthetic copolymer of styrene and butadiene monomers. It resembles natural rubber in most respects, but is somewhat lower in cost and has poorer gum strength and less resilience than natural rubber.

12.3.1.7 **POLYACRYLATE (POLYACRYLIC) RUBBER.** Polyacrylate rubber is characterized by good oil resistance at temperatures up to 350 F. Applications for polyacrylate include oil seals, O-rings, and diaphragms. It is not recommended for dynamic seals unless spring-loaded. Its strength, compression set, and water resistance are inferior; however, it has an outstanding resistance to hot oil, oxidation, ozone, and sunlight.

12.3.1.8 **POLYSULFIDE RUBBER.** Polysulfide rubbers have limited applications due to relatively poor physical properties, although they do have excellent resistance to most solvents. They are used extensively in sealing aircraft fuel tanks and pressurized cabins.

12.3.1.9 **FLUOROSILICONE RUBBER.** Fluorosilicone rubbers combine most of the extended temperature range of silicones with greater resistance to oils and aromatic fuels. Having low physical properties and poor wear resistance, they usually are not recommended for dynamic applications.

12.3.1.10 **KEL-F ELASTOMER.** Kel-F elastomer is a fluorinated polymer having excellent resistance to fuming acids, strong bases, peroxides, solvents, and oils. Its physical properties include good tensile strength, abrasion resistance, and high tear strength. It also has low compression set. Applications include hose, tubing, diaphragms, gaskets and O-rings.

12.3.1.11 **VITON RUBBER.** Viton is a linear copolymer of vinylidene fluoride and hexafluoro-propylene, containing about 65 percent fluorine. It has excellent resistance to many petroleum oils, synthetic lubricants, silicate esters, and fuels covering a temperature range of -40°F to +500°F.

12.3.1.12 POLYURETHANE ELASTOMERS. Urethane elastomers show excellent resistance to abrasion, better by a factor of ten than many of the conventional elastomers. They may be produced in a hardness range from Shore A10 (softer than an eraser) to Shore D80 (harder than a bowling ball). Urethane elastomers have high strength and a higher load-bearing capacity than conventional elastomers of comparable hardness. They can best be classified as elastoplastics, since they have the desirable properties of both elastomers and plastics.

12.3.1.13 CHLOROSULFONATED POLYETHYLENE (HYPALON). Hypalon has a useful temperature range of -65°F to +250°F. It has good acid resistance but its mechanical properties, compression, and permanent set characteristics are less than desirable for dynamic and many static sealing applications.

12.3.1.14 ETHYLENE PROPYLENE RUBBER. Ethylene propylene is a relatively low cost copolymer of ethylene and propylene. The copolymer has an excellent combination of qualities, including thermal resistance, abrasion resistance, and low compression set, together with good physical properties, good low temperature flexibility, and compatibility with a wide range of fluids. The latter include acids, alcohol, and polar solvents. It is not compatible with petroleum-based fluids.

12.3.2 Plastics

A plastic is one of many high polymeric substances, including both natural and synthetic products but excluding rubbers. Plastic materials are used extensively in rocket system fluid components as seal materials, for they generally possess superior chemical resistance to elastomers. Because of the relatively hard unyielding character (compared to elastomers) of most plastic materials, they require careful workmanship and often special techniques to obtain good seals. Plastics are broadly divided into two major subdivisions, thermosetting and thermoplastics. Thermoplastics are resins which may be softened repeatedly without undergoing a change in chemical composition, while thermosetting resins undergo a chemical change (crosslinking) with application of heat and pressure and cannot be resoftened. Thermoplastics are the plastic materials most commonly used in valves for seal and diaphragm applications. A good general reference on plastics is the "Plastics Encyclopedia" published annually by Hildreth Press, Inc. An information center established by the Department of Defense for compiling and disseminating data of plastics is Plastec, the Plastics Evaluation Center, at Picatinny Arsenal, Dover, New Jersey. Table 12.3.2 presents representative properties of basic plastics used in fluid component applications.

12.3.2.1 POLYETHYLENE. Polyethylene is a strong, flexible, wax-like plastic with a hard surface, available as a clear, transparent, translucent, or opaque material. It is highly resistant to chemicals and has near-zero moisture absorption. Polyethylenes range from low density to high density classes, each class constituting a different family of resins. The properties of the material are dependent upon the density: increasing density increases rigidity, temperature, resistance, and load-carrying ability. The higher density materials generally have somewhat better chemical resistance. Polyethylene has a melting point of about 240°F. It responds to irradiation through improvements of its mechanical properties, including improved solvent resistance and improved high temperature characteristics.

12.3.2.2 TEFLON. Teflon is a trademark of the du Pont Company, applied to a variety of polymers of tetrafluoroethylene. The two most common Teflon materials used in fluid component applications are Teflon TFE (tetrafluoroethylene) and the newer Teflon FEP (fluorinated ethylene-propylene copolymer) which unlike the TFE resin can be processed by conventional plastic injection molding and extruding equipment. Teflon TFE is not a true thermoplastic and must be molded by a compacting and sintering process. Teflon is a tough wax-like solid, white to gray in color. A major reason for its widespread use as a seal material is its outstanding chemical inertness. Teflon is used in both static and dynamic sealing applications and as a diaphragm material. One characteristic of Teflon that must be considered by the designer for any application is the tendency of the material to flow under load. Care must be taken in the use of Teflon to contain the material adequately so that dimensional changes do not occur as a result of flow under compressive loads. The "cold flow" characteristics of Teflon can be improved by the addition of inert filler materials such as fiberglass and asbestos. One of Teflon's unusual characteristics is its extremely low coefficient of friction (see Table 12.7a). This property makes it useful as a journal-bearing material.

Teflon is commercially available in a variety of granular grades for molding and ram extrusion, a special grade for paste extrusion of thin wall sections, and in water dispersions for coating formulations.

Granular grades of TFE resin differ from each other in particle size and shape. In general, the finest grade produces the least porous parts with the highest physical and electrical properties, but are more difficult to process because of low bulk density, high compression ratio, high shrinkage, and poor flowability.

ASTM Spec 1457-62T designates three types of granular TFE powders (I, II, and IV) which correspond to the following commercial grades:

a)	ASTM I	Teflon 1	Halon G-10	
b)	ASTM II	Teflon 5	Halon G-50	Fluon G-1
c)	ASTM IV	Teflon 7	Halon G-80	Fluon G-4

TFE past-extrusion grades are granular powders precipitated from dispersions and used with a lubricant such as naphtha for extrusion of thin wall tubings and wire coating

Table 12.3.1. General Propert
(References 19-227, 65-28, 8

ELASTOMER TYPES	Natural Rubber (NR)	Isobutylene Isoprene Rubber (IIR)	Nitrile Butadiene Rubber (NBR)	Chloroprene Rubber (CR)	Silicone Rubber	Polysulfide Rubber	Polyurethane Elastomer	Chlorosulfonated Polyethylene
COMMON NAME	Crude rubber	Butyl or GR-I	Buna-N	Neoprene or GR-N				
TRADE NAME (Alphabet letters in parentheses refer to Manufacturers' Listing in right-hand column)		Hycar 2202(a) Enjay Butyl(b) Polysar Butyl(c)	Chemigum(d), Butaprene(e), Paracril(f), Polysar (c) Krynac(c), Hycar(a), Herecol(g)	Neoprene(h)	Silastic(i), Silicone rubber(j,k)	Thiokol(l)	Thiokolthanes(l), Multrathanes(u), Chemigum SL(d), Genthane(q), Adiprene(l)	Hypalon(h) Hypalon 20(h)
ASTM-SAE DESIGNATION (1)	Type R	Type R	Type S, Class SB	Type S, Class SC	Type T, Class TA	Type S, Class SA	Type S, Class SB	Type S, Class SC
RELATIVE COST - BUNA S = 100	115	125	140	125	1000	250	500	170
PHYSICAL AND MECHANICAL PROPERTIES								
Durometer Range (Shore A)	30-100	40-75	20-100	40-95	45-80	20-80	A10 to D80	45-95
Specific Gravity (base elastomer)	0.93	0.92	1.00	1.23	1.17-1.46	1.34	1.05	1.10
Density, lb/in^3 (base elastomer)	0.033	0.033	0.036	0.044	0.045	0.048	0.039	0.040
Tensile Strength, psi								
Pure gum	2500-3500	2500-3000	500-900	3000-4000	Under 400	> 1000	> 5000	4000 (Max.)
Reinforced	3500-4500	2500-3000	3000-4500	3000-4000	600-1500			3500 (Max.)
Elongation, %								
Pure gum	750-850	750-950	450-700	800-900	Under 200	450-650	540-750	600
Reinforced	550-650	650-850	450-650	500-600	200-800			500
Thermal Conductivity, Btu/hr/sq ft/(°F/ft) (2)	0.082	0.053	0.143	0.112	0.13			0.065
Coefficient of Therm. Exp., cubical, $in^3/in^3/°F$ (3)	37×10^{-5}	32×10^{-5}	39×10^{-5}	34×10^{-5}	45×10^{-5}			27×10^{-5}
Electrical insulation	Good	Good	Poor	Fair	Excellent	Fair	Fair	Fair
Rebound								
Cold	Excellent	Bad	Good	Very good	Very good	Good	Bad	Good
Hot	Excellent	Very good	Good	Very good	Very good	Good	Good	Good
Compression set	Good	Fair	Good	Fair to good	Good to excellent	Poor	Excellent	Fair to good
RESISTANCE PROPERTIES								
Temperature:								
Tensile strength at 250°F, psi (4)	2500	1000	700	1500	850	700	1800	500
Tensile strength at 400°F, psi (5)	125	350	130	180	400	Under 25	200	200
Elongation at 250°F, % (4)	500	250	120	320	350	140	250	60
Elongation at 400°F, % (5)	80	80	20	10	200	Under 25	140	20
Low temperature brittle point, °F	-90	-80	-65	-45	-90 - -200	-60	-63	-80
Low temperature range of rapid stiffening, °F	-20 - -50	0 - -20	+30 - -20	+10 - -20	-60 - -120	-15	-10 - -30	-30 - -50
Drift, room temperature	Very good	Fair	Very good	Fair	Poor to excellent	Poor	Good to excellent	Fair
Drift, elevated temperature - 158-212°F	Good	Fair	Good	Fair to good	Excellent	Poor	Excellent	Fair to good
Heat aging, 212°F	Good	Good	Good	Good	Excellent	Good	Fair	Good
Max. recomm. continuous service temp., °F	180	300	250	240	480	250	240	325
Min. recomm. service temp., °F	-60	-50	-60	-40	-178	-60	-65	-40
Mechanical:								
Tear resistance	Very good	Good	Fair	Good	Poor	Poor	Excellent	Fair to good
Abrasion resistance	Excellent	Good	Excellent	Good to excellent	Poor	Poor	Excellent	Good to excellent
Impact resistance (fatigue)	Very good	Fair	Fair	Good	Poor	Poor	Good	Fair to good
Chemical:								
Sunlight aging	Poor	Very good	Poor	Very good	Excellent	Very good	Excellent	Excellent
Weather resistance	Fair	Excellent	Good	Excellent	Excellent	Excellent	Excellent	Excellent
Oxidation	Good	Very good	Good	Excellent	Excellent	Very good	Very good	Excellent
Acids								
Dilute	Fair to good	Excellent	Good	Excellent	Very good	Good	Fair	Excellent
Concentrated	Fair to good	Excellent	Good	Good	Good	Good	Poor	Excellent
Alkali	Fair to good	Very good	Fair to good	Good	Fair to excellent	Good	Poor to fair	Good
Alcohol	Good	Excellent	Excellent	Fair	Good	Good	Good	Good
Petroleum products, resistance (6)	Poor	Poor	Excellent	Good	Poor to Fair	Excellent	Good	Fair
Coal tar derivatives, resistance (7)	Poor	Poor	Good	Poor	Poor	Excellent	Poor	Good
Chlorinated solvents, resistance (8)	Good	Good	Poor	Poor	Good	Good	Fair to good	Poor to fair
Hydraulic oils								
Silicates	Poor	Poor	Fair to good	Good	Poor	Poor to fair		Good
Phosphates	Poor to fair	Good	Poor	Poor	Good	Poor		Poor
Water swell resistance	Excellent	Excellent	Excellent	Good	Good	Excellent	Good	Good
Permeability to gases	Good	Outstanding	Good	Very good	Good	Excellent	Good	Very good

A

. General Properties of Elastomers
(References 19-227, 65-28, 65-29, 479-1)

	Chlorosulfonated Polyethylene	Vinylidene Fluoride Hexafluoropropylene	Fluorosilicone	Polytrifluorochloroethylene	Styrene Butadiene Rubber (SBR)	Polybutadiene (BR)	Ethylene Propylene Rubber (E. P.)	Polyacrylate (Polyacrylic)
					Buna-S or GR-S			Acrylic
(f), (e), (d),	Hypalon(h) Hypalon 20(h)	Viton(h) Fluorel(o)	Silastic LS-53(i)	Kel-F(o)	Philprene(r), Naugapol(f), Synpol(t), Copo(s)	Diene(p) Philprene-Cis-4(r)	Nordel(h), Royalene, Enjay EPR(b) Dutral N(m), Olethene(n)	Thiacril(l) Hycar 4021(a) Acrylon(h) Polysar Krynac 880(s)
S8	Type S, Class SC	Type 1, Class TB	Type T	Type T, Class TA	Type R	Type R	Type R	Type T, Class TB
	170	3000		3000	100	115	100	400
	45-95	55-90	50-6C	55-90	40-100	45-80	30-90	40-90
	1.10	1.4-1.85	1.41-1.46	1.4-1.85	0.94	0.94	0.86	1.10
	0.040	0.051-0.067	0.051	0.051-0.067	0.034	0.034	0.031	0.040
	4000 (Max.)	> 2000	1000	350-600	200-3000	200-1000		250-400
	3500 (Max.)				2500-3000	2030-3000	2000-3400	1000-2500
	600	> 350	200	300-800	400-600	400-1000		450-750
	500				300-600	410-600	400-600	150-450
	0.065		0.13		0.143			
	27×10^{-5}	27×10^{-5}	45×10^{-5}		37×10^{-5}	37.5×10^{-5}		
	Fair	Excellent	Good	Excellent	Good	Good	Excellent	Fair
	Good	Good	Very good		Good	Excellent	Good	Fair
	Good	excellent	Very good		Good	Excellent	Good	Very good
	Fair to good	Good to excellent	Good	Good to excellent	Good	Fair	Fair	Fair to good
	500	300-800		300-800	1200	1200	2000	1300
	200	130-300		150-250	170	170	400	225
	60	100-350		100-350	250	250	300-500	400
	20	50-100		50-100	60	60	0-120	150
	-80	+10 -60	-90	+10 - -40	-80	-100	-90	-10
	-30 - -50	+20 - -30		+20 - -30	0 - -50	-20 - -60	-20 - -50	+35 - -10
ellent	Fair	Good		Good	Very good	Good	Fair	Fair
	Fair to good	Good to excellent	Excellent	Good to excellent	Good	Good	Fair	Fair
	Good	Excellent	Excellent	Excellent	Good	Fair	Excellent	Excellent
	325	450	500	400	180	200	300	350
	-40	-50	-90	-60	-60		-60	-20
	Fair to good	Poor to good		Poor to good	Fair	Good	Poor	Fair
	Good to excellent	Good	Poor	Good	Excellent	Excellent	Good	Fair to good
	Fair to good	Poor to good	Poor	Poor to good	Very good	Good	Good	Poor
	Excellent	Excellent	Excellent	Excellent	Poor	Poor	Good	Excellent
	Excellent	Excellent	Excellent	Excellent	Fair	Fair	Excellent	Excellent
	Excellent	Excellent	Excellent	Excellent	Good	Good	Excellent	Excellent
	Excellent	Excellent	Excellent	Excellent	Fair to good		Good	Fair
	Excellent	Good	Very good	Excellent	Fair to good		Good	Fair
	Good	Poor to fair	Fair to excellent	Poor to fair	Fair to good	Fair to good	Good to excellent	Poor
	Good	Excellent	Poor to good	Excellent	Fair	Fair to good	Poor	Poor
	Fair	Good to excellent	Excellent	Good to excellent	Poor	Poor	Poor	Excellent
	Good	Excellent		Excellent	Poor	Poor	Fair	Poor
	Poor to fair	Good		Good	Good	Poor	Poor	Poor
	Good	Good		Good	Poor to fair	Poor	Fair to good	Good
	Poor	Poor		Poor	Poor	Poor to fair	Good to excellent	Poor
	Good	Good	Excellent	Good	Excellent	Excellent	Good to excellent	Fair
	Very good	Excellent	Fair	Excellent	Good	Good	Good	Good

NOTES (from first column):

(1) Data recorded is that most generally associated with various elastomer types described. Significant change can be made in many instances by special compounding. In an effort to arrive at an optimum balance of properties, evaluation compounds may deviate from values given here.

(2) Type R - Non-oil resistant
Type S - Resistant to petroleum chemicals
Type T - Temperature resistant
Class SB - Low volume swell
Class SC - Medium volume swell
Class TA - High and low temperature
Class TB - Hot air and oil.

(3) Linear coefficient of expansion is 1/3 volumetric coefficient.

(4) Values would be subject to the balance required with reference to other properties. Samples aged at this temperature just long enough to attain ambient.

(5) Samples aged eight hours before being tested at this temperature. Eight hours is not usually long enough for aging to become a factor, however, most elastomers except silicones and fluoroelastomers degrade rapidly at this temperature range.

(6) Petroleum products & esters, gasoline, Stoddard Solvent, mineral spirits, etc.), straight carbon chain.

(7) Coal tar derivatives (benzol, toluol, xylol) normally have higher solvent power than petroleum products. Referred to as benzene ring or ring hydrocarbons.

(8) Chlorinated solvents (carbon tetrachloride, trichlorethylene, perchlorethylene), high solvency power.

MANUFACTURER LISTING

(a) B. F. Goodrich Chemical Co.
(b) Enjay Chemical Company
(c) Polymer Corp. Ltd.
(d) Goodyear Tire and Rubber Co.
(e) Firestone Tire and Rubber Co.
(f) Naugatuck Chemical
(g) Hercules and Chemical Co.
(h) E. I. duPont de Nemours
(i) Dow Corning Corp.
(j) General Electric
(k) Union Carbon and Carbide
(l) Thiokol Chemical Corp.
(m) Montecatini Soc. Gen.
(n) Avisun Corp.
(o) Minnesota Mining and Mfg. Co.
(p) Firestone Synthetic Rubber Corp.
(q) The General Tire and Rubber Co.
(r) Phillips Petroleum Co.
(s) CoPolymer Rubber and Chemical Corp.
(t) Texas-U.S. Chemical Co.
(u) Mobay Chemical Co.

B

COMMON NAME	Polyamide Type 6/6 Nylon	High-density Polyethylene	Unmodified Polypropylene	Polytetrafluoroethylene	Flexible Unfilled Polyvinyl Chloride
TRADE NAME (Alphabet letters in parentheses refer to Manufacturers' Listing in right-hand column)	Zytel(a)	Alathon(a), Dylan(b) Marlex(d)	Moplen(c) Pro-fax(e)	Fluorosint(f, g) Teflon TFE(a) Halon TFE(n)	Tygon(h) Opalon(i)
PHYSICAL AND MECHANICAL PROPERTIES					
Specific gravity	1.09-1.14	0.941-0.965	0.900-0.915	2.13-2.22	1.16-1.35
Tensile strength, psi	7000-12,000	3100-5500	4800-5000	2000-4500	1500-3500
Elongation, %	25-200	15-100	200-700	200-400	200-450
Tensile modulus, 10^5 psi	2.6-4.0	0.6-1.5	1.6-2.0	0.58	
Compressive strength, psi	7200-13,000	3200	6000-8000	1700	900-1700
Flexural strength, psi	8000-13,000	1000	6000-8000		
Impact strength, ft-lb per in. of notch	1.0	1.5-2.0	0.6-6.0	3.0	Varies with amount of plasticizer
Hardness, Rockwell	R111-R118	D60-D70 (Shore)	R85-R110	D50-D65 (Shore)	A50-A100 (Shore)
Thermal conductivity, Btu/hr/sq.ft/(°F/in)	1.7	3.2-3.6	0.95	1.75	0.87-1.16
Specific heat, $Btu/lb_m/°F$	0.4	0.55	0.46	0.25	0.3-0.5
Coeff. of linear exp. in/in/°F × 10^{-5}	5.5	8.3-16.7	6-9	5.5	4-14
Volume resistivity, ohm-cm	$(0.45-4) \times 10^{14}$	$10^{15} - 10^{16}$	6.5×10^{16}	$> 10^{18}$	$10^{11} - 10^{13}$
Clarity	Translucent to opaque	Transparent to opaque	Translucent, transparent, opaque	Opaque	Transparent to opaque
PROCESSING PROPERTIES					
Molding qualities	Excellent	Excellent	Excellent		Good
Injection molding temperature, °F	470-720	300-600	390-580		320-385
Mold Shrinkage, in.-per-in.	0.015	0.02-0.05	0.01-0.25		0.010-0.050
Machining qualities	Excellent	Excellent	Excellent	Excellent	
RESISTANCE PROPERTIES					
Mechanical abrasion and wear Tabor CS 17 Wheel mg, loss/1000 cycles	6-8	2.57	18-28		
Temperature:					
Flammability	Self-extinguishing	Very slow	Slow to self-extinguishing	None	Slow to self-extinguishing
Low temperature brittle point, °F		-76 to -200	0	-420	
Resistance to heat, °F (continuous)	270-300	250	250-320	550	150-175
Deflection temperature under load, °F	300-360 (66 psi)	140-180 (66 psi)	210-240 (66 psi)	250 (66 psi)	
Chemical:					
Effect of sunlight	Discolors slightly	Unprotected material crazes; requires black; weather resist. grade avail.	Unprotected material crazes; requires black; weather resist. grade avail.	None	Slight
Effect of weak acids	Resistant	Very resistant	Very resistant	None	None
Effect of strong acids	Attacked	Attacked slowly by oxidizing acids	Attacked slowly by oxidizing acid	None	None to slight
Effect of weak alkalies	None	Very resistant	None	None	None
Effect of strong alkalies	None	Very resistant	Very resistant	None	None
Effect of organic solvents	Resistant to common solvents	Resistant (below 80°C)	Resistant (below 80°C)	None	Resists alcohols, aliph. hydrocarbons, oils; soluble in ketones and esters; swells in aromatic hydrocarbons

THE FOLLOWING NOTES REFER TO POLYESTER AND POLYMIDE COLUMNS:

* Type T

** Film

ISSUED: MARCH 1967
SUPERSEDES: OCTOBER 1965

Table 12.3.2. General Properties of Plastics
(References 1-279, 19-238, 510-1)

...modified ...propylene	Polytetrafluoroethylene	Flexible Unfilled Polyvinyl Chloride	Fluorinated Ethylene Propylene	Polychlorotrifluoroethylene	Polyvinylidene Fluoride	Polyvinyl Fluoride	Polyester	Polyvinylidene Chloride	Polyimide	Rigid Polyvinyl Chloride	Chlorinated Polyether
(e) (e)	Fluorosint(f,g) Teflon TFE(e) Halon TFE(n)	Tygon(h) Opalon(l)	Teflon FEP(e)	Kel-F(j)	Kynar(l)	Tedlar(e)	Mylar(e)	Saran(k)	Kapton H (e) Kapton F (e) Vespel (e) Polymer SP-1 (e)		Hypalon(e)
.915	2.13-2.22	1.16-1.35	2.14-2.17	2.1-2.2	1.76-1.77	1.38	1.38-1.395	1.20-1.68	1.42	1.35-1.45	1.4
00	2000-4500	1500-3500	2700-3100	4500-6000	7000	7000-18,000	23,000; 40,000*	8000-20,000	25,000**; 10,500	5000-9000	6000
	200-400	200-450	250-330	30-250	100-300	115-250	100; 50*	20-14[illegible]	70**; 6-8	2.0-40	130
	0.58		0.5	1.5-3	1.2	3.2	5.5	0.5-0.8	4.3	3.5-6	1.5
00	1700	900-1700	2200	32,000-80,000	10,000			2000-2700	24,400	8000-13,000	9000
00				7400-9300				4200-6200	14,000	10,000-16,000	5000
	3.0	Varies with amount of plasticizer	No break	0.8-5.0	3.[illegible]			0.3-1.0	0.9	0.4-20	0.4
0	D50-D65 (Shore)	A50-A100 (Shore)	R25	R110-R115	D80 (Shore)			M50-M65	H85-95	D70-D90 (Shore)	R100
	1.75	0.87-1.16	1.75	0.9	0.9		1.035	0.9	2.2	0.9-2.0	1.2
	0.25	0.3-0.5	0.28	0.22	0.33			0.32	0.27	0.2-0.28	
	5.5	4-14	4.7-5.8	5-15	6.7	2.8	1.5	10.5	28-35	2.8-10.3	6.6
16	$> 10^{18}$	$10^{11} - 10^{15}$	$> 2 \times 10^{18}$	1.2×10^{18}	2×10^{14}	3×10^{13}	10^{18}	$10^{12} - 10^{15}$	10^{18}	10^{16}	$1.5 - 10^{16}$
ent, ent,	Opaque	Transparent to opaque	Transparent to translucent	Transparent to translucent	Transparent to translucent	Transparent	Transparent	Transparent to opaque	Opaque	Transparent to opaque	Translucent to opaque
		Good	Excellent	Excellent	Excellent			Excellent		Fair to good	Good
		320-385	625-760	440-600	450-550			300-400		300-400	440-465
25		0.010-0.050	0.03-0.06	0.005-0.010	0.030			0.005-0.025		0.001-0.004	0.004-0.008
	Excellent		Excellent	Excellent	Excellent			Good		Excellent	Excellent
				0.01	17.6					90-120	
self- ...hing	None	Slow to self-extinguishing	None	None	Self-extinguishing	Slow to self-extinguishing	Slow to self-extinguishing	Self-extinguishing		Self-extinguishing	Self-extinguishing
	-420		-420	-400	-80	-100				+10 - -20	
	550	150-175	400	350-390	300	220-250	300	150-200 (dry) 300 (wet)	500	120-160	250
(66 psi)	250 (66 psi)			258 (66 psi)	300 (66 psi) 195 (264 psi)			1[illegible]0-150		130-165	300 (66 psi)
...ted material ...equires ...eather ...rade avail.	None	Slight	None	None	Slight bleaching on long exposure	Excellent	Type W excellent; others moderate	Good	Degrades after prolonged exposure	Darkens on prolonged intense exposure	Slight
...istant	None	None	None	None	None			None	None	None	None
... slowly ...zing acids	None	None to slight	None	None	Attacked by fuming sulfuric	None	Slight	None		None	Attacked only by oxidizing a...
	None	None	None	None	None			Resistant		None	None
...istant	None	None	None	None	None	None	Slight	None to slight	Attacked	None	None
...0°C)	None	Resists alcohols, aliph. hydrocarbons, oils, soluble in ketones and esters; swells in aromatic hydrocarbons	None	Halogenated compounds cause slight swelling	Resists most solvents	None	None	Slight to moderate	Resistant to most organic solvents	Resists alcohols, aliph. hydrocarbons, oils; soluble in ketones and esters; swells in aromatic hydrocarbons	Resists most solvents

B

...of Plastics
...1)

...vinylidene Fluoride	Polyvinyl Fluoride	Polyester	Polyvinylidene Chloride	Polyimide	Rigid Polyvinyl Chloride	Chlorinated Polyether	Acetal
...r(l)	Tedlar(a)	Mylar(a)	Saran(k)	Kapton H (a) Kapton F (a) Vespel (a) Polymer SP-1 (a)		Hypalon(a)	Delrin (a) Celcon (m)
...-1.77	1.38	1.38-1.395	1.20-1.68	1.42	1.35-1.45	1.4	1.425
...	7000-18,000	23,000; 40,000*	8000-20,000	25,000**; 10,500	5000-9000	6000	10,000
...300	115-250	100; 50*	20-140	70**; 6-8	2.0-40	130	15
	3.2	5.5	0.5-0.8	4.3	3.5-6	1.5	4.1
...00			2000-2700	24,400	8000-13,000	9000	5,200 (1% deformation)
			4000-6200	14,000	10,000-16,000	5000	14,100
			[illegible]-1.0	0.9	0.4-20	0.4	1.4
...(Shore)			M50-M65	H85-95	D70-D90 (Shore)	R100	R120
		1.035	0.9	2.2	0.9-2.0	1.2	1.6
...			0.32	0.27	0.2-0.28		0.35
	2.8	1.5	10.5	28-35	2.8-10.3	6.6	4.5
...0^{14}	3×10^{13}	10^{18}	$10^{12} - 10^{15}$	10^{18}	10^{16}	$1.5 - 10^{16}$	6×10^{14}
...parent to ...cent	Transparent	Transparent	Transparent to opaque	Opaque	Transparent to opaque	Translucent to opaque	Translucent to opaque
...llent			Excellent		Fair to good	Good	Excellent
...50			300-400		300-400	440-465	380-440°F
...			0.005-0.025		0.001-0.004	0.004-0.008	
...llent			Good		Excellent	Excellent	Excellent
					90-120		
...guishing	Slow to self-extinguishing	Slow to self-extinguishing	Self-extinguishing		Self-extinguishing	Self-extinguishing	
	-100				+10 - -20		
	220-250	300	150-200 (dry) 300 (wet)	500	120-160	250	180-220
...6 psi) ...64 psi)			130-150		130-165	300 (66 psi)	
...bleaching ...g exposure	Excellent	Type W excellent; others moderate	Good	Degrades after prolonged exposure	Darkens on prolonged intense exposure	Slight	Chalks slightly
			None	None	None	None	Resistant to some
...ed by ...g sulfuric	None	Slight	None		None	Attacked only by oxidizing acids	Attacked
			Resistant		None	None	Resistant to some
	None	Slight	None to slight	Attacked	None	None	Attacked
...s most ...ts	None	None	Slight to moderate	Resistant to most organic solvents	Resists alcohols, aliph. hydrocarbons, oils; soluble in ketones and esters; swells in aromatic hydrocarbons	Resists most solvents	Resistant to most solvents

MANUFACTURERS' LISTING

(a) E. I. duPont de Nemours
(b) Koppers Co.
(c) Chemore Corp.
(d) Phillips Chemical Co.
(e) Hercules Powder Co., Inc.
(f) Polymer Corp. of Pennsylvania
(g) Polypenco, Inc.
(h) United States Stoneware Co.
(i) Monsanto Chemical Co.
(j) Minnesota Mining and Mfg. Co.
(k) Dow Chemical Co.
(l) Pennsalt Chemicals Corp.
(m) Celanese Polymer Co.
(n) Allied Chemical Corp.

ASTM Grade III, Class 1 and 2, corresponds to these and is available in such commercial products as Teflon 6 and 6C and Fluon CD-1, CD-3W, and CD-3. Aqueous dispersions of TFE resins are available as Teflon 30, 30B, and 41BX from duPont.

The granular grades of TFE corresponding to ASTM I, II, and IV are all essentially the same resin differing only in mechanical treatment affecting size and shape. The ASTM Type III paste extrusion grades and the dispersions from which they are made are of lower molecular weight than the granular resins and are of much finer fundamental particle size. The lower molecular weight results in substantially different physical properties and calls for different processing technology.

Properties of TFE moldings are determined in part by crystallinity level which can be controlled by proper rate of cooling from gel during processing. This is easier to control with molded than extruded TFE. For maximum properties, molded material should be specified, however ram extruded TFE is less expensive for small diameters and is available in longer lengths for screw machining.

12.3.2.3 KEL-F. Kel-F is a polymer of chlorotrifluoroethylene (CTF) possessing chemical and physical characteristics quite similar in nature to those of Teflon. A Kel-F molecule differs from Teflon TFE in that one in four fluorine atoms are replaced by a chlorine atom. The introduction of the chlorine atom into the molecule contributes to the transparency, moldability, and rigidity characteristics of the plastic, while the fluorine atoms are responsible for its chemical inertness, thermal stability, and zero moisture absorption. Kel-F can be molded by conventional thermoplastic forming techniques. It is impervious to corrosive chemicals and highly resistant to most organic solvents. Swelling may occur in the presence of highly halogenated and aromatic compounds, but the reaction is reversable.

12.3.2.4 POLYAMIDE (NYLON). Nylon is among the strongest of the thermoplastics. Nylons have good abrasion resistance, low friction characteristics, and relatively high heat resistance. However, they do have a relatively high moisture absorption and the properties are sensitive to moisture content. Nylons are resistant to most solvents and particularly resistant to petroleum oils and greases. The heat distortion point of nylon is exceeded among plastics only by that of Teflon. Its chemical inertness is generally inferior to polyethylene, Kel-F, Teflon, and vinyls. Nylon is used in fluid components as a valve seal material and for journal-bearing applications.

12.3.2.5 POLYPROPYLENE. Polypropylene, like polyethylene, is a polyolefin having properties similar to high density polyethylenes. Polypropylene has excellent resistance to creep and environmental stress cracking. It also has low water absorption and low permeability to water vapor and solvents. Polypropylene has greater rigidity, strength, and heat resistance than polyethylene. It is resistant to most acids, alkalies, and saline solutions, even at elevated temperatures, and is also resistant to organic solvents and polar substances. Above 175°F, polypropylene is soluble in aromatic substances such as toluene and xylene, and chlorinated hydrocarbons such as trichloroethylene.

12.3.2.6 POLYVINYL CHLORIDE. Polyvinyl chloride (PVC) materials can be formulated to provide a diverse combination of properties and generally are tough, with high inherent strength and abrasion resistance and exceptional chemical resistance. PVC materials can be broadly classified as either rigid or flexible. Rigid, or unplasticized, PVC has a Shore hardness range from B75 to B85, similar to hard rubber. Flexible, or plasticized, PVC has elastomeric properties, typically in the Shore hardness range of A50 to A100. Polyvinyl chloride materials have found only limited application in aerospace valves due primarily to the fact that the more corrosion-resistant rigid PVC is rather hard and unyielding, making it generally unsuitable for sealing applications. The addition of plasticizers in order to obtain rubber-like characteristics frequently seriously degrades the chemical resistance.

12.3.2.7 POLYVINYLIDENE CHLORIDE. Vinylidene chloride is most often used when copolymerized with vinyl chloride. The polyvinylidene chloride copolymer is best known as a film or fiber, although it can also be molded and extruded. In the extruded form, it is used as a valve seal material. It has excellent resistance to all acids and most common alkalies.

12.3.2.8 POLYVINYLIDENE FLUORIDE. Polyvinylidene fluoride is a material which is characterized by toughness, which means the material has a good combination of tensile strength, elongation, impact, and abrasion resistance. Polyvinylidene fluoride is resistant to most corrosive chemicals and organic compounds including acids, alkalies, strong oxidizers, and halogens.

12.3.2.9 POLYVINYL FLUORIDE. Polyvinyl fluoride is characterized by an exceptional resistance to ultraviolet light, solvents, chemicals, and oils, while at the same time remaining tough, flexible, and dimensionally stable over a broad temperature range. It is available as both a clear and pigmented film.

12.3.2.10 POLYESTER. The term polyester covers a large number of plastics, most of which are laminating and casting resins; however, the polyester used most commonly in fluid component applications is polyethylene terephthalate film. The most widely known polyester film is Mylar, manufactured by the du Pont Company. This material is a clear, tough film having good elongation and impact resistance. It has low moisture absorption, is dimensionally stable under extremes of temperatures and humidity, and has excellent resistance to acids, greases, oils, and organic solvents. The material is useful for diaphragm applications, retaining sufficient flexibility in thin sections to be used for diaphragm applications down to liquid hydrogen temperatures (−423°F).

12.3.2.11 CHLORINATED POLYETHER. Chlorinated polyether has an outstanding combination of mechanical properties and chemical resistance. It has good load-bearing characteristics and excellent abrasion resistance.

12.3.2.12 POLYIMIDES. Polyimides are characterized by physical toughness and resistance to abrasion, radiation, and many chemicals. They are characterized as extremely heat-stable polymers, with useful temperatures as high as 1000°F, and are available in either molded shapes or thin films, all offering unique high temperature properties. Polyimide film has excellent mechanical properties throughout a wide range, from liquid helium temperatures (−423°F) to as high as 1000°F. It ranks among the toughest polymeric films, having a high tensile strength, high impact strength, and high resistance to tear initiation.

12.3.2.13 ACETAL. Acetal is a hard thermoplastic which looks and feels something like nylon. The material is highly crystalline, making it one of the strongest and stiffest plastics. Acetal is also characterized by high heat resistance, excellent fatigue life, low frictional characteristics, good creep resistance, and resistance to organic solvents. Abrasion resistance of acetals, although not as good as that of nylons, is better than that of many thermoplastics. Acetal is available in two basic types: a homopolymer (du Pont's Delrin) and a copolymer (Celanese's Celcon). Typical applications of acetal include gears, bearings, valve seats, and fittings.

12.4 PROPERTIES OF METALS

Because extensive published material exists on metal properties, this handbook includes only tables giving representative data indicating the range of usefulness of various classes of metal or alloy groups. Readily available sources on detailed metals data include the following:

1. METALS HANDBOOK — VOL I: PROPERTIES AND SELECTION OF METALS
8th ed., Metals Park, Ohio, American Society for Metals, 1961, 1300 pp.

2. METALS HANDBOOK — VOL II: HEAT TREATING, CLEANING AND FINISHING
8th ed., Metals Park, Ohio, American Society for Metals, 1964, 708 pp.

3. METALLIC MATERIALS AND ELEMENTS FOR AEROSPACE VEHICLE STRUCTURES
Research and Technology Division (MAAE), Wright-Patterson Air Force Base, Ohio 45433, MIL-HDBK-5A, February 8, 1966 (see latest change notice). (For sale by Supt. of Documents, U.S. Govt. Printing Office, Washington D.C., 20402.)

4. AEROSPACE STRUCTURAL METALS HANDBOOK — VOL I: FERROUS ALLOYS*
Syracuse Univ. Research Inst., Syracuse, N. Y., Contr. AF 33(616)-1184, Proj. No. 7381, Task No. 738103, March 1967, 620 pp.

5. AEROSPACE STRUCTURAL METALS HANDBOOK — VOL II: NON-FERROUS LIGHT METAL ALLOYS*
Syracuse Univ., Research Inst., Syracuse, N.Y., Contr. AF 33(615)-1184, Proj. No. 7381, Task No. 738103, March 1967, 464 pp.

6. AEROSPACE STRUCTURAL METALS HANDBOOK — VOL IIA: NON-FERROUS HEAT RESISTANT ALLOYS*
Syracuse Univ., Research Inst., Syracuse, N.Y., Contr. AF 33(615)-1184, Proj. No. 7381, Task No. 738103, March 1967, 380 pp.

7. MATERIALS SELECTOR ISSUE
Materials Engineering, V. 70, No. 5, Mid-October 1969, 516 pp., (annual).

8. CRYOGENIC MATERIALS DATA HANDBOOK
The Martin Co., Denver, Colo., Contr. AF 33(657)-9161, Proj. No. 7391, Task No. 738106, ML-TDR-64-280, OTS PB 171 809 (Rev.), Supplement 3, AD 633 388, March 1966, 745 pp.

9. Defense Metals Information Center, Battelle Memorial Institute, Columbus, Ohio (branch office, Los Angeles, California). Publications include Reports, Memoranda, Reviews of Recent Developments, and Technical Notes.

10. Military Standardization Handbooks, designated as MIL-HDBK-XXX, prepared for Department of Defense by Army Materials Research Agency, Watertown Arsenal, Watertown, Mass. 02172.

12.4.1 Ferrous Metals

Typical ranges of property values for ferrous alloys are presented in the following tables. All property data are for room temperature unless otherwise specified.

*More recent editions of Aerospace Structural Metals Handbook are available from Mechanical Properties Data Center, 13919 W. Bay Shore Dr., Traverse City, Michigan 49684.

Table 12.4.1e. Properties of Age Hardenable Stainless Steels (Specific Material Types: Stainless W, 17-4 PH, 17-7 PH, PH 15-7Mo, AM 350, AM 355)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.276 - 0.282	65-29
Modulus of Elasticity in Tension, psi	$28 - 29.4 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	812×10^3	65-29
Tensile Strength, psi	$195 - 240 \times 10^3$	65-29
Yield Strength, psi	$173 - 225 \times 10^3$	65-29
Creep Strength, psi	40 - 1000 (800°F)	65-29
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb	4 - 19	65-29
Hardness, Rockwell		
Elongation, percent	3 - 19	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	8.87 - 10.4 (212°F	65-29
Coefficient of Thermal Expansion, in/in/°F	$5.5 - 6.4 \times 10^6$ (68 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.11	286-7
Melting Point, °F	2500 - 2550	65-29
Electrical Resistivity, microhm-in	29 - 39	65-29

Table 12.4.1b. Properties of Alloy Steels – Hardening Grades, Wrought (Specific Material Types: 4130, 3140, 4140, 4340, 8630, 4150, 8740)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.282	65-29
Modulus of Elasticity in Tension, psi	29 - 30 x 10^6	65-29
Specific Strength, in. (yield strength/density)	821 x 10^3	65-29
Tensile Strength, psi	98 - 301 x 10^3	65-29
Yield Strength, psi	89 - 250 x 10^3	65-29
Creep Strength, psi	16 - 32 x 10^3 (1100°F); 110 x 10^3 (700°F)	236-7
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb	9 - 108	65-29
Hardness, Brinell	202 - 578	65-29
Elongation, percent	10 - 28	65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	21.7 - 38.5 x 10^{-6}	65-29
Coefficient of Thermal Expansion, in/in/°F	6.3 - 8.6	65-29
Specific Heat, Btu/lb °F	0.10 - 0.12	65-29
Melting Point, °F	2600 - 2760	65-29
Electrical Resistivity, microhm-in	$\frac{0°F}{5}$ $\frac{1100°F}{30}$	286-7

Table 12.4.1c. Properties of Austenitic Stainless Steels, Wrought (Specific Materials. Types: 201, 202, 301, 302, 303, 304, 304L, 305, 309, 309S, 310, 310S, 316, 316L, 316ELC, 321, 347)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.28 - 0.29	65-29
Modulus of Elasticity in Tension, psi	$28 - 29 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	463×10^3	65-29
Tensile Strength, psi	$85 - 115 \times 10^3$ Annealed $110 - 185 \times 10^3$ Cold Worked	65-29
Yield Strength, psi	$30 - 55 \times 10^3$ Annealed $75 - 140 \times 10^3$ Cold Worked	65-29
Creep Strength, psi	$15 - 35 \times 10^3$ (1000°F)	65-29
Endurance Limit, psi	$30 - 40 \times 10^3$ Annealed 80×10^3 Cold Worked	65-29
Impact Strength, Notched Izod, ft/lb	80 - 110	65-29
Hardness, Brinell	150 - 170 Annealed 240 Cold Worked	65-29
Elongation, percent	45 - 60 Annealed 8 - 60 Cold Worked	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	8.3 - 9.4 (212°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	$8.7 - 9.6 \times 10^{-6}$ (32 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.12 (32 - 212°F)	65-29
Melting Point, °F	2500 - 2650	65-29
Electrical Resistivity, microhm-in	27 - 31	65-29

Table 12.4.1d. Properties of Carbon Steels — Hardening Grades (Specific Material Types: C1030, C1040, C1050, C1060, C1080, C1095, C1137, C1141, C1144)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.282	65-29
Modulus of Elasticity in Tension, psi	$29 - 30 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	502×10^3	65-29
Tensile Strength, psi	75 - 237	65-29
Yield Strength, psi	58 - 188	65-29
Creep Strength, psi		
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb	5 - 22	65-29
Hardness, Brinell	174 - 495	65-29
Elongation, percent	6 - 33	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^oF/ft)$	27	65-29
Coefficient of Thermal Expansion, $in/in/^oF$	6.7 - 8.4	65-29
Specific Heat $Btu/lb\ ^oF$	0.10 - 0.11	65-29
Melting Point, ^oF	2700 - 2775	65-29
Electrical Resistivity, microhm-in	56 - 75	65-29

ISSUED FEBRUARY 1970
SUPERSEDES: OCTOBER 1965

Table 12.4.1e. Properties of Ferritic Stainless Steels, Wrought (Specific Material Types: 405, 430, 530F, 446)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.27 - 0.28	65-29
Modulus of Elasticity in Tension, psi	29×10^6	65-29
Specific Strength, in. (yield strength/density)	286×10^3	65-29
Tensile Strength, psi	$65 - 85 \times 10^3$ Annealed $75 - 90 \times 10^3$ Cold Worked	65-29
Yield Strength, psi	$35 - 55 \times 10^3$ Annealed $45 - 80 \times 10^3$ Cold Worked	65-29
Creep Strength, psi	6000 - 8500 (1000°F)	65-29
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb	2 - 25 Annealed	65-29
Hardness, Brinell	137 - 185 Annealed 185 Cold Worked	65-29
Elongation, percent	20 - 30 Annealed 15 - 25 Cold Worked	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	10 - 15 (212°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	6×10^{-6} (32 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.11 - 0.12 (32 - 212°F)	65-29
Melting Point, °F	2600 - 2800	65-29
Electrical Resistivity, microhm-in	23 - 27	65-29

Table 12.4.1f. Properties of High Temperature Steels, Wrought (Specific Material Types: Martensitic Stainless, 422, 1420WM, 1415NW, 1430MV, Low Alloy, Chromalloy, 17-22 AS)

PROPERTY	VALUES		REF.
	Others	Chromalloy, 17-22AS	
Density, lb/in^3	0.281 - 0.285		65-29
Modulus of Elasticity in Tension, psi	$29 - 31.5 \times 10^6$		65-29
Specific Strength, in. (yield strength/density)			
Tensile Strength, psi	$160 - 235 \times 10^3$ (H&T)	$140 - 150 \times 10^3$ (H&T)	65-29
Yield Strength, psi	$125 - 186 \times 10^3$ (H&T)	$117 - 127 \times 10^3$ (H&T)	65-29
Creep Strength, psi	$\frac{1000^\circ F}{50\text{-}70 \times 10^3}$ $\frac{1200^\circ F}{17\text{-}35 \times 10^3}$	$\frac{600^\circ F}{130\text{-}150 \times 10^3}$ $\frac{1350^\circ F}{3\text{-}26 \times 10^3}$	286-7
Endurance Limit, psi			
Impact Strength, Notched Izod, ft/lb	10 - 20 (H&T)	30 (H&T)	65-29
Hardness, Rockwell			
Elongation, percent	8 - 17		65-29
Thermal Conductivity, $Btu/ft^2/hr/(^\circ F/ft)$	15 - 18 ($800^\circ F$)	$\frac{70^\circ F}{18.8}$ $\frac{1200^\circ F}{16.5}$	65-29, 286-7
Coefficient of Thermal Expansion, $in/in/^\circ F$ ($70 - 1000^\circ F$)	$6.3 - 6.5 \times 10^{-6}$	$7.8 - 7.9 \times 10^{-6}$	65-29
Specific Heat, $Btu/lb\ ^\circ F$	0.11		286-7
Melting Point, $^\circ F$	2600 - 2700	2700 - 2750	65-29
Electrical Resistivity, microhm-in	$\frac{700^\circ F}{24.3}$ $\frac{800^\circ F}{23.9}$ $\frac{900^\circ F}{23.7}$		286-7

Table 12.4.1g. Properties of Iron Base Superalloys (Cr-Ni), Wrought (Specific Material Types: 19-9DL, Unitemp 212, W545, Discaloy, D-979, A-286, V-57, 16-25-6, Incoloy 901)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.285 - 0.296	65-29
Modulus of Elasticity in Tension, psi	$28.2 - 30.0 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	495 x 10	65-29
Tensile Strength, psi	$142 - 150 \times 10^3$ (Disaloy, A-286, 16-25-6), 114×10^3 (19-9 DL), $172 - 204 \times 10^3$ (others)	65-29
Yield Strength, psi	$100 - 146 \times 10^3$, 71×10^3 (19-9 DL)	65-29
Creep Strength, psi	$19 - 30 \times 10^3$ A-286 and 16-25-6 (1200°F)	65-29
Endurance Limit, psi	$38 - 60 \times 10^3$ (1200°F, 10^8 cycles)	65-29
Impact Strength, Notched Izod, ft/lb	15 (16-25-6), 64 (A-286) 30-46 (19-9 DL, Unitemp 212, W545, Discaloy)	65-29
Hardness, Brinell	24 - 32 (Aged)	286-7
Elongation, percent	15 - 25, 41 (19-9 DL)	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	12.2 - 15 (212 - 1800°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	$9.2 - 10.7 \times 10^{-6}$ (80 - 1400°F)	65-29
Specific Heat, Btu/lb °F	0.10 - 0.11	65-29
Melting Point, °F	2200 - 2700	65-29
Electrical Resistivity, microhm-in	36 - 41 (120 - 212°F)	65-29

Table 12.4.1h. Properties of Iron Base Superalloys (Cr-Ni-Co), Cast, Wrought (Specific Material Types: Multimet, N-155, Refractaloy 26, S-590)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.296 - 0.301	65-29
Modulus of Elasticity in Tension, psi	28.8 - 31.1 x 10^6	65-29
Specific Strength, in. (yield strength/density)	307 x 10^3	65-29
Tensile Strength, psi	118 - 154 x 10^3	65-29
Yield Strength, psi	58 - 91 x 10^3	65-29
Creep Strength, psi	15 - 37 x 10^3 (1350°F)	65-29
Endurance Limit, psi	33 - 37 x 10^3 (1500°F, 10^8 cycles)	65-29
Impact Strength, Notched Izod, ft/lb	12 - 18 65 (Multimet, N-155)	65-29
Hardness, Rockwell		
Elongation, percent	19 - 40	65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)		
Coefficient of Thermal Expansion, in/in/°F	8.0 - 9.1 x 10^{-6} (70 - 1000°F)	65-29
Specific Heat, Btu/lb °F	0.10 (70 - 212°F)	65-29
Melting Point, °F	2350 - 2500	65-29
Electrical Resistivity, microhm-in		

Table 12.4.1I. Properties of Martensitic Stainless Steels, Wrought (Specific Material Types: 403, 410, 414, 416, 420, 431, 440A, 440B, 440C, 501, 502)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.28	65-29
Modulus of Elasticity in Tension, psi	29×10^6	65-29
Specific Strength, in. (yield strength/density)	782×10^3	65-29
Tensile Strength, psi	$65 - 125 \times 10^3$ Annealed $90 - 285 \times 10^3$ H&T	65-29
Yield Strength, psi	403, 410, 416, 420, 501, 502 - $25\text{-}65 \times 10^3$ Annealed $60\text{-}145 \times 10^3$ H&T 414, 431, 440A, B, C - $60\text{-}105 \times 10^3$ Annealed $95\text{-}275 \times 10^3$ H&T	65-29
Creep Strength, psi	9.2×10^3 (1000°F)	65-29
Endurance Limit, psi	$35 - 40 \times 10^3$ Annealed	65-29
Impact Strength, Notched Izod, ft/lb	440A, B, C: 2 Annealed; 2-4 H&T Others: 50-90 Annealed; 20-75 H&T	65-29
Hardness, Brinell	150 - 250 Annealed 180 - 580 H&T	65-29
Elongation, percent	440A, B, C: 14-20 Annealed; 2-[illegible] H&T Others: 14-35 Annealed; 8-30 H&T	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	10 - 20 (212°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	$5.5 - 6.5 \times 10^{-6}$ (32 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.11 (32 - 212°F)	65-29
Melting Point, °F	2500 - 2800	65-29
Electrical Resistivity, microhm-in	16 - 27	65-29

Table 12.4.1j. Properties of Ultra High Strength Steels, Wrought (Specific Material Types: Modified H-11, MX-2, 300-M, D-6A, 4340, 25 Ni, 20Ni, 18-Ni) Vascojet 1000, Unimach 1,

PROPERTY	VALUES		REF.
Density, lb/in^3	0.275 - 0.295		65-29
Modulus of Elasticity in Tension, psi	24 - 30 x 10^6		65-29
Specific Strength, in. (yield strength/density)	880 x 10^3		65-29
Tensile Strength, psi	275 - 320 x 10^3 (H&T)		65-29
Yield Strength, psi	240 - 290 x 10^3 (H&T)		65-29
Creep Strength, psi	700°F 200-270 x 10^3	1000°F 35-95 x 10^3	286-7
Endurance Limit, psi	110 - 140 x 10^3 (10^6 cycles)		65-29
Impact Strength, Notched Izod, ft/lb	15 - 23 (H&T)		65-29
Hardness, Rockwell	60		286-7
Elongation, percent	6 - 12 (H&T)		65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	16 - 17		286-7
Coefficient of Thermal Expansion, in/in/°F	5.6 - 7.4 x 10^{-6}		65-29
Specific Heat, Btu/lb °F	0.11		286-7
Melting Point, °F	2500 - 2600		286-7
Electrical Resistivity, microhm-in			

ISSUED: MARCH 1967
SUPERSEDES: OCTOBER 1965

12.4.2 Nonferrous Metals

Typical ranges of property values for nonferrous alloys are presented in the following tables. Values given are for room temperature unless otherwise indicated.

Table 12.4.2a. Properties of Aluminum and Its Alloys, Cast (Specific Material Types: 10B, A10B, 40E)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.10	65-29
Modulus of Elasticity in Tension, psi	$10 - 10.6 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	240×10^3	65-29
Tensile Strength, psi	$21 - 35 \times 10^3$ as cast	65-29
Yield Strength, psi	$14 - 24 \times 10^3$ as cast	65-29
Creep Strength, psi		
Endurance Limit, psi	Cast: 13,500 Wrought: 24,000	547-5
Impact Strength, Notched Izod, ft/lb		
Hardness, Brinell	55 - 75 as cast	65-29
Elongation, percent	2.0 - 3.0 as cast	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^{\circ}F/ft)$	70 - 82	65-29
Coefficient of Thermal Expansion, $in/in/^{\circ}F$	$12 - 14 \times 10^{-6}$ (68 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.22 - 0.23	65-29
Melting Point, °F	910 - 1195	65-29
Electrical Resistivity, microhm-in	1.1 - 2.5	65-29

Table 12.4.2b. Properties of Aluminum and Its Alloys, Cast (Specific Material Types: 355, C355, 356, A356, 327)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.10	65-29
Modulus of Elasticity in Tension, psi	$10 - 10.6 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	400×10^3	65-29
Tensile Strength, psi	$40 - 50 \times 10^3$ Solution Treated and Aged	65-29
Yield Strength, psi	$27 - 40 \times 10^3$ Solution Treated and Aged	65-29
Creep Strength, psi	400°F: $9 - 10 \times 10^3$; 600°F: $2 - 3.5 \times 10^3$	286-6
Endurance Limit, psi	$10 - 13 \times 10^3$ (5×10^6 cycles) Solution Treated and Aged	65-29
Impact Strength, Notched Izod, ft/lb		
Hardness, Brinell	80 - 100 Solution Treated and Aged	65-29
Elongation, percent	2 - 10 Solution Treated and Aged	65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	60 - 90 Solution Treated and Aged	65-29
Coefficient of Thermal Expansion, in/in/°F	12×10^{-6} (68 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.22 - 0.23	65-29
Melting Point, °F	910 - 1195	65-29
Electrical Resistivity, microhm-in	1.6 - 1.9	286-8

Table 12.4.2c. Properties of Aluminum and Its Alloys, Wrought (Specific Material Types: 5052, 5056, 5083, 5086, 5454, 6061)

PROPERTY	VALUES		REF.
Density, lb/in^3	0.095 - 0.098		65-29
Modulus of Elasticity in Tension, psi	$10.0 - 10.3 \times 10^6$		65-29
Specific Strength, in. (yield strength/density)	510×10^3		65-29
	5052, 5056, 5083, 5086, 5454	6061	
Tensile Strength, psi	$28 - 45 \times 10^3$ Annealed $38 - 60 \times 10^3$ Hard	18×10^3 Annealed	65-29
Yield Strength, psi	$13 - 22 \times 10^3$ Annealed $26 - 50 \times 10^3$ Hard	8×10^3 Annealed	65-29
Creep Strength, psi	200°F: 20×10^3 400°F: 5×10^3	200°F: 40×10^3 500°F: $6-18 \times 10^3$	286-8
Endurance Limit, psi	$16 - 20 \times 10^3$ Annealed $20 - 22 \times 10^3$ Hard	9×10^3 Annealed	65-29
Impact Strength, Notched Izod, ft/lb			
Hardness, Brinell	47 - 65 Annealed 70 - 100 Hard	30 Annealed	65-29
Elongation, percent	22 - 35 Annealed 7 - 15 Hard		65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	68 - 80		65-29
Coefficient of Thermal Expansion, in/in/°F	$13.1 - 13.4 \times 10^{-6}$ (68 - 212°F)		65-29
Specific Heat, Btu/lb °F	0.22 - 0.23 (212°F)		65-29
Melting Point, °F	1055 - 1200		65-29
Electrical Resistivity, microhm-in	1.9 - 2.3	1.5	65-29

Table 12.4.2d. Properties of Aluminum and Its Alloys, Wrought (Specific Material Types: 2014, 2024, 2219, 7075, 7079, 7178)

PROPERTY	VALUES		REF.
Density, lb/in^3	0.098 - 0.103		65-29
Modulus of Elasticity in Tension, psi	$10.0 - 10.6 \times 10^6$		65-29
	2014, 2024, 2219	7075, 7079, 7178	
Specific Strength, in. (yield strength/density)	600×10^3	780×10^3	65-29
Tensile Strength, psi	$25-27 \times 10^3$ Annealed $62-70 \times 10^3$ Heat Treated	33×10^3 Annealed $78-88 \times 10^3$ Heat Treated	65-29
Yield Strength, psi	$10-14 \times 10^3$ Annealed $42-60 \times 10^3$ Heat Treated	15×10^3 Annealed $68-78 \times 10^3$ Heat Treated	65-29
Creep Strength, psi	13×10^3 (400°F)	6×10^3 (400°F)	65-29
Endurance Limit, psi	13×10^3 Annealed $15-20 \times 10^3$ Heat Treated	23×10^3 Heat Treated	65-29
Impact Strength, Notched Izod, ft/lb			
Hardness, Brinell	$45-47 \times 10^3$ Annealed $105-130 \times 10^3$ Heat Treated	60×10^3 Annealed 145-160 Heat Treated	65-29
Elongation, percent	18 - 20 Heat Treated	15 - 17 Annealed 11 - 14 Heat Treated	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	100 - 111	70	65-29
Coefficient of Thermal Expansion, in/in/°F	$12.4 - 13.1 \times 10^{-6}$ (68 - 212°F)		65-29
Specific Heat, Btu/lb °F	0.23		65-29
Melting Point, °F	890 - 1205		65-29
Electrical Resistivity, microhm-in	1.5 - 2.2		65-29

Table 12.4.2e. Properties of Aluminum and Its Alloys, Wrought (Specific Material Types: 1060, 1100, 3003, 3004)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.098 - 0.099	65-29
Modulus of Elasticity in Tension, psi	10×10^6	65-29
Specific Strength, in. (yield strength/density)	370×10^3	65-29
Tensile Strength, psi	$10 - 26 \times 10^3$ Annealed $19 - 41 \times 10^3$ Hard	65-29
Yield Strength, psi	$4 - 10 \times 10^3$ Annealed $18 - 36 \times 10^3$ Hard	65-29
Creep Strength, psi		
Endurance Limit, psi	$3 - 14 \times 10^3$ Annealed $6.5 - 16 \times 10^3$ Hard	65-29
Impact Strength, Notched Izod, ft/lb		
Hardness, Brinell	19 - 45 Annealed 35 - 77 Hard	65-29
Elongation, percent	20 - 45 Annealed 5 - 15 Hard	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^\circ F/ft)$	93.8 - 135	65-29
Coefficient of Thermal Expansion, $in/in/^\circ F$	$12.9 - 13.3 \times 10^{-6}$ (68 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.22 (212°F)	65-29
Melting Point, °F	1165 - 1215	65-29
Electrical Resistivity, microhm-in	1.1 - 1.6	65-29

Table 12.4.2f. Properties of Beryllium Copper, Wrought

PROPERTY	VALUES		REF.
Density, lb/in^3	0.296 - 0.298		65-29
Modulus of Elasticity in Tension, psi	19×10^6		65-29
Specific Strength, in. (yield strength/density)	Solution Annealed: $84 - 118 \times 10^3$	Annealed and Heat Treated: $436 - 506 \times 10^3$	65-29
Tensile Strength, psi	$60 - 80 \times 10^3$	$165 - 185 \times 10^3$	65-29
Yield Strength, psi	$25 - 35 \times 10^3$	$130 - 150 \times 10^3$	65-29
Creep Strength, psi			
Endurance Limit, psi	$30 - 40 \times 10^3$ (10^8 cycles)		65-29
Impact Strength, Notched Izod, ft/lb			
Hardness, Rockwell	Solution Annealed: 50 - 65B	Annealed and Heat Treated: 36 - 41C	65-29
Elongation, percent	35 - 50	3 - 12	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^{\circ}F/ft)$	100 - 110 (Heat Treated)		65-29
Coefficient of Thermal Expansion, $in/in/^{\circ}F$	9.3×10^{-6} (68 - 572°F)		65-29
Specific Heat, Btu/lb °F	0.10 (86 - 212°F)		65-29
Melting Point, °F	1600 - 1800		65-29
Electrical Resistivity, microhm-in	1.90 - 2.29		65-29

Table 12.4.2g. Properties of Cobalt Base Super Alloys, Cast, Wrought (Specific Material Types: HS-21, HS-31, X-40, NIVCO, J1650, SM 302, HS 151, WI 52)

PROPERTY	VALUES		REF.
Density, lb/in^3	0.30 - 0.33		65-29
Modulus of Elasticity in Tension, psi	$30 - 36 \times 10^6$		65-29
Specific Strength, in. (yield strength/density)	384×10^3		65-29
Tensile Strength, psi	$100 - 170 \times 10^3$		65-29
Yield Strength, psi	$75 - 115 \times 10^3$		65-29
Creep Strength, psi	1500°F: $16 - 25 \times 10^3$	2000°F: $4 - 6.5 \times 10^3$	286-9
Endurance Limit, psi	$35 - 50 \times 10^3$ (10^6 cycles)		65-29
Impact Strength, Notched Izod, ft/lb	6 - 30		65-29
Hardness, Rockwell	C30 - C40		65-29
Elongation, percent	2 - 15		65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	12 - 16 (1000°F)		65-29
Coefficient of Thermal Expansion, in/in/°F	$8 - 9 \times 10^{-6}$ (70 - 1500°F)		65-29
Specific Heat, Btu/lb °F	0.09 x 0.12		65-29
Melting Point, °F	2400 - 2550		65-29
Electrical Resistivity, microhm-in	9 - 38		65-29

Table 12.4.2h. Properties of Cobalt Base Super Alloys, Wrought (Specific Material Types: L-186, V-36, Haynes Alloy 25, L-605)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.30 - 0.33	65-29
Modulus of Elasticity in Tension, psi	$30 - 35 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	365×10^3	65-29
Tensile Strength, psi	$101 - 165 \times 10^3$ Solution Treated and Aged	65-29
Yield Strength, psi	$67 - 113 \times 10^3$ Solution Treated and Aged	65-29
Creep Strength, psi	$\frac{1350^\circ F}{30 - 40 \times 10^3}$ $\frac{1800^\circ F}{5 - 12 \times 10^3}$	286-9
Endurance Limit, psi	$30 - 40 \times 10^3$ (10^8 cycles @ $1200^\circ F$)	65-29
Impact Strength, Notched Izod, ft/lb		
Hardness, Rockwell		
Elongation, percent	20 - 60	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^\circ F/ft)$	12.0 ($1700^\circ F$)	65-29
Coefficient of Thermal Expansion, $in/in/^\circ F$	$9.1 - 9.4 \times 10^{-6}$ ($70 - 1800^\circ F$)	65-29
Specific Heat, $Btu/lb\ ^\circ F$	0.09 - 0.12 ($70 - 1300^\circ F$)	65-29
Melting Point, $^\circ F$	2300 - 2550	65-29
Electrical Resistivity, microhm-in	36 - 75	65-29

Table 12.4.2l. Properties of Magnesium Alloys, Wrought (Specific Material Types: AZ31B-F, AZ61A-F, AZ80A-T5, ZK60A-T5, (P)ZK60B-T5, ZK21A-F, ZE10A-H24, AZ31B-H24, HK31A-H24, HM21A-T8, HM31A-T5)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.064 - 0.065	65-29
Modulus of Elasticity in Tension, psi	6.5×10^6	65-29
Specific Strength, in. (yield strength/density)	625×10^3	65-29
Tensile Strength, psi	$35 - 50 \times 10^3$	65-29
Yield Strength, psi	$20 - 40 \times 10^3$	65-29
Creep Strength, psi	$1.0 - 20.0 \times 10^3$ (300°F)	65-29
Endurance Limit, psi	$16 - 25 \times 10^3$ (10^8 cycles)	65-29
Impact Strength, Notched Izod, ft/lb	1.0 - 5.0	65-29
Hardness, Brinell	45 - 80	65-29
Elongation, percent	6 - 19	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	30 - 80	65-29
Coefficient of Thermal Expansion, in/in/°F	$14 - 16 \times 10^{-6}$	65-29
Specific Heat, Btu/lb °F	0.245	65-29
Melting Point, °F	900 - 1200	65-29
Electrical Resistivity, microhm-in	1.75 - 5.90	65-29

Table 12.4.2j. Properties of Molybdenum and Its Alloys, Wrought (Specific Material Types: Molybdenum, Mo.0.5 Ti, TZM)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.37	65-29
Modulus of Elasticity in Tension, psi	46×10^6	65-29
Specific Strength, in. (yield strength/density)	284×10^3	65-29
Tensile Strength, psi	$95 - 125 \times 10^3$	65-29
Yield Strength, psi	$82 - 105 \times 10^3$	65-29
Creep Strength, psi	1800°F: $10\text{-}30 \times 10^3$; 4400°F: 0.4×10^3; Mo. 0.5 Ti, 1800°F: $70\text{-}80 \times 10^3$; TZM, 2400°F: $2\text{-}30 \times 10^3$	286-9, 554-1
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb		
Hardness, VHN	250 - 325 Cold Worked	65-29
Elongation, percent	15 - 20	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	67 - 84 (212°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	3×10^{-6}	65-29
Specific Heat, Btu/lb °F	0.61 - 0.65	65-29
Melting Point, °F	4750	65-29
Electrical Resistivity, microhm-in	2.9	65-29

Table 12.4.2k. Properties of Nickel and Its Alloys, Cast (Specific Material Types: Nickel 210 (Nickel), Inconel 610 (In[illegible]el), Inconel 705 (S Inconel), Monel 411 (Monel), and Monel 505 (S Monel)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.292 - 0.312	65-29
Modulus of Elasticity in Tension, psi	$19 - 25 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	381×10^3	65-29
Tensile Strength, psi	$30 - 145 \times 10^3$ Annealed and Aged 1[illegible]0 - 190×10^3 Annealed and Age Hardened	65-29
Yield Strength, psi	$12 - 65 \times 10^3$ Annealed $90 - 120 \times 10^3$ Annealed and Age Hardened	65-29
Creep Strength, psi		
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb	4 Monel 505 60 - 70 others	65-29
Hardness, Brinell	80 - 380	
Elongation, percent	1 - 4 Inconel 705 and Monel 505 10 - 45 others	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^{\circ}F/ft)$	9 - 34 @ 212°F	65-29
Coefficient of Thermal Expansion, in/in/°F	$8.9 - 9.1 \times 10^{-6}$ (70 - 1400°F)	65-29
Specific Heat, Btu/lb °F	0.11 - 0.13 (80 - 750°F)	65-29
Melting Point, °F	2300 - 2600	65-29
Electrical Resistivity, microhm-in	20.9 - 25.7 Monel 411 and Monel 505, 4.6 - 8.2 others	65-29

NICKEL
NICKEL ALLOYS

Table 12.4.21. Properties of Nickel and Its Alloys, Wrought (Specific Material Types: Nickel 200 (A Nickel) and 201 (Nickel), Duranickel 301 (Duranickel), Monel 400 (Monel), Monel K-500 (K Monel)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.298 - 0.231	65-29
Modulus of Elasticity in Tension, psi	$26 - 30 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	218×10^3	65-29
Tensile Strength, psi	$50 - 105 \times 10^3$ Annealed $130 - 190 \times 10^3$ Annealed and Age Hardened	65-29
Yield Strength, psi	$12 - 30 \times 10^3$ Annealed Ni 200 and 201 $25 - 65 \times 10^3$ Annealed for others listed	65-29
Creep Strength, psi	900°F: $30 - 70 \times 10^3$ 1100°F: $20 - 35 \times 10^3$	286-9
Endurance Limit, psi	50×10^3 (10^8 cycles) Cold Drawn	65-29
Impact Strength, Notched Izod, ft/lb	26 - 120 +	286-9
Hardness, Rockwell	55 - 90B Annealed	65-29
Elongation, percent	25 - 60 Annealed	65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	10 - 36 (80 - 212°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	$7.2 - 7.8 \times 10^{-6}$ (80 - 212°F)	65-29
Specific Heat, Btu/lb °F	0.1[illegible]3 - 0.130 (80 - 212°F)	65-29
Melting Point, °F	2370 - 2635	65-29
Electrical Resistivity, microhm-in	3.3 - 3[illegible] Nickel 200 and 201 18.3 - 22.[illegible] others	65-29

Table 12.4.2m. Properties of Nickel Base Super Alloys, Cast, Wrought (Specific Material Types: Inconel X-750, 713, and 700; Inco 718; Hastelloy B, C, and X; Udimet 500 and 700; Waspaloy; Nicrotung; René 41; Unitemp 1753; M252; IN-100)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.28 - 0.32	65-29
Modulus of Elasticity in Tension, psi	$26 - 33.5 \times 10^6$	65-29
Specific Strength, in. (yield strength/density)	513×10^3	65-29
Tensile Strength, psi	$162 - 205 \times 10^3$ Solution Treated and Aged	65-29
Yield Strength, psi	$92 - 170 \times 10^3$ Solution Treated and Aged $105 - 120 \times 10^3$ (Cast)	65-29
Creep Strength, psi	1200°F: $35 - 70 \times 10^3$; 1650°F: $8 - 12 \times 10^3$	286-9
Endurance Limit, psi	$37 - 60 \times 10^3$ (10^7 cycles) - 1300°F	65-29
Impact Strength, Notched Izod, ft/lb	21 - 62	65-29
Hardness, Brinell	187 - 241 Solution Treated	65-29
Elongation, percent	6 - 60	65-29
Thermal Conductivity, $Btu/ft^2/hr/(°F/ft)$	10 - 14.6	65-29
Coefficient of Thermal Expansion, in/in/°F	$7.8 - 9.8 \times 10^{-6}$	65-29
Specific Heat, Btu/lb °F	0.10	65-29
Melting Point, °F	2300 - 2600	65-29
Electrical Resistivity, microhm-in	46.5 - 58.2	65-29

Table 12.4.2n. Properties of Oxygen-Free Copper (99.95 Percent copper), Wrought

PROPERTY	VALUES		REF.
Density, lb/in^3	0.323		65-29
Modulus of Elasticity in Tension, psi	Annealed	Hard	65-29
	31×10^3	108×10^3	
Specific Strength, in. (yield strength/density)	32×10^3	50×10^3	65-29
Tensile Strength, psi	10×10^3	50×10^3	65-29, 547-6
Yield Strength, psi		45×10^3	547-6
Creep Strength, psi			
Endurance Limit, psi	$30\text{-}35 \times 10^3$ (10^8 cycles)	$35\text{-}40 \times 10^3$ (10^8 cycles)	65-29
Impact Strength, Notched Izod, ft/lb			
Hardness, Rockwell	50 - 65B	36 - 41C	65-29
Elongation, percent	35 - 50	3 - 12	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^\circ F/ft)$	226		65-29
Coefficient of Thermal Expansion, $in/in/^\circ F$	9.8×10^{-6} (68 - 572°F)		65-29
Specific Heat, Btu/lb °F	0.092		65-29
Melting Point, °F	1981		65-29
Electrical Resistivity, microhm-in	0.673		65-29

ISSUED: MARCH 1967
SUPERSEDES: OCTOBER 1965

Table 12.4.2o. Properties of Titanium and Its Alloys (Specific Material Types: Unalloyed, 5Al-2.5Zn, 5Al-5Sn-5Zr, 8Al-1Mo-1V, 7Al-4Mo, 6Al-6V-2Sn, 6Al-4V, 2Fe-2Cr-2Mo, 8Mn, 13V-11Cr-3Al)

PROPERTY	VALUES	REF.
Density, lb/in^3	0.158 - 0. 75	65-29
Modulus of Elasticity in Tension, psi	15 - 18 x 10^6	65-29
Specific Strength, in. (yield strength/density)	1400 x 10^3	65-29
Tensile Strength, psi	60 - 170 x 10^3 Annealed 145 - 240 x 10^3 Heat Treated	65-29
Yield Strength, psi	40 - 150 x 10^3 Annealed 135 - 220 x 10^3 Heat Treated	65-29
Creep Strength, psi	80,000	65-29
Endurance Limit, psi	60 - 90 x 10^3 (10^7 cycles)	65-29
Impact Strength, Notched Izod, ft/lb	15 - 25 (20 - 100 unalloyed)	65-29
Hardness, Rockwell	25 - 40 C	65-29
Elongation, percent	1 - 12 Annealed 8 - 25 Heat Treated	65-29
Thermal Conductivity, Btu/ft^2/hr/(°F/ft)	4.1 - 9.8 (212°F)	65-29
Coefficient of Thermal Expansion, in/in/°F	4.9 - 7.1 x 10^{-6} (68 - 1650°F)	65-29
Specific Heat, Btu/lb °F	0.118 - 0.135 (212°F)	65-29
Melting Point, °F	2730 - 3040	65-29
Electrical Resistivity, microhm-in	22 - 69	65-29

Table 12.4.2p. Properties of Gold, Unalloyed, Wrought

PROPERTY	VALUES		REF.
Density, lb/in^3	0.698		65-29
Modulus of Elasticity in Tension, psi	11×10^6		65-29
	Annealed	Cold Rolled	
Specific Strength, in. (yield strength/density)	11.5×10^3	63.0×10^3	65-29
Tensile Strength, psi	22×10^3	54×10^3	65-29
Yield Strength, psi	8×10^3	44×10^3	65-29
Creep Strength, psi			
Endurance Limit, psi	16×10^3		533-1
Impact Strength, Notched Izod, ft/lb			
Hardness, Vickers	60 - 125	60 - 125	65-29
Elongation, percent	48	2.5	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^{\circ}F/ft)$	172 (212°F)		65-29
Coefficient of Thermal Expansion, in/in/°F	7.9×10^{-6}		65-29
Specific Heat, Btu/lb °F	0.031		65-29
Melting Point, °F	1945		65-29
Electrical Resistivity, microhm-in	0.861 (32°F)		65-29

Table 12.4.2q. Properties of Platinum, Unalloyed, Wrought

PROPERTY	VALUES		REF.
Density, lb/in^3	0.775		65-29
Modulus of Elasticity in Tension, psi	25×10^6		65-29
	Annealed	Cold Rolled	
Specific Strength, in. (yield strength/density)	2.58×10^3	34.8×10^3	65-29
Tensile Strength, psi	$18 - 21 \times 10^3$	$28 - 30 \times 10^3$	65-29
Yield Strength, psi	$2 - 5.5 \times 10^3$	27×10^3	65-29
Creep Strength, psi			
Endurance Limit, psi			
Impact Strength, Notched Izod, ft/lb			
Hardness, Vickers	40	100	65-29
Elongation, percent	30 - 40	2.5 - 3.5	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^oF/ft)$	42 (212^oF)		65-29
Coefficient of Thermal Expansion, $in/in/^oF$	4.9×10^{-6}		65-29
Specific Heat, $Btu/lb\ ^oF$	0.031		65-29
Melting Point, ^oF	3217		65-29
Electrical Resistivity, microhm-in	3.87 (32^oF)		65-29

Table 12.4.2r. Properties of Silver, Unalloyed, Wrought

PROPERTY	VALUES	REF.
Density, lb/in^3	0.379	65-29
Modulus of Elasticity in Tension, psi	11.0×10^6	65-29
Specific Strength, in. (yield strength/density)	21,100 Annealed 116,000 Cold Worked	65-29
Tensile Strength, psi	22,000 Annealed 54,000 Cold Worked	65-29
Yield Strength, psi	8,000 Annealed 44,000 Cold Worked	65-29
Creep Strength, psi		
Endurance Limit, psi		
Impact Strength, Notched Izod, ft/lb		
Hardness, Vickers	25 - 35 Annealed	65-29
Elongation, percent	48 Annealed 2.5 Cold Worked	65-29
Thermal Conductivity, $Btu/ft^2/hr/(^oF/ft)$	242 (20 - 212^oF)	65-29
Coefficient of Thermal Expansion, $in/in/^oF$	10.9×10^{-6}	65-29
Specific Heat, $Btu/lb\,^oF$	0.056	65-29
Melting Point, ^oF	1761	65-29
Electrical Resistivity, microhm-in	0.626	65-29

ISSUED FEBRUARY 1970
SUPERSEDES: OCTOBER 1965

12.5 PROPELLANT CHEMICAL COMPATIBILITY

The highly reactive nature of most liquid rocket propellants makes propellant chemical compatibility a major consideration in the selection of materials for many aerospace fluid components. Propellant chemical compatibility involves a wide variety of mechanisms such as material loss, swelling, dissolving, and propellant breakdown, as well as a host of variables such as proximity of different materials, surface-to-volume ratios, stress levels, geometry, surface finish, contaminants, fluid velocity, and energy sources (e.g., impact). Because laboratory compatibility tests seldom include all factors likely to affect propellant compatibility in any given application, material compatibility data should generally be regarded as only a guide, with the final materials selection based on prototype tests simulating actual service conditions.

In many rocket system development programs where propellant compatibility is an important consideration, materials compatibility studies are very often too limited and begun too late in the development program to be most effective. Testing is often limited to so-called "preliminary screening tests" which serve to eliminate poor candidates but do not give sufficient information as to how good the remaining materials are, and under what conditions they may be used. Attempts to correlate compatibility test results from a variety of sources is often difficult, if not impossible, due to a wide variety of methods used in conducting compatibility tests and in reporting results. Lack of accepted compatibility test standards results, for instance, in one experimenter reporting that a certain polymer is satisfactory based on his technique of measuring physical property changes after propellant outgassing, while another experimenter may report the same material to be incompatible based on his property measurements before outgassing.

Another factor leading to discrepancies in the compatibility literature is the degree of conservativeness used in interpreting test results. For instance, some materials which are listed in the literature as being incompatible with a certain propellant due to an extremely conservative interpretation of laboratory test results have been demonstrated to be satisfactory for numerous applications based on actual service experience. An example of such conservatism would be the conclusion that all metals containing a certain alloying constituent are incompatible with a propellant based on a test which shows that this alloying constituent, by itself in finely divided form, accelerates decomposition of the propellant.

The compatibility tables included in this section list materials for various fluid component elements which show, based on laboratory test experience and limited serviceability data, a *good probability* of being resistant to chemical attack under temperature and pressure conditions normally associated with the propellant in question. The data presented do not take into account unusual circumstances such as temperature well in excess of the normal boiling point of the propellant, contamination, and conditions of severe impact beyond those normally encountered in a typical propellant feed system. In addition to noting materials that are generally considered to be compatible for the applications noted, a few materials are indicated which should be definitely avoided either due to severe attack or rapid breakdown of the propellants.

12.5.1 Aerozine-50

Aerozine-50, which is a mixture of UDMH and hydrazine, does not present any significant problems in storage and handling as there are varieties of metals and nonmetals compatible with the propellant.

12.5.1.1 COMPATIBILITY WITH METALS. Of the two constituents in Aerozine-50, hydrazine places more restrictions on the selection of metals; thus metals compatible with hydrazine can safely be used with Aerozine-50. Aerozine-50 is not corrosive to most metals at ordinary temperatures, and small amounts of absorbed water do not seem to increase the corrosion. Of the common structural alloys used in aerospace fluid component applications, only magnesium alloys are considered unsuitable for Aerozine-50 service. One of the reasons for mixing UDMH and hydrazine is that the addition of UDMH greatly reduces the tendency towards catalytic decomposition of hydrazine, while providing a fuel that has better performance characteristics than UDMH alone. In spite of the fact that Aerozine-50 is far more resistant to catalytic breakdown than hydrazine alone, it is advisable to avoid materials that are known to be decomposition catalysts for hydrazine, particularly under elevated temperature conditions. Catalytic materials which should be avoided in the presence of Aerozine-50 at temperatures above its boiling point (160°F) are iron oxides (rust) and copper oxides. It is important to note that although numerous references repeat the statement that Aerozine-50 should not be used with alloys containing molybdenum in quantities greater than 0.5 percent, there is no published test data to support this conclusion; in fact, laboratory tests and extensive field experience by a number of users of Aerozine-50 have definitely shown that molybdenum-bearing alloys such as 316 stainless steel, AM-355, and A-286 are perfectly satisfactory for use with Aerozine-50 under service temperatures encountered normally.

12.5.1.2 COMPATIBILITY WITH NONMETALS. In contrast to metals, UDMH is more severe on nonmetals than is hydrazine, although both constituents are highly effective solvents. Teflon and graphite are two nonmetals which are chemically resistant to Aerozine-50. High density polyethylene and Kel-F are satisfactory for Aerozine-50 if not used in a highly stressed condition, since both materials are subject to stress cracking in contact with the fuel. Nylon materials, although gradually degraded, are useful for Aerozine-50 service for periods up to several months. A number of specific formulations of ethylene propylene rubber and butyl rubber are suitable for static and dynamic seal applications in Aerozine-50. Mylar is rapidly attacked by Aerozine-50; however, limited test data indicates that the material may be satisfactory for component applications exposed to fuel vapors. Nitroso is not compatible.

Aerozine-50, however, limited test data indicates that the material may be satisfactory for component applications exposed to fuel vapors.

Valve Bodies
Stainless steels 303, 304L, 316, 321, 347; aluminum alloys 2219, 6061, 3003, 5456, 7075, 2024; titanium alloys B120-VCA, A110-AT.

Springs
Stainless steels 301, 321, 347, 17-4PH, 17-7PH; alloy steel A-286: Ni Span C; Inconel-X.

Stems
Stainless steels 321, 347, 410, 403, AM 355, 17-4PH, 17-7PH; alloy steel 8630; Haynes Stellite 25.

Bellows
Stainless steels 303, 321, 347; Inconel-X; Berylco 25.

Bearings
Stainless steels 301, 301N, 403, 410, 440C.

Valving Units (seats and poppets)
Stainless steels 303, 347; aluminum 1100; Teflon; Zytel 101; 31 nylon; polypropylene: Haynes Stellite 25, 6K, 21; titanium carbide, tungsten carbide.

Seals
Aluminum 1100: Teflon; butyl rubber compounds 823-70 (Parco), 805-70 (Parco), 1357 (Goshen), B480-7 (Parker), B496-7 (Parker), 9257 (Precision); propylene; polyethylene; Kel-F; ethylene propylene rubber compounds, EPR132, E515-8 (Parker), 721-80 (Stillman), 724-90 (Stillman).

Packing
Teflon, Kel-F.

Lubricants
Teflon coatings and carbon graphite; UDMH Lube; LOX Safe, Microseal 100-1.
Fluorinated lubricants unsatisfactory.

Bolts, Nuts, and Screws
Stainless steels 303, 321, 347, AM 355, AM 350, 17-4PH, 17-7PH.

Thread Sealants and Antiseize Compounds
Unsintered Teflon; Redel UDMH Sealant, LOX Safe; Reddy Lube 100, 200; Drilube 822.

Coatings
Chrome plate, nickel, anodize.

Diaphragms
Teflon, butyl rubber, Berylco 25, ethylene propylene rubber. Mylar satisfactory for vapor exposure but unsuitable for liquid.

12.5.2 Ammonia

Ammonia is a highly reactive reducing agent which is alkaline in nature. Due to the possibility of forming explosive compounds, ammonia should not be brought in contact with the following chemicals: mercury, chlorine, iodine, bromine, calcium, silver oxide, or hypochlorite.

12.5.2.1 COMPATIBILITY WITH METALS. Very few metals are completely incompatible with ammonia; however, ammonia becomes more corrosive with increasing water content. Moist ammonia corrodes copper, copper alloys, tin, and zinc.

12.5.2.2 COMPATIBILITY WITH NONMETALS. A variety of elastomers, plastics, and lubricants are compatible with ammonia.

Valve Bodies
Stainless steels 302, 304, 316; alloy steels 4340, 4320, 4130; aluminum alloys 2024, 356, 6061, 7075, 5052.

Springs
Stainless steels 302, 304; carbon steel 1075; Inconel.

Stems
Stainless steel 430.

Bellows
Stainless steels 302, 304; Inconel.

Bearings
Stainless steel 430.

Valving Units (seats and poppets)
Stainless steels 304, 316; alloy steel 4340; Teflon; Kel-F.

Seals
Teflon, Kel-F, polyethylene, ethylene propylene rubber, butyl rubber, Neoprene, nitrile silicone.

Packing
Teflon, Kel-F, asbestos.

Lubricants
Fluorolube, dry films, silicone greases, refrigeration-grade petroleum oil.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347, 17-7PH.

Thread Sealants and Antiseize Compounds
Fluorolube, silicone greases, Teflon tape.

Coatings
Gold, nickel, chrome plate.

Diaphragms
Teflon, ethylene propylene rubber, polyethylene, Neoprene, stainless steels.

12.5.3 Chlorine Pentafluoride and Trifluoride

Chlorine pentafluoride and trifluoride are strong oxidizing agents which react vigorously with most organic substances at room temperature and with most metals at elevated temperatures. Like fluorine, system cleanliness is of extreme importance in handling these oxidizers, since small amounts of contamination, including water, grease, and other organic materials, can cause a local hot spot which may raise the temperature of an adjacent metal to its kindling temperature, causing it to burn. Chlorine pentafluoride and trifluoride systems therefore must be carefully descaled, degreased, passivated, and dried. See reference 35-19 for compatibility data.

CAUTION: Careful consideration should be given to the selection of materials when components are designed for use with fluorinated oxidizers. Materials selection should be based on previous satisfactory material compatibility tests under static, dynamic and environmental conditions which are applicable to the specific design contemplated.

12.5.3.1 COMPATIBILITY WITH METALS. The corrosion resistance of all metals used with chlorine pentafluoride and trifluoride depends upon the formation of a passive metallic fluoride film which protects the metal from further attack. The ability of some metals such as Monel, copper, brass, nickel, aluminum, magnesium, carbon steel, and stainless steel to form passive metal fluoride films makes them resistant to attack by chlorine trifluoride. However, in the presence of contaminants such as grease, oil, paint, or other organic materials, chlorine trifluoride will ignite most metals including those listed above. Among the metals suitable for chlorine trifluoride service, Monel and nickel are preferred because of their resistance to hydrogen fluoride and hydrazine chloride, which are formed by the reaction of chlorine trifluoride with water Hastelloy C and nickel 200 are the only metals presently known to have proven resistant to chlorine pentafluoride contaminated with moisture. Titanium, columbium, tantalum, and molybdenum are metals which are rapidly attacked by chlorine trifluoride. Soft aluminum and copper are both compatible, and are used extensively as gasket and seal materials for chlorine trifluoride service.

12.5.3.2 COMPATIBILITY WITH NONMETALS. Chlorine trifluoride attacks most polymeric materials, many of which ignite on contact with the oxidizer. For gas exposure and nonflow liquid exposure to chlorine trifluoride, Teflon and Kel-F are satisfactory static seal materials. As with metals, however, small amounts of contamination such as grease or absorbed water can cause a violent reaction between Teflon and chlorine trifluoride, resulting in complete vaporization of the plastic. TFE Teflon is superior to FEP Teflon for CTF applications. Proposed applications of nonmetallics with chlorine pentafluoride should be experimentally and thoroughly investigated (Reference 35-19).

12.5.3.3 LUBRICANTS. The use of the standard petroleum-base lubricants is prohibited. Fluorinated hydrocarbons may react violently with chlorine trifluoride. Pure molybdenum disulfide (MoS_2) with no binder has been found to be a satisfactory lubricant in some applications; however, because this material is commonly used with incompatible binders, MoS_2 lubricants should be used with caution. No completely satisfactory lubricant is known.

Valve Bodies
Stainless steels 304, 304ELC, 321, 347, AM 350; Monel, K-Monel; aluminum alloys 356T6, M517, 6061, 5052, 3001, 2024, 7075, Tens 50; magnesium alloy AZ31B.
Titanium unsatisfactory.

Springs
Stainless steels 302, 304ELC, 321, 347, AM 350; alloy steel A-286; Inconel, Inconel-X, Inconel-W; K-Monel.

Stems
Stainless steels 321, 347, 410, 403, 422, AM 350; alloy steel A-286; K-Monel; Rɔné 41.

Bellows
Stainless steels 304ELC, 321, 347; Monel, K-Monel.

Bearings
Stainless steels 301, 501N; aluminum 6061; hard anodized copper.

Valving Units (seats and poppets)
Stainless steels 321, 347, 410, 403, 422; Monel; copper; aluminum 1100; titanium carbide.

Seals
Beryllium copper, aluminum 1100, brass, copper, lead, 50-50 tin-indium alloy and tin, Teflon (non-flow), Kel-F (non-flow).

Packing
Copper, pure tin, Teflon.

Lubricants
Molybdenum disulfide.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347, AM 350; alloy steel A-286; Monel, K-Monel; Inconel-X.

Thread Sealants and Antiseize Compounds
Unsintered Teflon and Permatex Nos. 2 and 3 applied to all but the first two threads of the male fitting.

Coatings
Hard nickel plate, chrome plate, anodized (aluminum).

Diaphragms
Stainless steels 304ELC, 321, 347; Monel, K-Monel; beryllium copper.

12.5.3A Diborane

Very little data on the compatibility of diborane with materials have been published. Because of the close chemical relationship between diborane and pentaborane, it has been suggested that, when other information is lacking, materials be selected for diborane service that are known to be recommended for use with pentaborane. These two propellants are similar in reactivity; both are hydrolyzed by water, are pyrophoric, and are very toxic.

12.5.3A.1 COMPATIBILITY WITH METALS. A general comment has been made that diborane seems to be safe with all the common metals; metal oxides, on the other hand, are probably not inert to it. 300 series stainless steels, low carbon steels, nickel, Monel, and brass have been used for tanks, piping, valves, fittings, etc., in chemical process plants handling diborane. No specific reference has been found to the use of titanium or aluminum alloys. Lead has also been reported as unaffected by diborane.

12.5.3A.2 COMPATIBILITY WITH NON-METALS. Teflon, Kel-F, Saran, a packing of asbestos/graphite/copper, and a lubricant mixture of Vaseline/paraffin/graphite, have been used successfully in contact with gaseous diborane at ambient temperatures. Natural rubbers and most synthetic elastomers are probably not compatible, but one source indicates that Saran and 50-50 polyethylene-polyisobutylene are unaffected. No data were found on the compatibility of ceramics with diborane. Glyptol apparently can be used.

Valve Bodies
Stainless steels 304, 316, 321, 347, alloy steel 4130, brass, Monel, K-Monel.

Springs
Stainless steels 302, 304, 321, 347, 17-7 PH, K-Monel.

Stems
Alloy steel 17-7 PH, K-Monel.

Bellows
Stainless steels 304, 321, 347, Monel, K-Monel, Nickel

Bearings
Alloy steel 4130, brass

Valving Units (seats and poppets)
Stainless steels 304, 316, 321, 347, 17-7 PH, K-Monel, Polytetrafluoroethylene (Teflon, etc.), Kel-F.

Seals
Polytetrafluoroethylene (Teflon, etc.), Kel-F.

Packing
Polytetrafluoroethylene (Teflon, etc.), Kel-F, lead, asbestos, graphite copper.

Lubricants
Mixture of vaseline, paraffin, and graphite.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347, 17-7 PH, alloy steel 4130, Monel, K-Monel.

Thread Sealant and Antiseize Compounds
Mixture of vaseline, paraffin, and graphite.

Coatings
Nickel.

Diaphragms
Stainless steels 304, 321, 347, polytetrafluoroethylene (Teflon, etc.) (Mylar is unsuitable).

12.5.4 Fluorine

Fluorine is the most powerful oxidizing agent available for rocket propulsion. Its extreme reactivity is demonstrated by the fact that it will combine under suitable conditions with all materials except the inert gases. Cleanliness is a key to the successful handling of fluorine, since it reacts violently with water and organic substances such as grease, oil, or polymers. Local hot spots caused by reaction of contaminants with fluorine can lead to violent failure of any encasing material. The tendency for system failures resulting from local hot spots can be minimized through the use of construction materials having high thermal conductivity which resist ignition by rapidly dissipating heat. See References 36-38 and 183-10.

CAUTION: Careful consideration should be given to the selection of materials when components are designed for use with fluorinated oxidizers. Materials selection should be based on previous satisfactory material compatibility tests under static, dynamic and environmental conditions which are applicable to the specific design contemplated.

12.5.4.1 COMPATIBILITY WITH METALS. Although fluorine reacts with practically all metals, the formation of a passive fluoride film on many metals makes them useful for fluorine service. The density and adherence of the protective metallic fluoride film is a measure of the relative value of the base metal for service with fluorine. The effectiveness of the film is based on the solubility of the metal fluorides in liquid fluorine. It is believed that as the protective film builds up and the rate of corrosion slows down, an equilibrium between the reaction rate and solubility of the film is reached, resulting in a relatively steady corrosion rate. Service data indicates that fluorides of nickel, copper, chromium, and iron are relatively insoluble in liquid fluorine. Monel and nickel, in particular, form a dense and extremely tough coating which is invisible by contrast to the green and powdery coating of iron fluoride. Stainless steels exhibit satisfactory performance in liquid fluorine. Generally, the presence of silicon in steel makes it more susceptible to fluorine attack. Several of the lightweight metals such as alloys of aluminum, titanium, and magnesium are known to produce protective films in liquid fluorine. Of these, titanium probably exhibits the lowest corrosion rate; however, tests have shown that titanium is impact sensitive in fluorine. Soft copper and aluminum are recommended as gasket materials for fluorine service. Important factors to consider in selecting metals for use in liquid fluorine systems are flow rates, system water contamination, and mechanical properties of materials at the low temperatures experienced with liquid fluorine. High flow rates tend to remove fluoride coatings, increasing corrosive action. This is particularly true with metals that are less resistant to fluorine, such as low alloy steels which develop coatings that are either very brittle or are porous and powdery. In addition to increasing corrosion rates, flaking of coatings may result in contamination of the propellant, creating the usual hazards of particulate contamination in systems having contamination-sensitive valves, etc. Of all the metals showing resistance to fluorine attack, Monel is generally preferred, for in addition to being compatible with fluorine it is resistant to hydrofluoric acid, a common contaminant in fluorine systems formed by the reaction of fluorine and water.

12.5.4.2 COMPATIBILITY WITH NONMETALS. Very few nonmetals are resistant to fluorine attack. Of the polymers normally used for seal and gasket applications, only Teflon and Kel-F have been found suitable for contact with fluorine. Even these materials, however, are attacked by liquid fluorine under dynamic flow conditions.

12.5.4.3 COMPATIBILITY WITH LUBRICANTS. Fluorine reacts with organic, aqueous, or siliceous materials normally considered inert. Silicones and standard petroleum-based lubricants, therefore, are not compatible with fluorine. Pure molybdenum disulfide (MoS_2) with no binder is satisfactory for some lubricant applications in fluorine. There are, however, no reliable lubricants for fluorine service.

Valve Bodies
Stainless steels 304, 304ELC, 316, 321, 347; Monel, K-Monel; bronze, aluminum alloys 356T6, M517, 359T6, 2024, 7075, 6061, 5052, 3001, Tens 50; magnesium alloys HK31, AS1.

Springs
Stainless steels 301, 304ELC, 321, 347; Inconel, Inconel-X, Inconel-W; K-Monel.

Stems
Stainless steels 321, 347, 403, 410, 422; K-Monel; René 41, PH15-7 Mo.

Bellows
Stainless steels 304ELC, 321, 347; Monel, K-Monel; Inconel-X.

Bearings
Stainless steels 301, 301N; aluminum 6061; hard anodized copper.

Valving Units (seats and poppets)
Stainless steels 321, 347, 403, 410, 422; Monel; copper; aluminum 1100; brass.

Seals
Beryllium-copper, aluminum 1100, brass, copper, lead, 50-50 tin-indium alloy and tin.

Packing
Copper, pure tin.

Lubricants
(See text)

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347; Monel, K-Monel; Inconel-X.

Thread Sealants and Antiseize Compounds
Unsintered Teflon tape and Permatex Nos. 2 and 3 applied to all but the first two threads of the male fitting; for use with fluorine gas only.

Coatings
Hard nickel plate, chrome plate, anodize (aluminum).

Diaphragms
Stainless steels 304ELC, 321, 347; Monel, K-Monel; Beryllium copper.

12.5.5 Hydrazine

Hydrazine tends to be unstable in the presence of certain materials which act as decomposition catalysts, particularly at elevated temperatures. Therefore, in selecting materials for hydrazine service, not only must the effect of hydrazine on the material be considered, but equally important is the effect of the material on the rate of hydrazine decomposition. Anhydrous hydrazine is a powerful reducing agent, particularly with acids, oxidizers, and various organic substances.

12.5.5.1 COMPATIBILITY WITH METALS. Hydrazine is compatible with a number of common structural alloys including titanium, aluminum alloys, stainless steels, and nickel alloys. Metals not recommended for hydrazine service due to chemical attack include magnesium and zinc. The major problem with selecting materials for handling hydrazine is the tendency for hydrazine to decompose in the presence of certain metal oxides such as iron oxide, cobalt oxide, copper oxide, manganese oxide, and lead oxide. The effectiveness of hydrazine decomposition catalysts increases with increasing temperature conditions. Because of the problem with metal oxides, particular care should be taken in selecting metals for hydrazine service, particularly where these metals can be subjected to air oxidation, i.e., where prolonged exposure to air cannot be avoided prior to contact with hydrazine. Ferrous and copper alloys should only be used where air oxidation can be avoided. Gold is another material which tends to act as a hydrazine decomposition catalyst. Numerous references state that molybdenum-bearing alloys, in particular alloys containing more than 0.5 percent molybdenum, should be avoided for hydrazine service because of catalytic decomposition; however, the basis for this conclusion in terms of supporting use and test data is not documented. On the other hand, numerous agencies have used molybdenum-bearing alloys such as 316 stainless steel (3 percent Mo) in a variety of applications for hydrazine service including valves, pumps, piping, etc. without incident. Brass and bronze are not compatible. Erosion of 17-4 PH adjacent to nickel has been observed.

12.5.5.2 COMPATIBILITY WITH NONMETALS. A number of polymers including both elastomers and plastics have been found to be suitable for hydrazine service. Nonmetals can also cause catalytic decomposition of hydrazine to varying degrees, however, for most feed system component applications of polymers the wetted area is small (in static seal applications for instance) and/or exposure time is short so that propellant decomposition becomes a relatively minor compatibility consideration. Additives and/or contaminants found in nonmetals which could influence hydrazine decomposition rate include metal oxides and carbon. Applications where propellant decomposition could be significant in selecting and determining the purity of nonmetals include positive expulsion diaphragms and bladders which have large wetted areas and often involve extended propellant exposure times. The presence of trace quantities of iron as an impurity in Teflon, or the carbon black commonly used in elastomers could limit the usefulness of these polymers for hydrazine diaphragms or bladders particularly at temperatures in excess of normal ambient.

Plastics generally suitable for hydrazine service include Teflon, nylon (Zytel 101), Kynar, Kel-F and high density polyethylene. Elastomers which have been used successfully in hydrazine include butyl rubbers, neoprene, silicone rubber, and ethylene propylene rubber. In applications extremely sensitive to volume swell (e.g. poppet seals in small solenoid valves) certain ethylene propylene rubber compounds (EPR and EPT) have shown superior dimensional stability in hydrazine. It should be noted that minor variations in compounding, curing and purity can profoundly influence swell and compression set characteristics, hence, not only must one carefully select the right rubber compound but rigorous quality standards for a given compound must be maintained lest propellant compatibility be degraded. Non-metallic materials unsatisfactory for hydrazine service include Mylar, Nitroso rubber, and fluorinated rubbers, i.e., fluorosilicone rubber, Kel-F elastomers and Viton.

Valve Bodies
Stainless steels 304, 304L, 316, 321, 347; aluminum alloys 6061, 3003, 4043, 2024, 356T6, Tens 50; titanium 6Al-4V, B120VCA.

Springs
Stainless steels 301, 321, 347, AM 350, AM 355, 17-7PH; alloy steel A-286; Inconel, Inconel-X.

Stems
Stainless steels 321, 347, 403, 410, AM 350, AM 355, 17-7PH: alloy steel 8630.

Bellows
Stainless steels 305, 321, 347; Inconel, Inconel-X.

Bearings
Stainless steels 301, 301N, 403, 410, 440C.

Valving Units (seats and poppets)
Stainless steels 303, 321, 347, 440C, AM 350: Teflon; aluminum 1100; stellite 21; nylon; Kynar.

Seals
Teflon; aluminum 1100; butyl rubber compounds 805-70 (Parco), 613-75 (Stillman), 823-70 (Parco), B-480-7 (Parker), 60-61 (Bell), 9257 (Precision); propylene, polyethylene; Hypalon; Cis-4 polybutadiene; ethylene propylene rubber compounds EPL 132, EPT 10, E515-8 (Parker), 721-80 (Stillman), 724-90 (Stillman), 3015 (Uniroyal), Silicone rubber.

Packing
Teflon, Kel-F.

Lubricants
Teflon coatings and carbon graphite; DC-11, Krytox 240 fluorine grease.

Bolts, Nuts, and Screws
Stainless steels 303, 321, 347, 17-7PH; Inconel-X.

Thread Sealants and Antiseize Compounds
Unsintered Teflon; Redel UDMH Sealant, LOX Safe (exterior use only).

Coatings
Chrome plate, anodize (aluminum and magnesium), nickel plate.

Diaphragms
Stainless steels 304, 321, 347; Teflon; butyl rubber; SER, ethylene propylene rubber E515-8 (Parker), SR 721-80 (Stillman), SR 722-70 (Stillman), SR 724-90 (Stillman).

12.5.6 Hydrogen Peroxide

The compatibility of hydrogen peroxide, H_2O_2, with materials is primarily determined by the degree of decomposition; H_2O_2 decomposes to some degree with all materials. Compatibility is a function of not only the material selected, but also of cleanliness and surface preparation of the material. H_2O_2 breaks down into oxygen and water, and in a closed system the evolution of oxygen results in pressure buildup.

12.5.6.1 COMPATIBILITY WITH METALS. Aluminum and some of its alloys, tantalum, and zirconium, are the only metals considered compatible for long term contact with hydrogen peroxide. Stainless steels and nickel, however, are satisfactory for many component applications where long term continuous exposure is not a requirement. The most widely used aluminum alloy is 1060 aluminum. The presence of copper in aluminum alloys greatly reduces their compatibility.

12.5.6.2 COMPATIBILITY WITH NONMETALS. Many nonmetallic materials cause rapid decomposition of hydrogen peroxide and are rapidly attacked by, or form explosive peroxide and are rapidly attacked by, or form explosive mixtures with, the propellant. Fluorinated polymers including Teflon, Kel-F, and Viton are compatible.

Valve Bodies
Stainless steels 304, 304ELC, 316, 321, 347; aluminum alloys 1060, 1260, 5052, 5652, 6061, B-356; titanium.

Springs
Stainless steels 302, 304, 17-7PH.

Stems
Stainless steel 17-7PH.

Bellows
Stainless steels 304, 321, 347.

Bearings
6061 Al.

Valving Units (seats and poppets)
Stainless steels 321, 347.

Seals
Viton A, Teflon, Kel-F, polyethylene, silicone rubber.

Packing
Teflon, Kel-F.

Lubricants
Fluorolubes.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347, 17-7PH.

Thread Sealants and Antiseize Compounds
Teflon tape.

Coatings
Nickel plating.

Diaphragms
Stainless steels 304, 321, 347; Teflon.

12.5.7 Liquid Hydrogen

Liquid hydrogen is chemically inert to most structural materials, therefore, chemical compatibility is not a problem in the selection of materials for hydrogen service. The compatibility consideration in selecting materials for hydrogen service is the low temperature environment. Embrittlement of some materials at low temperatures requires selection of materials on the basis of structural properties such as yield strength, tensile strength, ductility, impact, and notch sensitivity. The materials selected for hydrogen service must also be metallurgically stable so that phase changes in the crystalline structure will not occur with time or temperature cycling. It is known, for instance, that body-centered cubic materials such as low alloy steels undergo a transition from ductile to brittle behavior at low temperatures; therefore, such metals are generally not suitable for structural applications at cryogenic temperatures. The face-centered cubic metals such as the austenitic stainless steels normally do not show a transition from ductile to brittle behavior at low temperature. For this reason, these types of materials are desirable for use in cryogenic applications. For high pressures of extended duration, embrittlement due to hydrogen diffusion into metals such as low alloy steels and titanium should be considered. Due to low temperature brittleness, very few nonmetals are satisfactory at liquid hydrogen temperatures. However, Kel-F, Teflon, and Mylar are suitable for certain applications. Some elastomeric materials such as silicone rubber can be used for static seals at low temperatures where the seal is given a high initial compression loading.

Valve Bodies
Stainless steels 301, 302, 304, 310, 316, 321, 347; K-Monel; Hastalloy B; aluminum alloys 2014T6, 6061T6, 5456H-24, 5052, 2024, 5154, 5086; titanium; alloy steel N-155.

Springs
Stainless steels 301, 321, 347; alloy steel A-286; K-Monel; Inconel, Inconel-X.

Stems
Stainless steels 321, 347; alloy steel A-286; Haynes No. 25; K-Monel; Inconel-X.

Bellows
Stainless steels 321, 347; K-Monel; Inconel-X.

Bearings
Stainless steels 440C, 52100, 410.

Valving Units (seats and poppets)
Stainless steels 321, 347; Teflon; Kel-F; copper; aluminum 1100; Monel; stellite 21; nylon.

Seals
Stainless steels 321, 347; Teflon; Kel-F; silicone rubber (static seals); aluminum 1100.

Packing
Teflon, Kel-F.

Lubricants
Teflon coatings and molybdenum disulfide.
Halogenated oils may be used for installation only.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347; alloy steel A-286; Inconel-X.

Thread Sealants and Antiseize Compounds
LOX Safe.

Diaphragms
Mylar, Teflon.

12.5.8 Liquid Oxygen (LOX)

Liquid oxygen is a strong cryogenic oxidizer. Materials selection with oxygen must be based on low temperature characteristics as well as on chemical compatibility considerations.

12.5.8.1 COMPATIBILITY WITH METALS. Most metals are not chemically affected by liquid oxygen; therefore, as in the case of liquid hydrogen, low temperature considerations dictate selection of compatible metals. An exception is titanium, which can result in explosive reactions with liquid oxygen under conditions of sufficient impact. In spite of the LOX-titanium reaction, titanium has been used successfully in applications where the material would not be subjected to impact conditions. Such an application is on the Titan I missile, where titanium spheres containing helium pressurant are located inside the liquid oxygen tanks.

12.5.8.2 COMPATIBILITY WITH NONMETALS. Many organic materials detonate, sometimes violently, when subjected to impact in the presence of liquid oxygen. Many common plastics, elastomers, and lubricants react under conditions of impact with such violence that the reaction constitutes a hazard. A generally accepted impact test criteria for compatibility of nonmetals with liquid oxygen is no detonations out of 20 trials at an impact level of 70 foot-pounds. Since the 70 foot-pound level is, to a large extent, quite arbitrary, materials with threshold levels considerably below 70 foot-pounds may be suitable for certain applications, specifically where conditions of impact are highly unlikely. Certain nonmetals react violently in the presence of gaseous oxygen, and thus should be judiciously avoided for LOX service. Nylon is one of these materials. Nonmetals generally found useful for liquid oxygen service are Teflon, Kel-F, and Mylar. Some elastomers, including silicone rubber, have been used successfully in liquid oxygen static seal applications. Viton has less impact sensitivity than any other elastomer.

Valve Bodies
Stainless steels 304, 310, 316, 321, 347; K-Monel; Hastalloy B; aluminum alloys 2014T6, 6061T6, 5456H-24, 5154, 5052, 5086, 356T6, 6061; alloy steel N-155.

Springs
Stainless steels 321, 347; alloy steel A-286; K-Monel; Inconel, Inconel-X.

Stems
Stainless steels 321, 347; alloy steel A-286; Haynes No. 25; Inconel-X.

Bellows
Stainless steels 304, 321, 347; K-Monel; Inconel-X

Bearings
Stainless steels 440C, 52100.

Valving Units (seats and poppets)
Stainless steels 321, 347; Teflon; Kel-F; aluminum 1100; Monel.

Seals
Stainless steels 321, 347; Teflon; Kel-F; aluminum 1100.

Packing
Teflon, Kel-F.

Lubricants
Teflon coatings and molybdenum disulfide.
Halogenated oils may be used for installation only.

Bolts, Nuts, and Screws
Stainless steels 321, 347; alloy steel A-286; Inconel-X.

Thread Sealants and Antiseize Compounds
LOX Safe.

ISSUED FEBRUARY 1970
SUPERSEDES: OCTOBER 1965

Coatings
Chromium, nickel, anodize (aluminum).

Diaphragms
Stainless steels 321, 347; Teflon; beryllium copper, Mylar.

12.5.9 Monomethylhydrazine (MMH)

12.5.9.1 COMPATIBILITY WITH METALS. MMH has decomposition characteristics similar to those of hydrazine. Iron rust, for example, is known to result in spontaneous ignition of MMH. Due to the similarity in catalytic and decomposition activity between MMH and hydrazine, those metals satisfactory for hydrazine are generally considered satisfactory for MMH.

12.5.9.2 COMPATIBILITY WITH NONMETALS. In general, MMH attacks organic materials more readily than does hydrazine. There is very little actual test data, however, on its compatibility with nonmetallic materials. Teflon, polyethylene, EPR butyl and Cis 1-4 polybutadiene are nonmetals considered to be serviceable with MMH.

12.5.9.3 COMPATIBILITY WITH LUBRICANTS. Because of MMH's solvent properties, no completely suitable lubricant has yet been found, but experience with hydrazine and UDMH suggests that Dow Corning 11 Compound (silicone), Fluorolube GR-470, and Kel-F grease may be used. Teflon coatings can also be used for some lubricant applications. Dupont's Krytox 240 fluorinated grease is compatible.

Valve Bodies
Stainless steels 304, 304L, 321, 347, 17-7PH; aluminum alloys 3003, 5052, 6061, Tens 50, 356T6.

Springs
Stainless steels 301, 321, 347, 17-7PH.

Stems
Stainless steels 321, 347, 403, 410, AM 350, AM 355, 17-4PH, 17-7PH.

Bellows
Stainless steels 303, 321, 347; Inconel, Inconel-X.

Bearings
Stainless steels 301, 301N, 403, 410, 440.

Valving Units (seats and poppets)
Stainless steels 303, 321, 347; Teflon; polypropylene; nylon.

Seals
Teflon, polypropylene, nylon, polyethylene, silicone, EPR, butyl, polybutadiene.

Packing
Teflon, Kel-F.

Lubricants
Teflon coatings, Dow Corning 11 Compound (silicone), Fluorolube GR-470, Kel-F grease. Krytox 240 fluorinated grease.

Bolts, Nuts, and Screws
Stainless steels 303, 321, 347, 17-4PH, 17-7PH; Inconel-X.

Thread Sealants and Antiseize Compounds
Unsintered Teflon tape.

Coatings
Chrome plate.

Diaphragms
Stainless steels 304, 321, 347; Teflon.

Braze Alloys
Permabraze 130 (82% Au, 18% Ni)

12.5.10 Fuming Nitric Acid

Fuming nitric acid is a highly corrosive oxidizing agent. It will vigorously attack many metals and will react with many organic materials spontaneously, causing fire.

12.5.10.1 COMPATIBILITY WITH METALS. A number of aluminum alloys and stainless steels are compatible with fuming nitric acid although in both material classes there are specific alloys which are incompatible. To reduce corrosion rates, hydrofluoric acid (HF) is added to nitric acid as an inhibitor.

12.5.10.2 COMPATIBILITY WITH NONMETALS. Polyethylene and Kel-F elastomers are nonmetals suitable for nitric acid service.

Valve Bodies
Stainless steels 301, 302, 304, 304ELC, 316, 321, 347; aluminum alloys 1060, 2024, 6061; titanium 75A.

Springs
Stainless steels 301, 321, 347.

Stems
Stainless steels 410, 430.

Bellows
Stainless steels 301, 321, 347.

Bearings
Stainless steels 301, 301N, 410, 430.

Valving Units (seats and poppets)
Stainless steels 410, 430; Haynes Stellite Nos. 1, 6, 25.

Seals
Buna N, Teflon, Kel-F, Kel-F elastomer, Hypalon, nylon.

Packing
Teflon, Kel-F.

Lubricants
Fluorolube, DC-11.

Bolts, Nuts, and Screws
Stainless steels 304, 304ELC.

Thread Sealants and Antiseize Compounds
Teflon tape.

Coatings
Chromium, gold.

Diaphragms
Stainless steels 304, 321, 347; Teflon.
Mylar unsuitable.

12.5.11 Nitrogen Tetroxide

Nitrogen tetroxide (NTO) is a strong oxidizer and a potent solvent. It is the most widely used storable propellant oxidizer and as a result there is a quantity of published nitrogen tetroxide materials compatibility information. The most complete single source of N_2O_4 compatibility data is given in Reference 81-4.

12.5.11.1 COMPATIBILITY WITH METALS. Dry nitrogen tetroxide is compatible with many metals and alloys used in fluid components; however, water contamination of nitrogen tetroxide causes the formation of nitric acid which is corrosive to many metals. The difficulty of being assured that no moisture is introduced into a nitrogen tetroxide system normally dictates that materials be selected not only for compatibility with anhydrous N_2O_4, but also with dilute nitric acid. As aluminum alloys and anodized aluminum coatings are attacked by nitric acid, great care must be exercised in maintaining absolute system dryness if these materials are to be used with N_2O_4. In general, aluminum alloys are suitable materials for use with dry nitrogen tetroxide. The degree of alloying constituents in aluminum alloys significantly affects the corrosion resistance of these materials in the presence of N_2O_4. Zinc-bearing 7075 aluminum corrodes much faster than copper-bearing 2024 which, in turn, has a higher corrosion rate than either 5052 or 3003 alloys. 3003, being the purest aluminum of this group, exhibits the lowest corrosion rate. The difference in corrosion rates, however, does not seem to be significant unless the water content in the N_2O_4 exceeds 0.3 percent. Nickel and nickel alloys constitute another group of materials which, although compatible with anhydrous nitrogen tetroxide, should be used with caution because of attack by nitric acid. Other alloys which show resistance to anhydrous N_2O_4 but which are generally avoided due to incompatibility with acids are magnesium alloys and copper alloys. Titanium alloys are compatible with N_2O_4 with certain limitations. Titanium alloys are susceptible to stress corrosion in N_2O_4, a particular problem in applications involving relatively long exposure (hours rather than minutes) at high stress levels, e.g., storage vessels. There is some indication that the stress corrosion is related to contaminations found in N_2O_4. The addition of an inhibitor to the N_2O_4 may be a possible solution to the stress corrosion problem. Titanium is susceptible to localized reaction with N_2O_4 under conditions of extreme impact, however, the reaction does not propagate, as is the case with titanium and oxygen under impact conditions. In spite of these limitations there are numerous suitable applications for titanium in N_2O_4 valves and other fluid components. At temperatures above 275°F 6Al4V is more satisfactory than the 300 series stainless steels which tend to cause a gelatinous, viscous deposit to form.

12.5.11.2 COMPATIBILITY WITH NONMETALS. No polymeric materials are completely satisfactory for extended N_2O_4 service. Plastics showing reasonable compatibility with N_2O_4 are Teflon TFE and FEP. Teflon absorbs N_2O_4, and such permeability must be considered in the use of this material in any given application. One solution, in applications such as gaskets, is the use of a glass filler in Teflon which not only reduces its cold flow tendency but also greatly improves its resistance to permeation by N_2O_4. Teflon FEP shows considerably lower permeability rates to N_2O_4 than the Teflon TFE. High density polyethylene and Kynar, a vinylidene fluoride material, are also useful for N_2O_4 service. Kel-F can be used with N_2O_4 providing the application accounts for the fact that it suffers a considerable loss in tensile strength due to absorption of N_2O_4. The only elastomers which appear to be useful for dynamic applications in N_2O_4 are certain butyl rubbers and certain compounds of ethylene propylene rubber. Wide variations exist from compound to compound within these materials classifications, requiring great care in the selection of a particular compound. No elastomers are satisfactory for long term service in N_2O_4, as even the best elastomers deteriorate under continuous exposure. Viton and fluorosilicone elastomers can be used for N_2O_4 service in such applications as static seals. In the free state, fluorosilicones swell as much at 300 percent in contact with N_2O_4; however, they retain their physical properties. Nitroso rubber has been used satisfactorily in some applications despite severe permanent set problems.

12.5.11.3 COMPATIBILITY WITH LUBRICANTS. Lubricants which have been used to varying degrees of success in N_2O_4 include silicone greases, Kel-F grease, molybdenum disulfide, Microseal, and flake graphite. Dupont's Krytox 240 fluorinated grease is compatible.

Valve Bodies

Stainless steels 302, 304, 316, 321, 347.
Aluminum alloys (anhydrous only, attached by dilute nitric acid formed by combining NTO with water) 6061, 356T6, Tens 50, 3003, 2024.
Titanium alloys (should be used with caution if high impact loads could occur).

Springs

Stainless steels 301, 304, 321, 347, AM 350, AM 355, 17-4PH, 17-7PH; alloy steels 8630, A-286; Inconel, Inconel-X; Ni Span C.

Stems

Stainless steels 321, 347, 403, 410; alloy steels A-286, 8630; René 41.

Bellows

Stainless steels 303, 321, 347; Inconel-X.

Bearings

Stainless steels 301, 301N, 410, 430, 440C.

Valving Units (seats and poppets)

Stainless steels 303, 347, 403, 410; Teflon; Haynes Stellite 25; aluminum 1100; vinylidene fluoride; polyvinyl fluoride. Nylon unsuitable.

Seals

Teflon; Kel-F 300, aluminum 1100, irradiated polyethylene, vinylidene fluoride, polyvinyl fluoride.

Packing

Teflon, Kel-F 300.

Lubricants

Teflon coatings, flake graphite, molybdenum disulfide, Kel-F 90, Microseal 100-1, silicone greases, Krytox 240 fluorinated grease.

Bolts, Nuts, and Screws

Stainless steels 303, 321, 347, AM 350, AM 355, 17-4PH, 17-7PH; alloy steel A-286; Inconel-X.

Thread Sealants and Antiseize Compounds
Unsintered Teflon; Redel N_2O_4 Thread Sealant; LOX Safe; Reddy Lube 100, 200; Drilube 822.

Coatings
Chrome plate (free of pin holes), gold.
Avoid cadmium.

Diaphragms
Stainless steels 304, 321, 347; alloy steel 17-7PH.
Mylar satisfactory for vapor exposure but unsuitable for liquid.

Braze Alloys
Permabraze 130 (82% Au, 18% Ni).

12.5.12 Oxygen Difluoride

Oxygen difluroide apparently reacts to some degree with all materials of construction. Metals burn in OF_2 with a hot, intense flame if they are ignited by being raised to their kindling temperatures. OF_2 reacts spontaneously with many inorganic and organic substances (but ignition is not reliable so latent hazards can exist in an OF_2 system). Reactions of explosive suddenness occur when some materials, including certain metals, ice, and even fluorocarbon plastics, are subjected to impact in the presence of OF_2.

This reactivity means that systems for OF_2 service must be thoroughly cleaned, dried, then de-activated, for a spot of matter which is spontaneously ignited may heat the substrate (of normally compatible material) to its kindling temperature and thus start a nearly uncontrollable fire. Cleaning and drying removes unwanted substances and contamination such as dirt, grease, scale, moisture, solid particles, etc. Cleaning involves degreasing, descaling and flushing. The cleaning and drying process normally should be followed by the so-called passivation process which further de-activates the surface by causing a controlled reaction to occur which fluorinates the surface without the generation of excessive temperatures. Passivation is usually accomplished by cautiously introducing dilute fluorine because OF_2 apparently is not as reliable a reactant as fluorine.

Passivating of an OF_2 system also results in the development of fluoride films which are capable of protecting the surfaces from progressive corrosion. These films start to form immediately upon contact with fluorine-bearing reactants (F_2, OF_2, ClF_3, etc.), however the rate of formation and the uniformity of the films are variable because the process is affected by local conditions (concentration of reactant, presence of moisture and other contaminants, temperatures, etc.). Once formed, these films may be transparent or appear as tarnish-like stains or deposits. Some fluorides turn white if contacted by moisture, others flake or dust under certain circumstances. Uniformity and tenacity are desired in the films as this minimizes the depth of corrosion and the chances of malfunctions due to the presence of fluoride particles in the system.

CAUTION: Careful consideration should be given to the selection of materials when components are designed for use with fluorinated oxidizers. Materials selection should be based on previous satisfactory material compatibility tests under static, dynamic and environmental conditions which are applicable to the specific design contemplated.

12.5.12.1 COMPATIBILITY WITH METALS. The amount of compatibility data is limited but there appears to be no major problem except with those metals which are impact sensitive (lead, tantalum, titanium alloys, magnesium alloys, etc.). Metals recommended for OF_2 service exhibit considerable resistance to corrosion if a stable fluoride film is maintained. Aluminum alloys seem to be pitted slightly, stainless steels are mostly resistant, nickel and copper and their alloys are slightly pitted or stained respectively. Some of these materials are, however, susceptible to increased corrosion and pitting if the OF_2 is contaminated with water (type 316 stainless steel is worst in this regard, followed by Type 1100 and 2024 aluminum alloys, and also measurably affected are type 347 stainless steel, columbium, brass 70-30, and the Cufenloys). The presence of fluorocarbons seems to increase corrosion rates.

A jet of OF_2 may erode or pit metals; type 1100 aluminum is not very resistant to this condition, but Monel 400A, Type 316 and 347 stainless steel, nickel 200, Type 2014 aluminum alloy, columbium, type 6061 aluminum alloy, and Cufenloy 40 show good to fair resistance. Aluminum alloys 2014, 2219, and 6061 are subject to intergranular corrosion adjacent to welds but no metals have been found to be prone to stress-corrosion cracking in OF_2. The 300-series stainless steels, copper, aluminum alloys, Monel, and nickel have been used for OF_2 service at temperatures up to +400°F. In addition to the metals mentioned above, the following have exhibited resistance to impact and corrosion: Types AM350, AM355, and 410 stainless steels, Type PH15-7MO steel, Inconel X, and Rene 41. (Embrittlement at low temperatures will rule out Type 410 stainless steel for some applications.)

12.5.12.2 COMPATIBILITY WITH NON-METALS. No polymers are known to be generally suitable for OF_2 service. Teflon, KelF-81, fluorosilicones, and vinyl silicone elastomers have been used under limited conditions but all of these are known or suspected to be impact sensitive. Limited tests have shown sintered alumina (Al_2O_3) and Pyrex glass to be insensitive to impact. At about +390°F the glass would be attacked by OF_2; no data are available concerning the high temperature suitability of alumina.

Valve Bodies
Stainless steels 304, 304ECL, 316, 321, 347; Monel, K-Monel; aluminum alloys 356T6, M517, 359T6, 6061, 5052, 3001, Tens 50; Cufenloy 10, Cufenloy 40, brass 70-30.

Springs
Stainless steels 304ELC, 316, 321, 347; Inconel, Inconel-X, Inconel-W; K-Monel.

Stems
Stainless steels 321, 316, 347, 403, 410, 422; K-Monel; Rene 41.

Bellows
Stainless steels 304ELC, 316, 321, 347; Monel, K-Monel; Inconel-X.

Bearings
Stainless steels 301, 301N; aluminum 6061; hard anodized copper.

Valving Units (seats and poppets)
Stainless steels 316, 321, 347, 403, 410, 422; Monel, copper; aluminum 1100; alumina.

Seals
Beryllium-copper, copper, aluminum, brass, 50-50 tin-indium alloy and tin; lead, Kel-F 81, Teflon (avoid impact), vinyl silicone, impact sensitive.

Packing
Copper, pure tin (corrodes rapidly), Teflon, Kel-F (impact sensitive).

Lubricants
Molybdenum disulfide.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347; Inconel-X; Monel; K-Monel.

Thread Sealants and Antiseize Compounds
Unsintered Teflon and Permatex Nos. 2 and 3 applied to all but the first two threads of the male fitting.

Coatings
Hard nickel plate, chrome plate, anodize (aluminum).

Diaphragms
Stainless steels 304ELC, 316, 321, 347; Monel, K-Monel; beryllium copper.
Mylar is unsatisfactory.

12.5.13 Pentaborane

Pentaborane reacts vigorously with many oxygen-containing materials such as water, air, and metal oxides, and it reacts with many reducible organic compounds. For this reason, considerable care should be exercised in the selection of materials to be used with pentaborane in order to avoid the use of any organic compounds containing a reducible functional group.

12.5.13.1 COMPATIBILITY WITH METALS. No metals are known to be incompatible with pentaborane at ordinary room temperatures and atmospheric pressure, although because of its strong reducing potential, pentaborane will reduce some metal oxides.

12.5.13.2 COMPATIBILITY WITH NONMETALS. Nearly all of the common rubbers swell when exposed to pentaborane. Nonmetals considered compatible with pentaborane include Teflon, polyethylene, polypropylene, Viton A, Kel-F, and fluorosilicone rubbers. Nonmetals found to be incompatible with pentaborane include nylon, Mylar, polyurethane, Neoprene, styrene, rubber, Buna-N, butyl rubber, and silicone.

Valve Bodies
Stainless steels 304, 316, 321, 347; alloy steel 4130.

Springs
Stainless steels 302, 304, 321, 347, 17-7PH; K-Monel.

Stems
Alloy steel 17-7PH; K-Monel.

Bellows
Stainless steels 304, 321, 347; Monel, K-Monel; Inconel.

Bearings
Alloy steel 4130.

Valving Units (seats and poppets)
Teflon, Kel-F, copper.

Seals
Teflon, Kel-F.

Packing
Teflon, Kel-F.

Lubricants
(See comment above.)

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347, 17-7PH; alloy steel 4130; Monel.

Thread Sealants and Antiseize Compounds
Teflon tape.

Coatings
(No specific coating has been reported acceptable.)

Diaphragms
Stainless steels 304, 321, 347; Teflon.

12.5.14 Perchloryl Fluoride

Perchloryl fluoride is much less reactive than either fluorine or chlorine trifluoride, its chemical behavior being much more like that of oxygen.

12.5.14.1 COMPATIBILITY WITH METALS. The corrosion resistance of metals with perchloryl fluoride depends largely on the moisture content of the propellant. Under moist conditions, 300 series stainless steels and high nickel alloys are recommended; however, dry perchloryl fluoride can be safely handled with aluminum alloys, nickel alloys, magnesium, copper, brass, bronze, carbon steel, lead, zinc, and stainless steels. Although not attacked by perchloryl fluoride under quiescent conditions, titanium is not recommended for perchloryl fluoride service due to reaction under impact conditions.

12.5.14.2 COMPATIBILITY WITH NONMETALS. Most organic polymers should be avoided, due to attack by perchloryl fluoride. Exceptions are the fluorinated plastics, Teflon, and Kel-F which can be used under certain service conditions, although these materials have a tendency to fail when subjected to heat, shock, or flow conditions.

12.5.14.3 COMPATIBILITY WITH LUBRICANTS. Perchloryl fluoride should never be brought into contact with petroleum greases, oils, pipe compounds, etc., or with conventional valve greases, oils, and pipe compounds. The only lubricants found to be suitable are the fluorocarbons, for example, Fluorolube.

Valve Bodies
Stainless steels 304, 310, 314, 316, 321; Hastelloy B, Hastelloy C; Monel; Durimet 20.

Springs
Stainless steels 304, 321.

ISSUED FEBRUARY 1970
SUPERSEDES: OCTOBER 1965

Stems
Stainless steel 321.

Bellows
Stainless steels 304, 321; Monel.

Bearings
(None)

Valving Units (seats and poppets)
Stainless steels 304, 321; Kel-F; Teflon.

Seals
Viton B, Teflon, Kel-F.

Packing
Teflon, Kel-F.

Lubricants
Fluorolubes.

Bolts, Nuts, and Screws
Stainless steels 304, 321; Monel.

Thread Sealants and Antiseize Compounds
Teflon tape.

Coatings
(None)

Diaphragms
Stainless steels 304, 321; Teflon.

12.5.15 RP-1

RP-1, like most hydrocarbon fuels, does not present any major compatibility problems.

12.5.15.1 COMPATIBILITY WITH METALS. RP-1 is compatible with most metals used in liquid rocket systems. Exceptions are copper alloys such as brass, bronze, and beryllium copper which should not be used in continual contact with hydrocarbon fuels due to a tendency to gum formation.

12.5.15.2 COMPATIBILITY WITH NONMETALS. A variety of nonmetals are satisfactory for RP-1 service. Kel-F, Teflon, Viton A, Neoprene, Buna-N, Mylar, and nylon are among the acceptable materials. Nonmetals which should be avoided in RP-1 service are butyl rubbers and silicones.

Valve Bodies
Stainless steels 304, 304L, 316, 321, 347; alloy steels 4130, 4340; titanium; Monel; aluminum alloys 2024, 7075, 356T6, 6061, 5052.

Springs
Stainless steels 304, 321, 347, 17-7PH; Inconel; carbon steel; alloy steel A-286.

Stems
Stainless steels 321, 347; alloy steels 4130, A-286.

Bellows
Stainless steels 304, 321, 347; Monel.

Bearings
Alloy steel 4130.

Valving Units (seats and poppets)
Stainless steels 304, 321, 347; nylon; Haynes 25; Kel-F; Teflon.

Seals
Teflon, Kel-F, Viton A, Buna N, Neoprene, polyethylene.

Packing
Teflon, Kel-F.

Lubricants
Fluorolubes, silicone grease, dry film lubes.

Bolts, Nuts, and Screws
Stainless steels 304, 321, 347, 17-7PH; Monel.

Thread Sealants and Antiseize Compounds
Unsintered Teflon tape.

Coatings
Cadmium, chromium, nickel.

Diaphragms
Stainless steels 304, 321, 347; Teflon; Mylar.

12.5.16 Unsymmetrical Dimethylhydrazine (UDMH)

Unsymmetrical dimethylhydrazine (UDMH), unlike hydrazine, is thermally stable at temperatures well above its boiling point. Substances which cause violent decomposition of hydrazine in the presence of air have little or no effect on UDMH.

12.5.16.1 COMPATIBILITY WITH METALS. UDMH is compatible with most common metals, including mild steel, 300 series stainless steels, nickel, aluminum alloys, magnesium alloys, and titanium alloys. Copper and brass are not recommended for use with UDMH due to chemical attack by the propellant.

12.5.16.2 COMPATIBILITY WITH NONMETALS. UDMH is a powerful solvent, causing swelling of many nonmetallic materials. Nonmetals found to be satisfactory for UDMH service include Teflon, polyethylene, nylon, Kel-F, Neoprene, and butyl rubber.

Valve Bodies
Stainless steels 303, 304, 316, 321, 347; aluminum alloys 6061, 3003, 356T6, 2024; brass; titanium A-556, 6A1 4V, B-120VCA; magnesium AZ-92-F, AZ-31B-0.

Springs
Stainless steels 301, 321, 347, 17-4PH, 17-7PH; alloy steel A-286; Inconel; Monel.

Stems
Stainless steels 321, 347, 403, 410, AM 350, AM 355, 17-4PH, 17-7PH; alloy steel 8630.

Bellows
Stainless steels 303, 321, 347; Inconel-X.

Bearings
Stainless steels 301, 301N, 403, 410, 440C; alloy steel 4130.

Valving Units (seats and poppets)
Stainless steels 303, 347, 410, AM 350, 17-4PH; Teflon;

aluminum 1100; polypropylene; copper; polyethylene; nylon; Kel-F.

Seals

Teflon; aluminum 1100; butyl rubber compounds 823-70 (Parco), B480-7 (Parker), ethylene propylene rubber E515-8 (Parker), 61375 (Stillman), 9257 (Precision); polypropylene; polyethylene; Cis4-polybutadiene; Kel-F.

Packing

Teflon, Kel-F.

Lubricants

Teflon coatings, carbon graphite, Apiezon L, Reddy Lube 200, Krytox 240 fluorinated grease.

Bolts, Nuts, and Screws

Stainless steels 303, 321, 347, AM 350, AM 355, 17-4PH, 17-7PH; Inconel-X.

Thread Sealants and Antiseize Compounds

Unsintered Teflon; Redel UDMH Sealant, LOX Safe (exterior use only).

Coatings

Chrome plate.

Diaphragms

Stainless steels 304, 321, 347; Teflon TFE and FEP. Mylar unsuitable.

12.6 PERMEABILITY

Permeability data for various materials to gases and to water are presented in the tables on the following pages. All the data given are for a temperature of 77°F (permeation rate normally varies exponentially with temperature variation). Permeation rate is directly proportional to pressure differential in the case of polymers, and varies as the square root of pressure differential in the case of metals.

Table 12.6a. Permeability of Metals to Gases at 77°F
(Reference 476-1)

System		Permeability Rate scc/sec/cm²/mm/atm 1/2
Gas	Metal	
H_2	Aluminum	7.5×10^{-11}
	Copper	2.6×10^{-14}
	Hastelloy B	2.7×10^{-13}
	Inconel	4.0×10^{-13}
	Iron	2.6×10^{-8}
	Kovar	4.1×10^{-13}
	Monel	5.9×10^{-11}
	Nickel	6.9×10^{-11}
	Niobium	8.7×10^{-5}
	Palladium	1.7×10^{-8}
	Platinum	2.1×10^{-14}
	Steel	
	Cold drawn	1.8×10^{-8}
	Low carbon	4.2×10^{-10}
	303 SS	4.6×10^{-13}
	304 SS	1.3×10^{-13}
	316, 321 SS	2.3×10^{-12}
	347 SS	9.2×10^{-13}
	410 SS	5.7×10^{-12}
N_2	Iron	4.3×10^{-19}
	Molybdenum	4.4×10^{-33}
CO	Iron	8.1×10^{-16}
O_2	Silver	1.5×10^{-17}

ISSUED: FEBRUARY 1970
SUPERSEDES: MARCH 1967

Table 12.6b. Permeability of Polymeric Materials to Gases at 77°F
(References 478-1 and 495-8)

Polymer	Permeability Rates 10^{-7} scc/sec/cm^2/mm/atm					
	H_2	He	N_2	O_2	CO_2	Air
Natural rubber	39	23	6.6	18	102	4.9
Butyl	4.9	5.6	0.22	0.90	3.8	0.2
Buna-S	30.5	17.5	4.8	13	94	2.5
Neoprene	10.3	3.4	0.89	3.0	19.5	1.0
Mylar A	0.445	0.74	0.0031	0.019	0.90	-
Nylon 6	-	-	0.0064	0.023	0.093	-
Teflon	18	530	2.4	7.6	-	-
Kel-F	0.74		0.0025	0.028	0.11	-
Silicone	-	-	-	-	-	115

Table 12.6c. Permeability of Polymeric Materials to Water at 77°F
(Reference 478-1)

Polymer	Permeation Rate 10^{-7} scc/sec/cm^2/mm/atm
Natural rubber	2600
Mylar A	98.8
Polyvinyl chloride	110
Nylon 6-6	53-516 (humidity dependent)
Kel-F	0.22

Note: A convenient and comprehensive new source of permeability data is Reference 152-12, "Permeability Data for Aerospace Applications", Illinois Institute of Technology Research Institute, Chicago, Contract No. NAS7-388, IITRI Project C8070, March 1968. Appendix A.2.23 of this handbook contains permeability conversion factors from this reference.

12.7 FRICTION COEFFICIENTS

The design of components which have parts in sliding contact frequently requires some estimate of the coefficient of friction at the sliding interface. Table 12.7 consists of a summary of published data considered to be of potential value to the fluid component designer.

Most comprehensive studies of friction coefficients have evaluated the effects of some or all of the following factors:

- Temperature
- Surface finishes
- Load
- Actual contact area
- Sliding velocity
- Oxide and other films
- Lubricant properties
- Metal structure.

Discrepancies between values in Table 12.7 for the same material combinations obtained from different sources may be due to such factors or to differences in experimental technique.

To supplement the available experimental data of Table 12.7, Figure 12.7, based on Rabinowicz's surface energy theory, has been included to assist in estimating friction coefficient values for material combinations not listed in Table 12.7. The extent of the shading in Figure 12.7 shows the probable range of values, there being about a 90 percent probability of getting a point within the shaded region.

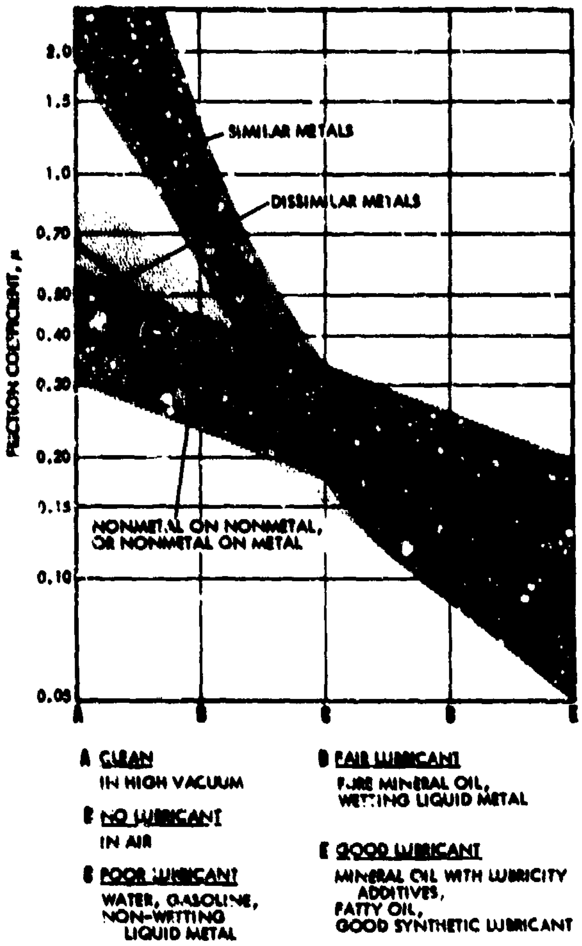

Figure 12.7. General Purpose Friction Chart
(Adapted with permission from "Product Engineering," 15 March 1965, vol. 36, no. 6, E. Rabinowicz, Copyright 1965, McGraw-Hill Publishing Company, Inc.)

Table 12.7. Static and Kinetic Friction Coefficients

(Numbers and letters in parentheses refer to information found at end of table.)

MATERIALS	STATIC		KINETIC		REMARKS
	DRY	LUBRICATED	DRY	LUBRICATED	
METALS					
Hard steel on hard steel	0.78 (A,1)	0.11 (A,1,a)	0.42 (A,2)	0.029 (A,5,h)	
	0.149 (G,43)	0.23 (A,1,b)	---	0.081 (A,5,c)	
	---	0.15 (A,1,c)	---	0.080 (A,5,i)	
	---	0.11 (A,1,d)	---	0.058 (A,5,j)	
	---	0.0075 (A,1,8,p)	---	0.084 (A,5,d)	
	---	0.0052 (A,1,8,h)	---	0.105 (A,5,k)	
	---	---	---	0.096 (A,5,l)	
	---	---	---	0.108 (A,5,m)	
	---	---	---	0.12 (A,5,a)	
	(weld on contact, A,20)	---	---	---	Degassed at elevated temperature at high vacuum
	0.39 (A,8)	---	---	---	Grease-free in air
	0.013 (A,21)	---	---	---	Clean and coated with solution of stearic acid
Mild steel on mild steel	0.74 (A,19)	---	0.57 (A,3)	0.09 (A,3,a)	
	---	---	---	0.19 (A,3,u)	
Steel on steel	---	0.09–0.1 (B,bb)	0.7 (E)	---	(E) low velocity
	---	0.13 (B,cc)			
	---	0.35 (B,dd)			
	---	0.43 (B,ee)			
	---	0.2 (B,ff)			
	---	---	0.34–0.22 (C,24)	0.1–.06 (C,24,gg)	(C) μ decreases as velocity increases from 0 to 6600 fpm
	---	---	---	0.16–0.09 (C,27,hh)	
	---	---	---	0.16–0.21 (C,27,ii)	
	0.3–0.35 (B,4)	0.27 (A,1,hhh)	0.36–0.24 (C,27)	0.30–39 (C,27,qq)	
	0.78 (A,1)	0.39 (A,1,iii)	---	0.28–0.06 (C,27,rr)	
	---	---	---	0.1–0.25 (E,l)	
	---	---	---	0.08–0.18 (B,34,ddd)	
Hard steel on babbitt (ASTM No. 1)	0.70 (A,11)	0.23 (A,1,b)	0.33 (A,6)	0.16 (A,1,b)	
	---	0.15 (A,1,c)	---	0.06 (A,1,c)	
	---	0.08 (A,1,d)	---	0.11 (A,1,d)	
	---	0.085 (A,1,e)			
Hard steel on babbitt (ASTM No. 8)	0.42 (A,11)	0.17 (A,1,b)	0.35 (A,11)	0.14 (A,1,b)	
	---	0.11 (A,1,c)	---	0.065 (A,1,c)	
	---	0.09 (A,1,d)	---	0.07 (A,1,d)	
	---	0.08 (A,1,e)	---	0.08 (A,11,h)	
Hard steel on babbitt (ASTM No. 10)	---	0.25 (A,1,b)	---	0.13 (A,1,b)	
	---	0.12 (A,1,c)	---	0.06 (A,1,c)	
	---	0.10 (A,1,d)	---	0.055 (A,1,d)	
	---	0.11 (A,1,e)			
Mild steel on cadmium silver	---	---	---	0.097 (A,2,f)	
Mild steel on phosphor bronze	---	---	0.34 (A,3)	0.173 (A,2,f)	
Mild steel on copper lead	---	---	---	0.145 (A,2,f)	
Mild steel on cast iron	---	0.183 (A,15,c)	0.23 (A,6)	0.133 (A,2,f)	
Mild steel on lead	0.95 (A,11)	0.5 (A,1,f)	0.95 (A,11)	0.3 (A,11,f)	
Nickel on mild steel	---	---	0.64 (A,3)	0.178 (A,3,k)	
Aluminum on mild steel	0.61 (A,8)	---	0.47 (A,3)		
Magnesium on mild steel	---	---	0.42 (A,3)		
Steel on lead-base bearing alloy	---	---	0.45–0.7 (B)	0.12 (B,f) 0.08 (B,h)	
Steel on lead-base matrix alloy	---	---	0.4–0.8 (B)	0.15 (B,f) 0.07 (B,h)	
Steel on lead	---	---	1.1 (B)		
Lead-base bearing alloy on steel	---	---	0.55–1.0 (B)	0.32 (B,f)	
Lead-base matrix alloy on steel	---	---	0.45–1.2 (B)	0.36 (B,f)	
Steel on tin-base bearing alloy	---	---	0.65–0.8 (B)	0.13 (B,f) 0.07 (B,h)	
Steel on tin-base matrix alloy	---	---	0.7–0.85 (B)	0.13 (B,f) 0.09 (B,h)	
Tin-base bearing alloy on steel	---	---	0.7–0.8 (B)		
Tin-base matrix alloy on steel	---	---	0.8–0.85 (B)		
Tin on cast iron	---	---	0.174 (G,43)		
Sintered bronze on steel	---	0.13 (B,f)			
Steel on indium	---	---	0.2–1.8 (E)	---	μ varies with velocity
Steel on lead	---	---	0.5–1.2 (F)	---	μ varies with velocity
Lead on steel	---	---	0.5 (E)	---	Low velocity
Steel on bronze	---	---	0.25 (E)	0.001–0.006 (G,44,jjj)	Lubricated kinetic value from thrust bearing
	---	---	0.152 (G,43)	0.0008 (G,44,jjj)	Lubricated kinetic value from thrust bearing
Cadmium on mild steel	---	---	0.46 (A,3)		
Copper on mild steel	0.53 (A,8)	---	0.36 (A,3)	0.18 (A,17,e)	
Brass on mild steel	0.51 (A,8)	---	0.44 (A,3)		
Copper-lead (dendritic) alloy on steel	0.22 (B)				
Copper-lead (non-dendritic) alloy on steel	0.22 (B)				
Phosphor-bronze alloy on steel	0.35 (B)				
Aluminum-bronze alloy on steel	0.45 (B)				
Brass on steel	0.35 (B)				
Constantan on steel	0.4 (B)				
Cast iron on steel	0.4 (B)				
White metal (tin base) on steel	0.8 (B)				
White metal (lead base) on steel	0.55 (B)				
Cast iron on soft steel	0.155 (G,45)				
Steel on aluminum	---	---	1.2 (B)	---	Steel slider on electrolytically-polished aluminum
Stainless steel on stainless steel	---	---	---	0.1 (B,z)	
Inconel X on Inconel X	---	0.6 (D,32,ww)	---	0.5–0.12 (D,32,ww)	μ decreased with time
	---	0.67 (D,32,xx)	---	0.35–0.25 (D,32,xx)	μ varied with time

Table 12.7. Static and Kinetic Friction Coefficients (Continued)

MATERIALS	STATIC		KINETIC		REMARKS
	DRY	LUBRICATED	DRY	LUBRICATED	
440C stainless steel on 440C stainless steel	—	—	—	0.16–0.28 (D,33,yy) 0.28 (D,33,zz) 0.21 (D,33,aaa) 0.13–0.22 (D,33,bbb) 0.20–0.29 (D,33,ccc)	4-ball tester; 700 fpm sliding velocity
Iron-chromium nickel alloy on iron-chromium nickel alloy	—	—	—	0.04–0.3 (D,33,yy) 0.24–0.31 (D,33,zz) 0.23–0.39 (D,33,aaa) 0.16–0.24 (D,33,bbb) 0.16–0.39 (D,33,ccc)	
M-1 tool steel on M-1 tool steel	—	—	—	0.08–0.14 (D,33,yy) 0.21 (D,33,zz) 0.11–0.18 (D,33,aaa)	
52100 steel on 52100 steel	—	—	0.38 (C,29,vac.)	0.52 (C,29,air)	390 fpm velocity
440C stainless steel on 440C stainless steel	—	—	0.41 (C,29,vac.)	0.41 (C,29,air)	
Cobalt-base alloy on cobalt-base alloy	—	—	0.28 (C,29,vac.)	0.38 (C,29,air)	
Nickel-chrome alloy on nickel-chrome alloy	—	—	0.37 (C,29,vac.)	0.22 (C,29,air)	
52100 steel on 52100 steel	—	—	0.2–0.48 (C,29)	—	μ varied as air pressure varies between atmospheric and 2.0 x 10^{-7} mm Hg
52100 steel on 52100 steel	—	—	0.2–weld (C,29)	—	Welding occurred after 30 minutes of rubbing at 390 fpm in 2.0 x 10^{-7} mm Hg vacuum
440C stainless steel on 440C stainless steel	—	—	—	0.14 (C,29,kk) 0.10 (C,29,ll) 0.09 (C,29,mm) 0.07 (C,29,nn) 0.15–0.32 (C,29,oo) 0.31 (C,29,ss) 0.10 (C,29,tt) 0.02 (C,29,uu, and vv)	390 fpm, vacuum environment 390 fpm, ambient air 390 fpm, vacuum environment
52100 steel on 52100 steel	—	—	0.43 (C,29)	0.21 (C,29,oo)	
440C stainless steel on 440C stainless steel	—	—	0.42 (C,29)	0.17 (C,29,oo)	
304 austenitic stainless steel on 304 austenitic stainless steel	—	—	—	0.4 (C,30,pp)	2300 fpm velocity
440C stainless steel on 440C stainless steel	—	—	—	0.38 (C,30,pp)	
52100 steel on 52100 steel	—	—	—	0.38 (C,30,pp)	
304 stainless steel on nickel	—	—	0.4-1.7 (E)	—	Oxide film formed at 1500°F persists on cooling
Brass on 52100 steel	—	—	0.23–0.53 (C,28)	—	μ varies over velocity range of 0 to 12,000 fpm
Bronze on 52100 steel	—	—	0.18–0.45 (C,28)	—	μ varies over velocity range of 0 to 19,000 fpm
Monel on 52100 steel	—	—	0.6–0.38 (C,28)	—	μ decreases as velocity increases from 0 to 16,000 fpm
Nichrome V on 52100 steel	—	—	0.4–0.25 (C,25)	—	μ decreases as velocity increases from 0 to 16,000 fpm
Grey cast iron on 52100 steel	—	—	0.6-0.43 (C,25)	—	μ varies over velocity range from 0 to 12,000 fpm
Nodular iron on 52100 steel	—	—	0.89–0.4 (C,25)	—	μ varies over velocity range from 0 to 14,000 fpm
Cast Inconel on M-10 tool steel	—	—	0.75–0.44 (C,26)	—	μ decreases as temperature increases to 1000°F
Cast Inconel on Inconel X	—	—	—	0.26–0.09 (C,28,jj)	μ decreases as temperature increases to 1000°F
Iron on iron	Gross seizure	—	1.2 (B,air, or O_2)	1.2 (B,aa)	Spectroscopically pure, outgassed in vacuum
Iron on iron	1.0 (B)	0.15–0.2 (B,y)	0.95 (E)	—	(E) low velocity
Cast iron on cast iron	1.10 (A,16)	—	0.15 (A,9)	0.070 (A,9,d)	
	—	—	—	0.064 (A,9,n)	
Bronze on cast iron	—	—	0.22 (A,9)	0.077 (A,9,n)	
Brass on cast iron	—	—	0.30 (A,6)		
Zinc on cast iron	0.85 (A,16)	—	0.21 (A,7)		
Magnesium on cast iron	—	—	0.25 (A,7)		
Copper on cast iron	1.05 (A,16)	—	0.29 (A,7)		
Tin on cast iron	—	—	0.32 (A,7)		
Lead on cast iron	—	—	0.43 (A,7)		
Aluminum on aluminum	1.05 (A,16)	—	1.4 (A,3)		
	—	—	1.35 (B)	—	Aluminum slider on electrolytically polished aluminum
	Gross seizure	—	1.9 (B,air, or O_2)	1.1 (B,aa)	Spectroscopically pure; outgassed in vacuum
	1.38 (B)	0.30 (B,y)	1.3 (E)	—	(E) low velocity
Copper on copper	Gross seizure	—	1.6 (B,air, or O_2)	1.6 (B,aa)	Spectroscopically pure; outgassed in vacuum
	—	—	4 (B, H_2, or N_2)	—	Spectroscopically pure; outgassed in vacuum
	1.0 (B) 1.21 (A,1)	0.08 (B,y) 0.76 (A,1,hhh)	1.2 (E)	—	(E) low velocity
	—	0.74 (A,1,iii)	0.4–1.4 (D,31)	—	μ increases with load
	—	—	0.6–1.2 (E)	—	μ varies with surface finish, low velocity
	—	0.09 (E,aaa)	1.0 (E)	0.09–0.32 (E,aaa)	μ increases as solid lubricant liquifies, then is desorbed
Gold on gold	Gross seizure	—	2.9 (B,air, or O_2)	2.5 (B,aa)	Spectroscopically pure; outgassed in vacuum
	—	—	4 (B, H_2, or N_2)	—	Spectroscopically pure; outgassed in vacuum
Chromium on chromium	0.41 (B)	0.34 (B,y)	0.6 (E)	—	(B) Surface freshly scraped in lubricated test; (E) low velocity
	—	—	0.35 (D,31)	—	μ constant with load
Lead on lead	—	—	1.2 (E)	—	Low velocity

Table 12.7. Static and Kinetic Friction Coefficients (Continued)

MATERIALS	STATIC DRY	STATIC LUBRICATED	KINETIC DRY	KINETIC LUBRICATED	REMARKS
Magnesium on magnesium	0.6 (A,22)	0.08 (A,22,y)			
	0.6 (B)	0.08 (B,y)	0.6 (B)	—	(B) low velocity
Nickel on nickel	1.10 (A,16)	—	0.53 (A,3)	0.12 (A,3,w)	
	Gross seizure (B)	—	3 (B,air, or O_2)	1.6 (B,aa)	Spectroscopically pure; outgassed in vacuum
	—	—	5 (B,H_2, or N_2)	—	Spectroscopically pure; outgassed in vacuum
	0.7 (B)	0.28 (B,y)	0.6 (B)	—	(B) low velocity
Platinum on platinum	1.2 (B)	0.25 (B,y)	0.9 (B)	—	(B) low velocity
	Gross seizure (B)	—	3 (B,air, or O_2)	3 (B,aa)	Spectroscopically pure; outgassed in vacuum
Silver on silver	Gross seizure (B)	—	1.5 (B,air, or O_2)	1.5 (B,aa)	Spectroscopically pure; outgassed in vacuum
	1.4 (B)	0.38 (B,y)	1.2 (B)	—	(B) low velocity
	0.65 (D,32)	—	0.68–0.72 (D,32)	—	μ varied with time
Titanium on titanium	—	—	0.4–0.68 (D)	—	Slight average decrease in μ (0.56–0.47) with increase in velocity
	—	—	0.6 (B)	—	Low velocity
Tin on tin	—	—	1.0 (D,31)	—	μ constant with load
	—	—	1.1 (B)	—	Low velocity
Tungsten on tungsten	—	—	0.30–1.2 (C,30)	0.40–0.38 (C,30,ggg)	
Zinc on zinc	0.6 (B)	0.04 (B,y)			
	—	—	0.6 (B)	—	(B) low velocity
Brass on brass	0.88 (A,1)	0.37 (A,1,iii)	0.4 (B)		
Molybdenum on molybdenum	Gross seizure (B)	—	0.8 (B,air, or O_2)	0.6 (B,aa)	Spectroscopically pure; outgassed in vacuum
Cadmium on cadmium	0.5 (B)	0.05 (B,y)	0.06 (B)		
Indium on indium	—	—	1.8 (B)	—	Low velocity
METALS AND NON-METALS					
Hard steel on graphite	0.21 (A,1)	0.09 (A,1,a)			
Carbon graphite on chrome-plated mild steel	—	—	0.083–0.36 (F)	—	For 3 graphite samples, μ decreased with increase (from ambient temperature to 400°F, velocity = 9063 fpm)
Graphite on steel	—	0.1 (B)			
Steel on graphite	0.1 (B)				
Tungsten carbide on steel	0.4–0.6 (B)	0.1–0.2 (B)			
Hard carbon on steel	0.14 (B)	0.11–0.14 (B)			
Stainless steel on Teflon	—	—	0.04 (C,36)	—	Teflon thickness of 0.010 inch required for 1 hour run at 2000 fpm
Stainless steel on polytrifluorochloroethylene	—	—	0.04 (C,36)		
Unimpregnated carbon on 304 stainless steel	—	—	0.22 (C,37)	0.20 (C,37,pp)	2000 fpm
Carbon impregnated with:					
phenolic on 304 stainless steel	—	—	0.32 (C,37)	0.19 (C,37,pp)	
paraffin on 304 stainless steel	—	—	0.12 (C,37)	0.21 (C,37,pp)	
chlorinated biphenyl on 304 s.s.	—	—	0.20 (C,37)	0.13 (C,37,pp)	
trichloroethylene on 304 stainless steel	—	—	0.21 (C,37)	0.18 (C,37,pp)	
fluorinated ethylene propylene on 304 s.s.	—	—	—	0.14 (C,37,pp)	
sulfur on 304 stainless steel	—	—	0.18 (C,37)	0.30 (C,37,pp)	
metal fluoride on 304 stainless steel	—	—	0.30 (C,37)	0.19 (C,37,pp)	
cadmium oxide on 304 stainless steel	—	—	0.18 (C,37)	0.04 (C,37,pp)	
Carbon impregnated with Teflon on 304 stainless steel	—	—	0.22 (C,37)	0.10 (C,37,pp)	(15% carbon – 85% Teflon)
	—	—	—	0.16 (C,37,ww)	
Graphite impregnated with nylon on 304 stainless steel	—	—	0.37 (C,37)	0.24 (C,37,pp)	(5% graphite – 95% nylon)
	—	—	—	0.13 (C,37,ww)	
Graphite impregnated with phenolfurfuryl on 304 stainless steel	—	—	0.13 (C,37)	0.10 (C,37,pp)	
	—	—	—	0.08 (C,37,pp)	
	—	—	—	0.06 (C,37,ww)	
Phenolic – impregnated carbon on 304 stainless steel	—	—	0.17 (C,38)	0.12 (C,38,pp)	
				0.26 (C,38,ww)	
Dense graphitic carbon on 304 stainless steel	—	—	—	0.04 (C,38,pp)	
	—	—	—	0.02 (C,38,ww)	
MoS_2 with: various binders on manganese-phosphated metal	—	—	0.034–0.074 (D,41)	—	0.0002–0.0008 inch thick coating at 72 in/min
epoxy binders on titanium	—	—	0.27–0.10 (D,42)		
graphite on titanium	—	—	0.13–0.08 (D,42)		
graphite with silicone binder on titanium	—	—	0.10–0.06 (D,42)		
Graphite with various binders on titanium	—	—	0.21–0.02 (D,42)		
Graphite on aluminum	—	—	0.07–0.8 (E)	—	μ remains constant near 0.1 up to 700°F, then increases
Stainless steel on boron carbide	—	—	0.01–2.5 (E)	—	μ = 0.08 at ambient, 2.5 at 1200°F, 0.01 at 1600°F
Steel on:					
asbestos fabric	0.27–0.38 (G,46)	—	—	—	μ constant with load
polymethylmethacrylate	—	—	0.52 (D,40)	—	μ constant with load
polystyrene	—	—	0.33–0.4 (D,40)	—	μ constant with load at lower loads
polytetrafluoroethylene	—	—	0.04–0.25 (D,40)	—	μ constant with load
polyethylene	—	—	0.7–0.2 (D,40)	—	μ decreases with increasing load
polyvinylchloride	—	—	0.40 (D,40)	—	μ constant with load
cork	0.458 (G,47)	0.694 (G,47,t)			
Fluted rubber on steel	—	—	—	0.08 (A,13,t)	
Teflon on steel	0.04 (B)	—	0.04 (B)	—	μ constant over temperature range
Steel on Teflon	—	—	0.08–0.14 (B)	—	μ increases with increase in temperature

Table 12.7. Static and Kinetic Friction Coefficients (Continued)

MATERIALS	STATIC		KINETIC		REMARKS
	DRY	LUBRICATED	DRY	LUBRICATED	
Polythene on steel	---	---	0.15 (B)		
Polystyrene on steel	---	---	0.4 (B)		
Steel on polythene	---	---	0.2 (B)		
Steel on polystyrene	---	---	0.30 (B)		
Perspex on steel	0.4–0.5 (B)	---	0.5 (B)	---	Perspex = a clear plastic similar to Plexiglas
Steel on Perspex	---	---	0.45 (B)		
Rubber on steel	---	---	4 (B)		
Steel on rubber	---	---	1 (B)		
Steel on nylon	0.3 (B)				
Brass on nylon	0.3 (B)				
Cast iron on leather	---	---	0.221 (O,48)		
Cast iron on leather	---	0.613 (O,43,i)	0.361 (O,43)	0.34–0.36 (O,43,i)	
Leather on cast iron	---	---	0.56 (A,9)	0.36 (A,9,i) 0.13 (A,9,n)	
Glass on nickel	0.78 (A,8)	---	0.56 (A,8)		
Copper on glass	0.68 (A,8)	---	---	0.08 (A,13,i)	
Linde LW-1 "Flame-Plated" coating on:					
GA "Meehanite"	---	---	0.11–0.68 (H)	0.63 (H,hkk)	LW-1 = WC + 7 to 10% C_o "Flame Plated" on metal base material
440C stainless steel	---	---	0.28–0.34 (H)		
52100 steel	---	---	0.27–0.46 (H)	0.17 (H,kkk)	
A 2 tool steel	---	---	0.31–0.41 (H)		
M 2 tool steel	---	---	0.30–0.47 (H)		
H 13 tool steel	---	---	0.31–0.38 (H)		
Nitralloy 135 M	---	---	0.48–0.58 (H)	0.27 (H,kkk)	
Haynes Alloy No. 25	---	---	0.28–0.38 (H)		
"Malcomized" 316 stainless steel	---	---	0.31–0.73 (H)		
annealized ( ) stainless steel	---	---	0.19–0.62 (H)	---	(poor combination)
chrome plating	---	---	0.34–0.42 (H)	0.2' (H,kkk)	(poor combination)
Inconel X	---	---	0.48–0.58 (H)	---	
Linde LW-5 "Flame-Plated" coating on:					
Haynes Alloy No. 25	---	---	0.19–0.35 (H)	---	LW-5 = 25% WC + 7% Ni + mixed W-Cr carbides "Flame Plated" on metal base material
440C stainless steel	---	---	0.30–0.40 (H)	---	
M 2 tool steel	---	---	0.32–0.50 (H)	---	
Linde LC-1 "Flame-Plated" coating on:					
440C stainless steel	---	---	0.38–0.46 (H)	---	LC-1 = 80% Cr_3C_2 + 15% Ni-Cr "Flame Plated" on metal base material
Haynes Alloy No. 25	---	---	0.17–0.22 (H)	---	
Haynes LT-1B	---	---	0.13–0.34 (H)	---	
Linde LC-5 "Flame-Plated" coating on:					
Haynes Alloy No. 25	---	---	0.19–0.22 (H)	---	LC-5 = 80% Cr_2 - O_3 + 20% Al_2O_3 "Flame Plated" on metal base material
440C stainless steel	---	---	0.26–0.30 (H)	---	
Linde LA-2 "Flame-Plated" coating on:					
Haynes Alloy No. 25	---	---	0.17–0.28 (H)	---	LA-2 = 99+% Al_2O_3 "Flame Plated" on metal base material
Malcomized 316 stainless steel	---	---	0.28–0.36 (H)	---	
Hard aluminum bronze	---	---	0.44 (H)	---	
Hastelloy C	---	---	0.10–0.32 (H)	---	
Inconel X	---	---	0.42–0.52 (H)	---	(poor combination)
440C stainless steel	---	---	0.27–0.36 (H)	---	
GA "Meehanite"	---	---	---	0.07 (H)	
NON-METALS					
Carbon on glass	---	---	0.18 (A,8)		
Nylon on nylon	---	---	0.3 (B)		
Teflon on Teflon	0.04 (B)	---	0.04 (B)	---	μ constant over temperature range (see also Reference 88-1)
Polythene on polythene	0.2 (B)	---	0.1 (B)		
Polystyrene on polystyrene	0.5 (B)	---	0.5–0.7 (B)	---	μ increases with increase in temperature
Perspex on Perspex	0.8 (B)	---	0.8 (B)		
Bakelite on Bakelite	---	---	0.3 (B)		
Graphite on graphite	0.5–0.8 (B)	0.[illegible] (B)	---	---	Dry couples outgassed
Diamond on diamond	0.1 (B)	0.05–0.1 (B)			
Sapphire on sapphire	---	0.2 (B)			
Hard carbon on carbon	0.16 (B)	0.12–0.14 (B)			
Phenolic-impregnated carbon on carbon	---	---	0.12 (C,38)	0.04 (C,38,pp)	2200 fpm
Nickel-bonded titanium carbide on columbium carbide	---	---	0.4 vac (C,29)	0.2 air (C,29)	
Phenolic-impregnated carbon on itself	---	---	---	0.08 (C,38,rr)	
Dense graphitic carbon on carbon	---	---	---	0.08 (C,38,ss)	
Dense graphitic carbon on itself	---	---	---	0.08 (C,38,tt)	
Titanium carbide on titanium carbide	---	---	0.1–0.4 (C,39)	0.08–0.7 (C,39,ggg)	
Titanium carbide on nickel-molybdenum cermet	---	---	---	0.04 (D,38,yy)	4 ball tester, 700 fpm
	---	---	---	0.14–0.3 (D,38,zz)	
	---	---	---	0.10 (D,38,aaa)	
Tungsten carbide on tungsten carbide	---	---	0.28–0.7 (C,39)	0.4–0.3 (C,39,ggg)	
	0.2–0.25 (B)	0.12 (B)	0.2 (B)	0.17 (B,38)	Similar results with several lubricants
Tungsten carbide on copper carbide	0.35 (A,23)				
Tungsten carbide on iron carbide	0.8 (A,23)				
Aluminum oxide on aluminum oxide	---	---	0.24		
Aluminum oxide on tungsten carbide	---	---	0.37		
Linde LW-1 "Flame-Plated" coating on:					
itself	---	---	0.33–0.46 (H)	---	LW-1 = WC + 7 to 10% C_o coating
carbon-graphite	---	---	---	0.04 (H,kkk)	LW-1 = WC + 7 to 10% C_o coating
Linde LW-5 "Flame-Plated" coating on:					
itself	---	---	0.21–0.34 (H)	---	LW-5 = 25% WC 7% Ni + mixed W-Cr carbides
"Kentanium" 161	---	---	0.11–0.23 (H)	---	"Kentanium" = TiC
"Kentanium" 162B	---	---	0.15–0.38 (H)	---	"Kentanium" = TiC
Linde LW-1	---	---	0.27–0.44 (H)	---	LW-1 = WC + 7 to 10% C_o
Linde LA-2	---	---	0.18–0.26 (H)	---	LA_2 = Al_2O_3
carbon-graphite	---	---	---	0.04 (H,kkk)	

Table 12.7. Static and Kinetic Friction Coefficients (Continued)

MATERIALS	STATIC		KINETIC		REMARKS
	DRY	LUBRICATED	DRY	LUBRICATED	
Linde LC-1 "Flame-Plated" coating on:					
itself	---	---	0.19–0.34 (H)	0.13 (H,kkk)	LC-1 = 85% Cr_3C_2 + 15% Ni-Cr
Linde LW-1	---	---	0.20–0.42 (H)	---	LW-1 = WC + 7 to 10% Co
Linde LA-2	---	---	0.18–0.27 (H)	0.08 (H,kkk)	$LA_2 = Al_2O_3$
Linde LW-5	---	---	0.22–0.42 (H)	---	LW-5 = 25% WC + 7% Ni + mixed W-C carbides
Linde LC-3 "Flame-Plated" coating on itself	---	---	0.22–0.28 (H)	0.09 (H,kkk)	LC-3 = 80% Cr_2O_3 + 20% Al_2O_3
Linde LA-2 "Flame-Plated" coating on:					
itself	---	---	0.27–0.24 (H)	0.08 (H,kkk)	LA-2 = 99+% Al_2O_3
"Kentanium" 161	---	---	0.18–0.35 (H)	---	"Kentanium" = TiC
"Kentanium" 138	---	---	0.23–0.39 (H)	---	"Kentanium" = TiC
Linde LW-1	---	---	0.28–0.30 (H)	---	LW-1 = WC + 7 to 10% Co
carbon-graphite	---	---	---	0.04 (H,kkk)	

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LUBRICANTS

(a) Oleic acid
(b) Atlantic spindle oil (light mineral)
(c) Castor oil
(d) Lard oil
(e) Atlantic spindle oil plus 2 percent oleic acid
(f) Medium mineral oil
(g) Medium mineral oil plus ½ percent oleic acid
(h) Stearic acid
(i) Grease (zinc oxide base)
(j) Graphite
(k) Turbine oil plus 1 percent graphite
(l) Turbine oil plus 1 percent stearic acid
(m) Turbine oil (medium mineral)
(n) Olive oil
(p) Palmitic acid
(q) Ricinoleic acid
(r) Dry soap
(s) Lard
(t) Water
(u) Rape oil
(v) 3-in-1 oil
(w) Octyl alcohol
(x) Triolein
(y) 1 percent lauric acid in paraffin oil
(z) Cholesterol
(aa) Water vapor
(bb) Extreme pressure mineral oil
(cc) Graphited mineral oil
(dd) Trichlorethylene
(ee) Alcohol
(ff) Glycerine
(gg) SAE 100:1
(hh) MoS_2 resin-bonded film, estimated thickness 0.005 in.
(ii) Rubbed graphite film, estimated thickness 0.0005 in.
(jj) Lead monoxide
(kk) Tin coating
(ll) Gold coating
(mm) Lead coating
(nn) Silver coating
(oo) Gallium coating
(pp) Liquid nitrogen
(qq) FeS film, approximately 1000A (4×10^{-6} in.) thick
(rr) $FeCl_3$
(ss) Ceramic-bonded MoS_2 film, 0.0002 to 0.0003 in. thick
(tt) Metal-matrix-bonded MoS_2 film, 0.0002 to 0.0003 in. thick
(uu) Silicone-resin-bonded MoS_2 film, 0.0002 to 0.0003 in. thick
(vv) Phenolic-epoxy-bonded MoS_2 film, 0.0002 to 0.0003 in. thick

LUBRICANTS (Continued)

(ww)	Silver plate
(xx)	Rolled film
(yy)	Metal-free phthalocyanine
(zz)	MoS_2 (see also listings hh, ss, tt, uu, and vv)
(aaa)	Lead oxide (litharge)
(bbb)	Flake graphite
(ccc)	Boron nitride
(ddd)	Stearates, metallic soaps
(eee)	Octadecyl (steryl) alcohol
(fff)	Liquid hydrogen
(ggg)	Sodium
(hhh)	Oxide film
(iii)	Sulphide film
(jjj)	Oil bath, viscosity 150 saybolt universal sec.
(kkk)	Water and abrasive dust

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12.2 PROPERTIES OF FLUIDS

Storable Propellants

34-14	486-1	773-1**	81-11**
54-19	491-1	35-18**	
81-4	492-1	35-20**	
331-1	287-4	35-21**	

Cryogenic Fluids

34-15	155-1	493-1*	509-3**
82-15	478-1	522-1*	35-19**
106-10	486-1	698-1**	35-20**
			35-21**
			476-7**

Water

132-1	508-1
409-2	564-1

Hydraulic Fluids

42-1	V-274

Gases

34-15	248-1	152-7*	522-1*
132-1	486-1	556-1*	563-1*
155-1	493-1	551-1*	698-1**

12.3 PROPERTIES OF POLYMERS

Elastomers

19-227	65-28
65-19	65-29
56-1	479-1

Plastics

1-279	65-31	104-4*
19-236	117-10	
65-30	310-1	

12.4 PROPERTIES OF METALS

65-29	286-4	286-5*	547-4*	547-7*
286-3	482-1	286-7*	547-5*	554-1*
136-7*	286-5*	547-3*	547-6*	

12.5 PROPELLANT CHEMICAL COMPATIBILITY

Aerozine-50

81-4	34-20	104-3*
34-9	131-25	131-40**

Ammonia

44-15	452-2
287-4	104-3*

Chlorine Trifluoride and Pentafluoride

34-9	131-25	104-3*	507-1**
35-6	287-4	35-19**	507-2**
44-15	452-2	131-40**	86-3**

Diborane

505-1**
131-40**

Fluorine

34-9	131-25	36-38**
44-15	287-4	183-10**
44-16	104-3*	

Hydrazine

34-9	117-13	131-40**
34-20	131-25	86-3**
35-4	287-4	
44-15	452-2	
81-4	104-3*	

Hydrogen Peroxide

44-15	452-2	35-18**
287-4	104-3*	86-3**

Liquid Hydrogen

34-9	155-1	44-32*
44-15	287-4	104-3*
131-25	452-2	

Liquid Oxygen

34-9	131-25	104-3*
44-15	287-4	
44-16	452-2	

Monomethylhydrazine

44-15	287-4	104-3*
131-25	452-2	131-40**

Fuming Nitric Acid

44-15	452-2
287-4	104-3*

Nitrogen Tetroxide

34-9	147-17	44-30*	86-3**
34-20	287-4	44-32*	
44-15	452-2	104-3*	
81-4	509-1	773-1**	
131-25	36-29*	131-40**	

Oxygen Difluoride

34-9	131-25	131-40**
44-15	104-3*	446-4**

Pentaborane

35-7	287-4	104-3*
44-15	452-2	131-40**
131-25	86-3**	

Perchloryl Fluoride

44-15	452-2	131-40**
131-25	486-1	

RP-1

452-2	104-3*

Unsymmetrical Dimethylhydrazine

34-9	131-25	104-3*
34-20	287-4	131-40**
44-15	452-2	

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476-2
153-12**

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ISSUED: FEBRUARY 1970
SUPERSEDES: MAY 1964

TABLES

Table

13.1 INTRODUCTION

The purpose of the Environments Section is to (1) describe the kinds of environments encountered in the rocket system, the atmosphere, and in space; and, (2) discuss the implications of environmental extremes on fluid component design. The rocket system environment is described with respect to pressure (Sub-Section 13.2), temperature (Sub-Section 13.5), and acceleration, shock, and vibration (Sub-Section 13.3). The atmospheric environment is discussed in Sub-Section 13.4. The space environment (Sub-Section 13.6) is characterized by vacuum, particle and electromagnetic radiation, meteoroids, and zero gravity, as well as the environments of the planets of the solar system.

Although not strictly an environment, corrosion which results from atmospheric or rocket system environments is discussed in Sub-Section 13.7. More detailed rocket propellant compatibility data is given in Sub-Section 12.5.

Environmental testing is treated specifically in Sub-Section 15.7.

13.2 PRESSURE

13.2 PRESSURE

13.2.1 Introduction

The subject of pressure can be divided into two major categories: (1) pressures above one atmosphere and, (2) pressures below one atmosphere. Pressures above one atmosphere are encountered primarily as system pressures (internal environment) while pressures encountered below one atmosphere include aerospace environmental pressures as well as vacuum system pressures. The range of pressures through which aerospace fluid components may be expected to operate in the near future varies from a low pressure approximately 10 [illegible] torr (mm Hg), which is the approximate pressure of interplanetary space, to a high internal system pressure of approximately 10,000 psi. Table 13.2.1 presents the range of pressures encountered, imposed either by the system or by the natural ambient environment. Since the design considerations for pressures below one atmosphere are treated under "Space Vacuum" in Sub-Topic 13.6.2, these paragraphs are limited to design considerations for pressure above one atmosphere.

13.2.2 Terminology Related to Pressures Above One Atmosphere

There is no general agreement as to a quantitative definition of the term *high pressure*. Beyond the fact that high pressure is in excess of one atmosphere, engineers working with hydraulic systems, pneumatic systems, or propellant feed systems have significantly differing concepts as to the magnitude implied by the term high pressure. For purposes of this handbook, the following definitions are used:

Low pressure: one atmosphere to 500 psia

Medium pressure: 500 to 3000 psia

High pressure: 3000 to 10,000 psia

Ultra high pressure: above 10,000 psia.

Pressure requirements for fluid components are usually specified in terms of operating pressure, proof pressure, and burst pressure. These terms are defined as follows:

Operating pressure: the maximum pressure to which a component will be subjected during normal, anticipated service.

Proof pressure: a pressure above the operating pressure which provides an operational safety margin. This safety margin accounts for factors such as unusual dynamic conditions, for instance pressure surges which can provide transient pressure conditions in excess of a normal operating pressure. A component must be designed to withstand sustained exposure to its proof pressure with no degradation in its operational performance. For airborne systems the proof pressure is often specified as 1.5 to 2 times the operating pressure. (See Sub-Topic 14.7.1.)

Burst pressure. a non-operational pressure level in excess of the proof pressure, established to provide an additional margin of safety to the component in the event of unscheduled pressures in excess of the operational level. The component must be capable of withstanding the burst pressure without structural failure to assure that there is no hazard to personnel or equipment. The component is not normally required to meet operational performance after exposure to its burst pressure. Burst requirements have been met if the component has not ruptured, even though leakage is excessive and other operational performance parameters are out of specification by virtue of some permanent damage to the component. Usual standards for ground equipment specify burst pressure at four times the operational level for non-shock conditions. On airborne equipment, burst pressures are commonly defined as two times the operational pressure. (See Sub-Topic 14.7.1.)

Airborne equipment design is commonly based on proof pressure. Minimum dimensions are determined using proof pressure loading and an allowable design stress to some fraction of the yield strength. Values of 50 to 90 percent of the yield stress are commonly employed for making design calculations, with the specific value being dependent

Table 13.2.1. Pressure Environments for Various Systems and Components

PRESSURE RANGE	SYSTEM OR ENVIRONMENT
Extreme Vacuum $10^{-[illegible]}$ — $10^{-[illegible]}$ mm Hg	Space environmental pressure; affects spacecraft operating in the space and lunar environment.
Ultra-High Vacuum $10^{-[illegible]}$ — $10^{-[illegible]}$ mm Hg	Includes the upper region of the ionosphere and the exosphere; affects missiles, space boosters, and spacecraft.
High Vacuum $10^{-[illegible]}$ — $10^{-[illegible]}$ mm Hg	Includes the ionosphere; affects missiles, space boosters, and spacecraft.
Medium Vacuum 1 — $10^{-[illegible]}$ mm Hg	Includes the upper region of the stratosphere; affects missiles.
Rough Vacuum 14.7 psi — 1 mm Hg	Includes the troposphere and lower stratosphere; affects missiles and aircraft.
1 Atmosphere 14.7 — 100 psi	Earth ambient atmospheric pressure; affects all components and systems. Hydraulic and pneumatic control systems, including attitude control for space vehicles. Regulated propellant pressurant on some missiles. Low pressure rotary pumps.
100 — 500 psi	Hydraulic and pneumatic control systems. Regulated pressurants for liquid bipropellant rocket engines for space vehicles.
500 — 1000 psi	Liquid rocket propulsion system for attitude control of lunar space vehicle. High pressure vane and gear pumps.
1000 — 2000 psi	Current and future fuel pressures for aircraft systems. Mobile equipment.
2000 — 3000 psi	Stored pressurants for space vehicles and missile propulsion systems. High pressure piston pumps.
3000 — 5000 psi	Stored pressurants for missile and spacecraft propulsion systems.
5000 — 6000 psi	Proposed high pressure hydraulic piston pumps. Maximum capability of available axial piston pumps
6000 — 10,000 psi	High pressure radial piston pump and actuators.
10,000 — 30,000 psi	Liquid springs and spring shocks used on suspension and landing gear systems. High energy shock absorbers and accumulators.
30,000 — 50,000 psi	Fluid compressibility devices for research and development.

on fabrication methods, material characteristics, surface finishes, and accuracies with which actual stresses can be predicted. Stresses must also be checked at the burst pressure to determine that the ultimate strength of the material has not been exceeded.

13.2.3 Design Considerations for High Pressure

In the following Detailed Topics, problem areas associated with the design of fluid components for high pressure are discussed.

13.2.3.1 LEAKAGE. Leakage, one of the most common problems with fluid components, is usually magnified as the pressure level is increased. Exceptions are certain pressure loaded, or pressure energized, seals which develop higher sealing forces as pressure levels increase. The subject of leakage is covered in the following sections of the handbook:

Sub-Topic 3.11.5, "Tortuous Passages." These paragraphs cover the basic theory of leakage flow and presents data on the permeation of gases through various materials.

Sub-Sections 6.3 and 6.4, "Seals," These paragraphs treat the subject of design and selection of static and dynamic seals.

13.2.3.2 DIMENSIONAL CHANGES. Dimensional changes occurring under pressure loading must be carefully considered in the design of components which must operate at elevated pressures. Strains resulting from pressure loading can adversely affect component operation, particularly where there are moving parts with very close tolerances.

13.2.3.3 ACTUATION FORCES. High pressure forces on unbalanced valving elements such as balls, butterflies, gates, and poppets result in the need for high actuation forces, particularly where fast actuation times are required. Spool valves and balanced poppet designs are commonly used for high pressure valves to avoid the problems experienced in unbalanced designs. Balanced poppets are discussed briefly in Detailed Topic 5.2.5.2.

13.2.3.4 STRUCTURAL INTEGRITY. The housing of a fluid component is essentially a pressure vessel which must be designed for structural integrity under pressure loading.

As the housing of fluid components often involves complex shapes, the designer must combine stress analysis and the use of adequate safety factors with laboratory testing. The stress analysis of fluid components is usually based on making simplifying assumptions as to shape, and analysing the component in terms of simple figures of revolution such as cylinders and spheres.

The walls of a pressure vessel in a form of a surface of revolution are stressed simultaneously in three mutually perpendicular axes (a triaxial state of stress) when the vessel is subjected to uniform internal or external pressure. These three stresses, illustrated in Figure 13.2.3.4 are defined as follows:

Meridional Stress (σ_m) : stress in a closed ended pressure vessel along a line representing the intersection of the vessel wall and a plane containing the axis of the vessel. In cylinders, meridional stress is synonymous with *longitudinal* and *axial* stress.

Circumferential or Hoop Stress (σ_c) : stress acting tangentially to the vessel wall in a plane normal to the axis of the vessel.

Radial Stress (σ_r) : stress acting across the wall thickness along a radius. This stress is a bearing stress resulting from the pressure against the vessel wall.

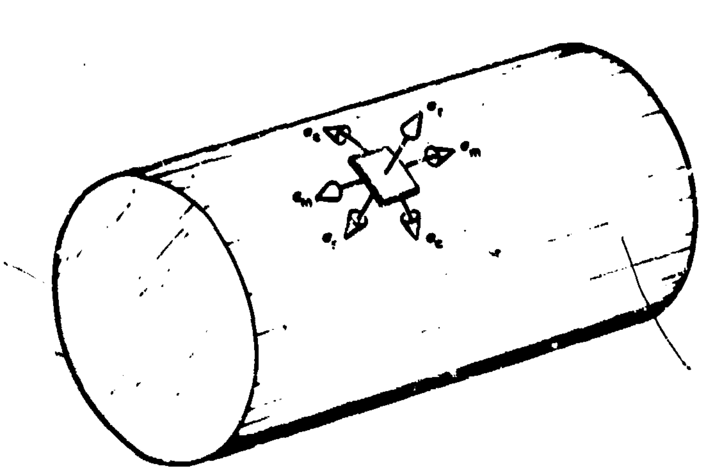


Figure 13.2.3.4. Stresses in a Pressure Vessel

Both meridional and circumferential stresses are called *diaphragm* or *membrane* stresses. The displacements caused by membrane stresses result in *bending* stresses which are significant in thick wall vessels. Additional stresses which are important, particularly in closed ended, stiffened, or non-uniform pressure vessels, are *discontinuity* stresses. These stresses are caused by bending shear due to abrupt changes in wall thickness or meridional slope and are superimposed upon membrane stresses.

Reference 461-1 presents formulas for determining discontinuity stresses in a variety of pressure vessel configurations.

If the wall thickness is small compared to the radius of curvature (less than one-tenth the radius) and there are no discontinuities such as sharp bends in meridional curves, bending of the vessel walls can be neglected, i.e., the membrane stresses across the wall thickness can be considered as uniformly distributed. A vessel satisfying these simplifying assumptions is defined as a thin wall vessel. Formulas for stress and radial displacement (ballooning) for thick and thin walled cylindrical and spherical pressure vessels are given in Table 13.2.3.4.

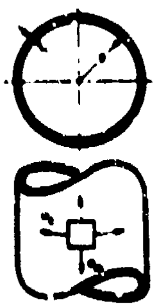
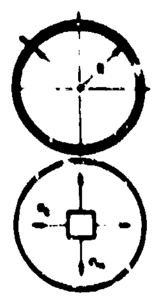
A comprehensive treatment of pressure-induced stresses may be found in Sub-Section 14.7.

13.3 ACCELERATION, SHOCK, AND VIBRATION

13.3.1 Introduction

Dynamic forces resulting from acceleration, shock, and vibration can present a challenge to the fluid component designer. This Sub-Section describes the dynamic environment to which aerospace fluid components are subjected, and design techniques for minimizing the effects of these environments. Shock and vibration environment will affect component design more than acceleration environment because of the resonance phenomenon associated with vibration. The actual dynamic loads on aerospace components cannot be predicted in many instances, because the shock and vibration conditions have complex wave forms which cannot be described by mathematical analysis. Because of these complex wave forms and resonance phenomena, the

Table 13.2.3.4. Formulas for Stresses and Deformations in Pressure Vessels
(Reference 4J1-1)

FORM OF VESSEL	THIN-WALLED VESSELS: LOADING	THIN-WALLED VESSELS: FORMULAS
 Cylindrical	Uniform internal (or external) pressure, p, lb_f/in^2	$\sigma_m = \frac{pR}{2t}$ $\sigma_t = \frac{pR}{t}$ Radial displacement $= \frac{R}{E}(\sigma_t - \nu\sigma_m)$ where ν = Poisson's ratio External collapsing pressure $p = \frac{t}{R}\left(\frac{\sigma_y}{1 + 4\frac{\sigma_y}{E}\left(\frac{R}{t}\right)^2}\right)$ where σ_y = compressive yield point of material. This formula is for nonelastic failure, and holds only when $\frac{pR}{t}$ > proportional limit.
 Spherical	Uniform internal (or external) pressure, p, lb_f/in^2	$\sigma_m = \sigma_t = \frac{pR}{2t}$ Radial displacement $= \frac{R\sigma_m}{E}(1 - \nu)$

development of fluid components depends more on testing than on design analysis. The sources of acceleration, vibration, and shock inputs include engine ignition shocks, combustion instability, rocket engine acoustic pressures, aerodynamic forces, fuel sloshing, engine burnout, and stage and satellite separation forces.

13.3.2 Dynamic Force Environment

13.3.2.1 DYNAMIC FORCES DURING MISSILE ASCENT. The initial launch phase of a space vehicle is characterized by engine ignition and an intense acoustical field from the rocket engine exhaust which is reflected from the ground to the launch vehicle. As the launch vehicle rises, its acoustical excitation diminishes until the vehicle approaches the speed of sound, at which time aerodynamic disturbances increase sharply. Once past sonic speed, aerodynamic excitation diminishes until stage separation, when the vehicle is subjected to shock forces resulting from exploding bolts and/or second stage engine ignition.

Typical flight acceptance specifications for several vehicle types are given in Tables 13.3.2.1a and 13.3.2.1b. These specifications are for illustration only and the values are not the values measured in flight.

13.3.2.2 DYNAMIC FORCES DURING SPACE FLIGHT. Vibration and shock may be more serious during space flight than during the launch phase. This would be particularly true in a mission which requires maneuvering for rendezvous or for transfer between orbits and/or soft landing by throttling. Sources of dynamic forces include maneuvering and landing engines (start, shutdown, random, and discrete

Table 13.3.2.4 (Continued)

THICK-WALLED VESSELS

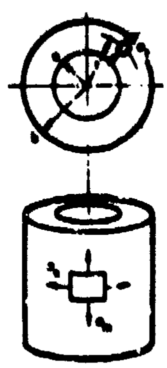
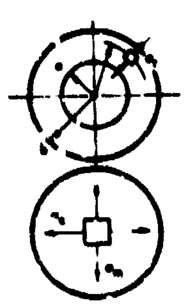
FORM OF VESSEL	LOADING	FORMULAE
	Uniform internal radial pressure, p, lb_f/in^2 (longitudinal pressure zero or externally balanced)	$\sigma_m = 0$ $\sigma_c = p\,\frac{a^2(b^2+r^2)}{r^2(b^2-a^2)}$ Max $\sigma_c = p\,\frac{b^2+a^2}{b^2-a^2}$ at inner surface $\sigma_r = p\,\frac{a^2(b^2-r^2)}{r^2(b^2-a^2)}$ Max $\sigma_r = p$ at inner surface Max $\sigma_s = p\,\frac{b^2}{b^2-a^2}$ at inner surface where σ_s = shear stress $\Delta a = p\,\frac{a}{E}\left(\frac{b^2+a^2}{b^2-a^2}+\nu\right)$ $\Delta b = p\,\frac{b}{E}\left(\frac{2a^2}{b^2-a^2}\right)$
Cylindrical	Uniform external radial pressure, p, lb_f/in^2	$\sigma_m = 0$ $\sigma_c = -p\,\frac{b^2(a^2+r^2)}{r^2(b^2-a^2)}$ Max $\sigma_c = -p\,\frac{2b^2}{b^2-a^2}$ at inner surface $\sigma_r = p\,\frac{b^2(r^2-a^2)}{r^2(b^2-a^2)}$ Max $\sigma_r = p$ at outer surface Max $\sigma_s = \frac{1}{2}$ Max σ_c at inner surface $\Delta a = -p\,\frac{a}{E}\left(\frac{2b^2}{b^2-a^2}\right)$ $\Delta b = -p\,\frac{b}{E}\left(\frac{a^2+b^2}{b^2-a^2}-\nu\right)$
	Uniform internal pressure, p, lb_f/in^2 in all directions	$\sigma_l = p\,\frac{a^2}{b^2-a^2}$ $\sigma_c = p\,\frac{a^2(b^2+r^2)}{r^2(b^2-a^2)}$ Max $\sigma_c = p\,\frac{b^2+a^2}{b^2-a^2}$ at inner surface $\sigma_r = p\,\frac{a^2(b^2-r^2)}{r^2(b^2-a^2)}$ Max $\sigma_r = p$ at inner surface $\Delta a = p\,\frac{a}{E}\left[\frac{b^2+a^2}{b^2-a^2}-\nu\left(\frac{a^2}{b^2-a^2}-1\right)\right]$ $\Delta b = p\,\frac{b}{E}\left[\frac{a^2}{b^2-a^2}(2-\nu)\right]$
	Uniform internal pressure, p, lb_f/in^2	$\sigma_m = \sigma_c = p\,\frac{a^3(b^3+2r^3)}{2r^3(b^3-a^3)}$ Max σ_m = Max $\sigma_c = p\,\frac{b^3+2a^3}{2(b^3-a^3)}$ at inner surface $\sigma_r = p\,\frac{a^3(b^3-r^3)}{r^3(b^3-a^3)}$ Max $\sigma_r = p$ at inner surface Max $\sigma_s = p\,\frac{3b^3}{4(b^3-a^3)}$ at inner surface $\Delta a = p\,\frac{a}{E}\left[\frac{b^3+2a^3}{2(b^3-a^3)}(1-\nu)+\nu\right]$ $\Delta b = p\,\frac{b}{E}\left[\frac{3a^3}{2(b^3-a^3)}(1-\nu)\right]$
Spherical	Uniform external pressure, p, lb_f/in^2	$\sigma_m = \sigma_c = -p\,\frac{b^3(a^3+2r^3)}{2r^3(b^3-a^3)}$ Max σ_m = Max $\sigma_c = -p\,\frac{3b^3}{2(b^3-a^3)}$ at inner surface $\sigma_r = p\,\frac{b^3(r^3-a^3)}{r^3(b^3-a^3)}$ Max $\sigma_r = p$ at outer surface $\Delta a = -p\,\frac{a}{E}\left[\frac{3b^3}{2(b^3-a^3)}(1-\nu)\right]$ $\Delta b = -p\,\frac{b}{E}\left[\frac{a^3+2b^3}{2(b^3-a^3)}(1-\nu)-\nu\right]$

Table 13.3.2.1a. Spacecraft Environment Test Parameters
(Reference 36-37)

LAUNCH VEHICLE	ACCELERATION*	VIBRATION**	
Atlas/Agena	1.5 times maximum combined thrust and lateral acceleration	Mode A:	5-250 cps, ±2.3g 250-400 cps, ±3.7g 400-2000 cps, ±7.5g
		Mode B:	5-250 cps, ±1.5g 250-400 cps, ±3.0g 400-2000 cps, ±7.5g
		Mode C:	20-150 cps, 0.023 g/cps 150-300 cps, increasing by 3 db/octave 300-2000 cps, 0.045 g^2/cps
		Mode D:	Resonance point between 60 and 75 cps, or 68 cps if no major torsional resonance, 96.6 rad/sec^2
Thrust Augmented Delta (TAD)	1.5 times maximum third stage thrust	Mode A:	10-50 cps, ±3.8g 10-500 cps, ±7.5g 500-2000 cps, ±21.0g
		Mode B:	10-18 cps, ±3.0g 18-500 cps, ±2.3g 500-2000 cps, ±4.0g
		Mode C:	20-2000 cps 0.07 g^2/cps 11.8 g-rms
		Mode D:	Not required
Scout	1.5 times maximum fourth stage thrust	Mode A:	10-53 cps, ±12 in/sec constant velocity 53-100 cps, ±10.5g 100-2000 cps, ±7.5g
		Mode B:	5-150 cps, ±1.5g 150-400 cps, ±3.0g 400-2000 cps, ±7.5g
		Mode C:	20-20,000 cps 0.07 g^2/cps 11.8 g-rms
		Mode D:	Not required

* g's vary with spacecraft weight

** Mode A: Sinusoidal — Thrust Axis (2 octaves/minute)
Mode B: Sinusoidal — Two Lateral Axes Mutually Perpendicular (2 octaves/minute)
Mode C: Random — Thrust and Lateral Axes (4 minutes, each axis)
Mode D: Torsional — Thrust (2 pulses)

ISSUED: FEBRUARY 1970
SUPERSEDES: MAY 1964

Table 13.3.2.1b. Spacecraft Design Environments for Titan III
(Reference 131-32)

DESIGN CONDITION	INTENSITY OR RATE	CYCLE OR TIME	REMARKS
1. Launch/Ascent Vibration			
a) For internal spacecraft components	0.016 g^2/cps to 0.25 g^2/cps increasing by 3 db/octave	20-300 cps	Overall level 19.5 RMS
	0.25 g^2/cps	300-1200 cps	
	0.25 g^2/cps to 0.09 g^2/cps with roll off at 6 db/octave	1200-2000 cps	
b) For external spacecraft	0.01 g^2/cps to 0.4 g^2/cps increasing at 3 db/octave	20-800 cps	Overall level 19.5 RMS applies to such components as sun sensors, solar panels, etc.
	0.4 g^2/cps	800-1500 cps	
	0.4 g^2/cps to 0.3 g^2/cps with roll off at 3 db/octave	1500-2000 cps	
2. Acceleration	4.5g peak	420 seconds	Applies to ascent phase; launch phase negligible
3. Shock	2500g peak	50-10,000 cps	For components near satellite/booster interface
	480g peak	80-10,000 cps	For components connected to interface by truss assembly

frequencies of thrust variation), touchdown, and rendezvous. It should be noted, however, that even though a fluid component used on the space vehicle may not be operating at the time of the earth launch, it will still be subjected to launch forces and must be designed to withstand the adverse effects of acceleration and vibration loads. Table 13.3.2.2 presents typical Apollo Lunar Module dynamic environment data.

13.3.2.3 DYNAMIC FORCES DURING RE-ENTRY. Accelerations well in excess of values during launch can be experienced by spacecraft during entry or re-entry to the planet's atmosphere.

Expected values for deceleration experienced during entry or re-entry to the planets are as follows (Reference 331-1):

Planet	Direct entry at escape velocity			Direct entry at orbital velocity			Entry by decay from satellite orbit
	$\theta = 5°$	20°	90°	$\theta = 5°$	20°	90°	
Venus	28.6	112	326	14.3	56	163	8.9
Earth	28.3	111	324	14.3	55.5	162	9.5
Mars	1.6	6.3	18.3	0.8	3.2	9.2	9.2

where θ is the re-entry angle with the horizontal, and decelerations are given in earth g's.

Table 13.3.2.2. Spacecraft Dynamic Design Environments for Saturn/Apollo Lunar Module
(Reference 131-31)

DESIGN CONDITION	INTENSITY OR RATE	CYCLE OR TIME	REMARKS
1. Random Vibration			
a) Launch/Ascent	0.010 g^2/cps to 0.06 g^2/cps linear increase	15-100 cps	30 minutes for each of three mutually perpendicular axes
	0.06 g^2/cps	100-1000 cps	
	0.06 g^2/cps to 0.015 g^2/cps linear decrease	1000-2000 cps	
b) Space Flight	0.012 g^2/cps to 0.04 g^2/cps linear increase	15-100 cps	10 minutes for each of three mutually perpendicular axes
	0.04 g^2/cps	100-200 cps	
	0.04 g^2/cps to 0.015 g^2/cps linear decrease	200-2000 cps	
c) Lunar Descent	0.0015 g^2/cps to 0.04 g^2/cps linear increase	10-50 cps	10 minutes for each of three mutually perpendicular axes
	0.04 g^2/cps	50-200 cps	
	0.04 g^2/cps to 0.0015 g^2/cps linear decrease		
2. Sinusoidal Vibration			
a) Launch/Ascent	0.20 inch Double Amplitude	5-10 cps	Sinusoidal vibration shall be superimposed on random vibration. The sinusoidal vibration shall be swept logarithmically from 5 to 2000 cps in 6 minutes for each of three mutually perpendicular axes.
	1.0g	10-18 cps	
	1.0 - 18g	18-78 cps	
	18g peak	78-200 cps	
	10g	200-2000 cps	
b) Space Flight	1.0g to 4.7g linear increase	5-150 cps	Sinusoidal vibration shall be superimposed on random vibration. Sinusoidal vibration shall be swept logarithmically from 5 to 2000 cps in 2 minutes for each of three mutually perpendicular axes.
	4.7g	150-200 cps	
	2.5g	200-1000 cps	
	2.0g	1000-2000 cps	
c) Lunar Descent	2.0 to 15.0g linear increase	10-50 cps	Sinusoidal vibration shall be superimposed on random vibration. The sinusoidal vibration shall be swept logarithmically from 5 to 2000 cps in 5 minutes for each of three mutually perpendicular axes.
3. Prelaunch Vibration	0.5 inch D.A.	5-7.2 cps	Vibration to be applied along three mutually perpendicular axes at 1/2 octave per minute.
	±1.3g	7.2-27.5 cps	
	0.036 inch D.A.	27.5-52 cps	
	6.0g	52-500 cps	
4. Acceleration	10g ± 2g	5 minutes in each direction	Each of three mutually perpendicular axes
5. Shock	±15g peak	10 ms	Modified shock-pulse to sawtooth, 15g peak, 10 to 12 ms rise time and 0 to 2 ms delay time; 3 shocks in each of three mutually perpendicular directions.

13.3.3 Acceleration

The basic effect of acceleration is a force equal to the product of the acceleration and the mass of the accelerated part. Acceleration forces are encountered in all vehicles which start from rest and achieve a given velocity. Acceleration forces are also inherent in shock and vibration. Acceleration testing is treated in Sub-topic 15.7.3.

Acceleration is commonly given in terrestrial g units where one g equals 32.1740 ft/sec². To obtain the forces of a body in a different gravitational field or under acceleration, the weight under a one g acceleration or terrestrial weight is multiplied by the number of g units. For example, a valve which weighs 10 pounds while at rest on earth weighs 100 pounds under an acceleration or gravitational field of 10 g's (321.74 ft/sec²).

Under acceleration loads, spring-loaded poppets may be unseated and electrical contacts may arc or close. The acceleration force must be less than the spring force to prevent a poppet from opening (assuming there is no pressure force on the valve poppet) or the electrical contact from closing. A simple design technique to avoid unseating a valve poppet under acceleration is to orient the valve closure such that the direction of acceleration is normal to the poppet. Assuming that the increased closing force does not adversely effect the opening of components such as relief valves and regulators, the component can be oriented such that acceleration forces tend to close the poppet. Component orientation to avoid valve opening in an acceleration field is illustrated in Figure 13.3.3a. An increase in the spring stiffness or a decrease in the weight of the spring plus poppet are also recommended design techniques to avoid movement of the valve poppet under acceleration loads.

Another means of nullifying the effect of acceleration on spring-loaded masses is to add a mass compensator, also known as a counterbalance. A schematic illustrating the application of a mass compensator is shown in Figure 13.3.3b. An application of this type of compensator in a pressure regulator controller is shown in Figure 5.4.5. The controller is extremely sensitive to small deviations in downstream regulated pressure and magnifies the error between actual and desired values of regulated pressure by means of a pilot valve. It is important that any movement of the controller mass be prevented when subjected to acceleration.

Since the acceleration loads will act through the center of gravity of the component, overhung moments should be avoided by locating the centroid of the attaching fasteners as close as possible to the component's center of gravity.

13.3.4 Shock

Shock, sometimes referred to as impulse or impact loading, may be defined as a suddenly applied load of short duration.

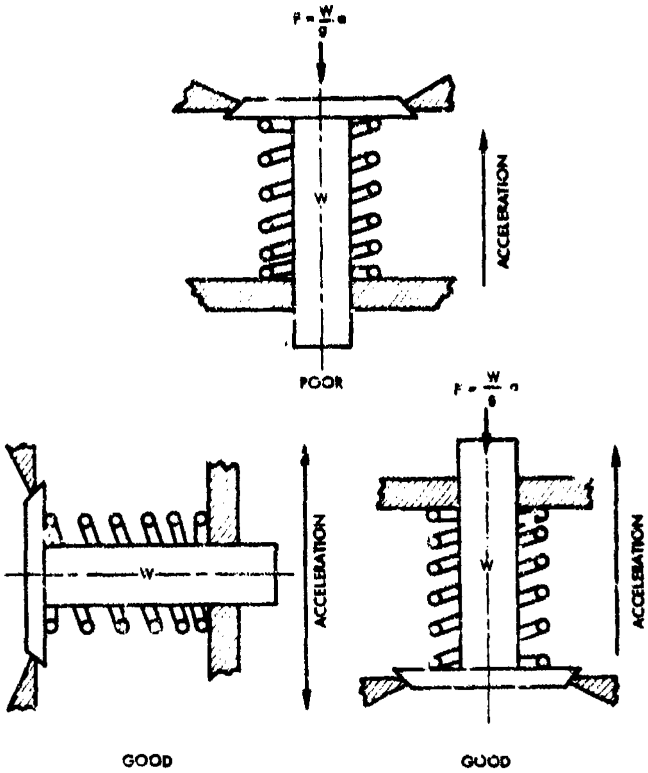


Figure 13.3.3a. Poppet Orientations to Avoid Valve Openings in an Acceleration Field

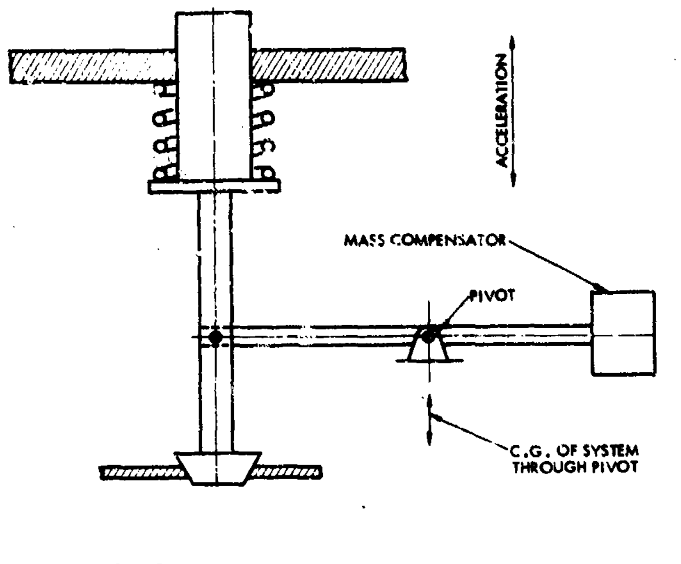


Figure 13.3.3b. A Counterbalanced Spring-Loaded Poppet Illustrating Mass Compensation

The magnitude of a shock load is usually high but the time duration of loading is relatively small. The characteristic of a shock load which makes it different from a static load is the time required for the force to rise from zero to a maximum, compared to the natural period of vibration of the structure. In general, the following statements will apply:

1) If the time of load application is less than one-half the natural period of the structure, it is definitely an impact load.

2) If the time of load application is greater than 3 times the natural period of the structure, it is definitely a static load.

The response of the structure under shock conditions has characteristics similar to those of systems under vibration. The initial deformation of the structure is large, and then is damped to a harmonic oscillation which finally goes to zero. The intensity of the response of a structure to a pulse loading will depend on how close the structural or natural frequency is to the forcing frequency. It should be noted that for anything other than a single element structure there may be a number of natural frequencies corresponding to various elements in the component.

A load factor of 2 applied to the equivalent static load caused by the impulse is normally considered good design practice. This is a safe load factor; however, in many cases it may be too high. A further discussion on shock loading and load factors is given in Sub-Section 7.3.

Example:

A valve which weighs 5 pounds under an acceleration of one g is subjected to an impact loading of 50 g's. Find the design load due to shock.

$$\text{Equivalent static load} = 5\,\frac{\text{lbs}}{\text{g}} \times 50\text{g} = 250 \text{ lbs}$$

$$\text{Design load} = 2 \times 250 \text{ lbs} = 500 \text{ lbs}$$

The load factor of 2 may have been applied to the static stress or to the g loading instead of the static load.

13.3.4.1 SOURCES OF SHOCK ENVIRONMENT. Sources of shock environment include:

a) Rocket engine ignition shock

b) Rocket engine combustion instability

c) Stage separation forces

d) Satellite separation forces

e) Recovery loads such as caused by parachute or water impact

f) Launching of rockets or projectiles from satellites

g) Impact loads due to meteoroid bombardment

h) Lunar landing impact loads

i) Pressure surges in fluid systems due to rapid opening or closing of a valve

j) Nuclear explosions near launch sites.

13.3.4.2 DESIGN TECHNIQUES FOR MINIMIZING THE EFFECTS OF SHOCK ENVIRONMENT. Eight design techniques are:

1) Locate the fluid component so that the sensitive elements do not coincide with the direction of greatest shock (See Figure 13.3.3a).

2) Mount the component so that the housing or brackets absorb the shock load.

3) For spring-loaded devices (e.g., valve closures) use lightweight parts or increase spring load to overcome expected shock loads.

4) Provide counterbalancing for sensitive spring-loaded masses such as the controller of a pressure regulator (Figure 13.3.3b).

5) Reduce excessive clearance in bearings and bushings.

6) Add rubber bumpers to cushion motion of plungers.

7) Provide damping to dissipate the impulse energy.

8) Provide shock isolation by mounting sensitive elements on energy absorbers. Shock testing is treated in Sub-topic 15.7.2.

13.3.5 Vibration

Vibration is a periodic or random displacement of a body from its equilibrium position. All bodies possessing mass and elasticity are subject to vibration along or transverse to any axis of the body. This Sub-Topic treats periodic vibration only, while random vibration is discussed in Sub-Section 7.3. Sub-Topic 15.7.1 treats vibration testing.

Vibration may be free or forced. Free vibration in an elastic system refers to a system free of impressed forces but under the action of forces inherent in the system itself. A freely vibrating system will vibrate at one or more of its natural frequencies. Forced vibration refers to a vibrating system under the excitation of an external force, i.e., a forcing function. The frequency of the exciting force is independent of the natural frequency of the system. When the frequency of the exciting force coincides with one of the natural frequencies, resonance will occur. Representative vibration environments are given in Table 13.3.5.

13.3.5.1 RESONANCE. When the frequency of the driving force is near the natural frequency of the structure, resonance will occur. When no damping is available in the system and when the driving frequency is equal to the natural frequency, the amplitude of vibration tends toward infinity. Avoiding resonance in a structure or in equipment is a primary objective of the designer; but, because of the complexity of equipment, it is not usually feasible to determine the resonant condition analytically. System and component testing must be done.

13.3.5.2 PERIODIC VIBRATIONS. The simplest form of periodic motion is simple harmonic motion represented by the sine and cosine functions. Consider the spring-mass system capable of free vibration, shown in Figure 13.3.5.2a.

Table 13.3.5. Representative Vibration Environments
(From References 13.-6 and 247-1)

SOURCE	VIBRATION ENVIRONMENT
Jet aircraft	Acoustical vibration due to jet wake and combustion turbulence. Frequency range up to 500 cps and maximum amplitude approximately 0.001 inch.
Piston engine aircraft	Engine vibration range up to 60 cps and maximum amplitude to 0.01 inch. Propeller vibrations range up to 100 cps with maximum amplitudes to 0.01 inch. Amplitudes of vibration vary with location in aircraft.
Ships	Engine vibration in diesel or reciprocating steam type range up to 15 cps with maximum amplitudes to 0.02 inch. Most vibrations are amplified. An amplification factor of 3 is usually acceptable.
Trucks	Suspension resonance of 1 cps with maximum amplitude of 5 inches. Structural resonance above 80 cps and maximum amplitude of 0.005 inch.
Passenger automobiles	Suspension resonance of 1 cps and maximum amplitude of 6 inches. Irregular transit vibrations due to road roughness above 20 cps and maximum amplitude of 0.002 inch.
Railroad trains	Broad and erratic frequency range. Isolation resonant frequency of 20 cps has been successful in railroad applications.
Rocket noise generated in exhaust stream	Usually most severe vibration environment in missiles. Results in random high amplitude vibrations during launch in atmosphere. Characterized by a broad spectral distribution coinciding with resonance frequencies of vehicle structure, skin, and equipment.
Space vehicles earth launch	Approximately 10 g's rms, 600 to 1600 cps. Acoustical noise in field of payload 150 decibels for 60 second duration.
Space vehicles low earth orbit	Vibration range to 1000 cps and up to 50 g's for 5 minute duration.
Space vehicles lunar orbit	Vibration range above 1000 cps and up to 50 g's for 10 minute duration.
Lunar launch	Vibration levels up to 15 g's with frequency spectrum greater than 1000 cps.
Lunar landing	Vibration levels up to 50 g's and frequency range from a few to several thousand cycles per second.

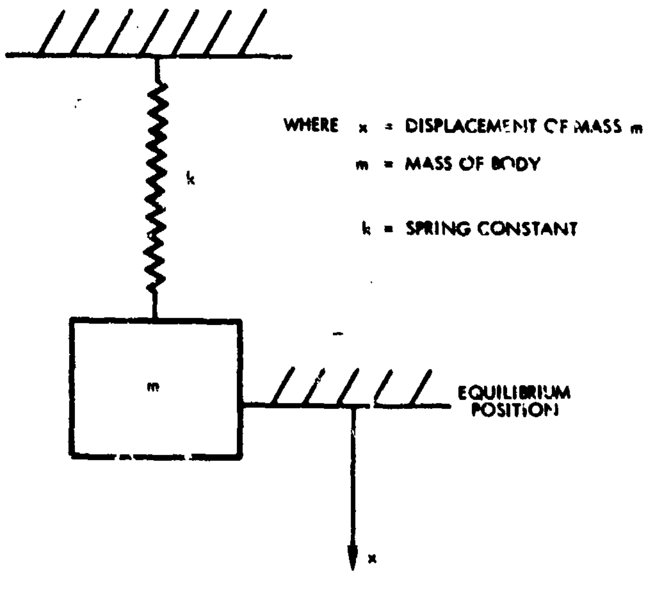


Figure 13.3.5.2a. Single Degree of Freedom Spring Mass System

If the mass, m, is displaced with a distance of $x = A$ and released, the system will vibrate with simple harmonic motion which may be represented by the following equations:

Displacement

$$x = A \cos \omega_n t \qquad \text{(Eq 13.3.5.2a)}$$

Velocity

$$\dot{x} = -A\omega_n \sin \omega_n t = +A\omega_n \cos\left(\omega_n t + \frac{\pi}{2}\right) \qquad \text{(Eq 13.3.5.2b)}$$

Acceleration

$$\ddot{x} = -A\omega_n^2 \cos \omega_n t = +A\omega_n^2 \cos(\omega_n t + \pi) = -x\omega_n^2 \qquad \text{(Eq 13.3.5.2c)}$$

where

$\omega_n = \sqrt{\frac{k}{m}}$ = the natural angular frequency, radians/sec

k = spring constant, lb_f/ft

m = mass, $lb_f\ sec^2/ft$

These equations can be represented by vectors rotating with velocity ($\omega_n t$) as shown in Figure 13.3.5.2b.

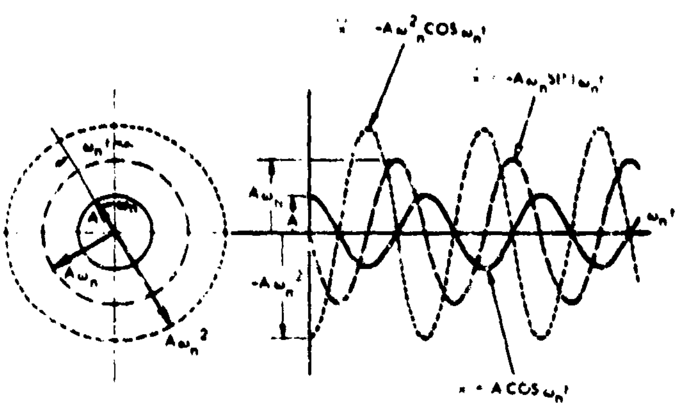


Figure 13.3.5.2b. Displacement, Velocity, and Acceleration in Harmonic Motion

The vector with magnitude, $A\omega_n$, represents the velocity and is 90° ahead of the displacement. The acceleration vector, $A\omega_n^2$, is 180° ahead of the displacement. These angles are called phase angles.

From the equations of motion, the following information may be obtained:

$\pm A$ = maximum displacement of mass, m

$\pm A\omega_n$ = maximum velocity of mass, m

$\pm A\omega_n^2$ = maximum acceleration of mass, m

The plus and minus signs indicate the motion in either direction from the equilibrium position.

Additional important relationships in vibration include:

Period of oscillation, seconds

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(Eq 13.3.5.2d)}$$

Frequency of oscillation, cycles per second (cps)

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{(Eq 13.3.5.2e)}$$

Acceleration at time t, sec

(Eq 13.3.5.2f)

$$Z = \pm 4\pi^2 f^2 A \cos 2\pi ft$$

Maximum acceleration, ft/sec²

$$Z_{max} = \pm 4\pi^2 f^2 A \qquad \text{(Eq 13.3.5.2g)}$$

Velocity at displacement x, ft/sec

$$V = \pm\sqrt{\frac{k}{m}}\sqrt{(A^2 - x^2)} \qquad \text{(Eq 13.3.5.2h)}$$

Example:

A valve weighs 1 lb_f and is vibrated sinusoidally at a frequency of 1000 cps at an amplitude of 0.0005 inch. Find the maximum force exerted by the valve.

$$a_{max} = 4\pi^2 f^2 A$$

$$= 4\pi^2 (1000)^2 \left(\frac{0.0005}{12}\right) = 1640 \text{ ft/sec}^2$$

$$F = ma$$

$$F = \frac{1}{32.2}(1640) = 51\ lb_f$$

13.3.5.3 REDUCING THE EFFECTS OF VIBRATION. The following sixteen design techniques will reduce the effects of vibration:

1) Reduce excessive clearances in bearings.

2) In rotating members, reduce or eliminate vibration forces by balancing or counterbalancing.

3) Provide isolation by mounting sensitive equipment on isolators.

4) If resonance occurs, it is often possible to change the natural frequency of the structure by modifying the mass or stiffness of the vibrating member.

5) If the vibration spectrum contains a large number of different frequencies and it is impractical to modify the natural frequency of the member, consider dissipating the energy with the use of dampers or damping materials (vibration mounts).

6) Vibration may be reduced by attaching an auxiliary mass to the system by a spring. The auxiliary mass vibrates and reduces the vibration of the system.

7) Avoid sharp bends, fillets, and cross-sectional area changes in notch-sensitive materials such as magnesium.

8) Avoid mounting components with large overhung moments.

9) Break up large areas, panels, etc., to minimize drumming or "oil canning."

10) To avoid poppet and electrical contact chatter, use ample spring forces (preload) or provide damping in spring-mass system. Appropriately mount sensitive elements of a component to avoid directional vibration.

11) Pins should be secured to prevent their motion along longitudinal axis.

12) Space tubing clamps at irregular intervals to prevent creation of large nodes which could vibrate at low frequencies.

13) Minimize excessive weld joint stress concentrations such as reducing the number of intermittent weld lengths. It is recommended that welds be at least 1½ inches long, spaced with at least 4 inches between welds. Welds should be full-depth to eliminate crevices. Subsequent heat treatment of welds to relieve residual stresses tends to increase fatigue strength.

14) Spot welds tend to be weak because of high stress concentrations in the junctions between the metals and are, therefore, not recommended for structural members supporting heavy equipment which may be subjected to shock and vibration.

15) Riveted members are more desirable than welded members because they provide interface friction and, therefore, damping between members. Cold-driven rivets should not be loaded in tension because of residual stress concentration at the formed head.

16) As bolts tend to loosen under vibration and shock, a means of locking must be provided. Slippage of the joint due to excessive clearance in bolt hole should be avoided by close tolerance bolts or dowel pins. Bolts made from materials with low yield strengths, such as 18-8 stainless steel, tend to stretch and loosen under shock loads even though they have a high ultimate strength. The fatigue strength of bolts may be increased by cold working such as rolling of thread, rolling of fillets near head, and shot peening the shank. Typical locking devices include threading lock wire through holes in the nuts or bolts fastened to the structure, friction nuts with a polymeric insert or distorted holes, friction bolts with a polymeric insert in the threaded portion, and lock washers. Lock washers are never used as locking devices when shock and vibration are present. Bolted structures provide friction damping between members and may be more desirable than a welded structure if damping is required.

REFERENCES

19-191	131-31*	360-1
35-1	131-32*	388-1
36-37*	247-1	388-2
131-6	331-1	388-3

*References added February 1970.

13.4 THE ATMOSPHERE

13.4.1 Physical Properties of the Atmosphere

The atmosphere is a gaseous envelope that surrounds the earth, extending from sea level to an altitude of several hundred miles. The altitude for near space has been somewhat arbitrarily set at 50 miles.

The earth's atmosphere is divided into five levels based on temperature variation. These levels are troposphere, stratosphere, mesosphere, thermosphere or ionisphere, and exosphere. The *troposphere* extends from sea level to 54,000 feet at the equator, decreasing to 28,000 feet at the poles, and is composed of approximately 79 percent nitrogen and 21 percent oxygen. A complete breakdown of all constituents is presented in Table 13.4.1a. With increasing altitude from sea level, the temperature diminishes from 60°F to −70°F. Above the troposphere is the *stratosphere*, which extends to approximately 65,000 feet and exists at a relatively constant temperature of −70°F. The *mesosphere* extends from nearly 65,000 feet to 300,000 feet, and its temperature increases from −70°F to +28°F, then decreasing to −134°F. The mesosphere is characterized by an ozone layer which absorbs the ultraviolet radiation from the sun. Above the mesosphere is the *thermosphere*, also called the ionosphere, which extends from approximately 300,000 feet to 1,000,000 feet. The temperature in this layer increases from −134°F to nearly 2200°F. The composition is primarily ionized atoms of the lighter gases. The last level is the *exosphere*, which extends into the space environment. At 2,320,000 feet the temperature of the widely separated gas molecules is approximately 2250°F. The temperature in space is discussed under Sub-Topic 13.6.5. Gas composition and concentrations are presented as a function of altitude under "Space Environments" in Table 13.6.2. The effects of altitude on pressure, temperature, and density are presented in Table 13.4.1b.

Table 13.4.1a. Normal Composition of Clean, Dry Atmospheric Air Near Sea Level
(Reference 349-1)

CONSTITUENT GAS, AND FORMULA	CONTENT, PERCENT BY VOLUME
Nitrogen (N_2)	78.084
Oxygen (O_2)	20.9476
Argon (Ar)	0.934
Carbon dioxide (CO_2)	0.0314
Neon (Ne)	0.001818
Helium (He)	0.000524
Krypton (Kr)	0.000114
Xenon (Xe)	0.0000087
Hydrogen (H_2)	0.00005
Methane (CH_4)	0.0002
Nitrous oxide (N_2O)	0.00005
Ozone (O_3)	Summer: 0 to 0.000007 Winter: 0 to 0.000002
Sulfur dioxide (SO_2)	0 to 0.0001
Nitrogen dioxide (NO_2)	0 to 0.000002
Ammonia (NH_3)	0 to trace
Carbon monoxide (CO)	0 to trace
Iodine (I_2)	0 to 0.000001

13.4.2 Moisture

The moisture content of the atmosphere is commonly expressed by the relative humidity, defined as the ratio of the actual vapor pressure of the water vapor contained in the air to the saturated vapor pressure of water vapor at the same temperature. Air with a constant water vapor content will experience a decrease in relative humidity with a rise in temperature. Another measure of atmospheric moisture content is the dew point. The dew point temperature, which is a function of the absolute quantity of moisture in the air, is the temperature to which the air must be lowered for water vapor to condense. Atmospheric moisture ranges from low relative humidity to precipitation, which can take the form of rain, snow, or hail. Sub-Topic 15.7.5 describes humidity testing.

13.4.2.1 EFFECTS OF MOISTURE ON FLUID COMPONENTS. Moisture may cause corrosion, especially in a salt atmosphere; short circuits between electrical conductors; and icing of components such as actuators and valve closures. Icing of vent valves and relief valves is a particularly critical problem in cryogenic systems.

13.4.2.2 DESIGN TECHNIQUES FOR AVOIDING THE ADVERSE EFFECTS OF MOISTURE. Nine design techniques for avoiding adverse effects of moisture are:

1) Provide proper surface coatings for materials to prevent corrosion.

2) Seal lubricated surfaces and moving parts.

3) Use drain holes, drip skirts, or rain masks where water may accumulate.

4) Use non-porous and non-absorbing materials for gaskets, insulation, etc.

5) Impregnate all capillary surfaces and edges with wax, moisture-resistant varnish, or resin.

6) Encapsulate or hermetically seal electrical windings.

7) Provide electric heating blankets for components susceptible to failures due to icing.

8) Design vent valves and relief valves so that in the closed position atmospheric air is prevented from coming in contact with seals and other moving parts.

9) Provide sufficient actuator forces to break ice accumulating on external seals.

13.4.3 Ozone

The ozone layer occurs in the mesosphere. Ozone is formed by the dissociation of molecular oxygen (O_2), caused by the photochemical process of ultraviolet radiation, and the uniting of the single atom of oxygen, O, with one molecule of O_2, forming a molecule of ozone, O_3. Ozone is also formed in the atmosphere from an electric discharge such as may occur during electrical storms or near electrical equipment.

The concentration of ozone ranges from 0.05 to 1 part per million by volume at sea level, and increases with increasing altitude to 10 ppm at 65,000 feet. This concentration remains constant to an altitude of approximately 90,000 feet. As the altitude increases to 160,000 feet, the ozone concentration gradually decreases to a value about the same as at sea level. In addition to naturally occurring ozone, ozone may also occur in urban areas as a result of the activities of man. During periods of moderate to severe conditions, ozone concentrations in the range of 25 to 50 ppm are measured in certain cities including Los Angeles and San Francisco.

Ozone causes cracking of natural rubber, butadiene-styrene (SBR), butadiene-acrylonitrile (NBR), and some other elastomers under stress. Ozone cracking resistance of an elastomer part is dependent on exposure temperatures, material strains, humidity, and ozone concentration. Polymers classified according to ozone resistance are presented in Table 13.4.3.

13.4.4 Sand and Dust

Sand is a siliceous particle ranging in size from 400 to 5000 microns in diameter. Dust consists of multiple composite particles, usually less than 15 or 20 microns in diameter. Dust particles may be electrically conductive and are usually soluble in water. (See Sub-Topic 15.7.4.)

Sand and dust damage is most severe in desert regions. Desert dust becomes airborne with slight winds and may remain suspended for hours as dust clouds, sometimes

Table 13.4.1b. Properties of the Atmosphere
(Reference 349-1)

ALTITUDE	TEMPERATURE	PRESSURE		DENSITY	
(THOUSANDS OF FT)	(°F)	(P, in. Hg)	$\left(\frac{P}{P_{sea\ level}}\right)$	(ρ, lb/ft³)	$\left(\frac{\rho}{\rho_{sea\ level}}\right)$
0 (sea level)	59.0	29.9213	1.0000	7.6474×10^{-2}	1.0000
1	55.434	28.8557	9.64389×10^{-1}	7.4262×10^{-2}	9.7107×10^{-1}
2	51.868	27.8212	9.29815×10^{-1}	7.2098×10^{-2}	9.4278×10^{-1}
3	48.303	26.8171	8.96255×10^{-1}	6.9984×10^{-2}	9.1513×10^{-1}
4	44.738	25.8426	8.63686×10^{-1}	6.7917×10^{-2}	8.8811×10^{-1}
5	41.173	24.8970	8.32085×10^{-1}	6.5898×10^{-2}	8.6170×10^{-1}
6	37.609	23.9798	8.01430×10^{-1}	6.3925×10^{-2}	8.3590×10^{-1}
7	34.045	23.0902	7.71698×10^{-1}	6.1998×10^{-2}	8.1070×10^{-1}
8	30.482	22.2276	7.42868×10^{-1}	6.0116×10^{-2}	7.8609×10^{-1}
9	26.918	21.3913	7.14920×10^{-1}	5.8278×10^{-2}	7.6206×10^{-1}
10	23.355	20.5806	6.87832×10^{-1}	5.6483×10^{-2}	7.3859×10^{-1}
15	5.546	16.8931	5.64587×10^{-1}	4.8137×10^{-2}	6.2946×10^{-1}
20	−12.255	13.7612	4.59912×10^{-1}	4.0773×10^{-2}	5.3313×10^{-1}
25	−30.047	11.1180	3.71577×10^{-1}	3.4306×10^{-2}	4.4859×10^{-1}
30	−47.831	8.90289	2.97544×10^{-1}	2.8637×10^{-2}	3.7473×10^{-1}
35	−65.606	7.06029	2.35962×10^{-1}	2.3751×10^{-2}	3.1058×10^{-1}
40	−69.700	5.55844	1.85769×10^{-1}	1.8895×10^{-2}	2.4708×10^{-1}
45	−69.700	4.37531	1.46227×10^{-1}	1.4873×10^{-2}	1.9449×10^{-1}
50	−69.700	3.44440	1.15116×10^{-1}	1.1709×10^{-2}	1.5311×10^{-1}
60	−69.700	2.13537	7.13664×10^{-2}	7.2589×10^{-3}	9.4919×10^{-2}
70	−67.424	1.32521	4.42898×10^{-2}	4.4787×10^{-3}	5.8565×10^{-2}
80	−61.977	8.27295×10^{-1}	2.76491×10^{-2}	2.7576×10^{-3}	3.6060×10^{-2}
90	−56.525	5.20011×10^{-1}	1.73793×10^{-2}	1.7100×10^{-3}	2.2360×10^{-2}
100	−51.096	3.29046×10^{-1}	1.09971×10^{-2}	1.0676×10^{-3}	1.3960×10^{-2}
150	19.403	4.01815×10^{-2}	1.34291×10^{-3}	1.1119×10^{-4}	1.4539×10^{-3}
200	−2.371	5.84575×10^{-3}	1.95371×10^{-4}	1.6957×10^{-5}	2.2174×10^{-4}
250	−107.84	6.0065×10^{-4}	2.0074×10^{-5}	2.263×10^{-6}	2.959×10^{-5}
300	−126.77	3.7268×10^{-5}	1.2489×10^{-6}	1.488×10^{-7}	1.946×10^{-6}
350	−24.53	3.3542×10^{-6}	1.1210×10^{-7}	1.012×10^{-8}	1.324×10^{-7}
400	233.94	6.3047×10^{-7}	2.1071×10^{-8}	1.164×10^{-9}	1.522×10^{-8}
450	731.10	2.4845×10^{-7}	8.3036×10^{-9}	2.600×10^{-10}	3.399×10^{-9}
500	1203.31	1.3799×10^{-7}	4.6117×10^{-9}	1.020×10^{-10}	1.333×10^{-9}
600	1647.18	5.9175×10^{-8}	1.9777×10^{-9}	3.350×10^{-11}	4.381×10^{-10}
700	1835.70	2.9221×10^{-8}	9.7661×10^{-10}	1.467×10^{-11}	1.918×10^{-10}
800	1964.33	1.5559×10^{-8}	5.1998×10^{-10}	7.138×10^{-12}	9.350×10^{-11}
900	2053.39	8.7376×10^{-9}	2.9202×10^{-10}	3.724×10^{-12}	4.870×10^{-11}
1000	2124.55	5.1288×10^{-9}	1.7141×10^{-10}	2.046×10^{-12}	2.675×10^{-11}
1500	2231.16	5.5234×10^{-10}	1.8460×10^{-11}	1.761×10^{-13}	2.303×10^{-12}
2000	2250.84	9.1699×10^{-11}	3.0647×10^{-12}	2.596×10^{-14}	3.395×10^{-13}
2300	2255.96	3.4819×10^{-11}	1.1637×10^{-12}	9.491×10^{-15}	1.241×10^{-13}

reaching an altitude of 8000 feet. During wind storms, dust particles penetrate almost any enclosure.

13.4.4.1 EFFECTS OF SAND AND DUST ON FLUID COMPONENTS

Increased friction between sliding surfaces, causing abrasion, excessive wear, and binding of parts.

Degradation of plastics and elastomers used for dynamic seals.

Clogging of orifices such as vent ports.

Contamination of lubricants.

Erosion of paints, coatings, glass, plastics, and surface finishes.

Dust may be hygroscopic; its presence on metallic surfaces may aggravate corrosion.

Short circuiting of electrical elements.

13.4.4.2 DESIGN TECHNIQUES FOR MINIMIZING SAND AND DUST

Seal all bearings.

Use dust shields, such as rubber boots, for exposed moving shafts.

Table 13.4.2. Elastomers According To Ozone Resistance
(Reference 86-1)

Inherently Ozone-Resistant Elastomers
- Acrylons
- Hypalon
- Vyram
- Hycar 4021
- LS-53
- Kel-F elastomer
- Poly FBA
- Silicone
- Viton A

Ozone Resistant (without antiozonant)* If Properly Compounded
- Brominated butyl
- Butyl
- Neoprene
- Urethane (Genthane S)

Ozone Resistant (with antiozonant)* If Properly Compounded
- Buna N
- Carboxylic Buna N
- Butadiene-styrene (SBR)
- Vinyl pyridine
- Natural rubber
- Synthetic cis 1-4 polyisoprene
- cis 1-4 polybutadiene
- Conventional polybutadiene
- Mercapan modified adducts of polybutadiene
- Polysulfide
- Urethane (Adiprene B,C)

*Antiozonant is a substance which inhibits cracking due to the action of air containing ozone when the elastomer is subjected to tension strains. The effect of an antiozonant may be lost after exposure to high vacuum at room or elevated temperature.

13.4.5 Fungus

Fungus is an organism which is encountered primarily in tropical climates and which feeds on organic matter (nutrients) such as wood, paper, cotton, cellulose, paints, plastics, rubber, etc. Even a coating of dust or dirt will support fungi growth (see Sub-Section 13.7). Fungus growth is often accompanied by a high moisture content. Fungus testing is treated in Sub-topic 15.7.7.

13.4.5.1 EFFECTS OF FUNGI ON FLUID COMPONENTS

Properties of polymers change due to plasticizer loss.

Surfaces etch.

Bonded joints delaminate.

Electrical apparatus may short circuit, caused by conductive moist elements of fungi.

13.4.5.2 DESIGN METHODS TO PREVENT DAMAGE FROM FUNGI

Use encapsulation or hermetic seals.

Avoid the use of moisture-absorbing materials.

Avoid materials which can be nutrients for fungi such as natural rubber, polysulfides, and some plasticizers.

13.4.6 Solar Radiation

Solar radiation (sunlight) probably accounts for more widespread destruction of polymeric materials than all the other climatic, chemical, or physical agents. Material degradation is caused by a photochemical reaction, the rate of which is influenced by the presence of moisture and oxygen. Approximately 100,000 calories per gram-mole are required for the photochemical activation of most materials.

Radiation is propogated in small units called photons, each photon containing one quantum of energy. The actual value of the energy in a quantum is given by Planck's equation

$$E = h\nu \qquad \text{(Eq 13.4.6)}$$

where $\nu = \frac{c}{\lambda}$ = frequency of the radiation

λ = wave length of the radiation

c = velocity of light

h = (Planck's Constant) = 6.554×10^{-27} erg-sec

Each absorbed photon or quantum of radiation energy causes one light-absorbing molecule of the absorbing material to be activated. Since there are 6.025×10^{23} (Avogadro's number) molecules contained in a gram-mole, it requires 6.025×10^{23} photons to activate a gram-mole. This unit of radiation is called an einstein.

Since the energy of the quantum is inversely proportional to the wave-length of the radiation, the sort wave length ultraviolet possesses much more energy per quantum than does the visible or infrared. The energy in various types of radiation and their wave lengths are given in Table 13.4.6. Because of the absorption properties of the upper atmosphere (ozone in particular), little energy from wave lengths shorter than 3000 Angstroms, Å, reaches the earth. The small fraction of ultraviolet radiation that does penetrate the atmosphere nevertheless accounts for widespread destruction of many materials. Solar radiation or sunshine testing is treated in Sub-Topic 15.7.8.

13.4.6.1 EFFECTS OF SOLAR RADIATION ON PLASTICS. The effects of radiation on plastics is influenced to a great extent by the presence or absence of other agents such as moisture and oxygen. In many cases, outdoor weathering is largely due to photochemical oxidation such as is found in the degradation of polyvinyl chloride. Severe degradation may occur in some plastics previously subjected to intensive drying. Because of the interrelated action of heat,

Table 13.4.6. Energy in Various Types of Radiation
(Reference 413-1)

DESCRIPTION	WAVE-LENGTH (Å)	FREQUENCY	ERG PER QUANTUM	CALORIES PER EINSTEIN
X-rays	1	3×10^{18}	1.98×10^{-8}	2.84×10^{8}
Ultraviolet	1,000	3×10^{15}	1.98×10^{-11}	284,500
Ultraviolet	2,000	1.5×10^{15}	9.92×10^{-12}	142,300
Ultraviolet	3,000	1×10^{15}	6.55×10^{-12}	94,840
Visible (violet)	4,000	7.5×10^{14}	4.97×10^{-12}	71,120
Visible (blue-green)	5,000	6×10^{14}	3.98×10^{-12}	57,000
Visible (orange)	6,000	5×10^{14}	3.27×10^{-12}	47,400
Visible (red)	7,000	4.3×10^{14}	2.81×10^{-12}	40,600
Visible (red)	8,000	3.7×10^{14}	2.48×10^{-12}	35,500
Near infrared	10,000	3×10^{14}	1.98×10^{-12}	28,450
Infrared	100,000	3×10^{13}	1.98×10^{-13}	2,845
Far infrared	1,000,000	3×10^{12}	1.98×10^{-14}	284

moisture, and oxygen on the ultraviolet degradation of plastics, no generalizations can be made. The cellulose esters tend to embrittlement and crazing, while certain plastics composed of polymeric cellulose esters discolor.

13.4.6.2 EFFECTS OF SOLAR RADIATION ON NATURAL AND SYNTHETIC RUBBER. As with plastics, the effects of radiation on natural and synthetic rubbers will be influenced by the presence of heat, moisture, and oxygen. Natural rubber, Buna N, and Buna S do not possess good sunlight resistance. Neoprene, polysulfide, Butyl, and silicone synthetics are superior when compared to sunlight aging of natural rubber.

REFERENCES

56-1

413-1

349-1

13.5 TEMPERATURE

13.5.1 The Environmental Temperature Range

The terrestial temperature environment consists of temperatures occurring both in the natural environment and temperature conditions induced by the system. The induced thermal environment generally accounts for the most severe temperature extremes. System applications representing the range of temperatures under which fluid components must be designed to operate are listed in Table 13.5.1.

13.5.2 Thermal Behavior of Materials

Materials react so differently between the environmental extremes, from nearly absolute zero to above 5000°F, that this subject is discussed under low and high temperature categories. Because of the importance of ductility, brittleness, and toughness in considering temperature effects on materials, these properties are defined as follows:

Ductility: A property which indicates the ability of a material to undergo plastic deformation without fracturing. There is no single method of testing which can be considered a measure of ductility, although percentage of elongation, reduction in cross-sectional area at the breaking point, and the notched/unnotched tensile strength ratio are commonly used as measures of a material's ductility. Temperature is a major influencing factor on the ductility of a

Table 13.5.1. Fluid Component Temperature Spectrum Based on System Application

TEMPERATURE RANGE (°F)	SYSTEM APPLICATION
−452	Systems using liquid helium for achieving extremely low temperatures.
−424 to −200	Cryogenic propellant systems for rocket propulsion and ground propellant loading systems; includes liquid hydrogen, oxygen, and fluorine.
−200 to −65	Components located in the vicinity of cryogenic systems or equipment. Systems and components operating in lunar and Martian environments.
−65 to +160	Range of earth ambient atmospheric and geological environments from arctic cold to desert heat.
+160 to 400	Space exploration vehicles, aircraft hydraulic systems, and liquid rocket hot gas pressurization systems.
400 to 700	Nuclear power generation equipment. High temperature hydraulic flight control systems.
700 to 1000	Proposed pneumatic and hydraulic control systems for aircraft and missiles. Space equipment operating in Venus atmosphere.
1000 to 1200	Turbine exhaust temperatures. Hot gas control systems.
1200 to 3000	Solid and liquid propellant gas generator systems.
3000 to 7000	Thrust chamber hot gas tap off for secondary injection thrust vector control systems.

material. For a number of materials there is a transition temperature known as the "ductility transition temperature" above which materials are ductile, and below brittle. When a material becomes brittle, the ultimate tensile strength and the yield strength have the same value. Ductility is a factor in measuring toughness of a material and an important factor in a material's capability for redistribution of stresses. In a ductile material, if the stress in some localized region exceeds the yield strength, the material deforms in the highly stressed area, thus reducing the localized stress and redistributing the stresses in the part. Because it is very difficult to avoid localized stress concentrations in highly stressed components, ductility considerations are extremely important.

Brittleness: the opposite of ductility; a nonductile material is a brittle material.

Toughness: a measure of the energy a material can absorb before breaking. Ductility and strength are the major factors determining the degree of toughness in a given material. A ductile material with the same strength as a brittle material will absorb more energy before fracture occurs. In a static tensile test, absorbed energy is related to the area under a stress-strain curve. Impact tests are a direct measure of the energy a material will absorb prior to fracture. The Charpy or Izod tests are standard impact tests.

13.5.2.1 SELECTION OF MATERIALS FOR LOW TEMPERATURE SERVICE

Metals. The strength of most metals is higher at low temperatures than at room temperature while, with few exceptions, the ductility of a metal decreases with decreasing temperature. As toughness, measured by impact strength, is a function of both strength and ductility, toughness increases for some materials and decreases for others as temperature decreases, depending on whether strength or ductility predominate. Ductility and toughness are the most often used criteria in the selection of materials for low temperature service. There are, however, some applications where ductility is not a satisfactory criterion for selecting materials for low temperature service. Coil springs having good surface finishes and having no local stress risers, act completely as elastic members and do not need to be ductile.

Unfortunately, there is no single accepted index which will satisfactorily predict whether or not a material will behave satisfactorily for low temperature service. The various indices that have been employed in the selection of low temperature materials include the elongation of a tensile specimen, the beam impact energy test, toughness measured by the notch impact test, and the notch tensile test. Elongation data based on unnotched uniaxial tensile tests can be misleading, since the effects of multiaxial stresses caused by local stress risers (notches) and the effects of varying strain rates are not considered.

Notch tensile tests are conducted at low strain rate, while the Charpy V-notch impact test combines high strain rates with sharp notches. The notched-to-unnotched tensile strength ratio is gaining favor as an index of embrittlement which is useful as a criteria for selecting metals for low temperature service. In utilizing notched tensile data, it is important to relate the values of the test stress concentration factor, K_t, which range from 3 to 18. Higher values of K_t will favor ductile materials. A comparison of methods of measuring ductility is shown in Figure 13.5.2.1a.

Low temperature fatigue data showing the effects of notch sensitivity and stress-concentration factors are extremely limited. Generally, the endurance limit of metals increases as the temperature decreases; however, some materials exhibiting extremely brittle behavior at low temperatures experience a reduction in endurance limit in notched specimens. For optimum fatigue life it is good design practice to avoid sharp notches, rough surfaces, and sharp reductions in sections.

In general, face centered cubic metals, FCC, have less tendency toward embrittlement at low temperatures than other

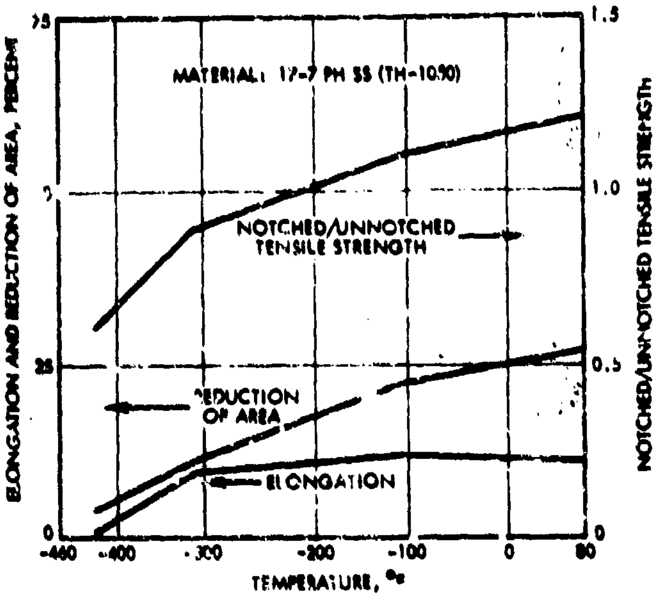


Figure 13.5.2.1a. Comparison of Various Methods of Testing a Metal's Ductility with Changing Temperature

(Material in S.S. 17-7 PH Sheet; HT (TH-1050); from Reference 91-1.)

structures, such as body centered cubic metals. Copper, nickel, aluminum and its alloys, cobalt base alloys, and the austenitic stainless steels are examples of FCC metals which remain ductile at cryogenic temperatures. The precipitation hardening stainless steels, carbon steels, magnesium alloys, beryllium, and some titanium alloys are not recommended for low temperature structures based on ductility criteria. While most common metals suitable for low temperature applications are of FCC structure, some titanium alloys having hexagonal close-packed, HCP, structure remain moderately ductile at low temperatures. Where impact loads are low, a number of inherently brittle materials such as ferritic stainless steels, 440C, have been used successfully in cryogenic valve seat and poppet applications.

Polymers. Elastomers are characterized by large deformability, non-linear stress-strain curves, high hysteresis, and large variations of stiffness with temperature and rate of loading.

The stiffness of an elastomer will gradually increase as the temperature decreases, until a temperature known as the *first order transition temperature* or freezing point, at which point stiffness increases sharply with further decrease in temperature. Stiffness continues to increase rapidly until a second transition point is reached where further decrease in temperature causes little increase in stiffness. This point is known as the *second order transition point* or glass transition temperature, T_g, as the material becomes hard and rigid like glass. All polymeric materials pass through the glass transition before reaching −150°F.

At some temperatures, dependent on test techniques, a test specimen becomes brittle or will shatter on sudden bending or impact. The temperatures at which this condition occurs (dependent on specific testing conditions such as rate of load application) is called the brittle temperature or brittle point. The brittle point bears no definite relation to the stiffness curve due primarily to the difference in time scale between stiffness and impact tests.

Stiffness of elastomers at low temperatures is illustrated in Figure 13.5.2.1b. Table 13.5.2.1 lists the glass transition temperatures for several common polymers.

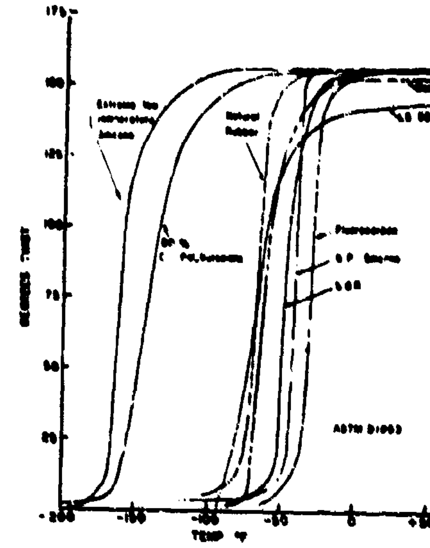


Figure 13.5.2.1b. Stiffening of Elastomers at Low Temperature

(From Reference 55-1.)

Some elastomers contain large amounts of special plasticizers to improve flexibility at low temperatures and to depress the brittle point of the material. Under prolonged exposure to low temperatures (below −40 F) these plasticizers, which are soluble in the elastomers at room temperature, may be ejected out of solution, thereby lowering flexibility above the brittle point and raising the brittle point several degrees.

The low temperature limits of a polymer are dependent on the particular application for which it is to be used. For example, elastomers can be used in static seal applications at temperatures well below the stiffening temperatures

Table 13.5.2.1. Polymer Glass Transitions Temperatures

(Reference 46-20)

MATERIAL	TEMPERATURE GLASS TRANSITION, T_g, °F
Polyisobutylene	−101
Natural rubber (Hevea)	−99
Polyurethane	−81
Polystyrene	+212
Polymethylmethacrylate (Plexiglass)	+221
Polyvinyl chloride	+165
Butadiene styrene rubber	27
Silicone rubber	−112
Polytetrafluoroethylene (Teflon)	+77

where elastic response is not required. Investigation of the use of elastomers as static seals for cryogenic service by the National Bureau of Standards has shown that if the seal is initially compressed above 50 to 70 percent, the sealing force will not go to zero at the brittle point but will level off at some constant value. Figure 13.5.2.1c shows force-temperature curves for an elastomer after various degrees of initial compression measured in percent squeeze.

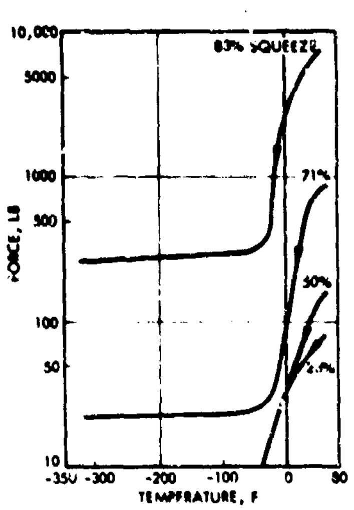


Figure 13.5.2.1c. Force-Temperature Diagram

(From "Applied Cryogenic Engineering," R. W. Vance and W. M. Duke, 1962, John Wiley and Sons, New York.)

Very few polymers are useable at cryogenic temperatures in applications where flexing of the material is required. Exceptions are fluorocarbon plastics such as Teflon and Kel-F, which can be used in cryogenic applications for lip seals and diaphragms which require only a limited amount of flexing. Also it is possible to maintain flexibility at cryogenic temperatures if a polymer can be used in sufficiently thin sections. Mylar, for example, has been used successfully as a diaphragm material in liquid hydrogen valves where a high degree of flexibility was achieved at temperatures as low as -400 F. Because of the low temperature limitations of polymers, it is common design practice to isolate flexing elements such as diaphragms and dynamic seals used in actuators from the low temperature environment by techniques described in "Heat Transfer," under Detailed Topics 2.3.1.2 and 2.3.2.1.

13.5.2.2 SELECTION OF MATERIALS FOR HIGH TEMPERATURE SERVICE. Whereas ductility is usually the limiting criterion in selecting materials for low temperature service, strength is usually the limiting factor in selecting materials for high temperature service. Another important limiting factor for high temperature materials is the reaction of the material to the environment, i.e., oxidation.

Metals. High temperature limitations of metals are based on considerations of *strength*, a function of temperature alone, *creep*, a function of time and temperature, *oxidation*, and *melting point*.

Strength: Most structural metals retain their useful strength at temperatures *up to 400°F*. A reduction in strength level of some of the lighter metals must be compensated for by increased thickness of the part. From *400°F to 1000°F* cast irons, low alloy steels, magnesium thorium alloys, aluminum alloys, ferritic stainless steels, and some titanium alloys are used at reduced stress levels. The martensitic tool steels, steel alloys containing high amounts of vanadium, tungsten, and molybdenum, and austenitic stainless steels offer good structural properties at these temperatures. Figure 13.5.2.2a shows the tensile strength of various alloy systems as a function of temperature.

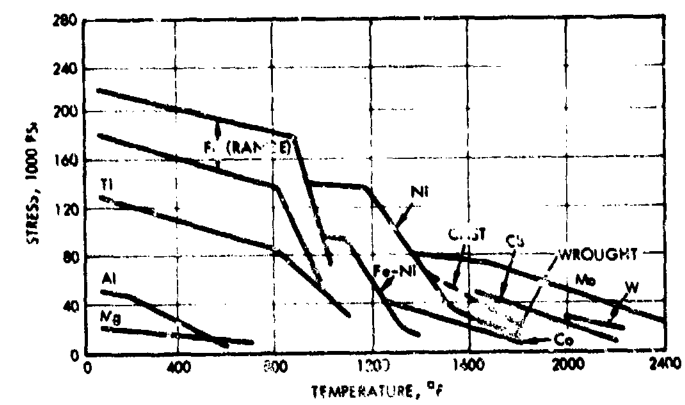


Figure 13.5.2.2a. Tensile Strength of Various Alloy Systems as a Function of Increasing Temperature

(From "Metal Progress," L. P. Jahnke and R. G. Frank, December 1958, Copyright 1958 by the American Society for Metals, Metals Park, Ohio.)

From *1000°F to 1200°F* the nickel base alloys are best for structural parts. Cold-worked precipitation-strengthened nickel base alloys such as Inconel "X" and René 41 have high strengths in the 600°F to 1200°F range.

Over the range from *1200°F to 2100°F* the super-alloys are considered primary engineering materials. These include the non-heat-treated chromium-nickel-iron-steels, which exhibit good strength and good oxidation resistance to 1600°F. Heat-treated chromium-nickel-iron may be used to 1000°F for continuous service. From 1900°F to 2100°F the cast cobalt alloys have excellent oxidation resistance and high strength, and have been used for turbine blades, nozzles and valves. The nickel base alloys have been used to 2100°F for furnace parts, exhaust stacks, and combustion chambers.

Refractory metals are promising materials for structural applications *above 2000°F*. Refractory metals may be defined as those metals which have melting points exceeding 3270 F. The refractory metals include titanium, zirconium, niobium, tantalum, chromium, molybdenum, tungsten, vanadium, rhodium, and iridium.

Refractory metals exhibit good radiation resistance at elevated temperatures, are weldable, and show high wear resistance. However, refractory metals are characterized by poor high temperature oxidation resistance (Figure 13.5.2.2b). At present it is necessary to rely for oxidation protection on coatings which are in themselves still in the development stage. Silicide base coatings have been considered as coatings for molybdenum, niobium, and tantalum. Aluminides and chromium-titanium-silicon compositions show promise as coatings for niobium alloys.

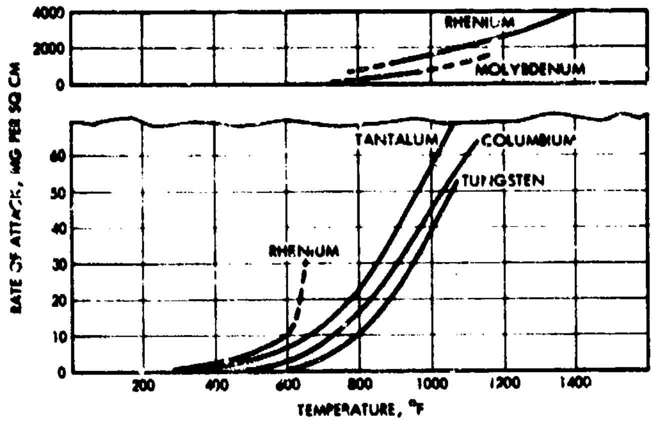


Figure 13.5.2.2b. Oxidation Rate of Refractory Metals.
(There is a rapid increase at temperatures above 800°F. Molybdenum and rhenium, which have very high oxidation rates due to the high vapor pressure of their oxides, are shown in the upper portion of the figure. From "Metal Progress," L. P. Jahnke and R. G. Frank, December 1958, Copyright 1958 by the American Society for Metals, Metals Park, Ohio.)

Creep: Creep is an important consideration where extended service is required at elevated temperatures. Creep may be defined as an increase in strain in a material under a constant static load at a given temperature. The total amount of creep varies with time, while the rate of creep is a function of temperature and stress level. When a metal is stressed, it undergoes initial plastic adjustments occurring at points of stress along grain boundaries. After these initial adjustments, creep sets in and continues until a reduction of cross-sectional area can no longer support the load and rupture occurs. Figure 13.5.2.2c illustrates typical strain rate (creep) curves. It can be seen from the curve that the straight line portion of the curve represents a steady creep rate which increases with temperature and stress. The time before failure by rupture is decreased as temperature is increased. *Creep rate* is defined as the strain per time during the period of steady elongation. It is equal to the slope of straight portion of the curves shown in Figure 13.5.2.2c. Creep strength is defined as the stress level which will produce a given strain over a fixed time interval at a given temperature. The creep strength of several materials is shown in Table 13.5.2.2a. The stress rupture strength is the stress level which will cause rupture of the material within a given time interval at a given temperature. The stress rupture strength of several high temperature alloys is presented in Table 13.5.2.2b.

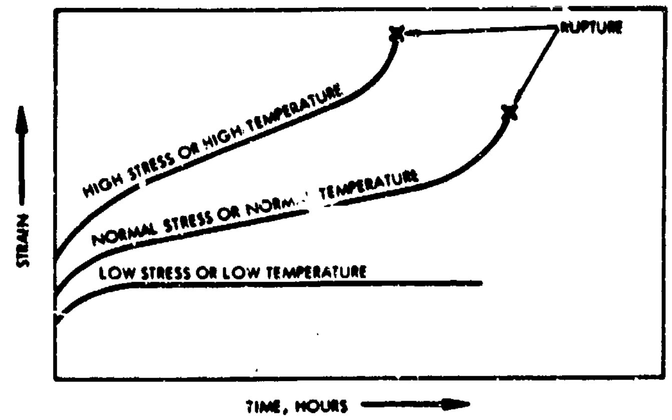


Figure 13.5.2.2c. Typical Creep Curves
(The straight line portion of the curves controls the useful life of the material.)

When available, stress time curves (Figure 13.5.2.2d) are valuable in designing for a certain total strain over the life of a component for a given operating temperature level. The family of curves shifts downwards for increasing temperature level. The designer, however, should be aware of the other variables such as oxidation and phase changes which will also affect the service life of the component.

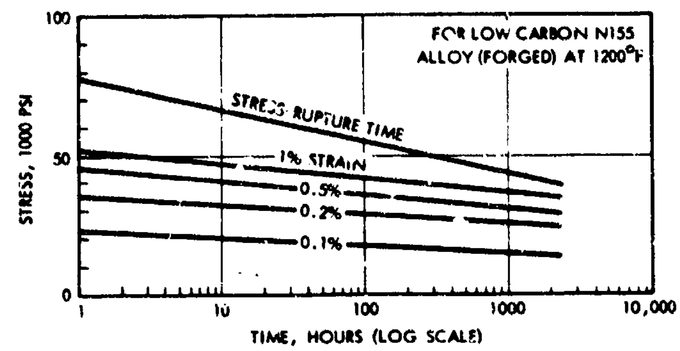


Figure 13.5.2.2d. Stress-Time Curves at High Temperatures
(Adapted from J. W. Freeman, E. E. Reynolds, and A. E. White, "High Temperature Alloys Developed for Aircraft Turbochargers and Gas Turbines," ASTM Symposium on Materials for Gas Turbines, 52, 1945.)

Oxidation: In high temperature applications, a metal's resistance to oxidation is determined primarily by the properties of the scale formed on its surface. Data on some materials show that oxidation proceeds according to a parabolic relationship between the thickness or weight of the oxide film and the time. However, for long exposure at high temperatures, the increased thickness of film is subject to rupture or cracking, especially if cyclic stresses are imposed on the part and spalling or flaking of the oxide occurs. Thermal cycling can also cause increased compressive stresses in the oxide film because of the different coefficient of expansion of the underlying material. The combined effects of corrosion and stress have been responsible for many high temperature structural failures. Localized corrosive attack produces notches which act as stress risers. Carbon steel oxidizes readily in air at 1000°F. When the temperature is raised to 1300°F-1400°F, corrosion is favored at the grain boundaries. When a surface tensile stress is present, a localized corrosion attack occurs, known as stress corrosion, or corrosion fatigue if the stress is cyclic. Additional information on oxidation corrosion is given in Sub-Section 13.7.

Melting Points: The absolute temperature limit for any solid material as a structural member is determined by its melting point. The melting points of several types of metals and ceramics are shown in Table 13.5.2.2c. The relationship between useful strength and melting point for several metals is shown in Table 13.5.2.2d.

Non-metals. Non-metals cover an extremely wide range of materials, varying from plastics and elastomers useful only up to a few hundred degrees Fahrenheit to ceramics and graphite useful to several thousand degrees Fahrenheit.

Plastics and Elastomers: In addition to strength, important properties to consider in the selection of plastics and elastomers for elevated temperature application are elongation, compression set, hardness, and chemical degradation.

Sustained high temperatures cannot be tolerated by polymers. Many rubbers oxidize and either soften or embrittle at temperatures above 300°F. Fluorinated polymers have the highest service temperature (approximately 600°F) of the plastic and elastomeric materials used in valve seal and diaphragm applications. Table 13.5.2.2e presents the maximum service temperatures for a number of commonly used plastic and elastomer materials.

The deformation of an elastomer under a constant force varies inversely with the absolute temperature. This effect can cause an O-ring to seize around a shaft as temperature increases or cause the elastic properties of the material to change with temperature. An irreversible process of an elastomer is referred to as aging, and at elevated temperatures some materials will take a permanent compression set, causing leakage in a seal or loss of elastic properties. Creep resulting from scission of polymer bond either by radiation or oxidation is also considered an irreversible process.

High Temperature Non-metals: Refractory materials such as oxides, carbides, nitrides, silicides, borides, beryllides, aluminides, zirconides, germanides, and chromides show good strength and oxidation resistance up to 2000°F. Oxides are the most widely used ceramics at present and are known for their low thermal shock and mechanical impact resistance. They are considered in the temperature range from 2000°F to 2500°F. The carbides have exceptionally high melting points, falling in the range of 4500°F to 7000°F. However, with the exception of SiC, they show poor oxidation resistance at temperatures exceeding 1800°F. At temperatures near 3000°F, SiC oxides rapidly. Reliable and conclusive data on these materials and their applications at high temperatures are limited.

Graphite is a high temperature, non-structural material which has been used for such applications as thrust vector control system components and solid propellant rocket nozzle inserts. Its strength increases with temperature to approximately 5000°F. For shapes of constant thickness and relatively high purity, graphite has excellent thermal shock resistance. Above 2500°F graphite has the highest strength-weight ratio of all the high temperature materials.

13.5.2.3 DESIGN CONSIDERATIONS FOR OPERATION OVER A WIDE TEMPERATURE RANGE. In addition to the effects of high and low temperatures on materials, the designer must consider the effects of temperature changes.

Dimensional Stability

Coefficient of Thermal Expansion or (Contraction). When the temperature of a material is altered, expansion or contraction results in a volume change.

Where materials having different thermal expansion coefficients are in contact, thermal stresses and subsequent dimensional changes can occur. When a component must be capable of operation over a wide temperature range, care must be taken to assure that differential expansion (contraction) will not result in detrimental dimensional changes, particularly where moving parts and close tolerances are involved. Where elastomers or plastics are used for seals, and increase or decrease in temperature will affect the sealing forces in a closure because of a corresponding increase or decrease in the seal volume with relation to the groove. This is caused by the fact that all polymeric materials have a much higher thermal expansion coefficient than metals. In some elastomeric materials at certain rates of cooling, the rate of return of the material at the lower temperature is less than the rate of shrinkage due to cooling. Consequently, the sealing force may be lessened to the point where leakage occurs. Thermal expansion or contraction of a part can result in increased friction or seizure, relief of pre-stressed bolts causing loosening of the nut, leakage in a valve closure, and a shift in calibration of linkage-controlled equipment. The coefficient of thermal expansion of several materials are given in Table 13.5.2.3a.

Thermal Gradients: Thermal gradients (non-uniform heating or cooling of a component) can result from large differences in mass distribution. Large masses of material take longer to change temperature due to their larger heat capacity. If a component must operate under non-equilibrium temperature conditions, the designer should carefully consider non-uniform dimensional changes resulting from thermal gradients. Such effects can be minimized by designing components with uniform wall thicknesses and avoiding large masses of material. Where operation takes place only after temperature is stabilized, the only concern is that thermal gradients do not result in stresses which exceed allowable yield stresses. Even in a part having uniform

Table 13.5.2.2a. Creep Strength of Metals

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," vol. 58, mid-October 1963, p. 37, Copyright 1963 by Reinhold Publishing Corp., New York)

MATERIAL, FORM, CONDITION	STRESS (1000 PSI) FOR 0.01 PERCENT CREEP PER 1000 HR AT INDICATED TEMP (°F)					STRE… PE…
Up to 800°F	**300**	**400**	**500**	**600**	**800**	**300**
Non-Ferrous Metals						
Copper .... wrought (annealed)	3–8	1.5–5	0.4–2.6	—	—	—
Nonleaded brasses ..... wrought (annealed)	0.9–19	2–11	0.3–23	—	—	25
Bronzes .... wrought (annealed)	14–23	5–10	2-5	—	—	—
Cupro-nickel .... wrought (water quenched, aged)	25–40	15–30	8–30	—	—	—
Aluminum 2024-T .... sheet	22	9.5	2.5	1.5	—	30
Aluminum 7075-T .... sheet	12	4	2.5	1.5	—	16
Titanium (commercial) .... sheet (annealed)	—	38	—	32	10	37
Ti-6Al-4V .... sheet (annealed)	—	—	—	—	—	—
Ti-7Al-4Mo ...... bar or forging (annealed)	—	—	—	—	—	—
Above 800°F	**1000**	**1100**	**1200**	**1500**	**1000**	**1000**
Carbon and Low Alloy Steels						
Low carbon steel .... wrought, cast	1.8	—	0.1	—	—	3.3–5
Carbon-molybdenum steels ... wrought, cast	5–7	3	1	—	—	10–12
Chromium-molybdenum steels (0.5-3 %) .... wrought, cast	6–12	2–4	1–2.5	—	—	10–20
Chromium steels						
4-6% .... wrought, cast	6-7	2.5–3.5	1–2	—	—	8–11
6-10% .... wrought, cast	5–9	2.5–4	1–2	—	—	8–12
Stainless Steels						
Martensitic chromium steels (403, 410, 416, 420) .... wrought	8	3.5	1.3	—	—	9.2
Ferritic chrominum steels (405, 430, 440) .... wrought	4.2–7	2.3–4.5	1.0–1.6	—	—	6–8.5
Nickel-chromium steels						
304, 316, 321, 347 .... wrought	12–17	7.5–11.5	4.5–7	1-2	—	17-25
309 .... wrought	—	—	4	0.5	—	15.9
310, 314 .... wrought	17	13	8	2	—	17
Heat Resistant Cast High Alloys						
Iron-chromium alloys (HA, HC, HD) .... cast	—	—	—	—	—	—
Iron-chromium-nickel alloys (HE, HF, HH, HI, HK, HL) .... cast	—	—	—	—	—	—
Nickel-chromium alloys (HN, HT, HU, HW, HX) .... cast	—	—	—	—	—	—
Superalloys						
Inconel X	—	—	—	—	—	—
19-9 DL	—	—	—	—	—	—
Hastelloy X	—	—	—	—	—	—
N-155	—	—	—	—	—	—
S-816	—	—	—	—	—	—

*at 1400°F

**at 1350°F

Table 13.5.2.2a. Creep Strength of Metals

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," vol. 58 no. 5, mid-October 1963, p. 37, Copyright 1963 by Reinhold Publishing Corp., New York)

...TION	STRESS (1000 PSI) FOR 0.01 PERCENT CREEP PER 1000 HR AT INDICATED TEMP (°F)					STRESS (1000 PSI) FOR 0.1 PERCENT CREEP PER 1000 HR AT INDICATED TEMP (°F)				
	300	400	500	600	800	300	400	500	600	800
...ght (annealed)	3–8	1.5–5	0.4–2.6	—	—	—	—	—	—	—
...ght (annealed)	0.9–19	2–11	0.3–23	—	—	25	5–9	1–3	—	—
...ght (annealed)	14–23	5–10	2–5	—	—	—	—	—	—	—
...ght (water quenched, aged)	25–40	15–30	8–30	—	—	—	22	13	—	—
.........sheet	23	9.5	2.5	1.5	—	30	13	3	2	—
.........sheet	12	4	2.5	1.5	—	16	6	3	2	—
...eet (annealed)	—	38	—	32	10	37	40	37	32	13
...eet (annealed)	—	—	—	—	—	—	—	—	30	—
...ing (annealed)	—	—	—	—	—	—	—	—	35	18
	1000	1100	1200	1500	1800	1000	1100	1200	1500	1800
...wrought, cast	1.8	—	0.1	—	—	3.3–5	—	0.5	—	—
...wrought, cast	5–7	3	1	—	—	10–12	4	2	—	—
...wrought, cast	6–12	2–4	1–2.5	—	—	10–20	3–8	2–4.5	—	—
...wrought, cast	6–7	2.5–3.5	1–2	—	—	8–11	5–6.5	2–3.5	—	—
...wrought, cast	5–9	2.5–4	1–2	—	—	8–12	4–6	2.5–3	—	—
.....wrought	8	3.5	1.3	—	—	9.2	4.2	2	—	—
.....wrought	4.2–7	2.3–4.5	1.0–1.6	—	—	6–8.5	3–5	1.5–2.2	—	—
.....wrought	12–17	7.5–11.5	4.5–7	1–2	—	17–25	12–18.2	7–12.7	1.2–2.8	—
.....wrought	—	—	4	0.5	—	15.9	11.6	8	1.0	—
.....wrought	17	13	8	2	—	17	13–14	9	1–2.5	—
.........cast	—	—	—	—	—	—	—	—	1.2–2.5*	0.7–1.9
.........cast	—	—	—	—	—	—	—	—	3.5–7*	2–4.3
.........cast	—	—	—	—	—	—	—	—	6–8.5*	3–5
....	—	—	—	—	—	—	—	64	12.3	9.0
....	—	—	—	—	—	—	—	20	7.1	2.4
....	—	—	—	—	—	—	—	—	—	—
....	—	—	—	—	—	—	—	18.4**	10.3	—
....	—	—	—	—	—	—	—	42	11.5	5.8

B

Table 13.5.2.2b. Stress-Rupture Strength of High Temperature Alloys

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," mid-October 1963, p. 38, Copyright by Reinhold Publishing Corp., New York)

10 HR								
1200 F		1500 F		1800 F		1200 F		
ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	A
Waspaloy	130	Inconel 713[a]	70	Mo-0.5 Ti[d]	65	1753	115	Nicrot
M-252	120	René 41	65	Columbium[d]	53	Waspaloy	110	Incone
Incoloy 901[b]	110	U-500	62	Molybdenum[d]	30	Inconel 700	100	1753 .
W-545	95	1753	60	Inconel 713[a]	24	U-212	100	René 4
Inconel X	92	Waspaloy	58	GMR-235[a,b]	16	M-252	98	Udime
Refractaloy 26	92	Inconel 700[b]	55	1753	16	D979	94	Incone
S-816	83	GMR-235[a,b]	52	V-36[b,c]	13	W-545	90	Waspa
A-286	80	M-252	43	X-40[a]	13	GMR-235[a]	86	GMR-2
Inco 702[a,b]	75	Inconel X	38	HS-21[a]	12.5	Incoloy 901	85	M-252
Hastelloy B	71.5	Refractaloy 26	36	M-252	12	Refractaloy 26	80	D-979
Discaloy[e]	70	X-40[a]	33	HS-25	11.5	HS-25	70	S-816
HS-21[a]	70	S-816	33	Inconel 700[b]	9	S-816	65	X-40[a]
Refractaloy 70	70	HS-25	30	N-155	8.8	A-286	63	S-816[a]
Hastelloy C	69	Hastelloy B[a]	29	Hastelloy X[a]	8	Refractaloy 70	56	Refrac
Nivco	66	V-36[b,c]	29	HK[a]	6.5	S-816[a]	56	HS-25
N-155	62	S-590	28	HH[a]	6	Discaloy	55	V-36 .
S-590	62	HS-21[a]	27	HT[a]	5.8	Hastelloy C	55	HS-21
X-40[a]	61	Hastelloy B	26	Inco 702[a,b]	4.2	Inconel 702	55	S-590
Hastelloy X	58	Hastelloy C	26			Nivco	54	Incoloy
16-25-6	55	N-155	26			HS-21[a]	52	N-155
N-155[a]	52	Hastelloy C[a]	25			Hastelloy B[a]	51	N-155[a]
19-9DL	50	N-155[a]	25			Hastelloy B	50	Refrac
HH[a]	46	16-25-6	24			N-155	50	Hastel
HT[a]	41	Inco 702[a,b]	23			S-590	50	Hastel
HF[a]	37	A-286	22			Hastelloy C[a]	49.5	Hastel
HK[a]	35	Hastelloy X	22			N-155[a]	49	Hastel
Hastelloy X[a]	34	Hastelloy X[a]	20			Hastelloy X	44	Incone
		19-9DL	20			X-40[a]	44	Hastel
		HT[a]	16.5			16-25-6	44	Discal
		HH, HK[a]	16			19-9DL	44	A-286
		HF[a]	13			HH[a]	35	16-25-
						Hastelloy X[a]	32	19-9DL
						HT[a]	32	HT[a]
						Hastelloy X[a]	32	HH[a]
						HF[a]	30	HK[a]
						HK[a]	25	HF[a]

[a]Cast [b]Estimated [c]Sheet

[d]Annealed or recrystallized [e]Stress relieved

A

Table 13.5.2.2b. Stress-Rupture Strength of High Temperature Alloys

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," vol. 58, no. 5, mi i October 1963, p. 38, Copyright by Reinhold Publishing Corp., New York)

10 HR					100 HR					
	1500 F		1800 F		1200 F		1500 F		1800 F	
STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI	ALLOY	STRESS, 1000 PSI
190	Inconel 713	70	Mo-0.5 Ti[d]	65	1753	115	Nicrotung[a]	65	Mo-0.5 Ti-	
120	René 41	65	Columbium[d]	53	Waspaloy	110	Inconel 713C[a]	55	0.07 Zr[e]	70
110	U-500	62	Molybdenum[d]	30	Inconel 700	100	1753	47	Mo-0.5 Ti[e]	62
95	1753	60	Inconel 713[a]	24	U-212	100	René 41	45	Mo-0.5 Ti-	
92	Waspaloy	58	GMR-235[a, b]	16	M-252	98	Udimet 500	45	0.07 Zr[d]	40
92	Inconel 700[b]	55	1753	16	D979	94	Inconel 700	43	Columbium[d]	36
85	GMR-235[a, b]	52	V-36[a, c]	13	W-545	90	Waspaloy	40	Mo-0.5 Ti[d]	28
80	M-252	48	X-40[a]	13	GMR-235[a]	86	GMR-235[a]	38	Molybdenum[e]	22
75	Inconel X	38	HS-21[a]	12.5	Incoloy 901	85	M-252	37	Nicrotung[a]	22
71.5	Refractaloy 26	36	M-252	12	Refractaloy 26	80	D-979	36	Inconel 713C[a]	16
70	X-40[a]	33	HS-25	11.5	HS-25	70	S-816	29	Udimet 700	16
70	S-816	33	Inconel 700[b]	9	S-816	65	X-40[a]	29	GMR-235[a]	13
70	HS-25	30	N-155	8.8	A-286	63	S-816[a]	28	Udimet 500	12
69	Hastelloy B[a]	29	Hastelloy X[a]	8	Refractaloy 70	56	Refractory 26	27	Molybdenum[d]	11.5
66	V-36[b, c]	29	HK[a]	6.5	S-816[a]	56	HS-25	24	X-40[a]	11.3
62	S-590	28	HH[a]	6	Discaloy	55	V-36	24	S-816[a]	11
62	HS-21[a]	27	HT[a]	5.8	Hastelloy C	55	HS-21[a]	22	HS-21[a]	9.[illegible]
61	Hastelloy B	26	Inco 702[a, b]	4.2	Inconel 702	55	S-590	22	V-36	9
58	Hastelloy C	26			Nivco	54	Incoloy 901	20	Waspaloy	8
55	N-155	26			HS-21[a]	52	N-155	20	HS-25	7.5
52	Hastelloy C[a]	25			Hastelloy B[a]	51	N-155[a]	19	Inconel 700	5.6
50	N-155[a]	25			Hastelloy B	50	Refractaloy 70	19	N-155	5
46	16-25-6	24			N-155	50	Hastelloy C[a]	18.5	HT, HK[a]	4.5
41	Inco 702[a, b]	23			S-590	50	Hastelloy B[a]	18	HH[a]	4.0
37	A-286	22			Hastelloy C[a]	49.5	Hastelloy C	18	Inconel 702	3.1
35	Hastelloy X	22			N-155[a]	49	Hastelloy B	17		
34	Hastelloy X[a]	20			Hastelloy X	44	Inconel 702	16		
	19-9DL	20			X-40[a]	44	Hastelloy X	15.5		
	HT[a]	16.5			16-25-6	44	Discaloy	15		
	HH, HK[a]	16			19-9DL	44	A-286	14		
	HF[a]	13			HH[a]	35	16-25-6	13.5		
					Hastelloy X[a]	32	19-9DL	13		
					HT[a]	32	HT[a]	12		
					Hastelloy X[a]	32	HH[a]	11.5		
					HF[a]	30	HK[a]	10.5		
					HK[a]	25	HF[a]	9		

[a]Cast [b]Estimated [c]Sheet

[d]Annealed or recrystallized [e]Stress relieved

B

Table 13.5.2.2c. Melting Points of Metals and Ceramics*

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," vol. 58, no. 5, mid-October 1963, p. 24. Copyright 1963 by Reinhold Publishing Corp., New York)

MATERIAL	HIGH**	LOW	MATERIAL	HIGH**	LOW	MATERIAL	HIGH**	LOW
Tungsten	6170	—	Heat resistant alloys			Gold	1945	—
Thoria	6000	—	(cast)	2750	2350	Aluminum bronzes		
Tantalum	5425	—	High temperature			(cast)	1937	1880
Magnesia	5070	—	steels	2750	2660	Commercial bronze	1910	1870
Osmium	4890	—	Stainless steels (cast)	2750	2550	Leaded brasses	1900	1610
Molybdenum alloys	4750	4730	Wrought irons	2750	—	Tin bronzes (cast),		
Calcia and zirconia	4710	—	Cobalt	2723	—	leaded	1830	1570
Columbium alloys	4620	4100	Cr-Ni-Fe superalloys	2664	2225	Beryllium copper	1800	1600
Beryllia	4695	—	Austenitic stainless			Tin bronzes (cast),		
Ruthenium	4530	—	steels	2650	2500	high leaded	1800	1700
Iridium	4450	—	Nickel and its alloys	2635	2300	Tin and aluminum		
Molybdenum disilicide	3775	3595	Low expansion nickel			brasses	1780	1590
Rhodium	3571	—	alloys	2606	2600	Silver	1761	—
Silicon nitride	3452	—	Nickel-base superalloys	2600	2310	Aluminum silicate		
Hafnium	3400	—	Cobalt-base superalloys	2570	1600	glass	1675	—
Alumina cermets	3362	—	Age hardenable			Borosilicate glass	1500	1300
Zirconium and its			stainless steels	2550	2500	Soda-lime glass	1330	1285
alloys	3355	3300	Cr-Ni-Co-Fe			Aluminum and its		
Platinum	3224	—	superalloys	2470	2350	alloys	1215	935
Thorium	3180	—	Beryllium	2341	—	Magnesium alloys	1200	830
Vanadium	3110	—	Cupro-nickels	2260	2020	Aluminum and its		
Fused silica glass	3050	—	Austenitic nodular			alloys (cast)	1195	910
Titanium and its			irons	2250	—	Lead silicate glasses	1160	1075
alloys	3040	2730	Chromium copper	2147	—	Tin-lead-antimony		
Boron nitride	>3000	—	Uranium	2071	—	alloys	792	358
Palladium	2829	—	Heat resistant nodular			Zinc and its alloys	792	727
Martensitic stainless			irons	2150	2050	Soft lead	623	617
steels	2800	2500	Nickel silvers	2030	1870	Hard lead alloys	610	490
96% silica glass	2800	—	Silicon bronzes	1990	1780	Pewter	565	475
Ferritic stainless steels	2790	2600	Coppers	1981	1949	Lead-base babbitts	540	460
Carbon steels	2775	2700	Phosphor bronzes	1970	1550	White metal	475	—
Low alloy steels	2760	2600	Gliding, 95%	1950	1920	Hard tin	443	—

*Values represent high and low sides of a range of *typical* values.
**Temperature, °F

wall thickness, rapid temperature changes (thermal shock) may cause distortion. Warpage may occur when the surfaces of a hot part cool unevenly. Such distortion may occur in a valve body which cools faster on the outside surface than on the inside. Thermal gradients can be minimized by using materials having relatively high thermal conductivities.

Residual Stresses: When residual stresses are relieved, a part can change its dimensions as much as several ten-thousandths of an inch over a few months duration. The change may be an increase or a decrease in warping, depending on the machining and cold working process. Dimensional changes of some materials due to cyclic cooling and heating are given in Table 13.5.2.3b. Holding parts at temperature for long periods of time, and temperature cycling to subzero temperatures are methods used to alleviate residual stresses in parts.

Microstructure Changes: Metallurgical changes have been the cause of many mechanical failures. Constant temperature changes, such as precipitation in an age-hardening alloy or transformation of an unstable phase, may cause large dimensional changes, resulting in seizure of moving parts and failure of the component. A common example of metallurgical change is in the quench-hardenable steels, where any retained austenite transforms to martensite. Martensitic transformation produces large dimensional changes causing steel to grow as much as 140×10^{-6} inches per inch for each volume percent of austenite. If an appreciable amount of unstable austenite is retained in a heat-treated part, further cooling to subzero temperatures will induce dimensional changes long after the part is in service. High carbon and high alloy steels may retain austenite, whereas low carbon and low alloy steels do not. An alloy steel should be stabilized by quenching several times from

Table 13.5.2.2d. Highest Temperatures at Which Today's Best Heat-Resistant Alloys Can Be Used

(Reference 20-17)

BASE METAL	MELTING POINT (°F)	TEMPERATURE FOR USEFUL STRENGTH OF BEST ALLOYS* (°F)	PERCENT OF MELTING POINT†
	Light alloys		
Mg	1200	650	67
Al	1220	550	60
Ti	3100	1200	46
	Superalloys		
Fe (Mart.)	2800	1350	56
Fe (Aust.)	2800	1600	63
Ni	2650	1960	78
Co	2720	1900	74
	Refractory alloys		
Cb	4470	2200	54
Mo	4760	2650	59
W	6170	2550	45

*Withstands 10,000 psi for 100 hours
†Percent of absolute melting point at which alloy is useful

Table 13.5.2.2e. Maximum Service Temperature of Plastics and Elastomers*

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," vol. 58, no. 5, mid-October 1963, p. 24, Copyright 1963 by Reinhold Publishing Corp., New York)

MATERIAL	HIGH	LOW
Butadiene-acrylonitrile foams	210	—
Rubber hydrochloride film	205	—
Acrylics	200	140
Polystyrenes, glass-filled	200	190
PVC-nitrile rubber blend film	200	—
Urethane foams, flexible	200	—
Modified polystyrenes	190	120
Acetal	185	—
Polystyrene foamed-in-place, rigid	185	—
Natural rubber	180	—
Neoprene foams	180	—
Polystyrenes, GP	180	140
Polyvinyl chloride film, nonrigid	180	150
Styrene-butadiene rubber	180	—
Epoxies (cast), GP	175	—
Prefoamed polystyrene, rigid	175	165
Polyvinyl formal	165	130
Butadiene-styrene foams	160	—
Natural rubber foam	160	—
Cellulose nitrate	140	120
Epoxies (cast), resilient	122	—
Polyvinyl butyral	115	—
Prefoamed cellulose acetate, rigid	350	200
Alkyds, GP	345	295
Alkyds, elec	300	—
Allyls (cast)	300	—

MATERIAL	HIGH	LOW
Butyl rubber	300	—
Diallyl phthalate, orlon-filled	300	—
Nylons 66 and 610	300	225
Phenolic foamed-in-place, rigid	300	—
Polypropylene film	300	—
Rubber phenolics	300	212
Plastics laminates, GP	295	245
Polyester (cast), rigid	295	245
Polyvinylidene chloride film	290	—
Melamines, fabric-filled	250	—
Melamines, shock res	250	—
Nitrile rubber	250	—
Nylons 6 and 11	250	200
Polyethylene film	250	200
Polysulfide rubber	250	—
Neoprene rubber	240	—
Urethane rubber	240	—
Polyvinyl chloride	220	140
Methylstyrenes	212	210
Vinylidene chloride	212	170
Melamines, GP	210	—
Silicones (molded)	>700	>600
TFE film	585	566
Silicone rubber	550	—
Plastics laminates, low pressure	500	250
TFE fluorocarbons	500	—
Polyester film	490	—
Diallyl phthalate	450	300
Fluorinated acrylic rubber	450	—
Phenolics, shock and ht res	450	250
Viton rubber	450	—
Cellulosic films	400	140
Epoxies (cast), ht res	400	—
FEP fluorocarbons	400	—
Melamines, glass-filled	400	300
Nylon, glass-filled	400	300
Phenolics (molded), shock and heat	400	350
Plastics laminates, elec	400	160
Urethane foamed-in-place, rigid	400	—
CFE film	395	300
Melamines, cellulose or mineral-filled	395	205
CFE fluorocarbons	380	—
Nylon 6 film	380	—
Alkyds, high str	350	—
Phenolics (molded), GP	350	300

*Values represent high and low sides of a range of typical values.

high to subzero temperatures to insure all the austenite is stabilized. Valve seats and poppets fabricated from austenitic or semi-austenitic stainless steels with room seating surfaces lapped at room temperature have resulted in leakage problems at cryogenic temperatures due to martensitic transformation and subsequent dimensional changes. A solution to such a situation is temperature cycling prior to final lapping to insure no further dimensional changes.

Table 13.5.2.3a. Coefficient of Thermal Expansion[a]

(10^{-6} in./in./°F)

(Reprinted with permission from Materials Selector Issue, "Materials in Design Engineering," vol. 58, no. 5, mid-October 1963, p. 23, Copyright 1963 by Reinhold Publishing Corp., New York)

MATERIAL	HIGH	LOW
Silicone rubber	670	—
Nitrile rubber	390	—
Neoprene rubber	340	—
Butyl rubber	320	—
Polypropylene	170	—
Polyethylenes, medium and high density	167	83
Polyethylenes, low density	110	89
Vinylidene chloride	87.8	—
Nylons 6 and 11	71	46
Phenolics (cast)	66	33
Nylons 66 and 610	55	—
TFE fluorocarbons	55	—
Acrylics and epoxies (cast)	50	30
CFE fluorocarbons	38.8	—
Silicones (molded)	32.2	4.5
Phenolics (molded)	25	8.3
Zinc and its alloys[c]	19.3	10.8
Nylon, glass-filled	17	12.5
Lead and its alloys[c]	16.3	14.4
Magnesium alloys[b]	16	14
Epoxies (molded)	14	—
Aluminum and its alloys[c]	13.7	11.7
Tin and its alloys[c]	13	—
Tin and aluminum brasses[c]	11.8	10.3
Plain and leaded brasses[c]	11.6	10
Silver[c]	10.9	—
Cr-Ni-Fe superalloys[d]	10.5	9.2
Stainless steels (cast)[d]	10.4	6.4
Tin bronzes (cast)[c]	10.3	10
Austentic stainless steels[c]	10.2	9
Phosphor silicon bronzes[c]	10.2	9.6
Coppers[c]	9.8	—
Nickel-base superalloys[d]	9.8	7.7
Aluminum bronzes (cast)[c]	9.5	9
Cobalt-base superalloys[d]	9.4	6.8
Beryllium copper[c]	9.3	—
Cupro-nickels and nickel silvers[c]	9.3	9
Nickel and its alloys[d]	9.2	6.8
Cr-Ni-Co-Fe superalloys[d]	9.1	8
Alloy steels[d]	8.6	6.3
Carbon free-cutting steels[d]	8.4	8.1
Alloy steels (cast)[d]	8.3	8
Age hardenable stainless steels[c]	8.2	5.5
Gold[c]	7.9	—
High temperature steels[d]	7.9	6.3
Titanium carbide cermet[d]	7.5	4.3
Wrought irons[c]	7.4	—
Titanium and its alloys[d]	7.1	4.9
Cobalt[d]	6.8	—
Martensitic stainless steels[c]	6.5	5.5
Nitriding steels[d]	6.5	—
Ferritic stainless steels[c]	6	5.8
Gray irons (cast)[c]	6	—
Low expansion nickel alloys[c]	5.5	1.5
Columbium and its alloys	4.1	3.8
Titanium carbide[d]	4.1	—
Tungsten carbide cermet[c]	3.9	2.5
Alumina ceramics[c]	3.7	3.1
Zirconium and its alloys[b]	3.6	3.1
Molybdenum and its alloys	3.1	2.7
Borosilicate glasses[c]	2.5	1.8
Aluminum silicate glass[c]	2.3	—
Tungsten[b]	2.2	—
Invar (free cutting)	0.8	—
Silica glasses[c]	0.5	0.3
Silica, vitreous[c]	0.28	—

[a] Values represent high and low sides of a range of typical values. Values for plastic materials are for a range of temperatures between −22 and 86F (ASTM D696).
[b] Value at room temperature only.
[c] Value for a temperature range between room temperature and 212-750 F.
[d] Value for a temperature range between room temperature and 1000-1800 F.
[e] Value for a temperature range between room temperature and 2200-2475 F.

Stresses Due to Externally Restrained Parts. When a part is heated uniformly, with the edges rigidly supported or clamped, free expansion of the part is prevented and stresses are induced. The linear expansion due to temperature change is

(Eq 13.5.2.3a)

$$\Delta L = \alpha\,(\Delta T)\,L$$

where ΔL = total increase (or decrease) in length, in.

α = coefficient of linear expansion, in. per in. per °F

ΔT = change in temperature, °F

L = original length, in.

If the member is clamped so it cannot expand (or contract), the effect is the same as though a compressive force is applied of sufficient magnitude to produce a compression (or tension) of ΔL inches. The stress is given by

(Eq 13.5.2.3b)

$$\sigma = E\,\frac{\Delta L}{L} = \alpha E\,(\Delta T)$$

where E is the modulus of elasticity.

Table 13.5.2.3b. Dimensional Changes of Materials After 10 Cycles from 300 F to −100°F

(Reprinted with permission from "Product Engineering," McSherry, vol. 31, no. 38, September 1960, Copyright 1960 by McGraw-Hill Publishing Company, New York)

MATERIALS	DIMENSIONAL CHANGE
1020 steel, annealed	-15×10^{-6} in./in.
4140 steel, annealed	+5
4340, Rc 56	+10
52100, Rc 64	+15
303 stainless steel, quench-annealed	−40
303 stainless steel, stress-relieved	−15
Invar, stress-relieved	10
Invar, cold drawn	−20
2024-T6 aluminum	+15
7075-T6 aluminum	−20
Ti 75A titanium	+20

Thermal stresses may occur in cylindrical shells when there is a temperature gradient in the radial direction. If the edges are assumed clamped and at large distances from the end, the stress is

(Eq 13.5.2.3c)

$$\sigma = \frac{\alpha}{2}\,\frac{E\,\Delta T}{(1-\nu)}$$

where ν is Poisson's ratio.

The temperature difference, ΔT, is likely to be greater for thick walls than for thin ones and, therefore, greater stresses can be expected for thick-walled cylinders. Near the end of the shell the thermal stress is maximum and acts at the outer surface of the pipe in the circumferential direction, given by

(Eq 13.5.2.3d)

$$\sigma_{max} = \frac{\alpha E\,\Delta T}{2(1-\nu)}\left(\frac{1+\sqrt{1-\nu^2}}{\sqrt{3}} - \nu\right)$$

For $\nu = 0.3$, this stress is approximately 25 percent greater than the stress in Equation (13.5.2.3c). From this it is evident that in a brittle material, if a crack occurs due to temperature difference, ΔT, it will start at the edge and proceed in an axial direction.

Under a temperature increase, two materials having different coefficients of thermal expansion, rigidly fastened or welded together throughout their length, will tend to expand different amounts; however, must expand equally. The material having the higher coefficient will be subjected to compressive stresses, while the other material will be in tension; thereby the composite part will assume a curvature. This principle is used in the manufacture of bimetallic thermostats.

Thermal Compensation Techniques. Due to dimensional and physical property changes in materials as a function of temperature, techniques for temperature compensation are often required for fluid components which must operate over a wide temperature range. Cryogenic ballistic missile systems often require temperature compensation for components such as regulators and relief valves which rely on a constant force loading spring for satisfactory performance. In such spring-loaded components, spring modulus changes caused by temperature variations can be compensated for by mechanically varying the spring deflection, thereby keeping the spring load constant. The load change in a spring is a function of both modulus change and dimensional change due to thermal expansion. This combined effect of modulus and expansion coefficient is called the thermoelastic coefficient, or coefficient of stiffness. Some of the techniques used for temperature compensation include:

a) Design for configuration symmetry.
b) Make components from a single material, or several materials having the same expansion coefficient.
c) Use bimetallic elements.
d) Use gas expansion or contraction elements.

Selection of compensation techniques is dependent upon functional requirements, weight and space limitations, and environmental compatibility. In the design of aerospace fluid components, judicious material selection or use of bimetallic devices are the techniques most commonly used. Methods employing the vapor pressure liquids, or expansion of liquids or gases confined in bellows, are more bulky and tend to be unsuitable for use in equipment subjected to environmental extremes.

Where the temperature range varies between −50° and 200°F, it is often possible to fabricate the springs from constant modulus alloys of which Ni Span-C is typical. This alloy exhibits a very small change in elastic modulus within this temperature range and, therefore, can be used directly in springs, eliminating the need for compensation. Dimensional changes as a function of temperature, however, must still be considered with constant modulus alloys since they have a relatively high coefficient of thermal expansion. For temperature changes exceeding the useful range of the constant modulus alloys, temperature compensation must be used. The constant modulus alloys are usually undesirable if temperature compensation is required because of the nonlinear character of their temperature modulus curves.

For high spring rate applications where the effects of thermal expansion and elastic modulus changes are of the same order of magnitude, judicious selection of materials can be an effective means of temperature compensation. The estimated temperature range and the material in each element of the spring load circuit must be established. The net change in spring length is then determined. Invar is a useful material for many applications in temperature-compensating circuits due to its extremely low coefficient of expansion. Used in combination with stainless steel or aluminum alloys having relatively high coefficients of expansion, significant deflections are obtainable.

Bimetallic devices are useful for compensation in applications involving lower spring rates, where the equivalent dimensional effect of elastic modulus change is relatively large compared to the effects of thermal expansion. Bimetallic elements can be in a variety of shapes including flat disc, flat strips, dished washers, U shapes, V shapes, and spirals. The particular shape chosen depends upon the nature of the force and/or displacement desired.

Generally, it is simpler to utilize the deflection of the bimetallic element rather than the force. These elements are capable of providing a temperature compensation over wide ranges of temperature. Temperature compensation elements in pressure switches, relief valves, and regulators have been successfully employed over a range of −300°F to +500°F. Elements are also available which will provide satisfactory compensation over the range of −100°F to +1000°F. Design of most bimetal devices is similar to spring design, where the spring changes configuration with temperature when it is in its free condition. Because of the complex elastic properties of such design and development, testing is required to fully establish the characteristics of a new design. Catalog data is available giving the characteristics of commercially available compensating elements. The transient response of temperature compensators must be considered in some applications. Devices subjected to sudden and extreme temperature changes will operate off their calibration range until thermal equilibrium is established, even though compensation is provided. If this behavior is unacceptable, means must be provided to give rapid or equal conduction of heat to the active elements. Such techniques include keeping masses small, using materials having high thermal conductivities, and keeping heat transfer paths from the temperature changing medium to the thermally active elements as short as possible.

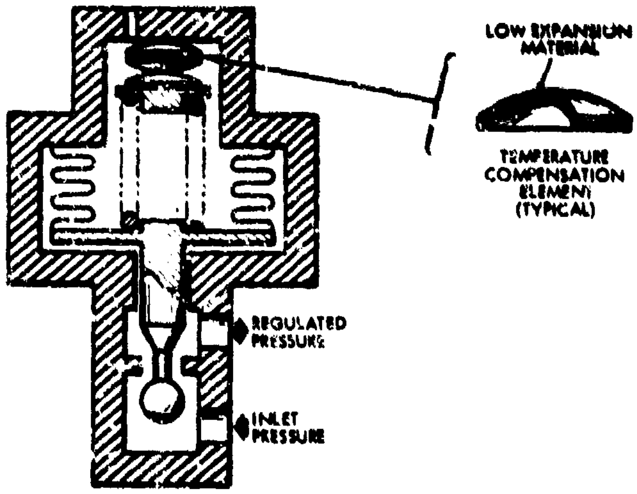


Figure 13.5.2.3a. Temperature Compensated Pressure Regulators

A simple temperature compensated pressure regulator is illustrated in Figure 13.5.2.3a. In the design illustrated, compensation for changes in spring modulus is achieved by stacking dished bimetallic washers. The deflection achieved by the bimetallic elements compensates for the change in spring force as a result of modulus and dimensional changes in the spring.

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13.6 SPACE ENVIRONMENTS

13.6.1 The Space Environment

The space environment is characterized by high vacuum, particle and electromagnetic radiation, meteoroids, and zero gravity; the environments of planets within the solar system represent widely varying temperature and pressure extremes and a variety of atmospheres and gravitational conditions. This section describes the environments of space and the planets, and the effects of the space environments on fluid components.

13.6.2 The Space Vacuum

The vacuum of space consists of a low-density gas mixture, consisting primarily of hydrogen and helium. The estimated gas pressure in interplanetary space is approximately 10^{-16} mm Hg; in interstellar space, pressures lower than 10^{-25} mm Hg may be encountered. The pressure spectrum of space is given in Table 13.6.2, including gas temperature, composition, and concentration. The best vacuum obtainable in a laboratory ranges from 10^{-10} mm Hg to 10^{-11} mm Hg; however, 10^{-10} mm Hg is considered practical for the best commercial vacuum systems.

The following problems of fluid component design are associated with operation under high vacuum conditions: sublimation and evaporation of materials, cold welding, friction, and wear.

Table 13.6.2. Gas Pressures and Concentration in Space
(Adapted from "Chemical Engineering Progress, Symp. Ser.) L. D. Jaffe, vol. 59, no. 40, Copyright 1963 by The American Institute of Chemical Engineers, New York)

ALTITUDE	PRESSURE (mm Hg)	TEMPERATURE (°F)	CONCENTRATION (MOLECULES, ATOMS, OR IONS/CM³)	COMPOSITION
Sea level	$760 \approx 10^{3}$	−40 to +105	2.5×10^{19}	78% N_2, 21% O_2, 1% Ar
100,000 feet	$9 \approx 10^{1}$	−40	4×10^{17}	N_2, O_2, Ar
125 miles	10^{-6}	10^{3}	10^{10}	N_2, O, O_2, O^+
500 miles	10^{-9}	10^{3}	10^{6}	O, He, He^+ O^+, H
4000 miles	10^{-11}	10^{3}	10^{3}	H^{+}, H, He^+
14000 miles	$<10^{-13}$	10^{3} to 10^{4}	10^{1} to 10^{3}	85% H^+ 15% H^{++}

13.6.2.1 SUBLIMATION OF METALS IN VACUUM. The effects of vacuum on the sublimation rate of metals can be calculated from the Langmuir Equation, assuming that none of the molecules leaving the surface return to it.

$$G = \frac{P_v}{17.14}\sqrt{\frac{M}{T}} \qquad \textbf{(Eq 13.6.2.1)}$$

where
- G = weight loss rate per unit area of exposed surface, gm cm^{-2} sec^{-1}
- P_v = vapor pressure of metal at temperature T, mm Hg
- M = molecular weight of the metal in the gas phase
- T = absolute temperature, °K

From Equation (13.6.2.1) it is seen that weight loss rate increases directly with increasing vapor pressure. Table 13.6.2.1 presents a list of several metals and their corresponding sublimation rates for different temperatures. Cadmium, which is often used for plating parts, is seen from these data to be a poor material for use in high vacuum. Metals that sublimate from a warm surface will have a tendency to plate out on cooler surfaces, possibly causing electrical short-circuiting, change of surface emissivities, or change in optical properties of mirrors and lenses. Sublimation of the base material can be retarded by the use of surface coatings with low-vapor pressures, for example inorganic coatings such as oxides.

Table 13.6.2.1. Sublimation of Metals in High Vacuum
(Reference 35-11)

ELEMENT	TEMPERATURE, °F, AT WHICH GIVEN SUBLIMATION RATE OCCURS (FROM EQUATION (13.6.2.1))*			MELTING POINT, °F
	10^{-5} CM/YR (3.94×10^{-6} IN./YR)	10^{-3} CM/YR (0.000394 IN./YR)	10^{-1} CM/YR (0.0394 IN./YR)	
Cadmium	100	170	250	610
Zinc	160	260	350	790
Magnesium	230	340	470	1200
Silver	890	1090	1300	1760
Aluminum	1020	1260	1400	1220
Beryllium	1140	1300	1540	2330
Copper	1160	1400	1650	1980
Gold	1220	1480	1750	1940
Chromium	1300	1600	1840	3410
Iron	1420	1650	1920	2800
Nickel	1480	1720	2000	2650
Titanium	1690	1960	2280	3040
Molybdenum	2580	2960	3450	4730
Tantalum	3250	3700	4200	5400
Tungsten	3400	3900	4500	6300

*To convert sublimation rate G in gm/cm² sec to cm/sec, divide G by density in gm/cm³.

13.6.2.2 PLASTICS AND ELASTOMERS IN THE SPACE VACUUM. Because the Langmuir Equation is not applicable to the organic materials of engineering interest, experimental data of weight loss of organic materials are necessary. The weight loss exhibited by organic polymers in vacuum is usually the result of the evaporation of relatively lower molecular weight fractions, unreacted additives, contaminants, adsorbed and absorbed gases, moisture, etc. The loss of these additives and contaminates, however can change important properties of the polymers. For example, the loss of a plasticizer by evaporation in a vacuum environment will produce a more rigid or brittle part with a corresponding decrease in elongation and increase in tensile and flexure strength. Electrical components, such as capacitors, may change in value if the insulating materials used in their construction lose moisture or other contaminates which are trapped during their manufacture.

The rate of weight loss at a given pressure and temperature varies as a function of time. The initial weight loss is usually high and is due to the loss of adsorbed and absorbed gases, water, and other contaminants. During this stage, the total weight loss may be as great as 3 percent for some polymers. This relatively high initial weight loss will drop to a very low value when the loss of weight is due primarily to degradation of the basic polymer.

In general, polymers of relatively high molecular weight, such as Teflon, do not evaporate or vaporize in vacuum, but when supplied with sufficient thermal energy they decompose or depolymerize. These polymers have such low vapor pressures that the thermal energy required to cause evaporation exceeds that required to break the chemical bonds of the polymer. Many polymers of engineering importance do not sublime or evaporate in high vacuum environments, and the thermal stability of these polymers should be at least as good in high vacuum as in the earth atmosphere.

Weight loss of several high purity polymers are given in Table 13.6.2.2. The weight loss data are given as 10 percent per year at some temperatures. Normally a 1 percent or 2 percent weight loss is not considered detrimental to materials for engineering applications; however, 10 percent weight loss can result in considerable change in the engineering properties of organic materials. Table 13.6.2.2 should be used with caution, since much of the data are of questionable quality Teflon, Mylar, Viton A, and Neoprene are materials which show promise for space vacuum exposures (Reference 331-1).

In general, the following eight points should be noted:

1) High molecular weight polymers apparently do not evaporate or sublime in vacuum.

2) The thermal stability of these polymers should be at least as good in vacuum as in air.

3) The weight loss exhibited by engineering plastics in vacuum is the result of the evaporation of relatively lower molecular weight fractions, unreacted additives, contaminants, etc.

Table 13.6.2.2. Decomposition of Polymers in High Vacuum[a]

(Reprinted with permission from "Chemical Engineering Progress," Symp. Series, L. D. Jaffe, vol. 59, no. 40, Copyright 1963, The American Institute of Chemical Engineers, New York)

POLYMER	TEMPERATURE[b] FOR 10% WEIGHT LOSS PER YEAR		QUALITY OF DATA[c]
	F	C	
Acrylonitrile	240	120	A
Alkyd	200-300	90-150	C-E
Benzyl	540	280	B
Butadiene	490	250	B
Butadiene-Acrylonitrile (NBR rubber)	300-450	150-230	B-D
Butadiene-Styrene (SBR rubber)	460	240	B
Carbonate	350	180	D
Cellulose	350	180	A
Cellulose, oxidized	100	40	B
Cellulose acetate	370	190	A
Cellulose acetate butyrate	340	170	C
Cellulose nitrate	100	40	C
CFE	490	250	A
CFE-Vinylidene Fluoride	500	260	A
Epoxy	100-460	40-240	B-C
Ester	100-460	40-240	B-C
Ethylene, high density	560	290	A
Ethylene, low density	460-540	240-280	A
Ethylene Terephthalate (Mylar, Dacron)	400	200	A
Isobutylene	400	200	B
Isobutylene-Isoprene (Butyl rubber)	250	120	D
Isoprene	380	190	B
Linseed oil	200	90	C
Melamine	380	190	E
Methyl acrylate	100-300	40-150	A-C
Methyl methacrylate	220-390	100-200	A
Methyl phenyl silicone resin	>710	>380	B
Methyl styrene	350-420	180-220	A
Neoprene (chloroprene)	200	90	C
Nylon	80-410	30-210	A
Phenolic	270-510	130-270	B-D
Propylene	370-470	190-240	A
Rubber, natural	380	190	B
Silicone elastomer	400	200	D
Styrene	270-420	130-220	A
Styrene, crosslinked	440-490	230-250	A
Styrene-Butadiene	270	130	C
Sulfide	100	40	C
TFE	710	380	A
Trivinyl benzene	560	290	A
Urethane	150-300	70-150	C
Vinyl acetate	320	160	A
Vinyl alcohol	310	150	B
Vinyl butyral	180	80	C
Vinyl chloride	190	90	A
Vinyl fluoride	460	240	B
Vinylidene fluoride	510	270	A
Vinylidene fluoride-Hexafluoropropene	490	250	A
Vinyl toluene	400	200	B
Xylene	540	280	B

[a] Based on data in the literature as tabulated by Jaffe and Rittenhouse
[b] All temperature values are approximate
[c] In descending order of quality from A to E

4) Weight loss rate and amount of weight loss are greatest early in the test period when the materials at or near the surface evaporate. These loss factors decrease subsequently to a rate determined principally by diffusion rates through the polymer to the surface.

5) Rigid plastics are, in general, preferred over flexible, elastomeric materials.

6) Materials with minimum number and quantity of additives and modifiers are preferred.

7) Complete cure of the plastics must be obtained by extended time and/or elevated temperature post-curing to ensure the elimination of unreacted, low molecular fractions in the product.

8) Those materials exhibiting high loss rates but considered necessary for use on space vehicles because of special desirable properties should be preconditioned in vacuum at elevated temperature to reduce, as much as possible, the potential loss of the material to space.

13.6.2.3 LUBRICANTS IN SPACE VACUUM. Conventional lubricants are generally not suitable for use in the space vacuum because of their high vapor pressure which results in loss of fluid by evaporation. Even if the rate of evaporation of a fluid lubricant is acceptable, the vapors may condense on cooler surfaces such as lenses, relay contacts, or other sensitive components essential to the operation of the equipment within the spacecraft. Other problems associated with using a lubricant in a vacuum are (1) the absence of oxygen — essential to forming a metallic soap, and (2) poor thermal conductivity due to the absence of convective gases, resulting in high thermal gradients due to friction. Also, the lack of absorbents in space will prevent the use of such bearing materials as graphite, which depend on absorbed water vapor for its lubricating properties. The problem of vacuum lubrication is treated in Detailed Topic 6.8.2.6.

13.6.2.4 **COLD WELDING.** Cold welding, often referred to as pressure bonding, may be defined as the joining of two solid metallic elements without the use of heat to produce a liquid or melt phase at the interface. In the earth's atmosphere, metal parts possess a natural surface oxide coating and their surfaces are normally contaminated. The detailed mechanisms involved in cold welding of metallic elements is at present not well understood; however, it is generally accepted that the surface contamination and oxide layers play an important part in preventing bonding of materials in static or dynamic contact.

Solid metal surfaces are normally neither perfectly clean nor perfectly smooth. Under normal atmospheric conditions, oxygen molecules are adsorbed and react with the metal atoms to form oxides. On top of this oxide layer a condensed adsorbed moisture layer is formed. The moisture layer varies in thickness with the relative humidity of the atmosphere. These surface layers are often referred to as surface contaminants, illustrated in Figure 13.6.2.4.

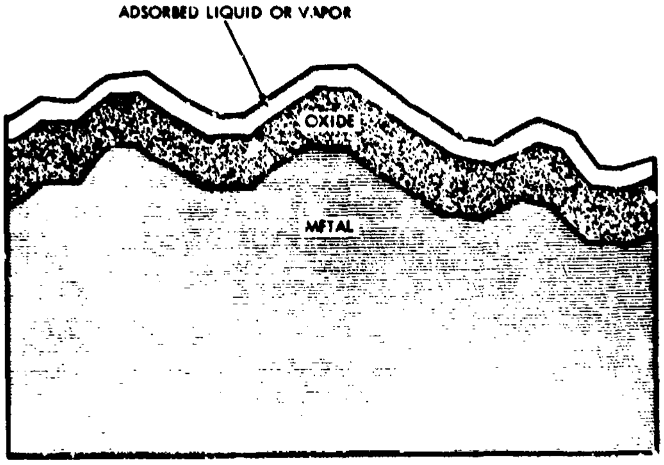

Figure 13.6.2.4. A Solid Surface Showing the Oxide Layer and Adsorbed Liquid Contaminants

The conventional process of welding two metals requires heat in sufficient quantity to remove the surface oxide layers and reduce the metal at the interface to a liquid phase. It should be noted that the liquid phase does not promote the joining of two metals but only allows complete contact of the surfaces. It is only the removal of the surface oxides that allows the metals to be joined.

If the oxide films removed from the metal surfaces are not quickly replaced, the metals will cold weld when in static or dynamic contact. Because surfaces are not perfectly smooth, the real area of contact between parts is limited to the contact of surface asperities. Under the high bearing pressures that can occur at these points, brittle oxide layers are fractured because they cannot conform to the changing surface contours. If the metal surfaces remain free of contaminants (oxides, moisture, etc.) metal-to-metal contact occurs and a metallic junction is formed. Continued removal of oxide layers eventually results in appreciable welding, and seizure of the parts may occur.

Under vacuum conditions, adsorbed moisture cannot exist on a surface and oxide layers may be removed by sublimation. Other removal mechanisms include the removal of surface films by micrometeoroid erosion and sputtering (see Sub-Topic 13.6.4). At present, data on these removal mechanisms are limited or unknown.

Although there is little useful design data on cold welding, limited experimental results indicate that:

a) The degree of cold welding may be a function of solubility between mating materials as indicated by phase diagrams. The results of one investigation of various clean metals coupled under static pressure in a vacuum showed joining of the following soluble couples: iron/aluminum, copper/silver, nickel/copper, and nickel/molybdenum. No joining occurred between the following insoluble couples: copper/molybdenum, silver/molybdenum, silver/iron, and silver/nickel (Reference 286-2).

b) Hard materials which have good wear resistance also show resistance to cold welding. Limited data on 52100 steel, a common ball and roller bearing material, indicates that this material is relatively resistant to cold welding in a vacuum. (Reference 131-22.)

c) Although some materials are less susceptible to cold welding than others, it is advisable, whenever possible, to provide lubrication when sliding surfaces are exposed to vacuum conditions.

13.6.3 Radiation in Space

Radiation may be defined as the emission and propagation of energy through either space or a material medium. The space radiation environment is composed of cosmic rays, electromagnetic radiation, Van Allen Belt radiation, auroral particles, and solar flare particles.

13.6.3.1 **RADIATION TYPES.** Radiation types may be generally classified as either electromagnetic (zero rest mass) or particulate (finite rest mass). Electromagnetic radiation includes ultraviolet light, X-rays, and gamma rays (photons). Particulate radiation consists of electrons, protons, neutrons, alpha particles, and a small number of higher atomic number particles. These particles are defined as follows:

Alpha Particle (α)*:* a positively-charged particle identical to all properties of the nucleus of a helium atom, consisting of two protons and two neutrons.

Beta Particle (β)*:* a negatively or positively charged electron emitted from a nucleus with an energy range of approximately 1 Mev.

Electromagnetic Radiation: radiation having wave lengths from approximately 10^{-1} to 10^{-11} cm.

Photon: the generic term for high energy electromagnetic radiation. Photons of nuclear origin are called gamma rays, and photons of atomic origin are called X-rays. Photons have wavelike properties, but occur as discrete energy pulses. The energy of a photon is inversely proportional to its wave length.

Bremsstrahlung: the secondary radiation induced by charged particles which are accelerated by another charged particle such as a nucleus. The Bremsstrahlung photons are X-rays having energies near that of high energy electrons, but which are more penetrating than the electrons themselves. Also called *free-free radiation.*

Cosmic Rays: high energy particles or electromagnetic radiation originating in interstellar space.

Electron (e): an elementary particle of rest mass m = 9.107 × 10[illegible] grams, and a charge of 4.802 × 10[illegible] statcoulomb; its charge may be positive or negative. A negative electron is called a negatron, but the term electron is often used. A positive electron is called a positron. Negative electrons occurring in space are designated by e^-.

Gamma Rays (γ): electromagnetic radiation having wave lengths from approximately 10[illegible] to 10[illegible] cm. Gamma rays are highly penetrating, and are emitted by a nucleus in its transition from a higher to a lower energy state.

Proton (p^+): a positively charged particle of mass number 1 and a charge equal in magnitude to the electron. It is the nucleus of a hydrogen atom.

X-Ray. electromagnetic radiation having wave lengths of approximately 10[illegible] cm. X-rays are highly penetrating and are usually formed by bombarding a metallic target in a high vacuum with a particle. X-rays are often called Roentgen rays.

13.6.3.2 RADIATION ENERGY. Radiation energy terms are defined as follows:

ev (electron volt): unit of energy necessary to accelerate an electron across a potential difference of one volt (equivalent to 1.6×10^{-12} ergs).

kev: thousand electron volts.

Mev: million electron volts.

Bev: billion electron volts.

Hard and Soft: designations for approxima[illegible] on energies. Hard X-rays have energies greater t[illegible]veral kev and have great penetration, while soft X[illegible] lower energies and are less penetrating.

13.6.3.3 RADIATION FLUX. Radiation fl[illegible] are defined as follows:

Flux: Flux defines the number of particles, photons, or energy passing through a given area in a specified time, usually given in particles/cm² sec, photons/cm² sec, or Mev/cm² sec. Flux may also be specified in terms of the number of particles per unit time passing through an area on the surface of a sphere enclosed by a solid angle. The units are particles/cm² sec steradian where a steradian is defined as the solid angle which encloses a surface on a sphere equal in area to the radius of the sphere squared.

Integrated Flux: the total particles/cm² in any given time period.

Omnidirectional Flux: the number of particles of a particular type that would transverse a test sphere of one centimeter square cross-sectional area in one second (particles/cm² sec).

Unidirectional Flux: the flux arriving at a test sphere per unit solid angle from any particular direction having units of particles/cm² sec steradian. If the incident radiation is isotropic, the unidirectional flux equals the omnidirectional flux divided by 4π (there are 4π steradians in a sphere). A flux of particles incident on a plain surface is one-fourth the omnidirectional flux, or is equal to the unidirectional flux divided by 16π. The results of laboratory radiation damage experiments are usually quoted in units of particles/cm² required to produce some effect in a given sample. To obtain the omnidirectional flux corresponding to a laboratory flux, the laboratory flux should be multiplied by four.

13.6.3.4 RADIATION DOSE. Radiation dosage can be expressed either in terms of the exposure dose, which is a measure of the radiation field to which a material is exposed, or in terms of the absorbed dose, which is a measure of the energy absorbed by the radiated material.

Adsorbed dose units:

Erg/gram: the energy expressed in ergs absorbed by a gram of the irradiated material.

Rad: an absorbed dose defined as 100 ergs of radiation energy of any type absorbed per gram of any irradiated material.

Exposure Dose Units:

Roentgen (r): an exposure dose defined as the quantity of X- and gamma-radiation which will produce one electrostatic unit of electrical charge in 1 cc of dry air at standard conditions of temperature and pressure. This amount of energy gives an absorbed dose of 87.7 ergs of energy per gram of air.

ergs/gram carbon, ergs/gram(C): an indirect measure of a gamma radiation field based on an absorbed dose using carbon as a standard. One roentgen, r, of gamma rays is equivalent to approximately 87.7 ergs/gram carbon.

Dose Rate: the rate of energy delivered or absorbed, e.g., r/month, r/year, rad/day.

13.6.3.5 RADIATION MEASUREMENTS. Flux measurement and dose measurement are outlined as follows:

Flux Measurement. Radiation flux measurements are made with particle sensors which depend on the conversion of radiation ionization to an electrical signal. Various types of particle detectors and their applications are:

Detector	Basis	Chief Uses
Geiger counter	Ionization	e^-, γ, p^+ gross counting
Ionization chamber	Ionization	γ, α gross counting
Proportional chamber	Ionization	e^-, p^+ gross counting
Bare multiplier	Electron multiplication	e^- gross counting
Scintillation counter	Light production	e^-, γ, p^+ energy analysis
Solid-State counter	Ionization	p^+, α energy analysis
Cerekov counter	Light production	p^+ high-energy detection

Dose Measurement. Radiation dosimeters measure the total exposure to ionizing radiation. Four types of dosimeters and their dose ranges are:

Photographic films	10^{-2}–10^{+3} r
Plastics	10^{4}–10^{9} r
Glasses	10 –10^{7} r
Chemical dosimeters	50 –10^{8} r

13.6.3.6 SPACE RADIATION ZONES. The space radiation environment is characterized by the earth radiation zone (Van Allen Belts), the auroral zone, and the interplanetary zone. Types of radiation found in space include electrons, protons, cosmic rays, and electromagnetic radiation, consisting of ultra-violet rays, X-rays, and gamma rays.

Geomagnetic Coordinates. Normally, it is convenient to plot the radiation intensity in the earth's radiation zone in geomagnetic rather than geographic coordinates. The origin of these coordinate systems coincide, but the geomagnetic axis is tilted by 11.5 degrees with respect to the axis of rotation of the earth.

The Earth Radiation Zone. The earth radiation zone is characterized by magnetically-trapped electrons and protons. This zone, often referred to as the Van Allen Belts, is made up of two concentric belts, the inner belt and the outer belt. The inner belt extends approximately 4000 miles, with intensity reaching a maximum at 1800 to 2000 miles above the geomagnetic equator. The inner belt is sometimes referred to as the hard belt, and contains high energy protons of energies to 700 Mev, with electrons in the 20 kev to 1 Mev range. The outer belt extends about 8000 to 37,000 miles, where the region of high intensity is at 10,000 to 15,000 miles and up. This belt, called the soft belt, consists primarily of electrons from 20 kev to 5 Mev and some protons over 60 Mev. The isointensity contours for electrons and protons based on data from Explorer 12 are shown in Figure 13.6.3.6.

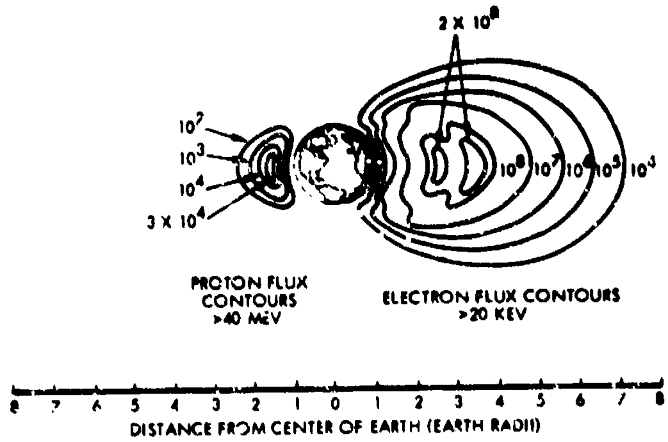

Figure 13.6.3.6. Isointensity Contours for Electrons and Protons

(Based on data from Explorer 12. To some extent this distribution varies with time. From "Space/Aeronautics," F. L. Bouquet, Jr., May 1963, Copyright 1963 by Conover Mast Publications, Inc., New York.)

Radiation dosages in the belts, including the energy penetration to a given depth or range, are presented in Table 13.6.3.6.

The Auroral Zone. The auroral zone is located between approximately 60 and 65 degrees geomagnetic latitude. The auroral displays are produced by low energy (less than 200 kev) electrons entering the atmosphere. Protons may also be present. The auroral particles are easily stopped and, consequently, do not present a serious radiation problem.

The Interplanetary Zone. Radiation in interplanetary space consists of an energetic cosmic flux and pulses of radiation associated with solar flares. The distribution and frequency of the solar flares follow the well known eleven year sun spot cycle. The largest flares, consisting of relativistic (Bev) protons, are extremely rare; only nine have been observed in the last 23 years. The smallest flares occur as often as eight times per day. In addition to these sources of interplanetary radiation, there also exists a continuous ejection of low energy particles, primarily protons and electrons from the sun, known as the solar wind. The distribution of the solar wind particles is believed to obey the inverse square law with the sun acting as a point source.

Table 13.6.3.6. Radiation Dosage in Space
(Reference 166-9)

RADIATION	ENERGY (ev)	PENETRATING RANGE (gm/cm²)**	DOSAGE AT VARIOUS PENETRATION DEPTHS (erg/gm-yr)		
			EXTREME SURFACE	THROUGH 10^{-3} gm/cm²	THROUGH 4 gm/cm²
Inner radiation belt					
Protons	$10^4(?)$–7×10^8	$10^{-4}(?)$–10^3	$10^{11}(?)$	10^{11}	10^7
Electrons	$<2\times10^4$–1×10^6	10^{-5}–10^0	$10^{11}(?)$	10^{11}	0
Bremsstrahlung	$<2\times10^4$–1×10^6	10^{-1}–10^1	$10^7(?)$	10^7	10^7–10^8
Total			$10^{11}(?)$	10^{11}	10^7–10^8
Due principally to*			e	e	γ, p
Outer radiation belt					
Electrons	2×10^4–5×10^6	10^{-5}–10^0	10^{14}–10^{15}	10^{12}–10^{15}	10^8
Bremsstrahlung	2×10^4–5×10^6	10^{-1}–10^1	10^7–10^9	10^7–10^9	10^6–10^8
Total			10^{12}–10^{17}	10^{12}–10^{15}	10^6–10^8
Due principally to*			e	e	γ
Solar flare high energy particles					
Protons	2×10^7–10^9	10^0–10^2	10^5–10^6	10^5–10^6	10^4–10^5
Electrons	$\sim5\times10^4$	10^{-3}	10^7–$10^9(?)$	10^7–$10^9(?)$	0
Bremsstrahlung	$\sim5\times10^4$	10^0–10^1	10^2–$10^4(?)$	10^2–$10^4(?)$	10^2–$10^4(?)$
Total			10^7–$10^9(?)$	10^7–$10^9(?)$	10^4–10^5
Due principally to*			e(?)	e(?)	p(?)
Solar flare low energy particles					
Protons	5×10^3–2×10^6	$10^{-6}(?)$–$10^{-3}(?)$	0	0	0
Electrons	2×10^{-1}–10^3	10^{-6}	0	0	0
Bremsstrahlung	2×10^{-1}–10^1	$10^{-6}(?)$–10^3	0	0	0
Total			0	0	0
Steady solar emission					
Protons	10^0–10^3	10^{-9}–$10^{-6}(?)$	0	0	0
Electrons	10^0	10^{-6}	0	0	0
Bremsstrahlung	10^0	10^{-7}	0	0	0
Total			0	0	0
Cosmic rays					
Protons	10^8–10^{19}	$>10^{-1}$	10^2–10^3	10^2–10^3	10^2–10^3

*e = electron; p = proton; γ = bremsstrahlung photon

**To convert to penetration depth in cm divide by the material density in gm/cm³.

Cosmic rays of galactic origin consist of protons (~93 percent) and alpha particles (~7 percent) along with smaller amounts of heavier elements. The energy of the protons is in the range of 500 Mev to 20 Bev. Although energies are quite high, the free space flux of particles is 2.5 particles/cm^2 sec. Since this flux is small, radiation damage due to cosmic rays usually needs to be considered only in very long space flights.

Radiation dosages in the interplanetary scene, including the solar flares and cosmic rays, are presented in Table 13.6.3.6. The values given represent the approximate range or depth of penetration in materials, including the effect of shielding materials.

13.6.3.7 RADIATION EFFECTS ON MATERIALS. An important factor in determining the effect of radiation on materials is the range or penetrating power. Particles are less penetrating than photons, and particles which are highly charged and/or relatively large are less penetrating than electrically neutral and/or relatively small particles. Gamma rays or X-rays are highly penetrating, while alpha particles which are relatively massive and highly charged penetrate only a small distance before stopping. The comparative penetrating power of various types of radiation at different energy levels—in terms of penetration depth through a gas (air), liquid (water), and a solid (aluminum)—is listed in Table 13.6.3.7a. For charged particles, the penetration range listed in Table 13.6.3.7a is the thickness required to reduce the intensity essentially to zero. For gamma rays, the thickness is that required to reduce the intensity to half the incident value.

Table 13.6.3.7a. Comparative Penetrating Power of Charged Particles
(Reference 131-9)

RADIATION TYPE	ENERGY (Mev)	PENETRATING RANGE (INCHES)		
		AIR	WATER	ALUMINUM
Alpha	1	0.2	0.0002	0.0001
	10		0.01	0.004
	100		0.4	0.14
Proton	1	0.9	0.001	0.0005
	10		0.04	0.014
	100		2.3	0.75
	300		24.0	7.9
Electrons	1	104	0.14	0.06
	3		0.58	0.21
Gamma ray	1		4.5*	1.7*
	5		9.1*	3.7*

*Thickness necessary to reduce the intensity by 0.5.

Metals are considerably more resistant to radiation damage than most other classes of solids, showing little damage due to radiation except when exposed to the radiation in the heart of nuclear reactors (fast neutron exposure).

Organic materials, as a class, are the least stable in a radiation field. Radiation damage to organic materials is dependent upon the total energy absorbed and sometimes upon the radiation intensity; damage is usually not dependent upon the type of radiation. Radiation damage to polymers may occur because of the removal of a bonded electron leading to bond rupture, free radicals, discoloration, etc. Polymers may be degraded by a loss in mechanical strength, an increase in vapor pressure and viscosity, and a reduction in molecular weight. Most elastomeric materials are not satisfactory for use beyond a gamma dosage of 10^{10} ergs/gm(C). Natural rubber is the most radiation resistant of the elastomers. Styrene-butadiene rubber is the most resistant synthetic elastomer. Silicones and fluorine-based polymers are below average in radiation resistance. Table 13.6.3.7b presents the radiation resistance data of some plastics and elastomers. It is important to note that the data shown in Table 13.6.3.7b is directly applicable to radiation exposure in the presence of air. Limited radiation testing of some polymers, including Teflon, in a vacuum environment indicates that radiation damage is reduced considerably. This is explained by the fact that the presence of an oxidizer in the environment causes oxidation of ionized polymers which results in greater alteration of the molecular structure than in a chemically inert (vacuum) environment.

Lubricants, in general, are affected by radiation exposures at $10^{[illegible]}$ r. The effects noted are a decrease in initial viscosity, followed by an increase in foaming, increasing acidity, and decreasing oxidation stability. Petroleum lubricants are the most stable, and little change is noted at $10^{[illegible]}$ r radiation exposure. Fluorinated materials are not recommended for use in a radiation environment because the resulting acid which is liberated is highly corrosive. Conventional metal soap greases harden and solidify under continued radiation exposure. Aromatic hydrocarbons change very little for radiation exposures to $10^{[illegible]}$ r.

Ceramic materials, in general, are not seriously affected by radiation. Glass transparency may be reduced or become opaque. Explosives such as diozodinitrophenol and nitroglycerin are damaged when exposed to a radiation dose of $10^{[illegible]}$ and $10^{[illegible]}$ r, respectively.

The tolerable radiation dose for any given material may depend strongly on its application. For example, a material which loses tensile strength may still be useful as an insulation material, while its use as a seal material may be adversely affected.

13.6.3.8 RADIATION SHIELDING. Three important factors to consider in determining radiation shielding requirements are:

1) The nature and properties of space radiation
2) Mission durations and space flight paths
3) The tolerable dosage for a particular component or material, dependent upon the function of the component or material.

Table 13.6.3.7b. Radiation Tolerance for Various Materials
(Reference 66-8)

MATERIAL	THRESHOLD DAMAGE*	25 PERCENT DAMAGE**	REMARKS
Elastomers			
Natural rubber	2 × 10^[illegible] ergs/gram (C)	2.5 × 10^[illegible] ergs/gram (C)	Damage refers to overall properties
Natural rubber	2.4 × 10^[illegible]	1.5 × 10^[illegible]	Damage refers to tensile strength
Polyurethane rubber	9 × 10^[illegible]	1 × 10^[illegible]	
Styrene-butadiene (SBR)	2 × 10^[illegible]	1 × 10^[illegible]	Damage refers to overall properties
Styrene-butadiene (SBR)	—	3 × 10^[illegible]	Damage refers to tensile strength
Nitrile rubber (NBR)	—	7 × 10^[illegible]	Compression set degrades
Nitrile rubber (NBR)	—	1.5 × 10^[illegible]	Tensile strength increases by 25 percent
Neoprene rubber	4.5 × 10^[illegible]	—	Hardness begins to change
Hypalon (chlorosulfonated polyethylene)	4.5 × 10^[illegible]	—	Tensile strength begins to increase
Acrylic rubber	9 × 10^[illegible]	—	
Silicone rubber	—	10^[illegible]	
Viton A	—	9 × 10^[illegible]	Hardness increases 25 percent, elongation decreases 80 percent
Kel-F 5500	—	10^[illegible]	
Polysulfide rubber	2.5 × 10^[illegible]	—	Great damage at 4.5 × 10^[illegible] erg/gram (C)
Butyl rubber	—	10^[illegible]	
Plastics			
Amino resins	7.0 × 10^[illegible]	10^[illegible]	
Cellulosics	—	2 × 10^[illegible]	
Epoxies	9.5 × 10^[illegible]	—	
Polyethylene	2 × 10^[illegible]	9 × 10^[illegible]	
Teflon	2 × 10^[illegible]	3.5 × 10^[illegible]	
Phenolics	—	10^[illegible]	
Nylon sheet	9 × 10^[illegible]	5 × 10^[illegible]	25 percent decrease in elongation and impact strength
Silicones	10^[illegible]	—	Glass reinforced silicone laminates reach threshold at approximately 10^[illegible] erg/gram (C)
Polyvinyl chloride (PVC)	2 × 10^[illegible]	10^[illegible]	Liberates hydrogen chloride

*Threshold damage is the amount of radiation exposure required to change at least one physical property of the material.
**25 percent damage is the amount of radiation exposure required to change a physical property of the material by 25 percent.

At present it is possible to obtain only approximate shield requirements because of the lack of data for radiation interactions with materials, and lack of knowledge of radiation dosage in space as a function of time.

Radiation shielding should be designed to optimize weight and maintain an acceptable dose rate to the shielded element; it is left to the designer to determine how much radiation the shielded element may receive. A great amount of the material in a space vehicle is used for structural purposes. The structure then provides a free shield, since it must be used regardless of the radiation environment. Structural housing should be designed to provide as much shielding as possible for the weaker elements such as seals, etc., and still maintain an optimum strength-to-weight ratio. Radiation dosages through different shielding thicknesses in gm/cm^2 produced in the various radiation zones in space are shown in Table 13.6.3.6.

13.6.4 Meteoroids

The space environment includes a class of material particles of stony and iron-nickel compositions. The density of these materials ranges from 0.05 gm/cm for dust ball meteoroids to 2-10 gms/cm^3 for stony and metallic (iron-nickel) particles. These particles have been classed as meteoroids, micrometeoroids, meteorites, dust, cosmic particles, etc., according to size. However, for purposes of this discussion the term *meteoroid* will be used to describe the stony and iron-nickel type of materials in space.

It is believed that meteoroids are of two origins — asteroids and comets. Meteoroids of asteroidal origin constitute approximately 10 percent of the total influx of the particles that enter the earth's atmosphere. Most meteoroidal material in the solar system is believed to be associated with comets. Approximately 90 percent of the total flux into the earth's atmosphere is thought to be of cometary origin. A comet is composed of a low density mass consisting of loose particles. As dissipation of the comet occurs and small particles are released, they take up the orbit of the original comet, with the exception of the perturbation forces of other planets which tend to widen their path.

It is generally agreed that the velocity of meteoroids falls in the range of 11-72 km/sec (7-45 mi/sec), with large concentrations in the 20 km/sec (12 mi/sec) and 40 km/sec (25 mi/sec) ranges.

Important parameters of the meteoroid environment that must be considered in design are probability of meteoroid hits, meteoroid damage, and protection against meteoroid damage.

13.6.4.1 PROBABILITY OF METEOROID HITS. The probability of a particle in space striking a component is a function of the meteoric flux and the exposed area of the component. The number of meteoroid impacts per square foot per day on a structural surface for an earth-orbiting space vehicle is plotted in Figure 13.6.4.1. It can be seen from this figure that it is probable that one meteoroid of mass ranging from 10 [illegible] to 10 [illegible] grams will impact a body one square foot in area per day.

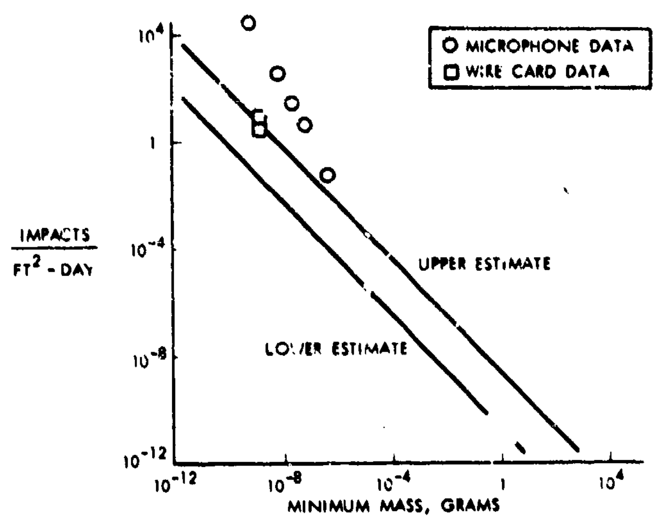

Figure 13.6.4.1. Meteoroidal Impacts on Structural Surfaces
(From Reference 476-1.)

13.6.4.2 METEOROID DAMAGE. Meteoroid damage ranges from surface erosion which can change optical properties to complete puncture of spacecraft structural materials. Even though puncture does not occur, damage can result from spalling (fragmentation of the inner surface) causing structural damage and the generation of harmful contaminants in the system; or deformation of the inner surface which could restrict flow in fluid lines or impair the operation of close fitting moving parts such as bearings, sleeves, and spools. Also, localized heating caused by meteoroidal impact could result in reaction between a strong oxidizer such as fluorine and the heated surface. Figure 13.6.4.2a illustrates inner surface damage caused by meteoroid impact. Data on the degree of damage caused by meteoroid impact is generally vague due to uncertainties in meteoroid composition, mass density, shape, and velocities. Although a number of relationships have been developed relating puncture depth to meteoroid and target material properties such as density, hardness, and heat of fusion, as well as geometric factors, experimental verification of these relationships is extremely limited. A prime limitation is the inability to achieve representative meteoroid velocity. One relationship for hypervelocity impact at normal incidence which shows good agreement with the limited available experimental data is given by Bjork (Reference 2-2).

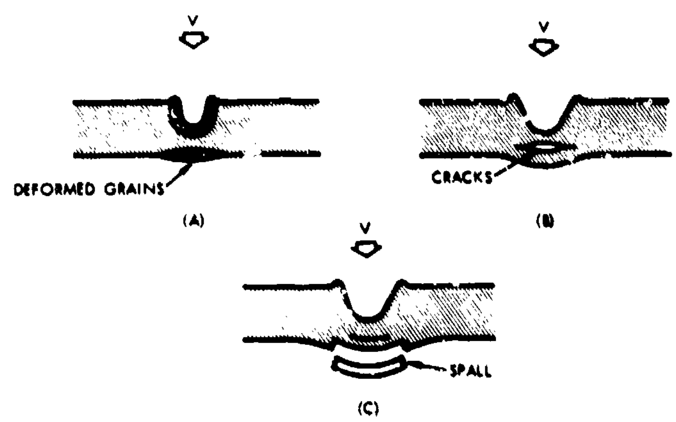

Figure 13.6.4.2a. Rear Surface Damage by Hypervelocity Particles in Relatively Thick Targets

$$d = C(mV)^{1/3} \qquad \textbf{(Eq 13.6.4.2)}$$

where
- d = depth and radius of a hemispherical crater, cm
- m = particle mass, gm
- V = impact velocity, km/sec
- C = constant for a given combination of particle and target materials
  - C = 1.04 for aluminum hitting aluminum
  - C = 0.606 for iron hitting iron
  - C = 1.3 for lead hitting lead

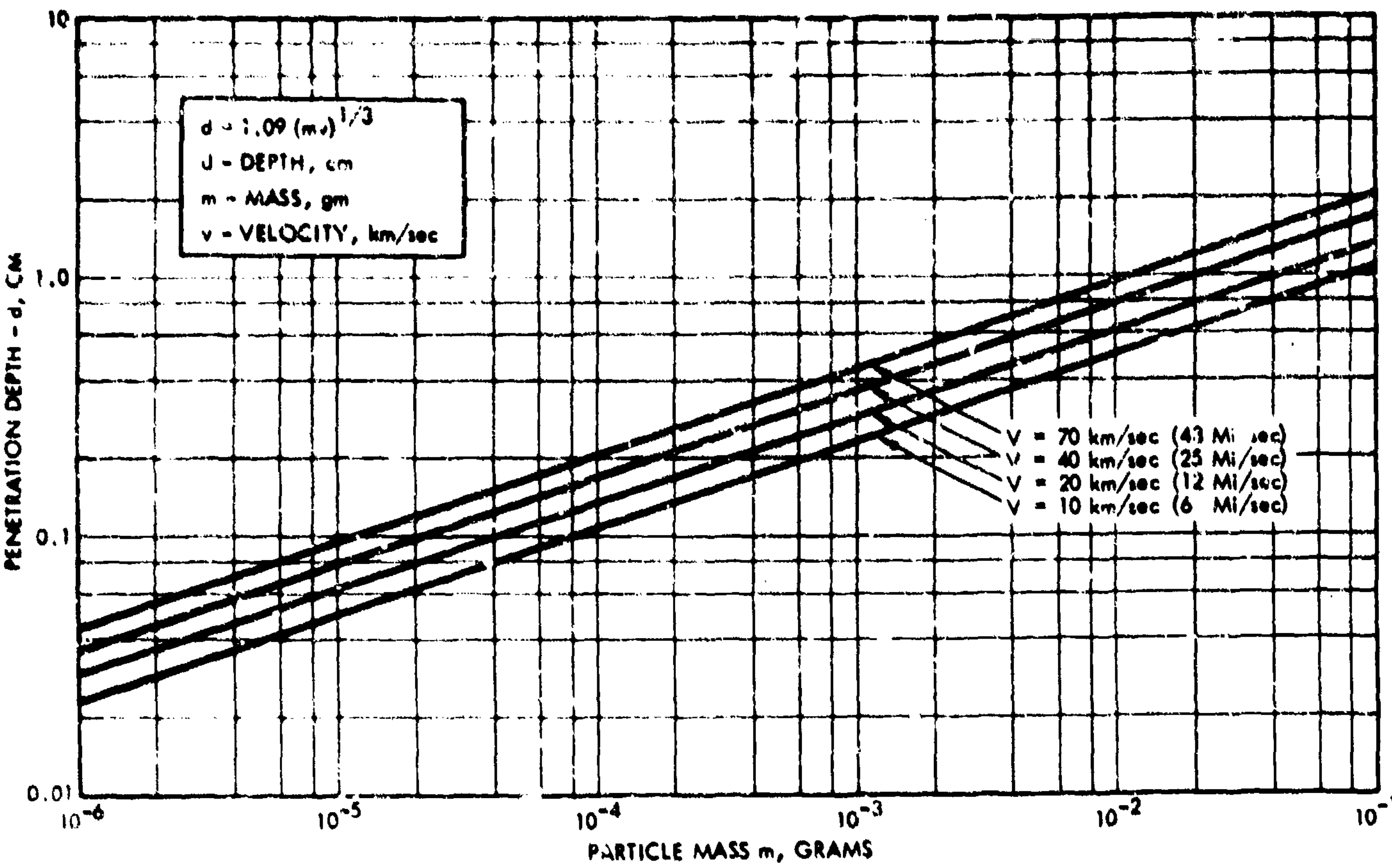

Figure 13.6.4.2b. Penetration of Hypervelocity Particles at Various Impact Velocities, Based on Bjork's Equation for Aluminum on Aluminum

This equation for aluminum hitting aluminum is plotted in Figure 13.6.4.2b.

13.6.4.3 PROTECTION AGAINST METEOROID DAMAGE. In general, meteoroids do not present a serious hazard to fluid components because there is sufficient wall thickness in most components to prevent penetration by the small particles most likely to be encountered; larger particles occur so rarely that the probability of being hit is very low. Reference 34-7 states that particles smaller than 10[illegible] gms present no penetration hazard and particles larger than 10[illegible] gms occur too rarely to be considered a hazard worthy of consideration. Meteoroids most likely to present a significant penetration hazard have masses from 10[illegible] to 10[illegible] grams. A technique for meteoroid protection which has received a considerable amount of study is the use of meteoroid bumpers. A bumper is a sacrificial protective shield mounted with a space between and the surface to be protected. This space can either be left void or be provided with a filler which acts as an energy absorber. Fragmented meteoroid particles and bumper spall are dispersed so that they have insufficient energy to penetrate protected components. If the scattered fragments are so spread out that they act independently, the energy transmitted at each point of impact is reduced in proportion to the number of fragments. The relative thickness and corresponding relative weight of materials for equivalent meteoroid protection is given in Table 13.6.4.3. The use of multiple plates for bumper shielding reduces the penetrating capability of particles, as compared with a single plate having the same total thickness. The relative weights for various bumper configurations necessary to preclude puncture are illustrated in Figure 13.6.4.3. An extensive bibliography on meteoroids is presented in References 131-3, 131-19, and 131-21.

13.6.5 Temperature in Space

13.6.5.1 THE SPACE MEDIA. Since interplanetary space consists of widely separated gas molecules, the concept of temperature environment in space is quite different from the concept of temperature as an environment in the atmosphere. Due to the extremely low density of the interplanetary gas mixture, it is necessary to consider temperature in terms of kinetic theory of gases, i.e., the relationship

Table 13.6.4.3. Material Thickness and Corresponding Weight to Provide Equivalent Meteoroid Protection
(Reference 476-1)

MATERIAL	DENSITY	RELATIVE THICKNESS	RELATIVE WEIGHT
Aluminum	0.098	1.00	1.00
Magnesium	0.065	1.32	0.88
Beryllium	0.066	0.72	0.48
Filament wound plastic	0.077	1.61	1.24
Fused silica	0.079	1.05	0.85
6Al-4V titanium	0.160	0.72	1.18
17-7PH steel	0.276	0.49	1.38

between motion of a gas and its temperature. This relationship is given by the following:

$$\frac{1}{2} mV^2 = \frac{3}{2} kT \qquad \text{(Eq 13.6.5.1)}$$

where
m = mass of the gas molecule, lb_m
V = velocity of the gas molecule, ft/sec
k = Boltzmann's molecular gas constant, $lb_m\ ft^2/sec^2\ {}^\circ R$
T = absolute temperature, °R

Although the velocity of a molecule in space is not known with any degree of accuracy, gas temperatures of several thousand degrees have been predicted based on kinetic temperature. The fact that these high temperature gas molecules are so widely scattered, however, means that they have a negligible effect on the temperature of a space vehicle due to the small amount of heat energy involved. The temperature of a space vehicle, therefore, is determined not by the temperature of the surrounding atmosphere, but rather as a result of radiation from other sources, such as the sun and radiation, to the heat sink of space.

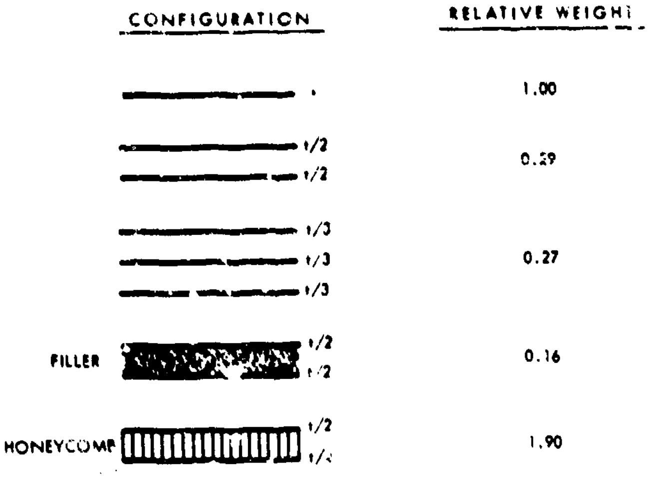

Figure 13.6.4.3. Relative Weights for Various Bumper Configurations Necessary to Provide Meteoroid Protection
(From Reference 34-7)

13.6.5.2 THERMAL SOURCES. The primary external source of thermal energy for a spacecraft traveling within the solar system is direct radiation from the sun. Heat energy derived from the total electromagnetic spectrum of the sun is a heat flux of 442 Btu/ft² hr at a distance of 1 au* (astronomical unit). The heat flux intensity varies inversely as the square of the distance from the sun.

The steady-state temperature of a body in space can be determined by equating the radiant energy emitted from the body at thermal equilibrium to the total energy absorbed by the body.

$$A_T \epsilon_R \sigma T_R^4 = A_{P,S} a_S H_S + \Sigma A_P a_R B H_R + \Sigma A_P a_E T^4 + Q \qquad \text{(Eq 13.6.5.2a)}$$

Energy emitted from body	Energy absorbed directly from sun	Energy absorbed by body from planets, reflected	Energy absorbed by body from planets, emitted

where
A_T = total surface area of body, ft²
ϵ_R = emissivity of body
σ = Stefan-Boltzman constant, 0.1713×10^{-8} Btu/hr ft² °R⁴
T_R = steady-state temperature of the body, °R
$A_{P,S}$ = projected area of the body toward the sun, ft²
a_S = solar absorptivity of body
H_S = intensity of solar radiation, Btu/ft²-hr ($H_S = \frac{442}{R^2}$; R = distance from sun in astronomical units)
A_P = projected area of body toward planet, ft²
a_R = absorptivity of body of reflected energy from planet (function of spectral distribution from planet; for the earth, value nearly the same as solar absorptivity)
B = Albedo of planet
Q = Heat from other sources, including heat generated by spacecraft, Btu/hr
a_E = absorptivity of body of emitted energy from planet
T = effective black body temperature of planet, °R

Because the energy contribution from the planets is relatively small, a close approximation of body temperature, neglecting heat generated by the spacecraft, is given by

*1 au is a unit of distance equal to the mean distance of the earth from the sun.

(Eq 13.6.5.2b)

$$A_T \epsilon_B \sigma T_B^4 \quad A_{p,s} a_s H_s$$

$$\text{or} \qquad T_B \left(\frac{A_T}{A_{p,s}}\right)^{\frac{1}{4}} \quad \left[\left(\frac{a_s}{\epsilon_B \sigma}\right) H_s\right]^{\frac{1}{4}}$$

Equation (13.6.5.2b) is plotted in Figure 13.6.5.2 with H_s converted to distance from the sun (H_s $\frac{442}{R^2}$; R distance from the sun, au), for various values of the ratio a_s/ϵ_B. (See Detailed Topic 2.2.3.3 for a table of a_s/ϵ_B ratios). The planets' distances from the sun are also indicated in this figure, although the effect of radiation from the planets is not included in the curves.

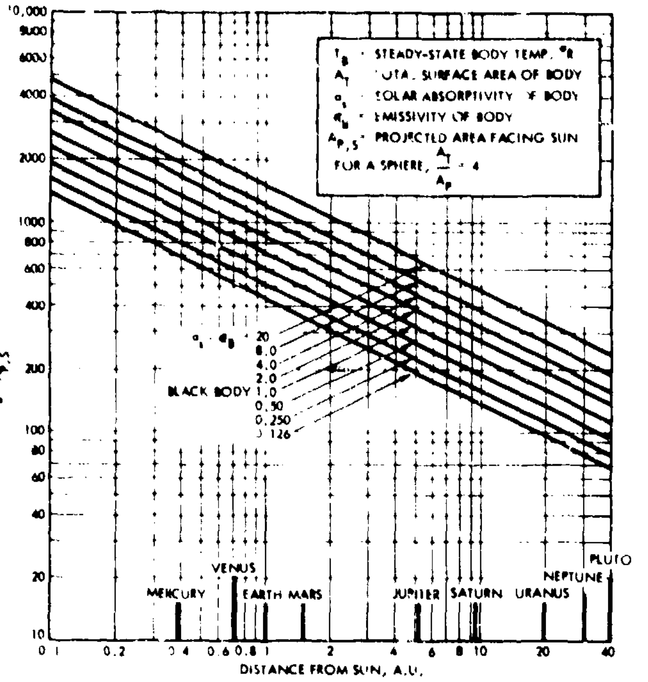

Figure 13.6.5.2. Equilibrium Body Temperature Versus Distance from the Sun, as a Function $\frac{a_s}{\epsilon_B}$

Another source of heat for bodies in close proximity to a planet is their reflected and emitted energy. The emission and reflectivity of heat energy from a planet depends on its temperature and emissive properties, and on its albedo (reflective properties). The albedo represents the percentage of incident energy reflected by a planet. The average albedo, B, for several solar system bodies is given in the following table:

Table 13.6.5.2. Albedoes of Solar System Bodies

(From "Handbook of Astronautical Engineering," H. H. Koelle, ed., Copyright 1961 by McGraw-Hill Book Company, New York)

Earth	0.36	Venus	0.76	Uranus	0.66
Moon	0.07	Jupiter	0.51		
Mars	0.15	Saturn	0.50		

The only other important source of heat is that generated by the equipment housed within the space vehicle.

13.6.5.3 THERMAL SINKS. The heat sinks for a vehicle in space include the space sphere and components within the vehicle. The space sphere may be considered to be a black body at a temperature of approximately 4° Kelvin. This temperature is a result of starlight, otherwise the temperature of the space sphere would be considered absolute zero.

13.6.5.4 TEMPERATURE CONTROL. Temperature control in space is obtained primarily through radiation heat transfer by the use of reflecting materials or reflection devices such as rotating louvers. Minimizing absorbed radiant energy can be achieved by the use of super insulation materials and coatings having low a/ϵ ratios. The use of super insulation materials to reduce heat transfer is discussed in Detailed Topic 2.3.2.1. A table of a/ϵ ratios for a variety of materials is given in Detailed Topic 2.2.3.3.

13.6.6 Zero Gravity

The absence of gravity is impossible to duplicate except for short periods of time. At present, testing in a zero gravity field is very difficult and expensive; therefore, very little information is available on this environmental parameter.

It is generally believed that no significant effects on materials will be encountered due to the zero gravity conditions in space. Designs dependent on weights and liquid-liquid or liquid-vapor separating to some predictable orientation will be useless in the zero gravity field. The behavior of contained liquid in this field may depend on the wettability of the container walls. Liquids which do not wet the container wall tend to contract to a spherical shape, leave the walls, and become suspended in space. Liquids which do wet the wall will tend to spread out over the wall, leaving a gas pocket in the center. Transfer or flow of fluids such as propellants and lubricants must be made without depending on gravity. Venting a gas from a liquid vapor phase requires techniques to prevent loss of the liquid. Fluid heat transfer must depend on mechanisms other than convection, such as film boiling, conduction, and diffusion.

Other forces will act on a space vehicle whereby the problems associated with zero gravity are alleviated to some degree. Forces that may produce an artificial gravity force such as orbit transfer or correction forces, spinning or tumbling of the spacecraft, and solar radiation pressures, may alleviate the problems of zero gravity to some degree.

13.6.7 Planetary Environment

The bodies within the solar system shown in Figure 13.6.7 present a wide range of environmental conditions. Although detailed information on the environments of the planets is extremely limited at present, some of the basic characteristics of the planets and the earth's moon are given in Table 13.6.7.

Table 13.6.7. Characteristics of the Solar System

(Adapted from reference 131-30, corrected to reflect preliminary Mariner 6 and 7 data)

BODY	SEMI-MAJOR AXIS TO SUN (AU)*	PERIOD EARTH-YEARS (EARTH=1)	MEAN DIAMETER (EARTH =1)	MASS (EARTH=1)	NUMBER OF NATURAL SATELLITES	EQUATORIAL SURFACE GRAVITY (Earth=1)
Sun	--	--	109.2	3×10^5	-	28
Mercury	0.387	0.241	0.379	0.055	0	0.380
Venus	0.723	0.616	0.956	0.815	0	0.893
Earth	1.000	1.00	1.00	1.00	1	1.00
Mars	1.524	1.88	0.535	0.108	2	0.377
Jupiter	5.203	11.9	11.14	317.9	12	2.54
Saturn	9.539	29.5	9.47	95.1	10	1.06
Uranus	19 25	84.0	3.69	14.5	5	1.07
Neptune	30.04	164.8	3.50	17.0	2	1.4
Pluto	39.64	247.7	1.1?	0.8?	0	0.7?
Earth's Moon	---	0.075	0.272	0.012	0	0.165

BODY	SURFACE ESCAPE VELOCITY (Earth=1)	SURFACE TEMP (°F)	SURFACE ATMOSPHERIC PRESSURE (in atmospheres)	ATMOSPHERIC COMPOSITION
Sun	55.0	≈11,500	—	—
Mercury	0.371	750	≪1	traces of heavy gases
Venus	0.915	800	16?	93% CO_2 ; possibly N_2 trace of water vapor
Earth	1.00	60	1	78% N_2, 20% O_2
Mars	0.449	90 to -190	0.01	90 - 100% CO_2 remainder unknown, but upper limit for N_2 is possibly 3%
Jupiter	5.38	-220	≫1	
Saturn	3.26	-270	?	
Uranus	1.97	-340	?	NH_3, CH_4, H_2, He
Neptune	2.24	-360	?	
Pluto	0.85?	-370	?	heavy gases?
Earth's Moon	0.212	-243 to 260	10^{-17}	traces of very heavy gases

*1 AU = 92,959,670 miles

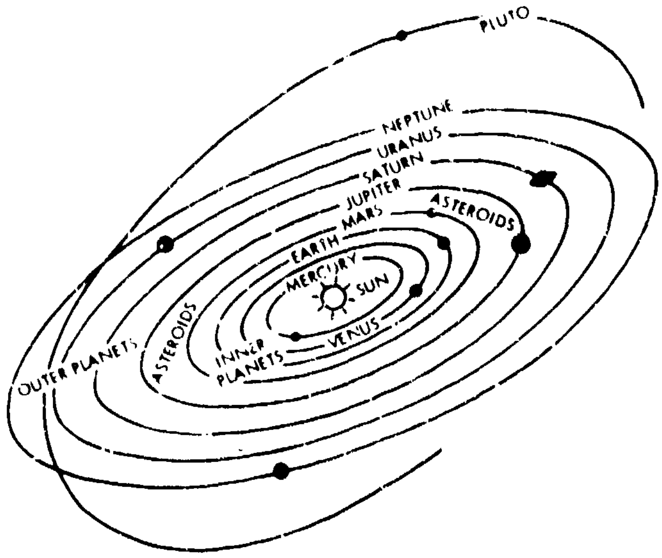

Figure 13.6.7. The Solar System

Table 13.6.8. Space Mission Durations
(Reference 107-6)

SPACE MISSION	NOMINAL DURATION
Earth orbit 300 n mi orbit	90 minutes
Lunar landing, one way	2½ days
Lunar reconnaissance mission, no landing, no lunar orbit	5 days
Lunar landing, earth-return	1 to 2 weeks
Close solar probe, one way instrumented	4½ months
Mars landing, one way	9 months
Mars reconnaissance mission, no landing, no martian orbit	12 months
Venus reconnaissance, planetary orbit and return to earth	1¼ years
Mercury reconnaissance, planetary orbit and return to earth	1½ years
Mars reconnaissance, planetary orbit and return to earth	2½ years
Jupiter reconnaissance planetary orbit and return to earth	3⅔ years
Saturn reconnaissance, planetary orbit and return to earth	4½ years

13.6.8 Time in Space

As the durations of space missions increase, space environmental effects become increasingly more important to fluid component designers since most of the adverse effects of the space environment are a function of time. The probability of meteoroid penetration and the degree of meteoroidal erosion of material surfaces will increase with exposure to time. Radiation dosage is cumulative with time. The effects of the high vacuum on the sublimation and evaporation of materials are also time dependent. The length of time required to accomplish several representative space missions is presented in Table 13.6.8.

Recent successful manned and unmanned near-earth spaceflight (earth orbit, lunar landing, etc.) and unmanned planetary reconnaissance has prompted the tentative planning of longer duration missions. Examples are manned earth-orbiting space stations for 10 year durations and unmanned *Grand Tour* reconnaissance missions to fly by the outer planets. Grand Tour missions to Jupiter, Saturn, Uranus, and Neptune or Pluto have been proposed for the late 1970s and would require mission durations of 6 to 12 years.

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34-6	107-6	166-9
35-11	131-2	174-5
47-23	131-3	286-2
65-3	131-6	422-1
65-5	131-19	423-1
77-9		476-1

ISSUED: FEBRUARY 1970
SUPERSEDES: MAY 1964

13.7 CORROSION

13.7.1 Chemical Process

Corrosion is the deterioration and loss of material due to a chemical reaction between the material and its environment. Corrosion usually involves both a chemical solution and an oxidation-reduction process.

The natural chemical reaction between most metals and their environment may be represented by:

Metal + Environment $\longrightarrow$ Metal Compound + Energy
(Fe, Al, etc.) + (H_2S, O_2, H_2O, NO_3, etc.) $\longrightarrow$
(Sulfide, Oxide, etc.) + (Heat)

The rate of corrosion is governed by a number of factors, some of the most common being:

a) formation of surface films
b) oxygen concentration
c) hydrogen ion activity (pH)
d) presence of other ions
e) temperature
f) polarization
g) electrical resistance of electrolyte
h) static or cyclic stress conditions
i) rate of flow of environment over material
j) presence of dissimilar metals
k) surface configuration.

Reaction times may vary from extremely slow to very fast and may not occur at all unless initiated with a certain "activation" energy.

Chemical solution involves the dissociation of a material, resulting in molecules or ions going into solution with the environment. Figure 13.7.1 illustrates the dissociation of iron into solution as ions and excess electrons are produced in the metal. In general, the factors which influence chemical solution are:

a) The size of the molecule or ion. Small molecules and ions usually dissolve more readily.
b) Structural similarity of solvent and solute. Organic materials are most soluble in organic solvents, and metals are most soluble in other liquid metals.
c) More than one solute. The presence of two solutes may produce greater solubility than the presence of only one.
d) Temperature. The rate of solution increases with temperature.

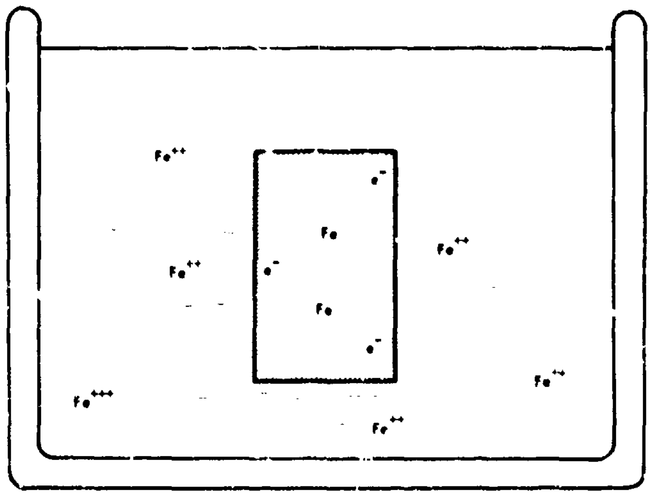

Figure 13.7.1. Dissociation of Iron into Solution

13.7.2 Oxidation and Reduction

Oxidation involves the loss of electrons from an atom, whereas reduction involves the gain of electrons. It should be noted that the presence of oxygen is not necessary for oxidation. Oxidation corrosion of metals is frequently considered as being a reaction involving an anode and a cathode. The anode supplies electrons (oxidation) and the cathode receives electrons (reduction).

Oxidation may occur at any temperature and becomes increasingly important with higher temperatures. When metals are exposed to an oxidizer such as oxygen or fluorine, an oxide (fluoride) layer or scale will form at the surface and the reaction process is retarded. For oxidation to continue, the metal must migrate across the oxide layer to the surface, or the oxidizer must diffuse through to the base metal surface as illustrated in Figure 13.7.2. The relationship between the growth of this oxide layer to time and temperature is important as a basis for determining the resistance of the metal to oxidation.

If the iron shown in Figure 13.7.2 is in a water solution, rust is formed according to the reaction

$$4Fe + 3O_2 + 6H_2O \rightarrow 4Fe(OH)_3 \quad \text{(Ferris hydroxide)}$$

and the half reactions are:

Oxidation $Fe \rightarrow Fe^{++} + 2e$ Anode reaction

Reduction $H_2O + 1/2\, O_2 + 2e \rightarrow 2OH^-$ Cathode reaction

The ease with which electrons are removed and, therefore, the corrosion rate will depend on the environment. Electrons are readily removed from iron when oxygen and water are present, and readily removed from aluminum when chlorine is present.

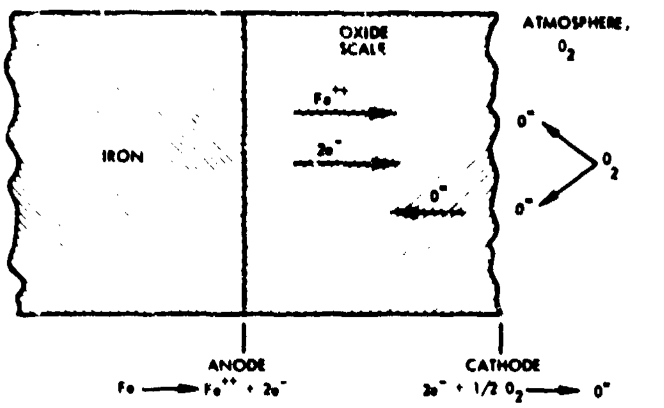

Figure 13.7.2. Oxidation-Reduction Process

13.7.3 Galvanic Corrosion

Galvanic corrosion occurs when two dissimilar metals are coupled in presence of an electrolyte. Galvanic corrosion also occurs when electrodes of the same metal contact with different electrolytes or the same electrolyte but at different strengths. The extent of galvanic corrosion will depend on the type of metal and the electrical resistance of the electrolyte.

13.7.3.1 ELECTRODE POTENTIAL. When metals go into solution as ions, excess electrons which are liberated remain in the metal and the metal acquires a negative charge. Equilibrium is reached when the metal ions and electrons recombine at the same rate at which they form. The potential drop resulting from the production of ions in the solution and electrons in the metal is known as the *electrode potential.*

The tendency of a metal to corrode in a solution is related to the electrode potential between the metal surface and its ions in solution. Because the potential is influenced by temperature, concentration, velocity, etc., standardization is employed in measuring the electrode potential. The value of the electrode potential of a metal and a solution is usually measured with reference to a standard hydrogen electrode which is taken to be zero. Table 13.7.3.1a lists the electromechanical series of metals. The metals higher in the series are termed *anodic* to the metals below them, and the more noble metals are called *cathodic* to the metals above them.

Table 13.7.3.1a. Electrochemical Series

ANODIC (LEAST NOBLE) END
Lithium
Potassium
Sodium
Barium
Magnesium
Beryllium
Aluminum
Manganese
Zinc
Chromium
Iron ($Fe \rightarrow Fe^{++}$)
Cadmium
Nickel
Tin
Lead
Iron ($Fe \rightarrow Fe^{+++}$)
Hydrogen
Copper
Silver
Palladium
Mercury
Platinum
Gold
CATHODIC (MOST NOBLE) END

The relative position in the series between metals should not be used to predict whether one metal will displace another in solution, since the actual values prevailing in a specific solution may cause a change in the relative position. Each environment must be considered separately. Table 13.7.3.1b gives the relative position of metals in sea water, called the *galvanic series.*

13.7.3.2 ELECTROLYTES. Electrolytes are ionic solutions of acids, alkalies, or salts which conduct electrical currents, with electrical conductivity resulting from the free ions available in the electrolyte. Water ionizes slightly, forming hydrogen ions, H^+, and hydroxl ions, OH^-. Acidic solutions have a greater hydrogen ion concentration and alkaline solutions have an increased hydroxyl ion, OH^-, concentration. Salts are the reaction products of acids and alkalies. They are highly ionized and give essentially neutral solutions. All are conductive due to the free motion of the ions in solution.

13.7.3.3 GALVANIC CELLS. A galvanic cell consists of two electrodes, one which supplies electrons (anode) and the other which receives electrons (cathode). If an electrical contact is made between the two electrodes, the greater potential at the anode will force electrons and metallic ions to flow to the cathode. Corrosion always occurs at the anode

Table 13.7.3.1b. Galvanic Series in Sea Water
(Reference V-201)

CORRODED END (ANODE)

Magnesium
Magnesium alloys

Zinc
Galvanized steel
Galvanized iron

Aluminum (5052, 3004, 3004, 3003, 1100, 6053 in this order)

Cadmium
Aluminum (2117, 2017, 2024 in this order)
Mild steel
Wrought iron
Cast iron

Stainless steel, type 410 (active)*

50-50 lead tin solder

Stainless steel, type 304 (active)
Stainless steel, type 316 (active)

Lead
Tin

Muntz metal
Manganese bronze
Naval brass

Nickel (active)
Inconel (active)

Yellow brass
Aluminum bronze
Red brass
Copper
Silicon bronze

Nickel (passive)**
Inconel (passive)

Monel

Stainless steel, type 304 (passive)
Stainless steel, type 316 (passive)

PROTECTED END (CATHODE)

*Active: metal surface without a protective film.
**Passive: metal with a protective film (such as an oxide film).

because it is at a higher electrical potential than the cathode. Galvanic cells may be classified as (1) composition cells, (2) concentration cells, and (3) stress cells. These three types of cells require an anode, a cathode, and an electropotential (electrolyte) between them.

Composition Cells. A composition cell may be established between any two dissimilar metals in the presence of an electrolyte (Figure 13.7.3.3a). Examples of composition cells include (1) platings or coatings on a base metal where the plating is not continuous, (2) threaded fasteners and their associated parts, (3) solder and the parent material, (4) shaft and bearings supports, and (5) connection between dissimilar pipe or tubing. Dissimilar metals are defined for aircraft and aircraft parts in Table 13.7.3.3 per military standard MS 33586.

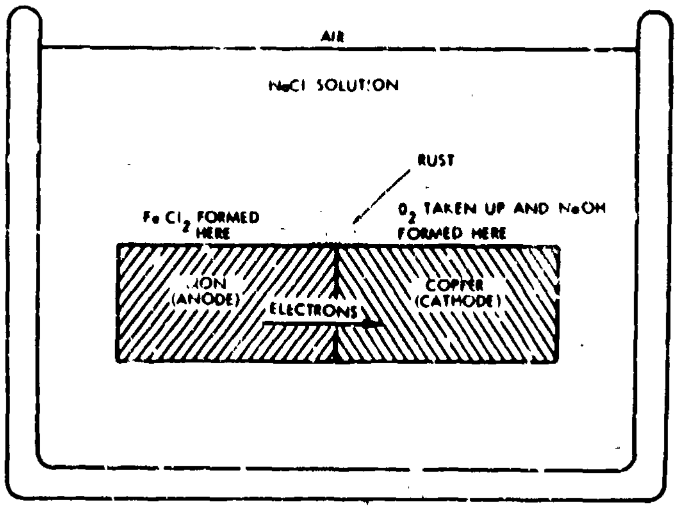

Figure 13.7.3.3a. A Composition Cell

Table 13.7.3.3. Grouping of Similar and Dissimilar Metals and Their Alloys*

GROUP I	GROUP II	GROUP III	GROUP IV
Magnesium and its alloys Aluminum alloys 5052, 5056, 5356, 6061 6063	Cadmium, zinc, aluminum, and their alloys	Iron, lead, tin and their alloys (except stainless steels)	Copper, chromium, nickel, silver, gold, platinum, titanium, cobalt, rhodium and their alloys; stainless steels and graphite

*Metals in the same group are considered similar to one another, and metals in different groups are considered dissimilar to one another.

A composition cell may be created on a sheet of cadmium-plated or galvanized steel where the plating has been scratched exposing the steel base metal. The cadmium or zinc (galvanized) coating acts as the anode, and any corrosion that occurs is on the coated surface, with the steel protected, (Table 13.7.3.1b). A tin coating on steel, however, will provide protection only as long as the coating is continuous. If the tin coating is broken, exposing the steel, (Figure 13.7.3.3b), the steel will become the anode, tin the cathode, and the steel is subject to corrosion (Table

13.7.3.1b). In order that the base alloy be protected, the cladding for aluminum alloys is always selected so that the coating provides a higher electrical potential than the core.

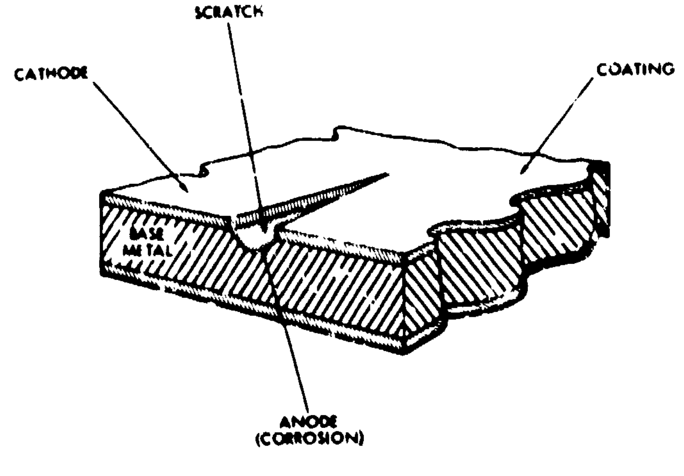

Figure 13.7.3.3b. A Composition Cell Formed by Scratching Tin-Coated Steel

Concentration Cells. When the electrolyte is not homogeneous, the less concentrated areas of the part become the anode, causing current to flow from the metal-to-solution at the point where the concentration is low, and from solution-to-metal when the concentration is high. Concentration cell corrosion usually takes place in hidden and secluded areas such as in crevices and beneath scale and other deposits. Two common types of concentration cells are metal-ion cells and oxygen cells.

In a metal-ion cell, variations in metal-ion concentrations in solution immediately adjacent to the metal surface cause differences in electrical potential, hence galvanic corrosion. The lower the metal-ion concentration, the greater the tendency for the metal to dissolve; in other words, the higher will be its solution potential measured in volts. Metal-ion concentration cells are commonly associated with differences in velocity between two points on a metallic surface where metal ions may be removed continuously at one point while accumulating at another.

Oxygen concentration cells are similar in nature to metal-ion cells, with variations in concentration of dissolved oxygen causing differences in electrical potential. Moist metal surfaces in contact with air provide conditions favorable for oxygen concentration cells.

Inaccessible areas such as cracks, crevices, interfaces between parts in contact (such as washers and fasteners), and those areas covered by dirt may be at a lower oxygen concentration than more accessible areas. This is because the liquid (electrolyte) cannot obtain oxygen as readily in the inaccessible areas. Dirt or other surface contaminations are responsible for localized pitting, because the contamination restricts the access of oxygen. Figure 13.7.3.3c illustrates the effect of corrosion due to concentration cells.

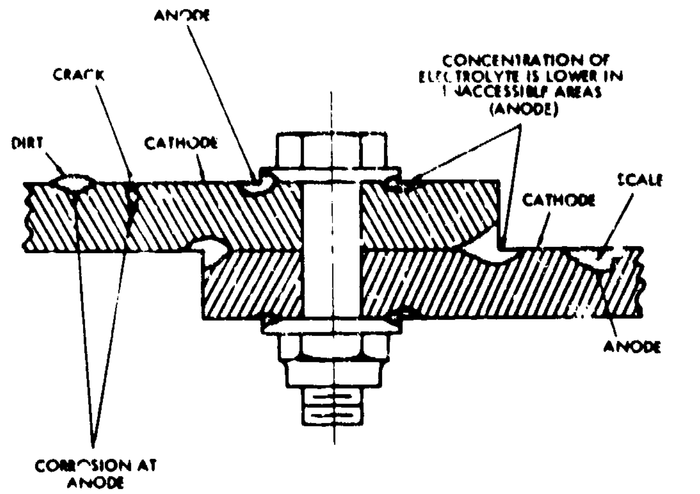

Figure 13.7.3.3c. Concentration Cells Due to Dirt, Cracking, or Scale

Stress Cells. When a metal is stressed such as due to cold working, stress corrosion may occur. A common example of stress corrosion due to cold working is at the bend of sheet metal or wire and at the point and head of a nail that is cold formed. The cold worked area is the anode and the stress-free area is the cathode.

The atoms within the grain boundaries of a metal matrix have a higher energy that the atoms within the grain. In a corrosive environment, the grain boundary acts as the anode and the grain the cathode. Because of the greater boundary area in a fine grained metal, it is expected that this type of grain structure will have a higher corrosive rate than a coarse grained metal.

13.7.4 Polarization

Polarization may be defined as the production of counter-emf, i.e., it opposes the corrosion potential by products formed or by concentration changes resulting from passage of current through an electrolytic cell. Polarization can alter the potential of the anode in the cathodic direction, and the potential of the cathode in the anodic direction, thereby reducing the potential of the electrodes. Anodic polarization can be caused by the accumulation near the anode of metal ions going into solution, retarding the further dissolution of metal or even forming protective oxide films of poor conductivity on the anode. Cathodic polarization is the delay in the absorption of the arriving electrons due to the inadequate speed at which the cations are discharged, or due to the inadequate rate of supply of oxidant to the cathode. A type of polarization which is irreversible is the formation of an adherent layer of salt or oxide on the anode when current first flows. This is a form of passivation.

13.7.5 Electrode Control

If in a solution of low resistivity the cathode is polarized and the anode is not, the current flow will be controlled by the cathode. Under this condition, the amount of corrosion or weight loss of the anode is independent of the anode area, but the intensity of attack will increase with decreasing area. If the cathodic area is reduced, the total galvanic corrosion of the anode is proportionally reduced, or if the area of the cathode is increased the total corrosion of the anode is proportionally increased. Such a system is considered to be under cathodic control.

If the anode polarizes and the cathode does not, the system is under anodic control and the conditions are reversed from those under cathodic control.

If both the anode and the cathode polarize in solutions of low resistivity, the current flow will be controlled by both electrodes and the areas of both electrodes will affect the galvanic corrosion of the anode. If neither electrode polarizes, the current flowing (and thereby the corrosion) will be controlled by the resistance of the electrolyte and metallic path.

Diffusion control is probably of greater importance than any other factor in determining corrosion rates. If a metal is totally immersed in a solution and the area of the metal is sufficiently large, the rate of corrosion is controlled by the rate at which the oxidant diffuses through the liquid-air interface. The total corrosion loss will not be increased, even if the size of the specimen is increased. When connecting another metal to the specimen so that a galvanic cell is made, the rate of oxidant diffusion through the liquid air interface is not increased. Therefore, the corrosion of the specimen will not increase over that occurring on the uncoupled specimen.

13.7.6 Corrosion Fatigue

Stress cycling and the simultaneous effect of corrosion on metals is known as corrosion fatigue. The contributing factors to corrosion fatigue are corrosion pitting and crack propagation. The combined action of corrosion and cyclic stresses may produce pitting and crack formation. Consequently, propagation of the crack due to cycling occurs. The mechanism is identical with that of a failure from fatigue in which the stress concentration effects are the results of corrosion. (See Sub-Section 14 5.)

The rate of corrosion of metal surfaces is usually controlled by the properties of the surface film. Under cyclic stressing, surface films may rupture exposing the base metals to further corrosive action. Secondary corrosion products may be formed which clog the pits and retard diffusion of the oxidant, forming a concentration cell.

Surface coatings may be effective in limiting corrosion fatigue, but if they are to be used to prevent corrosion fatigue, they must adhere to the base metal, withstand deformation which the base metal undergoes without rupture, and should preferably be anodic to the base metal

13.7.7 Intergrannular Corrosion

Intergranular corrosion is a condition of localized corrosion at the grain boundaries of a metal matrix. It may cause complete failure of a part or welded joint, and may occur in some stainless steels and nickel alloys.

If a high alloy 18-8 stainless steel is heated at 950 to 1400 F and held for a short period of time, the carbon may precipitate as chromium carbides. In this case, galvanic cells may be established on a microscopic scale, or the carbon may form chromium carbide which depletes the grain boundaries of chromium and reduces the corrosion protection locally.

Intergranular corrosion may be prevented in stainless steels by the following techniques:

a) Quenching to avoid carbon precipitation

b) Selection of steel with less than 0.03 percent carbon

c) Selection of steel with high chromium content

d) Selection of steel containing strong carbide formers.

The last technique involves the addition of titanium, columbium, and tantalum. The carbon will then precipitate as titanium carbide, columbium carbide, or tantalum carbide at high temperatures and will not deplete the chromium from the steel. This technique is often used for stainless steels in welded structure.

13.7.8 Fretting Corrosion

Fretting corrosion is corrosion at the interface between two contacting surfaces, accelerated by relative vibration between them of sufficient amplitude to produce relative motion. The slipping movements at the interface of the highly loaded metal surfaces destroy the continuity of the protective films and corrosion may advance at a rapid rate.

Fatigue failures traceable to fretting corrosion include aircraft engine parts such as connecting rods, knuckle pins, splined shafts, clamped and bolted flanges, and couplings. In interference press fits, the products of fretting corrosion may accumulate in the corroded region to such extent that it is difficut to disassemble the contacting parts. Splines, ball and roller bearings, and gears have been known to fail because of a loss of materials by fretting corrosion. Fretting corrosion has been observed between such materials as paper and steel, wood and steel, glass and steel, and between many combinations of metals and alloys.

Prevention of fretting may be accomplished by prevention of slipping. The latter may be done by increasing the load at the interface sufficiently to prevent relative motions, or by increasing the friction between the interface by roughing the surfaces. Corrosive attack by fretting is more pronounced in soft steels than in hard steels. Therefore, it is common practice to increase the hardness of one or both contacting steel surfaces. Lubricating the contacting surfaces to prevent air from contacting them will greatly retard fretting corrosion. Fretting corrosion is also retarded when the contacting surfaces have been treated to induce residual compressive stress by such mechanical operations as shot peening, surface rolling, or surface treatment such as nitriding and carbonizing.

13.7.9 Stress Corrosion Cracking

Stress corrosion cracking results from the interaction of a *sustained* tension stress and a corrosive attack, and may result in brittle failure of a ductile material. Stress corrosion cracking has only been observed in those metals where surface tensile stresses were present. Surface tensile stresses may result from externally applied loads and from residual stresses, examples of which are:

Applied Loads: Dead loading such as propellant storage tanks, pressure differentials, and thermal expansion of restrained parts.

Residual Stresses: Deformation of metal during assembly of parts such as rivets, bolts and press fits; phase changes within metal; and unequal cooling of metal sections from high temperatures.

Stress corrosion cracking is always at right angles to the direction of stress. Stress corrosion cracks may cross grains (transgranular) or may follow the grain boundaries (intergranular). In aluminum alloys, the stress corrosion cracks are usually intergranular. Figure 13.7.9 compares the resistance to stress corrosion cracking of rolled bar and rod in some aluminum alloys. Aluminum alloys 2024-T62 or T851 and 7075-T73 are alloys recommended where greater strength and resistance to stress corrosion cracking is desired. Sodium chloride and tropical environments are known to cause cracking in aluminum alloys. Stress corrosion cracking is most commonly encountered in aqueous solutions; however, it may occur in liquid metals, molten salts, and organics. Cracking failures are not limited to metals; for instance, some examples of nonmetallics experiencing cracking failures include polyvinyl chloride, plastics in water and glass in air. Red fuming nitric acid, chlorinated hydrocarbons, and nitrogen tetroxide (N_2O_4) have been known to cause stress corrosion cracking in titanium alloys.

The introduction of a bleaching process to remove the impurity NO by producers of nitrogen tetroxide (N_2O_4) for the purpose of providing a purer product led to serious stress corrosion problems with 6Al-4V titanium N_2O_4. Whether free oxygen, chloride ions (NOCl is another impurity in N_2O_4), or some other specie provides the corrosive environment for titanium has not been determined; however, the addition of NO to N_2O_4 has been found to prevent stress corrosion.

Table 13.7.9 lists environments in which stress corrosion cracking has been observed.

13.7.10 Corrosion by Propellants

Rocket propellants present a wide range of corrosive environments. Corrosion can result from contact of the propellant with the component containing it, and from the combination of spilled and vaporized propellant and atmospheric moisture coming in contact with external surfaces. There are cases where the external environment contaminated by propellant leakage, spills, or vented vapors is more corrosive than exposure to the propellant itself. N_2O_4, for instance, is compatible with most aluminum alloys in the anhydrous form in which it is used as a rocket oxidizer; however, when vapors caused by N_2O_4 leakage combine with atmospheric moisture, highly corrosive nitric acid is formed which can damage exposed surfaces.

For the same reason, care must be taken when selecting materials for components which must be subjected to water flushing after propellant exposure. Although a certain material may be compatible with the propellant, it may not be compatible with the residual formed after the propellant has combined with water.

Other important factors influencing the suitability of materials from a propellant corrosion standpoint are temperature, service time, dissimilar metals, surface oxides, and propellant purity.

Table 13.7.10 shows corrosion resistance of some common metals and alloys to rocket propellants under room temperature conditions. A more detailed tabulation of propellant compatibility data is given in Section 12.0, "Materials."

13.7.11 Corrosion by Lubricants

Metals are not corroded by pure hydrocarbon lubricants but may be corroded by impurities or lubricant additives which develop oil oxidation products. The oil oxidation products may attack cadmium base and copper-lead bearing metals. Copper and copper alloys may be attacked by oxidized oil or by any active sulphur compounds which may be present. Lubricants seldom attack steel and iron but when corrosion does occur, it is usually the result of the use of additives. Lubricants, however, do protect metals against corrosion caused by an environment containing moisture.

13.7.12 Corrosion by Atmosphere

Most metals can be oxidized by atmospheric oxygen. The exceptions include the noble metals such as gold, platinum, and silver. The water content and the amount and type of pollutants contained in the atmosphere have a direct influence on the degree of corrosion. A moisture film must be formed on the metal surfaces for corrosion to start. Pollutants dissolve in the moisture film and raise the electrical conductivity of the electrolyte, allowing higher corrosion currents.

Pollutants can be solids or gases. The solids include dirt, soot, nitrogen oxides, and salts such as sodium chloride, ammonium sulfate, and ammonium chloride. All the solid pollutants may be hygroscopic, i.e., collecting and holding moisture, which allow prolonged corrosive action. The gaseous pollutants include sulfur dioxide, hydrogen sulfide, carbon dioxide, hydrogen chloride, ammonia, and ozone. The most damaging gaseous element is sulfur dioxide.

Dry rural and unpolluted atmospheres are less corrosive than industrial and marine atmospheres. Marine atmos-

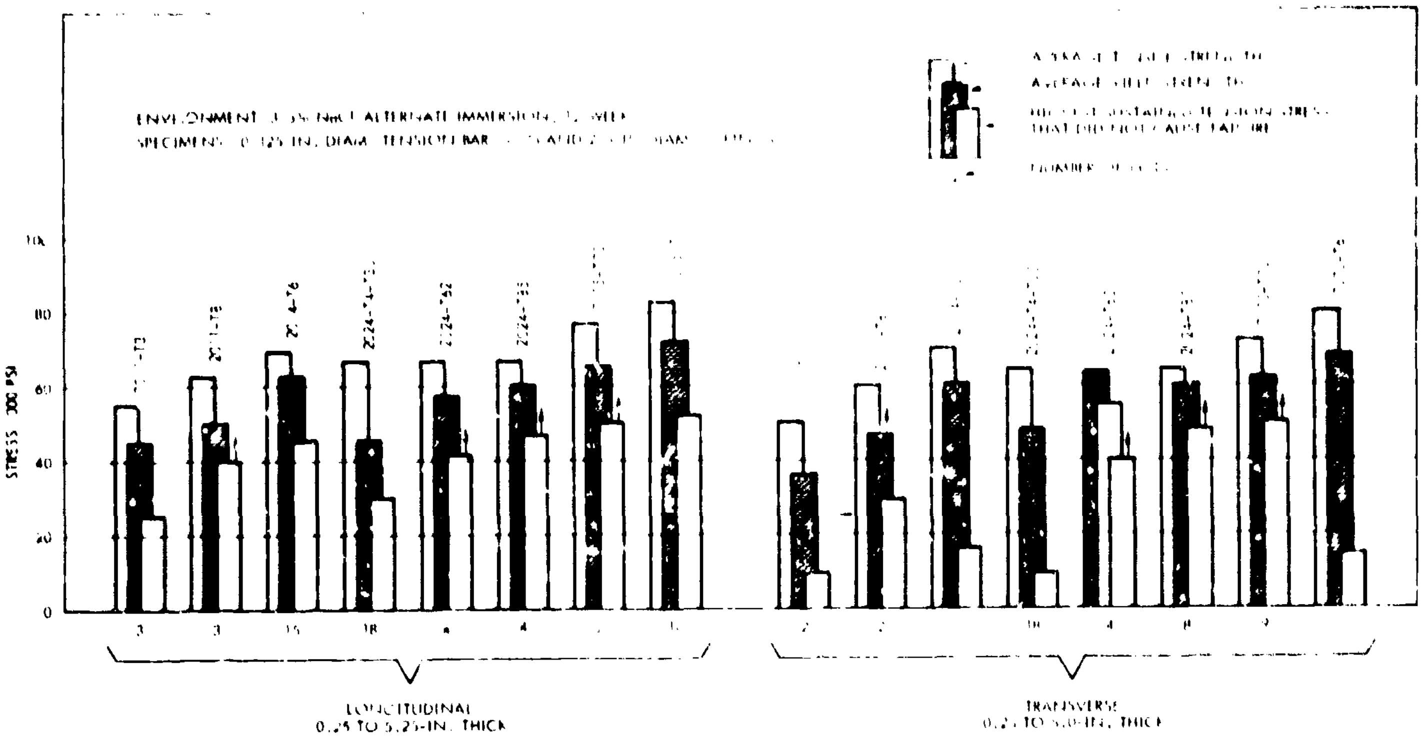

Figure 13.7.9. Resistance of Aluminum Alloys to Stress Corrosion Cracking
(From "Metal Progress," D. O. Sprowls and R. H. Brown, May [illegible]. Copyright [illegible] by the American Society for Metals, Metals Park, Ohio.)

pheres contain salts, such as ammonium sulfate and ammonium chloride, which are corrosive because of the acids produced by hydrolysis. Industrial areas often contain alkalies, but while most metals are resistant to their corrosive action, zinc and aluminum are particularly vulnerable to attack by alkali, even in relatively weak concentrations.

The corrosion products formed on iron and zinc are hygroscopic and aggravate corrosion. Aluminum corrosion products efficiently protect the metal and the rate of corrosion action greatly decreases with increasing time of exposure. Table 13.7.12 shows the relative corrosivity of open hearth iron exposed to atmospheres at different locations around the world.

13.7.13 Corrosion by Sea Water

Sea water is a good electrolytic conductor, favoring local cell action and having a high sodium ion concentration which promotes the development of alkalinity at cathodic areas. The high total ion concentration in sea water leads to the establishment of ion concentration gradients and a high content of chloride ions, which then leads to the breakdown of a passivated coating.

Table 13.7.10. Corrosion Resistance of Several Metals and Alloys to Rocket Propellants at Room Temperature

S — Satisfactory
U — Unsatisfactory

METALS	FUELS:			OXIDIZERS:			
	N_2H_4	KEROSENE	UDMH	N_2O_4 (DRY)	N_2O_4 (WET)	ClF_3	RED FUMING NITRIC ACID
Aluminum	S	S	S	S	U	S	S
Stainless Steel	S	S	S	S	S	S	S
Magnesuim	U	S	U	S	U	S	U
Mild Steel	U	S	S	S	U	S	U
Nickel	S	S	S	S	U	S	U
Copper	U	S	S	S	U	S	U

Table 13.7.12. Relative Corrosivity of Atmospheres at Different Locations
(As determined using open hearth iron specimens, 2 × 4 × 1/8 inch)
(Reprinted with permission from "Symposium on Corrosion Fundamentals," Brasunas and Stansbury, eds. Copyright 1956 by University of Tennessee Press, Knoxville, Tennessee)

LOCATIONS	TYPE OF ATMOSPHERE	AVERAGE WEIGHT LOSS, GRAMS (IN ONE YEAR)	RELATIVE CORROSIVITY
Khartoum, Sudan	Dry inland	0.16	1
Abisco, North Sweden	Unpolluted	0.46	3
Singapore, Malaya	Tropical marine	1.36	9
Daytona Beach, Florida	Rural	1.62	11
State College, Pennsylvania	Rural	3.75	25
South Bend, Pennsylvania	Semi-Rural	4.27	29
Miraflores, Canal Zone (Pacific Side)	Tropical marine	4.5	31
Kure Beach, North Carolina (800 feet from ocean)	Marine	5.78	38
Sandy Hook, New Jersey	Marine, semi-industrial	7.34	50
Kearny, New Jersey	Industrial marine	7.75	52
Vandergrift, Pennsylvania	Industrial	8.34	56
Pittsburgh, Pennsylvania	Industrial	9.65	65
Frodingham, British Isles	Industrial	14.81	100
Daytona Beach, Florida	Marine	20.43	138
Kure Beach, North Carolina (80 feet from ocean)	Marine	70.49	475

Galvanic action in sea water is reduced by the precipitated deposits and by the growth of marine organisms. Calcium, magnesium, and strontium which are present in sea water tend to precipitate as carbonates on cathodic surfaces, and marine growths tend to be distributed over anodic and cathodic surfaces.

13.7.14 Corrosion by Micro Organisms

Micro organisms may contribute to corrosion by directly influencing the rate of galvanic reaction, changing the surface film of a metal by their metabolism, or creating a corrosive environment or an electrolytic concentration cell on the surface of a metal. Micro organisms must have available certain inorganic and organic chemical elements such as oxygen, carbon, nitrogen, hydrogen, or sulfur which are necessary to their metabolic processes. A number of organisms, however, are able to develop in environments devoid of coupled organic nutrients. Aerobic micro organisms readily grow in an environment containing oxygen, whereas the anaerobic micro organisms develop in environments in which the dissolved oxygen concentration approaches zero. Anaerobic micro organisms are of the sulfate-reducing type in that hydrogen sulfate results as a byproduct of bacteria reduction. Sulfides may be formed in contact with metals, and hydrogen may be obtained from cathodic surfaces

Other varieties of organisms such as fungi, algae, protozoa, diatoms, and bryozoa may contribute to corrosion by establishing a microbiological film capable of maintaining concentration gradients of the dissolved salts and gases of the electrolyte in contact with the metal. Coatings of paint, asphalt, concrete, and also cathodic protection have been successfully used in combating corrosion resulting from these organisms.

13.7.15 Corrosion Prevention

Methods for preventing corrosion include protective coatings, use of inhibitors, cathodic protection, and proper design techniques.

13.7.15.1 **PROTECTIVE COATINGS.** Protective coatings include metal, chemical conversion, organic, and ceramic coatings. The coating may act as a mechanical insulator from the action of the environment, or it may include an inhibitor or passivator increasing the protective effect of the coating. Metallic coatings may be classified as either anodic or cathodic depending on their electrode potential relative to that of the metal being protected.

Cathodic coatings can only protect the base metals when they are free from cracks, scratches and pores; otherwise the base metals, being anodic, will experience rapid corrosion of the exposed areas (Figure 13.7.3.3b). Zinc and cadmium are frequently used as coatings on steels and are less noble than, or anodic, to the steel. These types of coatings

ISSUED: FEBRUARY 1970
SUPERSEDES: MAY 1964

may be considered to be sacrificial coatings, since the coatings will corrode rather than the steel. Most other coatings are more noble than steel and do not provide sacrificial protection. To protect the base metal, these coatings must provide a continuous non-porous area. The more noble coatings include those metals cathodic to steel such as nickel, chrome, the precious metals, tin, and lead.

Organic coatings can provide high corrosion resistance. However, they cannot be used when the service conditions involve high temperatures, abrasive wear, or periodic resurfacing. Other organic coatings include paints, ceramics and glass, porcelain enamels, and rust preventatives. Zinc, manganese, and iron phosphate coatings are commonly used to retard corrosion and serve as a good base for organic coatings.

Conversion coatings are widely used for aluminum and magnesium. They are achieved by chemically converting the metal surface into a metal compound (oxide, phosphate, or chromate). The coating is then integral with the surface and will not chip or peel. The appearance and durability of the coating depends upon the alloy and surface preparation. Conversion coatings can be classed either as anodic, which are aluminum oxide films formed by an electrochemical process; or non-electrolytic chemical conversion types, which are oxide, phosphate, or chromate films formed by chemical action only. The anodic coatings require dipping in an electrolytic bath, while the chemical types can be achieved by either immersion, spraying, or brushing.

The electrolyte for anodic coatings is usually within sulphuric acid (the most common) or chromic acid. Chromic acid coatings are usually less than 0.5 mils thick, while sulphuric acid coatings of over 10 mils are possible to achieve. Hard, thick anodic coatings achievable with sulfuric electrolyte are specified when the highest resistance to abrasion, erosion, and corrosion is needed. These coatings are produced commercially by the Martin Hard Coating, Alumilite Hard Coating, Sanford, or Hardas processes.

Common chemical conversion coatings are oxide, phosphate, and chromate coatings. They are cheaper and easier to apply than anodic coatings, but are considerably softer and thinner, ranging from 0.01 to 4 millionths of an inch.

Aluminum conversion coatings are relatively stable in the atmosphere and in weak acidic solutions in the pH range from 4.5 to 7.0. In strong acidic and alkaline solutions, conversion coatings on aluminum are attacked. These coatings give a good base for paint coatings, the anodic coating giving increased adhesion and life.

13.7.15.2 INHIBITORS. An inhibitor is any substance which decreases the corrosion rate of a metal when added in the proper amounts to the environment of the metal. Additives which tend to increase polarization at the anode or cathode areas are known, respectively, as anodic or cathodic inhibitors. Corrosion inhibitors retarding the anodic reaction cause anodic polarization.

With increasing anodic polarization, the overall corrosion of a metal diminishes. If the corrosion is controlled by the cathodic reactions, the corrosion current and therefore, the amount of corrosion, is not affected by decreasing the anodic areas by polarization. In this case the same amount of corrosion must be distributed over a smaller anodic area resulting in an intensified localized attack or pitting. Anodic inhibitors must be employed in a sufficient amount to assure complete inhibition and avoid pitting. If anodic inhibitors are added in insufficient quantities, they will often diminish the area affected more rapidly than they diminish the total corrosion and thus actually increase the intensity of attack. It is for this reason that anodic inhibitors have been called dangerous inhibitors. Use of anodic inhibitors should be avoided in places containing corners or crevices where replacement is difficult, at points where dirt, sludge, or contamination are likely to collect, or where fittings, valves, and connectors are used.

Soluble hydroxides, chromates, phosphates, silicates, and carbonates used to decrease the corrosion rate of metals and alloys in an aqueous media are examples of anodic inhibitors.

Cathodic inhibitors do not prevent corrosion as well as anodic inhibitors, for they interfere with the reduction of hydrogen ions and the reduction of oxygen to hydroxyl ions. The cathodic reaction causes cathodic polarization. The cathodic areas are not attacked during corrosion and, consequently, cathodic inhibitors do not lead to intensified or localized attack. If corrosion is controlled by cathodic reaction, a decrease in the cathodic area due to partial polarization will result in an overall decrease in corrosion. If, on the other hand, the corrosion is controlled by anodic reactions, the decrease of the cathodic area will increase the cathode current density, but will have no effect on the overall corrosion. Cathodic inhibitors are safer than anodic inhibitors, being less likely to intensify attack if added in insufficient amounts.

Passivators are special kinds of inhibitors which reduce the electrochemical potential of a metal in a more stable direction. Passivity has been attributed to the formation of a thin protective surface film. Anodic inhibitors are more likely to act as passivators, whereas cathodic inhibitors seldom do.

13.7.15.3 CATHODIC PROTECTION. Cathodic protection is the reduction or prevention of corrosion by the use of sacrificial anodes or impressed current. Examples of sacrificial anodes include aluminum or magnesium used on underground pipelines, zinc plates on ship hulls, and magnesium bars in hot water tanks. Normally, sacrificial anodes are designed so that they can be easily replaced. Platings such as zinc or cadmium coatings on steel are common examples of the use of sacrificial anodes that corrode in preference to the base metal

By impressing a voltage upon a metal which otherwise would be the anode, electrons are supplied so that the metal becomes the cathode and corrosion is prevented. Figure 13.7.15.3 illustrates the application of an impressed voltage.

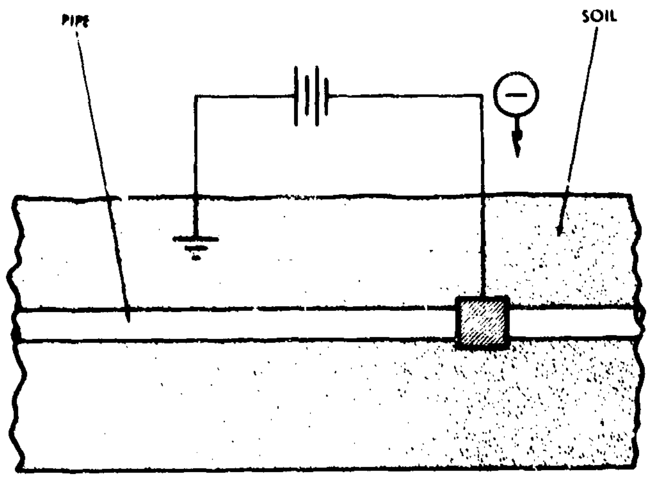

Figure 13.7.15.3. Use of an Impressed Voltage for Cathodic Protection of an Underground Pipe

13.7.15.4 DESIGN TECHNIQUES. Four design techniques that may be used to prevent or alleviate corrosion are:

1) *Avoid Galvanic Couples.* Insulate dissimilar metals to discouple metals electrically or to increase the electrolytic resistance. Use more noble metals, or metals close in the galvanic series (Table 13.7.3.3). Use only one metal in the design.

2) *Avoid Small Anodic Areas Compared to Cathode.* Utilize alloys which are cathodic to the main structure, especially fasteners such as rivets and screws. Avoid use of anodic-cathodic couples which do not polarize.

3) *Avoid Concentration Cells.* Eliminate crevices by use of welds. Use fluorocarbon gaskets between surfaces in contact.

4) *Avoid Corrosive Environments.* Provide drip skirt or design part so that solutions are not collected because of geometry (capillarity, etc.).

13.7.16 Corrosion Measurements

Major methods for measuring the amount and intensity of corrosion include visual observation, loss or gain in weight, loss in dimension, change in electrical resistance, and change in physical properties. No one method has been found acceptable for determining the amount and influence of corrosion on specimens. The more common terms in use for measuring corrosion rates are:

mdd: milligrams loss per square decimeter of exposed area per day

ipy: inches per year

mpy: mils per year

ipm: inches per month.

13.7.17 Corrosion Testing

Corrosion tests are useful in studying the mechanism of corrosion and in determining the environments to which a metal can be exposed. The salt spray test is a performance test for metals with or without protective coatings and serves as a measure of quality. For most aerospace components, the salt spray test is mandatory, and is made in accordance with MIL-E-5272 specification. Other tests include total immersion tests where the metal is immersed in a solution; alternate immersion tests where the test specimen is cyclically immersed in the corroding solution; high temperature tests; galvanic coupled tests; high humidity and condensation tests; soil corrosion tests; atmospheric exposure tests, and sea water corrosion tests.

An accelerated corrosion test, one that could be used to select the most suitable material in a corrosive environment, has been sought after by many investigators. Attempts to develop such a test have failed primarily because certain corrosive conditions have been intensified in order to cause severe corrosion in a short period of time, thereby changing the nature of environment. There is also a difference in the behavior of metals in various environments, which an accelerated test would find difficult to interpret in terms of the actual service of materials.

REFERENCES

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48-6	357-1	V-201
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TABLES

14.1 INTRODUCTION

14.1 INTRODUCTION

This section of the handbook contains fundamental equations, data, criteria, and procedures applicable to the structural design of fluid components. It is presumed that the reader has a working knowledge of strength of materials and of mechanical engineering design; therefore, emphasis has been placed upon presenting fundamental design information normally obtained from a variety of sources. The application of stress and deflection analysis techniques to aerospace fluid components is treated from two points of view:

a) What does the designer need in order to properly design or select a component which will structurally meet functional requirements?

b) How can the designer determine when a comprehensive analysis by a stress analyst should be performed?

To complement the material contained in this section, Appendix A.4, Section Properties and Moments of Inertia, will be added to the handbook.

14.1.1 Symbols and Units

The symbols and units used in this section are defined on the foldout sheet at the end of this section. The basic symbols common to the aerospace industry and used in such documents as MIL-HDBK-5A (Reference 547-12) are used throughout this section. A number of different symbols, especially for denoting stress, are presently found in engineering practice. Some of these designations are shown in Table 14.1.1

Table 14.1.1. Alternate Notation for Normal and Shear Stress and Strain

	Stress					Strain	
	A*	B**	C	D	E	A*	B**
Normal	f_x	σ_x	σ_{xx}	X_x	S_{xx}	e_x	ϵ_x
	f_y	σ_y	σ_{yy}	Y_y	S_{yy}	e_y	ϵ_y
	f_z	σ_z	σ_{zz}	Z_z	S_{zz}	e_z	ϵ_z
Shear	f_{sxy}	τ_{xy}	σ_{xy}	X_y	S_{xy}	e_{xy}	γ_{xy}
	f_{syz}	τ_{yz}	σ_{yz}	Y_z	S_{yz}	e_{yz}	γ_{yz}
	f_{szx}	τ_{zx}	σ_{zx}	Z_x	S_{zx}	e_{zx}	γ_{zx}

*Most commonly used in aerospace work and used in this section of the handbook

**Most commonly used in general engineering

14.2 APPLICATION TO COMPONENT DESIGN

14.2 APPLICATION TO COMPONENT DESIGN

A distinguishing characteristic of the successful component designer is his appreciation of precisely how detailed an analysis should be performed of stresses and deflections in a component. This sub-section endeavors to complement this appreciation by presenting some basic criteria relating to the application of stress analysis to the design of aerospace fluid components.

14.2.1 Basic Design Criteria

14.2.1.1 LOAD DETERMINATION. The analysis of stresses and deflections of any structure can be no more accurate than the estimate of the condition of loading. Evaluation of all the load considerations listed below can help to preclude failures:

Pressure Loads. Are both yield strength at proof pressure and ultimate strength at burst pressure evaluated? Are peak pressures from surges and transients such as *water hammer* considered?

Vibration Loads. Are vibration loads under the worst anticipated conditions added to any static and dynamic loads which may occur simultaneously? Do vibration loads present a fatigue problem (especially for otherwise steady-load applications)?

Acceleration Loads. Is the influence of acceleration upon mating hardware as well as upon the component element itself considered?

Thermal Expansion Loads. Is the entire thermal transient of the system superimposed over other system loads? (In many rocket engine applications the most severe thermal conditions for fluid components occur during post-fire heat soak.) Is thermal expansion (contraction) of mating components treated?

Thermal Stresses. Are thermal stresses resulting from steep thermal gradients also evaluated?

Eccentric Loads. Has the influence of eccentric load application or the misalignment of component elements been considered?

Progressive Deflections. Preliminary design analysis starts with the unstrained configuration; has the possibility of load amplification resulting from the deformed (loaded) condition been considered?

Internal Loads. Are internal forces from springs, bolts, pressurized bellows, and diaphragms evaluated?

Residual Stresses. Are all residual stresses considered, such as those from shrink or press fits, case hardening, grinding, lapping, shot peening, surface rolling, and autofrettage?

Combined Loads. Are *all* simultaneously occurring loads considered as combined rather than individual loads?

Transient Loads. Have all transient load conditions been evaluated to ensure that stress or deflection at maximum-load conditions is known? (Note that in some applications with large temperature changes it may be necessary to determine margins of safety at high temperature, low strength conditions even though loads are not of maximum value at the high temperature.)

14.2.1.2 FAILURE CRITERIA. A satisfactory fluid component design should evaluate the four potential failure modes:

a) Rupture failure (exceeding ultimate strength)

b) Elastic failure (exceeding yield strength)

c) Instability failure (buckling)

d) Stiffness (rigidity/flexibility) failure (functional rather than structural failure).

The calculation of pressure vessel burst pressure using material ultimate tensile strength is an example of the use of the rupture failure criterion, whereas the calculation of proof pressure using yield strength is an example of the use of the elastic failure criterion. This distinction applies to the usual *ductile* materials of construction; *brittle* materials show essentially no differentiation between the yield and rupture points on a stress-strain curve. Table 14.2.1.2 lists factors influencing the type of failure; in general, brittle materials always exhibit brittle failure (i.e. shear or diagonal tension), whereas ductile materials can fail by brittle fracture under certain conditions. Stiffness failure refers to the deformation of the component structure in response to the applied load with the result that the component fails to perform its required function even though no rupture or plastic deformation occurs. The most common examples of such stiffness failure are elastic deflections under loads which result in seal leakage, excessive power requirements, or poor response characteristics.

Rupture (Ultimate) and Elastic (Yield) Failure Criteria. The stress, f, at a point is completely defined by nine components of stress or by three principal stresses as follows:

Table 14.2.1.2. Factors Influencing Type of Failure for Metals

Conducive to Ductile Failure	Conducive to Brittle Failure
High shear stresses	High tensile stresses
Low tensile stresses	Low shear stresses
High temperature	Low temperature
Slow rate of loading	High rate of loading
	High stress concentrations (notches, holes, etc.)
	Repeated loads (fatigue)*

*Although the formation of fatigue cracks is actually the result of plastic action, the final failure will generally be of a brittle nature (see Sub-Section 14.5).

$$f = \begin{bmatrix} f_x & f_{sxy} & f_{sxz} \\ f_{syx} & f_y & f_{syz} \\ f_{szx} & f_{szy} & f_z \end{bmatrix} = \begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{bmatrix}$$

where

f = internal stress (usually expressed in psi)

and where the subscripts x, y, z identify normal stress components; sxy, sxz, syx, syz, szx, szy identify shear stress components; and 1, 2, 3, identify principal stresses.

A failure criterion can be simply a mathematical relation among these stress components which states that failure may be expected when a certain combination of the stress components attains a critical value. Such a simple mathematical formula based on the stress state alone is called a phenomenological theory. For uniaxial tension or compression, or for pure shear, the failure criterion is simply the limiting value of the corresponding material strength (ultimate for rupture and yield for elastic failure). For combined stresses, however, the most common phenomenological theories used today are (Reference 152-7):

a) *Maximum Tensile Stress Theory* (also called Rankine Theory). Application: brittle materials. Principal stress mathematical form:

$$f_1 \geq F_t \qquad \text{(Eq 14.2.1.2a)}$$

where

f_1 = principal stress in the f_1 direction, psi

F_t = failure stress or allowable stress in simple tension, either ultimate tensile strength, F_{tu}, or yield tensile strength, F_{ty}, psi.

b) *Maximum Tensile Strain Theory* (also called St. Venant's Theory). Application: brittle materials. Principal stress mathematical form:

$$f_1 - \mu(f_2 + f_3) \ge F_t \qquad \text{(Eq 14.2.1.2b)}$$

where

μ = Poisson's ratio

f_2 = principal stress in the f_2 direction

f_3 = principal stress in the f_3 direction

c) *Maximum Shear Stress Theory* (also called Coulomb Theory or Tresca Criterion). Application: ductile materials. Principle stress mathematical form:

$$f_1 - f_3 \ge F_t \qquad \text{(Eq 14.2.1.2c)}$$

d) *Maximum Strain Energy Theory.* Application: plastically flowing solids. Principal stress mathematical form:

(Eq 14.2.1.2d)

$$\left[f_1^2 + f_2^2 + f_3^2 - 2\mu(f_1 f_2 + f_2 f_3 + f_3 f_1)\right]^{1/2} \ge F_t$$

e) *Distortion Energy Theory* (also called Shear Energy Theory and von Mises-Hencky Theory). Application: ductile materials. Principal stress mathematical form:

(Eq 14.2.1.2e)

$$\left[f_1^2 + f_2^2 + f_3^2 - (f_1 f_2 + f_2 f_3 + f_3 f_1)\right]^{1/2} \ge F_{ty}$$

Note: The distortion energy theory is primarily applicable to yield strength failure. The distortion energy theory may be used for ultimate failure with brittle materials, provided one of the physical bases other than distortion energy is used, as, for example, the limiting value of octahedral shear stress (Reference 661-1).

f) *Internal Friction Theory.* Application: brittle materials. Principal stress mathematical form:

$$f_1 - f_3\left(\frac{1 - \sin\alpha}{1 + \sin\alpha}\right) \ge F_t \qquad \text{(Eq 14.2.1.2f)}$$

where

$\alpha = \tan^{-1} f$ (f = coefficient of friction; this theory is postulated on the assumption of a frictional resistance to sliding on a plane of shear.)

g) *Mohr Theory.* Application: brittle materials (allows adjustable ratio of ultimate compressive strength, F_{cu}, to ultimate tensile strength, F_{tu}). Mathematical form:

$$F_{su} = \frac{F_{tu} F_{cu}}{F_{tu} - F_{cu}} \qquad \text{(Eq 14.2.1.2g)}$$

where

F_{su} = ultimate stress in pure shear, psi

F_{cu} = ultimate compressive stress, psi

By the Mohr theory, failure is defined by the envelope to the Mohr's circles representing failure for different states of stress. The internal friction theory is a special case of Mohr's theory.

h) *Octahedral Shear Theory.* Application: brittle materials, biaxial stress, ultimate strength. Mathematical form:

$$f_1^2 - f_1 f_2 + f_2^2 \ge F_{tu} \qquad \text{(Eq 14.2.1.2h)}$$

A comparison of the shear strengths computed by the first six failure theories is given by Marin (Reference 661-1) for the typical case of $F_{ty} = 0.8\,F_{cy}$ and $\mu = 0.30$. For shear stress, $f_1 = -f_2 = f_{sy}$, and the following shear strength values corresponding to the various theories are obtained:

a) Maximum tensile stress theory: $f_{sy} = F_{ty}$

b) Maximum tensile strain theory: $f_{sy} = 0.80\,F_{ty}$

c) Maximum shear stress theory: $f_{sy} = 0.77\,F_{ty}$

d) Maximum strain energy theory: $f_{sy} = 0.62\,F_{ty}$

e) Distortion energy theory: $f_{sy} = 0.577\,F_{ty}$

f) Internal friction theory: $f_{sy} = 0.55\,F_{ty}$

The choice of theory for a particular design application depends largely upon the nature of the combined stress. An example of the biaxial stress case is shown in Figure 14.2.1.2a, from which it may be seen that the three theories coincide fairly closely in the first and third stress quadrants. In the second and fourth quadrants only the maximum shear stress and distortion energy theories should be used. There is considerable theoretical and experimental evidence that the distortion energy theory, while not the

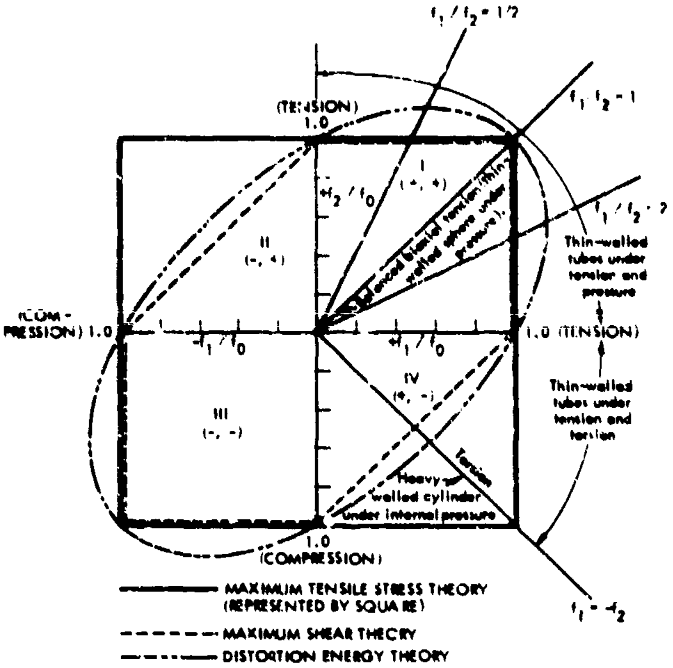


Figure 14.2.1.2a. Stress Quadrants in Two-Dimensional Cases
(Adapted with permission from Reference 596-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

most conservative, is the most fundamental of the strength theories (Reference 598-1). The following example demonstrates the use of the various failure theories.

Example. A closed cylinder 2.0 inches in diameter with a wall thickness of 0.020 inch is subjected to an internal pressure of 500 psi. Neglecting Poisson's ratio, what are the stress levels using the maximum tensile stress, maximum shear stress and distortion energy theories?

$$\text{(hoop)}\ f_h = f_1 = \frac{pr}{t}$$

$$\text{(longitudinal)}\ f_\ell = f_2 = \frac{pr}{2t}$$

$$\text{(radial)}\ f_r = f_3 = 0$$

Because the principal stresses are both tensile (first quadrant of Figure 14.2.1 2a), the maximum tensile stress and maximum shear stress theories give the same results:

$$F_y = f_1 = \frac{pr}{t} = \frac{500 \times 1.0}{0.020} = 25{,}000 \text{ psi}$$

The distortion energy theory solution is expressed by

$$F_y = \left[f_1^2 + f_2^2 - f_1 f_2\right]^{1/2}$$

$$= \left[\left(\frac{pr}{t}\right)^2 + \left(\frac{pr}{2t}\right)^2 - \left(\frac{pr}{t}\right)\left(\frac{pr}{2t}\right)\right]^{1/2}$$

$$= \frac{\sqrt{3}\,pr}{2t} = \frac{\sqrt{3} \times 500 \times 1.0}{2 \times 0.020} = 21{,}600 \text{ psi}$$

In this particular example the distortion energy theory results in a stress level of only 86 percent of that obtained with the maximum tensile stress and maximum shear stress theories.

Instability Failure Criteria. (Note: This treatment of instability failure criteria has been largely adapted from MIL-HDBK-5A, Reference 547-12.) Practically all structural members, particularly those made from thin material, are subject to failure through instability. In general, instability can be classed as either primary or local. For example, the failure of a tube under compression may occur either through lateral deflection of the tube as a column (primary instability) or by collapse of the tube wall at a stress lower than that required to produce a general column failure. It is obviously necessary to consider both types (primary and local) of instability failure, unless it is apparent that the critical load for one type is definitely less than that for the other type.

Instability failures may occur in either the elastic range (below the proportional limit) or in the plastic range (above the proportional limit). To distinguish between these two types of action, it is not uncommon to refer to them as *elastic instability failures* and *plastic instability failures*. It is important to note that instability failures are not usually associated with the ultimate stresses of the material. This point also has a bearing on the choice of material for a given type of construction, as the strength-weight ratio will be determined from different physical characteristics when this type of failure can be expected. For materials which have a very small spread between the proportional limit and the yield stress, the plastic instability failure occurs in such a narrow range that it is not very important, but in materials which have a considerable spread between these two properties, the plastic instability failure may be as important as the elastic instability failure.

In studying any structural member it is important to avoid confusing the different types of failure, particularly where instability is expected to be important. In general, most members should be first investigated from the standpoint of exceeding allowable stress (failures of material). They should then be checked separately for their resistance to primary instability failure. Members which are suspected of being weak in resisting local instability should also be checked for this third possible type of failure. Whichever type of failure gives the lowest strength should be used as the criterion in design.

Primary Instability Failure. A column may fail through primary instability by bending laterally or by twisting about some axis parallel to its own axis. The latter type of primary failure is particularly common to columns having unsymmetrical open sections. The twisting failure of a closed-section column is precluded by its inherently high torsional rigidity. Most fluid component members are of a closed-section form and therefore are more susceptible to lateral bending instability than to twisting column failure.

The Euler formula for long columns which fail by lateral bending is given by

$$F_c = \frac{c\pi^2 E}{\left(\frac{L}{\rho}\right)^2} \qquad \text{(Eq 14.2.1.2i)}$$

or

$$F_c = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2}$$

where

F_c = allowable compressive stress, psi

c = fixity coefficient, dimensionless

E = modulus of elasticity, psi

L = column length, in.

ρ = radius of gyration, in. ($\rho = \sqrt{I/A}$)

L' = $L/\sqrt{c}$, in.

In this equation the term L'/ρ is the *effective slenderness ratio*. Figure 14.2.1.2b shows Euler curves of allowable compressive stress versus effective slenderness ratio. If the applied compressive stress, f_c, exceeds the allowable compressive stress, F_c, determined from Figure 14.2.1.2b, the column may be expected to fail by elastic instability. In Equation (14.2.1.2i), the value to be used for the restraint coefficient, c, depends on the degree of end fixation. The true significance of the restraint coefficient is best under-

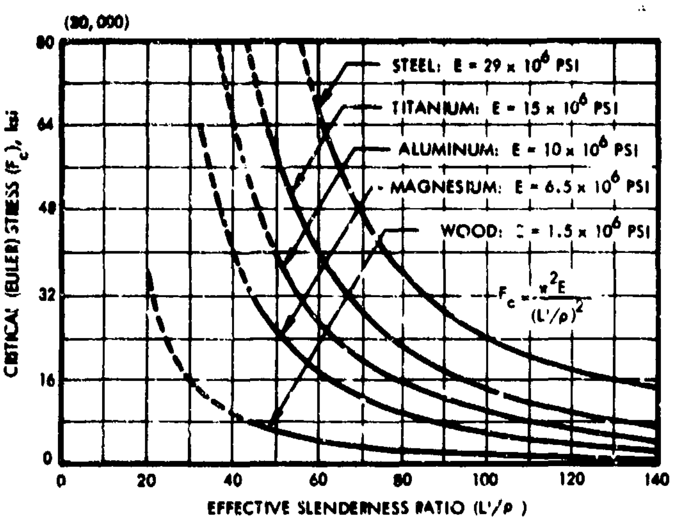


Figure 14.2.1.2b. Euler Column Curves
(Adapted with permission from Reference 720-1, "Strength of Materials", F. R. Shanley, McGraw-Hill, 1957)

stood by considering the end restraint as modifying the effective column length, as indicated in Equation (14.2.1.2i). For a pin-ended column having zero end restraint, c = 1.0 and L′ = L. A fixity coefficient of 2 corresponds to a reduction of the effective length to $1/\sqrt{2}$ or 0.707 times the total length. Typical values of c are:

a) Pin ends, free but guided: c = 1

b) Both ends fixed: c = 4

c) One end fixed, other free but guided: c = 2

d) One end fixed, other free: c = 0.25

If the length of a column is reduced below a certain critical value, failure in lateral bending will occur at loads below those predicted by the Euler formula. This is due in great part to a reduction in the effective value of E, caused primarily by changes in the slope of the stress-strain diagram when the stress is above the proportional limit and, secondarily, by unavoidable eccentricities. In this region, the test results show more scatter than in the Euler range, and empirical formulae for predicting the allowable column stress are often adopted. When a definite eccentricity exists, the critical column loads are reduced due to the combined effects of axial load and bending. Special formulae for such cases can be found in sources such as References 1-343, 461-2, 533-1, 598-1, 720-1, and 734-1. Short-column failure can also be expressed by the modified Euler formula in which the elastic modulus is replaced by an effective modulus E′, as in the following equation:

$$F_c = \frac{\pi^2 E'}{\left(\frac{L'}{\rho}\right)^2} \qquad \text{(Eq 14.2.1.2j)}$$

where

F_c = allowable compressive stress, psi

E' = effective modulus of elasticity, psi

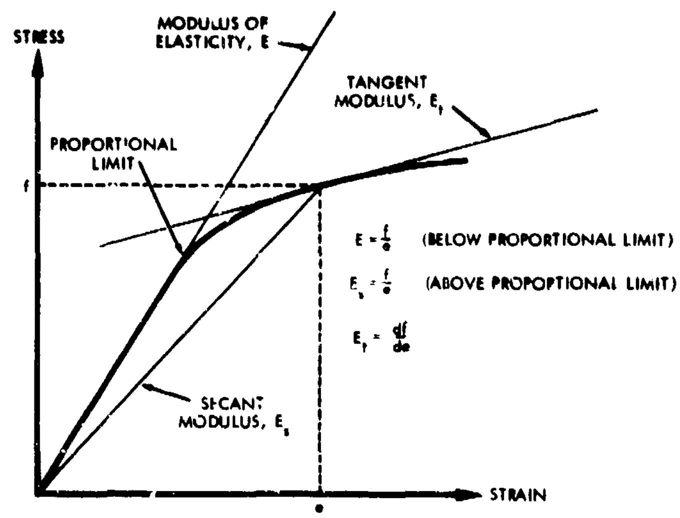


Figure 14.2.1.2c. Relationship Between Modulus of Elasticity, Secant Modulus, and Tangent Modulus

L′/ρ = slenderness ratio, dimensionless

This equation has come to have much practical importance in recent years in determining the short-column curve; note that an effective modulus equal to the *tangent modulus* can usually be used to compute failing stresses. The value of the tangent modulus at any given compressive stress, F_c can be determined from stress-strain curves for the material, as shown in Figure 14.2.1.2c. The tangent modulus may be thought of as a measure of the instantaneous resistance against an increase in strain (Reference 720-1).

The upper limit of the allowable column stress for primary failure is called the column-yield stress and is designated F_{co}. It can be determined by extending the short-column curve to a point corresponding to zero length, ignoring any tendency of the curve to rise rapidly or pickup for very short lengths. The short-column curve used in determining F_{co} should be obtained from tests on specimens having geometrical proportions such that local failure is precluded except for very low values of L′/ρ. When the column-yield stress is reached, the walls of the column will tend to buckle unless restrained by extreme shortness or by the application of lateral restraining forces. In some cases, however, if the specimen has not been allowed to buckle, the stress above this value may be increased considerably. Because of the danger of buckling when the column-yield stress is approached, the latter should be considered as the limiting stress for all columns. The column-yield stress is determined mainly by the nature of the compressive stress-strain diagram of the material. When the material has a definite yield point in compression, this value may be assumed for the column-yield stress. However, few aerospace materials have a sharply defined yield point. In such cases, it is usually possible to determine the column-yield stress as a function of either the tensile or compressive yield stress. Column-yield stresses for various materials are given in Reference 547-12.

Local Instability Failure. The upper limit of the allowable column stress for local failure is called the crushing or crippling stress and is designated F_{cc}. The crushing stresses of round tubes subject to plastic failure generally can be expressed by a modified form of the equation for the buckling of a thin-walled cylinder in compression, as given below:

$$F_{cc} = \frac{K\sqrt{EE'}}{D/t} \qquad \text{(Eq 14.2.1.2k)}$$

where

F_{cc} = allowable crushing or crippling stress, psi

K = a constant (see text)

E = modulus of elasticity, psi

E' = effective modulus of elasticity, psi

D = diameter of tube, in.

t = thickness of tube, in.

The effective modulus, E', can be determined from the basic column curve for primary failure by the method given above for short columns. As the value of the effective modulus corresponds to a given value of stress, it usually is convenient to: assume a value of F_{cc}; compute the corresponding value of E'; and substitute these values into Equation (14.2.1.2k) and solve for D/t. This latter value is the D/t at which crushing will occur at the assumed stress. Values of the constant, K, usually must be determined empirically, but Shanley (Reference 720-1) lists the theoretical value of K as 1.2 while showing test data yielding a range of 0.4 to 0.65. As noted above, Equation (14.2.1.2k) applies to plastic failure, i.e., for stresses above the proportional limit. In the case of thin-walled tubes which fail locally at stresses below the proportional limit, the initial eccentricities are likely to be relatively larger and the constant should be suitably reduced.

Additional Column Data. The references at the end of this section include numerous tables of elastic stability for various column sections as well as a variety of other data useful to the designer. Reference 598-1 includes a comprehensive discussion of plastic column instability, and Reference 735-1 presents an excellent basic treatment of the techniques for determining column dimensions.

Buckling of Thin-Walled Vessels by Internal Pressure. Reference 598-1 treats situations when tubes subjected to internal pressure can fail by buckling. One case is hydraulic tubing loaded by a piston as shown in Figure 14.2.1.2d. The force P due to the internal pressure is

$$P = \pi r^2 p \qquad \text{(Eq 14.2.1.2l)}$$

where

P = axial force resulting from pressure, lb_f

r = internal radius of cylinder, in.

p = internal pressure, psi

As the pressure is increased a critical value of P at which buckling occurs is reached. Because of the piston arrangement, the tube walls do not support axial loads in the example of Figure 14.2.1.2d. However, the bending moment is carried by the tube walls, and since buckling is due to the lateral displacement which results from the bending moment, the critical load for the tube is the same as for a column which carries the axial load.

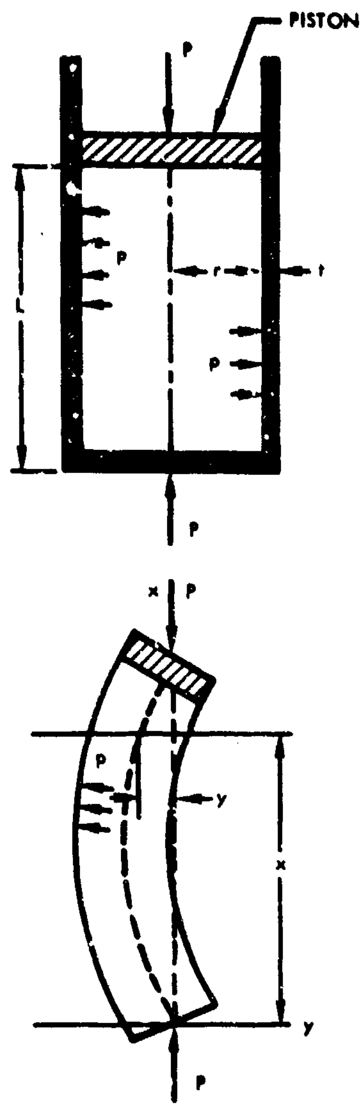


Figure 14.2.1.2d. Buckling of Thin-Walled Cylinder Under Internal Pressure from Piston Action
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

The Euler buckling load is

$$P_c = \frac{\pi^2 EI}{L^2} \qquad \text{(Eq 14.2.1.2m)}$$

where

P_c = critical axial (compressive) force, lb_f

E = modulus of elasticity, psi

I = moment of inertia, in^4

L = length, in.

Therefore the critical internal pressure, from Equation (14.2.1.2l) is

$$P_{cr} = \frac{\pi^2 Etr}{L^2} \qquad \text{(Eq 14.2.1.2n)}$$

where

P_{cr} = critical internal pressure, psi

t = cylinder wall thickness, in.

Bending Instability Failure. Failures of round tubes when subjected to bending are usually of the plastic-instability type. In such cases, the criterion of strength is the *modulus of rupture* as derived from test results through the use of Equation (14.2.1.2o).

$$F_b = \frac{My}{I} = \frac{M}{Z} \qquad \text{(Eq 14.2.1.2o)}$$

where

F_b = modulus of rupture in bending, psi

M = applied bending moment, in-lb_f

y = distance from neutral axis to given (outermost) fiber, in.

I = moment of inertia, in^4

Z = section modulus = I/y, in^3 (Note: a distinction is often made between *plastic section modulus*, Z, and *elastic section modulus*, S; Z is used for both moduli in this text and in MIL-HDBK-5.)

Bending instability failure will occur when

$$f_b \geq F_b$$

where

f_b = internal or calculated primary bending stress, psi

It should be noted that the modulus of rupture as calculated by the method of Cozzone (References 9-7 or 734-1) is not applicable to this type of failure and will yield unconservative results if used.

Torsional Instability Failure. Similarly, round tubes in torsion will fail by plastic instability when the shear stress, f_s, exceeds the modulus of rupture in torsion, F_{st}, when the latter is derived from test results using the equation

$$F_{st} = \frac{Ty}{J} = \frac{T}{Z_p} \qquad \text{(Eq 14.2.1.2p)}$$

where

F_{st} = modulus of rupture in torsion, psi

T = torque, in-lb_f

y = distance from neutral axis to given (outermost) fiber, in.

J = I_p = polar moment of inertia, in^4

Z_p = J/y = polar section modulus, in^3

Combined Loading Instability Failure. In practice most instability and buckling analyses of combined loading is based upon the *margin of safety* as described in Detailed Topic 14.2.1.3.

Stiffness (Rigidity/Flexibility) Failure Criteria. There are numerous failure criteria other than the ultimate, yield, and buckling criteria discussed above (for example, Marin, Reference 661-1, also treats theories of resilience, fracture strength, ductility and toughness). The primary additional criteria of importance to the fluid component designer, however, are the stiffness (rigidity/flexibility) failure criteria. These criteria, unlike the stress criteria discussed above, are based upon the extent to which elements of the component structure will *resist deflection* (rigidity) or *deflect* (flexibility) under applied load without buckling or exceeding the yield strength of the material. Because no structural damage necessarily occurs in these modes of failure, these failure criteria apply only if component functional performance is impaired. Evaluation of stiffness criteria is essential to good component design and is a major consideration in the selection of material and configuration. Unlike the stress criteria, stiffness criteria usually require evaluation of deflection of the structural member *in relation to other component parts.* For this reason no phenomenological theories are usually employed in applying stiffness failure criteria to actual design, although Reference 661-1 presents stiffness theories based on the distortion energy theory or bulk modulus for evaluating the relative stiffness of materials under combined stresses. In this application bulk modulus, K, is given by the equation

$$K = \frac{E}{3(1 - 2\mu)} \qquad \text{(Eq 14.2.1.2q)}$$

where

K = bulk modulus, psi

E = modulus of elasticity, psi

μ = Poisson's ratio, dimensionless

The practical application of stiffness criteria to component design requires the use of appropriate deflection/deformation equations (Sub-Topics 14.3.3 through 14.3.6) to analyze the behavior of component structural members under load with respect to component functioning. Some common stiffness considerations are:

a) Flange distortion (see Figure 5.12.4.4)

b) Component distortion which relieves seal preloads, resulting in seal leakage (Sub-Sections 6.3 and 6.4)

c) Shaft deflections resulting in binding, leakage, bearing or seal damage, or poor response (Sub-Section 14.12)

d) Distortion of component housings which cause binding shafts, sleeves, etc.

e) Component distortions which preclude proper mating of valving elements and valve seats (especially in poppet valves) (Sub-Section 6.2)

f) Progressive distortions, wherein the elastic deflection of one element results in loads which distort mating elements thereby permitting further distortion of the first element

g) Connector tension element design (the bolts of a bolted flange connector must continue to exert a minimum compressive load on the sealing element when the connector is subjected to various pressure and bending loads)

h) All of the elastic deflections of an actuator linkage must be added to fitting and bearing clearances when evaluating hysteresis in actuator systems.

Some basic stiffness ground rules to consider in all fluid component applications are:

a) Are mating part deflections or motions known or specified? (If not it may be much safer to assume logical values of deflection rather than to assume perfect rigidity of mating hardware.)

b) Thermal, vibration, and pressure-caused deflections of complex structures may be extremely difficult to analyze even with sophisticated computer techniques, especially during transients such as rocket engine start-up and shut-down. Accordingly it may be advisable to locate fluid components such as valves and regulators away from such structures, or mount the components with flexures or other devices which will minimize the loads imparted to the component.

c) If difficulty is encountered in obtaining requisite component stiffness or rigidity with a selected high-strength material, evaluate the feasibility of using lower strength (perhaps lighter) material of heavier wall thickness (with a high value of E suitable for carrying the load with less deflection).

14.2.1.3 FACTOR OF SAFETY AND MARGIN OF SAFETY. The term *factor of safety* (F.S.) is used synonymously with *safety factor* and *design factor* to indicate the ratio of estimated strength to computed stress or the ratio of estimated load-carrying capacity to maximum computed operational load. A *margin of safety* (M.S.) is used to describe the additional strength of the structure over that required and is defined by

$$M.S = F.S. - 1$$

The factor of safety is applied to either stresses or loads in the design phase when dimensions are determined and materials are selected. After the design is completed, stresses are analyzed in more detail and the margin of safety is calculated. In many aerospace activities the margin of safety is formally determined by a group of stress analysts who review the design with more sophisticated techniques than those used by the designer in selecting materials and establishing critical dimensions. In this discussion, the application of factors of safety and margins of safety as well as some considerations regarding criteria for establishing values of factor of safety are treated.

Application of Factor of Safety to Stress. It is common in machine design to apply a factor of safety to stresses, whereas in aircraft design a factor of safety is often applied to loads. It is recommended that the fluid component designer give careful consideration to the latter technique, which is discussed below. Application of a factor of safety to stress alone will influence only rupture or yield failure, ignoring buckling and stiffness failure modes. The following equations relate allowable stress, F, to factor of safety, F.S., and appropriate yield or ultimate material properties, F_y or F_u. If buckling or instability criteria are used to determine allowable stresses, then the margin of safety is applied to combined loads by using *stress ratios*.

Ductile Materials — Static Loading — Yield Criteria

Tension:

$$F.S. = \frac{F_{ty}}{f_t} \quad or \quad F_t = \frac{F_{ty}}{F.S.} \qquad \text{(Eq 14.2.1.3a)}$$

where

F.S. = factor of safety, dimensionless

F_{ty} = tensile yield stress, psi

f_t = calculated tensile stress, psi

F_t = allowable tensile stress, psi

Compression*:

$$F.S. = \frac{F_{cy}}{f_c} \quad or \quad F_c = \frac{F_{cy}}{F.S.} \qquad \text{(Eq 14.2.1.3b)}$$

where

F_{cy} = compressive yield stress, psi

f_c = calculated compressive stress, psi

F_c = allowable compressive stress, psi

Shear:

$$F.S. = \frac{F_{sy}}{f_s} \quad or \quad F_s = \frac{F_{sy}}{F.S.} \qquad \text{(Eq 14.2.1.3c)}$$

where

F_{sy} = yield stress in pure shear, psi

f_s = calculated shearing stress, psi

F_s = allowable shearing stress, psi

Bearing:

$$F.S. = \frac{F_{bry}}{f_{br}} \quad or \quad F_{br} = \frac{F_{bry}}{F.S.} \qquad \text{(Eq 14.2.1.3d)}$$

where

F_{bry} = bearing yield stress, psi

f_{br} = calculated bearing stress, psi

F_{br} = allowable bearing stress, psi

*Caution: These compression expressions apply *only* if instability is not a factor.

Ductile Materials — Static Loading — Ultimate (Rupture) Criteria

Tension: Same as Equation (14.2.1.3a) except substitute ultimate tensile stress, F_{tu}, for F_{ty}.

Compression: Same as Equation (14.2.1.3b) except substitute ultimate compressive stress, F_{cu}, for F_{cy}.

Shear: Same as Equation (14.2.1.3c) except substitute ultimate stress in pure shear, F_{su}, for F_{sy}.

Bearing: Same as Equation (14.2.1.3d) except substitute ultimate bearing stress, F_{bru}, for F_{bry}.

Brittle Materials — Static Loading — Ultimate Criteria. Essentially the same approach is used as that applied to ductile materials in static loading using ultimate failure criteria, except that the respective calculated stresses, f, are multiplied by the static stress concentration factor, K_t (Sub-Section 14.6).

Fatigue. Where alternating stresses make fatigue failure a possibility, use of the factor of safety becomes somewhat more complex because the fatigue strength reduction factor, K_f, is applied only to the alternating stress. The fatigue safety factor, S_f, is discussed in Sub-Section 14.5.

Ductile Materials — Combined Static Loading — Instability Criteria. For combined-loading conditions in which failure is caused by buckling or instability, no general theory exists which will apply in all cases. It is convenient, however, to represent such conditions by the use of *stress ratios* which can be considered as nondimensional coefficients denoting the fraction of the allowable stress or strength which is utilized or which can be developed under special conditions. For simple stresses, the stress ratio can be expressed as:

$$R = \frac{f}{F} \qquad \text{(Eq 14.2.1.3e)}$$

where

R = stress ratio, dimensionless

f = applied stress, psi

F = allowable stress, psi

The margin of safety as usually expressed is given by the equation:

$$M.S. = \frac{1}{R} - 1.0 \qquad \text{(Eq 14.2.1.3f)}$$

For combined loadings, the general conditions for failure can be expressed by equations of the following type:

$$R_{1a}^{x} + R_{2a}^{y} + R_{3a}^{z} + \ldots = 1.0 \qquad \text{(Eq 14.2.1.3g)}$$

In this equation, R_{1a}, R_{2a}, and R_{3a} may denote, for instance, the allowable stress ratios for compression, bending, and shear, and the exponents x, y, and z define the general relationship of the quantities. This equation may be interpreted as indicating that failure will occur only when the sum of the stress ratios is equal to or greater than 1.0 (if x:y:z = 1). An advantage of this method is that the formula yields correct results when only one loading condition is present. Consequently, it tends to give good results when any one loading condition predominates. It also permits test data to be plotted in nondimensional form, which is a decided advantage.

In many cases it is convenient to deal directly with load ratios rather than stress ratios. The load ratio is simply the ratio of the applied load to the allowable load and is equal to the corresponding stress ratio.

Considering only two loading conditions, such as bending and compression, R_a can be plotted against R_b. When all three conditions exist, the equation represents an interaction surface, which can be plotted as a family of curves. Typical curves corresponding to various exponents are shown in Figure 14.2.1.3. The general significance of Equation (14.2.1.3g) and Figure 14.2.1.3 is that the addition of a second loading condition will lower the percentage of the allowable stress which may be utilized in the original loading condition. If the exponents approach infinity, the curve of Figure 14.2.1.3 will approach the lines $R_1 = 1.0$ and $R_2 = 1.0$, indicating that the two loading conditions have no affect on each other.

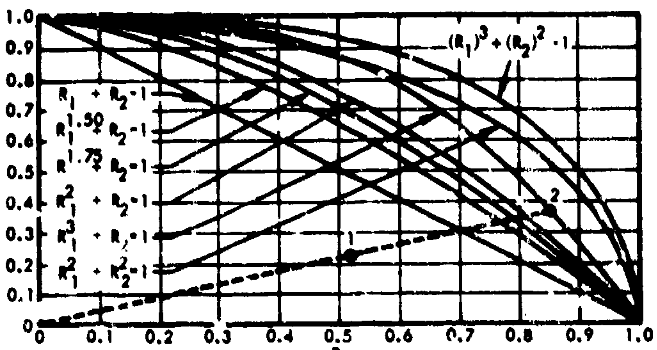


Figure 14.2.1.3. Typical Stress Ratio Interaction Curves for Combined Loading Conditions
(Reference 5-7-12)

When only two stress ratios are involved and when the two different applied stresses remain in constant proportion, the margin of safety of the member may be determined from Figure 14.2.1.3 by the following method:

a) Locate the point on the chart representing the applied values of R_1 and R_2 computed from the applied stresses (illustrated as point 1 in Figure 14.2.1.3).

b) Draw a straight line through this point and the origin (shown as a diagonal dotted line in Figure 14.2.1.3).

c) Extend this line to intersect the proper stress-ratio curve (corresponding to the condition under consideration) at point 2.

d) Read the allowable values R_{1a} and R_{2a} as the abscissa and ordinate, respectively, of point 2.

e) The *factor of utilization or strength ratio*, U, is obtained as the ratio of the applied to the allowable value of either stress ratio, as follows:

$$U = \frac{R_1}{R_{1a}} = \frac{R_2}{R_{2a}} \qquad \text{(Eq 14.2.1.3h)}$$

f) The true margin of safety, then, can be computed from the following equation:

$$M.S. = \frac{1}{U} - 1 \qquad \text{(Eq 14.2.1.3h)}$$

Note that, when the following stress-ratio expressions are used, the margins of safety can be computed as indicated:

For $R_{1a} + R_{2a} = 1$

$$M.S. = \frac{1}{(R_1 + R_2)} - 1$$

For $R_{1a}^2 + R_{2a}^2 = 1$

$$M.S. = \frac{1}{\sqrt{R_1^2 + R_2^2}} - 1$$

Other M.S. formulae can be determined for the more complicated stress-ratio expressions.

The general formula for the margin of safety stated analytically for interaction equations where any or all of x, y, and z are 1 or 2 but no other value (except one term may be missing) is as follows:

$$M.S. = \frac{2}{R + \left[(R')^2 + 4(R'')^2\right]^{1/2}} - 1$$

where

R' = the sum of all first-power ratios

R'' = the sum of all second-power ratios.

Table 14.2.1.3a gives all combinations.

The practical application of Equation (14.2.1.3g) is shown in the following examples:

Example 1: Round Tubes in Bending and Compression. In the case of combined bending and compression, it is necessary to consider the effects of secondary bending, that is, bending produced by the axial load acting in conjunction with the lateral deflection of the column. In general, Equation (14.2.1.3g) can be used in the following form for safe values:

$$\frac{f_b'}{F_b} + \frac{f_c}{F_c} = 1.0$$

or

$$R_b + R_c = 1.0$$

$$M.S. = \frac{1}{R_b + R_c} - 1$$

where

f_b' = maximum bending stress, including effects of secondary bending, psi

F_b = bending modulus of rupture, psi

f_c = axial compressive stress, psi

F_c = allowable compressive stress, psi

In no case shall the axial compressive stress, f_c, exceed the allowable, F_c, for a simple column.

Example 2: Round Tubes in Bending and Torsion. For round tubes, Equation (14.2.1.3g) can be used in the following forms for safe values:

$$\left(\frac{f_b}{F_b}\right)^2 + \left(\frac{f_s}{F_{st}}\right)^2 = 1.0$$

or

$$R_b^2 + R_s^2 = 1.0$$

$$M.S. = \frac{1}{\sqrt{(R_b)^2 + (R_s)^2}} - 1$$

where

f_b = bending stress, psi

F_b = bending modulus of rupture, psi

f_s = shear stress, psi

F_{st} = torsional modulus of rupture, psi

Table 14.2.1.3a. Margin of Safety Expressions for Various Buckling Stress Ratio Interaction Formulas

Interaction Formula	Margin of Safety
$R_{1a} + R_{2a}^2 = 1.0$	$\frac{2}{R_1 + \sqrt{R_1^2 + 4R_2^2}} - 1$
$R_{1a} + R_{2a} + R_{3a} = 1.0$	$\frac{1}{R_1 + R_2 + R_3} - 1$
$R_{1a} + R_{2a} + R_{3a}^2 = 1.0$	$\frac{2}{R_1 + R_2 + \sqrt{(R_1 + R_2)^2 + 4R_3^2}} - 1$
$R_{1a} + R_{2a}^2 + R_{3a}^2 = 1.0$	$\frac{2}{R_1 + \sqrt{R_1^2 + 4(R_2^2 + R_3^2)}} - 1$
$R_{1a}^2 + R_{2a}^2 + R_{3a}^2 = 1.0$	$\frac{1}{\sqrt{R_1^2 + R_2^2 + R_3^2}} - 1$

Example 3: Round Tubes in Bending, Compression and Torsion. The bending stresses should include the effects of secondary bending due to compression. The following empirical equation will serve as a working basis:

$$\left(\frac{f'_b}{F_b}\right)^2 + \left(\frac{f_s}{F_{st}}\right)^2 = \left(1 - \frac{f_c}{F_c}\right)^2$$

$$\mathrm{M.S.} = \frac{1}{R_c + \sqrt{(R_b)^2 + (R_s)^2}} - 1$$

where

f'_b = maximum bending stress, including effects of secondary bending, psi

F_b = modulus of rupture in bending, psi (from Eq 14.2.1.2o)

f_s = internal (or calculated) shear stress, psi

F_{st} = modulus of rupture in torsion, psi (from Eq 14.2.1.2p)

f_c = internal (or calculated) compressive stress, psi

F_c = allowable compressive stress, psi (from Eq 14.2.1.2i)

R_c = compressive stress ratio, dimensionless

R_b = bending stress ratio, dimensionless

R_s = shear stress ratio, dimensionless

In no case may the axial compressive stress, f_c, exceed the allowable compressive stress, F_c.

Application of Factor of Safety to Load. If the factor of safety, F.S., is used with the maximum applied load, P, the margin of safety may be determined for the buckling and stiffness failure criteria as well as for the stress failure criteria. In aircraft structural design and in the design of many aerospace fluid components, the calculated maximum load is referred to as the *limit load* (P_{LL}).

$$P_{LL} = F_{LL}\, P_{DL} \qquad \text{(Eq 14.2.1.3j)}$$

where

P_{LL} = limit load, lb_f

F_{LL} = limit factor, dimensionless

P_{DL} = design load (maximum expected load), lb_f

Values for *limit factor*, F_{LL}, usually vary from 1.0 for well-defined loads to about 1.5 for estimated loads such as aerodynamic loads and duct misalignment loads. Typical limit and ultimate factors are tabulated in Table 14.2.1.3b for a manned spacecraft rocket propulsion system. The limit factor is applied to each source of loading, such as pressure, acceleration, and vibration. In addition, the nature of loads (or stresses) is specified, such as combined, axial, or pressure. The limit load obtained for each source of loading is then multiplied by the ultimate factor of safety (ultimate factor) or yield factor of safety, as appropriate.

Ultimate:

$$P_u = (P_{LL})\,(\mathrm{F.S.}_u) \qquad \text{(Eq 14.2.1.3k)}$$

where

P_u = ultimate load, lb_f

$\mathrm{F.S.}_u$ = ultimate factor of safety, dimensionless (Note $\mathrm{F.S.}_u$ = 1.5 in most aircraft applications; References 19-275 and 662-1)

Yield:

$$P_y = (P_{LL})\,(\mathrm{F.S.}_y) \qquad \text{(Eq 14.2.1.3l)}$$

where

P_y = yield load, lb_f

$\mathrm{F.S.}_y$ = yield factor of safety, dimensionless (Note: $\mathrm{F.S.}_y$ = 1.0 in most aircraft applications; References 19-275 and 662-1)

Table 14.2.1.3c lists some typical proof and burst pressures currently specified for fluid components. It may be noted from Table 14.2.1.3b that in many cases there is no apparent correlation between material yield and ultimate strength and between proof and burst pressures specified for pressure vessels. Some materials such as titanium have a yield strength very near their ultimate strength and will therefore usually be designed to burst pressure (ultimate strength) requirements. Other materials, such as aluminum alloys, have a yield stress far below their ultimate stress and will consequently usually be designed for yield strength. It is essential that all pressurized component element margins be evaluated both for yield stress at proof pressure and ultimate stress at burst pressure.

14.2.2 Preliminary Design

Preliminary structural design of aerospace fluid components, which may be interpreted to encompass many or few steps, will most frequently include the following:

a) Sketch or drawing of the system, showing the component in relation to other system elements

b) Assembly drawing of the component

c) Sketch or drawing of major component elements

d) Estimate of loads, both internal and external (Detailed Topic 14.2.1.1)

e) Estimate of most probable failure criteria, i.e., will stress considerations (such as hoop stress in high-pressure applications) predominate or will other failure criteria control the design? (see Detailed Topic 14.2.1.2)

f) Preliminary selection of materials, i.e., will the part be of aluminum, stainless steel, titanium, or some very high strength alloy? Will it be cast, bar, sheet, or forged? Will it be used in a heat-treated or annealed condition? Compatibility should be considered at this time also. It is not imperative that the specific alloy and treatment

Table 14.2.1.3b. Typical Safety Factor Specification for a Manned Spacecraft Rocket Propulsion System

Structural Factors of Safety for Applied and Self-Generated Loads and Environments*

Load Source	Load Type	Limit Factors**	Ultimate Factors**
Load	(c)	1.0	1.5
Accelerations	(c)	1.0	1.5
Shock	(c)	1.0	1.5
Vibration (g^2/cps)	(c)	$(1.3)^2$	$(1.5)^2$
Vibration (g and D.A.)	(c)	1.3***	1.5
Pressure vessels	(p)	1.0****	2.0****
Pressure vessels	(c)	1.0	1.5
Acoustics (db)	(c)	1.0	1.0
Temperature	(c)	1.0	1.0 (T) 1.5 (L)

*These factors are to be applied to mission level structural loads (applied and self-generated loads) conservatively selected to represent the maximum severity expected. If transport or other environments provide more severe loads, these loads should be used in lieu of mission loads.

**There shall be no structural damage which will prevent successful completion of the mission, upon application of mission level loads multiplied by the limit factor and no structural failure when multiplied by the ultimate factor.

***For prelaunch vibration, limit factor = 1.0

****For propellant valves, fluid lines, and hydraulic valve actuators, the proof pressure is 2.0 times maximum operating pressure, and burst pressure is 3.0 times maximum operating pressure.

(c) = combined loadings (applied and self-generated) shall be considered

(p) = pressure only

(T) = applies to temperature

(L) = applies to thermal stress/loads

be specified at this time, but even preliminary sizing of component elements requires knowledge of basic material characteristics. The generalized data presented in Section 12.0 of this handbook will often be adequate for preliminary material selection.

g) Calculation or estimate of basic dimensions, such as wall thickness and shaft diameter, by simple stress equations such as those contained in Sub-Section 14.3. (Note: Functional requirements, such as flow rate, fix primary dimensions, such as line diameter.)

h) Evaluation of the component's capability to meet specification constraints such as weight and envelope while satisfying functional requirements.

i) Consideration of manufacturing operations which could degrade design properties of the preliminary design. Such manufacturing considerations include method of assembly, need for disassembly or reassembly, effects of welding, and provisions for inspection.

These steps do not include the obvious necessity for attention to component functional requirements which provide critical inputs for the structural design. In practice, functional design is usually carried out at the same time as the initial phases of preliminary structural design. During the first stages of preliminary design, critical functional parameters such as fluid flow rate, pressure drop, response time, and stroke are firmly established. It is at this stage that system interaction is primarily considered (water hammer, vibration transmissibility, etc.) with design refinements added as necessary in the detail design phase.

14.2.3 Detail Design

Detail design usually starts after approval of the preliminary design and results in a complete set of working drawings and manufacturing procedures or procurement specifications for every part of the component. In most instances, all of the design stress analysis will be performed by the designer. (In larger organizations, all detail designs are reviewed by stress analysts prior to release of the working drawings to the fabricating or procuring organization.) The information contained in this section of the handbook should provide the designer or stress analyst with the basic equations and criteria for performing most stress and

Table 14.2.1.3a. Typical Examples of Aerospace Proof and Burst Pressures

Application	Proof*	Burst*
Apollo lunar module spacecraft propulsion system	2.0	3.0
Classified unmanned spacecraft propellant tanks	1.5	2.0
Classified unmanned spacecraft lines and fittings**	2.0	4.0
Classified unmanned spacecraft valves, regulators, etc.	2.0	4.0
Classified unmanned spacecraft valves, regulators, etc.	1.5	2.0
Surveyor spacecraft propellant		
Tanks*** (not pressurized fully in presence of personnel)	1.15	1.25
Lines, valves, fittings, etc.	—	4.0
Agena gas storage tanks***	1.2	1.6
Saturn SIV-B gas storage tanks***	1.5	2.5
Discoverer spacecraft gas storage tanks***	1.6	2.2
Titan III gas storage tanks***	1.6	2.0
Aircraft pneumatic systems (MIL-P-5518C)		
Lines, fittings, and hose	2.0	4.0
Air reservoirs	2.0	4.0
Actuating cylinders and components	1.5	2.5
JPL Specification 30285 for spacecraft Flight equipment pressure systems	1.5	****

*Pressures in multiples of maximum expected operating pressure

**Eastern Test Range Safety Manual requirement

***Titanium tanks

****Burst pressure equals operating pressure times safety factor safety. Safety factors specified as 2.2 for pressure vessels and 2.0 for valves, regulators, fittings, tubing, etc.

deflection calculations. An important element of component detail design is recognition that a small minority of cases require comprehensive stress analysis, such as that discussed in Sub-Topic 14.2.4. Good practice demands that all phases of the detail design be properly documented with neat, legible, well-organized engineering statements and calculations. No superfluous phrasing is required or desired. A common error is cryptic, incomplete calculations with inadequate explanations of assumptions, references, or sources of values used. It should always be possible to review detail design calculations without having to ask such questions as:

a) Where did that equation come from?

b) Where did that value of yield strength come from?

c) What is the basis for that factor of safety?

During detail design, it is particularly important that the influence of stress concentrations or stress raisers such as fillets and radii be considered.

14.2.4 Design Analysis

14.2.4.1 COMPREHENSIVE STRESS ANALYSIS. The most obvious form of design analysis, comprehensive stress analysis, evaluates the capability of a particular design to meet structural requirements; often it is directed toward establishing or verifying the margin of safety. Preliminary design and detail design comprise analysis of the stresses and deflections in component members for the purpose of establishing dimensions and specifying material properties for these members; design analysis verifies whether that design will work. From another viewpoint, the designer has the opportunity and responsibility to adjust dimensions and materials to maintain stresses and deflections within allowable limits, whereas the stress analyst uses these established dimensions and properties to verify load carrying capacity. Most frequently, design analysis is performed by someone other than the designer; usually a stress analyst. Comprehensive stress analysis cannot be performed on a preliminary design, since it is necessary that the effect of details such as stress raisers (fillets, holes, radii), alloy and processing specifications, and manufacturing techniques be considered. Although in many cases the analysis may be simple and straightforward, using techniques and data such as those described in this handbook, in other cases more comprehensive analysis is required which entails such techniques as:

a) Sophisticated (often computer-aided) analysis of stresses and deflections in shells, bellows, complex flexures, piping systems, etc.

b) Experimental evaluation of strains (from which stress levels may be inferred) in components where configuration complexity or load complexity preclude accurate calculation of stresses. Techniques include brittle or photoelastic coating, photoelastic models, and electrical strain gage testing.

The justification for performing a comprehensive stress analysis of an aerospace fluid component is usually based upon one or more of the following considerations:

a) Unexplained failures in test or service

b) Low level of confidence in the accuracy of simple stress analysis for a complex application

c) Critical applications without similar precedent (such as many of the larger Saturn feed system components)

d) Design optimization, wherein critical weight requirements or very low factors of safety necessitate several design iterations

e) Reduction of instrumentation and data analysis requirements for component structural testing

14.2.4.2 COMPREHENSIVE MATERIALS ANALYSIS. Another aspect of design analysis which is frequently overlooked is that of comprehensive materials analysis. Again, the designer usually requires specialized assistance, but in this instance it is the materials specialist rather than the stress analyst whose aid must be sought. This is easily done in the large aerospace corporations and government agencies, but the component manufacturer frequently must rely upon the customer and/or material source for the necessary information. Many apparently sound designs have failed in aerospace applications as a result of unpredicted material property changes in service. Similarly, breakdowns in quality-assurance procedures have resulted in failures because a part did not receive a particular heat treatment or other process before going into service. A comprehensive materials analysis should ensure that, as a minimum, all of the items listed in Table 14.2.4.2 have been properly accounted for.

Table 14.2.4.2 Check List for Comprehensive Materials Evaluation (Properties Affecting Structural Performance Only)

1. Are property values (F_u, F_y, E, μ, etc.) used in design calculations actually obtained by the specified manufacturing procedures? (Particularly, is the material used in the heat-treated or annealed condition?) If any property values have been extrapolated from those of similar alloys, are the assumptions valid?

2. If *any* process is specified for the material (such as heat treating and nitriding) does 100 percent inspection assure that the finished product has received the benefit of the process?

3. Are property values used in design calculations *minimum* values, such as those in MIL-HDBK-5? If average or typical values have been used, has suitable allowance been made?

4. If fatigue loading is possible, is the endurance limit or fatigue limit used in design properly estimated from tensile data if fatigue data are not available?

5. Have the possibilities of stress corrosion cracking, hydrogen embrittlement, and corrosion fatigue been thoroughly evaluated?

6. Have *all* compatibility considerations been evaluated, including fluid medium, mating materials, environmental media (especially salt spray and propellant spill), cleaning media, and test media? Do compatibility evaluations take temperature during exposure into account?

7. Has the affect of stress concentrations been considered, especially as related to material notch sensitivity?

14.3 FUNDAMENTAL STRESS/DEFLECTION EQUATIONS

14.3 FUNDAMENTAL STRESS/DEFLECTION EQUATIONS

The equations presented below are those most frequently used in structural design and stress analysis. Many are presented elsewhere in this section with more comprehensive descriptions of their use, but are also shown here for ready reference.

The sign conventions generally accepted in their use are that quantities associated with tensile action (load, stress, strain, etc.) are considered as positive, and quantities associated with compressive action are considered as negative. When compressive action is of primary interest, however, it is sometimes convenient to consider the associated quantities to be positive.

14.3.1 Simple Unit Stresses

Tension:

$$f_t = \frac{P}{A} \qquad \text{(Eq 14.3.1a)}$$

where

f_t = internal (or calculated) average tensile stress, psi

P = applied load (total, not unit), lb_f

A = area of cross section, in^2

Compression:

$$f_c = \frac{P}{A} \qquad \text{(Eq 14.3.1b)}$$

where

f_c = internal (or calculated) average compressive stress, psi

P = applied load (total, not unit), lb_f

A = area of cross section, in^2

Bending:

$$f_b = \frac{My}{I} = \frac{M}{Z} \qquad \text{(Eq 14.3.1c)}$$

where

f_b = internal (or calculated) local primary bending stress, psi

M = bending moment, in lb_f

y = distance from neutral axis to given fiber, in.

I = moment of inertia about the neutral axis, in^4

Z = section modulus, I/y, in^3

Average direct shear stress:

$$f_s = \frac{V}{A} \qquad \text{(Eq 14.3.1d)}$$

where

f_s = internal (or calculated) average shearing stress, psi

V = shear force, lb_f

A = area of cross section which is in shear, in^2

Longitudinal or transverse shear stress:

$$f_s = \frac{VQ}{Ib} \qquad \text{(Eq 14.3.1e)}$$

where

f_s = internal (or calculated) average shearing stress, psi

V = shear force, lb_f

Q = static moment of a cross section = $\int y\,dA$, in^3

I = moment of inertia, in^4

b = width of cross section, in.

Shear stress in round tubes due to torsion:

$$f_s = \frac{Ty}{I_p} = \frac{Ty}{J} \qquad \text{(Eq 14.3.1f)}$$

where

f_s = internal (or calculated) average shearing stress, psi

T = applied torsional moment, in-lb_f

y = distance from neutral axis to given fiber, in. (Note: the symbol C is commonly used in structural analysis.)

I_p = J = polar moment of inertia, in^4

Shear stress due to torsion in thin-walled structures of closed section:

$$f_s = \frac{T}{2At} \qquad \text{(Eq 14.3.1g)}$$

where

f_s = internal (or calculated) average shearing stress, psi

T = applied torsional moment, in-lb$_f$

A = area enclosed by median line of the section, in^2

t = wall thickness, in.

Biaxial ratio:

$$f_A = Bf_H; \quad f_T = Bf_L \qquad \text{(Eq 14.3.1h)}$$

where

f_A = axial stress, psi

B = biaxial ratio, dimensionless

f_H = hoop stress, psi

f_T = transverse (grain direction) stress, psi

f_L = longitudinal (grain direction) stress, psi

14.3.2 Combined Stresses

Compression and bending:

$$f_n = f_c + f_b \qquad \text{(Eq 14.3.2a)}$$

where

f_n = internal (or calculated) normal stress, psi

f_c = internal (or calculated) compressive stress, psi

f_b = internal (or calculated) primary bending stress, psi

Compression, bending, and torsion:

$$f_{smax} = \sqrt{f_s^2 + \left(\frac{f_n}{2}\right)^2} \qquad \text{(Eq 14.3.2b)}$$

where

f_{smax} = maximum internal shearing stress, psi

f_s = internal (or calculated) shearing stress, psi

f_n = internal (or calculated) normal stress, psi

$$f_{nmax} = \left(\frac{f_n}{2}\right) + f_{smax} \qquad \text{(Eq 14.3.2c)}$$

where

f_{nmax} = maximum internal normal stress, psi

f_n = internal (or calculated) normal stress, psi

f_{smax} = maximum internal shearing stress, psi

14.3.3 Axial Deflections

Unit (average) deformation or strain:

$$e = \frac{\delta}{L} \times 100 \qquad \text{(Eq 14.3.3a)}$$

where

e = percent elongation, dimensionless

δ = deflection, in.

L = length, in.

Stress/strain ratio (this equation applies when E is to be found from tests in which f and e are measured):

$$E = \frac{f}{e} \qquad \text{(Eq 14.3.3b)}$$

where

E = modulus of elasticity, average ratio of stress to strain below proportional limit, psi

f = internal (or calculated) stress, psi

e = percent elongation, dimensionless

Deflection calculation with a known value of E:

$$\delta = \frac{PL}{AE} \qquad \text{(Eq 14.3.3.c)}$$

where

δ = deflection, in.

P = applied load (total, not unit), lb$_f$

L = length (total over which strain is summed to obtain deflection), in.

A = area of loaded cross section, in^2

E = modulus of elasticity in tension, psi

14.3.4 Bending Deflections

Beam deflection equations are given in Sub-Section 14.9.

Change of slope per unit length of beam:

$$\frac{di}{dx} = \frac{M}{EI} \qquad \text{(Eq 14.3.4)}$$

where

$\frac{di}{dx}$ = change of slope per unit length of beam, radians per unit length

M = applied bending moment, in-lb$_f$

E = modulus of elasticity, psi

I = moment of inertia about the neutral axis, in^4

14.3.5 Torsional Deflections

Basic equation:

$$\frac{d\phi}{dx} = \frac{T}{GJ} \quad \text{(Eq 14.3.5a)}$$

where

$\frac{d\phi}{dx}$ = change of angular deflection or twist per unit length of member, radians per unit length

T = applied torsional moment, in-lb$_f$

G = modulus of rigidity, psi

J = torsion constant = I_p for circular section, in^4

For torque T/GJ constant over length L:

$$\phi = \frac{TL}{GJ} \quad \text{(Eq 14.3.5b)}$$

where

ϕ = total angular deflection, radians

T = applied torsional moment, in-lb$_f$

G = modulus of rigidity, psi

J = torsion constant = I_p for circular section, in^4

14.3.6 Biaxial Elastic Deformation

Poisson's ratio in uniaxial loading:

$$\mu = \frac{\text{unit lateral deformation}}{\text{unit axial deformation}} \quad \text{(Eq 14.3.6a)}$$

where

μ = Poisson's ratio, dimensionless

$$Ee_x = f_x - \mu f_y \quad \text{(Eq 14.3.6b)}$$

where

E = modulus of elasticity, psi

e_x = unit strain in the x direction, in/in.

f_x = internal stress in the x direction, psi

f_y = internal stress in the y direction, psi

μ = Poisson's ratio, dimensionless

$$Ee_y = f_y - \mu f_x \quad \text{(Eq 14.3.6c)}$$

where

E = modulus of elasticity, psi

e_y = unit strain in the y direction, in/in.

f_y = internal stress in the y direction, psi

f_x = internal stress in the x direction, psi

μ = Poisson's ratio, dimensionless

Biaxial elastic modulus:

$$E_{biaxial} = \frac{E}{(1 - \mu B)} \quad \text{(Eq 14.3.6d)}$$

where

E = modulus of elasticity, psi

B = biaxial ratio, dimensionless

μ = Poisson's ratio, dimensionless

14.3.7 Basic Column Formula

Euler formula for long columns:

$$F_c = \frac{c\pi^2 E}{\left(\frac{L}{\rho}\right)^2} = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2} \quad \text{(Eq 14.3.7)}$$

where

F_c = allowable compressive stress, psi

c = fixity coefficient, dimensionless (see Detailed Topic 14.2.1.2)

E = modulus of elasticity, psi

L = length, in.

ρ = radius of gyration, in.

$L' = L/\sqrt{c}$

14.3.8 Polar Moment of Inertia for Circular Sections

Solid shafts:

$$J = I_p = \frac{\pi D^4}{32} \quad \text{(Eq 14.3.8a)}$$

Table 14.3.8. Formulas for Stress and Strain due to Pressure on or Between Elastic Bodies
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1954)

Notation: f_c = unit compressive stress; f_s = unit shear stress; f_t = unit tensile stress; a = radius of circular contact area for cases 1, 2, and 3; b = width of rectangular contact area for cases 4, 5, and 6; c = major semiaxis and d = minor semiaxis of elliptical contact area for cases 7 and 8; y = combined deformation of both bodies at each contact, along axis of load; μ = Poisson's ratio; E = modulus of elasticity. Subscripts 1 and 2 refer to bodies 1 and 2, respectively. All dimensions in inches, all forces in pounds

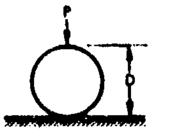
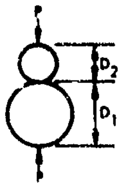
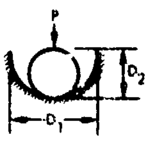
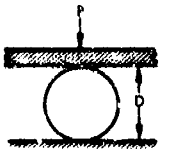
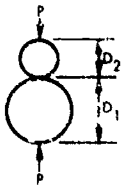
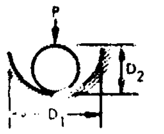
Conditions and Case No.	Formulas for dimensions of contact area and for a maximum stress
1. Sphere on a flat plate. P = total load 	$a = 0.721\sqrt[3]{PD\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}$ Max $f_c = 0.918\sqrt[3]{\frac{P}{D^2\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]^2}}$ If $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = 0.3$, $a = 0.881\sqrt[3]{\frac{PD}{E}}$, Max $f_c = 0.616\sqrt[3]{\frac{PE^2}{D^2}}$, Max $f_t = 0.133$ (Max f_c), $y = 1.55\sqrt[3]{\frac{P^2}{E^2D}}$ Max $f_s = \frac{1}{3}$(Max f_c), at depth $\frac{1}{2}a$ below surface of plate (approximate values)
2. Sphere on a sphere. P = total load 	$a = 0.721\sqrt[3]{P\left(\frac{D_1D_2}{D_1+D_2}\right)\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}$ Max $f_c = 0.918\sqrt[3]{P\frac{\left(\frac{D_1+D_2}{D_1D_2}\right)^2}{\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]^2}}$, $y = 1.04\sqrt[3]{\frac{P^2(D_1+D_2)}{D_1D_2}\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)^2}$ If $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = 0.3$, $a = 0.881\sqrt[3]{\frac{P}{E}\frac{D_1D_2}{D_1+D_2}}$, Max $f_c = 0.616\sqrt[3]{PE^2\left(\frac{D_1+D_2}{D_1D_2}\right)^2}$, Max $f_s = \frac{1}{3}$ (Max f_c). Max $f_t = 0.133$ (Max f_c), $y = 1.55\sqrt[3]{\frac{P^2}{E^2}\frac{(D_1+D_2)}{D_1D_2}}$
3. Sphere in spherical socket. P = total load 	$a = 0.721\sqrt[3]{P\frac{D_1D_2}{D_1-D_2}\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}$ Max $f_c = 0.918\sqrt[3]{P\frac{\left(\frac{D_1-D_2}{D_1D_2}\right)^2}{\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]^2}}$ If $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = 0.3$, $a = 0.881\sqrt[3]{\frac{P}{E}\frac{D_1D_2}{D_1-D_2}}$, Max $f_c = 0.616\sqrt[3]{PE^2\left(\frac{D_1-D_2}{D_1D_2}\right)^2}$, Max $f_s = \frac{1}{3}$ (Max f_c). Max $f_t = 0.133$ (Max f_c), $y = 1.55\sqrt[3]{\frac{P^2}{E^2}\frac{(D_1-D_2)}{D_1D_2}}$
4. Cylinder between flat plates p = load per linear in. = P/L 	$b = 1.6\sqrt{pD\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}$ Max $f_c = 0.798\sqrt{\frac{p}{D\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}}$ Total compression of cylinder between two plates is: $\Delta D = 4p\left(\frac{1-\mu^2}{\pi E}\right)\left(\frac{1}{3}+\log_e\frac{2D}{b}\right)$ If $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = 0.3$, $b = 2.15\sqrt{\frac{pD}{E}}$, Max $f_c = 0.591\sqrt{\frac{pE}{D}}$ For E = 30,000,000, $\mu_1 = \mu_2 = 0.25$, $b = 0.0_3394\sqrt{pD}$, Max $f_c = 3190\sqrt{\frac{p}{D}}$, Max $f_s = 958\sqrt{\frac{p}{D}}$ at depth $0.393b$ below surface of plane (Approximate formula) Mutual approach of remote points in two plates $= 4p\frac{1-\mu^2}{\pi E}\log_e\frac{\pi EL}{p(1-\mu^2)}$
5. Cylinder on cylinder. Axes parallel. p = load per linear in. 	$b = 1.6\sqrt{p\frac{D_1D_2}{D_1+D_2}\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}$ Max $f_c = 0.798\sqrt{\frac{p\frac{D_1+D_2}{D_1D_2}}{\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}}$ If $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = 0.3$, $b = 2.15\sqrt{\frac{p}{E}\frac{D_1D_2}{D_1+D_2}}$, Max $f_c = 0.591\sqrt{pE\frac{D_1+D_2}{D_1D_2}}$, $y = \frac{2(1-\mu^2)}{E}\frac{p}{\pi}\left(\frac{2}{3}+\log_e\frac{2D_1}{b}+\log_e\frac{2D_2}{b}\right)$
6. Cylinder in circular groove. p = load per linear in. 	$b = 1.6\sqrt{p\frac{D_1D_2}{D_1-D_2}\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}$ Max $f_c = 0.798\sqrt{\frac{p\frac{D_1-D_2}{D_1D_2}}{\left[\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right]}}$ If $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = 0.3$, $b = 2.15\sqrt{\frac{p}{E}\frac{D_1D_2}{D_1-D_2}}$, Max $f_c = 0.591\sqrt{pE\frac{D_1-D_2}{D_1D_2}}$

Table 14.3.8. Formulas for Stress and Strain due to Pressure on or Between Elastic Bodies (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

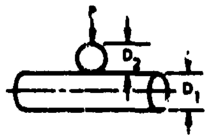
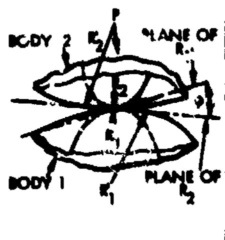
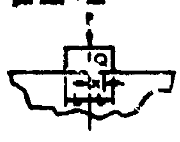
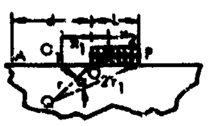
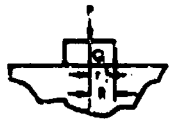
Conditions and Case No.	Formulas for dimensions of contact area and for a maximum stress
7. Cylinder on cylinder, axes at right angles. P = total load 	$b = \alpha\sqrt[3]{P\frac{D_1D_2}{D_1+D_2}\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]}$, $d = \beta b$, Max $f_c = \frac{1.5P}{\pi b d}$ $\quad y = \lambda\sqrt[3]{\frac{P^2}{\left(\frac{E_1}{1-\nu_1^2}+\frac{E_2}{1-\nu_2^2}\right)^2}\frac{(D_1+D_2)}{D_1D_2}}$ where α and β and λ depend on ratio $\frac{D_1}{D_2}$ and have values as follows: $\frac{D_1}{D_2}$: 1, 1½, 2, 3, 4, 6, 10 α: 0.908, 1.045, 1.150, 1.350, 1.505, 1.767, 2.175 β: 1, 0.765, 0.632, 0.482, 0.400, 0.308, 0.221 λ: 2.080, 2.060, 2.025, 1.980, 1.875, 1.770, 1.613 If $E_1 = E_2 = 30{,}000{,}000$, $\nu_1 = \nu_2 = 0.25$, $b = 0.00297\alpha\sqrt[3]{P\frac{D_1D_2}{D_1+D_2}}$ For these values of E and ν and for values of $\frac{D_1}{D_2}$ between 1 and 8, Max $f_c = \frac{11{,}700}{\left(\frac{R_1}{R_2}\right)^{0.271}}\sqrt[3]{\frac{P}{R_2^2}}$, where $R_1 = \frac{1}{2}D_1$, $R_2 = \frac{1}{2}D_2$ (approximate Formula)
8. General case of two bodies in contact. P = total pressure 	At point of contact minimum and maximum radii of curvature are R_1 and R_1' for Body 1, R_2 and R_2' for Body 2. Then $\frac{1}{R_1}$ and $\frac{1}{R_1'}$ are principal curvatures of Body 1, and $\frac{1}{R_2}$ and $\frac{1}{R_2'}$ of Body 2, and in each body the principal curvatures are mutually perpendicular. The plane containing curvature $\frac{1}{R_1}$ in Body 1 makes with the plane containing curvature $\frac{1}{R_2}$ in Body 2 the angle ϕ. Then: Max $f_c = \frac{1.5P}{\pi c d}$, $c = \alpha\sqrt[3]{\frac{P\Delta}{K}}$, $d = \beta\sqrt[3]{\frac{P\Delta}{K}}$, and $y = \lambda\sqrt[3]{\frac{P^2}{K^2\Delta}}$, where $\Delta = \frac{4}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1'}+\frac{1}{R_2'}}$ and $K = \frac{8}{3}\frac{E_1E_2}{E_2(1-\nu_1^2)+E_1(1-\nu_2^2)}$ α and β are given by the following table, where $\theta = \arccos\frac{1}{4}\Delta\sqrt{\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)^2+\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)^2+2\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)\cos 2\phi}$ θ: 0°, 10°, 20°, 30°, 35°, 40°, 45°, 50°, 55°, 60°, 65°, 70°, 75°, 80°, 85°, 90° α: ∞, 6.612, 3.778, 2.731, 2.397, 2.136, 1.926, 1.754, 1.611, 1.486, 1.378, 1.284, 1.202, 1.128, 1.061, 1.00 β: 0, 0.319, 0.408, 0.493, 0.530, 0.567, 0.604, 0.641, 0.678, 0.717, 0.759, 0.802, 0.846, 0.893, 0.944, 1.00 λ: —, 0.851, 1.220, 1.453, 1.550, 1.637, 1.709, 1.772, 1.828, 1.875, 1.912, 1.944, 1.967, 1.985, 1.996, 2.00
9. Rigid knife-edge across edge of semi-infinite plate. Load p lb. per linear in. 	At any point Q, $f_c = \frac{2p\cos\theta}{\pi r}$
10. Rigid block of width $2b$ across edge of semi-infinite plate. Load p lb. per linear in. 	At any point Q on surface of contact, $f_c = \frac{p}{\pi\sqrt{b^2-x^2}}$ (For loading on block of finite width and influence of distance of load from corner see Ref. 48)
11. Uniform pressure p lb. per sq. in. over length L across edge of semi-infinite plate 	At any point O_1 outside loaded area, $y = \frac{2p}{\pi E}\left[(L+x_1)\log_e\frac{d}{L+x_1} - x_1\log_e\frac{d}{x_1}\right] + pL\left(\frac{1-\nu}{\pi E}\right)$ At any point O_2 inside loaded area, $y = \frac{2p}{\pi E}\left[(L-x_2)\log_e\frac{d}{L-x_2} + x_2\log_e\frac{d}{x_2}\right] + pL\left(\frac{1-\nu}{\pi E}\right)$ Where y = deflection relative to a remote point A distant d from edge of loaded area At any point Q, $f_c = 0.318\,p(\alpha + \sin\alpha)$ $f_s = 0.318\,p\sin\alpha$

Table 14.3.8. Formulas for Stress and Strain due to Pressure on or Between Elastic Bodies (Continued)
(Adapted with permission from Reference 481-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Conditions and Case No.	Formulas for dimensions of contact area and for a maximum stress
12. Rigid cylindrical die of radius R on surface of semi-infinite body, total load P lb. 	$y = \frac{P(1-\nu^2)}{2RE}$ At any point Q on surface of contact $f_z = \frac{P}{2\pi R\sqrt{R^2 - r^2}}$ Max $f_z = \infty$ at edge Min $f_z = \frac{P}{2\pi R^2}$ at center
13. Uniform pressure p lb. per sq. in. over circular area of radius R on surface of semi-infinite body 	Max $y = \frac{2pR(1-\nu^2)}{E}$ at center y at edge $= \frac{4pR(1-\nu^2)}{\pi E}$ Max $f_s = 0.33\ p$ at point 0.638R below center of loaded area.
14. Uniform pressure p lb. per sq. in. over square area of sides 2b on surface of semi-infinite body 	Max $y = \frac{2.24pb(1-\nu^2)}{E}$ at center $y = \frac{1.12pb(1-\nu^2)}{E}$ at corners Average $y = \frac{1.90pb(1-\nu^2)}{E}$

where

J = torsion constant = I_p for circular section, in^4

D = diameter, in.

Thick-walled tube:

$$J = I_p = \frac{\pi\left(D_o^4 - D_i^4\right)}{32} \qquad \text{(Eq 14.3.8b)}$$

where

D_o = outside diameter, in.

D_i = inside diameter, in.

Thin-walled tube:

$$J = I_p = 2\pi r^3 t = \frac{\pi d^3 t}{4} \qquad \text{(Eq 14.3.8a)}$$

where

r = average radius, in.

t = wall thickness, in.

d = average diameter, in.

14.3.9 Bearing or Contact (Hertz) Stresses and Deflections

The basic expressions describing stresses and deflections resulting from two bodies in contact are summarized in Table 14.3.9.

14.3.10 Thermal Stress

Thermal stresses introduced into an externally constrained member by a change in temperature may be calculated from the basic equation:

$$f_T = \frac{\Delta T \alpha E}{K} = K' \Delta T \alpha E \qquad \text{(Eq 14.3.10)}$$

where

f_T = thermal stress, psi

ΔT = temperature change, °F

α = coefficient of thermal expansion, in/in/°F

E = modulus of elasticity, psi

K = constant dependent upon configuration and constraint (Table 14.3.10a), dimensionless

K' = 1/K, dimensionless

Thermal stresses in various flat plates may be calculated using the equations in Table 14.3.10b.

Table 14.3.10a. Thermal Stress Constant for Externally Constrained Bodies
(Adapted with permission from Reference 73-241, "Design News," 8 January 1964, vol. 19, no. 1, R. Welder)

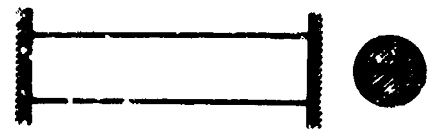
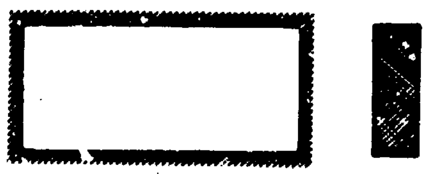
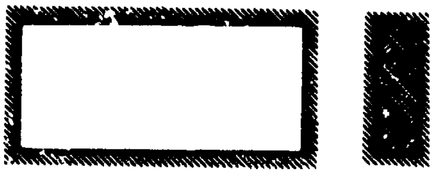
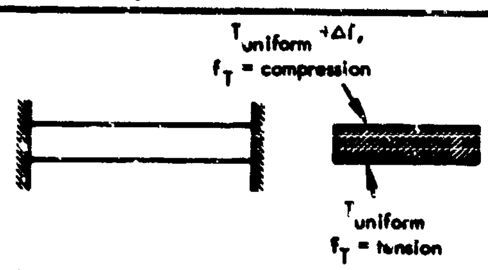
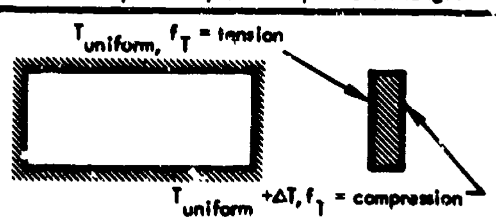
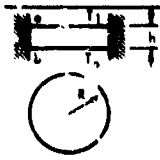
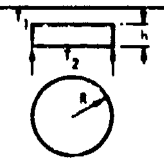
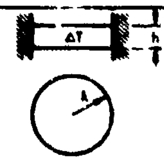
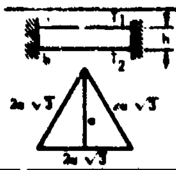
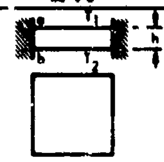
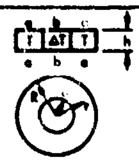
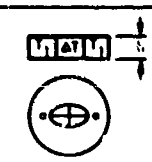
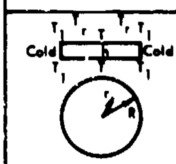
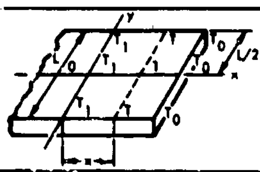
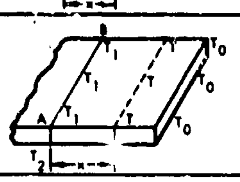
End conditions	K
 A uniform straight bar restrained at the ends, subjected to a temperature change throughout.	1
 A uniform flat plate restrained at the edges, subjected to a temperature change throughout.	0.7
 A solid body of any form restrained to the same form and volume, subjected to a temperature change throughout.	0.4
 A uniform bar of rectangular cross-section restrained at the ends with one face subjected to a uniform temperature, the other face subjected to a uniform temperature plus a temperature change.	2
 A uniform flat plate of any shape restrained at the edges with one face subjected to a uniform temperature, the other face subjected to a uniform temperature plus a temperature change.	1.4

Table 14.3.10b. Thermal Stresses in Various Plates
(Adapted with permission from Reference 886-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

	CIRCULAR PLATE, CLAMPED AT EDGE $(f)_a = \frac{-\alpha(\Delta T)E}{2(1-\mu)}$ $(f)_b = \frac{\alpha(\Delta T)E}{2(1-\mu)}$ $\delta = 0$
	CIRCULAR PLATE, SIMPLY SUPPORTED $f = 0$ $\delta = \frac{4\alpha(\Delta T)R^2}{8h}$
	CIRCULAR PLATE, CLAMPED AT EDGE $f = \frac{-\alpha E(\Delta T)}{1-\mu}$ $\delta = 0$
	TRIANGULAR PLATE, CLAMPED AT EDGE $(f_{max})_a = \frac{-3\alpha(\Delta T)E}{4}$ AT CORNERS $(f_{max})_b = \frac{3\alpha(\Delta T)E}{4}$ AT CORNERS
	SQUARE PLATE, CLAMPED AT EDGE $(f_{max})_a = \frac{-\alpha E(\Delta T)}{2}$ $(f_{max})_b = \frac{\alpha E(\Delta T)}{2}$
	CIRCULAR PLATE WITH UNIFORM CIRCULAR AREA OF ΔT AT CENTER $(f_r)_a = \frac{-\alpha E(\Delta T)}{2}\left(\frac{a}{r}\right)^2$ $(f_h)_a = \frac{\alpha E(\Delta T)}{2}\left(\frac{a}{r}\right)^2$ $(f_r)_b = (f_h)_b = \frac{\alpha E(\Delta T)}{2}$
	CIRCULAR PLATE WITH UNIFORM ELLIPTICAL AREA OF ΔT AT CENTER $(f_h)_{max} = (f_h)_a = \frac{\alpha E(\Delta T)}{(1 + b/a)}$
	CIRCULAR PLATE WITH UNIFORM RADIAL TEMPERATURE CHANGE $(f_r) = \alpha E\left[\frac{1}{R^2}\int_0^R (T_r - T_1)r\,dr - \frac{1}{r^2}\int_0^r (T_r - T_1)r\,dr\right]$ $(f_h) = \alpha E\left[-(T_r - T_1) + \frac{1}{R^2}\int_0^R (T_r - T_1)r\,dr + \frac{1}{r^2}\int_0^r (T_r - T_1)r\,dr\right]$
	RECTANGULAR PLATE WITH UNIFORM LONGITUDINAL TEMPERATURE CHANGE $(f_x)_{y=L/2} = E\alpha(T - T_0)$; max x = 0 $(f_y)_{y=0} = -E\alpha(T - T_0)$ $(f_y)_{y=L/2} = 0$
	RECTANGULAR PLATE WITH UNIFORM TEMPERATURE CHANGE LONGITUDINALLY AND THROUGH THICKNESS $(f_x)_{max\ A,B} = \frac{E\alpha}{2}\left[T_1 + T_2 - 2T_0 + \frac{1-\mu}{(3+\mu)}(T_1 - T_2)\right]$

14.4 CREEP AND STRESS RUPTURE

14.4.1 Basic Creep

Creep is defined as the slow (time-dependent) deformation of a material over long periods while under an applied load. It is usually regarded as an elevated-temperature phenomenon, although some materials creep at room temperature or below. If permitted to continue indefinitely, creep terminates in rupture. Since creep in service is usually typified by complex conditions of loading and temperature, the number of possible stress-temperature-time profiles is infinite. For economic reasons, creep data for general design use are usually obtained under conditions of constant uniaxial load and constant temperature. Creep data are sometimes obtained under conditions of cyclic uniaxial load and constant temperature to provide data necessary for dynamic creep evaluation. It is recognized that when significant creep appears likely to occur it may be necessary to test the material under actual service conditions because of difficulties in extrapolating from the simple to the complex stress-temperature-time conditions.

Damage incurred in a material as a result of creep (including effects resulting from elevated-temperature exposure) is cumulative and often permanent. This damage may involve the tempering or annealing of hardened materials and the initiation and growth of cracks and voids (initially of microscopic size) within a material. Its effects are often recognizable as a reduction in short-time strength properties or ductility, both at room and at elevated temperatures. Thus, designing under conditions of creep must take into account not only the time-dependent deformation that characterizes creep but also material damage that may result from creep.

14.4.2 Dynamic Creep

Dynamic creep is the permanent deformation that can occur during fatigue loading at elevated temperature while a *tensile* mean stress exists. Dynamic creep is often evident in comparing constant-life diagrams (Sub-Section 14.5) for the same material at various elevated temperatures. The combined effects of creep and fatigue are often analyzed by plotting a Goodman-type diagram (S_a versus S_m) using S_c, the *creep-limited static stress*, in lieu of S_u. This stress corresponds either to some acceptable creep elongation after a certain number of hours or to creep rupture (stress rupture) at the end of the required life.

14.4.3 Creep-Rupture Curve

The results of tests of materials under a constant load and temperature are usually plotted as strain versus time to rupture. A typical plot of creep-rupture data is shown in Figure 14.4.3. The strain indicated in this curve includes both the instantaneous deformation due to loading and the plastic strain due to creep.

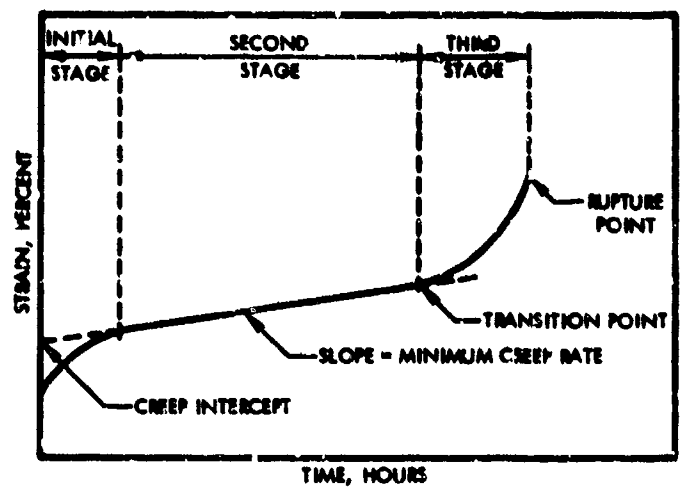


Figure 14.4.3. Typical Creep-Rupture Curve
(Reference [illegible]-12)

14.5 FATIGUE

14.5 FATIGUE

Fatigue failure is failure brought about by repeated reversal, removal or fluctuation of the applied load. *Low-cycle fatigue* has been arbitrarily defined by various investigators to apply to failures occurring in less than 1000 to 100,000 load cycles, with 10,000 cycles representing the most common limit. *Thermal fatigue* is essentially low-cycle fatigue resulting from alternating thermal expansion and contraction. *Corrosion fatigue* is the simultaneous action of corrosion and alternating stress (Sub-Topic 13.7.6).

14.5.1 Nature of Fatigue Failure

Fatigue has been the most common source of failure of space hardware (Reference 456-6). It has been estimated that 90 percent of all *rupture failures* in service are caused by fatigue and 90 percent of all *fatigue failures* are caused by improper design (Reference 598-1). Although much remains to be learned about the precise mechanism of fatigue failure, the following three phases are commonly considered to be essential elements:

a) Crack initiation

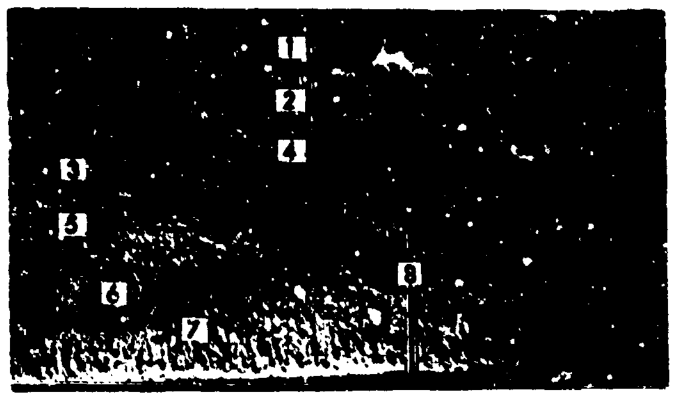
b) Crack propagation

c) Final unstable (brittle) rupture.

It is usually possible to identify a fatigue failure from evidence of these three distinct phases on the fracture surface. Figure 14.5.1 shows the fatigue failure purposely induced in a laboratory sample of D6-AC steel rocket pressure-vessel material. The sample shown in Figure 14.5.1 is one of many subjected to notch beam bending fatigue in a study of fracture toughness testing (Reference 131-35). The initial flaw was purposely introduced by arc burning or by ultrasonic machining a slot 0.100-inch long by 0.020-inch deep by 0.010-inch wide. Repeated simple (not reversed) bending during a fatigue test produced a stress cycle comprised of zero-to-tension at the flaw location.

A stress level of 125 ksi was repeatedly induced until the fatigue crack appeared well started (propagation), then a level of 100 ksi was maintained until the crack attained the desired length. Final rupture was achieved in simple tension, as is apparent from the appearance of the fracture area outside of the "beach marks". Final tensile failure occurred at stresses well below the average ultimate tensile stress for the material.

14.5.1.1 CRACK INITIATION. Crack initiation is almost invariably the result of some *stress concentration*, such as a notch or inclusion in a highly stressed region. Such a stress concentration may be very difficult to identify in a highly polished specimen tested in the laboratory for the purpose of establishing fatigue life characteristics of a material, but is often obvious when analyzing a part which has failed in service (The initial cracks formed in the laboratory specimen shown in Figure 14.5.1 are so small as to be almost invisible in the photograph.) Fatigue cracks are usually initiated at a free surface.



1. ULTRASONICALLY-INDUCED INITIAL FLAW, (SLOT)
2. FATIGUE CRACK WHICH PROPAGATED WITH STRESS AT 125 KSI
3. FIRST BEACH MARK (OCCURRED WHEN THE FATIGUE STRESS LEVEL WAS REDUCED FROM 125 KSI TO 100 KSI)
4. FATIGUE CRACK WHICH PROPAGATED WITH STRESS AT 100 KSI
5. FATIGUE CRACK FRONT (SECOND BEACH MARK)
6. NARROW, DARK BORDER IS HEAT-STAINED SLOW GROWTH
7. BRIGHT BORDER IS ALSO SLOW GROWTH
8. RAPID FRACTURE REGION

Figure 14.5.1. Fatigue Crack in Fracture Area of D6-AC Steel Specimen
(Reference 131-35)

14.5.1.2 CRACK PROPAGATION. Crack propagation consists of a complex series of microscopic and macroscopic structural changes induced by repeated load fluctuation which results in crack growth until the intact or undamaged area can no longer sustain the load and so ruptures or fails suddenly. Stage I growth consists of fine-scale crack propagation along primary slip planes, followed by Stage II growth at right angles to the principal tensile stress. The transition is governed by the magnitude of the tensile stress; the lower the magnitude of this stress, the larger the extent of the first stage of growth. For this reason Stage I growth is favored in torsion testing because the tensile component at right angles to the Stage I crack is low. If the tensile stresses are high enough, Stage I may not be observed at all, as in sharply notched specimens, and growth occurs entirely in the second mode (Reference 719-1).

During Stage II growth the crack advances a finite increment in each loading cycle, even for propagation rates as low as 10^{-8} inches per cycle (Reference 363-3). A striation on the fracture surface with each load cycle provides a record of the passage of the fatigue crack front. If the striations are formed at high strain amplitudes, as in low-cycle fatigue, the striations will be visible to the naked eye as the classic "thumbnail" or "beach marks" having a smooth, velvety appearance (Reference 456-8). With low-strain amplitude, high-cycle fatigue, the fatigue zone will appear smooth to the naked eye, but microscopic examination will reveal similar striations. Figure 14.5.1.2 shows how the appearance of fatigue failures may vary as a function of load application. The basic process of crack propagation is not limited to crystalline solids but has also been observed in noncrystalline polymeric materials. In high-cycle fatigue most of the lifetime of unnotched specimens is spent in slipband formation and Stage I crack growth, with Stage II crack growth occurring in but a small portion of the total lifetime (Reference 719-1). Recent work has significantly increased the understanding of the precise mechanism of fatigue crack propagation (References 363-3 and 719-1).

14.5.1.3 FINAL RUPTURE. Final rupture of the remaining uncracked area resembles the fracture of a brittle material, even though the actual material is considered to be ductile. Fracture usually occurs by shear rupture on shear planes inclined 45 degrees to the tensile axis. Thus the extent of Stage II crack growth is also governed by the toughness of the material because this determines the critical-sized crack that can exist before causing final instability at a given peak stress.

Although localized plastic flow is known to be an essential element of fatigue failure, it is significant to note that in most instances failure occurs without prior indications of plastic deformation. True fatigue failure invariably takes place below a stress level which would result in rupture under static load conditions. Much of fatigue analysis is based upon the idea that there exists a stress level (endurance limit or fatigue limit) below which the material can withstand an infinite number of load cycles without failing. However, it is known that some materials (such as aluminum) do not exhibit a fatigue limit.

14.5.2 Fatigue Data Correlation

The four curves of constant fatigue life in Figure 14.5.2a represent different approaches proposed to establish a

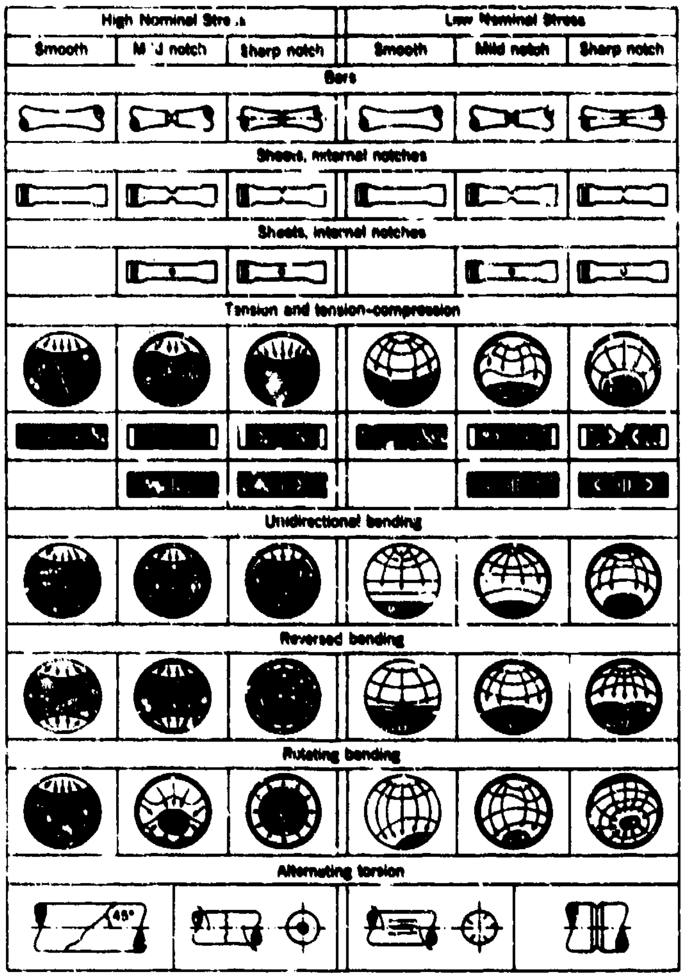
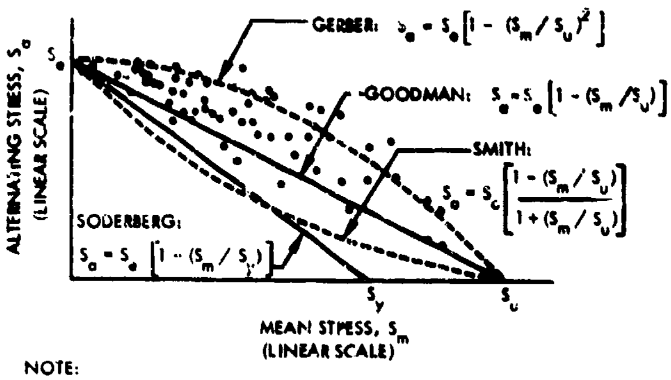


Figure 14.5.1.2. Appearance of Fatigue Failures Resulting from Various Loadings
(Adapted with permission from Reference 719-1, "Fracture of Structural Materials", A. S. Tetelman and A. J. McEvily, Jr., Wiley and Sons, 1967)



NOTE:
ALTERNATING UNIAXIAL STRESS WITH TENSILE MEAN STRESS.
DATA POINTS ARE FOR STEEL AND ALUMINUM.

Figure 14.5.2a. Goodman, Gerber, Smith, and Soderberg Diagrams
(Adapted with permission from Reference 719-1, "Engineering Considerations of Stress, Strain and Strength", R. C. Juvinall, McGraw-Hill, 1967)

relationship between alternating stress (S_a) and mean stress (S_m) when only minimal material characteristics (S_e, S_u, and S_y) are available, such as that presented in Sub-Section 12.4. An S_a-S_m diagram can be constructed for any given fatigue life. This will be necessary for materials not possessing a fatigue limit or for fatigue at elevated temperatures or in a corrosive environment. It may be seen from Figure 14.5.2a that the Goodman line is simple to plot and approximates reasonably well the bottom of the scatter band (Reference 718-1). The Goodman line is commonly used in the United States and is recommended when a generalized curve must be used for steel, aluminum, or titanium. The more conservative line proposed by Smith is often preferred for magnesium alloys and cast iron, whose test results often fall below the Goodman lines. The less conservative Gerber parabola is the standard for fluctuating fatigue stress analysis in Germany, but may be considered a logical alternative to the Goodman line only if many of the lower data points are attributed to extraneous testing factors. The Soderberg line is an approximation which precludes the possibility of failure by yielding but is very conservative and therefore not recommended.

A preferred approach which avoids exceeding the yield stress of ductile materials is illustrated in Figure 14.5.2b, wherein the fatigue fracture criterion of the Goodman line is combined with that of static yielding. Theoretically no combination of alternating and mean stresses falling below line ABC should result in fatigue fracture or plastic deformation. In the absence of fatigue limit (endurance limit) data, it is common to assume that S_e for complete reversal is one-third F_{tu} in the construction of Goodman diagrams (Reference 533-1).

The preceding approximations are adequate for correlating minimal fatigue data with design stresses for most fluid component applications. For highly critical applications involving alternating stresses, however, it is essential that comprehensive fatigue data such as that shown in Figure 14.5.2c be known. The S-N diagram provides such data as a plot of stress against number of cycles to failure at various stress ratios. A particularly useful form of S-N curves is the *constant-lifetime diagram* or *fatigue-strength diagram*, wherein alternating and mean stresses are related for various constant lifetimes as illustrated in Figure 14.5.2d. Constant-lifetime diagrams are constructed for failure criterion or for various dynamic creep criteria at elevated temperature. The curves of constant lifetime are estimated from actual tests involving various stress ratios. The S_m and S_a coordinates are turned 45 degrees from those of Figures 14.5.2a and b, and maximum and minimum stress coordinates have been added for convenience. The curves are intermediate between Goodman lines and Gerber parabolas.

Fatigue data used in constructing such curves are usually obtained from one of the following tests:

a) Axial loading

b) Plate bending

c) Rotating-beam bending

d) Torsion.

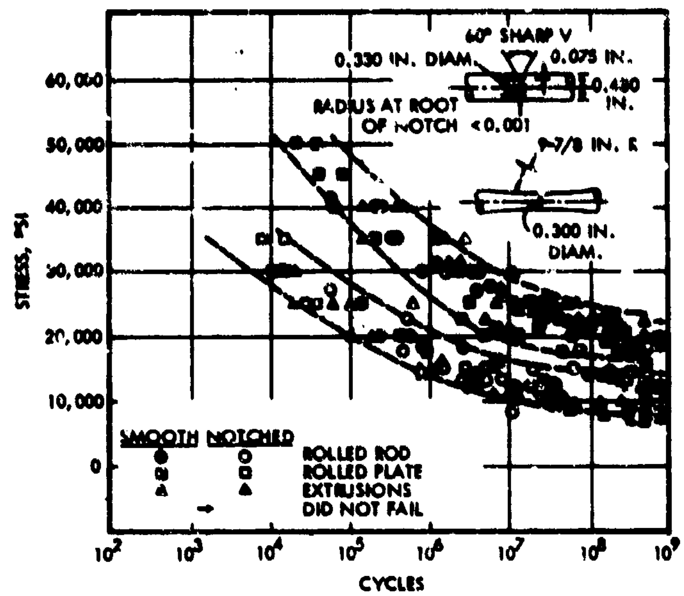


Figure 14.5.2a. Rotating-Beam Fatigue Data for 2024-T4 Aluminum Alloy
(Reference 547-12)

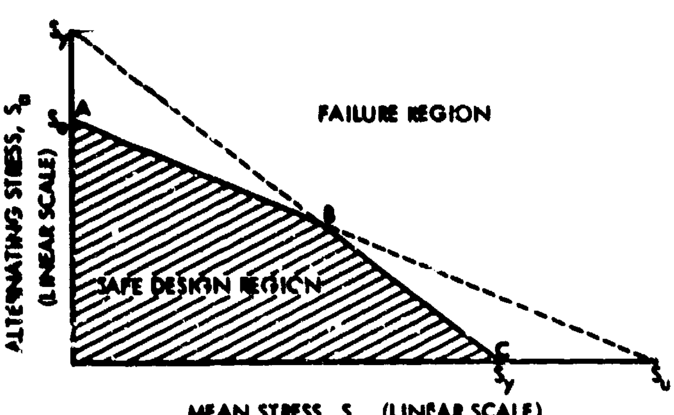


Figure 14.5.2b. Preferred Approach to Goodman Diagrams

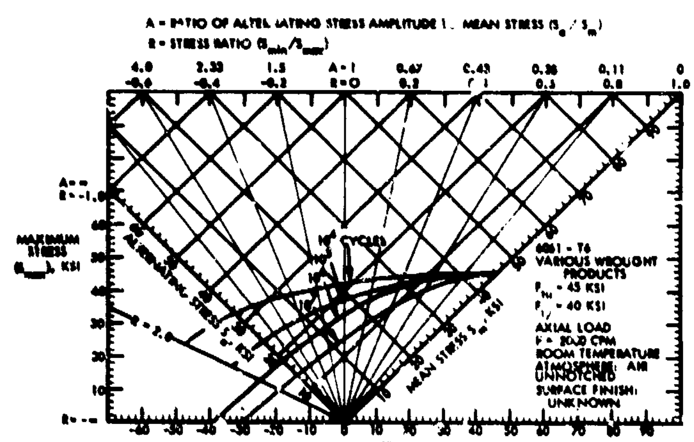


Figure 14.5.2d. Typical Constant Lifetime or Fatigue Strength Diagram
(Reference 547-12)

14.5.3 Factors Influencing Fatigue

The relationships between stress applications and fatigue life discussed above apply to carefully-controlled tests on precisely-prepared specimen. In the design of fluid components or other machine elements for fatigue, it is necessary to consider the many environmental, load application, configuration, and material factors which can influence the actual fatigue life of a component in service.

14.5.3.1 VACUUM ENVIRONMENT. Although different researchers have shown contradictory results, it is generally agreed that fatigue life in a vacuum environment is longer than in air, and that the influence is upon crack propagation rather than initiation. Although there is some disagreement as to whether this difference in fatigue life is due to oxide formations at the crack tip in air or reweldment of cracked material upon the compression cycle in vacuum, the important design consideration is that fatigue data obtained in air may be considered conservative for vacuum application (References 47-96, 398-6, and 719-1).

14.5.3.2 CORROSIVE ENVIRONMENT. The fatigue life of metals which can readily form oxide coatings is increased when specimens are tested in a vacuum or an inert atmosphere (Reference 398-6). The combined action of corrosion and fatigue gives rise to failures which characteristically occur much more quickly than would be anticipated from a consideration of the two effects acting separately. *Corrosion fatigue* (Sub-Topic 13.7.6) is quite distinct from *stress-corrosion cracking* (Sub-Topic 13.7.9) and occurs most markedly in those metals having low corrosion resistance. Corrosion fatigue failures may often be identified by the presence of many cracks which often give a serrated appearance to the fracture (Reference 718-1). Essentially it may be assumed that if a component will be subjected to fatigue loading in an environment corrosive to the material of construction, the fatigue life of the component may be less than that indicated by data from fatigue tests conducted on the material in air or vacuum.

14.5.3.3 TEMPERATURE. In general, variations in fatigue limit with temperature follow the variations in tensile strength of the same material, thereby tending to preserve a fixed ratio of fatigue strength to tensile strength. Figure 14.5.3.3a illustrates how the shape of the S-N curve for a typical steel varies with temperature. Most materials having a definite fatigue limit at normal temperatures lose this characteristic at high temperatures. Figure 14.5.3.3b illustrates the influence of cryogenic temperatures on the fatigue life of four materials of interest in liquid rocket engine systems.

14.5.3.4 STRESS AMPLITUDE

Alternating Stress. The most important factor in determining fatigue lifetime is the magnitude of the external stress or strain amplitude. In the absence of mean stress, the alternating stress (S_a) and cycle life (N) below approximately 10^5 cycles may be expressed as

$$S_a^a N = \text{constant} \quad \text{(Eq 14.5.3.4a)}$$

where

S_a = alternating stress, psi

N = cycle life, cycles

a = 8 to 15

Thus a 2 percent reduction in alternating stress can lead to as much as a 30 percent reduction in fatigue (Reference 719-1).

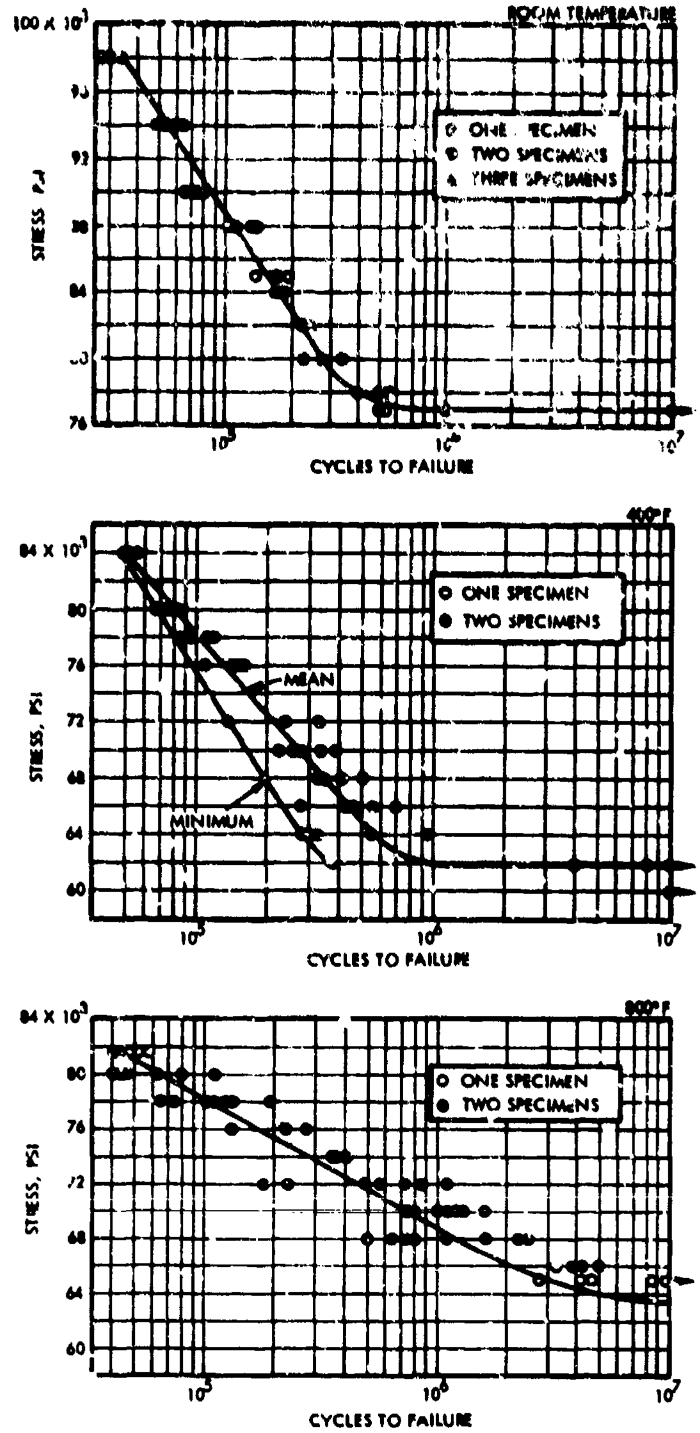


Figure 14.5.3.3a. S-N Curves for SAE 4130 Steel at Various Temperatures
(Reference 146-18)

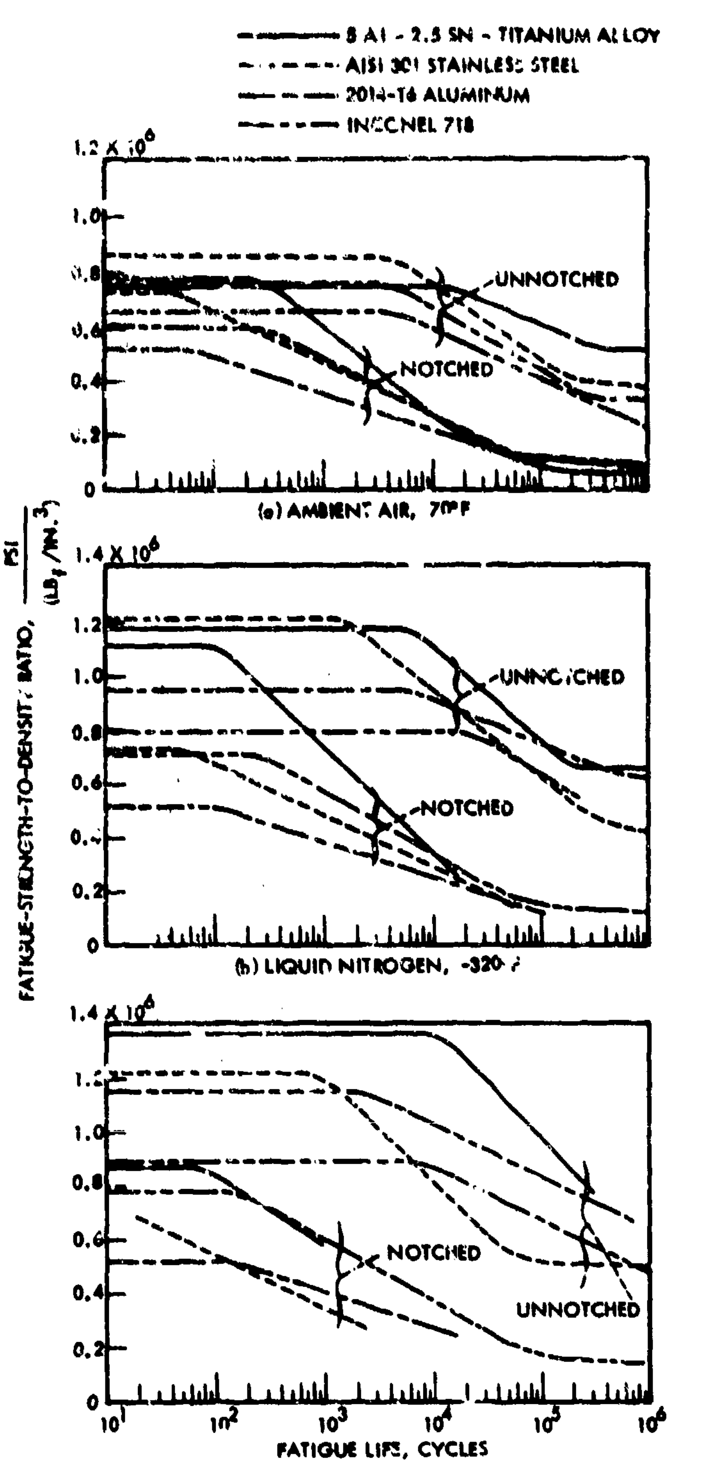


Figure 14.5.3.3b. Comparison of Fatigue-Strength-to-Density Ratios of Four Materials at Three Test Temperatures
(Reference 36-68)

Mean Stress. A structure stress-cycled about some mean stress other than zero has different fatigue characteristics from one cycled about zero mean stress. The precise reason for this is not known, but it is believed due to hysteresis effects caused by plastic flow that change the fatigue characteristics on each cycle (Reference 598-1). A mean tensile stress decreases fatigue life, whereas a mean compressive stress increases fatigue life. Since distortion energy (or octahedral shear stress) is the same for tension and compression, distortion energy is not a valid criterion for the influence of mean stress (Reference 716-1).

For low-cycle fatigue it is preferable to use strain amplitude rather than stress amplitude. This arises from the fact that gross plastic straining occurs in low-cycle fatigue, and, especially for materials with relatively little strain hardening, strain is a more sensitive measure than stress or fatigue damage. For this reason the Coffin-Manson relation is used for low-cycle fatigue:

$$N^n = \frac{c_f}{e_p} \qquad \text{(Eq 14.5.3.4b)}$$

where

c_f and n = material constants

e_p = the plastic strain amplitude.

Typical values of n are found experimentally from 0.2 to 0.6. If insufficient data are available to establish the value of n for a given material, it is customary to use n = 0.5, based on an analysis by Coffin of a large amount of low-cycle fatigue data. If a monotonic tensile test is considered as a 1/4-cycle fatigue test, the above equation shows that, with n = 0.5 and $e_p = e_f$ (the strain at fracture in the tensile test), $c_f = e/2$. Thus, in the absence of fatigue data an approximate design formula for low-cycle fatigue is:

$$N = \left(\frac{e_f}{2e_p}\right)^2 \qquad \text{(Eq 14.5.3.4c)}$$

14.5.3.5 FREQUENCY OF ALTERNATING STRESS. Cyclic frequency has very little affect upon fatigue limit in the usual range of testing frequencies below 200 cps. For steel there is a slight increase in fatigue limit with increase in frequency, reaching a maximum at from 1200 to 1800 cps, beyond which there is a decrease. There is also evidence of a 2-to-1 reduction in crack propagation rate with an increase in frequency of 1 to 100 for aluminum, but a much smaller change in crack propagation rate in vacuum indicates that corrosion fatigue is probably more responsible than the change in frequency (Reference 363-3).

14.5.3.6 SIZE. Most fatigue data are gained from rotating-beam specimens of 0.3-inch diameter. Although most test data indicate little size effect up to about 2-inch diameter, there is evidence that fatigue limit decreases with increase in size (Figure 14.5.3.6). Proposed explanations for a size effect include:

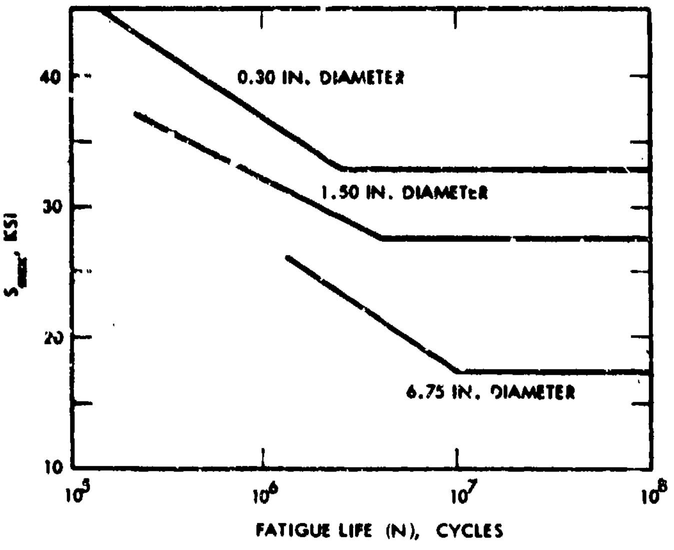


Figure 14.5.3.6. Effect of Size on the Fatigue Strength of Steel
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

a) A statistical size effect related to the probability of finding a critical flow in the most highly stressed regions (including the fact that a large specimen has more volume subjected to a high-stress level than does a small one)

b) A change in metallurgical structure and properties as a function of absolute size

c) A notch-size effect related to the steepness of the stress gradient at the root of a notch as a function of notch radius (Reference 719-1).

Whatever the reason, there is evidence to suggest that a size factor of from 0.6 to 0.75 should be applied to the fatigue limit of component elements exceeding 4 inches in diameter, and a factor of 0.9 should be applied to elements between 0 to 4 inches in diameter.

14.5.3.7 SHAPE. Reversed bending tests show a lower fatigue limit for rectangular and diamond cross sections than for circular cross sections (Figure 14.5.3.7) The reduction in fatigue limit for rectangular cross sections is approximately 0.9 for steels and somewhat lower for aluminum. This difference is due in part to the stress concentration effect associated with sharp corners on rectangular or diamond cross sections.

14.5.3.8 SURFACE FINISH. In general, a highly polished surface gives the highest fatigue life, although there is evidence suggesting that the uniformity of finish is more important than the finish itself. A single scratch on a highly polished surface would probably lead to a fatigue life somewhat lower than for a surface containing an even distribution of scratches. Typical trend data are shown in Figure 14.5.3.8 for steel. Surface finish influence is very closely related to other conditions, such as residual stresses resulting from cold working in the finishing process. Polished, ground, and machined surfaces give significantly higher endurance strengths when the surface markings produced in finishing are parallel to the loading. The effect of surface finish on steel parts subjected to less than 1000

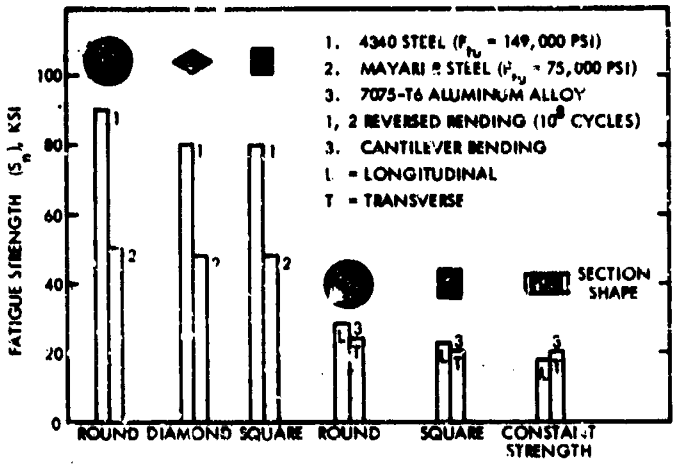


Figure 14.5.3.7. Effect of Section Shape on Fatigue Strength
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

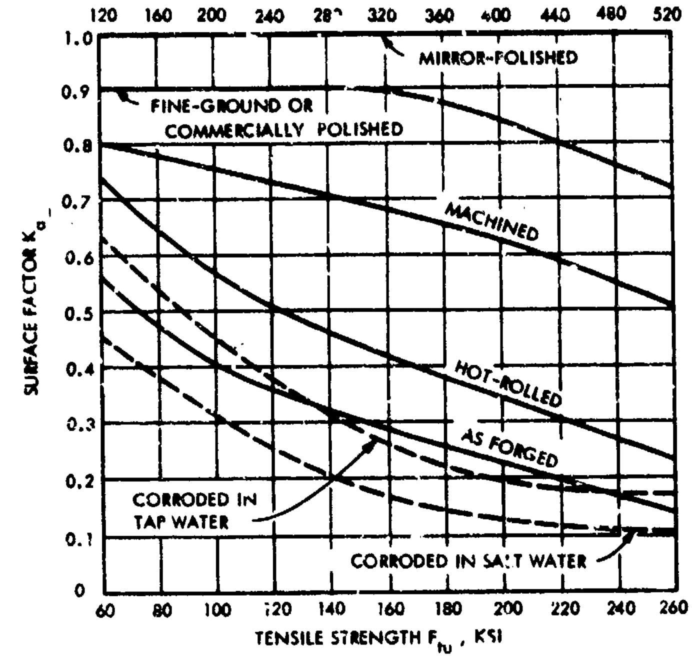


Figure 14.5.3.8. Reduction of Fatigue Strength Due to Surface Finish for Steel Parts
(Adapted with permission from Reference 716-1, "Engineering Considerations of Stress, Strain and Strength", R. C. Juvinall, McGraw-Hill, 1967)

cycles is generally considered negligible, therefore no correction is made for surface finish (Reference 716-1). Decarburizing reduces the fatigue limit of a surface. Case hardening and nitriding improve fatigue life. Heat treating usually decreases fatigue strength (Reference 19-270). Usually no fatigue surface correction factor is applied to nonferrous metals such as aluminum and magnesium. The harder materials with uniformly fine-grain structure are most susceptible to fatigue weakening by surface roughness Anodizing aluminum causes a reduction in fatigue properties.

14.5.3.9 MATERIALS. High ductility (particularly notch ductility) and good impact strength are important for finite-life fatigue applications, but have little affect on fatigue limits. Conversely, hardness and tensile strength are important in establishing fatigue limits, but may not be important in finite-life fatigue depending on the mode of cyclic straining.

14.5.3.10 STRESS CONCENTRATIONS. Fatigue cracks almost always start at stress concentrations, and the notch sensitivity of a material is a primary indicator of the material's ability to resist fatigue failure. The difference between notched and unnotched fatigue specimens is apparent in Figure 14.5.2c. The sharpness of the stress concentration or radius of the notch apex can be of significance, especially when corrosion factors are present. In general the presence of any kind of stress raiser lowers the fatigue life of any component member. Stress concentration factors are treated in Sub-Section 14.6.

14.5.3.11 STRAIN HARDENING, STRESS HISTORY, AND CUMULATIVE DAMAGE. Strain hardening is indicated in Figure 14.5.3.11a by the variation in shape of the stress-strain hysteresis loop during the first loading cycles before attaining a saturation level of hardening. The curve drawn through the saturation stress for each plastic strain amplitude describes a cyclic stress-plastic strain curve. This cyclic load-deflection curve may be higher (for strain-hardening materials) or lower (for strain-softening materials) than that obtained in monotonic loading. Three variations in this characteristic are shown in Figure 14.5.3.11b. The stress-range/life characteristics of aluminum is essentially independent of prior working history.

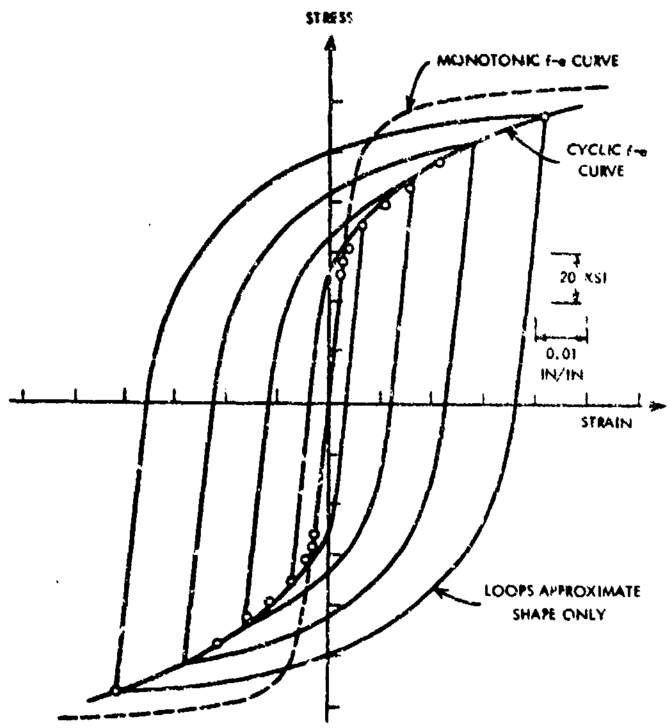


Figure 14.5.3.11a. Monotonic and Cyclic Stress-Strain Behavior of SAE 4340 Steel
(Adapted with permission from Reference 719-1, "Fracture of Structural Materials," A. S. Tetelman and A. J. McEvily, Jr., Wiley and Sons, 1967)

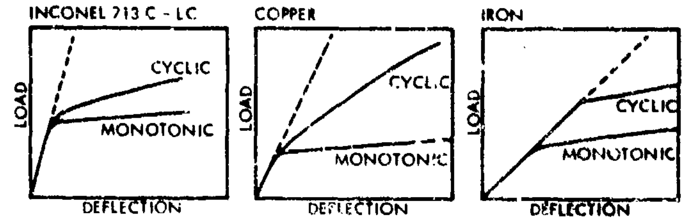


Figure 14.5.3.11b. Cyclic and Monotonic Load-Deflection Curves for Three Metals

Structures which have been hardened, such as cold-worked parts, may be softened when subjected to cyclic loads. Figure 14.5.3.11c indicates that as strain cycling is continued the annealed material hardens and the cold-worked material softens under the influence of large alternating strains. A beneficial residual stress system in a cold-worked surface may be rendered ineffective by cyclic loading into the plastic range (Reference 719-1).

Stressing a member n times to some value of stress f above the endurance limit but below the S-N curve usually reduces the remaining fatigue life of the member (Figure 14.5.3.11d). In actual practice most aerospace fluid components are subjected to a wide variety of loads in random sequence. One method used as a guide to the cumulative effects of random loading is based on Miner's rule, which states that failure will occur when the sum of the ratio of the number of cycles at the ith stress level, n_i, to the constant amplitude lifetime at that level, N_i, summed over all i stress levels is equal to unity, or

$$\sum_{i=1}^{i=i} \frac{n_i}{N_i} = 1 \qquad \text{(Eq 14.5.3.11)}$$

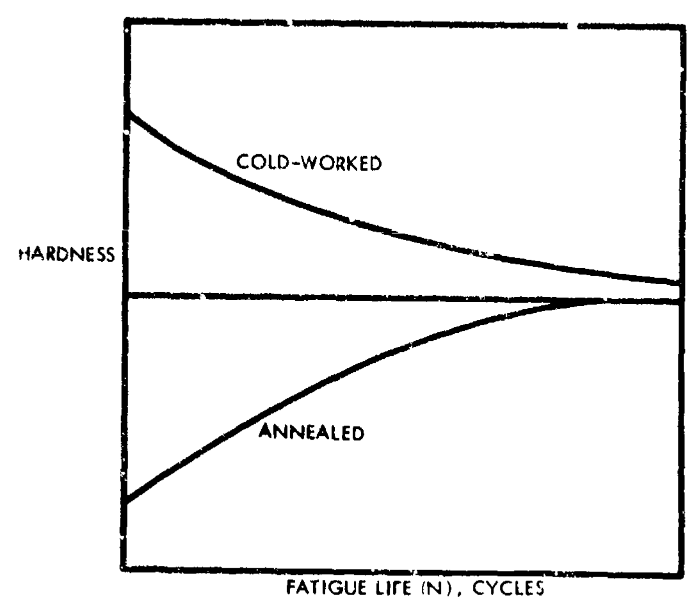


Figure 14.5.3.11c. Influence of Cycling on the Hardness of Annealed and Cold Worked Copper Specimens
(Adapted with permission from Reference 719-1, "Fracture of Structural Materials," A. S. Tetelman and A. J. McEvily, Jr., Wiley and Sons, 1967)

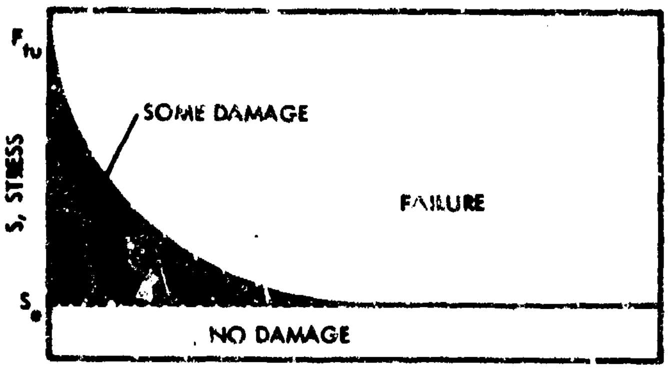


Figure 14.5.3.11d. Typical S-N Diagram Showing Cumulative Damage Region

It is generally considered that overstressing above the endurance limit for periods shorter than necessary to produce failure at that stress (the shaded area of Figure 14.5.3.11d) reduces the endurance limit in a subsequent test. Similarly, understressing below the endurance limit may increase it (Reference 132-1). The results of many fatigue tests, especially at elevated temperatures, indicate that the frequently-accepted assumption that damage is proportional to cycle ratio (Eq 14.5.3.11) errs on the unsafe side under certain conditions (Reference 146-18). When stress levels are progressively increased or progressively decreased, tests have shown $\Sigma n/N$ to vary from 0.18 to 23 (Reference 716-1). Figure 14.5.3.11e illustrates the manner in which the sequence of load application can affect fatigue life.

For materials with a definite endurance limit, there is evidence that understressing for many cycles just below the endurance limit will result in a specimen actually stronger in fatigue than when new. This phenomenon is usually attributed to strain strengthening of highly localized vulnerable regions or to strain aging. Remarkable increases in the

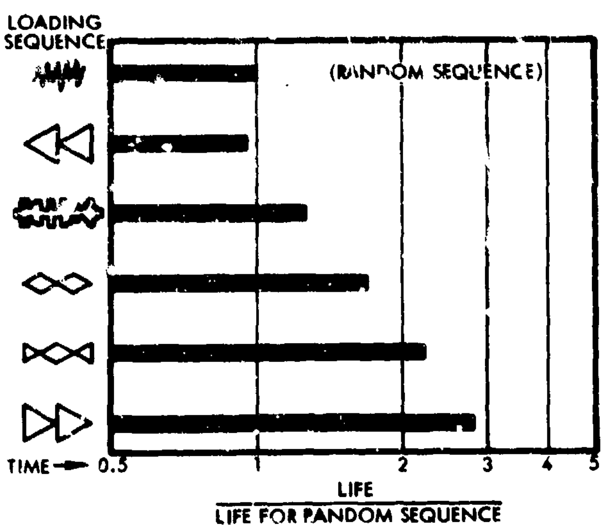


Figure 14.5.3.11e. Effect of Loading Sequence on Fatigue Life for 7075-T6 Aluminum

(Adapted with permission from Reference 719-1)

endurance limit of SAE 1045 steel have been obtained by applying successive periods of 10^7 stress cycles beginning just below the normal endurance limit and raising the stress in small increments. This process, known as coaxing, has proven feasible with SAE 2340 steel but ineffective with 7075-T6 and 2025-T6 aluminum alloys and some other materials. Since at best the coaxing treatment requires many millions of accurately-controlled stress cycles, its application has been confined largely to laboratory experiments (Reference 716-1). Although obviously unpracticable for most fluid component applications, coaxing might conceivably find application in highly optimized aerospace fluid systems if the technique were to be proven effective with high-strength alloys. Data are not available to show whether high endurance limits obtained through coaxing are susceptible to subsequent reduction from high amplitude alternating strains as shown in Figure 14.5.3.11c for copper.

14.5.4 Predicting Fatigue Life and Endurance Limits

No standard for obtaining working fatigue-stress relations has been universally accepted. Fatigue life prediction requires correlation of loading with fatigue resistance of the material, and the approach employed is largely dependent upon which of the following types of data are available:

a) Constant lifetime fatigue-strength diagrams

b) Endurance limit or fatigue limit

c) Static tensile strength only.

14.5.4.1 CONSTANT LIFETIME FATIGUE-STRENGTH DIAGRAMS. These diagrams provide the most comprehensive data on fatigue characteristics and should be used when available. MIL-HDBK-5A (Reference 547-12) currently includes constant lifetime diagrams of both unnotched and notched specimens of the following materials:

a) AISI 4340 steel bar at F_{tu} = 125, 150, 200, and 260 ksi, with 150 ksi data at 600, 800, and 1000°F as well as at room temperature.

b) 2014-T4, 2014-T6, and 7075-T6 aluminum alloys (wrought)

c) Ti-6Al-4V bar and sheet

d) M-252 alloy at 1500°F

e) Udimet 500 alloy bar at 1200 and 1650°F.

These curves may also be applied to similar alloys; for example, the AISI 4340 diagram may also be used with AISI 2330, 4130, and 8330 alloys.

14.5.4.2 ENDURANCE LIMIT OR FATIGUE LIMIT. For materials which demonstrate a fatigue limit or endurance limit, S_e, this value is usually tabulated with summaries of material properties such as those in Sub-Section 12.4 of this handbook. This fatigue limit is that value of totally reversed stress below which the material can theoretically withstand an infinite number of stress cycles.

Loading (Stressing). Unless otherwise specified, most tabulations of fatigue (endurance) limit or S-N curves such as Figure 14.5.2c are based on fully reversed (usually rotating) bending. In the absence of specific data, the fatigue limit for various loading conditions may be approximated by multiplying reversed or rotating bending fatigue limits by the following factors:

a) *Reversed axial loads:* 0.9 no bending; 0.6 to 0.85 indeterminate bending (note: the lower values should be used when a probability of eccentric loading exists).

b) *Reversed torsional loads:* 0.58 ductile metals; 0.8 cast iron.

Where loads are not totally reversed, as in prestressed members such as threaded connectors or bolted joints, the fatigue limit may be expected to be higher than that tabulated for fully reversed loading. Recommended practice, however, is to use the tabulated fatigue limit with no correction for the less severe service loading.

Size, Shape and Proportion

a) Multiply fatigue limit values obtained on 0.3-inch diameter rotating-beam specimens by the appropriate factor listed below:

Diameter	*Reverse Bending*	*Axial*	*Torsional*
D≤0.4 in.	1.0	1.0	2.0
0.4 in.<D<4.0 in.	0.9	1.0	0.9
4.0 in.<D	0.6 - 0.75	0.6 - 0.75	0.6 - 0.75

b) Use force-flow diagrams to analyze stress distribution

c) Members should be sized and positioned to carry distributed loads rather than to have individual members carry concentrated loads.

14.5.4.3 PREDICTION FROM STATIC TENSILE PROPERTIES. When data on fatigue characteristics are not available, it is common practice to estimate fatigue life by means of some form of the Goodman or Soderberg diagram, as described in Sub-Topic 14.5.2.

14.5.5 Designing to Prevent Fatigue Failure

The preceding discussion of fatigue refers to many of the techniques which may be employed to minimize the probability of fatigue failure. The following summary of design considerations can be used to assist in the evaluation of fatigue-sensitive designs.

The simplest way to decrease fatigue stress is to increase the size of critical sections, but this approach has obvious disadvantages. For the simple situation in which all mean stresses are zero

$$S_f = \frac{S_n}{K_f S_a} \qquad \text{(Eq 14.5.5a)}$$

where

S_f = fatigue factor of safety, dimensionless

S_n = fatigue strength, psi

K_f = fatigue strength reduction factor, dimensionless

S_a = alternating stress amplitude, psi

Good fatigue proportions require that both S_f and K_f be fixed, and that nominal alternating-stress amplitude be adjusted such that for all elements of the component

$$\frac{S_a}{S_n} = \frac{1}{S_f K_f} = \text{constant} \qquad \text{(Eq 14.5.5b)}$$

Fatigue strength, S_n, is determined as described in Sub-Topics 14.5.2 and 14.5.4. Equations (14.5.5a) and (14.5.5b) are also used when mean stresses are not zero.

14.5.5.1 LOAD EVALUATION. It is imperative in fluid component design that load estimation account for pressure fluctuations. In rocket propulsion systems it is not unusual to observe high frequency pressure fluctuations (500 to 1000 cps) of a peak-to-peak amplitude nearly equal to the system pressure. Under some circumstances this loading may occur at all times during engine operation. Fatigue analysis must account for maximum service life of the component. For example, if a rocket engine must be capable of sustaining twelve full-duration firings of 600 seconds and operate with a 1000 cps pressure fluctuation, the accumulated cycles will be:

$$12 \times 600 \times 1000 = 7{,}200{,}000 \text{ cycles}$$

This is exclusive of strain cycles resulting from proof pressure testing, flow testing, and other component-level tests. In addition it is common practice during rocket engine development to reuse certain components on several engines during development, thereby subjecting the component to far more load cycles than engine service life would indicate. Accordingly, safe design practice is to stress all components below the fatigue limit, where a fatigue limit exists for the material. This can necessitate extremely conservative design with those nonferrous alloys which evidence no true fatigue limit and for which only fatigue strength at N = 10^6 cycles is provided. See Figure 14.5.5.1 for means of adjusting stress distributions to avoid fatigue failure.

14.5.5.2 STRESS CONCENTRATIONS (See Sub-Section 14.6)

a) *Minimize all stress concentrations*

b) Use maximum radii and fillets

c) Ensure tangential blend of radii and plane surfaces (Figure 14.5.5.2a)

d) Where stress concentrations cannot be eliminated, minimize effect by relieving adjacent areas (Figures 14.5.5.2b, c, and d)

e) Locate necessary stress concentrations in areas of low nominal stress

f) Locate welds away from cross-sectional discontinuities.

g) Ensure that welds are full penetration (both by specification and by providing adequate accessibility for performing the welding operation)

h) Evaluate stress concentrations associated with shrink or press fits

i) When undercutting fillets, avoid increasing nominal stress

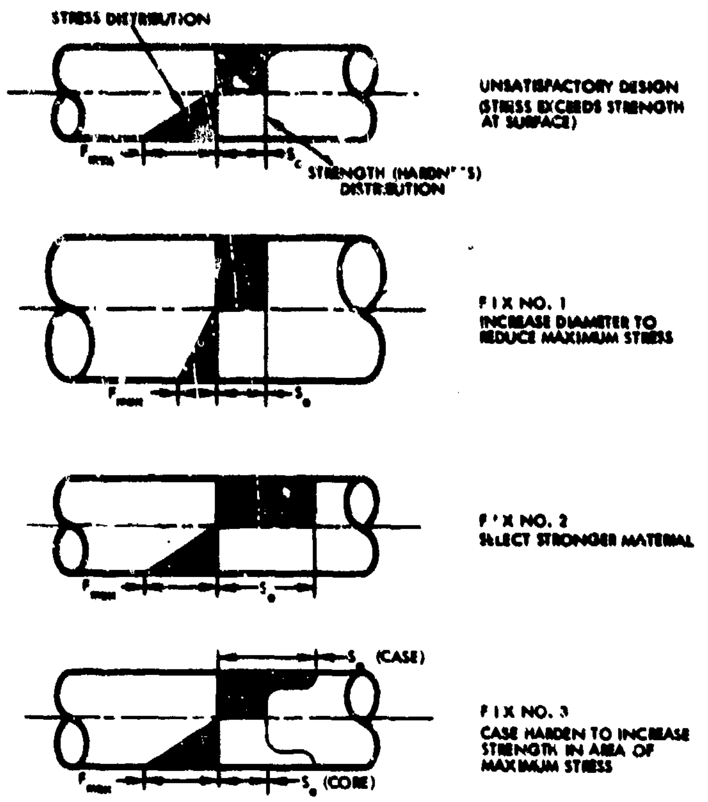


Figure 14.5.5.1. Techniques of Adjusting Stress Distribution in a Round Bar Under Bending or Torsion

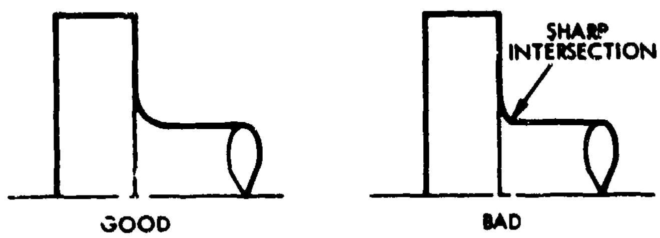


Figure 14.5.5.2a. Tangential (Good) and Sharp (Bad) Fillets

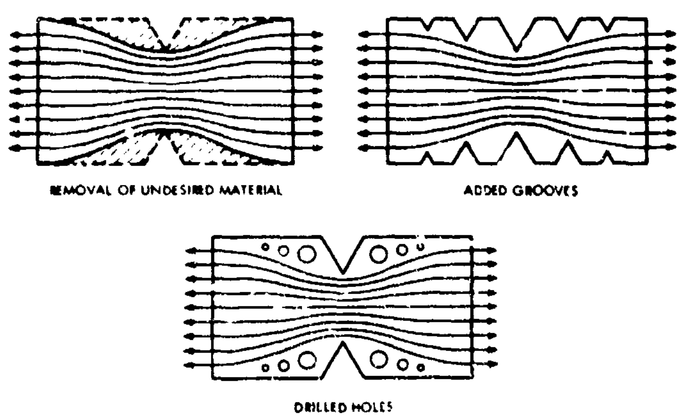


Figure 14.5.5.2b. Means of Reducing the Stress Concentration in a Notched Flat Plate
(Adapted with permission from Reference 716-1)

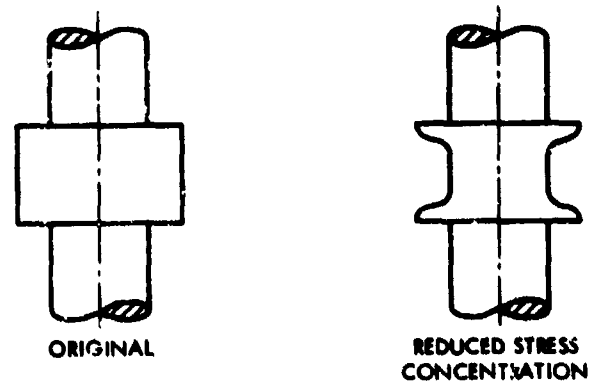


Figure 14.5.5.2c. Narrow Collars Reduce Stress Concentration
(Adapted with permission from Reference 716-1, "Engineering Considerations of Stress, Strain and Strength," R. C. Juvinall, McGraw-Hill, 1967)

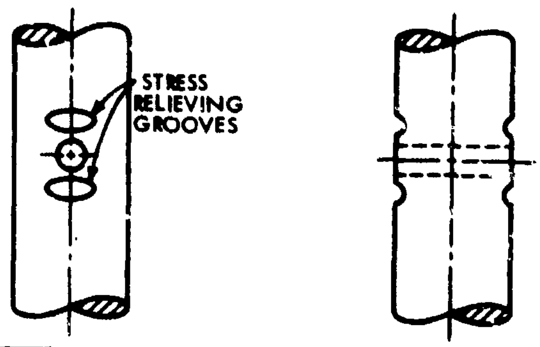


Figure 14.5.5.2d. Grooves Reduce Stress Concentration Around Hole
(Adapted with permission from Reference 716-1, "Engineering Considerations of Stress, Strain and Strength," R. C. Juvinall, McGraw-Hill, 1967)

Figure 14.5.5.2e illustrates several ways of reducing stress concentrations in a stepped shaft. Figure 14.5.5.2f shows a bolt and nut designed to resist fatigue by minimizing stress concentrations.

The *fatigue strength-reduction factor*, K_f, is obtained from the following equation

$$K_f = 1 + (K_t - 1)q \qquad \text{(Eq 14.5.5.2)}$$

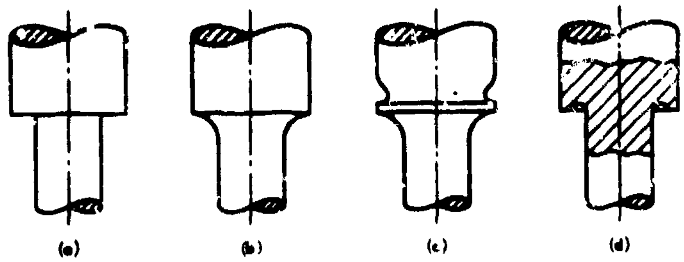
where

K_f = fatigue strength-reduction factor, dimensionless

K_t = theoretical stress concentration factor, dimensionless (Sub-Topic 14.6.3)

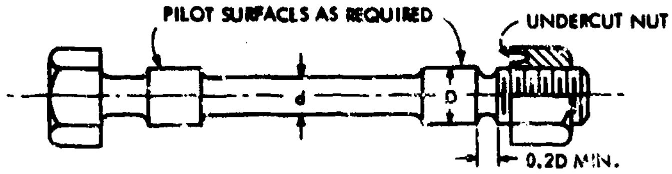
q = notch sensitivity, dimensionless (from Figure 14.5.5.2g)

In estimating q it may be noted that wrought copper, nickel, magnesium, and titanium alloys have roughly the same notch sensitivities as wrought steels of the same ultimate strength. Wrought metals are more notch sensitive than cast metals, but wrought metals tested in the transverse direction have approximately the same notch sensitivity as cast metals of the same ultimate strength (Reference 1-409). The procedure is:



(a) SEVERE STRESS CONCENTRATION; (b) USE LARGE RADIUS IF POSSIBLE; (c) ADDED GROOVE GIVES FURTHER BENEFIT; (d) UNDERCUT SHOULDER HELPS IF MODIFICATIONS (b) AND (c) CANNOT BE USED.

Figure 14.5.5.2e. Reducing Stress Concentrations in a Stepped Shaft
(Adapted with permission from Reference 716-1)



NOTE: THREADS TO BE ROLLED; DIAMETER d TO BE SLIGHTLY LESS THAN THREAD ROOT DIAMETER; ALL FILLET RADII TO BE LARGE.

Figure 14.5.5.2f. Reduction of Stress Concentrations in a Bolt and Nut Design
(Adapted with permission from Reference 716-1)

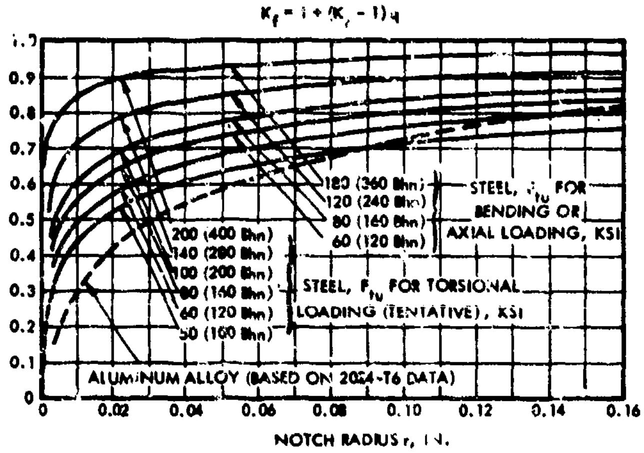


Figure 14.5.5.2g. Notch Sensitivity Factor, q
(Adapted with permission from Reference 716-1, "Engineering Considerations of Stress, Strain, and Strength," R. C. Juvinall, McGraw-Hill, 1967). (Curves attributed to Peterson (Adapted with permission from Reference 737-1, "Stress Concentration Design Factors," R. E. Peterson, Wiley and Sons, 1953)

a) First determine K_t from the geometry of the part

b) Determine q on the basis of the material

c) Calculate K_f from Equation (14.5.5.2)

d) Divide the endurance limit (or fatigue limit) of the material by K_f.

14.5.5.3 SURFACE FINISH AND TREATMENT

a) For steel, obtain surface factor, K_s, for fatigue limit from Figure 14.5.3.8, bearing in mind that these curves represent the bottom of data scatter bands. These curves are used only for cycle life greater than 1000 cycles; for cycle life less than 1000 cycles, $K_s = 1$. Figure 6.3.2.13 (from Section 6.0 of this handbook) may be of value in modifying K_s from Figure 14.5.3.8 if a less conservative value of K_s is desired, based upon the maximum depth of surface irregularities.

b) For aluminum, magnesium, and most nonferrous materials, a surface factor of unity is normally used (Reference 731-1), although there are indications that the application of some surface factor less than unity may be appropriate. For hard nonferrous materials of uniformly fine-grain structure, it may be desirable to regard surface roughness as a special case of geometric stress concentrations (Reference 716-1).

c) Application of Figure 14.5.3.8 surface finish data should be undertaken with the realization that most aerospace fluid components, especially in small sizes, are finished to a much smoother surface than that associated with the corresponding commercial operation. Commercial polishing, for example, encompasses roughness heights from 0.5 to 32 microinches.

d) Fine-grinding or polishing of fatigue-critical locations, especially those susceptible to tool marks, may improve fatigue life.

e) Shot peening or other techniques which form residual compressive stresses in the stressed surface should be considered for specific applications. (The shot peening of the interior of titanium tanks for N_2O_4 service is an example, although the primary purpose was to prevent stress corrosion cracking rather than fatigue.)

f) Case hardening can be used in some instances to increase local strength without increasing size or weight, as shown in Figure 14.5.5.1. Shafts in bending or torsion, wherein stress is a maximum at the outer surface, are examples of such an application. In such cases, the stress gradient must be plotted and superimposed on the fatigue limit gradient to ascertain minimum margin of safety.

14.5.5.4 MATERIALS

a) For low-cycle fatigue applications (below 10,000 cycles for this purpose), select materials with high notch ductility and good impact strength.

b) For high-cycle fatigue applications, select materials with high fatigue (endurance) limits. Roughly, above 100,000 cycles, it may be assumed that fatigue strength increases in direct proportion to ultimate strength or hardness (Reference 1-314). Fatigue strength increases with ultimate strength or hardness only to about 200,000 psi ultimate strength. In general, for higher strength materials there is little increase in fatigue strength.

c) Avoid materials with sharp ductile-brittle transition temperatures, especially for cryogenic service.

d) For low-cycle fatigue applications, reduce the range of cyclic elastic strain by selecting materials with high modulus of elasticity, E, and reduce cyclic plastic strain by selecting materials with high cyclic yield strength.

e) For most applications strain-hardening materials are preferred over strain-softening materials.

f) Know whether fatigue (endurance) limit values used are minimum or average. When only average values of S_e are available, this average value should be multiplied by 0.75 in the absence of other data.

g) To avoid a 10 to 20 percent reduction in fatigue limit, specify the direction of grain flow for forgings, extrusions, and other directionally oriented materials to ensure that cyclic stresses act in the longitudinal rather than transverse direction.

14.5.5.5 RESONANT FREQUENCY. Vibration of a member at natural frequency can lead to rapid accumulation of many high amplitude fatigue cycles and subsequent failure. The probability of such resonant fatigue failures can be minimized by the following:

a) Design for maximum stiffness by providing for maximum moment of inertia in bending and by using ribs or flanges, as appropriate. This will both increase natural frequency and reduce amplitude of resonant vibration.

b) Reduce effective length of the member by providing supports which restrain motion in the direction of vibratory motion.

c) Employ damping by such means as: surrounding or filling the member with liquid, substituting riveted joints for welded joints in large components, optimizing hysteresis associated with cyclic shear strain in viscoelastic gaskets and adhesives, and utilizing a material with a high specific damping energy (this usually means use of a magnetic material such as 403 stainless steel to take advantage of magnetoelastic hysteresis) (Reference 1-409).

14.6 STRESS CONCENTRATION FACTORS

14.6 STRESS CONCENTRATION FACTORS

The elementary stress formulas of Sub-Section 14.3 are based on members having a constant section or section with gradual change in contour. The presence of shoulders, grooves, holes, keyways, threads, etc., result in increased localized stress or stress concentrations, a measure of which is the *stress concentration factor*, K, defined for normal stress (tension or bending) as:

$$K = K_t = \frac{f_{max}}{f} \qquad \text{(Eq 14.6a)}$$

where

K = stress concentration factor, dimensionless

K_t = theoretical stress concentration factor, dimensionless

f_{max} = maximum or effective stress, psi

f = calculated stress based upon load and area only, psi

or for shear stress (tension)

$$K = K_{ts} = \frac{f_{smax}}{f_s} \qquad \text{(Eq 14.6b)}$$

where

K_{ts} = theoretical shear stress concentration factor, dimensionless

f_{smax} = maximum or effective shear stress, psi

f_s = calculated shear stress based upon load and area only, psi.

From Sub-Section 14.5, the fatigue strength-reduction factor, K_f, is determined from the theoretical stress concentration factor, K_t, and the notch sensitivity factor, q, in the formula:

$$K_f = 1 + (K_t - 1)q \qquad \text{(Eq 14.6c)}$$

Where no q data are available, it is suggested the theoretical stress concentration factor, K_t, be used alone. If the notch sensitivity factor is not used, the error will be on the safe side (Reference 737-1).

14.6.1 Ductile Materials

Ductile materials exhibit stress-strain curves as shown in Figure 14.6.1. Ordinarily, a ductile member with a steady stress (uniaxial) does not lose strength due to the presence of a notch. If, however, the part is loaded with a steady stress and subjected to shock loading, or subjected to high or low temperatures, or if the part has sharp discontinuities, the material may behave in the manner of brittle materials. If there is doubt, the stress concentration factor should be applied. Where a part is loaded with a steady stress with an alternating stress superimposed, the stress concentration factor is usually applied to the alternating component only (see Sub-Section 14.5).

14.6.2 Brittle Materials

Brittle materials exhibit stress-strain curves such as that shown in Figure 14.6.2. In the design of members of brittle materials the stress concentration factor should always be used. Where steady stresses and alternating stresses are superimposed, the stress concentration factor is applied to both.

14.6.3 Stress Concentration Factor Charts

In using the following charts the maximum normal or shear stress, f_{max} or f_{smax} is found by multiplying the nominal stress, f, as found by the elementary formulas in Sub-Section 14.3, by the stress concentration factor, K, as follows

$$f_{max} = Kf \qquad \text{(Eq 14.6.3a)}$$

$$f_{smax} = Kf_s \qquad \text{(Eq 14.6.3b)}$$

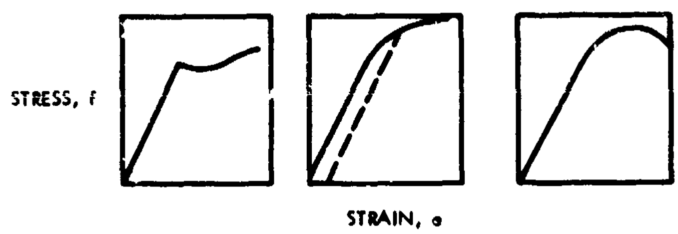


Figure 14.6.1. Stress-Strain Behavior of Ductile Materials

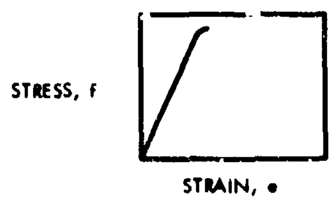


Figure 14.6.2. Stress-Strain Behavior of Brittle Materials

Additional factors for a large number of situations may be found in Peterson's classic book on the subject, Reference 737-1. Detailed mathematical approaches to stress concentrations may be found in the numerous references listed at the end of this section.

14.6.3.1 FLAT PLATES WITH HOLES. Figures 14.6.3.1a through h give stress concentration factors for flat plates with holes.

14.6.3.2 CLEVIS OR TRUNNION CONNECTORS. In many applications load is transmitted from one member to another by pin or trunnion connections through circular holes as shown in Figure 14.6.3.2a. In this case the inner surface of the hole is subjected to a high bearing stress which results in a stress concentration effect. Some results based on photoelastic tests are shown in Figure 14.6.3.2b for close-fitting pins. Curve A is based on a consideration of net section

$$f_{net} = \frac{P}{(w - a)h}$$

where

f_{net} = net internal stress, psi

P = applied force, lb_f

w, a, and h = dimension in Figure 14.6.3.7a.

Whereas curve B is based on bearing area

$$f_{bearing} = \frac{P}{ah}$$

14.6.3.3 STEPPED CYLINDERS. Figure 14.6.3.3 gives stress concentration factors for a stepped cylinder with shoulder fillets.

14.6.3.4 BARS WITH SHALLOW FILLET GROOVES. Figures 14.6.3.4a through e give stress concentration factors for flat and round bars with shallow fillets or grooves.

14.6.3.5 CIRCULAR SHAFTS WITH TRANSVERSE HOLES. Figures 14.6.3.5a, b, and c give stress concentration factors for shafts with transverse holes.

14.6.3.6 STEPPED BARS AND SHAFTS WITH CIRCULAR FILLETS. Figures 14.6.3.6a through g give stress concentration factors for stepped flat bars and circular shafts with fillets.

14.6.3.7 CIRCULAR SHAFTS WITH GROOVES. Figures 14.6.3.7a through d give stress concentration factors for circular shafts with circular grooves.

14.6.3.8 BARS WITH NOTCHES. Figures 14.6.3.8a through f give stress concentration factors for flat bars with notches.

14.6.3.9 CIRCULAR SHAFT WITH KEYWAY. Figure 14.6.3.9 gives stress concentration factors for a circular shaft with a longitudinal keyway. Note the radius in the keyway.

14.6.3.10 DIAMETRALLY-LOADED RINGS. Figure 14.6.3.10 gives stress concentration factors for both internally and externally-loaded rings with loading in two places only.

14.6.3.11 U-SHAPED MEMBER. Figure 14.6.3.11 gives stress concentration factors for a flat U-shaped member.

14.6.3.12 FLAT BAR WITH PROTRUSION. Figure 14.6.3.12 gives stress concentration factors for a flat bar with a protrusion.

14.6.3.13 FLANGE IN BENDING. Figure 14.6.3.13 gives stress concentration factors for a straight, infinitely long flange in bending.

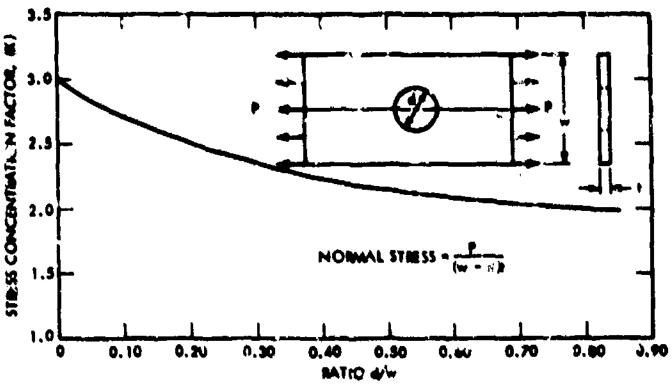


Figure 14.6.3.1a. Effect of Hole Size on Stress Concentration Factor
(Adapted with permission from Reference 598-1, Engineering Design, J. H. Faupel, Wiley, 1964)

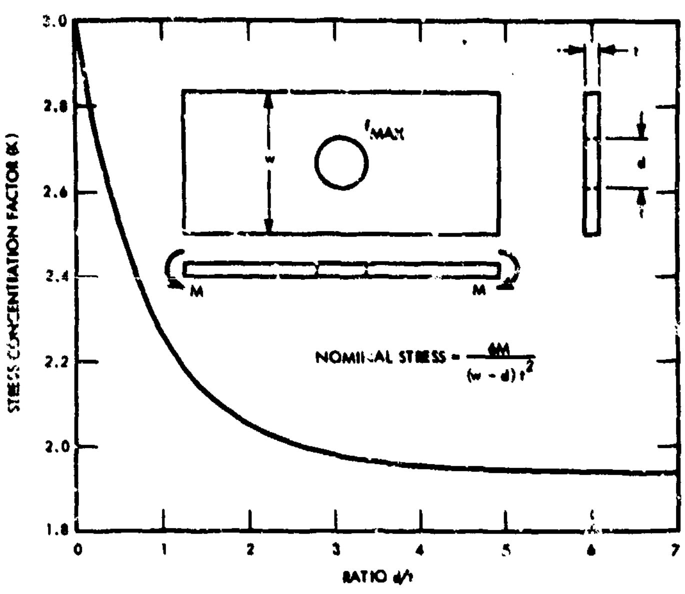
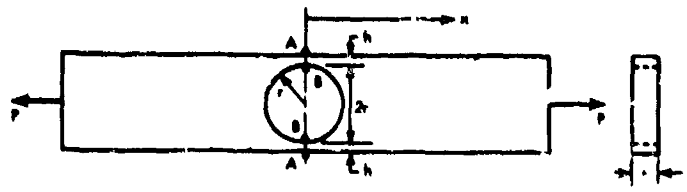


Figure 14.6.3.1b. Stress Concentration Factor for Bending of Plate With Hole
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)



f_x (OUTER EDGE) = $\frac{4Pr}{t}\left(\frac{x}{2rh+x^2}\right)^2$

f_x (INNER EDGE) = $\frac{2Pr}{t}\left(\frac{2rh-x^2}{(2rh+x^2)^2}\right)$

$(f_x)_{MAX} = f_x$ (INNER EDGE) A, $x = 0$, $(f_x)_{MAX} = \frac{P}{th}$

STRESS CONCENTRATION FACTOR (K) = 2
(SEE FIGURE 14.6.3.1e)
x = DISTANCE FROM HOLE CENTERLINE

Figure 14.6.3-1c. Stresses in a Strip With a Large Hole
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

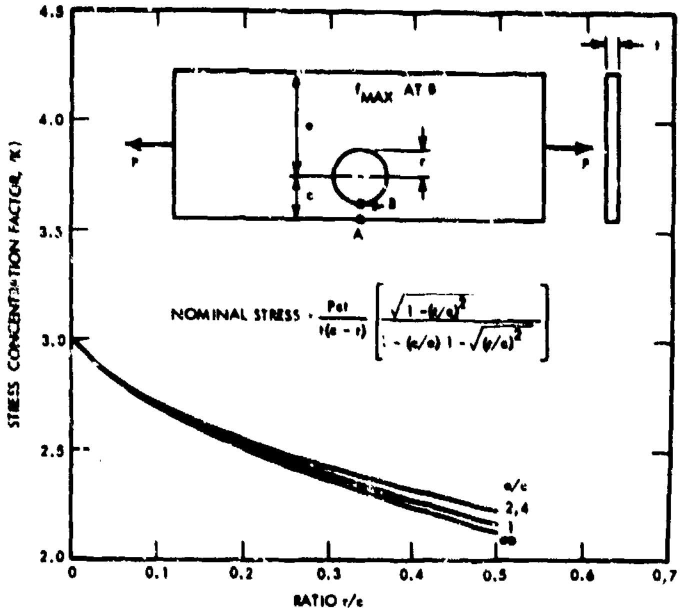


Figure 14.6.3.1e. Stress Concentration Factor for Tensioning of a Plate with an Eccentric Hole
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

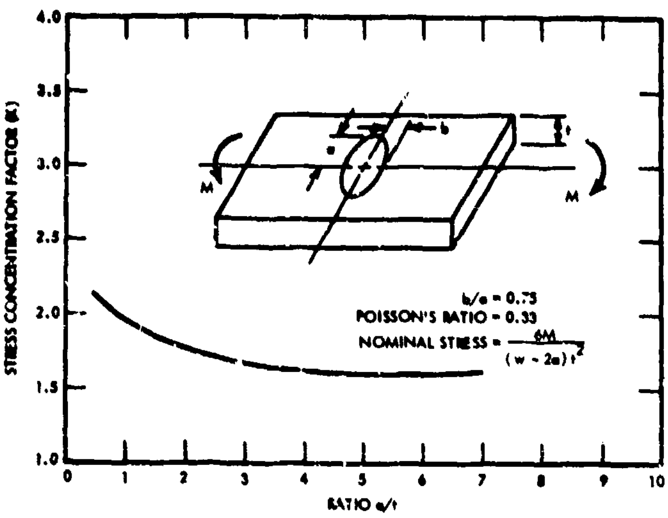


Figure 14.6.3.1d. Stress Concentration Factor for Bending of Plate with an Elliptical Hole
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

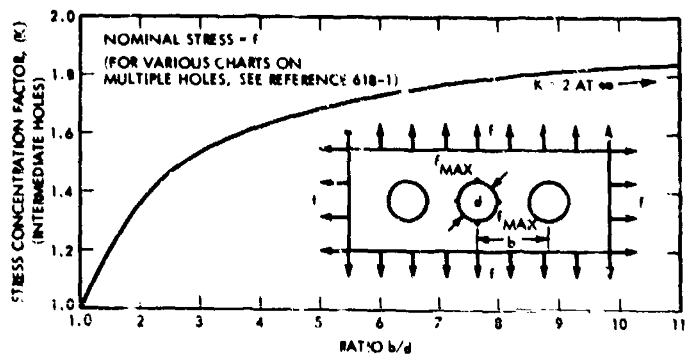


Figure 14.6.3.1f. Stress Concentration Factor for Biaxial Stressing of a Plate Containing a Row of Holes
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

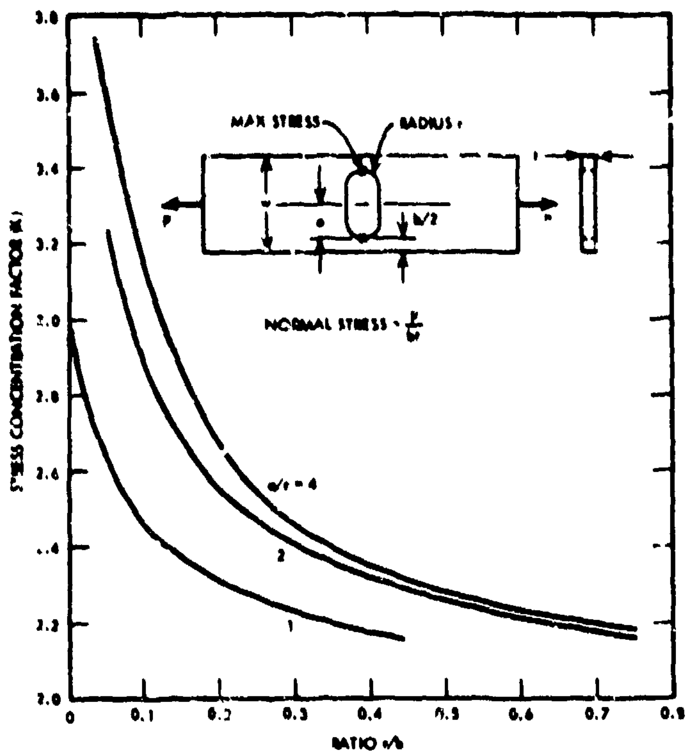


Figure 14.6.3.1g. Stress Concentration for a Slotted Bar
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

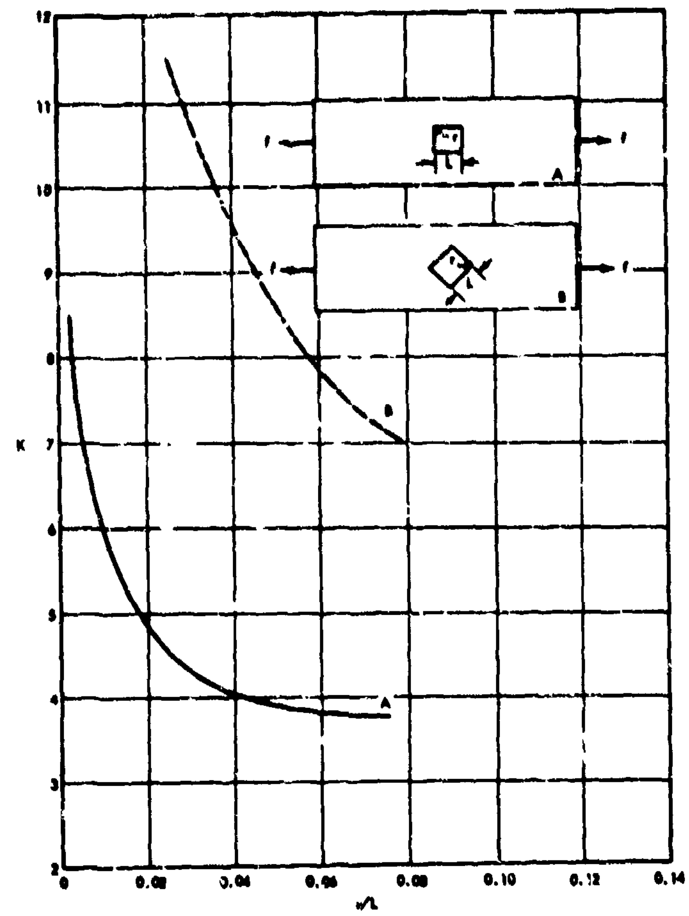


Figure 14.6.3.1h. Stress Concentration Factor for a Plate with Square- or Diamond-Shaped Holes
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

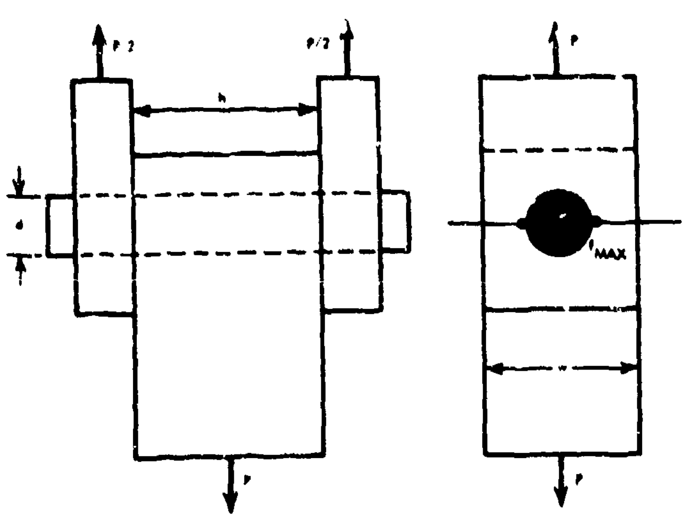


Figure 14.6.3.2a. Clevis Pin or Trunnion Connection
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

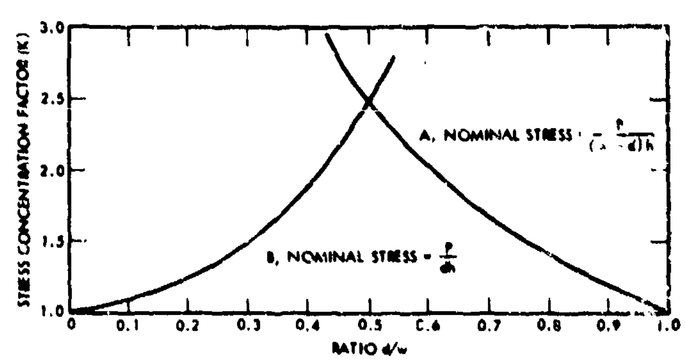


Figure 14.6.3.2b. Stress Concentration in Clevis Pin or Trunnion
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel Wiley, 1964)

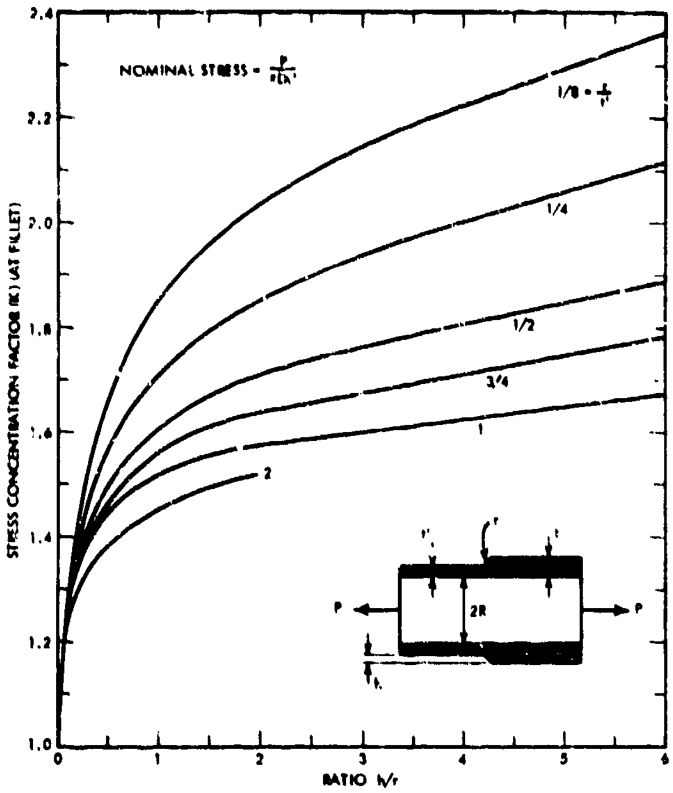


Figure 14.6.3.3. Stress Concentration Factors for a Stepped Cylinder with Shoulder Fillets
(Adapted with permission from Reference 538-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

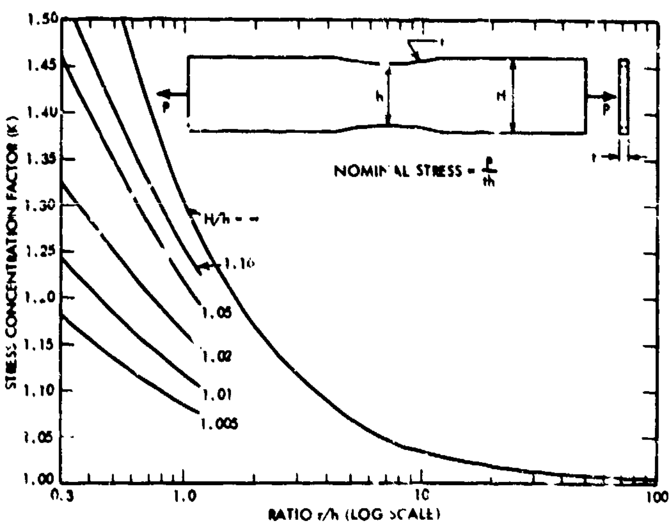


Figure 14.6.3.4a. Stress Concentration Factor for Tensioning of a Flat Bar With Shallow Fillet Groove
(Adapted with permission from Reference 598-1)

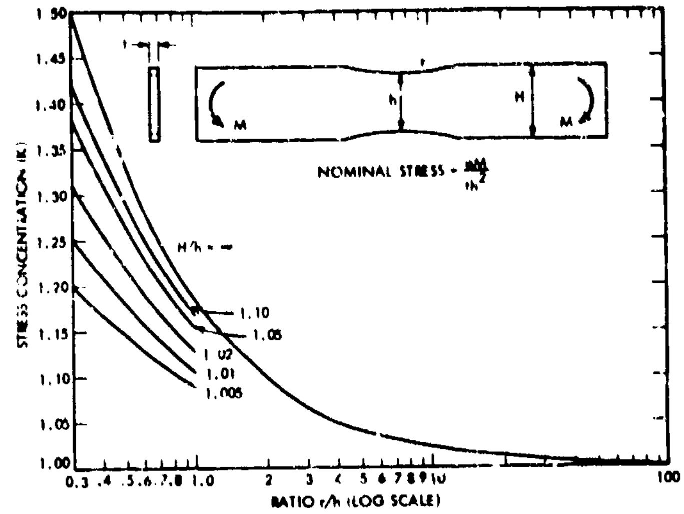


Figure 14.6.3.4b. Stress Concentration Factor for Bending of a Flat Bar with a Shallow Fillet Groove
(Adapted with permission from Reference 598-1)

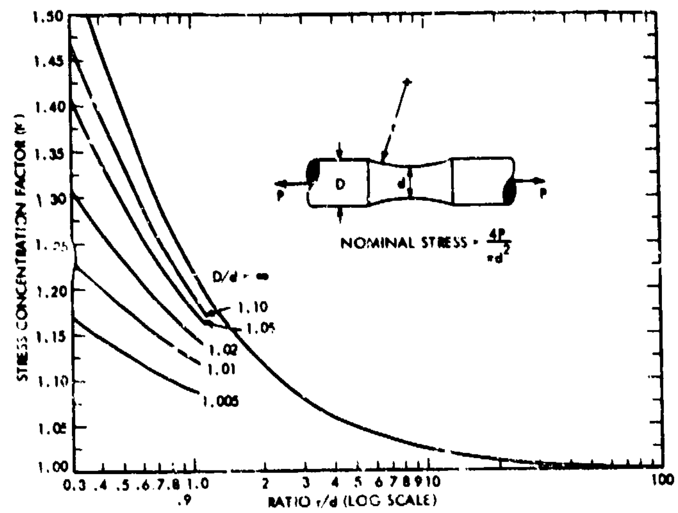


Figure 14.6.3.4c. Stress Concentration Factor for Tensioning of a Solid Round Bar with a Shallow Fillet Groove
(Adapted with permission from Reference 598-1)

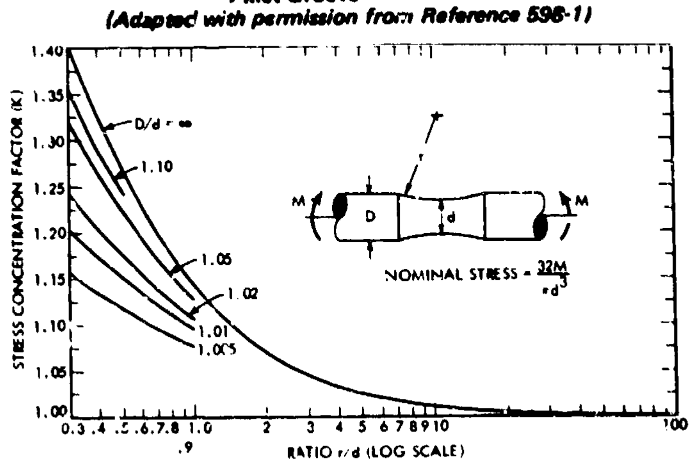


Figure 14.6.3.4d. Stress Concentration Factor for Bending of a Solid Round Bar with a Shallow Fillet Groove
(Adapted with permission from Reference 598-1)

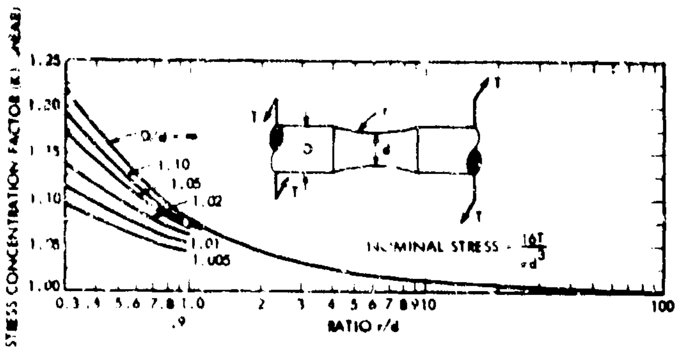


Figure 14.6.3.4e. Stress Concentration Factor for Torsion of a Solid Round Bar with a Shallow Fillet Groove
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

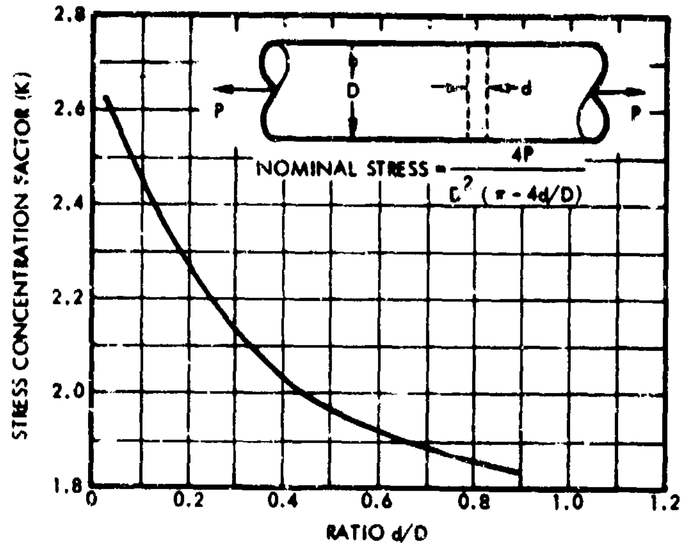


Figure 14.6.3.5a. Stress Concentration Factor for Tensioning of a Solid Round Bar with a Small Transverse Hole
(Adapted with permission from Reference 598-1)

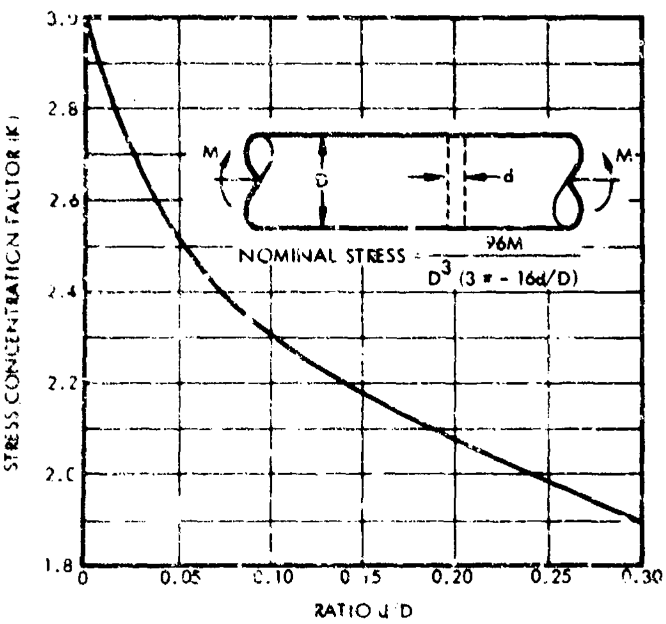


Figure 14.6.3.5b. Stress Concentration Factor for Bending of a Solid Round Bar with a Small Transverse Hole
(Adapted with permission from Reference 599-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

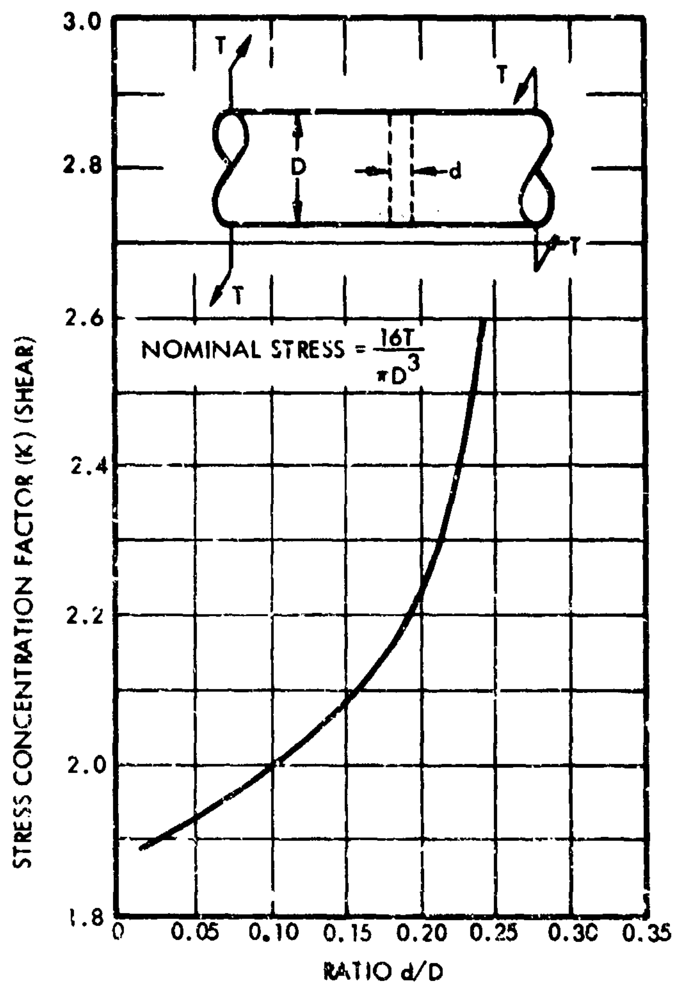


Figure 14.6.3.5c. Stress Concentration Factor for a Solid Round Bar in Shear
(Adapted with permission from Reference 598-1)

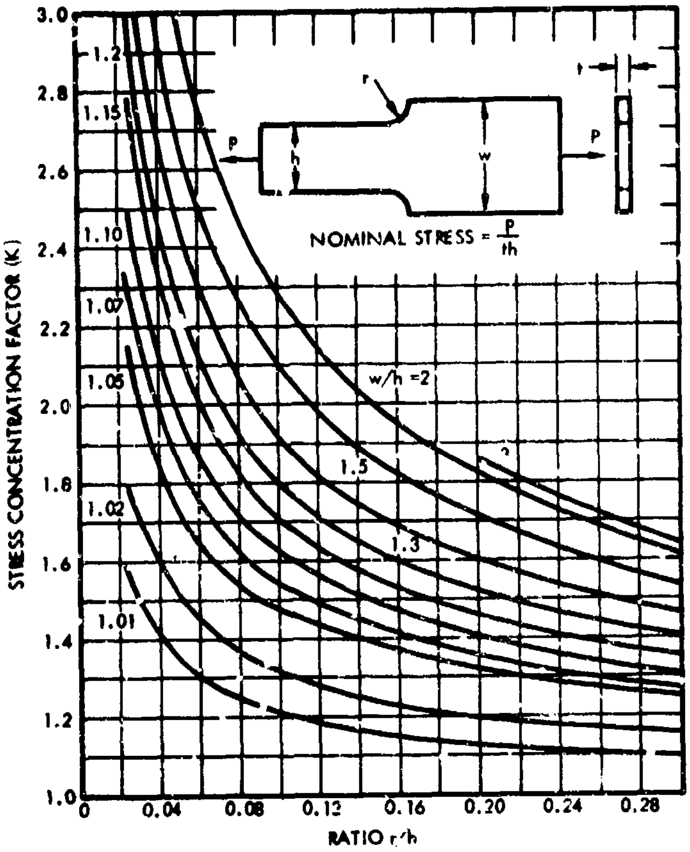


Figure 14.6.3.6a. Stress Concentration Factor for Tensioning of a Flat Bar With Circular Fillets
(Adapted with permission from Reference 598-1)

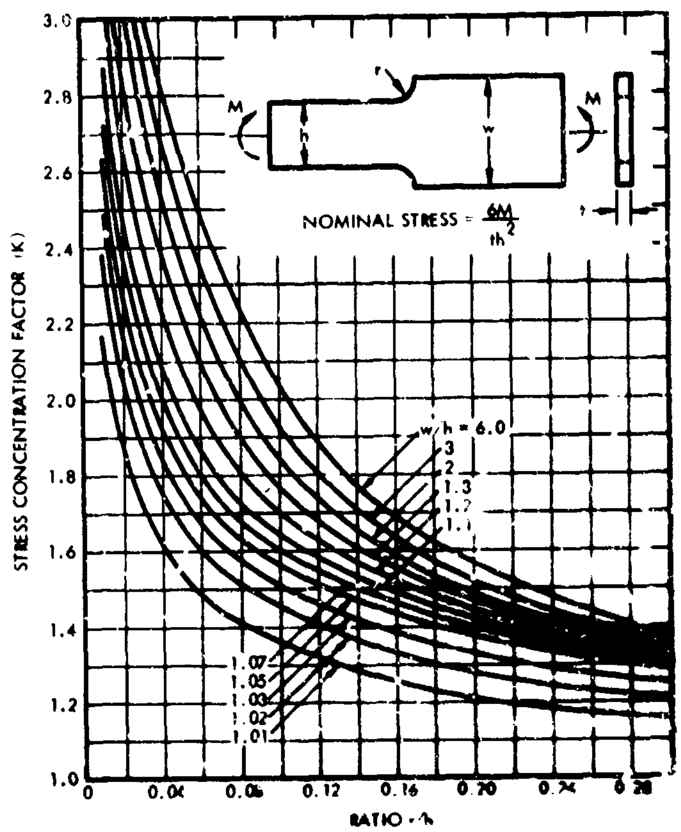


Figure 14.6.3.6b. Stress Concentration Factor for Bending of a Flat Bar with Circular Fillets
(Adapted with permission from Reference 598-1)

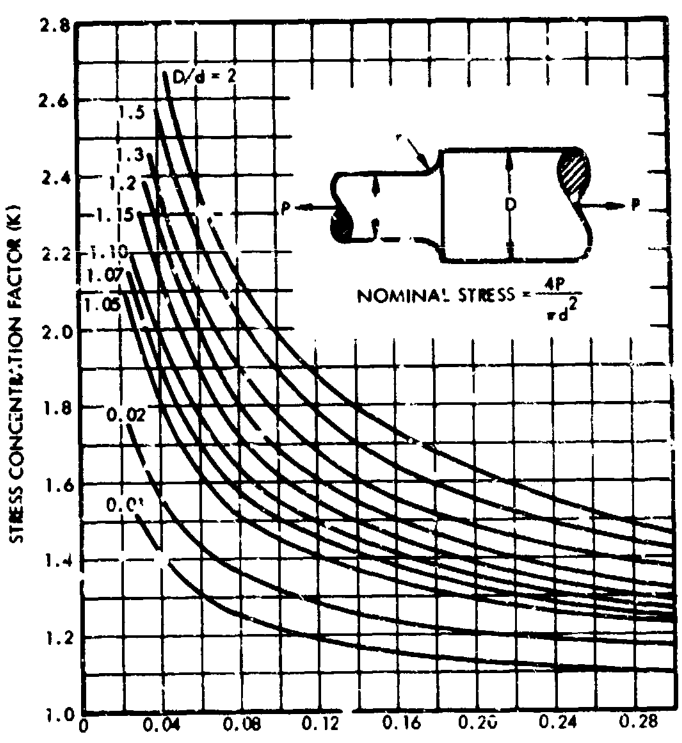


Figure 14.6.3.6c. Stress Concentration Factor for Tensioning of a Solid Round Bar with Shoulder Fillet
(Adapted with permission from Reference 598-1)

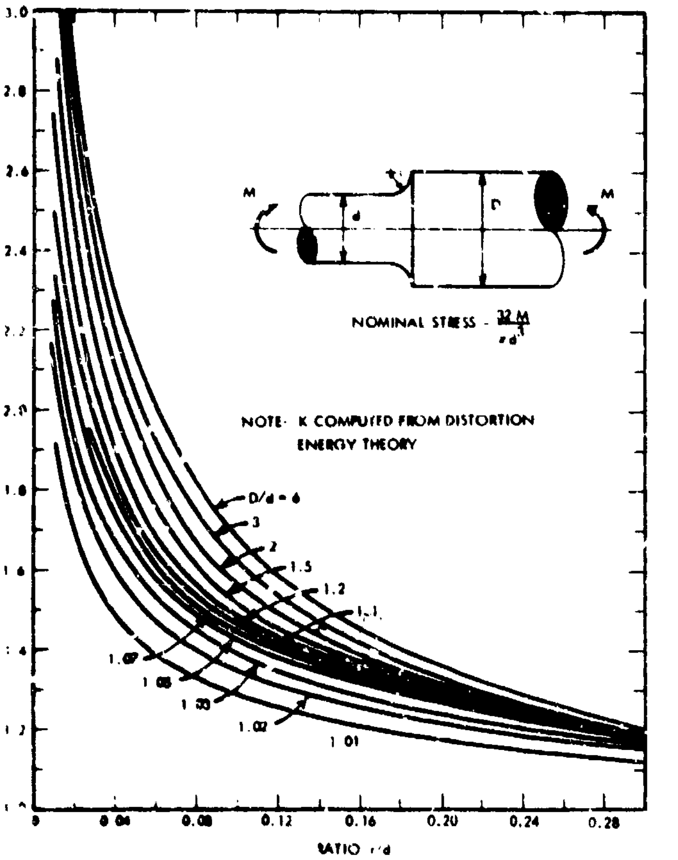


Figure 14.6.3.6d. Stress Concentration Factor for Bending of a Solid Round Bar with Shoulder Fillets
(Adapted with permission from Reference 5?8-1)

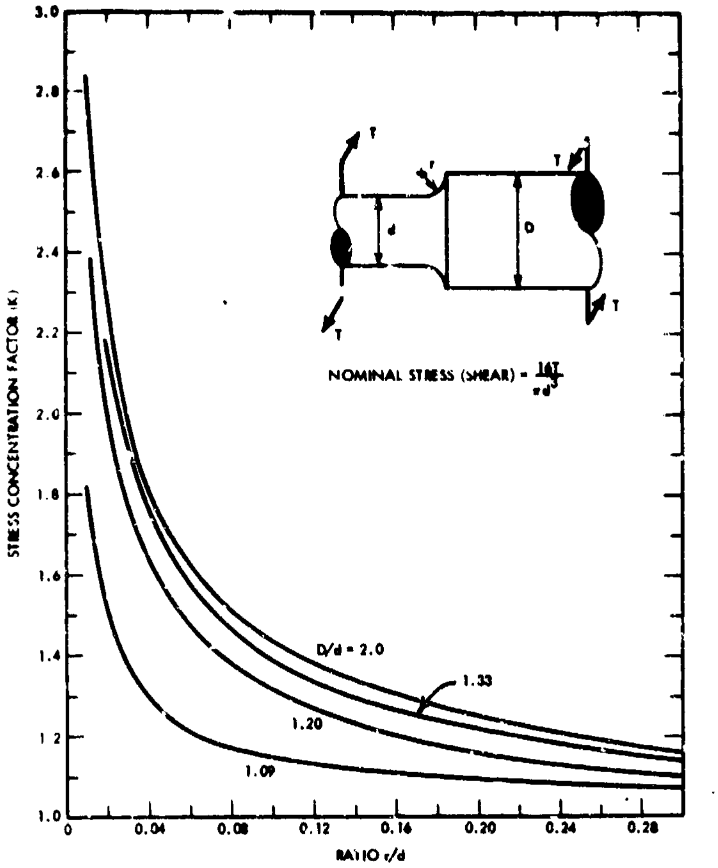


Figure 14.6.3.6e. Stress Concentration Factor for Torsion of a Solid Round Bar with Circular Fillets
(Adapted with permission from Reference 598-1)

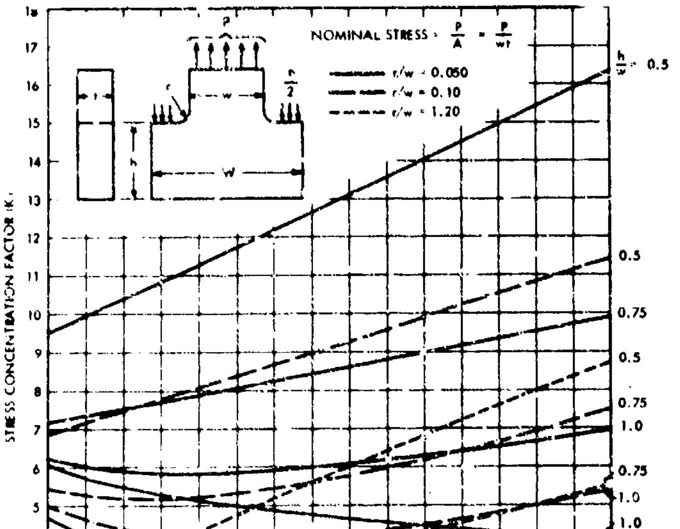


Figure 14.6.3.6f. Stress Concentration Factor for a Flat Bar with Circular Fillets
(Adapted with permission from Reference ?16-1)

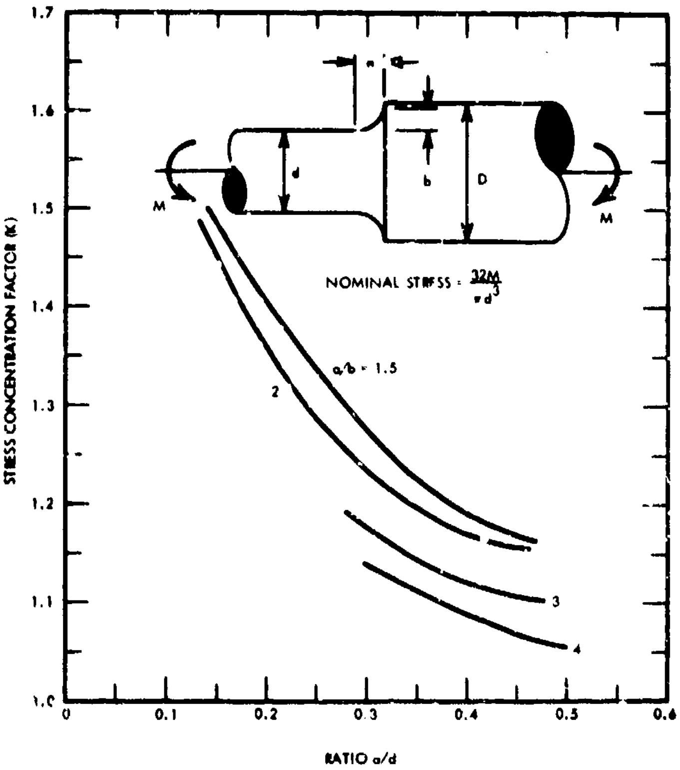


Figure 14.6.3.6g. Stress Concentration Factor for Bending of a Solid Round Bar with Elliptical Fillet
(Adapted with permission from Reference 598-1)

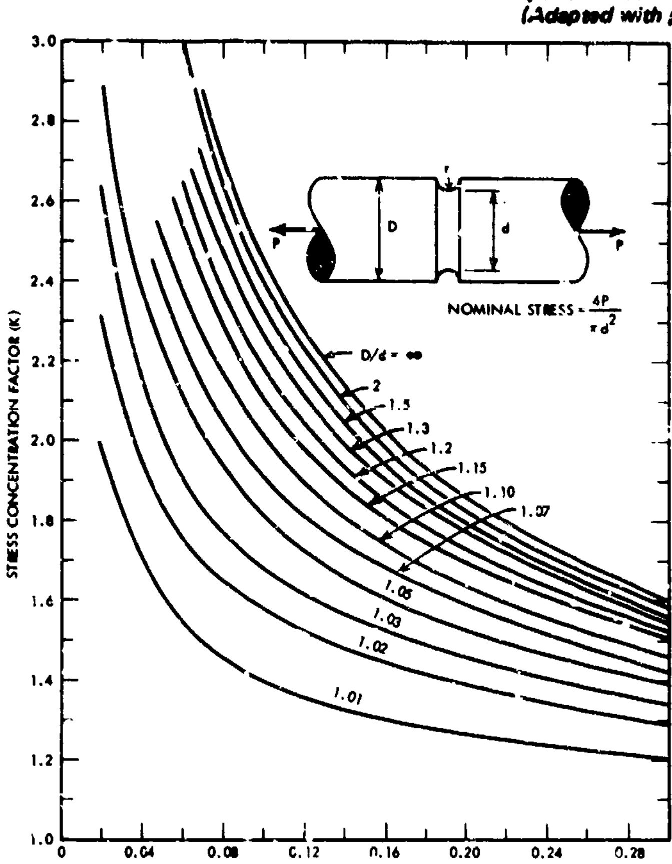


Figure 14.6.3.7a. Stress Concentration Factor for Tensioning of a Solid Round Bar with a Circular Groove
(Adapted with permission from Reference 598-1)

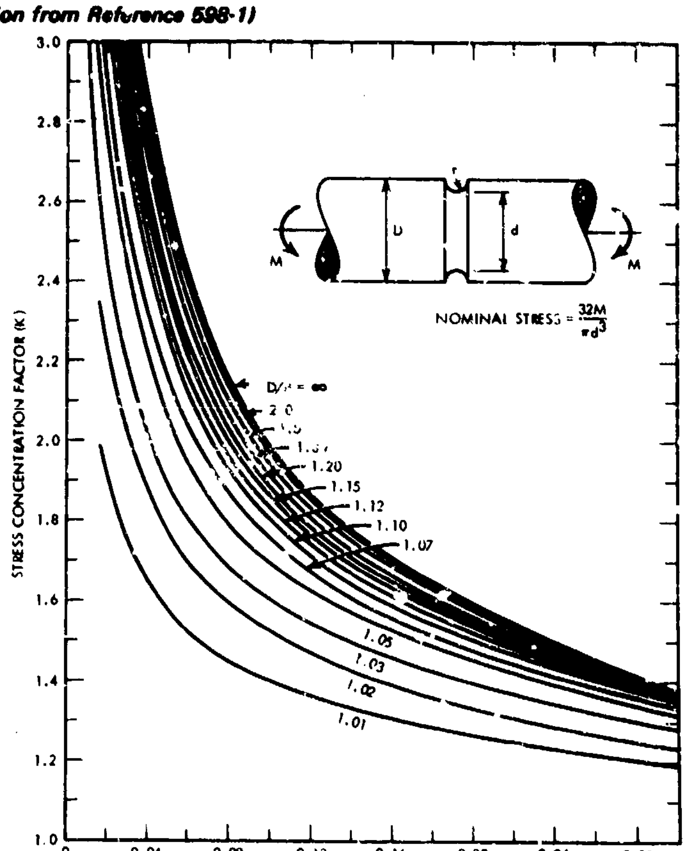


Figure 14.6.3.7b. Stress Concentration Factor for Bending of a Solid Round Bar with a Circular Groove
(Adapted with permission from Reference 598-1)

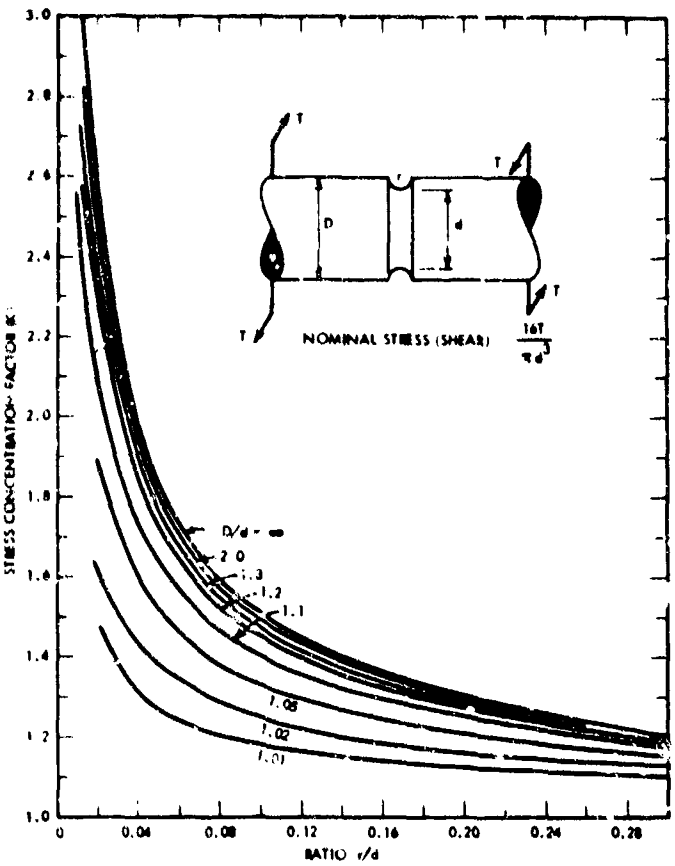


Figure 14.6.3.7c. Stress Concentration Factor for Torsion of a Solid Round Bar with a Circular Groove
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

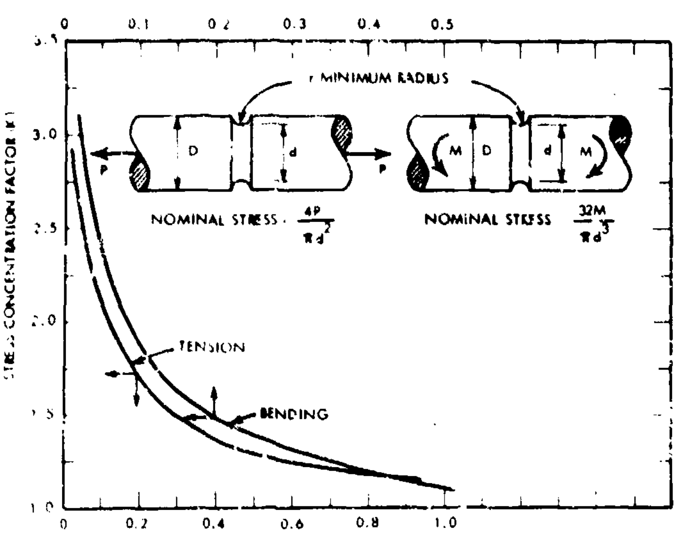


Figure 14.6.3.7d. Stress Concentration Factor for Tensioning and Bending of a Solid Round Bar with Hyperbolic Notch Groove
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

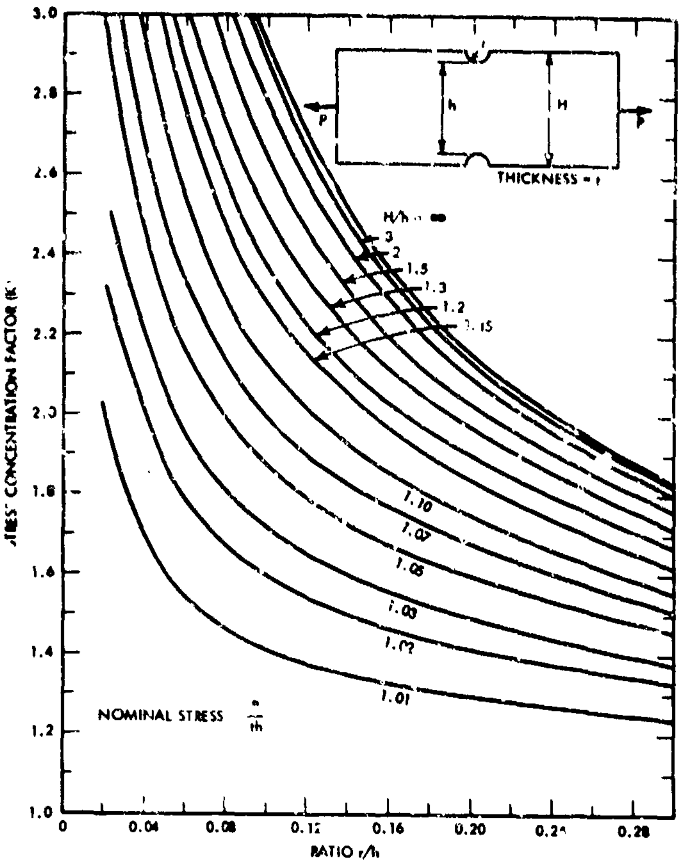


Figure 14.6.3.8a. Stress Concentration Factor for Tensioning of a Notched Flat Bar
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

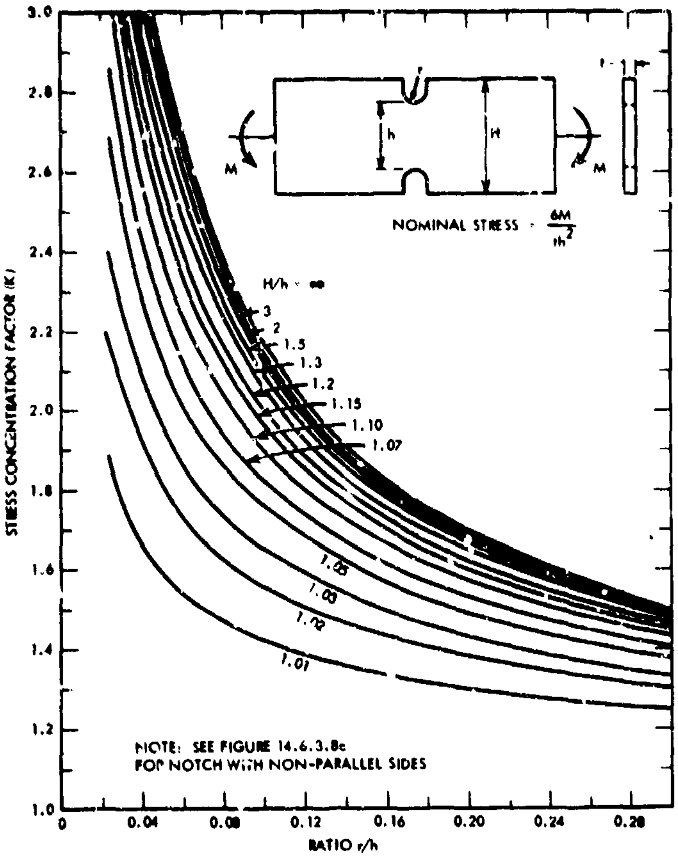


Figure 14.6.3.8b. Stress Concentration Factor for Bending of a Notched Flat Bar
(Adapted with permission from Reference 59R-1)

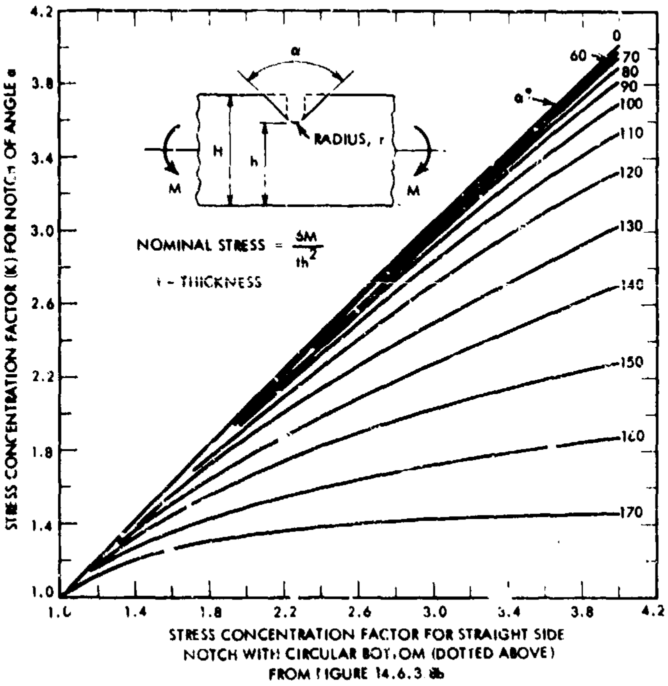


Figure 14.6.3.8c. Effect of Notch Angle on Stress Concentration Factor
(Adapted with permission from Reference 59C-1)

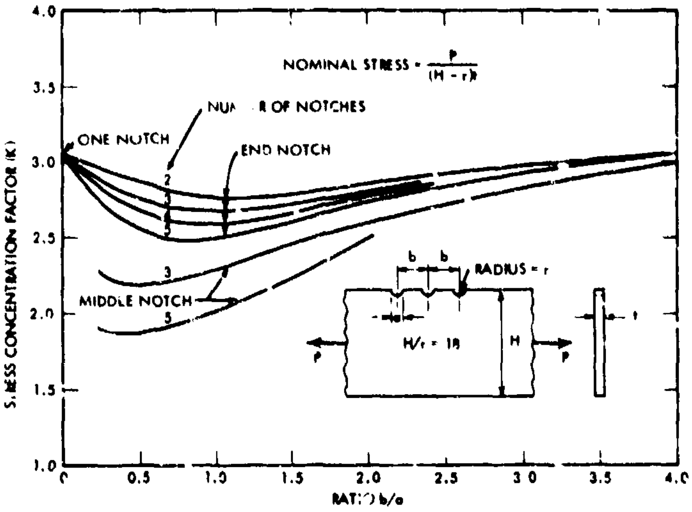


Figure 14.6.3.8d. Effect of Multiple Notches in a Flat Plate
(Adapted with permission from Reference 59B-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

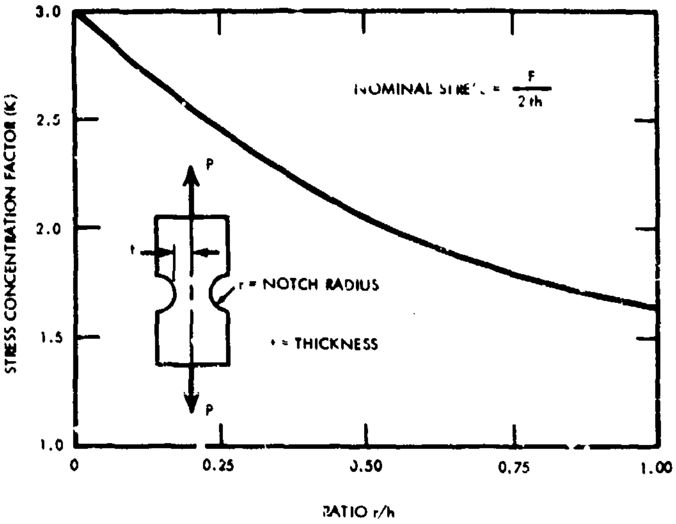


Figure 14.6.3.8e. Stress Concentration Factor for Tensioning of a Notched Flat Bar
(Adapted with permission from Reference 59B-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

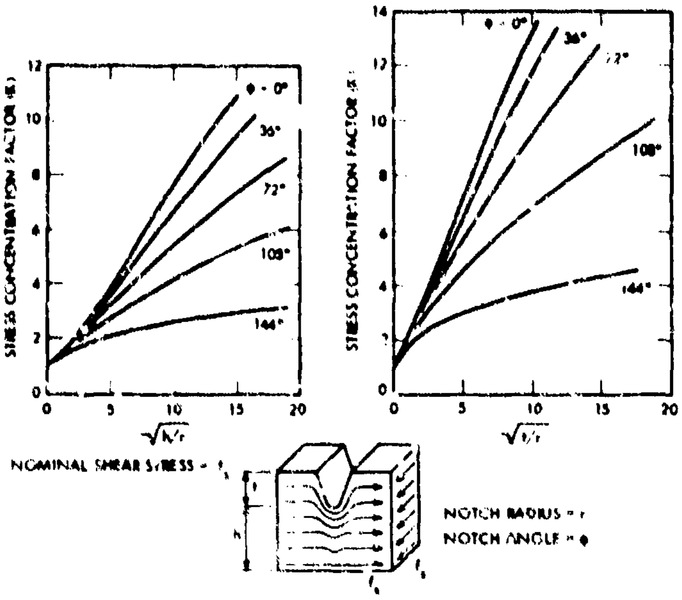


Figure 14.6.3.8. Stress Concentration in the Presence of Pure Shear
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

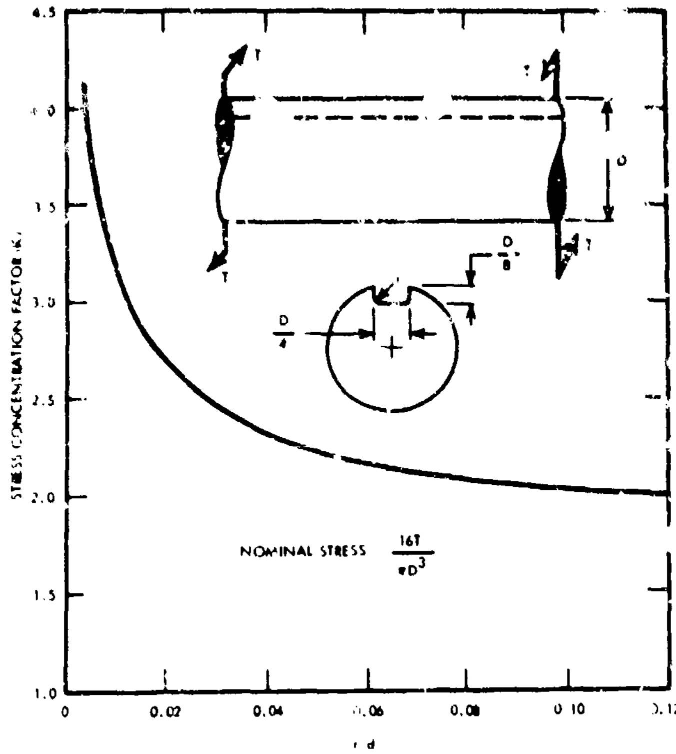


Figure 14.6.3.9. Stress Concentration Factors for the Straight Portion of a Keyway in a Solid Circular Bar Subjected to Torsion
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

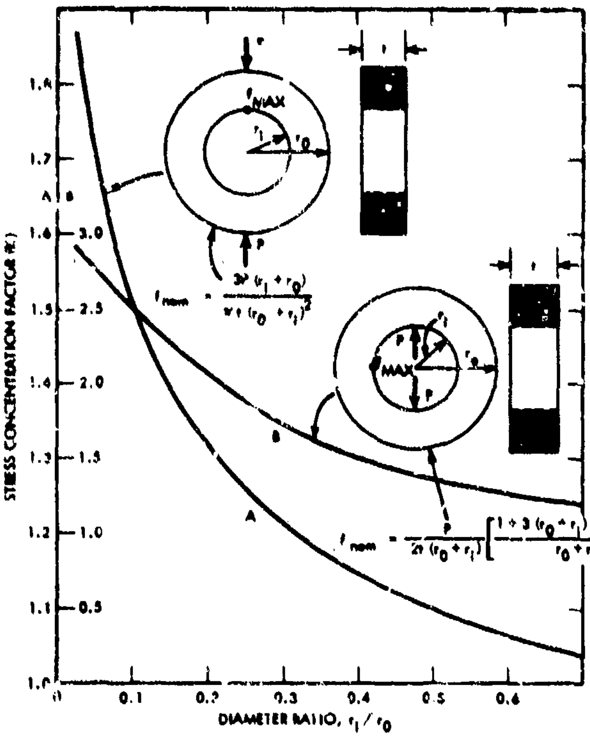


Figure 14.6.3.10. Stress Concentration Factors for Diametrally Loaded Rings
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

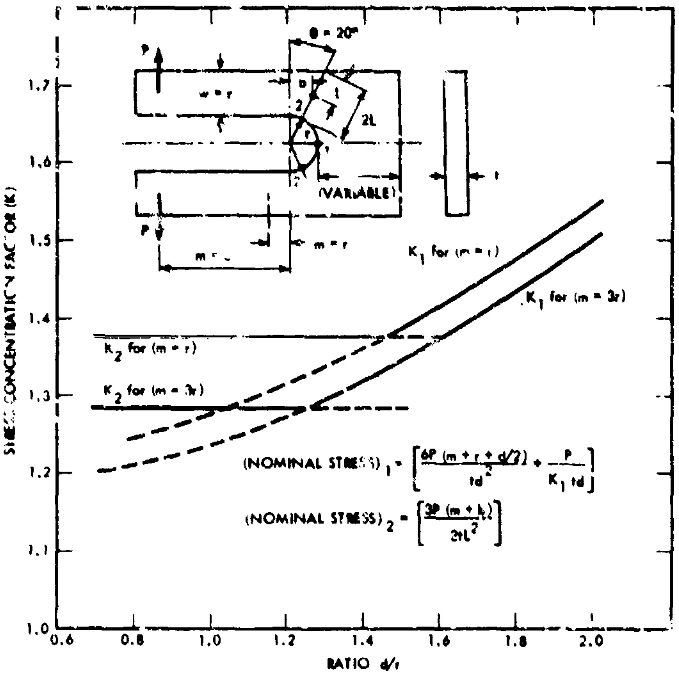


Figure 14.6.3.11. Stress Concentration Factor for Lateral Loading of a U-Shaped Member
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

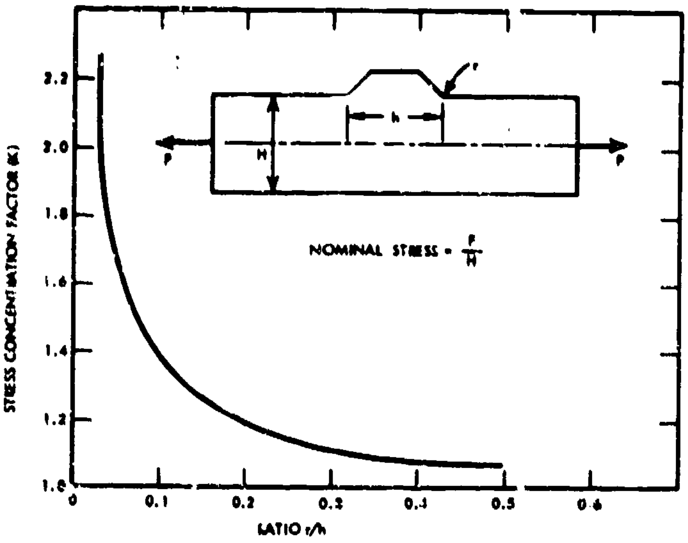


Figure 14.6.3.12. Stress Concentration Factor for Tensioning of a Flat Bar with a Protrusion
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

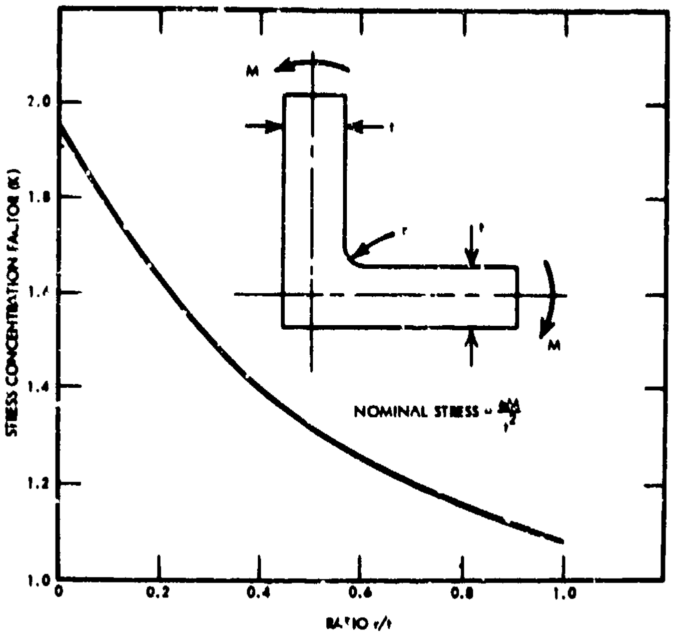


Figure 14.6.3.13. Stress Concentration Factor for Bending of a Flange
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

14.7 PRESSURE VESSELS AND PRESSURE STRESSES

14.7 PRESSURE VESSELS AND PRESSURE STRESSES

The determination of pressure stresses in a valve, line, or other fluid component is an integral part of the component design. Pressure stresses, in combination with thermal, structural, and mounting stresses, usually determine the material thicknesses required by a structure. Normally, only a simplified analysis is performed by the component designer. This analysis would include the calculation of hoop stresses in cylinders and spheres, and bending stresses in flat plates used in closures. Stress concentration factors based on published data and experience would then be applied to account for the discontinuity stresses at changes in sections. This approach is adequate for most aerospace fluid components. However, when weight becomes a critical factor and the above procedure does not yield a satisfactory design, a more sophisticated analysis is required. This may be performed by a stress analyst or by the designer himself. It is the purpose of this section to present an analytic technique which may be used for a more accurate analysis than the rudimentary one noted above. Tables and curves of critical parameters are presented to ease the calculations as much as possible. The excellent summary of basic pressure vessel equations presented by Roark (Reference 461-2) is reproduced with permission as Table 14.7 to provide a consolidated ready reference.

14.7.1 Design Criteria

Design criteria normally established for a component include maximum values of size, weight, and leakage as well as minimum values for power and response time. In this section we are only concerned with the criteria affecting pressure induced stress in the component — primarily size and weight. In order to assure satisfactory operation and structural integrity over the required pressure range, samples of most fluid components are proof-pressure and burst-pressure tested (Section 15.0). Proof pressures are normally set at 1.5 to 2 times the working pressure, while burst pressures are set at 2 to 4 times the working pressure (see Table 14.2.1.3c). Components should not exhibit any signs of permanent deformation or functional impairment following a proof pressure test and may yield but not rupture during the burst pressure test. The properties of the component material will determine whether the proof or burst condition will govern the design. It is advisable to compute the component wall thicknesses on both no yield-at-proof and no fail-at burst criteria and to use the more conservative of the two values in the design.

Deflections due to applied loads may cause seal unloading and subsequent leakage and/or component distortion with an accompanying binding of moving parts. Deflection rather than stress may govern the actual component design and should be computed for any component where it could affect function or mounting. Stiffness criteria are discussed in Detailed Topic 14.2.1.3.

Buckling as a result of fluid pressure is often the most important design criterion for thin-walled components. The necessity for evaluating a component's resistance to buckling is obvious for those vessels designed for operation with internal pressure lower than external pressure, but is too often overlooked for the following two equally important situations:

1) A component which is normally internally pressurized is subject to buckling if it is evacuated during some intermediate process (i.e. the evacuation of certain spacecraft propellant tanks preliminary to filling with propellant).
2) Buckling of thin-walled components may result from *internal* pressure. This phenomenon is frequently the primary failure criterion for thin-walled vessels which are of a "flattened" configuration, such as non-circular section toruses and torispherical or elliptical end closures.

Except as noted, the following assumptions apply to the equations presented in this sub-section (Reference 152-7):

1) A brittle material is perfectly elastic up to its ultimate strength. When it fractures, according to the maximum principal (tensile) stress theory, it does so without appreciable yielding.
2) A ductile material is perfectly elastic up to the yield point; thereafter it yields at constant maximum shear stress (Tresca theory); no strain hardening occurs.
3) The temperature is low enough so that creep is negligible.
4) Temperature and stress have no effect on elastic moduli and the yield point; the coefficient of thermal expansion is negligible.
5) The Bauschinger effect does not occur (reduction in yield point due to previous plastic flow in the reverse direction).
6) The strains are small compared with the dimensions of the vessel.
7) Important stress raisers are absent.

Table 14.7. Formulas for Stresses and Deflection in Pressure Vessels.
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Notation for thin vessels: p = unit pressure (lb. per sq. in.); f_l = meridional membrane stress, positive when tensile (lb. per sq. in.); f_h = hoop membrane stress, positive when tensile (lb. per sq. in.); f_{lb} = meridional bending stress, positive when tensile on convex surface (lb. per sq. in.); f_{hb} = hoop bending stress, positive when tensile at convex surface (lb. per sq. in.); f_h' = hoop stress due to discontinuity, positive when tensile (lb. per sq. in.); f_s = shear stress (lb. per sq. in.); V_o, V_x = transverse shear normal to wall, positive when acting as shown (lb. per linear in.); M_o, M_x = bending moment, uniform along circumference, positive when acting as shown (in.-lb. per linear in.); x = distance measured along meridian from edge of vessel or from discontinuity (in.); R_1 = mean radius of curvature of wall along meridian (in.); R_2 = mean radius of curvature of wall normal to meridian (in.); R = mean radius of circumference (in.); t = wall thickness (in.); E = modulus of elasticity (lb. per sq. in.); μ = Poisson's ratio; $D = \dfrac{Et^3}{12(1-\mu^2)}$; $\lambda = \sqrt[4]{\dfrac{3(1-\mu^2)}{R_2^2t^2}}$; radial displacement positive when outward (in.); θ = change in slope of wall at edge of vessel or at discontinuity, positive when outward (radians); y = vertical deflection, positive when downward (in.). Subscripts 1 and 2 refer to parts into which vessel may be imagined as divided, e.g., cylindrical shell and hemispherical head. General relations: $f_{lb} = \dfrac{6M}{t^2}$ at surface; $f_s = \dfrac{V}{t}$

Thin vessels--membrane stresses f_l (meridional) and f_h (hoop)

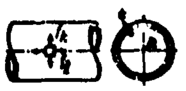
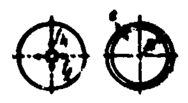
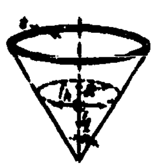
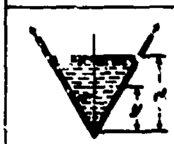
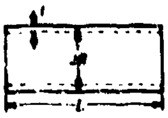
Form of vessel	Manner of loading and Case No.	Formulas
 Cylindrical	1. Uniform internal (or external) pressure p, lb. per sq. in.	$f_l = \frac{pR}{2t}$ $f_h = \frac{pR}{t}$ Radial displacement $= \frac{R}{E}(f_h - \mu f_l)$. Internal bursting pressure $p_u = 2s_u \frac{b-a}{b+a}$ (Here s_u = ultimate tensile strength, a = inner radius, b = outer radius) External collapsing pressure $p' = \frac{t}{R}\left(\frac{s_y}{1 + 4\frac{s_y}{E}\left(\frac{R}{t}\right)^2}\right)$, where s_y = compressive yield point of material (Ref. 1). This formula is for *nonelastic* failure, and holds only when $\frac{p'R}{t}$ > proportional limit.
 Spherical	2. Uniform internal (or external) pressure p, lb. per sq. in.	$f_l = f_h = \frac{pR}{2t}$ Radial displacement $= \frac{R_2}{E}(1-\mu)$
	3. Uniform internal (or external) pressure p, lb. per sq. in., tangential edge support	$f_l = \frac{pR}{2t\cos\alpha}$ $f_h = \frac{pR}{t\cos\alpha}$ Change in side slope $\Delta\alpha = \frac{2pR\sin\alpha(1-\frac{1}{2}\mu)}{Et}$
	4. Same as Case 3 but vertical edge support	At supported edge $f_l = \frac{pR}{2t}\cos\alpha$ $f_h = p\left[-U\sqrt{\frac{R^3\sin^2\alpha}{2t^3\cos\alpha}} + \frac{(1-\mu/2)R}{t\cos\alpha}\right]$ Radial displacement $\Delta R_2 = \frac{p}{E}\left[-U\sqrt{\frac{R^3\sin^2\alpha}{2t^3\cos\alpha}} + \frac{(1-\frac{1}{2}\mu)R^2}{t\cos\alpha}\right]$ Rotation $\theta = \frac{p}{E}\left[-\frac{U^2}{2}\frac{R^2\tan\alpha}{t^2} + \frac{3R\tan\alpha}{2t\cos\alpha}\right]$ ($U = \sqrt[4]{12(1-\mu)}$. Signs here are for internal pressure)
Conical	 5. Filled to depth d with liquid of sp. wt. γ lb. per cu. in., tangential edge support	At any level y above bottom $f_l = \frac{\gamma y}{2t}\frac{\tan\alpha}{\cos\alpha}\left(d - \frac{2}{3}y\right)$. Max $f_l = \frac{3\gamma d^2\tan\alpha}{16t\cos\alpha}$ when $y = \frac{3}{4}d$ $f_h = \frac{(d-y)\gamma \cdot y\tan\alpha}{t\cos\alpha}$. Max $f_h = \frac{\gamma d^2\tan\alpha}{4t\cos\alpha}$ when $y = \frac{1}{2}d$

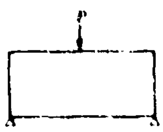
Table 14.7. Formulas for Stresses and Deflection in Pressure Vessels (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Form of vessel	Manner of loading and Case No.	Formulas
 Any figure of revolution, tangential edge support	6 Uniform internal (or external) pressure p lb per sq in	$f_1 = \frac{pR_2}{2t}$ $f_2 = \frac{pR_2}{2t}\left(2 - \frac{R_2}{R_1}\right)$ Radial displacement $= \frac{R}{E}(f_2 - \nu f_1)$ $p = t\left(\frac{f_1}{R_1} + \frac{f_2}{R_2}\right)$

Thin vessels — local and discontinuity stresses f_1 (meridional bending stress), f_{hb} (circumferential bending stress), f_h' (membrane hoop stress)

Case 7 — Radial load P uniformly distributed over small area A, approximately square or round, located near mid-span. Cylinder with open ends.





Max stresses are circumferential stresses at center of loaded area and can be found from following table. Values given are for $L/R = 8$ but may be used for L/R ratios between 3 and 40

Values of $f_{hb}(t^2/P)$

R/t \ A/R^2	0.0004	0.0016	0.0036	0.0064	0.010	0.0144	0.0196	0.0256	0.0324	0.040	0.0576	0.090	0.160	0.25
300	1.475	1.11	0.906	0.780	0.678	0.600	0.522	0.450	0.390	0.345	0.294	0.196	0.130	0.078
100		1.44	1.20	1.034	0.918	0.840	0.780	0.606	0.600	0.540	0.444	0.342	0.260	0.190
50			1.44	1.284	1.11	1.006	0.900	0.846	0.756	0.720	0.600	0.480	0.360	0.264
15										0.960	0.800	0.780	0.600	0.468

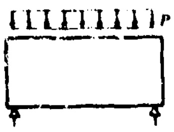
Values of $f_h(Rt/P)$

R/t \ A/R^2	0.0004	0.0016	0.0036	0.0064	0.010	0.0144	0.0196	0.0256	0.0324	0.040	0.0576	0.090	0.160	0.25
300	85	53.5	49	44.5	40	35.5	32	28	24	21	16	11	6	4
100		33.5	30.5	27.6	25	22.5	20	17.5	15	13	10	7	4.2	3.0
50					9.6	9	8.8	6.0	7.7	7.8	6.5	5.0	4.1	3.1
15										3.26	3.0	2.4	2.0	1.56

For A very small (nominal point loading) at point of load

$$f_h = \frac{0.4P}{t^2},\quad f_{hb} = \frac{2.4P}{t^2},\quad y = \frac{P}{Et}\left[0.48\left(\frac{L}{R}\right)^{1}\left(\frac{R}{t}\right)^{1.22}\right]$$

Case 8 — Uniform load of p lb per linear in over entire length of top element



At top center: Max hoop stress $f_h = -0.492A^{\frac14}pR^{\frac12}/t^{\frac32}$

Max circ bndg stress $f_{hb} = \pm 1.217A^{\frac14}pR^{\frac12}/t^{\frac32}$

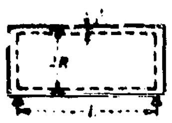
Max long. stress $f_l = -0.1188A^{\frac14}pR^{\frac12}/t^{\frac32}$

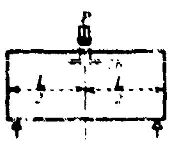
Max $y = 0.0305A^{\frac14}\frac{R^{\frac32}}{Et^{\frac32}}$

At quarter points of span, $y_q = 0.774$ max y

Here $A = 12(1 - \nu^2)$

Case 9 — Center load P concentrated on very short length $2b$. Cylinder with closed ends and end support.





At top center: Max hoop stress $f_h = -0.130A^{\frac14}PR^{\frac14}/t^{\frac74}$

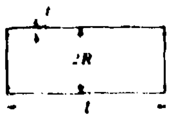
Max circ bndg stress $f_{hb} = \pm A^{\frac14}PR^{\frac14}/t^{\frac74}$

Max long. stress $f_l = -0.150A^{\frac14}PR^{\frac14}/t^{\frac74}$

Max $y = 0.0620A^{\frac14}\frac{PR^{\frac34}}{Et^{\frac74}}$

Here $A = 12(1 - \nu^2)$

Case 10 — Diametrically opposite and equal concentrated loads

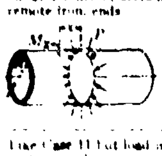


$y = 6.5\frac{P}{Et}\left(\frac{R}{t}\right)^{1.5}\left(\frac{L}{R}\right)^{-\frac34}$ for $\frac{L}{R}$ from 1 to 18

For $L/R > 18$ maximum stresses and deflection are approximately same as for Case 7.

For load at extreme end maximum stresses are approximately four times as great as for loading near mid-span.

Case 11 — Uniform radial pressure of p lb per linear in of circumference of section remote from ends



Max $M = \frac{p}{4\lambda}$ at load, Max $f_{lb} = \frac{3p}{2\lambda t^2}$

$M_x = $ Max $M\, e^{-\lambda x}(\cos\lambda x - \sin\lambda x)$

Radial displacement $= \frac{pR^2\lambda}{2Et}$

Max hoop stress $f_h = \frac{pR\lambda}{2t}$ at load

Case 12 — Like Case 11 but load is uniform radial pressure of p lb per sq in on circumferential band of width $2b$. Cylinder with open ends.

Max $M_x = \frac{p}{2\lambda^2}e^{-b\lambda}\sin b\lambda$

Radial displacement $= -\frac{pR^2}{Et}(1 - e^{-b\lambda}\cos b\lambda)$

Table 14.7. Formulas for Stresses and Deflection in Pressure Vessels (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

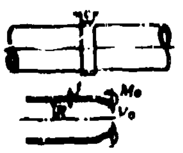
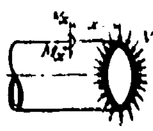
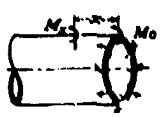
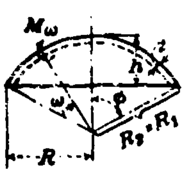
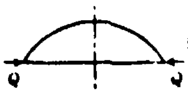
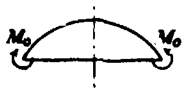
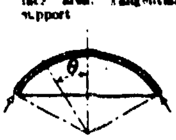
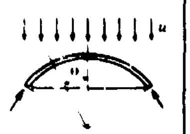
Form of vessel	Manner of loading and Case No.	Formulas
 Cylindrical, with reinforcing rings of cross-sectional area A	13. Uniform internal (or external) pressure of p lb. per sq. in	$M_0 = \left(\frac{p}{2\lambda^2}\right)\left(\frac{A}{A + bt + 2t/\lambda}\right)$ ($M_0 = p/2\lambda^2$ for rigid ring or disk) $V_0 = 2M_0\lambda$ ($V_0 = p/\lambda$ for rigid ring or disk) Max long. bending stress $= \frac{-6M_0}{t^2}$ at edge of ring. Formulas for Cases 14 and 15 may be used to find M_x, f_{lb} and V_x at other sections. Ring pressure per linear in. of circumference $= 2V_0$. The above formulas are valid if rings are spaced so far apart that the influence of one does not extend to the next. This spacing $\geq \frac{4}{\lambda}$. The sum of the direct and bending longitudinal stresses $f_l + f_{lb}$ exceeds the normal hoop stress f_h unless the ring spacing $< \frac{\pi}{2}\sqrt{Rt}$
Cylindrical (Long)	14. Uniform radial shear or pressure V_0 lb. per linear in. of circumference at end 	$M_x = \frac{1}{\lambda}V_0 e^{-\lambda x}\sin\lambda x$ Max $M = 0.322\frac{V_0}{\lambda}$ at $x = \frac{\pi}{4\lambda}$ $V_x = V_0 e^{-\lambda x}(\cos\lambda x - \sin\lambda x)$ $f_{lb} = \frac{6M_x}{t^2}$; Max $f_{lb} = \frac{1.932\,V_0}{\lambda t^2}$; $f_{hb} = \mu f_{lb}$ $f_s = \frac{V}{t}$ Hoop stress $f_h = \frac{-2V_0}{t}(\lambda R e^{-\lambda x}\cos\lambda x)$; Max $f_h = \frac{-2V_0}{t}\lambda R$ at end Radial displacement $= \frac{-V_0}{2D\lambda^3}$ $\theta = \frac{-V_0}{2D\lambda^2}$
	15. Uniform radial moment M_0 in.-lb. per linear in. of circumference at end 	$M_x = M_0 e^{-\lambda x}(\cos\lambda x + \sin\lambda x)$; Max $M = M_0$ at end; $V_x = 2\lambda M_0 e^{-\lambda x}\sin\lambda x$; Max $V = 0.644\lambda M_0$ at $x = \frac{\pi}{4\lambda}$ $f_{lb} = -\frac{6M_x}{t^2}$; Max $f_{lb} = -\frac{6M_0}{t^2}$; $f_{hb} = \mu f_{lb}$ $f_s = \frac{V}{t}$ Hoop stress $f_h = \frac{2\lambda^2 R M_0 e^{-\lambda x}(\cos\lambda x - \sin\lambda x)}{t}$; Max $f_h = \frac{2M_0}{t}\lambda^2 R$ at end Radial displacement $= \frac{M_0}{2D\lambda^2}$; $\theta = \frac{M_0}{\lambda D}$
 	16. Uniform radial force Q lb. per linear in. at edge  	$M_\omega = C\frac{R_2}{2\beta}\frac{e^{-\beta\omega}}{\sqrt{\sin(\phi-\omega)}}[K_1\cos(\beta\omega+\psi) + \sin(\beta\omega+\psi)]$ Max M at $\omega = \frac{\pi}{4\lambda R_1}$ $f_l = -\frac{C}{t}\cot(\phi-\omega)\frac{e^{-\beta\omega}}{\sqrt{\sin(\phi-\omega)}}\sin(\beta\omega+\psi)$ Max $f_l = \frac{Q\cos\phi}{t}$ at edge $f_h = \frac{C}{t}\frac{\beta e^{-\beta\omega}}{2\sqrt{\sin(\phi-\omega)}}[2\cos(\beta\omega+\psi) - (K_1+K_2)\sin(\beta\omega+\psi)]$ Max $f_h = -\frac{Q}{t}\left(\frac{1}{K_1}+\frac{K_1+K_2}{2}\right)\beta\sin\phi$, at edge Radial displacement $= -\frac{Q}{Et}(\beta R_2\sin^2\phi)\left(K_2+\frac{1}{K_1}\right)$ $\theta = -\frac{Q}{Et}(2\beta^2\sin\phi)\left(\frac{1}{K_1}\right)$ Here $\beta = \sqrt[4]{3(1-\mu^2)\left(\frac{R_2}{t}\right)^2}$; $K_1 = 1 - \frac{1-2\mu}{2\beta}\cot(\phi-\omega)$, $K_2 = 1 - \frac{1+2\mu}{2\beta}\cot(\phi-\omega)$ $C = Q(\sin\phi)^{\frac{3}{2}}\left(\frac{\sqrt{1+K_1^2}}{K_1}\right)$; $\psi = \tan^{-1}(-K_1)$
	17. Uniform radial moment M_0 in.-lb. per linear in. at edge  	M_ω, f_l, and f_h as for Case 14 except $C = \frac{2M_0\beta(\sin\phi)^{\frac{1}{2}}}{R_2K_1}$ and $\psi = 0$ Max $M = M_0$; max $f_h = \frac{M_0}{tR_2}\left(\frac{2\beta^2}{K_1}\right)$ at edge Radial displacement $= \frac{M_0}{Et}\left(\frac{2\beta^2\sin\phi}{K_1}\right)$ $\theta = \frac{M_0}{Et}\left(\frac{4\beta^3}{R_2K_1}\right)$ Here β, K_1, and K_2 are same as for Case 14
Spherical shell. Formulas for Cases 16 and 17 are also applicable to cone, or similar figure of revolution	18. Loaded by own weight of w lb. per sq. in. of surface area. Tangential support  	$f_l = -\frac{Rw}{t(1+\cos\theta)}$ Max $f_l = -\frac{Rw}{t(1+\cos\phi)}$ at edge ($\theta = \phi$) $f_h = \frac{Rw}{t}\left(\frac{1}{1+\cos\theta} - \cos\theta\right)$ Max compressive $f_h = \frac{Rw}{2t}$ at $\theta = 0$ Max tensile $f_h = \frac{Rw}{t}\left(\frac{1}{1+\cos\phi} - \cos\phi\right)$ when $\phi > 51.83°$

Table 14.7. Formulae for Stresses and Deflection in Pressure Vessels (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Form of vessel	Manner of loading and Case No.	Formulae

19. Like Case 18, but load is w lb. per sq. in. of horizontal projected area



$$f_l = \frac{wR}{2t}$$

$$f_h = \frac{wR\cos 2\theta}{2t}$$

20. Load P concentrated on small circular area of radius r_0 at pole. Vertical support, edge neither held nor fixed

Max deflection $y = A\frac{PR_1}{Et^2}$

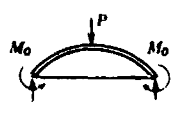
Max membrane stress $f_l = f_h = B\frac{P}{t^2}$ at pole

Max bending stress $f_{lb} = f_{hb} = C\frac{P}{t^2}$ at pole

Here A, B, and C are numerical coefficients that depend on $\nu = \sqrt[4]{12(1-\mu^2)}\left(\frac{r_0}{\sqrt{Rt}}\right)$ and have values as tabulated below:

ν	0	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4
A	0.424	0.418	0.410	0.405	0.381	0.354	0.330	0.305	0.280
B	0.212	0.209	0.205	0.202	0.190	0.177	0.165	0.152	0.140
C	∞	1.74	1.33	0.923	0.693	0.536	0.421	0.332	0.263

21. Point load at pole, edge fixed but not held



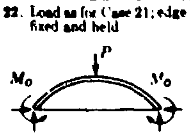
Max deflection $y = A\frac{PR^2}{16\pi D}$

Edge moment $M_0 = B\frac{P}{4\pi}$

Here A and B are numerical coefficients that depend on $\alpha = 2\sqrt[4]{3(1-\mu^2)}\sqrt{\frac{h}{t}}$ and have values as tabulated below:

α	0	1	2	3	4	5	6	7	8	9	10
A	1	0.996	0.935	0.754	0.406	0.321	0.210	0.148	0.111	0.085	0.069
B	1	0.995	0.932	0.746	0.498	0.324	0.234	0.192	0.168	0.153	0.140

22. Load as for Case 21; edge fixed and held

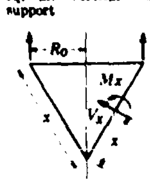


Formulas for y and M_0 same as for Case 18 but A and B have values as tabulated below:

α	0	1	2	3	4	5	6	7	8	9	10
A	1	0.985	0.817	0.515	0.320	0.220	0.161	0.122	0.095	0.075	0.061
B	1	0.975	0.690	0.191	−0.080	−0.140	−0.117	−0.080	−0.059	−0.034	−0.026

Spherical shell

23. Uniform internal (or external) pressure, p lb. per sq. in. Vertical edge support



$$M_x = \left(\frac{n}{n_0 k}\sqrt{2}\,x_0^2\sin^2\alpha\right)p$$

$$V_x = \frac{n}{n_0 k^2}(2j^2 x_0^2\sin^2\alpha)\left[\cos\frac{k_0-k}{\sqrt{2}} + \left(\frac{2\sqrt{2}}{k}-1\right)\sin\frac{k_0-k}{\sqrt{2}}\right]p$$

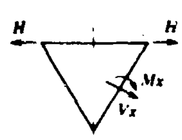
$$f_l = -\frac{V_x\tan\alpha}{t} + \frac{px}{2t}\tan\alpha \qquad \Delta R = \left[-\frac{\sqrt{2x}}{Dj^2}\frac{n}{n_0}\left(\frac{x_0^2\sin^2\alpha\cos\alpha}{2}\right) + \frac{(1-\nu/2)R^2}{Et\cos\alpha}\right]p$$

$$f_{lb} = \frac{6M_x}{t^2} \qquad f_h = \left(\frac{\Delta R}{R}\right)E + \mu f_l$$

$$j = \sqrt[4]{\frac{12(1-\mu^2)}{t^2\tan^2\alpha}} \qquad k = 2j\sqrt{x} \qquad n = \frac{e^{k/\sqrt{2}}}{\sqrt{2\pi k}} \qquad \theta = \left(\frac{-tR_0\tan\alpha}{2D} + \frac{3}{2E}\frac{R_0}{t}\frac{\tan\alpha}{\cos\alpha}\right)p$$

(Subscript 0 denotes that the term in question has value corresponding to $x = x_0$)

24. Uniform outward (or inward) radial force, H lb. per linear in. of circumference at edge



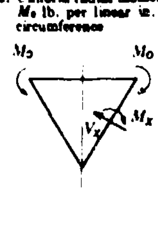
$$M_x = \frac{n}{n_0 k}(2\sqrt{2}\,x_0\cos\alpha)H$$

$$V_x = \frac{n}{n_0 k^2}(2x_0\cos\alpha)\left[\cos\frac{k_0-k}{\sqrt{2}} + \left(\frac{2\sqrt{2}}{k}-1\right)\sin\frac{k_0-k}{\sqrt{2}}\right]H$$

$$\Delta R = \frac{\sqrt{2x}}{Dj^2}\frac{n}{n_0}(x_0\cos\alpha)H$$

$$f_l = \frac{V_x\tan\alpha}{t} \qquad f_{lb} = \frac{6M_x}{t^2} \qquad f_h = \left(\frac{\Delta R}{R}\right)E + \mu f_l \qquad \theta = \left(\frac{\sqrt{12(1-\mu^2)}}{E}\frac{R_0}{t^2}\right)H$$

25. Uniform radial moment, M_0 lb. per linear in. of circumference



$$M_x = \frac{n}{n_0 k}\left[\sqrt{x_0}\,j\left(2-\frac{4\sqrt{2}}{k_0}\right)\sin\frac{k_0-k}{\sqrt{2}} + 2\sqrt{x_0}\,j\cos\frac{k_0-k}{\sqrt{2}}\right]M_0$$

$$V_x = \frac{n}{n_0 k^2}(4\sqrt{2}\sqrt{x_0}\,j^3)\left[\left(1-\frac{\sqrt{2}}{k_0}-\frac{\sqrt{2}}{k}+\frac{4}{k_0 k}\right)\sin\frac{k_0-k}{\sqrt{2}} + \left(\frac{\sqrt{2}}{k_0}-\frac{\sqrt{2}}{k}\right)\cos\frac{k_0-k}{\sqrt{2}}\right]M_0$$

$$\Delta R = -\frac{\sqrt{2x}}{Dj^3}\frac{n}{n_0}\cos\alpha\left\{\sqrt{x_0}\,j\left[\left(\frac{2}{\sqrt{k_0}}-\frac{\sqrt{2}}{2}\right)\cos\frac{k_0-k}{\sqrt{2}} + \frac{\sqrt{2}}{2}\sin\frac{k_0-k}{\sqrt{2}}\right]\right\}M_0$$

$$f_l = -\frac{V_x\tan\alpha}{t} \qquad f_{lb} = \frac{6M_x}{t^2} \qquad f_h = \left(\frac{\Delta R}{R}\right)E + \mu f_l \qquad \theta = \left(\frac{[12(1-\mu^2)]^{\frac{3}{4}}}{E}\sqrt{\frac{2R_0}{t^5\cos\alpha}}\,M_0\right)$$

Conical shell

Table 14.7. Formulae for Stresses and Deflection in Pressure Vessels (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

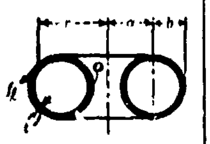
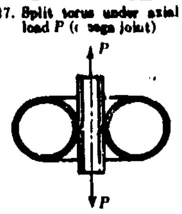
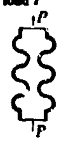
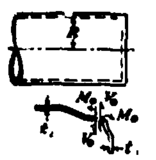
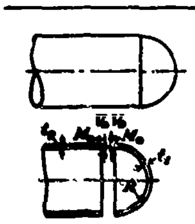
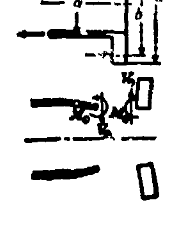
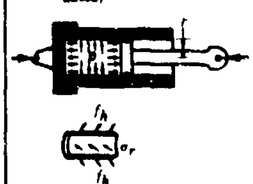
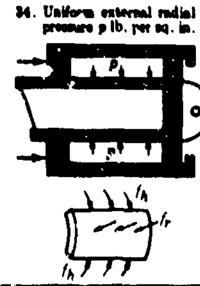
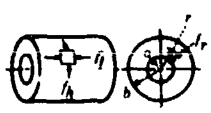
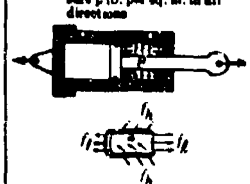
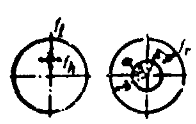
Form of vessel	Manner of loading and Case No.	Formulae
	26. Complete torus under uniform internal pressure p lb. per sq. in	$f_l = \frac{pb}{t}\left(\frac{r+a}{2r}\right)$ Max $f_l = \frac{pb}{t}\left(\frac{2a-b}{2a-2b}\right)$ at 0 $f_h = \frac{pb}{2t}$ (uniform throughout)
Torus 	27. Split torus under axial load P (omega joint) 	Stretch $= \frac{10.88Pb\sqrt{1-\mu^2}}{\pi E t^2}$ Max merid. bndg. stress $f_{lb} = \frac{2.99P}{2\pi ta\sqrt[4]{1-\mu^2}}\sqrt[3]{\frac{ab}{t^2}}$ (near 0) Max circ. mem. stress $f_h = \frac{2.15P\sqrt[4]{1-\mu^2}}{2\pi ta}\sqrt[3]{\frac{ab}{t^2}}$ (tensile, at 0)
	28. Corrugated tube under axial load P 	Stretch $= \frac{1.813Pbn\sqrt{1-\mu^2}}{\pi E t^3}$ Max merid. bndg. stress $f_{lb} = \frac{1.63P}{2\pi ta\sqrt[4]{1-\mu^2}}\sqrt[3]{\frac{ab}{t^2}}$ Max circ. mem. stress $f_h = \frac{0.925P\sqrt[4]{1-\mu^2}}{2\pi ta}\sqrt[3]{\frac{ab}{t^2}}$ (compressive) Here n = number of semicircular corrugations (5 in figure shown)
	29. Same as Case 28 except loaded only by uniform internal pressure, p lb. per sq. in.	Stretch = 0 Max merid. bndg. stress $f_{lb} = \frac{0.955p}{\sqrt[4]{1-\mu^2}}\sqrt[3]{\left(\frac{ab}{t^2}\right)^2}$ Max. circ. mem. stress. $f_h = 0.955p\sqrt[4]{1-\mu^2}\sqrt[3]{\left(\frac{ab}{t^2}\right)^2}$
 Cylinder with flat head	30. Uniform internal (or external) pressure of p lb per sq. in.	$M_0 = \dfrac{\frac{pR^2\lambda_2^2D_1}{4D_1(1+\mu)} + \frac{2pR^2\lambda_2^2Et_1D_2}{Et_2(1-\mu)[Et_1+2RD_2\lambda_2^3(1-\mu)]}}{2\lambda_2 + \frac{2R\lambda_2^2D_2}{D_1(1+\mu)} - \frac{\lambda_2 Et_1}{Et_1+2D_2\lambda_2^3R(1-\mu)}}$ $V_0 = M_0\left(2\lambda_2 + \frac{2R\lambda_2^2D_2}{D_1(1+\mu)}\right) - \frac{pR^2\lambda_2^2D_2}{4D_1(1+\mu)}$ Here D_1 refers to flat head; D_2 and λ_2 refer to cylinder Stress in cylinder is found by superposing the stresses due to p (Case 1), V_0 (Case 14), and M_0 (Case 15)
 Cylindrical with hemispherical head	31. Uniform internal (or external) pressure p lb. per sq. in.	$M_0 = pRt_1\left\{\dfrac{[c(2-\nu)-(1-\nu)]\frac{\sqrt{3(1-\mu)^2}}{12(1-\mu^2)}}{(1-c^3) - \frac{2(1+c^{\frac{3}{2}})(1+c^{\frac{1}{2}})}{1-c^2}}\right\}$ $V_0 = 2M_0\lambda_1\left(\frac{c^{\frac{3}{2}}+1}{c^2-1}\right)$ where $c = \frac{t_1}{t_2}$ and λ_1 refers to hemispherical head If $t_1 = t_2$, $M_0 = 0$ and $V_0 = \frac{p}{8\lambda_1}$ Stress in cylinder is found by superposing the stresses due to p (Case 1), V_0 (Case 14), and M_0 (Case 15) Stress in head is found by superposing the stresses due to p (Case 2), V_0 (Case 16), and M_0 (Case 17)
 Flanged and bolted pipe	32. Uniform internal pressure, p lb. per sq. in.; longitudinal tension P lb.	$V_0 = \dfrac{\left(s^2 - \frac{h^2}{2t}T_1\right)(t+0.2235/T_1)p - 2T_2(h+0.5[illegible]s)P}{1.860/t + T_1\left[h^2\left(2+0.1160\frac{s}{t}T_1\right) + 1.6103sh + 0.866s^2\right]}$ $M_0 = \dfrac{(h^2T_1 + 1.86st)V_0 + hT_2P - 0.5tp\left(s^2 - \frac{h^2}{2t}T_1\right)}{1.3T_1h - 3.464t}$ where $s = \sqrt{at}$; $T_1 = \frac{t^2(3a^2+5d^2)}{h^3(d^2-a^2)}$; $T_2 = \frac{3.58t^2}{h^3(d^2-a^2)}\left[\frac{d^2}{3}\log_e\frac{b}{a} + 0.1(b^2-a^2)\right]$ Long. bending stress in cylinder: $f_{lb} = \frac{6M_0}{t^2}$. Radial bending stress in flange: $f_{lb} = \frac{6}{h^2}\left(M_0 - \frac{1}{2}V_0h\right)$ Long. direct stress in cylinder: $f_l = \frac{P + p\pi(\frac{1}{2}a - \frac{1}{2}t)^2}{\pi at}$. Radial direct stress in flange: $f_l = \frac{V_0}{h} + p$ Max long. stress in cylinder $= f_{lb} + f_l$ (tension at outer surface, at junction with flange). Tangential bending stress in flange: $f_{hb} = f_{lb} + \frac{0.80}{h^2(d^2-a^2)}\left[d^2\left(-15M_0 + 7.5hV_0 + 1.492P\log_e\frac{b}{a}\right) + 0.4475P(b^2-a^2)\right]$ Tangential hoop stress in flange: $f_h = \frac{h^2}{4t^2}T_1(V_0 + hp)$ Max radial stress in flange $= f_{lb} + f_l$ (compression at outer face at junction with cylinder) Max tangential stress in flange $= f_{hb} + f_h$ (tension at inner face at junction with cylinder)

Table 14.7. Formulas for Stresses and Deflection in Pressure Vessels (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Notation for thick vessels: f_l = meridional wall stress, positive when acting as shown (lb. per sq. in.); f_h = hoop wall stress, positive when acting as shown (lb. per sq. in.); f_r = radial wall stress, positive when acting as shown (lb. per sq. in.), a = inner radius of vessel (in.); b = outer radius of vessel (in.); r = radius from axis to point where stress is to be found (in.); Δa = change in inner radius due to pressure, positive when representing an increase (in.); Δb = change in outer radius due to pressure, positive when representing an increase (in.). Other notation same as that used for thin vessels.

Thick vessels—wall stresses f_l (longitudinal), f_h (circumferential) and f_r (radial)

Form of vessel	Manner of loading and Case No.	Formulas
	33. Uniform internal radial pressure p lb. per sq. in. (longitudinal pressure zero or externally balanced) 	$f_l = 0$ $f_h = p\frac{a^2(b^2+r^2)}{r^2(b^2-a^2)}$. Max $f_h = p\frac{b^2+a^2}{b^2-a^2}$ at inner surface $f_r = p\frac{a^2(b^2-r^2)}{r^2(b^2-a^2)}$. Max $f_r = p$ at inner surface; max $f_s = p\frac{b^2}{b^2-a^2}$ at inner surface $\Delta a = p\frac{a}{E}\left(\frac{b^2+a^2}{b^2-a^2}+\mu\right)$; $\Delta b = p\frac{b}{E}\left(\frac{2a^2}{b^2-a^2}\right)$
	34. Uniform external radial pressure p lb. per sq. in. 	$f_l = 0$ $f_h = -p\frac{b^2(a^2+r^2)}{r^2(b^2-a^2)}$. Max $f_h = -p\frac{2b^2}{b^2-a^2}$, at inner surface $f_r = p\frac{b^2(r^2-a^2)}{r^2(b^2-a^2)}$. Max $f_r = p$ at outer surface; max $f_s = \frac{1}{2}$ max f_h at inner surface $\Delta a = -p\frac{a}{E}\left(\frac{2b^2}{b^2-a^2}\right)$; $\Delta b = -p\frac{b}{E}\left(\frac{a^2+b^2}{b^2-a^2}-\mu\right)$
 Cylindrical	35. Uniform internal pressure p lb. per sq. in. in all directions 	$f_l = p\frac{a^2}{b^2-a^2}$. f_h and f_r same as for Case 33 $\Delta a = p\frac{a}{E}\left[\frac{b^2+a^2}{b^2-a^2}-\mu\left(\frac{a^2}{b^2-a^2}-1\right)\right]$; $\Delta b = p\frac{b}{E}\left[\frac{a^2}{b^2-a^2}(2-\mu)\right]$; $p_u = s_u \log_e\frac{b}{a}$
	36. Uniform internal pressure p lb. per sq. in.	$f_l = f_h = p\frac{a^3(b^3+2r^3)}{2r^3(b^3-a^3)}$. Max f_l = max $f_h = p\frac{b^3+2a^3}{2(b^3-a^3)}$ at inner surface $f_r = p\frac{a^3(b^3-r^3)}{r^3(b^3-a^3)}$. Max $f_r = p$ at inner surface; max $f_s = p\frac{3b^3}{4(b^3-a^3)}$ at inner surface $\Delta a = p\frac{a}{E}\left[\frac{b^3+2a^3}{2(b^3-a^3)}(1-\mu)+\mu\right]$; $\Delta b = p\frac{b}{E}\left[\frac{3a^3}{2(b^3-a^3)}(1-\mu)\right]$. Yield pressure $p_y = \frac{2\sigma_y}{3}\left(1-\frac{a^3}{b^3}\right)$
 Spherical	37. Uniform external pressure p lb. per sq. in.	$f_l = f_h = -p\frac{b^3(a^3+2r^3)}{2r^3(b^3-a^3)}$. Max f_l = max $f_h = -p\frac{3b^3}{2(b^3-a^3)}$ at inner surface $f_r = p\frac{b^3(r^3-a^3)}{r^3(b^3-a^3)}$. Max $f_r = p$ at outer surface $\Delta a = -p\frac{a}{E}\left[\frac{3b^3}{2(b^3-a^3)}(1-\mu)\right]$; $\Delta b = -p\frac{b}{E}\left[\frac{a^3+2b^3}{2(b^3-a^3)}(1-\mu)-\mu\right]$

When fluid components are used over a wide range of temperatures, as is the case with units used for cryogenic fluids or those attached directly to rocket engine, the material properties must be considered over the entire temperature range. Many materials exhibit phase changes with resulting changes in properties. This was illustrated in the case of carbon steels by the brittle failure of certain Liberty ships of welded construction and by similar failures in fluid components in cryogenic service.

Compatibility of the fluid with the component material must also be considered, as corrosion will weaken the structure and may cause premature failures. The compatibility of the component material with the mount material, insulation, and external atmosphere, if any, must also be considered.

14.7.2 Membrane Stress Analysis

The following equations are based principally on a thin-wall or membrane analysis. This results in calculated stress levels which are approximately 10 percent lower than indicated by thick-wall theory at a wall thickness equal to 20 percent of the inner radius. Equations and curves for thick-walled cases are presented for certain specific configurations in Sub-Topic 14.7.3. This treatment of membrane stress analysis has been largely excerpted from Reference 152-7.

14.7.2.1 MEMBRANE STRESSES. For shells of revolution, a free-body diagram (Figure 14.7.2.1a) may be drawn of any section to facilitate identifying the membrane stresses. Many fluid components may be treated as shells having the form of a *surface of revolution*. A surface of revolution is obtained by rotating a plane curve about an axis lying in the plane of the curve. This curve is called the *meridian*, and its plane is a *meridian plane*. If the shell of revolution is a vertically-oriented body with the axis of rotation vertical, then the meridian planes are vertical planes containing both the axis of rotation and the meridian. The *membrane stresses* are a function of the pressure load and the radii of curvature. Since the sum of the forces must balance for equilibrium, one arrives at the general form:

$$p = \frac{N_\ell}{r_1} + \frac{N_h}{r_2} \qquad \text{(Eq 14.7.2.1a)}$$

where

p = applied pressure load, psi

N_h = membrane hoop stress resultant, lb_f/in*

N_ℓ = membrane longitudinal stress resultant, lb_f/in*

r_1 = radius of curvature in the vertical (meridian) planes

r_2 = radius of curvature in *normal planes*, i.e., planes perpendicular to both the meridian plane and the shell

*Note: membrane stress resultants are expressed in lb_f per lineal inch and are sometimes called *membrane forces*.

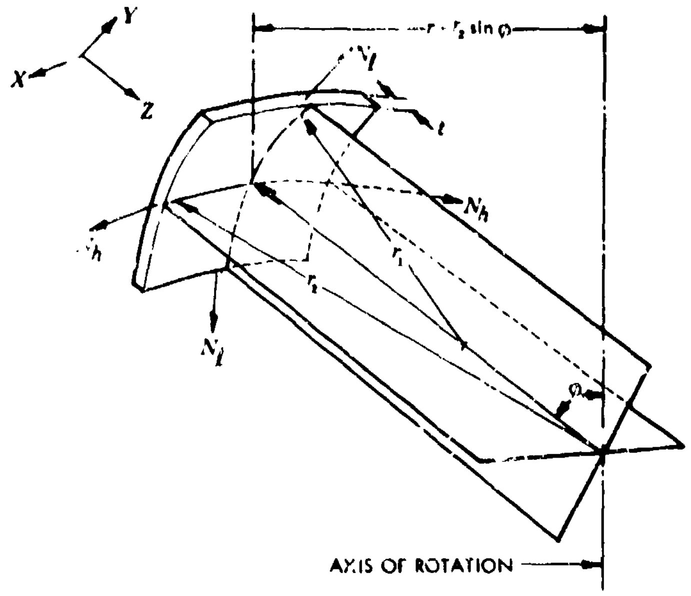


Figure 14.7.2.1a. Membrane Free-Body Diagram

The relationship between membrane stresses and actual stresses is given by:

$$f_h = \frac{N_h}{t}; \; f_\ell = \frac{N_\ell}{t} \qquad \text{(Eq 14.7.2.1b)}$$

where

t = membrane thickness, in.

f_h = hoop stress, psi

f_ℓ = longitudinal stress, psi

With any shell of revolution, the intersection of the surface with planes perpendicular to the axis of rotation are parallel circles and are called *parallels*. The radius of any parallel is denoted as *r* and is defined by

$$r = r_2 \sin\phi \qquad \text{(Eq 14.7.2.1c)}$$

where

ϕ = the angle in the meridian plane between the axis of rotation and the normal plane (Figure 14.7.2.1a).

In membrane stress analysis the axes X, Y, and Z for any given point on a shell at revolution are defined such that the X and Y axes lie in the plane tangent to the shell and the Z axis is normal to the shell. It may be seen from Figure 14.7.2.1a that the Z axis is the intersection of the meridian plane and the normal plane. Hoop stress resultants correspond to the X direction and longitudinal stress resultants correspond to the Y direction. The X, Y, and Z axes associated with a point on the shell are not to be confused with the X, Y, and Z axes associated with a complete shell of revolution wherein the Z axis is the axis of rotation and the X and Y axes lie in planes perpendicular to the axis of rotation.

A membrane under internal pressure, p, as represented by the free-body diagram shown in Figure 14.7.2.1a may be analyzed for $\Sigma F_z = 0$ for equilibrium which gives:

$$2\pi N_\ell r_2 \sin^2\phi = \pi p (r_2 \sin\phi)^2 \quad \text{(Eq 14.7.2.1d)}$$

or

$$N_\ell = \frac{pr_2}{2} \quad \text{(Eq 14.7.2.1e)}$$

And Equation (14.7.2.1e) along with Equation (14.7.2.1a) may be solved for N_h as follows:

$$N_h = pr_2\left(1 - \frac{r_2}{2r_1}\right) \quad \text{(Eq 14.7.2.1f)}$$

Note that if $2r_1 \le r_2$ in Equation (14.7.2.1f), the hoop stresses will be compressive. Notice also that for a cylinder $r_1 \to \infty$ and $N_h = pr_2$, $f_h = N_h/t$, yielding the familiar form for cylindrical hoop stress:

$$t = \frac{pr}{f_h} \quad \text{(Eq 14.7.2.1g)}$$

Equations for determining r_1 and r_2 for various shells of revolution are shown in Table 14.7.2.1. For shells of revolution with negative curvature, r_1 is replaced by $-r_1$ and the same analysis follows (see Figure 14.7.2.1b).

14.7.2.2 DISPLACEMENTS. Many occasions arise where the hoop displacements of a membrane are required. For the usual biaxial stress state encountered in pressure vessels, Hooke's law is:

$$e_h = \frac{1}{E}(f_h - \mu f_\ell) \quad \text{(Eq 14.7.2.2a)}$$

And the hoop strain e_h is defined by:

$$\left.\begin{aligned} e_h &= \frac{\Delta\ell}{\ell} = \frac{2\pi(r+\delta) - 2\pi r}{2\pi r} = \frac{\delta}{r} \\ \therefore\ \delta &= e_h r \end{aligned}\right\} \quad \text{(Eq 14.7.2.2b)}$$

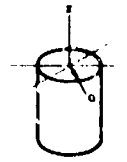
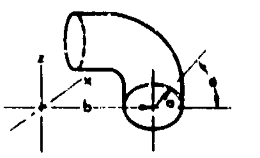
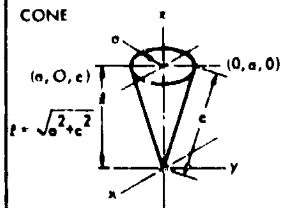
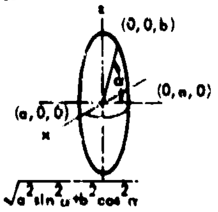
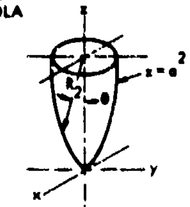
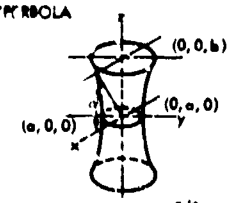
In terms of N_h and N_ℓ

$$\delta = \frac{r}{Et}(N_h - \mu N_\ell) \quad \text{(Eq 14.7.2.2c)}$$

Stress and displacement equations for some surfaces of revolution are shown in Table 14.7.2.2.

14.7.2.3 DISCONTINUITY STRESSES. Because of the differing stiffnesses between a membrane and a head or between a membrane and a part, *discontinuity stresses* develop and must be considered in design. Figures 14.7.2.3a and 14.7.2.3b illustrate some typical discontinuity problems. From continuity, the radial displacement of the

Table 14.7.2.1. r_1 and r_2 for Shells of Revolution
(Reference 152-7)

CYLINDER  $r_1 = \infty$, $r_2 = a$	TORUS  $r_1 = \frac{2a(b + a\cos\phi)}{a(b + 2a\cos\phi) + b}$, $r_2 = \frac{2a(b + a\cos\phi)}{a(b - 2a\cos\phi) - b}$
CONE  $r_1 = \infty$, $r_2 = \frac{\ell a z}{c^2}$	ELLIPSE  $r_1 = \frac{a}{b}\sqrt{a^2\sin^2\alpha + b^2\cos^2\alpha}$; $r_2 = \frac{(a^2\sin^2\alpha + b^2\cos^2\alpha)^{3/2}}{ab}$
PARABOLA  $r_1 = \frac{\sin\theta}{r}\cos^2\theta$, $r_2 = \frac{r}{\sin\theta}$	HYPERBOLA  $r_1 = \frac{(a^2\cosh^2\alpha + b^2\sinh^2\alpha)^{3/2}}{ab}$; $r_2 = \frac{a}{b}\sqrt{a^2\cosh^2\alpha + b^2\sinh^2\alpha}$

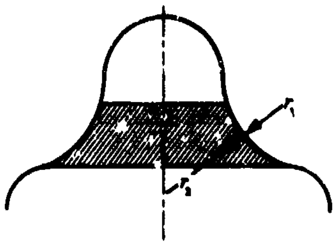
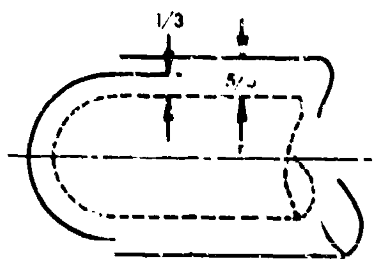


Figure 14.7.2.1b. Shell of Revolution With r_1 Negative in the Shaded Area
(Reference 152-7)

Table 14.7.2.2. f_h, f_l and δ for Common Shells of Revolution

SHELL	f_h	f_l	δ
CYLINDER—HOOP STRESS ONLY	$\frac{pr}{t}$	0	$\frac{pr^2}{Et}$
CYLINDER—HOOP AND LONGITUDINAL STRESS	$\frac{pr}{t}$	$\frac{pr}{2t}$	$\frac{pr^2}{Et}\left(1 - \frac{\mu}{2}\right)$
SPHERE	$\frac{pr}{2t}$	$\frac{pr}{2t}$	$\frac{pr^2}{2Et}(1 - \mu)$
ELLIPSE—AT EQUATOR	$\frac{pa}{t}\left(1 - \frac{a^2}{2b^2}\right)$	$\frac{pa}{2t}$	$\frac{pa^2}{2Et}\left(2 - \mu - \frac{a^2}{b^2}\right)$



$$\delta_{Sphere} = \frac{1}{3}\left(\frac{pr^2}{Et}\right)$$

$$\delta_{Cylinder} = \frac{5}{6}\left(\frac{pr^2}{Et}\right)$$

Figure 14.7.2.3a. Discontinuity Stresses for a Steel (μ = 1/3) Cylindrical Pressure Vessel with a Hemispherical Head
(Reference 152-7)

cylinder must match that of the head. These discontinuities produce bending and compressive stresses in the membrane which must be added to the membrane stresses as computed by Equation (14.7.2.1a). These stresses are included in the hoop stress by the addition of the compressive (or tensile) stress, f_c, acting on the radius, and the bending stress, f_b, acting around the circumference. Therefore, the total circumferential (hoop) stresses become:

$$f_h' = f_h \pm f_c \pm f_b \qquad \textbf{(Eq 14.7.2.3a)}$$

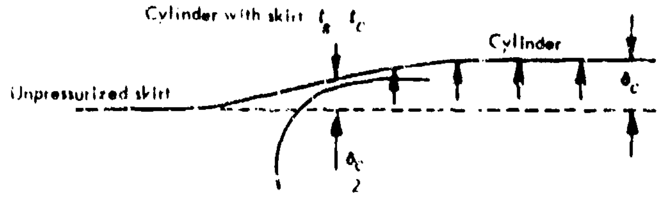


Figure 14.7.2.3b. Discontinuity Stresses for a Cylinder with a Skirt and a Hemispherical Head
(Reference 152-7)

where f_b = Mc/I. However, because of the lateral restrictions of a plate,

$$I = \frac{t^3}{12(1 - \mu^2)}$$

$$c = \frac{t}{2}$$

therefore,

$$f_b = \frac{6M}{t^2(1 - \mu^2)}$$

The longitudinal stress contains the membrane stress plus the discontinuity stress found by using the elastic foundation beam formulae.

A beam on an elastic foundation may be thought of as a beam held in equilibrium under load by an infinite number of springs, each with the spring constant k. Figure 14.7.2.3c shows this analogy roughly.

The solution to the differential equation for various loading conditions usually encountered in pressure vessel design is as follows.

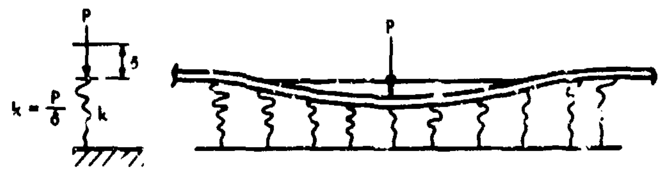


Figure 14.7.2.3c. Beam on an Elastic Foundation
(Reference 152-7)

Case 1. Infinitely long beam, single concentrated load deflection:

$$y = \frac{P\beta}{2k} A_{\beta x} \qquad \textbf{(Eq 14.7.2.3b)}$$

where

y = deflection in.

P = load, lb_f

$$\beta = \sqrt[4]{\frac{k}{4EI}} = \frac{\sqrt[4]{3(1-\mu^2)}}{\sqrt{rt}} \qquad k = \frac{Et}{r^2} \qquad \text{(Eq 14.7.2.3c)}$$

$A_{\beta x}$ = coefficient from Table 14.7.2.3

and the slope is

$$\theta = -\frac{P\beta^2}{k} B_{\beta x} \qquad \text{(Eq 14.7.2.3d)}$$

the moment is

$$M = \frac{P}{4\beta} C_{\beta x} \qquad \text{(Eq 14.7.2.3e)}$$

the shear is

$$V = -\frac{P}{2} D_{\beta x} \qquad \text{(Eq 14.7.2.3f)}$$

and at x = 0 the maximum values become

$$\left. \begin{aligned} y_{max} &= \frac{P\beta}{2k} \\ M_{max} &= M_o = \frac{P}{4\beta} \\ V_{max} &= -\frac{P}{2} \end{aligned} \right\} \qquad \text{(Eq 14.7.2.3g)}$$

These results are summarized in Figure 14.7.2.3d and the coefficients for x are listed in Table 14.7.2.3.

Case 2. Infinitely long beam, uniformly distributed load (Figure 14.7.2.3e), where the deflection is

$$y_o = \frac{q}{2k}(2 - D_{\beta a} - D_{\beta b})$$

the slope is

$$\theta = \frac{q\beta}{2k}(A_{\beta a} - A_{\beta b})$$

the moment is

$$M = \frac{q}{4\beta^2}(B_{\beta a} + B_{\beta b}) \qquad \text{(Eq 14.7.2.3h)}$$

and the shear is

$$V = \frac{q}{4\beta}(C_{\beta a} - C_{\beta b})$$

Table 14.7.2.3. Functions A_x, B_x, C_x and D_x
(Reference 152-7)

FUNCTIONS $A_{\beta x}$, $B_{\beta x}$, $C_{\beta x}$ AND $D_{\beta x}$

βx	$A_{\beta x}$	$B_{\beta x}$	$C_{\beta x}$	$D_{\beta x}$
0	1.0000	0	1.0000	1.0000
0.1	0.9907	0.0903	0.8100	0.9003
0.2	0.9651	0.1627	0.6398	0.8024
0.3	0.9267	0.2189	0.4888	0.7077
0.4	0.8784	0.2610	0.3564	0.6174
0.5	0.8231	0.2908	0.2415	0.5323
0.6	0.7628	0.3099	0.1431	0.4530
0.7	0.6997	0.3199	0.0599	0.3798
π/4	0.6448	0.3224	0	0.3224
0.8	0.6354	0.3223	−0.0093	0.3131
0.9	0.5712	0.3185	−0.0657	0.2527
1.0	0.5083	0.3096	−0.1108	0.1986
1.1	0.4476	0.2967	−0.1457	0.1510
1.2	0.3899	0.2807	−0.1716	0.1091
1.3	0.3355	0.2626	−0.1897	0.0729
1.4	0.2849	0.2430	−0.2011	0.0419
1.5	0.2384	0.2226	−0.2068	0.0158
π/2	0.2709	0.2709	−0.2709	0
1.6	0.1959	0.2018	−0.2077	−0.0059
1.7	0.1576	0.1812	−0.2047	−0.0235
1.8	0.1234	0.1610	−0.1985	−0.0376
1.9	0.0932	0.1415	−0.1899	−0.0484
2.0	0.0667	0.1230	−0.1798	−0.0563
2.1	0.0439	0.1057	−0.1675	−0.0618
2.2	0.0244	0.0895	−0.1548	−0.0652
2.3	0.0080	0.0748	−0.1416	−0.0668
3π/4	0	0.0671	−0.1342	−0.0671
2.4	−0.0056	0.0613	−0.1282	−0.0669
2.5	−0.0166	0.0492	−0.1149	−0.0658
2.6	−0.0254	0.0383	−0.1019	−0.0636
2.7	−0.0320	0.0287	−0.0895	−0.0608
2.8	−0.0369	0.0201	−0.0777	−0.0573
2.9	−0.0403	0.0132	−0.0666	−0.0534
3.0	−0.0423	0.0070	−0.0563	−0.0493
3.1	−0.0431	0.0019	−0.0469	−0.0450
π	−0.0432	0	−0.0432	−0.0432
3.2	−0.0431	−0.0024	−0.0383	−0.0407
3.3	−0.0422	−0.0058	−0.0306	−0.0364
3.4	−0.0408	−0.0085	−0.0237	−0.0323
3.5	−0.0389	−0.0106	−0.0177	0.0283

βx	$A_{\beta x}$	$B_{\beta x}$	$C_{\beta x}$	$D_{\beta x}$
3.6	−0.0366	−0.0121	−0.0124	−0.0245
3.7	−0.0341	−0.0131	−0.0079	−0.0210
3.8	−0.0314	−0.0137	−0.0040	−0.0177
3.9	−0.0286	−0.0140	−0.0006	−0.0147
5π/4	−0.0278	−0.0140	0	−0.0139
4.0	−0.0258	−0.0139	0.0019	−0.0120
4.1	−0.0231	−0.0136	0.0040	−0.0095
4.2	−0.0204	−0.0131	0.0057	−0.0074
4.3	−0.0179	−0.0125	0.0070	−0.0054
4.4	−0.0155	−0.0117	0.0079	−0.0038
4.5	−0.0132	−0.0108	0.0085	−0.0023
4.6	−0.0111	−0.0100	0.0089	−0.0011
4.7	−0.0092	−0.0091	0.0090	0.0001
3π/2	−0.0090	−0.0090	0.0090	0
4.8	−0.0075	−0.0082	0.0089	0.0007
4.9	−0.0059	−0.0073	0.0087	0.0014
5.0	−0.0046	−0.0065	0.0084	0.0019
5.1	−0.0033	−0.0057	0.0080	0.0023
5.2	−0.0023	−0.0049	0.0075	0.0026
5.3	−0.0014	−0.0042	0.0069	0.0028
5.4	−0.0006	−0.0035	0.0064	0.0029
7π/4	0	−0.0029	0.0058	0.0029
5.5	0.0000	−0.0029	0.0058	0.0029
5.6	0.0005	−0.0023	0.0052	0.0029
5.7	0.0010	−0.0018	0.0046	0.0028
5.8	0.0013	−0.0014	0.0041	0.0027
5.9	0.0015	−0.0010	0.0036	0.0026
6.0	0.0017	−0.0007	0.0031	0.0024
6.1	0.0018	−0.0004	0.0026	0.0022
6.2	0.0019	−0.0002	0.0022	0.0020
2π	0.0019	0	0.0019	0.0019
6.3	0.0019	+0.0001	0.0018	0.0018
6.4	0.0018	0.0003	0.0015	0.0017
6.5	0.0018	0.0004	0.0012	0.0015
6.6	0.0017	0.0005	0.0009	0.0013
6.7	0.0016	0.0006	0.0006	0.0011
6.8	0.0015	0.0006	0.0004	0.0010
6.9	0.0014	0.0006	0.0002	0.0008
7.0	0.0013	0.0006	0.0001	0.0007
9π/4	0.0012	0.0006	0	0.0006

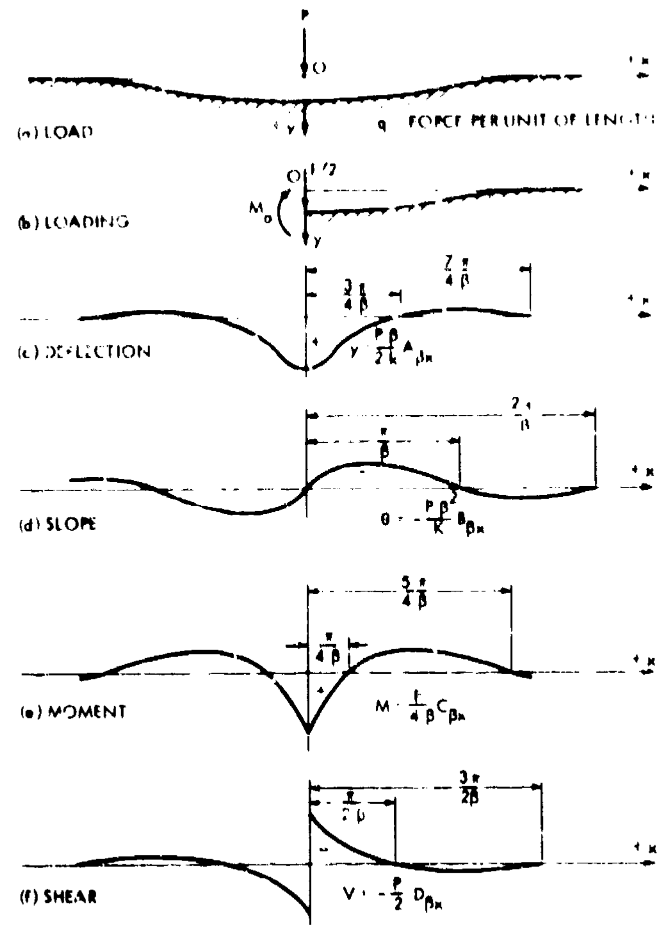


Figure 14.7.2.3d. Loading, Deflection, Slope, Moment and Shear in a Beam on an Elastic Foundation
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

Case 3. Infinitely long beam, single moment or couple (Figure 14.7.2.3f), where the deflection is

$$y = \frac{M_o\beta^2}{k} B_{\beta x}$$

the slope is

$$\theta = \frac{M_o\beta^3}{k} C_{\beta x}$$

the moment is

$$M = \frac{M_o}{2} D_{\beta x}$$

and the shear is

$$V = \frac{M_o\beta}{2} A_{\beta x}$$

(Eq 14.7.2.3i)

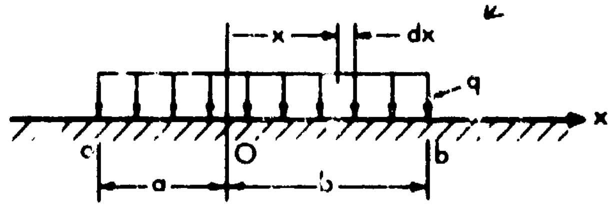


Figure 14.7.2.3e. Uniformly Distributed Load Over a Portion of a Beam on an Elastic Foundation
(Reference 152-7)

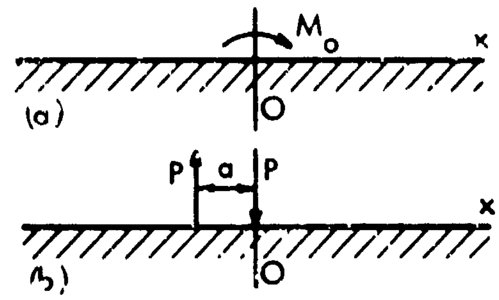


Figure 14.7.2.3f. Single Moment Acting on a Beam on an Elastic Foundation
(Reference 152-7)

Case 4. Semi-infinite beam (Figure 14.7.2.3g) where the deflection is

$$y = \frac{2P\beta}{k} D_{\beta x} - \frac{2M_o\beta^2}{k} C_{\beta x}$$

the slope is

$$\theta = \frac{2B\beta^2}{k} A_{\beta x} + \frac{4M_o\beta^3}{k} D_{\beta x}$$

the moment is

$$M = -\frac{P}{\beta} B_{\beta x} + M_o A_{\beta x}$$

the shear is

$$V = -PC_{\beta x} - 2M_o B_{\beta x}$$

(Eq 14.7.2.3j)

and at x = 0 the maximum values become:

$$y_{max} = \frac{2P\beta}{k} - \frac{2M_o\beta^2}{k}$$

$$\theta_{max} = -\frac{2P\beta^2}{k} + \frac{4M_o\beta^3}{k}$$

(Eq 14.7.2.3k)

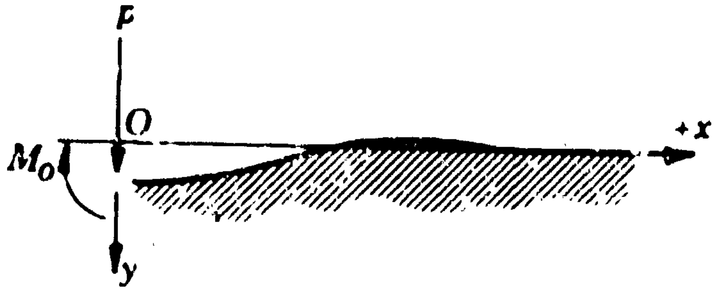


Figure 14.7.2.3g. Semi-Infinite Beam on an Elastic Foundation (Reference 152-7)

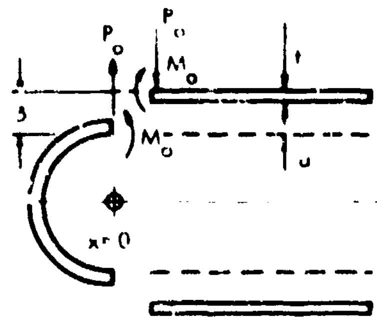


Figure 14.7.2.3h. Discontinuity at Cylinder to Hemispherical Head Junction with Internal Pressure p (Reference 152-7)

Now with the above equations and superposition, discontinuity problems may be considered. Also, the examination of these equations leads to the following results for local load on a cylindrical shell.

a) The load is distributed as hoop stresses from deflection and by longitudinal bending.

b) The load may be considered negligible beyond the distance $x = 2.45\sqrt{rt}$

As an example of the application of the previous development, consider the problem of a cylindrical vessel with a hemispherical head shown in Figure 14.7.2.3h.

From Table 14.7.2.2, stresses and deflections are as follows:

for a cylinder

$$\left.\begin{aligned} f_{hc} &= \frac{pr}{t} \\ f_{\ell c} &= \frac{pr}{2t} \\ \delta_c &= \frac{pr^2}{2tE}(2-\mu) \end{aligned}\right\} \quad \text{(Eq 14.7.2.3l)}$$

for a hemisphere

$$\left.\begin{aligned} f_{hs} &= \frac{pr}{2t} \\ f_{\ell s} &= \frac{pr}{2t} \\ \delta_s &= \frac{pr^2}{2Et}(1-\mu) \end{aligned}\right\} \quad \text{(Eq 14.7.2.3m)}$$

The difference in displacement is $\delta = \delta_c - \delta_s = pr^2/2Et$. If the thickness of the cylinder and hemisphere are equal ($t_c = t_s$) then the deflections caused by the shear load V_o are equal and therefore continuity is satisfied if the edge moment $M_o = 0$ and $V_o = \delta/2$. Substituting these values for M_o and v into Equation (14.7.2.3g), the following results:

$$\frac{\delta}{2} = \frac{2V_o\beta}{k} D_{\beta x} \quad \text{(Eq 14.7.2.3n)}$$

From Table 14.7.2.3, $D_{\beta x} = 1$ at $x = 0$, and using the value of k from Equation (14.7.2.3c), a solution for V_o is:

$$V_o = \frac{P}{8\beta} \quad \text{(Eq 14.7.2.3o)}$$

Consequently the total hoop and longitudinal stress for the cylinder at any x becomes:

$$\left.\begin{aligned} f_h^* &= \frac{pr}{t} - \frac{pr}{4t} D_{\beta x} \pm \frac{3\mu p}{4t^2\beta^2} B_{\beta x} \\ f_\ell^* &= \frac{pr}{2t} \pm \frac{3p}{t^2 4\beta^2} B_{\beta x} \end{aligned}\right\} \quad \text{(Eq 14.7.2.3p)}$$

If the ratio of the stresses with discontinuity versus the stresses without discontinuity is considered as a stress concentration factor, equations may be developed and plotted for differing geometries. One such plot is for the junction between a cylinder and an elliptical head and is summarized in Figures 14.7.2.3i and 14.7.2.3j.

14.7.2.4 INFLUENCE COEFFICIENTS FOR LONG CYLINDERS. Solutions to various basic problems of the type just considered have been tabulated as influence coefficients and are shown in Table 14.7.2.4.

14.7.2.5 INFLUENCE COEFFICIENTS FOR SHORT CYLINDERS. If the cylinder is short, a load at one end will produce an influence at the other end which cannot be ignored. This effectively entails adding a correction factor to the solution of the differential equation for the infinitely long cylinder. As it turns out, if the length of a cylinder, x, is less than $2\pi/\beta$, this effort must be considered. Figure 14.7.2.5 summarizes these corrections.

14.7.2.6 PRESSURE VESSEL HEADS OR END CLOSURES. So far the various head shapes have been ignored; however, this paragraph will outline some properties of interest of the more common head shapes. Important: see also Detailed Topic 14.7.2.10, Membrane Buckling.

Elliptical Heads. Figure 14.7.2.6a describes an elliptical head in terms of the stresses and deflections. One disadvantage of an elliptical head is that once the depth, b, is selected the edge displacement, δ, is fixed and if

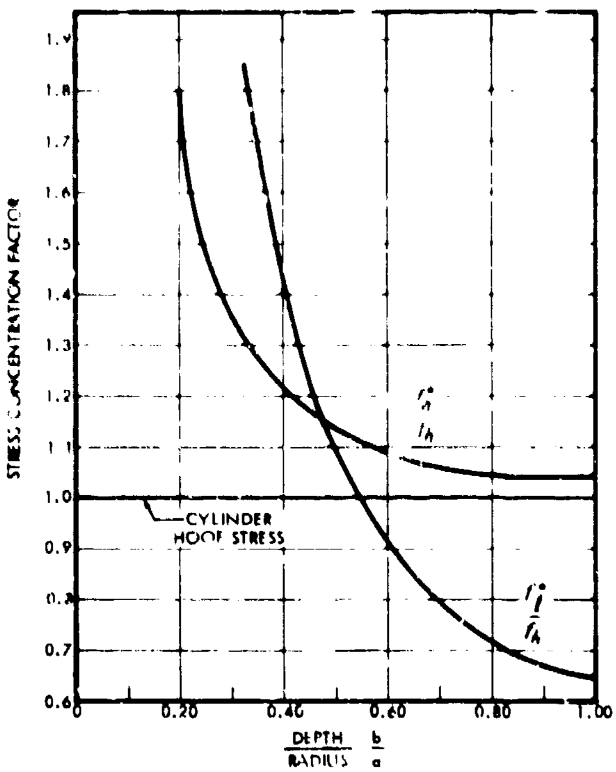


Figure 14.7.2.3i. Stress Concentration Factors in Cylinders Connected to Elliptical Heads
(Reference 152-7)

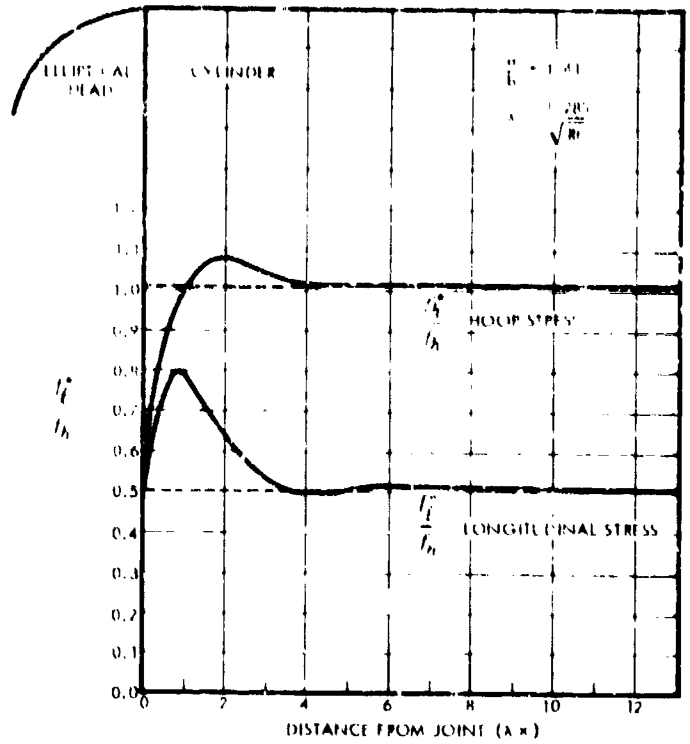
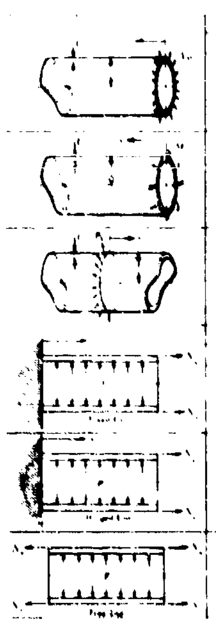


Figure 14.7.2.3j. Stress Distribution Near Elliptical Head to Cylinder Joint
(Reference 152-7)

Table 14.7.2.4. Influence Coefficients for Long Cylinders
(Reference 152-7)



[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

$b/a < 0.707$ the edge will move inward under pressure causing an increase in the discontinuity stresses. Figures 14.7.2.6b and 14.7.2.6c show the distribution of forces for a typical elliptical head, while Figures 14.7.2.6d and 14.7.2.6e show deformations experienced by elliptic shells under internal pressure loads.

Cassinian Heads. An advantage of the Cassinian head is that the dome and edge displacements may be chosen as required, which gives the designer a greater flexibility than with conventional shapes. Figure 14.7.2.6f describes some properties of Cassinian heads. The hemisphere and ellipse are special cases of Cassinian curves.

Elasseno Heads – Constant Shear Strength. This type of head is designed to produce the same shearing stress at each point. But this is not a good situation if the head material has low ductility and, as with the elliptical head, the edge will move inward under pressure. Figure 14.7.2.6g summarizes the various head shapes discussed.

Flat Heads. A listing of stresses in flat plates under pressure is given in Sub-Section 14.10. Curves of stress concentration factors for various flat head configurations are shown in Figures 14.7.2.6h, i, j, and k.

Conical Heads. Table 14.7 includes stresses and strains in cones under internal or external pressure.

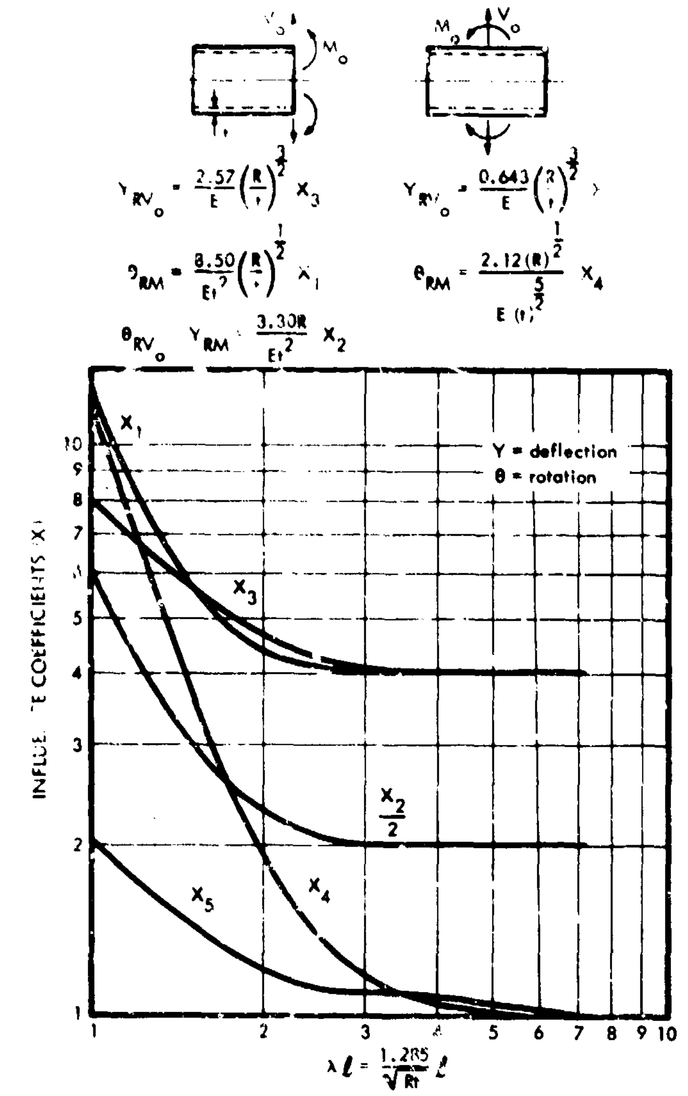


Figure 14.7.2.5. Influence Coefficients for Short Cylindrical Shells
(Reference 152-7)

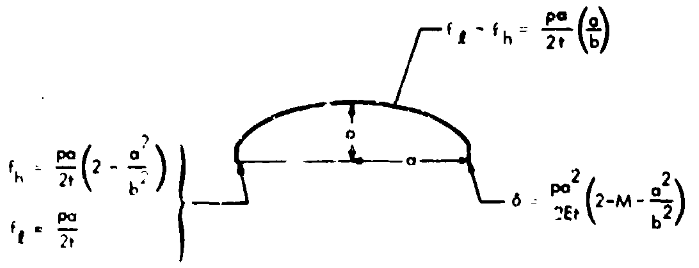


Figure 14.7.2.6a. Elliptical Head
(Reference 152-7)

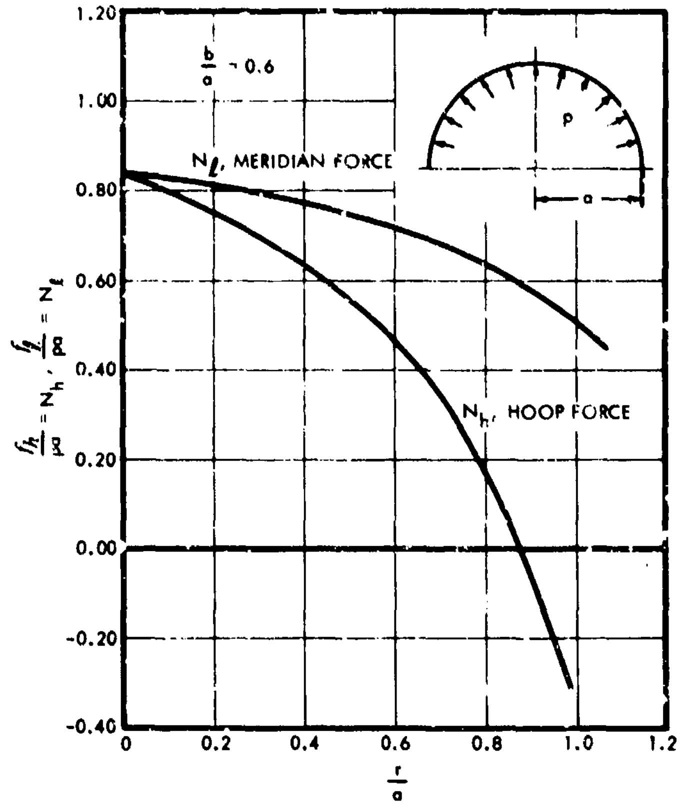


Figure 14.7.2.6b. Membrane Forces for a Typical Elliptical Head (b/a = 0.6)
(Reference 152-7)

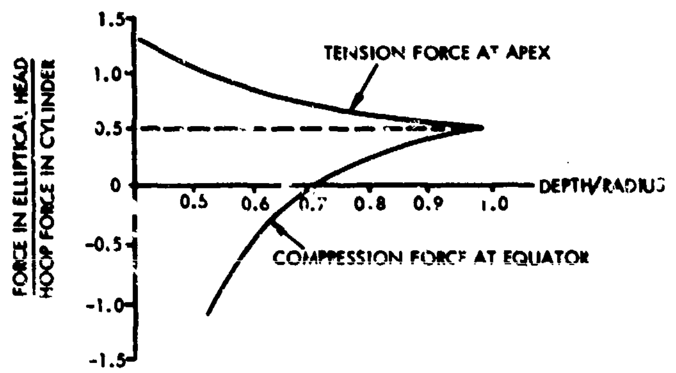
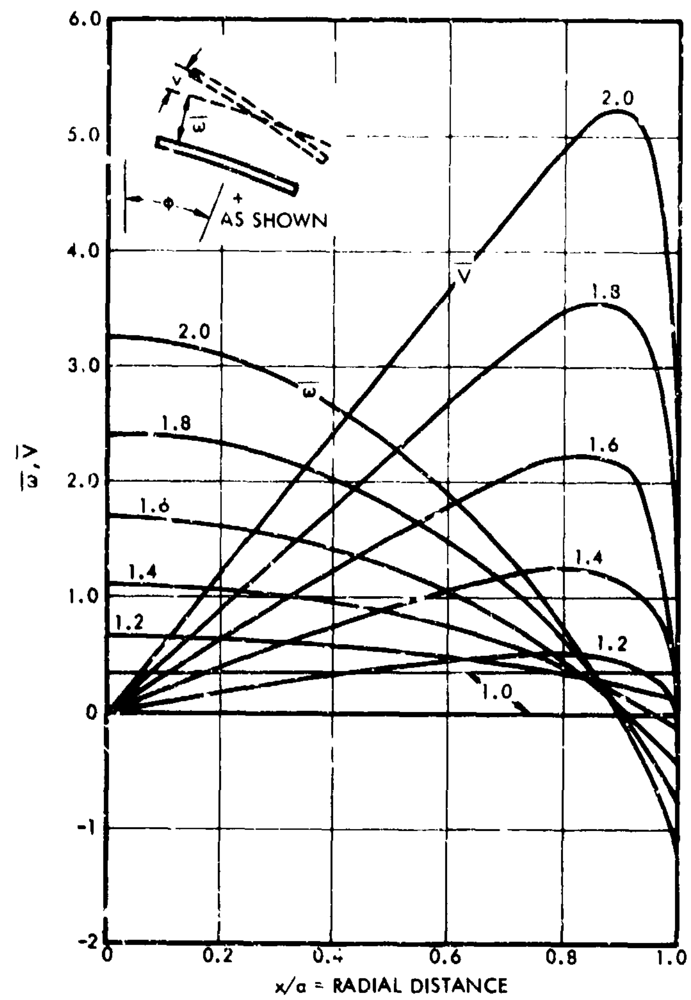


Figure 14.7.2.6c. Membrane Forces in Elliptical Heads Versus b/a
(Reference 152-7)

14.7.2.7 OPENINGS IN SHELLS. Consider first the case of a membrane with an unreinforced opening as shown in Figure 14.7.2.7a. By equilibrium, N_ℓ is found to be:

$$2\pi r N_\ell \sin\phi = \pi r^2 p - \pi r_o^2 p$$

or

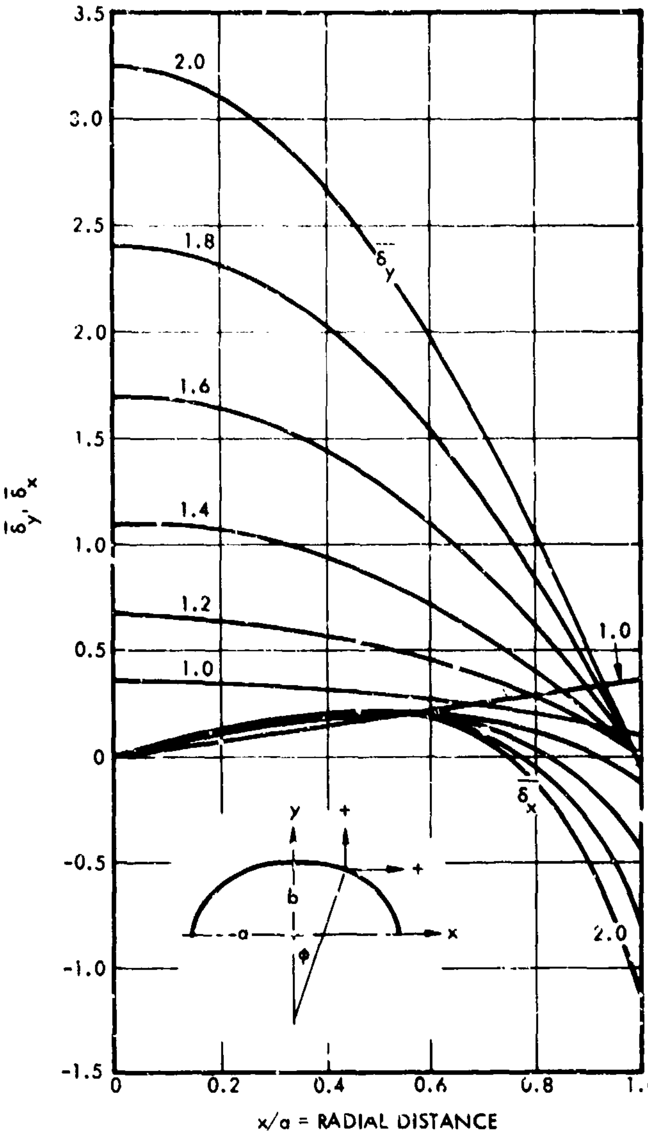


NOTE:

$\omega = \bar{\omega}\left(\frac{pa^2}{Et}\right)$: NORMAL DEFLECTION OF SHELL SURFACE

$V = \bar{V}\left(\frac{pa}{Et}\right)$: ROTATION OF MERIDIAN ON SHELL SURFACE

Figure 14.7.2.9d. Membrane Deformation of Elliptical Shells Subjected to Internal Pressure
(Reprinted with permission from Aerojet-General Solid Rocket Design Manual)



NOTE:

$\delta_x = \bar{\delta}_x\left(\frac{pa^2}{Et}\right)$: DISPLACEMENT OF SHELL IN x DIRECTION

$\delta_y = \bar{\delta}_y\left(\frac{pa^2}{Et}\right)$: DISPLACEMENT OF SHELL IN y DIRECTION

Figure 14.7.2.9e. Membrane Deformation of Elliptical Shells Subjected to Internal Pressure
(Reprinted with permission from Aerojet-General Solid Rocket Design Manual)

$$N_\ell = \frac{pr_2}{2}\left[1 - \left(\frac{r_o}{r_2 \sin\phi}\right)^2\right] \qquad \text{(Eq 14.7.2.7a)}$$

and then, with Equation (14.7.2.1a), the relationship for N_h becomes:

$$N_h = \frac{pr_2}{2}\left\{2 - \frac{r_2}{r_1}\left[1 - \left(\frac{r_o}{r_2 \sin\phi}\right)^2\right]\right\} \qquad \text{(Eq 14.7.2.7b)}$$

When $r = r_o = r_2 \sin\phi$ (at the edge of the hole) Equations (14.7.2.7a) and (14.7.2.7b) reduce to the following:

$$\left.\begin{aligned} N_\ell &= 0 \\ N_h &= pr_2 \end{aligned}\right\} \qquad \text{(Eq 14.7.2.7c)}$$

Thus the hoop stress is twice that of the longitudinal stress if no hole is present.

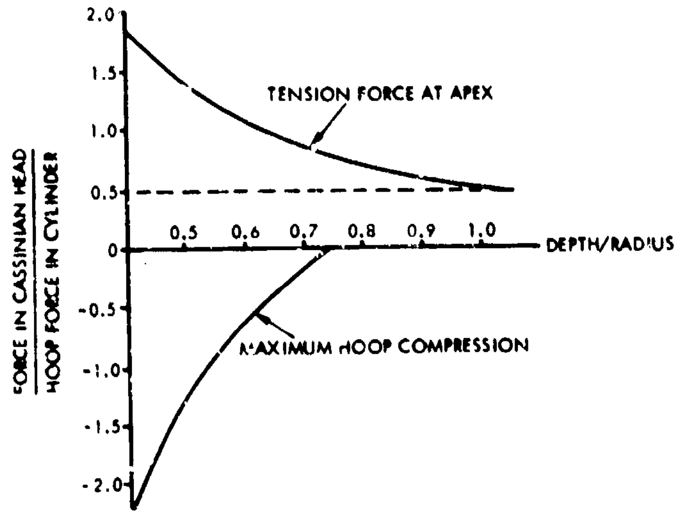


Figure 14.7.2.6f. Tensile and Compressive Forces in Cassinian Heads
(Reference 152-7)

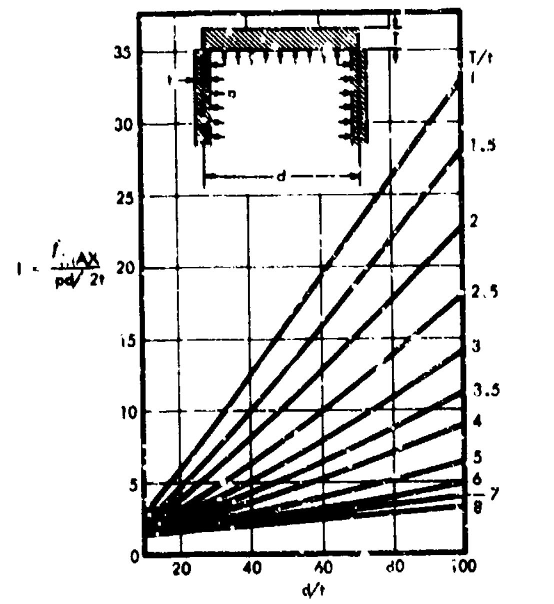


Figure 14.7.2.6h. Discontinuity Stress at Edge of Flat Head (Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain", W. Griffel, Frederick Ungar Pub. Co., 1966)

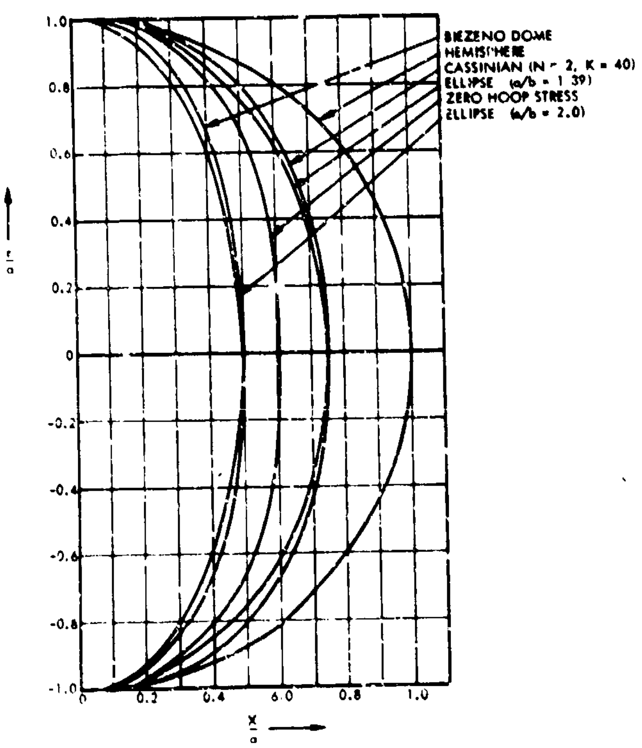


Figure 14.7.2.6g. Comparison of Head Shapes
(Reference 152-7)

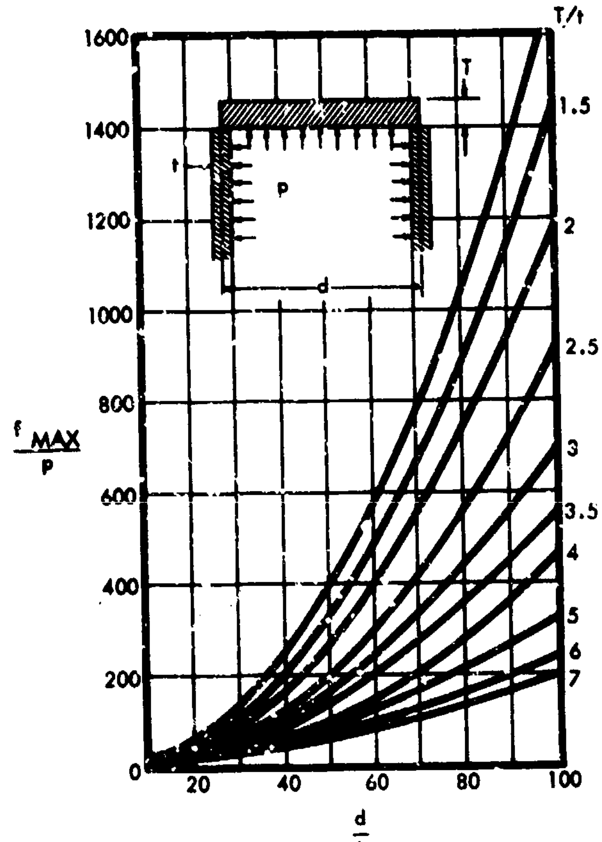


Figure 14.7.2.6i. Discontinuity Stress at Edge of Flat Head (Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain", W. Griffel, Frederick Ungar Pub. Co., 1966)

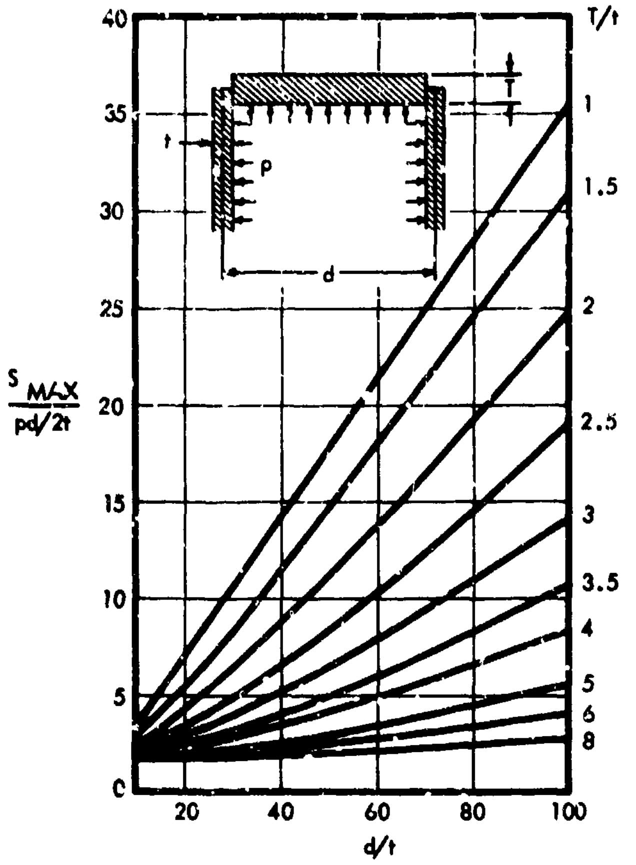


Figure 14.7.2.6j. Discontinuity Stress at Edge of Flat Head *(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Co., 1966)*

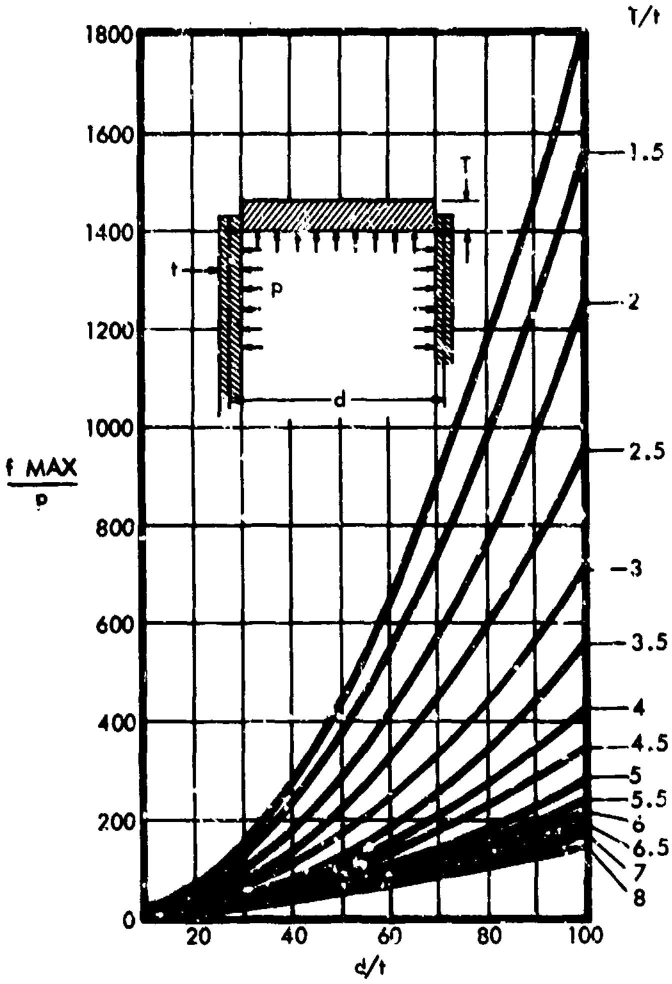


Figure 14.7.2.6k. Discontinuity Stress at Edge of Flat Head *(Adapted with permission from Reference 598-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Co., 1966)*

By the previous analysis, the effect of a hole in a membrane is shown to be a stress concentration. Figure 14.7.2.7b shows how a hole affects the stress in a membrane.

To compensate for this increase of stress near a hole, reinforcement must be added around the hole. As a general rule of thumb the reinforcement is placed within the limits of non-negligible stress concentration. This is shown graphically in Figure 14.7.2.7c. For an ideal amount of reinforcement the displacement of the reinforcing ring must be matched to that of the membrane. These displacement forces are shown in Figure 14.7.2.7d.

The displacements are as follows:

$$\left.\begin{aligned} \delta_{ring} &= \frac{F r_o}{A_R E_R} \\ \delta_{sphere} &= \frac{N_\ell r_o}{E_s t}(1-\mu) \end{aligned}\right\} \quad \text{(Eq 14.7.2.7d)}$$

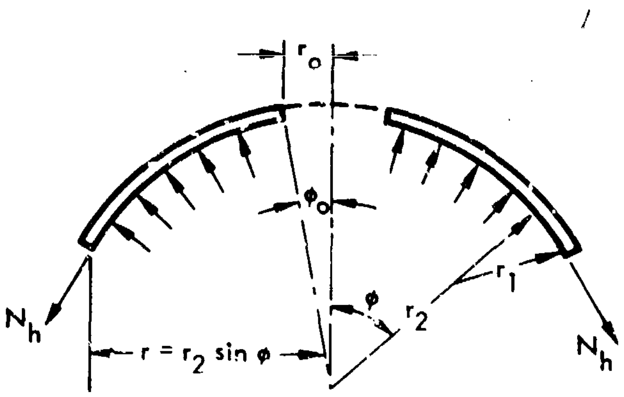


Figure 14.7.2.7a. Unreinforced Opening in a Membrane with Internal Pressure p *(Reference 152-7)*

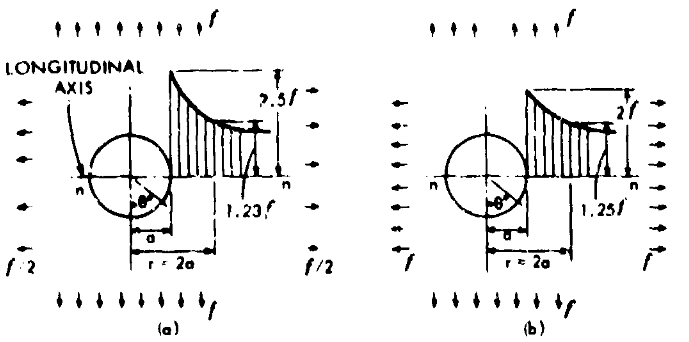


Figure 14.7.2.7b. Variation in Stress in the Region of a Circular Hole in (a) Cylinder, (b) Sphere Subjected to Internal Pressure
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications", J. F. Harvey, Van Nostrand, 1963)

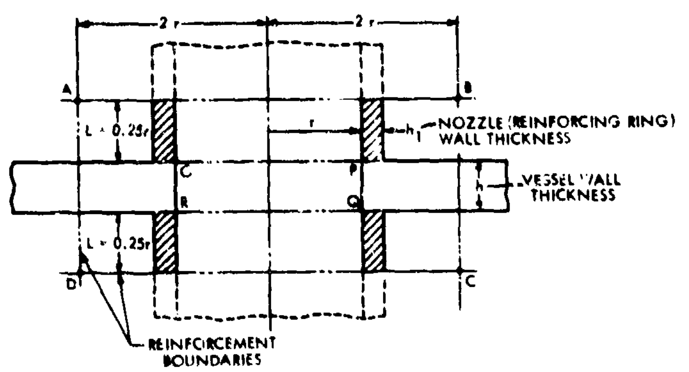


Figure 14.7.2.7c. Reinforcement Boundaries for Circular Openings in Cylindrical and Spherical Vessels
(Adapted with permission from Reference 628-1)

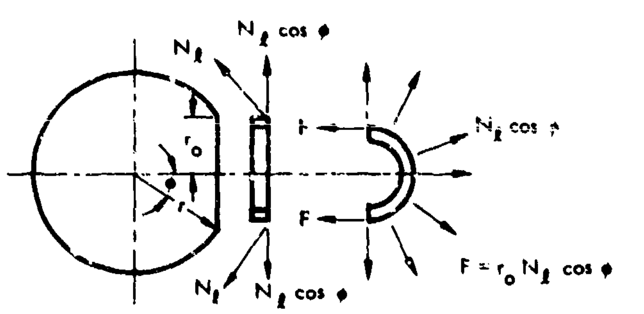


Figure 14.7.2.7d. Equilibrium Forces Around a Reinforced Hole
(Reference 152-7)

then since $f_R = f_S$ for continuity,

$$\frac{N_\ell \cos\phi \, r_o^2}{A_R E_R} = \frac{N_\ell r_o}{E_S t}(1-\mu) \qquad \text{(Eq 14.7.2.7e)}$$

or

$$A_R = \frac{E_S}{E_R}\left(\frac{r_o t \cos\phi}{1-\mu}\right) \qquad \text{(Eq 14.7.2.7f)}$$

If, as is usually the case, $E_R = E_S$, then

$$A_R = \frac{r_o t \cos\phi}{1-\mu} \qquad \text{(Eq 14.7.2.7g)}$$

Many times, however, the area given by Equation (14.7.2.7g) is too small to be practical, i.e., not enough area is provided for studs, etc. In cases such as this, *rigid* reinforcing rings are used. Figures 14.7.2.7e and 14.7.2.7f show the effect of reinforcement location on discontinuity stress in the membrane. When rigid rings are used as hole reinforcing, the displacements must be equal to preserve continuity. Figure 14.7.2.7g graphically describes a typical case where, for continuity, the points 0″ and 0′ are brought together by applying moments and forces to the ring and shell. The addition of the moments and forces to preserve continuity must, to preserve equilibrium, be such that their magnitudes are equal and their directions opposite. Figure 14.7.2.7b shows these forces in a ring to membrane junction.

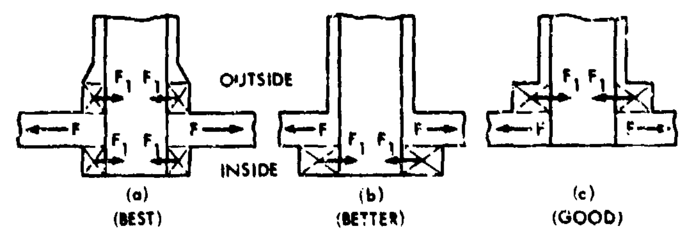


Figure 14.7.2.7e. Diagrammatic Location of Nozzle Opening Reinforcement, (a) Balanced, (b) Unbalanced Inside, (c) Unbalanced Outside
(Adapted with permission from Reference 628-1)

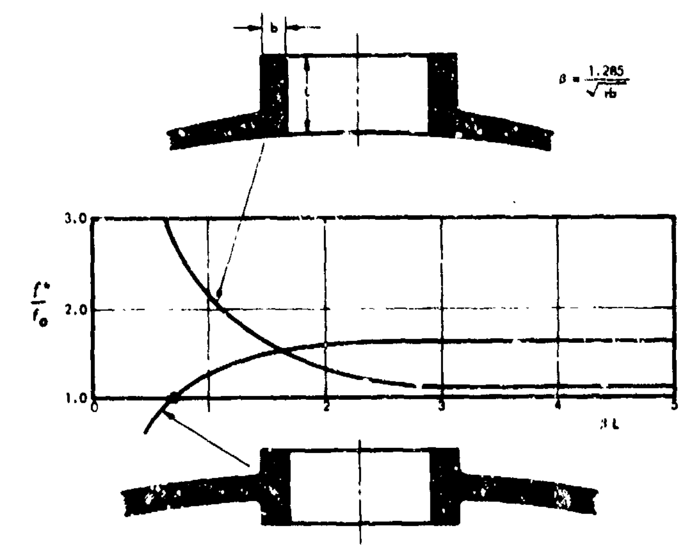


Figure 14.7.2.7f. Comparison of Symmetrical Versus Unsymmetrical Reinforcing Rings
(Reference 152-7)

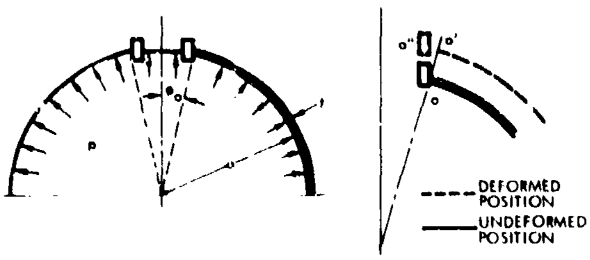


Figure 14.7.2.7g. Spherical Shell with a Reinforced Opening *(Reference 152-7)*

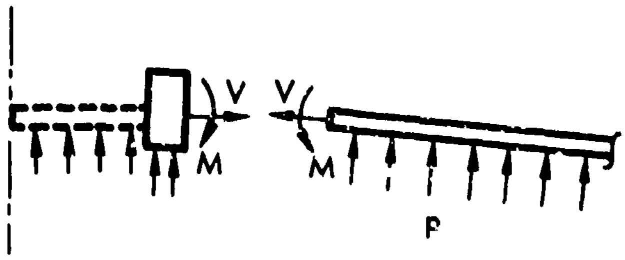


Figure 14.7.2.7h. Discontinuity Moments and Forces *(Reference 152-7)*

To facilitate the analysis of a ring-membrane system of the type previously described, influence coefficients (displacements and rotations due to unit values of M and V) are defined as follows:

Membrane:

$\theta_{MM}, u_{MM} \equiv$ rotation and displacement due to a unit moment

$\theta_{MV}, u_{MV} \equiv$ rotation and displacement due to a unit force

$\theta_{MP}, u_{MP} \equiv$ rotation and displacement due to a unit pressure

Ring:

$\theta_{RM}, u_{RM} \equiv$ rotation and displacement due to a unit moment

$\theta_{RV}, u_{RV} \equiv$ rotation and displacement due to a unit force

$\theta_{RP}, u_{RP} \equiv$ rotation and displacement due to a unit pressure

where the signs of θ, u, M, and V are established as follows:

a) M and V are positive if they act in the direction shown in Figure 14.7.2.7h

b) θ is positive counterclockwise

c) u is positive if the membrane displacement is increased

d) The membrane displacement is positive outward.

Since these influence coefficients are unit values they must be multiplied by the loads to give the true rotation or displacement. The total values of rotation and displacement become:

Membrane:

total displacement:

$$u_M = u_{MP}P - u_{MM}M - u_{MV}V \qquad \text{(Eq 14.7.2.7h)}$$

total rotation:

$$\theta_M = \theta_{MM}M + \theta_{MV}V - \theta_{MP}P \qquad \text{(Eq 14.7.2.7i)}$$

Ring:

total displacement:

$$u_R = u_{RV}V + u_{RP}P - u_{RM}M \qquad \text{(Eq 14.7.2.7j)}$$

total rotation:

$$\theta_R = \theta_{RP}P + \theta_{RV}V - \theta_{RM}M \qquad \text{(Eq 14.7.2.7k)}$$

Compatibility then requires that:

$$\left.\begin{array}{l} u_R = u_M \\ \text{and} \\ \theta_R = \theta_M \end{array}\right\} \qquad \text{(Eq 14.7.2.7l)}$$

By substituting Equations (14.7.2.7h) through (14.7.2.7k) into Equation (14.7.2.7l) gives the following equations for M and V.

$$M = \frac{P\left[(u_{MP} - u_{RP})(\theta_{RV} - \theta_{MV}) + (\theta_{MP} + \theta_{RP})(u_{RV} + u_{MV})\right]}{(\theta_{MM} + \theta_{RM})(u_{RV} + u_{MV}) - (u_{MM} - u_{RM})(\theta_{MV} - \theta_{RV})} \qquad \text{(Eq 14.7.2.7m)}$$

$$V = \frac{P\left[(u_{MP} - u_{RP})(\theta_{MM} + \theta_{RM}) - (\theta_{MP} + \theta_{RP})(u_{MM} - u_{RM})\right]}{(\theta_{MM} + \theta_{RM})(u_{RV} + u_{MV}) - (u_{MM} - u_{RM})(\theta_{MV} - \theta_{RV})} \qquad \text{(Eq 14.7.2.7n)}$$

Hemispherical Heads. Reference 152-7 lists the following displacements and rotations for hemispherical heads.

Displacements:

$$\left.\begin{array}{l} u_{MV} = \dfrac{W_c}{E\phi_o} \\[2ex] u_{MM} = \dfrac{X_c}{Et\phi_o} \\[2ex] u_{MP} = \dfrac{a^2 (1 - \mu) \sin\phi_o}{2Et} \end{array}\right\} \qquad \text{(Eq 14.7.2.7o)}$$

Rotations:

$$\left.\begin{aligned} \theta_{MV} &= \frac{X_c}{Et\phi_o} \\ \theta_{MM} &= \frac{X_d}{Et^2\phi_o} \\ \theta_{MP} &= 0 \end{aligned}\right\} \quad \text{(Eq 14.7.2.7p)}$$

where W_c, X_c and X_d are functions of ξ_o. Figure 14.7.2.7l shows these relations.

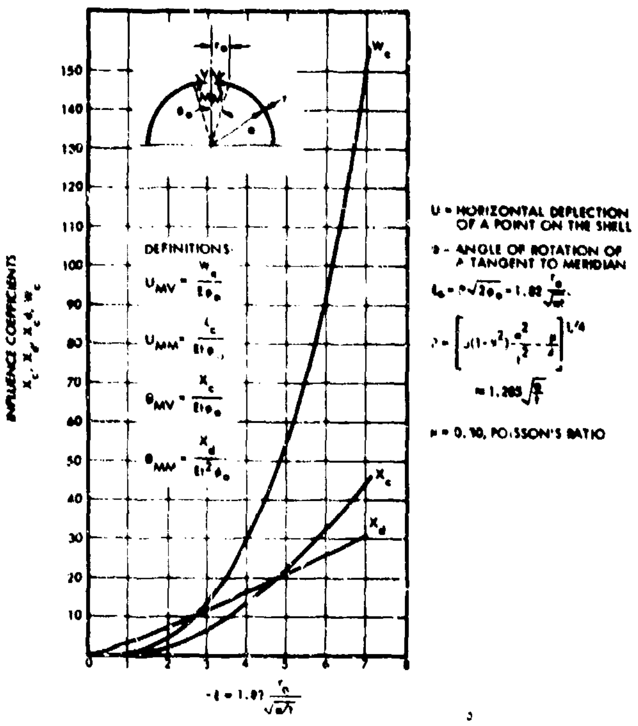


Figure 14.7.2.7l. Influence Coefficients for a Small Opening ($0° < \phi_o < 10°$) *(Reference 152-7)*

For heads other than hemispherical, Equations (14.7.2.7o) and (14.7.2.7p) become:

$$\left.\begin{aligned} U_{MP} &= \frac{PR_1 \sin\phi_o}{2Et} \left[R_1 (2-\nu) 1 - \frac{R_1^2}{R_2}\right] \\ \theta_{MP} &\neq 0 \end{aligned}\right\} \quad \text{(Eq 14.7.2.7q)}$$

and the relationship for ξ_o becomes:

$$\xi_o = 1.82 \frac{R_o}{\sqrt{R_1 t}} \quad \text{(Eq 14.7.2.7r)}$$

With these results, Equation (14.7.2.7c) becomes, for the longitudinal stress (if the reinforcing ring has an area greater than ideal, the longitudinal stresses at point o are larger than the hoop stresses):

$$S_2 = \frac{T_2}{t} + \frac{V}{t\cos\phi_o} \pm \frac{6M}{t^2} \quad \text{(Eq 14.7.2.7s)}$$

where the plus sign before the last term is for the outer surface of the membrane and the negative sign for the inner surface. Table 14.7.2.7 summarizes the influence coefficients for various reinforcement types.

In placing the reinforcement, there is the choice of placing the ring either completely inside, completely outside, or symmetrical with respect to the membrane. Figures 14.7.2.7j through 14.7.2.7p show the effect of this ring geometry on stresses.

The treatment of openings at points other than the apex of a head is extremely complicated, and when a problem of this type arises it is wise to consult directly with a stress analyst. Figure 14.7.2.7q shows the stress distribution around a typical off-apex reinforced opening.

14.7.2.8 CONCENTRATED LOADS ON MEMBRANES. Many times a membrane will be subjected to loads which are concentrated in a small area. Here the membrane forces are extremely high and, if bending stresses were neglected, would approach infinity. Usually it is safe to assume that the effects of a concentrated load are negligible in a region defined by the radius r, where r = 1.9 Rt (Reference 152-7). Figure 14.7.2.8 shows the stress concentration encountered when a membrane is reinforced with a rigid insert, while Table 14.7 contains some common types of concentrated loads and their associated stresses and deflections. These stresses and deflections must be added to those due to other types of loading.

14.7.2.9 SPHERICAL MEMBRANES. For the spherical membrane shown in Figure 14.7.2.9, the following equations give the deflection, y, and edge moment, M_o.

$$y = A \frac{PR^2\, 3(1-\mu^2)}{4\pi Et^3} \quad \text{(Eq 14.7.2.9a)}$$

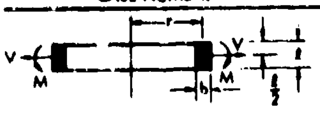
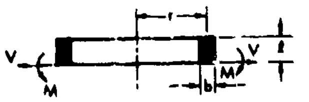
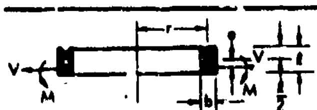
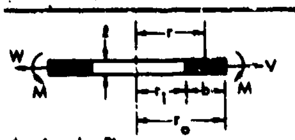
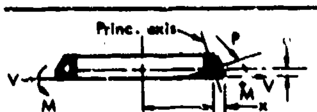
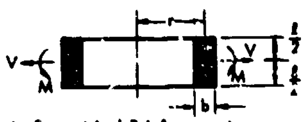
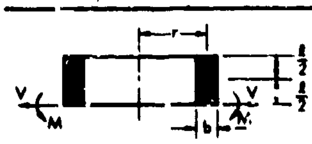
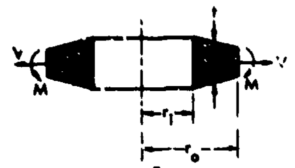
$$M_o = B \frac{P}{4\pi} \quad \text{(Eq 14.7.2.9b)}$$

where A and B are coefficients that depend upon

$$\alpha = 2\left[3(1-\mu^2)\right]^{1/4} \sqrt{\frac{h}{t}}$$

and are tabulated in Table 14.7.2.9. Other equations for stress and deflection in spherical shells are included in Table 14.7.

Table 14.7.2.7. Formulas for Deflections and Rotations

RING GEOMETRY, LOADING AND CASE NUMBER	DEFLECTIONS	ROTATIONS	LIMITATIONS FOR FORMULAE
 1. Symmetrical Ring	$U_{RV} = \frac{Vr^2}{EA}$, $(A = b\ell)$ $U_{RM} = 0$	$\theta_{RV} = 0$ $\theta_{RM} = \frac{Mr}{EI_x}$, $\left(I_x = \frac{b\ell^3}{12}\right)$	$\frac{b}{r} << 1$ $\ell < 1.56\sqrt{rb}$
 2. Unsymmetrical Ring	$U_{RV} = \frac{4Vr^2}{EA}$ $U_{RM} = \frac{6Mr^2}{EA\ell}$	$\theta_{RV} = \frac{Vr^2\ell}{2EI_x}$ $\theta_{RM} = \frac{Mr^2}{EI_x}$	$\frac{b}{r} << 1$ $\ell < 1.56\sqrt{rb}$
 3. Partially Unsymmetrical Ring	$U_{RV} = \frac{Vr^2}{EA}\left[1 + 12\left(\frac{e}{\ell}\right)^2\right]$ $U_{RM} = \frac{12Mr^2}{EA\ell}\left(\frac{e}{\ell}\right)$	$\theta_{RV} = \frac{Vr^2 e}{EI_x}$ $\theta_{RM} = \frac{Mr^2}{EI_x}$	$\frac{b}{r} << 1$ $\ell < 1.56\sqrt{rb}$
 4. Annular Ring	$U_{RV} = \frac{Vr_o}{\ }\left[\frac{(1+\mu)\left(\frac{r_i}{r_o}\right)^2 + (1-\mu)}{1-\left(\frac{r_i}{r_o}\right)^2}\right]$ $U_{RM} = 0$	$\theta_{RV} = 0$ $\theta_{RM} = \frac{12Mr}{E\ell^3 \ln\left(\frac{r_o}{r_i}\right)^2}$	
 5. Compact Ring With Oblique Principal Axes	$U_{RV} = \frac{Vr^2}{EA} + \frac{Vr^2e^2}{E}\left[\frac{1}{I_x - I_{xy}^2/I_y}\right]$ $U_{RM} = \frac{Mr^2e}{E}\left[\frac{1}{I_x - I_{xy}^2/I_y}\right]$	$\theta_{RV} = \frac{Vr^2e}{E}\left[\frac{1}{I_x - I_{xy}^2/I_y}\right]$ $\theta_{RM} = \frac{Mr^2}{E}\left[\frac{1}{I_x - I_{xy}^2/I_y}\right]$	$x << r$
 6. Symmetrical Reinforcement	$U_{RV} = \frac{Vr^2\beta}{2Eb} X_5$ $U_{RM} = 0$ $\beta = \frac{1.285}{\sqrt{rb}}$ For $\mu = 0.3$	$\theta_{RV} = 0$ $\theta_{RM} = \frac{Mr^2\beta^3}{Eb} X_4$	$\frac{b}{r} \leq 0.2$
 7. Unsymmetrical Reinforcement	$U_{RV} = \frac{Vr^2\beta}{2Eb} X_3$ $U_{RM} = \frac{Mr^2\beta^2}{Eb} X_2$	$\theta_{RV} = \frac{Vr^2\beta^2}{Eb} X_2$ $\theta_{RM} = \frac{Mr^2\beta^3}{Eb} X_1$	$\frac{b}{r} \leq 0.2$
 $t = \alpha r^n$ at $r = r_i$, $t = t_i$ at $r = r_o$, $t = t_o$ 8. Tapered Ring	$U_{RV} = \frac{(1-\mu^2) r_o^{1-n} V}{E\left(r_i^{m'_2} r_o^{m'_1} - r_i^{m'_1} r_o^{m'_2}\right)}$ $O\left\{\frac{r_i^{m'_2} r_o^{m'_1}}{m'_1 + \mu} - \frac{r_i^{m'_1} r_o^{m'_2}}{m'_2 + \mu}\right\}$ $U_{RM} = 0$	$\theta_{RV} = 0$ $\theta_{RM} = \frac{12(1-\mu^2) r_o^{1-3n}}{E\alpha^3\left(r_i^{m_2} r_o^{m_1} - r_i^{m_1} r_o^{m_2}\right)}$ $O\left\{\frac{r_i^{m_2} r_o^{m_1}}{m_1 + \mu} - \frac{r_i^{m_1} r_o^{m_2}}{m_2 + \mu}\right\}$	NOTE: When $\ell < 1.56\sqrt{rb}$ (6) and (7) reduce to cases (1) and (2) respectively. *Coefficients X_1 and X_3 are given in Figure 14.7.2.5.

$$m_1 = \frac{-3n + \sqrt{9n^2 - 4(3n\mu - 1)}}{2}, \quad m_2 = \frac{-3n - \sqrt{9n^2 - 4(3n\mu - 1)}}{2}$$

$$m'_1 = \frac{-n + \sqrt{n^2 - 4(n\mu - 1)}}{2}, \quad m'_2 = \frac{-n - \sqrt{n^2 - 4(n\mu - 1)}}{2}$$

$$n = \text{Taper}\left(n = \frac{\ln t_o - \ln t_i}{\ln r_o - \ln r_i}\right)$$

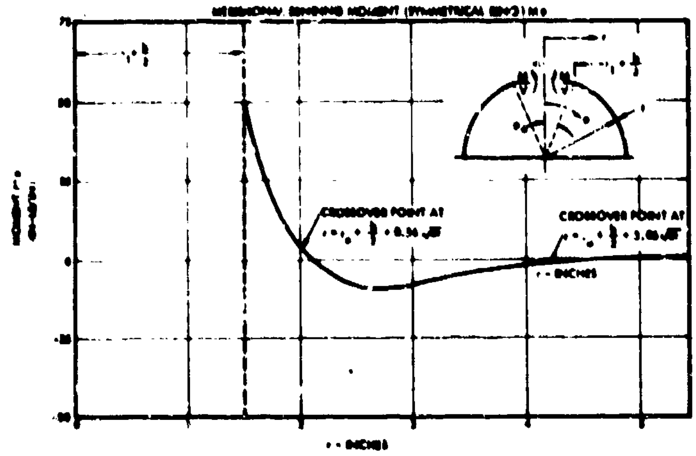


Figure 14.7.2.7j. Longitudinal Bending Moment M_2 for a Symmetric Ring
(Reference 152-7)

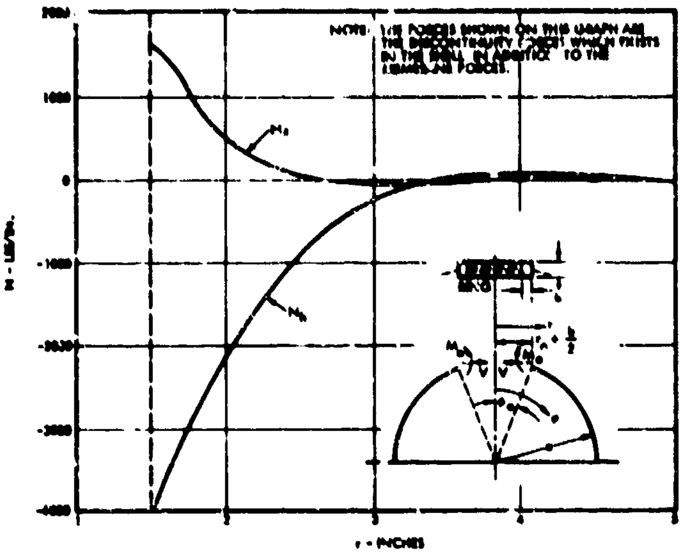


Figure 14.7.2.7k. Longitudinal and Hoop Forces in a Symmetrical Ring
(Reference 152-7)

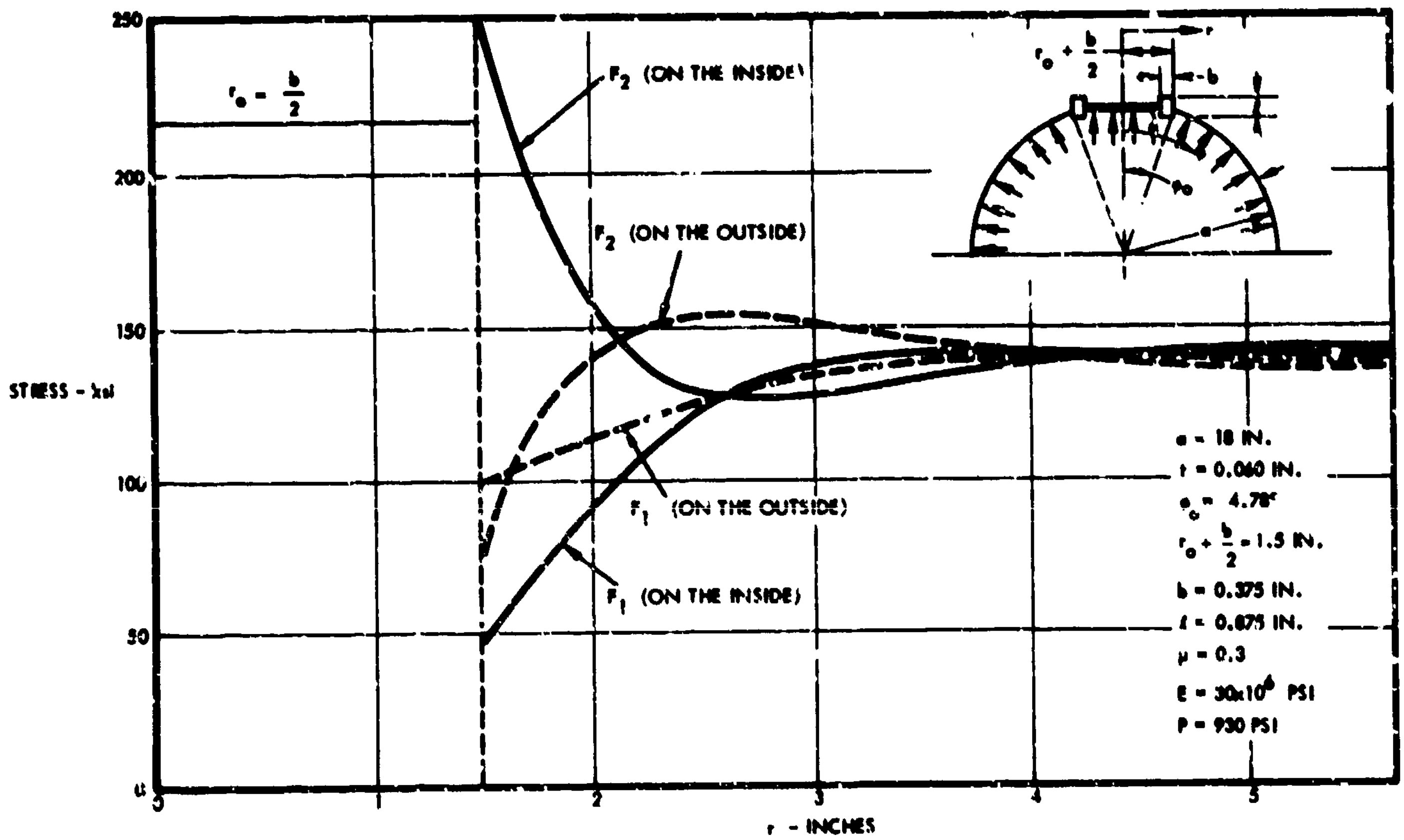


Figure 14.7.2.7l. Resultant Stresses Due to Moment, Horizontal Forces V, and Membrane Forces for a Symmetrical Ring
(Reference 152-7)

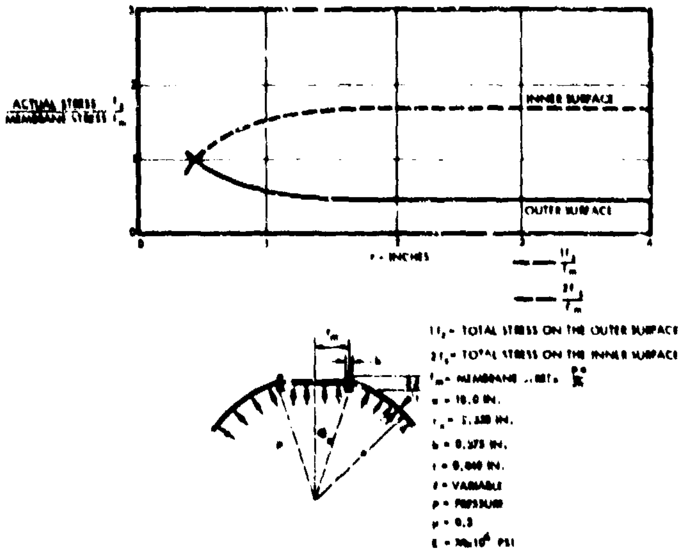


Figure 14.7.2.7m. Stresses at the Ring-Membrane Junction for a Symmetrical Ring
(Reference 152-7)

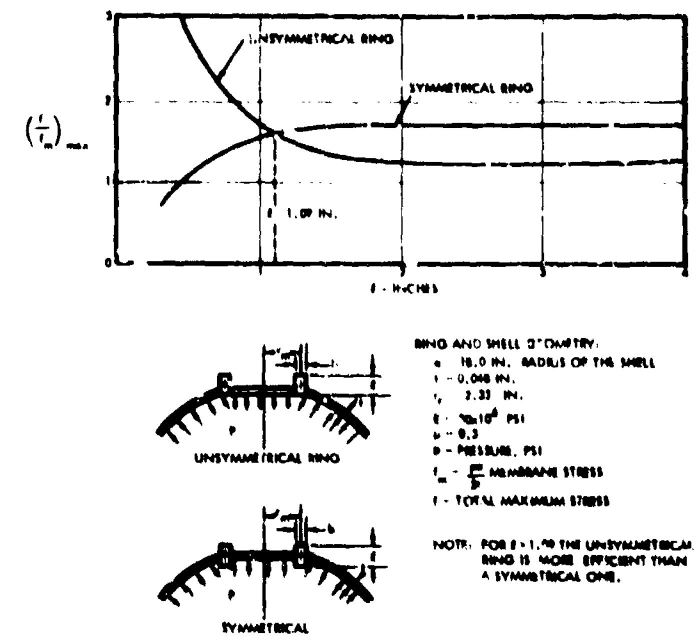


Figure 14.7.2.7o. Comparison of the Maximum Stresses at the Ring-Membrane Junction
(Reference 152-7)

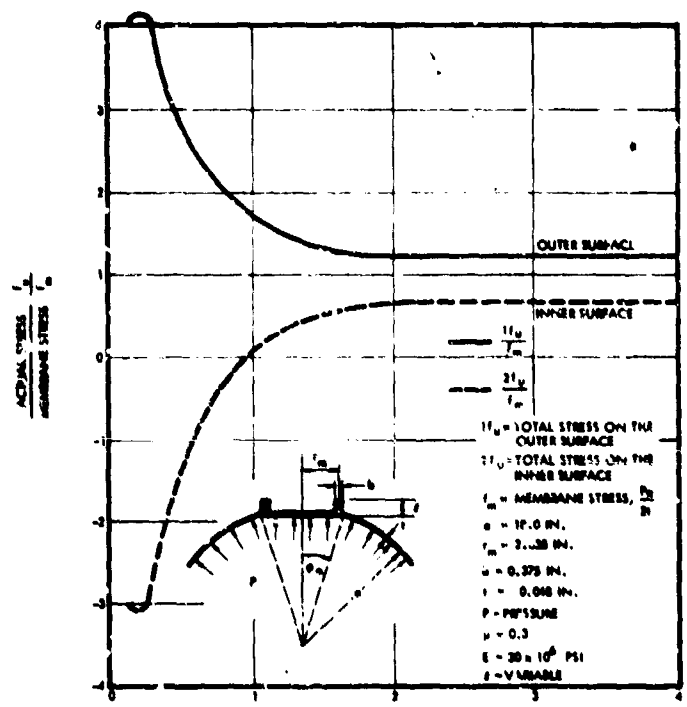


Figure 14.7.2.7n. Stresses at the Ring-Membrane Junction for an Unsymmetrical Ring
(Reference 152-7)

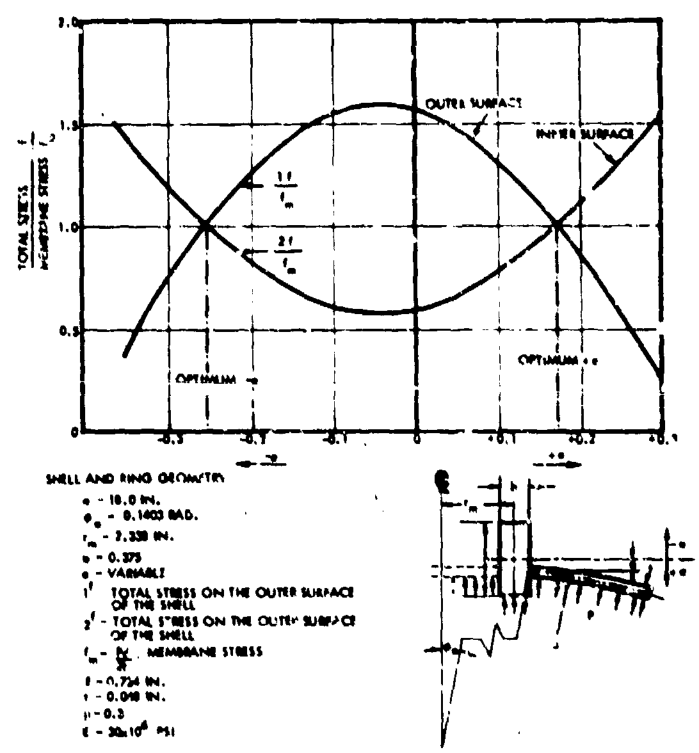


Figure 14.7.2.7p. Stresses at the Ring Membrane Junction Versus Eccentricity
(Reference 152-7)

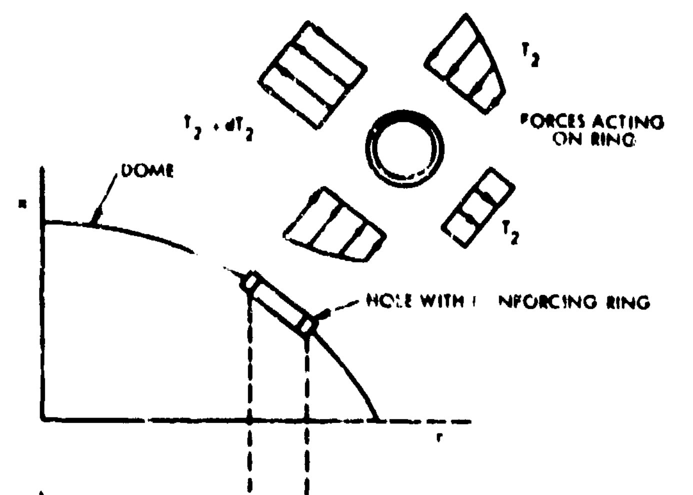


Figure 14.7.2.7q. Off-Apex Opening in a Head of Arbitrary Shape (Reference 152-7)

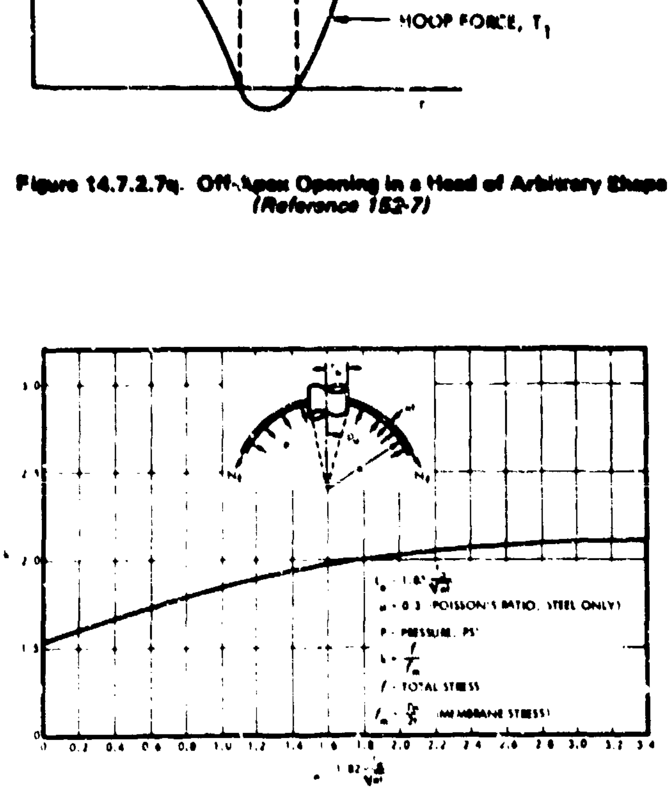


Figure 14.7.2.8. Stress Concentration in a Membrane Reinforced with a Rigid Insert (Reference 152-7)

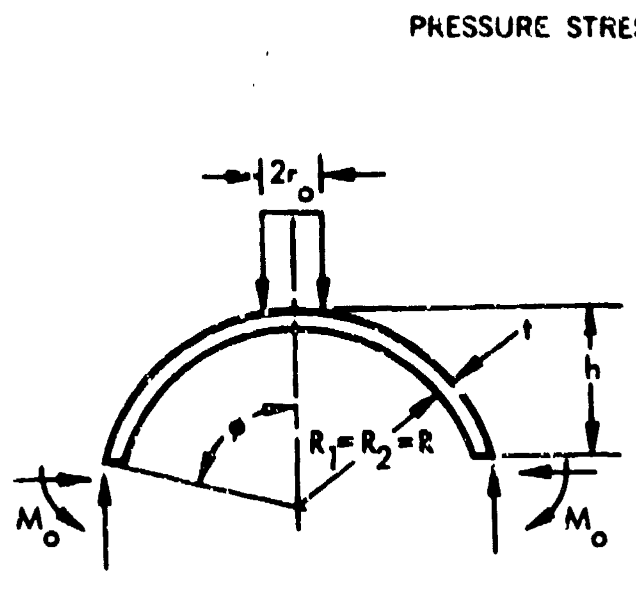


Figure 14.7.2.9. Spherical Membrane with Load P Concentrated over a Circular Area with Radius R_o (Edges fixed and rigid) (Reference 152-7)

14.7.3 Pressure Stresses in Heavy-Walled Components

The analysis of pressure stresses in heavy-walled components (those having wall thickness greater than 10 percent of the inner radius) becomes extremely complicated, since both bending and shear stresses become significant in most cases. Rather than supplying deviations for these, a series of tables and curves have been collected, presenting stress/deflection data or equations to determine them for most cases of interest. These have been separated into three basic shapes of interest: cylinders, spheres, and flat plates.

14.7.3.1 CYLINDERS. The stresses and deflections due to internal pressure in heavy-walled cylinders are presented in Table 14.7 while Figure 14.7.3.1a presents a comparison of stresses calculated on a thick and thin-wall cylinder basis. A chart relating shear stress to pressure and diameter is presented in Figure 14.7.3.1b while Figures 14.7.3.1c, d, and e show the relationship of hoop stress, strain, and yield pressure of a cylinder to its diameter ratio.

Principal stresses and maximum shear stresses for a range of cylinder diameter ratios at both inner and outer diameters are presented in Figures 14.7.3.1f, g, h and i. For cylinders with an eccentric bore as shown in Figure 14.7.3.1j, the maximum stress is the hoop stress at A with the restriction that $a < r_i/2$ and is found from Reference 598-1:

(Eq 14.7.3.1)

$$(f_h)_A = p\left[\frac{2r_o^2\left(r_o^2 + r_i^2 - 2r_i a - a^2\right)}{\left(r_i^2 + r_o^2\right)\left(r_o^2 - r_i^2 - 2r_i a - a^2\right)} - 1\right]$$

Tubes of elliptic and oval cross sections are occasionally used in fluid component fabrication and, therefore, curves relating their pressure stresses to tube geometry are presented in Figures 14.7.3.1k and 14.7.3.1l, respectively.

14.7.3.2 SPHERES. Stress and deflection equations for heavy-walled spheres for internal and external pressure loading are shown in Table 14.7.

Table 14.7.2.8. Spherical Membrane Moment and Deflection Coefficients

α	0	1	2	3	4	5	6	7	8	9	10
A	1	0.985	0.817	0.515	0.320	0.220	0.161	0.122	0.095	0.075	0.061
B	1	0.975	0.690	0.191	-0.080	-0.140	-0.117	-0.080	-0.059	-0.034	-0.026

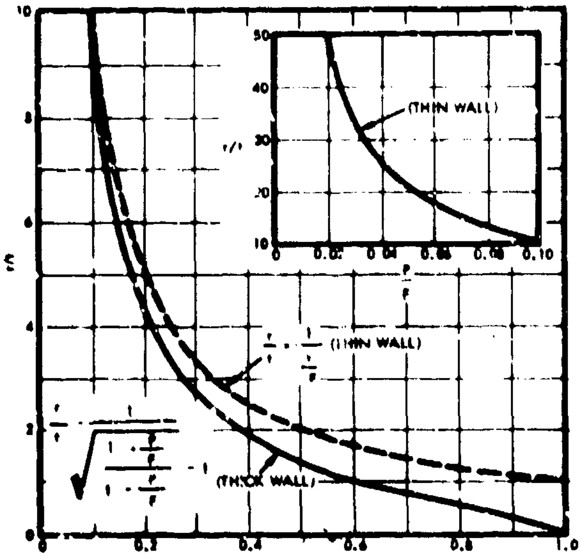


Figure 14.7.2.1a. Plots of Basic Thick and Thin-Wall Equations for Determination of Cylinder Wall Thickness. Large Chart is Applicable for Values of r/t Less Than 10; Inset Diagram for Values of r/t Greater Than 10
(Adapted with permission from Reference 1-383, "Machine Design", 4 April 1957, Vol. 29, No. 7, B. Saelman)

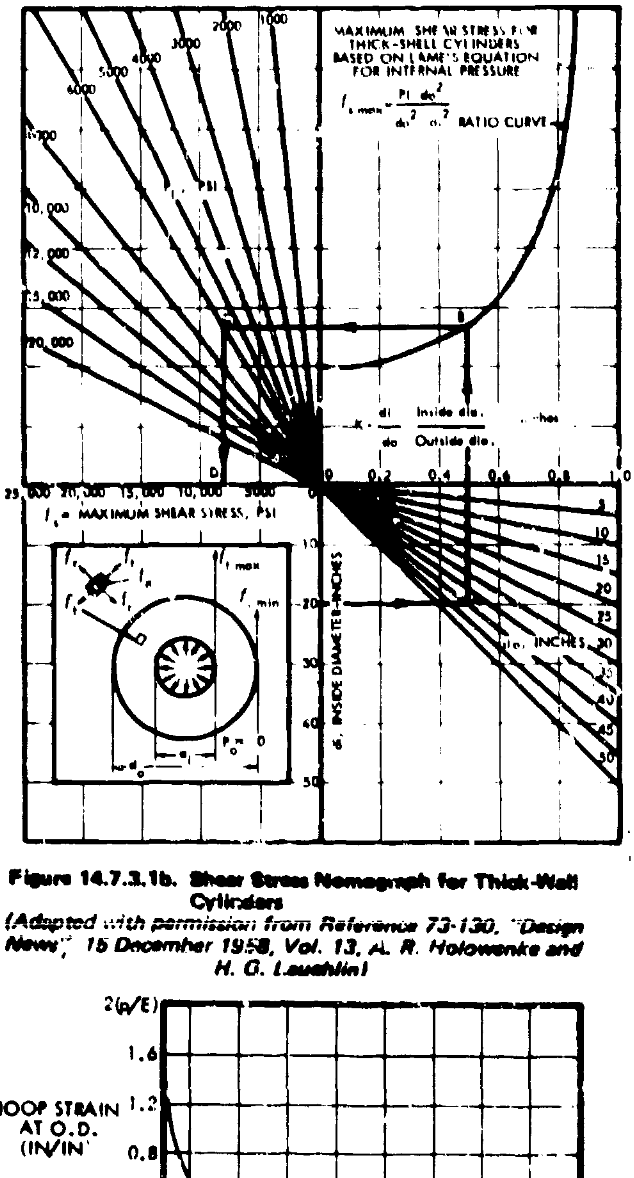


Figure 14.7.3.1b. Shear Stress Nomograph for Thick-Wall Cylinders
(Adapted with permission from Reference 73-130, "Design News", 15 December 1958, Vol. 13, A. R. Holowenko and H. G. Laughlin)

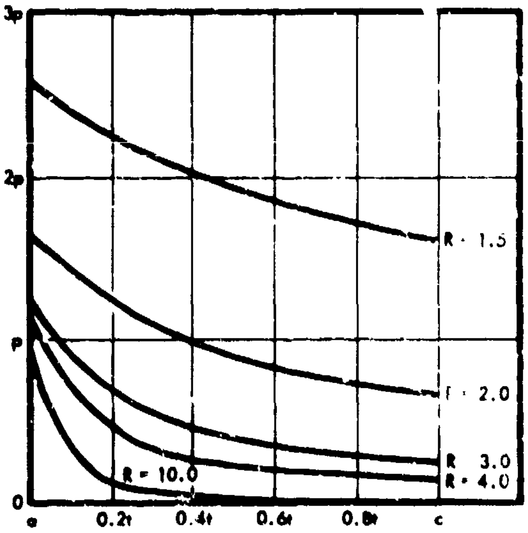


Figure 14.7.3.1c. Distribution of Hoop Stress in Cylinder in Elastic Range
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

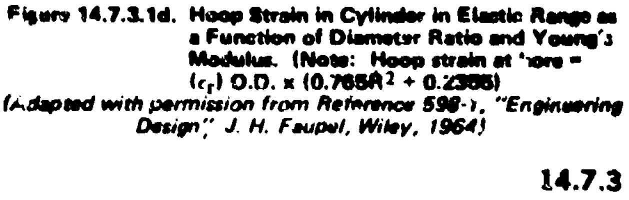


Figure 14.7.3.1d. Hoop Strain in Cylinder in Elastic Range as a Function of Diameter Ratio and Young's Modulus. (Note: Hoop strain at bore = (ϵ_r) O.D. x (0.765R^2 + 0.235))
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

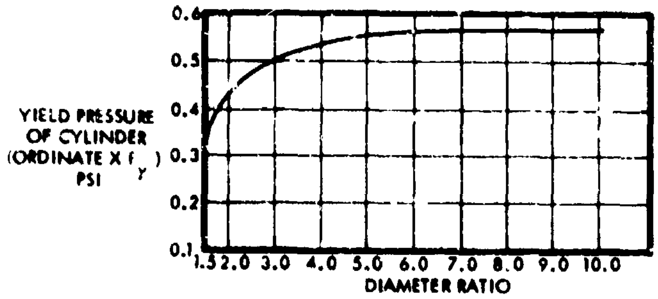


Figure 14.7.3.1e. Yield Pressure of Cylinder as a Function of Diameter Ratio and Yield Strength of Material
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

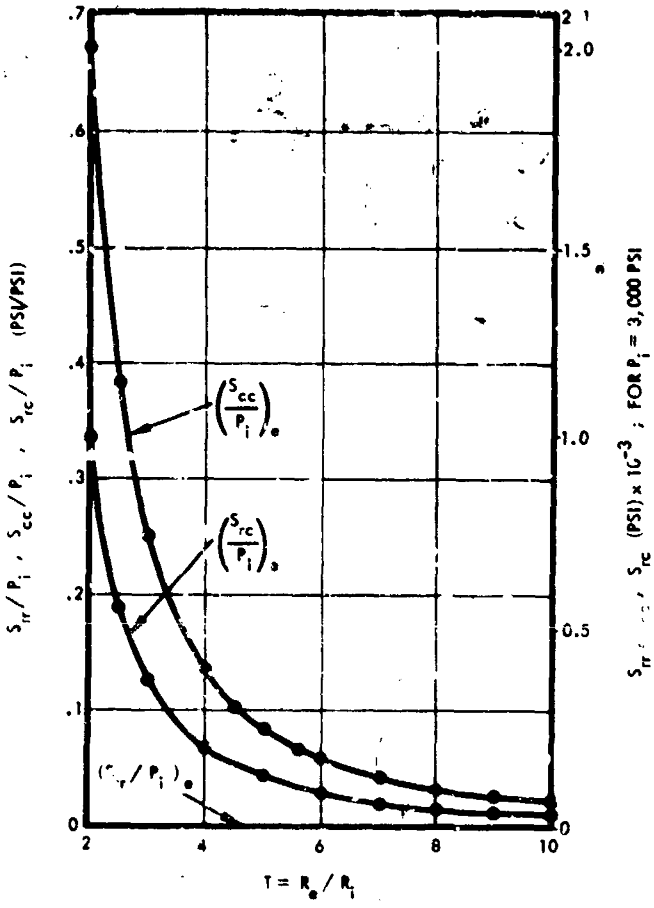


Figure 14.7.3.1f. Principal Stresses and Maximum Shear Stress at the External Surface—Internal Pressure Only, Open-End or Closed-End Cylinders. $2 \leq R_e/R_i \leq 10$
(Adapted with permission from Reference 376-8, "Stress Analysis of Pressurized Cylinders," R. E. Little and C. Bagci, Oklahoma State University, 1965)

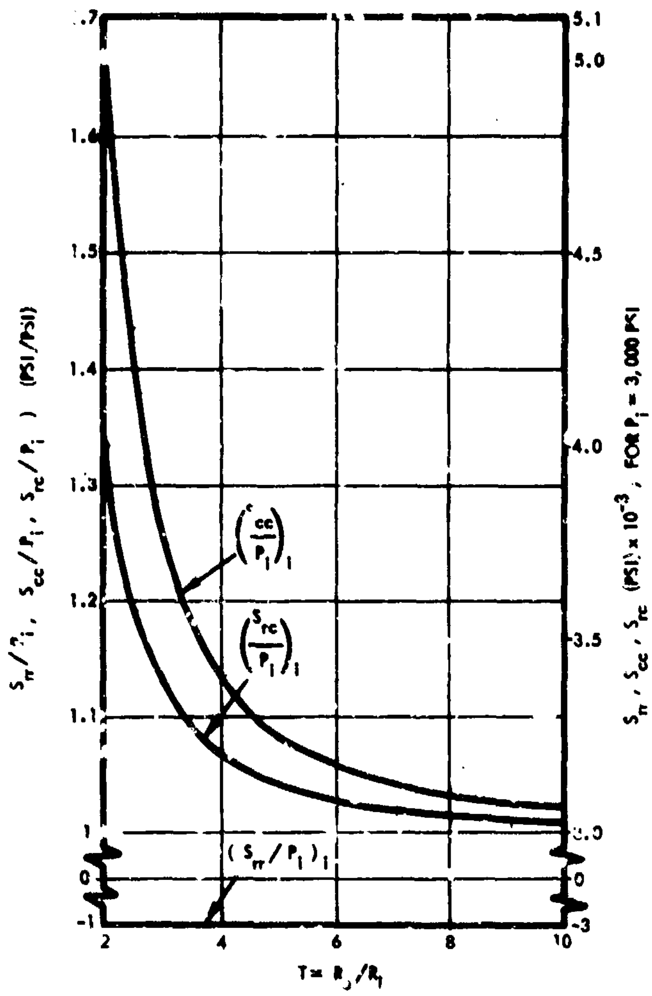


Figure 14.7.3.1g. Principal Stresses and Maximum Shear Stress at the Internal Surface—Internal Pressure Only, Open-End or Closed-End Cylinders $2 \leq R_e/R_i \leq 10$
(Adapted with permission from Reference 376-8, "Stress Analysis of Pressurized Cylinders," R. E. Little and C. Bagci, Oklahoma State University, 1965)

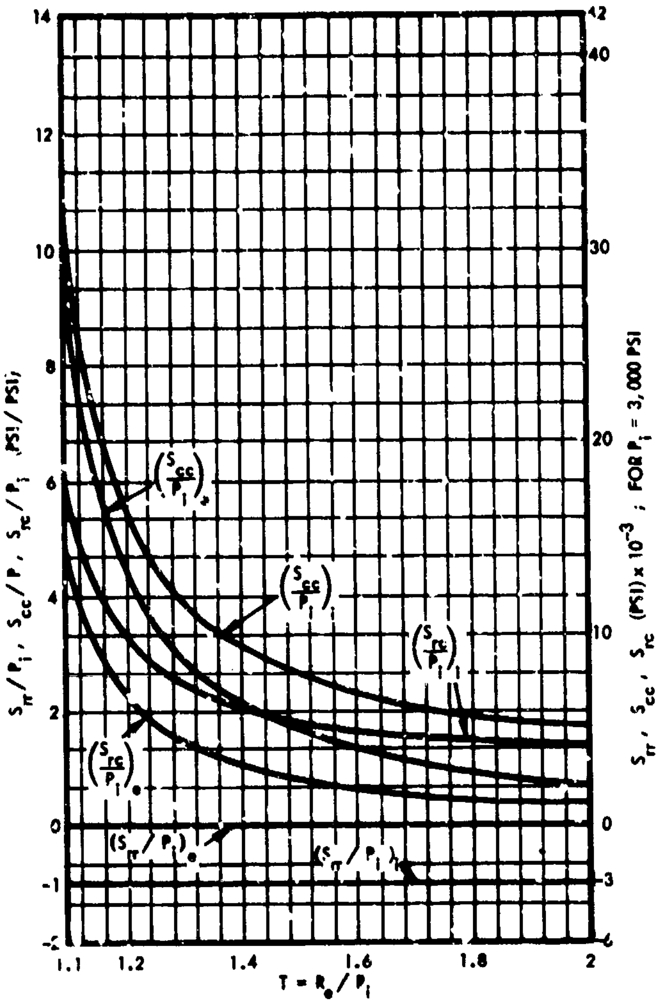


Figure 14.7.3.1h. Principal Stresses and Maximum Shear Stress at Both the Internal and External Surfaces—In-Pressure Only, Open-End or Closed-End Cylinders. $1.1 \leq R_e/R_i < 2$
(Adapted with permission from Reference 376-8, "Stress Analysis of Pressurized Cylinders", R. E. Little and C. Bagci, Oklahoma State University, 1965)

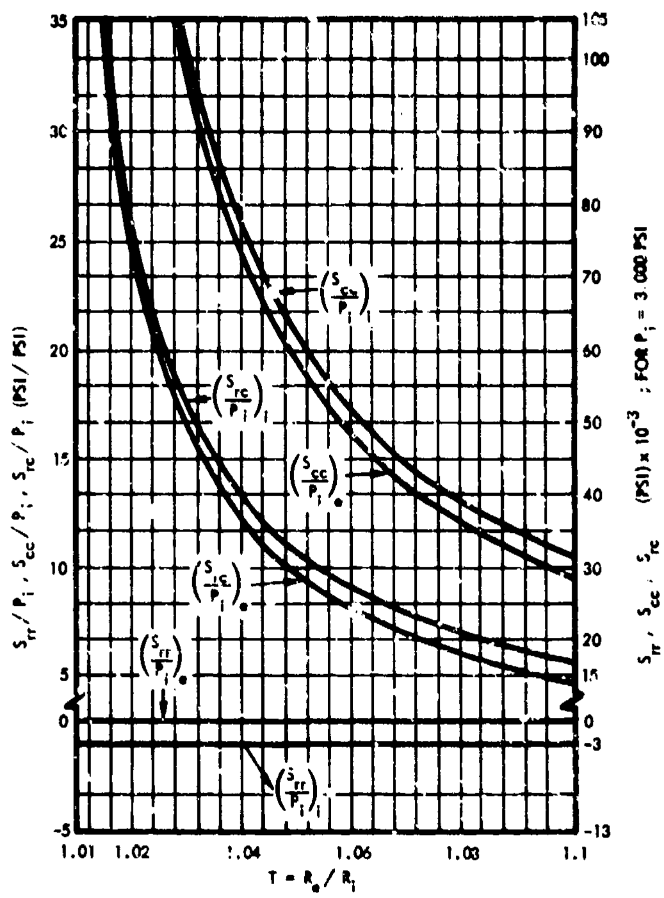


Figure 14.7.3.1i. Principal Stresses and Maximum Shear Stress at Both the Internal and External Surfaces—Internal Pressure Only, Open-End or Closed-End Cylinders. $1 \leq R_e/R_i < 1.1$
(Adapted with permission from Reference 376-8, "Stress Analysis of Pressurized Cylinders", R. E. Little and C. Bagci, Oklahoma State University, 1965)

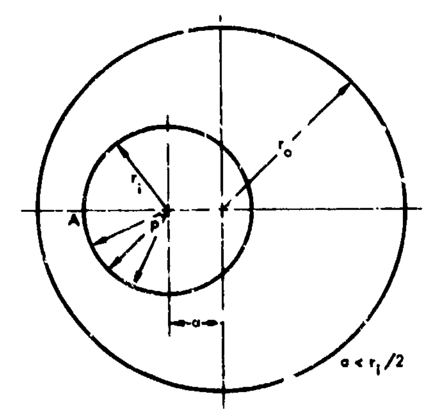


Figure 14.7.3.1j. Heavy-Walled Cylinder with Eccentric Bore Under Pressure
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

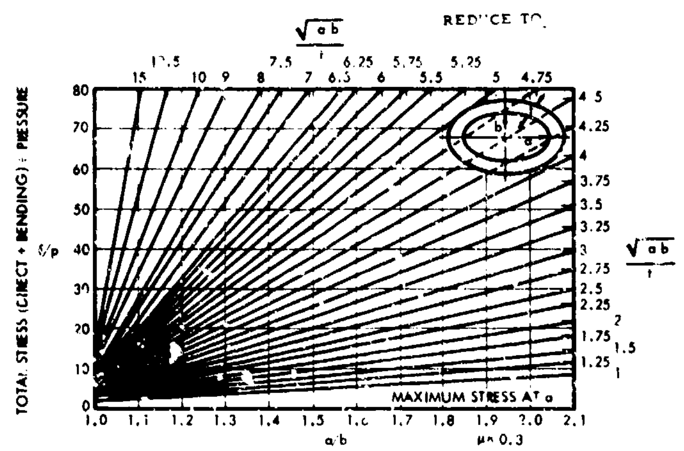


Figure 14.7.3.1k. Maximum Stresses in Internally Pressurized Elliptical Tubes
(Adapted with permission from Reference 598-1, "Engineering Design", J. H. Faupel, Wiley, 1964)

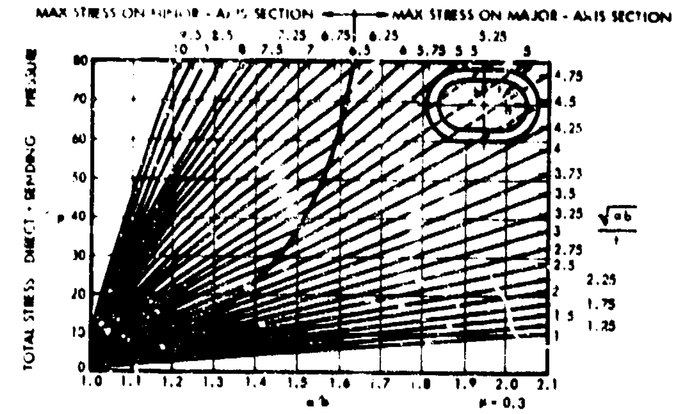


Figure 14.7.3.11. Maximum Stresses in Internally Pressurized Oval Tubes
(Adapted with permission from Reference 5.8-1, "Engineering Design," J. H. Faupel, Wiley, 1964)

14.7.4 Buckling of Thin-Walled Components

14.7.4.1 CRITICAL EXTERNAL PRESSURES. Table 14.7.4.1 presents equations for calculating the *critical external pressure* (P_{cr}) which may be expected to induce buckling in shells of various configurations (Reference 183-12).

14.7.4.2 BUCKLING OF CYLINDERS. Reference 147-20 includes a large selection of buckling data for pressurized and unpressurized cylinders which are presented below.

Axial Compression, Unstiffened Cylinders

Unpressurized. The design-allowable buckling stress for a circular cylinder subjected to axial compression is given by

$$\frac{F_{cr}}{\eta} = C_c \frac{Et}{R} \qquad \text{(Eq 14.7.4.2a)}$$

where

F_{cr} = allowable buckling stress, psi

η = plasticity correction term = 1 for elastic buckling (for inelastic buckling see Reference 147-20)

C_c = buckling stress coefficient, dimensionless

E = modulus of elasticity, psi

t = wall thickness, in.

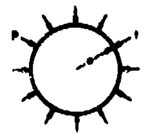
r = mean radius of cylinder, in.

The range of applicability of Equation (14.7.4.2a) is dependent upon the curvature parameter, Z, defined as

$$Z = \frac{L^2 (1 - \mu^2)}{rt} \qquad \text{(Eq 14.7.4.2b)}$$

Table 14.7.4.1. Equations for Calculating Critical Buckling Pressure for Various Shell Configurations
(Reference 147-20)

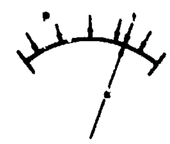
SPHERE



$$P_{cr} = \frac{2Et^2}{a^2 \sqrt{3(1-\mu^2)}}$$

(ZOELLY)

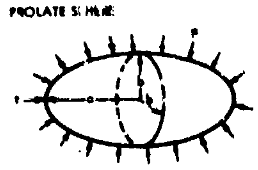
SPHERICAL CAP



$$P_{cr} = 0.8 \frac{2Et^2}{a^2 \sqrt{3(1-\mu^2)}}$$

(HUANG)

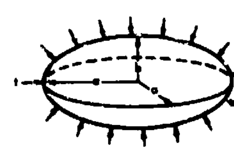
PROLATE SPHERE



$$P_{cr} = \frac{2Et^2}{\sqrt{3(1-\mu^2)}} \frac{1}{2a^2 - b^2}$$

(MUSHTARI-GALIMOV)

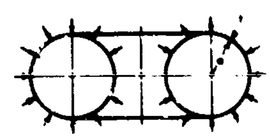
OBLATE SPHERE



$$P_{cr} = \frac{2Et^2}{\sqrt{3(1-\mu^2)}} \frac{b^2}{a^4}$$

(MUSHTARI-GALIMOV)

TORUS



$$P_{cr} = \frac{2Et^2}{a^2 \sqrt{3(1-\mu^2)}} \text{ (ESTIMATE)}$$

(STEIN AND McELMAN)

where

L = length, in.

μ = Poisson's ratio, dimensionless

Z = curvature parameter, dimensionless

For simply supported cylinders with the curvature parameter Z > 25 and for clamped edge cylinders with Z > 80 (i.e., in the long-cylinder domain), the design curve of Figure 14.7.4.2a presents the buckling-stress coefficient, C_c, for an unpressurized cylinder in axial compression as a function of the radius-to-thickness ratio, r/t. Very long cylinders must be checked for Euler-column buckling (Detailed Topic 14.2.1.2 and Sub-Topic 14.3.7).

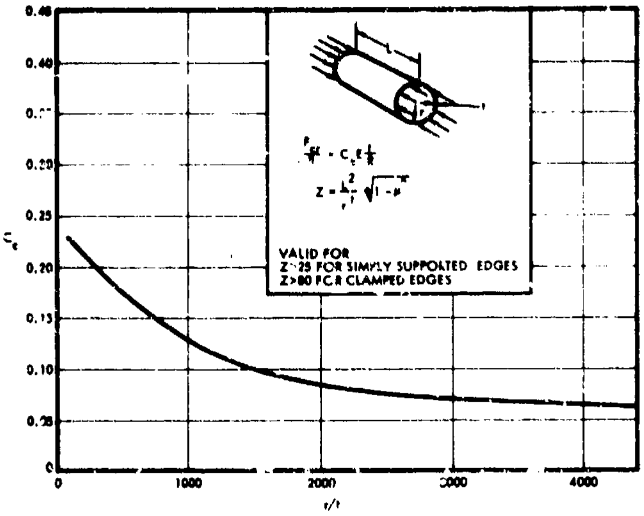


Figure 14.7.4.2a. Buckling-Stress Coefficient, C_c, for Unstiffened Unpressurized Circular Cylinders Subjected to Axial Compression
(Reference 147-20)

Pressurized. The buckling stress of long cylinders under internal pressure and axial compression may be determined by using Figure 14.7.4.2b in conjunction with Figure 14.7.4.2a. Figure 14.7.4.2b presents a curve that allows the calculation of the increase in buckling stress as a function of pressure and geometry only.

The design allowable buckling stress is

$$\frac{F_{cr}}{\eta} = (C_c + \Delta C_c)\frac{Et}{r} \qquad \text{(Eq 14.7.4.2c)}$$

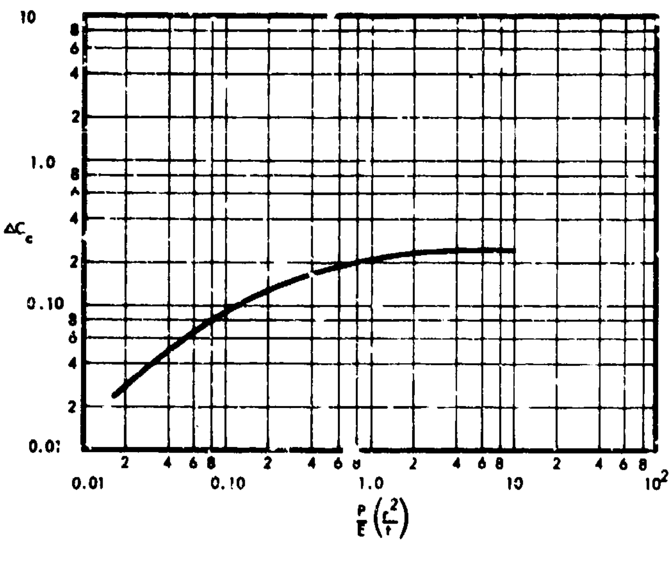


Figure 14.7.4.2b. Increase in Axial-Compressive Buckling-Stress Coefficient of Cylinders Due to Internal Pressure
(Reference 147-20)

where C_c is obtained from Figure 14.7.4.2a, and ΔC_c is obtained from Figure 14.7.4.2b. The pressurized cylinder is capable of resisting a total compressive load, P_{cr}, which may be obtained from the equation

$$P_{cr} = 2\pi r\, F_{cr} t + \pi r^2 p \qquad \text{(Eq 14.7.4.2d)}$$

It should be noted that the pressurized design curve in Figure 14.7.4.2b is valid only for long cylinders. Very long cylinders must be checked for buckling as Euler columns.

Shear or Torsion, Unstiffened Cylinders

Unpressurized. The design-allowable shear buckling stress of thin-walled circular cylinders subjected to torsion is given by

$$\frac{F_{scr}}{\eta} = C_s \frac{Et}{RZ^{1/4}} \qquad \text{(Eq 14.7.4.2e)}$$

where

F_{scr} = allowable shear buckling stress, psi

η = plasticity correction term = 1 for elastic buckling

C_s = shear buckling-stress coefficient, dimensionless (from Figure 14.7.4.2c for simply supported and fixed-edge cylinders with a curvature parameter $Z > 100$)

Pressurized. The shear buckling stress of long thin-walled cylinders subjected to internal pressure and torsion may be determined by using Figure 14.7.4.2d in conjunction with Figure 14.7.4.2c. Figure 14.7.4.2d presents curves that allow the calculation of the increase in buckling stress as a function of pressure and geometry only. The design-allowable shear buckling stress is given by

$$\frac{F_{scr}}{\eta} = (C_s + \Delta C_s)\frac{Et}{rZ^{1/4}} \qquad \text{(Eq 14.7.4.2f)}$$

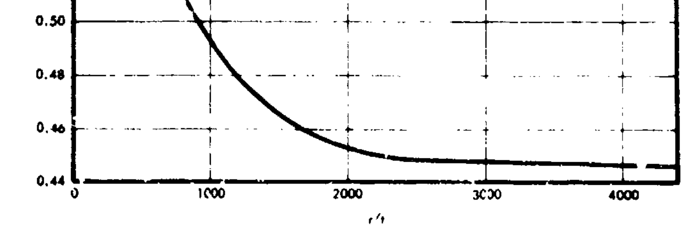


Figure 14.7.4.2c. Buckling-Stress Coefficient, C_s, for Unstiffened Unpressurized Circular Cylinders Subjected to Torsion
(Reference 147-20)

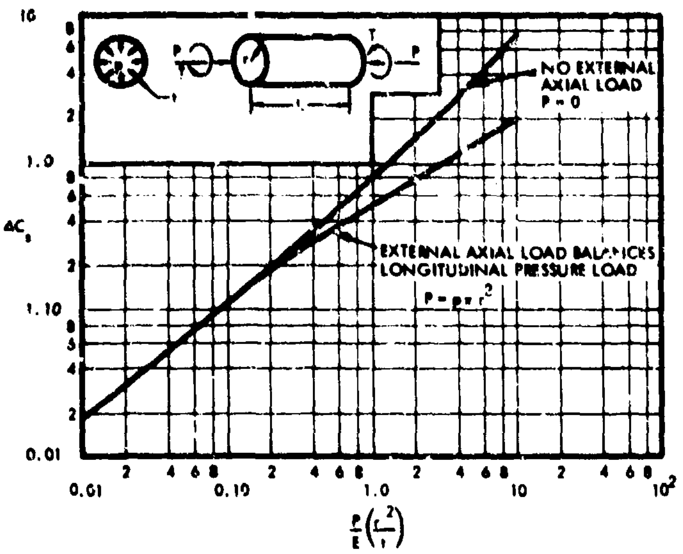


Figure 14.7.4.2d. Increase in Torsional Buckling-Stress Coefficient of Cylinders Due to Internal Pressure
(Reference 147-20)

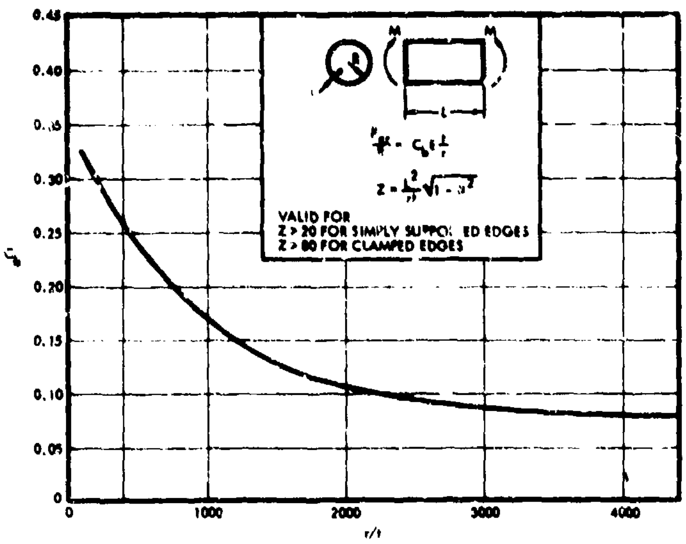


Figure 14.7.4.2e. Buckling-Stress Coefficient C_b, for Unstiffened Unpressurized Circular Cylinders Subjected to Bending
(Reference 147-30)

where C_r is obtained from Figure 14.7.4.2c and ΔC_s is obtained from Figure 14.7.4.2d.

Two curves are presented in Figure 14.7.4.2d for calculating the increment in critical stress caused by pressurization. One curve, labeled "no external axial load," should be used for calculating the critical stress of a cylinder subjected to torsion and internal pressure only. The second curve, labeled "external axial load balances longitudinal pressure load," should be used to calculate the critical stress of a cylinder subjected to torsion and internal pressure plus an external axial compression load equal to the internal pressure load $\pi r^2 p$, acting on the heads of the cylinder. It should be noted that the pressurized design curves of Figure 14.7.4.2d are valid only for long cylinders.

Bending, Unstiffened Cylinders

Unpressurized. The design-allowable buckling stress for a thin-walled circular cylinder subjected to bending is given by

$$\frac{F_{cr}}{\eta} = C_b \frac{Et}{r} \qquad \textbf{(Eq 14.7.4.2g)}$$

where

F_{cr} = maximum allowable stress due to the bending moment (e.g., the outer fiber stress), psi

C_b = buckling stress coefficient, dimensionless (from Figure 14.7.4.2e for simply supported cylinders having a curvature parameter $Z > 20$ and for clamped edge cylinders with $Z > 80$)

η = plasticity correction term = 1 for elastic buckling.

If the stresses are elastic, the allowable moment is

$$M_{cr} = \pi r^2 F_{cr} t \qquad \textbf{(Eq 14.7.4.2h)}$$

Pressurized. The buckling stress of long cylinders subjected to internal pressure and bending may be determined by using Figure 14.7.4.2f in conjunction with Figure 14.7.4.2e. Figure 14.7.4.2f presents curves that allow the calculation of the increase in critical stress as a function of pressure and geometry only. The design-allowable buckling stress is

$$\frac{F_{cr}}{\eta} = (C_b + \Delta C_b) \frac{Et}{r} \qquad \textbf{(Eq 14.7.4.2i)}$$

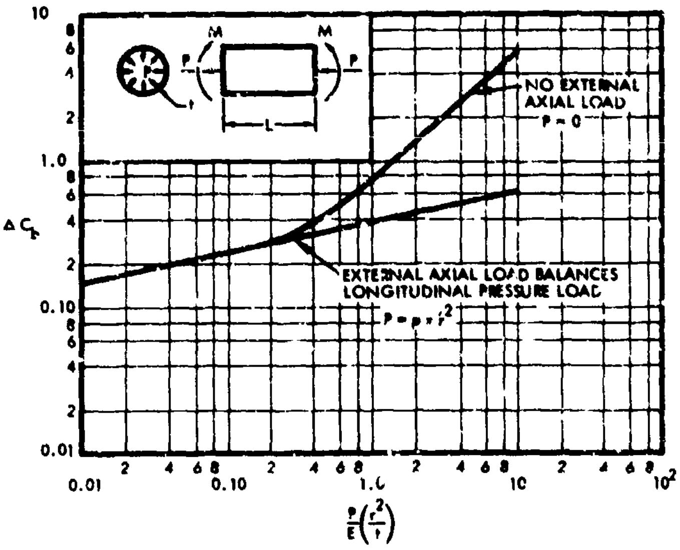


Figure 14.7.4.2f. Increase in Bending Buckling-Stress Coefficient of Cylinders Due to Internal Pressure
(Reference 147-20)

where C_b is obtained from Figure 14.7.4.2e and ΔC_b is obtained from Figure 14.7.4.2f.

Two curves for calculating the increment in critical stress caused by pressurization are presented in Figure 14.7.4.2f. The curve labeled "no external axial load" should be used to calculate the critical stress of a cylinder subjected to bending and internal pressure only. The curve labeled "external axial load balances longitudinal pressure load" should be used to calculate the critical stress of a cylinder subjected to bending and internal pressure plus an external axial compression load equal to the internal pressure load, $\pi r^2 p$, acting on the heads of the cylinder. If the curve for no axial load is used and the stresses are elastic, the design-allowable moment is

$$M_{cr} = \pi r^2 \left[F_{cr}t + \frac{pr}{2}\right] \qquad \text{(Eq 14.7.4.2j)}$$

It should be noted that the pressurized design curves in Figure 14.7.4.2f are valid only for long cylinders.

External Pressure, Unstiffened Cylinders. If a cylindrical shell with simply supported edges is subjected to uniform external pressure, p, the design-allowable buckling stress in the circumferential direction is

$$\frac{F_{cr}}{\eta} = K_p \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{L}\right)^2 \qquad \text{(Eq 14.7.4.2k)}$$

where

F_{cr} = allowable buckling stress, psi

K_p = buckling coefficient, dimensionless (from Figure 14.7.4.2g)

E = modulus of elasticity, psi

t = wall thickness, in.

L = length, in.

μ = Poisson's ratio, dimensionless

η = plasticity correction term, dimensionless (see below)

The buckling coefficient, K_p, and a definition of the geometrical parameters are given in Figure 14.7.4.2g. For elastic buckling, η = 1 is used. For moderate length cylinders $(100 < Z < 11\ R^2/t^2)$ in the inelastic range, Reference 147-20 suggests

$$\eta = \frac{E_s}{E} \sqrt{\left(\frac{E_t}{E_s}\right)}^{1/2} \left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right) \qquad \text{(Eq 14.7.4.2l)}$$

where

E_s = secant modulus, psi

E_t = tangent modulus, psi

For long cylinders, (e.g., $L^2/r^2 > 11\ r/t$) the design-allowable buckling stress is

$$\frac{F_{cr}}{\eta} = \frac{\gamma E}{4(1-\mu^2)} \left(\frac{t}{r}\right)^2 \qquad \text{(Eq 14.7.4.2m)}$$

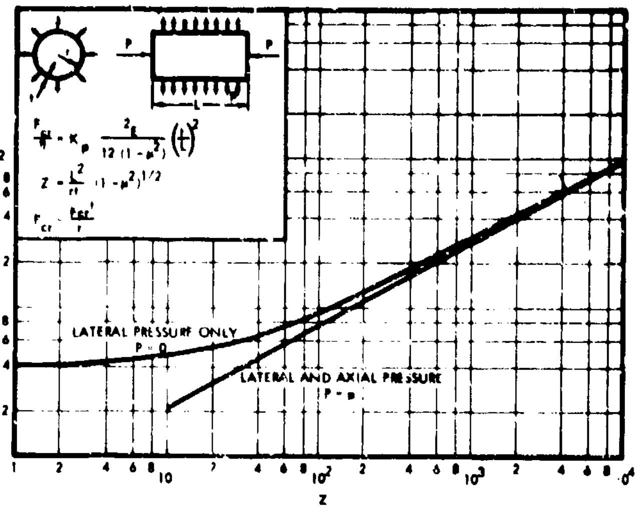


Figure 14.7.4.2g. Buckling Coefficients for Circular Cylinders Subjected to External Pressure
(Reference 147-20)

The factor, γ, was introduced to reduce the theory to a design value. NASA SP8007, Buckling of Thin-Walled Circular Cylinders, recommends γ = 0.9. For inelastic buckling, Reference 147-20 suggests

$$\eta = \frac{E_s}{E}\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right) \qquad \text{(Eq 14.7.4.2n)}$$

The design-allowable pressure may be obtained from the formula

$$p_{cr} = \frac{F_{cr}t}{r} \qquad \text{(Eq 14.7.4.2o)}$$

The pressure, p_{cr}, is the design-allowable pressure for complete buckling of the shell (e.g., when buckles have formed all the way around the cylinder). For some values of the parameters (large r/t and/or large initial imperfections), single buckles will occur at pressures less than p_{cr}, but complete buckling will occur at higher pressures. Therefore, for some applications these results should be used with caution.

The plasticity correction factors recommended in this section were obtained primarily for the case of lateral pressure, but they are probably sufficiently accurate for the case of lateral and axial pressure (Reference 147-20).

Combined Loading, Unstiffened Cylinders. The criterion for structural failure of a member under combined loading is frequently expressed in terms of a stress-ratio equation, $R_1^x + R_2^y + R_3^z = 1$ (Detailed Topic 14.2.1.2). The subscripts denote the stress due to a particular kind of loading (compression, shear, etc.), and the exponents (usually empirical) express the general relationship of the quantities for failure of the member. The stress-ratio, R, is most easily understood if it is defined first for a particular loading condition. In combined compression and torsion loading $(R_c^x + R_s^z = 1)$, the stress-ratio, R_c, is defined as the ratio of compressive stress at which buckling occurs under the combined loading to the compressive stress at which buckling occurs under compression alone. In general, the stress-ratio is the ratio of the allowable value of the stress caused by a particular kind of load in a combined loading condition to the allowable stress for the same kind of load when it is acting alone. A curve drawn from such a stress-ratio equation is termed a stress-ratio interaction curve. In simple loadings, the term *stress-ratio* is used to denote the ratio of applied to allowable stress.

Combined Torsion and Axial Loading. A semi-empirical interaction curve for circular cylinders under combined torsion and axial loading is given in Figure 14.7.4.2h. F_{cr} is found from Equation (14.7.4.2a) and F_{scr} from Equation (14.7.4.2f). In Figure 14.7.4.2h the curves for r/t ratios of 600, 800, and 1000 were determined by test. Curves for r/t of 1500 and 2000 were drawn by extrapolation.

Bending and Torsion. Test results indicate that a conservative estimate of the interaction for cylinders under combined bending and torsion may be obtained from Figure 14.7.4.2i; F_{cr} is found from Equation (14.7.4.2g) and F_{scr} from Equation (14.7.4.2e).

Axial Compression and Bending. Test data indicate that the linear interaction for the case of cylinders under combined axial compression and bending, shown in Figure 14.7.4.2j, may be used. The buckling stress due to bending alone may be found from Equation (14.7.3.2g), and the buckling stress under axial compression alone may be found in Equation (14.7.4.2a).

Axial Compression and External Pressure. Limited test data for cylinders subjected to axial compression and external lateral and axial pressure indicate that the linear interaction curve presented in Figure 14.7.4.2k may be used for design. F_{cr} is found from Equation (14.7.4.2a) and p_{cr} from Equation (14.7.4.2o).

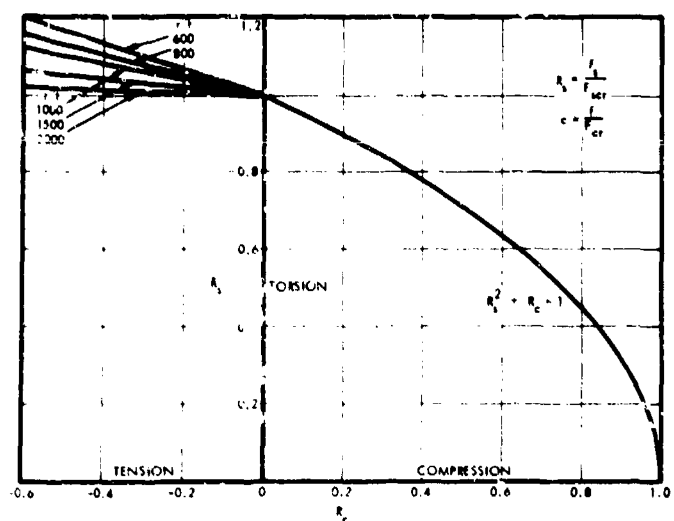


Figure 14.7.4.2h. Buckling Stress Interaction Curve for Unstiffened Circular Cylinders under Combined Torsion and Axial Loading
(Reference 147-20)

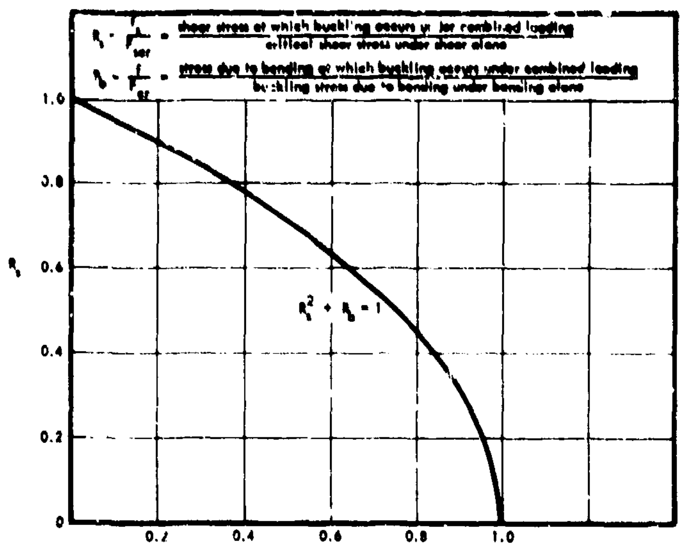


Figure 14.7.4.2i. Buckling Stress Interaction Curve for Unstiffened Circular Cylinders Under Combined Bending and Torsion
(Reference 147-20)

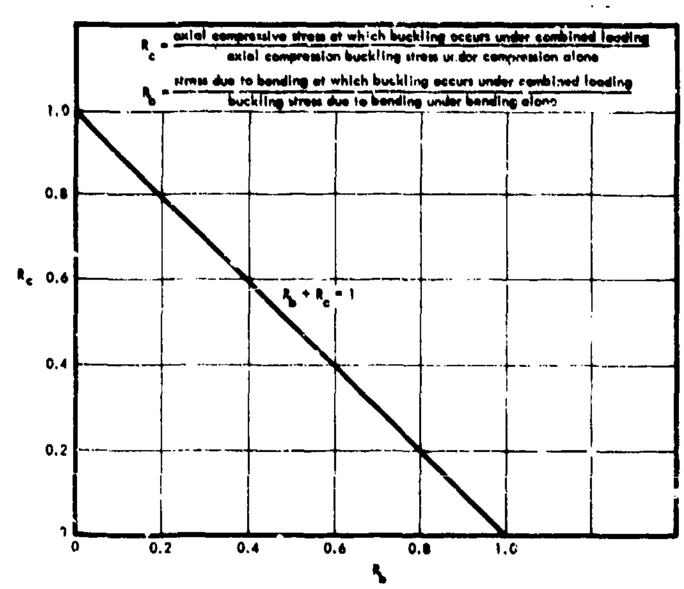


Figure 14.7.4.2j. Buckling Stress Interaction Curve for Unstiffened Circular Cylinders Under Combined Axial Compression and Bending
(Reference 147-20)

14.7.4.3 BUCKLING FROM INTERNAL PRESSURE. Thin-wall components, particularly those of noncircular cross section, often fail by buckling induced by internal pressure. In particular, components which have been "flattened" to meet packaging requirements are subject to this mode of failure. Figure 14.7.4.3 presents the results of buckling pressure tests performed in a variety of scale model torospherical head configurations.

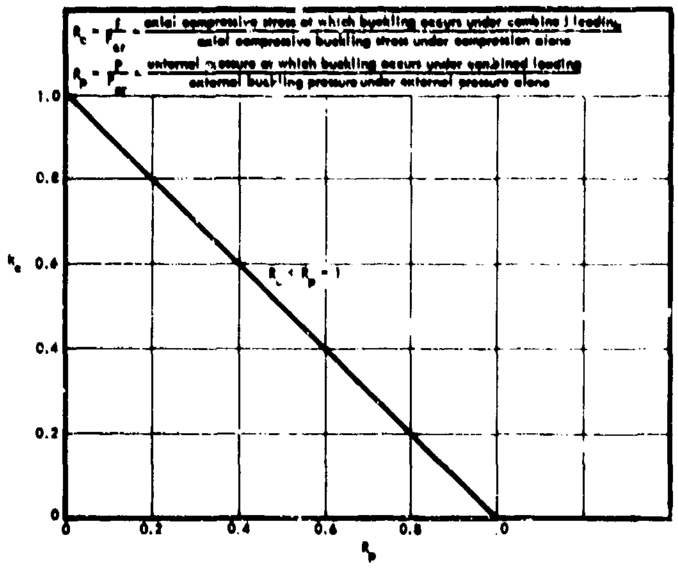
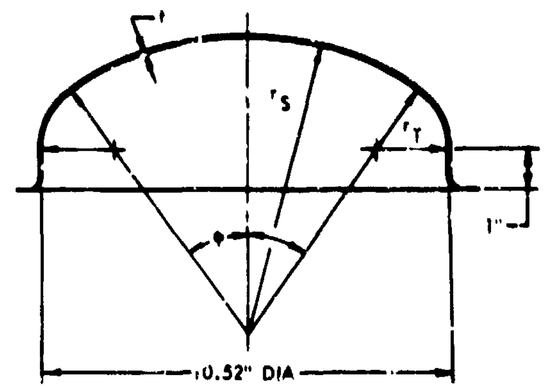


Figure 14.7.4.2k. Buckling Stress Interaction Curve for Unstiffened Circular Cylinders Under Combined External Pressure and Axial Compression
(Reference 147-20)

The state-of-the-art in analysis of buckling due to internal pressure is presently being rapidly advanced. The fluid component designer is cautioned to evaluate all critical thin-wall components for this possible mode of failure. If high strength materials are employed, note from Figure 14.7.4.3 that critical buckling pressure is a function of E, r, and t rather than F.



CONFIGURATION NO.	r_S	r_T	φ
A	7.80"	1.82"	35° JUPITER
B	9.40"	1.82"	27°
C	12.40"	1.82"	18.9°
D	8.52"	.88	35°

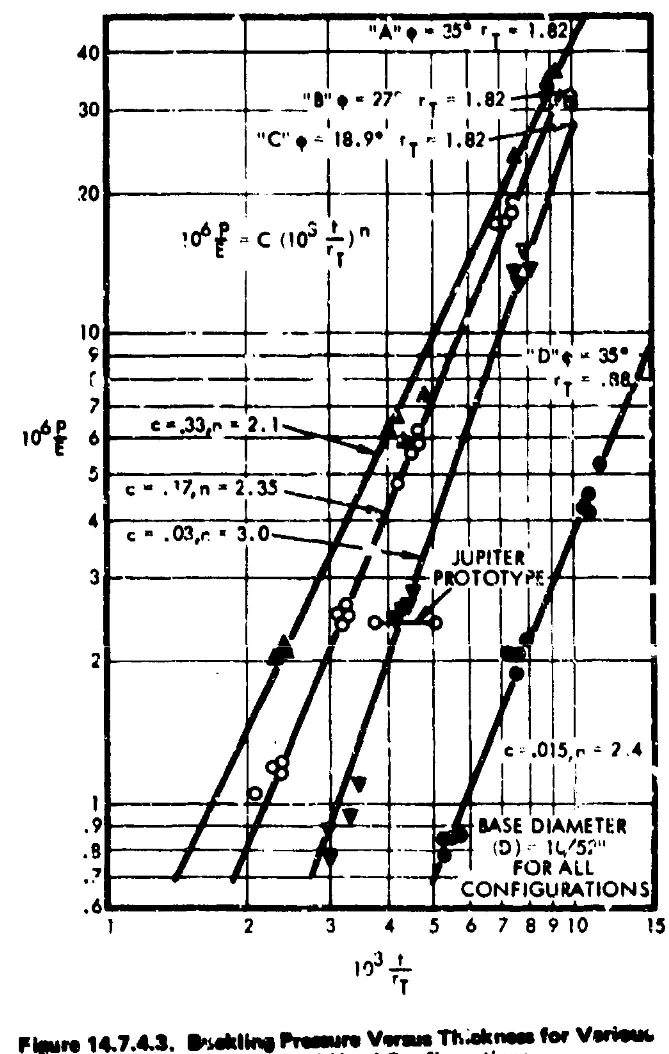


Figure 14.7.4.3. Buckling Pressure Versus Thickness for Various Torospherical Head Configurations
(Adapted with permission from Reference 586-5, "Experimental Mechanics," August 1964, pgs. 217-222, J. Adachi and M. Benicek)

14.8 PIPING, TUBING, AND DUCTING

14.8 PIPING, TUBING, AND DUCTING

The basic elements of stresses and deflections in pressurized cylindrical members are treated in Sub-Section 14.7, and bellows joints are discussed in Sub-Sections 5.13 and 6.6. Flanges and other connector elements are discussed in Sub-Section 5.12. This sub-section treats the basic analysis of line systems, including elbows, branches, combined loading, and flexibility considerations, but does not evaluate code piping design. Piping systems for most facilities come under various federal, state, and insurance company codes which set minimum standards and offer general guidance. Information for such design may be obtained from standard reference sources such as the applicable codes themselves, handbooks such as *Mark's* (Reference 132-1), basic piping texts (such as Reference 369-1), the ASME Code for Pressure Piping, USA Standards (published by the United States of America Standards Institute), and the catalogs and bulletins published by major fabricators of piping components and systems.

14.8.1 Bends and Elbows

A bend or elbow of constant radius is essentially a section of a torus and behaves under pressure according to the expressions for stress and deflection presented in Sub-Topic 14.7.2. In a torus of uniform thickness, both of the principal stresses are tensile, with the hoop stress greater and reaching a maximum at the crotch where failure would be expected to occur first (Reference 628-1). Figure 14.8.1a shows the variation in hoop stress around a cross section through an elbow. Figure 14.8.1b shows the variation in this stress (at the crotch point of maximum intensity) with the radius of bend centerline, from which it is seen that this stress becomes large for small bend radii. Conventional pipe or tube bends are made by pushing or pulling the pipe or tubing around a form of the required radius. The operation is usually performed cold when the size is small and/or the bend radius generous; when the size is large and/or the bend radius sharp, hot forming is done. The natural redistribution of metal which occurs during bending, thinning at the outside and thickening at the inside, is a compensating factor of the same order as the acting stress; hence, the requirement that conventional pipe bends be made of thicker material to adjust for thinning during bending is seldom warranted for the ratios of R/r customarily used. In fact, this and other associated factors, such as strain hardening, usually result in pressure failures in the straight portion of pipes or tubes. This is illustrated in the pressure tests of the tube bend of Figures 14.8.1c, d, and e, showing the rupture in the straight cylindrical portion under internal pressure. When failure was forced to occur in the torus portion (Figure 14.8.1f), the rupture took place on the centerline of the bend where the stress is the same as that in a straight cylinder and, incidentally, where the material tensile strength had been increased the least by strain hardening from the fabrication process. The stabilizing effect of the double curvature in the torus region accounted for a 20 percent increase in bursting pressure. For this particular torus subjected to external pressure, the collapse pressure was 93 percent higher (Figure 14.8.1g) than that for a cylinder of the same size and thickness (Reference 628-1).

When hoop stress resulting from internal pressure is the only major consideration, the wall thickness of a bend or elbow is given by

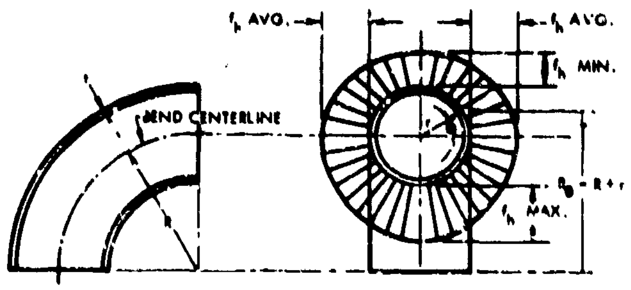


Figure 14.8.1a. Variation in Hoop Stress in a Bend
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

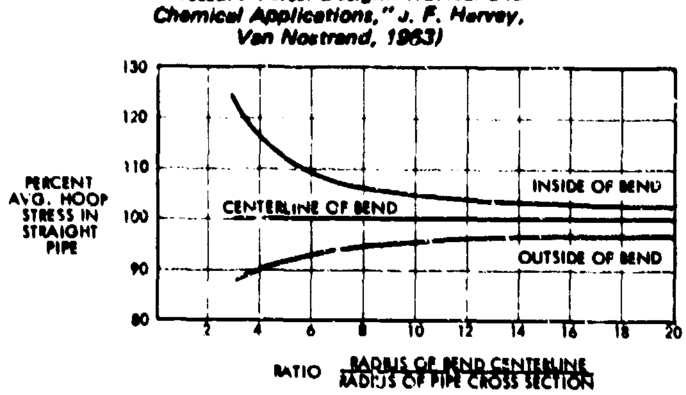


Figure 14.8.1b. Variation in Hoop Stress with Bend Radii
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

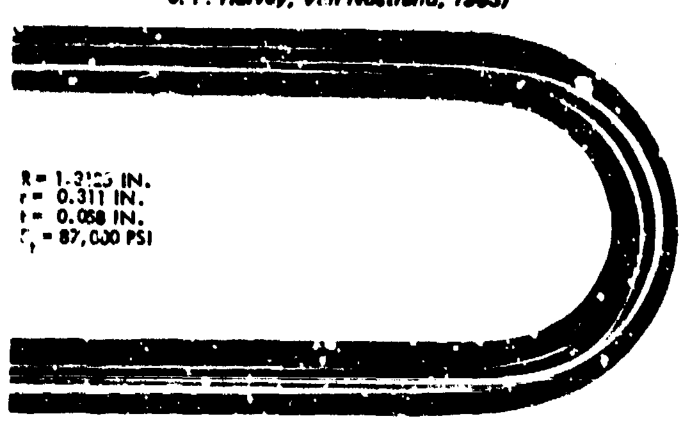


Figure 14.8.1c. Tube With 180° Bend
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

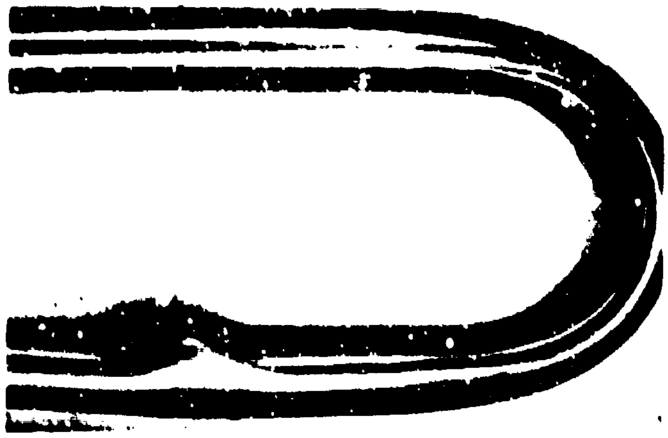


Figure 14.8.1d. Tube Failure at Internal Pressure of 17,800 psi, Showing Gross Deformation and Rupture in the Cylindrical Section
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

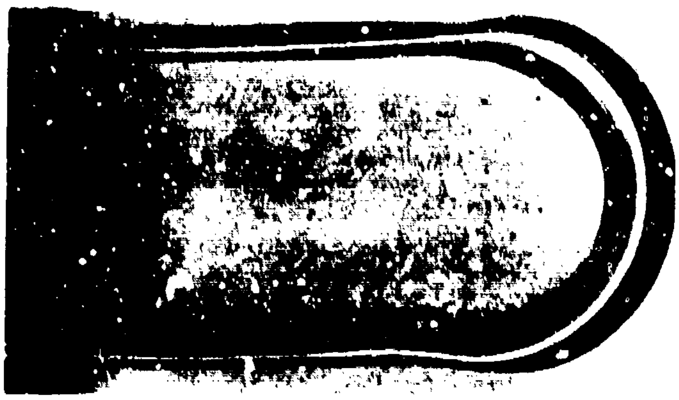


Figure 14.8.1e. Tube Failure at External Pressure of 8,800 psi, Showing Collapse of the Cylindrical Section
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

Figure 14.8.1f. Rupture of Torus Section at Internal Pressure of 21,000 psi
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

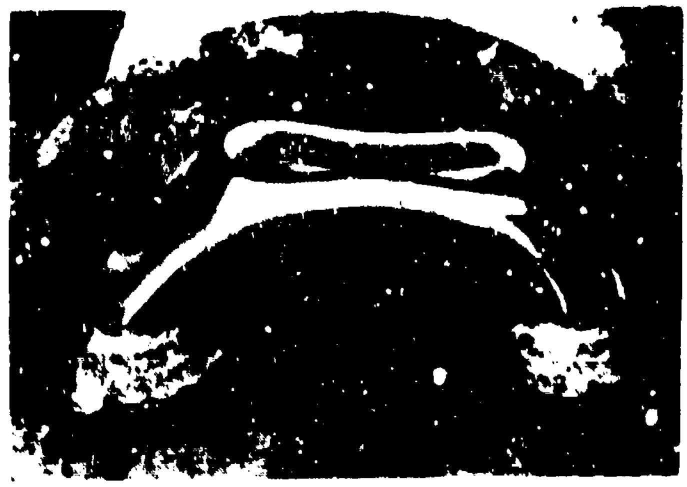


Figure 14.8.1g. Collapse of Torus Section at External Pressure of 12,200 psi
(Adapted with permission from Reference 628-1, "Pressure Vessel Design: Nuclear and Chemical Applications," J. F. Harvey, Van Nostrand, 1963)

$$t = \frac{prC}{F} \qquad \text{(Eq 14.8.1a)}$$

where

- t = wall thickness, in.
- p = internal pressure, psi
- r = tube radius, in.
- C = correction factor from Table 14.8.1
- F = allowable stress, psi

This is the simple formula for hoop stress in a thin-walled cylinder, modified by the correction factor C (Reference 73-111). C is the product of the length factor, C_ℓ, and the load factor C_p. Table 14.8.1 gives values of C_ℓ, C_p, and C for several values of the radius ratio, R/r. Figure 14.8.1h provides the values of C for any point about the circumference of a section through the elbow for specific radius ratios between 2 and 8. Figure 14.8.1i provides the maximum value of C (at R_i) and the minimum value (at R_o) for any radius ratio between 2 and 8. At and near radius R, the correction factor is C = 1.0 and the wall thickness is the same as for a straight pipe, while for R_o the thickness is reduced, and for R_i it is increased.

Table 14.8.1 1. Wall Thickness Correction Coefficients for Elbows
(Adapted with permission from Reference 73-111, "Design News," 23 November 1966, vol. 21, no. 24, H. W. Hamm)

	At R_i			At R_o		
$\frac{R}{r}$	C_ℓ	C_p	$C = C_\ell C_p$	C_ℓ	C_p	$C = C_\ell C_p$
2	2.0	0.83	1.66	0.66	1.18	0.78
3	1.5	0.88	1.32	0.75	1.12	0.84
4	1.33	0.89	1.18	0.80	1.11	0.89
5	1.25	0.91	1.14	0.83	1.09	0.91

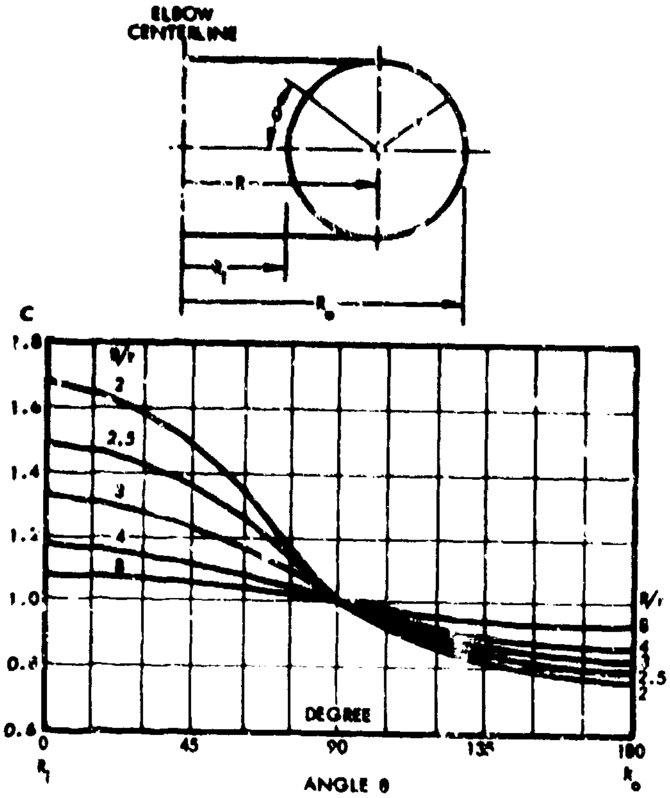


Figure 14.8.1h Elbow Wall Thickness Correction Factor C (Adapted with permission from Reference 73-111, "Design News," 23 November 1966, vol. 21, no. 24, H. W. Hamm)

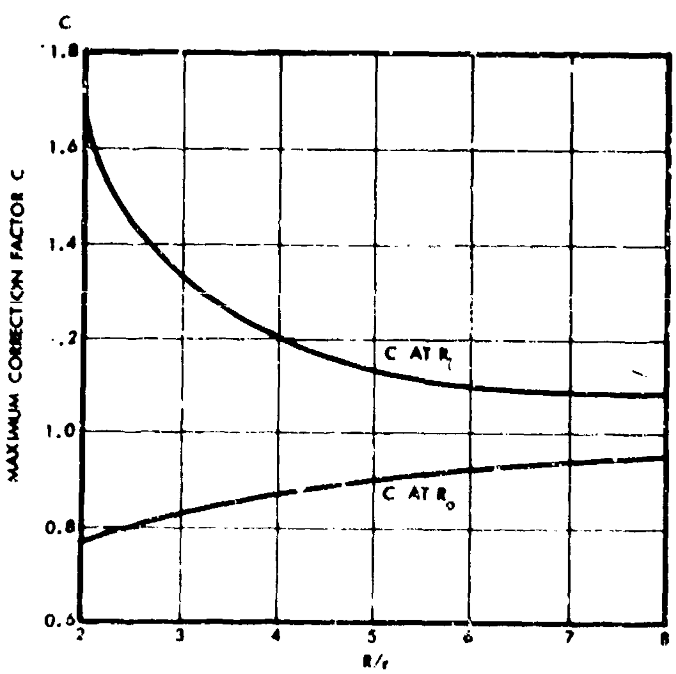


Figure 14.8.1i. Maximum Elbow Wall Thickness Correction Factor C (Adapted with permission from Reference 73-111, "Design News," 23 November 1966, vol. 21, no. 24, H. W. Hamm)

The complete analysis of any bend or elbow must take into consideration not only the pressure load discussed above, but also any bending or torsional loads. Reference 44-24 presents an excellent design procedure for a family of threaded connectors consisting of unions, elbows, tees, and crosses. The design procedures are programmed for digital computer solution and are representative of the state-of-the-art in computer-aided design of aerospace fluid components. The following analysis of combined stresses in a flanged elbow has been adapted from Reference 44-24. The elementary stresses at a point in the member on certain planes passing through the point can be determined. None of these stresses, in general, will be the maximum stress at the point. It is important, therefore, that the relations between the stresses at a point on different planes passing through the point be found.

For any combination of stresses at a point in a stressed body, three mutually perpendicular planes passing through the point can be formed on which only normal stresses exist; the normal stresses on these planes on which no shearing stress occurs are the principal stresses. The maximum and minimum normal stresses at a point are principal stresses. These stresses are used to determine the fatigue-stress condition. They are also important in the determination of maximum shear stress at a point in the body.

The first step is to compute the elementary stresses developed by the fluid pressure and the bending moment. Subsequent stress calculations include determination of stress-concentration factors, principal normal stresses, and maximum shear stresses.

The stress relationships being considered in this analysis are indicated in Figure 14.8.1j. Views A, B, and C represent a point of stress on a plane perpendicular to the axis passing through the point. Elementary stresses developed by fluid pressure and bending are indicated in each view.

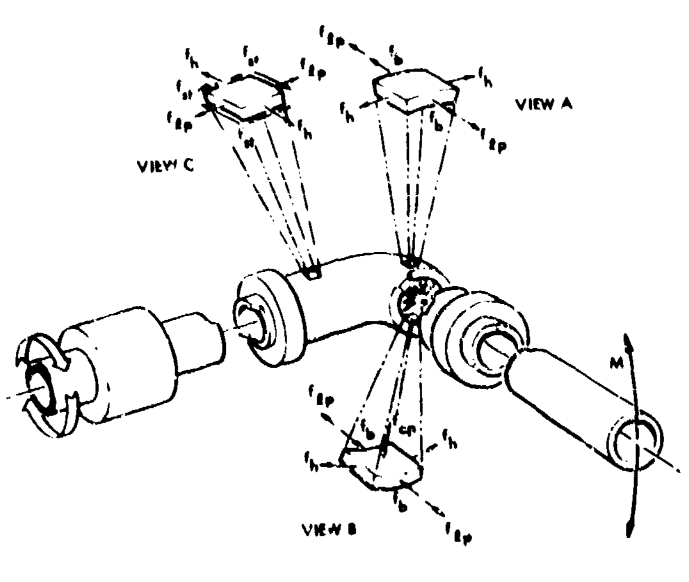


Figure 14.8.1j. Elementary Stresses Developed at a Point Due to Pressure and Bending (Reference 44-24)

Compute Elementary Stresses Due to Pressure. As indicated in Figure 14.8.1j, there are three elementary stresses created by the pressure. The compressive stress, f_{cp} in the inside surface equals the fluid pressure in magnitude. Generally, this stress can be neglected except in high-pressure systems.

A tensile hoop stress is also created which may be computed by the Lamé equation for thick-walled cylinders

$$f_H = \frac{P(D_o^2 + D_i^2)}{D_o^2 - D_i^2} \qquad \text{(Eq 14.8.1b)}$$

where

f_H = tensile hoop stress, psi

P = internal pressure, psi

D_o = outside diameter, in.

D_i = inside diameter, in.

The third elementary stress due to pressure is the longitudinal stress. It is also a tensile stress and acts at 90 degrees to the hoop stress. To compute this stress:

$$f_{\ell p} = \frac{PD_i^2}{D_o^2 - D_i^2} \qquad \text{(Eq 14.8.1c)}$$

where

$f_{\ell p}$ = longitudinal tensile stress due to pressure, psi

Compute Elementary Stresses Due to Bending Moment. Two stresses are created by the bending moment: a bending stress, f_b, and a torsional shear stress, f_{st}. When calculating the bending and torsional stresses, it is necessary to take into consideration the nonconcentricity of the inside and outside diameters of the fluid passage. Because the flanges add reinforcement to the passage walls in withstanding the stresses created by fluid pressure, a consideration of nonconcentricity was not required for the calculation of the hoop and the longitudinal stresses.

A bending stress, f_b, is developed by the action of the bending moment, M. Since M is a fully reversed moment as shown, the bending stress at the stressed point alternates from a tensile stress to a compressive stress.

The magnitude of the stress is:

$$f_b = \frac{M}{Z} \qquad \text{(Eq 14.8.1d)}$$

where

f_b = bending stress, psi

M = bending moment, in-lb

Z = section modulus, in^3

A torsional shear stress, f_s, results from application of the bending moment as indicated in Figure 14.8.1j, view C:

$$f_{st} = \frac{4M}{\pi t(D_o^2 + D_i^2)} \qquad \text{(Eq 14.8.1e)}$$

where

f_{st} = torsional shear stress, psi

t = wall thickness, in.

Select Stress-Concentration Factors. In a flanged fitting such as this particular elbow, one of the most severe stress conditions in terms of fatigue exists at the fillet blending the flanges to the body. Such sections are best handled by applying stress-concentration factors. These factors are selected on the basis of the ratio of the fillet radius to the outside diameter of the body, r_f/D_o.

For the purposes of this analysis it was decided that if r_f/D_o was less than 0.02 the stress-concentration factor, K_b, for bending stress should be 1.9 and the stress concentration factor K_t for torsional stress should be 2.2. If r_f/D_o was greater than 0.02, appropriate stress-concentration factors may be approximated from Figures 14.6.3.6d and e.

Compute Principal Stresses. In Figure 14.8.1j, view A, the bending stress and the longitudinal pressure stress are additive. On the basis of the methods described by Timoshenko in Reference 538-4, three principal stresses are calculated.

$$f_1 = \left[f_h + \frac{(f_{\ell p} + K_b f_b)}{2}\right] + \left[\left(\frac{f_h - (f_{\ell p} + K_b f_b)}{2}\right)^2 + (K_t f_{st})^2\right]^{1/2} \qquad \text{(Eq 14.8.1f)}$$

$$f_2 = \left[\frac{f_h + (f_{\ell p} + K_b f_b)}{2}\right] - \left[\left(\frac{f_h - (f_{\ell p} + K_b f_b)}{2}\right)^2 + (K_t f_{st})^2\right]^{1/2} \qquad \text{(Eq 14.8.1g)}$$

$$f_3 = f_{cp} \qquad \text{(Eq 14.8.1h)}$$

In view B the bending stress and the longitudinal pressure stress are not additive, since f_b is a compressive stress. In this case the principal stresses, following Timoshenko's method are:

$$f_1' = \left[\frac{f_h + (f_{\ell p} - K_b f_b)}{2}\right] + \left[\left(\frac{f_h - (f_{\ell p} - K_b f_b)}{2}\right)^2 + (K_t f_{st})^2\right]^{1/2} \quad \text{(Eq 14.8.1i)}$$

$$f_2' = \left[\frac{f_h + (f_{\ell p} - K_b f_b)}{2}\right] - \left[\left(\frac{f_h - (f_{\ell p} - K_b f_b)}{2}\right)^2 + (K_t f_{st})^2\right]^{1/2} \quad \text{(Eq 14.8.1j)}$$

$$f_3' = f_{cp} \quad \text{(Eq 14.8.1k)}$$

Compute Shear Stresses. Shear stresses are associated with each set of stress conditions from which the principal stresses were determined. These shear stresses are computed by means of the maximum shear stress theory (Eq 14.3.11.4). One set of shear stresses is calculated from the first set of principal stresses (f_1, f_2, and f_3), thus:

$$f_{s1} = \frac{f_1 - f_3}{2} \quad \text{(Eq 14.8.1l)}$$

$$f_{s2} = \frac{f_2 - f_1}{2} \quad \text{(Eq 14.8.1m)}$$

$$f_{s3} = \frac{f_3 - f_2}{2} \quad \text{(Eq 14.8.1n)}$$

A second set of shear stresses is calculated from the second set of principal stresses (f_1', f_2', and f_3'), thus:

$$f_{s1}' = \frac{f_1' - f_3'}{2} \quad \text{(Eq 14.8.1o)}$$

$$f_{s2}' = \frac{f_2' - f_1'}{2} \quad \text{(Eq 14.8.1p)}$$

$$f_{s3}' = \frac{f_3' - f_2'}{2} \quad \text{(Eq 14.8.1q)}$$

Reference 14-24 presents detailed step-by-step procedures for using the stresses calculated above in fatigue analysis and design iterations. Essentially the basic techniques outlined in Sub-Section 14.5 utilizing a modified Goodman diagram and selected allowable values of material fatigue properties are used to establish the margin of safety associated with these calculated principal stresses.

14.8.2 Bending Loads

Stress Levels and Wall Thickness. The elementary stress formula for the elastic bending of a round tube is (Reference 1-318):

$$f_b = \frac{MR_o}{I} \quad \text{(Eq 14.8.2a)}$$

where

f_b = applied bending stress, psi

M = bending moment, in-lb

R_o = outside radius, in.

I = moment of inertia, in^4

$$I = \frac{\pi}{64}\left[D_o^4 - (D_o - 2t)^4\right] \quad \text{(Eq 14.8.2b)}$$

where

D_o = $2R_o$, in.

t = wall thickness, in.

The minimum practicable wall thickness will depend on the method used in manufacturing the tube, such as extruding and machining, drawing, forging and machining. For tubes in bending, it has been found that four ranges of D/t exist, each corresponding to a different mode of failure (Reference 1-318):

a) $0 \leq D_o/t \leq 10$. Failure in plastic bending and no local instability.

b) $10 \leq D_o/t \leq 20$. Failure in plastic bending with local instability exhibiting a single transverse fold.

c) $20 \leq D_o/t \leq 2000$. Failure by local instability exhibiting one or more inward diamond-shaped buckles. Failure is in the plastic range at lower values, and is apparently locally plastic at the higher values.

d) $D_o/t > 2000$. Failure by elastic instability in the form of inward diamond-shaped buckles.

Reference 1-318 indicates that when bending loads only are considered a minimum weight is usually obtained with a value of D/t between 90 and 100.

14.8.3 Torsion Loads

Pure torsion loads on a straight tube or duct result in twisting, with each section rotating about the longitudinal axis. Within the elastic range plane sections remain plane and radii remain straight. A shear stress, f_s, exists at any point in the plane of the section; the magnitude of this shear is proportional to the distance from the center of the section, and its direction is perpendicular to the radius drawn through the point. Accompanying this shear stress is an equal longitudinal shear stress on a radial plane and equal tensile and compressive stresses at 45 degrees. The twisting deformation is measured in terms of the angle of twist (radians) representing the angular change of a radius in the section under consideration, as shown in Figure 14.8.3. This angle of twist may be expressed by the general equation

$$\theta = \frac{T\ell}{JG} \qquad \text{(Eq 14.8.3a)}$$

where

- θ = angle of twist, radians
- T = twisting moment, lb_f-in
- ℓ = length of the member, in.
- J = Torsion constant (polar moment of inertia, Ip, in the case of round tubes), in^4
- G = modulus of rigidity lb_f/in^2

For a hollow pipe or tube of inner radius, r_i, and outer radius, r_o,

$$\theta = \frac{2T\ell}{\pi(r_o^4 - r_i^4)\,G} \qquad \text{(Eq 14.8.3b)}$$

The shear stress at any point q a distance r from the center of the section may be expressed by

$$f_s = \frac{Tr}{J} \qquad \text{(Eq 14.8.3c)}$$

The shear stress becomes a maximum at the surface:

$$f_{smax} = \frac{Tr_o}{J} \qquad \text{(Eq 14.8.3d)}$$

where

r_o = radius to outer surface

Figure 14.8.3. Straight Bar of Uniform Circular Section Under Pure Torsion
(Adapted with permission from Reference 461-2, "Formulas For Stress and Strain," R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

For the hollow pipe or tube

$$f_{smax} = \frac{2Tr_o}{\pi(r_o^4 - r_i^4)} \qquad \text{(Eq 14.8.3e)}$$

14.8.4 Combined Loads

In actual practice a line seldom sees only a single pure form of loading such as pressure only, bending only, or torsion only. Actual stresses and strains are the result of a combination of any or all loads. Where bending exists in a closed-end cylinder under pressure (having a longitudinal component of stress due to pressure), the combined longitudinal tensile stresses may well exceed the hoop stress due to pressure and thereby govern the selection of the tube diameter, wall thickness, and/or material.

If stress is the primary concern, the combined stresses may be obtained by simply adding bending, tension, and (longitudinal pressure) stresses, as in the discussion of elbows (Sub-Topic 14.8.1). If instability is a consideration, as in the case of thin wall sections, the stress ratio approach discussed in Detailed Topic 14.2.1.2 should be used.

14.9 BEAMS

14.9 BEAMS

The following introductory discussion of loaded beam problems is adapted largely from Reference 73-127. When solving problems of loaded beams supported in any manner, the *bending moment* is assumed to be:

$$\pm M = EI\left(\frac{d^2y}{dx^2}\right) \qquad \text{(Eq 14.9)}$$

where

M = bending moment, in-lb

E = modulus of elasticity, psi

I = moment of inertia, in^4

x = coordinate of a point measured from one end of the beam, in.

y = deflection, in.

The usual method for finding the *elastic line* (center line of the beam) is to integrate twice to obtain the equation of the deflection curve, $y = f(x)$. This is known as the double-integration method. There are other methods, some of which are:

1) Graphical
2) Area-moment
3) Conjugated beam
4) Castigliano's theorem
5) Virtual work
6) Finite difference
7) Laplace transform
8) Maclaurin series
9) Fourier series
10) Macaulay's method
11) Hetenyi-Niedenfuhr method
12) Theorem of three moments
13) Slope deflection

Of these techniques, the simplest is Macaulay's method (Reference 618-1). It is especially suited for discontinuous loading such as simultaneous action of concentrated loads, moments, and uniform loads acting only on part of a beam. However, despite its simplicity, Macaulay's method has one important limitation — it does not apply to statically indeterminate beams. The Hetenyi-Niedenfuhr method is clear, easy, and faster than other known methods. The particular advantage of the Hetenyi-Niedenfuhr method is that the equations for statically indeterminate beams are as easily set up as for statically determinate beams; also, it is applicable to both continuous and discontinuous loads. An abstract of this method is in Reference 73-127.

14.9.1 Beams Under Bending Loads Only

14.9.1.1 TABULATED BEAM DEFLECTION DATA. The bulk of available data on beam deflections under bending loads only pertains to small deflections within the elastic range. Table 14.9.1.1 gives formulae for reaction and vertical shear loads, bending moments, deflections, and end slopes of beams supported and loaded in various ways. The following assumptions apply to these formulae:

1) The beam is of homogeneous material with the same modulus of elasticity in tension and compression.
2) The beam is straight or nearly so; if slightly curved, the curvature is at least 10 times the depth.
3) The cross section is uniform.
4) The beam has at least one longitudinal plane of symmetry.
5) All loads and reactions are perpendicular to the axis of the beam and lie in the same plane, which is a longitudinal plane of symmetry.
6) The beam is long in proportion to depth.
7) The maximum stress does not exceed the proportional limit.

14.9.1.2 NOMOGRAPH FOR MAXIMUM DEFLECTION DUE TO ANY NUMBER OF LOADS. A simplified approach is available for determining the maximum deflection of any of the five cases described in Figure 14.9.1.2a by means of the nomograph reproduced in Figure 14.9.1.2b from Griffel's *Handbook of Formulas for Stress and Strain* (Reference 618-1). The nomograph is given in terms of the ℓ/L ratio, which defines the point of application of the load.

The nomograph also allows fast, accurate solution of multiload beams of any material and cross section. For beams of a circular cross section, the moment of inertia will be found directly on the nomograph as a function of diameter, d.

Example: Find the maximum end deflection of the steel beam shown in Figure 14.9.1.2c. $E = 30 \times 10^6$ psi and $I = 800\ in^4$.

Solution: To find $y_1 \ldots y_4$, first calculate values of the ratio ℓ/L for each load, then determine each value of y from the nomograph. Determination of the value y_2 is given in detail as follows:

1) Locate the intersection of $\ell/L = 0.2$ and the curve for case 1.
2) Project horizontally to the right to reference line 1. (For end load start at point T.)
3) Align this intersection with $E = 30 \times 10^6$, intersecting reference line 2.
4) Align this intersection with $W = 5000$, intersecting reference line 3 extended.
5) Align this intersection with $L = 120$, intersecting reference line 4.
6) Align this intersection with $I = 800$, intersecting $y_2 = 0.083$ inch.

Deflections y_1, y_3, and y_4 are found in a similar manner. The total deflection is shown in Table 14.9.1.2.

Table 14.9.1.1. Shear, Moment, and Deflection Formulae for Beams
(Adapted with permission from Reference 618-1)

Notation: W = load (lb.); w = unit load (lb. per linear in.). M is positive when clockwise; V is positive when upward; y is positive when upward. Constraining moments, applied couples, loads, and reactions are positive when acting as shown. All forces are in pounds, all moments in inch-pounds; all deflections and dimensions in inches. θ is in radians and $\tan\theta = \theta$

Statically Determinate Cases

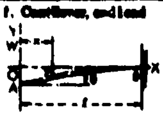
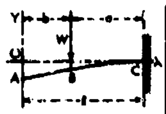
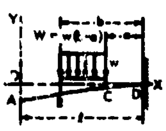
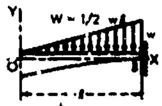
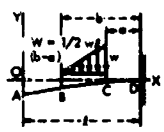
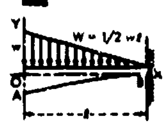
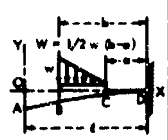
Loading, support, and reference number	Reactions R_1 and R_2, vertical shear V	Bending moment M and maximum bending moment	Deflection y, maximum deflection, and end slope θ
1. Cantilever, end load 	$R_2 = +W$ $V = -W$	$M = -Wx$ Max $M = -Wl$ at B	$y = -\frac{1}{6}\frac{W}{EI}(x^3 - 3l^2x + 2l^3)$ Max $y = -\frac{1}{3}\frac{Wl^3}{EI}$ at A $\theta = +\frac{1}{2}\frac{Wl^2}{EI}$ at A
2. Cantilever, intermediate load 	$R_2 = +W$ (A to B) $V = 0$ (B to C) $V = -W$	(A to B) $M = 0$ (B to C) $M = -W(x - b)$ Max $M = -Wa$ at C	(A to B) $y = -\frac{1}{6}\frac{W}{EI}(-a^3 + 3a^2l - 3a^2x)$ (B to C) $y = -\frac{1}{6}\frac{W}{EI}[(x-b)^3 - 3a^2(x-b) + 2a^3]$ Max $y = -\frac{1}{6}\frac{W}{EI}(3a^2l - a^3)$ $\theta = +\frac{1}{2}\frac{Wa^2}{EI}$ (A to B)
3. Cantilever, uniform load 	$R_2 = +W$ $V = -\frac{W}{l}x$	$M = -\frac{1}{2}\frac{W}{l}x^2$ Max $M = -\frac{1}{2}Wl$ at B	$y = -\frac{1}{24}\frac{W}{EIl}(x^4 - 4l^3x + 3l^4)$ Max $y = -\frac{1}{8}\frac{Wl^3}{EI}$ $\theta = +\frac{1}{6}\frac{Wl^2}{EI}$ at A
4. Cantilever, partial uniform load 	$R_2 = +W$ (A to B) $V = 0$ (B to C) $V = -\frac{W}{b-a}(x - l + b)$ (C to D) $V = -W$	(A to B) $M = 0$ (B to C) $M = -\frac{1}{2}\frac{W}{b-a}(x-l+b)^2$ (C to D) $M = -\frac{1}{2}W(2x - 2l + a + b)$ Max $M = -\frac{1}{2}W(a + b)$ at D	(A to B) $y = -\frac{1}{24}\frac{W}{EI}[4(a^2 + ab + b^2)(l - x) - a^3 - a^2b - ab^2 - b^3]$ (B to C) $y = -\frac{1}{24}\frac{W}{EI}\left[6(a+b)(l-x)^2 - 4(l-x)^3 + \frac{(l-x-a)^4}{b-a}\right]$ (C to D) $y = -\frac{1}{12}\frac{W}{EI}[3(a+b)(l-x)^2 - 2(l-x)^3]$ Max $y = -\frac{1}{24}\frac{W}{EI}[4(a^2 + ab + b^2)l - a^3 - a^2b - ab^2 - b^3]$ at A $\theta = +\frac{1}{6}\frac{W}{EI}(a^2 + ab + b^2)$ (A to B)
5. Cantilever, triangular load 	$R_2 = +W$ $V = -\frac{W}{l^2}x^2$	$M = -\frac{1}{3}\frac{W}{l^2}x^3$ Max $M = -\frac{1}{3}Wl$ at B	$y = -\frac{1}{60}\frac{W}{EIl^2}(x^5 - 5l^4x + 4l^5)$ Max $y = -\frac{1}{15}\frac{Wl^3}{EI}$ at A $\theta = +\frac{1}{12}\frac{Wl^2}{EI}$ at A
6. Cantilever, partial triangular load 	$R_2 = +W$ (A to B) $V = 0$ (B to C) $V = -W\frac{(x-l+b)^2}{(b-a)^2}$ (C to D) $V = -W$	(A to B) $M = 0$ (B to C) $M = -\frac{1}{3}\frac{W(x-l+b)^3}{(b-a)^2}$ (C to D) $M = -\frac{1}{3}W(3x - 3l + b + 2a)$ Max $M = -\frac{1}{3}W(b + 2a)$ at D	(A to B) $y = -\frac{1}{60}\frac{W}{EI}[(5b^2 + 10ba + 15a^2)(l - x) - 4a^3 - 3a^2b - 2ab^2 - b^3]$ (B to C) $y = -\frac{1}{60}\frac{W}{EI}\left[(20a + 10b)(l-x)^2 - 10(l-x)^3 + 5\frac{(l-x-a)^4}{b-a} - \frac{(l-x-a)^5}{(b-a)^2}\right]$ (C to D) $y = -\frac{1}{6}\frac{W}{EI}[(2a+b)(l-x)^2 - (l-x)^3]$ Max $y = -\frac{1}{60}\frac{W}{EI}[(5b^2 + 10ba + 15a^2)l - 4a^3 - 3a^2b - 2ab^2 - b^3]$ at A $\theta = +\frac{1}{12}\frac{W}{EI}(3a^2 + 2ab + b^2)$ (A to B)
7. Cantilever, triangular load 	$R_2 = +W$ $V = -W\left(\frac{2lx - x^2}{l^2}\right)$	$M = -\frac{1}{3}\frac{W}{l^2}(3lx^2 - x^3)$ Max $M = -\frac{2}{3}Wl$ at B	$y = -\frac{1}{60}\frac{W}{EIl^2}(-x^5 - 15l^4x + 5lx^4 + 11l^5)$ Max $y = -\frac{11}{60}\frac{Wl^3}{EI}$ at A $\theta = +\frac{1}{4}\frac{Wl^2}{EI}$ at A
8. Cantilever, partial triangular load 	$R_2 = +W$ (A to B) $V = 0$ (B to C) $V = -W\left[1 - \frac{(l-a-x)^2}{(b-a)^2}\right]$ (C to D) $V = -W$	(A to B) $M = 0$ (B to C) $M = -\frac{1}{3}W\left[\frac{3(x-l+b)^2}{b-a} - \frac{(x-l+b)^3}{(b-a)^2}\right]$ (C to D) $M = -\frac{1}{3}W(-3l + 2b + 3x + a)$ Max $M = -\frac{1}{3}W(2b + a)$ at D	(A to B) $y = -\frac{1}{60}\frac{W}{EI}[(5a^2 + 10ab + 15b^2)(l - x) - a^3 - 2a^2b - 3ab^2 - 4b^3]$ (B to C) $y = -\frac{1}{60}\frac{W}{EI}\left[\frac{(l-x-a)^5}{(b-a)^2} - 10(l-x)^3 + (10a + 20b)(l-x)^2\right]$ (C to D) $y = -\frac{1}{6}\frac{W}{EI}[(a + 2b)(l-x)^2 - (l-x)^3]$ Max $y = -\frac{1}{60}\frac{W}{EI}[(5a^2 + 10ab + 15b^2)l - a^3 - 2a^2b - 3ab^2 - 4b^3]$ at A $\theta = +\frac{1}{12}\frac{W}{EI}(a^2 + 2ab + 3b^2)$ (A to B)

Table 14.9.1.1. Shear, Moment, and Deflection Formulas for Beams (Continued)
(Adapted with permission from Reference 618-1)

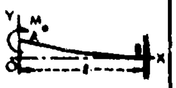
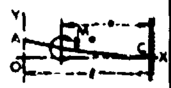
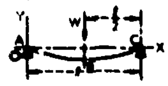
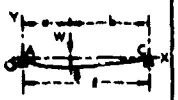
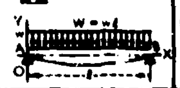
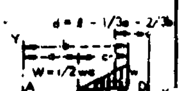
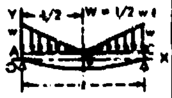
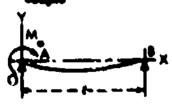
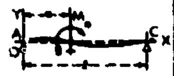
Loading, support, and reference number	Reactions R_1 and R_2, vertical shear V	Bending moment M and maximum bending moment	Deflection y, maximum deflection, and end slope θ
9. Cantilever, end couple 	$R_B = 0$ $V = 0$	$M = M_0$ Max $M = M_0$ (A to B)	$y = \frac{1}{2}\frac{M_0}{EI}(l^2 - 2lx + x^2)$ Max $y = +\frac{1}{2}\frac{M_0 l^2}{EI}$ at A $\theta = -\frac{M_0 l}{EI}$ at A
10. Cantilever, intermediate couple 	$R_B = 0$ $V = 0$	(A to B) $M = 0$ (B to C) $M = M_0$ Max $M = M_0$ (B to C)	(A to B) $y = \frac{M_0 a}{EI}\left(l - \frac{1}{2}a - x\right)$ (B to C) $y = \frac{1}{2}\frac{M_0}{EI}[(x - l + a)^2 - 2a(x - l + a) + a^2]$ Max $y = \frac{M_0 a}{EI}\left(l - \frac{1}{2}a\right)$ at A $\theta = -\frac{M_0 a}{EI}$ (A to B)
11. End supports, center load 	$R_A = +\frac{1}{2}W$ $R_C = +\frac{1}{2}W$ (A to B) $V = +\frac{1}{2}W$ (B to C) $V = -\frac{1}{2}W$	(A to B) $M = +\frac{1}{2}Wx$ (B to C) $M = +\frac{1}{2}W(l - x)$ Max $M = +\frac{1}{4}Wl$ at B	(A to B) $y = -\frac{1}{48}\frac{W}{EI}(3l^2x - 4x^3)$ Max $y = -\frac{1}{48}\frac{Wl^3}{EI}$ at B $\theta = -\frac{1}{16}\frac{Wl^2}{EI}$ at A, $\theta = +\frac{1}{16}\frac{Wl^2}{EI}$ at C
12. End supports, intermediate load 	$R_A = +W\frac{b}{l}$ $R_C = +W\frac{a}{l}$ (A to B) $V = +W\frac{b}{l}$ (B to C) $V = -W\frac{a}{l}$	(A to B) $M = +W\frac{b}{l}x$ (B to C) $M = +W\frac{a}{l}(l - x)$ Max $M = +W\frac{ab}{l}$ at B	(A to B) $y = -\frac{Wbx}{6EIl}[2l(l - x) - b^2 - (l - x)^2]$ (B to C) $y = -\frac{Wa(l - x)}{6EIl}[2lb - b^2 - (l - x)^2]$ Max $y = -\frac{Wab}{27EIl}(a + 2b)\sqrt{3a(a + 2b)}$ at $x = \sqrt{\frac{1}{3}a(a + 2b)}$ when $a > b$ $\theta = -\frac{1}{6}\frac{W}{EI}\left(bl - \frac{b^3}{l}\right)$ at A; $\theta = +\frac{1}{6}\frac{W}{EI}\left(2bl + \frac{b^3}{l} - 3b^2\right)$ at C
13. End supports, uniform load 	$R_A = +\frac{1}{2}W$ $R_B = +\frac{1}{2}W$ $V = \frac{1}{2}W\left(1 - \frac{2x}{l}\right)$	$M = \frac{1}{2}W\left(x - \frac{x^2}{l}\right)$ Max $M = +\frac{1}{8}Wl$ at $x = \frac{1}{2}l$	$y = -\frac{1}{24}\frac{Wx}{EIl}(l^3 - 2lx^2 + x^3)$ Max $y = -\frac{5}{384}\frac{Wl^3}{EI}$ at $x = \frac{1}{2}l$ $\theta = -\frac{1}{24}\frac{Wl^2}{EI}$ at A $\theta = +\frac{1}{24}\frac{Wl^2}{EI}$ at B
14. End supports, partial uniform load $d = l - 1/2b - 1/2c$ 	$R_A = W\frac{d}{l}$ $R_D = \frac{W}{l}\left(a + \frac{1}{2}c\right)$ (A to B) $V = R_A$ (B to C) $V = R_A - W\frac{x - a}{c}$ (C to D) $V = R_A - W$	(A to B) $M = R_A x$ (B to C) $M = R_A x - W\frac{(x - a)^2}{2c}$ (C to D) $M = R_A x - W(x - \frac{1}{2}a - \frac{1}{2}b)$ Max $M = W\frac{d}{l}\left(a + \frac{cd}{2l}\right)$ at $x = a + \frac{cd}{l}$	(A to B) $y = \frac{1}{24EI}\left\{4R_A(x^3 - l^2x) + Wx\left[\frac{8d^3}{l} - \frac{2bc^2}{l} + \frac{c^3}{l} + 2c^2\right]\right\}$ (B to C) $y = \frac{1}{24EI}\left\{4R_A(x^3 - l^2x) + Wx\left[\frac{8d^3}{l} - \frac{2bc^2}{l} + \frac{c^3}{l} + 2c^2\right] - W\frac{(x - a)^4}{c}\right\}$ (C to D) $y = \frac{1}{24EI}\left\{4R_A(x^3 - l^2x) + Wx\left[\frac{8d^3}{l} - \frac{2bc^2}{l} + \frac{c^3}{l}\right] - 4W(x - \frac{1}{2}a - \frac{1}{2}b)^3 + W(2b^3 - c^3)\right\}$ $\theta = \frac{1}{24EI}\left[-4R_A l^2 + W\left(\frac{8d^3}{l} - \frac{2bc^2}{l} + \frac{c^3}{l} + 2c^2\right)\right]$ at A; $\theta = \frac{1}{24EI}\left[16R_A l^2 - W\left(24d^2 - \frac{8d^3}{l} + \frac{2bc^2}{l} - \frac{c^3}{l}\right)\right]$ at D
15. End supports, triangular load 	$R_A = \frac{1}{3}W$ $R_B = \frac{2}{3}W$ $V = W\left(\frac{1}{3} - \frac{x^2}{l^2}\right)$	$M = \frac{1}{3}W\left(x - \frac{x^3}{l^2}\right)$ Max $M = 0.128Wl$ at $x = l\left(\frac{\sqrt{3}}{3}\right) = 0.5774l$	$y = -\frac{1}{180}\frac{Wx}{EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$ Max $y = -0.01304\frac{Wl^3}{EI}$ at $x = 0.5193l$ $\theta = -\frac{7}{180}\frac{Wl^2}{EI}$ at A; $\theta = +\frac{8}{180}\frac{Wl^2}{EI}$ at B.
16. End supports, partial triangular load $d = l - 1/3a - 2/3b$ 	$R_A = W\frac{d}{l}$ $R_D = W\frac{l - d}{l}$ (A to B) $V = +R_A$ (B to C) $V = R_A - \left(\frac{x - a}{c}\right)^2 W$ (C to D) $V = R_A - W$	(A to B) $M = R_A x$ (B to C) $M = R_A x - W\frac{(x - a)^3}{3c^2}$ (C to D) $M = R_A x - \frac{1}{3}W(3x - a - 2b)$ Max $M = W\frac{d}{l}\left(a + \frac{2}{3}c\sqrt{\frac{d}{l}}\right)$ at $x = a + c\sqrt{\frac{d}{l}}$	(A to B) $y = \frac{1}{6EI}\left\{R_A(x^3 - l^2x) + Wx\left[\frac{d^3}{l} + \frac{1}{6}c^2\left(1 - \frac{b}{l}\right) + \frac{17}{270}\frac{c^3}{l}\right]\right\}$ (B to C) $y = \frac{1}{6EI}\left[R_A(x^3 - l^2x) - \frac{1}{10}W\frac{(x - a)^5}{c^2} + Wx\left(\frac{d^3}{l} + \frac{1}{6}c^2 - \frac{1}{6}c^2\frac{b}{l} + \frac{17}{270}\frac{c^3}{l}\right)\right]$ (C to D) $y = \frac{1}{6EI}\left\{R_A(x^3 - l^2x) - W\left[\left(x - \frac{1}{3}a - \frac{2}{3}b\right)^3 - d^3\frac{x}{l} - \frac{1}{6}bc^2\left(1 - \frac{x}{l}\right) + \frac{17}{270}c^3\left(1 - \frac{x}{l}\right)\right]\right\}$ $\theta = \frac{1}{6EI}\left[-R_A l^2 + W\left(\frac{d^3}{l} + \frac{1}{6}c^2 + \frac{17}{270}\frac{c^3}{l} - \frac{1}{6}\frac{bc^2}{l}\right)\right]$ at A $\theta = \frac{1}{6EI}\left[2R_A l^2 + W\left(\frac{d^3}{l} + \frac{17}{270}\frac{c^3}{l} - \frac{1}{6}\frac{bc^2}{l} - 3d^2\right)\right]$ at D

Table 14.9.1.1. Shear, Moment, and Deflection Formulas for Beams (Continued)
(Adapted with permission from Reference 618-1)

Loading, support, and reference number	Reactions R_1 and R_2, vertical shear V	Bending moment M and maximum bending moment	Deflection y, maximum deflection, and end slope θ
17. End supports, triangular load 	$R_1 = \frac{1}{2}W$ $R_2 = \frac{1}{2}W$ (A to B) $V = \frac{1}{2}W\left(1 - 4\frac{x^2}{l^2}\right)$ (B to C) $V = -\frac{1}{2}W\left(1 - 4\frac{(l-x)^2}{l^2}\right)$	(A to B) $M = \frac{1}{6}W\left(3x - 4\frac{x^3}{l^2}\right)$ (B to C) $M = \frac{1}{6}W\left[3(l-x) - 4\frac{(l-x)^3}{l^2}\right]$ Max $M = \frac{1}{6}Wl$ at B	(A to B) $y = \frac{1}{6}\frac{Wx}{EIl^2}\left(\frac{1}{2}l^2x^2 - \frac{1}{5}x^4 - \frac{5}{16}l^4\right)$ Max $y = -\frac{1}{60}\frac{Wl^3}{EI}$ at B $\theta = -\frac{5}{96}\frac{Wl^2}{EI}$ at A; $\theta = +\frac{5}{96}\frac{Wl^2}{EI}$ at C
18. End supports, triangular load 	$R_1 = \frac{1}{2}W$ $R_2 = \frac{1}{2}W$ (A to B) $V = \frac{1}{2}W\left(\frac{l-2x}{l}\right)^2$ (B to C) $V = -\frac{1}{2}W\left(\frac{2x-l}{l}\right)^2$	(A to B) $M = \frac{1}{2}W\left(x - 2\frac{x^2}{l} + \frac{4}{3}\frac{x^3}{l^2}\right)$ (B to C) $M = \frac{1}{2}W\left[(l-x) - 2\frac{(l-x)^2}{l} + \frac{4}{3}\frac{(l-x)^3}{l^2}\right]$ Max $M = \frac{1}{6}Wl$ at B	(A to B) $y = \frac{1}{12}\frac{W}{EI}\left(x^3 - \frac{x^4}{l} + \frac{2}{5}\frac{x^5}{l^2} - \frac{3}{8}l^2x\right)$ Max $y = -\frac{3}{320}\frac{Wl^3}{EI}$ at B $\theta = -\frac{1}{32}\frac{Wl^2}{EI}$ at A; $\theta = +\frac{1}{32}\frac{Wl^2}{EI}$ at B
19. End supports, end couple 	$R_1 = -\frac{M_0}{l}$ $R_2 = +\frac{M_0}{l}$ $V = R_1$	$M = M_0 + R_1x$ Max $M = M_0$ at A	$y = \frac{1}{6}\frac{M_0}{EI}\left(3x^2 - \frac{x^3}{l} - 2lx\right)$ Max $y = -0.0642\frac{M_0l^2}{EI}$ at $x = 0.422l$ $\theta = -\frac{1}{3}\frac{M_0l}{EI}$ at A; $\theta = +\frac{1}{6}\frac{M_0l}{EI}$ at B
20. End supports, intermediate couple 	$R_1 = -\frac{M_0}{l}$ $R_2 = +\frac{M_0}{l}$ (A to C) $V = R_1$	(A to B) $M = R_1x$ (B to C) $M = R_1x + M_0$ Max $-M = R_1a$ just left of B Max $+M = R_1a + M_0$ just right of B	(A to B) $y = \frac{1}{6}\frac{M_0}{EI}\left[\left(6a - 3\frac{a^2}{l} - 2l\right)x - \frac{x^3}{l}\right]$ (B to C) $y = \frac{1}{6}\frac{M_0}{EI}\left[3a^2 + 3x^2 - \frac{x^3}{l} - \left(2l + 3\frac{a^2}{l}\right)x\right]$ $\theta = -\frac{1}{6}\frac{M_0}{EI}\left(2l - 6a + 3\frac{a^2}{l}\right)$ at A; $\theta = +\frac{1}{6}\frac{M_0}{EI}\left(l - 3\frac{a^2}{l}\right)$ at C $\theta = \frac{M_0}{EI}\left(a - \frac{a^2}{l} - \frac{l}{3}\right)$ at B

Statically Indeterminate Cases

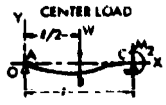
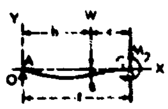
Loading, support, and reference number	Reactions R_1 and R_2, constraining moments M_1 and M_2, and vertical shear V	Bending moment M and maximum positive and negative bending moments	Deflection y, maximum deflection, and end slope θ
21. One end fixed, one end supported CENTER LOAD 	$R_1 = \frac{5}{16}W$ $R_2 = \frac{11}{16}W$ $M_2 = \frac{3}{16}Wl$ (A to B) $V = +\frac{5}{16}W$ (B to C) $V = -\frac{11}{16}W$	(A to B) $M = \frac{5}{16}Wx$ (B to C) $M = W\left(\frac{1}{2}l - \frac{11}{16}x\right)$ Max $+M = \frac{5}{32}Wl$ at B Max $-M = -\frac{3}{16}Wl$ at C	(A to B) $y = \frac{1}{96}\frac{W}{EI}(5x^3 - 3l^2x)$ (B to C) $y = \frac{1}{96}\frac{W}{EI}\left[5x^3 - 16\left(x - \frac{l}{2}\right)^3 - 3l^2x\right]$ Max $y = -0.009317\frac{Wl^3}{EI}$ at $x = 0.4472l$ $\theta = -\frac{1}{32}\frac{Wl^2}{EI}$ at A
22. One end fixed, one end supported INTERMEDIATE LOAD 	$R_1 = \frac{1}{2}W\left(\frac{3b^2l - b^3}{l^3}\right)$ $R_2 = W - R_1$ $M_2 = \frac{1}{2}W\left(\frac{b^3 + 2bl^2 - 3b^2l}{l^2}\right)$ (A to B) $V = +R_1$ (B to C) $V = R_1 - W$	(A to B) $M = R_1x$ (B to C) $M = R_1x - W(x - l + b)$ Max $+M = R_1(l - b)$ at B; max possible value $= 0.174\,Wl$ when $b = 0.634l$ Max $-M = -M_2$ at C; max possible value $= -0.1927\,Wl$ when $b = 0.5773l$	(A to B) $y = \frac{1}{6EI}[R_1(x^3 - 3l^2x) + 3Wb^2x]$ (B to C) $y = \frac{1}{6EI}[R_1(x^3 - 3l^2x) + W(3b^2x - (x - l + b)^3)]$ If $b < 0.586l$, max y is between A and B at: $x = l\sqrt{1 - \frac{2l}{3l - b}}$ If $b > 0.586l$, max y is at: $x = \frac{l(l^2 + b^2)}{3l^2 - b^2}$ If $b = 0.586l$, max y is at B and $= -0.0098\frac{Wl^3}{EI}$, max possible deflection $\theta = \frac{1}{4}\frac{W}{EI}\left(\frac{b^3}{l} - b^2\right)$ at A
23. One end fixed, one end supported UNIFORM LOAD 	$R_1 = \frac{3}{8}W$ $R_2 = \frac{5}{8}W$ $M_2 = \frac{1}{8}Wl$ $V = W\left(\frac{3}{8} - \frac{x}{l}\right)$	$M = W\left(\frac{3}{8}x - \frac{1}{2}\frac{x^2}{l}\right)$ Max $+M = \frac{9}{128}Wl$ at $x = \frac{3}{8}l$ Max $-M = -\frac{1}{8}Wl$ at B	$y = \frac{1}{48}\frac{W}{EIl}(3lx^3 - 2x^4 - l^3x)$ Max $y = -0.0054\frac{Wl^3}{EI}$ at $x = 0.4215l$ $\theta = -\frac{1}{48}\frac{Wl^2}{EI}$ at A

Table 14.9.1.1. Shear, Moment, and Deflection Formulas for Beams (Continued)
(Adapted with permission from Reference 618-1)

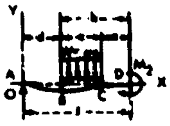
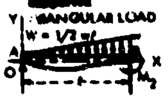
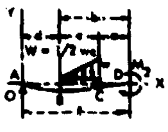
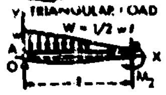
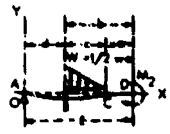
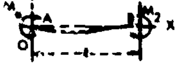
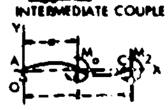
Loading, support, and reference number	Reactions R_1 and R_2, constraining moments M_1 and M_2, and vertical shear V	Bending moment M and maximum positive and negative bending moments	Deflection y, maximum deflection, and end slope θ
24. One end fixed, one end supported PARTIAL UNIFORM LOAD 	$R_1 = \frac{1}{8}\frac{W}{l^3}$ [illegible] $R_2 = W - R_1$ $M_2 = -R_1 l + \frac{1}{2}W($[illegible]$)$ (A to B) $V = R_1$ (B to C) $V = R_1 - W\left(\frac{x-d}{c}\right)$ (C to D) $V = R_1 - W$	(A to B) $M = R_1 x$ (B to C) $M = R_1 x - W\frac{(x-d)^2}{2c}$ (C to D) $M = R_1 x - W(x - d - \frac{1}{2}c)$ Max $+M = R_1\left(d + \frac{1}{2}\frac{R_1}{W}c\right)$ at $x = d + \frac{R_1}{W}c$ Max $-M = -M_2$	(A to B) $y = \frac{1}{EI}\left[R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x\right) + Wx(\frac{1}{2}a^2 + \frac{1}{2}ac + \frac{1}{6}c^2)\right]$ (B to C) $y = \frac{1}{EI}\left[R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x\right) + Wx\left(\frac{1}{2}a^2 + \frac{1}{2}ac + \frac{1}{6}c^2\right) - W\frac{(x-d)^4}{24c}\right]$ (C to D) $y = \frac{1}{EI}\left\{R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x + \frac{1}{3}l^3\right) + \right.$ [illegible] $\left.\right\}$ $\theta = -\frac{1}{EI}\left[\frac{1}{2}R_1l^2 - W\left(\frac{1}{2}a^2 + \frac{1}{2}ac + \frac{1}{6}c^2\right)\right]$ at A
25. One end fixed, one end supported TRIANGULAR LOAD 	$R_1 = \frac{1}{5}W$ $R_2 = \frac{4}{5}W$ $M_2 = \frac{2}{15}Wl$ $V = W\left(\frac{1}{5} - \frac{x^2}{l^2}\right)$	$M = W\left(\frac{1}{5}x - \frac{1}{3}\frac{x^3}{l^2}\right)$ Max $+M = 0.06Wl$ at $x = 0.4472l$ Max $-M = -M_2$	$y = \frac{1}{60}\frac{W}{EIl}\left(2l^2x^3 - l^4x - \frac{x^5}{l}\right)$ Max $y = -0.00477\frac{Wl^3}{EI}$ at $x = l\sqrt{\frac{1}{5}}$ $\theta = -\frac{1}{60}\frac{Wl^2}{EI}$ at A
26. One end fixed, one end supported PARTIAL TRIANGULAR LOAD 	$R_1 = $ [illegible] $R_2 = W - R_1$ $M_2 = -R_1 l + \frac{1}{3}W($[illegible]$)$ (A to B) $V = R_1$ (B to C) $V = R_1 - W\left(\frac{x-d}{c}\right)^2$ (C to D) $V = R_1 - W$	(A to B) $M = R_1 x$ (B to C) $M = R_1 x - \frac{1}{3}W\frac{(x-d)^3}{c^2}$ (C to D) $M = R_1 x - W(x - d - \frac{2}{3}c)$ Max $+M = R_1\left(d + \frac{2}{3}c\sqrt{\frac{R_1}{W}}\right)$ at $x = d + c\sqrt{\frac{R_1}{W}}$ Max $-M = -M_2$	(A to B) $y = \frac{1}{EI}\left[R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x\right) + \right.$ [illegible] $\left.\right]$ (B to C) $y = \frac{1}{EI}\left[R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x\right) + \right.$ [illegible] $\left.\right]$ (C to D) $y = \frac{1}{EI}\left\{R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x + \frac{1}{3}l^3\right) + \right.$ [illegible] $\left.\right\}$ $\theta = -\frac{1}{EI}\left[\frac{1}{2}R_1l^2 - W\left(\frac{1}{2}a^2 + \frac{1}{3}ac + \frac{1}{12}c^2\right)\right]$ at A
27. One end fixed, one end supported TRIANGULAR LOAD 	[illegible] $M_2 = \frac{7}{60}Wl$ $V = W\left(\frac{11}{40} - \frac{2x}{l} + \frac{x^2}{l^2}\right)$	$M = W\left(\frac{11}{40}x - \frac{x^2}{l} + \frac{1}{3}\frac{x^3}{l^2}\right)$ Max $+M =$ [illegible] Wl at $x =$ [illegible] Max $-M = -\frac{7}{60}Wl$ at B	$y = \frac{1}{120}\frac{W}{EIl}\left(11lx^3 - 3l^3x - 10x^4 + 2\frac{x^5}{l}\right)$ Max $y = -$[illegible]$\frac{Wl^3}{EI}$ at $x =$ [illegible] $\theta = -\frac{1}{40}\frac{Wl^2}{EI}$ at A
28. One end fixed, one end supported PARTIAL TRIANGULAR LOAD 	$R_1 = \frac{1}{20}\frac{W}{l^3}$ [illegible] $R_2 = W - R_1$ $M_2 = -R_1 l + \frac{1}{3}W($[illegible]$)$ (A to B) $V = R_1$ (B to C) $V = R_1 - \frac{2(x-d)}{c}W + \frac{(x-d)^2}{c^2}W$ (C to D) $V = R_1 - W$	(A to B) $M = R_1 x$ (B to C) $M = R_1 x - \frac{(x-d)^2}{c}W + \frac{(x-d)^3}{3c^2}W$ (C to D) $M = R_1 x - W(x - d - \frac{1}{3}c)$ Max $+M = R_1\left(d + c - c\sqrt{1 - \frac{R_1}{W}}\right) -$ $\frac{1}{3}Wc\left(1 - \sqrt{1 - \frac{R_1}{W}}\right)^2\left(2 + \sqrt{1 - \frac{R_1}{W}}\right)$ at $x = d + c\left(1 - \sqrt{1 - \frac{R_1}{W}}\right)$ Max $-M = -M_2$ at D	(A to B) $y = \frac{1}{EI}\left\{R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x\right) + \right.$ [illegible] $\left.\right\}$ (B to C) $y = \frac{1}{EI}\left\{R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x\right) + \right.$ [illegible] $\left.\right\}$ (C to D) $y = \frac{1}{EI}\left\{R_1\left(\frac{1}{6}x^3 - \frac{1}{2}l^2x + \frac{1}{3}l^3\right) - \right.$ [illegible] $\left.\right\}$ $\theta = -\frac{1}{EI}\left[\frac{1}{2}R_1l^2 - W\left(\frac{1}{2}a^2 + \frac{2}{3}ac + \frac{1}{4}c^2\right)\right]$ at A
29. One end fixed, one end supported END COUPLE 	$R_1 = -\frac{3}{2}\frac{M_0}{l}$ $R_2 = +\frac{3}{2}\frac{M_0}{l}$ $M_2 = \frac{1}{2}M_0$ $V = -\frac{3}{2}\frac{M_0}{l}$	$M = \frac{1}{2}M_0\left(2 - 3\frac{x}{l}\right)$ Max $+M = M_0$ at A Max $-M = -\frac{1}{2}M_0$ at B	$y = \frac{1}{4}\frac{M_0}{EI}\left(2x^2 - \frac{x^3}{l} - lx\right)$ Max $y = -\frac{1}{27}\frac{M_0l^2}{EI}$ at $x = \frac{1}{3}l$ $\theta = -\frac{1}{4}\frac{M_0l}{EI}$ at A
30. One end fixed, one end supported INTERMEDIATE COUPLE 	$R_1 = -\frac{3}{2}\frac{M_0}{l}\left(\frac{l^2 - a^2}{l^2}\right)$ $R_2 = +\frac{3}{2}\frac{M_0}{l}\left(\frac{l^2 - a^2}{l^2}\right)$ $M_2 = \frac{1}{2}M_0\left(1 - 3\frac{a^2}{l^2}\right)$ (A to B) $V = R_1$ (B to C) $V = R_1$	(A to B) $M = R_1 x$ (B to C) $M = R_1 x + M_0$ Max $+M = M_0\left[1 - \frac{3a(l^2 - a^2)}{2l^3}\right]$ at B (to right) Max $-M = -M_2$ at C (when $a < 0.275l$) Max $-M = R_1 a$ at B (to left) (when $a > 0.275l$)	(A to B) $y = \frac{M_0}{EI}\left[\frac{l^2 - a^2}{4l^3}(3l^2x - x^3) - (l - a)x\right]$ (B to C) $y = \frac{M_0}{EI}\left[\frac{l^2 - a^2}{4l^3}(3l^2x - x^3) - lx + \frac{1}{2}(x^2 + a^2)\right]$ $\theta = \frac{M_0}{EI}\left(a - \frac{1}{4}l - \frac{3}{4}\frac{a^2}{l}\right)$ at A

Table 14.1.1.1. Shear, Moment, and Deflection Formulas for Beams (Continued)
(Adapted with permission from Reference 818-1)

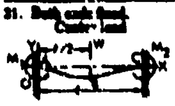
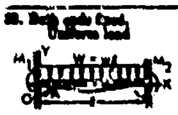
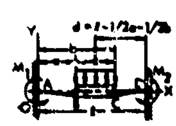
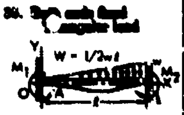
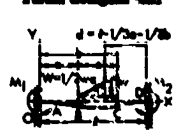
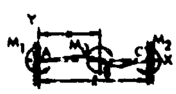
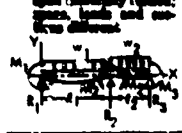
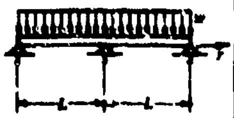
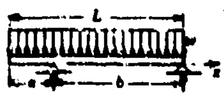
Loading, support, and reference number	Reactions R_1 and R_2, constraining moments M_1 and M_2, and vertical shear V	Bending moment M and maximum positive and negative bending moments	Deflection y, maximum deflection, and end slope θ
31. Both ends fixed. Center load 	$R_1 = \frac{1}{2}W \quad R_2 = \frac{1}{2}W$ $M_1 = \frac{1}{8}Wl \quad M_2 = \frac{1}{8}Wl$ (A to B) $V = +\frac{1}{2}W$ (B to C) $V = -\frac{1}{2}W$	(A to B) $M = \frac{1}{8}W(4x - l)$ (B to C) $M = \frac{1}{8}W(3l - 4x)$ Max $+M = \frac{1}{8}Wl$ at B Max $-M = -\frac{1}{8}Wl$ at A and C	(A to B) $y = -\frac{1}{48}\frac{W}{EI}(3lx^2 - 4x^3)$ Max $y = -\frac{1}{192}\frac{Wl^3}{EI}$ at B
32. Both ends fixed. Intermediate load 	$R_1 = \frac{Wb^2}{l^3}(3a + b) \quad R_2 = \frac{Wa^2}{l^3}(3b + a)$ $M_1 = W\frac{ab^2}{l^2} \quad M_2 = W\frac{a^2b}{l^2}$ (A to B) $V = R_1$ (B to C) $V = R_1 - W$	(A to B) $M = -W\frac{ab^2}{l^2} + R_1x$ (B to C) $M = -W\frac{ab^2}{l^2} + R_1x - W(x - a)$ Max $+M = -W\frac{ab^2}{l^2} + R_1a$ at B; max possible value $= \frac{1}{8}Wl$ when $a = \frac{1}{2}l$ Max $-M = -M_1$ when $a < b$; max possible value $= -0.1481\,Wl$ when $a = \frac{1}{3}l$ Max $-M = -M_2$ when $a > b$; max possible value $= -0.1481\,Wl$ when $a = \frac{2}{3}l$	(A to B) $y = \frac{1}{6}\frac{Wb^2x^2}{EIl^3}(3ax + bx - 3al)$ (B to C) $y = \frac{1}{6}\frac{Wa^2(l - x)^2}{EIl^3}[(3b + a)(l - x) - 3bl]$ Max $y = -\frac{2}{3}\frac{W}{EI}\frac{a^3b^2}{(3a + b)^2}$ at $x = \frac{2al}{3a + b}$ if $a > b$ Max $y = -\frac{2}{3}\frac{W}{EI}\frac{a^2b^3}{(3b + a)^2}$ at $x = l - \frac{2bl}{3b + a}$ if $a < b$
33. Both ends fixed. Uniform load 	$R_1 = \frac{1}{2}W \quad R_2 = \frac{1}{2}W$ $M_1 = \frac{1}{12}Wl \quad M_2 = \frac{1}{12}Wl$ $V = \frac{1}{2}W\left(1 - \frac{2x}{l}\right)$	$M = \frac{1}{2}W\left(x - \frac{x^2}{l} - \frac{l}{6}\right)$ Max $+M = \frac{1}{24}Wl$ at $x = \frac{1}{2}l$ Max $-M = -\frac{1}{12}Wl$ at A and B	$y = \frac{1}{24}\frac{Wx^2}{EIl}(2lx - l^2 - x^2)$ Max $y = -\frac{1}{384}\frac{Wl^3}{EI}$ at $x = \frac{1}{2}l$
34. Both ends fixed. Partial uniform load 	$R_1 = \frac{1}{4}\frac{W}{l^2}\left(12d^2 - 8\frac{d^3}{l} + 2\frac{bc^2}{l} - \frac{c^3}{l} - c^2\right)$ $R_2 = W - R_1$ $M_1 = -\frac{1}{24}\frac{W}{l}\left(24\frac{d^3}{l} - 6\frac{bc^2}{l} + 3\frac{c^3}{l} + 4c^2 - 24d^2\right)$ $M_2 = \frac{1}{24}\frac{W}{l}\left(24\frac{d^3}{l} - 6\frac{bc^2}{l} + 3\frac{c^3}{l} + 2c^2 - 48d^2 + 24dl\right)$ (A to B) $V = R_1$ (B to C) $V = R_1 - W\frac{x - a}{c}$ (C to D) $V = R_1 - W$	(A to B) $M = -M_1 + R_1x$ (B to C) $M = -M_1 + R_1x - \frac{1}{2}W\frac{(x - a)^2}{c}$ (C to D) $M = -M_1 + R_1x - W(x - l + d)$ Max $+M$ is between B and C at $x = a + \frac{cR_1}{W}$ Max $-M = -M_1$ when $a < l - b$ Max $-M = -M_2$ when $a > l - b$	(A to B) $y = \frac{1}{6EI}(R_1x^3 - 3M_1x^2)$ (B to C) $y = \frac{1}{6EI}\left(R_1x^3 - 3M_1x^2 - \frac{1}{4}W\frac{(x - a)^4}{c}\right)$ (C to D) $y = \frac{1}{6EI}[R_2(l - x)^3 - 3M_2(l - x)^2]$
35. Both ends fixed. Triangular load 	$R_1 = \frac{3}{20}W \quad R_2 = \frac{7}{20}W$ $M_1 = \frac{1}{15}Wl$ $M_2 = \frac{1}{10}Wl$ $V = W\left(\frac{3}{20} - \frac{x^2}{l^2}\right)$	$M = W\left(\frac{3}{20}x - \frac{1}{15}l - \frac{1}{3}\frac{x^3}{l^2}\right)$ Max $+M = 0.0429\,Wl$ at $x = 0.548l$ Max $-M = -\frac{1}{10}Wl$ at B	$y = \frac{1}{60}\frac{W}{EI}\left(3x^3 - 2lx^2 - \frac{x^5}{l^2}\right)$ Max $y = -0.001309\frac{Wl^3}{EI}$ at $x = 0.525l$
36. Both ends fixed. Partial triangular load 	$R_1 = \frac{W}{l^2}\left(3d^2 - \frac{1}{6}c^2 + \frac{1}{6}\frac{bc^2}{l} - \frac{17}{270}\frac{c^3}{l} - 2\frac{d^3}{l}\right)$ $R_2 = W - R_1$ $M_1 = -\frac{W}{l}\left(\frac{d^3}{l} + \frac{1}{6}c^2 + \frac{31}{270}\frac{c^3}{l} - \frac{1}{6}\frac{bc^2}{l} - d^2\right)$ $M_2 = \frac{W}{l}\left(\frac{d^3}{l} + \frac{1}{18}c^2 + \frac{31}{270}\frac{c^3}{l} - \frac{1}{6}\frac{bc^2}{l} - 2d^2 + dl\right)$ (A to B) $V = R_1$ (B to C) $V = R_1 - W\frac{(x - a)^2}{c^2}$ (C to D) $V = R_1 - W$	(A to B) $M = -M_1 + R_1x$ (B to C) $M = -M_1 + R_1x - W\frac{(x - a)^3}{3c^2}$ (C to D) $M = -M_1 + R_1x - W(x - l + d)$ Max $+M$ at $x = a + c\sqrt{\frac{R_1}{W}}$ Max $-M = M_1$ when $d > \frac{l}{2}$, M_2 when $d < \frac{l}{2}$	(A to B) $y = \frac{1}{6EI}(R_1x^3 - 3M_1x^2)$ (B to C) $y = \frac{1}{EI}\left(\frac{1}{6}R_1x^3 - \frac{1}{2}M_1x^2 - \frac{1}{60}W\frac{(x - a)^5}{c^2}\right)$ (C to D) $y = \frac{1}{6EI}\{R_1(x^3 - 3lx + 2l^3) - 3M_1(l - x)^2 + W[3d^2x + d^3 - 3dl^2 - (x - l + d)^3]\}$
37. Both ends fixed. Intermediate couple 	$R_1 = -6\frac{M_0}{l^3}(al - a^2) \quad R_2 = 6\frac{M_0}{l^3}(al - a^2)$ $M_1 = -\frac{M_0}{l^2}(4la - 3a^2 - l^2)$ $M_2 = \frac{M_0}{l^2}(2la - 3a^2)$ $V = R_1$	(A to B) $M = -M_1 + R_1x$ (B to C) $M = -M_1 + R_1x + M_0$ Max $+M = M_0\left(4\frac{a}{l} - 9\frac{a^2}{l^2} + 6\frac{a^3}{l^3}\right)$ just right of B Max $-M = M_0\left(4\frac{a}{l} - 9\frac{a^2}{l^2} + 6\frac{a^3}{l^3} - 1\right)$ just left of B	(A to B) $y = -\frac{1}{6EI}(3M_1x^2 - R_1x^3)$ (B to C) $y = \frac{1}{6EI}[(M_0 - M_1)(3x^2 - 6lx + 3l^2) - R_1(3l^2x - x^3 - 2l^3)]$ Max $+y$ at $x = \frac{2M_1}{R_1}$ if $a > \frac{1}{3}l$ Max $-y$ at $x = l - \frac{2M_2}{R_2}$ if $a < \frac{1}{3}l$
38. Continuous beam, each span uniformly loaded; spans, loads and sections different. 	$\frac{M_1l_1}{I_1} + 2M_2\left[\frac{l_1}{I_1} + \frac{l_2}{I_2}\right] + \frac{M_3l_2}{I_2} = -\frac{w_1l_1^3}{4I_1} - \frac{w_2l_2^3}{4I_2}$ (Theorem of Three Moments: I_1 and I_2 refer to 1st and 2d spans. Equation gives M_2 when M_1 and M_3 are known, or can be written for each pair of spans of a continuous beam and resulting equations solved. M_2 acts on span 1, M_2' on span 2.)	Superpose cases 13 and 19	Superpose cases 13 and 19

Table 14.9.1.1. Shear, Moment, and Deflection Formulas for Beams (Continued)
(Adapted with permission from Reference 618-1)

Additional Cases from Reference 618-1

Loading, Support, And Reference Number	Reactions R_1 And R_2 And Vertical Shear	Bending Moment M And Maximum Positive And Negative Bending Moments	Deflection y, Maximum Deflection, And End Slope
39. Beam On Three Supports, Uniform Load 	$R_1 = \frac{3}{8}wL$ $R_2 = \frac{10}{8}wL$ $R_3 = \frac{3}{8}wL$ $V = \frac{wL}{8}\left(3 - 8\frac{x}{L}\right)$	$M = \frac{wLx}{8}\left(3 - 4\frac{x}{L}\right)$ $\text{Max} + M = \frac{9}{128}wL^2 \text{ at } x = \frac{3}{8}L$ $\text{Max} - M = \frac{wL^2}{8} \text{ at } x = L$	$y = -\frac{wL^3x}{48EI}\left(1 - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}\right)$ $\text{Max } y = -\frac{1}{185}\frac{wL^4}{EI} \text{ at } x = 0.4215L$ $\theta_1 = \theta_3 = \frac{wL^3}{48EI}$
40. Simple Supports With One Overhang, Uniform Load 	$R_2 = \frac{W}{L-a}\left(\frac{L}{2} - a\right)$ $R_1 = R_2 + W$ $(A \text{ to } B): V = -\frac{W}{L}x$ $(B \text{ to } C): V = -\frac{W}{L}x + R_1$	$(A \text{ to } B): M = -\frac{W}{L}\frac{x^2}{2}$ $(B \text{ to } C): M = -\frac{W}{L}\frac{x^2}{2} + R_1(x - a)$	$(A \text{ to } B): y = \frac{1}{24}\frac{W}{EIL}\left[-x^4 + \frac{(L^4 - a^4)x}{b} - 2L^2bx + a^4 - \frac{(L^4 - a^4)a}{b} + 2L^2ab\right]$ $(B \text{ to } C): y = \frac{1}{24}\frac{W}{EIL}\left[-x^4 + \frac{2L^2(x-a)^3}{b} + \frac{(L^4 - a^4)x}{b} - 2L^2bx + a^4 - \frac{(L^4 - a^4)a}{b} + 2L^2ab\right]$

Shear and Deflection for Additional Cases from Reference 533-1

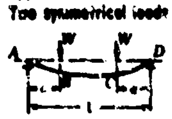
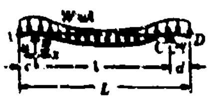
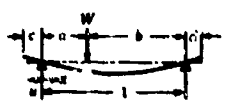
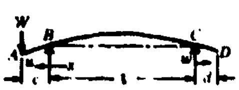
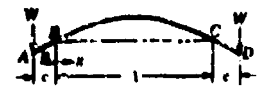
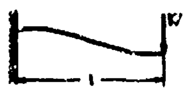
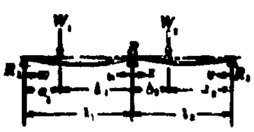
Loading, support, and reference number	Reactions R_1 and R_2, vertical shear V	Deflection y, maximum deflection, and end slope θ
41. Supported at both ends. Two symmetrical loads 	$R_A = R_D = W$ $(A \text{ to } B)$ and $(C \text{ to } D)$ $V = W$ $(B \text{ to } C)$ $V = 0$	$(A \text{ to } B)$ and $(C \text{ to } D)$ $y = -\frac{Wx}{6EI}(3al - 3a^2 - x^2)$ $(B \text{ to } C)$ $y = -\frac{Wa}{6EI}(3lx - 3x^2 - a^2)$ $\text{Max } y = -\frac{Wa}{24EI}(3l^2 - 4a^2)$ $\theta_A = -\frac{Wa}{2EI}(l - a)$
42. Both ends overhanging, supports unsymmetrical. Uniform load 	$R_B = \frac{W}{2l}(l^2 - d^2 + c^2)$ $R_C = \frac{W}{2l}(l^2 + d^2 - c^2)$	$(A \text{ to } B)$ $y = -\frac{Wu}{24EIL}[2l(d^2 + 3c^2) + 6c^2u - u^2(4c - u) - l^3]$ $(B \text{ to } C)$ $y = -\frac{Wx(l - x)}{24EIL}\left\{x(l - x) + l^2 - 2(d^2 + c^2) - \frac{2}{l}[d^2x + c^2(l - x)]\right\}$ Deflection at end: $y = -\frac{Wc}{24EIL}[2l(d^2 + 3c^2) + 3c^3 - l^3]$
43. Both ends overhanging supports, load at any point between 	$R_B = \frac{Wb}{l}$ $R_C = \frac{Wa}{l}$	Between supports same as case 13; for overhang $y = \frac{Wabu}{6EIl}(l + b)$ Max y same as case 1 Max y at end $y = \frac{Wabx}{6EIl}(l + a)$
44. Both ends overhanging supports, single overhanging load 	$R_B = \frac{W(c + l)}{l}$ $R_C = \frac{Wc}{l}$	$(A \text{ to } B)$ $y = -\frac{Wu}{6EI}(3cu - u^2 + 2cl)$ $(B \text{ to } C)$ $y = -\frac{Wcx}{6EIl}(l - x)(2l - x)$ $(C \text{ to } D)$ $y = -\frac{Wclu}{6EI}$ $y_A = -\frac{Wc^2}{3EI}(c + l)$ $y_D = -\frac{Wcld}{6EI}$ $\text{Max } y = \frac{Wcl^2}{15.59EI} \text{ at } x = 0.42265l$

Table 14.9.1.1. Shear, Moment, and Deflection Formulas for Beams (Continued)
(Adapted with permission from Reference 618-1)

Loading, support, and reference number	Reactions R_1 and R_2, vertical shear V	Deflection y, maximum deflection, and end slope θ
45. Both ends overhanging supports, symmetrical overhanging loads 	$R_1 = R_2 = W$ *(A TO B): $V = -W$* *(B TO C): $V = 0$* *(C TO D): $V = +W$*	(A to B) and (C to D) $y = -\frac{Wx}{6EI}[3c(l+x) - x^2]$ (B to C) $y = -\frac{Wcx}{2EI}(l - x)$ $y_A = y_D = -\frac{Wc^2}{6EI}(3l + 2c)$ Max $y = \frac{Wcl^2}{8EI}$
46. Fixed at one end, free but guided at the other Uniform load 	$R_1 = W$	$y = -\frac{Wx^2}{24EIl}(2l - x)^2$ Max $y = -\frac{Wl^3}{24EI}$
47. Fixed at one end, free but guided at the other With load 	$R_1 = W$	$y = -\frac{Wx^2}{12EI}(3l - 2x)$ Max $y = -\frac{Wl^3}{12EI}$
48. Continuous beam with two unequal spans. Unequal loads at any point of each 	$R_1 = \frac{W_1 b_1 - m}{l_1}$ $R_2 = \frac{W_1 a_1 + m}{l_1} + \frac{W_2 a_2 + m}{l_2}$ $R_3 = \frac{W_2 b_2 - m}{l_2}$ $m = -\frac{1}{2(l_1 + l_2)}\left[\frac{W_1 a_1 b_1}{l_1}(l_1 + a_1) + \frac{W_2 a_2 b_2}{l_2}(l_2 + a_2)\right]$	(R_1 to W_1) $y = \frac{x}{6EI}\left[(l_1 - a_1)(l_1 + a_1)R_1 - \frac{R_1 x^2}{l_1}\right]$ (R to W_1) $y = \frac{v}{6EIl_1}[W_1 a_1 b_1(l_1 + a_1) - W_1 a_1 v^2 - m(2l_1 - v)(l_1 - v)]$ (R to W_2) $y = \frac{v}{6EIl_2}[W_2 a_2 b_2(l_2 + a_2) - W_2 a_2 v^2 - m(2l_2 - v)(l_2 - v)]$ (R_3 to W_2) $y = \frac{x}{6EI}\left[(l_2 - a_2)(l_2 + a_2)R_3 - \frac{R_3 x^2}{l_2}\right]$ $y_{W_1} = \frac{a_1 b_1}{6EIl_1}[2a_1 b_1 W_1 - m(l_1 + a_1)]$ $y_{W_2} = \frac{a_2 b_2}{6EIl_2}[2a_2 b_2 W_2 - m(l_2 + a_2)]$

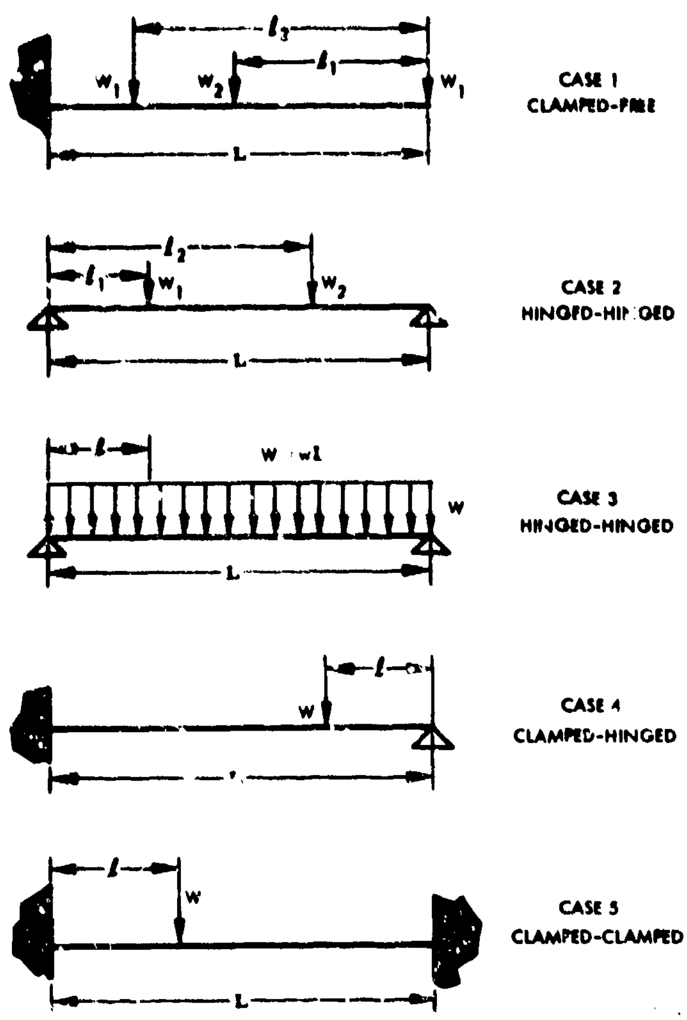


Figure 14.9.1.2a. Loading Diagrams of Beams Treated in Section 14.9.1.2
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

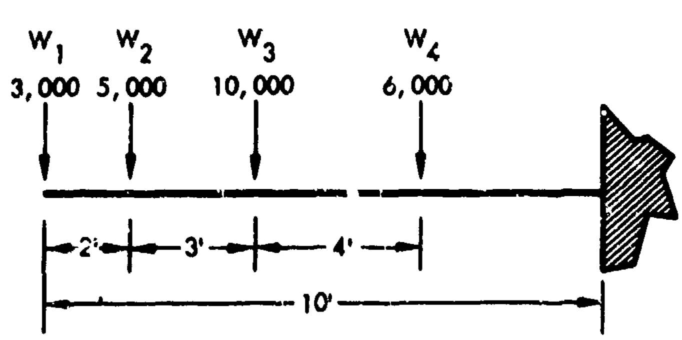


Figure 14.9.1.2b. Loading Diagram for Section 14.9.1.2 Example
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Unger Publishing Company, 1966)

Table 14.9.1.2. Deflections in Multi-Load Beam Example
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Company, 1966)

Load (lb)	Ratio ℓ/L	Deflection (in.)
3,000	0	$y_1 = 0.079$
5,000	0.2	$y_2 = 0.083$
10,000	0.2	$y_3 = 0.065$
6,000	0.3	$y_4 = 0.009$
	Total deflection	$y = 0.232$ in.

14.9.1.3 LARGE ELASTIC DEFLECTION OF BEAMS. The solution for *large* deflection of cantilever beams, such as a cantilever spring (Figure 14.9.1.3a) cannot be obtained from elementary beam theory because the basic assumptions no longer hold true. Specifically, the elementary theory neglects the square of the first derivative in the demoninator of the curvature formula (Eq 14.9.1.3) and is therefore invalid for beams of large deflections.

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \qquad \text{(Eq 14.9.1.3)}$$

where

R = radius of curvature = $\frac{EI}{M}$, in.

Also, no provision is made in the formula for the shortening of the moment arm as the loaded end of the cantilever deflects.

Figure 14.9.1.3b is reproduced from Reference 598-1 and aids in solution for the deflection by both the elementary formula and the exact formula.

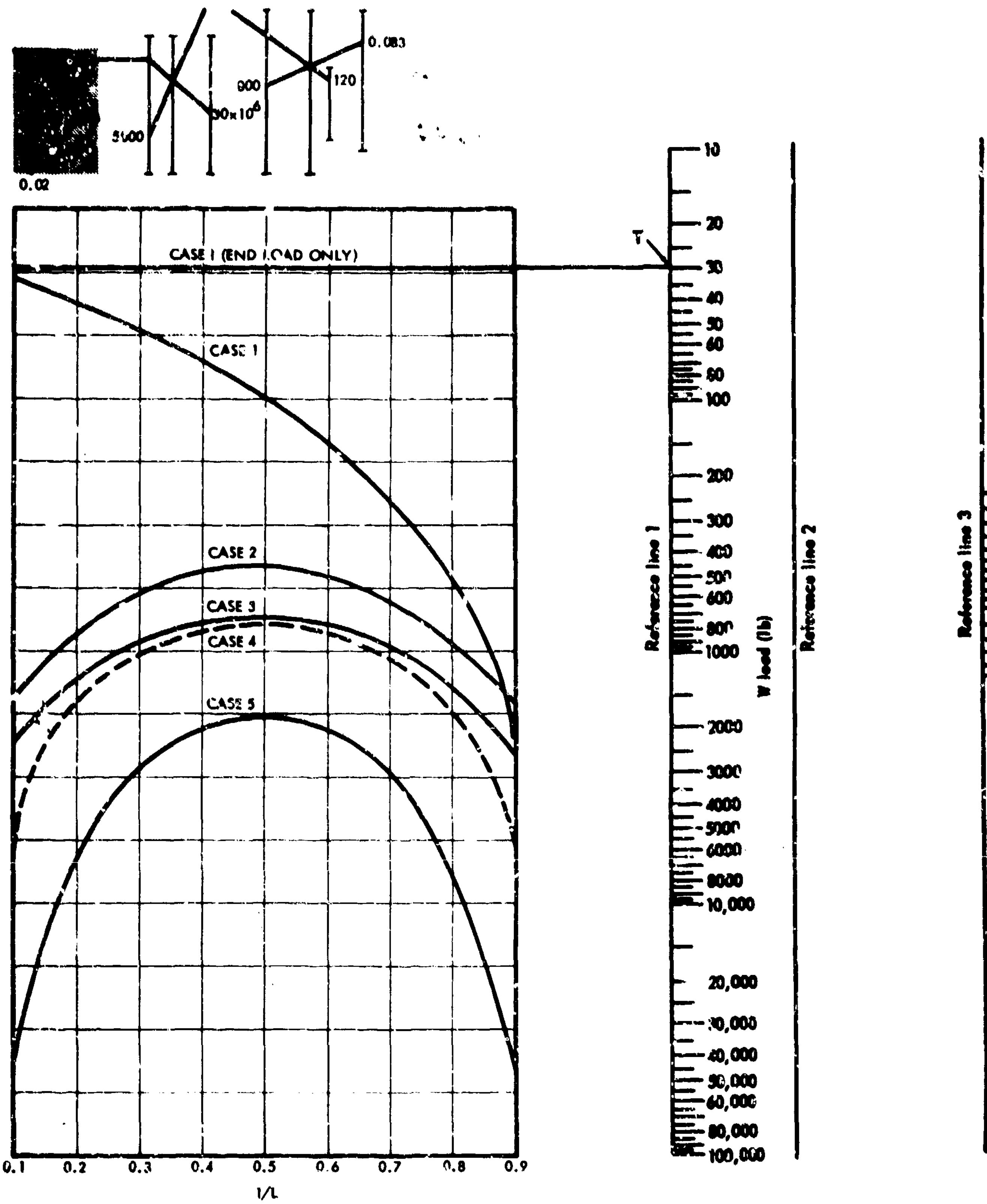
The following example illustrates the errors which may be introduced when elementary beam equations are used to determine bending stresses for flexible beams capable of large elastic deformations.

Example: Given an end-loaded flat cantilever spring (Figure 14.9.1.3a) with $b = 0.40$ inch, $h = 0.030$ inch, $L = 3$ inches, $\delta = 2$ inches, $E = 30 \times 10^6$ psi, $\mu = 0.3$, and $I = 9 \times 10^{-7}$ in^4. Find $L - \Delta$ and P by both the elementary and exact methods and compute corresponding bending stresses.

Solution:

Elementary method:

Case 1, Table 14.9.1.1



ISSUED: NOVEMBER 1968

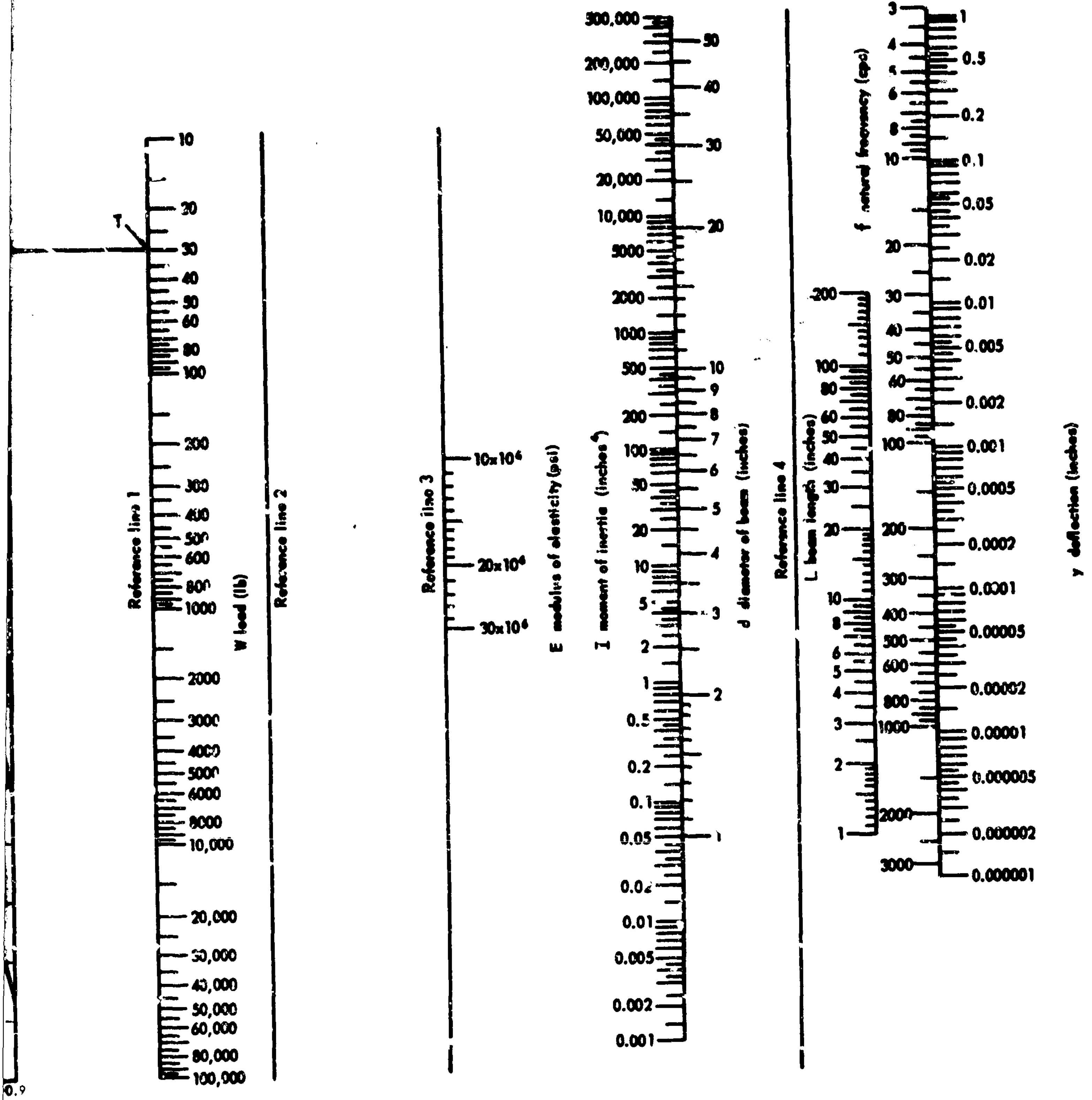


Figure 14.9.1.2b. Nomograph for Maximum Deflection Due to Any Number of Loads
(Adapted with permission from Reference G18-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

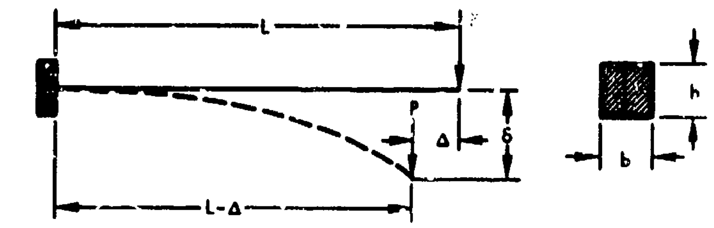


Figure 14.9.1.3a. Large Elastic Deflection of a Cantilever Beam
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley Company, 1964)

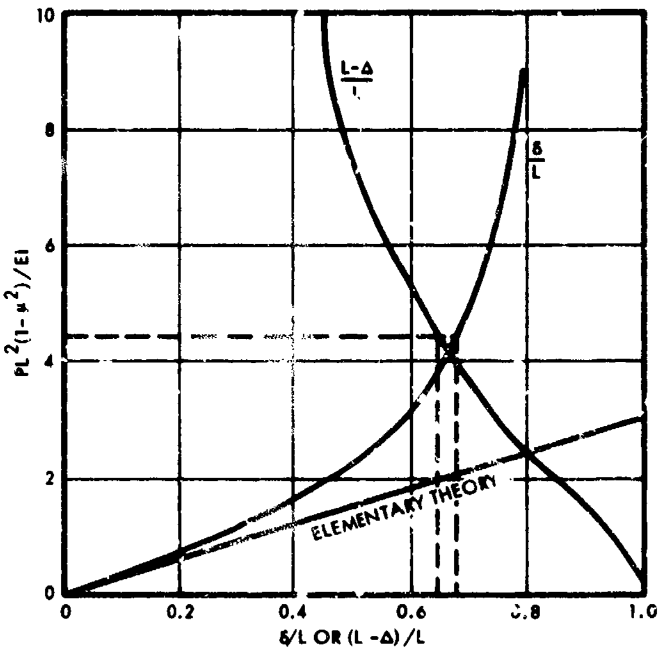


Figure 14.9.1.3b. Chart for Large Elastic Deflection Values of a Cantilever Beam
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley Company, 1964)

Maximum deflection = $\delta = \dfrac{PL^3}{3EI}$

Maximum bending moment = PL

Bending stress = $f_b = \dfrac{Mh}{2I}$

$$P = \frac{(3)(30 \times 10^6)(9 \times 10^{-7})(2)}{(3)^3} = 6 \text{ lb}$$

$M = (6)(3) = 18$ in-lb

$$f_b = \frac{(18)(0.030)}{(2)(9 \times 10^{-7})} = 300{,}000 \text{ psi}$$

Exact method:

$\delta/L = 2/3 = 0.67$

From Figure 14.9.1.3b $PL^2 \dfrac{(1-\mu^2)}{EI} = 4.5$ at $\delta/L = 0.67$

$\Delta = 3 - (0.64)(3) = 1.08$ in.

$M = P(L - \Delta) = (14.85)(1.92) = 25.95$ in-lb

$$f_b = \frac{(25.95)(0.030)}{(2)(9 \times 10^{-7})} = 432{,}000 \text{ psi}$$

Error in using elementary equations is

$$\frac{432{,}000 - 300{,}000}{300{,}000}(100) = 44\%$$

which is unacceptable for even rudimentary stress calculations.

It may be seen from Figure 14.9.1.3b that the exact method coincides fairly well with elementary theory up to $\delta/L = 0.1$, but diverges rapidly above $\delta/L = 0.5$.

14.9.1.4 DEFLECTION OF SHORT BEAMS. Another instance where the elementary formulas for bending stress can yield large errors is the short beam where the span is relatively short in comparison to the depth of the beam (Reference 73-138).

When a simply supported beam is subjected to a uniformly distributed load of magnitude w (Figure 14.9.1.4a), it can be shown by the theory of elasticity that

$$f_x = \frac{w}{2I}(L^2 - x^2)\,y + \frac{2wy^3}{15I} \quad \textbf{(Eq 14.9.1.4a)}$$

where

- f_x = internal stress in the x direction, psi
- w = unit load, lb_f/unit length
- I = moment of inertia, in^4
- L = one-half length of beam, in.
- x = position from center of beam along length, in.
- y = distance from neutral axis to extreme fiber, equal to one half the depth, in. (for beam of rectangular cross section)

The first term gives the stress provided by the elementary theory and the second term, which is independent of x, gives the correction. It is assumed in the derivation that the normal loading at the ends of the beam is the same as the magnitude of the normal stresses at the ends. At the center, x = o and the moment is maximum.

$$M_{max} = \frac{wL^2}{2} \quad \textbf{(Eq 14.9.1.4b)}$$

where

M = bending moment, in-lb.

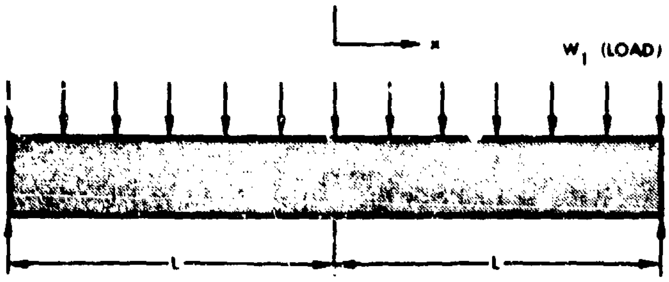


Figure 14.9.1.4a. (Simply Supported) Short Beam
(Adapted with permission from Reference 73-138, "Design News," 1 November 1968, vol. 13, B. Saelman)

and

(Eq 14.9.1.4c)

$$f_{max} = \frac{My}{I} + \frac{2wy^3}{15I} = \frac{wy^3}{I}\left[1/2\left(\frac{L}{y}\right)^2 + \frac{2}{15}\right]$$

The ratio of the stress by the exact method to that given by the elementary theory is

$$\frac{\text{Exact } f_{max}}{\text{Elementary } f_{max}} = 1 + \frac{4}{15\left(\frac{L}{y}\right)^2} \qquad \text{(Eq 14.9.1.4d)}$$

A plot of L/c versus the stress ratio is given in Figure 14.9.1.4b. It can be seen from Equation (14.9.1.4d) that the difference between the exact bending stress and that calculated from elemental theory exceeds one percent when the ratio of beam length to beam depth is below 5.2.

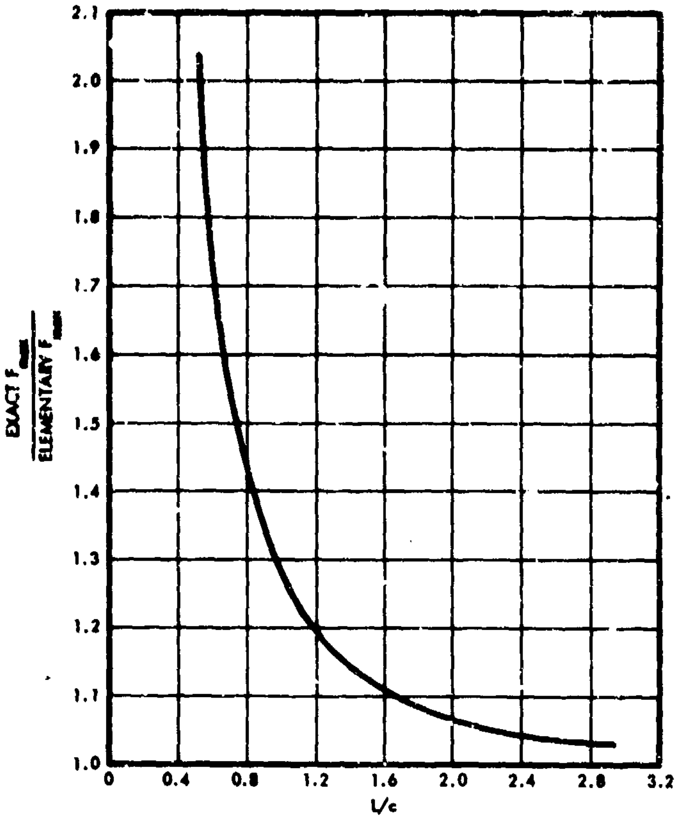


Figure 14.9.1.4b. Comparison of Stress Calculation Methods for Short Beam Bending
(Adapted with permission from Reference 73-138, "Design News," 1 November 1968, vol. 13, B. Saelman)

14.9.2 Beams Under Combined Loading

The following discussion of beams under combined axial and transverse loads includes a simplified approximate method and has been reproduced with permission from Reference 618-1 to supplement the classic tabulation of formulae reproduced in Table 14.9.2a from Reference 461-2. Reference 618-1 also includes a detailed treatment of the accurate method of analyzing beams under combined loading, including tabulations of constants and nomographs.

Analysis of the deformation of beams under simultaneous axial and transverse loading can become extremely complex. Axial tension tends to straighten the beam, thus counteracting the bending moments produced by the transverse load. On the other hand, axial compression may greatly increase the bending moment, slope, and deflection of the beam. Any of these effects may also be produced when the transverse load is replaced by an externally applied couple.

A beam under combined loading cannot be analyzed by simply superimposing the effects of axial and lateral loads or externally applied moments. The method of solution must take into account the simultaneous effect of these loads.

Two methods of analysis may be used in determining the total fiber stress in such members. One method, which is approximate in nature, assumes that the elastic curve of the deflected member is similar in form to the curve for a similar member under the action of transverse loads. The moment due to the deflection is estimated on this assumption and combined with the moment due to transverse loads. The other method, which is an exact one, makes use of the differential equation of the elastic curve and applies to relatively long and slender members.

The criterion for using either of the two methods is the critical Euler load. The approximate method may be used: for cantilever beams if $P > 0.135\ EI/L^2$; for beams with end support if $P > 0.5\ EI/L^2$; and for beams with fixed ends if $P > 2\ EI/L^2$. The precise method (Table 14.9.2) should be used for cantilevers if $P > 0.8\ EI/L^2$; for beams with end supports if $P > 3\ EI/L^2$; and for beams with fixed ends if $P > 4\ EI/L^2$.

As stated above, the approximate method of solution is based on the assumption that the elastic curve for the member with the axial load removed is similar in form to the curve when the axial load is in place. It is also assumed that the deflection and moment under combined loading are proportional to the deflection and moment for a similar member subjected only to transverse loading. Therefore the formula that applies here is appropriate only for beams in which the maximum bending moment and maximum deflection occur at the same section.

Table 14.9.2a. Formulas for Beams Under Combined Axial and Transverse Loading
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain, R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Notation: M = bending moment (in.-lb.) due to the combined loading, positive when clockwise, negative when counterclockwise; M_1 and M_2 are applied external couples (in.-lb.) positive when acting as shown; y = deflection (in.), positive when upward, negative when downward; θ = slope of beam (radians) to horizontal, positive when upward to the right; $j = \sqrt{\frac{EI}{P}}$ where E = modulus of elasticity, I = moment of inertia (in.⁴) of cross section about horizontal central axis, P = axial load (lb.); $U = \frac{l}{j}$; W = transverse load (lb.), w = transverse unit load (lb. per linear in.). All dimensions are in inches, all forces in pounds, all angles in radians

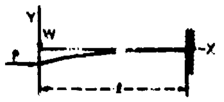
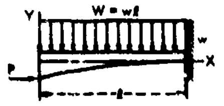
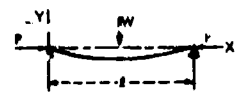
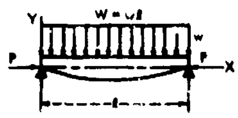
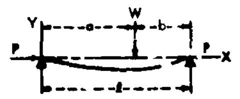
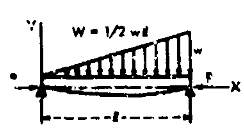
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments
1. Cantilever beam under axial compression and transverse end load 	Max $M = -Wj \tan U$ at $x = l$ Max $y = -\frac{W}{P}(j \tan U - l)$ at $x = 0$ $\theta = \frac{W}{P}\left(\frac{1 - \cos U}{\cos U}\right)$ at $x = 0$
2. Cantilever beam under axial compression and uniform transverse load 	Max $M = -wj[j(1 - \sec U) + l \tan U]$ at $x = l$ Max $y = -\frac{wj}{P}\left[j\left(1 + \frac{1}{2}U^2 - \sec U\right) + l(\tan U - U)\right]$ at $x = 0$ $\theta = \frac{w}{P}\left[\frac{l}{\cos U} - j\frac{1 - \cos 2U}{\sin 2U}\right]$
3. Beam on end supports under axial compression and transverse center load 	Max $M = \frac{1}{2}Wj \tan \frac{1}{2}U$ at $x = \frac{1}{2}l$ Max $y = -\frac{1}{2}\frac{Wj}{P}\left(\tan \frac{1}{2}U - \frac{1}{2}U\right)$ at $x = \frac{1}{2}l$ $\theta = -\frac{W}{2P}\left(\frac{1 - \cos \frac{1}{2}U}{\cos \frac{1}{2}U}\right)$ at $x = 0$
4. Beam on end supports under axial compression and uniform transverse load 	Max $M = wj^2(\sec \frac{1}{2}U - 1)$ at $x = \frac{1}{2}l$ Max $y = -\frac{wj^2}{P}\left(\sec \frac{1}{2}U - 1 - \frac{1}{8}U^2\right)$ at $x = \frac{1}{2}l$ $\theta = -\frac{wj}{P}\left[-\frac{1}{2}U + \frac{1 - \cos U}{\sin U}\right]$ at $x = 0$
5. Beam on end supports under axial compression and intermediate transverse load 	Moment equation: $x = 0$ to $x = a$: $M = \frac{Wj \sin \frac{b}{j} \sin \frac{x}{j}}{\sin U}$; Max M at $x = \frac{1}{2}\pi j$ if $\frac{1}{2}\pi j < a$ Moment equation: $x = a$ to $x = l$: $M = \frac{Wj \sin \frac{a}{j} \sin \frac{l - x}{j}}{\sin U}$; Max M at $x = \left(l - \frac{1}{2}\pi j\right)$ if $\left(l - \frac{1}{2}\pi j\right) > a$ If $\frac{1}{2}\pi j > a$ and $(l - \frac{1}{2}\pi j) < a$, Max M is at $x = a$ Deflection equation: $x = 0$ to $x = a$: $y = \frac{Wj}{P}\left(\frac{\sin \frac{b}{j} \sin \frac{x}{j}}{\sin U} - \frac{bx}{jl}\right)$ Deflection equation: $x = a$ to $x = l$: $y = \frac{Wj}{P}\left(\frac{\sin \frac{a}{j} \sin \frac{l - x}{j}}{\sin U} - \frac{a(l - x)}{jl}\right)$ $\theta = -\frac{W}{P}\left(\frac{b}{l} + \frac{\sin \frac{a}{j}}{\tan U} - \cos \frac{a}{j}\right)$ at $x = 0$ $\quad \theta = \frac{W}{P}\left(\frac{a}{l} - \frac{\sin \frac{a}{j}}{\sin U}\right)$ at $x = l$
6. Beam on end supports under axial compression and triangular transverse load 	Moment equation: $x = 0$ to $x = l$; $M = wj^2\left(\frac{\sin \frac{x}{j}}{\sin U} - \frac{x}{l}\right)$ Max M at $x = j$ arc cos $\left(\frac{\sin U}{U}\right)$ Deflection equation: $x = 0$ to $x = l$; $y = -\frac{w}{P}\left(\frac{x^3}{6l} + \frac{j^2 \sin \frac{x}{j}}{\sin U} - \frac{jx}{U} - \frac{1}{6}lx\right)$ $\theta = -\frac{w}{P}\left(\frac{j}{\sin U} - \frac{j}{U} - \frac{l}{6}\right)$ at $x = 0$ $\quad \theta = \frac{w}{P}\left(-\frac{j}{\tan U} + \frac{j}{U} - \frac{l}{3}\right)$ at $x = l$

Table 14.9.2a. Formulas for Beams Under Combined Axial and Transverse Loading (Continued)
(Adapted with permission from Reference 451-2, "Formulas for Stress and Strain, R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

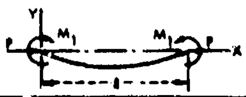
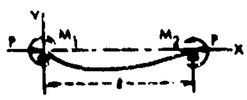
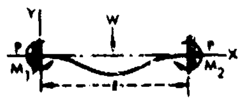
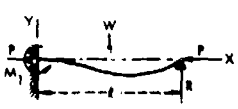
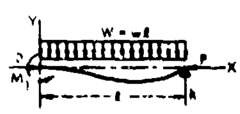
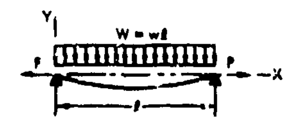
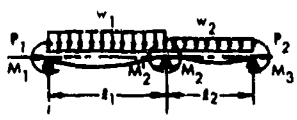
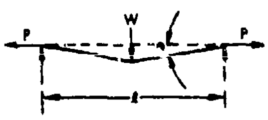
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, and slope, and constraining moments
7. Beam under axial compression and equal and opposite end couples 	Max $M = M_1 \sec \frac{1}{2}U$ at $x = \frac{1}{2}l$ Max $y = -\frac{M_1}{P}\left(\frac{1-\cos\frac{1}{2}U}{\cos\frac{1}{2}U}\right)$ at $x = \frac{1}{2}l$ $\theta = -\frac{M_1}{Pj}\tan\frac{1}{2}U$ at $x = 0$
8. Beam on end supports under axial compression and unequal end couples 	Moment equation: $x = 0$ to $x = l$: $M = \left(\frac{M_2 - M_1\cos U}{\sin U}\right)\sin\frac{x}{j} + M_1\cos\frac{x}{j}$ Max M at $x = j \arctan\left(\frac{M_2 - M_1\cos U}{M_1 \sin U}\right)$ Deflection equation: $x = 0$ to $x = l$: $y = \frac{1}{P}\left[M_1 + (M_2 - M_1)\frac{x}{l} - (M_2 - M_1\cos U)\frac{\sin\frac{x}{j}}{\sin U} - M_1\cos\frac{x}{j}\right]$ $\theta = \frac{1}{P}\left[\frac{M_2 - M_1}{l} - \frac{M_2 - M_1\cos U}{j\sin U}\right]$ at $x = 0$ $\theta = \frac{1}{P}\left[\frac{M_2 - M_1}{l} - \frac{M_2 - M_1\cos U}{j\sin U}\cos U + \frac{M_1}{j}\sin U\right]$ at $x = l$
9. Beam with fixed ends under axial compression and transverse center load 	$M_1 = M_2 = \frac{1}{2}Wj\left(\frac{1-\cos\frac{1}{2}U}{\sin\frac{1}{2}U}\right)$ At $x = \frac{l}{2}$ $M = \frac{1}{2}Wj\left(\tan\frac{1}{4}U - \frac{1-\cos\frac{1}{2}U}{\sin\frac{1}{2}U\cos\frac{1}{2}U}\right)$ Max $y = -\frac{Wj}{2P}\left[\tan\frac{1}{2}U - \frac{1}{2}U - \frac{(1-\cos\frac{1}{2}U)^2}{\sin\frac{1}{2}U\cos\frac{1}{2}U}\right]$
10. Beam with fixed ends under axial compression and uniform transverse load 	$M_1 = M_2 = wj^2\left(1 - \frac{\frac{1}{2}U}{\tan\frac{1}{2}U}\right)$ At $x = \frac{l}{2}$ $M = wj^2\left(\frac{\frac{1}{2}U}{\sin\frac{1}{2}U} - 1\right)$ Max $y = -\frac{wj^2}{P}\left[-\left(1 - \frac{\frac{1}{2}U}{\tan\frac{1}{2}U}\right)\left(\frac{1-\cos\frac{1}{2}U}{\cos\frac{1}{2}U}\right) + \sec\frac{1}{2}U - \frac{1}{8}U^2 - 1\right]$
11. Beam with one end fixed, other end supported, under axial compression and transverse center load 	Max $M = M_1 = \frac{Wl}{2}\left[\frac{j\tan U(\sec\frac{1}{2}U - 1)}{j\tan U - l}\right]$ $R = \frac{1}{2}W - \frac{M_1}{l}$ Moment equation: $x = \frac{l}{2}$ to $x = l$: $M = M_1\left(\frac{\sin\frac{x}{j}}{\tan U} - \cos\frac{x}{j}\right) + Wj\left(\sin\frac{1}{2}U\cos\frac{x}{j} - \frac{\sin\frac{1}{2}U\sin\frac{x}{j}}{\tan U}\right)$ Deflection equation: $x = \frac{l}{2}$ to $x = l$: $y = -\frac{1}{P}\left\{M_1\left(1 - \frac{x}{l} + \frac{\sin\frac{x}{j}}{\tan U} - \cos\frac{x}{j}\right) - Wj\left[\frac{l-x}{2j} + \frac{\sin\frac{1}{2}U\sin\frac{x}{j}}{\tan U} - \sin\frac{1}{2}U\cos\frac{x}{j}\right]\right\}$
12. Beam with one end fixed, other end supported, under axial compression and uniform transverse load 	Max $M = M_1 = wlj\left[\frac{\tan U(\tan\frac{1}{2}U - \frac{1}{2}U)}{\tan U - U}\right]$ $R = \frac{1}{2}wl - \frac{M_1}{l}$ Moment equation: $x = 0$ to $x = l$: $M = M_1\left(\cot U\sin\frac{x}{j} - \cos\frac{x}{j}\right) + wj^2\left[\frac{\sin\frac{x}{j}}{\sin U}(1 - \cos U) + \cos\frac{x}{j} - 1\right]$ Deflection equation: $x = 0$ to $x = l$: $y = -\frac{1}{P}\left[M_1\left(1 - \frac{x}{l} + \cot U\sin\frac{x}{j} - \cos\frac{x}{j}\right) - wj^2\left(\cot U\sin\frac{x}{j} - \frac{\sin\frac{x}{j}}{\sin U} - \cos\frac{x}{j} + \frac{lx - x^2}{2j^2} + 1\right)\right]$
13. Same as Case 1 (cantilever with end load) except that P is tension	Max $M = -Wj\tanh U$ at $x = l$ Max $y = -\frac{W}{P}(l - j\tanh U)$ at $x = 0$
14. Same as Case 2 (cantilever with uniform load) except that P is tension	Max $M = -wj[l\tanh U - j(1 - \operatorname{sech} U)]$ at $x = l$ Max $y = -\frac{wj}{P}\left[j\left(1 - \frac{1}{2}U^2 - \operatorname{sech} U\right) - l(\tanh U - U)\right]$ at $x = 0$
15. Same as Case 3 (end supports, center load) except that P is tension	Max $M = \frac{1}{2}Wj\tanh\frac{1}{2}U$ at $x = \frac{1}{2}l$ Max $y = -\frac{W}{P}\left(\frac{1}{4}l - \frac{1}{2}j\tanh\frac{1}{2}U\right)$ at $x = \frac{1}{2}l$
16. Same as Case 4 (end supports, uniform load) except that P is tension	Max $M = wj^2(1 - \operatorname{sech}\frac{1}{2}U)$ Max $y = -\frac{w}{P}\left[\frac{1}{8}l^2 - j^2\left(1 - \operatorname{sech}\frac{1}{2}U\right)\right]$
17. Same as Case 9 (fixed ends, center load) except that P is tension	$M_1 = M_2 = \frac{1}{2}Wj\left(\frac{\cosh\frac{1}{2}U - 1}{\sinh\frac{1}{2}U}\right)$, Max $+M = \frac{1}{2}Wj\left(\frac{1 - \cosh\frac{1}{2}U}{\sinh\frac{1}{2}U\cosh\frac{1}{2}U} + \tanh\frac{1}{2}U\right)$ at $x = \frac{1}{2}l$ Max $y = -\frac{Wj}{2P}\left[\frac{1}{2}U - \tanh\frac{1}{2}U - \frac{(1 - \cosh\frac{1}{2}U)^2}{\sinh\frac{1}{2}U\cosh\frac{1}{2}U}\right]$

Table 14.9.2a. Formulas for Beams Under Combined Axial and Transverse Loading (Continued)
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain, R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Manner of loading and support	Formulas for maximum bending moment, maximum deflection, and slope, and constraining moments
18. Same as Case 10 (fixed ends, uniform load) except that P is tension	$M_1 = M_2 = -\frac{wl^2}{U^2}\left(\frac{\frac12 U - \tanh\frac12 U}{\tanh\frac12 U}\right)$, Max $+M = \frac{wl^2}{U^2}\left(1 - \frac{\frac12 U}{\sinh\frac12 U}\right)$ at $x = \frac12 l$ Max $y = -\frac{wl^4}{8EIU^4}\left[\frac{4U(1-\cosh\frac12 U)}{\sinh\frac12 U} + U^2\right]$ at $x = \frac12 l$
19. Beam with ends pinned to rigid supports so horizontal displacement is prevented. Uniform transverse load and unknown axial tension 	$\frac{E^2A^2h^8}{w^2l^8}U^7 = \frac{1}{24}U^3 - \frac{5}{4}U + \frac{5}{2}\tanh\frac{U}{2} + \frac{U}{4}\tanh^2\frac{U}{2}$ where $h = \sqrt{\frac{I}{A}}$ This equation is solved for U, and P determined therefrom When $C = \frac{wl^4}{16EAh^4}$ is small (less than 4), $P = \frac{68}{690}\frac{EIC^2}{l^2}\left(1 - \frac{62}{2835}C^2\right)$ When C is large (greater than 15), $P = \frac{4EI}{l^2}\left[\left(\frac{C^2}{6}\right)^{\frac13} - 2\right]$ When P has been found by one of the above formulas, M and y may be found by the formulas of Case 16
20. Continuous beam, spans 1 and 2 unequal and unequally loaded 	$\frac{M_1 l_1}{I_1}\left(\frac{U_1\csc U_1 - 1}{U_1^2}\right) + \frac{M_3 l_2}{I_2}\left(\frac{U_2\csc U_2 - 1}{U_2^2}\right)$ $+ M_2\left[\frac{l_1}{I_1}\left(\frac{1 - U_1\cot U_1}{U_1^2}\right) + \frac{l_2}{I_2}\left(\frac{1 - U_2\cot U_2}{U_2^2}\right)\right]$ $= -\frac{w_1 l_1^3}{I_1}\left(\frac{\tan\frac12 U_1 - \frac12 U_1}{U_1^3}\right) + \frac{w_2 l_2^3}{I_2}\left(\frac{\tan\frac12 U_2 - \frac12 U_2}{U_2^3}\right)$ (Theorem of Three Moments: Subscripts with P, l, w, I, and U refer to first and second spans. M_2 acts on span 1, M_2' on span 2)
21. Same as Case 19 except ends fixed as well as held to prevent horizontal displacement	$2\pi^4EIy + \frac18\pi^4EAy^3 = \frac12 wl^4$ (Here y = Max y; A = cross-section area) $P = \frac14\pi^2y^2\frac{EA}{l^2}$ Solve first equation for y, then second equation for P; then solve for M_1, M_2, and Max M by formulas for Case 18
22. Same as Case 19 except load is W concentrated at center	$\frac12\pi^4EIy + \frac18\pi^4EAy^3 = Wl^3$ (Here y = Max y; A = cross-section area) $P = \frac14\pi^2y^2\frac{EA}{l^2}$ Solve first equation for y, then second equation for P; then solve for Max M by formulas for Case 15
23. Same as Case 21 except load is W concentrated at center	$2\pi^4EIy + \frac18\pi^4EAy^3 = Wl^3$ (Here y = Max y; A = cross-section area) $P = \frac14\pi^2y^2\frac{EA}{l^2}$ Solve first equation for y, then second equation for P; then solve for M_1, M_2, and Max M by formulas for Case 9
24. Same as Case 19 except beam is perfectly flexible like a cable or chain and has natural length l	Max $y = l\left(\frac{3wl}{64EA}\right)^{\frac13}$ (Here A = cross-section area) $P = \frac18\frac{wl^2}{\text{Max } y}$
25. Same as Case 24 except load is W concentrated at center 	$\tan\theta - \sin\theta = \frac12\frac{W}{EA}$ (Here A = cross-section area) $P = \frac12 W\cot\theta$

For any condition of loading

$$f_{max} = \frac{P}{A} \pm \frac{M}{Z} \qquad \text{(Eq 14.9.2a)}$$

where

f_{max} = maximum stress in the extreme fiber, psi

P = axial load, lb_f

A = cross-sectional area of the beam, in^2

M = maximum bending moment due to the combined effect of the axial and transverse loads, $in\text{-}lb_f$

Z = I/y = section modulus, in^3

The plus sign is used for the fibers in which the direct stress P/A and the bending stress M/Z are alike (compression-compression, tension-tension). The minus sign applies to fibers in which direct and bending stresses are not alike. M in Equation (14.9.2a) is given with sufficient accuracy by the following equation:

$$M = \frac{M'}{1 \pm \frac{(KPL^2)}{EI}} \qquad \text{(Eq 14.9.2b)}$$

where

- M' = the maximum moment in beam when P is removed, in-lb_f
- K = a constant depending for its value upon the loading and end conditions for the beam in question
- P = axial load, lb_f
- L = length of the beam, in.
- E = modulus of elasticity, psi
- I = moment of inertia of the section about the central axis normal to the plane of bending, in^4

When P is tension, use a plus sign in the denominator of Equation (14.9.2b), and when P is compression, use a minus sign.

Values of the constant K are given in Table 14.9.2b.

Approximate equations for calculating the maximum bending moment, slope, and deflection of beams subjected to simultaneous axial and transverse loads for several cases are given in Table 14.9.2c as reproduced from Reference 73-245. The error in these equations is less than 1 percent when Q does not exceed the value given as the Q-limit, $Q = PL^2/EI$.

Table 14.9.2b. Values of Constant "K" in Equation 14.9.2b
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Unger Pub. Company, 1966)

Cantilever, end load	K = 1/3
Cantilever, uniform load	K = 1/4
End supports, center load	K = 1/12
End supports, uniform load	K = 5/48
Equal and opposite end couples	K = 1/2
Fixed ends, center load	K = 1/24
	K = 1/32 (for end moments)
Fixed ends, uniform load	K = 1/16 (for center moments)

14.9.3 Beam Deflection Due To Shear

The deflection of beams due to bending alone as discussed in Sub-Topic 14.9.1 ignores the deflection due to shear because it is negligible in most instances. In many applications wherein beam theory is used to calculate deflections of fluid component elements (such as flange or a valve seat support) the beam is short relative to its depth. In such beams the shear stress may be high with correspondingly high shear deflection; to neglect shear deflection in these instances may lead to appreciable error. Reference 1-315 includes the following information which will assist the designer in determining deflection due to shear.

Two types of beams are considered; cantilever beams and simply-supported beams, each with a concentrated load or with a uniformly distributed load (Table 14.9.1.1, cases 1, 3, 11, and 13). Three types of cross sections are treated: rectangular, round, and thin-walled round. Maximum deflection for each type of loading is given in Table 14.9.3. The first term in the equation is the deflection due to bending, and the deflection due to shear is expressed by the remaining quantity. For example, for a cantilever beam with concentrated end load, the bending deflection is $PL^3/3EI$ and the shear deflection is

$$\frac{P\ell^3}{3EI}\left[K\left(\frac{E}{G}\right)\left(\frac{d}{\ell}\right)^2\right] \qquad \text{(Eq 14.9.3a)}$$

Table 14.9.3. Beam Shear Deflection Equations
(Adapted with permission from Reference 1-315, "Machine Design," 9 February 1966, vol 38, no. 19, H. H. Mabie)

Beam Type	Load Type	Deflection, δ
Cantilever	Concentrated at end	$\frac{P\ell^3}{3EI}\left[1 + K\frac{E}{G}\left(\frac{d}{\ell}\right)^2\right]$
Cantilever	Uniform	$\frac{W\ell^3}{8EI}\left[1 + K\frac{E}{G}\left(\frac{d}{\ell}\right)^2\right]$
Simply supported	Concentrated at center	$\frac{P\ell^3}{48EI}\left[1 + K\frac{E}{G}\left(\frac{d}{\ell}\right)^2\right]$
Simply supported	Uniform	$\frac{5W\ell^3}{384EI}\left[1 + K\frac{E}{G}\left(\frac{d}{\ell}\right)^2\right]$

Note: K is a form factor depending upon shape of cross section and type of loading

Four series of curves, one for each type of loading, are presented in Figures 14.9.3a, b, c and d from which one can easily determine deflection due only to shear. The ratio E/G, taken as 2.6, is an average value suitable for most metals. The influence of E/G may be evaluated from Figure 14.9.3e wherein the deflection factor is computed for rectangular cross-section beams (K = 3/10) for various values of E/G.

Example: Given a cantilever beam of rectangular cross section with d/ℓ = 0.5 and with a concentrated load at the end, find the proportionate amount of total deflection which is due to shear.

Solution: For a rectangular section (K = 3/10) the shear deflection factor from Figure 14.9.3a is

$$K\left(\frac{E}{G}\right)\left(\frac{d}{\ell}\right)^2 = 0.195. \qquad \text{(Eq 14.9.3b)}$$

Table 14.9.3a. Approximate Equations for Beams Under Combined Axial and Transverse Loads
(Adapted with permission from Reference 73-248, "Design News," 13 May 1964, vol. 19, no. 10, S. Kravitz)

L = LENGTH OF BEAM, IN.
W = TOTAL TRANSVERSE LOAD, LB_f
P = AXIAL LOAD, LB_f
E = YOUNG'S MODULUS OF ELASTICITY, PSI
I = MOMENT OF INERTIA, $IN.^4$
Q = PL^2/EI, DIMENSIONLESS
M = BENDING MOMENT, LB_f - IN.
θ = SLOPE, RADIANS
y = DEFLECTION, IN.

1.

EQUATION	Q - LIMIT
$M = WL\left(1 + \frac{Q}{3} + \frac{2Q^2}{15}\right)$ AT B	0.56
$\theta = \frac{WL^2}{2EI}\left(1 + \frac{5Q}{12} + \frac{61Q^2}{360}\right)$ AT A	0.53
$y = \frac{WL^3}{3EI}\left(1 + \frac{2Q}{5} + \frac{17Q^2}{105}\right)$ AT A	0.53

2.

EQUATION	Q - LIMIT
$M = WL\left(1 - \frac{Q}{3} + \frac{2Q^2}{15}\right)$ AT B	0.35
$y = \frac{WL^3}{3EI}\left(1 - \frac{2Q}{5} + \frac{17Q^2}{105}\right)$ AT A	0.34

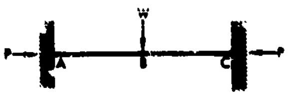
3.

EQUATION	Q - LIMIT
$M = \frac{WL}{4}\left(1 + \frac{Q}{12} + \frac{Q^2}{120}\right)$ AT B	2.2
$\theta = \frac{WL^2}{16EI}\left(1 + \frac{5Q}{48} + \frac{61Q^2}{5760}\right)$ AT A OR C	2.1
$y = \frac{WL^3}{48EI}\left(1 + \frac{Q}{10} + \frac{17Q^2}{1680}\right)$ AT B	2.1

4.

EQUATION	Q - LIMIT
$M = \frac{WL}{4}\left(1 - \frac{Q}{12} + \frac{Q^2}{120}\right)$ AT B	2.5
$y = \frac{WL^3}{48EI}\left(1 - \frac{Q}{10} + \frac{17Q^2}{1680}\right)$ AT B	2.0

5.



EQUATION	Q - LIMIT
$M = \frac{WL}{8}\left(1 \quad \frac{Q}{48} + \frac{Q^2}{1920}\right)$ AT B	9.0
$y = \frac{WL^3}{192EI}\left(1 + \frac{Q}{40} + 0.000633\,Q^2\right)$ AT B	8.7

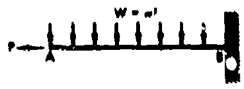
6.

EQUATION	Q - LIMIT
$M = \frac{WL}{8}\left(1 - \frac{Q}{48} + \frac{Q^2}{1920}\right)$ AT B	9.0
$y = \frac{WL^3}{192EI}\left(1 - \frac{Q}{40} + 0.000633\,Q^2\right)$ AT B	8.6

7.

EQUATION	Q - LIMIT
$M = \frac{WL}{2}\left(1 + \frac{Q}{4} + \frac{7Q^2}{72}\right)$ AT B	0.61
$\theta = \frac{WL^2}{6EI}\left(1 + \frac{9Q}{20} + \frac{31Q^2}{148}\right)$ AT A	0.52
$y = \frac{WL^3}{8EI}\left(1 + \frac{7Q}{18} + \frac{113Q^2}{720}\right)$ AT A	0.53

8.

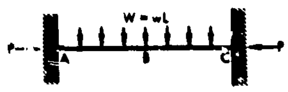


EQUATION	Q - LIMIT
$M = \frac{WL}{2}\left(1 - \frac{Q}{4} + \frac{7Q^2}{72}\right)$ AT B	0.41
$y = \frac{WL^3}{8EI}\left(1 - \frac{7Q}{18} + \frac{113Q^2}{720}\right)$ AT A	0.34

9.

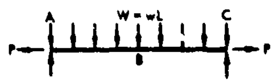
EQUATION	Q - LIMIT
$M = \frac{WL}{12}\left(1 - \frac{Q}{60} + \frac{Q^2}{1260}\right)$ AT A AND C	4.4
$M = \frac{WL}{24}\left(1 - \frac{7Q}{240} + \frac{31Q^2}{40320}\right)$ AT B	7.1
$y = \frac{WL^3}{384EI}\left(1 - \frac{Q}{40} + 0.000633Q^2\right)$ AT B	9.3

10.



EQUATION	Q - LIMIT
$M = \frac{WL}{12}\left(1 + \frac{Q}{60} + \frac{Q^2}{1260}\right)$ AT A AND C	5.8
$M = \frac{WL}{24}\left(1 + \frac{7Q}{240} + \frac{31Q^2}{40320}\right)$ AT B	8.6
$y = \frac{WL^3}{384EI}\left(1 + \frac{Q}{40} + 0.000633Q^2\right)$ AT B	6.0

11.



EQUATION	Q - LIMIT
$M = \frac{WL}{8}\left(1 - \frac{5Q}{48} + \frac{61Q^2}{5740}\right)$ AT B	2.1
$y = \frac{5WL^3}{384EI}\left(1 - \frac{61Q}{600} + \frac{277Q^2}{26880}\right)$ AT B	2.0

12.

EQUATION	Q - LIMIT
$M = \frac{WL}{8}\left(1 + \frac{5Q}{48} + \frac{61Q^2}{5760}\right)$ AT B	2.1
$\theta = \frac{WL^2}{24EI}\left(1 + \frac{Q}{10} + \frac{17Q^2}{1680}\right)$ AT A AND C	2.1
$y = \frac{5WL^3}{384EI}\left(1 + \frac{61Q}{600} + \frac{277Q^2}{26880}\right)$ AT B	2.1

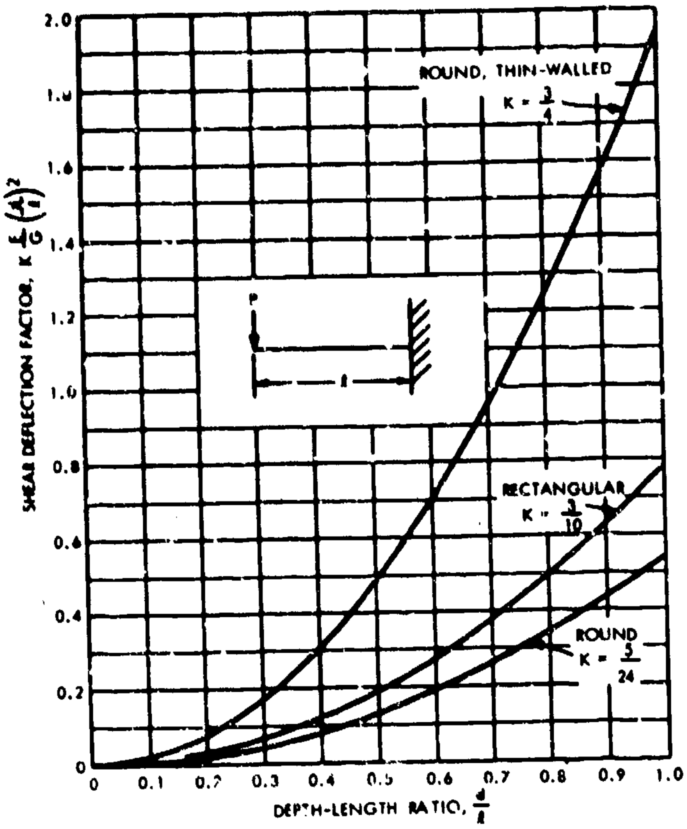


Figure 14.9.3a. Beam Deflection Due to Shear; Cantilever with Concentrated End Load
(Adapted with permission from Reference 1-315, "Machine Design," 9 February 1956, vol. 28, no. 3. H. H. Mabie)

From Table 14.9.3 the total deflection of the beam is

$$\delta = \frac{P\ell^3}{3EI}(1 + 0.195) \qquad \textbf{(Eq 14.9.3a)}$$

Thus, neglecting the shear deflection would result in an error of approximately 20 percent.

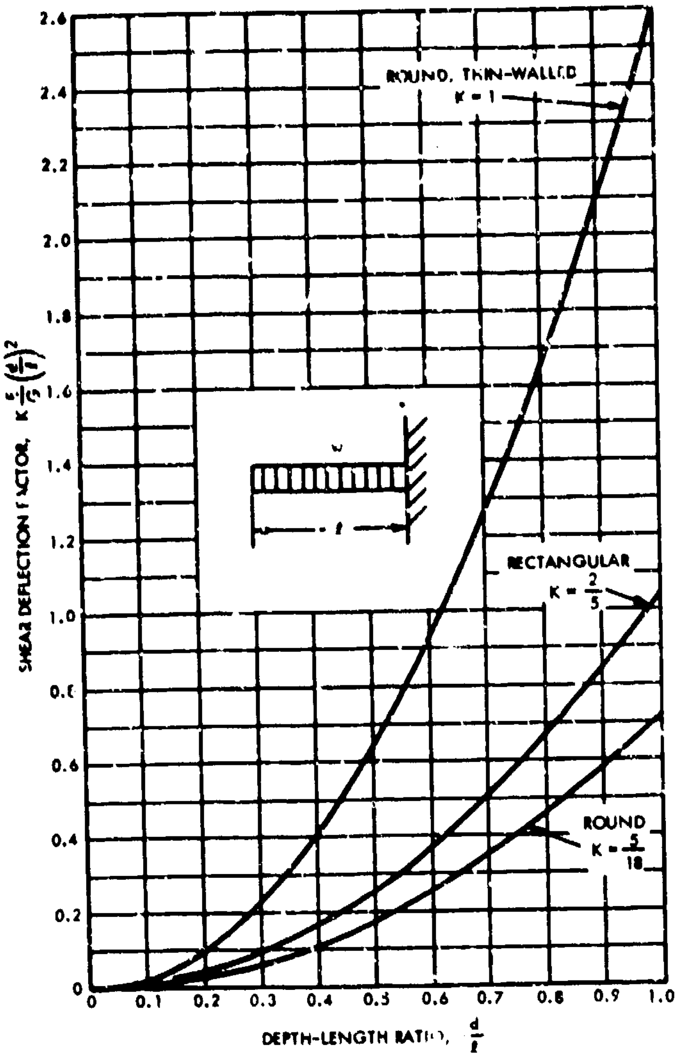


FIG. NO. 14.9.3b
PAGE NO.
REDUCE TO 53 %

Figure 14.9.3b. Beam Deflection Due to Shear; Cantilever with Uniform Load
(Adapted with permission from Reference 1-315, "Machine Design," 9 February 1956, vol. 28, no. 3, H. H. Mabie)

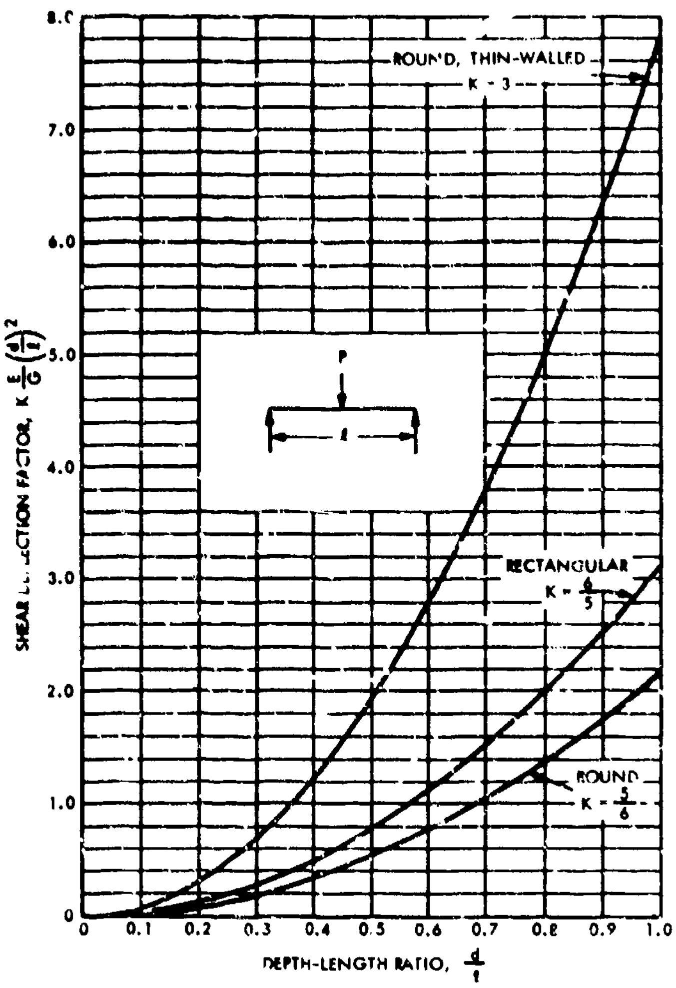


Figure 14.9.3c. Beam Deflection Due to Shear Simply Supported Beam with Concentrated Center Load
(Adapted with permission from Reference 1-315, "Machine Design," 9 February 1956, vol. 28, no. 3, H. H. Mabie)

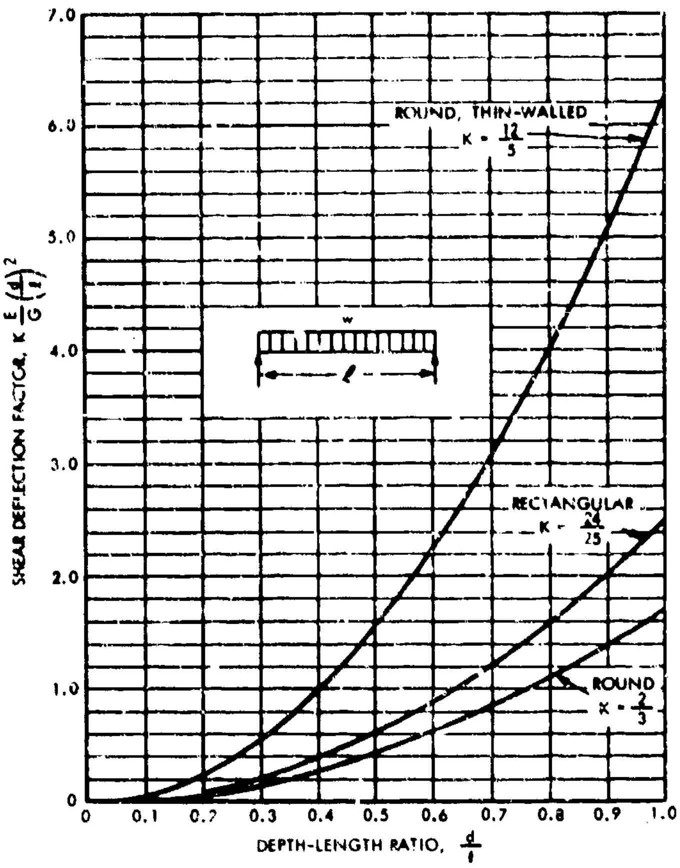


Figure 14.9.3d. Beam Deflection Due to Shear; Simply Supported Beam with Uniform Load
(Adapted with permission from Reference 1-315, "Machine Design," 9 February 1956, vol. 28, no. 3, H. H. Mabie)

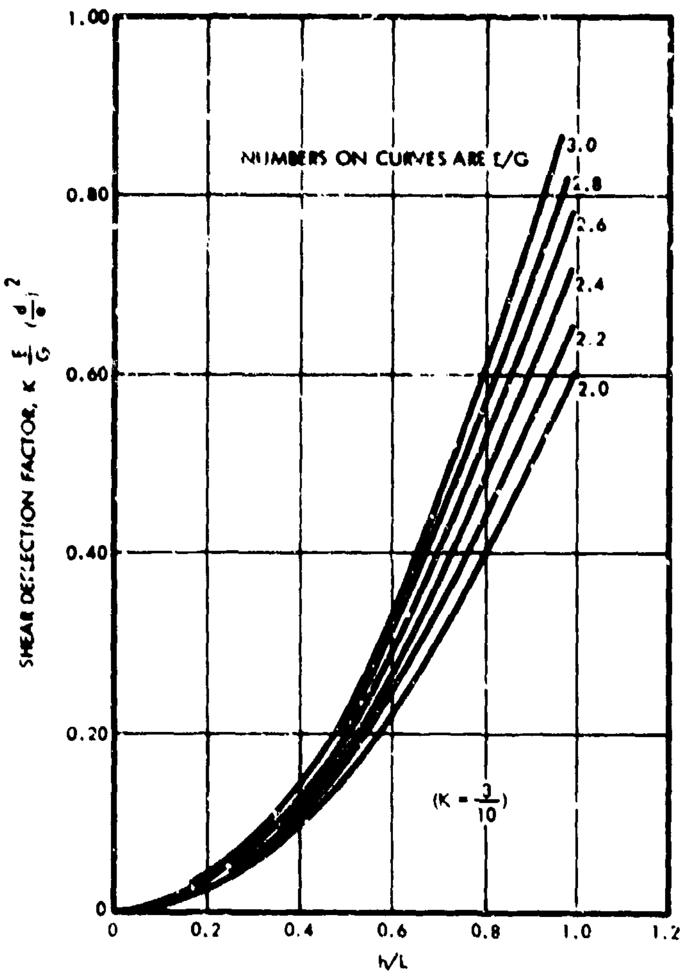


Figure 14.9.3e. Design Chart for Rectangular Beams with Transverse Shear
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley Company, 1964)

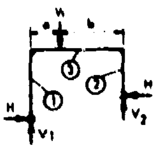
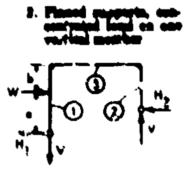
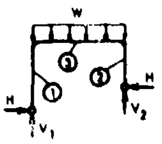
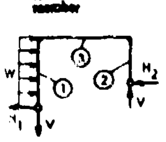
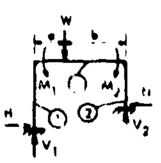
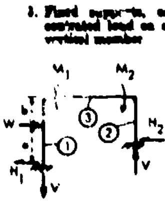
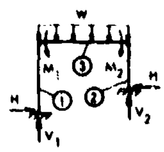
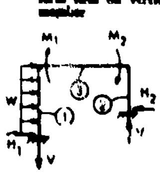
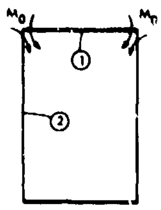
14.9.4 Curved Beams

Curved beams are not often encountered in fluid components, and therefore this subject is not covered in detail in this section. For special requirements, Reference 461-2 covers thirty-five cases of loading in plane of curvature for circular rings and arches. Reference 19-277 covers 21 similar cases and, in addition, covers 7 cases of transversely loaded circular rings and partial rings. Perhaps the most comprehensive treatment of curved beams may be found in Blake's recent text *Design of Curved Members for Machines* (Reference 744-1), devoted exclusively to this subject.

14.9.5 Reaction Formulae for Rigid Frames

By superposition, the beam formulae in Table 14.9.1.1 can be made to apply to combinations of beams, such as rigid frames. The formulae in Table 14.9.5 have been derived in that way (Reference 461-2).

Table 14.9.5. Reaction Formulas for Rigid Frames
(Adapted with permission from Reference 461-2, "Formulas for Stress and Strain, R. J. Roark, McGraw-Hill Book Company, Inc., 1965)

Loading, support, and reference number	Formulas for statically indeterminate forces and moments
1. Pinned supports, concentrated load on horizontal member 	$H = \frac{3}{8}W \frac{[illegible]}{[illegible]}$ $V_1 = \frac{Wb + H(L_1 - L_2)}{L_3}$
2. Pinned supports, concentrated load on one vertical member 	$H_1 = W \frac{[illegible]}{[illegible]}$ $V = \frac{Wa - H_2(L_1 - L_2)}{L_3}$
3. Pinned supports, uniform load on horizontal member 	$H = W \frac{[illegible]}{[illegible]}$ $V_1 = \frac{\frac{1}{2}WL_3 + H(L_1 - L_2)}{L_3}$
4. Pinned supports, uniform load on vertical member 	$H_2 = \frac{1}{8}W \frac{[illegible]}{[illegible]}$ $V = \frac{\frac{1}{2}WL_1 - H_2(L_1 - L_2)}{L_3}$
5. Fixed supports, concentrated load on horizontal member 	[illegible]
6. Fixed supports, concentrated load on one vertical member 	[illegible]
7. Fixed supports, uniform load on horizontal member 	[illegible]
8. Fixed supports, uniform load on vertical member 	[illegible]
9. Rectangular frame or tube under uniform outward (or inward) pressure w 	$M_0 = \frac{1}{12} w \frac{\frac{L_1^3}{I_1} + \frac{L_2^3}{I_2}}{\frac{L_1}{I_1} + \frac{L_2}{I_2}}$

14.10 FLAT PLATES

14.10 FLAT PLATES

The analysis of stresses, deflections, and slopes in flat plates is relatively complex. The graphical presentation of stress and deflection plate solutions and a simple tabular technique for determining slope of circular plates have been adapted with permission from Griffel's *Handbook of Formulas for Stress and Strain* (Reference 618-1) to provide an expeditious means of solving a variety of flat plate problems. These presentations are based on several assumptions which are described below. The designer requiring more comprehensive equations, including the effect of Poisson's ratio, is referred to Reference 461-2. Where numerous calculations must be performed, the designer might well be interested in the comprehensive series of nomographs by H.A. Magnus which were published over a period of months in *Design News* during 1957 and included in the 1958 *Design Data Manual* by the same publisher, Rogers Publishing Company, Inglewood, Colorado. Reference 618-1 also contains a detailed discussion of the effect of Poisson's ratio on stresses in flat plates, as well as stress and deflection data on annular plates of linearly varying thickness.

14.10.1 Graphical Data for Stresses and Deflections in Flat Plates

Stress and deflection coefficients for rectangular and circular plates under 27 types of loading and edge conditions and for a wide range of plate dimensions are presented in Detailed Topic 14.10.1.1 and 14.10.1.2. The following assumptions apply to these data:

1) The plate is flat, of uniform thickness, not more than one-quarter of the smallest transverse dimension
2) Maximum deflection is no more than one-half the plate thickness
3) Force loads are normal to the plane of the plate
4) The plate is not stressed beyond the elastic limit at any point.

In all cases, Poisson's ratio was taken as 0.3, a value used for steel. A considerable change of this value will only slightly change the stress and deflection. As the thickness of plate is small, the additional deflection due to shear is negligible.

14.10.1.1 STRESS AND DEFLECTION FORMULAE FOR FLAT RECTANGULAR PLATES. Table 14.10.1.1 contains stress and deflection formulae for flat rectangular plates, utilizing the following equations:

$$f = \frac{CwL^2}{t^2} \quad \text{for cases 1 to 13} \qquad \text{(Eq 14.10.1.1a)}$$

$$f = \frac{C_{14} w a_1 b_1}{t^2} \quad \text{for case 14} \qquad \text{(Eq 14.10.1.1b)}$$

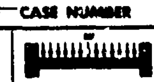
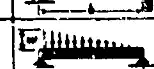
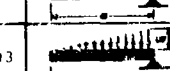
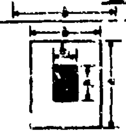
$$y = \frac{KwL^4}{Et^3} \quad \text{for cases 1 to 13} \qquad \text{(Eq 14.10.1.1c)}$$

where

f = maximum unit stress at surface of plate, psi

C = stress loading-support factor for rectangular plates, dimensionless (see Figures 14.10.1.1a and b)

Table 14.10.1.1. Stress and Deflection of Rectangular Plates
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Company, 1966)

LOADING AND EDGE CONDITIONS			STRESS		DEFLECTION		L
CASE NUMBER			C	FIG.	K	FIG.	
1		ALL EDGES FIXED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_1	14.10.1.1a	K_1	14.10.1.1b	b
2		LONG EDGES FIXED, SHORT EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_2		K_2		
3		ONE LONG EDGE CLAMPED, OTHER THREE EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_3		K_3		
4		ALL EDGES SUPPORTED. HYDROSTATIC PRESSURE VARYING ALONG LENGTH.	C_4		K_4		
5		ALL EDGES SUPPORTED. HYDROSTATIC PRESSURE VARYING ALONG BREADTH.	C_5		K_5		
6		SHORT EDGES FIXED, LONG EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_6		K_6		
7		ALL EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_7		K_7		
8		ONE SHORT EDGE FREE, OTHER THREE EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_8		K_8		
9		ONE SHORT EDGE CLAMPED, OTHER THREE EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	C_9		K_9		
10		ONE LONG EDGE FREE, OTHER THREE EDGES SUPPORTED. UNIFORM LOAD OVER ENTIRE SURFACE.	10		10		a
11		ALL EDGES SUPPORTED. DISTRIBUTED LOAD IN FORM OF A TRIANGULAR PRISM	11		11		b
12		ONE SHORT EDGE FREE, OTHER THREE EDGES SUPPORTED. DISTRIBUTED LOAD VARYING LINEARLY ALONG LENGTH.	12		12		b
13		ONE LONG EDGE FIXED, OTHER THREE EDGES SUPPORTED. DISTRIBUTED LOAD VARYING LINEARLY ALONG BREADTH.	13		13		a
14		ALL EDGES SUPPORTED. UNIFORMLY DISTRIBUTED LOAD OVER SHADED PORTION.	$f = \frac{C_{14} w a_1 b_1}{t^2}$ FIGURES 14.10.1.1a, d, e				

C_{14} = see Figures 14.10.1.1c, d, and e

w = unit applied load, psi

L = length, in. (see Table 14.10.1.1)

t = plate thickness, in.

a_1, b_1 = plate dimensions, in. (see Table 14.10.1.1, case 14)

y = vertical deflection, in.

K = deflection loading-support factor for rectangular plates, dimensionless (see Figures 14.10.1.1a and b)

E = modulus of elasticity, psi

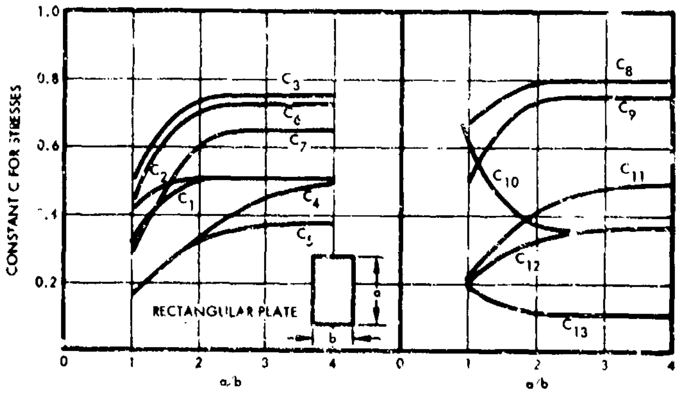


Figure 14.10.1.1a. Stress Constants for Rectangular Plates *(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)*

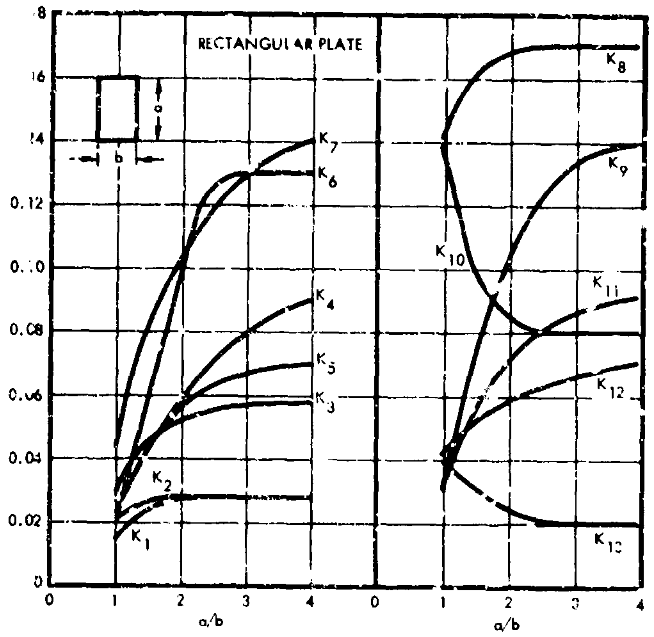


Figure 14.10.1.1b. Deflection Constants for Rectangular Plates *(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)*

Example: Find the maximum deflection and maximum stress in a rectangular plate simply supported along its four edges and with a uniformly distributed load w = 6 psi over the entire surface of the plate. The dimensions of the plate are: a = 50 inches, b = 36 inches, t = 1/2 inch. Modulus of elasticity, E = 30 x 10^6 psi.

Solution: The loading and edge conditions correspond to those of Table 14.10.1.1, case 7. Therefore for a/b = 1.39 we find from the curves of Figures 14.10.1.1a and b that C_7 = 0.43 and K_7 = 0.079. Also, from Table 14.10.1.1, L = b. Substituting these numbers into Equations (14.10.1.1a) and (14.10.1.1c) the maximum stress and deflection are:

$$f = \frac{0.46 \times 6 \times 36^2}{(1/2)^2} = 14{,}980 \text{ psi}$$

$$y = \frac{0.079 \times 6 \times 36^4}{30 \times 10^6 \times (1/2)^3} \quad 0.200 \text{ in.}$$

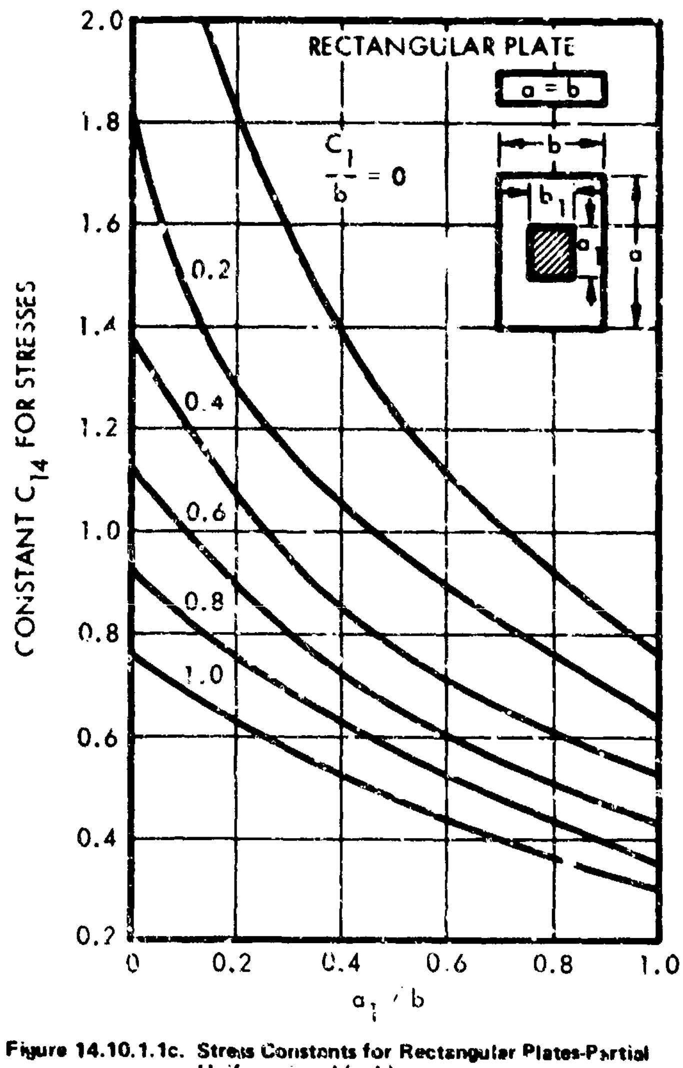


Figure 14.10.1.1c. Stress Constants for Rectangular Plates-Partial Uniform Load (a=b) *(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)*

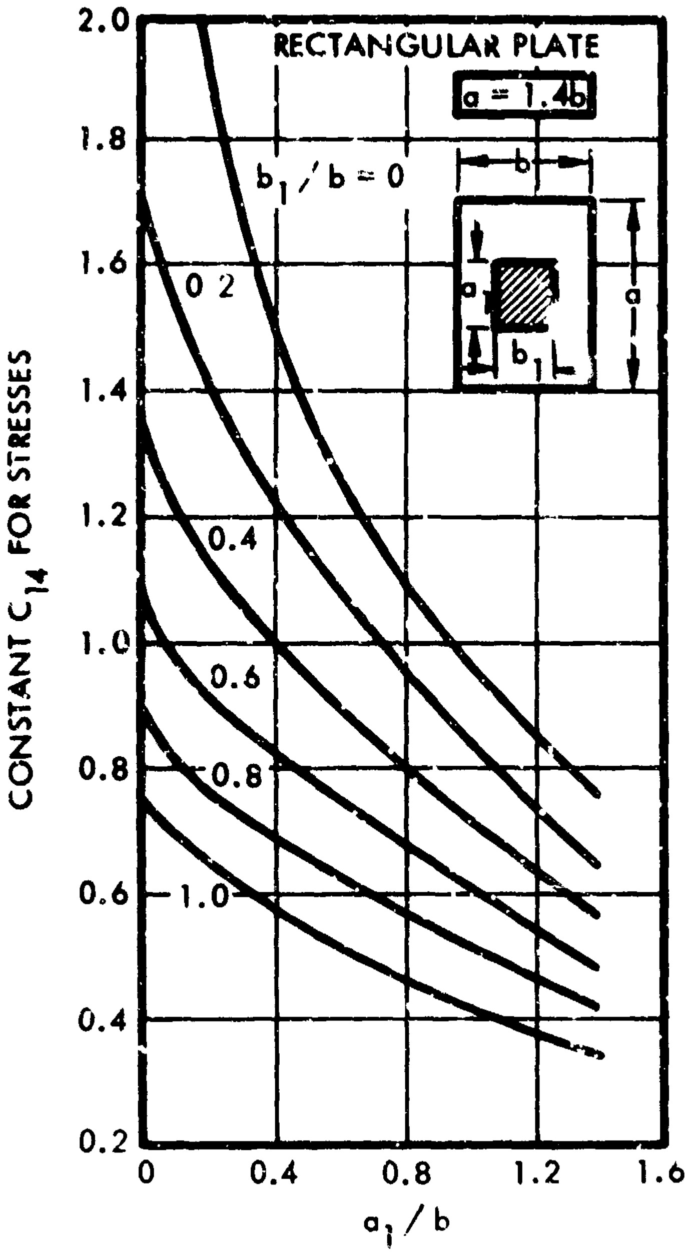


Figure 14.10.1.1d. Stress Constants for Rectangular Plates-Partial Uniform Load (a=1.4b)
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

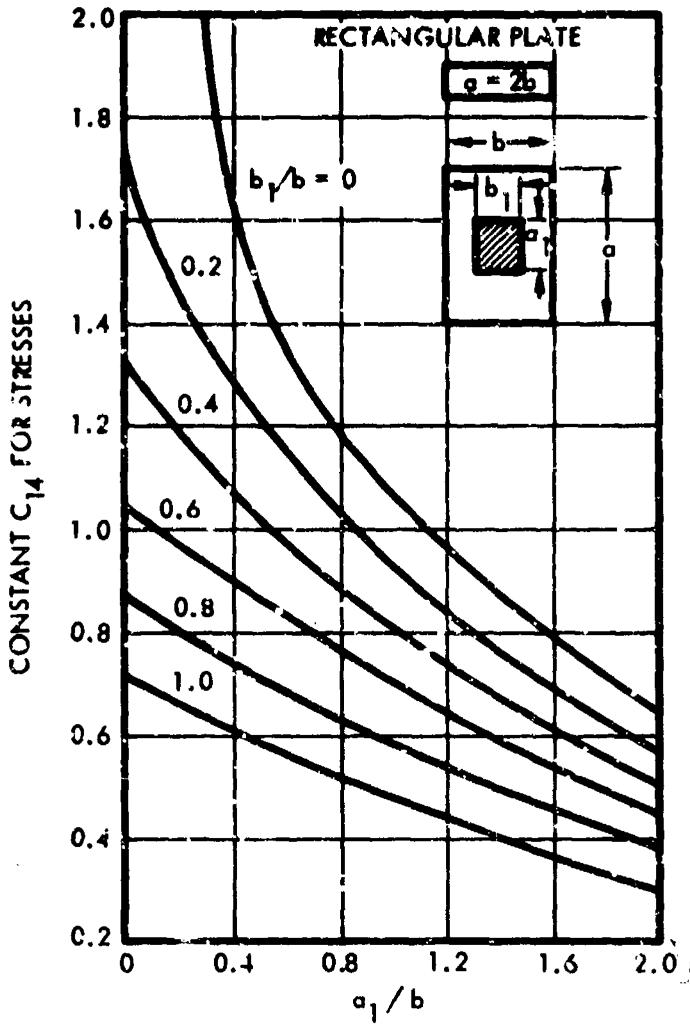


Figure 14.10.1.1e. Stress Constants for Rectangular Plates-Partial Uniform Load (a=2b)
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

14.10.1.2 STRESS AND DEFLECTION FORMULAE FOR FLAT CIRCULAR PLATES. Table 14.10.1.2 contains stress and deflection formulae for flat circular plates, utilizing the following equations:

$$f = \frac{\lambda W}{t^2} \quad \text{(Eq 14.10.1.2a)}$$

$$y = \frac{\beta W R^2}{E t^2} \quad \text{(Eq 14.10.1.2b)}$$

where:

f = maximum unit stress at surface of plate, psi

λ = stress loading-support factor for circular plates, dimensionless (see Figures 14.10.1.2a, b, c, and d)

W = total applied load, lb_f

t = plate thickness, in.

y = vertical deflection, in.

β = deflection loading-support factor for circular plates dimensionless (see Figures 14.10.1.2a, b, c, and d)

R = outside radius of plate, in.

E = modulus of elasticity, psi

Table 14.10.1.2. Stress and Deflection of Circular Plates
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Unger Pub. Company, 1966)

Case Number	Loading and Edge Conditions	Total Applied Load (W)	Deflection β	Deflection Fig.	Stress λ	Stress Fig.
1	Edge supported, uniform load over concentric circular area of radius r.	$w\pi r^2$	β_6	14.10.1.2a	λ_6	14.10.1.2c
2	Edge fixed, uniform load over concentric circular area of radius r.	$w\pi r^2$	β_2		λ_2	
3	Outer edge supported, uniform load over entire actual surface.	$w\pi(R^2 - r^2)$	β_4		λ_4	
4	Outer edge supported, loaded along inner edge.	$w\ (R^2 - r^2)$	β_7		λ_7	
5	Inner edge supported, uniform load over entire actual surface.	$w\ (R^2 - r^2)$	β_1		λ_1	
6	Outer edge fixed and supported, uniform load over entire actual surface.	$w\ (R^2 - r^2)$	β_3		λ_3	
7	Outer edge fixed and supported, loaded along inner edge.	W	β_5		λ_5	
8	Outer edge fixed and supported, inner edge fixed, uniform load over entire actual surface.	$w\ (R^2 - r^2)$	β_8	14.10.1.2b	λ_8	14.10.1.2d
9	Outer edge fixed and supported, inner edge fixed, loaded along inner edge.	W	β_{10}		λ_{10}	
10	Inner edge fixed and supported, uniform load over entire actual surface.	$w\ (R^2 - r^2)$	β_{12}		λ_{12}	
11	Inner edge fixed and supported, loaded along outer edge.	W	β_{13}		λ_{13}	
12	Outer edge supported, inner edge fixed, uniform load over entire actual surface.	$w\ (R^2 - r^2)$	β_{11}		λ_{11}	
13	Both edges fixed, balanced loading (piston).	$w\ (R^2 - r^2)$	β_9		λ_9	

Example: Find the maximum deflection and maximum stress in a circular plate simply supported along the edge and with a uniformly distributed load w = 3 psi over the entire surface of the plate. The dimensions of the plate and Young's modulus are: R = 10 inches, t = 0.2 inch, E = 30 x 10^6 psi.

Solution: The loading edge conditions correspond to those of Table 14.10.1.2, case 1, where $\beta = \beta_6$, $\lambda = \lambda_6$, $W = w\pi R^2 = 942$ pounds. Since R/r = 1 (load uniformly distributed over entire surface of the plate) the curves of Figures 14.10.1.2a and c show that $\beta_6 = 0.210$ and $\lambda_6 = 0.40$. Substituting these numbers into Equations (14.10.1.2a) and (14.10.1.2b), the maximum stress and deflection are:

$$f = \frac{0.40 \times 942}{(0.2)^2} = 9{,}400 \text{ psi}$$

$$y = \frac{0.21 \times 942 \times 10^2}{30 \times 10^6 \times (0.2)^3} = 0.082 \text{ in.}$$

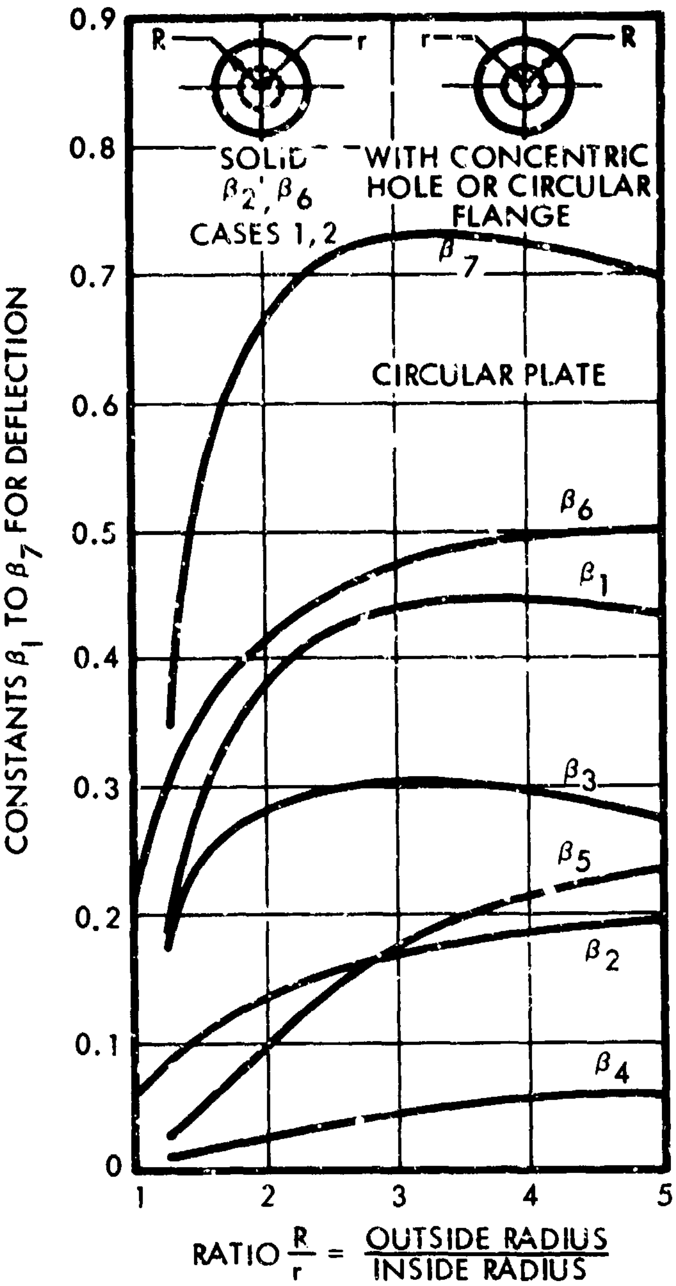


Figure 14.10.1.2a. Deflection Constants for Circular Plates Cases 1 to 7
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

14.10.2 Slope for Circular Plates

The slope for circular plates can be easily determined by using the load support factors given in Tables 14.10.2a, b, c and d in conjunction with the following equations:

$$\theta = \frac{CWa}{Et^3} \qquad \textbf{(Eq 14.10.2a)}$$

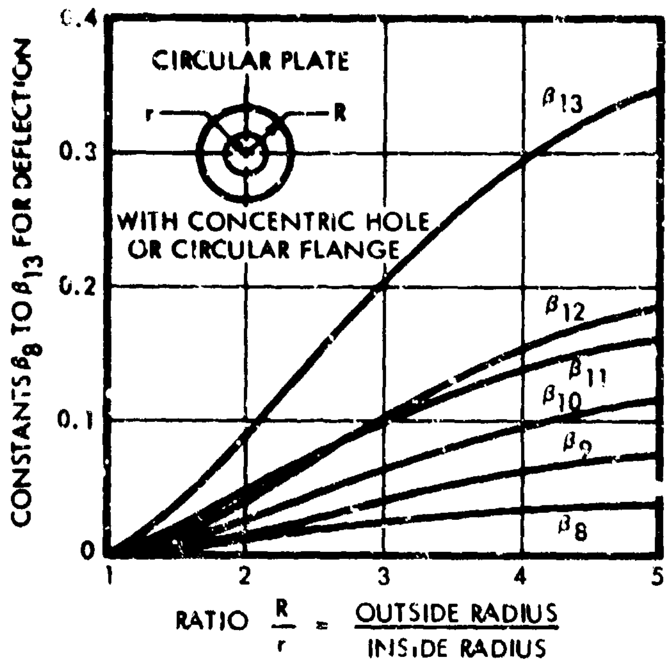


Figure 14.10.1.2b. Deflection Constants for Circular Plates Cases 8 to 13
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

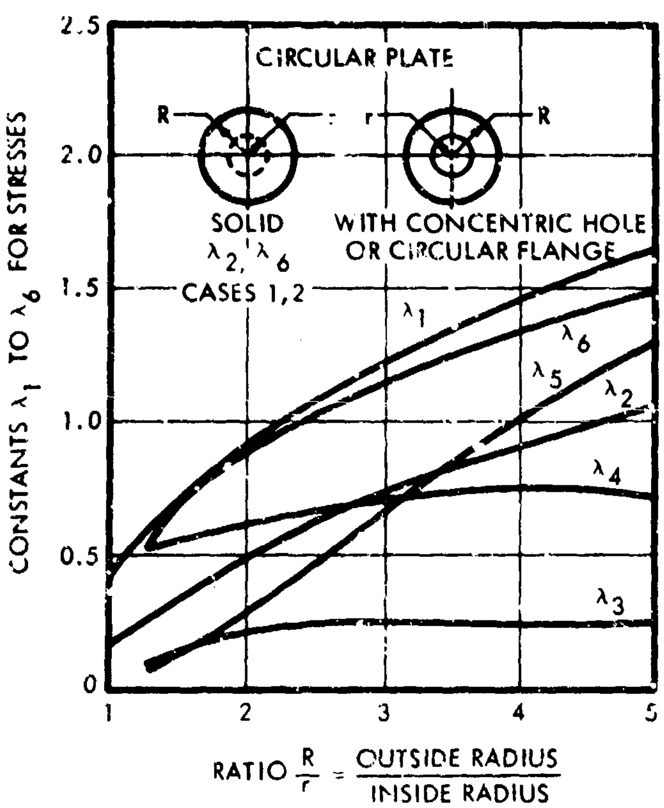


Figure 14.10.1.2c. Stress Constants for Circular Plates Cases 1 to 6
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

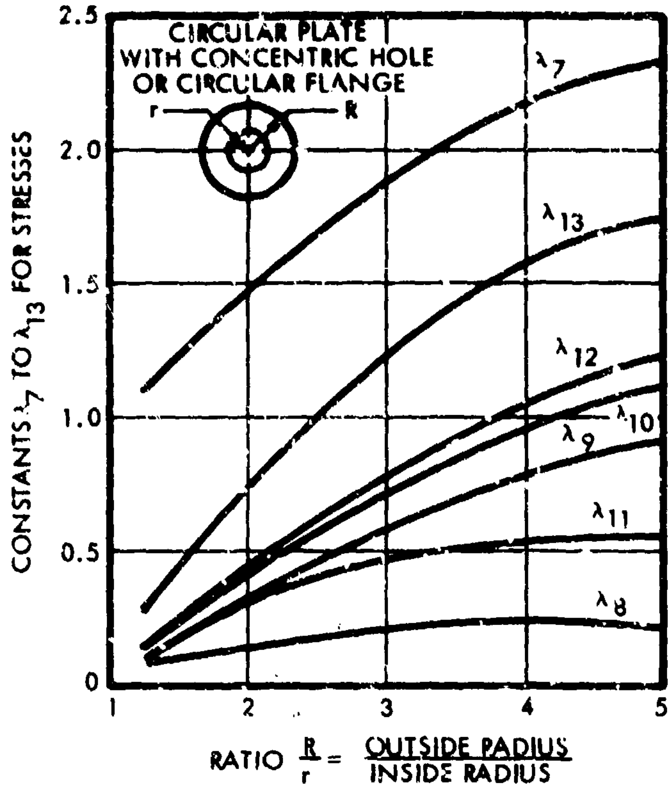


Figure 14.10.1.2d. Stress Constants for Circular Plates Cases 7 to 13
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

$$\theta = \frac{C_1 Ma}{Et^3} \qquad \text{(Eq 14.10.2b)}$$

where

θ = slope of plate measured from horizontal, radians

C = loading-support factor (Table 14.10.2a and b)

C_1 = loading-support factor (Table 14.10.2c and d)

W = total applied load, lb_f

a = outside radius of plate, in.

E = modulus of elasticity, psi

t = plate thickness, in.

M = end moment, in-lb

By superposition the load factors may be made to cover a wide variety of loadings not specifically considered. Equation (14.10.2a) solves for slopes of plates loaded with a total load, W, and Equation (14.10.2b) solves for slopes of plates with end moments. In cases where a plate is loaded with a uniformly distributed load w (psi), compute the total load as $W = w\pi a^2$ or $W = w\pi(a^2 - b^2)$. The C and C_1 factors in the tables are based on a Poisson's ratio of 0.3, a

Table 14.10.2a. Load Support Factors for Circular Plates (Cases 1 to 8)
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Company, 1966)

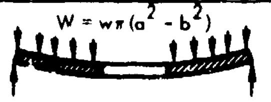
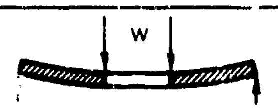
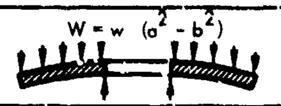
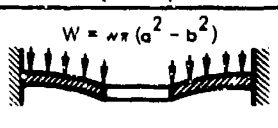
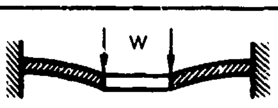
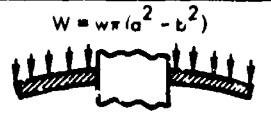
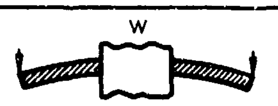
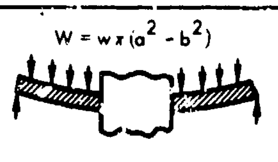
		C (for equation 14.10.2a)						
Manner of loading and case number		a/b = 1.25	1.5	2	3	4	5	Location of slope
1. Outer edge supported. Uniform load over entire actual surface.		0.786 0.919	0.679 0.731	0.640 0.617	0.441 0.447	0.401 0.355	0.379 0.290	At outer edge At inner edge
2. Outer edge supported. Uniform load along inner edge.		1.646 1.758	1.470 1.650	1.237 1.475	1.006 1.238	0.895 1.082	0.832 0.932	At outer edge At inner edge
3. Inner edge supported. Uniform load over entire actual surface.		0.884 1.017	0.794 0.910	0.606 0.862	0.565 0.791	0.492 0.726	0.448 0.642	At outer edge At inner edge
4. Outer edge fixed and supported. Uniform load over entire actual surface.		0.013	0.035	0.069	0.096	0.096	0.089	At inner edge
5. Outer edge fixed and supported. Uniform load along inner edge.		0.045	0.115	0.269	0.448	0.510	0.522	At inner edge
6. Inner edge fixed and supported. Uniform load over entire actual surface.		0.012	0.040	0.097	0.177	0.223	0.252	At outer edge
7. Inner edge fixed and supported. Uniform load along outer edge.		0.046	0.105	0.239	0.403	0.499	0.544	At outer edge
8. Outer edge supported, inner edge fixed. Uniform load over entire actual surface.		0.091	0.123	0.197	0.260	0.297	0.306	At outer edge

Table 14.10.2b. Load Support Factors for Circular Plates (Cases 9 and 10)
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Company, 1966)

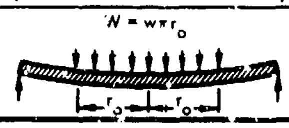
Manner of loading and case number		C (for equation 14.10.2a)	Note
9. Edges supported. Uniform load over entire surface		0.318	For cases 1–10: only the total load "W" should be used (See example I)
10. Edges supported Uniform load over concentric circular area of radius r_o.		0.636	

Table 14.10.2c. Load Support Factors for Circular Plates (Cases 11 to 14)
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub Company, 1966)

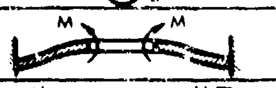
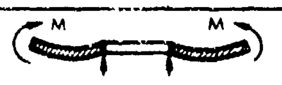
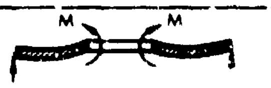
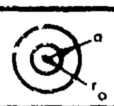
		C_1 (for equation 14.10.2b)						
Manner of loading and case number		a/b = 1.25	1.50	2	3	4	5	Location of slope
11. Outer edge fixed. Uniform moment along inner edge.		2.20	3.16	3.86	3.31	3.12	2.72	At inner edge
12. Inner edge fixed. Uniform moment along outer edge.		2.30	3.84	5.67	6.94	7.32	3.17	At outer edge
13. Inner edge supported. Uniform moment along outer edge.		51.00	23.00	16.40	11.60	10.23	9.61	At outer edge
		53.30	27.80	15.60	[illegible].78	6.24	4.86	At inner edge
14. Outer edge supported. Uniform moment along inner edge.		42.70	18.75	7.81	2.93	1.56	0.94	At outer edge
		44.90	22	11.27	5.52	4.08	3.17	At inner edge

Table 14.10.2d. Load Support Factors for Circular Plates (Cases 15 to 17)
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Pub. Company, 1966)

		C_1 (for equation 14.10.2b)																
Manner of loading and case number		r_o/a = 0	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	Location of slope
15. No supports. Uniform edge moment.		8.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	At edge
16. Edges supported. Central couple (trunnion loading)		—	0.71	0.92	1.22	1.60	2.00	2.50	3.53	5.60	8.54	12.00	16.30	24.00	41.40	82.00	162.00	At center
17. Edges fixed. Central couple (trunnion loading)		—	0.87	1.23	1.68	2.31	3.10	4.00	5.45	8.20	12.40	18.00	28.50	44.00	77.90	156.00	314.00	At center

value generally used for steel and aluminum. For other values, however, there will be little change in slope. The assumptions listed in Detailed Topic 14.10.4.1 apply here also.

Example 1: A circular, aluminum plate 20 inches in diameter and 0.12-inch thick has a concentric hole 4 inches in diameter. The plate is fixed and supported along the outer edge and loaded with a uniformly distributed load of 3 psi over the entire surface. Determine the slope.

Solution: The plate parameters are a = 10 inches, b = 2 inches, t = 0.12 inch, E = 10×10^6 psi w = 3 psi and a/b = 10/2 = 5. From Table 14.10.2a, case 4, C = 0.089. The total load on the plate is $W = w\pi(a^2 - b^2) = 904$ pounds. Incorporating these values into Equation (14.10.2a):

$$\theta = \frac{(0.089)\,(904)\,(10)}{(10 \times 10^6)\,(0.12)^3} = 0.05 \text{ radians}$$

Example 2: A circular, solid steel plate 20 inches in diameter and 0.20-inch thick has no supports. A uniform edge moment of 12.45 in-lb per inch is applied. Find the slope of the plate.

Solution: The plate parameters are a = 10 inches, t = 0.20 inch, E = 30×10^6 psi and M = 12.45 in-lb per inch. This plate corresponds to case 15 in Table 14.10.2d for which the load support factor is 8. Using Equation (14.10.2b):

$$\theta = \frac{(8)\,(12.45)\,(10)}{(30 \times 10^6)\,(0.20)^3} = 0.004 \text{ radians}$$

14.10.3 Large Deflections of Circular Plates Without Holes (Reference 618-1)

When the deflection becomes larger than about half the thickness, the stresses of the middle surface cannot be ignored. These stresses enable the plate to carry part of the load as a diaphragm in direct tension. Under such conditions, the plate is stiffer than indicated by ordinary theory. Stresses for a given load are less, and stresses for a given deflection are generally greater than the ordinary theory indicates.

Consider a circular plate whose edge is clamped so that rotation and radial displacement are prevented at the edge. The plate is uniformly loaded to the extent that the maximum deflection is large relative to the thickness of the plate. The radial membrane stress at the edge is due to the tensile forces which must be applied radially to prevent edge displacement. The deflection is not a linear function of the load.

The deflection of the circular plate can be determined from Figure 14.10.3a. The stresses at the edge and center of the plate can be obtained from Figure 14.10.3b. It will be noted that the dimensionless ordinates and abscissas in Figures 14.10.3a and b make it possible to use the curves for plates of many dimensions provided that other conditions are the same.

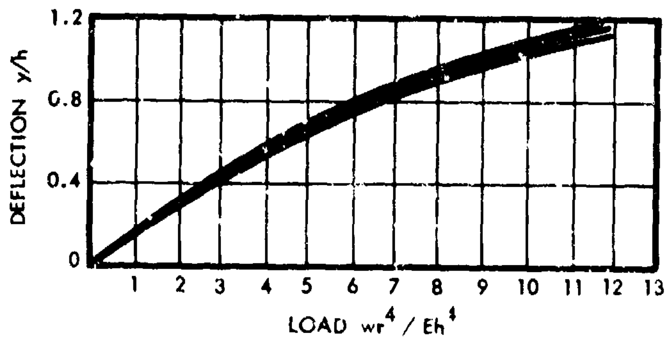


Figure 14.10.3a. Large Deflection of Circular Plate with Edge Clamped
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

It can be seen from Figure 14.10.3a that variations in Poisson's ratio have very little effect on the behavior of plates.

Example: Given a plate of thickness t = 0.02 inch, radius r = 2 inches, and load w = 3 psi. Let E = 30 x 10^6 psi and let μ = 0.3. Determine the deflection of the plate and find the bending and membrane stresses at the edge and at the center of the plate.

Solution: From Figure 14.10.3a for wr^4/Et^4 = 10 and μ = 0.3 we obtain y/t = 1.055; y = (1.055) (0.02) = 0.0211 inch. The stresses are determined from Figure 14.10.3b as follows:

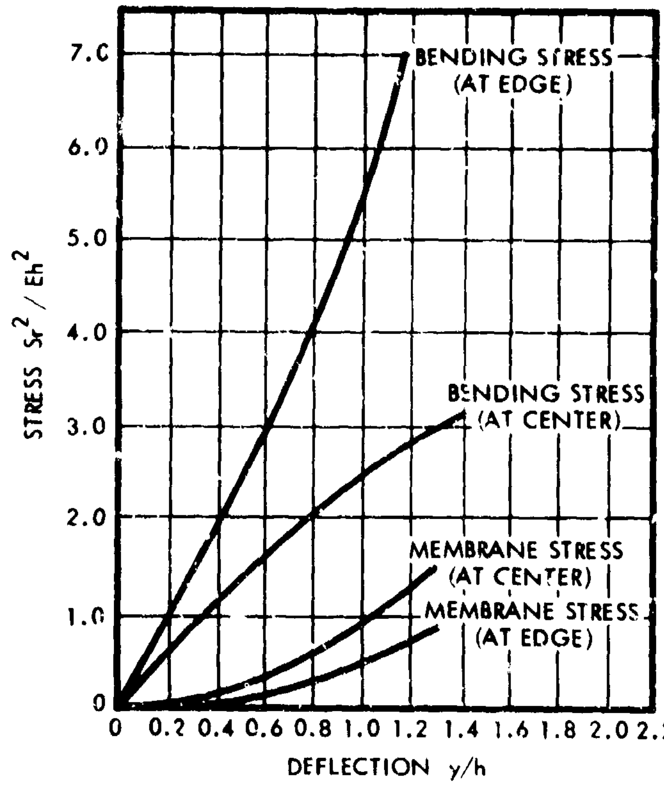


Figure 14.10.3b. Edge/Center Stresses in Circular Plate with Edge Clamped
(Adapted with permission from Reference 618-1, "Handbook of Formulas for Stress and Strain," W. Griffel, Frederick Ungar Publishing Company, 1966)

Bending stresses:

$$\text{Edge: } \frac{fr^2}{Et^2} = 5.85;\ f = 17{,}550 \text{ psi}$$

$$\text{Center: } \frac{fr^2}{Et^2} = 2.57;\ f = 7{,}710 \text{ psi}$$

Membrane stresses:

$$\text{Edge: } \frac{fr^2}{Et^2} = 0.56;\ f = 1{,}680 \text{ psi}$$

$$\text{Center: } \frac{fr^2}{Et^2} = 1.07;\ f = 3{,}210 \text{ psi}$$

Faupel (Reference 598-1) presents deflection load curves similar to Figure 14.10.3a for simply-supported and vertically-restrained circular plates as well as fixed-edge circular plates. These are reproduced here, by permission, as Figures 14.10.3c, d, and e. Note the similarity between Figures 14.10.3a and e.

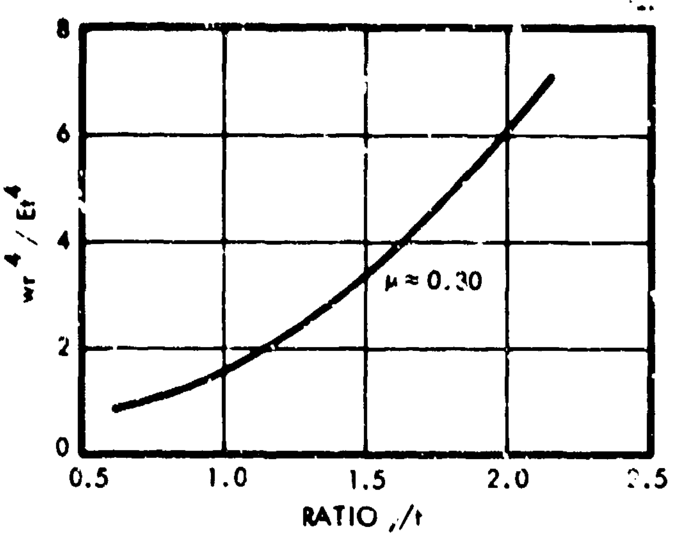


Figure 14.10.3a. Large Elastic Deflections of Simply Supported Circular Plate
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley Company, 1964)

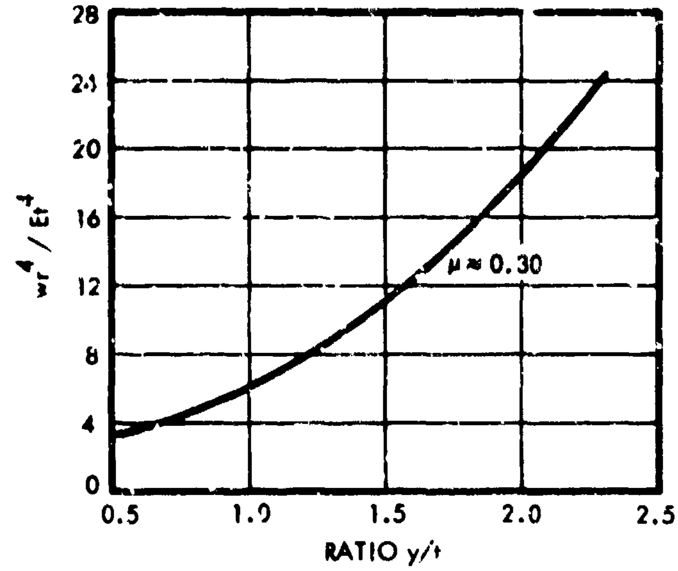


Figure 14.10.3e. Large Elastic Deflections of Circular Plate with Fixed Edge
(Adapted with permission from Reference 550-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

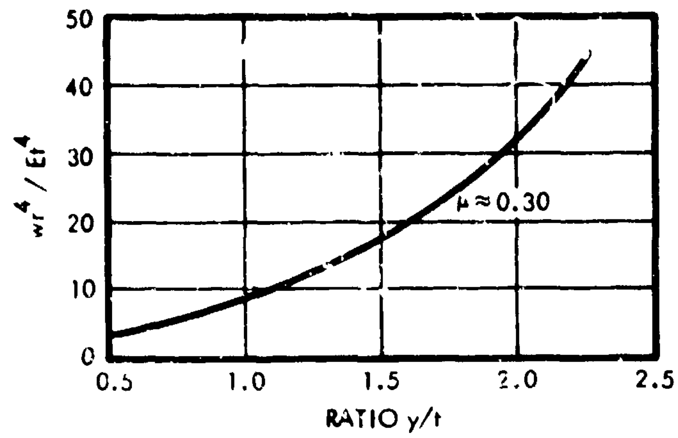


Figure 14.10.3d. Large Elastic Deflections of Circular Plate with Edge Restrained in Vertical Plane
(Adapted with permission from Reference 598-1, "Engineering Design," J. H. Faupel, Wiley Company, 1964)

14.11 FLEXURES

Reference 650-1 contains an excellent treatment of flexure plates by A. G. Thorpe II of Westinghouse Electric Corporation. Much of the following discussion of force deflection relationships is adapted from this reference. Design data on a variety of simple and complex flexures may be found in Reference 1411.

An arrangement that permits a structure to move freely in one direction is shown schematically in Figure 14.11a. In this design, the pivots are loaded in compression. The upper member is free to move in the horizontal direction through a limited distance dependent upon stress conditions in the pivots. To hold the structure in the deflected position, a transverse force 2Q is required. The magnitude and direction of the force 2Q depends upon the load 2P and the stiffness of the flexure pivots under column loading.

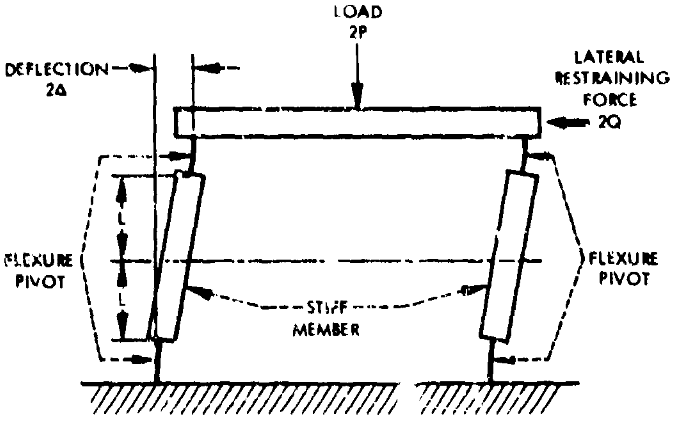


Figure 14.11a. Support in which Pivots and Stiff Members are Loaded Axially in Compression
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

When the pivots are relatively flexible, the restraining force 2Q is in the direction indicated in Figure 14.11a. When the pivots are relatively stiff, this force is opposite to the direction shown. With proper design of the pivots, this lateral force may be made to vanish, thus creating a condition of neutral stability or zero restoring force. This condition, however, can be achieved only under a constant vertical force which may not always be present in practice.

A suspension in which flexure pivots act in tension rather than compression is shown in Figure 14.11b. This suspension also may be designed to have zero restoring force or neutral stability under a constant vertical force. Since the pivots are in tension, they do not buckle as pivots loaded in compression may under loads exceeding the design condition. (For buckling or instability criteria, see Detailed Topic 14.2.1.2.)

The general configuration of the arrangement shown in Figure 14.11b sometimes lends itself to space requirements that cannot be satisfied by the configuration of Figure 14.11a.

Where the variation in vertical loading is large and it is desirable to approach as nearly as possible the condition of neutral stability, the arrangement shown in Figure 14.11c may be used. With flexure pivots of negligible spring force and for small deflections, the restoring tendency of the upper member will balance the overturning tendency of the lower member and the structure will move in a plane.

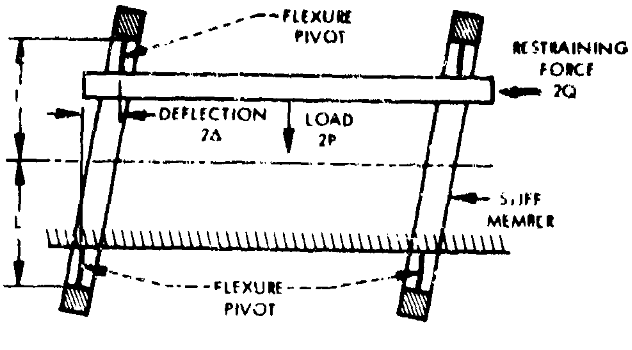


Figure 14.11b. Suspension in which Flexure Pivots are Loaded Axially in Tension and the Stiff Members in Compression
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

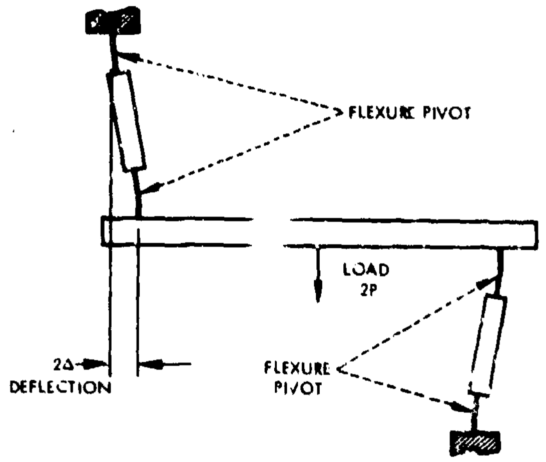


Figure 14.11c. Flexure Pivot with One Tension Member and One Compression Member
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

The configurations shown in Figures 14.11a and b are adapted to the support of large heavy structures and machinery. By providing "universal" flexure pivots, the structure can be made free to move in any direction in a plane. Other universal arrangements may be employed where necessary to limit rotation to acting about a point. Where the loading is light, a rod or wire may be substituted for the plates to provide the universal feature.

The manner in which flexure pivots can be used in the design of balances and control linkages to transmit a force through a bell crank are shown in Figure 14.11d. The pivot fastened to the foundation consists of two sets of plates: one for transmitting the vertical component of force and the other for transmitting the horizontal component.

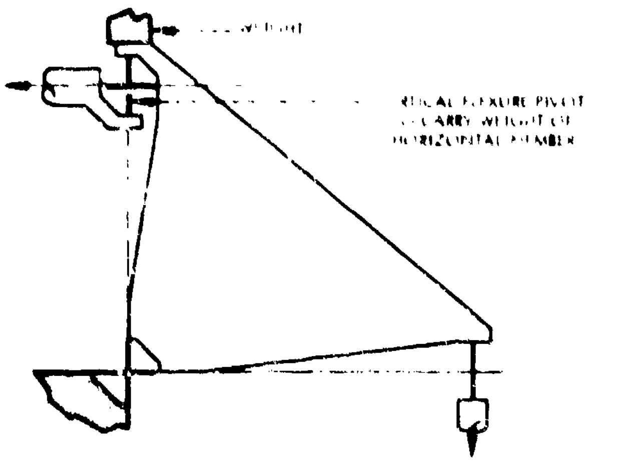


Figure 14.11d. Flexure Pivot Arrangement for Transmitting Force Through a Bell Crank
(Adapted with permission from Reference 650-1, "Spring Design and Application", N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

Where small spring forces may be detrimental to proper performance, as in very sensitive balance systems, a weight may be attached to the top of the bell crank to act as an inverted pendulum that counterbalances the spring restoring forces of the flexure pivots.

Another less common arrangement is shown in Figure 14.11e. This type of flexure pivot construction may be dictated by space requirements or by requirements imposed by a movable wall (as in a duct). The center of rotation of this pivot lies outside of the upper boundary of the structure. Such *cross spring* pivots have been used to advantage in instrument applications, and wind tunnel balances.

For small deflections, the analysis of the flexure pivot is the same as that for simple beam columns as covered by textbooks in advanced strength of materials, Reference 333-2. The assumptions made are linear, and the stiff member connected to the flexure pivot has infinite bending stiffness, thus all bending is required to take place in the flexure pivot.

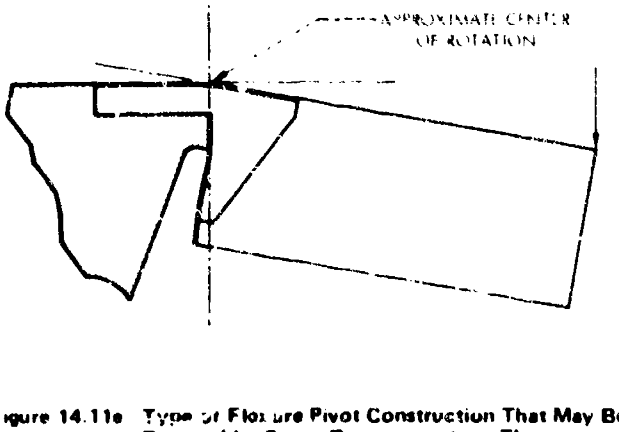


Figure 14.11e Type of Flexure Pivot Construction That May Be Dictated by Space Requirements or Those Imposed By a Movable Wall
(Adapted with permission from Reference 650-1, "Spring Design and Application" N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

Where axial forces are imposed on the flexure pivot, the maximum stress in the flexure pivot is

$$f_{max} = \frac{P}{A} + \frac{M_{max}}{Z} \qquad \text{(Eq 14.11)}$$

where

f_{max} = maximum tensile or compressive stress in flexure, psi

P = axial force, lb_f

A = cross-sectional area of flexure, in^2

M_{max} = maximum bending moment, in-lb

Z = section modulus of flexure, in^3

In design studies, it is important that the stress be checked at the point of maximum moment. Since a flexure pivot acting in compression can buckle as a column if the loads are high enough, the designer should be careful to allow an ample margin of safety. (See Detailed Topic 14.2.1.2 for buckling instability criteria.)

If the flexure pivot is to be subjected to variable bending, it is important that the fatigue strength of the material be taken into account. The techniques discussed in Sub-Section 14.5, such as the modified Goodman diagram, should be used to account for the effect of the steady stress upon the fatigue limit. Consideration should also be given to the effect of stress concentration at the ends of the flexure pivot with the provision that ample fillets are to be employed.

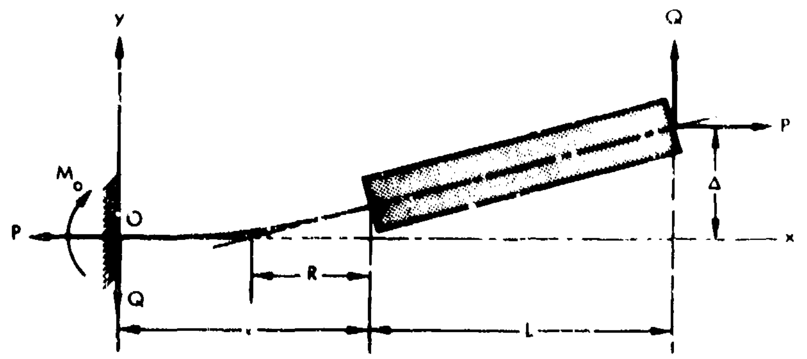
Fillets at the ends of flexure pivots alter somewhat the effective length, ℓ. For fillet radii approximately equal to the thickness of the flexure pivot, a good rule-of-thumb is to consider the effective length to be the distance between fillet centers plus the fillet radius at one end. For simplicity, it is generally preferable to use identical flexure pivots at each end of the stiff member, thus making L equal half the length of the stiff member.

Formulae and equations are given in Table 14.11 for the solution of the most commonly used cases. To simplify numerical work and to aid in visualizing the effects of the different variables, the curves shown in Figures 14.11f through 14.11n have been constructed to cover a range sufficient for most problems. The design formulae given can be used to obtain increased accuracy or for problems beyond the range of the curves.

Note that in the formulae in which the coefficient k appears k varies as the square root of P; therefore, these formulae are not linear with respect to the axial force P. This nonlinear relation is typical for all beam column problems and complicates the analysis when the axial load is variable, in which event care must be exercised to make sure that the most severe condition is determined.

The most common form of flexure pivot is a plate of rectangular cross section. When the plate width is greater than three times the plate length, the flexural rigidity relation $E/(1-\mu^2)$ should be used instead of E. For intermediate width-to-length ratios, if additional accuracy of calculation is necessary, reference can be made to literature, such as "The Anticlastic Curvature of Rectangular Beams and Plates", D. G. Ashwell, Journal of the Royal Aeronautical Society, November 1950, pp. 708-715.

CASE



$k = \sqrt{\frac{P}{EI}}$

$\frac{dy}{dx} = \Delta\left\{C_1 k \sinh kx - \frac{1+C_1}{L+\ell}\cosh kx + \frac{1+C_1}{L+\ell}\right\}$

$C_1 = \frac{M_o}{P\Delta} = \frac{k\ell \coth k\ell + \frac{\ell}{L}}{(k\ell)^2\left(1+\frac{1}{\ell/L}\right) + \frac{\ell}{L}(k\ell \coth k\ell - 1)}$

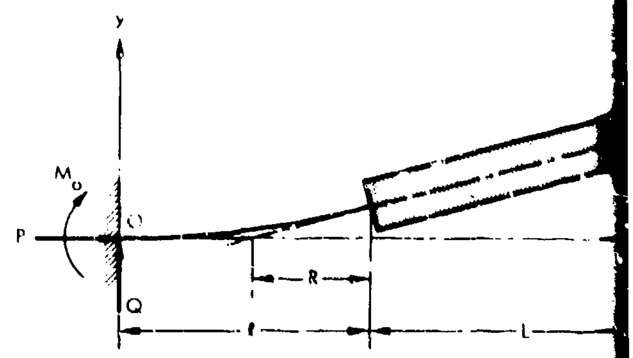
$R = \frac{1}{C_1 k \sinh k\ell - \frac{1+C_1}{L+\ell}(\cosh k\ell - 1)} - L$

$Q = P\Delta \frac{1+C_1}{L+\ell}$

M_o = Maximum moment

$y = \Delta\left\{C_1 \cosh kx - \frac{1+C_1}{k(L+\ell)}\sinh kx + \frac{1+C_1}{L+\ell}x - C_1\right\}$

CASE 2



$k = \sqrt{\frac{P}{EI}}$

$\frac{dy}{dx} = \Delta\left\{C_2 \right.$

$C_2 = \frac{M_o}{P\Delta} = \frac{k\ell \cot k\ell + \frac{\ell}{L}}{(k\ell)^2\left(1+\frac{1}{\ell/L}\right) + \frac{\ell}{L}(1 - k\ell \cot k\ell)}$

$R =$

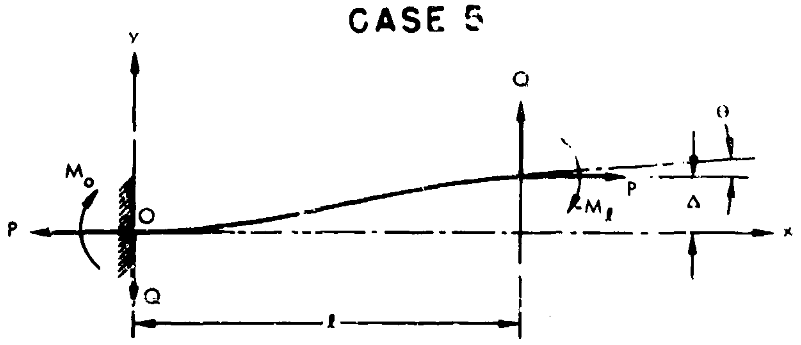
$Q = P\Delta \frac{1-C_2}{L+\ell}$

$\left(\frac{x}{\ell}\right)_{M_{max}} =$

$y = \Delta\left\{-\frac{1-C_2}{k(L+\ell)}\sin kx - C_2 \cos kx + \frac{1-C_2}{L+\ell}x + C_2\right\}$

$\frac{M_{max}}{P\Delta} =$

CASE 5



$k = \sqrt{\frac{P}{EI}}$

$\frac{dy}{dx} = \frac{1}{P}\left\{-Q\cosh kx + M_o k \sinh kx + Q\right\}$

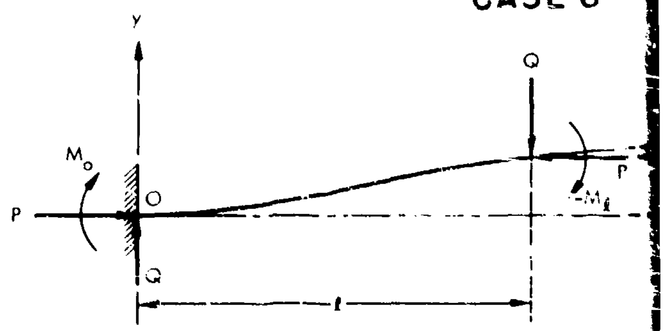
$M_o = P\left\{\frac{\theta}{k \sinh k\ell} + \frac{1}{k}\left(\tanh\frac{k\ell}{2}\right)\left(\frac{\theta \tanh\frac{k\ell}{2} - k\Delta}{2\tanh\frac{k\ell}{2} - k\ell}\right)\right\}$ $M_x = Py - Qx + M_o$

$Q = P\left\{\frac{\theta \tanh\frac{k\ell}{2} - k\Delta}{2\tanh\frac{k\ell}{2} - k\ell}\right\}$

$M_\ell = P\Delta - Q\ell + M_o$

$y = \frac{1}{P}\left\{-\frac{Q}{k}\sinh kx + M_o \cosh kx + Qx - M_o\right\}$

CASE 6



$k = \sqrt{\frac{P}{EI}}$

$\frac{dy}{dx} = \frac{1}{P}\left\{ \right.$

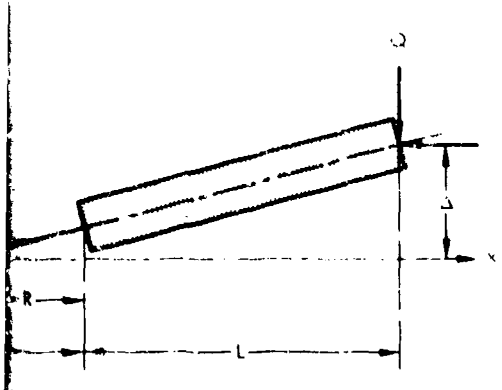
$M_o = P\left\{\frac{\theta}{k \sin k\ell} - \frac{1}{k}\left(\tan\frac{k\ell}{2}\right)\left(\frac{\theta \tan\frac{k\ell}{2} - k\Delta}{2\tan\frac{k\ell}{2} - k\ell}\right)\right\}$ $M_x = -Py +$

$Q = P\left(\frac{\theta \tan\frac{k\ell}{2} - k\Delta}{2\tan\frac{k\ell}{2} - k\ell}\right)$

$M_\ell = P\Delta$

$y = \frac{1}{P}\left\{-\frac{Q}{k}\sin kx - M_o \cos kx + Qx + M_o\right\}$

CASE 2



$$\frac{dy}{dx} = \Delta \left\{ C_2 k \sin kx - \frac{1-C_2}{L+\ell} \cos kx + \frac{1-C_2}{L+\ell} \right\}$$

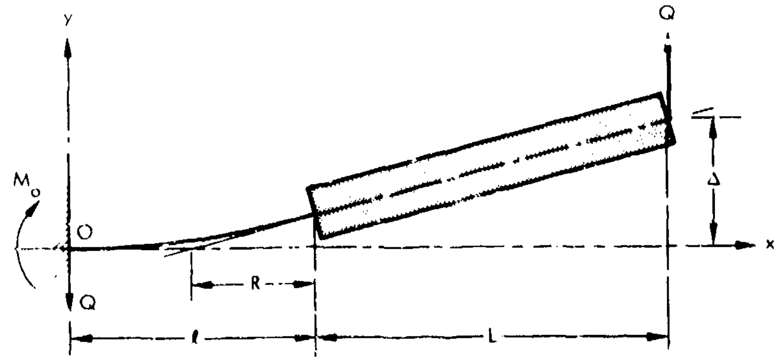
$(1-k\ell \coth \ell)$

$$R = \frac{1}{\frac{1-C_2}{L+\ell}(1-\cos k\ell) + C_2 k \sin k\ell} - L$$

$$\left(\frac{x}{\ell}\right)_{M_{max}} = \frac{1}{k\ell} \tan^{-1} \frac{1-C_2}{C_2 k (L+\ell)}$$

$\ldots kx + \frac{1-C_2}{L+\ell} x + C_2 \}$ $\quad \frac{M_{max}}{P\Delta} = \frac{\sqrt{\ldots \ell^2 + (\frac{\ell}{L})^2}}{\sin k\ell \{ (k\ell)^2 (1+\frac{\ell}{L}) + \frac{\ell}{L} \ldots k\ell \cot k\ell) \}}$

CASE 3

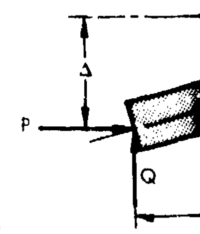


$M_o = Q(L+\ell)$ $\qquad R = \frac{L\ell + \frac{2}{3}\ell^2}{\ell + 2L}$

$$Q = \frac{EI\Delta}{\ell(L^2 + L\ell + \frac{\ell^2}{3})}$$

$$y = \frac{Q}{EI}\left(\frac{L+\ell}{2}x^2 - \frac{x^3}{6}\right)$$

$$\frac{dy}{dy} = \frac{Q}{EI}\left\{(L+\ell)x - \frac{x^2}{2}\right\}$$



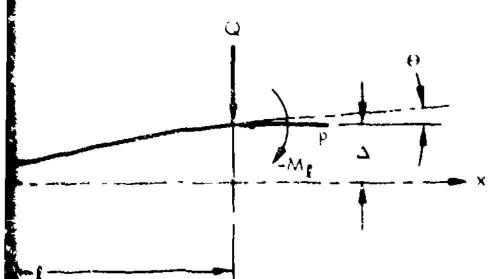
$k = \sqrt{\frac{P}{EI}}$

$C_4 = \frac{M_o}{P\Delta} = \frac{k\ell \ldots}{(k\ell)^2 (\frac{1}{\ldots} \ldots}$

$Q = P\Delta \frac{1-C_4}{L-\ell}$

$y = \Delta \{ C_4 \cosh kx - \ldots$

CASE 6



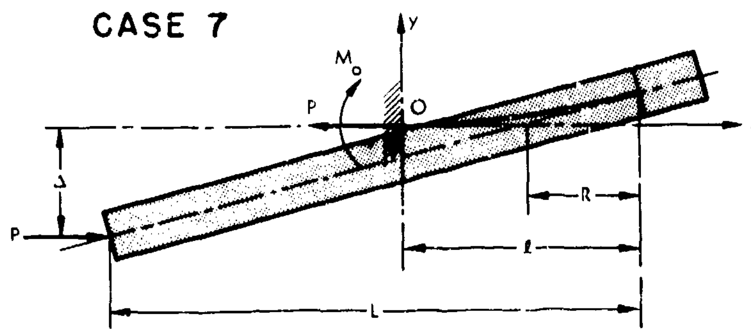
$$\frac{dy}{dx} = \frac{1}{P}\left\{ Q \cos kx + M_o k \sin kx + Q \right\}$$

$\ldots \ell)\left(\frac{\theta \tan \frac{k\ell}{2} - k\Delta}{2\tan\frac{k\ell}{2} - k\ell}\right)\Big)$ $\qquad M_x = -Py + Qx + M_o$

$$M_\ell = -P\Delta + Q\ell + M_o$$

$\ldots x + Qx + M_o \}$

CASE 7

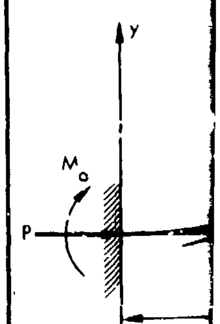


$k = \sqrt{\frac{P}{EI}}$ $\qquad M_{max} = M_\ell = P\Delta \cosh k\ell$

$Q = 0$ (Neutral stability) $\qquad R = L - \frac{1}{k \sinh k\ell}$

Proportions dictated by: $\frac{\ell}{L} = k\ell \tanh k\ell$

$$C_7 = \frac{M_o}{P\Delta} = 1$$



$k = \sqrt{\frac{P}{EI}}$

$Q = 0$ (N…

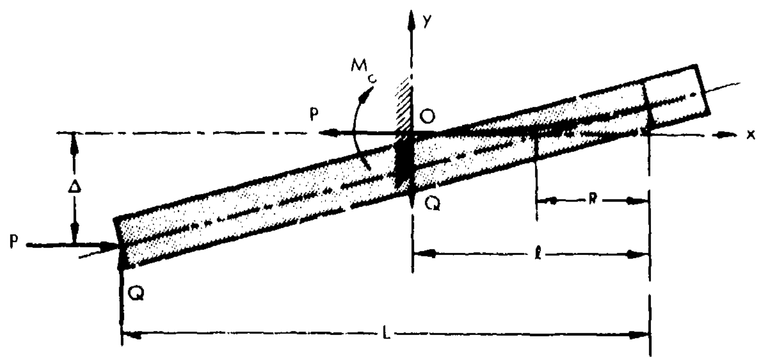
Proporti…

$C_8 = \frac{M\ldots}{P\ldots}$

B

Table 14.11. Flexure Pivot Equations
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis (ed.), McGraw-Hill Book Company, Inc., 1961)

CASE 4



$$k = \sqrt{\frac{P}{EI}}$$

$$\frac{dy}{dx} = \Delta\left\{C_4 k \sinh kx - \frac{1-C_4}{L-\ell}\cosh kx + \frac{1-C_4}{L-\ell}\right\}$$

$$C_4 = \frac{M_o}{P\Delta} = \frac{k\ell \coth k\ell - \frac{\ell}{L}}{(k\ell)^2\left(\frac{1}{\ell/L}-1\right) + \frac{\ell}{L}(k\ell \coth k\ell - 1)}$$

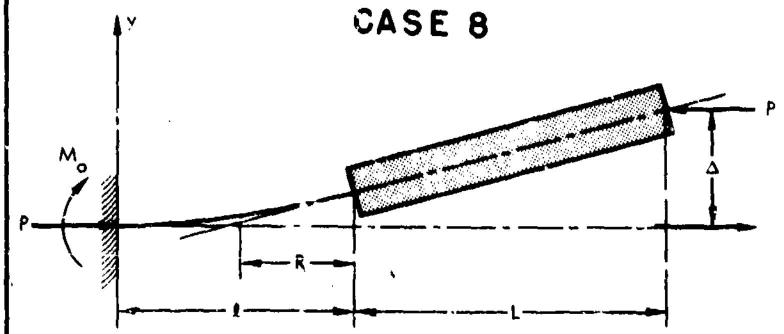
$$R = L - \frac{1}{C_4 k \sinh k\ell - \frac{1-C_4}{L-\ell}(\cosh k\ell - 1)}$$

$$Q = P\Delta\,\frac{1-C_4}{L-\ell}$$

$$\frac{M_\ell}{P\Delta} = \frac{k\ell}{\sinh k\ell\left[(k\ell)^2\left(\frac{1}{\ell/L}-1\right) + \frac{\ell}{L}(k\ell \coth k\ell - 1)\right]}$$

$$y = \Delta\left\{C_4 \cosh kx - \frac{1-C_4}{k(L-\ell)}\sinh kx + \frac{1-C_4}{L-\ell}x - C_4\right\}$$

CASE 8



$$k = \sqrt{\frac{P}{EI}}$$

$$R = \frac{1}{k \sin k\ell} - L$$

$Q = 0$ (Neutral stability)

Proportion dictated by: $\frac{\ell}{L} = k\ell \tan k\ell$

$$C_8 = \frac{M_o}{P\Delta} = 1 = \frac{M_{max}}{P\Delta}$$

C_n = moment constants
E = modulus of elasticity, psi
I = principal moment of inertia of flexure pivot cross section with respect to axis of bending, in.^{4}
k = $\sqrt{P/EI}$, in.$^{-1}$
l = length of flexure pivot, in.
L = length of stiff member, in.
M = bending moment, in. lb
P = axial force, tension or compression, lb
Q = lateral force, lb
R = distance locating center of rotation of flexure pivot, in.
x = distance along longitudinal axis, in.
y = distance along transverse axis, in.
θ = angle of rotation of stiff member, radians
Δ = deflection at end of stiff member, in.
μ = Poisson's ratio
f = tensile or compressive stress in pivot, psi
A = cross sectional area of flexure pivot, in.^{2}
Z = section modulus of flexure pivot, in.^{3}

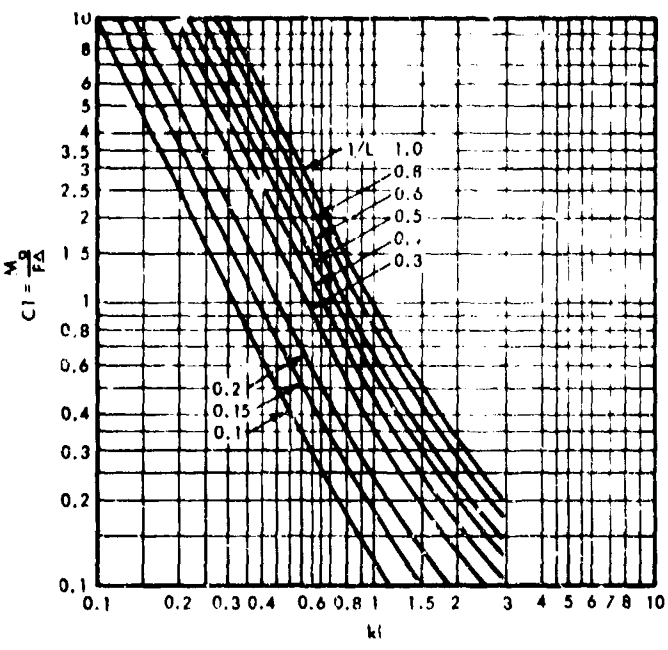


Figure 14.11f. Chart for Obtaining Value of End Moment M_o, Case 1 When Pivot and Stiff Member are Both in Axial Tension and a Lateral Restraining Force is Provided

(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

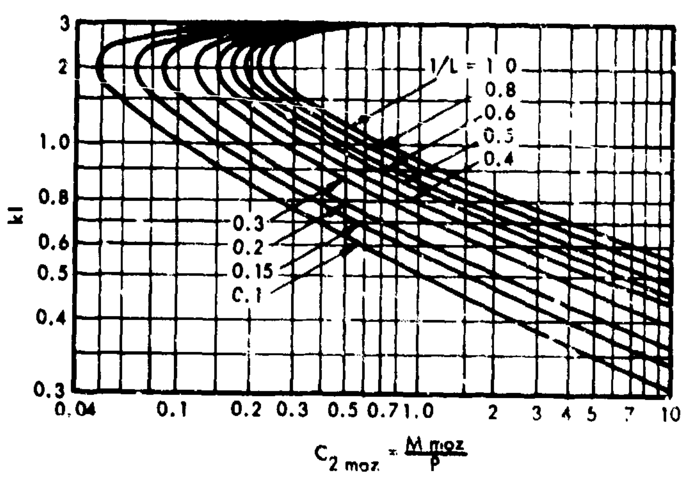


Figure 14.11h. Chart for Obtaining Value of Maximum Moment, Case 2, When Pivot and Stiff Member are Both in Axial Compression and a Lateral Restraining Force is Provided

(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

The development of the design formulae for cases 1, 2, 3, 4, 7 and 8 discussed below are for a single flexure pivot only, but may be applied to the configurations shown in Figures 14.11a, b, and c by determining the length, L, to the point of zero moment in the stiff member. Note that if the two flexure pivots at each end of the stiff connecting member, Figures 14.11a, b, and c, are equal in stiffness and length, there will be no bending moment at the mid-point of the stiff member.

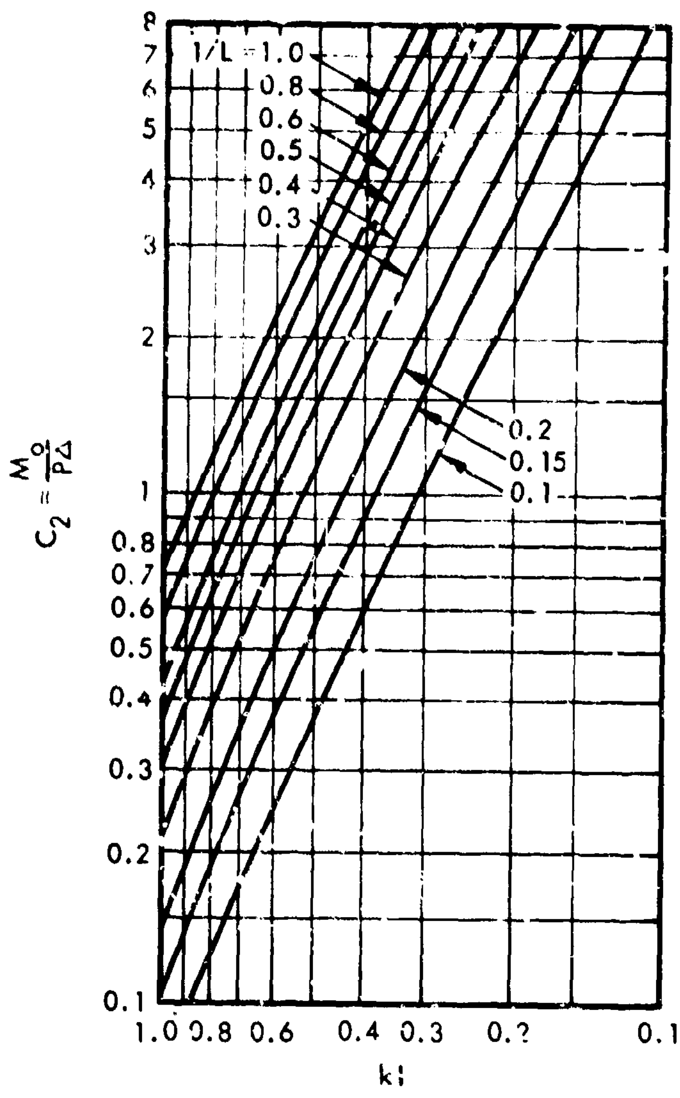


Figure 14.11g. Chart for Obtaining Value of End Moment M_o, Case 2 When Pivot and Stiff Member are Both in Axial Compression and a Lateral Restraining Force is Provided

(Adapted with permission from Reference 650-1)

In deriving each equation for end moment, M_o, and transverse force, Q, for cases 1, 2, 3, 4, 7, and 8, the slope at the end of the flexure pivot joining the stiff member was equated to the end deflections Δ minus the deflection y_l at the end of the flexure pivot divided by the length L of the stiff member.

Case 1: Single Flexure Pivot and Stiff Member Both in Axial Tension with Lateral Restraining Force. This configuration is similar to that of the upper pivot shown in Figure 14.11c but with a lateral restraining force. When designing pivots of this type, the axial force, P, the deflection, Δ, and the lengths, ℓ and L, are generally the prescribed conditions. It is necessary, therefore, to design the flexure pivot to stay within safe stress limits at the point of maximum moment, which, in this case, is the end moment, M_o. The curves in Figure 14.11f for C_1 give values of $M_o/P\Delta$ for this case as a function of the variables k, ℓ, and L.

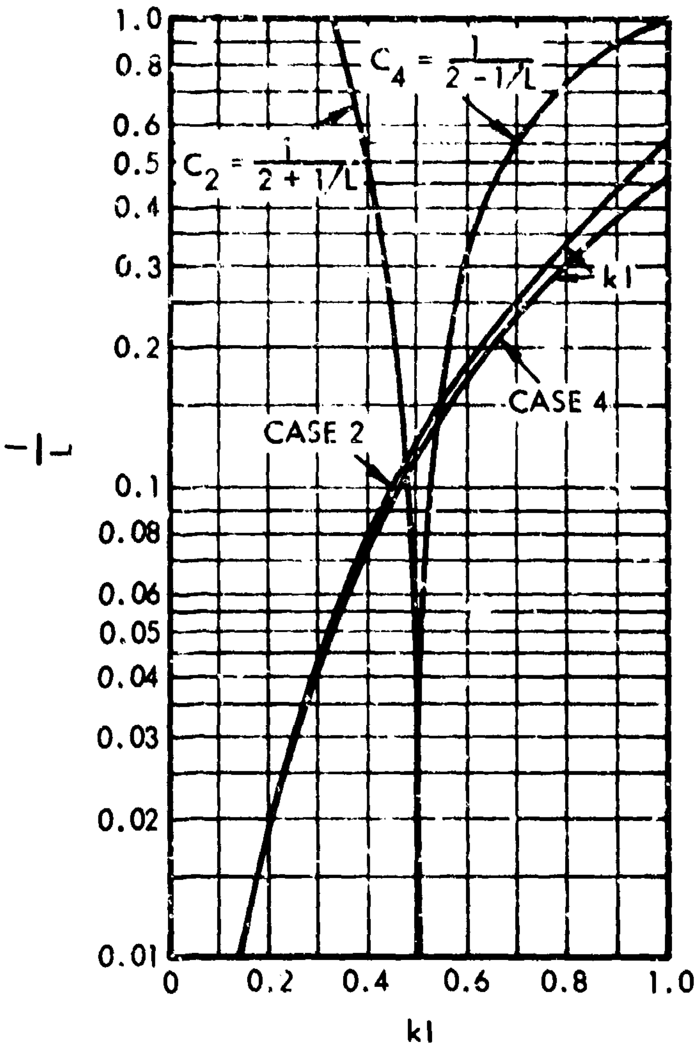


Figure 14.11i. Relations for Cases 2 and 4 for M_0 equal to M_L, or That Provide Equal Moments at Each End of the Flexure Pivot
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

Case 2: Single Flexure Pivot and Stiff Member Both in Axial Compression with Lateral Restraining Force. This configuration is similar to that shown in Figure 14.11a. When designing pivots of this type, the axial force, P, the deflection, Δ, and the lengths, ℓ and L, are generally the prescribed conditions. In this case, the point of maximum moment lies at the distance x from the origin. The curves in Figure 14.11g for C_2 gives values of $M_0/P\Delta$ for this case as a function of the variables k, ℓ, and L.

The value of the coefficient C_{2max} $(M_{max}/P\Delta)$ is given in Figure 14.11h as a function of k, ℓ, and L, to facilitate obtaining the value of the maximum bending moment M_{max}. Note that for values of $k\ell$ exceeding approximately 2.2, the maximum bending moment rises rapidly with increasing $k\ell$ and approaches infinity at values of $k\ell$ slightly over π. Since k is proportional to the square root of P, a small increase in the axial load will cause large additional bending moments at values of $k\ell$ exceeding 2.2 and may lead to failure.

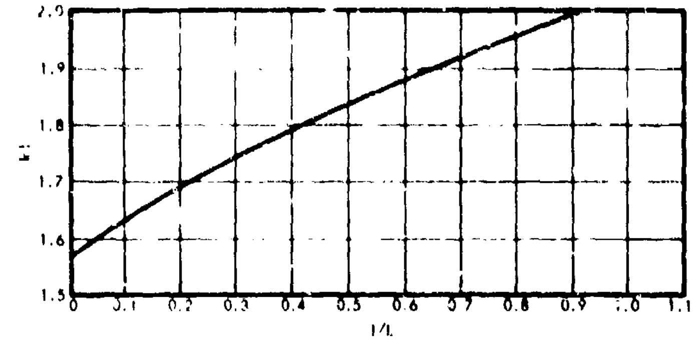


Figure 14.11j. Design Conditions for Case 2 to Produce a Zero End Moment in the Pivot
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

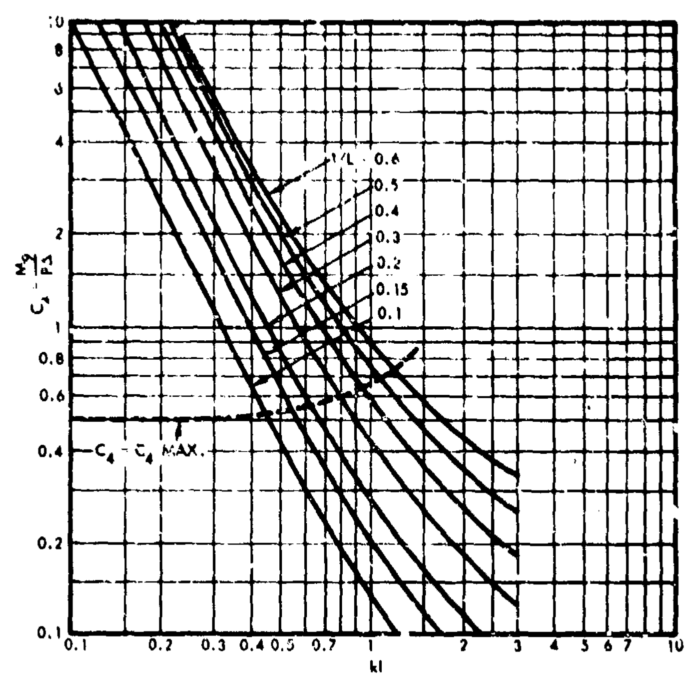


Figure 14.11k. Chart for Obtaining Value of End Moment, Case 4
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

The curves of Figure 14.11i show the relation between ℓ/L and $k\ell$ that will provide equal moments at each end of the flexure pivot. This criterion for design should enable the designer to approach a design of minimum weight since the level of bending stress will be nearly constant throughout the length of the flexure pivot.

The curve of Figure 14.11j indicates the relation between ℓ/L and $k\ell$ when the end moment for this case becomes zero. At values of $k\ell$ above the curve, the end moment reverses, becoming negative; thus, the flexure pivot will have a point of reverse curvature. It is good practice to avoid values of $k\ell$ that will cause the end moment to be negative since the assembly acquires considerably more flexibility and has less resistance to incidental transverse forces.

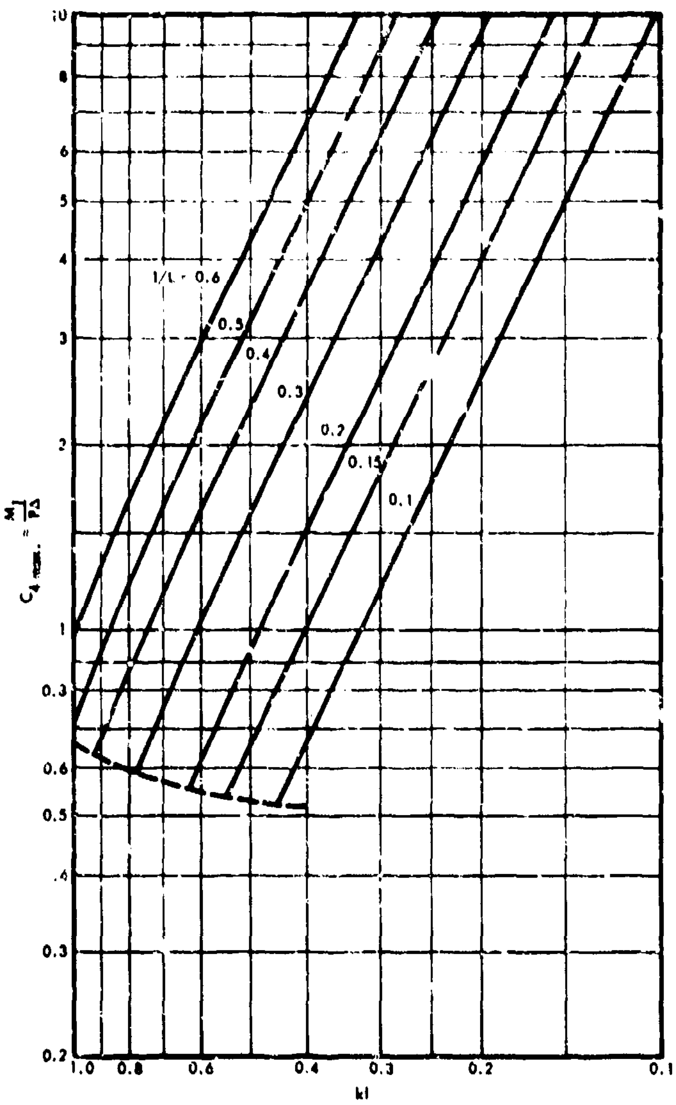


Figure 14.11l. Chart for Obtaining the Maximum Moment, Case 4, When at the Deflected End of Flexure Pivot
(Adapted with permission from Reference 650-1. "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

Case 3: Single Flexure Pivot with Zero Axial Force and with Lateral Restraining Force. This configuration is similar to that shown in Figure 14.11b, but the axial force in the flexure pivot is zero.

Case 4: Single Flexure Pivot in Tension, Stiff Member in Compression, with Lateral Restraining Force. This configuration is similar to that shown in Figure 14.11b. From the curves shown in Figure 14.11k for the coefficient C_4, the value of the end moment for this case can be obtained. Note that below the dashed line, values of C_4 also determine maximum bending moment in the flexure pivot; but for values above this line, the maximum moment occurs at the other end. Figure 14.11l is constructed for convenience in obtaining the maximum moment in the flexure pivot for regions above the dashed line of Figure 14.11i.

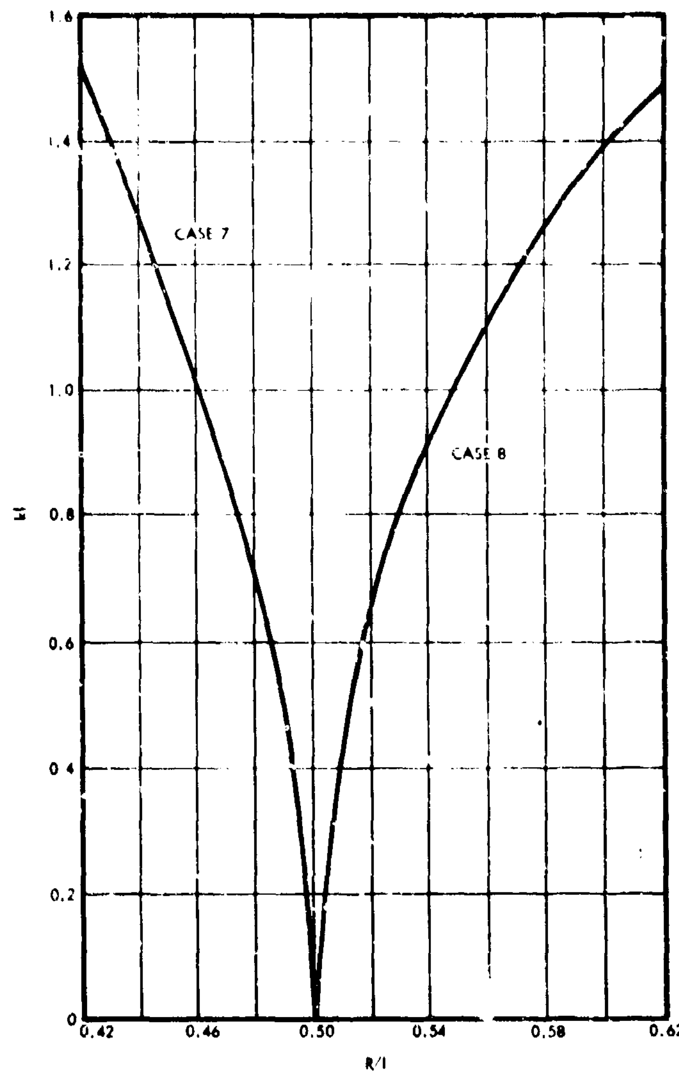


Figure 14.11m. Design Relations, Cases 7 and 8, That Provide Neutral Stability or Zero Lateral Force
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

The curves in Figure 14.11i give the relation between ℓ/L and kℓ that will provide equal moments at each end of the flexure pivot.

Cases 5 and 6: Flexure Pivot Without Stiff Member. This configuration is similar to that shown in Figure 14.11e.

Case 7: Single Flexure Pivot in Tension, Stiff Member in Compression, with No Lateral Restraining Force. This configuration is similar to that shown in Figure 14.11b as designed for neutral stability. The curve in Figure 14.11m for this case gives the relations between k, ℓ, and L that provide neutral stability. Note that $M_0/P\Delta$ equals unity is the necessary condition for zero lateral force. Reference should be made to Figure 14.11l to obtain the maximum moment.

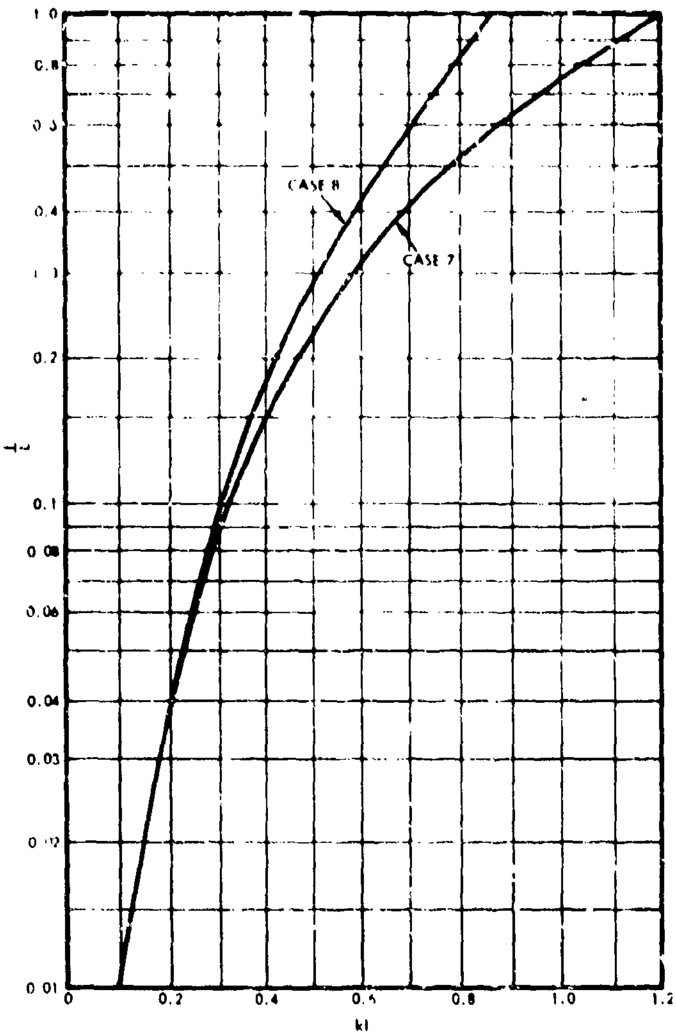


Figure 14.11n. Chart for Obtaining Center of Rotation, Cases 7 and 8, For Various Design Conditions
(Adapted with permission from Reference 650-1, "Spring Design and Application," N. P. Chironis, McGraw-Hill Book Company, Inc., 1961)

The curve in Figure 14.11n gives the variation in centers of rotation for various values of kℓ for this case. Note that as k increases, the center of rotation moves away from the point of fixity.

Case 8: Single Flexure Pivot and Stiff Member Both in Compression, with No Lateral Restraining Force. This configuration is similar to that shown in Figure 14.11a as designed for neutral stability. The curve in Figure 14.11m for this case gives the relations between k, ℓ, and L that provide neutral stability. Note that $M_0/P\Delta$ equals unity is the necessary condition for zero lateral force. Reference should be made to Figure 14.11h to obtain the maximum moment for this case.

The curve in Figure 14.11n gives the variation in centers of rotation for various values of kℓ for this case. Note that as k increases, the center of rotation moves toward the point of fixity.

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*Note: This temporary list of references identifies source material specified in Section 14.0 and will not be found in the handbook Bibliography. Revision D, to be published shortly, will contain a completely revised Bibliography and will incorporate a comprehensive list of references for Section 14.0.

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SYMBOL	QUANTITY	UNIT
A	Area of cross section	in^2
A	Ratio of stress amplitude to mean stress (S_a/S_m), fatigue	dimensionless
a	Amplitude	in.
a	Plate dimension	in.
B	Slenderness ratio factor	dimensionless
B	Biaxial ratio	dimensionless
b	Width of cross section	in.
b	Plate dimension	in.
C	Circumference	in.
C	Configuration or damping parameter	dimensionless
C	Correction factor	dimensionless
C	Stress loading support factor for flat plates	dimensionless
C_c	Buckling stress coefficient	dimensionless
C_m	Moment constant	dimensionless
C_s	Surface factor, fatigue	dimensionless
C_s	Shear buckling stress coefficient	dimensionless
c	Fixity coefficient for columns	dimensionless
c	Distance from neutral axis to extreme fiber	in.
D	Diameter	in.
d	Depth or height	in.
d	Mathematical operator denoting differential	dimensionless
E	Modulus of elasticity in tension	psi
E	Average ratio of stress to strain for stress below proportional limit	psi
e	Elongation	percent
e	Unit deformation or strain	in./in.
e	Eccentricity	in.
e	The minimum distance from a hole center line to the edge of sheet	in.
E'	Effective modulus of elasticity	psi
E_c	Modulus of elasticity in compression	psi
E_c	Average ratio of stress to strain below proportional limit	psi
E_s	Secant modulus	psi
E_t	Tangent modulus	psi
F	Allowable stress	psi
f( )	A function of ()	—
f	Internal (or calculated) stress	psi
f	Coefficient of friction	dimensionless
F_b	Allowable bending stress	psi
F_b	Modulus of rupture in bending	psi
f_b	Internal (or calculated) primary bending stress	psi
f_b'	Internal (or calculated) precise bending stress	psi
F_{be}	Endurance limit in bending	psi
f_{br}	Internal (or calculated) bearing stress	psi

SYMBOL	QUANTITY	UNIT
F_{bru}	Ultimate bearing stress	psi
F_{bry}	Bearing yield stress	psi
F_c	Allowable compressive stress	psi
f_c	Internal (or calculated) compressive stress	psi
F_{cc}	Allowable crushing or crippling stress (upper limit of column stress for local failure)	psi
F_{co}	Column yield stress (upper limit of column stress for primary failure)	psi
F_{cp}	Proportional limit in compression	psi
F_{cr}	Allowable buckling stress	psi
F_{cu}	Ultimate compressive stress	psi
F_{cy}	Compressive yield stress at which permanent strain equals 0.002 (from tests of standard specimens)	psi
f_d	Diaphragm stress	psi
F_{LL}	Limit factor	dimensionless
F_n	Allowable normal stress	psi
f_n	Internal (or calculated) normal stress	psi
f_s	Internal (or calculated) shearing stress	psi
f_t	Internal (or calculated) tensile stress	psi
f_z	Hertz (or contact stress)	psi
F_{se}	Endurance limit in torsion	psi
F_{scr}	Allowable shear buckling stress	psi
F_{sp}	Proportional limit in shear	psi
F_{st}	Modulus of rupture in torsion	psi
F_{su}	Ultimate stress in pure shear (this value represents the average shearing stress over the cross section)	psi
F_{sy}	Yield stress in pure shear	psi
F_t	Allowable tensile stress	psi
f_t	Thermal stress	psi
F_{tp}	Proportional limit in tension	psi
F_{tu}	Ultimate tensile stress (from tests of standard specimens)	psi
F_{ty}	Tensile yield stress at which permanent strain equals 0.002 (from tests of standard specimens)	psi
F.S.	Factor of safety	dimensionless
G	Modulus of rigidity	psi
G	Modulus of elasticity in shear	psi
h	Height or depth, especially the distance between centroids of chords of beams and trusses	in.
I	Moment of inertia	in^4
i	Slope (due to bending) of neutral plane of a beam (1 radian = 57.3°)	radians
I_p	Polar moment of inertia	in^4
J	Torsion constant ($=I_p$ for round tubes)	in^4
j	Stiffness factor = EI/P	in^2

SYMBOL
K
K
K
K'
K
K_f
K_p
K_t
K_{ts}
L
l or ℓ
M
m
M_a
M.S.
N
n
n
P
P
P_c
P_{DL}
P_{LL}
P_u
P_y
Q
Q
q
q
R
R
R
R
r
S
S
S
S
S_{max}
S_{min}
S_r

SYMBOL	QUANTITY	UNIT
K	A constant, generally empirical	–
K	Bulk modulus	psi
K	Short cylinder influence coefficient	dimensionless
K	Deflection loading support factor for rectangular plates	dimensionless
K	Stress concentration factor (see K_t)	dimensionless
K_f	Fatigue strength reduction factor	dimensionless
K_p	Buckling coefficient	dimensionless
K_t	Theoretical stress concentration factor (see K)	dimensionless
K_{ts}	Theoretical shear stress concentration factor	dimensionless
L	Length	in.
l or ℓ	Length	in.
M	Applied moment or couple, usually a bending moment	in-lb_f
m	Mass	lb_m = lb_f sec^2/in.
M_a	Allowable bending moment	in-lb_f
M.S.	Margin of safety	dimensionless
N	Fatigue life or cycles to failure, fatigue	dimensionless
n	Cycles applied in fatigue testing	dimensionless
n	The shape parameter for the standard compression stress-strain curve	dimensionless
P	Applied load (total, not unit, load)	lb_f
p	Pressure	psi
P_c	Allowable load (columns)	lb_f
P_{DL}	Design load (maximum expected load)	lb_f
P_{LL}	Limit load	lb_f
P_u	Ultimate load	lb_f
P_y	Yield load	lb_f
Q	Static moment of a cross section	in-lb_f
Q	Lateral force (flexures)	lb_f
q	Notch sensitivity (fatigue)	dimensionless
q	A point	dimensionless
R	Radius of curvature	in.
R	Outside radius of flat plate	in.
R	Distance locating center of rotation flexure pivot	in.
R	Stress ratio (S_{min}/S_{max}), fatigue	dimensionless
r	Radius	in.
S	Shear force (also V)	lb_f
S	Nominal stress, fatigue	psi
S	S basis for mechanical-property values	—
S_a	Alternating stress amplitude, fatigue	psi
S_{max}	Maximum stress; highest algebraic value of stress in the stress cycle (tensile +; compressive -), fatigue	psi
S_{min}	Minimum stress; lowest algebraic value of stress in the stress cycle (tensile +; compressive -), fatigue	psi
S_r	Range of stress (S_{max} - S_{min}), fatigue	psi

SYMBOL	QUANTITY	UNIT
S_c	Creep-limited static stress, fatigue	psi
S_e	Fatigue limit (or endurance limit)	psi
S_f	Fatigue safety factor = S_n/K_fS_a	dimensionless
S_m	Mean stress amplitude, fatigue	psi
S_n	Fatigue strength (S_n = S_e for n = ∞	psi
S_t	Stiffness	lb_f-in^2
ST	Short transverse grain direction	—
S_u	Ultimate stress, unspecified loading, fatigue	psi
s_y	Yield stress, unspecified loading, fatigue	psi
T	Applied torsional moment or torque	in-lb_f
T	Transverse grain direction	—
T	Membrane stress	lb_f/in
t	Thickness	in.
T_a	Allowable torsional moment	in-lb_f
U	Factor of utilization	dimensionless
V	Shearing force, total (also S)	lb_f
v	Shear force per unit area	psi
W	Weight, or total distributed load	lb_f
w	Unit weight, unit load	lb_f
w	Dimension (usually width)	in.
x	Value of an individual measurement	—
$\bar{x}$	Average value of individual measurements	—
y	Deflection (due to bending) of elastic curve of a beam	in.
y	Distance from neutral axis to given fiber	in.
Z	Section modulus, I/y	in^3
Z	Curvature parameter	dimensionless
Z_p	Polar section modulus = I_p/y (for round tubes)	in^3
α	Coefficient of thermal expansion, mean	in/in/°F
δ	Deflection	in.
Δ	Deflection (flexures)	in.
Δ	Change in any value	same as initial value
ϕ	Angular deflection	radians
ϕ	Notch angle	degrees
ρ	Radius of gyration	in.
μ	Poisson's ratio	dimensionless
ω	Density	lb_f/in^3
ω	Frequency	radians/sec
ω_n	Natural frequency	radians/sec
'	In general, denotes an "effective" or "precise" value	—
θ	Slope or angle of rotation	radians
η	Stress loading-support factor for circular plates	dimensionless
β	Deflection loading-support factor for circular plates	dimensionless

SUBSCRIPTS

A	Axial
a	Allowable
b	Bending
br	Bearing
c	Compression
cr	Critical
e	Euler's formula
e	Endurance
f	Fillet
H	Hoop
i	Inner
L	Lateral or Limit
l or ℓ	Longitudinal
m	Mean stress
n	Normal
o	Outer
p	Polar
p	Proportional limit
r	Radial
s	Shear
T	Thermal
t	Tension, Theoretical
u	Ultimate
y	Yield
z	Hertz

Note Concerning Stress Units

Stress is designated either as psi or ksi, where

psi = lb_f/in^2

ksi = psi x 1000

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15.1 INTRODUCTION

The success of any program that involves hardware is dependent on the accuracy of the *testing* accomplished on the system and components. The importance of testing in a fluid component development program is reflected in the large percentage of program time and funding normally allocated to testing. This section of the handbook discusses factors influencing the *costs* and *schedules* associated with testing, describes the basic *functional* and *environmental* tests, and discusses the unique *test requirements* of certain fluid components.

15.2 TEST PURPOSE

Five basic reasons for testing a device or system are to:

1) Provide empirical design data
2) Determine functional capabilities
3) Evaluate the ability to operate in the service environment
4) Determine design limits
5) Determine if production units are of the same quality as qualification units.

The tests used to make these determinations are called:

a) Development tests
b) Design verification tests
c) Prequalification tests
d) Qualification tests, airworthiness tests
e) Preproduction, pilot model, pilot lot tests
f) System integration tests
g) Production acceptance tests
h) Production monitoring tests, quality verification tests
i) Reliability tests

The extent of testing is usually a compromise between testing necessary to assure reliability and the time, money, and facilities available to conduct that testing. This tradeoff is especially difficult to make in components to be utilized in space vacuum and zero gravity because of the cost associated with environmental simulation.

Similarly, production quantities and applications significantly influence the relative effort devoted to various testing categories. For example, a pressure regulator for a single-mission deep-space probe, such as a Mariner spacecraft, may have to be as reliable as a comparable regulator mass-produced for aircraft application. Numerous samples of the aircraft regulator will be subjected to virtually all of the tests listed above, with the result that the units actually used in service will be subjected only to relatively simple production acceptance tests. However, the spacecraft regulator program may call for only one, two, or three development models, but the actual flight item will be subjected to very comprehensive acceptance tests.

15.2.1 Development Tests

Development tests are conducted on initial preprototype components to check out basic design parameters during the development process. Development tests are used to verify such factors as flow area, pressure drop, electrical power drain, and functional operation. Development tests should verify all requirements necessary to produce a complete set of engineering drawings describing a component capable of meeting its specification requirements. The model used for such tests is usually called a *breadboard*, *boiler plate*, or *engineering model* and is produced specifically for those tests.

The tests should provide data to finalize a new design or to modify an existing design to comply with new requirements. Adjustment, rework, repair, and retest are normal functions of a development test. Specifications should require that all activities, as well as details of all repairs and adjustments, be documented for future correlation with the production unit.

15.2.2 Design Verification Tests

Design verification tests (DVT) should be conducted on prototype hardware before proceeding to production drawings and actual fabrication of production hardware. Test requirements, toward which the manufacturer must design, should be itemized in the component specification. Design verification tests are planned to prove that a component can meet all of its functional requirements and the most critical of its environmental requirements. Component design verification tests allow system tests to be started with maximum assurance that components can perform their system function prior to performing time-consuming life or reliability tests. Some organizations combine development tests with design verification tests.

15.2.3 Prequalification Tests

Prequalification tests (also called design approval tests, preliminary flight rating tests or PFRT, flight certification tests, and type approval tests) are conducted on production hardware prior to flight testing to determine whether the article fabricated by production tooling and techniques will perform as capably when fabricated as a prototype. These tests should include most functional and environmental requirements and some life-cycle tests. The tests should prove that the production hardware can meet all the required parameters for the length of time required by the flight test program. Special *stress to failure* tests are sometimes included as part of prequalification testing. These tests, which can be destructive, are designed to establish margins of safety over minimum design requirements. In some organizations prequalification tests are combined with design verification tests.

15.2.4 Qualification Tests

Qualification tests (also called flight acceptance tests) are normally formal demonstrations (in contrast to evaluations) with production hardware and are the final test requirements to be met by the component. It is important that qualification test requirements be realistic and not simply be included because it was done before. A primary difference between formal qualification tests and other tests is that the test qualifications are used to demonstrate rather than evaluate the product. They should consist of all the steps taken in prequalification tests, as well as the following:

1) The component tested should be randomly selected, representative production-type hardware made entirely with the manufacturer's production tooling and processes.

2) The number of samples tested should be adequate to prove that the components are statistically capable of meeting their reliability requirements. (Usually 3 to 5 is a minimum number.)

3) The tests should be repeated at various undefined points during the production phase of the program to assure that the last components produced meet the same standards as the first.

15.2.5 Preproduction, Pilot Model, or Pilot Lot Tests

When an extensive production run of products is anticipated, tests are often conducted prior to commencing a full-scale production run to check the conformance of the preproduction or pilot units to specific test requirements. These tests are called preproduction tests, pilot model tests, or pilot lot tests. The individual tests may consist of any or all of the tests in the categories of functional, environmental, or reliability testing. Preproduction tests are considered mandatory for products made in lots when the end of one lot and the beginning of another are separated by significant time periods or when a source of supply is changed. Any defects that might occur in the product are thereby detected and corrected.

15.2.6 System Integration Tests

System integration tests are conducted to evaluate the compatibility of the components with system requirements and serve to evaluate and optimize checkout and operating procedures. Although a component may have been correctly designed to fulfill its own function, its compatibility with related equipment and its performance as part of an integrated system must be demonstrated. Compatibility includes proper interfacing with mating flanges and connectors.

15.2.7 Production Acceptance Tests

Production acceptance tests are nondestructive tests performed on deliverable production hardware to assure that it is equivalent in design and manufacture to those components which have previously completed the formal qualification and/or prequalification test programs. Although these tests are of a quality control nature, they are an integral part of the step by step program to ensure a satisfactory end product. During early hardware production, acceptance tests may include limited environmental testing. Testing of this nature is commonly called production environmental testing (PET). These tests usually start on a 100 percent basis; then, as confidence in the quality is achieved, the number of parts tested is reduced to a sampling. The PET testing is ultimately dropped, with subsequent acceptance testing limited to the normal perfunctory bench-type functional tests. Although some minimal testing is usually maintained on a 100 percent basis, production acceptance testing on a sampling basis is sometimes performed. The test sample may be selected at random from the production run on the basis of one per given number of units produced. Acceptance tests in this category usually consist of nondestructive tests of a relatively abbreviated nature to check critical performance parameters or structural integrity. Such sampling acceptance tests are frequently considered to be of limited value.

15.2.8 Production Monitoring Tests

Production monitoring tests are conducted at prescribed intervals to subject the product to more intensive or extensive conditions than are encountered in the normal production acceptance test. These tests may be either destructive or nondestructive and are performed on a sampling basis.

15.2.9 Reliability Tests

Reliability tests are performed to determine the probability that a component will fulfill its intended function. Components which are cyclic in operation are usually tested for a number of operating cycles until failure, and components which operate continuously are usually tested to determine the mean time to failure (MTTF) or mean time between failures (MTBF). (See Sub-Topics 11.2.7 and 11.2.8.) Cyclic tests can usually be repeated with sufficient frequency to simulate the operating cyclic life in a reasonably short test period. On the other hand, continuous life tests may be difficult to simulate, particularly on components designed to operate thousands of hours in normal service.

Limit testing, or performance margin testing, determines the margin of safe operation over the specified design conditions. Limit tests are conducted by progressively increasing the severity of a test parameter, such as temperature, until the component fails. The margin of safe operation over the design conditions is a measure of the component's functional reliability.

15.3 TEST COSTS AND SCHEDULES

15.3.1 General

Component testing has a significant influence on the overall cost and schedule of any development effort. The preparation of the test schedules and cost estimates as well as a discussion of factors influencing test cost are treated in this sub-topic.

15.3.2 Review of Test Specification and Requirements

When the request for proposal (RFP) defines the test requirements in detail, the task of preparing a test plan, cost estimate, and schedule is facilitated. If the specification is general in nature, it will be necessary to propose a logical test program, including those tests which are believed to be required based on experience. In reviewing the specification, it is helpful to have a preprinted form, sometimes called a *compliance tabulation.* Each paragraph and subparagraph is listed on the form and checked off in the appropriate column. Any noncompliance is explained. There should be no reluctance on the part of the reviewer to question the logic of any given test procedure. If the test procedure, as received, has any aspects that appear to be impractical or excessively expensive compared to the information that will be derived, this fact should be pointed out to the requestor. If this cannot be done, the suggested change should be included in the proposal under the section on deviations. Figure 15.3.2 shows an excerpt from a sample specification review form.

PARAGRAPH NUMBER	TITLE	CONFORM		COMMENT	BY
		YES	NO		
[illegible]	[illegible]	X			
[illegible]	[illegible]	X			
[illegible]	[illegible]		X	[illegible]	[illegible]

Figure 15.3.2. Typical Specification Review Form.

15.3.3 Preparation of Cost Estimates

A test cost estimate lists the major headings of the test, e.g., shock vibration, leakage. Estimates of the individual test costs are then made and include such items as fixtures, materials (for example, liquid nitrogen or other test media), and any special equipment that might be needed for the task. In making the estimate, the hours for the various labor categories are recorded, as are the hours for the equipment required. Estimates are made for material costs and other off-site charges, and the man-hours, equipment hours, and dollar expenditures are totaled at the bottom of the sheet. The appropriate burden factors are then applied; this yields the total cost of the test program.

15.3.4 Preparation of Test Schedule

After the individual tests have been defined, a schedule is made for the overall test program. Bar charts are normally used, as they are particularly well suited for presenting this type of information. Characteristically, test schedules are made with a very high degree of optimism; tests are generally viewed in an ideal situation, and rarely is sufficient time allocated for the contingencies and problems that are sure to arise. Due consideration should be given to the possibility of test failure, equipment problems, conflict with other programs, personnel availability, and other factors that may cause schedules and costs to vary.

15.3.5 Factors Influencing Test Cost

General Factors. The cost of any test program is obviously affected by the complexity of the test, the number of pieces to be tested, and the equipment and facilities required. There are less obvious factors which may affect the cost, and consideration of these factors in advance of the testing will increase the accuracy of the estimate and may serve to reduce the overall program expenditure.

Schedule. An unrealistic schedule for completion can significantly increase the cost of the test program by requiring premium pay for personnel, for vendors of test fixtures, and for commercial test laboratories performing some or all of the test program. Hidden overhead costs associated with the activity are also generated in the form of additional burden on the purchasing and liaison personnel who will be directly affected by the increased expediting effort required. Conversely, a long drawn-out schedule can result in tying up expensive test equipment and instrumentation. This is especially true in development testing, where the cost of taking down and setting up tests must be compared with that of leaving specimens set up while analyzing data.

In-House versus Commercial Laboratory. A *make or buy decision* is often in order in conducting a test program. That is, it may be more economical to have the testing done by a commercial laboratory than to do it in-house. This is true if testing in-house involves capital expenditures for equipment that has little subsequent application, or if lack of personnel and/or equipment causes a schedule slippage. If the program is conducted in-house, consideration should be given to the possibility of renting or leasing equipment that is needed. In some areas, complete instrumentation rental services are available which provide single pieces of equipment, such as an accelerometer, or the total equipment and personnel needed for a test series. When off-site expenditures of this type are made, the total cost of the job is known in advance, which simplifies cost control.

In addition to commercial testing laboratories, extensive environmental and functional test facilities of many of the large aerospace firms are frequently available to component manufacturers when conditions permit.

Unnecessary Tests. Very often tests are specified which will produce little or no information with respect to the product's ability to meet its functional requirements. Many such tests seem to be specified out of habit; that is, such tests were required on components in the past and the practice is carried over to present equipment. Examples of such tests are: sand and dust, humidity, and fungus tests on hermetically sealed units; and acceleration tests on components which can obviously suffer no ill effects from the environment or for which the effects are easily calculated. Usually, an examination of the design will indicate whether such tests are needed or not. If the examination indicates that the test may not be worthwhile, a suggested deletion in the deviations section of the proposal should be made.

Changes to Test Programs. It is frequently necessary to make changes to a test program after it has been started. Often these are minor changes and involve relatively little cost. However, in normal practice the test engineer in charge of conducting the program is not authorized to incur any additional charges without approval of his company's contracting department. Should the requirement for a change occur at night or on a weekend when such approval is difficult or impossible to obtain, very large expenses can be incurred due to the stoppage of the program while awaiting approval. It may, therefore, be advisable on some programs to authorize a test engineer to expend a limited amount of money to circumvent the shutdown of a test program under extenuating policy.

15.3.6 Instrumentation and Data Requirements

The data required in any test should be very carefully considered, since either too little or too much data can seriously affect the test program. If insufficient data are taken, the entire test may be worthless. If superfluous data are taken the cost will be excessive both because of the effort required to obtain the data in the first place and the additional effort required to reduce the data. The parameters to be considered in determining the instrumentation and data requirements of any given test procedure are discussed below.

As a first step, determine what is to be measured (force, temperature, pressure, etc). While this may seem to be an obvious statement not infrequently a parameter is omitted from a test procedure, necessitating a rerun of the test when the missing factor is eventually discovered. Any parameter that could possibly affect the outcome of the test should be considered. For example, if meteorological data such as barometric pressure, ambient temperature, and time of the day could have a bearing on the test results, their recording should be specified.

The accuracy of the specified recordings will affect the test costs. In the early phases of the development test, it may be satisfactory to use commercial pressure gauges to obtain readings which would be completely unsuitable for use in a subsequent qualification test. The accuracy required should always be specified (*full scale* or *test point*, as well as the percent) and steps must be taken to ensure that the accuracy specified is actually being obtained. Very frequently, the accuracies specified are completely incompatible with the instrumentation equipment specified in the same test procedure. NBS Technical Note 262 (Reference 82-21) contains 66 charts which describe the accuracy obtainable by conventional measures. An accuracy analysis of the final test computation should also be required (i.e. root mean square sum).

After the test parameters to be measured have been determined, a schematic of the setup should be made. This schematic may vary from a hand sketch to an elaborate drawing, depending upon the complexity of the test program. A schematic serves two basic purposes: it helps the test planner design the setup and provides the test lab with the necessary information for making the proper setup.

A typical schematic for a pressure drop test is shown in Figure 15.3.6. The schematic should show sufficient detail to permit construction of the setup and to provide direction concerning the instrumentation quantities and qualities required. If the instrumentation list or notes become too extensive to include conveniently on a drawing, the list may be supplied as an appendix. Typical notes that might be supplied with Figure 15.3.6 are as follows:

1) Pressure tap diameter is 0.040 Deburr inside of hole.
2) Gage numbers 1 and 2 to have 1/4% full-scale accuracy. All other gages may have 1% full-scale accuracy.
3) Location of upstream control valve optional.
4) Bypass and shutoff valve to be located as close as possible to flow section.
5) Flow section (furnished) is 0.065 wall, 3.37 ID, 3.50 OD carbon steel. It is adapted to test specimen by ring-clamp flange connectors. Opposite ends fitted with ASA 150 lb slip-on flange.
6) Upon completion of setup, prior to test, leak check all fittings.

The latter statement regarding leak check of the fittings is of extreme importance, yet it is often overlooked in test

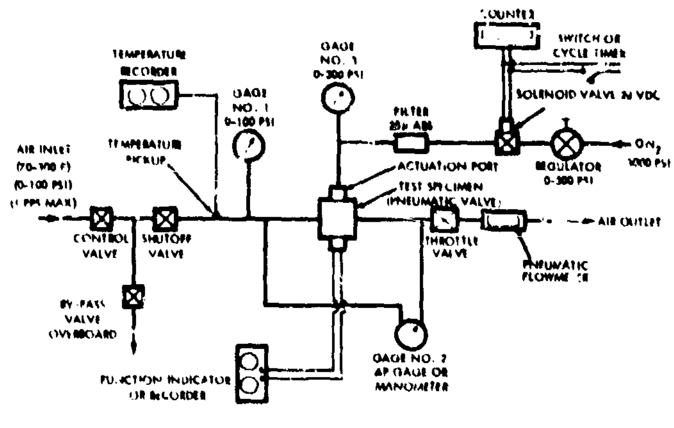


Figure 15.3.6. Typical Test Schematic

laboratories. Of particular importance are the fittings associated with the ΔP measurement as a very slight leakage at these fittings can cause serious errors in the test data.

15.3.6.1 RECORDING OF DATA. The method of recording data will vary with the requirements and the nature of the test. In some cases it is acceptable to record the data by hand on a reproducible data sheet. When such a recording is appropriate, a basic requirement is accurate and legible writing. It should not be transcribed on a typewriter, as the transcription merely adds to the cost and injects the possibility of transcription errors. This method of recording is permissible when steady-state conditions can be maintained long enough to permit recording the levels of all parameters, as in a valve pressure drop test at constant flow rate. When it is necessary to read and record several instruments simultaneously, an ordinary camera can be very useful. Pictures are taken as required during the test and the readings of the various instruments may be recorded on preprinted data sheets as time permits. The photographs may be included in the permanent record of the data. A calendar and a clock should be included in the pictures to show the time and date. It is also advisable to include a test number and run number in the picture.

Polaroid cameras are frequently used to take pictures of traces appearing on oscilloscope screens. An example would be a picture of a solenoid valve electrical trace. This procedure is accurate, economical, and very useful when the event being recorded occurs in a rather brief period of time. The photograph serves as a permanent record and may be reproduced and used in the test report. Movie cameras are used extensively for recording data, and the procedure in some cases is far simpler and more economical than one which requires numerous individual instrumentation points. It is usually necessary to incorporate a narrative account in the test report of what was observed during the film sequence. A copy of the film is usually supplied with the test report, but it may be inconvenient or unnecessary for all recipients of the report to review the actual sequence of events. Closed circuit video cameras used in conjunction with video tape are finding increasing use in recording test data. The instant playback feature of the video tape can be extremely useful during development programs.

The multichannel strip chart recorder or oscillograph has been in use for many years in recording data, and numerous models and types are available. The particular units specified for any given test will depend on such factors as the number of channels required per test, the width of the channel (which affects the readability), the frequency response, the ease of setup and calibration, and the process required to develop the trace. Most modern oscillograph traces are developed immediately by light intensification. This type of recording is particularly well-suited to test programs which have numerous parameters that must be recorded simultaneously, especially if a study of interaction between the various parameters is necessary. An example of such an application is the testing of high-response solenoid or torque-motor propellant shutoff valves, wherein simultaneous recording of current, voltage, and pressures must be performed.

Close liaison should be maintained with the test laboratory when determining the amount of data required. For example, if seven channels of information are specified and only six channel recorders are available, the cost of setting up a second recorder to obtain the data for the seventh channel may be excessive. A reexamination of the requirements may indicate that either the seventh channel be eliminated or the data be acquired by some other method.

There are special forms of strip chart recorders design to sample data points on a multiplexing basis. A typical example of multiplexing requirement is a program involving numerous temperature measurements over long periods of time. The recorder may be set up to sample and record temperature readings on a 1-second interval. Since each datum reading is a point on the chart, a very large number of points over a long period of time may be recorded on a relatively short length of paper.

Magnetic tape may be used to record an extremely large volume of data in a very convenient and compact form. Data then may be played back a few channels at a time on available recording channels. Also, the time base may be slowed down to obtain a higher effective frequency response on the recorder.

15.3.6.2 DATA REDUCTION. The cost of data reduction should be very carefully considered when the basic instrumentation plan is being made. The reduction required in some of the techniques described above can be extremely tedious and costly and subject to error. When the data reduction procedure is considered, it may be found that an entirely different recording process is indicated. For example, when many data points are recorded on numerous channels of a strip recorder and many of the values must be digitized on a data sheet, the overall cost might be less if the data were recorded on magnetic tape. The tape can then be processed by a computer that can provide a printout in a readily usable form. This plan should be considered even when in-house computer facilities are not available, as there are numerous commercial facilities available capable of performing this work. The overall cost may be significantly less than it would be for a manual reduction. When photographs are used to take pictures of numerous instruments, it may be tedious and expensive to record the reading of each of those instruments; and it may not be necessary to record every reading. For example, it may be only necessary to know that a pressure indicated on a gauge did not exceed or fall below certain limits. These limits could be inscribed upon the face of the gauge, as long as the indication is within the specified requirements; no recording would be made and this fact could simply be noted on the data sheet.

It must be remembered, however, that should anomalies develop during subsequent test or operation, all test records and data will probably be closely examined to ascertain causes.

15.4 TEST PLANS AND PROCEDURES

15.4.1 Test Plans

A distinction is made between test plans and test procedures. The test plan contains general statements regarding a specific program, defining what is to be tested and to what extent. It is normally submitted with a proposal and has the same headings as the test procedure discussed below. A typical paragraph from a test plan might read: "A leakage test shall be performed to assure compliance with the detail specification." The related paragraph in the test procedure would specify precisely how the testing is to be done.

15.4.2 Test Procedure

The test procedure is a completely self-contained document which contains all information necessary for the successful performance of a specified test program. The various sections are listed below:

1.0 Purpose
2.0 References
3.0 Test Schedule (not always required)
4.0 Test Conditions and Test Equipment
5.0 Requirements and Procedures
6.0 Test Witnesses
7.0 Test Reports

The following paragraphs present a brief description of each section and a sample writeup.

15.4.2.1 PURPOSE. The purpose section presents the reason for conducting the test program and notes the name and part number of the equipment being tested along with the number of parts to be tested.

> *1.0 Purpose.* The purpose of this procedure is to present the detailed testing methods to be used during a qualification test program on three fuel hose assemblies, Rubber Hose Inc., Part Number 6159268, as specified in References 2.1 through 2.3, in accordance with Reference 2.4.

15.4.2.2 REFERENCES. Each applicable military specification, customer specification, and drawing is listed here along with the document name, number, title, and latest revision letter and date.

> *2.0 References.*
>
> 2.1 TRW Test Plan Number 12345, dated 10 July 1968.
>
> 2.2 Military Standard MIL-STD-810A, dated 23 June 1964, title: Environmental Test Methods for Aerospace and Ground Equipment.
>
> 2.3 TRW Detailed Specification Number 6155A48, dated 14 August 1967, title: Hose Assembly, Fuel.
>
> 2.4 TRW Drawing Number 6159268, Revision 2, dated 20 July 1967, title: Hose Assembly, Fuel.
>
> 2.5 TRW Purchase Order Number X64592A68.

15.4.2.3 TEST SCHEDULE. The test schedule presents the sequence of testing of all units in a clear, logical manner.

3.0 Test Schedule.

Item No.	Test Title	Reference Para No.
1.0	PERFORMANCE TESTS (all units)	5.1
1.1	Examination of Product	5.1.1
1.2	Proof Pressure	5.1.2
1.3	Leakage	5.1.3
2.0	VIBRATION (unit 1)	5.2
2.1	Plus 1.3 above	5.1.1 to 5.1.3
3.0	SALT SPRAY (unit 2)	5.3
3.1	Plus 1.3 above	5.1.1 to 5.1.3
4.0	SHOCK (unit 3)	5.4
4.1	Plus 1.3 above	5.1.1 to 5.1.3
5.0	BURST (all units)	5.5

15.4.2.4 TEST CONDITIONS AND TEST EQUIPMENT. This section presents the ambient conditions to be maintained throughout the test program, the test media to be used, and a complete instrumentation listing (by test).

4.0 Test Conditions and Test Equipment.

4.1 Ambient Conditions. Unless otherwise noted, the ambient conditions throughout the test program shall be an atmospheric pressure of 29.92 ±0.5 inches of mercury absolute, a temperature of 75 ±15°F, and a relative humidity of less than 90 percent.

4.2 Test Media. Where specified in this procedure, the test media shall conform to the following:

Gaseous nitrogen per MIL-P-27401C
Potable water
Hydrazine per MIL-P-26536

4.3 Test Equipment. The following equipment, or equivalent, shall be used during the test program. (Note: the procedure to be used for instrument calibration shall be described or an appropriate quality control document should be specified.) In addition it is frequently desirable to specify instrumentation accuracy as well as range.

		Test Equipment	Range
4.3.1	Performance Tests		
4.3.1.1	Examination of Product	Vernier calipers	0 to 6 inches
		Scale	0 to 10 pounds
4.3.1.2	Proof Pressure Test	Pressure gauge	0 to 10,000 psig
		Head pump	0 to 1500 psig
4.3.1.3	Leakage Test	Pressure gauges	0 to 500 psig
4.3.2	Vibration Test	Vibration system	5 to 2000 cps, 3500 lb;
		Accelerometers	0 to 30 g
		Oscillograph	6 channel recording
		Amplifier	

Plus equipment listed in 4.3.1.1 through 4.3.1.3, above.

4.3.3	Salt Spray Test	Chamber	5 x 7 x 9 ft

Plus equipment listed in 4.3.1.1 through 4.3.1.3, above.

4.3.4	Shock Test	Shock machine	100 lb specimen capacity
		Accelerometers	0 to 100 g
		Oscillograph	20 cps response 8 channel
4.3.5	Burst Test	Pressure gauge	0 to 5000 psig
		Head pump	0 to 10,000 psig

15.4.2.5 REQUIREMENTS AND PROCEDURES. The format of this important section should permit each test writeup to start on a separate page to facilitate changes, additions, and deletions. Each test is further broken down into requirements and procedures sections. Immediately following each test writeup, the figures and tables, if applicable, are presented, and a sample data sheet is included. The applicable paragraph of the customer's specification should be noted. This format facilitates review by the customer and is easily corrected, if necessary.

The above procedure lends itself readily to the preparation of a test report at the conclusion of the test program. The test procedure and test report outlines are compared below to illustrate this point.

Test Report		Test Procedure	
1.0	Purpose	1.0	Purpose
2.0	References	2.0	References
3.0	Summary	3.0	Test Schedule
4.0	Test Conditions and Test Equipment	4.0	Test Conditions and Test Equipment
5.0	Requirements, Procedures and Results	5.0	Requirements and Procedures

As seen above, only the titles of 3.0 and 5.0 change between the procedure and report formats. This allows full use of the procedure in preparing the final report and reduces the time required to transmit the report to the customer.

It is advisable to write each test report section as the work is completed rather than attempting to write the entire report at the conclusion of the test program. This procedure ensures a faster delivery of the final report, and flaws may be detected in the data in time to permit a rerun if necessary.

A sample test procedure writeup is presented below. All other tests would be described in a similar manner.

5.0 Requirements and Procedures.
5.1 Performance Tests.
5.1.1 Examination of Product.

5.1.1.1 (Reference 2.2, Paragraph 6.2) Requirements. The specimen is a pressure regulator which shall be packaged in a sealed container and shall conform to drawing XXX. The weight of the unit shall not exceed 4 pounds.

5.1.1.2 Procedure. The specimen shall be visually examined to determine conformance to the general requirements for workmanship, markings, and identification. The critical dimensions noted in Reference 2.3 shall be recorded. The weight of the unit shall be recorded.

5.1.2 Functional Test

5.1.2.1 Requirements. In this paragraph the requirements for a complete functional test should be described. Such tests usually include leakage measurements, response, and current. It may be desirable to also describe a limited functional test to be used following certain environmental tests where a complete functional test would not be warranted.

5.1.2.2 Procedure. This section contains a detailed description of the tests to be conducted. It must be complete enough to permit the technicians to set up the tests with proper instrumentation and other equipment, conduct the tests, and obtain the necessary data. Illustrations clearly showing the test setup should be provided and, where possible, the drawings should specify the particular test equipment to be used. Generous use should be made of notes, both on the drawings and in the text of the specific procedure. If a standard test is to be conducted, it is helpful to reproduce the section in its entirety from the appropriate document and include the copy in the procedure. This eliminates errors and saves considerable writing time.

15.4.2.6 TEST WITNESSES. This paragraph identifies, by title only, the witnesses to be notified prior to the beginning of a test and specifies the amount of time available before each test. It should also clearly state that the tests will proceed as scheduled whether or not the witnesses appear, since costs could mount greatly if the program were delayed for lack of a witness. This paragraph should also specify how failures will be reported and define the course of action to be taken in the event of a failure. This action could be a predetermined agreement to continue or repeat the test, or to stop the test entirely pending further instructions. A written notice of failure or deviation from the specification should be supplied in every case.

15.4.2.7 TEST REPORTS. This paragraph describes the report to be supplied at the conclusion of the test, including the date it will be supplied, its general format, the number of copies, and whether or not a reproducible copy will be supplied.

15.5 MECHANICAL FUNCTIONAL TESTS

15.5.1 General

As the title implies, the functional test is designed to determine whether the component performs according to specification. Such testing can range from a 2-minute pressure switch check to determine if it actuates and deactuates within the prescribed limits at room temperature, to a 4 to 8-hour flow test on a regulator using actual service fluids, controlled temperature conditions, a programmed flow rate, and varying ambient pressures.

The functional test is normally conducted after each environmental test to determine that the component continues to meet the requirements. A complete functional test after each environmental test is not always necessary and may be undesirable, as some functional tests are degrading if the test levels are sufficiently high. Judgment on the part of the design or test engineer is required to determine how many tests are required to establish satisfactory operation of the unit. When the number of cycles accrued in functional testing becomes a significant portion of the total design life cycle, it is conventional to reduce the number of cycles in the life test by a like amount. A discussion of techniques used in conducting the normally-encountered mechanical functional tests follows.

15.5.2 Examination of Product

Conventionally, the first test to be conducted is an examination of the test article. The extent of the examination will vary with different products, ranging from a cursory visual evaluation of condition and verification of identification to a complete disassembly and dimensional and functional check. Any disassembly should be done either by the manufacturer or with a manufacturer's representative present. If the article has been subjected to a normal receiving inspection procedure, the data obtained may constitute the examination of product. However, the test engineer may elect to verify some of the findings prior to the start of tests

15.5.3 Proof and Burst Pressure Tests

Proof and burst pressure tests (both overpressure tests) are discussed together because of the similarity of the procedures. Proof and burst pressures are also discussed in Sub-Topic 13.2.2. The proof pressure test, which is performed to establish the structural integrity of the part and ensure that it will be completely functional after it is subjected to abnormal pressure, is usually the first functional test conducted on a unit. Proof pressure factors normally vary from one and a half to two times the working pressure, depending on the usage and individual specification. Burst tests, conducted to establish the margin of safety, fall into two classes:

1) The test specimen is tested to some arbitrary level normally ranging from two to four times the working pressure. The unit is not expected to operate after this test has been conducted, but it should not break and should not leak when the pressure is reduced to the working pressure level.

2) The pressure is raised until a structural failure occurs, and the point of rupture obtained in this test may be used to determine the margin of safety that exists.

15.5.3.1 SAFETY PRECAUTIONS. Very close attention should be given to proper safeguards while the test is being conducted, especially if the test fluid is a gas. Some methods of providing safeguards are discussed below.

Protective Wire Mesh Cages. For tests involving relatively low volume and/or pressures, the use of a wire mesh enclosure may be economical and convenient. Such a cage may range in size from a small bench-mounted box to a large walk-in box. Mounting the latter on casters may provide added convenience.

Protective Glass Shield. When close visual observation of a specimen is required, an enclosure of shatter-proof glass may be used. Use of mirrors permits viewing from several angles and helps to reduce viewing hazards.

Sand Bags and Other Heavy Barriers. Sand bags may be used to provide very economical protection for *one-shot* tests, as the material is inexpensive and the structure easily erected by unskilled labor. They are not suitable for long-term use, since the structure is unsightly and the bags tend to rot rather quickly, especially in strong sunlight. An efficient barrier consisting of 2 x 12-inch tongue and groove lumber may be erected very economically, and may be disassembled, moved, and reused (see Figure 15.5.3.1). The amount of protection afforded by the barrier may be varied by adjusting the width of the enclosure. Railroad ties may also be used to advantage in some cases.

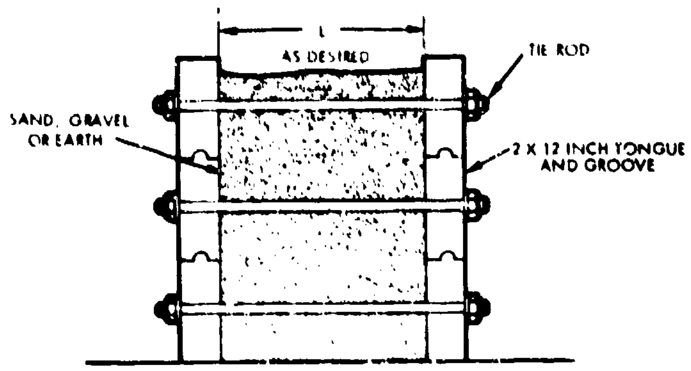


Figure 15.5.3.1. Blast Protection Device

Closed Circuit Television. Closed circuit television may be used to good advantage in monitoring hazardous tests such as proof and burst, and the equipment is relatively inexpensive. In some cases television would permit use of a less expensive test facility, i.e. less secure, as ultimate protection for personnel would not be required.

15.5.3.2 TEST MEDIUM. The test medium used to conduct the proof pressure test may affect the results of the external leakage test, and this point should be considered in selecting the pressurant. A pressure vessel (e.g., tank or valve body) may contain tiny capillary leak paths which tend to close under the application of liquid pressure. When the pressure is removed, the liquid tends to remain in the cavity, thus sealing off potential leak paths. If an external leak test is conducted immediately after a proof pressure test, an erroneous indication of zero leakage may result. If such a possibility exists for a given component, gas should be used as the pressurant or the specimen should be carefully dried, preferably at elevated temperature and in vacuum, prior to the leakage test. The test fluid should always be compatible with the test specimen.

Hydrostatic tests are usually preferred for pressure vessels because of the significantly less severe reaction to rupture. Because of energy stored in compressed gas, failure of the unit charged with gas will produce an explosion and blast effect comparable to a bomb of similar size, whereas a rupture in a properly conducted hydrostatic test may result in a split or crack, after which the pressure falls to zero almost instantly. However, any entrapped gas, especially at extreme pressures, can cause a lethal explosion, the violence of which will be in proportion to the amount of gas in the vessel. Although hydrostatic tests are much safer than pneumatic tests, they should be conducted in a closed chamber to stop flying fragments. This chamber may be something as simple as a plywood box or cover.

15.5.3.3 TEST PROCEDURE. The setup for an overpressure test (proof or burst) requires a suitable facility, test medium, source of pressure, and instrumentation. Prior to installation in a test setup, it may be desirable to record various dimensions of the test specimen not previously obtained. These dimensions may be compared to those taken after the test to determine any permanent set which may have occurred. In conducting burst tests on pressure vessels, it is often helpful to paint a numbered grid on the vessel to assist in reassembling the fragments after the burst. In some cases, measurement of strain and permanent set of a pressure vessel is mandatory. One such example is the requirement by the Interstate Commerce Commission that such data be recorded during the hydrostatic proof pressure testing of compressed gas cylinders. The method used during these tests is illustrated in Figure 15.5.3.3a. To conduct the test, the valve is removed from the cylinder and the cylinder is filled with water. A flange containing a suitable nipple is then threaded into the neck of the cylinder, and the cylinder is lowered into a larger tank which is also filled with water. The flange is secured by a clamp, and pressure is applied to the cylinder. As the pressure in the cylinder rises, the cylinder expands slightly, displacing water from the outer cylinder into an external manometer. The manometer is calibrated to indicate the maximum strain permissible and is also marked to indicate the maximum permissible permanent set. Any indication of excess strain or excess set is cause for rejection.

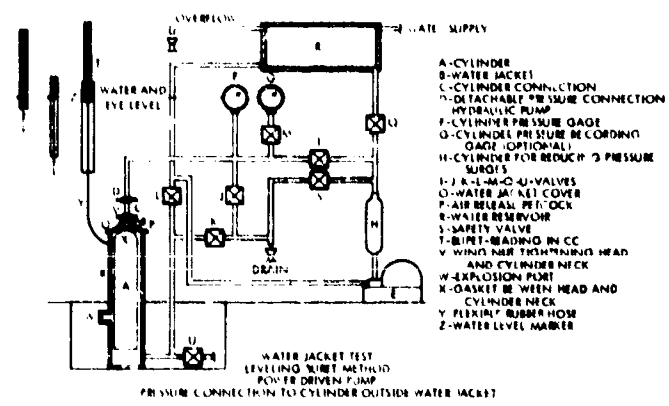


Figure 15.5.3.3a. Hydrostatic Test of Pressure Vessel
(Adapted with permission from "Methods of Hydrostatic Testing of Compressed Gas Cylinders", Compressed Gas Association, Inc., New York, New York)

Unless some information regarding possible residual set is obtained in a proof test, there is no guarantee that the unit will meet the proof pressure requirements upon a second application of pressure. This fact should be carefully considered before a second application of proof pressure is made to the unit.

A typical overpressure test setup and data sheet are shown in Figure 15.5.3.3b. Note that the vent valve is shown on the top of the unit. It is very important that no trapped air remain in the test specimen during a hydrostatic test; even a small volume of air at extreme pressures can drastically change the characteristics of the explosion should a failure occur. Test specimens of odd configurations should be turned and rotated to allow trapped air to rise to the high point and be bled off, or the unit may be evacuated with a vacuum pump and then allowed to fill with the test medium.

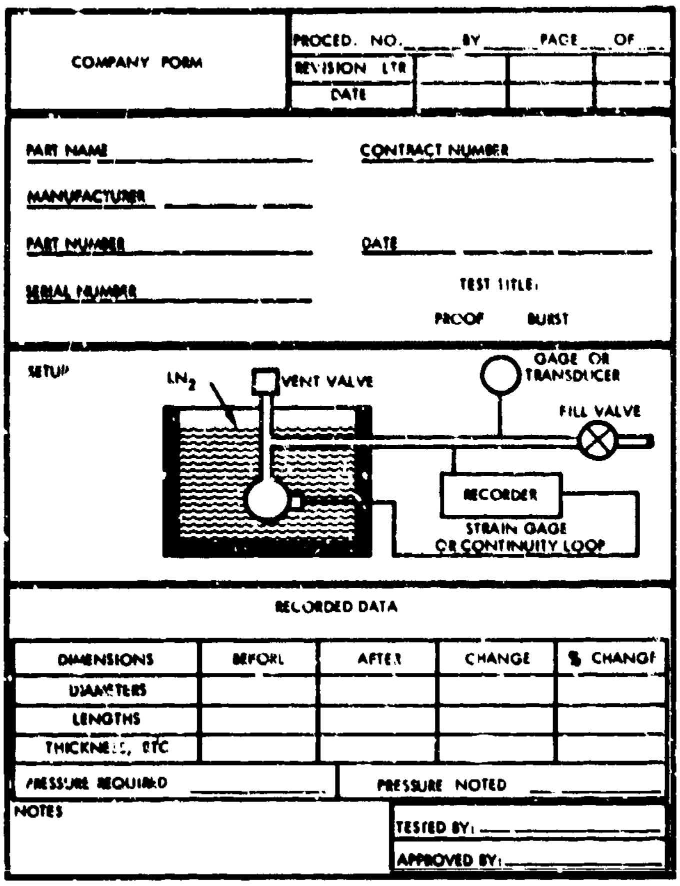


Figure 15.5.3.3b. Typical Data Sheet – Proof or Burst Test

Pressure data may be obtained by visual observation of the gauge or by means of a transducer and a strip chart recorder. Strain may be recorded by use of bonded strain gauges or by the hydrostatic displacement technique described above. If the intent of the burst test is to take the unit to destruction, the precise point of failure may easily be recorded by use of a continuity circuit on the test specimen. The point of failure is indicated on the chart when the circuit is broken. If the pressure is being recorded, the trace will indicate the point of failure by pressure decay.

In some cases it is practical to provide a cryogenic environment for the specimen under test as shown in Figure 15.5.3.3b by immersing the unit in a tank of LN_2 or other suitable cryogen. This technique is of value when the necessary test pressure exceeds that which is available with the cryogenic test fluid, but usually requires gas as a pressurizing fluid. Gas pressurant must also not condense at cryogenic temperatures, so gaseous helium is generally used. Because the strength of many materials is significantly improved at low temperature, it is necessary to either test at the proper temperature or to make allowance for the decreased strength if the test is conducted at ambient temperatures. If the specimen is an insulated tank the immersion method may not be used, and filling and testing with cryogenic test fluid is required.

15.5.3.4 POST-TEST EXAMINATION. At the conclusion of a proof pressure test, the unit should be examined for evidence of distortion, permanent set, or other modes of failure. A similar examination is made at the conclusion of a burst test. In the event that the unit ruptures during the burst test, the fragments should be collected to determine the nature of the failure, with particular emphasis on examination of welded areas.

15.5.4 Leakage

Leakage rates vary from molecular flow at one extreme to a rate of several cubic feet per minute at the other. Molecular flow rates are encountered with static seals, shutoff valves, and diffusion phenomena. The larger flow rates are encountered in special valves for ground support equipment where relatively large leak rates are inherent in design (such as the bleed leakage encountered in a servo operated valve). A high leakage rate that is tolerable may be used because it permits more economical construction. The method used to detect or measure leakage will depend on the test medium and on the rate of leakage. Some of the specific methods for detecting or measuring leakage are discussed below. Most of the leak measurement techniques described in Detailed Topic 15.5.4.2 may also be used for leak detection.

15.5.4.1 LEAK DETECTION. An excellent compendium of commercially available leak detectors is contained in a report titled "Characteristics and Sources of Commercially Available Leak Detectors" by A. J. Bialous of the General Electric Company (Reference 46-42). This report published in June 1967, lists 22 different types of leak detectors, provides data sheets describing the operation and performance characteristics of each, and lists such data as the manufacturer's name and address, availability of the various units, price, and delivery. A discussion of those methods that are applicable to the leakage rate levels being considered is presented below.

Immersion. A form of a test which requires no special equipment involves a total immersion of the test specimen. The specimen should be immersed to a depth just sufficient to cover the entire surface. In some cases, immersion to a further depth would affect the reading due to the static head of fluid over the leak area. Water is a convenient and inexpensive test fluid to use with this procedure but has some disadvantages which may dictate the use of another fluid. For example, water may rust or corrode the test specimen or cause electrical problems and, in some cases, is difficult to remove at the conclusion of the test. Alcohol or Freon is sometimes used in lieu of water to eliminate these problems; these fluids have the added advantage of forming smaller bubbles and are therefore more sensitive. They can also be used at a lower temperature than water. External leakage testing of cryogenic apparatus may be accomplished by total immersion in liquid nitrogen. One of the difficulties normally encountered with this procedure is that the nitrogen usually boils rather violently for a considerable period of time before stabilization is reached. Furthermore, if there is any continuous input of heat, as from an electrical system or warm fluids flowing through the unit, the boiling will continue and make the detection of the leak impossible. This problem may be avoided by fitting the container with a plastic lid which has sufficient strength to maintain a slight positive pressure of several psi over the LN_2. The containment will change the boiling point of the liquid, causing the boiling to stop, which will then permit detection of the leak.

A method which is used to test electronic components (resistors) will be briefly discussed to illustrate a method which is potentially applicable to aerospace fluid components. In this test, the resistors are heated by immersing them in a heated bath of liquid. The heat from the bath causes the air or gas trapped inside the component to expand, and a differential pressure of approximately 3.5 psi can be achieved with water at 203°F (95°C) and 6.2 psi with silicon oil at 338°F (170°C). In an alternate method, the components are submerged in a liquid and a vacuum is applied to the bath which can be made to create a differential pressure of almost one atmosphere. In this procedure, care must be taken not to reduce the pressure to such a low value that the liquid would boil and make detection of leaks impossible.

Bubble Test. A bubble test is conducted by applying a special liquid to the area to be tested; if a leak is present, small bubbles form in the liquid. Ordinary soap solution may be used; however, the advantage of some commercial products is their compatibility with normally encountered fluid system media such as oxygen. In addition, most proprietary liquids leave no residue and are likely to be more consistent in operation than a soap solution. Solutions are available that operate to -65°F with a leak detection sensitivity of 10^{-5} sccs.

Sonic Leak Detector. Sonic leak detectors are very useful for disclosing leaks in pressurized (not evacuated) fluid systems. These units employ a highly directional probe which is aimed at the pipe or tubing being inspected. If a leak exists, the noise of the leakage is amplified by the unit, and the signal is transmitted to a head set or an external speaker. Portable units are available which permit rapid inspection of pipelines. These units do not permit quantitative measurements of leakage. (See Detailed Topic 5.17.6.3.)

Halogen Leak Detector. The halogen leak detector is used extensively in the production testing of commercial equipment such as household refrigerators. However, it is included here because the characteristics of the method are useful in testing certain aerospace components or equipment, e.g., any component tested with Freon.

The detector uses a red hot platinum or ceramic filament which emits positive ions. The presence of small traces of halogen vapors markedly increases the emission of positive ions. The increase in emission causes a change in a meter reading or actuates relays which, in turn, may be connected to various audio amplifiers and other signalling equipment. Freon is a liquid containing both chlorine and fluorine (halogens) and may be readily used as a tracer gas.

One of the most valuable characteristics of the halogen method is its extreme sensitivity, which is on the order of one part per billion of halogen in air. This corresponds to a leakage rate of 1×10^{-9} sccs.

The equipment for conducting this test is simple, portable, and relatively inexpensive (prices range from $100 to $1000). General Electric and Deveco Engineering are two manufacturers.

The greatest advantage of the heated anode halogen leak detector is that the detector operates in air, unlike a mass spectrometer which requires a high vacuum and the associated expensive vacuum-producing equipment. A high level of skill is not required for operation of the unit and personnel may be trained to use it in a short period of time. The unit may be very advantageously used in detecting small leaks in hydraulic components. The hydraulic oil used in such components tends to clog very small leaks which may thus escape detection for a long period of time. However, halogen-containing gases are soluble in this oil and will diffuse through the leak path and be identified by the detector.

The chief disadvantage of this system is that the detector will respond to any gas which contains a halogen compound. Examples of compounds which could contaminate the area are solder fluxes, cleaning compounds, and aerosol container propellants. Provisions must be made to ensure that such backgrounds are eliminated from the test area. The halogens used in testing are of a very high molecular weight, and therefore the diffusion rate is very slow. If a large system is to be tested, provisions should be made to ensure that sufficient time has been allowed for migration of the tracer gas to all parts of the system; otherwise, the system must be evacuated prior to injection of the tracer gas. Also, because of the low diffusion rate, the tracer gas may be trapped in cavities and give a false reading during a subsequent test, even though the latter test is conducted many hours after the initial test. Finally, as the element of the unit operates at both a high temperature and a high voltage, care must be exercised to ensure that the element is never inserted into an explosive atmosphere. Proper protection for personnel should be provided through proper grounding.

Chemical Indicators (Dye). Numerous chemical compounds which detect leakage or defects by color indication are on the market. Some dyes are used internally and consist of either water or oil-soluble compounds which are used with the normal system operating fluid. In the event of a leak, a stain will appear externally at the location. The sensitivity of these dyes can be extended by the use of an ultraviolet or a fluorescent light. Other chemicals are available for spraying or applying to the external surface of a specimen, and if a crack or hole exists the chemical will penetrate the defect, which is then readily detected by the use of a fluorescent or ultraviolet light. Care must be taken to assure that the fluid being used is compatible with the flow medium of the unit being tested.

Chemical Indicators (Reagent). Chemical reagent-type detectors are available for detecting leaks of specific gases. In some units a color change is effected in a sensor, and this change in color may be detected by a photoelectric cell which is used to actuate another device such as an alarm or ventilating system. Another apparatus involves the use of reagent tubes which are available for many different specific gases. A sample is drawn through the tube, and the change in color is compared to a calibrated chart to approximate the concentration. The sensitivity of this device ranges from 40 to 250,000 ppm.

Odorizing Agent. An odorizing agent which helps detect leaks may be added to a gas system. A familiar application of this method is found in gas used for domestic service, which is odorized to warn householders of leaks.

15.5.4.2 LEAK MEASUREMENT. Most of the following leak measurement techniques may also be used for leak detection.

Flow Meters. Many suitable flow meters, as described in Sub-Section 5.17, may be used for leakage measurement. The tapered glass tube and float types are frequently convenient to give an instantaneous reading; a sufficient range of size is available to handle most of the normally-encountered leakage situations. This type of meter is more convenient than an orifice meter for measuring leakage rates since it is more compact and simpler to set up.

Pressure Change. The system or component may be either pressurized or evacuated and the gas leakage calculated by:

$$Q = \frac{V\Delta P}{T}$$

where

Q = volumetric leakage

V = system volume

ΔP = pressure change during test

T = duration of the test

Any consistent system of units may be used.

The accuracy of the procedure is high in the pressurized mode; however, in the evacuated mode, possible outgassing of the specimen during the test may reduce the accuracy.

Two difficulties are encountered with the pressure change procedure. The first is accurately determining the volume of the system or the component, and the second is determining what temperature changes, if any, have taken place during the test. In some cases a small component may be filled with a known volume or weight of liquid. On a large system, known leakage may be added. From this calibrated leak and the changes in the system pressure that result, the internal volume of the system may be calculated.

Temperature errors may be minimized in several ways. In a small system, the test may be conducted in a temperature-controlled area. Also, the system or unit may be instrumented at various points to provide temperature data. On a large system, a reference volume may be installed at one or more points within the system; pressure changes in the reference volume may be determined and the temperature of the test specimen may be calculated.

The pressure change technique has been used to test extremely large systems such as a 190-foot diameter sphere of 3.0 million cubic feet volume containing a nuclear reactor. The procedure has been prepared as a specification by the Standards Committee of the American Nuclear Society for Leakage Rate Testing of Containment Structures of Nuclear Reactors (Reference 46-41).

Weight Change. Weight of the fluid may be used as a measure of leakage. If the fluid is a gas, the weight change of the vessel is determined. If the fluid is a liquid, the change in the weight of the vessel may be determined or the effluent may be collected over a period of time and the flow rate calculated.

Liquid Level Change. Change in liquid level over a period of time may be used to determine leak rate if the liquid holding vessel can be calibrated. In a vertical cylindrical tank, a plastic tube may be connected to the top and bottom; changes in liquid level may be observed in the tube. When using this procedure it may be necessary to consider the change of pressure on the component along with the change of liquid level. If the tank is pressurized from an external source, the change in head may be insignificant as compared with the total pressure.

Volume Displacement. Volumetric displacement of a fluid is a common method of leakage measurement. One commercial device, described in Reference 46-42, incorporates a precision adjustable piston. The device is connected to the test specimen, and after a known period of time, the piston is adjusted to restore the original system pressure. The displacement of the piston is read on a dial as a volume in cubic inches. The volumetric change is then converted to a leakage rate. The claimed adjustment ranges from 10^{-6} to 0.88 cubic inch with an accuracy of 0.1 psia. A less sophisticated method of measuring leakage by volumetric displacement involves the use of a conventional buret and a suitable tubing connection. Two methods of using these burets are described.

For leakage of approximately 100 scch a conventional method of displacing water from an inverted buret is satisfactory using a 100 milliliter buret. However, for smaller leakage, tubes ranging from 1 to 10 milliliters are used to reduce the test time. With the smaller tubing, a problem is encountered with large gas bubbles which tend to stick at the base of the tube. This difficulty may be overcome by using a leveling buret procedure in which the gas is introduced at the top of the tube and level is maintained by manually adjusting the height of the reservoir as shown in Figure 15.5.4.2a. This method yields an accuracy of approximately ±5 with readings as low as 1 scch.

Water displacement is a common method of measuring gas leakage. A plastic tube is connected to a convenient part on the component; the other end of the plastic tube is inserted in an inverted buret which is filled with water. External leakage of a valve or component may be measured in this manner by enveloping the component in a plastic bag to which the plastic tube is secured by tape. The other end of the plastic tube is inserted in the buret. This method may be used in hazardous areas. The amount of water displaced may be measured when it is convenient or safe.

A similar procedure may be used to check the leakage of a flanged joint. The flanged joint is sealed with masking tape through which the plastic tube is inserted and the measurement conducted as before. If a quantitative reading is not required, the flange may be taped and the leakage observed through a small hole in the tape to which a leak detector solution has been applied.

Capillary Tube Flow Measurement. One of the common testing methods uses a horizontal capillary tube as illustrated in Figure 15.5.4.2b. A 1.5 millimeter glass capillary

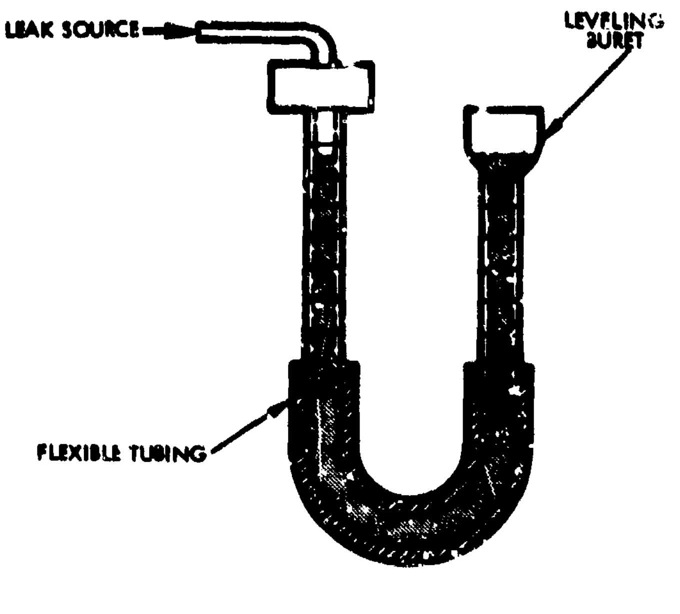


Figure 15.5.4.2a. Leakage Test Method, Using Leveling Buret

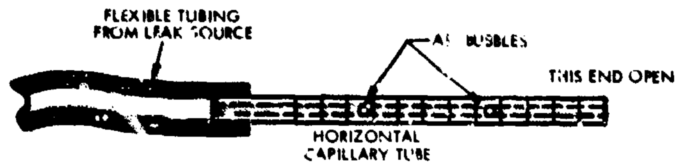


Figure 15.5.4.2b. Leak Test Method, Using Leveling Buret

may be used to measure leakage rates from 10^{-2} to 1 sccs and a 5 millimeter glass capillary may be used for measuring leakage rates from 10^{-4} to 10^{-2} sccs. In operation, the capillary is filled with water and shaken to generate one or more liquid plugs within the tube. The tube is held in a horizontal position and leakage is determined by timing the movement of the plug (or air bubble) between the calibrated marks along the tube. The inside of the tube should be clean to minimize errors; errors may be reduced further by coating the inside of the tube with an organo silicone compound. This compound prevents the water from wetting the glass.

In using any of the above procedures, it is mandatory that either temperature changes be kept to a minimum or the temperatures recorded so that the necessary corrections may be effected.

Bubbleometer. Another form of bubble test involves the use of a *Bubbleometer*, manufactured by the Bubble O Meter Company of Temple City, California. This inexpensive device consists of a glass tube having three diameters along its length, which in effect gives three sensitivity ranges. See Figure 15.5.4.2c. The rubber bulb at the bottom is filled with soap solution, and when the bulb is squeezed, one or more films will form in the tube. The rate of movement of these films provides an accurate indication of the leak rate. A nomograph is provided with the device to facilitate conversion to volumetric flow units.

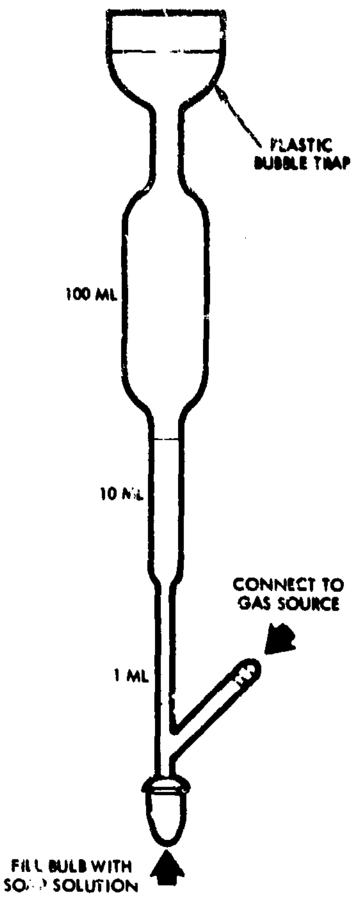


Figure 15.5.4.2c. Leak Test Method, Using Bubbleometer
(Courtesy of Bubble O Meter Company, Temple City, California)

Mass Spectrometer. The mass spectrometer (see Detailed Topic 5.17.7.2) ionizes molecules and separates them according to their mass in a magnetic and electrostatic field. In leak testing, the mass spectrometer is used as a detector for a tracer gas, usually helium. When a leak occurs, an increase in current in the spectrometer tube results. The current is read out by a meter calibrated in terms of mass flow. The mass spectrometer is one of the most commonly used leak testing devices and has a sensitivity of one part of helium per ten million parts of gas.

Gas molecules from a sample drawn from the unit are ionized by a bombarding beam of electrons. Resultant positive ions are accelerated by the influence of high voltage into a magnetic field. The direction of the magnetic field is perpendicular to the plane of the main path of the ions. Under the influence of this magnetic field, ions of different masses travel on arcs of different radii; ions of a preselected mass may be directed and collected on an electrode. From the electrode the ions leak to ground through a high value resistor. The current generated by this passage of ions is amplified and read out on a meter calibrated in suitable units.

Helium is used as a trace gas for four principal reasons:

1) **Helium's low molecular weight permits it to diffuse through a leak with greater ease than any other gas except hydrogen**

2) **Helium occurs in the atmosphere to an extent of only one part in 200,000, minimizing background effects**

3) **There is little possibility that an ion from another gas would give an indication that would be mistaken for helium because of the great difference between the weight of helium and that of any normally-encountered gas**

4) **Helium's low molecular weight makes possible a simple construction for the mass spectrometer.**

There are numerous procedures for leak detection with the mass spectrometer, but basically the procedure is either to sense outward leakage from a pressurized system or inward leakage to a system that has been evacuated. A pressurized tank which contains a source of helium and is probed on the outside is an example of the former procedure; a direct connection to an evacuated vessel which is sprayed on the outside with helium is an example of the latter procedure. Television picture tubes are normally tested by connecting a detector to the evacuated tube after which helium is sprayed over the outside of the tube. Any indication on the detector is evidence of a leak. See Figure 15.5.4.2d.

Relatively little skill and training are involved in the use of the mass spectrometer on routine production tests. This presumes that skilled personnel are available for maintenance and checkout. On more complex test setups, a skilled operator is required to cope with such variables as changes in temperature, sensitivity, and the original calibration.

Radioisotope Procedures. A procedure known as *rad'flo* has been developed for the mass production testing of small electrical components such as sealed transistors and relays. Since the initial expenditure for the equipment is high and it is more suited to high production volumes, the method is not likely to find a great deal of application in the manufacture or testing of aerospace fluid components. However, it is mentioned and described because the technique could be applicable in special situations.

The procedure consists of putting the components to be tested inside a tank which is then sealed and evacuated to about 2 torr. Krypton 85, which has been diluted with air, is then pumped into the tank under pressure. The radioactive gas diffuses into any leaks in the components, and after a prescribed soaking period from a few minutes to several hundred hours, the krypton is pumped out of the tank and stored for reuse. The tank is then flushed with air, and the components are removed and scanned for radiation.

Under the proper operating conditions, the sensitivity of the technique may be as high as 10^{-13} sccs, and this high sensitivity constitutes one of the chief advantages of the method. A more complete discussion of the technique is contained in Reference 350-8.

15.5.4.3 LEAKAGE MEASUREMENT CORRELATION.

Most leakage testing is conducted using an inert gas, usually helium or nitrogen, to perform the actual leakage measurement. It is often desirable to know what the leakage characteristic of the component will be with another fluid, either a gas or a liquid, under various

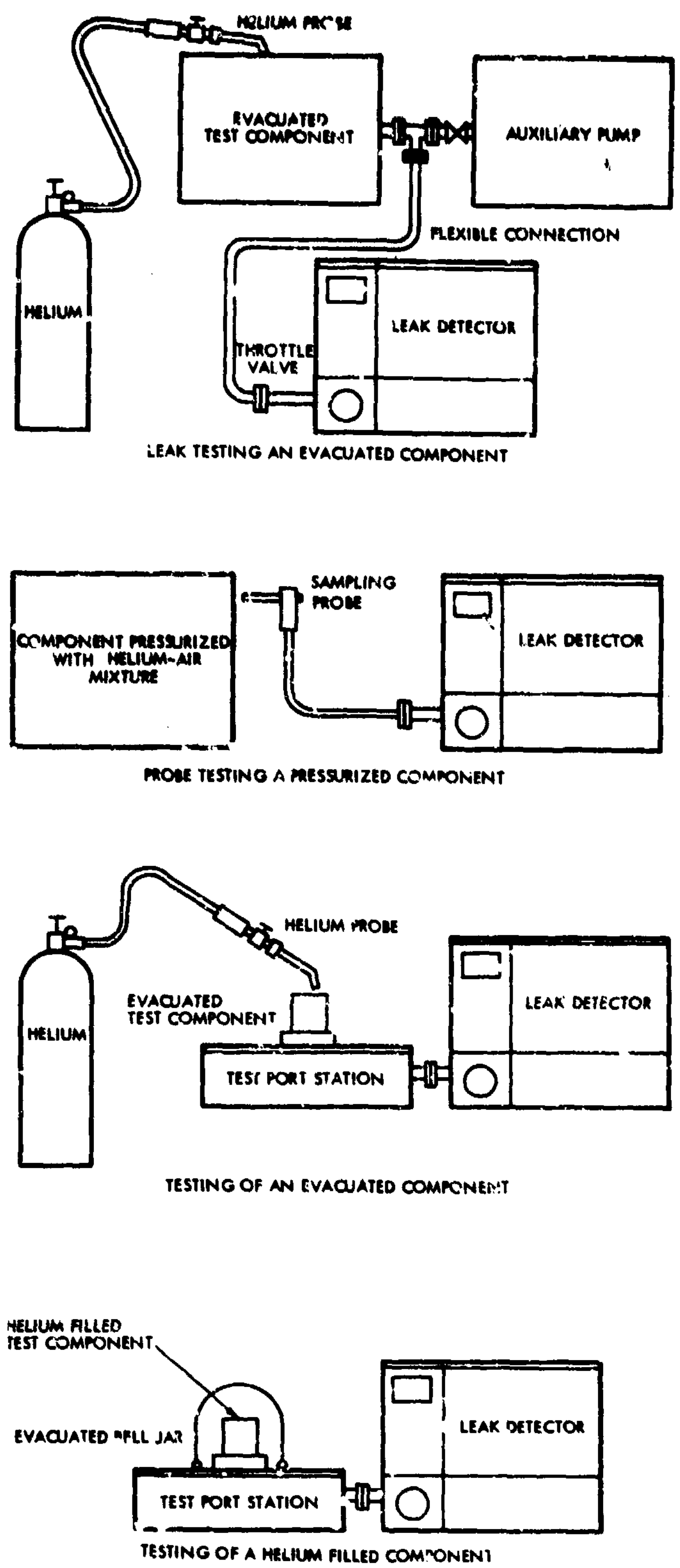


Figure 15.5.4.2d. Mass Spectrometer Gas Leak Detector
(Courtesy of Consolidated Electro Dynamics Corporation, Pasadena, California)

operating conditions. Accurate prediction of operating fluid leakage based upon leak test data is extremely difficult for the following reasons:

a) The geometry of the leak is usually unknown.

b) The actual leakage rate is a function of the flow regime or flow regimes, which are in turn a function of fluid properties, leak path geometry, and pressure.

c) The geometry of the leak path may be changed with pressure.

d) With liquids, surface tension effects may cause complete plugging of the leak path resulting in true zero leakage.

e) Numerous other phenomena such as wall wetting, surface absorption, chemisorption, fluid evaporation, polarization, and seal permeation, may all combine to influence actual results.

Zero Leakage. The first serious attempt at providing a criterion for zero liquid leakage and for correlating gaseous leakage flow test data was provided in Reference 12-10. Reference 12-10 defines *zero liquid leakage* as that value of liquid leak or flow rate at which the surface tension of the liquid has just overcome the pressure acting on the liquid and no flow occurs, assuming a given pressure and leak path diameter. Zero *gas leakage* as such does not exist as far as laboratory measurements have thus far been able to determine because of the limitations of laboratory instruments. Therefore an arbitrary curve was constructed as shown in Figure 15.5.4.3a for use as a specification standard.

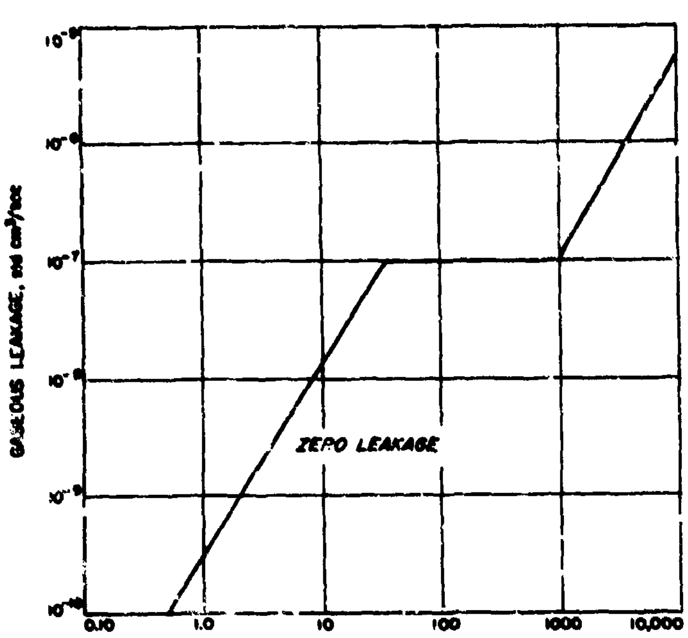


Figure 15.5.4.3a. Zero Gas Leakage Arbitrary Definition (Reference 12-10)

Figure 15.5.4.3a is a straight, sloped curve with a discontinuity at the leakage value of 1×10^{-7} std cm^3/sec at which point the line is translated but maintains its original slope. The lower portion of the curve is based on the basic point of 1 std cm^3/yr at 1 atm differential pressure. Other points making up that portion of the curve were obtained from the correlation of the 1 std cm^3/yr with equivalent flow rates at the other pressures using the fluid conversion graph. However, the knowledge of future propulsion system requirements dictated that the maximum acceptable equivalent leakage, as originally constructed, was too great at the higher pressures. Hence, the arbitrary decision was made to shift a part of the curve upward at 1×10^{-7} std cm^3/sec.

Gas/Liquid Leakage Correlation. Two basic methods are presently available for predicting liquid leakage based upon gaseous leak test data. Both methods are valid *only* in the *laminar flow regime*, which many authors have shown to be the predominate flow regime for leaks ranging from 10^{-1} to 10^{-6} atm cc/sec (Reference 46-41). Both of these techniques have been studied by the General Electric Company under NASA contracts and are presented here in very abbreviated form. Further details may be obtained from References 12-10, 46-41, and 46-15.

Figure 15.5.4.3b shows a simplified version of the laminar conversion graph from Reference 12-10 with a sample problem to illustrate its use. The guidelines with a slope of 2:1 located on the right-hand side represent the gas flow equations; while the guidelines having a 1:1 slope located on the left-hand side represent the liquid flow equation. Correlating one fluid to the other requires drawing lines parallel to the guidelines. The discontinuity found between the gas flow and liquid flow guidelines is only the result of the original drawing style. However, a transition between gas and liquid flow is illustrated in the nomograph. The gradual bend represents exhaustion to atmospheric conditions. A sharp or sudden change from liquid to gas, or vice versa, represents exhaustion to vacuum.

Working the sample problem shown in Figure 15.5.4.3b will illustrate the application of the graph to predict the equivalent liquid propellant leak rate from measured gaseous test helium leakage at a seal. Assume the following conditions:

1) Gaseous helium leakage past a seal at 8×10^{-6} std cm^3/sec.

2) Helium test pressure of 1 atmosphere (atm) or 14.7 psia with vacuum on the downstream side of the leak, the ΔP across the leak being 14.7 psia.

3) Liquid system flow pressure of 10 atm or 147 psia with vacuum externally, or 147 psia ΔP across the leak.

4) Liquid viscosity of 8×10^{-1} centipoises (cp).

To solve the problem of predicting the equivalent liquid leak rate for the known gaseous leakage at the seal, the following plots are made on the graph:

1) 8×10^{-6} std cm^3/sec helium leakage is located on the right-hand ordinate (gas flow).

2) A horizontal line is drawn intersecting the helium viscosity value along the abscissa (gas viscosity).

3) A line parallel with the 2:1 slope is extended from the helium viscosity value until 1 atm pressure is intersected along the Δ pressure abscissa.

4) This pressure point is connected with the liquid system pressure (10 atm) by a horizontal line.

5) A line is drawn parallel with the 1:1 slope graph lines from the previously found liquid pressure value until the

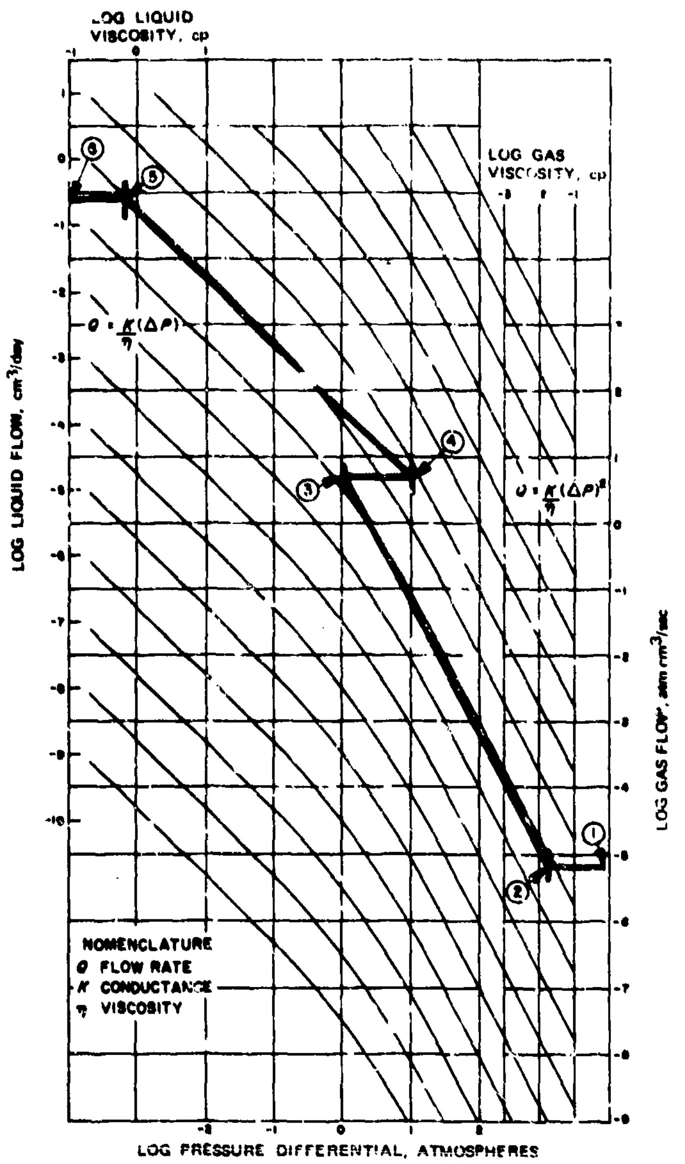


Figure 15.5.4.3b. Simplified Fluid Flow Conversion Graph
(Reference: 12-10 and 46-41; originally copyrighted 1956 by the General Electric Company Schenectady, New York)

intersection is made with the value for the liquid viscosity along the liquid viscosity abscissa

6) A horizontal line is drawn from the intersection of liquid flow with liquid viscosity to find the predicted liquid leakage along the log liquid flow ordinate: 3×10^{-1} cm^3/day.

Figure 15.5.4.3b provides a graphical solution for correlation at any pressure. Any temperature may be considered merely by selecting that corresponding value of viscosity (Reference 12-10).

A gas-liquid leakage nomograph from Reference 46-45 is presented in Figure 15.5.4.3c. This nomograph also applies only to laminar flow leakage but is of practical value because the leakage of a gas from atmospheric pressure to vacuum is often dominated by the laminar flow mode in typical hardware leaks. Although certain definite units are assigned to the axes, the nomograph is really much more flexible. For example, the viscosities could be read in poise units rather than centipoise units and the pressure axis could be read in atmospheres if the gas leakage axis is read in atm cm^3/sec. Table 15.5.4.3 shows comparative leak rates. It is emphasized that this procedure will be accurate only in laminar flow leaks. Should the measured leakage be molecular rather than laminar, the error introduced in the calculation will predict a greater liquid leakage than will actually be found. The procedure may therefore be used with confidence, since any error will add a margin of safety into the results.

Table 15.5.4.3. Comparative Leak Rates

Leakage Rate Unit	Relative Magnitude*
3 std atm cc/10 years	0.95
1×10^{-8} std atm cc/sec	1.00
1 std atm cc/year	3.17
1 std atm in^3/year	52.00
1 micron ft^3/hour	1034.00
1 std atm cc/hour	27,800.00
1 micron ft^3/minute	62,200.00
1 micron liter/sec	131,600.00
1 std atm in^3/hour	455,000.00
1 micron ft^3/sec	3,730,000.00
1 std atm cc/sec	100,000,000.00
1 torr liter/sec	131,600,000.00
1 std atm in^3/sec	163,900,000.00

*1×10^{-8} std atm cc/sec selected as unity for comparison purposes.

NOTE: Appendix A, Table A.2.31 contains basic leak rate conversion factors.

The following restrictions apply to the use of the above method:

1) The leakage is the result of a finite hole or holes and not the result of permeation.

2) The gas flow is laminar, i.e. the flow through the leak is in the range of 1 to 10^{-6} atm cc/sec or is made up of a number of leaks in that flow range.

3) The calculations should at best be considered accurate to only a factor of two. Error in both the measurements and the deviations from the flow equations preclude more accurate solution.

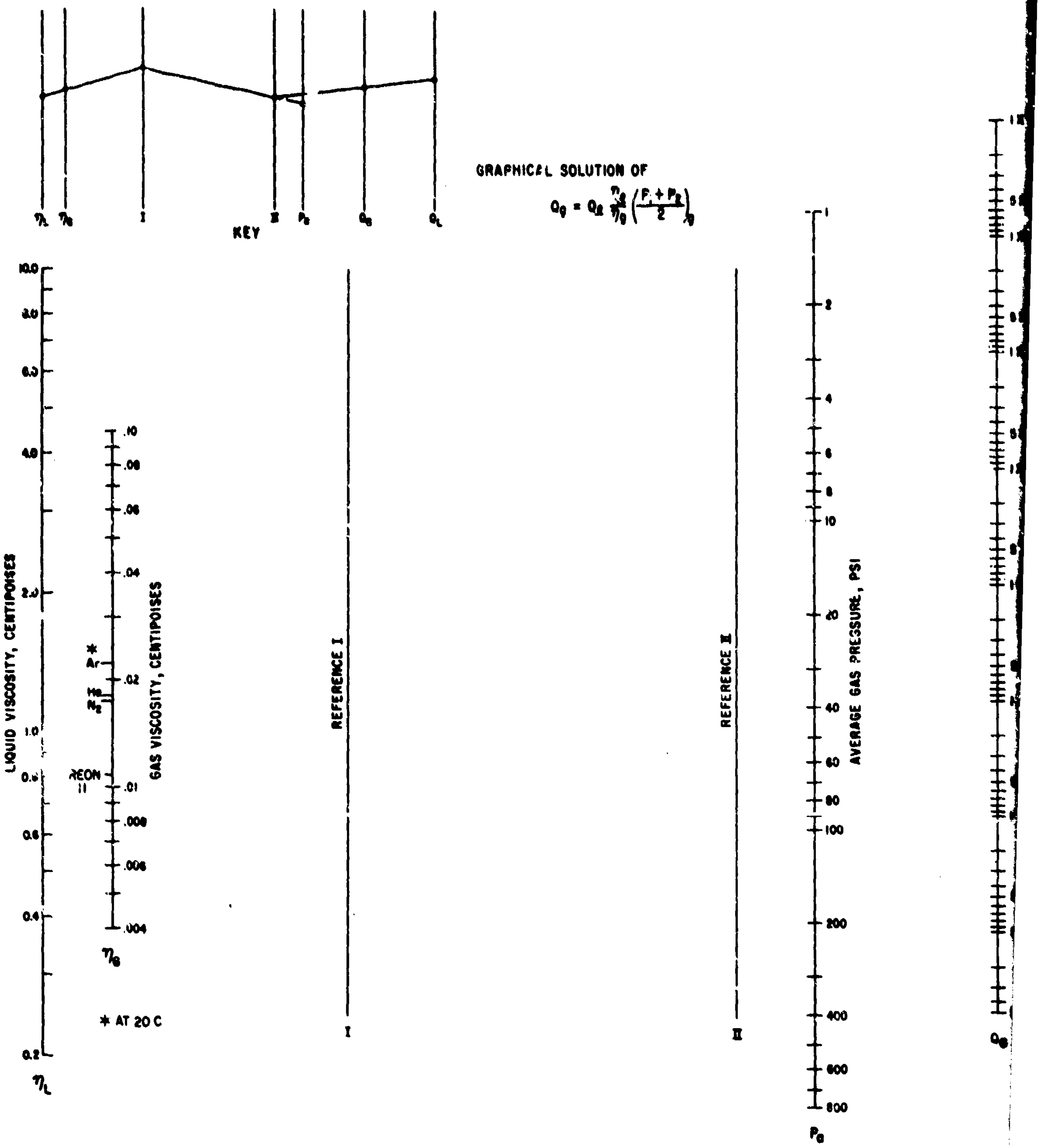


Figure 15.5.4.3c. Gas/Liquid Leakage Conversion Nomograph
(Reference 46-45)

GAS/LIQUID LEAK CORRELATION NOMOGRAPH

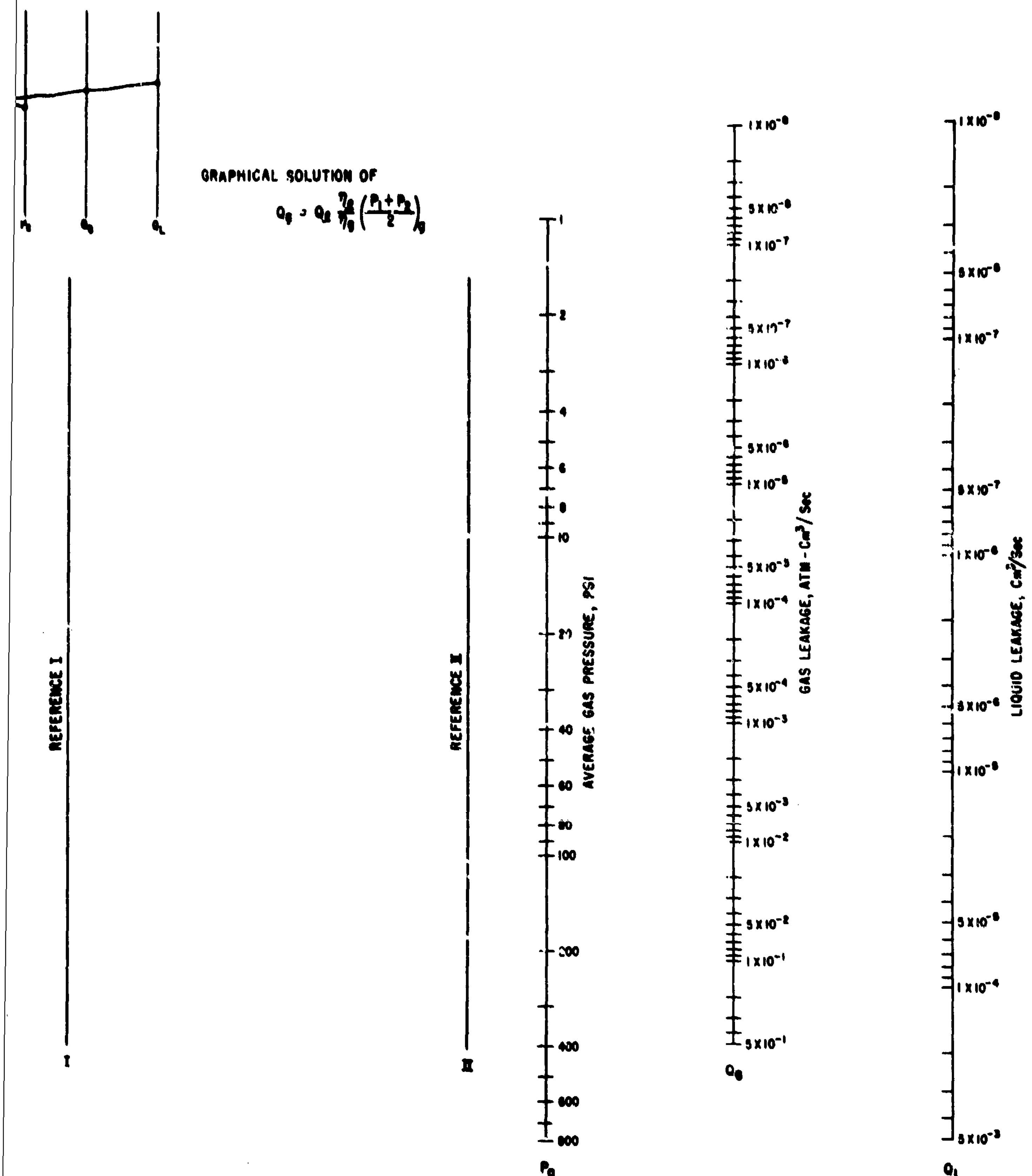


Figure 15.5.4.3c. Gas/Liquid Leakage Conversion Nomograph
(Reference 46-48)

B

If gas and liquid laminar flow equations are combined, the ratio of gas to liquid leakage through the same leak at the same pressure is

$$\frac{Q}{Q_v} = \frac{\eta_{liq.} P_a}{\eta_{gas}} \qquad \text{(Eq 15.5.4.3)}$$

This is a convenient method of determining what liquid leakage will exist if the gas leakage is measured at the same pressure.

Experiments were recently performed (Reference 46-41) to check the validity of the above correlation. Liquid leakage was measured for leaks with previously measured gas leak rates. It was found that leaks having a gas conductance in the 10^{-3} atm. cc/sec range had a liquid leakage approximately one-half that predicted by Equation (15.5.4.3). Leaks in the 10^{-4} atm cc/sec range leaked liquid at a rate approximately one-tenth the rate predicted by theory. Liquid leakage in leaks in the 10^{-5} atm cc/sec atm range were approximately one-twentieth of that predicted by the above equation.

Based on the above, it would appear that these methods of correlation will produce conservative answers, i.e. the actual liquid leakage will always be smaller than that predicted by theory.

It is believed that the liquid flow is lower than calculated for two reasons:

1) No correction was made for any molecular flow component of the measured gas leakage.

2) Physical adsorption completely immobilized a layer of liquid adjacent to the leak wall and therefore reduced the apparent leak diameter.

15.5.5 Flow and Pressure Drop

15.5.5.1 INTRODUCTION. The primary reason for running a flow ΔP test is to determine that the component or system will pass the specified rate of flow within the allowable pressure drop limits. However, there are other reasons for running a flow test. Erosion of the component housing or of a valve seat may be important for extended durations of flow; this is particularly true in components flowing gases such as hydrogen or helium at sonic velocities. The sonic velocity of helium is approximately 3300 ft/sec at 70°F (the velocity of an M1 rifle bullet is 2700 ft/sec). If the gas contains any particulate contamination, considerable erosion may result in a relatively short time. Dynamic loads on elements of the component, i.e., the disc of a butterfly valve, may be of interest during a flow test. Flutter problems may be encountered or the torque on the disc may be in excess of that being provided by the valve operator, which will affect the performance of the valve. Dynamic forces caused by the flow of the fluid may dislodge a seal which would not be affected by static pressure. Only by flow testing may water hammer effects of rapid valve opening or closing be measured.

15.5.5.2 EQUIPMENT REQUIRED. Equipment requirements vary with the size and nature of the component. A pressurized medium, sufficient controls to regulate the flow, and adequate instrumentation are generally required.

15.5.5.3 SETUP AND PROCEDURE. It is extremely important in conducting pressure drop tests to observe the requirements for pressure tap location and geometry, as large errors may result if the taps are not properly located and designed. This matter has been the subject of analysis and test for many years, and the recommendations set forth should be followed precisely. The usual standard employed is an ASME publication entitled "Fluid Meters -- Their Theory and Application" (Reference 68-1). This report specifies in detail the design and location of the taps. When using water as a test medium, care should be taken to increase the downstream pressure enough to suppress cavitation. Also, generally 3 tests are made — one at nominal flow and at 10 percent above and 10 percent below nominal flow. If the flow factors (computated) do not show a square power relationship, then cavitation has altered the results. There are some exceptions to this rule such as convoluted flow sections.

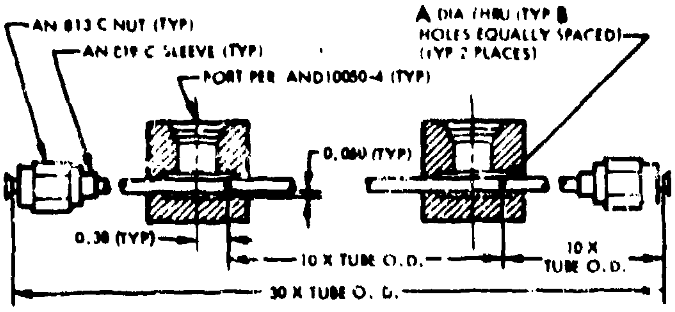
If a pressure drop across a given component is critical, it is usually necessary to conduct a tare test prior to conducting the main flow test. This test is conducted by installing a dummy spool piece of precisely the same length and port size as the component to be tested. A test at the specified flow rate is conducted and the pressure drop across the spool piece (tare value) is noted. The test specimen is then installed and a gross pressure drop test is conducted. The tare value is subtracted from the gross value obtained in the latter test to give the net pressure drop across the specimen. If a tare test is not conducted, the indicated pressure drop will be higher than the actual because of the pressure drop across the additional length of tubing required to accommodate the pressure taps. In many low-velocity systems the pressure drop across the spool piece is negligible.

An alternate method of measuring pressure drop eliminates the need for conducting the tare test and is very useful in many test situations. The method is completely described in SAE ARP 868 for the specific case of measuring pressure drop through aircraft fuel system components. The method involves the use of double piezometer tubes as shown in Figure 15.5.5.3a. Typical tubes are constructed in accordance with the drawing shown in Figure 15.5.5.3b (courtesy of Accessory Products Co.). As can be seen from the drawing and the sketch, the distances between the drilled holes in the piezometer ring and between the piezometer rings and the test specimen are equal. It then follows

$$\begin{aligned}
\Delta P_3 &= \Delta P_2 - \Delta P_1 \\
\Delta P &= \Delta P_1 - \Delta P_3 \\
&= \Delta P_1 - (\Delta P_2 - \Delta P_1) \\
&= 2\Delta P_1 - \Delta P_2
\end{aligned}$$

15.5.6 Crack and Reseat

Crack and reseat pressure tests are used with vent and relief valves (Sub-Section 5.5), as well as on-off pressure regulators (Sub-Section 5.4). Cracking pressure is sometimes defined as the pressure at which flow of any magnitude in excess of the allowable leakage is observed as pressure is increased and is sometimes expressed as a percentage of full rated flow. Reseat is usually defined as the pressure at which the specification leakage rate is not exceeded as pressure is decreased. Because the reseat pressure may vary, depending upon the magnitude of flow above crack leakage, the specification and procedure should be specific whether

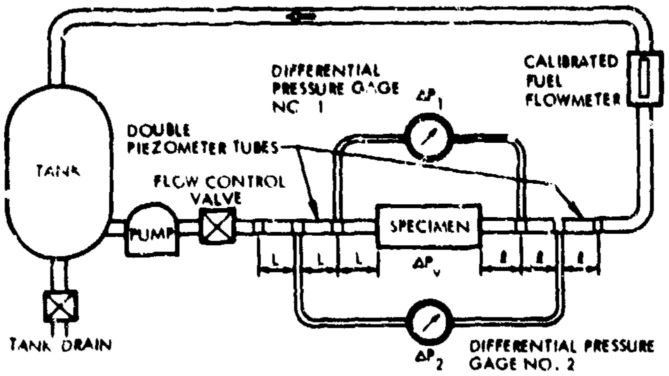


TUBE O.D.	A DIA	B HOLES
1/4	0.040	4
3/8	0.060	4
1/2	0.060	6
5/8	0.098	6
3/4	0.078	6
1	0.098	6

TUBE O.D.	A DIA	B HOLES
1-1/4	0.098	6
1-1/2	0.098	6
1-3/4	0.098	6
2	0.093	6
2-1/2	0.098	6
3	0.098	6

NOTE: BORE AND HOLES MUST BE CLEAN AND FREE OF BURRS.

Figure 15.5.6.3a. Pressure Drop Test Setup
(Reprinted with permission from Aerospace Recommended Practice 868, Society of Automotive Engineers, New York, New York)



L = 10 DIAMETERS OF INLET SIZE TUBE
ℓ = 10 DIAMETERS OF OUTLET SIZE TUBE
ΔP_1 = STATIC PRESSURE DROP READ ON GAGE NO. 1
ΔP_2 = STATIC PRESSURE DROP READ ON GAGE NO. 2
ΔP_v = COMPONENT STATIC PRESSURE DROP = $2\Delta P_1 - \Delta P_2$

Figure 15.5.6.3b. Double Piezometer Tube Detail
(Reprinted with permission from Aerospace Recommended Practice 868, Society of Automotive Engineers, New York, New York)

or not full rated flow is required between crack pressure and reseat pressure tests.

Determining the reseat points may be accomplished by slowly raising or lowering the pressure and measuring the flow rate. If it is inconvenient to measure the flow rate, an alternate method may be used in which a mercury manometer is connected to the upstream port of the valve. The manometer will be steady and will read the static upstream pressure prior to crack. As the valve begins to crack, the slight flow will cause a perceptible drop in the static pressure. If the system pressure is too high to permit use of a manometer, a pressure transducer may be used. A suppressed scale should be employed with the readout covering only the range of valve operation. Snap action relief valves which employ belleville springs have a high opening and closing rate and must be tested with high response transducers. When acceptance testing check valves subsequent to a proof and leak test, it is important to ignore the first cracking pressure and to use the second actuation.

Hydraulic relief valves are specified for pressure at *rated flow* and *reseat*. Cracking pressure is unimportant since reseat is lower than cracking and reseating must be accomplished at a pressure which is higher than maximum pump compensator pressure.

15.5.7 Response

Response may be defined as the time required for an element to react to a signal. For example, response of a solenoid valve is defined as the time required for the valve to change from one mode to another upon command (see Detailed Topic 6.9.3.7). In a pressure regulator it is the time required for the regulator to achieve steady-state pressure after a step change has been made, e.g., startup of a system (see Detailed Topic 5.4.3.4). Response time for explosive valves is characteristically short and difficult to measure accurately, as discussed in Detailed Topic 5.7.8 2.

In measuring the response of a solenoid valve, use of transducers and recording equipment is generally required because of the very short intervals of time involved (a range of 2 to 50 ms is normally encountered). If the valve has position switches, a trace which indicates applied voltage and another which indicates position of the poppet or valving element is all that is needed. The response should not be measured by use of the position switches alone as this will neglect the time required for the solenoid coil to generate the necessary magnetic flux which may be a significant portion of the total. If the valve does not have position switches, a trace may be made of the applied voltage and downstream pressure. When conducting this test, it is convenient to use a recorder which has a quick change speed mechanism which permits a momentary high velocity of the paper. This velocity need be maintained for only a second or so and facilitates reading of the values. The response time may also be determined by monitoring the current on a triggering oscilloscope. Time is from initial rise of current trace to the dip in current trace (solenoid movement). An example of this technique is described in Reference 58-5 wherein solenoid valve response characteristics were evaluated for experiments subsequently performed on the Environmental Research Satellite (ERS) series satellites.

Actuation times, both opening and closing, were measured on the solenoid current trace. The trace was photographed as it appeared on an oscilloscope screen. Figure 15.5.7a is a schematic of the test apparatus. The test valves were cycled 5 to 10 cps with the pulse timer. The oscilloscope was synchronized with the timer to provide a steady image for photographing. Figure 15.5.7b simulates such a typical current trace from which opening and closing times were measured. The portion designated as Δt_1 represents the time from closing of the solenoid circuit to the start of poppet movement. As the poppet moves, counter emf produces a negative slope in the current trace, representing Δt_2. Buildup of solenoid field strength to overcome poppet spring and frictional forces, plus the time for total travel of the poppet, is defined as the opening time of the valve ($\Delta t_1 + \Delta t_2$). The closing time is that required for collapse of the solenoid field and return of the poppet to the closed position (Δt_3). Present practice entails photographing the oscilloscope trace of solenoid field current for valve opening response only, as shown in Figure 15.5.7c. Accurate representation of valve closing time requires photographing the trace of solenoid field voltage rather than current, as shown in Figure 15.5.7d.

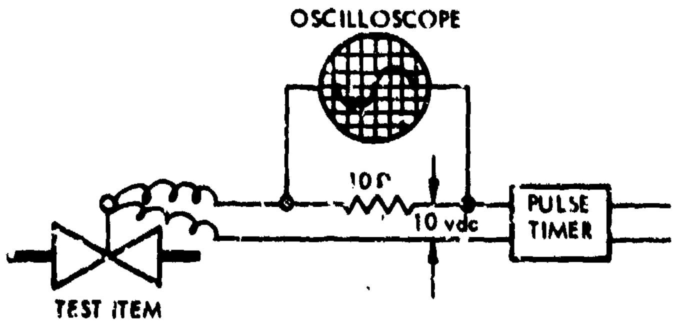


Figure 15.5.7a. Solenoid Valve Response Test Schematic for Obtaining Oscilloscope Current Trace
(Reference 58-5)

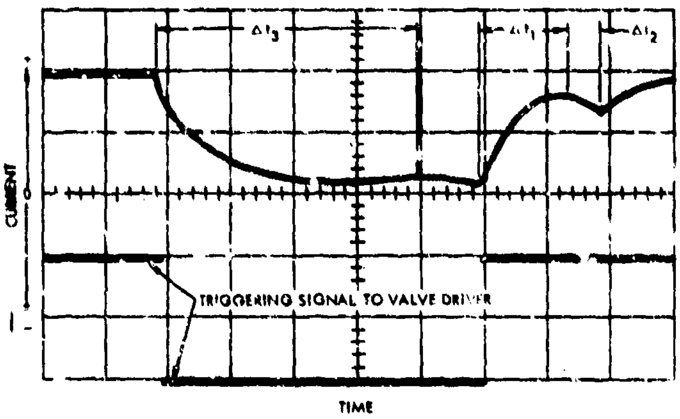


Figure 15.5.7b. Solenoid Valve Response as shown by Oscilloscope Current Trace
(Reference 58-5)

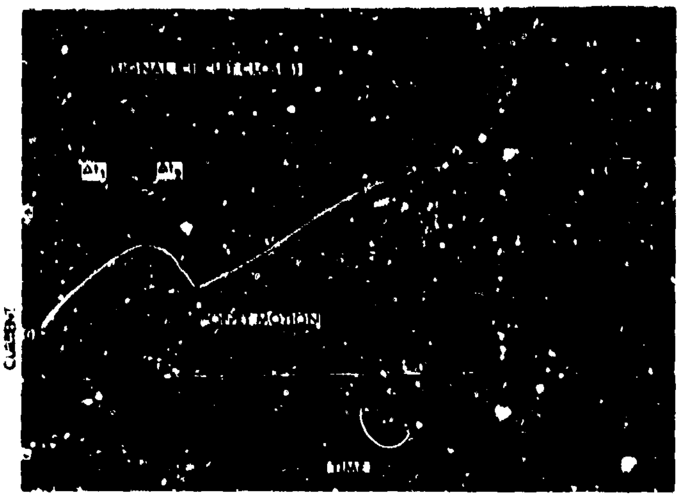


Figure 15.5.7c. Photograph of Oscilloscope Current Trace of Solenoid Valve Opening

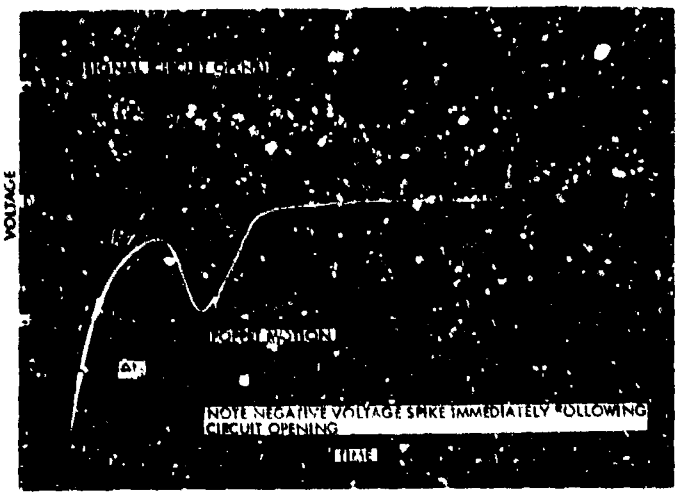


Figure 15.5.7d. Photograph of Oscilloscope Voltage Trace of Solenoid Valve Closing

The procedure for measuring the response of a regulator is similar, with the exception that no applied voltage is usually required. An exception to this is found when a regulator has a solenoid-operated pilot section which is integral with the main unit. In this event, the response is generally taken from the instant the voltage is applied until the regulator reaches steady state.

15.5.8 Pressure Regulation

Pressure regulation may be defined as a process of reducing some upstream, usually variable, high pressure to a fixed downstream pressure of lower value. The lower value is called the set point of the regulator. The tolerance band which specifies the values permissible both above and below the set point are always given. In conducting pressure regulation tests, it is extremely important to simulate both the upstream and downstream plumbing very accurately. Adequate flow must be provided upstream of the regulator, or the regulator will tend to oscillate. The downstream ullage must be neither too large nor too small, or erroneous results will be obtained. If the ullage is too small, the start transient may indicate a sharp spike on the pressure recording which will indicate an out-of-specification condition. This will happen when the response of the regulator is too slow to close the main valve in the unit before an overpressurized condition results. Conversely, if the ullage is too large in the test setup, the regulation control effected by the unit may appear to be better than it actually is. For example, the start transient may appear to be very smooth and within specification, but when the regulator is installed in the actual system an overpressurized condition may result for an unacceptably long period of time.

Recording of data during a pressure regulation test is ideally done on a multichannel strip recorder since the several outputs are all available on the same time base and the interaction between them can be readily analyzed. The parameters normally measured include upstream pressure, temperature, flow rate, and regulated pressure. It is common practice to use a suppressed scale for the regulated pressure trace to improve the readout. For example, if a regulator is regulating between 725 and 750 psi, the channels indicating the regulated pressure would be calibrated to show the 25 psi which is of interest instead of the entire range of zero to 750 psi.

15.5.9 Force

It is often necessary to measure the force applied to a component or, conversely, the force exerted by a component such as an actuating cylinder or pressurized diaphragm. Various methods of making these measurements are discussed.

An axially applied load may be measured with a spring scale, and either tensile or compressive loads may be measured. Laboratory quality scales are available which will measure loads ranging from a few grains to 50 pounds or more. Care must be taken to ensure that the axis of the scale is in line with the load being measured. The accuracy of these scales ranges from 1 to 2 percent of full scale.

Proving rings are used to measure axial loads of a relatively high magnitude. These rings are usually toroidal steel members with attachment fixtures or hooks at opposite points. The measured load is inferred from the deflection which is displayed on a dial indicator. When used within the elastic range, these rings have an accuracy of 0.1 percent of full scale. Calibration of the rings is accomplished with deadweights.

Deadweights are sometimes convenient for measuring force; the range of this method is limited only by the ability to determine the value of the weights being used. These values may range from a few milligrams to many tons. For the smaller values, the weights used for laboratory balances or for deadweight testing are convenient, readily available standards. For large loads, lead castings, steel or cast iron blocks, and concrete blocks may be prepared for a particular application. Their weight is determined by a suitable method and the data is permanently affixed to the mass by tag, stamping, stencil, etc. Weight of a large load may be determined on a certified truck scale, such as those operated by state highway departments. Water and sand provide convenient deadweight material since they are both inexpensive and easy to handle. If sand is used over a long period of time, care must be taken to maintain the moisture content at a constant value, since the weight will be drastically altered by either the addition or evaporation of water. Simple lever systems may be used advantageously with deadweights to increase or decrease the effect of the load. Use of geometric shapes having easily defined centroids will facilitate the computations if a lever system is used.

The load cell provides a very convenient and accurate means of measuring loads, and cells are available in a range from ounces to tons. As the output of the cell is a voltage, it may be displayed in several forms. One special application of the load cell is in a spring tester in which deflection of the spring and the load are measured simultaneously. The load-deflection voltages are fed to an X-Y plotter which prints out a stress-strain curve for the part.

The same tester may be used to determine the effective area of a capsule or diaphragm by introducing a known pressure into the cavity and measuring the resulting force. The pressure can be read with a high degree of precision with either a precision gauge or the output indicated on a deadweight tester. Load cells are calibrated by use of deadweights, and a calibration is usually performed before and after a test. A simple setup permits calibration of such a cell by remote control as often as desired and in a very short time. The setup consists of a suitable deadweight attached to the cell by a cord and pully arrangement. The weight rests on either an electric or pneumatic actuator. To apply the load to the cell, the actuator is driven downward until it separates from the weight, and the output of the cell for this load is recorded. The actuator is then raised, lifting the weight and thus removing the load. Such a setup has been used to measure thrust in a firing test of a small rocket engine being run in a vacuum chamber.

15.5.10 Torque

Torque is a torsional moment about a center produced by equal and opposite tangential forces and may be measured with devices as simple as a torque wrench. In critical applications it should be borne in mind that a torque wrench not only produces a couple or pure torsion on the part being measured but also imposes a transverse force component. This component is usually insignificant with respect to the torque and is ignored.

The accuracy of a torque measurement is dependent to a very large degree on several difficult-to-control factors. A film of lubricant on a part which should be tested dry can make a difference in the reading of approximately 25 percent. The fit and condition of the mating surfaces can also exert a very large influence on the values obtained. Care must be taken to ensure that the allowable limits of the material are not exceeded. The limits could be exceeded, for example, by applying a torque specified for a dry nut and bolt assembly to an assembly that was lubricated.

The purpose of a torque measurement on a bolted assembly is to permit calculation of the load in the member. A more accurate means of determining this load is to measure the strain in a bolt, as the variables mentioned above are eliminated. Special hollow bolts are available which permit the use of a depth micrometer to measure the strain. Some bolts incorporate a plug in the hollow center, and the strain is measured by noting the change in height of the plug above the bolt head.

Torque may also be measured with good accuracy by use of a lever and weights if the distance from the axis of rotation to the centroid of the weights is known. If the line of action between the lever and the weights varies from 90 degrees, the actual resultant of the downward force must be calculated.

A spring may be substituted for the weights if less accuracy is tolerable. Loss of accuracy is caused by the difficulty of applying a steady load and taking a reading when motion is impending.

15.5.11 Life Cycle

The purpose of the life cycle test is to ensure that the unit can be operated a sufficient number of times to fulfill its service function. Two types of tests may be specified. In one, an arbitrary number of cycles is selected, and in the other the unit is simply cycled to failure. The latter test is conducted more often in reliability studies to determine the margin that exists in a particular component. The life cycle test is generally conducted at the end of a program, and no other testing should be expected of this unit with the possible exception of the burst test. Sometimes it is advisable to run this test in two halves with vibration between the first and second halves.

15.5.11.1 EQUIPMENT REQUIRED. Most life cycle tests are accomplished with the use of automatic cycling equipment. This equipment is relatively inexpensive and is normally found in most testing laboratories. It may consist of either electronic or electromechanical devices which can be arranged to actuate one or more control valves, solenoids, or other circuits in almost any combination of on-off duty cycles. A solenoid-operated counter is usually included which automatically records the number of cycles accumulated.

If functional performance of a unit is to be monitored in a cycling test (e.g., if temperatures and pressures are to be recorded), the data are normally taken on a strip chart or multichannel recorder in which the paper is operated at the lowest possible speed. The strip chart is a convenient, permanent record of the various operating parameters and also indicates the number of cycles. Should a failure occur during the test, such a recording not only pinpoints the area of the failure, but often gives an indication of the cause, e.g., a slowly rising temperature or pressure prior to the failure.

15.5.11.2 TEST PROCEDURE. It is not possible to give a specific test procedure for a life cycle test as the requirements will vary from component to component.

One of the most important considerations in conducting a life cycle test is the simulation of the actual operating conditions of the unit. Poor simulation can cause a unit to fail the life cycle test when it would have passed under realistic test conditions and vice versa. Therefore, in designing the life cycle test procedure and setups, due consideration should be given to the unit's actual operating conditions. It may be necessary, for instance, to simulate the actual plumbing line sizes both upstream and downstream of the part to ensure that transients such as water hammer are neither too severe nor too mild. If a test fluid other than the fluid to be used in service is involved, the possible effects of a substitution shall be examined. For instance, if a valve or actuator is designed to operate on dry nitrogen or dry air and is cycled with shop air, the results may be dramatically different as the shop air tends to be moist and oily and will serve to lubricate the part, whereas the service fluid will carry no lubrication. Water is a convenient and inexpensive test fluid, but may have more or less lubricity than the service fluid and be more or less corrosive to the unit than the service fluid. The ambient temperature in which the test is conducted should reasonably approximate the test conditions. The wear resulting from a unit operating at 160 degrees is likely to be different from the wear that will occur on a unit cycled at laboratory ambient temperatures. The ambient pressure should be approximated within reasonable limits, especially if the unit is to operate at altitude, because the heat transfer characteristics will be more severe at altitude than at sea level. Where time span per cycle is a factor, testing at a higher temperature will sometimes give equivalent results in a shorter elapsed time. The rate of cycling does not necessarily have to be the same as the rate the unit will experience in service, but if a change is made to shorten the test time, the possible effects should be considered. One such effect is overheating in an actuator, switch, or motor. The operating range of the unit should be considered, and in some cases, if it is not properly simulated, the results will be erroneous. For example, if a pressure switch were to be cycled between 600 and 700 psi (its normal working range), the recorded actuation points may be different than if it is cycled between 0 and 700 psi because of the hysteresis normally present in most pressure switches. The application of force to a component, for example to the hand wheel of a manual valve, should be properly simulated to avoid misleading results. Such a force would normally be a torque, but if a moment arm were used that imposed radial loads on the valve shaft, abnormal wear would occur on the shaft, bearings, and packing.

The above examples are presented to suggest a line of thought when life cycling is being considered and are by no means all inclusive.

The number of cycles to be used in conducting a life cycle test should be based on the actual service required from the part. A part that will be cycled four or five times in its service life probably should not be tested to a million cycles. On the other hand, the number of actual operations in the service application should not be taken as a criterion. Quite often a part will experience more cycles in acceptance and checkout testing than it will on an actual mission, and these cycles should also be included in the determination of the number of life cycles to be used in the test.

The functional test should be performed periodically during the course of a life cycle test to determine that the unit is still operating properly.

15.6 ELECTRICAL FUNCTION TESTS

15.6.1 Dielectric Strength

The purpose of the dielectric strength test (also called a dielectric withstanding voltage test) is to prove that a component can withstand a momentary overpotential resulting from switching, surges, or other phenomenon.

15.6.1.1 DEFINITION. A dielectric is a medium in which the energy required to establish an electric field (voltage stress) is recoverable in whole or in part as electric energy (Reference 659-1). For example, when a voltage is established across a dielectric, such as the insulating medium between two plates of a capacitor, a displacement or charging current results. This charging current is recovered when the charge is removed from the capacitor plates. The dielectric properties of a medium relate to its ability to sustain a static electric field in distinction to its insulation leakage properties which relate to its ability to conduct steady current. The dielectric strength of a material is usually given as a voltage gradient, i.e.. volts per mil, volts per millimeter, or kilovolts per centimeter (Reference 659-1). It is very important to note that dielectric strength for a given material is determined under carefully controlled conditions; in actual use the materials may not duplicate these theoretical values. This is because the dielectric strength is affected by sharp corners, small radii, contamination, moisture film, humidity, occlusions, or other factors that tend to induce electric flashover or physical breakdown of the material.

15.6.1.2 PRECAUTIONS IN TESTING. The limitations of the instrumentation should be considered when any given reading is being evaluated. For example, if a voltmeter having a specified accuracy of ±5 percent of full scale is being used to measure 1000 volts full scale, the observed reading may have an error of 50 volts. If, for instance, a failure is encountered at 960 volts, use of a more accurate voltmeter may be justified.

A careful examination of the part should be made for particles, films, or other types of contamination which would tend to produce premature flashover or breakdown.

Humidity of the ambient air can be a factor in breakdown and should not exceed the conditions of the test specification. If humidity is not specified, a recommended value is 50 percent. Previous history and *soak* in a humid environment will profoundly affect test results obtained with many solid dielectrics.

15.6.1.3 SPECIFICATION REQUIREMENTS. If dielectric test criteria are being established, realistic values should be selected for the test. Conventionally, 500 to 1000 volts are normally specified for aerospace components. However, the lowest limit that can be used and still provide an adequate margin of safety should be specified, as excessive test values will result in over-design or raise the rejection rate and therefore increase the cost. When applicable, altitude should be specified.

Since the dielectric test tends to degrade the equipment, repeated applications of the maximum specified test voltage may result in component failure. Dielectric test is not repeated after acceptance testing unless voltage is reduced to 75 percent of the specified maximum.

15.6.1.4 EQUIPMENT REQUIRED. The equipment required for conducting a dielectric test is self-contained and includes a power supply, voltage control, and suitable voltmeter and milliammeter.

15.6.1.5 TEST PROCEDURE. A generalized test procedure for conducting this test is described in MIL-Standard 202C, Method 301.

15.6.2 Insulation Resistance

The purpose of this test is to measure the resistance offered by the insulating members of the component part to an impressed direct voltage. This test is not to be considered the equivalent of a dielectric strength test and is in fact different in nature and intent.

15.6.2.1 EFFECTS OF INSULATION FAILURE. Excessive current leakage will lead to deterioration of the insulation by heating and may eventually form a carbonized track or path leading to total breakdown. Excessive leakage can also disturb the operation of circuits by forming feedback loops. Low insulation resistance is often an indication of a low residual operating life.

15.6.2.2 FACTORS AFFECTING INSULATION RESISTANCE MEASUREMENTS. There are many factors that affect insulation resistance measurements including temperature, humidity, altitude (ambient pressure), residual charge, charging current, time constant of the instrument, the measured circuit and the test voltage employed, and the deviation of the test voltage. Some components will exhibit a high initial leakage current which in reality is a charging current which decreases with time. Therefore, sometimes it is necessary to wait until steady-state conditions are achieved before making the measurement.

15.6.2.3 EQUIPMENT REQUIRED. Conventional equipment, such as a megohm meter, milliammeter, and a voltage source ranging from 100 to 10[illegible] volts, is required. The test apparatus is normally found in most test laboratories.

15.6.2.4 PROCEDURE. MIL-Standard 202C, Method 302, gives general instructions for conducting this test.

15.6.3 DC Resistance

The resistance of components is measured with a bridge circuit; the general instructions are given in MIL-Standard 202C, Method 303. Resistance of contacts is tested in a slightly different manner, as described in MIL-Standard 202C, Method 307.

15.6.4 Capacitance

Capacitance measurements are required on capacitors, and occasionally it is necessary to measure the capacitance of other bits of equipment such as lead wires and cables. This measurement is commonly made with a capacitance bridge which consists essentially of a number of precision capacitors which may be selected by a switching arrangement. The test specimen is connected to the proper terminals of the bridge and the capacitance is measured by balancing the unknown capacitance to a known capacitance. The accuracy of this device will range from 1 to 0.0001 percent, and the time required to conduct the test will vary depending upon the accuracy being sought. A measurement in the order of 1 percent could be typically made in a few minutes, whereas a measurement requiring the utmost accuracy may take a half hour or more.

Capacitance may also be measured with an impedance meter which usually has an accuracy of 2 to 10 percent. These meters are less expensive than the capacitance bridge and are easier and quicker to use. The accuracy required by the specification will determine which type of equipment to use.

Both the bridge and the impedance meters incorporate a frequency generator.

15.6.5 Inductance

Inductance is measured by an inductance bridge which differs from a capacitance bridge in that it incorporates resistances which may be coupled with various capacitances. The inductance is measured at a fixed frequency, and the device essentially compares the unknown inductance to a reference resistance and capacitance. The time required for making this measurement is in minutes.

15.6.6 Magnetic Flux

There are several procedures available for measuring magnetic flux and magnetic flux density. Flux density may be measured by use of a flux density meter employing the Hall effect. The area to be measured is searched with a small probe, and a direct readout on the meter is obtained.

A more precise, less convenient method employs a rotating coil. When the coil is inserted into the magnetic field it rotates and generates a voltage which is proportional to the flux. The voltage is applied to a meter which again provides a direct readout of flux density.

Total flux may be read by a flux gate magnetometer which employs a search coil which is moved through a magnetic field. The readout is on a meter which is an integrating device and reads the total flux in a circuit. The meter incorporates a stop mechanism which prevents the dial from returning to zero. At the completion of a particular reading, the data are recorded and the meter is then reset to zero.

15.6.7 Electromagnetic Interference (EMI)

EMI tests are performed to measure and determine the electromagnetic interference characteristics (*emission* and *susceptibility*) of electronic, electrical, and electromechanical equipment. EMI testing is primarily used with electronic equipment or with equipment which generates significant radio frequency interference, such as automobile or truck engines. Such testing is described in detail in MIL-STD-461, MIL-STD-462, and MIL-STD-463. Several specialized EMI tests are performed on certain aerospace fluid components, however, such as the following:

a) Measurement of the magnetic field generated by solenoid or torque-motor actuated valves on spacecraft. Some spacecraft, such as Pioneer and OGO series, require that the magnetic fields generated by such components be almost totally neutralized.

b) Determination of the susceptibility of electroexplosive devices (EED) such as squib valves to actuation by spurious radio frequency (RF) signals. The Range Safety Manuals of the Air Force Eastern Test Range and Western Test Range (AFETRM 127-1 and AFWTRM 127-1) specify determining the RF susceptibility of such devices. This susceptibility is defined as the magnitude of the smallest electric field (expressed as an RF *field intensity* or RF *field strength*) capable of producing the no-fire current or no-fire power in an EED. (This no-fire current is the current sensitivity at which no more than one EED per thousand will fire with a confidence of 95 percent.)

c) Fluid components which utilize any electronic circuitry should be tested to determine the susceptibility of that circuitry to actuation or malfunction as a result of the anticipated electromagnetic environment in the immediate vicinity.

15.6.7.1 TESTING. The testing of a component for EMI can range from a fairly simple procedure requiring ordinary equipment commonly found in most test laboratories to a very complex test program involving special equipment, special screen rooms, and highly skilled specialists to conduct the test. The complexity is a function of both the complexity of the unit to be tested and the requirements of the specification. The new military standards (461, 462, and 463), which supersede MIL-STD-826, require more sophisticated tests such as magnetic field measurement (Reference 647-4).

15.6.7.2 EQUIPMENT REQUIRED. The test equipment needed falls into two broad categories--emission measuring equipment used to measure interference generated by the test sample, and susceptibility test equipment used to subject the test sample to interference. Emission-measuring equipment includes EMI meters, monitoring devices, current probes, feed-through capacitors, and antennas. Susceptibility test equipment includes signal generators, power amplifiers, spike generators, isolation transformer, and antennas (Reference 647-4).

15.6.7.3 TEST PROCEDURE. Test methods are described in considerable detail in MIL-STD-462. As stated in the standard, it would be impractical to attempt to define a procedure that suits every conceivable case. Therefore, an emission test procedure will be described here in very general terms to present an overall picture of the effort involved.

After the testing and display equipment has been set up, the first test to be conducted would be a recording of the background noise to determine its level for the test. It may be necessary to change the location of the test to conduct the test at a different time such as at night or on the weekend when the interfering equipment would not be operating. It is also necessary to have some knowledge of the nature of the background interference, as in some cases the background noise can either add to the apparent emission of the component under test, which would make it appear to be worse than it is, or it can subtract from the apparent emission, which would make it look better than it is. This analysis is normally performed with equipment known as a correlation detector.

If the testing to be accomplished is of an infrequent nature or if the item is relatively complex and the permissible levels of emission are low, it would probably be better to have the work done in a commercial laboratory where the necessary equipment and technical help are available. If the test results should indicate that the emission interference levels are out of specification limits, it is highly advisable to seek expert help in designing any changes into the suppression circuitry as this task cannot be accomplished economically on a trial and error basis.

15.6.8 Chatter Monitor

Chatter is a momentary opening and closing of contacts, such as are found in pressure switches, relays, and indicating switches. Chatter may be measured or observed by use of a galvanometer and a strip chart recorder. This method may be used where chatter exists for a considerable period of time or where only a rough indication of chatter is desired. For an accurate measurement, an oscilloscope is normally employed, often in conjunction with a Polaroid camera which provides a permanent record of the contact actuation. Chatter monitor tests are often required in conjunction with vibration tests.

15.7 ENVIRONMENTAL TESTS

15.7.1 Vibration Test

Of all the tests performed on systems and components, probably none is more important than that of vibration. This is evidenced by the fact that more failures occur in vibration testing than in any other environmental test. For this reason and the fact that the subject is technically complex, the somewhat detailed discussion of the subject comprising Detailed Topic 15.7.1.1 has been excerpted from *Vibration Fundamentals* by Ling Electronics (Reference 657-3). In addition, Sub-Section 7.3 includes such applicable information as equations for calculating natural frequencies and stiffness factors for some common systems. Sub-Topic 13.3.5 treats various vibration environments and suggests design techniques for reducing the effects of vibration.

15.7.1.1 VIBRATION FUNDAMENTALS AND SPECIFICATIONS

Sinusoidal Vibration. Sinusoidal motion is the motion of a shaker table constrained to move only up and down in response to a driving voltage which varies sinusoidally with time. The table displacement from its rest position is shown in Figure 15.7.1.1a.

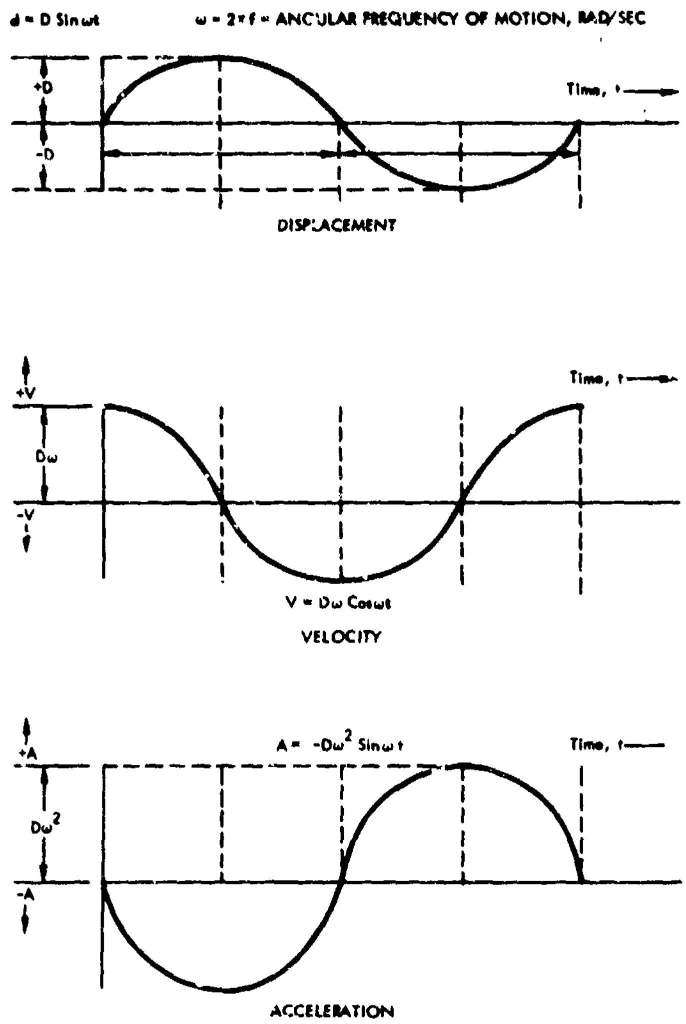


Figure 15.7.1.1a. Sinusoidal Displacement, Velocity, and Acceleration
(Reprinted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Ling Altec, Inc.)

The most important relationship in the preceding is the one between acceleration and displacement. Dropping the sign (acceleration may be either positive or negative), the absolute value of acceleration is proportional to the product of displacement and the square of frequency.

$$A = D\,(2\pi f)^2 \qquad \text{(Eq 15.7.1.1a)}$$

where

A = acceleration, in/sec^2

D = displacement, inches

f = frequency, cps

In terms of g ($g = A/g_c$ where $g_c = 386$ in/sec^2), the peak acceleration is:

$$A_g = \frac{4\pi^2 D f^2}{386} = \pm 0.102 D f^2 \qquad \text{(Eq 15.7.1.1b)}$$

where

A_g = peak acceleration, in/sec^2

If D_{da} is now defined to be double amplitude (da) or peak-to-peak displacement, the peak acceleration in g is:

$$A_g = 0.051\, D_{da} f^2 \qquad \text{(Eq 15.7.1.1c)}$$

This relationship should be well understood. If a constant 0.01 inch da is required, then, as frequency is increased, the acceleration rises with the square of frequency and is equal to $0.00051\, f^2$. A frequency of 100 cps gives an acceleration of 5.1 g. Doubling the frequency to 200 cps produces four times the acceleration, or 20.4 g. Doubling the frequency again to 400 cps produces four times 20.4 g or 81.6 g. It is evident from Equation (15.7.1.1c) that at low frequencies, displacement governs the maximum acceleration obtainable, while at high frequencies, acceleration governs the maximum displacement obtainable. The crossover frequency is that frequency for which the maximum allowable acceleration and displacement are simultaneously present. The relationship between displacement, acceleration, and frequency is plotted in Figure 15.7.1.1b.

Resonance. Resonance (as pertaining to vibration testing) is a characteristic possessed by all objects in varying degrees. A weight on a spring (pulled down and released) will oscillate at a resonant frequency or natural frequency determined by the mass and the spring constant. The duration of these oscillations is determined by the damping in the oscillating system. The more damping, the sooner the mass will come to rest. Without damping the mass and spring would oscillate forever. One of the main purposes of vibration testing is to detect resonances in the test specimen, for it is at the resonant frequencies that most damage can occur. The equation used to determine natural (resonant) frequency follows:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} \qquad \text{(Eq 15.7.1.1d)}$$

where

f_n = natural frequency, cps

ISSUED: FEBRUARY 1970
SUPERSEDES: NOVEMBER 1968

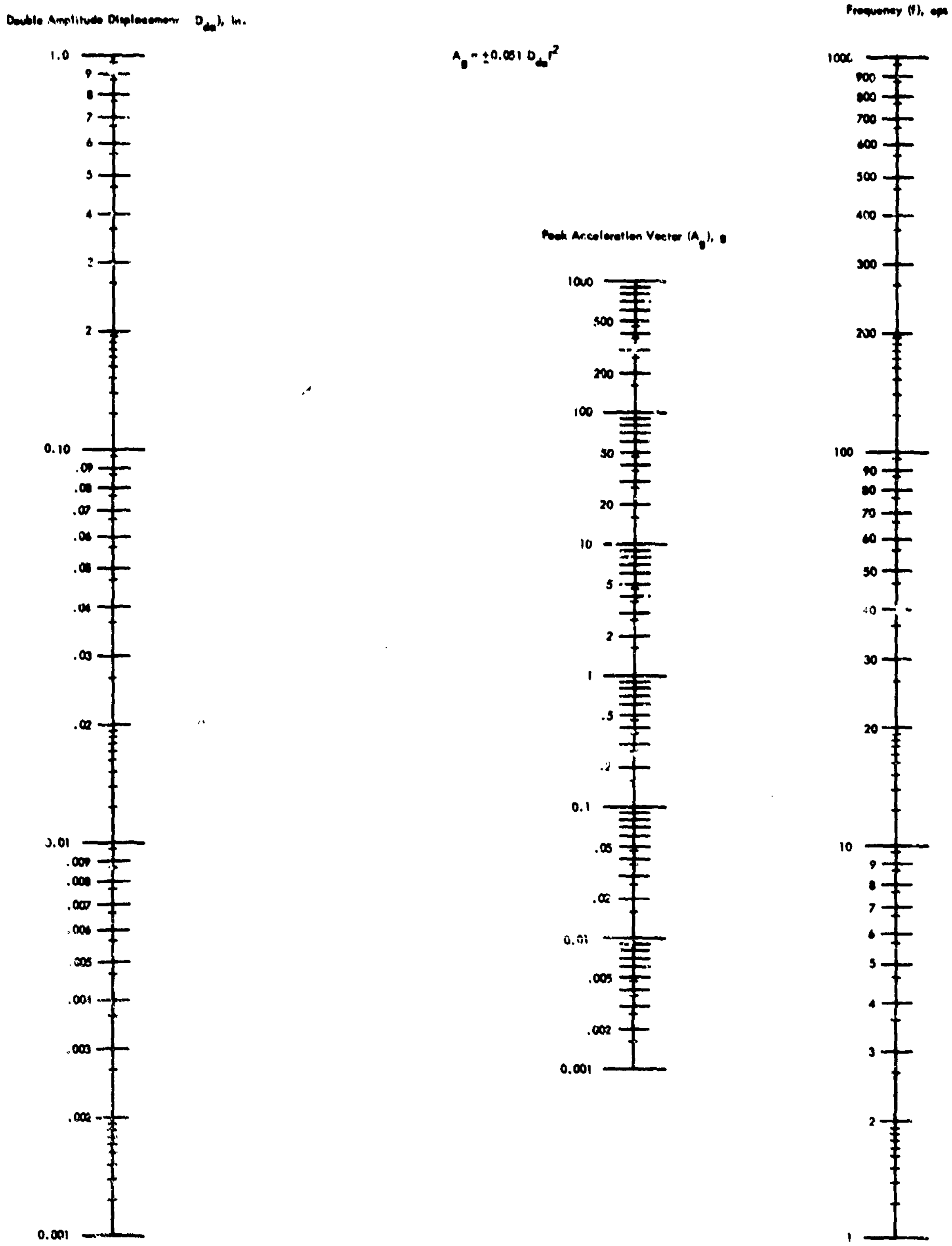


Figure 15.7.1.1b. Nomograph of Displacement versus Acceleration and Frequency
(Reprinted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Altec, Inc.)

K = spring constant, $lb_f/in.$

$$M = \text{mass, } lb_m = \frac{lb_f}{386 \text{ in/sec}^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} \qquad \text{(Eq 15.7.1.1e)}$$

where

g = local acceleration of gravity, 386 in/sec²

Δ = static deflection, inch (due to force of gravity on M)

In dealing with electromechanical devices excited by electronic equipment, it is important that mechanical/electrical analogies be understood. The input impedance of a shaker is important when driven electrically. A clear understanding of this impedance and the associated resonance phenomena requires that analogies be used. In vibration work, the inverse or shunt electrical analogs are commonly used as shown in Table 15.7.1.1a.

Table 15.7.1.1a. Inverse of Shunt Electrical Analogs for Vibration Work
(Adapted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Ling Altec, Inc.)

MECHANICAL		INVERSE ELECTRICAL	
Mass	$M = \frac{W}{g}$	Capacitance	C
Damping	R	Conductance	G
Compliance	C	Inductance	L
Force	F	Current	I
Velocity	V	Voltage	E

Just as the mass and compliant spring oscillate in the mechanical sense (damping by friction), in the electrical sense, an inductance, capacitance, and resistance connected together in a circuit will have a resonant frequency. The damping is determined by the resistance (reciprocal of conductance) of the circuit.

The energy analogs between mechanics and electricity are as shown in Table 15.7.1.1b.

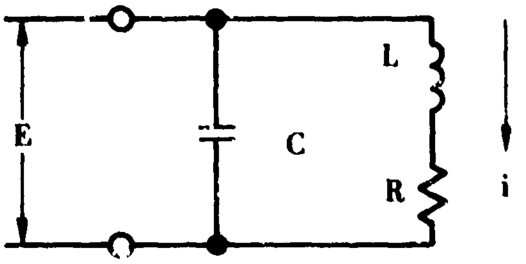
Table 15.7.1.1b. Energy Analogs between Mechanics and Electricity
(Adapted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Ling Altec, Inc.)

QUANTITY	MECHANICAL RELATION	INVERSE ELECTRICAL RELATION
Stored energy, mass	$\frac{1}{2}(MV^2)$	$\frac{1}{2}(CE^2)$
Stored energy, spring	$\frac{1}{2}(CF^2)$	$\frac{1}{2}(LI^2)$
Damping loss	RV^2	GE^2

The above analogs are derived from the differential equations that describe both the mechanical and electrical systems. These relationships are useful in that a person experienced with electrical circuits but not mechanical circuits can transform the latter into his frame of reference and vice versa. They are also useful when dealing with electromechanical devices.

In dealing with resonance phenomena, it is helpful to use a dimensionless parameter known as Q which describes the amount of damping in the system in relation to the stored energy of the system. A high Q system has low damping. While there are many definitions for Q, the easiest one to understand is that defining Q as the ratio of total stored-energy to energy lost per angular cycle. This definition literally applies to high Q cases, but if stored energy is interpreted to be the average stored energy during the period of one full cycle, the definition can still apply to low Q. Thus, the Q of the system by this definition is:

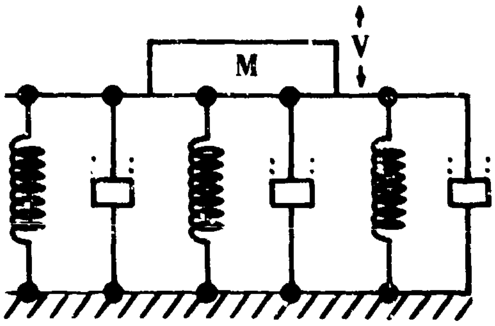
(Eq 15.7.1.1f)



$$Q = \frac{\omega L}{R}$$

In a mechanical case, if we have a mass, a spring, and a damper in oscillation as shown:

(Eq 15.7.1.1g)



$$Q = \frac{\omega C}{R}$$

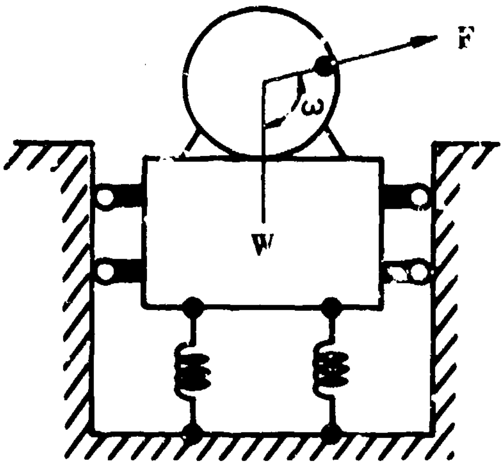
where

C = total spring compliance

R = total damping

ω = angular frequency

In the mechanical case of forced vibrations, the situation is more complicated. The damping factor is important when interest lies in the amplitude of vibrations at or near resonance. Consider the driven system shown, where a motor of weight W is suspended on springs having an eccentric mass on its shaft. As the motor turns ω radians per second, centrifugal force produces an excitation to the system. Although constrained in the lateral direction, it can move in the vertical direction. Damping is provided by friction between the rollers and the constraining sides. If the spring constant is k, then we can compute the amplitude of forced vibration as a function of the system damping and the exciting frequency. The maximum displacement is given by the impression:



$$D_m = D_s \frac{1}{\sqrt{(1-m^2)^2 + 4\zeta^2 m^2}} \qquad \text{(Eq 15.7.1.1h)}$$

where

D_s = static displacement due to the eccentric mass, inches

$$m = \frac{\omega}{\left(\frac{Kg}{W}\right)^{1/2}}$$

ζ = damping factor given by $\frac{c}{2}\left(\frac{g}{WK}\right)^{1/2}$

If one plots a family of curves of this maximum displacement as frequency is varied, they appear as shown in Figure 15.7.1.1c. As the damping factor (ζ) is increased in relation to mass of the system, the amplitude of the resonance buildup is decreased markedly, although the amplitude at frequencies well away from resonance is not greatly affected.

The sharpness of a resonance curve in either an electrical or mechanical system is related to the Q in the system, which in turn depends on the amount of energy dissipation or damping present. It is characteristic of a high Q system that for a given amplitude of oscillation, less energy is required to excite that oscillation than for a low Q system. This is an important point in vibration work because this means that relatively little excitation is required at a high Q resonance point to produce very high stresses in the specimen.

Random Vibration. Random vibration is important because in most aerospace applications the excitation forces are not at discrete points in a frequency spectrum but rather exist over a wide, continuous band of multiple frequencies. In order to more closely simulate the actual

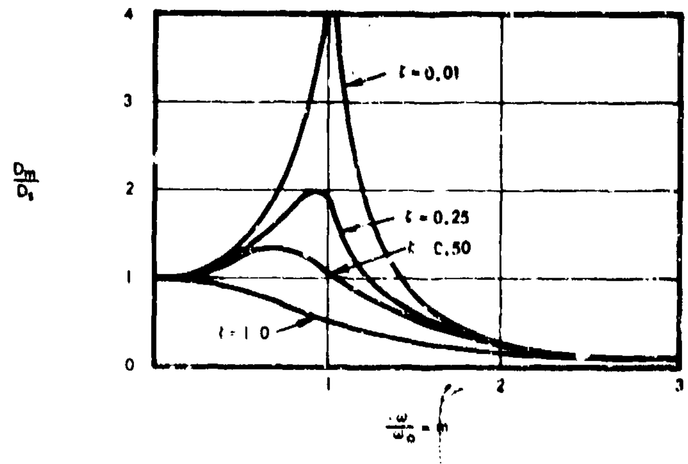
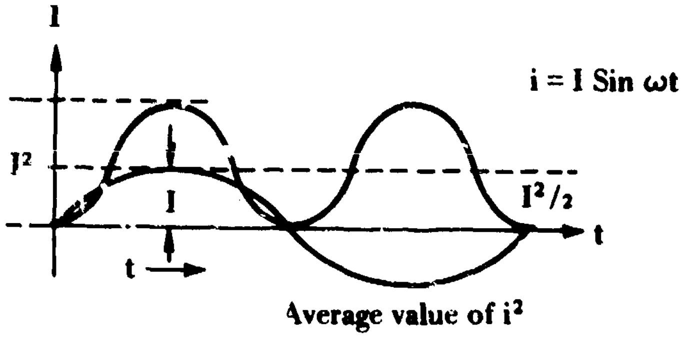


Figure 15.7.1.1c. Damping Factor Characteristic
(Adapted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Ling Altec, Inc.)

environment the equipment will encounter, random, rather than periodic, forces must be generated in vibration test equipment. The sinusoidal cases already discussed have dealt largely with peak or peak-to-peak amplitudes. In random work, it is more convenient and meaningful to deal with root-mean-square (rms) values of such quantities as displacement and acceleration. The rms value of an electrical or mechanical quantity is related to energy. The rms value of a sinusoidal alternating current of peak value I is the equivalent direct current which will produce the same amount of heat in a dissipating element as the alternating current. The power dissipated by resistance (r) with a current (i) flowing through it is given by $p = i^2r$. The instantaneous power dissipated varies as the square of the current (or voltage). It can be seen that the average of i^2 wave is $i^2/2$.



The square root of this average is the rms or effective value of the current i: $I/\sqrt{2} = 0.707\ I$. In the case of a complex wave containing many frequencies and amplitudes, to find the rms value it is necessary to square the amplitude of the wave at every point in time, find the average value of the squared wave over a given period of time, and extract the square-root of this average value. In practice, this is done by a true rms meter. An ordinary peak or average reading meter is not good enough to find the true rms value of a random wave or one containing many frequencies.

It is not possible to specify the frequency of a random wave system, because all frequencies are present simultaneously, or at least all frequencies within some specified bandwidth are present. In order to conveniently cope with calculations and experimental work involving random quantities, the concept known as *acceleration density* is used in the same way that a total summation of acceleration density over a frequency spectrum yields the mean-square value of the acceleration. The units for acceleration density are g^2 per cps, analogous to i^2 or e^2 per cycle in the case of electrical quantities. The acceleration density is literally defined as:

$$g = \lim_{B \to 0} \frac{a^2}{B} \qquad \text{(Eq 15.7.1.1i)}$$

where

- a = rms value of the random acceleration, in/sec^2
- B = bandwidth or range of frequencies under consideration, cps

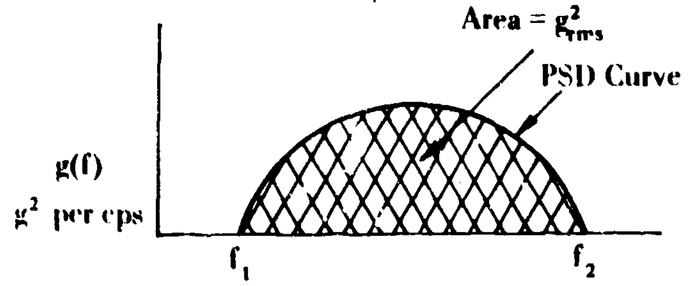
If the bandwidth (B) is made to approach zero cps, the acceleration density given by Equation (15.7.1.1i) is that of a single component frequency. A plot of the acceleration density of each component frequency gives a curve of g^2 per cps versus frequency over the frequency spectrum of interest. This is known as the *power spectral density* (PSD) curve. It is apparent that a complete description of a random vibration test requires a specification of the PSD to be used. When the PSD is flat (that is, all frequency components are present at an equal energy level), the random motion is referred to as *white noise*.

In certain situations, the total rms vibration level has an absolute significance. For example, the mechanical power dissipated in a structural member undergoing oscillating elastic deformation is directly proportional to the mean-square (rms^2) value of the oscillating strain, regardless of the waveform of the oscillation. The total rms vibration level represented by a particular PSD may be determined by making a total summation of all the increments of acceleration density over the entire bandwidth. This is the mean-square acceleration, and the square root of this area is the root-mean-square acceleration, where g_o is the acceleration density in g^2 per cps

$$g^2_{rms} = \sum_{f_1}^{f_2} g_o \Delta f \qquad \text{(Eq 15.7.1.1j)}$$

A more formal definition of rms acceleration in the random situation is:

$$g_{rms} = \left[\int_{f_1}^{f_2} g(f)\, df \right]^{1/2} \qquad \text{(Eq 15.7.1.1k)}$$



For a flat or white noise spectrum, $g(f) = g_o$.

$$g_{rms} = \sqrt{g_o (f_2 - f_1)} = \sqrt{g_o B} \qquad \text{(Eq 15.7.1.1l)}$$

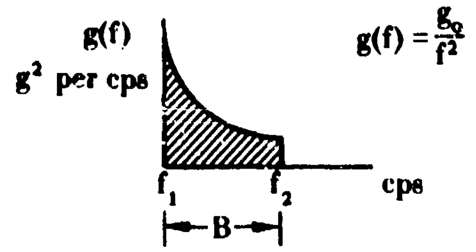
where

- B = bandwidth, cps
- g_o = acceleration density, g^2/cps.

If we have g_o given as 0.1 g^2 per cps and B given as 1000 cps, the rms value of acceleration is $(0.1 \times 1000)^{1/2}$ = 10 g. A nomograph of this relation is given in Figure 15.7.1.1d.

There are many possible PSD curves, other than the flat spectrum, which might be used. It is well to point out that even though the frequency spectrum of a random signal is not flat, a signal is no less random. A truly random acceleration follows what is known as a Gaussian, or normal, distribution of instantaneous accelerations. The Gaussian distribution means that the probability of occurrence of instantaneous acceleration is such that the acceleration will be less than the rms level 68 percent of the time, less than twice the rms level 95 percent of time, and less than three times the rms level 99.7 percent of the time. In practical equipment, there are limits to the peak accelerations (and displacements) obtainable. The accepted standard for random vibration systems is a capacity to produce peak accelerations equal to three times the rated rms accelerations. Thus in practice, 99.7 percent of a true Gaussian distribution is realized. To determine the random rms g rating of a particular shaker amplifier system, refer to the plotted curve of system limits. These curves are determined by a combination of amplifier input limit, plate dissipation, output transformer, and shaker heat limit for various mass load conditions.

Occasionally, other types of PSD curves are specified. In all cases, the important thing to remember is that the mean-square acceleration in a given bandwidth is equal to the area under the PSD curve covering that bandwidth. The root-mean-square (rms) acceleration is the square root of that area. Sometimes a PSD curve is given in the form:



For this case, the rms acceleration is given by:

$$g_{rms} = \sqrt{\frac{g_o B}{f_1 f_2}} \qquad \text{(Eq 15.7.1.1m)}$$

For the special case where f_1 and f_2 are very close together and approximately equal to frequency f_oB, it reduces to a small value of Δf

$$g_{rms} = \sqrt{\frac{g_o \Delta f}{f_o}} \qquad \text{(Eq 15.7.1.1n)}$$

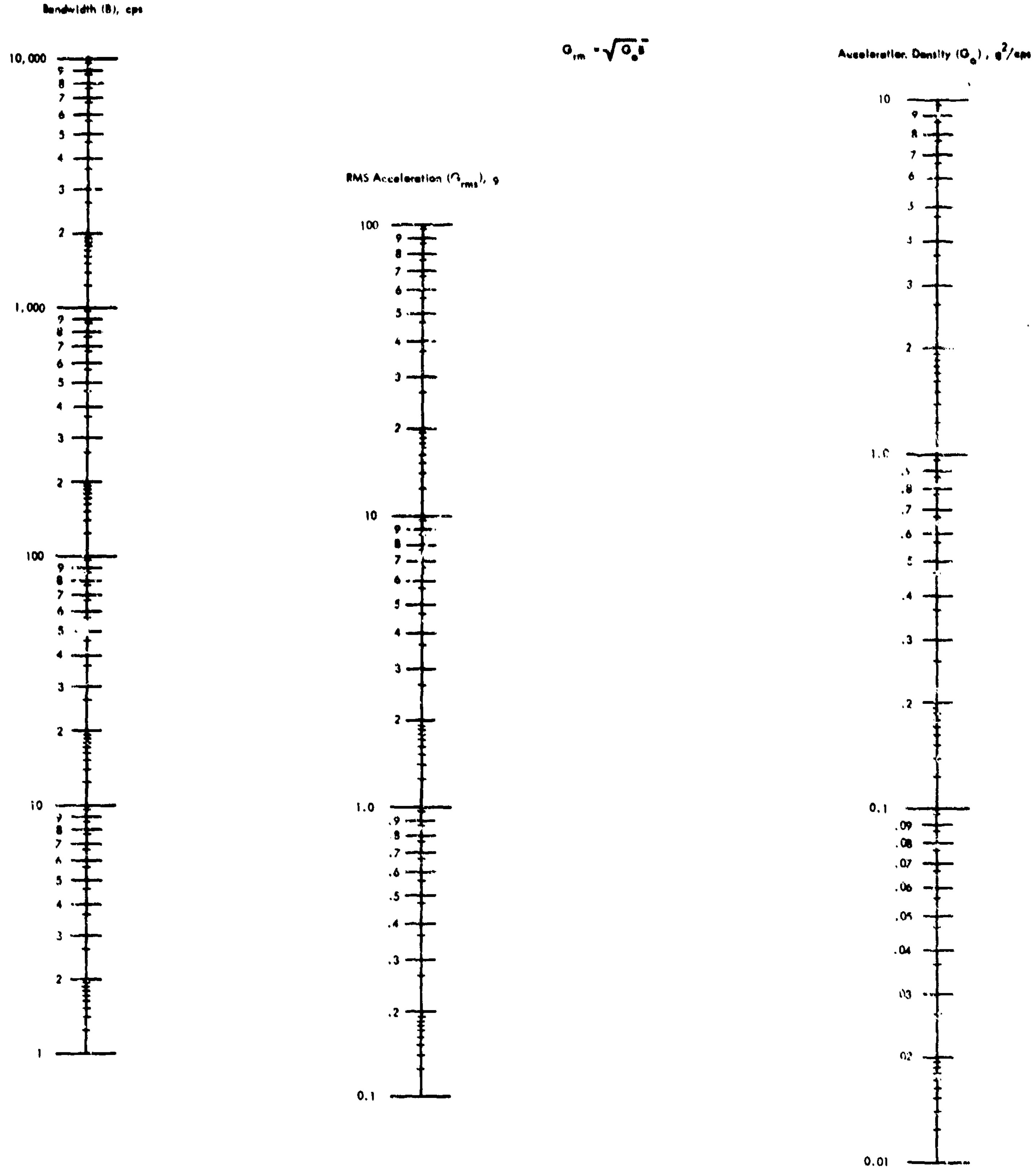
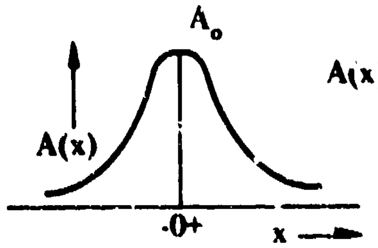


Figure 15.7.1.1d. Nomograph for Flat Power Spectral Density
(Adapted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Ling Altec, Inc.)

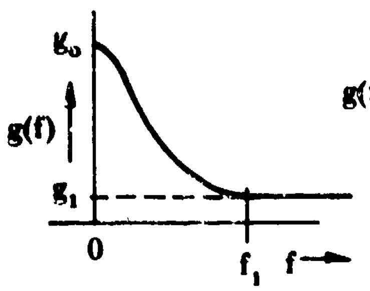
Another type of PSD curve sometimes used is derived from probability theory (from the Gaussian error curve). It is used because the Gaussian error curve is a close approximation to the amplitude versus frequency response of many electrical and mechanical filters. These are encountered in practice, especially over the frequency range where response is significant.

PSD curve from a Gaussian error curve is:



$$A(x) = A_o e^{-bx^2}$$

Another PSD curve is given by:



$$g(f) = g_o e^{-(kf/f_1)^2}$$

Where e = 2.71, the base of natural or Naperian logarithms, and the constant (k) is found from the value of $g(f) = g_1$ at some frequency f_1:

$$k = \left[\mathrm{Ln}\,(g_o/g_1)\right]^{1/2} \qquad \text{(Eq 15.7.1.1o)}$$

The rms value of acceleration for this case is:

$$g_{rms} = \sqrt[4]{\frac{\pi}{\mathrm{Ln}\left(\frac{g_o}{g_1}\right)}} \times \sqrt{\frac{g_o f_1}{2}} \qquad \text{(Eq 15.7.1.1p)}$$

This is the result for infinite bandwidth. As long as g_o/g_1 is fairly large (corresponding to a bandwidth sufficient that g_1 is fairly small compared to g_o), g_{rms} will not differ appreciably from that given in Equation (15.7.1.1p).

For example:

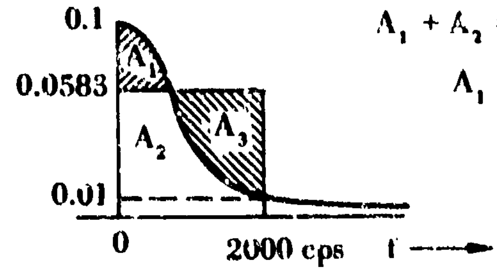
$g_o = 0.1\ g^2$ per cps

$f_1 = 2000$ cps

$g_1 = 0.01\ g^2$ per cps

Equation (15.7.1.1p) gives: $g_{rms} = 10.8$

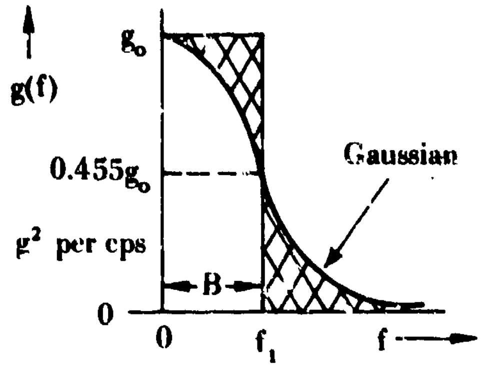
Using the values in the example for the Gaussian section, an equivalent PSD for a flat spectrum to 2000 cps (f_1) is 0.0583 g^2 per cps. The energy (g^2_{rms}) in both cases is identical, but the distribution in the frequency spectrum has been altered as shown:



$$A_1 + A_2 = A_3 + A_2$$

$$A_1 = A_3$$

In the case where the g_{rms} is required to be the same for a flat PSD spectrum and the Gaussian spectrum, using the maximum PSD of the Gaussian spectrum, the bandwidth B is at the frequency where the PSD is 45.5 percent of the Gaussian maximum. In the figure below, the area under the Gaussian spectrum (beyond the frequency f_1) is just equal to the cross-hatched area in the rectangular spectrum below the frequency f_1. This relationship is useful in estimating rms levels from Gaussian type PSD curves; it is easily proved by equating Equations (15.7.1.1l) and (15.7.1.1p) with $B = f_1$ and solving for g_o/g_1.



There are cases where the mean-square acceleration is not especially significant. In a dynamic fatigue failure, the mean-square amplitude of vibration may be a useful measure for comparing similar random vibrations; an absolute criterion for failure may require other statistical information. Failures forced at resonant frequencies with sine wave testing tell relatively little about the ability of the specimen to withstand an actual missile environment where the vibration excitation is very much like noise.

To obtain a meaningful value of the spectral density (relatively stationary with time), it is necessary to average the mean-square output over a time which is compared with the reciprocal of the bandwidth. The required averaging time depends on the desired confidence limits. It can be shown that the measured value of the spectral density (averaged over a frequency band Δf for a time T) will be within ±1 db of the long time value for only about half the time if $T\Delta f = 5$. If it is desired that the measured value be within these limits for at least 95 percent of the time, it is necessary that $T\Delta f > 50$.

Random Displacement. To determine testing equipment limitations, it is necessary to understand how to figure displacement.

Recalling that displacement and acceleration are related by $A_g = 0.10 \times Df^2$ (Eq 15.7.1.1b), then $D = 9.77\ A_g/f^2$. If A_g is in rms units, so is D. In a small bandwidth (Δf and power spectral density g_o given in g per cps), the mean-square displacement (D_m) in the bandwidth of f is:

$$D_m^2 = (9.77)^2\, g_o \frac{\Delta f}{f^4} = 95.5\, g_o \frac{\Delta f}{f^4}$$

(Eq 15.7.1.1q)

To obtain the mean-square displacement over the band frequencies extending from f_1 to f_2, it is necessary to sum up the individual areas represented by the above equation;

$$D_m^2 = 95.5 \sum_{f_1}^{f_2} \frac{g_o \Delta f}{f^4} \qquad \text{(Eq 15.7.1.1r)}$$

To obtain the maximum double amplitude, or peak-to-peak displacement (D_{da}), it is necessary to multiply the rms value by 3 for peak displacement and again by 2 for peak-to-peak displacement. If a perfectly flat PSD curve is considered extending from f_1 to f_2 and assuming $f_2 \gg f_1$, then:

$$D_{da} = 34\left(\frac{g_o}{f_1^3}\right)^{1/2} \qquad \text{(Eq 15.7.1.1s)}$$

A more realistic situation however is not to assume a rectangular PSD curve, but to assume one which attenuates at some finite rate below the frequency of f_1. This is because no real filter can produce the ideal rectangular spectrum. Assuming the PSD curve to attenuate 24 db/octave (acceleration density in g^2/cps and falling to $(1/16)^2$ or 1/256 of its f_1 value at one-half the frequency f_1), then:

$$D_{da} = 42.8\left(\frac{g_o}{f_1^3}\right)^{1/2} \qquad \text{(Eq 15.7.1.1t)}$$

where

D_{da} = double amplitude, inches

g_o = acceleration density, g^2 rms/cps

f_1 = cutoff frequency, cps

Equation (15.7.1.1t) is used by Ling in preference to Equation (15.7.1.1s) because it is more conservative and more closely represents the true situation where a sharp low-frequency cutoff is not obtainable. A nomograph of Equation (15.7.1.1t) is given in Figure 15.7.1.1e. (Ling offers a vibration slide rule which may also be used to solve this equation.) As an example, assume a shaker limited to 0.5 inch da displacement. To use an acceleration density of 0.4 g^2/cps, what is the lowest frequency of the spectrum, i.e. what cutoff frequency must be used, for a 24-db-per-octave filter? Laying a straight-edge between 0.4 on the g_o scale and 0.5 on the D_{da} scale, the intersection on the f_1 scale is 14.3 cps. If the shaker were capable of 1.0 da displacement, the frequency could be decreased to 9.02 cps. As another example, for a low-frequency cutoff of 5 cps and a shaker limited to a 1.0 da displacement, acceleration density cannot exceed 0.068 g^2/cps.

Where a simultaneous combination of random and sine wave vibration exists, the total displacement is found by adding the maximum sine wave total displacement to the random double amplitude displacement determined in Equation (15.7.1.1t).

Specifications. The following specifications are widely used in military vibration testing:

1) MIL-E-5272, "Environmental Testing, Aeronautical, and Associated Equipment, General Specifications for"

2) MIL-STD-167, "Mechanical Vibrations of Shipboard Equipment"

3) MIL-E-8189, "Electronic Equipment, Guided Missiles, General Specifications for"

4) MIL-E-4970, "Environmental Testing, Ground Support Equipment, General Specifications for"

5) AFSC Manual 80-6

3) MIL-STD-810, "Environmental Test Methods".

15.7.1.2 VIBRATION LEVELS. The appropriate level of vibration to be used for testing a particular part is often unknown. Other uncertainties are the type of vibration to be used: sine, random, acoustic, or a combination of all three; and the axes through which the specimen is to be tested. If the specified levels are too low (a rare occurrence), the part may fail on the vehicle. If the levels are too high, an unnecessary penalty in dollars and weight is imposed.

Vibration levels are determined in several ways, each having advantages and disadvantages. A description of the procedures for determining these levels follows.

Environmental Prediction. In the case of a component being supplied for a vehicle not yet built, the vibration levels to be expected must be predicted based on the general design of the structure and information available from similar vehicles. A large degree of uncertainty as to the validity of the levels must necessarily be present when this procedure is used.

Data Acquisition. In this procedure, actual data is acquired from an existing vehicle. If the vehicle is an airplane, the data acquisition is relatively inexpensive and can be quite reliable, as the instrumentation can be carried and monitored on-board. In the case of the space vehicle, the problem is more complex because the amount of data that can be acquired is limited by factors such as the weight that can be carried and the relatively short launch duration.

Extrapolation of Data. If the vehicle to be built is similar to a previous vehicle, vibration levels may be inferred from data taken on the previously tested vehicle. This data will tend to be more accurate than data calculated for a new vehicle but is still subject to severe limitations in accuracy.

Zoning. Information from any of the above procedures may be used to arbitrarily designate zones within a vehicle.

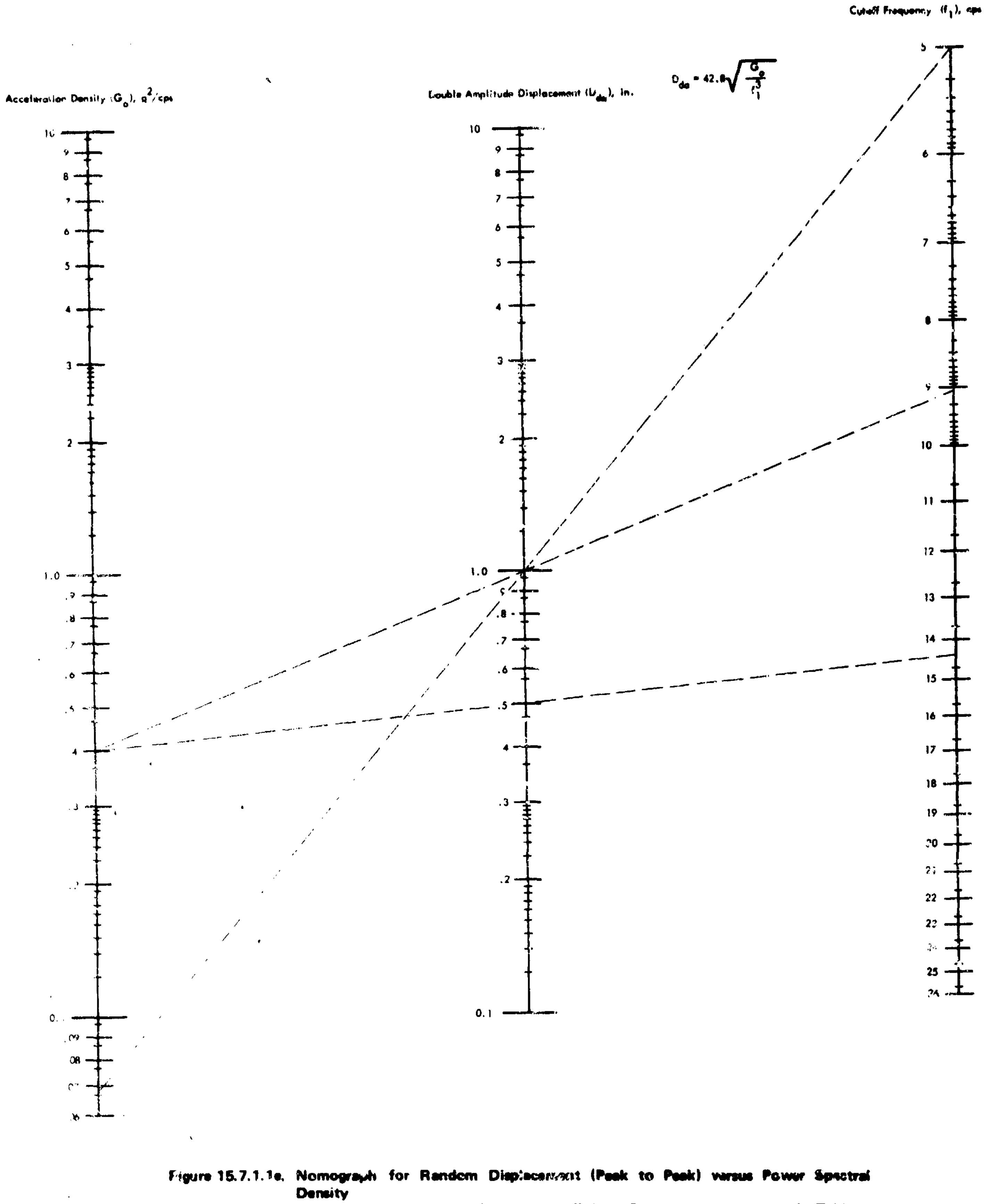


Figure 15.7.1.1e. Nomograph for Random Displacement (Peak to Peak) versus Power Spectral Density

(Adapted with permission from "Vibration Fundamentals", Ling Electronics, a Division of LTV Ling Altec, Inc.)

In this procedure, every piece of equipment in a given zone is considered to be subject to the same level of vibration.

It is apparent that the range of uncertainty associated with specified levels in any given vibration test specification may be quite large. The specification writer should consider the uncertainties in the various methods of setting test levels and should make every effort to ensure that they are realistic. The design and test engineer should be aware that the values set in any specification may be in error when applied to a particular component or system. With this awareness, a requirement that is unrealistic may be detected, reevaluated, and modified.

15.7.1.3 TYPES OF VIBRATION TESTS

Development Tests. Development tests are conducted on components to obtain basic data regarding the design. These tests are often more informal than qualification or acceptance tests as the usual instrumentation and inspection constraints do not apply. In many cases, the component can be mounted to a simple fixture, and its action observed under arbitrarily selected levels of vibration. A strobe light which may be made to flash in synchronization with the vibrator is a useful tool for observing the action of the component. If vibration equipment is available in-house, many tests may be run in a matter of minutes and involve only the test engineer and perhaps the vibration machine operator, as contrasted with the formal test which may involve considerable time because of the requirements for fixture design, fixture checkout, instrumentation, witnesses, and so forth.

Design Verification Test (DVT). A design verification test is conducted on the component to verify the design of the completed unit (see Sub-Topic 15.2.2). In this test the unit is subjected to the complete vibration requirements as noted in the specification. This test may be more severe than in the qualification test of the specification. In some instances, after a component has successfully completed a rigorous DVT, the qualification test requirements may be greatly reduced based on the information obtained during the DVT. One reason for the severe DVT is that a failure occurring in DVT may be corrected with fewer reporting and contractual implications than a failure that occurs in a qualification test.

Acceptance Testing. Vibration tests are often specified as part of the acceptance test for a component. The vibration levels are usually set much lower than the levels used in qualification testing. The purpose of the test is to discover any discrepancy that may have occurred in manufacture, such as a cold solder joint or a bad weld. High quality electric relays are often subjected to low level vibration testing as a routine manufacturing procedure.

15.7.1.4 TYPES OF EQUIPMENT

Mechanical Vibrators. Motor-driven mechanical vibrators use a scotch yoke, simple crank, or similar mechanism to impart reciprocating motion to a table or to a specimen. The acceleration level and frequency inputs can be varied by changing displacement or the rotational speed of the motor or both. With proper controls, they may be programmed to vary the inputs. These vibrators are relatively inexpensive and are used in testing small components for which the requirements of frequency and acceleration are not too exacting.

Simple Shakers. Electromagnetic shakers normally used in industry for such tasks as vibrating paper stock, IBM cards, or other materials may be used to good advantage where it is not required to meet stringent MIL Spec requirements. With a 60 cps input the device produces a clean, pure half cycle. Machines are available that will produce accelerations from 1 g to about 100 g and the level is normally controlled by a simple rheostat on the machine. If more precise g levels are desired, a Variac controlling the power line voltage may be used. These machines are ideally suited for performing preliminary tests before submitting a specimen to the much more expensive electrodynamic vibration programs.

Electrodynamic Vibrators. The most common vibrator in use in the aerospace industry is the electrodynamic vibrator which consists of a coil moving in direct proportion to an input voltage. These vibrators have a force output ranging from 2 to 35,000 pounds. They may be programmed to produce a sinusoidal waveform, random vibration, or to duplicate any pattern from a magnetic tape.

Electrohydraulic Shakers. The electrohydraulic shaker (or hydrashaker) consists of a hydraulically-driven piston or actuator controlled by a servovalve which receives a signal from a recorded tape. This type of shaker is useful where very high force-pound outputs are required and where the frequency does not exceed approximately 500 cps. The force output of these shakers is about 100,000 pounds.

Acoustic Vibrators — Noise Generators. There are three types of facilities in general use for conducting acoustical tests. These are progressive wave tubes, standing wave tubes, and reverberant wave chambers. Progressive waves, standing waves, and diffused fields will exist as a function of frequency range of these facilities.

1) *Progressive Wave Tube.* A free progressive wave in a medium free of boundary defects propagates with the velocity of sound. In a progressive wave tube, the acoustic source is coupled to a suitable test section by an acoustic horn. Reflections are avoided by a termination placed at the end of the test section.

2) *Standing Wave Tube.* A standing wave tube is a device containing a periodic wave having a fixed distribution in space which is the result of interference of progressive waves of the same frequency and kind. The standing wave tube is terminated by a hard or semi-hard reflecting surface which causes waves characterized by the existence of pressure nodes or partial nodes and anti-nodes fixed in space.

3) *Reverberant Wave Chamber.* A reverberant wave chamber is an enclosure containing a diffused sound field in which the time average of the mean-square sound pressure is everywhere the same, and the flow of energy in all directions is assumed to be equal.

While acoustic testing is becoming increasingly important, it still must be regarded as a specialized field limited to large prime contractors, governmental agencies, and private laboratories. MIL-STD 515-1 and MIL-STD 515-5 give general information on this test.

There are numerous accessories which may be required in addition to the basic shaker and control equipment. Some of these accessories are as follows:

1) An instrumentation quality tape recorder and playback system are used to record output from the accelerometers and to play this recording back to the shaker at a later date or to program vibration levels to the machine

It may also be used to play output records back to an oscillograph for visual interpretation.

2) An intercom system may be required between the shaker room and the control room. The head set serves a dual purpose in that it provides ear protection from the high levels of noise and unrestricted communication. Generally it is important that the operator be next to the component when performing frequency excursions or resonance tests. Actual visual observation, touch, and careful listening for intermittent sounds are extremely important.

3) A slip table is necessary for horizontal vibration testing of large items. A slip table consists of a specially-formulated heavy block on which the fixture slides. An oil film is maintained between the block and the fixture to decrease friction.

4) Sound-proofing of the control room often helps to minimize operator fatigue and reduce errors.

5) Closed circuit television may be necessary for viewing some setups which the operator may be unable to see or which may be hazardous if viewed at too close a range.

6) A stroboscopic light system for viewing the test item will be very useful. The flashing rate of the strobe can be controlled in the same manner as the input to the shaker.

7) An oscilloscope for viewing motion waveforms. (A Polaroid camera for recording these waveforms is often required.)

8) A multichannel recording oscillograph for making permanent records of multiple signals, such as those required during the conduct of a functional test, will be mandatory on many programs.

9) An assortment of accelerometers will be found to be necessary. These devices come in various ranges and are used for monitoring the levels at various points on a fixture and for controlling the shaker power supply. Amplifiers and readout meters for these additional accelerometers will be required.

Fixture Design. In all but the simplest cases it is best to have the fixture designed by a person with extensive experience in the field. While the basic principles of design apply to fixtures as to any other device, there is considerable art involved in executing a design relatively free of unwanted resonances and amplification. It is important that the center of gravity of the test specimen be precisely determined and installed directly over the axis of the vibrator to prevent unwanted couples. Generally tests are conducted separately in the X, Y, and Z axes. Ease of mounting and ease of changing of axes are important from a cost standpoint because time charges for this equipment generally continue as long as the specimen is on the table.

Fixture Scan. If the fixture is complex it is good practice to mount it without the test specimen or with a simulated test specimen on the vibrator, and subject it to a scan through the frequency to be used during the test. Should any resonances be observed during the scan, the fixture should be modified prior to the test. If such modification is not feasible, analysis of the specimen test data should consider fixture resonance.

15.7.1.5 LOCATION OF ACCELEROMETERS. Location of the accelerometer controlling the vibratory input is very important. Some specifications state the location for this accelerometer, but others do not. There can be a significant difference in the input to the specimen depending on the specific location used. For example, if the accelerometer is mounted on the vibrator head itself, the input indicated by the accelerometer may not approximate the actual input to the test specimen if the attachments are not properly designed. If the attachments fit loosely, for example, a decoupling will occur, and in this case, only a fraction of the energy being supplied by the shaker head will be transmitted to the part. If such a difference does exist, it may be readily measured by mounting an accelerometer on the shaker head and another accelerometer on the part itself. In a similar manner, the amplification factor of the fixture at any point may be determined by mounting another accelerometer at any desired location.

15.7.1.6 FUNCTIONAL TEST AND COMBINED ENVIRONMENTS. It is often necessary to conduct functional tests on equipment while vibrating, and the functional tests often include combined environments. For example, vibration may be combined with low or high pressure and low or high temperature. If fluid flow is involved, flex lines are required for input and output of the fluid. If the fluid media or any other aspect of the test is hazardous, it is necessary to conduct the test at a site with adequate protection for equipment and personnel. Such tests are usually more economically conducted by commercial laboratories which normally have such facilities. An environment such as high or low temperature is provided by surrounding the specimen with a special environmental chamber. The bottom of the chamber consists of a flexible diaphragm through which the motion of the shaker may be transmitted.

15.7.2 Shock Test

15.7.2.1 PURPOSE. Mechanical shock tests are conducted to determine that the specimen will perform satisfactorily in service under the expected shock loads.

15.7.2.2 EQUIPMENT REQUIRED. The device used for conducting shock tests will depend on the requirements of the particular specification. The equipment includes machines having platforms upon which the specimen may be mounted. After mounting, the test specimen is allowed to free fall to a sand bed or to lead cones which may be shaped to provide the proper rise time for the shock wave. Other devices impart shock by means of a hydraulic or pneumatic ram. An electromagnetic vibrator may be programmed to apply shock to a specimen. Often the same fixture that has been designed for the vibration tests may be used for shock tests.

15.7.2.3 PROCEDURE. Specific methods for conducting the various shock tests are described in MIL-STD-516. Additional information regarding both equipment and instrumentation may be found in the following USA Standards Institute bulletins:

1) S2.1-1961, "Specification for the Design, Construction and Operation of Variable Duration, Medium-Impact Shock-Testing Machine for Lightweight Equipment"

2) S2.2-1959, "Methods for the Calibration of Shock and Vibration Pick-Ups"

3) S2.3-1964, "Specifications for a High-Impact Shock Machine for Electronic Devices"

4) S2.4-1960 (R 1966), "Method for Specifying the Characteristics of Auxiliary Equipment for Shock and Vibration Measurement".

15.7.3 Acceleration Test

15.7.3.1 PURPOSE. Acceleration tests are conducted to determine the effects of acceleration on the performance of the component. The test may be accomplished on a centrifuge which imparts a constant acceleration to the test specimen, or it may be accomplished on a sled which imparts a linear acceleration including start and stop transients. Most acceleration testing is performed on centrifuges since the acceleration may be maintained indefinitely and these facilities are readily available. Linear acceleration facilities are very few in number, and test time on those that do exist is not generally available; therefore, this discussion is limited to centrifugal testing.

15.7.3.2 JUSTIFICATION FOR TEST. Acceleration tests have been specified for many test programs in the past without due consideration to the value of the results obtained. Acceleration levels specified are usually low because they are related to vehicle acceleration. Experience has shown that actual failure because of constant acceleration is rare. It is also likely that a unit that fails in the relatively mild environment of an acceleration test fails in the same manner in the vibration test. In addition, the effect of the acceleration on many components, such as regulators or solenoid valves, may be calculated from the known mass of the movable parts. Thus, there is little justification for specifying the test for conventional components such as most valves, regulators, pressure switches, and disconnects. There are exceptions; an example is level control valves which are acceleration sensitive. It should not require more than a cursory examination of the design and function of the unit to determine whether an acceleration test is justified. In the case of components where the orientation of the fluid might be difficult to assess or the effect of the orientation might be difficult to determine, the test is justified and should be conducted.

15.7.3.3 ACCELERATION TESTING FOR INSPECTION. In special cases acceleration may be used as an inspection tool. In one case, circular magnets for use in a solenoid valve were found to be subject to cracking, and the incipient flaw was difficult or impossible to detect. A fixture was built which consisted essentially of a small high-speed motor to which the magnets were attached. The magnets were then 100 percent inspected by spinning them to a preselected speed. The cracked magnets disintegrated under the acceleration loads.

15.7.3.4 EQUIPMENT. The equipment consists of a flat disc or a boom mounted on a motor or engine-driven shaft that can be rotated at varying speeds. The radius of the disc may vary from a few feet to 40 feet or more, and a wide range of rotational speeds is usually provided. Hydraulic drives provide the most versatile and vibration-free drive units.

Auxiliary equipment includes power and instrumentation electrical slip rings, swivel joints that permit flow of gas from a stationary source to the specimen and back to a receiver, and TV cameras for viewing the test specimen.

Special equipment may be required for some tests. For example, a large test setup which offers too much wind resistance to permit attaining the required speed on a given table could be equipped with a streamlined fairing to reduce the power requirements to an acceptable level.

Rapid changes in acceleration by varying the table speed are not possible on a centrifuge because of the inertia of the system. However, a very rapid change may be effected by changing the position of the specimen on the table. Diagrams of two methods are shown in Figure 15.7.3.4. In one setup, the test specimen is mounted on a pivoted arm which can be rotated through an arc by an actuator (pneumatic, hydraulic, spring, and latch arrangements). Initially, the test specimen may be positioned near the center of the table, and when the table is brought up to speed the arm can be quickly moved to reposition the specimen at the outer rim in a higher acceleration field. If the mass of the arm and the specimen is significant with respect to that of the table, it may be necessary to consider the effect of the momentary reduction in speed as the moment of inertia of the rotating mass is changed. A variation of the procedure involves the use of a carriage mounted on rails and a suitable pulley or actuator system for moving the carriage from the center of the table to the outward position (see Figure 15.7.3.4). This setup is more applicable to heavy units which might be difficult to control on a pivoted arm. In either setup, an analysis should be made to determine if the rate of acceleration change will produce a Coriolis effect that would cause additional loads having a magnitude and direction in excess of the allowable limits.

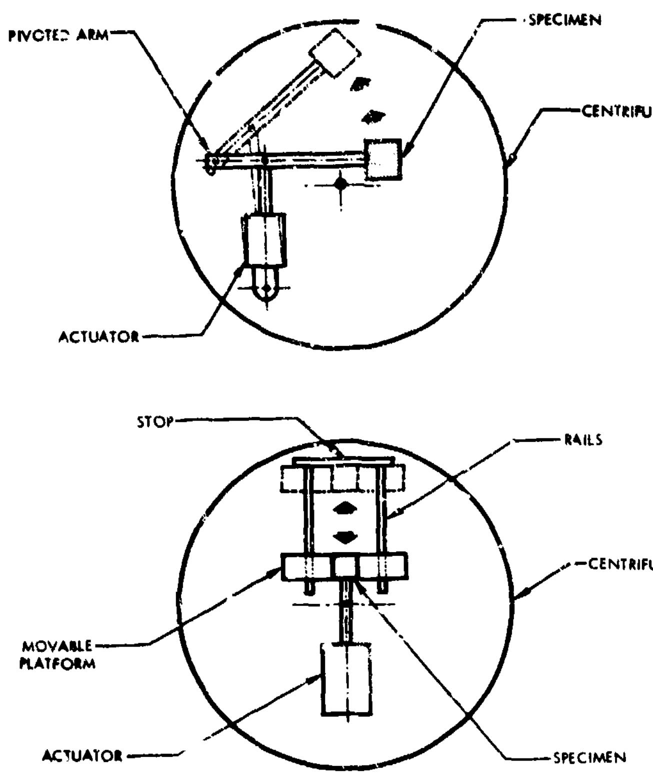


Figure 15.7.3.4. Devices for Creating Rapid Changes of Acceleration

15.7.3.5 COMBINED ENVIRONMENTS. The acceleration environment may be combined with several other environments in a single test, e.g., temperature, altitude, and vibration. An elaborate test of this kind was conducted involving an Atlas missile gas pressure regulator, which was simultaneously subjected to acceleration, vibration, and programmed change of pressure (altitude) while regulating pressure in a tank in which the ullage was also changing at a programmed rate. The helium gas temperature was varied from cryogenic levels to 400°F during the 5-minute run. The vibration was accomplished by mounting a modified MB C-25 shaker on the centrifuge.

A flexible diaphragm in a specially-constructed environmental chamber provided vibration input to the test specimen. Flow of helium gas on and off the table was done through special rotating swivels. The example is given to illustrate possible techniques, but it should not be inferred that such a complex test is necessarily desirable. The test setup is costly, and when a failure does occur it may be difficult to determine which environment caused the failure, thus complicating the task of correcting the design. However, there have been documented instances where a component successfully passed vibration and acceleration tests individually and failed when the environments were combined. Such component failure may be suspected where the side loads produced by acceleration can change the wear or actuation characteristics of the unit during vibration. Switches and relays are particularly susceptible to malfunction under the combined environment, as small changes in the friction forces can produce large changes in the operating characteristics. A design analysis should indicate the degree of potential failure. Sub-Topic 13.3.3 discusses design techniques to minimize the probability of failure due to acceleration.

15.7.3.6 TEST PROCEDURE. When the required acceleration of the specimen is known, the combination of radius and speed required may be determined to define the position of the specimen on the table. Angular acceleration is defined by the equation

$$a = \frac{v^2}{r} \qquad \text{(Eq 15.7.3.6a)}$$

where

- a = acceleration, ft/sec^2
- v = tangential velocity, ft/sec
- r = radius, ft

The force required to restrain the specimen is

$$F = ma = \frac{w}{g}\left(\frac{v^2}{r}\right) = \frac{w}{g}\left(\frac{\pi^2 N^2 r}{900}\right) \qquad \text{(Eq 15.7.3.6b)}$$

where

- F = force, lb$_f$
- m = mass, lb$_m$
- g = local acceleration of gravity (32.2 ft/sec^2)
- N = rotating speed of table rpm

From this expression, the acceleration can be expressed in terms of rpm and table radius,

$$a = \frac{\pi^2 N^2 r}{900} \qquad \text{(Eq 15.7.3.6c)}$$

If the radius for a particular test is fixed, the constants may be grouped, $a = KN^2$. Nomographs and slide rules are available from equipment manufacturers to facilitate calculations and are useful when numerous tests of different specimens are to be conducted.

15.7.3.7 PRECAUTIONS IN TESTING. Acceleration is directly proportional to the radius, and a specific acceleration may be obtained at only those points in a cylindrical plane of specific radius on the specimen. Thus, there will be a radial gradient in acceleration across the specimen, and if the dimensions of the test article are large in relation to the radius of the table, the results may be unacceptable. The solution is to use a larger radius or to position the specimen such that the affected portion (e.g., a valve poppet) will experience the desired acceleration.

15.7.3.8 TEST TECHNIQUES. By the use of electrical slip rings and rotary swivels, functional tests may be accomplished on a rotating table in much the same manner as they are on the bench.

When flow rates are low, as in the case of leakage measurement, the test may be more conveniently conducted without swivels by including a suitable pressure source in the test setup on the table. A remotely-operated solenoid valve may be used to admit pressure to the setup, and leakage can be collected in a plastic bag or it may be measured by the water displacement method using burets mounted near the center of the table.

If slip rings are used on the centrifuge, several precautions must be observed. Squib firing circuits and instrumentation circuits should be widely separated to prevent possible interference. If an oscillograph is to be operated off the table, it is advisable to put the amplifier on the table and amplify the signal before it goes to the slip rings. If the amplifier is mounted off the table, it will also amplify the slip ring noise, which may make the data unintelligible.

15.7.4 Sand and Dust Test

Sand and dust tests are specified for equipment likely to be operated in sandy and dusty environments. Military vehicles, including aircraft, are examples of such equipment, and components that are vented or exposed to the atmosphere and intended for use on such equipment should be subjected to the sand and dust tests in a qualification test program. If the unit is hermetically sealed, it should not be subjected to such a test because the only effect will be to abrade the surface finish. Aerospace equipment normally should not be subjected to this testing because of the well-defined procedures employed for packaging, handling, and installation, unless characteristics of the component are such that entry of external contamination is highly probable, e.g., relief valves without dust barriers or filters. The contamination sensitivity is established by this test. Effective barriers may be devised for many such valves during the design and may be in the form of plastic blow-out plugs or elastic bands that blow off when the valve is activated. For missiles and other equipment kept in controlled or semi-controlled environments at launch sites, it is not likely that the test would provide any useful results.

Requirements for the equipment and for conducting the tests are described in Method 510 of MIL-STD-810. The sand and dust test is relatively inexpensive. A commercial laboratory will charge approximately $125 for a normal specimen. The time required to conduct the test, which is usually nonoperating, is approximately 12 hours (exclusive of setup time). The cost of conducting this or any of the following environmental tests may be greatly affected by the size of the unit. If the unit fits a standard environmental chamber or test facility, the tests will be relatively inexpensive. Obviously, however, if a special facility has to be built to accommodate the unit, the costs will rise accordingly.

15.7.5 Humidity Test

15.7.5.1 INTRODUCTION. Humidity tests determine potential failures of a component because of moisture and absorption, adsorption, or penetration. In most cases it is a necessary environmental test, as indicated by the fact that the failure rate is relatively high. Hermetically-sealed units of corrosion-resistant material (e.g., an all-welded absolute pressure regulator which has no access whatever to the ambient atmosphere, or an absolute pressure switch of similar construction) do not require humidity tests.

15.7.5.2 EFFECTS OF HUMIDITY ON THE COMPONENT. Results of component exposure to humidity are:

1) A very thin film of water forms on the object.

2) Vapor penetrates into the object by various mechanisms.

3) If electromagnetic energy is present, there is an absorption of energy in the surrounding vapor which will have the effect of loading an inductive element.

4) Rust or corrosion may result.

The formation of water film and the penetration of the unit are of greater significance than the electromagnetic effect. When the film forms on the surface of the unit, it rapidly becomes ionized and conducting, thus providing a conducting path and capacitance effect because of the high dielectric constant. The practical effect is to cause a change in insulation resistance, surface resistance, inductance, and capacitance. The surface arcing resistance is lowered. Penetration of liquid or vapor into an organic material causes dimensional changes, lowering flexural strength and, in some cases, improving impact strength, which may degrade performance. It also adversely affects the electrical characteristics of the material.

A film of pure water on a surface is significantly corrosive only when contaminated with impurities such as salt or acids. As the levels of such contaminants vary with location, the effects of humidity on the surfaces will also vary.

15.7.5.3 MECHANISMS OF ENTRY. Water can enter the component by diffusion through a material which forms a part of the equipment and by entry through a hole in the sealing of the equipment. Moisture can permeate most organic materials, since the molecular spacing is larger than the diameter of the water molecule. It is important to note that a component that is apparently hermetically sealed may not, in fact, present an adequate barrier to moisture and the adverse effects noted above should be anticipated. Temperature or ambient pressure cycling during exposure to humidity produces breathing which results in entry of moisture through openings not otherwise objectionable.

15.7.5.4 EQUIPMENT REQUIRED. Humidity chambers are generally available in sizes ranging from 2 to 64 cubic feet and costing from $3000 to $8000 depending upon the programming equipment included. These chambers may be set up to program a test specimen through a procedure as specified in MIL-STD-810, Method 507. As operation of the specimen is usually not required during this test, little or no technical attention is required during the 10 day test period. A commercial laboratory will perform such a test for approximately $100, including a report.

15.7.6 Salt Spray Test

The salt spray test is used for all components exposed to a salt atmosphere such as is normally found near oceans. The tests identify potential corrosion problems resulting from the use of dissimilar metals or of noncorrosion-resistant materials. The test should be specified if a problem from corrosion could arise; however, if the material of a hermetically-sealed unit is known to be corrosion resistant, the test probably could be deleted without risk. Entry of salt atmosphere through small openings may not occur during the accelerated test because a cake of salt seals the opening. Normal salt atmosphere would not obstruct the openings and could therefore be detrimental to internal parts.

15.7.6.1 EQUIPMENT REQUIRED. Commercially available chambers are built to withstand corrosive salt spray. A 3-cubic foot chamber will cost between $500 and $1000 depending upon the control equipment supplied. The volume of testing would determine whether or not such a purchase should be made. The test is relatively inexpensive in commercial laboratories, ranging from $50 to $75.

The test is conducted in accordance with Method 509 of MIL-STD-810. The elapsed time for the testing is 48 hours. At the conclusion of the test, the part is rinsed in tap water, then inspected and tested 48 hours after the rinse.

15.7.7 Fungus Test

The fungus test determines if the materials of the component will support a fungal growth. The test should be performed on equipment to be used in the tropics or in damp areas. It is rarely justified on normal aerospace components which usually consist of non-nutrient materials.

15.7.7.1 EQUIPMENT REQUIRED. The fungus test is usually conducted in a commercial laboratory using the services of a bacteriologist to prepare the four groups of fungi required. The duration of the test is 28 days, which may cause a schedule problem if a limited number of test samples is available for a given test program. The test is performed per Method 508 of MIL-STD-810. The cost for this test is approximately $125.

15.7.8 Sunshine Test

Sunshine tests are specified for nonmetallic material, such as rubber and plastic, which will deteriorate after long exposure to sunshine. Many common plastics, for example garden hoses, tend to become stiff and brittle after exposure to sunshine, and this effect will often be associated with loss or change of color. Rubber products also tend to become hard and brittle, and this effect is greatly accelerated in the presence of air with a high ozone content. There is probably no justification for running such a test on a metal component.

A chamber incorporating arc lamps capable of supplying energy in wavelengths above 7800 angstrom units and additional lamps capable of supplying energy in wavelengths below 3800 angstrom units is specified by MIL-STD-810, Method 505. The duration of the test is 48 hours.

If duplication of results is not a requirement, information on the effects of sunshine may be obtained by exposing the specimen to the sun in an area where the sunshine is fairly constant (e.g., desert regions of Arizona, Nevada, and California).

15.7.9 Rain Test

A rain test is conducted to determine if water penetration will result from a simulated heavy rainfall. This test is rarely conducted on present day equipment, and it is doubtful that a rain test would expose discrepancies not discovered by the humidity test. Examination of the detailed design should indicate whether the test is required.

Method 506 of MIL-STD-810 describes the equipment needed and the procedure for conducting the rain test. The duration of the test is 2 hours, and the normal laboratory charge is approximately \$125. If rigorous documentation of the test is not required, the same results may be obtained in-house by directing an ample spray of water from an ordinary shower head or hose-type nozzle on the test specimen.

15.7.10 Explosion Test

An explosion test is conducted to determine that operation of the unit will not cause an explosion due to an electrical arc or sparks igniting fumes or vapor that might be present. A procedure for performing the test is given in Method 511 of MIL-STD-810.

15.7.10.1 EQUIPMENT REQUIRED. Equipment required for the explosion test consists of a suitable chamber or tank, a vacuum pump, provisions for raising the temperature of the tank, and provisions for injecting a specified explosive atmosphere into the chamber. The chamber is usually fitted with a hatch held in place by atmospheric pressure since the pressure in the chamber is less than atmospheric during the test. If an explosion does occur, the hatch blows off before significant pressure can build up inside the chamber. Several tests at different conditions of temperature, pressure, and fuel-air mixture may be required.

15.7.10.2 TEST PROCEDURE. The test specimen is installed in the chamber, and the test conditions are established (proper temperature, vacuum, and fuel-air mixture). The specimen is then operated. If no explosion occurs the unit is deemed non-hazardous in an explosive atmosphere.

This test should be conducted by a commercial laboratory because it requires special equipment and trained personnel, and its conduct within city limits is often prohibited by ordinances.

15.7.11 Temperature-Altitude (Thermal Vacuum) Test

The combined environment of temperature and altitude is imposed on components which may be adversely affected by the combination but which may not be affected by the environments imposed individually. The test is particularly applicable to equipment dependent upon convection cooling which is ineffective at altitude because of the lower density of the air. The test is also applicable to any equipment that might tend to outgas or sublimate and have deleterious effects upon other equipment in a space vehicle, such as fogging of optical systems.

15.7.11.1 TEST PROCEDURE. Two general procedures are described in MIL-STD-810 for conducting temperature-altitude tests. Method 504 is intended primarily for electronic equipment. This procedure requires approximately 35 hours in the test chamber, exclusive of functional cycles required between the various settings of temperature and altitude. Because close monitoring is required and because numerous functional tests are conducted, it is advisable to conduct this test in-house, provided that a suitable chamber is available.

Method T517 is a tentative method entitled "Space Simulation," and is to be applicable to space components in general. It specifies very low ambient pressure (10^{-7} torr) and solar heating. The length of the test is a function of the mission time. The conditions of the test are difficult to meet if there will be any significant outgassing from the components under test.

15.7.11.2 EQUIPMENT REQUIRED. The equipment required for Method 504 is a conventional altitude chamber with a capability of programming altitude from 0 to 100,000 feet. Equipment for maintaining temperatures between -62° and +185°C is also required. As the test procedure requires recording the temperature and the altitude at numerous points, a strip-type recording instrument should be included as part of the equipment.

Method T517 requires much more elaborate equipment to provide the low vacuum solar radiation and black-coated cryogenic shrouds capable of maintaining liquid nitrogen temperatures. No estimates of the outside cost of such a test can be made because of the wide variations in the duration of the test according to Method T517.

15.7.12 Low Pressure Test

The effects of low pressure can be loss of pressurization fluid, rupture or distortion of pressurized containers caused by a change in differential pressure, damage caused by reduced heat transfer capability, and damage resulting from electrical arcing.

15.7.12.1 TEST PROCEDURE. Method 500 of MIL-STD-810 describes two low pressure procedures. One is for ground equipment subjected to operation at a high altitude or shipped by air, and the other is for equipment designed to be used on aerospace vehicles. Ground equipment is subjected to an absolute pressure of 3.44 inches of mercury, equivalent to 50,000 feet above sea level, and is maintained at this pressure for 1 hour. The pressure is then increased to an equivalent of 10,000 feet above sea level, at which pressure the test item is operated. In this test the temperature is uncontrolled. In the second procedure, the pressure is reduced to the lowest value for which the equipment is designed and the temperature is maintained at -65°F. These conditions are maintained for 1 hour, after which the equipment is operated. Then with the equipment still operating, the pressure is increased to the prevailing room ambient level.

15.7.12.2 EQUIPMENT REQUIRED. For small components, a standard bell jar arrangement is satisfactory. For larger components, or where a low temperature is also required, a conventional altitude chamber is used.

A commercial laboratory will conduct a test to MIL-STD-810, Method 500, for about $125.

15.7.13 High Temperature Test

The purpose of this test is to determine if the component will be damaged by exposure to high temperatures. Examples of possible damage are: permanent set of packing and gaskets, distortion of seals and valve seats, a change in size or actual chemical composition, or overheating of electrical coils causing insulation damage.

The test temperature has been established at 160°F. This value was arrived at by considering 125°F to be the maximum ambient temperature normally encountered. To this was added 35°F to account for solar radiation and increases in temperature because of operation. These criteria were based on aircraft requirements and are not necessarily applicable to space vehicles.

15.7.13.1 EQUIPMENT REQUIRED. Any chamber suitable to house the equipment and provide the proper temperature may be used for this test.

15.7.13.2 TEST PROCEDURE. The component is placed in a chamber in which the temperature is raised to 160°F and maintained for a period of not less than 48 hours. At the end of this period, the temperature of the unit is adjusted to the maximum operating temperature. When the temperature has been stabilized, the equipment is operated.

15.7.14 Low Temperature Test

The purpose of the low temperature test is to determine if the component will be damaged as a result of storage and operation at low temperature. Difficulties to be expected are differential contractions of metal parts or loss of resiliency in packing, gaskets, and seat material. The lowest test temperature specified in MIL-STD-502 for this test is -80°F. This low temperature is specified for storage and transport only for equipment to be operated in the United States. In temperature-controlled areas, the specified temperature is +35°F.

15.7.14.1 EQUIPMENT REQUIRED. Any standard chamber capable of maintaining the minimum temperature required may be used for this work.

15.7.14.2 PROCEDURES. The equipment is installed in a chamber and maintained at 80°F for 48 hours. At the conclusion of this exposure, the item may be removed and inspected for damage, then reinstalled in the chamber and stabilized at its design operating temperature. A functional test is conducted at design-operating temperature to determine what effects, if any, have been caused by the low temperature.

15.7.15 Temperature Shock Test

The purpose of the temperature shock test is to determine the effects of sudden changes in temperature on ground or aerospace equipment. The chief effects to be expected are cracking or rupturing of materials due to sudden expansion or contraction (thermal stresses are discussed in Section 14.0, Stress Analysis).

15.7.15.1 EQUIPMENT REQUIRED. Two chambers are required, each capable of maintaining a specified temperature. In addition, material handling equipment such as a forklift or crane may be required for heavy equipment.

15.7.15.2 TEST PROCEDURES. Test items are placed in a test chamber and maintained at 185°F for 4 hours, then transferred (within a minimal time such as 5 minutes) to a -40°F chamber. The equipment is maintained at this temperature for 4 hours and is then returned to the high temperature chamber within 5 minutes. Three such cycles are conducted. At the end of the third cycle a functional test is conducted at room ambient conditions.

15.7.16 Shipping Shock Test

Shipping shock tests are designed to ensure that the product will arrive at its destination in an undamaged condition. Imposing the requirement causes the manufacturer to design the shipping container in such a manner that the component is not likely to fail the shipping shock test. If the type of shipping container and packaging are known in advance, it is sometimes possible to delete the requirements for a shipping shock test because the results may be safely predicted. For example, if a small component (such as a valve weighing 1 pound) is properly packaged in a 1 gallon metal container and surrounded by resilient packing material, the valve is certain to be undamaged in any reasonable shipping shock test.

If time permits, realistic data may be acquired on the ability of a component system and package to survive shipping shock by simply routing a test sample via common carrier to one or more destinations and return. The condition of the package and component will provide a typical picture of what may be expected during the actual shipment. If desired, the specimen may be instrumented with a recording device that will preserve a history of the shock encountered during the shipment.

JPL Technical Report No. 32-876, dated 15 March 1966, entitled "The Dynamic Environment of Spacecraft Surface Transportation" (Reference 12-23), describes a test program involving the shipment of Ranger spacecraft from California to Cape Kennedy. The vehicle was instrumented to record shock and vibration data. The article provides useful information on instrumentation techniques for this type of test. If a standard test is to be conducted, e.g., to MIL-STD-516 Procedure I, the cost will be approximately $175.

15.7.17 Combined Environments Test

The ultimate judge of the adequacy of any component or system is the vehicle in which the components and systems are to be used. That is, only in an actual flight are all of the environments and operating conditions present in precisely the correct levels and proportions. A goal in testing is to approximate these conditions as precisely as possible in the test program. However, it is a monetary and physical impossibility to completely duplicate all of the conditions; therefore, necessary compromises should be made to consider those conditions that are believed to be most likely to produce adverse results on the system or component.

Considerations of interaction between various parameters are given below. The discussion is not all-inclusive because the number of combinations possible is almost infinite. The intent is more to stimulate thought that will result in specifying the necessary combinations in the test procedure.

Considering the parameter of pressure, including both very high and very low pressure, the following observations may be made: In conducting the proof or burst test on a vessel, both the fluid used in service and its temperature should be considered in the test. If the fluid is corrosive in nature, it may attack the material of the vessel in the stressed condition (at proof or burst levels), whereas there may be no attack in the unstressed condition. This fact, therefore, should influence the conduct of either a compatibility test or a stress level test such as proof or burst pressure. Similarly, temperature must be considered. A low temperature may be either beneficial or detrimental to the properties of a particular material. Elevated temperatures usually degrade the properties of a material. Therefore, if a component is to be used at either extreme of temperature, these extremes should be employed during any test involving pressure. At very low pressures, electrical discharge may occur and heat transfer by means of conduction may disappear entirely. Both of these phenomena could affect the results of a test conducted on a solenoid coil.

It is common practice to combine vibration tests with various environmental conditions such as temperature and altitude, and it is usually required that the part be functioning during these tests. The reason for the requirement is that a component may successfully pass a severe vibration test in a nonfunctioning mode and fail the vibration test at significantly lower levels of input while operating. An example might be a solenoid valve or regulator in which the poppet is held firmly against the seat in the closed position. When the valve is open the poppet may be supported in a less rigid manner with the result that the vibration may damage the seat due to a transverse motion which causes a scuffing action.

The purpose of the test, and the time and money available for the test, will dictate the extent to which the environments are combined. In development testing, for instance, combined environments are not generally used because of the difficulty in determining which environment caused a particular failure. On the other hand, in qualification testing it is desirable to combine as many environments as possible within the limits of time and money. Beyond a point, however, the complexity of the test becomes so great that diminishing returns are received for the efforts expended. Cost analysis and good judgment must be employed in devising the test procedure involving combined environments.

15.8 SPECIFIC COMPONENT TESTS

Some components have inherent characteristics that require special mention with respect to the test techniques to be used.

15.8.1 Solenoid Valves

The usual tests conducted on solenoid valves, such as leakage, power, and response, are described elsewhere under the appropriate headings. However, two precautions should be mentioned in testing solenoid valves. In conducting cycling tests, it is important to use a cycling rate consistent with the design duty cycle of the valve if the duty cycle is rated as being other than continuous. For a valve designed for intermittent operation, the cycling rate should be such that overheating of the coil will not become a problem. Similarly, testing performed in a temperature-altitude condition should take into account the fact that heat transfer in a vacuum is different from heat transfer in air, and if the design has not provided for adequate heat transfer by other means such as conduction, overheating and burnout is a possibility and should be anticipated. If the design of a valve includes the flowing medium as part of the heat transfer mechanism, the same medium should be used in the test to avoid the possibility of overheating and burnout. Heat transfer is treated in Section 2.0 of this handbook, and solenoids are discussed in Detailed Topic 6.9.3.7.

15.8.2 Explosive Valves

Because of the nature of explosive valves (Sub-Section 5.6 of this handbook) very little testing can be accomplished prior to use beyond the conventional circuit continuity tests conducted on the squib itself. For a normally-closed valve, a conventional leakage measurement may be made in the usual manner. For a normally-open valve, verification of the pressure drop could be made if deemed necessary. However, it is not likely that the pressure drop would vary significantly from valve to valve of this type. Therefore, once determined, it should not be necessary to repeat this test on a routine basis. After a normally-open valve is fired, a leakage test may be conducted; conversely, after a normally-closed valve is fired, a pressure drop test may be conducted, if required. As some ordnance valves generate considerable particles during operation, a particle count taken on the downstream portion of the valve may be warranted. An examination of the locking mechanism used to hold the valve open or closed after actuation may be advisable depending on the design of the particular valve. The squib (explosive actuator) is usually evaluated on the basis of all-fire and no-fire tests conducted on samples from each batch or lot.

15.8.3 Relief Valves

The operational testing of relief valves may be done in accordance with Sub-Topics 15.5.5 and 15.5.6. Sub-Section 5.5 describes the characteristics of relief valves, and the pecularities in testing unique to these valves are discussed below.

The upstream plumbing and, in particular, the ullage should closely simulate the conditions the valve will experience in service. If the flow rate is relatively high, the time available for an actual run will be short due to limitations of tankage. Close coordination of all test functions is necessary to obtain as much data as possible in the time available. Care must be given in the test setup to allow for any possible mechanical reaction in the plumbing or fixtures as a result of valve discharge forces. With high flow valves this reaction can be large, and if the valve is not properly secured, damage to the equipment or injury to personnel can result. If [illegible]alve under test discharges into a duct or pipe, it may [illegible]sible to affix a tee to the end of the pipe to neutralize any reaction. Similarly, the noise and air blast from such a valve can reach dangerous proportions, and proper protection for personnel should be provided. If the valve is to be tested in an environmental chamber or any other form of enclosure, provision must be made for proper venting of the gas vented during valve operation. The ports that are often provided in environmental chambers may be inadequate to relieve a sudden overpressurization; a very small rise in internal pressure in such a chamber may produce destructive forces because of the large areas involved. It may be desirable to employ a blowout wall construction for testing any valve that could operate during the course of a test. These precautions are especially important in the case of a burst disc, which is a special form of relief valve, because of the suddenness with which this valve operates. It operates with no warning and does not reseat as the pressure drops but continues to flow until the tank or source is either empty or shut off.

15.8.4 Pressure Switches

Pressure switches (described in Sub-Section 5.16) are unique in many ways with respect to testing, and unless the problems peculiar to switches are understood, completely erroneous results will be obtained

In testing a pressure switch for actuation point, the rate of pressure application should be specified. In many designs, the orifice which admits the pressure to the sensing element is extremely small, and if gas pressure is applied too rapidly, the sensing capsule or element will not have time to fill. Therefore, when an actuation indication is eventually observed, the indicated reading will be too high. In switches which have large amounts of friction, a similar error may be encountered from a rapid application of pressure. Failure of a pressure switch to function correctly at the specified rate of pressure rise when the switch actuates at the design pressure when pressure is raised slowly may indicate that the problem is either a small orifice or high friction.

If the switch is an absolute pressure switch, the test gauge scale reading should be offset to account for the local ambient barometric pressure. Failure to take this into account will result in an error of approximately 15 psi at sea level.

Most switches make some audible sound upon actuation, and this sound is sometimes taken as an indication that the switch has operated. The switch may have operated, but there may be a circuit defect which would not be detected by sound; therefore, lights or a meter should always be used to ensure that no circuit problem exists.

Many switch designs have an inherent problem known in the switch industry as *first cycle stick*. This refers to the fact that the first operation of the switch, especially after it has undergone an extreme temperature change, will be different from subsequent operations. This is caused by the moveable elements within the switch repositioning themselves after an extreme temperature change or after being brought from a pressure that was initially zero. If a switch is likely to have this peculiarity, the test specification should state that any data taken in the first cycle of operation may be different, to differentiate it from subsequent data. The switching pressures for certain designs of units may shift if, for example, the pressure is not continuously increasing but has a slight decrease before finally reaching the actuation point.

Another peculiarity of some pressure switches is a phenomenon known as *dead-breaking*. This term describes a condition in which the contacts of a switch move to an intermediate position which is neither on nor off. When observing the actuation of the switch, especially at extreme temperature, the indicating lights should be carefully observed to ensure that the time delay is within specified requirements.

When life cycling a pressure switch, the unit should be cycled within the normal operating range, rather than from zero to the operating range and then back to zero. The reason for this is that the actuation pressures of some switches will be different when they are cycled to zero and then back to the operating point, and this does not usually simulate the application condition.

An automatic life cycling test setup is shown in Figure 15.8.4. The system consists of a regulator, solenoid valve, two manual bleed valves, a gauge, and the test specimen. The regulator is used to control the maximum pressure to a safe limit, and the two manual valves may be simultaneously adjusted to control pressure rate of change to achieve any desired cycling rate. The solenoid-operated counter automatically records each cycle of operation. If desired, the pressure gauge may be replaced with a pressure transducer with the output fed to a strip chart recorder.

Chatter, especially under vibration conditions, is particularly important with pressure switches. Chatter monitor tests are discussed in Sub-Topic 15.6.8.

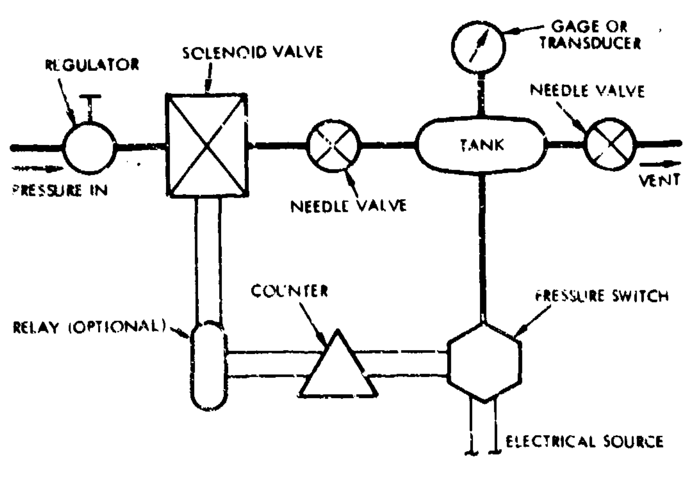


Figure 15.8.4. Pressure Switch Cycling Setup

15.8.5 Filters

Filters (Sub-Section 5.10) are very commonly used in fluid systems but require test procedures unlike those employed for other components. Some of the tests may be readily conducted in any conventional laboratory by personnel possessing ordinary skills in the testing field. Other tests not only demand special equipment, clean room facilities, and tight fluid cleanliness levels, but also require experienced and skilled technicians to obtain suitable results. In general, a normal acceptance test for a filter could be completely performed by most organizations with clean room facilities. However, such tests as the *initial cleanliness test* or extensive *qualification tests* will require special equipment and techniques as well as experienced and skilled technicians to execute them properly.

In addition to the conventional tests run on any pressure vessel, such as leakage, proof pressure, etc., there are three unique tests which may be conducted on filters or filter elements. These tests are:

1) *Initial Cleanliness.* Although this test is also conducted on most other aerospace components as a routine matter, the conduct of the initial cleanliness test on a filter requires special equipment and techniques and is therefore regarded as being unique with respect to filters.

2) *Filtration Rating.* This test determines the pore size distribution (maximum and average) of the filter medium and is therefore an indication of the size of particles that will pass through the filter under carefully controlled conditions.

3) *Contaminant Capacity.* This test is a measure of the useful service life of a filter. It provides an indication of the amount and effectiveness of the filter medium furnished by the manufacturer and determines whether the filter envelope is properly sized for the intended mission duty cycle by demonstrating that the initial clean pressure drop at room flow does not increase to an unacceptable value as a result of adding a specified amount of a standardized contaminant upstream of the filter.

The individual tests are illustrated in Figure 15.8.5 and are discussed in greater detail under the headings below.

15.8.5.1 ACCEPTANCE TESTING. The various types of tests that should be conducted on each filter furnished under contract are listed below. All of these tests should be conducted under environmentally controlled conditions since any contamination introduced into the downstream side of the filter element would eventually end up in the system fluid and any contamination introduced into the upstream side would reduce its useful service life.

Examination of Product. Each filter should be examined for dimensional compliance with the drawing requirements in order to assure ease of assembly into the component or fluid system. In cases where weight is critical, each unit should be weighed individually and the measured value recorded. The downstream side of each filter element and housing should be examined under 30 to 40 power stereoscopic magnification to make certain that no loose burrs, wires, or other particles are present. In addition the amount of surface area should be verified since the contaminant capacity cannot be verified without conducting a destructive-type test.

Initial Cleanliness. The initial cleanliness of a filter is determined by repetitive cycles of first subjecting the filter to an ultrasonic vibration field and then flowing a predetermined volume of fluid (usually 100 or 500 ml) through the filter until the total specified sample amount (usually 500 or 2,500 ml) has passed through the filter. The effluent is passed through a downstream membrane-type filter and examined under a microscope. Specific details for conducting this test are contained in ARP 599. Particle counting techniques and methods are described in ARP 598. It should be noted here that any filter cleanliness evaluation test method which does not employ ultrasonic energy (such as the *flow-through* or *vibraflush* methods) do not provide sufficiently high energy levels to release built-in contaminants. As a consequence these tests are practically meaningless except where gross contamination is present. A full definition of the test parameters should include the following:

a) Whether the test is to be conducted from the outside to the inside of the filter element, in the reverse direction, or in both directions. This is a function of the flow direction and system application of the filter.

b) The total volume of the sampling fluid on which the count is to be based and the incremental volumes withdrawn.

c) Whether the procedure of ARP 599 or some modified method should be used.

d) The allowable number of particles in each of at least 3 samples (see MIL-STD-1246A for classes).

This test should always be conducted as the final acceptance test since it is a cleanliness verification test. For this reason, the filter must be dry at the start of the test and the first fluid passing through it must be part of the total sample as specified in ARP 599.

Initial Bubble Point Test (Absolute Rating). This test is fully defined in ARP 901. It measures the air pressure at which the first bubble emits from the wetted filter medium, which is an indication of the diameter of the largest sphere that can be inscribed in the pore structure. The test can be conducted in any number of fluids if the fluid is capable of wetting the filter medium and if the surface tension, temperature, and depth of immersion are measured and correlated to a standard bubble point constant (see Figure 15.8.5).

Boiling Pressure Test (Average Pore Size). The boiling pressure test is similar to and an extension of the initial bubble point test described above for determining the maximum pore size of a filter. Air is injected under a specimen submerged in a liquid of known surface tension, usually alcohol. Air pressure is increased slowly until air bubbles *boil* at the surface. A graph of air flow rate versus pressure will show that pressure increases with increases in flow rate up to a saturation region where no appreciable increase in pressure is required for an increase in flow rate. At the saturation pressure, air is passing through a representative number of pores (Reference 6-211). The average pore diameter is obtained from an equation similar to the one used for the initial bubble point test described in ARP 901 (see Figure 15.8.5 and Reference 37-12).

Clean Pressure Drop. This test is important particularly in the case of liquid rocket propellant feed systems where excessive pressure differentials caused by the filter can have

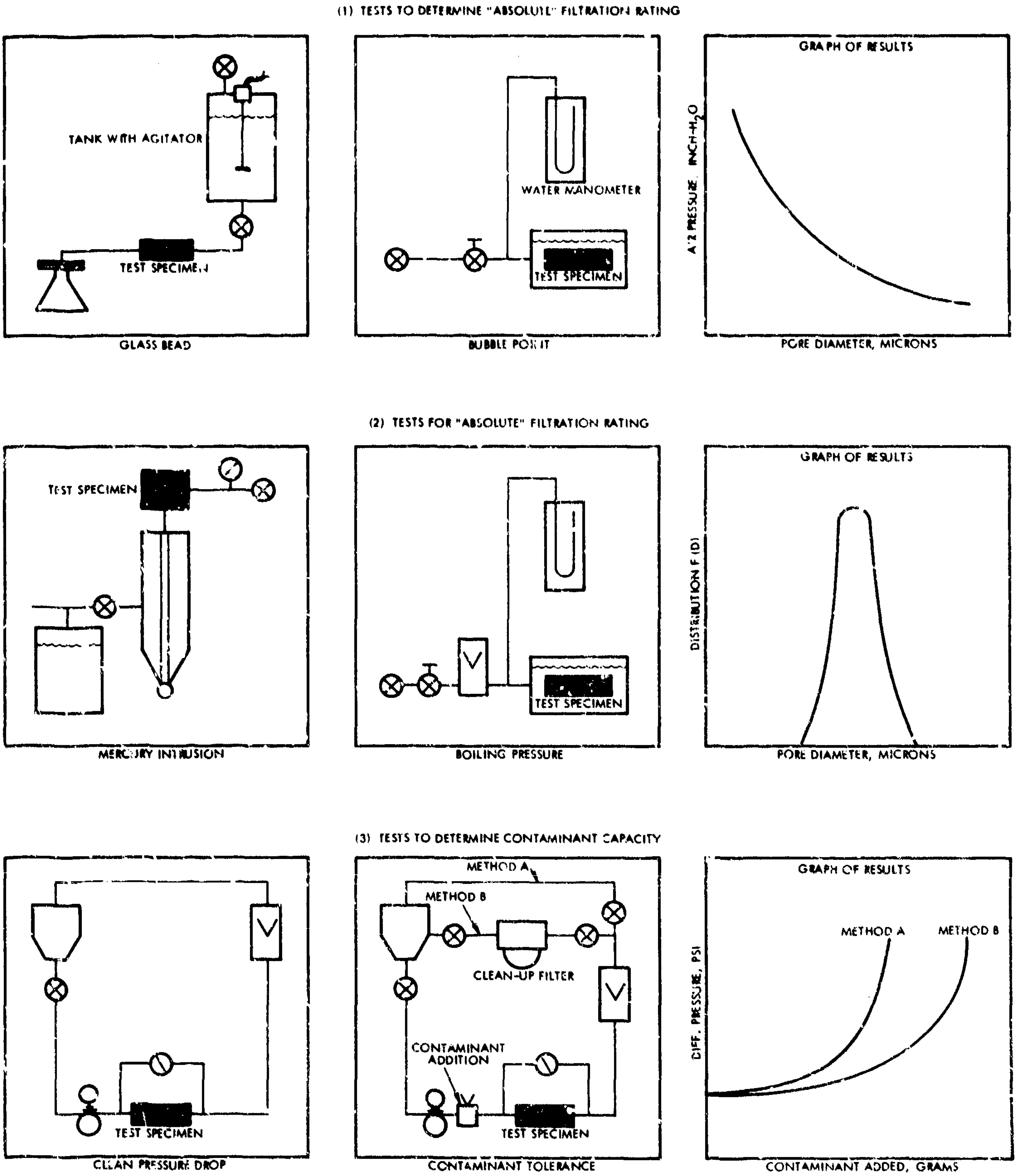


Figure 15.8.5. Standard Test Methods for the Determination of Filter Performance

a serious impact on fuel/oxidizer mixture ratio, specific impulse, etc. In such critical cases it is suggested that each filter delivered under contract is tested for pressure drop at maximum rated flow. The test methods are fully described in ARP 24B.

Contaminant Capacity. There is no nondestructive test method available to verify the contaminant capacity or useful service life of a filter. This characteristic can only be determined by verifying that the surface area provided in the filter is equal to that of the filter used for qualification testing. Control should be exercised on the basis of a configuration management plan.

15.8.5.2 QUALIFICATION TESTING. Qualification testing includes all of the tests described under acceptance testing and also requires the tests described below. Environmental tests such as shock, vibration, acceleration, and others described in Sub-Section 15.7 are not discussed here as they are not basically different from such tests that would be run on any other component.

Filtration Performance

a) *Absolute Filtration Rating.* This characteristic is usually expressed in terms of the *absolute micron rating* which is defined as the largest hard spherical particle that can pass through the filter under blow-down conditions. This test determines the maximum pore opening in a filter medium (see Figure 15.8.5). The tests are based on the filtration of an artificial contaminant under specified test conditions. Spherical glass beads are used to provide positive identification and obtain consistent results. The effluent fluid containing the artificial contaminants which have passed through the test filter is collected. This fluid is filtered through a membrane filter, and the particles retained on the membrane surface are scanned microscopically and examined. The largest particle is measured, and this defines the *absolute rating* of the filter in microns. This microscopic measurement is two-dimensional, but since the particles are essentially spherical only one dimension is considered. The test procedure must define the glass bead mixture to be employed, the quantity of glass beads to be added, and the flow rate during the blow-down test (see Table 15.8.5.2 for recommended glass bead mixtures). The test results are expressed in microns and can be correlated to the initial bubble point test described under Acceptance Testing (Detailed Topic 15.8.5.1).

Average Filtration Rating

a) *Gravimetric Efficiency.* In order to provide a measure of the filter's ability to remove particles in sizes smaller than the *absolute rating*, a number of tests are employed which determine its efficiency in percent by the weight of removing a predetermined amount of glass beads introduced under blow-down conditions similar to those described for the absolute rating tests (see Figure 15.8.5). The test procedure must again define the glass bead mixture, the amount to be added, the type of fluid, the flow rate during the blow-down test, and the efficiency percent required for the specified glass bead mixture (see Table 15.8.5.2 for recommended glass bead mixtures). The results cannot be correlated directly to the average filtration rating since the measurements are made gravimetrically rather than microscopically and are expressed in percent of removal for a specified glass bead mixture rather than in microns.

Table 15.8.5.2. Recommended Contaminants for Filter Tests

ABSOLUTE MICRON RATING OF SPECIMEN	CONTAMINANT TYPE AND GRADE	STANDARD AMOUNT ADDED (GRAMS)
Recommended Contaminants for Absolute Micron Rating Tests		
6 to 16	F-9 glass beads	0.05
17 to 24	F-12 glass beads	0.05
25 to 48	F-13 glass beads	0.05
49 to 64	F-15 glass beads	0.05
65 to 96	F-19 glass beads	0.10
97 to 128	F-17 glass beads	0.10
129 to 160	F-16 glass beads	0.10
161 to 260	F-18 glass beads	0.10
Recommended Contaminants for Efficiency Tests		
6 to 12	F-9 glass beads	The amount of contaminant added shall be 167 mg/GPM of rated flow unless otherwise specified.
13 to 18	F-12 or Fram 10/20 glass beads	
19 to 40	F-13 glass beads	
41 to 60	F-15 glass beads	
61 to 90	F-19 glass beads	
91 to 120	F-17 glass beads	
121 to 150	F-16 glass beads	
151 to 200	F-18 glass beads	

b) *Efficiency by Particle Size (Glass Bead Test).* This test method again employs glass beads but in a series of narrowly classified bands with a maximum range per band of 5 microns. The particle size at which a 50 percent removal (by microscopic count) is effected is considered to be the *average filtration rating* of the filter since it corresponds to the mean pore size of the filter medium. This test procedure is quite elaborate and time consuming. When used, it must specify the quantity and size range of the glass bead bands introduced as well as the fluid type and flow rate under blow-down conditions.

c) *Efficiency by Particle Size (Mercury Instrusion Test).* In this test mercury is forced into the pores of a small test specimen within an evacuated chamber. As the mercury pressure in the chamber increases, mercury enters first the largest pores of the filter medium, then pores of decreasing diameter until all pores are completely filled. Pore size distribution is determined by using an equation which is derived from the balance of pressure and surface tension forces for mercury on steel. The maximum (peak value) of the pore size distribution function occurs at the average pore size. This test can only be performed on flat specimens. Once it has been conducted for each type and grade of filter medium employed, it need not be repeated as long as dimensional controls are maintained within specified limits over the dimensions which control the pore size of a filter medium (such as mesh count and wire diameter in the case of wire cloth).

Contaminant Capacity. Contaminant capacity tests measure the amount of artificial contaminant which can be added upstream of a filter under rated flow conditions before the initial (clean) differential pressure reaches a specified maximum value. This maximum value can be a function of system design considerations such as maximum permissible flow or pressure decay, or the opening pressure of an upstream relief valve. Generally, however, it can be assumed that due to the asymptotic shape of the pressure drop buildup curve (see Figure 15.8.5), the useful life of a filter is expended when the differential pressure reaches a value five times that of the clean differential pressure. This test is conducted by adding artificial contaminant upstream of the filter under specified test conditions. Pressure drop across the filter is measured at a constant flow rate and increments of contaminant are added periodically until the specified differential pressure has been reached. The test is sensitive to several test variables such as size distribution of the contaminant, velocity or specific flow rate through the filter medium, type of fluid, frequency and weight of contaminant added, and presence of a *clean-up filter*. Standardized Arizona road dust which simulates natural airborne dirt is normally used for this test. There are two basic methods of conducting contaminant capacity tests which are shown as method A and method B in Figure 15.8.5. The basic difference is the use of a system cleanup filter in method B. All available test data show that the indicated contaminant capacity of a filter tested with a cleanup filter in the test setup is greater than that of an identical filter tested without a cleanup filter in the system. This is because the smaller contaminant which is originally passed by the test filter is subsequently removed by the cleanup filter and is not reintroduced into the test specimen on the second or subsequent passes. As a result, a contaminant capacity test conducted with a cleanup filter (method B) would simulate conditions in an open-end system (such as propellant feed system) while method A would simulate a recirculating system without a downstream system cleanup filter.

Contaminant Transmissions. A logical extension of the contaminant capacity test, which is not frequently conducted, is the contaminant transmission test. This test requires the installation of a sampling probe just downstream of the filter and the taking of effluent samples of approximately 100 to 500 ml concurrently with the addition of each increment of AC dust. The test results that can be obtained in this manner provide a ready comparison with the spherical glass bead test and make it possible to establish the size of three-dimensional particles whose longest (rather than smaller) dimensions would be reported in normal contamination control sampling methods. As a result a ratio of *largest spherical* to *largest three-dimensional* particle passed can be developed for each filter, filter medium, and test condition; in addition, the effects of pressure differential buildup, actual flow rate, and pumping conditions on contaminant transmission can be analyzed.

Recleanability. Another frequently-conducted test consists of first loading the test filter with the test contaminant up to the maximum allowable pressure differential, then cleaning the filter element, and finally reloading it for a total of 10 different cycles.

This test is of interest in hydraulic oil applications and requires that the unit meet the original initial pressure drop after recleaning and be capable of demonstrating a contaminant capacity 90 to 100 percent of that of the original run during each cycle. Before specifying this test, however, the influence of the type of contaminant encountered under actual operating conditions, which most probably will not be as easily removed as AC road dust, must be considered.

Most importantly, however, while it is possible to restore the initial clean pressure differential and contaminant capacity with repetitive ultrasonic and chemical cleaning cycles, it has been proven that to restore the initial cleanliness level to anything approaching the original count is almost impossible. While this may be of little consequence in a recirculating system, it could not be tolerated in critical applications such as a rocket propellant feed system.

Collapse Pressure. Another extension of the contaminant capacity test is the filter element collapse pressure test which is conducted by adding sufficient additional contaminant at the end of the contaminant capacity test to raise the differential pressure to the value specified. At the conclusion of this test the filter element is examined for evidence of damage or rupture and frequently recleaned and subjected to another initial bubble point test for comparison with the original values to verify structural integrity of the filter medium.

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*Note: This temporary list of references identifies source material specified in Section 15.0 and which will not be found in the handbook Bibliography. Revision D, to be published shortly, will contain a completely revised Bibliography and will incorporate a list of references for Section 15.0.

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16.1 INTRODUCTION

This section of the handbook summarizes the technology of fluidics for the fluid component designer and introduces the basic language of this new technology to facilitate use of the reference literature on fluidics. Basic fluidic devices such as fluid amplifiers, interface devices, and sensors are discussed in terms of operating principles, configurations, performance characteristics, and applications. Equally important is the presentation of realistic potential advantages and limitations of fluidics as related to aerospace applications. In order to assist the designer in exploiting the advantages, information is presented on the analysis, design, and fabrication of fluidic components and on the definition, performance, and means of specifying basic fluidic elements and systems for given applications. Terminology and symbology are defined in Sub-Sections 16.2 and 16.3, respectively.

16.1.1 The Role of Fluidics

A fluidic system is one in which sensing, control, signal processing, or amplification functions are performed through the use of fluid dynamic phenomena, i.e., no moving mechanical parts (Reference 23-72). The role of fluidics in the evolution of fluid systems is analogous to the role of electronics in the evolution of electrical systems. A fluid system with one fluidic device is called a fluidic system just as an electrical system with one electronic device (vacuum tube or transistor) is called an electronic system.

Conventional control of sophisticated hydraulic and pneumatic fluid power circuits and of flow control systems (such as rocket propellant feed systems) has been based primarily on the use of electrical and electronic signals for sensing, data transmission, and amplification. Although fluid power control devices and systems have been used for many years in a wide range of applications, heretofore they could not be employed effectively at low power levels because they required devices with moving mechanical parts. Therefore, readily available electrical and electronic devices and circuits were preferred for low power level control functions such as sensing, signal transmission, switching, and amplification. The role of fluidics is not limited to these low power level control functions, but encompasses the entire range of fluid power and propellant feed systems.

With the growing availability of a wide variety of fluidic control elements, power elements, and interface transducers, a new generation of fluid control systems is being developed which offers improved reliability potential through the elimination of moving mechanical parts.

16.1.2 Advantages and Limitations of Fluidics

Fluidics offers unique capabilities which are leading to a new generation of valves and controls for aerospace systems. Practically any fluid can be used, and some fluidic elements will operate equally well on either gases or liquids (although not with both liquids and gases present at the same time). While fluidics promises potential weight savings in many cases, the primary advantage may occur when fluids are used for performing all functions, and components such as sensors, logic devices, and amplifiers can be conveniently coupled together directly. Such a fluidic system eliminates the need for interface devices, i.e., the transducers between the electrical and fluid portions of a system. This simplifying characteristic, as well as a wide range of available fabrication materials including high-temperature alloys and ceramics, makes fluidic systems capable of operating in extreme temperature, radiation, vibration, and shock environments. An obvious advantage is that there are no moving parts to seize or wear out.

Aerospace applications of fluidics have been limited mostly by the necessity for continuous fluid flow (unless moving-part valves are employed) and the lack of data concerning fluidic systems designed to operate with exotic fluids such as liquid rocket propellants.

16.1.3 The Basis for Fluidics

The operation of fluidic elements is based on various fluid flow phenomena such as wall attachment (Coanda Effect), jet deflection, turbulent mixing, momentum exchange, vortex generation, turbulent diffusion, boundary layer separation, and transition from laminar to turbulent flow. Many of these phenomena are familiar; wall attachment, for example, is observed when water spills over the edge, reattaches and runs down the side of a tipped glass. The deflection of the exhaust jet of a rocket engine by the perpendicular injection of a secondary fluid involves a more complex combination of jet deflection, momentum exchange, and boundary layer separation (see Sub-Section 4.5 of this handbook, Secondary Injection Thrust Vector Control Systems). The jet pump is another common device which is an example of the application of fluidic principles.

Most of the fluid dynamic phenomena now being applied in fluidic devices have been known for many years. Several significant events may be cited in the progression of discoveries and descriptions of fluid dynamic phenomena which preceded the initial recognition of fluidics as a discrete technology. In 1904 Prandtl, while investigating flow separation in a wide-angle diffuser, discovered that flow separation could be varied by applying suction at the boundary layer of the diffuser (Reference 198-1). By installing control ports at each side of the diffuser, he found that when suction was applied to one side of the diffuser, the discharge fluid would adhere to that side (Figure 16.1.3a). When suction was applied at the control ports on both sides of the diffuser, the discharge flow expanded and filled the entire diffuser. Prandtl could have made the first fluidic logic element by installing an output duct on each side of the diffuser.

The valvular conduit (Figure 16.1.3b), invented by Tesla in 1916, has been acclaimed as the first pure fluid device with no moving parts. This device is actually a fluidic diode which offers low resistance to flow in one direction and a large resistance in the opposite direction (Reference 580-5).

During the 1930's, Coanda observed that when a free jet was introduced near an adjacent curved or flat plate, the jet would adhere to the plate and follow the plate even though the new flow path diverged as much as 45 degrees from the original flow direction. This phenomenon is explained by the fact that the emerging jet stream entrains molecules of fluid in adjacent space due to the large velocity gradient at the edge of the jet (Figure 16.1.3c). Near the adjacent plate, the entrained fluid is not easily replaced, whereas on the opposite side of the jet the entrained fluid is easily replaced by ambient fluid. This condition results in the formation of a low-pressure bubble or vortex and the development of a transverse pressure gradient across the jet, which bends the jet toward and eventually against the adjacent plate. The Coanda Effect is of major importance to fluidic technology.

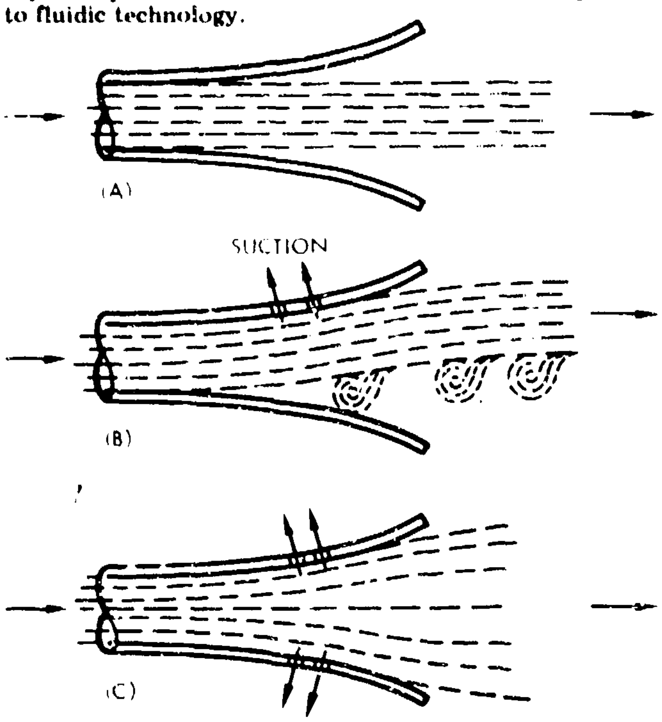


Figure 16.1.3a. Prandtl Diffuser

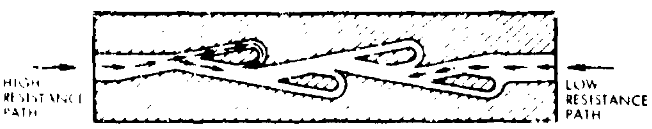


Figure 16.1.3b. Tesla's Valvular Conduit

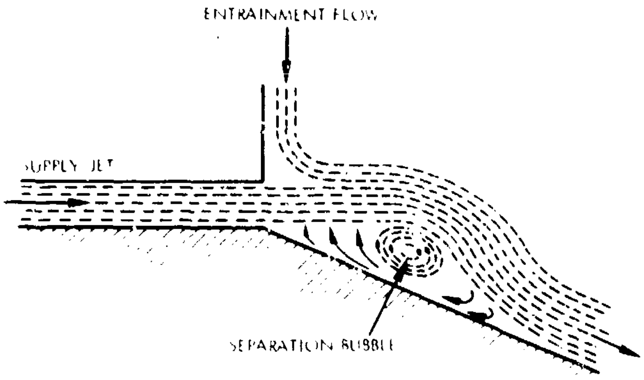


Figure 16.1.3c. The Coanda Effect

16.1.4 The Advent of Fluidic Technology

The primary impetus to fluidics as a technology was provided by R. E. Bowles, W. M. Horton, and R. W. Warren of the Army's Harry Diamond Laboratories, Washington, D.C. Together they explored fluid systems as a means of overcoming environmental problems in artillery fuzing systems. These three scientists developed a number of devices in 1959, which were unveiled in 1960. Horton invented the stream interaction proportional amplifier, Bowles and Warren developed the flip-flop and digital switch, and the three subsequently disclosed and patented a number of fundamental devices and applications in the field.

16.2 FLUIDIC STANDARDS AND TERMINOLOGY

16.2.1 Fluidic Standards

Fluidic terminology, nomenclature, graphical symbology, and definitions used in this handbook are based primarily on MIL-STD-1306 (Reference 447-10) and SAE ARP 993, Fluidic Technology (Reference 23-72), including material for the proposed revision, ARP 993A.

The first set of symbols for fluidic circuitry (References 241-15 and 46-48) was presented by General Electric Company personnel at the October 1962 Fluid Amplification Symposium held at the Harry Diamond Laboratories, Washington, D.C. Since then, the National Fluid Power Association (NFPA) and the Fluidics Panel of the Society of Automotive Engineers (SAE) Committee A-6 (Aerospace Fluid Power and Control Technologies) have done a great deal of work in defining fluidic standards. Both organizations have agreed on most standards with minor symbology differences, some favoring SAE in the case of military, aerospace, and vehicular applications (i.e., SAE ARP 993) and others favoring NFPA in the case of industrial and commercial applications (Reference 462-3). This handbook endeavors to follow MIL-STD-1306 (Reference 447-10), which is based primarily on SAE ARP 993.

Section 3.0, Fluid Mechanics, of this handbook provides a source of fundamental theory and equations of flow. The important properties and parameters of fluid mechanics pertaining to fluidics (in particular, fluid pressure, fluid flow, and fluid resistance) are given general treatment in References 1-298, 1-299, 1-300, and 1-301, and a more detailed treatment in References 532-1 and 770-1.

Terms common to fluidics are defined below in Sub-Topic 16.2.2. Symbols used in fluidics are discussed in Sub-Section 16.3 and are defined in foldouts at the end of this section. In each of these areas MIL-STD-1306 has been used as the primary standard, with supplementary terms and symbols selected as required to complement Section 3.0 of this handbook.

16.2.2 Terminology

Active — Adjective to describe an amplifying or switching device whose operation depends upon a separate supply source of power in addition to the signal power.

Actuator	A component device or system which provides a mechanical actuation in response to some input signal.
Amplifier	An active device or component which provides a variation in output signal having a potential power level variation which is usually greater than that of the impressed input control signal variation. The variation in output signal bears a specified functional relationship to the input control signal variation.
Analog	Adjective to describe a general class of components or circuits in which all signals may vary continuously (as opposed to signals which may only vary in discrete increments).
Aspect ratio, nozzle (σ)	Ratio of nozzle depth to nozzle width.
Bandwidth	The operating frequency range of a device as defined by the minimum (usually zero or steady state) and maximum operating frequencies. An indication of maximum operating frequency is the frequency at which the output signal lags the control signal by 45 degrees for a specified load and control amplitude.
Bias	Magnitude of input signal to null or provide zero output signal for differential amplifiers; signal magnitude required to establish operating point for single-ended amplifiers.
Boundary layer amplifier	An amplifier which utilizes the separation-point control of a power stream from a curved or plane surface to modulate the output.
Capacitor	A passive fluid element which produces a pressure within itself which lags the inflow rate by 90 degrees phase.
Circuit	An array of interconnected components and elements which performs a desired function; for example, an integrator, counter, or operational amplifier.
Closed amplifier	A fluidic amplifier which has no communication with an independent reference, i.e., the interaction region is not vented.
Coanda Effect	The wall attachment phenomenon. See Detailed Topic 16.4.1.2 and Sub-Topic 16.4.2.
Digital	The general class of devices or circuits whose output is a discontinuous function of its input.
Direct impact modulator	See Detailed Topic 16.4.6.5.
Double-leg elbow amplifier	See Detailed Topic 16.4.6.2.
Edgetone amplifier	See Detailed Topic 16.4.6.4.
Element	The general class of devices in their simplest form, used to make up fluidic components and circuits, for example, resistors, capacitors, flip-flops, and jet deflection amplifiers.
Fan-in	The number of control signals (push-pull or single ended) accepted by a logic gate, which can effect the desired change in state of the logic gate.
Fan-out	The number of components which can be driven by a single component; all components are to be operated at the same supply pressure. Also, components are to be of similar size and have similar switch points. Fan-out value relates to steady-state operation unless the corresponding frequency is given.
Flip-flop	A bistable flueric component (reset-set) which changes state with the proper reset-set input of sufficient amplitude and width. It exhibits "memory" (remains in a particular state) once it has switched, without requiring a continual input signal.
Flow amplifier	An amplifier designed primarily for amplifying flow signals.
Flow diverter	A digital flueric amplifier with no memory designed primarily for high pressure recovery. It operates on the jet interaction principle. See Figure 16.4.2b.
Flow recovery, output	The maximum output mass-flow rate divided by the supply mass-flow rate. Generally given as a percentage.
Flueric	An adjective sometimes applied to those fluidic components and systems which perform sensing, logic, amplification, and control functions, but which use no moving mechanical elements whatsoever to perform the desired function.
Fluerics	The area within the field of fluidics in which fluid components and systems perform sensing, logic, amplification, and control functions without the use of moving mechanical parts.
Fluidic	An adjective denoting a device or system in which some sensing, control, signal processing, and/or amplification functions are performed through the use of fluid dynamic phenomena (no moving mechanical parts).
Fluidic component	A fluidic device, distinguished from an element by virtue of the fact that it is composed of more than one element.

Fluidics — The general field of fluid devices and systems and the associated peripheral equipment used to perform sensing, logic, amplification, and control functions.

Focused jet amplifier — See Detailed Topic 16.4.5.5

Frequency response — Usually given in the form of frequency response curves of the variation of output/input amplitude ratio and phase as a function of frequency.

Gain, flow (analog) — Average gain; the slope of a straight line drawn through an input flow versus output flow curve, so that deviations from the measured curve up to the maximum output level are minimized. Deviations should be based on net area. If other than maximum output level is used for the average gain definition, the range should be noted. Measured curve is to be for either low output pressure recovery (resulting from instrumentation) or a value which provides maximum flow gain.

Gain, flow (digital) — Ratio of output flow change to input flow change (from quiescent) required for switching to occur.

Gain, flow (incremental, analog) — The slope of the output flow versus the input flow curve at the operating point of interest.

Gain, power (analog) — Average power gain; ratio of the change in output power to the change in input power; the average value over operating range up to maximum output level unless the range is stated.

Gain, power (digital) — Ratio of the change in output power to the change in input power (from quiescent) for switching to occur.

Gain, power (incremental, analog) — The slope at the operating point of an input/output power curve.

Gain, pressure (analog) — Average gain; the slope of a straight line drawn through a measured input pressure versus output pressure curve so that deviations from the measured curve up to the maximum output level are minimized. Deviations should be based on net area. If other than the maximum output level is used for the average gain definition, the range used should be noted. Gage pressure values should be used. The measured curve is to be for either zero output flow or a value which provides maximum pressure gain (see Figure 16.2.2a).

Gain, pressure (incremental, analog) — Incremental gain; the slope of the measured input pressure versus output pressure curve at the operating point of interest (see Figure 16.2.2a).

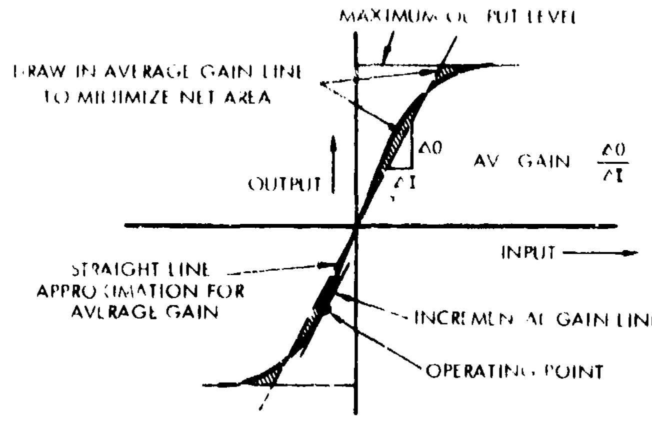


Figure 16.2.2a Pressure Gain
(Reference 447-10)

Gain, pressure (digital) — Ratio of measured output pressure change to input pressure change (from quiescent) required for switching to occur. All control ports except the one under consideration should be maintained at the quiescent pressure level. Output flow should be zero or a value which results in maximum pressure gain. If gain value is for other than steady-state conditions, the test frequency should be stated.

Hydraulic diameter — The ratio of the cross-sectional area of a flow passage to one-fourth the wetted perimeter of the passage.

Hysteresis, analog amplifier — Total width of hysteresis loop expressed as a percent of peak-to-peak saturation input signal. Measurements must be at frequencies below those where dynamic effects become significant (see Figure 16.2.2b). Measurements to be made at the widest point on the curve.

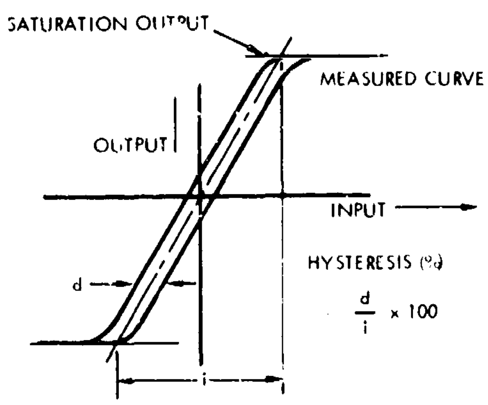


Figure 16.2.2b. Analog Amplifier Hysteresis
(Reference 447-10)

Hysteresis, digital amplifier — Width of the hysteresis loop as measured on an input/output curve and expressed as a percentage of the supply conditions; for example, flow hysteresis is the hysteresis loop width (measured on an input/output flow curve), divided by the supply flow (see Figure 16.2.2c).

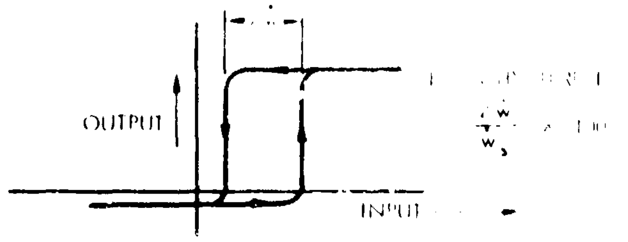


Figure 16.2.2c Digital Amplifier Hysteresis
(Reference 447 10)

Impact modulator — See Detailed topic 16.4.6.5.

Impedance, input — The ratio of pressure change to flow change measured at an input port. Numerical value may depend on operating point, since input pressure-flow curve may not be linear. For active elements, the power source should be connected for measurements.

Impedance, output — The ratio of pressure change to flow change, measured at an output port. Numerical value may depend on operating point, since output pressure-flow curve may not be linear.

Induction amplifier — See Detailed Topic 16.4.6.3.

Inductor — A passive fluidic element which, because of fluid inertance, has a pressure drop across it which leads the through flow by 90 degrees phase.

Jet-deflection amplifier (also beam-deflection amplifier, stream interaction proportional amplifier, jet-on-jet proportional amplifier) — See Sub-Topic 16.4.3.

Linearity deviation, output — Deviation of the measured curve from the straight-line average gain approximation: the ratio of the deviation to the peak-to-peak output range (range should be stated if other than maximum output level) expressed as a percentage (see Figure 16.2.2d).

Logic elements (also logic gates) — The general category of digital components which provide logic functions; for example, AND, OR, NOR, and NAND. They can gate or inhibit signal transmission with the application, removal, or other combinations of input signals.

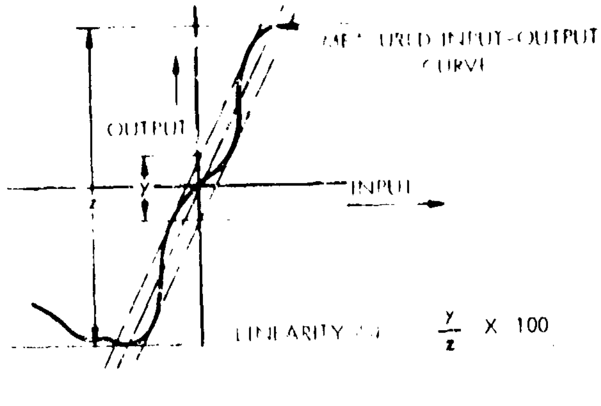


Figure 16.2.2d. Output Linearity
(Reference 447-10)

Memory — The capability of a logic gate to retain the state of its output signal corresponding to the most recently applied control signal after the control signal is removed.

Passive — The general class of devices which operate on the signal power alone.

Power amplifier — An amplifier designed primarily to provide maximum power gain.

Pressure amplifier — An amplifier designed primarily to amplify pressure signals.

Pressure recovery, output — The difference between the maximum output pressure and the local vent pressure divided by the difference between the supply pressure and the pressure in the interaction region. For closed amplifiers, the control port pressure should be used as the reference pressure.

Relaxation oscillator — See Detailed Topic 16.4.7.2.

Resistor — A passive fluidic element which, because of viscous losses, produces a pressure drop as a continuous function of the flow through it.

Response time — The time interval between the application of an input step signal and the resulting output signal. The time measurement for the response to the input step signal is to be made when the output signal reaches a level which is 63 percent of the final output value. (See Figure 16.2.2e.)

Reynolds number — A dimensionless parameter of fluid flow which often indicates the ratio of inertial-to-viscous forces:

$$N_R = \frac{\bar{u} d_h}{\nu}$$

where d_h = hydraulic diameter, $\bar{u}$ = mean velocity of the fluid, and ν = kinematic viscosity.

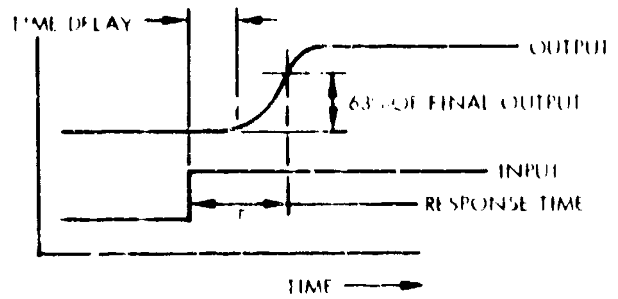


Figure 16.2.2e. Response Time and Time Delay
(Reference 447-10)

SI — An abbreviation indicating the international system of units.

Saturation — The maximum output value regardless of input magnitude (see Figure 16.2.2f).

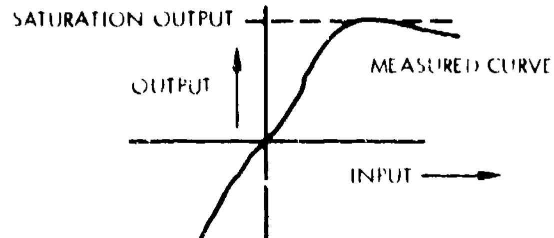


Figure 16.2.2f. Saturation
(Reference 447-10)

Sensor, fluidic — A fluidic device which senses a basic quantity such as rate, position, acceleration, pressure, or temperature, in terms of a fluid quantity such as pressure or flow rate.

Signal-to-noise ratio (SNR) (analog amplifier) — Ratio of maximum (saturation value) output signal amplitude to maximum noise amplitude (at output). Signal and noise data should be RMS values.

Signal-to-noise ratio (digital amplifier) — Ratio of the amplitude of the output signal to the peak-to-peak maximum noise signal. Maximum noise signal is to be measured when the port is active and inactive. The greater value of the two is used in calculating the SNR.

Transducer — A device which converts signals from one medium to an equivalent signal in a second medium.

Time delay — The time from the initiation of an input signal until the first discernible change in the output, caused by this input signal (see Figure 16.2.2e).

Transport delay — Time required for a fluid particle to travel from the input control port region to the output receiver region.

Transverse impact modulator — See Detailed Topic 16.4.6.5.

Truth table — A table depicting the function of a logic element; all possible combinations of input signals are tabulated along with the corresponding state of the output signal (see Detailed Topic 16.8.4.2).

Turbulence amplifier — See Detailed Topic 16.4.5.1.

Vented amplifier — A fluidic amplifier which utilizes vents to establish a reference pressure in the interaction region.

Vortex amplifier — A fluidic amplifier which utilizes the pressure drop across a controlled vortex for the modulating principle (see Sub-Topic 16.4.4).

Wall attachment amplifier — See Sub-Topic 16.4.2.

16.3 FLUIDIC SYMBOLS AND UNITS

16.3.1 Units, Dimensions, and Symbols

The system of units used in fluid mechanics work in this handbook is the unit force-mass system, which provides a compromise between the absolute and gravitational systems. This system is described in Sub-Topic 3.2.1 of this handbook. MIL-STD-1306 (Reference 447-10) provides both the international system (SI) and English (standard) units, with the recommendation that all current and future work should be documented and reported in SI units. Table 16.3.1 lists the symbols used in this section of the handbook, both the SI and English units, and dimensions for these symbols. Table 16.3.1 is located on a foldout sheet at the end of this section. Conversion tables for the primary quantities are given in Appendix A.

16.3.2 Graphical Symbols

Graphical symbols enable the circuit designer to depict clearly and concisely in drawings and schematics the function to be performed or the operating principle of the device employed to perform the function. An integrated set of symbology which satisfies these two basic needs has been extracted from SAE ARP 993A (Reference 23-72).

Functions of fluidic devices are defined by symbols enclosed within square envelopes. Operating principles of fluidic devices are defined by symbols enclosed within round envelopes. The difference in envelopes is specifically intended to emphasize the difference in purpose of the symbols as shown below:

Functional Symbol — Operating Principle Symbol

By definition, the symbols are intended to show the following:

a) The functional symbol depicts a function which may be performed by a single fluidic element or by an interconnected circuit containing multiple elements.

b) The operating principle symbol depicts the fluid-dynamic phenomenon in the interaction region which is employed to perform the function.

In the cases where no operating principle is shown, it is implied that, at present, no single operating principle or interaction region is adequate to perform the function. In these cases a combination of operating principles or interaction regions is required to represent the function. For convenience, graphical symbols have been listed in Table 16.3.2 and incorporated as foldouts at the end of this section.

16.4 FLUIDIC DEVICES

Various fluidic devices which perform a variety of circuit functions are available. A clear understanding of their operation, performance characteristics, and limitations is essential to the successful application of these devices and to the analysis, design, and test of circuits. This sub-section describes the various fluid interaction phenomena that form the basis of fluidic technology and shows how these phenomena are utilized in practical fluidic devices. (Adapted from Reference 37-25.)

16.4.1 Basic Device Phenomena

All active fluidic devices have at least four basic functional parts: a supply port, an output port, one or more control ports, and an interaction region (Figure 16.4.1). These parts have been respectively compared to the cathode, plate, control grid, and interelectrode region in a vacuum tube. In the fluidic device, the supply jet (fluid stream) is introduced into the interaction region and directed toward the output port or receiver. The degree of pressure and flow recovery in the receiver is influenced by the details of the device configuration. When a control flow is introduced into the interaction region it modifies the direction and distribution of the supply flow, so that a change in output results at the receiver. Since the change in output energy is usually achieved with a much smaller incremental change in control energy, useful amplification results.

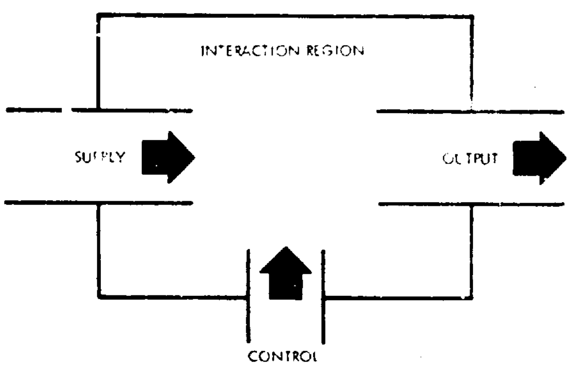


Figure 16.4.1. Basic Fluidic Device

The various fluid interaction mechanisms presently used in fluidic devices can be divided into three basic categories:

a) Jet interaction — where a supply jet is essentially unconstrained by surfaces (other than top and bottom plates) in the interaction region and the control flow directly modulates the supply flow.

b) Surface interaction — where the presence of an adjacent surface is essential to the control action.

c) Vortex flow — where the existence of a vortex field in the interaction region is essential to the device function.

16.4.1.1 JET INTERACTION. In jet interaction devices, control action is achieved through the direct effect of control flow on the source jet. Included in this category are beam deflection, impact modulation, and controlled turbulence effects.

Beam deflection is illustrated in Figure 16.4.1.1a, where the vector direction of flow from the supply jet is varied by flow from one or more control jets which are oriented at approximately 90 degrees to the source jet. For the small modulation angles normally used in practical fluidic devices, the angular deflection or modulation angle is essentially a linear function of the control momentum such that, for a given properly designed receiver, the beam deflection effect can be utilized to develop a linear proportional amplifier.

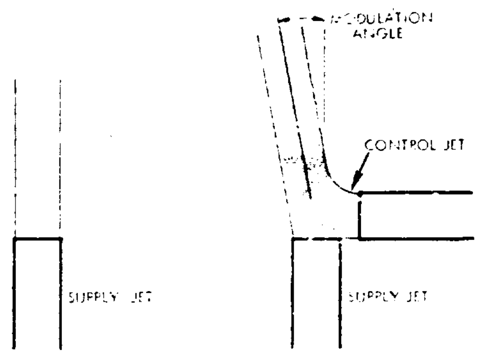


Figure 16.4.1.1a. Beam Deflection

Impact modulation (Figure 16.4.1.1b) is achieved by the use of two axially opposed supply jets which provide a planar impact region. The shape and location of the impact region can be varied by modifying one of the supply jets. This is accomplished by introducing a control flow into or transversely across one of the supply jets, which will either increase or decrease the momentum of the jet such that the impact region is axially displaced. Consequently, when an appropriate receiver is located near the impact region, transverse radial flow from the impact region into the receiver can be modulated by the control flow.

The controlled turbulence effect is illustrated in Figure 16.4.1.1c, where a supply flow is ejected from a nozzle into a disturbance-free medium. Under the proper conditions, the jet flow will remain laminar for a considerable distance downstream from the nozzle and then abruptly become turbulent. When control flow is introduced near the exit of the supply jet, it disturbs the supply jet, causing the point of turbulent breakdown to move axially upstream toward the supply jet nozzle. Since the energy recoverable from the source jet is much greater in the laminar region than in the turbulent region, a receiver located between the uncontrolled turbulence point and the controlled turbulence point will sense a significant change in energy level when control flow is introduced or shut off.

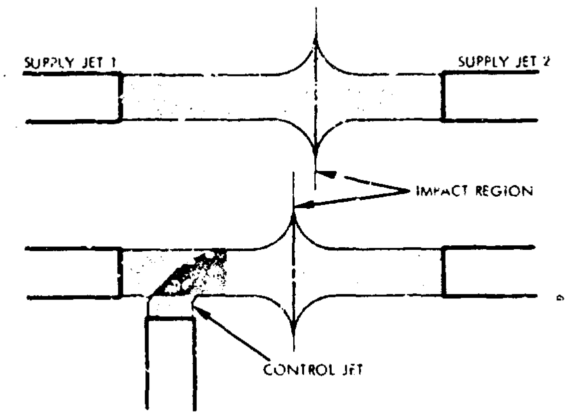


Figure 16.4.1.1b. Impact Modulation

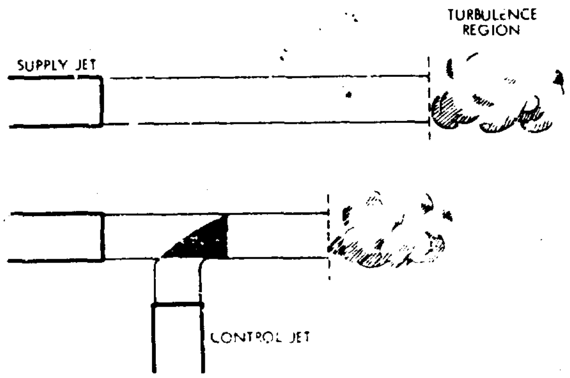


Figure 16.4.1.1c. Controlled Turbulence

16.4.1.2 SURFACE INTERACTION. The function of some devices depends upon the influence of an adjacent surface on the supply flow. The most important effects are:

a) The attachment of a stream to a surface, and

b) The separation of flow from a curved surface.

Although in each case the control function is provided by a control flow, the surface supports, and is essential to, device operation.

The Coanda Effect is the primary fluid dynamic phenomenon influencing the performance of a wall attachment device. To understand the mechanism of attachment, consider a supply jet emerging into the area bounded on one side by a wall perpendicular to the jet and on the other side by an angled wall oriented approximately 30 degrees from the supply jet centerline (Figure 16.4.1.2a). The emerging jet entrains ambient fluid because of high shear at the edge of the jet. This entrained fluid is not easily replaced by ambient fluid on the angled wall side of the jet, so that a transverse static pressure gradient is formed across the jet which bends the jet and forces it to attach to the angled wall. A low pressure vortex region (or bubble) is formed between the jet and the point of attachment. Within the bubble, fluid is entrained near the supply nozzle and replenished by separated flow near the point of attachment. The attached jet may be detached from the surface by injecting control flow into the low pressure separation bubble. The stability of wall attachment plus the ability to detach and shift the jet make this an extremely useful effect in digital fluidic devices.

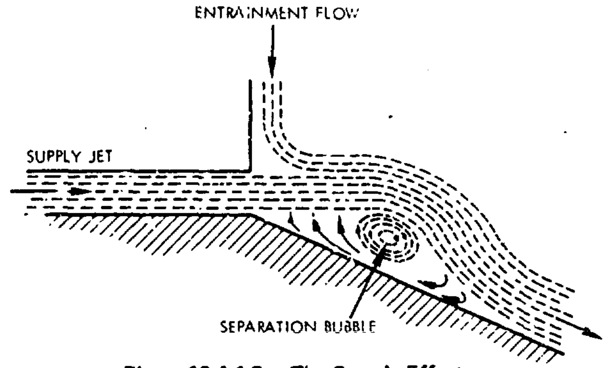


Figure 16.4.1.2a. The Coanda Effect
(Reference 131-40)

The separation effect is based on the tendency of a supply flow to follow an adjacent gradually curved surface as long as the pressure gradient is larger than the momentum vector (Figure 16.4.1.2b). When the radius of curvature of the surface is sharply reduced, momentum will predominate at some point downstream and the flow will separate from the surface. Control flow injected upstream of the separation point will influence the point of separation by reducing the pressure gradient across the jet and thus change the angle at which the flow leaves the curved surface. Several fluidic devices use this effect to modulate the source flow in one or more receivers downstream of the controlled separation region.

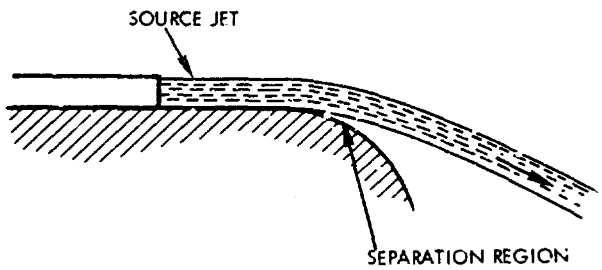
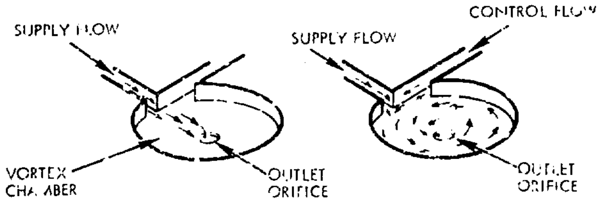


Figure 16.4.1.2b. Separation Effect

16.4.1.3 VORTEX FLOW. In vortex controlled devices, supply flow is introduced radially at the circumference of a shallow cylindrical chamber (Figure 16.4.1.3). With no control present, the supply flow enters the vortex chamber and proceeds radially inward with minimal resistance and flows out through the centrally located outlet orifice. The



(a) NO CONTROL FLOW (b) WITH CONTROL FLOW

Figure 16.4.1.3. Vortex Flow Effects
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

supply port is generally much larger than the outlet orifice, so that the outlet flow rate is determined by the area of the exit orifice and system pressures. When control flow is injected tangentially into the chamber, the source and control flows combine, and the resultant flow develops a degree of swirl dependent on the relative magnitudes of the source and control flow momenta. This development of a forced vortex field within the chamber varies the pressure gradient across the chamber such that the magnitude as well as the pattern of the source flow is altered. Since the vortex field provides a variable resistance, it can substantially reduce or throttle flow and thus provides a unique control function in fluidic devices (Reference 37-19).

16.4.2 Wall Attachment Amplifiers

The Coanda Effect is the primary fluid dynamic phenomenon influencing the performance of wall attachment devices. These devices provide a fairly high speed of response, average efficiency, and relatively high fan-out. Versatility and relatively good performance in a number of applications are strong recommendations for their continued widespread use. Several fluidic counterparts of the basic electronic logic elements have been developed, including a flip-flop, monostable switch, OR-NOR element, half adder, and a pulse converter.

The configuration of the basic two-dimensional wall attachment device is shown in Figure 16.4.2a. This device utilizes two walls set back from the nozzle, control ports, and channels to define two downstream outputs. Because of the Coanda Effect, the device is bistable, i.e., a turbulent free jet emerging from the supply port can be made to stably attach to either wall. Operating characteristics of the unit can be varied by many parameters including pressure, flow, and the physical relationships in the interaction region. The effect of varying some of the physical parameters, loading, and the power jet pressure is illustrated in Figure 16.4.2b.

The switching mechanism in a bistable wall attachment device is illustrated in Figure 16.4.2c. Presuming the jet is initially attached to the lower wall, fluid is injected through the lower control port into the vortex bubble. When the rate of injected fluid exceeds the rate at which fluid is removed by entrainment, the pressure on the lower edge of the jet will increase. As this pressure becomes greater than the pressure on the upper edge of the jet, the pressure differential is reversed and the jet will detach, cross over to and attach to the upper wall, and remain attached even after the lower control flow is removed.

The flip-flop (bistable wall attachment device) and a few examples of the many logic elements which utilize a combination of wall attachment and stream interaction principles are shown in Figure 16.4.2d. The digital states of each device can be followed by referring to the accompanying truth table. In the OR-NOR gate the interaction region is geometrically biased so that the supply flow will stably attach only to the adjacent wall leading to output 2. When control flow is supplied to either or both of the control ports, the power stream is shifted to output 1 by stream interaction. When the controls are removed, the power stream returns to output 2.

The AND gate and half adder (exclusive OR) elements also shown in Figure 16.4.2d are passive elements, i.e., devices that operate on the signal power alone. In the AND gate, control 1 will appear at output 3 if control 2 is not present, and control 2 will appear at output 1 if control 1 is not present. These stable output states are achieved by wall attachment in the respective output ducts. When controls 1 and 2 appear together (presuming equal control pressures), they combine by stream interaction to produce a signal at output 2. The half adder function differs from the AND gate in that both controls appear at output 2, control 1 by wall attachment, and control 2 by deflection in the opposite cusp. The two controls combine to give a signal at output 1 as above.

The elimination of load sensitivity has been a major problem in the design of wall attachment devices, since it is impractical to design interconnecting impedances for each separate case. Consequently, most wall attachment elements are vented or bled off to a suitable sink so that over an appreciable operating range each element is automatically matched to its applied load. This approach, although inefficient in terms of input fluid power considerations, is quite adequate in many circuit applications where total fluid power is not critical. The configurations shown in Figure 16.4.2e are representative of the many methods utilized to decrease load sensitivity in a wall attachment device.

The size of a wall attachment device with a given aspect ratio is established by the width of the power nozzle, i.e., a 10-mil element refers to one which has a power nozzle width of 0.010 inch. All of the internal amplifier dimensions are then defined as ratios of the power nozzle width. These devices are normally designed with aspect ratios (height to width ratio of the power nozzle) from 1:1 to 4:1.

In considering wall attachment amplifier performance, variation of the size (in the range from 10 to 25 mils) or variation of the aspect ratio (in the range of 1:1 to 4:1) does not appreciably affect performance. The following performance figures are based on air data; however, present data indicate that performance with water or other low viscosity liquid should be similar except that time response may be considerably lower.

Response time of a digital element is defined as the time delay between the application of an input signal and the resulting change of output signal when the device is subjected to a step input large enough to switch the flow. The load must be specified if the element is load sensitive. Changes in design and operating conditions will affect the switching speed of elements, but typical response times for small wall attachment elements range from 0.1 to 2 milliseconds. Faster switching speeds have been achieved with high supply pressures (50 psig) and with geometries which produce instabilities; however, this is achieved at the expense of power consumption and reliability.

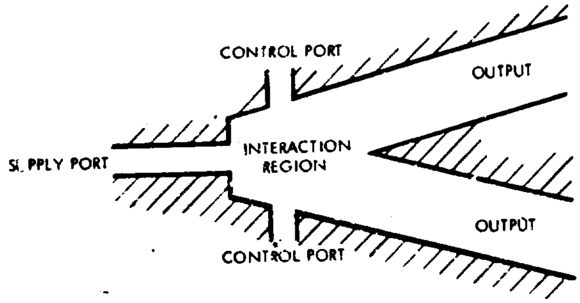


Figure 16.4.2a. Basic Wall Attachment Devices
(Reference 131-40)

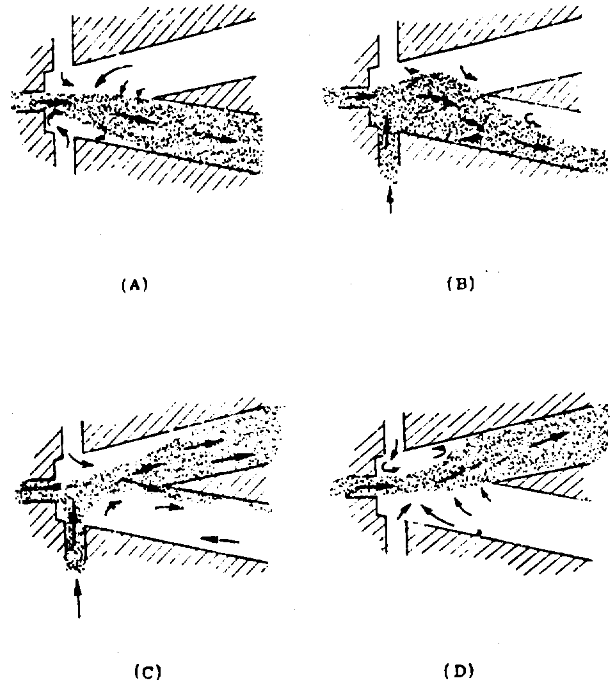


Figure 16.4.2c. Switching Mechanism in Bistable Wall Attachment Device
(Reference 131-40)

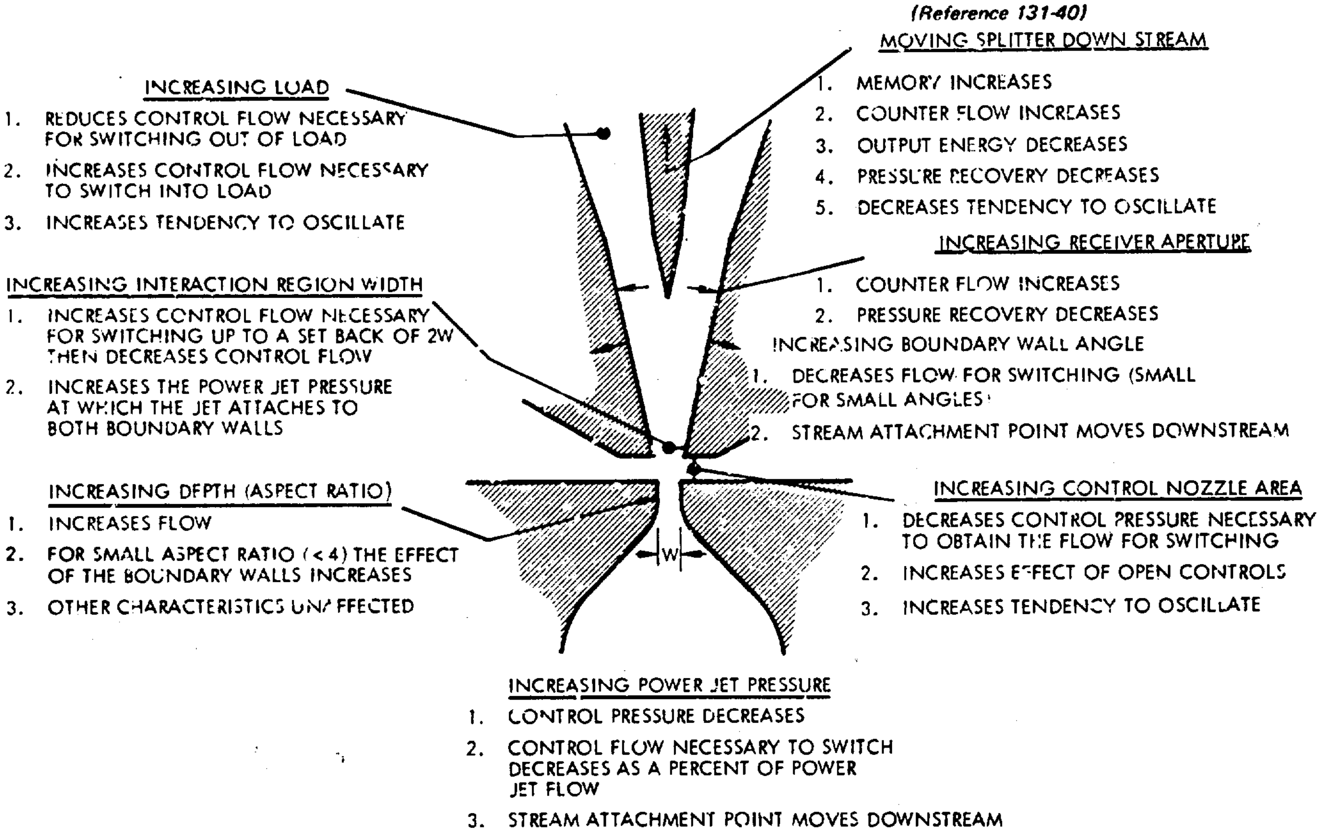
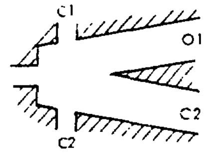


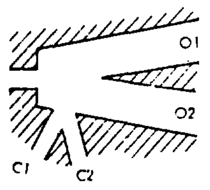
Figure 16.4.2b. Effects of Increasing Dimensions of Wall Attachment Device

1. FLIP FLOP



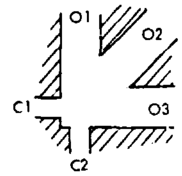
C1	C2	O1	O2
1	0	0	1
0	0	0	1
0	1	1	0
0	0	1	0

2. OR-NOR GATE



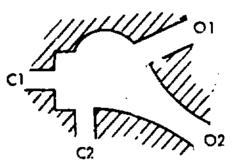
C1	C2	O1	O2
0	0	0	1
1	0	1	0
0	0	0	1
0	1	1	0
0	0	0	1
1	1	1	0

3. AND GATE (PASSIVE)



C1	C2	O1	O2	O3
1	0	0	0	1
0	1	1	0	0
0	0	0	0	0
1	1	0	1	0

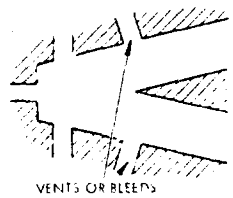
4. HALF ADDER (PASSIVE)



C1	C2	O1	O2
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	0

Figure 16.4.2d. Wall Attachment Logic Elements
(Reference 131-40)

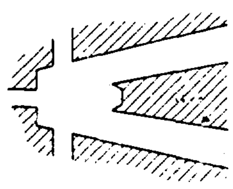
1.



VENTED OUTPUTS

VENTS OR BLEEDS IN THE OUTPUT CHANNELS ISOLATE THE INTERACTION REGION FROM DOWNSTREAM CONDITIONS.

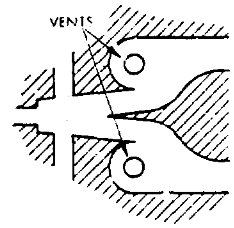
2.



CUSPED SPLITTER

THE CUSP IN THE SPLITTER GENERATES A LATCHING VORTEX ON THE PASSIVE OUTPUT SIDE OF THE POWER JET. CONSEQUENTLY, THE PRESSURE, FLOW, AND POWER RECOVERY OF THE DEVICE ARE CONSIDERABLY INCREASED. IN ADDITION, THE ACTIVE AND PASSIVE OUTPUT PORTS ARE EFFECTIVELY DECOUPLED.

3.



LATCHED VORTEX VENT

THIS VENT CONFIGURATION ALLOWS IMPEDANCE MATCHING OVER A WIDE RANGE OF LOAD CONDITIONS, FROM ZERO LOAD UP TO AND INCLUDING REVERSE FLOW INTO THE AMPLIFIER. COMPRESSION PULSES PROPAGATED BACK INTO THE AMPLIFIER INTERACTION REGION ARE ALSO ATTENUATED.

Figure 16.4.2e. Methods of Reducing Load Sensitivity
(Reference 131-40)

Response times of 0.5 to 1 millisecond are state of the art for 10-mil elements. With more efficient element geometries, the response time of a 10-mil element should eventually decrease to about 0.2 millisecond. However, it must be kept in mind that response time is strongly influenced by the transport delay, which is the interval between the issuance of a particle of fluid from the control port nozzle and the arrival of the particle at the output of an element.

Power recovery in nonvented wall attachment devices is presently a maximum of about 50 percent at a pressure recovery of 60 percent. Maximum pressure recovery is about 85 percent at low flow recovery, and maximum flow recovery is about 90 percent at low pressure recoveries. Typically, for state of the art vented-wall attachment devices, pressure recovery is about 50 percent at near-zero output flow and flow recovery is about 85 percent at near-zero output pressure.

Fan-in capability of wall attachment devices is limited by the configuration, i.e., there is a practical limit to the number of control input ports which can be present in the interaction region. A fan-in of four is considered state of the art, and potentially this should increase to about eight.

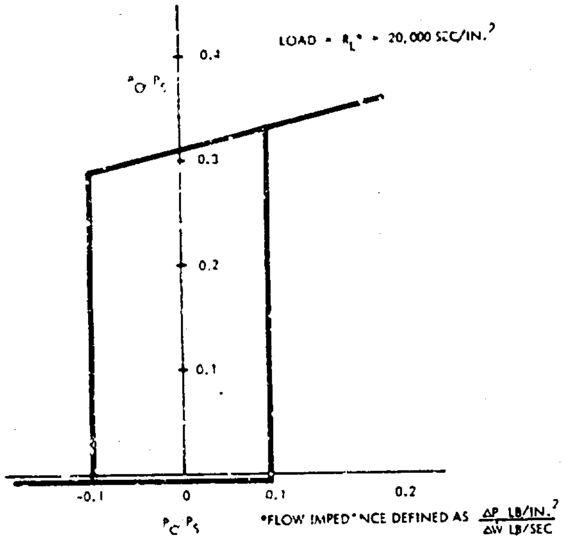
Fan-out is defined as the number of digital elements which can be controlled from the output of a single identical element operating at a common power nozzle pressure. Fan-out of up to 16 (Reference 532-1) has been reported; however, present practical fan-out capability is 2 to 6.

Typical pressure and flow gains range from 1 to 15, depending on fan-out. Control pressures of 2 to 15 percent of supply pressure are normally required to switch a device. However, switching pressure can increase about 50 percent with increase in fan-out from 1 to 4. Present state of the art for a 10-mil element is a switching pressure 5 to 10 percent of supply for a fan-out of 1. The performance of bistable wall attachment devices is summarized in Table 16.4.2. Typical performance curves of three commercially available wall attachment elements are shown in Figures 16.4.2f, 16.4.2g, and 16.4.2h.

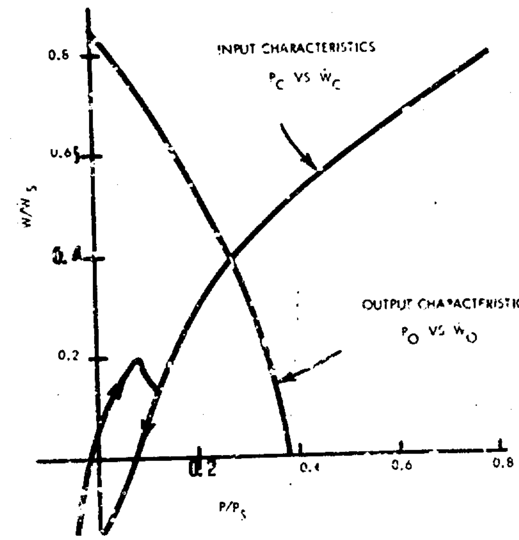
16.4.3 Beam Deflection Amplifier

Of the many possible types of proportional fluidic amplifiers, the beam deflection amplifier is the most widely used. There are several practical reasons for this, but the most important are: better technical design information available in the current literature and the advantages provided by a two-dimensional planar configuration. This planar configuration makes it easier to hold the tight tolerances required, provides the simplest method of cascading amplifiers for high gain, and facilitates the development of practical integrated circuits. The beam deflection amplifier fulfills many critical system functions and is particularly suited to proportional control functions such as sensors, stabilization systems, speed control, temperature control, pressure control, and analog computation.

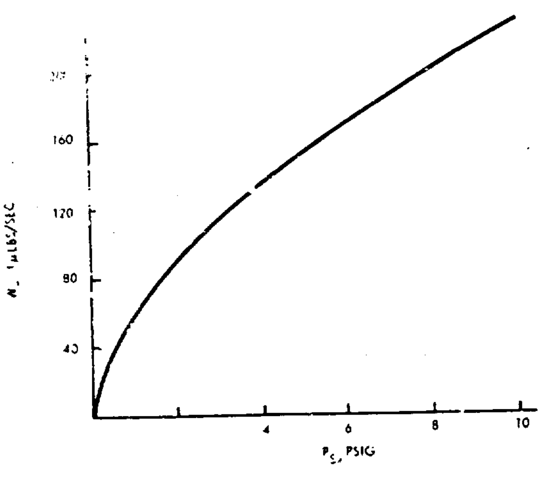
TYPICAL SWITCHING CHARACTERISTICS



INPUT AND OUTPUT PRESSURE - FLOW CHARACTERISTICS

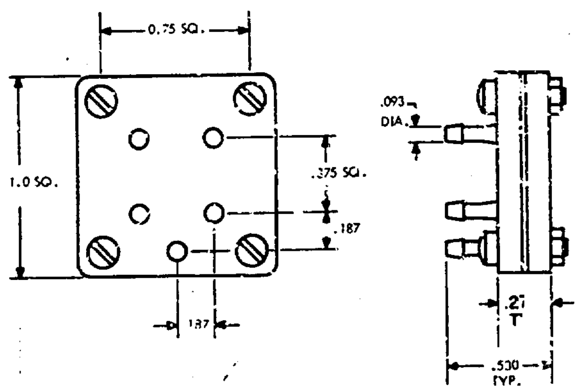


SUPPLY PRESSURE - FLOW CHARACTERISTICS



1 µ LB/SEC = 1 X 10^{-6} LB/SEC;
1000 µ LB/SEC = 0.80 SCFM

OUTLINE DIMENSIONS

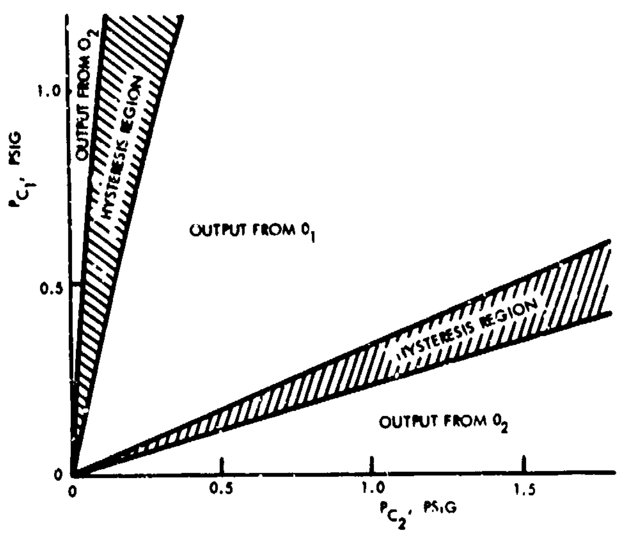


CONSTRUCTION

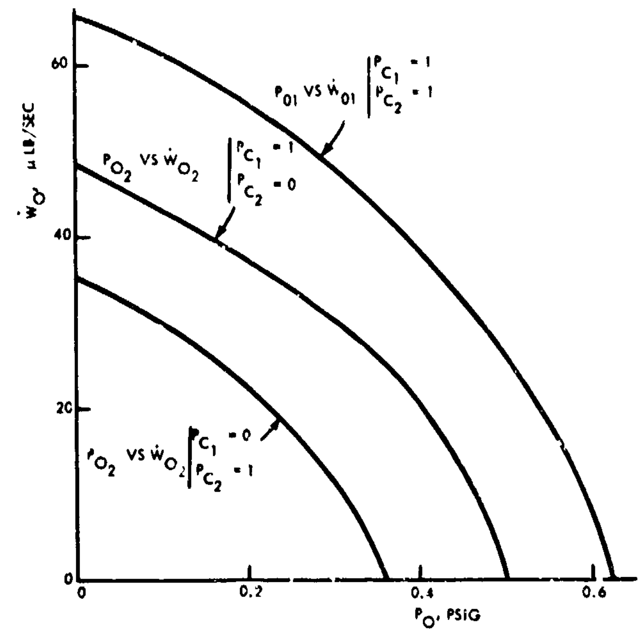
POWER INPUT NOZZLE - 0.020 IN. SQUARE
ELEMENT MATERIALS
LAMINATIONS - STAINLESS STEEL
COVER AND BASE - ALUMINUM (ANODIZED)
FITTINGS - STAINLESS STEEL
ALSO AVAILABLE WITHOUT TUBING FITTINGS FOR MANIFOLD MOUNTING

Figure 16.4.21. Model DF34 Flip-Flop
(Courtesy of General Electric Company, Schenectady, New York)

SWITCHING DOMAN

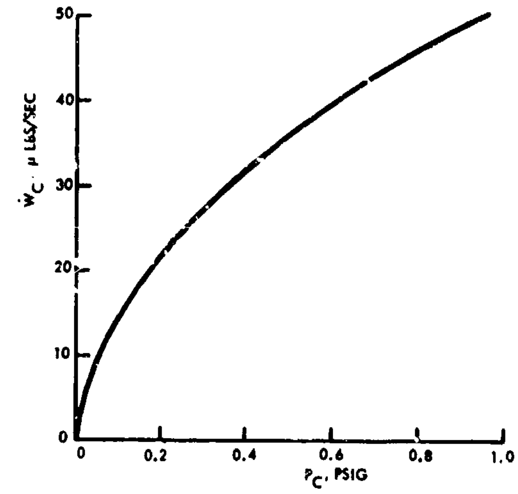


OUTPUT PRESSURE - FLOW CHARACTERISTICS



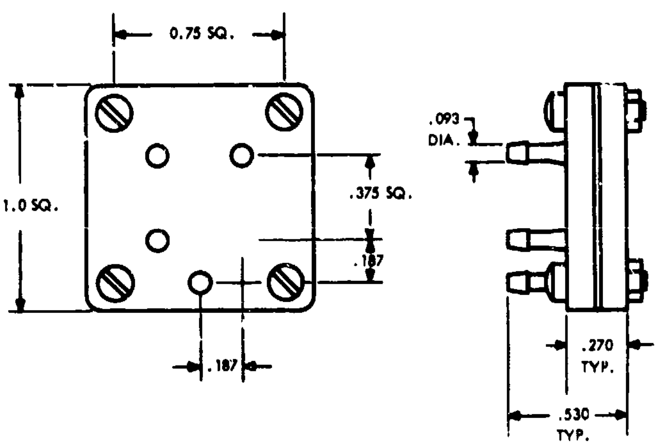
INPUT PRESSURE - FLOW CHARACTERISTICS

OTHER CONTROL PORT $P_C = 0$



1 μ LB/SEC = 1 X 10^{-6} LB/SEC:
1000 μ LB/SEC = 0.80 SCFM

OUTLINE DIMENSIONS

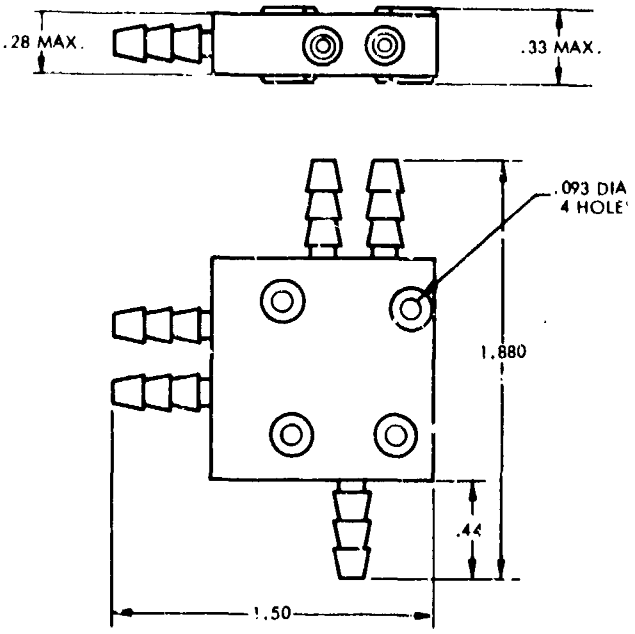


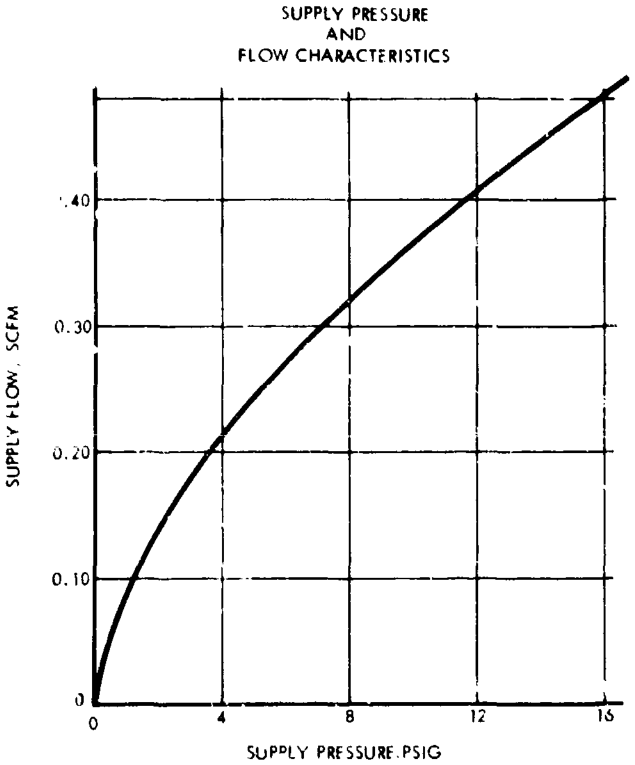
CONSTRUCTION
INPUT NOZZLES--0.020 IN. SQUARE
ELEMENT MATERIALS
LAMINATIONS--STAINLESS STEEL
COVER AND BASE--ALUMINUM (ANODIZED)
FITTINGS--STAINLESS STEEL
ALSO AVAILABLE WITHOUT TUBE FITTINGS FOR MANIFOLD MOUNTING.

Figure 16.4.[illegible]. OR/NOR Performance
(Courtesy of General Electric Company, Schenectady, New York)

OPERATING CHARACTERISTICS
FUNCTION. TWO-INPUT MONOSTABLE FLIP-FLOP
OPERATING MEDIUM: GASEOUS FLUIDS
OPERATING PRINCIPLE: WALL ATTACHMENT
TEMPERATURE RANGE. -140°F TO +270°F.

	MAXIMUM	NOMINAL	MINIMUM
INPUT PRESSURE	15 PSIG	2.5 PSIG	1.0 PSIG
POWER CONSUMPTION		1.1 WATTS	
PRESSURE RECOVERY (BLOCKED)		42%	
FLOW RECOVERY (UNBLOCKED)		125%	
FREQUENCY RESPONSE		800 CPS	
RESPONSE TIME		0.0004 SEC	
SWITCHING PRESSURE		0.35 PSI MAX	





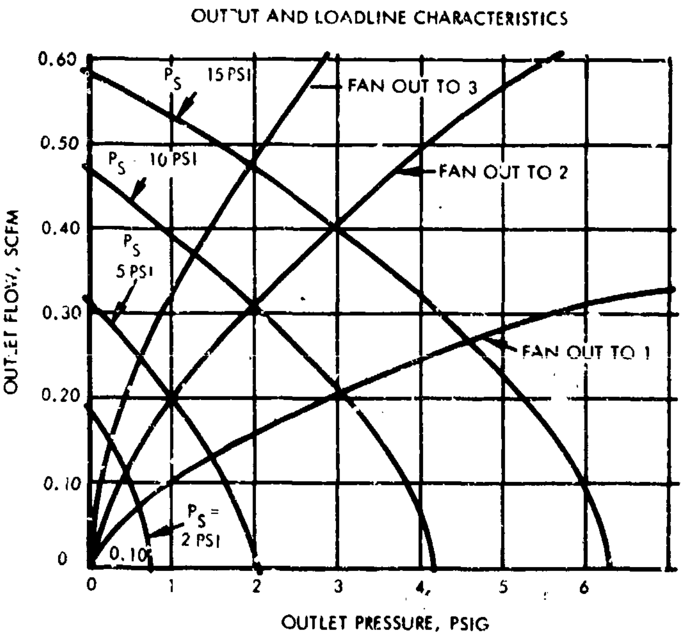


Figure 16.4.2h. Two Input Monostable Digital Amplifier Performance
(Courtesy of Aviation Electric Limited, Montreal, Canada)

Table 16.4.2. Performance of Bistable Wall Attachment Devices
(Reference 131-40)

Parameter	Performance Range
Power nozzle size (mil)	5 — 40
Aspect ratio	0.8 — 4
Supply pressure (psig)	0.1 — 40
Switching pressure* (% of supply)	5 — 15
Fan-in	1 — 4
Fan-out	1 — 6
Nonvented devices:	
Pressure recovery at $Q_o = 0$ (%)	60
Flow recovery at $P_o \to 0$ (%)	70
Power recovery (%)	40
Vented devices:	
Pressure recovery at $Q_o = 0$ (%)	25 — 40
Flow recovery at $P_o \to 0$ (%)	40 — 70
Power recovery (%)	20 — 30
Pressure gain	1 — 15
Flow gain	5 — 20
Response time (milliseconds)	0.1 — 2

*For fan-out of 1, i.e., with the control port of an identical device as a load.

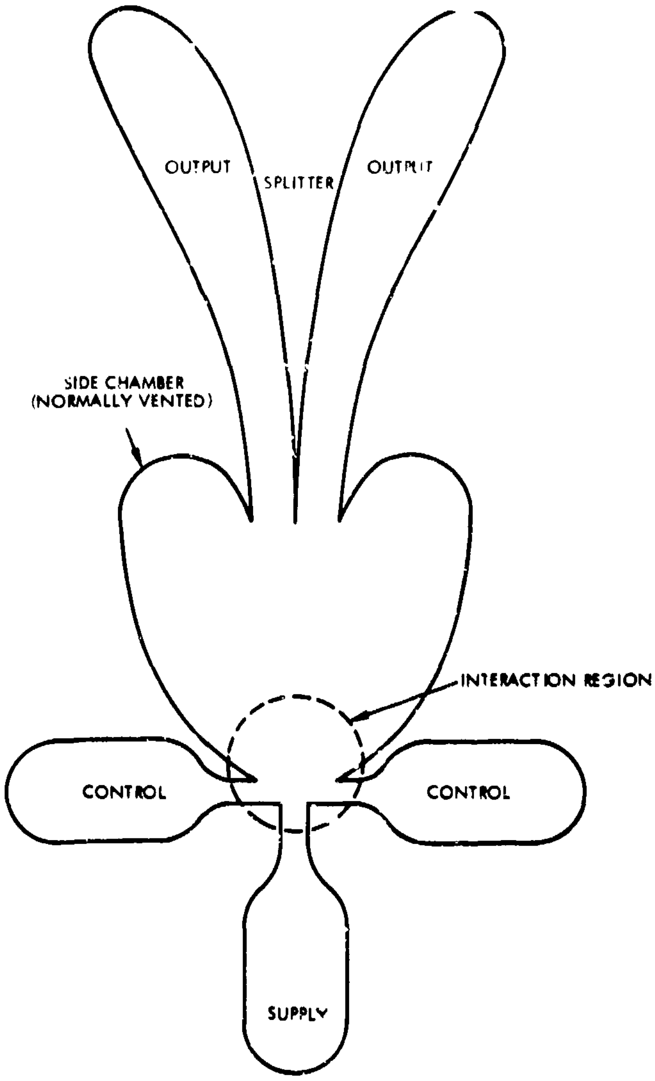


Figure 16.4.3a. Beam Deflection Device Configuration

In the beam deflection device (Figure 16.4.3a), the supply jet emerges and flows across the interaction region and is divided at the splitter. When there is no control flow or when the control pressures and flows are equal, the supply jet is not deflected (i.e., remains axially centered) and equal flows issue from each output port. Control flow is directed into the interaction region from nozzles on each side of the supply jet and approximately perpendicular to its centerline. If one control force is made greater than the other, the supply jet is deflected away from the centerline in the direction of the weaker force and a greater portion of the jet enters the output receiver on that side. If the amplifier is properly designed, the change in output power is greater than the change in input control power.

The deflecting force of the control streams may be either a pressure force or a momentum force; both forces are present to some degree in all beam deflection amplifiers. In general, momentum forces predominate when the controls are set back several supply nozzle widths from the supply jet, and the pressure forces predominate when the control nozzle is close to the edge of the supply stream.

As the supply jet proceeds through the interaction region, the shape of its velocity profile becomes approximately Gaussian, and the control jet flows do not appreciably alter its shape. As the supply jet moves further downstream, the profile broadens and decreases in centerline velocity. At some distance downstream, the resulting jet stream is divided and collected in the output apertures. There is an optimum size and position for the output apertures. They must be far enough downstream to take full advantage of the supply jet deflection, and far enough upstream to recover an appreciable portion of the supply jet pressure. Typically, in a nonvented amplifier without a center dump, the output apertures are located about 10 power nozzle widths downstream and are about 1.5 nozzle widths wide.

For proportional operation, beam deflection devices are specifically designed to prevent wall attachment. This is done by omitting adjacent walls in the vicinity of the supply nozzle, as shown in Figure 16.4.3b. The shape and dimensions of the cutout areas also have considerable influence on the performance of the amplifier. Any fluid not collected in the outputs can be reflected from the walls

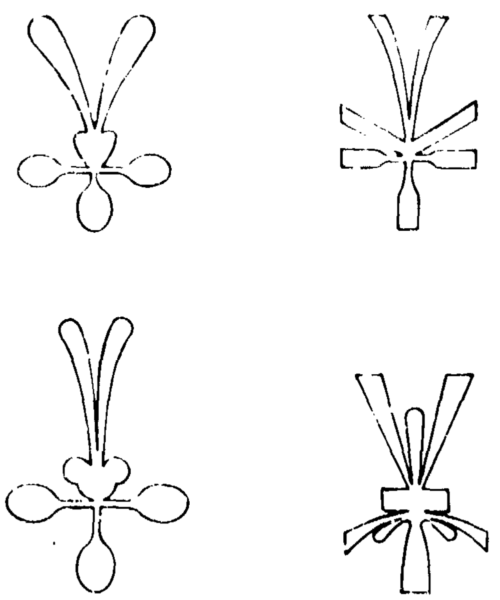


Figure 16.4.3b. Proportional Amplifier – Interaction Region Shapes *(Reference 131-40)*

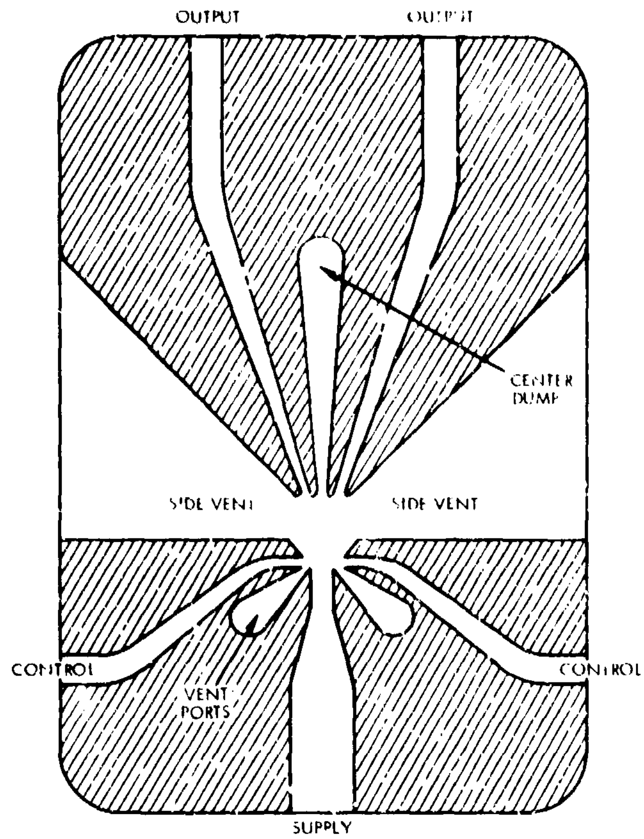


Figure 16.4.3c. Center Dump Proportional Amplifier

of the cutout back toward the supply jet to produce a feedback effect, which could result in unstable operation, oscillation, and reduced gain. In vented amplifiers, the cutouts on both sides are vented to atmosphere or, in some cases, to a constant-pressure reservoir. This tends to equalize the static pressure across the supply jet downstream of the interaction region, and, since the venting region is always at a relatively constant pressure, it also provides overflow ports for the excess fluid in the supply stream and for back flow due to output port loading. In the closed beam deflection amplifier, although there are no vent ports, the side chambers are still connected together to equalize the pressure across the power jet. Without vents, all of the flow must leave through the output ports, raising the pressure in the side chambers, in the interaction region, etc., so that with a blocked load the device no longer functions.

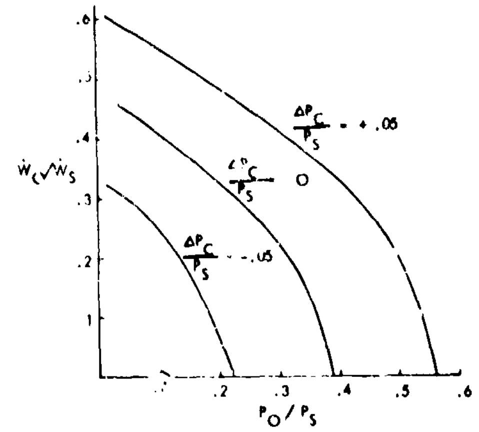
The vented beam deflection amplifier overcomes many of the shortcomings of the closed amplifier in that it provides: higher pressure gain, the ability to operate with blocked loads, and stability under most load conditions. Many of these amplifiers also utilize a vent (center dump) between the two output ports (Figure 16.4.3c). Since a considerable segment of the higher velocity portion of the supply jet is vented out the center dump, the pressure gain and pressure recovery of the center dump amplifier are somewhat lower than the standard vented design. However, the center dump provides several advantages, including: increased stability with blocked loads and repeatable zero-balance conditions (differential output pressure equal to zero) over a wide range of power jet pressure, which is a necessity in high-gain staged units.

The size of beam deflection amplifiers is determined by the width of the power nozzle and the aspect ratio. All of the internal amplifier dimensions are then defined as ratios of the power nozzle width, as in wall attachment devices. Normally these devices are made with an aspect ratio of 1 to 2 for best performance.

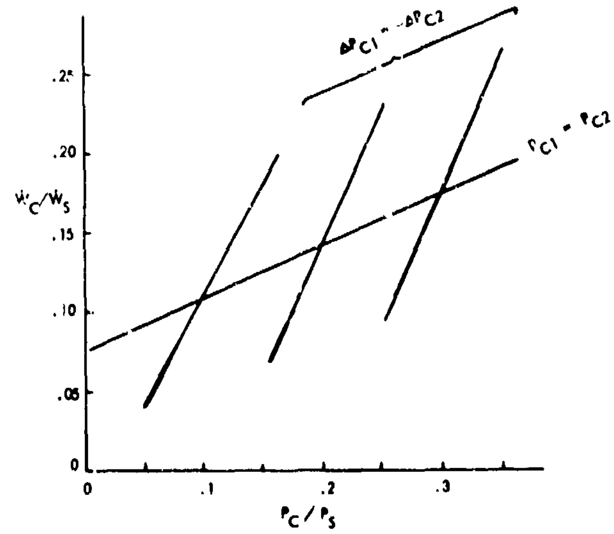
The operating performance of an amplifier is generally described by gain, bandwidth, efficiency, and signal-to-noise ratio. Pressure gain is usually the parameter of general interest, but it should be understood that maximum pressure gain is achieved at the expense of flow gain, power gain, linearity, etc. Consequently, a useful amplifier must be a compromise among all these parameters. The present theoretical maximum pressure gain of a beam deflection amplifier is about 20, presuming all the control energy is converted into momentum flux and the power jet is at near-zero position. When the control pressure forces are included, the pressure gain should increase substantially, however a maximum has not been established. Flow gain in beam deflection amplifiers depends on a number of things, including the downstream distance and width of the output apertures, output loading, and control bias level. Normally, flow gain ranges from about 2 to 10, depending on bias level and loading.

Signal-to-noise ratio is perhaps the most important criterion in beam deflection amplifiers. In most designs, pressure gain is usually sacrificed in favor of reduced pressure noise. A signal-to-noise ratio of greater than 100 should be sought in high-power single-stage amplifiers and as high as possible in elements suitable for staging (200 to 400 has been achieved). In staged high gain amplifiers, noise is influenced by power jet pressures, interconnections, output loading, vent configurations, and many other criteria. Consequently, it is necessary to use integrated circuits when interconnecting beam deflection amplifiers to achieve high gain, because interconnection by conventional means (tubing) is not practical.

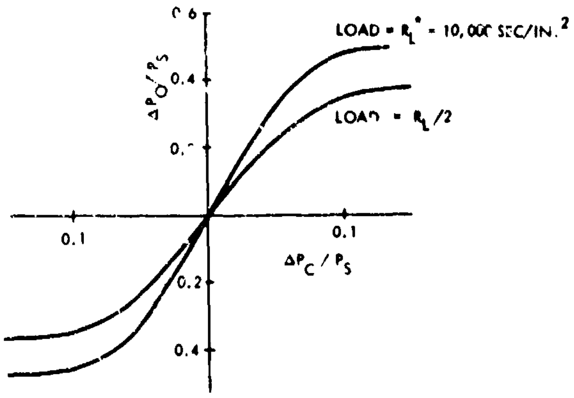
OUTPUT CHARACTERISTICS



INPUT CHARACTERISTICS

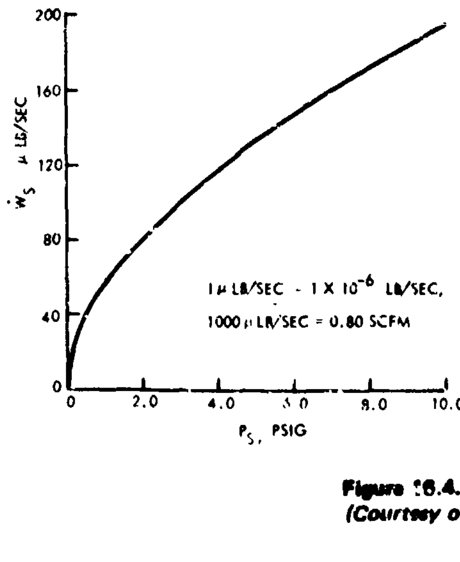


GAIN CHARACTERISTICS

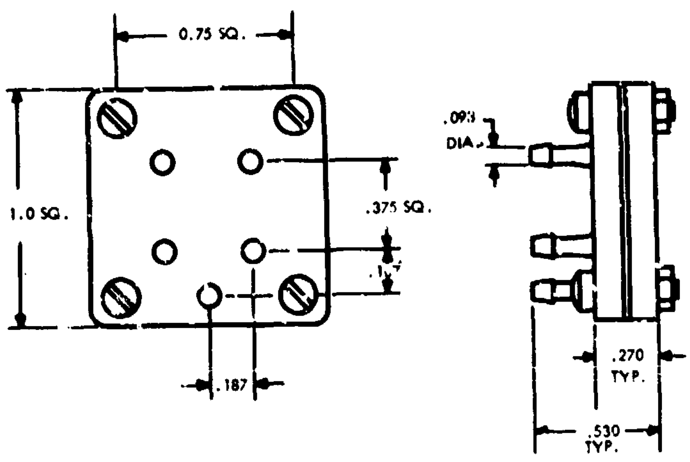


* FLOW IMPEDANCE DEFINED AS $\frac{\Delta P \text{ LB/IN.}^2}{\Delta \dot{W} \text{ LB/SEC}}$

SUPPLY PRESSURE - FLOW CHARACTERISTICS



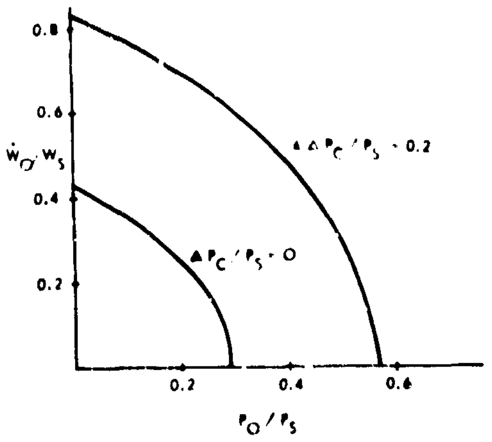
OUTLINE DIMENSIONS



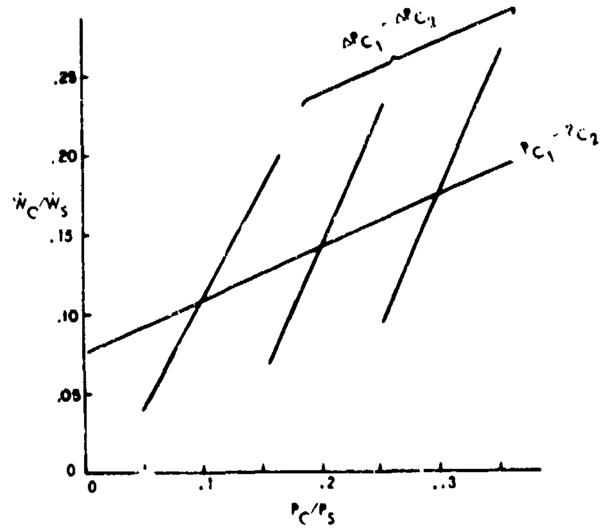
CONSTRUCTION
POWER NOZZLE - 0.020 IN. SQUARE.
ELEMENT MATERIALS
LAMINATIONS - STAINLESS STEEL
COVER AND BASE - ALUMINUM (ANODIZED)
FITTINGS - STAINLESS STEEL
ALSO AVAILABLE WITHOUT TUBING FITTINGS FOR MANIFOLD MOUNTING.

Figure 16.4.3d. Model AW32 Proportional Amplifier Performance
(Courtesy of General Electric Company, Schenectady, New York)

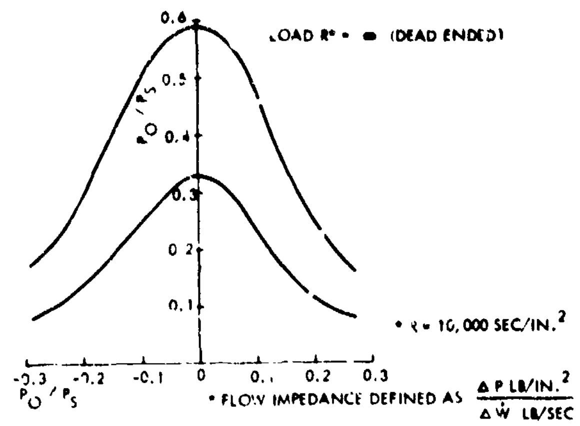
OUTPUT CHARACTERISTICS



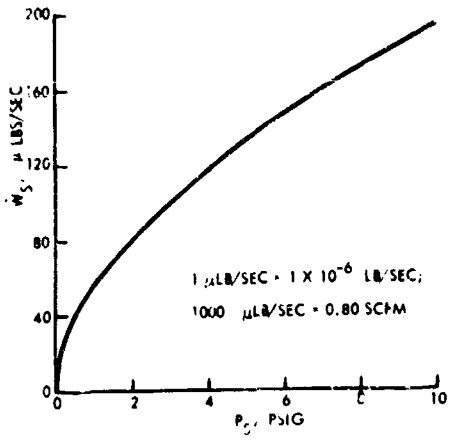
INPUT CHARACTERISTICS



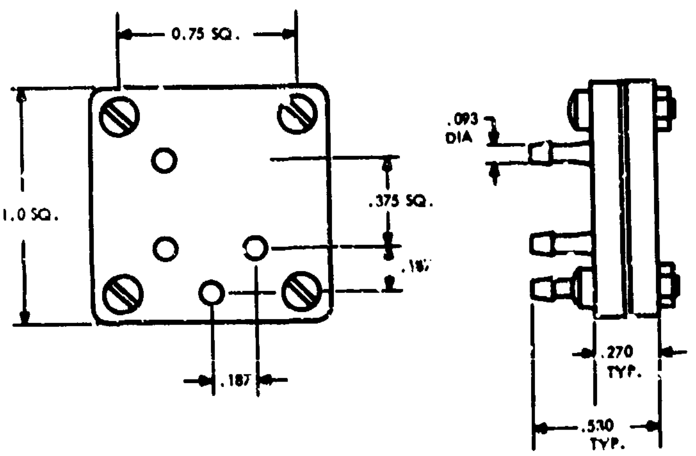
INPUT - OUTPUT CHARACTERISTICS



SUPPLY PRESSURE - FLOW CHARACTERISTICS



OUTLINE DIMENSIONS



<u>CONSTRUCTION</u>
THE POWER NOZZLE IS 0.020 IN. SQUARE.
ELEMENT MATERIALS ARE:
LAMINATIONS - STAINLESS STEEL
COVER AND BASE - ALUMINUM (ANODIZED)
FITTINGS - STAINLESS STEEL
ALSO AVAILABLE WITHOUT TUBING FITTINGS FOR MANIFOLD MOUNTING.

Figure 16.4.3a. Model AW32 Rectifier Performance
(Courtesy of General Electric Company, Schenectady, New York)

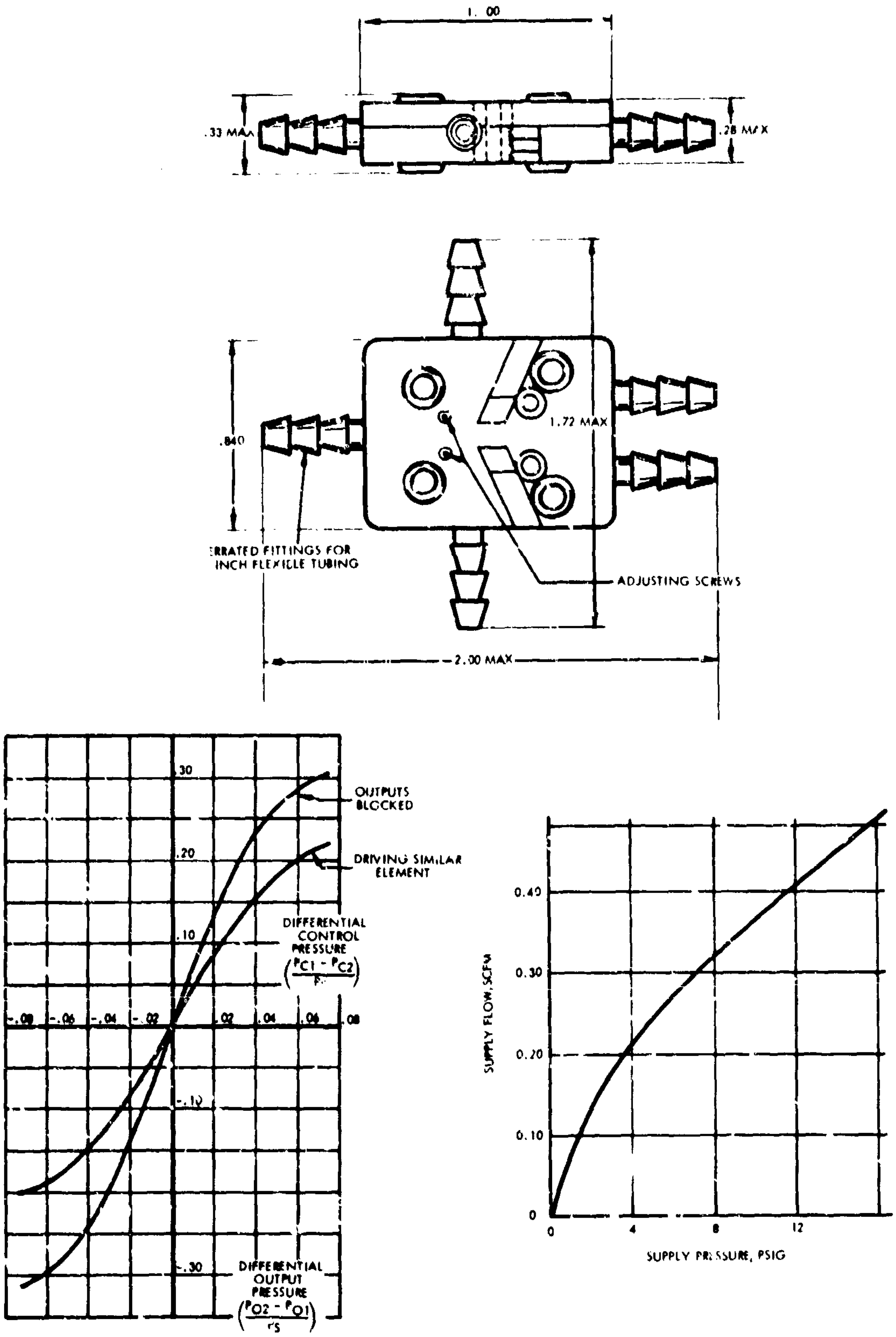


Figure 16.4.3f. Trimmable Proportional Amplifier
(Courtesy of Aviation Electric Limited, Montreal, Canada)

The present state of the art performance of beam deflection amplifiers is summarized in Table 16.4.3. Typical performance curves of three commercially available amplifiers are shown in Figures 16.4.3d through 16.4.3f.

Table 16.4.3. Performance of Beam Deflection Proportional Amplifiers

(Reference 131-40)

Parameter	Performance Range
Power nozzle size (mil)	5 — 40
Aspect ratio	1 — 3
Supply pressure (psig)	0.1 — 100
Pressure gain:	
No load	2 — 15
With equivalent load	1 — 10
With signal-to-noise ratio > 100	1 — 8
Flow gain	10
Power gain	50 — 100
Pressure recovery at Q_0 = 0 (%)	35
Flow recovery at P_0 → 0 (%)	40
Power recovery (%)	20 — 30
Linearity of best straight line (%)	2 — 5
Linear range of supply pressure (%)	±25
Maximum range of supply pressure (%)	±30
Operating frequency-gas (kc)	0 — 2

16.4.4 Vortex Devices

The common amplification mechanism which is responsible for the operation of vortex devices is the conservation of angular momentum. This amplification takes place in a shallow two-dimensional vortex chamber, as shown in Figure 16.4.4. As discussed in Detailed Topic 16.4.1.3, swirl is imparted to a radial supply flow by a tangential control jet which is introduced at the periphery of the vortex chamber. The amount of swirl and the method used to generate it depend on the particular vortex device used. As the flow proceeds toward the center of the vortex chamber, the tangential velocity of a fluid molecule (Figure 16.4.4) must increase, since angular momentum must be conserved. The velocity increase is inversely proportional to the radial location. If the ratio of the outer to inner radius is very large, the corresponding increase in the tangential velocity will also be great. In practical vortex devices operating on real viscous fluids, the maximum amplification is limited by nonlinearities within the flow field. One of the more important flow distortions is caused by the degradation of the tangential velocity in the boundary layer at the end walls of the vortex chamber. Savino and Keshock (Reference 241-14) describe the nonlinearities in the vortex flow field on the basis of detailed velocity profile measurements. At high tangential velocities, the flow is carried through the vortex chamber mainly along the cover plates and a recirculation flow takes place in the center of the chamber. The flow leaves the chamber through the exit port or sink.

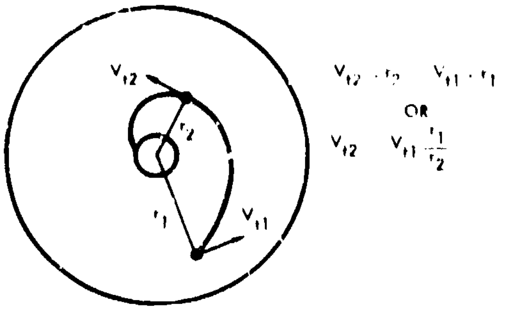


Figure 16.4.4. Two-Dimensional Vortex Chamber
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

The line sink is a line into which fluid is flowing; in a practical sense it is a finite-size sink (output port) instead of a line. The fact that real flow cannot disappear at the sink and must be discharged in the axial direction causes many three-dimensional problems which have been studied by Donaldson and others (Reference 771-3). This three-dimensional aspect of the flow field results in much analytical difficulty. Since flow nonlinearities are extremely difficult to describe mathematically, articles on vortex analysis are usually based upon many simplifying assumptions. Sub-Topics 16.4.4.1 and 16.4.4.2 have been adapted from a paper by E. A. Mayer (Reference 760-1).

16.4.4.1 VORTEX DIODE. One of the simplest flow control devices is the ball check valve, which closes off the flow stream when flow is in one direction and moves out of the flow path where flow is in the opposite direction. In a device that has no moving parts, it is easy to keep the flow resistance low in one direction, but quite difficult to provide high resistance in the opposite direction without continued venting of fluid. The vortex diode (Figure 16.4.4.1) has a circular chamber with a tangential inlet and an axial sink for flow in the high resistance direction. Flow through the tangential inlet produces a high pressure loss because of the swirling flow in the vortex chamber. In the opposite direction, flow enters through the axial sink and passes through the vortex chamber without swirl, so that the pressure loss is much lower. The most common performance index for a fluidic diode is the ratio of flow in the easy direction to the flow in the high resistance direction measured at a given upstream pressure. These devices are presently limited to a flow ratio of less than 10. The most common flow ratio is in the range of 3 to 5. Heim reported a flow ratio of 6.6 which was achieved by careful shaping of the tangential inlet port to the vortex chamber (Reference 771-3). In spite of the low flow ratios, the vortex diode has been found useful in many applications where a flow ratio of 3 has been sufficient.

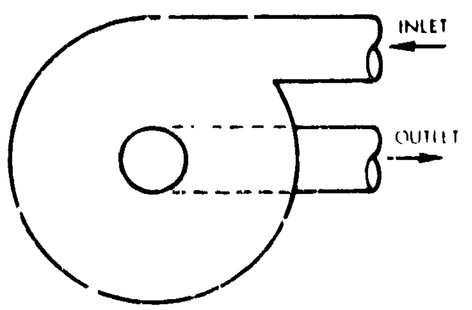


Figure 16.4.4.1. Vortex Diode – High Resistance Flow Direction

16.4.4.2 NONVENTED VORTEX AMPLIFIER. The vortex amplifier in its simplest form is known as a nonvented vortex amplifier. As shown in Figure 16.4.4.2a, it is similar in construction to the vortex diode, except that a third opening has been added for a supply stream. In this form, the vortex amplifier can be easily constructed in a two-dimensional configuration, and although performance is not optimized, it is adequate for many applications.

A basic problem with the single supply and single control inlet (Figure 16.4.4.2a) is the asymmetry of the device. By introducing the supply flow around the circumference of the vortex chamber in a "button" configuration, as shown in Figure 16.4.4.2b, control flow can be introduced at several points to ensure axisymmetric mixing of the supply and control flows. It is important to note that the control ports must be within the supply flow annulus to prevent the control momentum from being dissipated by a free expansion into the vortex chamber before mixing occurs. This device is rather complex and, consequently, rather difficult to make except in a three-dimensional configuration.

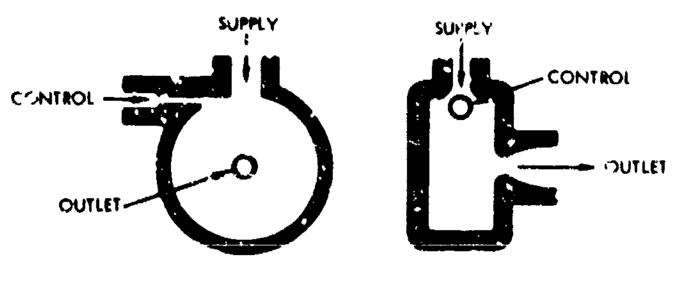


Figure 16.4.4.2a. Nonvented Vortex Amplifier Configuration
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

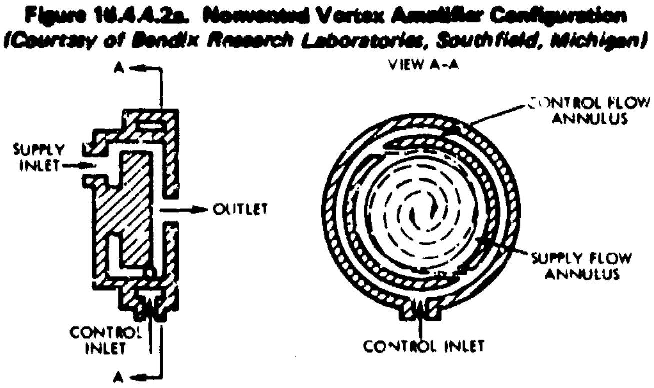


Figure 16.4.4.2b. Nonvented Vortex Amplifier – Button Configuration
(Courtesy of General Electric Company, Schenectady, N. Y.)

Perhaps the most practical compromise is the dual-inlet, dual-control configuration used by General Electric (Figure 16.4.4.2c). Performance of the device is almost as good as the button configuration, yet it can be made in a two-dimensional configuration. The two controls and two supplies can be connected externally or manifolded together in a cover plate.

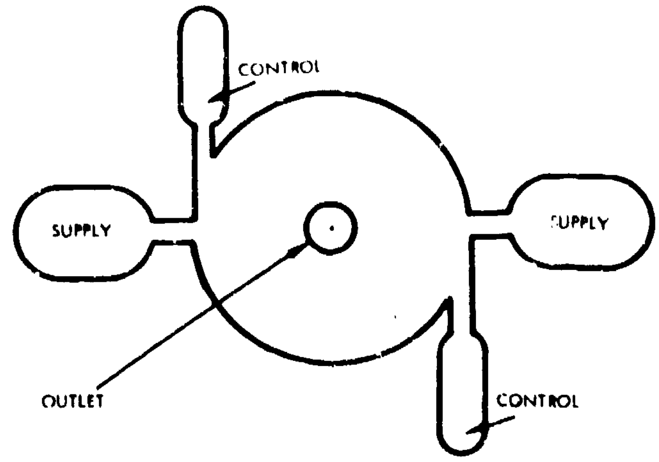


Figure 16.4.4.2c. Dual Nonvented Vortex Amplifier Configuration

The dual-exit nonvented vortex amplifier (Figure 16.4.4.2d) was introduced by the Bendix Corporation. Dual exits provide an increase in performance of about 70 percent over a single exit vortex amplifier, i.e., the maximum flow capacity of the amplifier is increased 70 percent with identical control flows. The primary reason is that with the establishment of a vortex in the spin chamber, the core region has a static pressure which is nearly zero irrespective of the number of output ports. For optimum performance, the output ports are normally of different sizes.

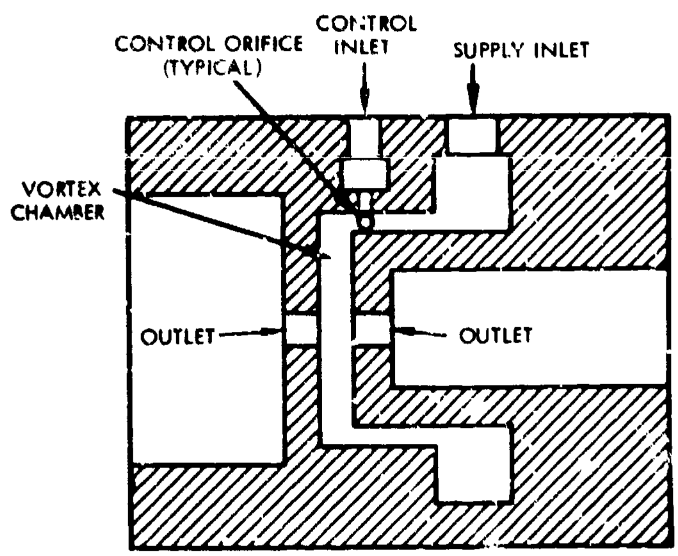


Figure 16.4.4.2d. Dual Exit Nonvented Vortex Amplifier
(Courtesy of General Electric Company, Schenectady, N. Y.)

Two types of performance curves can be used to describe the input/output characteristic of the nonvented vortex amplifier: the constant supply pressure curve and the constant control pressure curve. The curves in Figure 16.4.4.2a show the flow-turndown characteristics of the nonvented vortex amplifier for several values of supply pressure. The characteristic curves fall between two limiting envelope curves: the maximum flow curve at zero tangential control flow and the minimum flow curve corresponding to zero supply flow. At zero tangential flow, the outlet flow is maximum and follows typical orifice flow as determined by the outlet orifice of the amplifier. Pressure losses within the amplifier are negligible for this condition. If control pressure is applied and the supply pressure is at some constant value, P_s, the control signal causes flow within the chamber to swirl. As the control pressure is increased, the tangential control flow imparts greater swirl to the flow through the amplifier, and the increasing pressure loss through the vortex flow field reduces total flow through the amplifier. Minimum flow occurs at the point of zero supply flow: at that point, all of the flow passing through the nonvented vortex amplifier is supplied from the control port. Because these devices cannot completely eliminate outlet flow, the use of nonvented vortex amplifiers is limited to applications not requiring complete flow shutoff.

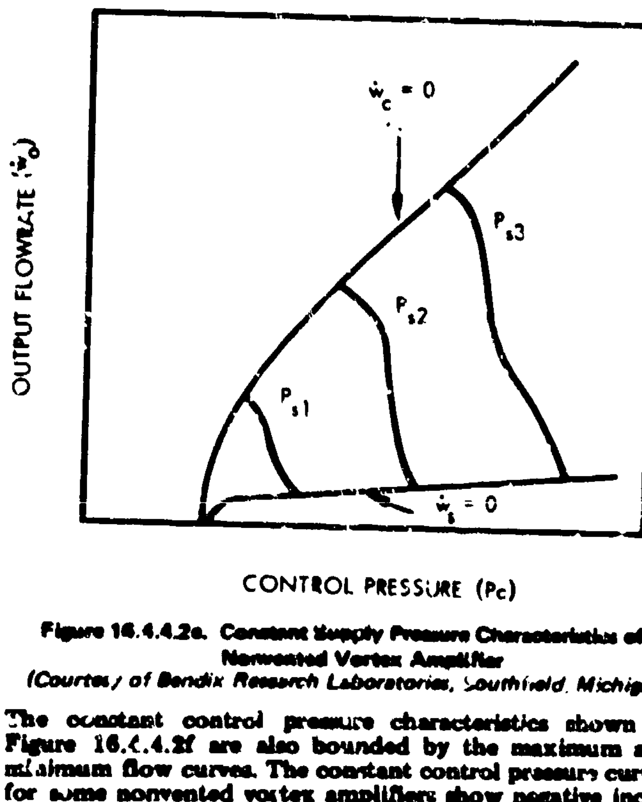


Figure 16.4.4.2a. Constant Supply Pressure Characteristics of a Nonvented Vortex Amplifier
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

The constant control pressure characteristics shown in Figure 16.4.4.2f are also bounded by the maximum and minimum flow curves. The constant control pressure curves for some nonvented vortex amplifiers show negative incremental resistance; in those cases, a small increase in the supply pressure reduces the outlet flow through the amplifier. Negative resistance is undesirable in flow control applications but is useful in vortex oscillator circuits. A simpler and more widely used performance criterion for nonvented vortex amplifiers is the turndown ratio. This is defined as the ratio of the maximum and minimum outlet flows at a constant supply pressure. At maximum outlet flow, $\dot w_o$, the control pressure, P_c, is generally equal to or less than the supply pressure, P_s, and normally the supply flow, $\dot w_s$, is zero at minimum outlet flow.

$$\text{Turndown Ratio} = \left.\frac{\dot w_o \ (\text{when } P_c \le P_s)}{\dot w_o \ (\text{when } \dot w_s = 0)}\right|_{P_s = \text{constant}}$$

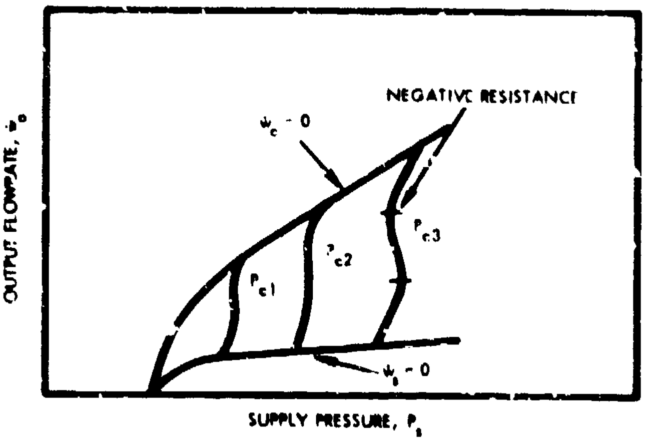


Figure 16.4.4.2f. Constant Control Pressure Characteristics of a Nonvented Vortex Amplifier
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

A nonvented vortex amplifier can be optimized for maximum turndown by utilizing relatively small control port areas and chamber-to-outlet hole diameter ratios of less than 8; however, a bistable hysteretic device will result. Normal useful designs are nonhysteretic with minimum noise in the high gain region. To accomplish this:

1) Chamber to outlet hole diameter ratio is usually between 8 and 12.
2) Outlet hole to control port area should be about 8.
3) Chamber length is usually greater than one-half the exit hole diameter.

In addition to the physical configuration of the nonvented vortex amplifier, the turndown ratio is affected significantly by the size of the control port as shown in Figure 16.4.4.2g for one amplifier configuration. The indicated decreasing turndown ratio for a constant control flow results from the reduction in fluid velocity at the control ports when larger control port areas are used.

Since the control pressure must be higher than the supply pressure, a nonvented vortex amplifier cannot operate with full line pressure on its load unless a separate high pressure control source is available. An important criterion often overlooked when specifying amplifier performance is the control to supply pressure ratio at maximum turndown (when $\dot w_s \to 0$). This ratio varies from about 1.05 to 2, depending on how the amplifier is optimized. Because of the importance of the control to supply pressure ratio, nonvented amplifier performance is widely published as a

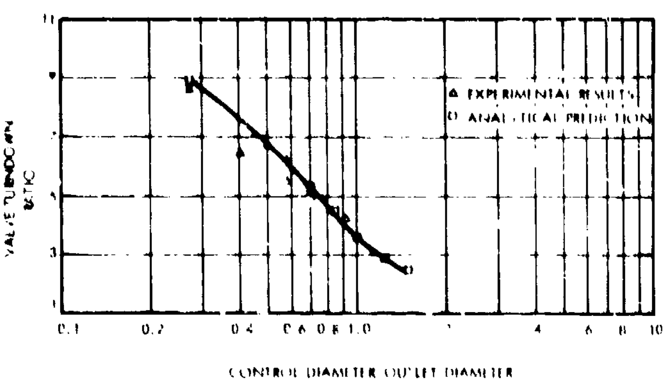


Figure 16.4.4.2g. Flow Turndown as a Function of Relative Control Port Size

(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

function of the normalised output flow (output flow/maximum supply flow) and the normalised control pressure (control pressure/supply pressure). The normalised performance curve of a commercially available amplifier illustrates this point (Figure 16.4.4.2h). The performance curve is generally drawn for a constant supply pressure. However, nonvented vortex amplifier performance is quite insensitive to fluid density, so that when the output port is sonic throughout the operating range, one normalised curve will accurately define performance over a wide range of supply pressures.

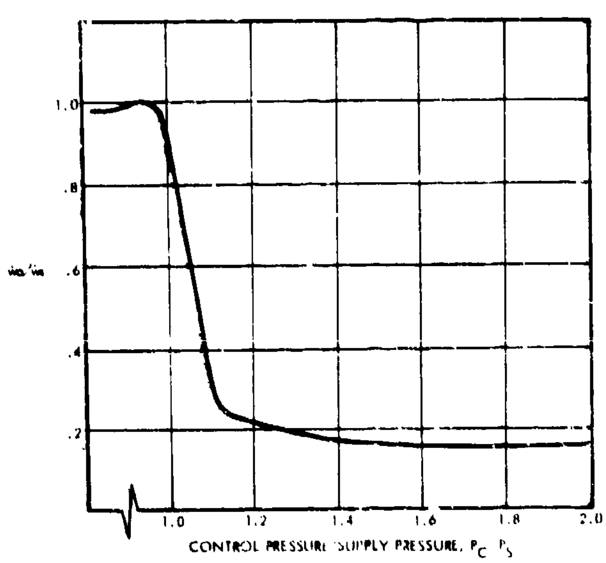


Figure 16.4.4.2h. Nonvented Vortex Amplifier Normalized Performance Characteristic

(Courtesy of General Electric Company, Schenectady, New York)

Present nonvented vortex amplifier performance is summarized in Table 16.4.4.2. Amplifier chamber diameters are assumed to be about 1 inch. Wall effects become a factor below a chamber diameter of 1 inch and performance will be lower than estimated in the table. The outlet orifice is also presumed to be sonic, although performance will improve somewhat with a subsonic orifice. A sufficient number of control nozzles are also required to ensure minimum pressure drop and uniform mixing in the vortex chamber.

A nonvented vortex amplifier can operate with any type of fluid. It has been used with gases, water, hydraulic fluids, liquid propellants, and liquid metals. The most efficient operation is obtained with low viscosity fluids such as air and water; modulation range is reduced with the higher viscosity fluids. Units have been built in sizes ranging from 0.072-inch to 9-inch chamber diameter.

One type of nonvented vortex amplifier, the vortex throttle, utilises a gas to control fluids such as water or liquid propellants. Turndown ratios of up to 50 can be expected with the vortex throttle. This device should find wide application in liquid throttling applications where the mixing of small percentages of gas with the controlled liquid is either beneficial or at least not detrimental. For instance, low molecular weight gases have been used to stabilise combustion in several small deep-throttling bipropellant rocket engines.

Table 16.4.4.2. Nonhypergolic Vortex Valve Performance

(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

Fluid	P_c/P_s	Turndown Ratio	
		Single Exit	Double Exit
Air	1.05	2.5	4*
Air	1.2	5.5	8
Air	1.5	8	11
Air	2	9	12*
Water	1.05	U	U
Water	1.2	7	U
Water	1.5	10	U
Water	2	12	U
Hydraulic Oil (MIL-H-5606)	1.05	2.35	3.02
Hydraulic Oil (MIL-H-5606)	1.2	4.7	6.08
Hydraulic Oil (MIL-H-5606)	1.5	7.45	9.55
Hydraulic Oil (MIL-H-5606)	2.0	10*	13*

U – Information presently unavailable.
* – Predicted from experimental data.

16.4.4.3 VENTED VORTEX AMPLIFIER. The nonvented vortex amplifier has a limited range of flow modulation. To overcome this limitation, a vented vortex amplifier utilises a receiver tube which is located and displaced axially away from the vortex chamber outlet orifice as shown in Figure 16.4.4.3a. With no control flow to the vortex amplifier, the flow exiting from the vortex chamber is in the form of a well-defined axial jet. This flow is recovered in the receiver tube, and the recovery characteristic is similar to that achieved in the receiver of a jet pipe valve. As control flow is applied, a vortex is generated and the flow out of the exit orifice forms into a hollow conical shape such that some of the flow is diverted to the exhaust. When a sufficiently strong vortex is generated, all of the exiting flow fans out to miss the receiver tube. This then produces a diverting action with full modulation of the receiver flow down to zero.

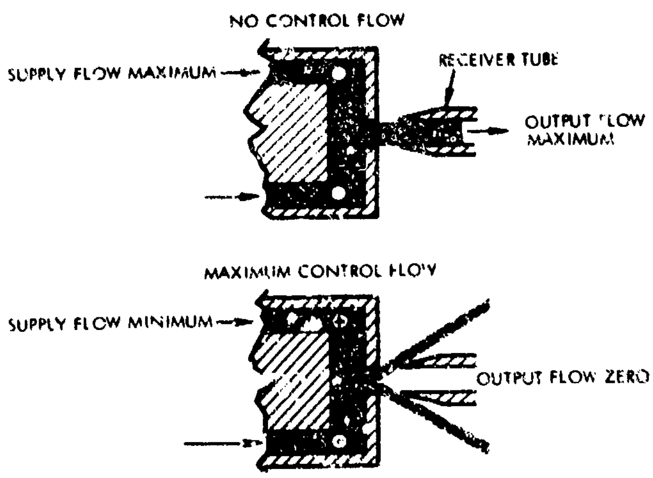


Figure 16.4.4.3a. Vented Vortex Amplifier Operation
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

The vented vortex amplifier is often used as a pressure amplifier (Figure 16.4.4.3b). For this application, the diameters of the vortex chamber outlet hole and the receiver tube are about the same, and the receiver is usually located approximately one tube radius axially downstream of the chamber outlet. The gain and efficiency of the receiver output can be controlled by changing the tube diameter and the axial distance between the receiver and vortex chamber outlet. Reducing the diameter or increasing the axial distance will decrease the power efficiency but improve the power gain.

Other important aspects of a properly designed vented vortex amplifier are as follows:

1) Incremental gain is virtually independent of load over a wide range, so that this type of amplifier has a very low incremental output impedance and can be cascaded effectively with virtually no gain loss.

2) Increased vent pressure does not degrade performance.

3) Load output pressure is virtually independent of vent pressure over a considerable range.

The performance of vented vortex amplifiers is hard to define and is best described by the incremental pressure gain and the power efficiency at the load. The incremental pressure gain is the change in load pressure for an incremental change in control pressure in the linear range of the amplifier. Power is defined as the product of pressure and flow. Load power efficiency is the percentage ratio of delivered load power to output power, or the product of pressure recovery and flow recovery. Although pressure recovery can be extremely high (95 percent) under blocked load conditions with no turndown, the flow recovery is zero. Large vented vortex amplifiers (output flow of 0.5 lb/sec) have been operated under certain conditions with flow recovery of about 95 percent and pressure recovery of about 30 percent.

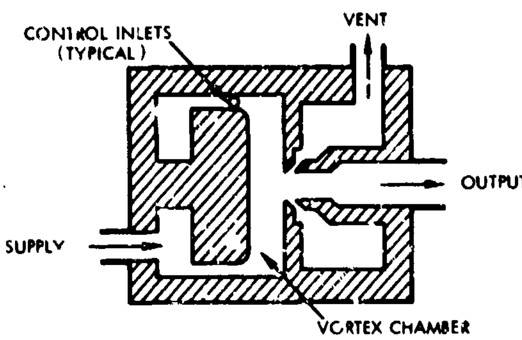


Figure 16.4.4.3b. Vortex Pressure Amplifier Configuration
(Reference 131-40)

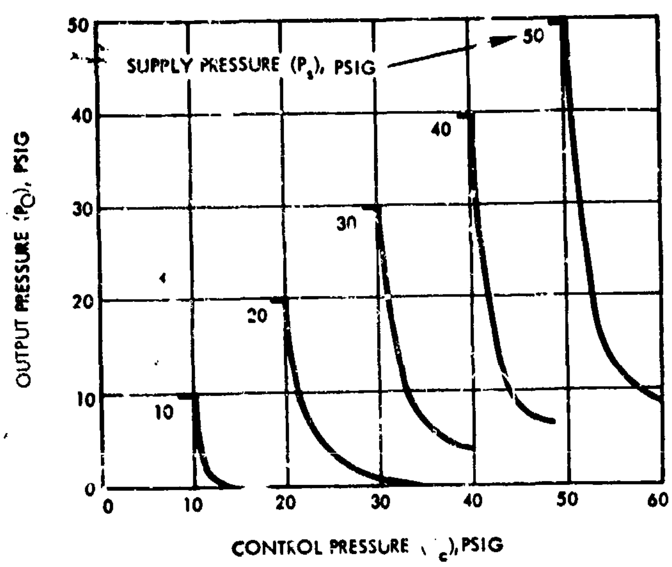


Figure 16.4.4.3c. Typical Pressure Gain Characteristic Vented Vortex Amplifier
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

Typical pressure gain characteristics for the vented vortex amplifier at several supply pressures and under blocked load conditions are shown in Figure 16.4.4.3c. It should be noted that the control pressure must exceed the supply pressure level before the characteristic turndown pressure recovery curves are achieved, i.e., no incremental gain is exhibited until the control pressure exceeds supply pressure. The gain shown by the characteristic curves, i.e., the change in load pressure for an incremental change in control pressure, is about 10. Pressure gains of several thousand have been reported for the vented vortex amplifier. However, these high gains are not useful since they occur only at a single point and under blocked load conditions. Power efficiency for an amplifier which provides useful pressure gain is generally about 50 percent for gases and 65 percent for liquids.

Error detection circuitry is readily implemented with vented vortex amplifiers since there is sufficient room around the outer periphery of the vortex chamber to accommodate a large number of control ports which can be arranged to either aid or oppose each other. For instance, the Bendix Research Laboratories have demonstrated the use of up to 16 separate summing control ports on a 1-inch amplifier. This is compared to a typical beam deflection amplifier where it is very difficult to sum more than two pairs of control parts without significant loss in gain and pressure recovery.

16.4.5 Logical NOR Amplifiers

The NOR function is the most basic and universal logic concept. In simple terms, the NOR gate provides an output signal when no control signals are present. Using the NOR element, all other logic functions such as AND, OR, NAND, NOT, and flip-flop can be obtained by the interconnection of two or more elements (see Table 16.8.4.5b). This type of fluidic device has found wide acceptance in the design of relatively low power digital circuits. See Sub Topic 16.8.4 for more comprehensive treatment of digital circuit design and explanation of logic functions such as NOR, AND, etc.

16.4.5.1 TURBULENCE AMPLIFIER. The turbulence amplifier (Reference 1-304) consists of a supply tube and an output tube precisely aligned in a vented cavity, and one or more control input tubes perpendicular to the power tube axis. The power jet is introduced into the vented cavity as a laminar stream so that in the absence of control flow much of the original jet power can be recovered at the output. When one or more of the control flows are introduced perpendicular to the power stream, as shown in Figure 16.4.5.1a, the jet becomes turbulent before reaching the receiver and the output pressure drops sharply.

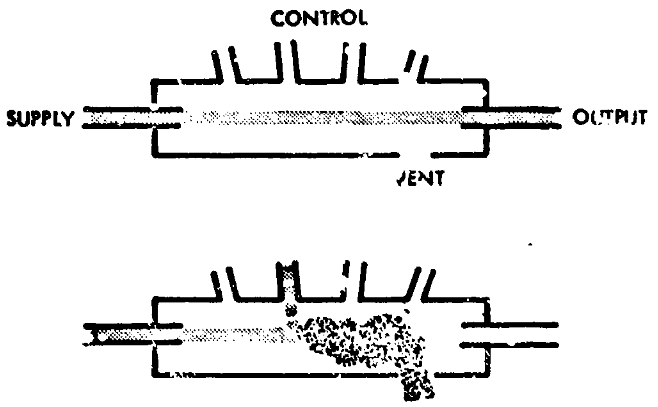


Figure 16.4.5.1a. Turbulence Amplifier Configuration and Principle of Operation
(Reference 131-40)

The operating principle of the turbulence amplifier is the transition of flow from the laminar to turbulent condition. In a practical turbulence amplifier, a number of conditions determine the transition point, such as the relative length and smoothness of the supply tube, supply flow turbulence at the entrance to the tube, and the absolute size of the supply tube. Perhaps the two most effective means of changing turbulence amplifier performance characteristics are variation of the supply pressure and adjustment of the distance between the power nozzle and the receiver. Figure 16.4.5.1b illustrates that a submerged laminar jet has three distinct ranges: an initial laminar segment, a transition range from laminar to turbulent, and a final turbulent segment. A pitot tube, such as the receiver of a turbulence amplifier, will show high pressure recovery if positioned in the first segment and low pressure recovery if positioned in the turbulent zone. As the supply pressure is increased, the velocity of the jet increases and turbulence occurs closer to the supply nozzle. For high supply pressures, the initial laminar-flow zone is almost completely eliminated, while at lower pressures it may extend as far as one hundred supply tube diameters downstream.

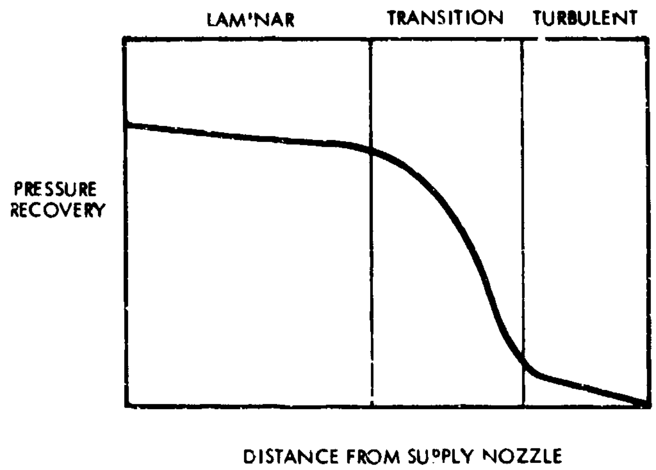


Figure 16.4.5.1b. Submerged Laminar Jet Operating Ranges

In a turbulence amplifier, the receiver is normally located in the laminar zone, near the transition zone. Placing the receiver closer to the transition zone reduces the initial flat segment of the turbulence amplifier characteristics (quadrant 3, Figure 16.4.5.1c); however, small increases in supply pressure may shift the transition zone and cause false output changes. Typically, a 0.03-inch diameter supply tube is located 30 diameters from the output tube.

The high gain characteristic exhibited by the turbulence amplifier in the transition region is very useful for digital applications. The device is not used as a proportional amplifier because, in the transistion region, the output signal-to-noise ratio is very low and the output pressure is very sensitive to small changes in the supply pressure. However, with no control input, the output is a relatively high pressure signal (although just a few inches of water), and with a sufficiently high control signal the output pressure is negligible.

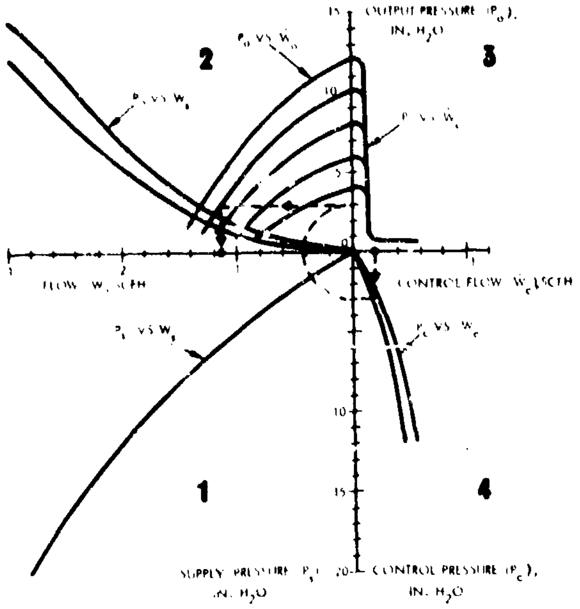


Figure 16.4.5.1c. Typical Turbulence Amplifier Output Characteristics
(Courtesy of Howie Corporation, Norristown, Pennsylvania)

Typical static characteristics of a turbulence amplifier are shown in Figure 16.4.5.1c. The plots shown in the graph completely describe the static characteristics of this device. The curve in the lower left quadrant (1) indicates the supply flow over a range of supply pressures. Output pressure versus supply flow and a family of curves for output pressure versus output flow are plotted in the upper left quadrant (2). Output pressure versus control flow is plotted in the upper right quadrant (3) and control pressure versus control flow in the lower right quadrant (4). The plotting of the curves on one axis enables fan-out to be determined for any operating pressure. First, note the minimum control pressure and flow required for turnoff. Then, using the minimum control pressure, check the flow available at that pressure on the proper P_o versus Q_o curve. Maximum fan-out is determined by dividing the Q_o available by the Q_o required. The method is illustrated on the graph by a dotted line.

The turbulence amplifier has a typical fan-in of 4 to 6 and a fan-out of about 6 to 10. Supply pressure range is 0.1 to 1 psig, and typical power consumption is about 60 milliwatts at 0.5 psig. The response time or switching time is in the range of 1 to 2 milliseconds. Turn-on time is generally about twice as long as the turn-off time because of the relatively long interval required to reestablish laminar flow after the transition to turbulent conditions. The amplifier also has the advantage of excellent control-output isolation.

Some of the prominent disadvantages of this device are:

1) It is sensitive to sound and vibration in the 5000 cps range.

2) A closely regulated supply pressure is required.

3) The output pressure recovery and absolute pressure level are low.

4) The signal-to-noise ratio is low.

Application of the three-dimensional turbulence amplifier in aerospace systems does not appear practical, because of the inability to integrate circuits as well as some of the disadvantages cited. An adaptation of this device, called the planar turbulence amplifier (Reference 73-263), appears practical since it can be assembled in modular circuits. The performance of the device is similar to that of the regular turbulence amplifier, except that it has an improved output pressure recovery of about 60 percent at 0.25 psig supply and should be less sensitive to sound and vibration.

16.4.5.2 FLOW INTERACTION NOR AMPLIFIER. The flow interaction NOR amplifier is a relatively new concept which operates somewhat like the turbulence amplifier. The general configuration of this amplifier is shown in Figure 16.4.5.2a. Laminar flow is developed in the long supply nozzle, and the issuing supply jet remains laminar through the interaction cavity and reaches the output receiver. The jet flows adjacent to a flat plate (top wall) through the interaction cavity, and the presence of the wall reduces the effects of ambient noise. With control flow present, the jet is deflected to the side and also away from the top plate to reduce the output pressure. A portion of the deflected supply jet recirculates in the interaction cavity and acts as a positive feedback, i.e., the resultant swirling flow in the interaction cavity disrupts the jet to further decrease the output from the unit.

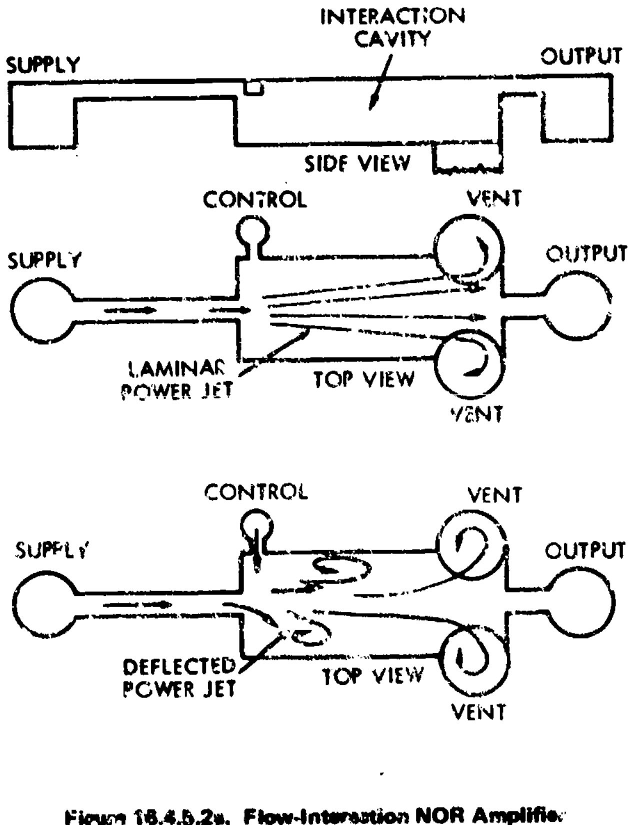


Figure 16.4.5.2a. Flow-Interaction NOR Amplifier
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

This device has a fan-in and fan-out capability of four. Supply pressure range is 1 to 1.6 psig and power consumption is about 23 milliwatts at 1.5 psig. Typical control-output pressure characteristics at 1.5 psig supply pressure are shown in Figure 16.4.5.2b. The response or switching time varies from 1.5 to 3 milliseconds for fan-outs of 1 to 4. Commercially available units are fabricated in 22 amplifier modules.

Although this device is adaptable to integrated circuits, it suffers from the same disadvantages of the turbulence amplifier (i.e., a well-regulated supply is required, a low pressure level, and low signal-to-noise ratio).

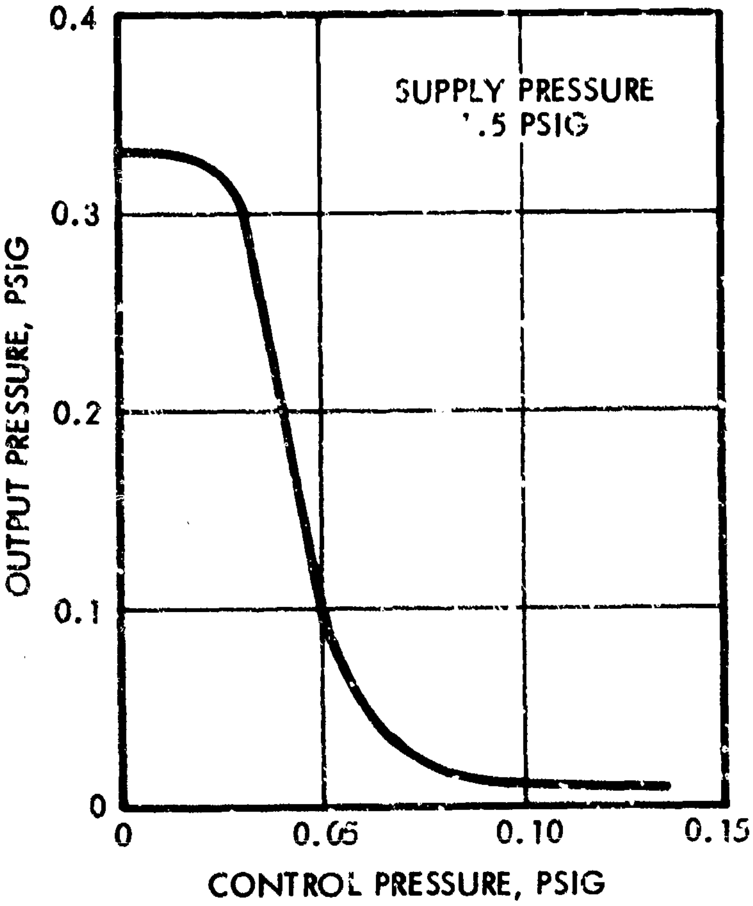


Figure 16.4.5.2b. Typical Input-Output Characteristics for Flow-Interaction NOR Amplifier
(Reference 131-42)

16.4.5.3 TWO-DIMENSIONAL LAMINAR NOR AMPLIFIER. The operation of this device depends on the deflection of a laminar supply jet rather than the laminar turbulent transition which is characteristic of the turbulence amplifier. In the laminar NOR amplifier (Figure 16.4.5.3), the laminar supply jet moves directly across the interaction region to the output port. The adjacent side wall and the top and bottom walls are a strong stabilizing influence on the supply jet. With control flow present, the supply flow is deflected into the vent port. The vent holes in the straight wall immediately upstream of the output port are required to decrease the static pressure buildup along the wall when the output port is blocked. This results in improved blocked output performance as well as in a significant increase in the blocked output pressure recovery.

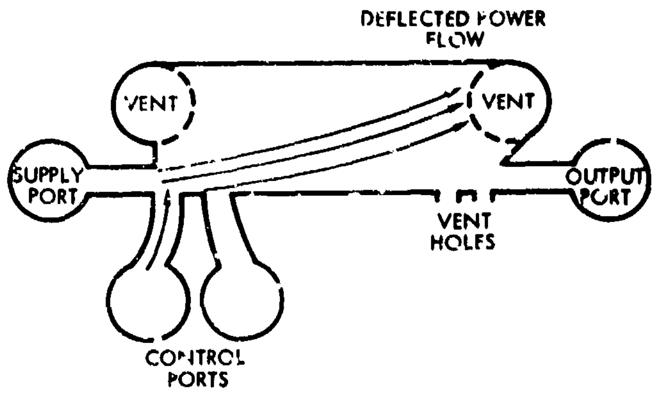


Figure 16.4.5.3. Two-Dimensional Laminar NOR Unit
(Reference 131-42)

One unfortunate aspect of the basic geometry is the mismatching of the two controls. The second control, located closer to the output port, causes a decrease of about 20 percent in the maximum fan-out of the device. Extremely low fluid velocity is also inherent in the laminar jet. Response time measurements are not presently available, but about 10 milliseconds is expected, which is relatively slow compared with other logical NOR units. However, the switching time is more than adequate for most applications and in most cases should be outweighed by the compactness and low power consumption of the unit.

The laminar NOR amplifier utilizes a 20 x 20 mil power nozzle and consumes about 2 milliwatts of fluid power at a supply pressure of 0.1 psig. Performance curves are not presently available, but the output pressure recovery is about 50 percent of the supply. Present units have a fan-in and fan-out capability of 2.

These excellent low power elements are adaptable to miniaturized integrated circuits and should find widespread application in aerospace systems, particularly in start-run-shutdown sequencing and other digital circuits. Reliability estimates should be high because of the relatively large (20 x 20 mil) power nozzle.

16.4.5.4 IMPACT MODULATOR NOR AMPLIFIER. This device utilizes the jet interaction-impact modulation effect discussed in Detailed Topic 16.4.1.1. In the impact modulator NOR amplifier (Figure 16.4.5.4a), two submerged jets emerge from opposed supply nozzles along the same axis so that they impact and form a radial jet. The location of the radial jet is determined by the momentum of each of the two impinging jets. A concentric orifice is placed between the two supply nozzles such that the radial jet is enclosed in an output chamber which is separated from a vented chamber. When a transverse control jet is applied to the supply jet in the vented chamber, it reduces the axial momentum of the jet so that the radial jet moves into the vented chamber and the output pressure is drastically reduced.

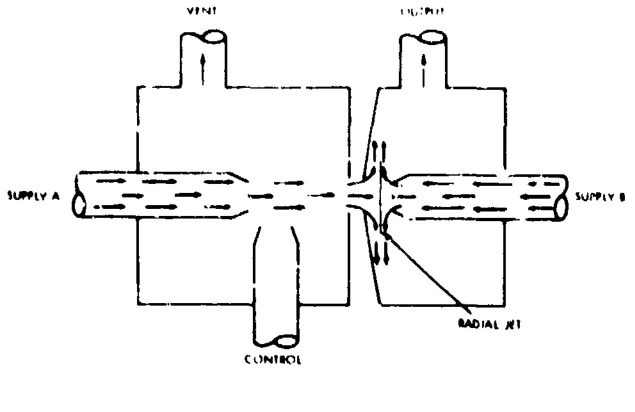


Figure 16.4.5.4a. Impact Modulator NOR
(Reference 131-42)

This device is particularly well suited to logic applications because of its high input and output impedances and high pressure gain. The control signals are applied in a completely vented chamber, such that control flow is independent of output pressure and flow, and there is complete isolation between individual control inputs. Output changes do not affect input pressure and flow because of the concentric orifice separating the output and vented chambers.

The impact modulator NOR amplifier has a fan-in of 4 and fan-out of 11. Supply pressure range is 0.25 to 6 psig and power consumption is 0.2 watt at 1 psig. Typical control-output pressure characteristics at 1 psig supply pressure are shown in Figure 16.4.5.4b. The response or switching time averages about 300 microseconds at 1 psig supply, which is good when compared with other fluidic logic elements of similar power drain. However, switching time varies drastically as a function of fan-out, and turn-off time is several times longer than the turn-on time (Figure 16.4.5.4c)

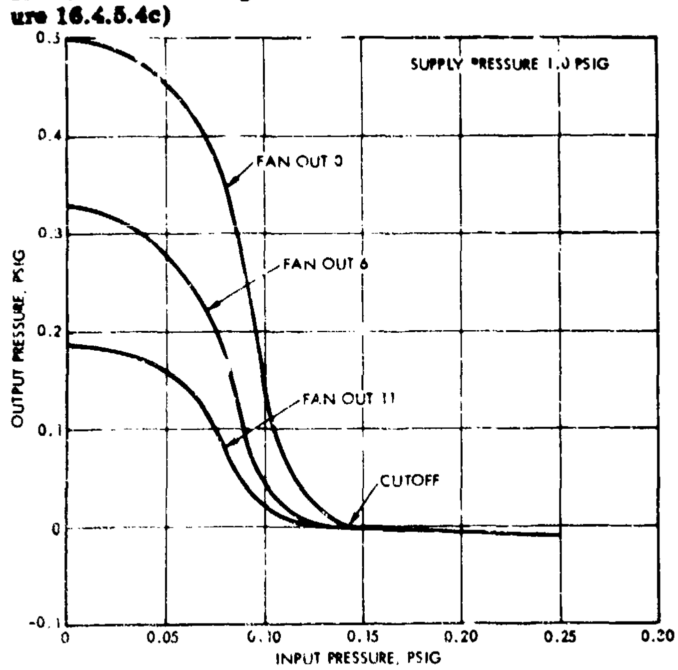


Figure 16.4.5.4b. Impact Modulator Typical Input-Output Characteristic
(Reference 131-42)

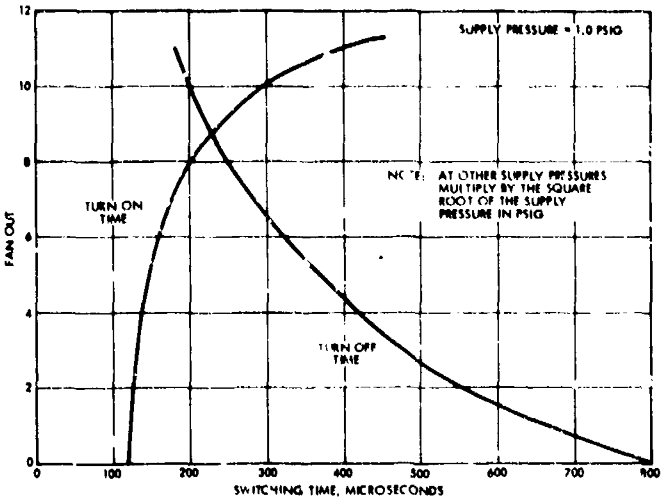


Figure 16.4.5.4c. Impact Modulator NOR Switching Time
(Reference 131-42)

In spite of the excellent performance characteristics exhibited by this device, the relatively complex three-dimensional configuration will limit its application in aerospace systems. Injection molding is the only logical manufacturing technique at present which limits the fabrication of elements for high-temperature operation. Current size is about 3/4 inch in diameter by 1-1/4 inches long, and the design is not adaptable to integrated circuit blocks. Miniaturization or fabrication in a two-dimensional configuration does not appear practical or functionally feasible.

16.4.5.5 FOCUSED JET AMPLIFIER. Operation of the focused jet amplifier is based on the tendency of an inwardly directed annular jet to adhere to the upper surface of a flow separator by wall attachment and to form a focused jet which is collected at the output tube (Figure 16.4.5.5a). When an annular control signal is applied, it prevents attachment of the power jet, and the flow is directed away from the output tube so that the output is greatly reduced.

The operating pressure range of this device is 3 to 14 psig. However, a large volume flow is required because of the large aspect ratio. Switching speeds range from 0.3 to 0.6 millisecond. Typical fan-in is 4 and fan-out is 6. The switching of the jet is not completely snap action, but the device is not suited for proportional operation.

The high flow requirements with no significant performance gains will limit the application of this device. In addition, the axisymmetric configuration (Figure 16.4.5.5b) is considerably more expensive to manufacture than a two-dimensional logical NOR element.

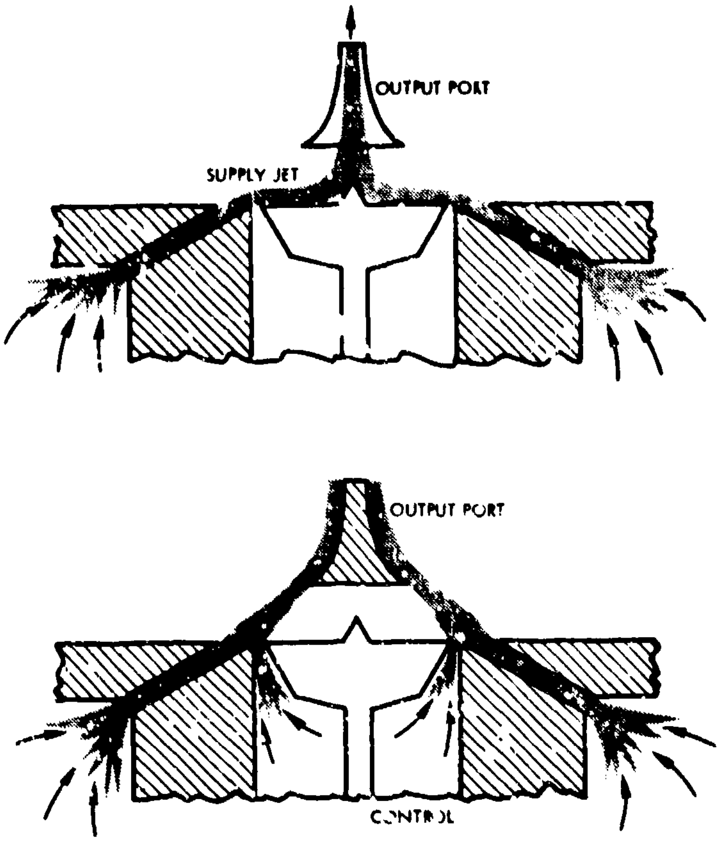


Figure 16.4.5.5a. Focused Jet Amplifier Configuration and Operation
(Adapted with permission from Reference 1-307, "Machine Design", August 18, 1966, D.L. Letham, Copyright 1966 by the Penton Publishing Company)

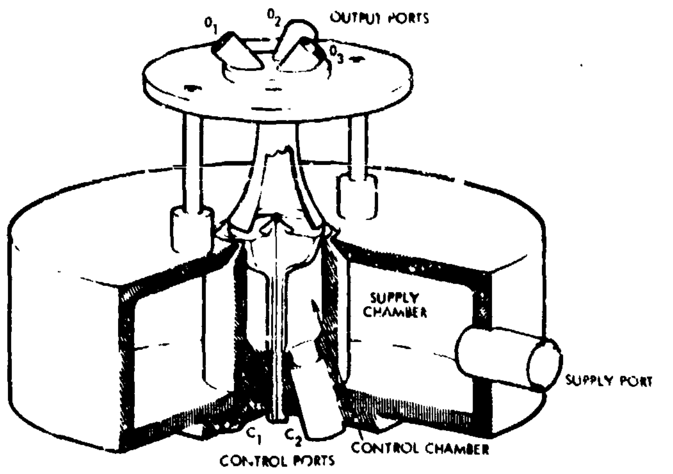


Figure 16.4.5.5b. Focused Jet Amplifier Typical Cross Section
(Adapted with permission from Reference 1-307, "Machine Design", August 18, 1966, D. L. Letham, Copyright 1966 by the Penton Publishing Company)

16.4.6 Special Devices

16.4.6.1 BOUNDARY LAYER AMPLIFIER. This two-dimensional device uses the principle of forced separation of a stream flowing over a curved surface. With no control flow (Figure 16.4.6.1a), the supply flow adheres to the adjacent curved surface until well downstream of the control duct so that the flow is directed to the vent. When control flow is injected into the boundary layer of the curved surface, the point of separation moves upstream and the supply flow is directed into the output duct.

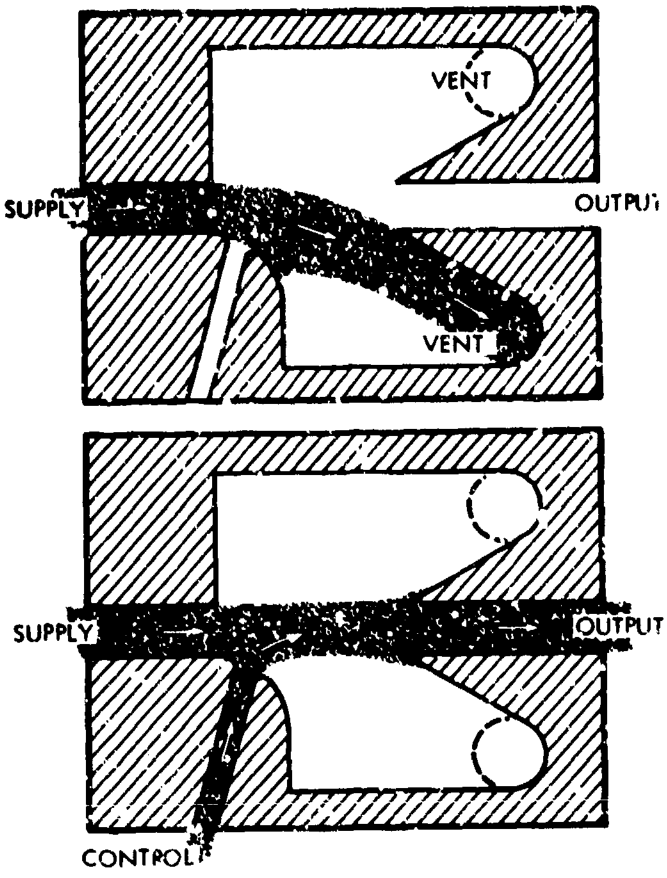


Figure 16.4.6.1a. Boundary-Layer Amplifier Operation
(Reference 131-40)

A prototype model of a boundary layer amplifier is shown in Figure 16.4.6.1b. The bias flow is required to force the power jet to unlock and return to the off position when control flow is removed. An "island" is necessary in this device to eliminate output hysteresis effects. The number and location of the control slots have a significant effect on the characteristics of the amplifier.

The boundary layer amplifier is used primarily when a high input impedance power amplifier is required. At relatively low pressures (about 5 psig), typical pressure gains are 2 to 3, flow gains 20 to 30, and power gains 60 to 80. This device is limited by its low pressure gain, complexity of construction, and relatively slow response time.

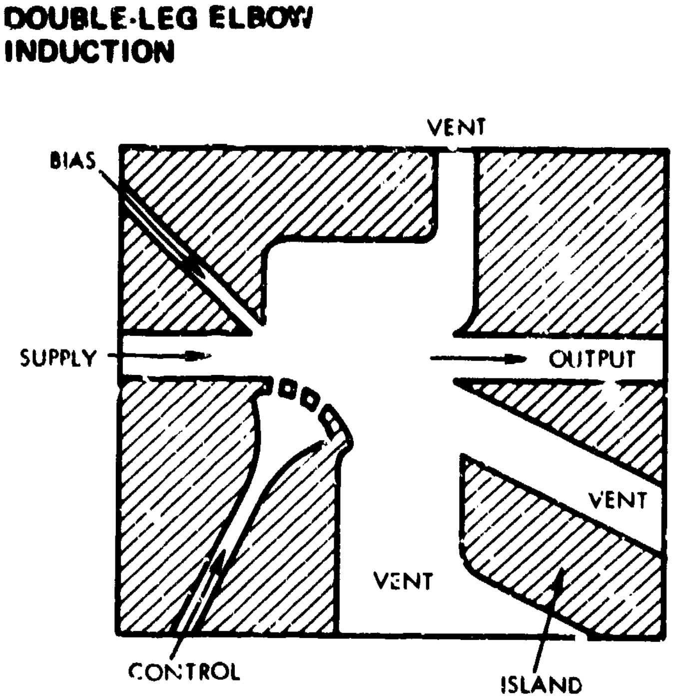


Figure 16.4.6.1b. Boundary Layer Amplifier Configuration
(Reference 131-40)

16.4.6.2 DOUBLE-LEG ELBOW AMPLIFIER. This device is essentially a more complex version of the boundary layer amplifier except that it has two output ducts (Figure 16.4.6.2). With no control flow, the momentum flux in the active leg is low near the outlet of the passive leg, hence the combined flow is directed into the left output duct. When a control flow is applied, the point at which the flow in the active leg separates from the channel wall moves upstream so that the momentum distribution across the flow changes and the combined flow is directed toward the right output duct. The action is proportional, since the proportion of the power stream which flows into either of the output ports depends upon the momentum distribution of the combined active and passive flows.

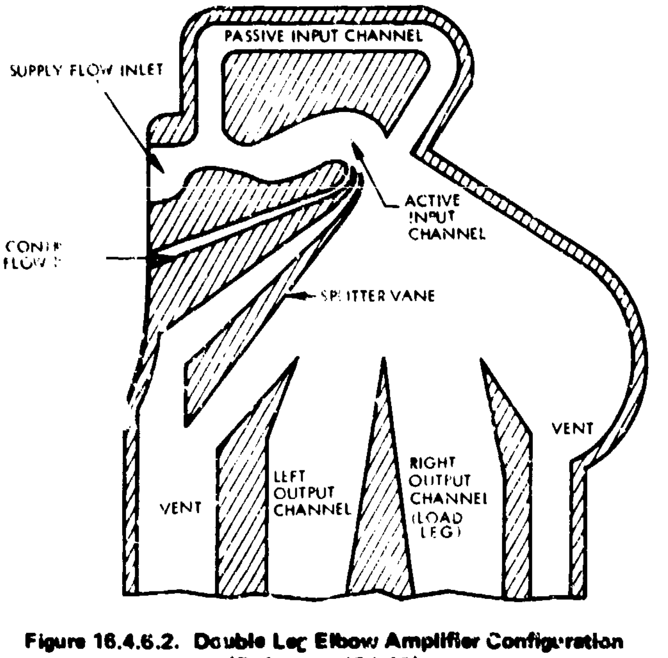


Figure 16.4.6.2. Double Leg Elbow Amplifier Configuration
(Reference 131-40)

The double-leg elbow amplifier provides very high flow amplification at low pressures and low operating frequencies. Maximum gains under static conditions are flow gain 300, pressure gain 3, and power gain 500. Typical flow gain is 200 with a corresponding power gain of 40. Performance of the device drops drastically as the operating frequency is increased, i.e., down 3 db at 10 cps and down 10 db at 40 cps. (NOTE: Performance of fluid amplifiers is usually measured in terms of the amplitude ratio (output pressure/input pressure) similar to that presented for evaluating servo system performance described in Sub-Topic 7.2.3 of this handbook. It is common practice, especially in the fields of electronics, automatic control, and acoustics, to express the logarithms of an amplitude ratio in units known as decibels (db). One decibel is equal to 20 times the logarithm to the base 10 of the output/input amplitude ratio. Reference 770-1 presents a particularly clear and comprehensive description of such terminology.)

16.4.6.3 INDUCTION AMPLIFIER. This device is essentially a back-to-back arrangement of two airfoils as shown in Figure 16.4.6.3. When the power jet is on, the flow will adhere (by assymetry or a bias pressure) to one of the airfoil shaped boundaries downstream of the power nozzle. With flow originally coming out of the left output duct, a control signal must be applied to the right control duct to switch the flow. The stream from the right control duct adheres to the outside wall of the right output duct. Since the control flow is tangential to the power flow in the interaction region, the transverse pressure gradient is reversed, which causes the power jet to switch and flow through the right output duct.

Except for the switching principle, the characteristics of this device are similar to those of the bistable wall attachment amplifier. No performance information is specifically available on this device, although it performs somewhat like a boundary layer amplifier. Typical pressure gains are 2 to 3, flow gains 20 to 30, and power gains 60 to 80.

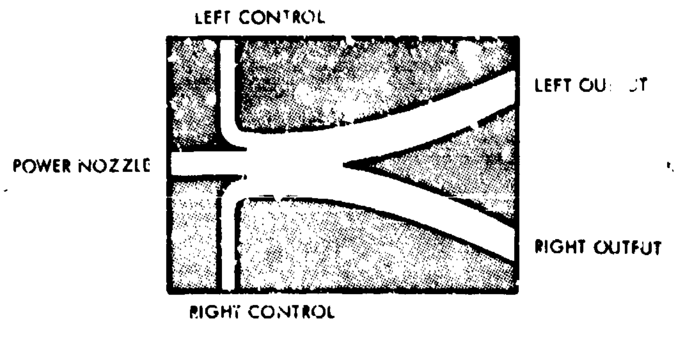


Figure 16.4.6.3. Induction Amplifier Configuration
(Reference 131-40)

16.4.6.4 EDGETONE AMPLIFIER. The edgetone amplifier is a high-speed planar flip-flop which uses a fluid dynamic phenomenon called the edgetone effect. To understand this effect, consider a fluid jet impinging on a wedge. Under the proper conditions, the jet will continuously oscillate back and forth across the wedge tip, alternately shedding vortices on each side. In the edgetone amplifier, as shown in Figure 16.4.6.4, the power jet stably oscillates between the wedge-shaped splitter and the cusp at the entrance to the output duct in use until a signal is applied to the control duct to switch the flow.

A relatively small signal is required to switch the output because the power jet is oscillating. The primary functional difference in the edgetone amplifier from a typical wall attachment bistable element is the 0.1 millisecond or less switching time for a typical device. Since it is not in widespread use, little information about this device is presently available.

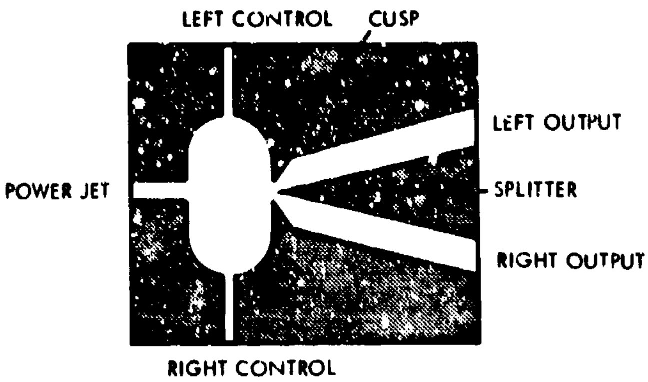


Figure 16.4.6.4. Edgetone Amplifier Configuration
(Reference 131-40)

16.4.6.5 IMPACT MODULATOR. The impact modulator is a proportional amplifier concept that uses two axially opposed power jets to provide a planar impact region. Its operation depends on varying the axial momentum of one of the supply jets to vary the position of the planar impact region. There are two versions of this device: the transverse and the direct impact modulator.

In the transverse impact modulator, Figure 16.4.6.5a, when a perpendicular control signal is applied, the momentum of the left supply jet is decreased and the impact region moves to the left. This results in decreased output flow and pressure, and since the output pressure decreases with increase in control pressure, the device has negative gain.

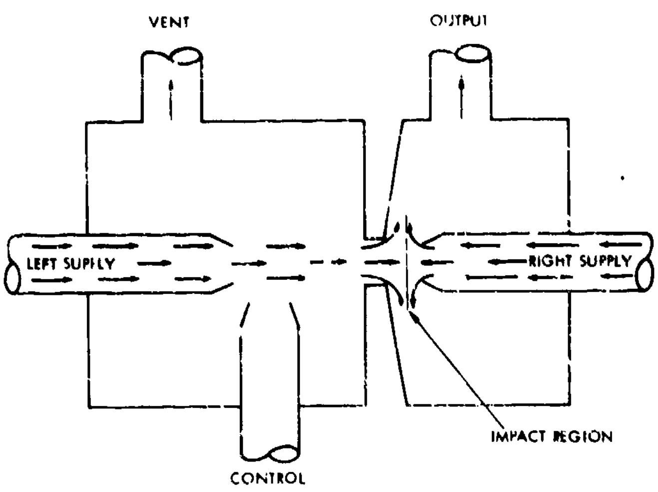


Figure 16.4.6.5a. Transverse Impact Modulator
(Reference 131-42)

In the direct impact modulator, Figure 16.4.6.5b, when a concentric control signal is applied, the momentum of the left supply jet is increased and the impact region moves to the right. This results in increased output flow and pressure, and since the output pressure increases with increased control pressure, the device has positive gain.

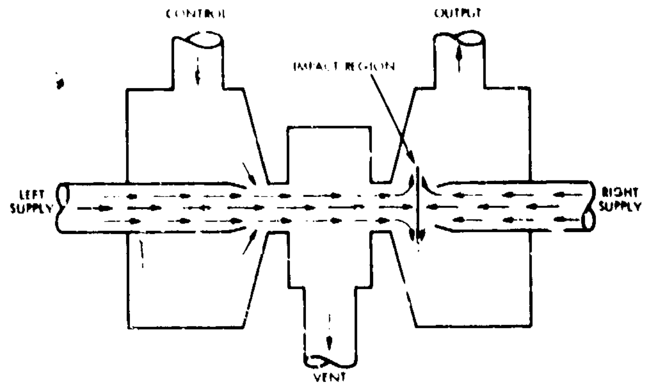


Figure 16.4.6.5b. Direct Impact Modulator

Typical performance of the transverse impact modulator is maximum flow gain of 5 to 30 and no-load pressure gain of 20 to 40. A characteristic curve is shown in Figure 16.4.6.5c. Optimized four-element cascades have given pressure gains of about 12,000, which reduces the average pressure gain per stage to about 10.5. This is necessary to ensure output linearity and proper interstage impedance matching.

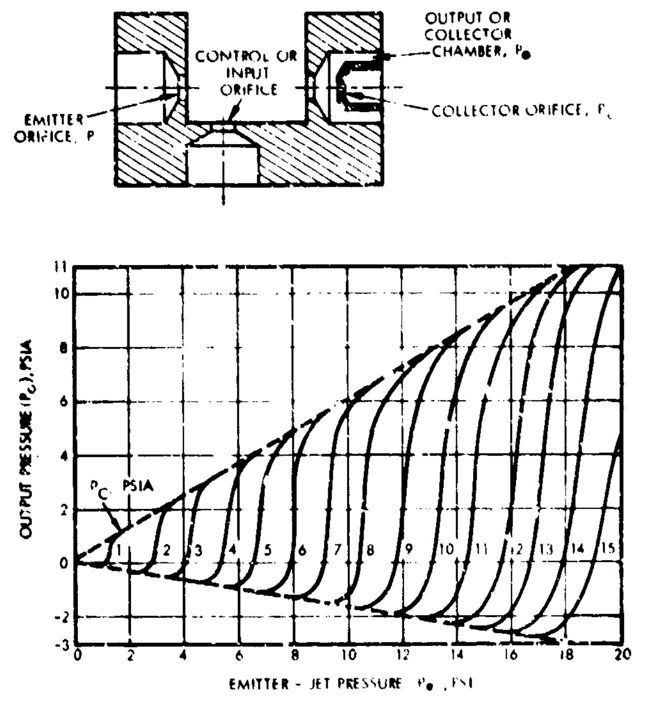


Figure 16.4.6.5c. Transverse Impact Modulator Performance
(Courtesy of Johnson Service Company, Milwaukee, Wisconsin)

The direct impact modulator is a significant improvement over the transverse impact modulator. Pressure gains up to 200 have been reported for this device, and the input impedance is variable and can be adjusted to approach infinity. Unloaded frequency response is reportedly quite high (300 to 400 cps); however, this has not been related to a particular element size and response will decrease markedly when the element is loaded. Signal-to-noise ratio information is not generally available for either device but ranges from 60 to 300.

The difficulty of obtaining reproducible characteristics from one device to another is one of the major obstacles to the development of impact modulators. This is primarily due to the three-dimensional concentric nozzle configurations which are expensive to manufacture, except by injection molding. Some effort is being expended in developing a two-dimensional version of the direct impact modulator, although if successful, lower pressure gains are expected. Impact modulators are attractive for proportional control application, however additional development is necessary before the true value of these devices can be assessed.

16.4.7 Oscillators

Fluidic oscillators depend on feedback for operation just like any other type. These devices have been utilized in timer circuits, temperature sensors (see Sub-Topic 16.6.2), pressure references, and analog-to-digital converters.

16.4.7.1 WALL ATTACHMENT OSCILLATORS. An oscillator can be constructed utilizing the wall attachment principle and output feedback loops as shown in Figure 16.4.7.1a. When the supply flow is turned on initially, the supply jet will attach to either the left or right wall and flow out the respective output tube as in a normal wall attachment device. Presuming the power jet is initially attached to the right wall, part of the power stream is returned to the right control via the external feedback loop so that the power jet is switched to the left wall when the right control pressure reaches the correct switching pressure. The process repeats itself on each side so that the stream oscillates at a frequency which depends on the sum of the transit time of the fluidic signal through the feedback path and the power jet switching time.

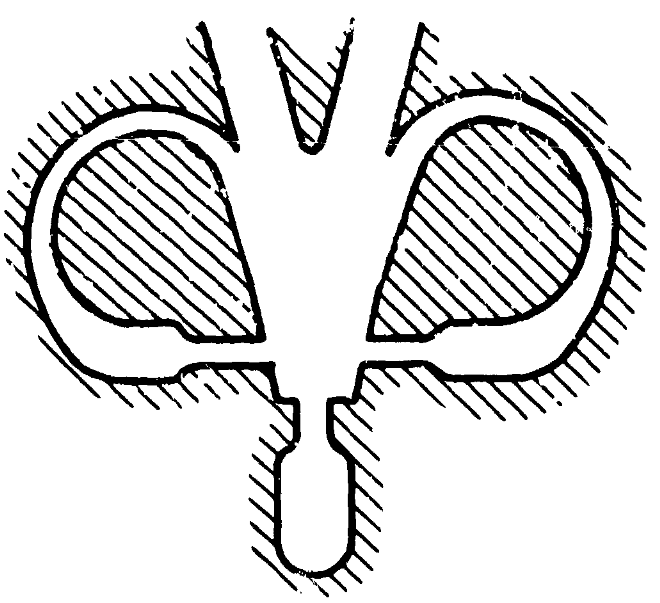


Figure 16.4.7.1a. External Feedback Oscillator

Another type of wall attachment oscillator is the coupled control oscillator which utilizes a feedback loop joining the two control ports (Figure 16.4.7.1b). Assuming that the power jet is about to attach to the right wall, a rarefaction wave, due to suddenly increased entrainment at the right control port, travels around the control passage and is reflected at the left control port. The reflected wave, a compression, then travels back to the right control port causing the jet to switch to the left wall and the process is repeated.

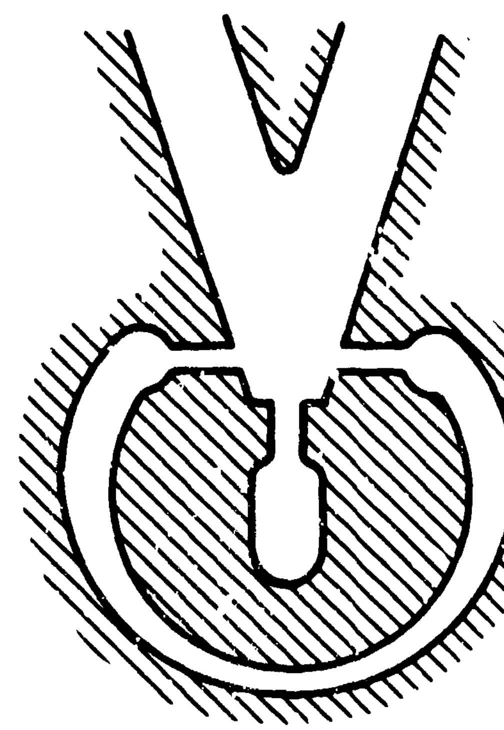


Figure 16.4.7.1b. Coupled Control Oscillator

16.4.7.2 RELAXATION OSCILLATOR. This oscillator was developed by the Harry Diamond Laboratories for use in pneumatic timers and logic circuits that must operate under severe environmental conditions (Reference 241-8). By installing a lumped R-C-R (resistance-capacitance-resistance) network in the feedback loop of the basic wall attachment oscillator (Figure 16.4.7.1a), this device can be made relatively insensitive to temperature and pressure.

The relaxation oscillator (Figure 16.4.7.2) has demonstrated less than ±2 percent frequency change over a pressure range from 6 to 30 psig. A frequency variation of less than 1 percent was also obtained over a temperature range from 77 to 175°F at constant pressure. Careful design is necessary if pressure and temperature insensitivity are required together.

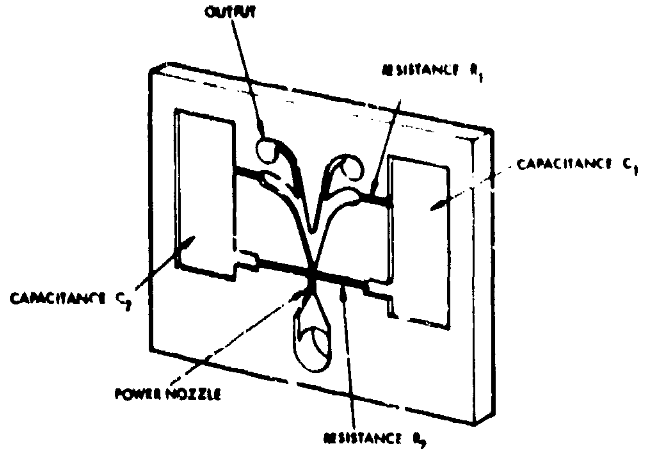


Figure 16.4.7.2. Relaxation Oscillator Configuration

16.4.7.3 PRESSURE CONTROLLED OSCILLATOR. The pressure controlled oscillator (PCO) is a special form of the external-feedback oscillator shown in Figure 16.4.7.1a, with an output frequency that varies as an approximately linear function of the supply pressure. This is accomplished by varying the R-L-C (resistance-inductance-capacitance) components in the feedback loop to effect the necessary phase shift. A PCO can use either wall attachment (Figure 16.4.7.1a) or stream interaction to achieve the gain necessary for oscillation.

One type of stream interaction PCO operates with a gain of 30 cps/inch of water pressure as shown in Figure 16.4.7.3. This particular oscillator only has a useful range of about 80 cps, but an important advantage is that it can operate at very low pressures with excellent resolution (800 cps/psi). A PCO is used for analog-to-digital conversion in fluidic frequency-modulated systems and as a pressure reference.

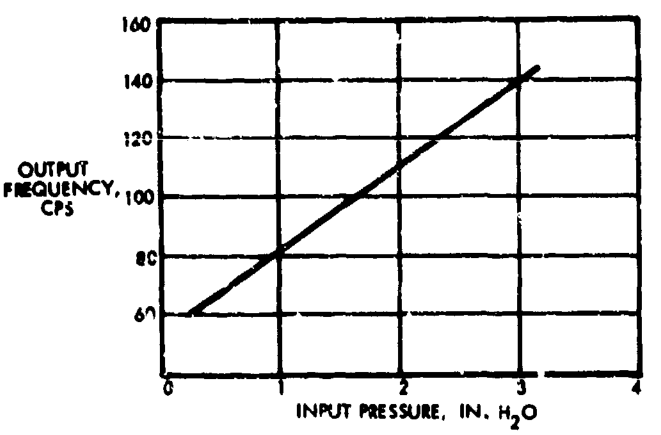


Figure 16.4.7.3. Pressure Controlled Oscillator

16.4.7.4 TURBULENCE AMPLIFIER OSCILLATOR. An oscillator which uses a turbulence amplifier and depends on external circuitry for its operation is shown in Figure 16.4.7.4. A portion of the flow which enters the output tube is diverted into the return path, as shown in (a) of Figure 16.4.7.4. When this flow reaches the end of the return path, it impinges on the main power jet (see (b) of Figure 16.4.7.4), causing turbulence and a resulting decrease in the pressure in the output tube. Consequently, flow along the return path decreases or stops, the power stream regains laminarity, and the cycle repeats.

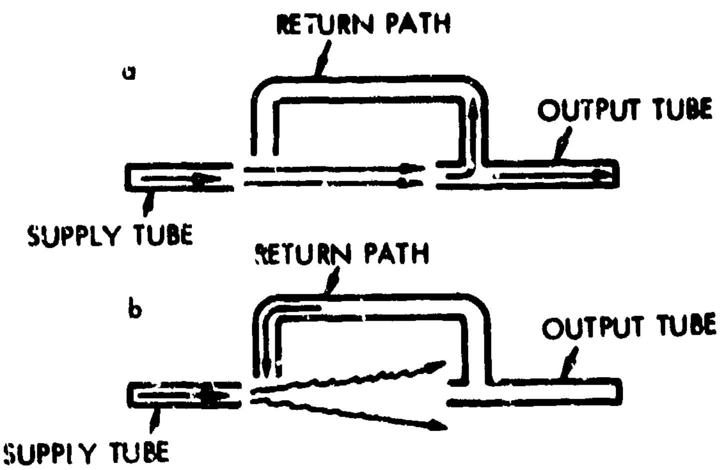


Figure 16.4.7.4. Turbulence Amplifier Oscillator
(Adapted with Permission from Reference 1-413, "Machine Design", November 10, 1965, D. L. Letham, Copyright 1965 by the Penton Publishing Company)

16.4.7.5 TUNING FORK FLUIDIC OSCILLATOR. A precision oscillator has been developed recently (Reference 6-252) which consists of a temperature-compensated tuning fork, a load-sensitive fluidic flip-flop, control transmission lines, and a feedback transmission line. This device has a frequency accuracy of ±0.002 percent at room temperature and ±0.05 percent over a temperature range of -35 to +200°F when operated at 400 cps. Although hybrid in nature, this tuning fork oscillator offers the possibility of such an extremely accurate frequency reference that it should not be overlooked.

Operation of the oscillator is illustrated in Figure 16.4.7.5. The supply stream emerges from an aperture in one tine (control) of a tuning fork and is alternately switched to the two downstream channels as the control tine oscillates. The two downstream channels are used as the control inputs to a load-insensitive flip-flop which oscillates accordingly. The fluid pulse train emerging from one output channel is fed back and impinged on the other tine of the tuning fork which maintains oscillation of the fork at its natural frequency. The other flip-flop output channel is used as the output signal. Since the device uses the air pulse only to apply sufficient energy to the tuning fork to sustain oscillation, the oscillator is considerably less sensitive to variations in the speed of sound due to temperature changes which compromise the accuracy of a typical fluidic sonic oscillator.

The tuning fork is high-Q device (i.e., it loses a minimum of energy due to damping and mounting effects) and it can be driven with relatively low power inputs. Temperature compensation of the tuning fork can be accomplished by special alloys, heat treatment, or bimetal construction. The trend is toward small high-frequency forks, since higher frequencies result in smaller amplitudes and better accuracy and also minimize the effects of acceleration and vibration.

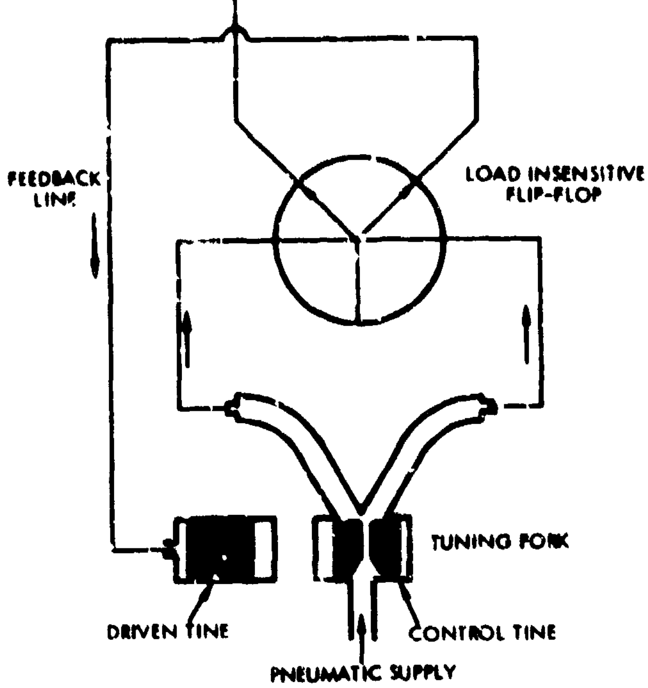


Figure 16.4.7.8. Fluidic Tuning Fork Oscillator
(Reference 131-40)

16.4.8 Moving Part Devices

Fluidic devices are generally associated with control and logic functions operating with low power level signals, whereas ordinary moving-part valves are thought of in terms of controlling higher power level flows. Many kinds of moving mechanical part devices are used as elements for control, such as spools, poppets, flappers, nozzles, diaphragms, balls, force-balance levers, bellows, pivoting jets, tape, and helical springs. Although they are industrially oriented, some of these devices are briefly considered here because they help accomplish some functions even at low power levels. They may find application in aerospace systems where it is simply impossible or uneconomical to operate fluidic elements because of limited space, weight, and energy.

Many moving part devices are also being used in fluidic systems to boost a low pressure signal (a fraction of a psi) to a useful working pressure, i.e., sufficient to operate a valve actuator. These can be either gas to gas, gas to hydraulic fluid, or even gas to liquid propellant interfaces.

16.4.8.1 MOVING PART LOGIC

Shuttle Valve. The simplest mechanical control valve is the shuttle (Figure 16.4.8.1a). It transmits the higher of two inputs (A or B) and blocks the other. Except where the two inputs balance exactly, the logic mode is OR, i.e., either one input or the other produces an output.

Ball Valve. A control pressure of the same magnitude as the supply will shut off the supply because the upper ball has the larger area (Figure 16.4.8.1b). This device performs the logic NOT function, i.e., there is an output only when there is not an input.

Tilting Spring. If there are no input signals to the diaphragm, then the tightly coiled spring acts as a closed valve, and output pressure is built up (Figure 16.4.8.1c). The logic form is NOR, i.e., if neither A nor B nor C are pressurized, there will be an output.

Disc Valve. A simple disc can perform the same function as the shuttle valve described above (Figure 16.4.8.1a). Both the shuttle valve and the disc valve can be used to perform a bistable or flip-flop function as shown in Figure 16.4.8.1d.

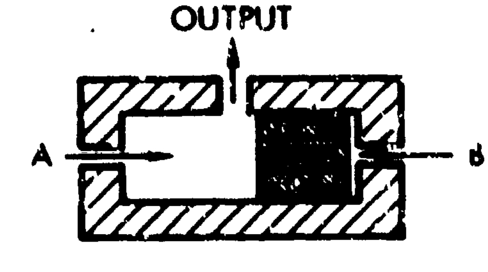


Figure 16.4.8.1a. Shuttle Valve Logic OR Function

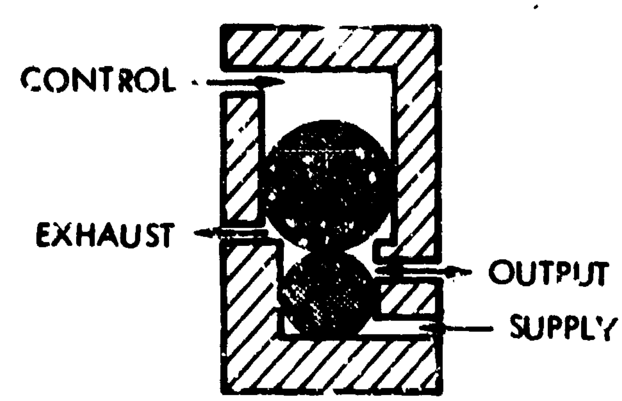


Figure 16.4.8.1b. Ball Valve Logic NOT Function

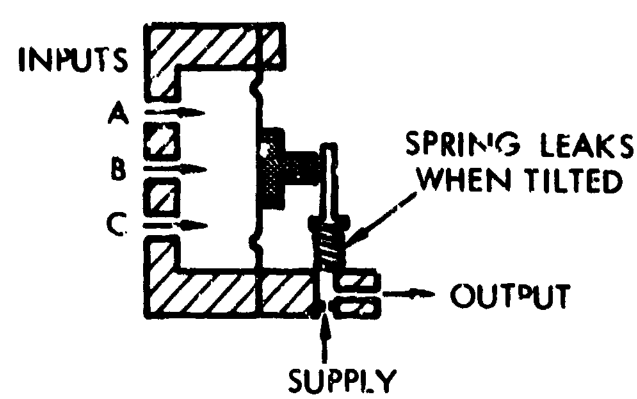


Figure 16.4.8.1c. Tilting Spring Logic NOR Function
(Courtesy of Techne, Ltd., Dunford, Cambridge, England)

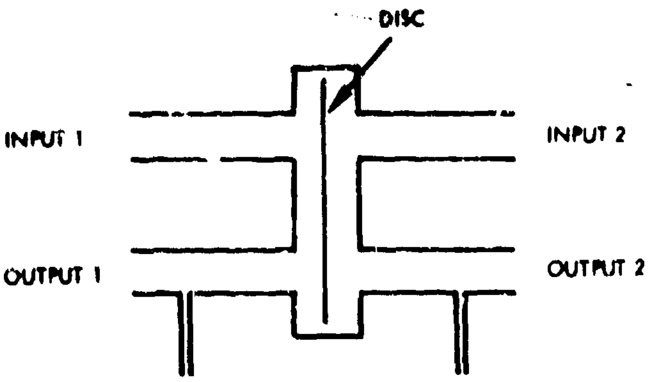


Figure 16.4.8.1d. Disc Valve Flip-Flop Function

Liquid Bead. This device is based on the surface tension of a glycerine ball in an hour-glass shaped cavity (Figure 16.4.8.1e). When a control (input) signal is applied to the side of the cavity with the bead, the bead will squeeze through the neck into the opposite side. Thus the device acts as a flip-flop with nondestructive memory. The classification of this device as a moving part is arbitrary and is justified on the basis that it involves a fixed quantity of high density liquid as the active element.

Vacuum-Actuated Diaphragm. When the control port is at atmospheric pressure (zero input) the diaphragm is forced against the output port (Figure 16.4.8.1f). The output port is then drawn down to a low vacuum through the orifice. When a control signal is applied, the diaphragm lifts away from the output port and the output builds up to atmospheric pressure. Since an output (vacuum) occurs when the control signal is off, this device performs a logic NOR function.

Diaphragm NOT Module. This logic element functions by producing an output only when there is no input at A (Figure 16.4.8.1g).

Diaphragm AND Module. An output is produced by this element only if inputs A and B are pressurized. Input A moves the diaphragm down which also pushes the actuator down to seal off the exhaust and connect the output to input B (Figure 16.4.8.1h).

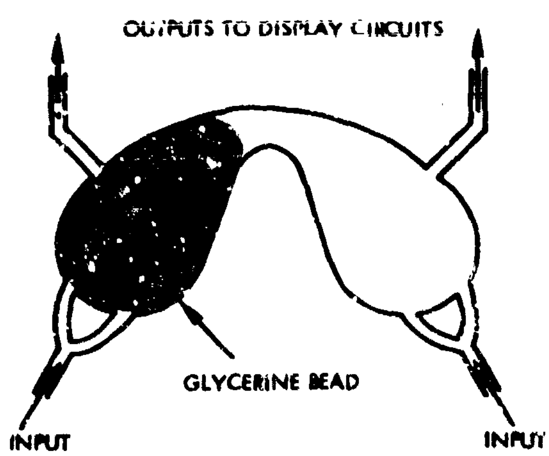


Figure 16.4.8.1e. Liquid Bead Flip-Flop
(Courtesy of Conrac Corporation, Malvern, Pennsylvania)

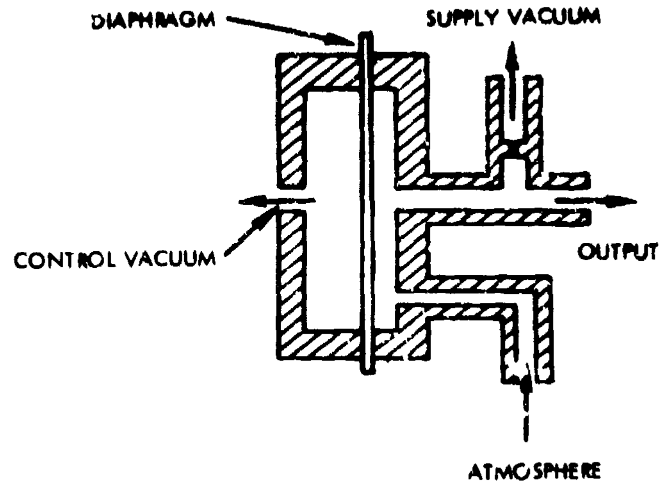


Figure 16.4.8.1f. Vacuum Logic Element Logic NOR Function
(Courtesy of Robertshaw Controls Company, Goshen, Indiana)

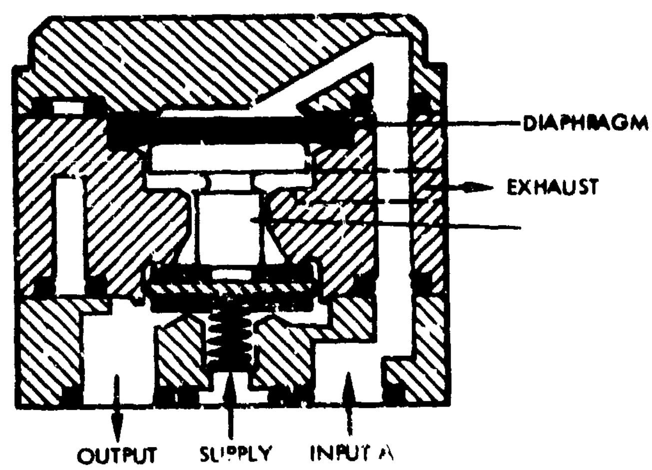


Figure 16.4.8.1g. NOT Module
(Courtesy of the Aro Corporation, Bryan, Ohio)

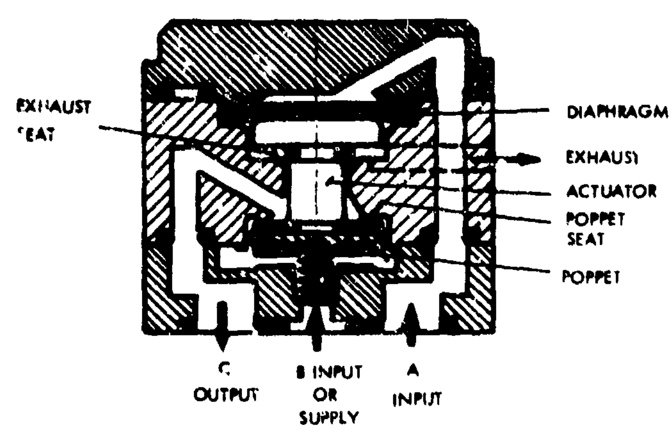


Figure 16.4.8.1h. AND Module
(Courtesy of the Aro Corporation, Bryan, Ohio)

16.4.8.2 POWER INTERFACES

Diaphragm Valve. This device has three diaphragms but no sliding parts (Figure 16.4.8.2a). The fluidic control signal actuates the upper (sensing) diaphragm and seals off the chamber above the middle diaphragm. The pressure then builds up and forces both the middle and lower (power) diaphragms downward, opening the valve. It operates on air signal pressures as low as 1.5 inches of water and controls supplies up to 100 psig.

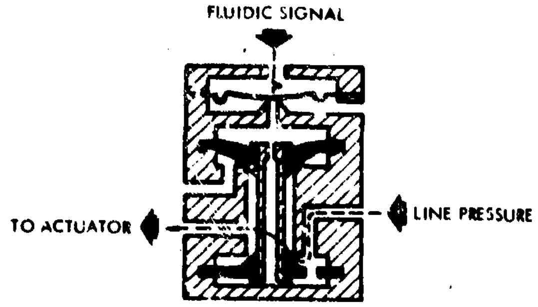


Figure 16.4.8.2a. Diaphragm Valve
(Courtesy of Northeast Fluidics Incorporated, Bethany, Connecticut)

Diaphragm-Piloted Spool. This type of valve can be controlled by pressures as low as 1 to 4 inches of water (Figure 16.4.8.2b). It can operate both as an air-to-air or an air-controlled hydraulic valve at supply pressures up to 100 psig.

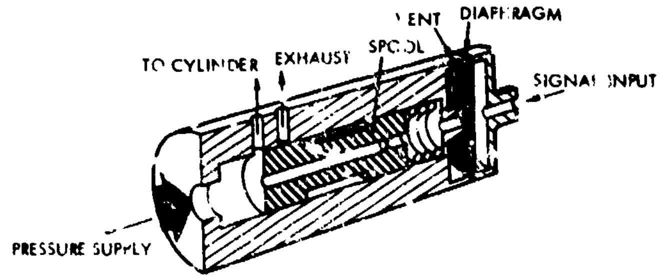


Figure 16.4.8.2b. Diaphragm Piloted Spool

Fluidic Input Servovalve. This servovalve was developed by Hydraulic Research Company to operate on a differential input pressure in the range of ±1 to ±5 psi. The input stage is comprised of two small pressure capsules which convert the fluidic input signal into a force which acts upon the flapper of the hydraulic amplifier. Figure 16.4.8.2c illustrates the operating principles of the valve in schematic form. The input arm, flapper, and feedback spring are an integrated unit. This unit is suspended by transverse webs so that it rotates through a very small angle about a pivot axis normal to the view shown. An O-ring is employed at the pivot point of the flapper to provide fluid isolation between return system oil in the nozzle area and atmosphere at the input capsules. An input signal pressure unbalance therefore produces a lateral force on the input arm which moves the flapper closer to one nozzle and farther away from the other. An unbalance is thus created in the spool end chamber pressures, causing the spool to be displaced from its neutral or null position. As the spool moves, the feedback spring is deflected in a direction which opposes the torque induced by the fluid input signal. This action restores the flapper to its neutral position between the nozzles and causes the end chamber pressures to again be approximately in balance. At this point spool movement ceases and the spool holds this position until the input signal changes. Closed loop positional control of the spool is thereby achieved where the spool displacement is directly proportional to the input differential pressure signal.

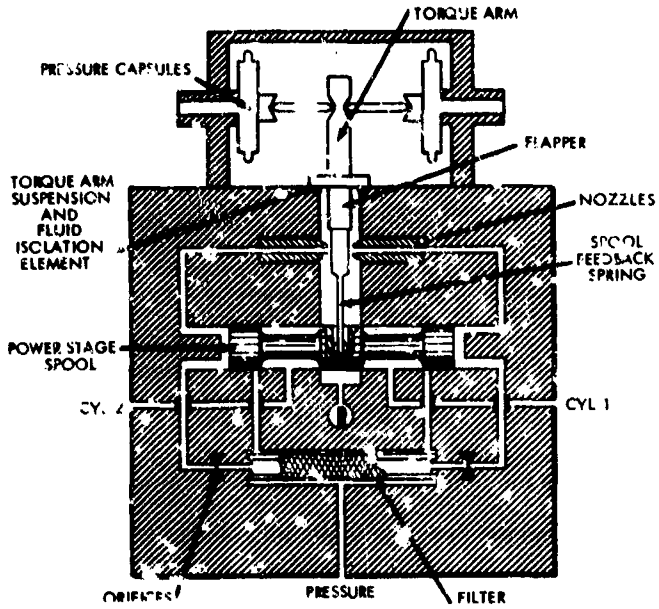


Figure 16.4.8.2c. Fluidic Input Servovalve Schematic
(Courtesy of Hydraulic Research and Manufacturing Company, Burbank, California)

16.5 FLUID INTERFACES

The ideal fluidic application is one where a single fluid medium can be used for all control functions and where the need to translate information from one medium to another is eliminated. However, the bulk of present fluidic applications are of the hybrid variety in which interface elements are required. Interface elements encompass a broad range of transducers (electrical, mechanical, etc.) required in applications where system inputs and outputs are nonfluid. Little original work has been done to develop new devices in this area, and most interface elements are adaptations of commercially-available hardware for hydraulic and pneumatic control.

16.5.1 Electrical-to-Fluidic Transducers

There is a wide variety of electrical-to-fluidic (E-F) transducers in general use. In an E-F transducer an electrical signal produces a mechanical movement of an element into the active area of a fluidic device. For example, an E-F transducer for on/off or digital operation can be a solenoid valve and for proportional control a torque motor driven flapper valve. Several variations of this type of transducer in conjunction with fluidic elements are shown in Figures 16.5.1a through 16.5.1c. The torque motor driven E-F transducer has a bandwidth of about 300 cps and the bandwidth for the piezoelectric ceramic disc E-F transducer is between 1000 and 2000 cps.

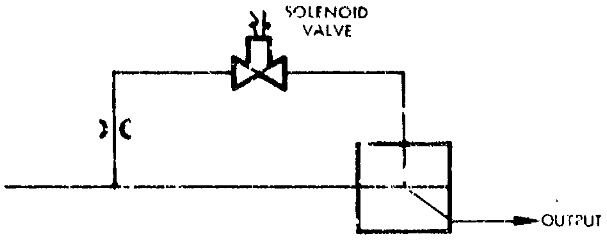


Figure 16.5.1a. Solenoid Valve E-F Transducer

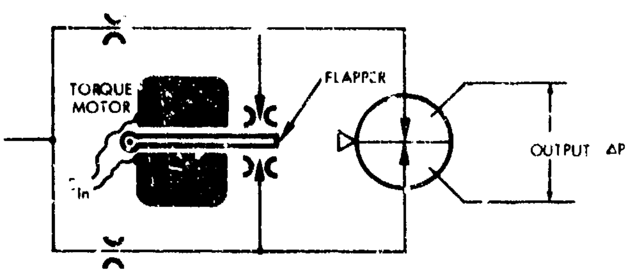


Figure 16.5.1b. Torque Motor Driven E-F Transducer

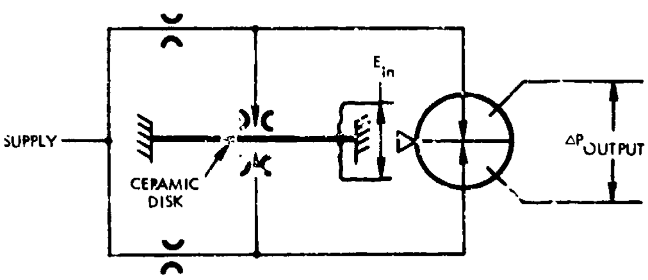


Figure 16.5.1c. Piezoelectric Bimorph Disc E-F Transducer

Acoustic power (in the audic range) from an E-F transducer has been used to switch digital fluidic devices. The effect of the acoustic power is due to several factors. Sound increases the turbulence of a supply jet, causing its velocity profile to change. This change, coupled with the second order effects of acoustic streaming and radiation pressure, causes the jet of a wall attachment amplifier to switch to the opposite output if the acoustic wave is applied directly into the separation bubble. A turbulence amplifier can also be switched to the NOR condition by means of sound waves.

Several practical E-F transducers have been made which utilize the secondary effects of acoustic power, i.e., acoustic streaming and radiation pressure (References 73-264, 73-273 and 131-42). The device shown in Figure 16.5.1d utilizes an electrically induced magnetic field to position or oscillate a diaphragm which varies the differential pressure across a proportional amplifier. A piezoelectric ceramic disc can also be used in place of the electromagnetic driver and diaphragm. Each of these devices is capable of producing a relatively low pressure pneumatic signal in the range from steady state to about 2000 cps.

Current investigators are experimenting with E-F transducers in which an electrical signal is converted directly into a fluid signal (References 95-29, 131-40 and 131-42). Heat has also been used to control the separation point in a boundary layer amplifier and to switch the flow in a diffuser. Some of these concepts are illustrated in Figure 16.5.1e. Considerably more electric power than equivalent pneumatic power is required to operate these devices.

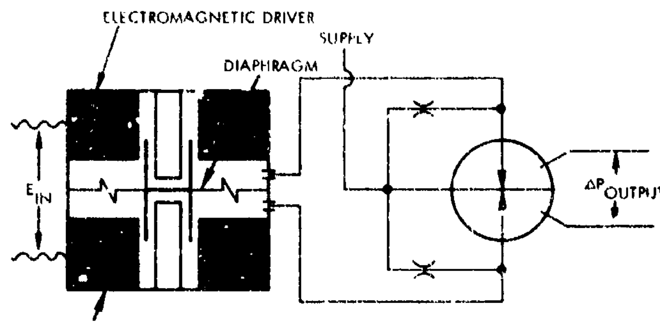


Figure 16.5.1d. E-F Transducer-Diaphragm Oscillator Type

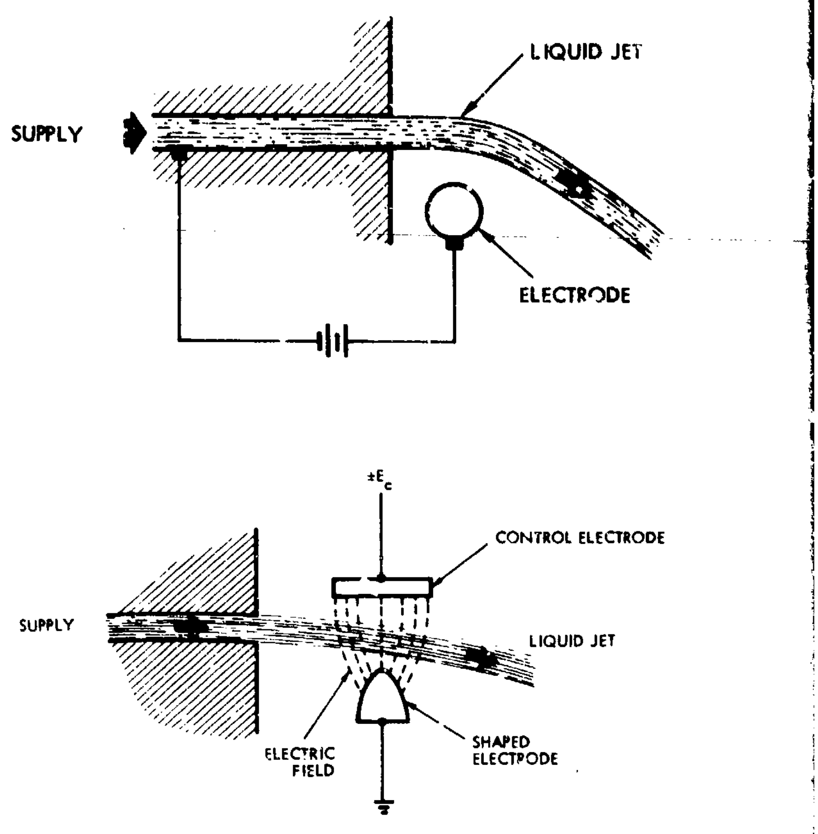


Figure 16.5.1e. E-F Transducer Experimental Phenomena
(Reference 131-42)

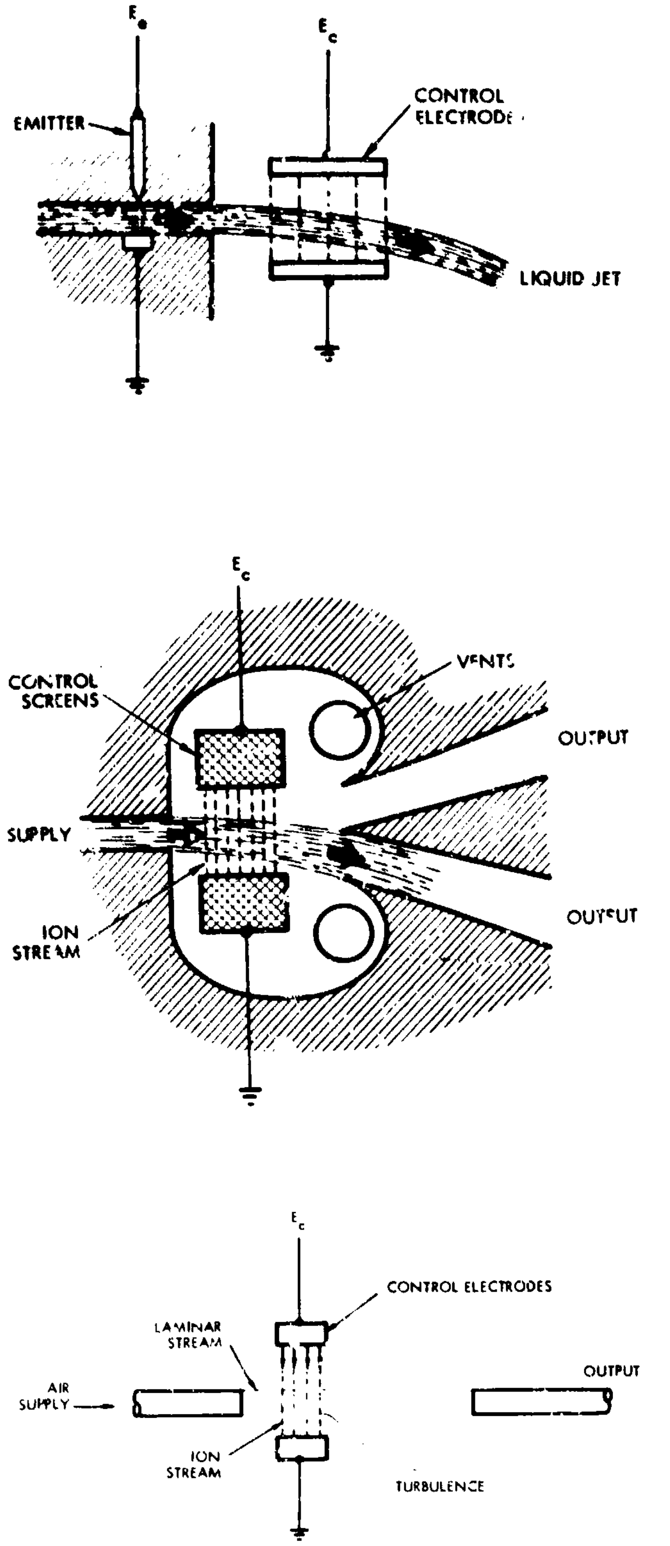


Figure 16.5.1e(c). E-F Transducer Experimental Phenomena
(Reference 131-42)

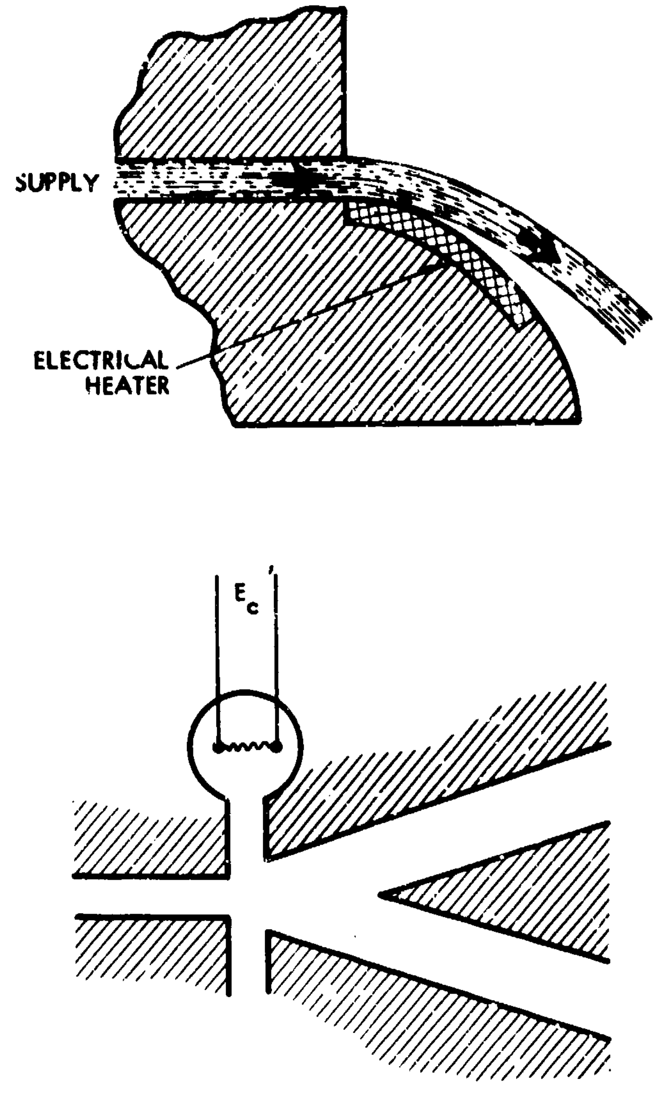


Figure 16.5.1e(d). E-F Transducer Experimental Phenomena
(Reference 131-42)

16.5.2 Fluidic-to-Electrical Transducers

Many fluidic-to-electrical (F-E) transducers are possible, but the most widely used are simple pressure switches, pressure transducers, and hot-wire probes. Because of the additional transducer volume involved, most pressure switches and many pressure transducers are limited to application in systems with a bandwidth of less than 100 cps. Flush mounted piezoelectric pressure transducers and the newer semiconductor strain gage elements, which have been made in extremely small sizes (0.10-inch sensing diameter) are capable of operating in components with bandwidths in excess of 20,000 cps. Thermistor or hot-wire probes have also been installed in the control and output channels of fluidic devices to indicate the presence or absence of flow.

One type of differential, cooled filament, F-E transducer (Figure 16.5.2a) consists of two heater filaments or hot-films which are installed in the output ducts of a proportional amplifier and connected in a bridge circuit. The bridge output voltage is then proportional to the differential cooling of the two sensors. Another type of differential F-E transducer utilizes a small semiconductor or wire strain element mounted between the output legs of a proportional amplifier (Figure 16.5.2b). A transducer of this type, with a close-coupled strain element, will provide a sensitive and accurate output signal directly proportional to the amplifier differential output pressure. Both of these devices are capable of bandwidths of better than 20,000 cps, depending on how closely they can be coupled to the fluidic element.

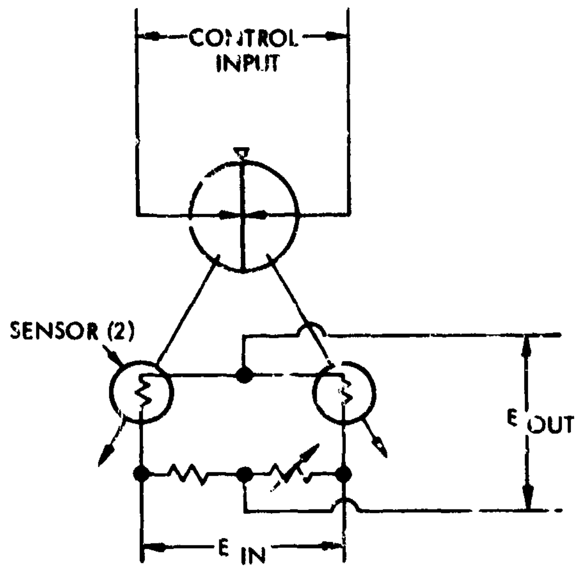


Figure 16.5.2a. Fluid to Electrical Transducer With Hot Film Sensors

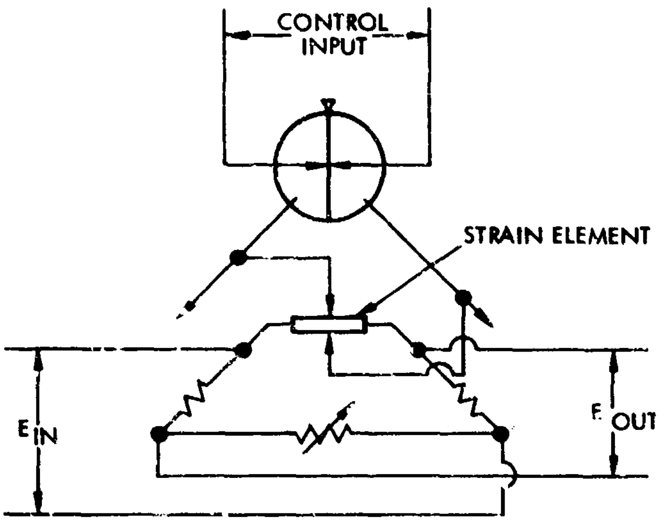


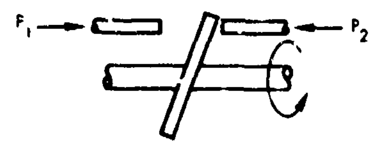
Figure 16.5.2b. Fluid to Electrical Transducer With Strain Element Sensor

16.5.3 Mechanical-to-Fluidic Transducers

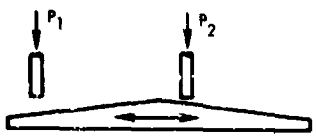
Mechanical-to-fluidic (M-F) transducers are normally used to detect linear and angular displacement. One of the simplest M-F transducers is a pressure divider, where the exit is a variable orifice controlled by the operation of a flapper. The flapper is either a translating member or a rotating cam attached to the mechanical device. Another commercially available M-F transducer is the interruptable jet. This is essentially a turbulence amplifier in which the turbulence-inducing element is an object which intrudes into the jet stream. The jet can be interrupted at any point in the length of the stream. The interruptable jet can sense a mechanical intrusion into the laminar stream with a repeatability of better than 0.0001 inch. In spite of such sensitivity, the force exerted on the intruding element by the interruptable jet is negligible. For digital circuitry, the concept of the traditional player-piano roll would permit the use of complex programmed inputs to digital circuitry. One version of this concept uses standard punched cards as the input signal or programming device.

Many fluidic systems require a differential pressure signal at the interface between the transducer and the system input. The M-F transducers illustrated in Figure 16.5.3 are conceived to perform this function. In devices (a), (b), and (c) the output nozzles (P_1 and P_2) are each supplied from a constant pressure source through a choked orifice. As the displacing member moves closer to one nozzle and further away from the other, the resulting changes in back pressure are reflected in the differential pressure signal P_1 minus P_2. The transducer in (d) functions in a similar manner, except that the change in orifice area is accomplished within the device itself.

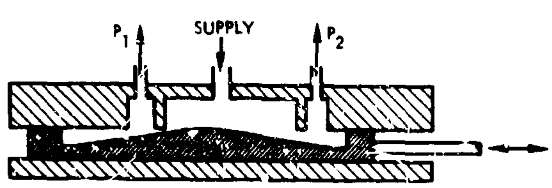
(a) ANGULAR DISPLACEMENT WITH CAM



(b) ANGULAR DISPLACEMENT WITH WOBBLE PLATE



(c) LINEAR DISPLACEMENT WITH UP-DOWN RAMP



(d) LINEAR DISPLACEMENT WITH VARIABLE ORIFICES

Figure 16.5.3. Mechanical-to-Fluidic Transducer Concepts With Differential Output Pressure

16.5.4 Fluidic-to-Mechanical Transducers

Output pressures of many fluidic devices are relatively low; however, these pressures can be amplified fluidically or can be used directly to drive a variety of devices. A typical application would be the control of a valve with a diaphragm, piston, or a geared gas turbine actuator. The differential output of a fluidic device may also be used to position a spool valve in a power circuit. There devices are generally adaptations of existing pneumomechanical devices, some of which are discussed in Detailed Topic 16.4.8.2.

16.6 FLUIDIC SENSORS

Fundamental to all control is the sensing of system variables. The output of a sensor is a function of a system variable such as temperature, position, angular rate, or acceleration. Whether a device is called a sensor or an interface element is often a matter of opinion. For example, many of the mechanical-to-fluidic transducers discussed in Sub-Topic 16.5.3 could be called sensors because they "sense" the physical position of an object and provide an output which is a function of the sensed position. Available information on fluidic sensors is rather scarce because many of the devices are either classified or proprietary. The fluidic devices discussed in this section are representative of the sensors which have been reported in current literature and those which are novel in terms of fluidic principles.

16.6.1 High-Impedance Pressure Sensor

Many fluidic circuits require the detection of various pressure levels. These pressure signals are transmitted into the circuits for processing and are eventually utilized externally. In some situations the fluid producing the control input data may be toxic, corrosive, dirty, or hot, so that it may not be desirable to have the fluid enter the fluidic circuits. This is especially true where continued exposure to internal contamination could render a system inoperative or where human exposure to a toxic exhaust gas could be harmful. One high-impedance fluidic pressure sensor provides a means by which pressure levels can be detected without flowing the sensed media into the sensor (Reference 68-100).

A two-dimensional configuration of the high-impedance pressure sensor is shown in Figure 16.6.1a. It is essentially a bistable wall-attachment amplifier with a bypass channel from the supply to one control port. This control port is designated as the control input and the opposite control port is then designated as the bias input. In operation, the supply fluid is bypassed to the control input channel where it impinges on the far wall causing the stream to split (Figure 16.6.1b). A relatively small portion of the stream is entrained by the power jet in the interaction region, and the remaining portion is discharged through the control channel. The bias input is adjusted to cause the power jet to initially attach to the opposite or right wall. When the control input is restricted by either a physical blockage or a control signal of the proper magnitude, the power jet will switch to the left output port (Figure 16.6.1c). A variable-bias resistor is used to adjust the sensitivity of the sensor and, consequently, the control pressure level at which the supply stream switches (Figure 16.6.1d).

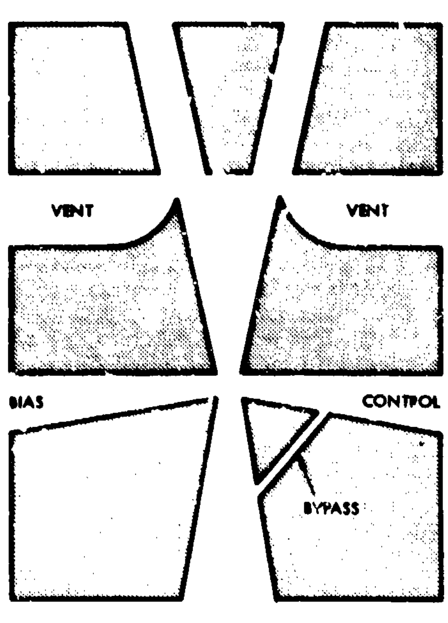


Figure 16.6.1a. High Impedance Pressure Sensor
(Reference 131-42)

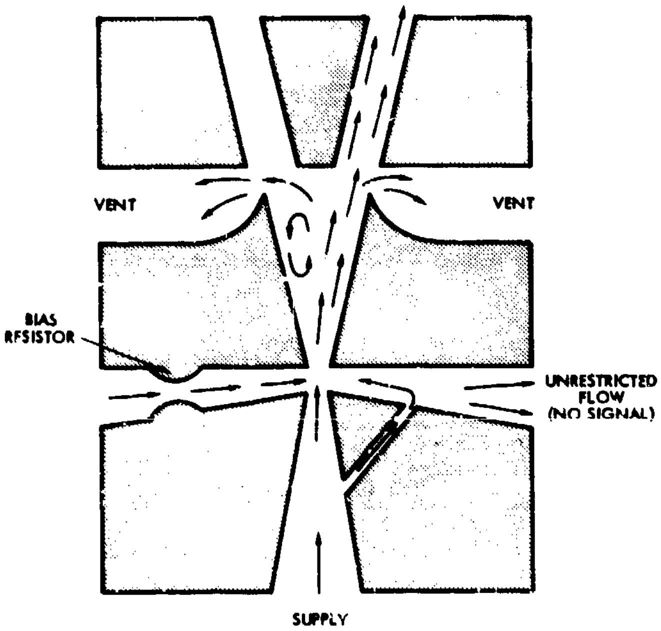


Figure 16.6.1b. High-Impedance Sensor With No Control Signal
(Reference 131-42)

The pressure sensor can be modified for use at high altitudes or in outer space as shown in Figure 16.6.1e. This is accomplished by interconnecting the vents and the bias input and discharging the flow through a common orifice. The sensor will then function independently of atmospheric back pressure when the vent pressure is high enough to choke the vent orifice.

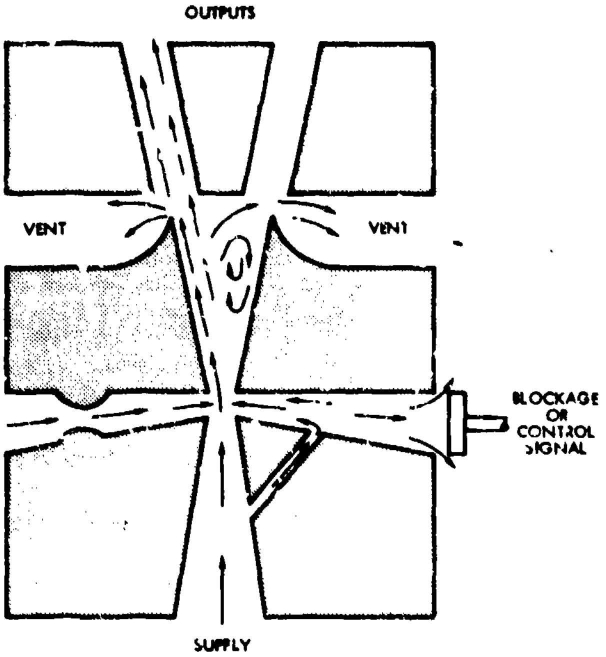


Figure 16.6.1c. High-Impedance Sensor With Control Signal (Reference 131-42)

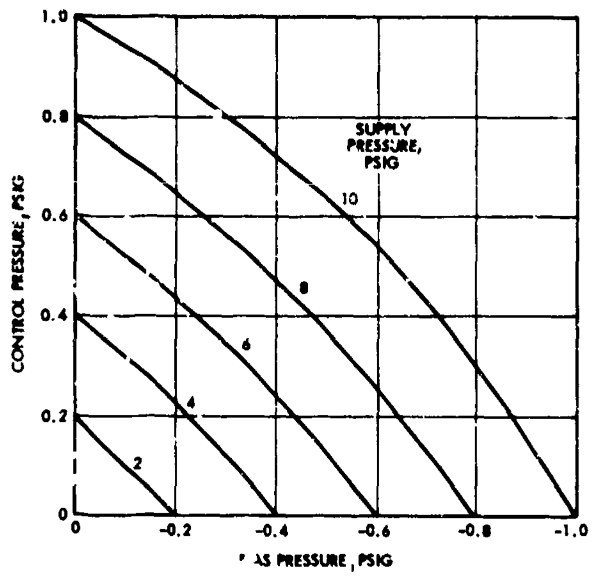


Figure 16.6.1d. Pressure Sensor Control or Switching Pressure (Reference 131-42)

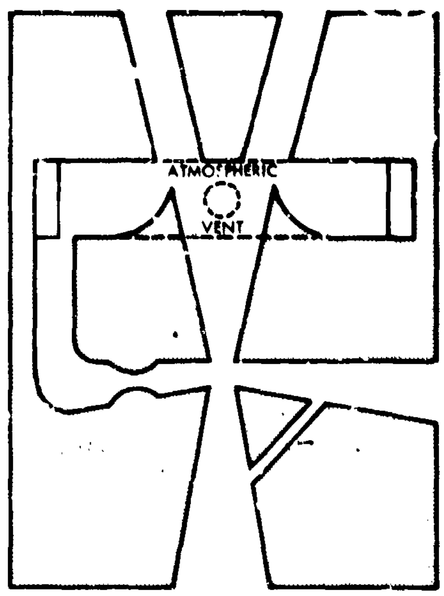


Figure 16.6.1e. Pressure Sensor for High Altitude Application (Reference 131-42)

16.6.2 Temperature Sensors

There is currently an intense interest in the development of fluidic sensing devices for measuring gas temperature (References 19-248, 47-38, and 674-4). Although many of these vary in configuration and design, their basic operation depends on the fact that acoustic velocity is a function of gas temperature. Sizes range from a probe-size (miniature) sensor which can be fitted inside turbine stator blades to larger units which are used to measure temperature inside the fuel cells of a nuclear reactor. Theoretically, the sensors will operate in virtually any environment as long as the minimum flow velocity necessary for operation is maintained. The temperature range for a given device is determined by the liquification temperature of the working gas at low temperatures and the melting point of the sensor material at elevated temperatures.

Frequency of a fluidic oscillator (Sub-Topic 16.4.7 and Figure 16.6.2a) depends on the length of the external feedback path and the length of time required for an acoustic pulse to travel the length of this path. This time depends on the acoustic pulse velocity of the speed of sound in the supply fluid, which in turn varies with the fluid temperature. If the switching time for the active fluidic element in the oscillator is assumed to be zero, the oscillator frequency is:

$$f = \frac{u_c}{2\ell} \qquad \text{(Eq 16.6.2a)}$$

where

f = frequency, cps

u_c = acoustic velocity of the gas, m/s

ℓ = length of conduit, m

Since the speed of sound in a compressible fluid is equal to $\sqrt{kRT}$,

$$f = \frac{\sqrt{kRT}}{2\ell} \text{ or } f = K_1 \sqrt{T} \qquad \text{(Eq 16.6.2b)}$$

where

k = specific heat ratio, dimensionless

R = gas constant, N-m/Kg-°K

T = static temperature, °K

and K_1 is an arbitrary constant depending on the oscillator configuration and the working fluid. Thus, the oscillator frequency is a function of the square root of the absolute temperature, which is the basis of the fluidic temperature sensor.

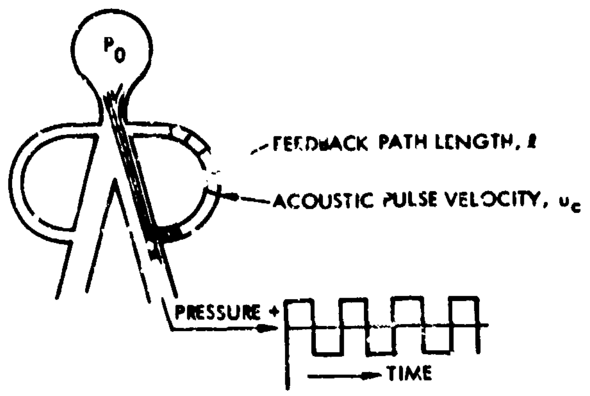


Figure 16.6.[illegible] [illegible] Temperature Sensor

Oscillator temperature sensors are built by Honeywell in sizes ranging from 1/4 x 3/8 x 0.09-inch thick to 2 x 2 x 1/2-inch thick, which operate from about 17 kilocycles down to 2 kilocycles, respectively. A calibration curve for a typical 2 x 2-inch sensor is shown [in] Figure 16.6.2b. Inlet fluid pressures must be sufficient [to st]art and sustain sensor oscillation, normally 3 or 4 psig. Ultimate sensor accuracy is ±0.2 percent which is achieved after the sensor exit nozzle is choked.

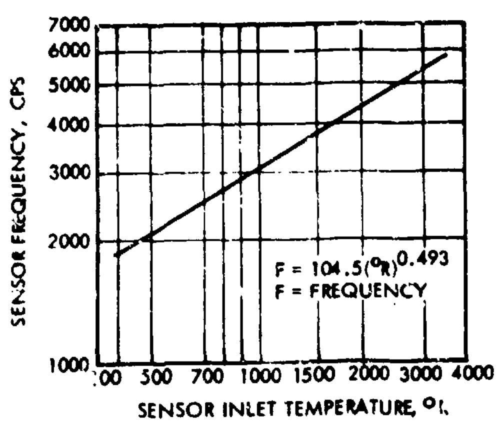


Figure 16.6.2b. Temperature Sensor Calibration Curve
(Courtesy of Honeywell, Incorporated, Minneapolis, Minnesota)

The sensor has a response time of less than 1 second, which is influenced by the time required to purge the sensor of operating fluid and by the time required for the heat transfer to reach steady state conditions. Signal-to-noise ratio varies from 5 to 20 depending on the inlet pressure.

16.6.3 Vortex Rate Sensor

Vortex rate sensors exemplify the high amplification available in a vortex flow field (Reference 57-21). A typical sensor is shown in Figure 16.6.3a. Supply fluid flows through the inertial coupling element, through the vortex chamber, and out the vent. The coupling element is usually porous material, but uniformly spaced vanes have also been used.

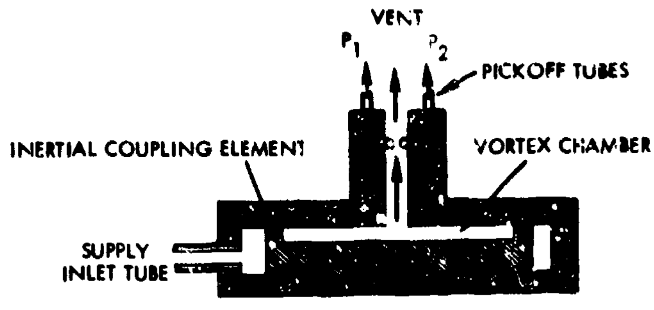


Figure 16.6.3a. Vortex Rate Sensor
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

When the angular rate is zero, the supply fluid passes through the coupling ring and flows radially toward the vent. With the application of an angular rate, a tangential velocity is imparted to the supply fluid by the coupling element which is amplified in the vortex chamber due to the conservation of angular momentum. This increased (amplified) velocity is detected with an aerodynamic pickoff located in the drain. The pickoff, usually in the form of a very small probe, measures the induced vorticity in terms of the angle of attack that the high-velocity flow makes with the probe. Figure 16.6.3b illustrates the function of one of the simpler pick-off configurations. The pressure differential generated across the probes is directly proportional to the applied angular rate.

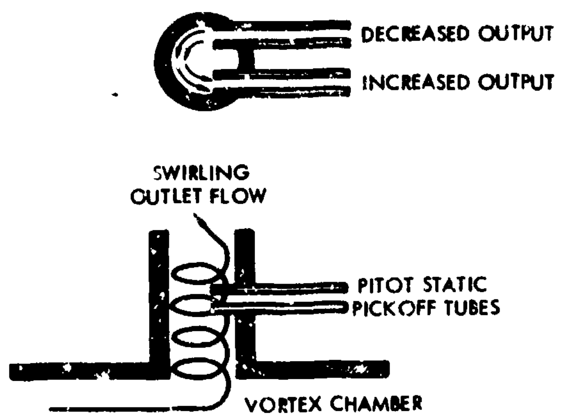


Figure 16.6.3b. Aerodynamic Rate Sensor Pickoff
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

The performance of vortex rate sensors is usually discussed in terms of sensitivity, threshold, size, and response.

a) *Sensitivity* is defined as the output pressure generated per degree per second and is primarily a function of the diameter of the sensor and pickoff design.

b) *Threshold* refers to the minimum rate that can be observed above the accustic noise produced as the flow enters and passes over the pickoff.

c) *Response* is primarily a function of transport time (the time it takes a particle of fluid to pass from the coupler to the pickoff) and is determined by the diameter and flow rate through a given device.

d) *Saturation* refers to the maximum rate that can be measured within the linear range of the device.

The performance characteristics for a typical device are summarized in Table 16.6.3.

Table 16.6.3. Vortex Rate Sensor Characteristics

Diameter:	4.0 inches
Supply pressure:	20.0 psig
Flow rate:	150 cc/sec
Gas:	air
Sensitivity:	0.02 psi/deg/sec
Response time:	20.0 milliseconds
Noise amplitude:	0.002 psi

One of the many varied applications anticipated for the vortex rate sensor is the utilization of the device in the fluidic missile control system. Most applications are in the experimental phase and range from missile attitude control to light aircraft controls.

16.7 SYSTEM APPLICATION

The most appealing advantages offered by fluidics to aerospace systems are: no moving parts, environmental insensitivity, simplicity, and ruggedness — all of which make for high reliability expectations. Other considerations are potential weight and volume savings and, to a lesser degree, reduced system fabrication cost. These advantages are qualified by whether the comparison is with conventional fluid power controls or with electronics. Perhaps the most important points are:

1) Any pressurized fluid (such as stored gas, combustion products, and liquid propellant) can be used as a power source for a fluidic system. This is a distinct advantage if it can eliminate the need for electric power when electric power is not readily available.

2) In systems where parameters such as pressure, flow, temperature, and angular rate are sensed and used as control signals, fluidics does not require the conversion of these signals into mechanical motions as would be required in conventional controls.

A Fluidic Component Rating Analysis Chart from Reference 131-42 is presented in Table 16.7. It is intended as a general reference for the systems engineer in defining the state of the art of fluidic devices, interfaces, and sensors relative to propellant compatibility, functional parameters, and specific environments. Reliability ratings assigned to the various combinations of fluidic components, propellants, and parameters in the chart have the following definitions:

RATING	*DEFINITION*
1	Poor—a serious problem exists for which there is no satisfactory solution.
2	Fair—a problem exists, but a remedy may be available.
3	Satisfactory—i.e., within the state of the art.
U	Information upon which to make a judgment is unavailable.

The purposes of this section are to consider the problems and limitations of fluidic system applications, to define important application criteria, and to present some typical system applications.

16.7.1 Problems and Limitations

16.7.1.1 OPERATIONAL PROBLEMS. Fluid filtration, power source performance, and element interchangeability are the areas in which operational problems are most frequently encountered. Conventional 10 to 40 micron (nominal) filters have been found adequate for most applications; however, in atmospherically vented circuits, care must be taken to avoid the aspiration of contaminants from the environment. The use of liquid propellants in fluid circuits necessitates other considerations, such as propellant compatibility, rheopectic or thixotropic behavior (see Sub-Topic 3.3.4 and Reference 131-41), and contamination of the environment. Many available power sources do not deliver a constant-pressure supply, and component selections and the circuit design must compensate for this. Only conventional mechanical pressure regulators are available at the present time, but good fluidic pressure regulators are expected in the future. Monopropellant gas generation systems and closed cycle power supplies hold the answers for future aerospace application.

Unless absolutely necessary, miniature devices with small nozzle dimensions must be avoided to ensure low sensitivity to variations in operating conditions, fabrication, and contamination. Many new fluidic element designs are less sensitive to geometry variations, and manufacturing techniques are also being improved constantly. Instrumentation is presently inadequate; consequently, it is difficult to verify system performance and, more important, to pinpoint malfunctions. Concentration on satisfying the need for special-purpose instrumentation should help; however, in the long run, the most promising solution is self-contained miniaturized instrumentation.

16.7.1.2 ANALYTICAL TECHNIQUES. Earlier development and a large percentage of the current development of fluidic elements and systems have been done on an empirical basis, although current macroscopic mathematical models have provided useful results. This reflects the difficulty of mathematically analyzing device steady-state operation and the even more formidable problems encountered in representing dynamic phenomena. Major efforts are underway in industry, government agencies, and universities (notably MIT) to formulate and to commit to practice the tools and techniques required to facilitate the analytical design of fluidic components and systems.

16.7.1.3 FLUIDIC DEVICE PROBLEMS. The formulation of an analytical model is complicated by the fact that fluid flow phenomena are very sensitive to several interrelated variables. For example, wall attachment amplifier performance is influenced by many factors which include Reynolds number, the ratio of control jet velocity to power jet velocity, several aspects of element geometry, size, surface roughness, upstream perturbations, and downstream loading. The complete determination of a suitable model will require a better understanding of turbulent jets and interaction flows. Only very crude models resulting from the use of simplifying assumptions are available in most cases. Marked improvements in fluidic technology should result when it becomes possible to readily solve the partial differential equations for turbulent fluid flow. Jet and solid surface interactions, the solution of pressure and velocity transients, and the stability of a free jet in the presence of acoustic disturbances are examples of critical problems which need to be solved in order to optimize device design.

16.7.1.4 SYSTEM DESIGN. With all fluidic elements, it is necessary to cope with similar considerations in order to achieve successful interconnection into circuits and systems. These considerations include: the effects of nonlinearities and dynamics in active and passive circuit elements and in connecting passageways; loading due to temporary and permanent instrumentation; noise generation, propagation, and amplification; temperature and pressure sensitivity; and impedance matching at critical places. When these considerations are ignored, instabilities may occur, signals may prematurely saturate, energy may be wasted, and excessive stages of amplification (with the accompanying complications of noise amplification) may be needed. Accordingly, small changes in element and line geometries, mean operating pressures, or mean flow rates in lines and passages can cause significant changes in performance.

Obtaining optimum system performance requires a compromise between gain, stability, and response time. Each fluidic element must be carefully matched to its load to obtain maximum signal power transfer and to provide sufficient signal power to drive successive stages. Besides providing maximum power transfer, the matching of line and port impedance minimizes the reflection of waves at the junctions of lines and ports and reduces the likelihood of premature saturation. In general, fluidic device static pressure flow curves are very useful in achieving proper matching. Approximately linear operation is usually achieved for small swings about a chosen operating point. The dynamic response characteristics are such that for a small device, static performance can be assumed up to about 400 cps.

Power supply regulation and reliability are necessary prerequisites to proper system performance. Fluid supply lines should be large enough to avoid excessive losses. If the flow area of supply lines and connectors is too small, undesirable pressure drops can occur so that supply pressures at individual element power nozzles will be less than specified. The most serious consequence of small flow areas is the greatly increased loss in each bend and fitting due to eddies and turbulences which lead to greater losses in straight sections. Pressure losses are treated in Sub-Section 3.9 of this handbook.

In the design of analog systems:

1) Problems exist in matching component characteristics because of the inherent load sensitivity of analog devices and because of the variation of component characteristics with operating point.

2) Noise is a major problem, particularly in high gain circuits where staging is required and the noise is generated in the first or second stage.

3) Most systems are nearly proportional, i.e., the fluidic elements operate at a frequency and output level continuously related to the input signal, so that there are no discontinuities and only very slight amounts of higher harmonics in the output signal.

4) Carrier techniques including both AM and FM can be applied to minimize problems of noise, drift, and bias in critical applications.

Regarding digital system design:

1) Early advances were achieved by means of cut-and-try developmental work, leading to empirical results. Theoretical work has not yielded many results that are directly useful in design.

2) Digital elements are normally less sensitive to noise and load conditions than corresponding analog elements.

3) Interstage matching of element characteristics is not as critical for analog systems.

4) Vented digital amplifiers are the most widely used in digital circuits. However, a vented circuit may not be the best choice where maximum power transfer is desired (such as in most aerospace applications). See Sub-Topic 16.4.5 relating to problems with turbulence amplifiers, as an extreme case.

16.7.2 System Application Criteria

16.7.2.1 PERFORMANCE WITH VARIOUS FLUIDS

Gases. The gases commonly available in aerospace systems include ram air, pressurants, propellant boiloff, and combustion products. Any of these gases may be used as a working fluid for fluidic devices. Particulate contamination, such as metallic particles, can cause erosion in fluidic elements, particularly when in hydrogen and helium, because of the high sonic velocities. Ice crystals formed from impurities such as water vapor and carbon dioxide in the gases can clog orifices and filters. Normal care to ensure that systems are clean and dry combined with the use of adequate filters (usually 10 to 20 micron nominal rating) should obviate most problems.

FLUIDIC SYSTEM APPLICATION

Table 16.7. Fluidic Component Rating

RATING CHARACTERISTICS:

- 1 - POOR
- 2 - FAIR
- 3 - GOOD
- U - UNAVAILABLE INFORMATION

APPLICATION DESIGNATIONS:

- DC - DIGITAL CONTROL
- PC - PROPORTIONAL CONTROL
- PM - PRESSURE MODULATION
- FM - FLOW MODULATION

NOTES:

- (a) LIQUIFIED PETROLEUM GAS - BUTENE, ETHYLENE, METHANE, PROPANE
- (b) FLOX IS ALSO CONSIDERED A SPACE STORABLE

PERFORMANCE WITH PROPELLANTS

APPLICATION			GASES: HYDROGEN	HELIUM	NITROGEN	OXYGEN	ARGON	CO_2	PROPELLANT BOILOFF	COMBUSTION PRODUCTS TO 5000°F	EARTH STORABLES: MON 10	NITROGEN TETROXIDE	AEROZINE 50	HYDRAZINE	MONOMETHYLHYDRAZINE	PENTABORANE	UDMH
DC	DIGITAL DEVICES	WALL ATTACHMENT - VENTED	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3
DC, FM	DIGITAL DEVICES	WALL ATTACHMENT - CLOSED	3	3	3	3	3	3	3	3	2	3	3	2	2	2	2
DC	DIGITAL DEVICES	TURBULENCE	3	3	3	3	3	3	2	2	2	2	2	2	2	3	2
DC, FM	DIGITAL DEVICES	INDUCTION	3	3	3	3	3	3	2	2	2	3	2	2	2	3	2
DC	DIGITAL DEVICES	EDGETONE	3	3	3	3	3	3	2	2	1	1	1	1	1	2	1
DC	DIGITAL DEVICES	FOCUSED JET	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2
PC, FM, PM	PROPORTIONAL DEVICES	STREAM INTERACTION	3	3	3	3	3	3	3	3	2	2	3	2	2	2	2
ALL	PROPORTIONAL DEVICES	VORTEX AMPLIFIER	3	3	3	3	3	3	3	3	[illegible]	2	3	3	3	2	3
ALL	PROPORTIONAL DEVICES	VORTEX VALVE	3	3	3	3	3	3	3	3	2	2	3	3	3	[illegible]	3
PC, PM, FM	PROPORTIONAL DEVICES	VENTED JET	3	3	3	3	3	3	3	3	2	2	2	2	2	3	2
DC, FM	PROPORTIONAL DEVICES	FLOW DIVERTER	3	3	3	3	3	3	2	2	2	3	3	2	2	2	2
PC, PM, FM	PROPORTIONAL DEVICES	BOUNDARY LAYER	3	3	3	2	2	2	2	2	1	1	1	1	1	2	1
PC, PM, DC	PROPORTIONAL DEVICES	IMPACT MODULATOR	3	3	3	3	3	3	2	2	1	1	1	1	1	2	1
PC, PM	PROPORTIONAL DEVICES	THROAT INJECTION	3	3	3	3	[illegible]	2	2	3	2	3	3	3	3	2	3
PC, PM, DC	PROPORTIONAL DEVICES																
DC, PC	PASSIVE ELEMENTS	ORIFICE	3	3	3	3	3	3	3	3	[illegible]	3	3	3	3	3	3
DC, PC	PASSIVE ELEMENTS	LAMINAR RESISTOR	3	3	3	3	2	2	2	[illegible]	2	3	3	3	3	3	3
PC	PASSIVE ELEMENTS	INDUCTOR	2	2	3	2	2	[illegible]	2	2	2	3	3	[illegible]	3	3	3
PC, DC	PASSIVE ELEMENTS	CAPACITOR	3	3	3	3	3	3	2	2	2	[illegible]	3	2	2	3	2
PC, DC	PASSIVE ELEMENTS	DIODE	3	3	3	3	3	3	3	2	2	2	3	3	3	2	3
DC, PC	INTERFACE DEVICES	ELECTRICAL TO FLUID	3	3	3	2	2	2	2	U	2	2	2	2	2	2	2
DC, PC	INTERFACE DEVICES	FLUID TO ELECTRICAL	2	2	2	2	2	2	1	U	U	U	U	U	U	U	U
DC, PC	INTERFACE DEVICES	MECHANICAL TO FLUID	3	3	3	2	2	2	U	U	2	2	3	3	2	2	2
DC, PC	INTERFACE DEVICES	FLUID TO MECHANICAL	3	3	3	2	2	2	U	U	2	2	3	3	2	2	2
DC, PC	INTERFACE DEVICES	PRESSURE SENSOR	3	3	3	3	3	3	3	3	3	[illegible]	3	3	3	3	[illegible]
DC, PC	INTERFACE DEVICES	TEMPERATURE SENSOR	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
DC, PC	INTERFACE DEVICES	FLOW SENSOR	3	3	3	3	3	3	3	3	2	3	3	3	3	2	3

APPLICATION			SPACE STORABLES: CHLORINE PENTAFLUORIDE	CHLORINE TRIFLUORIDE	NITROGEN TRIFLUORIDE	NITRYL FLUORIDE	OXYGEN DIFLUORIDE	PERCHLORYL FLUORIDE	TETRAFLUOROHYDRAZINE	AMMONIA	DIBORANE	HYDRAZINE AE	LPG[a]
DC	DIGITAL DEVICES	WALL ATTACHMENT - VENTED	2	2	2	2	2	2	2	2	3	U	2
DC, FM	DIGITAL DEVICES	WALL ATTACHMENT - CLOSED	2	2	2	2	2	2	2	2	2	2	2
DC	DIGITAL DEVICES	TURBULENCE	2	2	2	2	2	2	2	2	2	U	2
DC, FM	DIGITAL DEVICES	INDUCTION	2	2	2	2	2	2	2	2	1	U	2
DC	DIGITAL DEVICES	EDGETONE	1	1	1	1	1	1	1	1	1	U	2
DC	DIGITAL DEVICES	FOCUSED JET	2	2	2	2	2	2	2	2	2	2	2
PC, FM, PM	PROPORTIONAL DEVICES	STREAM INTERACTION	2	2	2	2	2	2	2	2	2	U	2
ALL	PROPORTIONAL DEVICES	VORTEX AMPLIFIER	2	2	2	2	2	2	2	2	2	U	2
ALL	PROPORTIONAL DEVICES	VORTEX VALVE	2	2	2	3	2	2	2	2	2	U	2
PC, PM, FM	PROPORTIONAL DEVICES	VENTED JET	2	2	2	3	2	2	2	2	2	U	2
DC, FM	PROPORTIONAL DEVICES	FLOW DIVERTER	2	2	2	2	2	2	2	2	2	2	2
PC, PM, FM	PROPORTIONAL DEVICES	BOUNDARY LAYER	1	1	1	1		1	1	1	1	1	2
PC, PM, DC	PROPORTIONAL DEVICES	IMPACT MODULATOR	1	1	1	1	1	1	1	1	1	1	2
PC, PM	PROPORTIONAL DEVICES	THROAT INJECTION	2	2	2	2	2	2	2	2	2	2	2
PC, PM, DC	PROPORTIONAL DEVICES												
DC, PC	PASSIVE ELEMENTS	ORIFICE	1-2	2	1-2	2	2	2	2	2	2	3	3
DC, PC	PASSIVE ELEMENTS	LAMINAR RESISTOR	1-2	2	1-2	2	2	2	1-2	2	1-2	3	3
PC	PASSIVE ELEMENTS	INDUCTOR	1-2	2	1-2	2	2	2	1-2	2	1-2	3	3
PC, DC	PASSIVE ELEMENTS	CAPACITOR	2	2	2	3	2	2	2	2	2	2	3
PC, DC	PASSIVE ELEMENTS	DIODE	2	2	[illegible]	3	2	2	2	2	2	U	2
DC, PC	INTERFACE DEVICES	ELECTRICAL TO FLUID	1-2	1-2	1-2	2	2	2	1-2	2	1-2	2	2
DC, PC	INTERFACE DEVICES	FLUID TO ELECTRICAL	U	U	U	U	U	U	U	U	U	U	U
DC, PC	INTERFACE DEVICES	MECHANICAL TO FLUID	U	U	U	U	U	U	U	U	2	2	2
DC, PC	INTERFACE DEVICES	FLUID TO MECHANICAL	U	U	U	U	U	U	U	U	2	2	2
DC, PC	INTERFACE DEVICES	PRESSURE SENSOR	U	U	U	U	U	U	U	U	2	2	2
DC, PC	INTERFACE DEVICES	TEMPERATURE SENSOR	U	U	U	U	U	U	U	U	2	2	2
DC, PC	INTERFACE DEVICES	FLOW SENSOR	U	U	U	U	U	U	U	U	2	U	2

t Rating Analysis Chart (Reference 131-42)

UFC[a]	HARD CRYOGENICS: FLOX[b] 80F2 + 20% LO2	HARD CRYOGENICS: LIQUID FLUORINE	HARD CRYOGENICS: LIQUID OXYGEN	HARD CRYOGENICS: LIQUID HYDROGEN	METALLIZED GELS: N_2H_4	METALLIZED GELS: UDMH	METALLIZED GELS: MMH	NON-METALLIZED GELS: N_2O_4	NON-METALLIZED GELS: ClF_3	NON-METALLIZED GELS: MAF-MIXED AMINE FUELS
2	2	2	2	3	1	1	1	1	1	1
2	2	2	2	3	2	2	2	2	2	2
2	2	2	2	2	1	1	1	1	1	1
2	1	2	2	2	1	1	1	2	2	2
2	1	1	1	2	1	1	1	1	1	1
2	2	2	2	2	1	1	1	1	1	1
2	2	2	2	3	1	1	1	1	1	1
2	2	2	2	3	1-2	1-2	1-2	1-2	1-2	1-2
2	2	2	3	3	2	2	2	2	2	2
2	2	2	3	3	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2
2	1	1	1	2	1	1	1	1	1	1
2	1	1	1	2	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2
3	2	1-2	3	3	2	2	2	2	2	2
3	1-2	1-2	3	3	1	1	1	1	1	1
3	1-2	1-2	3	3	1	1	1	2	2	2
3	2	2	3	3	2	2	2	2	2	2
2	2	2	3	3	2	2	2	2	2	2
2	1-2	1-2	2	2	1-2	1-2	1-2	2	2	2
U	U	U	2	2	U	U	U	U	U	U
2	U	U	2	2	U	U	U	U	U	U
2	U	U	2	2	U	U	U	U	U	U
2	U	U	3	3	2	2	2	2	2	2
2	U	U	3	3	2	2	2	2	2	2
3	U	U	2	2	2	[illegible]	2	2	2	2

TEMPERATURE - HIGH TO 1200°F	TEMPERATURE 1500°F TO 2500°F	TEMPERATURE - LOW TO (-) 420°F	PRESSURE - LOW TO 10 PSIA	PRESSURE TO 100 PSIA	PRESSURE TO 1000 PSIA	GAS PRESSURE TO 5000 PSIA	TIME RESPONSE - GASES 1000 CPS	TIME RESPONSE - LIQUIDS >100 CPS	VIBRATION AND SHOCK	POWER REQUIREMENTS	OPERATING LIFE > 10^6 CYCLES	CONTAMINATION	LEAKAGE INC. (ENTER) > 10^{-5} SCCS He	STERILIZATION 300°F(OR SONIC - 6 CYCLES)	SPACE MAINTENANCE	SIGNAL TO NOISE RATIO > 10B	ON BOARD NUCLEAR POWER SOURCES	SPACE RADIATION: VAN ALLEN BELT	SPACE RADIATION: SOLAR FLARE	SPACE RADIATION: INDUCED BREMSTRAHLUNG	VACUUM	ZERO G	METEOROIDS	IN 1 OF COMBUSTION (AND GROUND)
3	2	3	3	3	2	1	3	2	3	2	3	2-3	3	3	U	2	3	3	3	[illegible]	2	[illegible]	3	3
3	3	2-3	3	3	2-3	2	2	2	3	2	3	3	3	3	U	2	3	3	3	1	2	3	3	3
2	1	2	2	1	1	1	1	1	2	3	2	1	1	3	U	1	3	3	2	3	1	3	2	3
1	2	2	3	3	2	1	1-2	1	2	1	3	2	2	3	U	2	3	3	3	3	1	3	3	3
1	1	1	3	3	2	1	3	2	1	2	2	2	2	3	U	1	2	3	2	2	1	3	2	2
2	2	2	3	2	1	1	2	2	2	1	3	2	2	3	U	1	3	3	3	3	1	3	3	3
2	1	2	3	3	1	1	2	2	2	2	3	2	3	3	U	3	3	3	3	3	1	3	3	3
3	3	2	3	3	2	2	2	1	2	2	3	3	3	3	U	1	3	3	3	3	2	3	3	
3	3	3	3	1	3	3	2	2	3	2	3	3	3	2	U	2	3	3	3	3	3	3	3	3
3	2	2	2	3	3	2	2	2	2	2	3	2	2	3	U	2	3	[illegible]	3	3	1	3	3	3
3	3	2	3	3	2	1	2	2	3	2	2	3	3	3	U	2	3	3	3	3	3	3	3	3
3	2	2	3	3	2	1	3	2	2	2	3	2	2	3	U	2	3	3	3	3	1	3	3	3
2	2	2	3	2-3	1	1	3	2	2	2-3	2	2	2	3	U	2	3	3	3	3	1	3	3	3
3	3	3	3	3	2	2	2	2	2	2	3	3	3	3	U	2	3	3	3	3	2	3	3	3
3	3	3	3	3	3	3	3	3	3	NA	3	3	3	3	U	2	3	3	2	3	3	3	3	3
3	3	3	3	2	2	1	1	1	3	NA	2	2	3	3	U	2	3	3	3	3	3	3	3	3
3	3	3	3	2	2	1	NA	NA	3	NA	2	2	3	3	U	2	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	NA	NA	3	NA	3	2	2	3	U	2	3	3	3	3	3	2	2	3
3	2	2	3	3	2	2	2	2	3	NA	3	2	2	3	U	2	3	3	3	3	3	3	3	3
2	1	2	3	3	2	2	2	2	2	2	2	2	2	2	U	2-3	2	2	2	3	2	3	2	3
2	1	2	3	3	2	2	2	2	2	2	2	2	2	2	U	2	2	[illegible]	2	3	2	3	2	3
U	U	U	3	3	2	2	2	2	2	2	3	2	2	3	1	3	3	3	3	3	3	3	2	3
U	U	U	3	3	2	2	2	2	2	2	3	2	2	3	1	2	3	3	3	3	3	3	2	4
3	2	2	3	3	3	3	2	2	2	3	2	2	2	2	U	2	3	3	3	3	3	3	3	3
3	3	2	2	3	3	2	2	2	3	1	3	2	3	2	U	2	3	3	3	3	3	3	3	3
3	2	2	3	3	3	2	2	2	2	2	3	2	3	3	U	2	3	3	3	3	2	2	3	3

B

Gases such as hydrogen, helium, and high temperature combustion products are notoriously difficult to seal and will often leak through exceedingly small openings such as are found in connectors and static seals. Once a leak starts, the erosive effects of these gases can be quite significant. Boiloff gases from oxidizers such as N_2O_4 and LF_2, as well as most combustion product gases, are highly corrosive and adequate provisions must be made to ensure compatibility with construction materials. Monopropellant hydrazine gas generator systems supply a relatively clean gas which should find wide application as a working fluid for fluidic systems.

Liquids. Water, hydraulic oils, and virtually any liquid propellant (including cryogenics) may be used as the working fluid in a fluidic system. In most cases more serious problems are encountered with liquids than with gases, primarily because of difficulties with materials compatibility and the lack of design data and elements for liquid operation. As with gases, cleanliness of liquid flow media must be maintained to avoid problems. Of particular significance is the case of cryogenic fluids that may become contaminated with gases whose freezing points are higher than the storage temperature of the cryogenic fluid.

The development of gaseous fluidic systems has progressed much more rapidly than liquid systems. Although fluidic elements can be successfully operated with any of the liquid propellants, present component configurations were designed for gas operation, and consideration must be given to redesigning elements for liquid operation. Liquid elements are slower to respond than similarly-sized pneumatic elements, and higher density working fluids require higher input power for system operation. Also, dissolved gases tend to come out of solution in the low pressure regions formed at abrupt changes in cross section or direction. Small elements pose other problems which can be related to the flow regime required for proper operation. Some elements require either laminar or turbulent flow conditions to perform their function while others rely on a laminar to turbulent transition. A specific element tested with air and then with hydraulic fluid showed that supply pressures of 0.2 and 360 psig, respectively, were required to attain the same Reynolds number for the two cases.

Gelled Propellants. Both metallized and nonmetallized gels are characterized by thixotropic properties, i.e., the viscosity decreases with increasing shear rate and stress decreases with time at constant shear. As the gel flows through lines and components, the shear becomes greater, the viscosity becomes less, and the gel behaves more like a low viscosity liquid.

Gelled propellants, especially metallized gels with metal particle sizes ranging from 5 to 50 microns, are obviously not applicable to miniaturized fluidic systems. In addition, the properties of gelled propellants can present several problems in larger fluidic devices. Pressure drops through lines and elements are higher than those of comparable liquids and are unpredictable because the viscosity varies. Flow through nozzles can cause evaporation of the liquid phase of the gel, which leaves a solid matrix as a residue that can hinder or restrict the flow. The abrasive action of metal particles can cause erosion of nozzles and passages. The compatibility of gelled propellants with the materials of construction is generally comparable to the base liquid propellant.

16.7.2.2 OPERATING TEMPERATURE. Fluidic devices can be made to operate with some fluids at any given temperature, limited only by the materials available of construction. Elements have been operated with liquid hydrogen, and a vortex valve has been operated with 5500°F working fluid (Reference 37-11). Digital elements can be operated over broad temperature ranges; however, analog devices are quite sensitive to temperature variation. This sensitivity is caused by such things as viscosity variations, sonic velocity changes, and orifice and nozzle size variations because of thermal expansion or contraction. Differential circuits can be used to compensate for small temperature changes, and temperature sensitive gain changing networks are required for compensation over broad temperature ranges.

16.7.2.3 OPERATING PRESSURE. The primary problems anticipated with high pressure levels are structural strength and seals. The minimum pressure requirement at the current state of the art of computational elements operating with gases is about 0.5 to 10 psig. Where required, digital logic can operate successfully at 0.1 psig or less. Power elements operating on liquid have been tested at pressures up to several thousand psig. Back pressure regulation or a constant pressure sump may be necessary to maintain acceptable Reynolds numbers if elements are required to operate over a wide range of ambient or vent conditions. For elements operating on gases, overall pressure ratios seldom exceed 1.3:1.

16.7.2.4 RESPONSE TIME. The response time of fluidic elements refers to the time delay between the application of an input step signal and the time at which the resulting output signal reaches a level which is 63 percent of the final value. Response time of a specific fluidic element is primarily influenced by the transport time of a fluid molecule through the device. With gases, this transit time is normally figured to be equivalent to a velocity of 1 inch per millisecond.

State-of-the-art response time of small fluidic elements operated on gases is presently about 1 millisecond. An important consideration is that the response time of most fluidic elements increases as fluid density increases. This is illustrated in Figure 16.7.2.4, where it can be seen that liquid-operated elements tend to be an order of magnitude slower than gas-operated elements (References 23-70 and 761-2). The figure also shows that elements will operate faster as they are made smaller.

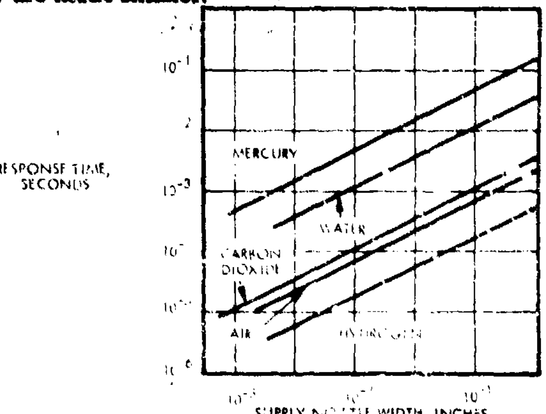


Figure 16.7.2.4. Component Response Times With Various Fluids
(Adapted with permission from Reference 23-70, "Hydraulic Fluidics", S.A.E. Paper No. 670736, C. K. Taft, September 1967)

16.7.2.5 POWER REQUIREMENTS. Fluidic elements require a continuously flowing supply of working fluid for normal operation so that, in logic control circuitry, the individual component supplies can add up to a sizeable power drain. For power functions in applications with low duty cycles and long missions, fluidic elements consume a little more power than conventional control components. Power consumptions should be considered carefully even if a plentiful supply of working fluid, such as gas turbine or rocket engine bleed gas, is available. If a stored gas supply is required for the fluidic system, the power drains of fluidic logic and analog elements should be in the low milliwatt range.

Power consumption in state-of-the-art fluidic computational devices ranges from 0.02 to several watts of fluid power as shown in Figure 16.7.2.5. One example of the new low-power logic indicated in Figure 16.7.2.5 is the two-dimensional laminar NOR amplifier (Detailed Topic 16.4.5.3) which was developed at the Harry Diamond Laboratories.

The most logical approaches to reduce power consumption are miniaturization and reduced supply pressure. Extreme miniaturization (below 10 mil widths) compromises element reliability and complicates element and circuit fabrication. This justifies the present trend toward lower supply pressures.

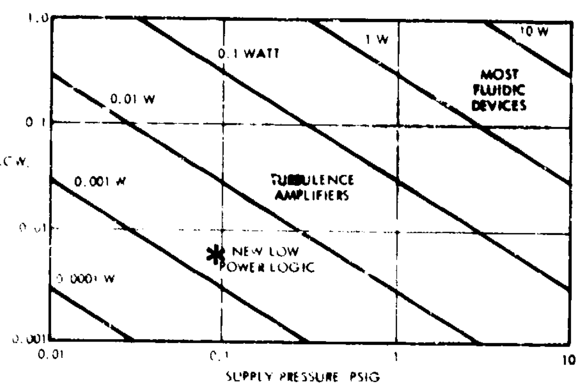


Figure 16.7.2.5. Power Consumption of Fluidic Devices

16.7.2.6 OPERATING LIFE. The actual required operating life of a fluidic element can vary from a few cycles to several hundred thousand cycles depending on mission requirements. Since there are no moving parts that can wear out, the operating life of an element is not usually a problem. The most significant effects on operating life are materials compatibility, seals, erosive action of the working fluid, and the environment.

16.7.2.7 LEAKAGE. For a basic two-dimensional fluidic element sealed with a cover plate, consider the leakage paths across the seal layer. In a cold gas or liquid system some leakage can be tolerated across the seal without adversely affecting component performance. However, in a hot gas system even minute leaks can cause severe erosion in the seal layer which will soon develop into a leak and ultimate component malfunction. Nonvented leakage of this type is generally hard to detect unless it is external to the component vent ports. Manufacturing techniques and careful inspection of seals during component assembly are perhaps the best methods of maintaining the integrity of the seals. As in conventional fluid systems, leakage can result in severe loss of fluid media, fire and explosions, and, in some cases, toxicity hazards to personnel.

16.7.2.8 SIGNAL-TO-NOISE RATIO. Noise is defined as the peak-to-peak pressure fluctuations on the signals of a fluidic device so that in high-gain circuits the signal-to-noise ratio becomes a comparative measure of element performance. Of primary concern are element geometry, fabrication method, and operating conditions.

There are several fluidic elements potentially capable of operating with relatively high signal-to-noise ratios (>200). Some element geometries are much noisier than others; however, these devices are generally of the digital variety. The turbulence amplifier is particularly susceptible to external vibration and shock, and the edgetone amplifier generates considerable internal noise since the device is purposely designed unstable to enhance switching speed.

16.7.2.9 STERILIZATION. Complete sterilization of all components on a spacecraft may be necessary for planetary missions or flyby missions to the planets. One method of sterilization involves a soak at temperatures up to 300°F for 60 hours, which is repeated for six cycles. A mixture of 12 percent ethylene oxide and 88 percent Freon is also used in a spray for surface sterilization. In a fluidic system, the materials of construction, the methods of fabrication and assembly, and the subsequent handling required must all be considered.

16.7.2.10 CONTAMINATION. Fluidic elements can be designed to be contamination insensitive by utilizing large nozzle widths (0.025 inch). For aerospace application this is inconsistent with the normal requirements for low power systems. Therefore, 0.005 to 0.010-inch nozzle widths are considered more practical for gas systems with normal filtration and contamination control during assembly. Estimates as to the smallest practical power nozzle width for liquid-operated systems range from 0.007 to 0.025 inch. The decision on width must be tempered by the required operational life and the fluid properties as well as by the fluid contamination level.

16.7.2.11 SPACE MAINTENANCE. The maintenance of manned and unmanned spacecraft is a requirement that will involve new designs, techniques, and procedures. In-flight maintenance will be necessary during space travel or in orbiting space stations, and major repairs may be required on vehicles which have landed on the moon or other celestial bodies.

Fluidic elements will not be interchangeable because integrated circuitry should be employed in spacecraft applications. Maintenance or replacement then must be considered on a subsystem basis. The problem then becomes one of the usual difficulties which would be imposed on an astronaut who must connect and disconnect conventional fittings or even specially-designed quick-disconnect fittings.

16.7.2.12 SPACE ENVIRONMENTS. The space environment is characterized by radiation, vacuum, zero gravity, and meteoroids as discussed in detail in Section 15.0 of this handbook. Fluidic systems containing no moving parts or electrical or magnetic elements are particularly insensitive to the effects of radiation and zero gravity. A vacuum environment is primarily significant insofar as it influences venting characteristics and increases leakage potential by increasing the difference between system and ambient pressures. The meteoroid hazard is essentially the same for a fluidic system as for any other fluid system of comparable size. Fluidic systems for in-space applications are inherently unaffected by the high acceleration and vibration levels associated with rocket launch transients, and may also be designed to function during launch transients.

16.7.3 Typical Applications

16.7.3.1 VORTEX AMPLIFIER CONTROLLED SITVC. A successful demonstration of a vortex amplifier controlled secondary injection thrust vector control (SITVC) system has been made on a solid propellant rocket motor (References 37-5, 37-11, 768-1). The vortex amplifiers utilized in this program have the capability of modulating a 750 psia, 1 lb/sec flow of 16 percent aluminized, 5500°F solid propellant gas, with a demonstrated operating time of 50 seconds. The rocket motor controlled during the test was a NASA-furnished model EM-72, which is a 22-inch end burner containing 400 pounds of propellant. The motor is capable of producing approximately 6800 pounds of thrust with a mass flow rate of 30 lb/sec for 13 seconds. For comparison, conventional SITVC systems are discussed in Sub-Section 4.5 of this handbook.

The SITVC (Figure 16.7.3.1a) system consisted of a pilot stage which provided push-pull control of two SITVC hot gas vortex amplifiers. The pilot stage contained a torque motor powered flapper-nozzle valve which, in turn, controlled two additional vortex amplifiers. A 2000°F solid propellant gas generator (SPGG) supplied gas to the pilot stage. The two SITVC vortex amplifiers were supplied with gas from an auxiliary 5500°F SPGG. The SITVC vortex amplifiers were installed on the horizontally positioned EM-72 rocket, such that one amplifier injected in the engine thrust nozzle vertical plane and the other in the horizontal plane.

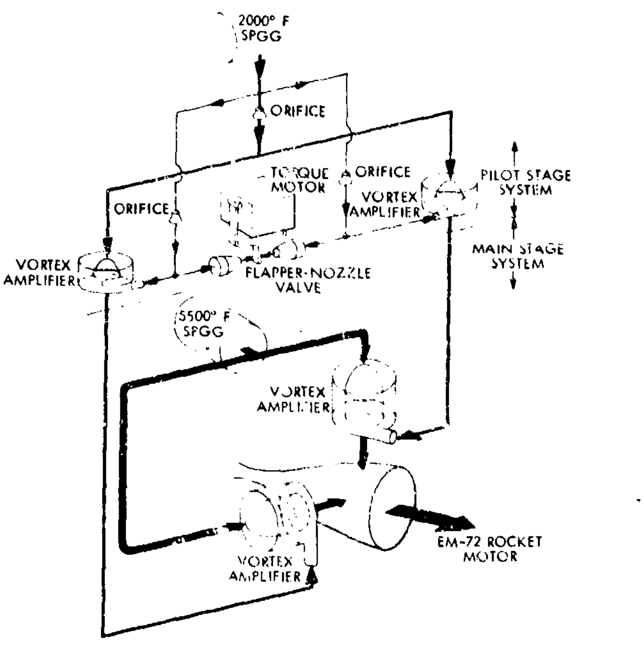


Figure 16.7.3.1a. Schematic of Vortex Valve Controlled SITVC System
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

The results of the hot gas tests conducted at Allegany Ballistics Laboratory, Cumberland, Maryland, in October 1967 (Reference 37-11), showed that the vortex amplifier controlled SITVC system produced side forces of up to 4 percent of the main engine thrust. The SITVC system materials and structure were able to control and handle the flow of aluminized 5500°F gas for over 50 seconds with little component degradation. The need for fast response and high reliability in extreme environments suggests the application of vortex amplifiers (Figure 16.7.3.1b). The overall frequency response of the SITVC system showed a phase lag of 28 degrees at about 30 cps.

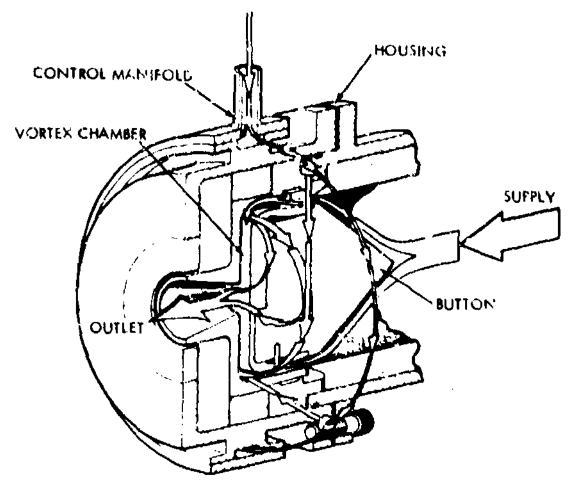


Figure 16.7.3.1b. 5500°F Vortex Valve
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

A possible method of implementing a vortex amplifier controlled SITVC system on a buried nozzle solid propellant rocket engine is shown in Figure 16.7.3.1c. The power stage vortex amplifiers will modulate bleed gas directly from the rocket motor combustion chamber and inject it into the nozzle for thrust vector control. A similar type system is possible on a liquid rocket engine using an auxiliary solid or liquid propellant gas generator.

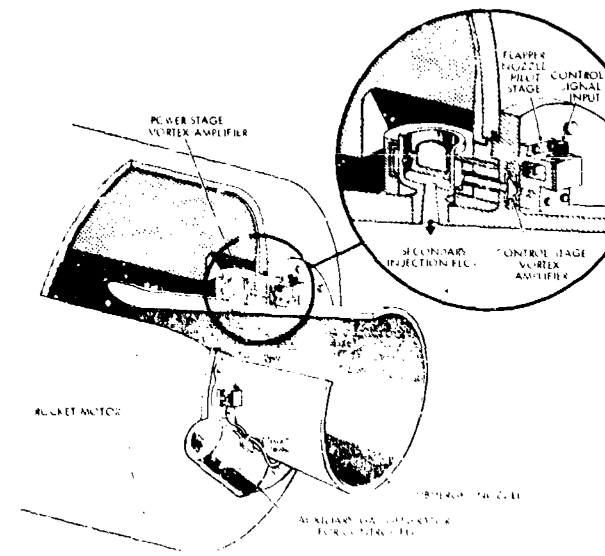


Figure 16.7.3.1c. Conceptual Vortex Valve SITVC System – Buried Nozzle Installation
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

16.7.3.2 FLUIDIC STALL SENSOR. An application of the high-impedance fluidic pressure sensor (see Sub-Topic 16.6.1) is its use as a stall sensor on aerodynamic surfaces such as airplane wings and helicopter rotors (Reference 241-10). Stall on a wing is the condition where the attached flow encounters an adverse pressure gradient and detaches or separates.

A small probe similar to a pitot tube is positioned just above the boundary layer on a wing and pointed about 15 degrees aft of perpendicular to the flow. As shown in Figure 16.7.3.2, the attached flow aspirates air from the sensor which prevents foreign matter from entering the system. When stall occurs, the flow over the sensor becomes highly turbulent and then reverses, so that flow is no longer aspirated from the sensor but is resisted. Some of the flow from the bypass (Figure 16.6.1c) then enters the control and the bistable fluid amplifier is switched. This operates a red plastic piston which becomes visible in the cockpit of the plane. A row of these indicators is connected to a row of stall sensor systems on the wing. As stall becomes progressively worse, more red pistons become visible to form a lengthening red line which indicates to the pilot that his lift or margin of safety is decreasing.

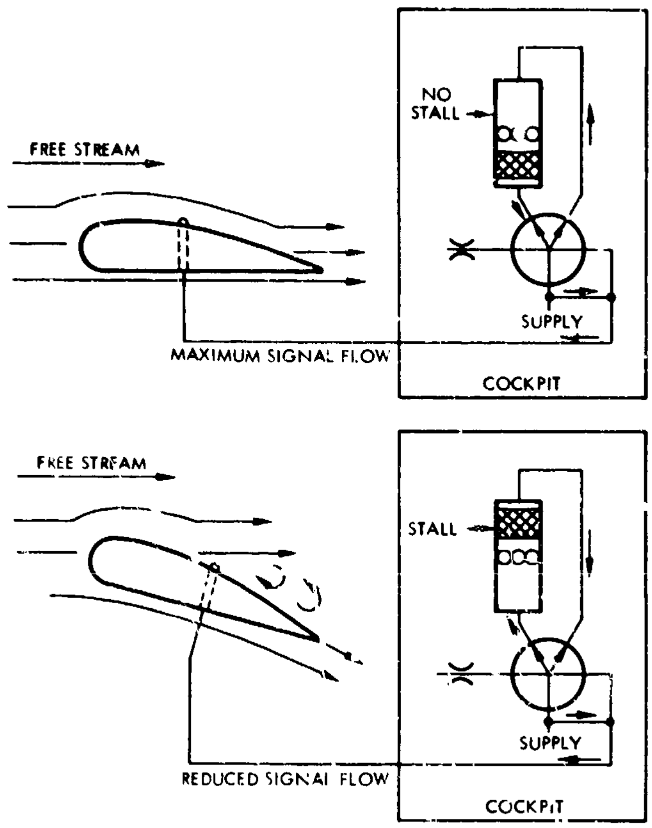


Figure 16.7.3.2. Flueric Stall Sensor Indicating (A) No Stall (Piston Up); (B) Stall (Piston Down)

16.7.3.3 FLUIDIC BOW THRUSTER. A new concept in bow thruster design for ships utilizes a marine diverter valve which operates on the principle shown in Figure 16.7.3.3a.

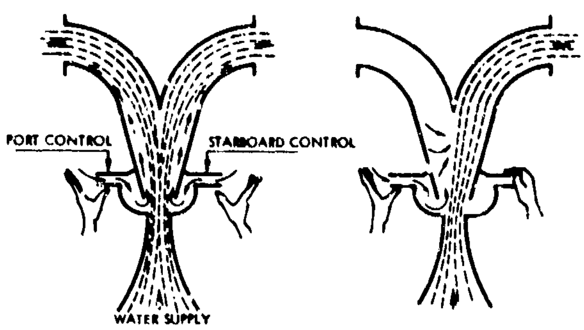


Figure 16.7.3.3a. Marine Diverter Valve Principle

In the neutral position, air is entrained from both the port and starboard controls, and the supply is equally divided between the two outputs. When the starboard control port is blocked, the supply of entrained air on that side is cut off. Continued air entrainment by the water jet produces an area of reduced pressure near the mouth of the closed control port. The resultant pressure difference across the jet causes it to move to the blocked side, as shown. Once the sea-water jet has locked on the one wall, both control ports may be closed and the jet will continue to flow through the starboard discharge leg. The jet can be switched to the port side by opening the starboard control and closing the port control.

The initial prototype bow thruster was successfully tested on a 35-ton barge. Several large two-stage diverter units (Figure 16.7.3.3b) have been built for experimental use by the U.S. Navy. Servo operated control ports allow full diversion of the thruster jet to either side of a ship. Lateral forces and steerage for the ship are possible even when the ship is stationary.

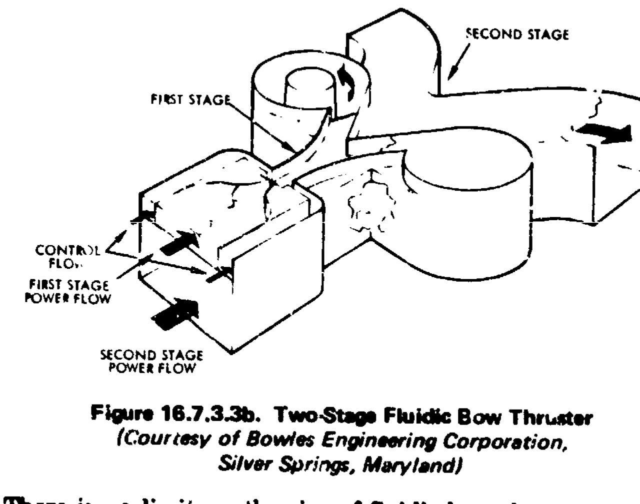


Figure 16.7.3.3b. Two-Stage Fluidic Bow Thruster
(Courtesy of Bowles Engineering Corporation, Silver Springs, Maryland)

There is no limit on the size of fluidic bow thrusters which can be designed. Thrust levels are determined by the size of the pump supplying the unit. Because of their geometry and relatively small size, fluidic thrusters can be located in the extremity of the bow or stern. Also, since the unit can usually be located deep within the vessel's hull, with its pump taking bottom suction, the thrust remains operable at very shallow draft and is unaffected by surface ice or floating debris.

16.7.3.4 FLUIDIC POWER AMPLIFIER. This prototype fluidic system was developed to operate a displacement actuator on a nuclear rocket control drum drive unit (Reference 37-10). The system utilizes a low power pneumatic input signal, its output characteristics are similar to those of a four-way open-centered servovalve, and it incorporates frequency-variant load pressure feedback. Having no moving parts, it should be particularly advantageous for operation in cryogenic, high temperature, and radiation environments.

The fluidic system for operating a displacement actuator is shown schematically in Figure 16.7.3.4a. Three fluidic components are used: the vortex amplifier, the jet proportional amplifier, and the confined-jet amplifier. The power stage basically consists of two vortex pressure amplifiers, controlled in a push-pull operating mode. An increase in control pressure P_{c1} diverts the flow leaving the output orifice of one vortex pressure amplifier and reduces the recovered load flow and pressure. A simultaneous reduction of P_{c2} converges the flow leaving the opposite vortex amplifier and increases the load flow and pressure recovery. The result is a differential pressure ($P_2 - P_1$) across the load and a load flow depending on the load force requirements.

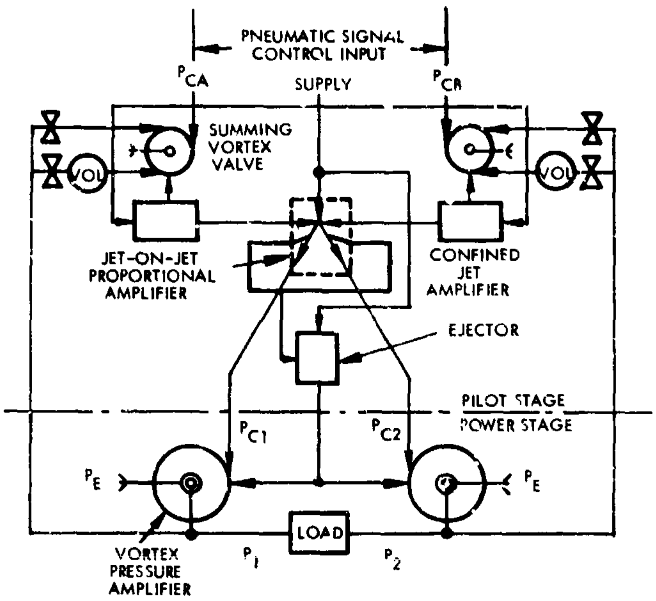


Figure 16.7.3.4a. Fluidic Power Amplifier Schematic
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

When the power stage is used to drive a load, such as a two-way piston actuator or gear motor, it is necessary for one pressure amplifier to exhaust the flow from the low pressure side of the actuator when the actuator is moving. The backflow is exhausted back through the flow receiver and then out through the area between the vortex chamber outlet and the receiver entrance. The resistance to backflow is reduced when the control pressure is increased because the vortex chamber outlet flow is reduced and is diverted to exhaust.

The pilot stage includes a jet-on-jet proportional amplifier, an ejector, two confined-jet amplifiers, and two summing vortex amplifiers. The jet-on-jet proportional amplifier provides the differential control flow to the power stage supply vortex amplifiers. The exhaust flow from the jet-on-jet proportional amplifier is recovered in the supply flow to the power stage which increases the flow recovery of the system.

Two summing vortex amplifiers are used to introduce the servovalve input signals. The control input to these units has a pressure bias so that an input signal consists of lowering control pressure to one unit and raising it to the other. To achieve dynamic load pressure feedback, each summing vortex amplifier also includes two opposing control ports which are connected to the servovalve output ports through a frequency-sensitive pneumatic filter. The summing vortex amplifiers control the vent flow (and thus the output pressure) of the confined-jet amplifiers which supply the control ports of the jet-on-jet proportional amplifier. This system is capable of accepting a low-level differential control signal and optimally controlling a displacement actuator. Performance of the prototype system is summarized in Table 16.7.3.4, and the output characteristics are shown in Figure 16.7.3.4b. Predicted performance of an optimized design based on state-of-the-art components is shown in Figure 16.7.3.4c.

Table 16.7.3.4. Fluidic Power Amplifier Performance Using Nitrogen
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

Item	Power Stage Vortex Pressure Amplifier
1) Supply pressure	148 N/cm² (215 psia)
2) Exhaust pressure	34.5 N/cm² (50 psia)
3) Flow recovery	50%
4) Rated input signal	10 N/cm² (14.5 psi)
5) Input signal pressure bias	53.7 N/cm² (76.7 psia)
6) Total input power	10.5 watts
7) Rated no-load flow	3.0 gm/sec (0.0067 psia)
8) Pressure recovery	67 N/cm² (98 psi)
9) Linearity deviation	19%
Gain variation	2 times
10) Stability	9 N/cm² (13.1 psi)
11) Transient response	0.110 sec 0.190 sec
12) Frequency response	20° @ 5 cps
Phase shift	90° @ 45 cps
Amplitude ratio	±1.7 db
13) Threshold	1%
14) Hysteresis	3%

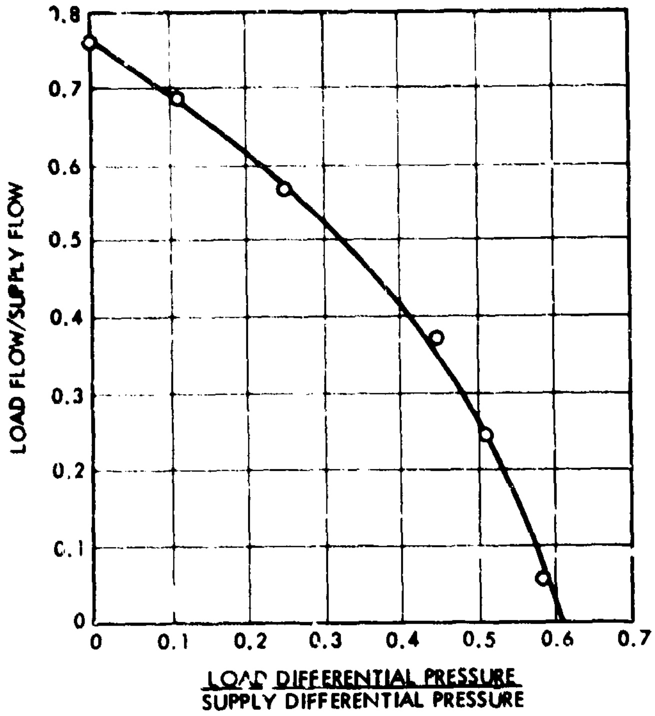


Figure 16.7.3.4b. Fluidic Servovalve – Output Characteristics
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

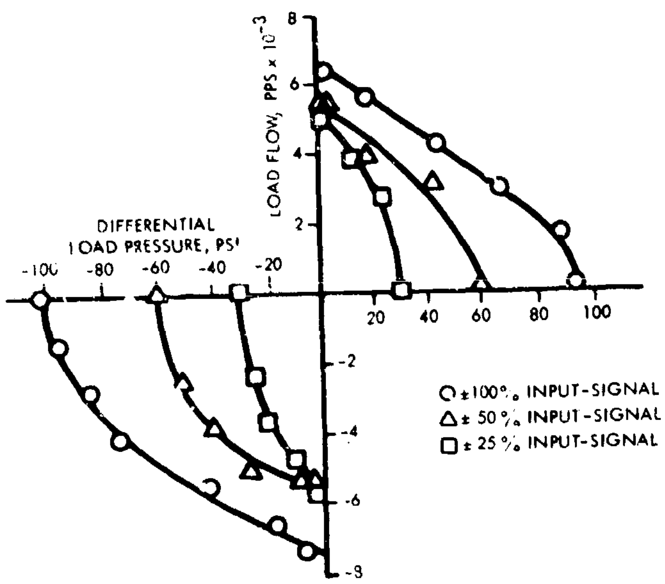


Figure 16.7.3.4c. Predicted Performance Potential
(Courtesy of Bendix Research Laboratories, Southfield, Michigan)

16.7.3.5 TIM ROLL CONTROL SYSTEM. The first successful use of a fluidic control system in a missile flight was in September 1964 on the Test Instrumentation Missile (TIM) which is a modified version of the Little John Rocket (References 332-29 and 564-15). Stored cold nitrogen gas was used as the power source, and supersonic bistable reaction amplifiers were used as the control moment producers. The stabilization fins on the aft end of the missile were purposely canted to provide a disturbance torque of 5 ft-lb and a roll rate of about 100 deg/sec. During the flight test, the control system operated correctly since the reaction jets were on in a direction to oppose the impressed roll rate. This point was proven by a measured increase in the missile roll rate after the control system supply nitrogen was exhausted.

A schematic of the TIM control system is shown in Figure 16.7.3.5a. The power section of the system was controlled by a high flow dome-type regulator which was activated at launch by a solenoid valve. A combination of proportional, bistable, and supersonic fluidic devices was used with a vortex rate sensor to form the roll control function. As seen in Figure 16.7.3.5b, the system utilizes a bistable rate damper with an integrator to minimize the roll attitude drift. Integration is accomplished with a resistor-capacitor combination (first order lag circuit) with a long time constant. The system operates in a limit cycle or bang-bang mode at a frequency and amplitude dependent upon the system's threshold and the time delays of the various components. During the limit cycle, the bistable moment producer output and vehicle acceleration are square waves with a frequency of approximately 3 cps. Vehicle rate is determined by the integration of this wave.

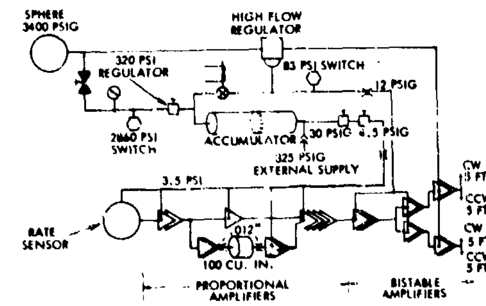


Figure 16.7.3.5a. TIM Fluidic Control System
(Adapted with permission from Reference 332-29, "The Development and Flight of a Pure Fluid Missile Control System," AIAA/IGCC Paper No. A66-10020, B. J. Clayton and W. M. Posingies, August 1965)

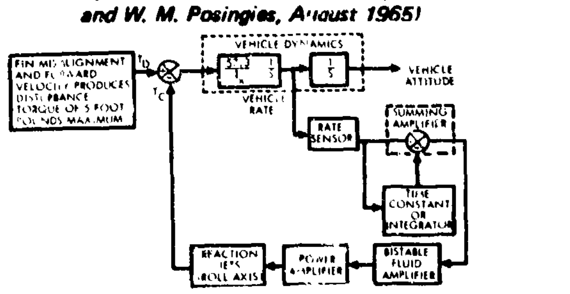


Figure 16.7.3.5b. TIM Control System Block Diagram
(Adapted with permission from Reference 332-29, "The Development and Flight of a Pure Fluid Missile Control System", AIAA/IGCC Paper No. A66-10020, B. J. Clayton and W. M. Posingies, August 1965)

16.7.3.6 THRUST REVERSING SEQUENCE CONTROL. This fluidic system was developed to perform the sequencing functions in an aircraft turbojet engine thrust reversing system (Reference 756-9). The thrust reversing system is shown schematically in Figure 16.7.3.6a. It is essentially a two-position system (one for direct thrust and one for reversing) with interlocks and detectors, with the following requirements:

a) When the *reversing thrust command* occurs it unlocks the obstacles, places the obstacles in reversing position if unlocking has taken place, and locks the obstacles in the reversing position.

b) When a *direct thrust command* occurs it unlocks the obstacles, places the obstacles in direct position if unlocking has taken place, and locks the obstacles in direct position.

c) All the above functions may be realized with a simple pulse command.

d) Any breakdown will prohibit further function.

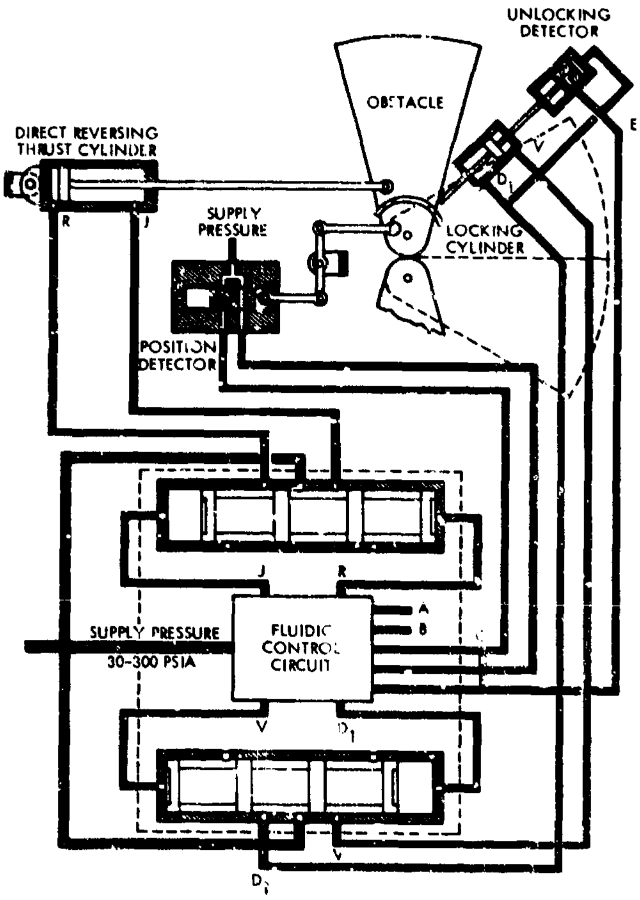


Figure 16.7.3.6a. Schematic of Thrust Reversing System
(Adapted with permission from Reference 756-9, "Fluidic Thrust Reversing Control System", 3rd Cranfield Fluidics Conference Paper J1, J. P. Champagnon and A. A. Thiney, May 1968)

A schematic of the fluidic control circuit is shown in Figure 16.7.3.6b. The input and output signals are defined below and are also indicated in Figure 16.7.3.6a.

The input signals are:

A — reversing thrust command

B — direct thrust command

C — direct thrust sensor

D — reversing thrust sensor

E — unlocking detector

The output signals are:

R — reversing

J — direct

D_1 — unlocking

V — locking

The system actuator is operated on turbine bleed air at absolute pressures ranging from 30 to 300 psia depending on the running condition of the turbine. It was also found practical to operate the fluidic circuit in this supply pressure range by maintaining the supply to vent pressure ratio P_s/P_v relatively constant. This was accomplished (Figure 16.7.3.6c) by installing the fluidic circuit in an enclosed chamber and venting the chamber to the atmosphere through a fixed orifice.

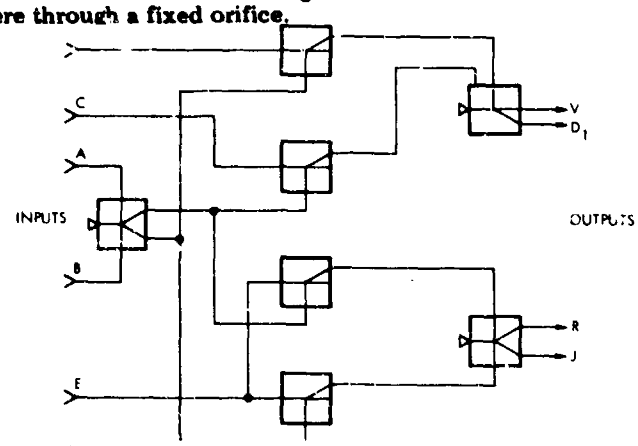


Figure 16.7.3.6b. Fluidic Control Circuit System
(Adapted with permission from Reference 756-9, "Fluidic Thrust Reversing Control System", 3rd Cranfield Fluidics Conference, Paper J1, J. P. Champagnon and A. A. Thiney, May 1968)

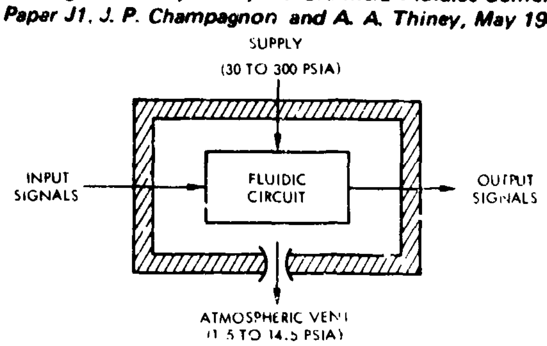


Figure 16.7.3.6c. Fluidic System Operating Conditions
(Adapted with permission from Reference 756-9, "Fluidic Thrust Reversing Control System", 3rd Cranfield Fluidics Conference, Paper J1, J. P. Champagnon and A. A. Thiney, May 1968)

This application is unique in that the fluidic circuit is integrated in one monolithic block using the Corning Fotoceram fabrication process. All inputs, outputs, and supply to the circuit are located on one face of the block which is polished prior to mounting on the thrust reverser power input (Figure 16.7.3.6c). The block is held in place with three screws, and the vent chamber is simply a formed metal cap installed over the circuit block. This system was developed in France and is typical of the trend in Europe toward utilizing fluidics because of the price advantage as well as the obvious intrinsic advantages.

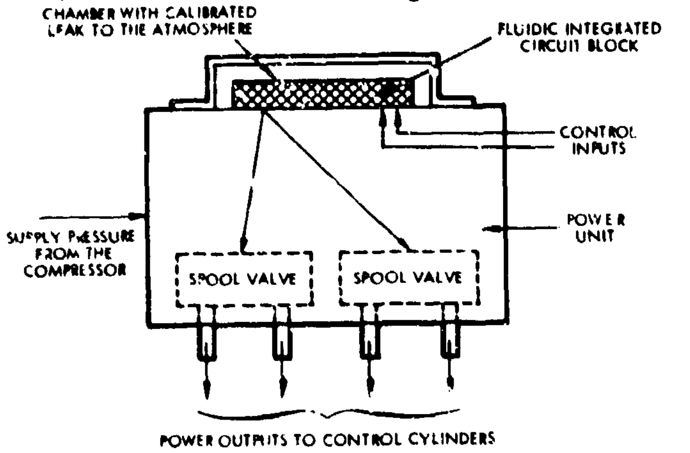


Figure 16.7.3.6c. Mounting of Fluidic Circuit on Power Unit
(Adapted with permission from Reference 756-9. "Fluidic Thrust Reversing Control System", 3rd Cranfield Fluidics Conference, Paper J1, J. P. Champagnon and A. A. Thiney, May 1968)

16.7.3.7 VTOL AIRCRAFT CONTROLS. An experimental attitude control system is under development for the hover control of turbojet-powered VTOL (vertical take-off and landing) aircraft which makes extensive use of fluidic devices (Reference 332-31). Two systems are being developed, a thrust modulation system for pitch axis control and a bleed air powered reaction jet system for roll control. Sensing, amplification, and actuation within these systems are all accomplished fluidically. Rate sensing is accomplished with a vortex rate sensor, and attitude information is provided by a gimballess attitude sensor. Thrust modulation is accomplished with a fluidic engine control system and large bistable fluidic amplifiers are used to power the reaction jets. The combined system has been breadboarded for test on a special-purpose single degree of freedom simulator with two J-85 turbojet engines and equivalent roll attitude control reaction jets.

Block diagrams of the pitch and roll control systems for the baseline configuration are shown in Figures 16.7.3.7a and 16.7.3.7b. The only change from these block diagrams for simulator operation is the reduction in number of engines being controlled to two. Corresponding fluidic mechanization schematics are shown in Figures 16.7.3.7c and 16.7.3.7d. Systems are being implemented using fluidic devices with no mechanical linkages or electronics and a minimum of moving parts. Power for operation is obtained from compressor discharge bleed air of the simulator engines.

Rate signals for both axes are obtained from individual vortex rate sensors. Attitude information is obtained from a two-axis pneumatically-driven attitude sensor. A separate attitude sensor is used in each axis, however, due to the

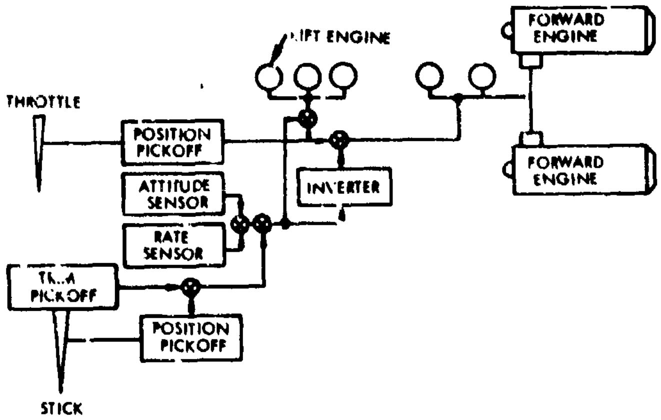


Figure 16.7.3.7a. Pitch Axis Control System
(Adapted with permission from Reference 332-31, "A Fluidic Approach to Control of VTOL Aircraft", AIAA/JACC Guidance and Control Conference, J. L. Haugen, August 1966)

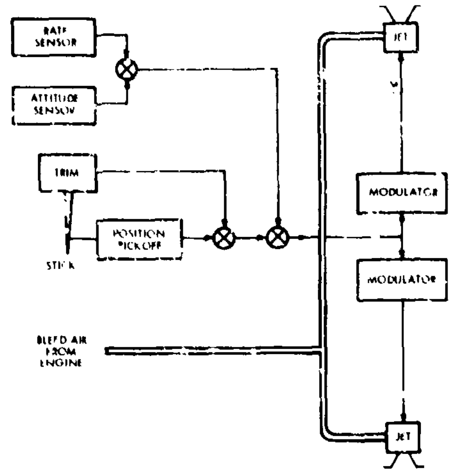


Figure 16.7.3.7b. Roll Axis Control System
(Adapted with permission from Reference 332-31, "A Fluidic Approach to Control of VTOL Aircraft", AIAA/JACC Guidance and Control Conference, J. L. Haugen, August 1966)

separation of operation of these axes of control on the simulator. The pressure differential signals from these sensors are amplified and summed using proportional fluid amplifiers.

In the pitch axis, the resultant pressure differentials are transmitted in analog form through 3/16-inch diameter tubing to the individual engines. At the engines, the attitude control signals are summed with the collective throttle commands and feedback signals from the fluidic engine control. The summed differential pressure is applied across a flapper nozzle by means of bellows to control the fuel-metering valve and to vary engine thrust.

In the roll axis, torque output is achieved with reaction jets powered by air bleed from the engine compressors. These reaction jets are, for the fluidic system, bistable fluid amplifiers with supersonic power nozzles. In order to achieve proportional control, these jets are pulse width modulated. Therefore, the analog error signals drive a pair

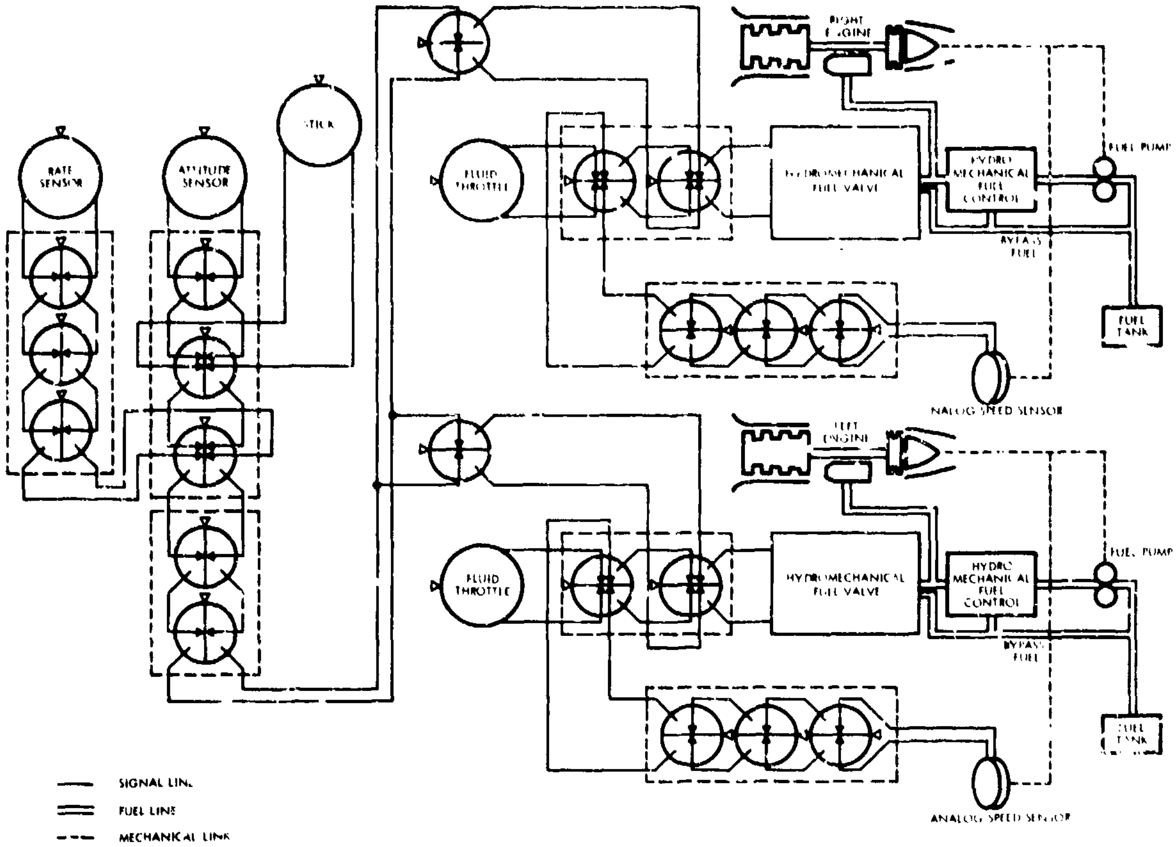


Figure 16.7.3.7c. Pitch Axis Fluid Schematic
(Adapted with permission from Reference 332-31, "A Fluidic Approach to Control of VTOL Aircraft", AIAA/JACC Guidance and Control Conference, J. L. Haugen, August 1966)

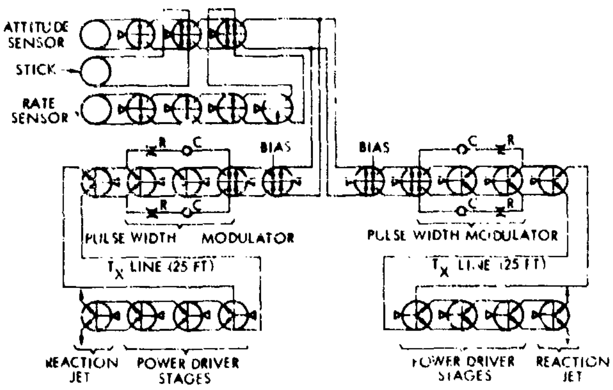


Figure 16.7.3.7d. Roll Axis Fluid Schematic
(Adapted with permission from Reference 332-31, "A Fluidic Approach to Control of VTOL Aircraft", AIAA/JACC Guidance and Control Conference, J. L. Haugen, August 1966)

of pulse width modulators operating at a nominal frequency of 40 cps. Signals are then transmitted to the wing tip reaction jets as pulse trains using 3/16-inch diameter tubing. The signal level is raised at the wing tip by a series of bistable supersonic power amplifiers to the level necessary for reaction jet triggering.

On the actual VTOL aircraft, reaction jets on both wings would normally be biased in the downward direction to conserve thrust. With an error signal present, one jet is deflected upward and the other remains biased downward yielding a torque couple. The length of time the jet is deflected upward is a function of error signal magnitude. For very small error signals, the jet points upward for the minimum modulator pulse width. For large error signals, the modulator goes hardover and the jet is directed upward continuously until the error signal is reduced.

This bistable method of roll control has a number of advantages over the more conventional systems using area-modulated reaction jets:

a) Reaction control is achieved without any moving parts

b) The continuous flow through both ducts minimizes duct dynamics, resulting in lighter ducts

c) The bistable jets give high response; they can be switched in excess of 100 cps if desired

d) Since both halves of the bleed air duct are flowing full at all times, the duct need have only one-half the area of one for a conservative system (control thrust always pointed downstream)

e) Redundancy is easy to implement.

The chief disadvantage of this system is the fact that thrust is not always conserved, i.e., the level control system is flowing constantly even when there is no attitude error to be corrected. However, a maximum of one-half of the thrust can be deflected upward at any time. Also, in a typical VTOL aircraft going through transition between vertical and horizontal flight, the average thrust command will only be approximately 10 percent of the peak command. Therefore, a maximum of 5 percent of the integrated roll impulse will be lost.

16.7.3.8 ROCKET ENGINE SEQUENCE CONTROL. A sequence control for a large, pump-fed, regeneratively cooled, cryogenic liquid rocket engine was breadboarded and successfully tested with helium (Reference 564-19). The sequence control system is required to start and shut down the rocket engine upon command, to monitor the progress of the start, and to activate engine shutoff automatically in the event of malfunction. In addition, a prestart logic circuit ensures that the engine is in the proper state prior to acceptance of the start signal.

The time-based sequence requirements for the control are shown in Figure 16.7.3.8a. Prior to start, the conditions of the components to be controlled are: the electrical ignition spark exciter system is off; both main propellant valves are closed; the start valve, a valve which controls the application of high pressure gas to the turbines during start, is closed; the engine pneumatic regulator is off; and the fuel bypass valve, a valve which bypasses fuel around the thrust chamber cooling tube bundle during the start, is open. All valves are pneumatically actuated by four-way valves which are sequenced by the engine controller.

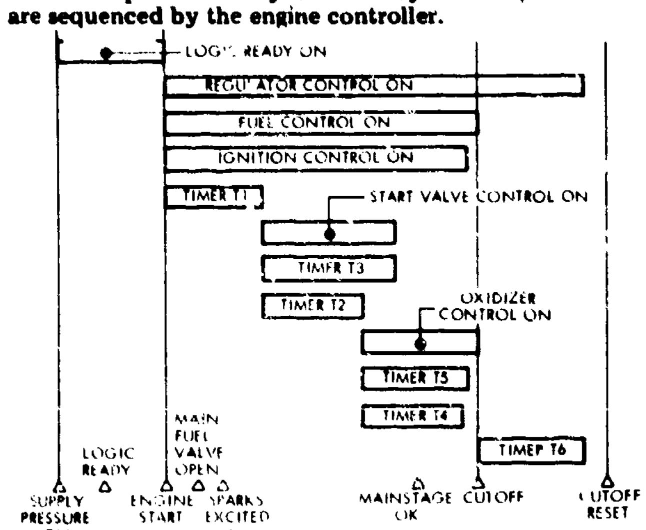


Figure 16.7.3.8a. Logic Sequence Chart
(Adapted with permission from Reference 564-19, "Journal of Spacecraft and Rockets", April 1968, vol. 5, No. 4, S. E. Milleman, AIAA)

Activation of the logic-supply pressure initiates the logic-ready circuit. The engine start signal is applied to the controller after the logic-ready signal is received from the controller. This initiates the engine pneumatic regulator control, the engine fuel control, the engine ignition control, and timer T1. The regulator control puts the engine pressure regulator on line, providing the means of actuating the engine valves. The fuel control causes the main fuel valve to open. Prior to expiration of timer T1, a check is accomplished to ensure that the main fuel valve has been opened and that the engine spark exciters are activated. If neither of these have occurred when timer T1 expires, the engine is shut down by an internal cutoff circuit. Assuming cutoff does not occur, the engine thrust chamber has sparks excited, low-pressure fuel, and no oxidizer. When timer T1 expires, the start valve is opened, the main oxidizer valve is opened slightly, and two more timers (T2 and T3) are started. The turbopumps are accelerated by high-pressure gas that has been released by the start valve. Ignition occurs in the thrust chamber as the oxidizer enters. When timer T2 expires, the oxidizer control and timers T4 and T5 are activated. The oxidizer control causes the main oxidizer valve to go to full open and thrust begins to build up. Just after this, timer T3 expires and the start valve closes, shutting down the initial source of turbine power. A new source is now available and is obtained by the bleeding of combustion products from the thrust chamber.

When the thrust buildup reaches a predetermined value, a mainstage OK signal is received by the control to indicate that a satisfactory start has been achieved. If this signal has not been received when timer T4 expires, the cutoff circuit is activated and the engine automatically shuts down. If it has been received at this time, the fuel bypass valve is closed. When timer T5 expires, the ignition system is turned off and the start sequence is complete.

Upon receipt of the cutoff signal, the fuel and oxidizer controls are deactivated, causing both engine propellant valves to be closed, and timer T6 is activated. When timer T6 expires, the regulator is shut down, and the engine cutoff sequence is complete. This method of sequencing provides the valves with sufficient pneumatic power to close during cutoff.

The fluidic sequence control system is shown schematically in Figure 16.7.3.8b. It was implemented with 37 OR gates and 6 bistable elements which are used in the timer circuits. The timers are adjustable in the following ranges: 0.2 to 1.0 second, 0.5 to 0.2 second, and 2 to 8 seconds, with a typical timing accuracy of ±10 percent. System supply pressure is 20 psig, with a supply flow of 5.8 scfm.

Although the system has proved feasible, power drains are excessively high; however, this can be substantially reduced by the utilization of low power logic elements. A reduction in the number of logic elements is also possible by utilizing elements with higher fan-in and fan-out and through the use of logic elements other than the OR, in particular a passive AND.

16.7.3.9 FLIGHT SUIT CONTROL SYSTEM. A prototype automatic temperature control system for liquid-cooled flight suits was developed by Honeywell for the Navy's Aerospace Crew Equipment Laboratory (Reference 6-231). The system incorporates direct sensing and control of skin temperature which is accomplished in each of four zones by the flow modulation and mixing of cold and warm fluid supplies in response to a sensor signal. The main components of the control system are mounted on a web-like undergarment which the pilot wears under his outer flight suit. The flight suit control system functioned extremely well during tests with a human subject both at rest and at various levels of activity. This example is particularly interesting because the fluidic system uses liquid media throughout.

The complete system consists of four skin temperature sensors, four fluid control modules, a bias control (for

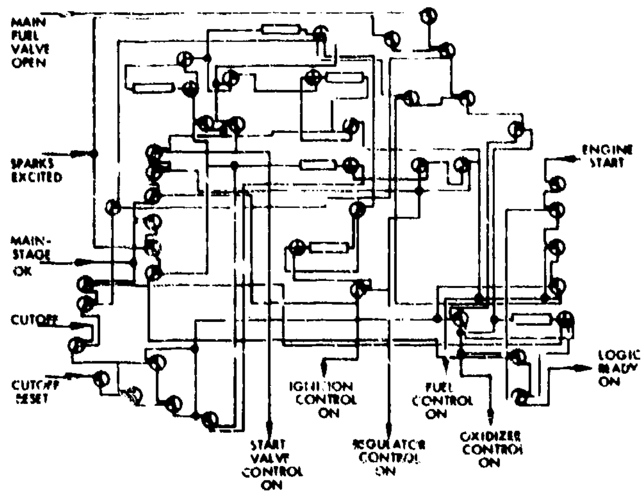


Figure 16.7.3.8b. Schematic of Fluidic Control Logic
(Adapted with permission from Reference 564-19, "Journal of Spacecraft and Rockets", April 1968, vol. 5, no. 4, S. E. Milleman, AIAA)

set-point adjustment), and the necessary interconnecting tubing (Figure 16.7.3.9a). Control zones with skin temperature sensors are located on the upper legs and upper arms. The torso is integrated into the arm zones with each half of the torso controlled by the adjacent arm zone. A separate control module is provided for each zone. An external refrigeration unit supplies cold, constant-temperature fluid to the bias control, sensors, and control modules. A heating unit, also externally located, raises coolant temperature (and hence, skin temperature) when the pilot's physical activity is too low to supply natural body heat. One main bias valve changes coolant temperature to all four zones simultaneously. Individual bias valves enable each bias pressure to be balanced against the sensor for that zone.

In each zone, the toluene fill of the sensor bulb expands against the valve diaphragm when the skin temperature rises (Figure 16.7.3.9b). The diaphragm moves the ball closer to the valve seat, which drops the pressure at the signal amplifier's right control port, so that the cold fluid flow in the right output leg is increased. This action — cold fluid flowing into the signal amplifier's right leg (Figure 16.7.3.9c) — increases pressure at the respective left control ports of the cold and warm diverters, thus increasing cold power jet flow into the suit coolant line while simultaneously decreasing the warm power jet flow into the suit coolant line. This cools the skin in the control zone area.

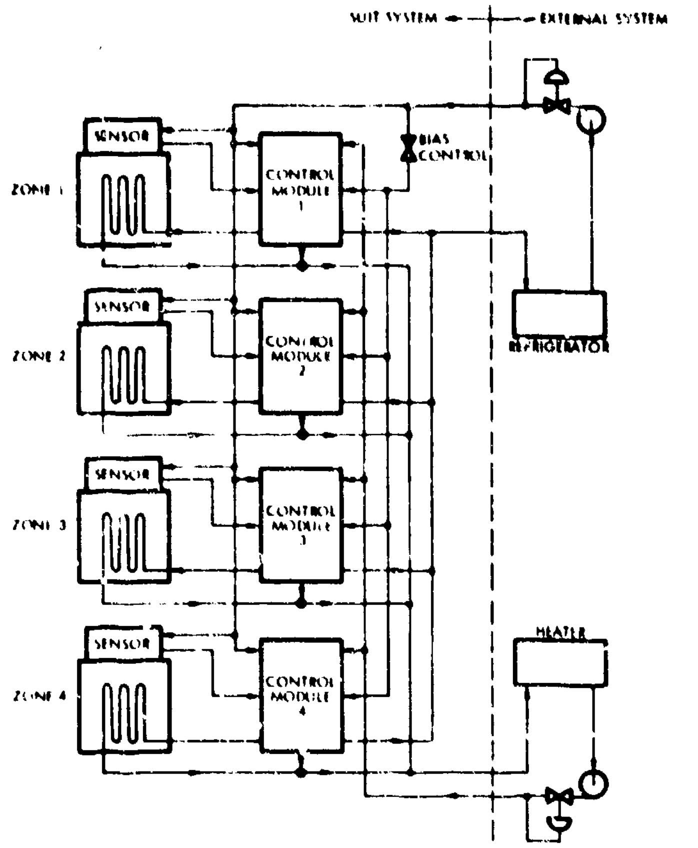


Figure 16.7.3.9a. Flight Suit Temperature Control System
(Adapted with permission from Reference 6-231, "Hydraulics and Pneumatics", November 1968, vol. 21, no. 11, E. G. Zoerb)

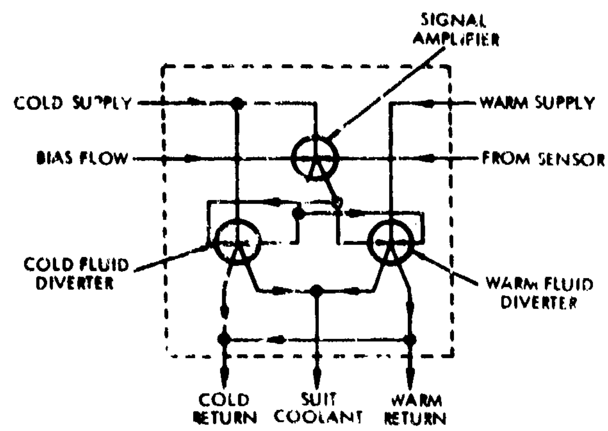


Figure 16.7.3.9c. Control Module Circuit

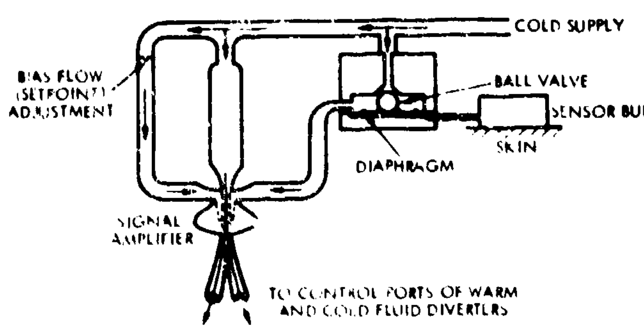


Figure 16.7.3.9b. Sensor-Signal Amplifier Circuit
(Adapted with permission from Reference 6-231, "Hydraulics and Pneumatics", November 1968, vol. 21, no. 11, E. G. Zoerb)

16.8 ANALYSIS AND DESIGN

16.8.1 Introduction

This sub-section provides a guide to the analysis and design of fluidic control systems. Reference information is provided to facilitate the design of beam deflection, wall attachment, and vortex amplifiers, and the use of passive circuit elements, i.e., resistors, capacitors, inductors, and lines. An introduction to control circuit design is also presented. Digital circuit design, including binary arithmetic, logic symbols, and digital logic operators are covered in some detail. The important fluidic operational amplifier circuits are also covered, including the implementation of a number of dynamic shaping networks. Finally, computer-aided design techniques based on programmed solutions on analog, digital, and hybrid computers are covered.

16.8.2 Basic Circuit Elements and Components

Circuit elements and components are the least common denominators in the fluidic field which are interconnected to form circuits. The designer considering the development of a fluidic system must realize that most active fluidic devices have been brought to fruition by cut-and-try methods. Many of the basic geometries have been tested over a broad range of sizes and operating pressures. Comprehensive analysis has led to useful empirical formulae and design criteria which define the interdependence of the supply and control jets upon each other and their mutual dependence on the interaction region geometry, aspect ratio, output configuration, and loading. Only computers can cope with the mathematics involved with the purely analytical design of fluidic components, and some progress has been made in this area.

Passive circuit elements, such as restrictors, lines, capacitors, and inductors, are generally needed when assembling fluidic elements in digital or analog circuits. Since these elements have been used in hydraulic and pneumatic circuitry for a long time, their performance is well documented. Passive elements produce no gain and consequently require no separate power supply. Mass flow is considered analogous to current, and pressure analogous to voltage. A fluid impedance produces a pressure drop as a function of the flow through it. Algebraic representation of the impedance may be composed of either real or pure imaginary parts or both. Section 3.0 of this handbook summarizes basic fluid mechanics and provides data on the performance of passive elements such as the pressure drop through orifices.

Simple orifices are generally used to provide a fluid resistance. When orifices are used with low pressure gases or incompressible fluids, their pressure-flow characteristics follow the square law relationship and hence are nonlinear. Tube resistors fabricated from metal or glass capillaries and porous plugs can provide essentially linear characteristics but may also have significant inductance. The primary problem with present laminar fluid resistors is a narrow linear range which must be carefully selected.

In a capacitor the pressure drop lags the flow by a phase angle of 90 degrees. Ideally, only compressible fluids show capacitive effects, and in low pressure designs capacitive effects on most liquids are neglected. In circuits employing compressible fluids, the analog of the electrical capacitor is simply a volume. Shunt capacitance is the only type that can be obtained without moving parts. The series capacitance (i.e., coupling capacitor) requires a diaphragm; the same effect can be accomplished fluidically by the use of differentiating circuits employing operational amplifiers.

In an inductor, the pressure drop leads the flow by a phase angle of 90 degrees. An acceptable fluid inductor can be made from long tubes; however, in most cases fluid inductance can be neglected with respect to resistance.

16.8.2.1 WALL ATTACHMENT AMPLIFIER. Design parameters of importance in the wall attachment amplifier are as follows:

a) Aspect ratio

b) Relative sizes of the supply and control nozzles

c) Setback of the control nozzles

d) Setback, angle, and length of the attachment wall

e) Length of the interaction region

f) Location and shape of the splitter

g) Location and area of the [illegible]

h) Output area

i) Relative area and location of the outputs and vents.

References 532-1 and 68-92 are excellent sources of information regarding the effects of varying the above parameters. Other references are 749-10, 749-11, 756-8, and 756-7.

16.8.2.2 BEAM DEFLECTION AMPLIFIER. Most of the design parameters indicated for the wall attachment amplifier apply to the beam deflection amplifier. The primary difference is that the side walls are removed in the beam deflection amplifier to prevent wall attachment. However, the size and shape of the interaction region is important. This device is usually required to provide pressure gain but must also provide some flow or power gain if it is to provide a usable output. Of particular importance in component design is the relation of the pressure and flow gains to jet deflection angle, downstream distance of a receiver, and receiver width. References 68-92, 68-95, 68-101, 184-11, 532-1, 748-1, 748-2, and 757-2 provide design information for beam deflection amplifiers.

16.8.2.3 VORTEX AMPLIFIERS. Some of the important design parameters for vortex amplifiers are discussed in Sub-Topic 16.4.4. References 37-10, 37-22, 37-26, 68-92, 68-97, 95-3', 757-4, and 757-5 provide additional design information relative to vortex amplifiers. Specific attention should be given to the work done by D. N. Wormley (Reference 95-31) which presents a simplified design procedure for nonvented vortex amplifiers operating in the incompressible flow regime, and by E. A. Mayer (Reference 68-92) which presents an experimental curve that accurately describes the nonlinear flow characteristics of the nonvented vortex amplifier over a wide range of operating conditions.

The following sections, Detailed Topics 16.8.2.4 through 16.8.2.9, are adapted from a paper by J. N. Shinn (Reference 760-1).

16.8.2.4 ORIFICE RESISTANCE. An orifice is often used for a resistor because it is so easily constructed. The resistance value of an orifice used with gases generally is calculated from incompressible relations since adequate accuracy is obtained over typical operating ranges. The weight flow rate through the orifice is obtained from

$$\dot{w} = C_d \rho A u \qquad \text{(Eq 16.8.2.4a)}$$

The coefficient of discharge, C_d may typically vary from 0.8 to 1.0, but will be assumed here as unity for simplification. The velocity, u, is obtained from the incompressible relation

$$u = \left(2g \frac{\Delta P}{\rho}\right)^{1/2} \qquad \text{(Eq 16.8.2.4b)}$$

where ΔP is the pressure drop across the orifice. Combining the previous two equations and using the perfect gas law, $\rho = P/(R_g T)$, the weight flow expression becomes

$$\dot{w} = A\left(\frac{2gP\Delta P}{R_g T}\right)^{1/2} \qquad \text{(Eq 16.8.2.4c)}$$

This expression will provide adequate accuracy (3 percent maximum error) for compressible fluids if ΔP is limited to about one-half the value for P and if the value used for P is the absolute pressure downstream of the orifice (the solution of course becomes exact as $\Delta P \rightarrow 0$). The pressure-flow curve for the orifice is thus parabolic as illustrated in Figure 16.8.2.4. The incremental resistance is the slope of the curve at the operating point of interest (e.g., point B in Figure 16.8.2.4) and becomes

$$R = \frac{\partial(\Delta P)}{\partial \dot{w}} = \frac{1}{A}\left(\frac{2R_g T\Delta P}{gP}\right)^{1/2} \qquad \text{(Eq 16.8.2.4d)}$$

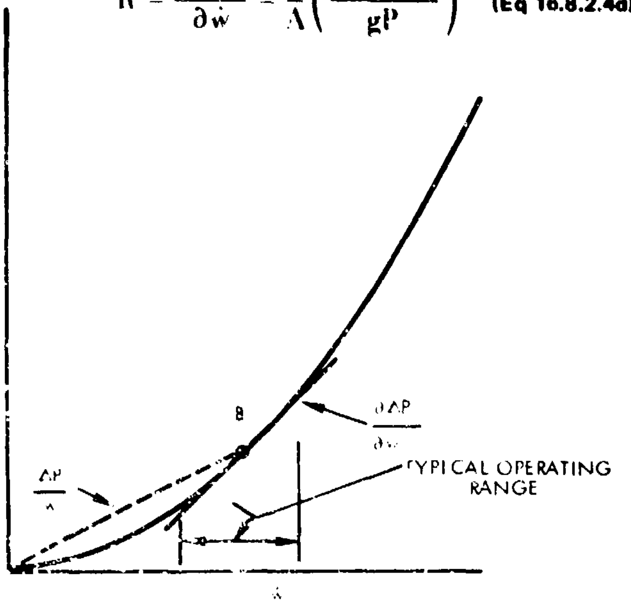


Figure 16.8.2.4. Orifice Pressure - Flow Characteristics
(Adapted with permission from Reference 760-1 "Applying Fluidics to Control Systems: Digital and Analog", W. E. BeVier, Union College, Schenectady, N. Y., 1968)

For a given gas and temperature the resistance would be linear if it were only a function of the flow area, A, but the terms ΔP and P result in nonlinearities. Equation (16.8.2.4d) is an expression for the incremental resistance, i.e., the small signal resistance suitable for a specific operating range about point B as shown in Figure 16.8.2.4. It also is of interest to identify a steady-state or operating-point resistance to calculate bias flows and pressures. From the above weight-flow relation this resistance value is

$$R_0 = \frac{\Delta P}{\dot{w}} = \frac{1}{2A}\left(\frac{2R_g T\Delta P}{gP}\right)^{1/2} \qquad \text{(Eq 16.8.2.4e)}$$

Thus R_0, the slope of the line from the origin to the operating point B, is one-half the incremental resistance for orifice flow.

16.8.2.5 LAMINAR RESISTANCE. Resistance for fluid circuits also can be provided with laminar flow in long, small-diameter passages. Calculation of resistance using incompressible flow relations provide adequate accuracy for cases if the pressure drop is not large compared to the absolute pressure level. Laminar flow requires that the Reynolds number (based on passage hydraulic diameter) be somewhat less than 2000. Assuming that a fully developed laminar flow exists over the flow length, ℓ, the expression for weight flow rate (from Poiseuille's law) is

$$\dot{w} = \frac{AD^2\rho}{32\mu\ell}\Delta P \qquad \text{(Eq 16.8.2.5a)}$$

and

$$R = \frac{\partial(\Delta P)}{\partial \dot{w}} = \frac{32\mu\ell}{AD^2\rho} \qquad \text{(Eq 16.8.2.5b)}$$

Substituting $\rho = P/(R_g T)$ the expression becomes

$$R = \frac{32\mu\ell R_g T}{AD^2 P} \qquad \text{(Eq 16.8.2.5c)}$$

In Equation (16.8.2.5c) A is the flow area and D is the hydraulic diameter, that is

$$D_h = \frac{4A}{kw} \qquad \text{(Eq 16.8.2.5d)}$$

where kw is the wetted perimeter. The value used for P in Equation (16.8.2.5c) should be the average pressure (absolute) in the resistor and ΔP should be somewhat less than P. The actual characteristic for laminar flow is nonlinear since the average gas density also increases as ΔP increases. This nonlinearity is not nearly as severe as in the case for orifice flow as shown qualitatively in Figure 16.8.2.5. Equation (16.8.2.5c) provides good results for circular cross sections or rectangular cross sections that are nearly square. If the cross section approaches a slit, the slit flow relation (see Reference 140-1)

$$R = \frac{12\mu\ell R_g T}{AD^2 P} \qquad \text{(Eq 16.8.2.5e)}$$

results in a more accurate resistance expression. Thus, the numerical constant 32 (Equation (16.8.2.5c)) will take on intermediate values and will approach the value 12 (Equation (16.8.2.5e)) as the cross sectional shape of the laminar flow path is changed from circular or square to a thin slit. Although a laminar resistor is far more linear than an orifice, it is also considerably more temperature sensitive. Using air as an example fluid, the viscosity of air can be expressed by the empirical relation

$$\mu = 3.11 \times 10^{-11}\ T^{0.71}\ \text{lb-sec/in}^2 \qquad \text{(Eq 16.8.2.5f)}$$

$$(360^\circ R \text{ to } 2000^\circ R)$$

Substitution of this relation and the value of R_g for air (640 in/°R) into Equation (16.8.2.5c) results in

$$R = \frac{6.4 \times 10^{-7}\ \ell T^{1.71}}{AD_h^2 P} \qquad \text{(Eq 16.8.2.5g)}$$

The resistance of a laminar orifice therefore is proportional to absolute temperature to the 1.71 power while orifice resistance is a function of temperature to the 0.5 power. The temperature sensitivity of both types of resistors are not negligible for circuits which must operate over wide temperature ranges. The effects of this temperature sensitivity, however, can be minimized by using differential circuitry and by using operational amplifier techniques which result in overall gain proportional to the ratio of resistance values rather than in absolute values. In addition, the temperature-sensitivity effects generally are offset by temperature effects of other components; for example, the input impedance of a proportional amplifier also increases with temperature.

16.8.2.6 LINEAR RESISTANCE. From Figure 16.8.2.5, one quickly concludes that a linear resistance should be obtainable from flow which is not quite pure laminar flow. An approximate analysis (Reference 68-92) of compressible flow and test results confirm that linear resistance is possible over a relatively large range of pressure drops. The analysis concludes that the length-to-area ratio which provides linear resistance is:

$$\frac{\ell}{A} = \frac{0.0034P}{\mu R_g T} \qquad \text{(Eq 16.8.2.6)}$$

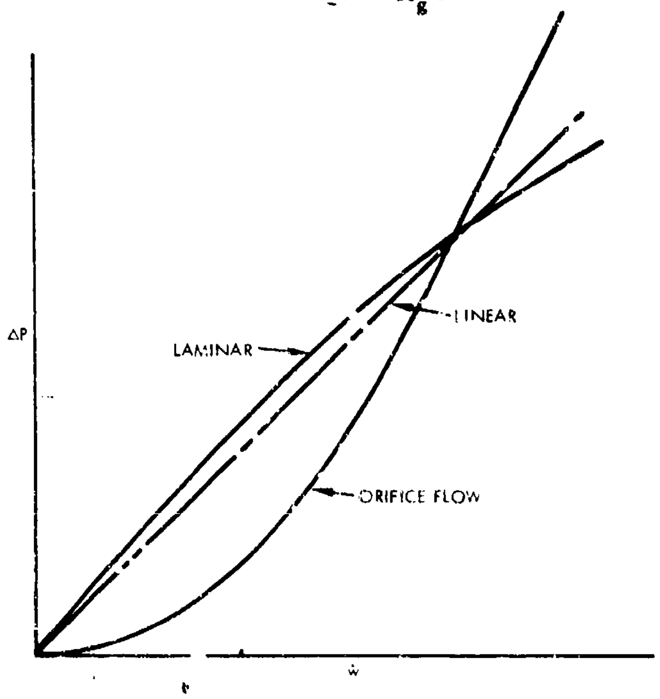


Figure 16.8.2.5. Comparison of Orifice, Laminar and Linear Flow (*Adapted with permission from Reference 760-1, "Applying Fluidics to Control Systems: Digital and Analog", W. E. DeVier, Union College, Schenectady, N. Y., 1968*)

Experimental results show that, to obtain linear resistance, the length should be approximately 10 percent longer than indicated by Equation (16.8.2.5c). For air at 528°R and the downstream pressure, P, at atmospheric pressure, the ℓ/D which resulted in linear resistance was approximately 300. Constant resistance (within 1.5 percent) was attainable with pressure drops as high as 7.5 psi. Since the flow is very nearly ideal laminar flow, Equation (16.8.2.5c) can be used to predict the resistance value of a flow channel with adequate accuracy. The reader is cautioned that the validity of Equation (16.8.2.6) is predicated on laminar flow; if the pressure drop is sufficiently large (i.e., Reynolds number > 2000) turbulent flow will occur and laminar flow relations are no longer applicable.

16.8.2.7 FLUIDIC RESISTOR CONSIDERATIONS. Most resistors used in fluidic circuitry are laminar (near-linear) type elements. The flow passages generally are made from photoetched metal laminates, glass, or a photosensitive plastic. The most commonly used units of fluidic resistance are sec/in^2 and the value quoted for a particular device generally is based on air at 14.7 psia and 68°F. As an example calculation, using Equation (16.8.2.4d), the resistance value of a 0.01-inch diameter orifice operating with a 1.0 psi pressure drop is

(Eq 16.8.2.7a)

$$R = \frac{1}{A}\left(\frac{2R_g T \Delta P}{gP}\right)^{1/2} = \frac{1}{(0.785)(10^{-4})} \cdot \left[\frac{2\,(640\ \text{in}/^\circ\text{R})\,(528^\circ\text{R})\,(1.0\ \text{lb/in}^2)}{(386\ \text{in/sec}^2)\,(14.7\ \text{lb/in}^2)}\right]^{1/2}$$

or

$$R = (1.39)(10^5) = 140{,}000\ \text{sec/in}^2$$

The operating-point resistance, as shown by Equation (16.8.2.4e) is one-half this value, i.e., 70,000. The above calculation was based on a flow coefficient $C_d = 1$. Depending on the upstream conditions, the entrance geometry, and the length-to-diameter of the orifice, the value of C_d can vary considerably and may result in a measured resistance 20 to 30 percent higher than that calculated.

As a comparison, the resistance of a laminar resistor with a 0.015 x 0.015-inch cross sectional dimension and a length of 3.3 inches is found from Equation (16.8.2.5g) to be

(Eq 16.8.2.7b)

$$R = \frac{(6.4)(10^{-7}\,\ell T^{1.71})}{A D_h^2 P}$$

$$= \frac{(6.4)\,(10^{-7}\ \text{lb-sec/in}^\circ\text{R})\,(3.3\ \text{in.})\,(528^{1.71}\,^\circ\text{R})}{(2.25)\,(10^{-4}\ \text{in}^2)\,(2.25)\,(10^{-4}\ \text{in}^2)\,(14.7\ \text{lb/in}^2)}$$

or

$$R = (1.40)\,(10^5)\ \text{or}\ 140{,}000\ \text{sec/in}^2$$

16.8.2.8 CAPACITORS. In fluidic circuits which use gas as the operating medium, the gas compressibility results in energy storage analogous to that of a capacitor in electronic circuitry. Hence, the fluidic capacitor is simply a volume for gas storage and can be used in conjunction with resistors to form first order lags with single time constants. Since liquids are essentially incompressible, much larger volumes are required to produce significant capacitance, and an accumulator or other moving parts device is generally used to provide the required energy storage in a smaller space. It is important to note that there is a capacitance associated with every element of volume in a fluidic circuit. The discussion below is concerned only with capacitance-associated gases since they are by far the most common fluidic operating media.

The capacitor in fluidics provides the function of a shunt-to-ground capacitance; the series or coupling capacitor has no analog in fluidic circuitry. A simplified analysis for determining the expression for calculating capacitance assumes adiabatic flow in a fixed volume (see Figure 16.8.2.8) so that the energy equation is

$$\dot{w}_s (c_p T_s) = \left[\frac{d}{dt} \rho_1 V_1 (c_v T_1)\right] \quad \text{(Eq 16.8.2.8a)}$$

where

$c_p T_s$ = the specific enthalpy of the supply

$c_v T_1$ = the specific internal energy of the volume or capacitance.

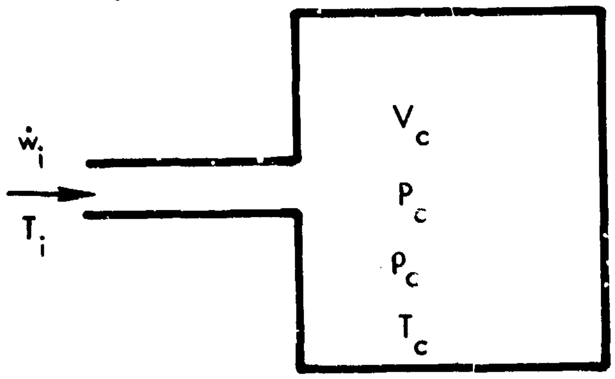


Figure 16.8.2.8. Fluidic Capacitance Model
(Adapted with permission from Reference 760-1, "Applying Fluidics to Control Systems: Digital and Analog", W. E. BeVier, Union College, Schenectady, N. Y., 1968)

From the perfect gas law

$$\rho_1 T_1 = \frac{P_1}{R_g} \quad \text{(Eq 16.8.2.8b)}$$

By substituting Equation (16.8.2.8b) in Equation (16.8.2.8a) and with V_1 and c_v constant

$$\dot{w}_1 c_p T_1 = \left(\frac{c_v V_1}{R_g}\right)\frac{d}{dt}(P_1) \quad \text{(Eq 16.8.2.8c)}$$

Rearranging and using LaPlace notation

$$\dot{w}_s = \left(\frac{c_v V_1}{c_p R_g T_s}\right) s P_1 \quad \text{(Eq 16.8.2.8d)}$$

Since $c_p/c_v = k$

$$P_1 = \frac{k R_g T_s}{V_1}\left(\frac{\dot{w}_s}{s}\right) \quad \text{(Eq 16.8.2.8e)}$$

or

$$P_1 = \left(\frac{1}{C_1 s}\right)\dot{w}_s \quad \text{(Eq 16.8.2.8f)}$$

where

$$C_1 = \frac{V_1}{k R_g T_s} \quad \text{(Eq 16.8.2.8g)}$$

The above derivation is based on gravimetric rather than volumetric analysis. This expression for capacitance assumes no heat transfer, i.e., an adiabatic process, which is approached for rapid pressure changes within the capacitance volume. If the pressure changes occur very slowly, an isothermal process is approached and the expression for capacitance

$$C_1 = \frac{V_1}{R_g T_s} \quad \text{(Eq 16.8.2.8h)}$$

is more valid. Practical experience with air (room temperature) in fluidic circuits has shown that a value of 1.2 in place of k is the best compromise. For air at room temperature

$$C_{air} = \frac{V\ in^3}{(1.2)(640\ in/^\circ R)(528^\circ R)} \quad \text{(Eq 16.8.2.8i)}$$

$$= (2.48)(10^{-6}\ V\ in^2)$$

Thus a typical 1.0 in^3 capacitor would have a capacitance value of 2.48×10^{-6} in^2. When combined with the typical laminar resistor, Equation (16.8.2.7b) would result in a time constant value of

$$\tau = RC$$

$$= (1.4)(10^5\ sec/in^2)(2.48)(10^{-6}\ in^2)$$

$$= 0.35\ sec$$

It is interesting to note that if the nonlinear orifice-type resistor is used with the capacitor, the time constant varies as the pressure in the capacitor approaches final value. In fact, the resistance of the orifice approaches zero (hence $\tau \rightarrow 0$) as the pressure drop across it becomes zero, as shown by the slope of the curve at the origin in Figure 16.8.2.4.

16.8.2.9 INDUCTORS. The fluid in fluidic passageways has inertial properties or inertance which can result in significant dynamic characteristics. The inertial effects, which are present for both compressible and incompressible fluids, result in characteristics similar to inductance in electrical circuits. Consider a line of length ℓ and cross sectional area A as shown in Figure 16.8.2.9. From Newton's second law

$$\text{Force} = \text{mass x acceleration}$$

$$(P_1 - P_2)A = \rho A \ell \left(\frac{d\bar{u}}{dt}\right) \quad \text{(Eq 16.8.2.9a)}$$

and since

$$\rho A \bar{u} = \frac{\dot{w}}{g} \quad \text{(Eq 16.8.2.9b)}$$

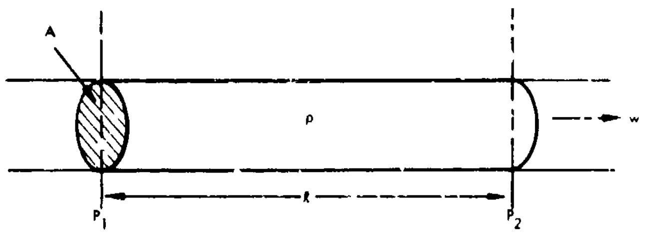


Figure 16.8.2.9. Fluidic Inductance Model
(Adapted with permission from Reference 760-1, "Applying Fluidics to Control Systems: Digital and Analog", W. F. BeVier, Union College, Schenectady, N. Y., 1968)

then

$$(P_1 - P_2)A = \frac{\ell}{g}\left(\frac{d\dot{w}}{dt}\right) \quad \text{(Eq 16.8.2.9c)}$$

or

$$\Delta P = \frac{\ell}{Ag}\left(\frac{d\dot{w}}{dt}\right) \quad \text{(Eq 16.8.2.9d)}$$

Using LaPlace notation gives

$$\Delta P = Ls\Delta\dot{w} \quad \text{(Eq 16.8.2.9e)}$$

where

$$L = \frac{\ell}{gA} \quad \text{(Eq 16.8.2.9f)}$$

Thus the inductance, L, of a line is directly proportional to its length and inversely proportional to the cross sectional flow area. In practice, fluidic inductance without corresponding resistance is impossible to obtain. Laminar or linear resistors have inductive properties which must be considered in high response circuitry. In fact it is often convenient (for filtering, etc.) to generate a time constant with the resistive inductor. As a typical example, the resistor in Equation (16.8.2.7b) would have an inductance value of

$$L = \frac{\ell}{gA} = \frac{3.3 \text{ in.}}{(386 \text{ in/sec}^2)(0.015 \text{ in.})^2} = 38 \text{ sec}^2/\text{in}^2$$

and the time constant of the resistor would be

$$\tau = \frac{L}{R} = \frac{38 \text{ sec}^2/\text{in}^2}{(140)(10^3 \text{ sec/in}^2)} = 0.27 \text{ msec}$$

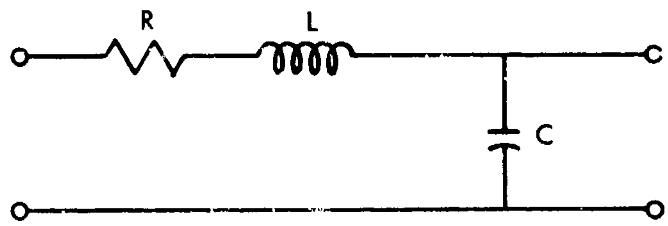


Figure 16.8.2.10. Lumped-Parameter Model of Line

It is noted that the expression derived above for inductance is based on gravimetric rather than volumetric analysis.

16.8.2.10 LINES. The lines which interconnect fluidic elements usually have more dynamic effect on the circuit than the response of the individual elements (Reference 74-53). The line has both capacitive and inductive reactance as illustrated in the simple lumped-parameter model shown in Figure 16.8.2.10. For a long line a distributed parameter analysis, which is beyond the scope of this discussion, must be used (see References 45-1 and 770-1). A line is defined as short if its length is small (e.g., 10 percent) compared to the wavelength of the maximum frequency of interest. At 100 cps the wavelength is 130 inches (propagation velocity 13,000 in/sec) and thus the characteristics of a 13-inch line can be approximated with a lumped parameter model as shown in Figure 16.8.2.10a. The inductance and capacitance can be calculated from Equations (16.8.2.9f) and (16.8.2.8i), respectively.

A simplified technique for the interconnection of digital fluidic amplifiers which should be considered if it becomes necessary to reduce signal attenuation in the interconnecting lines between fluidic circuit elements is discussed in Reference 36-74.

16.8.3 Control Circuit Design

It is a well-recognized fact that interconnecting fluidic devices into circuits and systems is a problem in the field of fluidics. It is very important to examine these difficulties carefully. Many of them have been faced to a certain degree by those working in the fields of mechanics, electronics, and hydraulics. Nonlinearity is one such difficulty. It is apparent that transistors, mechanical linkages, and servo-valves are nonlinear. A second difficulty concerns continuity. Both electric and hydraulic circuits must satisfy certain limited continuity principles. Kirchoff's law is a statement of continuity for electric circuits. A third difficulty results from the large number of relevant variables. Electric circuit and hydraulic system performance are affected by the variation of numerous parameters including temperature, aging, and bulk modulus. Even considering these difficulties, many electronic and hydraulic systems have been produced and are operating satisfactorily.

Therefore, it is apparent that a person working in fluidics has something to learn from the fields of mechanics, electronics, and hydraulics. He should adapt his thinking to take advantage of the analogous system design methods already developed in those fields. In other words, a thorough understanding of fluidic system operation is difficult and will remain so for many years. Approximate methods based on those used in other fields yield good results in the great majority of cases and provide tremendous insight into the operation of the system. As a result, these techniques will accelerate the advance of fluidics technology by making it possible for a circuit design engineer to use fluidic devices now, without an elaborate education in fluid mechanics (Reference 765-1).

The material in Detailed Topics 16.8.3.1 through 16.8.3.10 was adapted from a paper by C.A. Belsterling (Reference 771-2).

16.8.3.1 THE SYSTEMS APPROACH. In the design of any system of interconnected components, it is necessary to take into account the effect of one component upon the other — that is, of cross-coupling. This statement is true whether the components are electronic, mechanical, hydraulic, acoustic, or fluidic.

The most practical systematic procedure used in control system design is the so-called *black-box* method. This technique requires that each component be isolated from all other components in the system and then be subjected to a few simple tests under typical operating conditions. These processes are normally performed by the manufacturer before he ships a component to a user. For example, the vacuum tube manufacturer supplies a set of characteristic curves and dynamic parameters for each tube he markets, and the servovalve manufacturer supplies output pressure-flow characteristics. By using the same proven approach, all the mathematical tools now used in electronics and hydraulics can be applied to fluidic circuit analysis and design.

For most practical cases, it is possible to describe the total behavior of any fluidic device using the three characteristics illustrated in Figure 16.8.3.1. Input characteristics define the particular load that an input signal sees when it is applied at the input ports. Transfer characteristics determine precisely what happens to the output when an input signal is applied. Output characteristics explain the manner in which the output signal is affected when an external load is connected at the output ports. For static and large signal analysis, these three characteristics are most conveniently defined graphically, since graphs consider device nonlinearities without a need for complex mathematics.

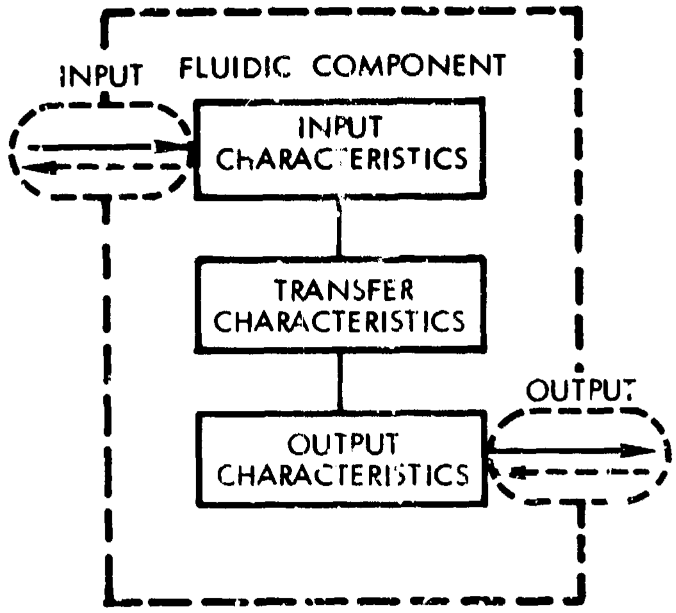


Figure 16.8.3.1. Signal Flow Characteristics of any Fluidic Component
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The term *static* is used to define those cases in which time is a negligible variable in the analysis of performance, such as in biasing and in the response to a slowly-changing signal. The term *large signal* is used to explain those cases in which the signal distortion due to the nonlinear characteristics of the device is important in the analysis of performance, such as in the response to signals subject to large excursions. Exactly what constitutes a negligible error in the analysis must be decided by the control systems engineer.

For dynamic and small signal analyses, these characteristics are instead described in terms of equivalent electric circuits, primarily because well-developed linear circuit theory is directly applicable to the calculation of performance. The term *dynamic* is used to define those cases in which the effects of energy storage cannot be neglected in the analysis of performance, such as in the response to transient or high-frequency sinusoidal signals. The term *small signal* is used for those cases in which the signal excursion is small enough to permit the assumption of linear characteristics around the operating point, without introducing unreasonable errors in the analysis of performance, such as in the response to incremental sinusoidal signals used in frequency response analysis. Exactly what can be considered negligible or reasonable errors in the analysis must again be decided by the control systems engineer.

16.8.3.2 STATIC CHARACTERISTICS. Typical fluidic component characteristics can be illustrated for the most common analog fluidic amplifier, the vented jet-interaction amplifier, which is shown in Figure 16.8.3.2a. Note that there are two control ports and two output ports which are operated in a differential mode.

Graphically, the input characteristics of a single input port are plots of the control flow versus the pressure applied at each control port shown in Figure 16.8.3.2b. In most vented amplifiers the input characteristics are practically independent of output loading, but this may not be the case for other types such as the closed configuration. Note that there are two separate curves. The locus of bias points

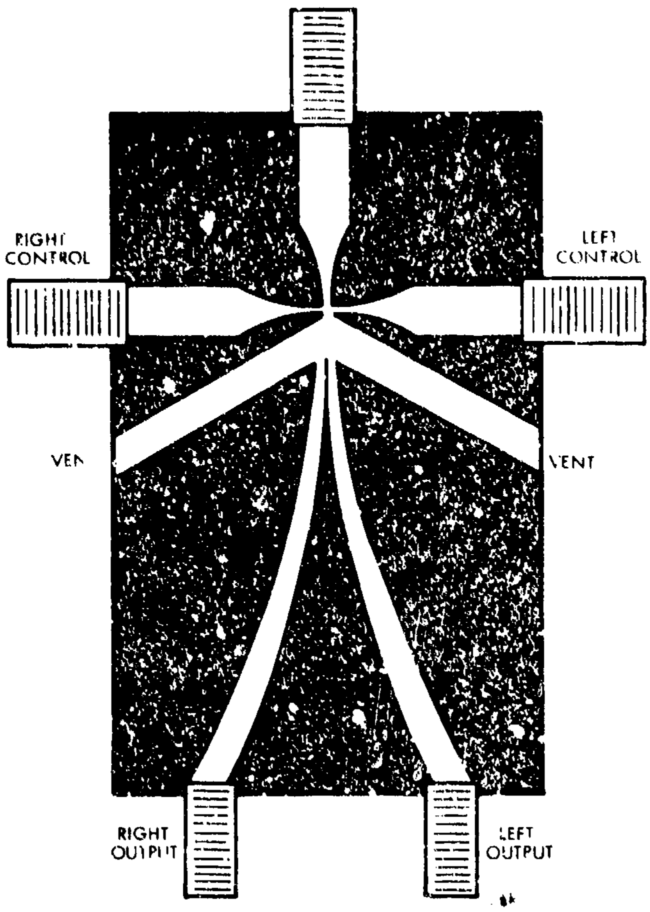

Figure 16.8.3.2a. Description of a Typical Vented Jet-Interaction Amplifier
(Adapted with permission from Reference 771-2, "Fluidics Quarterly", January 1968, C. A. Belsterling, Copyright 1968 by Fluid Amplifier Associates)

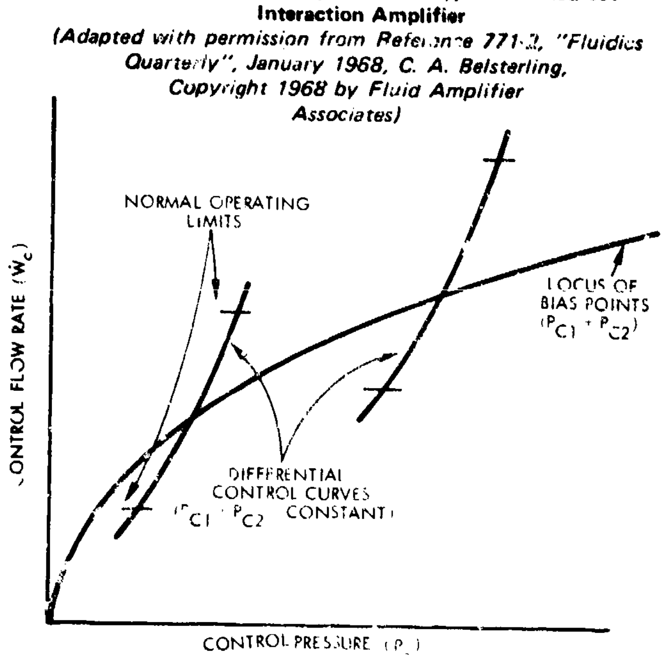

Figure 16.8.3.2b. Typical Static Input Characteristics
(Adapted with permission from Reference 771-2, "Fluidics Quarterly", January 1968, C. A. Belsterling, Copyright 1968 by Fluid Amplifier Associates)

is the curve generated when both control pressures are equal. Their equality must be considered when designing for no-signal matching (or biasing). The differential control curves, on the other hand, are generated when one control port pressure is increased and the other decreased the same amount, keeping the average of the two always at a fixed bias level. This condition must be considered when analyzing the effect of a differential signal. Note that a differential control curve can be generated at any particular level of bias pressure.

The transfer characteristics shown in Figure 16.8.3.2c define the gain of the amplifier and are presented graphically as a family of curves of output pressure versus control pressure, with load as the parameter. Note that it is normal for the pressure gain to decrease as the load impedance is reduced (opened from blocked conditions) and that beyond saturation a reversal of slope can occur.

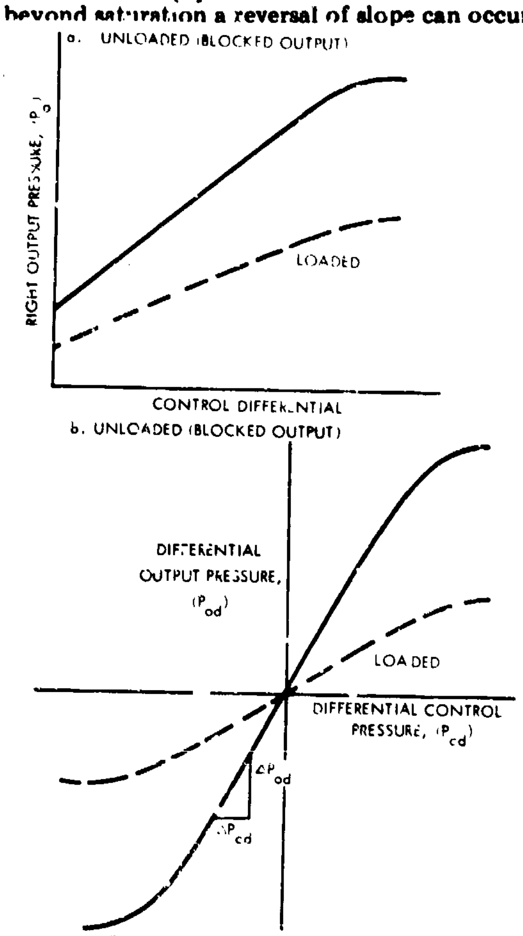

Figure 16.8.3.2c. Typical Transfer Curves
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The output characteristics shown in Figure 16.8.3.2d are plots of the output flow versus output pressure as the load is varied from near-zero impedance (relatively large flow) to near-infinite impedance (blocked output port). Because the output characteristics are also a function of the control signal, a complete graphical description of the output characteristics requires a family of curves of output flow versus output pressure with control pressure (or control flow) as the parameter. Note that the transfer characteristics and the output characteristics are only slightly different; both of them report the output behavior, under load, in response to an input signal. Therefore, only one of these two sets of curves is required to define performance. The output characteristics are preferred because they are more convenient for analyzing the problems of cascading components.

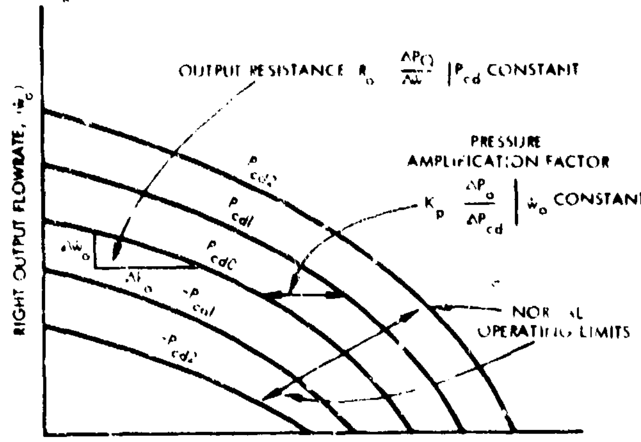

Figure 16.8.3.2d. Typical Static Output Characteristics
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Digital fluidic component characteristics are illustrated for the most common type, the vented wall-attachment amplifier, shown in Figure 16.8.3.2e.

Figure 16.8.3.2f graphically demonstrates that the input characteristics of a single input port are a plot of the control flow versus the pressure applied at each control port. In this case, as with the proportional vented amplifier, the input characteristics are practically independent of output loading. The most striking feature of the input characteristic is its abrupt discontinuity. This occurs at the point of switching: the pressure has increased sufficiently to detach the power stream from the adjacent wall and allows the stream to reattach to the opposite wall. The impedance for the control port is thereby increased. Note that the curve exhibits considerable hysteresis due to the latching effect of wall attachment. In other words, the curve of increasing control pressure to the point of switching is different from the curve of decreasing control pressure to the point of reattaching. Each curve is dependent on the pressure applied on the opposite control port.

The switching characteristic is shown in Figure 16.8.3.2g. Utilizing effective vents that prevent the feedback of output pressure in to the interaction region, the switching characteristics can be illustrated as a family of curves with load as a parameter. Note that it is normal for the output pressure to decrease as load impedance is reduced (opened from blocked conditions).

The output characteristics shown in Figure 16.8.3.2h are plots of the output flow versus the output pressure as the load is varied from near-zero impedance to near-infinite impedance. Since the output characteristics are a function of the control signal, a complete graphic description of

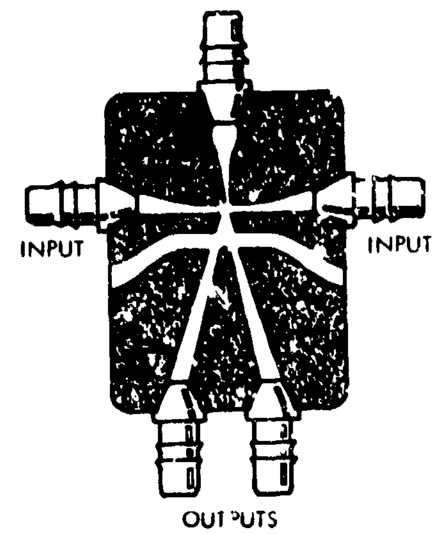

Figure 16.8.3.2e. Typical Wall-Attachment Amplifier
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the output characteristics of the digital amplifier can be given by two curves: one relationship for the condition in which the power stream is deflected into the output leg being measured and the other describing the behavior if the power stream is deflected away from the output leg measured. Note again that the switching and output characteristics of the digital amplifier in determining the output response under load to an input signal are only

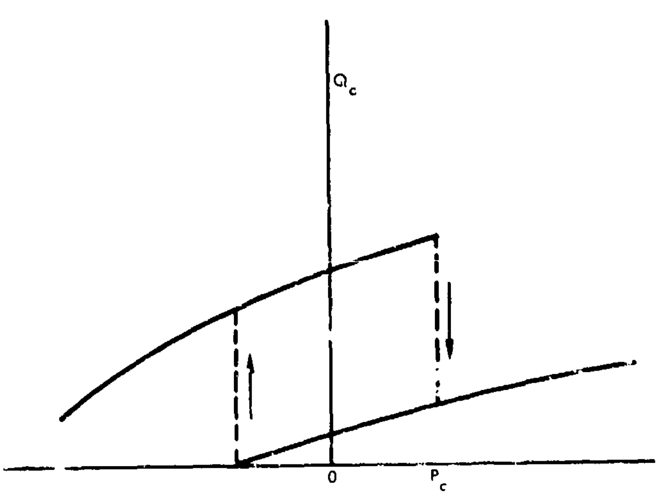

Figure 16.8.3.2f. Input Characteristics of Typical Wall-Attachment Amplifier
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slightly different. Therefore in this case, only one of these two sets of curves is required to define performance. The output characteristics are preferred because they are more convenient for analyzing the problems of cascading components.

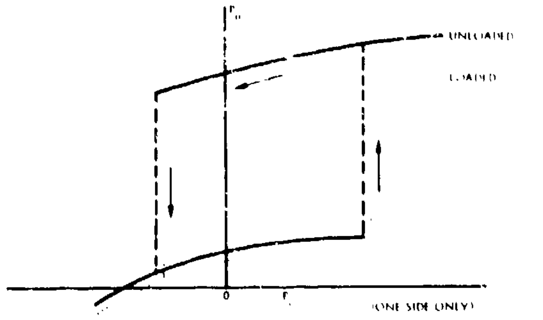

Figure 16.8.3.2g. Switching Characteristic of Typical Wall-Attachment Amplifier
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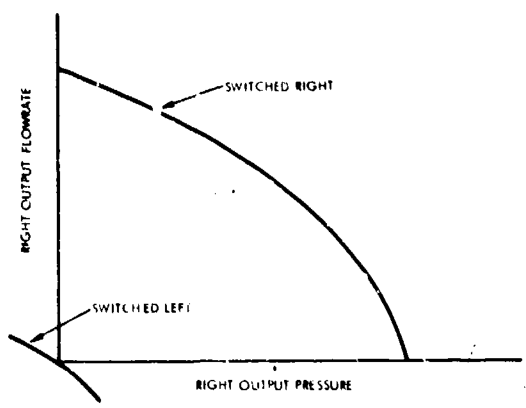

Figure 16.8.3.2h. Output Characteristics of Typical Wall-Attachment Amplifier
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The characteristic curves of active fluidic devices are, of course, a function of supply pressure. Therefore, to be complete it is necessary to have a set of input and output characteristics for every allowable supply pressure. This can be done by providing input and output curves for a number of supply pressures and interpolating when necessary, but it is more convenient to provide a single set of input and output characteristics normalized with respect to supply pressure and supply flow. In this case, it is necessary to provide another characteristic curve (essentially the power nozzle characteristic shown in Figure 16.8.3.2i) defining how supply flow varies with supply pressure.

In summary, the static operating characteristics of an active fluidic device under normal operating conditions can be described by only three sets of curves. They consist of input characteristics (normalized) including bias and differential curves, output characteristics (normalized) with an

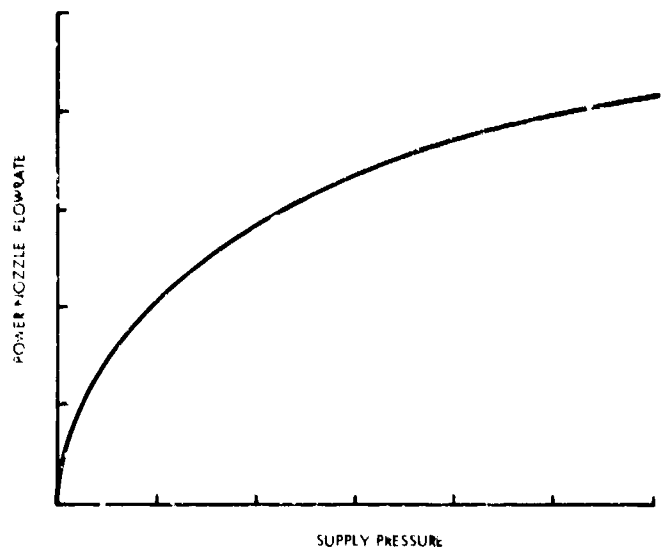

Figure 16.8.3.2i. Typical Power Nozzle Characteristic of Fluidic Amplifier
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input signal as the parameter, and the power nozzle characteristic. For passive digital logic and elements such as resistors, capacitors, and inductors, only the input and output characteristics are required. When interfaces are utilized, only the output and power nozzle characteristics are required for transducers and sensors, and only the input characteristics (not normalized) are required for actuators.

16.8.3.3 EQUIVALENT ELECTRIC CIRCUITS. The analog jet-interaction fluidic amplifier is represented as an equivalent electric circuit in Figure 16.8.3.3a. The input characteristics are described in terms of simple impedances between the two control ports or between a control port and return. The transfer characteristics are represented by a pressure generator and network whose output is a function of the net pressure appearing at the control nozzle. The output characteristics are shown as simple series and shunt impedances directly coupled to the load impedance. These elements of the electrical equivalent circuits can all be calculated from the graphical characteristics, circuit dimensions, and conditions at the bias (quiescent, no-signal) operating point.

The digital fluidic amplifier also can be represented as an equivalent electric circuit (Figure 16.8.3.3b). The input characteristics are described in terms of nonlinear impedances between a control port and return which are controlled by output conditions. The switching characteristics are illustrated by a pressure generator with infinite gain and an output-controlled reference diode at the input to the pressure generator. The output characteristics are represented as simple linear series and shunt impedances directly coupled to the load impedance. These elements of the electrical equivalent circuits can again be calculated from the graphical characteristics, circuit dimensions, and conditions at the bias operating point.

In summary, the small-signal and dynamic characteristics of a fluidic device can be represented by an equivalent electric circuit. This circuit would contain various linear and nonlinear impedances and a generator.

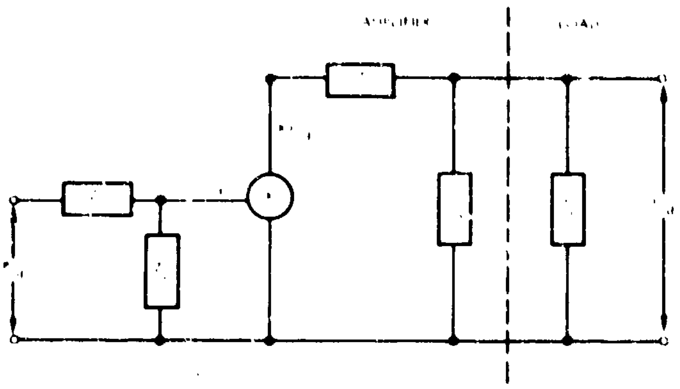

Figure 16.8.3.3a. Generalized Small-Signal Equivalent Circuit of a Vented Jet-Interaction Amplifier
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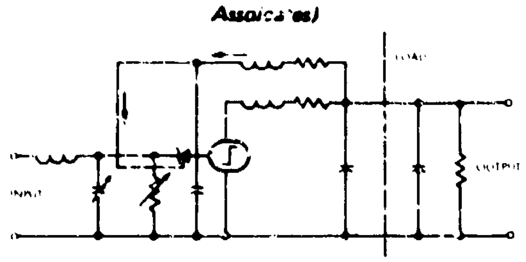

Figure 16.8.3.3b. Equivalent Electric Circuit of Wall-Attachment Amplifier
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16.8.3.4 PERFORMANCE PARAMETERS AND CIRCUIT ELEMENTS. Performance parameters can be utilized for two basic purposes: to describe the behavior of a device under static or dynamic conditions (such as pressure gain) and to provide the data necessary to calculate behavior from basic information (such as input resistance). The performance parameters most pertinent to fluidic control systems are defined in the following paragraphs.

Output resistance, R_o, is the ratio of a change in output pressure to a change in output flowrate for a fixed control signal, that is

$$R_o = \left.\frac{\Delta P_o}{\Delta \dot{w}_o}\right|_{P_{cd}\ \text{constant}} \qquad \textbf{(Eq 16.8.3.4a)}$$

With reference to the static output characteristics of a typical amplifier shown in Figure 16.8.3.4a, the output resistance is simply a slope of one of the family of curves. Thus the output characteristic curves define the output resistance under all static conditions, but because the characteristic curves are not linear, the actual output resistance is quite variable. Therefore in determining the appropriate numerical value, the resistance must be calculated at the point at which the amplifier is operated when connected in a circuit.

The pressure amplification factor for amplifiers, K_p, is the ratio of the change in output pressure to the change in control pressure when the output flow is constant. That is

$$K_p = \left.\frac{\Delta P_o}{\Delta P_{cd}}\right|_{\dot{w}_o\ \text{constant}} \qquad \textbf{(Eq 16.8.3.4b)}$$

In effect, this is the maximum pressure gain an amplifier can deliver if there are no loading effects (zero amplifier output resistance). With reference to the output characteristic curves for a typical amplifier shown in Figure 16.8.3.4a, one can see that the amplification factor is a function of the horizontal distance between the output resistance curves. Since the curves are neither linear nor evenly spaced, it is evident that the pressure amplification factor is quite variable. Therefore in determining the appropriate numerical value for K_p, calculations must be made in the vicinity of the point at which the amplifier operates in a circuit.

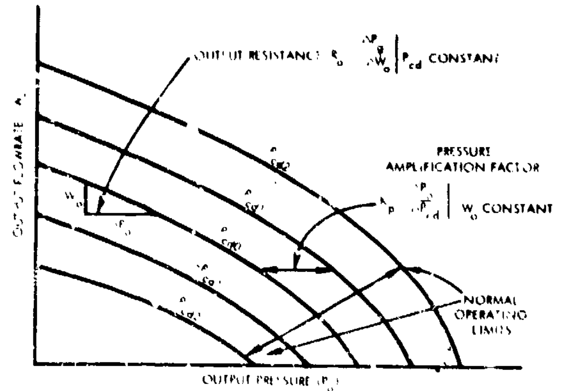

Figure 16.8.3.4a. Definition of Parameters from Output Characteristics
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Pressure gain, G_p, is the ratio of the change in output pressure to the change in control pressure when the fluidic amplifier is operating in a particular circuit with a particular load. For a differential analog amplifier

$$G_p = \frac{\Delta P_{od}}{\Delta P_{cd}} \qquad \textbf{(Eq 16.8.3.4c)}$$

The transfer curve for a typical differential amplifier is shown in Figure 16.8.3.4b. By the above definition, the pressure gain is the slope of the transfer curve. Since the curve is not linear, the point at which the amplifier operates in a circuit must be specified in calculating a numerical value for pressure gain.

For a digital amplifier, the definition of pressure gain changes. In this case pressure gain refers to the ratio of the change in output pressure to the change in control pressure required for switching to occur. That is

$$G_p = \frac{\Delta P_{od}\ (sw)}{\Delta P_{cd}\ (sw)} \qquad \textbf{(Eq 16.8.3.4d)}$$

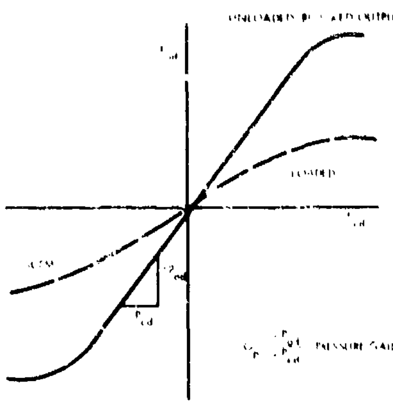

Figure 16.8.3.4b. Definition of Parameters from Transfer Characteristics
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According to this definition, it should be recognized that the gain of a digital device can be infinite if it has negligible hysteresis.

Input resistance, R_c, is the ratio of the change in control pressure to the change in control flow when the bias pressure is held constant. That is

$$R_c = \left.\frac{\Delta P_c}{\Delta \dot{w}_c}\right|_{(P_{cl} + P_{cr})\ \text{constant}} \qquad \text{(Eq 16.8.3.4e)}$$

With reference to the typical differential amplifier static input characteristics (Figure 16.8.3.4c), the input resistance is simply a function of the slope of the differential curve. Since the curves are nonlinear, the numerical value of the input resistance must be calculated at the point at which the amplifier operates when connected in a circuit.

Because of the compressibility of the operating fluid, there is an equivalent capacitor, C, formed by every element of volume under pressure in the fluidic circuit. As a result, the change of pressure at every point is delayed until there is sufficient flow to satisfy the conditions of compressibility at the new pressure level. The effect is analogous to an electrical shunt capacitor and can be treated as such in equivalent circuit analysis. The equivalent capacitance of a fluidic device can be determined from the equations in Detailed Topic 16.8.2.8. Since passages are seldom uniform and the pressure is not the same in every section, each must be calculated as a separate element and then added together to arrive at a total circuit capacitance. The pressure used in the calculation of the equivalent capacitance must, of course, correspond to the point at which the device operates in a circuit. In digital wall-attachment devices, the major portion of the effective capacitance is due to "charging" the attachment bubble.

Because of the inertance of the operating fluid, there is an equivalent inductor, L, formed by every element of mass in the fluid circuit. As a result, the change in flow at every point is delayed until sufficient forces can build up and accelerate the flow to the new level. The effect is analogous

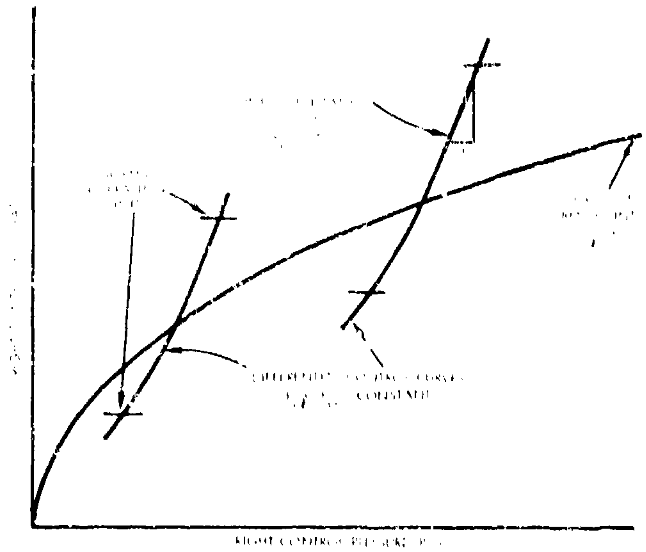

Figure 16.8.3.4c. Definitions of Parameters from Input Characteristics
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to an electrical series inductor and can be treated as such in an equivalent circuit analysis. The equivalent inductance of a fluidic device can be determined from the equations in Detailed Topic 16.8.2.9. Since the area of the passages of fluidic circuits is seldom uniform and the density is not the same in every section, each must be calculated as a separate element and then added together to arrive at a total circuit inductance. The pressure used to calculate mass density must, of course, correspond with the point at which the device operates in a circuit.

16.8.3.5 LARGE-SIGNAL ANALYSIS AND MATCHING. Like most electronic and hydraulic components, the characteristics of fluidic components are nonlinear. When operated at extremes or driven by a large signal, the performance parameters are not constant and, consequently, the output will be a distorted reproduction of the input signal. In the cases where the effects of these nonlinearities are significant and the effect of time is less important, the graphical method of performance analysis is most convenient because, as mentioned previously, the system designer can account for the nonlinearities without the use of complex mathematics.

The importance of the effects of nonlinearities may be determined from a preliminary analysis of the magnitude of the signal excursion and from the degree to which the output characteristics deviate from linearity over this excursion. An estimate of the contribution to error of time-dependent circuit parameters at the expected signal frequencies must also be added.

Consider the manner in which the jet-interaction amplifier behaves in a circuit. Figure 16.8.3.5a compares the fluidic amplifier with both the transistor and the spool valve. Note that the amplifier is equivalent to a differential connection of transistors or to a spool valve. In these cases, it is normally necessary to analyze the behavior of each leg and take the difference in output signals. For an analysis of coupling fluidic devices, consider a differential fluidic amplifier with a passive load having the characteristics

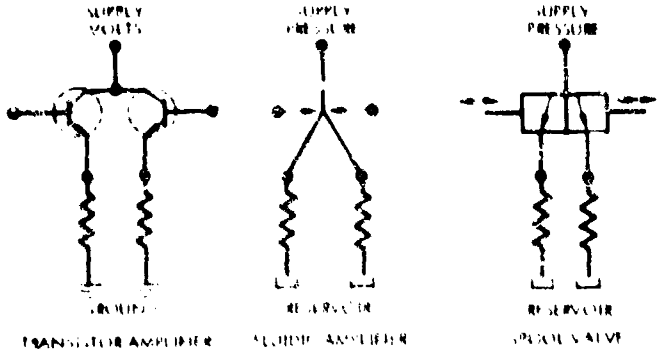

Figure 16.8.3.5a. Electronic-Fluidic-Hydraulic Circuit Analogies

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shown in Figure 16.8.3.5b. The output characteristics of the amplifier would appear as shown in (a) of Figure 16.8.3.5b. In determining what will happen when the amplifier is connected to the passive load, the characteristics of each side must be considered.

The circuit designer must realize, first, that the output characteristics demonstrate the way in which the amplifier will behave with any load. In fact, the curves are plotted from the performance of the amplifier for a number of loads, from open outlet port (zero impedance) to blocked outlet (infinite impedance). Second, he must then consider that the load characteristic is a single line. In other words, there is only one flow level for each pressure applied. Third, he must be aware that if the load is connected to the amplifier, their pressures and flows are the same. The amplifier output pressure is identical to the passive load pressure and the amplifier output flow is identical to the passive load flow. Therefore, the combined behavior of an amplifier with passive load can be found simply by plotting their characteristics on the same graph as shown in (c) of Figure 16.8.3.5b. The passive load characteristics are superimposed on the amplifier output characteristics as a load line. Since pressures and flows must be identical in both components, the points of intersection of the curves can be the only operating points.

Consider as a second case the cascading of two differential fluidic amplifiers (Figure 16.8.3.5c). For the driving amplifier, it would be essential to have a set of output characteristics, as shown in (a) of Figure 16.8.3.5c, and for the driven amplifier, a set of input characteristics, as shown in (b) of Figure 16.8.3.5c.

When the output of the driver is connected to the input of the driven amplifier, the output pressure and flow of the first must equal the input pressure and flow of the second. That is, the only stable operating conditions are those in which the output pressure and flow of the driver amplifier coincide with the input pressure and flow of the driven amplifier. These points are easily found by superimposing the input characteristics of the driven amplifier as a load line on the output characteristics of the driver as shown in (c) of Figure 16.8.3.5c. Digital devices also require that the operating points occur where the input characteristics of the driven amplifier and the output characteristics of the driver coincide (Figure 16.8.3.5d).

In summary, the load line concept can be generalized in the following manner: Whenever two fluidic components are connected together, the coupled behavior can be determined by superimposing the appropriate characteristic

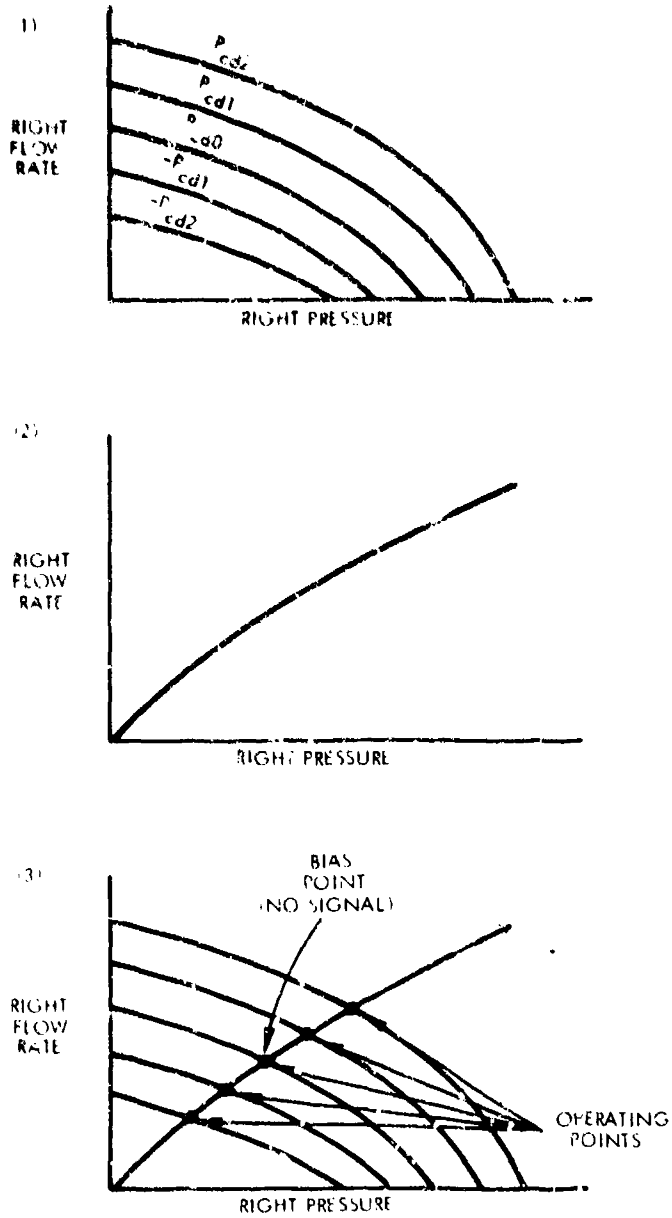

Figure 16.8.3.5b Coupling a Differential Fluidic Amplifier to a Passive Load

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curves for the two components. The only stable operating points are those at which the characteristics intersect.

16.8.3.6 CALCULATION OF THE TRANSFER (GAIN) CURVE. Once the operating conditions have been defined by the superposition of characteristic curves, the static transfer curve can be calculated. Referring again to the example shown in (c) of Figure 16.8.3.5c, it is first necessary to determine the bias (or quiescent) point. This is given by the intersection of the zero control curve of the driver amplifier and the passive load characteristics or the bias curve of the driven amplifier. At this point there will be pressure and flow when there is no signal into the driver amplifier

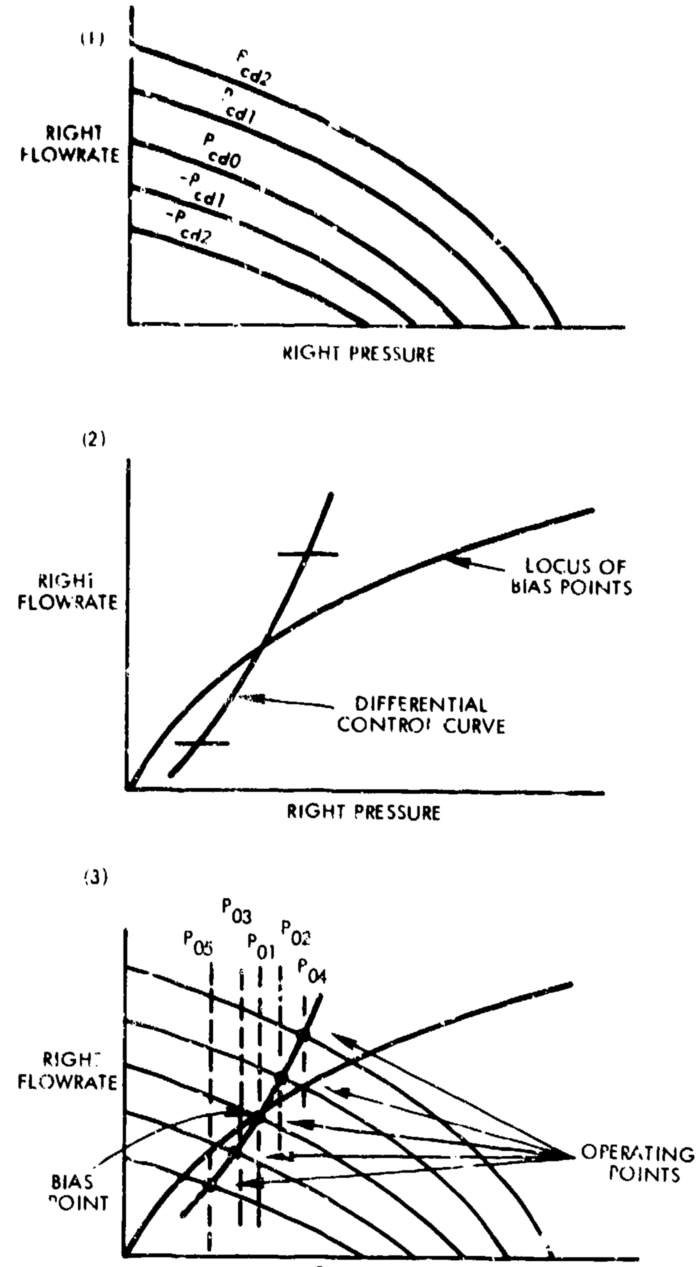

Figure 16.8.3.5c. Coupling Two Differential Fluidic Amplifiers
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When the differential amplifier receives an input signal, one output port pressure increases while the other decreases. Since this condition applies here, it is appropriate to use the differential curve for the "incremental" load line for changes about the operating bias point. To plot the differential pressure gain curve for the driver amplifier loaded with the second differential amplifier, increments of P_{cd} are taken. Where $P_{cd} = 0$, the output pressure of the right port is P_{01} and the output pressure of the left (if the amplifier is perfectly balanced) is also P_{01}. Therefore, the differential output P_{od} is zero. When $P_{cd} = +1$, the right output is P_{02}, the left is P_{03}, and the difference is $P_{od} = +2$. When $P_{cd} = -1$, the right output is P_{03}, the left is P_{02}, and the difference is $P_{od} = -2$. Continuing this

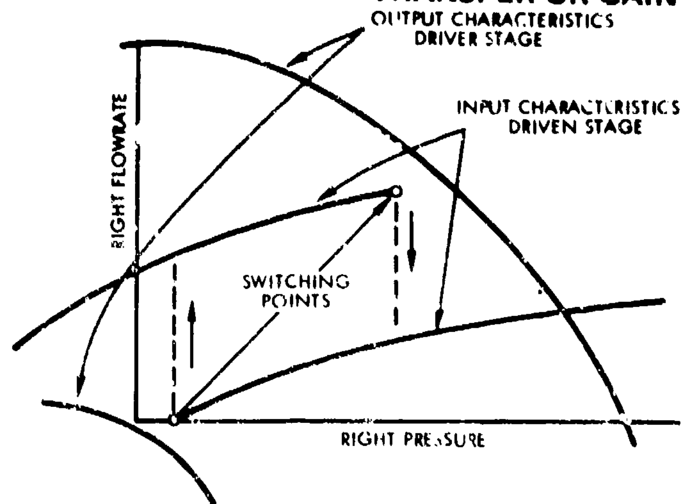

Figure 16.8.3.5d. Coupling Two Wall-Attachment Amplifiers
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procedure of taking increments of P_{cd} and calculating from the curves, the value of P_{od} leads to a complete transfer (gain) curve as shown in Figure 16.8.3.6. This defines the pressure gain of the driver amplifier only when it has the driven amplifier as a load.

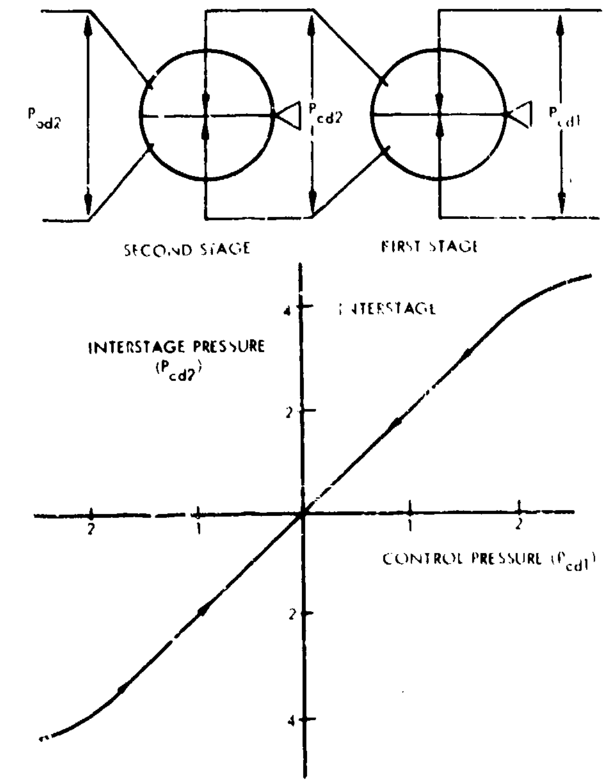

Figure 16.8.3.6. Pressure Transfer Curve of Differential Amplifier Loaded with Second Differential Stage
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16.8.3.7 STATIC MATCHING OF CASCADED FLUIDIC COMPONENTS. Having introduced the load line method for determining the performance of cascaded fluidic components, it is necessary to note that the ideal case was assumed in the illustration given; that is, no matching problems arose. In this section, the more probable occurrence of matching problems is considered.

The objectives in properly cascading fluidic components are:

a) Providing proper gains

b) Matching operating bias points

c) Matching operating ranges.

Proper gains are usually the most important. However, the designer may want primarily flow gain, leaving pressure gain and power gain as secondary considerations. On the other hand, he may want to optimize pressure gain. Operating bias (quiescent) points are established by the desired pressures and flows in the component with no signal applied. Operating ranges are those ranges of pressures and flows over which the component can be operated with good results.

As an example of the matching problem which must be considered, suppose an existing vortex rate sensor with output characteristics as shown in Figure 16.8.3.7a is used to measure rates-of-turn from 10 degrees per second counterclockwise to 10 degrees per second clockwise. Amplification of the rate sensor output using available off the-shelf amplifiers is desirable.

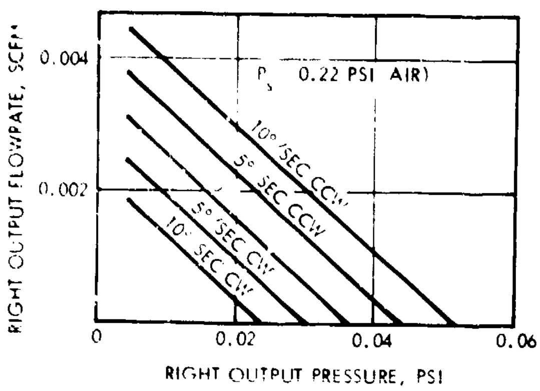

Figure 16.8.3.7a. Static Output Characteristics of Vortex Rate Sensors
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Vented jet-interaction amplifiers are available in a limited number of standard sizes. Because vortex rate sensors are inherently low-pressure and high output impedance devices, an amplifier of high input impedance is needed to match the high output impedance of the rate sensor. This requirement implies that an amplifier with small control nozzles, and therefore an amplifier of small overall size, should be used. Figure 16.8.3.7b shows the input characteristics of a typical small jet-interaction amplifier with a 0.010 x 0.025 inch power nozzle. Note that the preferred bias operating point is 10 percent of supply pressure, and

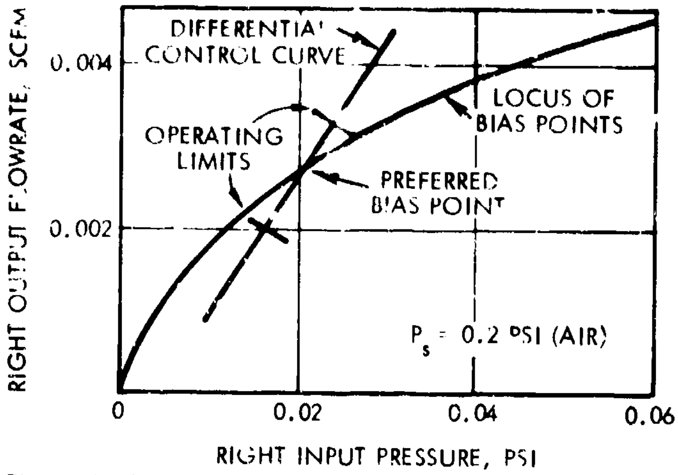

Figure 16.8.3.7b. Static Input Characteristics of Small Vented Jet-Interaction Amplifier
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the linear range of amplification is about ±5 percent of supply pressure.

Following the procedures for determining the operating characteristics of the interconnected rate sensor and amplifier, the input characteristics of the amplifier are superimposed as a load line on the output characteristics of the rate sensor as shown in Figure 16.8.3.7c. Note that preferred operating bias points and operating ranges do not match. It is apparent that taking increments of rate-of-turn to plot the transfer curve yields relatively small increments of input pressure. This result occurs because the input characteristic of the amplifier is relatively steep compared to the rate sensor output characteristics. In other words, the impedance match is poor.

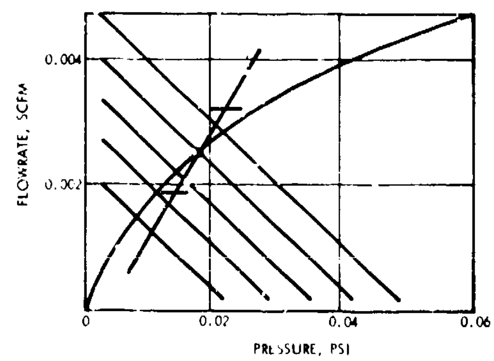

Figure 16.8.3.7c. Superposition of Static Characteristics of Vortex Rate Sensor and Vented Jet-Interaction Amplifier
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If it is necessary to optimize pressure sensitivity of the amplifier rate sensor circuit, an amplifier input characteristic with relatively low slope (high resistance) as illustrated in Figure 16.8.3.7d is required. Then if increments of rate-of-turn are taken to determine the resulting increments of amplifier input pressure, a vastly increased pressure sensitivity is found.

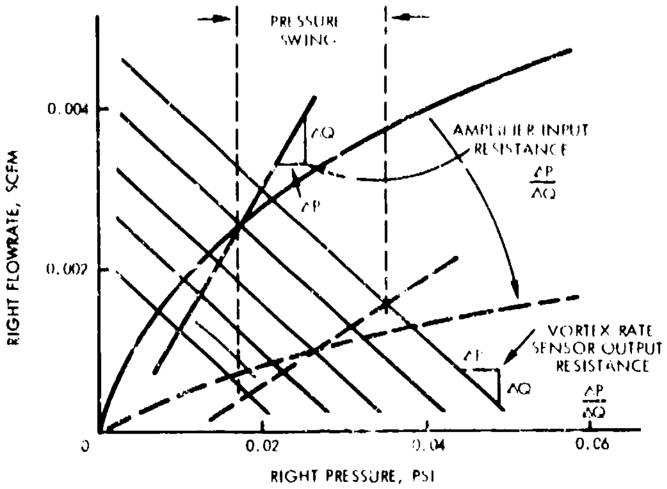

Figure 16.8.3.7d. Impedance Matching for High Static Pressure Gains
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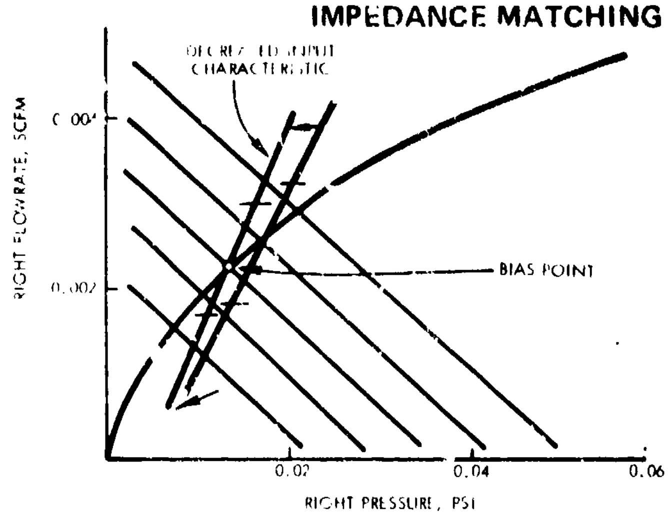

Figure 16.8.3.7f. Matching Operating Points by Reducing Amplifier Supply Pressure
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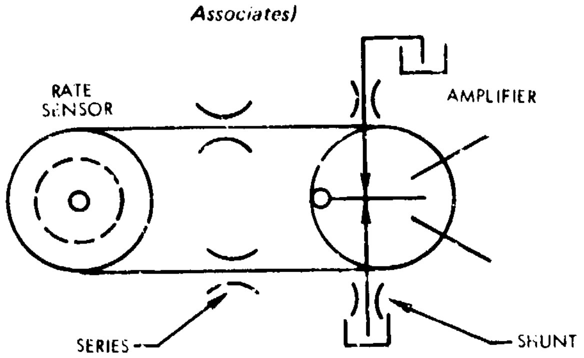
Since the preferred bias point for the amplifier does not coincide with the zero rate-of-turn of the rate sensor (Figure 16.8.3.7c), one of at least three methods can be used to correct the mismatch: The output bias level of the rate sensor could be increased by raising supply pressure as in Figure 16.8.3.7e; the amplifier supply pressure could be reduced, maintaining the input bias at 10 percent of the supply as in Figure 16.8.3.7f; the effective load line of the rate sensor could be shifted by the addition of resistors in series or in parallel with the amplifier input as in Figure 16.8.3.7g. (The third method is obviously not suitable for correcting the type of mismatch illustrated in the example problem.)



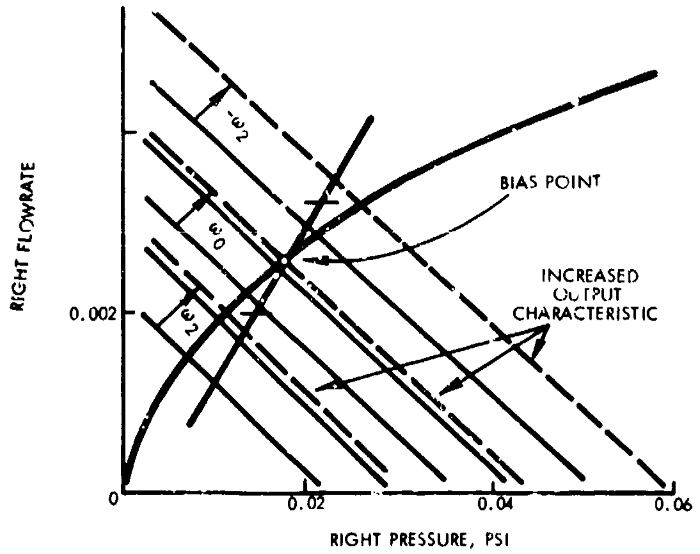


Figure 16.8.3.7e. Matching Operating Points by Raising Rate Sensor Output Bias
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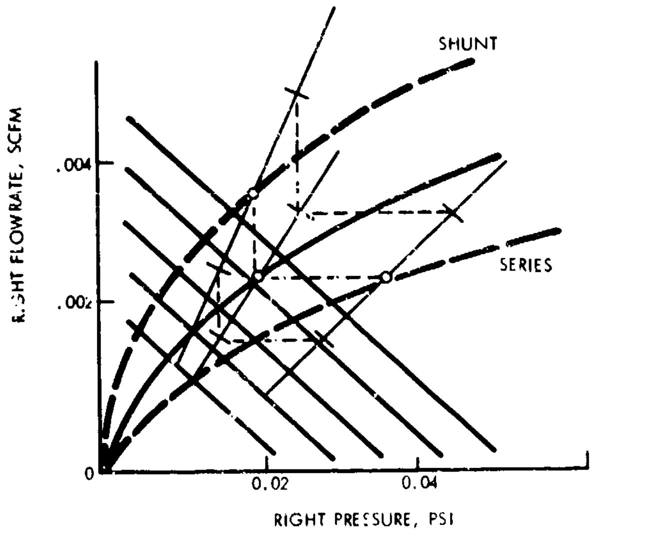

Figure 16.8.3.7g. Matching Operating Points by Adding Restrictors in Each Side of the Circuit
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With reference to Figure 16.8.3.7c, it is apparent that there is also a mismatch of optimum operating ranges. The rate sensor is capable of over-driving the amplifier into its nonlinear range. Again, there are at least three corrective techniques to be investigated: add series or shunt resistance in the differential circuit, change the output bias of the rate sensor, or change the amplifier supply pressure. Figure 16.8.3.7h shows that resistances across the differential lines will affect the slope and length of the differential load line but not the operating point. It is evident that two of these steps are also used to match the operating bias points; therefore, the effect of one upon the other must be considered. Although static matching of fluidic components requires a series of compromises based on a thorough understanding of component behavior, it is possible to match the components in a logical and straightforward way.

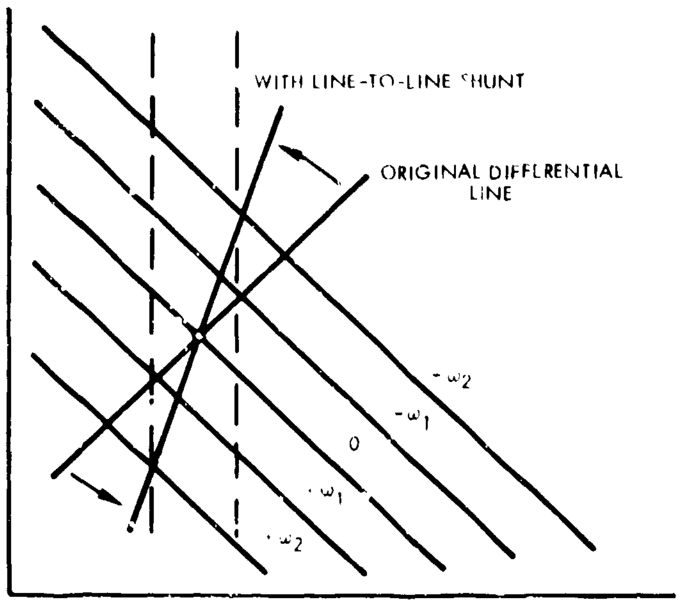

Figure 16.8.3.7h. Matching Operating Ranges by Adding Restrictor Between Differential Lines

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The same matching procedure also can be used for digital components. As long as the switching portion of the input characteristic falls within the boundaries of the two output characteristic curves of the driving component, the devices are roughly matched; the driving amplifier is capable of switching the driven amplifier. However, for maximum efficiency or exact matching of the range capability of the driving amplifier with that of the driven amplifier, it is necessary to match the input and output characteristic curves as shown in Figure 16.8.3.7i.

In most digital circuits, one objective is to provide output signal fan-out from a given stage for the purpose of driving multiple stages in parallel. Therefore in matching digital devices, one can often connect a number of similar units in parallel across the output of the driving component until the total range capability of the driving component is achieved. Range matching; however, is somewhat separate from bias or operating point matching because it may have to be accomplished using padding restrictors.

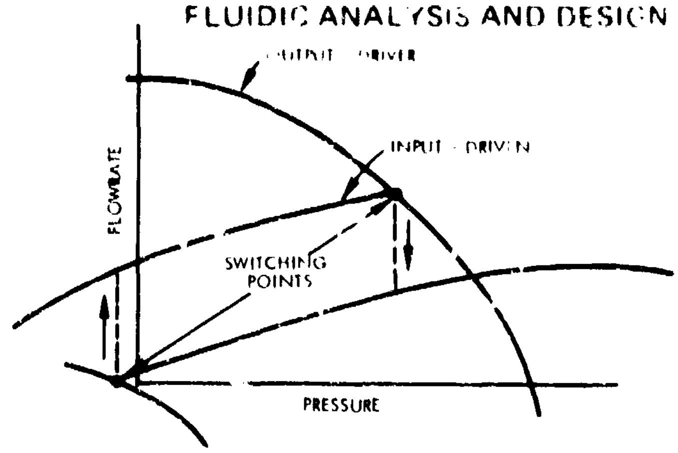

Figure 16.8.3.7i. Matching Operating Ranges of Wall-Attachment Amplifiers

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16.8.3.8 DYNAMIC AND SMALL-SIGNAL ANALYSIS. Graphic methods of performance analysis are general but valid in all situations if complete static and dynamic data are available. However, for small analog signals, the graphs are inaccurate. In the case of analog circuits, a more exact and convenient method of calculating performance is accomplished by linearizing parameters around the operating bias point and employing them in an equivalent electrical circuit. This approach has been widely used in all forms of engineering analysis, including electronics, acoustics, pneumatics, hydraulics, and mechanics.

In digital circuit analysis, the equivalent electrical circuit is also of value despite the fact that linearizing lumped parameters for such large signals is a gross oversimplification. Specifically, its value lies in estimating transient response and in gaining considerable insight into the dynamics of the circuit.

The process of developing an equivalent electrical circuit for a fluidic component can be a difficult analytical task. Fortunately, useful analog mathematical models have been developed through comprehensive experimental tests. To date, this is the only known approach which has produced useful results.

The equivalent circuit for a proportional vented jet-interaction amplifier is shown in Figure 16.8.3.8a. At high frequencies where resistive elements no longer satisfactorily describe amplifier behavior, several time delays including those due to transit time, wave propagation, and the presence of reactive circuit elements such as volume capacitance must be considered. The element in series with the input circuit, $2L_c$, is due to inertance in the line to the control nozzle. The shunt elements, $2R_c$ and $C_c/2$, are the effective nozzle resistance and volume capacitance of the control line. The equivalent generator, $2K_p$, contains a delay factor, e^{-st_d}, which includes wave propagation and transit times in the total path from the control port to the load terminals. The output circuit contains a series inductor ($2L_o$), a resistor ($2R_o$), and a shunt volume capacitor ($C_o/2$). If the lines to the load are short, the load volume capacitance and the load resistance ($2R_\ell$) parallel both. The transfer function for this amplifier contains an attenuation due to the output circuit resistor network, a gain factor equal to twice the amplification factor, a time delay, and several quadratic factors resulting from the combination of time constants in the input and output networks.

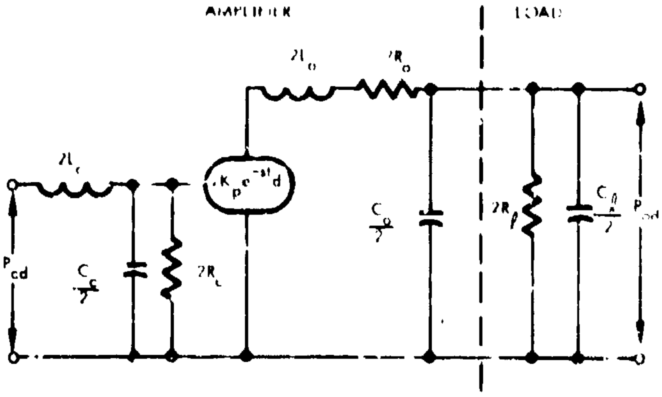

Figure 16.8.3.8a. Equivalent Electrical Circuit for Vented Jet-Interaction Amplifier Valid to 400 cps
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The equivalent circuit for a digital vented wall-attachment amplifier is shown in Figure 16.8.3.8b. The element in series with the input circuit is the effective inductance, $2L_c$, due to inertance in this line. The shunt elements, $2R_c$ and $C_c/2$, are the effective control nozzle resistance and the effective volume capacitance. These elements are at least double-valued, depending on the state of the output circuit. Therefore, there is a feedback loop, containing some dynamics due to conditions in the interaction region, which changes the effective input impedance values when the power stream switches from one wall to the other.

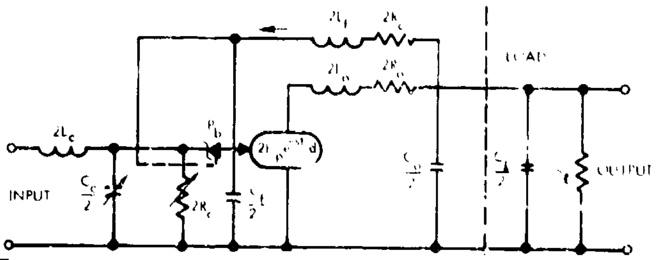

Figure 16.8.3.8b. Equivalent Electrical Circuit of Wall-Attachment Amplifier
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The equivalent generator, $2K_p$, effectively acts as a pressure switch triggered at a level determined by feedback-controlled reference diodes. It contains a delay factor, e^{-st_d}, which includes wave propagation and transit times in the total path from the control port to the output ports. The output circuit is similar to that of the proportional amplifier and contains series inductance, shunt capacitance, and series resistance. If the loads are closely coupled, the load impedances are directly in parallel with the amplifier capacitance. The dynamic response contains a nonlinear second-order term due to the input circuit, a time delay due to transit time, and a linear second-order term due to the output circuit.

16.8.3.9 CASCADING EQUIVALENT CIRCUITS. If fluidic components are cascaded (one becomes the load on the other), their equivalent electrical circuits are cascaded in a similar way. Figure 16.8.3.9 illustrates the connection of a vented jet-interaction amplifier to the output of a vortex rate sensor for the purpose of amplifying the signal. The cascading of the equivalent circuits simply involves connecting the output terminals of the rate sensor circuit to the input terminals of the amplifier circuit.

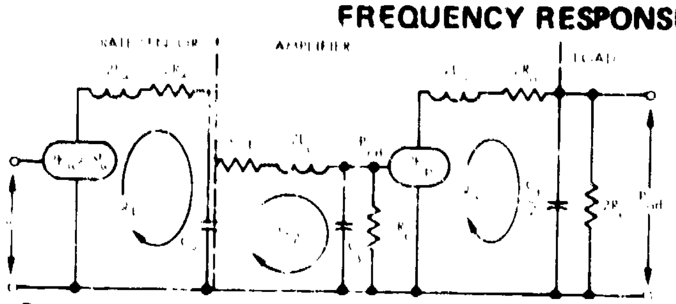

Figure 16.8.3.9. Cascading Equivalent Circuits of Vortex Rate Sensor and Jet Interaction Amplifier
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16.8.3.10 DERIVATION OF THE TRANSFER FUNCTION. The derivation of the transfer function for the cascaded fluidic components involves the straightforward analysis of the equivalent electrical circuits by well known mathematical models. Specifically, it involves loop analysis of each set of coupled circuits using the LaPlace transform notation and combination of the results into a single transfer function. The transfer function describes the small signal static and dynamic behavior of the cascaded rate sensor and amplifier. To calculate the behavior in numerical form, it is first necessary to evaluate each of the equivalent circuit parameters contained in the transfer function.

16.8.3.11 CALCULATING FREQUENCY RESPONSE. The procedure for calculating the frequency response of cascaded fluidic components is to:

a) Generate the coupled equivalent circuit

b) Derive the transfer function

c) Calculate the performance parameters

d) Substitute them into the transfer function.

The result would be a numerical equivalent of the transfer function containing the LaPlace transform variable, s, from which the frequency response can be calculated by substituting

$$s = j\omega = j2\pi f \qquad \textbf{(Eq 16.8.3.11)}$$

A typical frequency response plot of an amplifier with load is shown in Figure 16.8.3.11.

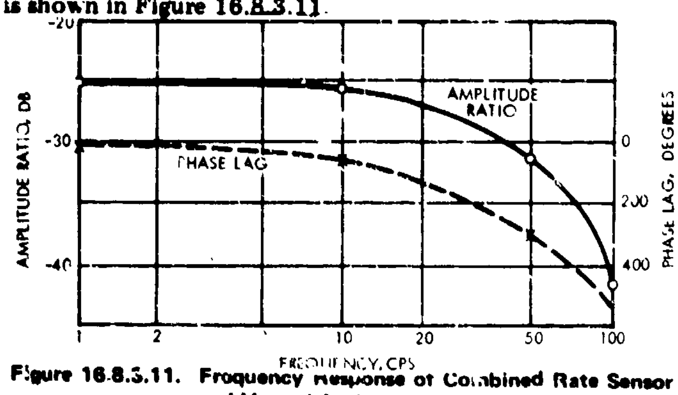

Figure 16.8.3.11. Frequency Response of Combined Rate Sensor and Vented Jet-Interaction Amplifier (Bode Diagram)
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16.8.4 Digital Circuit Design

The black box approach and static matching techniques described in Sub-Topic 16.8.3 apply to the interconnection of any fluidic element. However, most digital fluidic devices are designed with high input and output impedances, so that control inputs are isolated from each other and function independently of the outputs. Fluidic circuit design incorporating this type of element can be a very straightforward task. The principles outlined below have evolved for use with pneumatic, hydraulic, electrical, electronic, mechanical, and optical controls and should apply equally well to fluidic digital circuit design.

Binary arithmetic is the operating arithmetic for all modern computing and logic devices with binary-to-decimal conversion used only where the number is desired in familiar decimal form. The reasons for the universal use of binary techniques are the simplicity of the system for all arithmetic manipulations, such as addition and multiplication, and the ease with which the desired function can be implemented with any two-state device, such as a simple switch of any type.

Symbolic logic notation is the language used to express binary arithmetic functions and logical decision-making functions. The equations used to express the problem (and its solution) are in the form of symbolic logic. Thus logic is the language of binary arithmetic. To understand binary systems (and their implementation with logic devices), it is necessary to know binary arithmetic and symbolic logic.

16.8.4.1 BINARY ARITHMETIC. Binary arithmetic uses only two digits. For convenience the digits chosen are 0 and 1. No other digits are used in binary arithmetic, and all numbers are expressed with these two symbols, as shown below:

Decimal Number	*Binary Number*
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010

The progression of numbers is by powers of 2, as follows:

$$1 = 1 = 2^0$$
$$10 = 2 = 2^1$$
$$100 = 4 = 2^2$$
$$1000 = 8 = 2^3$$
$$10000 = 16 = 2^4$$
$$100000 = 32 = 2^5$$

Thus the binary number

$$10101 = 16 + 4 + 1 = 21$$

Similarly, numbers to the right of a decimal point can be found from negative powers of 2:

$$0.1 = 2^{-1} = 1/2$$
$$0.01 = 2^{-2} = 1/4$$
$$0.001 = 2^{-3} = 1/8$$

Therefore any decimal number can be expressed in binary form, and vice versa. Note that in binary all numbers can have only the two chosen marks. Numbers appear like this: 11101, 111011, 1101111, 1111111.

16.8.4.2 ADDITION WITH BINARY NUMBERS. Binary is the easiest set of numbers in which to perform arithmetic; this is the reason why the binary system is used in computers and logical networks. In decimal addition, we are forced to remember that 9 plus 8 are 17; that 5 plus 6 are 11; and so on. In binary, we need remember only the following two simple rules:

a) Rule 1: 0 plus 1 is 1

b) Rule 2: 1 plus 1 is 0 and carry a 1 to the next column left.

With these two rules we can perform addition. Thus, if we are to add 2 and 3 (binary 10 and 11)

```
 10 = 2
 11 = 3
 ──
  1
 0    carry 1 left
 1    the carry
 ───
 101 = 5
```

Let us add two bits (binary digits), which we shall call A and B. As each bit can be a 0 or a 1, there are four possible combinations of A and B, as shown in the input column of the following truth table:

Input A	Input B	Output S	Output C	S exists when	C exists when
0	0	0	0		
0	1	1	0	$\bar{A}B$	
1	0	1	0	$A\bar{B}$	
1	1	0	1		AB

Note that a sum (S) exists when either A or B is 1; a carry (C) exists only when both A and B are 1. These situations are expressed in the language of logic in the last two columns.

16.8.4.3 SYMBOLIC LOGIC NOTATION. In the language of logic, each bit is called A, B, etc., if it is a 1: it is called $\bar{A}$, $\bar{B}$, etc., if it is a zero, with the bar representing the word not. That is,

$$\bar{A} = \text{not } A$$

$$\bar{B} = \text{not } B$$

From the truth table above, if the inputs are A = 0 and B = 1, then a sum exists. This (not A and B) is written

$$S = \bar{A} B$$

Similarly, if A = 1, and B = 0, a sum exists. This (A and not B) is written

$$S = A \bar{B}$$

We can now say that an adder is a device that develops an output signal when:

$$S = (\text{A and not B}) \text{ or } (\text{B and not A})$$

$$= A\bar{B} \text{ or } B\bar{A}$$

16.8.4.4 THE AND, OR CONCEPT. In 1854, the book *Laws of Thought* by George Boole was published. Boole proposed the theory that the logical relationship between objects can be expressed in terms of the concepts AND and OR; that is, given objects A and B, then the only relations involving A and B can be expressed as (1) A AND B, meaning that both must exist at one time, or (2) A OR B, meaning that one can exist without the other. The expression A AND B is usually written A · B or AB; A OR B is written A + B, with the + sign meaning OR. Note that it does not mean addition.

The simplicity of binary techniques has caused the arithmetic based on Boole's postulates to become a powerful tool for designing circuits involving switching and interlocking relations. Today, Boolean algebra (symbolic logic arithmetic) is used in the design of most digital circuitry. The extent of the field can be inferred from the fact that more than 60 manufacturers now offer electronic modules (called gates), that perform the simple AND and OR functions, described as follows:

Consider two binary variables (A, B). As there are two conditions for each variable and two possible relations (AND, OR), there are only eight possible relationships (called logic expressions):

$A + B$	(A OR B)	(1)
AB	(A AND B)	(2)
$A\bar{B}$	(A AND not B)	(3)
$B\bar{A}$	(B AND not A)	(4)
$\bar{A}\bar{B}$	(not A AND not B)	(5)
$\bar{A} + \bar{B}$	(Not A OR not B)	(6)
$A + \bar{B}$	(A OR not B)	(7)
$B + \bar{A}$	(B OR not A)	(8)

Logic expressions can be factored, multiplied together, etc., by following a few obvious (logical) rules, as follows:

$$AA = A \qquad (9)$$

$$A + B = B + A \qquad (10)$$

$$A = A \qquad (11)$$

$$A(B + C) = AB + AC \qquad (12)$$

$$A + BC = (A + B)(A + C) \qquad (13)$$

$$A + AB = A \qquad (14)$$

$$A + AB + AC + AD = A \qquad (15)$$

$$\overline{AB} = \bar{A} + \bar{B} \qquad (16)$$

$$\overline{A + B} = \bar{A}\bar{B} \qquad (17)$$

Most of the above rules are obvious. A AND A must be A; if A must exist in the expression A + AB, then A alone is all that is needed. Expressions (16) and (17) follow from the single most powerful manipulative technique, one that permits any logic expression using AND relations to be converted into another expression using OR relations — the DeMorgan inversion.

The two basic logic operations are AND and OR. Expressions involving AND logic can be changed into OR logic, and vice versa, by use of DeMorgan's theorem, also known as the involution law, inversion postulate, or dualization law. DeMorgan's theorem states that a logic expression can be inverted simply by (1) inverting each term of the expression, (2) changing each AND element into an OR element, and (3) changing each OR element into an AND element. For example, to invert the expression A + B:

$$A + B \text{ inverted} = \overline{A + B} = \bar{A}\bar{B}$$

To invert AB:

$$\overline{AB} = \bar{A} + \bar{B}$$

To invert the expression $(\overline{A + B}) + (AB)$

$$\overline{(\overline{A + B}) + (AB)} = (A + B)(\overline{AB}) = (A + B)(\bar{A} + \bar{B}) = A\bar{B} + B\bar{A}$$

Note in the above that each term in parentheses is considered an entity. To invert C = AB, we get

$$\bar{C} = \overline{AB} = \bar{A} + \bar{B}$$

To invert C = A + B

$$\bar{C} = \overline{A + B} = \bar{A}\bar{B}$$

If we reinvert the last expression

$$\bar{\bar{C}} = C = \overline{\bar{A}\bar{B}} = A + B$$

Note here that $\overline{AB} = \bar{A} + \bar{B}$.

Even the most complicated digital computer in existence is based on only the simple logical functions of addition and comparison. All multiplication is but a series of additions; all decisions are made by comparing some number (the result) with some predetermined number. Thus addition and comparison are the two basic functions of all digital circuits, and the two simple symbolic expressions for the half-adder and comparator appear time and time again in all digital and logical circuits.

$$\text{Half-Adder:} \quad S = A\bar{B} + B\bar{A}$$

$$\text{Comparator:} \quad S = AB + \bar{A}\bar{B}$$

AND and OR are the two basic logical relations. All other expressions are but variations of the AND and OR relations and can be expressed in AND, OR terms by the DeMorgan technique, including the common NOR logic.

16.8.4.5 DIGITAL LOGIC OPERATORS. The basic building block used in the design of logic-type circuitry is the operator or gate. A gate is defined as a device having several inputs, and designed so that there is an output when and only when a certain definite set of input conditions are met. Digital logic utilizes the three basic operators used in Boolean algebra: AND, OR, and NOT. In addition, there are three more operators which are useful combinations of the basic operators: NOR, NAND, and exclusive OR. The last operator is the flip-flop which is actually a memory function.

Control systems make decisions based on information, but automatic systems are generally lacking in value judgement. This means we must define our operators precisely so there can never be a doubt about their exact meaning. The accepted logic definitions are very similar to the common language definitions and are not difficult to remember.

In system design, the easiest way to think of operators is as black boxes. A black box takes information in and gives a decision out. The exact means used to convert information into a decision is not important but it is necessary to know what decision the black box makes. This is accomplished by naming the black box after the operator that describes its decision, so that several black boxes are available, which are named AND, OR, NOT, NOR, NAND, exclusive OR, and flip-flop.

The information inputs must be in Yes or No form. For example, the part is in position or it is not; in position is Yes, not in position is No. Then the black box operators may be defined in terms of the inputs and output as shown in Table 16.8.4.5a.

Table 16.8.4.5a. Black Box Definitions

Black Box Name (Operators)	Information (Inputs)	Decision (Output)
AND	All yes	Yes
	One or more no	No
OR	One or more yes	Yes
	All no	No
NOT	No	Yes
	Yes	No
NOR	All no	Yes
	One or more yes	No
NAND	One or more no	Yes
	All yes	No
Exclusive OR	One yes, one no	Yes
	Both yes	No
	Both no	No
Flip-Flop	Last input yes	Yes
	Last input no	No

Several sets of standard symbols have been adopted to facilitate the ready identification of the digital logic operators. The symbols defined in MIL-STD-806 (Reference 447-9) are shown in the digital logic cross reference chart (Table 16.8.4.5b) along with abbreviated function descriptions.

16.8.4.6 DESIGN PROCESS. Fluidic circuit design can proceed at any level, depending on the complexity of the circuit involved. Complex circuits can be converted into Boolean functions and simplification techniques used to minimize the amount of circuitry involved. Minimization can be accomplished by computer techniques or, in the simpler cases by the Veitch-diagram or Harvard-chart methods (Reference 772-1). The elementary form of the Boolean functions or operators may then be converted into standard digital logic operators and a circuit drawn using standard logic or fluidic device symbols (Table 16.8.4.5b).

Digital circuit theory is well established and can easily be applied to fluidic circuit design. However, where power drain is not particularly significant the design of simple digital circuits can be accomplished directly with fluidic device symbols. NOR logic can also be used to design simple circuits for many applications which can have economic advantages in that all of the connective logic can be accomplished with a single logic element (see Table 16.8.4.5b).

16.8.5 Fluidic Operational Amplifiers

The basic building blocks used in fluidic operational amplifiers are staged proportional amplifiers or high-gain blocks (References 23-69, 750-1). The high-gain block can provide linear forward gains as high as 10,000. In general push-pull or differential techniques are used in analog fluidic circuitry for increased linearity and power utilization. This also allows simple sign inversion by crossing over connections. When this gain block is connected into feedback networks consisting of fluidic linear resistors and capacitors, a number of very desirable performance characteristics can be obtained. Steady-state or dc characteristics available with these operational amplifier techniques are:

a) Fixed gain where the load and supply pressure vary

b) Accurate signal summation

c) Signal limiting

d) Isolation amplifier

e) Adjustable gain amplifier.

The dynamic or ac characteristics which can be obtained are:

a) Flat frequency response

b) Lag-lead

c) Lead-lag

d) Simple lag

e) Notch network.

The following material (Detailed Topics 16.8.5.1 through 16.8.5.7) was adapted from a paper by M. C. Doherty (Reference 52-77).

16.8.5.1 REVIEW OF OPERATIONAL AMPLIFIER TECHNIQUES. Before proceeding in detail into each of these functions it is worthwhile to review operational amplifier techniques which have been developed in the electronics field. A simple first order analysis shows the advantage of employing these techniques. The usual analogies are used to analyze equivalent fluidic circuits. A basic

Table 16.8.4.1b. Digital Logic Cross Reference Chart

LOGIC OPERATORS	AND	OR	NOT (INVERTER)	NOR	NAND	EXCLUSIVE OR	FLIP-FLOP
FUNCTIONAL DESCRIPTION	OUTPUT IF ALL CONTROL INPUTS ARE ON	OUTPUT IF ANY CONTROL INPUT IS ON	OUTPUT ONLY IF INPUT IS OFF	OUTPUT IF ALL CONTROL INPUTS ARE OFF	NO OUTPUT IF ALL CONTROL INPUTS ARE ON	OUTPUT IF ONE OF TWO INPUTS IS ON	RETAINS OUTPUT CONDITION CORRESPONDING TO LAST INPUT
MIL-STD-806 SYMBOL			(NEVER USED ALONE)				F-F
RELAY LOGIC							CR1, CR2
BOOLEAN ALGEBRA OPERATOR	$(A)\cdot(B)$	$(A)+(B)$	$\overline{(A)}$	$\overline{(A)+(B)}$	$\overline{A\cdot B}$	$A\cdot\bar{B}+\bar{A}\cdot B$	
NOR LOGIC							
FLUIDIC DEVICE SYMBOL	C1, C3; 02, 01	C_1, C_3; 02, 01	C1; 02, 01	C1, C3; 02, 01	C1, C3; 02, 01	C1, C3; 02, 01	C1, C2; 02, 01
TRUTH TABLE	C1 C2 01: 0 0 0; 1 0 0; 0 1 0; 1 1 1	C_1 C_3 01: 0 0 0; 1 0 1; 0 1 1; 1 1 1	C1 02: 0 1; 1 0	C1 C3 02: 0 0 1; 1 0 0; 0 1 0; 1 1 0	C1 C3 02: 0 0 1; 1 0 1; 0 1 1; 1 1 0	[illegible]	[illegible]

*DEPEND ON PREVIOUS STATE

operational amplifier circuit is shown in Figure 16.8.5.1. It consists of an input resistor, R_i, a feedback resistor, R_f, a high gain amplifier with a gain of $-K$, and its inherent input resistance, R_c. Current or flow is designated as $\dot{W}$ and pressure as P. Applying Kirchhoff's law for the flow into the summing junction,

$$\dot{W}_1 + \dot{W}_2 = \dot{W}_3$$

Rewriting in terms of resistance and pressure gives

$$\frac{P_i - P_g}{R_i} + \frac{P_o - P_g}{R_f} = \frac{P_g}{R_c} \qquad \text{(Eq 16.8.5.1a)}$$

P_g can be eliminated using the amplifier gain relationship

$$P_o = -KP_g \qquad \text{(Eq 16.8.5.1b)}$$

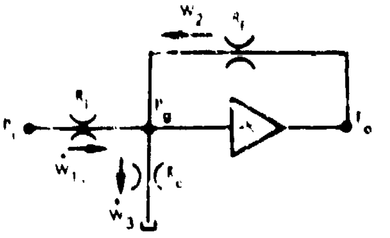


Figure 16.8.5.1. Basic Operational Amplifier Circuit
(Adapted with permission from Reference 52-77, "Applying Fluidic Operational Amplifiers", ISA Paper, M. C. Doherty, October 1968)

Equations (16.8.5.1a) and (16.8.5.1b) can be combined and reduced to the form

$$\frac{P_o}{P_i} = -\frac{R_f}{R_i}\left[\frac{K\left(\dfrac{1}{1 + \dfrac{R_f}{R_i} + \dfrac{R_f}{R_c}}\right)}{1 + K\left(\dfrac{1}{1 + \dfrac{R_f}{R_i} + \dfrac{R_f}{R_c}}\right)}\right] \qquad \text{(Eq 16.8.5.1c)}$$

Equation (16.8.5.1c) can be rewritten in the familiar control system terms with the substitution of

$$G = K$$

for the forward gain of the loop, and

$$H = \frac{1}{1 + \dfrac{R_f}{R_i} + \dfrac{R_f}{R_c}}$$

for the feedback gain (attenuation). Therefore

$$\frac{P_o}{P_i} = -\frac{R_f}{R_i}\left[\frac{GH}{1 + GH}\right] \qquad \text{(Eq 16.8.5.1d)}$$

The significance of Equation (16.8.5.1d) is that the transfer function is determined primarily by the passive input and feedback resistors when the loop gain GH is large. Variations in the gain of the active amplifier due to loading condition or supply pressure changes are practically eliminated from the transfer function. These characteristics can be expanded so that signal summation is accomplished with the addition of parallel input resistors and frequency shaping functions accomplished by including capacitors in various parts of the circuit.

Of course, as with any closed loop, stability must be considered. The open loop transfer function, GH, must be attenuated below unity gain before 180 degrees of phase shift are accumulated. The characteristics of the gain block approximate a pure time delay without attenuation in the frequency range of interest (less than 1000 cps). For stability purposes the gain block transfer function should be refined to include this term as

$$G = Ke^{-\tau_1 s}$$

where τ_1, the transport lag, has an approximate value of 5.5×10^{-4} seconds and s is the LaPlace operator. Clearly, some attenuation must be added to the loop for stability. Small pneumatic capacitors added to the amplifier output provide this attenuation. The open loop transfer function thus becomes

$$GH = K\left(\frac{1}{1 + \dfrac{R_f}{R_i} + \dfrac{R_f}{R_c}}\right)\left(\frac{1}{1 + \tau_2 s}\right)\left(e^{-\tau_1 s}\right) \qquad \text{(Eq 16.8.5.1e)}$$

where τ_2 is the RC time constant of the stabilizing volumes. The closed loop transfer function is not limited in frequency response directly by the break frequency of the stabilizing volumes. The lag in this expression appears as

$$\frac{\Delta P_o}{\Delta P_i} = -\frac{R_f}{R_i}\left(\frac{GH}{1 + GH}\right)\left(\frac{1}{1 + \dfrac{\tau_2}{1 + GH}s}\right)\left(e^{-\tau_1 s}\right) \qquad \text{(Eq 16.8.5.1f)}$$

Thus the closed-loop lag time constant is the stabilizing lag, τ_2, reduced by the quantity (1 + GH). This is the frequency where the open loop transfer function has unity gain. In normal system design, the small phase lag contributed by the operational amplifier is neglected and the simple transfer function of Equation (16.8.5.1d) is employed.

16.8.5.2 FLAT RESPONSE AMPLIFIER. This is the simplest and most common application of fluidic operational amplifiers. The principal requirements are that the transfer function be an accurate and constant amplification independent of the input frequency, supply pressure variations, load conditions, and input null level (bias level). Figure 16.8.5.2a shows plotter traces of pressure-gain characteristics of a model FS-12 amplifier with supply pressure and load variations. Typical frequency response is shown in Figure 16.8.5.2b. The gain is frequency independent or flat out to approximately 200 cps.

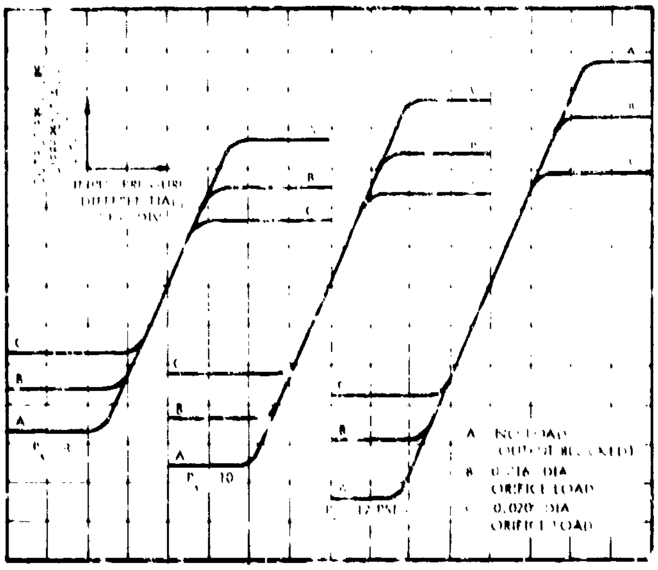


Figure 16.8.5.2a Model FS-12 Operational Amplifier Performance
(Courtesy of General Electric Company, Schenectady, New York)

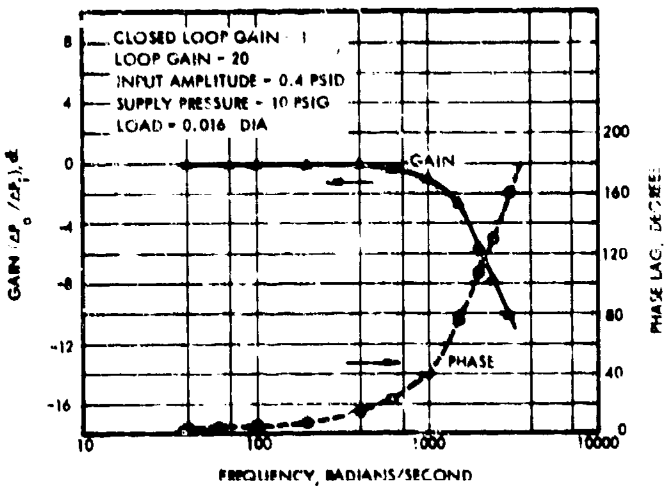


Figure 16.8.5.2b. Operational Amplifier Frequency Response
(Courtesy of General Electric Company, Schenectady, New York)

This type operational amplifier is used in many fluidic control circuits where sensing, computation, and logic are accomplished at very low power levels and then amplified to a higher power level actuator. Large power amplification can cause inaccuracies so that often a closed loop is required. The FS-12 can provide a block of the amplification and the summing junction for this type of loop.

A feasibility demonstrator of a fluidic main fuel control for a J79 turbojet engine represents a typical application of this function (Reference 68-98). A fuel valve position loop, shown in Figure 16.8.5.2c, provides fuel flow proportional to a low-pressure fluidic input signal. The fixed-gain amplifier is used to provide an amplification of 45 and to sum the feedback position signal of the rotary fuel metering valve with the input signal. The position feedback transducer consists of an eccentric cam on the fuel valve shaft and a flapper nozzle sensor.

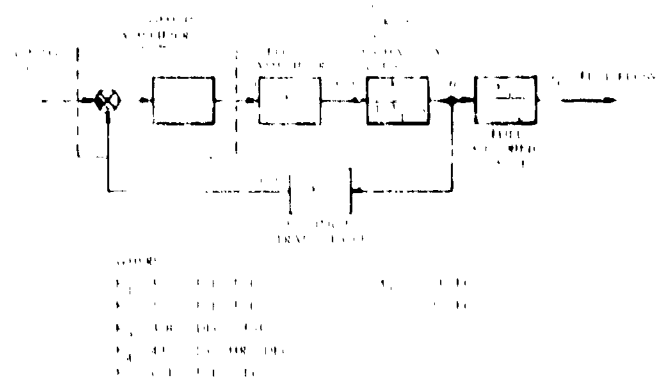


Figure 16.8.5.2c. FS-12 Summing Amplifier in J79 Turbojet Engine Control
(Courtesy of General Electric Company, Schenectady, New York)

The transfer function for the summing amplifier can be developed in a manner similar to the single-input amplifier of Equation (16.8.5.1d)

$$\Delta P_o = \Delta P_a\left(\frac{R_f}{R_a}\right)\left(\frac{GH}{1+GH}\right) + \Delta P_b\left(\frac{R_f}{R_b}\right)\left(\frac{GH}{1+GH}\right)$$

or

$$\Delta P_o \approx \Delta P_a\left(\frac{R_f}{R_a}\right) + \Delta P_b\left(\frac{R_f}{R_b}\right) \qquad \text{(Eq 16.8.5.2)}$$

Typical pressure-gain plots for this type of amplifier are shown in Figure 16.8.5.2d. Each input channel is shown to have a gain of 45. Summing accuracy is demonstrated by the plot of a single input signal summed with itself and subtracted from itself. The gains for these two cases are 90 and zero which agree with the predicted gain from Equation (16.8.5.2). Another application of the fixed-gain operational amplifier to jet-engine or gas-turbine fuel control is as a variable signal limiter. Figure 16.8.5.2a indicates how the output of the amplifier has very flat saturations resulting in a limited output pressure when the input signal is very large. The saturation level is a function of supply pressure at a constant load. This characteristic has been utilized by inserting the operational amplifier in the fuel control loop and supplying it with pressure proportional to compressor discharge pressure (CDP). In this manner the amplifier calls for fuel flow proportional to speed error during steady-state conditions and limits fuel flow as a function of CDP during transient accelerations when large speed errors exist, which protects the turbine against over-temperature and the compressor from stall. This function is very complex to mechanize using conventional hydromechanical components.

The characteristic high input impedance of the FS-12 operational amplifier has been frequently used to uncouple or isolate a pneumatic sensor from a fluidic circuit either to minimize the loading effects on the sensor, or because of a high dc level on the sensor output. An operational amplifier has been designed for this purpose which has an input impedance 100 times greater than that of a typical 0.020 x 0.020-inch power nozzle fluidic amplifier.

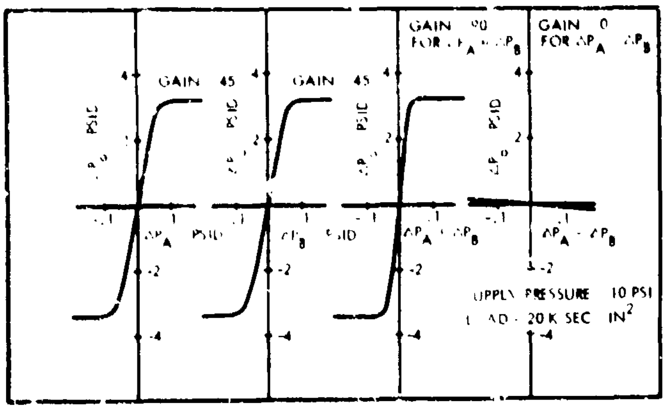


Figure 16.8.5.2d. Pressure Gain Characteristics, FS-12 Summing Amplifier
(Courtesy of General Electric Company, Schenectady, New York)

A recent additional feature of the operational amplifiers has been the use of a variable feedback resistor. Pressure gain performance plots of this type device are shown in Figure 16.8.5.2e. All the features of the fixed-gain amplifier are maintained and the gain is adjustable with an external knob over a 10 to 1 range. This amplifier also has the capability of summing three input signals. In almost all controls work it is desirable to have some external gain adjustment to compensate for design inaccuracies. The adjustable-gain operational amplifier provides this function for fluidic control systems.

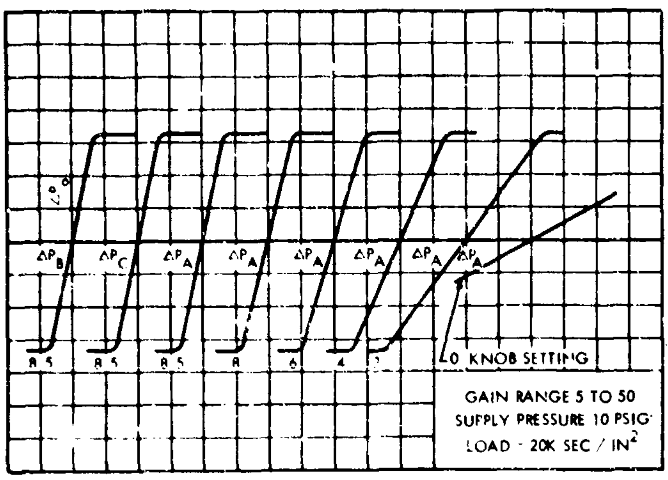


Figure 16.8.5.2e. Pressure-Gain Characteristics, Variable-Gain Amplifier (Model FV-52)
(Courtesy of General Electric Company, Schenectady, New York)

16.8.5.3 INTEGRATION. Integration is the most difficult frequency dependent function to mechanize in fluidics because of the absence of the series capacitor. Whether the connections to a fixed-volume capacitor are in series or parallel, the resulting transfer function is that of a shunt capacitor to ground. This precludes the use of the analogous electronic circuit to obtain integration, an operational amplifier with a series capacitor as the feedback.

Proportional-plus-integral action, which approximates integration, has been mechanized in fluidics by using positive feedback. A capacitor to lag a positive-feedback path and an equal but unlagged negative-feedback path result in a lag-lead circuit having the following transfer function

$$\frac{\Delta P_o}{\Delta P_i} = K\tau_2 \left[\frac{(1+\tau_1 s)}{(1+\tau_2 s)}\right] \quad \text{(Eq 16.8.5.3a)}$$

where K is the integrating rate or the gain at 1 radian/sec, τ_2 is the lag or integrating-time constant, and τ_1 the lead-time constant. The lag-time constant can be up to 60 seconds so that the approximation

$$1 + \tau_2 s \simeq \tau_2 s \qquad \text{for } \tau_2 s \gg 1$$

the transfer function becomes

$$\frac{\Delta P_o}{\Delta P_i} \simeq \frac{K}{S} + \tau_1 \quad \text{(Eq 16.8.5.3b)}$$

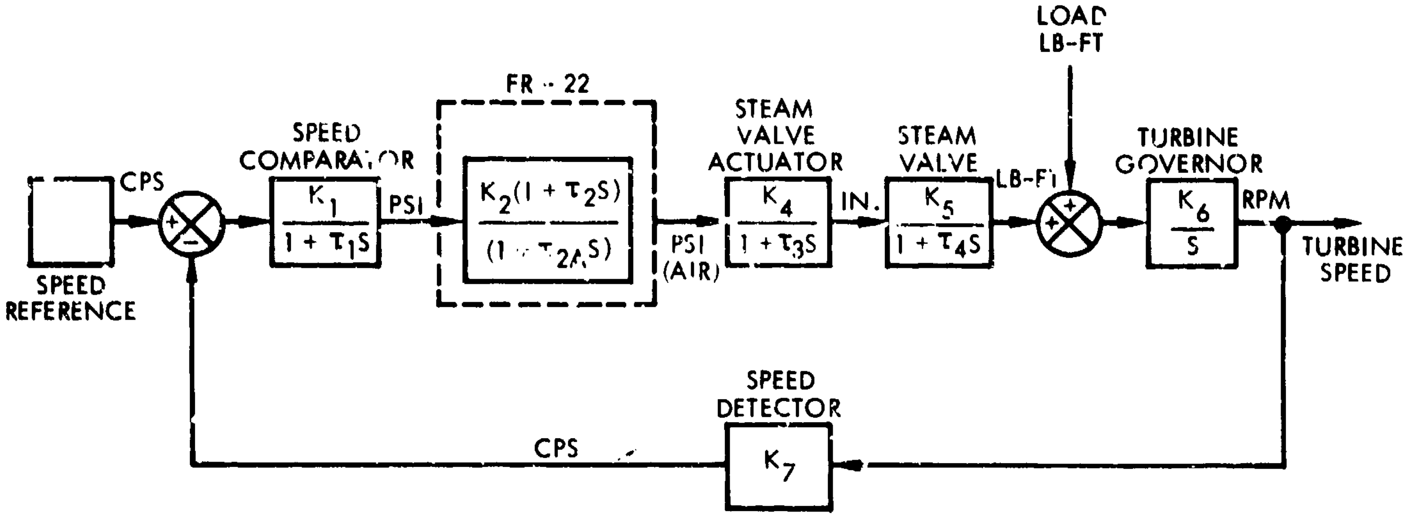
This is proportional-plus-integral control action. It is used in closed-loop control systems subjected to sustained disturbances of load variations to eliminate steady state error or droop. The lead term of Equation (16.8.5.3a) could be eliminated with the addition of a simple lag in series, but this is rarely required because pure integral control tends to produce instability.

16.8.5.4 LAG-LEAD. A typical application for a fluidic lag-lead circuit is in an isochronous governor for a shipboard steam turbine. The block diagram of this control is shown in Figure 16.8.5.4a. A fluidic speed error signal is treated to produce a proportional-plus-integral action pressure signal. This signal is amplified and drives the main steam valve. The actual control has governed a simulated turbine-generator in accordance with the military specification for shipboard governors, MIL-G-22410. Frequency response data for the lag-lead circuit are shown in Figure 16.8.5.4b.

16.8.5.5 LEAD-LAG. Many high-performance analog control circuits require derivative action to compensate for either the dynamics of the load or other control components. To produce a lead-lag circuit or proportional-plus-derivative action, a capacitor is inserted in the feedback path of Figure 16.8.5.1. A typical application is a fluidic position control loop for a rocket engine actuator as shown in Figure 16.8.5.5a. The lead-lag circuit accepts the position error signal from a flapper-nozzle valve and drives a 1000 psi hot gas actuator. The dynamic response characteristics of the lead-lag are shown in Figure 16.8.5.5b. This device produces 65 degrees of phase lead to compensate for the compliance of the actuator.

16.8.5.6 SIMPLE LAG. Occasionally only moderate lag is required in fluidic systems. If a passive RC (resistance-capacitance) lag is employed it introduces attenuation or requires large volumes. When this is undesirable the operational amplifier circuit of Figure 16.8.5.1 can be modified by the addition of a capacitor in front of the input resistor to provide this function. This circuit would provide up to a one second lag with a gain of five using small volumes on a compact hardware module. The circuit has been utilized to compensate for lags in engines or actuators, or in series with the lag-lead circuit to cancel out the lead term. Accurate summing can also be performed by this device.

Two lag circuits are being used in a fluidic carrier approach power-compensator control system under development for



WHERE

$K_1 = 0.125$ PSI / CPS		τ_1	= 0.02 SEC
$K_2 = 200$ PSI / PSI		τ_2	= 0.5 "
$K_4 = 0.1$ IN / PSI		τ_{2A}	= 25 "
$K_5 = 7250$ LB-FT / IN		τ_3	= 0.02 "
$K_6 = 0.046$ RPM / SEC-LB-FT		τ_4	= 0.011 "
$K_7 = 0.333$ CPS / RPM			

Figure 16.8.5.4a. FR-22 Lag-Lead in Shipboard Steam Turbine Governor
(Courtesy of General Electric Company, Schenectady, New York)

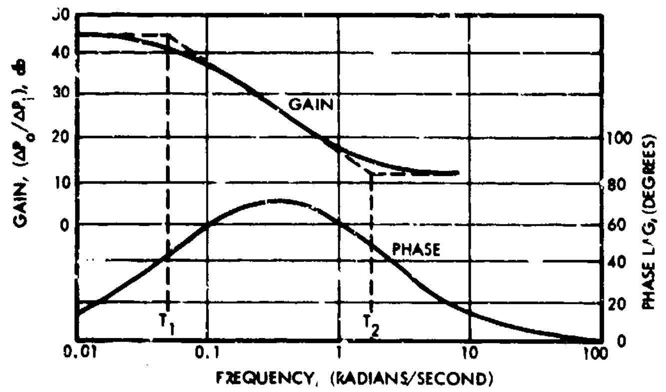
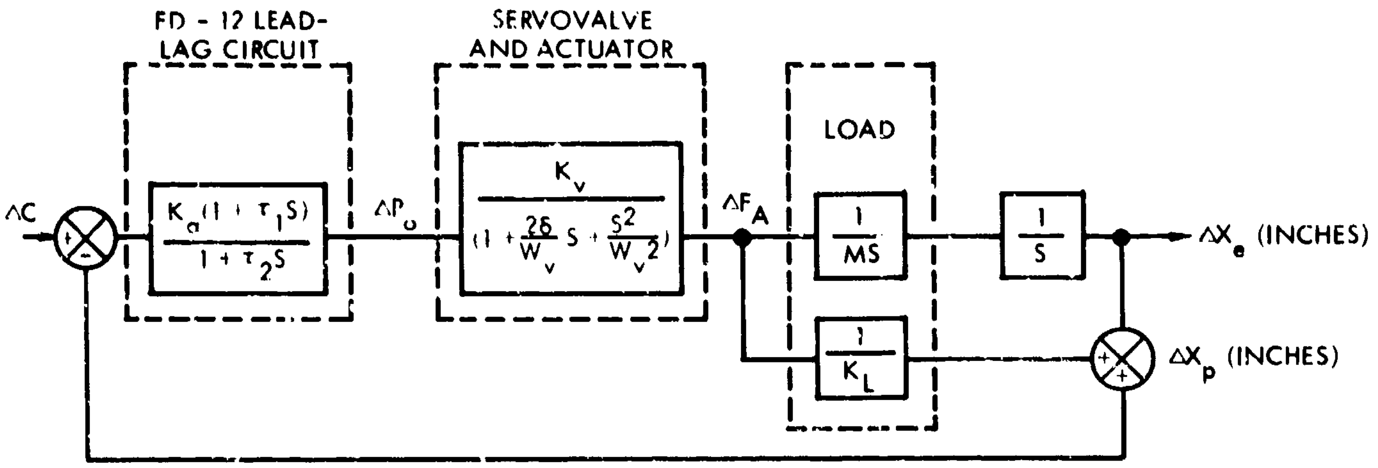


Figure 16.8.5.4b. Frequency Response of FR-22 Lag-Lead Network
(Courtesy of General Electric Company, Schenectady, New York)



K_L	13,700 LBS / IN	W_v	100 RAD / SEC
M	2.12 LB-SEC^2 / IN	τ_1	1/10 SEC
K_v	100 LBS / PSI	τ_2	1/300 SEC
K_a	5.4 PSI / IN		

Figure 16.8.5.5a. FD-12 Lead-Lag Circuit in Rocket Engine Actuator Loop
(Courtesy of General Electric Company, Schenectady, New York)

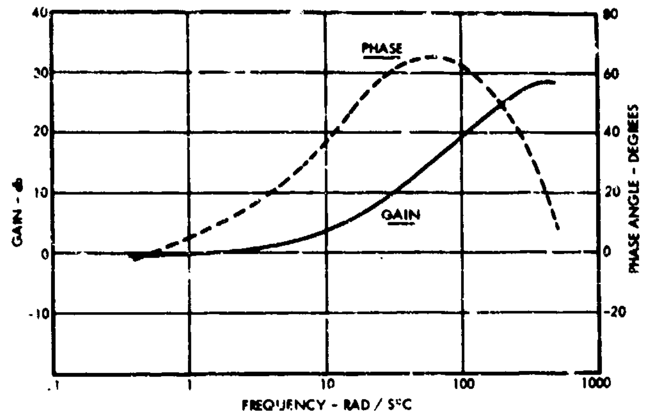


Figure 16.8.5.5b. Frequency Response of FD-12 Lead-Lag Network
(Courtesy of General Electric Company, Schenectady, New York)

a naval aircraft. The primary input is the aircraft angle of attack, which is sensed and modified by the following transfer function

$$\frac{\Delta P_o}{\Delta P_i} = \frac{1.23}{S} + \frac{9}{1 + 0.75S}$$

The fluidic circuit has been mechanized as shown by the block diagram of Figure 16.8.5.6. Two lag circuits and one lag-lead circuit provide this precise dynamic transfer function in three simple blocks.

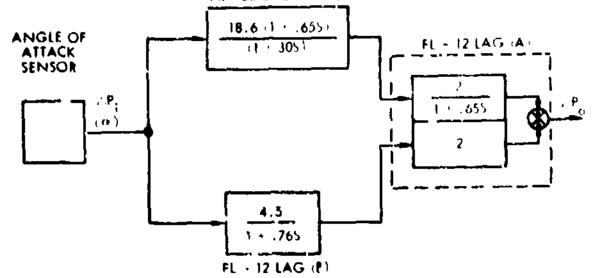


Figure 16.8.5.6. FL-12 Lag Circuits in Naval Aircraft Carrier Landing Control
(Courtesy of General Electric Company, Schenectady, New York)

16.8.5.7 NOTCH. The lag-lead and lead lag functions have been combined in a single amplifier by providing both lagged negative and lagged positive feedback around a gain block (Figure 16.8.5.7). The capacitors in the negative and positive feedback paths are of different volumes. The ratio of their size determines the location and magnitude of the notch. It can be used as a filter or to provide proportional-plus-integral-plus-derivative control action.

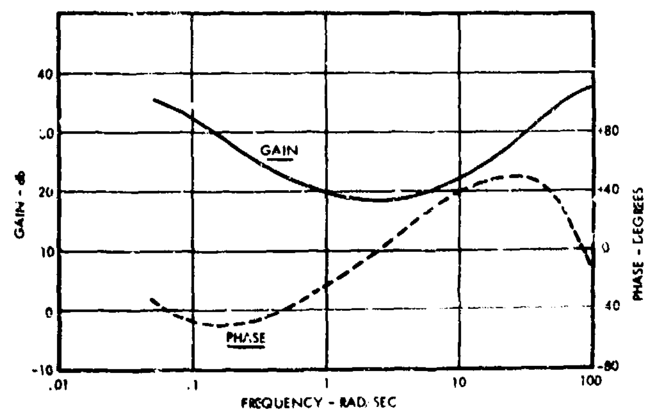


Figure 16.8.5.7. Frequency Response of a Notch Network
(Courtesy of General Electric Company, Schenectady, New York)

16.8.6 Formal Analysis

This section briefly outlines some analytical techniques and tools that could be used to help synthesize and design fluidic systems from a component level and to reduce the time, cost, and uncertainty involved with present repetitive cut-and-try methods. Analysis should be based on component performance data (characteristic curves) which are either made available by the component manufacturer or derived from suitable laboratory testing which is coordinated with the analytical effort. Implementation of analytical techniques will support the system designer in specifying, monitoring, and verifying fluidic component operation in both linear and/or nonlinear system applications. In addition, these techniques should be useful in the specification of the type of testing required for proper component checkout, which will help to ensure successful system design.

16.8.6.1 ANALYTICAL TECHNIQUES. In order to perform a useful analysis of fluidic systems, it will be necessary to model the operation of the components and associated connecting passageways involved from a dynamic as well as a static viewpoint. There are several ways that the dynamic and static analysis of fluidic systems can be approached. Two possible ways are:

1) Fluid dynamic analysis of the detailed complex flow phenomena involved
2) Fluid circuit analysis analogous to the approach used for electronic circuit design.

The fluid dynamic analysis approach has yielded very little practical information for fluidic system designers to date due to the nonlinearities involved in the governing partial-differential fluid flow equations (Navier-Stokes). This is true both at the component and system levels. Thus, the first approach is not recommended as it does not appear to be applicable to overall fluidic system design at the present time.

A logical area for fluidic system modeling lies in the second approach, i.e., the application of fluid circuit theory as outlined in Reference 532-1 and further discussed in Reference 745-1 in connection with a dynamic analysis design philosophy for fluidic systems. Each of these references point out the logical adaptation of equivalent electrical circuit theory utilizing a mix of lumped and distributed parameters to perform fluidic system modeling. Circuit theory is applicable to the interconnecting lines between circuit elements and is covered in detail in Reference 68-92. It should also be applicable to fluidic components themselves in a manner similar to that used in Reference 765-1 in defining small signal dynamics of various proportional amplifiers by means of derived equivalent circuits which is an adaptation of electronic design techniques.

In areas where application of circuit theory becomes untractable, it will be necessary to use the black box technique (see Sub-Topic 16.8.3), based on extensive fluidic component testing (both static and dynamic), to establish the required analytical transfer functions for the fluidic device in question. Binary logic design (see Sub-Topic 16.8.4) should be based on standard techniques, such as Boolean algebra, which have been developed and used in the design of digital computers. These techniques will be helpful in optimizing the selection and combination of bistable fluidic components such as flip-flops, OR-NOR, and AND-NAND gates into such digital devices as adders, counters, timers, multivibrators, and shift registers, to be used for either sensing, logic, or control functions.

16.8.6.2 ANALYTICAL TOOLS. The analytical tools needed to perform fluidic circuit analysis fall into two categories:

1) *Analytical Techniques* — Large-signal nonlinear problems using graphical techniques and small-signal problems using linearizing approximations
2) *Computer Aided Design (CAD) Techniques* — Based on programmed solutions generated on analog, digital, or

hybrid computers, used to solve either small or large-signal linear or nonlinear problems.

In general, the application of purely analytical techniques to the design of fluidic systems will be limited to relatively simple circuits. Techniques for performing analyses for both small signals based on equivalent linear circuits and large signals based on graphical analysis are covered in Reference 765-1 and Sub-Topic 16.8.3, Control Circuit Design. The techniques outlined in detail in Reference 765-1 are similar to the standard procedures used for both single and multiple stage electronic circuit design. The only new facet that has been added is the generation of equivalent circuits which include the time delays associated with signal propagation.

The application of computer analysis is recommended for the design of relatively large, complex, fluidic systems to achieve a more rapid turn-around time than presently possible. This approach should also minimize the costs associated with the system design, development, and test cycle.

There are two digital programs, ECAP and SCEPTRE, which are presently being used to aid electronic circuit designers in the design and development of complex electronic circuits. Use of these programs helps minimize the costs associated with the breadboarding and testing of actual hardware prior to finalizing a design. Since both of these programs are circuit analysis oriented, they can also be gainfully used for computer analysis of complex fluidic systems utilizing fluidic component performance data to characterize and model the equivalent networks.

ECAP (Reference 94-7) is basically oriented to handle small signal or linear circuits. To use the program, an equivalent linear circuit is first established in which any representation of such components as diodes and transistors can be used, provided it can be modeled with conventional linear passive circuit elements, voltage and current sources, and current sensing switches. The matrix approach is fundamental (solution is not dependent on a transfer function approach) and information on basic network branches (circuit topology) are key entries to the computer. The input to the program is user-oriented, i.e., no translational language is needed. ECAP can perform DC, AC, and transient analysis and has options for sensitivity, standard deviation, and worst-case analysis. The latter options are useful in establishing component tolerance criteria which are compatible with overall system performance specifications. Reference 765-1 provides equivalent circuits for fluidic components which can be used in ECAP to perform AC analysis (provide system frequency response) with suitable modifications to include fluidic component and circuit connection time delays.

The SCEPTRE program (see Reference 94-6) was written to allow the designer to perform both DC and transient analysis of large nonlinear electronic networks. The input format of this program again basically describes the topology of the circuit and the discrete circuit elements. However, unlike ECAP, these circuit elements may be nonlinear and/or linear. The input format for nonlinearities can be either tables or equations. In addition, active models can be built up from passive elements and stored in a library and recalled for use as needed. Thus, SCEPTRE should facilitate fluidic circuit modeling and analysis for large signal cases provided that the fluidic time delays are accounted for. A typical area of application would be monostable or bistable switching devices used either as relays or to implement logic operations. Modeling involving component cascading would be simplified through the use of the stored model feature. Impedance matching and/or stage isolation in the case of cascaded analog and digital components would also be facilitated through the use of SCEPTRE.

On-line computers can be used to help optimize the design and checkout of both synchronous and nonsynchronous digital logic circuitry which involve fluidic switching hardware. This analytical approach would be especially useful to ensure the acceptable operation of switching networks operating at relatively high speeds.

Timing problems can arise in this case due to inherent fluidic system time delays and the effect of signal noise modification of the signal pulse width which could cause a loss of signal. Examples of the problems involved and proposed solutions are given in Reference 68-92. Additional information on computer-aided design may be found in Section 8.0 of this handbook.

16.9 FABRICATION AND MATERIALS

16.9.1 Basic Elements

Fluidic devices can be made by a wide variety of manufacturing processes and in almost any type of rigid material. Techniques for the fabrication of these devices are well known and not difficult. The most important consideration is that the performance and characteristics of a fluidic device are closely related to its geometric shape, so that in fabricating fluidic devices intricate shapes must be held to precise dimensions.

The size of fluidic devices varies widely because of the many different types and also because of the different uses for the same device. For example, fluidic elements used as logic gates in aerospace applications are miniaturized to minimize power consumption. A similar device used as a switch to divert flow in a pipeline is much larger. Because of this wide diversity in the size required, quantities involved, tolerances required, and materials used, there is no singularly best fabrication technique for fluidic devices.

Manufacturing processes in common use include the casting, thermoforming, photoetching, and molding of plastics; chemical milling, photoetching, electrical discharge machining, electroforming, die casting, and powder metallurgy of metals; and photoetching, ultrasonic machining, and electron beam machining of ceramics. This listing is only representative of the wide range of choices available.

The environmental tolerance required of a fluidic element is the primary consideration in the choice of material. Fluidic devices must have sufficient strength to withstand both structural and hydraulic forces without undue distortion. Surface hardness of the material must also be considered, particularly if the working fluid carries abrasive particles. Wear in stream-interaction devices is critical in the nozzles and on the splitter. Other factors, such as operating temperature and compatibility with the working fluid, also enter into the selection.

Injection molding of thermoplastic material appears to offer the cheapest method of fabricating large quantities of fluidic elements. However, these elements are limited to operation at near room temperature conditions and with noncorrosive media. In industrial applications, injection molded devices should provide long-term reliable operation, particularly in digital systems.

There are several fabrication methods which are suitable for two-dimensional elements for aerospace applications. The important methods are discussed below.

16.9.1.1 COMPRESSION MOLDING. This is perhaps the most economical production method for manufacturing parts from thermosetting materials. Tolerances can be held as close as required for most fluidic elements. Fillers are used to add stiffness, control shrinkage, and reduce the coefficient of thermal expansion. Maximum operating temperature is about 400°F for the best filled thermosetting plastic elements, and filled epoxy elements are limited to about 300°F.

16.9.1.2 PHOTOETCHING CERAMICS. This process was originally developed by the Corning Glass Works to prepare substrates for electronic circuits and has been adapted to the manufacture of fluidic elements. A high contrast negative is placed upon a thin sheet of Fotoform glass which is a silicate glass containing a photosensitizing ingredient such as the cesium radical, Ce^{+3}. In the presence of ultraviolet light, the exposed glass absorbs the ultraviolet radiation, creating a contact print in depth. The glass is then heated to about 1200°F so that colloidal particles of crystallized lithium metasilicate appear as a white opal image, which is formed in the exposed areas of the glass. When the glass sheet is immersed in a hydrofluoric acid bath, the exposed areas dissolve 20 to 30 times faster than the clear unexposed areas of the glass.

Further processing converts the Fotoform glass to a higher-strength partially-crystalline material called Fotoceram. The finished Fotoceram elements offer several important advantages which are normally associated with ceramic material, i.e., high dimensional stability, low moisture absorption, good shock resistance, and operating temperatures approaching 1000°F. This process can produce intricate two-dimensional elements down to a nozzle width of 0.005 inch. An important consideration in circuit fabrication is that both the Fotoform and Fotoceram plates can be thermally laminated to form a monolithic structure.

16.9.1.3 PHOTOETCHING METALS. This process has recently become very important in the manufacture of fluidic elements for aerospace applications. Essentially the process removes metal by the chemical etching of preferentially exposed surfaces. The process is presently limited to metal sheets no thicker than about 0.020-inch, because the dimensional tolerances that can be achieved increase with increasing metal thickness. A 0.005-inch wide channel can be etched through a 0.001-inch thick stainless steel with a tolerance of 0.00025-inch or about ±5 percent. This same 0.005-inch wide channel would have a tolerance of ±20 percent if etched in a 0.005-inch thickness of the same material.

In the fabrication of two-dimensional fluidic elements, several laminations of etched sheets are required to provide the required aspect ratio. Photoetching can be used with the following metals (presented in the order of increasing difficulty): copper, nickel, carbon steel, stainless steel, aluminum, titanium, and molybdenum. Operating temperature depends primarily on the metal used and the method selected for sealing the laminated sheets.

16.9.1.4 OTHER METHODS. Many new methods are being considered for the fabrication of fluidic elements. Techniques such as electron and laser beam machining may eventually make it possible to pack 1000 fluidic elements in one cubic inch. Coining techniques may soon make it possible to manufacture interconnected fluidic elements by indexing a die and stamping in the right location. However, much work still needs to be done in the sealing of fluidic elements, particularly those for use with high temperature working fluids. To date, diffusion bonding and furnace brazing have been used with moderate success in the sealing of photoetched metal elements, but more efficient sealing methods are required if the inherent reliability of fluidic elements is to be approached.

One way of overcoming the sealing problem is a ceramic molding process where a polystrene mold is made from a metal master. Ceramic is then molded around the polystyrene and fired at about 2000°F. The polystyrene is vaporized, leaving a one-piece ceramic device. Although the process is complex, it eliminates the basic problem of sealing a two-dimensional element with a cover plate.

16.9.2 Integrated Circuits

The interconnection of fluidic elements with fittings and tubing is neither practical nor reliable enough for aerospace circuits. One trend in aerospace systems is to group circuit elements on a functional basis in rectangular or circular two-dimensional planar arrays or modules (Figure 16.9.2a). This allows the incorporation of the maximum number of interconnections within the module. Power supplies, vent connections, and interconnections between modules can then be accomplished by interspersing manifolds between the modules (Figure 16.9.2b). The number of circuit modules that can be stacked is limited because the supply and exhaust ports as well as the circuit interconnections must all be ported through the stacked circuit blocks, and a point of diminishing returns is eventually reached.

Another method is to bring all the connections out to the edge of the module. Modules can then be stacked on edge in between manifolds which provide the fluid power supplies and circuit interconnections. As shown in the physical concept of a rocket engine fluidic controller (Figure 16.9.2c), this makes for a convenient arrangement in that sensors, interfaces, and compensating volumes can be located close to the circuit modules.

For smaller fluidic circuits it may be more convenient to fabricate the manifold and interconnections in a single block (Figure 16.9.2d). Then the fluidic elements, sensors, and interfaces are externally attached to the manifold block. This method is more convenient for prototype applications and allows the modification or replacement of circuit elements when required.

16.10 TEST EQUIPMENT

16.10.1 Introduction

A block diagram concept of a fluidic control system for aerospace application which contains several sensing, computation, and control actuation functions is shown in Figure 16.10.1. It indicates that some system instrumentation will be self-contained, i.e., designed as an integral part of the control system and used both for operational instrumentation and ground test. System inputs are also provided for diagnostic instrumentation and for fluid or electrical perturbation of the system during ground test. Sensors and techniques selected for checkout of a fluid control system should provide pertinent test data without disturbing normal system operation. The sensors should be

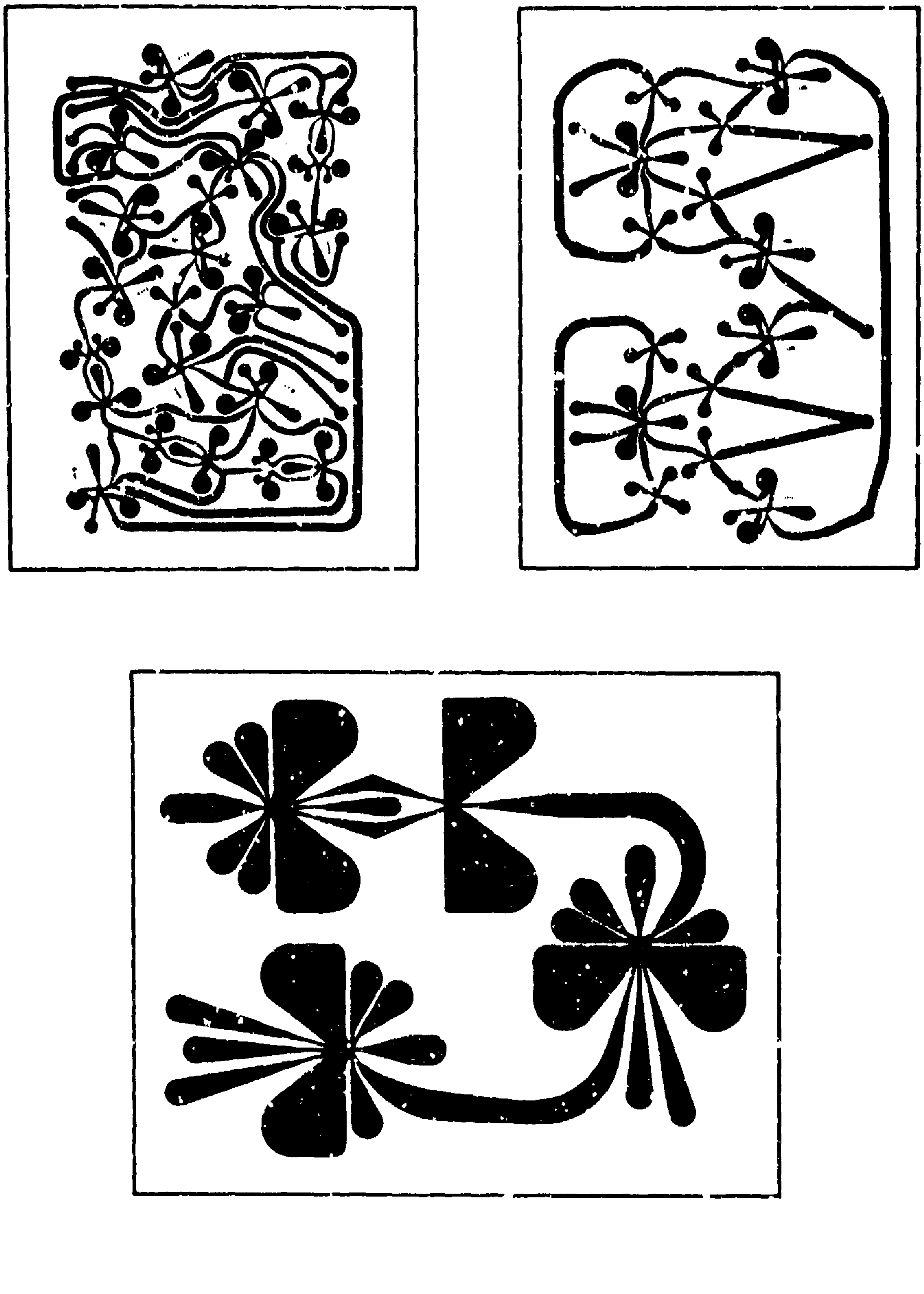


Figure 16.9.2a. Silhouettes of Circuit Modules – Planar Arrays
(Reference 131-42)

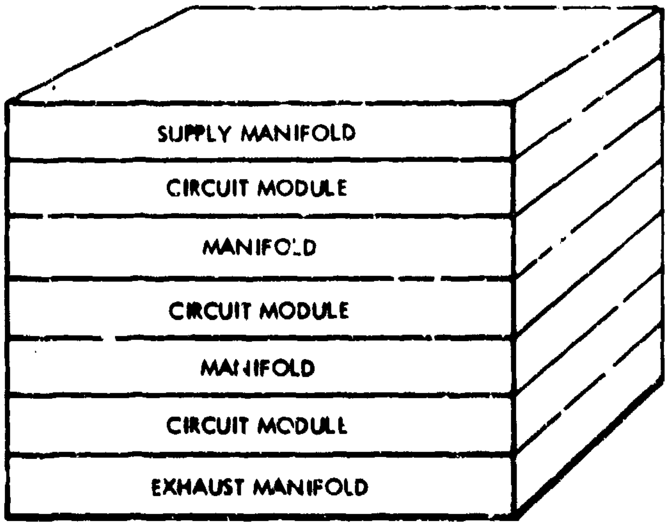


Figure 16.9.2b. Stacked Integrated Fluidic Circuits (Reference 131-42)

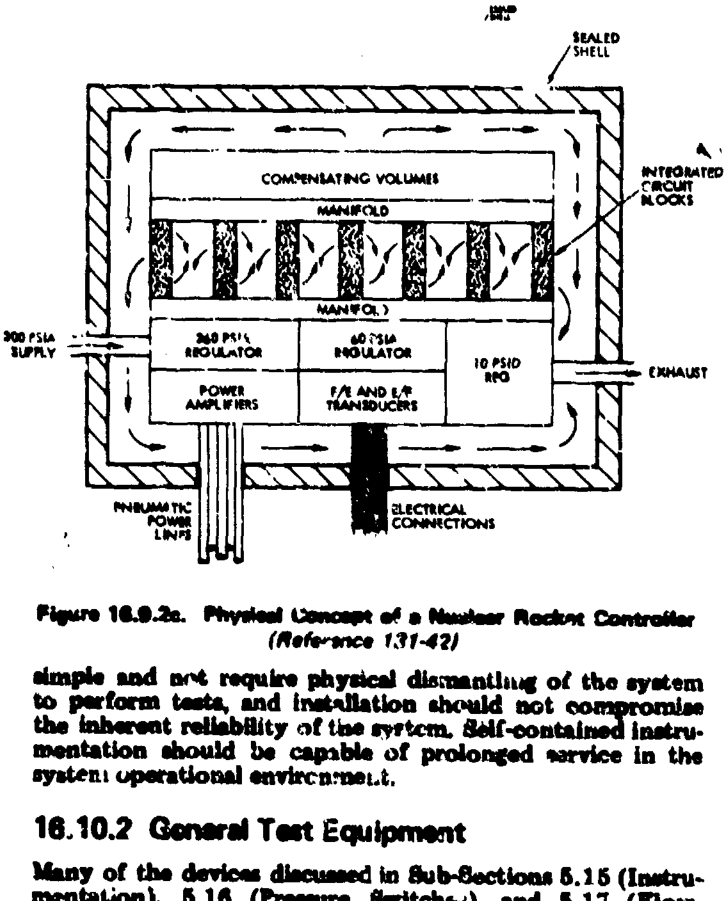


Figure 16.9.2c. Physical Concept of a Nuclear Rocket Controller (Reference 131-42)

simple and not require physical dismantling of the system to perform tests, and installation should not compromise the inherent reliability of the system. Self-contained instrumentation should be capable of prolonged service in the system operational environment.

16.10.2 General Test Equipment

Many of the devices discussed in Sub-Sections 5.15 (Instrumentation), 5.16 (Pressure Switches), and 5.17 (Flowmeters), as well as Section 15.0 (Component Testing) of this handbook are useful in the test of fluidic systems. Other instruments are also available for use in aerodynamics and thermodynamics which can be used directly in fluidics, particularly for steady-state pressure and flow measurements. In addition, many of the E-F and F-E transducers described in Sub-Section 16.5 (Fluidic Interfaces) can also

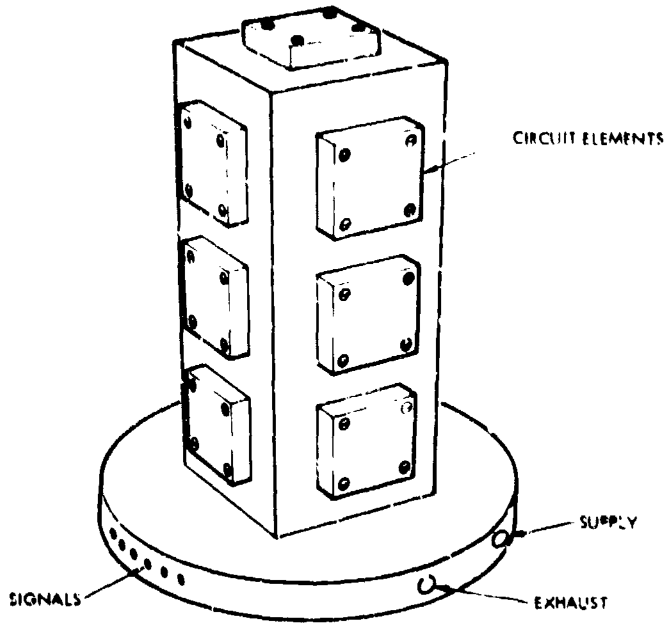


Figure 16.9.2d. Single Manifold Integrated Fluidic Circuit (Reference 131-42)

serve as excellent means of perturbating and monitoring a fluidic system during ground test.

16.10.3 Specialized Test Equipment

Some of the specialized sensors and techniques applicable to the checkout of a fluidic system are discussed in the following detailed topics.

16.10.3.1 CONSTANT-TEMPERATURE ANEMOMETER. Hot-wire and hot-film anemometers have long been valuable tools for making physical measurements in gas streams. The basic measurement is the rate of heat loss from the hot-wire or film to the gas stream. For the majority of applications, this heat loss has been interpreted in terms of velocity for a constant-temperature, constant-pressure air stream. The hot-wire is also sensitive to temperature, density, and composition fluctuations, and has been used specifically for temperature and composition measurements in low-velocity flows. For high velocity flows, density changes become an important parameter in the heat loss equation.

Hot-wire and hot-film sensors will operate reliably at temperatures up to 1000°F and, if required, can be used in environments up to 1500°F. The hot-wire sensor with suitable electronics has a bandwidth of 0 to 5000 cps at near zero velocities, which increases as a function of the mean velocity to approximately 50,000 cps at approximately 300 ft/sec. The electronic output can be made proportionately linear to velocity or mass flow. In a small, low-velocity passage, the sensor can also be compensated to provide an output linear with gage, absolute, or differential pressure.

Subminiature, quartz-coated, hot-film sensors are considered most suitable for application to fluidic system instrumentation. These sensors provide unequalled stability for flow measurements in gases and liquids, and are very rugged, contamination-resistant, and easy to handle. In gases, they have a bandwidth of 0 to 35,000 cps. A single hot-film sensor with simplified electronics will provide good accuracy over a dynamic flow range greater than 1000 to 1 and over a bandwidth of 0 to 3000 cps.

Hot-film sensors are made in several size sensor holders, down to approximately 0.020 inch. The 1/16-inch diameter sensor would be a good standard size for application to system instrumentation. A standard-type fitting, as shown in Figure 16.10.3.1a, could be used for ease of installation to the proper depth, and a standard plug can be used to seal the transducer installation when not in use. The sensor end could be transverse, single-ended, or flush and as rugged as possible, commensurate with minimum flow disturbance in operational channels. Several types of sensor ends are illustrated in Figure 16.10.3.1b.

The primary uses of the basic hot-wire and hot-film sensors are:

1) Both types are excellent for high response measurement of velocity and mass flow.

2) A single sensor can be used for flow switching indication with simple circuitry (digital information). As shown in Figure 16.10.3.1c, the sensor is placed in the base circuit of a high-speed switching transistor, Q_1. With low flow in the channel, Q_1 is off and Q_2 is on, and the relay is energized. When a preset minimum flow is sensed in the flow channel, the hot-wire sensor resistance decreases and the voltage at A goes more positive so that Q_1 turns on and Q_2 turns off, de-energizing the relay.

3) A hot-wire makes an excellent pressure transducer, whenever a small flow can be bled into or out of a flow channel without affecting performance. This transducer utilizes a hot-wire sensor inside a very small capillary tube (0.004-inch diameter) approximately 1-inch long. The tube is sized so that the flow velocity is a direct function of the pressure difference between the flow channel and an outside reference pressure. A differential pressure measurement can be made in a bypass, low-velocity, duct between the output legs of an amplifier, but direction must be sensed by other means.

16.10.3.2 THERMISTOR SENSOR. A thermistor sensor can be installed on the end of a probe with a diameter of 1/16-inch or less. Depending on size the sensor would have a response time of 1 millisecond or better and could be used in place of the hot-wire sensor for flow-switching indication, as shown previously in Figure 16.10.3.1c.

16.10.3.3 PRESSURE TRANSDUCERS. Many pressure transducers currently available have excessively large pressure cavity volumes as well as large volumetric displacements in the operational pressure range. These transducers are still useful for low frequency (<100 cps) measurements if properly installed in fluidic devices. For high frequency measurements, the flush-mounted miniature types are the most practical. The semiconductor strain gage pressure transducer is considered to be the best choice where high sensitivity, good accuracy, and high frequency response are required. Natural frequencies as high as 100 kilocycles have been reported to date with this type of transducer. They are available in sizes as small as 0.1-inch in diameter by 0.002-inch thick, in ranges from 0-0.1 to 0-10,000 psig or psid. The differential-pressure type can be adapted for installation within an integrated fluidic system circuit block. The operational temperature range is -125°F to +350°F for semiconductor strain gage elements, and considerably higher for wire strain gage types.

Quartz pressure transducers with electrostatic charge amplifiers are capable of higher response than the semiconductor strain gage type. However, the incremental increase in response to approximately 130,000 cps is made at the sacrifice of sensitivity. The quartz transducer must be increased in size (0.37-inch diameter sensor) for low-pressure, high-sensitivity measurements, and the resonant frequency is reduced to 60,000 cps for this larger size.

16.10.3.4 PRESSURE SWITCHES. Several types of miniature pneumatic pressure switches are commercially available for use in checkout of a digital fluidic system. These may be used if the additional volume and slow response (<100 cps) do not compromise system performance.

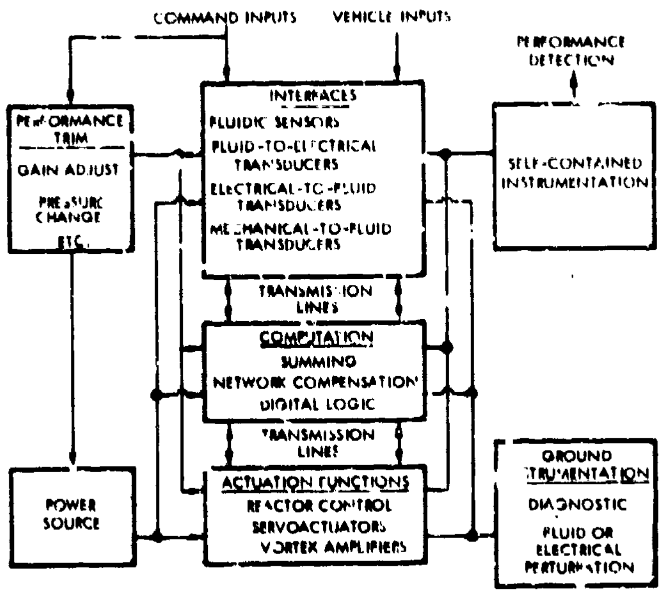


Figure 16.10.1. Fluidic Control System Concept

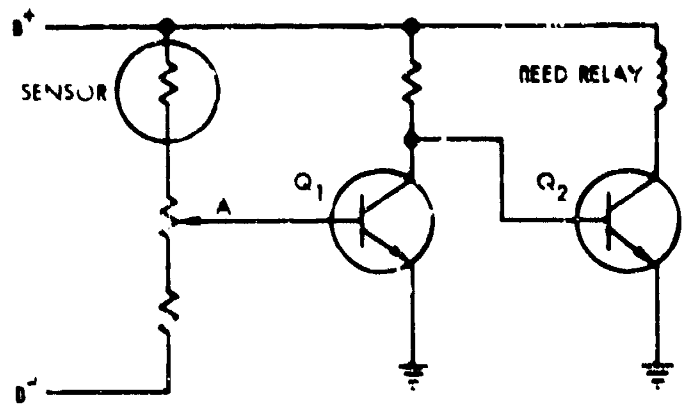


Figure 13.10.3.1c. Voltage Trip Circuit For Hot-Film Flow Sensor

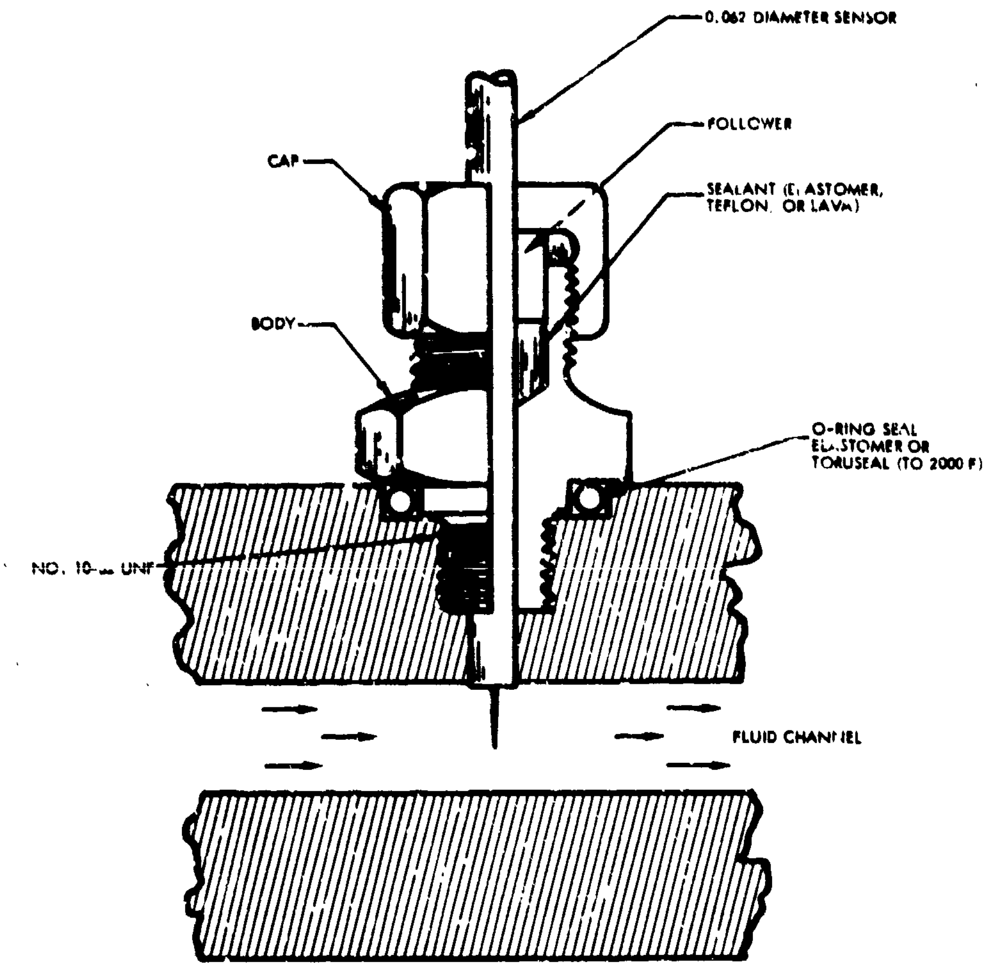


Figure 13.10.4.1a. Universal Transducer Fitting

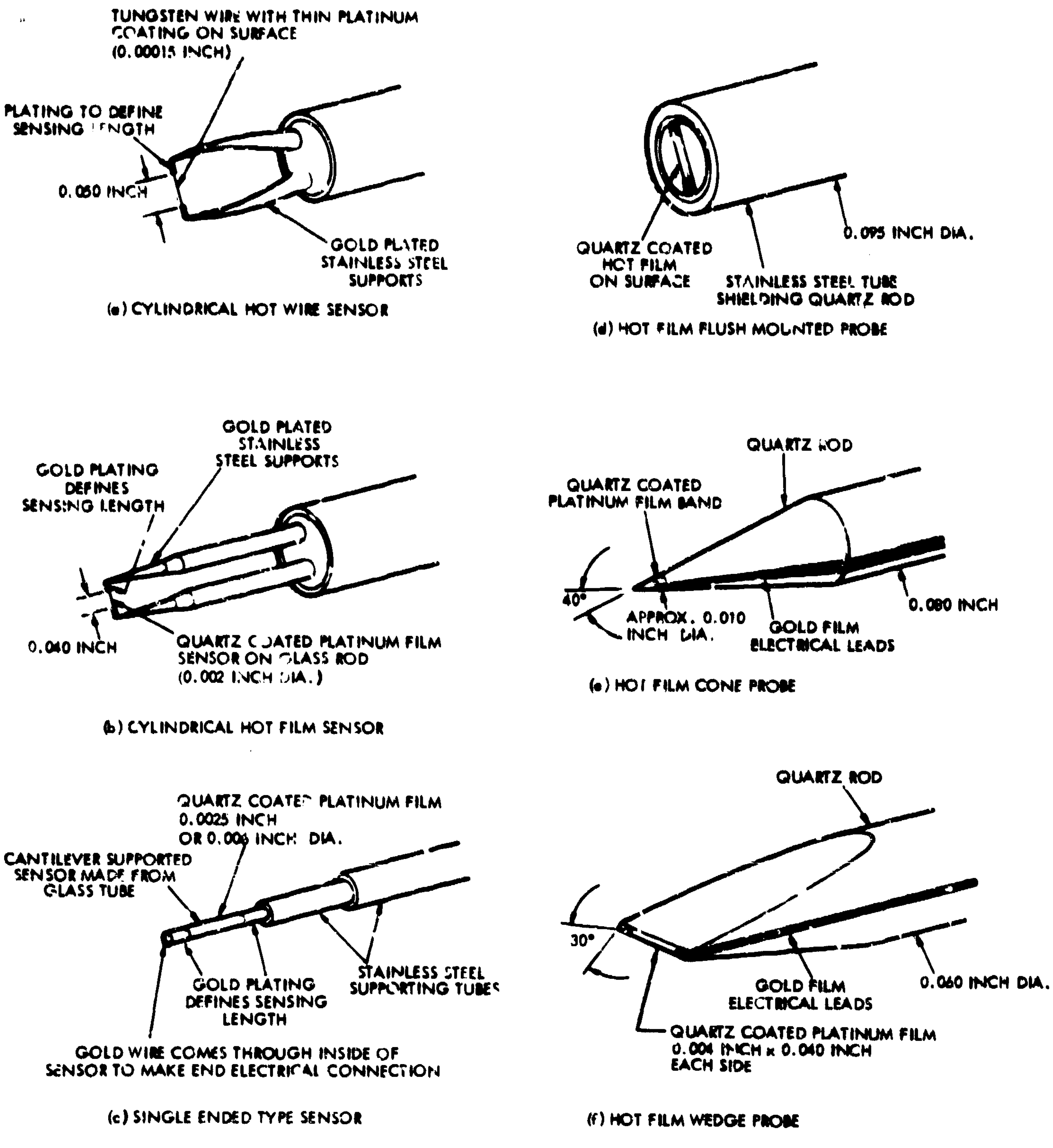


Figure 16.10.3.1b. Types of Hot-Wire and Hot-Film Sensor Ends
(Courtesy of Thermo-Systems Inc., Saint Paul, Minnesota)

REFERENCES

Symbol	Quantity	International System (SI) Units	Standard* Units	Dimension
. . .	Angle	radians, rad	degrees, deg	
a	Acceleration	m/s^2	in/sec^2	L/t^2
A	Area	m^2	in^2	L^2
C	Fluid capacitance (weight rate of flow)	in^2	in^2	L^2
f	Frequency	hertz, Hz	cycles/sec, cps	t^{-1}
F	Force	newton, N	pound, lb_f	F
g	Local acceleration of gravity	m/sec	ft/sec^2 or in/sec^2	L/t^2
g_c	Conversion constant = 32.2 in the expression $F = \frac{1}{g_c} ma$	---	$\frac{lb_m}{lb_f}$ ft/sec^2	ML/Ft^2
G	Power gain, average	dimensionless		
G_f	Flow gain, average	dimensionless		
G_{f_i}	Flow gain, incremental	dimensionless		
G_i	Power gain, incremental	dimensionless		
G_p	Pressure gain, average	dimensionless		
G_{p_i}	Pressure gain, incremental	dimensionless		
κ	Specific heat ratio	dimensionless		
K	An arbitrary constant	dimensionless		
L	Fluid inertance (weight rate of flow)	s^2/m^2	sec^2/in^2	t^2/h^2
m	Mass	kilogram, kg	lb_m	M
$\dot{m}$	Mass flow rate	kg/s	lb_m/sec	M/t
M	Mach number	dimensionless		
N_R	Reynolds number	dimensionless		
N_s	Strouhal number	dimensionless		
p	Pressure	N/m^2	lb_f/in^2	F/L^2
p_o	Pressure, total	N/m^2	lb_f/in^2	F/L^2
q	Pressure, dynamic	N/m^2	lb_f/in^2	F/L^2
Q	Volumetric flow rate	m^3/s	in^3/sec	L^3/t
R	Fluid resistance (weight rate of flow)	s/m^2	sec/in^2	t/L^2
R_g	Gas constant	$N\text{-}m/kg\text{-}^\circ K$ or $m^2/s^2/^\circ K$	$in\text{-}lb_f/lb_m\text{-}^\circ R$ or $in^2/sec^2/^\circ R$	LF/MT or L^2/t^2T
s	LaPlace operator	1/s	1/sec	t^{-1}
S/N	Signal-to-noise ratio	dimensionless		

Symbol	
t	Time
T	Temperature,
T_o	Temperature,
u	Velocity, gene
$\bar{u}$	Velocity, mea
u_c	Velocity, acou
V	Volume
w	Weight
$\dot{w}$	Weight flow ra
W	Power
Z	Fluid impedan
α	Acceleration,
β	Bulk modulus
γ	Weight density
ℓ	Length
η	Efficiency
ρ	Mass density
σ	Nozzle aspect
μ	Viscosity, abs
ν	Viscosity, kin
ω	Velocity, angu
c	Control
cd	Control, differ
co	Control, quies
i	Input, or incre
ℓ	Load
o	Output, or tot
od	Output, differ
s	Supply

*The English gravitational system units of MIL-STD-1306 (Reference 447-10) and SAE ARP 993A (Reference 23-72) have been replaced here by the unit fo handbook. See sub-section 1.5 of this handbook.

A

Table 16.3.1. Pertinent Symbols and Their Units
(Adapted from MIL-STD-1306 and SAE ARP 993A)

Symbol	Quantity	International System (SI) Units	Standard* Units	Dimension
t	Time	second, s	second, sec	t
T	Temperature, static	degrees Kelvin, °K	degrees Rankine, °R	T
T_o	Temperature, total	degrees Kelvin, °K	degrees Rankine, °R	T
u	Velocity, general	m/s	in/sec	L/t
$\bar{u}$	Velocity, mean	m/s	in/sec	L/t
u_c	Velocity, acoustic (speed of sound)	m/s	in/sec	L/t
V	Volume	m^3	in^3	L^3
w	Weight	N	lb_f/sec	F
$\dot{w}$	Weight flow rate	N/s	lb_f/sec	F/t
W	Power	N-m/s	lb_f-in/sec	FL/t
Z	Fluid impedance	s/m^2	sec/in^2	t/L^2
α	Acceleration, angular	rad/s^2	rad/sec^2	t^{-2}
β	Bulk modulus of liquid	N/m^2	lb_f/in^2	F/L^2
γ	Weight density	N/m^3	lb_f/in^3	F/L^3
ℓ	Length	meter, m	inch, in.	L
η	Efficiency	dimensionless		
ρ	Mass density	kg/m^3	lb_m/in^3	M/L^3
σ	Nozzle aspect ratio	dimensionless		
μ	Viscosity, absolute	$N\text{-}s/m^2$	$lb\text{-}sec/in^2$	Ft/L^2
υ	Viscosity, kinematic	m^2/s	in^2/sec	L^2/t
ω	Velocity, angular	rad/s	rad/sec	t^{-1}

General Subscripts

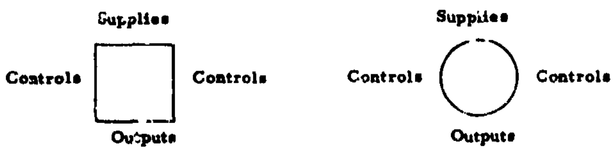
c	Control
cd	Control, differential
co	Control, quiescent
i	Input, or incremental when used with gain (G)
ℓ	Load
o	Output, or total when used with temperature (T_o)
od	Output, differential
s	Supply

*...3-72) have been replaced here by the unit force-mass system to provide compatibility with Section 3.0, Fluid Mechanics, of this

TABLE 16.3.2

General Conventions

The relative port locations for the symbols are patterned in the following manner:



All symbols may be oriented in 90-degree increments from the position shown.

Specific ports are identified by the following nomenclature:

Supply port - S

Control port - C

Output port - O

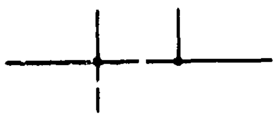
The nomenclature shown on the graphic symbols need not be used on schematic diagrams. It is primarily intended to correlate the function of each port with the truth table.

Supply ports can be either active or passive. An inverted triangle, ∇, denotes a supply source connected to the supply port (active device).

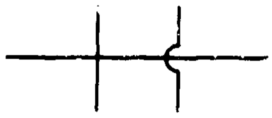
An arrowhead on the control line inside the symbol envelope indicates continual flow is required to maintain state (no memory, no hysteresis):

Indicates no memory

(a) Interconnecting fluid lines shall be shown with a dot at the point of interconnection:



(b) Crossing fluid lines are to be shown without dots:



A small + on the output of a bistable device indicates initial or start-up flow condition.

Logic Notation

A . B = A "and" B

A + B = A "or" B

$\bar{A}$. $\bar{B}$ = "not" A and "not" B

Port Marking

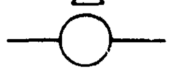
Port nomenclature shown on schematics need not be used on schematic diagrams; the nomenclature may be useful, however, in correlating test data and specification data with the physical device.

Fluidic Impedances

General Resistance - Fixed 

Linear Resistance - Fixed 

Nonlinear Resistance - Fixed 

Capacitance - Fixed 

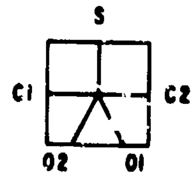
Inductance - Fixed 

Diode 

Bistable Digital Devices

(a) Flip Flop

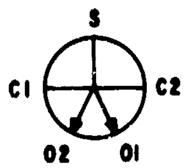
Functional Symbol



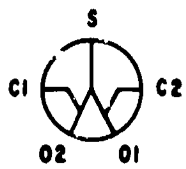
Truth Table

C1	C2	O1	O2
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1

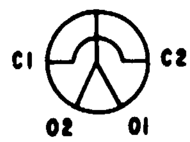
Operating Principle Symbols



Wall Attachment



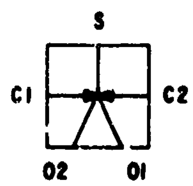
Induction



Edgetone

(b) Digital Amplifier

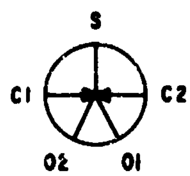
Functional Symbol



Truth Table

C1	C2	O1	O2
1	0	1	0
0	1	0	1
0	0	Undefined	
1	1	Undefined	

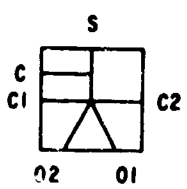
Operating Principle Symbol



Jet Interaction

(c) Binary Counter

Functional Symbol

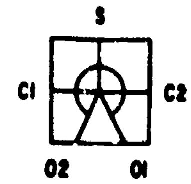


Truth Table

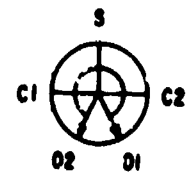
C	C1	C2	O1	O2
0	1	0	1	0
0	0	0	1	0
0	0	1	0	1
0	0	0	0	1
1	0	0	1	0
0	0	0	1	0
1	0	0	0	1
0	0	0	0	1

(d) Multivibrator

Functional Symbol



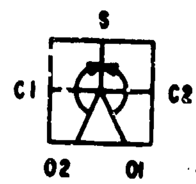
Operating Principle Symbol



Wall Attachment

(e) Oscillator (Sine Wave)

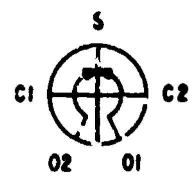
Functional Symbol



C1-C

O1-O

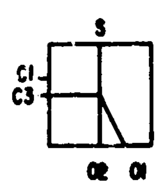
Operating Principle Symbol

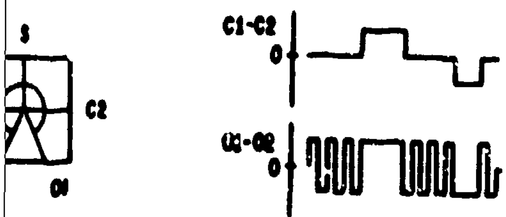


Monostable Digital Devices

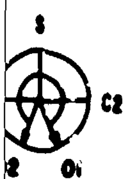
(a) OR-NOR

Functional Symbol



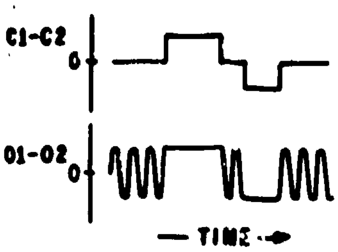


ymbol

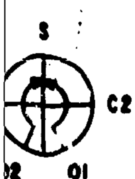


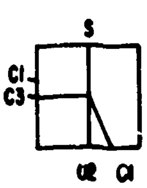
l Attachment

)



ymbol

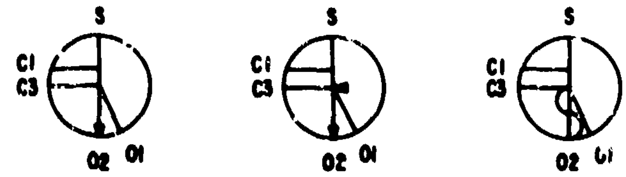




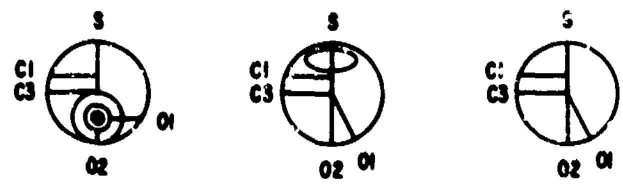
Truth Table

C1	C3	O1	O2
0	0	0	1
1	0	1	0
0	1	1	0

Operating Principle Symbols



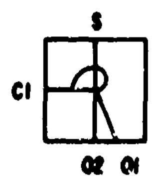
Wall Attachment | Wall Attachment Internal Fluid Bias | Turbulence

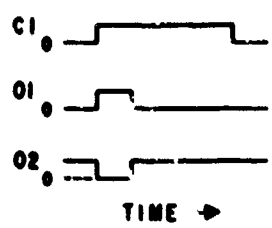


Vortex | Focused Jet | Jet Interaction Geometrical Bias

(b) One-Shot

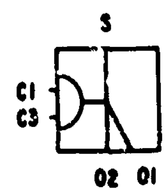
Functional Symbol





(c) AND-NAND

Functional Symbol

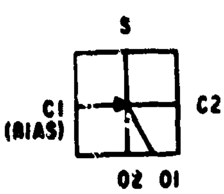


Truth Table

C1	C3	O1	O2
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

(d) Schmitt Trigger

Functional Symbol

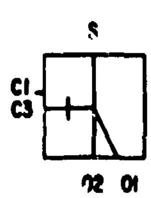


Truth Table

	O1	O2
C1 > C2	1	0
C1 < C2	0	1
C1 = C2	Undefined	

(e) Exclusive OR

Functional Symbol



Truth Table

C1	C2	O1	O2
0	0	0	1
1	0	1	0
0	1	1	0
1	1	0	1

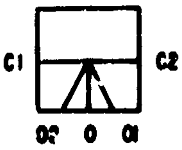
B

TABLE 16.3.2 (CONTI

Passive Digital Devices

(a) AND - 2/3 AND

Functional Symbol



Truth Table

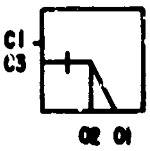
C1	C2	O1	O2	O
1	0	1	0	0
0	1	0	1	0
1	1	0	0	1
0	0	0	0	0

Operating Principle Symbols

Passive
Jet Interaction

(b) Exclusive OR-AND

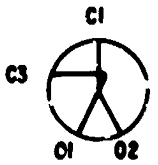
Functional Symbol



Truth Table

C1	C3	O1	O2
1	0	1	0
0	1	1	0
1	1	0	1
0	0	0	0

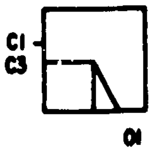
Operating Principle Symbols



Passive
Jet Interaction

) OR

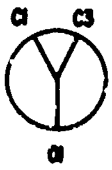
Functional Symbol



Truth Table

C1	C3	O1
0	0	0
1	0	1
0	1	1
1	1	1

Operating Principle Symbols

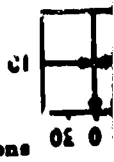


Passive
Jet Interaction

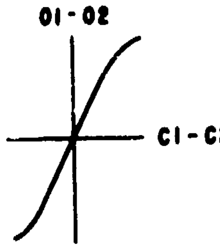
Analog Fluidic Devices

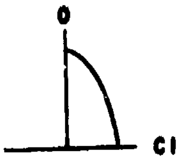
(a) Proportional Amplifiers

Functional Symbol

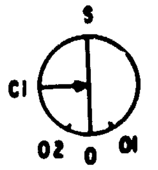


Functions



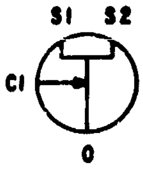


Operating Principle Symbols



Single Input
Jet Interaction

D
Jet



Transverse
Impact Modulator

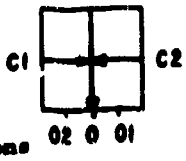
Impa

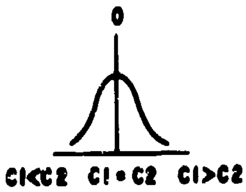
2 (CONTINUED)

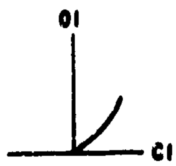
rs

al

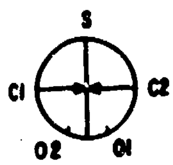
ctions



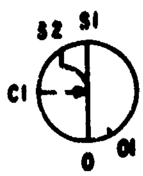




ymbols

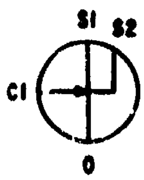


Differential
Jet Interaction

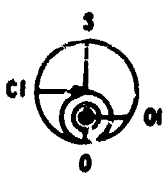


Separation
Point Control

nt
ow



Direct
Impact Modulator

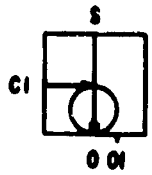


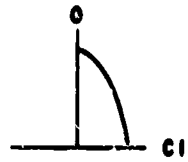
Vortex

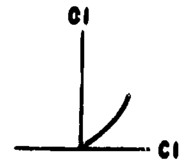
rse
ulator

(b) Throttling Valve

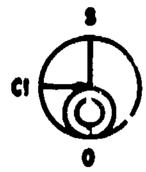
Functional Symbol







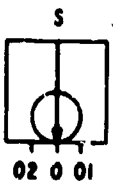
Operating Principle Symbol



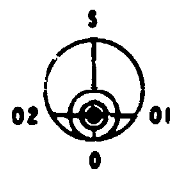
Vortex

(c) Rate Sensor

Functional Symbol



Operating Principle Symbol



Vortex

APPENDIX A

TABLE OF CONTENTS

A.0 GREEK ALPHABET

Α	α	alpha	Η	η	eta	Ν	ν	nu	Τ	τ	tau
Β	β	beta	Θ	θ	theta	Ξ	ξ	xi	Υ	υ	upsilon
Γ	γ	gamma	Ι	ι	iota	Ο	ο	omicron	Φ	φ	phi
Δ	δ	delta	Κ	κ	kappa	Π	π	pi	Χ	χ	chi
Ε	ε	epsilon	Λ	λ	lambda	Ρ	ρ	rho	Ψ	ψ	psi
Ζ	ζ	zeta	Μ	μ	mu	Σ	σ	sigma	Ω	ω	omega

A.1 ENGINEERING DATA

A.1.1 Atomic Weights and Numbers

PERIODIC CHART

IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIII			IB	IIB	IIIA	IVA	VA	VIA	VIIA	Inert gases
1 H 1.008																1 H 1.008	2 He 4.003
3 Li 6.940	4 Be 9.013											5 B 10.82	6 C 12.010	7 N 14.008	8 O 16.000	9 F 19.00	10 Ne 20.183
11 Na 22.997	12 Mg 24.32											13 Al 26.97	14 Si 28.06	15 P 30.98	16 S 32.066	17 Cl 35.457	18 A 39.944
19 K 39.096	20 Ca 40.08	21 Sc 45.10	22 Ti 47.90	23 V 50.95	24 Cr 52.01	25 Mn 54.93	26 Fe 55.85	27 Co 58.94	28 Ni 58.69	29 Cu 63.54	30 Zn 65.38	31 Ga 69.72	32 Ge 72.60	33 As 74.91	34 Se 78.96	35 Br 79.916	36 Kr 83.7
37 Rb 85.48	38 Sr 87.63	39 Y 88.92	40 Zr 91.22	41 Nb 92.91	42 Mo 95.95	43 Tc (99)	44 Ru 101.7	45 Rh 102.91	46 Pd 106.7	47 Ag 107.88	48 Cd 112.41	49 In 114.76	50 Sn 118.70	51 Sb 121.76	52 Te 127.61	53 I 126.92	54 Xe 131.3
55 Cs 132.91	56 Ba 137.36	57 *La 138.92	72 Hf 178.6	73 Ta 180.88	74 W 183.92	75 Re 186.31	76 Os 190.2	77 Ir 193.1	78 Pt 195.23	79 Au 197.2	80 Hg 200.61	81 Tl 204.39	82 Pb 207.21	83 Bi 209.00	84 Po 210.	85 At (210)	86 Rn 222
87 Fr (223)	88 Ra 226.05	89 †Ac 227.0															

* Lanthanum series...........	58 Ce 140.13	59 Pr 140.92	60 Nd 144.27	61 Pm (147)	62 Sm 150.43	63 Eu 152.0	64 Gd 156.9	65 Tb 159.2	66 Dy 162.46	67 Ho 164.94	68 Er 167.2	69 Tm 169.4	70 Yb 173.04	71 Lu 174.99
† Actinium series............	90 Th 232.12	91 Pa 231	92 U 238.07	93 Np (237)	94 Pu (239)	95 Am (241)	96 Cm (242)	97 Bk (243)	98 Cf (244)	99 (E) 253	100 (Fm) 254	101 (Mv) 256	102 (No) (254)	103 (Lw) (257)

Numbers in parenthesis indicate atomic mass of most stable known isotope.
Source: Fisher Scientific Company.

The CHEMICAL ELEMENTS with their ATOMIC WEIGHTS and NUMBERS

(Based on Carbon 12 according to latest information available and Commission on Atomic Weights, International Union of Pure and Applied Chemistry)

Name	Symbol	Atomic Number	Atomic Weight	Name	Symbol	Atomic Number	Atomic Weight	Name	Symbol	Atomic Number	Atomic Weight
Actinium	Ac	89	(227)	Hafnium	Hf	72	178.49	Potassium	K	19	39.102
Aluminum	Al	13	26.98	Helium	He	2	4.003	Praseodymium	Pr	59	140.91
Americium	Am	95	(243)	Holmium	Ho	67	164.93	Promethium	Pm	61	(147)
Antimony	Sb	51	121.75	Hydrogen	H	1	1.00797	Protactinium	Pa	91	(231)
Argon	Ar	18	39.948	Indium	In	49	114.82	Radium	Ra	88	(226)
Arsenic	As	33	74.92	Iodine	I	53	126.90	Radon	Rn	86	(222)
Astatine	At	85	(210)	Iridium	Ir	77	192.2	Rhenium	Re	75	186.2
Barium	Ba	56	137.34	Iron	Fe	26	55.85	Rhodium	Rh	45	102.91
Berkelium	Bk	97	(249)	Krypton	Kr	36	83.80	Rubidium	Rb	37	85.47
Beryllium	Be	4	9.012	Lanthanum	La	57	138.91	Ruthenium	Ru	44	101.1
Bismuth	Bi	83	208.98	Lawrencium	Lw	103	(257)	Samarium	Sm	62	150.35
Boron	B	5	10.81	Lead	Pb	82	207.19	Scandium	Sc	21	44.96
Bromine	Br	35	79.909	Lithium	Li	3	6.939	Selenium	Se	34	78.96
Cadmium	Cd	48	112.40	Lutetium	Lu	71	174.97	Silicon	Si	14	28.09
Calcium	Ca	20	40.08	Magnesium	Mg	12	24.31	Silver	Ag	47	107.870
Californium	Cf	98	(251)	Manganese	Mn	25	54.94	Sodium	Na	11	22.990
Carbon	C	6	12.011	Mendelevium	Md	101	(256)	Strontium	Sr	38	87.62
Cerium	Ce	58	140.12	Mercury	Hg	80	200.59	Sulfur	S	16	32.064
Cesium	Cs	55	132.91	Molybdenum	Mo	42	95.94	Tantalum	Ta	73	180.95
Chlorine	Cl	17	35.453	Neodymium	Nd	60	144.24	Technetium	Tc	43	(99)
Chromium	Cr	24	52.00	Neon	Ne	10	20.183	Tellurium	Te	52	127.60
Cobalt	Co	27	58.93	Neptunium	Np	93	(237)	Terbium	Tb	65	158.92
Copper	Cu	29	63.54	Nickel	Ni	28	58.71	Thallium	Tl	81	204.37
Curium	Cm	96	(247)	Niobium	Nb	41	92.91	Thorium	Th	90	232.04
Dysprosium	Dy	66	162.50	Nitrogen	N	7	14.007	Thulium	Tm	69	168.93
Einsteinium	Es	99	(254)	Nobelium	No	102	(254)	Tin	Sn	50	118.69
Erbium	Er	68	167.26	Osmium	Os	76	190.2	Titanium	Ti	22	47.90
Europium	Eu	63	152.0	Oxygen	O	8	15.999	Tungsten	W	74	183.85
Fermium	Fm	100	(253)	Palladium	Pd	46	106.4	Uranium	U	92	238.03
Fluorine	F	9	19.00	Phosphorus	P	15	30.974	Vanadium	V	23	50.94
Francium	Fr	87	(223)	Platinum	Pt	78	195.09	Xenon	Xe	54	131.30
Gadolinium	Gd	64	157.25	Plutonium	Pu	94	(242)	Ytterbium	Yb	70	173.04
Gallium	Ga	31	69.72	Polonium	Po	84	(210)	Yttrium	Y	39	88.91
Germanium	Ge	32	72.59					Zinc	Zn	30	65.37
Gold	Au	79	197.0					Zirconium	Zr	40	91.22

Note: Atomic Weight in parentheses indicates the mass number of the most stable isotope.

A.1.2 Numerical Prefixes

Prefix	Order	Symbol
atto —	10^{-18}	a
femto —	10^{-15}	f
pico —	10^{-12}	p
nano —	10^{-9}	n
micro —	10^{-6}	μ
milli —	10^{-3}	m
centi —	10^{-2}	c
deci —	10^{-1}	d
deka —	10	da
hecto —	10^{2}	h
kilo —	10^{3}	k
mega —	10^{6}	M
giga —	10^{9}	G
tera —	10^{12}	T

A.1.3 Physical Constants

Avogadro's Number, N_o $= 6.0228 \times 10^{23}$ atoms/mole

Planck's Constant, h $= 6.624 \times 10^{-34}$ joule sec

Stefan-Boltzmann Constant $= 1.713 \times 10^{-9}$ Btu/(hr)(ft^2)($^\circ$R^4)

Universal Gas Constant, R_o $= 1544$ ft-lb$_f$/$^\circ$R

Velocity of light in a vacuum, c $= 186,282$ miles/sec

Joules' Constant, J $= 778$ ft-lb$_f$/Btu

A.1.4 Mathematical Tables

These tables were reprinted with permission from "Handbook of Engineering Fundamentals," Eshbach, John Wiley and Sons, 1952.

A.1.4.1 CERTAIN CONSTANTS CONTAINING e AND π

$e = 2.7182818285 \qquad M = \log_{10} e = 0.4342944819$

$\pi = 3.1415926536 \qquad M^{-1} = \log_e 10 = 2.3025850930$

Multiples of π

$n\pi$	Value	Logarithm
π	3.141593	0.497150
2π	6.283185	0.798180
3π	9.424778	0.974271
4π	12.566371	1.099210
5π	15.707963	1.196120

Powers of π

$\pi^{\pm n}$	Value	Logarithm
π^2	9.869604	0.994300
$1/\pi^2$	0.101321	$\bar{1}.005700$
π^3	31.006277	1.491450
$1/\pi^3$	0.032252	$\bar{2}.508550$

Fractions of π

π/n	Value	Logarithm
$\pi/2$	1.570796	0.196120
$\pi/3$	1.047198	0.020029
$\pi/4$	0.785398	$\bar{1}.895090$
$\pi/180$	0.017453*	$\bar{2}.241877$

Reciprocals of π

n/π	Value	Logarithm
$1/\pi$	0.318310	$\bar{1}.502850$
$2/\pi$	0.636620	$\bar{1}.803880$
$3/\pi$	0.954930	$\bar{1}.979971$
$180/\pi$	57.295780†	1.758123

Roots of π

$\pi^{\pm 1/n}$	Value	Logarithm
$\sqrt{\pi}$	1.772454	0.248575
$1/\sqrt{\pi}$	0.564190	$\bar{1}.751425$
$\sqrt[3]{\pi}$	1.464592	0.165717
$1/\sqrt[3]{\pi}$	0.682784	$\bar{1}.834283$

Powers of e

e^n	Value	Logarithm
e	2.718282	0.434294
e^{-1}	0.367879	$\bar{1}.565706$
e^2	7.389056	0.868589
e^{-2}	0.135335	$\bar{1}.131411$
$e^{1/2}$	1.648721	0.217147

* Number of radians per degree.

† Number of degrees per radian.

A.1.4.2 FACTORIALS

n	$n! = 1 \cdot 2 \cdot 3 \ldots n$	$1/n!$
1	1	1.
2	2	0.5
3	6	0.166667
4	24	0.416667×10^{-1}
5	120	0.833333×10^{-2}
6	720	0.138889×10^{-2}
7	5,040	0.198413×10^{-3}
8	40,320	0.248016×10^{-4}
9	362,880	0.275573×10^{-5}
10	3,628,800	0.275573×10^{-6}
11	$399,168 \times 10^{2}$	0.250521×10^{-7}
12	$479,002 \times 10^{3}$	0.208768×10^{-8}
13	$622,702 \times 10^{4}$	0.160590×10^{-9}
14	$871,783 \times 10^{5}$	0.114707×10^{-10}
15	$130,767 \times 10^{7}$	0.764716×10^{-12}
16	$209,228 \times 10^{8}$	0.477948×10^{-13}
17	$355,687 \times 10^{9}$	0.281146×10^{-14}
18	$640,237 \times 10^{10}$	0.156192×10^{-15}
19	$121,645 \times 10^{12}$	0.822064×10^{-17}
20	$243,290 \times 10^{13}$	0.411032×10^{-18}

A.1.4.3 INCHES TO DECIMALS OF A FOOT

In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.
1/16	.0052	5/16	.0260	9/16	.0469	13/16	.0677
1/8	.0104	3/8	.0313	5/8	.0521	7/8	.0729
3/16	.0156	7/16	.0364	11/16	.0573	15/16	.0781
1/4	.0208	1/2	.0417	3/4	.0625	1	.0833

In.	Ft.	In.	Ft.	In.	Ft.
1	.0833	5	.4167	9	.7500
2	.1667	6	.5000	10	.8333
3	.2500	7	.5833	11	.9167
4	.3333	8	.6667	12	1.0000

A.1.4.4 POWERS AND ROOTS OF NUMBERS, CIRCUMFERENCES AND AREAS OF CIRCLES. The following table lists decimal equivalents, squares, cubes, square roots, cube roots, three-halves powers, fifth roots, reciprocals, and circumference and area of circles.

Number, N		N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
Fraction	Decimal								Circum.	Area
1/64	.015625	0.000244	$.381\times10^{-5}$	.1250	.2500	.00195	.4353	64.0	.04909	.00019
1/32	.03125	.000977	$.305\times10^{-4}$	.1768	.3150	.00552	.5000	32.0	.09818	.00077
3/64	.046875	.002197	$.103\times10^{-3}$	.2165	.3606	.01015	.5422	18.8235	.14726	.00173
1/16	.0625	.003906	$.244\times10^{-3}$	.2500	.3969	.01563	.5743	16.0	.19635	.00307
5/64	.078125	.006104	$.477\times10^{-3}$	.2793	.4275	.02184	.6006	12.80	.24544	.00479
3/32	.09375	.008789	$.824\times10^{-3}$	.3062	.4543	.02871	.6229	10.6667	.29452	.00690
	.10	.010	.00100	.3162	.4642	.03162	.6310	10.0	.31416	.00785
7/64	.109375	.01196	.001308	.3307	.4782	.03617	.6424	9.1429	.34361	.00939
1/8	.125	.01563	.001953	.3536	.5000	.04419	.6598	8.0	.39270	.01227
9/64	.140625	.01978	.002782	.3750	.5200	.05273	.6755	7.1111	.44179	.01554
5/32	.15625	.02441	.003814	.3953	.5386	.06176	.6899	6.40	.49087	.01917
11/64	.171875	.02954	.005077	.4146	.5560	.07126	.7031	5.8182	.53996	.02320
3/16	.1875	.03516	.006592	.4330	.5724	.08119	.7155	5.3333	.58905	.02761
	.20	.040	.0080	.4472	.5848	.08944	.7248	5.0	.62832	.03142
13/64	.203125	.04126	.008381	.4507	.5878	.09155	.7270	4.9231	.63814	.03241
7/32	.21875	.04785	.01047	.4677	.6025	.10231	.7379	4.5714	.68722	.03758
15/64	.234375	.05493	.01287	.4841	.6166	.11347	.7481	4.2667	.73631	.04314
1/4	.250	.0625	.01563	.5000	.6300	.12500	.7579	4.0	.78540	.04909
17/64	.265625	.07056	.01874	.5154	.6426	.13690	.7671	3.7647	.83448	.05542
9/32	.28125	.07910	.02225	.5303	.6552	.14916	.7759	3.5556	.88357	.06213
19/64	.296875	.08813	.02615	.5449	.6671	.16176	.7844	3.3684	.93266	.06922
	.30	.090	.0270	.5477	.6694	.16432	.7860	3.3333	.94248	.07069
5/16	.3125	.09766	.03052	.5590	.6786	.17469	.7925	3.2000	.98175	.07670
21/64	.328125	.10767	.03533	.5728	.6897	.18796	.8002	3.0476	1.0308	.08456
11/32	.34375	.11816	.04062	.5863	.7005	.20154	.8077	2.9091	1.0799	.09281
23/64	.359375	.12915	.04641	.5995	.7110	.21544	.8149	2.7826	1.1290	.10143
3/8	.375	.14063	.05273	.6124	.7211	.22964	.8219	2.6667	1.1781	.11045
25/64	.390625	.15259	.05961	.6250	.7310	.24414	.8286	2.5600	1.2272	.11984
	.40	.16	.064	.6325	.7368	.25298	.8326	2.50	1.2566	.12566
13/32	.40625	.16504	.06705	.6374	.7406	.25894	.8351	2.4615	1.2763	.12962
27/64	.421875	.17798	.07508	.6495	.7500	.27402	.8415	2.3704	1.3254	.13979
7/16	.4375	.19141	.08374	.6614	.7592	.28938	.8476	2.2857	1.3744	.15033
29/64	.453125	.20532	.09304	.6732	.7681	.30503	.8536	2.2069	1.4235	.16126
15/32	.46875	.21973	.10300	.6847	.7768	.32093	.8594	2.1333	1.4726	.17257
31/64	.484375	.23462	.11364	.6960	.7854	.33711	.8650	2.0645	1.5217	.18427
1/2	.50	.2500	.12500	.7071	.7937	.35355	.8706	2.0	1.5708	.19635
33/64	.515625	.26587	.13709	.7181	.8019	.37025	.8759	1.9394	1.6199	.20881
17/32	.53125	.28223	.14993	.7289	.8099	.38721	.8812	1.8824	1.6690	.22166
35/64	.546875	.29907	.16355	.7395	.8178	.40442	.8863	1.8286	1.7181	.23489
9/16	.5625	.31641	.17798	.7500	.8255	.42188	.8913	1.7778	1.7671	.24850
37/64	.578125	.33423	.19323	.7604	.8331	.43957	.8962	1.7297	1.8162	.26250
19/32	.59375	.35254	.20932	.7706	.8405	.45751	.9010	1.6842	1.8653	.27688
	.60	.3600	.21600	.7746	.8434	.46476	.9029	1.6667	1.8850	.28274
39/64	.609375	.37134	.22628	.7806	.8478	.47569	.9057	1.6410	1.9144	.29165
5/8	.625	.39063	.24414	.7906	.8550	.49410	.9103	1.6000	1.9635	.30680
41/64	.640625	.41040	.26291	.8004	.8621	.51275	.9148	1.5610	2.0126	.32233
21/32	.65625	.43066	.28262	.8101	.8690	.53162	.9192	1.5238	2.0617	.33824
43/64	.671875	.45142	.30330	.8197	.8759	.55072	.9235	1.4884	2.1108	.35454
11/16	.6875	.47266	.32495	.8292	.8826	.57005	.9278	1.4545	2.1598	.37122
	.70	.4900	.34300	.8367	.8879	.58566	.9312	1.4286	2.1991	.38485
45/64	.703125	.49438	.34761	.8385	.8892	.58959	.9320	1.4222	2.2089	.38828
23/32	.71875	.51660	.37131	.8478	.8958	.60935	.9361	1.3913	2.2580	.40574
47/64	.734375	.53931	.39605	.8570	.9022	.62933	.9401	1.3617	2.3071	.42357
3/4	.750	.56250	.42188	.8660	.9086	.64952	.9441	1.3333	2.3562	.44179
49/64	.765625	.58618	.44879	.8750	.9148	.66992	.9480	1.3061	2.4053	.46038
25/32	.78125	.61035	.47684	.8839	.9210	.69053	.9518	1.2800	2.4544	.47937
51/64	.796675	.63501	.50602	.8927	.9271	.71135	.9555	1.2549	2.5035	.49874
	.80	.6400	.51200	.8944	.9283	.71554	.9564	1.2500	2.5133	.50265
13/16	.8125	.66016	.53638	.9014	.9331	.73238	.9593	1.2308	2.5525	.51849
53/64	.828125	.68579	.56792	.9100	.9391	.75361	.9630	1.2075	2.6016	.53862
27/32	.84375	.71191	.60067	.9186	.9449	.77505	.9666	1.1852	2.6507	.55914
55/64	.859375	.73853	.63467	.9270	.9507	.79666	.9702	1.1636	2.6998	.58004
7/8	.875	.76563	.66992	.9354	.9565	.81849	.9737	1.1429	2.7489	.60132
57/64	.890625	.79321	.70645	.9437	.9621	.84051	.9771	1.1228	2.7980	.62299
	.90	.81000	.72900	.9487	.9655	.85435	.9792	1.1111	2.8274	.63617
29/32	.90625	.82129	.74429	.9520	.9677	.86272	.9805	1.1034	2.8471	.64504
59/64	.921875	.84985	.78346	.9601	.9733	.88513	.9839	1.0847	2.8962	.66747
15/16	.9375	.87891	.82397	.9682	.9787	.90773	.9872	1.0667	2.9452	.69029
61/64	.953125	.90845	.86587	.9763	.9841	.93053	.9905	1.0492	2.9943	.71349
31/32	.96875	.93848	.90915	.9843	.9895	.95349	.9937	1.0323	3.0434	.73708
63/64	.984375	.96899	.95385	.9922	.9948	.97666	.9969	1.0159	3.0925	.76104

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
1.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.00000000	3.1416	.7854
1.125	1.2656	1.4238	1.0606	1.0400	1.1932	1.0238	.88888888	3.5343	.9940
1.25	1.5625	1.9531	1.1180	1.0772	1.3975	1.0456	.80000000	3.9270	1.2272
1.375	1.8906	2.5996	1.1726	1.1120	1.6123	1.0658	.72727272	4.3197	1.4849
1.5	2.25	3.3750	1.2247	1.1447	1.8371	1.0845	.66666666	4.7124	1.7671
1.625	2.6406	4.2910	1.2748	1.1757	2.0715	1.1020	.61538462	5.1051	2.0739
1.75	3.0625	5.3594	1.3229	1.2051	2.3150	1.1186	.57142857	5.4978	2.4053
1.875	3.5156	6.5918	1.3693	1.2331	2.5675	1.1340	.53333333	5.8905	2.7612
2.	4.0000	8.0000	1.4142	1.2599	2.8284	1.1487	.50000000	6.2832	3.1416
2.125	4.5156	9.5957	1.4577	1.2856	3.0977	1.1627	.47058823	6.6759	3.5466
2.25	5.0625	11.3906	1.5000	1.3104	3.3750	1.1761	.44444444	7.0686	3.9761
2.375	5.6406	13.3965	1.5411	1.3342	3.6601	1.1889	.42105263	7.4613	4.4301
2.5	6.2500	15.6250	1.5811	1.3572	3.9529	1.2011	.40000000	7.8540	4.9087
2.625	6.8906	18.0879	1.6202	1.3795	4.2530	1.2129	.38095238	8.2467	5.4119
2.75	7.5625	20.7969	1.6583	1.4011	4.5604	1.2242	.36363636	8.6394	5.9396
2.875	8.2656	23.7637	1.6956	1.4219	4.8748	1.2352	.34782609	9.0321	6.4918
3.	9.0000	27.0000	1.7321	1.4422	5.1962	1.2457	.33333333	9.4248	7.0686
3.125	9.7656	30.5176	1.7678	1.4620	5.5243	1.2559	.32000000	9.8175	7.6699
3.25	10.5625	34.3281	1.8028	1.4813	5.8590	1.2658	.30769231	10.2102	8.2958
3.375	11.3906	38.4434	1.8371	1.5000	6.2003	1.2754	.29629629	10.6029	8.9462
3.5	12.2500	42.8750	1.8708	1.5183	6.5479	1.2847	.28571429	10.9956	9.6211
3.625	13.1406	47.6348	1.9039	1.5362	6.9018	1.2938	.27586207	11.3883	10.3206
3.75	14.0625	52.7344	1.9365	1.5536	7.2619	1.3026	.26666666	11.7810	11.0447
3.875	15.0156	58.1855	1.9685	1.5707	7.6279	1.3112	.25806452	12.1737	11.7932
4.	16.0000	64.0000	2.0000	1.5874	8.0000	1.3195	.25000000	12.5664	12.5664
4.125	17.0156	70.1895	2.0310	1.6038	8.3779	1.3277	.24242424	12.9591	13.3640
4.25	18.0625	76.7656	2.0616	1.6198	8.7616	1.3356	.23529412	13.3518	14.1863
4.375	19.1406	83.7402	2.0916	1.6355	9.1510	1.3434	.22857143	13.7445	15.0330
4.5	20.2500	91.1250	2.1213	1.6510	9.5460	1.3510	.22222222	14.1372	15.9043
4.625	21.3906	98.9317	2.1506	1.6661	9.9465	1.3584	.21621622	14.5299	16.8001
4.75	22.5625	107.1719	2.1795	1.6810	10.3524	1.3656	.21052632	14.9226	17.7205
4.875	23.7656	115.8574	2.2079	1.6956	10.7637	1.3728	.20512821	15.3153	18.6655
5.	25.0000	125.0000	2.2361	1.7100	11.1803	1.3797	.20000000	15.7080	19.6350
5.125	26.2656	134.6113	2.2638	1.7241	11.6022	1.3866	.19512195	16.1006	20.6289
5.25	27.5625	144.7031	2.2913	1.7380	12.0295	1.3933	.19047619	16.4933	21.6475
5.375	28.8906	155.2871	2.3184	1.7517	12.4614	1.3998	.18604651	16.8860	22.6906
5.5	30.2500	166.3750	2.3452	1.7652	12.8987	1.4063	.18181818	17.2787	23.7583
5.625	31.6406	177.9785	2.3717	1.7784	13.3409	1.4126	.17777777	17.6714	24.8505
5.75	33.0625	190.1094	2.3979	1.7915	13.7880	1.4188	.17391304	18.0641	25.9672
5.875	34.5156	202.7793	2.4238	1.8044	14.2400	1.4250	.17021277	18.4568	27.1085
6.	36.0000	216.0000	2.4495	1.8171	14.6969	1.4310	.16666666	18.8496	28.2743
6.125	37.5156	229.7832	2.4749	1.8297	15.1586	1.4369	.16326531	19.2422	29.4647
6.25	39.0625	244.1406	2.5000	1.8420	15.6250	1.4427	.16000000	19.6349	30.6796
6.375	40.6406	259.0840	2.5249	1.8542	16.0961	1.4484	.15686275	20.0276	31.9190
6.5	42.2500	274.6250	2.5495	1.8663	16.5718	1.4542	.15384615	20.4203	33.1831
6.625	43.8906	290.7754	2.5739	1.8781	17.0522	1.4596	.15094339	20.8130	34.4716
6.75	45.5625	307.5469	2.5981	1.8899	17.5370	1.4651	.14814815	21.2057	35.7847
6.875	47.2656	324.9512	2.6220	1.9015	18.0264	1.4705	.14545454	21.5984	37.1223
7.	49.0000	343.0000	2.6458	1.9129	18.5203	1.4758	.14285714	21.9911	38.4845
7.125	50.7656	361.7051	2.6693	1.9245	19.0186	1.4810	.14035088	22.3838	39.8712
7.25	52.5625	381.0781	2.6926	1.9354	19.5212	1.4863	.13793103	22.7765	41.2825
7.375	54.3906	401.1309	2.7157	1.9465	20.0283	1.4913	.13559322	23.1692	42.7183
7.5	56.2500	421.8750	2.7386	1.9574	20.5396	1.4963	.13333333	23.5619	44.1786
7.625	58.1406	443.3223	2.7613	1.9683	21.0552	1.5012	.13114754	23.9546	45.6635
7.75	60.0625	465.4844	2.7839	1.9789	21.5751	1.5061	.12903226	24.3473	47.1730
7.875	62.0156	488.3731	2.8063	1.9895	22.0992	1.5110	.12698413	24.7400	48.7069
8.	64.0000	512.0000	2.8284	2.0000	22.6274	1.5157	.12500000	25.1327	50.2655
8.125	66.0156	536.3770	2.8504	2.0104	23.1598	1.5204	.12307692	25.5254	51.8485
8.25	68.0625	561.5156	2.8723	2.0206	23.6963	1.5251	.12121212	25.9181	53.4562
8.375	70.1406	587.4277	2.8940	2.0308	24.2369	1.5297	.11940298	26.3108	55.0883
8.5	72.2500	614.1250	2.9155	2.0408	24.7816	1.5342	.11764706	26.7035	56.7450
8.625	74.3906	641.6192	2.9368	2.0508	25.3301	1.5387	.11594203	27.0962	58.4262
8.75	76.5625	669.9219	2.9580	2.0606	25.8828	1.5431	.11428571	27.4889	60.1320
8.875	78.7656	699.0430	2.9791	2.0704	26.4394	1.5475	.11267605	27.8816	61.8623
9.	81.0000	729.0000	3.0000	2.0801	27.0000	1.5518	.11111111	28.2743	63.6173
9.125	83.2656	759.7988	3.0207	2.0897	27.5645	1.5561	.10958904	28.6670	65.3966
9.25	85.5625	791.4531	3.0414	2.0992	28.1328	1.5604	.10810811	29.0597	67.2006
9.375	87.8906	823.9746	3.0619	2.1086	28.7050	1.5646	.10666666	29.4524	69.0291
9.5	90.2500	857.3750	3.0822	2.1179	29.2810	1.5687	.10526316	29.8451	70.8822
9.625	92.6406	891.6660	3.1024	2.1272	29.8608	1.5728	.10389610	30.2378	72.7597
9.75	95.0625	926.8594	3.1225	2.1363	30.4444	1.5769	.10256410	30.6305	74.6619
9.875	97.5156	962.9658	3.1425	2.1454	31.0317	1.5809	.10126582	31.0232	76.5886

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Circle ($N = D$) Area
10	100	1000	3.1623	2.1544	31.623	1.5849	.10000000	31.4159	78.5398
11	121	1331	3.3166	2.2240	36.483	1.6154	.09090909	34.5575	95.0332
12	144	1728	3.4641	2.2894	41.569	1.6438	.08333333	37.6991	113.0973
13	169	2197	3.6056	2.3513	46.873	1.6703	.07692308	40.8407	132.7323
14	196	2744	3.7417	2.4101	52.384	1.6952	.07142857	43.9823	153.9380
15	225	3375	3.8730	2.4662	58.095	1.7188	.06666667	47.1239	176.7146
16	256	4096	4.0000	2.5198	64.000	1.7411	.06250000	50.2654	201.0619
17	289	4913	4.1231	2.5713	70.093	1.7623	.05882353	53.4070	226.9801
18	324	5832	4.2426	2.6207	76.367	1.7826	.05555556	56.5486	254.4690
19	361	6859	4.3589	2.6684	82.819	1.8020	.05263158	59.6902	283.5287
20	400	8000	4.4721	2.7144	89.443	1.8206	.05000000	62.8319	314.1593
21	441	9261	4.5826	2.7589	96.235	1.8384	.04761905	65.9734	346.3606
22	484	10648	4.6904	2.8020	103.19	1.8556	.04545455	69.1150	380.1327
23	529	12167	4.7958	2.8439	110.30	1.8722	.04347826	72.2566	415.4756
24	576	13824	4.8990	2.8845	117.58	1.8882	.04166667	75.3982	452.3893
25	625	15625	5.0000	2.9240	125.00	1.9037	.04000000	78.5398	490.8739
26	676	17576	5.0990	2.9625	132.57	1.9186	.03846154	81.6813	530.9292
27	729	19683	5.1962	3.0000	140.30	1.9332	.03703704	84.8229	572.5553
28	784	21952	5.2915	3.0366	148.16	1.9473	.03571429	87.9645	615.7522
29	841	24389	5.3852	3.0723	156.17	1.9610	.03448276	91.1061	660.5198
30	900	27000	5.4772	3.1072	164.32	1.9744	.03333333	94.2478	706.8583
31	961	29791	5.5678	3.1414	172.60	1.9873	.03225806	97.3893	754.7676
32	1024	32768	5.6569	3.1748	181.02	2.0000	.03125000	100.5309	804.2477
33	1089	35937	5.7446	3.2075	189.57	2.0123	.03030303	103.6725	855.2986
34	1156	39304	5.8310	3.2396	198.25	2.0244	.02941176	106.8141	907.9203
35	1225	42875	5.9161	3.2711	207.06	2.0362	.02857143	109.9557	962.1127
36	1296	46656	6.0000	3.3019	216.00	2.0477	.02777778	113.0972	1017.8760
37	1369	50653	6.0828	3.3322	225.06	2.0589	.02702703	116.2388	1075.2101
38	1444	54872	6.1644	3.3620	234.25	2.0699	.02631579	119.3804	1134.1149
39	1521	59319	6.2450	3.3912	243.56	2.0807	.02564103	122.5220	1194.5906
40	1600	64000	6.3246	3.4200	252.98	2.0913	.02500000	125.6637	1256.6371
41	1681	68921	6.4031	3.4482	262.52	2.1016	.02439024	128.8052	1320.2543
42	1764	74088	6.4807	3.4760	272.19	2.1118	.02380952	131.9468	1385.4424
43	1849	79507	6.5574	3.5034	281.97	2.1218	.02325581	135.0884	1452.2012
44	1936	85184	6.6332	3.5303	291.86	2.1315	.02272727	138.2300	1520.5308
45	2025	91125	6.7082	3.5569	301.87	2.1411	.02222222	141.3716	1590.4313
46	2116	97336	6.7823	3.5830	311.99	2.1506	.02173913	144.5131	1661.9025
47	2209	103823	6.8557	3.6088	322.22	2.1598	.02127660	147.6547	1734.9445
48	2304	110592	6.9282	3.6342	332.55	2.1689	.02083333	150.7963	1809.5574
49	2401	117649	7.0000	3.6593	343.00	2.1779	.02040816	153.9379	1885.7410
50	2500	125000	7.0711	3.6840	353.55	2.1867	.02000000	157.0796	1963.495
51	2601	132651	7.1414	3.7084	364.21	2.1954	.01960784	160.2211	2042.820
52	2704	140608	7.2111	3.7325	374.98	2.2039	.01923077	163.3627	2123.716
53	2809	148877	7.2801	3.7563	385.85	2.2124	.01886792	166.5043	2206.183
54	2916	157464	7.3485	3.7798	396.82	2.2206	.01851852	169.6459	2290.221
55	3025	166375	7.4162	3.8030	407.89	2.2288	.01818182	172.7875	2375.829
56	3136	175616	7.4833	3.8259	419.07	2.2369	.01785714	175.9292	2463.009
57	3249	185193	7.5498	3.8485	430.33	2.2448	.01754386	179.0706	2551.758
58	3364	195112	7.6158	3.8709	441.72	2.2526	.01724138	182.2122	2642.079
59	3481	205379	7.6811	3.8930	453.19	2.2603	.01694915	185.3538	2733.970
60	3600	216000	7.7460	3.9149	464.76	2.2679	.01666667	188.4956	2827.433
61	3721	226981	7.8102	3.9365	476.43	2.2755	.01639344	191.6370	2922.466
62	3844	238328	7.8740	3.9579	488.19	2.2829	.01612903	194.7786	3019.070
63	3969	250047	7.9373	3.9791	500.05	2.2902	.01587302	197.9202	3117.245
64	4096	262144	8.0000	4.0000	512.00	2.2974	.01562500	201.0618	3216.991
65	4225	274625	8.0623	4.0207	524.05	2.3045	.01538462	204.2034	3318.307
66	4356	287496	8.1240	4.0412	536.19	2.3116	.01515152	207.3449	3421.194
67	4489	300763	8.1854	4.0615	548.42	2.3186	.01492537	210.4865	3525.652
68	4624	314432	8.2462	4.0817	560.74	2.3254	.01470588	213.6281	3631.680
69	4761	328509	8.3066	4.1016	573.16	2.3322	.01449275	216.7697	3739.280
70	4900	343000	8.3666	4.1213	585.66	2.3389	.01428571	219.9115	3848.451
71	5041	357911	8.4261	4.1408	598.26	2.3456	.01408451	223.0529	3959.191
72	5184	373248	8.4853	4.1602	610.94	2.3522	.01388889	226.1945	4071.503
73	5329	389017	8.5440	4.1793	623.71	2.3587	.01369863	229.3361	4185.386
74	5476	405224	8.6023	4.1983	636.57	2.3651	.01351351	232.4777	4300.839
75	5625	421875	8.6603	4.2172	649.52	2.3714	.01333333	235.6193	4417.864
76	5776	438976	8.7178	4.2358	662.55	2.3777	.01315789	238.7608	4536.459
77	5929	456533	8.7750	4.2543	675.68	2.3840	.01298701	241.9024	4656.625
78	6084	474552	8.8318	4.2727	688.88	2.3901	.01282051	245.0440	4778.361
79	6241	493039	8.8882	4.2908	702.17	2.3962	.01265823	248.1856	4901.669

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Circle ($N = D$) Area
80	6400	512000	8.9443	4.3089	715.54	2.4022	.01250000	251.327	5026.548
81	6561	531441	9.0000	4.3267	729.00	2.4082	.01234568	254.469	5152.998
82	6724	551368	9.0554	4.3445	742.54	2.4141	.01219512	257.610	5281.016
83	6889	571787	9.1104	4.3621	756.17	2.4200	.01204819	260.752	5410.607
84	7056	592704	9.1652	4.3795	769.88	2.4258	.01190476	263.894	5541.770
85	7225	614125	9.2195	4.3968	783.66	2.4315	.01176471	267.035	5674.501
86	7396	636056	9.2736	4.4140	797.53	2.4373	.01162791	270.177	5808.805
87	7569	658503	9.3274	4.4310	811.49	2.4429	.01149425	273.318	5944.679
88	7744	681472	9.3808	4.4480	825.52	2.4485	.01136364	276.460	6082.124
89	7921	704969	9.4340	4.4647	839.62	2.4540	.01123596	279.602	6221.138
90	8100	729000	9.4868	4.4814	853.82	2.4595	.01111111	282.743	6361.725
91	8281	753571	9.5394	4.4979	868.09	2.4650	.01098901	285.885	6503.882
92	8464	778688	9.5917	4.5144	882.44	2.4705	.01086957	289.026	6647.618
93	8649	804357	9.6437	4.5307	896.86	2.4758	.01075269	292.168	6792.909
94	8836	830584	9.6954	4.5468	911.36	2.4810	.01063830	295.309	6939.778
95	9025	857375	9.7468	4.5629	925.95	2.4863	.01052632	298.451	7088.219
96	9216	884736	9.7980	4.5789	940.61	2.4915	.01041667	301.593	7238.230
97	9409	912673	9.8489	4.5947	955.34	2.4966	.01030928	304.734	7389.812
98	9604	941192	9.8995	4.6104	970.15	2.5018	.01020408	307.876	7542.962
99	9801	970299	9.9499	4.6261	985.04	2.5069	.01010101	311.017	7697.688
100	10000	1000000	10.0000	4.6416	1000.0	2.5119	.01000000	314.159	7853.982
101	10201	1030301	10.0499	4.6570	1015.0	2.5169	.00990099	317.301	8011.85
102	10404	1061208	10.0995	4.6723	1030.1	2.5219	.00980392	320.442	8171.28
103	10609	1092727	10.1489	4.6875	1045.3	2.5268	.00970874	323.584	8332.29
104	10816	1124864	10.1980	4.7027	1060.6	2.5317	.00961538	326.725	8494.87
105	11025	1157625	10.2470	4.7177	1075.9	2.5365	.00952381	329.867	8659.01
106	11236	1191016	10.2956	4.7326	1091.3	2.5413	.00943396	333.009	8824.73
107	11449	1225043	10.3441	4.7475	1106.8	2.5461	.00934579	336.150	8992.02
108	11664	1259712	10.3923	4.7622	1122.4	2.5509	.00925926	339.292	9160.88
109	11881	1295029	10.4403	4.7769	1138.0	2.5556	.00917431	342.433	9331.32
110	12100	1331000	10.4881	4.7914	1153.7	2.5604	.00909091	345.575	9503.32
111	12321	1367631	10.5357	4.8059	1169.5	2.5649	.00900901	348.716	9676.89
112	12544	1404928	10.5830	4.8203	1185.3	2.5695	.00892857	351.858	9852.03
113	12769	1442897	10.6301	4.8346	1201.2	2.5740	.00884956	355.000	10028.75
114	12996	1481544	10.6771	4.8488	1217.2	2.5786	.00877193	358.141	10207.03
115	13225	1520875	10.7238	4.8629	1233.2	2.5831	.00869565	361.283	10386.89
116	13456	1560896	10.7703	4.8770	1249.4	2.5876	.00862069	364.424	10568.32
117	13689	1601613	10.8167	4.8910	1265.5	2.5920	.00854701	367.566	10751.31
118	13924	1643032	10.8628	4.9049	1281.8	2.5964	.00847458	370.708	10935.88
119	14161	1685159	10.9087	4.9187	1298.1	2.6008	.00840336	373.849	11122.02
120	14400	1728000	10.9545	4.9324	1314.5	2.6052	.00833333	376.991	11309.73
121	14641	1771561	11.0000	4.9461	1331.0	2.6095	.00826446	380.132	11499.01
122	14884	1815848	11.0454	4.9597	1347.5	2.6138	.00819672	383.274	11689.86
123	15129	1860867	11.0905	4.9732	1364.1	2.6181	.00813008	386.416	11882.29
124	15376	1906624	11.1355	4.9866	1380.8	2.6223	.00806452	389.557	12076.28
125	15625	1953125	11.1803	5.0000	1397.5	2.6265	.00800000	392.699	12271.84
126	15876	2000376	11.2250	5.0133	1414.4	2.6307	.00793651	395.840	12468.98
127	16129	2048383	11.2694	5.0265	1431.2	2.6349	.00787402	398.982	12667.68
128	16384	2097152	11.3137	5.0397	1448.2	2.6390	.00781250	402.124	12867.96
129	16641	2146689	11.3578	5.0528	1465.2	2.6431	.00775194	405.265	13069.81
130	16900	2197000	11.4018	5.0658	1482.2	2.6472	.00769231	408.407	13273.23
131	17161	2248091	11.4455	5.0788	1499.4	2.6513	.00763359	411.548	13478.22
132	17424	2299968	11.4891	5.0916	1516.6	2.6553	.00757576	414.690	13684.77
133	17689	2352637	11.5326	5.1045	1533.8	2.6593	.00751880	417.831	13892.91
134	17956	2406104	11.5758	5.1172	1551.2	2.6633	.00746269	420.973	14102.61
135	18225	2460375	11.6190	5.1299	1568.6	2.6673	.00740741	424.115	14313.88
136	18496	2515456	11.6619	5.1426	1586.0	2.6712	.00735294	427.256	14526.72
137	18769	2571353	11.7047	5.1551	1603.6	2.6751	.00729927	430.398	14741.14
138	19044	2628072	11.7473	5.1676	1621.1	2.6790	.00724638	433.539	14957.12
139	19321	2685619	11.7898	5.1801	1638.8	2.6829	.00719424	436.681	15174.67
140	19600	2744000	11.8322	5.1925	1656.5	2.6867	.00714286	439.823	15393.80
141	19881	2803221	11.8743	5.2048	1674.3	2.6906	.00709220	442.964	15614.50
142	20164	2863288	11.9164	5.2171	1692.1	2.6944	.00704225	446.106	15836.77
143	20449	2924207	11.9583	5.2293	1710.0	2.6981	.00699301	449.247	16060.60
144	20736	2985984	12.0000	5.2415	1728.0	2.7019	.00694444	452.389	16286.01
145	21025	3048625	12.0416	5.2536	1746.0	2.7057	.00689655	455.531	16512.99
146	21316	3112136	12.0830	5.2656	1764.1	2.7094	.00684932	458.672	16741.54
147	21609	3176523	12.1244	5.2776	1782.2	2.7131	.00680272	461.814	16971.67
148	21904	3241792	12.1655	5.2896	1800.5	2.7168	.00675676	464.955	17203.36
149	22201	3307949	12.2066	5.3015	1818.8	2.7204	.00671141	468.097	17436.62

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Area
150	22500	3375000	12.2474	5.3133	1837.1	2.7241	.00666667	471.238	17671.46
151	22801	3442951	12.2882	5.3251	1855.5	2.7277	.00662252	474.380	17907.86
152	23104	3511808	12.3288	5.3368	1874.0	2.7314	.00657895	477.522	18145.84
153	23409	3581577	12.3693	5.3485	1892.5	2.7349	.00653595	480.663	18385.38
154	23716	3652264	12.4097	5.3601	1911.1	2.7385	.00649351	483.805	18626.50
155	24025	3723875	12.4499	5.3717	1929.7	2.7420	.00645161	486.946	18869.19
156	24336	3796416	12.4900	5.3832	1948.4	2.7455	.00641026	490.088	19113.43
157	24649	3869893	12.5300	5.3947	1967.2	2.7490	.00636943	493.230	19359.28
158	24964	3944312	12.5698	5.4061	1986.0	2.7525	.00632911	496.371	19606.68
159	25281	4019679	12.6095	5.4175	2004.9	2.7560	.00628931	499.513	19855.65
160	25600	4096000	12.6491	5.4288	2023.9	2.7594	.00625000	502.654	20106.19
161	25921	4173281	12.6886	5.4401	2042.9	2.7629	.00621118	505.796	20358.30
162	26244	4251528	12.7279	5.4514	2061.9	2.7663	.00617284	508.938	20611.99
163	26569	4330747	12.7671	5.4626	2081.0	2.7697	.00613497	512.079	20867.24
164	26896	4410944	12.8062	5.4737	2100.3	2.7731	.00609756	515.221	21124.06
165	27225	4492125	12.8452	5.4848	2119.5	2.7765	.00606061	518.362	21382.46
166	27556	4574296	12.8841	5.4959	2138.8	2.7799	.00602410	521.504	21642.43
167	27889	4657463	12.9228	5.5069	2158.1	2.7832	.00598802	524.646	21903.96
168	28224	4741632	12.9615	5.5178	2177.5	2.7865	.00595238	527.787	22167.07
169	28561	4826809	13.0000	5.5288	2197.0	2.7898	.00591716	530.929	22431.75
170	28900	4913000	13.0384	5.5397	2216.5	2.7931	.00588235	534.070	22698.00
171	29241	5000211	13.0767	5.5505	2236.1	2.7964	.00584795	537.212	22965.82
172	29584	5088448	13.1149	5.5613	2255.8	2.7997	.00581395	540.353	23235.21
173	29929	5177717	13.1529	5.5721	2275.5	2.8029	.00578035	543.495	23506.18
174	30276	5268024	13.1909	5.5828	2295.2	2.8061	.00574713	546.637	23778.71
175	30625	5359375	13.2288	5.5934	2315.0	2.8094	.00571429	549.778	24052.81
176	30976	5451776	13.2665	5.6041	2334.9	2.8126	.00568182	552.920	24328.49
177	31329	5545233	13.3041	5.6147	2354.8	2.8158	.00564972	556.061	24605.73
178	31684	5639752	13.3417	5.6252	2374.8	2.8189	.00561798	559.203	24884.55
179	32041	5735339	13.3791	5.6357	2394.9	2.8221	.00558659	562.345	25164.94
180	32400	5832000	13.4164	5.6462	2415.0	2.8252	.00555556	565.486	25446.90
181	32761	5929741	13.4536	5.6567	2435.1	2.8284	.00552486	568.628	25730.42
182	33124	6028568	13.4907	5.6671	2455.3	2.8315	.00549451	571.769	26015.52
183	33489	6128487	13.5277	5.6774	2475.5	2.8346	.00546448	574.911	26302.19
184	33856	6229504	13.5647	5.6877	2495.9	2.8377	.00543478	578.053	26590.43
185	34225	6331625	13.6015	5.6980	2516.3	2.8408	.00540541	581.194	26880.25
186	34596	6434856	13.6382	5.7083	2536.7	2.8438	.00537634	584.336	27171.63
187	34969	6539203	13.6748	5.7185	2557.2	2.8469	.00534759	587.477	27464.58
188	35344	6644672	13.7113	5.7287	2577.7	2.8499	.00531915	590.619	27759.11
189	35721	6751269	13.7477	5.7388	2598.3	2.8529	.00529101	593.761	28055.20
190	36100	6859000	13.7840	5.7489	2619.0	2.8560	.00526316	596.902	28352.87
191	36481	6967871	13.8203	5.7590	2639.7	2.8590	.00523560	600.044	28652.10
192	36864	7077888	13.8564	5.7690	2660.4	2.8619	.00520833	603.185	28952.91
193	37249	7189057	13.8924	5.7790	2681.2	2.8649	.00518135	606.327	29255.29
194	37636	7301384	13.9284	5.7890	2702.1	2.8679	.00515464	609.468	29559.24
195	38025	7414875	13.9642	5.7989	2723.0	2.8708	.00512821	612.610	29864.76
196	38416	7529536	14.0000	5.8088	2744.0	2.8738	.00510204	615.752	30171.85
197	38809	7645373	14.0357	5.8186	2765.0	2.8767	.00507614	618.893	30480.51
198	39204	7762392	14.0712	5.8285	2786.1	2.8796	.00505051	622.035	30790.74
199	39601	7880599	14.1067	5.8383	2807.2	2.8825	.00502513	625.174	31102.55
200	40000	8000000	14.1421	5.8480	2828.4	2.8854	.00500000	628.318	31415.92
201	40401	8120601	14.1774	5.8578	2849.7	2.8883	.00497512	631.460	31730.87
202	40804	8242408	14.2127	5.8675	2871.0	2.8911	.00495050	634.601	32047.39
203	41209	8365427	14.2478	5.8771	2892.3	2.8940	.00492611	637.743	32365.47
204	41616	8489664	14.2829	5.8868	2913.7	2.8968	.00490196	640.884	32685.13
205	42025	8615125	14.3178	5.8964	2935.2	2.8997	.00487805	644.026	33006.36
206	42436	8741816	14.3527	5.9059	2956.7	2.9025	.00485437	647.168	33329.16
207	42849	8869743	14.3875	5.9155	2978.2	2.9053	.00483092	650.309	33653.53
208	43264	8998912	14.4222	5.9250	2999.8	2.9081	.00480769	653.451	33979.47
209	43681	9129329	14.4568	5.9345	3021.5	2.9109	.00478469	656.592	34306.98
210	44100	9261000	14.4914	5.9439	3043.2	2.9137	.00476190	659.734	34636.05
211	44521	9393931	14.5258	5.9533	3065.0	2.9165	.00473934	662.875	34966.71
212	44944	9528128	14.5602	5.9627	3086.8	2.9192	.00471698	666.017	35298.94
213	45369	9663597	14.5945	5.9721	3108.7	2.9220	.00469484	669.159	35632.73
214	45796	9800344	14.6287	5.9814	3130.6	2.9247	.00467290	672.300	35968.09
215	46225	9938375	14.6629	5.9907	3152.5	2.9274	.00465116	675.442	36305.03
216	46656	10077696	14.6969	6.0000	3174.5	2.9302	.00462963	678.583	36643.54
217	47089	10218313	14.7309	6.0092	3196.6	2.9329	.00460829	681.725	36983.61
218	47524	10360232	14.7648	6.0185	3218.7	2.9356	.00458716	684.867	37325.26
219	47961	10503459	14.7986	6.0277	3240.9	2.9383	.00456621	688.008	37668.48

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Area
220	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	691.150	[illegible]
221	48841	10793861	14.8661	6.0459	3285.4	2.9436	.00452489	694.291	38359.63
222	49284	10941048	14.8997	6.0550	3307.7	2.9463	.00450450	697.433	38707.56
223	49729	11089567	14.9332	6.0641	3330.1	2.9489	.00448430	700.575	39057.07
224	50176	11239424	14.9666	6.0732	3352.5	2.9516	.00446429	703.716	39408.14
225	50625	11390625	15.0000	6.0822	3375.0	2.9542	.00444444	706.858	39760.78
226	51076	11543176	15.0333	6.0912	3397.5	2.9568	.00442478	709.999	40115.00
227	51529	11697083	15.0665	6.1002	[illegible]	2.9594	.00440529	713.141	40470.78
228	51984	11852352	15.0997	6.1091	3442.7	2.9620	.00438596	716.283	40828.14
229	52441	12008989	15.1327	6.1180	3465.4	2.9646	.00436681	719.424	41187.07
230	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
231	53361	12326391	15.1987	6.1358	3510.9	2.9698	.00432900	725.707	41909.63
232	53824	12487168	15.2315	6.1446	3533.7	2.9723	.00431034	728.849	42273.27
233	54289	12649337	15.2643	6.1534	3556.6	2.9749	.00429185	731.990	42638.48
234	54756	12812904	15.2971	6.1622	3579.5	2.9774	.00427350	735.132	43005.26
235	55225	12977875	15.3297	6.1710	3602.5	2.9800	.00425532	738.274	43373.61
236	55696	13144256	15.3623	6.1797	3625.5	2.9825	.00423729	741.415	43743.54
237	56169	13312053	15.3948	6.1885	3648.6	2.9850	.00421941	744.557	44115.03
238	56644	13481272	15.4272	6.1972	3671.7	2.9875	.00420168	747.698	44488.09
239	57121	13651919	15.4596	6.2058	3694.8	2.9900	.00418410	750.840	44862.73
240	[illegible]	[illegible]	[illegible]	[illegible]	3718.0	[illegible]	[illegible]	[illegible]	[illegible]
241	58081	13997521	15.5242	6.2231	3741.3	2.9950	.00414938	757.123	45616.71
242	58564	14172488	15.5563	6.2317	3764.6	2.9975	.00413223	760.265	45996.06
243	59049	14348907	15.5885	6.2403	3788.0	3.0000	.00411523	763.406	46376.98
244	59536	14526784	15.6205	6.2488	3811.4	3.0025	.00409836	766.548	46759.47
245	60025	14706125	15.6525	6.2573	3834.9	3.0049	.00408163	769.690	47143.52
246	60516	14886936	15.6844	6.2658	3858.4	3.0074	.00406504	772.831	47529.16
247	61009	15069223	15.7162	6.2743	3881.9	3.0098	.00404858	775.973	47916.36
248	61504	15252992	15.7480	6.2828	3905.5	3.0123	.00403226	779.114	48305.13
249	62001	15438249	15.7797	6.2912	3929.2	3.0147	.00401606	782.256	48695.47
250	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
251	63001	15813251	15.8430	6.3080	3976.6	3.0195	.00398406	788.539	49480.87
252	63504	16003008	15.8745	6.3164	4000.4	3.0219	.00396825	791.681	49875.92
253	64009	16194277	15.9060	6.3247	4024.3	3.0243	.00395257	794.822	50272.55
254	64516	16387064	15.9374	6.3330	4048.1	3.0267	.00393701	797.964	50670.75
255	65025	16581375	15.9687	6.3413	4072.0	3.0291	.00392157	801.105	51070.52
256	65536	16777216	16.0000	6.3496	4096.0	3.0314	.00390625	804.247	51471.85
257	66049	16974593	16.0312	6.3579	4120.0	3.0338	.00389105	807.389	51874.76
258	66564	17173512	16.0624	6.3661	[illegible]	3.0362	.00387597	810.530	52279.24
259	67081	17373979	16.0935	6.3743	4168.2	3.0385	.00386100	813.672	52685.29
260	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
261	68121	17779581	16.1555	6.3907	4216.6	3.0432	.00383142	819.955	53502.11
262	68644	17984728	16.1864	6.3988	4240.8	3.0455	.00381679	823.097	53912.87
263	69169	18191447	16.2173	6.4070	4265.1	3.0478	.00380228	826.238	54325.21
264	69696	18399744	16.2481	6.4151	4289.5	3.0501	.00378788	829.380	54739.11
265	70225	18609625	16.2788	6.4232	4313.9	3.0524	.00377358	832.521	55154.59
266	70756	18821096	16.3095	6.4312	4338.3	3.0547	.00375940	835.663	55571.63
267	71289	19034163	16.3401	6.4393	[illegible]	3.0570	.00374532	838.805	55990.25
268	71824	19248832	16.3707	6.4473	4387.3	3.0593	.00373134	841.946	56410.44
269	72361	19465109	16.4012	6.4553	4411.9	3.0616	.00371747	845.088	56832.20
270	[illegible]	[illegible]	16.4317	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
271	73441	19902511	16.4621	6.4713	4461.2	3.0662	.00369004	851.371	57680.43
272	73984	20123648	16.4924	6.4792	4485.9	3.0684	.00367647	854.512	58106.90
273	74529	20346417	16.5227	6.4872	4510.7	3.0707	.00366300	857.654	58534.94
274	75076	20570824	16.5529	6.4951	4535.5	3.0729	.00364964	860.796	58964.55
275	75625	20796875	16.5831	6.5030	4560.4	3.0752	.00363636	863.937	59395.74
276	76176	21024576	16.6132	6.5108	4585.3	3.0774	.00362319	867.079	59828.49
277	76729	21253933	16.6433	6.5187	4610.2	3.0796	.00361011	870.220	60262.82
278	77284	21484952	16.6733	6.5265	4635.2	3.0818	.00359712	873.362	60698.71
279	77841	21717639	16.7033	6.5343	4660.2	3.0840	.00358423	876.504	61136.18
280	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
281	78961	22188041	16.7631	6.5499	4710.4	3.0885	.00355872	882.787	62015.82
282	79524	22425768	16.7929	6.5577	4735.6	3.0907	.00354610	885.928	62458.00
283	80089	22665187	16.8226	6.5654	4760.8	3.0928	.00353357	889.070	62901.75
284	80656	22906304	16.8523	6.5731	4786.0	3.0950	.00352113	892.212	63347.07
285	81225	23149125	16.8819	6.5808	4811.3	3.0972	.00350877	895.353	63793.97
286	81796	23393656	16.9115	6.5885	4836.7	3.0994	.00349650	898.495	64242.43
287	82369	23639903	16.9411	6.5962	4862.1	3.1015	.00348432	901.636	64692.46
288	82944	23887872	16.9706	6.6039	4887.5	3.1037	.00347222	904.778	65144.07
289	83521	24137569	17.0000	6.6115	4913.0	3.1058	.00346021	907.920	65597.34

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Circle ($N = D$) Area
290	84100	[illegible]	17.0294	6.6191	[illegible]	[illegible]	[illegible]	911.061	[illegible]
291	84681	24642171	17.0587	6.6267	4964.1	3.1101	.00343643	914.203	66508.30
292	85264	24897088	17.0880	6.6343	4989.7	3.1123	.00342466	917.345	66966.19
293	85849	25153757	17.1172	6.6419	5015.4	3.1144	.00341297	920.486	67425.65
294	86436	25412184	17.1464	6.6494	5041.1	3.1165	.00340136	923.627	67886.68
295	87025	25672375	17.1756	6.6569	5066.8	3.1186	.00338983	926.769	68349.28
296	87616	25934336	17.2047	6.6644	5092.6	3.1207	.00337838	929.911	68813.45
297	88209	26198073	17.2337	6.6719	5118.4	3.1228	.00336700	933.052	69279.19
298	88804	26463592	17.2627	6.6794	5144.3	3.1249	.00335570	936.194	69746.50
299	89401	26730899	17.2916	6.6869	5170.2	3.1270	.00334448	939.335	70215.38
300	90000	[illegible]	17.3205	6.6943	5196.2	3.1291	[illegible]	942.477	[illegible]
301	90601	27270901	17.3494	6.7018	5222.2	3.1312	.00332226	945.619	71157.86
302	91204	27543608	17.3781	6.7092	5248.2	3.1333	.00331126	948.761	71631.45
303	91809	27818127	17.4069	6.7166	5274.3	3.1354	.00330033	951.902	72106.62
304	92416	28094464	17.4356	6.7240	5300.4	3.1374	.00328947	955.043	72583.36
305	93025	28372625	17.4642	6.7313	5326.6	3.1395	.00327869	958.185	73061.66
306	93636	28652616	17.4929	6.7387	5352.8	3.1416	.00326797	961.327	73541.54
307	94249	28934443	17.5214	6.7460	5379.1	3.1436	.00325733	964.468	74022.99
308	94864	29218112	17.5499	6.7533	5405.4	3.1456	.00324675	967.610	74506.01
309	95481	29503629	17.5784	6.7606	5431.7	3.1477	.00323625	970.751	74990.60
310	96100	[illegible]	17.6068	6.7679	5458.1	3.1497	[illegible]	973.894	75476.76
311	96721	30080231	17.6352	6.7752	5484.5	3.1518	.00321543	977.035	75964.50
312	97344	30371328	17.6635	6.7824	5511.0	3.1538	.00320513	980.176	76453.80
313	[illegible]	30664297	17.6918	6.7897	5537.5	3.1558	.00319489	983.318	76944.67
314	[illegible]	30959144	17.7200	6.7969	5564.1	3.1578	.00318471	986.459	77437.12
315	99225	31255875	17.7482	6.8041	5590.7	3.1598	.00317460	989.601	77931.13
316	99856	31554496	17.7764	6.8113	5617.3	3.1618	.00316456	992.743	78426.72
317	100489	31855013	17.8045	6.8185	5644.0	3.1638	.00315457	995.884	78923.88
318	101124	32157432	17.8326	6.8256	5670.7	3.1658	.00314465	999.026	79422.60
319	101761	32461759	17.8606	6.8328	5697.5	3.1678	.00313480	1002.167	79922.90
320	[illegible]	[illegible]	[illegible]	[illegible]	5724.3	[illegible]	.00312500	1005.310	[illegible]
321	103041	33076161	17.9165	6.8470	5751.2	3.1718	.00311526	1008.450	80928.21
322	103684	33386248	17.9444	6.8541	5778.1	3.1737	.00310559	1011.592	81433.22
323	104329	33698267	17.9722	6.8612	5805.0	3.1757	.00309598	1014.734	81939.80
324	104976	34012224	18.0000	6.8683	5832.0	3.1777	.00308642	1017.875	82447.96
325	105625	34328125	18.0278	6.8753	5859.0	3.1796	.00307692	1021.017	82957.68
326	106276	[illegible]	18.0555	6.8824	5886.1	3.1816	.00306748	1024.158	83468.97
327	[illegible]	[illegible]	18.0831	6.8894	5913.2	3.1835	.00305810	1027.300	83981.84
328	[illegible]	[illegible]	18.1108	6.8964	5940.3	3.1855	.00304878	1030.442	84496.28
329	108241	35611289	18.1384	6.9034	5967.5	3.1874	.00303951	1033.585	85012.28
330	108900	[illegible]	18.1659	6.9104	5994.7	3.1894	[illegible]	1036.726	[illegible]
331	109561	36264691	18.1934	6.9174	6022.0	3.1913	.00302115	1039.866	86049.01
332	110224	36594368	18.2209	6.9244	6049.3	3.1932	.00301205	1043.008	86569.73
333	110889	[illegible]	[illegible]	6.9313	6076.7	3.1951	.00300300	1046.149	87092.02
334	111556	[illegible]	[illegible]	6.9382	6104.1	3.1970	.00299401	1049.291	87615.88
335	112225	[illegible]	18.3030	6.9451	6131.5	3.1989	.00298507	1052.433	88141.31
336	112896	37933056	18.3303	6.9521	6159.0	3.2009	.00297619	1055.574	88668.31
337	113569	38272753	18.3576	6.9589	6186.5	3.2028	.00296736	1058.716	89196.88
338	114244	38614472	18.3848	6.9658	6214.1	3.2047	.00295858	1061.857	89727.03
339	114921	38958219	18.4120	6.9727	6241.7	3.2066	.00294985	1064.999	90258.74
340	115600	[illegible]	18.4391	6.9795	[illegible]	3.2085	.00294118	1068.142	[illegible]
341	116281	39651821	18.4662	6.9864	6297.0	3.2103	.00293255	1071.282	91326.88
342	116964	40001688	18.4932	6.9932	6324.7	3.2122	.00292398	1074.424	91863.31
343	117649	40353607	18.5203	7.0000	6352.4	3.2141	.00291545	1077.565	92401.31
344	118336	40707584	18.5472	7.0068	6380.2	3.2160	.00290698	1080.707	92940.88
345	119025	41063625	18.5742	7.0136	6408.1	3.2178	.00289855	1083.849	93482.02
346	119716	41421736	18.6011	7.0203	6436.0	3.2197	.00289017	1086.990	94024.73
347	120409	41781923	18.6279	7.0271	6463.9	3.2216	.00288184	1090.132	94569.01
348	121104	42144192	18.6548	7.0338	6491.9	3.2234	.00287356	1093.273	95114.86
349	121801	42508549	18.6815	7.0406	6519.9	3.2253	.00286533	1096.415	95662.28
350	[illegible]	[illegible]	18.7083	7.0473	6547.9	3.2271	.00285714	1099.557	[illegible]
351	123201	43243551	18.7350	7.0540	6576.0	3.2289	.00284900	1102.698	96761.84
352	123904	43614208	18.7617	7.0607	6604.1	3.2308	.00284091	1105.840	97313.97
353	124609	43986977	18.7883	7.0674	6632.3	3.2326	.00283286	1108.981	97867.68
354	125316	44361864	18.8149	7.0740	6660.5	3.2345	.00282486	1112.123	98422.96
355	126025	44738875	18.8414	7.0807	6688.7	3.2363	.00281690	1115.264	98979.80
356	126736	45118016	18.8680	7.0873	6717.0	3.2381	.00280899	1118.406	99538.22
357	127449	45499293	18.8944	7.0940	6745.3	3.2399	.00280112	1121.548	100098.21
358	128164	45882712	18.9209	7.1006	6773.7	3.2417	.00279330	1124.689	100659.77
359	128881	46268279	18.9473	7.1072	6802.1	3.2435	.00278552	1127.831	101222.90

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
360	129600	46656000	18.9737	7.1138	6830.5	3.2453	.00277778	1130.972	101787.60
361	130321	47045881	19.0000	7.1204	6859.0	3.2471	.00277008	1134.114	102353.87
362	131044	47437928	19.0263	7.1269	6887.5	3.2489	.00276243	1137.256	102921.72
363	131769	47832147	19.0526	7.1335	6916.1	3.2507	.00275482	1140.397	103491.13
364	132496	48228544	19.0788	7.1400	6944.7	3.2525	.00274725	1143.539	104062.12
365	133225	48627125	19.1050	7.1466	6973.3	3.2543	.00273973	1146.680	104634.67
366	133956	49027896	19.1311	7.1531	7002.0	3.2561	.00273224	1149.822	105208.80
367	134689	49430863	19.1572	7.1596	7030.7	3.2579	.00272480	1152.964	105784.49
368	135424	49836032	19.1833	7.1661	7059.5	3.2596	.00271739	1156.105	106361.76
369	136161	50243409	19.2094	7.1726	7088.3	3.2614	.00271003	1159.247	106940.60
370	136900	50653000	19.2354	7.1791	7117.1	3.2632	.00270270	1162.388	107521.01
371	137641	51064811	19.2614	7.1855	7146.0	3.2649	.00269542	1165.530	108102.99
372	138384	51478848	19.2873	7.1920	7174.9	3.2667	.00268817	1168.671	108686.54
373	139129	51895117	19.3132	7.1984	7203.8	3.2684	.00268097	1171.813	109271.66
374	139876	52313624	19.3391	7.2048	7232.8	3.2702	.00267380	1174.955	109858.35
375	140625	52734375	19.3649	7.2112	7261.8	3.2719	.00266667	1178.096	110446.62
376	141376	53157376	19.3907	7.2177	7290.9	3.2737	.00265957	1181.238	111036.45
377	142129	53582633	19.4165	7.2240	7320.0	3.2754	.00265252	1184.379	111627.86
378	142884	54010152	19.4422	7.2304	7349.2	3.2772	.00264550	1187.521	112220.83
379	143641	54439939	19.4679	7.2368	7378.3	3.2789	.00263852	1190.663	112815.38
380	144400	54872000	19.4936	7.2432	7407.6	3.2806	.00263158	1193.804	113411.49
381	145161	55306341	19.5192	7.2495	7436.8	3.2824	.00262467	1196.946	114009.18
382	145924	55742968	19.5448	7.2558	7466.1	3.2841	.00261780	1200.087	114608.44
383	146689	56181887	19.5704	7.2622	7495.5	3.2858	.00261097	1203.229	115209.27
384	147456	56623104	19.5959	7.2685	7524.8	3.2875	.00260417	1206.371	115811.67
385	148225	57066625	19.6214	7.2748	7554.2	3.2892	.00259740	1209.512	116415.64
386	148996	57512456	19.6469	7.2811	7583.7	3.2909	.00259067	1212.654	117021.18
387	149769	57960603	19.6723	7.2874	7613.2	3.2926	.00258398	1215.795	117628.30
388	150544	58411072	19.6977	7.2936	7642.7	3.2943	.00257732	1218.937	118236.98
389	151321	58863869	19.7231	7.2999	7672.3	3.2960	.00257069	1222.079	118847.24
390	152100	59319000	19.7484	7.3061	7701.9	3.2977	.00256410	1225.220	119459.06
391	152881	59776471	19.7737	7.3124	7731.5	3.2994	.00255754	1228.362	120072.46
392	153664	60236288	19.7990	7.3186	7761.2	3.3011	.00255102	1231.503	120687.42
393	154449	60698457	19.8242	7.3248	7790.9	3.3028	.00254453	1234.645	121303.96
394	155236	61162984	19.8494	7.3310	7820.7	3.3045	.00253807	1237.786	121922.07
395	156025	61629875	19.8746	7.3372	7850.5	3.3061	.00253165	1240.928	122541.75
396	156816	62099136	19.8997	7.3434	7880.3	3.3078	.00252525	1244.070	123163.00
397	157609	62570773	19.9249	7.3496	7910.2	3.3095	.00251889	1247.211	123785.82
398	158404	63044792	19.9499	7.3558	7940.1	3.3111	.00251256	1250.353	124410.21
399	159201	63521199	19.9750	7.3619	7970.0	3.3128	.00250627	1253.494	125036.17
400	160000	64000000	20.0000	7.3681	8000.0	3.3145	.00250000	1256.636	125663.71
401	160801	64481201	20.0250	7.3742	8030.0	3.3161	.00249377	1259.778	126292.81
402	161604	64964808	20.0499	7.3803	8060.1	3.3178	.00248756	1262.919	126923.48
403	162409	65450827	20.0749	7.3864	8090.2	3.3194	.00248139	1266.061	127555.73
404	163216	65939264	20.0998	7.3925	8120.3	3.3211	.00247525	1269.202	128189.55
405	164025	66430125	20.1246	7.3986	8150.5	3.3227	.00246914	1272.344	128824.93
406	164836	66923416	20.1494	7.4047	8180.7	3.3243	.00246305	1275.486	129461.89
407	165649	67419143	20.1742	7.4108	8210.9	3.3260	.00245700	1278.627	130100.42
408	166464	67917312	20.1990	7.4169	8241.2	3.3276	.00245098	1281.769	130740.52
409	167281	68417929	20.2237	7.4229	8271.5	3.3292	.00244499	1284.910	131382.19
410	168100	68921000	20.2485	7.4290	8301.9	3.3309	.00243902	1288.052	132025.43
411	168921	69426531	20.2731	7.4350	8332.3	3.3325	.00243309	1291.193	132670.24
412	169744	69934528	20.2978	7.4410	8362.7	3.3341	.00242718	1294.335	133316.63
413	170569	70444997	20.3224	7.4470	8393.2	3.3357	.00242131	1297.477	133964.58
414	171396	70957944	20.3470	7.4530	8423.7	3.3373	.00241546	1300.618	134614.10
415	172225	71473375	20.3715	7.4590	8454.2	3.3389	.00240964	1303.760	135265.20
416	173056	71991296	20.3961	7.4650	8484.8	3.3406	.00240385	1306.901	135917.86
417	173889	72511713	20.4206	7.4710	8515.4	3.3422	.00239808	1310.043	136572.10
418	174724	73034632	20.4450	7.4770	8546.0	3.3438	.00239234	1313.185	137227.91
419	175561	73560059	20.4695	7.4829	8576.7	3.3454	.00238663	1316.326	137885.29
420	176400	74088000	20.4939	7.4889	8607.4	3.3470	.00238095	1319.468	138544.24
421	177241	74618461	20.5183	7.4948	8638.2	3.3485	.00237530	1322.609	139204.76
422	178084	75151448	20.5426	7.5007	8669.0	3.3501	.00236967	1325.751	139866.85
423	178929	75686967	20.5670	7.5067	8699.8	3.3517	.00236407	1328.893	140530.51
424	179776	76225024	20.5913	7.5126	8730.7	3.3533	.00235849	1332.034	141195.74
425	180625	76765625	20.6155	7.5185	8761.6	3.3549	.00235294	1335.176	141862.54
426	181476	77308776	20.6398	7.5244	8792.5	3.3565	.00234742	1338.317	142530.92
427	182329	77854483	20.6640	7.5302	8823.5	3.3580	.00234192	1341.459	143200.86
428	183184	78402752	20.6882	7.5361	8854.5	3.3596	.00233645	1344.601	143872.38
429	184041	78953589	20.7123	7.5420	8885.6	3.3612	.00233100	1347.742	144545.46

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Area
430	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
431	185761	80062991	20.7605	7.5537	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
432	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
433	[illegible]	[illegible]	[illegible]	[illegible]	9010.1	[illegible]	[illegible]	[illegible]	[illegible]
434	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
435	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
436	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
437	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
438	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
439	192721	84604519	[illegible]	7.6001	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
440	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
441	194481	85766121	[illegible]	7.6117	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
442	195364	[illegible]	[illegible]	7.6174	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
443	196249	[illegible]	[illegible]	[illegible]	9324.1	[illegible]	[illegible]	[illegible]	[illegible]
444	197136	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
445	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
446	198916	[illegible]	[illegible]	7.6403	9419.0	3.3874	[illegible]	[illegible]	[illegible]
447	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
448	200704	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
449	201601	[illegible]	[illegible]	[illegible]	9514.2	3.3919	[illegible]	[illegible]	[illegible]
450	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
451	203401	91733851	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
452	[illegible]	[illegible]	[illegible]	7.6744	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
453	[illegible]	[illegible]	[illegible]	7.6801	9641.5	[illegible]	.00220751	[illegible]	161170.77
454	206116	93576664	21.3073	7.6857	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
455	[illegible]	94196375	[illegible]	7.6914	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
456	[illegible]	94818816	21.3542	7.6970	[illegible]	[illegible]	[illegible]	[illegible]	163312.35
457	[illegible]	95443993	21.3776	7.7026	9769.5	[illegible]	[illegible]	[illegible]	[illegible]
458	209764	96071912	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
459	210681	96702579	21.4243	7.7138	[illegible]	[illegible]	.00217865	1441.990	[illegible]
460	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
461	212521	97972181	21.4709	7.7250	9898.1	[illegible]	.00216920	1448.273	[illegible]
462	213444	98611128	21.4942	7.7306	9930.3	3.4113	.00216450	1451.415	[illegible]
463	214369	99252847	21.3174	7.7362	9962.6	3.4128	.00215983	1454.556	[illegible]
464	215296	99897344	21.5407	7.7418	9994.8	3.4143	.00215517	[illegible]	[illegible]
465	216225	100544625	[illegible]	7.7473	10027.	3.4158	.00215054	[illegible]	[illegible]
466	217136	101194696	21.5870	7.7529	10060.	3.4173	.00214592	[illegible]	[illegible]
467	218089	101847563	21.6102	7.7584	10092.	3.4187	.00214133	1467.123	171286.70
468	219024	102503232	21.6333	7.7639	10134.	3.4202	.00213675	1470.264	172021.05
469	219961	103161709	21.6564	7.7695	10157.	3.4217	.00213220	1473.406	172756.96
470	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
471	221841	104487111	21.7025	7.7805	10222.	3.4246	.00212314	1479.689	174233.51
472	222784	105154048	21.7256	7.7860	10255.	3.4260	.00211864	1482.830	174974.14
473	223729	105823817	21.7486	7.7915	10287.	3.4275	.00211416	1485.972	175716.34
474	224676	106496424	21.7715	7.7970	10320.	3.4289	.00210970	1489.114	176460.12
475	225625	107171875	21.7945	7.8025	10352.	3.4304	.00210526	1492.255	177205.46
476	226576	107850176	21.8174	7.8079	10385.	3.4318	.00210084	1495.397	177952.37
477	227529	108531333	21.8403	7.8134	10418.	3.4332	.00209644	1498.538	178700.86
478	228484	109215352	21.8632	7.8188	10450.	3.4347	.00209205	1501.680	179450.91
479	229441	109902239	21.8861	7.8243	10483.	3.4361	.00208768	1504.822	180202.54
480	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
481	231361	111284641	21.9317	7.8352	10549.	3.4390	.00207900	1511.105	181710.50
482	232324	111980168	21.9545	7.8406	10582.	3.4404	.00207469	1514.246	182466.84
483	233289	112678587	21.9773	7.8460	10615.	3.4418	.00207039	1517.388	183224.75
484	234256	113379904	22.0000	7.8514	10648.	3.4433	.00206612	1520.530	183984.23
485	235225	114084125	22.0227	7.8568	10681.	3.4447	.00206186	1523.671	184745.28
486	236196	114791256	22.0454	7.8622	10714.	3.4461	.00205761	1526.813	185507.90
487	237169	115501303	22.0681	7.8676	10747.	3.4475	.00205339	1529.954	186272.10
488	238144	116214272	22.0907	7.8730	10780.	3.4489	.00204918	1533.096	187037.86
489	239121	116930169	22.1133	7.8784	10813.	3.4504	.00204499	1536.238	187805.19
490	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
491	241081	118370771	22.1585	7.8891	10880.	3.4532	.00203666	1542.521	189344.57
492	242064	119095488	22.1811	7.8944	10913.	3.4546	.00203252	1545.662	190116.62
493	243049	119823157	22.2036	7.8998	10946.	3.4560	.00202840	1548.804	190890.24
494	244036	120553784	22.2261	7.9051	10980.	3.4574	.00202429	1551.945	191665.43
495	245025	121287375	22.2486	7.9105	11013.	3.4588	.00202020	1555.087	192442.18
496	246016	122023936	22.2711	7.9158	11046.	3.4602	.00201613	1558.229	193220.51
497	247009	122763473	22.2935	7.9211	11080.	3.4616	.00201207	1561.370	194000.41
498	248004	123505992	22.3159	7.9264	11113.	3.4630	.00200803	1564.512	194781.89
499	249001	124251499	22.3383	7.9317	11147.	3.4643	.00200401	1567.653	195564.93

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Area
500	[illegible]	[illegible]	[illegible]	[illegible]	11180	[illegible]	[illegible]	[illegible]	[illegible]
501	251001	[illegible]	[illegible]	[illegible]	11214	3.4671	[illegible]	[illegible]	[illegible]
502	[illegible]	[illegible]	[illegible]	[illegible]	11247	[illegible]	[illegible]	[illegible]	[illegible]
503	[illegible]	[illegible]	[illegible]	[illegible]	11281	3.4699	[illegible]	[illegible]	[illegible]
504	[illegible]	[illegible]	[illegible]	[illegible]	11315	3.4713	[illegible]	[illegible]	[illegible]
505	[illegible]	[illegible]	[illegible]	[illegible]	11348	[illegible]	[illegible]	[illegible]	[illegible]
506	[illegible]	[illegible]	[illegible]	[illegible]	11382	3.4740	[illegible]	[illegible]	[illegible]
507	[illegible]	[illegible]	[illegible]	[illegible]	11416	[illegible]	[illegible]	[illegible]	[illegible]
508	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
509	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.4781	[illegible]	[illegible]	203481.74
510	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
511	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
512	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
513	[illegible]	[illegible]	[illegible]	[illegible]	11619	[illegible]	[illegible]	[illegible]	[illegible]
514	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
515	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
516	[illegible]	[illegible]	[illegible]	[illegible]	11721	3.4876	[illegible]	[illegible]	[illegible]
517	[illegible]	[illegible]	[illegible]	[illegible]	11755	[illegible]	[illegible]	[illegible]	[illegible]
518	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
519	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.4917	[illegible]	[illegible]	211553.63
520	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
521	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
522	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
523	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
524	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
525	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
526	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.5010	.00190114	[illegible]	[illegible]
527	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
528	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
529	[illegible]	[illegible]	[illegible]	[illegible]	13167	[illegible]	[illegible]	1661.901	[illegible]
530	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
531	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	221451.65
532	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
533	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.5103	[illegible]	[illegible]	[illegible]
534	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
535	[illegible]	[illegible]	23.1301	[illegible]	12375	[illegible]	[illegible]	[illegible]	[illegible]
536	[illegible]	[illegible]	23.1517	8.1231	12410	3.5143	[illegible]	[illegible]	225641.75
537	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
538	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
539	290521	156590819	23.2164	8.1382	12514	3.5182	[illegible]	1693.315	[illegible]
540	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
541	292681	158340421	23.2594	8.1483	12583	3.5208	.00184843	[illegible]	229871.12
542	293764	[illegible]	[illegible]	8.1533	12618	3.5221	.00184502	[illegible]	[illegible]
543	294849	160103007	23.3024	8.1583	12653	3.5234	.00184162	[illegible]	231573.86
544	295936	160989184	[illegible]	8.1633	[illegible]	3.5247	.00183824	1709.025	[illegible]
545	297025	161878625	23.3452	8.1683	12723	3.5260	.00183486	1712.167	[illegible]
546	298116	162771336	[illegible]	8.1733	12758	3.5273	[illegible]	[illegible]	[illegible]
547	299209	163667323	[illegible]	8.1783	12793	3.5286	.00182815	1718.450	[illegible]
548	300304	164566592	[illegible]	8.1833	12828	3.5299	[illegible]	1721.591	235858.21
549	301401	165469149	23.4307	8.1882	12863	3.5311	.00182149	1724.733	236719.79
550	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
551	303601	167284151	23.4734	8.1982	12934	3.5337	.00181488	1731.016	238447.67
552	304704	[illegible]	23.4947	8.2031	12969	3.5350	.00181159	1734.158	239313.96
553	305809	169112377	23.5160	8.2081	13004	3.5363	.00180832	1737.299	240181.83
554	306916	170031464	23.5372	8.2130	13040	3.5376	.00180505	1740.441	241051.36
555	308025	170953875	23.5584	8.2180	13075	3.5388	.00180180	1743.582	241922.27
556	309136	171879616	23.5797	8.2229	13110	3.5401	.00179856	1746.724	242794.85
557	310249	172808693	[illegible]	8.2278	13146	3.5414	.00179533	1749.866	243668.99
558	311364	173741112	23.6220	8.2327	13181	3.5426	.00179211	1753.007	244544.71
559	312481	174676879	23.6432	8.2377	13217	3.5439	.00178891	1756.149	245422.00
560	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
561	314721	176558481	23.6854	8.2475	13287	3.5464	.00178253	1762.432	247181.30
562	315844	177504328	23.7065	8.2524	13323	3.5477	.00177936	1765.574	248063.39
563	316969	178453547	23.7276	8.2573	13359	3.5490	.00177620	1768.715	248946.87
564	318096	179406144	23.7487	8.2621	13394	3.5502	.00177305	1771.857	249832.01
565	319225	180362125	23.7697	8.2670	13430	3.5515	.00176991	1774.990	250718.73
566	320356	181321496	23.7908	8.2719	13466	3.5527	.00176678	1778.140	251607.01
567	321489	182284263	23.8118	8.2768	13501	3.5540	.00176367	1781.282	252496.87
568	322624	183250432	23.8328	8.2816	13537	3.5553	.00176056	1784.423	253388.30
569	323761	184220009	23.8537	8.2865	13573	3.5565	.00175747	1787.565	254281.29

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
570	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
571	326041	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
572	327184	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
573	328329	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
574	329476	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
575	330625	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
576	331776	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
577	332929	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
578	334084	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
579	335241	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
580	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
581	337561	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
582	338724	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
583	339889	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
584	341056	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
585	342225	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
586	343396	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
587	344569	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
588	345744	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
589	346921	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
590	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
591	349281	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
592	350464	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
593	351649	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
594	352836	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
595	354025	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
596	355216	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
597	356409	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
598	357604	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
599	358801	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
600	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
601	361201	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
602	362404	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
603	363609	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
604	364816	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
605	366025	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
606	367236	[illegible]	24.6171	[illegible]	14918	3.6016	[illegible]	[illegible]	[illegible]
607	368449	[illegible]	[illegible]	8.4670	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
608	369664	[illegible]	[illegible]	8.4716	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
609	370881	[illegible]	24.6779	8.4763	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
610	372100	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
611	373321	228099131	24.7184	8.4856	15103	3.6075	.00163666	1919.511	293203.63
612	374544	229220928	24.7386	8.4902	15140	3.6087	.00163399	1922.653	294166.17
613	375769	230346397	24.7588	8.4948	15177	3.6099	.00163132	1925.795	295128.28
614	376996	231475544	24.7790	8.4994	15214	3.6111	.00162866	1928.936	296091.97
615	378225	232608375	24.7992	8.5040	15252	3.6122	.00162602	1932.078	297057.22
616	379456	233744896	24.8193	8.5086	15289	3.6134	.00162338	1935.219	298024.05
617	380689	234885113	24.8395	8.5132	15326	3.6146	.00162075	1938.361	298992.44
618	381924	236029032	24.8596	8.5178	15363	3.6158	.00161812	1941.503	299962.41
619	383161	237176659	24.8797	8.5224	15400	3.6169	.00161551	1944.644	300933.93
620	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
621	385641	239483061	24.9199	8.5316	15475	3.6192	.00161031	1950.927	302881.77
622	386884	240641848	24.9399	8.5362	15513	3.6204	.00160772	1954.069	303857.98
623	388129	241804367	24.9600	8.5408	15550	3.6216	.00160514	1957.211	304835.80
624	389376	242970624	24.9800	8.5453	15588	3.6227	.00160256	1960.352	305815.20
625	390625	244140625	25.0000	8.5499	15625	3.6239	.00160000	1963.494	306796.16
626	391876	245314376	25.0200	8.5544	15663	3.6250	.00159744	1966.635	307778.69
627	393129	246491883	25.0400	8.5590	15700	3.6262	.00159490	1969.777	308762.79
628	394384	247673152	25.0599	8.5635	15738	3.6274	.00159236	1972.919	309748.47
629	395641	248858189	25.0799	8.5681	15775	3.6285	.00158983	1976.060	310735.71
630	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
631	398161	251239591	25.1197	8.5772	15850	3.6309	.00158479	1982.343	312714.92
632	399424	252435968	25.1396	8.5817	15888	3.6320	.00158228	1985.485	313706.88
633	400689	253636137	25.1595	8.5862	15926	3.6331	.00157978	1988.626	314700.40
634	401956	254840104	25.1794	8.5907	15964	3.6343	.00157729	1991.768	315695.50
635	403225	256047875	25.1992	8.5952	16002	3.6354	.00157480	1994.910	316692.17
636	404496	257259456	25.2190	8.5997	16040	3.6366	.00157233	1998.051	317690.42
637	405769	258474853	25.2389	8.6043	16077	3.6377	.00156986	2001.193	318690.23
638	407044	259694072	25.2587	8.6088	16115	3.6389	.00156740	2004.334	319691.61
639	408321	260917119	25.2784	8.6132	16153	3.6400	.00156495	2007.476	320694.56

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = D)	
								Circum.	Area
640	[illegible]	[illegible]	[illegible]	8.6177	[illegible]	3.6411	[illegible]	[illegible]	[illegible]
641	410881	263374721	25.3180	8.6222	16229	3.6423	.00156006	2013.759	322705.18
642	412164	264609288	25.3377	8.6267	16267	3.6435	.00155763	2016.902	323712.85
643	413449	265847707	25.3574	8.6312	16305	3.6446	.00155521	2020.042	324722.09
644	414736	267089984	25.3772	8.6357	16343	3.6457	.00155280	2023.184	325732.89
645	416025	268336125	25.3969	8.6401	16381	3.6468	.00155039	2026.326	326745.27
646	417316	269586136	25.4165	8.6446	16419	3.6479	.00154799	2029.467	327759.22
647	418609	270840023	25.4362	8.6490	16457	3.6490	.00154560	2032.609	328774.74
648	419904	272097792	25.4558	8.6535	16495	3.6502	.00154321	2035.750	329791.83
649	421201	273359449	25.4755	8.6579	16534	3.6513	.00154083	2038.892	330810.49
650	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
651	423801	275894451	25.5147	8.6668	16610	3.6536	.00153610	2045.175	332852.53
652	425104	277167808	25.5343	8.6713	16648	3.6547	.00153374	2048.317	333875.90
653	426409	278445077	25.5539	8.6757	16687	3.6558	.00153139	2051.458	334900.85
654	427716	279726264	25.5734	8.6801	16725	3.6569	.00152905	2054.600	335927.36
655	429025	281011375	25.5930	8.6845	16764	3.6580	.00152672	2057.741	336955.43
656	430336	282300416	25.6125	8.6890	16802	3.6592	.00152439	2060.883	337985.10
657	431649	283593393	25.6320	8.6934	16840	3.6603	.00152207	2064.025	339016.35
658	432964	284890312	25.6515	8.6978	16879	3.6614	.00151976	2067.166	340049.13
659	434281	286191179	25.6710	8.7022	16917	3.6625	.00151745	2070.308	341083.53
660	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
661	436921	288804781	25.7099	8.7110	16994	3.6647	.00151286	2076.591	343156.95
662	438244	290117528	25.7294	8.7154	17033	3.6658	.00151057	2079.733	344196.09
663	439569	291434247	25.7488	8.7198	17071	3.6669	.00150830	2082.874	345236.69
664	440896	292754944	25.7682	8.7241	17110	3.6680	.00150602	2086.016	346278.91
665	442225	294079625	25.7876	8.7285	17149	3.6691	.00150376	2089.157	347322.70
666	443556	295408296	25.8070	8.7329	17187	3.6702	.00150150	2092.299	348368.07
667	444889	296740963	25.8263	8.7373	17226	3.6713	.00149925	2095.441	349415.00
668	446224	298077632	25.8457	8.7416	17265	3.6724	.00149701	2098.582	350463.51
669	447561	299418309	25.8650	8.7460	17304	3.6735	.00149477	2101.724	351513.59
670	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
671	450241	302111711	25.9037	8.7547	17381	3.6757	.00149031	2108.007	353618.45
672	451584	303464448	25.9230	8.7590	17420	3.6768	.00148810	2111.148	354673.24
673	452929	304821217	25.9422	8.7634	17459	3.6779	.00148588	2114.290	355729.60
674	454276	306182024	25.9615	8.7677	17498	3.6790	.00148368	2117.432	356787.54
675	455625	307546875	25.9808	8.7721	17537	3.6801	.00148148	2120.573	357847.04
676	456976	308915776	26.0000	8.7764	17576	3.6812	.00147929	2123.715	358908.11
677	458329	310288733	26.0192	8.7807	17615	3.6823	.00147710	2126.856	359970.75
678	459684	311665752	26.0384	8.7850	17654	3.6834	.00147493	2129.998	361034.97
679	461041	313046839	26.0576	8.7893	17695	3.6845	.00147275	2133.140	362100.75
680	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
681	463761	315821241	26.0960	8.7980	17771	3.6866	.00146843	2139.423	364237.04
682	465124	317214568	26.1151	8.8023	17810	3.6877	.00146628	2142.564	365307.54
683	466489	318611987	26.1343	8.8066	17850	3.6888	.00146413	2145.706	366379.69
684	467856	320013504	26.1534	8.8109	17889	3.6899	.00146199	2148.848	367453.24
685	469225	321419125	26.1725	8.8152	17928	3.6909	.00145985	2151.989	368528.45
686	470596	322828856	26.1916	8.8194	17967	3.6920	.00145773	2155.131	369605.2[illegible]
687	471969	324242703	26.2107	8.8237	18007	3.6931	.00145560	2158.272	370683.59
688	473344	325660672	26.2298	8.8280	18046	3.6942	.00145349	2161.414	371763.51
689	474721	327082769	26.2488	8.8323	18085	3.6953	.00145138	2164.556	372845.00
690	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
691	477481	329939371	26.2869	8.8408	18164	3.6974	.00144718	2170.[illegible]	375012.70
692	478864	331373888	26.3059	8.8451	18204	3.6985	.00144509	2173.980	376098.91
693	480249	332812557	26.3249	8.8493	18243	3.6995	.00144300	2177.122	377186.66
694	481636	334255384	26.3439	8.8536	18283	3.7006	.00144092	2180.263	378276.03
695	483025	335702375	26.3629	8.8578	18322	3.7016	.00143885	2183.405	379366.95
696	484416	337153536	26.3818	8.8621	18362	3.7027	.00143678	2186.547	380459.44
697	485809	338608873	26.4008	8.8663	18401	3.7038	.00143472	2189.688	381553.50
698	487204	340068392	26.4197	8.8705	18441	3.7049	.00143266	2192.830	382649.13
699	488601	341532099	26.4386	8.8748	18480	3.7059	.00143062	2195.971	383746.33
700	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
701	491401	344472101	26.4764	8.8833	18560	3.7080	.00142653	2202.255	385945.44
702	492804	345948408	26.4953	8.8875	18600	3.7091	.00142450	2205.396	387047.36
703	494209	347428927	26.5141	8.8917	18640	3.7101	.00142248	2208.538	388150.84
704	495616	348913664	26.5330	8.8959	18679	3.7112	.00142045	2211.679	389255.98
705	497025	350402625	26.5518	8.9001	18719	3.7123	.00141844	2214.821	390362.52
706	498436	351895816	26.5707	8.9043	18759	3.7135	.00141643	2217.963	391470.72
707	499849	353393243	26.5895	8.9085	18799	3.7144	.00141443	2221.104	392580.49
708	501264	354894912	26.6083	8.9127	18839	3.7154	.00141243	2224.246	393691.82
709	502681	356400829	26.6271	8.9169	18879	3.7165	.00141044	2227.387	394804.72

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[6]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Area
710	504100	357911000	[illegible]	8.9211	18919	3.7175	[illegible]	[illegible]	[illegible]
711	505521	359425431	26.6646	8.9253	18959	3.7185	.00140647	2233.670	397035.26
712	506944	360944128	26.6833	8.9295	18999	3.7196	.00140449	2236.812	398152.89
713	508369	362467097	26.7021	8.9337	19039	3.7206	.00140252	2239.954	399272.33
714	509796	363994344	26.7208	8.9378	19079	3.7217	.00140056	2243.095	400391.84
715	511225	365525875	26.7395	8.9420	19119	3.7227	.00139860	2246.337	401515.18
716	512656	367061696	26.7582	8.9462	19159	3.7238	.00139665	2249.378	402639.08
717	514089	368601813	26.7769	8.9503	19199	3.7248	.00139470	2252.520	403764.56
718	515524	370146232	26.7955	8.9545	19239	3.7258	.00139276	2255.662	404891.60
719	516961	371694959	26.8142	8.9587	19280	3.7269	.00139082	2258.803	406020.22
720	518400	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	407150.41
721	519841	374805361	26.8514	8.9670	19360	3.7290	.00138696	2265.086	408282.17
722	521284	376367048	26.8701	8.9711	19400	3.7300	.00138504	2268.228	409415.50
723	522729	377933067	26.8887	8.9752	19440	3.7310	.00138313	2271.370	410550.40
724	524176	379503424	26.9072	8.9794	19481	3.7321	.00138122	2274.511	411686.87
725	525625	381078125	26.9258	8.9835	19521	3.7331	.00137931	2277.653	412824.91
726	527076	382657176	26.9444	8.9876	19562	3.7341	.00137741	2280.794	413964.52
727	528529	384240583	26.9629	8.9918	19602	3.7351	.00137552	2283.936	415105.71
728	529984	385828352	26.9815	8.9959	19643	3.7362	.00137363	2287.078	416248.46
729	531441	387420489	27.0000	9.0000	19683	3.7372	.00137174	2290.219	417392.79
730	[illegible]	[illegible]	[illegible]	9.0041	19724	[illegible]	[illegible]	[illegible]	[illegible]
731	534361	390617891	27.0370	9.0082	19764	3.7392	.00136799	2296.502	419686.15
732	535824	392223168	27.0555	9.0123	19805	3.7403	.00136612	2299.644	420835.19
733	537289	393832837	27.0740	9.0164	19845	3.7413	.00136426	2302.785	421985.79
734	538756	395446904	27.0924	9.0205	19886	3.7423	.00136240	2305.927	423137.97
735	540225	397065375	27.1109	9.0246	19927	3.7433	.00136054	2309.069	424291.72
736	541696	398688256	27.1293	9.0287	19967	3.7443	.00135870	2312.210	425447.04
737	543169	400315553	27.1477	9.0328	20008	3.7454	.00135685	2315.352	426603.94
738	544644	401947272	27.1662	9.0369	20049	3.7464	.00135501	2318.493	427762.40
739	546121	403583419	27.1846	9.0410	20090	3.7474	.00135318	2321.635	428922.43
740	[illegible]	[illegible]	[illegible]	9.0450	20130	3.7484	[illegible]	[illegible]	[illegible]
741	549081	406869021	27.2213	9.0491	20171	3.7494	.00134953	2327.918	431247.21
742	550564	408518488	27.2397	9.0532	20212	3.7504	.00134771	2331.060	432411.95
743	552049	410172407	27.2580	9.0572	20253	3.7514	.00134590	2334.201	433578.27
744	553536	411830784	27.2764	9.0613	20294	3.7524	.00134409	2337.343	434746.16
745	555025	413493625	27.2947	9.0654	20335	3.7534	.00134228	2340.485	435915.62
746	556516	415160936	27.3130	9.0694	20376	3.7545	.00134048	2343.626	437086.64
747	558009	416832723	27.3313	9.0735	20417	3.7555	.00133869	2346.768	438259.24
748	559504	418508992	27.3496	9.0775	20458	3.7565	.00133690	2349.909	439433.41
749	561001	420189749	27.3679	9.0816	20499	3.7575	.00133511	2353.051	440609.14
750	562500	421875000	27.3861	9.0856	[illegible]	[illegible]	[illegible]	[illegible]	441786.47
751	564001	423564751	27.4044	9.0896	20581	3.7395	.00133156	2359.334	442965.35
752	565504	425259008	27.4226	9.0937	20622	3.7605	.00132979	2362.476	444145.80
753	567009	426957777	27.4408	9.0977	20663	3.7615	.00132802	2365.617	445327.83
754	568516	428661064	27.4591	9.1017	20704	3.7625	.00132626	2368.759	446511.42
755	570025	430368875	27.4773	9.1057	20745	3.7635	.00132450	2371.900	447696.59
756	571536	432081216	27.4955	9.1098	20787	3.7645	.00132275	2375.042	448883.32
757	573049	433798093	27.5136	9.1138	20828	3.7655	.00132100	2378.184	450071.63
758	574564	435519512	27.5318	9.1178	20869	3.7665	.00131926	2381.325	451261.51
759	576081	437245479	27.5500	9.1218	20910	3.7675	.00131752	2384.467	452452.96
760	577600	[illegible]	27.5681	9.1258	20952	3.7685	[illegible]	[illegible]	[illegible]
761	579121	440711081	27.5862	9.1298	20993	3.7694	.00131406	2390.750	454840.57
762	580644	442450728	27.6043	9.1338	21035	3.7704	.00131234	2393.892	456036.73
763	582169	444194947	27.6225	9.1378	21076	3.7714	.00131062	2397.033	457234.46
764	583696	445943744	27.6405	9.1418	21117	3.7724	.00130890	2400.175	458433.77
765	585225	447697125	27.6586	9.1458	21159	3.7734	.00130719	2403.316	459634.64
766	586756	449455096	27.6767	9.1498	21200	3.7744	.00130548	2406.458	460837.08
767	588289	451217663	27.6948	9.1537	21242	3.7754	.00130378	2409.600	462041.10
768	589824	452984832	27.7128	9.1577	21283	3.7764	.00130208	2412.741	463246.69
769	591361	454756609	27.7308	9.1617	21325	3.7774	.00130039	2415.883	464453.84
770	592900	456533000	27.7489	9.1657	21367	3.7784	.00129870	[illegible]	465662.57
771	594441	458314011	27.7669	9.1696	21408	3.7793	.00129702	2422.166	466872.87
772	595984	460099648	27.7849	9.1736	21450	3.7803	.00129534	2425.307	468084.74
773	597529	461889917	27.8029	9.1775	21492	3.7813	.00129366	2428.449	469298.18
774	599076	463684824	27.8209	9.1815	21533	3.7822	.00129199	2431.591	470513.19
775	600625	465484375	27.8388	9.1355	21575	3.7832	.00129032	2434.732	471729.77
776	602176	467288576	27.8568	9.1894	21617	3.7842	.00128866	2437.874	472947.92
777	603729	469097433	27.8747	9.1933	21658	3.7852	.00128700	2441.015	474167.65
778	605284	470910952	27.8927	9.1973	21700	3.7861	.00128535	2444.157	475388.94
779	606841	472729139	27.9106	9.2012	21742	3.7871	.00128370	2447.299	476611.86

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$) Circum.	Area
780	608400	474552000	27.9285	9.2052	21784	3.7881	.00128205	2450.440	477836.26
781	609961	476379541	27.9464	9.2091	21826	3.7890	.00128041	2453.582	479062.29
782	611524	478211768	27.9643	9.2130	21868	3.7900	.00127877	2456.723	480289.83
783	613089	480048687	27.9821	9.2170	21910	3.7910	.00127714	2459.865	481518.97
784	614656	481890304	28.0000	9.2209	21952	3.7920	.00127551	2463.007	482749.49
785	616225	483736625	28.0179	9.2248	21994	3.7929	.00127389	2466.148	483981.98
786	617796	485587656	28.0357	9.2287	22036	3.7939	.00127226	2469.290	485215.84
787	619369	487443403	28.0535	9.2326	22078	3.7949	.00127065	2472.431	486451.22
788	620944	489303872	28.0713	9.2365	22120	3.7959	.00126904	2475.573	487688.28
789	622521	491169069	28.0891	9.2404	22162	3.7969	.00126743	2478.715	488926.85
790	624100	493039000	28.1069	9.2443	22205	3.7978	.00126582	2481.858	490166.99
791	625681	494913671	28.1247	9.2482	22247	3.7987	.00126422	2484.998	491408.71
792	627264	496793088	28.1425	9.2521	22289	3.7997	.00126263	2488.139	492651.99
793	628849	498677257	28.1603	9.2560	22331	3.8006	.00126103	2491.281	493896.85
794	630436	500566184	28.1780	9.2599	22373	3.8016	.00125945	2494.422	495143.28
795	632025	502459875	28.1957	9.2638	22416	3.8025	.00125786	2497.564	496391.27
796	633616	504358336	28.2135	9.2677	22458	3.8035	.00125628	2500.706	497640.84
797	635209	506261573	28.2312	9.2716	22500	3.8044	.00125471	2503.847	498891.98
798	636804	508169592	28.2489	9.2754	22543	3.8054	.00125313	2506.989	500144.69
799	638401	510082399	28.2666	9.2793	22585	3.8064	.00125156	2510.130	501398.97
800	640000	512000000	28.2843	9.2832	22627	3.8073	.00125000	2513.274	502654.82
801	641601	513922401	28.3019	9.2870	22670	3.8083	.00124844	2516.414	503912.25
802	643204	515849608	28.3196	9.2909	22712	3.8092	.00124688	2519.555	505171.24
803	644809	517781627	28.3373	9.2948	22755	3.8102	.00124533	2522.697	506431.80
804	646416	519718464	28.3549	9.2986	22797	3.8111	.00124378	2525.838	507693.94
805	648025	521660125	28.3725	9.3025	22840	3.8121	.00124224	2528.980	508957.64
806	649636	523606616	28.3901	9.3063	22883	3.8130	.00124069	2532.122	510222.92
807	651249	525557943	28.4077	9.3102	22925	3.8139	.00123916	2535.263	511489.77
808	652864	527514112	28.4253	9.3140	22968	3.8149	.00123762	2538.405	512758.19
809	654481	529475129	28.4429	9.3179	23010	3.8158	.00123609	2541.546	514028.18
810	656100	531441000	28.4605	9.3217	23053	3.8168	.00123457	2544.690	515299.74
811	657721	533411731	28.4781	9.3255	23096	3.8177	.00123305	2547.829	516572.87
812	659344	535387328	28.4956	9.3294	23138	3.8186	.00123153	2550.971	517847.57
813	660969	537367797	28.5132	9.3332	23181	3.8196	.00123001	2554.113	519123.84
814	662596	539353144	28.5307	9.3370	23224	3.8205	.00122850	2557.254	520401.68
815	664225	541343375	28.5482	9.3408	23267	3.8215	.00122699	2560.396	521681.10
816	665856	543338496	28.5657	9.3447	23310	3.8224	.00122549	2563.537	522962.08
817	667489	545338513	28.5832	9.3485	23352	3.8234	.00122399	2566.679	524244.63
818	669124	547343432	28.6007	9.3523	23395	3.8243	.00122249	2569.821	525528.76
819	670761	549353259	28.6182	9.3561	23438	3.8252	.00122100	2572.962	526814.46
820	672400	551368000	28.6356	9.3599	23481	3.8262	.00121951	2576.104	528101.73
821	674041	553387661	28.6531	9.3637	23524	3.8271	.00121803	2579.245	529390.56
822	675684	555412248	28.6705	9.3675	23567	3.8280	.00121655	2582.387	530680.97
823	677329	557441767	28.6880	9.3713	23610	3.8290	.00121507	2585.529	531972.95
824	678976	559476224	28.7054	9.3751	23653	3.8299	.00121359	2588.670	533266.50
825	680625	561515625	28.7228	9.3789	23696	3.8308	.00121212	2591.812	534561.62
826	682276	563559976	28.7402	9.3827	23740	3.8317	.00121065	2594.953	535858.32
827	683929	565609283	28.7576	9.3865	23783	3.8327	.00120919	2598.095	537156.58
828	685584	567663552	28.7750	9.3902	23826	3.8336	.00120773	2601.237	538456.41
829	687241	569722789	28.7924	9.3940	23869	3.8345	.00120627	2604.378	539757.82
830	688900	571787000	28.8097	9.3978	23912	3.8355	.00120482	2607.520	541060.79
831	690561	573856191	28.8271	9.4016	23955	3.8364	.00120337	2610.661	542365.34
832	692224	575930368	28.8444	9.4053	23999	3.8373	.00120192	2613.803	543671.46
833	693889	578009537	28.8617	9.4091	24042	3.8382	.00120048	2616.944	544979.15
834	695556	580093704	28.8791	9.4129	24085	3.8391	.00119904	2620.086	546288.40
835	697225	582182875	28.8964	9.4166	24128	3.8401	.00119760	2623.228	547599.23
836	698896	584277056	28.9137	9.4204	24172	3.8410	.00119617	2626.369	548911.63
837	700569	586376253	28.9310	9.4241	24215	3.8419	.00119474	2629.511	550225.61
838	702244	588480472	28.9482	9.4279	24259	3.8428	.00119332	2632.652	551541.15
839	703921	590589719	28.9655	9.4316	24302	3.8437	.00119190	2635.794	552858.26
840	705600	592704000	28.9828	9.4354	24346	3.8446	.00119048	2638.936	554176.94
841	707281	594823321	29.0000	9.4391	24389	3.8456	.00118906	2642.077	555497.20
842	708964	596947688	29.0172	9.4429	24432	3.8465	.00118765	2645.219	556819.02
843	710649	599077107	29.0345	9.4466	24476	3.8474	.00118624	2648.360	558142.42
844	712336	601211584	29.0517	9.4503	24520	3.8483	.00118483	2651.503	559467.39
845	714025	603351125	29.0689	9.4541	24563	3.8492	.00118343	2654.644	560793.92
846	715716	605495736	29.0861	9.4578	24607	3.8501	.00118203	2657.785	562122.03
847	717409	607645423	29.1033	9.4615	24650	3.8510	.00118064	2660.927	563451.71
848	719104	609800192	29.1204	9.4652	24694	3.8519	.00117925	2664.068	564782.96
849	720801	611960049	29.1376	9.4690	24738	3.8528	.00117786	2667.210	566115.78

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
850	**722500**	**614125000**	**29.1548**	**9.4727**	**24782**	**3.8528**	**.00117647**	**2670.354**	**567450.17**
851	724201	616295051	29.1719	9.4764	24825	3.8547	.00117509	2673.493	568786.14
852	725904	618470208	29.1890	9.4801	24869	3.8556	.00117371	2676.635	570123.67
853	727609	620650477	29.2062	9.4838	24913	3.8565	.00117233	2679.776	571462.77
854	729316	622835864	29.2233	9.4875	24957	3.8574	.00117096	2682.918	572803.45
855	731025	625026375	29.2404	9.4912	25000	3.8582	.00116959	2686.059	574145.69
856	732736	627222016	29.2575	9.4949	25044	3.8592	.00116822	2689.201	575489.51
857	734449	629422793	29.2746	9.4986	25088	3.8601	.00116686	2692.343	576834.90
858	736164	631628712	29.2916	9.5023	25132	3.8610	.00116550	2695.484	578181.85
859	737881	633839779	29.3087	9.5060	25176	3.8619	.00116414	2698.626	579530.38
860	**739600**	**636056000**	**29.3258**	**9.5097**	**25220**	**3.8628**	**.00116279**	**2701.770**	**580880.48**
861	741321	638277381	29.3428	9.5134	25264	3.8637	.00116144	2704.909	582232.15
862	743044	640503928	29.3598	9.5171	25306	3.8646	.00116009	2708.051	583585.39
863	744769	642735647	29.3769	9.5207	25352	3.8655	.00115875	2711.192	584940.20
864	746496	644972544	29.3939	9.5244	25396	3.8664	.00115741	2714.334	586296.59
865	748225	647214625	29.4109	9.3281	25440	3.8673	.00115607	2717.475	587654.54
866	749956	649461896	29.4279	9.5317	25485	3.8682	.00115473	2720.617	589014.07
867	751689	651714363	29.4449	9.5354	25529	3.8691	.00115340	2723.759	590375.16
868	753424	653972032	29.4618	9.5391	25573	3.8700	.00115207	2726.900	591737.83
869	755161	656234909	29.4788	9.5427	25617	3.8708	.00115075	2730.042	593102.06
870	**756900**	**658503000**	**29.4958**	**9.5464**	**25661**	**3.8717**	**.00114943**	**2733.185**	**594467.87**
871	758641	660776311	29.5127	9.5501	25706	3.8726	.00114811	2736.325	595835.25
872	760384	663054848	29.5296	9.5537	25750	3.8735	.00114679	2739.466	597204.20
873	762129	665338617	29.5466	9.5574	25794	3.8744	.00114548	2742.608	598574.72
874	763876	667627624	29.5635	9.5610	25839	3.8753	.00114416	2745.750	599946.81
875	765625	669921875	29.5804	9.5647	25883	3.8762	.00114286	2748.891	601320.47
876	767376	672221376	29.5973	9.5683	25927	3.8771	.00114155	2752.033	602695.70
877	769129	674526133	29.6142	9.5719	25972	3.8780	.00114025	2755.174	604072.50
878	770884	676836152	29.6311	9.5756	26016	3.8789	.00113895	2758.316	605450.88
879	772641	679151439	29.6479	9.5792	26061	3.8797	.00113766	2761.458	606830.82
880	**774400**	**681472000**	**29.6648**	**9.5828**	**26105**	**3.8806**	**.00113636**	**2764.600**	**608212.34**
881	776161	683797841	29.6816	9.5865	26150	3.8815	.00113507	2767.741	609595.42
882	777924	686128968	29.6985	9.5901	26194	3.8823	.00113379	2770.882	610980.08
883	779689	688465387	29.7153	9.5937	26239	3.8832	.00113250	2774.024	612366.31
884	781456	690807104	29.7321	9.5973	26283	3.8841	.00113122	2777.166	613754.11
885	783225	693154125	29.7489	9.6010	26328	3.8850	.00112994	2780.307	615143.48
886	784996	695506456	29.7658	9.6046	26373	3.8859	.00112867	2783.449	616534.42
887	786769	697864103	29.7825	9.6082	26417	3.8868	.00112740	2786.590	617926.93
888	788544	700227072	29.7993	9.6118	26462	3.8877	.00112613	2789.732	619321.01
889	790321	702595369	29.8161	9.6154	26507	3.8885	.00112486	2792.874	620716.66
890	**792100**	**704969000**	**29.8329**	**9.6190**	**26551**	**3.8894**	**.00112360**	**2796.018**	**622113.80**
891	793881	707347971	29.8496	9.6226	26596	3.8902	.00112233	2799.157	623512.68
892	795664	709732288	29.8664	9.6262	26641	3.8911	.00112108	2802.298	624913.04
893	797449	712121957	29.8831	9.6298	26686	3.8920	.00111982	2805.440	626314.98
894	799236	714516984	29.8998	9.6334	26730	3.8929	.00111857	2808.581	627718.49
895	801025	716917375	29.9166	9.6370	26775	3.8937	.00111732	2811.723	629123.56
896	802816	719323136	29.9333	9.6406	26820	3.8946	.00111607	2814.865	630530.21
897	804609	721734273	29.9500	9.6442	26865	3.8955	.00111483	2818.006	631938.43
898	806404	724150792	29.9666	9.6477	26910	3.8963	.00111359	2821.148	633348.22
899	808201	726572699	29.9833	9.6513	26955	3.8972	.00111235	2824.289	634759.58
900	**810000**	**729000000**	**30.0000**	**9.6549**	**27000**	**3.8981**	**.00111111**	**2827.431**	**636172.51**
901	811801	731432701	30.0167	9.6585	27045	3.8989	.00110988	2830.573	637587.01
902	813604	733870808	30.0333	9.6620	27090	3.8998	.00110865	2833.714	639003.09
903	815409	736314327	30.0500	9.6656	27135	3.9007	.00110742	2836.856	640420.73
904	817216	738763264	30.0666	9.6692	27180	3.9015	.00110619	2839.997	641839.95
905	819025	741217625	30.0832	9.6727	27225	3.9024	.00110497	2843.139	643260.73
906	820836	743677416	30.0998	9.6763	27270	3.9032	.00110375	2846.281	644683.09
907	822649	746142643	30.1164	9.6799	27316	3.9041	.00110254	2849.422	646107.01
908	824464	748613312	30.1330	9.6834	27361	3.9050	.00110132	2852.564	647532.51
909	826281	751089429	30.1496	9.6870	27406	3.9059	.00110011	2855.705	648959.58
910	**828100**	**753571000**	**30.1662**	**9.6905**	**27451**	**3.9067**	**.00109890**	**2858.847**	**650388.22**
911	829921	756058031	30.1828	9.6941	27497	3.9076	.00109769	2861.988	651818.43
912	831744	758550528	30.1993	9.6976	27542	3.9084	.00109649	2865.130	653250.21
913	833569	761048497	30.2159	9.7012	27587	3.9093	.00109529	2868.272	654683.56
914	835396	763551944	30.2324	9.7047	27632	3.9101	.00109409	2871.413	656118.48
915	837225	766060875	30.2490	9.7082	27678	3.9110	.00109290	2874.555	657554.98
916	839056	768575296	30.2655	9.7118	27723	3.9118	.00109170	2877.696	658993.04
917	840889	771095213	30.2820	9.7153	27769	3.9127	.00109051	2880.838	660432.68
918	842724	773620632	30.2985	9.7188	27814	3.9135	.00108932	2883.980	661873.88
919	844561	776151559	30.3150	9.7224	27859	3.9144	.00108814	2887.121	663316.66

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
920	846400	778688000	30.3315	9.7259	27905	3.9152	.00108696	2890.265	664761.01
921	848241	781229961	30.3480	9.7294	27930	3.9161	.00108578	2893.464	666206.92
922	850084	783777448	30.3645	9.7329	27996	3.9169	.00108460	2896.546	667654.41
923	851929	786330467	30.3809	9.7364	28042	3.9178	.00108342	2899.688	669103.47
924	853776	788889024	30.3974	9.7400	28087	3.9186	.00108225	2902.829	670554.10
925	855625	791453125	30.4136	9.7435	28133	3.9194	.00108108	2905.971	672006.30
926	857476	794022776	30.4302	9.7470	28179	3.9203	.00107991	2909.112	673460.08
927	859329	796597983	30.4467	9.7505	28224	3.9212	.00107875	2912.254	674915.42
928	861184	799178752	30.4631	9.7540	28270	3.9220	.00107759	2915.396	676372.33
929	863041	801765089	30.4795	9.7575	28315	3.9229	.00107643	2918.537	677830.82
930	864900	804357000	30.4959	9.7610	28361	3.9237	.00107527	2921.679	679290.87
931	866761	806954491	30.5123	9.7645	28407	3.9246	.00107411	2924.820	680752.50
932	868624	809557568	30.5287	9.7680	28453	3.9254	.00107296	2927.962	682215.69
933	870489	812166237	30.5450	9.7715	28499	3.9262	.00107181	2931.103	683680.46
934	872356	814780504	30.5614	9.7750	28544	3.9271	.00107066	2934.245	685146.80
935	874225	817400375	30.5778	9.7785	28590	3.9279	.00106952	2937.387	686614.71
936	876096	820025856	30.5941	9.7819	28636	3.9288	.00106838	2940.528	688084.19
937	877969	822656953	30.6105	9.7854	28682	3.9296	.00106724	2943.670	689555.24
938	879844	825293672	30.6268	9.7889	28728	3.9304	.00106610	2946.811	691027.86
939	881721	827936019	30.6431	9.7924	28774	3.9313	.00106496	2949.953	692502.05
940	883600	830584000	30.6594	9.7959	28820	3.9321	.00106383	2953.095	693977.82
941	885481	833237621	30.6757	9.7993	28866	3.9329	.00106270	2956.236	695455.15
942	887364	835896888	30.6920	9.8028	28912	3.9338	.00106157	2959.378	696934.06
943	889249	838561807	30.7083	9.8063	28958	3.9346	.00106045	2962.519	698414.53
944	891136	841232384	30.7246	9.8097	29004	3.9354	.00105932	2965.661	699896.58
945	893025	843908625	30.7409	9.8132	29050	3.9363	.00105820	2968.803	701380.19
946	894916	846590536	30.7571	9.8167	29096	3.9371	.00105708	2971.944	702865.38
947	896809	849278123	30.7734	9.8201	29142	3.9379	.00105597	2975.086	704352.14
948	898704	851971392	30.7896	9.8236	29189	3.9388	.00105485	2978.227	705840.47
949	900601	854670349	30.8058	9.8270	29235	3.9396	.00105374	2981.369	707330.37
950	902500	857375000	30.8221	9.8305	29281	3.9404	.00105263	2984.511	708821.84
951	904401	860085351	30.8383	9.8339	29327	3.9413	.00105152	2987.652	710314.88
952	906304	862801408	30.8545	9.8374	29374	3.9421	.00105042	2990.794	711809.50
953	908209	865523177	30.8707	9.8408	29420	3.9429	.00104932	2993.935	713305.68
954	910116	868250664	30.8869	9.8443	29466	3.9438	.00104822	2997.077	714803.43
955	912025	870983875	30.9031	9.8477	29513	3.9446	.00104712	3000.218	716302.76
956	913936	873722816	30.9192	9.8511	29559	3.9454	.00104603	3003.360	717803.66
957	915849	876467493	30.9354	9.8546	29605	3.9462	.00104493	3006.502	719306.12
958	917764	879217912	30.9516	9.8580	29652	3.9471	.00104384	3009.643	720810.16
959	919681	881974079	30.9677	9.8614	29698	3.9479	.00104275	3012.785	722315.77
960	921600	884736000	30.9839	9.8648	29743	3.9487	.00104167	3015.928	723822.96
961	923521	887503681	31.0000	9.8683	29791	3.9495	.00104058	3019.068	725331.70
962	925444	890277128	31.0161	9.8717	29838	3.9503	.00103950	3022.210	726842.02
963	927369	893056347	31.0322	9.8751	29884	3.9512	.00103842	3025.351	728353.91
964	929296	895841344	31.0483	9.8785	29931	3.9520	.00103734	3028.493	729867.37
965	931225	898632125	31.0644	9.8819	29977	3.9528	.00103627	3031.634	731382.40
966	933156	901428696	31.0805	9.8854	30024	3.9536	.00103520	3034.776	732899.01
967	935089	904231063	31.0966	9.8888	30070	3.9544	.00103413	3037.918	734417.18
968	937024	907039232	31.1127	9.8922	30117	3.9553	.00103306	3041.059	735936.93
969	938961	909853209	31.1288	9.8956	30164	3.9561	.00103199	3044.201	737458.24
970	940900	912673000	31.1448	9.8990	30210	3.9569	.00103093	3047.342	738981.13
971	942841	915498611	31.1609	9.9024	30257	3.9577	.00102987	3050.484	740505.59
972	944784	918330048	31.1769	9.9058	30304	3.9585	.00102881	3053.625	742031.62
973	946729	921167317	31.1929	9.9092	30351	3.9593	.00102775	3056.767	743559.22
974	948676	924010424	31.2090	9.9126	30398	3.9602	.00102669	3059.909	745088.39
975	950625	926859375	31.2250	9.9160	30444	3.9610	.00102564	3063.050	746619.13
976	952576	929714176	31.2410	9.9194	30491	3.9618	.00102459	3066.192	748151.44
977	954529	932574833	31.2570	9.9227	30538	3.9626	.00102354	3069.333	749685.32
978	956484	935441352	31.2730	9.9261	30585	3.9634	.00102249	3072.475	751220.78
979	958441	938313739	31.2890	9.9295	30633	3.9642	.00102145	3075.617	752757.80
980	960400	941192000	31.3050	9.9329	30679	3.9650	.00102041	3078.758	754296.40
981	962361	944076141	31.3209	9.9363	30726	3.9658	.00101937	3081.900	755836.59
982	964324	946966168	31.3369	9.9396	30773	3.9666	.00101833	3085.041	757378.30
983	966289	949862087	31.3528	9.9430	30820	3.9674	.00101729	3088.183	758921.61
984	968256	952763904	31.3688	9.9464	30867	3.9682	.00101626	3091.325	760466.48
985	970225	955671625	31.3847	9.9497	30914	3.9691	.00101523	3094.466	762012.93
986	972196	958585256	31.4006	9.9531	30961	3.9699	.00101420	3097.608	763560.95
987	974169	961504803	31.4166	9.9565	31008	3.9707	.00101317	3100.749	765110.54
988	976144	964430272	31.4325	9.9598	31055	3.9715	.00101215	3103.891	766661.70
989	978121	967361669	31.4464	9.9632	31102	3.9723	.00101112	3107.033	768214.44

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = D) Circum.	Area
990	980100	[illegible]	31.4643	[illegible]	[illegible]	3.9731	.00101010	3110.176	[illegible]
991	982081	973242271	31.4802	9.9699	31197	3.9739	.00100908	3113.316	771324.61
992	984064	976191488	31.4960	9.9733	31244	3.9747	.00100806	3116.457	772882.06
993	986049	979146657	31.5119	9.9766	31291	3.9755	.00100705	3119.599	774441.07
994	988036	982107784	31.5278	9.9800	31339	3.9763	.00100604	3122.740	776001.66
995	990025	985074875	31.5436	9.9833	31386	3.9771	.00100503	3125.882	777563.82
996	992016	988047936	31.5595	9.9866	31433	3.9779	.00100402	3129.024	779127.54
997	994009	991026973	31.5753	9.9900	31480	3.9787	.00100301	3132.165	780692.84
998	996004	994011992	31.5911	9.9933	31528	3.9795	.00100200	3135.307	782259.71
999	998001	997002999	31.6070	9.9967	31575	3.9803	.00100100	3138.458	783820.15
1000	1000000	1000000000	31.6228	10.0000	31623	3.9811	.00100000	3141.593	785398.16

A.2. CONVERSION FACTORS*

A.2.1 Length (L)

Multiply → by; ↓ To Obtain	Centimeters	Feet	Inches	Kilometers	Nautical miles	Meters	Microns	Mils	Miles	Millimeters	Yards
Centimeters	1	30.48	2.540	10^5	1.853×10^5	100.	10^{-4}	2.540×10^{-3}	1.609×10^5	0.1	91.44
Feet	3.281×10^{-2}	1.	8.333×10^{-2}	3281	6080	3.281	3.281×10^{-6}	8.333×10^{-5}	5280.	3.281×10^{-3}	3.
Inches	0.3937	12.	1.	3.937×10^4	7.296×10^4	39.37	3.937×10^{-5}	0.001	6.336×10^4	3.937×10^{-2}	36.
Kilometers	10^{-5}	3.048×10^{-4}	2.540×10^{-5}	1.	1.853	0.001	10^{-9}	2.540×10^{-8}	1.609	10^{-6}	9.144×10^{-4}
Nautical Miles	5.395×10^{-6}	1.645×10^{-4}	1.371×10^{-5}	0.5396	1.	5.396×10^{-4}	5.397×10^{-10}	1.371×10^{-8}	0.8684	5.397×10^{-7}	4.934×10^{-4}
Meters	0.01	0.3048	2.540×10^{-2}	1000.	1853	1.	10^{-6}	2.540×10^{-5}	1609	0.001	0.9144
Microns	10^4	3.048×10^5	2.540×10^4	10^9	1.853×10^9	10^6	1.	25.40	1.609×10^9	1000.	9.144×10^5
Mils	393.7	1.200×10^4	1000.	3.937×10^7	7.296×10^7	3.937×10^4	3.937×10^{-2}	1.	6.337×10^7	39.37	3.600×10^4
Miles	6.214×10^{-6}	1.894×10^{-4}	1.578×10^{-5}	0.6214	1.1516	6.214×10^{-4}	6.214×10^{-10}	1.578×10^{-8}	1.	6.214×10^{-7}	5.682×10^{-4}
Millimeters	10.	304.8	25.40	10^6	1.853×10^6	1000.	10^{-3}	2.540×10^{-2}	1.609×10^6	1.	914.4
Yards	1.094×10^{-2}	0.3333	2.778×10^{-2}	1094	2027	1.094	1.094×10^{-6}	2.778×10^{-5}	1760.	1.094×10^{-3}	1.

*All conversion Tables A.2.1 through A.2.14 were reprinted or adapted with permission from "Handbook of Engineering Fundamentals," Eshbach, pp. 1-148 to 1-158, and 1-165, 1-166, 1952, John Wiley and Sons.

A.2.2 Area (L^2)

Multiply → by ↘ to Obtain ↓	Acres	Circular mils	Square centimeters	Square feet	Square inches	Square kilometers	Square meters	Square miles	Square millimeters	Square yards
Acres	1			2.296×10^{-5}		247.1	2.471×10^{-4}	640		2.066×10^{-4}
Circular mils		1	1.973×10^{5}	1.833×10^{8}	1.273×10^{6}		1.973×10^{9}		1973	
Square centimeters		5.067×10^{-6}	1	929.0	6.452	10^{10}	10^{4}	2.590×10^{10}	0.01	8361
Square feet	4.356×10^{4}		1.076×10^{-3}	1	6.944×10^{-3}	1.076×10^{7}	10.76	2.788×10^{7}	1.076×10^{-5}	9
Square inches	6,272,640	7.854×10^{-7}	0.1550	144	1	1.550×10^{9}	1550	4.015×10^{9}	1.550×10^{-3}	1296
Square kilometers	4.047×10^{-3}		10^{-10}	9.290×10^{-8}	6.452×10^{-10}	1	10^{-6}	2.590	10^{-12}	8.361×10^{-7}
Square meters	4047		0.0001	9.290×10^{-2}	6.452×10^{-4}	10^{6}	1	2.590×10^{6}	10^{-6}	0.8361
Square miles	1.562×10^{-3}		3.861×10^{-11}	3.587×10^{-8}		0.3861	3.861×10^{-7}	1	3.861×10^{-13}	3.228×10^{-7}
Square millimeters		5.067×10^{-4}	100	9.290×10^{4}	645.2	10^{12}	10^{6}		1	8.361×10^{5}
Square yards	4840		1.196×10^{-4}	0.1111	7.716×10^{-4}	1.196×10^{6}	1.196	3.098×10^{6}	1.196×10^{-6}	1

A.2.3 Volume (L^3)

Multiply → by ↘ to Obtain ↓	Bushels (dry)	Cubic centimeters	Cubic feet	Cubic inches	Cubic meters	Cubic yards	Gallons (liquid)	Liters	Pints (liquid)	Quarts (liquid)
Bushels (dry)	1		0.8036	4.651×10^{-4}	28.38			2.838×10^{-2}		
Cubic centimeters	3.524×10^{4}	1	2.832×10^{4}	16.39	10^{6}	7.646×10^{5}	3785	1000	473.2	946.4
Cubic feet	1.2445	3.531×10^{-5}	1	5.787×10^{-4}	35.31	27	0.1337	3.531×10^{-2}	1.671×10^{-2}	3.342×10^{-2}
Cubic inches	2150.4	6.102×10^{-2}	1728	1	6.102×10^{4}	46,656	231	61.02	28.87	57.75
Cubic meters	3.524×10^{-2}	10^{-6}	2.832×10^{-2}	1.639×10^{-5}	1	0.7646	3.785×10^{-3}	0.001	4.732×10^{-4}	9.464×10^{-4}
Cubic yards		1.308×10^{-6}	3.704×10^{-2}	2.143×10^{-5}	1.308	1	4.951×10^{-3}	1.308×10^{-3}	6.189×10^{-4}	1.238×10^{-3}
Gallons (liquid)		2.642×10^{-4}	7.481	4.329×10^{-3}	264.2	202.0	1	0.2642	0.125	0.25
Liters	35.24	0.001	28.32	1.639×10^{-2}	1000	764.6	3.785	1	0.4732	0.9464
Pints (liquid)		2.113×10^{-3}	59.84	3.463×10^{-2}	2113	1616	8	2.113	1	2
Quarts (liquid)		1.057×10^{-3}	29.92	1.732×10^{-2}	1057	807.9	4	1.057	0.5	1

A.2.4 Plane Angle

To Obtain ↓ / Multiply → By	Degrees	Minutes	Quadrants	Radians	Revolutions (Circumferences)	Seconds
Degrees	1	1.667 $\times 10^{-2}$	90	57.30	360	2.778 $\times 10^{-4}$
Minutes	60	1	5400	3438	2.16 $\times 10^{4}$	1.667 $\times 10^{-2}$
Quadrants	1.111 $\times 10^{-2}$	1.852 $\times 10^{-4}$	1	0.6366	4	3.087 $\times 10^{-6}$
Radians	1.745 $\times 10^{-2}$	2.909 $\times 10^{-4}$	1.571	1	6.283	4.848 $\times 10^{-6}$
Revolutions * (Circumferences)	2.778 $\times 10^{-3}$	4.630 $\times 10^{-5}$	0.25	0.159	1	7.716 $\times 10^{-7}$
Seconds	3600	60	3.24 $\times 10^{5}$	2.063 $\times 10^{5}$	1.296 $\times 10^{6}$	1

A.2.5 Time (t)

To Obtain ↓ / Multiply → By	SECOND	MINUTE	HOUR	DAY	MONTH (30-Day)	YEAR (365 days)
Seconds	1	60	3600	86,400	2.592 $\times 10^{6}$	31.536 $\times 10^{6}$
Minute	1.667 $\times 10^{-2}$	1	60	1440	43,200	525,600
Hour	2.778 $\times 10^{-4}$	1.667 $\times 10^{-2}$	1	24	720	8760
Day	1.157 $\times 10^{-5}$	6.944 $\times 10^{-4}$	4.167 $\times 10^{-2}$	1	30.	365
Month (30 Day)	3.856 $\times 10^{-7}$	2.283 $\times 10^{-5}$	1.370 $\times 10^{-3}$	0.033	1	12
Year (365 day)	3.17 $\times 10^{-8}$	1.9 $\times 10^{-6}$	1.142 $\times 10^{-4}$	2.74 $\times 10^{-3}$	8.33 $\times 10^{-2}$	1

A.2.6 Linear Velocity (Lt^{-1})

to Obtain ↓ / by / Multiply →	Centimeters per second	Feet per minute	Feet per second	Kilometers per hour	Kilometers per minute	Knots*	Meters per minute	Meters per second	Miles per hour	Miles per minute
Centimeters per second	1	0.5080	30.48	27.78	1667	51.48	1.667	100	44.70	2682
Feet per minute	1.969	1	60	54.68	3281	101.3	3.281	196.8	88	5280
Feet per second	3.281 $\times 10^{-2}$	1.667 $\times 10^{-2}$	1	0.9113	54.68	1.689	5.468 $\times 10^{-2}$	3.281	1.467	88
Kilometers per hour	0.036	1.829 $\times 10^{-2}$	1.097	1	60	1.853	0.06	3.6	1.609	96.54
Kilometers per minute	0.0006	3.048 $\times 10^{-4}$	1.829 $\times 10^{-2}$	1.667 $\times 10^{-2}$	1	3.088 $\times 10^{-2}$	0.001	0.06	2.682 $\times 10^{-2}$	1.609
Knots*	1.943 $\times 10^{-2}$	9.868 $\times 10^{-3}$	0.5921	0.5396	32.38	1	3.238 $\times 10^{-2}$	1.943	0.8684	52.10
Meters per minute	0.6	0.3048	18.29	16.67	1000	30.88	1	60	26.82	1609
Meters per second	0.01	5.080 $\times 10^{-3}$	0.3048	0.2778	16.67	0.5148	1.667 $\times 10^{-2}$	1	0.4470	26.82
Miles per hour	2.237 $\times 10^{-2}$	1.136 $\times 10^{-2}$	0.6318	0.6214	37.28	1.152	3.728 $\times 10^{-2}$	2.237	1	60
Miles per minute	3.728 $\times 10^{-4}$	1.892 $\times 10^{-4}$	1.136 $\times 10^{-2}$	1.036 $\times 10^{-2}$	0.6214	1.919 $\times 10^{-2}$	6.214 $\times 10^{-4}$	3.728 $\times 10^{-2}$	1.667 $\times 10^{-2}$	1

* Nautical miles per hour.

A.2.7 Angular Velocity (t^{-1})

to Obtain ↓ / by / Multiply →	Degrees per second	Radians per second	Revolutions per minute	Revolutions per second
Degrees per second	1	57.30	6	360
Radians per second	1.745 $\times 10^{-2}$	1	0.1047	6.283
Revolutions per minute	0.1667	9.549	1	60
Revolutions per second	2.778 $\times 10^{-3}$	0.1592	1.667 $\times 10^{-2}$	1

A.2.8 Linear Acceleration (Lt^{-2})

Multiply → by ↘ to Obtain ↓	Centimeters per second per second	Feet per second per second	Kilometers per hour per second	Meters per second per second	Miles per hour per second
Centimeters per second per second	1	30.48	27.78	100	44.70
Feet per second per second	3.281×10^{-2}	1	0.9113	3.281	1.467
Kilometers per hour per second	0.036	1.097	1	3.6	1.609
Meters per second per second	0.01	0.3048	0.2778	1	0.4470
Miles per hour per second	2.237×10^{-2}	0.6818	0.6214	2.237	1

* The (standard) acceleration due to gravity (g_0) = 980.7 cm per sec per sec, = 32.17 feet per sec per sec = 35.30 km per hour per sec = 9.807 meters per sec per sec = 21.94 miles per hour per sec.

A.2.9 Angular Acceleration (t^{-2})

Multiply → by ↘ to Obtain ↓	Radians per second per second	Revolutions per minute per minute	Revolutions per minute per second	Revolutions per second per second
Radians per second per second	1	1.745×10^{-3}	0.1047	6.283
Revolutions per minute per minute	573.0	1	60	3600
Revolutions per minute per second	9.549	1.667×10^{-2}	1	60
Revolutions per second per second	0.1592	2.778×10^{-4}	1.667×10^{-2}	1

A.2.10 Mass (M) and Weight

Multiply by → / to Obtain ↓	Grains	Grams	Kilograms*	Milligrams*	Ounces †	Pounds †	Tons (long)	Tons (metric)	Tons (short)
Grains	1	15.43	1.543 $\times 10^{4}$	1.543 $\times 10^{-2}$	437.5	7000			
Grams	6.481 $\times 10^{-2}$	1	1000	0.001	28.35	453.6	1.016 $\times 10^{6}$	10^{6}	9.072 10^{5}
Kilograms	6.481 $\times 10^{-5}$	0.001	1	10^{-6}	2.835 $\times 10^{-2}$	0.4536	1016	1000	907.2
Milligrams	64.81	1000	10^{6}	1	2.835 $\times 10^{4}$	4.536 $\times 10^{5}$	1.016 $\times 10^{9}$	10^{9}	9.072 $\times 10^{8}$
Ounces †	2.286 $\times 10^{-3}$	3.527 $\times 10^{-2}$	35.27	3.527 $\times 10^{-5}$	1	16	3.584 $\times 10^{4}$	3.527 $\times 10^{4}$	3.2 $\times 10^{4}$
Pounds †	1.429 $\times 10^{-4}$	2.205 $\times 10^{-3}$	2.205	2.205 $\times 10^{-6}$	6.250 $\times 10^{-2}$	1	2240	2205	2000
Tons (long)		9.842 $\times 10^{-7}$	9.842 $\times 10^{-4}$	9.842 $\times 10^{-10}$	2.790 $\times 10^{-5}$	4.464 $\times 10^{-4}$	1	0.9842	0.8929
Tons (metric)		10^{-6}	0.001	10^{-9}	2.835 $\times 10^{-5}$	4.536 $\times 10^{-4}$	1.016	1	0.9072
Tons (short)		1.102 $\times 10^{-6}$	1.102 $\times 10^{-3}$	1.102 $\times 10^{-9}$	3.125 $\times 10^{-5}$	0.0005	1.120	1.102	1

* These same conversion factors apply to the *gravitational* units of force having the corresponding names. The dimensions of these units when used as gravitational units of force are MLT^{-2}; see table for *Force*.

† Avoirdupois pounds and ounces.

A.2.11 Density or Mass per Unit Volume (ML^{-3})

Multiply by → / To Obtain ↓	Grams per cubic centimeter	Kilograms per cubic meter	Pounds per cubic foot	Pounds per cubic inch	Pounds per mil foot*
Grams per cubic centimeter	1.	0.001	1.602×10^{-2}	27.68	2.937×10^{6}
Kilograms per cubic meter	1000.	1.	16.02	2.768×10^{4}	2.937×10^{9}
Pounds per cubic foot	62.43	6.243×10^{-2}	1.	1728	1.833×10^{8}
Pounds per cubic inch	3.613×10^{-2}	3.613×10^{-5}	5.787×10^{-4}	1.	1.062×10^{5}
Pounds per mil foot*	3.405×10^{-7}	3.405×10^{-10}	5.456×10^{-9}	9.425×10^{-6}	1.

* Unit of volume is a volume one foot long and one circular mil in cross-section area.

A.2.12 Force (MLt^{-2}) or (F)

Multiply → by ↘ to Obtain ↓	Dynes	Grams	Joules per cm	Joules per meter	Kilograms	Pounds	Poundals
Dynes	1	980.7	10^7	10^5	9.807×10^5	4.448×10^5	1.383×10^4
Grams	1.020×10^{-3}	1	1.020×10^4	102.0	1000	453.6	14.10
Joules per cm	10^{-7}	9.807×10^{-5}	1	.01	9.807×10^{-2}	4.448×10^{-2}	1.383×10^{-3}
Newtons or joules per meter	10^{-5}	9.807×10^{-3}	100	1	9.807	4.448	0.1383
Kilograms	1.020×10^{-6}	0.001	10.20	0.1020	1	0.4536	1.410×10^{-2}
Pounds	2.248×10^{-6}	2.205×10^{-3}	22.48	0.2248	2.205	1	3.108×10^{-2}
Poundals	7.233×10^{-5}	7.093×10^{-2}	723.3	7.233	70.93	32.17	1

* Conversion factors between absolute and gravitational units apply only under standard acceleration due to gravity conditions.

A.2.13 Pressure or Force per Unit Area ($ML^{-1}t^{-2}$) or (FL^{-2})

Multiply → by ↘ to Obtain ↓	Atmospheres (1)	Baryes or dynes per square centimeter (2)	Centimeters of mercury at 0°C (3)	Inches of mercury 0°C (3)	Inches of water at 4°C	Kilograms per square meter (4)	Pounds per square foot	Pounds per square inch	Tons (short) per square foot	Newtons per square meter	Torr or millimeters of mercury	Micron
Atmospheres (1)	1	9.869×10^{-7}	1.316×10^{-2}	$3.342 \times 10^{[illegible]}$	2.458×10^{-3}	9.678×10^{-5}	4.725×10^{-4}	6.804×10^{-2}	0.9450	9.869×10^{-6}	1.316×10^{-3}	1.316×10^{-6}
Baryes or dynes per square centimeter (2)	1.013×10^6	1	1.333×10^4	3.386×10^4	2.491×10^{-3}	98.07	478.8	6.895×10^4	9.576×10^5	10	1333	1.333
Centimeters of mercury at 0°C (3)	76.00	7.501×10^{-5}	1	2.540	0.1866	7.356×10^{-3}	3.591×10^{-2}	5.171	71.83	7.501×10^{-4}	0.1	10^{-4}
Inches of mercury at 0°C	29.92	2.953×10^{-5}	0.3937	1	7.355×10^{-2}	2.896×10^{-3}	1.414×10^{-2}	2.036	[illegible]	2.953×10^{-4}	3.937×10^{-2}	3.937×10^{-5}
Inches of water at 4°C	406.8	4.015×10^{-4}	5.354	13.60	1	3.937×10^{-2}	0.1922	27.68	384.5	4.015×10^{-3}	0.5354	5.354×10^{-4}
Kilograms per square meter (4)	1.033×10^4	1.020×10^{-2}	136.0	345.3	25.40	1	4.882	703.1	9765	0.1020	13.60	1.360×10^{-2}
Pounds per square foot	2116	2.089×10^{-3}	27.85	70.73	5.202	0.2048	1	144	2000	2.089×10^{-2}	2.785	2.785×10^{-3}
Pounds per square inch	14.70	1.450×10^{-5}	0.1934	0.4912	3.613×10^{-2}	1.422×10^{-3}	6.944×10^{-3}	1	13.89	1.450×10^{-4}	1.934×10^{-2}	1.934×10^{-5}
Tons (short) per square foot	1.058	1.044×10^{-6}	1.392×10^{-2}	3.536×10^{-2}	2.601×10^{-3}	1.024×10^{-4}	0.0005	0.0720	1	1.044×10^{-5}	1.392×10^{-3}	1.392×10^{-6}
Newtons per square meter	1.013×10^5	10^{-1}	1.333×10^3	3.386×10^3	2.491×10^{-4}	9.807	47.88	6.895×10^3	9.576×10^4	1	133.3	0.1333
Torr or millimeters of mercury	760	7.501×10^{-4}	10	25.40	1.868	7.356×10^{-2}	0.3591	51.71	718.3	7.501×10^{-3}	1	10^{-3}
Micron	$7.60 \times 10^{[illegible]}$	0.7501	10^4	2.540×10^4	1868	73.56	359.1	5.171×10^4	7.183×10^5	7.501	1000	1

(1) Definition: One atmosphere (standard) = 76 cm of mercury or 760 mm of mercury at 0°C.

(2) Sometimes called a bar. However, by international agreement, 1 bar = 0.98692 atmosphere (see ref. 284-1).

(3) To convert height h of a column of mercury at t degrees Centigrade to the equivalent height h_o at 0°C use $h_o = h\left\{1 - \frac{(m - l)t}{1 + mt}\right\}$ where m = 0.0001815 and $l = 18.4 \times 10^{-6}$ if the scale is engraved on brass; $l = 8.5 \times 10^{-6}$ if on glass. This assumes the scale is correct at 0°C; for other cases (any liquid) see International Critical Tables, Vol 1, 68.

(4) 1 gram per sq cm = 10 kilograms per sq m.

A.2.14 Energy, Work, and Heat (ML^2t^{-2}) or (FL)

Multiply → by ↘ to Obtain ↓	British thermal units †	Centimeter-grams	Ergs or centimeter-dynes	Foot-pounds	Horsepower-hours	Joules ‡ or watt-seconds	Kilogram-calories †	Kilowatt-hours	Meter-kilograms	Watt-hours
British thermal units †	1	9.297×10^{-8}	9.486×10^{-11}	1.285×10^{-3}	2545	9.486×10^{-4}	3.969	3413	9.297×10^{-3}	3.413
Centimeter-grams	1.076×10^{7}	1	1.020×10^{-3}	1.383×10^{4}	2.737×10^{10}	1.020×10^{4}	4.269×10^{7}	3.671×10^{10}	10^{5}	3.671×10^{7}
Ergs or centimeter-dynes	1.055×10^{10}	980.7	1	1.356×10^{7}	2.684×10^{13}	10^{7}	4.186×10^{10}	3.6×10^{13}	9.807×10^{7}	3.6×10^{10}
Foot-pounds	778.3	7.233×10^{-5}	7.367×10^{-8}	1	1.98×10^{6}	0.7376	3087	2.655×10^{6}	7.233	2655
Horsepower-hours	3.929×10^{-4}	3.654×10^{-11}	3.722×10^{-14}	5.050×10^{-7}	1	3.722×10^{-7}	1.559×10^{-3}	1.341	3.653×10^{-6}	1.341×10^{-3}
Joules ‡ or watt-seconds	1054.8	9.807×10^{-5}	10^{-7}	1.356	2.684×10^{6}	1	4186	3.6×10^{6}	9.807	3600
Kilogram-calories †	0.2520	2.343×10^{-8}	2.389×10^{-11}	3.239×10^{-4}	641.3	2.389×10^{-4}	1	860.0	2.343×10^{-3}	0.8600
Kilowatt-hours	2.930×10^{-4}	2.724×10^{-11}	2.778×10^{-14}	3.766×10^{-7}	0.7457	2.778×10^{-7}	1.163×10^{-3}	1	2.724×10^{-6}	0.001
Meter-kilograms	107.6	10^{-5}	1.020×10^{-8}	0.1383	2.737×10^{5}	0.1020	426.9	3.671×10^{5}	1	367.1
Watt-hours	0.2930	2.724×10^{-8}	2.778×10^{-11}	3.766×10^{-4}	745.7	2.778×10^{-4}	1.163	1000	2.724×10^{-3}	1

† Mean calorie and Btu used throughout. One gram-calorie = 0.001 kilogram-calorie; one Ostwald calorie = 0.01 kilogram-calorie.

The IT cal, 1000 international steam-table calories, has been defined as the 1/860th part of the international kilowatthour (see *Mechanical Engineering*, Nov., 1935, p. 710). Its value is very nearly equal to the mean kilogram-calorie, 1 IT cal = 1.00037 kilogram-calories (mean). 1 Btu = 251.996 IT cal.

‡ Absolute joule, defined as 10^7 ergs. The international joule, based on the international ohm and ampere, equals 1.0003 absolute joules.

Multiply → by ↘ To Obtain ↓	Cal/gm	Joules/gm or Watt sec/gm	Btu/lb
Cal/gm	1.	0.239	0.556
Joules/gm or Watt sec/gm	4.184	1.	2.326
Btu/lb	1.799	0.430	1.

A.2.15 Thermal Conductivity ($MLt^{-3}T^{-1}$) and Thermal Resistivity ($M^{-1}L^{-1}t^{3}T$)

Multiply → by / ↓ To Obtain	$\frac{W\ cm}{cm^2\ °K}$	$\frac{W\ in}{in^2\ °F}$	$\frac{Cal\ cm}{sec\ cm^2\ °K}$	$\frac{Btu\ in}{hr\ ft^2\ °F}$	$\frac{Btu\ ft}{hr\ ft^2\ °F}$	$\frac{Btu\ in}{sec\ in^2\ °F}$	$\frac{Btu\ in}{hr\ in^2\ °F}$
$\frac{W\ cm}{cm^2\ °K}$	1.	0.7087	4.1858	1.442×10^{-3}	1.730×10^{-2}	747.4	0.2076
$\frac{W\ in}{in^2\ °F}$	1.411	1.	5.9[illegible]	2.035×10^{-3}	2.442×10^{-2}	1055	0.2930
$\frac{Cal\ cm}{sec\ cm^2\ °K}$	0.2389	0.1693	1.	3.445×10^{-4}	4.135×10^{-3}	178.5	4.960×10^{-2}
$\frac{Btu\ in}{hr\ ft^2\ °F}$	693.4	491.4	2903	1.	12.	5.184×10^{5}	144.
$\frac{Btu\ ft}{hr\ ft^2\ °F}$	57.79	40.95	241.9	8.33×10^{-2}	1.	4.319×10^{4}	12.
$\frac{Btu\ in}{sec\ in^2\ °F}$	1.338×10^{-3}	9.480×10^{-4}	5.604×10^{-3}	1.929×10^{-6}	2.315×10^{-5}	1.	2.778×10^{-4}
$\frac{Btu\ in}{hr\ in^2\ °F}$	4.816	3.413	20.16	6.940×10^{-3}	8.333×10^{-2}	3600.	1.

Factors given are for thermal conductivity. To Find thermal resistivity, take reciprocal of indicated factor.

A.2.16 Specific Heat ($L^2t^{-2}T^{-1}$)

Multiply → by / ↓ To Obtain	Gm cal/gm °K or Gm cal/gm °C	Joules/gm °K or Joules/gm °C	Joules/lb °F or Joules/lb °R	Kw hours/kg °C or Kw hours/kg °K	Kw hours/lb °F or Kw hours/lb °R	Btu/lb °F or Btu/lb °R	Btu/lb °C or Btu/lb °K
Gm cal/gm °K or Gm cal/gm °C	1.	0.2390	9.478×10^{-4}	859.8	3413	1.001	0.5556
Joules/gm °K or Joules/gm °C	4.184	1.	3.966×10^{-3}	3597	1.428×10^{4}	4.188	2.325
Joules/lb °F or Joules/lb °R	1055	252.1	1.	9.068×10^{5}	3.600×10^{6}	1056	586.1
Kw hours/kg °C or Kw hours/kg °K	1.163×10^{-3}	2.779×10^{-4}	1.102×10^{-6}	1.	3.967	1.164×10^{-3}	6.459×10^{-4}
Kw hours/lb °F or Kw hours/lb °R	2.930×10^{-4}	7.003×10^{-5}	2.778×10^{-7}	0.2520	1.	2.935×10^{-4}	1.630×10^{-4}
Btu/lb °F or Btu/lb °R	0.9996	0.2388	9.471×10^{-4}	858.2	3407	1.	0.5555
Btu/lb °C or Btu/lb °K	1.800	0.4302	1.706×10^{-3}	1547	6130	1.801	1.

A.2.17 Viscosity

To Obtain ↓ / Multiply By →	ABSOLUTE						
	$\frac{lb_f\ sec}{in^2}$	$\frac{lb_f\ sec}{ft^2}$	$\frac{lb_m}{sec\ ft}$	$\frac{lb_m}{hr\ ft}$	$\frac{slug}{sec\ ft}$	Poise $\frac{dyne.\ sec}{cm^2}$	Centipoise
$\frac{lb_f\ sec}{in^2}$	1	6.95×10^{-3}	2.16×10^{-4}	5.99×10^{-8}	6.95×10^{-3}	1.453×10^{-5}	1.453×10^{-7}
$\frac{lb\ sec}{ft^2}$	144	1	3.11×10^{-2}	8.63×10^{-6}	1	2.09×10^{-3}	2.09×10^{-5}
$\frac{lb_m}{sec\ ft}$	4640	32.17	1	2.78×10^{-4}	32.17	6.72×10^{-2}	6.72×10^{-4}
$\frac{lb_m}{hr\ ft}$	1.67×10^{7}	1.16×10^{5}	3600	1	1.16×10^{5}	242	2.42
$\frac{slug}{sec\ ft}$	144	1	3.11×10^{-2}	8.63×10^{-6}	1	2.09×10^{-3}	2.09×10^{-5}
Poise $\frac{dyne.\ sec}{cm^2}$	6.885×10^{4}	478.5	14.88	4.135×10^{-3}	478.5	1	0.01
Centipoise	6.885×10^{6}	47850	1488	0.4135	47850	100	1

To Obtain ↓ / Multiply By →	KINEMATIC		
	$\frac{ft^2}{sec}$	Stoke $\frac{cm^2}{sec}$	Centistoke
$\frac{ft^2}{sec}$	1	1.076×10^{-3}	1.076×10^{-5}
Stoke $\frac{cm^2}{sec}$	929	1	0.01
Centistoke	9.29×10^{4}	100	1

A.2.18 TEMPERATURE

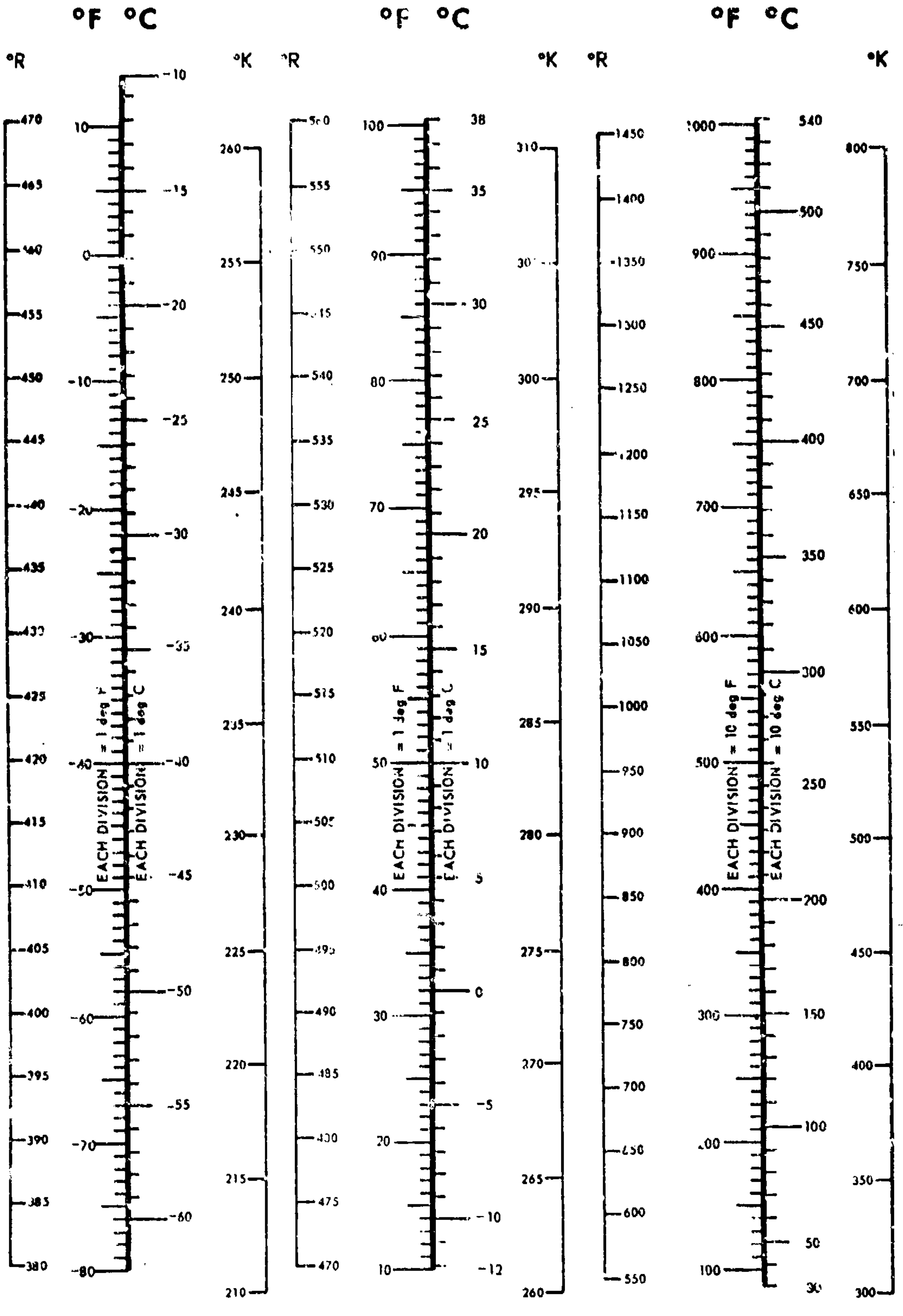


Figure A.2.18. Temperature Conversion Chart.
(Courtesy of Aerojet-General Corporation, Azusa, California)

A.2.18 Temperature*

A.2.18.1 TEMPERATURE CONVERSION, C TO F*

C	0	−10	−20	−30	−40	−50	−60	−70	−80	−90
−200	−328	−346	−364	−382	−400	−418	−436	−454	—	—
−100	−148	−166	−184	−202	−220	−238	−256	−274	−292	−310
0	32	14	−4	−22	−40	−58	−76	−94	−112	−130

C	0	10	20	30	40	50	60	70	80	90
0	32	50	68	86	104	122	140	158	176	194
100	212	230	248	266	284	302	320	338	356	374
200	392	410	428	446	464	482	500	518	536	554
300	572	590	608	626	644	662	680	698	716	734
400	752	770	788	806	824	842	860	878	896	914
500	932	950	968	986	1004	1022	1040	1058	1076	1094
600	1112	1130	1148	1166	1184	1202	1220	1238	1256	1274
700	1292	1310	1328	1346	1364	1382	1400	1418	1436	1454
800	1472	1490	1508	1526	1544	1562	1580	1598	1616	1634
900	1652	1670	1688	1706	1724	1742	1760	1778	1796	1814
1000	1832	1850	1868	1886	1904	1922	1940	1958	1976	1994
1100	2012	2030	2048	2066	2084	2102	2120	2138	2156	2174
1200	2192	2210	2228	2246	2264	2282	2300	2318	2336	2354
1300	2372	2390	2408	2426	2444	2462	2480	2498	2516	2534
1400	2552	2570	2588	2606	2624	2642	2660	2678	2696	2714
1500	2732	2750	2768	2786	2804	2822	2840	2858	2876	2894
1600	2912	2930	2948	2966	2984	3002	3020	3038	3056	3074
1700	3092	3110	3128	3146	3164	3182	3200	3218	3236	3254
1800	3272	3290	3308	3326	3344	3362	3380	3398	3416	3434
1900	3452	3470	3488	3506	3524	3542	3560	3578	3596	3614
2000	3632	3650	3668	3686	3704	3722	3740	3758	3776	3794
2100	3812	3830	3848	3866	3884	3902	3920	3938	3956	3974
2200	3992	4010	4028	4046	4064	4082	4100	4118	4136	4154
2300	4172	4190	4208	4226	4244	4262	4280	4298	4316	4334
2400	4352	4370	4388	4406	4424	4442	4460	4478	4496	4514
2500	4532	4550	4568	4586	4604	4622	4640	4658	4676	4694
2600	4712	4730	4748	4766	4784	4802	4820	4838	4856	4874
2700	4892	4910	4928	4946	4964	4982	5000	5018	5036	5054
2800	5072	5090	5108	5126	5144	5162	5180	5198	5216	5234
2900	5252	5270	5288	5306	5324	5342	5360	5378	5396	5414
3000	5432	5450	5468	5486	5504	5522	5540	5558	5576	5594
3100	5612	5630	5648	5666	5684	5702	5720	5738	5756	5774
3200	5792	5810	5828	5846	5864	5882	5900	5918	5936	5954
3300	5972	5990	6008	6026	6044	6062	6080	6098	6116	6134
3400	6152	6170	6188	6206	6224	6242	6260	6278	6296	6314
3500	6332	6350	6368	6386	6404	6422	6440	6458	6476	6494
3600	6512	6530	6548	6566	6584	6602	6620	6638	6656	6674
3700	6692	6710	6728	6746	6764	6782	6800	6818	6836	6854
3800	6872	6890	6908	6926	6944	6962	6980	6998	7016	7034
3900	7052	7070	7088	7106	7124	7142	7160	7178	7196	7214
C	0	10	20	30	40	50	60	70	80	90

C	F
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2
10	18.0

Example I:

Convert 312 C to the Fahrenheit Scale. From the body of the table, 310 C equals 590 F. From the right hand margin, 2 C = 3.6 F. Summing, we have 310 C + 2 C = 312 C equals 590 F + 3.6 F = 593.6 F

A.2.18.2 TEMPER[illegible]

(All d[illegible]

F	0	−10	−20
−400	−240.0	−245.5	−251.1
−300	−184.4	−190.0	−195.5
−200	−128.8	−134.4	−140.0
−100	−73.3	−78.8	−84.4
0	−17.7	−23.3	−28.8

F	0	10	20
0	−17.7	−12.2	−6.6
100	37.7	43.3	48.8
200	93.3	98.8	104.4
300	148.8	154.4	160.0
400	204.4	210.0	215.5
500	260.0	265.5	271.1
600	315.5	321.1	326.6
700	371.1	376.6	382.2
800	426.6	432.2	437.7
900	482.2	487.7	493.3
1000	537.7	543.3	548.8
1100	593.3	598.8	604.4
1200	648.8	654.4	660.0
1300	704.4	710.0	715.5
1400	760.0	765.5	771.1
1500	815.5	821.1	826.6
1600	871.1	876.6	882.2
1700	926.6	932.2	937.7
1800	982.2	987.7	993.3
1900	1037.7	1043.3	1048.8
2000	1093.3	1098.8	1104.4
2100	1148.8	1154.4	1160.0
2200	1204.4	1210.0	1215.5
2300	1260.0	1265.5	1271.1
2400	1315.5	1321.1	1326.6
2500	1371.1	1376.6	1382.2
2600	1426.6	1432.2	1437.7
2700	1482.2	1487.7	1493.3
2800	1527.7	1543.3	1548.8
2900	1593.3	1598.8	1604.4
3000	1648.8	1654.4	1660.0
3100	1704.4	1710.0	1715.5
3200	1760.0	1765.5	1771.1
3300	1815.5	1821.1	1826.6
3400	1871.1	1876.6	1882.2
F	0	10	20

Example II

Convert 250 F to the Centigrade Scale. [illegible] 250 F [illegible] Adding, we obtain 250 F [illegible] equals [illegible]

*Tables A.2.18.1 and A.2.18.2 were reprinted with permission from "Gas Tables," Keenan and Kaye, pp. 196-198, 1948, John Wiley and Sons.

ISSUED: MARCH 1967
SUPERSEDES: MAY 1964

A.2.18.2 TEMPERATURE CONVERSION, F TO C*

(All decimals are recurring decimals)

F	0	10	20	30	40	50	60	70	80	90		
400	240.0	245.5	251.1	256.6	262.2	267.7	—	—	—	—		
300	184.4	190.0	195.5	201.1	206.6	212.2	217.7	223.3	228.8	234.4		
200	128.8	134.4	140.0	145.5	151.1	156.6	162.2	167.7	173.3	178.8		
100	73.3	78.8	84.4	90.0	95.5	101.1	106.6	112.2	117.7	123.3		
0	17.7	23.3	28.8	34.4	40.0	45.5	51.1	56.6	62.2	67.7		
F	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**F**	**C**
0	17.7	12.2	6.6	1.1	4.4	10.0	15.5	21.1	26.6	32.2		
100	37.7	43.3	48.8	54.4	60.0	65.5	71.1	76.6	82.2	87.7	1	0.5
200	93.3	98.8	104.4	110.0	115.5	121.1	126.6	132.2	137.7	143.3	2	1.1
300	148.8	154.4	160.0	165.5	171.1	176.6	182.2	167.7	193.3	198.3	3	1.6
400	204.4	210.0	215.5	221.1	226.6	232.2	237.7	243.3	248.8	254.4	4	2.2
500	260.0	265.5	271.1	276.6	282.2	287.7	293.3	298.3	394.4	310.0	5	2.7
600	315.5	321.1	326.6	332.2	337.7	343.3	348.8	354.4	360.0	365.5	6	3.3
700	371.1	376.6	382.2	387.7	393.3	398.3	404.4	410.0	415.5	421.1	7	3.8
800	426.6	432.2	437.7	443.3	448.8	454.4	460.0	465.5	471.1	476.6	8	4.4
900	482.2	487.7	493.3	498.8	504.4	510.0	515.5	521.1	526.6	532.2	9	5.0
1000	537.7	543.3	548.8	554.4	560.0	565.5	571.1	576.6	582.2	587.7		
1100	593.3	598.8	604.4	610.0	615.5	621.1	626.6	632.2	637.7	643.3		
1200	648.8	654.4	660.0	665.5	671.1	676.6	682.2	687.7	693.3	698.8		
1300	704.4	710.0	715.5	721.1	726.6	732.2	737.7	743.3	748.8	754.4		
1400	760.0	765.5	771.1	776.6	782.2	787.7	793.3	798.8	804.4	810.0		
1500	815.5	821.1	826.6	832.2	837.7	843.3	848.8	854.4	860.0	865.5		
1600	871.1	876.6	882.2	887.7	893.3	898.8	904.4	910.0	915.5	921.1		
1700	926.6	932.2	937.7	943.3	948.8	954.4	960.0	965.5	971.1	976.6		
1800	982.2	987.7	993.3	998.8	1004.4	1010.0	1015.5	1021.1	1026.6	1032.2		
1900	1037.7	1043.3	1048.8	1054.4	1060.0	1065.5	1071.1	1076.6	1082.2	1087.7		
2000	1093.3	1098.8	1104.4	1110.0	1115.5	1121.1	1126.6	1132.2	1137.7	1143.3		
2100	1148.8	1154.4	1160.0	1165.5	1171.1	1176.6	1182.2	1187.7	1193.3	1198.8		
2200	1204.4	1210.0	1215.5	1221.1	1226.6	1232.2	1237.7	1243.3	1248.8	1254.4		
2300	1260.0	1265.5	1271.1	1276.6	1282.2	1287.7	1293.3	1298.8	1304.4	1310.0		
2400	1315.5	1321.1	1326.6	1332.2	1337.7	1343.3	1348.8	1354.4	1360.0	1365.5		
2500	1371.1	1376.6	1382.2	1387.7	1393.3	1398.8	1404.4	1410.0	1415.5	1421.1		
2600	1426.6	1432.2	1437.7	1443.3	1448.8	1454.4	1460.0	1465.5	1471.1	1476.6		
2700	1482.2	1487.7	1493.3	1498.8	1504.4	1510.0	1515.5	1521.1	1526.6	1532.2		
2800	1527.7	1543.3	1548.8	1554.4	1560.0	1565.5	1571.1	1576.6	1582.2	1587.7		
2900	1593.3	1598.8	1604.4	1610.0	1615.5	1621.1	1626.6	1632.2	1637.7	1643.3		
3000	1648.8	1654.4	1660.0	1665.5	1671.1	1676.6	1682.2	1687.7	1693.3	1698.3		
3100	1704.4	1710.0	1715.5	1721.1	1726.6	1732.2	1737.7	1743.3	1748.8	1754.4		
3200	1760.0	1765.5	1771.1	1776.6	1782.2	1787.7	1793.3	1798.8	1804.4	1810.0		
3300	1815.5	1821.1	1826.6	1832.2	1837.7	1843.3	1848.8	1854.4	1860.0	1865.5		
3400	1871.1	1876.6	1882.2	1887.7	1893.3	1898.8	1904.4	1910.0	1915.5	1921.1		
F	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**		

Absolute scales:

T_r (°Rankine) t_f (°Fahrenheit) +459.69

T (°Kelvin) t_c (°Centrigrade) +273.16

F	0	10	20	30
3500	1926.6	1932.2	1937.7	1943.3
3600	1982.2	1987.7	1993.3	1998.8
3700	2037.7	2043.3	2048.8	2054.4
3800	2093.3	2098.8	2104.4	2110.0
3900	2148.8	2154.4	2160.0	2165.5
4000	2204.4	2210.0	2215.5	2221.1
4100	2260.0	2265.5	2271.1	2276.6
4200	2315.5	2321.1	2326.6	2332.2
4300	2371.1	2376.6	2382.2	2387.7
4400	2426.6	2432.2	2437.7	2443.3
4500	2482.2	2487.7	2493.3	2498.8
4600	2537.7	2543.3	2548.8	2554.4
4700	2593.3	2598.8	2604.4	2610.0
4800	2648.8	2654.4	2660.0	2665.5
4900	2704.4	2710.0	2715.5	2721.1
5000	2760.0	2765.5	2771.1	2776.6
5100	2815.5	2821.1	2826.6	2832.2
5200	2871.1	2876.6	2882.2	2887.7
5300	2926.6	2932.2	2937.7	2943.3
5400	2982.2	2987.7	2993.3	2998.8
5500	3037.7	3043.3	3048.8	3054.4
5600	3093.3	3098.8	3104.4	3110.0
5700	3148.8	3154.4	3160.0	3165.5
5800	3204.4	3210.0	3215.5	3221.1
5900	3260.0	3265.5	3271.1	3276.6
6000	3315.5	3321.1	3326.6	3332.2
6100	3371.1	3376.6	3382.2	3387.7
6200	3426.6	3432.2	3437.7	3443.3
6300	3482.2	3487.7	3493.3	3498.8
6400	3537.7	3543.3	3548.8	3554.4
6500	3593.3	3598.8	3604.4	3610.0
6600	3648.8	3654.4	3660.0	3665.5
6700	3704.4	3710.0	3715.5	3721.1
6800	3760.0	3765.5	3771.1	3776.6
6900	3815.5	3821.1	3826.6	3832.2
7000	3871.1	3876.6	3882.2	3887.7
7100	3926.6	3932.2	3937.7	3943.3
7200	3982.2	3987.7	3993.3	3998.8
7300	4037.7	4043.3	4048.8	4054.4
7400	4093.3	4098.8	4104.4	4110.0
7500	4148.8	4154.4	4160.0	4165.5
7600	4204.4	4210.0	4215.5	4221.1
7700	4260.0	4265.5	4271.1	4276.6
7800	4315.5	4321.1	4326.6	4332.2
7900	4371.1	4376.6	4382.2	4387.7
F	**0**	**10**	**20**	**30**

B

F	0	10	20	30	40	50	60	70	80	90
3500	1926.6	1932.2	1937.7	1943.3	1948.8	1954.4	1960.0	1965.5	1971.1	1976.6
3600	1982.2	1987.7	1993.3	1998.8	2004.4	2010.0	2015.5	2021.1	2026.6	2032.2
3700	2037.7	2043.3	2048.8	2054.4	2060.0	2065.5	2071.1	2076.6	2082.2	2087.7
3800	2093.3	2098.8	2104.4	2110.0	2115.5	2121.1	2126.6	2132.2	2137.7	2143.3
3900	2148.8	2154.4	2160.0	2165.5	2171.1	2176.6	2182.2	2187.7	2193.3	2198.8
4000	2204.4	2210.0	2215.5	2221.1	2226.6	2232.2	2237.7	2243.3	2248.8	2254.4
4100	2260.0	2265.5	2271.1	2276.6	2282.2	2287.7	2293.3	2298.8	2304.4	2310.0
4200	2315.5	2321.1	2326.6	2332.2	2337.7	2343.3	2348.8	2354.4	2360.0	2365.5
4300	2371.1	2376.6	2382.2	2387.7	2393.3	2398.8	2404.4	2410.0	2415.5	2421.1
4400	2426.6	2432.2	2437.7	2443.3	2448.8	2454.4	2460.0	2465.5	2471.1	2476.6
4500	2482.2	2487.7	2493.3	2498.8	2504.4	2510.0	2515.5	2521.1	2526.6	2532.2
4600	2537.7	2543.3	2548.8	2554.4	2560.0	2565.5	2571.1	2576.6	2582.2	2587.7
4700	2593.3	2598.8	2604.4	2610.0	2615.5	2621.1	2626.6	2632.2	2637.7	2643.3
4800	2648.8	2654.4	2660.0	2665.5	2671.1	2676.6	2682.2	2687.7	2693.3	2698.8
4900	2704.4	2710.0	2715.5	2721.1	2726.6	2732.2	2737.7	2743.3	2748.8	2754.4
5000	2760.0	2765.5	2771.1	2776.6	2782.2	2787.7	2793.3	2798.8	2804.4	2810.0
5100	2815.5	2821.1	2826.6	2832.2	2837.7	2843.3	2848.8	2854.4	2860.0	2865.5
5200	2871.1	2876.6	2882.2	2887.7	2893.3	2898.8	2904.4	2910.0	2915.5	2921.1
5300	2926.6	2932.2	2937.7	2943.3	2948.8	2954.4	2960.0	2965.5	2971.1	2976.6
5400	2982.2	2987.7	2993.3	2998.8	3004.4	3010.0	3015.5	3021.1	3026.6	3032.2
5500	3037.7	3043.3	3048.8	3054.4	3060.0	3065.5	3071.1	3076.6	3082.2	3087.7
5600	3093.3	3098.8	3104.4	3110.0	3115.5	3121.1	3126.6	3132.2	3137.7	3143.3
5700	3148.8	3154.4	3160.0	3165.5	3171.1	3176.6	3182.2	3187.7	3193.3	3198.8
5800	3204.4	3210.0	3215.5	3221.1	3226.6	3232.2	3237.7	3243.3	3248.8	3254.4
5900	3260.0	3265.5	3271.1	3276.6	3282.2	3287.7	3293.3	3298.8	3304.4	3310.0
6000	3315.5	3321.1	3326.6	3332.2	3337.7	3343.3	3348.8	3354.4	3360.0	3365.5
6100	3371.1	3376.6	3382.2	3387.7	3393.3	3398.8	3404.4	3410.0	3415.5	3421.1
6200	3426.6	3432.2	3437.7	3443.3	3448.8	3454.4	3460.0	3465.5	3471.1	3476.6
6300	3482.2	3487.7	3493.3	3498.8	3504.4	3510.0	3515.5	3521.1	3526.6	3532.2
6400	3537.7	3543.3	3548.8	3554.4	3560.0	3565.5	3571.1	3576.6	3582.2	3587.7
6500	3593.3	3598.8	3604.4	3610.0	3615.5	3621.1	3626.6	3632.2	3637.7	3643.3
6600	3648.8	3654.4	3660.0	3665.5	3671.1	3676.6	3682.2	3687.7	3693.3	3698.8
6700	3704.4	3710.0	3715.5	3721.1	3726.6	3732.2	3737.7	3743.3	3748.8	3754.4
6800	3760.0	3765.5	3771.1	3776.6	3782.2	3787.7	3793.3	3798.8	3804.4	3810.0
6900	3815.5	3821.1	3826.6	3832.2	3837.7	3843.3	3848.8	3854.4	3860.0	3865.5
7000	3871.1	3876.6	3882.2	3887.7	3893.3	3898.8	3904.4	3910.0	3915.5	3921.1
7100	3926.6	3932.2	3937.7	3943.3	3948.8	3954.4	3960.0	3965.5	3971.1	3976.6
7200	3982.2	3987.7	3993.3	3998.8	4004.4	4010.0	4015.5	4021.1	4026.6	4032.2
7300	4037.7	4043.3	4048.8	4054.4	4060.0	4065.5	4071.1	4076.6	4082.2	4087.7
7400	4093.3	4098.8	4104.4	4110.0	4115.5	4121.1	4126.6	4132.2	4137.7	4143.3
7500	4148.8	4154.4	4160.0	4165.5	4171.1	4176.6	4182.2	4187.7	4193.3	4198.8
7600	4204.4	4210.0	4215.5	4221.1	4226.6	4232.2	4237.7	4243.3	4248.8	4254.4
7700	4260.0	4265.5	4271.1	4276.6	4282.2	4287.7	4293.3	4298.8	4304.4	4310.0
7800	4315.5	4321.1	4326.6	4332.2	4337.7	4343.3	4348.8	4354.4	4360.0	4365.5
7900	4371.1	4376.6	4382.2	4387.7	4393.3	4398.8	4404.4	4410.0	4415.5	4421.1
F	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**

F	C
1	0.5
2	1.1
3	1.6
4	2.2
5	2.7
6	3.3
7	3.8
8	4.4
9	5.0

−90
—
−234.4
−178.9
−123.3
−67.7

90	F	C
32.2		
87.7	1	0.5
143.3	2	1.1
198.8	3	1.6
254.4	4	2.2
310.0	5	2.7
365.5	6	3.3
421.1	7	3.8
476.6	8	4.4
532.2	9	5.0
587.7		
643.3		
698.8		
754.4		
810.0		
865.5		
921.1		
976.6		
1032.2		
1087.7		
1143.3		
1198.8		
1254.4		
1310.0		
1365.5		
1421.1		
1476.6		
1532.2		
1587.7		
1643.3		
1698.8		
1754.4		
1810.0		
1865.5		
1921.1		
90		

59.69
273.16

C

A.2.19 Liquid-Gas Conversion *

NORMAL-HYDROGEN - EQUILIBRIUM-HYDROGEN - HELIUM

		Pounds	Tons	SCF Gas	Gallons Liquid	Cu. Ft. Liquid	Liters Liquid
1 POUND	Normal-Hydrogen			192.0	1.686	0.2258	6.381
	Equilibrium-Hydrogen			192.0	1.693	0.2263	6.409
	Helium	1.0	0.0005	96.71	0.9593	0.1282	3.631
1 TON	Normal-Hydrogen			383,950.	3,379.	451.7	12,762.
	Equilibrium-Hydrogen			383,950.	3,386.	452.7	12,819.
	Helium	2000.0	1.0	193,424.	1,919.	256.5	7,262.
1 SCF GAS	Normal-Hydrogen	0.005209	0.000002605		0.008781	0.001176	0.03288
	Equilibrium-Hydrogen	0.005209	0.000002605	1.0	0.008820	0.001179	0.03339
	Helium	0.01034	0.000005170		0.009919	0.001326	0.03755
1 GAL. LIQUID	Normal-Hydrogen	0.5919	0.0002960	113.6			
	Equilibrium-Hydrogen	0.5906	0.0002953	113.4	1.0	0.133680	3.78533
	Helium	1.042	0.0005212	100.8			
1 CU. FT. LIQUID	Normal-Hydrogen	4.428	0.002214	850.1			
	Equilibrium-Hydrogen	4.418	0.002209	848.2	7.48052	1.0	28.3162
	Helium	7.798	0.003899	754.2			
1 LITER LIQUID	Normal-Hydrogen	0.1564	0.00007819	30.02			
	Equilibrium-Hydrogen	0.1560	0.00007801	29.99	0.264178	0.0353154	1.0
	Helium	0.2754	0.0001377	26.63			

Liquid quantities refer to conditions at 14.6960 psia pressure and the following temperatures:
Normal-Hydrogen: 20.39°K or —423.0°F [75% orthohydrogen, 25% parahydrogen (gas)]
Equilibrium-Hydrogen: 20.27°K or —423.2°F [0.21% orthohydrogen, 99.79% parahydrogen (liquid)]
Helium: 4.216°K or —452.1°F • SCF Gas meas. at 70°F @ 14.6960 psia
All values are consistent with standards adopted by the Compressed Gas Association on June 19, 1952

CARBON DIOXIDE

	Pounds	Tons	SCF Gas	Gallons Liquid	Cu. Ft. Liquid	Liters Liquid	Cu. Ft. Solid
1 Pound	1.0	0.0005	8.7291	0.1181	0.01578	0.4470	0.01025
1 Ton	2000.0	1.0	17,458.	236.1	31.57	894.05	20.5
1 SCF Gas	0.1146	0.0000573	1.0	0.01353	0.001809	0.05123	0.001175
1 Gal. Liquid	8.469	0.004235	73.93	1.0	0.133680	3.78533	0.08681
1 Cu. Ft. Liquid	63.36	0.03168	553.1	7.48052	1.0	28.3162	0.6494
1 Liter Liquid	2.237	0.0011185	19.53	0.264178	0.0353154	1.0	0.02293
1 Cu. Ft. Solid	97.56	0.04878	851.6	11.52	1.540	43.61	1.0

SCF Gas measured at 70°F. and 14.7 psia.
Liquid CO_2 measured at 1.7°F. and 314.7 psia.
Solid CO_2 measured at normal sublimation temperature of —109.4°F

* Tables A.2.19 and A.2.20 are reprinted by courtesy of Air Reduction Pacific Company, Vernon, California.

OXYGEN • NITROGEN • ARGON

		Pounds	Tons	SCF Gas	Gallons Liquid	Cu. Ft. Liquid	Liters Liquid
1 POUND	Oxygen			12.08	0.1050	0.01403	0.3973
	Nitrogen	1.0	0.0005	13.80	0.1481	0.01982	0.5612
	Argon			9.671	0.08600	0.01150	0.3255
1 TON	Oxygen			24,160.	209.9	28.06	794.6
	Nitrogen	2000.0	1.0	27,605.	296.5	39.64	1,122.3
	Argon			19,342.	172.0	22.99	651.1
1 SCF GAS	Oxygen	0.08281	0.00004141		0.008691	0.001162	0.03290
	Nitrogen	0.07245	0.00003623	1.0	0.01074	0.001436	0.04066
	Argon	0.1034	0.00005170		0.008893	0.001189	0.03366
1 GAL. LIQUID	Oxygen	9.527	0.004764	115.1			
	Nitrogen	6.745	0.003373	93.11	1.0	0.133680	3.78533
	Argon	11.63	0.005814	112.5			
1 CU. FT. LIQUID	Oxygen	71.27	0.03564	860.6			
	Nitrogen	50.46	0.02523	696.5	7.48052	1.0	28.3162
	Argon	86.98	0.04349	841.2			
1 LITER LIQUID	Oxygen	2.517	0.001259	30.38			
	Nitrogen	1.782	0.0008910	24.60	0.264178	0.0353154	1.0
	Argon	3.072	0.001535	29.71			

Liquid quantities refer to conditions at 14.6960 psia pressure and the following temperatures:
Oxygen: 90.19°K or —297.33°F • Nitrogen 77.395°K or —320.36°F • Argon: 87.29°K or —302.55°F
SCF Gas measured at 70F @ 14.6960 psia
All values are consistent with standards adopted by the Compressed Gas Association on June 19, 1962

A.2.20 Water Content in Gases

CONVERSION TABLE FOR MOISTURE CONTENT IN GASES

(at Standard Temperature and Pressure)

To Convert "B" to "A" Multiply by:	"A"	"B"	To Convert "A" to "B" Multiply by:
10^4	PPM (V/V)	volume %	10^{-4}
$(MW/1.8) \times 10^3$	"	weight %	$(1.8\ MW) \times 10^{-3}$
MW/18	"	PPM (W W)	(18 MW)
10^3	"	Ml liter	10^{-3}
1.25×10^3	"	Mg liter	8.04×10^{-4}
35.4	"	Ml cu.ft.	2.83×10^{-2}
43.8	"	Mg/cu.ft.	2.28×10^{-2}
2.86×10^3	"	Grains/cu.ft.	3.50×10^{-4}
$(MW/1.8) \times 10^2$	"	Mg/gram	$(1.8\ MW) \times 10^{-2}$
$(MW/8.2) \times 10^3$	"	Gram/pound	$(8.2\ MW) \times 10^{-3}$
$(MW\ 1.26) \times 10^1$	"	Grain/pound	$(1.26\ MW) \times 10^{-1}$
$(MW/1.8) \times 10^5$	"	Pound/pound	$(1.8\ MW) \times 10^{-5}$
20	"	Pound/MMCF	5×10^{-2}

Note MW — Molecular Weight of the gas involved.
PPM (V V) — Parts Per Million on a volume basis
PPM (W W) — Parts Per Million on a weight basis

TABLE OF MOLECULAR WEIGHTS

Acetylene	26.036	Neon	20.183
Argon	39.944	Nitrogen	28.016
Carbon Dioxide	44.01	Nitrous Oxide	44.02
Helium	4.00?	Oxygen	32.000
Hydrogen	2.016	Xenon	131.3
Krypton	83.70		

MW — Molecular Weight of the gas involved.
PPM (V/V) — Parts Per Million on a volume basis
PPM (W/W) — Parts Per Million on a weight basis

MOISTURE CONTENT IN GASES
DEW POINT VERSUS ppm

D.P.	ppm	D.P.	ppm	D.P.	ppm
—130°F	0.1	—73°F	13.3	—38°F	14
—120	0.25	—72	14.3	—37	15
—110	0.63	—71	15.4	—36	16
—105	1.00	—70	16.6	—35	17
—104	1.08	—69	17.9	—34	18
—103	1.18	—68	19.2	—33	19
—102	1.29	—67	20.6	—32	21
—101	1.40	—66	22.1	—31	22
—100	1.53	—65	23.6	—30	23
— 99	1.66	—64	25.6	—29	25
— 98	1.81	—63	27.5	—28	26
— 97	1.96	—62	29.4	—27	28
— 96	2.15	—61	31.7	—26	30
— 95	2.35	—60	34.0	—25	3'
— 94	2.54	—59	36.5	—24	33
— 93	2.76	—58	39.0	—23	35
— 92	3.00	—57	41.8	—22	37
— 91	3.28	—56	44.3	—21	40
— 90	3.53	—55	48.0	—20	42
— 89	3.84	—54	51	—19	44
— 88	4.15	—53	55	—18	47
— 87	4.50	—52	59	—17	50
— 86	4.78	—51	62	—16	53
— 85	5.3	—50	67	—15	56
— 84	5.7	—49	72	—14	59
— 83	6.2	—48	76	—13	63
— 82	6.6	—47	82	—12	66
— 81	7.2	—46	87	—11	70
— 80	7.8	—45	92	—10	74
— 79	8.4	—44	98	— 9	78
— 78	9.1	—43	105	— 8	92
— 77	9.8	—42	113	— 7	87
— 76	10.5	—41	119	— 6	92
— 75	11.4	—40	128	— 5	97
— 74	12.3	—39	136	— 4	102

A.2.21 Leakage Flow (ML^4t^{-3}) or (FLt^{-1})

(Note: This leakage flow is in pressure x volume/time units.
For Standard Volumetric Leakage (SCIM, etc.) use Table A.2.22).

Multiply → by ↘ ↓ To Obtain	Atm ft^3 per min	Atm cc per sec	Atm in^3 per min	Micron ft^3 per hr (mfh)	Micron liter per sec (Lusec)	Torr liter per sec
Atm ft^3 per min	1.	2.110×10^{-3}	5.79×10^{-4}	2.19×10^{-8}	2.79×10^{-6}	2.79×10^{-9}
Atm cc per sec	4.72×10^{2}	1.	0.273	1.04×10^{-5}	1.32×10^{-3}	1.32
Atm in^3 per min	1728	3.66	1.	3.78×10^{-5}	4.80×10^{-3}	4.80×10^{-6}
Micron ft^3 per hr (mfh)	4.56×10^{7}	9.67×10^{4}	2.64×10^{4}	1.	127	0.127
Micron liter per sec (Lusec)	3.59×10^{5}	760	208	7.87×10^{-3}	1.	10^{-3}
Torr liter per sec	3.59×10^{8}	.760	2.08×10^{5}	7.87	1000	1.

A.2.22 Volumetric Flow Rate (L^3t^{-1})

Multiply → by ↘ ↓ To Obtain	Cubic Feet per Second (SCFS)	Cubic Feet per Minute (SCFM)	Cubic Feet per Hour (SCFH)	Cubic Inches per Second (SCIS)	Cubic Centimeters per Second (SCCS)	Cubic Inches per Minute (SCIM)	Liters per Minute (LPM)	U.S. Gallons per Minute (GPM)	Imperial Gallons per Minute (IGPM)
Cubic Feet per Second (SCFS)	1.	0.01667	2.778×10^{-4}	5.787×10^{-4}	3.531×10^{-5}	9.64×10^{-6}	5.886×10^{-4}	2.228×10^{-3}	2.676×10^{-3}
Cubic Feet per Minute (SCFM)	60.	1.	0.01667	0.03472	2.119×10^{-3}	5.79×10^{-4}	0.03531	0.1337	0.1605
Cubic Feet per Hour (SCFH)	3600.	60.	1.	2.083	0.1271	0.03472	2.119	8.021	9.632
Cubic Inches per Second (SCIS)	1728.	28.80	0.4800	1.	0.06102	0.01667	1.0171	3.850	4.624
Cubic Centimeters per Second (1) (SCCS)	28,317.	471.9	7.866	16.387	1.	0.273	16.667	63.08	75.77
Cubic Inches per Minute (SCIM)	103,680.	1728.	28.80	60.	3.667	1.	61.02	231.0	277.8
Liters per Minute (LPM)	1699.	28.32	0.4720	0.9832	0.0600	0.0164	1.	3.785	4.546
U.S. Gallons per Minute (GPM)	448.8	7.4805	0.1247	0.2597	0.01585	4.33×10^{-3}	0.2642	1.	1.2009
Imperial Gallons per Minute (IGPM)	373.74	6.229	0.1038	0.2163	0.01320	3.605×10^{-3}	0.21993	0.8327	1.

(1) To convert cc per sec to cc per hour, day, month, year, etc., apply time conversion from Table A.2.5

A.2.23 Permeability ($L^4F^{-1}t^{-1}$) (Volume·Thickness/Area·Time·ΔPressure)

Reference 152-12 ("Permeability Data for Aerospace Applications," Illinois Institute of Technology Research Institute, Chicago, Contract No. NAS7-388, IITRI Project C6070, March 1968) presents a useful compilation of permeability data and arbitrarily adopts the following unit system as a "standard"

$$\text{Unit}_p \quad \frac{\text{cc (S.T.P.) cm}}{\text{cm}^2\ \text{sec Bar}} \quad \frac{\text{Volume Thickness}}{\text{Area-Time-}\Delta\text{Pressure}}$$

This is the volume of permeant in cubic centimeters at standard temperature and pressure per square centimeter of area per second per Bar Δp per centimeter thickness of membrane. The abbreviation scc is used for cc (S.T.P.).

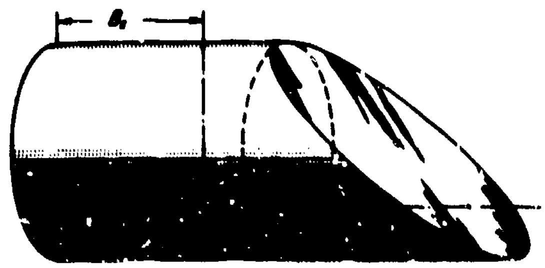
This unit system is comprised solely of cgs units. Any of the systems in use could have been used as a standard. However, this system was selected because it is self consistent.

Table A.2.23 presents conversion factors to other unit systems. Some unit systems are not convertible, e.g., metal permeability is frequently reported in units inversely proportional to the square root of the Δ-pressure. These systems are listed in the table, with the comment 'Not Convertible.'

Table A.2.23.

Units System	Multiplication Factor for Converting to "Standard" Units	Units System	Multiplication Factor for Converting to "Standard" Units
$\frac{\text{scc cm}}{\text{cm}^2\ \text{sec cm Hg}}$	7.501×10^{1}	$\frac{\text{scc mm}}{\text{cm}^2\ \text{hr atm}^{1/2}}$	Not convertible
$\frac{\text{scc cm}}{\text{cm}^2\ \text{sec mm Hg}}$	7.501×10^{2}	$\frac{\text{scc mm}}{\text{cm}^2\ \text{min atm}^{1/2}}$	Not convertible
$\frac{\text{scc mm}}{\text{cm}^2\ \text{sec cm Hg}}$	7.501	$\frac{\text{scc mil}}{100\ \text{in.}^2\ \text{day 17.3 psi}}$	3.82×10^{-9}
$\frac{\text{scc mm}}{\text{m}^2\ \text{day atm}}$	1.142×10^{10}	$\frac{\text{mg mil}}{\text{in.}^2\ \text{day atm}}$	Not convertible
$\frac{10^5\ \text{ft}^3\ \text{mil}}{\text{min ft}^2\ \text{atm}}$	1.273×10^{8}	$\frac{\text{mg mil}}{\text{in.}^2\ \text{hr atm}}$	Not convertible
$\frac{\text{scc mm}}{\text{cm}^2\ \text{sec atm}}$	9.8692×10^{-2}	$\frac{\text{mg mil}}{\text{in.}^2\ \text{hr}}$	Not convertible
$\frac{\text{scc mil}}{100\ \text{in.}^2\ \text{day atm}}$	1.197×10^{-11}	$\frac{\text{scc mil}}{100\ \text{m}^2\ \text{day 17.7 psi}}$	2.408×10^{-14}
$\frac{\text{scc cm}}{\text{cm}^2\ \text{sec atm}}$	9.8692×10^{-1}	$\frac{\text{mg}}{\text{cm}^2\ \text{hr}}$	Not convertible
$\frac{\text{scc mil}}{\text{cm}^2\ \text{day atm}}$	2.901×10^{-8}	$\frac{\text{mg}}{\text{in.}^2\ \text{hr}}$	Not convertible
$\frac{\text{scc mil}}{\text{in.}^2\ \text{hr lb in.}^{-2}}$	1.023×10^{5}	$\frac{\text{scc mil}}{\text{in.}^2\ \text{day atm}}$	2.901×10^{-12}
$\frac{\text{liter mm}}{\text{cm}^2\ \text{sec mm Hg}}$	7.501×10^{4}	$\frac{\text{gm mil}}{100\ \text{in.}^2\ \text{day atm}}$	Not convertible
$\frac{\text{scc mm}}{\text{cm}^2\ \text{hr atm}}$	$2.7?1 \times 10^{-5}$	$\frac{\text{scc}}{\text{hr in.}^2}$	Not convertible
$\frac{\text{scc mm}}{\text{cm}^2\ \text{sec atm}^{1/2}}$	Not convertible	$\frac{\text{lb mil}}{100\ \text{in.}^2\ \text{day atm}}$	Not convertible
$\frac{\text{lb}}{\text{in.}^2\ \text{hr psi}}$	Not convertible	$\frac{\text{scc mil}}{\text{cm}^2\ \text{sec atm}^{1/2}}$	Not convertible
$\frac{\text{lb}}{\text{in.}^2\ \text{hr}}$	Not convertible	$\frac{\text{scc mil}}{\text{cm}^2\ \text{hr atm}^{1/2}}$	Not convertible

A.3 VOLUME AND CENTER OF GRAVITY EQUATIONS*



VOLUME equations are included for all cases. Where the equation for the CG (center of gravity) is not given, you can easily obtain it by looking up the volume and CG equations for portions of the shape and then combining values. For example, for the shape above, use the equations for a cylinder, Fig 1, and a truncated cylinder, Fig 10 (subscripts C and T, respectively, in the equations below). Hence taking moments

$$B_s = \frac{V_C B_C + V_T(B_T + L_C)}{V_C + V_T}$$

or

$$B_s = \frac{\left(\frac{\pi}{4} D^2 L_C\right)\left(\frac{L_C}{2}\right) + \frac{\pi}{8} D^2 L_T\left(\frac{3}{16} L_T + L_C\right)}{\frac{\pi}{4} D^2 L_C + \frac{\pi}{8} D^2 L_T}$$

$$B_s = \frac{L_C^2 + L_T\left(\frac{5}{16} L_T + L_C\right)}{2L_C + L_T}$$

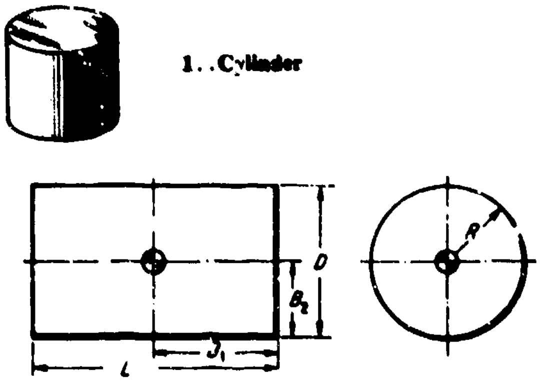
In the equations to follow, angle θ can be either in degrees or in radians. Thus θ (rad) $= \pi\theta/180$ (deg) $= 0.01745\,\theta$ (deg). For example, if $\theta = 30$ deg in Case 3, then $\sin\theta = 0.5$ and

$$B = \frac{2R\,(0.5)}{3\,(30)\,(0.01745)} = 0.637R$$

Symbols used are: B = distance from CG to reference plane, V = volume, D and d = diameter, R and r = radius, H = height, L = length.—*Nicholas P. Chironis*

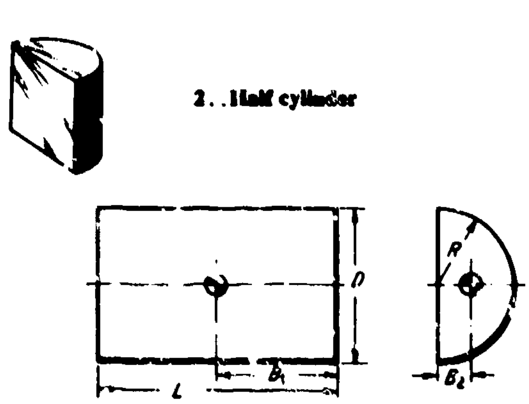
CYLINDERS

1. Cylinder



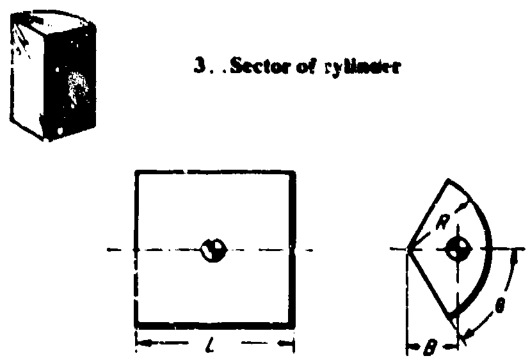
$V = \frac{\pi}{4} D^2 L = 0.7854 D^2 L$ $B_1 = L/2$ $B_2 = R$

2. Half cylinder



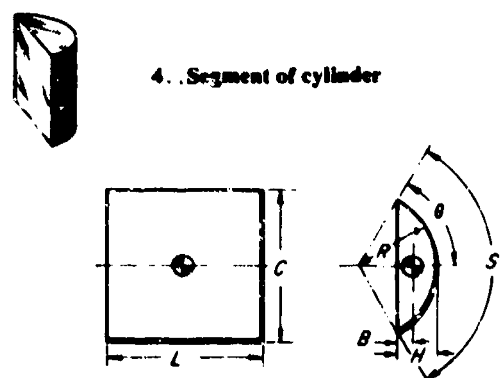
$V = \frac{\pi}{8} D^2 L = 0.3927 D^2 L$ $B_1 = L/2$ $B_2 = \frac{4R}{3\pi} = 0.4244R$

3. Sector of cylinder



$V = \theta R^2 L$ $B = \frac{2R\sin\theta}{3\theta}$

4. Segment of cylinder



$V = LR^2\left(\theta - \frac{1}{2}\sin 2\theta\right)$

$V = 0.5L\,[RS - C(R - H)]$

$B = \frac{4R\sin^3\theta}{6\theta - 3\sin 2\theta}$

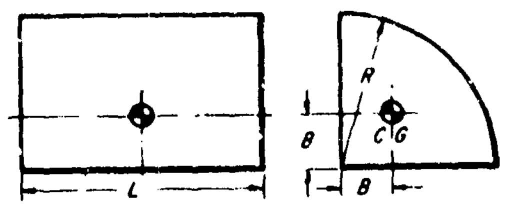
$S = 2R\theta$

$H = R(1 - \cos\theta)$

$C = 2R\sin\theta$

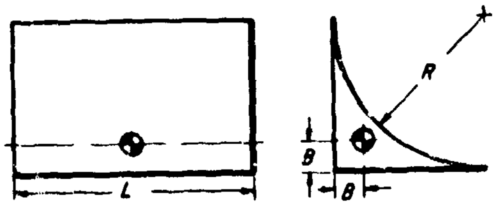
*Volume and C. G. Equations are reprinted with permission from *Product Engineering*, March 18, 1963, F. W. Jenkins,

5. Quadrant of cylinder



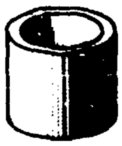
$V = \frac{\pi}{4} R^2 L = 0.7854R^2L \qquad B = \frac{4R}{3\pi} = 0.4244R$

6. Fillet or spandrel

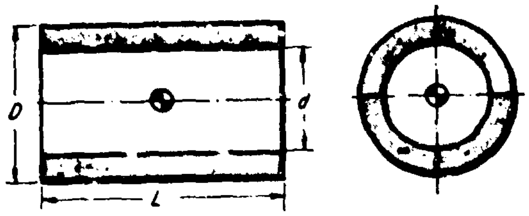


$$V = \left(1 - \frac{\pi}{4}\right) R^2 L = 0.2146R^2L$$

$$B = \frac{10 - 3\pi}{12 - 3\pi} R = 0.2234R$$

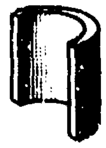


7. Hollow cylinder

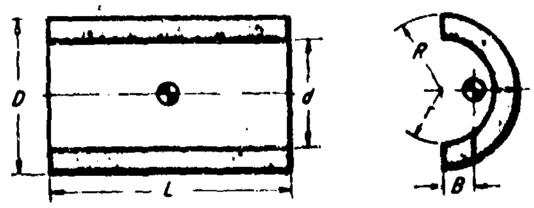


$$V = \frac{\pi L}{4} (D^2 - d^2)$$

CG at center of part



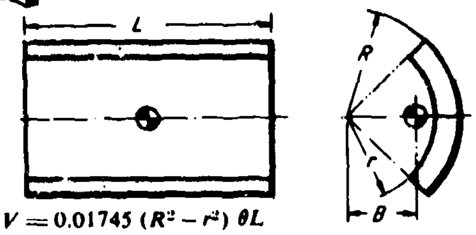
8. Half hollow cylinder



$$V = \frac{\pi L}{8} (D^2 - d^2)$$

$$B = \frac{4}{3\pi}\left[\frac{R^3 - r^3}{R^2 - r^2}\right]$$

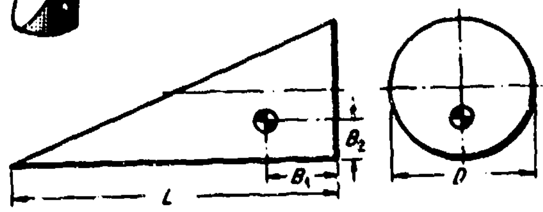
9. Sector of hollow cylinder



$$V = 0.01745\,(R^2 - r^2)\,\theta L$$

$$B = \frac{38.1972\,(R^3 - r^3)\sin\theta}{(R^2 - r^2)\,\theta}$$

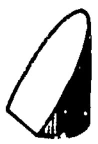
10. Truncated cylinder (with full circle base)



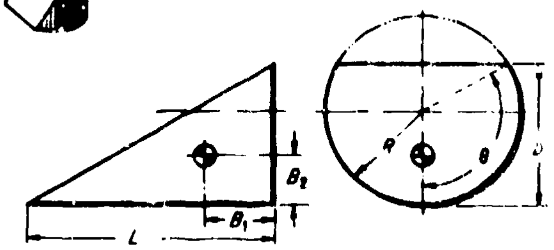
$$V = \frac{\pi}{8} D^2 L = 0.3927D^2L$$

$B_1 = 0.3125L$

$B_2 = 0.375D$



11. Truncated cylinder
(with partial circle base)



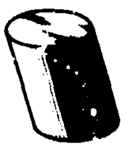
$$h = R\,(1-\cos\theta)$$

$$V = \frac{R^3L}{h}\left[\sin\theta - \frac{\sin^3\theta}{3} - \theta\cos\theta\right]$$

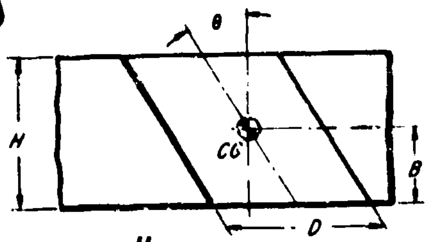
$$B_1 = \frac{L\left[\frac{\theta\cos^2\theta}{2} - \frac{5\sin\theta\cos\theta}{8} + \frac{\sin^3\theta\cos\theta}{12} + \frac{\theta}{8}\right]}{\left[1-\cos\theta\right]\left[\sin\theta - \frac{\sin^3\theta}{3} - \theta\cos\theta\right]}$$

$$B_2 = \frac{2R\left[-\frac{\theta\cos\theta}{2} + \frac{\sin\theta}{2} - \frac{\theta}{8} + \frac{\sin\theta\cos\theta}{8} - N\right]}{\left[\sin\theta - \frac{\sin^3\theta}{3} - \theta\cos\theta\right]}$$

where $N = \frac{5\sin^3\theta}{6} - \frac{\sin^3\theta\cos\theta}{12}$



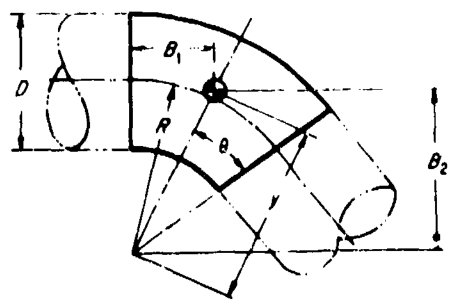
12. Oblique cylinder
(or circular hole at oblique angle)



$$V = \frac{\pi}{4}D^2\frac{H}{\cos\theta} = 0.7854D^2H\sec\theta$$

$B = H/2 \quad r = \frac{d}{2}$

13. Bend in cylinder

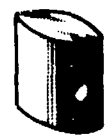


$$V = \frac{\pi^2}{360}D^2R\theta = 0.0274D^2R\theta$$

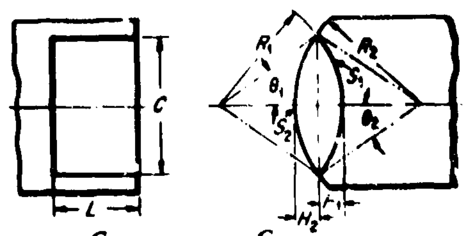
$$y = R\left[1 + \frac{r^2}{4R^2}\right]$$

$B_1 = y\tan\theta$

$B_2 = y\cot\theta$



14. Curved groove in cylinder



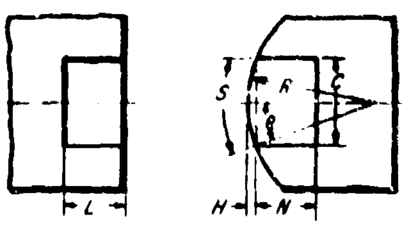
$\sin\theta_1 = \frac{C}{2R_1} \quad \sin\theta_2 = \frac{C}{2R_2} \quad S = 2R\theta$

$H_1 = R_1\,(1-\cos\theta_1) \quad H_2 = R_2\,(1-\cos\theta_2)$

$$V = L\left[R_1^2\left(\theta_1 - \frac{1}{2}\theta_1\sin 2\theta_1\right) + R_2^2\left(\theta_2 - \frac{1}{2}\theta_2\sin 2\theta_2\right)\right]$$

Compute CG of each part separately

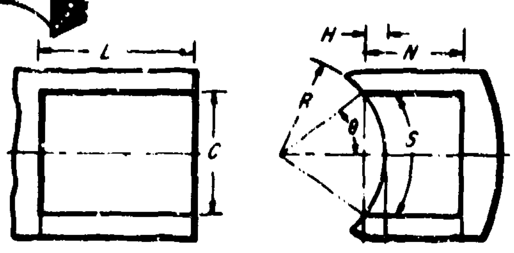
15. . Slot in cylinder



$H = R(1 - \cos\theta)$ $\sin\theta = \frac{C}{2R}$ $S = 2R\theta$

$V = L\left[CN + R^2\left(\theta - \frac{1}{2}\sin 2\theta\right)\right]$

16. . Slot in hollow cylinder

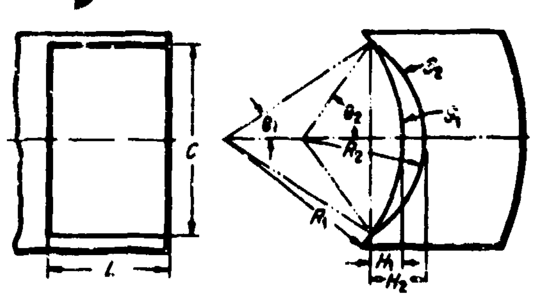


$S = 2R\theta$ $\sin\theta = \frac{C}{2R}$ $H = R(1 - \cos\theta)$

$V = L\left[CN - R^2\left(\theta - \frac{1}{2}\sin 2\theta\right)\right]$

$V = L\left(CN - 0.5\left[RS - C(R - H)\right]\right)$

17. . Curved groove in hollow cylinder

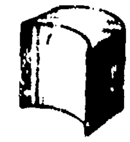


$\sin\theta_1 = \frac{C}{2R_1}$ $\sin\theta_2 = \frac{C}{2R_2}$ $S = 2R\theta$

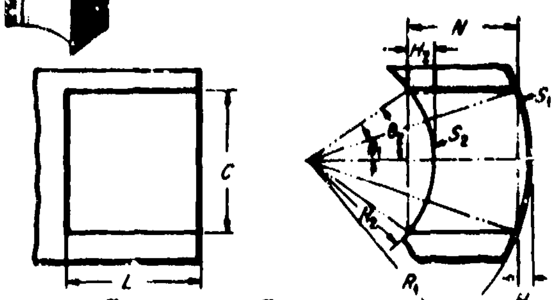
$H_1 = R_1(1 - \cos\theta_1)$ $H_2 = R_2(1 - \cos\theta_2)$

$V = L\left(\left[R_2^2\left(\theta_2 - \frac{1}{2}\sin 2\theta_2\right)\right] - \left[R_1^2\left(\theta_1 - \frac{1}{2}\sin 2\theta_1\right)\right]\right)$

$V = \frac{L}{2}\left(\left[R_2S_2 - C(R_2 - H_2)\right] - \left[R_1S_1 - C(R_1 - H_1)\right]\right)$



18. . Slot through hollow cylinder

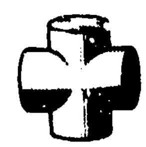


$\sin\theta_1 = \frac{C}{R_1}$ $\sin\theta_2 = \frac{C}{R_2}$ $S = 2R\theta$

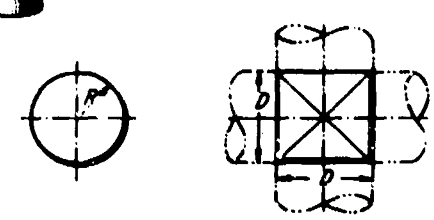
$H_1 = R_1(1 - \cos\theta_1)$ $H_2 = R_2(1 - \cos\theta_2)$

$V = L\left(CN + \left[R_1^2\left(\theta_1 - \frac{1}{2}\sin 2\theta_1\right)\right] - \left[R_2\left(\theta_2 - \frac{1}{2}\sin\theta_2\right)\right]\right)$

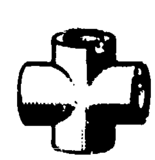
$V = L\left(CN + 0.5\left[R_1S_1 - C(R_1 - H_1)\right] - 0.5\left[R_2S_2 - C(R_2 - H_2)\right]\right)$



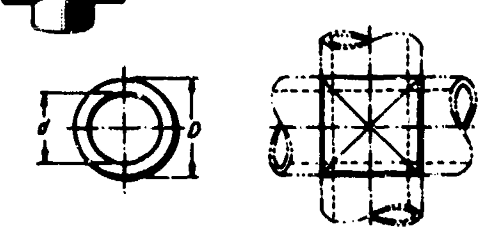
19. . Intersecting cylinder (volume of junction box)



$V = D^3\left(\frac{\pi}{2} - \frac{2}{3}\right) = 0.9041D^3$



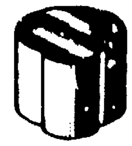
20. . Intersecting hollow cylinders (volume of junction box)

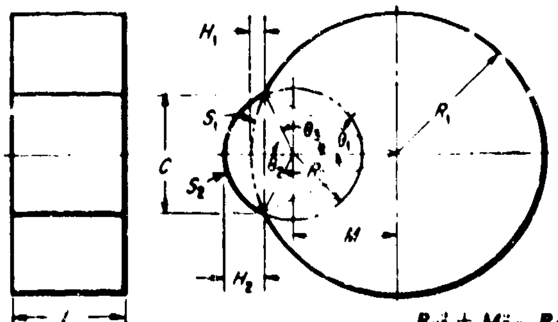


$V = \left(\frac{\pi}{2} - \frac{2}{3}\right)(D^3 - d^3) - \frac{\pi}{2}d^2(D - d)$

$V = 0.9041(D^3 - d^3) - 1.5708\,d^2(D - d)$

21. . Intersecting parallel cylinders
($M < R_1$)



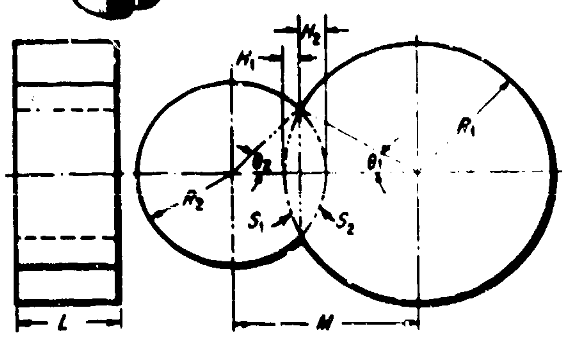


$\theta_2 = 180° - \theta_3 \quad \cos\theta_3 = \dfrac{R_2^2 + M^2 - R_1^2}{2MR_2}$

$\cos\theta_1 = \dfrac{R_1^2 + M^2 - R_2^2}{2MR_1} \qquad H_1 = R_1(1 - \cos\theta_1)$
$S_1 = 2R_1\theta$

$V = L\left(R_1^2 + \left[R_2^2\left(\theta_2 - \frac{1}{2}\sin 2\theta_2\right]\left[R_1^2\left(\theta_1 - \frac{1}{2}\sin 2\theta_1\right]\right)\right.\right.$

22. . Intersecting parallel cylinders
($M > R_1$)

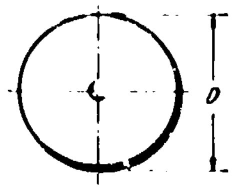


$H_1 = R_1(1 - \cos\theta_1) \qquad \cos\theta_1 = \dfrac{R_1^2 + M^2 - R_2^2}{2MR_1}$
$S_1 = 2R_1\theta_1$

$$V = L\left(\left[\pi(R_1^2 + R_2^2)\right] - \left[R_1^2\left(\theta_1 - \frac{1}{2}\sin 2\theta_1\right)\right] - \left[R_2^2\left(\theta_2 - \frac{1}{2}\sin 2\theta_2\right)\right]\right)$$

SPHERES

23. . Sphere

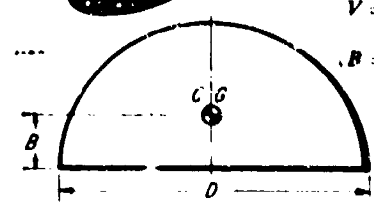


$V = \dfrac{\pi D^3}{6} = 0.5236D^3$

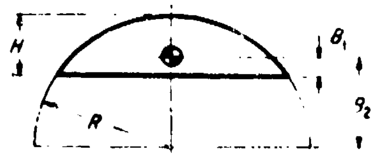
24. . Hemisphere

$V = \dfrac{\pi D^3}{12} = 0.2618D^3$

$B = 0.375R$



25. . Spherical segment

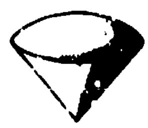


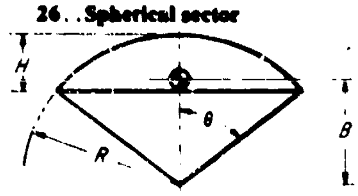
$V = \pi H^2\left(R - \dfrac{H}{3}\right)$

$B_1 = \dfrac{H(4R - H)}{4(3R - H)}$

$B_2 = \dfrac{3(2R - H)^2}{4(3R - H)}$

26. . Spherical sector

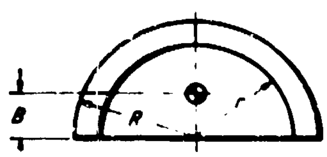




$V = \dfrac{2\pi}{3}R^2H = 2.0944R^2H$

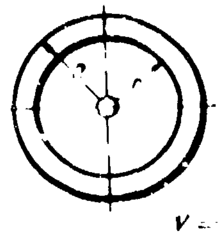
$B = 0.375(1 + \cos\theta)R = 0.375(2R - H)$

27. . Shell of hollow hemisphere



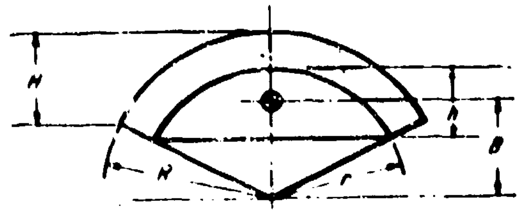
$V = \dfrac{2\pi}{3}(R^3 - r^3) \qquad B = 0.375\left(\dfrac{R^4 - r^4}{R^3 - r^3}\right)$

28. . Hollow sphere



$$V = \frac{4\pi}{3}(R^3 - r^3)$$

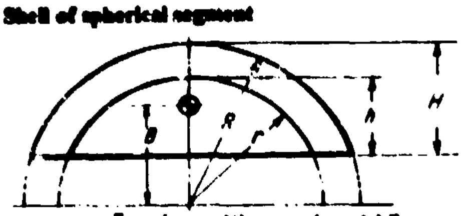
29. . Shell of spherical sector



$$V = \frac{2\pi}{3}(R^2H - r^2h)$$

$$B = 0.375\left\{\frac{[R^2H\,(2R-H)] - [r^2h\,(2r-h)]}{R^2H - r^2h}\right\}$$

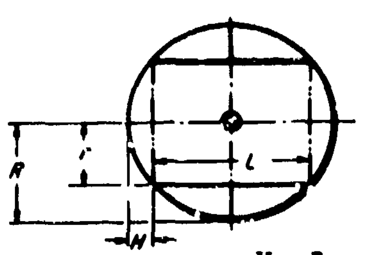
30. . Shell of spherical segment



$$V = \pi\left[H^2\left(R - \frac{H}{3}\right) - h^2\left(r - \frac{h}{3}\right)\right]$$

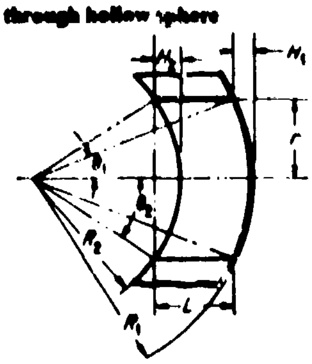
$$B = \frac{3}{4}\left[\frac{\left(R - \frac{H}{3}\right)\frac{H^2\,(2R-H)^2}{3R-H} - \left(r - \frac{h}{3}\right)\frac{h^2\,(2r-h)^2}{3r-h}}{H^2\left(R - \frac{H}{3}\right) - h^2\left(r - \frac{h}{3}\right)}\right]$$

31. . Circular hole through sphere



$$V = \pi\left[r^2L + 2H^2\left(R - \frac{H}{3}\right)\right] \qquad H = R - \sqrt{R^2 - r^2} \qquad L = 2(R - H)$$

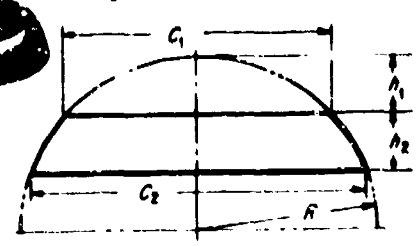
32. . Circular hole through hollow sphere



$$V = \pi\left\{r^2L + H_1^2\left(R_1 - \frac{H_1}{3}\right) - H_2^2\left(R_2 - \frac{H_2}{3}\right)\right\}$$

$\sin\theta_1 = r/R_1 \qquad \sin\theta_2 = r/R_2 \qquad H = R\,(1 - \cos\theta)$

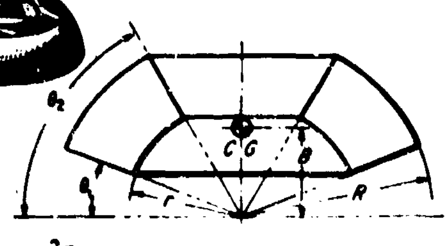
33. . Spherical zone



$$V = \pi\left\{\left[H^2\left(R - \frac{H}{3}\right)\right] - \left[h_1^2\left(R - \frac{h}{3}\right)\right]\right\}$$

$$V = \frac{\pi h_2}{6}\left[\frac{3}{4}C_1^2 + \frac{3}{4}C_2^2 + h_2^2\right]$$

34. . Conical hole through spherical shell

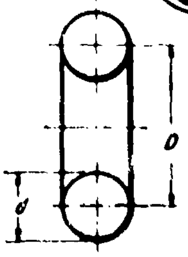


$$V = \frac{2\pi}{3}(R^3 - r^3)(\sin\theta_2 - \sin\theta_1)$$

$$B = \frac{0.375\,(R^4 - r^4)\,(\sin\theta_2 + \sin\theta_1)}{R^3 - r^3}$$

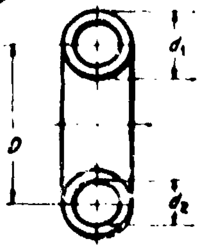
RINGS

35. Torus



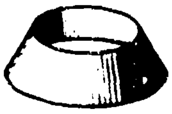
$$V = \frac{1}{4}\pi^2 d^2 D = 2.467 d^2 D$$

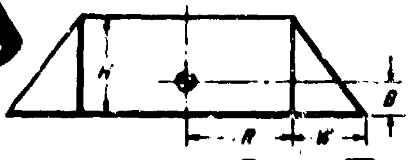
36. Hollow torus



$$V = \frac{1}{4}\pi^2 D (d_1^2 - d_2^2)$$

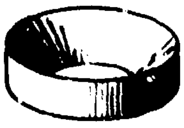
37. Bevel ring

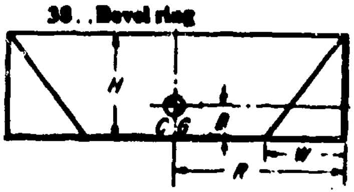




$$V = \pi\left(R + \frac{1}{3}W\right) WH \qquad B = H\left[\frac{\frac{R}{3} + \frac{W}{12}}{R + \frac{W}{3}}\right]$$

38. Bevel ring

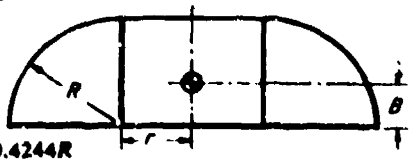




$$B > \frac{H}{3}$$

$$V = \pi\left(R - \frac{1}{3}W\right) WH \qquad B = H\left[\frac{\frac{R}{3} - \frac{W}{12}}{R - \frac{W}{3}}\right]$$

39. Quarter torus

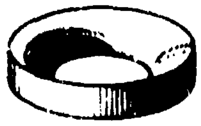


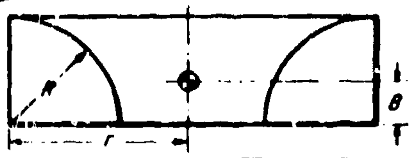
$$B < 0.4244R$$

$$V = \frac{\pi^2 R^2}{2}\left(r + \frac{4R}{3\pi}\right) = 4.9348R^2 (r + 0.4244R)$$

$$B = \frac{4R}{3\pi}\left[\frac{r + \frac{3R}{8}}{r + \frac{4R}{3\pi}}\right] = \frac{0.4244Rr + 0.1592R^2}{r + 0.4244R}$$

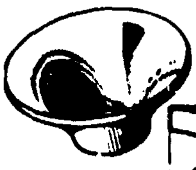
40. Quarter torus

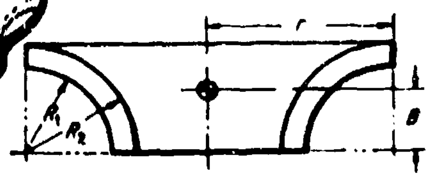




$$V = \frac{\pi^2 R^2}{2}\left[r - \frac{4R}{3\pi}\right] \qquad B = \frac{4R}{3\pi}\left[\frac{r - \frac{3R}{8}}{r - \frac{4R}{3\pi}}\right]$$

41. Curved shell ring

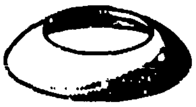


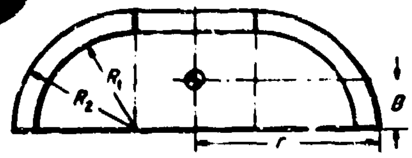


$$V = 2\pi\left\{r - \frac{4}{3\pi}\left[\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right]\right\}\frac{\pi}{4}(R_2^2 - R_1^2)$$

$$B = \frac{4}{3\pi}\left[\frac{R_2^3\left(r - \frac{3}{8}R_2\right) - R_1^3\left(r - \frac{3}{8}R_1\right)}{(R_2^2 - R_1^2)\left\{r - \frac{4}{3\pi}\left[\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right]\right\}}\right]$$

42. Curved shell ring

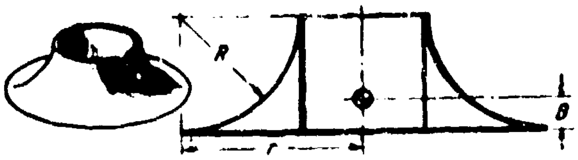




$$V = \frac{\pi^2}{2}\left[r(R_2^2 - R_1^2) + \frac{4}{3\pi}(R_2^3 - R_1^3)\right]$$

$$B = \frac{2}{\pi}\left[\frac{\frac{2r}{3}(R_2^3 - R_1^3) + \frac{1}{4}(R_2^4 - R_1^4)}{r(R_2^2 - R_1^2) + \frac{4}{3\pi}(R_2^3 - R_1^3)}\right]$$

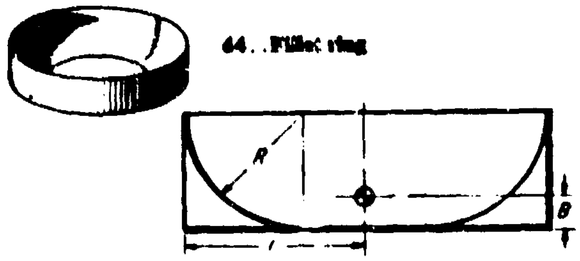
43. . Fillet ring



$$V = 2\pi R^2\left[\left(1-\frac{\pi}{4}\right)r-\frac{R}{6}\right]$$

$$B = R\left[\frac{\left(\frac{5}{6}-\frac{\pi}{4}\right)r-\frac{R}{24}}{\left(1-\frac{\pi}{4}\right)r-\frac{R}{5}}\right]$$

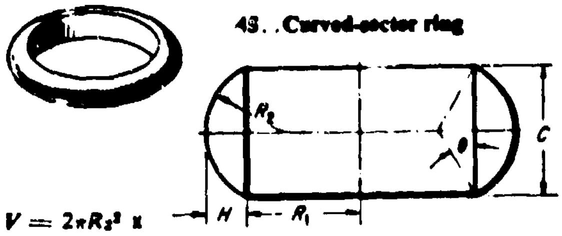
44. . Fillet ring



$$V = 2\pi R^2\left[\left(1-\frac{\pi}{4}\right)r-\left(\frac{5}{6}-\frac{\pi}{4}\right)R\right]$$

$$B = R\left[\frac{\left(\frac{5}{6}-\frac{\pi}{4}\right)r-\left(\frac{19}{24}-\frac{\pi}{4}\right)R}{\left(1-\frac{\pi}{4}\right)r-\left(\frac{5}{6}-\frac{\pi}{4}\right)R}\right]$$

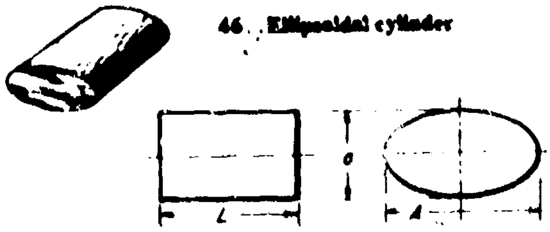
45. . Curved-sector ring



$$V = 2\pi R_2^3 \times \left[R_1+\left(\frac{4\sin 3\theta}{6\theta-3\sin 2\theta}-\cos\theta\right)R_2\right]\left[\theta-0.5\sin 2\theta\right]$$

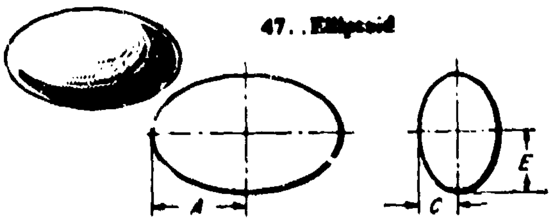
MISCELLANEOUS

46. . Ellipsoidal cylinder



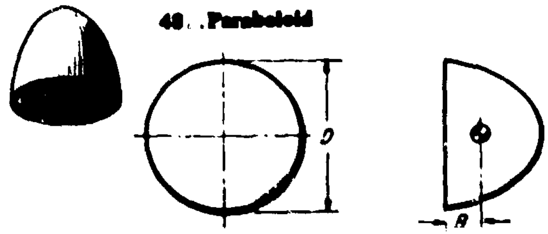
$$V = \frac{\pi}{4}AaL$$

47. . Ellipsoid



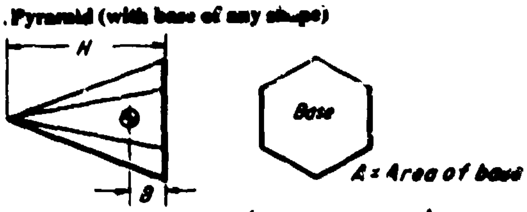
$$V = \frac{4}{3}\pi ACE$$

48. . Paraboloid



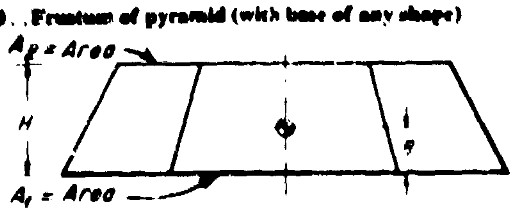
$$V = \frac{\pi}{8}HD^2 \qquad B = \frac{1}{3}H$$

49. . Pyramid (with base of any shape)



$$V = \frac{1}{3}AH \qquad B = \frac{1}{4}H$$

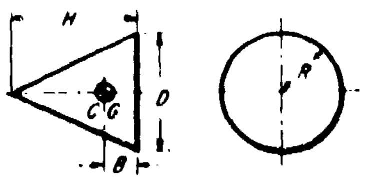
50. . Frustum of pyramid (with base of any shape)



$$V = \frac{1}{3} H (A_1 + \sqrt{A_1 A_2} + A_2)$$

$$B = \frac{H (A_1 + 2\sqrt{A_1 A_2} + 3A_2)}{4 (A_1 + \sqrt{A_1 A_2} + A_2)}$$

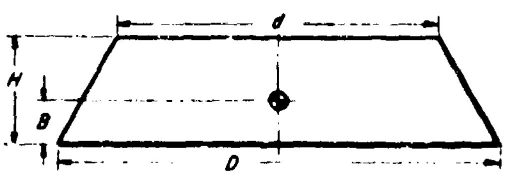
51. . Cone



$$V = \frac{\pi}{12} D^2 H$$

$$B = \frac{1}{4} H$$

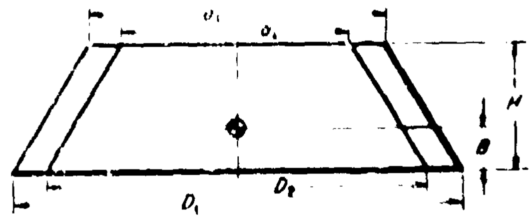
52. . Frustum of cone



$$V = \frac{\pi}{12} H (D^2 + Dd + d^2)$$

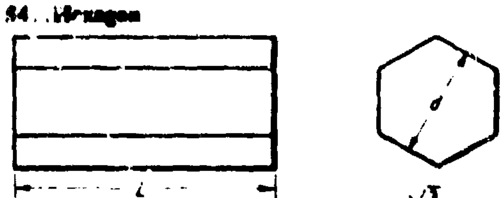
$$B = \frac{H (D^2 + 2Dd + 3d^2)}{4 (D^2 + Dd + d^2)}$$

53. . Frustum of hollow cone



$$V = 0.2618H [(D_1^2 + D_1 d_1 + d_1^2) - (D_2^2 + D_2 d_2 + d_2^2)]$$

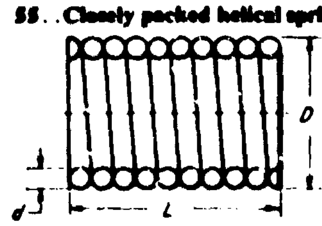
54. . Hexagon



$$V = \frac{\sqrt{3}}{2} d^2 L$$

$$V = 0.866 d^2 L$$

55. . Closely packed helical springs



$$V = \frac{\pi^2 dL}{4} (D - d)$$

$$V = 2.4674 (D - d)$$

A.4 [illegible]

In the tables that follow there are equations for the determination of section properties for plane areas. Equations for areas, centroidal distances, area moments of inertia and radii of gyration for use in stress calculations are provided. Equations for weight moments of inertia are also provided in the tables.

An introductory section contains basic definitions, nomenclature, an example of the derivation of equations for calculating typical moments of inertia and centroidal distances through the use of calculus. The derivations illustrate the method for determining information which is not included in the tables.

The introductory text also presents the method for transferring a moment of inertia for an area from its centroidal axis to any parallel non-centroidal axis in the same plane.

Sectional data for certain common structural shapes such as channels, I-beams, angles and T sections are omitted because these are normally found in a construction manual such as the AISC steel construction manual.

This entire subsection, Appendix A.4, has been adapted directly from Reference [illegible], Handbook of Equations for Mass and Area Properties of Various Geometrical Shapes, NAVWEPS Report [illegible], NOTS TP [illegible], [illegible] A. Myers, April 1962 [illegible]. Most of the tabulated data were in turn adapted from Weight Handbook, Volume I, Society of Weight Engineers, Inc.

NOMENCLATURE

An effort has been made to typify symbols for thickness, height and base width for all plane areas considered. However, in some instances, additional nomenclature has been introduced for descriptive purposes. It is advisable to refer to the diagram associated with each item.

- A — Area, in
- I — Moment of inertia. The unit for a plane area moment of inertia is in^4. The unit for a weight moment of inertia is lb_f in^2.
- I_z — Polar moment of inertia
- I_x — Moment of inertia about the x axis
- I_y — Moment of inertia about the y axis
- I_z — Moment of inertia about the z axis
- $\bar{I}_x$ — Moment of inertia about the centroidal x-axis
- $\bar{I}_y$ — Moment of inertia about the centroidal y-axis
- $\bar{I}_z$ — Moment of inertia about the centroidal z-axis
- I_{xy} — Product of inertia in the x-y plane
- I_{xz} — Product of inertia in the x-z plane
- I_{yz} — Product of inertia in the y-z plane
- k_x — Radius of gyration about the x axis
- k_y — Radius of gyration about the y axis
- k_z — Radius of gyration about the z-axis
- l — Length, in
- $\bar{x}$ — Centroidal distance along the x axis, in
- $\bar{y}$ — Centroidal distance along the y axis, in
- $\bar{z}$ — Centroidal distance along the z-axis, in

SUMMARY OF PLANE AREA PROPERTIES

Centroid by Integration

$$\bar{y} = \frac{\int y\,dA}{A} \qquad \bar{x} = \frac{\int x\,dA}{A}$$

Centroid by Area Moment Summation

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + \cdots + a_n y_n}{a_1 + a_2 + \cdots + a_n}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + \cdots + a_n x_n}{a_1 + a_2 + \cdots + a_n}$$

Moments of Inertia

$$I_x = \int y^2\,dA$$

$$I_y = \int x^2\,dA$$

$$I_z = I_p = \int r^2\,dA = I_x + I_y$$

Transfer of Axes

$$I_x = \bar{I}_x + Ady^2$$

$$I_y = \bar{I}_y + Adx^2$$

$$I_z = \bar{I}_z + Ad^2 = I_x + I_y$$

$$k^2 = \bar{k}^2 + d^2$$

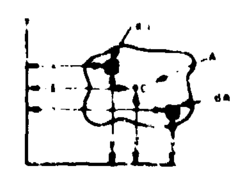
Radius of Gyration

$$k = \sqrt{I/A}$$

CENTROID OF AN AREA

The centroid of an area is represented by a point whose distance from any axis times the total area is equal to the first moment of the area with respect to that axis.

The first moment of an area, often referred to as the statical moment, is the algebraic sum of the moments of the differential parts of the area, with the product of the differential area and the perpendicular distance from the differential area to the axis in question representing the moment of each segment.



Letting Q represent the first moment of the area, the resulting mathematical expressions define the moment of area A

$$Q_x = \int y\,dA, \qquad Q_y = \int x\,dA$$

and, for the centroid,

$$A\bar{y} = \int y\,dA, \qquad A\bar{x} = \int x\,dA$$

or

$$\bar{y} = \frac{\int y\,dA}{A}, \qquad \bar{x} = \frac{\int x\,dA}{A}$$

CENTROID OF A COMPOSITE AREA

Composite areas have centroids, the coordinates of which may be determined by applying the basic definition to the total area as follows

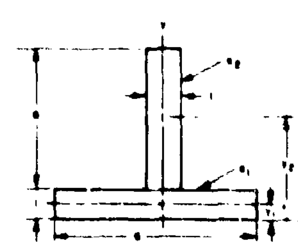
$$A\bar{y} = \Sigma ay \quad \text{and} \quad A\bar{x} = \Sigma ax$$

or

$$\bar{y} = \frac{\Sigma ay}{A} \quad \text{and} \quad \bar{x} = \frac{\Sigma ax}{A}$$

where x and y are the perpendicular distances from their respective axes to the areas in question, a is the area of each individual part, and A is the total area of the composite body.

Example.

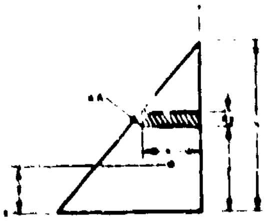


$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{A} = \frac{[illegible]}{[illegible]} = 2.25 \text{ in. (ans.)}$$

$\bar{x} = 0$, by symmetry.

CENTROID EXAMPLES

TRIANGLE



$A = bh/2$

$A\bar{y} = \int y\,dA$

$dA = x\,dy$

$A\bar{y} = \int xy\,dy$

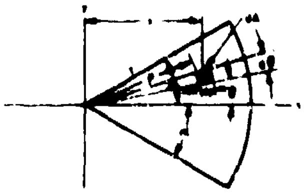
From similar triangles

$$\frac{x}{h - y} = \frac{b}{h}, \quad \text{or} \quad x = \frac{b(h - y)}{h}$$

$$A\bar{y} = \frac{b}{h}\int_0^h (h - y)y\,dy = \frac{bh^2}{6}$$

$$\bar{y} = \frac{bh^2/6}{bh/2} = \frac{h}{3} \quad \text{(ans.)}$$

CIRCULAR SECTOR



$A\bar{x} = \int x\,dA$

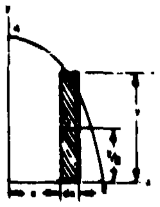
$x = \rho \cos\theta$

$dA = \rho\,d\theta\,d\rho$

$$A\bar{x} = \int_0^r\int_{-\alpha}^{\alpha} \rho \cos\theta\,\rho\,d\rho\,d\theta = \frac{2r^3 \sin\alpha}{3}$$

$$\bar{x} = \frac{(2/3)r^3 \sin\alpha}{A} = \frac{(2/3)r^3 \sin\alpha}{r^2\alpha} = \frac{2r \sin\alpha}{3\alpha} \quad \text{(ans.)}$$

HALF PARABOLA



Parabola: $y = 4 - x^2$

$dA = y\,dx$

$$A = \int_0^2 y\,dx = \int_0^2 (4 - x^2)\,dx = \frac{16}{3}$$

$$Q_x = \int y\,dA = \int_0^2 \left(\frac{y}{2}\right)(y)\,dx = \int_0^2 \frac{y^2}{2}\,dx = \frac{1}{2}\int_0^2 (4 - x^2)^2\,dx = \frac{128}{15}$$

$$Q_y = \int x\,dA = \int_0^2 xy\,dx = \int_0^2 (4x - x^3)\,dx = 4$$

$$\bar{x} = \frac{Q_y}{A} = \frac{4}{16/3} = \frac{3}{4}, \qquad \bar{y} = \frac{Q_x}{A} = \frac{128/15}{16/3} = \frac{8}{5} \quad \text{(ans.)}$$

Note. When summing the elemental strips about the x-axis as shown in the diagram, the moment arm is equal to y/2. If the summation is made with respect to the y-axis, the moment arm is equal to x (not x/2).

MOMENTS OF INERTIA OF A PLANE AREA

The moment of inertia of a plane area about an axis is sometimes referred to as the second moment, since the first moment Q is multiplied by the differential moment arm.

$I_x = \int y^2\,dA$

$I_y = \int x^2\,dA$

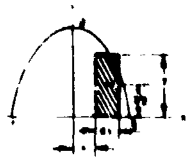
where the elements are integrated over the entire body.

The moment of inertia of the body about the z axis is

$I_z = J_z = \int r^2\,dA$

and, since $r^2 = x^2 + y^2$, $J_z = I_x + I_y$ where J_z is known as the polar moment of inertia of the body.

Example.



The moment of inertia of the parabola $y = 9 - x^2$ about the y-axis is calculated as follows:

$dA = y\,dx$

$x^2 = 9 - y$

$y = 9 - x^2$

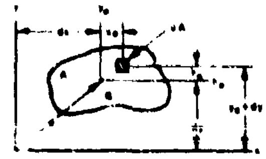
$I_y = \int x^2\,dA$

$= 2\int_0^3 (x^2)(9 - x^2)\,dx$

$= 2\int_0^3 (9x^2 - x^4)\,dx$

$$= 2\left[3x^3 - \frac{x^5}{5}\right]_0^3 = \frac{324}{5} \quad \text{(ans.)}$$

TRANSFER OF AXES ON A PLANE AREA



$I_x = \int (y_0 + d_x)^2\,dA$

$I_x = \int (y_0^2 + 2y_0 d_x + d_x^2)\,dA$

$= \int y_0^2\,dA + 2d_x\int y_0\,dA + d_x^2\int dA$

Since the second term in the equation above is zero, the resulting integrations leave

$I_x = \bar{I}_x + Ad_x^2$

and similarly,

$I_y = \bar{I}_y + Ad_y^2$

The sum of these two equations (from $J_z = I_x + I_y$) gives

$J_z = \bar{J}_z + Ad^2$

which is the polar moment of inertia of the body when transferred through the distance d.

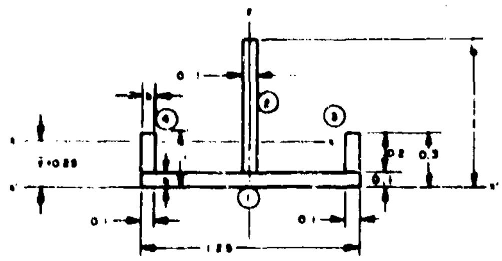
RADIUS OF GYRATION

By definition, $k = \sqrt{I/A}$. Substituting into the moment of inertia equations yields

$k^2 = \bar{k}^2 + d^2$

where $\bar{k}$ is the radius of gyration about a centroidal axis parallel to the axis about which k applies, the axes being separated by d.

MOMENT OF INERTIA OF A COMPOSITE AREA



$$\bar{y} = \frac{[(1.25)(0.1)](0.05) + (0.1)(0.9)(0.55) + (2)(0.1)(0.2)(0.2)}{(1.25)(0.1) + (0.1)(0.9) + (2)(0.1)(0.2)}$$

$$= \frac{0.06375}{0.255} = 0.25 \text{ in.} \quad \text{(ans.)}$$

An alternative method for obtaining the moment of inertia of a composite area and the centroidal distance, d, as in the diagram above, is given in Table 1.

TABLE 1. COMPUTATION TABLE FOR THE MOMENT OF INERTIA AND CENTROIDAL DISTANCE OF A COMPOSITE AREA

Part	Area, in²	b, in.	h, in.	[illegible], in²	Moment, $bh\frac{h}{2}$, in³	[illegible], in³	h^3, in³	[illegible], in⁴
1	0.125	1.25	0.01	0	0.006	0.001	0	0.00001
2	0.09	0.1	1.0	0.01	0.049	1.000	0.001	0.03333
3	0.02	0.1	0.09	0.01	0.004	0.027	0.001	0.00087
4	0.02	0.1	0.09	0.01	0.004	0.027	0.001	0.00087
	A = 0.255				M = 0.06375		I_x' = 0.035	

The centroidal distance may be found by either method. By definition,

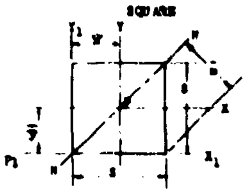
$$d = \frac{M}{A} = \frac{0.06375}{0.255} = 0.25 = \bar{y}$$

$$I_x = I_{x'} - Ad^2$$

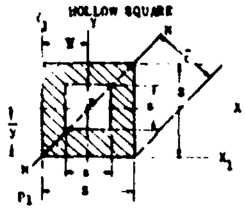
$$= 0.035 - 0.255(0.25)^2$$

$$= 0.0191 \text{ in}^4 \text{ (ans.)}$$

SQUARE

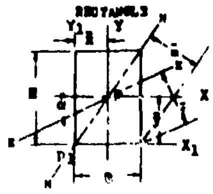


HOLLOW SQUARE



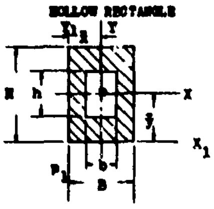
	SQUARE	HOLLOW SQUARE
AREA	S^2	$S^2 - s^2$
CENTROID (in.)	$\bar{x} = \bar{y} = \frac{S}{2}$ $\bar{n} = \frac{S}{\sqrt{2}} = 0.7075$	$\bar{x} = \bar{y} = \frac{S}{2}$ $\bar{n} = \frac{S}{\sqrt{2}} = 0.7075$
WEIGHT MOMENT OF INERTIA (lb-in²)	$I_x = I_y = \frac{WS^2}{12}$ $I_{x_1} = I_{y_1} = \frac{WS^2}{3}$ $I_p = I_x + I_y = \frac{WS^2}{6}$ $I_{p_1} = I_{x_1} + I_{y_1} = \frac{2WS^2}{3}$	$I_x = I_y = \frac{W(S^2 + s^2)}{12}$ $I_{x_1} = I_{y_1} = \frac{W(4S^2 + s^2)}{12} = I_x + \frac{3}{12}WS^2$ $I_p = I_x + I_y = \frac{W(S^2 + s^2)}{6}$ $I_{p_1} = \frac{W(4S^2 + s^2)}{6} = 2 I_{x_1}$
AREA MOMENT OF INERTIA (in⁴)	$I_N = I_x = I_y = \frac{S^4}{12}$ $I_{x_1} = I_{y_1} = \frac{S^4}{3}$ $I_p = I_x + I_y = \frac{S^4}{6}$ $I_{p_1} = I_{x_1} + I_{y_1} = \frac{2S^4}{3}$	$I_N = I_x = I_y = \frac{S^4 - s^4}{12}$ $I_{x_1} = I_{y_1} = \frac{4S^4 - 3S^2s^2 - s^4}{12}$ $I_p = \frac{S^4 - s^4}{6}$ $I_{p_1} = \frac{4S^4 - 3S^2s^2 - s^4}{6}$
RADIUS OF GYRATION (in.)	$k_N = k_x = k_y = 0.289S$ $k_{x_1} = k_{y_1} = 0.577S$ $k_p = 0.408S$ $k_{p_1} = 0.816S$	$k_N = k_x = k_y = 0.289\sqrt{S^2 + s^2}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{S^2 - s^2}} = \frac{1}{6}\sqrt{3(4S^2 + s^2)}$ $k_{y_1} = \sqrt{\frac{I_{y_1}}{S^2 - s^2}} = \frac{1}{6}\sqrt{3(4S^2 + s^2)}$ $k_p = 0.408\sqrt{S^2 + s^2}$ $k_{p_1} = \sqrt{\frac{I_{p_1}}{S^2 - s^2}} = \frac{1}{6}\sqrt{6(4S^2 + s^2)}$

RECTANGLE



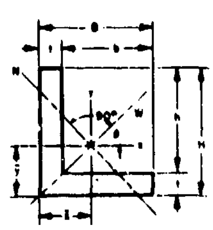
	RECTANGLE
AREA	BH
CENTROID (in.)	$\bar{x} = \frac{B}{2}$ $\bar{y} = \frac{H}{2}$ $\bar{l} = \frac{B\sin\alpha + H\cos\alpha}{2}$ $\bar{n} = \frac{BH}{\sqrt{B^2 + H^2}}$
WEIGHT MOMENT OF INERTIA (lb-in²)	$I_x = \frac{WH^2}{12}$ $I_y = \frac{WB^2}{12}$ $I_p = \frac{W}{12}(H^2 + B^2)$ $I_{x_1} = \frac{WH^2}{3}$ $I_{y_1} = \frac{WB^2}{3}$ $I_{p_1} = \frac{W}{3}(H^2 + B^2)$
AREA MOMENT OF INERTIA (in⁴)	$I_x = \frac{BH^3}{12}$ $I_y = \frac{HB^3}{12}$ $I_p = \frac{BH}{12}(H^2 + B^2)$ $I_N = \frac{B^3H^3}{6(B^2 + H^2)}$ $I_{x_1} = \frac{BH^3}{3}$ $I_{y_1} = \frac{HB^3}{3}$ $I_{p_1} = \frac{BH}{3}(H^2 + B^2)$ $I_z = \frac{BH(B^2\sin^2\alpha + H^2\cos^2\alpha)}{12}$
RADIUS OF GYRATION (in.)	$k_x = 0.289H$ $k_y = 0.289B$ $k_p = 0.289\sqrt{H^2 + B^2}$ $k_N = \frac{BH}{\sqrt{6(B^2 + H^2)}}$ $k_{x_1} = 0.577H$ $k_{y_1} = 0.577B$ $k_{p_1} = 0.577\sqrt{H^2 + B^2}$ $k_z = \sqrt{\frac{B^2\sin^2\alpha + H^2\cos^2\alpha}{12}}$

HOLLOW RECTANGLE



	HOLLOW RECTANGLE
AREA	BH - bh
CENTROID (in.)	$\bar{x} = \frac{B}{2}$ $\bar{y} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA (lb-in²)	$I_x = \frac{W}{12}\left[\frac{BH^3 - bh^3}{BH - bh}\right]$ $I_y = \frac{W}{12}\left[\frac{HB^3 - hb^3}{BH - bh}\right]$ $I_{x_1} = I_x + \frac{WH^2}{4}$ $I_{y_1} = I_y + \frac{WB^2}{4}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA (in⁴)	$I_x = \frac{BH^3 - bh^3}{12}$ $I_y = \frac{HB^3 - hb^3}{12}$ $I_{x_1} = \frac{BH^3}{3} - \frac{bh(3H^2 + h^2)}{12}$ $I_{y_1} = I_y + \frac{(BH - bh)B^2}{4}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION (in.)	$k_x = \sqrt{\frac{BH^3 - bh^3}{12(BH - bh)}}$ $k_y = \sqrt{\frac{HB^3 - hb^3}{12(BH - bh)}}$ $k_p = \sqrt{\frac{I_p}{BH - bh}}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{BH - bh}}$ $k_{y_1} = \sqrt{\frac{I_{y_1}}{BH - bh}}$ $k_{p_1} = \sqrt{\frac{I_{p_1}}{BH - bh}}$

ANGLE



$\tan 2\theta = \dfrac{2I_{xy}}{I_y - I_x}$

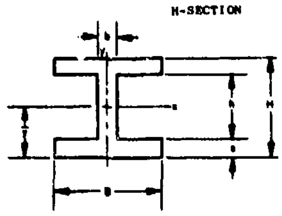
I_{xy} = product of inertia about x-x and y-y

$I_{xy} = \pm\dfrac{(bBhHt)}{4(B + h)}$

I_{xy} is negative when the heel of the angle, with respect to the center of gravity, is in the first or third quadrant; positive when it is in the second or fourth quadrant

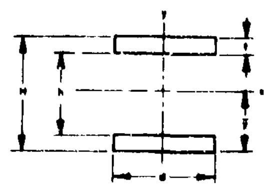
AREA	$t(B + h)$
CENTROID	$\bar{x} = \dfrac{B^2 + ht}{2(B + h)}$ $\quad \bar{y} = \dfrac{H^2 + bt}{2(B + h)}$
WEIGHT MOMENT OF INERTIA ($lb\text{-}in^2$)	$I_x = \dfrac{m}{3}\left[\dfrac{t(H - \bar{y})^3 + B\bar{y}^3 - b(\bar{y} - t)^3}{t(H + B - t)}\right]$ $I_y = \dfrac{m}{3}\left[\dfrac{t(B - \bar{x})^3 + H\bar{x}^3 - h(\bar{x} - t)^3}{t(H + B - t)}\right]$ $I_N = \dfrac{m(I_x \sin^2\theta + I_y \cos^2\theta + I_{xy}\sin 2\theta)}{t(H + B - t)}$ $I_W = \dfrac{m(I_x \cos^2\theta + I_y \sin^2\theta - I_{xy}\sin 2\theta)}{t(H + B - t)}$
AREA MOMENT OF INERTIA (in^4)	$I_x = 1/3\left[t(H - \bar{y})^3 + B\bar{y}^3 - b(\bar{y} - t)^3\right]$ $I_y = 1/3\left[t(B - \bar{x})^3 + H\bar{x}^3 - h(\bar{x} - t)^3\right]$ $I_N = I_x \sin^2\theta + I_y \cos^2\theta + I_{xy}\sin 2\theta$ $I_W = I_x \cos^2\theta + I_y \sin^2\theta - I_{xy}\sin 2\theta$
RADIUS OF GYRATION	$k = \sqrt{I/A}$

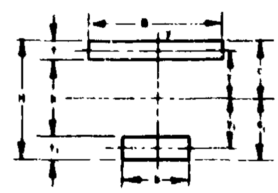
H-SECTION



AREA	$BH - h(B - b)$
CENTROID	$\bar{y} = H/2 \quad \bar{x} = 0$
WEIGHT MOMENT OF INERTIA	$I_x = \dfrac{m\left[BH^3 - h^3(B - b)\right]}{12\left[BH - h(B - b)\right]}$ $I_y = \dfrac{m(hb^3 + 2aB^3)}{12\left[BH - h(B - b)\right]}$
AREA MOMENT OF INERTIA	$I_x = \dfrac{BH^3 - h^3(B - b)}{12}$ $I_y = \dfrac{hb^3 + 2aB^3}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{\dfrac{BH^3 - h^3(B - b)}{12\left[BH - h(B - b)\right]}}$ $k_y = \sqrt{\dfrac{hB^3 + 2aB^3}{12\left[BH - h(B - b)\right]}}$

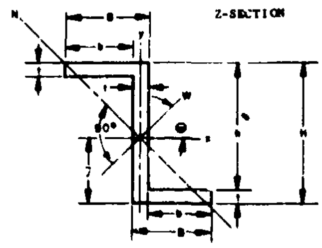
EQUAL RECTANGLES | UNEQUAL RECTANGLES

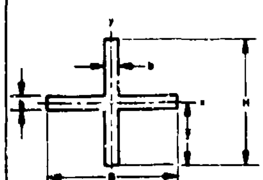




	EQUAL RECTANGLES	UNEQUAL RECTANGLES
AREA	$B(H - h)$	$Bt + bt_1$ [illegible] $H -$ [illegible]
CENTROID	$\bar{y} = H/2$	$c = \dfrac{(1/2)Bt^2 + bt_1\left[H - (1/2)t_1\right]}{Bt + bt_1}$
WEIGHT MOMENT OF INERTIA ($lb\text{-}in^2$)	$I_x = \dfrac{m(H^3 - h^3)}{12(H - h)}$ $I_y = \dfrac{mB^2}{12}$	$I_x = \dfrac{m(Bt^3 + 12Bty^2 + bt_1^3 + 12bt_1y_1^2)}{12(Bt + bt_1)}$ $I_y = \dfrac{m(tB^3 + t_1b^3)}{12(Bt + bt_1)}$
AREA MOMENT OF INERTIA (in^4)	$I_x = \dfrac{B(H^3 - h^3)}{12}$ $I_y = \dfrac{B^3(H - h)}{12}$	$I_x = \dfrac{Bt^3}{12} + Bty^2 + \dfrac{bt_1^3}{12} + bt_1y_1^2$ $I_y = \dfrac{tB^3 + t_1b^3}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{\dfrac{H^3 - h^3}{12B(H - h)}}$ $k_y = \sqrt{\dfrac{B^3(H - h)}{12B(H - h)}} = 0.289B$	$k_x = \sqrt{\dfrac{Bt^3 + 12Bty^2 + bt_1^3 + 12bt_1y_1^2}{12(Bt + bt_1)}}$ $k_y = \sqrt{\dfrac{tB^3 + t_1b^3}{12(Bt + bt_1)}}$

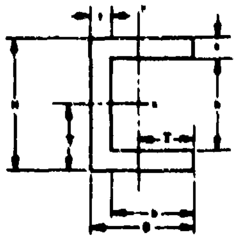
Z-SECTION | CROSSED RECTANGLES





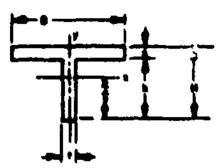
	Z-SECTION	CROSSED RECTANGLES
AREA	$t(H + 2b)$	$Hb + h(B - b)$
CENTROID	$\bar{y} = H/2 \quad \bar{x} = 0$	$\bar{y} = H/2 \quad \bar{x} = B/2$
WEIGHT MOMENT OF INERTIA	$\tan 2\theta = -\dfrac{(Ht - t^2)(B^2 - Bt)}{I_x - I_y}$ $I_x = \dfrac{m\left[BH^3 - b(H - 2t)^3\right]}{12\left[t(H + 2b)\right]}$ $I_y = \dfrac{m\left[H(B + b)^3 - 2b^3h - 6bB^2h\right]}{12t(H + 2b)}$	$I_x = \dfrac{m\left[bH^3 + h^3(B - b)\right]}{12\left[Hb + h(B - b)\right]}$ $I_y = \dfrac{m\left[hB^3 + b^3(H - h)\right]}{12\left[Hb + h(H - b)\right]}$
AREA MOMENT OF INERTIA	$I_x = \dfrac{BH^3 - b(H - 2t)^3}{12}$ $I_y = \dfrac{H(B + b)^3 - 2b^3h - 6B^2bh}{12}$ $I_N = I_x \sin^2\theta + I_y \cos^2\theta + I_{xy}\sin 2\theta$ $I_W = I_x \cos^2\theta + I_y \sin^2\theta - I_{xy}\sin 2\theta$	$I_x = \dfrac{bH^3 + h^3(B - b)}{12}$ $I_y = \dfrac{hB^3 + b^3(H - h)}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$	$k_x = \sqrt{\dfrac{bH^3 + h^3(B - b)}{12\left[Hb + h(B - b)\right]}}$ $k_y = \sqrt{\dfrac{hB^3 + b^3(H - h)}{12\left[Hb + h(B - b)\right]}}$

CHANNEL OR U-SECTION



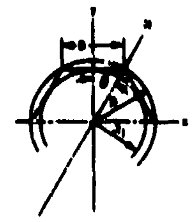
AREA	$BH - h(B - t) = A$
CENTROID	$\bar{y} = \frac{H}{2}$ $\quad \bar{x} = B - \frac{2B^2 s + ht^2}{2BH - 2h(B - t)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{w\left[BH^3 - h^3(B - t)\right]}{12\left[BH - h(B - t)\right]}$ $I_y = \frac{w}{3}\left[\frac{2sB^3 + ht^3 - 3A(B - \bar{x})^2}{BH - h(B - t)}\right]$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3 - h^3(B - t)}{12}$ $I_y = \frac{2sB^3 + ht^3}{3} - A(B - \bar{x})^2$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{BH^3 - h^3(B - t)}{12\left[BH - h(B - t)\right]}}$ $k_y = \sqrt{\frac{2sB^3 + ht^3 - 3A(B - \bar{x})^2}{3\left[BH - h(B - t)\right]}}$

T-SECTION



AREA	$Bs + ht$
CENTROID	$\bar{y} = H - \frac{H^2 t + s^2(B - t)}{2(Bs + ht)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{w}{3}\left[\frac{t\bar{y}^3 + B(H - \bar{y})^3 - (B - t)(H - \bar{y} - s)^3}{Bs + ht}\right]$ $I_y = \frac{w}{12}\left(\frac{sB^3 + ht^3}{Bs + ht}\right)$
AREA MOMENT OF INERTIA	$I_x = \frac{t\bar{y}^3 + B(H - \bar{y})^3 - (B - t)(H - \bar{y} - s)^3}{3}$ $I_y = \frac{sB^3 + ht^3}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$

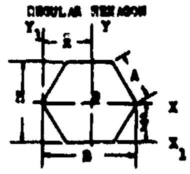
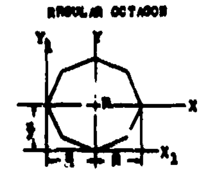
REGULAR POLYGON



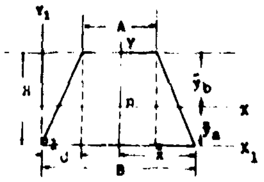
n = number of sides

$\theta = \frac{180°}{n}$ $\quad B = 2\sqrt{R^2 - R_1^2}$

AREA	$\frac{nB^2 \cot\theta}{4} = \frac{nR^2 \sin 2\theta}{2} = nR_1^2 \tan\theta$
CENTROID	$\bar{x} = \bar{y} = 0$
WEIGHT MOMENT OF INERTIA	$R = \frac{B}{2 \sin\theta}$ $\quad R_1 = \frac{B}{2 \tan\theta}$ $I_y = I_x = \frac{w(6R^2 - B^2)}{24}$ $= \frac{w(12R_1^2 + B^2)}{48}$
AREA MOMENT OF INERTIA	$I_y = I_x = \frac{A(6R^2 - B^2)}{24} = \frac{A(12R_1^2 + B^2)}{48}$
RADIUS OF GYRATION	$k_y = k_x = \sqrt{\frac{6R^2 - B^2}{24}} = \sqrt{\frac{12R_1^2 + B^2}{48}}$

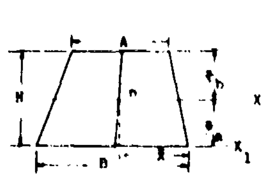
	REGULAR HEXAGON	REGULAR OCTAGON
		
AREA	$0.866\ H^2$	$2.8284\ R^2$
CENTROID	$\bar{x} = \frac{B}{2} = A \quad \bar{y} = \frac{H}{2}$	$\bar{x} = \bar{y} = R$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = 0.0694wH^2 = 0.0521wB^2$ $I_{x_1} = 0.3194wH^2$ $I_p = 0.1388wH^2 = 0.1042wB^2$ $I_{y_1} = 0.4028wH^2 = 0.3021wB^2$	$I_x = I_y = 0.2286wR^2$ $I_{x_1} = I_{y_1} = 1.[illegible]wR^2$ $I_p = 0.4512wR^2$
AREA MOMENT OF INERTIA	$I_x = I_y = 0.0601H^4$ $I_{x_1} = 0.2766H^4$ $I_{y_1} = 0.3488H^4$ $I_p = 0.1203H^4$	$I_x = I_y = 0.6381R^4$ $I_{x_1} = I_{y_1} = 3.1669R^4$ $I_p = 1.2751R^4$
RADIUS OF GYRATION	$k_x = k_y = 0.2635H = 0.2282B$ $k_{x_1} = 0.5652H$ $k_{y_1} = 0.63[illegible]H$ $k_p = 0.3727H$	$k_x = k_y = 0.4750R$ $k_{x_1} = k_{y_1} = 1.1071R$ $k_p = 0.672R$

ISOSCELES TRAPEZOID



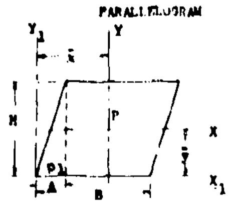
AREA	$\frac{H(A+B)}{2}$
CENTROID	$\bar{x} = \frac{B}{2}$ $\quad \bar{y}_a = \frac{H(B+2A)}{3(B+A)}$ $\quad \bar{y}_b = \frac{H(A+2B)}{3(A+B)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}\left(1 + \frac{2AB}{(A+B)^2}\right)$ $I_{x_1} = \frac{WH^2(2A+C)}{6(A+C)}$, or $\frac{WH^2(3A+B)}{6(A+B)}$ $I_y = \frac{W}{24}(A^2+B^2)$ $I_{y_1} = \frac{W}{24}(A^2+7B^2)$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA	$I_x = \frac{H^3(A^2+4AB+B^2)}{36(A+B)}$ $I_{x_1} = \frac{H^3(A+B)(2A+C)}{12(A+C)}$ $I_y = \frac{H(A+B)(A^2+B^2)}{48}$ $I_{y_1} = \frac{H(A+B)(A^2+7B^2)}{48}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = H\frac{\sqrt{2(B^2+4AB+A^2)}}{6(A+B)}$ $k_{x_1} = \sqrt{\frac{H^2(2A+C)}{A+C}}$ $k_y = \sqrt{\frac{A^2+B^2}{24}}$ $k_{y_1} = \sqrt{\frac{A^2+7B^2}{24}}$ $k_p = \sqrt{\frac{2I_p}{H(A+B)}}$ $\quad k_{p_1} = \sqrt{\frac{2I_{p_1}}{H(A+B)}}$

OBLIQUE TRAPEZOID



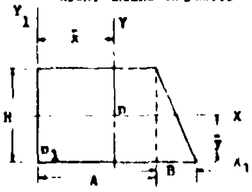
AREA	$\frac{H(A+B)}{2}$
CENTROID	$\bar{x}$ is on a line connecting mid-points of sides A & B. $\bar{y}_a = \frac{H(B+2A)}{3(A+B)}$ $\quad \bar{y}_b = \frac{H(2B+A)}{3(A+B)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}\left(1 + \frac{2AB}{(A+B)^2}\right)$ $I_{x_1} = \frac{WH^2}{6}\left(\frac{3A+B}{A+B}\right)$
AREA MOMENT OF INERTIA	$I_x = \frac{H^3(A^2+4AB+B^2)}{36(A+B)}$ $I_{x_1} = \frac{H^3(B+3A)}{12}$
RADIUS OF GYRATION	$k_x = H\sqrt{\frac{2(A^2+4AB+B^2)}{6(A+B)}}$ $k_{x_1} = H\sqrt{\frac{3A+B}{6(A+B)}}$

PARALLELOGRAM



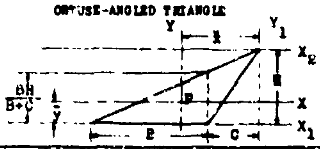
AREA	BH
CENTROID	$\bar{x} = \frac{A+B}{2}$ $\quad \bar{y} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{12}$ $I_{x_1} = \frac{WH^2}{3}$ $I_y = \frac{W}{12}(A^2+B^2)$ $I_{y_1} = \frac{W}{6}(2A^2+2B^2+3AB)$ $I_p = \frac{W}{12}(A^2+B^2+H^2)$ $I_{p_1} = \frac{W(2A^2+2B^2+3AB+2H^2)}{6}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{12}$ $I_{x_1} = \frac{BH^3}{3}$ $I_y = \frac{BH(A^2+B^2)}{12}$ $I_{y_1} = \frac{BH}{6}(2A^2+2B^2+3AB)$ $I_p = \frac{BH}{12}(A^2+B^2+H^2)$ $I_{p_1} = \frac{BH}{6}(2A^2+2B^2+3AB+2H^2)$
RADIUS OF GYRATION	$k_x = 0.289H$ $k_{x_1} = 0.577H$ $k_y = 0.289\sqrt{A^2+B^2}$ $k_{y_1} = 0.408\sqrt{2A^2+2B^2+3AB}$ $k_p = 0.289\sqrt{A^2+B^2+H^2}$ $k_{p_1} = 0.408\sqrt{2A^2+2B^2+3AB+2H^2}$

RIGHT-ANGLED TRAPEZOID



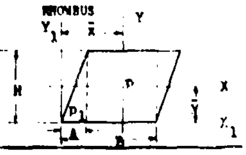
AREA	$\frac{H}{2}(2A+B)$
CENTROID	$\bar{x} = \frac{3A^2+3AB+B^2}{3(2A+B)}$ $\quad \bar{y} = \frac{H(3A+B)}{3(2A+B)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2(6A^2+6AB+B^2)}{18(2A+B)^2}$ $I_{x_1} = \frac{WH^2(4A+B)}{6(2A+B)}$ $I_y = I_{y_1} - W\bar{x}^2$ $I_{y_1} = W\left[\frac{6A^2(2A+3B)+B^2(12A+5B)}{18(2A+B)}\right]$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA	$I_x = \frac{H^3(6A^2+6AB+B^2)}{36(2A+B)}$ $I_{x_1} = \frac{H^3(4A+B)}{12}$ $I_y = I_{y_1} - \frac{H(3A^2+3AB+B^2)^2}{18(2A+B)}$ $I_{y_1} = \frac{H[6A^2(2A+3B)+B^2(12A+5B)]}{36}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = \frac{0.236H}{(2A+B)}\sqrt{6A^2+6AB+B^2}$ $k_{x_1} = 0.408H\sqrt{\frac{4A+B}{2A+B}}$ $k_y = \sqrt{\frac{2I_y}{H(2A+B)}}$ $k_{y_1} = \sqrt{\frac{6A^2(2A+3B)+B^2(12A+5B)}{18(2A+B)}}$ $k_p = \sqrt{\frac{2I_p}{H(2A+B)}}$ $\quad k_{p_1} = \sqrt{\frac{2I_{p_1}}{H(2A+B)}}$

OBTUSE-ANGLED TRIANGLE



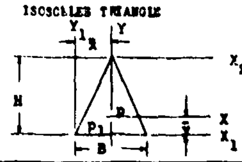
AREA	$\frac{BH}{2}$
CENTROID	$\bar{x} = \frac{B+2C}{3}$ $\quad \bar{y} = \frac{H}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_y = \frac{W}{18}(B^2+BC+C^2)$ $I_{y_1} = \frac{W(B^2+3BC+3C^2)}{6}$ $I_p = \frac{W(H^2+B^2+BC+C^2)}{18}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x_1} = \frac{BH^3}{12}$ $I_{x_2} = \frac{BH^3}{4}$ $I_y = \frac{BH}{36}(B^2+BC+C^2)$ $I_{y_1} = \frac{BH}{12}(B^2+3BC+3C^2)$ $I_p = \frac{BH}{36}(H^2+B^2+BC+C^2)$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x_1} = 0.408H$ $k_{x_2} = 0.707H$ $k_y = 0.236\sqrt{B^2+BC+C^2}$ $k_{y_1} = 0.408\sqrt{B^2+3BC+C^2}$ $k_p = 0.236\sqrt{H^2+B^2+BC+C^2}$

RHOMBUS



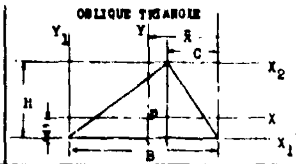
AREA	BH
CENTROID	$\bar{x} = \frac{A+B}{2}$ $\quad \bar{y} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{12}$ $I_{x_1} = \frac{WH^2}{3}$ $I_y = \frac{W(A^2+B^2)}{12}$ $I_{y_1} = \frac{W(2A^2+2B^2+3AB)}{6}$ $I_p = \frac{WB^2}{6}$ $I_{p_1} = \frac{WB(3A+4B)}{6}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{12}$ $I_{x_1} = \frac{BH^3}{3}$ $I_y = \frac{BH(A^2+B^2)}{12}$ $I_{y_1} = \frac{BH(2A^2+2B^2+3AB)}{6}$ $I_p = \frac{1}{6}B^3H$ $I_{p_1} = \frac{B^2H(3A+4B)}{6}$
RADIUS OF GYRATION	$k_x = 0.289H$ $k_{x_1} = 0.577H$ $k_y = 0.289\sqrt{(A^2+B^2)}$ $k_{y_1} = 0.408\sqrt{2A^2+2B^2+3AB}$ $k_p = 0.408B$ $k_{p_1} = 0.408\sqrt{B(3A+4B)}$

ISOSCELES TRIANGLE

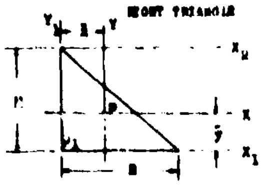
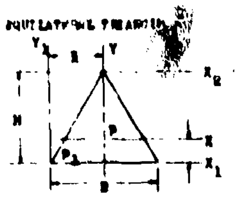


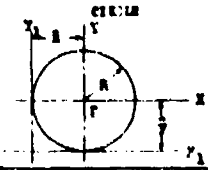
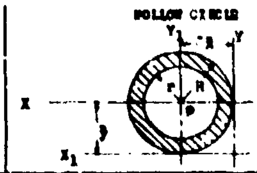
AREA	$\frac{BH}{2}$
CENTROID	$\bar{x} = \frac{B}{2}$ $\quad \bar{y} = \frac{H}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_y = \frac{WB^2}{24}$ $I_{y_1} = \frac{7WB^2}{24}$ $I_p = \frac{W(4H^2+3B^2)}{72}$ $\quad I_{p_1} = \frac{W(7B^2+4H^2)}{24}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x_1} = \frac{BH^3}{12}$ $I_{x_2} = \frac{BH^3}{4}$ $I_y = \frac{B^3H}{48}$ $I_{y_1} = \frac{7B^3H}{48}$ $I_p = \frac{4BH^3+3B^3H}{144}$ $\quad I_{p_1} = \frac{4BH^3+7B^3H}{48}$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x_1} = 0.408H$ $k_{x_2} = 0.707H$ $k_y = 0.204B$ $k_{y_1} = 0.540B$ $k_p = 0.118\sqrt{4H^2+3B^2}$ $k_{p_1} = 0.204\sqrt{4H^2+7B^2}$

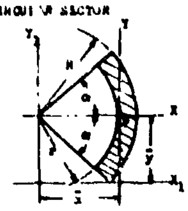
OBLIQUE TRIANGLE

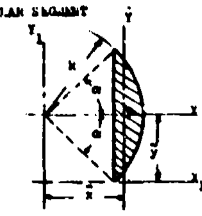


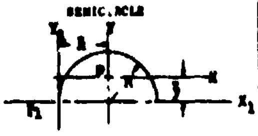
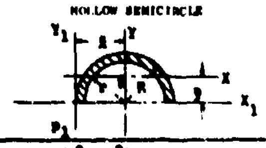
AREA	$\frac{BH}{2}$
CENTROID	$\bar{y} = \frac{H}{3}$ $\quad \bar{x} = \frac{B+C}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_y = \frac{W}{18}(B^2+C^2-BC)$ $I_p = \frac{W}{18}(H^2+B^2+C^2-BC)$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x_1} = \frac{BH^3}{12}$ $I_{x_2} = \frac{BH^3}{4}$ $I_y = \frac{BH}{36}(B^2+C^2-BC)$ $I_p = \frac{BH}{36}(H^2+B^2+C^2-BC)$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x_1} = 0.408H$ $k_{x_2} = 0.707H$ $k_y = 0.236\sqrt{B^2+C^2-BC}$ $k_p = 0.236\sqrt{H^2+B^2+C^2-BC}$

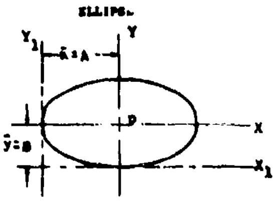
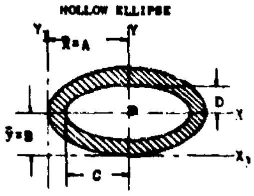
	RIGHT TRIANGLE	EQUILATERAL TRIANGLE
		
AREA	$\frac{BH}{2}$	$\frac{BH}{2}$
CENTROID	$\bar{x} = \frac{B}{3}$ $\bar{y} = \frac{H}{3}$	$\bar{x} = \frac{B}{2}$ $\bar{y} = \frac{H}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_y = \frac{WB^2}{18}$ $I_{y_1} = \frac{WB^2}{6}$ $I_p = \frac{W(H^2+B^2)}{18}$ $I_{p_1} = \frac{W(B^2+H^2)}{6}$	$I_x = I_y = \frac{WH^2}{24}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_{y_1} = \frac{7WB^2}{24}$ $I_p = \frac{WB^2}{12}$ $I_{p_1} = \frac{5WB^2}{12}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x_1} = \frac{BH^3}{12}$ $I_{x_2} = \frac{BH^3}{4}$ $I_y = \frac{B^3H}{36}$ $I_{y_1} = \frac{B^3H}{12}$ $I_p = \frac{BH(B^2+H^2)}{36}$ $I_{p_1} = \frac{BH(B^2+H^2)}{12}$	$I_x = I_y = \frac{B^3H}{48}$ $I_{x_1} = \frac{B^3H}{36}$ $I_{x_2} = \frac{BH^3}{4}$ $I_{y_1} = \frac{7B^3H}{48}$ $I_p = \frac{B^3H}{24}$ $I_{p_1} = \frac{5B^3H}{24}$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x_1} = 0.408H$ $k_{x_2} = 0.707H$ $k_y = 0.236B$ $k_{y_1} = 0.408B$ $k_p = 0.236\sqrt{B^2+H^2}$ $k_{p_1} = 0.408\sqrt{B^2+H^2}$	$k_x = 0.204B$ $k_{x_1} = 0.354B$ $k_y = 0.204B$ $k_{x_2} = 0.707H$ $k_{y_1} = 0.540B$ $k_p = 0.289B$ $k_{p_1} = 0.456B$

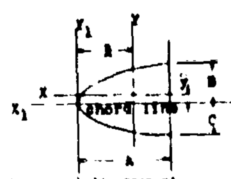
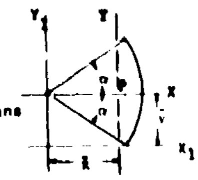
	CIRCLE	HOLLOW CIRCLE
		
AREA	$0.7854\ D^2$	$\pi(R^2 - r^2)$
CENTROID	$\bar{x} = \bar{y} = R$	$\bar{x} = \bar{y} = R$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{WD^2}{16} = \frac{WR^2}{4}$ $I_{x_1} = I_{y_1} = 1.25\ WR^2$ $I_p = \frac{WD^2}{8} = \frac{WR^2}{2}$	$I_x = I_y = \frac{W(R^2+r^2)}{4}$ $I_p = \frac{W(R^2+r^2)}{2}$ $I_{x_1} = I_{y_1} = \frac{W(5R^2+r^2)}{4}$
AREA MOMENT OF INERTIA	$I_x = I_y = 0.0491D^4$ $I_{x_1} = I_{y_1} = 0.2454D^4$ $I_p = 0.0982D^4$	$I_x = I_y = \frac{\pi(R^4 - r^4)}{4}$ $I_p = \frac{\pi(R^4 - r^4)}{2}$ $I_{x_1} = I_{y_1} = \frac{\pi(5R^4-4R^2r^2-r^4)}{4}$
RADIUS OF GYRATION	$k_x = k_y = \frac{D}{4}$ $k_{x_1} = k_{y_1} = 0.5590D = 1.118R$ $k_p = 0.3536D$	$k_x = k_y = \frac{1}{2}\sqrt{R^2+r^2}$ $k_p = \sqrt{\frac{R^2+r^2}{2}}$ $k_{x_1} = k_{y_1} = \frac{1}{2}\sqrt{5R^2+r^2}$

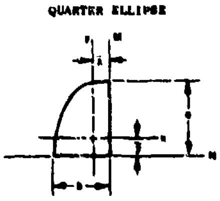
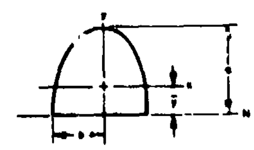
	HOLLOW CIRCULAR SECTOR	
	α in radians	
AREA	$(R^2 - r^2)\alpha$	
CENTROID	$\bar{x} = \frac{2\,SIN\,\alpha\,(R^3-r^3)}{3\,\alpha\,(R^2-r^2)}$ $\bar{y} = R\,SIN\,\alpha$	
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{4}(R^2+r^2)\left(1 - \frac{SIN\,\alpha\,COS\,\alpha}{\alpha}\right)$ $I_{x_1} = I_x + WR^2SIN^2\alpha$ $I_y = I_{y_1} - W\left[\frac{2SIN\alpha\,(R^3-r^3)}{3\alpha\,(R^2-r^2)}\right]^2$	$I_{y_1} = \frac{W}{4}(R^2+r^2)\left(1 + \frac{SIN\,\alpha\,COS\,\alpha}{\alpha}\right)$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = \frac{\alpha}{4}(R^4-r^4)\left(1 - \frac{SIN\alpha\,COS\alpha}{\alpha}\right)$ $I_{x_1} = I_x + \alpha(R^4-r^2R^2)\,SIN^2\alpha$ $I_y = I_{y_1} - \frac{1}{\alpha(R^2-r^2)}\left[\frac{2SIN\alpha(R^3-r^3)}{3}\right]^2$	$I_{y_1} = \frac{\alpha}{4}(R^4-r^4)\left(1 + \frac{SIN\alpha\,COS\alpha}{\alpha}\right)$ $I_p = I_x + I_y$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{R^2+r^2}{4}\left[1 - \frac{SIN\,\alpha\,COS\,\alpha}{\alpha}\right]}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{(R^2-r^2)\,\alpha}}$ $k_y = \sqrt{\frac{I_y}{(R^2-r^2)\,\alpha}}$	$k_{y_1} = \sqrt{\left[\frac{R^2+r^2}{4}\right]\left[1 + \frac{SIN\,\alpha\,COS\,\alpha}{\alpha}\right]}$ $k_p = \sqrt{\frac{I_p}{(R^2-r^2)\alpha}}$

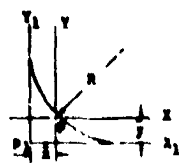
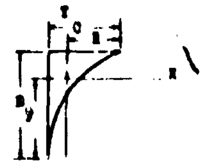
	CIRCULAR SEGMENT	
	α in radians A = area	
AREA	$\frac{R^2}{2}(2\,\alpha - SIN\,2\,\alpha)$	
CENTROID	$\bar{x} = \frac{4R\,SIN^3\,\alpha}{3(2\,\alpha - SIN\,2\,\alpha)}$ $\bar{y} = R\,SIN\,\alpha$	
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WR^2}{4}\left[1 - \frac{2\,SIN^3\,\alpha\,COS\,\alpha}{3\,(\alpha - SIN\,\alpha\,COS\,\alpha)}\right]$ $I_{x_1} = I_x + WR^2SIN^2\alpha$ $I_y = I_{y_1} - W\left[\frac{2R\,SIN^3\,\alpha}{3\,(\alpha - SIN\,\alpha\,COS\,\alpha)}\right]^2$	$I_{y_1} = \frac{WR^2}{4}\left[1 + \frac{2\,SIN^3\,\alpha\,COS\,\alpha}{\alpha - SIN\,\alpha\,COS\,\alpha}\right]$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = \frac{AR^2}{4}\left[1 - \frac{2}{3}\left[\frac{SIN^3\,\alpha\,COS\,\alpha}{\alpha - SIN\,\alpha\,COS\,\alpha}\right]\right]$ $I_{x_1} = I_x + \frac{R^4}{2}(2\,\alpha - SIN\,2\,\alpha)\,(SIN^2\,\alpha)$ $I_y = I_{y_1} - \frac{4R^6\,SIN^6\,\alpha}{9A}$	$I_y = \frac{AR^2}{4}\left[1 + \frac{2\,SIN^3\,\alpha\,COS\,\alpha}{\alpha - SIN\,\alpha\,COS\,\alpha}\right]$ $I_p = I_x + I_y$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{R^2}{4}\left[1 - \frac{2\,SIN^3\,\alpha\,COS\,\alpha}{3\,(\alpha - SIN\,\alpha\,COS\,\alpha)}\right]}$ $k_y = \sqrt{\frac{2I_y}{R^2\,(2\,\alpha - SIN\,2\alpha)}}$ $k_p = \sqrt{\frac{2I_p}{R^2\,(2\,\alpha - SIN\,2\,\alpha)}}$	

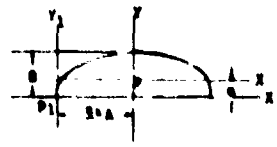
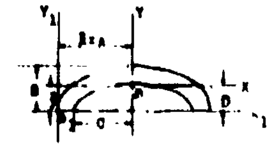
	SEMICIRCLE	HOLLOW SEMICIRCLE
		
AREA	$0.3927\ D^2 = 1.571R^2$	$\frac{\pi(R^2 - r^2)}{2}$
CENTROID	$\bar{x} = R \quad \bar{y} = 0.2122D = 0.4244R$	$\bar{x} = R \quad \bar{y} = 0.4244\left(R + \frac{r^2}{R + r}\right)$
WEIGHT MOMENT OF INERTIA	$I_x = 0.06987\ WR^2 = 0.01747\ WD^2$ $I_{x_1} = 0.25\ WR^2 = 0.0625\ WD^2$ $I_y = 0.25\ WR^2 = 0.0625\ WD^2$ $I_{y_1} = 1.25\ WR^2 = 0.3125\ WD^2$ $I_p = 0.3199\ WR^2 = 0.0800\ WD^2$ $I_{p_1} = 1.50\ WR^2 = 0.3750\ WD^2$	$I_x = \frac{W(R^2 + r^2)}{4} - W\bar{y}^2 \quad I_y = \frac{W(R^2+r^2)}{4}$ $I_p = I_x + I_y$ $I_{x_1} = \frac{W(R^2 + r^2)}{4} \quad I_{y_1} = \frac{W(5R^2 + r^2)}{4}$ $I_{p_1} = \frac{W(3R^2 + r^2)}{2}$
AREA MOMENT OF INERTIA	$I_x = 0.1098\ R^4 = 0.00686\ D^4$ $I_{x_1} = 0.3927\ R^4 = 0.02454\ D^4$ $I_y = 0.3927\ R^4 = 0.02454\ D^4$ $I_{y_1} = 1.9635\ R^4 = 0.12272\ D^4$ $I_p = 0.5025\ R^4 = 0.03140\ D^4$ $I_{p_1} = 2.3562\ R^4 = 0.14726\ D^4$	$I_x = \frac{\pi}{8}(R^4 - r^4) - \frac{\pi(R^2 - r^2)}{2}\bar{y}^2$ $I_{x_1} = \frac{\pi}{8}(R^4 - r^4)$ $I_y = \frac{\pi}{8}(R^4 - r^4)$ $I_{y_1} = \frac{\pi(R^2 - r^2)(5R^2 + r^2)}{8}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.264R = 0.132D$ $k_{x_1} = 0.5R = 0.25D$ $k_y = 0.5R = 0.25D$ $k_{y_1} = 1.118R = 0.559D$ $k_p = 0.566R = 0.2830D$ $k_{p_1} = 1.225R = 0.6124D$	$k_x = \sqrt{\frac{2I_x}{\pi(R^2 - r^2)}} \quad k_{y_1} = \sqrt{\frac{2I_{y_1}}{\pi(R^2 - r^2)}}$ $k_{x_1} = \sqrt{\frac{R^2 + r^2}{4}} \quad k_p = \sqrt{\frac{2I_p}{\pi(R^2 - r^2)}}$ $k_y = \sqrt{\frac{R^2 + r^2}{4}} \quad k_{p_1} = \sqrt{\frac{2I_{p_1}}{\pi(R^2 - r^2)}}$

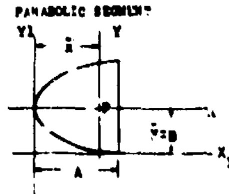
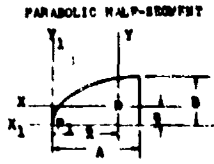
	ELLIPSE	HOLLOW ELLIPSE
		
AREA	πAB	$\pi(AB - CD)$
CENTROID	$\bar{x} = A \quad \bar{y} = B$	$\bar{x} = A \quad \bar{y} = B$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WB^2}{4}$ $I_{x_1} = 1.25\ WB^2$ $I_y = \frac{WA^2}{4}$ $I_{y_1} = 1.25\ WA^2$ $I_p = \frac{W(A^2 + B^2)}{4}$	$I_x = \frac{W}{4}\left[\frac{AB^3 - CD^3}{AB - CD}\right]$ $I_{x_1} = \frac{W}{4}\left[\frac{AB^3 - CD^3}{AB - CD}\right] + WB^2$ $I_y = \frac{W}{4}\left[\frac{A^3B - C^3D}{AB - CD}\right]$ $I_{y_1} = \frac{W}{4}\left[\frac{A^3B - C^3D}{AB - CD}\right] + WA^2$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = \frac{\pi AB^3}{4} = 0.7854\ AB^3$ $I_{x_1} = 1.25\ \pi AB^3 = 3.927\ AB^3$ $I_y = \frac{\pi A^3B}{4} = 0.7854\ A^3B$ $I_{y_1} = 1.25\ \pi A^3B = 3.927\ A^3B$ $I_p = \frac{\pi AB(A^2 + B^2)}{4}$	$I_x = \frac{\pi}{4}(AB^3 - CD^3)$ $I_{x_1} = \frac{\pi}{4}(AB^3 - CD^3) + \pi(AB - CD)(B^2)$ $I_y = \frac{\pi}{4}(A^3B - C^3D)$ $I_{y_1} = \frac{\pi}{4}(A^3B - C^3D) + \pi(AB - CD)(A^2)$ $I_p = I_x + I_y$
RADIUS OF GYRATION	$k_x = \frac{B}{2}$ $k_{x_1} = 1.118\ B$ $k_y = \frac{A}{2}$ $k_{y_1} = 1.118\ A$ $k_p = \frac{\sqrt{A^2 + B^2}}{2}$	$k_x = \sqrt{\frac{AB^3 - CD^3}{4(AB - CD)}}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{\pi(AB - CD)}}$ $k_y = \sqrt{\frac{A^3B - C^3D}{4(AB - CD)}}$ $k_{y_1} = \sqrt{\frac{I_{y_1}}{\pi(AB - CD)}} \quad k_p = \sqrt{\frac{I_p}{\pi(AB - CD)}}$

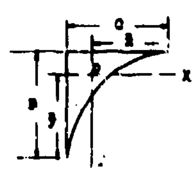
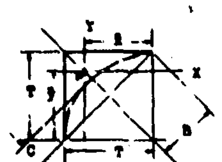
	NOSE RIB	CIRCULAR SECTOR
	 based on parabolic segment	α in radians 
AREA	$\frac{2}{3}A(B + C)$	$R^2\alpha$
CENTROID	$\bar{x} = 0.6A \quad \bar{y} = 0.373(B - C)$	$\bar{x} = \frac{2}{3}\left[\frac{R\sin\alpha}{\alpha}\right] \quad \bar{y} = R\sin\alpha$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{120}(15B^2 + 26BC + 19C^2)$ $I_{x_1} = \frac{W}{5}(B^2 - BC + C^2)$ $I_y = 0.0686WA^2$ $I_{y_1} = 0.4286WA^2$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$	$I_x = \frac{WR^2}{4\alpha}(\alpha - \sin\alpha\cos\alpha)$ $I_{x_1} = I_x + WR^2\sin^2\alpha$ $I_y = \frac{WR^2}{4\alpha}\left(\alpha - \frac{16\sin^2\alpha}{9\alpha} + \frac{\sin 2\alpha}{2}\right)$ $I_{y_1} = \frac{WR^2}{4\alpha}(\alpha + \sin\alpha\cos\alpha)$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = \frac{A(B + C)}{480}(15B^2 + 26BC + 15C^2)$ $I_{x_1} = 0.1333(AB + AC)(B^2 - BC + C^2)$ $I_y = 0.0457\ A^3(B + C)$ $I_{y_1} = 0.2857\ A^3(B + C)$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$	$I_x = \frac{R^4}{4}(\alpha - \sin\alpha\cos\alpha)$ $I_{x_1} = \frac{R^4}{4}(\alpha - \sin\alpha\cos\alpha) + R^4\alpha\sin^2\alpha$ $I_y = \frac{R^4}{4}\left(\alpha - \frac{16\sin^2\alpha}{9\alpha} + \frac{\sin 2\alpha}{2}\right)$ $I_{y_1} = \frac{R^4}{4}(\alpha + \sin\alpha\cos\alpha)$ $I_p = \frac{R^4}{4}\left(2\alpha - \frac{16\sin^2\alpha}{9\alpha}\right)$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{3I_x}{2A(B + C)}}$ $k_y = \sqrt{\frac{3I_y}{2A(B + C)}}$	$k_x = \frac{R}{2}\sqrt{1 - \frac{\sin\alpha\cos\alpha}{\alpha}}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{R^2\alpha}}$ $k_y = \frac{R}{2}\sqrt{1 + \frac{\sin\alpha\cos\alpha}{\alpha} - \frac{16\sin^2\alpha}{9\alpha^2}}$ $k_{y_1} = \frac{R}{2}\sqrt{1 + \frac{\sin\alpha\cos\alpha}{\alpha}}$ $k_p = \frac{R}{2}\sqrt{2 - \frac{16\sin^2\alpha}{9\alpha^2}}$

	QUARTER ELLIPSE	HALF ELLIPSE
		
AREA	$\pi ab/4$	$\pi ab/2$
CENTROID	$\bar{y} = \frac{4a}{3\pi} \quad \bar{x} = \frac{4b}{3\pi}$	$\bar{y} = \frac{4a}{3\pi} \quad \bar{x} = 0$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{4ma^2}{\pi}\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_y = \frac{4mb^2}{\pi}\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_N = \frac{ma^2}{4}$ $I_M = \frac{mb^2}{4}$	$I_x = \frac{2ma^2}{\pi}\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_y = \frac{mb^2}{4}$ $I_N = \frac{ma^2}{4}$
AREA MOMENT OF INERTIA	$I_x = a^3b\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_y = ab^3\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_N = \frac{\pi a^3b}{16}$ $I_M = \frac{\pi ab^3}{16}$	$I_x = a^3b\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_y = \frac{\pi ab^3}{8}$ $I_N = \frac{\pi a^3b}{8}$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$ $k_N = \sqrt{I_N/A}$ $k_M = \sqrt{I_M/A}$	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$ $k_N = \sqrt{I_N/A}$

	CIRCULAR COMPLEMENT	ELLIPTIC COMPLEMENT
		
AREA	$0.2146\ R^2$	$0.2146\ BC$
CENTROID	$\bar{x} = \bar{y} = 0.2234\ R$	$\bar{x} = \frac{C}{1.288} = 0.7766C$ $\bar{y} = \frac{B}{1.288} = 0.7766B$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = 0.038\ WR^2$ $I_{x_1} = I_{y_1} = 0.085\ WR^2$ $I_p = 0.070\ WR^2$ $I_{p_1} = 0.170\ WR^2$	$I_x = 0.038\ WB^2$ $I_y = 0.038\ WC^2$ $I_p = 0.038\ W(B^2 + C^2)$
AREA MOMENT OF INERTIA	$I_x = I_y = 0.0075\ R^4$ $I_{x_1} = I_{y_1} = 0.0183\ R^4$ $I_p = 0.0151\ R^4$ $I_{p_1} = 0.0366\ R^4$	$I_x = 0.0075B^3C$ $I_y = 0.0075BC^3$ $I_p = 0.0075\ BC(B^2 + C^2)$
RADIUS OF GYRATION	$k_x = k_y = 0.186\ R$ $k_{x_1} = k_{y_1} = 0.292\ R$ $k_p = 0.265\ R$ $k_{p_1} = 0.412\ R$	$k_x = 0.186B$ $k_y = 0.148C$ $k_p = \sqrt{0.038(B^2 + C^2)}$

	SEMIELLIPSE	HOLLOW SEMIELLIPSE
		
AREA	$\frac{\pi AB}{2}$	$\frac{\pi(AB-CD)}{2}$
CENTROID	$\bar{x} = A$ $\bar{y} = 0.424\ B$	$\bar{x} = A$ $\bar{y} = \frac{4}{3\pi}\left[\frac{AB^2-CD^2}{AB-CD}\right]$
WEIGHT MOMENT OF INERTIA	$I_x = 0.070\ WB^2$ $I_{x_1} = 1.25\ WB^2$ $I_y = 0.25\ WA^2$ $I_{y_1} = 1.25\ WA^2$ $I_p = \frac{W(A^2 + 0.2802B^2)}{4}$ $I_{p_1} = \frac{W(5A^2 + B^2)}{4}$	$I_x = \frac{W}{4}\left(\frac{AB^3-CD^3}{AB-CD}\right) - W\frac{4}{3\pi}\left[\frac{AB^2-CD^2}{AB-CD}\right]^2$ $I_{x_1} = \frac{W}{4}\left(\frac{AB^3-CD^3}{AB-CD}\right)$ $I_y = \frac{W}{4}\left(\frac{A^3B-C^3D}{AB-CD}\right)$ $I_{y_1} = \frac{W}{4}\left(\frac{A^3B-C^3D}{AB-CD}\right) + WA^2$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = 0.1098\ AB^3$ $I_{x_1} = 0.3927\ AB^3$ $I_y = 0.3927\ A^3B$ $I_{y_1} = 1.9635\ A^3B$ $I_p = AB\ (0.3927\ A^2 + 0.1098\ B^2)$ $I_{p_1} = AB\ (1.9635\ A^2 + 0.3927\ B^2)$	$I_x = \frac{\pi}{8}(AB^3-CD^3) - \frac{\pi(AB-CD)}{2}\left[\frac{4(AB^2-CD^2)}{3\pi(AB-CD)}\right]^2$ $I_{x_1} = \frac{\pi}{8}(AB^3-CD^3)$ $I_y = \frac{\pi}{8}(A^3B-C^3D)$ $I_{y_1} = \frac{\pi}{8}(A^3B-C^3D) + \frac{\pi A^2(AB-CD)}{2}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.2643\ B$ $k_{x_1} = \frac{B}{2}$ $k_y = \frac{A}{2}$ $k_{y_1} = 1.118\ A$ $k_p = \sqrt{\frac{A^2}{4} + \frac{B^2}{4} - \frac{16B^2}{9\pi^2}}$ $k_{p_1} = \sqrt{\frac{2I_{p_1}}{\pi AB}}$	$k_x = \sqrt{\frac{2I_x}{\pi(AB-CD)}}$ $k_{x_1} = \sqrt{\frac{AB^3-CD^3}{4(AB-CD)}}$ $k_y = \sqrt{\frac{2I_y}{\pi(AB-CD)}}$ $k_{y_1} = \sqrt{\frac{2I_{y_1}}{\pi(AB-CD)}}$ $k_p = \sqrt{\frac{2I_p}{\pi(AB-CD)}}$ $k_{p_1} = \sqrt{\frac{2I_{p_1}}{\pi(AB-CD)}}$

	PARABOLIC SEGMENT	PARABOLIC HALF-SEGMENT
		
AREA	$\frac{4}{3}AB$	$\frac{2AB}{3}$
CENTROID	$\bar{x} = 0.6A$ $\bar{y} = B$	$\bar{x} = 0.6A$ $\bar{y} = 0.375B$
WEIGHT MOMENT OF INERTIA	$I_x = 0.2WB^2$ $I_{x_1} = 1.2\ WB^2$ $I_y = 0.0686WA^2$ $I_{y_1} = 0.4286WA^2$ $I_p = I_x + I_y$	$I_x = 0.0594WB^2$ $I_{x_1} = 0.2WB^2$ $I_y = 0.0686WA^2$ $I_{y_1} = 0.4286WA^2$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA	$I_x = 0.2667AB^3$ $I_{x_1} = 1.6AB^3$ $I_y = 0.0914A^3B$ $I_{y_1} = 0.5714A^3B$ $I_p = I_x + I_y$	$I_x = 0.0396AB^3$ $I_{x_1} = 0.1333AB^3$ $I_y = 0.0437A^3B$ $I_{y_1} = 0.2857A^3B$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.4472B$ $k_{x_1} = 1.095B$ $k_y = 0.2619A$ $k_{y_1} = 0.6547A$ $k_p = \sqrt{\frac{3I_p}{4AB}}$	$k_x = 0.2437B$ $k_{x_1} = 0.4472B$ $k_y = 0.2619A$ $k_{y_1} = 0.6547A$ $k_p = \sqrt{\frac{3I_p}{2AB}}$ $k_{p_1} = \sqrt{\frac{3I_{p_1}}{2AB}}$

	COMPLEMENT OF HALF PARABOLA	PARABOLIC FILLET IN RIGHT ANGLE
		
AREA	$\frac{1}{3}BC$	$\frac{T^2}{6}$
CENTROID	$\bar{x} = \frac{3}{4}C$ $\bar{y} = \frac{7}{10}B$	$\bar{x} = \bar{y} = \frac{4}{5}T$ $a = \frac{T}{\sqrt{2}}$ $c = \frac{T}{8\sqrt{2}}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{37}{700}WB^2$ $I_y = \frac{3}{80}WC^2$ $I_p = \frac{W}{20}\left(\frac{37}{35}B^2 + \frac{3C^2}{4}\right)$	$I_x = I_y = \frac{3WT^2}{100}$ $I_p = \frac{3WT^2}{50}$
AREA MOMENT OF INERTIA	$I_x = \frac{37}{2100}B^3C$ $I_y = \frac{1}{80}BC^3$ $I_p = \frac{BC}{80}\left(\frac{37}{105}B^2 + \frac{C^2}{4}\right)$	$I_x = I_y = \frac{11}{2100}T^4$ $I_p = \frac{T^4}{100}$
RADIUS OF GYRATION	$k_x = 0.230\ B$ $k_y = 0.194\ C$ $k_p = \sqrt{\frac{3I_p}{BC}}$	$k_x = k_y = 0.173\ T$ $k_p = 0.2449\ T$

REVISION	DATE	NEW MATERIAL ADDED
A	October 1965	Sub-Section 5.12 Sub-Section 6.2 Sub-Section 6.3 Sub-Section 6.4 Sub-Section 9.0 Sub-Section 12.0

ERRATA

Para. 5.2.4.1
"Values" (not valves) of a. can range . . .

Para. 10.6.2.7
Line 3, spelling should be critical

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1.6-1	5.17.5-4
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3.16-4	13.4.5-1
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REVISION RECORD

REVISION	DATE	NEW MATERIAL ADDED	
B	March 1967	5.7.8-3	6.9.3-10A
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V-327 THE ELECTRADA CORPORATION, STILLMAN DIVISION
Culver City, California

V-329 ARMSTRONG CORK COMPANY, INDUSTRY PRODUCTS DIVISION
Lancaster, Pennsylvania

MANUFACTURER'S SOURCE LIST V-330 TO V-426

V-330 **COOK AIRTOMIC DIVISION, DOVER CORPORATION**
Los Angeles, California

V-338 **THE GARLOCK PACKING COMPANY**
Palmyra, New York

V-339 **AEROQUIP CORPORATION, AIRCRAFT DIVISION**
Jackson, Michigan

V-342 **PORTER SEAL COMPANY**
Glendale, California

V-343 **SEALS, EASTERN INC.**
Red Bank, New Jersey

V-344 **TETRAFLUOR, INC.**
Inglewood, California

V-348 **BERTEA PRODUCTS**
Pasadena, California

V-350 **PURE CARBON COMPANY, INC.**
St. Marys, Pennsylvania

V-351 **FLODAR CORPORATION**
Cleveland, Ohio

V-354 **SERVOTRONICS, INC.**
Buffalo, New York

V-355 **NATIONAL UTILITIES, DIVISION OF NUCO INDUSTRIES, INC.**
Monrovia, California

V-356 **MICROMETRICAL MANUFACTURING COMPANY**
Ann Arbor, Michigan

V-357 **TAYLOR FORGE AND PIPE WORKS**
Bellwood, Illinois

V-374 **ECKEL VALVE COMPANY**
San Fernando, California

V-375 **SMIRRA DEVELOPMENTS COMPANY**
Los Angeles, California

V-422 **CARLETON CONTROLS CORPORATION**
Buffalo, New York

V-424 **WINTEC DIVISION**
Cemarc Corporation
Inglewood, California

V-425 **STERER ENGINEERING AND MANUFACTURING COMPANY**
Los Angeles, California

V-426 **CEMARC DIVISION**
Cemarc Corporation
Inglewood, California

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