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AFML-TR-70-58

Volume III

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**ADVANCED COMPOSITES DATA
FOR
AIRCRAFT STRUCTURAL DESIGN**

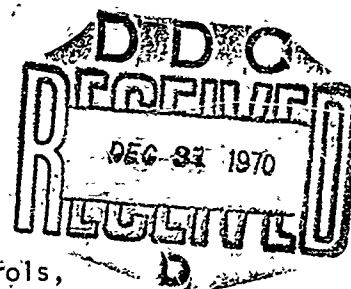
Volume III: Theoretical Methods

L. M. Lackman
T. H. Lin
D. Y. Konishi
J. W. Davidson

**LOS ANGELES DIVISION
NORTH AMERICAN ROCKWELL**

TECHNICAL REPORT AFML-TR-70-58, VOLUME III

DECEMBER 1970



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**AIR FORCE MATERIALS LABORATORY
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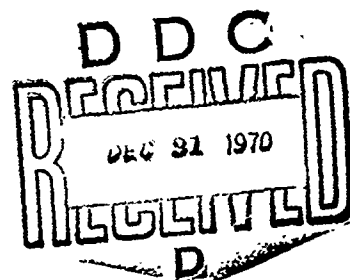
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AFML-TR-70-58
Volume III

**ADVANCED COMPOSITES DATA
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AIRCRAFT STRUCTURAL DESIGN**

Volume III: Theoretical Methods

L. M. Lackman
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FOREWORD

This report was prepared by the Los Angeles Division of North American Rockwell Corporation under Contract F33615-68-C-1489 for the Advanced Composites Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. Mr. R. L. Rapson (AFML/LC) was the Air Force Project Engineer and Dr. L. M. Lackman was the North American Rockwell Program Manager.

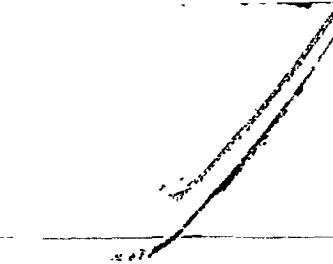
The authors of Volume III are Dr. L. M. Lackman, Mr. D. Y. Konishi, Dr. T. H. Lin, and Mr. J. W. Davidson, who must be credited collectively for this volume, since, for the most part, the author of each section was supported substantially by some or all of the others. In addition, Dr. A. Caputo contributed to the development of the thermoelastic relationships reported in section VII. Mr. G. H. Arvin, as Editor in Chief, was responsible for the organization and continuity of this volume.

This report was submitted by the authors on 14 May 1970 for publication as a technical report.

This report has been reviewed and is approved.



Robert C. Tomashot
Technical Area Manager
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ABSTRACT

This volume summarizes that portion of the program concerned with the development of theoretical methods of analysis for orthotropic laminated composite structure. An appendix to this volume presents a number of computer programs that were generated in conjunction with these theoretical developments.

The main body of this volume is comprised of independent sections devoted to the following topics:

Extensional and flexural elastic constants

Flat plates

Orthotropic sandwich plates and shells

Stiffened skin construction

Minimum weight analysis

Thermoelastic relationships

Joint analysis

The appendix to this volume presents, for each of the included computer programs, an abstract, data input instructions, output description, and a typical example problem. The listed programs cover the following subjects:

Calculation of laminate extensional and flexural elastic constants.

Honeycomb sandwich panels under in-plane biaxial loading, in-plane shear loading, normal pressure, or certain combinations of these.

Hat- and Z-stiffened panel stability and stiffness.

Laminate thermoelastic analysis.

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LIST OF SYMBOLS

a	- length dimension (in.), especially rectangular panel
A_{ij} ($i, j=1, 2, 6$)	- extensional rigidities (lb/in.)
b	- width dimension (in.), especially width of compression panel normal to load
b_s	- stiffener spacing (in.)
c	- honeycomb sandwich core depth (in.)
D_{ij} ($i, j=1, 2, 6$)	- flexural rigidities (lb/in.)
$DQ_x = cG'_{cx}$	- honeycomb sandwich shear stiffness along X axis
$DQ_y = cG'_{cy}$	- honeycomb sandwich shear stiffness along Y axis
d_s	- width of stiffener element (in.)
E_L	- Young's modulus of laminae parallel to filament direction (lb/in. ²)
E_T	- Young's modulus of laminae transverse to filament direction (lb/in. ²)
E_x	- Young's modulus of laminate along X reference axis (lb/in. ²)
E_y	- Young's modulus of laminate along Y reference axis (lb/in. ²)
f_s	- width of stiffener element (in.)
G_{LT}	- shear modulus of laminae in TL plane (lb/in. ²)
G_{xy}	- shear modulus of laminate in XY plane (lb/in. ²)

G'_{cx}	- shear modulus of honeycomb sandwich core in XZ plane (lb/in. ²)
G'_{cy}	- shear modulus of honeycomb sandwich core in YZ plane (lb/in. ²)
h_i	- thickness of i^{th} lamina (in.)
h_s	- stiffener depth (in.)
I_1	- invariant defined by equation 83
I_2	- invariant defined by equation 84
k	- (1) buckling coefficient (2) coefficient of thermal conductivity (BTU/ft ² /hr/in./°F)
k_x, k_y	- plate compression buckling coefficients
k_s	- plate shear buckling coefficient
L	- cylinder, beam, or column length (in.)
M	- mass per unit area of laminate
m	- number of half-wave lengths
M_x, M_y, M_{xy}	- bending and twisting moment components (in.-lb/in.)
N	- number of laminae in a laminate
n	- (1) number of half-wave lengths (2) number of laminae in a laminate (3) shear stiffness ratio $\frac{DQ_y}{DQ_x}$
N_x, N_y, N_{xy}	- force components (lb/in.)
P	- applied load (lb)
p_l	- number of plies in a set which are at 0°

p_2	- number of plies in a set which are at 90°
p_θ	- half the number of plies in a set which are at $\pm\theta^\circ$
Q_x, Q_y	- shear force parallel to Z axis of sections of a plate perpendicular to the X and Y axes, respectively (lb/in.)
Q_{ij}, T_{ij} (i,j=1,2,6)	- lamina elastic constants in natural coordinate system
$\bar{Q}_{ij}$ (i,j=1,2,6)	- lamina elastic constants in laminate coordinate system
R	- radius
r	- honeycomb core shear modulus ratio $\frac{G'_{cx}}{G'_{cy}}$
T	- temperature ($^\circ\text{F}$)
t	- thickness (in.)
t_f	- honeycomb sandwich face sheet thickness (in.)
t_s, t_{s_1}, t_{s_2}	- stiffener thickness (in.)
u, v, w	- components of the displacement vector (in.)
u_o, v_o, w	- components of the displacement vector at the midsurface of the laminate (in.)
V	- (1) shear force (lb) (2) sandwich shear stiffness parameter $\left(\frac{\pi}{b}\right)^2 \frac{\sqrt{D_{11}D_{22}}}{DQ_x}$
α	- (1) coefficient of thermal expansion (in./in./ $^\circ\text{F}$) - (2) the quantity $\sqrt{E_x/E_y}$ (used in section IV only)
α_L	- lamina coefficient of thermal expansion along L axis (in./in./ $^\circ\text{F}$)

- α_T - lamina coefficient of thermal expansion along T axis (in./in./°F)
- α_X^C - laminate coefficient of thermal expansion along X axis (in./in./°F)
- α_Y^C - laminate coefficient of thermal expansion along Y axis (in./in./°F)
- α_{xy}^C - laminate shear distortion coefficient of thermal expansion (in./in./°F)
- β - the quantity $\alpha\nu_{yx} + 2\gamma$ (used in section IV only)
- γ - (1) shear strain (in./in.)
- (2) ratio $\frac{D_{66}}{\sqrt{D_{11}D_{22}}}$
- Δ - difference (used as a prefix to quantitative symbols)
- ϵ - strain (in./in.)
- η - honeycomb core shear modulus ratio $\frac{G'_{cy}}{G'_{cx}}$
- θ - angular orientation of a lamina in a laminate, i.e., the angle between L and X axes (°)
- λ_{xy} - product of $\nu_{xy} \nu_{yx}$
- ν - Poisson's ratio
- ν_{LT} - Poisson's ratio relating contraction in the T direction due to extension in the L direction
- ν_{TL} - Poisson's ratio relating contraction in the L direction due to extension in the T direction

ν_{xy}	- Poisson's ratio relating contraction in the y direction due to extension in the x direction
ν_{yx}	- Poisson's ratio relating contraction in the x direction due to extension in the y direction
ρ	- density (lb/ft ³)
ρ'_c	- honeycomb sandwich core density (lb/ft ³)
ρ_f	- density of honeycomb sandwich face sheets (lb/ft ³)
σ	- applied normal stress (lb/in. ²)
τ	- applied shear stress (lb/in. ²)
σ_{ij}, τ_{ij}	- stress in j direction on surface whose outer normal is in i direction (i,j = 1,2,3 or x,y,z) (lb/in. ²)
ϕ	- ratio $\frac{N_y}{N_x}$
ψ	- the quantity $\frac{\sqrt{E_x E_y}}{1 - \nu_{xy} \nu_{yx}}$

SUBSCRIPTS AND SUPERSCRIPTS

c	- composite laminate (as distinguished from separate laminae)
f	- face sheet
L, T, Z	- laminae natural orthogonal coordinates
s	- stiffener
X, Y, Z	- general coordinate system, laminate coordinate system

Section I

INTRODUCTION

The purpose of this program was to take the first step toward the generation and presentation of basic engineering data necessary to perform high-confidence-level structural design of primary aircraft structures utilizing advanced composite materials. The program was limited to an in-depth generation of basic material allowables for one boron/epoxy and several graphite/epoxy material systems, and the determination of basic structural element response for the boron/epoxy system alone. The boron portion of this program was conducted in conjunction with a concurrent General Dynamics/Forth Worth (GD/FW) program which was funded under Air Force Contract F33615-68-C-1474. The boron/epoxy material system highlighted by both these programs was Narmco 5505, furnished by the supplier as 3-inch prepreg tape. This is a composite material consisting of collimated 4-mil boron filaments, 208 per inch of tape width, embedded in a matrix of Narmco 2387 epoxy resin, and supported on a 1-mil layer of 104 glass scrim cloth. The graphite portion of this program is being conducted independent of any other program, and will be described in fuller detail in a later volume of this report. Additional data for all these materials, as well as for other filament/matrix material systems, were obtained from published Government, industry, and technical journal reports, and were used to augment the data generated in this program.

This program was composed of three major work task areas:

- Task I - Generation of Composite Material Design Allowables
- Task II - Structural Element Test Program and Analysis Evaluation
- Task III - Development of Advanced Composite Structural Design Manual for Aircraft

Task I is divided into two distinct areas of effort by the separate boron/epoxy and graphite/epoxy programs. The purpose of the boron portion of task I was to complement the basic material design allowable activities conducted by GD/FW (reference 12) and to develop acceptable laminate fabrication and inspection procedures. The boron effort was divided into the following work areas: The establishment of program coordination procedures for the North American Rockwell Corporation and General Dynamics related programs; the accomplishment of a limited material development program; the generation of basic allowables for the constituent materials; establishment of the accuracy of current analytical procedures for predicting certain basic allowables; and the development, where reliable techniques were lacking, of prediction techniques for these basic material allowables.

The graphite portion of task I consists primarily of a screening and characterization of several graphite/epoxy material systems and will be delineated in a later volume of this report.

The purpose of task II, which was concerned solely with boron/epoxy material, was to generate data on basic structural elements which form the building blocks from which aircraft structures are designed. A minimum evaluation of structural elements was conducted, including one basic laminate and one elevated temperature. Factors which were considered in the detail design of the structural elements included laminate orientations, panel proportions and edge restraints, effectiveness of typical forms of panel stabilization, evaluation of cutouts, and thermal gradient effects. One or more elements were selected for each primary and/or combined load applications. The test program included local and general instability of flat panels and natural frequency determinations. The results of this test program were compared to predicted response, failure mode, and strength techniques for basic structural elements.

The task III work area was originally centered on the development of an advanced composite structural design manual for aircraft structures. The first effort of this task involved revision and refinement of the Aircraft Division of the Intermediate Draft of the Structural Design Guide developed by the Southwest Research Institute, San Antonio, Texas, under Air Force Contract AF33(615)-5142. The completely revised and reorganized Aircraft Division resulting from this phase of effort was published in the Final Draft of the Design Guide in November 1968 under Contract AF33(615)-68-C-1241. Soon thereafter, a review of the Final Draft by a select industry group led to a decision by AFML to reorganize the entire Design Guide for the First Edition, which was then assigned to NR/LAD under Contract F33615-69-C-1368.

Subsequent phases of task III of this program, in light of the foregoing developments, consisted of the preparation of the Aircraft System Applications chapter of the First Edition of the Design Guide as well as the preparation of data generated by tasks I and II of this program for incorporation into the various technical function-oriented chapters. Task III also included the incorporation into the Design Guide of data generated by the concurrent GD program.

The bulk of the basic material allowables for the 5505 material system was generated by the General Dynamics contract. This concurrent and integrally related contract was coordinated with the Los Angeles Division program effort through scheduled periodic coordination meetings. These meetings insured the continuous flow of pertinent program data between the two contractors.

This report is divided into four separate volumes, in each of which the subject areas of interest comprise an independent segment of the overall program. Each volume is a self-contained document, complementing the other three

volumes but not dependent upon them for coherence or continuity. The titles of the four volumes are:

Volume I - Material and Basic Allowable Development - Boron/Epoxy

Volume II - Structural Element Behavior - Test and Analytical Determination

Volume III - Theoretical Methods

Volume IV - Material and Basic Allowable Development - Graphite/Epoxy

Volume II contains the analytical derivation of the behavioral equations and relationships characteristic of anisotropic composites. It is divided into a number of independent sections, each of which treats a particular structural element concept or some separate facet of advanced composites application to structural design.

Section II is fundamental in scope and concerns the extensional and flexural elastic constants of filamentary composite laminae and laminates. Sections III, IV, and V cover the behavioral characteristics of flat plates, orthotropic sandwich plates and shells, and stiffened skin construction, respectively. Section IV discusses the general approach to minimum weight design, with detailed attention to zee- and hat-stiffened elements, while Section VII is devoted to the thermoelastic relationships of composites. Section VIII concerns the special topic of composite panel joints, and includes separate treatment of several types of joint.

Laminate ply orientations are described and specified in this report by use of the laminate orientation code defined in the Structural Design Guide for Advanced Composite Applications.

SECTION II

EXTENSIONAL AND FLEXURAL ELASTIC CONSTANTS

STRESS-STRAIN RELATIONS OF A LAMINA IN GENERALIZED PLANE STRESS

Consider the fiber reinforcement to be unidirectional. Referring to a set of orthogonal axes, L, T, and Z, with L parallel to the fiber direction and Z = 0 coinciding with the middle surface of the lamina, then the stress-strain relation of the lamina shown in figure 1 may be represented by

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \frac{\gamma_{LT}}{2} \end{Bmatrix} \quad (1)$$

where σ_L , σ_T are direct stresses, σ_{LT} is the shear stress. ϵ_L , ϵ_T are direct strains, and γ_{LT} is the shear strain.

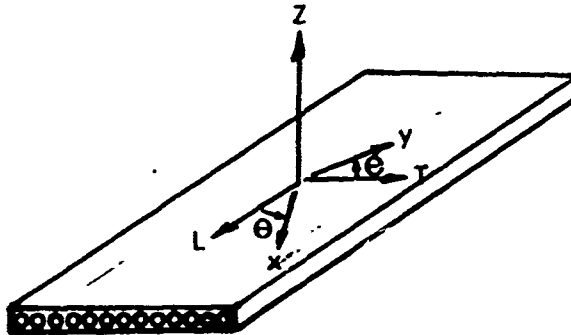


Figure 1. Typical Lamina

$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}}, \quad Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{12} = Q_{21} = \frac{E_T \nu_{LT}}{1 - \nu_{LT}\nu_{TL}} = \frac{E_L \nu_{TL}}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{66} = G_{LT}$$

E_L , E_T denote the Young's modulus in L and T directions, respectively, G_{LT} denotes the shear modulus, and ν_{LT} and ν_{TL} denote the major and minor Poisson's ratios, respectively.

Let X, Y, and Z be a set of rectangular axes where the X and Y axes are obtained by rotating L and T axes through an angle θ as shown in figure 2.

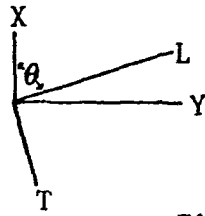


Figure 2. Lamina and Laminar Axes

The stress-strain relation referring to X, Y axes is obtained by transforming equation 1 as

$$\begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_{XY} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_X \\ \epsilon_Y \\ \gamma_{XY} \end{Bmatrix} \quad (2)$$

where

$$\bar{Q}_{11} = Q_{11} m^4 + 2(Q_{12} + 2Q_{66}) m^2 n^2 + Q_{22} n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) m^2 n^2 + Q_{12}(m^4 + n^4) = \bar{Q}_{21}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66}) m^3 n + (Q_{12} - Q_{22} + 2Q_{66}) mn^3 = \bar{Q}_{61}$$

$$\bar{Q}_{22} = Q_{11} n^4 + 2(Q_{12} + 2Q_{66}) m^2 n^2 + Q_{22} m^4$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66}) mn^3 + (Q_{12} - Q_{22} + 2Q_{66}) m^3 n = \bar{Q}_{62}$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) m^2 n^2 + Q_{66}(m^4 + n^4)$$

$$m = \cos \theta$$

$$n = \sin \theta$$

It should be noted that the stress-strain relation referring to the lamina axes (equation 1) is orthotropic, i.e., no coupling between the extensional and shearing stresses and strains, while the stress-strain relation referring to the laminate axes (equation 2) is generally not orthotropic.

SECTIONAL FORCES AND MOMENTS OF A LAMINATE IN TERMS OF DISPLACEMENTS

From Kirchhoff's assumption, normals to the middle plane of the laminate remain normal to the middle surface of the laminate during deformation, and the normal stress on planes parallel to the middle surface may be neglected in comparison with other stress components. Let u , v and w be the displacement along the X , Y and Z axes, and let u_0 , v_0 and w_0 be the corresponding displacement of points on the middle plane. We have

$$\begin{aligned} u &= u_0 - z \frac{\partial w}{\partial x} \\ v &= v_0 - z \frac{\partial w}{\partial y} \\ w &= w_0 \end{aligned} \quad (3)$$

For small strain,

$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (4)$$

Substituting equation 4 into equation 2, we have

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = [\bar{Q}_{ij}]_i \begin{Bmatrix} \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} \\ \left(\frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \right) - 2z \frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \quad (5)$$

where the subscript "i" denotes the i^{th} lamina in the laminate. The notations of the laminate are shown in figure 3. For a laminate of N laminae, the section forces and moments shown in figure 4 are defined as

$$\{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}\} = \sum_{i=1}^N \int_{h_i}^{h_{i+1}} \left(\sigma_x^i, \sigma_y^i, \sigma_{xy}^i, z\sigma_x^i, z\sigma_y^i, z\sigma_{xy}^i \right) dz \quad (6)$$

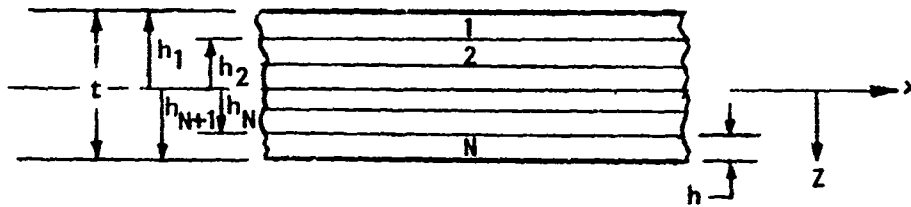


Figure 3. Laminate of N Laminae

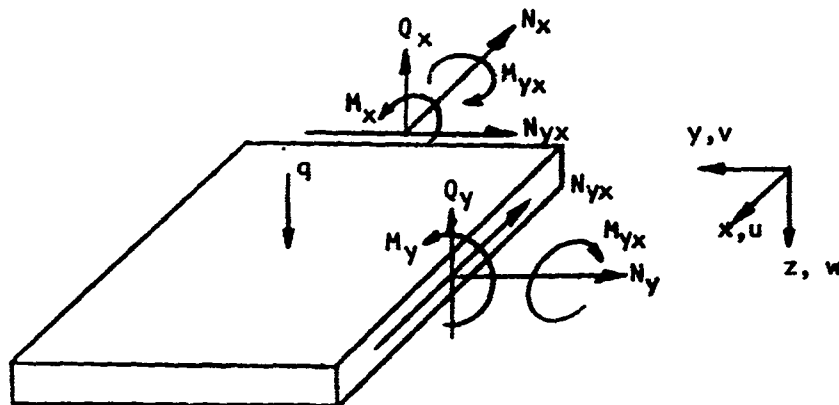


Figure 4. Positive Direction of Internal Forces and Moments

The sign conventions for those section forces and moments are shown in figure 4. The laminae are now considered to be symmetrically laid with respect to the middle plane of the laminate. Performing the integration as shown in equation 6, we have

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A_{ij}] \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ (u_{0,y} + v_{0,x}) \end{Bmatrix} \quad (7a)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = - [D_{ij}] \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{2\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (7b)$$

LAMINATE EXTENSIONAL CONSTANTS

Since all the laminae are of the same thickness, we have from figure 3

$$h = \frac{t}{N} \quad (8a)$$

$$h_i = (i-1)h - \frac{t}{2}$$

$$h_{i+1}^3 - h_i^3 = \frac{h^3}{3} \left[(3i^2 - 3i + 1) + \frac{3N}{2}(1-2i) + \frac{3N^2}{4} \right] \quad \text{for } i=1, 2, \dots, N \quad (8b)$$

$$= \frac{1}{3} \left(\frac{t}{N} \right)^3 \left[3 \left(i - \frac{N+1}{2} \right)^2 + \frac{1}{4} \right]$$

and

$$\begin{Bmatrix} A_{ij} \\ D_{ij} \end{Bmatrix} = \sum_{i=1}^N \bar{Q}_{ij}^{(i)} \begin{Bmatrix} h \\ \frac{h^3}{3} \left[3 \left(i - \frac{N+1}{2} \right)^2 + \frac{1}{4} \right] \end{Bmatrix} \quad (9)$$

Let us now further consider only the cases where the number of laminae with $+\theta$ is equal to that with $-\theta$, i.e., the layup is balanced. Then

$$A_{16} = A_{61} = A_{26} = A_{62} = 0 \quad (10)$$

and equation 7a reduces to

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (11)$$

The stress-strain relation corresponding to equation 11 for a homogeneous plate is given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{1}{t} \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (12)$$

Comparing this with equation 1, it is seen that the equivalent homogeneous elastic constants of the laminate are then

$$\nu_{yx} = A_{12}/A_{11}$$

$$\nu_{xy} = A_{12}/A_{22}$$

$$E_x = \frac{A_{11} A_{22} - A_{12}^2}{t A_{22}}$$

$$E_y = \frac{A_{11} A_{22} - A_{12}^2}{t A_{11}}$$

$$G_{xy} = \frac{A_{66}}{t}$$

(13)

In addition, since the laminate is considered to be laid up symmetrically relative to its midsurface, equation 9 reduces to

$$\begin{Bmatrix} A_{ij} \\ D_{ij} \end{Bmatrix} = \sum_{i=\frac{N}{2}+1}^N \begin{Bmatrix} 2h \\ \frac{2h^3}{3} \left[3 \left(i - \frac{N+1}{2} \right)^2 + \frac{1}{4} \right] \end{Bmatrix} \bar{Q}_{ij}^{(i)} \quad (14)$$

D_{ij} 's OF SOME COMMONLY USED LAMINATES

Let us now consider symmetric laminates composed of q sets of plies with each set having p_1 plies at 0° , $2p_\theta$ plies at $\pm\theta^\circ$, and p_2 plies at 90° as shown in figure 5.

Let

$$\Phi(i) = 3 \left(i - \frac{N+1}{2} \right)^2 + \frac{1}{4} \quad (15)$$

and

$$r = p_1 + 2p_\theta + p_2$$

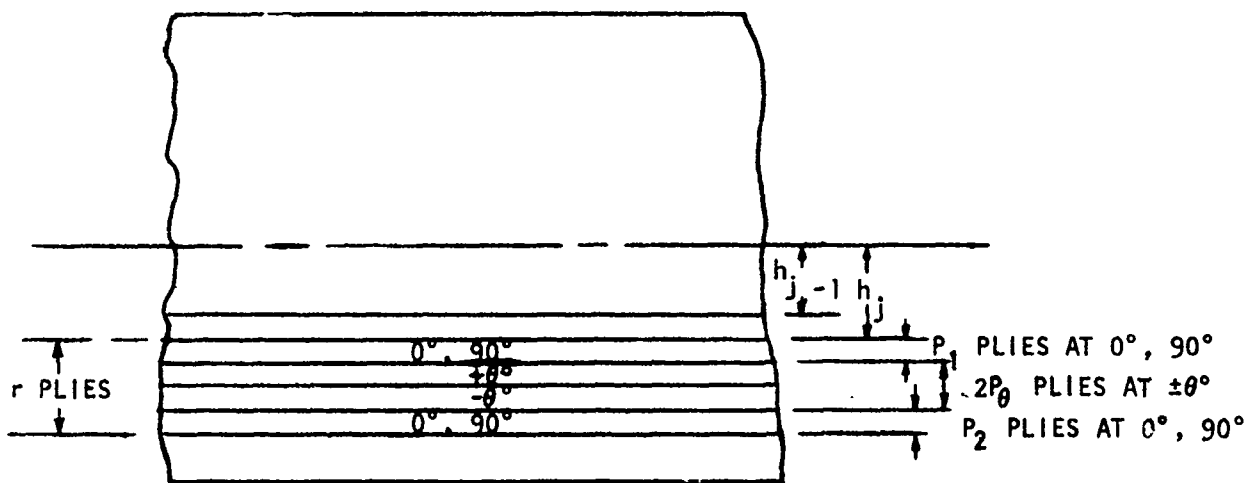


Figure 5. Typical Symmetrical Laminate

then

$$\begin{aligned}
D_{11} &= \frac{2}{3} h^3 \sum_{k=1}^{q/2} \left\{ \left(\bar{Q}_{11} \right)_{00} \sum_{i=\left(\frac{N}{2}+1\right)+(k-1)r}^{\left(\frac{N}{2}+1\right)+(k-1)r+p_1-1} \phi(i) \right. \\
&\quad + \left(\bar{Q}_{11} \right)_{\theta 0} \sum_{i=\left(\frac{N}{2}+1\right)+(k-1)r+p_1+2p_\theta-1}^{\left(\frac{N}{2}+1\right)+(k-1)r+p_1+2p_\theta-1} \phi(i) \\
&\quad \left. + \left(\bar{Q}_{11} \right)_{900} \sum_{i=\left(\frac{N}{2}+1\right)+(k-1)r+p_1+2p_\theta}^{\left(\frac{N}{2}+1\right)+kr-1} \phi(i) \right\} \\
D_{11} &= \frac{2}{3} h^3 \sum_{k=1}^{q/2} \left\{ \left(\bar{Q}_{11} \right)_{00} \sum_{j=1}^{p_1} \phi \left[\frac{N}{2} + j + (k-1)r \right] \right. \\
&\quad + \left(\bar{Q}_{11} \right)_{\theta 0} \sum_{j=1}^{2p_\theta} \phi \left[\frac{N}{2} + j + (k-1)r + p_1 \right] \\
&\quad \left. + \left(\bar{Q}_{11} \right)_{900} \sum_{j=1}^{p_2} \phi \left[\frac{N}{2} + j + (k-1)r + p_1 + 2p_\theta \right] \right\} \quad (16) \\
&= \frac{2}{3} h^3 \sum_{k=1}^{q/2} \left\{ \left(\bar{Q}_{11} \right)_{00} \sum_{j=1}^{p_1} \phi \left(j + kr + r_1 \right) \right. \\
&\quad \left. + \left(\bar{Q}_{11} \right)_{\theta 0} \sum_{j=1}^{p_2} \phi \left(j + kr + r_2 \right) + \left(\bar{Q}_{11} \right)_{900} \sum_{j=1}^{p_2} \phi \left(j + kr + r_3 \right) \right\}
\end{aligned}$$

where

$$\begin{aligned} r_1 &= \frac{N}{2} - r \\ r_2 &= \frac{N}{2} - 2p_\theta - p_2 \\ r_3 &= \frac{N}{2} - p_2 \end{aligned} \quad (17)$$

Letting

$$\begin{aligned} C_A &= \sum_{k=1}^{q/2} \sum_{j=1}^{p_1} \phi(j + kr + r_1) \\ C_B &= \sum_{k=1}^{q/2} \sum_{j=1}^{2p_\theta} \phi(j + kr + r_2) \\ C_C &= \sum_{k=1}^{q/2} \sum_{j=1}^{p_2} \phi(j + kr + r_3) \end{aligned} \quad (18)$$

then

$$D_{11} = \frac{2}{3} h^3 \left[C_A (\bar{Q}_{11})_0 + C_B (\bar{Q}_{11})_\theta + C_C (\bar{Q}_{11})_{90} \right] \quad (19)$$

Expanding ϕ in equation 18 and noting that

$$\begin{aligned} \sum_{n=1}^N n^2 &= \frac{N}{6} (N+1) (2N+1) \\ \sum_{n=1}^N n &= \frac{N}{2} (N+1) \end{aligned} \quad (20)$$

$$\begin{aligned}
C_A &= \frac{q p_1}{4} \left\{ (p_1+1) \left[2p_1 + 1 + 3(2r_1 - N - 1) \right] + \frac{3}{2} (2r_1 - N - 1)^2 + \frac{1}{2} \right\} \\
&\quad + 3rp_1 (p_1 + 2r_1 - N) \frac{q}{2} \frac{\left(\frac{q}{2} + 1\right)}{2} + 3p_1 r^2 \frac{q}{2} \frac{\left(\frac{q}{2} + 1\right)(q+1)}{6} \\
&= \frac{q p_1}{8} \left[2(p_1+1) (2p_1 + 6r_1 - 3N - 2) + 3(2r_1 - N - 1)^2 \right. \\
&\quad \left. + r(q+2) (3p_1 + 6r_1 - 3N + r + qr) + 1 \right]
\end{aligned} \tag{21}$$

By replacing r_1 by r_2 and p_1 by $2p_\theta$ in the preceding equation, we obtain

$$\begin{aligned}
C_B &= \frac{qp_\theta}{4} \left[2(2p_\theta+1) (4p_\theta+6r_2-3N-2) + 3(2r_2-N-1)^2 \right. \\
&\quad \left. + r(q+2) (6p_\theta+6r_2-3N+r+qr) + 1 \right]
\end{aligned} \tag{22}$$

By replacing r_1 by r_3 and p_1 by p_2 in equation 21, we obtain

$$\begin{aligned}
C_C &= \frac{qp_2}{8} \left[2(p_2+1) (2p_2+6r_3-3N-2) + 3(2r_3-N-1)^2 \right. \\
&\quad \left. + r(q+2) (3p_2+6r_3-3N+r+qr) + 1 \right]
\end{aligned} \tag{23}$$

The preceding results are also applicable to D_{12} , D_{16} , D_{22} , D_{26} , and D_{66} . The resulting expressions are

$$\begin{aligned}
(D_{11}, D_{12}, D_{16}, D_{22}, D_{26}, D_{66}) &= \frac{2}{3} h^3 \left\{ \left[(\bar{Q}_{11})_0, (\bar{Q}_{12})_0, 0, (\bar{Q}_{22})_0, 0, (\bar{Q}_{66})_0 \right] C_A \right. \\
&\quad + \left[(\bar{Q}_{11})_\theta, (\bar{Q}_{12})_\theta, (\bar{Q}_{16})_\theta, (\bar{Q}_{22})_\theta, (\bar{Q}_{26})_\theta, (\bar{Q}_{66})_\theta \right] C_B \\
&\quad \left. + \left[(\bar{Q}_{11})_{90}, (\bar{Q}_{12})_{90}, 0, (\bar{Q}_{22})_{90}, 0, (\bar{Q}_{66})_{90} \right] C_C \right\}
\end{aligned} \tag{24}$$

Equation 24 represents the closed-form sums for the flexural stiffnesses of a laminate composed of q repeated sets of $\begin{bmatrix} 0 & / \pm \theta & / 90 \\ p_1 & p_\theta & p_2 \end{bmatrix}$ laid up

symmetrically relative to the midsurface of the laminate. These stiffnesses are required to conduct analyses of orthotropic laminated plates. The equivalent extensional constants are obtained by combining equations 13 and 14 in the appropriate manner.

For cases when a more general laminate layup is under consideration, equation 9, which is valid for any laminate ply orientations, should be utilized.

CALCULATIONS OF EXTENSIONAL AND FLEXURAL CONSTANTS

A computer program, AC-2, has been developed to perform the calculations required to determine the extensional elastic constants and flexural stiffnesses for a balanced laminate laid up symmetrically relative to its midsurface. This program incorporates the closed-form solution for D_{ij} 's described by equation 24 and, as such, is very rapid for thick as well as thin laminates. The elastic constants, as defined by equation 13, for several laminate families of Narmco 5505 as obtained from AC-2 are plotted in figures 6 through 17 for room temperature (RT) and 350° F. The elastic constants of each lamina in its natural coordinate system (L, T, Z) were obtained from reference 2.

The elastic constants that can be found from these curves are E_x , E_y , G_{xy} , ν_{xy} , and ν_{yx} . To assist the designer further in choosing the desired laminate orientation, lines were drawn which represent both "% at 0°" and "% at 90°," even though only one need be given to derive the other.

Also, it should be noted that the G_{xy} curves are independent of the "% at 0°" versus the "% at 90°" plies. That is, as long as the "% at ±0°" values are known, then the remaining relative percentage of the 0° versus the 90° plies does not affect G_{xy} . Furthermore, considering the number of plies at 0°, ±0°, and 90° to be n_1 , n_2 , and n_3 , respectively, then it is seen that

1. No curve is given for E_y for $[0_{n_1}/\pm 45_{n_2}/90_{n_3}]$ laminate orientations. This is because E_y for these orientations can be found by using the curves for E_x after interchanging the "% at 0°" and "% at 90°" labels.
2. The curves for $[0_{n_1}/\pm 30_{n_2}/90_{n_3}]$ laminate orientations are not given, but the curves for the $[0_{n_1}/\pm 60_{n_2}/90_{n_3}]$ laminate orientations can be used for $[0_{n_1}/\pm 30_{n_2}/90_{n_3}]$ properties if the following are considered:
 - a. E_x for $[0_{n_1}/\pm 30_{n_2}/90_{n_3}]$ is found by interchanging the "% at 0°" and "% at 90°" labels and using the E_y curves for $[0_{n_1}/\pm 60_{n_2}/90_{n_3}]$.

- b. E_y for $[0_{n1}/\pm 30_{n2}/90_{n3}]$ is found by interchanging the "% at 0°" and "% at 90°" labels and using the E_x curves for $[0_{n1}/\pm 60_{n2}/90_{n3}]$.
- c. ν_{xy} for $[0_{n1}/\pm 30_{n2}/90_{n3}]$ is found by interchanging the "% at 0°" and "% at 90°" labels and multiplying the ν_{xy} value from the $[0_{n1}/\pm 60_{n2}/90_{n3}]$ curves by the ratio E_x/E_y for $[0_{n1}/\pm 30_{n2}/90_{n3}]$.

The program also computes the flexural stiffnesses of the laminate assuming it to be a homogeneous orthotropic medium and thus utilizing the extensional constants, e.g., $D_{11} = E_x t^3/12(1-\nu_{xy}\nu_{yx})$, and compares these stiffnesses to those obtained by the rigorous method, i.e., equation 14. This output permits both the determination of how thick a laminate of a certain orientation must be before the stiffnesses D_{16} and D_{26} approach zero, and determination of the error associated with the use of flexural stiffnesses D_{11} , D_{12} , D_{22} , and D_{66} based on the extensional constants rather than equation 14.

The latter data permit an assessment of when the classical solutions for the response behavior of homogeneous orthotropic plate and shell elements are valid (for the orientation in question), i.e., cases where the ratios D_{16}/D_{11} and D_{26}/D_{11} approach zero. A further discussion of the significance of the stiffness terms D_{16} and D_{26} and the ratios D_{16}/D_{11} and D_{26}/D_{11} in obtaining static and dynamic response solutions to solid laminated plates is contained in section III.

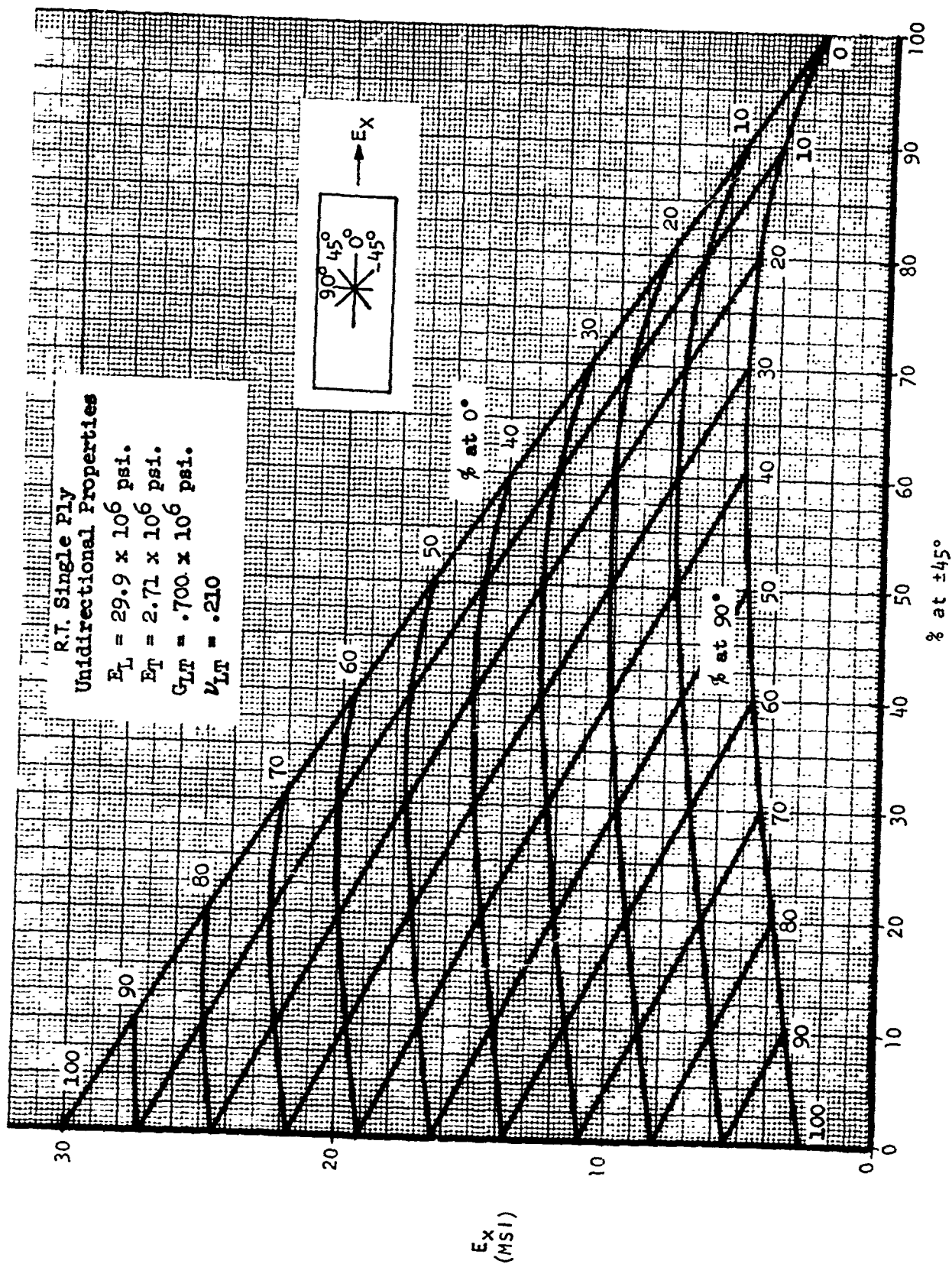


Figure 6. Laminate E_x vs Percent Laminates at 0°, $\pm 45^\circ$, and 90° for Narmco 5505 at RT

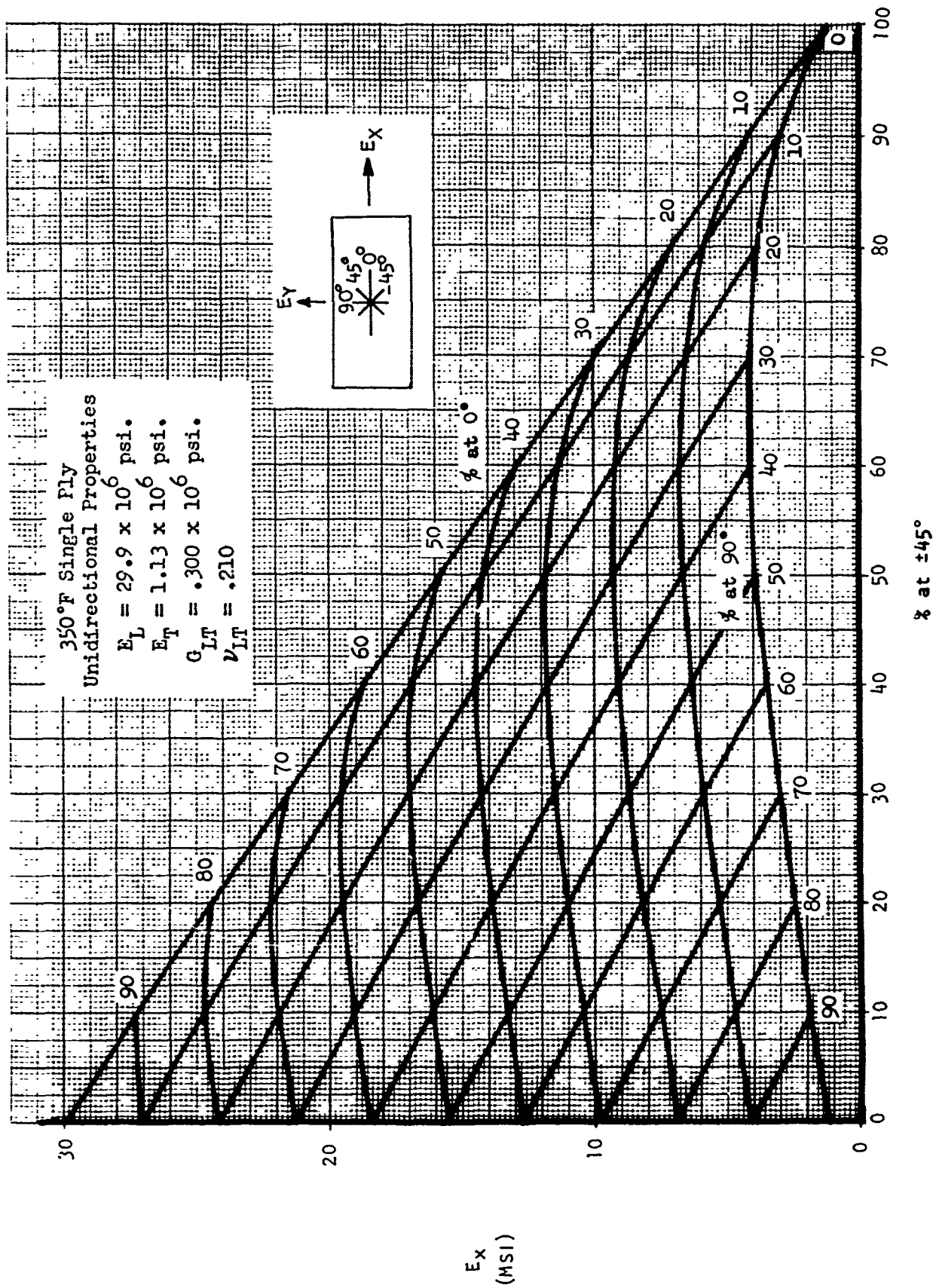


Figure 7. Laminate E_x vs Percent Laminae at 0° , $\pm 45^\circ$, and 90° for Narmco 5505 at 350° F

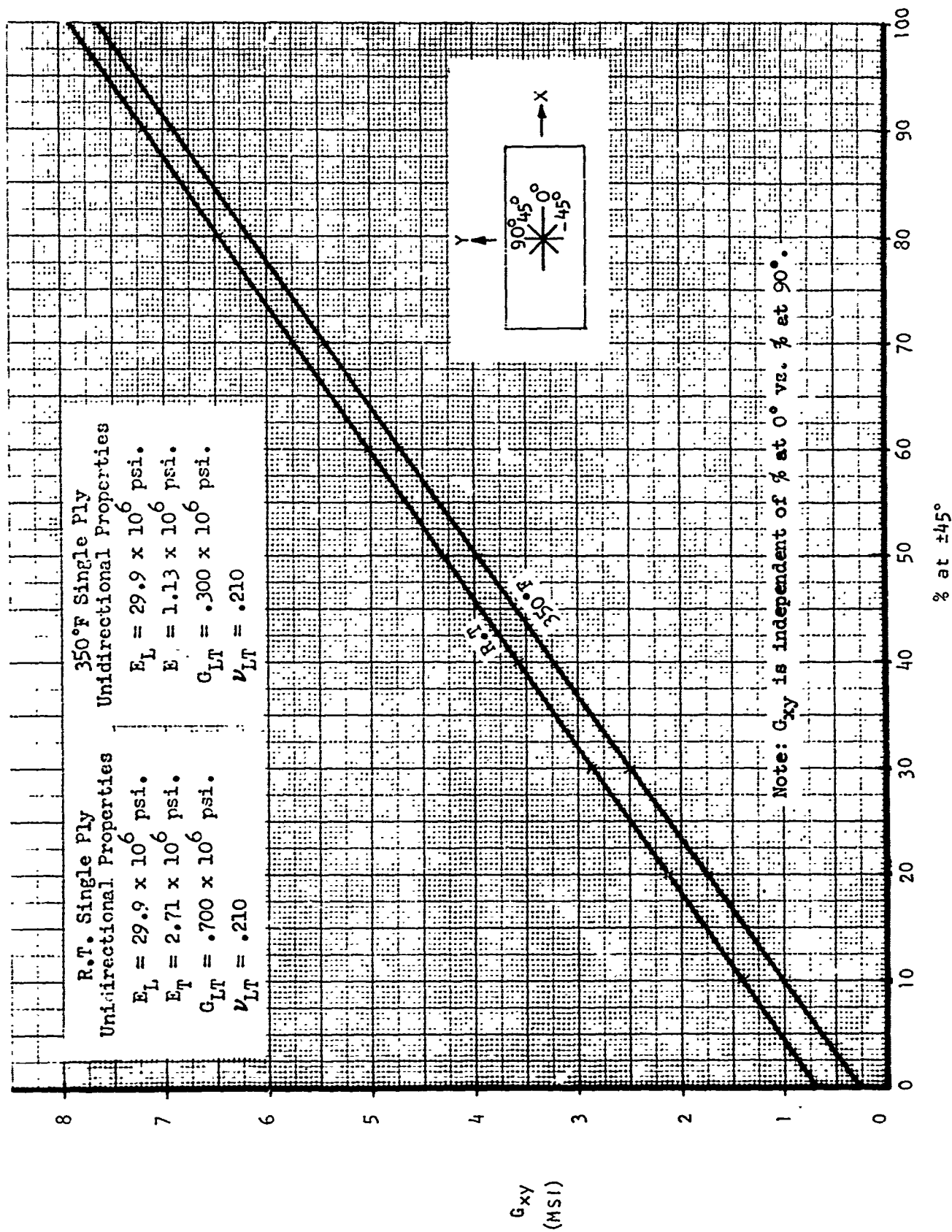


Figure 8. Laminate G_{xy} vs Percent Laminae at 0° , $\pm 45^\circ$, and 90° for Narmco 5505 at RT and 350° F

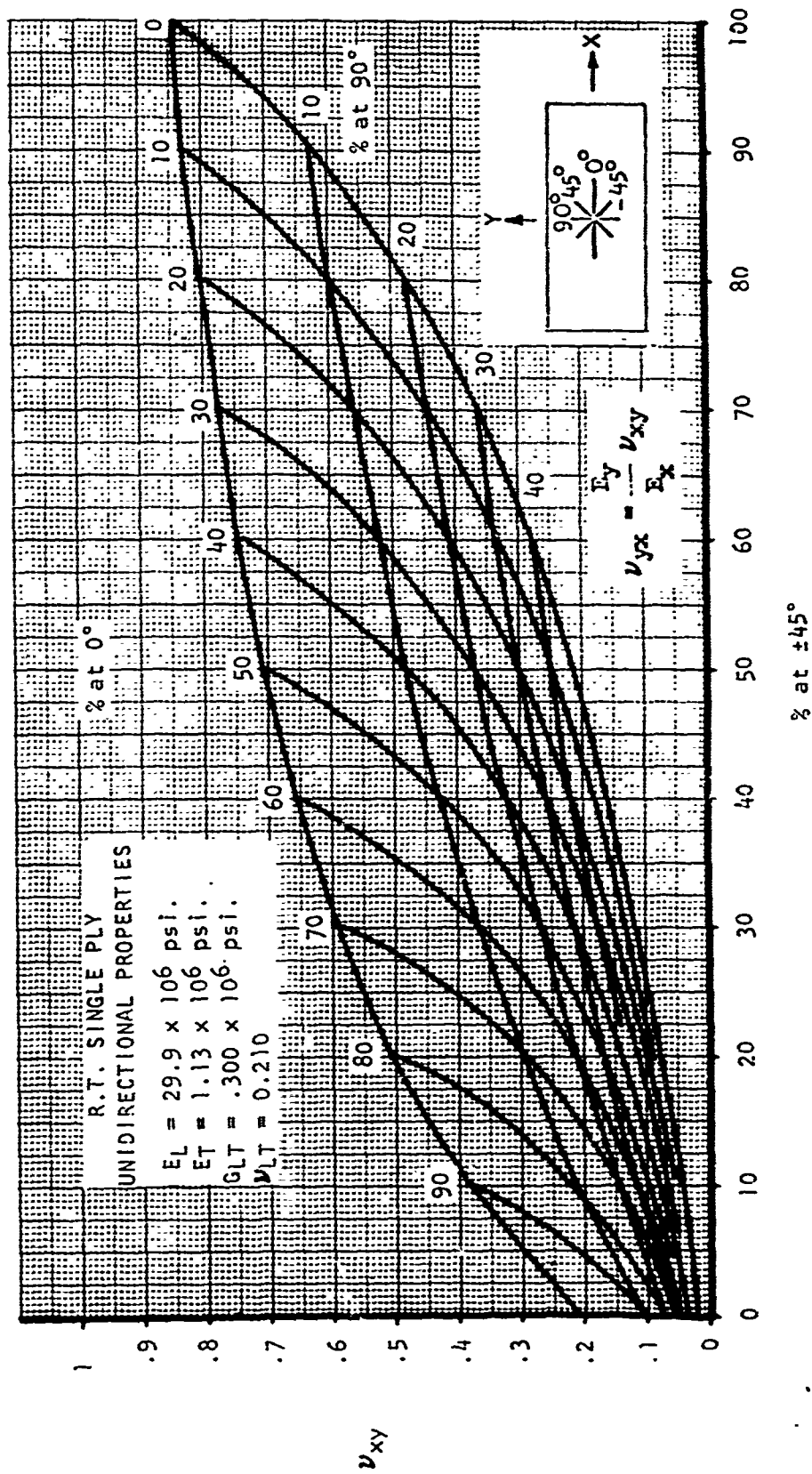


Figure 9. Laminate ν_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT

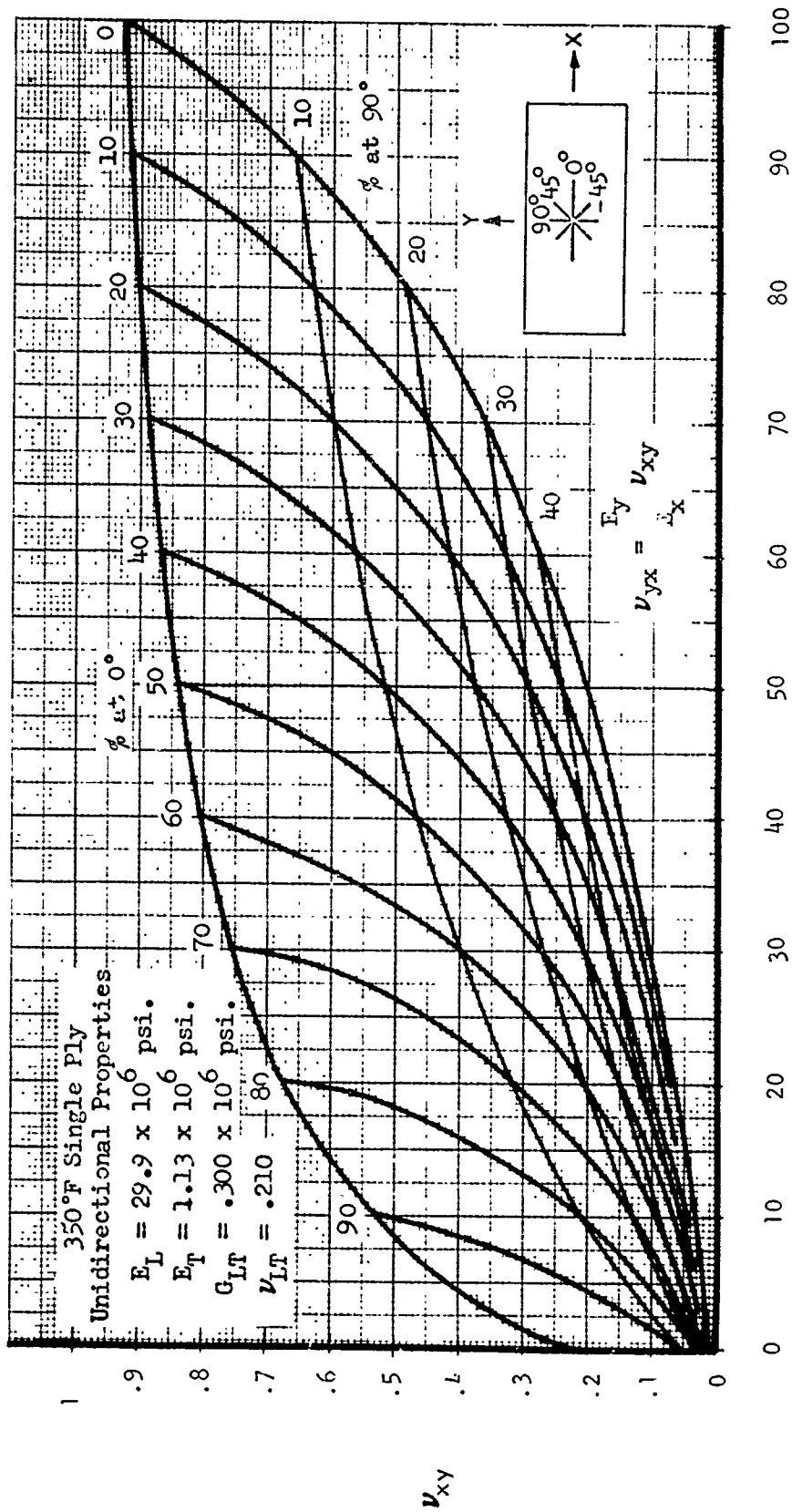


Figure 10. Laminate ν_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at 350° F

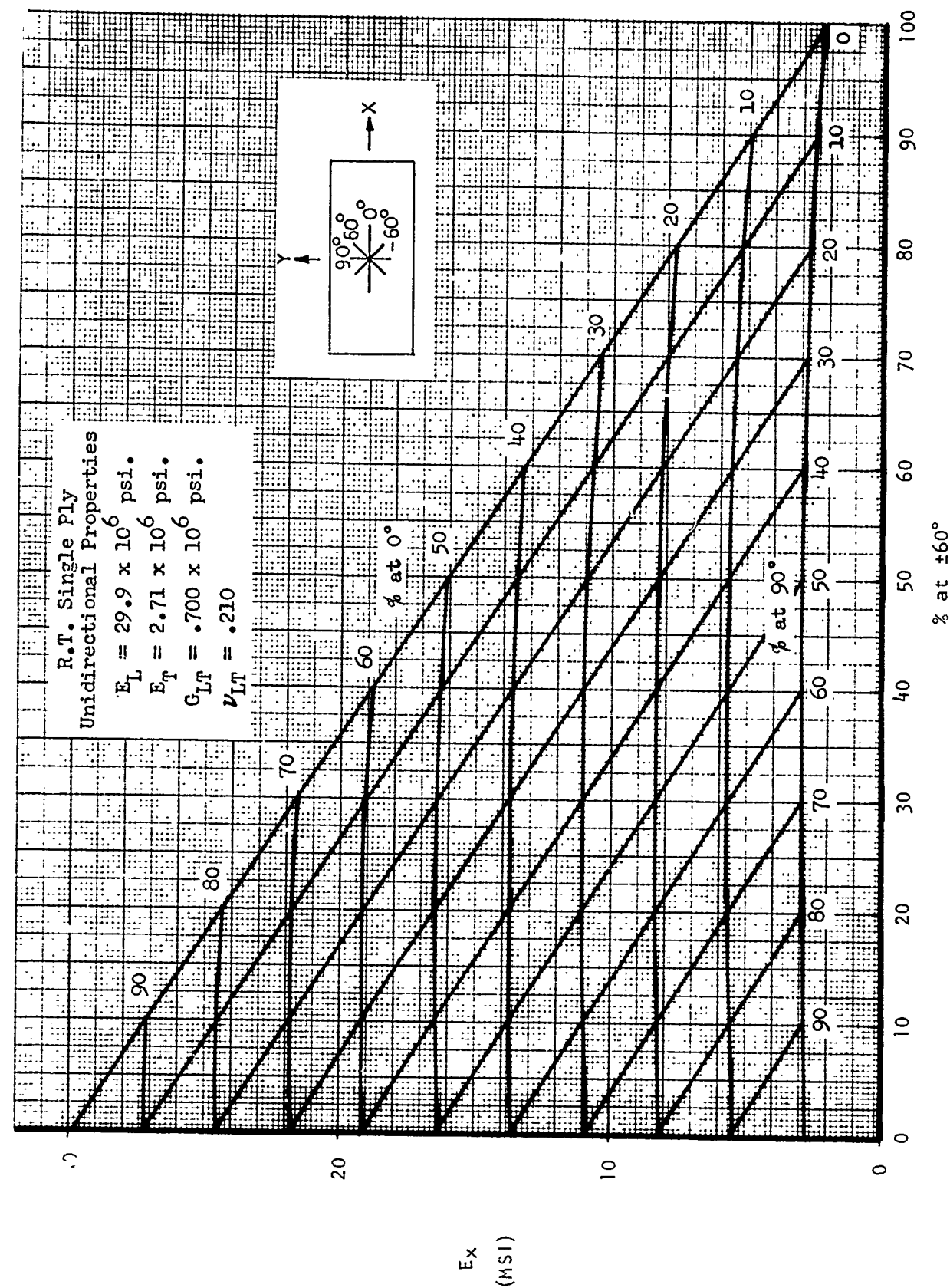


Figure 11. Laminate E_x vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT

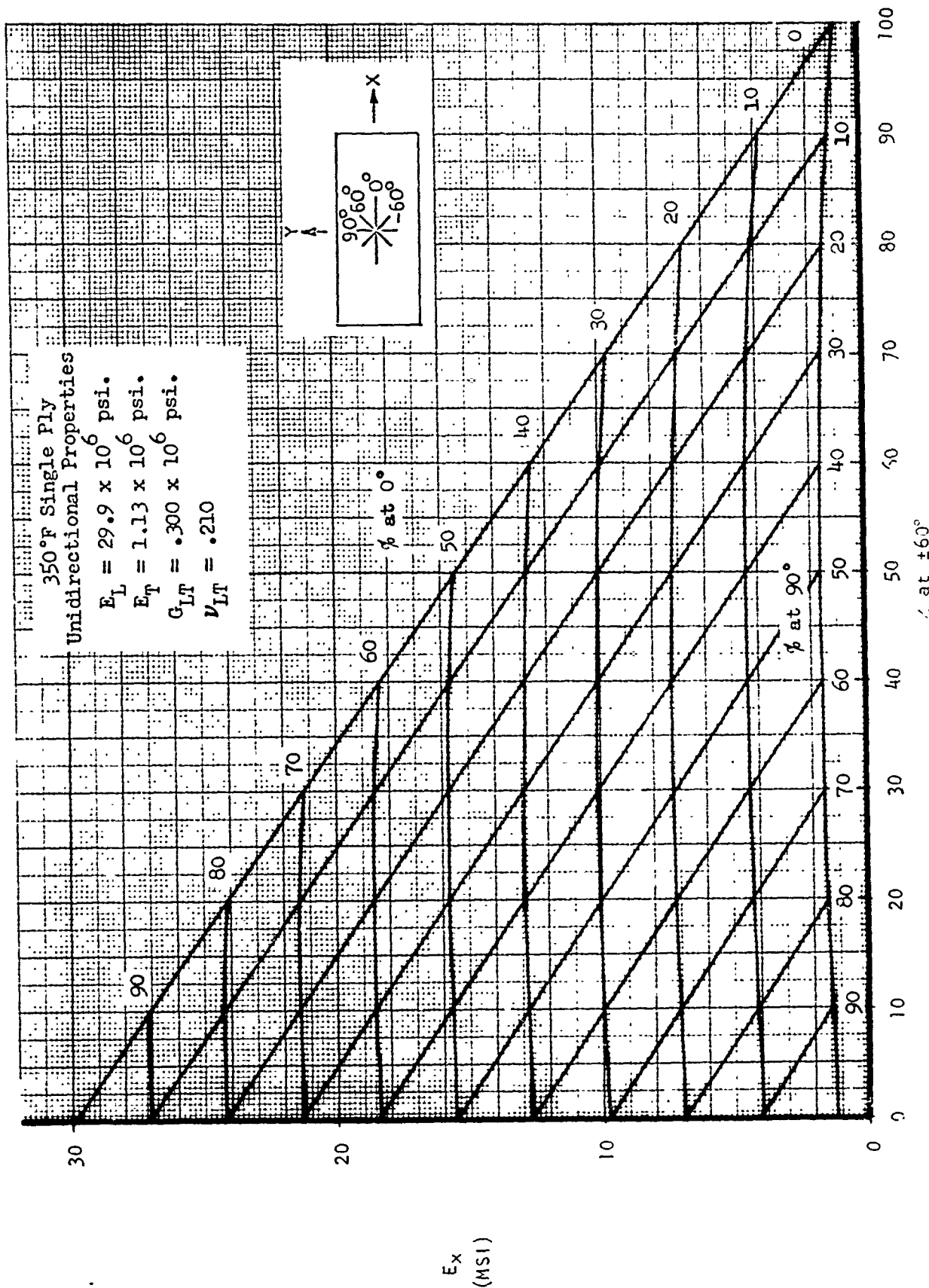


Figure 12. Laminate E_x vs Percent Laminar at 0° , $\pm 60^\circ$, and 90° for Narmco 5505 at 350° F

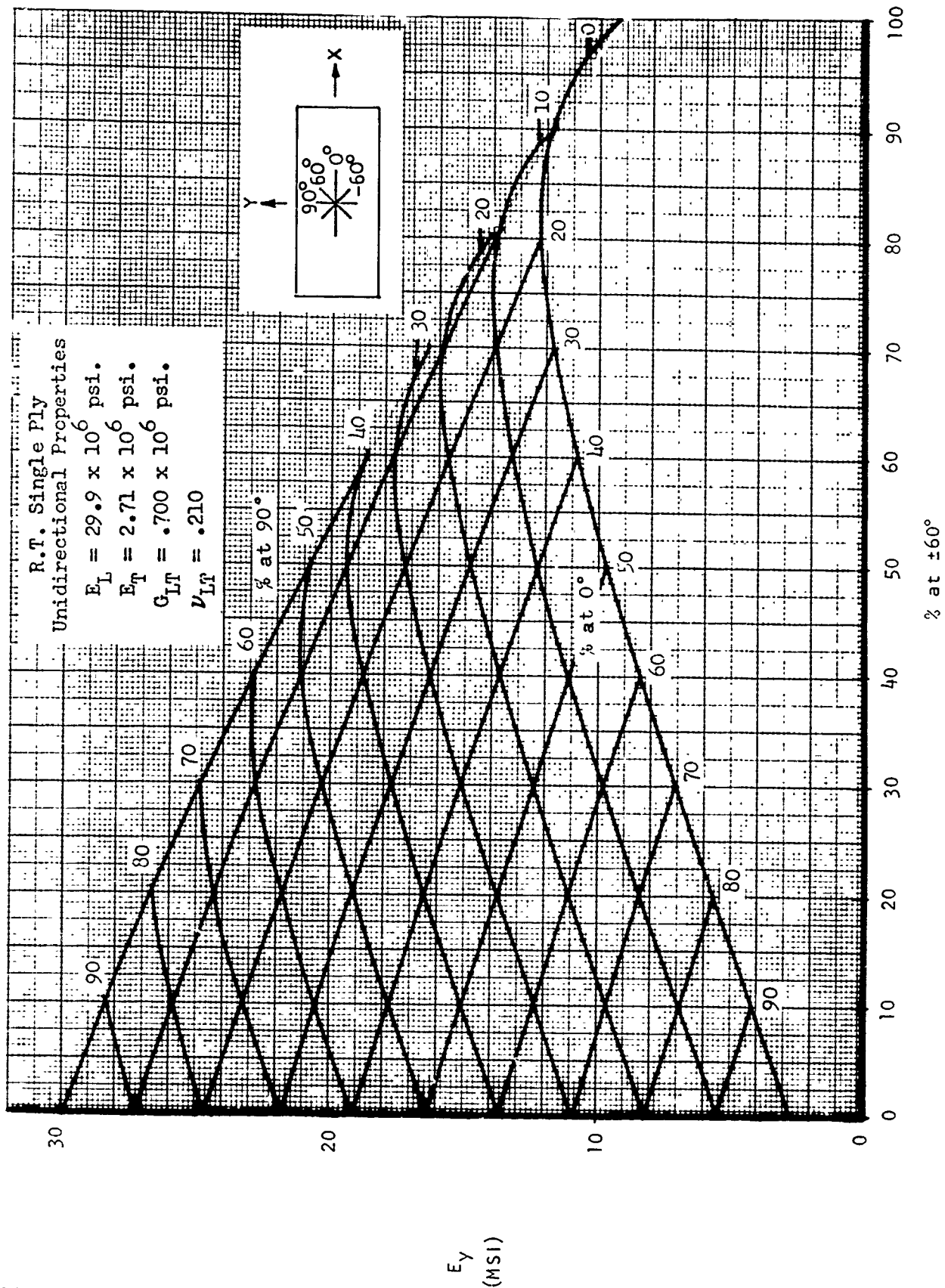


Figure 13. Laminate E_y vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT

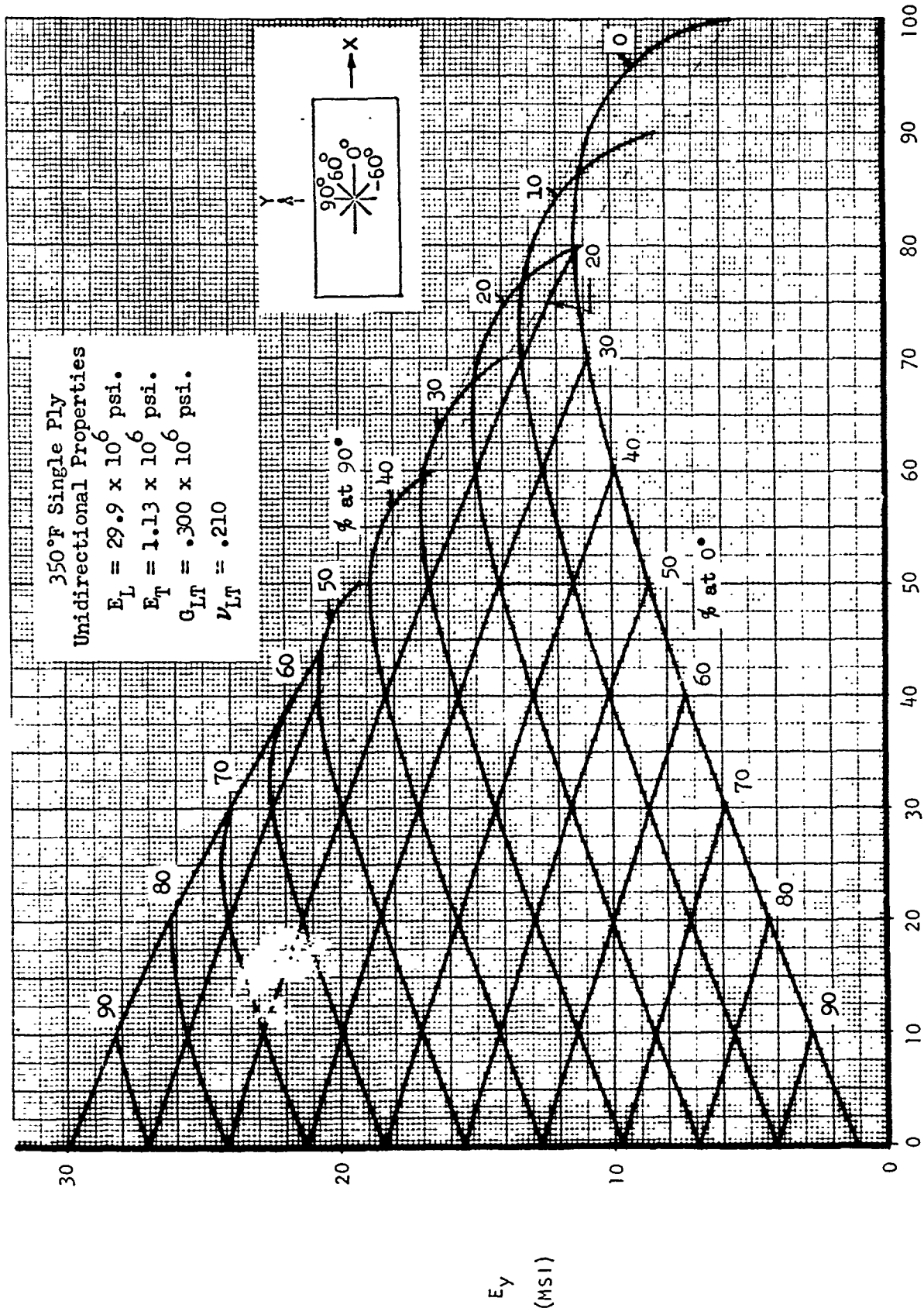


Figure 14. Laminate E_y vs Percent Laminae at 0° , $\pm 60^\circ$, and 90° for Narmco 5505 at 350° F

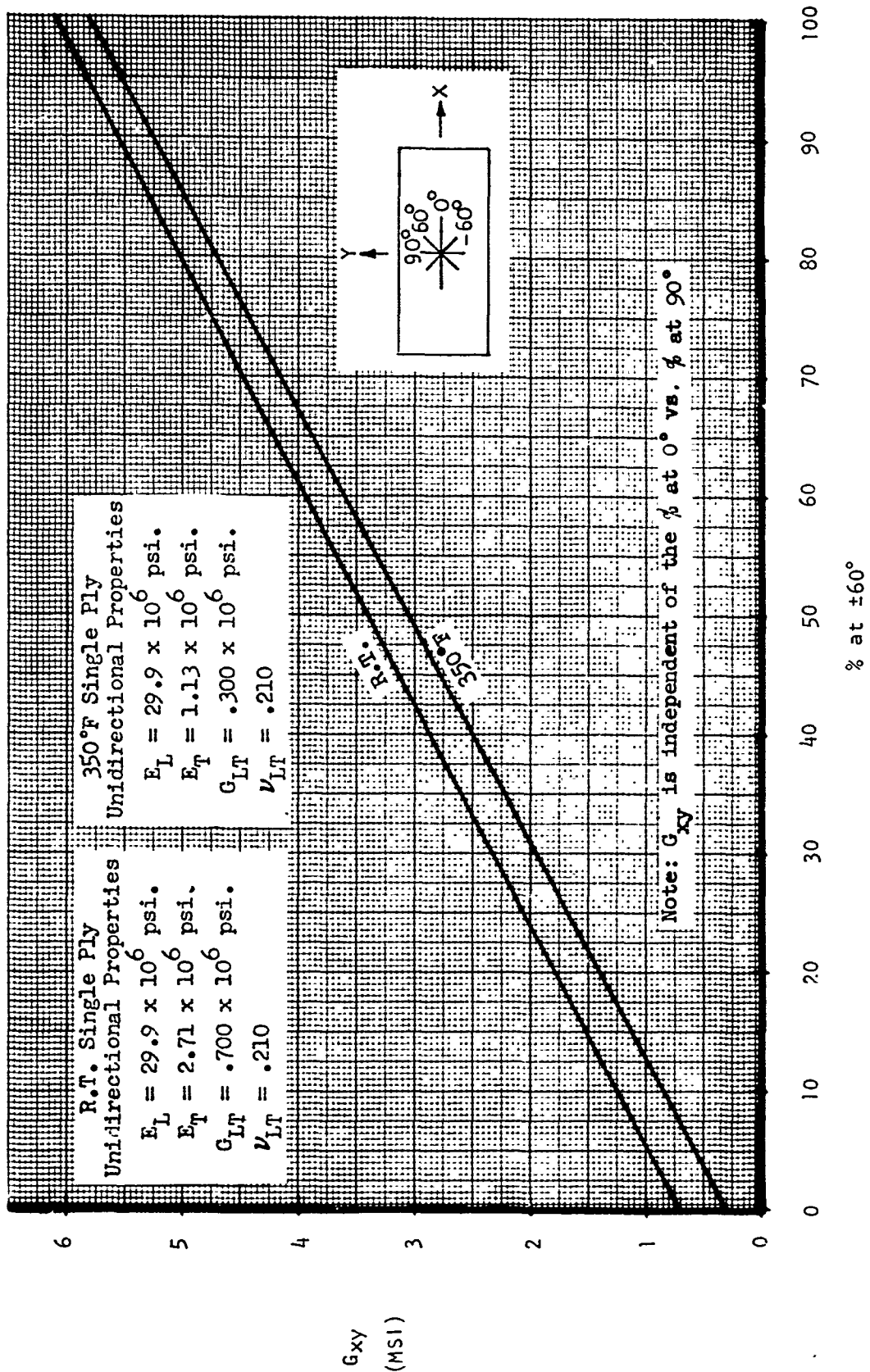


Figure 15. Laminate G_{xy} vs Percent Laminae at 0° , $\pm 60^\circ$, and 90° for Narmco 5505 at RT and 350° F

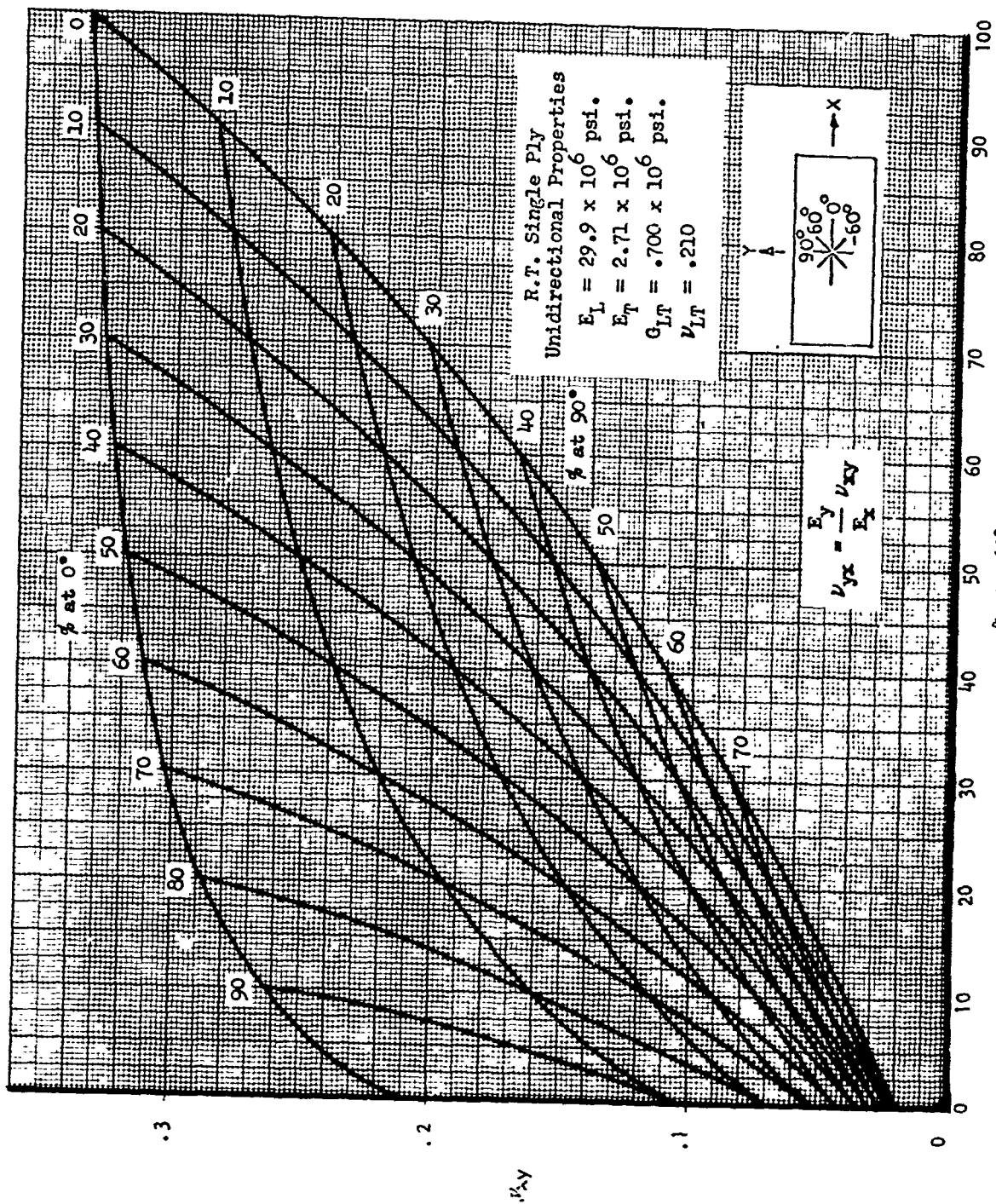


Figure 16. Laminate ν_{xy} vs Percent Laminae at 0° , $\pm 60^\circ$, and 90° for Narmco 5505 at RT

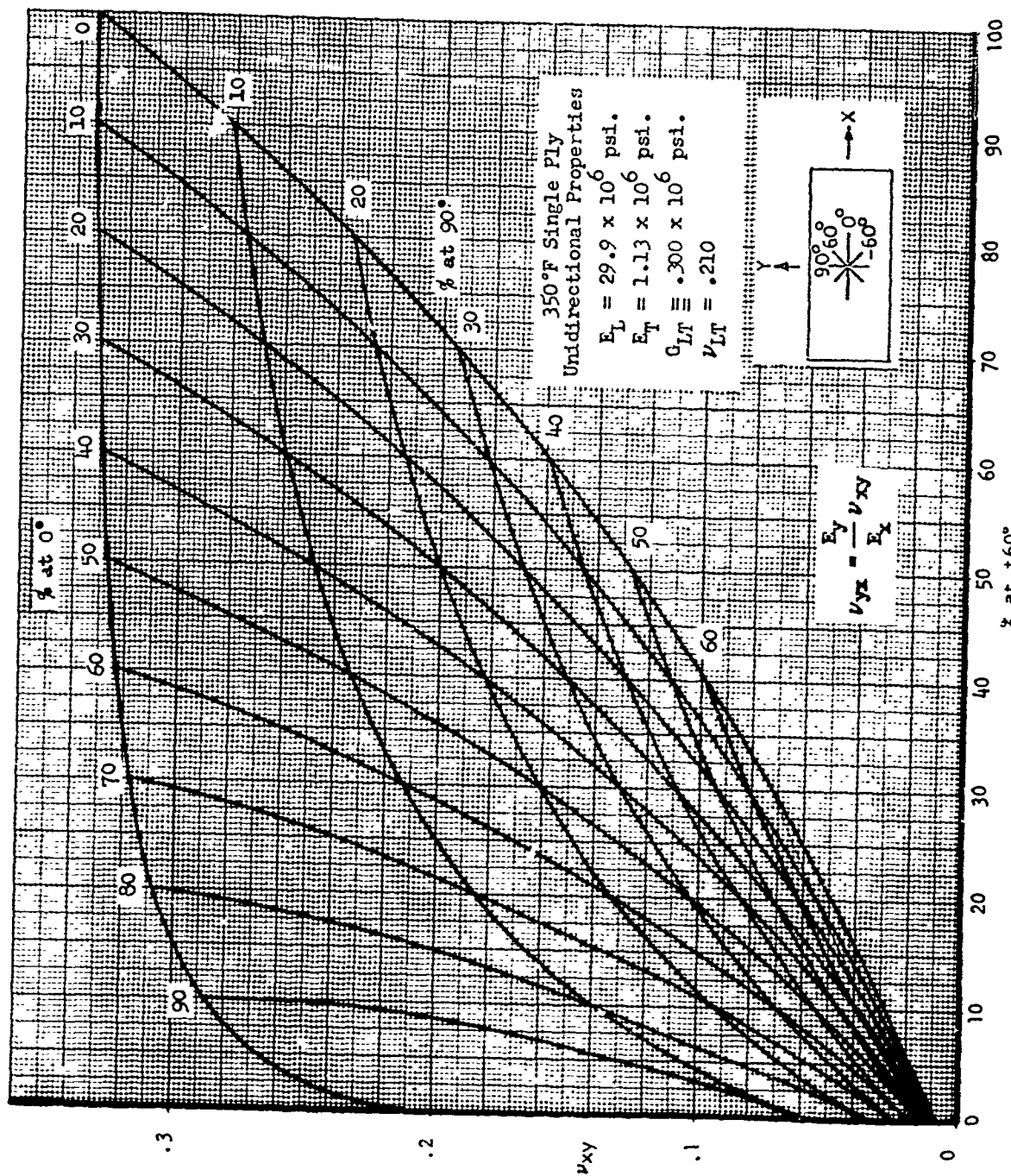


Figure 17. Laminate ν_{xy} vs Percent Laminae at 0° , $\pm 60^\circ$, and 90° for Narmco 5505 at 350° F

SECTION III

FLAT PLATES

EQUATIONS OF MOTION

The equations of motion for the plate element shown in figure 18 are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho t \frac{\partial^2 u}{\partial t^2} \left(\sum F_x = m a_x \right) \quad (25a)$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = \rho t \frac{\partial^2 u}{\partial t^2} \left(\sum F_y = m a_y \right) \quad (25b)$$

$$\frac{\partial M_y}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = 0 \left(\sum M_y = 0 \right) \quad (25c)$$

$$\frac{\partial M_x}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \left(\sum M_x = 0 \right) \quad (25d)$$

$$q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho t \frac{\partial^2 w}{\partial t^2} \left(\sum F_z = m a_z \right) \quad (25e)$$

where the plate rotary inertia is considered to be negligible, ρ is the mass density of the plate, t is the plate thickness, the force and moment components are defined by equation 6, and the condition which each equation enforces for the element is indicated within the parentheses. Also, all plies are considered to have the same density, which is generally the case. The shear forces Q_x and Q_y can be eliminated from these expressions by combining equations 25c, 25d, and 25e to yield the relation

$$\begin{aligned} N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 M_x}{\partial x^2} \\ + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho t \frac{\partial^2 w}{\partial t^2} - q \end{aligned} \quad (26)$$

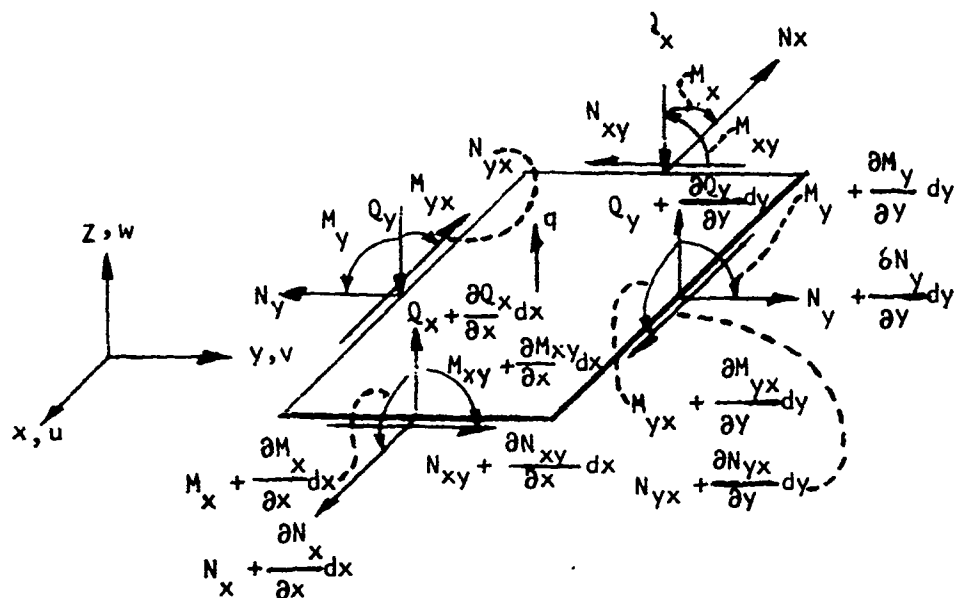


Figure 18. Plate Element Subjected to Force and Moment Components

If, as before, the discussion is limited to a laminate with a ply layup which is symmetric relative to its midsurface, then combining equations 7b and 25e and neglecting in-plane inertia terms yields

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad (27a)$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \quad (27b)$$

$$\begin{aligned} D_{11} \frac{\partial^4 w}{\partial x^4} + 4 D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2 (D_{12} + 2 D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} \\ + 4 D_{26} \frac{\partial^4 w}{\partial x \partial y^3} + D_{22} \frac{\partial^4 w}{\partial y^4} = - \rho t \frac{\partial^2 w}{\partial t^2} + q \\ + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \quad (27c)$$

This equation is based on the assumption that the normal deflections of the plate are small compared to its thickness, and the laminate ply orientation is symmetric relative to its midsurface. The latter assumption permits the uncoupling of the bending and membrane force and displacement components.

An examination of equation 27c indicates that except for cases when the laminate layup is composed totally of 0- and 90-degree plies, the D_{16} and D_{26} terms are nonzero. However, for thick laminates composed of many repeating sets of plies, the D_{16} and D_{26} stiffness terms tend toward zero.

This can be seen in figures 19, 20, 21, and 22, which plot the ratio of these stiffnesses to the D_{11} and D_{22} stiffnesses for 50-percent volume fraction boron/epoxy laminates. If we consider the average ply thickness to be 0.005 inch, we see that for the laminate orientations considered, the ratio $|D_{16}/D_{11}|$ or $|D_{26}/D_{11}|$ is less than 7 percent of D_{11} for thicknesses of 0.20 inch (40 plies) or greater. In addition, for thick laminates, the flexural stiffnesses can be defined in terms of the extensional constants for the basic ply set which is repeated to form the laminate. This is because each basic set of thick laminates undergoes nearly a uniform strain field during bending, as shown in figure 23, while this is not true for thin laminates.

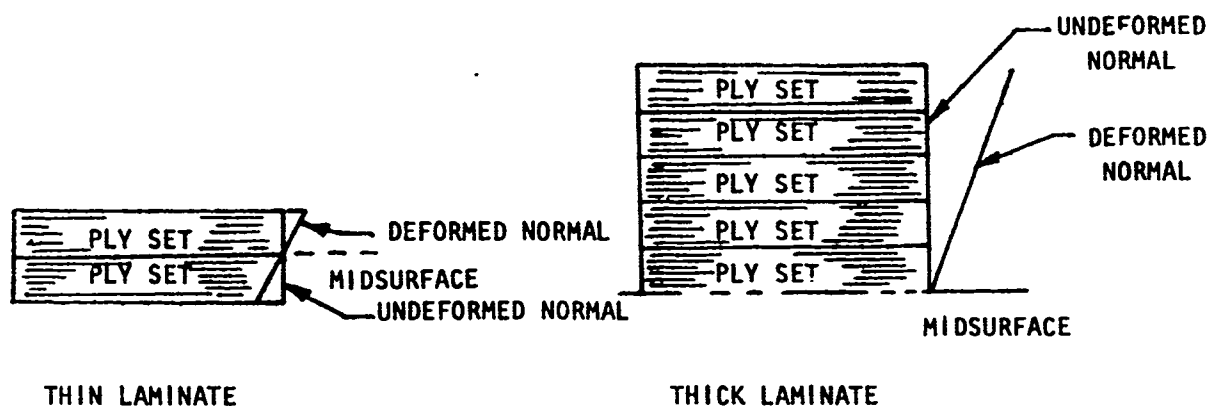


Figure 19. Thin and Thick Laminate Deformation

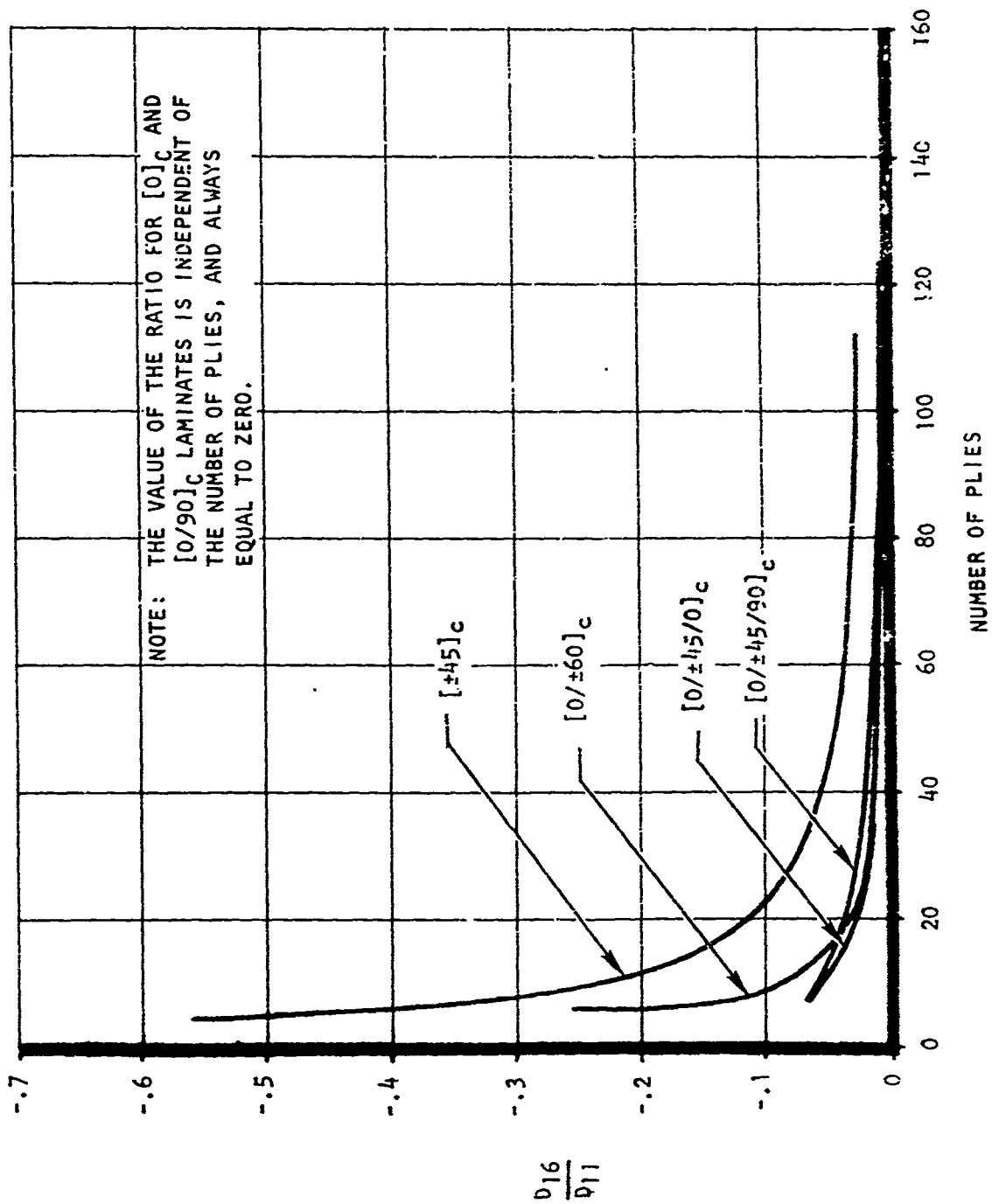


Figure 20. D_{16}/D_{11} vs Number of Plies for Several Laminate Orientations

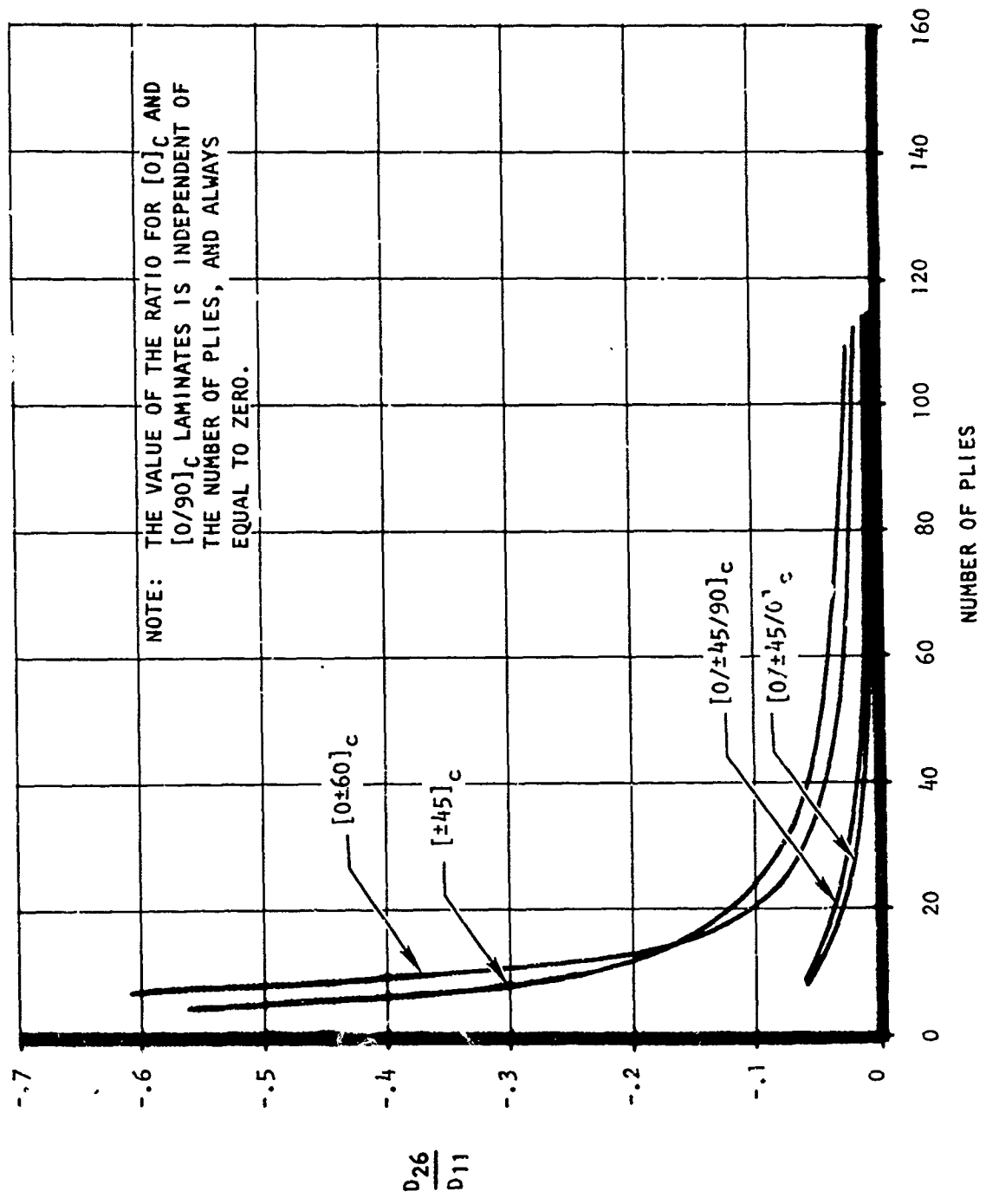


Figure 21. D_{26}/D_{11} vs Number of Plies for Several Laminate Orientations

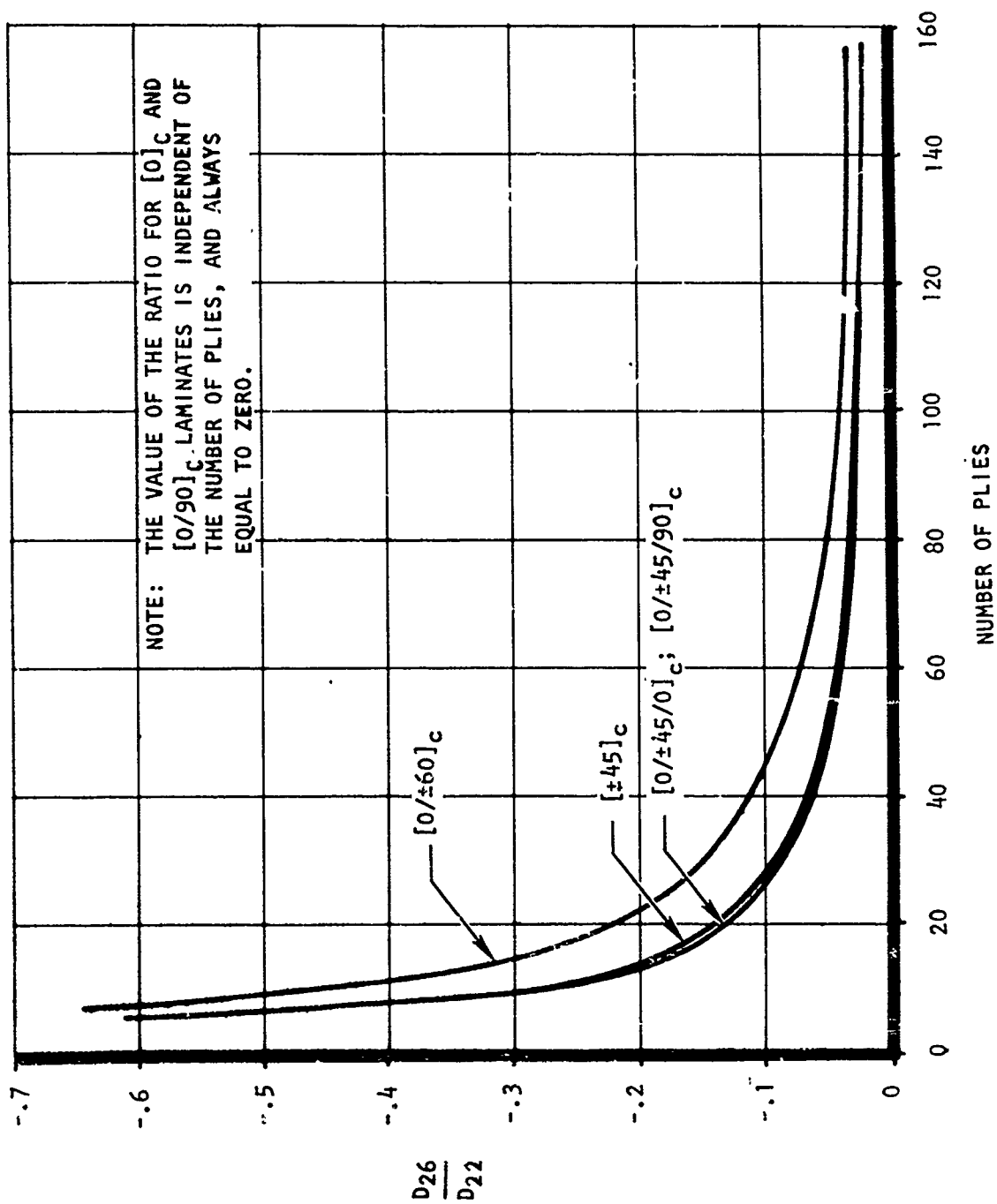


Figure 22. D_{26}/D_{22} vs Number of Plies for Several Laminate Orientations

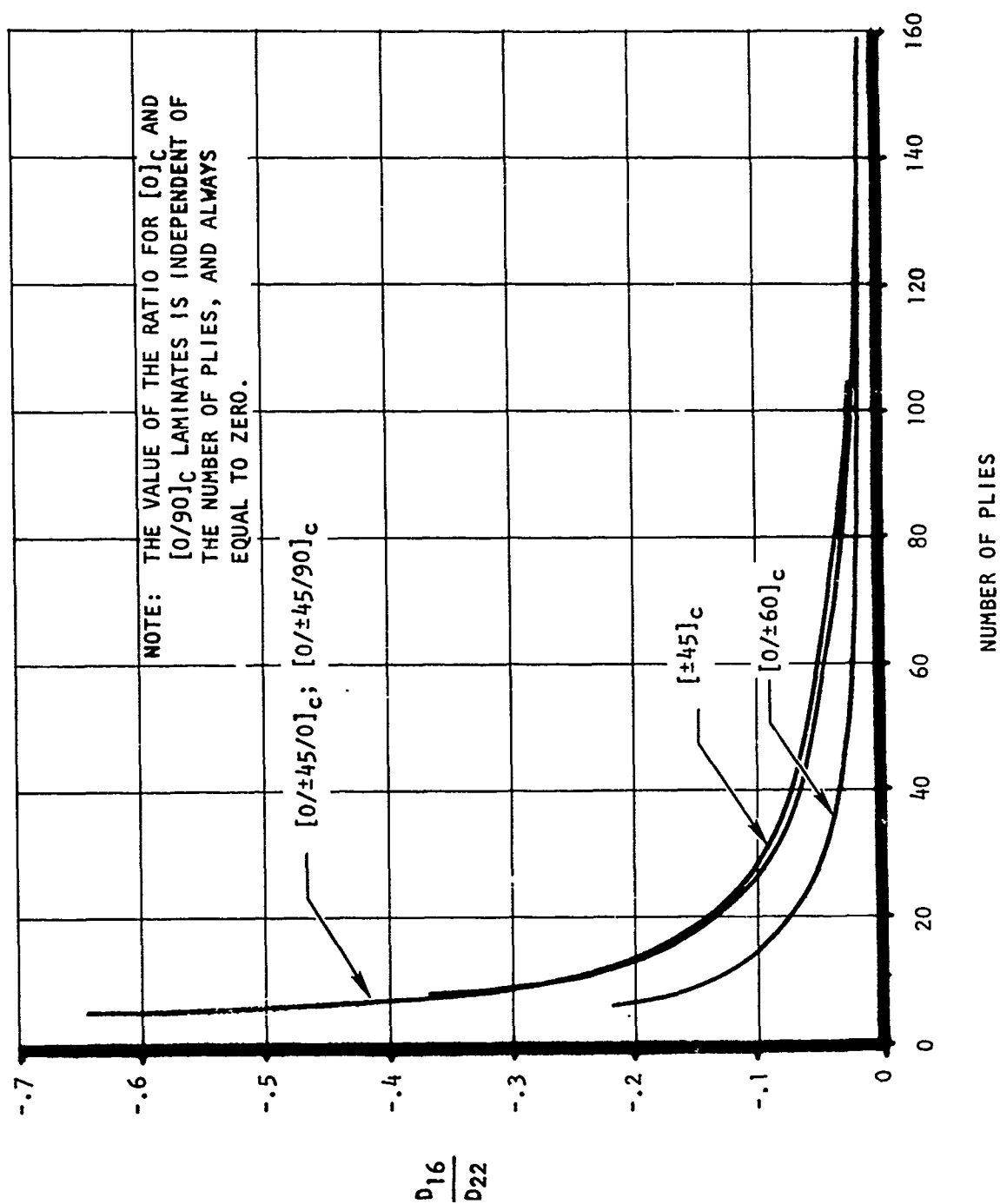


Figure 23. D_{16}/D_{22} vs Number of Plies for Several Laminate Orientations

Thus, for thick laminates and zero temperature change, the nonzero flexural stiffness can be defined by

$$\begin{aligned}
 D_{11} &= \frac{E_x t^3}{12 (1 - \nu_{xy} \nu_{yx})} \\
 D_{22} &= \frac{E_y t^3}{12 (1 - \nu_{xy} \nu_{yx})} \\
 D_{12} &= D_{21} = \frac{\nu_{yx} E_x t^3}{12 (1 - \nu_{xy} \nu_{yx})} \\
 D_{66} &= \frac{G_{xy} t^3}{12}
 \end{aligned} \tag{28}$$

where the elastic constants for each laminate are obtained from equation 13. Equation 27c reduces to the following classical governing equation for specially orthotropic plates

$$\begin{aligned}
 D_1 \frac{\partial^4 w}{\partial x^4} + 2 D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = - \rho t \frac{\partial^2 w}{\partial t^2} + q \\
 + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y}
 \end{aligned} \tag{29}$$

where

$$\begin{aligned}
 D_1 &= D_{11} \\
 D_3 &= D_{12} + 2 D_{66} \\
 D_2 &= D_{22}
 \end{aligned}$$

A broad class of closed-form orthotropic plate solutions to equation 29 are available in the literature, and are valid for the analysis of thick, laminated, filamentary composite structures.

PRESSURE PANELS

The classical theory (reference 11) for the deflection and moments of a uniformly loaded, simply supported, specially orthotropic laminated plate can be obtained from equation 29 by setting

$$N_x = N_y = N_{xy} = \frac{\partial^2 w}{\partial t^2} = 0$$

and the plate's normal deflection w in the form

$$w = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (30)$$

and the uniform normal pressure by

$$q = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{16 q}{\pi^2 mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (31)$$

then substituting equations 30 and 31 into equation 29 and solving for A_{mn} . This procedure results in the following relationship for the plate's normal deflection:

$$w = 16 \frac{q b^4}{\pi^6} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (32)$$

where

$$A_{mn} = \frac{16 q b^4}{\pi^6 mn \left[D_{11} \left(\frac{mb}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{mb}{a} \right)^2 + D_{22} n^4 \right]}$$

The stresses of any i^{th} ply at a distance z from the midsurface of the laminate, which correspond to the deflection field described by equation 32, can be obtained from equation 5 as

$$\sigma_x^{(i)} = \frac{-E_y^{(i)} z}{1 - \nu_{xy}^{(i)} \nu_{yx}^{(i)}} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \pi^2 \left\{ \frac{m^2}{a^2} + \nu_{yx}^{(k)} \frac{n^2}{b^2} \right\} \cdot \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (33)$$

$$\sigma_y^{(i)} = \frac{-E_y^{(i)} z}{1 - \nu_{xy}^{(i)} \nu_{yx}^{(i)}} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \pi^2 \left\{ \frac{n^2}{b^2} + \nu_{xy}^{(k)} \frac{m^2}{a^2} \right\} \cdot \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (34)$$

for 0- and 90-degree plies. For plies of other orientations, the contributions of $\bar{Q}_{26}$ and $\bar{Q}_{16}$ must be considered.

For the case when in-plane compression forces N_x and N_y are also present, A_{mn} becomes

$$A_{mn} = \frac{16 q b^4}{\pi^6 mn \left\{ \left[D_{11} \left(\frac{mb}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{mnb}{a} \right)^2 + D_{22} n^4 \right] - \left(\frac{b}{\pi} \right)^2 \left[N_x \left(\frac{mb}{a} \right)^2 + N_y n^2 \right] \right\}} \quad (35)$$

UNIAXIAL AND BIAXIAL COMPRESSION

The classical solution for the buckling of specially orthotropic, simply supported plates is obtained from equation 29 by setting

$$N_{xy} = q = \frac{\partial^2 w}{\partial t^2} = 0$$

and taking w in the form

$$w = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (36)$$

where m and n denote the number of half-wave lengths along the X and Y axes, respectively.

Noting that the forces N_x and N_y are compression, we obtain from equation 29

$$(N_x)_{cr} = \frac{k_x \pi^2 D_{11}}{b^2} \quad (37)$$

where

$$k_x = \left\{ \frac{\left[m^4 \left(\frac{b}{a} \right)^4 + 2 \left(\frac{D_{12}}{D_{11}} + 2 \frac{D_{66}}{D_{11}} \right) m^2 n^2 \left(\frac{b}{a} \right)^2 + \frac{D_{22}}{D_{11}} n^4 \right]}{\left(\frac{mb}{a} \right)^2 + \phi n^2} \right\}$$

$$\phi = \frac{N_y}{N_x}$$

and m and n are the half-wave length integers which minimize k_x . For the case when only N_x is present, i.e., $\phi = 0$, and the plate is very long ($a/b \gg 1$), we have, after noting that k_x is minimized in this case for $n = 1$,

$$\frac{dk_x}{dm} = 2m \left(\frac{b}{a} \right)^2 - 2 \frac{\left(\frac{D_{22}}{D_{11}} \right)}{m^3 \left(\frac{b}{a} \right)^2} = 0$$

or $(k_x)_{\min}$ occurs when

$$m = \left(\frac{D_{22}}{D_{11}} \right)^{1/4} \left(\frac{a}{b} \right)$$

when m is a whole number and we see for a long plate that k_x becomes

$$k_x = 2 \left\{ 1 + \frac{(D_{12} + 2 D_{66})}{\sqrt{D_{11} D_{22}}} \right\} \quad (38)$$

Figures 24 and 25 are plots of N_{xcr} for $[0/\pm 45/0]_{2S}$ laminates at room temperature and 350° F as a function of panel aspect ratio, panel width, and ratio of N_y to N_x . These curves were obtained from equation 37.

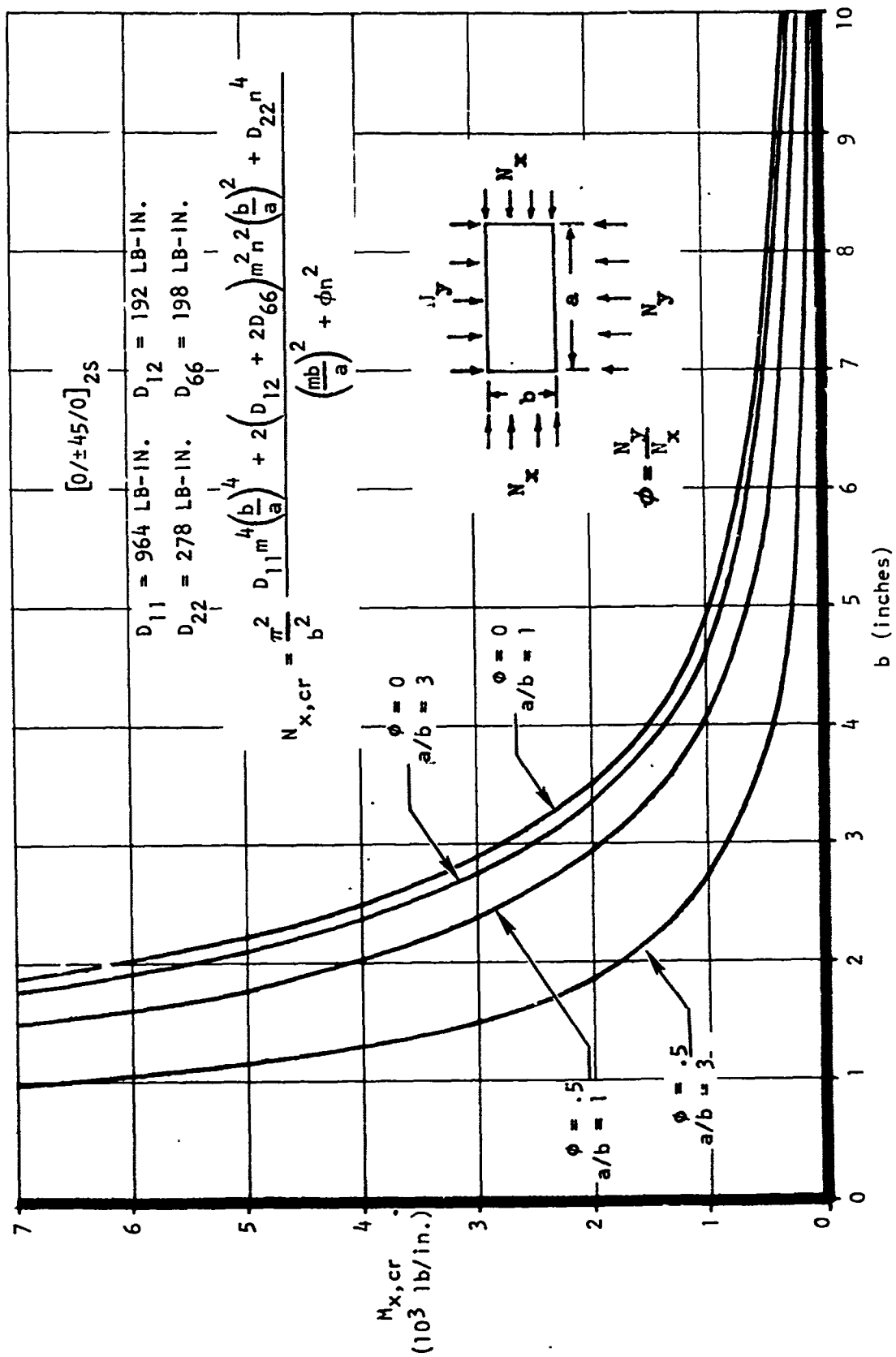


Figure 24. Buckling Load $N_{x,cr}$ vs Width b for a Simply Supported Plate With N_y (RT)

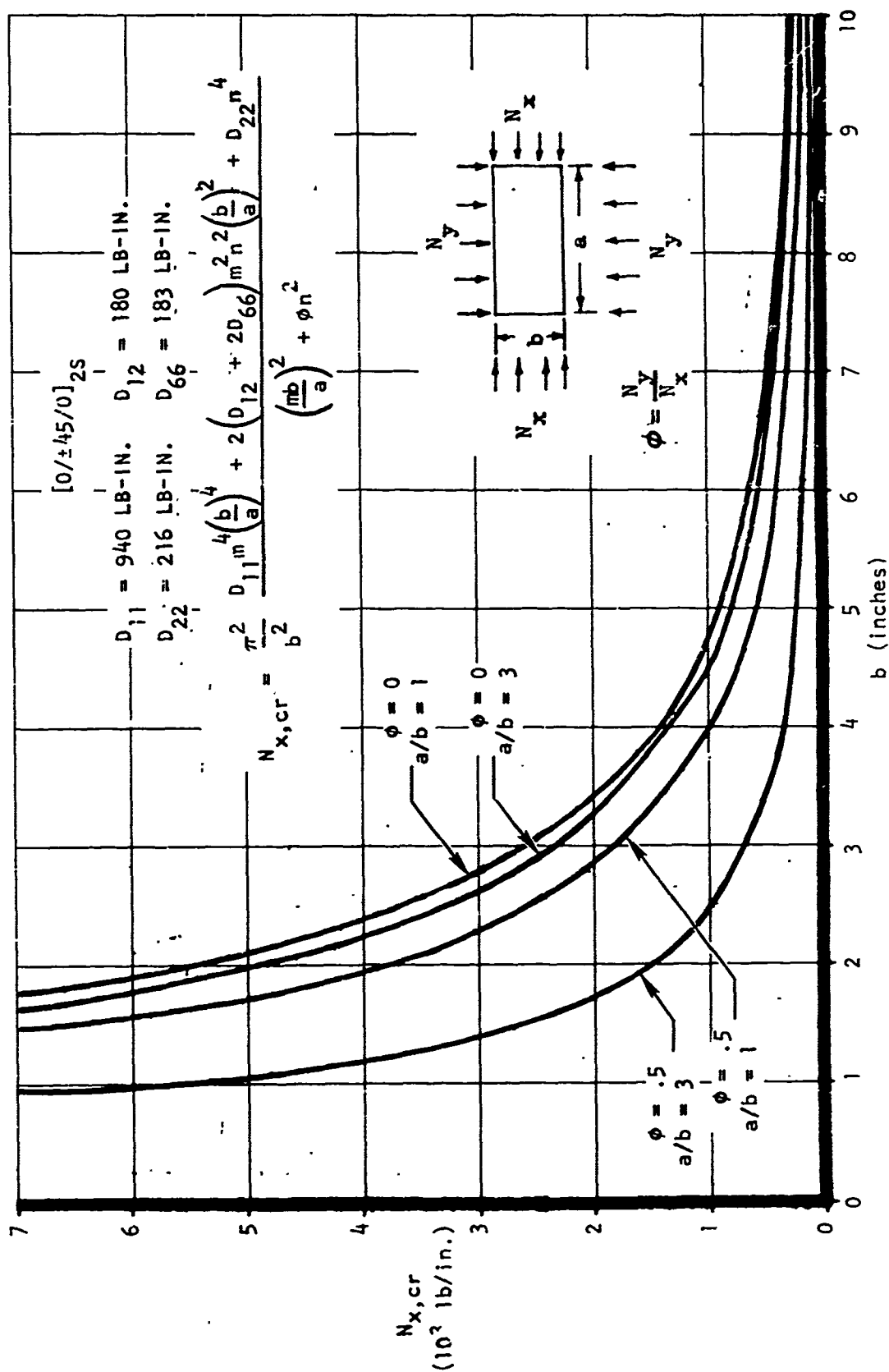


Figure 25. Buckling Load $N_{x,cr}$ vs Width b for a Simply Supported Plate With N_y (350° F)

NATURAL FREQUENCIES

If we seek the fundamental frequencies associated with various m and n mode shapes for a simply supported plate, w is taken in the form

$$w(x, y, t) = e^{i\omega_{mn}t} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (39)$$

where ω_{mn} is the angular velocity in radians per second. Substitution of equation 39 into equation 29 and setting the in-plane forces and normal pressure q to zero yields

$$\omega_{mn} = \left\{ \frac{D_1 \left(\frac{m\pi}{a} \right)^4 + 2 D_3 \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 + D_2 \left(\frac{n\pi}{b} \right)^4}{\rho t} \right\}^{\frac{1}{2}} \quad (40)$$

SECTION IV

ANALYSIS OF ORTHOTROPIC SANDWICH PLATES AND SHELL STRUCTURES

CONSTITUTIVE RELATIONS

In solid plates and shells, shear strain is generally neglected. But in sandwich plates and shells, shear strain in core needs to be considered. The sandwich element is composed of two face plates and a core as shown in figure 26. The distance between the centroids of the two faces is

$$h = c + t_f \quad (41)$$

Consider a cylindrical sandwich shell element as shown in figure 27. Let X, Y, Z be a set of orthogonal axes with X along the genatrix of the cylinder, Y tangent to the cylindrical surface, and Z normal to the cylindrical surface, and u, v, w be displacements along these three axes, respectively. Let N_x, N_y, N_{xy} , denote the sectional forces; M_x, M_y, M_{xy} the sectional moments; and Q_x, Q_y , the transverse shear forces per unit length of the element. This element is subject to a distributed lateral load q per unit area.

The equations of motion of this element are written as

$$\frac{\partial M_y}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = I \frac{\partial^2 \beta_x}{\partial t^2} \quad (42)$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I \frac{\partial^2 \beta_y}{\partial t^2} \quad (43)$$

$$q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - \frac{N_y}{R} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = M \frac{\partial^2 w}{\partial t^2} \quad (44)$$

where I denotes the rotary moment of inertia, M the mass per unit area of the element, R the radius, t the time, β_x and β_y rotations as shown in figure 28.

Let ρ_f and ρ_c denote the mass densities of the face and the core, respectively.

$$I \cong \rho_f t_f \frac{(c^2 + ct_f)}{2} + \rho_c \frac{c^3}{12} \quad (45)$$

$$M = 2t_f \rho_f + c \rho_c \quad (46)$$

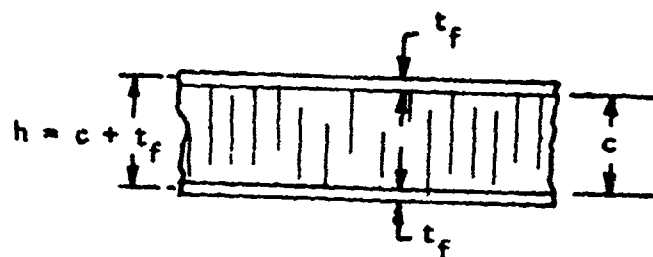


Figure 26. Section of a Sandwich Plate

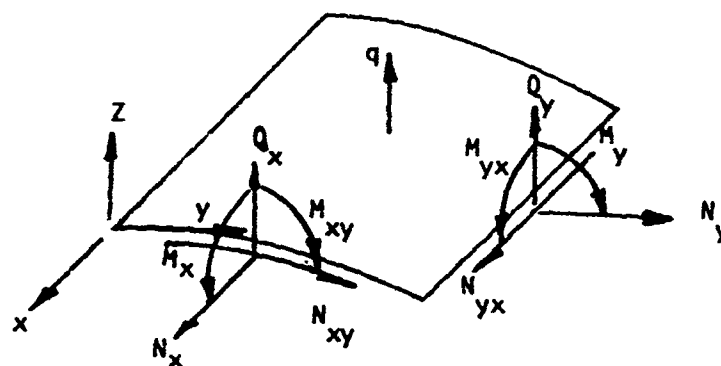


Figure 27. Positive Direction of Sectional Forces and Moments

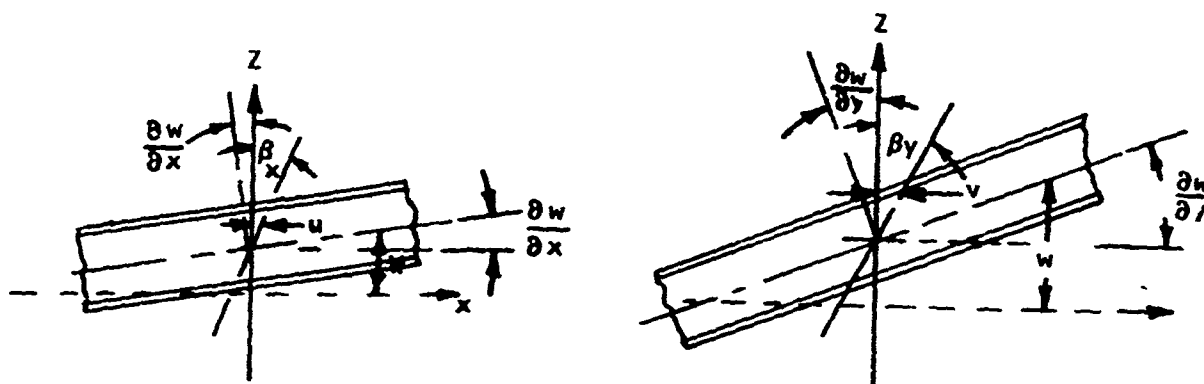


Figure 28. Rotations Due to Shear

NATURAL FREQUENCIES

If we seek the fundamental frequencies associated with various m and n mode shapes for a simply supported plate, w is taken in the form

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where ω_{mn} is the angular velocity in radians per second. Substitution of equation 39 into equation 29 and setting the in-plane forces and normal pressure q to zero yields

$$\omega_{mn} = \left\{ \frac{D_1 \left(\frac{m\pi}{a} \right)^4 + 2 D_3 \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 + D_2 \left(\frac{n\pi}{b} \right)^4}{\rho t} \right\}^{\frac{1}{2}} \quad (40)$$

SECTION IV

ANALYSIS OF ORTHOTROPIC SANDWICH PLATES AND SHELL STRUCTURES

CONSTITUTIVE RELATIONS

In solid plates and shells, shear strain is generally neglected. But in sandwich plates and shells, shear strain in core needs to be considered. The sandwich element is composed of two face plates and a core as shown in figure 26. The distance between the centroids of the two faces is

$$h = c + t_f \quad (41)$$

Consider a cylindrical sandwich shell element as shown in figure 27. Let X, Y, Z be a set of orthogonal axes with X along the genatrix of the cylinder, Y tangent to the cylindrical surface, and Z normal to the cylindrical surface, and u, v, w be displacements along these three axes, respectively. Let N_x, N_y, N_{xy} denote the sectional forces; M_x, M_y, M_{xy} the sectional moments; and Q_x, Q_y the transverse shear forces per unit length of the element. This element is subject to a distributed lateral load q per unit area.

The equations of motion of this element are written as

$$\frac{\partial M_y}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = I \frac{\partial^2 \beta_x}{\partial t^2} \quad (42)$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I \frac{\partial^2 \beta_y}{\partial t^2} \quad (43)$$

$$q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - \frac{N_y}{R} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = M \frac{\partial^2 w}{\partial t^2} \quad (44)$$

where I denotes the rotary moment of inertia, M the mass per unit area of the element, R the radius, t the time, β_x and β_y rotations as shown in figure 28.

Let ρ_f and ρ'_c denote the mass densities of the face and the core, respectively.

$$I \cong \rho_f t_f \frac{(c^2 + ct_f)}{2} + \rho'_c \frac{c^3}{12} \quad (45)$$

$$M = 2t_f \rho_f + c \rho'_c \quad (46)$$

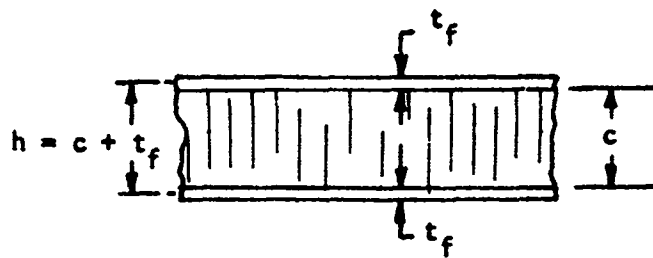


Figure 26. Section of a Sandwich Plate

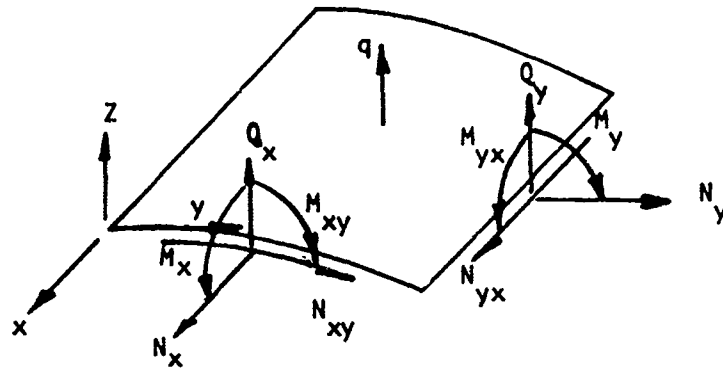


Figure 27. Positive Direction of Sectional Forces and Moments

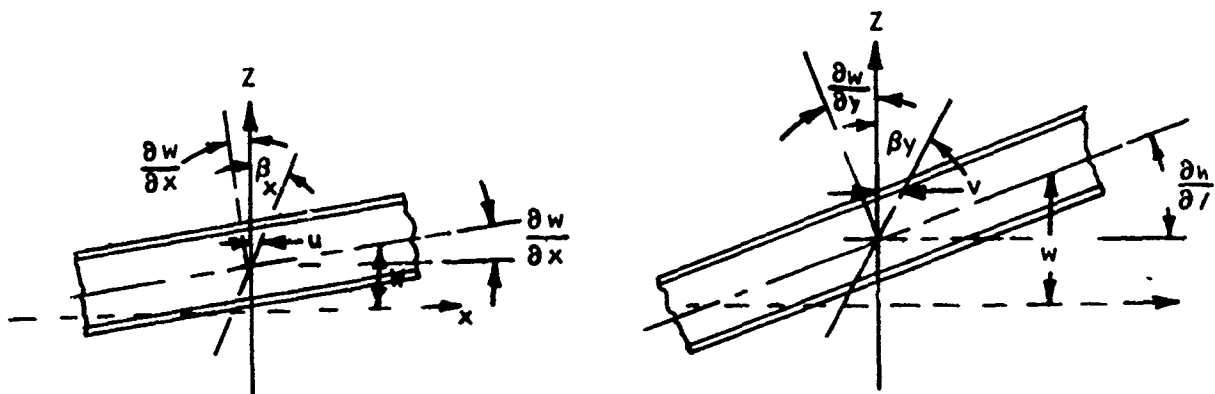


Figure 28. Rotations Due to Shear

Referring to figure 28, we see that the u and v components can be taken in the form

$$u(z) = \beta_x z; \quad v(z) = \beta_y z \quad (47)$$

Assume the bending moment to be taken solely by the faces and shear solely by the core, then

$$\tau_{yz} = G'_{cy} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) = G'_{cy} \left(\beta_y + \frac{\partial w}{\partial y} \right) \quad (48)$$

$$\tau_{xz} = G'_{cx} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = G'_{cx} \left(\beta_x + \frac{\partial w}{\partial x} \right)$$

$$\begin{aligned} Q_y &\cong \int_{-\frac{c}{2}}^{\frac{c}{2}} \tau_{yz} dz = G'_{cy} \left(\beta_y + \frac{\partial w}{\partial y} \right) \\ Q_x &\cong \int_{-\frac{c}{2}}^{\frac{c}{2}} \tau_{xz} dz = G'_{cx} \left(\beta_x + \frac{\partial w}{\partial x} \right) \end{aligned} \quad (49)$$

The face sheets are considered to be specially orthotropic and thus their stress-strain relation is given by

$$\begin{aligned} \sigma_{xx} &= \frac{E_x}{1-\nu_{xy}\nu_{yx}} (e_x + \nu_{yx} e_y) = \frac{E_x}{1-\nu_{xy}\nu_{yx}} \left(\frac{\partial u}{\partial x} + \nu_{yx} \frac{\partial v}{\partial y} \right) \\ &\cong \frac{E_x}{1-\nu_{xy}\nu_{yx}} \frac{c}{2} \left(\frac{\partial \beta_x}{\partial x} + \nu_{yx} \frac{\partial \beta_y}{\partial y} \right) \end{aligned} \quad (50)$$

Similarly,

$$\sigma_{yy} \cong \frac{E_y}{1-\nu_{xy}\nu_{yx}} \frac{c}{2} \left(\frac{\partial \beta_y}{\partial y} + \nu_{xy} \frac{\partial \beta_x}{\partial x} \right) \quad (51)$$

$$\tau_{xy} \cong G_{xy} \frac{c}{2} \left(\frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \right) \quad (52)$$

The section moments are obtained by integration.

$$M_x = \int_{\frac{c}{2}}^{\frac{c}{2}+t_f} \sigma_{xx} z dz + \int_{-(\frac{c}{2}+t_f)}^{-\frac{c}{2}} \sigma_{xx} z dz = D_{11} \left(\frac{\partial \beta_x}{\partial x} + \nu_{yx} \frac{\partial \beta_y}{\partial y} \right) \quad (53)$$

where

$$D_{11} = \frac{E_x c^2 t_f}{2(1 - \nu_{xy} \nu_{yx})} \left(1 + \frac{t_f}{c}\right)$$

Similarly,

$$M_y = D_{22} \left(\frac{\partial \beta_y}{\partial y} + \nu_{xy} \frac{\partial \beta_x}{\partial x} \right) \quad (54)$$

where

$$D_{22} = D_{11} \frac{E_y}{E_x}$$

$$M_{xy} = D_{66} \left(\frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \right) \quad (55)$$

where

$$D_{66} = t_f \frac{G_{xy}}{2} (c^2 + t_f c)$$

Substituting equations 53, 54, and 55 into equations 42 and 43 we obtain

$$D_{11} \left(\frac{\partial^2 \beta_x}{\partial x^2} + \nu_{yx} \frac{\partial^2 \beta_y}{\partial x \partial y} \right) + D_{66} \left(\frac{\partial^2 \beta_y}{\partial y^2} + \frac{\partial^2 \beta_x}{\partial x \partial y} \right) - Q_x = I \frac{\partial^2 \beta_x}{\partial t^2} \quad (56)$$

$$D_{22} \left(\frac{\partial^2 \beta_y}{\partial y^2} + \nu_{xy} \frac{\partial^2 \beta_x}{\partial x \partial y} \right) + D_{66} \left(\frac{\partial^2 \beta_x}{\partial x \partial y} + \frac{\partial^2 \beta_y}{\partial x^2} \right) - Q_y = I \frac{\partial^2 \beta_y}{\partial t^2} \quad (57)$$

Now let us consider the case of a simply supported rectangular plate.

Let

$$w = e^{i\omega t} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (58)$$

$$Q_x = e^{i\omega t} B_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (59)$$

$$Q_y = e^{i\omega t} C_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \quad (60)$$

$$q = e^{i\omega t} q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (61)$$

From equations 49, 58, 59, and 60, we find

$$\beta_y = e^{i\omega t} \left(\frac{C_{mn}}{cG'_{cy}} - \frac{n\pi}{b} A_{mn} \right) \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \quad (62)$$

$$\beta_x = e^{i\omega t} \left(\frac{B_{mn}}{cG'_{cx}} - \frac{m\pi}{a} A_{mn} \right) \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (63)$$

Substituting equations 59, 62, and 63 into equation 56 and simplifying yields

$$\begin{aligned} B_{mn} & \left\{ 1 + \frac{D_{11}}{cG'_{cx}} \bar{m}^2 + \frac{D_{66}}{cG'_{cx}} \bar{n}^2 - I \frac{\omega^2}{cG'_{cx}} \right\} \\ & + C_{mn} \left\{ \frac{\nu_{yx}}{cG'_{cy}} D_{11} \bar{m} \bar{n} + \frac{D_{66}}{cG'_{cy}} \bar{m} \bar{n} \right\} \\ & - A_{mn} D_{11} \left\{ \bar{m}^3 + \nu_{yx} D_{11} \bar{m}^2 + 2D_{66} \bar{m} \bar{n}^2 - I \omega^2 \bar{m} \right\} = 0 \end{aligned} \quad (64)$$

where

$$\bar{m} = \frac{m\pi}{a} \text{ and } \bar{n} = \frac{n\pi}{b} \quad (65)$$

Equation 64 can be written in the following compacted form:

$$a_{11} A_{mn} + a_{12} B_{mn} + a_{13} C_{mn} = 0 \quad (66)$$

where a_{11} , a_{12} , and a_{13} are the coefficient of A_{mn} , B_{mn} , and C_{mn} , respectively.

Similarly, substituting equations 59, 62, and 63 into equation 57, we obtain

$$a_{21} A_{mn} + a_{22} B_{mn} + a_{23} C_{mn} = 0 \quad (67)$$

where

$$\begin{aligned}
 a_{21} &= D_{22} \bar{n}^3 + \left(2D_{66} + \nu_{xy} D_{22} \right) \bar{n} \bar{m}^2 - \bar{n} I \omega^2 \\
 a_{22} &= \frac{\bar{m} \bar{n}}{cG'_{cy}} \left(D_{22} \nu_{xy} + D_{66} \right) \\
 a_{23} &= - \frac{1}{cG'_{cy}} \left(cG'_{cy} + D_{22} \bar{n}^2 + D_{66} \bar{m}^2 - I \omega^2 \right)
 \end{aligned} \tag{68}$$

A plate can be considered as a circular shell with infinite radius. R in equation 44 becomes infinity. Let $N_{xy} = 0$. Then eliminating Q_x and Q_y in equation 44 by equations 56 and 57, we obtain

$$\begin{aligned}
 q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} - I \frac{\partial^3 \beta_x}{\partial x \partial t^2} + D_{11} \left(\frac{\partial^3 \beta_x}{\partial x^3} + \nu_{yx} \frac{\partial^3 \beta_y}{\partial x^2 \partial y} \right) \\
 + 2 D_{66} \left(\frac{\partial^3 \beta_x}{\partial x \partial y^2} + \frac{\partial^3 \beta_y}{\partial x^2 \partial y} \right) + D_{22} \left(\frac{\partial^3 \beta_y}{\partial x^3} + \nu_{xy} \frac{\partial^3 \beta_x}{\partial x \partial y^2} \right) - I \frac{\partial^3 \beta_y}{\partial t^2 \partial y} = M \frac{\partial^2 w}{\partial t^2}
 \end{aligned} \tag{69}$$

Substituting equations 58, 62, and 63 into the preceding equation yields

$$q_{mn} - \left(a_{31} + N_x \bar{m}^2 + N_y \bar{n}^2 \right) A_{mn} + a_{32} B_{mn} + a_{33} C_{mn} = 0 \tag{70}$$

where

$$\begin{aligned}
 a_{31} &= - \bar{m}^2 I \omega^2 + D_{11} \bar{m}^4 + \bar{m}^2 \bar{n}^2 \left(\nu_{yx} D_{11} + 4D_{66} + \nu_{xy} D_{22} \right) \\
 &\quad + D_{22} \bar{n}^4 - I \omega^2 \bar{n}^2 - \omega^2 M \\
 a_{32} &= \frac{\bar{m}}{cG'_{cx}} \left[- I \omega^2 + D_{11} \bar{m}^2 + \bar{n}^2 \left(2D_{66} + \nu_{xy} D_{22} \right) \right] \\
 a_{33} &= \frac{\bar{n}}{cG'_{cy}} \left[- I \omega^2 + D_{22} \bar{n}^2 + \bar{m}^2 \left(2D_{66} + \nu_{yx} D_{11} \right) \right]
 \end{aligned} \tag{71}$$

From equations 66, 67, and 70, we obtain

$$C_{mn} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{12}a_{23} - a_{22}a_{13}} A_{mn} \quad (72)$$

$$B_{mn} = \frac{a_{13}a_{21} - a_{11}a_{23}}{a_{12}a_{23} - a_{22}a_{13}} A_{mn} \quad (73)$$

$$A_{mn} = \frac{q_{mn} (a_{22}a_{13} - a_{12}a_{23})}{(a_{31} + m^2 N_x + n^2 N_y) (a_{22}a_{13} - a_{12}a_{23}) + a_{32} (a_{13}a_{21} - a_{11}a_{23}) + a_{33} (a_{11}a_{22} - a_{12}a_{21})} \quad (74)$$

Let us define the following nondimensional ratios:

$$\alpha = \sqrt{\frac{E_x}{E_y}}, \quad r = \frac{G'_{cx}}{G'_{cy}}$$

$$\beta = \alpha v_{yx} + 2\gamma, \quad \gamma = \frac{D_{66}}{\sqrt{D_{11}D_{22}}} \quad (75)$$

$$v = \left(\frac{\pi}{b}\right)^2 \frac{\sqrt{D_{11}D_{22}}}{cG'_{cx}}$$

$$k_x = \frac{N_x \left(\frac{b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}, \quad k_y = \frac{N_y \left(\frac{b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}$$

Then the coefficients of equations 66, 67, and 70 may be written as

$$a_{11} = - \left(\frac{\pi}{b}\right)^3 \sqrt{D_{11}D_{22}} \left[\left(\frac{mb}{a}\right)^3 \alpha + mn^2 \left(\frac{b}{a}\right) \beta - \frac{\text{Im} \left(\frac{\omega b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}} \left(\frac{b}{a}\right) \right]$$

$$a_{12} = 1 + m^2 \alpha v \left(\frac{b}{a}\right)^2 + n^2 \gamma v - \frac{I\omega^2}{cG'_{cx}}$$

$$\begin{aligned}
a_{13} &= m \left(\frac{b}{a} \right) v (\beta - \gamma) r \\
a_{21} &= \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} \left[\frac{n^3}{\alpha} + m^2 \beta \left(\frac{b}{a} \right)^2 - \frac{\ln \left(\frac{\omega b}{\pi} \right)^2}{\sqrt{D_{11} D_{22}}} \right] \\
a_{22} &= -m \left(\frac{b}{a} \right) v (\beta - \gamma) \\
a_{23} &= -1 - r v \left[\frac{n^2}{\alpha} + \gamma m^2 \left(\frac{b}{a} \right)^2 \right] + \frac{l \omega^2 r}{c G'_{CX}} \\
a_{31} &= \left(\frac{\pi}{b} \right)^4 \sqrt{D_{11} D_{22}} \left\{ m^2 \left(\frac{b}{a} \right)^2 \left[\alpha m^2 \left(\frac{b}{a} \right)^2 + 2 r^2 \beta \right] + \frac{n^4}{\alpha} \right. \\
&\quad \left. - \frac{\omega^2 \left(\frac{b}{\pi} \right)^2}{\sqrt{D_{11} D_{22}}} \left[\left(\frac{m b}{a} \right)^2 I + n^2 I + M \left(\frac{b}{\pi} \right)^2 \right] \right\} \\
a_{32} &= \frac{\pi}{b} \left[m^3 \left(\frac{b}{a} \right)^3 \alpha v + m^2 \beta v \left(\frac{b}{a} \right) - \frac{l \omega^2}{c G'_{CX}} m \left(\frac{b}{a} \right) \right] \\
a_{33} &= \frac{\pi}{b} \left[n^3 r \frac{v}{\alpha} + m^2 n r \beta \left(\frac{b}{a} \right)^2 v - \frac{l \omega^2 n r}{c G'_{CX}} \right]
\end{aligned} \tag{76}$$

COMPRESSION STABILITY

When the loads N_x and N_y reach critical levels, the denominator of equation 74 vanishes. This yields

$$\begin{aligned}
\bar{m}^2 N_x + \bar{n}^2 N_y &= \\
&\frac{- \left(a_{33} (a_{12} a_{21} - a_{11} a_{22}) + a_{32} (a_{11} a_{23} - a_{13} a_{21}) \right)}{a_{12} a_{23} - a_{22} a_{13}} = -a'_{31}
\end{aligned} \tag{77}$$

Let N_x and N_y be positive for compression; the signs of N_x , N_y are changed in the preceding equation, giving

$$\bar{m}^2 N_x + \bar{n}^2 N_y = a_{31} + a'_{31}$$

From the last two equations of equation 75 and the preceding, we have

$$\begin{aligned} \sqrt{D_{11}D_{22}} \left[\left(\frac{m\pi}{a} \right)^2 k_x \left(\frac{\pi}{b} \right)^2 + \left(\frac{n\pi}{b} \right)^2 k_y \left(\frac{\pi}{a} \right)^2 \right] &= a_{31} + a'_{31} \\ \sqrt{D_{11}D_{22}} \left(\frac{\pi}{b} \right)^2 \left[\left(\frac{\pi}{a} \right)^2 m^2 k_x + \left(\frac{\pi}{b} \right)^2 n^2 k_y \right] &= a_{31} + a'_{31} \\ m^2 \left(\frac{b}{a} \right)^2 k_x + n^2 k_y &= \frac{(a_{31} + a'_{31}) b^4}{\pi^4 \sqrt{D_{11}D_{22}}} = \bar{a}_{31} + \bar{a}'_{31} \end{aligned} \quad (78)$$

where

$$\bar{a}_{31} = \frac{a_{31} b^4}{\pi^4 \sqrt{D_{11}D_{22}}} \quad \text{and} \quad \bar{a}'_{31} = \frac{a'_{31} b^4}{\pi^4 \sqrt{D_{11}D_{22}}} \quad (79)$$

For compressive loading along the x-axis only.

$$k_x = \left(\frac{a}{b} \right)^2 \frac{\bar{a}_{31} + \bar{a}'_{31}}{m^2} \quad (80)$$

For compressive loading along the y-axis only.

$$k_y = \frac{\bar{a}_{31} + \bar{a}'_{31}}{n^2} \quad (81)$$

For a constant ratio of $N_y/N_x = \phi$,

$$k_x = \frac{\bar{a}_{31} + \bar{a}'_{31}}{m^2 \left(\frac{b}{a} \right)^2 + \phi n^2} \quad (82)$$

The critical buckling load is obtained (equations 80, 81, and 82) by finding the integers m and n which give minimum values of k_x or k_y . These values for the cases of uniaxial and biaxial compression are shown in figures 29 through 36.

$$(N_x)_{cr} = \frac{k_x \pi^2 t_f c (c + t_f)}{2b^2}$$



$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$$

$$\alpha = \sqrt{E_x / E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c) (1 + t_f/c)}{2 G_c' (b/c)^2}$$

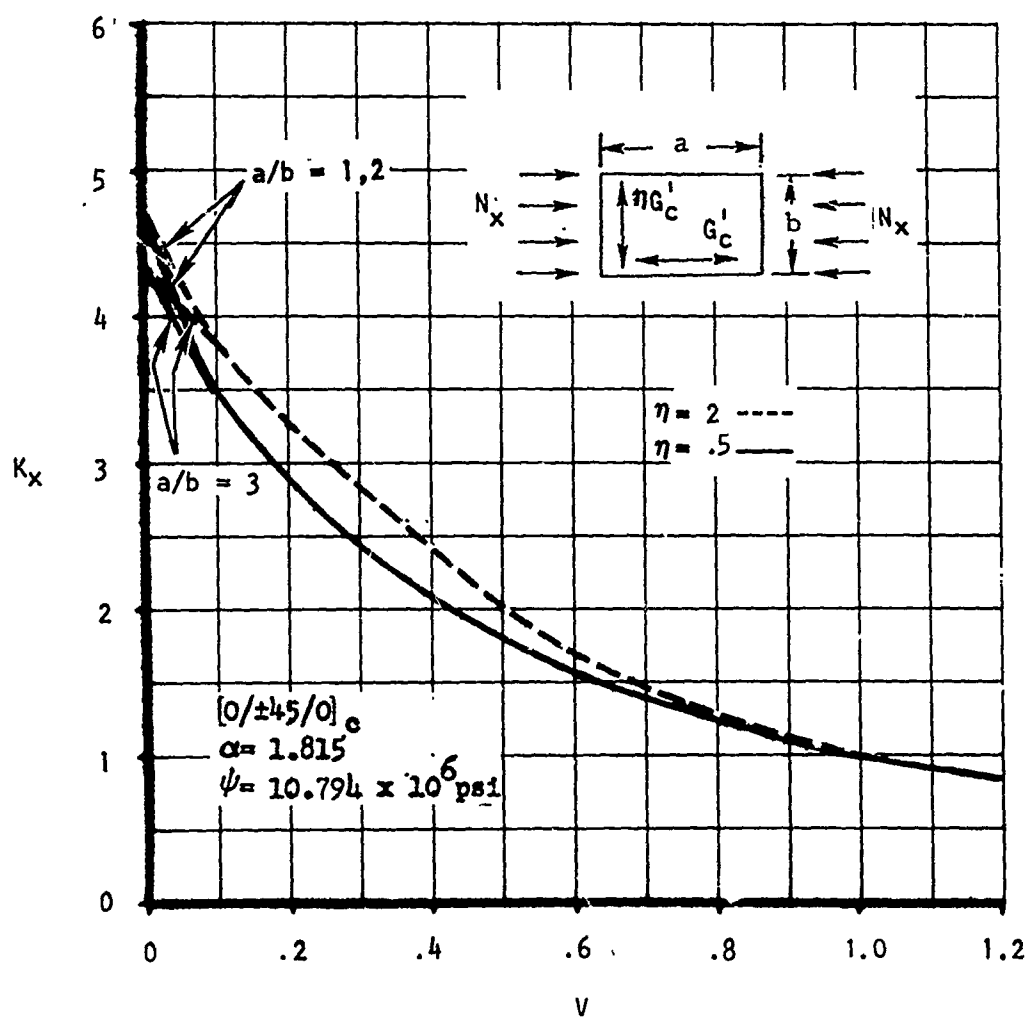


Figure 29. Buckling Coefficient K_x vs V for Simply Supported Flat Sandwich Panels (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c + t_f)}{2b^2}$$



$$\psi = \sqrt{E_x E_y / (1 - \nu_{xy} \nu_{yx})}$$

$$\alpha = \sqrt{E_x E_y}$$

$$\nu = \frac{\pi^2 \psi (t_f/c) (1 + t_f/c)}{2 G'_c (b/c)^2}$$

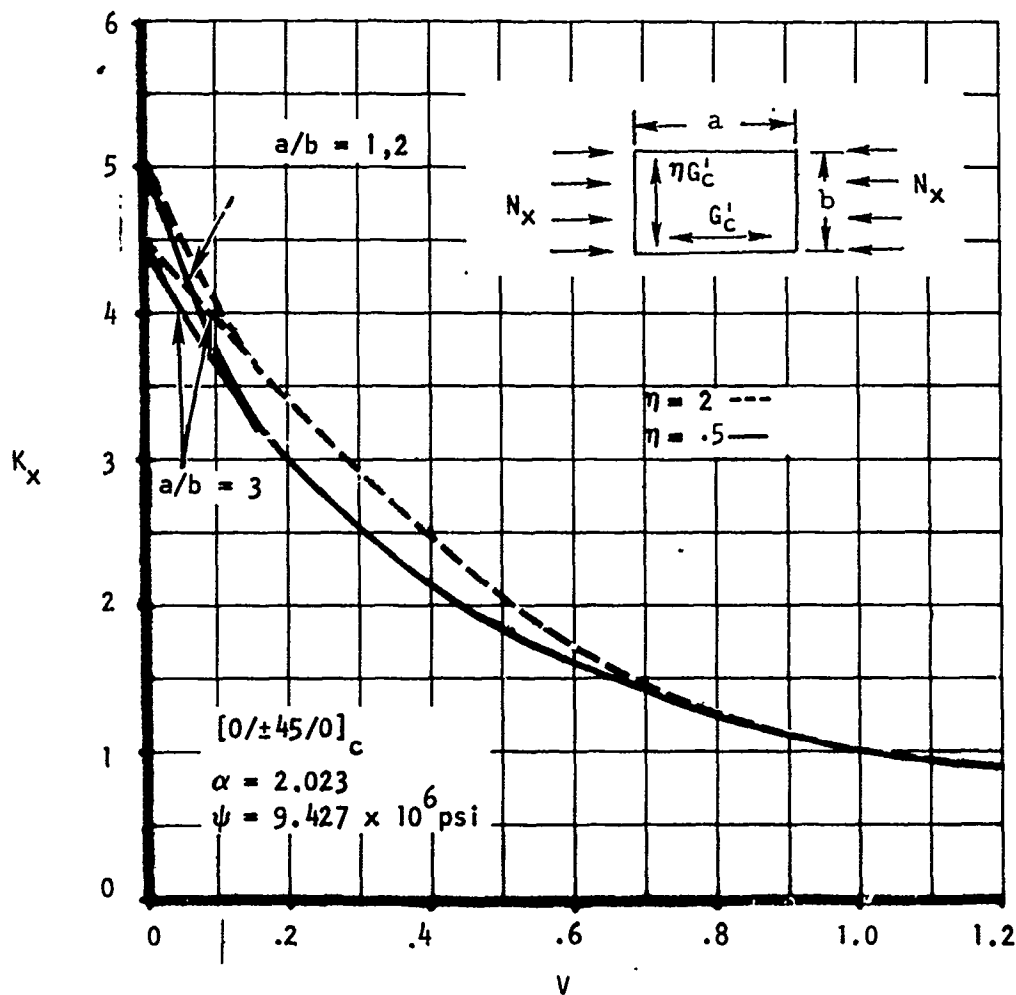


Figure 30. Buckling Coefficient K_x vs F for Simply Supported Flat Sandwich Panels (350° F)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c + t_f)}{2b^2}$$



$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$$

$$\alpha = \sqrt{E_x / E_y}$$

$$\nu = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2 G'_c (b/c)^2}$$

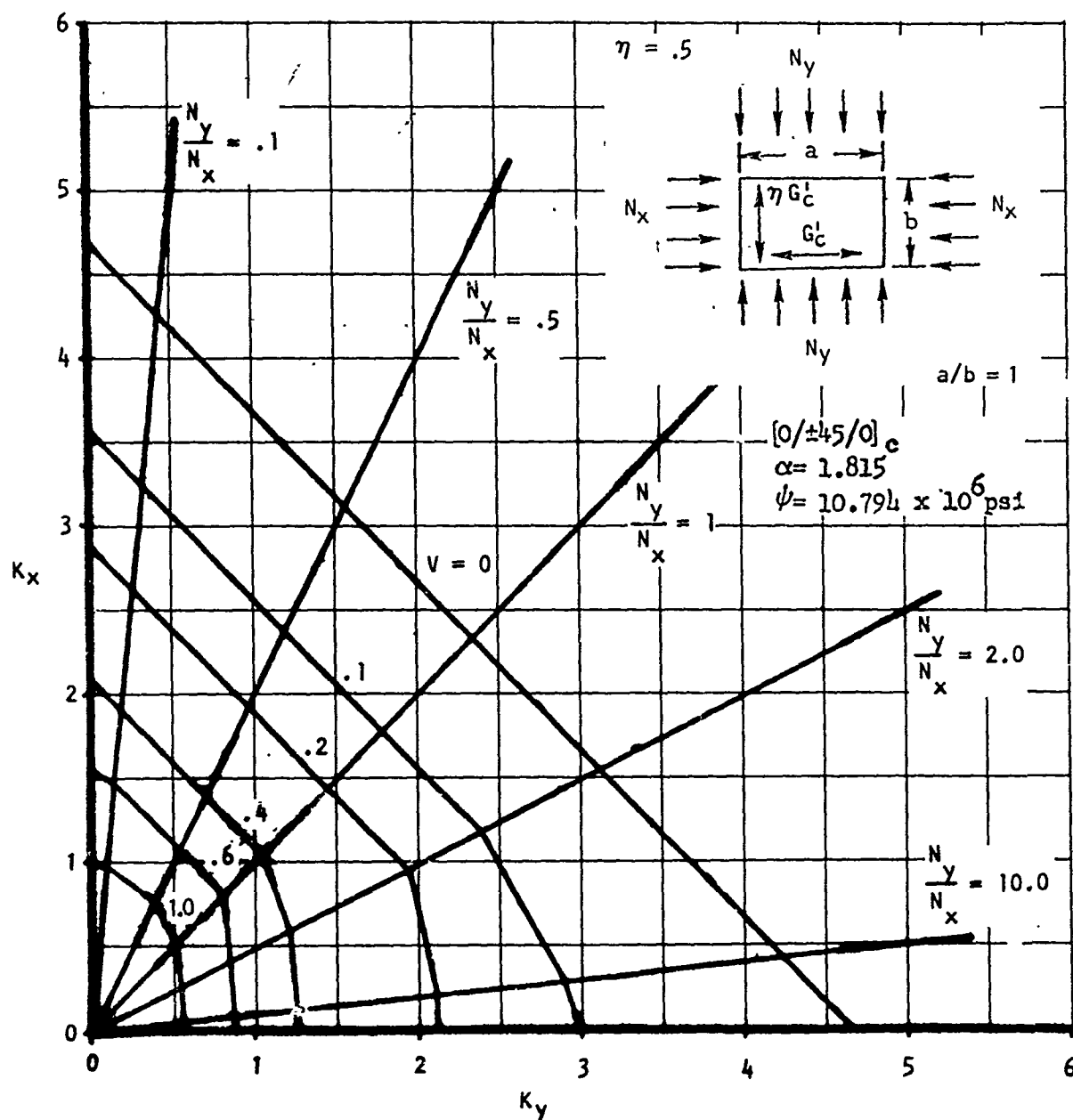


Figure 31. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of $a/b = 1$ (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 / t_f c (c + t_f)}{2b^2}$$



$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$$

$$\alpha = \sqrt{E_x / E_y}$$

$$\gamma = \frac{\pi^2 \psi (t_f / c) (1 + t_f / c)}{2 G_c^2 (b / c)^2}$$

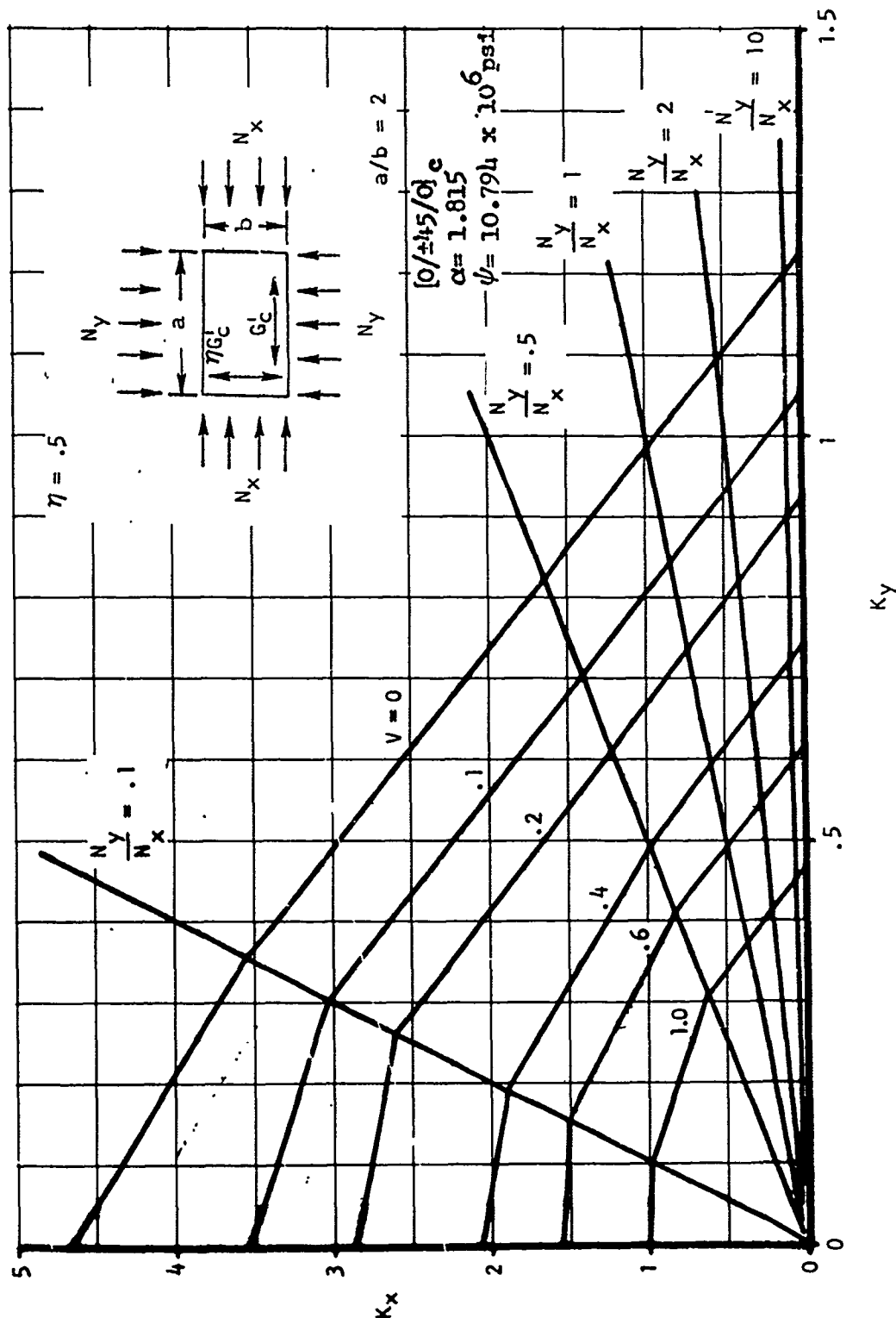


Figure 32. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of $a/b = 2$ (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c + t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$$

$$\alpha = \sqrt{E_x / E_y}$$

$$\nu = \frac{\pi^2 \psi (t_f / c) (1 + t_f / c)}{2 G_c' (b / c)^2}$$

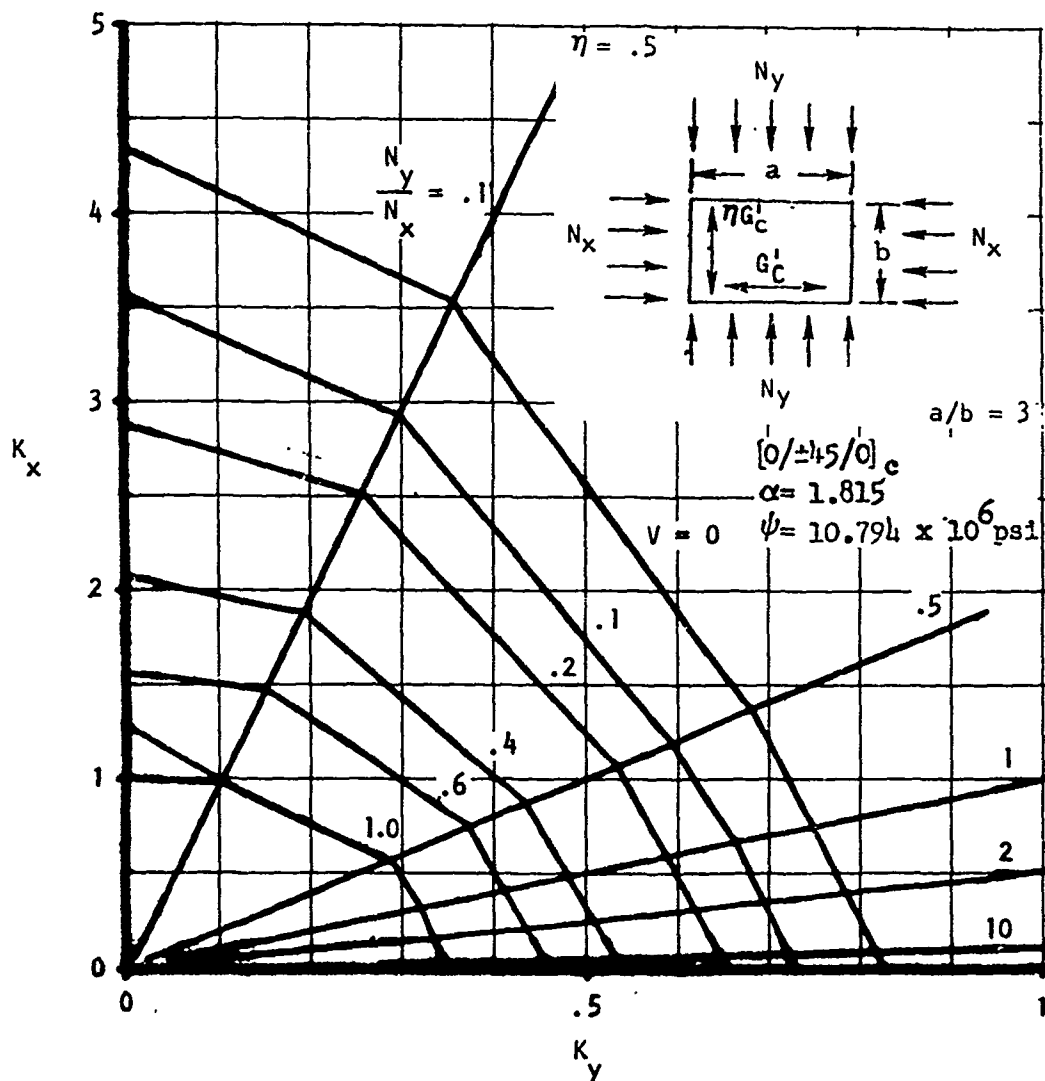


Figure 33. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of $a/b = 3$ (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 t_f c (c + t_f)}{2b^2}$$



$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$$

$$\alpha = \sqrt{E_x / E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c) (1 + t_f/c)}{2 G'_c (b/c)^2}$$

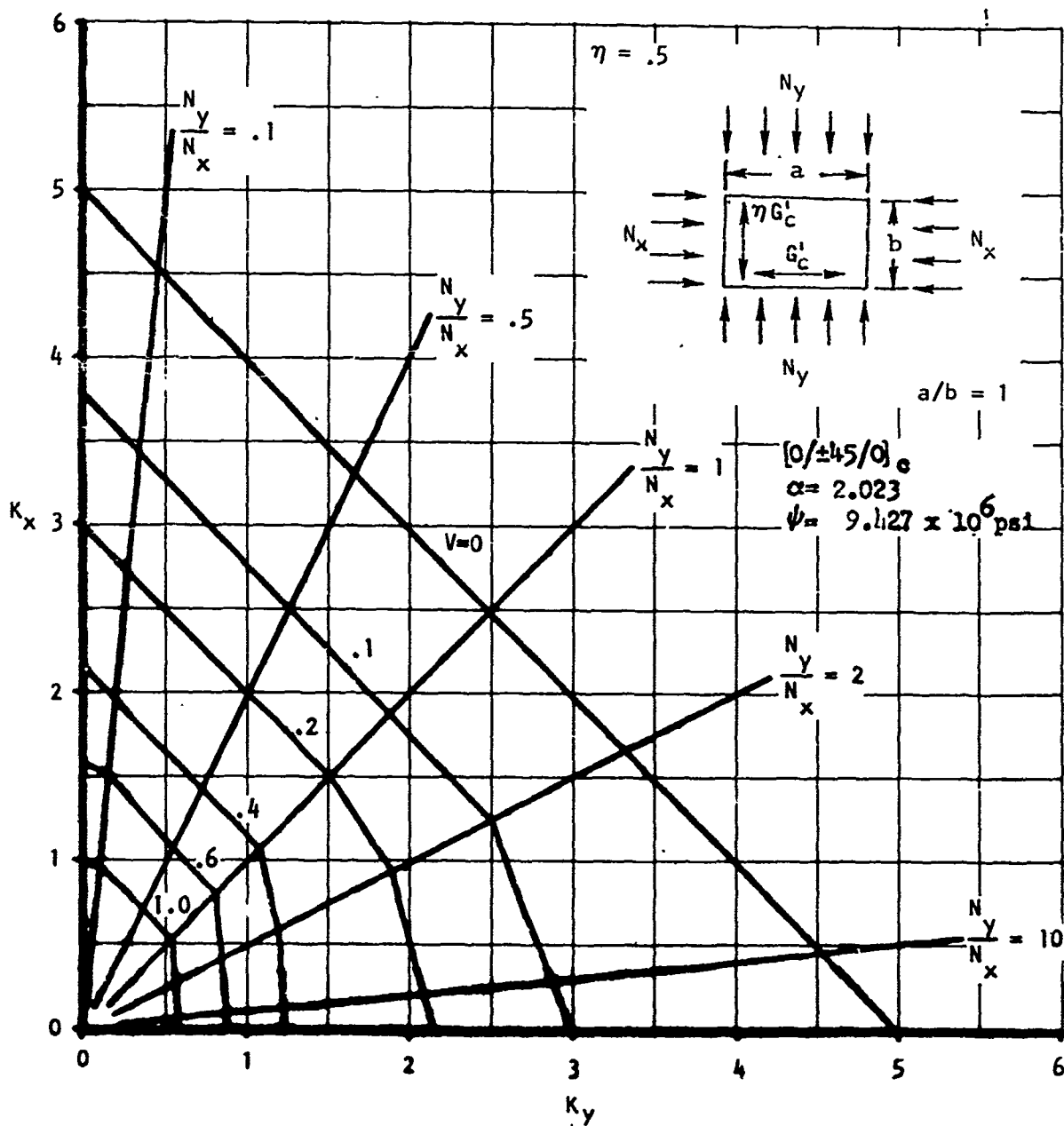


Figure 34. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of $a/b = 1$ (350° F)

$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx}) \quad V = \frac{\pi^2 \psi (t_f/c)(1 + t_f/c)}{2 G_c^2 (b/c)^2}$$

$$\alpha = \sqrt{E_x/E_y}$$

$$(N_x)_{cr} = \frac{k_x \pi^2 D_{xx} (c + t_f)}{2b^2}$$

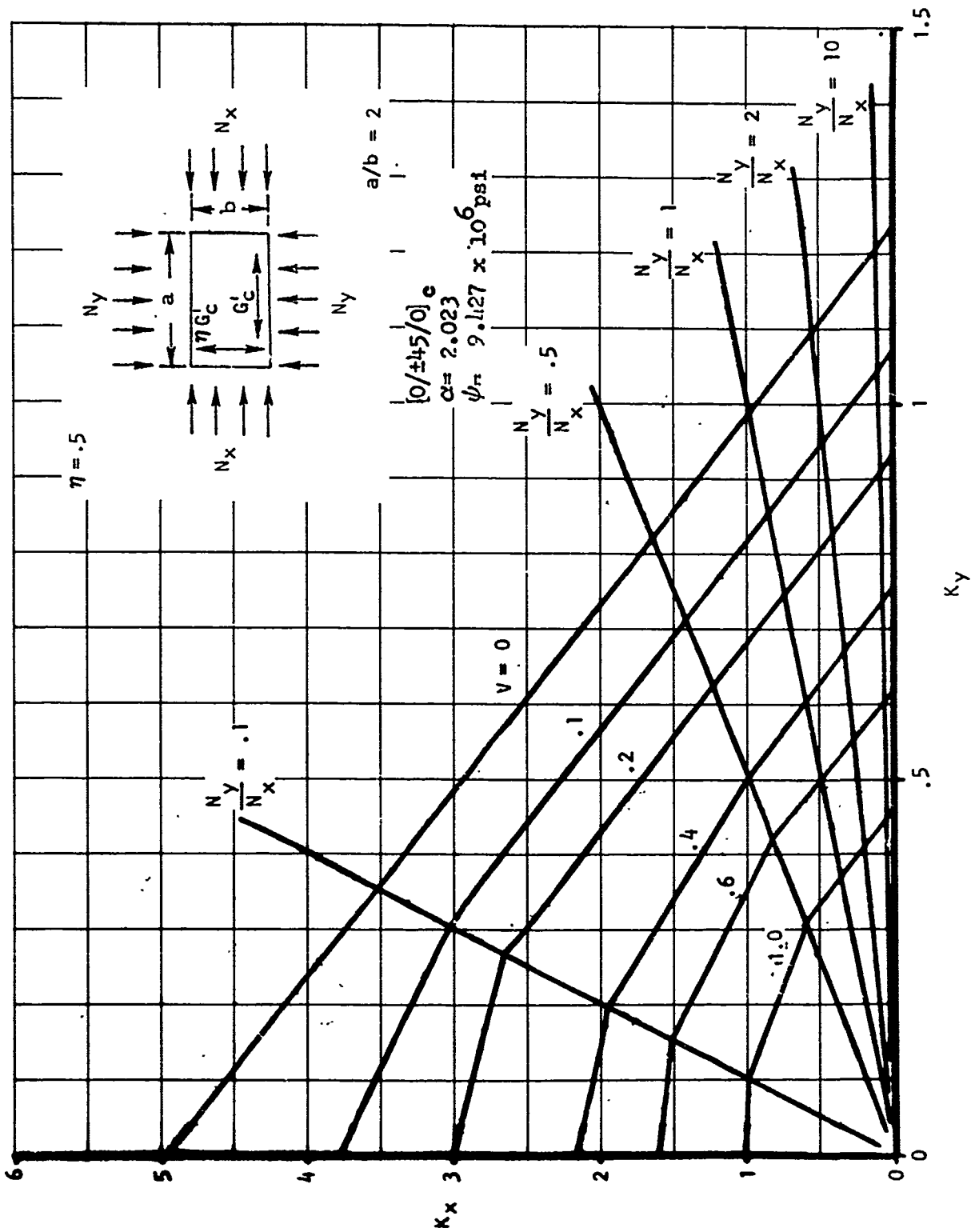


Figure 35. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of $a/b = 2$ (350° F)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c + t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$$

$$\alpha = \sqrt{E_x / E_y}$$

$$\nu = \frac{\pi^2 \psi (t_f / c) (1 + t_f / c)}{2 G'_c (b / c)^2}$$

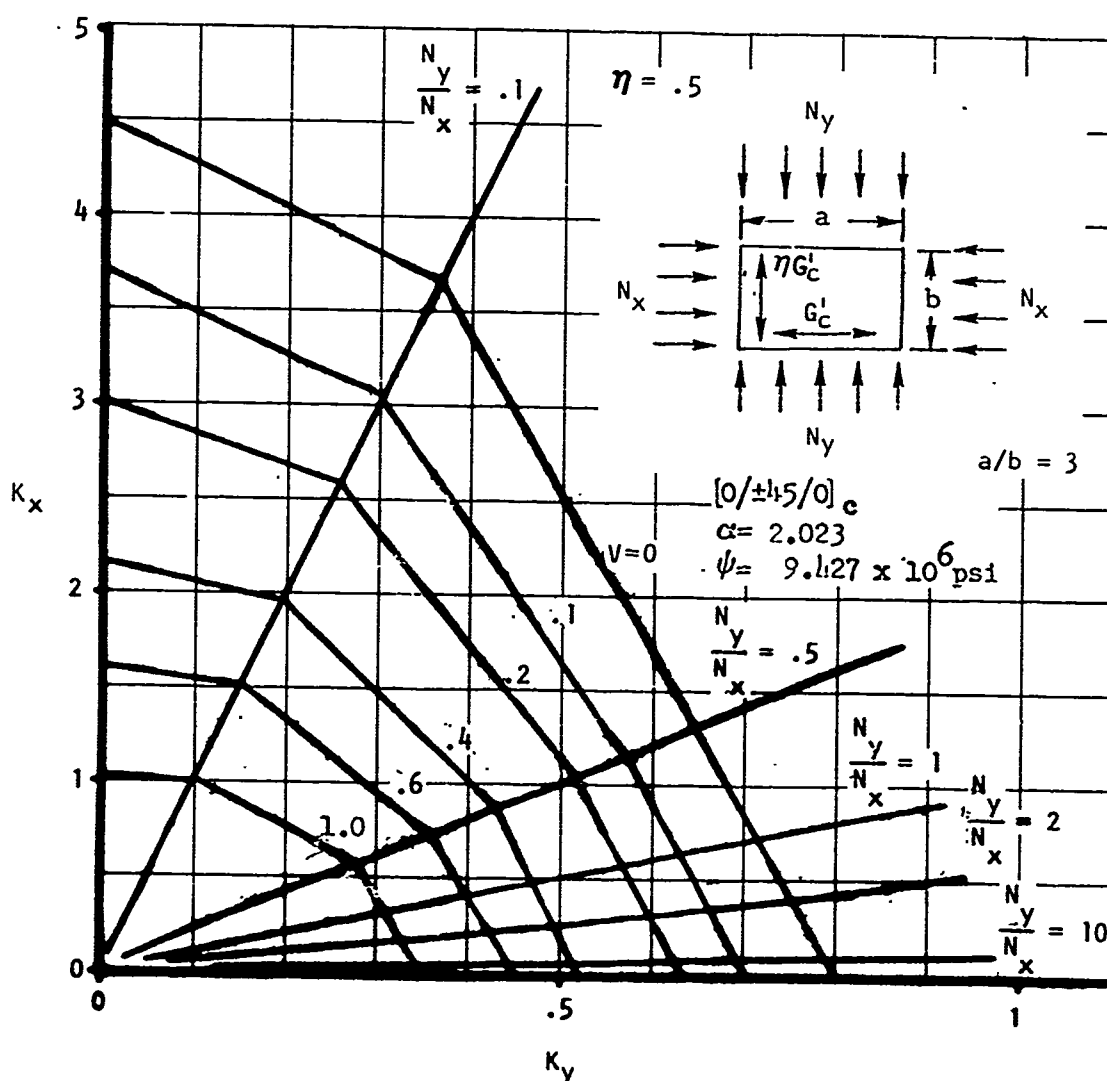


Figure 36. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of $a/b = 3$ (350° F)

In a similar manner, the natural frequencies ω for any m, n mode shape can be obtained by solving equation 78 for given k_x and k_y . As pointed out by previous investigators (reference 3), there are three resonant frequencies for each mode instead of a single resonant frequency for solid plate. Two of these resonances are associated with the two rotations, β_x and β_y , and one is associated with lateral deflection ω . For the cases where the core is isotropic ($G'_{cx} = G'_{cy}$), the three resonant frequencies reduce to two for each mode of vibration.

For filament-reinforced face sheets, the following expressions are invariant with respect to the rotation of the reference axes in the plane of the plate (reference 4).

$$I_1 = \frac{E_x}{1-\nu_{xy}\nu_{yx}} \left(1 + \frac{E_y}{E_x} + 2\nu_{yx} \right) = \frac{E_L}{1-\nu_{LT}\nu_{TL}} \left(1 + \frac{E_T}{E_L} + 2\nu_{TL} \right) \quad (83)$$

where L and T denote the longitudinal and transverse directions of the filaments.

$$I_2 = G_{xy} - \frac{\nu_{yx}E_x}{(1-\nu_{xy}\nu_{yx})} = G_{LT} - \frac{\nu_{TL}E_L}{1-\nu_{LT}\nu_{TL}} \quad (84)$$

Since the a_{ij} 's ($i, j = 1, 2, 3$) depend on the five face-sheet elastic constants through D_{11} , D_{22} , D_{66} , ν_{yx} , and ν_{xy} , the critical values of N_x and N_y depend on these five variables. The buckling constants k_x and k_y are the ratios of N_x and N_y divided by

$$\sqrt{D_{11}D_{22}} / \left(\frac{b}{\pi} \right)^2,$$

and hence are defined by four independent variables. (Only four of the five elastic constants E_x , E_y , ν_{xy} , ν_{yx} , G_{xy} are independent because of the auxiliary relationship $E_x\nu_{yx} = \nu_{xy}E_y$.) By using the invariant relations (equations 83 and 84), k_x and k_y may be expressed in terms of I_1 , I_2 , and two variables. Let

$$\psi = \sqrt{E_x E_y} / (1-\nu_{xy}\nu_{yx}) \quad (85)$$

We can express γ and β in terms of α and ψ as

$$\gamma = \frac{1}{2} \left\{ \frac{I_1}{\psi} - \alpha - \frac{1}{\alpha} + \frac{2I_2}{\psi} \right\} \quad (86)$$

$$\beta = \frac{3}{2} \left(\frac{I_1}{\psi} - \alpha - \frac{1}{\alpha} \right) + \frac{2I_2}{\psi} \quad (87)$$

For a given composite face sheet on a given core with given c , G_{cx} , and G_{cy} , the variation of critical k_x and k_y may be plotted against the properties of the core for different values of α and ψ . This reduces greatly the computation and presentation of the critical values of k_x and k_y for different orientations of the face plates.

A computer program, AC-5, entitled "Honeycomb Panel Stability Under In-plane Biaxial Loading," has been developed to perform the calculations indicated by equations 80 and 81. This program searches and finds the half-wave-length numbers m and n which minimize the buckling load for a given panel geometry and face sheet material. The AC-5 program was utilized to generate the stability curves for the $[0_2/\pm 45]_C$ laminate orientations used for the structural elements tested in this program and reported in volume II.

For the Narmco 5505 system, the values of the material invariants used were

$$\begin{aligned} \left. \begin{aligned} I_1 &= 33.88 \text{ Msi} \\ I_2 &= 0.128 \text{ Msi} \end{aligned} \right\} & \text{for room temperature} \\ \\ \left. \begin{aligned} I_1 &= 31.54 \text{ Msi} \\ I_2 &= 0.066 \text{ Msi} \end{aligned} \right\} & \text{for } 350^\circ \text{ F} \end{aligned}$$

Figures 29 and 30 give the uniaxial buckling loads at room temperature and 350° F , respectively, for the $[0_2/\pm 45]_C$ orientation. Figures 31 through 36 give the buckling loads for the same orientation at room temperature and 350° F for various combinations of longitudinal and transverse in-plane loads and aspect ratios.

UNIFORMLY DISTRIBUTED PRESSURE AND EDGE COMPRESSION

We now consider that the previously discussed panel is subjected to a uniform lateral load q as shown in figure 37. The pressure q can be expanded into the following Fourier sine series:

$$q = \sum_m \sum_n q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (88)$$

where

$$q_{mn} = 16q/\pi^2 mn \quad \text{for odd values of } m \text{ and } n$$

$$q_{mn} = 0 \quad \text{for } m \text{ or } n \text{ even}$$

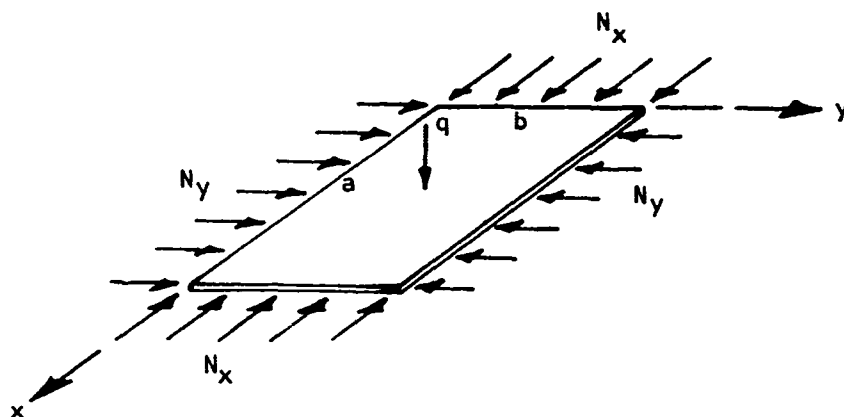


Figure 37. Sandwich Panel Subjected to Normal Load and Edge Compression

In this case, the loading is static and the radial velocity is zero, i.e., $\omega = 0$; thus, the panel deflection is expressed as

$$W = \sum_{m=1,3}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (89)$$

and we obtain from equations 74 and 88 that A_{mn} takes the form, with N_x and N_y taken to be positive for compression:

$$A_{mn} = \frac{16 q b^4 \bar{A}_{mn}}{\pi^6 mn \sqrt{D_{11} D_{22}}} \quad (90)$$

where

$$\bar{A}_{mn} = \frac{(\bar{a}_{12}\bar{a}_{23} - \bar{a}_{22}\bar{a}_{13})}{\left[\bar{a}_{31} + R_x k_x \left(\frac{mb}{a} \right)^2 + R_y k_y n^2 \right] \bar{a}_{22} (\bar{a}_{13} - \bar{a}_{12}\bar{a}_{23}) + \bar{a}_{32} (\bar{a}_{13}\bar{a}_{21} - \bar{a}_{11}\bar{a}_{23}) + \bar{a}_{33} (\bar{a}_{11}\bar{a}_{22} - \bar{a}_{12}\bar{a}_{21})} \quad (91)$$

and

$$\left. \begin{aligned}
 \bar{a}_{11} &= a_{11} / \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} \\
 \bar{a}_{12} &= a_{12} \\
 \bar{a}_{13} &= -a_{13} \\
 \bar{a}_{21} &= a_{21} / \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} \\
 \bar{a}_{22} &= a_{22} \\
 \bar{a}_{23} &= -a_{23} \\
 \bar{a}_{31} &= -a_{31} / \left(\frac{\pi}{b} \right)^4 \sqrt{D_{11} D_{22}}
 \end{aligned} \right\} \quad (92)$$

From equations 53, 62, and 63, we find

$$\begin{aligned}
 M_x &= D_{11} \left(\frac{\partial \beta_x}{\partial x} + \nu_{yx} \frac{\partial \beta_y}{\partial y} \right) \\
 &= \sum_m \sum_n D_{11} \left\{ \left[\frac{B_{mn} \left(\frac{m\pi}{a} \right)}{c G_{cx}} - \left(\frac{m\pi}{a} \right)^2 A_{mn} \right] \right. \\
 &\quad \left. + \nu_{yx} \left[\frac{C_{mn} \left(\frac{n\pi}{b} \right)}{c G_{cy}} - \left(\frac{n\pi}{b} \right)^2 A_{mn} \right] \right\} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (95)
 \end{aligned}$$

Equations 72 and 73 can be recast in the form

$$\begin{aligned}
 C_{mn} &= A_{mn} \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} \frac{(\bar{a}_{11} \bar{a}_{22} - \bar{a}_{12} \bar{a}_{21})}{(\bar{a}_{22} \bar{a}_{13} - \bar{a}_{12} \bar{a}_{23})} \\
 &= \lambda_2 \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} A_{mn} \quad (94)
 \end{aligned}$$

and

$$\begin{aligned} B_{mn} &= A_{mn} \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} \frac{(\bar{a}_{11} \bar{a}_{23} - \bar{a}_{13} \bar{a}_{21})}{(\bar{a}_{22} \bar{a}_{13} - \bar{a}_{12} \bar{a}_{23})} \\ &= \lambda_1 \left(\frac{\pi}{b} \right)^3 \sqrt{D_{11} D_{22}} A_{mn} \end{aligned} \quad (95)$$

$$\bar{a}_{32} = -a_{32}/(\pi/b)$$

$$\bar{a}_{33} = a_{33}/(\pi/b)$$

$$R_x = N_x/N_{x_{cr}}$$

$$R_y = N_y/N_{y_{cr}}$$

we obtain the following desired form for M_x by combining equations 90, 93, 94, and 95:

$$\begin{aligned} M_x &= -\frac{\alpha 16 q b^2}{\pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\bar{A}_{mn}}{mn} \left\{ \left[\frac{mb}{a} \lambda_1 V - \left(\frac{mb}{a} \right)^2 \right] \right. \\ &\quad \left. + \nu_{yx} \left[r V h \lambda_2 - n^2 \right] \right\} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \end{aligned} \quad (96)$$

In a similar manner, we find that

$$M_y = - \frac{16 q b^2}{\alpha \pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \left\{ \left[n \lambda_2 r V - n^2 \right] \right. \\ \left. + \nu_{xy} \left[\lambda_1 m \frac{b}{a} - \left(\frac{mb}{a} \right)^2 \right] \right\} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (97)$$

$$M_{xy} = \frac{16 q b^2 \gamma}{\pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \bar{A}_{mn} \left\{ \frac{\lambda_1 V}{m} + \frac{\lambda_2 r V (b/a)}{n} \right. \\ \left. - 2 (b/a) \right\} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \quad (98)$$

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} \\ = - \frac{16 q b}{\pi^3} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \bar{A}_{mn} \left\{ \left(\frac{b}{a} \right) \frac{1}{n} \left[(\beta - 2\gamma) (n \lambda_2 r V - n^2) \right. \right. \\ \left. \left. + \alpha \left(\lambda_1 m \frac{b}{a} V - \frac{m^2 b^2}{a^2} \right) \right] + \gamma n \left[\frac{\lambda_1 V}{m} + \lambda_2 \frac{r V (b/a)}{n} \right. \right. \\ \left. \left. - 2 \left(\frac{b}{a} \right) \right] \right\} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (99)$$

$$Q_y = \frac{\partial M}{\partial y} + \frac{\partial M_{yx}}{\partial x}$$

$$= \frac{-16 q b}{\pi^3} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \bar{A}_{mn} \left\{ \gamma \frac{mb}{a} \left[\frac{\lambda_1 V}{m} + \lambda_2 \frac{rV (b/a)}{n} - 2 \left(\frac{b}{a} \right) \right] \right. \\ \left. + \frac{1}{m} \left[\frac{(n\lambda_2 rV - n^2)}{\alpha} + (\beta - 2\gamma) \left(\lambda_1 m \frac{b}{a} V - m^2 \frac{b^2}{a^2} \right) \right] \right\} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \quad (100)$$

where the following auxiliary relationships were used to reduce the preceding equations to their final form:

$$\left. \begin{aligned} \nu_{xy} &= \nu_{yx} \frac{E_x}{E_y} = \nu_{yx} \alpha^2 \\ \nu_{yx} \alpha &= \beta - 2\gamma \end{aligned} \right\} \quad (101)$$

The stresses in the panel are then determined from the relationship

$$\left. \begin{aligned} \tau_{xz} &= Q_x/c, \tau_{yz} = Q_y/c \\ \sigma_x &= \pm \frac{M_x}{t_f(c+t_f)} - \frac{N_x}{2t_f} \\ \sigma_y &= \pm \frac{M_y}{t_f(c+t_f)} - \frac{N_y}{2t_f} \\ \tau_{xy} &= \pm \frac{M_{xy}}{t_f(c+t_f)} \end{aligned} \right\} \quad (102)$$

and, as mentioned before, the in-plane forces N_y and N_x are taken to be positive if they are compressive.

A computer program AC-7 has been developed to perform the calculations to obtain the normal deflections and internal moments and shears of simply supported honeycomb sandwich panels with orthotropic core and faces subjected to normal pressure and in-plane forces. Tables I and II contain data obtained from AC-7 for the structural elements tested in this program and reported in

volume II of this report. Figure 38 is a plot of the maximum normal deflection for panels with face sheets composed of $[0_2/\pm 45]_C$ laminates and no in-plane forces. These data were obtained from AC-7.

TABLE I. INTERNAL MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED PANELS WITH NO IN-PLANE LOADS*

a/b	Temperature	M_x/qb^2 x/a = 0.5 y/b = 0.5	M_y/qb^2 x/a = 0.5 y/b = 0.5	Q_x/qb x/a = 0.0 y/b = 0.5	Q_y/qb x/a = 0.5 y/b = 0.0
1.0	RT	0.073	0.027	0.396	0.262
	350° F	0.072	0.026	0.396	0.254
2.0	RT	0.094	0.077	0.485	0.408
	350° F	0.104	0.071	0.512	0.395
3.0	RT	0.092	0.104	0.496	0.461
	350° F	0.107	0.099	0.535	0.451

* Laminate orientation - $[0_2/\pm 45]_C$

Material - Narmco 5505

$$\frac{G'_{cx}}{G'_{cy}} = \frac{DQ_x}{DQ_y} = 2.0$$

$$V = \frac{\pi^2 (t_f/c) (1+t_f/c) \psi}{2G_c' (b/c)^2}$$

$$\xi = \frac{t_f c (c+t_f) \psi}{2}$$

$$\psi = \sqrt{E_x E_y / (1 - \nu_{xy} \nu_{yx})}$$

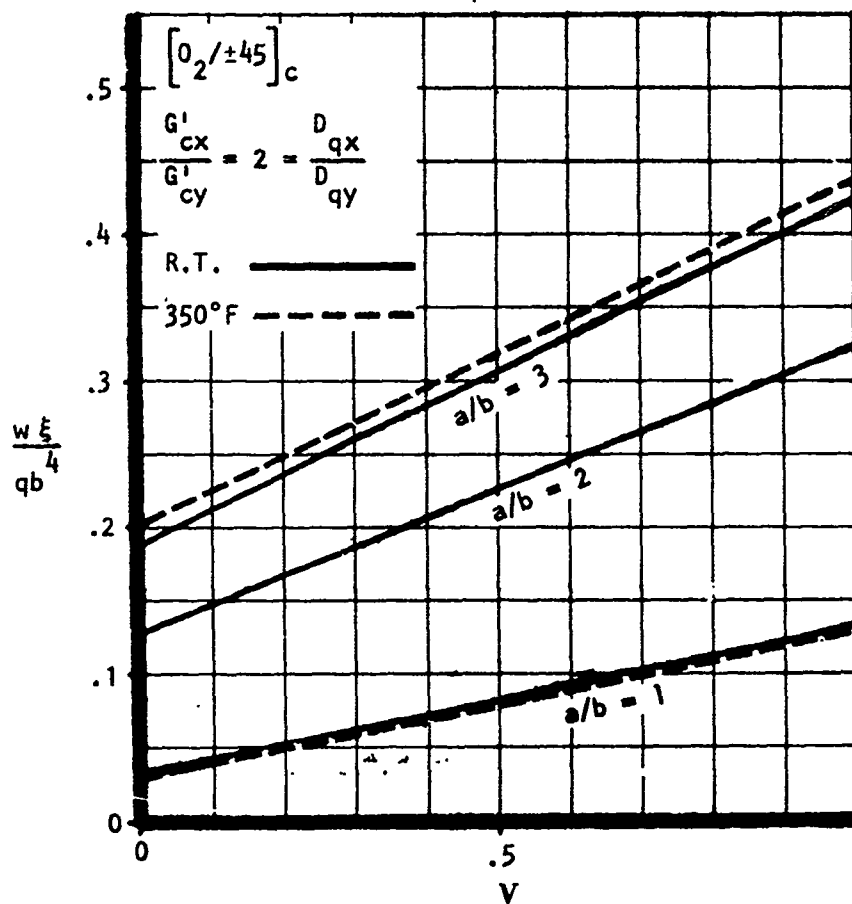
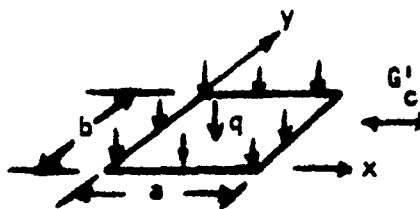


Figure 38. Parameter $w \xi / qb^4$ vs V for Simply Supported Honeycomb Sandwich Panels

TABLE II. MAXIMUM NORMAL DEFLECTION AND INTERNAL MOMENTS AND SHEARS FOR SIMPLY SUPPORTED PANELS* WITH $N_x/(N_x)_{cr} = 0.333$

a/b	Temperature	$w\xi/qb^4$ x/a = 0.5 y/b = 0.5	M_x/qb^2 x/a = 0.5 y/b = 0.5	M_y/qb^2 x/a = 0.5 y/b = 0.5	Q_x/qb x/a = 0.0 y/b = 0.5	$Q_y/2b$ x/a = 0.5 y/b = 0.0
1.0	RT	0.00581	0.107	0.044	0.562	0.356
	350° F	0.00547	0.111	0.040	0.573	0.342
2.0	RT	0.0290	0.121	0.149	0.850	0.594
	350° F	0.0308	0.148	0.145	0.864	0.597

*Laminate orientation - $[0_2/\pm 45]_C$

Material - Narmco 5505

$$\frac{G'_{cx}}{G'_{cy}} = \frac{DQ_x}{DQ_y} = 2.0$$

$$\xi = \sqrt{D_{11} D_{22}}$$

SHEAR STABILITY OF FLAT SANDWICH PANELS

A computerized method has been developed to predict the shear stability allowable of flat sandwich panels with orthotropic face sheets and core. The face sheets are assumed to be similar, and the shear loading is in the plane of the panel. The analytical development is given in reference 5, and only the final governing equations are presented, along with the listing of the computer program and a sample problem solution.

Consider the sandwich panel to lie in the X-Y plane with the Z-axis perpendicular to the face sheets, as shown in figure 39. The critical shear buckling load is given by

$$(N_{xy})_{cr} = K_s \frac{\pi^2 \psi t_f (c+t_f)^2}{2b^2} = (N_s)_{cr} \quad (103)$$

where

$$\psi = \frac{\sqrt{E_x E_y}}{1 - \nu_{xy} \nu_{yx}}$$

and the other terms are defined in the figure.

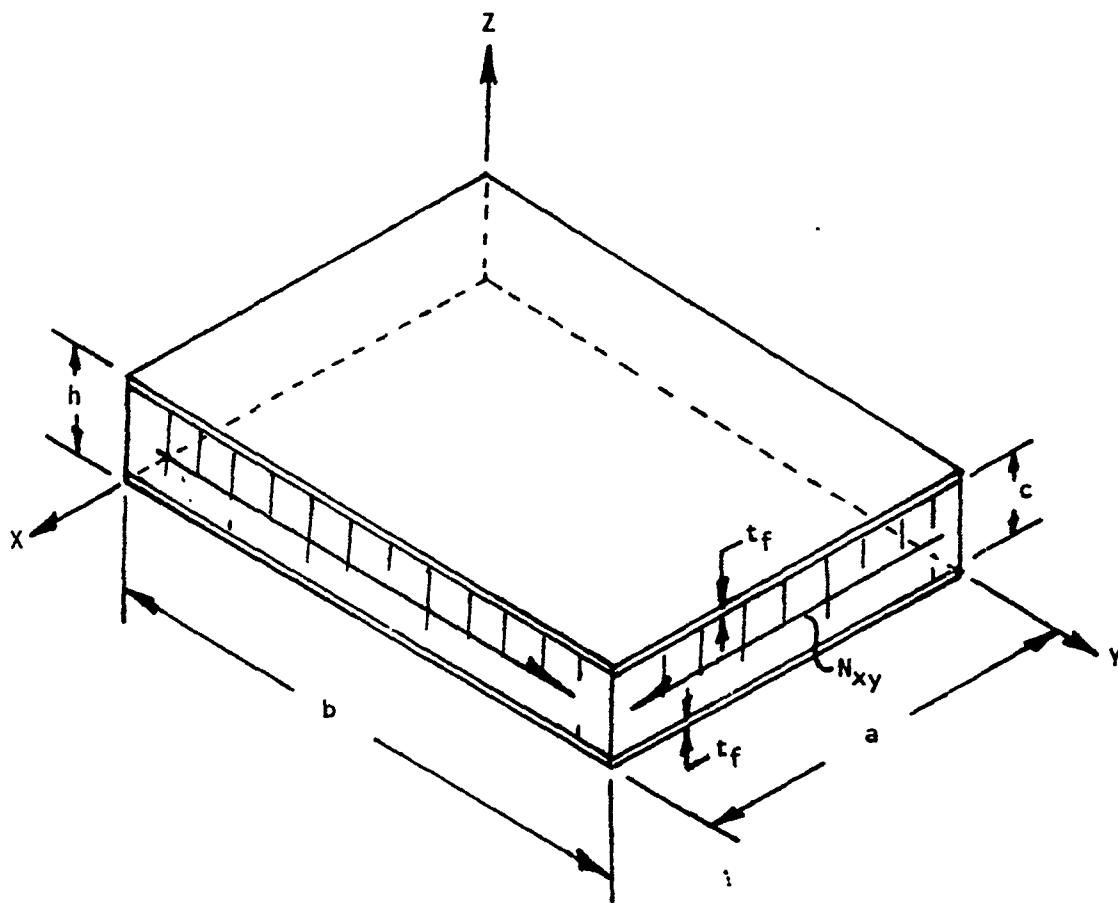


Figure 39. Typical Honeycomb Sandwich Panel Subjected to In-plane Shear Loading

K_S is a constant that is found by the solution of the following governing equation that is derived using energy principles and is

$$\left| \frac{1}{K_S} A_{mn} C_{mn} + \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} H_{mnrs} C_{rs} \right| = 0 \quad (104)$$

where

$$A_{mni} = (2V_{mni} + V_{mn}) \left(\frac{b}{a} \right)^3$$

$$H_{mnrs} = \begin{cases} \frac{32 mnrs}{\pi^2 (m^2 - r^2) (n^2 - s^2)}, & \text{if } m + r \text{ and } n + s \text{ are odd} \\ 0, & \text{otherwise.} \end{cases}$$

$$V_{mni} = \frac{1}{6} \left(\frac{t_f}{c + t_f} \right)^2$$

$$V_{mn} = \frac{K_{mn} + \left(\frac{n^2 a^2}{b^2} S_x + m^2 S_y \right) F_{mn}}{1 + L_{mn} S_x + S_x S_y F_{mn}}$$

$$F_{mn} = 1 - \beta^2 + \gamma K_{mn}$$

$$K_{mn} = \alpha m^4 + 2\beta \frac{m^2 n^2 a^2}{b^2} + \frac{n^4 a^4}{\alpha b^4}$$

$$L_{mn} = \alpha m^2 + \gamma \frac{n^2 a^2}{b^2} + r \left(\frac{n^2 a^2}{\alpha b^2} + \gamma m^2 \right)$$

$$\alpha = \sqrt{\frac{E_x}{E_y}}$$

$$\beta = \alpha \nu_{yx} + 2\gamma$$

$$\gamma = G_{xy} / \psi$$

$$r = \frac{G'_{cx}}{G'_{cy}}$$

$$S_x = \frac{c\pi^2 4t_f}{2a^2 G'_{cx}}$$

$$S_y = rS_x$$

G'_{cx} , G'_{cy} = core shear moduli in the x and y direction, respectively.
Using the contracted notation that

$$i = n + q(m - 1)$$

$$j = s + q(r - 1)$$

where q is the limit of m, n, r, and s, the resulting governing equation becomes

$$\left| \bar{K} A_i C_i + \sum_{j=1}^q H_{ij} C_j \right| = 0 \quad (105)$$

where

$$\bar{K} = \frac{1}{K_s}$$

Dividing through by A_i and rearranging terms results in

$$\bar{K} C_i = - \sum_{j=1}^q \frac{H_{ij} C_j}{A_i} \quad (106)$$

which in matrix notation becomes

$$\left| \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} C \end{bmatrix} - \bar{K} \begin{bmatrix} C \end{bmatrix} \right| = 0$$

where

$$B_{ij} = - \frac{H_{ij}}{A_i} \quad (107)$$

It is noticed that the last equation is in the form of the standard eigenvalue problem. For this type of problem, a modified matrix iteration process (reference 7) will yield the highest eigenvalue, which means that we have the lowest value of K_s since

$$K_s = \frac{1}{K}$$

To facilitate the solution for $\bar{K}$, it is pointed out in reference 5 that the matrix $[B]$ can be separated into even and odd terms of C_{mn} , i.e., $[B]$ even for $m + n$ even and $[B]$ odd for $m + n$ odd. Then, $[B]$ even and $[B]$ odd can each be solved independently for K_s even and K_s odd, and the minimum of these two values would be the same value, K_s , that would result if the original $[B]$ matrix were utilized to find K_s . This procedure greatly reduces computing time.

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SECTION V

STIFFENED SKIN CONSTRUCTION

CONSTITUTIVE RELATIONSHIPS

The analysis in this section for an N-ply flat sheet stiffened with Z or hat stiffeners parallels the work performed by Lackman and Ault (reference 6), but is extended to account for a heterogeneous sheet rather than the homogeneous sheet treated by these authors. We now seek to analyze the typical stiffened sheet shown in figure 40 when the coordinate origin is located in the midplane of the sheet.

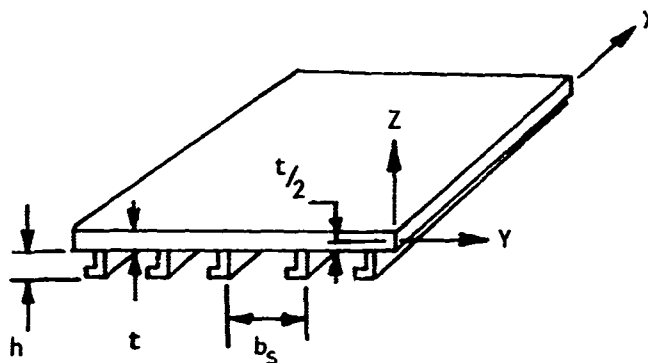


Figure 40. Typical Stiffened Sheet

From equation 5 we have the following relationship between the stress and displacement components of the i^{th} ply of the stiffened sheet shown in figure 40:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = [\bar{Q}_{ij}]_i \begin{Bmatrix} \frac{\partial u_o}{\partial x} - z \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial v_o}{\partial y} - z \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial v_o}{\partial x} + \frac{\partial u_o}{\partial y} - 2z \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

And if we further consider each sheet to be composed of ply orientations which are symmetric relative to its midsurface, then we have from equation 7 the following relationship between the sheet in-plane forces:

$$\begin{Bmatrix} N_x^f \\ N_y^f \\ N_{xy}^f \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \partial u_0 / \partial x \\ \partial v_0 / \partial y \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}$$

where

$$A_{ij} = \sum_1^N \left(\bar{Q}_{ij}^i \right) (h_{i+1} - h_i)$$

and $\bar{Q}_{ij}$ is defined in equation 2.

The axial force per unit width, N_x , along the X axis of the stiffened sheet is obtained from the relationship

$$b_s N_x = \int_{-b_s/2}^{b_s/2} \left\{ \underbrace{\int_{-(h_s + t/2)}^{-t/2} \sigma_x dz}_{\text{stiffener}} + \int_{-t/2}^{t/2} \sigma_x dz \right\} dy = N_x^s + b_x N_x^f \quad (108)$$

when the N_x^f and N_x^s denote the force per unit width carried by face and stiffener, respectively. If we now consider N_x^s is of the form

$$N_x^s = K_1 \frac{\partial u_0}{\partial x} + K_2 \frac{\partial^2 w}{\partial x^2} \quad (109)$$

then, by combining equations 109 and 7 with equation 108, we obtain, after integrating,

$$N_x = \left(A_{11} + \frac{K_1}{b_s} \right) \frac{\partial u_o}{\partial x} + A_{12} \frac{\partial v_o}{\partial y} + A_{16} \left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{\partial y} \right) + \frac{K_2}{b_s} \frac{\partial^2 w}{\partial x^2} \quad (110)$$

Noting that in the Y direction only the sheet carries the N_y and N_{yx} in-plane loads, we obtain from equation 7

$$\begin{aligned} N_y = N_y^f &= A_{12} \frac{\partial u_o}{\partial x} + A_{22} \frac{\partial v_o}{\partial y} + A_{26} \left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \right) \\ N_{xy} = N_{xy}^f &= A_{16} \frac{\partial u_o}{\partial x} + A_{26} \frac{\partial v_o}{\partial y} + A_{66} \left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \right) \end{aligned} \quad (111)$$

From equation 7b, we have the following relationship between the moment components of the sheet and the curvature of the stiffened sheet, namely:

$$\begin{Bmatrix} M_x^f \\ M_y^f \\ M_{xy}^f \end{Bmatrix} = - \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial x^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

where

$$D_{ij} = \sum_{i=1}^N \frac{\bar{Q}_{ij}}{3} \left(h_{i+1}^3 - h_i^3 \right) \quad (i)$$

If we further consider the bending moment component in the X direction carried by the stiffener to be of the form

$$M_x^s = C_1 \frac{\partial u_0}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2} \quad (112)$$

then we find

$$\begin{aligned} b_s M_x &= \int_{-b_s/2}^{b_s/2} \left\{ \underbrace{\int_{-(h_s + t/2)}^{t/2} \sigma_x z dz}_{\text{stiffener}} + \int_{-t/2}^{t/2} \sigma_x z dz \right\} dy \\ &= M_x^s + b_s M_x^f \end{aligned} \quad (113)$$

Thus, the bending moment components for the stiffened sheet become

$$\begin{aligned} M_x &= \frac{1}{b_s} \left(C_1 \frac{\partial u_0}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2} \right) - \left(D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 w}{\partial y^2} \right. \\ &\quad \left. + 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \right) \end{aligned} \quad (114)$$

$$M_y = - \left(D_{12} \frac{\partial^2 w}{\partial x^2} + D_{22} \frac{\partial^2 w}{\partial y^2} + 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \right) = M_y^f \quad (115)$$

$$M_{xy} = - \left(D_{16} \frac{\partial^2 w}{\partial x^2} + D_{26} \frac{\partial^2 w}{\partial y^2} + 2D_{66} \frac{\partial^2 w}{\partial x \partial y} \right) = M_{xy}^f \quad (116)$$

If we invoke the condition that no additional in-plane forces N_x , N_y , or N_{xy} are introduced by the plate displacement components u_0 , v_0 , and w , then by setting the in-plane force components equal to zero, we find

$$\begin{aligned} \frac{\partial v_0}{\partial y} &= \frac{K'_2 \left[A_{12} - \frac{A_{16}A_{26}}{A_{66}} \right] \frac{\partial^2 w}{\partial x^2}}{\left[K'_1 + A_{11} - \frac{A_{16}^2}{A_{66}} \right] \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right] - \left[A_{12} - A_{16} \frac{A_{26}}{A_{66}} \right]^2} \\ &= \bar{\alpha} \frac{\partial^2 w}{\partial x^2} \end{aligned} \quad (117)$$

$$\begin{aligned} \frac{\partial u_0}{\partial x} &= \frac{K'_2 \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right] \frac{\partial^2 w}{\partial x^2}}{- \left[K'_1 + A_{11} - \frac{A_{16}^2}{A_{66}} \right] \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right] + \left[A_{12} - A_{16} \frac{A_{26}}{A_{66}} \right]^2} \\ &= \bar{\beta} \frac{\partial^2 w}{\partial x^2} \end{aligned} \quad (118)$$

where

$$K'_2 = K_2/b_s$$

$$K'_1 = K_1/b_s$$

$$\bar{\alpha} = -\Delta \left[A_{12} - \frac{A_{16}A_{26}}{A_{66}} \right]$$

$$\bar{\beta} = \Delta \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right]$$

$$\Delta = K_2' / \left\{ \left[K_1' + A_{11} - A_{16}^2 / A_{66} \right] \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right] + \left[A_{12} - \frac{A_{16} A_{26}}{A_{66}} \right]^2 \right\}$$

Substitution of equation 118 into 114 yields

$$M_x = \left(\frac{C_1 \bar{\beta}}{b_s} + \frac{C_2}{b_s} - D_{11} \right) \frac{\partial^2 w}{\partial x^2} - D_{12} \frac{\partial^2 w}{\partial y^2} - 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \quad (119)$$

If the plate is now subjected to an in-plane compression force $\bar{N}_x$ and if we neglect the changes in the in-plane forces due to the plate's buckling displacements, the governing equilibrium equation for the plate shown in figure 18 is obtained from equation 26 as

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \bar{N}_x \frac{\partial^2 w}{\partial x^2} \quad (120)$$

Substitution of equations 115, 116, and 119 into equation 120 yields

$$\begin{aligned}
 D_1 \frac{\partial^4 w}{\partial x^4} + 4 D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2 (D_{12} + 2 D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} \\
 + 4 D_{26} \frac{\partial^4 w}{\partial x \partial y^3} + D_{22} \frac{\partial^4 w}{\partial y^4} = -\bar{N}_x \frac{\partial^2 w}{\partial x^2} :
 \end{aligned}
 \quad (121)$$

where

$$D_1 = (-C'_1 \bar{\beta} - C'_2 + D_{11})$$

$$C'_1 = C_1/b_s$$

$$C'_2 = C_2/b_s$$

By comparing equations 121 and 27c, it can be seen that the solution to any anisotropic, laminated, stiffened plate can be obtained from known solutions of equation 27c simply by substituting D_1 for D_{11} . For example, for the case of a simply supported wide-column construction, we take w in the form

$$w = A \sin \frac{\pi x}{L} \quad (122)$$

and we find by substituting the chosen form of w into equation 12 that the critical buckling load is

$$\left(\bar{N}_x \right)_{cr} = \frac{\pi^2 D_1}{L^2} \quad (123)$$

i.e., the classical solution has been modified by replacing D_{11} with D_1 .

ZEE STIFFENERS

To utilize equation 121 for Z-stiffened sheets, the values of C_1' , C_2' , K_1' , and K_2' must be established. Thus, we now consider the typical Z-stiffener shown in figure 41. Referring to equation 108, we see that

$$\begin{aligned}
 N_x^S = & \int_{-\left(\frac{t_s}{2} + f_s\right)}^{-t_s/2} \int_{-\left(h_s + \frac{t}{2} - t_{s1}\right)}^{-t/2} \sigma_x dz dy + \int_{-t_s/2}^{t_s/2} \int_{-\left(h_s + \frac{t}{2}\right)}^{-t/2} \sigma_x dz dy \\
 & + \int_{t_s/2}^{\frac{t_s}{2} + d_s} \int_{-\left(\frac{t}{2} + t_{s1}\right)}^{-t/2} \sigma_x dz dy
 \end{aligned} \quad (124)$$

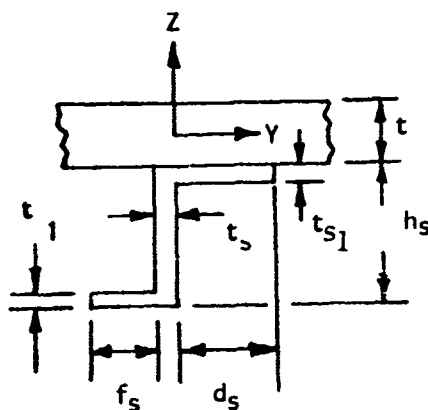


Figure 41. Typical Zee Stiffener

With the assumption that the stiffener only resists loading in its stiffening direction, we see from equation 5 that

$$\sigma_x = E_x^S \left[\frac{\partial u_0}{\partial x} - z \frac{\partial^2 w}{\partial x^2} \right] \text{ for } -\left(\frac{t}{2} + h_s\right) \leq z \leq -\frac{t}{2} \quad (125)$$

where E_x^S is the modulus of the stiffener in its stiffening direction.

Combining equations 124 and 125 and performing the indicated integration results in

$$N_x^s = K_1 \frac{\partial u_0}{\partial x} + K_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$\begin{aligned} K_1 &= E_x^s \left(f_s t_{s1} + t_s h_s + t_{s1} d_s \right) \\ K_2 &= E_x^s \left[f_s t_{s1} \left(h_s + \frac{t}{2} - \frac{t_{s1}}{2} \right) + \frac{t_s h_s}{2} (h_s + t) \right. \\ &\quad \left. + \frac{d_s t_{s1}}{2} (t_{s1} + t) \right] \end{aligned} \quad (126)$$

In a similar manner, we find that

$$M_x^s = \int_{-\frac{b_s}{2}}^{\frac{b_s}{2}} \int_{-\frac{t}{2} + h_s}^{-t/2} \sigma_x z dz dy = C_1 \frac{\partial u_0}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$\begin{aligned} C_1 &= -E_x^s \left\{ f_s t_{s1} \left(h_s + \frac{t}{2} - \frac{t_{s1}}{2} \right) + \frac{h_s t_s}{2} (h_s + t) \right. \\ &\quad \left. + \frac{d_s t_{s1}}{2} (t + t_{s1}) \right\} \end{aligned} \quad (127)$$

$$\begin{aligned}
C_2 = & - E_x^s \left\{ t_{s1} f_s \left[h_s \left(h_s + t - t_{s1} \right) + t_{s1} \left(\frac{t_{s1}}{3} - \frac{t}{2} \right) + \frac{t^2}{4} \right] \right. \\
& + t_{s1} d_s \left(\frac{t^2}{4} + \frac{t t_{s1}}{2} + \frac{t_{s1}^2}{3} \right) \\
& \left. + h_s t_s \left(\frac{h_s^2}{3} + \frac{h_s t}{2} + \frac{t^2}{4} \right) \right\} \quad (127)
\end{aligned}$$

HAT STIFFENERS

In the case of hat stiffeners, we consider the typical stiffener shown in figure 42. With the use of figure 42, we find that

$$\begin{aligned}
N_x^s = & \int_{-(f_s + d_s + t_s)}^{-(f_s + t_s)} \int_{-(t/2 + t_s)}^{-t/2} \sigma_x dz dy + \int_{(f_s + t_s)}^{-f_s} \int_{-(t/2 + h_s)}^{-t/2} \sigma_x dz dy \\
& + \int_{-f_s}^{f_s} \int_{-(t/2 + h_s)}^{-(t/2 + h_s - t_s)} \sigma_x dz dy + \int_{f_s}^{(f_s + t_s)} \int_{-(t/2 + h_s)}^{-t/2} \sigma_x dz dy \\
& + \int_{(f_s + t_s)}^{(f_s + t_s + d_s)} \int_{-(t/2 + h_s)}^{t/2} \sigma_x dz dy \quad (128)
\end{aligned}$$

Substitution of equation 125 into 128 and performing the indicated integration results in

$$N_x^s = K_1 \frac{\partial u_o}{\partial x} + K_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$\begin{aligned} K_1 &= 2E_x^s t_s (d_s + h_s + f_s) \\ K_2 &= E_x^s t_s \left[d_s (t_s + t) + h_s (h_s + t) \right. \\ &\quad \left. + 2 f_s \left(h_s + \frac{t}{2} - \frac{t_s}{2} \right) \right] \end{aligned} \quad (129)$$

Also

$$\int_{-\frac{b_s}{2}}^{\frac{b_s}{2}} \int_{-(t/2 + h_s)}^{-t_h} \sigma_x z \, dz \, dy = C_1 \frac{\partial u_o}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$\begin{aligned}
 C_1 &= -t_s E_x^s \left[d_s (t_s + t) + h_s (h_s + t) \right. \\
 &\quad \left. + 2 f_s \left(\frac{t}{2} + h_s - \frac{t_s}{2} \right) \right] \\
 C_2 &= -2 t_s E_x^s \left\{ d_s \left(\frac{t_s^2}{3} + \frac{t t_s}{2} + \frac{t^2}{4} \right) + f_s \left[h_s (h_s + t - t_s) \right. \right. \\
 &\quad \left. \left. + t_s \left(\frac{t_s}{3} - \frac{t}{2} \right) + \frac{t^2}{4} \right] + h_s \left[\frac{t^2}{4} + \frac{t h_s}{2} + \frac{h_s^2}{3} \right] \right\} \quad (130)
 \end{aligned}$$

SECTION VI

MINIMUM WEIGHT ANALYSIS

GENERAL

A computer program, AC-12, was developed to calculate D_1 for hat- and Z-stiffened sheets, as well as the buckling parameter $N_x L^2$ of equation 115 for wide columns.

For the case of axial compression, the program equates the local buckling loads of the stiffened plate elements to insure that these elements fail simultaneously. Then, by equating this local panel strength to the panel strength in general instability, one may obtain a minimum weight concept for a given load requirement according to the simultaneous failure mode hypothesis for minimum weight structures.

The local buckling allowables are obtained from the following considerations:

1. For elements with three sides simply supported and one of the unloaded sides free, the buckling load can be obtained from

$$N_x = k_7 t_s^3 / f_s^2 \quad (131)$$

for Z-stiffeners with k_7 obtained by solving the transcendental equation presented in reference 6.

2. For elements with all four sides simply supported, we have from equation 37 that

$$N_x = k_f t_f^3 / b_s^2 \quad (152)$$

$$N_x = k_s t_s^3 / h_s^2$$

where

$$k_f = \frac{\pi^2 E_x^f}{6 \left(1 - \nu_{xy}^f \nu_{yx}^f\right)} \left[\left(\frac{E_y^f}{E_x^f}\right)^{1/2} + 2 \frac{G_{xy}^f}{E_x^f} \left(1 - \nu_{xy}^f \nu_{yx}^f\right) + \nu_{yx}^f \right]$$

$$k_s = \frac{\pi^2 E_x^s}{6 \left(1 - \nu_{xy}^s \nu_{yx}^s\right)} \left[\left(\frac{E_y^s}{E_x^s}\right)^{1/2} + 2 \frac{G_{xy}^s}{E_x^s} \left(1 - \nu_{xy}^s \nu_{yx}^s\right) + \nu_{yx}^s \right]$$

and the symbols s and f denote stiffener and face sheet, respectively.

ZEE STIFFENERS

For simultaneous buckling of the Z-stiffened elements, we require

$$k_f \frac{t_s^2}{f_s^2} = k_s \frac{t_s^2}{h_s^2} \quad (133)$$

or

$$\left(f_s/h_s\right) = \left(k_f/k_s\right)^{1/2} = \beta_1$$

while for simultaneous buckling of the face and stiffener elements, we require

$$\left(t_s/h_s\right) \left(b_s/t_f\right) = \left(\frac{E_x^s}{E_x^f} \cdot \frac{k_f}{k_s}\right)^{1/2} = \alpha_1 \quad (134)$$

Equating the applied load in the stiffener to the allowable stiffener load yields

$$\frac{\bar{N}_x E_x^s t_s}{E_x^f t_f + E_x^s \frac{(2f_s + h_s)}{b_s} t_s} = \frac{k_s t_s^3}{h_s^2} \quad (135)$$

where $\bar{N}_x$ is the applied compressive force to the panel. With the use of these relationships, we find

$$h_s = b_s \left\{ \left(\frac{\bar{N}_x E_x^s b_s^2}{k_s \alpha_1^2 t_f^2} - E_x^f t_f \right) / \left[(2\beta_1 + 1) \alpha_1 E_x^s t_f \right] \right\}^{1/2}$$

$$t_s = \alpha_1 h_s t_f / b_s \quad (136)$$

$$f_s = \beta_1 h_s$$

For a given applied force $\bar{N}_x$ and b_s , computer program AC-12 utilizes the relations of equation 136 to calculate $(N_x L^2)_{\text{allowable}}$, and the optimum configuration is

$$\frac{(N_x L^2)}{L^2} = \bar{N}_x$$

HAT STIFFENERS

In a similar manner, we find for hat stiffeners that $f_s \approx h_s/2$ and

$$h_s = b_s \left\{ \left(\frac{N_x E_x^s b_s^2}{k_s \alpha_1^2 t_f^2} - t_f E_x^f \right) / \left[(\beta_2 + 1.5) 2 \alpha_1 E_x^s t_f \right] \right\}^{1/2}$$

$$f_s = h_s/2 \quad (137)$$

$$t_s = \alpha_1 t h_s / b_s$$

$$d_s = \beta_2 h_s$$

Using these relationships, program AC-12 calculates the desired properties for the hat-stiffened sheet in a manner identical to the Z-stiffened sheet.

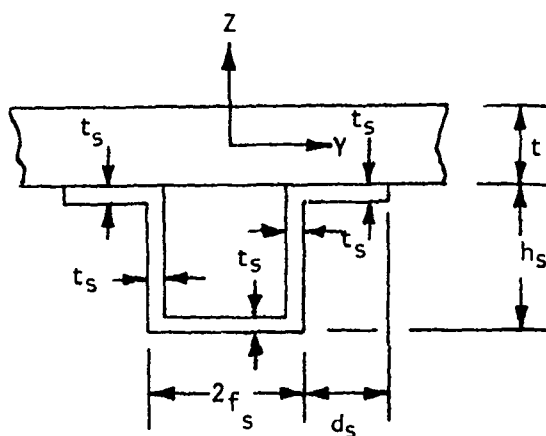


Figure 42. Typical Hat Stiffener

SECTION VII

THERMOELASTIC RELATIONSHIPS

In this analysis, it is assumed that the elastic and thermal response of each ply of an N-ply laminate is known. The quantities specifying the elastic and thermal response of a ply may be obtained from the data presented in volume I. The thermoelastic stress-strain relation for the i^{th} layer in a state of plane stress (stress components normal to the L-T plane are zero) is given by

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = \begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12} & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix}_i \begin{Bmatrix} \epsilon_L - \alpha_L \Delta T \\ \epsilon_T - \alpha_T \Delta T \\ \gamma_{LT} \end{Bmatrix}_i$$

where

$$\left. \begin{aligned} T_{11} &= E_L / (1 - \nu_{LT} \nu_{TL}) = E_L T_{22} / E_T \\ T_{12} &= \nu_{TL} E_L / (1 - \nu_{LT} \nu_{TL}) = \nu_{LT} E_T (1 - \nu_{LT} \nu_{TL}) \\ T_{33} &= G_{LT} \\ \gamma_{LT} &= 2 \epsilon_{LT} \end{aligned} \right\} \quad (138)$$

Figure 43 shows the laminate in question.

Here, ν_{TL} denotes the magnitude of the strain in the longitudinal direction due to a unit strain in the transverse direction; ν_{LT} denotes the magnitude of the strain in the transverse direction due to a unit strain in the longitudinal direction.

To obtain the stress-strain relations for a lamina for the case where the applied stress makes an angle θ_i to the natural lamina axis, equation 138 must be subjected to a coordinate transformation. The case under consideration is illustrated in figure 44.

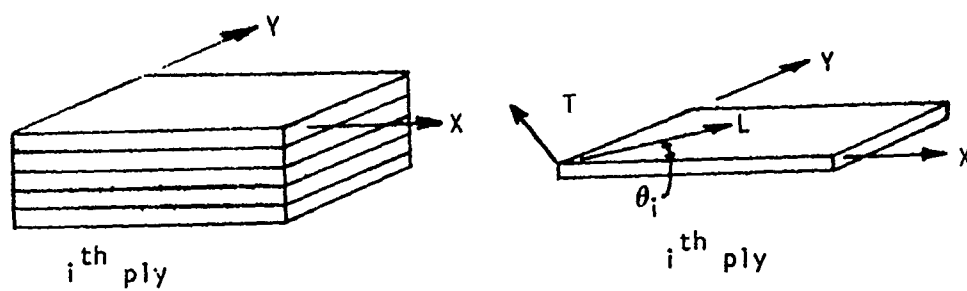


Figure 43. Coordinates of i^{th} Ply in Natural Lamina and Laminate Systems

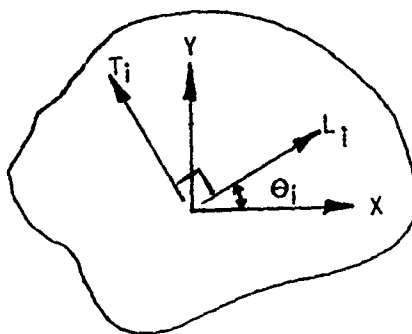


Figure 44. Relationship Between Laminae and Laminate Coordinate Systems

The stress-strain relations for the typical i^{th} ply of the laminate shown in figure 43 is assumed to be of the form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = \begin{bmatrix} B \end{bmatrix}_i \begin{Bmatrix} \epsilon_x - \alpha_x \Delta T \\ \epsilon_y - \alpha_y \Delta T \\ \epsilon_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}_i \quad (139)$$

$$\text{or} \quad \{\sigma\}_i = [B]_i \{\epsilon\}_i - [B]_i \{\alpha\}_i \Delta T$$

where the matrix $[B]_i$ of order 3×3 is yet to be determined. It is well known that the ply stress and strain components transform from one coordinate system (X, Y, Z) to another coordinate system (L, T, Z) according to the transformation law of a tensor of order two (reference 1). Hence,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = \begin{bmatrix} D \end{bmatrix}_i \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i, \quad \begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \epsilon_{LT} \end{Bmatrix}_i = \begin{bmatrix} D \end{bmatrix}_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix}_i, \quad (140)$$

$$\begin{Bmatrix} \alpha_L \\ \alpha_T \\ \frac{\alpha_{LT}}{2} \end{Bmatrix}_i = \begin{bmatrix} D \end{bmatrix}_i \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \frac{\alpha_{xy}}{2} \end{Bmatrix}_i$$

where

$$[D]_i = \begin{bmatrix} m_i^2 & n_i^2 & 2 m_i n_i \\ n_i^2 & m_i^2 & -2 m_i n_i \\ -m_i n_i & m_i n_i & m_i^2 - n_i^2 \end{bmatrix} \quad (141)$$

and $n_i = \sin \theta_i, \quad m_i = \cos \theta_i$

Substituting equation 140 into 138 and performing the appropriate matrix manipulations, we find that $[B]_i$ of equation 139 is

$$[B]_i = [D]_i^{-1} [T]_i [D] \quad (142)$$

By performing the indicated multiplication for equation 142, it is found that the elements of the $[B]_i$ matrix are

$$\begin{aligned} B_{11} &= T_{11} m^4 + 2 (T_{12} + 2 T_{33}) m^2 n^2 + T_{22} n^4 \\ B_{12} &= (T_{11} + T_{22} - 4 T_{33}) m^2 n^2 + T_{12} (m^4 + n^4) = B_{21} \\ B_{13} &= (T_{11} - T_{12} - 2 T_{33}) n m^3 + (T_{12} - T_{22} + 2 T_{33}) n^3 m = B_{31} \\ B_{22} &= T_{11} n^4 + 2 (T_{12} + 2 T_{33}) n^2 m^2 + T_{22} m^4 \\ B_{23} &= (T_{11} - T_{12} - 2 T_{33}) n^3 m + (T_{12} - T_{22} + 2 T_{33}) n m^3 = B_{32} \\ B_{33} &= (T_{11} + T_{22} - 2 T_{12} - 2 T_{33}) n^2 m^2 + T_{33} (m^4 + n^4) \end{aligned} \quad (143)$$

We will now consider a laminate consisting of n plies with natural axes making various angles θ_i , with respect to the reference (X, Y) axes of the laminate. We impose the restriction that the plies are arranged symmetrically with respect to the center plane of the laminate. Because of this restriction, there is no coupling between in-plane loading and out-of-plane deformations.

The laminate plate shown in figure 43 is assumed to be subjected to applied stresses (in-plane) and temperature changes (uniform) which do not result in curvature of the plies. For a laminate of thickness t , the stress resultants are defined by

$$N_x = \int_{-t/2}^{t/2} \sigma_x dz, \quad N_y = \int_{-t/2}^{t/2} \sigma_y dz, \quad N_{xy} = \int_{-t/2}^{t/2} \sigma_{xy} dz \quad (144)$$

Integrating equation 139 across the laminate thickness and assuming the stress components to be constant across each lamina and the strain components not to vary with z , we obtain

$$\{N\} = [\bar{B}] \{\epsilon\} - \Delta T \{C\} \quad (145)$$

where

$$\begin{aligned} N_x &= \sum_{i=1}^n \sigma_{x_i} h_i, & N_y &= \sum_{i=1}^n \sigma_{y_i} h_i, & N_{xy} &= \sum_{i=1}^n \sigma_{xy_i} h_i \\ \bar{B}_{jk} &= \sum_{i=1}^n (B_{jk})_i h_i \\ C_x &= \sum_{i=1}^n (\alpha_x B_{11} + \alpha_y B_{12} + \alpha_{xy} B_{13})_i h_i \end{aligned} \quad (146)$$

$$C_y = \sum_{i=1}^n \left(\alpha_x B_{21} + \alpha_y B_{22} + \alpha_{xy} B_{23} \right)_i h_i \quad (146)$$

$$C_{xy} = \sum_{i=1}^n \left(\alpha_x B_{31} + \alpha_y B_{32} + \alpha_{xy} B_{33} \right)_i h_i$$

and h_i denotes the thickness of the i th ply.

The coefficients of thermal expansion α_x^c , α_y^c , α_{xy}^c for the n -ply laminate are obtained from equation 145 by setting the applied in-plane resultants equal to zero. Hence,

$$\begin{Bmatrix} \alpha_x^c \\ \alpha_y^c \\ \alpha_{xy}^c \end{Bmatrix} = \{\alpha^c\} = \frac{\{\epsilon\}}{\Delta T} = [\bar{B}]^{-1} \{C\} \quad (147)$$

Then the general stress-strain relation for an n -layered laminate subjected to extensional deformation takes on the form

$$\{N\} = [\bar{B}]\{\epsilon\} - [\bar{B}]\{\alpha\} \Delta T = [\bar{B}] \{ \epsilon - \alpha^c \Delta T \} \quad (148)$$

Therefore, for any combination of applied in-plane loadings and uniform temperature changes, the corresponding strain field for the laminate is obtained from equation 148.

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} = \{\epsilon\} = [\bar{B}]^{-1} \{N\} + \Delta T \{\alpha^c\} = \{\epsilon\}_i, \text{ for } i = 1, 2, \dots, N \quad (149)$$

To calculate the stresses in the i^{th} ply due to the strain field of equation 149 for all plies, equation 138 is employed; hence,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = [B]_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} - [B]_i \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_i \Delta T \quad (150)$$

These stresses are in the direction of the laminate reference axes X, Y. The stresses in each ply with respect to the natural axes of the ply are obtained from

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = [T]_i [D]_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} - [T]_i [D]_i \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_i \Delta T \quad (151)$$

For the case when only a uniform temperature change is present, we find from equations 149 and 151 that the i^{th} ply stresses become

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = [T]_i [D]_i \left[\begin{Bmatrix} \alpha_x^c \\ \alpha_y^c \\ \alpha_{xy}^c \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_i \right] \Delta T \quad (152)$$

or the ply stresses in its natural coordinate system are proportional to the differences between the laminate coefficients of thermal expansion, as defined by equation 147, and the ply coefficients of thermal expansion.

A computer program, AC-40, has been written to perform these calculations for $\{\alpha^c\}$ for any crossplied laminates. The program listing and sample data and output sheets are presented in another section of volume III. Correlations of this theory with available test data and typical design curves are described in volume I of this report.

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SECTION VIII JOINT ANALYSIS

SINGLE-LAP ADHESIVE JOINTS

The derivation of the single-lap shear joint stress distribution equation for relatively flexible adhesive layers is shown in reference 8 and will not be repeated here.

The following assumptions were used:

1. The laminate and adhesive are isotropic and elastic.
2. The laminate deformation is cylindrical.
3. The adhesive is relatively flexible.
4. The transverse normal strain and shear strains in the laminate are negligible in comparison with those of the adhesive.

The results are

$$\bar{f}_a^s = \frac{N_x}{4L_a} \left[\beta(1 + 3\alpha) \frac{\cosh \frac{2\beta x}{L_a}}{\sinh \beta} + 3(1 - \alpha) \right]$$

where

$$\beta = \frac{L_a}{t} \left[\frac{2(1 - \nu_{xy}^2) G_a t}{E_x t_a} \right]^{1/2}$$

$$\alpha = \frac{1}{1 + 2\sqrt{2} \tanh \left[\frac{L_a \sqrt{3(1 - \nu_{xy}^2) N_x}}{2\sqrt{2} E_x t} \right]}$$
(153)

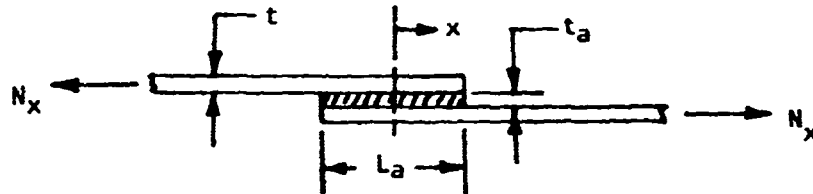


Figure 45. Typical Single Lap Joint

The axial stress distribution is also given in the reference but will not be presented here since it was not used in our analysis. The maximum shear stress occurs at $x = \pm L_a/2$:

$$\bar{f}_a^s = \frac{1}{4} [\beta(1+3\alpha) \coth \beta + 3(1-\alpha)] \frac{N_x}{L_a}$$
(154)

CONTINUOUS TAPER SCARF JOINT

The derivation of the continuous taper scarf joint stress distribution is presented. The following assumptions are made:

1. The adhesive is isotropic and elastic.
2. The adhesive thickness is small.
3. The joint is everywhere in a plane state of stress.
4. The stress state is not a function of y .

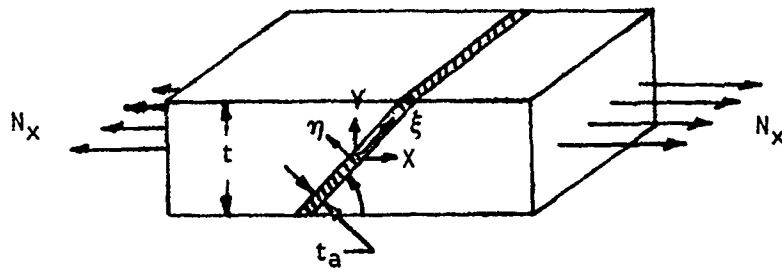


Figure 46. Typical Scarf Joint

From assumption 2 we see that for compatibility the strains at the bondline in the ξ direction for both the laminate and adhesive must be equal.

$$\epsilon_{\xi}^a = \epsilon_{\xi} \quad (155)$$

From assumption 4 we see that

$$\sigma_{\eta}^a = \sigma_{\eta} = \sigma_x \sin^2 \phi$$

Also $\tau_{\xi\eta} = \sigma_x \sin \phi \cos \phi$

$$\sigma_{\xi} = \sigma_x \cos^2 \phi$$

The constitutive equations yield

$$\epsilon_{\xi}^a = \frac{1}{E_a} (\sigma_{\xi}^a - \nu_a \sigma_{\eta}^a) = \frac{1}{E_a} (\sigma_{\xi}^a - \nu_a \sigma_x \sin^2 \phi) \quad (156a)$$

$$\epsilon_{\xi} = \frac{1}{E_x} (\sigma_{\xi} - \nu_{xy} \sigma_{\eta}) = \frac{\sigma_x}{E_x} (\cos^2 \phi - \nu_{xy} \sin^2 \phi) \quad (156b)$$

Substituting equations 156a and 156b into equation 155 yields

$$\sigma_{\xi}^a = \sigma_x \left[\nu_a \sin^2 \phi + \frac{E_a}{E_x} (\cos^2 \phi - \nu_{xy} \sin^2 \phi) \right]$$

The shearing stress is

$$\tau_{\xi\eta} = \sigma_x \sin \phi \cos \phi$$

Using Tresca's maximum shear criteria, we find that

$$\bar{f}_{a,\max}^s = \left[\left(\frac{\sigma_{\xi}^a - \sigma_{\eta}^a}{2} \right)^2 + \left(\tau_{\xi\eta}^a \right)^2 \right]^{1/2}$$

$$= \frac{N_x}{t} \left\{ \frac{1}{4} \left[\sin^2 \phi \left(1 - \nu_a + \nu_{xy} \frac{E_a}{E_x} \right) - \frac{E_a}{E_x} \cos^2 \phi \right]^2 + \sin^2 \phi \cos^2 \phi \right\}^{1/2} \quad (157)$$

DOUBLE-LAP ADHESIVE JOINT

The derivation of the double-lap shear joint stress distribution is presented. The assumptions are:

1. The adherend and adhesive are isotropic and elastic.
2. The adherend displacements, u_1 and u_2 , are small and a function of x only.
3. Only shearing deformation in the adhesive is considered.

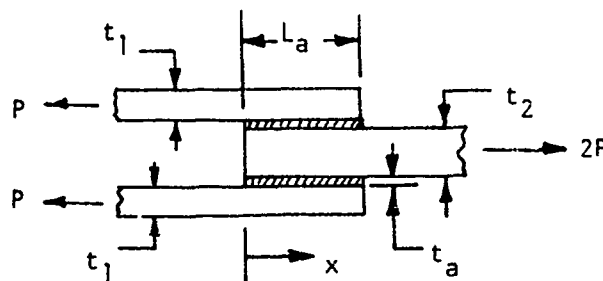


Figure 47. Typical Double Lap Joint

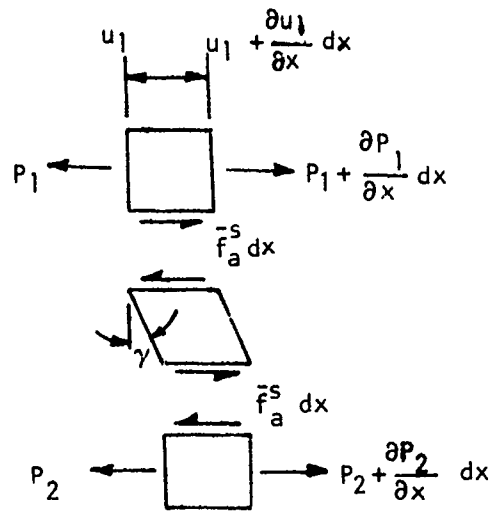


Figure 48. Typical Load Transfer Through Adhesive

Considering assumptions 2 and 3,

$$\gamma = \frac{u_2 - u_1}{t_a} \quad (158)$$

and the constitutive equation

$$\bar{f}_a^s = G_a \gamma = \frac{G_a}{t_a} (u_2 - u_1) \quad (159)$$

where

G_a = shear modulus of the adhesive.

Equilibrium for the upper element requires

$$\bar{f}_a^s dx + \frac{\partial P_1}{\partial x} dx = 0 = \left[\frac{G_a}{t_a} (u_2 - u_1) + \frac{\partial P_1}{\partial x} \right] dx \quad (160)$$

The incremental elongation of the adherends can be expressed as

$$\left(\frac{\partial P_1}{\partial x} dx \right) \frac{dx}{t_1 E_1} = \frac{\partial u_1}{\partial x} dx \quad (161a)$$

$$2 \left(\frac{\partial P_2}{\partial x} dx \right) \frac{dx}{t_2 E_2} = \frac{\partial u_2}{\partial x} dx \quad (161b)$$

where E_1 and E_2 are the Young's moduli of extension of the adherends. Noting assumption 2, we see that total derivatives can be used. Differentiating equation 160 and using equations 161a and 161b, we obtain

$$\frac{d^2 P_1}{dx^2} + \frac{G_a}{t_a} \left[2 \frac{dP_2}{dx} - \frac{dP_1}{dx} \right] = 0$$

Equilibrium for the joint requires that the condition $P = P_1 + P_2$ be satisfied; hence,

$$\frac{d^2 P_1}{dx^2} - \frac{G_a}{t_a} \left[\frac{1}{t_1 E_1} + \frac{2}{t_2 E_2} \right] P_1 = - \frac{2 G_a}{t_a t_2 E_2} P \quad (162)$$

The procedure followed is that of reference 9. However, there is a difference in the signs of the last two terms. The solution of this differential equation is

$$P_1 = A \sinh \omega x + B \cosh \omega x + \beta P \quad (163)$$

$$\omega = \sqrt{\frac{G_a}{t_a} \left(\frac{1}{t_1 E_1} + \frac{2}{t_2 E_2} \right)}$$

$$\frac{1}{\beta} = \left(1 + \frac{t_2 E_2}{2 t_1 E_1} \right)$$

A, B = undetermined coefficients.

The boundary conditions are

$$P_1(0) = P \quad (164a)$$

$$P_1(L_a) = 0 \quad (164b)$$

Solving equation 163 for the undetermined coefficients yields

$$P_1 = \left[-\frac{(1-\beta) \cosh \omega L_a + \beta}{\sinh \omega L_a} \sinh \omega x + (1-\beta) \sinh \omega x + \beta \right] P$$

From equation (160)

$$\bar{f}_a^S = -\frac{dP_1}{dx} = \left[\frac{(1-\beta) \cosh \omega L_a + \beta}{\sinh \omega L_a} \cosh \omega x - (1-\beta) \sinh \omega x \right] \omega P \quad (165)$$

Practical considerations show that

$$\omega L_a > 5$$

$$0 < \beta < 1$$

Thus we see that

$$(1-\beta) \cosh L_a \gg \beta$$

$$\cosh \omega L_a \cong \sinh \omega L_a$$

and equation (165) becomes

$$\bar{f}_a^S \cong (1-\beta) \left[\frac{\cosh \omega L_a}{\sinh \omega L_a} \cosh \omega x - \sinh \omega x \right] \omega P$$

This equation attains a maximum at either $x = 0$ or $x = L_a$, depending on β . Evaluating at $x = 0$, we get

$$\bar{f}_a^S(0) = (1-\beta)\omega P$$

Evaluating equation (165) at $x = L_a$

$$\begin{aligned}\bar{f}_a^S(L_a) &= \left[(1-\beta) \left(\frac{\cosh^2 \omega L_a - \sinh^2 \omega L_a}{\sinh \omega L_a} \right) + \beta \coth \omega L_a \right] \omega P \\ &= \left[(1-\beta) \frac{1}{\sinh \omega L_a} + \beta \coth \omega L_a \right] \omega P\end{aligned}$$

Therefore,

$$\bar{f}_a^S(L_a) \cong \beta \omega P$$

Letting $P = N_x/2$, the equations become

$$\bar{f}_a^S(0) = (1-\beta) \omega L_a \cdot \frac{N_x}{2L_a} = (1-\beta) \omega N_x/2 \quad (166a)$$

$$\bar{f}_a^S(L_a) = \beta \omega L_a \frac{N_x}{2L_a} = \beta \omega N_x/2 \quad (166b)$$

The maximum shear stress becomes

$$\bar{f}_{a, \max}^S = \beta' \omega L_a \frac{N_x}{2L_a} = \beta' \omega N_x/2 \quad (167)$$

where β' is the larger of β or $1-\beta$

DOUBLE-LAP VARIABLE STIFFNESS ADHESIVE JOINT

The derivation of the double-lap two-adhesive joint, as shown by figure 49, is presented in the following paragraphs.

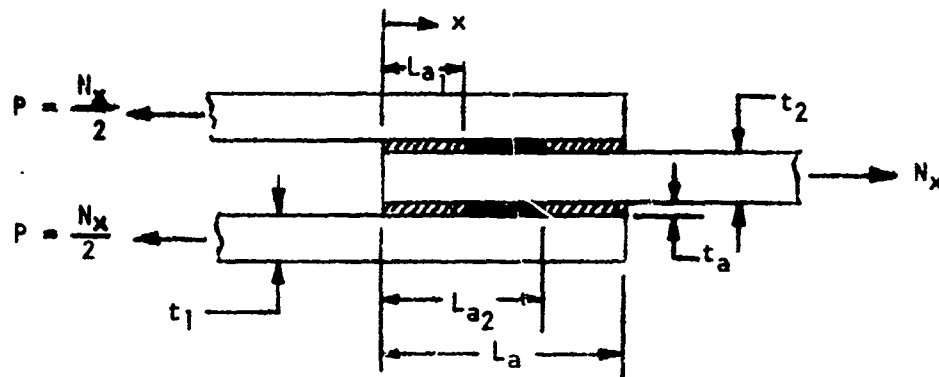


Figure 49. Typical Variable Stiffness Adhesive Double Lap Joint

The procedure used here will essentially follow Szepe's (reference 9). However, his equations were modified in reference 10 and developed in the previous section; thus, from equation 162 and figure 58 we have

$$P_{1,xx} - \frac{G_a}{t_a} \left(\frac{1}{t_1 E_1} + \frac{2}{t_2 E_2} \right) P_1 = -2 \frac{G_a}{t_a t_2 E_2} P \quad (168)$$

The solution to this equation is

$$P_1 = A \sinh \omega x + B \cosh \omega x + \beta P \quad (169)$$

where

$$\omega^2 = \frac{G_a}{t_a} \left(\frac{1}{t_1 E_1} + \frac{2}{t_2 E_2} \right)$$

$$\beta = \frac{2 t_1 E_1}{2 t_1 E_1 + t_2 E_2}$$

Considering the three regions designated as one, two, and three, respectively, as

$$0 \leq x \leq L_{a1}; \quad L_{a1} \leq x \leq L_{a2}; \quad L_{a2} \leq x \leq L_a$$

we get

$$\begin{aligned} P_{11} &= A_1 \sinh \omega_1 x + B_1 \cosh \omega_1 x + \beta P & 0 \leq x \leq L_{a1} \\ P_{12} &= A_2 \sinh \omega_2 x + B_2 \cosh \omega_2 x + \beta P & L_{a1} \leq x \leq L_{a2} \\ P_{13} &= A_3 \sinh \omega_1 x + B_3 \cosh \omega_1 x + \beta P & L_{a2} \leq x \leq L_a \end{aligned} \quad (170)$$

noting that the shearing stress in the adhesive is given by

$$\begin{aligned} \bar{f}_a^s &= P_{1,x} \\ \bar{f}_a^{s1} &= \omega_1 A_1 \cosh \omega_1 x + \omega_1 B_1 \sinh \omega_1 x \\ \bar{f}_a^{s2} &= \omega_2 A_2 \cosh \omega_2 x + \omega_2 B_2 \sinh \omega_2 x \\ \bar{f}_a^{s3} &= \omega_1 A_3 \cosh \omega_1 x + \omega_1 B_3 \sinh \omega_1 x \end{aligned} \quad (171)$$

The unknown coefficients $A_1, A_2, A_3, B_1, B_2,$ and B_3 can be obtained from the known boundary conditions,

$$\begin{aligned} P_{11}(0) &= P \\ P_{11}(L_{a1}) &= P_{12}(L_{a1}) \end{aligned} \quad (172)$$

$$\frac{f_a^{s1}(L_{a1})}{G_{a1}} = \frac{f_a^{s2}(L_{a1})}{G_{a2}}$$

$$P_{12}(L_{a2}) = P_{13}(L_{a2})$$

$$\frac{f_a^{s2}(L_{a2})}{G_{a2}} = \frac{f_a^{s3}(L_{a2})}{G_{a1}}$$

$$P_{13}(L_a) = 0$$

Substitution yields the following equations

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ S_{11} & C_{11} & -S_{21} & -C_{21} & 0 & 0 \\ C_{11} & S_{11} & -\frac{\omega_1}{\omega_2}C_{21} & -\frac{\omega_1}{\omega_2}S_{21} & 0 & 0 \\ 0 & 0 & S_{22} & C_{22} & -S_{12} & -C_{12} \\ 0 & 0 & C_{22} & S_{22} & -\frac{\omega_2}{\omega_1}C_{12} & -\frac{\omega_2}{\omega_1}S_{12} \\ 0 & 0 & 0 & 0 & S_{13} & C_{13} \end{bmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{Bmatrix} = P \begin{Bmatrix} 1-\beta \\ 0 \\ 0 \\ 0 \\ 0 \\ -\beta \end{Bmatrix} \quad (173)$$

The solutions of these equations are

$$A_1 = \begin{Bmatrix} -S_{11} S_{21} + \frac{\omega_1}{\omega_2} C_{11} C_{21} \\ -S_{11} C_{21} + \frac{\omega_1}{\omega_2} C_{11} S_{21} \\ 0 \end{Bmatrix}^T \begin{Bmatrix} A_2 \\ B_2 \\ A_3 \end{Bmatrix} \quad (174)$$

$$B_1 = (1 - \beta) P \quad (175)$$

$$\begin{Bmatrix} A_2 \\ B_2 \\ A_3 \end{Bmatrix} = -\frac{P}{\lambda} \begin{bmatrix} \lambda_{22} \lambda_{33} - \frac{\omega_1}{\omega_2} \lambda_{21} \lambda_{23} & -\lambda_{12} \lambda_{33} & \lambda_{12} \lambda_{23} \\ \frac{\omega_1}{\omega_2} \lambda_{22} \lambda_{23} - \lambda_{21} \lambda_{33} & \lambda_{11} \lambda_{33} & -\lambda_{11} \lambda_{23} \\ \frac{\omega_1}{\omega_2} (\lambda_{21}^2 - \lambda_{22}^2) & \frac{\omega_1}{\omega_2} (\lambda_{12} \lambda_{22} - \lambda_{11} \lambda_{21}) & \lambda_{11} \lambda_{22} - \lambda_{21} \lambda_{12} \end{bmatrix} \begin{Bmatrix} 1-\beta \\ \beta C_{12} \\ \beta S_{12} \end{Bmatrix} \quad (176)$$

$$B_3 = -\frac{S_{13}}{C_{13}} A_3 - \frac{\beta}{C_{13}} P \quad (177)$$

where

$$S_{11} = \sinh \omega_1 L_{a_1}$$

$$C_{11} = \cosh \omega_1 L_{a_1}$$

$$S_{12} = \sinh \omega_1 L_{a_2}$$

$$C_{12} = \cosh \omega_1 L_{a_2}$$

$$S_{21} = \sinh \omega_2 L_{a1}$$

$$C_{21} = \cosh \omega_2 L_{a1}$$

$$S_{22} = \sinh \omega_2 L_{a1}$$

$$C_{22} = \cosh \omega_2 L_{a2}$$

$$S_{13} = \sinh \omega_1 L_a$$

$$C_{13} = \cosh \omega_1 L_a$$

$$\lambda_{11} = -C_{11} S_{21} + \frac{\omega_1}{\omega_2} S_{11} C_{21}$$

$$\lambda_{12} = -C_{11} C_{21} + \frac{\omega_1}{\omega_2} S_{11} S_{21}$$

$$\lambda_{21} = S_{22} C_{13}$$

$$\lambda_{22} = C_{22} C_{13}$$

$$\lambda_{23} = C_{12} S_{13} - S_{12} C_{13}$$

$$\lambda_{33} = S_{12} S_{13} - C_{12} C_{13}$$

$$\lambda = \lambda_{11} \left(\lambda_{22} \lambda_{33} - \frac{\omega_1}{\omega_2} \lambda_{21} \lambda_{23} \right) - \lambda_{12} \left(\lambda_{21} \lambda_{33} - \frac{\omega_1}{\omega_2} \lambda_{22} \lambda_{23} \right)$$

The loading of the laminate, $P_1(x)$ and $P_2(x)$, where $P_1(x) + P_2(x) = 2P$, and the adhesive $\bar{f}_a^s(x)$ can now be found. To check these equations, we examine the case where a single adhesive is considered, i.e.,

$$P_{11}(x) \text{ for } 0 \leq x \leq L_{a1} ; \quad P_{13}(x) \text{ for } L_{a1} \leq x \leq L_a$$

$$L_{a2} = L_{a1} = L_a/2$$

For this case, the equations reduce to those of reference 3. We examine next the equations for

$$P_{12}(x) \text{ for } 0 \leq x \leq L_a \text{ and } L_{a1} = 0 ; \text{ with } L_{a2} = L_a$$

For this case, the equations also reduce to those of reference 3.

To avoid computational difficulties which occur, the equations can be written in the following form. Let $\tilde{\omega} = \frac{\omega_1}{\omega_2}$.

$$\begin{bmatrix} S_{11} & C_{11} & -S_{21} & -C_{21} & 0 & 0 \\ -C_{11} & -S_{11} & \tilde{\omega}C_{21} & \tilde{\omega}S_{21} & 0 & 0 \\ 0 & 0 & S_{22} & C_{22} & -S_{12} & -C_{12} \\ 0 & 0 & -C_{22} & -S_{22} & C_{12}/\tilde{\omega} & S_{12}/\tilde{\omega} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{13} & C_{13} \end{bmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1-\beta \\ -\beta \end{Bmatrix} \quad (178)$$

Noting the following relationships,

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh (x \pm y) = \sinh x \cdot \cosh y \pm \cosh x \cdot \sinh y$$

$$\cosh (x \pm y) = \cosh x \cdot \cosh y \pm \sinh x \cdot \sinh y$$

we obtain the following:

$$\begin{Bmatrix} A_1 \\ B_1 \end{Bmatrix} = \begin{bmatrix} C_{41} & - (1-\tilde{\omega})C_{11}C_{21} & S_{41} & - (1-\tilde{\omega})C_{11}S_{21} \\ S_{41} & + (1-\tilde{\omega})S_{11}C_{21} & C_{41} & + (1-\tilde{\omega})S_{11}S_{21} \end{bmatrix} \begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} \quad (179a)$$

$$\begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} = \frac{1}{\tilde{\omega}} \begin{bmatrix} C_{42} & + (1-\tilde{\omega})S_{12}S_{22} & -S_{42} & + (1-\tilde{\omega})C_{12}S_{22} \\ -S_{42} & + (1-\tilde{\omega})S_{12}S_{22} & C_{42} & - (1-\tilde{\omega})C_{12}C_{22} \end{bmatrix} \begin{Bmatrix} A_3 \\ B_3 \end{Bmatrix} \quad (179b)$$

$$\left[\begin{array}{c|c} -S_{44} + (1-\tilde{\omega})(S_{11}C_{55} - S_{12}C_{66}) + (1-\tilde{\omega})^2 S_{11}S_{12}S_{24} & \\ \hline S_{13} & \\ \hline \hline C_{44} + (1-\tilde{\omega})(S_{11}S_{55} - C_{12}C_{66}) + (1-\tilde{\omega})^2 S_{11}C_{12}S_{24} & \\ \hline C_{13} & \end{array} \right] \begin{Bmatrix} A_3 \\ B_3 \end{Bmatrix} = P \begin{Bmatrix} \tilde{\omega}(1-\beta) \\ -\beta \end{Bmatrix} \quad (179c)$$

where	$C_{41} = \cosh [(\omega_2 - \omega_1)C_1]$	$S_{41} = \sinh [(\omega_2 - \omega_1)C_1]$
	$C_{42} = \cosh [(\omega_2 - \omega_1)C_2]$	$S_{42} = \sinh [(\omega_2 - \omega_1)C_2]$
	$C_{44} = \cosh [(\omega_2 - \omega_1)(C_2 - C_1)]$	$S_{44} = \sinh [(\omega_2 - \omega_1)(C_2 - C_1)]$
	$C_{55} = \cosh [\omega_2 C_1 - (\omega_2 - \omega_1)C_2]$	$S_{24} = \sinh [\omega_2 (C_2 - C_1)]$
	$C_{66} = \cosh [\omega_2 C_2 - (\omega_2 - \omega_1)C_1]$	$S_{55} = \sinh [\omega_2 C_1 - (\omega_2 - \omega_1)C_2]$

It should be noted that the equation leading to the solution of A_3 , B_3 is still almost singular, and double-precision arithmetic must be used on a computer.

These equations can now be used to determine the loading in the laminate and the shear stress in the adhesive. Figure 50 shows a typical distribution of $\bar{f}_a^s$. The discontinuities of the shearing stress at $x/L_a = 0.25, 0.75$ are violations of equilibrium principles. However, considering the following:

1. only six free constants are available to satisfy boundary conditions,
2. finite element solution of a similar single-lap variable joint using an energy approach (reference 12) also shows an abrupt change in stress,
3. the thinness of the adhesive with respect to the joint members,

it is obvious that this solution is basically correct although the actual stresses would be more like the dotted line shown.

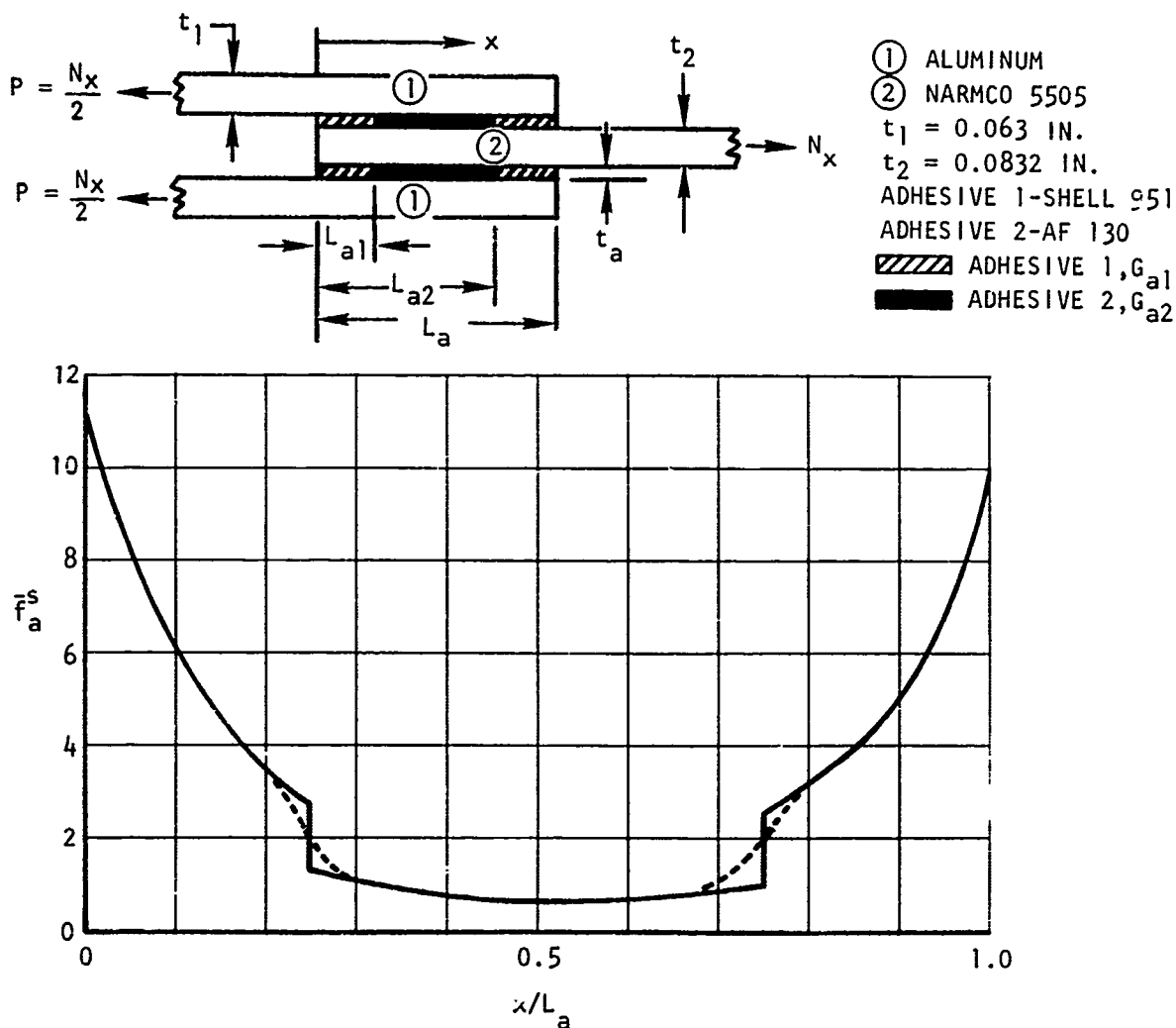


Figure 50. Double-Lap Variable Adhesive - Typical Distribution of $\bar{f}_a^s$ versus x/L_a

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APPENDIX

APPENDIX COMPUTER PROGRAMS

FOREWORD

This appendix contains a number of computer programs that were generated during the theoretical development efforts of this contract. They are quite general in applicability and are presented here separately from the main body of this volume to facilitate their usage. For each program, there are presented an abstract, the input data format, the output data format, and an example problem. The following is a list of the program numbers and titles.

<u>No.</u>	<u>Title</u>
AC-2	Extensional and Flexural Constants for Balanced Symmetrical Laminates of the Form $[0,90/\pm\theta/0,90]_{nS}$
AC-5	H/C Sandwich Panel Stability Under In-Plane Biaxial Loading
AC-7	Analysis of Simply Supported Orthotropic Sandwich Flat Panels Under Pressure or Pressure Plus In-Plane Loads
AC-11	H/C Sandwich Panel Stability for In-Plane Shear Loading
AC-12	Hat- and Z-Stiffened Panel Stability and Stiffness
AC-40	N-Ply Laminate Thermoelastic Analysis

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AC-40 - N-PLY LAMINATE THERMOELASTIC ANALYSIS

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COMPUTER PROGRAM AC-2

AC-2

EXTENSIONAL AND FLEXURAL CONSTANTS FOR
BALANCED SYMMETRICAL LAMINATES OF THE FORM

$$[0,90/\pm\theta/0,90]_{ns}$$

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-2

ABSTRACT

Fortran program deck No. AC-2 is a program to calculate the laminate flexural and extensional constants; the honeycomb (H/C) sandwich parameters α and ψ ; and the magnitudes of the flexural rigidity elements D_{16} and D_{26} relative to the D_{11} and D_{22} rigidities by using, as input data, the average mechanical property values for an individual unidirectional lamina and the geometric properties of the total laminate.

The program is somewhat restricted as to the type of laminate configuration that can be analyzed. The restrictions are:

1. The laminate layup is symmetric relative to its midplane.
2. The layups must be balanced with respect to $+\theta$ and $-\theta$ plies (i.e., there must be a $+\theta$ ply for every $-\theta$ ply).
3. The ply orientations only include one angle θ other than 0 and 90 degrees.
4. The laminate is made up of an even number of identical sets of ply orientations.
5. Every ply is assumed to have the same nominal ply thickness.
6. Every ply is assumed to have the same average mechanical properties.

The method used to find the "exact" flexural constants differs from the current state-of-the-art method in that no summations are involved. The summations are replaced by algebraic total sums. This means that, for a laminate with many plies, the computing time will be shortened considerably. Also, the computer results for a laminate with many plies will be more accurate because of the reduction of roundoff errors.

The "exact" flexural constants mentioned come from the standard state-of-the-art method. The program, however, also solves for the laminate flexural constants that result from the laminate extensional constants. The latter flexural constants are only approximations and consider the rigidities D_{16} and D_{22} to be zero. However, if a large number of plies is considered (i.e., a large number of sets), the "exact" and extensional constant methods will yield virtually the same results.

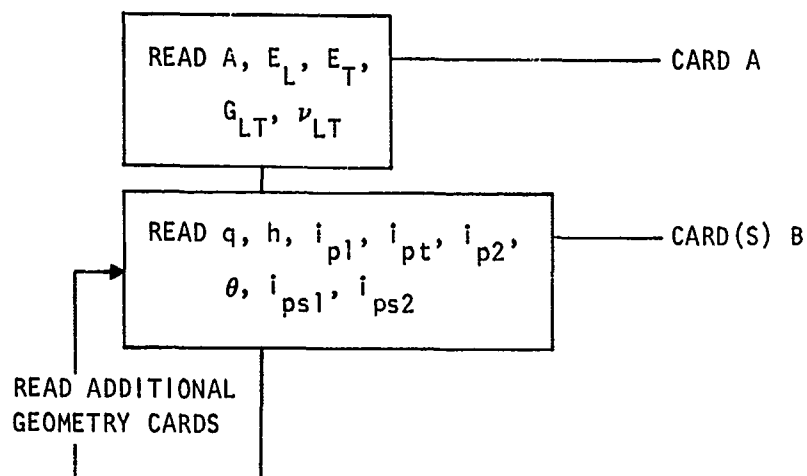
PROGRAM INPUT DATA

The input data are in two phases. The first phase (data card A) consists of giving the computer the average mechanical properties of a single unidirectional lamina. These properties are E_L (longitudinal Young's modulus), E_T (transverse Young's modulus), G_{LT} (Shear modulus), and ν_{LT} (Poisson's ratio). An indicator (A) is also part of the data on the first card. Its purpose is to tell the computer which kind of average mechanical property data are being used, (i.e., $A = 1.0$ for average compression data, $A = 0.0$ for average tension-compression data, and $A = -1.0$ for average tension data). The correct order of the data on the first data card is A, E_L , E_T , G_{LT} , and ν_{LT} . The input format is (5F12.0). An example phase I green sheet is shown following the text in this section. Note that all five input values need decimal points.

The second phase of input data (remaining data cards B) consists of one data card for each laminate configuration (set geometry) to be analyzed. The information needed on each card is IQ (the total number of ply sets), H (the nominal ply thickness), IP1 (the number of inner plies in each set), IPT (the number of $\pm\theta$ plies in each ply set), IP2 (the number of outer plies in each set), THETA (the ply angle associated with IPT), IPS1 (indicator for IP1), and IPS2 (an indicator for IP2). The format for phase II input is (I4, F8.0, 3I4, F12.0, 2I4). This means that the integers are IQ, IP1, IP2, IPS1, and IPS2, and the numbers requiring decimals are H and THETA. An example phase II green sheet is shown in this section following the text. Note that for the integer numbers (no decimals), the format calls for them to be against the right side of the allowed width indicated on the sample green sheets. For integer numbers, a zero is placed by the computer wherever there is a blank on the data card.

The total number of sets indicator IQ must always be an even integer (2, 4, 6) because of the assumed symmetry of the layup. The number IP1 is the total number of 0 or 90 degree plies nearest the centerline of symmetry of the laminates as shown in figure 1. It can be zero. IPT is the number of $\pm\theta$ plies in each set. For example, if IPT = 3, then there are three $\pm\theta$ pairs in each ply set (i.e., six plies). It can be zero. IP2 is like IP1 except that it is the total number of 0 or 90 degree plies in each set that are toward the outside or surface of the laminate. It also can be zero. THETA is the plus and minus angle of the middle plies. The program assumes that for an input of $+\theta$, the $+\theta$ ply starts closest to the centerline. If the opposite is desired, then the THETA value must be input as a minus THETA. IPS1 indicates the angle of the inner plies. If it is zero, then the inner plies of each set are 0 degrees, and if it is 90, then the inner plies are at 90 degrees. It cannot take any value other than 0 or 90. IPS2 is like IPS1 except that it refers to the outer plies in each set.

The input data flow chart for AC-2 is:



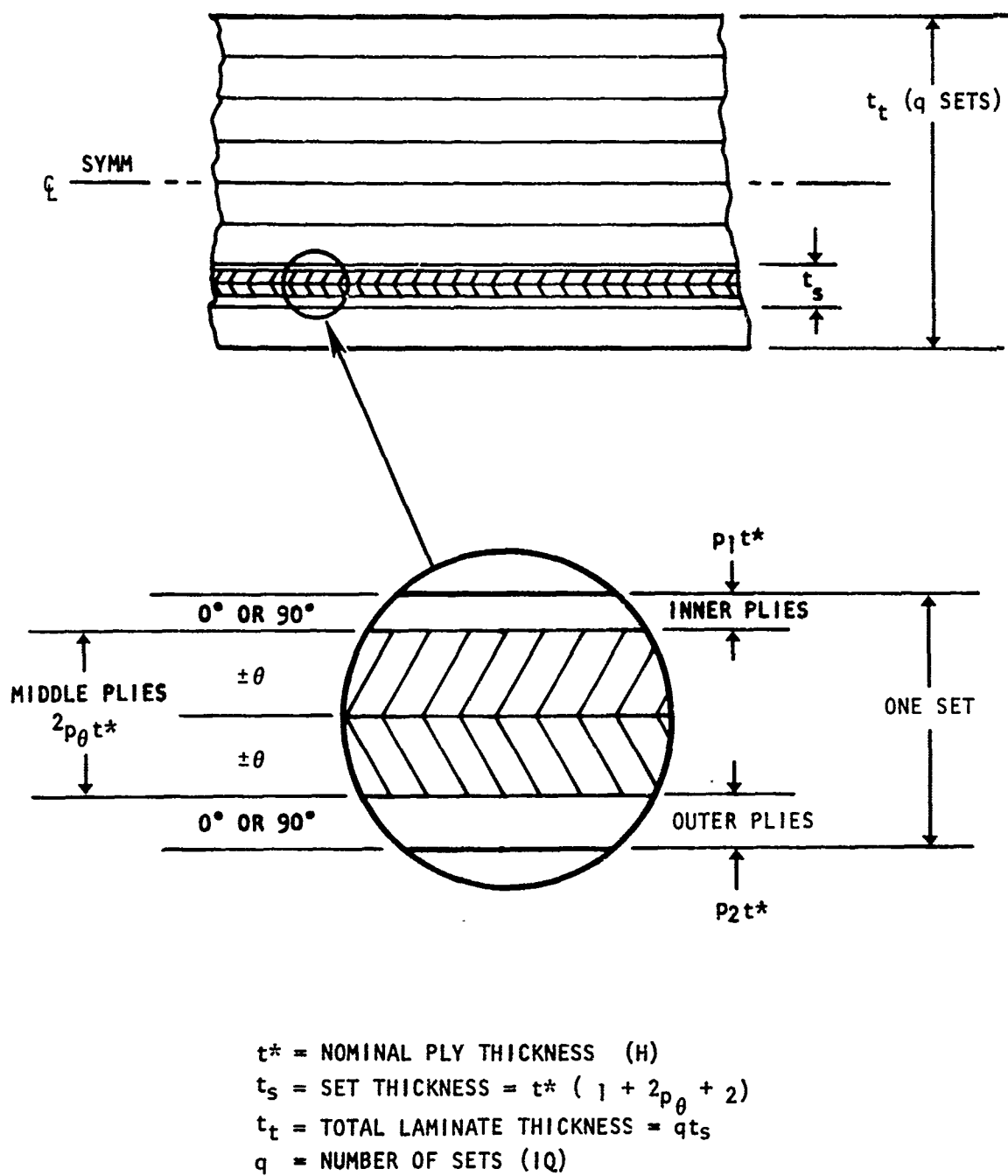


Figure 1. AC2 Laminate Mechanical Properties Program Figure for Ply Set Orientation

```

C
C --- PROGRAM AC2 ... EXTENSIONAL AND FLEXURAL CONSTANTS FOR BALANCED, ---
C SYMMETRICAL LAMINATES OF THE FORM (0/+THETA/90)NS
C
C
C THE INPUT DATA IS AS FOLLOWS.....
C THE FIRST DATA CARD MUST CONTAIN THE LAMINATE ELASTIC CONSTANTS IN
C THE FOLLOWING FORMAT...(5F12.0)
C THE LAMINATE ELASTIC CONSTANTS ARE A,EL,ET,GLT,ULT WHERE.....
C A IS A CONTROL CARD THAT INDICATES WHICH TYPE OF ELASTIC
C CONSTANTS ARE TO BE USED. A=-1.0 IS FOR COMPRESSION
C LOADINGS, A=0.0 IS FOR TENSION-COMPRESSION LOADINGS, AND
C A=1.0 IS FOR TENSION LOADINGS.
C EL IS THE LAMINA LONGITUDINAL YOUNGS MODULUS ...
C ET IS THE LAMINA TRANSVERSE YOUNGS MODULUS
C GLT IS THE LAMINA SHEAR MODULUS
C ULT IS THE LAMINA POISSONS RATIO
C
C THE SECOND AND FOLLOWING DATA CARDS MUST CONTAIN THE SET
C DESCRIPTIONS IN THE FOLLOWING FORMAT...(14,F8.0,3I4,F12.0,2I4)
C THE SET DESCRIPTIONS INCLUDE Q,H,IP1,IPT,IP2,THETA,IPS1, AND IPS2
C WHERE.....
C IQ IS THE TOTAL NUMBER OF SETS IN THE LAMINATE ( IT MUST ALWAYS
C BE AN EVEN INTEGER)
C H IS THE NOMINAL PLY THICKNESS (FLOAT. PT.)
C IP1 IS THE NUMBER OF INNER PLYS IN EACH IQ SET IN THE ZERO OR
C DEGREE DIRECTION DEPENDING ON IPS1 (INTEGER)
C IPT IS THE NUMBER OF MIDDLE PLYS IN EACH Q SET IN THE PLUS AND
C MINUS THETA DEGREE DIRECTION (INTEGER)
C IP2 IS THE NUMBER OF OUTER PLYS IN EACH IQ SET IN THE ZERO OR
C DEGREE DIRECTION DEPENDING ON IPS2 (INTEGER)
C THETA IS THE PLUS AND MINUS ANGLE OF THE P2 PLYS (FLOAT. PT.)
C IPS1 IS AN INDICATOR TO DETERMINE IF THE ANGLES ASSOCIATED WITH
C P1 ARE ZERO OR NINETY DEGREES. IF IPS1 IS ZERO THEN THESE ANGLES
C ARE ZERO DEGREES. IF IPS1 IS 90 THEN THESE ANGLES ARE 90 DEGREES
C IPS2 IS THE SAME AS IPS1 EXCEPT THAT IPS2 REFERS TO P2
C
C
C DIMENSION D(6),QPR(4,3),DC(3)
C READ(5, 999) A,EL,ET,GLT,ULT
C 999 FORMAT(5F12.0)
C 2000 READ(5,1000) IQ,H,IP1,IPT,IP2,THETA,IPS1,IPS2
C 1000 FORMAT(14,F8.0,3I4,F12.0,2I4)
C
C IF(A)7,5,3
C 3 WRITE(6,4)
C 4 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR TENSION LOADINGS')
C GO TO 14
C 5 WRITE(6,6)
C 6 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS')
C GO TO 14
C 7 WRITE(6,8)
C 8 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR COMPRESSION LOADINGS')
C 14 WRITE(6,9) EL,ET,GLT,ULT
C 9 FORMAT('0 EL='E12.5,5X'ET='E12.5,5X'GLT='E12.5,5X'MULT='F8.4//')
C
C
C Q=IQ
C P1=IP1
C PT=IPT
C P2=IP2

```

```

RK=P1+2*PT+P2
XN=O*RK
R1=XN/2-RK
R2=R1+P1
R3=XN/2-P2
T=XN*H
IPT2=2*PT
C
I=0
PN=P1
R=R1
100 I=I+1
V=O*PN/8*(2*(PN+1)*(2*PN+6*R-3*XN-2)+3*(2*R-XN-1)**2+RK*(Q+2)*(3*
1PN+6*R-3*XN+RK*(1+Q))+1)
IF(I-2) 110,120,130
110 DC(I)=V
PN=2*PT
R=R2
GO TO 100
120 DC(I)=V
PN=P2
R=R3
GO TO 100
130 DC(I)=V
C
C=COSD(THETA)
S=SIND(THETA)
C2S2=(C*S)**2
S2=S**2
C2=C**2
C2TS2=C2*S2
C4=C**4
S4=S**4
C4PS4=C4+S4
C
UTL=(ET/EL)*ULT
PSI=1-ULT*UTL
Q11=EL/PSI
Q12=Q11*UTL
Q22=ET/PSI
Q66=GLT
C
QPR(1,2)=Q11*C4+2*(Q12+2*Q66)*C2S2+Q22*S4
QPR(2,1)=Q12
QPR(2,2)=(Q11+Q22-4*Q66)*C2S2+Q12*(C4+S4)
QPR(2,3)=Q12
QPR(3,2)=Q11*S4+2*(Q12+2*Q66)*C2S2+Q22*C4
QPR(4,1)=Q66
QPR(4,2)=(Q11+Q22-2*Q12-2*Q66)*C2S2+Q66*(C4+S4)
QPR(4,3)=Q66
IF(IPS1-90) 140,145,145
140 QPR(1,1)=Q11
QPR(3,1)=Q22
GO TO 150
145 QPR(1,1)=Q22
QPR(3,1)=Q11
150 IF(IPS2-90) 155,160,160
155 QPR(1,3)=Q11
QPR(3,3)=Q22
GO TO 165
160 QPR(1,3)=Q22
QPR(3,3)=Q11

```

```

165 CONTINUE
C
AN=.6666666*(T/XN)**3.0
DO 170 I=1,4
  O(I)=0.0
  DO 170 J=1,3
    170 D(I)=D(I)+QPR(I,J)*DC(J)*AN
C
  Q16T=(Q11-Q12-2*Q66)*C**3*S+(Q12-Q22+2*Q66)*C*S**3
  Q26T=(Q11-Q12-2*Q66)*C*S**3+(Q12-Q22+2*Q66)*C**3*S
C
  CON=2*QPT*(T/XN)**3*(-PT+XN/2-RK/4*(Q+2))
  D(5)=Q16T*CON
  D(6)=Q26T*CON
C
  RATIO1=D(5)/D(1)
  RATIO2=D(5)/D(2)
  RATIO3=D(6)/D(1)
  RATIO4=D(6)/D(2)
C
  A1=P1*QPR(1,1)+2*PT*QPR(1,2)+P2*QPR(1,3)
  A2=P1*QPR(2,1)+2*PT*QPR(2,2)+P2*QPR(2,3)
  A3=P1*QPR(3,1)+2*PT*QPR(3,2)+P2*QPR(3,3)
  A6=P1*QPR(4,1)+2*PT*QPR(4,2)+QPR(4,3)*P2
C
  UYX=A12/A1
  UXY=A12/A2
  GXY=A6/RK
  EX=(A1*A2-A12**2)/A2/RK
  EY=(A1*A2-A12**2)/A1/RK
  ALF=(EX/EY)**.5
  PSII=(EX*EY)**.5/(1.0-UXY*UYX)
  R=1-UXY*UYX
  A11=FX*(1.0+EY/EX+2.0*UYX)/B
  A12=GXY-FX*UYX/B
C
  WRITE(6,575)
  575 FORMAT('0...SET DESCRIPTION')
  WRITE(6,576)IO,H,T
  576 FORMAT('0 THERE ARE'14 ' SETS OF PLIES WITH A NOMINAL PLY THICK
  INFSS OF'F7.4' INCHES.'// THE TOTAL LAMINATE THICKNESS IS'F10.4'
  2 INCHFS.'//)
C
  IF(IPSI-90) 577,579,579
C
  577 WRITE(6,578)IP1
  578 FORMAT(' NUMBER OF INNER PLIES AT 0 DEGREES ='13)
C
  GO TO 585
  579 WRITE(6,580) IP1
  580 FORMAT(' NUMBER OF INNER PLIES AT 90 DEGREES ='13)
C
  585 WRITE(6,590) THETA,IP2
  590 FORMAT(' NUMBER OF MIDDLE PLIES AT (+ AND -)'F7.2' DEGREES ='13)
C
  IF(IPS2-90) 600,620,620
C
  600 WRITE(6,610)IP2
  610 FORMAT(' NUMBER OF OUTER PLIES AT 0 DEGREES ='13//)
  GO TO 635
C
  620 WRITE(6,630)IP2

```

```

630 FORMAT('  NUMBER OF OUTER PLIES AT 90 DEGREES ='13//)
C
635 WRITE(6,64C)
640 FORMAT('O...LAMINATE FLEXURAL CONSTANTS//')
C
      WRITE(6,650) D(1),D(2),D(3),A1,A12,D(4),D(5),D(6),A2,A6
650 FORMAT('D11 ='E12.5,3X'D12 ='E12.5,3X'D22 ='E12.5,7X'A11 ='E12.5,
13X'A12 ='E12.5/'D66 ='E12.5,3X'D16 ='E12.5,3X'D26 ='E12.5,7X'A22 =
2'E12.5,3X'A66 ='E12.5//')
C
      WRITE(6,660)
660 FORMAT('O...RATIOS BETWEEN FLEXURAL CONSTANTS//')
C
      WRITE(6,670) RATIO1,RATIO2,RATIO3,RATIO4
670 FORMAT('D16/D11 ='E12.5,5X'D16/D12 ='E12.5,5X'D26/D11 ='E12.5,5X'D
126/D12 ='E12.5//')
C
      WRITE(6,680)
680 FORMAT('C...LAMINATE EXTENSIONAL CONSTANTS')
C
      WRITE(6,13) EX,EY,GXY,UXY,UYX,ALF,PSI1,A11,A12
13 FORMAT('O'4X'EX='E12.5,5X'EY='E12.5,5X'GXY='E12.5,5X'MUXY='F7.4,5X
1'MUYX='F7.4/' '1X'ALPHA=SQRT(EX/EY)'E12.5,13X'PSI=SQRT(EX+EY)/(1-
2MUYX*MUYX)'E12.5/' '3X'A11=EX*(1+EY/EX+2*MUYX)'E12.5,5X'A12=GXY-
3EX*MUYX/(1-MUYX*MUYX)'E12.5//')
C
      BC=T*0.3/12/B
      D11A=BC*EX
      D12A=UYX*EX*BC
      D22A=BC*EY

      D66A=GXY*B*BC
C
      WRITE(6,700)
700 FORMAT('C...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL
1CONSTANTS//')
C
      WRITE(6,710) C11A,D12A,D22A,D66A
710 FORMAT('D11E ='E12.5,5X'D12E ='E12.5,5X'D22E ='E12.5,5X'D66E ='E12
1.5)
C
      GO TO 2000
      END

```


NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A	Card A	A = Lamina data type (-1.0 = comp, 0.0 = ten-comp, 1.0 = ten.)
13	EL		EL = Longitudinal Young's modulus for one ply
25	ET		ET = Transverse Young's modulus for one ply
37	GLT		GLT = Shear modulus for one ply
49	ULT		ULT = Poisson's ratio for one ply
61			
1	I Q	Card B	IQ = Total number of sets (even integer)
13	I P 1		H = nominal ply thickness (Pl. Pt.)
25	I P T		IP1 = Number of inner plies in each IQ set (integer)
37	T H E T A		IP2 = Number of plus theta plies in each IQ set (integer)
49	I P 8 1		IP2 = Number of outer plies in each IQ set (integer)
61	I P 8 2		THETA = $\pm$ Angle of the middle plies (must be negative if the + theta degree ply is closest to the laminate surface...
1			...Pl. Pt.)
13			IP81 = Angle indicator for the inner plies. Make 0 for 0° and 90 for 90°. (integer can only be 0 or 90)
25			IP82 = Same as IP81 except that it refers to the outer IP2 plies
37			
49			
61			
1			
13			
25			
37			
49			
61			

EXAMPLE PROBLEM

AC-2 LAMINATE FLEXURAL AND EXTENSIONAL PROPERTIES

Consider the three laminates: $[0/\pm 45/0]_S$, $[0_2/\pm 45/90_2]_S$, and $[0/\pm 60]_S$. AC-2 is used to find the elastic and flexural constants from the single-ply properties which, for room temperature, are:

$$\begin{aligned}E_L &= 29.9 \times 10^6 \text{ psi} \\E_T &= 2.71 \times 10^6 \text{ psi} \\G_{LT} &= 0.70 \times 10^6 \text{ psi} \\\nu_{LT} &= 0.210\end{aligned}$$

The data sheets for these three laminates, as well as the output printouts, are shown at the end of this section.

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 0	Card A	A = 0.0 for average tension-compression data)
13	2 9 9 0 0 0 0 0 .		XL
20	2 7 1 0 0 0 0 .		ET
37	7 0 0 0 0 0 .		GLT
49	2 1 0		ULT
61		1 0	
1	2 . 0 0 5 2	Card B	Number of sets
13	1 1 1		IP1
29	4 5 . 0		IP2
37	0 0		Theta
49			Angle of IP1 plies
61		80	Angle of IP2 plies
1		2 0	[0/±45/0]s
13	4 . 0 0 5 2	Card B	
29	2 1 1		
37	4 5 . 0		
49	0 0 9 0		
61		80	
1		3 0	[0 ₂ /±45/90 ₂]2s
13	2 . 0 0 5 2	Card B	
29	1 1 1		
37	6 0 . 0		
49	0 0 0		
61		80	
1		4 0	[0/±60]s

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.299C0E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.0416 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 1
NUMBER OF MIDDLE PLIES AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 0 DEGREES = 1

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12928E 03 D12 = 0.20975E 02 D22 = 0.32043E 02 A11 = 0.78382E 08 A12 = 0.16685E 08
D66 = 0.21746E 02 D16 = -0.76769E 01 D26 = -0.76769E 01 A22 = 0.23784E 08 A66 = 0.17199E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 = -0.59380E-01 D16/D12 = -0.36601E 00 D26/D11 = -0.59380E-01 D26/D12 = -0.36601E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.16669E 08 EY= 0.50580E 07 GXY= 0.42998E 07 MUXY= 0.7015 MUYX= 0.2129
ALPHA=SQRT(EY/EY)= 0.18154E 01 PSI=SQRT(EX+EY)/(1-MUXY*MUYX)= 0.10794E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GX-EY*MUYX/(1-MUXY*MUYX)= 0.12662E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.11756E 03 D12E = 0.25024E 02 D22E = 0.35671E 02 D66E = 0.25795E 02

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 FT= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 4 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.1248 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 2
NUMBER OF MIDDLE PLIES AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 90 DEGREES = 2

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.15476E 04 D12 = 0.45968E 03 D22 = 0.30215E 04 A11 = 0.83824E 08 A12 = 0.17827E 08
D66 = 0.48052E 03 D16 = -0.46062E 02 D26 = -0.46062E 02 A22 = 0.83824E 08 A66 = 0.18599E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 = -0.29764E-01 D16/D12 = -0.10020E 00 D26/D11 = -0.29764E-01 D26/D12 = -0.10020E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.13339E 08 EY= 0.13339E 08 GXY= 0.30998E 07 MUXY= 0.2127 MUYX= 0.2127
ALPHA= SORT(EX/EY)= 0.10000E 01 PSI= SORT(EX*EY/(1-MUXY*MUYX))= 0.13971E 08
A11= EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12= GXY-FX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.22630E 04 D12E = 0.48128E 03 D22E = 0.22630E 04 D66E = 0.50211E 03

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.0312 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 1
NUMBER OF MIDDLE PLIES AT (+ AND -) 60.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 0 DEGREES = 0

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12919E 02 D12 = 0.14606E 02 D22 = 0.43626E 02 A11 = 0.38312E 08 A12 = 0.12513E 08
D66 = 0.14932E 02 D16 = -0.31417E 01 D26 = -0.10155E 02 A22 = 0.38312E 08 A66 = 0.12899E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 = -0.24319E 00 D16/D12 = -0.21509E 00 D26/D11 = -0.78608E 00 D26/D12 = -0.69527E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.11408E 08 EY= 0.11408E 08 GXY= 0.42998E 07 MUXY= 0.3266 MUYX= 0.3266
ALPHA=SQRT(EX/EY)= 0.10000E 01 PSI=SQRT(EX+EY)/(1-MUXY*MUYX)= 0.12771E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GXV-EX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.32322E 02 D12E = 0.10557E 02 D22E = 0.32322E 02 D66E = 0.10882E 02

COMPUTER PROGRAM AC-5

H/C SANDWICH PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING

AC-5

ADVANCED FILAMENTARY COMPOSITE MECHANICAL
PROPERTIES COMPUTER PROGRAM DECK NO. AC-5

ABSTRACT

Fortran program deck No. AC-5 is a program to calculate the general stability of a honeycomb (H/C) panel with orthotropic core and face sheets under biaxial loading. The loading ratio ϕ ($\phi = N_y/N_x$) can be zero, plus, or minus. Thus, AC-5 will handle panels under uniaxial compression, biaxial tension-compression, and biaxial compression-compression. A listing of AC-5 is shown following the text in this section.

PROGRAM INPUT DATA

The input data are shown on the sample data sheets in this section. The data, except for the title card, are read in with a 6F12.0 format per card. The definitions of the input data areas follow:

Card A:

TIT = Any desired one-card title in columns 1-72.
If no title is desired, use a blank card.

Card B:

TNAØB = Total number of (a/b)'s to be considered; maximum of 20.
ALF = $\text{Alpha} = \sqrt{E_x/E_y}$ (from program AC-5).
TNN = Total number of half-waves in the y-direction; no limit.
TNM = Total number of half-waves in the x-direction; no limit.
TNPHI = Total number of load ratios ϕ ; maximum of 20
(from program AC-5).
TNV = Total number of V's; maximum of 20.

Card C:

AI1 = Invariant 1 = $E_x(1 + E_y/E_x + 2 \nu_{yx})/(1 - \nu_{xy}\nu_{yx})$;
can be found from program AC-5.
AI2 = Invariant 2 = $G_{xy} - E_x \nu_{yx}/(1 - \nu_{xy}\nu_{yx})$; can be found
from program AC-5.

P = Orthotropic core shear modulus ratio; G'_{cx}/G'_{cy} .

TNPSI = Total number of ψ ; maximum of 20.

Card(s) D:

AØB(I) = Panel aspect ratios $(a/b)_i$. There are six of these to a card up to a maximum of 20 (i.e., four cards).

Card(s) E:

PHI(I) = Load ratios, $\phi = N_y/N_x$. There are six of these to a card up to a maximum of 20. These cards are to be left out if TNPHI = 0.0.

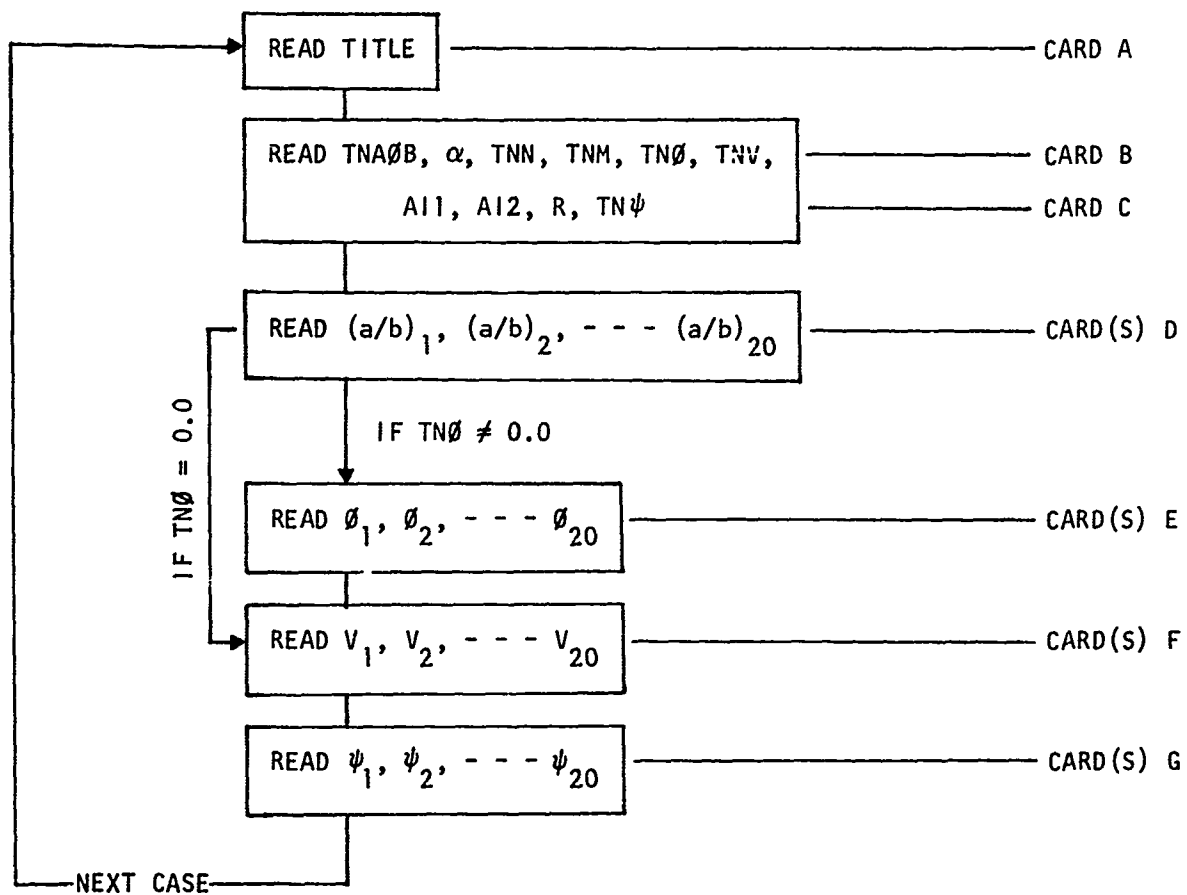
Card(s) F:

V(I) = Core shear parameters, where $V = \pi^2 \psi t_f (c + t_f) / (2G'_{cx} b^2)$. There are six of these to a card up to a maximum of 20.

Card(s) G:

PSI(I) = Modulus ratios found from AC-2 where $\psi = \sqrt{E_x E_y / (1 - \nu_{xy} \nu_{yx})}$. There are six of these to a card up to a maximum of 20.

An input data flow chart is included. Note that additional cases are simply stacked one behind the other.



OUTPUT

The output, as shown in this section, gives the title, the core shear ratio R, gamma, beta, alpha, psi, and V. It gives k_x and k_y for uniaxial compression in the x and y directions, respectively, for zero phi ($\phi = N_y/N_x$) and any other values of phi as given in the input data.

```

C
C PROGRAM AC5 ... H/C PANEL STABILITY UNDER IN-PLANE BIAxIAL LOADING
C
C THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
C A11 = INVARIANT 1 = EX/(1-UMXY*UMYX)*(1+EY/EX+2*UMYX)
C A12 = INVARIANT 2 = GXY - UMYX*EX/(1-UMXY*UMYX)
C ALF = SORT(EX/EY)
C BETA = B = ALF*UMYX + 2*GAMMA
C D11 = EX*TF*C**2/(2.*(1-UMXY*UMYX))
C D22 = D11*EY/EX
C D66 = TF*C**2*GXY/2
C DOX = C*GCX
C DOY = C*GCY
C GAMMA = G = GXY*(1-UMXY*UMYX)/SORT(EX*EY)
C IA = TOTAL NUMBER OF A/B
C IC = TOTAL NUMBER OF HALF WAVELENGTHS IN Y-DIRECTION
C ID = TOTAL NUMBER OF HALF WAVELENGTHS IN X-DIRECTION
C IE = TOTAL NUMBER OF PHI, WHERE PHI = NY/NX
C IF = TOTAL NUMBER OF V WHERE V = P1**2*SORT(D11)*D22/(B**2*DOX)
C PSI = SORT(EX*EY)/(1-UMXY*UMYX)
C R = DOX/DOY
C
C DIMENSION AOB(20), PHI(20), V(20), AK(20), APSI(20), TIT(18),
C IAAK(20), AAKAY(20), AKAY(20)
C 1 READ(5,100) (TIT(I),I=1,18)
C 100 FORMAT(18A4)
C READ(5,110) TNAOB, ALF, TNN, TNM, TNPHI, TNV, A11, A12, R, TNPSI
C 110 FORMAT(6F12.0)
C A=ALF
C IA=TNAOB
C IC=TNN
C ID=TNM
C READ(5,110) (AOB(I),I=1,IA)
C IF=TNPHI
C IF(IE) 108,108,105
C 105 READ(5,110) (PHI(I),I=1,IE)
C 108 IP=TNV
C READ(5,110) (V(I),I=1,IP)
C L2=TNPSI
C READ(5,110) (APSI(I),I=1,L2)
C DO 300 L3=1,L2
C PSI=APSI(L3)
C GAMMA= A11/(2.*PSI) + A12/PSI -.5*A -.5/A
C RFTA= 1.5*A11/PSI + 2.*A12/PSI -1.5*A -1.5/A
C R=RFTA
C G=GAMMA
C WRITE(6,120)
C 120 FORMAT('1 BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROP
C IIC SANDWICH PLATES - FLAT')
C WRITE(6,100) (TIT(I),I=1,18)
C WRITE(6,130) R, G, B, A, PSI
C 130 FORMAT('10.5X'G'1CX/6'CY='F5.2,5X'GAMMA='F8.4,5X'BETA='F8.3,5X'ALP
C IHA='F6.3,5X'PSI='F11.0)
C DO 300 I=1,IP
C WRITE(6,135) V(I)
C 135 FORMAT(14H0 DATA FOR V=F8.4)
C IF(IE) 138,140,138
C 138 WRITE(6,139) (PHI(LL),LL=1,IE)
C 139 FORMAT('126X'PHI='F5.1,6X'PHI='F5.1,6X'PHI='F5.1,6X'PHI='F5.1,6X'
C IPHI='F5.1)
C 140 CONTINUE

```

```

WRITE(6,142)
142 FORMAT('O'2X'A/B'5X'KX'6X'KY'5X'KX(1)/KY(1)'4X'KX(2)/KY(2)'4X'KX(3
1)/KY(3)'4X'KX(4)/KY(4)'4X'KX(5)/KY(5)')
DO 300 J=1,1A
  AB1=ANB(J)
  AB2=AB1**2
  AB3=AB1**3
  AB4=AB1**4
  AAKX= 1.0E+16
  AAKY=AAKX
  IF(IE) 145,150,145
145 DO 148 JI=1,IF
  AAKAY(JI)=AAKX
148 AAK(JI)=AAKX
150 CONTINUE
  DO 250 M=1,1D
  OM=M
  OM2=OM**2
  OM3=OM**3
  OM4=OM2**2
  DO 250 N=1,1C
  ON=N
  ON2=ON**2
  ON3=ON*ON2
  ON4=ON2**2
  A11= (A*OM3/AB3 +OM*ON2*B/AB1)*(-1.)
  A12= 1. + OM2*A*V(I)/AB2 +ON2*G*V(I)
  A13=(-1.)*(OM*ON*V(I)*R*(H-G)/AB1)
  A21=(ON3/A +OM*OM**2*B/AB2)
  A22= A13/R
  A23= 1.+R *(ON2/A +G*OM2/AB2)* V(I)
  A31=(-1.)*(A*OM4/AB4 +2.*B*OM2*ON2/AB2 + ON4/A)
  A32= -OM /AB1* (OM2*A/AB2 +ON2*B)*V(I)
  A33= OM*R*V(I)*(OM2/A + OM2*B/AB2)
  AP31=(A33*(A11*A22-A12*A21) +A32*(A13*A21-A11*A23))/ (A12*A23 -A22
1*A13)
  AP= AP31-A31
  AKX=AP/(OM2/AB2)
  AKY=AP/ON2
  IF(AKX-AAKX) 160,170,170
160 AAKX=AKX
170 CONTINUE
  IF(AKY-AAKY) 175,180,180
175 AAKY=AKY
180 CONTINUE
  IF(IE) 185,280,185
185 DO 195 L=1,IF
  AK(L)=AP/( OM2/AB2 +ON2*PHI(L))
  IF(AK(L)-AAK(L)) 190,195,195
190 AAK(L)=AK(L)
195 CONTINUE
  DO 198 L=1,IF
  AKAY(L)=AK(L)*PHI(L)
  IF(AKAY(L)-AAKAY(L)) 197,198,198
197 AAKAY(L)=AKAY(L)
198 CONTINUE
250 CONTINUE
280 WRITE(6,290) ANB(J), AAKX, AAKY, (AAK(L),AAKAY(L),L=1,IF)
290 FORMAT('F6.2,2F6.2, 5(F6.2/'F6.2 ')
300 CONTINUE
GO TO 1
END

```

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T	CARD A	TTT = Any desired one-card title in columns 1-72
13			
25			
37			
49			
61			
1	T N A O B	CARD B	TMAOB = Total number of (a/b)'s to be considered
13	A L F		ALF = $\sqrt{E_x/E_y}$
25	T N N		TNN = Total number of n
37	T N M		TNM = Total number of m
49	T N P H I		TNPHI = Total number of ϕ
61	T N V		TNV = Total number of v
1	A I 1	CARD C	AI1 = Invariant 1 = $E_x (1 + E_y/E_x + 2\nu_{yx}) / (1 - \nu_{xy} \nu_{yx})$
13	A I 2		AI2 = Invariant 2 = $G_{xy} - E_x \nu_{yx} / (1 - \nu_{xy} \nu_{yx})$
25	R		R = $D_{0x}/D_{0y} = G'_{cx}/G'_{cy}$
37	T N P S I		TNP SI = Total number of ψ
49			
61			
1	A O B (1)	CARD (S) D	AOB(1) = First panel aspect ratio (a/b) ₁
13	A O B (2)		AOB(2) = Second panel aspect ratio (a/b) ₂
25	.		.
37	.		.
49	.		.
61	.		(Maximum of 20 (a/b)'s)

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
PH I (1)	CARD(S) E	71	$\phi = N_y/N_x$
PH I (2)			ϕ_2
.			.
.			.
.			(Maximum of 20 ϕ)
V (1)	CARD(S) F	71	$V = \pi^2 \psi t_p (c+t_p)/(20' \alpha b^2)$
V (2)			V_1
.			V_2
.			.
.			(Maximum of 20 V 's)
PS I (1)	CARD(S) G	71	$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$
PS I (2)			ψ_1
.			ψ_2
.			.
.			(Maximum of 20 ψ 's)
		71	.
			.
			.
			.
			.
			.

EXAMPLE PROBLEM

AC-5 H/C PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING

The three laminates to be used in the example are $[0/\pm 45/0]_S$, $[0_2/\pm 45/90_2]_{2S}$, and $[0/\pm 60]_S$ at room temperature. The values for α , $AI1$, $AI2$, and ψ , E_x , E_y , G_{xy} , ν_{xy} , and ν_{yx} can be found from program AC-5.

The geometry of the panel is given as

$$\begin{aligned} c &= 0.5 \text{ inch}, & R &= D_{qx}/D_{qy} = 2.0 \\ a/b &= 1, 2, 3 & \phi &= 0, .1, .4, 1.0, & n, m &= 1, 2, \dots 6 \\ & & &2.0, 10.0 \\ b &= 15 \end{aligned}$$

The material properties of the core are such that

$$G'_{cx} = 126,000 \text{ psi } (8.1 \text{ lb/ft}^3 \text{ core})$$

Now, for $[0/\pm 45/0]_S$ laminate, $t_f = 8 \times .0052 = 0.0416 \text{ inch}$

$$\begin{aligned} \alpha &= 1.815, \quad AI1 = 33.884 \times 10^6, \quad AI2 = 0.12862 \times 10^6, \quad \psi = 10.794 \times 10^6 \\ V &= \pi^2 \psi t_f (c + t_f) / 2G'_{cx} b^2 = \frac{(9.8696)(10.794 \times 10^6)(0.0416)(0.500 + 0.0416)}{2(0.126 \times 10^{-6})(15)^2} \\ &= 0.04233 \end{aligned}$$

For $[0_2/\pm 45/90_2]_{2S}$ laminate, $t_f = 24 \times 0.0052 \times 0.1248 \text{ inch}$

$$\begin{aligned} \alpha &= 1.00, \quad AI1 = 33.884 \times 10^6, \quad AI2 = 0.12862 \times 10^6, \quad \psi = 13.971 \times 10^6 \\ V &= \frac{(9.8696)(13.971)(0.1248)(0.6248)}{2(0.126)(15)^2} = 0.18963 \end{aligned}$$

For $[0/\pm 60]_S$ laminate, $t_f = 6 \times 0.0052 = 0.0312 \text{ inch}$

$$\begin{aligned} \alpha &= 1.00, \quad AI1 = 33.894 \times 10^6, \quad AI2 = 0.12862 \times 10^6, \quad \psi = 12.771 \times 10^6 \\ V &= \frac{(9.8696)(12.771)(0.0312)(0.5312)}{2(0.126)(15)^2} = 0.03684 \end{aligned}$$

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	(0 / + - 4 5 / 0) S		TIT [0/415/0] S at Room Temperature
13	L A M I N A T E A T .		
15	R O O M T E M P E R A		
37	T U R E		
41			
61		1 0	
1	3 . 0		TNAOB
13	1 . 8 1 5		ALF
25	1 0 . 0		TNN
37	1 0 . 0		TNM
49	5 . 0	73	TNPHI
61	1 . 0	80	TNV
1	3 3 8 8 4 0 0 0 .	2 0	AI1
13	1 2 8 6 2 0 .		AI2
25	2 . 0		R
37	1 . 0		TNPSI
49		73	
61		80	
1	1 . 0	3 0	(a/b)1
13	2 . 0		(a/b)2
25	3 . 0		(a/b)3
37			
49		73	
61		80	
1		4 0	

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 1	PHI(1)
13	0 . 5	PHI(2)
25	1 . 0	PHI(3)
37	2 . 0	PHI(4)
49	1 0 . 0	PHI(5)
61	5 0	
1	0 . 0 4 2 3 3	V(1)
13		
25		
37		
49		
61		
1	1 0 7 9 4 0 0 0 .	PSI(1)
13		
25		
37		
49		
61		
1	(0 / 0 / + - 4 5 / 9 0	TIT
13	/ 9 0) 2 S L A M I N	[02/415/902]2S at Room Temperature
25	A T E A T R O O M	
37	T E M P E R A T U R E	
49		
61		

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3 . 0		TNAOB
13	1 . 0 0		ALP
25	1 0 . 0		TNN
37	1 0 . 0		TNN
49	5 . 0	73 80	TNPFI
61	1 . 0	9 0	TNV
1	3 3 8 8 4 0 0 0 .		AI1
13	1 2 8 6 2 0 .		AI2
25	2 . 0		R
37	1 . 0		TNPBI
49		73 80	
61		1 0 0	
1	1 . 0		AOB(1)
13	2 . 0		AOB(2)
25	3 . 0		AOB(3)
37			
49		73 80	
61		1 1 0	
1	0 . 1		PHI(1)
13	0 . 5		PHI(2)
25	1 . 0		PHI(3)
37	5 . 0		PHI(4)
49	1 0 . 0	73 80	PHI(5)
61		1 2 0	

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 1 8 9 6 3		V(1)
13			
25			
37			
49		73	
61		80	
		1 3 0	
1	1 3 9 7 1 0 0 0 .		PSI(1)
13			
25			
37			
49		73	
61		80	
		1 4 0	
1	(0 / + - 6 0) S L A		TIT [0.360]s at Room Temperature
13	M I N A T E A T R O		
25	O M T E M P E R A T U		
37	R E		
49		73	
61		80	
		1 5 0	
1	3 . 0		TNAOB
13	1 . 0		ALF
25	1 0 . 0		TNN
37	1 0 . 0		TNM
49	5 . 0		TNPHI
61	1 . 0	73	TNV
		80	
		1 6 0	

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3 3 8 8 4 0 0 0 .		AI1
13	1 2 8 6 2 0 .		AI2
25	2 . 0		R
37	1 . 0		TNPSI
49		73 80	
61		1 7 0	
1	1 . 0		AOB(1)
13	2 . 0		AOB(2)
25	3 . 0		AOB(3)
37			
49		73 80	
61		1 8 0	
1	0 . 1		PHI(1)
13	0 . 5		PHI(2)
25	1 . 0		PHI(3)
37	5 . 0		PHI(4)
49	1 0 . 0	73 80	PHI(5)
61		1 9 0	
1	0 . 0 3 6 8 4		V(1)
13			
25			
37			
49		73 80	
61		2 0 0	

[illegible]

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT (0/+45/0)S LAMINATE AT ROOM TEMPERATURE

G*CX/G*CY= 2.00 GAMMA= 0.3985 PHI= 1.184 ALPHA= 1.815 PSI= 10/940000.
DATA FOR V= 0.0423

A/R	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	4.17	3.91	3.79/ 0.38	2.78/ 1.39	2.09/ 2.09	1.39/ 2.78	0.38/ 3.79
2.00	4.17	1.17	3.35/ 0.34	1.56/ 0.78	0.94/ 0.94	0.52/ 1.02	0.11/ 1.44
3.00	4.06	0.70	3.31/ 0.33	1.29/ 0.65	0.71/ 0.71	0.37/ 0.75	0.08/ 0.76

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT (0/0/+45/90/90)2S LAMINATE AT ROOM TEMPERATURE

G*CX/G*CY= 2.00 GAMMA= 0.2219 PHI= 0.656 ALPHA= 1.000 PSI= 13971000.
DATA FOR V= 0.1896

A/R	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	2.27	2.19	2.06/ 0.21	1.51/ 0.76	1.13/ 1.13	0.76/ 1.51	0.21/ 2.06
2.00	2.27	1.00	2.06/ 0.21	1.33/ 0.67	0.80/ 0.80	0.64/ 0.89	0.10/ 0.97
3.00	2.27	0.84	2.06/ 0.21	1.44/ 0.67	0.75/ 0.75	0.40/ 0.79	0.08/ 0.63

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT (0/+60)S LAMINATE AT ROOM TEMPERATURE

G*CX/G*CY= 2.00 GAMMA= 0.4361 PHI= 1.000 ALPHA= 1.000 PSI= 1.771000.
DATA FOR V= 0.0368

A/R	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	3.60	3.60	3.28/ 0.33	2.40/ 1.20	1.80/ 1.80	1.20/ 2.40	0.33/ 3.28
2.00	3.60	1.64	3.28/ 0.33	1.92/ 0.96	1.15/ 1.15	0.60/ 1.20	0.14/ 1.41
3.00	3.60	1.15	3.28/ 0.33	1.87/ 0.94	1.03/ 1.03	0.54/ 1.09	0.11/ 1.13

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COMPUTER PROGRAM AC-7

ANALYSIS OF SIMPLY SUPPORTED ORTHOTROPIC SANDWICH FLAT
PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS

AC-7

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-7

ABSTRACT

This program calculates the deflection, moments, and shears in a flat orthotropic honeycomb (H/C) panel under pressure plus in-plane loads, given material and geometry parameters. The method is a series summation, and the series terms as well as the series sum are printed out to enable the user to monitor convergence.

The in-plane loads can be compression-compression, tension-compression, or tension-tension. For the first two cases, part of the input data, the buckling coefficients, can be found from program AC-5. For the latter case, tension-tension buckling coefficients are found as noted later in this program description.

PROGRAM INPUT DATA

The program uses as input the results of programs AC-2 and AC-5, as well as other items. The input items are:

Card A:

TIT = Any desired one-card title. The title is in columns 1-72.
If no title is desired, a blank card must be used.

Card B:

TNAØB = Total number of (a/b)'s to be read in (maximum of 20).

ALF = The alpha volume from program AC-2 ($\alpha = \sqrt{E_x/E_y}$).

TNM = Total number of half-waves in the X-direction (no limit).

TNV = Total number of core shear influence parameters V to be read in.

AI1 = Invariant 1 from program AC-2. It is $E_x(1 + E_y/E_x + 2\nu_{yx})/(1 - \nu_{xy}\nu_{yx})$.

AI2 = Invariant 2 from program AC-2. It is $G_{xy} - E_x\nu_{yx}/(1 - \nu_{xy}\nu_{yx})$.

Card C:

R_x = Ratio of N_x to critical N_x ($N_x/N_{x, cr}$); plus for compression and minus for tension.

R_y = Ratio of N_y to critical N_y ($N_y/N_{y, cr}$); plus for compression and minus for tension.

R = Core shear modulus ratio G'_{cx}/G'_{cy} .

TNPSI = Total number of ψ to be considered.

TNXØA = Total number of x/a to be considered.

TNYØB = Total number of y/b to be considered.

Card(s) D:

AØB(1) = Panel aspect ratio $(a/b)_1$.

AAKX(1) = Buckling coefficient K_x from:

1. AC-5 corresponding to given value of ϕ if N_x is compression

2. $K_x = N_x \left(\frac{b}{\pi} \right)^2 / \sqrt{D_{11} D_{22}}$ if N_x is tension.

AAKY(1) = Buckling coefficient K_y from:

1. AC-5 corresponding to given value of ϕ if N_y is compression.

2. $K_y = N_y \left(\frac{b}{\pi} \right)^2 / \sqrt{D_{11} D_{22}}$ if N_y is tension.

There are two sets of these parameters to a card (maximum of 20 sets).

Card(s) E:

$V(I)$ = Core shear parameters where $V = \pi^2 \psi t_f (c + t_f) / (2G'_{cx} b^2)$.

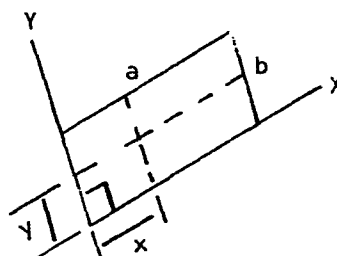
There are six of these to a card (maximum of 20).

Card(S) F:

APSI(I) = Modulus ratios ψ found from AC-2, where $\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$. There are six of these to a card (maximum of 20).

Card(S) G:

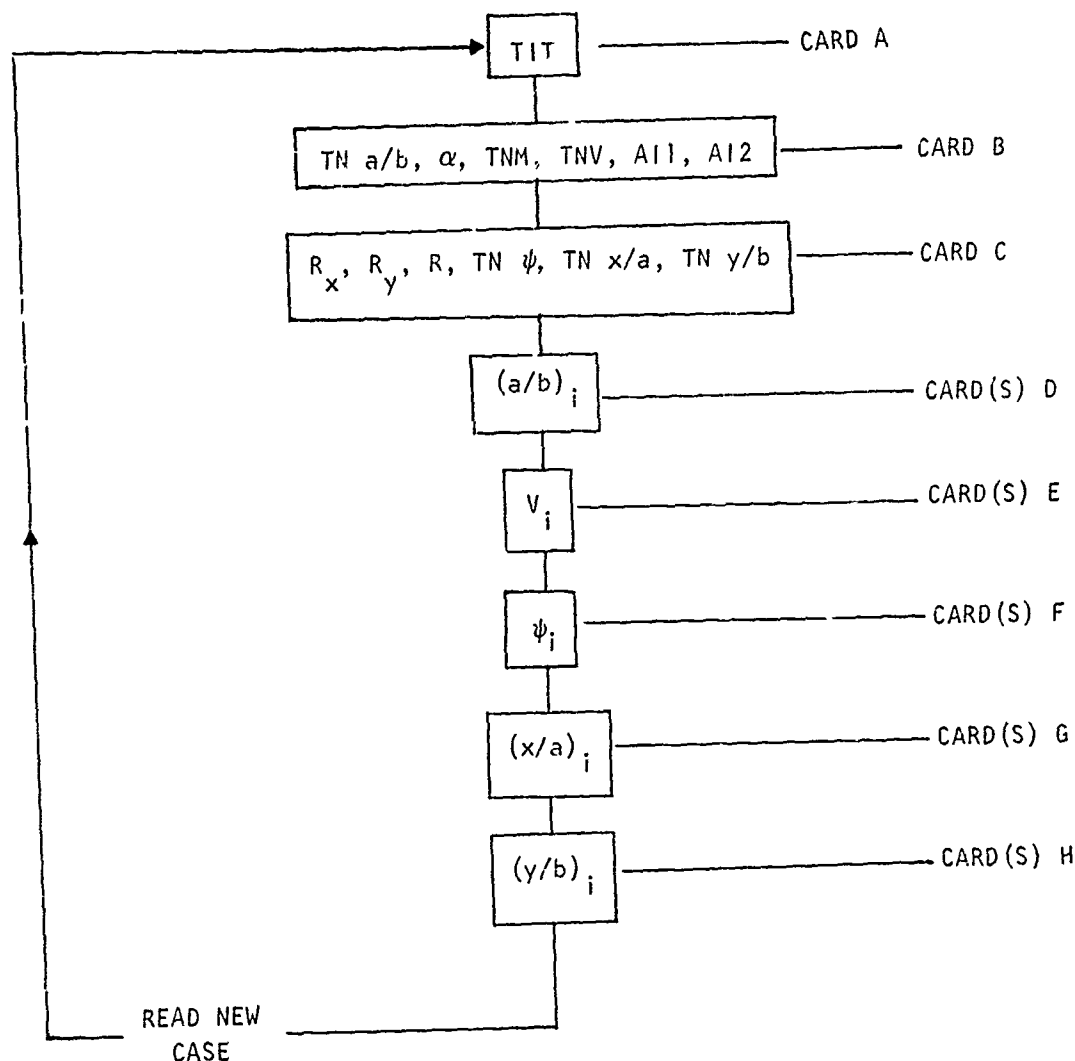
XØA(I) = Ratio of point on X-axis where results are to be given to the length of the panel, a , in the x-direction. There are six of these to a card (maximum of 20).



Card(s) H:

YØB(I) = Ratio of point on Y-axis where results are to be given to the width of the panel, b , in the y-direction. There are six of these to a card (maximum of 20).

Sample input data sheets are shown in this section following the text.



additional cases may be stacked one behind the other.

OUTPUT

The output, shown later in this section, gives the title and the parameters R , R_x , R_y , K_x , K_y , a/b , γ , β , α , and ψ . The series terms are printed for $m = n = 1, 3, 5, \dots$ TNM for the values of x/a and y/b given. The last term in the set can be used to find the various quantities, where:

1. Panel deflection at x/a and y/b

$$W = (\text{Col 1 value}) \cdot qb^4 / \sqrt{D_{11} D_{22}}$$

```

C
C PROGRAM AC7 ... ANALYSIS OF SIMPLY-SUPPORTED ORTHOTROPIC SANDWICH
C FLAT PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS
C
C THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
C
C A11 = INVARIANT 1 =  $EX \cdot (1 + \nu_{xy}/EX + \nu_{yx}/EY) / (1 - \nu_{xy}\nu_{yx})$ 
C A12 = INVARIANT 2 =  $G_{xy} - \nu_{xy}\nu_{yx}/(1 - \nu_{xy}\nu_{yx})$ 
C AKX = BUCKLING COEFFICIENT IN THE X-DIRECTION FOUND FROM PROGRAM AC5
C AKY = BUCKLING COEFFICIENT IN THE Y-DIRECTION FOUND FROM PROGRAM AC5
C ALFA = ALF =  $SORT(EX/EY)$ 
C BETA = B =  $ALFA \cdot \nu_{yx} + 2 \cdot GAMMA$ 
C D11 =  $EX \cdot T / ((\pi^2 / (2 \cdot (1 - \nu_{xy}\nu_{yx})))$ 
C D22 =  $D11 \cdot (EY/EX)$ 
C D66 =  $T \cdot (1 - \nu_{xy}\nu_{yx}) / 2$ 
C DOX =  $G_{xx}$ 
C DOY =  $G_{yy}$ 
C GAMMA =  $G = G_{xy} \cdot (1 - \nu_{xy}\nu_{yx}) / SORT(EX \cdot EY)$ 
C IA = TOTAL NUMBER OF A/R (PANEL ASPECT RATIOS)
C II = TOTAL NUMBER OF PHI, WHERE PHI =  $\pi Y/RX$ 
C IP = TOTAL NUMBER OF V, WHERE V =  $PI \cdot \pi \cdot SORT(D11 \cdot D22) / (EX \cdot Z \cdot DOX)$ 
C NI = TROMATION INTEGR FOR SERIES SUMMATION
C PSI =  $SORT(EX \cdot EY) / (1 - \nu_{xy}\nu_{yx})$ 
C R =  $DOX/DOY = G_{xx}/G_{yy}$ 
C RX =  $EX/DOX$ 
C RY =  $EY/DOY$ 
C XI =  $SORT(D11/D22)$ 
C
C DIMENSION AOR(20),XOA(20),YOB(20),PHI(20),V(20),OX(20),OY(20),
C 1 AFX(20),AFY(20),AMXY(20),T(20),AOX(20),AOY(20),SY(20),LY(20)
C 2,APSI(20),TIT(18),AAKX(20),AAKY(20)
C 1 READ(5,7) (TIT(I),I=1,18)
C 7 FORMAT(1PA6)
C READ(5,2) TMAOR,ALF,TMB,TNV,A11,A12,RX,RY,R,TNPS1,TNXOA,TNYOB
C 2 FORMAT(6F12.0)
C A=ALF
C O1=0.0166424
C IA=TMAOR
C NI=TMB
C MA=TNXOA
C MB=TNYOB
C 3 READ(5,2) (AOR(I),AAKX(I),AAKY(I),I=1,IA)
C 6 IP=TNV
C READ(5,2) (V(I),I=1,IP)
C L2=TNPS1
C READ(5,2) (APSI(I),I=1,L2)
C READ(5,2) (XOA(I),I=1,MA)
C READ(5,2) (YOB(I),I=1,MB)
C DO 100 L3=1,L2
C DO 100 J=1,IA
C AR1=AOR(J)
C AKX=AAKX(J)
C AKY=AAKY(J)
C AR2=AR1**2
C AR3=AR1**3
C AR4=AR1**4
C PSI=APSI(L3)
C GAMMA=(A11/(2.0*PSI) + A12/PSI - .5*A - .5/A)
C BETA=(1.5*A11/PSI + 2.0*A12/PSI - 1.5*A - 1.5/A)
C B=BETA
C G=GAMMA
C WRITE(6,8)
C 8 FORMAT('1')

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```

WRITE(6,7) (T11(I), I=1,18)
10 WRITE(6,11)
11 FORMAT('0 DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SU
SUPPORTED OR HETEROGENEOUS SANDWICH PLATES')
15 WRITE(6,16) P, G, B, A, PSI
16 FORMAT('005X'G'X/G, 'Y=F5.2, 5X'GAMMA=F8.4, 5X'BETA=F8.3, 5X'ALP
HA=F6.3, 5X'PSI=F11.0)
17 WRITE(6,18) AOB(J), RX, RY, AKX, AKY
18 FORMAT('005X'A/B =F5.2, 5X'NX/NXCR =F6.3, 5X'NY/NYCR =F6.3, 5X'KX
=F5.2, 5X'KY =F5.2)
25 DO 100 I=1,10
26 WRITE(6,27) V(I)
27 FORMAT('002X'...DATA FOR V =F8.4)
DO 100 J=1,NA
C1=3.1416*XB(A(J))
250 WRITE(6,251) XOA(J)
251 FORMAT(25H COEFFICIENTS FOR X/A=F6.3)
505 WRITE(6,506)
506 FORMAT('005X'G'X/G, 'Y=F5.2, 5X'GAMMA=F8.4, 5X'BETA=F8.3, 5X'ALP
HA=F6.3, 5X'PSI=F11.0)
DO 100 J2=1,MB
U(J2)=0
AMX(J2)=0
AMY(J2)=0
AMXY(J2)=0
AOX(J2)=0
AOY(J2)=0
199 CONTINUE
30 DO 90 I=1,MI,2
DO 90 N=1,NI,2
NM=M
NM=N
192 Z=510 (U(I), U(N))
Z=0.0 (U(I), U(N))
DO 200 K=1,MB
XY(K)=5.1N (YOB(K), 3.1416*NM)
(Y(K)-C0) (YOB(K), 3.1416*NM)
200 CONTINUE
U2=0.0
U3=0.0
U4=0.0
U5=0.0
U6=0.0
U7=0.0
U8=0.0
U9=0.0
U10=0.0
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U1000=0.0

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1=S/AB1-(OM**2/AB2) )
RMXY=G* 9.8696 *AMN*(UN*ALF1*S +ALF2*R*UM*S -2.*UM*(IN/AB1)
ROX=- 31.0065 *AMN*(OM*UN*(1./(IN*AB1))*((B-2.*G)*(UN*ALF2*R*S
1-ON2)+A*(ALF1*OM*S/AB1 -OM2/AB2)) +G*UN*(ALF1*S/OM +ALF2*R*S/(ON+
2AB1) -2./AB1))
ROY=- 31.0065 *AMN* (G*(OM*(ALF1*S/OM+ALF2*R*S/(UN*AB1)-2./AB1)
1*(1./AB1)+(1./OM)*((ON*ALF2*R*S-ON2)/A+(B-2.*G)*(ALF1*UM*S/AB1
2-OM2/AB2)))*OM*(ON
DO 300 J2=1,MR
AMX(J2)=AMX(J2)+RMX*SX*SY(J2)
AMY(J2)=AMY(J2)+RMY*SX*SY(J2)
AMXY(J2)=AMXY(J2)+RMXY*(X*CY(J2)
AOX(J2)=AOX(J2)+ROX*(X*SY(J2)
AOY(J2)=AOY(J2)+ROY*SX*CY(J2)
X(J2)=W(J2)+AMM* SX*SY(J2)
300 CONTINUE
N1=N
IF (N1)-(N1)90 ,400,90
C
C.....(END OF PICK UP AND TERM
C
400 IF (N1)401,403,401
401 N1=N
N1=N
GO TO 352
403 CONTINUE
500 WRITE(6,501) N,N
501 FORMAT(4H0 N=,I2,3X,2H N=,I2)
510 DO 520 J2=1,MR
511 WRITE(6,512)YOR(J2),W(J2),AMX(J2),AMY(J2),AMXY(J2),AOX(J2),AOY(J2)
512 FORMAT(5H0Y/R=F6.3,F9.6,F13.3,F14.3,F14.3,F13.3,F11.3)
520 CONTINUE
90 CONTINUE
100 CONTINUE
GO TO 1
END

```

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T	Card A	TIT = Any desired one-card title, cols. 1-72
13			
25			
37			
49			
61			
1	T N . 8	Card B	TNAB = Total number of a/b to be considered
13	A I .		ALF = Alpha = $\sqrt{E_x/E_y}$
25	T N M		TNM = Total number of M
37	T N V		TNV = Total number of V
49	A I . 1		AI1 = Invariant 1 = $E_x (1 + E_y/E_x + 2\psi_{xy}) / (1 - \psi_{xy}^2)$
61	A I . 2		AI2 = Invariant 2 = $G_{xy} - E_x \psi_{xy} / (1 - \psi_{xy}^2)$
1	R X	Card C	RX = N_x/N_{xcr}
13	R Y		RY = N_y/N_{ycr}
25	R		R = G_{cx}/G_{cy}
37	T N P S I		TNPSI = Total number of ψ
49	T N X Ø A		TNXØA = Total number of x/a
61	T N X Ø B		TNXØB = Total number of y/b
1	A Ø B C (1)	Card(s) D	a/b
13	A A K X (1)		k_x (1) k_x can be found from AC5
25	A A K Y (1)		k_y k_y can be found from AC5
37	A Ø B (2)		a/b
49	A A K X (2)		k_x (2)
61	A A K Y (2)		k_y (20 maximum)



NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	V (1)	Card(s) E	
13	V (2)		
25	V (3)		$V = \pi^2 \psi t_c (C + t_c) (2C^2 b^2)$
37			
49			
61			(20 Maximum)
1	A P S I (1)	Card(s) F	
13	A P S I (2)		
25	A P S I (3)		$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$
37			
49			
61			(20 Maximum)
1	X Ø A (1)	Card(s) G	
13	X Ø A (2)		
25	X Ø A (3)		
37			
49			
61			(20 Maximum)
1	Y Ø B (1)	Card(s) H	
13	Y Ø B (2)		
25	Y Ø B (3)		
37			
49			
61			(20 Maximum)

where

$$D_{11} = E_x t_f c^2 / [2 (1 - \nu_{xy} \nu_{yx})]$$

$$D_{22} = D_{11} E_y / E_x$$

q = pressure

2. Panel moment M_x at x/a and y/b

$$M_x = (\text{Col 2 value}) \cdot q b^2$$

3. Panel moment M_y at x/a and y/b

$$M_y = (\text{Col 3 value}) \cdot q b^2$$

4. Panel moment M_{xy} at x/a and y/b

$$M_{xy} = (\text{Col 4 value}) \cdot q b^2$$

5. Panel shear Q_x at x/a and y/b

$$Q_x = (\text{Col 5 value}) \cdot q b$$

6. Panel shear Q_y at x/a and y/b

$$Q_y = (\text{Col 6 value}) \cdot q b$$

EXAMPLE PROBLEM

An example problem is given to illustrate the use of the program. A honeycomb panel with $c = 0.5$ and $[0/\pm 45/0]_S$ face sheets at room temperature is loaded under pressure only and then with $N_y/N_x = 1.0$ for $N_x/N_{x,cr} = N_y/N_{y,cr} = 0.5$.

Many of the parameters on the data sheets shown are taken from the AC-5 example problem. The results are tabulated as follows:

Case	a/b	x/a	y/b	w	M_x	M_y	M_{xy}	Q_x	Q_y
Pressure only	1	0	0	0	0	0	-0.032 qb ²	0	0
		0	0.5	0	0	0	0	0.5.3 qb	0
		0.5	0	0	0	0	0	0	0.188 qb
		0.5	0.5	$0.003835 \frac{b^4}{a^4}$	$0.069 qb^2$	$0.028 qb^2$	0	0	0
	2	0	0	0	0	0	-0.056 qb ²	0	0
		0	0.5	0	0	0	0	0.461 qb	0
		0.5	0	0	0	0	0	0	0.394 qb
		0.5	0.5	$0.01366 \frac{b^4}{a^4}$	$0.094 qb^2$	$0.077 qb^2$	0	0	0
	3	0	0	0	0	0	-0.059 qb ²	0	0
		0	0.5	0	0	0	0	0.459 qb	0
		0.5	0	0	0	0	0	0	0.449 qb
		0.5	0.5	$0.01976 \frac{b^4}{a^4}$	$0.094 qb^2$	$0.104 qb^2$	0	0	0

Case	a/b	x/a	y/b	w	M_x	M_y	M_{xy}	Q_x	Q_y
Pressure plus N_x and N_y	1	0	0	0	0	0	-0.060 qb ²	0	0
		0	0.5	0	0	0	0	0.705 qb	0
		0.5	0	0	0	0	0	0	0.449 qb
		0.5	0.5	$0.007795 \frac{b^4}{q_x^4}$	$0.145 qb^2$	$0.059 qb^2$	0	0	0
	2	0	0	0	0	0	-0.107 qb ²	0	0
		0	0.5	0	0	0	0	0.811 qb	0
		0.5	0	0	0	0	0	0	0.457 qb
		0.5	0.5	$0.02781 \frac{b^4}{q_x^4}$	$0.202 qb^2$	$0.160 qb^2$	0	0	0
	3	0	0	0	0	0	-0.110 qb ²	0	0
		0	0.5	0	0	0	0	0.769 qb	0
		0.5	0	0	0	0	0	0	0.864 qb
		0.5	0.5	$0.04045 \frac{b^4}{q_x^4}$	$0.204 qb^2$	$0.218 qb^2$	0	0	0

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1 (0 / + - 4.5 / 0) . S .		TIT	
13 L A M I N A T E A T			
25 R Ø Ø M T E M P E R A			Start of First Case
37 T U R E . . . P R E S S U			
49 R E Ø N L Y	73		
61	1 0		
1 3 . 0		TNAØB	
13 1 . 8 1 5		ALF	
25 1 . 0 . . 0		TNA	
37 1 . 0		TIV	
49 3 . 3 8 8 4 0 0 0	73	AI1	
61 1 2 8 6 2 0	2 0	AI2	
1 0 . 0		RX	
13 0 . 0		RY	
25 2 . 0		R	
37 1 . 0		TNPSI	
49 2 . 0	73	TNØA	
61 2 . 0	3 0	TIVØB	
1 1 . 0		AØB(1)	
13 4 . 1 7		AAØX(1)	
25 3 . 9 1		AAØY(1)	
37 2 . 0		AØB(2)	
49 4 . 1 7	73	AAØX(2)	
61 1 . 1 7	4 0	AAØY(2)	

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3 . 0		AQB(3)
13	4 . 0 6		AAKX(3)
25	0 . 7 9		AAKY(3)
37			
49		73	
61		8	
		5.0	
1	0 . 0 4 2 3 3		V(1)
13			
25			
37			
49		73	
61		6.0	
1	1 0 7 9 4 0 0 0 .		APSI(1)
13			
25			
37			
49		73	
61		8	
		7.0	
1	0 . 0		XQA(1)
13	0 . 5		XQA(2)
25			
37			
49			
61		8.0	

NUMBER		IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	0.0		YQB(1)	
13	0.5		YQB(2)	
25				
37				
49				
61				
1	(0 / + - 4 5 1 0) S	9.0	TIT	
13	L A M I N A T E A T			
25	R Ø Ø M T E M P E R A			
37	T U R E , U N D E R			
49	P A E S S U R E P L U			
61	S N X A N D N Y	1.00		
1	3.0		THA2B	
13	1.8 1 5		ALF	
25	1.0.0		TNI	
37	1.0		THV	
49	3 3 8 8 4 0 0 0		AIL	
61	1 2 8 6 2 0	1.10	AI2	
1	0.5		RX	
13	0.5		RY	
25	2.0		F	
37	1.0		TAPS	
49	2.0		THA2A	
61	2.0	1.20	THY0B	

Start of Second Case

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1. C	A0B(1)
13	2. 0 9	AAKX(1)
25	2. 0 9	AAKY(1)
37	2. 0	A0B(2)
49	0. 9 4	AAKX(2)
61	0. 9 4	AAKY(2)
1	3. 0	A0B(3)
3	0. 7 1	AAKX(3)
25	0. 7 1	AAKY(3)
37		
49		
61		
1	0. 0 4 2 3 3	V(1)
13		
25		
37		
49		
61		
1	1. 0 7 9 4 0 0 0	APSI(1)
13		
25		
37		
49		
61		

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 0		XØA(1)
13	0 . 5		XØA(2)
25			
37			
49			
61		73 1 7 0	
1	0 . 0		YØB(1)
13	0 . 5		YØB(2)
25			
37			
49			
61		73 1 8 0	
1			
13			
25			
37			
49			
61		73	
1			
13			
25			
37			
49			
61		73	

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

$G^{\circ}CX/G^{\circ}CY = 2.00$
 $G^{\circ}ANMA = 0.3985$
 $\beta^{\circ}TA = 1.184$
 $\alpha^{\circ}HA = 1.815$
 $\psi^{\circ}SI = 10794000.$

$$\begin{array}{ccccccc} A/H = 1.00 & \cdot & NX/NXCR = 0.0 & & NY/NYLR = 0.0 & & KX = 4.17 & & KY = 3.91 \end{array}$$

...DATA FOR V = 0.0423
COEFFICIENTS FOR X/A = 0.0

M= 1	N= 1	W2X1/0#R**4	MX/Q#R**2	MY/Q#B**2	MX/Y/0#B**2	MX/Y/0#B	QY/Q#B
Y/R= 0.0	0.0	0.0	0.0	0.0	-0.028	0.0	0.0
Y/R= 0.500	0.0	0.0	0.0	0.0	0.000	0.328	0.0
M= 3	N= 3						
Y/R= 0.0	0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/R= 0.500	0.0	0.0	0.0	0.0	0.000	0.337	0.0
M= 5	N= 5						
Y/R= 0.0	0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/R= 0.500	0.0	0.0	0.0	0.0	0.000	0.367	0.0
M= 7	N= 7						
Y/R= 0.0	0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/R= 0.500	0.0	0.0	0.0	0.0	0.000	0.368	0.0
M= 9	N= 9						
Y/R= 0.0	0.0	0.0	0.0	0.0	-0.032	0.0	0.0
Y/R= 0.500	0.0	0.0	0.0	0.0	0.000	0.378	0.0

COEFFICIENTS FOR X/A = 0.500

M=1	N=1	W*XI/0**B**4	MX/0**B**2	MY/0**B**2	MY/0**B**2	QX/0**B	OY/0**B
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.188
Y/H=	0.500	0.003988	0.077	0.032	-0.000	-0.000	-0.000
M=3	N=3						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.218
Y/H=	0.500	0.003816	0.068	0.027	-0.000	-0.000	-0.000
M=5	N=5						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.238
Y/H=	0.500	0.003839	0.070	0.028	-0.000	-0.000	-0.000
M=7	N=7						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.244
Y/H=	0.500	0.003832	0.069	0.028	-0.000	-0.000	-0.000
M=9	N=9						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.251
Y/H=	0.500	0.003835	0.069	0.028	-0.000	-0.000	-0.000

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE ONLY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

$G^{+CX/CY} = 2.00$ $\gamma_{max} = 0.4985$ $KTA = 1.184$ $\alpha_{max} = 1.815$ $\psi = 10794000$

$A/R = 2.00$ $MX/MXCR = 0.0$ $NY/MYCR = 0.0$ $KX = 4.17$ $KY = 1.17$

DATA FOR $V = 0.0423$
COEFFICIENTS FOR $X/A = 0.0$

$M = X1/0.8884$		$MX/0.8884$	$MY/0.8884$	$MXY/0.8884$	$OX/0.8884$	$OY/0.8884$
$M = 1 \quad N = 1$						
$Y/R = 0.0$	0.0	0.0	0.0	-0.050	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.000	0.336	0.0
$M = 3 \quad N = 3$						
$Y/R = 0.0$	0.0	0.0	0.0	-0.055	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.000	0.395	0.0
$M = 5 \quad N = 5$						
$Y/R = 0.0$	0.0	0.0	0.0	-0.056	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.000	0.436	0.0
$M = 7 \quad N = 7$						
$Y/R = 0.0$	0.0	0.0	0.0	-0.056	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.000	0.448	0.0
$M = 9 \quad N = 9$						
$Y/R = 0.0$	0.0	0.0	0.0	-0.056	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.000	0.461	0.0

COEFFICIENTS FOR X/A= 0.500

		WXX1/0**4	MX/Q**2	MY/0**2	MXV/0**2	OX/O**2	OY/O**2
M= 1	N= 1						
Y/R= 0.0	0.0	0.0	0.0	0.0	0.000	0.0	0.348
Y/R= 0.500	0.014188	0.110	0.085	-0.000	-0.000	-0.000	-0.000
M= 3	N= 3						
Y/R= 0.0	0.0	0.0	0.0	0.0	0.000	0.0	0.352
Y/R= 0.500	0.013603	0.091	0.075	-0.000	-0.000	-0.000	-0.000
M= 5	N= 5						
Y/R= 0.0	0.0	0.0	0.0	0.0	0.000	0.0	0.384
Y/R= 0.500	0.013671	0.095	0.077	-0.000	-0.000	-0.000	-0.000
M= 7	N= 7						
Y/R= 0.0	0.0	0.0	0.0	0.0	0.000	0.0	0.385
Y/R= 0.500	0.013653	0.094	0.076	-0.000	-0.000	-0.000	-0.000
M= 9	N= 9						
Y/R= 0.0	0.0	0.0	0.0	0.0	0.000	0.0	0.394
Y/R= 0.500	0.013660	0.094	0.077	-0.000	-0.000	-0.000	-0.000

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNIFORM PRESSURE ONLY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

$G^*C^*X/G^*C^*Y = 2.00$ $GAMMA = 0.3985$ $BETA = 1.184$ $ALPHA = 1.415$ $PSI = 10794000$

$A/H = 3.00$ $MX/MXCR = 0.0$ $NY/MYCR = 0.0$ $KX = 4.06$ $KY = 0.79$

DATA FOR $V = 0.0423$

COEFFICIENTS FOR $X/A = 0.0$

M=1 N=1		$W^*X^2/0^*H^3$	$MX/0^*H^3$	$MY/0^*H^3$	$MY/0^*H^3$	$MX/0^*H^3$	$MY/0^*H^3$
$Y/H = 0.0$	0.0	0.0	0.0	0.0	-0.049	0.0	0.0
$Y/H = 0.500$	0.0	0.0	0.0	0.0	0.000	0.285	0.0
M=3 N=3							
$Y/H = 0.0$	0.0	0.0	0.0	0.0	-0.058	0.0	0.0
$Y/H = 0.500$	0.0	0.0	0.0	0.0	0.000	0.370	0.0
M=5 N=5							
$Y/H = 0.0$	0.0	0.0	0.0	0.0	-0.059	0.0	0.0
$Y/H = 0.500$	0.0	0.0	0.0	0.0	0.000	0.421	0.0
M=7 N=7							
$Y/H = 0.0$	0.0	0.0	0.0	0.0	-0.059	0.0	0.0
$Y/H = 0.500$	0.0	0.0	0.0	0.0	0.000	0.467	0.0
M=9 N=9							
$Y/H = 0.0$	0.0	0.0	0.0	0.0	-0.059	0.0	0.0
$Y/H = 0.500$	0.0	0.0	0.0	0.0	0.000	0.459	0.0

COEFFICIENTS FOR X/A = 0.500

	W*XI/0**R**4	MX/0**R**2	MY/0**R**2	MXV/0**R**2	QX/0**0	QY/0**H
M= 1 N= 1						
Y/R= 0.0 0.0	0.0	0.0	0.0	0.000	0.0	0.421
Y/R= 0.500 0.021051	0.116	0.117	0.117	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/R= 0.0 0.0	0.0	0.0	0.0	0.000	0.0	0.599
Y/R= 0.500 0.019594	0.087	0.101	0.101	-0.000	0.000	-0.000
M= 5 N= 5						
Y/R= 0.0 0.0	0.0	0.0	0.0	0.000	0.0	0.442
Y/R= 0.500 0.019796	0.096	0.105	0.105	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/R= 0.0 0.0	0.0	0.0	0.0	0.000	0.0	0.456
Y/R= 0.500 0.019745	0.092	0.103	0.103	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/R= 0.0 0.0	0.0	0.0	0.0	0.000	0.0	0.449
Y/R= 0.500 0.019763	0.094	0.104	0.104	-0.000	-0.000	-0.000

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

-----	GICX/GICY= 2.00	GAMMA= 0.3985	HTA= 1.184	ALPHA= 1.815	PSI= 1079400.

$$\Delta/R = 1.00 \quad N_X/N_{XCR} = 0.500 \quad N_Y/N_{YCR} = 0.500 \quad K_X = 2.09 \quad K_Y = 2.09$$

...DATA FOR V = 0.0423

COEFFICIENTS FOR $X/\Delta = 0.0$

$\mu = 1$	$N = 1$	$XY/XI/0 \times 10^{14}$	$MX/Q \times 10^{13} \times 2$	$MY/Q \times 10^{13} \times 2$	$XY/Q \times 10^{13}$	$QX/Q \times 10^{13}$	$QY/Q \times 10^{13}$
$Y/R = 0.0$	0.0	0.0	0.0	0.0	-0.055	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.0	0.000	0.656	0.0
$\mu = 3$	$N = 3$						
$Y/R = 0.0$	0.0	0.0	0.0	0.0	-0.059	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.0	0.000	0.661	0.0
$\mu = 5$	$N = 5$						
$Y/R = 0.0$	0.0	0.0	0.0	0.0	-0.060	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.0	0.000	0.694	0.0
$\mu = 7$	$N = 7$						
$Y/R = 0.0$	0.0	0.0	0.0	0.0	-0.060	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.0	0.000	0.695	0.0
$\mu = 9$	$N = 9$						
$Y/R = 0.0$	0.0	0.0	0.0	0.0	-0.060	0.0	0.0
$Y/R = 0.500$	0.0	0.0	0.0	0.0	0.000	0.705	0.0

COEFFICIENTS FOR $X/A = 0.500$

M=1 N=1		WXX1/0=8334	MX/0=8332	MY/0=8332	MX/0=8332	MY/0=8332	UX/0=8332	UY/0=8332
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.376
Y/R=	0.500	0.007989	0.154	0.065	-0.000	-0.000	-0.000	-0.000
M=3 N=3								
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.419
Y/R=	0.500	0.007774	0.143	0.058	-0.000	-0.000	-0.000	-0.000
M=5 N=5								
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.443
Y/R=	0.500	0.007800	0.145	0.060	-0.000	-0.000	-0.000	-0.000
M=7 N=7								
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.450
Y/R=	0.500	0.007792	0.144	0.059	-0.000	-0.000	-0.000	-0.000
M=9 N=9								
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.457
Y/R=	0.500	0.007795	0.145	0.059	-0.000	-0.000	-0.000	-0.000

0/+-45/0/0 LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY
 DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

$G'CX/G'CY = 2.00$ $GAMMA = 0.3985$ $BETA = 1.184$ $ALPHA = 1.815$ $PSI = 10794000.$
 $A/H = 2.00$ $NX/NXCR = 0.500$ $NY/NYCR = 0.500$ $KX = 1.94$ $KY = 0.94$

...DATA FOR $V = 0.0423$
 COEFFICIENTS FOR $X/A = 0.0$

		W*XI/O*RE*2		MX/O*RE*2		MY/O*RE*2		NXY/O*RE*2		OX/O*RE		OY/O*RE	
M= 1 N= 1													
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.101	0.0	0.0	0.0	0.0	0.0
Y/B=	0.500	0.0	0.0	0.0	0.0	0.0	0.0	0.000	0.674	0.0	0.0	0.0	0.0
M= 3 N= 3													
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.107	0.0	0.0	0.0	0.0	0.0
Y/B=	0.500	0.0	0.0	0.0	0.0	0.0	0.0	0.000	0.742	0.0	0.0	0.0	0.0
M= 5 N= 5													
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.107	0.0	0.0	0.0	0.0	0.0
Y/B=	0.500	0.0	0.0	0.0	0.0	0.0	0.0	0.000	0.785	0.0	0.0	0.0	0.0
M= 7 N= 7													
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.107	0.0	0.0	0.0	0.0	0.0
Y/B=	0.500	0.0	0.0	0.0	0.0	0.0	0.0	0.000	0.798	0.0	0.0	0.0	0.0
M= 9 N= 9													
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.107	0.0	0.0	0.0	0.0	0.0
Y/B=	0.500	0.0	0.0	0.0	0.0	0.0	0.0	0.000	0.811	0.0	0.0	0.0	0.0

COEFFICIENTS FOR X/A= 0.500

M= 1	N= 1	W=X1/0*H**4	MX/Q*H**2	MY/0*H**2	MXV/0*H**2	OX/0*H	OY/0*H
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.697
Y/B=	0.500	0.028423	0.220	0.170	-0.000	-0.000	-0.000
M= 3	N= 3						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.703
Y/H=	0.500	0.027754	0.198	0.154	-0.000	-0.000	-0.000
M= 5	N= 5						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.757
Y/H=	0.500	0.027824	0.203	0.161	-0.000	-0.000	-0.000
M= 7	N= 7						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.758
Y/H=	0.500	0.027817	0.201	0.160	-0.000	-0.000	-0.000
M= 9	N= 9						
Y/H=	0.0	0.0	0.0	0.0	0.000	0.0	0.748
Y/H=	0.500	0.027814	0.202	0.160	-0.000	-0.000	-0.000

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY = 2.00 GAMMA = 0.3985 G*TA = 1.184 ALPHA = 1.815 PSI = 10794000.

A/R = 3.00 NX/NXCR = 0.500 NY/NYCR = 0.500 KX = 0.71 KY = 0.71

...DATA FOR V = 0.0423
COEFFICIENTS FOR X/A = 0.0

	N*XI/O*B**4	NX/O*B**2	NY/O*B**2	MXY/O*B**2	QX/O*B	QY/O*B
M= 1 N= 1						
Y/R = 0.0 0.0	0.0	0.0	0.0	-0.098	0.0	0.0
Y/R = 0.500 0.0	0.0	0.0	0.0	0.000	0.568	0.0
M= 3 N= 3						
Y/R = 0.0 0.0	0.0	0.0	0.0	-0.108	0.0	0.0
Y/R = 0.500 0.0	0.0	0.0	0.0	0.000	0.674	0.0
M= 5 N= 5						
Y/R = 0.0 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/R = 0.500 0.0	0.0	0.0	0.0	0.000	0.729	0.0
M= 7 N= 7						
Y/R = 0.0 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/R = 0.500 0.0	0.0	0.0	0.0	0.000	0.751	0.0
M= 9 N= 9						
Y/R = 0.0 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/R = 0.500 0.0	0.0	0.0	0.0	0.000	0.769	0.0

COEFFICIENTS FOR X/A= 0.500

M= 1	N= 1	W ² X1/0#R**4	WX/0#R**2	Y/0#R**2	XY/0#R**2	UX/0#R	OY/0#R
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.840
Y/R=	0.500	0.042011	0.231	0.233	-0.000	-0.000	-0.000
M= 3	N= 3						
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.840
Y/R=	0.500	0.040266	0.197	0.215	-0.000	-0.000	-0.000
M= 5	N= 5						
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	-0.856
Y/R=	0.500	0.040487	0.206	0.218	-0.000	-0.000	-0.000
M= 7	N= 7						
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.850
Y/R=	0.500	0.040432	0.202	0.217	-0.000	-0.000	-0.000
M= 9	N= 9						
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.864
Y/R=	0.500	0.040451	0.204	0.218	-0.000	-0.000	-0.000

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COMPUTER PROGRAM AC-11

H/C SANDWICH PANEL STABILITY FOR IN-PLANE SHEAR LOADING

AC-11

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES COMPUTER
PROGRAM DECK NO. AC-11

ABSTRACT

This program calculates the general buckling load for a flat honeycomb (H/C) panel with orthotropic face sheets and core under in-plane shear. The program listing includes a modified eigenvalue solution subroutine which involves convergence of the geometric mean of succeeding eigenvalues. The face-sheet thicknesses are assumed to be equal.

PROGRAM INPUT DATA

The input data can be of two types, material property input or material parameter input. The input data format is all 6F12.0 except for the title card. Sample data sheets are included following the text in this section. The data description is as follows:

Card A:

TIT = any desired one-card title. The title is in columns 1-72.
If no title is desired, a blank card must be used.

Card B:

CNTL = Input data form indicator which is 0.0 for material property input (cards C₀ and D₀) and 1.0 for material parameter input (cards C₁ and D₁).

Q = Range of m and n (maximum of 13.0).

Card C₀: Used if CNTL = 0.0.

EX = Young's modulus in the x-direction.

EY = Young's modulus in the y-direction.

GXY = Shear modulus.

UMXY = Poisson's ratio ν_{xy} .

GCX = Core shear modulus in the x-direction (G'_{cx}).

GCY = Core shear modulus in the y-direction (G'_{cy}).

Card D₀: Used if CNTL = 0.0.

A = Length of panel in the x-direction, a.

B = Width of panel in the y-direction, b.

C = Core depth, c.

TF = One face-sheet thickness, t_f .

XNIT = Maximum number of iterations to be allowed (usually set at 100.0).

Card C₁: Used if CNTL = 1.0.

ALPHA = The alpha value from program AC-2 $\left(\alpha = \sqrt{E_x/E_y} \right)$.

PSI = Modulus ratio ψ found from AC-2 where $\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$.

V = Core shear parameters where $V = \pi^2 \psi t_f (c + t_f) / (2G'_{cx} b^2)$.

R = Core shear modulus ratio = G'_{cx}/G'_{cy} .

AI1 = Invariant 1 from AC-2. $I_1 = E_x (1 + E_y/E_x + 2\nu_{yx}) / (1 - \nu_{xy} \nu_{yx})$.

AI2 = Invariant 2 from AC-2. $I_2 = G_{xy} - E_x \nu_{yx} / (1 - \nu_{xy} \nu_{yx})$.

Card D₁: Used if CNTL = 1.0.

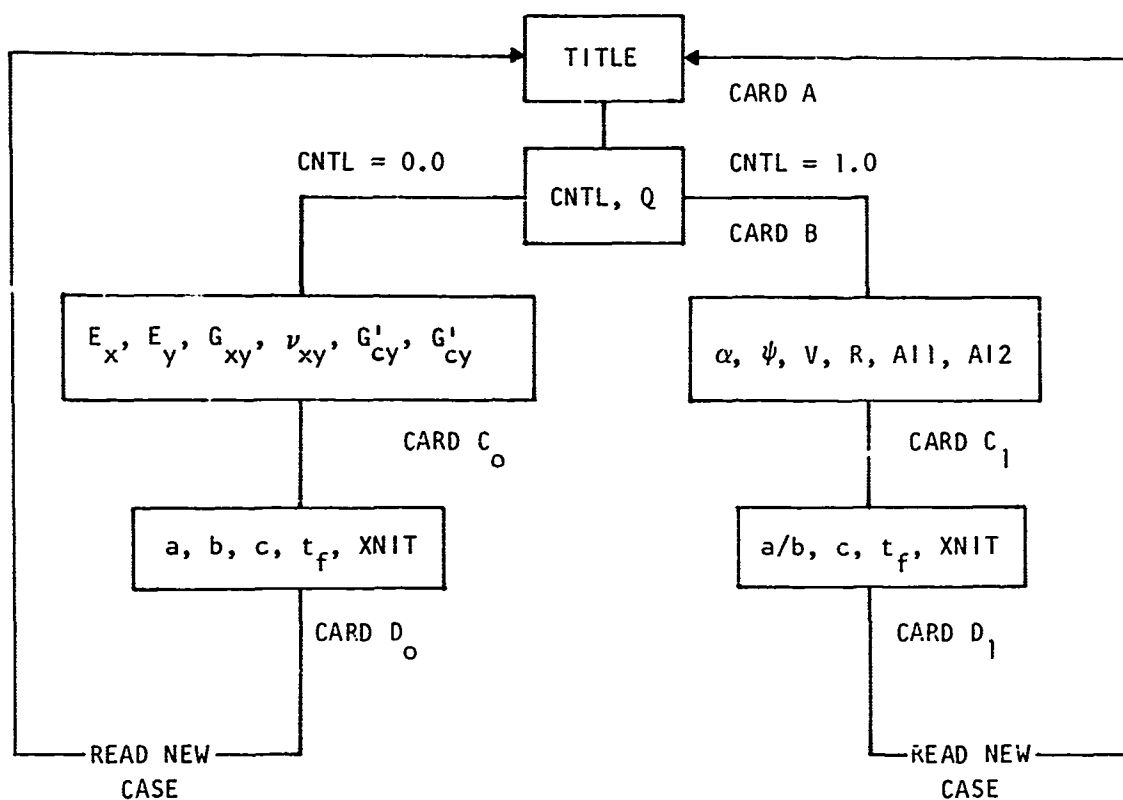
A/B = Panel aspect ratio a/b.

C = Core depth, c.

TF = Thickness of one face sheet, t_f .

XNIT = Maximum number of iterations to be allowed (usually set at 100).

The input data flow chart for AC-11 is:



Additional cases can be stacked one behind the other.

OUTPUT

The program output is of two types, depending on whether material property or material parameter input is desired. For material property input, the output includes the input data; the material parameters α , ψ , and V resulting from these data; the number of iterations (NIT) carried out for $k_{s,even}$ and $k_{s,odd}$; and the final buckling load per inch, $N_{s,cr}$. For material parameter input, the input data are printed, as well as the two values of NIT mentioned previously, and the final buckling load as a function of $1/b^2$, since for this input no b was introduced.

```

C
C PROGRAM AC11 ... H/C PANEL STABILITY FOR IN-PLANE SHEAR LOADING
C
C THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
C      A = PANEL WIDTH ALONG X-DIRECTION
C      A11 = INVARIANT 1 =  $EX*(1+EY/EX+2*UMYX)/(1-UMXY*UMYX)$ 
C      A12 = INVARIANT 2 =  $GXY - UMYX*EX/(1-UMXY*UMYX)$ 
C      ALPHA =  $SORT(EX/EY)$ 
C      AOB = PANEL ASPECT RATIO A/B
C      B = PANEL LENGTH ALONG Y-DIRECTION
C      C = CORF DEPTH
C      CNTL = CONTROL CARD FOR DESIGNATING TYPE OF INPUT TO BE USED
C              (=0 FOR MATL. PROPERTIES, OR =1 FOR MATL. PARAMETERS)
C      EIGVAL = EIGENVALUE ITERATION SOLUTION THAT TAKES THE SORT
C              OF THE ABS VALUE OF SUCCEEDING EIGENVALUES
C      GCX = CORF SHEAR MODULUS IN THE X-DIRECTION
C      GCY = CORF SHEAR MODULUS IN THE Y-DIRECTION
C      PSI =  $SORT(EX*EY)/(1-UMXY*UMYX)$ 
C      Q = NUMBER OF TERMS IN THE SERIES (MAXIMUM OF 13)
C      R =  $GCX/GCY$ 
C      TF = THICKNESS OF ONE FACE SHEET
C      TITLE = ANY DESIRED ONE-LINE TITLE IN COLS.1-72
C      V =  $PI**2*PSI*TF*(C+TF)/(2*GCX*B**2)$ 
C      XNIT = NUMBER OF ITERATIONS DESIRED IN THE SOLUTION
C
C DIMENSION M( 85, 85), TITLE(18), RB(2)
C 10 WRITE(6,50)
C 50 FORMAT(1H1)
C READ(5,100) TITLE
C 100 FORMAT(1HA4)
C WRITE(6,100) TITLE
C.....READ MATERIAL PROPERTIES
C READ(5,110) CNTL,0
C 110 FORMAT(6F12.0)
C IF(CNTL) 101,109,101
C.....READ PARAMETRIC PROPERTIES
C 101 READ(5,110) ALPHA,PSI,V,R,A11,A12,AOB,C,TF,XNIT
C CC1=A11/PSI-ALPHA-1.0/ALPHA
C RFTA=1.5*CC1+2.*A12/PSI
C GAMA=0.5*CC1+A12/PSI
C UMYX=ALPHA*(RFTA-2.0*GAMA)
C UMYX=UMYX/ALPHA**2
C AMDA=1.0-UMYX*UMYX
C EX=ALPHA*PSI*AMDA
C EY=EX/ALPHA**2
C GXY=A12+UMYX*EX/AMDA
C SX=V*(C+TF)/AMDA**2
C WRITE(6,115) ALPHA,PSI,V,R
C 115 FORMAT(1H04X,2BH THE MATERIAL PARAMETERS ARE, //10X,6HALPHA=F6.3,5X,
C 14HPSI=F12.5,5X,2HV=F7.4,5X,4HGCX/GCY=F6.3)
C WRITE(6,116) AOB,C,TF
C 116 FORMAT(1H0,4X,2BH THE GEOMETRY PROPERTIES ARE, //12X,4HA/B=F7.3,6X,
C 12HC=F7.4,10X,3HTF=F7.4)
C GO TO 131
C.....READ MATERIAL PROPERTIES
C 109 READ(5,110) EX,EY,GXY,UMYX,GCX,GCY
C.....READ PANEL GEOMETRY
C 112 READ(5,110) A,B,C,TF,XNIT
C AOB=A/B
C UMYX=EY/EX*UMYX
C ALPHA=(EX/EY)**.5
C AMDA=1.0-UMYX*UMYX

```

```

PSI=(FX*FY)**.5/AMDA
GAMA=GXY/PSI
BETA=ALPHA*UMYX+2.0*GAMA
P=C+TF
I=PSI*TF
D=I*P**2/2.0
SX=C*0.8*96/(P*A)**2/GCX
R=GCX/GCY
V=SX*P*AOR**2/C
C.....WRITE INPUT DATA
118 WRITE(6,120) FX,GCX,EY,GCY,GXY,UMXY,UMYX
120 FORMAT(1H0,4X,2HTHE MATERIAL PROPERTIES ARE //10X,15HFACE SHEET E
1X =E13.5,10X, 5HFIGURE GX =E13.5/21X,4HEY =E13.5,15X,4HGY =E13.5/20X
25H(XY =E13.5/19X,5HUMXY =F8.4/19X,6HUMYX =F8.4)
C
WRITE(6,130) A,B,C,TF,AOR
130 FORMAT(1H0,4X,2HTHE PANEL GEOMETRY IS //10X,16HPANEL LENGTH A =F8
1.3,5X,15HPANEL WIDTH B =F8.3/12X,14HCORE DEPTH C =F8.3, 5X,25HFACE
2 SHEET THICKNESS TF =F8.3/21X,5HA/B =F8.3)
WRITE(6,125) ALPHA,PSI,V,R
125 FORMAT(1H04X,2HTHE MATERIAL PARAMETERS ARE,//10X,6HALPHA=F6.3,5X,
14HPSI=E12.5,5X,2HV=F7.4,5X,8HGCX/GCY,F6.3)
C.....CALCULATE CONSTANTS
131 SY=R*SX
P=C+TF
I=PSI*TF
D=I*P**2/2.0
AOR2=AOR**2
AOR4=AOR2**2
IO=0+.001
C.....XLMN CONSTANTS
XLMN1=GAMA*AOR2
XLMN2= R /ALPHA*AOR2
XLMN3= R *GAMA
C.....XKMN CONSTANTS
XKMN1=2.0*BETA*AOR2
XKMN2=AOR4/ALPHA
C.....FMN CONSTANT
YFMN1=1.0-BETA**2
C.....VMN CONSTANTS
XVMN1=AOR2*SX
XVMN2=SX*SY
C.....DF CONSTANT
DF=TF**3/12.0*PSI
DEED=DF/D
C.....CALCULATE VARIABLE CONSTANTS AFTER CHECKING (M+N)
IP=1
134 J=0
NIT=XNIT
140 DO 200 M=1,10
A1=M
HA=3.74227*A1
A2=A1**2
A4=A2**2
IF(M-2*(H/21)) 145,150,145
145 NS=IP
GO TO 155
150 NS=3-IP
155 DO 200 N=NS,10,2
I=I+1
R1=M

```

```

      HR=HA*B1
      B2=B1**2
      B4=B2**2
C.....CALCULATE VARIABLE CONSTANTS
      XLMN =ALPHA*A2+XLMN1*B2+XLMN2*B2+XLMN3*A2
      XKMN =ALPHA*A4+XKMN1*A2+B2+XKMN2*B4
      FMN  = XFMN1+GAMA*XKMN
      VMN  =(XKMN+(XVMN1*B2+SY*A2)*FMN)/(1.0+XLMN*SX+XVMN2*FMN)
      VMN1 =DFND*XKMN
      AA=(2.0*VMN1+VMN)/AOB
C.....CHECK TO SEE IF (M+R) AND (N+S) ARE ODD OR EVEN
      J=0
      DO 190 IR=1,10
      RI=IR
      HR=HR+RI
      RI2=RI**2
      IF (IR-2*(IR/2)) 160,170,160
160  ISS=IP
      GO TO 180
170  ISS=3-IP
180  DO 190 IS=ISS,10,2
      SI=IS
      HC=HR*SI
      SI2=SI**2
      J=J+1
      IF (M+IR-2*((M+IR)/2)) 182,187,182
182  IF (N+IS-2*((N+IS)/2)) 185,187,185
185  H(I,J)=-HC / (A2-RI2)/(B2-SI2)/AA
      GO TO 190
187  H(I,J)=0.0
190  CONTINUE
200  CONTINUE
      IQ2=(IQ**2+2-IP)/2
      DELTA=.001
      CALL EIGHTH(IQ2,DELTA,H,BB(IP),NIT)
      WRITE(6,217) NIT
217  FORMAT(1H0,5X,4HNIT=16/)
      IF (IP-2)220,240,240,
220  IP=2
      GO TO 135
C.....PRINT OUT EVEN AND ODD KS WHICH ARE BB(1) AND BB(2)
240  BR(1)=BB(1)/AOB2
      BR(2)=BB(2)/AOB2
      IF (BR(1)-BB(2)) 245,250,250
245  XKS=BR(1)
      GO TO 255
250  XKS=BR(2)
255  IF (CNTL) 256,260,256
256  XKNXCR=XKS*9.8696*D
      WRITE(6,258) XKNXCR
258  FORMAT('0.5X'NSCR=XKS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) =
      IE13.5'/B**2')
      GO TO 280
260  XNCRS=XKS*9.8696*D/B**2
      WRITE(6,265) IO,BB(1),BB(2)
265  FORMAT(1H0,3RH***THE VALUES FOR KS WHEN THERE ARE 13,22H CMN COE
      EFFICIENTS ARE,/20X,10H KS EVEN =E13.5,10X, 9H KS ODD =E13.5)
      WRITE(6,270) XNCRS
270  FORMAT('0.5X'NSCR=XKS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) =
      IE13.5)
280  GO TO 10
      END

```

SUBROUTINE EIGMIN(NN,DELTA,BINV,ALAMIN,NIT)
 DIMENSION C(85),B1(85),BINV(85,85)

```

C
  II=1
  1 DO 2 I=1,NN
    C(I)=1.0
  2 CONTINUE
  N=1
  3 DO 5 I=1,NN
    B1(I)=0.0
  4 DO 5 J=1,NN
    B1(I)=B1(I)+BINV(I,J)*C(J)
  5 CONTINUE
  C1=ABS(B1(1))
  ALAMDA=C1
  8 DO 9 I=1,NN
    C(I)=B1(I)/C1
  9 CONTINUE
  IF(N-1) 11,10,11
  10 N=N+1
  BLAMDA=ALAMDA
  C4=(BLAMDA**ALAMDA)**.5
  GO TO 3
  11 C2=(ALAMDA*BLAMDA)**.5
  C3=ABS(C2-C4)
  IF(C3-DELTA) 14,14,12
  12 BLAMDA=ALAMDA
  C4=C2
  II=II+1
  IF(II-NIT) 13,14,14
  13 GO TO 3
  14 NIT=II
  ALAMIN=1.0/
  RETURN
  END

```

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T L E		Title = Any desired one card title - COL. 1-72
13			
25		Card A	
37			
49		73	
61		85	
1	C N T L		CNTL = Input data form indicator { 0.0 for matl. prop. 1.0 for matl. para.
13			
25	Q	Card B	Q = Range of M and N (Maximum of 13)
37			
49		73	
61		85	
1	E X		EX = Young's modulus in X-direction
13	E Y	IF CNTL = 0.0	EY = Young's modulus in Y-direction
25	G X Y	Card C	GXY = Shear
37	U M X Y		UMXY = Poissons ratio ν_{xy}
49	G C X	73	G'CX = Core shear modulus in X-direction - G'_{CX}
61	G C Y	85	G'CY = Core shear modulus in Y-direction - G'_{CY}
1	A		A = Length of panel in X-direction
13	B	IF CNTL = 0.0	B = Width of panel in Y-direction
25	C	Card D	C = Core depth
37	T F		TF = One face sheet thickness
49	X N I T	73	XNIT = Max. no. of iterations
61			

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A L P H A		$\text{ALPHA} = \sqrt{E_x/E_y}$
13	P S I	IF CNTL = 1.0 CARD C ₁	$\text{PSI} = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$
25	V		$V = \pi^2 \nu_{xy} t_f (c + t_f) / (2 G' b^2)$
37	R		$R = G' / G'_{cy}$
49	A I 1	73	$\text{AI1} = E_x (1 + E_y/E_x + 2 \nu_{yx}) / (1 - \nu_{xy} \nu_{yx})$
61	A I 2		$\text{AI2} = G_{xy} - E_x \nu_{yx} / (1 - \nu_{xy} \nu_{yx})$
1	A Ø B	IF CNTL = 1.0 CARD D ₁	$\text{AØB} = \text{Panel aspect ratio } a/b$
13	C		$C = \text{Core depth}$
25	T F		$\text{TF} = \text{One face sheet thickness}$
37	X N I T		$\text{XNIT} = \text{Max No. of iterations}$
49		73	
61			
1			
13			
25			
37			
49		73	
61			
1			
13			
25			
37			
49			
61		73	

EXAMPLE PROBLEM

Consider a honeycomb panel to be made of $[0/+45/0]_S$ face sheets with a core depth of 1/2 inch, $G'_{cx} = 126,000$ psi, $G'_{cx}/G'_{cy} = 2.0$, and an overall panel size of 15.0 x 15.0 inches. To illustrate the use of the program, both types of input will be used. The material parameters and material properties are taken from the AC-2 and AC-5 sample problems.

The data sheets are shown for both types of input near the end of this section. Note that for the material parameter input,

$$N_{s,cr} = 5.1021 \times 10^6 / b^2$$

and if $b = 15$, as in this example,

$$N_{s,cr} = 5.1021 \times 10^6 / (15)^2 = 22676 \text{ lb/in.}$$

as found from the output.

NUMBER		IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	(0 / + - 45 / 0) S		Title	
13	L A M I N A T E A T			
25	R Ø M T E M P E R A			
37	T U R E			
49		73		
61		1 0		
1	0 . 0		CNTL	
13	1 3 . 0		Q	
25				
37				
49		73		
61		2 . 0		
1	1 6 6 7 0 0 0 0 .		EX	
13	5 0 5 8 0 0 0 .		EY	
25	4 3 0 0 0 0 0 .		GXY	
37	. 7 0 1 5		UMXY	
49	1 2 6 0 0 0 .	73	GCX	
61	6 3 0 0 0 .	3 0	GCY	
1	1 5 0 . 0		A	
13	1 5 0 . 0		B	
25	0 . 5		C	
37	0 . 0 4 1 6		TF	
49	1 0 0 .	73	XMIT	
61		4 0		

NUMBER		IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	(0 / + - 4 5 / 0) S		TITLE	
13	L A M I N A T E A T			
25	R Ø M T E M P E R A			
37	T U R E			
49		73 . 80		
61		5.0		
1	1 . 0		CNTL	
13	1 3 . 0		Q	
25				
37				
49		73 . 80		
61		6.0		
1	1 . 8 1 5		ALPHA	
13	1 0 7 9 4 0 0 0 .		PSI	
25	. 0 4 2 3 3		V	
37	2 . 0		R	
49	3 3 8 8 4 0 0 0 .	80	AI1	
61	1 2 8 6 2 0	7 0	AI2	
1	1 . 0		AØB	
13	0 . 5		C	
25	. 0 4 1 6		TF	
37	1 0 0 .		XMIT	
49		73 . 80		
61		8.0		

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE

THE MATERIAL PROPERTIES ARE

FACE SHEET EX = 0.16670E 08
FY = 0.50580E 07
GXY = 0.43000E 07
MUXY = 0.7015
MUYX = 0.2128

CORE GX = 0.12600E 06
GY = 0.63000E 05

THE PANEL GEOMETRY IS

PANEL LENGTH A = 15.000 PANEL WIDTH B = 15.000
CORE DEPTH C = 0.500 FACE SHEET THICKNESS TF = 0.042
A/B = 1.000

THE MATERIAL PARAMETERS ARE,

ALPHA= 1.815 PSI= 0.10794E 08 V= 0.0423 GCX/GCY= 2.000

NIT= 7

NIT= 11

****THE VALUES FOR KS WHEN THERE ARE 13 CMN COEFFICIENTS ARE,

KS EVEN = 0.78484E 01 KS ODD = 0.84673E 01

NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) = 0.22670E 05

0/+45/0)S LAMINATE AT ROOM TEMPERATURE

THE MATERIAL PARAMETERS ARE,

ALPHA= 1.815 PSI= 0.10794E 08 V= 0.0423 GCX/GCY= 2.000

THE GEOMETRY PROPERTIES ARE

A/B= 1.000 C= 0.5000 TF= 0.0416

NIT= 7

NIT= 11

NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) = 0.51021E 07/B**2

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COMPUTER PROGRAM AC-12

HAT- AND Z-STIFFENED PANEL STABILITY
AND STIFFNESS

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-12

ABSTRACT

Computer program AC-12 is designed to provide the optimum minimum length and the resulting face sheet and stringer stresses for a hat- or Z-stiffened panel. These quantities are given as a function of load level and stiffener spacing as well as other geometrical and material properties. The loading is uniaxial in the x-direction.

The optimum minimum lengths of the panels are found by equating the local and Euler buckling equations and solving for the length. A program listing is given following the text in this section.

INPUT DATA

The input data, as shown on the sample data sheets in this section, are given more completely in the following listing:

Card A:

BMAT = Any desired one-card alphanumeric title limited to columns 1-72.

Card B: Face-sheet properties

$$A_{11} = E_x t / (1 - \nu_{xy} \nu_{yx})$$

$$A_{12} = \nu_{yx} A_{11}$$

A16 = Defined by equation 9 of this volume and is 0.0 for symmetric laminates.

$$A_{22} = E_y t / (1 - \nu_{xy} \nu_{yx}) = (E_y / E_x) A_{11}$$

A26 = Defined by equation 9 of this volume and is 0.0 for symmetric laminates.

$$A_{66} = G_{xy} t$$

Card C:

D11 = Flexural rigidity of the sheet in the x-direction (load direction).

$$AEXF = E_x^f = \text{Face-sheet modulus in the x-direction.}$$

T = Face-sheet thickness.

TP = Indicator for Z (TP = 0.0) and hat stiffeners.

AK7 = Buckling coefficient from equation 131 of this volume.

AKS = Stiffener buckling coefficient found from equation 132 of this volume.

Card D:

AKF = Face-sheet buckling coefficient defined by equation 132 of this volume.

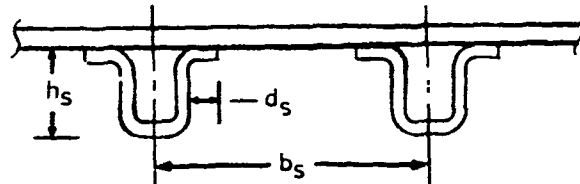
Card E:

AEXS = Young's modulus of the stiffener in the x-direction.

TNBS = Total number of stiffener spacing to be considered.

TNNX = Total number of N_x loadings to be considered.

DSØHS = Hat geometric parameter d_s/h_s (0.0 for Z-stiffener, i.e., when TP = 0.0), as defined by the following sketch:



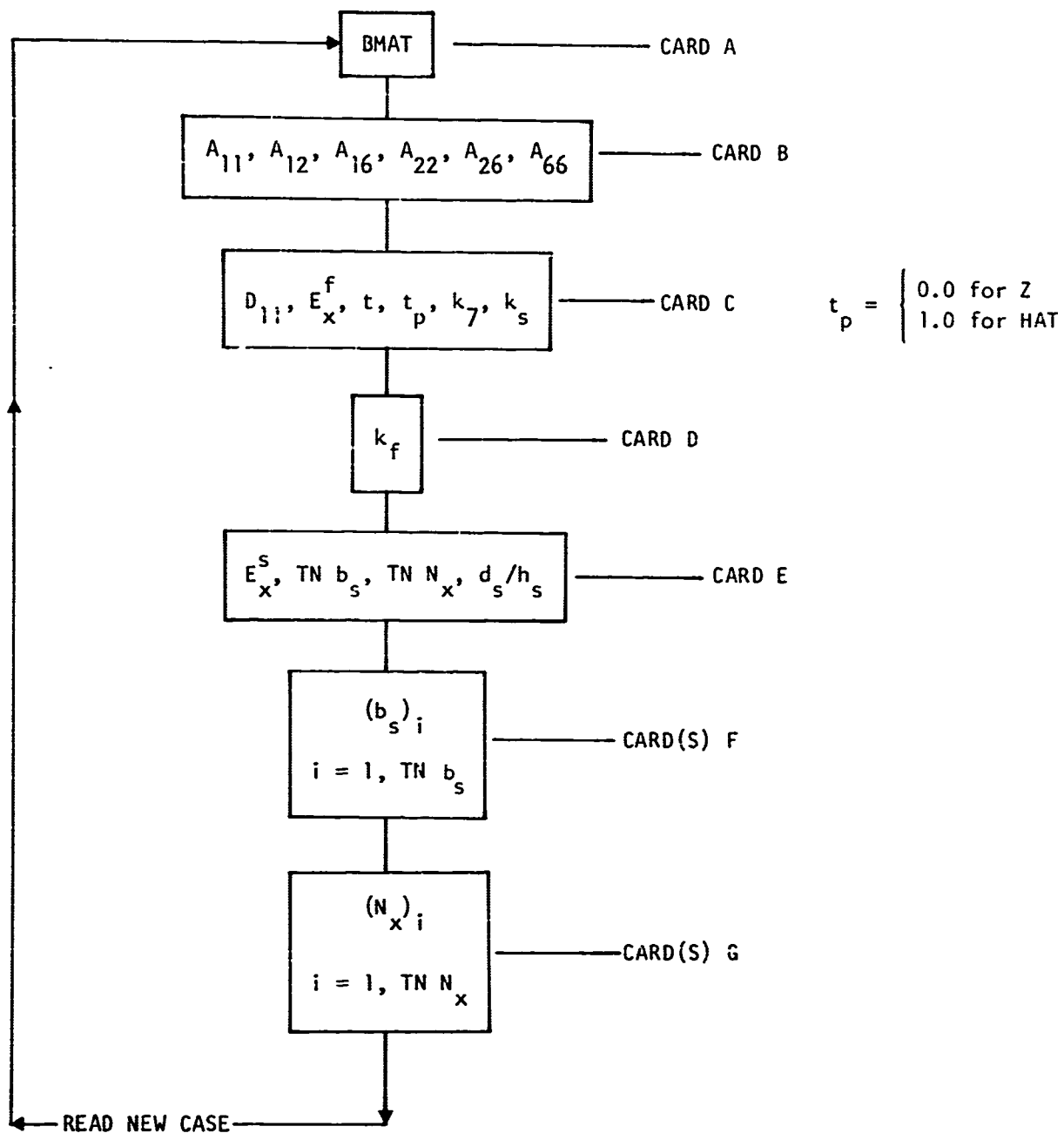
Card(s) F:

XBS(I) = Stiffener spacings $(b_s)_i$ (maximum of 30).

Card(s) G:

BNX(I) = Load levels to be considered, N_{x_i} (maximum of 30).

The input data flow chart for AC-12 is:




```

16 HS = (( ANX*AEFS/(AKS*(ALFA*T/RS)**2) -AEXF*T)*BS**2/(ALFA*T *AEFS
1*(2.*BETA +1.)) )**0.5
TS=(ALFA * HS * T)/RS
FS= BETA *HS
GO TO 20

```

C
C CALCULATION OF GEOMETRY OF HAT-STIFFENED ELEMENTS TO INSURE
C SIMULTANEOUS LOCAL BUCKLING

```

17 HS = (( ANX*AEFS/(AKS*(ALFA*T/RS)**2) -AEXF*T)*BS**2/(ALFA*T *AEFS
1*2.*(1.5+DSOHS)) )**0.5
DS= DSOHS*HS
TS= ALFA*HS *T/RS
FS=.5* HS
20 IF (TP)9,8,9
8 AK1=AEFS*TS*(2.*FS+HS)
AK2=AEFS*TS*(FS*(HS+.5*(T-TS))+.5*HS*(HS+T)+FS*.5*(TS+T))
C1=-AEFS*TS*(FS*(HS+.5*(T-TS))+.5*HS*(HS+T)+FS*.5*(TS+T))
C2=-AEFS*TS*(FS*(HS*(HS+T-TS) +TS*(TS/3.-T/2.))+T**2/4.) +FS*(T*
1*2/4. +T*TS/2.+TS**2/3.) +HS*(HS**2/3.+HS*T/2. +T**2/4.))
GO TO 10
9 AK1= 2.*AEFS*TS*(DS+HS+FS)
AK2=AEFS*TS*(DS*(TS+T)+HS*(HS+T) + 2.*FS * (HS +T/2. -TS/2.))
C1=- TS*AEFS *( DS*(TS+T) +HS*(HS+T) +2.*FS*(HS +T/2. -TS/2.))
C2= -AEFS *2.*TS *( FS*(HS*(HS+T-TS) +TS*(TS/3. -T/2.))+T**2/4. )
1 +DS*(TS**2/3. +T*TS/2. +T**2/4.)+HS*(HS**2/3. +HS*T/2. +T**2/4.))
10 DEL=(AK2/RS) / ( (A12-A16*A26/A66)**2 -(AK1/RS +A11 -A16**2/A66) *
1(A22-A26**2/A66))
BETA= DEL*(A22-A26**2/A66)
D1= -(C1/RS)*BETA-C2/RS + D11
ANXL2 = 9.8696*D1
XLCP=(ANXL2/ANY)**0.5

```

C
C DETERMINATION OF STIFFENER AND SHEET PROPERTIES

```

IF (TP)14,12,14
12 C3= AEXF*T + AEFS*TS*(2.*FS+ HS -2.*TS)/BS
GO TO 15
14 C3= AEXF*T + AEFS*TS*2.*(DS+HS+ FS)/BS
15 SIGF= ANX*AEFS/C3
SIGS= ANX*AEFS/C3
100 WRITE (6,101)RS,HS,TS,FS,D1,XLCP ,SIGS,SIGF,DS
101 FORMAT(F6.3,3F9.3,F14.2,F14.1,2F11.1,F16.3)
200 CONTINUE
GO TO 1
END

```

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	B M A T		BMAT = ANY ONE-LINE TITLE IN COLS. 1-72
13		A	
25			
37			
49		73	8
61			
1	A 1 1		$A11 = E_x^t / (1 - \nu_{xy} \nu_{yx})$
13	A 1 2	B	$A12 = \nu_{yx} A11$
25	A 1 6		A16 = Defined by Equ. 9 (0.0 for Symmetric Laminates)
37	A 2 2		$A22 = E_y^t / (1 - \nu_{xy} \nu_{yx})$
49	A 2 6	73	A26 = Defined by Equ. 9 (0.0 for Symmetric Laminates)
61	A 6 6		$A66 = G_{xy}$
1	D 1 1		D11 = Flexural rigidity of the sheet in the X-direction
13	A E X F		E_x^t = Face sheet modulus in the X-direction
25	T	C	t = Face sheet thickness
37	T P		TP = Indicator for $\bar{z}$ (TP = 0.3) and Hat (TP = 1.0)
49	A K T	73	K_T = Buckling coefficient found from Equ. 131
61	A K S		K_S = Stiffener buckling coefficient found from Equ. 132.
1	A K F		K_f = Face sheet buckling coefficient found from Equ. 132
13		D	
25			
37			
49		73	
61			

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A E X S	E	AEXS = Young's modulus of stiffener in the X-direction
13	T N B S		TNBS = Total number of stiffener spacings to be considered
25	T N N X		TNNX = Total number of loadings N _x to be considered
37	D S Ø H S		DSØHS = Hat geometric parameter (0.0 for Z - stiffener)
49		73	
61		80	
1	X B S (1)	F	XBS(1) = First stiffener spacing
13	X B S (2)		XBS(2) = Second stiffener spacing
25	X B S (3)		XBS(3) = Third stiffener spacing
37			
49		73	
61		80	(maximum of 30)
1	B N X (1)	G	BNX(1) = First applied load level
13	B N X (2)		BNX(2) = Second applied load level
25	B N X (3)		BNX(3) = Third applied load level
37			
49		73	
61		80	(Maximum of 30)
1			
13			
25			
37			
49		73	
61			

EXAMPLE PROBLEM

An example problem is given for a hat-stiffened and a Z-stiffened $[0/\pm 45/0]_S$ face sheet. The input values are given on the data sheets at the end of this section. For this laminate, from AC-2, we take the face-sheet properties as:

$$E_x = 16.67 \times 10^6 \text{ psi}$$

$$E_y = 5.06 \times 10^6 \text{ psi}$$

$$G_{xy} = 4.30 \times 10^6 \text{ psi}$$

$$\nu_{xy} = 0.7015$$

$$\nu_{yx} = (E_y/E_x) \nu_{xy} = 0.2129$$

The laminate thickness is 8 plies $\times$ 0.0052 inch/ply = 0.0416 inch.

The buckling coefficients k_7 , k_s , k_f , are found from equations 131 and 132 of this volume.

The results are given first for the Z-stiffened panels and then for the hat-stiffened panels. The panel critical lengths may be read directly from these results.

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2 0 9 0 0 0 0 0	CARD E	AEXS
13	6 0 0		TNBS
25	4 0 0		TNNX
37			
49			
61			
1	1 0 0	CARD F	XBS(1)
13	1 2 5		XBS(2)
25	1 5 0		XBS(3)
37	2 0 0		XBS(4)
49	3 0 0		XBS(5)
61	4 0 0		XBS(6)
1	3 0 0 0	CARD G	BNX(1)
13	5 0 0 0		BNX(2)
25	7 5 0 0		BNX(3)
37	9 9 9 9		BNX(4)
49			
61			
1	H A T - S T I F F E N E	CARD A	
13	P A N E L S W I T		
25	(0 / + - 4 5 / 0)		
37	F A C E S A T R		
49	Ø M T E M P E R A T		
61	U R E		

NUMBER		IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	8 0 9 0 0 0	CARD B	A11	
13	1 7 0 0 0 0		A12	
25	0 . 0		A16	
37	2 4 5 0 0 0		A22	
49	0 . 0		A26	
61	1 7 7 0 0 0		A66	
1	1 2 9 . 3	CARD C	D11	
13	1 6 6 7 0 0 0 0		AEXF	
25	0 . 0 4 1 6		T	
37	1 . 0		TP	
49	6 9 7 0 0 0		AK7	
61	1 8 0 0 0 0 0 0		AKS	
1	3 8 0 0 0 0 0 0	CARD D	AKF	
13				
25				
37				
49				
61				
1	2 9 9 0 0 0 0 0	CARD E	AEXS	
13	6 . 0		TNBS	
25	4 . 0		TNXX	
37	2 . 1 2 1		DSØHS	
49				
61				



NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	CARD P	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD C	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD E	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD F	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD G	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD H	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD I	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD J	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD K	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD L	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD M	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD N	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD O	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD P	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD Q	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD R	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD S	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD T	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD U	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD V	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD W	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD X	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			
1	CARD Y	XBS(1)	
13		XBS(2)	
25		XBS(3)	
37		XBS(4)	
49		XBS(5)	
61		XBS(6)	
1	CARD Z	BHX(1)	
13		BHX(2)	
25		BHX(3)	
37		BHX(4)	
49			
61			

ZEP-STIFFENED PANELS WITH (0/+45/0)S FACES AT ROOM TEMPERATURE

NX(LR/INC)= 3000.0 T= 0.0616

HS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.161	0.011	0.028	707.37	1.6	119171.8	66441.3	0.0
1.250	0.470	0.031	0.096	33642.80	10.4	78528.2	43781.5	0.0
1.500	0.826	0.066	0.162	162206.69	23.1	56954.5	30838.5	0.0
2.000	1.660	0.068	0.328	1207161.00	63.0	30872.5	17212.2	0.0
3.000	4.051	0.09	0.797	15387825.00	225.0	13578.3	7570.3	0.0
4.000	7.378	0.160	1.652	88648192.00	560.0	7579.6	4225.8	0.0

NX(LR/INC)= 5000.0 T= 0.0616

HS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.613	0.033	0.081	27076.11	7.4	124502.4	69613.3	0.0
1.250	0.772	0.040	0.152	167891.00	18.2	80361.7	46792.5	0.0
1.500	1.200	0.065	0.236	586217.25	36.0	55687.5	31047.2	0.0
2.000	2.278	0.092	0.468	3638562.00	86.7	31046.7	17308.2	0.0
3.000	5.367	0.166	1.052	43504768.00	293.0	13600.5	7582.6	0.0
4.000	9.639	0.195	1.807	267798400.00	699.6	7586.8	4228.7	0.0

NX(LR/INC)= 7500.0 T= 0.0616

HS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.598	0.068	0.118	96846.94	11.3	127380.7	71001.2	0.0
1.250	1.027	0.067	0.202	663716.69	26.2	81280.0	45315.7	0.0
1.500	1.566	0.083	0.304	1419846.00	43.2	56081.3	31255.6	0.0
2.000	2.863	0.110	0.563	8306704.00	105.1	31131.5	17356.6	0.0
3.000	6.619	0.179	1.302	98666736.00	359.0	13611.5	7565.5	0.0
4.000	11.875	0.260	2.337	658737920.00	857.6	7567.4	4230.1	0.0

NX(LR/INC)= 9000.0 T= 0.0616

HS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.739	0.060	0.165	196830.38	13.9	128023.7	71822.4	0.0
1.250	1.230	0.080	0.242	832252.25	28.7	81757.4	45581.8	0.0
1.500	1.828	0.099	0.360	2500816.00	60.6	56250.1	31360.8	0.0
2.000	3.367	0.135	0.650	15056129.00	121.9	31175.1	17380.9	0.0
3.000	7.602	0.207	1.512	175337600.00	416.0	13617.1	7591.9	0.0
4.000	13.751	0.278	2.706	603924864.00	990.5	7588.6	4230.0	0.0

COMPUTER PROGRAM AC-40

N-PLY LAMINATE THERMOELASTIC ANALYSIS

AC-40

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-40

ABSTRACT

This program calculates the thermal stresses and coefficients of thermal expansion in both the individual ply axes and the laminate axes, given the macromechanical properties of each ply. The program is now set up to handle boron/epoxy laminates at room temperature (R.T.), 220° F, and 350° F. If other materials or temperatures are involved, the form of input may be altered accordingly.

PROGRAM INPUT DATA

The program input is minimal and consists of only two cards per case, which are shown on the sample input sheets following the text. The data are described as follows:

Card A:

N = Number of plies per set in the laminate.

M = Material temperature indicator. 1 = R.T., 2 = 220° F, and
3 = 350° F

DT = Temperature increment.

Card B:

TH(I) = Angle of ply I. There are six of these to a card (maximum of 20).

Note that only one "set" of a laminate need be coded in. For example, for a $[0/\pm 60/90]_3$ laminate, only the 0°, +60°, -60°, 90°, and 90° plies need be input, giving a total N of five plies.

OUTPUT

The output consists of the D, B, α , and $\bar{B}$ matrices; the coefficients of thermal expansion in the laminate axes $\bar{\alpha}_x$, $\bar{\alpha}_y$, and $\bar{\alpha}_{xy}$; and the thermal stresses in the laminate axes, σ_x , σ_y , and τ_{xy} , plus the thermal stresses in the individual ply natural axes, σ_L , σ_T , and τ_{LT} .

The output is shown on the sample output pages following the text.

```

C
C _____ PROGRAM AC40...N-PLY LAMINATE THERMOELASTIC ANALYSIS PROGRAM _____
C
C INPUT DATA IS AS FOLLOWS.....
C      N = NUMBER OF PLYS
C      M = MATL. PROP. TEMP. INDICATOR....M=1 FOR R.T., M=2 FOR 220F,
C          M=3 FOR 350F
C      DT = TEMPERATURE INCREMENT IN DEGREES F
C      TH(I) = INDIVIDUAL PLY ANGLES THETA(I)
C
C      DIMENSION IL(20),ET(20),GLT(20),ULT(20),UTL(20),AL(20),AT(20),
1      H(20),A(20,3,3),SL(20),ST(20),SLT(20),TH(20)
C      DIMENSION T(20,3,3),FM(20),EM(20),R(20,3,3),D1(20,3,3),AX(20),
1      AXY(20),HR(3,3),SX(20),SXY(20),BI(3,3),AY(20)
C      DIMENSION D(20,3,3)
C      EQUIVALENCE (D(1),D1(1))
C      5 READ(5,10) N,M,DT
C      10 FORMAT(2I12, F12.0)
C TH(I) = ANGLE BETWEEN NATURAL AXES OF I-TH PLY AND LAMINATE AXES
15 READ(5,20) (TH(I),I=1,N)
20 FORMAT(6F12.0)
C PROPERTIES OF INDIVIDUAL LAMINA
40 DO 100 I=1,N
C CALCULATE ELEMENTS OF T MATRIX
IF(M=2)42,43,44
C ROOM TEMPERATURE PROPERTIES
42 EL(I)=29900000.
FT(I)=2710000.
GLT(I)=700000.
ULT(I)=.21
UTL(I)=ULT(I)*ET(I)/EL(I)
AL(I)=.00000232
AT(I)=.00001067
H(I)=.0052
GO TO 46
C 220 DEGREE PROPERTIES
43 EL(I)=29900000.
ET(I)=1420000.
GLT(I)=377100.
ULT(I)=.21
UTL(I)=ULT(I)*ET(I)/EL(I)
AL(I)=.000002414
AT(I)=.00001394
H(I)=.0052
GO TO 46
C 350 DEGREE PROPERTIES
44 EL(I)=29900000.
ET(I)=1130000.
GLT(I)=300000.
ULT(I)=.21
UTL(I)=ULT(I)*ET(I)/EL(I)
AL(I)=.00000283
AT(I)=.00001978
H(I)=.0052
46 T(I,1,1)=EL(I)/(1.-ULT(I)*UTL(I))
T(I,1,2)=EL(I)*UTL(I)/(1.-ULT(I)*UTL(I))
T(I,1,3)=0.
T(I,2,1)=T(I,1,2)
T(I,2,2)=FT(I)/(1.-ULT(I)*UTL(I))
T(I,2,3)=0.
T(I,3,1)=0.
T(I,3,2)=0.

```

```

      T(I,3,3)=GLT(I)
C CALCULATE ELEMENTS OF B MATRIX
      EM(I)=COSD(TH(I))
      EN(I)=SIND(TH(I))
50  B(I,1,1)=T(I,1,1)*FM(I)**4+2.*(T(I,1,2)+2.*T(I,3,3))*(EM(I)**2)*
      1(FN(I)**2)+T(I,2,2)*FN(I)**4
      B(I,1,2)=(T(I,1,1)+T(I,2,2)-4.*T(I,3,3))*(EM(I)**2)*(FN(I)**2)+T(I
      1,1,2)*(EM(I)**4+EN(I)**4)
      B(I,1,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EN(I)*EM(I)**3+(T(I,1,2)-
      1T(I,2,2)+2.*T(I,3,3))*FM(I)*FN(I)**3
      B(I,2,2)=T(I,1,1)*FN(I)**4+2.*(T(I,1,2)+2.*T(I,3,3))*(FN(I)**2)*
      1(EM(I)**2)+T(I,2,2)*EM(I)**4
      B(I,2,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EM(I)*EN(I)**3+(T(I,1,2)-
      1T(I,2,2)+2.*T(I,3,3))*EN(I)*EM(I)**3
      B(I,3,3)=(T(I,1,1)+T(I,2,2)-2.*T(I,1,2)-2.*T(I,3,3))*(FN(I)**2)*
      1(FM(I)**2)+T(I,3,3)*(EM(I)**4+FN(I)**4)
      B(I,2,1)=B(I,1,2)
      B(I,3,1)=B(I,1,3)
      B(I,3,2)=B(I,2,3)
C CALCULATE ELEMENTS OF INVERSE OF D MATRIX
70  DI(I,1,1)=EM(I)**2
      DI(I,1,2)=EN(I)**2
      DI(I,1,3)=-2.*EM(I)*EN(I)
      DI(I,2,1)=DI(I,1,2)
      DI(I,2,2)=DI(I,1,1)
      DI(I,2,3)=-DI(I,1,3)
      DI(I,3,1)=DI(I,2,3)/2.
      DI(I,3,2)=DI(I,1,3)/2.
80  DI(I,3,3)=EM(I)**2-EN(I)**2
C CALCULATE ELEMENTS OF ALPHA MATRIX
      AX(I)=DI(I,1,1)*AL(I)+DI(I,1,2)*AT(I)
      AY(I)=DI(I,2,1)*AL(I)+DI(I,2,2)*AT(I)
90  AXI(I)=DI(I,3,1)*AL(I)+DI(I,3,2)*AT(I)
100 CONTINUE
      WRITE(6,710)(I,DI(I,1,1),DI(I,1,2),DI(I,1,3),I,DI(I,2,1),DI(I,2,2)
      1,DI(I,2,3),I,DI(I,3,1),DI(I,3,2),DI(I,3,3),I=1,N)
710  FORMAT(1H1,30X,23HELEMENTS OF DI MATRIX  ///(10X,12,5X,E17.8,5X,E1
      17.8,5X,E17.8))
      WRITE(6,720)(I,B(I,1,1),B(I,1,2),B(I,1,3),I,B(I,2,1),B(I,2,2),B(I,
      12,3),I,B(I,3,1),B(I,3,2),B(I,3,3),I=1,N)
720  FORMAT(1H1,30X,22HELEMENTS OF B MATRIX  ///(10X,12,5X,E17.8,5X,E17
      1.8,5X,E17.8))
      WRITE(6,740)(I,AX(I),AY(I),AXI(I),I=1,N)
740  FORMAT(1H1,30X,26HELEMENTS OF ALPHA MATRIX  ///(10X,12,5X,E17.8,5X
      1,E17.8,5X,E17.8))
C CALCULATE PROPERTIES OF N-PLY LAMINATE
C CALCULATE ELEMENTS OF B-BAR MATRIX
      BB(1,1)=0.
      BB(1,2)=0.
      BB(1,3)=0.
      BB(2,1)=0.
      BB(2,2)=0.
      BB(2,3)=0.
      BB(3,1)=0.
      BB(3,2)=0.
      BB(3,3)=0.
120  DO 200 I=1,N
      DB1=B(I,1,1)*H(I)
      BB(1,1)=BB(1,1)+DB1
      DB2=B(I,1,2)*H(I)

```



```

      RR(1,2)=RR(1,2)+DB2
      DB3=R(1,1,3)*H(I)
      RR(1,3)=RR(1,3)+DB3
140  DB4=R(1,2,1)*H(I)
      RR(2,1)=RR(2,1)+DB4
      DB5=R(1,2,2)*H(I)
      RR(2,2)=RR(2,2)+DB5
      DB6=R(1,2,3)*H(I)
      RR(2,3)=RR(2,3)+DB6
      DB7=R(1,3,1)*H(I)
160  RR(3,1)=RR(3,1)+DB7
      DB8=R(1,3,2)*H(I)
      RR(3,2)=RR(3,2)+DB8
      DB9=R(1,3,3)*H(I)
      RR(3,3)=RR(3,3)+DB9
200  CONTINUE
      WRITE(6,760) RR(1,1),RR(1,2),RR(1,3),RR(2,1),RR(2,2),RR(2,3),RR(3,
1) ,RR(3,2),RR(3,3)
760  FORMAT(1H1,30X,26HELEMENTS OF B BAR MATRIX /// (17X,F17.8,5X,F17.8
1,5X,F17.8))
C CALCULATE ELEMENTS OF C MATRIX
      CX=0.
      DO 300 I=1,N
      DCX=(AX(I)*R(1,1,1)+AY(I)*R(1,1,2)+AXY(I)*R(1,1,3))*H(I)
      CX=CX+DCX
300  CONTINUE
      CY=0.
      DO 400 I=1,N
      DCY=(AX(I)*R(1,2,1)+AY(I)*R(1,2,2)+AXY(I)*R(1,2,3))*H(I)
      CY=CY+DCY
400  CONTINUE
      CXY=0.
      DO 500 I=1,N
      DCXY=(AX(I)*R(1,3,1)+AY(I)*R(1,3,2)+AXY(I)*R(1,3,3))*H(I)
      CXY=CXY+DCXY
500  CONTINUE
C CALCULATE ELEMENTS OF INVERSE OF R-BAR MATRIX
      DBR=RR(1,1)*(RR(2,2)*RR(3,3)-RR(2,3)*RR(3,2))+RR(1,3)*(RR(2,1)*RR(
13,2)-RR(2,2)*RR(3,1))+RR(1,2)*(RR(3,1)*RR(2,3)-RR(3,3)*RR(2,1))
      RI(1,1)=(RR(2,2)*RR(3,3)-RR(2,3)*RR(3,2))/DBR
520  RI(1,2)=-(RR(1,2)*RR(3,3)-RR(1,3)*RR(3,2))/DBR
      RI(1,3)=(RR(1,2)*RR(2,3)-RR(1,3)*RR(2,2))/DBR
      RI(2,1)=-(RR(2,1)*RR(3,3)-RR(2,3)*RR(3,1))/DBR
      RI(2,2)=(RR(1,1)*RR(3,3)-RR(1,3)*RR(3,1))/DBR
540  RI(2,3)=-(RR(1,1)*RR(2,3)-RR(1,3)*RR(2,1))/DBR
      RI(3,1)=(RR(2,1)*RR(3,2)-RR(2,2)*RR(3,1))/DBR
      RI(3,2)=-(RR(1,1)*RR(3,2)-RR(1,2)*RR(3,1))/DBR
560  RI(3,3)=(RR(1,1)*RR(2,2)-RR(1,2)*RR(2,1))/DBR
C CALCULATE ELEMENTS OF ALPHA-BAR MATRIX
      AXB=RI(1,1)*CX+RI(1,2)*CY+RI(1,3)*CXY
      AYB=RI(2,1)*CX+RI(2,2)*CY+RI(2,3)*CXY
580  AXYB=RI(3,1)*CX+RI(3,2)*CY+RI(3,3)*CXY
      WRITE(6,780) AXB,AYB,AXYB
780  FORMAT(1H1,30X,2HELEMENTS OF ALPHA BAR MATRIX//22X,11HALPHA BAR X
1,11X,11HALPHA BAR Y,11X,12HALPHA BAR XY//(17X,E17.8,5X,E17.8,5X,E1
27.8))
C CALCULATE STRAINS DEVELOPED IN LAMINATE DUE TO UNIFORM TEMPERATURE
C INCREASE DT
      EPX=DT*AXB
      EPY=DT*AYB
      EPXY=DT*AXYB

```

C CALCULATE STRESSES IN EACH PLY

700 DO 600 I=1,N

SX(I)=B(I,1,1)*(EPX-DT*AX(I))+B(I,1,2)*(EPY-DT*AY(I))+B(I,1,3)*
1(FPXY-DT*AXY(I))

S(I)=B(I,2,1)*(EPX-DT*AX(I))+B(I,2,2)*(EPY-DT*AY(I))+B(I,2,3)*
1(FPXY-DT*AXY(I))

SXY(I)=B(I,3,1)*(EPX-DT*AX(I))+B(I,3,2)*(EPY-DT*AY(I))+B(I,3,3)*
1(FPXY-DT*AXY(I))

A(I,1,1)=T(I,1,1)*D(I,1,1)+T(I,1,2)*D(I,2,1)+T(I,1,3)*D(I,3,1)

A(I,1,2)=T(I,1,1)*D(I,1,2)+T(I,1,2)*D(I,2,2)+T(I,1,3)*D(I,3,2)

A(I,1,3)=T(I,1,1)*D(I,1,3)+T(I,1,2)*D(I,2,3)+T(I,1,3)*D(I,3,3)

A(I,2,1)=T(I,2,1)*D(I,1,1)+T(I,2,2)*D(I,2,1)+T(I,2,3)*D(I,3,1)

A(I,2,2)=T(I,2,1)*D(I,1,2)+T(I,2,2)*D(I,2,2)+T(I,2,3)*D(I,3,2)

A(I,2,3)=T(I,2,1)*D(I,1,3)+T(I,2,2)*D(I,2,3)+T(I,2,3)*D(I,3,3)

A(I,3,1)=T(I,3,1)*D(I,1,1)+T(I,3,2)*D(I,2,1)+T(I,3,3)*D(I,3,1)

A(I,3,2)=T(I,3,1)*D(I,1,2)+T(I,3,2)*D(I,2,2)+T(I,3,3)*D(I,3,2)

A(I,3,3)=T(I,3,1)*D(I,1,3)+T(I,3,2)*D(I,2,3)+T(I,3,3)*D(I,3,3)

SL(I)=A(I,1,1)*EPX+A(I,1,2)*EPY+A(I,1,3)*EPXY-(T(I,1,1)*AL(I)+T(I,
1,2)*AT(I))*DT

ST(I)=A(I,2,1)*EPX+A(I,2,2)*EPY+A(I,2,3)*EPXY-(T(I,2,1)*AL(I)+T(I,
1,2)*AT(I))*DT

SLT(I)=A(I,3,1)*EPX+A(I,3,2)*EPY+A(I,3,3)*EPXY-(T(I,3,1)*AL(I)+T(I,
1,3,2)*AT(I))*DT

600 CONTINUE

WRITE(6,800)(I,SX(I),SY(I),SXY(I),I=1,N)

800 FORMAT(1H1,30X,16HTHERMAL STRESSES//8X,7HPLY NO., 8X,7HALPHA X,15
1X,7HALPHA Y,15X,8HALPHA XY//10X,12,5X,E17.8,5X,E17.8,5X,E17.8))

WRITE(6,900)(I,SL(I),ST(I),SLT(I),I=1,N)

900 FORMAT(1H1,30X,29HTHERMAL STRESSES NATURAL AXES//8X,7HPLY NO., 8X,
17HALPHA L,15X,7HALPHA T,15X,8HALPHA LT//10X,12,5X,E17.8,5X,E17.8,
25X,E17.8))

GO TO 5

END

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	N	A	N = Number of plies (20 max.)
13	M		M = Mat'l prop temp indicator { 1 = R.T. 2 = 220° F 3 = 350° F
25			
37	DT		DT = Temperature Increment
49		73	
61		80	
1	TH (1)	B	TH(1) = Angle of ply 1
13	TH (2)		TH(2) = Angle of ply 2
25	TH (3)		TH(3) = Angle of ply 3
37			
49		73	
61		80	
1			(Maximum of 20)
13			
25			
37			Start next case
49		73	
61		80	
1			
13			
25			
37			
49		73	
61		80	
1			
13			
25			
37			
49		73	
61		80	

EXAMPLE PROBLEM

To illustrate the use of AC-40, consider a $[0/\pm 45/0]_S$ laminate at room temperature with a $\Delta T = 10^\circ$. For this case, on card A, there are four plies per set, so $N = 4$. M is set to 1 for room temperatures properties, and DT is set to 10.0 since $\Delta T = 10^\circ$. For card B, only the angles 0.0 , $+45.0$, -45.0 , and 0.0 are input. The input data sheets and resulting output for this case are shown at the end of this section.

The results from the output printout are:

$$\bar{\alpha}_x = 2.363 \times 10^{-6} \text{ in./in./}^\circ\text{F}$$

$$\bar{\alpha}_y = 7.752 \times 10^{-6} \text{ in./in./}^\circ\text{F}$$

$$\bar{\alpha}_{xy} = 0.0$$

Ply	$\sigma_x/10^\circ \text{ F}$	$\sigma_y/10^\circ \text{ F}$	$\tau_{xy}/10^\circ \text{ F}$	$\sigma_L/10^\circ \text{ F}$	$\sigma_T/10^\circ \text{ F}$	$\tau_{LT}/10^\circ \text{ F}$
1	-3.7	-79.1	0	-3.7	-79.1	0
2	3.7	79.1	133.6	789.8	-137.1	-18.9
3	3.7	79.1	-133.6	789.8	-137.1	18.9
4	-3.7	-79.1	0	-3.7	-79.1	0

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	4	A	N
13	1		M
25	1 0 .		DT
37			
49			
61		73 80	
1	0 .	B	TH(1)
13	4 5 .		TH(2)
25	- 4 5 .		TH(3)
37	0 .		TH(4)
49			
61		73 80	
1		2 0	
13			
25			
37			
49			
61		73 80	
1			
13			
25			
37			
49			
61		73 80	
1			
13			
25			
37			
49			
61		73 80	

ELEMENTS OF ALPHA MATRIX

1	0.23199991E-05	0.10670000E-04	0.0
2	0.64950009E-05	0.64950009E-05	-0.41750000E-05
3	0.64950009E-05	0.64950009E-05	0.41750000E-05
4	0.23199991E-05	0.10670000E-04	0.0

ELEMENTS OF B BAR MATRIX

	0.4075A513E 06	0.86759750E 05	0.0
	0.86759750E 05	0.12367444E 06	0.0
	0.0	0.0	0.89434938E 05

ELEMENTS OF DI MATRIX

1	0.10000000E 01	0.0	0.0
1	0.0	0.10000000E 01	0.0
1	0.0	0.0	0.10000000E 01
2	0.50000018E 00	0.50000018E 00	-0.99999994E 00
2	0.50000018E 00	0.50000018E 00	0.99999994E 00
2	0.49999994E 00	-0.49999994E 00	0.0
3	0.50000018E 00	0.50000018E 00	0.99999994E 00
3	0.50000018E 00	0.50000018E 00	-0.99999994E 00
3	-0.49999994E 00	0.49999994E 00	0.0
4	0.10000000E 01	0.0	0.0
4	0.0	0.10000000E 01	0.0
4	0.0	0.0	0.10000000E 01

ELEMENTS OF B MATRIX

1	0.30019984E 08	0.57138356E 06	0.0
1	0.57138356E 06	0.27208750E 07	0.0
1	0.0	0.0	0.70000000E 06
2	0.91709110E 07	0.77709080E 07	0.68247770E 07
2	0.77709080E 07	0.91709110E 07	0.68247770E 07
2	0.68247770E 07	0.68247770E 07	0.78995250E 07
3	0.91709110E 07	0.77709080E 07	-0.68247770E 07
3	0.77709080E 07	0.91709110E 07	-0.68247770E 07
3	-0.68247770E 07	-0.68247770E 07	0.78995250E 07
4	0.30019984E 08	0.57138356E 06	0.0
4	0.57138356E 06	0.27208750E 07	0.0
4	0.0	0.0	0.70000000E 06

ELEMENTS OF ALPHA BAR MATRIX

ALPHA BAR X	ALPHA BAR Y	ALPHA BAR XY
0.23632065E-05	0.77521199E-05	0.0

THERMAL STRESSES

PLY NO.	ALPHA X	ALPHA Y	ALPHA XY
1	-0.37013893E 01	-0.79144974E 02	0.0
2	0.37009277E 01	0.79145752E 02	0.13361493E 03
3	0.37009277E 01	0.79145752E 02	-0.13361493E 03
4	-0.37013893E 01	-0.79144974E 02	0.0

THERMAL STRESSES NATURAL AXES

PLY NO.	ALPHA L	ALPHA T	ALPHA LT
1	-0.37014160E 01	-0.79144806E 02	0.0
2	0.78977881E 03	-0.13706204E 03	-0.18861176E 02
3	0.78977881E 03	-0.13706204E 03	0.18861176E 02
4	-0.37014160E 01	-0.79144806E 02	0.0

/10 00,200,1001,01,00,0000,3000
 /ETC NAME=AC2,LIST,DECK
 RPS FORTRAN D COMPILER

12/02/69

12/02/69

PROGRAM AC2 *** EXTENSIONAL AND FLEXURAL CONSTANTS FOR BALANCED,
 SYMMETRICAL LAMINATES OF THE FORM (0/+/-THETA/90)NS

THE INPUT DATA IS AS FOLLOWS.....
 THE FIRST DATA CARD MUST CONTAIN THE LAMINATE ELASTIC CONSTANTS IN
 THE FOLLOWING FORMAT... (A,FL,ET,GLT,ULT WHERE.....
 THE LAMINATE ELASTIC CONSTANTS ARE A,FL,ET,GLT,ULT WHERE.....
 A IS A CONTROL CARD THAT INDICATES WHICH TYPE OF ELASTIC
 CONSTANTS ARE TO BE USED. A=-1.0 IS FOR COMPRESSION
 LOADINGS, A=0.0 IS FOR TENSION-COMPRESSION LOADINGS, AND
 A=1.0 IS FOR TENSION LOADINGS.
 FL IS THE LAMINA LONGITUDINAL YOUNG'S MODULUS
 ET IS THE LAMINA TRANSVERSE YOUNG'S MODULUS
 GLT IS THE LAMINA SHEAR MODULUS
 ULT IS THE LAMINA POISSON'S RATIO

THE SECOND AND FOLLOWING DATA CARDS MUST CONTAIN THE SET
 DESCRIPTIONS IN THE FOLLOWING FORMAT... (I4,F8.0,3I4,F12.0,2I4)....
 THE SET DESCRIPTIONS INCLUDE OF, H, IP1, IP2, THETA, IPS1, AND IPS2
 WHERE.....
 IO IS THE TOTAL NUMBER OF SETS IN THE LAMINATE (IT MUST ALWAYS
 BE AN EVEN INTEGER)
 H IS THE NOMINAL PLY THICKNESS (FLOAT, PT.)
 IP1 IS THE NUMBER OF INNER PLYS IN EACH SET IN THE ZERO OR
 DEGREE DIRECTION DEPENDENT ON IPS1 (INTEGER)
 IPT IS THE NUMBER OF MIDDLE PLYS IN EACH 0 SET IN THE PLUS AND
 MINUS THETA DIRECTION (INTEGER)
 IPP IS THE NUMBER OF OTHER PLYS IN EACH 0 SET IN THE ZERO OR
 DEGREE DIRECTION DEPENDENT ON IPS2 (INTEGER)
 THETA IS THE PLUS AND MINUS ANGLE OF THE P2 PLYS (FLOAT, PT.)
 IPS1 IS AN INDICATOR TO DETERMINE IF THE ANGLES ASSOCIATED WITH
 P1 ARE ZERO OR NINETY DEGREES. IF IPS1 IS ZERO THEN THESE ANGLES
 ARE ZERO DEGREES. IF IPS1 IS 90 THEN THESE ANGLES ARE 90 DEGREES
 IPS2 IS THE SAME AS IPS1 EXCEPT THAT IPS2 REFERS TO P2

DIMENSION D(6),OPR(4,3),DC(3)
 READ(5, 999) A,EL,ET,GLT,ULT

S.0001
 S.0002


```

S.00003 999 FORMAT(5F12.0)
S.00004 2000 READ(5,1000) IG,IG,IP1,IP2,THETA,IPSI,IPSI
S.00005 1000 FORMAT(16,F8.0,314,F12.0,2/14)

C
S.00006 IF(A)7,5,3
S.00007 3 WRITE(6,6)
S.00008 4 FORMAT(11...LAMINATE ELASTIC CONSTANTS FOR TENSION LOADINGS)
S.00009 GO TO 16
S.00010 5 WRITE(6,6)
S.00011 6 FORMAT(11...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS)
S.00012 GO TO 16
S.00013 7 WRITE(6,6)
S.00014 8 FORMAT(11...LAMINATE ELASTIC CONSTANTS FOR COMPRESSION LOADINGS)
S.00015 16 WRITE(6,9) FL,FT,GLT,ULT
S.00016 9 FORMAT(10 F12.5,5X,ET=1F12.5,5X,GLT=1F12.5,5X,ULT=1F8.4///)

C
S.00017 ON(10
S.00018 P1=1P1
S.00019 PT=1PT
S.00020 P2=1P2
S.00021 RK=P1+2*PT+P2
S.00022 XN=0RK
S.00023 R1=XN/2-RK
S.00024 R2=P1+P1
S.00025 R3=XN/2-P2
S.00026 T=XN*H
S.00027 1PT2=2*PT

C
S.00028 1=0
S.00029 P1=P1
S.00030 R=R1
S.00031 100 1=1+1
S.00032 V=0.001/AN(2u(PN+1)+3u(2uPN+4uR-3uXN-2)+3u(2uR-XN-1)+u2+4K*(0+2)u(3+
1uN+6uR-3uXN+R*(1+u)+1)
S.00033 11-1-2) 110,120,130
S.00034 110 UC(1)=V
S.00035 P1=2*PT
S.00036 R=R2
S.00037 100 10 100
S.00038 120 UC(1)=V
S.00039 P1=P2
S.00040 R=R4
S.00041 GO TO 100

```

S.0042	130	DC(T)=V	
	C		
S.0043		C=COSD(THETA)	
S.0044		S=SIND(THETA)	
S.0045		C2S2=(C*S)*2	
S.0046		S2=S**2	
S.0047		C2=C**2	
S.0048		C21S2=C2*S2	
S.0049		C4=C**4	
S.0050		S4=S**4	
S.0051		C4PS4=C4*S4	
	C		
S.0052		UTL=(F/T+L)*ULT	
S.0053		PSI=1-ULT*UTL	
S.0054		O11=FL/PSI	
S.0055		O12=O11*UTL	
S.0056		Q22=E1/PSI	
S.0057		O66=GLT	
	C		
S.0058		OPR(1,2)=O11*(C4+2*(Q12+2*O66)*C2S2+O22*S4	
S.0059		OPR(2,1)=O12	
S.0060		OPR(2,2)=(O11+O22-4*O66)*C2S2+O12*(C4+S4)	
S.0061		OPR(2,3)=O12	
S.0062		OPR(3,2)=O11*S4+2*(Q12+2*O66)*C2S2+O22*C4	
S.0063		OPR(4,1)=O66	
S.0064		OPR(4,2)=(O11+O22-2*O12-2*O66)*C2S2+O66*(C4+S4)	
S.0065		OPR(4,3)=O66	
S.0066		IF(PSI=0) 140,145,145	
S.0067	140	OPR(1,1)=Q11	
S.0068		OPR(3,1)=O22	
		GO TO 150	
S.0069	145	OPR(1,1)=O22	
S.0070		OPR(3,1)=O11	
S.0071	150	IF(PS2=0) 155,160,160	
S.0072	155	OPR(1,3)=O11	
S.0073		OPR(3,3)=O22	
S.0074		GO TO 165	
S.0075	160	OPR(1,3)=Q22	
S.0076		OPR(3,3)=O11	
S.0077	165	C=O*TIME	
S.0078	C		
S.0079		AN=.6656666*(T/XN)**3.0	
S.0080		DO 170 I=1,4	
S.0081		D(I)=0.0	

```

S.0082      DA 170 J=1,3
S.0083      170 D(1)=D(1)+CPO(1,J)*DC(J)*AM
C
S.0084      016 T=(011-012-2*066)*C**3*S+(012-022+2*066)*C**S**3
S.0085      026 T=(011-012-2*066)*C**3*S+(012-022+2*066)*C**3*S
C
S.0086      CON=2*(016 T*(T/XN)**3*(-PT+XN/2-R2-0.5/4*(0+2))
S.0087      D(5)=016 T*CON
S.0088      D(6)=026 T*CON
C
S.0089      RATIO1=D(5)/D(1)
S.0090      RATIO2=D(5)/D(2)
S.0091      RATIO3=D(6)/D(1)
S.0092      P1104=D(6)/D(2)
C
S.0093      A1=P*OPR(1,1)+2*PT*OPR(1,2)+P2*OPR(1,3)
S.0094      A12=P1*OPR(2,1)+2*PT*OPR(2,2)+P2*OPR(2,3)
S.0095      A2=P1*OPR(3,1)+2*PT*OPR(3,2)+P2*OPR(3,3)
S.0096      A6=P1*OPR(4,1)+2*PT*OPR(4,2)+P2*OPR(4,3)*P2
C
S.0097      UYX=A12/A1
S.0098      UXY=A12/A2
S.0099      GXY=A6/RK
S.0100      EX=(A1*A2-A12**2)/A2/RK
S.0101      FY=(A1*A2-A12**2)/A1/RK
S.0102      AIF=(1/X+1/Y)**.5
S.0103      PS11=(FX*FY)**.5/(1.0-UXY*UYX)
S.0104      R=1-UYX*UYX
S.0105      A11=EX*(1.0+FY/FX+2.0-UYX)/R
S.0106      A12=EXY-FX*UYX/B
C
S.0107      WRITE(6,575)
S.0108      575 FORMAT(0...SET DESCRIPTION)
S.0109      WRITE(6,576)D,H,T
S.0110      576 FORMAT(0 THERE ARE 14 SETS OF PLIES WITH A NOMINAL PLY THICK-
        1 INFS OF 7.4 INCHES.// THE TOTAL LAMINATE THICKNESS IS 10.4
        2 INCHES.//)
C
S.0111      IF(IPS1-90) 577,579,579
C
S.0112      577 WRITE(6,578)IP1
S.0113      578 FORMAT(0 NUMBER OF INNER PLIES AT 0 DEGRS =13)
C
S.0114      GO TO 585

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S.0115 WRITE(6,500) 121
S.0116 FORMAT(10,NUMBER OF INNER PLIES AT 90 DEGREES='I3)
C
S.0117 WRITE(6,500) TOTAL,IP12
S.0118 FORMAT(10,NUMBER OF MIDDLE PLIES AT (+ AND -)'F7.2',DEGREES='I3)
C
S.0119 11(IP52-90) 600,020,620
C
S.0120 600 WRITE(6,610)IP2
S.0121 610 FORMAT(10,NUMBER OF OUTER PLIES AT 0 DEGREES='I3//)
S.0122 GO TO 645
C
S.0123 620 WRITE(6,630)IP2
S.0124 630 FORMAT(10,NUMBER OF OUTER PLIES AT 90 DEGREES='I3//)
C
S.0125 635 WRITE(6,640)
S.0126 640 FORMAT(10,...LAMINATE FLEXURAL CONSTANTS//)
C
S.0127 WRITE(6,650) D(1),D(2),D(3),A1,A12,D(4),D(5),D(6),A2,A6
S.0128 650 FORMAT(1011='F12.5,3X'D(12='F12.5,3X'D(22='F12.5,7X'A11='F12.5,
13X'A12='F12.5,7X'D(66='F12.5,3X'D(16='F12.5,3X'D(26='F12.5,7X'A22='
20F12.5,5X'A66='F12.5//)
C
S.0129 WRITE(6,660)
S.0130 660 FORMAT(10,...RATIOS BETWEEN FLEXURAL CONSTANTS//)
C
S.0131 670 WRITE(6,670) RATIO1,RATIO2,RATIO3,RATIO4
S.0132 670 FORMAT(1016/O11='F12.5,5X'O16/O22='F12.5,5X'O26/O11='F12.5,5X'O
126/O22='F12.5//)
C
S.0133 680 WRITE(6,680)
S.0134 680 FORMAT(10,...LAMINATE EXTENSIONAL CONSTANTS//)
C
S.0135 WRITE(6,13) EX,EY,GXY,UX,UYX,ALF,PSI1,A11,A12
S.0136 13 FORMAT(1014X'EX='F12.5,5X'EY='F12.5,5X'GXY='F12.5,5X'UXY='F7.4,5X
10MUYX='F7.4// 11X'ALPHA=SQRT(EX+EY)='F12.5,13X'PSI=SQRT(EX+EY)/(1-
2MUYX*MUYX)='F12.5// 13X'A11=EX*(1+EY/EX+2*MUYX)='F12.5,5X'A12=GXY-
3EX*MUYX/(1-MUYX*MUYX)='F12.5//)
C
S.0137 RC=T*23/12/R
S.0138 011A=RC*EX
S.0139 012A=UYX*EX*RC
S.0140 022A=RC*EY
S.0141 066A=GXY*RC

```

```

C
S.0142      WRITE(6,700)
S.0143      700 FORMAT(10.0,LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL
              ICONSTANTS//)
C
S.0144      WRITE(6,710)  D11A,D12A,D22A,D66A
S.0145      710 FORMAT(1011F = F12.5,5X'D12F = F12.5,5X'D66F = F12
              1.5)
C
S.0146      GO TO 2000
S.0147      END
      SIZE OF COMMON  00000      PROGRAM  06368
END OF COMPILE  AC2

```

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SPT DESCRIPTION

THERE ARE 2 SETS OF PLYS WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.0416 INCHES.

NUMBER OF UPPER PLYS AT 0 DEGREES = 1
NUMBER OF MIDDLE PLYS AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF LOWER PLYS AT 0 DEGREES = 1

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12928E 03 D12 = 0.20975E 02 D22 = 0.32043E 02 A11 = 0.78382E 08 A12 = 0.16662E 08
D66 = 0.21746E 02 D16 = -0.76769E 01 D26 = -0.76769E 01 A22 = 0.23784E 08 A66 = 0.17199E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 = -0.59380E -01 D16/D22 = -0.36601E 00 D26/D11 = -0.59380E -01 D26/D22 = -0.36601E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.16669E 08 EY= 0.50580E 07 GXY= 0.42998E 07 MUXY= 0.7015 MUXX= 0.2129
ALPHA= SORT(EX+EY) = 0.18154E 01 PSI= SORT(EX+EY)/(1-MUXY+MUXX) = 0.10794E -04
A11= EX*(1+EY/EX+2*MUXX) = 0.33886E 08 A12= GXY-EX*MUXX/(1-MUXY+MUXX) = 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11 = 0.11756E 03 D12 = 0.25024E 02 D22 = 0.35671E 02 D66 = 0.25705E 02

...LAMINATE PLASTIC CONSTANTS FOR TOW-CURVE LOADINGS

FL = 0.20000E 08 FI = 0.27100E 07 GLT = 0.70000E 08 MULTI = 0.2100

...SFT DESCRIPTION

THERE ARE 4 SFTS OF PLYS WITH A MINIMAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.1248 INCHES.

NUMBER OF INNER PLYS AT 0 DEGREES = 2
NUMBER OF MIDDLE PLYS AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF OUTER PLYS AT 90 DEGREES = 2

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.15476E 04 D12 = 0.45948E 03 D22 = 0.30215E 04 A11 = 0.83824E 08 A12 = 0.17827E 08
D66 = 0.48052E 03 D16 = -0.46062E 02 D26 = -0.46062E 02 A22 = 0.83824E 08 A66 = 0.18599E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 = -0.29764E-01 D16/D22 = -0.10020E 00 D26/D11 = -0.29764E-01 D26/D22 = -0.10020E 00

...LAMINATE EXTENSIONAL CONSTANTS

FX = 0.13339E 08 FY = 0.13339E 08 GXY = 0.20908E 07 MUXX = 0.2127 MUYY = 0.2127
ALPHA=SQRT(FX/FY) = 0.10000E 01 PSI=SQRT(EX/EY)/(1-MUXX-MUYY) = 0.13971E 08
A11=FX*(1+FY/FX+2*MUXX) = 0.33844E 08 A12=GXV-FX-MUXX/(1-MUXX-MUYY) = 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11F = 0.22630E 04 D12F = 0.48128E 03 D22F = 0.22630E 04 D66F = 0.50211E 03

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL = 0.29000E 08 $\nu_T = 0.27100E 07$ $G_{LT} = 0.70000E 06$ $\mu_{LT} = 0.2100$

...SFT DESCRIPTION

THERE ARE 2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.012 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 1
NUMBER OF MIDDLE PLIES AT (+ AND -) 60.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 0 DEGREES = 0

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12910E 02 D12 = 0.14606E 02 D22 = 0.43626E 02 A11 = 0.38312E 08 A12 = 0.12515E 06
D66 = 0.14932E 02 D16 = -0.31417E 01 D26 = -0.10154E 02 A22 = 0.38312E 08 A66 = 0.12899E 06

...RATIOS REFLECT FLEXURAL CONSTANTS

D16/D11 = -0.24410E 00 D16/D22 = -0.21509E 00 D26/D11 = -0.78608E 00 D26/D22 = -0.69527E 00

...LAMINATE EXTENSIONAL CONSTANTS

$E_X = 0.11408E 08$ $\nu_Y = 0.11408E 08$ $G_{XY} = 0.42998E 07$ $\mu_{UX} = 0.3266$ $\mu_{UYX} = 0.3266$
 $\alpha_{PMA} = \sqrt{E_X/E_Y} = 0.10000E 01$ $PSI = \sqrt{E_X E_Y} / (1 - \mu_{UX} \mu_{UYX}) = 0.12771E 08$
 $A11 = E_X (1 + \nu_Y/E_X + 2\mu_{UX}) = 0.33884E 08$ $A12 = G_{XY} - E_X \mu_{UYX} / (1 - \mu_{UX} \mu_{UYX}) = 0.12862E 06$

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.32322E 02 D12E = 0.10557E 02 D22E = 0.32322E 02 D66E = 0.10882E 02

12/15/66

12/05/61

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C      PRINCPAL AC'S ... D/C. PRINCL. STABILITY UNDER IN-PLANE BIAXIAL LOADING.
C
C      THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
C      A11 = INVARIANT 1 = EX/(1-UMXV*UMYX)*(1+EY/EX+2*UMYX)
C      A12 = INVARIANT 2 = GXY - UMYX*EX/(1-UMXV*UMYX)
C      ALF = SORT(EX/EY)
C      AETA = R = ALF*UMYX + 2*GAMMA
C      D11 = EX*TFSC**2 / (2.**(1-UMXV*UMYX))
C      D22 = D11*EY/EX
C      D66 = TFSC**2*GXY/2
C      D0X = C*GCMX
C      DQY = C*GCMY
C      GAMMA = G = GXY*(1-UMXV*UMYX)/SORT(EX*EY)
C      IA = TOTAL NUMBER OF A/R
C      IC = TOTAL NUMBER OF HALF WAVELENGTHS IN Y-DIRECTION
C      ID = TOTAL NUMBER OF HALF WAVELENGTHS IN X-DIRECTION
C      IE = TOTAL NUMBER OF PHI, WHERE PHI = EY/NX
C      IF = TOTAL NUMBER OF V WHERE V = PI**2*SORT(D11*D22)/(E**2*QX)
C      PSI = SORT(EX*EY)/(1-UMXV*UMYX)
C      R = D0X/D0Y
C
C      D1=EPSID* AOB(20), PHI(20), V(20), AK(20), APSI(20), TIT(16),
C      LAAR(20),AAKAY(20), AKAY(20)
C      1 PEAD(5,100) (TIT(1),I=1,18)
C      100 P=PPAT(18A4)
C      PEAD(5,110) TMAOB, ALF, TMA, TMM, TNPFI, TNV, A11, A12, R, TNPST
C      110 P=PPAT(6+12,0)
C      A=A1F
C      I=TNMAM
C      IC=TNM
C      10=TNM
C      PEAD(5,110) (AOB(1),I=1,1A)
C      IF=TNPHI
C      IE(IE) 108,108,105
C      105 READ(5,110) (PHI(1),I=1,1F)
C      108 IP=TNV
C      PEAD(5,110) (V(1),I=1,1P)
C      L2=TNPSI
C      READ(5,110) (APSI(1),I=1,L2)
C      0001
C      0002
C      0003
C      0004
C      0005
C      0006
C      0007
C      0008
C      0009
C      0010
C      0011
C      0012
C      0013
C      0014
C      0015
C      0016
C      0017

```

S.0018	DO 300 L3=1,L2	
S.0019	PSI=APSI(L3)	
S.0020	GAMMA= A11/(2.*PSI) + A12/PSI -.5*A - .5/A	
S.0021	HFTA= 1.5*A11/PSI + 2.*A12/PSI -1.5*A -1.5/A	
S.0022	B=HFTA	
S.0023	G=GAMMA	
S.0024	WRITE(6,120)	
S.0025	120 FORMAT(1) HUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROP	
	11C SANDWICH PLATES - FLAT	
S.0026	WRITE(6,100) (TIT(I),I=1,18)	
S.0027	WRITE(6,130) R2, G2, B2, A2, PSI	
S.0028	130 FORMAT(0,5X,G2,CX/G2,CY=,F5.2,5X,GAMMA=,F8.4,5X,BETA=,F8.3,5X,ALP	
	1HA=,F6.3,5X,PSI=,F11.0)	
S.0029	DO 300 I=1,IP	
S.0030	WRITE(6,135) V(I)	
S.0031	135 FORMAT(14H0 DATA FOR V=,F8.4)	
S.0032	IF(IE) 138,140,138	
S.0033	138 WRITE(6,139) (PHI(LL),LL=1,IE)	
S.0034	139 FORMAT(0,26X,PHI=,F5.1,6X,PHI=,F5.1,6X,PHI=,F5.1,6X,PHI=,F5.1,6X,	
	1PHI=,F5.1)	
S.0035	140 CONTINUE	
S.0036	WRITE(6,142)	
S.0037	142 FORMAT(0,2X,A/8,5X,KX(1),KY(1),KY(1),4X,KX(2)/KY(2),4X,KX(3	
	1)/KY(3),4X,KX(4)/KY(4),4X,KX(5)/KY(5))	
S.0038	DO 300 J=1,IA	
S.0039	AR1=AR(J)	
S.0040	AR2=AR1**2	
S.0041	AR3=AR1**3	
S.0042	AR4=AR1**4	
S.0043	AAKX= 1.0E+16	
S.0044	AAKY=AAKX	
S.0045	IF(IE) 145,150,145	
S.0046	145 DO 148 JJ=1,IE	
S.0047	AAKAY(JJ)=AAKX	
S.0048	148 AAK(JJ)=AAKX	
S.0049	150 CONTINUE	
S.0050	DO 250 N=1,IO	
S.0051	OM=M	
S.0052	OM2=OM**2	
S.0053	OM3=OM**3	
S.0054	OM4=OM2**2	
S.0055	DO 250 N=1,IC	
S.0056	ON=N	
S.0057	ON2=ON**2	

BUCKLE COEFFICIENTS FOR STAINY SUPPORTED ORTHOTROPIC SANDWICH PLATES - PL.1 (0/+45/0)S LAMINATE AT ROOM TEMPERATURE

GCX/GCY= 2.00 GAMMA= 0.3985 KETA= 1.186 ALPHA= 1.815 PSI= 10794000.

DATA FOR V= 0.0423

A/R	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	4.17	3.91	3.79/ 0.38	2.78/ 1.30	2.09/ 2.09	1.39/ 2.78	0.38/ 3.79
2.00	4.17	1.17	3.34/ 0.34	1.56/ 0.78	0.94/ 0.94	0.52/ 1.04	0.11/ 0.11
3.00	4.06	0.70	3.31/ 0.33	1.29/ 0.65	0.71/ 0.71	0.37/ 0.75	0.08/ 0.75

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT
(0/0/+45/90/90)25 LAMINATE AT ROOM TEMPERATURE

G¹CX/G¹CY = 2.00 GAMMA = 0.2219 BETA = 0.656 ALPHA = 1.000 PSI = 13971000.

DATA FOR V = 0.1896

A/A	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	2.27	2.19	2.06/ 0.21	1.51/ 0.76	1.13/ 1.13	0.76/ 1.51	0.21/ 2.06
2.00	2.27	1.00	2.06/ 0.21	1.33/ 0.67	0.80/ 0.80	0.44/ 0.89	0.10/ 0.97
3.00	2.27	0.64	2.06/ 0.21	1.34/ 0.67	0.75/ 0.75	0.40/ 0.79	0.08/ 0.83

PHI = 0.1

PHI = 0.5

PHI = 1.0

PHI = 2.0

PHI = 10.0

COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SAWTOOTH PLATES - FLAT (0/+60)S LONGITUDINAL ROOM TEMPERATURE

G*CX/G*CY = 2.00 GAMMA = 0.3367 K*TA = 1.000 ALPHA = 1.000 PSI = 1.7710000

DATA FOR V = 0.0368

A/R	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	3.60	3.60	3.28/ 0.33	2.40/ 1.20	1.80/ 1.80	1.20/ 2.40	0.55/ 3.28
2.00	3.60	1.44	3.28/ 0.33	1.02/ 0.96	1.15/ 1.15	0.64/ 1.28	0.14/ 1.41
3.00	3.60	1.15	3.28/ 0.33	1.87/ 0.66	1.03/ 1.03	0.54/ 1.09	0.11/ 1.15

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12/05/69
12/05/69

/ID 00,290,1041,01,DAVIDSON,3RR3
/ETC NAME=ACT,LIST,DECK
RPS FURTRAN D CUMPLER

C PROGRAM ACT *** ANALYSIS OF SIMPLY-SUPPORTED ORTHOTROPIC SANDWICH
C PLAT PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS
C
C THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS*****
C
C A11 = INVARIANT 1 =  $EX*(1+VY/EX+2*VUYX)/(1-MUXY*MUXX)$ 
C A12 = INVARIANT 2 =  $GXY - MUXY*EX/(1-MUXY*MUXX)$ 
C A13 = BUCKLING COEFFICIENT IN THE X-DIRECTION FOUND FROM PROGRAM ACS
C A14 = BUCKLING COEFFICIENT IN THE Y-DIRECTION FOUND FROM PROGRAM ACS
C
C ALPHA = ALF =  $SORT(EX/FY)$ 
C BETA = B =  $ALPHA*FUYX + 2*GAMMA$ 
C D11 =  $EX*TF*(2/(2*(1-MUXY*MUXX)))$ 
C D22 =  $D11*(FY/EX)$ 
C D66 =  $TF*C*(2*GXY/2)$ 
C DOX =  $C*GCX$ 
C DOY =  $C*GCY$ 
C GAMMA =  $G = GXY*(1-MUXY*MUXX)/SORT(EX*FY)$ 
C IA = TOTAL NUMBER OF A/R (PANEL ASPECT RATIO)
C IF = TOTAL NUMBER OF PHI, WHERE PHI =  $NY/NX$ 
C IP = TOTAL NUMBER OF V, WHERE V =  $PI*2*SORT(D11*D22)/(R*2*DOX)$ 
C
C PI = TRUNCATION INTEGER FOR SERIES SUMMATION
C PSI =  $SORT(EX*FY)/(1-MUXY*MUXX)$ 
C R =  $DOX/DOY = GCX/GCY$ 
C RX =  $NX/NXCR$ 
C RY =  $NY/NYCR$ 
C XI =  $SORT(D11*D22)$ 
C
S.0001 DTFPSI=PI DOX(20),XUA(20),YUH(20),PHI(20),W(20),OX(20),OY(20),
1 AAKX(20),AAKY(20),AAV(20),AAX(20),AOX(20),AOY(20),SY(20),Y(20)
2,APST(20),TIT(18),AAKX(20),AAKY(20)
1 READ(5,7) (TIT(I),I=1,18)
7 FORMAT(18A6)
2 FORMAT(6F12.0)
A=ALF
D1=0.0166474
IA=TNXUA
N1=TITL
NA=TNXUA
NR=TNXUH
S.0010
S.0011
S.0012 3 READ(5,2) (AOR(I),AAKX(I),AAKY(I),I=1,1A)

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S.0053      AMY(J2)=0
S.0054      A:XY(J2)=0
S.0055      AOX(J2)=0
S.0056      AOY(J2)=0
S.0057      199 CUM1THUF
S.0058      30 DO GO M=1,M1,2
S.0059      DO GO N=1,M,2
S.0060      MM=H
S.0061      ON=M
S.0062      SY=STM (OM=C1)
S.0063      CX=CUS (OM=C1)
S.0064      DO 200 K=1,MK
S.0065      SY(K)=STM (YOH(K)*3.1416*ON)
S.0066      CY(K)=CUS (YOH(K)*3.1416*ON)
S.0067      200 CUM1THUF
S.0068      M12=0:3,2
S.0069      M13=0:3,3
S.0070      M14=0:2,2
S.0071      M12=0:3,2
S.0072      M13=0:3,3
S.0073      M14=0:2,2
S.0074      A11=(A*OM3/AB3 +OM*OM2*AB/AB1)*(-1.)
S.0075      A12=1. + OM2*A*V(1)/AB2 +OM2*G*V(1)
S.0076      A13=(-1.)*(OM*OM*V(1)*R*(B-G)/AB1)
S.0077      A21=(OM3/A +OM*OM*2*A*AB/AB2)
S.0078      A22=A13/R
S.0079      A23=1+R *(OM2/A +G*OM2/AB2)*V(1)
S.0080      A31=(-1.)*(A*OM4/AB4 +2.*OM*OM2*OM2/AB2 + OM4/A)
S.0081      A32=-OM /AB1*(OM2*A/AB2 +OM2*B)*V(1)
S.0082      A33=OM*OM*V(1)*(OM2/A + OM*2*A/AB2)
S.0083      AFM=(A12*A23-A22*A13)/(A33*(A11*A22-A12*A21)+A32*(A13*A21-A11*A23)
      1+(-A31-AKX-RX*OM2/AB2-AKY-RY*OM2)*(A12*A23-A22*A13)*D1/(OM*ON)
S.0084      DO=A22*A13-A12*A23
S.0085      ALF2=(A11*A22-A12*A21)/DO
S.0086      ALF1=(A11*A23-A13*A21)/DO
S.0087      S=V(1)
S.0088      RMX=- 9.8696      *AMN=((OM*S*ALF1/AB1-OM2/AB2)*A+(B-2.*G)*R*S
      1+OM*AF2-OM2))
      RMY=- 9.8696      *AMN*1./A*(OM*AF2*R*S -OM2)+(B-2.*G)*(ALF1*OM
      1+S/AB1-OM*2/AB2)
S.0089      RMXY=G* 9.8696      *AMN*(OM*ALF1*S +ALF2*R*OM*S -2.*OM*ON/AB1)
S.0090      ROX=- 31.0065      *AMN*OM*OM*(1./((OM*AB1)*((B-2.*G)*((OM*ALF2*R*S
S.0091      1-OM2)+A*(ALF1*OM*S/AB1 -OM2/AB2) +G*ON*(ALF1*S/OM +ALF2*R*S/(OM
      2AB1) -2./AB1))

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S.0092	BOY=- 31.0065 *AMN* (G*OM*(ALF1*S/OM+ALF2*R*S/(ON*AB1))-2./AB1) 1*(1./AB1)+(1./OM)*((ON:ALF2*R*S-ON2)/A+(8-2.*G)*(ALF1*OM*S/AB1 2-OM2/AB2)))*OM*ON
S.0093	ON 300 J2=1,MB
S.0094	AMX(J2)=AMX(J2)+BMX*SX*SY(J2)
S.0095	AMY(J2)=AMY(J2)+BMY*SY*SY(J2)
S.0096	AMXY(J2)=AMXY(J2)+RMXY*CX*CY(J2)
S.0097	AOX(J2)=AOX(J2)+BOX*CX*SY(J2)
S.0098	AOY(J2)=AOY(J2)+BOY*SY*CY(J2)
S.0099	W(J2)=W(J2)+AMN* SX*SY(J2)
S.0100	300 CONTINUE
S.0101	OM1=M
S.0102	IF(OM1-OM)90 ,400,90
	C
	C.....LOOP TO PICK UP ANM TERM
	C
S.0103	400 IF(M-N)401,403,401
S.0104	401 OM=N
S.0105	ON=M
S.0106	GO TO 352
S.0107	403 CONTINUE
S.0108	500 WRITE(6,501) M,N
S.0109	501 FORMAT(4H0 M=,I2,3X,2H N=,I2)
S.0110	510 ON 520 J2=1,MB
S.0111	511 WRITE(6,512)YOR(J2),W(J2),AMX(J2),AMY(J2),AMXY(J2),AOX(J2),AOY(J2)
S.0112	512 FORMAT(5H0Y/R=F6.3,F9.6,F13.3,F14.3,F13.3,F13.3,F11.3)
S.0113	520 CONTINUE
S.0114	90 CONTINUE
S.0115	100 CONTINUE
S.0116	GO TO 1
S.0117	END
	SIZE OF COMMON 00000 PROGRAM 06380
	END OF COMPILATION AC7

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE ONLY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY = 2.00 GAMMA = 0.3985 BETA = 1.184 ALPHA = 1.815 PSI = 10794000.

A/R = 1.00 MX/MXR = 0.0 NY/MYR = 0.0 KX = 4.17 KY = 3.91

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A = 0.0

W=XI/0**R**4 MX/Q**R**2 MY/0**R**2 MXY/0**R**2 QX/Q**B QY/Q**B

N= 1 N= 1

Y/R = 0.0 0.0 0.0 0.0 0.0 -0.028 0.0 0.0 0.0

Y/R = 0.500 0.0 0.0 0.0 0.0 0.000 0.328 0.0 0.0

N= 3 N= 3

Y/R = 0.0 0.0 0.0 0.0 0.0 -0.031 0.0 0.0 0.0

Y/R = 0.500 0.0 0.0 0.0 0.0 0.000 0.337 0.0 0.0

N= 5 N= 5

Y/R = 0.0 0.0 0.0 0.0 0.0 -0.031 0.0 0.0 0.0

Y/R = 0.500 0.0 0.0 0.0 0.0 0.000 0.367 0.0 0.0

N= 7 N= 7

Y/R = 0.0 0.0 0.0 0.0 0.0 -0.031 0.0 0.0 0.0

Y/R = 0.500 0.0 0.0 0.0 0.0 0.000 0.368 0.0 0.0

N= 9 N= 9

Y/R = 0.0 0.0 0.0 0.0 0.0 -0.032 0.0 0.0 0.0

Y/R = 0.500 0.0 0.0 0.0 0.0 0.000 0.378 0.0 0.0

COEFFICIENTS FOR X/A = 0.500

Highly sensitive to input data. Check input data carefully. If input data is incorrect, the output will be incorrect.

W2X1/00000000		MX/00000000	MY/00000000	IXY/00000000	OX/0000	OY/0000
M= 1 N= 1						
Y/R=	0.0	0.0	0.0	0.000	0.0	0.188
Y/B=	0.500	0.003088	0.077	0.032	-0.000	-0.000
M= 3 N= 3						
Y/R=	0.0	0.0	0.0	0.000	0.0	0.218
Y/B=	0.500	0.003816	0.068	0.027	-0.000	-0.000
M= 5 N= 5						
Y/R=	0.0	0.0	0.0	0.000	0.0	0.258
Y/B=	0.500	0.003830	0.070	0.028	-0.000	-0.000
M= 7 N= 7						
Y/R=	0.0	0.0	0.0	0.000	0.0	0.244
Y/B=	0.500	0.003832	0.069	0.028	-0.000	-0.000
M= 9 N= 9						
Y/R=	0.0	0.0	0.0	0.000	0.0	0.251
Y/B=	0.500	0.003835	0.069	0.028	-0.000	-0.000

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE ONLY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY = 2.00 GAMMA = 0.3985 RFTA = 1.184 ALPHA = 1.815 PSI = 10794000.

A/H = 2.00 NX/NXCR = 0.0 NY/NYCR = 0.0 KX = 4.17 KY = 1.17

..DATA FOR V = 0.0423
COEFFICIENTS FOR X/A = 0.0

W*XI/O*B**4 MX/O*B**2 MY/O*B**2 MXY/O*B**2 OX/O*B OY/O*B

M = 1 N = 1

Y/R = 0.0 0.0

0.0 0.0 -0.050 0.0 0.0 0.0

Y/R = 0.500 0.0

0.0 0.0 0.000 0.336 0.0 0.0

M = 3 N = 3

Y/R = 0.0 0.0

0.0 0.0 -0.055 0.0 0.0 0.0

Y/R = 0.500 0.0

0.0 0.0 0.000 0.395 0.0 0.0

M = 5 N = 5

Y/R = 0.0 0.0

0.0 0.0 -0.056 0.0 0.0 0.0

Y/R = 0.500 0.0

0.0 0.0 0.000 0.436 0.0 0.0

M = 7 N = 7

Y/R = 0.0 0.0

0.0 0.0 -0.056 0.0 0.0 0.0

Y/R = 0.500 0.0

0.0 0.0 0.000 0.448 0.0 0.0

M = 9 N = 9

Y/R = 0.0 0.0

0.0 0.0 -0.056 0.0 0.0 0.0

Y/R = 0.500 0.0

0.0 0.0 0.000 0.461 0.0 0.0

COEFFICIENTS FOR X/A = 0.500

	$1/X1/0.00004$	$1/X/0.000002$	$1/Y/0.000002$	$1/XY/0.000002$	$0X/0.001$	$0Y/0.001$
$N=1$						
$Y/R=0.0$	0.0	0.0	0.0	0.000	0.0	0.348
$Y/R=0.500$	0.014188	0.110	0.085	-0.000	-0.000	-0.000
$N=3$						
$Y/R=0.0$	0.0	0.0	0.0	0.000	0.0	0.255
$Y/R=0.500$	0.013603	0.091	0.075	-0.000	-0.000	-0.000
$N=5$						
$Y/R=0.0$	0.0	0.0	0.0	0.000	0.0	0.284
$Y/R=0.500$	0.013671	0.095	0.077	-0.000	-0.000	-0.000
$N=7$						
$Y/R=0.0$	0.0	0.0	0.0	0.000	0.0	0.285
$Y/R=0.500$	0.013653	0.094	0.076	-0.000	-0.000	-0.000
$N=9$						
$Y/R=0.0$	0.0	0.0	0.0	0.000	0.0	0.306
$Y/R=0.500$	0.013660	0.094	0.077	-0.000	-0.000	-0.000

10/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE ONLY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

6. CX/CY = 2.00 GAMMA = 0.3985 BETA = 1.184 PSI = 10794000.

A/H = 3.00 PX/NXCR = 0.0 NY/NYCR = 0.0 KX = 4.06 KY = 0.79

***DATA FOR V = 0.0423

COEFFICIENTS FOR X/A = 0.0

W*XI/0*H**4 MX/0*H**2 MY/0*H**2 MXY/0*H**2 OX/0*H OY/0*H

M= 1 N= 1

V/H = 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

V/H = 0.500 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.285 0.0

M= 3 N= 3

V/H = 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

V/H = 0.500 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.370 0.0

M= 5 N= 5

V/H = 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

V/H = 0.500 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.421 0.0

M= 7 N= 7

V/H = 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

V/H = 0.500 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.442 0.0

M= 9 N= 9

V/H = 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

V/H = 0.500 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.459 0.0

COEFFICIENTS FOR X/A = 0.500

M=1 N=1		MAXI/04R44	MX/04R442	MY/04R442	MAX/04R442	MY/04R442	MX/04R442	MY/04R442	QX/Q44	QY/Q44
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R=	0.500	0.021051	0.116	0.117	0.117	0.117	0.117	0.117	0.000	0.000
M=3 N=3										
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R=	0.500	0.019594	0.087	0.101	0.101	0.101	0.101	0.101	0.000	0.000
M=5 N=5										
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R=	0.500	0.019796	0.096	0.105	0.105	0.105	0.105	0.105	0.000	0.000
M=7 N=7										
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R=	0.500	0.019745	0.092	0.103	0.103	0.103	0.103	0.103	0.000	0.000
M=9 N=9										
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R=	0.500	0.019763	0.094	0.104	0.104	0.104	0.104	0.104	0.000	0.000

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

$G^*CX/G^*CY = 2.00$ $GAMMA = 0.3985$ $BETA = 1.184$ $ALPHA = 1.815$ $PSI = 10794000.$

$A/R = 1.00$ $NX/NXCR = 0.500$ $NY/NYCR = 0.500$ $KX = 2.09$ $KY = 2.09$

..DATA FOR $V = 0.0423$

COEFFICIENTS FOR $X/A = 0.0$

$WXY1/Q^*R^*H^*4$ NX/Q^*H^*2 MY/Q^*H^*2 MX/Q^*B^*2 QY/Q^*B^*2

$M = 1$ $N = 1$

$Y/R = 0.0$ 0.0 0.0 0.0 -0.055 0.0 0.0

$Y/R = 0.500$ 0.0 0.0 0.0 0.000 0.656 0.0

$M = 3$ $N = 3$

$Y/R = 0.0$ 0.0 0.0 0.0 -0.059 0.0 0.0

$Y/R = 0.500$ 0.0 0.0 0.0 0.000 0.661 0.0

$M = 5$ $N = 5$

$Y/R = 0.0$ 0.0 0.0 0.0 -0.060 0.0 0.0

$Y/R = 0.500$ 0.0 0.0 0.0 0.000 0.694 0.0

$M = 7$ $N = 7$

$Y/R = 0.0$ 0.0 0.0 0.0 -0.060 0.0 0.0

$Y/R = 0.500$ 0.0 0.0 0.0 0.000 0.695 0.0

$M = 9$ $N = 9$

$Y/R = 0.0$ 0.0 0.0 0.0 -0.060 0.0 0.0

$Y/R = 0.500$ 0.0 0.0 0.0 0.000 0.705 0.0

COEFFICIENTS FOR $X/A = 0.500$

	MX/0*B**4	MY/0*B**2	MX/0*B**2	MY/0*B**2	MX/0*B**2	MY/0*B**2	MX/0*B	MY/0*B
N= 1 N= 1								
Y/R= 0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R= 0.500 0.007989	0.154	0.065	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
N= 3 N= 3								
Y/R= 0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R= 0.500 0.007774	0.143	0.058	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
N= 5 N= 5								
Y/R= 0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R= 0.500 0.007800	0.145	0.060	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
N= 7 N= 7								
Y/R= 0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R= 0.500 0.007792	0.144	0.059	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
N= 9 N= 9								
Y/R= 0.0 0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y/R= 0.500 0.007795	0.145	0.059	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

		W*XI/O*B**4	MX/Q*B**2	MY/O*B**2	MX/Y/O*B**2	QX/O*B	QY/O*B
M= 1 N= 1							
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.697
Y/B=	0.500	0.028423	0.220	0.170	-0.000	-0.000	-0.000
M= 3 N= 3							
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.703
Y/B=	0.500	0.027754	0.198	0.159	-0.000	-0.000	0.000
M= 5 N= 5							
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.737
Y/B=	0.500	0.027826	0.203	0.161	-0.000	-0.000	-0.000
M= 7 N= 7							
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.738
Y/B=	0.500	0.027807	0.201	0.160	-0.000	-0.000	-0.000
M= 9 N= 9							
Y/R=	0.0	0.0	0.0	0.0	0.000	0.0	0.748
Y/B=	0.500	0.027814	0.202	0.160	-0.000	-0.000	-0.000

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION, MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY = 2.00 GAMMA = 0.3985 BETA = 1.184 ALPHA = 1.815 PSI = 10794000.

A/R = 3.00 NX/NXCR = 0.500 NY/NYCR = 0.500 KX = 0.71 KY = 0.71

...DATA FOR V = 0.0423
COEFFICIENTS FOR X/A = 0.0

W*XI/Q*R**4 MX/Q*R**2 MY/Q*B**2 MXY/Q*B**2 QX/Q*B QY/Q*B

M= 1 N= 1

Y/R= 0.0 0.0 0.0 0.0 0.0 -0.098 0.0 0.0 0.0

Y/R= 0.500 0.0 0.0 0.0 0.0 0.000 0.568 0.0 0.0

M= 3 N= 3

Y/R= 0.0 0.0 0.0 0.0 0.0 -0.108 0.0 0.0 0.0

Y/R= 0.500 0.0 0.0 0.0 0.0 0.000 0.674 0.0 0.0

M= 5 N= 5

Y/R= 0.0 0.0 0.0 0.0 0.0 -0.110 0.0 0.0 0.0

Y/R= 0.500 0.0 0.0 0.0 0.0 0.000 0.729 0.0 0.0

M= 7 N= 7

Y/R= 0.0 0.0 0.0 0.0 0.0 -0.110 0.0 0.0 0.0

Y/R= 0.500 0.0 0.0 0.0 0.0 0.000 0.751 0.0 0.0

M= 9 N= 9

Y/R= 0.0 0.0 0.0 0.0 0.0 -0.110 0.0 0.0 0.0

Y/R= 0.500 0.0 0.0 0.0 0.0 0.000 0.769 0.0 0.0

COEFFICIENTS FOR X/A = 0.500

M= 1 N= 1		N*X1/0*8**4	NX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.840
Y/H=	0.500	0.042011	0.231	0.233	-0.000	-0.000	-0.000	-0.000
M= 3 N= 3		N*X1/0*8**4	NX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.810
Y/H=	0.500	0.040766	0.197	0.215	-0.000	-0.000	-0.000	-0.000
M= 5 N= 5		N*X1/0*8**4	NX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.856
Y/H=	0.500	0.040487	0.206	0.218	-0.000	-0.000	-0.000	-0.000
M= 7 N= 7		N*X1/0*8**4	NX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.850
Y/H=	0.500	0.040432	0.202	0.217	-0.000	-0.000	-0.000	-0.000
M= 9 N= 9		N*X1/0*8**4	NX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2	MX/0*8**2	MY/0*8**2
Y/R=	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.864
Y/H=	0.500	0.040451	0.204	0.218	-0.000	-0.000	-0.000	-0.000

/ID 00,290,1041,01,DAVIDSON,EXT3883
 /JOB TIME=10
 /FTC NAME=AC11,LIST,DECK
 RPS FORTRAN D COMPILER

12/05/69

12/05/69

C PROGRAM AC11 ... H/C PANEL STABILITY FOR IN-PLANE SHEAR LOADING

C THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....

C A = PANEL WIDTH ALONG X-DIRECTION

C A11 = INVARIANT 1 = $EX*(1+EY/FX+2*MUX)/(1-MUX*MUYX)$

C A12 = INVARIANT 2 = $GXY - MUYX*EX/(1-MUX*MUYX)$

C ALPHA = SORT(EX/EY)

C A08 = PANEL ASPECT RATIO A/R

C B = PANEL LENGTH ALONG Y-DIRECTION

C C = CORE DEPTH

C CNTL = CONTROL CARD FOR DESIGNATING TYPE OF INPUT TO BE USED

C (I=0 FOR MATL. PROPERTIES, OR =1 FOR MATL. PARAMETERS)

C EIGMIN = EIGENVALUE ITERATION SOLUTION THAT TAKES THE SORT

C OF THE ABS VALUE OF SUCCEEDING EIGENVALUES

C GCX = CORE SHEAR MODULUS IN THE X-DIRECTION

C GCY = CORE SHEAR MODULUS IN THE Y-DIRECTION

C PSI = SORT(EX*EY)/(1-MUX*MUYX)

C Q = NUMBER OF TERMS IN THE SERIES (MAXIMUM OF 13)

C R = GCX/GCY

C TF = THICKNESS OF ONE FACE SHEET

C TITLE = ANY DESIRED ONE-LINE TITLE IN COLS.1-72

C V = $PI*2*PSI*TF*(C+TF)/(2*GCX*B**2)$

C XNIT = NUMBER OF ITERATIONS DESIRED IN THE SOLUTION

S.0001 DIMENSION H(RS, RS), TITLE(18), RB(2)

S.0002 10 WRITE(6,50)

S.0003 50 FORMAT(1H1)

S.0004 READ(5,100) TITLE

S.0005 100 FORMAT(1HA4)

S.0006 WRITE(6,100) TITLE

C.....READ MATERIAL PROPERTIES

S.0007 READ(5,110) CNTL,0

S.0008 110 FORMAT(6F12,0)

S.0009 IF(CNTL) 101,109,101

C.....READ PARAMETRIC PROPERTIES

S.0010 101 READ(5,110) ALPHA,PSI,V,R,A11,A12,A08,C,TE,XNIT

S.0011 CCL=AL1/PSI-ALPHA-1.0/ALPHA

S.0012 BETA=1.5*CCL+2.*A12/PSI

S.0013 GAMA=0.5*CCL+A12/PSI

00000100

00000115

00000117

00000140

00000150

00000160

00000180

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S.0014      UMYX=ALPHA*(RETA-2.0*GAMA)
S.0015      UMYX=UMYX/ALPHA**2
S.0016      AMDA=1.0-UMYX*UMYX
S.0017      EX=ALPHA*PSI*AMDA
S.0018      EY=FX/ALPHA**2
S.0019      GXY=AI2+UMYX*EX/AMDA
S.0020      SX=VWC/(C+TF)/A0B**2
S.0021      WRITE(6,115) ALPHA,PSI,V,K
S.0022      115 FORMAT(1H04X,28HTHE MATERIAL PARAMETERS ARE, //10X,6HALPHA=F6.3,5X,
      14HPSI=E12.5,5X,2HV=F7.4,5X,8HGXY/GCY=F6.3)
S.0023      WRITE(6,116) A0B,C,TF
S.0024      116 FORMAT(1H04X,28HTHE GEUMETRY PROPERTIES ARE, //12X,4HA/B=F7.3,6X,
      12HG=F7.4,10X,2HTF=F7.4)
S.0025      GO TO 131
S.0026      C.....READ MATERIAL PROPERTIES
      109 READ(5,110) EX,EY,GXY,UMYX,GCX,GCY
S.0027      C.....READ PANEL GEOMETRY
      112 READ(5,110) A,B,C,TF,XNIT
S.0028      A0B=A/B
S.0029      UMYX=EY/EX*UMYX
S.0030      ALPHA=(EX/EY)**.5
S.0031      AMDA=1.0-UMYX*UMYX
S.0032      PSI=(EX*EY)**.5/AMDA
S.0033      GAMA=GXY/PSI
S.0034      RETA=ALPHA*UMYX*2.0*GAMA
S.0035      P=C*TF
S.0036      T=PSI*TF
S.0037      D=1/P**2/2.0
S.0038      SX=C*D*9.R696/(P*A)**2/GCX
S.0039      R=GCX/GCY
S.0040      V=SX*P*A0B**2/C
C.....WRITE INPUT DATA
S.0041      118 WRITE(6,120) EX,GCX,EY,GCY,GXY,UMYX,UMYX
S.0042      120 FORMAT(1H04X,28HTHE MATERIAL PROPERTIES ARE, //10X,15HFACE SHEET E
      1X=E13.5,10X,9HCURE GX=E13.5/21X,4HEY=E13.5/20X,5HGY=E13.5/20X,5HHA/B=F8.4)
      25HGXY=E13.5/19X,6HMUXY=F8.4/19X,6HMUYX=F8.4)
C
S.0043      WRITE(6,130) A,B,C,TF,A0B
S.0044      130 FORMAT(1H04X,28HTHE PANEL GEOMETRY IS, //10X,16HPANEL LENGTH A=F8.000000310
      1.3,5X,15HPANFL WINTH H=F8.3/12X,14HCURE DEPTH C=F8.3,5X,25HFACE000000315
      2 SHEET THICKNESS TF=F8.3/21X,5HA/B=F8.3)
S.0045      WRITE(6,125) ALPHA,PSI,V,K
S.0046      125 FORMAT(1H04X,28HTHE MATERIAL PARAMETERS ARE, //10X,6HALPHA=F6.3,5X,
      14HPSI=E12.5,5X,2HV=F7.4,5X,8HGXY/GCY=6.3)

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00000190
00000200
00000300
00000230
00000330
00000335
00000350
00000360
00000400
00000410
00000240
00000250
00000270
00000280
00000290
00000305
00000310
00000315
00000320

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S.0047      C.....CALCULATE CONSTANTS      00000325
131 SY=R*SYX
S.0048      P=C+TF
S.0049      T=PSI*TF
S.0050      D=T*P**2/2.0
S.0051      AOR2=AOR**2
S.0052      AOR4=AOR2**2
S.0053      IO=0+.001
S.0054      C.....XLMN CONSTANTS
XLMN1=GAMA*AOR2
S.0055      XLMN2=R.. /ALPHA*AOR2
S.0056      XLMN3= R *GAMA
S.0057      C.....XKMN CONSTANTS
XKMN1=2.0*BETA*AOR2
S.0058      XKMN2=AOR4/ALPHA
S.0059      C.....FMN CONSTANT
XFMN1=1.0-BETA**2
S.0060      C.....VMN CONSTANTS
XVMN1=ADH2*SYX
S.0061      XVMN2=SK*SY
S.0062      C.....DF CONSTANT
DF=TF**3/12.0*PSI
S.0063      C.....CALCULATE VARIABLE CONSTANTS AFTER CHECKING (N+N)
IP=1
S.0064      135 I=0
S.0065      NIT=XNIT
S.0066      140 DO 200 M=1,IO
S.0067      A1=M
S.0068      HA=3.24227*A1
S.0069      A2=A1**2
S.0070      A4=A2**2
S.0071      IF(M-2*(M/2)) 145,150,145
S.0072      145 NS=IP
S.0073      GN TO 155
S.0074      150 NS=3-IP
S.0075      155 DO 200 N=NS,IO,2
S.0076      I=I+1
S.0077      B1=N
S.0078      HR=HA*B1
S.0079      B2=B1**2
S.0080      B4=B2**2
S.0081      C.....CALCULATE VARIABLE CONSTANTS
XLMN =ALPHA*A2+XLMN1*B2+XLMN2*B2+XLMN3*A2
S.0082

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S.0083	XKMN	=ALPHA*AS+XKMN1*2*2+R2+XKMN2*2	00000000
S.0084	FMN	=X*MN1+GAHA*XKMN	00000010
S.0085	VMN	=(XKMN1+XKMN1*2+SY*2)*FMN/(1.0+XKMN1*2+XKMN2*2)	00000020
S.0086	VMN1	=DEFND*XKMN	00000030
S.0087	AA	=(2.0+VMN1+VMN)/A08	00000040
	C.....CHECK TO SEE IF (M+R) AND (N+S) ARE ODD OR EVEN		00000050
	J=0		00000060
S.0088	DO	190 IR=1,10	00000070
S.0089	RI=IR		00000080
S.0090	HR=HR+RI		00000085
S.0091	R12=RI*2		00000090
S.0092	IF	(R-2*(R/2)) 160,170,160	00000095
S.0093	160	ISS=IP	00000900
S.0094	GO	TO 180	00000910
S.0095	170	ISS=3-IP	00000920
S.0096	180	DO 190 ISS=ISS,10,2	00000925
S.0097	S1=15		00000930
S.0098	HC=HR-S1		00000932
S.0099	S12=S1*2		00000935
S.0100	J=J+1		00000940
S.0101	IF	(M+R-2*(M+R/2)) 182,187,182	00000942
S.0102	182	IF (N+S-2*(N+S/2)) 185,187,185	00000943
S.0103	185	H(I,J)=HC/(A2-R12)/(R2-S12)/2A	00000945
S.0104	GO	TO 190	00000947
S.0105	187	H(I,J)=0.0	00000948
S.0106	190	CONTINUE	00000950
S.0107	200	CONTINUE	00000955
S.0108	DO	102=(10*2+2-IP)/2	00000960
S.0109	DELTA=001		00000965
S.0110	CALL	EIGMIN(I02,DELTA,H,BH(IP),NIT)	00000966
S.0111	WRITE	(6,217) NIT	00000967
S.0112	217	FORMAT(1H0,5X,4HNT=16/)	00000970
S.0113	IF	IP-2(220,240,240)	00000980
S.0114	220	IP=2	00000990
S.0115	GO	TO 135	00000990
S.0116	C.....PRINT OUT EVEN AND ODD KS WHICH ARE B8(1) AND B8(2)		00001100
S.0117	240	B8(1)=B8(1)/A082	
S.0118	B8(2)=B8(2)/A082		
S.0119	IF	(B8(1)-B8(2)) 245,250,250	
S.0120	245	XKS=B8(1)	
S.0121	GO	TO 255	
S.0122	250	XKS=B8(2)	
S.0123	255	IF (CNTL) 256,260,256	
S.0124	256	XKNCR=XKS*9.8696*0	

```

S.0125      WRITE(6,25R) XKNXCR
S.0126      25R FORMAT(10,5X,NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/12 * B**2) *
              1E13.5/10**2)
S.0127      GO TO 280
S.0128      260 XNCRS=XKS*9.8486*D/B**2
S.0129      WRITE(6,265) IO,BH(1),BH(2)
S.0130      265 FORMAT(1H0,3H*****THE VALUES FOR KS WHEN THERE ARE 13,22H CMN COE00001120
              1FFICIENTS ARE,720X,10H KS EVEN =E13.5,10X, 9H KS ODD =E13.5) 00001130
S.0131      WRITE(6,270) XNCRS
S.0132      270 FORMAT(10,5X,NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/12 * B**2) *
              1E13.5)
S.0133      280 GO TO 10
S.0134      END
END OF COMPILATION AC11      SIZE OF COMMON 00000      PROGRAM 33640      00001150

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/FTC NAME=EIGMIN,LIST,DECK
BPS FOR TRAN D COMPILER
S.0001      SURROUTINE FIGMIN(NN,DELTA,BINV,ALAMIN,NIT)
S.0002      DIMENSION C( 85),R1( 85),BINV( 85,85)
C
S.0004      I1=1
S.0005      1 DO 2 I=1,NN
S.0006      C(I)=1.0
S.0007      2 CONTINUE
S.0008      N=1
S.0009      3 DO 4 J=1,NN
S.0010      R1(J)=0.0
S.0011      4 DO 5 J=1,NN
S.0012      R1(J)=R1(J)+BINV(I,J)*C(J)
S.0013      5 CONTINUE
S.0014      C1=ARS(R1(1))
S.0015      ALAMDA=C1
S.0016      8 DO 9 I=1,NN
S.0017      C(I)=R1(I)/C1
S.0018      9 CONTINUE
S.0019      IF (M-1) 11,10,11
S.0020      10 N=M+1
S.0021      BLAMDA=ALAMDA
S.0022      C4=(ALAMDA*ALAMDA)**.5
S.0023      11 C2=(ALAMDA*BLAMDA)**.5
S.0024      C3=ARS(C2-C4)
S.0025      IF (C3-DELTA) 14,14,12
S.0026      12 BLAMDA=ALAMDA
S.0027      C4=C2
S.0028      I1=I1+1
S.0029      IF (I1-NIT) 13,14,14
S.0030      13 GO TO 3
S.0031      14 NIT=I1
S.0032      ALAMIN=1.0/C2
S.0033      RETURN
S.0034      END
      SIZE OF COMMON 00000
      PROGRAM 01574
END OF COMPILATION FIGMIN

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12/05/69
12/05/60
00002100
00002120
00002125
00002130
00002140
00002150
00002160
00002170
00002180
00002190
00002200
00002210
00002220
00002260
00002270
00002280
00002290
00002300
00002310
00002320
00002330
00002360
00002365
00002366
00002370
00002375
00002390
00002400

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10/+45/0)S LAMINATE AT ROOM TEMPERATURE

THE MATERIAL PROPERTIES ARE

FACE SHEET FX = 0.16670E 08 CORE GX = 0.12600E 06
 FY = 0.50580F 07 GY = 0.63000E 05
 GXY = 0.43000E 07
 MUXY = 0.7015
 MUYX = 0.2128

THE PANEL GEOMETRY IS

PANEL LENGTH A = 15.000 PANEL WIDTH B = 15.000
 CORE DEPTH C = 0.500 FACE SHEET THICKNESS TF = 0.042
 A/B = 1.000

THE MATERIAL PARAMETERS ARE,

ALPHA= 1.815 PSI= 0.10794E 08 V= 0.0423 GCX/GCY= 2.000

NIT= 7

NIT= 11

****THE VALUES FOR KS WHEN THERE ARE 13 CMN COEFFICIENTS ARE,

KS EVEN = 0.78484E 01 KS ODD = 0.84673E 01

NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(12 * B**2) = 0.22670E 05

(0/+45/0)S LAMINATE AT ROOM TEMPERATURE

THE MATERIAL PARAMETERS ARE,

ALPHA= 1.815 PSI= 0.10794E 08 V= 0.0423 GCX/GCY= 2.000

THE GEOMETRY PROPERTIES ARE

A/B= 1.000 C= 0.5000 TF= 0.0416

NIT= 7

NIT= 11

NSCR=KS * PI**2 * PSI * YF * (C+TF)**2/(2 * B**2) = 0.51021E 07/B**2

```

12/10/69
12/10/69

/ID 00,290,1041,01,DAVIDSON,EXT3883
/FTC NAME=AC40,LIST,NFCK
RPS FORTRAN D COMPILER

C-----PROGRAM: AC40...N-PLY LAMINATE THERMOELASTIC ANALYSIS PROGRAM
C
C INPUT DATA IS AS FOLLOWS.....
C N = NUMBER OF PLYS
C M = MATL. PROP. TEMP. INDICATOR....M=1 FOR R.T., M=2 FOR 350F,
C M=3 FOR 350F
C DT = TEMPERATURE INCREMENT IN DEGREES F
C TH(I) = INDIVIDUAL PLY ANGLES THETA(I)

C
C DIMENSION EL(20),FT(20),GLT(20),SLT(20),ST(20),SLT(20),AT(20),
1 H(20),AL(20,3,3),SL(20),ST(20),SLT(20),TH(20)
C DIMENSION T(20,3,3),EM(20),EN(20),R(20,3,3),DI(20,3,3),AX(20),
1 AX(20),HR(3,3),SX(20),SY(20),SXY(20),BI(3,3),AY(20)
C DIMENSION D(20,3,3)
C EQUIVALENCE (D(1),DI(1))
5 READ(5,10) N,M,DT
10 FORMAT(2I12, F12.0)
C TH(I) = ANGLE BETWEEN NATURAL AXES OF I-TH PLY AND LAMINATE AXES
15 READ(5,20) (TH(I),I=1,N)
20 FORMAT(6F12.0)
C PROPERTIES OF INDIVIDUAL LAMINA
40 DO 100 I=1,N
C CALCULATE ELEMENTS OF T MATRIX
IF(M=2) 42,43,44
C ROOM TEMPERATURE PROPERTIES
42 EL(I)=2990000.
FT(I)=271000.
GLT(I)=700000.
ULT(I)=.21
UTL(I)=UL(I)*FT(I)/EL(I)
AL(I)=.00000732
AT(I)=.0001067
H(I)=.0052
GO TO 46
C 220 DEGREE PROPERTIES
43 EL(I)=2090000.
FT(I)=142000.
GLT(I)=377100.
ULT(I)=.21
UTL(I)=UL(I)*FT(I)/EL(I)

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S.0025      AL(I)=.000002414
S.0026      AT(I)=.00001394
S.0027      H(I)=.0052
S.0028      GO TO 46
C 350. DEGREE PROPERTIES
S.0029      44 EL(I)=29900000.
S.0030      FT(I)=1130000.
S.0031      GLT(I)=300000.
S.0032      ULT(I)=.21
S.0033      UTL(I)=UL(I)*ET(I)/EL(I)
S.0034      AL(I)=.00000283
S.0035      AT(I)=.00001978
S.0036      H(I)=.0052
S.0037      46 T(I,1,1)=EL(I)/(1.-ULT(I)*UTL(I))
S.0038      T(I,1,2)=EL(I)*UTL(I)/(1.-ULT(I)*UTL(I))
S.0039      T(I,1,3)=0.
S.0040      T(I,2,1)=T(I,1,2)
S.0041      T(I,2,2)=FT(I)/(1.-ULT(I)*UTL(I))
S.0042      T(I,2,3)=0.
S.0043      T(I,3,1)=0.
S.0044      T(I,3,2)=0.
S.0045      T(I,3,3)=GLT(I)
C CALCULATE ELEMENTS OF R MATRIX
S.0046      EM(I)=COSD(TH(I))
S.0047      EN(I)=SIND(TH(I))
S.0048      50 R(I,1,1)=T(I,1,1)*EM(I)**4+2.*T(I,1,2)+2.*T(I,1,3,3))*(EM(I)**2)*
1(FN(I)**2)+T(I,2,2)*EN(I)**4
S.0049      R(I,1,2)=(T(I,1,1)+T(I,1,2,2)-4.*T(I,3,3))*(EM(I)**2)*(FN(I)**2)+T(I,1,2,2)*
1,1,2)*(EM(I)**4+EN(I)**4)
S.0050      R(I,1,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EN(I)*EM(I)**3+T(I,1,2,2)*
1T(I,2,2)+2.*T(I,1,3,3)*FM(I)*EN(I)**3
S.0051      R(I,2,2)=T(I,1,1)*EN(I)**4+2.*T(I,1,2)+2.*T(I,1,3,3))*(EM(I)**2)*
1(FN(I)**2)+T(I,2,2)*EM(I)**4
S.0052      B(I,2,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EM(I)*EN(I)**3+T(I,1,2,2)*
1T(I,2,2)+2.*T(I,1,3,3)*FM(I)*EN(I)**3
S.0053      H(I,3,3)=(T(I,1,1)+T(I,1,2,2)-2.*T(I,1,2)-2.*T(I,3,3))*(EN(I)**2)*
1(FN(I)**2)+T(I,3,3)*(EM(I)**4+FN(I)**4)
S.0054      R(I,2,1)=R(I,1,2)
S.0055      B(I,3,1)=B(I,1,3)
S.0056      B(I,3,2)=B(I,2,3)
C CALCULATE ELEMENTS OF INVERSE OF D MATRIX
S.0057      70 DI(I,1,1)=EM(I)**2
S.0058      DI(I,1,2)=EN(I)**2
S.0059      DI(I,1,3)=-2.*EM(I)*EN(I)

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00010126
00010128
00010130
00010132
00010134
00010136
00010138
00010140
00010142
00010144
00010146
00010148
00010150
00010160
00010170
00010180
00010190
00010200
00010220
00010230
00010240
00010250
00010260
00010270
00010280
00010290
00010300
00010310
00010320
00010330
00010340
00010350
00010360
00010370
00010380
00010390
00010400
00010420
00010430
00010440

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S.0096      DR6=H(1,2,3)*H(1)
S.0097      RR(2,3)=RR(2,3)+DR6
S.0098      DR7=H(1,3,1)*H(1)
S.0099      RR(3,1)=RR(3,1)+DR7
S.0100      DR8=H(1,3,2)*H(1)
S.0101      RR(3,2)=RR(3,2)+DR8
S.0102      DR9=H(1,3,3)*H(1)
S.0103      RR(3,3)=RR(3,3)+DR9
S.0104      200 CONTINUE
S. 105      WRITE(6,760) RR(1,1),RR(1,2),RR(1,3),RR(2,1),RR(2,2),RR(2,3),RR(3,1),RR(3,2),RR(3,3)
S.0106      760 FORMAT(1H,30X,26HELEMENTS OF B BAR MATRIX /((17X,E17.8,5X,E17.80010861
1,5X,E17.8))
C CALCULATE ELEMENTS OF C MATRIX
CX=0.
DO 300 I=1,N
DCX=(AX(I,1)*B(1,1,1)+AY(I,1)*B(1,1,2)+AXY(I,1)*B(1,1,3))*H(1)
CX=CX+DCX
300 CONTINUE
CY=0.
DO 400 I=1,N
DCY=(AX(I,1)*B(1,2,1)+AY(I,1)*B(1,2,2)+AXY(I,1)*B(1,2,3))*H(1)
CY=CY+DCY
400 CONTINUE
CXY=0.
DO 500 I=1,N
DCXY=(AX(I,1)*B(1,3,1)+AY(I,1)*B(1,3,2)+AXY(I,1)*B(1,3,3))*H(1)
CXY=CXY+DCXY
500 CONTINUE
C CALCULATE ELEMENTS OF INVERSE OF H-BAR MATRIX
DR=RR(1,1)*(RR(2,2)*RR(3,3)-RR(2,3)*RR(3,2))-RR(1,2)*RR(2,1)*RR(3,3)
RR(1,1)=1/(RR(2,2)*RR(3,3)-RR(2,3)*RR(3,2))/DR
RR(1,2)=-1/(RR(1,2)*RR(3,3)-RR(1,3)*RR(3,2))/DR
RR(1,3)=-1/(RR(1,2)*RR(3,3)-RR(1,3)*RR(3,2))/DR
RR(2,1)=-1/(RR(2,1)*RR(3,3)-RR(2,3)*RR(3,1))/DR
RR(2,2)=1/(RR(1,1)*RR(3,3)-RR(1,2)*RR(3,1)-RR(2,1)*RR(3,2))/DR
RR(2,3)=-1/(RR(1,1)*RR(3,3)-RR(1,2)*RR(3,1)-RR(2,1)*RR(3,2))/DR
RR(3,1)=-1/(RR(2,1)*RR(3,1)-RR(2,2)*RR(3,1))/DR
RR(3,2)=-1/(RR(2,1)*RR(3,1)-RR(2,2)*RR(3,1))/DR
RR(3,3)=1/(RR(2,1)*RR(3,1)-RR(2,2)*RR(3,1))/DR
560 RR(3,3)=1/(RR(2,1)*RR(3,1)-RR(2,2)*RR(3,1))/DR
C CALCULATE ELEMENTS OF ALPHA-BAR MATRIX
AXB=B(1,1)*CX+B(1,2)*CY+B(1,3)*CXY
AYR=H(2,1)*CX+H(2,2)*CY+H(2,3)*CXY

```


ELEMENTS IIF ALPHA MATRIX

1	0.23199991E-05	0.10670000E-04	0.0
2	0.64950009E-05	0.64950009E-05	-0.41750000E-05
3	0.64950009E-05	0.64950009E-05	0.41750000E-05
4	0.23199991E-05	0.10670000E-04	0.0

ELEMENTS OF B BAR MATRIX

0.40758513E 06	0.86759750F 05	0.0
0.86759750F 05	0.12367444F 06	0.0
0.0	0.0	0.89434938E 05

ELEMENTS OF DI MATRIX

1	0.10000000E 01	0.0	0.0	0.0
1	0.0	0.0	0.10000000E 01	0.0
1	0.0	0.0	0.0	0.10000000E 01
2	0.50000018E 00	0.50000018E 00	0.50000018E 00	-0.99999994E 00
2	0.50000018E 00	0.50000018E 00	0.50000018E 00	0.99999994E 00
2	0.49999994E 00	-0.49999994E 00	0.0	0.0
3	0.50000018E 00	0.50000018E 00	0.50000018E 00	0.99999994E 00
3	0.50000018E 00	0.50000018E 00	0.50000018E 00	-0.99999994E 00
3	-0.49999994E 00	0.49999994E 00	0.0	0.0
4	0.10000000E 01	0.0	0.0	0.0
4	0.0	0.10000000E 01	0.0	0.0
4	0.0	0.0	0.10000000E 01	0.0

ELEMENTS OF B M. XIX

1	0.30019984F 08	0.57138356F 06	0.57138356F 06	0.0	0.70000000E 06
1	0.57138356F 06	0.27208750F 07	0.27208750F 07	0.0	0.68247770E 07
1	0.0	0.0	0.0	0.0	0.68247770E 07
2	0.91709110F 07	0.77709080F 07	0.77709080F 07	0.0	0.78995250E 07
2	0.77709080F 07	0.91709110F 07	0.91709110F 07	0.0	-0.68247770E 07
3	0.68247770F 07	0.68247770F 07	0.68247770F 07	0.0	-0.68247770E 07
3	0.91709110F 07	0.77709080F 07	0.77709080F 07	0.0	0.78995250E 07
3	0.77709080F 07	0.91709110F 07	0.91709110F 07	0.0	0.0
4	-0.68247770F 07	-0.68247770F 07	-0.68247770F 07	0.0	0.70000000E 06
4	0.30019984F 08	0.57138356F 06	0.57138356F 06	0.0	0.0
4	0.57138356F 06	0.27208750F 07	0.27208750F 07	0.0	0.70000000E 06
4	0.0	0.0	0.0	0.0	0.0

ELEMENTS OF ALPHA BAR MATRIX

ALPHA BAR X	ALPHA BAR Y	ALPHA BAR XY

0.23632065E-05	0.77521199E-05	0.0

THERMAL STRESSES

PLY NO.	ALPHA X	ALPHA Y	ALPHA XY
1	-0.37013893E 01	-0.79144974F 02	0.0
2	0.37009277E 01	0.79145752F 02	0.13361493E 03
3	0.37009277E 01	0.79145752F 02	-0.13361493E 03
4	-0.37013893E 01	-0.79144974F 02	0.0

THERMAL STRESSES NATURAL AXES

PLY NO.	ALPHA L	ALPHA T	ALPHA LT
1	-0.37014160E 01	-0.79144806E 02	0.0
2	0.78977881E 03	-0.13706204E 03	-0.18861176E 02
3	0.78977881E 03	-0.13706204E 03	0.18861176E 02
4	-0.37014160E 01	-0.79144806E 02	0.0

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A	Card A	A = Lamina data type (-1.0 = comp, 0.0 = ten-comp, 1.0 = ten.)
13	EL		EL = Longitudinal Young's modulus for one ply
25	ET		ET = Transverse Young's modulus for one ply
37	GLT		GLT = Shear modulus for one ply
49	ULT		ULT = Poisson's ratio for one ply
61			
1	I Q	Card B	IQ = Total number of sets (even integer)
13	I P 1		H = nominal ply thickness (Pl. Pt.)
25	T H E T A		IP1 = Number of inner plies in each IQ set (integer)
37	I P S 1		IP2 = Number of plus theta plies in each IQ set (integer)
49	I P S 2		IP2 = Number of outer plies in each IQ set (integer)
61			THETA = $\pm$ Angle of the middle plies (must be negative if the + theta degree ply is closest to the laminate surface... ...Pl. Pt.)
1			IP31 = Angle indicator for the inner plies. Make 0 for 0° and 90 for 90°. (integer can only be 0 or 90)
13			IP32 = Same as IP31 except that it refers to the outer IP2 plies
25			
37			
49			
61			
1			
13			
25			
37			
49			
61			

NUMBER			IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 0		Card A	A = 0.0 for average tension-compression data)
13	2 9 9 0 0 0 0 0 .			KL
25	2 7 1 0 0 0 0 .			EP
37	7 0 0 0 0 0 .			GLT
49	. 2 1 0			ULT
61			1 0	
1	2 . 0 0 5 2		Card B	Number of sets
13	1 1 1	1		IP1
25	4 5 . 0			Theta
37	0 0	0		Angle of IP1 plies
49				Angle of IP2 plies
61			2 0	[0/±45/0]s
1	4 . 0 0 5 2		Card B	
13	2 1 1	2		
25	4 5 5 . 0			
37	0 9 0			
49				
61			3 0	[0 ₂ /±45/90 ₂]2s
1	2 . 0 0 5 2		Card B	
13	1 1 1	0		
25	6 0 . 0			
37	0 0	0		
49				
61			4 0	[0/±60]s

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	CARD A	TIT = Any desired one-card title in columns 1-72
13		
25		
37		
49		
61		
1	CARD B	TNAOB = Total number of (a/b)'s to be considered
13		ALF = $\sqrt{E_x/E_y}$
25		TNN = Total number of n
37		TNM = Total number of m
49		TNPFI = Total number of ϕ
61		TNV = Total number of v
1	CARD C	AI1 = Invariant 1 = $E_x (1 + E_y/E_x + 2\nu_{xy}) / (1 - \nu_{xy} \nu_{yz})$
13		AI2 = Invariant 2 = $G_{xy} - E_x \nu_{xy} / (1 - \nu_{xy} \nu_{yz})$
25		$\dots = D_{xy}/D_{yz} = G'_{xy}/G'_{yz}$
37		TNPFI = Total number of ψ
49		
61		
1	CARD(S) D	AOB(1) = First panel aspect ratio (a/b) ₁
13		AOB(2) = Second panel aspect ratio (a/b) ₂
25		.
37		.
49		.
61		(Maximum of 20 (a/b)'s)

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	P H I (1)	CARD(S) E 73 80	$\phi = N_y/N_x$	
13	P H I (2)		$\phi(2) = \phi_2$	
25	.		.	
37	.		.	
49	.		.	
61	.		(Maximum of 20 ϕ)	
1	V (1)	CARD(S) F 73 80	$V(1) = V_1$	
13	V (2)		$V(2) = V_2$	
25	.		.	
37	.		.	
49	.		.	
61	.		(Maximum of 20 V 's)	
1	P S I (1)	CARD(S) G 73 80	$\psi = \sqrt{E_x E_y} / (1 - \nu_{xy} \nu_{yx})$	
13	P S I (2)		$\psi(2) = \psi_2$	
25	.		.	
37	.		.	
49	.		.	
61	.		(Maximum of 20 ψ 's)	
1				
13				
25				
37				
49				
61				

NUMBER		IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	(0 / + - 4 5 / 0) S	73	TIT	[0/415/0] S at Room Temperature
13	L A M I N A T E A T			
25	R O O M T E M P E R A			
37	T U R E			
49				
61		1 0		
1	3 . 0	73	TNAOB	
13	1 . 8 2 5		ALP	
25	1 0 . 0		TNN	
37	1 0 . 0		TNN	
49	5 . 0		TNPHI	
61	1 . 0	2 0	TNV	
1	3 3 8 8 4 0 0 0 .	73	AIL	
13	1 2 8 6 2 0 .		AI2	
25	2 . 0		R	
37	1 . 0		TNFSI	
49				
61		3 0		
1	1 . 0	73	(a/b) ₁	
13	2 . 0		(a/b) ₂	
25	3 . 0		(a/b) ₃	
37				
49				
61		4 0		

NUMBER		IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	0 . 1		PHI(1)	
13	0 . 5		PHI(2)	
25	1 . 0		PHI(3)	
37	2 . 0		PHI(4)	
49	1 0 . 0	80	PHI(5)	
61		5 0		
1	0 . 0 4 2 3 3		V(1)	
13				
25				
37				
49		80		
61		6 0		
1	1 0 7 9 4 0 0 0 .		PSI(1)	
13				
25				
37				
49		80		
61		7 0		
1	(0 / 0 / + - 4 5 / 9 0		TIT	[02/45/90]2S at Room Temperature
13	/ 9 0) 2 S L A M I N			
25	A T E A T R O O M			
37	T E M P E R A T U R E			
49		80		
61		8 0		

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3 . 0	TNAOB
13	1 . 0 0	ALF
25	1 0 . 0	TNN
37	2 0 . 0	TNN
49	5 . 0	TNPHI
61	1 . 0	TNY
1	3 3 8 8 4 0 0 0 .	ATL
13	1 2 8 6 2 0 .	AT2
25	2 . 0	R
37	1 . 0	TNPSI
49		
61	1 0 0	
1	1 . 0	AOB(1)
13	2 . 0	AOB(2)
25	3 . 0	AOB(3)
37		
49		
61	1 1 0	
1	0 . 1	PHI(1)
13	0 . 5	PHI(2)
25	1 . 0	PHI(3)
37	5 . 0	PHI(4)
49	1 0 . 0	PHI(5)
61	1 2 0	

NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 1 8 9 6 3		V(1)
13			
25			
37			
49			
61		80 1 3 0	
1	1 3 9 7 1 0 0 0 .		PSI(1)
13			
25			
37			
49		80	
61		1 4 0	
1	(0 / + - 6 0) S L A		TIT [0/260]g at Room Temperature
13	M I N A T E A T R O		
25	M O M T E M P E R A T U		
37	R E		
49		80	
61		1 5 0	
1	3 . 0		THA0B
13	1 . 0		ALF
25	1 0 . 0		TNN
37	1 0 . 0		TNM
49	5 . 0	80	TRPHI
61	1 . 0	1 6 0	TNV

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1 3 3 8 8 4 0 0 0 .	73 2 7 0	AI1	
12 1 2 8 6 2 0 .		AI2	
23 2 . 0		R	
24 1 . 0		TNPSI	
49			
51	73 2 7 0		
1 1 . 0	73 2 8 0	A08(1)	
12 2 . 0		A08(2)	
23 3 . 0		A08(3)	
24			
49			
51	73 2 8 0		
1 0 . 1	73 2 9 0	PHI(1)	
12 0 . 5		PHI(2)	
23 1 . 0		PHI(3)	
24 5 . 0		PHI(4)	
49 2 0 . 0		PHI(5)	
51	73 2 9 0		
1 0 . 0 3 6 8 4	73 2 0 0	V(1)	
12			
23			
24			
49			
51	73 2 0 0		

[illegible]

/ID 00,290,1041,01,DAVIDSON EXT3883
 /FTC NAME=AC12,LIST,DECK
 RPS FORTRAN D COMPILER

12/18/69

12/18/69

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C AC12...Z- AND HAT-STIFFENED PANEL STABILITY AND STIFFNESS
C
C THE INPUT DATA IS AS FOLLOWS.....
C BMAT = ANY DESIRED ONE-CARD TITLE IN COLS. 1-72
C A11 = EX*Y/(1-MUXY*NUYX)
C A12 = MUXY*A11
C A16 = DEFINED BY EQUATION 9 (0.0 FOR SYMMETRIC LAMINATES)
C A22 = EY*Y/(1-MUXY*NUYX) = A11*EY/EX
C A26 = DEFINED BY EQUATION 9 (0.0 FOR SYMMETRIC LAMINATES)
C A66 = GXY*T
C T = LAMINATE THICKNESS
C D11 = SHEET FLEXURAL STIFFNESS IN THE X-DIRECTION FROM
C PROGRAM AC2 OR EQUATION 9
C AEXF = FACE SHEET YOUNG'S MODULUS IN THE X-DIRECTION
C TP = INDICATOR FOR Z(TP=0.0) AND HAT(TP=1.0)
C AK7 = BUCKLING COEFFICIENT FOUND FROM EQUATION 131
C AKS = BUCKLING COEFFICIENT FOUND FROM EQUATION 132
C AKF = BUCKLING COEFFICIENT FOUND FROM EQUATION 132
C AFXS = YOUNG'S MODULUS EX OF THE STIFFENER
C TNBS = TOTAL NUMBER OF STIFFENER SPACINGS
C TNX = TOTAL NUMBER OF LOADINGS TO BE CONSIDERED
C DSOHS = HAT STIFFENER GEOMETRY RATIO
C BNX(1) = LOADINGS
C XBS(1) = STIFFENER SPACINGS
C
C DIMENSION BMAT(18),BNX(30),XBS(30)
C 1 READ(5,2) (BMAT(M),M=1,18)
C 2 FORMAT(18A4)
C 45 WRITE(6,46) (BMAT(M),M=1,18)
C 46 FORMAT(1H1,18A4)
C 4 READ(5,3)A11,A12,A16,A22,A26,A66,D11,AEXF,T,TP,AK7,AKS,AKF
C 3 FORMAT(6F12.0)
C
C READ IN STIFFENER PROPERTIES
C
C 5 READ(5,3) AFXS,TNBS,TNNX,DSOHS
C 7 1A=TNBS
C 18=TNX
C READ(5,3) (XBS(I),I=1,1A)
C READ(5,3) (BNX(I),I=1,18)

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S.0001
 S.0002
 S.0003
 S.0004
 S.0005
 S.0006
 S.0007

S.0008
 S.0009
 S.0010
 S.0011
 S.0012

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S.0013      DN 200 J=1,IR
S.0014      50 WRITE(6,51)NMX(J),T
S.0015      51 FORMAT(24H0)      NX(LB/INCH)=F7.1,7H      T=F7.4)
S.0016      40 WRITE(6,41)
S.0017      41 FORMAT(99H0) BS      HS      TS      FS      DS      01      L=0
      1RIT.      SIGS
S.0018      DN 200 I=1,IA
S.0019      ANX=HNX(J)
S.0020      RS=XBS(I)
S.0021      RFTA= (AK7/AKS)**.5
S.0022      ALFA = (AFXSAKF/(AFXFAKS))**.5
S.0023      IF(TP)17,16,17
C
C      LOCAL BUCKLING
C
C      CALCULATION OF GEOMETRY OF 2-STIFFENER ELEMENTS TO INSURE
C      SIMULTANEOUS LOCAL BUCKLING
C
S.0024      16 HS = (( ANX*AFXS/(AKS*(ALFA*T/BS)**2) -AEXE*T)*BS**2/(ALFA*T *AEXS
      1*(2.*BETA +1.)) )**.5
S.0025      TS=(ALFA * HS * T)/BS
S.0026      FS= BETA *HS
S.0027      GO TO 20
C
C      CALCULATION OF GEOMETRY OF HAT-STIFFENED ELEMENTS TO INSURE
C      SIMULTANEOUS LOCAL BUCKLING
C
S.0028      17 HS = (( ANX*AEXS/(AKS*(ALFA*T/RS)**2) -AEXE*T)*HS**2/(ALFA*T *AEXS
      1*(2.*1.5+DS*HS))**.5
S.0029      TS= NSDHS*HS
S.0030      TS= ALFA*HS *T/BS
S.0031      FS=.5* HS
S.0032      20 IF(TP)19,8,9
S.0033      R AK1=AEYS*TS*(2.*FS+HS)
S.0034      AK2=AFXS*TS*(FS*(HS+.5*T-TS)+.5*HS*(HS+T) +FS*.5*(TS+T))
S.0035      C1=-AFXS*TS*(FS*(HS+.5*T-TS)+.5*HS*(HS+T) +.5*FS*(TS+T))
S.0036      C2=-AFXS*TS*(FS*(HS*(HS+T-TS) +TS*(TS/3.-T/2.)+T**2/4.) +FS*(T
      1*(2/4. +T*TS/2.+TS**2/3.) +HS*(HS**2/3.+HS*T/2. +T**2/4.))
S.0037      GO TO 10
S.0038      9 AK1= 2.*AFXS*TS*(DS+HS+FS)
S.0039      AK2=AFXS*TS*(DS*(TS+T)+HS*(HS+T) +2.*FS*(HS +T/2. -TS/2.))
S.0040      C1=- TS*AEXS*( DS*(TS+T) +HS*(HS+T) +2.*FS*(HS +T/2. -TS/2.))
S.0041      C2= -AFXS *2.*TS *( FS*(HS*(HS+T-TS) +TS*(TS/3. -T/4.)+T**2/4. )
      1 +DS*(TS**2/3. +T*TS/2. +T**2/4.)+HC*(HS**2/3. +HS*T/2. +T**2/4.))

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S.0042      10 DEL=(AK2/BS) / ( (A12-A16*A26/A66)**2 -(AK1/BS +A11 -A16**2/A66) *
            1(A22-A26**2/A66))
S.0043      BETA= DEL*(A22-A26**2/A66)
S.0044      D1= -(C1/BS)*BETA-C2/BS + D11
S.0045      ANXL2 = 9.8696*D1
S.0046      XLCR=(ANXL2/ANX)**.5

C
C      DETERMINATION OF STIFFENER AND SHEET PROPERTIES
C
S.0047      IF(TP)14,12,14
S.0048      12 C3= AEXF*T + AEXS*TS*(2.*FS+ HS -2.*TS)/BS
S.0049      GO TO 15
S.0050      14 C3= AEXF*T + AEXS*TS*(2.*(DS+HS+ FS)/BS
S.0051      15 SIGF= ANX*AEXF/C3
S.0052      SIGS= ANX*AEXS/C3
S.0053      100 WRITE(6,101)BS,HS,TS,FS,D1,XLCR ,SIGS,SIGF,DS
S.0054      101 FORMAT(6.3,3F9.3,F14.2,F14.1,2F11.1,F16.3)
S.0055      200 CONTINUE
S.0056      GO TO 1
S.0057      END

END OF COMPILATION      SIZE OF COMMON      00000      PROGRAM      03644
AC12

```

ZEE-STIFFENED PANELS WITH (0/+45/0)S FACES AT ROOM TEMPERATURE

NX (LB/INCH) = 3000.0 T = 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.141	0.011	0.028	798.23	1.0	119171.8	66441.3	0.0	
1.250	0.479	0.031	0.094	33676.95	10.5	78528.2	43781.5	0.0	
1.500	0.824	0.044	0.162	163845.31	23.2	54954.5	30638.5	0.0	
2.000	1.669	0.068	0.328	1221920.00	63.4	30872.5	17212.2	0.0	
3.000	4.051	0.109	0.797	15562177.00	226.3	13578.3	7570.3	0.0	
4.000	7.378	0.149	1.452	89493424.00	542.6	7579.6	4225.8	0.0	
NX (LB/INCH) = 5000.0 T = 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.413	0.033	0.081	28253.52	7.5	124502.4	69413.3	0.0	
1.250	0.772	0.050	0.152	170285.88	18.3	80341.7	44792.5	0.0	
1.500	1.200	0.065	0.236	593462.00	34.2	55687.5	31047.2	0.0	
2.000	2.278	0.092	0.448	3695461.00	85.4	31044.7	17308.2	0.0	
3.000	5.347	0.144	1.052	44056144.00	294.9	13600.5	7582.6	0.0	
4.000	9.639	0.195	1.897	250316672.00	702.9	7584.8	4228.7	0.0	
NX (LB/INCH) = 7500.0 T = 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.598	0.048	0.118	98522.31	11.4	127350.7	71001.2	0.0	
1.250	1.027	0.067	0.202	452445.75	24.4	81280.0	45315.7	0.0	
1.500	1.546	0.083	0.304	1447846.00	43.6	56061.3	31255.6	0.0	
2.000	2.863	0.116	0.563	8545680.00	106.0	31131.5	17356.6	0.0	
3.000	6.619	0.179	1.302	99765376.00	362.3	13611.5	7588.8	0.0	
4.000	11.875	0.240	2.337	564602368.00	862.0	7587.4	4230.1	0.0	
NX (LB/INCH) = 9999.0 T = 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.739	0.060	0.145	201218.31	14.1	128823.7	71822.4	0.0	
1.250	1.230	0.080	0.242	851455.25	29.0	81757.4	45581.8	0.0	
1.500	1.828	0.099	0.360	2647926.00	51.1	56250.1	31360.8	0.0	
2.000	3.347	0.135	0.659	15340545.00	123.1	31175.1	17380.9	0.0	
3.000	7.682	0.207	1.512	177757312.00	418.9	13617.1	7591.9	0.0	
4.000	13.751	0.278	2.706	1004529664.00	995.8	7588.6	4230.9	0.0	

HAT-STIFFENED PANELS WITH (+/-45/0)S FACES AT ROOM TEMPERATURE

NX(LB/INCH)= 3000.0 T= 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.062	0.005	0.031	256.98	0.9	117951.9	65761.1	0.131	
1.250	0.210	0.014	0.105	5075.68	4.1	75489.1	42087.1	0.445	
1.500	0.361	0.020	0.181	25235.45	9.1	52423.1	29227.2	0.767	
2.000	0.732	0.030	0.366	209095.13	26.2	29488.0	16440.3	1.553	
3.000	1.777	0.048	0.889	3024526.00	99.8	13105.8	7306.8	3.769	
4.000	3.236	0.065	1.618	18428448.00	246.2	7372.0	4110.1	6.864	
NX(LB/INCH)= 5000.0 T= 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.181	0.015	0.090	4378.36	2.9	117952.0	65761.2	0.384	
1.250	0.339	0.022	0.169	26896.40	7.3	75489.3	42087.2	0.718	
1.500	0.526	0.028	0.263	99692.06	14.0	52423.1	29227.2	1.116	
2.000	0.999	0.040	0.500	683791.38	36.7	29488.0	16440.3	2.120	
3.000	2.345	0.063	1.173	8964109.00	133.0	13105.8	7306.8	4.975	
4.000	4.228	0.086	2.114	53027760.00	323.5	7372.0	4110.1	8.968	
NX(LB/INCH)= 7500.0 T= 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.262	0.021	0.131	15536.93	4.5	117951.9	65761.1	0.557	
1.250	0.451	0.029	0.225	76094.06	10.0	75489.1	42087.1	0.956	
1.500	0.678	0.037	0.339	258790.50	18.5	52423.2	29227.2	1.439	
2.000	1.256	0.051	0.628	1656045.00	46.7	29488.0	16440.3	2.664	
3.000	2.903	0.078	1.452	20805936.00	165.5	13105.8	7306.8	6.158	
4.000	5.209	0.105	2.604	121386912.00	399.7	7372.0	4110.1	11.048	
NX(LB/INCH)= 9999.0 T= 0.0416									
RS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS	
1.000	0.324	0.026	0.162	32884.21	5.7	117951.9	65761.1	0.687	
1.250	0.540	0.035	0.270	149326.00	12.1	75489.2	42087.1	1.145	
1.500	0.802	0.043	0.401	490950.56	22.0	52423.1	29227.2	1.701	
2.000	1.468	0.059	0.734	3048600.00	54.9	29488.0	16440.3	3.114	
3.000	3.370	0.091	1.685	37548640.00	192.5	13105.8	7306.8	7.148	
4.000	6.032	0.122	3.016	217602960.00	483.5	7372.0	4110.1	12.794	

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13. ABSTRACT This volume summarizes that portion of the program concerned with the development of theoretical methods of analysis for orthotropic laminated composite structure. An appendix to this volume presents a number of computer programs that were generated in conjunction with these theoretical developments. The main body of this volume is comprised of independent sections devoted to the following topics: extensional and flexural elastic constants, flat plates, orthotropic sandwich plates and shells stiffened skin construction, minimum weight analysis, thermoelastic relationships, and joint analysis. The appendix presents, for each computer program, an abstract, data input instructions, output description, and a typical example problem. The listed programs cover the following subjects: calculation of laminate extensional and flexural elastic constants; honeycomb sandwich panels under in-plane biaxial loading, in-plane shear loading, normal pressure, and certain combined loadings; hat- and Z-stiffened panel stability and stiffness; and laminate thermoelastic analysis.			

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Advanced Composite Structure						
Filamentary Composite Materials						
Filamentary Composite Structure						
Fiber-Reinforced Materials						
Composite Material						
Composite Structure						
Anisotropic Analysis						
Micromechanics						
Strength Properties of Composites						
Elastic Constants of Composites						
Thermoelastic Analysis						
Orthotropic Sandwich						

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