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**USER'S MANUAL: EXTENDED CAPABILITY
MAGNUS ROTOR AND BALLISTIC
BODY 6-DOF TRAJECTORY PROGRAM**

ALPHA RESEARCH, INC.

TECHNICAL REPORT AFATL-TR-70-40

MAY 1970

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USER'S MANUAL:

EXTENDED CAPABILITY MAGNUS
ROTOR AND BALLISTIC BODY
6-DOF TRAJECTORY PROGRAM

James E. Brunk

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FOREWORD

The extended Capability Magnus Rotor and Ballistic Body 6-DOF Trajectory Program has been prepared under Contract No. F08635-70-C-0012 with the Air Force Armament Laboratory, Eglin Air Force Base, Florida, by Alpha Research, Inc., Santa Barbara, California. The programmer at Alpha Research, Inc. was Mr. William Davidson. The program monitor for the Armament Laboratory was Mr. Edward S. Sears (ATRA). This effort was conducted during the period 6 October 1969 to 6 April 1970.

This program is a modification of several earlier computer programs prepared for magnus-rotor flight dynamics investigations. The original program was prepared for the U. S. Army Edgewood Arsenal under Contract No. DA-18-108-AMC-236(A). The original program was later adapted to the Air Force Armament Laboratory computer facilities by Air Force personnel, and was used by Alpha Research, Inc. in conjunction with Air Force Contracts Nos. F08635-67-C-0135 and F08635-69-C-0106.

The present modified program is written in General Fortran IV language compatible with third generation computers such as the GE 635, IBM 360, and CDC 6400.

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This technical report has been reviewed and is approved.



CHARLES K. ARPKE, Lt Colonel, USAF
Chief, Technology Division

ABSTRACT

A six-degrees-of-freedom trajectory program for quasi-symmetric rigid bodies is described. The equations of motion are developed such that either a body-fixed or fixed-plane moving coordinate system can be utilized. Provision is made for large angle and angular rate motions, such as are experienced by magnus rotor munitions.

The aerodynamic system permits the usual aeroballistic coefficients to be expressed as tabular functions of angle of attack and Mach number; in addition, the magnus force, magnus moment, and rolling moment coefficients can be tabular functions of the nondimensional spin parameter, $pd/2V$. Additional aerodynamic terms are provided to account for body-fixed aerodynamic asymmetries and/or control inputs, aerodynamic roll angle effects, flow asymmetry with respect to the angle of attack plane at zero spin, and lateral c.g. offset.

The computer program is written in General Fortran IV language compatible with CDC 6400, IBM 360, and GE 635 data processing machines. Included in the report are the program input and output formats, flow charts of the main program subroutines, and a complete program listing.

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LIST OF ABBREVIATIONS AND SYMBOLS

Symbol	Definition	Fortran Equivalent
C_x	Axial force coefficient	CX
C_N	Normal force coefficient	CN
C_M	Overturning moment coefficient	CM
C_{mq}	Damping derivative (α plane)	CMQ
C_{nr}	Damping derivative (Magnus plane)	CNR
$C_{m_{pr}}$	Angular velocity coupling derivative	CMPR
$C_{n_{pq}}$	Angular velocity coupling derivative	CNPQ
C_{N_p}	Magnus force coefficient	CNPA
C_l	Spin torque coefficient	CL
C_{l_p}	Spin damping coefficient	CLP
C_{M_p}	Magnus moment coefficient	CMPA
C_{y_0}	Trim force coefficient along y body-fixed axis	CYO
C_{z_0}	Trim force coefficient along z body-fixed axis	CZO
C_{m_0}	Trim moment coefficient about y body-fixed axis	CMO
C_{n_0}	Trim moment coefficient about z body-fixed axis	CNO
C_{SF_1}	Side force coefficient due to aerodynamic roll angle	CSF1
C_{N_1}	Normal force coefficient due to aerodynamic roll angle	CN1
C_{SF_3}	Side force coefficient due to asymmetric vortices	CSF3
C_{SM_1}	Side moment coefficient due to aerodynamic roll angle,	CSM1
C_{M_1}	Pitching moment coefficient due to aerodynamic roll angle,	CM1
C_{SM_3}	Side moment coefficient due to asymmetric vortices	CSM3
$C_{l_{\dot{\phi}1}}$	Roll moment coefficient due to aerodynamic roll angle,	CLPM1
$C_{l_{\dot{\phi}2}}$	Roll moment coefficient due to aerodynamic roll angle,	CLFM2

LIST OF ABBREVIATIONS AND SYMBOLS (CONTINUED)

Symbol	Definition	Units	Fortran Equivalent
a	Velocity of sound	ft/sec	VOS
d	Aerodynamic reference length (body diameter)	ft	DEE
g	Acceleration due to gravity	ft/sec ²	G
I _x	Axial moment of inertia	slug-ft ²	DIX
I _y	Transverse moment of inertia about y axis	slug-ft ²	DI
I _y = I	Transverse moment of inertia (fixed-plane axes)	slug-ft ²	DI
I _z	Transverse moment of inertia about z axis	slug-ft ²	DIZ
I _{xy}	Product of inertia	slug-ft ²	DIXY
m	Mass	slugs	DMM
M	Mach number		EM
p	Spin rate, angular velocity about x axis	rad/sec	P, Y(4)
pd/2V	Non-dimensional spin parameter		PDV
q	Angular velocity about y axis	rad/sec	Q, Y(5)
r	Angular velocity about z axis	rad/sec	R,Y(6)
S	Aerodynamic reference area	ft ²	S
t	Time	sec	TIME
Δt	Time increment used for integration	sec sec	TSTEP, TNEW
u	Axial velocity in direction of x axis	ft/sec	U, Y(1)
u _A	Aerodynamic velocity in direction of x axis	ft/sec	VA 1
v	Velocity in direction of y axis	ft/sec	V, Y(2)
v _A	Aerodynamic velocity in direction of y axis	ft/sec	VA 2
V	Total velocity	ft/sec	CAPV
V _A	Total aerodynamic velocity	ft/sec	CAPVA

LIST OF ABBREVIATIONS AND SYMBOLS (CONTINUED)

Symbol	Definition	Units	Fortran Equivalent
w	Velocity in direction of z axis	ft/sec	W, Y(3)
w_A	Aerodynamic velocity in direction of z axis	ft/sec	VA 3
X	Horizontal coordinate	ft	X, Y(7)
$\dot{X}$	Velocity in direction of X coordinate	ft/sec	XDOT
$\dot{X}_w$	Wind velocity in direction of X coordinate	ft/sec	WDX
Δy	c. g. lateral offset from axis of symmetry	ft	DY
Y	Horizontal coordinate	ft	Y, Y(8)
$\dot{Y}$	Velocity in direction of Y coordinate	ft	YDOT
$\dot{Y}_w$	Wind velocity in direction of Y coordinate	ft	WDY
Z	Vertical coordinate	ft	Z, Y(9)
$\dot{Z}$	Velocity in direction of Z coordinate	ft	ZDOT
α	Total angle of attack	radians degrees	ALPHA ALD
$\zeta_{1,2}$	Orientation of fins and wings, respectively	degrees radians	ZETD 1, 2 ZET 1, 2
$\eta_{1,2}$	Number of fins and wings, respectively		ETA 1, 2
θ	Euler angle	degrees	THETA
$\dot{\theta}$	Euler angle rate	rad/sec	THD
λ_0	Quaternion		Y(10)
λ_1	Quaternion		Y(11)
λ_2	Quaternion		Y(12)
λ_3	Quaternion		Y(13)
ξ	Orientation of cross velocity	radians	ZETA
ρ	Air density	slug/ft ³	RHO
ϕ	Euler angle	degrees	PHI
$\dot{\phi}$	Euler angle rate	rad/sec	PHD

LIST OF ABBREVIATIONS AND SYMBOLS (CONCLUDED)

Symbol	Definition	Units	Fortran Equivalent
Φ_1	Aerodynamic roll angle of fins	radians degrees	CAPHI 1 CP 1
Φ_2	Aerodynamic roll angle of wings	radians degrees	CAPHI 1 CP 2
ψ	Euler angle	degrees	PSI
$\dot{\psi}$	Euler angle rate	rad/sec	PSD

SECTION I

PROGRAM DESCRIPTION

A. INTRODUCTION

The present computer program has resulted from the need for more exact simulation of the motion of dispenser munitions. Particular attention has been given the simulation of the flight of magnus rotor bomb-lets. The program can also provide trajectory and motion simulation for most unpowered projectiles, missiles, and rockets.

The most significant features of the extended capability trajectory program are:

1. Choice of fixed-plane or body-fixed axes.
2. All-attitude motion prediction.
3. Adaptability to very large spin rates.
4. All basic aeroballistic coefficients are tabular functions of α and M . In addition, spin and magnus coefficients are tabular functions of $pd/2V$.
5. Inclusion of high order nonlinear damping terms.
6. Aerodynamic dependence upon roll angle is included.
7. Provision for aerodynamic and configurational asymmetries, including c. g. lateral offset.
8. Provision for wind.
9. Provision for elimination of high frequency (nutational) motion.

B. COORDINATE SYSTEMS

Either of two moving axes systems can be selected, a body-fixed coordinate system or a fixed-plane coordinate system. Both sets of moving coordinates have as their origin the center of mass of the body.

The inertial reference system is a set of XYZ non-rotating earth-fixed coordinates, with the origin at sea level. The orientation of the moving axes with respect to the inertial axes is given by three Euler angles: θ , ψ , and ϕ , as depicted in Figure 1.

Body-Fixed Coordinates The body-fixed coordinates are comprised of the right-handed orthogonal set of axes, x , y , and z .

Fixed-Plane Coordinates The fixed-plane coordinates are comprised of the right-handed orthogonal set of axes x , y' , and z' . In the fixed-plane system, the y' axis is initially in the XY plane and is so restrained as to stay in that plane. Consequently, $\phi = \dot{\phi} = 0$ for the fixed-plane coordinate system.

Since the body can rotate with respect to the $x y' z'$ axes, this axes system is restricted to bodies with rotational symmetry.

C. ATTITUDE REPRESENTATION

The orientation of the moving coordinates, for input and output purposes, is described by the Euler angles. For computational purposes, however, an angular orientation scheme based on quaternions is used. The quaternion system avoids the discontinuities which occur in the trigonometric functions and angular rates when the motion passes from quadrant to quadrant.

The scalar relations between the four parameter quaternion system and the three Euler angles are defined for the body-fixed and fixed-plane system as follows:⁽¹⁾

Body-fixed axes:

$$\lambda_0 = \cos(\phi/2) \cos(\theta/2) \cos(\psi/2) + \sin(\phi/2) \sin(\theta/2) \sin(\psi/2)$$

$$\lambda_1 = \sin(\phi/2) \cos(\theta/2) \cos(\psi/2) - \cos(\phi/2) \sin(\theta/2) \sin(\psi/2)$$

$$\lambda_2 = \cos(\phi/2) \sin(\theta/2) \cos(\psi/2) + \sin(\phi/2) \cos(\theta/2) \sin(\psi/2)$$

$$\lambda_3 = \cos(\phi/2) \cos(\theta/2) \sin(\psi/2) - \sin(\phi/2) \sin(\theta/2) \cos(\psi/2)$$

Note: x is body longitudinal axis
or axis of rotational
symmetry

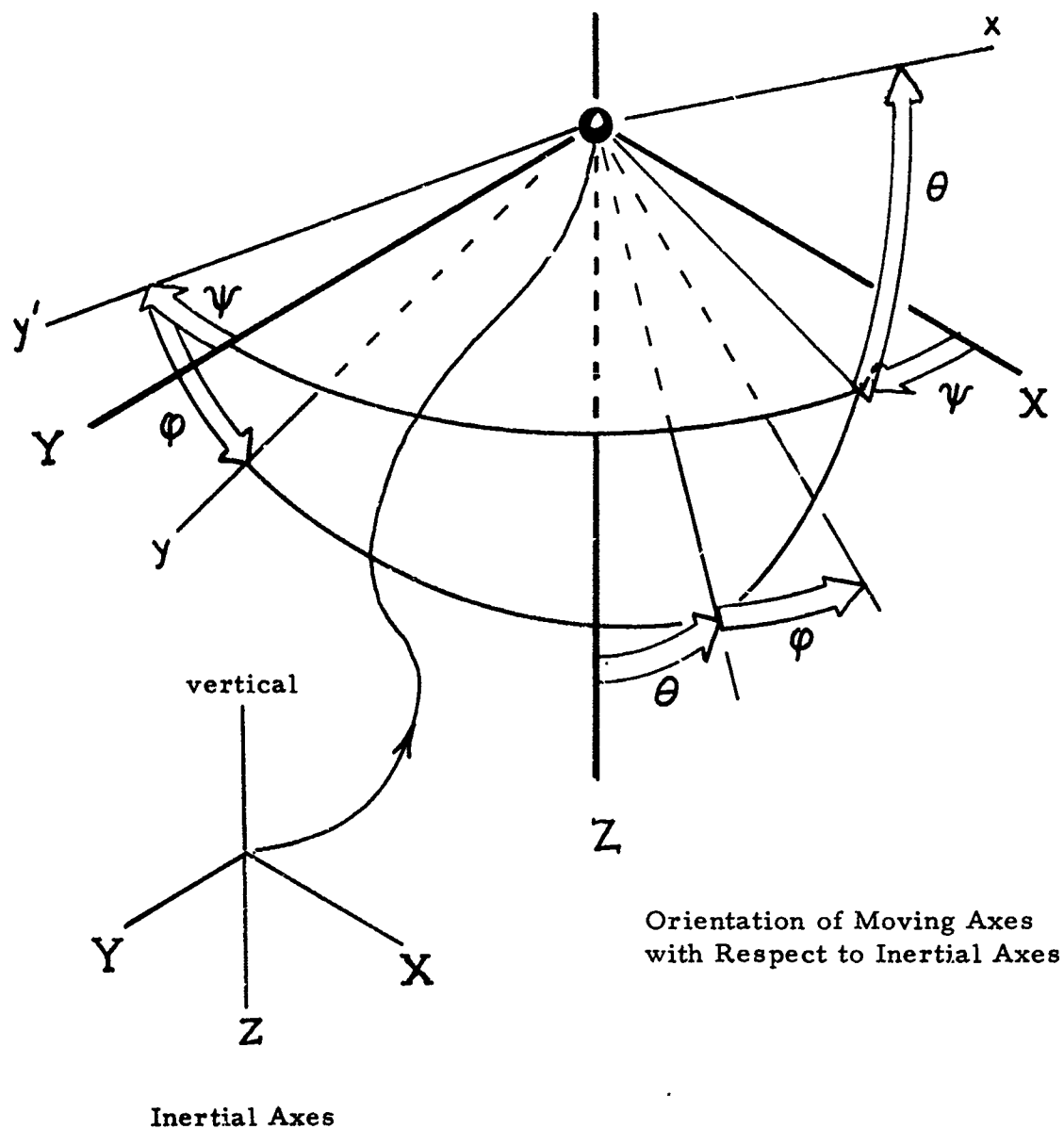


Figure 1. Coordinate Systems

Fixed-plane axes:

$$\lambda_0 = \cos(\theta/2) \cos(\psi/2)$$

$$\lambda_1 = -\sin(\theta/2) \sin(\psi/2)$$

$$\lambda_2 = \sin(\theta/2) \cos(\psi/2)$$

$$\lambda_3 = \cos(\theta/2) \sin(\psi/2)$$

Euler angles:

$$\sin \theta = 2(\lambda_0 \lambda_2 - \lambda_1 \lambda_3) \quad ; \quad -\pi/2 \leq \theta \leq \pi/2$$

$$\tan \theta = \frac{2(\lambda_0 \lambda_2 - \lambda_1 \lambda_3)}{\sqrt{[2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1)]^2 + [\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2]^2}}$$

$$\tan \psi = \frac{2(\lambda_1 \lambda_2 + \lambda_0 \lambda_3)}{\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2} \quad ; \quad -\pi < \psi \leq \pi$$

$$\tan \varphi = \frac{2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1)}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \quad ; \quad -\pi < \varphi \leq \pi$$

Rotation Matrices In the equations of motion, transformations from the moving axes to the fixed inertial axes are required. These can be expressed (for either axis system) by the general quaternion rotation matrix given below, where the subscripts F and M refer to the fixed inertial axes and the moving axis, respectively.

$$\begin{bmatrix} 0 \\ \bar{\xi}_{F_1} \\ \bar{\xi}_{F_2} \\ \bar{\xi}_{F_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2 & 2(\lambda_1 \lambda_2 - \lambda_0 \lambda_3) & 2(\lambda_1 \lambda_3 + \lambda_0 \lambda_2) \\ 0 & 2(\lambda_1 \lambda_2 + \lambda_0 \lambda_3) & \lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2 & 2(\lambda_2 \lambda_3 - \lambda_0 \lambda_1) \\ 0 & 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) & 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) & \lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2 \end{bmatrix} \begin{bmatrix} 0 \\ \bar{\xi}_{M_1} \\ \bar{\xi}_{M_2} \\ \bar{\xi}_{M_3} \end{bmatrix}$$

D. BASIC EQUATIONS OF MOTION

The equations of motion consist of 13 differential equations for the variables

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\lambda}_0 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = \text{Equations of Motion}$$

which must be integrated to obtain the desired motion solution and trajectory. The variables u , v , and w are the linear velocity components in the direction of the moving axes, and p , q , and r are the angular velocity components corresponding to the moving axes. Following standard notation, these components will be with respect to the x , y , z or x' , y' , z' , depending upon whether the moving axes are body-fixed or fixed-plane, respectively. It follows that p and u are identical in the two coordinate systems, but v , w , q , and r are not.

The derivation of the fixed-plane equations of motion must, of necessity, consider the simplified inertial tensor

$$\begin{bmatrix} I_x & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} = [I]_{F.P.}$$

The body-fixed equations of motion are more general.
Provision is made for bodies with an inertia tensor *

$$\begin{bmatrix} I_x & I_{xy} & 0 \\ I_{yx} & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} = [I]_{B.F.}$$

corresponding to the xy plane as a plane of mirror symmetry. This is sufficient for most simulation work.

From basic mechanics, we let Ω represent the angular velocity vector of the coordinate system with respect to the inertial system and ω represent the angular velocity of the body with respect to the moving coordinates, and the fundamental equations of motion become (for constant mass and inertia)

$$F = -mg + m\dot{V} + \Omega \times mV$$

$$M = I\dot{\omega} + \Omega \times [I]\omega$$

where $F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}_{B.F.} = \begin{bmatrix} F_x \\ F_{y'} \\ F_{z'} \end{bmatrix}_{F.P.} = \text{aerodynamic force}$

$$M = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{B.F.} = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{F.P.} = \text{aerodynamic moment}$$

$$g = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}_{B.F.} = \begin{bmatrix} g_x \\ g_{y'} \\ g_{z'} \end{bmatrix}_{F.P.} = \text{gravitational acceleration}$$

$$V = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_{B.F.} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_{F.P.} = \text{velocity}$$

* All products of inertia are positive quantities, $I_{xy} = I_{yx}$.

$$\Omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{B.F.} = \begin{bmatrix} r \tan \theta \\ q \\ r \end{bmatrix}_{F.P.} = \text{angular velocity of coordinates}$$

$$\omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{B.F.} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{F.P.} = \text{angular velocity of body}$$

$$\text{and } \tan \theta = \frac{2(\lambda_0 \lambda_2 - \lambda_1 \lambda_3)}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2}$$

Substitution of the appropriate quantities into the fundamental equations of motion results in the scalar equations for $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{p}$, $\dot{q}$, and $\dot{r}$. The differential equations for $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are obtained by transformation of the components u , v , w . Finally, the derivatives of the quaternions are obtained from (1)

$$\dot{\lambda} = \frac{1}{2} \lambda * \Omega$$

where $*$ denotes a non-commutative quaternion product,

$$a * b = (a_0 + a_1 i + a_2 j + a_3 k) (b_0 + b_1 i + b_2 j + b_3 k)$$

which can be expressed in matrix form as

$$a * b = \begin{bmatrix} (a_0) & (-a_1) & (-a_2) & (-a_3) \\ (a_1) & (a_0) & (-a_3) & (a_2) \\ (a_2) & (a_3) & (a_0) & (-a_1) \\ (a_3) & (-a_2) & (a_1) & (a_0) \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The resulting equations of motion are summarized in Tables I and II for the body-fixed and fixed-plane axes, respectively.

Note that a singularity exists in the fixed-plane equations of motion for $\theta = \pm \pi/2$, which precludes the use of these equations of motion when θ may approach $\pi/2$.

TABLE I. BODY-FIXED EQUATIONS OF MOTION

$$\dot{u} = r v - p \omega + 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) g + F_x / m$$

$$\dot{v} = p \omega - r u + 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) g + F_y / m$$

$$\dot{\omega} = q u - p v + (\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2) g + F_z / m$$

$$\begin{aligned} \dot{p} &= \frac{I_{xy}}{I_x} (\dot{q} - p r) - \left(\frac{I_x - I_y}{I_x} \right) q r + \frac{L}{I_x} \\ &= \frac{-[(I_x + I_y - I_z) I_{xy} r] p + [I_{xy}^2 + I_y (I_y - I_z)] q \cdot r + I_y \cdot L + I_{xy} M}{I_x I_y - I_{xy}^2} \end{aligned}$$

$$\begin{aligned} \dot{q} &= \frac{I_{xy}}{I_y} (\dot{p} + q \cdot r) - \left(\frac{I_x - I_z}{I_y} \right) p r + \frac{M}{I_y} \\ &= \frac{[(I_x + I_y - I_z) I_{xy} \cdot r] q - [I_{xy}^2 + I_x (I_x - I_z)] p \cdot r + I_{xy} \cdot L + I_x \cdot M}{I_x I_y - I_{xy}^2} \end{aligned}$$

$$\dot{r} = \frac{(p^2 - q^2) I_{xy} + (I_x - I_y) p \cdot q + N}{I_z}$$

$$\dot{X} = (\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2) u + 2(\lambda_1 \lambda_2 - \lambda_0 \lambda_3) v + 2(\lambda_1 \lambda_3 + \lambda_0 \lambda_2) \omega$$

$$\dot{Y} = 2(\lambda_1 \lambda_2 + \lambda_0 \lambda_3) u + (\lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2) v + 2(\lambda_2 \lambda_3 - \lambda_0 \lambda_1) \omega$$

$$\dot{Z} = 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) u + 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) v + (\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2) \omega$$

$$\dot{\lambda}_0 = \frac{1}{2} (-\lambda_1 p - \lambda_2 q - \lambda_3 r)$$

$$\dot{\lambda}_1 = \frac{1}{2} (\lambda_0 p - \lambda_3 q + \lambda_2 r)$$

$$\dot{\lambda}_2 = \frac{1}{2} (\lambda_3 p + \lambda_0 q - \lambda_1 r)$$

$$\dot{\lambda}_3 = \frac{1}{2} (-\lambda_2 p + \lambda_1 q + \lambda_0 r)$$

TABLE II. FIXED-PLANE EQUATIONS OF MOTION

$$\dot{u} = r v - g w + 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) g + F_x / m$$

$$\dot{v} = 2 w \frac{\lambda_1 \lambda_3 - \lambda_0 \lambda_2}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} - r u + \frac{F_y}{m}$$

$$\dot{w} = g u - 2 v r \frac{\lambda_1 \lambda_3 - \lambda_0 \lambda_2}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} + (\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2) g + \frac{F_z}{m}$$

$$\dot{p} = \frac{L}{I_x}$$

$$\dot{q} = r \left(2 r \frac{\lambda_1 \lambda_3 - \lambda_0 \lambda_2}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} - \frac{I_x}{I} p \right) + \frac{M}{I}$$

$$\dot{r} = -g \left(2 r \frac{\lambda_1 \lambda_3 - \lambda_0 \lambda_2}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} - \frac{I_x}{I} p \right) + \frac{N}{I}$$

$$\dot{X} = (\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2) u + 2(\lambda_1 \lambda_2 - \lambda_0 \lambda_3) v + 2(\lambda_1 \lambda_3 + \lambda_0 \lambda_2) w$$

$$\dot{Y} = 2(\lambda_1 \lambda_2 + \lambda_0 \lambda_3) u + (\lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2) v + 2(\lambda_2 \lambda_3 - \lambda_0 \lambda_1) w$$

$$\dot{Z} = 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) u + 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) v + (\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2) w$$

$$\dot{\lambda}_0 = -\frac{1}{2} \left(\lambda_2 g + \frac{\lambda_3 r}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \right)$$

$$\dot{\lambda}_1 = -\frac{1}{2} \left(\lambda_3 g + \frac{\lambda_2 r}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \right)$$

$$\dot{\lambda}_2 = \frac{1}{2} \left(\lambda_0 g + \frac{\lambda_1 r}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \right)$$

$$\dot{\lambda}_3 = \frac{1}{2} \left(\lambda_1 g + \frac{\lambda_0 r}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \right)$$

E. BASIC AERODYNAMIC SYSTEM

Basic Aeroballistic Coefficients The aerodynamic forces and moments are expressed in coefficient form, using an aeroballistic system consistent with that which is used for symmetric missiles.⁽²⁾ The basic aeroballistic coefficients are valid for either the body-fixed or fixed-plane coordinate systems, and only require that the configuration have trigonal or greater rotational symmetry. These basic aeroballistic coefficients are given below:

$$C_x (\vec{\alpha}, M) = \text{axial force coefficient}$$

$$C_N (\vec{\alpha}, M) = \text{normal force coefficient}$$

$$C_M (\vec{\alpha}, M) = \text{overturning moment coefficient}$$

$$C_{M_q} (\vec{\alpha}, M) = \text{pitch damping coefficient based on } \frac{qd}{2V}$$

$$C_{N_p} (\vec{\alpha}, M, \frac{pd}{2V}) = -C_{L_p} (\vec{\alpha}, M, \frac{pd}{2V}) = \text{magnus force coefficient for body-fixed and fixed-plane axes, respectively.}$$

$$C_{M_p} (\vec{\alpha}, M, \frac{pd}{2V}) = \text{magnus moment coefficient}$$

$$C_l (\vec{\alpha}, M, \frac{pd}{2V}) = \text{spin torque coefficient due to canted fins or vanes}$$

$$C_{l_p} (\vec{\alpha}, M, \frac{pd}{2V}) = \text{spin damping coefficient}$$

The above coefficients depend only upon the total angle of attack, $\vec{\alpha}$, and are independent of the components of the angle of attack.* These coefficients are also functions of Mach number, and in addition, C_{N_p} , C_{M_p} , C_l , and C_{l_p} are permitted to be functions of the nondimensional spin parameter, $pd/2V$.

The sense of the basic aeroballistic forces and moments is indicated in Figure 2. The aeroballistic forces and moments are transformed to the forces and moments corresponding to the moving coordinates

* The total angle of attack is the angle between x axis and the aerodynamic velocity vector.

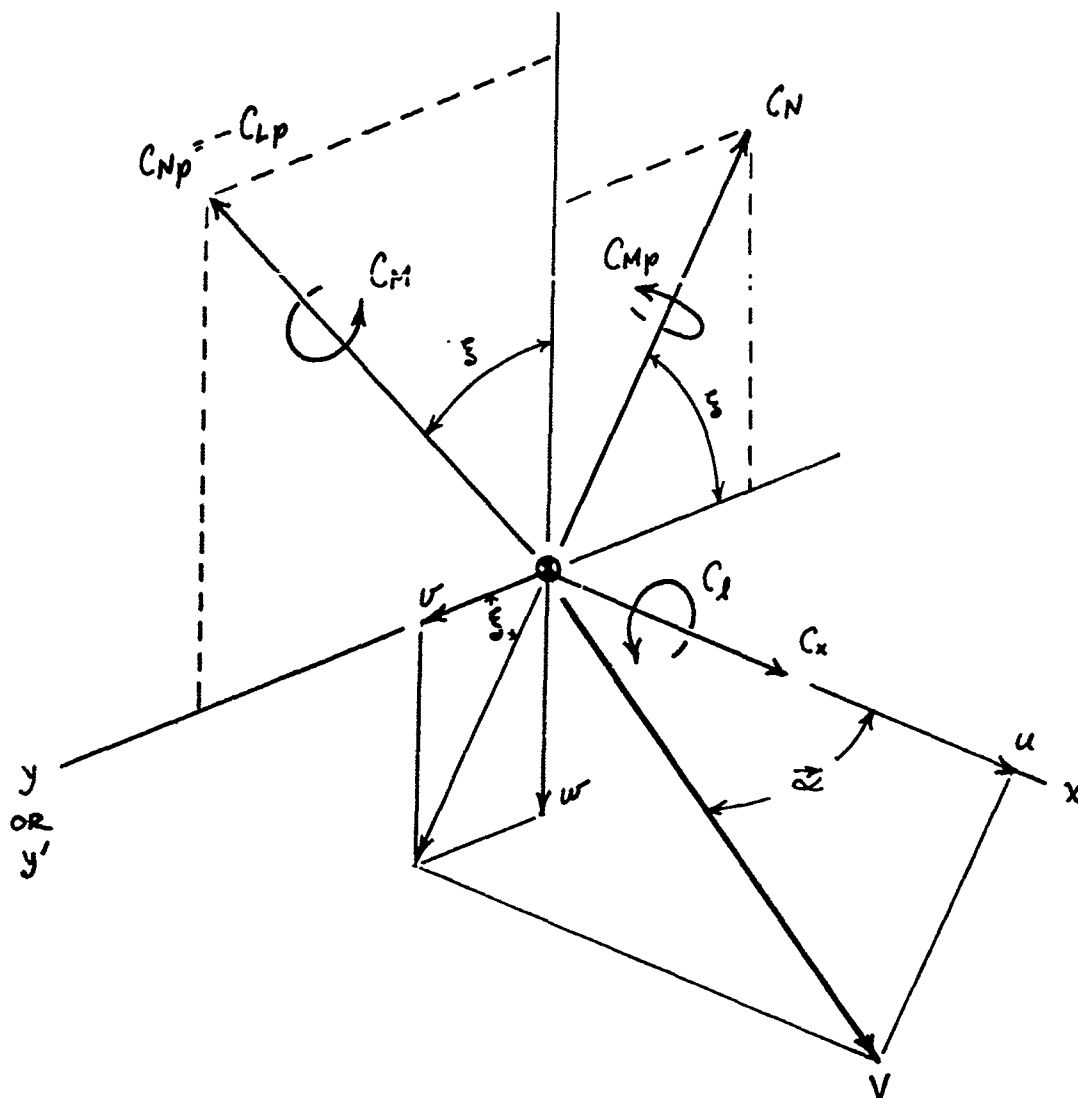


Figure 2. Aeroballistic Force and Moment Definitions

by the matrices

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} F_x \\ F_{y'} \\ F_{z'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (-\cos \xi) & (\sin \xi) \\ 0 & (-\sin \xi) & (-\cos \xi) \end{bmatrix} \begin{bmatrix} C_x \\ C_N \\ C_{N_p} \end{bmatrix} \cdot \frac{1}{2} \rho v^2 S$$

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (\sin \xi) & (\cos \xi) \\ 0 & (-\cos \xi) & (\sin \xi) \end{bmatrix} \begin{bmatrix} C_l \\ C_M \\ C_{M_p} \end{bmatrix} \cdot \frac{1}{2} \rho v^2 S d$$

where

$$\cos \xi = \frac{u}{\sqrt{u^2 + w^2}}$$

$$\sin \xi = \frac{w}{\sqrt{u^2 + w^2}}$$

Additional Damping Parameters In general, the cross-angular velocity does not coincide with the plane of the total angle of attack. For these nonplanar motions, it is now generally accepted that the aerodynamic damping can differ from that for a planar motion.⁽³⁾ The present trajectory program accounts for the nonplanar motion damping by dividing the cross-angular velocity into components q' and r' , which are coincident with, and normal to, the angle of attack plane, respectively. The nonplanar damping contribution due to r' is incorporated by use of an additional damping coefficient, C_{nr} . In a similar manner, the coupling coefficients, $C_{m_{pr}}$ and $C_{n_{pq}}$, are introduced. The sense of these coefficients is depicted in Figure 3.

The moments corresponding to the moving axes are determined by the double rotation matrices:

$$\begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} (\sin \xi) & (\cos \xi) \\ (-\cos \xi) & (\sin \xi) \end{bmatrix} \begin{bmatrix} (\sin \xi) & (-\cos \xi) \\ (\cos \xi) & (\sin \xi) \end{bmatrix} \begin{bmatrix} (\frac{qd}{2V}) \cdot C_{mq} \\ (\frac{rd}{2V}) \cdot C_{nr} \end{bmatrix} \cdot \frac{1}{2} \rho v^2 S d$$

$$\begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} (\sin \xi) & (\cos \xi) \\ (-\cos \xi) & (\sin \xi) \end{bmatrix} \begin{bmatrix} (\sin \xi) & (\cos \xi) \\ (-\cos \xi) & (\sin \xi) \end{bmatrix} \begin{bmatrix} (\frac{rd}{2V}) \cdot C_{m_{pr}} \\ (\frac{qd}{2V}) \cdot C_{n_{pq}} \end{bmatrix} \cdot \frac{1}{2} \rho v^2 S d$$

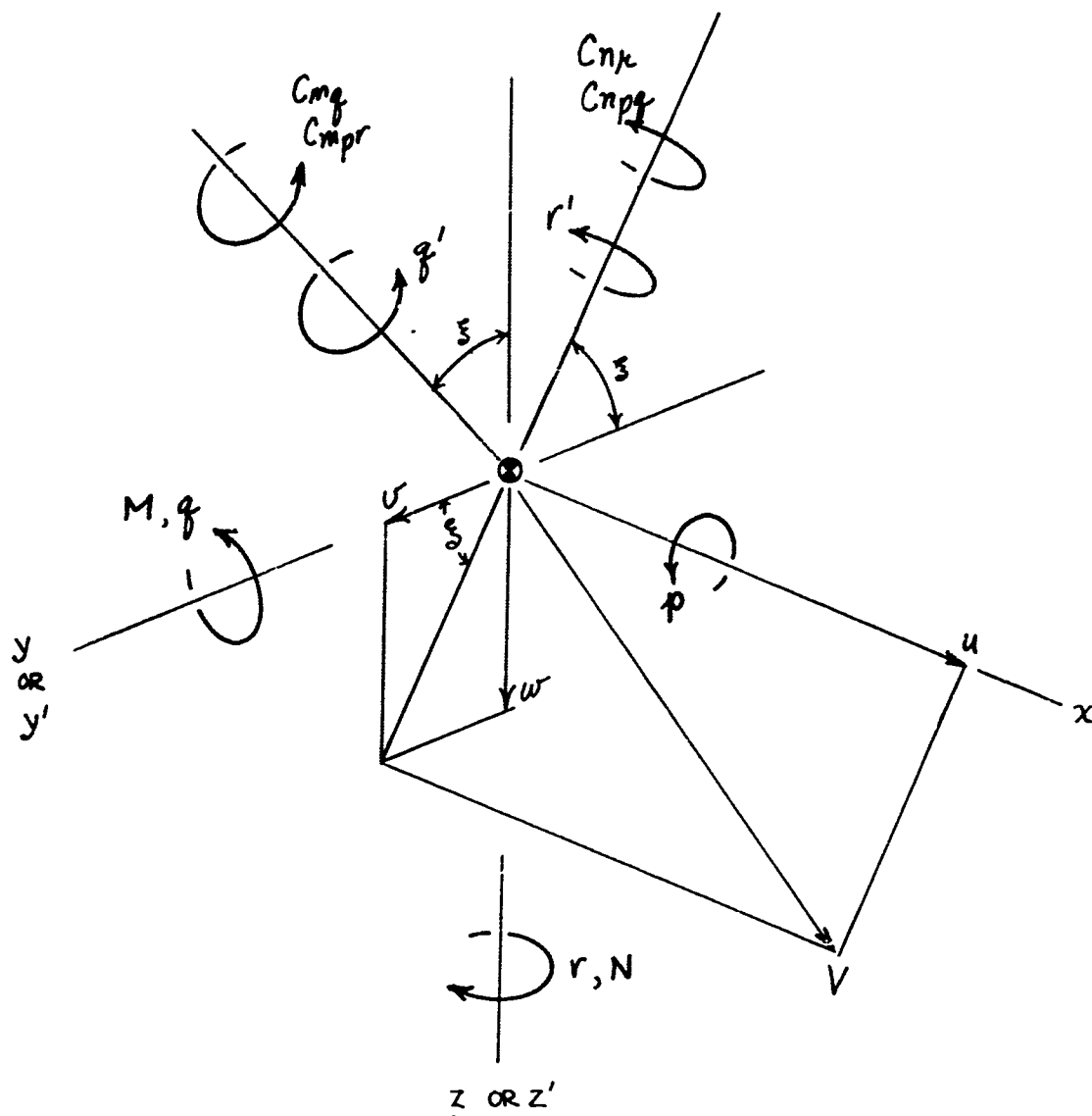


Figure 3. Damping Parameter Definitions

The damping coefficients, C_{n_r} , $C_{m_{pr}}$, and $C_{n_{pq}}$, are functions of total angle of attack and Mach number, as summarized below:

$C_{n_r} (\vec{\alpha}, M) =$ non planar damping coefficient

$C_{m_{pr}} (\vec{\alpha}, M) =$ cross damping moment - angle of attack plane

$C_{n_{pq}} (\vec{\alpha}, M) =$ cross damping moment - magnus plane.

These coefficients, together with the basic aeroballistic coefficients, comprise the aerodynamic coefficient system for the fixed-plane-axes system.

F. EXTENDED AERODYNAMIC SYSTEM

When the body-fixed-axes option is selected, additional aerodynamic coefficients are included to account for body-fixed asymmetries, windward meridian orientation (aerodynamic roll angle), flow asymmetry, and lateral displacement of the center of gravity. These coefficients are depicted schematically in Figure 4.

Body-Fixed Asymmetries Body-fixed aerodynamic forces and moments due to misalignment, cant, or asymmetry of body and/or lifting surfaces are accounted for by the coefficients

$$C_{y_0} (\vec{\alpha}, M)$$

$$C_{z_0} (\vec{\alpha}, M)$$

$$C_{m_0} (\vec{\alpha}, M)$$

$$C_{n_0} (\vec{\alpha}, M).$$

The aerodynamic effects of misalignment, cant, and asymmetry on the axial force and rolling moment are accounted for by the basic aeroballistic coefficients $C_x (\vec{\alpha}, M)$ and $C_l (\vec{\alpha}, M)$.

Aerodynamic Effects Due to Windward Meridian Orientation.

Bodies with wings and fins are subject to aerodynamic effects related to the orientation of the aerodynamic surfaces with respect to the windward meridian of the cross flow. The orientation of the aerodynamic surfaces with respect to the cross flow is specified by the aerodynamic roll angle Φ . This angle is defined as a clockwise rotation of the aerodynamic surfaces with respect to the cross flow, looking in the direction of the x axis, as described by the following sketch. The orientation of the symmetry planes

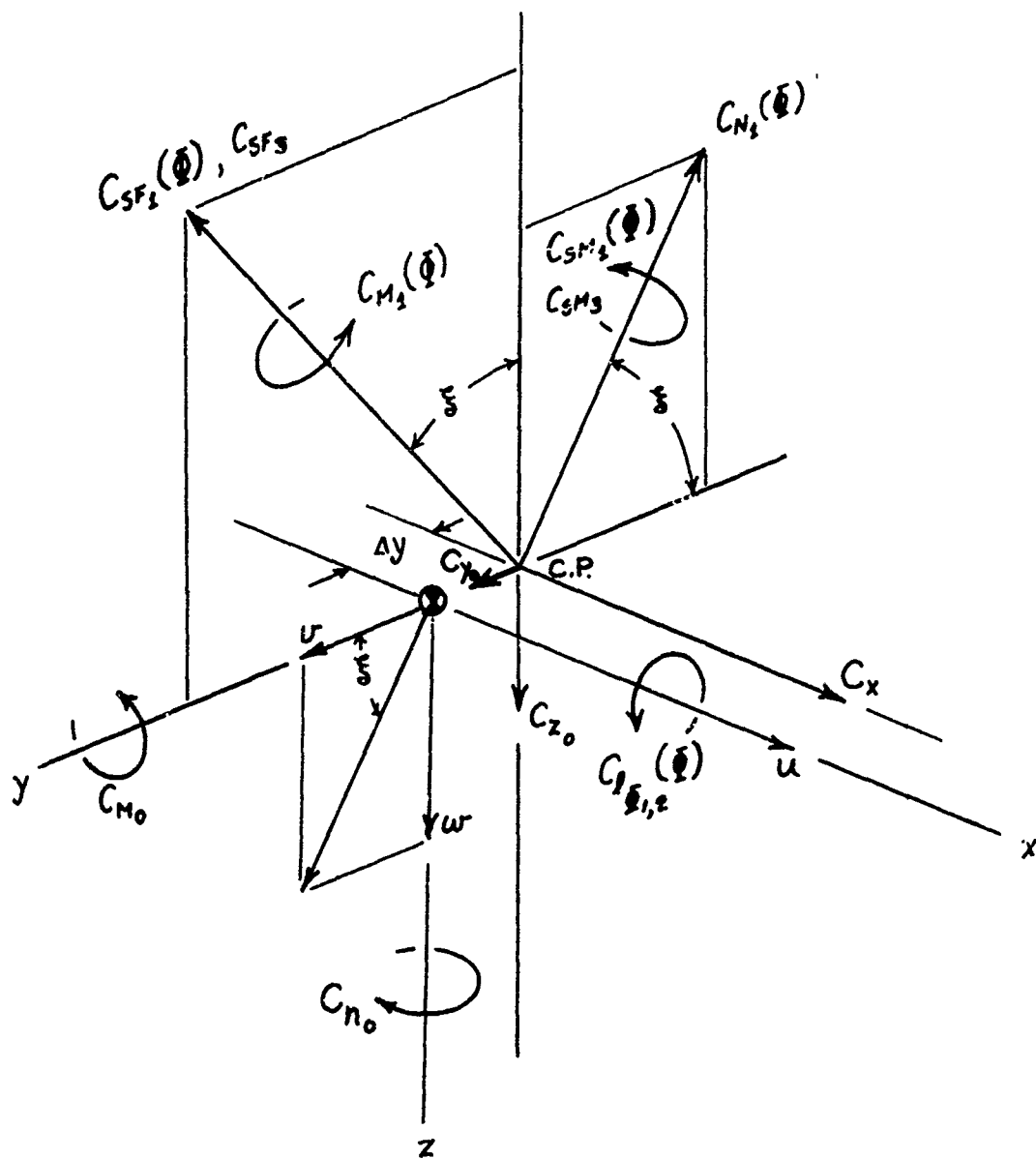
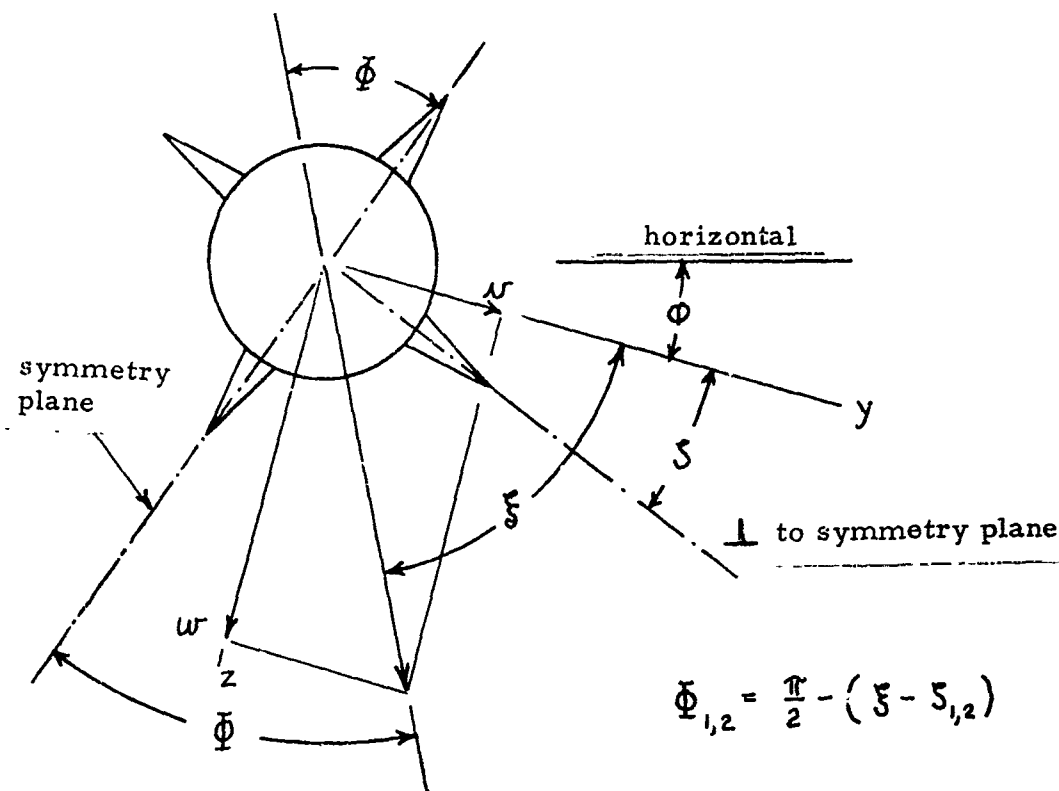


Figure 4. Additional Aerodynamic Forces and Moments for the Body-Fixed Axes Option

of the fins or wings with respect to arbitrary body axes is given by the phase angle ξ . This phase angle may be necessary if body-fixed asymmetries, such as lateral c.g. offset, are also present.



The most significant induced aerodynamic effects resulting from windward meridian are generally the induced rolling moment, the induced side force, and the induced side moment. The two moments often have a profound effect on vehicle stability, and are usually of greatest magnitude for 3 or 4-fin tail assemblies. Likewise, very large induced aerodynamic effects would be expected from a planar surface, such as a wing. In addition, the aerodynamic roll angle will have some effect on the normal force and pitching moment, which must be considered for completeness.

For a single set of aerodynamic surfaces of uniform size, the functional dependence of the aerodynamic forces and moments on the aerodynamic roll angle is given by the following coefficients and functional relationships:

$$C_{SF_1}(\alpha, M, \Phi) = C_{SF_1}(\alpha, M) \sin(\eta_1, \Phi_1)$$

$$C_{N_1}(\alpha, M, \Phi) = C_{N_1}(\alpha, M) \sin(\eta_1, \Phi_1)$$

$$C_{SM_1}(\alpha, M, \Phi) = C_{SM_1}(\alpha, M) \sin(\eta_1, \Phi_1)$$

$$C_{M_1}(\alpha, M, \Phi) = C_{M_1}(\alpha, M) \sin(\eta_1, \Phi_1)$$

$$C_{l_{\Phi_1}}(\alpha, M, \Phi) = C_{l_{\Phi_1}}(\alpha, M) \sin(\eta_1, \Phi_1)$$

where η = number of axially symmetric fins (i.e., cruciform fins, $\eta = 4$).

Although the dependence of C_{SF_1} , C_{N_1} , C_{SM_1} , C_{M_1} , and $C_{l_{\Phi_1}}$ on Φ could be more general, only the first harmonic is used. The principal reasons for this simplification is the usual lack of more definitive wind tunnel data (i.e., usual practice is to measure the near maximum effect of Φ by testing at $\Phi = \pi / 2\eta$).

Limited provision is made for a second set of aerodynamic surfaces with a different η . For the second set of aerodynamic surfaces only the coefficient $C_{l_{\Phi_2}}$ is provided.

Experience has shown that, in general, one set of aerodynamic surfaces will have aerodynamic induced effects which are much larger, and the complete force and moment coefficients should be used with this set of aerodynamic surfaces.

Combined Effect of Geometric Asymmetry and Windward Meridian Orientation The preceding paragraphs describe how provision has been made for both the body-fixed aerodynamic coefficients C_{y_0} , C_{z_0} , C_{m_0} , and C_{n_0} and windward meridian orientation. In practice, a single vehicle modification (or asymmetry) may lead to both effects simultaneously. And it is important to recognize, clearly, how the aerodynamic coefficients should be interpreted.

Consider a body with a portion of the nose sliced off as in Figure 5, such that the xz plane is a plane of mirror symmetry. Then, in addition to the body-fixed trim force $C_{z_0}(\alpha)$ and the normal force $C_N(\alpha)$ it is possible that induced aerodynamic normal and side forces exist. These are $C_{N_1}(\alpha, \Phi)$ and $C_{SF_1}(\alpha, \Phi)$, respectively, and have the vector orientations indicated by Figure 5. Note further that in Figure 5, which depicts a case for which $\eta = 1$, the first harmonic relationship used for C_{N_1} and C_{SF_1} requires that $C_{N_1} = C_{SF_1} \equiv 0$ at $\Phi = 0, \pi$.

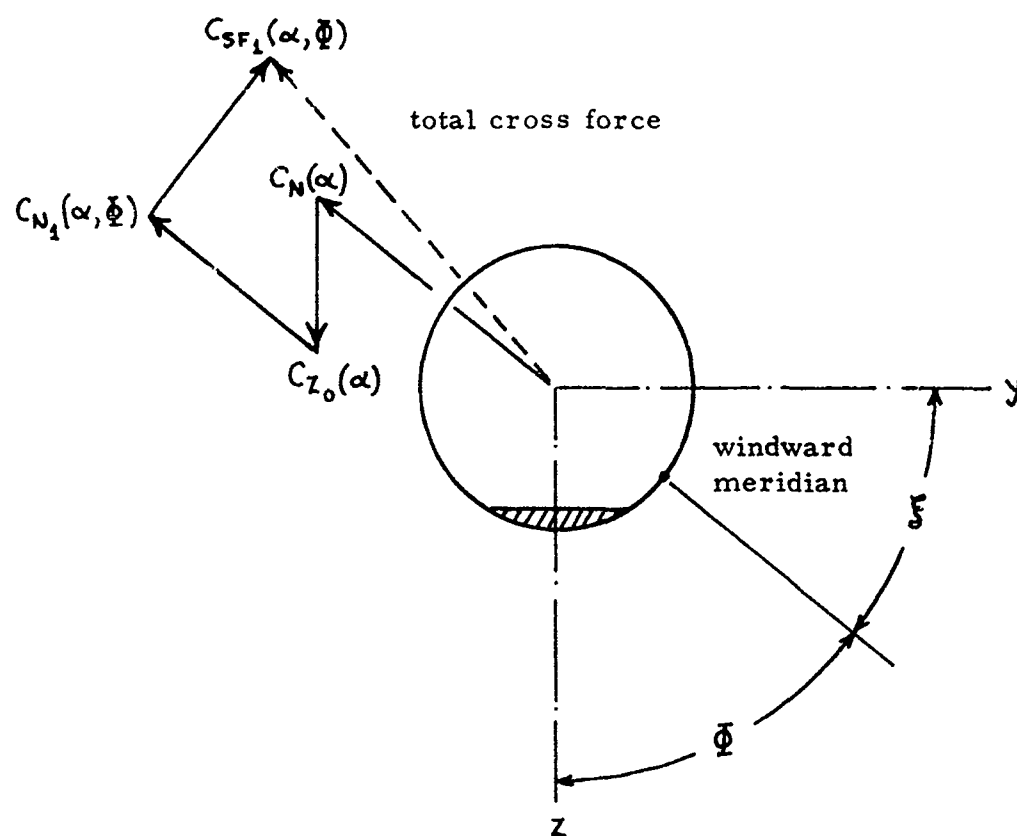


Figure 5. Aerodynamic Cross Force Due to Combined Effect of Geometric Asymmetry and Windward Meridian Orientation

At this point, it is very important to observe that the effect of the body-fixed trim force may be incorrectly interpreted as having a Φ dependence. For example, if the $y z$ axes are fixed and there is a rotation of the windward meridian, then there will occur a corresponding angular rotation between the vectors $C_N(\alpha)$ and $C_{z_0}(\alpha)$, such that the magnitude of the total cross force will vary with Φ . This effect is only due to the variation of the normal force with Φ , and is correctly accounted for in the body-fixed equations of motion, due to the fact that $F_y \propto C_N(\alpha) \cos \xi$ and $F_z \propto C_N(\alpha) \sin \xi$. Thus, in reducing wind tunnel force measurements to determine the effect of Φ , when configurational asymmetries are present, the trim force coefficients C_{y_0} and C_{z_0} must be subtracted first. Similar arguments apply to the aerodynamic moment coefficients.

Additional Aeroballistic Coefficients for Flow Asymmetry

The aerodynamic forces on a pure axi-symmetric body (i. e., body of revolution) with zero spin, are not always symmetric with respect to the angle of attack plane as postulated in the basic aeroballistic formulation. In particular, it has been noted that long pointed-nose bodies tend to have an asymmetric vortex-wake structure at large angles of attack for a range of Reynolds numbers and Mach numbers. Once established, the asymmetrical wake becomes reasonably stable, such that the resulting forces and moments can be considered steady. To include the above effects in the aerodynamic model, the following coefficients and functional relationships are added to the aeroballistic system employed with the body-fixed axes:

$$C_{SF_3}(\alpha, M)$$

$$C_{SM_3}(\alpha, M)$$

It will be noted that these coefficients have the same vector orientation as the magnus force and moment.

Effect of Lateral Displacement of Center of Gravity from the Longitudinal Reference Axis Additional aerodynamic moment terms are introduced into the equations of motion if the center of gravity is laterally displaced from the longitudinal reference axis.

These moments are:

$$\Delta C_l = -(\sum C_z)(\Delta y/d)$$

$$\Delta C_n = C_x(\alpha, M)(\Delta y/d)$$

Only a c. g. offset along the y body-fixed axis will be considered, because all other body-fixed forces and moments are introduced with complete generality. Also, the c. g. offset conforms to the xy plane, which is assumed to be a plane of mirror symmetry.

G. SUMMARY OF AERODYNAMIC FORCES AND MOMENTS

Table III summarizes the aerodynamic forces and moments, in scalar form, for each degree of freedom. Those terms which are added for the body-fixed axes option are enclosed by dashed lines.

H. ATMOSPHERIC MODEL AND WIND SIMULATION

Atmospheric Model Air density, ρ , and velocity of sound, a , are approximated for standard day conditions by the relations:

$$\rho = 0.0023769 [1 + 6.875 \times 10^{-6} (Z)]^{4.2561} \quad (-Z) < 36,000 \text{ ft.}$$

$$\rho = 0.0040 e^{4.806 \times 10^{-5} (Z)} \quad (-Z) > 36,000 \text{ ft.}$$

$$a = 968.46 - 0.004123 (-Z) \quad (-Z) < 36,000 \text{ ft.}$$

$$a = 968.46 \quad (-Z) > 36,000 \text{ ft.}$$

I. ATMOSPHERIC WIND SIMULATION

Atmospheric wind simulation is accomplished by introducing wind vectors $\dot{X}_w$, $\dot{Y}_w$, as a function of altitude $(-Z)$. The aerodynamic velocity components of the body with respect to the moving air mass are

$$(\dot{X} - \dot{X}_w), (\dot{Y} - \dot{Y}_w), \dot{Z}$$

and the corresponding aerodynamic velocity components along the body-axes are designated u_A , v_A , w_A . In the presence of wind, the aerodynamic forces and moments are redefined so as to be functions of

$$V_A = \sqrt{u_A^2 + v_A^2 + w_A^2}$$

$$\alpha = \cos^{-1} \frac{u_A}{V_A}$$

$$M = \frac{V_A}{a}$$

$$\xi = \tan^{-1} \frac{w_A}{v_A}$$

TABLE III. AERODYNAMIC FORCE AND MOMENT EXPANSIONS

$$\begin{aligned}
 F_x &= \frac{1}{2} \rho V^2 S \left\{ C_x(\vec{\alpha}, M) \right\} \quad : \quad \text{Body-Fixed or Fixed Plane Axes} \\
 &\quad \left\{ \quad \quad \quad \right\} \quad \text{Denotes Body-Fixed Axes Only} \\
 F_y &= F_{y'} = \frac{1}{2} \rho V^2 S \left\{ -C_N(\vec{\alpha}, M) - \left[C_{N_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right] \cos \xi + \left\{ \left[\frac{-C_{L_p}}{C_{N_p}}(\vec{\alpha}, M, \frac{p d}{2V}) \right] \left[\frac{p d}{2V} \right] + \left[C_{SF_2}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] + C_{SF_3}(\vec{\alpha}, M) \right] \sin \xi + C_{y_0}(\vec{\alpha}, M) \right\} \right. \\
 &\quad \left. - \left[C_{N_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right] \cos \xi - \left\{ \left[\frac{-C_{L_p}}{C_{N_p}}(\vec{\alpha}, M, \frac{p d}{2V}) \right] \left[\frac{p d}{2V} \right] + \left[C_{SF_2}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] + C_{SF_3}(\vec{\alpha}, M) \right] \cos \xi + C_{z_0}(\vec{\alpha}, M) \right\} \right. \\
 &\quad \left. - \left[C_{N_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right] \sin \xi - \left\{ \left[\frac{-C_{L_p}}{C_{N_p}}(\vec{\alpha}, M, \frac{p d}{2V}) \right] \left[\frac{p d}{2V} \right] + \left[C_{SF_2}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] + C_{SF_3}(\vec{\alpha}, M) \right] \cos \xi + C_{z_0}(\vec{\alpha}, M) \right\} \right. \\
 &\quad \left. + \frac{\Delta y}{d} \left\{ \left[C_N(\vec{\alpha}, M) + C_{N_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right] \sin \xi + \left[C_{N_p}(\vec{\alpha}, M, \frac{p d}{2V}) \right] \left[\frac{p d}{2V} \right] + C_{SF_2}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right. \right. \\
 &\quad \left. \left. + C_{SF_3}(\vec{\alpha}, M) \cos \xi - C_{z_0} \right\} \right. \\
 &\quad \left. + \left[C_{N_1}(\vec{\alpha}, M) + C_{N_2}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right] \sin \xi + \left[C_{N_p}(\vec{\alpha}, M, \frac{p d}{2V}) \right] \left[\frac{p d}{2V} \right] \cos \xi + \left[C_{M_q}(\vec{\alpha}, M) \left[\frac{q d}{2V} \right] \sin^2 \xi - \left[\frac{r d}{2V} \right] \cos \xi \sin \xi \right. \\
 &\quad \left. + \left[C_{N_p}(\vec{\alpha}, M) \left[\frac{q d}{2V} \right] \cos^2 \xi + \left[\frac{r d}{2V} \right] \sin \xi \cos \xi \right] + \left[C_{M_{pr}}(\vec{\alpha}, M) \left[\frac{p d}{2V} \right] \cos \xi \sin \xi + \left[\frac{r d}{2V} \right] \sin^2 \xi + \left[C_{N_{pq}}(\vec{\alpha}, M) \left[\frac{p d}{2V} \right] \sin \xi \cos \xi - \left[\frac{r d}{2V} \right] \cos^2 \xi \right] \right. \\
 &\quad \left. + \left[C_{SM_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] + C_{SM_2}(\vec{\alpha}, M) \cos \xi + C_{M_0}(\vec{\alpha}, M) \right] \right. \\
 &\quad \left. - \left[C_{M_1}(\vec{\alpha}, M) - C_{M_2}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] \right] \cos \xi + \left[C_{M_p}(\vec{\alpha}, M, \frac{p d}{2V}) \left[\frac{p d}{2V} \right] \sin \xi - \left[C_{M_q}(\vec{\alpha}, M) \left[\frac{q d}{2V} \right] \sin \xi \cos \xi - \left[\frac{r d}{2V} \right] \cos^2 \xi \right] \right. \\
 &\quad \left. + \left[C_{M_{pr}}(\vec{\alpha}, M) \left[\frac{p d}{2V} \right] \cos \xi \sin \xi + \left[\frac{r d}{2V} \right] \sin^2 \xi \right] - \left[C_{M_{pr}}(\vec{\alpha}, M) \left[\frac{p d}{2V} \right] \cos^2 \xi + \left[\frac{r d}{2V} \right] \sin \xi \cos \xi \right] + \left[C_{N_{pq}}(\vec{\alpha}, M) \left[\frac{p d}{2V} \right] \left[\frac{q d}{2V} \right] \sin^2 \xi - \left[\frac{r d}{2V} \right] \cos \xi \sin \xi \right. \\
 &\quad \left. + \left[C_{SM_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] + C_{SM_2}(\vec{\alpha}, M) \cos \xi + C_{M_0}(\vec{\alpha}, M) \right] \sin \xi + \left[C_{SM_1}(\vec{\alpha}, M) \sin[\eta_1 \Phi_1] + C_{SM_2}(\vec{\alpha}, M) \cos \xi + C_{M_0}(\vec{\alpha}, M) \right] \frac{\Delta y}{d} \right\}
 \end{aligned}$$

where

$$\sin \xi = \frac{\omega_A}{\sqrt{\nu_A^2 + \omega_A^2}}$$

$$\cos \xi = \frac{\nu_A}{\sqrt{\nu_A^2 + \omega_A^2}}$$

The above relations are of the same algebraic form as in the case of zero wind.

The transformation from the inertial wind aerodynamic velocities to the body-axes aerodynamic velocities is given by

$$\begin{bmatrix} \nu_A \\ \omega_A \end{bmatrix} = \begin{bmatrix} \lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2 & 2(\lambda_1\lambda_2 + \lambda_0\lambda_3) & 2(\lambda_1\lambda_3 - \lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2 - \lambda_0\lambda_3) & \lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2 & 2(\lambda_2\lambda_3 + \lambda_0\lambda_1) \\ 2(\lambda_1\lambda_3 + \lambda_0\lambda_2) & 2(\lambda_2\lambda_3 - \lambda_0\lambda_1) & \lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2 \end{bmatrix} \begin{bmatrix} \dot{X} - \dot{X}_w \\ \dot{Y} - \dot{Y}_w \\ \dot{Z} - \dot{Z}_w \end{bmatrix}$$

J. QUATERNION NORMALIZATION AND QUATERNION ERROR

The separate integration of each quaternion element, $\lambda_0 \rightarrow \lambda_3$, leads to the possibility of small errors, such that $\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \neq 1$. To insure normalization ($\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \equiv 1$), the following correction is made to each quaternion element after integration:

$$\lambda_i^* = \frac{\lambda_i}{\sqrt{\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

The above correction scheme is derived from the transformation relationships between the quaternions and the three Euler angles, which uniquely describe the moving-axis-system orientation. The method results in an exact normalized quaternion. However, in the case of the fixed-plane axes, the additional identity, $\lambda_0\lambda_1 + \lambda_2\lambda_3 \equiv 0$, which applies for this case, may not be satisfied. Consequently, the body-fixed axes should be used in preference to the fixed-plane axes whenever the rotational rates and frequencies permit.

The quaternion error, ϵ , is defined as the vector sum of the errors of the normalized quaternion elements after successive predict-correct or correct and re-correct integration operations: i. e.,

$$\epsilon = \sqrt{\sum \left[(\lambda_i)_{\text{SAVE}} - (\lambda_i)_o \right]^2}$$

where

$(\lambda_i)_{\text{SAVE}}$ = normalized value of λ_i after prediction (or correction)

$(\lambda_i)_o$ = normalized value of λ_i after correction (or re-correction)

K. INTEGRATION

Since the differential equations are quite complicated and lengthy, a refined integration scheme is employed. The basic system utilizes Milne's four-point method of prediction and Simpson's rule for correction. The Runge-Kutta method of third order accuracy is used to calculate the second through fourth ordinates needed to start Milne's method. These are described in detail in the flow charts, Section IV.

Options are also provided for the Adams and trapezoidal integration schemes, using four and three ordinates, respectively.

As a further means of reducing integration error, an additional option is provided for up to n corrections and re-corrections, depending upon the magnitude of the quaternion error, ϵ , as defined in the previous section. If $\epsilon > \epsilon_{\text{max}}$, additional corrections and re-corrections will be made until the number of corrections (NCOR) equals the specified maximum number of corrections (NCMAX).

The accumulative integration error for selected variables is provided in the optional program output, based on the errors computed in the last correction or re-correction.

L. OPTIONAL APPROXIMATE EQUATIONS OF MOTION FOR USE WHEN THE NUTATION IS DAMPED

Approximate equations of motion, with $\dot{q} = \dot{r} = 0$, may be selected at a particular time, if the fixed-plane axes option is exercised. In effect, this operation eliminates the high frequency oscillatory motion about the lateral axes, which is usually associated with the nutational mode.

The angular velocities q and r are determined by solution of the differential equations which result with $\dot{q} = \dot{r} = 0$. These modified equations are:

$$r = \frac{\frac{I_x}{I} p - \left[\left(\frac{I_x}{I} p \right)^2 - 8 \left(\frac{\lambda_1 \lambda_3 - \lambda_0 \lambda_2}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \right) \frac{M}{I} \right]^{1/2}}{4 \left(\frac{\lambda_1 \lambda_3 - \lambda_0 \lambda_2}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} \right)}$$

$$q = - \frac{N \cdot r}{M}$$

The minus sign is selected for the radical, corresponding to the slow precessional mode. The positive root represents a flat spin mode with large angular rates and is of little interest.

The optional equations of motion for $\dot{q} = \dot{r} = 0$ are initiated at a time when the nutational oscillations are known to be damped to an amplitude of a few degrees. The resulting motion predictions are valid to the extent that the nutational motion can be neglected. The approximations are invalid if the high frequency motion is undamped.

Because the angular rates will usually be small with $\dot{q} = \dot{r} = 0$, a larger integration interval can be utilized, and an option for this new time interval is included in the program input.

M. AUXILIARY FUNCTIONS

For purposes of program output, the Euler angle rates are specified, although these quantities do not enter into the equations of motion. The following relationships are utilized for computing the Euler angle time derivatives:

$$\dot{\psi} = \frac{2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) \cdot q + (\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2) \cdot r}{1 - [2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2)]^2}$$

$$\dot{\theta} = \frac{(\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2) \cdot q - 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) \cdot r}{\left[1 - [2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2)]^2 \right]^{\frac{1}{2}}}$$

$$\dot{\phi} = p - 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) \cdot \dot{\psi} \quad \text{body-fixed axes}$$

$$\dot{\phi} = \frac{2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) \cdot r}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2} - 2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) \cdot \dot{\psi} \quad \text{fixed-plane axes}$$

SECTION II

INPUT FORMAT AND USER'S INSTRUCTIONS

PHYSICAL MAGNETIC TAPE A2

SYSTF:1 INPUT (A2)

LEAD CARD SETUP

1. LEAD CARD 1 (ALWAYS)- RUN DESCRIPT , AXIS SYST AND PRINT OPTIONS

2. SUBSCRIPT CARD AND INTEGRATION OPTIONS

3. ALPHA VALUES

4. WACH VALUES

5. PD/2V VALUES

6. BASIC AERODYNALLISTIC COEFFICIENT TABLES
CX, CN, CH, CND, CYR, CMPP, CNPQ, CNP, CL , CLP, CMP

7. LEAD CARD 2
G, DMM, DIX, DI, DIZ, DIXY

8. LEAD CARD 3
S, DEE, THETA, PSI, Z, PHI

9. LEAD CARD 4
XDOT, YDOT, ZDOT, P, Q, R

10. LEAD CARD 5
TSTEP, INCPT, TMAX, ZSTOP, TCH, TNEW
(IF WIND OPTION SELECTED ON SUBSCRIPT CARD)

11. WIND CARDS
(IF BODY AXES)

12. LEAD CARD 6
(IF BODY AXES)

13. ADDITIONAL AERODYNAMIC COEFFICIENT TABLES (IF BODY AXES)
CYO, CZO, CMO, CNO, CSF1, CN1, CSF3, CSM1, CM1, CSM3, CLPH1, CLPH2

14. LEAD CARD 7
(OMIT IF NO CASE FOLLOWS)

SUBSCRIPT CARD AND INTEGRATION OPTIONS (OMIT IF COLUMN 1 OF LEAD CARD 7 EQUALS 0)			SCALING
COLUMN	ITEM		
1-2	IMAX	= NUMBER OF ANGLES OF ATTACK, (MAX OF 20)	I2
3-4	JMAX	= NUMBER OF HACH VALUES (MAX OF 5)	I2
5-6	KMAX	= NUMBER OF SPIN (PD/2V) VALUES (MAX OF 5)	I2
7-8	LMAX	= NUMBER OF WIND VALUES (MAX OF 10)	I2
9-10	NRK	= OPTION FOR INTEGRATION ROUTINE 2=TRAPEZOID, 3=ADAMS, 4 OR 0 =MILNES	I2
11-12	NCMAX	= MAXIMUM NUMBER OF CORRECTIONS	I2
13-24	EPSMAX	= MAXIMUM ERROR ON ABSOLUTE VALUE OF GENERAL NORMALIZED QUATERNION	F12.5

NOTE- IF IMAX, JMAX, KMAX, LMAX EQUALS ONE, NO INTERPOLATION
DONE IN THAT DIMENSION, HOWEVER, IF IMAX, ETC = 1,
CORRESPONDING ANGLE OF ATTACK VALUE CARD, ETC MUST
BE INCLUDED WITH ZERO VALUES

NOTE-EPSMAX = 0.0001 WILL BE SATISFACTORY FOR MOST CASES

NOTE- IF LMAX IS ZERO, DELETE WIND TABLES

LEAD CARD 1

COLUMN	ITEM	SCALING
1-3	IRUN = RUN NUMBER (IDENTIFICATION)	I3
4-5	BLANK	
6-71	HEADER = ANY LEGAL ALPHA-NUMERICAL DATA.	11A6
72	BLANK	
73	KA6 = OPTION FOR INTERMEDIATE O/P ON A6. (OPTIONAL PRINT OUT)	I1
	= 0 NO INTERMEDIATE O/P ON A6.	
	= 1 INTERMEDIATE O/P ON A6.	
74	BODFIX = OPTION FOR AXIS SYSTEM (T FOR BODY FIXED, F OR BLANK FOR FIXED PLANE)	L1
75-78	IDATE = DATE (MONTH, DAY)	I4

IRUN MUST BE RIGHT ADJUSTED.

NOTE THAT BASIC AEROBALLISTIC MAGNUS FORCE COEFFICIENT
IS NOT THE SAME FOR BODY-FIXED AND FIXED-PLANE AXES.
ALL OTHER COEFFICIENTS CAN BE USED INTERCHANGEABLY

LEAD CARD 2 (OMIT IF COLUMN 16 OF LEAD CARD 7 EQUALS 3)

COLUMN	ITEMS	SCALING
1-12	G = GRAVITATIONAL CONSTANT (FT/SEC**2)	F12.5
13-24	DMM = BODY MASS (SLUGS)	F12.5
25-36	DIX = AXIAL MOMENT OF INERTIA (SLUG-FT**2)	F12.5
37-48	DI = TRANSVERSE MOMENT OF INERTIA OR IY (SLUG-FT**2)	F12.5
49-60	DIZ = MOMENT OF INERTIA ABOUT Z AXIS (SLUG-FT**2)	F12.5
61-72	DIXY = PRODUCT OF INERTIA-IXY (SLUG-FT**2)	F12.5

LEAD CARD 3
(OMIT IF COLUMN 17 OF LEAD CARD 7 EQUALS 0)

COLUMN	ITEM	SCALING
1-12	S = AERODYNAMIC REFERENCE AREA (FT*2)	F12.5
13-24	DDE = AERODYNAMIC REFERENCE LENGTH (FT)	F12.5
25-36	THETA = INITIAL EULER ATTITUDE (DEG)	F12.5
37-48	PSI = INITIAL EULER ATTITUDE (DEG)	F12.5
49-60	Z = INITIAL VERTICAL COORDINATE (FT)	F12.5
61-72	PHI = INITIAL EULER ATTITUDE (DEG)	F12.5

Z MUST BE NEGATIVE.

LEAD CARD 4
(OMIT IF 18 OF LEAD CARD 7 EQUALS 0)

COLUMN	ITEM	SCALING
1-12	XDOT = INITIAL VELOCITY VECTOR (FT/SEC)	F12.5
13-24	YDOT = INITIAL VELOCITY VECTOR (FT/SEC)	F12.5
25-36	ZDOT = INITIAL VELOCITY VECTOR (VERTICAL) (FT/SEC)	F12.5
37-48	P = INITIAL ANGULAR VELOCITY (RAD/SEC)-SPIN	F12.5
49-60	Q = INITIAL ANGULAR VELOCITY (RAD/SEC)-PITCH	F12.5
61-72	R = INITIAL ANGULAR VELOCITY (RAD/SEC)-YAW	F12.5

LEAD CARD 5
(OMIT IF COLUMN 19 OF LEAD CARD 7 EQUALS 0)

COLUMN	ITEM	SCALING
1-12	TSTEP = COMPUTATION TIME INTERVAL (SEC)	F12.5
13-16	BLANK	
17-24	INCPT1 = PRINT-OUT FREQUENCY (INTERVALS BETWEEN PRINT)	18
25-36	TMAX = TIME STOP	F12.5
37-48	ZSTOP = COORDINATE STOP	F12.5
49-60	TCH = TIME TO BEGIN HOLDING 3 DOT AND R DOT EQUAL TO ZERO AND TO CHANGE INTEGRATION INTERVAL (SEC)	F12.5
61-72	TNEW = NEW INTEGRATION INTERVAL (SEC)	14
73-76	INCPT2 = PRINT OUT FREQUENCY (INTERVALS BETWEEN PRINT)	

INCPT MUST BE GREATER THAN ZERO AND RIGHT ADJUSTED.

LEAD CARD 6
(OMIT IF COLUMN 21 OF LEAD CARD 7 EQUALS 0)

COLUMN	ITEM	SCALING
1-12	DY = C.G. OFFSET (FT)	5F12.5
13-24	ZETD1 = FIN ORIENTATION (DEG)	5F12.5
25-36	ZETD2 = WING OR NOSE ORIENTATION (DEG)	5F12.5
37-48	ETA1 = NO. OF FINS (WINGS)	5F12.5
49-60	ETA2 = NO. OF WINGS (FINS)	5F12.5

LEAD CARD 7
(OMIT IF NO CASE FOLLOWS)

COLUMN	ITEM	SCALING
1	KRD(1) = OPTION TO READ IN NEW SUBSCRIPT CARD	I1
2	KRD(2) = OPTION TO READ IN NEW ANGLE OF ATTACK CARD	I1
3	KRD(3) = OPTION TO READ IN NEW MACH VALUE CARD	I1
4	KRD(4) = OPTION TO READ IN NEW SPIN VALUE CARD	I1
5	KRD(5) = OPTION TO READ IN NEW SET OF AXIAL FORCE COEFFICIENTS, CX	I1
6	KRD(6) = OPTION TO READ IN NEW SET OF NORMAL FORCE COEFFICIENTS, CN	I1
7	KRD(7) = OPTION TO READ IN NEW SET OF OVERTURNING MOMENT VARIATIONS, CM	I1
8	KRD(8) = OPTION TO READ IN NEW SET OF DAMPING DERIVATIVES, CMQ	I1
9	KRD(9) = OPTION TO READ IN NEW SET OF DAMPING DERIVATIVES, CNR	I1
10	KRD(10) = OPTION TO READ IN NEW SET OF DAMPING DERIVATIVES, CMPR	I1
11	KRD(11) = OPTION TO READ IN NEW SET OF DAMPING DERIVATIVES, CNPQ	I1
12	KRD(12) = OPTION TO READ IN NEW SET OF MAGNUS FORCES, CNP	I1
13	KRD(13) = OPTION TO READ IN NEW SET OF SPIN COEFFICIENTS, CL	I1
14	KRD(14) = OPTION TO READ IN NEW SET OF SPIN COEFFICIENTS, CLP	I1
15	KRD(15) = OPTION TO READ IN NEW SET OF MAGNUS MOMENTS, CMP	I1

16	KRD(16)=	OPTION TO	READ IN	NEW LEAD	CARD 2.	11
17	KRD(17)=	OPTION TO	READ IN	NEW LEAD	CARD 3.	11
18	KRD(18)=	OPTION TO	READ IN	NEW LEAD	CARD 4.	11
19	KRD(19)=	OPTION TO	READ IN	NEW LEAD	CARD 5.	11
20	KDD(20)=	OPTION TO	READ IN	NEW WIND	TABLES.	11
21	KDD(21)=	OPTION TO	READ IN	NEW DY, ZET1, ZET2,		11
			ETA1, ETA2, EPSMIN.			
22	KDD(22)=	OPTION TO	READ IN	NEW CYO.		11
23	KDD(23)=	OPTION TO	READ IN	NEW CZO.		11
24	KDD(24)=	OPTION TO	READ IN	NEW CMO.		11
25	KDD(25)=	OPTION TO	READ IN	NEW CNO.		11
26	KDD(26)=	OPTION TO	READ IN	NEW CSF1.		11
27	KDD(27)=	OPTION TO	READ IN	NEW CN1.		11
28	KDD(28)=	OPTION TO	READ IN	NEW CSF3.		11
29	KDD(29)=	OPTION TO	READ IN	NEW CSM1.		11
30	KDD(30)=	OPTION TO	READ IN	NEW CM1.		11
31	KDD(31)=	OPTION TO	READ IN	NEW CSM3.		11
32	KDD(32)=	OPTION TO	READ IN	NEW CLPH1.		11
33	KDD(33)=	OPTION TO	READ IN	NEW CLPH2.		11

= 1 INCLUDE NECESSARY CARD OR CARDS:
 = 0 OMIT CARD OR CARDS.

ANGLE OF ATTACK CARD(S)
 (OMIT IF COLUMN 2 OF LEAD CARD 7 EQUALS 0)
 COLUMN 1-72 TABA(I) = ANGLE OF ATTACK (RADIAN) 6 VALUES PER CARD
 (MAX OF 20 VALUES)
 I=1,IMAX
 SCALING
 6F12.5

MACH VALUE CARD
 (OMIT IF COLUMN 3 OF LEAD CARD 7 EQUALS 0)
 COLUMN 1-60 TABM(I) = MACH NUMBER. (MAX. OF 5 VALUES)
 I=1,JMAX
 SCALING
 5F12.5

SPIN VALUE CARD
 (OMIT IF COLUMN 4 OF LEAD CARD 7 EQUALS 0)
 COLUMN 1-60 TABAK(I) = SPIN VALUE. (MAX. OF 5 VALUES)
 I=1,KMAX
 SCALING
 5F12.5

BASIC AEROBALLISTIC COEFFICIENT TABLES

AXIAL FORCE COEFFICIENT CARD(S)
(UNIT IF COLUMN 5 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72

ITEM

CX(I,J) = AXIAL FORCE COEFFICIENT, 6 VALUES PER CARD, MAX. OF 100 VALUES.
(J=1, JMAX), I=1, IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE ONE COEFFICIENT FOR EACH MACH VALUE

SCALING 6F12.5

NORMAL FORCE COEFFICIENT CARD(S)
(OMIT IF COLUMN 6 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72

ITEM

CN(I,J) = NORMAL FORCE COEFFICIENT, 6 VALUES PER CARD, MAX. OF 100 VALUES.
(J=1, JMAX), I=1, IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING 6F12.5

OVERTURNING MOMENT VARIATIONS CARD(S)
(OMIT IF COLUMN 7 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72

ITEM

CM(I,J) = OVERTURNING MOMENT VARIATION, 6 VALUES PER CARD, MAX. OF 100 VALUES.
(J=1, JMAX), I=1, IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING 6F12.5

DAMPING DERIVATIVE CARD(S)
 (OMIT IF COLUMN 10 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CMPR(I,J)=DAMPING DERIVATIVE. 6 VALUES PER CARD.
 ITEM
 MAX OF 100 VALUES.
 ((J=J,JMAX),I=1,I MAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

 SCALING
 OF 12.5

DAMPING DERIVATIVE CARD(S)
 (OMIT IF COLUMN 11 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CNPQ(I,J)= DAMPING DERIVATIVE. 6 VALUES PER CARD.
 ITEM
 MAX OF 100 VALUES.
 ((J=1,JMAX),I=1,I MAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

 SCALING
 6F12.5

MAGNUS FORCE COEFFICIENT CARD(S)
 (OMIT IF COLUMN 12 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CNP(I,J,K)=MAGNUS FORCE COEFFICIENT, 6 VALUES PER
 ITEM
 CARD. MAX. OF 500 VALUES.
 ((K=1,KMAX),J=1,JMAX),I=1,IMAX)
 FOR EACH MACH VALUE THERE WILL BE ONE
 COEFFICIENT FOR EACH SPIN VALUE. THESE FORM
 A SUBTABLE. THERE WILL BE ONE SUBTABLE FOR
 EACH ANGLE OF ATTACK VALUE.

SCALING
 6F12.5

NOTE- IF BODY FIXED CNP
 IF FIXED PLANE CLP
 CNP=-CLP

SPIN COEFFICIENT CARD(S)
 (OMIT IF COLUMN 13 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CL(I,J,K)= SPIN COEFFICIENT, 6 VALUES PER CARD.
 ITEM
 MAX. OF 500 VALUES.
 ((K=1,KMAX),J=1,JMAX),I=1,IMAX)
 FOR EACH MACH VALUE THERE WILL BE ONE
 COEFFICIENT FOR EACH SPIN VALUE. THESE FORM
 A SUBTABLE. THERE WILL BE ONE SUBTABLE FOR
 EACH ANGLE OF ATTACK VALUE.

SCALING
 6F12.5

SPIN COEFFICIENT CARD(S)
 (OMIT IF COLUMN 14 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CLP(I,J,K)= SPIN COEFFICIENT. 6 VALUES PER CARD.
 ITEM
 MAX. OF 500 VALUES.
 ((K=1,KMAX),J=1,JMAX),I=1,IMAX)
 FOR EACH MACH VALUE THERE WILL BE ONE COEFFICIENT FOR EACH SPIN VALUE. THESE FORM A SUBTABLE. THERE WILL BE ONE SUBTABLE FOR EACH ANGLE OF ATTACK VALUE.

SCALING
 6F12.5

MAGNUS MOMENT CARD(S)
 (OMIT IF COLUMN 15 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CMP(I,J,K)=MAGNUS MOMENT. 6 VALUES PER CARD.
 ITEM
 MAX. OF 500 VALUES.
 ((K=1,KMAX),J=1,JMAX),I=1,IMAX)
 FOR EACH MACH VALUE THERE WILL BE ONE COEFFICIENT FOR EACH SPIN VALUE. THESE FORM A SUBTABLE. THERE WILL BE ONE SUBTABLE FOR EACH ANGLE OF ATTACK VALUE.

SCALING
 6F12.5

WIND TABLES (OPTIONAL)
 (OMIT IF COLUMN 20 OF LEAD CARD 7 EQUALS 0)
 (6 VALUES PER CARD, MAXIMUM OF 10)

COLUMN	ITEM	SCALING
1-72	TABZ(I) = VERTICAL COORDINATE, FT	6F12.5
1-72	WDX(I) = WIND VELOCITY IN DIRECTION OF X COORDINATE, FT/SEC	6F12.5
1-72	WDY(I) = WIND VELOCITY IN DIRECTION OF Y COORDINATE, FT/SEC	6F12.5

TABZ MUST BE IN POSITIVE ASCENDING ORDER-ALTITUDE CAN
 BE USED INSTEAD OF 7 COORDINATE IF DESIRED

MAXIMUM NUMBER OF WIND VALUES (VERTICAL COORDINATE) IS TFN

ADDITIONAL AERODYNAMIC COEFFICIENT TABLES

TRIM FORCE COEFFICIENT CARD(S)
(OMIT IF COLUMN 22 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CYO(I,J) = TRIM FORCE COEFFICIENT, Y BODY FIXED AXIS, SCALING
ITEM 6 VALUES PER CARD, MAX. OF 100 VALUES. 6F12.5
((J=1,JMAX), I=1=IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
ONE COEFFICIENT FOR EACH MACH VALUE.

TRIM FORCE COEFFICIENT CARD(S)
(OMIT IF COLUMN 23 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CZO(I,J) = TRIM FORCE COEFFICIENT, Z BODY FIXED AXIS, SCALING
ITEM 6 VALUES PER CARD, MAX. OF 100 VALUES. 6F12.5
((J=1,JMAX), I=1=IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
ONE COEFFICIENT FOR EACH MACH VALUE.

TRIM MOMENT COEFFICIENT CARD(S)
(OMIT IF COLUMN 24 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CNO(I,J) = TRIM MOMENT COEFFICIENT, ABOUT Y BODY-FIXED
 ITEM
 AXIS, 6 VALUES PER CARD.MAX.OF 100 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 SCALING 6F12.5
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

TRIM MOMENT COEFFICIENT CARD(S)
(OMIT IF COLUMN 25 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CNO(I,J) = TRIM MOMENT COEFFICIENT, ABOUT Z BODY-FIXED
 ITEM
 AXIS, 6 VALUES PER CARD.MAX OF 100 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 SCALING 6F12.5
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SIDE FORCE COEFFICIENT CARD(S)
 (OMIT IF COLUMN 26 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CSF1(I,J)= SIDE FORCE COEFFICIENT DUE TO AERODYNAMIC
 ITEM
 ROLL ANGLE OF FINS, 6 VALUES PER CARD.
 MAX. OF 100 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
 6F12.5

SIDE FORCE COEFFICIENT CARD(S)
 (OMIT IF COLUMN 27 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CN1(I,J)= NORMAL FORCE COEFFICIENT DUE TO AERODYNAMIC
 ITEM
 ROLL ANGLE OF FINS (WINGS), 6 VALUES PER
 CARD. MAX. OF 100 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
 6F12.5

SIDE FORCE COEFFICIENT CARD(S)
 (OMIT IF COLUMN 28 OF LEAD CARD 7 EQUALS 0)

 COLUMN 1-72 CSF3(I,J)= SIDE FORCE COEFFICIENT DUE TO ASYMMETRIC
 ITEM
 VORTICES, 6 VALUES PER CARD. MAX. OF 100
 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
 6F12.5

SIDE MOMENT COEFFICIENT CARD(S)
 (OMIT IF COLUMN 29 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CSM1(I,J)= SIDE MOMENT COEFFICIENT DUE TO AERODYNAMIC
 ITEM
 ROLL ANGLE OF FINS, 6 VALUES PER CARD.
 MAX. OF 100 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
 6F12.5

SIDE MOMENT COEFFICIENT CARD(S)
 (OMIT IF COLUMN 30 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CM1(I,J)=PITCH MOMENT COEFFICIENT DUE TO AERODYNAMIC
 ITEM
 ROLL ANGLE OF FINS (WINGS), 6 VALUES PER
 CARD. MAX. OF 100 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
 6F12.5

SIDE MOMENT COEFFICIENT CARD(S)
 (OMIT IF COLUMN 31 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CSM3(I,J)= SIDE MOMENT COEFFICIENT DUE TO ASYMMETRIC
 ITEM
 VORTICES, 6 VALUES PER CARD. MAX OF 100
 VALUES.
 ((J=1,JMAX),I=1=IMAX)
 FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
 ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
 6F12.5

ROLLING MOMENT COEFFICIENT CARD(S)
(OMIT IF COLUMN 32 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CLPH1(I,J)= ROLLING MOMENT COEFFICIENT DUE TO
ITEM
AERODYNAMIC ROLL ANGLE OF FINS, 6 VALUES
PER CARD. MAX.OF 100 VALUES.
((J=1,JMAX),I=1=IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
6F12.5

ROLLING MOMENT COEFFICIENT CARD(S)
(OMIT IF COLUMN 33 OF LEAD CARD 7 EQUALS 0)

COLUMN 1-72 CLPH2(I,J)= ROLLING MOMENT COEFFICIENT DUE TO
ITEM
AERODYNAMIC ROLL ANGLE OF WINGS,
6 VALUES PER CARD.MAX.OF 100 VALUES.
((J=1,JMAX),I=1=IMAX)
FOR EACH ANGLE OF ATTACK VALUE THERE WILL BE
ONE COEFFICIENT FOR EACH MACH VALUE.

SCALING
6F12.5

SECTION III

OUTPUT FORMAT

SYSTEM OUTPUT (B1-6)

LINE 1

WORD	ITEM		SCALING
1	TIME	= PRINT TIME (SEC)	F7.4
2	CAPV	= TOTAL VELOCITY (FT/SEC)	F12.1
3	Y(7)	= X RANGE COORDINATE (FT)	F12.1
4	Y(8)	= Y RANGE COORDINATE (FT)	F12.1
5	Y(9)	= Z COORDINATE, ALTITUDE (FT)	F12.1
6	Z(7)	= VELOCITY IN DIRECTION OF X RANGE	F12.1
		COORDINATE (FT/SEC)	
7	Z(8)	= VELOCITY IN DIRECTION OF Y RANGE	F12.1
		COORDINATE (FT/SEC)	
8	Z(9)	= VERTICAL VELOCITY (FT/SEC)	F12.1
9	ALD	= ANGLE OF ATTACK (DEG)	F7.2
10	CP1	= FIN OFFSET ANGLE (DEG)	F7.2
11	CP2	= WING OR NOSE OFFSET ANGLE (DEG)	F7.2
12	CAPVA	= AERODYNAMIC VELOCITY	F10.1
13	EM	= MACH NUMBER	F7.2

LINE 2

WORD	ITEM		SCALING
1	Y(1)	= VELOCITY IN DIRECTION OF X MOVING COORDINATE-BODY OR FIXED PLANE AXES (FT/SEC)	F12.1
2	Y(2)	= VELOCITY IN DIRECTION OF Y MOVING COORDINATE-BODY OR FIXED PLANE AXES (FT/SEC)	F12.1
3	Y(3)	= VELOCITY IN DIRECTION OF Z MOVING COORDINATE-BODY OR FIXED PLANE AXES (FT/SEC)	F12.2 F12.1
4	Y(4)	= SPIN RATE (RAD/SEC)	F12.2
5	Y(5)	= PITCH RATE (RAD/SEC)	F12.2
6	Y(6)	= YAW RATE (RAD/SEC)	F11.2
7	PHD	= ROLL ATTITUDE (DEG)	F11.2
8	THD	= PITCH ATTITUDE (DEG)	F11.2
9	PSD	= YAW ATTITUDE (DEG)	F11.2
10	EPSIL	= INTEGRATION ERROR IN QUATERNIONS	!PE17.6

SYSTEM OPTIONAL OUTPUT (A6-26)

LINE 1

SCALING
F9.4
F15.2
F15.2
F15.2
F15.6
F15.6
F15.6
F15.6

WORD	ITEM	
1	TIME	= PRINT TIME (SEC)
2	PHD	= ROLL ATTITUDE RATE (RAD/SEC)
3	THD	= PITCH ATTITUDE RATE (RAD/SEC)
4	PSD	= YAW ATTITUDE RATE (RAD/SEC)
5	Y(10)	= QUATERNION VALUE
6	Y(11)	= QUATERNION VALUE
7	Y(12)	= QUATERNION VALUE
8	Y(13)	= QUATERNION VALUE

LINE 2

SCALING
1PF15.6
E15.6
E15.6
E15.6
E15.6
E15.6
E15.6
E15.6

WORD	ITEM	
1	E(4)	= ERROR IN SPIN RATE (RAD/SEC)
2	E(5)	= ERROR IN PITCH RATE (RAD/SEC)
3	E(6)	= ERROR IN YAW RATE (RAD/SEC)
4	PDV	= NON-DIMENSIONAL SPIN VALUE
5	E(10)	= QUATERNION ERROR (ACCUMULATIVE)
6	E(11)	= QUATERNION ERROR (ACCUMULATIVE)
7	E(12)	= QUATERNION ERROR (ACCUMULATIVE)
8	E(13)	= QUATERNION ERROR (ACCUMULATIVE)

SECTION IV

PROGRAM FLOW CHARTS AND SUBROUTINE DESCRIPTION

Description of Labeled Common Names

Name	Description	Reference Section - Page	
<BTOF> Arguments of SETMAX			
XMAT(3, 3)	Rotational Matrix	I	4
<MPGIN> Input Related Only to Main Program			
XDOT(3)	$\dot{X}, \dot{Y}, \dot{Z}$	II	31
P	p	II	31
Q	q	II	31
R	r	II	31
ZALT	Z	II	31
PHI	φ	II	31
THETA	θ	II	31
PSI	ψ	II	31
TSTEP	Δt	II	32
INCPT1	Print cycle before $\dot{q} = \dot{r} = 0$	II	32
TMAX	$t_{\max}$	II	32
ZSTOP	$Z_{\min}$	II	32
TCH	$t_{\text{change}} (\dot{q} = \dot{r} = 0)$	II	32
TNEW	Δt_2	II	32
NRK	Integration code	II	29
NCMAX	Number of corrections	II	29
EPSMAX	$\epsilon_{\max}$	II	29
ZETD1	ζ_1 , deg.	II	32
ZETD2	ζ_2 , deg.	II	32
INCPT2	Print cycle after $\dot{q} = \dot{r} = 0$	II	32

Description of Labeled Common Names (Continued)

Name	Description	Reference Section - Page	
<ALLIN> Input Related to Both Main Program and Equations of Motion			
DIX	I_x	II	30
DI	I or I_y	II	30
DIZ	I_z	II	30
DIXY	I_{xy}	II	30
BODFIX	T/F Flag False if Fixed Plane	II	30
<EQMPG> Main Program Constants Needed In Equations of Motion			
QR	T/F Flag - if false, use simplified equations for q and r		
RAT	I_x/I_y	I	9
BODJX	I_{xy}/I_x	I	8
BODJY	I_{xy}/I_y	I	8
BODJZ	I_{xy}/I_z	I	8
BODEN	$1 - I_{xy}^2 / I_x I_y$	I	8
BODJ	$(I_x + I_y - I_z) I_{xy} / (I_x I_y)$	I	8
BODPD	$[I_{xy}^2 + I_y (I_y - I_z)] / (I_x I_y)$	I	8
BODQD	$[I_{xy}^2 - I_x (I_x - I_z)] / (I_x I_y)$	I	8
BODRD	$(I_x - I_y) / I_z$	I	8
ZET1	ζ_1 radians	I	21
ZET2	ζ_2 radians	I	21

Description of Labeled Common Names (Continued)

Name	Description	Reference Page	
< EQMIN > Input Related Only to Equations of Motion			
IMAX	≤ 20	II	29
JMAX	≤ 5	II	29
KMAX	≤ 5	II	29
LMAX	≤ 10	II	29
TABA (20)	(α)	II	35
TABM (5)	(M)	II	35
TABAR (5)	$(pd/2V)$	II	35
TABZ (10)	$(-Z)$	II	41
CX (20, 5)	C_x	II	36
CN (20, 5)	C_N	II	36
CM (20, 5)	C_M	II	36
CMQ (20, 5)	C_{mq}	II	37
CNR (20, 5)	C_{nr}	II	37
CMPR (20, 5)	C_{mpr}	II	38
CNPQ (20, 5)	C_{npq}	II	38
CNPA (20, 5, 5)	$C_{Np} = -C_{Lp}$	II	39
CL (20, 5, 5)	C_l	II	39
CLP (20, 5, 5)	C_{lp}	II	40
CMPA (20, 5, 5)	C_{Mp}	II	40
CYO (20, 5)	C_{y_0}	II	42
CZO (20, 5)	E_{z_0}	II	42
CMO (20, 5)	C_{m_0}	II	43
CNO (20, 5)	C_{n_0}	II	43
CSF1 (20, 5)	CSF_1	II	44
CN1 (20, 5)	C_{N_1}	II	44
CSF3 (20, 5)	CSF_3	II	44

Description of Labeled Common Names (Continued)

Name	Description	Reference Section - Page	
<EQMIN> (continued)			
CSM1 (20, 5)	C_{SM1}	II	45
CM1 (20, 5)	C_{M1}	II	45
CSM3 (20, 5)	C_{SM3}	II	45
CLPH1 (20, 5)	C_{l1}	II	46
CLPH2 (20, 5)	C_{l2}	II	46
WDX (10)	$\dot{X}_w$	II	41
WDY (10)	$\dot{Y}_w$	II	41
G	g	II	30
DMM	m	II	30
S	S	II	31
DEE	d	II	31
DY	Δy	II	32
ETA1	η_1	II	32
ETA2	η_2	II	32

<OUTIN> Input Related Only to Output

HEADER (11)	II	30
IRUN	II	30
IDATE	II	30
KA6	II	30

Description of Labeled Common Names (Continued)

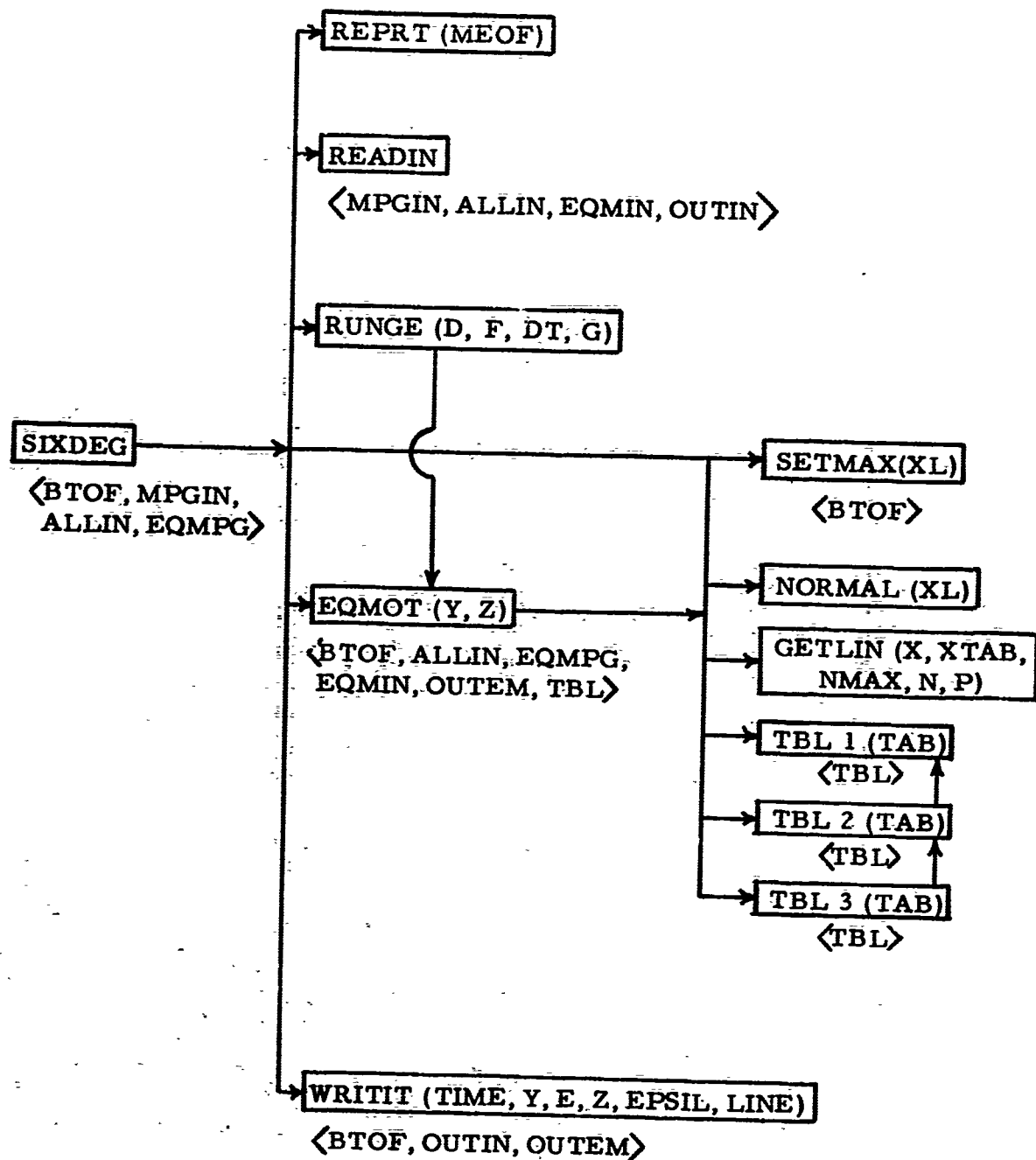
Name	Description	Reference Section - Page
<TBL> Arguments for TBL1, TBL2, TBL3 found from GETLIN		
N1	n_1 index such that $XT (n - 1) < X \leq XT (n)$	
N2	n_2 for $n = n_1$ or n_2 or n_3	
N3	n_3	
P1	1 ratio of $\frac{XT (n) - X}{XT (n) - XT (n - 1)}$	
P2	2	
P3	3 for $n = n_1$ or n_2 or n_3	

<OUTEM> Equations of Motion Generated Data for Output

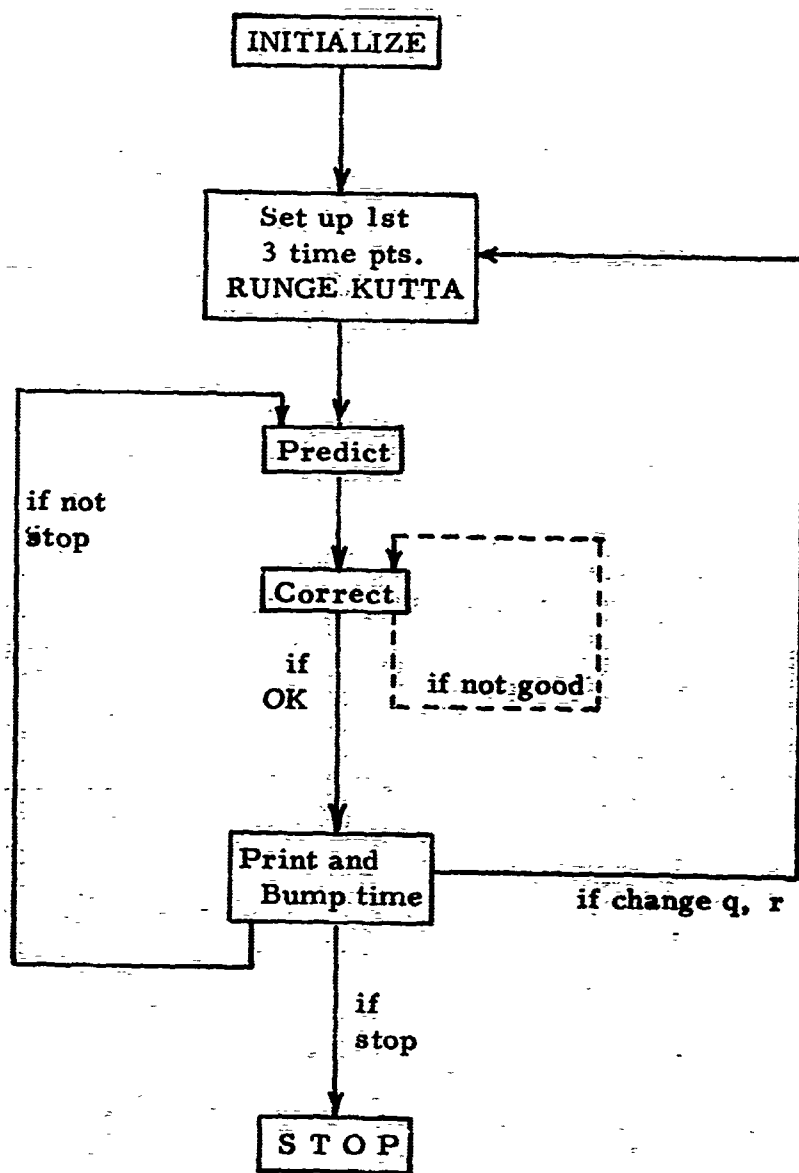
CAPHI1	$\Phi_1 = \pi/2 - (\xi - \xi_1)$	I	16
CAPHI2	$\Phi_2 = \pi/2 - (\xi - \xi_2)$	I	16
PDV	pd/2V	I	10
ALPHA	α	I	10
CAPVA	V_A	I	20
EM	Mach Number	I	10
PEE	p or $(-r \tan \theta)$	I	7, 25

Hierarchy of Programs and their Labeled Common

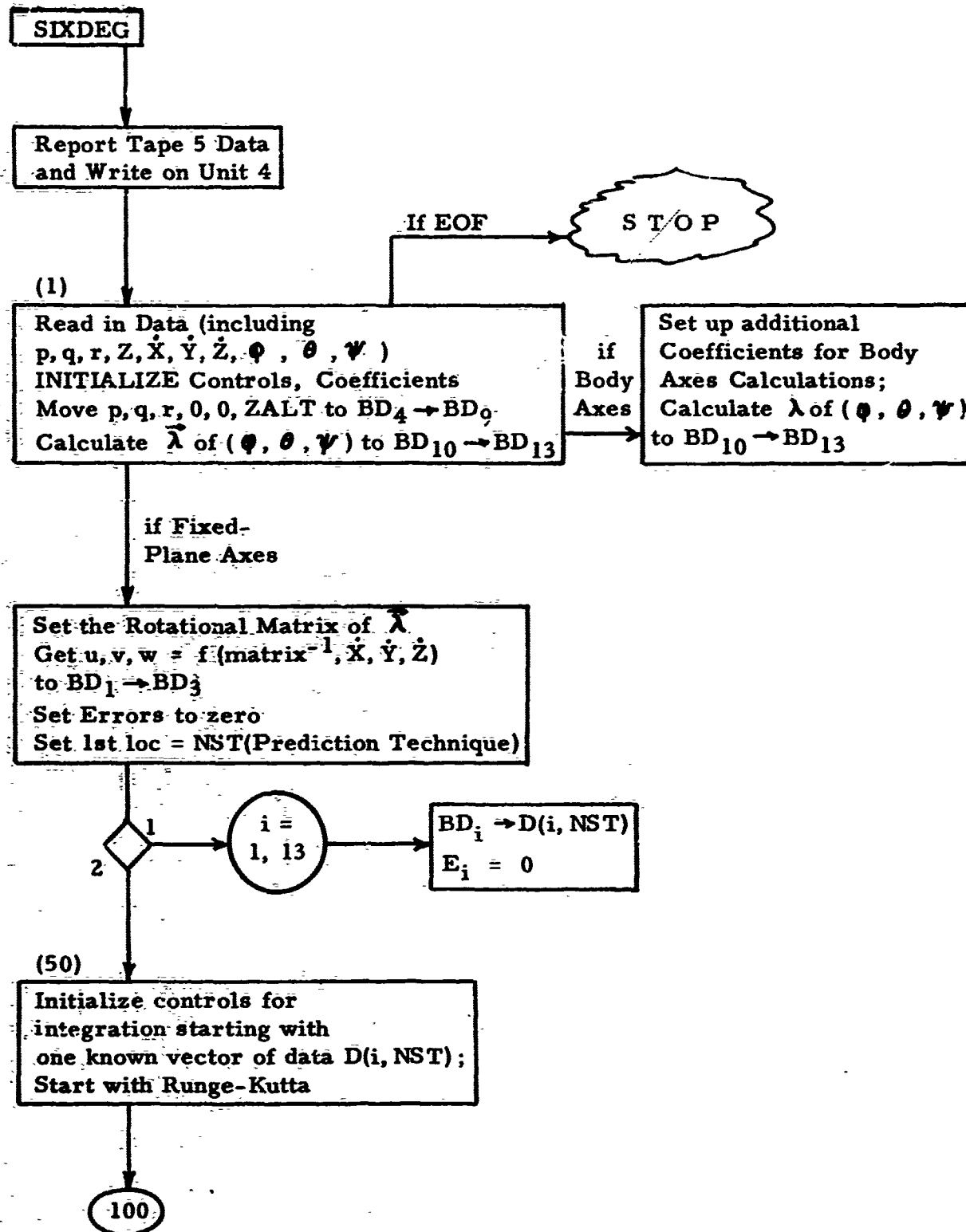
Key **Program Names** (Arguments) <Labeled Common Names>



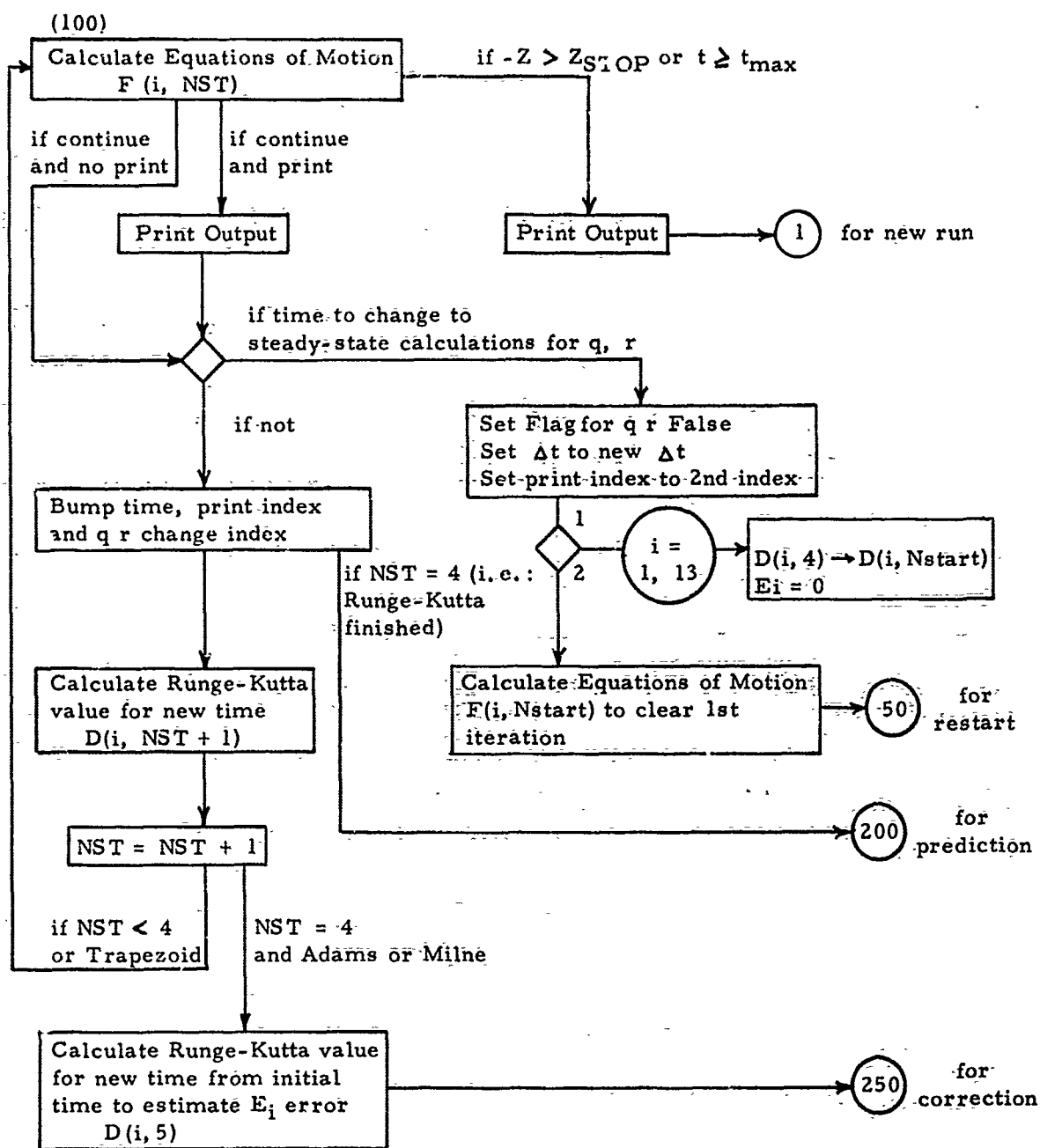
Simple Flow Chart of LOGIC for Single Run



Flow Chart of Main Program

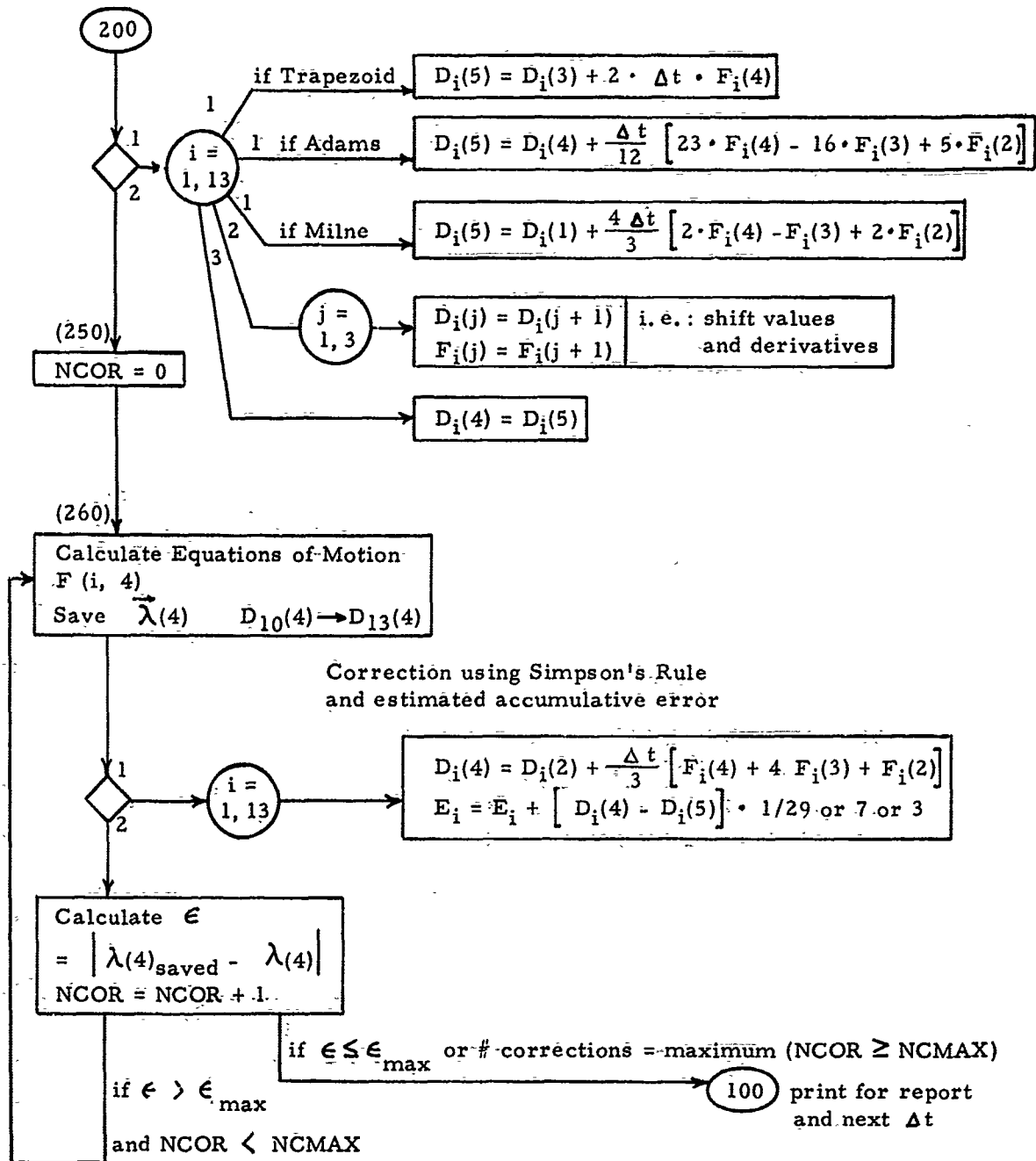


Main Program (continued)

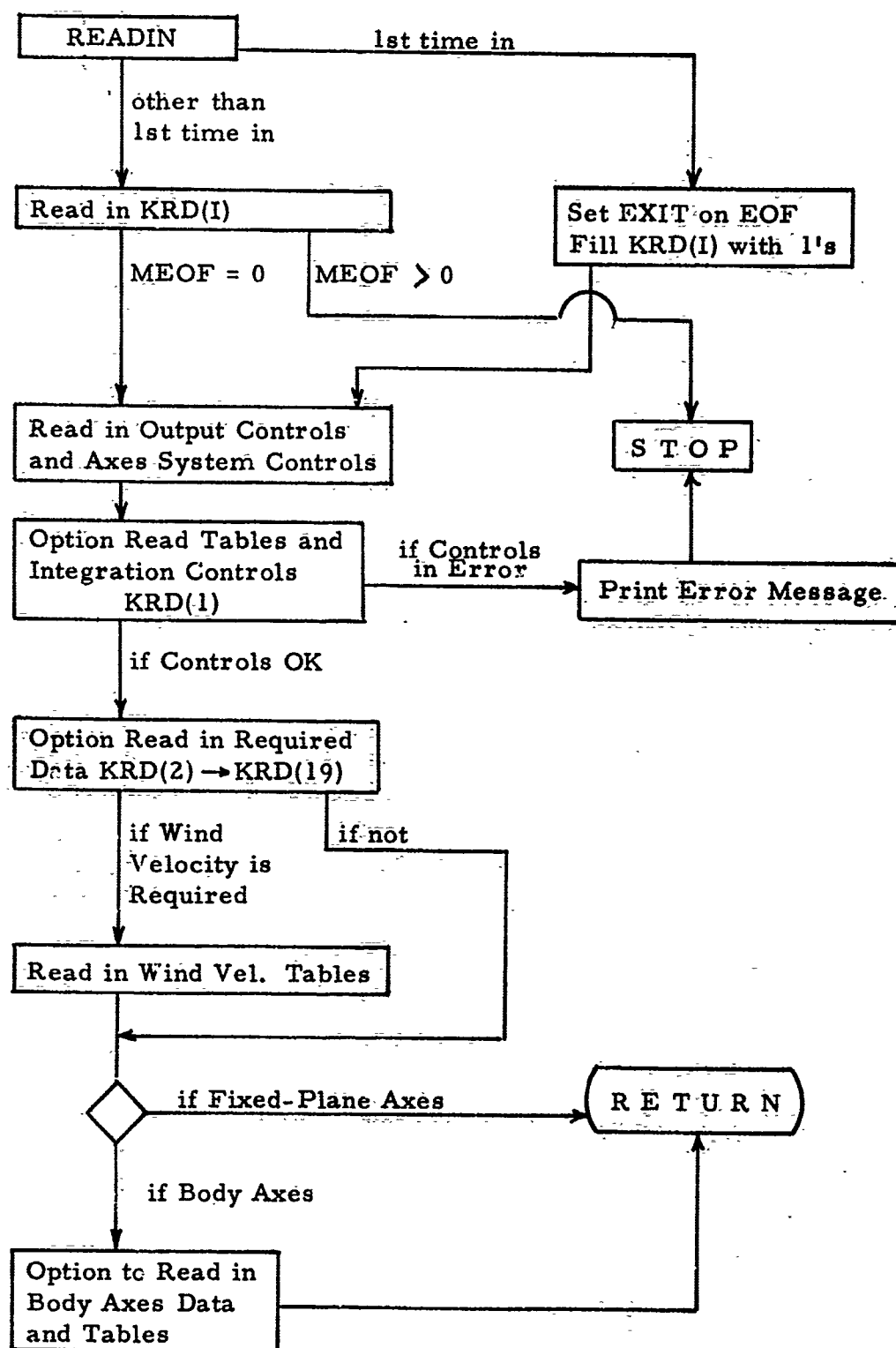


Main Program (concluded)

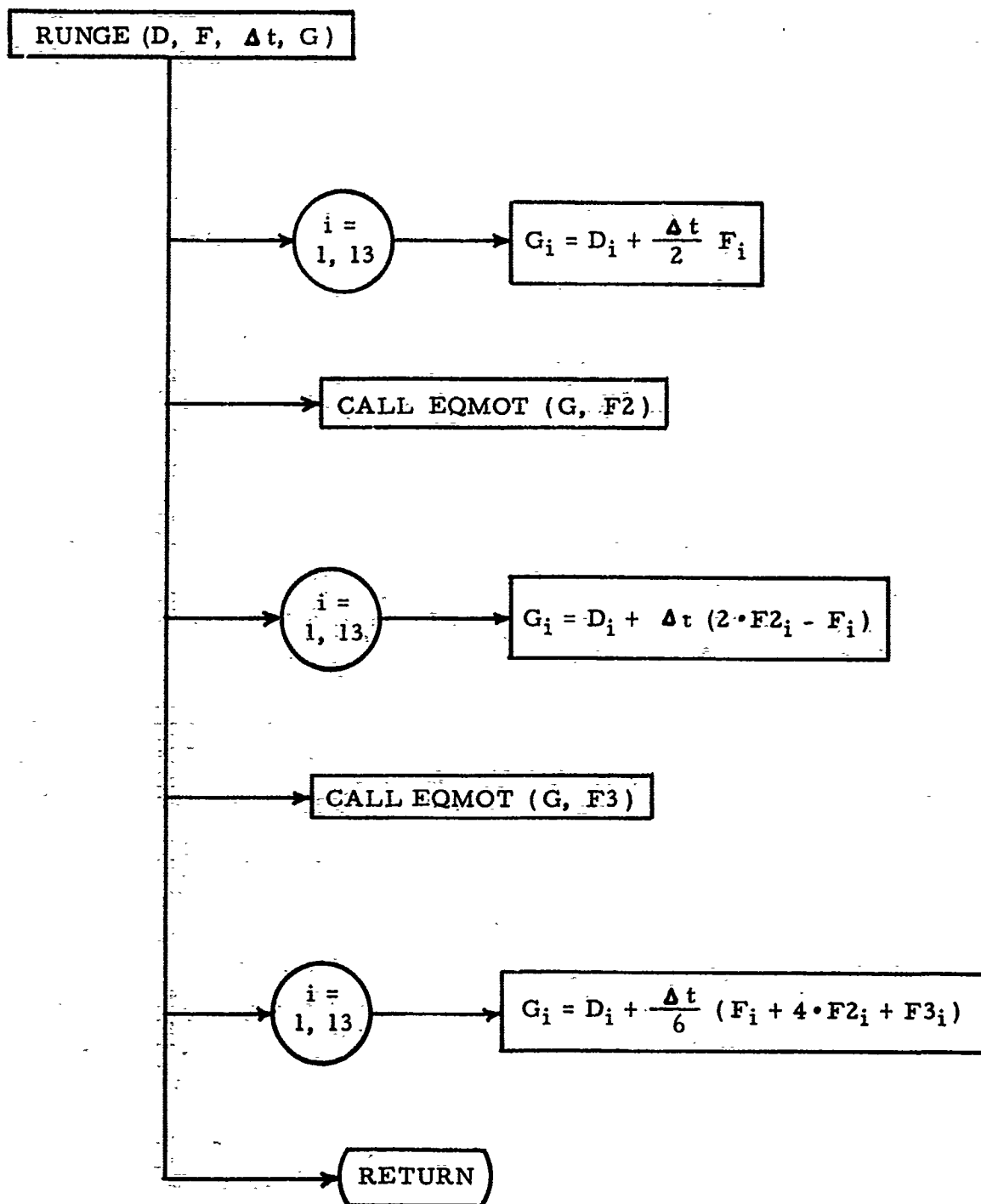
Prediction



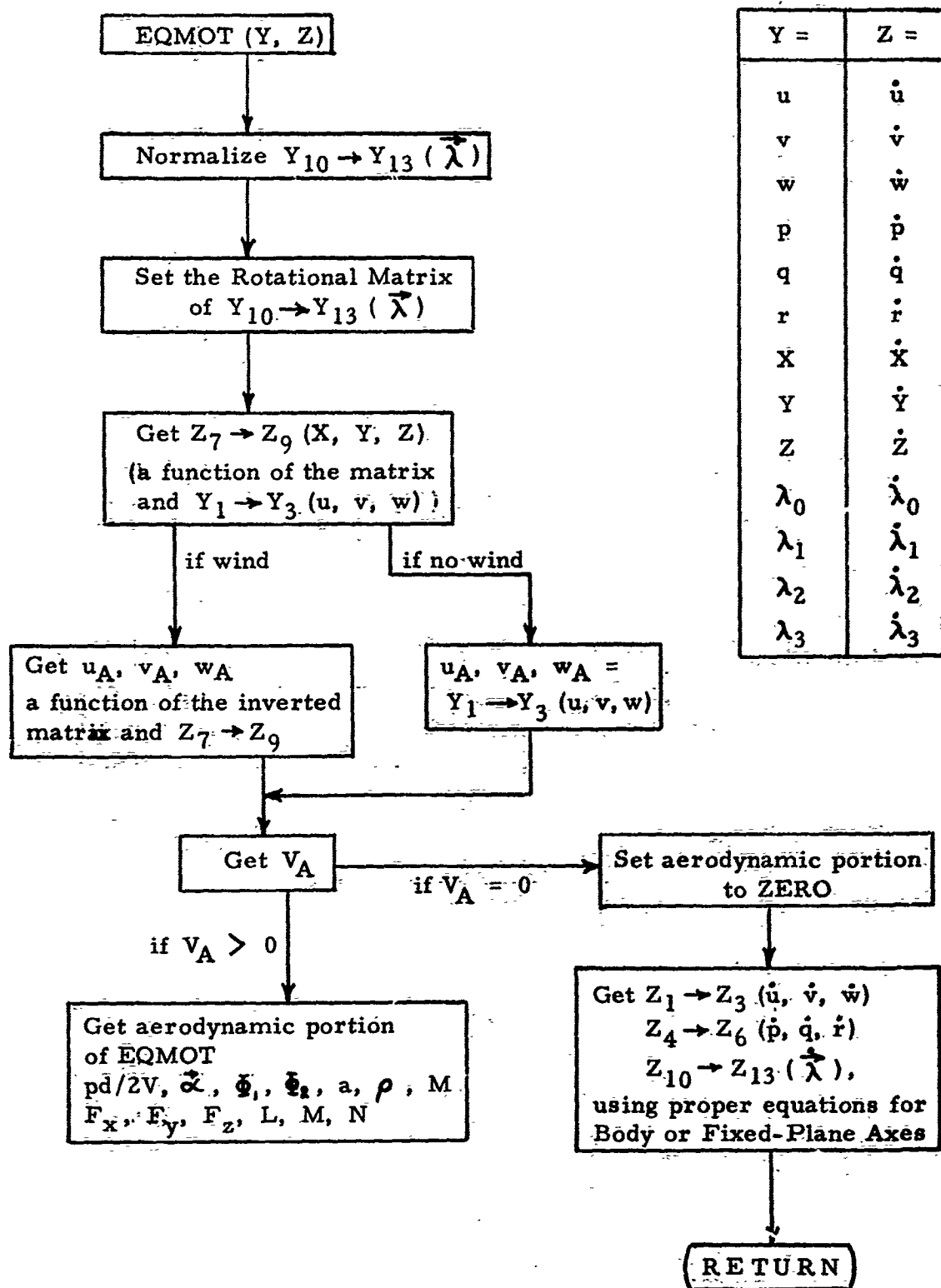
Flow Chart of READIN



Flow Chart of RUNGE



Flow Chart of EQMOT



$$A = [q \sin \xi - r \cos \xi] \left(\frac{d}{2V} \right)$$

$$B = [q \cos \xi + r \sin \xi] \left(\frac{d}{2V} \right)$$

$$C_{M\text{ADD}} = C_M(\vec{\alpha}, M) + \boxed{C_{M1}(\vec{\alpha}, M) \sin \eta_1 \left[\frac{\pi}{2} - \xi + \xi_1 \right]} + C_{mq}(\vec{\alpha}, M)$$

$$+ B \cdot C_{m_{pr}}(\vec{\alpha}, M) \cdot \frac{pd}{2V}$$

$$C_{M_p\text{ADD}} = C_{M_p}(\vec{\alpha}, M, \frac{pd}{2V}) \cdot \frac{pd}{2V} + B \cdot C_{nr}(\vec{\alpha}, M) + A \cdot C_{n_{pq}}(\vec{\alpha}, M) \cdot \frac{pd}{2V}$$

$$+ \boxed{C_{SM1}(\vec{\alpha}, M) \sin(\eta_1 [\frac{\pi}{2} - \xi + \xi_1]) + C_{SM3}(\vec{\alpha}, M)}$$

$$M = \text{CON} \cdot d * \left\{ \cancel{\boxed{C_{m_o}(\vec{\alpha}, M)}} + C_{M\text{ADD}} \cdot \sin \xi + C_{M_p\text{ADD}} \cdot \cos \xi \right\}$$

$$N = \text{CON} \cdot d * \left\{ \boxed{C_{n_o}(\vec{\alpha}, M)} - C_{M\text{ADD}} \cdot \cos \xi + C_{M_p\text{ADD}} \cdot \sin \xi \right\}$$

$$+ \boxed{\Delta_y \cdot F_x}$$

EQMOT Equations in More Detail for F_x, F_y, F_z, L, M, N

$\boxed{}$ denotes body-fixed axes option only

$$CON = \frac{1}{2} \rho v^2 S$$

$$F_x = CON * C_x(\vec{\alpha}, M)$$

$$C_{N_p}^{ADD} = \boxed{\frac{1}{2}} C_{N_p}(\vec{\alpha}, M, \frac{pd}{2V}) \cdot \frac{pd}{2V}$$

$$+ C_{SF_1}(\vec{\alpha}, M) \sin(\eta_1 [\frac{\pi}{2} - \xi + \xi_1]) + C_{SF_3}(\vec{\alpha}, M)$$

$$F_y = CON * \left\{ \boxed{C_{y_0}(\vec{\alpha}, M)} - \left[C_N(\vec{\alpha}, M) + \boxed{C_{N_1}(\vec{\alpha}, M) \sin \eta_1 [\frac{\pi}{2} - \xi + \xi_1]} \right] \cos \xi \right. \\ \left. + C_{N_p}^{ADD} \cdot \sin \xi \right\}$$

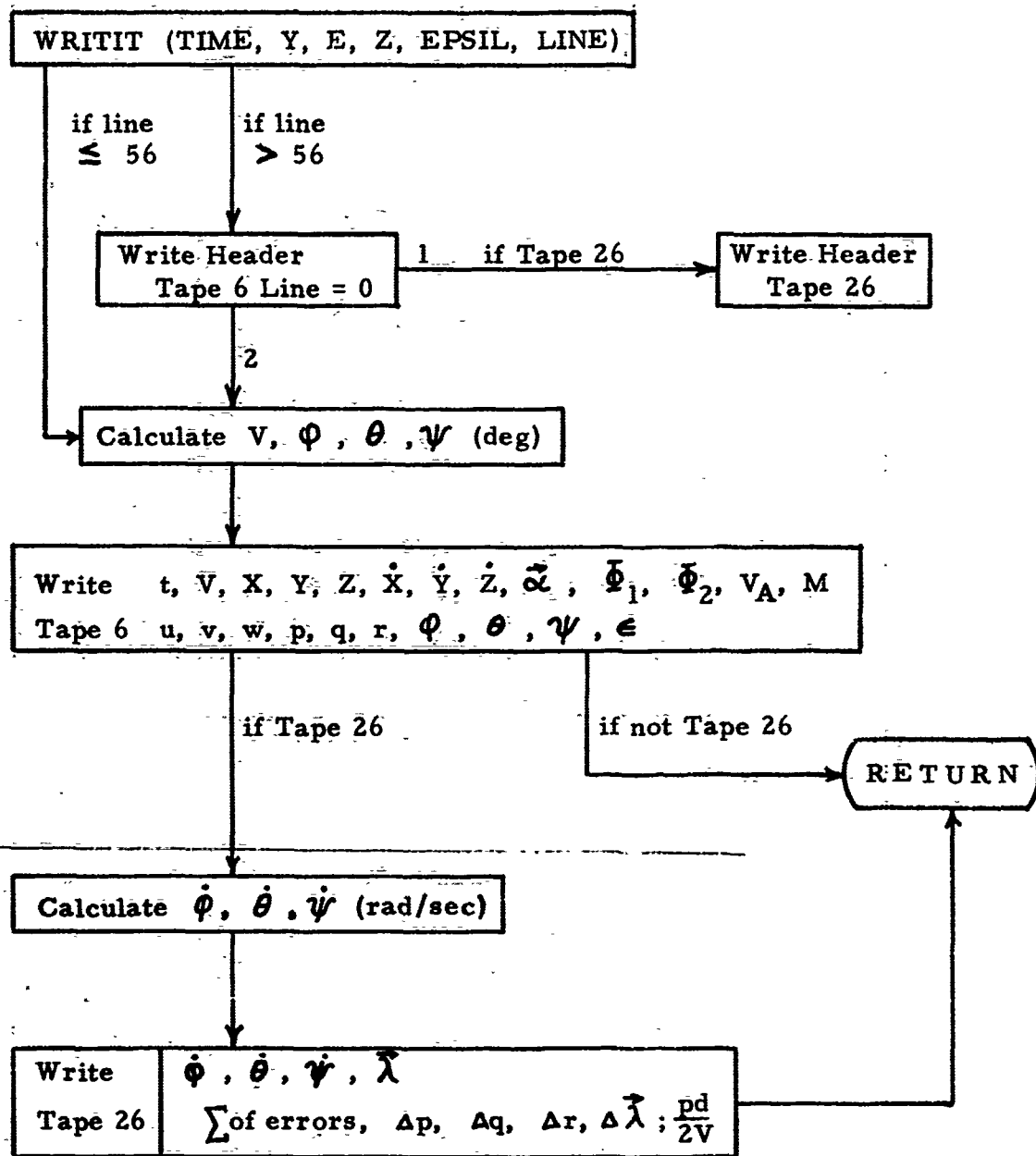
$$F_z = CON * \left\{ \boxed{C_{z_0}(\vec{\alpha}, M)} - \left[C_N(\vec{\alpha}, M) + \boxed{C_{N_1}(\vec{\alpha}, M) \sin \eta_1 [\frac{\pi}{2} - \xi + \xi_1]} \right] \sin \xi \right. \\ \left. - C_{N_p}^{ADD} \cdot \cos \xi \right\}$$

$$L = CON d * \left\{ C_l(\vec{\alpha}, M, \frac{pd}{2V}) + \frac{pd}{2V} \cdot C_{l_p}(\vec{\alpha}, M, \frac{pd}{2V}) \right.$$

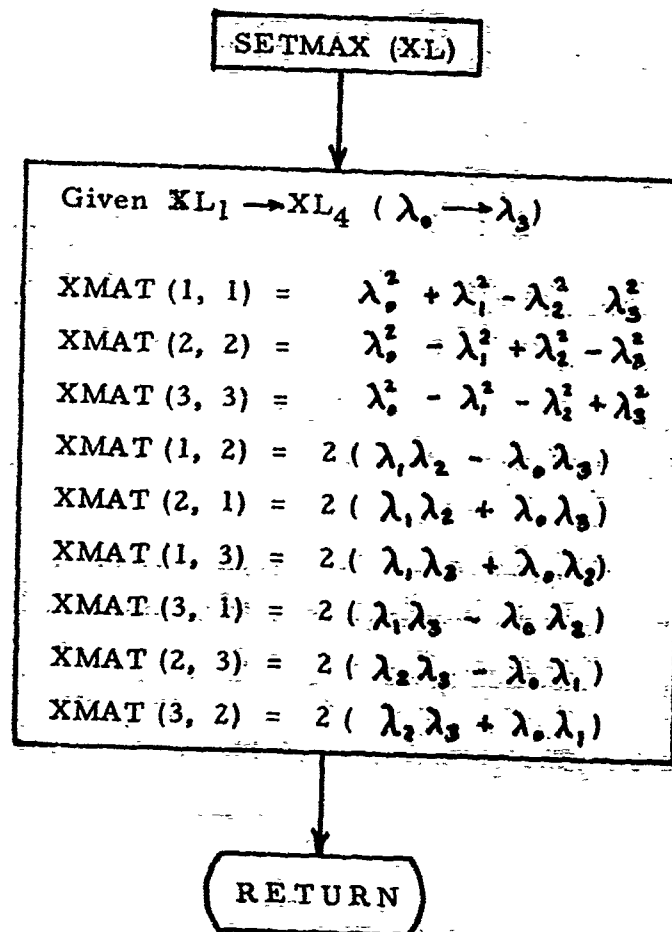
$$\left. + \boxed{C_{l_{\xi_1}}(\vec{\alpha}, M) \sin(\eta_1 [\frac{\pi}{2} - \xi + \xi_1]) + C_{l_{\xi_2}}(\vec{\alpha}, M) \sin(\eta_2 [\frac{\pi}{2} - \xi + \xi_2])} \right\}$$

$$- \boxed{\Delta_y \quad F_z}$$

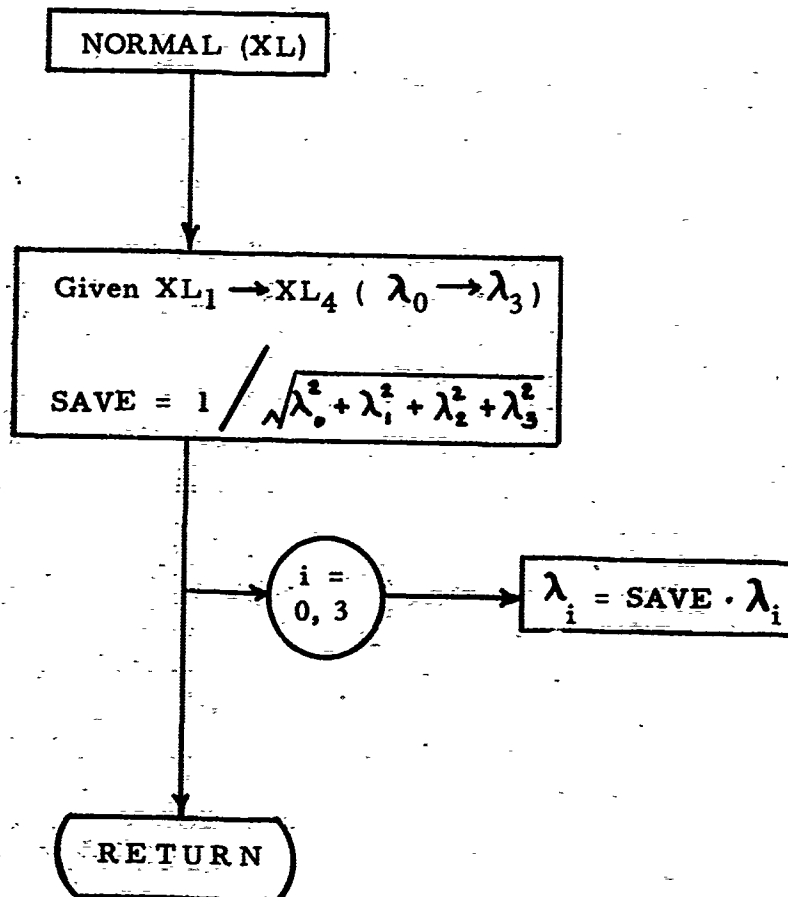
Flow Chart of WRITIT



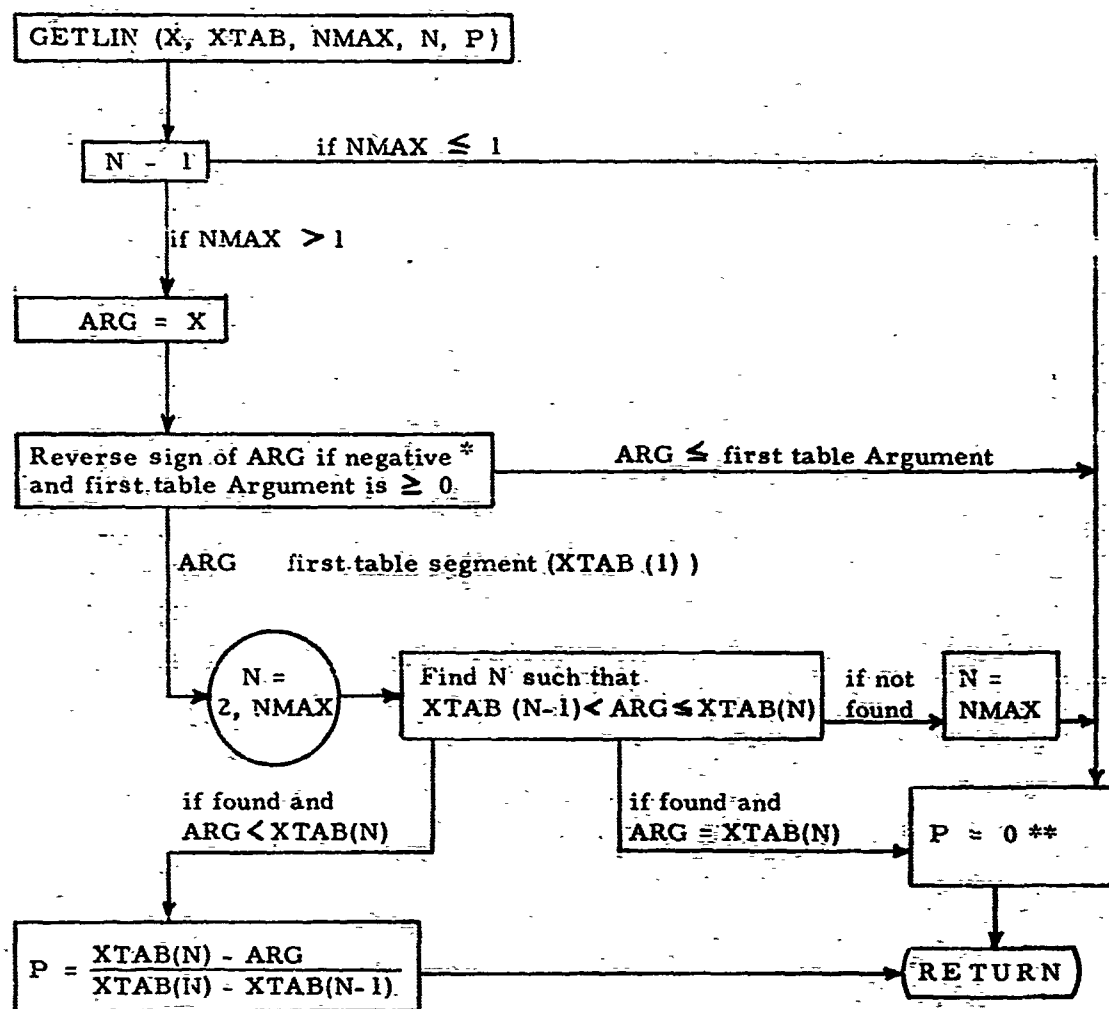
Flow Chart of SETMAX



Flow Chart of NORMAL



Flow Chart of GETLIN



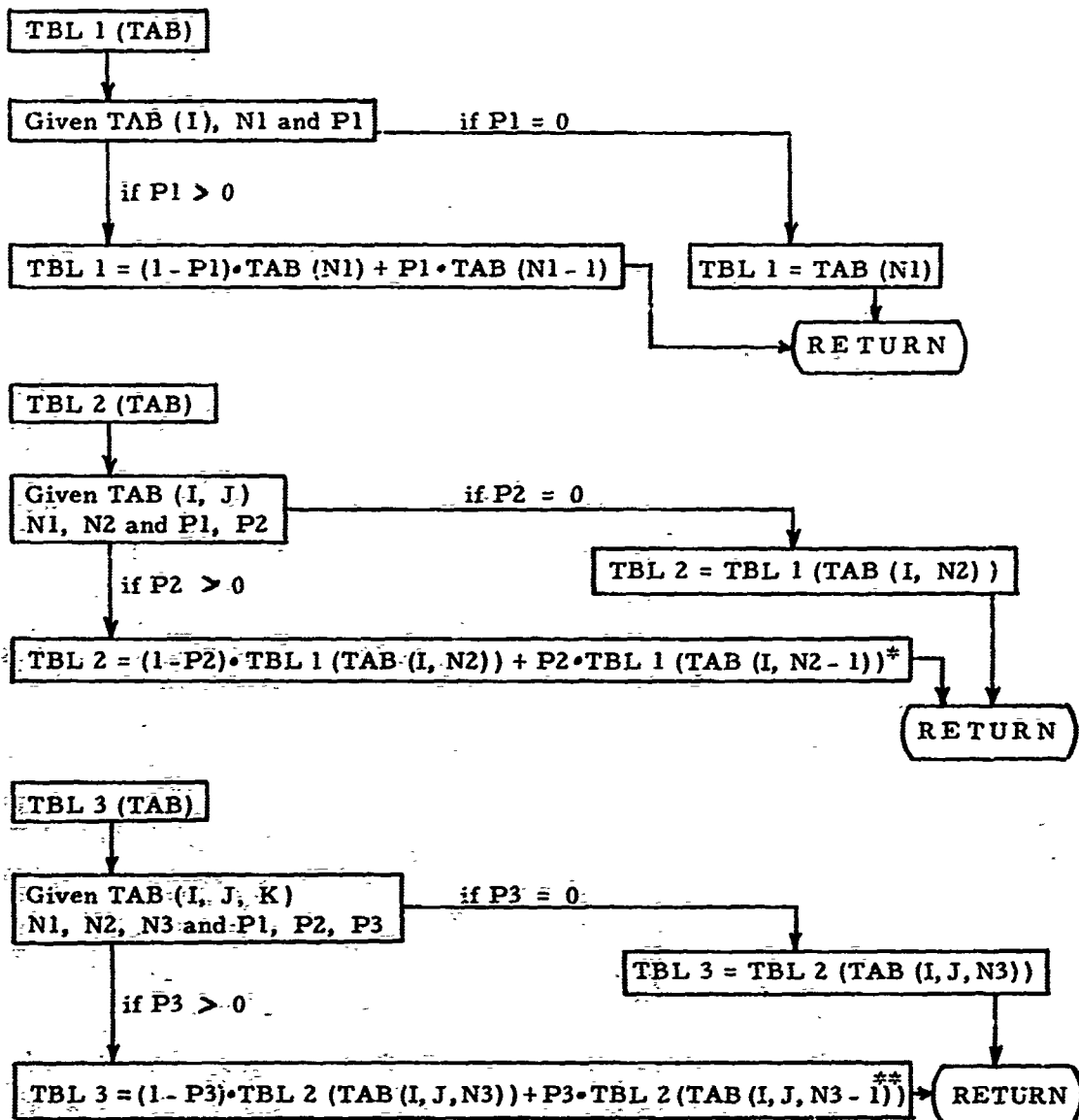
Note: N & P are used such that

$$Y = (1 - P) \cdot Y_N + P \cdot Y_{N-1} \quad \text{in TBL 1, 2 and 3}$$

* i.e., to handle negative altitude and negative p where only + arguments are in Table.

** i.e., saves interpolation.

Flow Chart of TBL 1, TBL 2, TBL 3



* $TBL\ 2 = (1 - P_2) [(1 - P_1) \cdot TAB(N_1, N_2) + P_1 \cdot TAB(N_1 - 1, N_2)]$

+ $P_2 [(1 - P_1) \cdot TAB(N_1, N_2 - 1) + P_1 \cdot TAB(N_1 - 1, N_2 - 1)]$

** $TBL\ 3 = (1 - P_3) [(1 - P_2) \{ (1 - P_1) \cdot TAB(N_1, N_2, N_3) + P_1 \cdot TAB(N_1 - 1, N_2, N_3) \}$

+ $P_2 \{ (1 - P_1) \cdot TAB(N_1, N_2 - 1, N_3) + P_1 \cdot TAB(N_1 - 1, N_2 - 1, N_3) \}]$

+ $P_3 [(1 - P_2) \{ (1 - P_1) \cdot TAB(N_1, N_2, N_3 - 1) + P_1 \cdot TAB(N_1 - 1, N_2, N_3 - 1) \}$

+ $P_2 \{ (1 - P_1) \cdot TAB(N_1, N_2 - 1, N_3 - 1) + P_1 \cdot TAB(N_1 - 1, N_2 - 1, N_3 - 1) \}]$

Compact Input Sheet

<u>N</u>			<u>No. of Columns</u>
-1	3511	KRD(I) I-1, 35	35
0	I3, 2X11A6, 1X11, L1, I4	I Run, HEADER, KA6, BODFIX, IDATE	78
1	6I2, F12.5	IMAX, JMAX, KMAX, LMAX, NRK, NCMAX, EPSMAX	24
2	6F12.5	TABA(I)	12-72
3		TABM(J)	
4		TABAR(K)	
5		CX (I, J) J then I	
6		CN (I, J) J then I	
7		CM (I, J) J then I	
8		CMQ (I, J) J then I	
9		CNR (I, J) J then I	
10		CMPR (I, J) J then I	
11		CMPQ (I, J) J then I	
12		CNPA (I, J, K) K then J then I	
13		CL (I, J, K) K then J then I	
14		CLP (I, J, K) K then J then I	
15		CMPA (I, J, K) K then J then I	
16		G, DMM, DIX, DI, DIZ, DIXY	72
17		S, DEE, THETA, PSI, Z, PHI	72
18		XDOT, YDOT, ZDOT, P, Q, R	72
19	F12.5, 4X18, 4F12.5, I4	TSTEP, INCPT1, TMAX, ZSTOP,	76
20	6F12.5	TCH, TNEW, INCPT2	
		TABZ(L)	12-72
		WDX(L)	12-72
		WDY(L)	12-72
21	6F12.5	DY, ZETD1, ZETD2	60
		ETA1, ETA2	
22		CYO (I, J) J then I	12-72
23		CZO (I, J) J then I	
24	Body	CMO (I, J) J then I	
25	Axis	CNO (I, J) J then I	
26	Only	CSF1 (I, J) J then I	
27		CN1 (I, J) J then I	
28		CSF3 (I, J) J then I	
29		CSM1 (I, J) J then I	
30		CM1 (I, J) J then I	
31		CSM3 (I, J) J then I	
32		CLPM1 (I, J) J then I	
33		CLPM2 (I, J) J then I	

N is a code for card type (helpful to punch in columns 79-80)

N > 0 all optional cards added runs (KRD(N) = 0 no, KRD(N) = 1 read)

N = 20 also optional on any run (LMAX = 0 no, LMAX > 0 read)

N = 0 required for each run.

N = -1 required for each additional run.

Cards 0 → 19 required for FIXED PLANE 1st run; 20 if LMAX > 0.

Cards 0 → 19, 21-33 required for BODY FIXED 1st run; 20 if LMAX > 0.

SECTION V

PROGRAM LISTING


```

0D(9)=ZAL7
SAVE=8.726646E-3*THETA
DUM(1)=COS(SAVE)
DUM(2)=SIN(SAVE)
SAVE=8.726646E-3*PSI
DUM(3)=COS(SAVE)
DUM(4)=SIN(SAVE)
BD(10)=DUM(1)*DUM(3)
BD(11)=-DUM(2)*DUM(4)
BD(12)=DUM(2)*DUM(3)
BD(13)=DUM(1)*DUM(4)
ZET1=0.
ZET2=0.
IF(.NOT.BODFIX) GO TO 10
EXTRA CALCULATIONS FOR BODY-FIXED AXIS
ZET1=1.745329E-2*ZETD1
ZET2=1.745329E-2*ZETD2
BODJX=DIXY/DIX
BODJY=DIXY/DI
BODJZ=DIXY/DIZ
BODEN=1.-BODJX*BODJY
SAVE=(DIX+DI-DIZ)/DIX
BODJ=SAVE*BODJY
RODPD=SAVE-BODEN
BODQD=SAVE*RAT-BODEN
BODRD=(DIX-DI)/DIZ
KOUNT=-1
IF(PHI.EQ.0.) GO TO 10
SAVE=8.726646E-3*PHI
DUM(1)=COS(SAVE)
DUM(2)=SIN(SAVE)
SAVE=BD(10)
BD(10)=SAVE*DUM(1)+BD(11)*DUM(2)
BD(11)=SAVE*DUM(2)+BD(11)*DUM(1)
SAVE=BD(12)
BD(12)=SAVE*DUM(1)+BD(13)*DUM(2)
BD(13)=-SAVE*DUM(2)+BD(13)*DUM(1)
10 CALL SETMAX(BD(10))

```

```

C      IF(.NOT.BODFIX) XMAT(3,2)=0.
C      GET U,V,W
C      DO 14 I=1,3
C      BD(I)=0.
C      DO 14 J=1,3
C      14 BD(I)=BD(I)+XMAT(J,I)*XDOT(J)
C      DO 19 I=1,13
C      E(I)=0.
C      19 D(I,NSTO)=BD(I)
C      INITIAL START OR RE-START
C      50 NINC=MAX0(1,INCPT)
C      NST=NSTO
C      EPSIL=0.
C      DTGOR=DT/3.
C      DTRP=2.*DT
C      DADM=DT/12.
C      DTMIL=4.*DTGOR
C      100 CALL EQMOT(D(1,NST),F(1,NST))
C      STOP OR CHANGE EQ TESTS
C      IF(D(9,NST).GT.ZSTOP.OR.TIME.GE.TMAX) GO TO 102
C      IF(NINC.LT.INCPT) GO TO 105
C      TIME TO PRINT
C      NINC=0
C      102 CALL WRITIT(TIME,D(1,NST),E,F(1,NST),EPSIL,LINE)
C      IF(NINC.GT.0) GO TO 1
C      105 IF(KNT.NE.KOUNT) GO TO 110
C      CHANGE TO STEADY-STATE EQUATIONS FOR Q,R AND RAISE DT (OLD GRNOT)
C      WRITE(6,2105)
C      2105 FORMAT(1X32(1H*),32HSIMPLIFIED EQUATIONS FOR Q AND R 32(1H*))
C      KNT=KNT+1
C      QR=.FALSE.
C      IF(TNEW.GT.0.) DT=TNEW
C      IF(INCPT2.GT.0) INCPT=INCPT2
C      DO 109 I=1,13
C      E(I)=0.
C      109 D(I,NSTO)=D(I,NST)
C      CALL EQMOT(D(1,NSTO),F(1,NSTO))
C      GO TO 50
C      ADVANCE TIME

```

```

110 TIME=TIME+DT
NINC=NINC+1
KNT=KNT+1
IF(NST,GE,4) GO TO 200
CONTINUE BUILDING UP FIRST 2-4 FUNCTIONS
CALL RUNGE(D(1,NST),F(1,NST),DT,D(1,NST+1))
NST=NST+1
IF(NST,LT,4,OR,NRK,EQ,2) GO TO 100
LAST RUNGE-KUTTA FUNCTION BUILT, CALCULATE FROM NSTO TO NST FOR ERRORS
CALL RUNGE(D(1,NSTO),F(1,NSTO),XINC*DT,D(1,5))
GO TO 250
C PREDICT
200 DO 249 I=1,13
GO TO (220,220,230,240),NRK
C TRAPAZOID
220 D(1,5)=D(1,3)+DTTRP*F(1,4)
GO TO 245
C ADAMS
230 D(1,5)=D(1,4)+DTADM*(23.*F(1,4)-16.*F(1,3)+5.*F(1,2))
GO TO 245
C MILNE
240 D(1,5)=D(1,1)+DTMIL*(2.*(F(1,4)+F(1,2))-F(1,3))
245 DO 247 J=1,3
D(1,J)=D(1,J+1)
247 F(1,J)=F(1,J+1)
249 D(1,4)=D(1,5)
C CORRECT WITH SIMPSON
250 NCOR=0
260 CALL EOMOT(D(1,4),F(1,4))
DO 269 I=1,13
IF(I,GE,10) DUM(I-9)=D(1,4)
269 D(1,4)=D(1,2)+DTCOR*(F(1,4)+4.*F(1,3)+F(1,2))
CALL NORMAL(D(10,4))
EPSIL=0.
DO 274 I=10,13
274 EPSIL=EPSIL+(D(1,4)-DUM(I-9))**2
EPSIL=SQRT(EPSIL)
NCOR=NCOR+1
CORRECT AGAIN IF ERROR IS TOO LARGE AND RE-CORRECTIONS ARE ALLOWED

```

```
IF(NCOR.LT.NCMAX.AND.EPSIL.GT.EPSMAX) GO TO 260
JO 279 I=1,13
279 E(I)=E(I)+CERR*(D(I,4)-D(I,5))
GO TO 100
END
```

SIXD
SIXD
SIXD
SIXD

```

S      FORTRAN NLSTOU,SYMTAB
S      INCODE  IBMF
C REPT  REPT
      SUBROUTINE REPT
      DIMENSION DATA(16)
      1 FORMAT (16A5)
      2 FORMAT (2H ,15,5X,16A5)
      3 FORMAT(20H1INPUT DATA -----/12H CARD NO. ,1H1,7X,2H10,8X,2H20REPR0050
      1,8X,2H30,8X,2H40,8X,2H50,8X,2H60,8X,2H70,8X,2H80//)
      4 FORMAT(2X,15A5,A3)
C***** KINDLY NOTE THAT COLS. 1-78 HAVE BEEN MOVED TO COLS. 3-80
C      IN ORDER TO AVOID RE-READING PROBLEMS OFF OF UNIT 4 *****
C      (THIS WILL CAUSE THE LAST 2 DIGITS OF DATE TO DISAPPEAR IN READIN. NOTE THAT
C      THE ROUTINE WRITIT PLACES IN A 70 ON THE OUTPUT OF DATE) *****
C      4 FORMAT(2X,15A5,A3)
      N = 0
      LINE = 56
C*****REPLACE BELOW*****
      CALL FLGEOF(5,IEOF)
      GO TO 20
C*****WITH*****
C      ASSIGN 10 TO IEOF
C      IF(EOF(IEOF)) 10,20,20
      10 END FILE 4
      END FILE 4
      REWIND 4
      RETURN
      20 REWIND 4
      40 READ (5,1) (DATA(I),I=1,16)
C*****DELETE CARD BELOW*****
C      IF(IEOF.GT.0) GO TO 10
C*****
      N = N + 1
      IF(LINE-56) 50,30,30
      30 WRITE (6,3)
      LINE = 3
      50 WRITE (6,2) N , (DATA(I),I=1,16)
      WRITE (4,4) (DATA(I),I=1,16)
      LINE = LINE + 1
      GO TO 40
      END

```

REPR0010
 REPR0020
 REPR0030
 REPR0040
 REPR0050
 REPR0060

REPR0070
 REPR0080
 REPR0090
 REPR0100

REPR0110
 REPR0120
 REPR0130
 REPR0140
 REPR0150
 REPR0160

REPR0161

REPR0170
 REPR0180
 REPR0190
 REPR0200
 REPR0210
 REPR0220
 REPR0230
 REPR0240
 REPR0250


```

IF(KRD(1).GT.0) READ(4,201) IMAX,JMAX,KMAX,LMAX,NRK,NCMAX,EPSSMAX READIN
IF(INAX.GT.20.OR.JMAX.GT.5.OR.KMAX.GT.5.OR.LMAX.GT.10) GO TO 8001 READIN
IF(KRD(2).GT.0) READ(4,202) (TABAR(I),I=1,IMAX)
IF(KRD(3).GT.0) READ(4,202) (TABM(I),I=1,JMAX)
IF(KRD(4).GT.0) READ(4,202) (TABAR(I),I=1,KMAX)
IF(KRD(5).GT.0) READ(4,202) ((CX(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(6).GT.0) READ(4,202) ((CN(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(7).GT.0) READ(4,202) ((CM(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(8).GT.0) READ(4,202) ((CMQ(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(9).GT.0) READ(4,202) ((CNR(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(10).GT.0) READ(4,202) ((CMPR(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(11).GT.0) READ(4,202) ((CNPQ(I,J),J=1,JMAX),I=1,IMAX)
IF(KRD(12).GT.0) READ(4,202) ((CNPA(I,J,K),K=1,KMAX),J=1,JMAX),
1I=1,IMAX)
IF(KRD(13).GT.0) READ(4,202) ((CL(I,J,K),K=1,KMAX),J=1,JMAX),
1I=1,IMAX)
IF(KRD(14).GT.0) READ(4,202) ((CLP(I,J,K),K=1,KMAX),J=1,JMAX),
1I=1,IMAX)
IF(KRD(15).GT.0) READ(4,202) ((CMPA(I,J,K),K=1,KMAX),J=1,JMAX),
1I=1,IMAX)
IF(KRD(16).GT.0) READ(4,202) G,DMM,DIX,D1,DIZ,DIXY READIN
IF(KRD(17).GT.0) READ(4,202) S,DEE,THETA,PSI,ZALT,PHI READIN
IF(KRD(18).GT.0) READ(4,202)(XDOT(I),I=1,3),P,Q,R READIN
IF(KRD(19).GT.0)READ(4,206)TSTEP,INCPT1,IMAX,ZSTOP,TCH,TNEW,INCPT2 READIN
IF(KRD(20).EQ.0.OR.LMAX.EQ.0) GO TO 10 READIN
READ(4,202) (TABZ(I),I=1,LMAX) READIN
READ(4,202) (WDX(I),I=1,LMAX) READIN
READ(4,202) (WDY(I),I=1,LMAX) READIN
10 IF(.NOT.HODFIX) RETURN READIN
IF(KRD(21).GT.0) READ(4,202) DY,ZETD1,ZETD2,ETA1,ETA2 READIN
IF(KRD(22).GT.0) READ(4,202) ((CYO(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(23).GT.0) READ(4,202) ((CZO(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(24).GT.0) READ(4,202) ((CMO(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(25).GT.0) READ(4,202) ((CNO(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(26).GT.0) READ(4,202) ((CSF1(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(27).GT.0) READ(4,202) ((CSF2(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(28).GT.0) READ(4,202) ((CSF3(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(29).GT.0) READ(4,202) ((CSM1(I,J),J=1,JMAX),I=1,IMAX) READIN
IF(KRD(30).GT.0) READ(4,202) ((CSM2(I,J),J=1,JMAX),I=1,IMAX) READIN

```

```

      IF(KRD(31).GT.0) READ(4,202) ((CSM3(I,J),J=1,JMAX),I=1,IJMAX)
      IF(KRD(32).GT.0) READ(4,202) ((CLPH1(I,J),J=1,JMAX),I=1,IJMAX)
      IF(KRD(33).GT.0) READ(4,202) ((CLPH2(I,J),J=1,JMAX),I=1,IJMAX)
      RETURN
8001 WRITE(6,E011)
9000 CALL EXIT
208 FORMAT(2X,35I1)
207 FORMAT(2X,I3,2X,I1A6,1X,I1,L1,A4)
201 FORMAT(2X,6I2,F12.5)
202 FORMAT(2X,6F12.5)
206 FORMAT(2X,F12.5,4X,I8,4F12.5,I4)
8011 FORMAT(1F,26H SUBSCRIPT VALUE TOO LARGE)
      END
      READIN
      READIN
      READIN
      READIN
      READIN
      READIN
      READIN
      READIN
      READIN
      READIN

```



```

S      FORTRAN MLSTOU,SYNTAX
S      INCODE IBMF
C EQMOT      EQMOT
      SUBROUTINE EQMOT(Y,Z)
C      SUBROUTINE TO DETERMINE THE DERIVATIVES OF THE 13 BASIC VARIABLES
COMMON TBL,ETOF,EQMIN,ALLIN,EQMPG,OUTEM
COMMON/TPL/ N1,N2,N3,P1,P2,P3
COMMON/ETOF/ XMAT(3,3)
COMMON /EQMIN/IMAX,JMAX,KMAX,LMAX,TAB(20),TARM(5),TABZ(1,COMMON
*0),CX(20,5),CN(20,5),CMQ(20,5),CNR(20,5),CMPR(20,5),CMPQ(COMMON
*20,5),CNFA(20,5,5),CL(20,5,5),CLP(20,5,5),CNPA(20,5,5),
*CYO(20,5),CZO(20,5),CMO(20,5),CFO(20,5),CSF1(20,5),CSF2(20,5),
*CSF3(20,5),CSM1(20,5),CSM2(20,5),CSM3(20,5),CLPH1(20,5),CLPH2(20,5,COMMON
*),WDX(10),WDY(10),G,PHM,S,DEE,DY,ETA1,ETA2
COMMON /ALLIN/DIX,DI,DIZ,DIXY, EODFIX
COMMON /EQMPG/ OR,PAT,BODJX,BODJY,BODJZ,BODEN,BODJ,BODPD,BODQD,
*BODFD,ZET1,ZET2
COMMON /OUTEM/ CAPH11,CAPH12,PDV,ALPHA,CAPVA,EM,PFE
DIMENSION Y(13),Z(13)
DIMENSION VA(3),XDOTA(3)
LOGICAL GR,BODFIX
CALL NORMAL(Y(10))
CALL SETMAX(Y(10))
IF(.NOT.EODFIX) XMAT(3,2)=0.
DO 3 I=1,3
Z(I+6)=0.
DO 3 J=1,3
3 Z(I+6)=Z(I+6)+XMAT(I,J)*Y(J)
IF(LMAX.GT.0) GO TO 6
DO 5 I=1,3
5 VA(I)=Y(I)
GO TO 10
6 CALL GETIN(Y(9),TABZ,LMAX,N1,P1)
XDOTA(1)=Z(7)-TBL1(WDX)
XDOTA(2)=Z(8)-TBL1(WDY)
XDOTA(3)=Z(9)
DO 7 I=1,3
VA(I)=0.

```

```

DO 7 J=1,3
  7 VA(I)=VA(I)+XMAT(J,I)*XDOTA(J)
  10 VASQ=VA(2)**2+VA(3)**2
    UASQ=VA(1)**2+VASQ
    ALPHA=0.
    IF(UASQ.GT.1.E-8) GO TO 12
    VELOCITY=0,SET OUTPUT TO ZERO AND SKIP AERODYNAMICS
    CAPVA=0.
    PDV=-999999.0
    CAPHI1=-ZET1
    CAPHI2=-ZET2
    ZETA=0.
    EM=0.
    FX=0.
    FY=0.
    FZ=0.
11  EMX=0.
    EMY=0.
    EMZ=0.
    IF(CR) GO TO 50
    Y(5)=0.
    Y(6)=0.
    GO TO 48
12  CAPVA=SQRT(UASQ)
    IF(VASQ.GT.1.E-8) GO TO 14
    SIDE VEL=0,SET FUNCTIONS OF ZETA TO 0.
    ZETA=0.
    SINZE=0.
    COSZE=0.
    GO TO 20
14  DEN=SQRT(VASQ)
    ALPHA=ATAN2(DEN,VA(1))
    SINZE=VA(3)/DEN
    COSZE=VA(2)/DEN
    ZETA=ATAN2(SINZE,COSZE)
    20 IF(36000.+Y(9).LE.0.) GO TO 22
    C LOW ALTITUDE
    VOS=1116.89+0.004123*Y(9)
    RHO=0.0023769*(1.+6.875E-6*Y(9))**4.2561

```

```

GO TO 25
C HIGH ALTITUDE
22 VOS=968.46
   RHO=0.004*EXP(46.46F-6*Y(9))
25 EM=CAPVA/VOS
   SET-UP TABLE VALUES
   PDV=.5*Y(4)*DEE/CAPVA
   CALL GETJN(ALPHA,TAPA,IMAX,N1,P1)
   CALL GETJN(EM,TARM,JMAX,N2,P2)
   CALL GETJN(PDV,TARAR,KMAX,N3,P3)
   CON=.5*RHO*UASQ*S
   FX=CON*TBL2(CX)
   CCN=TBL2(CN)
   CNPADD=-TBL3(CNPA)*PDV
   CAPH1=1.5707963-ZETA+ZET1
   CAPH2=1.5707963-ZETA+ZET2
   IF(.NOT.(RODFIX) GO TO 30
   SET1=SIN(ETA1*CAPH1)
   SET2=SIN(ETA2*CAPH2)
   CNPADD=-CNPADD+TBL2(CSF1)*SET1
   CCN=CCN+TBL2(CSF2)*SET1
30 FY=-CCN SIZE=CNPADD*SINZE
   IF(RODF FY=FY+TBL2(CY0)
   FY=CON*FY
   FZ=-CCN*SINZE-CNPADD*COSZF
   IF(RODFIX) FZ=FZ+TBL2(CZ0)
   FZ=CON*FZ
   CON=CON*DEE
   EMX=TBL3(CLP)+PDV*TBL3(CLP)
   IF(RODFIX) EMX=EMX+TBL2(CLPH1)*SET1+TBL2(CLPH2)*SET2
   EMX=EMX*CON
   IF(RODFIX.AND.DY.NE.0.) EMX=EMX-DY*FZ
   EMX=EMX/DIX
   SAVE2=.5*DEE/CAPVA
   SAVE1=(Y(5)*SINZE-Y(6)*COSZE)*SAVE2
   SAVE2=(Y(6)*SINZE+Y(5)*COSZE)*SAVE2
   CMADD=TBL2(CM)+SAVE1*TBL2(CMQ)+SAVE2*TBL2(CMPR)*PDV
   CNPADD=SAVE2*TBL2(CNR)+(TBL3(CNPA)+SAVE1*TBL2(CNPO))*PDV
   IF(RODFIX) CNPADD=CNPADD+TBL2(CSM1)*SET1+TBL2(CSM3)

```

```

IF(BODFIX) CMADD=CMADD+TBL2(CSM2)*SET1
EMY=CMADD*SINZE+CMADD*COSZF
IF(BODFIX) EMY=EMY+TRL2(CM0)
EMY=CON*EMY/DI
EMZ=-CMADD*COSZE+CMADD*SINZE
IF(BODFIX) EMZ=EMZ+TBL2(CM0)
EMZ=EMZ*CON
IF(.NOT.BODFIX) EMZ=EMZ/DI
IF(BODFIX) EMZ=(EMZ+DY*FX)/DI
45 IF(OR) GO TO 50
C SPECIAL CALCULATION FOR Q AND R
SAVE1=XMAT(3,1)/XMAT(3,3)*2.
Y(6)=((RAT*Y(4))-SORT(AMAX1(0.,(RAT*Y(4))*2-2.*SAVE1*EMY)))/SAVE1
Y(5)=-Y(6)*EMZ/EMY
48 Z(5)=0.
Z(6)=0.
CALCULATE EQUATIONS OF MOTION FOR U,V,W,P,Q,R AND THE QUATERNIANS
50 PEE=Y(4)
IF(.NOT.BODFIX) PEE=XMAT(3,1)/XMAT(3,3)*Y(6)
Z(1)=Y(2)*Y(6)-Y(3)*Y(5)+G*XMAT(3,1)+FX/DMM
Z(2)=Y(3)*PEE-Y(1)*Y(6)+G*XMAT(3,2)+FY/DMM
Z(3)=Y(1)*Y(5)-Y(2)*PEE+G*XMAT(3,3)+FZ/DMM
IF(BODFIX) GO TO 55
Z(4)=EMX
SAVE1=PEE-DIX/DI*Y(4)
Z(5)=EMY+SAVE1*Y(6)
Z(6)=EMZ-SAVE1*Y(5)
GO TO 60
55 Z(4)=((RODPD)*Y(5)-RODJ*PEE)*Y(6)+EMX+BODJX*EMY)/BODEN
Z(5)=((RODJ)*Y(5)-RODDJ*PEE)*Y(6)+EMY+RODJY*EMX)/RODEN
Z(6)=BODJZ*(PEE**2-Y(5)**2)+RODRD*PEE*Y(5)+EMZ
60 Z(10)=.5*(Y(11)*PEE-Y(12)*Y(5)-Y(13)*Y(6))
Z(11)=.5*(Y(10)*PEE-Y(13)*Y(5)+Y(12)*Y(6))
Z(12)=.5*(Y(13)*PEE+Y(10)*Y(5)-Y(11)*Y(6))
Z(13)=.5*(-Y(12)*PEE+Y(11)*Y(5)+Y(10)*Y(6))
RETURN
END

```



```

RETURN
100 FORMAT(1H1,31X,11A6/10X,11HRUN NUMBER ,13,90X,5HDATE-,A4,3H/70 / WRITIT
* 4X4HTIME, WRITIT
16X8HVELOCITY,8X1HX,11X1HY,11X1HZ,9X4HXDOT,8X4HYDOT,9X4HZDOT,1X WRITIT
*21H ALPHA CPH11 CPH127X8HVA MACH WRITIT
2 /4X5H(SEC),5X8H(FT/SEC),6X4H(FT),8X4H(FT),6X8H(FT/SEC)WRITIT
32(4X8H(FT/SEC)),3(7H (DEG)),3X8H(FT/SEC)/12X1HU11X1HV11X1HW11X1HPWRITIT
411X1HQ,11X1HR,9X3HPI8X WRITIT
* 5HTheta,7X3HPSI,8X5HEPSIL/9X8H(FT/SEC),4X8H(FT/SEC)WRITIT
5),4X8H(FT/SEC),3X9H(RAD/SEC),3X9H(RAD/SEC),3X9H(SEC),5X5H(DEG)WRITIT
6,6X5H(DEG)6X5H(DEG) // WRITIT
101 FORMAT(F9.4,7F12.1,1X 3F7.2,F10.1,F7.2/3X3F12.1,1X3F12.2, WRITIT
1 3F11.2,1PE17.6) WRITIT
103 FORMAT(1H1,31X,11A6/10X,11HRUN NUMBER ,13,90X,5HDATE-,A4,3H/70 / WRITIT
14X4HTIME12X5HPI10X5HTHDOT10X5HPSDOT9X21(1H*), WRITIT
*17HQUATERNION VALUES22(1H*)/4X5H(SEC)3X3(6X9H(RAD/SEC)),14X1H114X WRITIT
*1H214X1H314X1H4/12X5HP-ERR10X5HQ-ERR10X5HR-ERR10X5HPD/2V2X21(1H*),WRITIT
*17HQUATERNION ERRORS22(1H*)/4X3(6X9H(RAD/SEC)),22X1H114X1H214X1H3 WRITIT
*14X1H4//) WRITIT
104 FORMAT(F9.4, 3F15.2,4X4F15.6/4X1P8E15.6) WRITIT
END WRITIT

```

```

$  Fortran NLSTOU,SYMTAB
$  INCODE  IBMF
C  SETMAX  SETMAX
C  SUBROUTINE SETMAX(XL)
C  SUBROUTINE TO SET UP MATRIX FOR ROTATING BODY AXIS TO FIXED SPACE
C  XL=ARRAY OF QUATERNIONS (L0,L1,L2,L3) =L-BAR
C  XMAT=3X3 MATRIX--A FUNCTION OF L-BAR OR PHI,THETA,PSI
COMMON BT0F
COMMON/BT0F/ XMAT(3,3)
DIMENSION XL(4)
XMAT(1,2)=XL(1)**2
XMAT(1,3)=XL(2)**2
SAVE1 =XL(3)**2
SAVE2 =XL(4)**2
XMAT(1,1)=XMAT(1,2)+XMAT(1,3)-SAVE1-SAVE2
SAVE2=SAVE1-SAVE2
SAVE1=XMAT(1,2)-XMAT(1,3)
XMAT(2,2)=SAVE1+SAVE2
XMAT(3,3)=SAVE1-SAVE2
SAVE1=2.*XL(2)*XL(3)
SAVE2=2.*XL(1)*XL(4)
XMAT(1,2)=SAVE1-SAVE2
XMAT(2,1)=SAVE1+SAVE2
SAVE1=2.*XL(2)*XL(4)
SAVE2=2.*XL(1)*XL(3)
XMAT(1,3)=SAVE1+SAVE2
XMAT(3,1)=SAVE1-SAVE2
SAVE1=2.*XL(3)*XL(4)
SAVE2=2.*XL(1)*XL(2)
XMAT(2,3)=SAVE1-SAVE2
XMAT(3,2)=SAVE1+SAVE2
RETURN
END

```

```

$   FORTRAN NLSTOU,SYMTAB
$   INCODE  IBMF
C   NORMAL  NORMAL
      SUBROUTINE NORMAL(XL)
C   SUBROUTINE TO NORMALIZE LAMEDAS SO THEIR DOT PRODUCT=1
      DIMENSION XL(4)
      SAVE=0.
      DO 3 I=1,4
        3 SAVE=SAVE+XL(I)**2
      SAVE=1./SQRT(SAVE)
      DO 5 I=1,4
        5 XL(I)=XL(I)*SAVE
      RETURN
      END

```

NORMAL
 NORMAL
 NORMAL
 NORMAL
 NORMAL
 NORMAL
 NORMAL
 NORMAL
 NORMAL
 NORMAL


```

S      FORTRAN NLSTOU,SYMTAB
S      INCODE      IBMF
C      TBL1        FUNCTION TAB1
                  FUNCTION TBL1(TAB)
C      SINGLE DIMENSIONED INTERPOLATION GIVEN N1 AND P1
COMMON TBL
                  COMMON/TBL/ N1,N2,N3,P1,P2,P3
                  DIMENSION TAR(2)
                  TBL1=TAB(N1)
                  IF(P1.EQ.0.) RETURN
                  TBL1=TBL1-P1*(TBL1-TAB(N1-1))
                  RETURN
                  END

```

```

TBL1
TBL1
TBL1
COMMON
TBL1
TBL1
TBL1
TBL1
TBL1
TBL1

```

```

S      FORTRAN NLSTOU,SYMTAB
S      INCODE  IBMF
C TRL2  FUNCTION TAB2
C      DOUBLE DIMENSIONED INTERPOLATION GIVEN N1,N2, AND P1,P2
COMMON TBL
COMMON/TBL/ N1,N2,N3,P1,P2,P3
DIMENSION TAB(20,5)
TBL2=TBL1(TAB(1,N2))
IF(P2.EQ.0.) RETURN
TBL2=TBL2-P2*(TBL2-TBL1(TAB(1,N2-1)))
RETURN
END

```

```

TBL2
TBL2
TBL2
COMMON
TBL2
TBL2
TBL2
TBL2
TBL2
TBL2

```

```

S   FORTRAN NLSTOU,SYMTAB
S   INCODE IBMF
C   TRL3 FUNCTION TAB3
C   TRIPLE DIMENSIONED INTERPOLATION GIVEN N1,N2,N3 AND P1,P2,P3
COMMON TBL
COMMON/TBL/ N1,N2,N3,P1,P2,P3
DIMENSION TAB(20,5,5)
TBL3=TBL2(TAB(1,1,N3))
IF(P3.EQ.0.) RETURN
TBL3=TBL3-P3*(TBL3-TBL2(TAB(1,1,N3-1)))
RETURN
END

```

```

TBL3
TBL3
TBL3
COMMON
TBL3
TBL3
TBL3
TBL3
TBL3

```

SECTION VI

COMMENTS AND SPECIAL INSTRUCTIONS

A. MAGNUS ROTOR TRAJECTORY AND MOTION SIMULATION

In most instances, magnus rotor trajectories will be computed with fixed-plane axes. Where complete trajectory data are to be obtained (launch to impact) for flight times of several seconds or more, it will often be convenient to use the $\dot{q} = \dot{r} = 0$ option after about the first second of flight* (or after a time where the nutation is damped to a few degrees amplitude). For the investigation of autorotation initiation and the effects of dynamic unbalance, the body-fixed axes may be used.

For initiating magnus rotor trajectories, it is convenient to use the initial horizontal velocity component along the (-Y) axis, such that the Euler angles, θ and ψ , will approach zero if the rotor is in gliding flight attitude, $\alpha \rightarrow \pi/2$. In this manner, positive spin rates, corresponding to positive values of the spin torque coefficient $C_L(\alpha, M, \frac{p d}{2V})$, will result in an upward magnus lift force.

Special care must be used in selecting the initial conditions. Letting the initial flight path angle, γ , be specified by

$$\begin{aligned} \dot{X} &= 0 \\ -\dot{Y} &= V \cos \gamma \\ \dot{Z} &= V \sin \gamma \end{aligned} \quad 1)$$

then the initial Euler angles, θ and ψ for the fixed-plane axes, are related to the initial sideslip angle, β , and the orientation of the angle of attack plane, $\bar{\phi}$, by the following relations (see also Figure 6).

$$\sin(-\theta) = \cos \beta \sin \bar{\phi} \cos \gamma + \sin \beta \sin \gamma \quad 2)$$

$$\tan(-\psi) = \frac{-\cos \beta \sin \bar{\phi} \sin \gamma + \sin \beta \cos \gamma}{\cos \beta \cos \bar{\phi}} \quad 3)$$

$$\begin{array}{ll} \text{where} & -\frac{\pi}{2} < \psi < \frac{\pi}{2} \quad \text{for} \quad -\frac{\pi}{2} < \bar{\phi} < \frac{\pi}{2} \\ \text{and} & \frac{\pi}{2} < \psi < \frac{3\pi}{2} \quad \text{for} \quad \frac{\pi}{2} < \bar{\phi} < \frac{3\pi}{2} \end{array}$$

* Note: if the $\dot{q} = \dot{r} = 0$ option is not used with fixed plane axes, then TCH > TSTOP for input lead card 5.

If an initial coning motion is desired, such that the total angular momentum does not coincide with x axis, then the coning angle can be specified by, λ , and the equivalent cross-angular velocity determined by

$$\Omega_{\text{equiv.}} = \frac{p \frac{I_x}{I}}{\cos \lambda} \quad 4)$$

The angular velocity components with respect to the lateral fixed-plane axes are given in terms of $\Omega_{\text{eq.}}$ by the relationships

$$q = \Omega_{\text{eq.}} \left\{ \begin{aligned} &\cos(\beta - \lambda) [\cos \bar{\phi} \sin(-\psi) + \sin \bar{\phi} \sin \gamma \cos(-\psi)] \\ &- \sin(\beta - \lambda) \cos \gamma \cos(-\psi) \end{aligned} \right\} \quad 5)$$

$$\begin{aligned} r = \Omega_{\text{eq.}} \left\{ \begin{aligned} &\cos(\beta - \lambda) [-\cos \bar{\phi} \cos(-\psi) \sin(-\theta) \\ &+ \sin \bar{\phi} \sin \gamma \sin(-\psi) \sin(-\theta) + \sin \bar{\phi} \cos \gamma \cos(-\theta)] \\ &+ \sin(\beta - \lambda) [-\cos \gamma \sin(-\psi) \sin(-\theta) + \sin \gamma \cos(-\theta)] \end{aligned} \right\} \quad 6) \end{aligned}$$

When random values of $\bar{\alpha}$ and $\bar{\phi}$ are to be used for Monte Carlo simulation of impact patterns, relations 2) and 3) will be used extensively. In simulation of impact patterns, symmetry considerations can also be employed such that only the left or right hand side of the pattern need be determined. The symmetry considerations are applied to the initial conditions by restricting the angle of attack to $0 < \bar{\alpha} \leq \frac{\pi}{2}$.

B. BODY-FIXED AXES

Ballistic trajectories of slowly-spinning rockets, missiles, bombs, and projectiles will usually be computed using the body-fixed axes option. For ballistic-type trajectories it will be convenient to select the horizontal component of the initial velocity in the direction of the positive X axis, in order that first quadrant values can be used for the Euler angles, θ and ψ .

Supplementary calculations will be required to determine the initial Euler angles if only the velocity vector and the angle of attack parameters, $\bar{\alpha}$ and $\bar{\phi}$, are known or specified. The procedure involves finding the values of θ and ψ which satisfy the matrix equation

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} c\theta \cdot c\psi & c\theta \cdot s\psi & -s\theta \\ c\psi \cdot s\phi \cdot s\theta - c\phi \cdot s\psi & s\phi \cdot s\theta \cdot s\psi + c\phi \cdot c\psi & s\phi \cdot c\theta \\ c\psi \cdot c\phi \cdot s\theta + s\phi \cdot s\psi & c\phi \cdot s\theta \cdot s\psi - s\phi \cdot c\psi & c\phi \cdot c\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

subject to the constraints

$$\tilde{\alpha} = \tan^{-1} \frac{\sqrt{u^2 + w^2}}{v}$$

$$\xi = \tan^{-1} \frac{w}{v}$$

$$\phi_0 = \text{CONSTANT}$$

For special cases, closed form solutions exist, i. e., for $\dot{y} = \dot{z} = \phi_0 = 0$

$$\tan \theta = \frac{\tan \tilde{\alpha}}{\left[\frac{1}{\tan^2 \xi} + 1 \right]^{1/2}}$$

$$\cos \psi = \frac{\cos \theta}{\cos \tilde{\alpha}}$$

More general velocity vector orientations with $\phi_0 \neq 0$ require complicated numerical solutions. Obviously, these problems do not exist if the Euler angles can be selected apriori. However, for multiple simulations, involving a fixed velocity vector, the above procedure will normally be required. A nearly general solution for θ, ψ , subject only to $\dot{y} = 0$, i. e., $\tan \gamma = \dot{z}/\dot{x}$, is given by the relations

$$\sin \theta [-\sin \gamma] + \cos \theta [\cos \gamma \cos \psi] = \cos \tilde{\alpha}$$

$$\sin \theta \left[\frac{1}{\tan \psi} \right] + \cos \theta \left[\frac{\tan \gamma}{\sin \psi} \right] = \frac{\sin \phi_0 + \cos \phi_0 \tan \xi}{\sin \phi_0 \tan \xi - \cos \phi_0}$$

C. INTEGRATION INTERVAL

For magnus rotor motion simulations, it will generally be advisable to select an integration time interval (TSTEP) such that about 100 integrations are performed for each cyclic period. For fixed-plane axes, the highest frequency will be the nutation frequency, while for body-fixed axes the highest frequency will correspond to the spin rate. Since the nutation frequency is approximately $p I_x / I$, a large saving in computing time can be achieved using the fixed-plane axes if $I_x / I \ll 1$.

When it is anticipated that only a short segment of a motion history will involve high frequencies, localized integration improvement can be achieved by using the re-correction option. A quaternion error parameter EPSMAX of about 1.0×10^{-7} would then be used in conjunction with a value of NCMAX of 3 or 4.

REFERENCES

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13. ABSTRACT A six-degrees-of-freedom trajectory program for quasi-symmetric rigid bodies is described. The equations of motion are developed such that either a body-fixed or fixed-plane moving coordinate system can be utilized. Provision is made for large angle and angular rate motions, such as are experienced by magnus rotor munitions. The aerodynamic system permits the usual aeroballistic coefficients to be expressed as tabular functions of angle of attack and Mach number; in addition, the magnus force, magnus moment, and rolling moment coefficients can be tabular functions of the nondimensional spin parameter, $pd/2V$. Additional aerodynamic terms are provided to account for body-fixed aerodynamic asymmetries and/or control inputs, aerodynamic roll angle effects, flow asymmetry with respect to the angle of attack plane at zero spin, and lateral c. g. offset. The computer program is written in General Fortran IV language compatible with CDC 6400, IBM 360, and GE 635 data processing machines. Included in the report are the program input and output formats, flow charts of the main program subroutines, and a complete program listing.			

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