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ONE-DIMENSIONAL WAVE PROPAGATION IN BILINEAR MEDIA

R. O. Davis, Jr.

The Eric H. Wang Civil Engineering Research Facility

TECHNICAL REPORT NO. AFWL-TR-70-117

December 1970

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base
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FOREWORD

This report was prepared by the Eric H. Wang Civil Engineering Research Facility, University of New Mexico, Albuquerque, New Mexico, under Contract F29601-68-C-0009. The research was performed under Program Element 61102H, Project 5710, Subtask SB144, and was funded by the Defense Atomic Support Agency (DASA).

Inclusive dates of research were September 1968 through June 1970. The report was submitted 20 November 1970 by the Air Force Weapons Laboratory Project Officer, Captain Howard E. Selheimer (DEV). The former Project Officers were Dr. Henry Cooper and Captain John Thompson.

This technical report has been reviewed and is approved.

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ABSTRACT

An analytic investigation of one-dimensional shock wave propagation in bilinear hysteretic materials is described. Governing equations are solved by the method of characteristics, and solutions are obtained in the form of infinite series. The bilinear model is used to approximate the free-field response of a medium with curvilinear constitutive relationships. It is concluded that the bilinear model may be used with reasonable accuracy to approximate the response of soils subjected to airblast-type loading.

(Distribution Limitation Statement No. 3)

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ABBREVIATIONS AND SYMBOLS

A	attenuation factor
E_i	fitting error
F	functional of squared error in attenuation
G	functional of squared error in surface particle velocity
K	finite number exceeding $\|\kappa\|$
M	integer
M_1	confined modulus for loading
M_2	confined modulus for unloading
N	integer
P	pressure
R	secant modulus for curvilinear model
S	initial unloading tangent modulus for curvilinear model
U	shock velocity
Z_1	loading impedance
Z_2	unloading impedance
a, b, c	coefficients of Brode overpressure function
c	seismic velocity
$\bar{c}$	approximate seismic velocity
e	strain in curvilinear model
g_f	normalized overpressure function
m, n	integers
q_i	point designation in x-t plane
r_n	Riemann invariant
r_1, r_2	random numbers
s_n	Riemann invariant
t	time

t_D	positive-phase duration
t_{50}	half-load time
u	particle velocity
x	depth
x_s	depth to shock front
Δ	incremental value
ψ, Φ	arbitrary bilinear model parameters
α	downgoing characteristic designation
α_i	peak particle velocity measurement
α, β, γ	coefficients of exponential powers in Brode overpressure function
β	upgoing characteristic designation
ε	strain
ζ	bilinear model parameter = $(Z_2 - Z_1)/(Z_2 + Z_1)$
η	coefficient of exponential power in overpressure function
κ	$-\eta\xi$
μ	bilinear model parameter = Z_1/Z_2
ξ	$(x_s/U)(\mu - 1/\mu)$
ρ	density
ρ_0	initial density
σ	$1/\eta$
τ	nondimensionalized time
υ_i	surface particle velocity measurement

Subscript convention:

o	initial condition or configuration
f	variable evaluated at ground surface

SECTION I

INTRODUCTION

1. Background.

As hardness requirements in protective construction have become more severe, so have requirements for free-field response calculations. These response studies are based on a varying number of assumptions, depending on the required accuracy of calculation. At present the most definitive studies which can be made reduce the problem to one of two spatial dimensions and time, and incorporate complex layering and constitutive relationships. These studies consider not only airblast effects but also direct-induced shocks. Less sophisticated studies further reduce the problem to one spatial dimension (uniaxial strain) and time, and consider only airblast loading. Nonlinear constitutive relationships and layering may still be studied. Further simplification usually pertains to constitutive relationships. Although the more sophisticated approaches are rewarding in terms of completeness and applicability, they burden the user with excessive costs. Several hours are required to process many of the large codes. Furthermore, obtaining suitably reliable constitutive relationships from field samples is also very expensive. Complex field sampling procedures and special laboratory apparatus are required. One-dimensional analyses reduce computer costs considerably, but the problems inherent in obtaining constitutive relationships remain.

If further simplifications pertaining to the constitutive relationships are made, considerable savings result. The so-called bilinear model has received more attention than most of the numerous simplified constitutive relationships that have been proposed. This model offers ease of representation but retains the essential hysteretic nature common to all soils. It has been considered by numerous writers (refs. 1,2,3) and is the subject of this report.

2. Objectives.

The primary objective of this study was to evaluate the effectiveness of employing a bilinear constitutive relationship to represent a curvilinear hysteretic relationship. Of particular interest are comparisons of attenuation of peak particle velocity and peak pressure with depth. Also of interest are comparisons of wave shapes and residual displacements.

A minor objective of this study was the evaluation of a plane one-dimensional computer code called PLID, written by Air Force Weapons Laboratory (AFWL) personnel (ref. 4). This code has been designed specifically for calculation of free-field response due to nuclear overpressures. The code admits a wide variety of constitutive relationships and layering. Of special importance in this evaluation are comparisons between analytic and PLID solutions for bilinear materials.

SECTION II

BILINEAR ANALYTIC SOLUTION

1. Constitutive Relationships.

For the purposes of this report, a bilinear material is defined as a material which obeys a stress-strain relationship, in uniaxial strain, of the form shown in figure 1. In figure 1, M_1 and M_2 denote the slopes of the two lines. In problems to be considered here, loading will always occur in the form of a shock, increasing the pressure from zero to some positive compressive stress. Thus for loading, the bilinear material obeys

$$P = M_1 \varepsilon \tag{1}$$

where

P = pressure*
M_1 = loading modulus
ε = strain

Any decrease in pressure, ΔP, will obey

$$\Delta P = M_2 \Delta \varepsilon \tag{2}$$

where M_2 = unloading modulus. Reloading follows the unloading path until the loading curve is reached.

For plane one-dimensional wave propagation (i.e., uniaxial strain), the velocity of propagation of a shock wave, denoted by U, in a bilinear material, is given by

$$U = \sqrt{\frac{M_1}{\rho_0}} \tag{3}$$

where ρ_0 = initial mass density. The strain, ε, at any point in the material, is related to the density, ρ, at that point by

$$\varepsilon = 1 - \frac{\rho_0}{\rho} \tag{4}$$

*The terms pressure and stress will be used interchangeably in this report.

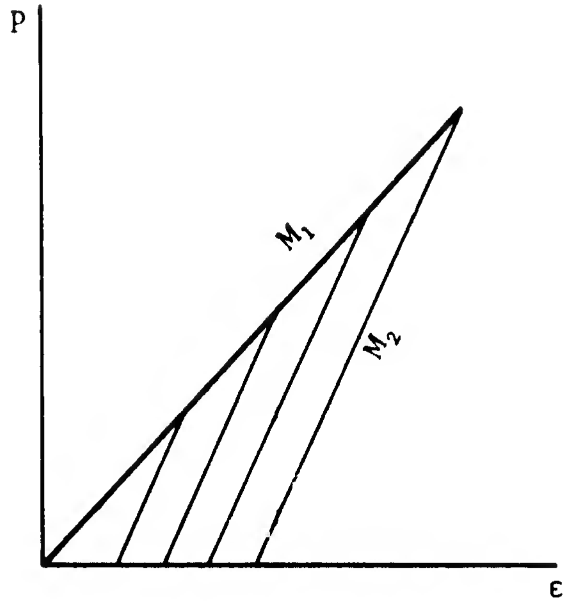


Figure 1. Bilinear Stress-strain Diagram

The conservation of mass jump equation (ref. 5, p. 121) relates the conditions immediately in front of and immediately behind the shock. If u and ρ denote the particle velocity and density directly behind the shock, the conservation of mass jump equation gives

$$\rho_0 U = \rho(U - u) \tag{5}$$

Here, the particle velocity in front of the shock is zero. Combining eqs. (4) and (5) gives

$$u = U\varepsilon \tag{6}$$

Combining eqs. (1), (3), and (6) gives the conservation of momentum jump equation

$$P = \rho_0 U u \tag{7}$$

The term, $\rho_0 U$, is frequently called the loading impedance, and will be denoted here by Z_1

$$Z_1 = \rho_0 U = \sqrt{\rho_0 M_1} \qquad (8)$$

where use of eq. (3) has been made. Thus eq. (7) can be written

$$P = Z_1 u \qquad (9)$$

The local sound velocity is defined as (ref. 5, p. 5)

$$c = \pm \left(\frac{dP}{d\rho}\right)^{\frac{1}{2}} \qquad (10)$$

Combining this relationship with eqs. (2) and (4) and simplifying will lead to

$$c = \pm(1 - \varepsilon)\sqrt{\frac{M_2}{\rho_0}} \qquad (11)$$

This is the velocity relative to the material at which small disturbances will propagate after the shock has passed. The plus and minus signs denote waves propagating in the same direction and the opposite direction as the shock, respectively. Note that c varies linearly with strain. A common assumption in one-dimensional wave propagation in soil and rock is that c is a constant, $\bar{c}$, taken to be (refs. 1,2,3,6)

$$\bar{c} = \sqrt{\frac{M_2}{\rho_0}} \qquad (12)$$

This assumption will be employed in certain portions of the following derivation.

2. Method of Characteristics.

Only problems concerning plane one-dimensional airblast loading of a bilinear half-space will be considered here. The airblast or overpressure load function is applied at the material surface (x = 0). It is assumed to consist of a shock to a maximum pressure, P_0, occurring at time, t = 0, followed by a monotonic decrease in pressure until zero pressure is reached. The overpressure load function will be designated as $P_f(t)$.

Graphically, the problem is best represented in the x-t plane. A schematic representation is shown in figure 2. In zone I, the material is at rest. In zone II, the shock has passed, and the material is deformed and in motion. The slope of the shock trace in the x-t plane is the shock velocity, U, given by

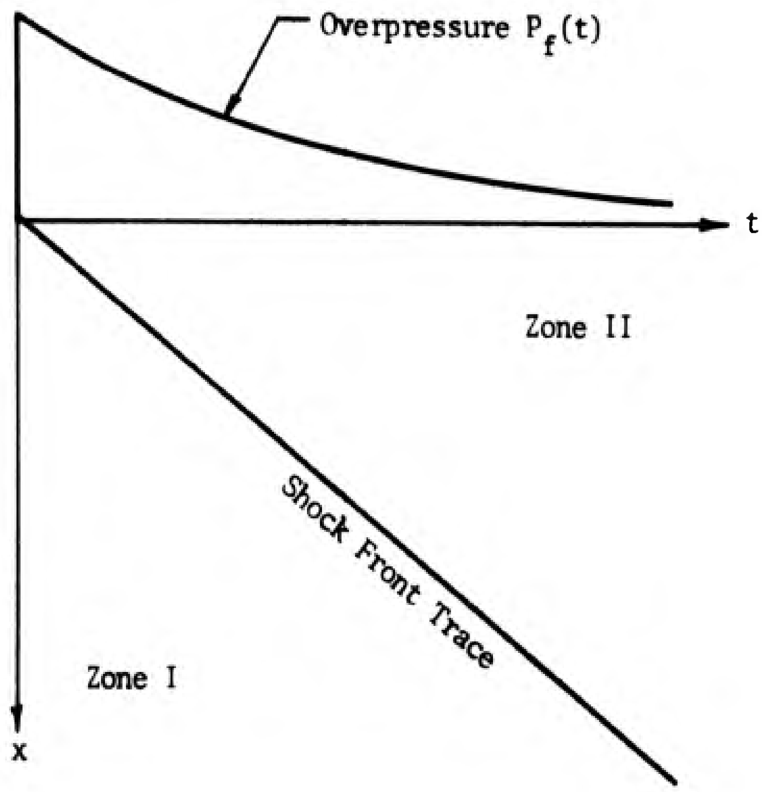


Figure 2. Schematic Representation of Problem in x-t Plane

eq. (3). In zone II, all disturbances, such as the pressure decay at the surface, propagate with velocity, c, given by eq. (11).

In zone II, the material obeys the set of partial differential equations (ref. 5, p. 37)

$$\rho u_x + u\rho_x + \rho_t = 0 \tag{13}$$

$$u_t + uu_x + \frac{c^2}{\rho}\rho_x = 0 \tag{14}$$

where ρ, u, and c represent the density, particle velocity, and sound velocity at a point, and subscripts denote partial differentiation. Zone II is bounded above by the material surface, where the boundary condition

$$P = P_f(t) \tag{15}$$

must be satisfied. It is bounded below by the shock front trace where the jump condition [eq. (9)] must be satisfied. The pressure and particle velocity at any point in zone II may be obtained by solving eqs. (13) and (14) so that eqs. (9) and (15) are satisfied.

To begin this solution, the differential eqs. (13) and (14), together with the differentials of the dependent variables, du and dρ, are written in matrix form, as follows (ref. 7, secs. 4,5, chap. 3; ref. 8, sec. 6.3)

$$\begin{bmatrix} u & 1 & \frac{c^2}{\rho} & 0 \\ \rho & 0 & u & 1 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{bmatrix} \begin{bmatrix} u_x \\ u_t \\ \rho_x \\ \rho_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ du \\ d\rho \end{bmatrix} \tag{16}$$

The characteristic directions in the x-t plane are determined by setting the determinant of the coefficients equal to zero. This leads to

$$\frac{dx}{dt} = u \pm c \tag{17}$$

The family of curves in zone II determined by eq. (17) may be visualized as lines along which all changes in pressure or particle velocity will propagate. The characteristic directions corresponding to the plus sign in eq. (17) will be denoted α characteristics. The characteristics corresponding to the minus sign will be denoted β characteristics.

For the system of equations [eq. (16)] to possess any solution at all on the characteristics, it is necessary that the augmented matrix

$$\begin{bmatrix} u & 1 & \frac{c^2}{\rho} & 0 & 0 \\ \rho & 0 & u & 1 & 0 \\ dx & dt & 0 & 0 & du \\ 0 & 0 & dx & dt & d\rho \end{bmatrix}$$

have at most a rank of three. Imposing this condition leads to two more differential equations

$$\text{on } \alpha: \quad \frac{c}{\rho}\, d\rho + du = 0 \tag{18}$$

$$\text{on } \beta: \quad \frac{c}{\rho}\, d\rho - du = 0 \tag{19}$$

where use of eq. (17) has been made. These two differential equations relate the dependent variables on the characteristics. Note that the independent variables, x and t, do not appear.

Equations (18) and (19) may be simplified by noting that

$$c^2 = \frac{dP}{d\rho} \tag{20}$$

This equation is obtained by squaring both sides of eq. (10). Substituting eq. (20) into eqs. (18) and (19) gives

$$\text{on } \alpha: \quad \frac{dP}{\rho c} + du = 0 \tag{21}$$

$$\text{on } \beta: \quad \frac{dP}{\rho c} - du = 0 \tag{22}$$

Combining eqs. (4) and (11) results in

$$\rho c = \sqrt{\rho_0 M_2} \tag{23}$$

Substituting this relationship into eqs. (21) and (22) leads to

$$\text{on } \alpha: \quad dP = -Z_2 du \tag{24}$$

$$\text{on } \beta: \quad dP = +Z_2 du \tag{25}$$

where

$$Z_2 = \rho c = \sqrt{\rho_0 M_2} \tag{26}$$

The constant, Z_2, is termed the unloading impedance.

Performing an indefinite integration of both sides of eqs. (24) and (25) gives

$$\text{on } \alpha: \quad P = -Z_2 u + 2r \tag{27}$$

$$\text{on } \beta: \quad P = +Z_2 u + 2s \tag{28}$$

where 2r and 2s = constants of integration called Riemann invariants. The values of 2r and 2s will be constant on any particular characteristic, but will change from one characteristic to another.

At this point, the problem consists of solving eq. (17) for the characteristic directions, plotting the characteristics, and then applying eqs. (27) and (28) on the characteristics. This must be done simultaneously, however, since the values of c and u in eq. (17) are not known until eqs. (27) and (28) have been solved. This task may be enormously simplified by the following two assumptions

$$|u| \ll c \tag{29}$$

$$\varepsilon \ll 1 \tag{30}$$

Both of these assumptions are frequently used in calculations involving airblast loading of soil and rock.

The assumption [eq. (29)] simplifies eq. (17) to

$$\frac{dx}{dt} = \pm c$$

and the assumption [eq. (30)] further reduces this to

$$\frac{dx}{dt} = \pm \bar{c} \tag{31}$$

where $\bar{c}$ is defined by eq. (12). Thus the characteristics are assumed to be linear with slopes $\pm\bar{c}$, and are independent of P and u. It is now possible to obtain relatively simple solutions for various overpressure functions.

3. <u>Solution at an Arbitrary Point.</u>

Consider an arbitrary point (x_o,t_o) in zone II of the x-t plane. One α characteristic and one β characteristic pass through this point (fig. 3). The Riemann invariants of these characteristics are denoted by $2r_o$ and $2s_o'$, respectively. Then from eqs. (27) and (28)

$$2r_o = P + Z_2 u \tag{32}$$

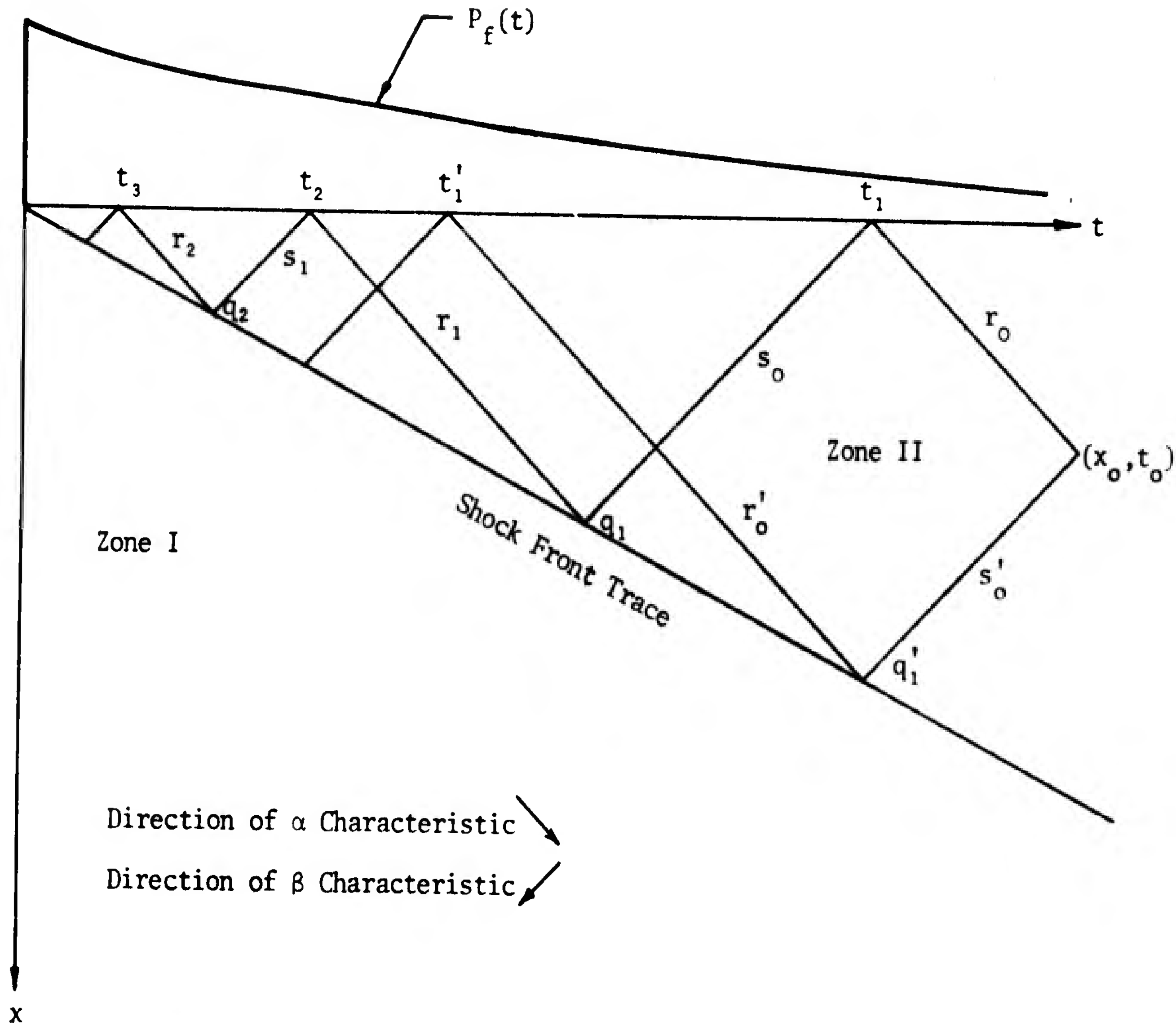


Figure 3. Characteristics Passing through Arbitrary Point (x_o, t_o)

$$2s_o' = P - Z_2u \tag{33}$$

where P and u = the pressure and particle velocity at the point (x_o,t_o). P and u may be found by solving eqs. (32) and (33) simultaneously after r_o and s_o' are determined.

The value of the Riemann invariant, r_o, may be found in the following manner. At the surface point marked t_1 in figure 3, the following equations apply

$$2r_o = P_f(t_1) + Z_2u_f(t_1)$$

$$2s_o = P_f(t_1) - Z_2u_f(t_1)$$

where s_o = the Riemann invariant associated with the β characteristic which intersects the surface at the point t_1 and $u_f(t_1)$ = the surface particle velocity at t_1. Adding these two equations gives

$$2r_o = 2P_f(t_1) - 2s_o \tag{34}$$

Next, at the point marked q_1 in figure 3, the following three equations apply

$$2r_1 = P_s(q_1) + Z_2u_s(q_1)$$

$$2s_o = P_s(q_1) - Z_2u_s(q_1)$$

$$P_s(q_1) = Z_1u_s(q_1)$$

Here, $P_s(q_1)$ and $u_s(q_1)$ denote the pressure and particle velocity immediately behind the shock at the point, q_1. The Riemann invariant, r_1, is associated with the α characteristic passing through q_1. The third of these equations is the conservation of momentum jump equation [eq. (9)]. Combining these three equations to eliminate $P_s(q_1)$ and $u_s(q_1)$ results in

$$2s_o = -\zeta(2r_1) \tag{35}$$

where

$$\zeta = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{36}$$

Substituting eq. (35) into eq. (34) gives

$$2r_o = 2P_f(t_1) + 2\zeta r_1 \tag{37}$$

This procedure can be repeated for r_1 to obtain

$$2r_1 = 2P_f(t_2) + 2\zeta r_2$$

and in general

$$2r_o = 2P_f(t_1) + 2\zeta P_f(t_2) + \cdots + 2\zeta^{n-1}P_f(t_n) + 2\zeta^n r_n \tag{38}$$

Passing to the limit as $n \to \infty$, the last term vanishes since $\zeta < 1$ and r_n is bounded for all n. Thus

$$r_o = \sum_{m=1}^{\infty} \zeta^{m-1} P_f(t_m) \tag{39}$$

A similar procedure can be carried out to find the other Riemann invariant, s_o'. It is found to be

$$s_o' = - \sum_{m=1}^{\infty} \zeta^m P_f(t_m') \tag{40}$$

Since the characteristics are assumed to be linear, it is an easy matter to express t_m and t_m' in terms of x_o and t_o. First for t_m

$$t_1 = t_o - \frac{x_o}{c}$$

and

$$t_m = \zeta^{m-1} t_1 = \zeta^{m-1} \left(t_o - \frac{x_o}{c}\right) \tag{41}$$

Similarly, for t_m'

$$t_1' = \zeta \left(t_o + \frac{x_o}{c}\right)$$

and

$$t_m' = \zeta^{m-1} t_1' = \zeta^m \left(t_o + \frac{x_o}{c}\right) \tag{42}$$

Combining eqs. (32) and (33) with eqs. (39) and (40) gives

$$P(x_o,t_o) = \sum_{m=1}^{\infty}\left\{\zeta^{m-1}P_f(t_m) - \zeta^m P_f(t_m')\right\} \tag{43}$$

and

$$u(x_o,t_o) = \frac{1}{Z_2}\sum_{m=1}^{\infty}\left\{\zeta^{m-1}P_f(t_m) + \zeta^m P_f(t_m')\right\} \tag{44}$$

Replacing t_m and t_m' in these two equations by eqs. (41) and (42) leads to a complete solution of the problem for any point in zone II, involving only the material parameters, ζ and Z_2, and the overpressure load function, $P_f(t)$. A short example will be given to illustrate the method of solution.

Consider the linear overpressure function

$$P_f(t) = \begin{cases} P_o(1 - t) & 0 \le t < 1 \\ 0 & 1 \le t \end{cases} \tag{45}$$

Let the material parameters be

$$Z_2 = 2000\ \frac{\text{lb}}{\text{ft}^2}\cdot\frac{\text{sec}}{\text{ft}}$$

$$\zeta = 1/3$$

Then for an arbitrary point (x_o,t_o), eq. (43) gives

$$P(x_o,t_o) = P_o\sum_{m=1}^{\infty}\left\{\left(\frac{1}{3}\right)^{m-1}\left[1 - \left(\frac{1}{3}\right)^{m-1}\left(t_o - \frac{x_o}{c}\right)\right] - \left(\frac{1}{3}\right)^{m}\left[1 - \left(\frac{1}{3}\right)^{m}\left(t_o + \frac{x_o}{c}\right)\right]\right\} \tag{46}$$

where it has been assumed that the point (x_o,t_o) lies above the shock front trace and that

$$t_o - \frac{x_o}{c} < 1$$

Equation (46) can be written

$$P(x_o,t_o) = P_o\left\{\sum_{m=1}^{\infty}\left[\left(\frac{1}{3}\right)^{m-1} - \left(\frac{1}{3}\right)^{m}\right]\right.$$

$$\left. + \sum_{m=1}^{\infty}\left[\left(\frac{1}{3}\right)^{2m}\left(t_o + \frac{x_o}{c}\right) - \left(\frac{1}{3}\right)^{2m-2}\left(t_o - \frac{x_o}{c}\right)\right]\right\} \qquad (47)$$

This expression can be simplified by cancellation of terms

$$P(x_o,t_o) = P_o\left\{1 - \left(t_o - \frac{x_o}{c}\right) + 2\,\frac{x_o}{c}\sum_{m=1}^{\infty}\left(\frac{1}{3}\right)^{2m}\right\} \qquad (48)$$

Noting that

$$\sum_{m=1}^{\infty}\left(\frac{1}{3}\right)^{2m} = \frac{1}{8}$$

obtains

$$P(x_o,t_o) = P_o\left(1 - t_o + 1.25\,\frac{x_o}{c}\right) \qquad (49)$$

The simplicity of this example is due to the very simple overpressure load function used. In general, for more complex overpressures, closed solutions cannot be obtained. For this reason a brief computer program was written to evaluate eqs. (43) and (44), taking only a finite number of terms. Both series converge very rapidly, however, and computations can be carried out by hand if necessary.

. Attenuation of Peak Pressure and Particle Velocity.

The attenuation of peak pressure and particle velocity at the shock front is of particular interest in free-field calculations.

Consider the point, q_o, lying at the shock front, in figure 4. The pressure, $P_s(q_o)$, and particle velocity, $u_s(q_o)$, immediately behind the shock at this point can be related by eqs. (9) and (27)

$$P_s(q_o) = Z_1 u_s(q_o) \qquad (50)$$

$$2r_o = P_s(q_o) + Z_2 u_s(q_o) \qquad (51)$$

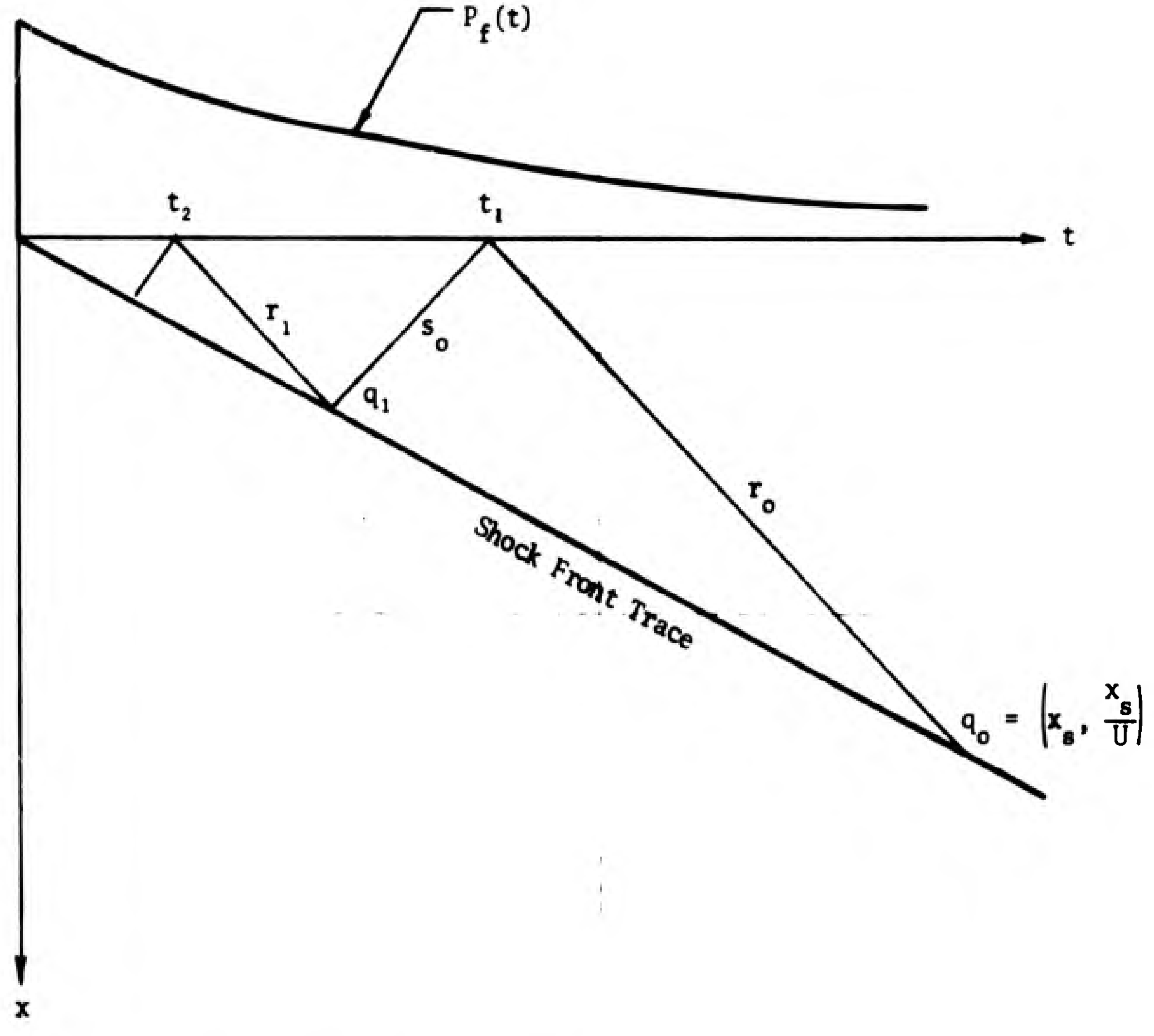


Figure 4. Characteristic Passing through Point on Shock Front Trace

Once the Riemann invariant, r_o, associated with the α characteristic through the point, q_o, is found, P_s and u_s may be determined from these two equations. An expression for r_o has already been determined. Rewriting eq. (39)

$$r_o = \sum_{m=1}^{\infty} \zeta^{m-1} P_f(t_m) \tag{52}$$

For the case considered here, the point, q_o, will have coordinates in the x-t plane of $\left(x_s, \frac{x_s}{U}\right)$, where x_s denotes depth to the shock. Thus, referring to figure 4

$$t_1 = \left(\frac{\mu - 1}{\mu}\right) \frac{x_s}{U} \tag{53}$$

where

$$\mu = \frac{\bar{c}}{U} = \frac{Z_2}{Z_1} \tag{54}$$

Then as before

$$t_m = \zeta^{m-1} t_1 = \zeta^{m-1} \left(\frac{\mu - 1}{\mu}\right) \frac{x_s}{U} \tag{55}$$

Solving eqs. (50) and (51) for P_s gives

$$P_s\left(x_s, \frac{x_s}{U}\right) = (1 - \zeta) \sum_{m=1}^{\infty} \zeta^{m-1} P_f(t_m) \tag{56}$$

Then if $P_f(t)$ is assumed to be of the form

$$P_f(t) = P_o g_f(t)$$

where P_o is the maximum surface overpressure and $g_f(t)$ is assumed to be a monotonically decreasing function of time, the normalized peak pressure will be

$$\bar{P}_s\left(x_s, \frac{x_s}{U}\right) = \frac{P_s}{P_o} = (1 - \zeta) \sum_{m=1}^{\infty} \zeta^{m-1} g_f(t_m) \tag{57}$$

The maximum surface particle velocity, u_o, will occur simultaneously with P_o and will be

$$u_o = \frac{P_o}{Z_1} \tag{58}$$

Thus the normalized peak particle velocity will be

$$\bar{u}_s \left(x_s, \frac{x_s}{U} \right) = \frac{u_s}{u_o} = \left(\frac{P_s}{Z_1} \right) \left(\frac{Z_1}{P_o} \right) = \bar{P}_s \left(x_s, \frac{x_s}{U} \right) \tag{59}$$

Therefore, the attenuation rates for peak pressure and peak particle velocity are identical. For convenience, define an attenuation factor, A, as

$$A = \bar{P}_s = \bar{u}_s \tag{60}$$

A particular overpressure load function will next be considered.

The most frequently employed theoretical overpressure is the so-called Brode wave (ref. 9). Defining t_D as the positive-phase duration of the overpressure and τ as the ratio, $\tau = t/t_D$, the Brode overpressure is given by

$$P_f(t) = P_o(ae^{-\alpha\tau} + be^{-\beta\tau} + ce^{-\gamma\tau})(1 - \tau) \tag{61}$$

for $0 \leq \tau \leq 1$. For times $t > t_D$, P_f is defined as identically zero. The constants, a, b, c, α, β, and γ, as well as t_D, are given in reference 8 for various peak overpressures, P_o, and various weapon yields. For example, for a 1-megaton weapon and P_o = 1,000 psi, reference 8 gives

$$P_f(t) = 1000(0.15e^{-2.9\tau} + 0.30e^{-21\tau} + 0.55e^{-130\tau})(1 - \tau)$$

Letting

$$\tau_m = \frac{t_m}{t_D} = \zeta^{m-1} \left(\frac{\mu - 1}{\mu} \right) \frac{x_s}{t_D U} \tag{62}$$

eq. (60) becomes for the Brode overpressure

$$A = (1 - \zeta) \sum_{m=1}^{\infty} \zeta^{m-1} \left(ae^{-\alpha\tau_m} + be^{-\beta\tau_m} + ce^{-\gamma\tau_m} \right) \left(1 - \tau_m \right) \tag{63}$$

Because of the very complex nature of the relationships between the constants, a, b, c, α, β, γ, and t_D, and the weapon yield and peak overpressure, a short computer program was written to evaluate A for various conditions. This program created plots of the attenuation factor, A, versus arrival time, x_s/U, for various values of μ. Five such plots are shown in figures 5 through 9. All represent a Brode overpressure due to a 1-megaton weapon yield. Peak overpressures of 100, 500, 1,000, 5,000, and 10,000 psi are shown. It has been pointed out (ref. 10) that attenuation curves for any particular weapon yield can be extrapolated to other weapon yields by cube root scaling. Attenuation factors for yields other than 1 megaton may be obtained from the figures in appendix I by multiplying the arrival time values, x_s/U, by the factor, $1/\sqrt[3]{MT}$, where MT = weapon yield in megatons.

One final case will be considered before the conclusion of this section. This is the case where the unloading modulus, M_2, is infinite. It is particularly noteworthy since, for many overpressure functions, closed form solutions may be obtained. Specifically, overpressures of the form

$$P_f = P_o(1 - t)\exp(-\eta t) \qquad \text{for } t < 1$$

$$P_f = 0 \qquad \text{for } t \geq 1 \qquad (64)$$

will be considered. Generalization to more complex functions will be apparent.

For the overpressure function [eq. (64)], the attenuation equation [eq. (60)] becomes

$$A = (1 - \zeta)\sum_{m=1}^{\infty} \zeta^{m-1}(1 - t_m)\exp(-\eta t_m) \qquad (65)$$

where t_m is given by eq. (55).

Combining eqs. (55) and (65) gives

$$A = (1 - \zeta)\sum_{m=1}^{\infty} \zeta^{m-1}(1 - \xi\zeta^{m-1})\exp(\kappa\zeta^{m-1}) \qquad (66)$$

where $\xi = \frac{x_s}{U}\left(\frac{\mu - 1}{\mu}\right)$ and $\kappa = -\eta\xi$

The exponential term of this equation can be expressed in series form

$$\exp(\kappa\zeta^{m-1}) = \sum_{n=0}^{\infty} \frac{\left(\kappa\zeta^{m-1}\right)^n}{n!}$$

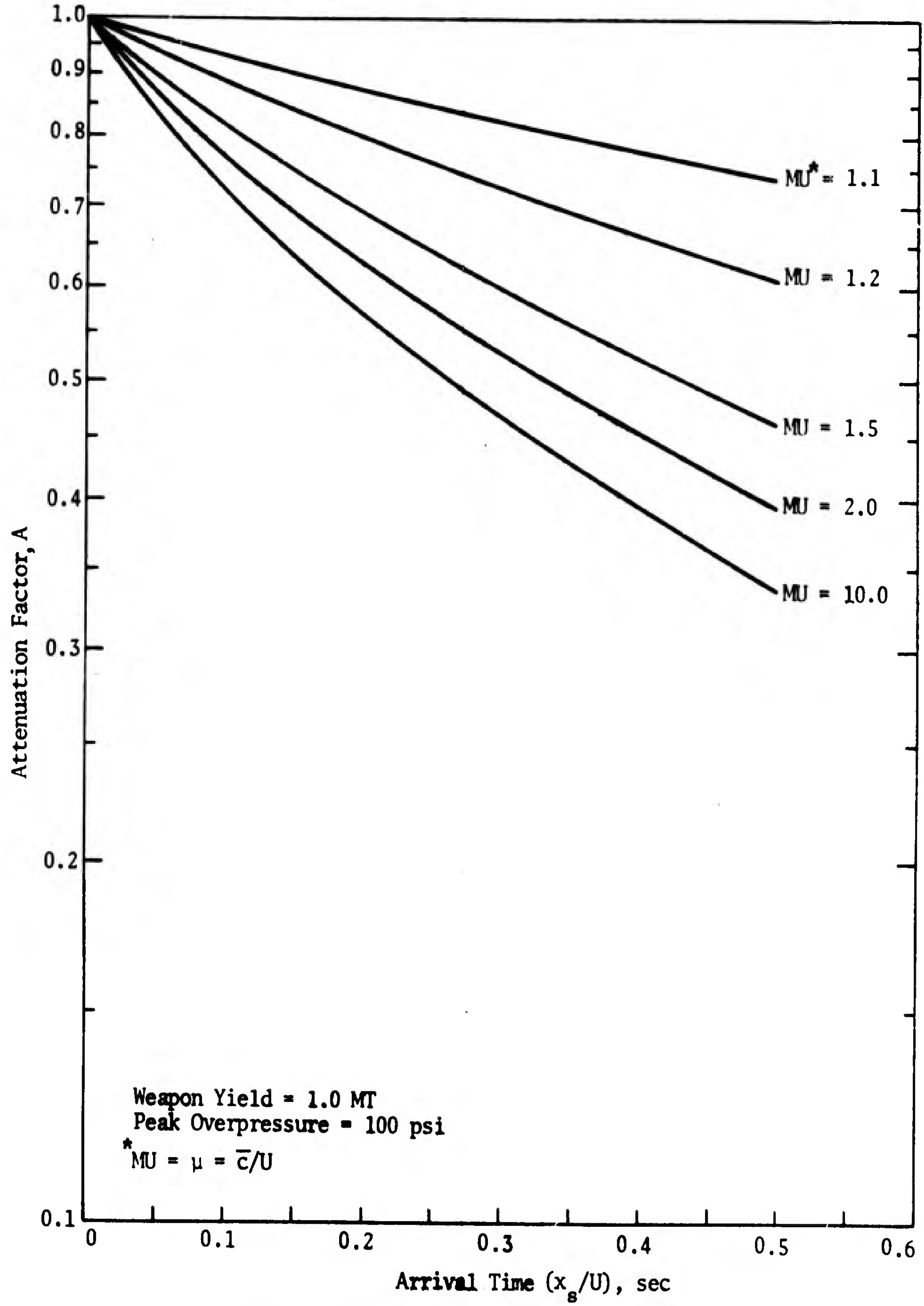


Figure 5. Attenuation Curves for P_o = 100 psi

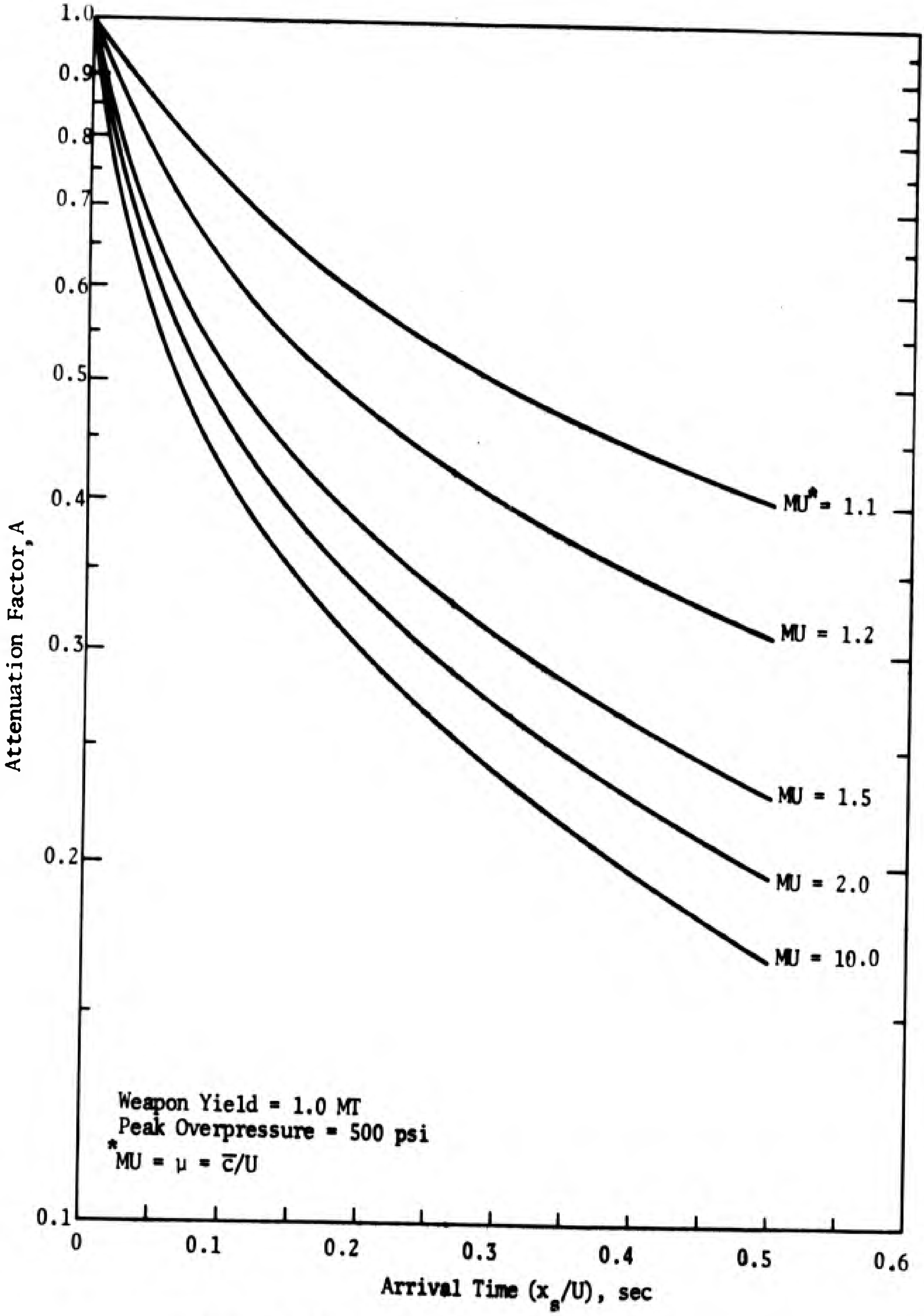


Figure 6. Attenuation Curves for P_o = 500 psi

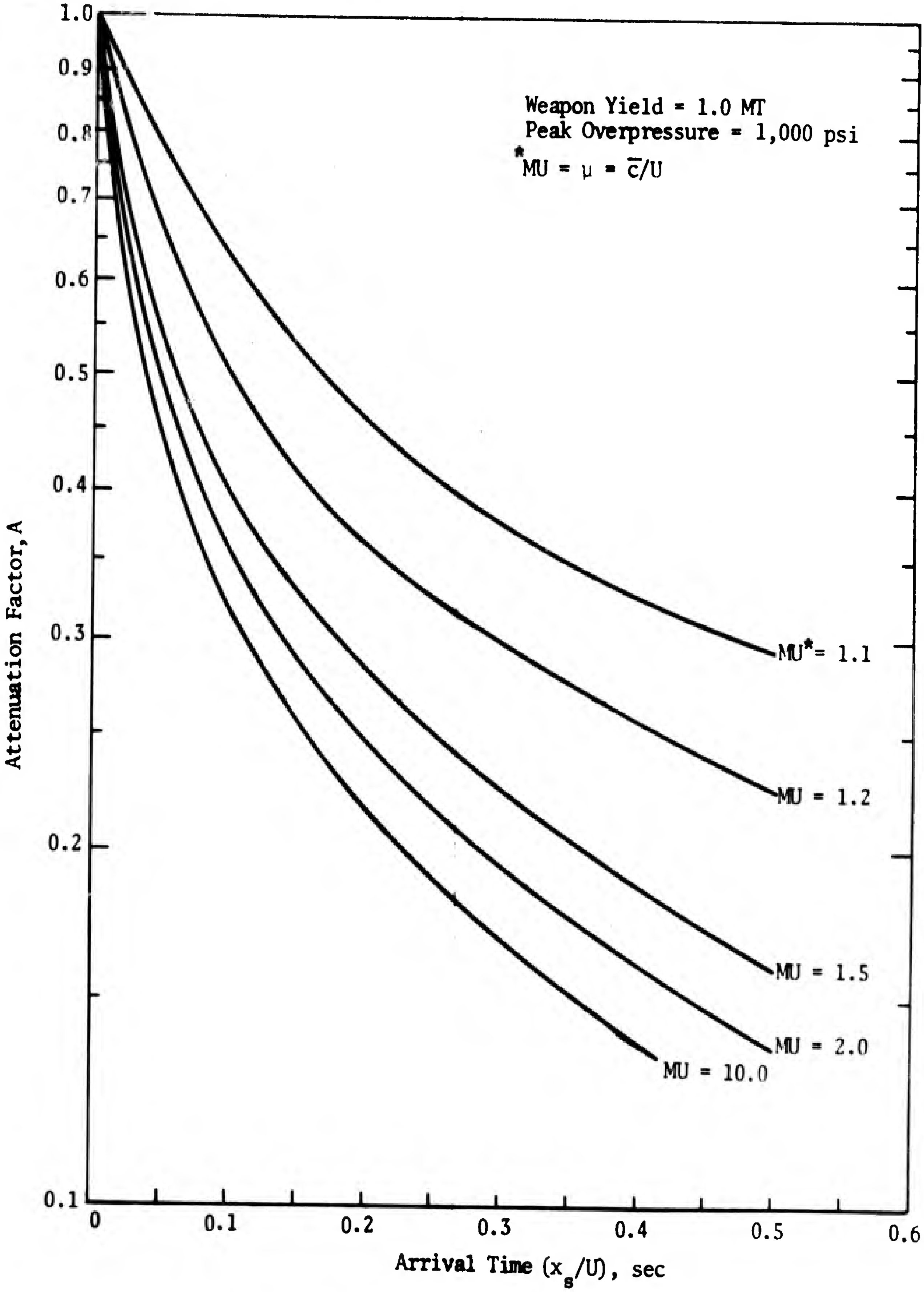


Figure 7. Attenuation Curves for P_o = 1,000 psi

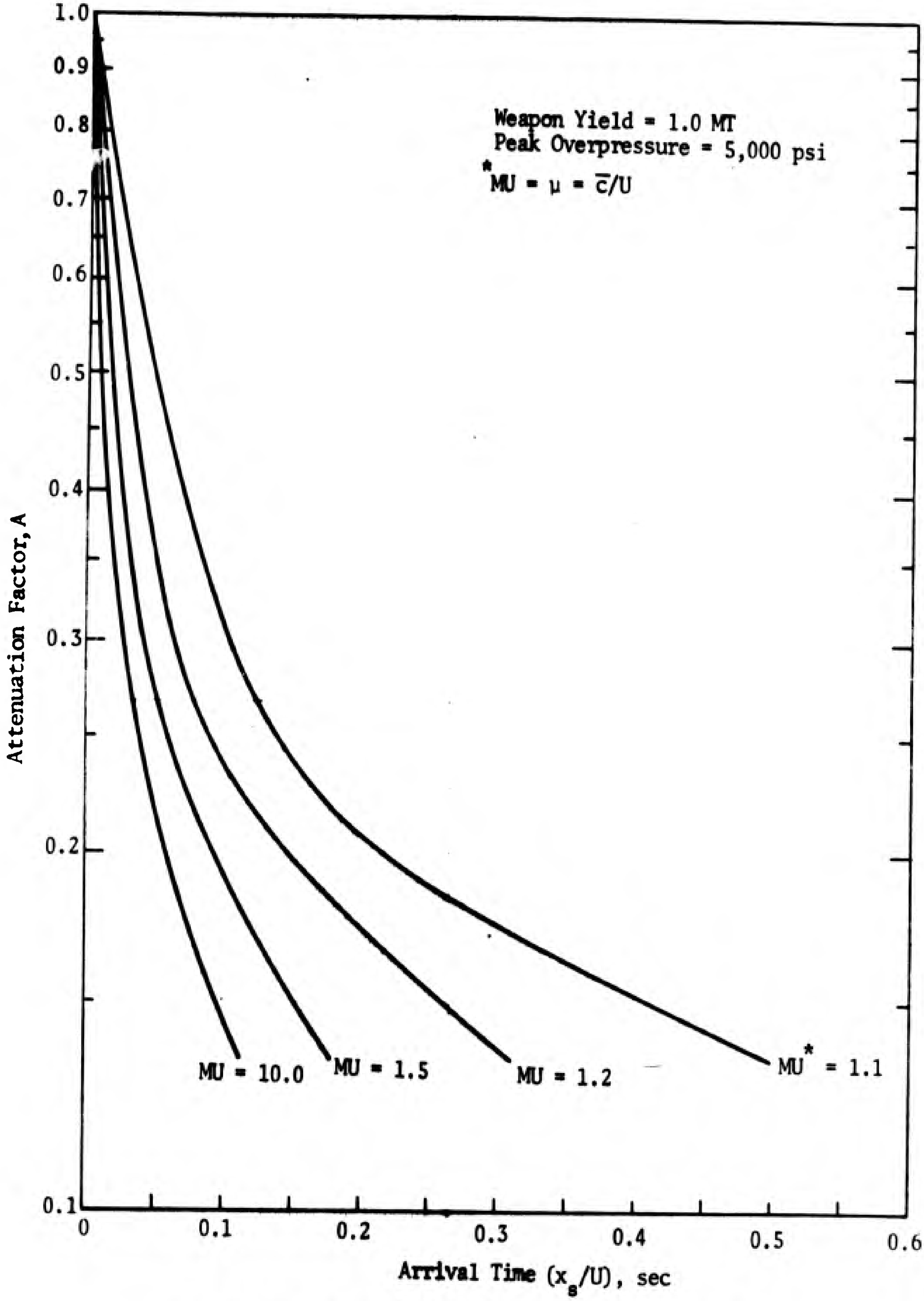


Figure 8. Attenuation Curves for P_o = 5,000 psi

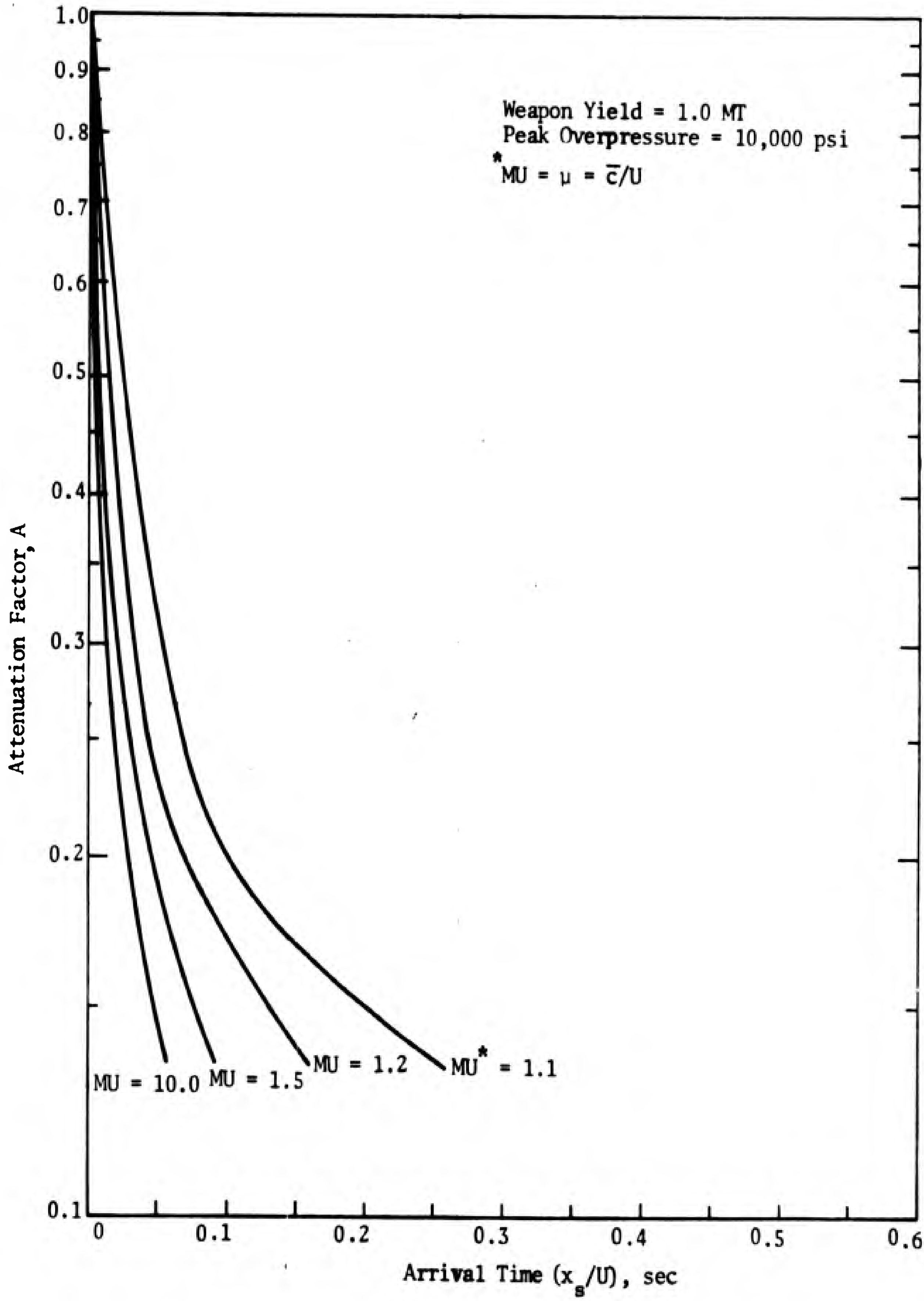


Figure 9. Attenuation Curves for P_o = 10,000 psi

Substituting this expression into eq. (66) and rearranging terms will give the iterated series

$$A = (1 - \zeta) \sum_{m=1}^{\infty} \left\{ \sum_{n=0}^{\infty} \left|\frac{\kappa^n}{n!}\right| \left|\zeta^{(m-1)(n+1)} - \xi\zeta^{(m-1)(n+2)}\right| \right\} \tag{67}$$

It is now convenient to reverse the order of summation. This step is legitimate if both of the single series over m and n converge and if the double series (not iterated)

$$\sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \left|\frac{\kappa^n}{n!}\right| \left|\zeta^{(m-1)(n+1)} - \xi\zeta^{(m-1)(n+2)}\right| \tag{68}$$

converges (ref. 11, pp. 80-81). Convergence of the two single series is trivially shown. To establish convergence of the double series [eq. (68)], it suffices to consider

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left|\frac{\kappa^n}{n!} \zeta^{m(n+1)}\right| \tag{69}$$

The terms of this series are dominated by $\frac{K^n}{n!} \zeta^m$ where $K \geq |\kappa|$. To show that the series of dominating terms converges, consider the quantity

$$s = \left| \frac{e^K}{1 - \zeta} - \sum_{m=0}^{M} \sum_{n=0}^{N} \frac{K^n}{n!} \zeta^m \right|$$

where M and N are integers. Using the series representation for e^K and noting that

$$\sum_{m=0}^{M} \zeta^m = \frac{1 - \zeta^{M+1}}{1 - \zeta}$$

will obtain

$$s = \frac{1}{1 - \zeta} \left| \sum_{n=0}^{\infty} \frac{K^n}{n!} - (1 - \zeta^{M+1}) \sum_{n=0}^{N} \frac{K^n}{n!} \right|$$

Combining terms gives

$$s = \frac{1}{1 - \zeta} \left| \sum_{n=N+1}^{\infty} \frac{K^n}{n!} + \zeta^{M+1} \sum_{n=0}^{N} \frac{K^n}{n!} \right|$$

The first term within the absolute value bars is simply the tail of the exponential series and it can be made arbitrarily small for sufficiently large N. Also since

$\zeta < 1$ and $\sum_{n=0}^{N} \frac{K^n}{n!}$ is bounded by e^K, the second term can be made small for sufficiently large M. Hence the double series [eq. (69)] must converge, and the order of the iterated series in eq. (67) may be reversed. When this is done and the summation over m is carried out explicitly, the following expression for A results

$$A = (1 - \zeta) \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \left[\frac{1}{1 - \zeta^{n-1}} - \frac{\xi}{1 - \zeta^{n+2}}\right]$$

$$= \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \left[\frac{1}{1 + \zeta + \zeta^2 + \cdots + \zeta^n} - \frac{\xi}{1 + \zeta + \zeta^2 + \cdots + \zeta^{n+1}}\right] \quad (70)$$

The unload modulus, M_2, being infinite, implies that ζ must equal 1. Equation (70) converges uniformly, thus passing to the limit as $\zeta \to 1$ gives

$$\tilde{A} = \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} \left[\frac{1}{n + 1} - \frac{\xi}{n + 2}\right] \quad (71)$$

where $\tilde{A}$ indicates the attenuation factor for $\zeta = 1$. For the special case where $\eta = 0$ [i.e., $P_f = P_o(1 - t)$], all of the terms of the above series, other than the first, must vanish. This gives

$$\tilde{A} = 1 - \frac{1}{2}\xi \qquad (\eta = 0)$$

Noting that $M_2 = \infty$ implies $\left(\frac{\mu - 1}{\mu}\right) = 1$; hence

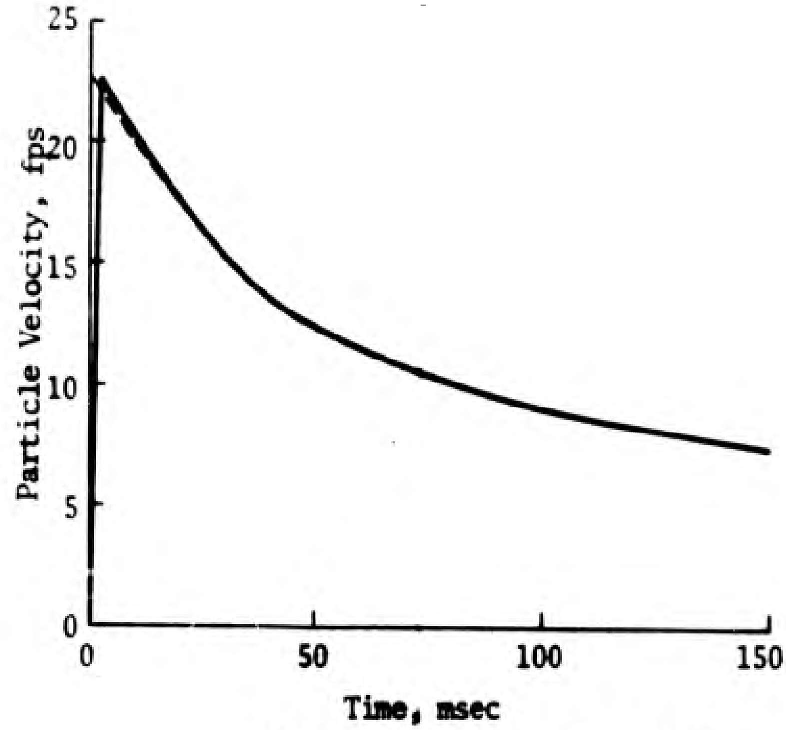
$$\tilde{A} = 1 - \frac{X_s}{2U} \qquad (\eta = 0) \quad (72)$$

Next for $\eta \neq 0$, eq. (71) can be written

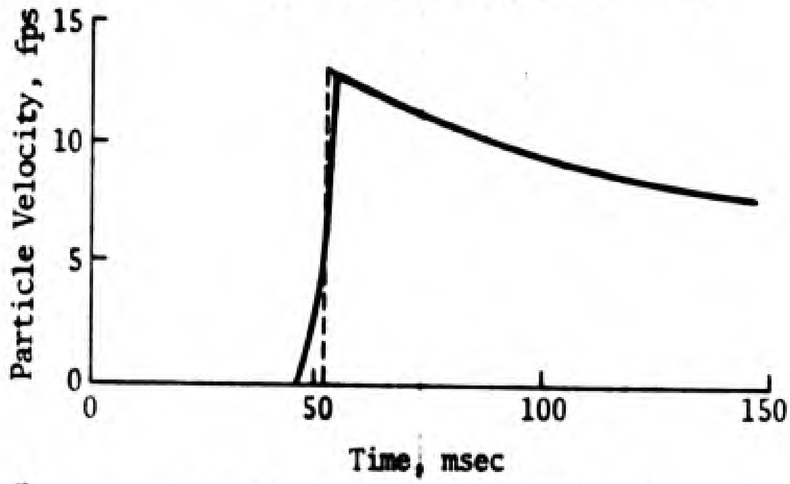
$$\tilde{A} = \frac{1}{\kappa} \sum_{n=0}^{\infty} \frac{\kappa^{n+1}}{(n + 1)!} - \xi \frac{d}{d\kappa} \left(\frac{1}{\kappa} \sum_{n=0}^{\infty} \frac{\kappa^{n+2}}{(n + 2)!}\right)$$

Both terms of this expression may be explicitly evaluated giving

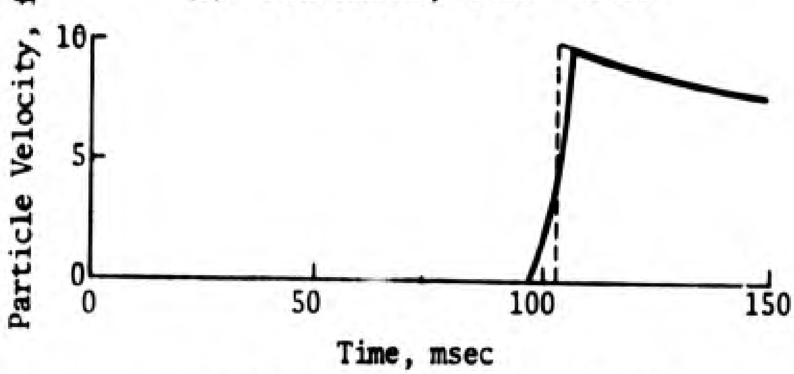
$$\tilde{A} = \frac{1}{\kappa} \left\{-1 - \frac{\xi}{\kappa} + e^{\kappa} \left[1 - \xi\left(1 - \frac{1}{\kappa}\right)\right]\right\}$$



(a) Time History at x = 0 ft



(b) Time History at x = 50 ft



(c) Time History at x = 100 ft

Figure 10. Comparison of PLID Solution with Analytic Solution

Replacing κ by $-\eta\xi$ and ξ by $\frac{x_s}{U}$ (since $\frac{\mu - 1}{\mu} = 1$) results in

$$\tilde{A} = \frac{U}{\eta x_s}\left[1 - \frac{1}{\eta} - \left(1 - \frac{1}{\eta} - \frac{x_s}{U}\right)\exp\left(-\eta\frac{x_s}{U}\right)\right] \tag{73}$$

Using this technique, expressions for pressure and particle velocity at arbitrary points may also be derived in closed form.

5. Comparisons with PLID.

As the PLID computer code was extensively used in calculations concerning curvilinear constitutive relationships, several PLID runs were made using bilinear models. The results of these runs were then compared with solutions determined from eqs. (43) and (44). In all cases, the PLID results were remarkably consistent with the analytic results. A typical example of this consistency is shown in figure 10. This figure shows particle velocity time histories for a 1-megaton, 500-psi Brode overpressure. The bilinear model parameters are M_1 = 20,000 psi and M_2 = 80,000 psi. Time histories at 0-, 50-, and 100-foot depths are shown. The solid and dashed lines depict the PLID and analytic solutions, respectively. These data were previously published in reference 12.

SECTION III

BILINEAR MODEL APPROXIMATION OF FREE-FIELD RESPONSE

1. Curvilinear Constitutive Relationships.

Dynamic stress-strain relationships for soil in uniaxial strain are characterized by a curvilinear, hysteretic shape (ref. 13). A typical example of this characteristic shape is shown in figure 11. The dominant features of this curve are the positive curvature of the load curve above a certain pressure and the very steep initial unloading curve producing a hysteresis loop.

All materials compressed in uniaxial strain exhibit stiffening at higher pressure levels. Soils, however, often show initial uniaxial strain yielding at lower pressures (not to be confused with the more commonly considered triaxial yielding). This yielding may occur to a varying extent in different soils. At low pressures (typically less than 100 psi) yielding will play an extremely important part in response calculations. As the pressure level of interest increases, however, the effects of yielding become overshadowed by higher level shocking effects.

Stress-strain relationships of the type shown in figure 11 are inherently complex and difficult to work with. At best, response calculations must be carried out numerically on a computer. The primary objective of this study was to examine the ramifications of replacing the curvilinear material description with a bilinear model. This subject has yet to be carefully investigated. The following assumptions, consistent with the theory developed in section II, apply:

(1) All overpressures consist of a shock followed by a monotonic decay in pressure.

(2) The overpressure is applied to a homogeneous semi-infinite half-space.

(3) The overpressure at any instant is the same at all points of the surface of the half-space.

It will be useful in this section to employ a pressure-particle velocity material description rather than stress-strain. Based on certain assumptions, the pressure-particle velocity (P - u) relationships are easily derived once the stress-strain relationships are known. It is assumed that the stress-strain loading curve is a Hugoniot (i.e., a locus of all final states which may

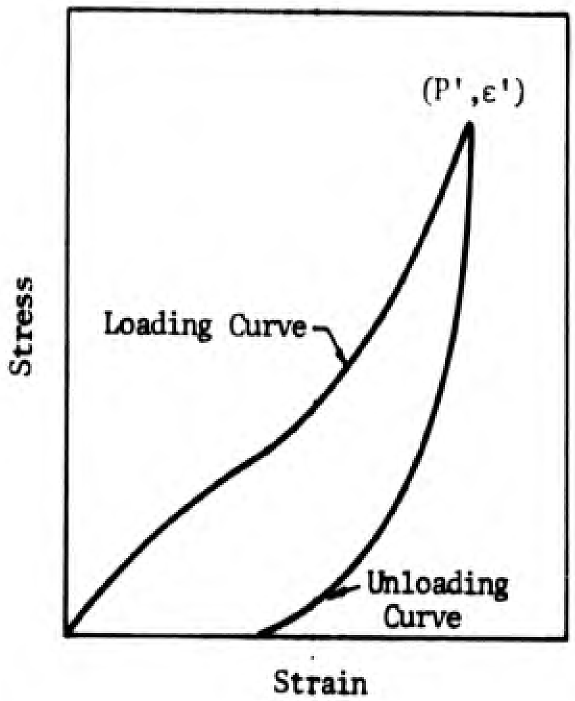


Figure 11. Typical Stress-Strain Curve

be attained by a single shock transition from the initial unstrained state). Furthermore, it is assumed that unloading occurs isentropically. The validity of these two assumptions depends upon the manner in which the stress-strain constitutive relationship was found.

The P - u Hugoniot is derived by combining eqs. (6) and (7) to eliminate the shock velocity, U. Solving for the particle velocity, u, gives

$$u = \left(\frac{P\varepsilon}{\rho_0} \right)^{\frac{1}{2}} \tag{74}$$

Thus, given values of P and ε, a corresponding value for u may be found. Once the P - u Hugoniot is known, a release isentrope may be constructed through any point (P',u') lying on the Hugoniot (fig. 12). This is accomplished by noting that

$$du = \pm \frac{dP}{\rho c} \tag{75}$$

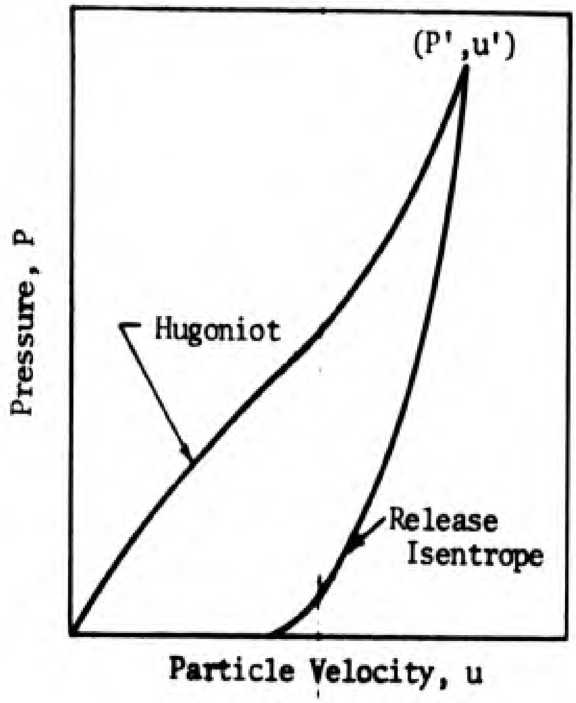


Figure 12. Pressure-Particle Velocity Curve

This equation is a restatement of eqs. (21) and (22) without the assumption that the release isentrope is linear. Equation (75) may also be viewed as the limiting case of eq. (7) where, in the limit, a very weak shock approaches an isentropic sound wave. A more complete discussion of this subject is contained in reference 14.

Substituting for c in eq. (75) from eq. (10) and integrating will give

$$u - u' = \int_{P'}^{P} \frac{1}{\rho} \left(\frac{d\rho}{dp} \right)^{\frac{1}{2}} dp \tag{76}$$

The density, ρ, may be eliminated from this equation by use of eq. (4). The following equation may be employed to obtain the P - u release isentrope through the point (P',u') from the P - ε isentrope through the point (P', ε')

$$u - u' = \int_{P'}^{P} \left(\frac{1}{\rho_0} \frac{d\varepsilon}{dp} \right)^{\frac{1}{2}} dp \tag{77}$$

2. Selection of "Best" Bilinear Model.

If the response of a curvilinear medium can be approximated by the bilinear model, then, in some sense, there should exist a particular bilinear model which makes the best approximation. This particular model will be termed the "best" bilinear model. The definition of the "best" bilinear model can be arbitrarily made if certain limitations are observed. First, both the curvilinear medium and the associated bilinear medium should be subjected to the same overpressure function; that is, the approximating bilinear medium response should be due to the same boundary conditions for which the curvilinear medium response is desired. Also, since the bilinear model is completely specified by two parameters, only two characteristics of the curvilinear medium response can be directly approximated. If the resulting bilinear medium response is then found to approximate other characteristics of the curvilinear medium response, the definition can be termed a success.

The following definition for the best bilinear model was adopted. The best bilinear model is the one which, in the least square sense, best represents both the peak particle velocity attenuation with depth and the surface particle velocity time history of the curvilinear medium. This definition was arbitrarily chosen with the hope that it would prove sufficient for the purposes of this study.

3. Method of Selection.

Implementation of the above definition can be easy or difficult depending on the shape of the overpressure function for which results are desired. For the present, it will be assumed that the overpressure is of the form

$$P_f(t) = P_o g_f(t)$$

where $g_f(t)$ is initially equal to 1 and decays monotonically for $t > 0$.

Let α_i, $i = 1, 2, \cdots, N$ represent N distinct peak particle velocity attenuation measurements obtained by subjecting a given curvilinear medium to a specified overpressure. Let υ_j, $j = 1, 2, \cdots, M$ represent measurements of surface particle velocity at M distinct times, for the same problem. Let x_i, $i = 1, 2, \cdots N$, and t_j, $j = 1, 2, \cdots M$ be the depths and times at which the measurements α_i and υ_j are taken. Then the problem of finding the best bilinear model reduces to simultaneously minimizing two functionals

$$F(\Psi) = \sum_{i=1}^{N} \left(\alpha_i - A(x_i)\right)^2 \tag{78}$$

$$G(\Phi) = \sum_{j=1}^{M} \left(\upsilon_j - u_f(t_j)\right)^2 \tag{79}$$

Here $A(x_i)$ represents the attenuation factor given by eq. (60) evaluated at x_i; $u_f(t_j)$ represents the surface particle velocity of the bilinear model at time t_j; and Ψ and Φ represent the two parameters of the bilinear model to be varied. The surface particle velocity, u_f, of the bilinear model can be determined as a special case of eq. (44)

$$u_f(t) = \frac{P_o}{Z_2}\left\{ g_f(t) + 2\sum_{m=1}^{\infty} \zeta^m g_f(\zeta^m t)\right\} \tag{80}$$

Letting $\Phi = Z_2$, the unloading impedance, eq. (79) may be minimized with respect to Z_2, holding ζ constant, to obtain

$$Z_2 = \frac{\sum_{j=1}^{M} \theta_j^2}{\sum_{j=1}^{M} \upsilon_j \theta_j} \tag{81}$$

where $\theta_j = P_o\left\{ g_f(t_j) + 2\sum_{m=1}^{\infty} \zeta^m g_f(\zeta^m t_j)\right\}$

In the light of this equation, the logical choice for Ψ, the second bilinear model parameter, is ζ. Thus using eqs. (57) and (60), eq. (78) becomes

$$F(\zeta) = \sum_{i=1}^{\infty}\left\{ \alpha_i - (1-\zeta)\sum_{m=1}^{\infty} \zeta^{m-1} g_f(t_m)\right\}^2 \tag{82}$$

where t_m is defined by eq. (55).

If $g_f(t)$ is a relatively simple function, so that the inner series of eq. (82) may be summed explicitly, then minimization can be carried out after the summation. This is the case when $g_f(t)$ represents a linear decay. In this instance, minimization of eq. (82) results in an equation for ζ which may be solved simultaneously with eq. (81) to completely determine the best bilinear model.

For more complex (and interesting) overpressures, the inner series of eq. (82) cannot, in general, be explicitly summed. Term-by-term differentiation can be justified for any bounded function, g_f; however, the resulting expression is extremely lengthy and contains multiples of infinite series. Simultaneous solution of this resulting expression with eq. (81) can only be performed numerically.

For this reason, a direct search technique was employed. A short computer program was written which would alternately estimate ζ and Z_2. Initial estimates for the two parameters were given to the program; then with one parameter held fixed, the other parameter estimate was improved. Equation (81) was used for improving the Z_2 estimate. A direct search routine was used to improve the ζ estimate from eq. (82). The estimates were alternately improved until convergence within specified tolerances occurred. The question of convergence was not considered analytically; however, in the majority of cases considered, convergence did occur. Convergence rates were markedly improved by overrelaxation.

Only overpressure functions of the form

$$g_f(t) = (1 - t)\exp(-\eta t) \tag{83}$$

were considered in studies involving this program. The single-exponential form was used rather than the three-exponential Brode form to economize computer run time. It was felt that the single-exponential form would still adequately represent the class of problems of interest. Also, it can be shown that solutions to the single exponential loading can be superposed to obtain solutions to particular Brode overpressures.

Several simulated problems were run to test the effectiveness of the solution method. In these problems, attenuation values, $A(x_i)$, and surface particle velocity values, $u_f(t_j)$, were generated for a particular bilinear model and overpressure function. These values were then corrupted according to the formulas

$$\alpha_i = A(x_i) + r_1|A(x_i)| \tag{84}$$

$$\upsilon_j = u_f(t_j) + r_2|u_f(t_j)| \tag{85}$$

to obtain simulated measurements of attenuation values, α_i, and surface particle velocities, υ_j. The numbers, r_1 and r_2, were normally distributed pseudorandom numbers with zero mean and variable standard deviation. By varying the standard deviation, a variable noise level could be superposed on bilinear values to obtain the measured values. It was deemed essential that the selection method for the best bilinear model be capable of filtering at least moderate sound levels and reacquiring very nearly the input bilinear parameters.

Simulation problems were run with input bilinear model parameters of $Z_2 = 6211.2\ \frac{\text{lb}}{\text{ft}^2} \cdot \frac{\text{sec}}{\text{ft}}$ and $\zeta = 1/3$. For a material density of 100 pcf, these values correspond to a shock velocity of U = 1,000 fps and a sound velocity of $\bar{c}$ = 2,000 fps. Simulation run results are summarized in tables I and II.

Table I

NOISE LEVEL EFFECTS*

Prob No.	Standard Deviation of		Computed U, fps	Percent Error, U	Computed $\bar{c}$, fps	Percent Error, $\bar{c}$
	r_1	r_2				
1	0.0	0.0	999.998	-0.0002	1999.663	-0.0166
2	0.05	0.05	980.933	-1.9067	1852.109	-7.3945
3	0.10	0.10	954.763	-4.5230	1580.064	-20.9968

*N = M = 50 for all runs.

Table II

SAMPLE SIZE EFFECTS*

Prob No.	Number of Measured Values		Computed U, fps	Percent Error, U	Computed $\bar{c}$, fps	Percent Error, $\bar{c}$
	N	M				
4	10	10	1108.475	+10.8457	3901.230	+95.0615
5	50	50	980.933	-1.9067	1852.109	-7.3945
6	100	100	1021.865	+2.1865	2137.978	+6.8989

*Standard deviation of both r_1 and r_2 = 0.05 for all runs.

Table I reflects noise level effects. Table II shows the effect of the number of measurements, N and M, employed. All values in table I were obtained using fifty simulated attenuation measurements and fifty simulated surface particle velocity measurements. The error in reacquiring the initial bilinear parameters clearly increased with increasing noise level. Experience with PLID indicated that the actual noise level which would be encountered in curvilinear material solutions could be maintained at less than the noise level corresponding to the 0.05 standard deviation results. The reacquisition accuracy at this level was considered adequate for the purposes of this study.

Table II shows the effect of increasing sample size. Accuracy was greatly improved by increasing the number of measurements used from ten to fifty. A second increase from fifty to one hundred measurements produced no noticeable change in accuracy. All simulation problems were run with the overpressure function

$$g_f(t) = (1 - t)\ \exp(-20t)$$

and a maximum overpressure, p_o = 1,000 psi. Figure 13 shows the particle velocity time history at the surface for problem no. 2 in table I. The solid line represents the given simulated data. The dashed line represents the response of the material using the reacquired bilinear parameters. The fit of computed to measured values shown in figure 13 is typical of all the simulation runs. The results of the simulation problems indicated that the method for selecting the best bilinear model was apparently adequate for attacking problems involving curvilinear media.

4. Peak Pressure Attenuation.

A frequent complaint concerning use of the bilinear model is that identical attenuation rates for both peak pressure and peak particle velocity are unrealistic. This, of course, is borne out by field data where greatly different pressure and particle velocity attenuation rates are encountered. This difference is due to the typically curvilinear Hugoniot for soil. If, however, the actual soil Hugoniot is known, the bilinear model may be used to approximate either pressure or particle velocity attenuation, and the attenuation rate for the other variable may then be deduced.

Suppose, for example, that attenuation of peak particle velocity with depth has been approximated for a particular soil and overpressure function. Both the peak pressure and the peak particle velocity always occur directly behind the shock front and are therefore related by the P - u Hugoniot for the soil. The Hugoniot is a unique function relating pressure and particle velocity, and thus it can be used to transform the particle velocity attenuation curve into an associated pressure attenuation curve. The validity of the resulting pressure attenuation curve will, of course, depend on the accuracy of the original particle velocity approximation and on the validity of the Hugoniot for the soil. In this way, however, a major criticism of the bilinear model can be avoided.

5. Results of Curvilinear Studies.

a. Problems Considered.

The derivation of the response equations for the bilinear model and the selection procedures for the determination of the best bilinear model which have

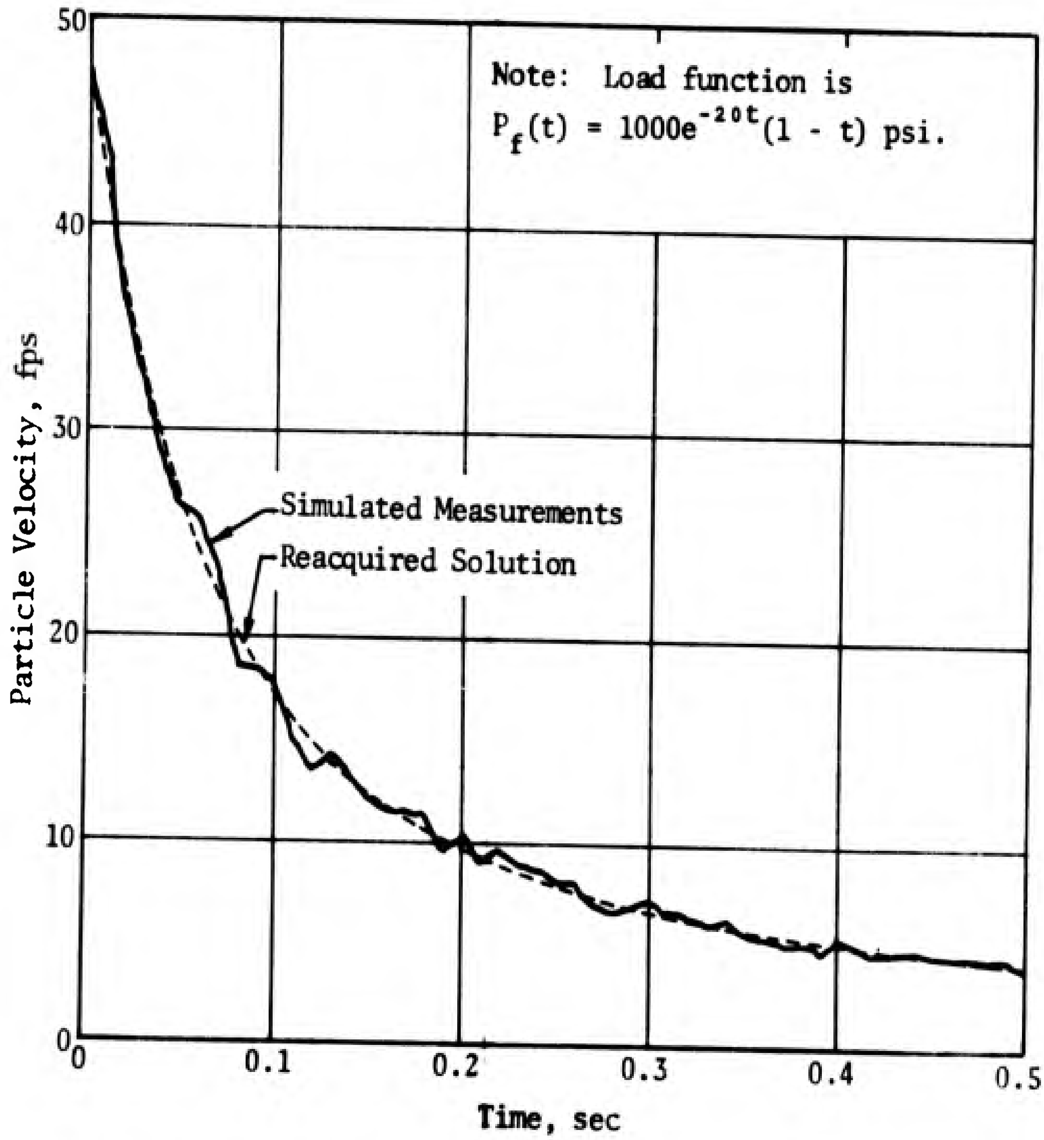


Figure 13. Surface Particle Velocity Time History

preceded this section are for the most part based on a rational mathematical development. In contrast to this development, the remainder of this report will be entirely empirical. The intent of this section is to demonstrate that the bilinear model may be used to approximate the response of an actual curvilinear soil material and to derive what might be called a "recipe" to determine the best bilinear model. This recipe will be based on a minimum of informatic ~ ncerning the curvilinear model.

The PLID code was used to generate attenuation and surface particle velocity measurements for 80 problems. Of these, the PLID results for 69 problems produced best bilinear models in the direct search program. The results from the remaining 11 PLID runs would not converge in the direct search program.

The direct search computer program was called BIFIT. BIFIT was equipped with a plot routine which produced Calcomp plots for each of the convergent problems. These plots are presented in the appendix.

In the problems considered, both the peak overpressure and the overpressure shape were varied. Also varied were the shapes of the curvilinear load and unload curves, as well as the initial material density. Table III summarizes the primary features of the curvilinear models which were considered. In table III, the values in the columns marked R and S are initial loading secant modulus values and the initial unloading tangent modulus values for the curvilinear model. Letting e denote the strain in the curvilinear model and e_t the curvilinear strain which occurs on loading to the peak overpressure P_o, the values of R and S are given by

$$R = \frac{P_o}{e_t} \tag{86}$$

and

$$S = \left.\frac{dP}{de}\right|_{e\,=\,e_t,\ \text{unloading}} \tag{87}$$

The column marked overpressure shape in table III refers to the Brode overpressure corresponding to the given overpressure for a 1-MT yield (ref. 7). The values in columns marked ρ_0 and P_o are the initial material density and peak overpressure respectively. Although several problems in table III appear to be identical (problems 1A and 1C for instance), the exact shapes of the curvilinear load and unload curves were varied. Also note that two sets of problems were numbered 11H, 12H, 13H, 14H. The only difference between these two problem sets was a change in the initial material density, ρ_0.

Table III

KEY TO PROBLEMS

Problem Identifier	ρ_0, slugs/ft^3	R, psi	S, psi	Overpressure Shape	P_o, psi
1A	3.51	53,500	172,250	100	500
2A	3.51	53,500	172,250	200	500
3A	3.51	53,500	172,250	500	500
4A	3.51	53,500	172,250	1000	500
5A	3.51	53,500	172,250	2000	500
1C	3.51	53,500	172,250	100	500
2C	3.51	53,500	172,250	200	500
3C	3.51	53,500	172,250	500	500
4C	3.51	53,500	172,250	1000	500
5C	3.51	53,500	172,250	2000	500
1D	[illegible].51	33,333	170,510	100	500
2D	3.51	33,333	170,510	200	500
3D	3.51	33,333	170,510	500	500
4D	3.51	33,333	170,510	1000	500
5D	3.51	33,333	170,510	2000	500
1E	3.51	33,333	172,380	100	500
2E	3.51	33,333	172,380	200	500
3E	3.51	33,333	172,380	500	500
4E	3.51	33,333	172,380	1000	500
5E	3.51	33,333	172,380	2000	500
1F	3.51	20,000	172,250	100	500
2F	3.51	20,000	172,250	200	500
3F	3.51	20,000	172,250	500	500
4F	3.51	20,000	172,250	1000	500
5F	3.51	20,000	172,250	2000	500
1G	3.51	20,000	172,250	100	500
2G	3.51	20,000	172,250	200	500
3G	3.51	20,000	172,250	500	500
4G	3.51	20,000	172,250	1000	500
5G	3.51	20,000	172,250	2000	500
7C	3.51	34,323	110,510	200	200
8C	3.51	53,500	172,250	500	500
9C	3.51	87,850	282,860	1000	1000
10C	3.51	152,150	498,880	2000	2000
7D	3.51	22,187	114,650	200	200
8D	3.51	33,333	172,250	500	500
9D	3.51	54,855	283,470	1000	1000
10D	3.51	95,662	494,350	2000	2000

Table III (Concl'd)

Problem Identifier	ρ_0, slugs/ft³	*R*, psi	*S*, psi	Overpressure Shape	p_o, psi
8E	3.51	33,333	172,250	500	500
9E	3.51	54,897	283,690	1000	1000
10E	3.51	94,300	487,310	2000	2000
7F	3.51	13,189	113,590	200	200
8F	3.51	20,000	172,250	500	500
9F	3.51	30,731	264,680	1000	1000
10F	3.51	52,393	451,250	2000	2000
6H	3.51	47,384	322,210	100	100
7H	3.51	64,979	441,860	200	200
8H	3.51	100,000	680,000	500	500
9H	3.51	149,240	1,014,800	1000	1000
10H	3.51	245,650	1,470,600	2000	2000
7J	3.51	5,963	65,358	200	200
8J	3.51	10,000	109,610	500	500
9J	3.51	15,929	174,590	1000	1000
10J	3.51	27,223	298,380	2000	2000
13C	3.51	53,500	285,870	500	500
14C	3.51	87,850	469,420	1000	1000
15C	3.51	152,150	813,000	2000	2000
11H	3.51	47,384	194,100	100	100
12H	3.51	64,979	266,180	200	200
13H	3.51	100,000	409,640	500	500
14H	3.51	142,240	611,340	1000	1000
12J	3.51	5,963	101,370	200	200
13J	3.51	10,000	170,000	500	500
14J	3.51	15,929	270,790	1000	1000
15J	3.51	27,223	462,790	2000	2000
11H	5.00	47,384	194,100	100	100
12H	5.00	64,979	266,180	200	200
13H	5.00	100,000	409,640	500	500
14H	5.00	142,240	611,340	1000	1000

b. Problem Results.

The Calcomp plots displayed in the appendix contain the following information. At the top of the first page is the problem identification and the date that computer processing of the problem began. Directly below this, the number of surface particle velocity data points, N, and the number of attenuation data points, M, are given. Then, to the right of the heading "material properties," the initial density, ρ_0, the bilinear shock velocity, U, the bilinear approximate seismic velocity, $\bar{c}$, and the bilinear parameter, ζ, are given. The material density is given in slugs/ft³ and both velocities have units of ft/sec. Below these values is given a set of numbers called "fitting errors" denoted E_i, i = 1,2,..., 8. The values of E_i for i = 1, 2, 3, and 4 provide a measure of the goodness of fit of the bilinear attenuation curve to the measured curvilinear attenuation data. These values of E_i are defined by

$$E_1 = \frac{M\sum_1^M \alpha_i A(x_i) - \left(\sum_1^M A(x_i)\right)\left(\sum_1^M \alpha_i\right)}{\sqrt{\left[M\sum_1^M \left[A(x_i)\right]^2 - \left[\sum_1^M A(x_i)\right]^2\right]\left[M\sum_1^M \left[\alpha_i\right]^2 - \left[\sum_1^M \alpha_i\right]^2\right]}}$$

$$E_2 = 1 - E_1^2$$

$$E_3 = \frac{1}{M-2}\sum_1^M \left(\alpha_i - A(x_i)\right)^2$$

$$E_4 = \sqrt{E_3}$$

Here E_1 corresponds to the coefficient of correlation and E_3 to the standard error of estimate in the attenuation fit. The values of E_i for i = 5, 6, 7, and 8 are analogous to those for i = 1, 2, 3, and 4 respectively, the only difference being that the surface particle velocity measurements are considered. Thus the fitting error E_5 is defined the same as E_1 except that M is replaced by N, α_i is replaced by υ_i, and $A(x_i)$ is replaced by $u_f(t_i)$.

Below the table of fitting errors, the curvilinear and bilinear displacements at five different depths are shown. These values provide the best measure of the goodness of fit of the bilinear approximation throughout the entire solution domain. Note that the problem results in the appendix are ordered identically to the entries in table III. That is, problem 1A is first, 1C is sixth, etc. The displacement results shown in the appendix may be separated into two groups. All displacement results up to and including problem 5F correspond to the displacement which occurred at the end of the positive-phase duration, t_D. In all problems following problem 5F, the displacements are those which occur after 200 msec. The final entry on the first page of the Calcomp plots is a plot of the surface particle velocity time histories for both the curvilinear and bilinear models. The curvilinear data are shown by the solid line while the bilinear data appear as crosses. A similar plot of the curvilinear and bilinear attenuation data versus depth is shown on the second page of the Calcomp plots. The overpressure used to generate the problem is described below this plot. Also shown below the attenuation plot is the time in seconds at which the overpressure has decayed to one-half the peak value, and the value of this time normalized by the positive-phase duration, t_D.

Finally, the third page of the Calcomp plots shows stress-strain curves for both the curvilinear model and the best bilinear model. The two bilinear moduli, M_1 and M_2, are given beneath this plot, as are the values of the curvilinear model Rayliegh line slope, R, the curvilinear Hugoniot nonlinearity factor, and the curvilinear maximum unload slope, S. The Hugoniot nonlinearity factor is defined as the ratio of the area between the Rayliegh line and the curvilinear Hugoniot to the total area beneath the Rayliegh line for the curvilinear model. The last entry on the third page, called zeta, is defined by the ratio R/S and should not be confused with the bilinear parameter, ζ.

Inspection of the plots in the appendix clearly shows in almost every case that the bilinear model solution provided an adequate approximation to the curvilinear model solution. This is especially evident when one compares the bilinear and curvilinear displacements at various depths. The excellent agreement in displacement data indicates that the selection criteria for the best bilinear model was evidently good. While the best bilinear model results compare favorably with the curvilinear model results, it is also clear from the stress-strain plots in the appendix that there is often little if any resemblance between the two models. Hence, some "recipe" is needed to select the best bilinear model for any particular curvilinear model.

To be truly useful, the recipe for the best bilinear model should be founded on as little information concerning the curvilinear model as possible. That is, a complete testing program should not be necessary to determine the best bilinear model for any particular soil. A great number of attempts were made to discover such a recipe. There is little information to guide one in a search such as this but a representation which is felt adequate was finally found. The only necessary information concerning the curvilinear model are the two moduli, R and S. The recipe takes the form of two correlations. The first relates the residual strain, ε_r, of the bilinear model to what will be called the gross residual strain, e_r, of the curvilinear model. These two strains are defined by

$$\varepsilon_r = P_o\left(\frac{1}{M_1} - \frac{1}{M_2}\right) \tag{88}$$

and

$$e_r = P_o\left(\frac{1}{R} - \frac{1}{S}\right) \tag{89}$$

The second correlation relates the curvilinear secant modulus, R, to a function of the bilinear unload modulus, M_2, and the moduli ratio, μ. This function will be called, C_2, and is defined as

$$C_2 = \frac{M_2}{0.54(\mu^2 - 1) + 0.054(\mu + 1)} \tag{90}$$

Values of e_r and ε_r, as well as R and C_2, are given for all problems in table IV.

The problems which have been considered can be conveniently divided into two subgroups. All the problems in tables III and IV up to and including problem 5G were run at a constant peak overpressure of 500 psi. This yielded the obvious advantage of isolating the effect of overpressure shape on the best bilinear model. All problems after problem 5G were run with a peak overpressure consistent with the overpressure shape used. In these problems, the curvilinear model changed for each overpressure considered. The first group of problems led to the discovery of the R - C_2 relation. Examination of table IV shows that C_2 is very nearly independent of the overpressure shape. The same cannot be said of the bilinear residual strain, ε_r. In fact, no combination of bilinear model parameters, other than C_2, could be found which was essentially independent of the overpressure shape. It is only in the second group of problems, where the true Brode overpressures are used, that the e_r - ε_r correlation is evident. Thus the best bilinear model recipe can only be applied to problems involving Brode overpressures. This does not seem too serious a restriction, however, since the Brode overpressure form is used in almost all ground-motion studies.

The data in table IV are shown in figures 14 and 15. Figure 14 shows the e_r - ε_r data, and figure 15 shows the R - C_2 data. In both figures, only those data from problems after problem 5G are shown. Both correlations are reasonably linear; however, log-log plots were used for figures 14 and 15 to display the data in more detail. The line in figure 14 is given by

$$\varepsilon_r = 0.941\, e_r \tag{91}$$

Table IV

RELATED CURVILINEAR AND BILINEAR PARAMETERS

Problem Identifier	e_r	ε_r	R, psi	C_2, psi
1A	.00644	.00622	53,500	135,690
2A	.00644	.00574	53,500	139,570
3A	.00644	.00519	53,500	140,350
4A	.00644	.00491	53,500	141,200
5A	.00644	.00468	53,500	140,180
1C	.00644	.00605	53,500	134,520
2C	.00644	.00589	53,500	135,630
3C	.00644	.00570	53,500	134,850
4C	.00644	.00552	53,500	139,880
5C	.00644	.00535	53,500	147,860
1D	.01207	.01267	33,333	68,079
2D	.01207	.01219	33,333	68,413
3D	.01207	.01139	33,333	68,059
4D	.01207	.01100	33,333	68,110
5D	.01207	.01066	33,333	68,137
1E	.01210	.01235	33,333	68,308
2E	.01210	.01211	33,333	68,209
3E	.01210	.01187	33,333	68,287
4E	.01210	.01194	33,333	69,640
5E	.01210	.01178	33,333	71,733
1F	.02210	.02322	20,000	37,386
2F	.02210	.02220	20,000	37,527
3F	.02210	.02035	20,000	37,198
4F	.02210	.01964	20,000	36,733
5F	.02210	.01920	20,000	36,023
1G	.02210	.02411	20,000	35,963
2G	.02210	.02400	20,000	35,858
3G	.02210	.02391	20,000	36,083
4G	.02210	.02340	20,000	37,348
5G	.02210	.02197	20,000	39,475
7C	.00402	.00413	34,323	89,097
8C	.00644	.00584	53,500	130,420
9C	.00785	.00670	87,850	207,200
10C	.00906	.00704	152,150	374,240
7D	.00727	.00720	22,187	47,222
8D	.01210	.01133	33,333	68,546
9D	.01470	.01414	54,855	102,830
10D	.01686	.01555	95,662	177,330

Table IV (Concl'd)

Problem Identifier	e_r	ε_r	R, psi	C_2, psi
8E	.01210	.01220	33,333	65,753
9E	.01469	.01381	54,897	103,520
10E	.01711	.01517	94,300	178,970
7F	.01340	.01346	13,189	24,992
8F	.02210	.01961	20,000	39,225
9F	.02876	.02716	30,731	54,440
10F	.03374	.02998	52,393	97,327
6H	.00180	.00192	47,384	89,345
7H	.00263	.00258	64,979	125,070
8H	.00426	.00396	100,000	190,350
9H	.00572	.00559	149,240	263,260
10H	.00678	.00721	245,650	390,990
7J	.03048	.03087	5,963	10,603
8J	.04544	.04021	10,000	17,992
9J	.05705	.05211	15,929	27,041
10J	.06676	.06587	27,223	39,519
13C	.00760	.00749	53,500	105,810
14C	.00925	.00891	87,850	162,440
15C	.01068	.00966	152,150	286,660
11H	.00160	.00156	47,384	104,540
12H	.00233	.00209	64,979	146,970
13H	.00378	.00324	100,000	223,040
14H	.00506	.00457	149,240	311,500
12J	.03157	.03286	5,963	10,097
13J	.04706	.04220	10,000	17,310
14J	.05909	.05503	15,929	25,745
15J	.06915	.06906	27,223	38,081
11H*	.00160	.00156	47,384	104,400
12H*	.00233	.00209	64,979	146,830
13H*	.00378	.00318	100,000	226,840
14H*	.00506	.00451	149,240	310,780

*Problems where ρ_0 = 5.0 slugs/ft³.

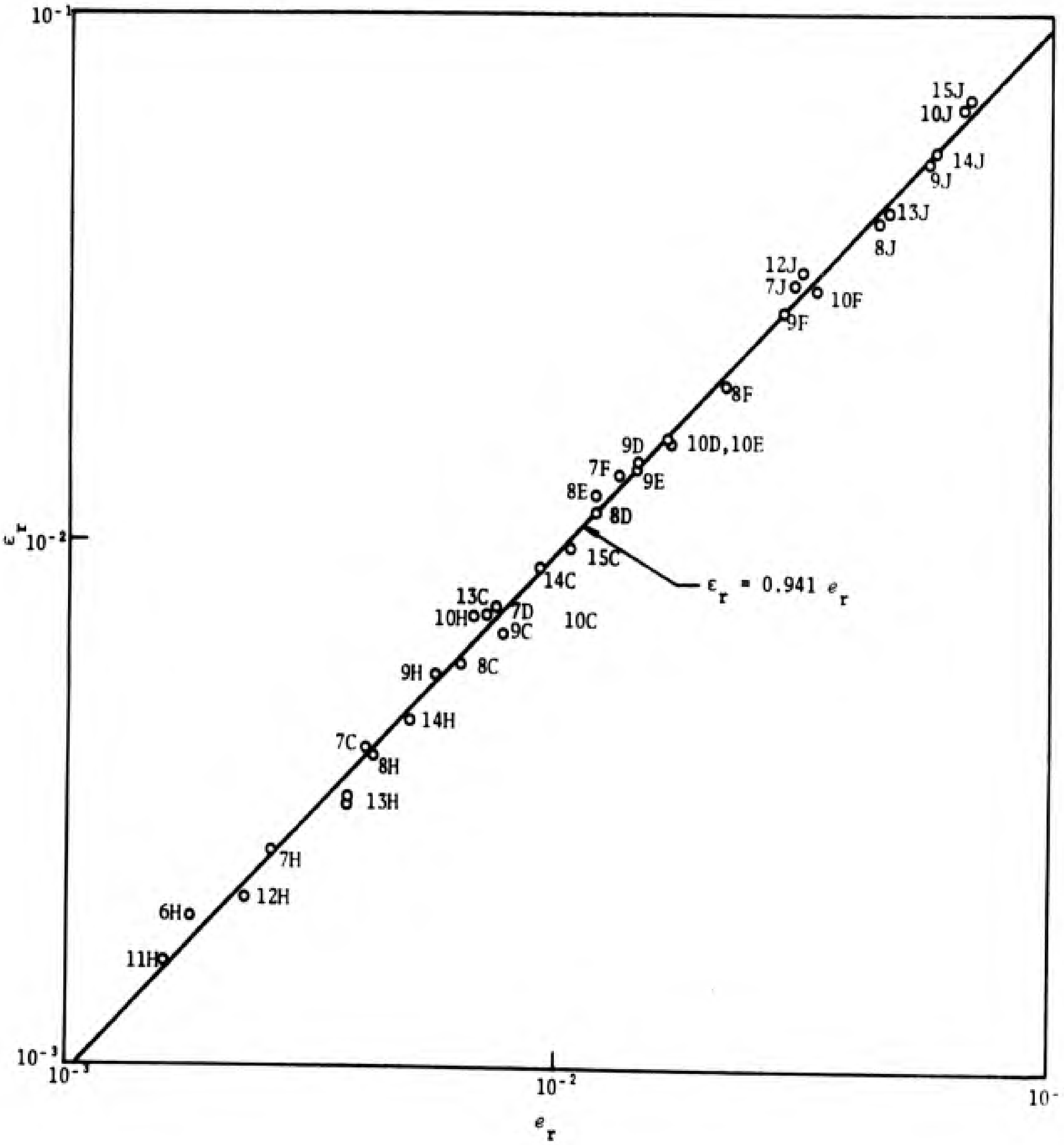


Figure 14. e_r - ε_r Correlation

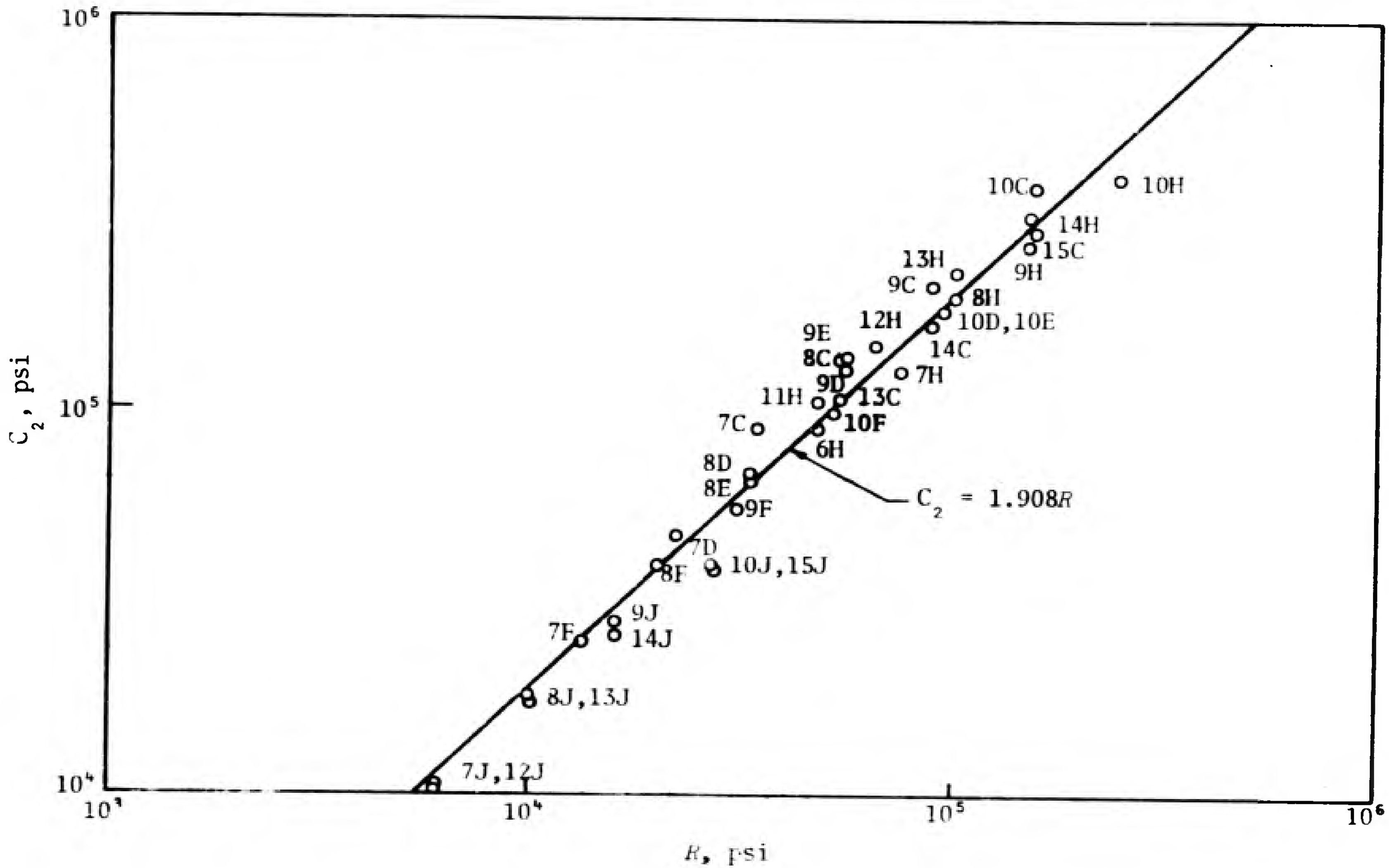


Figure 15. R - C_2 Correlation

In figure 15, the relation is given by

$$C_2 = 1.908R \tag{92}$$

Slightly better fits can be obtained by using small constant terms in eqs. (91) and (92). This was not done however, based on the assumption that $\varepsilon_r \to 0$ as $e_r \to 0$, and $M_2 \to 0$ as $R \to 0$.

The definitions of ε_r and C_2, eqs. (88) and (90), can be combined to solve for the bilinear parameter, μ [see eq. (54)].

$$\mu = 1 + \frac{0.054\varepsilon_r C_2}{P_o - 0.54\varepsilon_r C_2} \tag{93}$$

After μ has been obtained from this equation, the bilinear moduli, M_1 and M_2, may be obtained from

$$M_1 = \frac{P_o}{\varepsilon_r}\left(1 - \frac{1}{\mu^2}\right) \tag{94}$$

$$M_2 = \frac{P_o}{\varepsilon_r}(\mu^2 - 1) \tag{95}$$

This completely defines the bilinear model which, henceforth, will be called the recipe model.

Back calculating the recipe model using eqs. (91) through (95) for the problems in table III shows that in many cases the best bilinear model and the recipe model are dissimilar. Fortunately however, comparison of the model responses show excellent agreement, even when the two models are apparently very different. To better illustrate the apparent model disagreement accompanied by response agreement, figure 16 shows the best bilinear and recipe models for problem 7D. This problem is not typical of the dissimilar nature of the best bilinear and recipe models for all problems run, but is one of the most severe disagreements encountered. Note that even though the two models are grossly different, the responses are in reasonably good agreement. Similar response comparisons were made for all problems following problem 5G. In no case did the recipe model response differ greatly from the best bilinear model response. Although no complete explanation for this serendipitous finding will be offered here, it can be

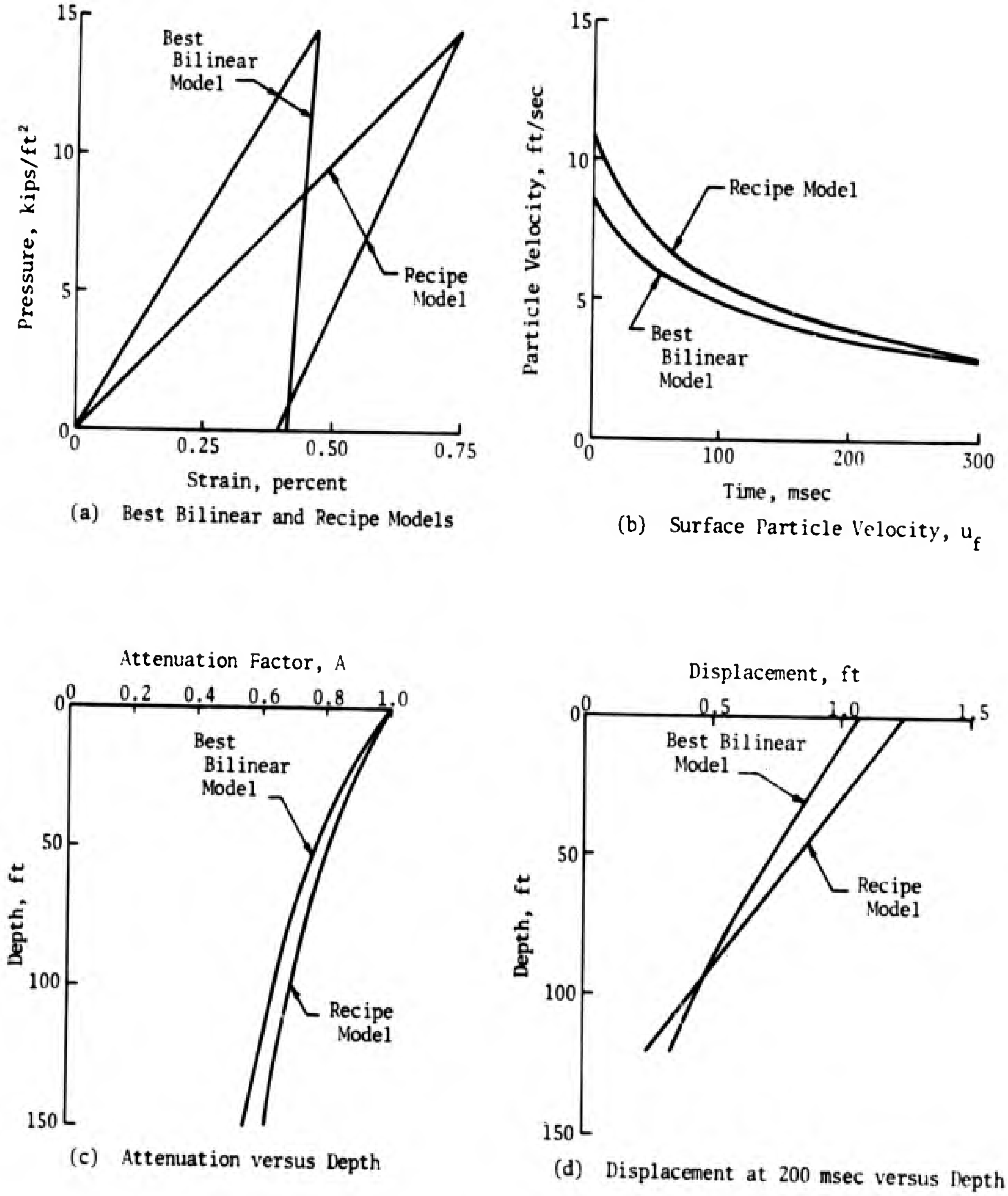


Figure 16. Comparison of Best Bilinear and Recipe Models for Problem 7D

attributed at least partially to two causes. First, both models are subject to the same overpressure function, and, second, the recipe and best bilinear models have nearly identical residual strains, ε_r, in every case. Evidently the value of ε_r is the single most important bilinear parameter.

On balance, the recipe presented here appears to provide an adequate representation for airblast-loaded soil. Its empirical nature must be stressed, however. The recipe is justified by nothing more than the quantity of problems which have been processed.

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SECTION IV

CONCLUSIONS

Within the structure of assumptions made in any one-dimensional free-field investigation, it is concluded that the bilinear model can be used to obtain reasonably accurate approximations at relatively low cost. This conclusion bears primarily upon two facts. First, it is well known that the assumption of uniaxial strain, plus the lack of knowledge of material properties, geometries, and overpressures, leaves considerable doubt in the results of any one-dimensional free-field study. Second, the results of section III indicate that the bilinear model incorporates the essential features necessary to approximate response of curvilinear hysteretic media.

The recipe presented in section III is relatively easy to use and requires very little information pertaining to the curvilinear media. It offers a quick and inexpensive method for investigating the uniaxial response of any soil to a Brode overpressure loading. However, its use should be restricted to homogeneous geometries and all studies employing it should be considered semiconclusive at best.

APPENDIX

CALCOMP PLOTS FROM THE BIFIT CODE

BEST BILINEAR MODEL

PROBLEM 1A OTTOWA SAND OCT 14, 1969

NUMBER OF DATA POINTS, N= 90 , M= 99

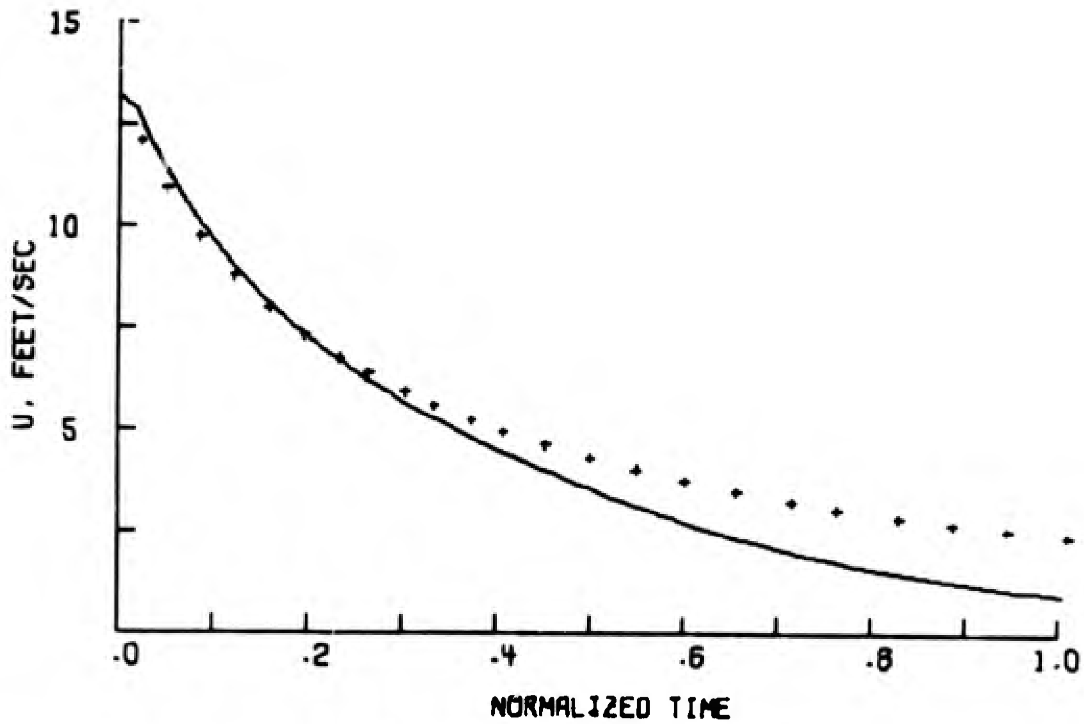
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.579404E+03
	SOUND VELOCITY =	3.197563E+03
	ZETA =	3.387420E-01

FITTING ERRORS

E_1 =	9.970460E-01	E_2 =	5.899256E-03
E_3 =	2.120380E-04	E_4 =	1.456152E-02
E_5 =	9.986947E-01	E_6 =	2.608961E-03
E_7 =	5.892009E-01	E_8 =	7.675942E-01

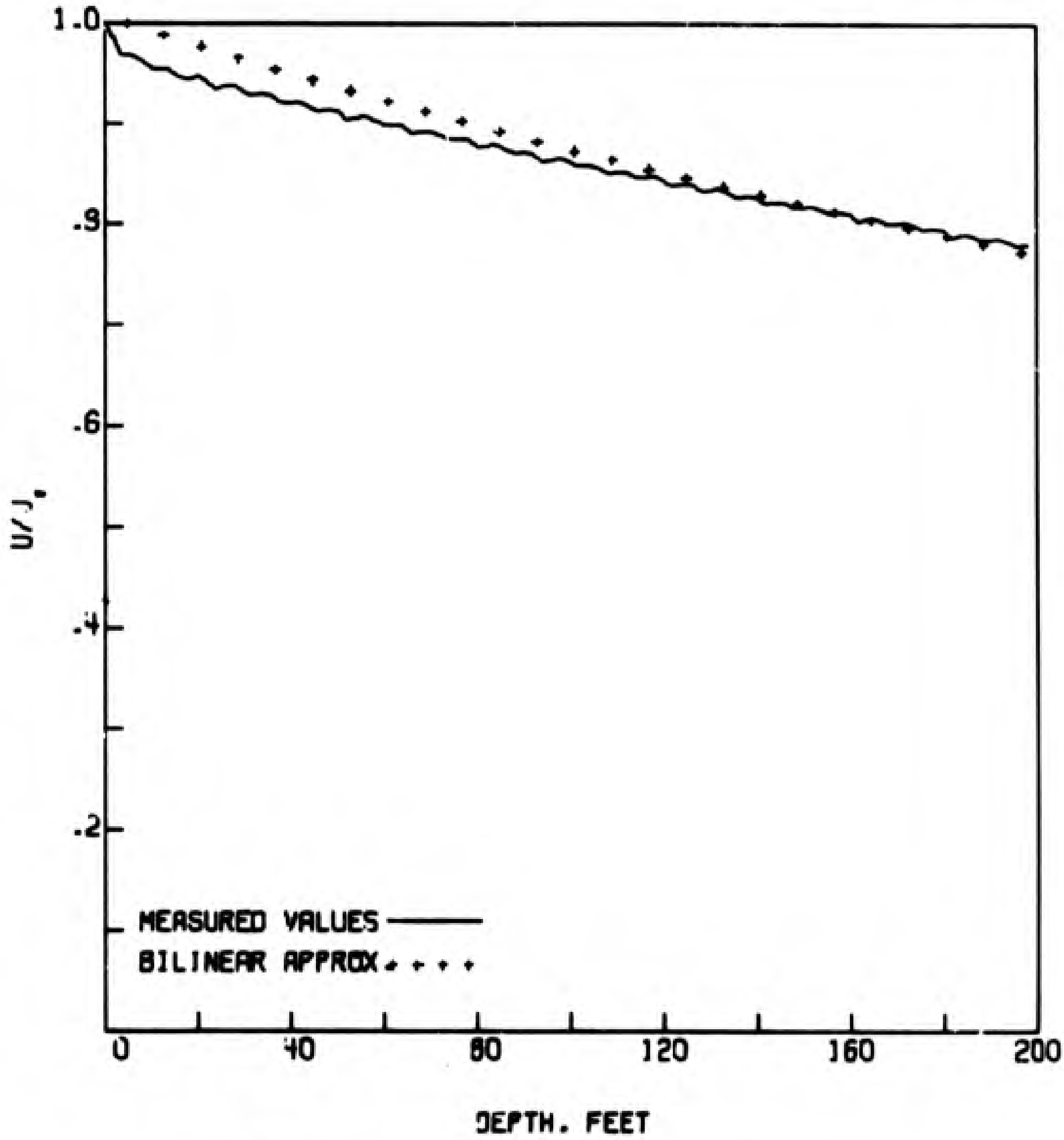
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	4.979	5.474
25	4.942	5.322
50	4.901	5.174
75	4.859	5.032
100	4.815	4.895



SURFACE PARTICLE VELOCITY

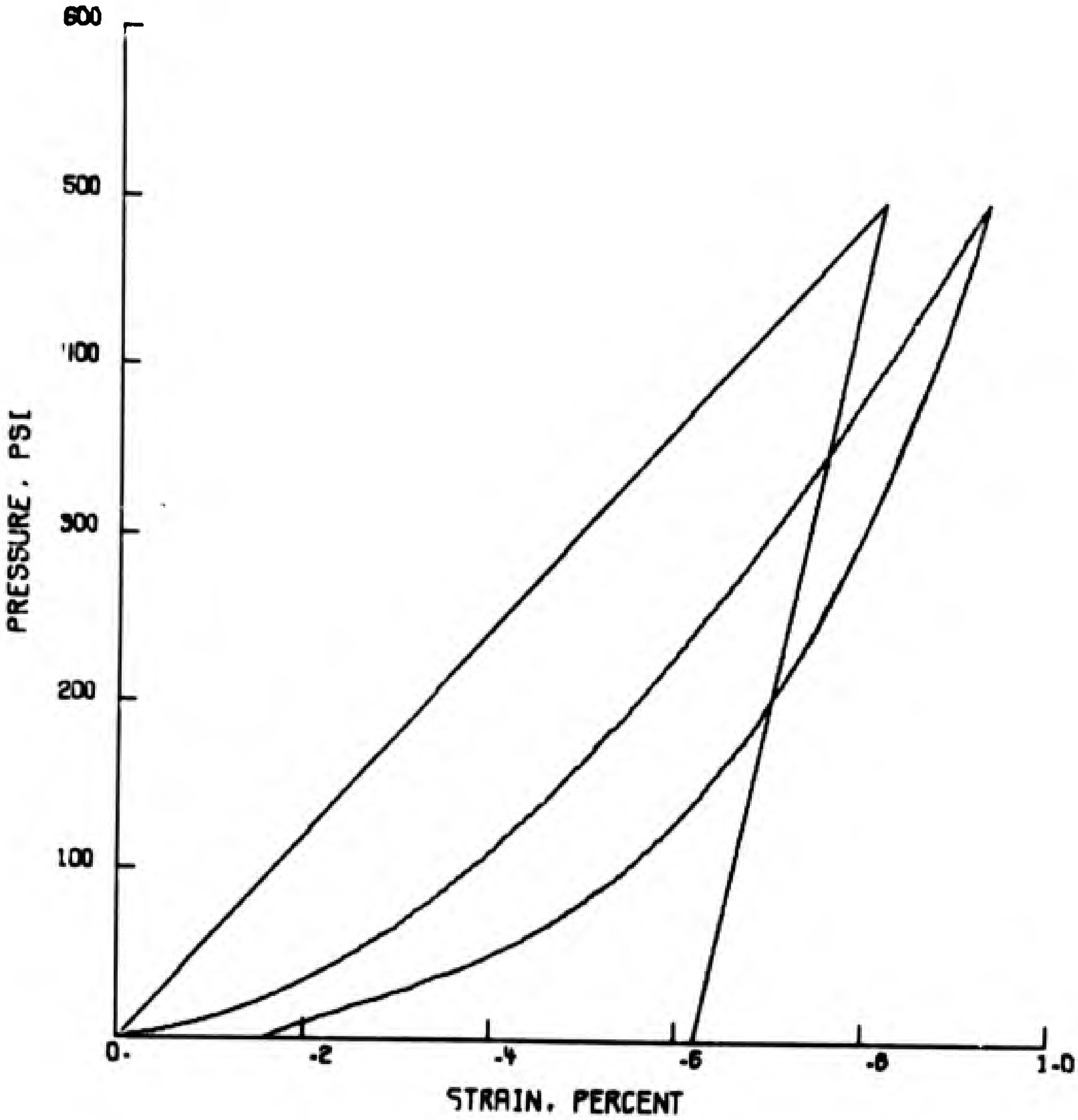
PROBLEM 1A OTTOWA SAND OCT 14. 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 100 PSI BRODE FORM. PMAX IS 500 PSI. TD IS 1.
HALF LOAD TIME = 1.304202E-01 SEC.
NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 1A   OTTOWA SAND   OCT 14. 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =6.080382E+04
M2(PSI) =2.492199E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745097E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 2A OTTOWA SAND OCT 19,1969

NUMBER OF DATA POINTS. N= 88 . M= 99

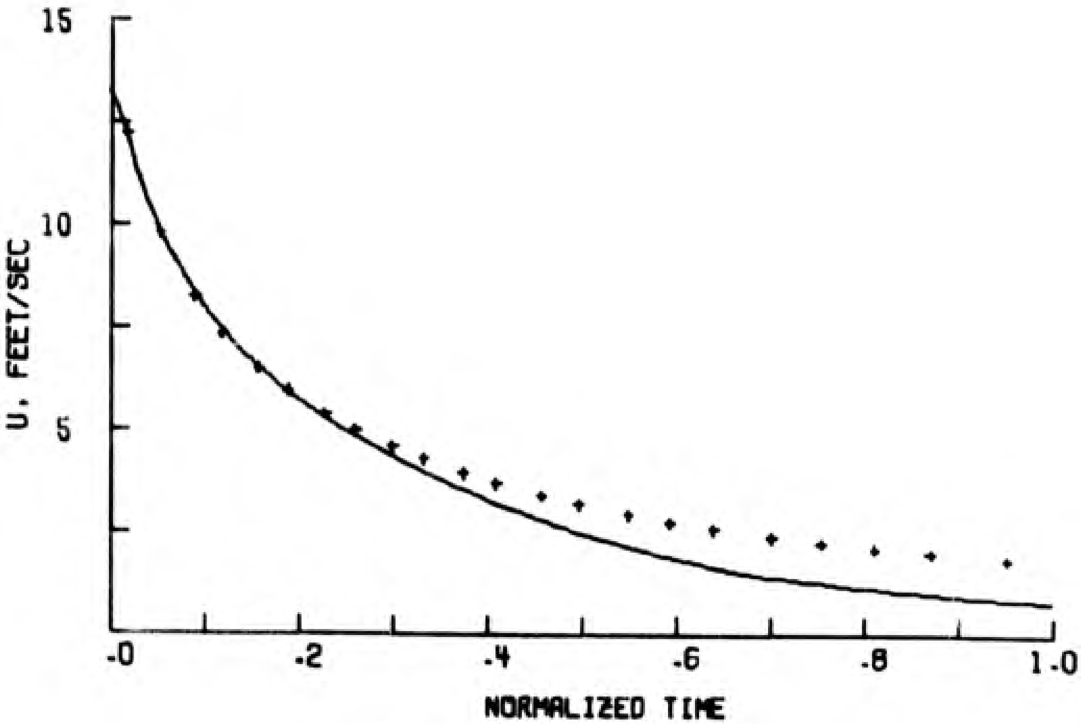
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.500665E+03
	SOUND VELOCITY =	2.469060E+03
	ZETA =	2.439450E-01

FITTING ERRORS

E_1 =	9.986123E-01	E_2 =	2.773479E-03
E_3 =	1.639006E-04	E_4 =	1.280237E-02
E_5 =	9.979310E-01	E_6 =	4.133751E-03
E_7 =	2.787797E-01	E_8 =	5.279959E-01

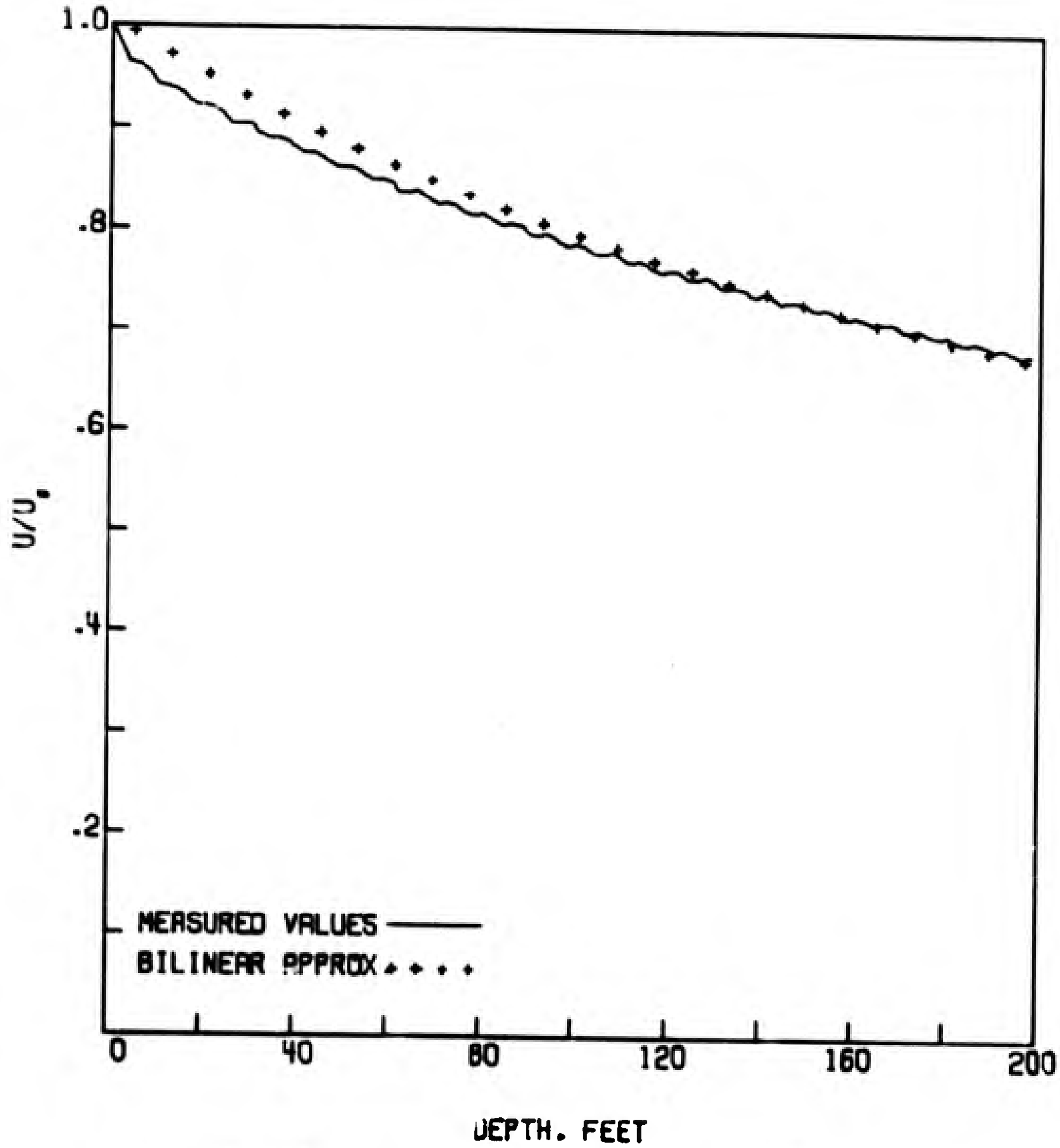
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	3.905	4.292
25	3.868	4.153
50	3.829	4.023
75	3.791	3.900
100	3.752	3.783



SURFACE PARTICLE VELOCITY

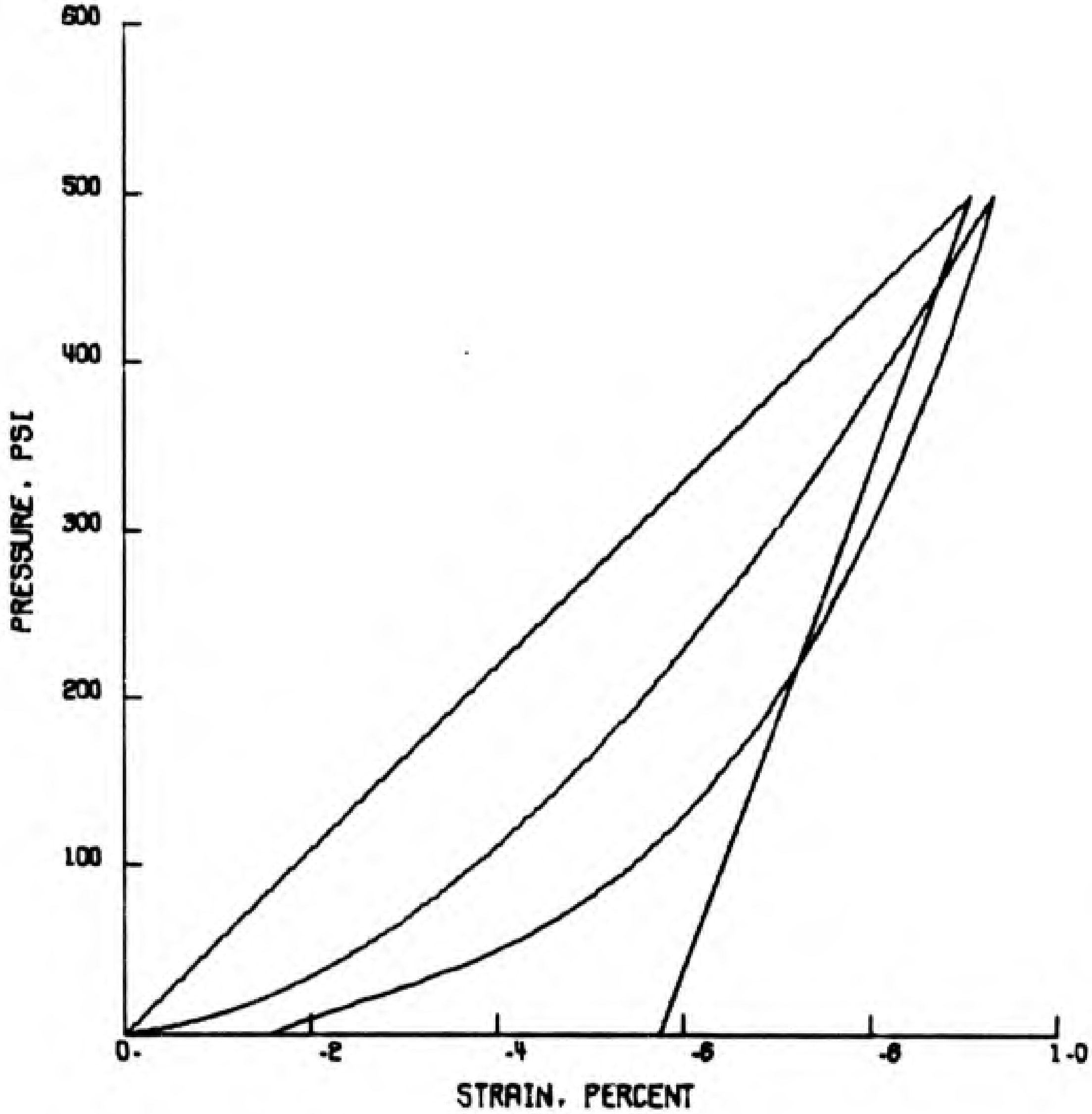
PROBLEM 2A OTTOWA SAND OCT 19.1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 200 PSI BRODE FORM. PMAX IS 500 PSI. TD IS 1.
HALF LOAD TIME = 7.710905E-02 SEC.
NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 2A OTTOWA SAND OCT 19.1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =5.489241E+04
M2(PSI) =1.485963E+05

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745097E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 3A OTTOWA SAND OCT 22. 1969

NUMBER OF DATA POINTS. N= 90 . M= 99

MATERIAL PROPERTIES

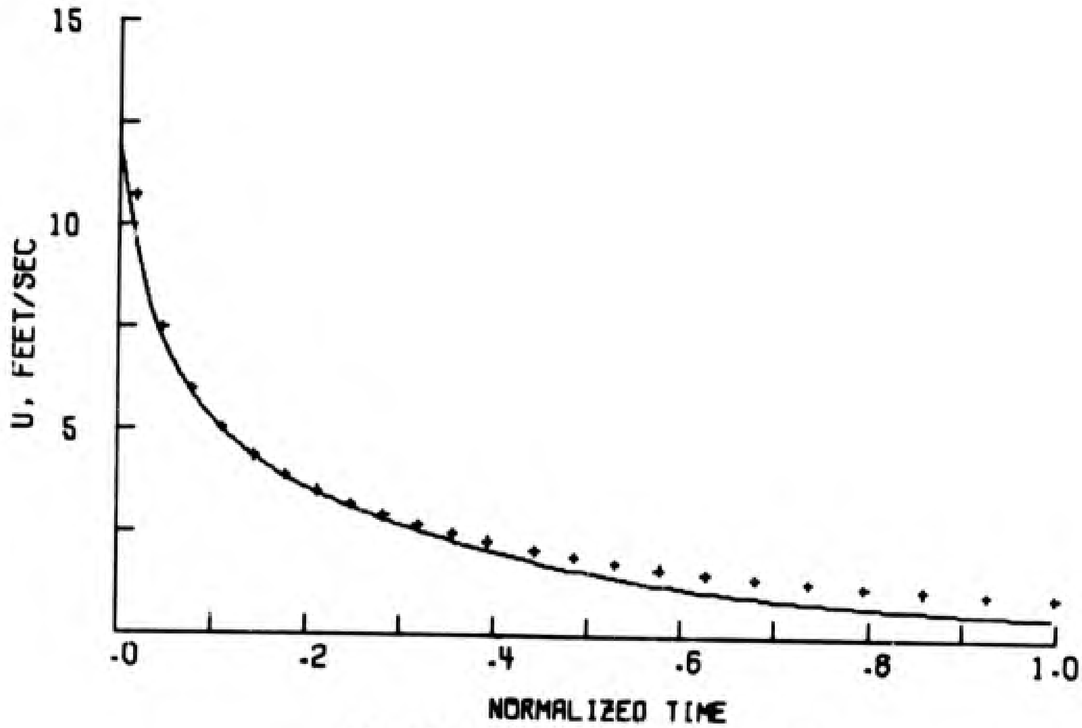
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.356942E+03
SOUND VELOCITY =	1.856329E+03
ZETA =	1.554140E-01

FITTING ERRORS

E_1 =	9.989563E-01	E_2 =	2.086251E-03
E_3 =	1.995744E-04	E_4 =	1.412708E-02
E_5 =	9.969104E-01	E_6 =	6.169605E-03
E_7 =	5.038354E-02	E_8 =	2.244628E-01

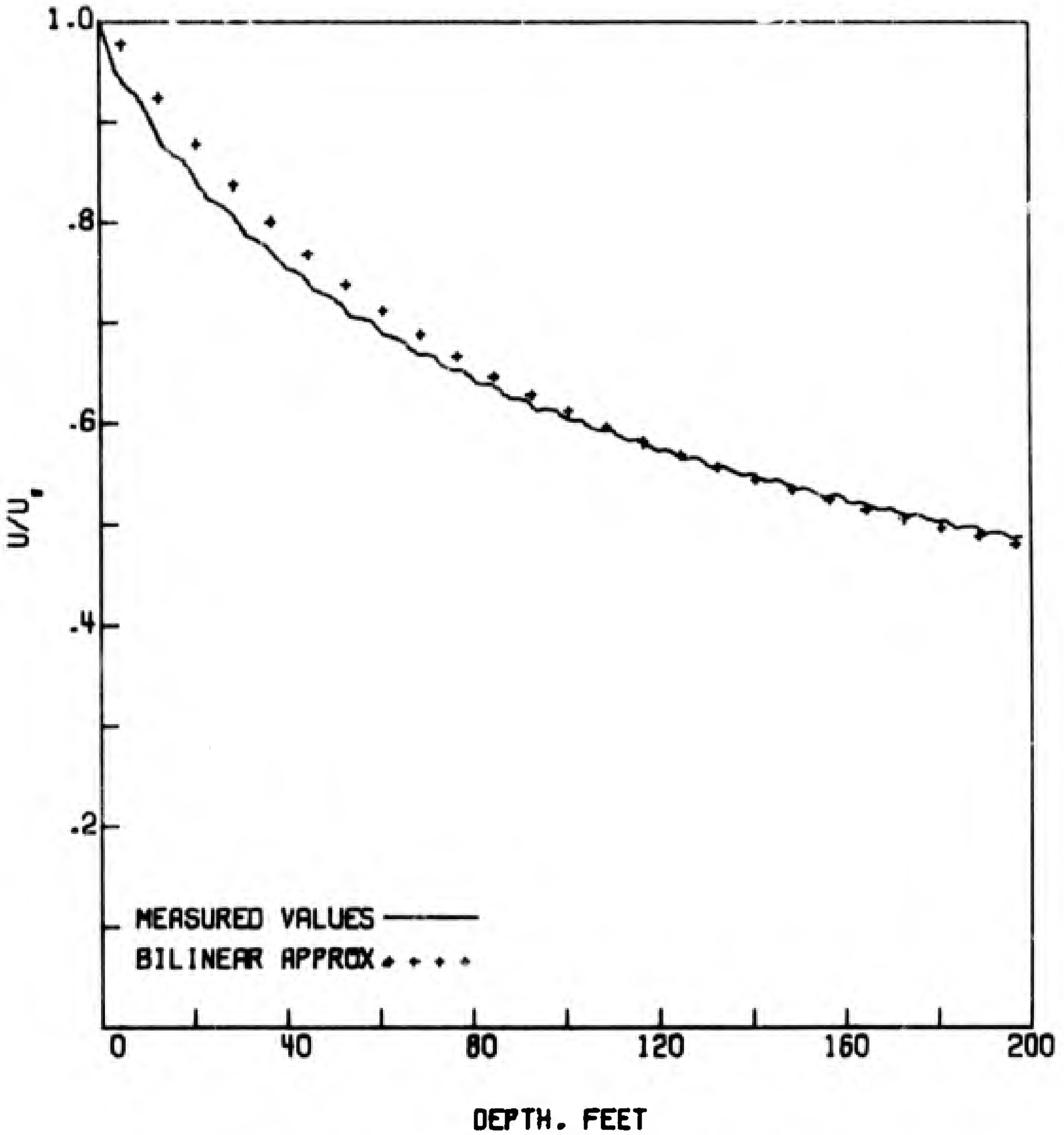
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	2.616	2.756
25	2.561	2.636
50	2.546	2.532
75	2.514	2.440
100	2.482	2.357



SURFACE PARTICLE VELOCITY

PROBLEM 3A OTTOWA SAND OCT 22. 1969



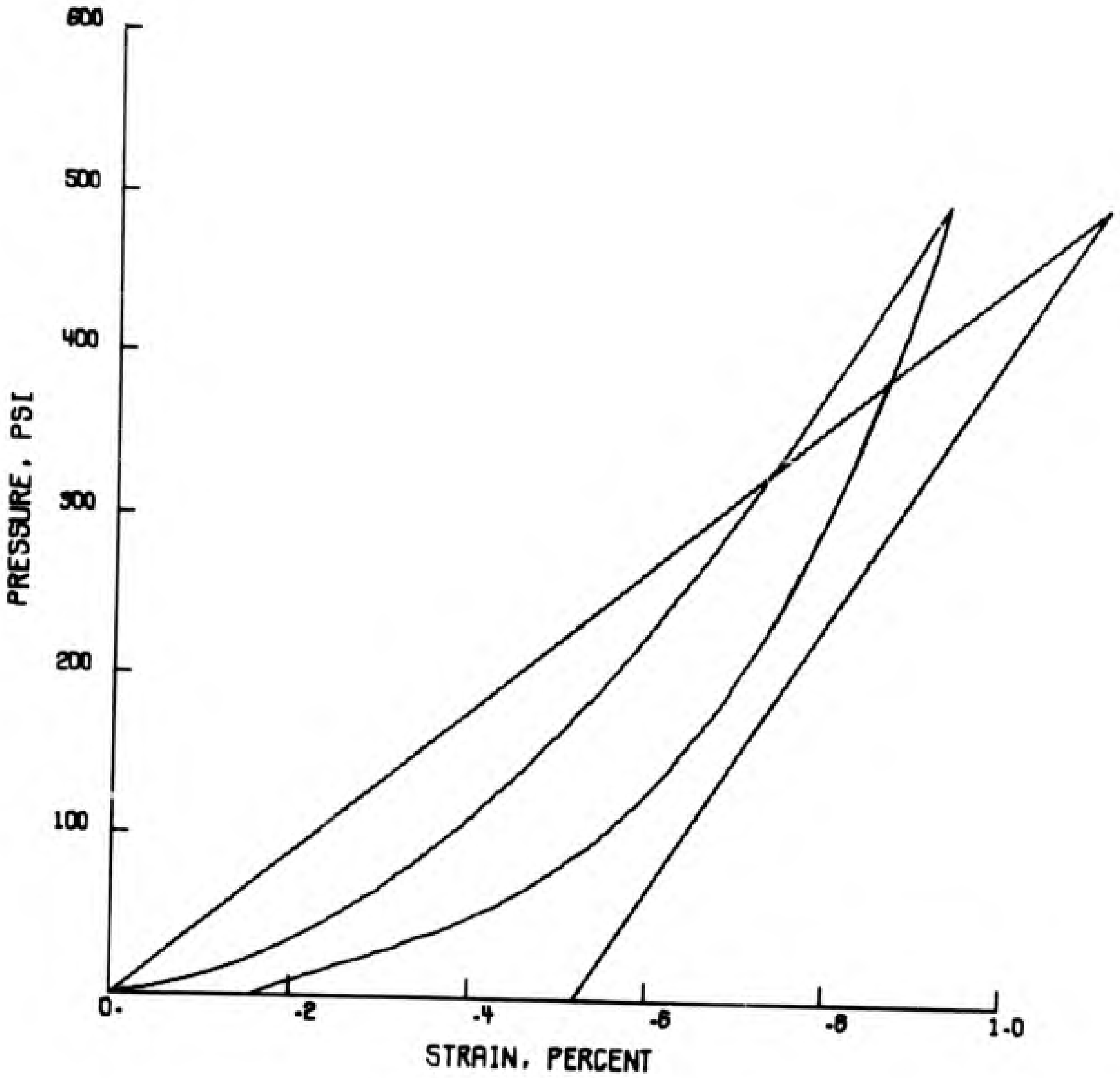
PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 500 PSI BRODE FORM. PMAX IS 500 PSI. TO IS 1.

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 3A OTTOWA SAND OCT 22. 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.468145E+04
M2(PSI) =8.399518E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745097E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 4A OTTOWA SAND

NUMBER OF DATA POINTS. N= 85 . M= 99

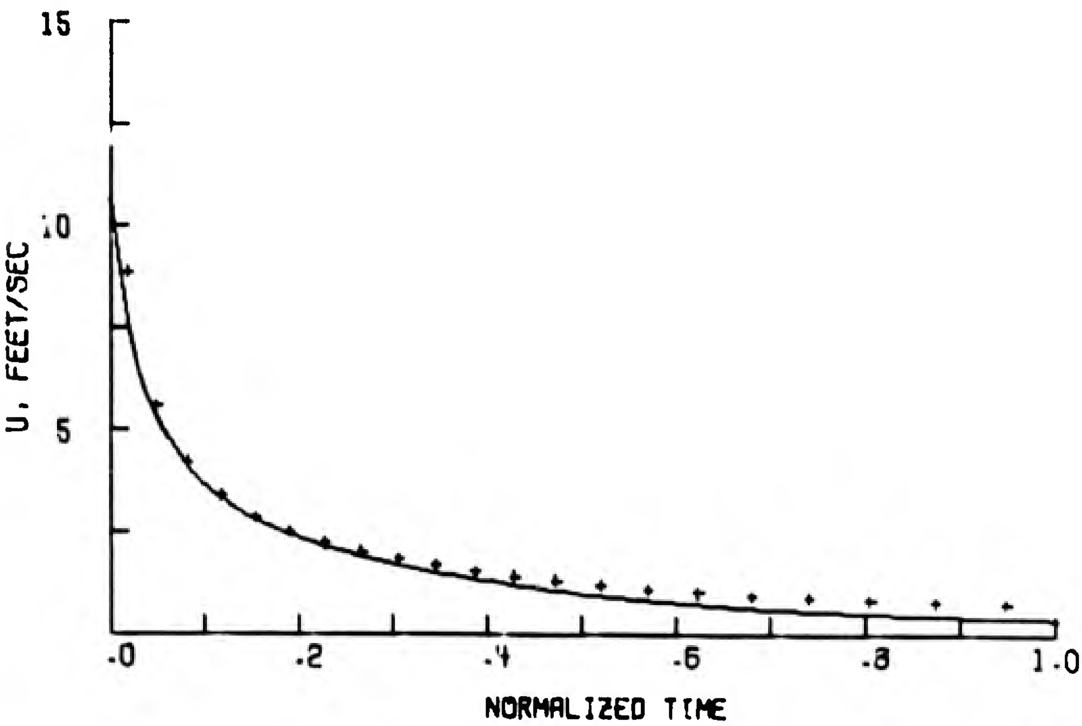
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.302395E+03
	SOUND VELOCITY =	1.689495E+03
	BETA =	1.293830E-01

FITTING ERRORS

E_1 =	9.985955E-01	E_2 =	2.807101E-03
E_3 =	3.124671E-04	E_4 =	1.767674E-02
E_5 =	9.977172E-01	E_6 =	4.560473E-03
E_7 =	1.877251E-02	E_8 =	1.370128E-01

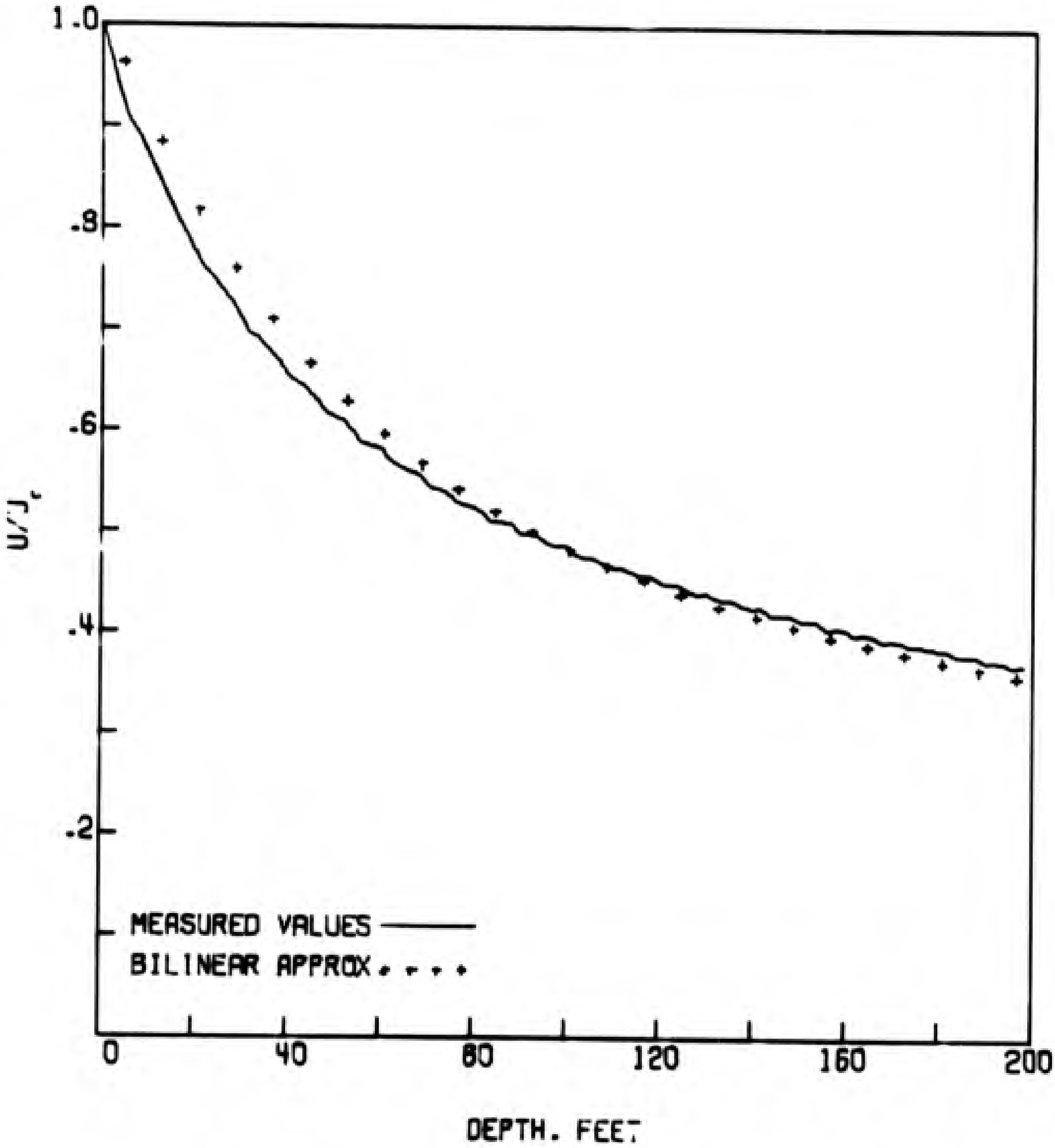
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	1.824	1.894
25	1.791	1.763
50	1.761	1.695
75	1.732	1.622
100	1.706	1.559



SURFACE PARTICLE VELOCITY

PROBLEM 4A OTTOWA SAND



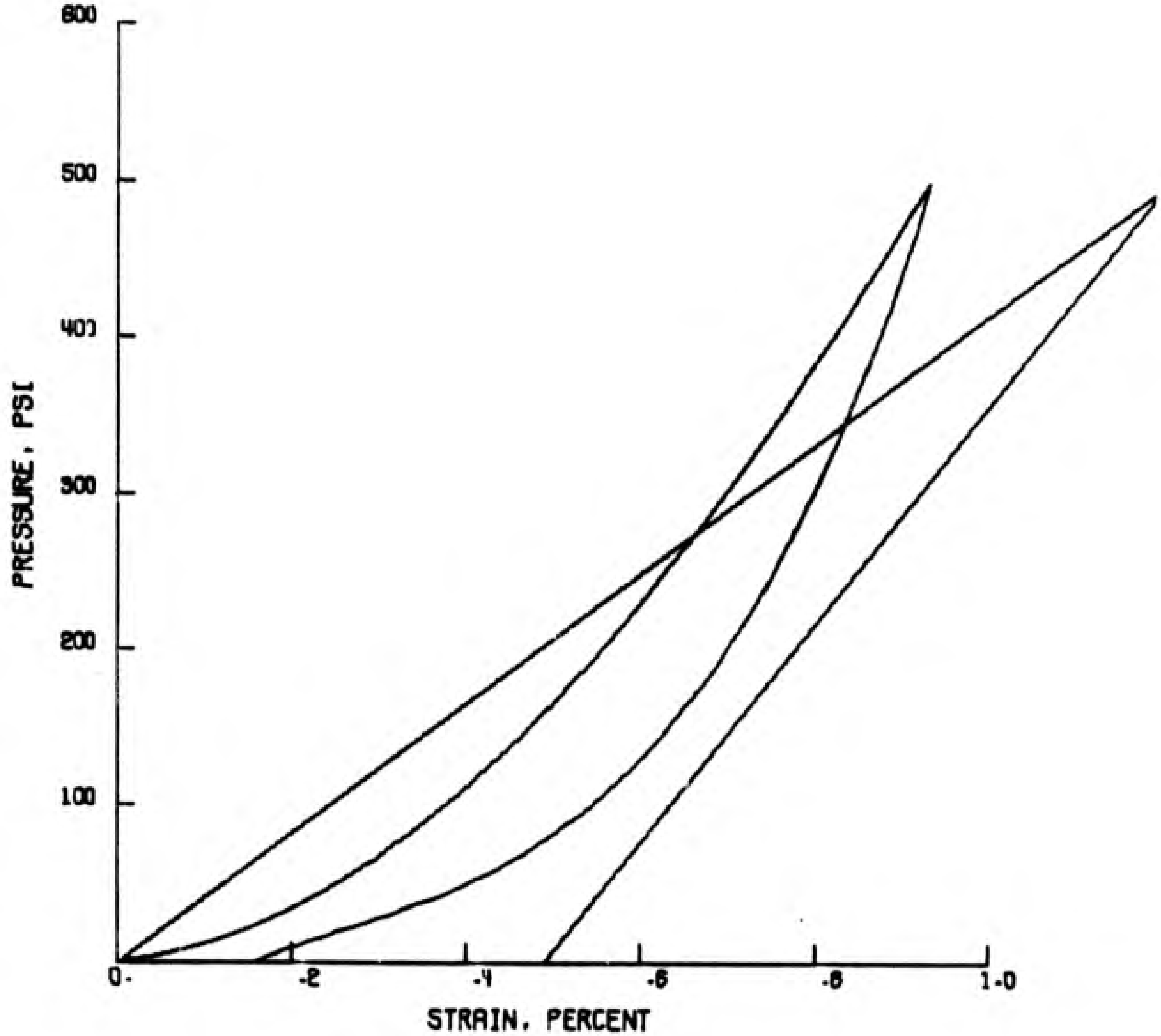
PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS ~~500~~ PSI BRODE FORM. PMAX IS 500 PSI. TD IS 1

HALF LOAD TIME = 1.285354E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 4A OTTOWA SAND



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.134569E+04
M2(PSI) =6.957585E+04

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04

HUGONIOT NONLINEARITY FACTOR = 2.745097E-01

MAXIMUM UNLOAD SLOPE = 1.722548E+05

ZETA = 3.105830E-01

BEST BILINEAR MODE

PROBLEM 5A   OTTOWA SAND   OCT 23.1969

NUMBER OF DATA POINTS. N= 79 . M= 99

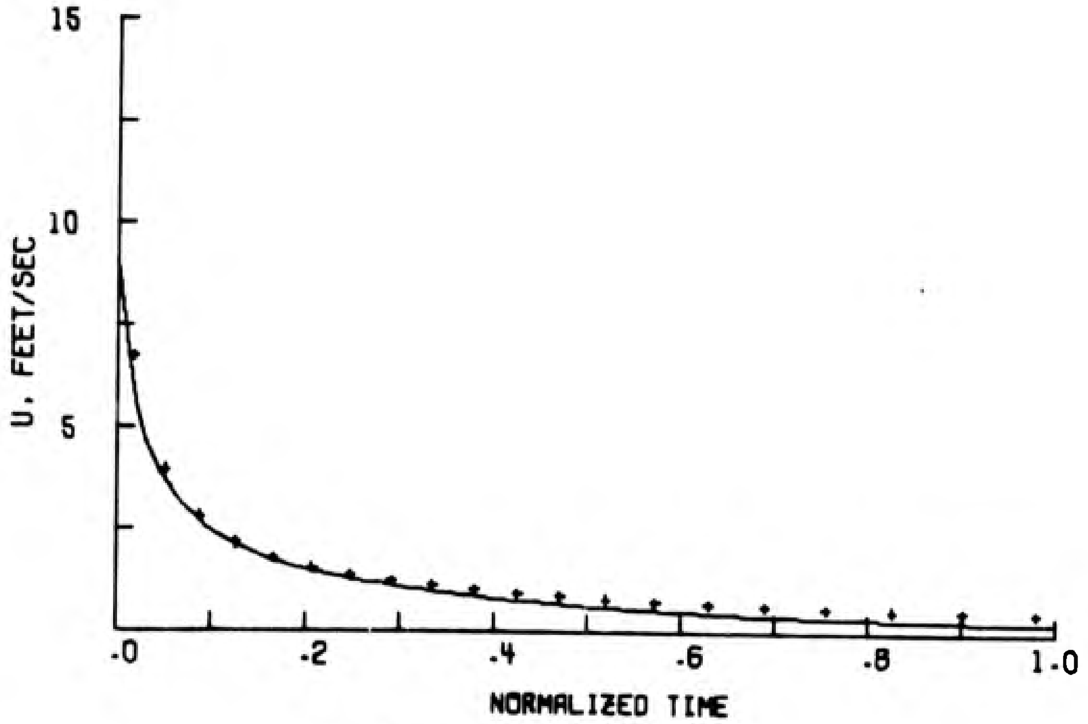
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.244242E+03
	SOUND VELOCITY =	1.547384E+03
	ZETA =	1.085900E-01

FITTING ERRORS

E_1 =	9.976685E-01	E_2 =	4.657545E-03
E_3 =	5.252871E-04	E_4 =	2.291914E-02
E_6 =	9.982077E-01	E_6 =	3.581357E-03
E_7 =	8.392275E-03	E_8 =	9.160936E-02

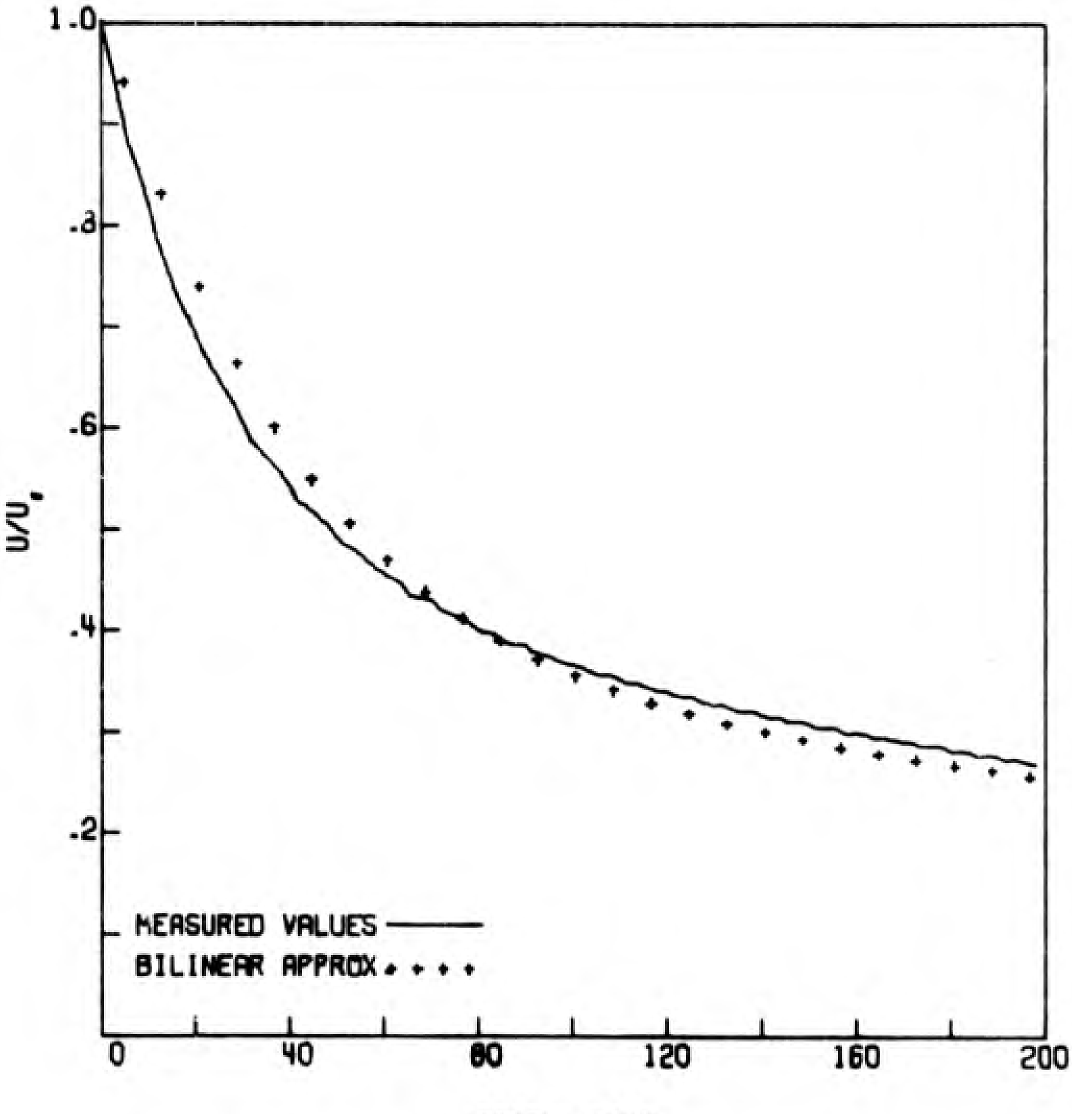
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	1.258	1.293
25	1.228	1.189
50	1.201	1.117
75	1.178	1.061
100	1.157	1.016



SURFACE PARTICLE VELOCITY

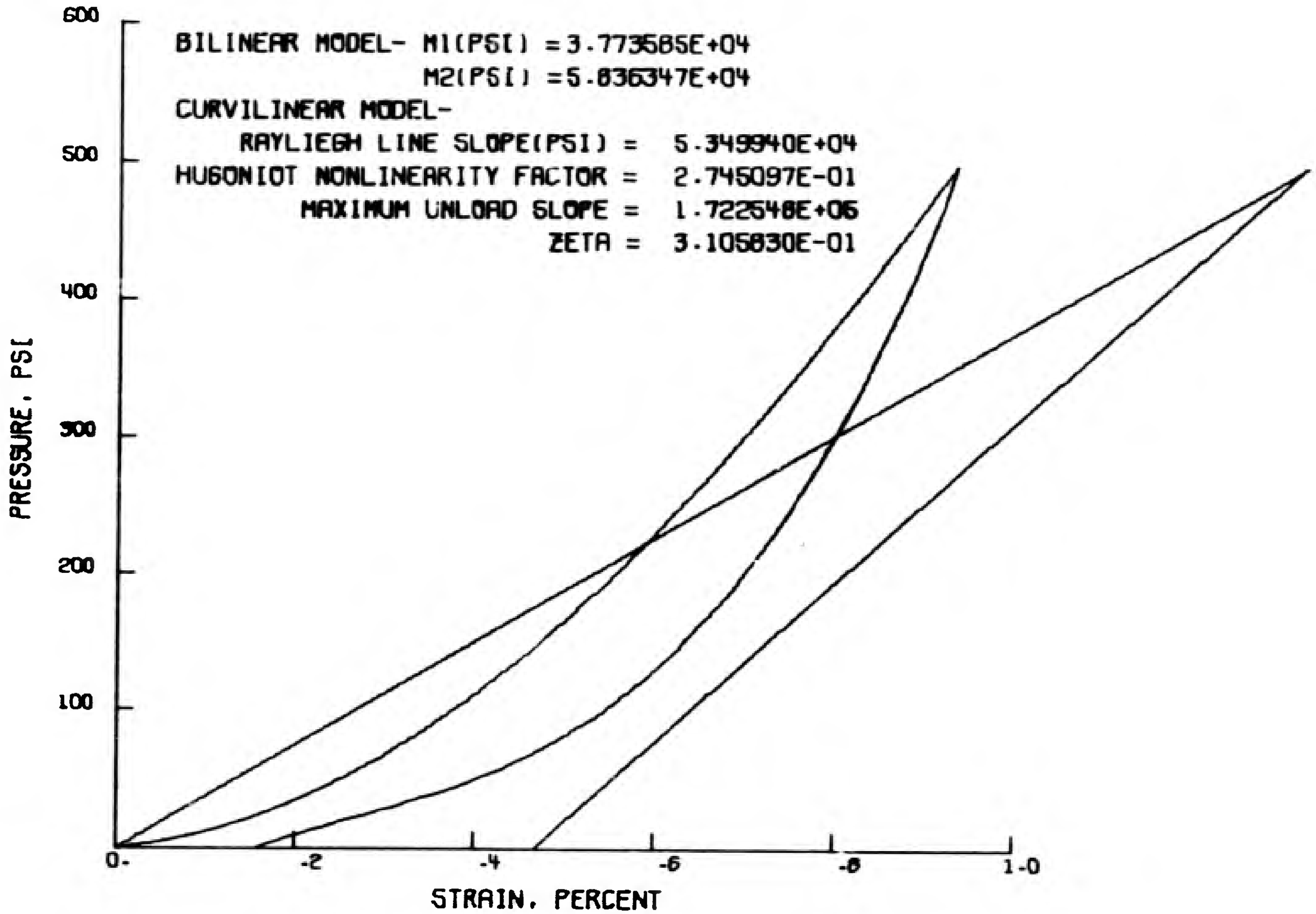
PROBLEM 5A OTTOWA SAND OCT 23.1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 2000 PSI. BRODE FORM PMAX IS 500 PSI TD=1.09
HALF LOAD TIME = 3.531115E-03 SEC.
NORMALIZED HALF LOAD TIME = 3.239565E-03

PROBLEM 5A OTTOWA SAND OCT 23.1969



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 1C -- 7 NOV 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

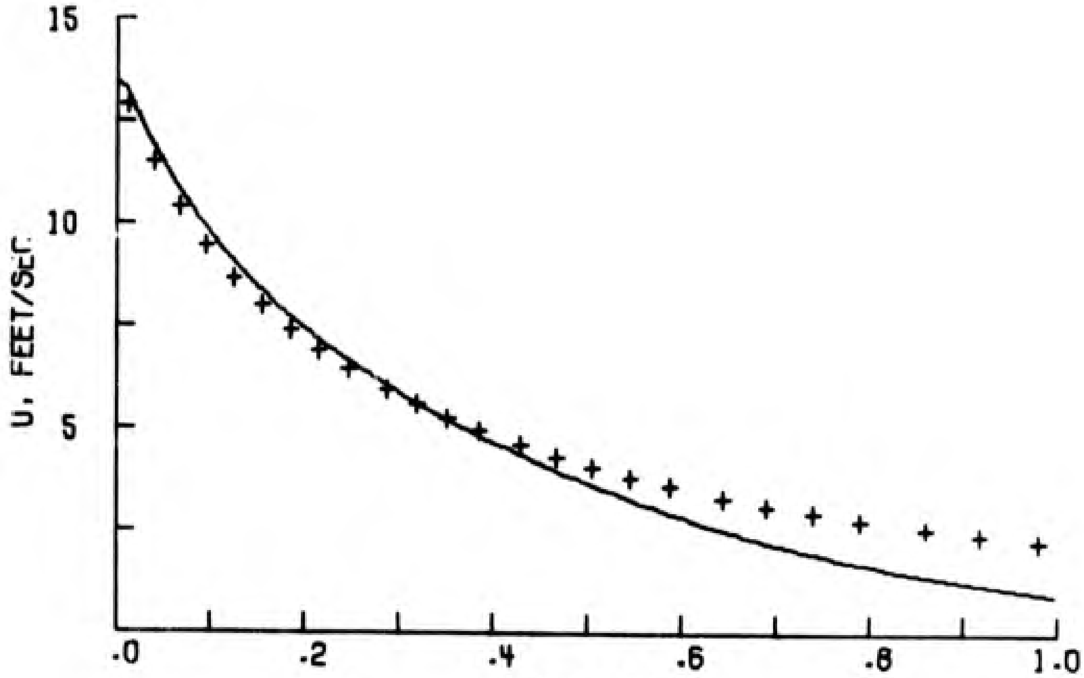
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.499605E+03
	SOUND VELOCITY =	2.582638E+03
	ZETA =	2.653035E-01

FITTING ERRORS

E_1 =	9.957359E-01	E_2 =	8.509935E-03
E_3 =	3.526022E-05	E_4 =	5.938032E-03
E_5 =	9.966960E-01	E_6 =	6.597027E-03
E_7 =	4.102132E-01	E_8 =	6.404789E-01

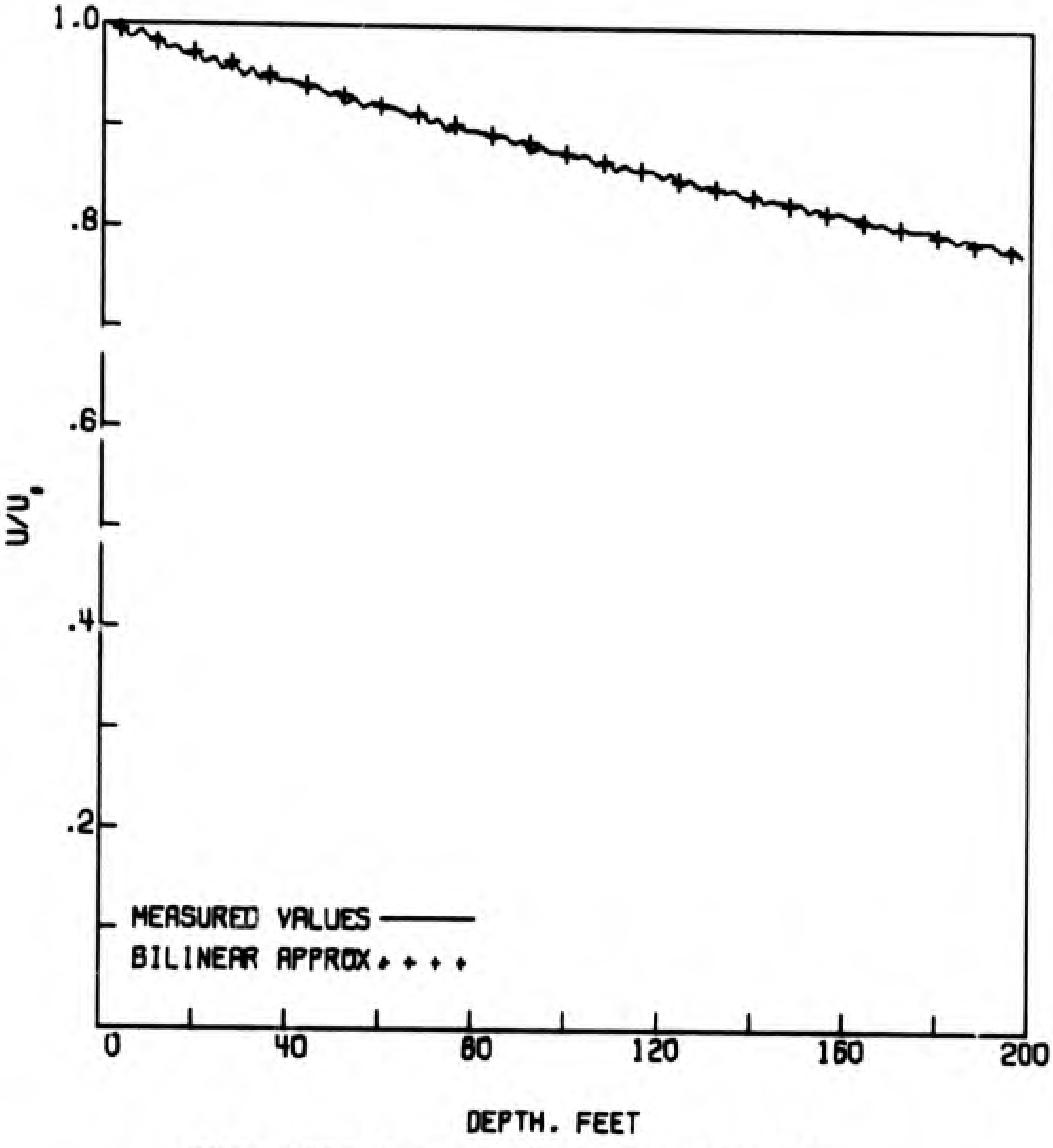
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	5.084	5.529
25	5.046	5.380
50	5.003	5.236
75	4.959	5.097
100	4.913	4.962



SURFACE PARTICLE VELOCITY

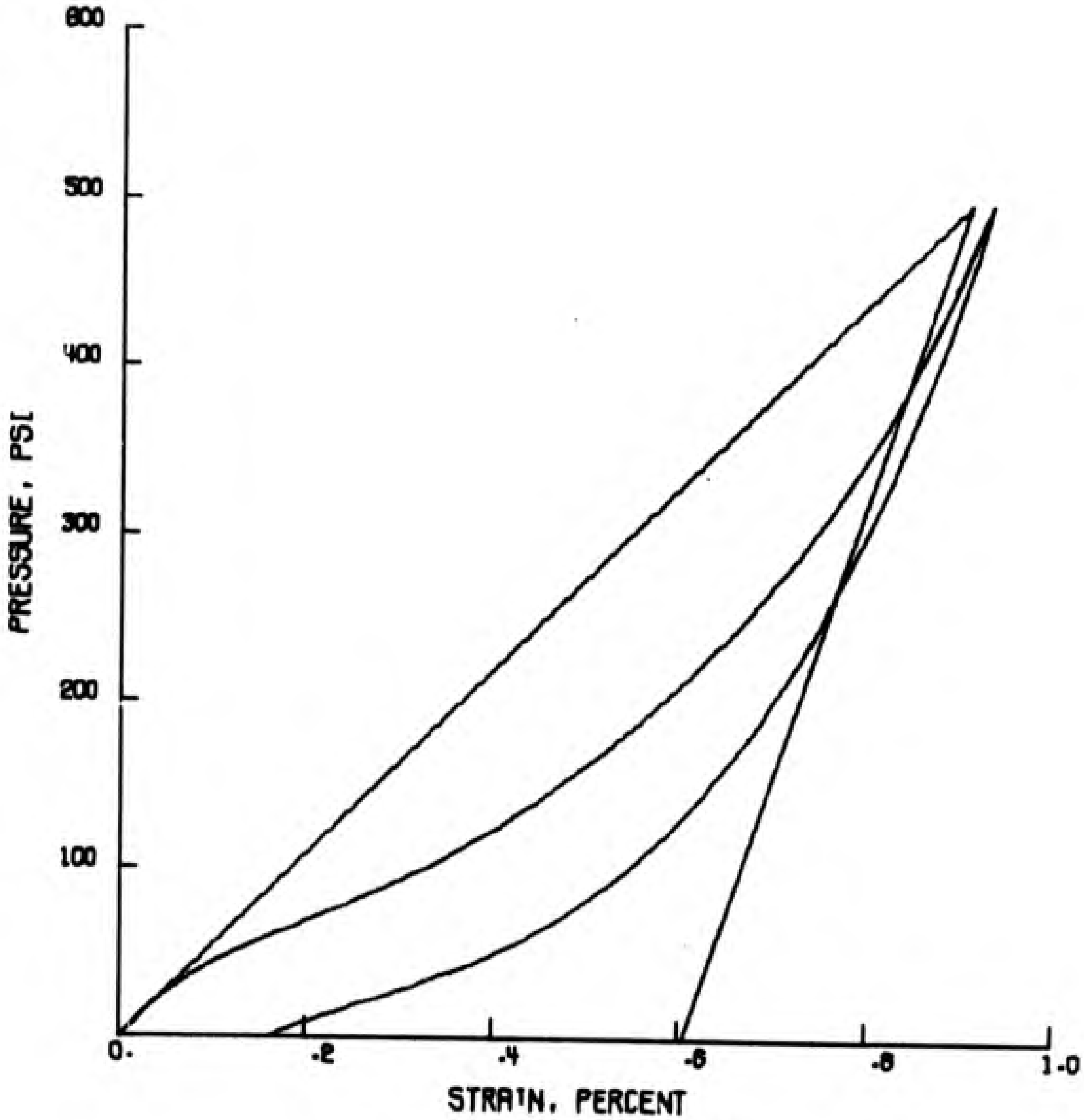
PROBLEM 1C -- 7 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 100 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.304202E-01 SEC.
NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 1[illegible] -- 7 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =5.481485E+04
M2(PSI) =1.625817E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 2C -- 7 NOV 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

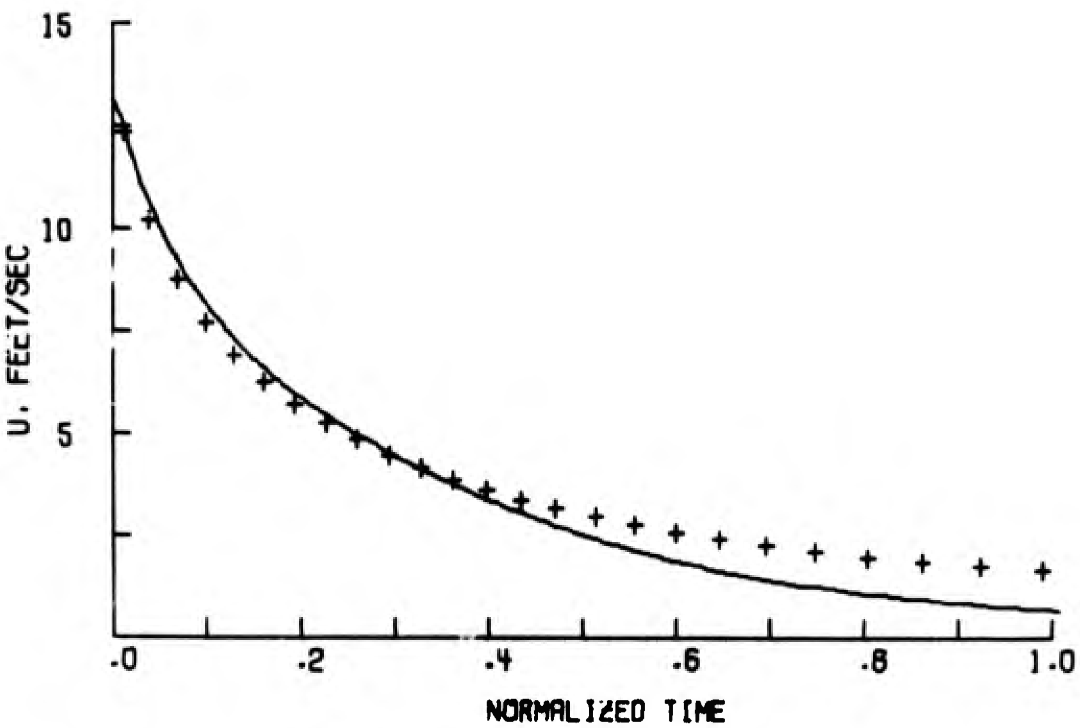
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.472974E+03
	SOUND VELOCITY =	2.398833E+03
	ZETA =	2.391295E-01

FITTING ERRORS

E_1 =	9.980900E-01	E_2 =	3.816410E-03
E_3 =	3.590933E-05	E_4 =	5.823172E-03
E_5 =	9.903714E-01	E_6 =	7.244094E-03
E_7 =	3.243837E-01	E_8 =	5.695469E-01

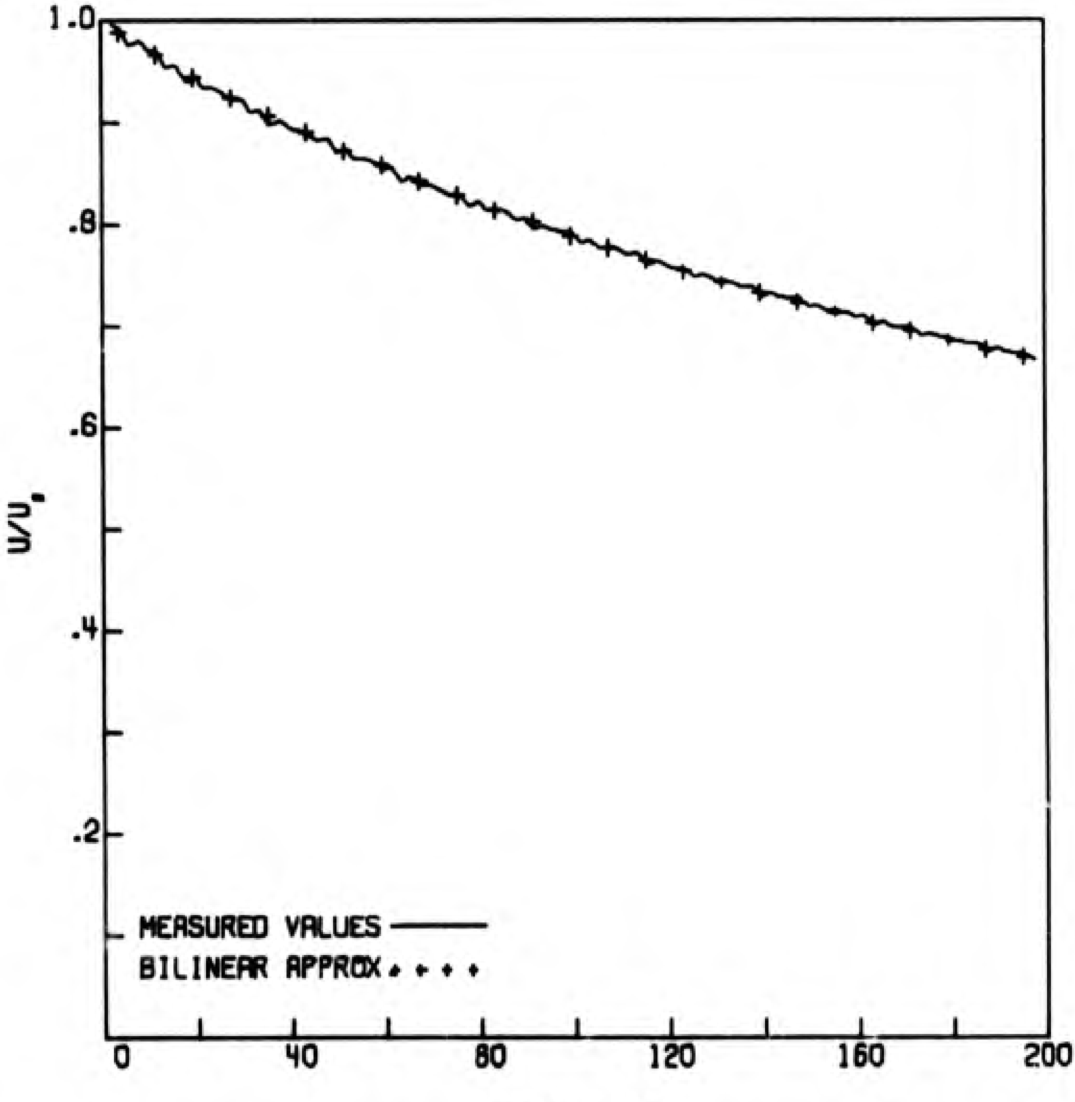
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	3.949	4.356
25	3.911	4.213
50	3.870	4.080
75	3.830	3.354
100	3.789	3.934



SURFACE PARTICLE VELOCITY

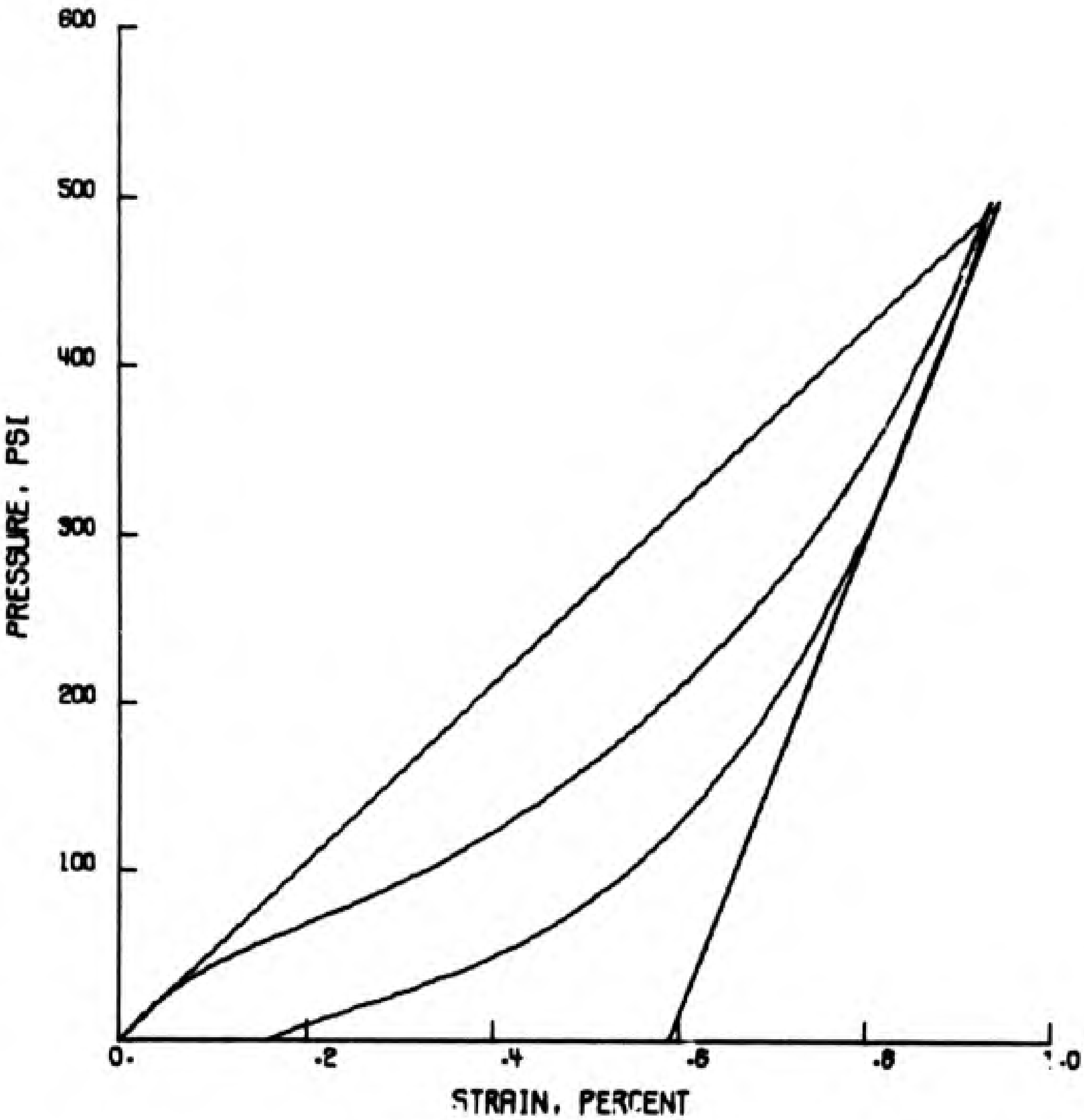
PROBLEM 2C -- 7 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 200 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 7.710905E-02 SEC.
NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 2C -- 7 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =5.288525E+04
M2(PSI) =1.402541E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 1.722518E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 3C -- 7 NOV 1969

NUMBER OF DATA POINTS. N= 95 . M= 99

MATERIAL PROPERTIES

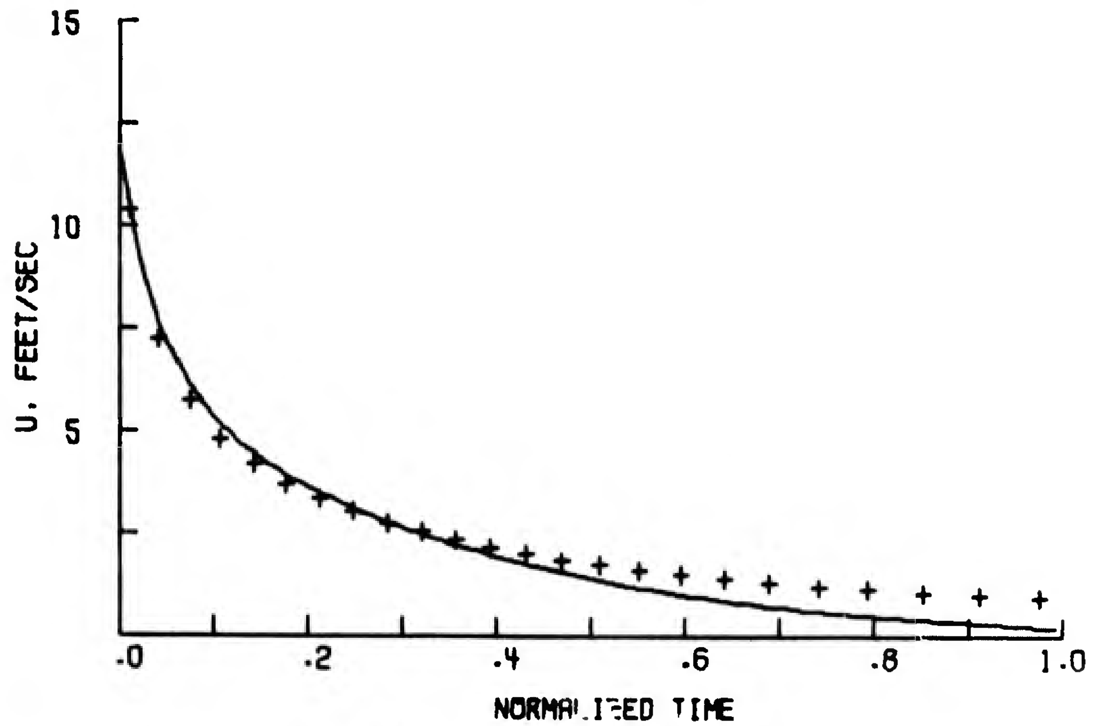
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.405726E+03
SOUND VELOCITY =	2.093839E+03
ZETA =	1.966280E-01

FITTING ERRORS

E_1 =	9.989717E-01	E_2 =	2.055607E-03
E_3 =	4.301951E-05	E_4 =	6.558926E-03
E_5 =	9.949362E-01	E_6 =	1.010186E-02
E_7 =	1.809833E-01	E_8 =	4.254213E-01

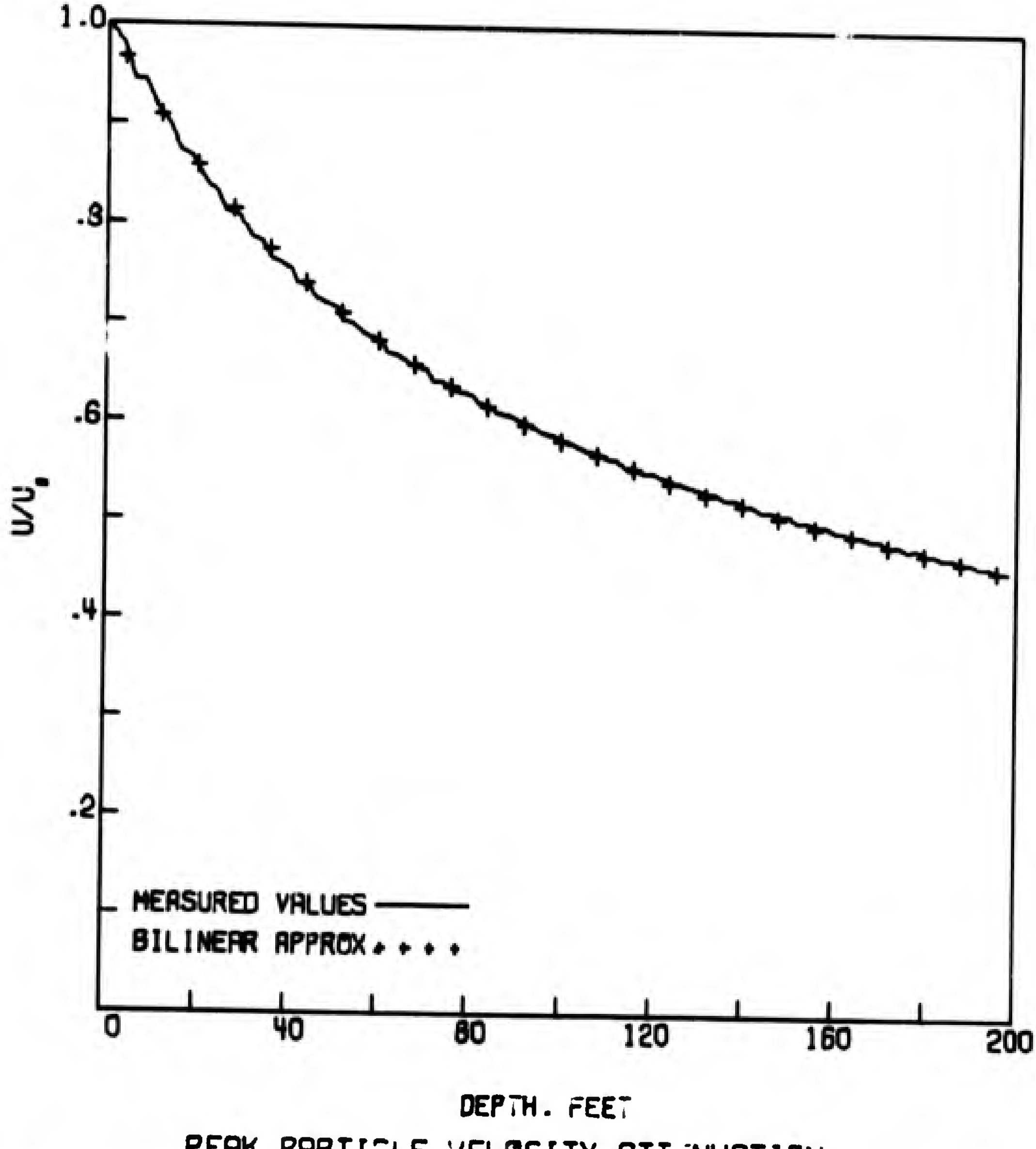
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	2.482	2.796
25	2.446	2.665
50	2.410	2.554
75	2.375	2.456
100	2.341	2.360



SURFACE PARTICLE VELOCITY

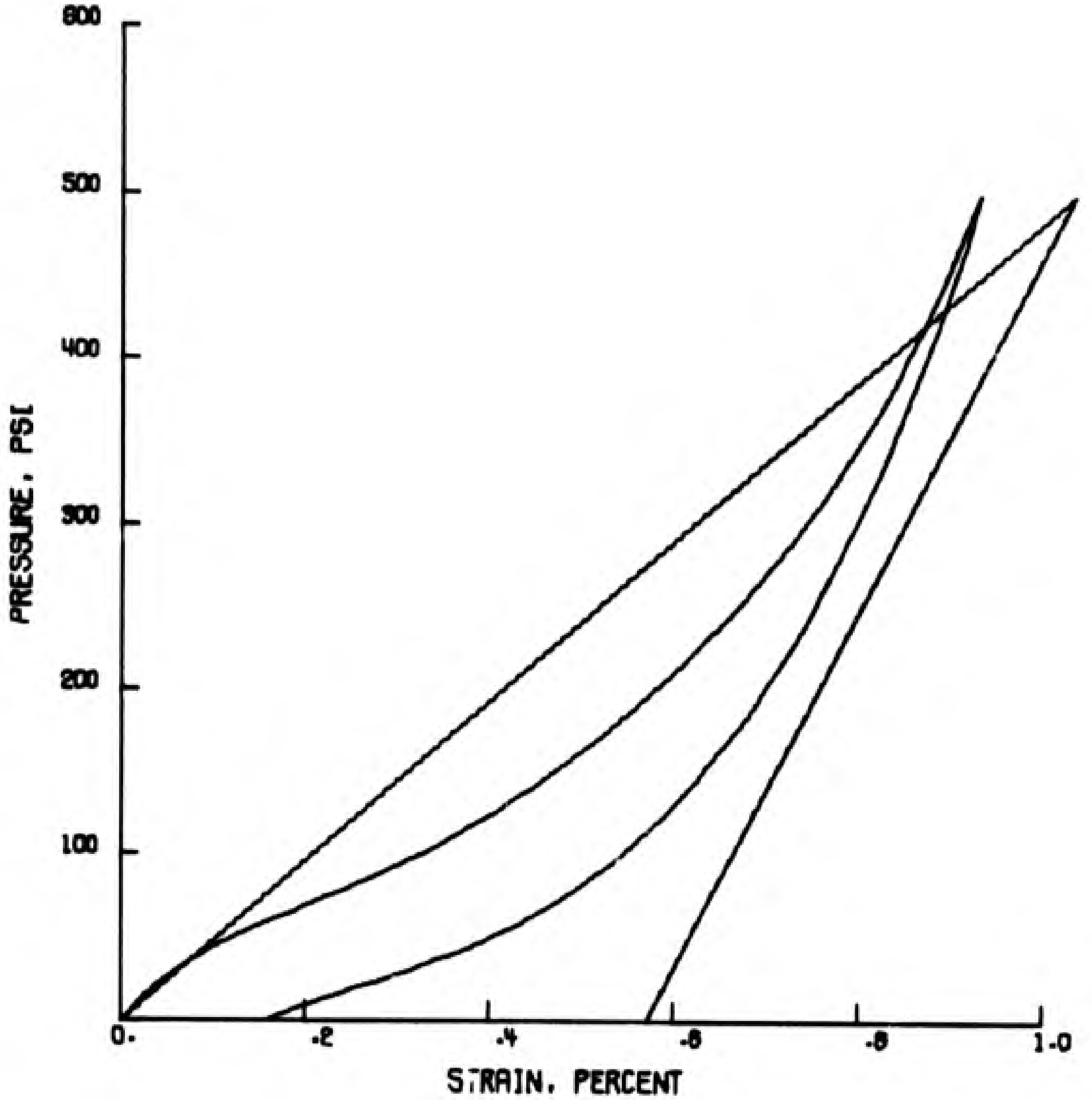
PROBLEM 3C -- 7 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 500 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 2.531964E-02 SEC.
NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 3C -- 7 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.816663E+04
M2(PSI) =1.068639E+05
CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 4C -- 7 NOV 1969

NUMBER OF DATA POINTS. N= 97 . M= 99

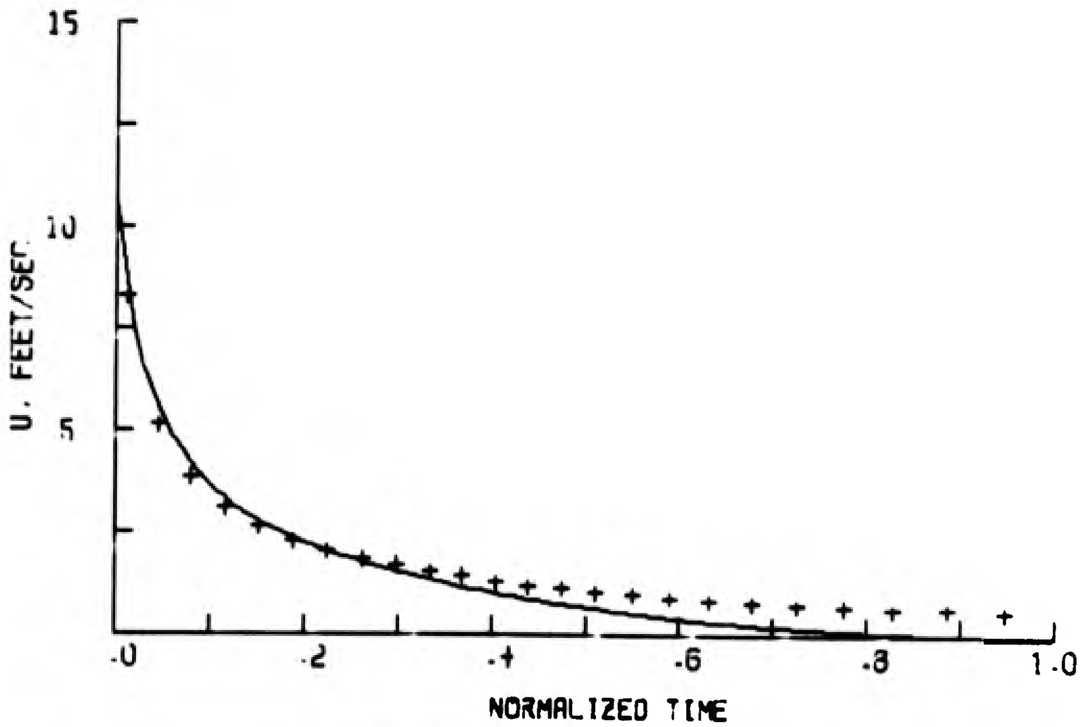
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.439425E+03
	SOUND VELOCITY =	2.164950E+03
	ZETA =	2.012900E-01

FITTING ERRORS

E_1 =	9.991677E-01	E_2 =	1.663996E-03
E_3 =	5.229563E-05	E_4 =	7.231572E-03
E_5 =	9.957937E-01	E_6 =	3.394953E-03
E_7 =	1.838E31E-01	E_8 =	4.287926E-01

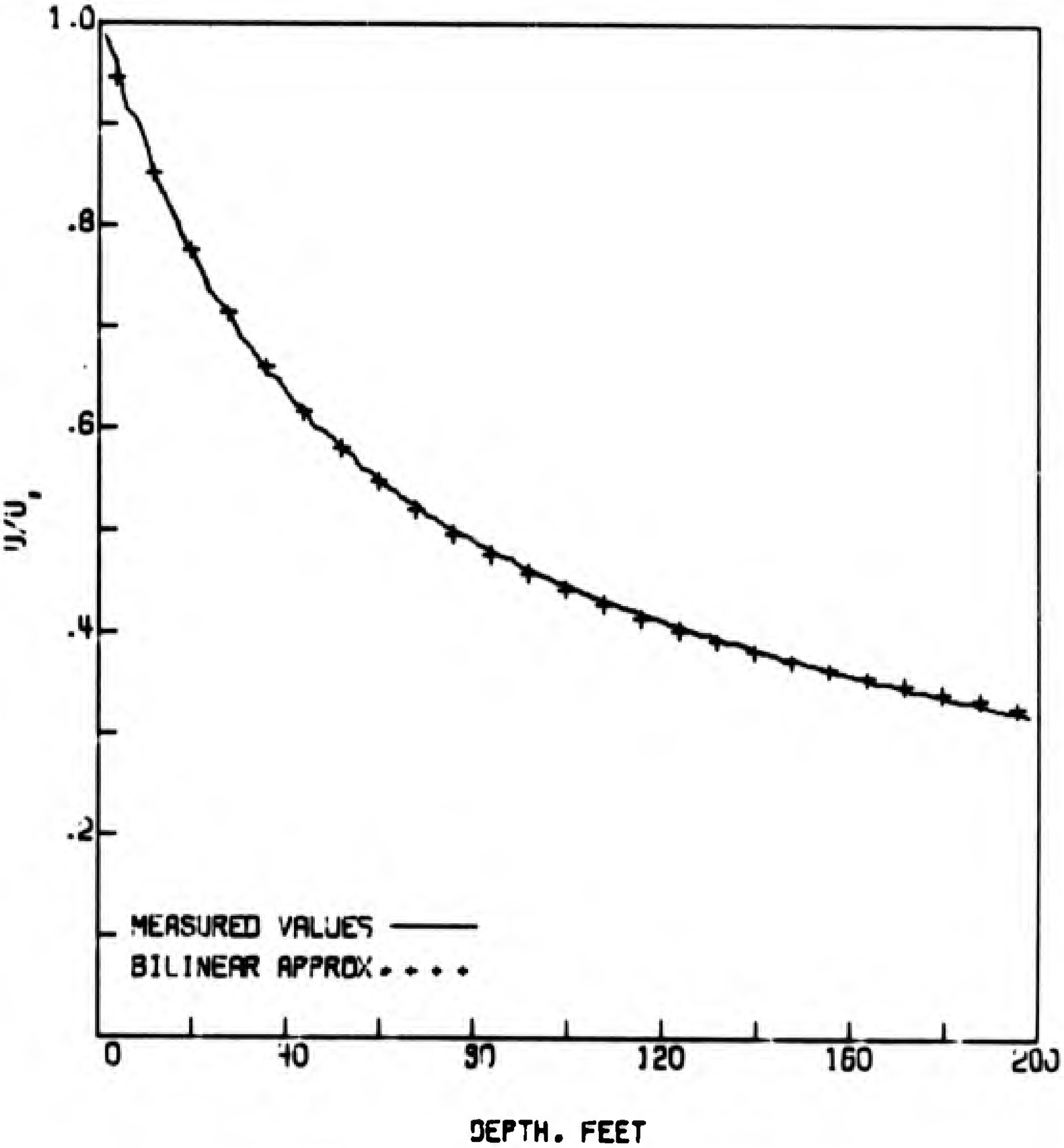
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	1.536	1.862
25	1.501	1.761
50	1.469	1.668
75	1.440	1.592
100	1.413	1.527



SURFACE PARTICLE VELOCITY

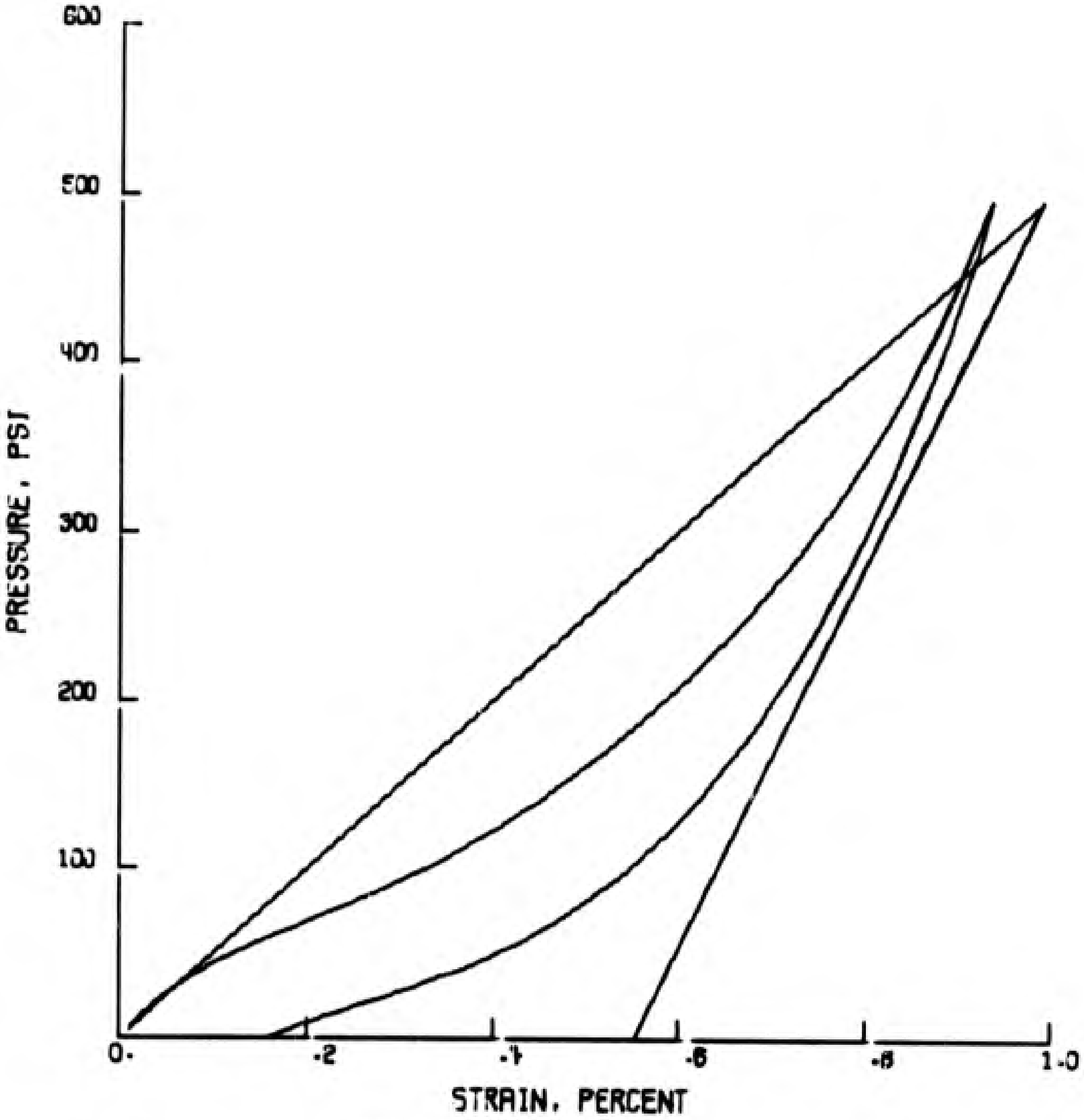
PROBLEM 4C -- 7 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 1000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.285354E-02 SEC.
NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 4C --- 7 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =5.050367E+04
M2(PSI) =1.142458E+05

CURVILINEAR MODEL--
RAYLIEGH LINE SLOPE(PSI) = 5.349040E+04
HUEONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM SC -- 7 NOV 1963

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

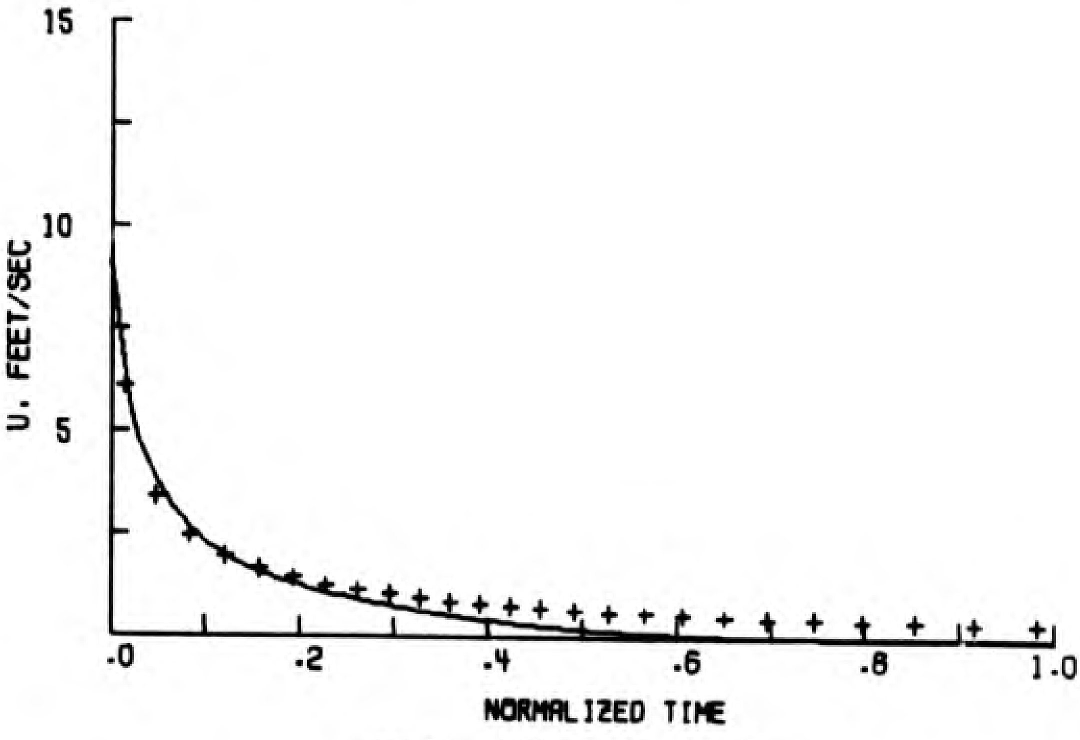
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.519937E+03
SOUND VELOCITY =	2.410611E+03
ZETA =	2.266030E-01

FITTING ERRORS

E_1 =	9.993203E-01	E_2 =	1.353002E-03
E_3 =	1.218887E-04	E_4 =	1.104032E-02
E_5 =	9.975042E-01	E_6 =	4.985314E-03
E_7 =	1.549254E-01	E_8 =	3.936056E-01

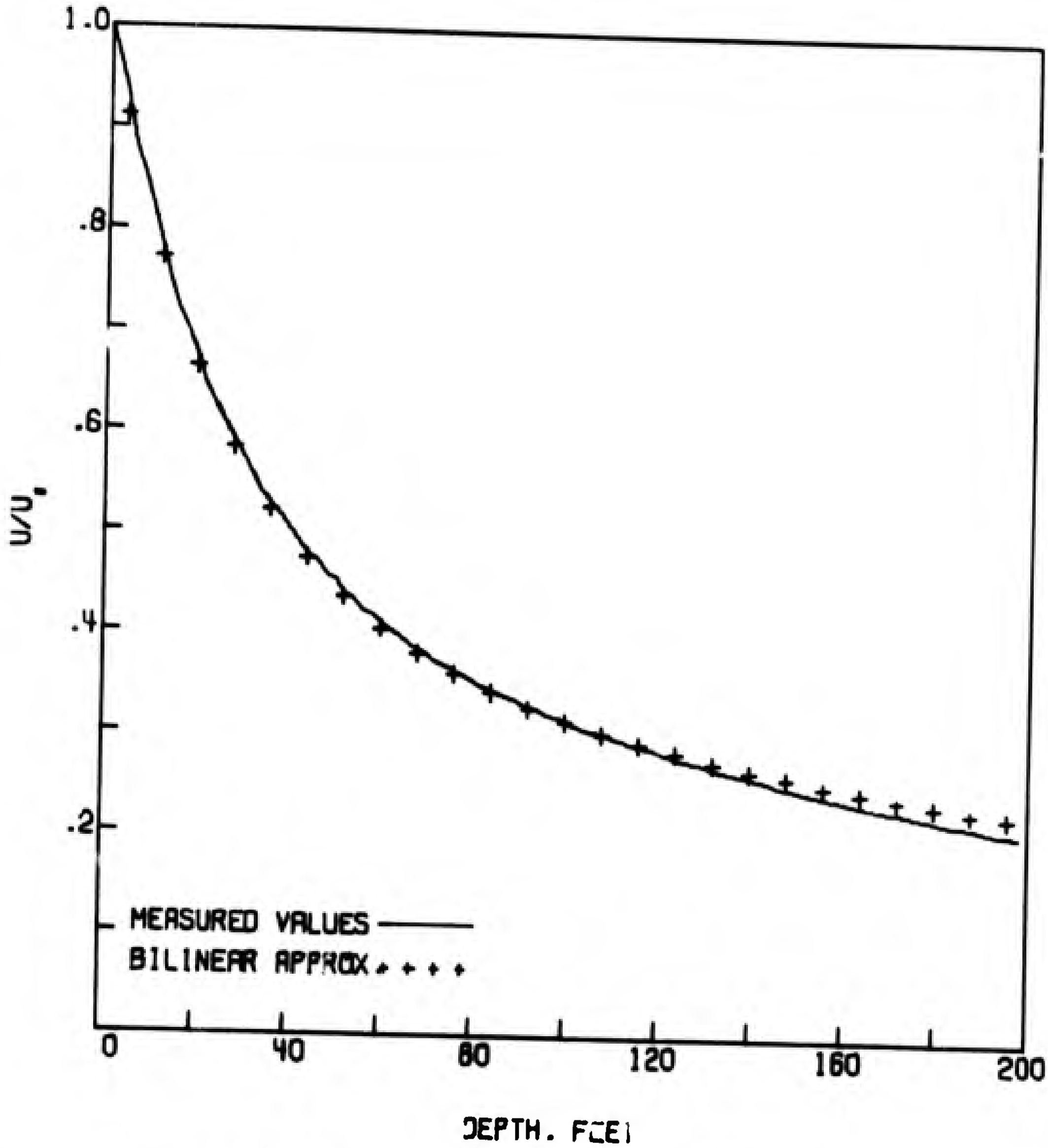
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	.909	1.236
25	.876	1.125
50	.848	1.054
75	.826	.999
100	.807	.954



SURFACE PARTICLE VELOCITY

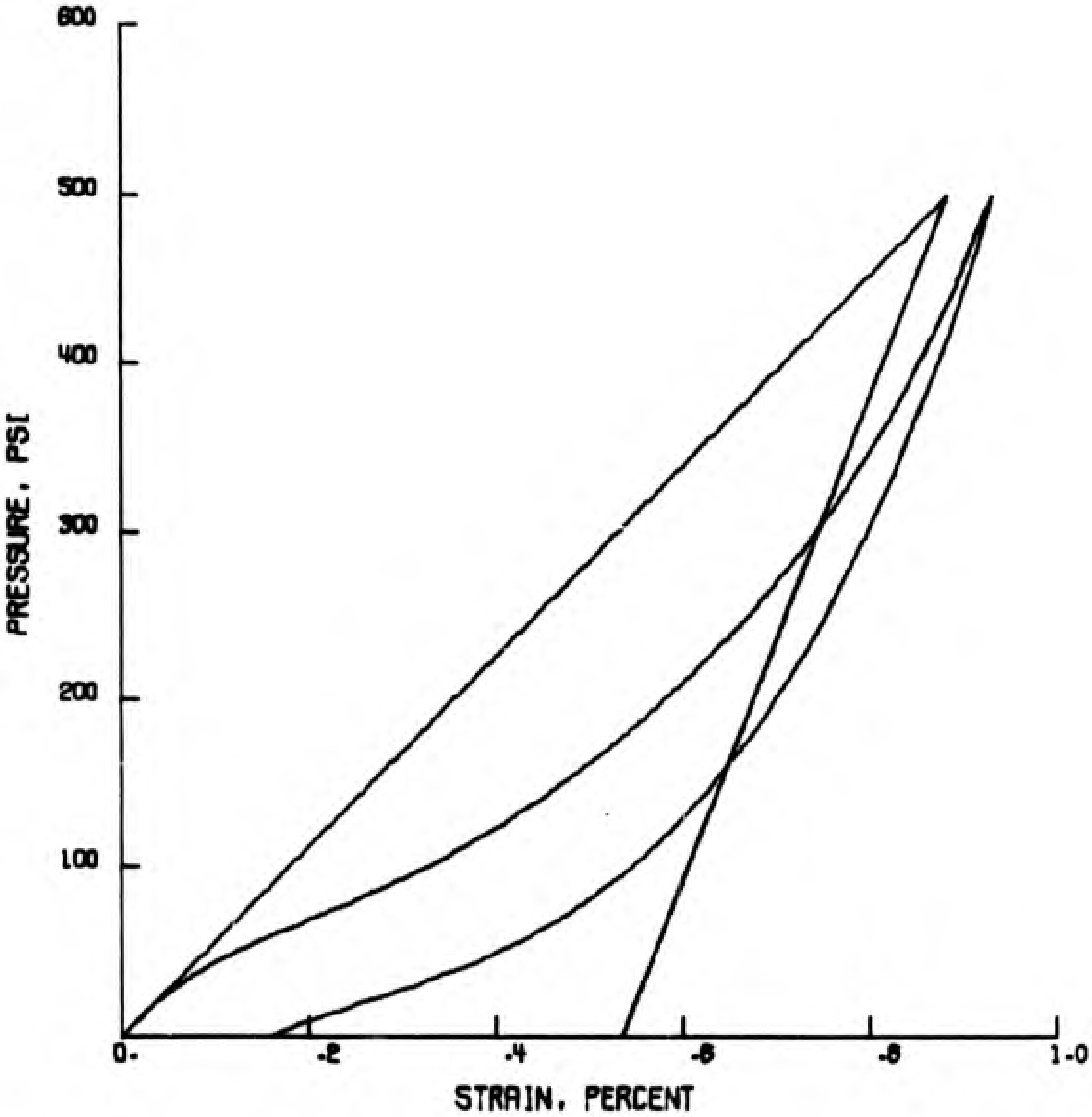
PROBLEM 5C -- 7 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 2000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 3.531115E-03 SEC.
NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM SC -- 7 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =5.631135E+04
M2(PSI) =1.416443E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 1D -- 10 NOV 1969

NUMBER OF DATA POINTS, N= 99 , M= 99

MATERIAL PROPERTIES

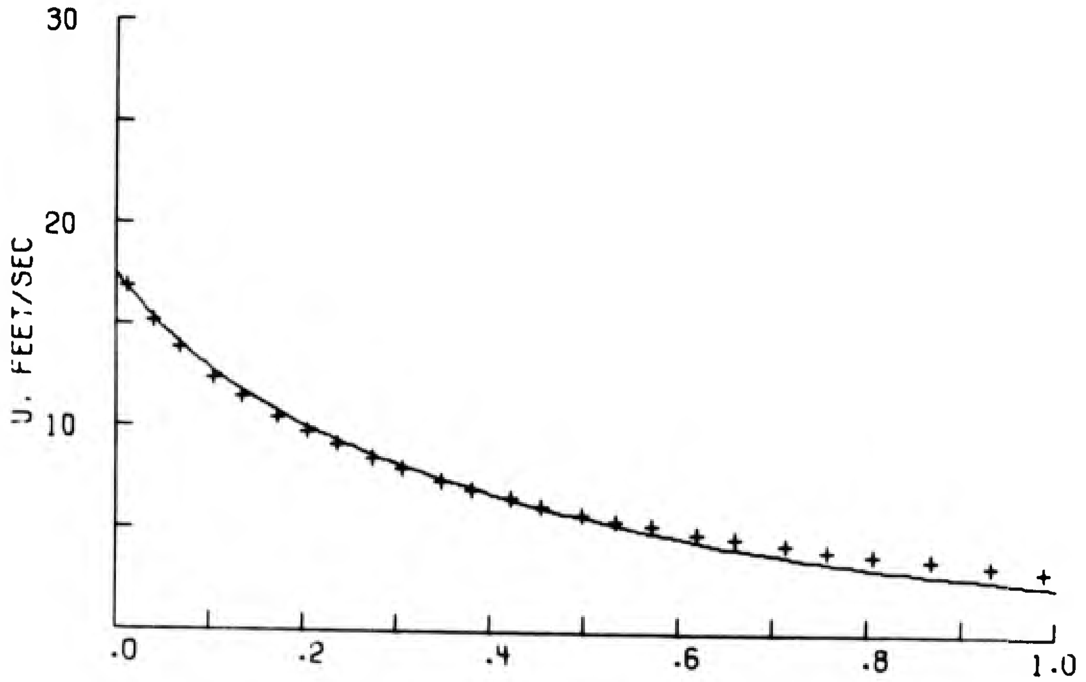
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.152647E+03
	SOUND VELOCITY =	2.721410E+03
	ZETA =	4.049405E-01

FITTING ERRORS

E_1 =	9.991651E-01	E_2 =	1.669106E-03
E_3 =	9.643425E-05	E_4 =	9.820094E-03
E_5 =	9.983980E-01	E_6 =	3.201505E-03
E_7 =	1.654526E-01	E_8 =	4.067586E-01

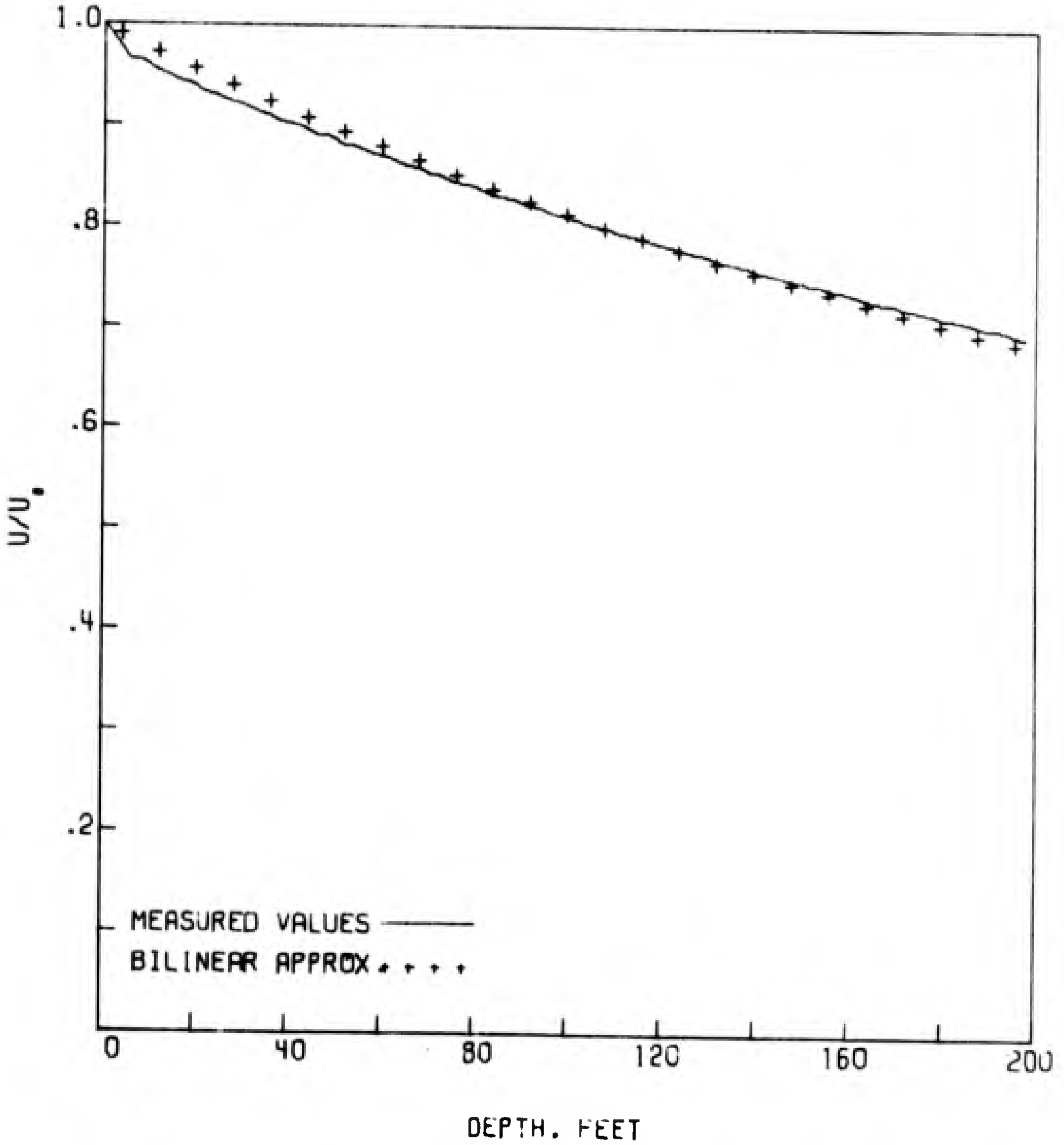
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	7.475	7.703
25	7.306	7.395
50	7.125	7.102
75	6.945	6.824
100	6.767	6.559



SURFACE PARTICLE VELOCITY

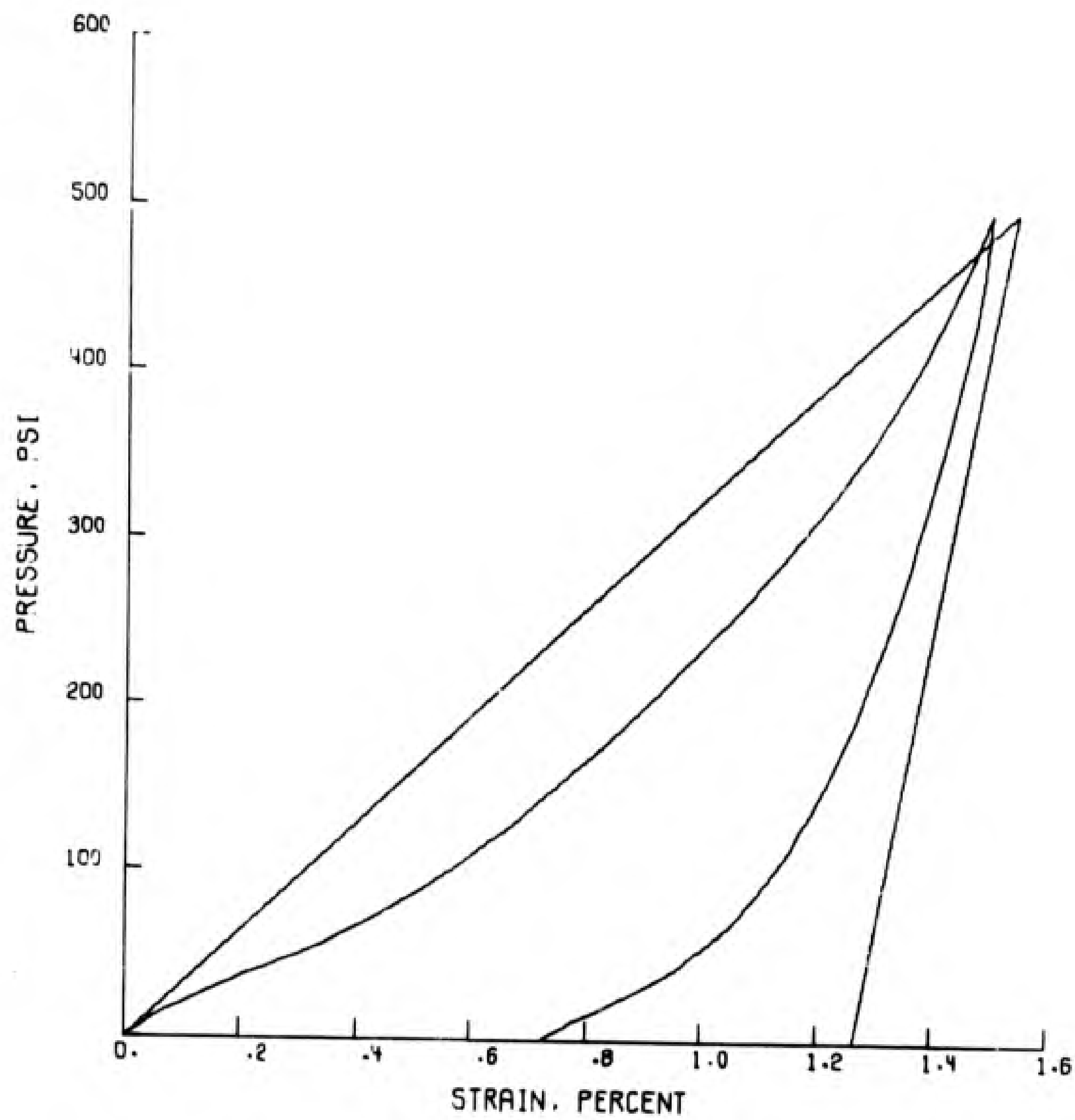
PROBLEM 1D --- 10 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 100 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.304202E-01 SEC.
NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 10 -- 10 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 3.238452E+04
M2(PSI) = 1.805230E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04
HUGONIOT NONLINEARITY FACTOR = 2.744445E-01
MAXIMUM UNLOAD SLOPE = 1.705140E+05
ZETA = 1.954874E-01

BEST BILINEAR MODEL

PROBLEM 2D -- 10 NOV 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

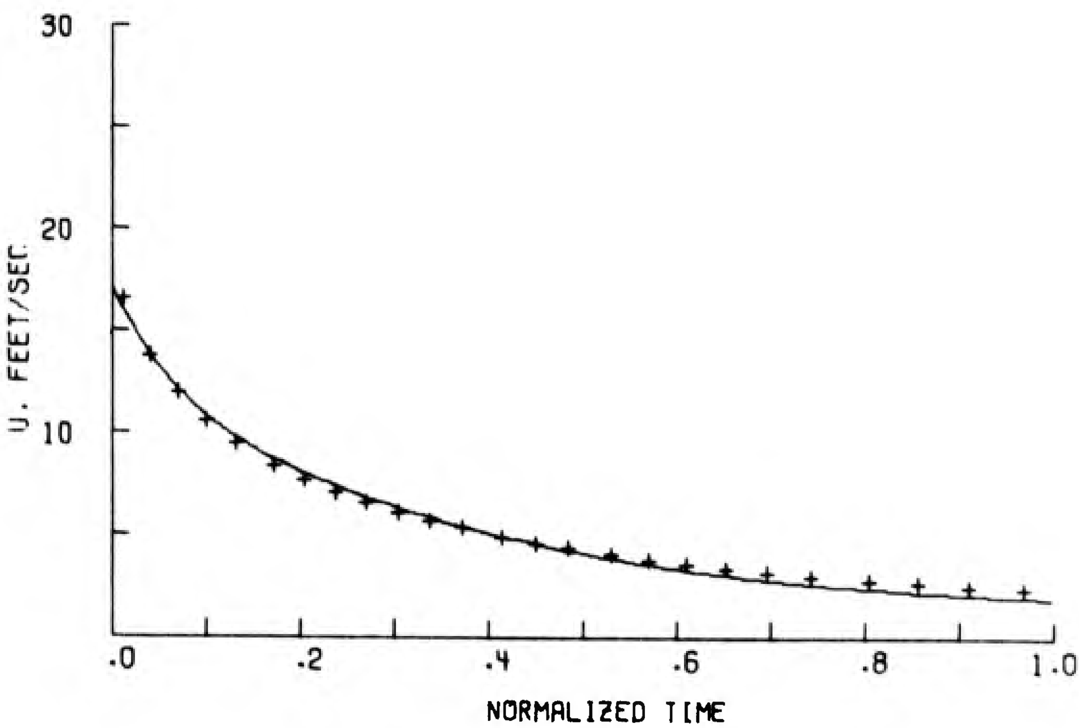
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.104762E+03
	SOUND VELOCITY =	2.108362E+03
	ZETA =	3.123440E-01

FITTING ERRORS

E_1 =	9.995330E-01	E_2 =	9.337918E-04
E_3 =	6.602570E-05	E_4 =	8.125620E-03
E_5 =	9.978579E-01	E_6 =	4.279631E-03
E_7 =	8.473019E-02	E_8 =	2.910845E-01

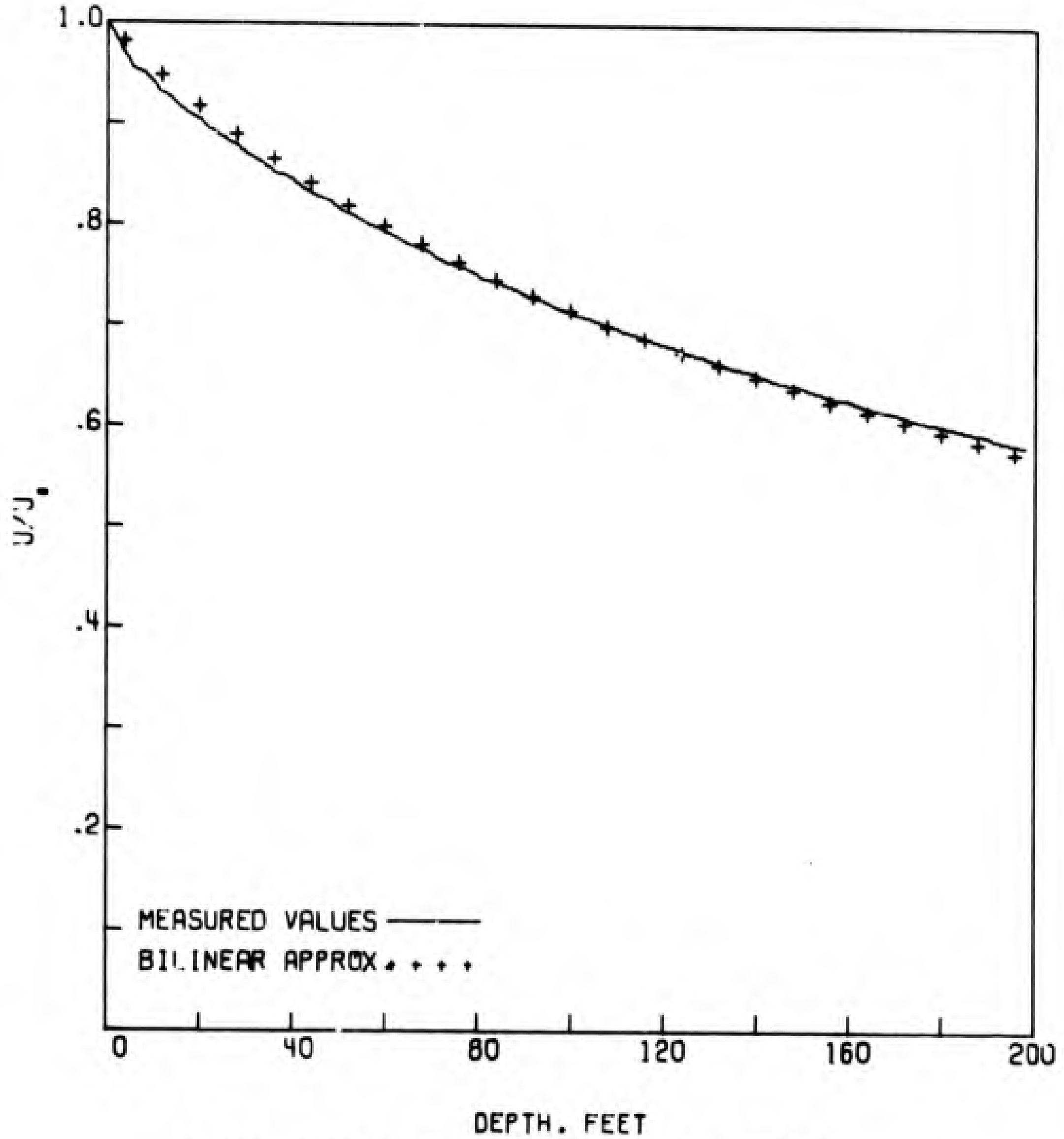
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	5.964	6.100
25	5.798	5.810
50	5.625	5.546
75	5.459	5.303
100	5.297	5.076



SURFACE PARTICLE VELOCITY

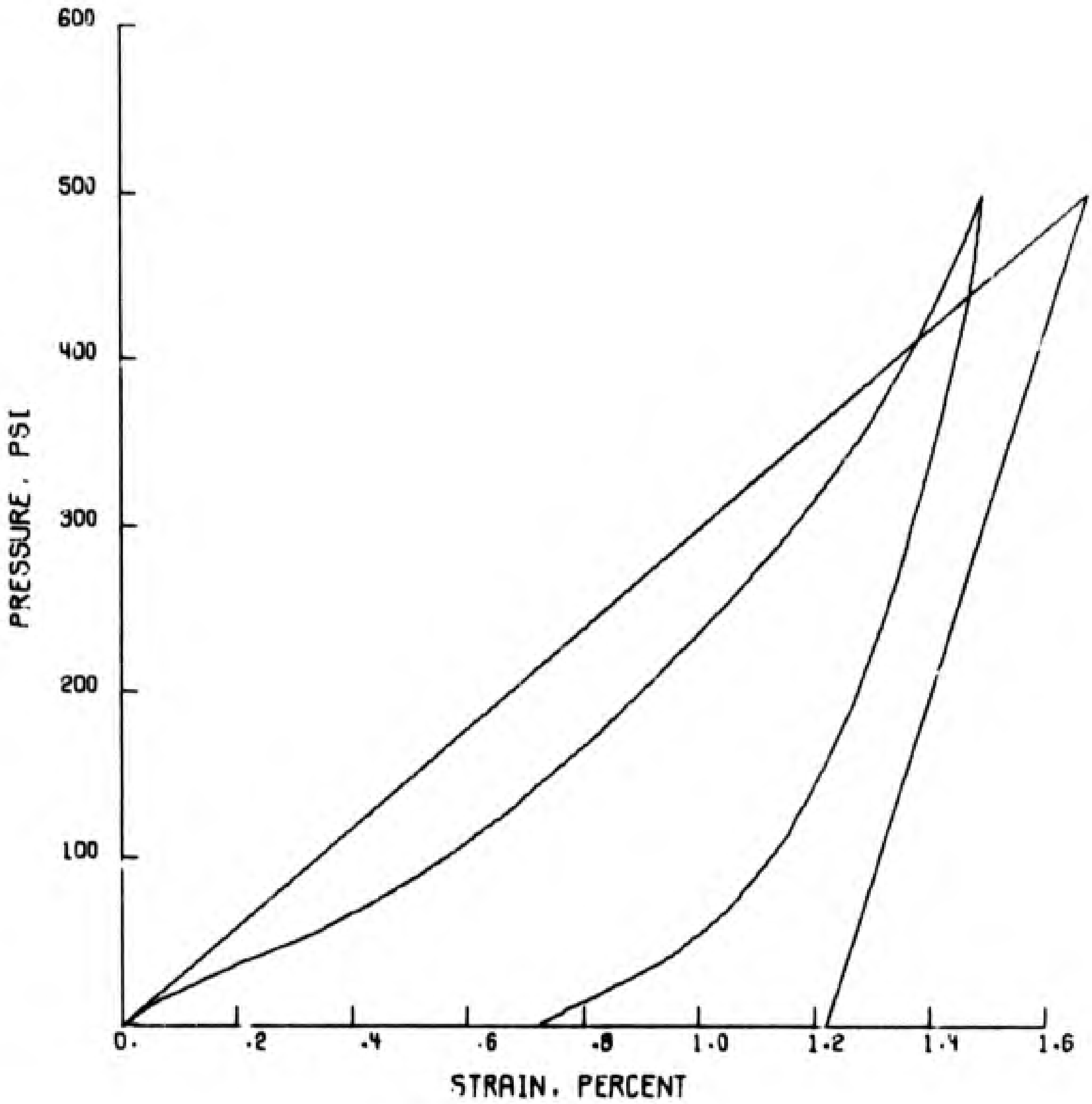
PROBLEM 2D -- 10 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 200 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 7.710905E-02 SEC.
NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 2D -- 10 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 2.974966E+04

M2(PSI) = 1.083515E+05

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04

HUGONIOT NONLINEARITY FACTOR = 2.744445E-01

MAXIMUM UNLOAD SLOPE = 1.705140E+05

ZETA = 1.954874E-01

BEST BILINEAR MODEL

PROBLEM 30 -- 10 NOV 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

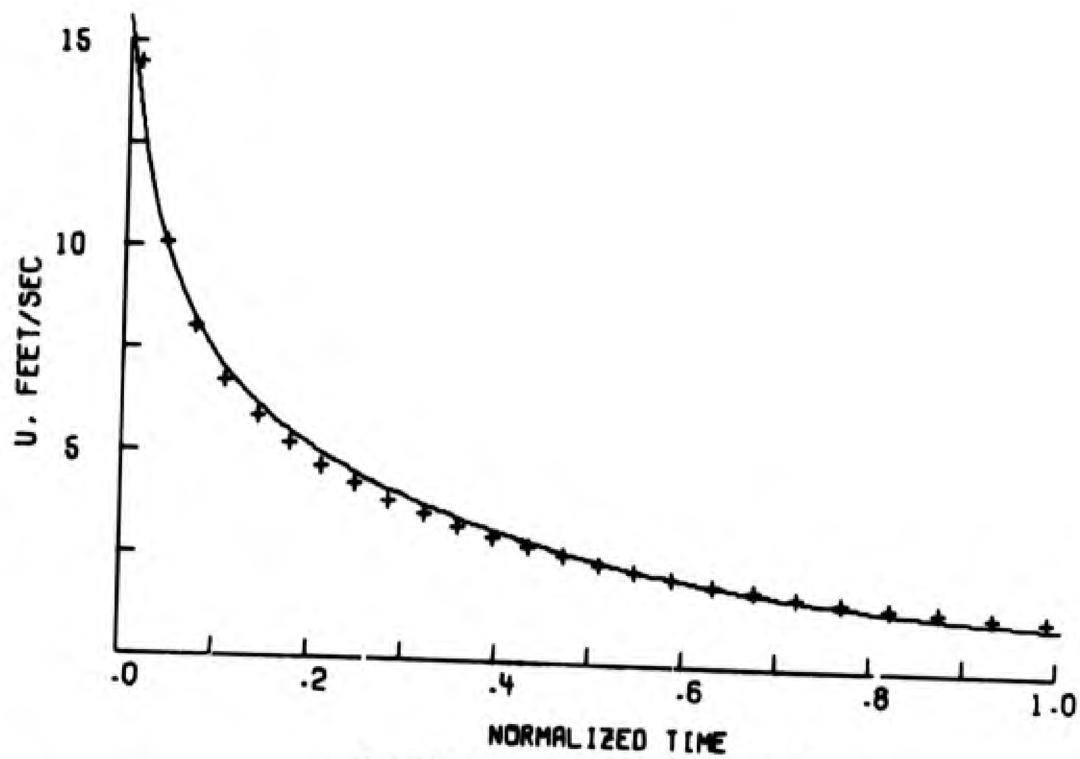
DENSITY	=	3.510000E+00
SHOCK VELOCITY	=	1.007408E+03
SOUND VELOCITY	=	1.524538E+03
ZETA	=	2.042420E-01

FITTING ERRORS

E_1 =	9.993909E-01	E_2 =	1.217794E-03
E_3 =	7.820043E-05	E_4 =	8.843101E-03
E_5 =	9.966889E-01	E_6 =	6.611307E-03
E_7 =	6.513693E-02	E_8 =	2.652174E-01

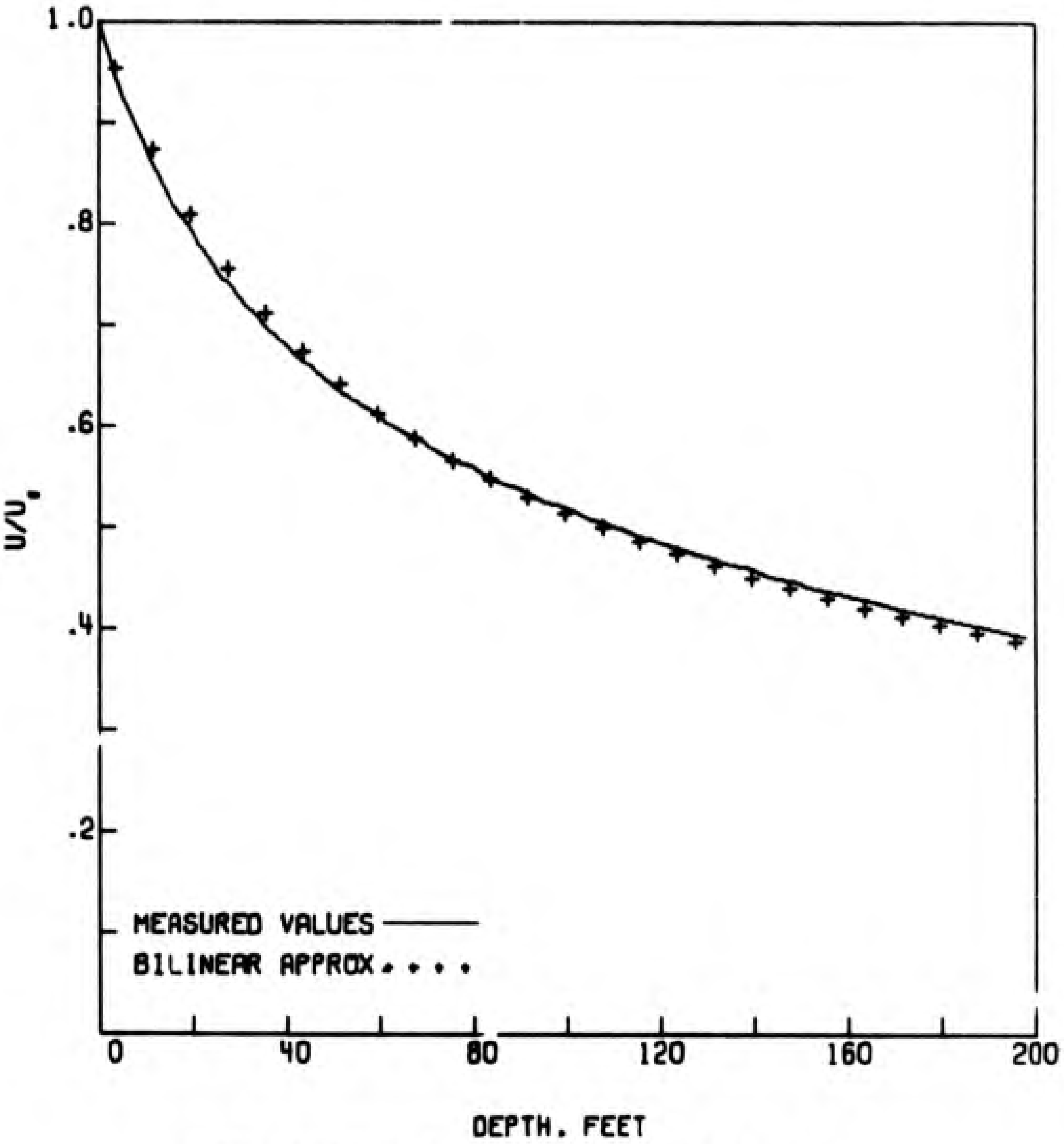
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	3.951	3.932
25	3.796	3.679
50	3.648	3.476
75	3.513	3.302
100	3.387	3.146



SURFACE PARTICLE VELOCITY

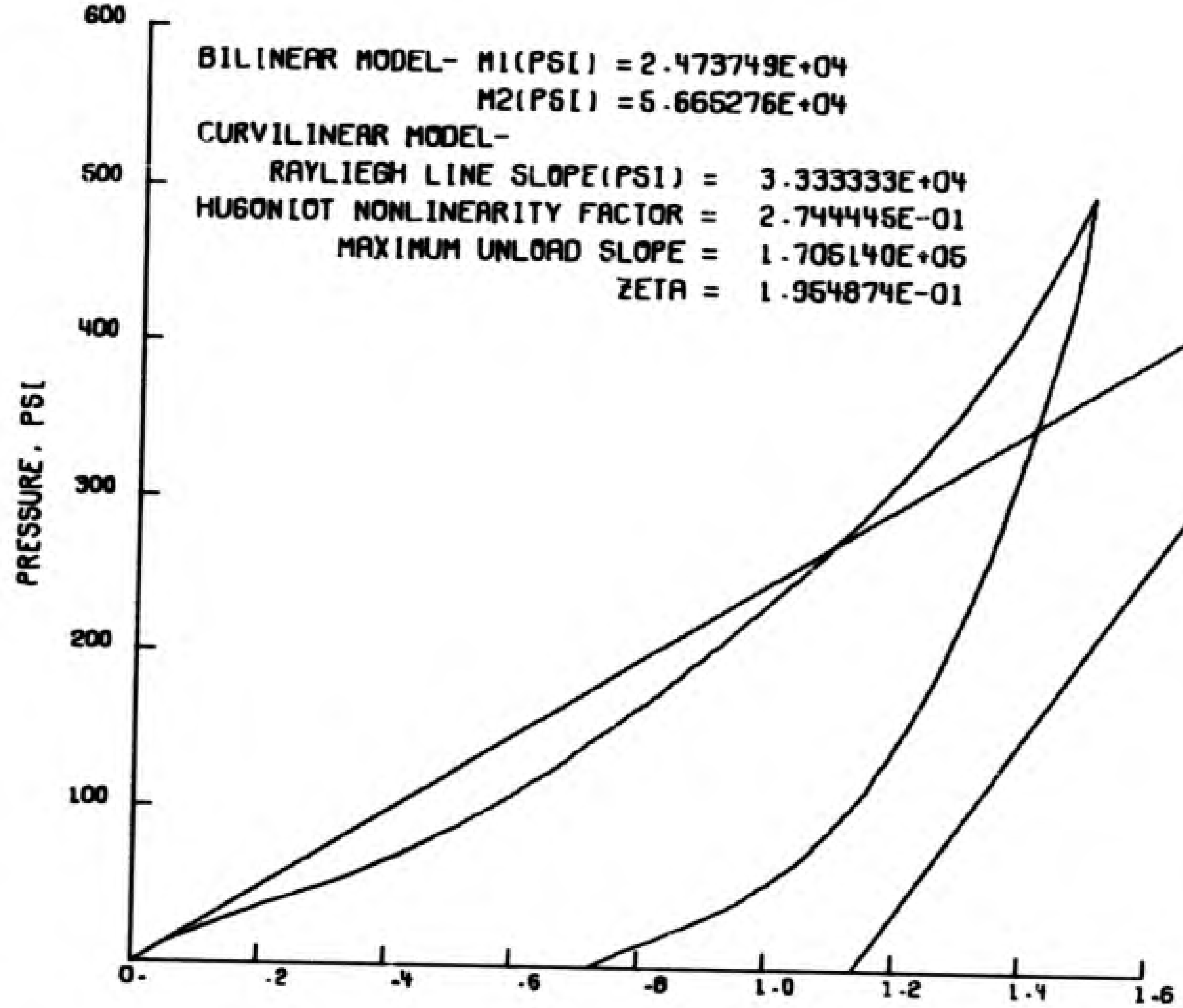
PROBLEM 30 -- 10 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 500 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 2.631964E-02 SEC.
NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 30 -- 10 NOV 1969



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 40 -- 10 NOV 1969

NUMBER OF DATA POINTS. N= 97 , M= 99

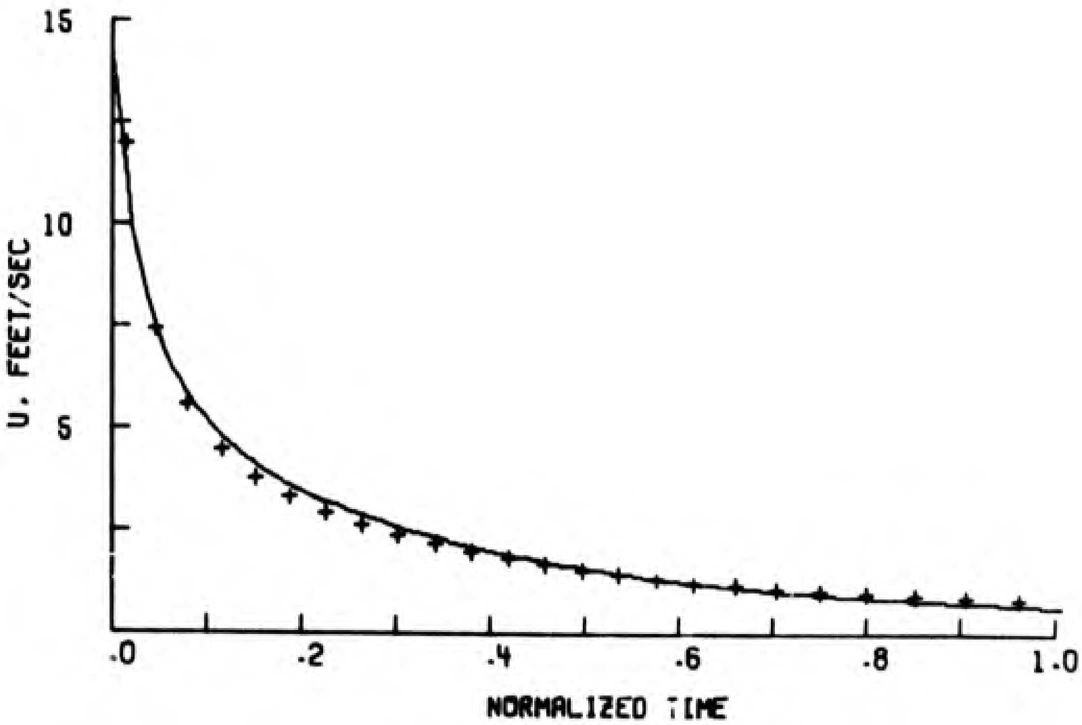
MATERIAL PROPERTIES		
	DENSITY =	3.610000E+00
	SHOCK VELOCITY =	9.717317E+02
	SOUND VELOCITY =	1.382743E+03
	ZETA =	1.745660E-01

FITTING ERRORS

E_1 =	9.989128E-01	E_2 =	2.173225E-03
E_3 =	1.038061E-04	E_4 =	1.018853E-02
E_5 =	9.967002E-01	E_6 =	6.588796E-03
E_7 =	4.376164E-02	E_8 =	2.091926E-01

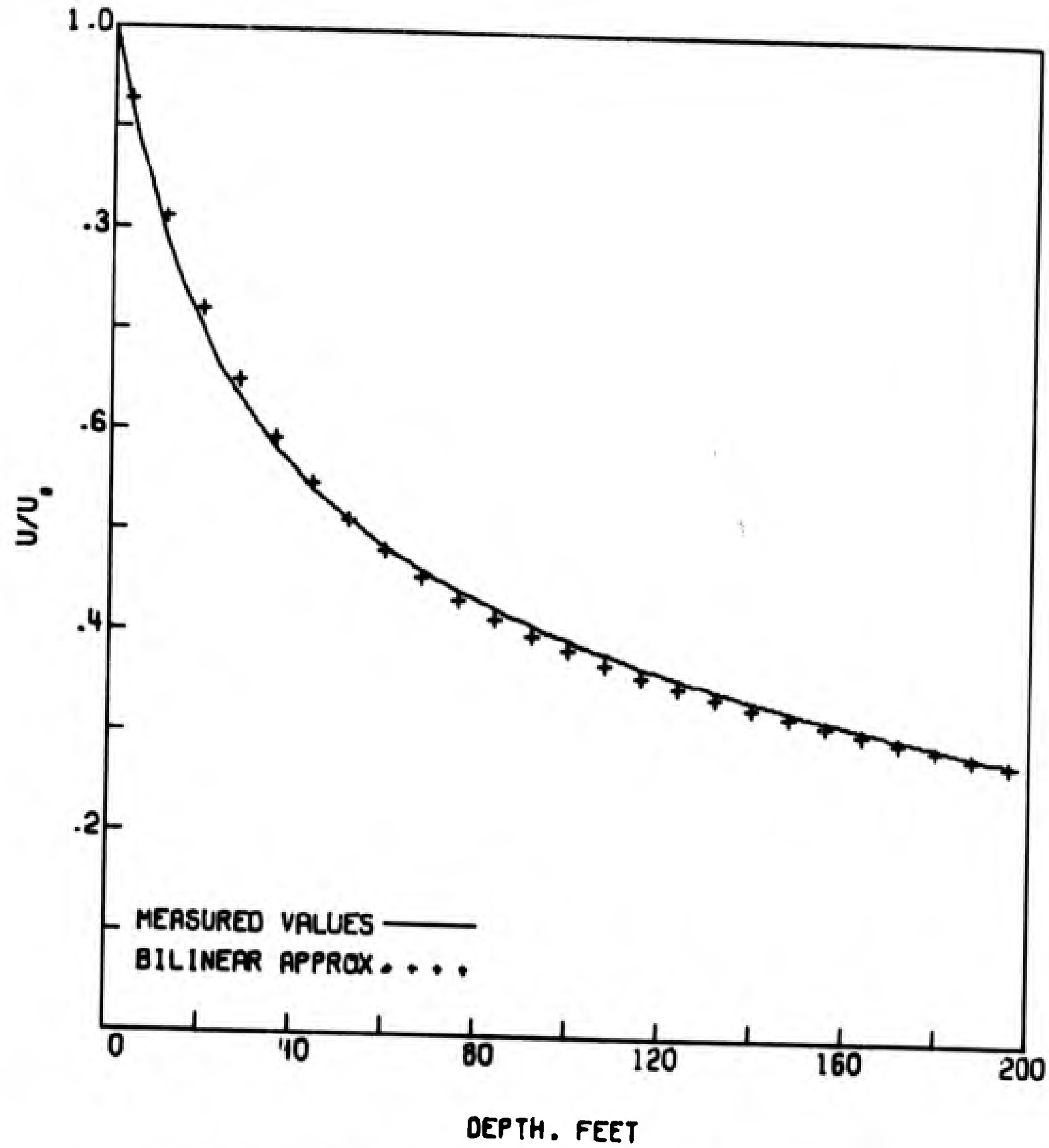
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	2.709	2.706
25	2.565	2.476
50	2.439	2.311
75	2.328	2.179
100	2.228	2.065



SURFACE PARTICLE VELOCITY

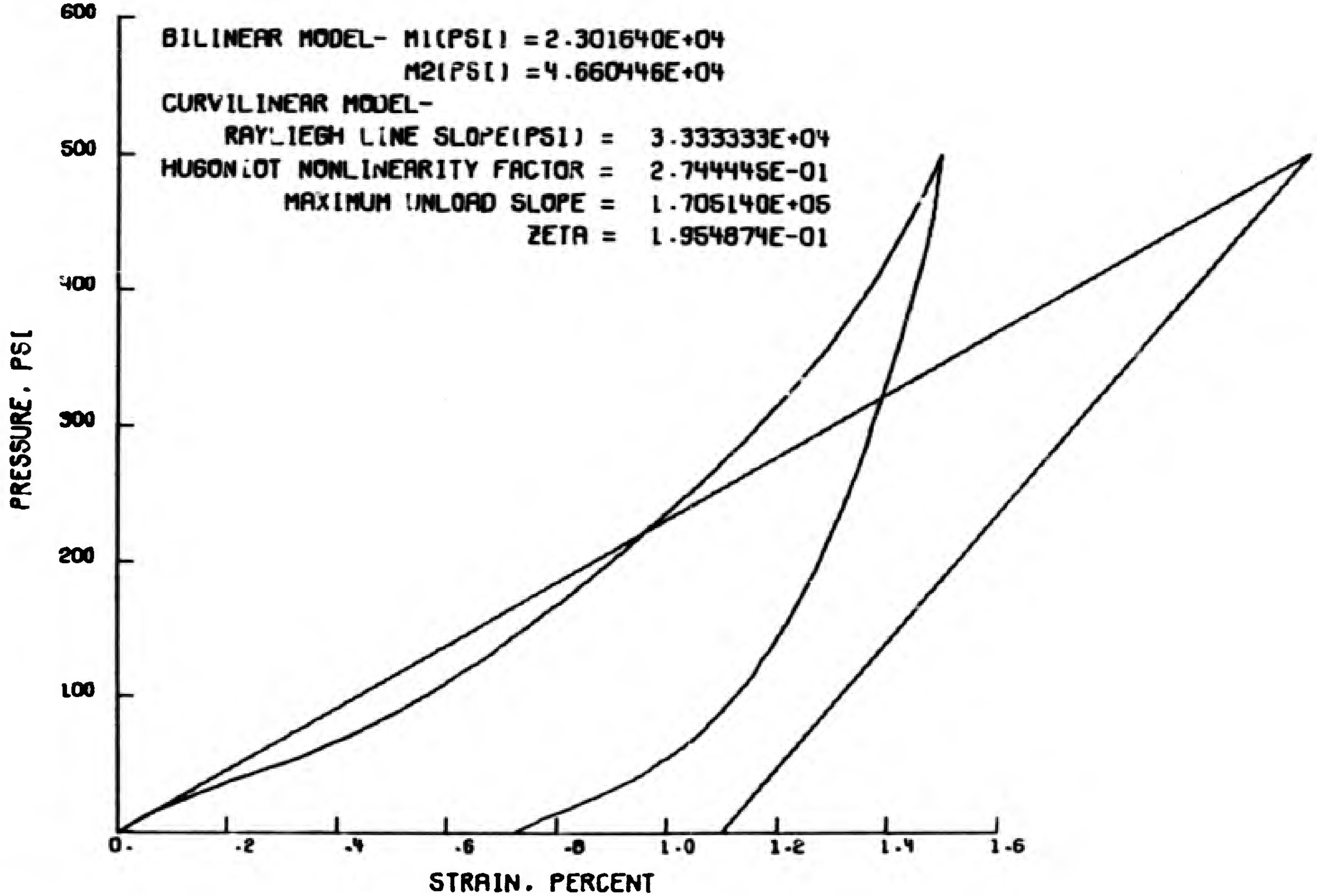
PROBLEM 40 -- 10 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 1000 PSI BRODE FORM PMAX=500 PSI TO=1.09
HALF LOAD TIME = 1.286354E-02 SEC.
NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 40 -- 10 NOV 1969



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 50 -- 10 NOV 1969

NUMBER OF DATA POINTS. N= 97 . M= 99

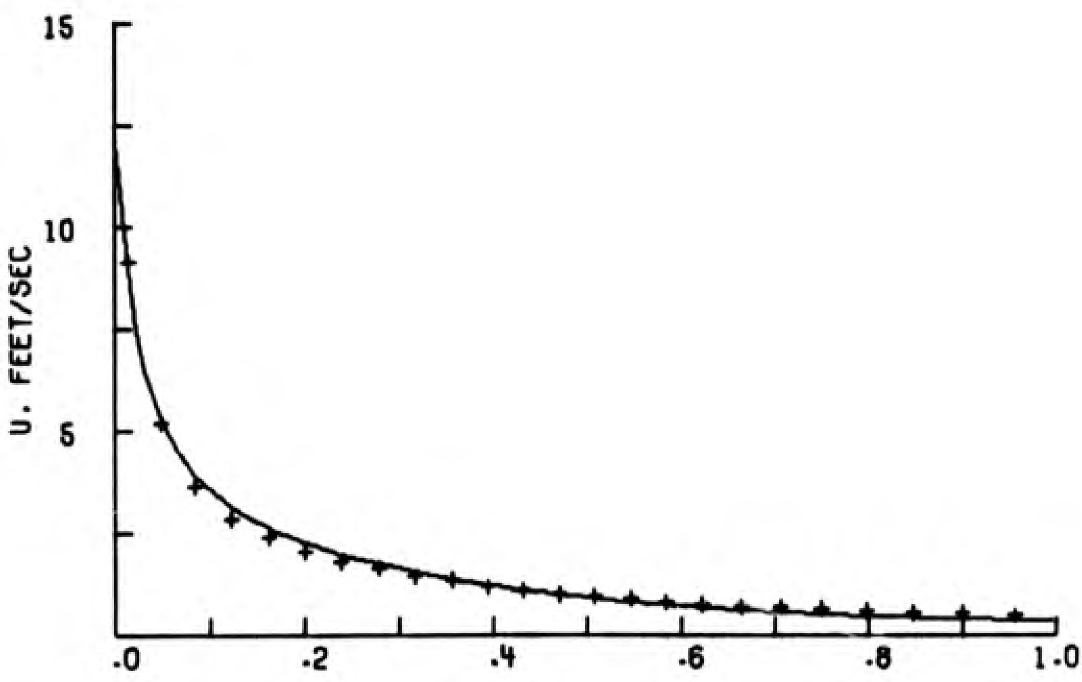
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	9.431348E+02
	SOUND VELOCITY =	1.286191E+03
	ZETA =	1.538526E-01

FITTING ERRORS

E_1 =	9.980984E-01	E_2 =	3.799611E-03
E_3 =	1.329663E-04	E_4 =	1.163106E-02
E_5 =	9.966453E-01	E_6 =	6.698065E-03
E_7 =	2.648354E-02	E_8 =	1.596356E-01

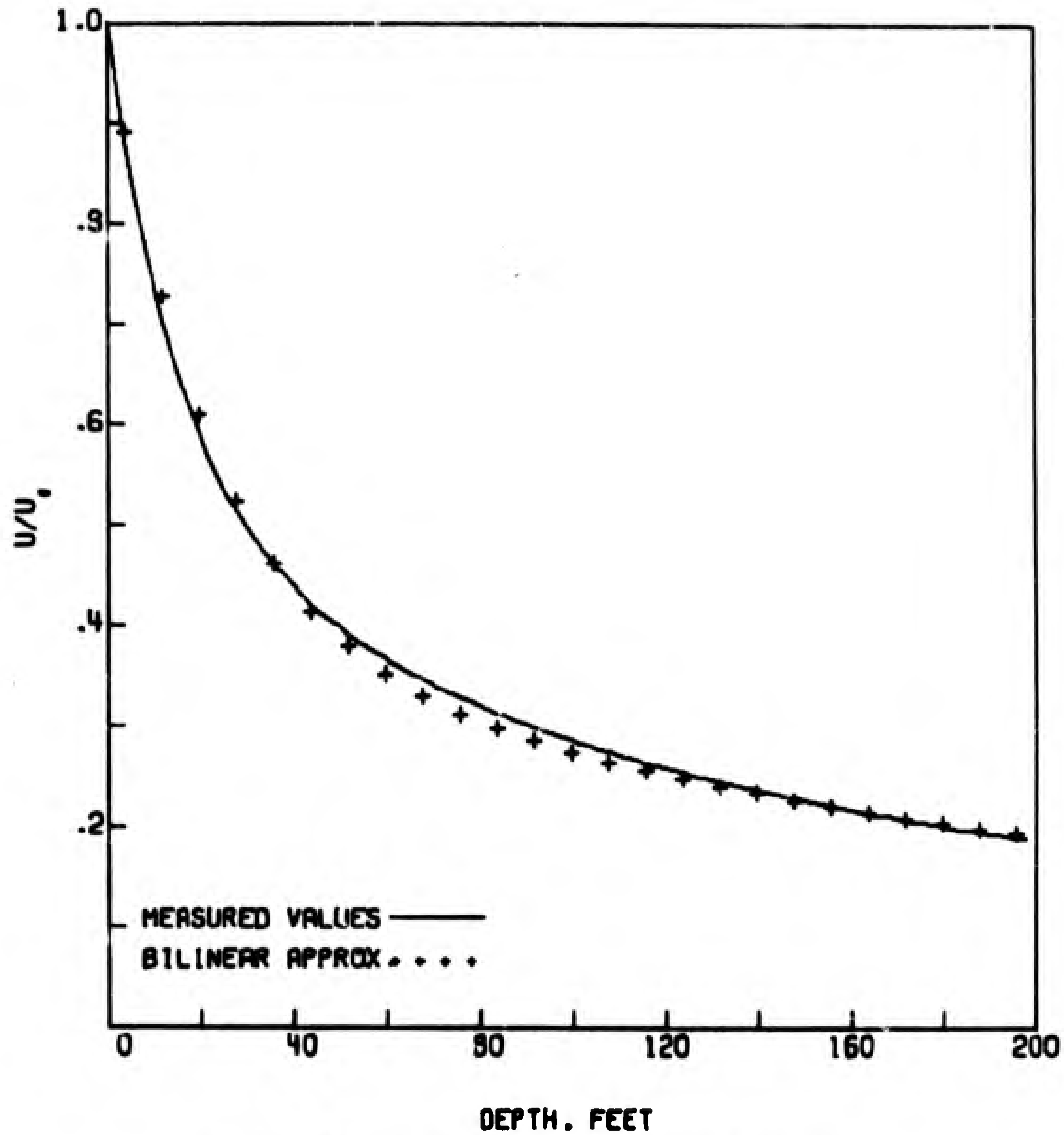
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	1.780	1.838
25	1.652	1.628
50	1.560	1.502
75	1.465	1.408
100	1.390	1.328



SURFACE PARTICLE VELOCITY

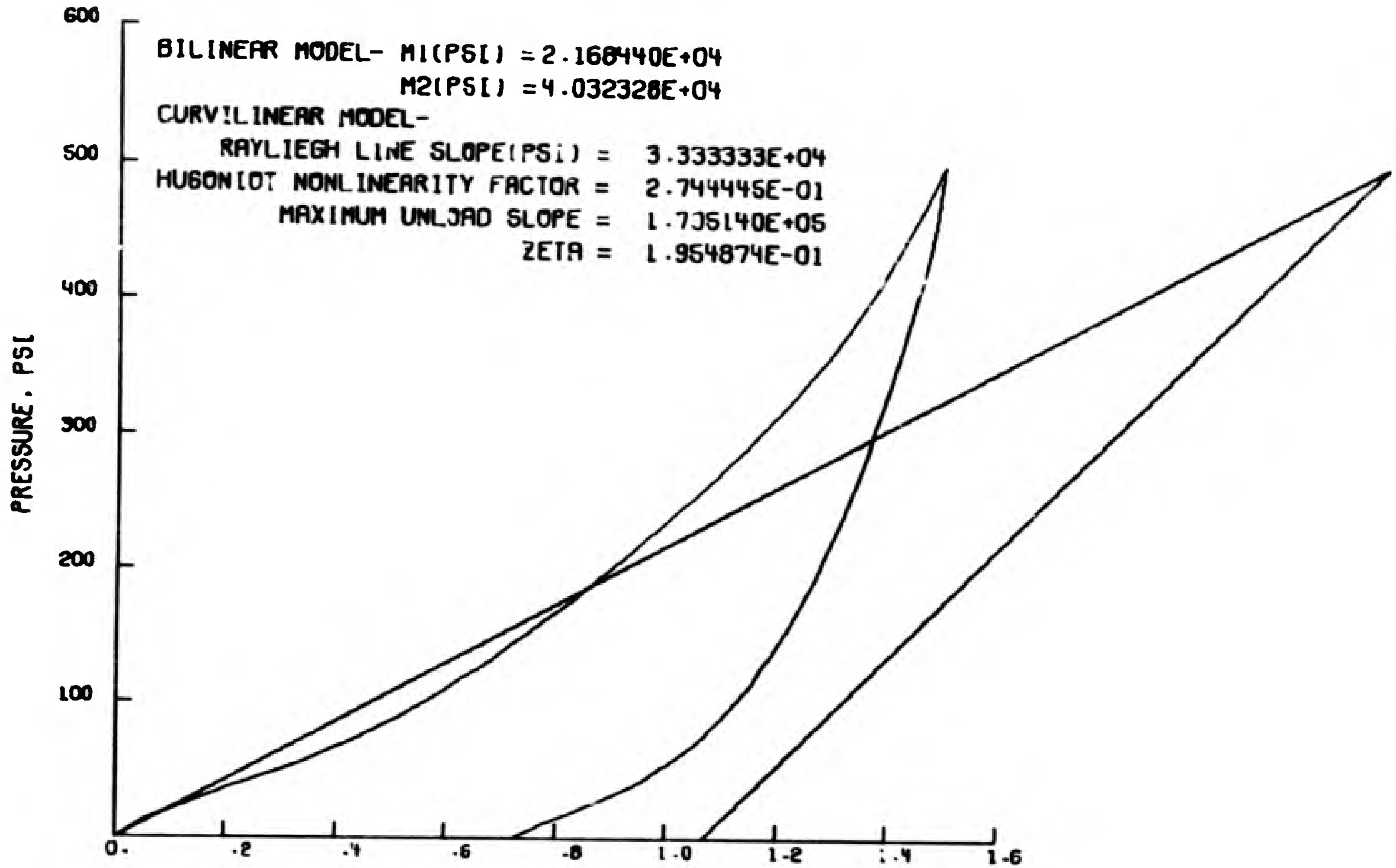
PROBLEM 50 -- 10 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 2000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 3.531116E-03 SEC.
NORMALIZED HALF LOAD TIME = 3.239556E-03

PROBLEM 5D -- 10 NOV 1969



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 1E -- 17 NOV 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

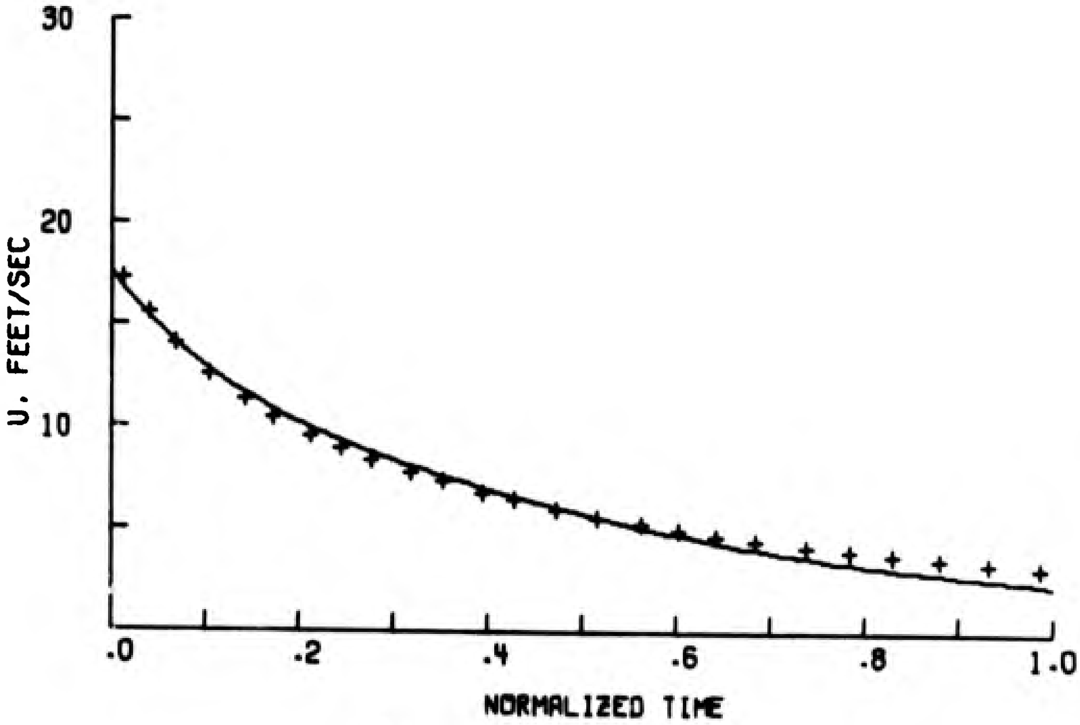
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.120814E+03
	SOUND VELOCITY =	2.270995E+03
	ZETA =	3.391055E-01

FITTING ERRORS

E_1 =	9.992111E-01	E_2 =	1.577078E-03
E_3 =	4.185746E-05	E_4 =	6.469734E-03
E_5 =	9.967594E-01	E_6 =	6.470691E-03
E_7 =	1.638394E-01	E_8 =	4.047708E-01

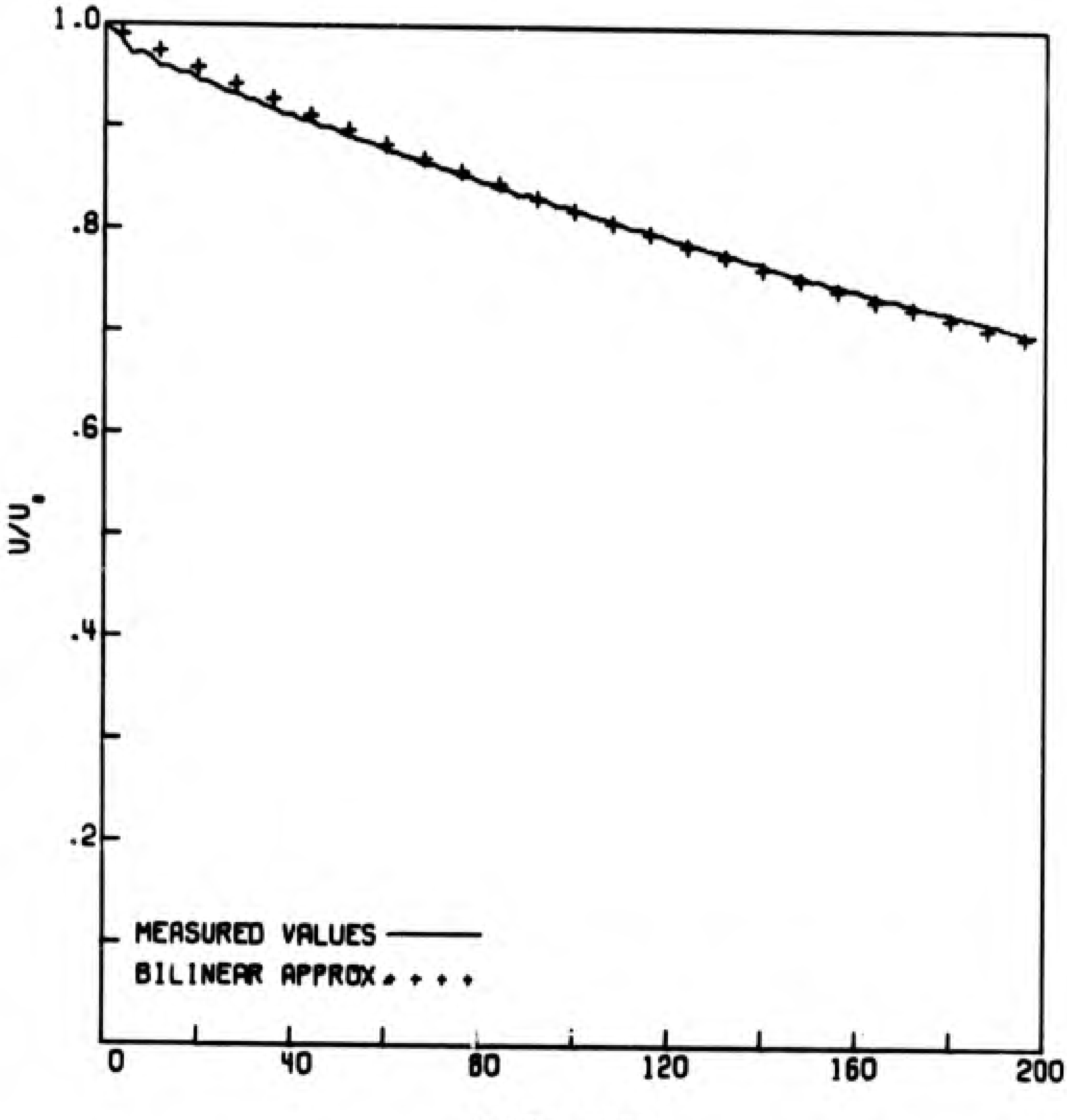
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	7.558	7.716
25	7.387	7.414
50	7.203	7.128
75	7.019	6.855
100	6.836	6.594



SURFACE PARTICLE VELOCITY

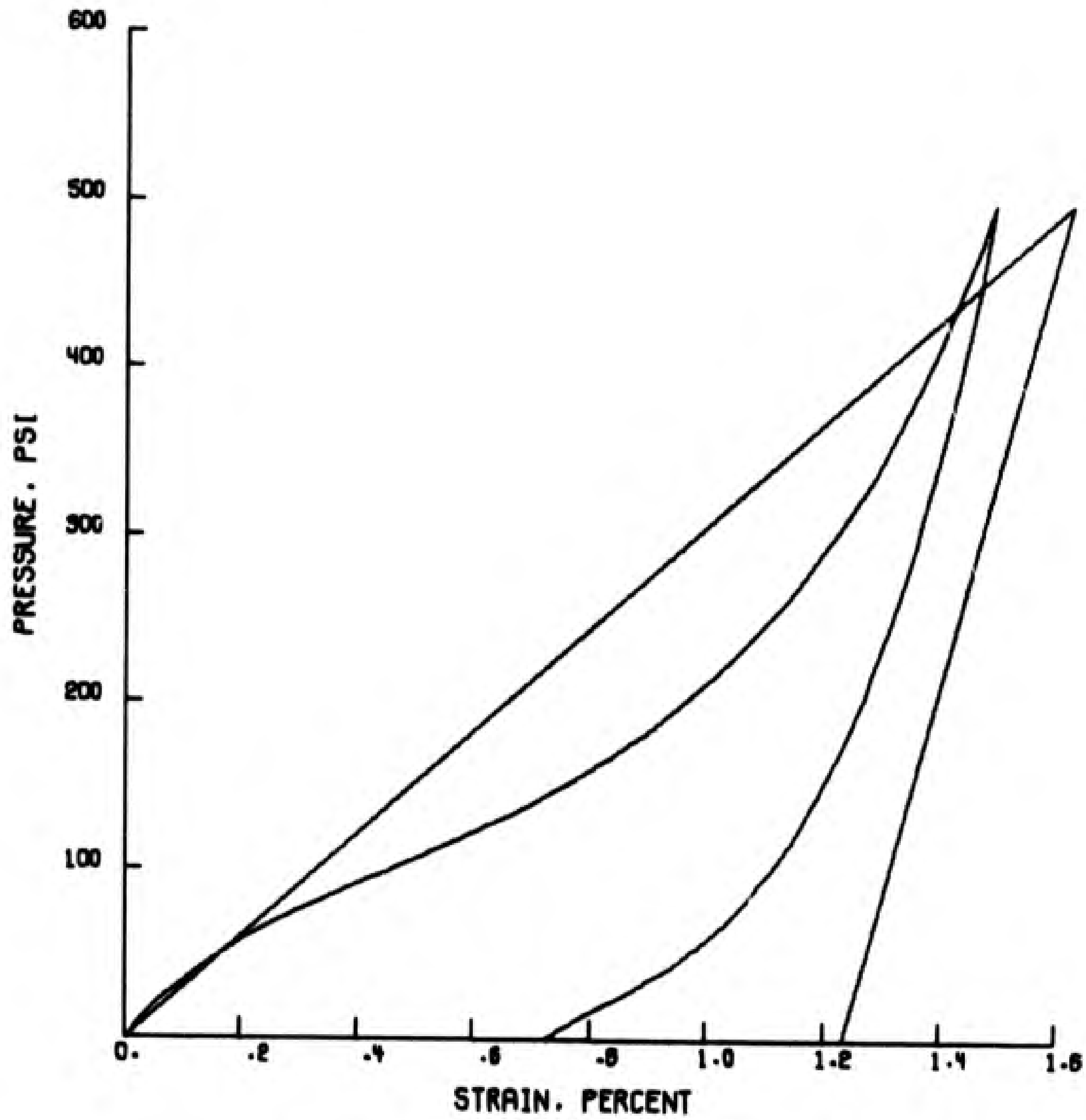
PROBLEM 1E -- 17 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 100 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.304202E-01 SEC.
NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 1E -- 17 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =3.062046E+04
M2(PSI) =1.257121E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04
HUGONIOT NONLINEARITY FACTOR = 2.744967E-01
MAXIMUM UNLOAD SLOPE = 1.723837E+05
ZETA = 1.933670E-01

BEST BILINEAR MODEL

PROBLEM 2E -- 17 NOV 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

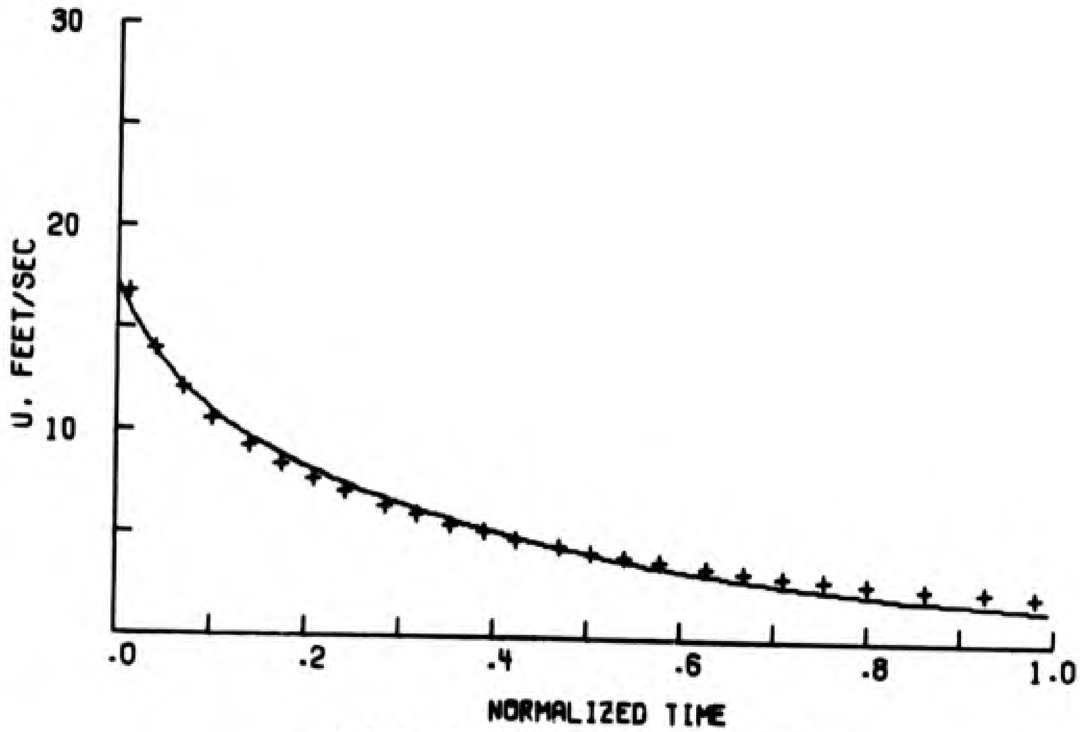
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.088937E+03
SOUND VELOCITY =	1.987890E+03
ZETA =	2.921690E-01

FITTING ERRORS

E_1 =	9.995863E-01	E_2 =	8.272785E-04
E_3 =	1.669963E-05	E_4 =	4.086519E-03
E_5 =	9.956962E-01	E_6 =	8.589148E-03
E_7 =	1.710408E-01	E_8 =	4.135708E-01

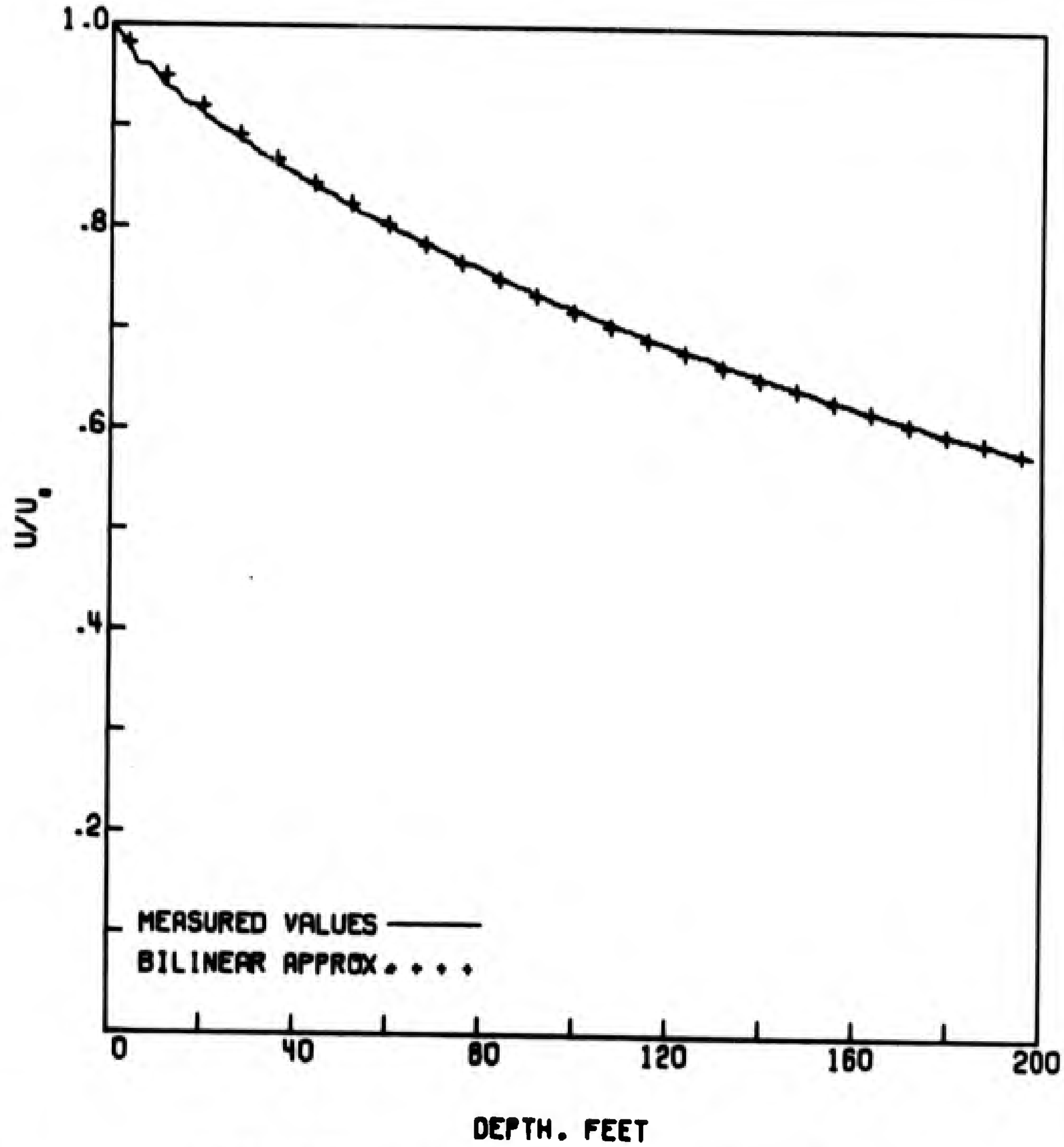
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	5.940	6.117
25	5.772	5.829
50	5.594	5.566
75	5.422	5.324
100	5.253	5.097



SURFACE PARTICLE VELOCITY

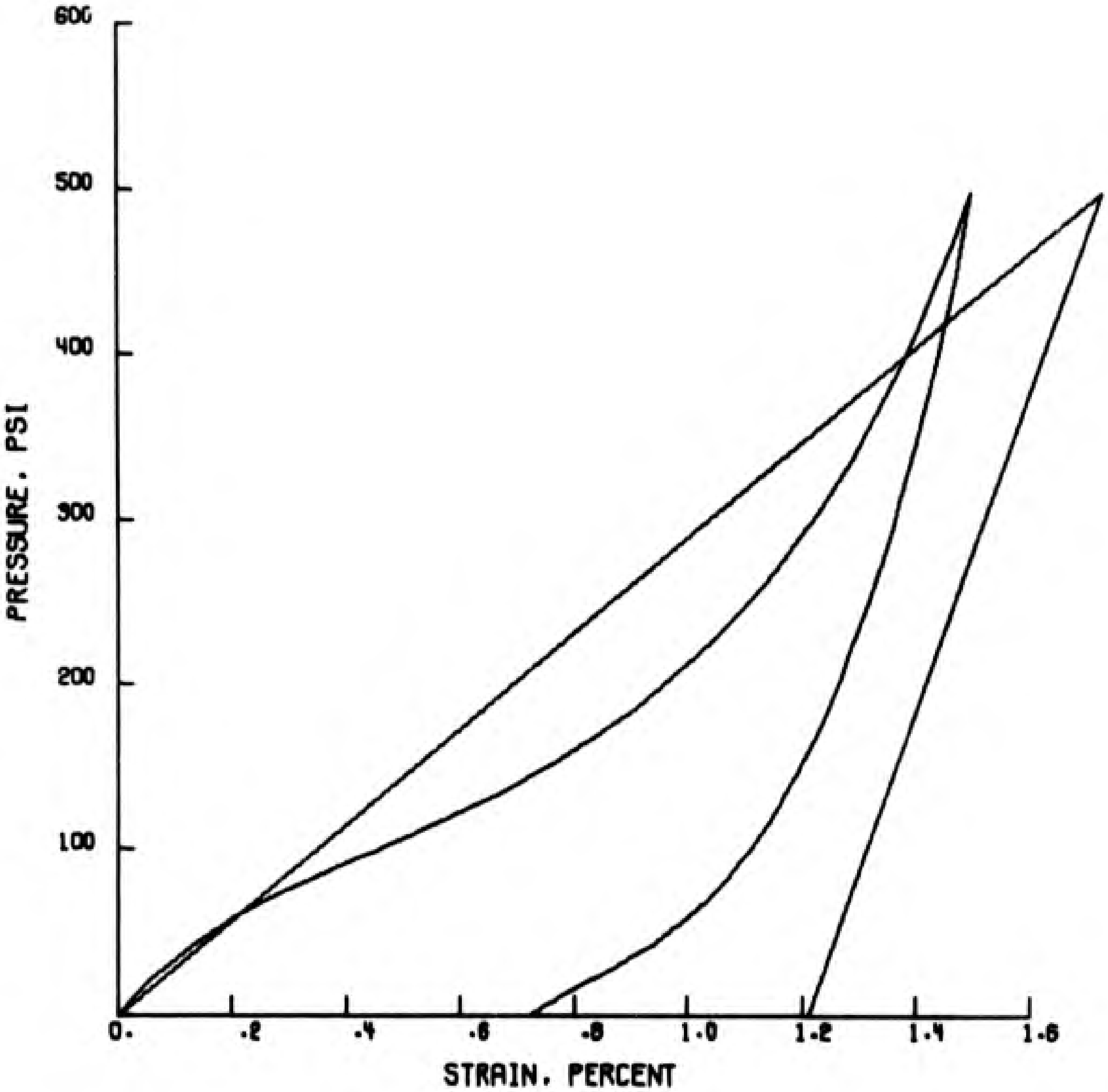
PROBLEM 2E -- 17 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 200 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 7.710905E-02 SEC.
NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 2E -- 17 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =2.890347E+04
M2(PSI) =9.632287E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04
HUGONIOT NONLINEARITY FACTOR = 2.744967E-01
MAXIMUM UNLOAD SLOPE = 1.723837E+05
ZETA = 1.933670E-01

BEST BILINEAR MODEL

PROBLEM 3E 24 NOV 1969

NUMBER OF DATA POINTS. N= 98 . M= 99

MATERIAL PROPERTIES

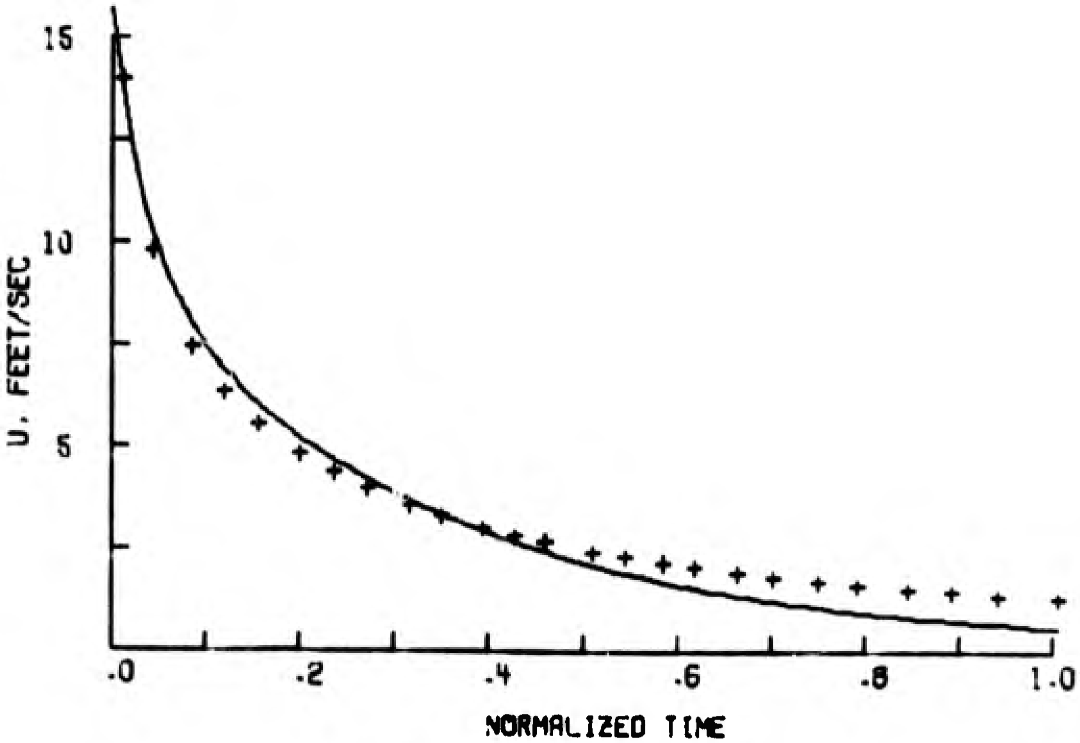
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.064003E+03
SOUND VELOCITY =	1.811803E+03
ZETA =	2.600315E-01

FITTING ERRORS

E_1 =	9.994466E-01	E_2 =	1.106736E-03
E_3 =	5.126685E-05	E_4 =	7.159319E-03
E_5 =	9.936373E-01	E_6 =	1.268495E-02
E_7 =	2.449184E-01	E_8 =	4.948924E-01

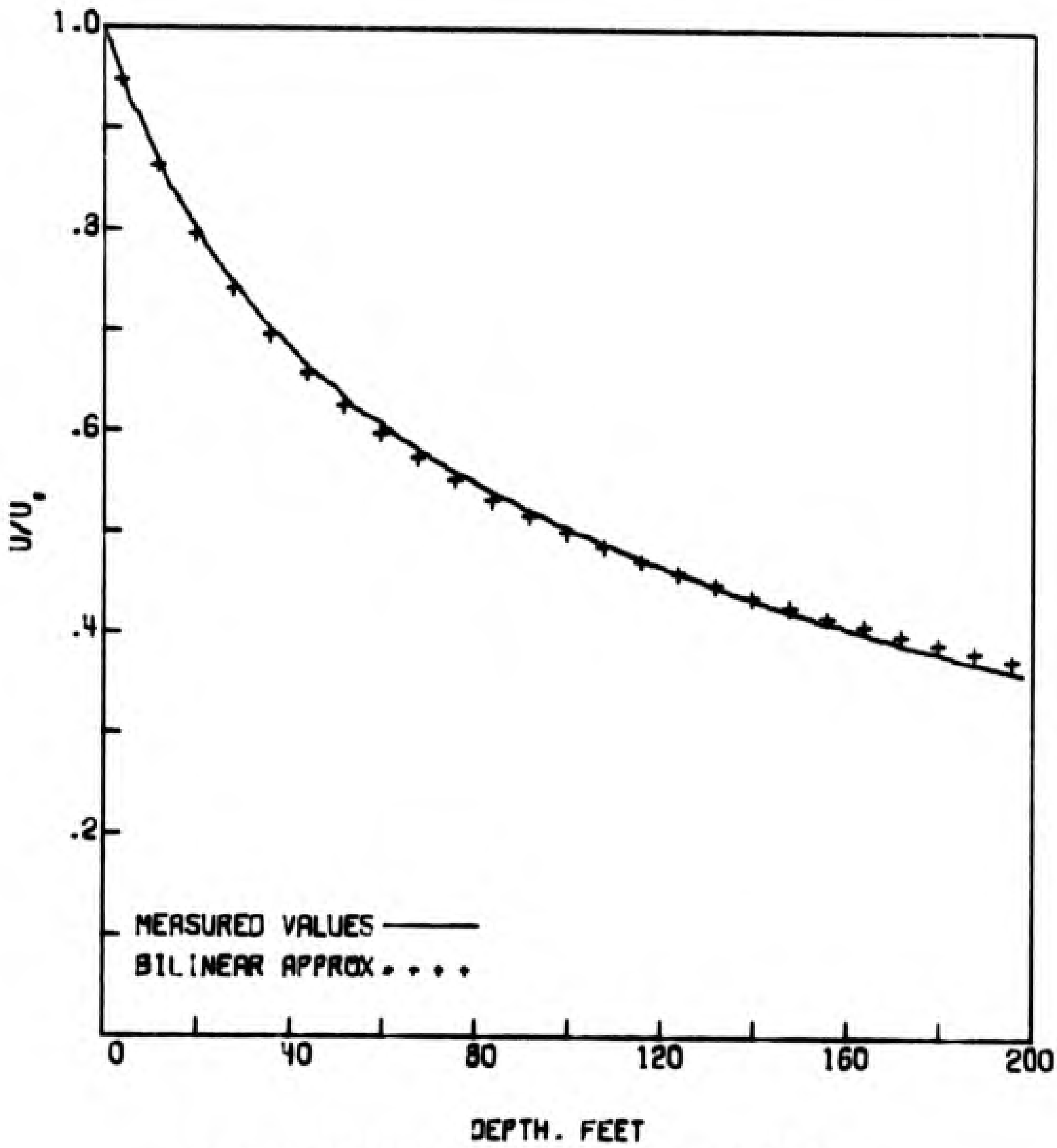
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	3.623	3.905
25	3.465	3.644
50	3.311	3.437
75	3.170	3.261
100	3.040	3.103



SURFACE PARTICLE VELOCITY

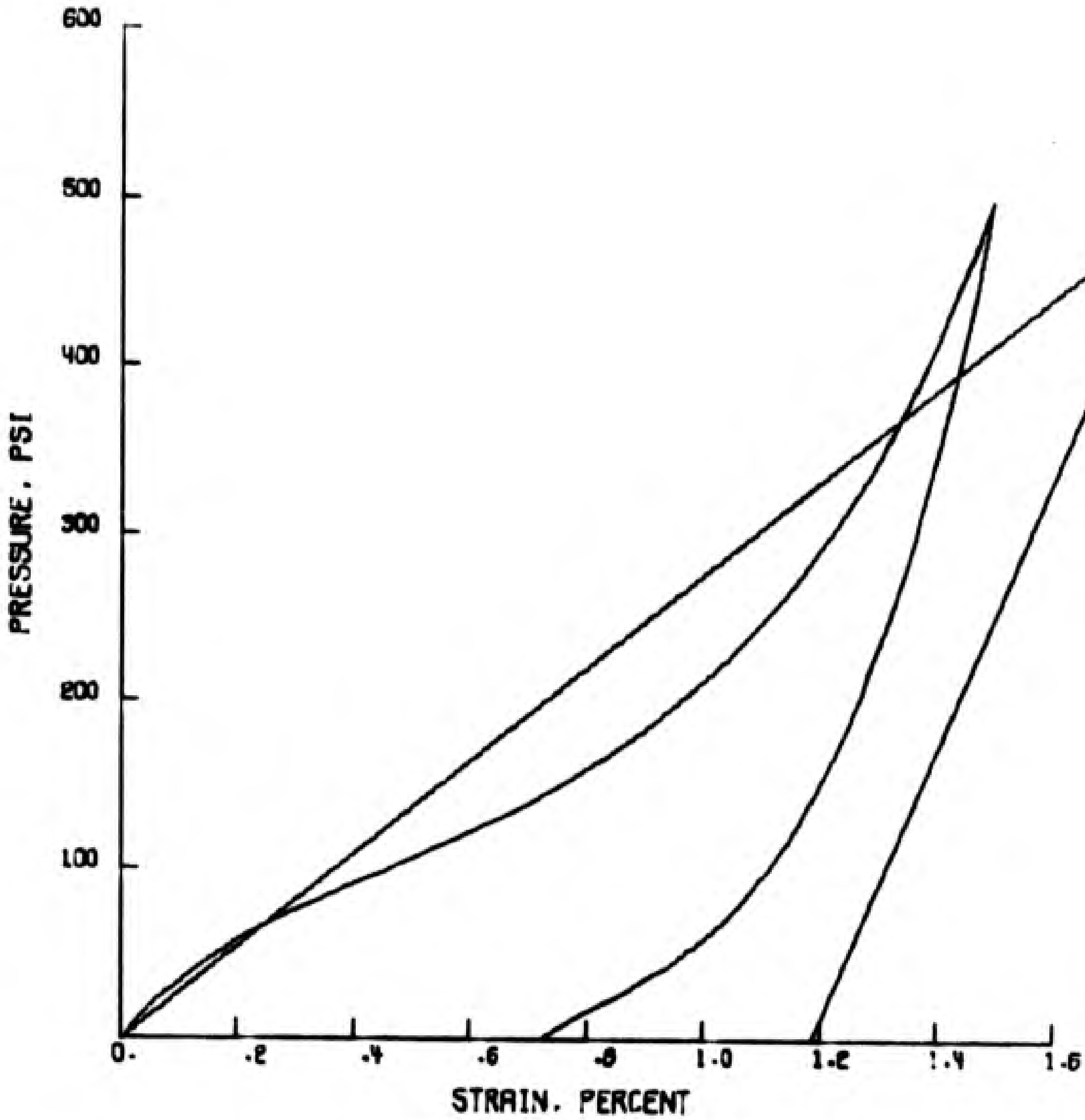
PROBLEM 3E 24 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 500 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 2.531964E-02 SEC.
NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 3E 24 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 2.759499E+04

M2(PSI) = 8.001411E+04

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04

HUGONIOT NONLINEARITY FACTOR = 2.744967E-01

MAXIMUM UNLOAD SLOPE = 1.723837E+05

ZETA = 1.933670E-01

BEST BILINEAR MODEL

PROBLEM 4E -- 17 NOV 1969

NUMBER OF DATA POINTS. N= 97 . M= 99

MATERIAL PROPERTIES

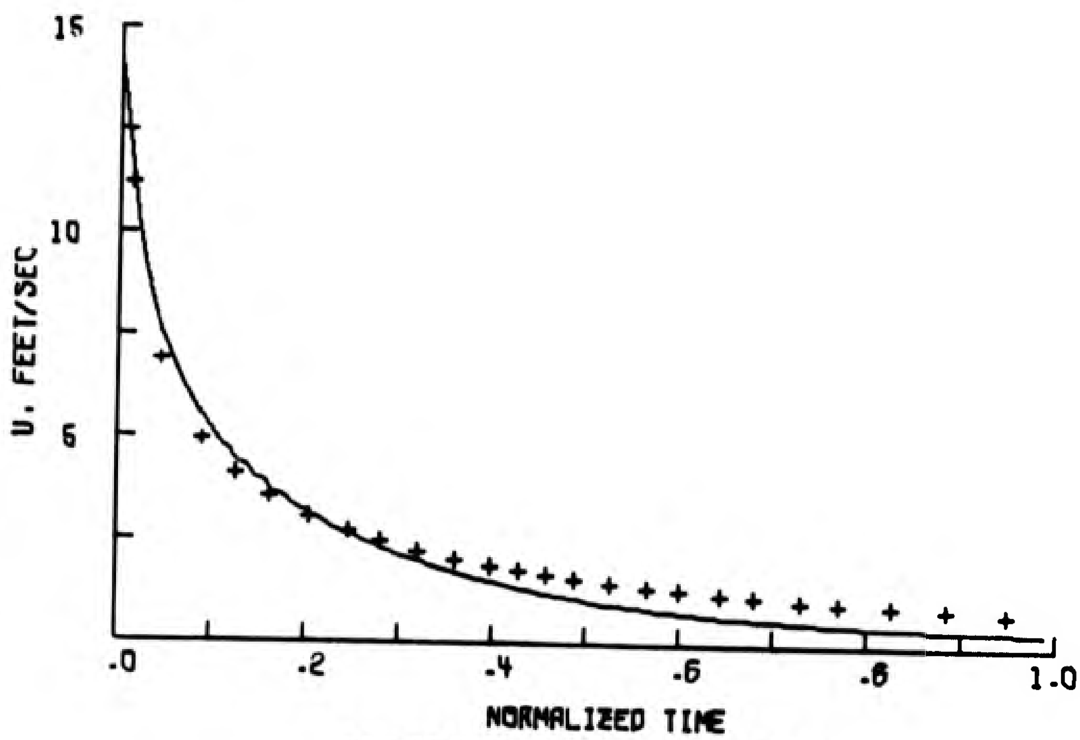
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.109801E+03
SOUND VELOCITY =	2.085388E+03
ZETA =	3.053300E-01

FITTING ERRORS

E_1 =	9.992981E-01	E_2 =	1.403382E-03
E_3 =	1.972538E-04	E_4 =	1.404471E-02
E_5 =	9.968892E-01	E_6 =	6.809979E-03
E_7 =	2.205226E-01	E_8 =	4.695984E-01

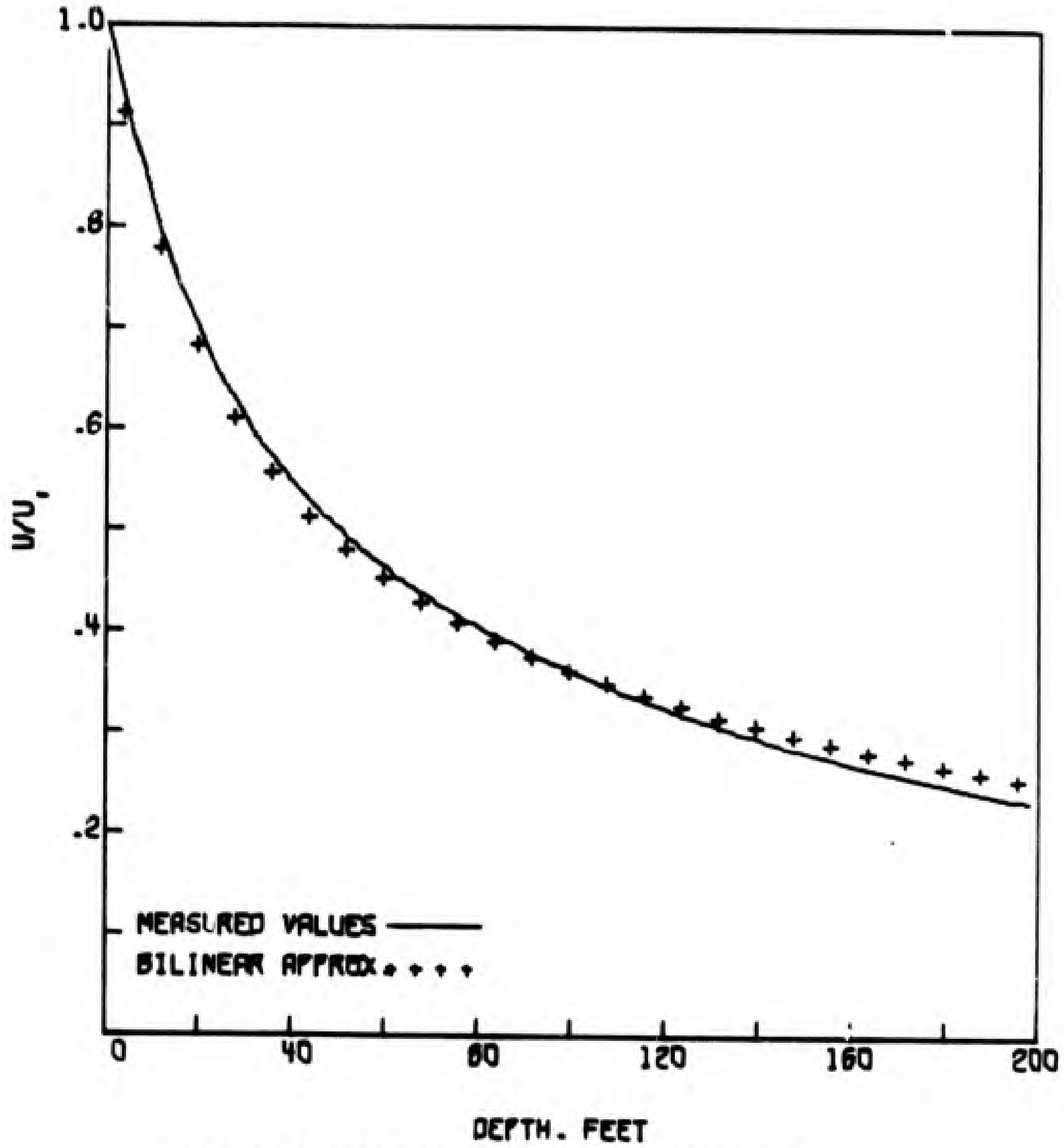
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	2.301	2.643
25	2.153	2.402
50	2.022	2.236
75	1.912	2.102
100	1.817	1.986



SURFACE PARTICLE VELOCITY

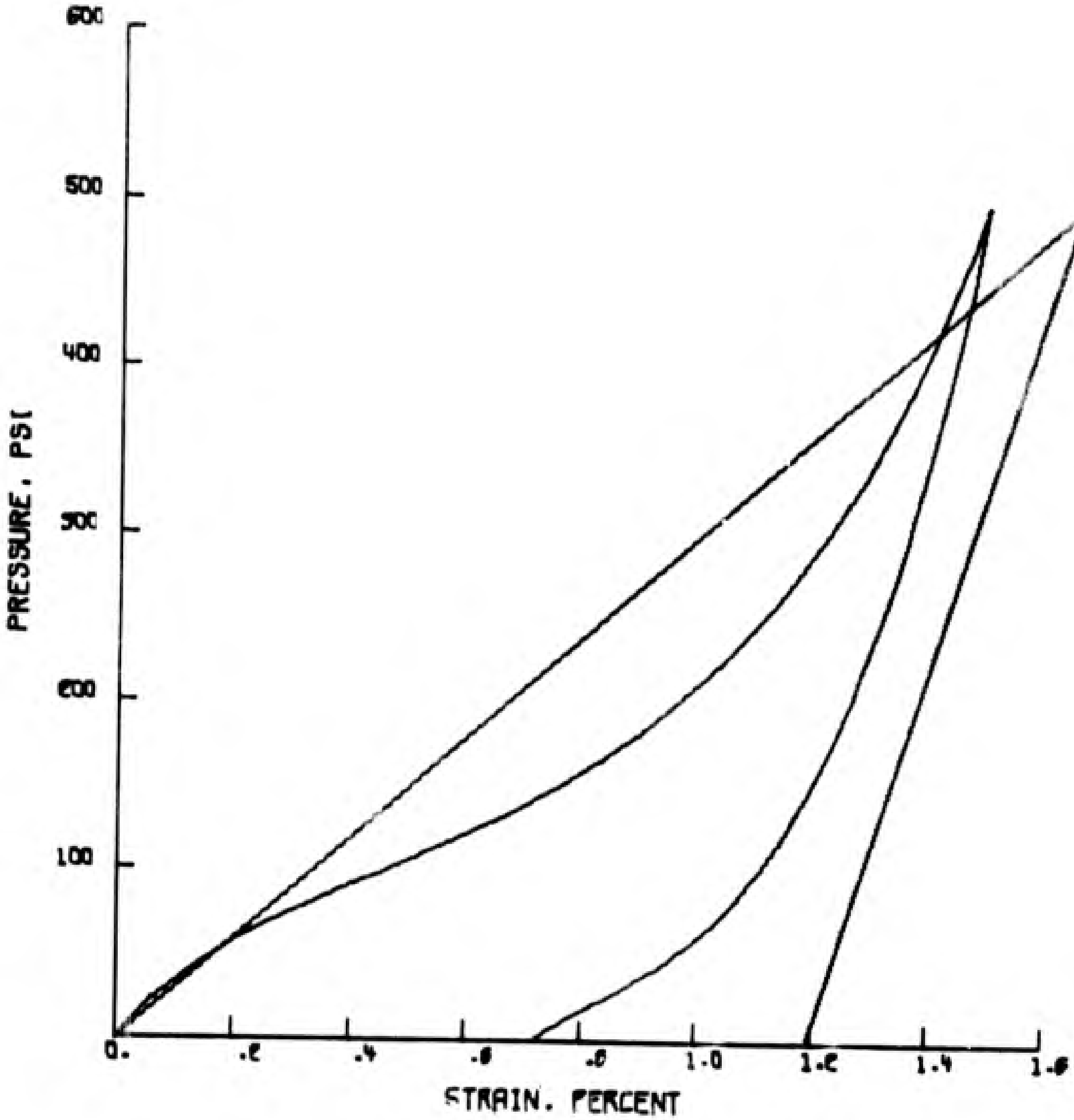
PROBLEM 4E -- 17 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 1000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.286364E-02 SEC.
NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 4E -- 17 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =3.002166E+04

M2(PSI) =1.060030E+06

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04

HUGONIOT NONLINEARITY FACTOR = 2.744967E-01

MAXIMUM UNLOAD SLOPE = 1.729837E+06

ZETA = 1.939670E-01

BEST BILINEAR MODEL

PROBLEM 5E -- 17 NOV 1969

NUMBER OF DATA POINTS. N= 97 . M= 99

MATERIAL PROPERTIES

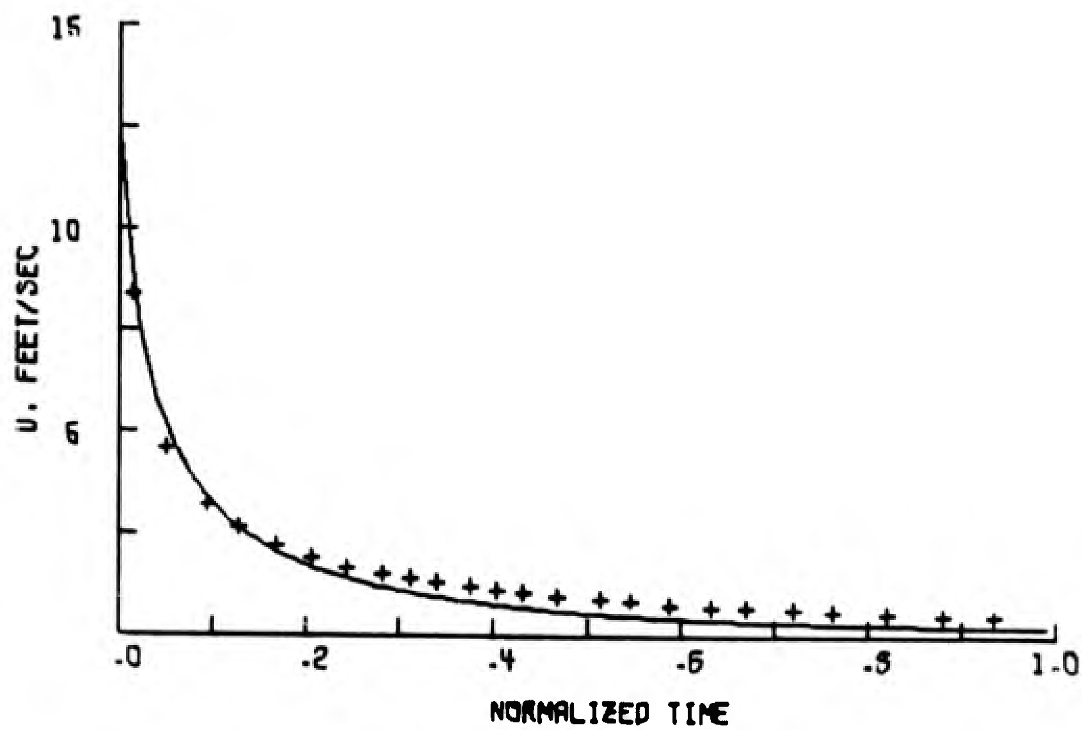
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.151181E+03
	SOUND VELOCITY =	2.355486E+03
	ZETA =	3.434330E-01

FITTING ERRORS

E_1 =	9.998278E-01	E_2 =	3.444376E-04
E_3 =	2.306278E-04	E_4 =	1.518643E-02
E_5 =	9.981416E-01	E_6 =	3.713270E-03
E_7 =	1.358730E-01	E_8 =	3.686095E-01

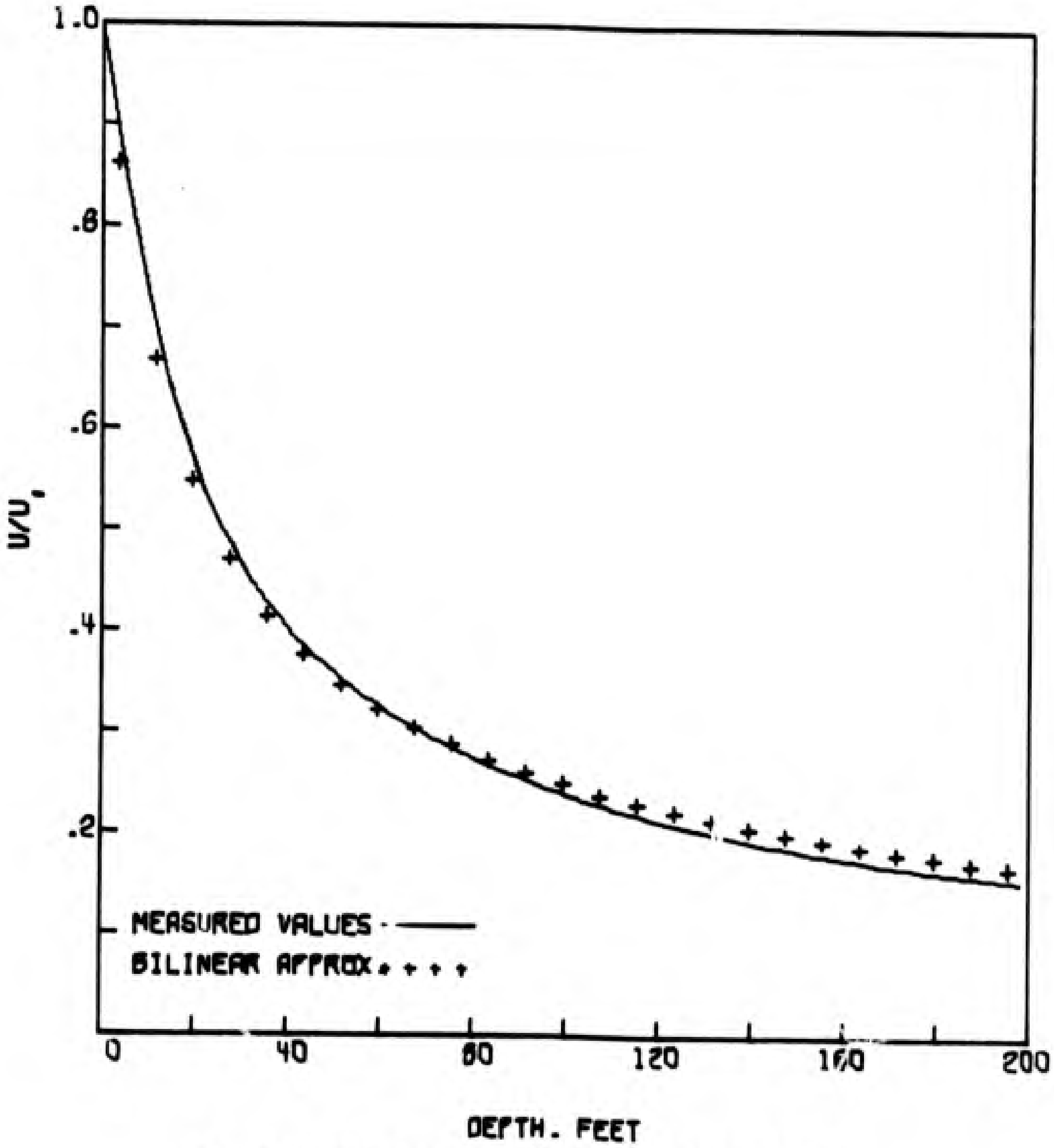
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	1.469	1.766
25	1.326	1.544
50	1.227	1.420
75	1.162	1.325
100	1.092	1.246



SURFACE PARTICLE VELOCITY

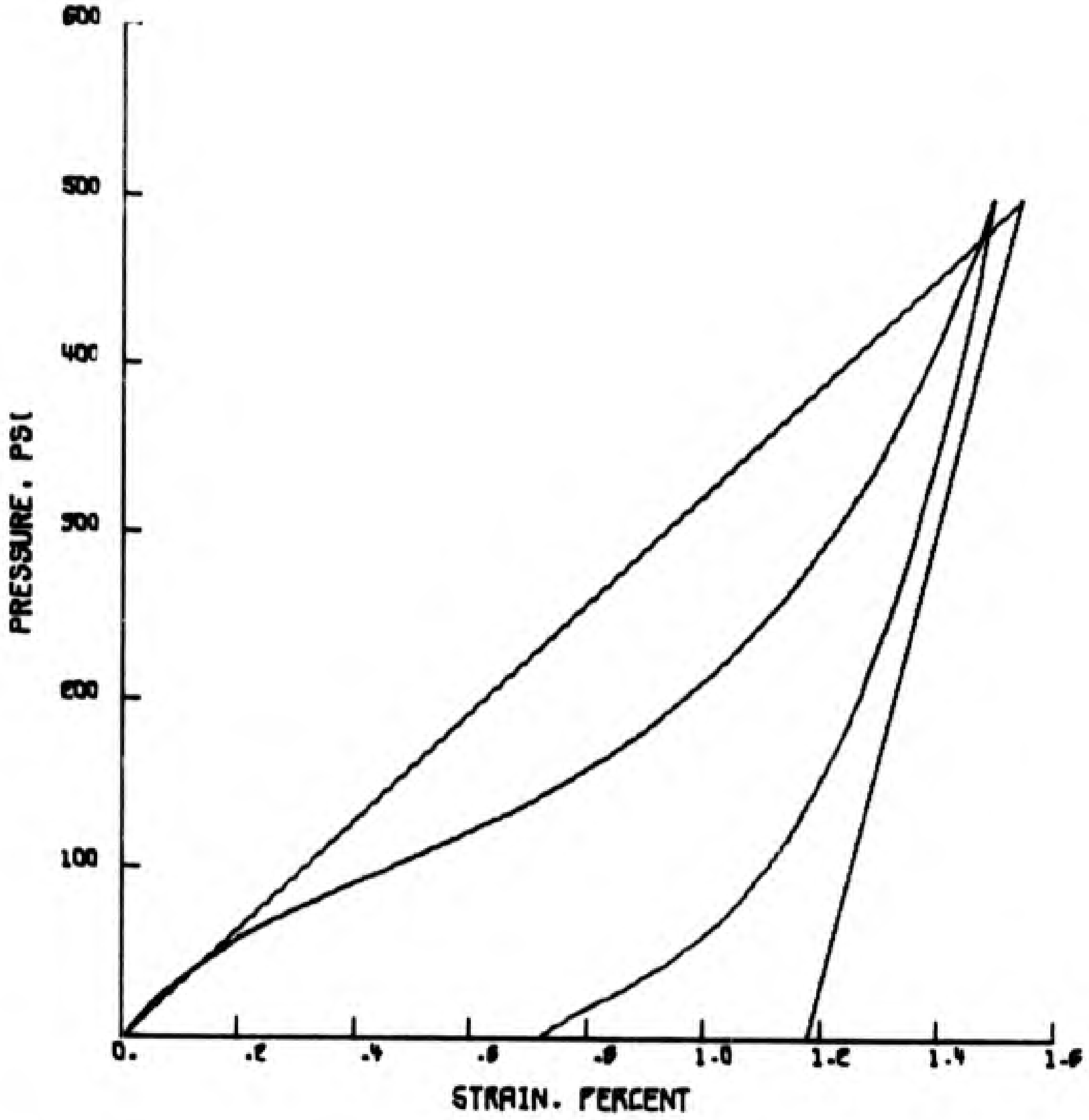
PROBLEM 5E -- 17 NOV 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 2000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 3.631116E-03 SEC.
NORMALIZED HALF LOAD TIME = 3.239566E-03

PROBLEM SE -- 17 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =3.230219E+04
M2(PSI) =1.352402E+06

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04

HUGONIOT NONLINEARITY FACTOR = 2.744967E-01

MAXIMUM UNLOAD SLOPE = 1.723837E+06

ZETA = 1.933670E-01

BEST BILINEAR MODEL

PROBLEM 1F NOV 13. 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

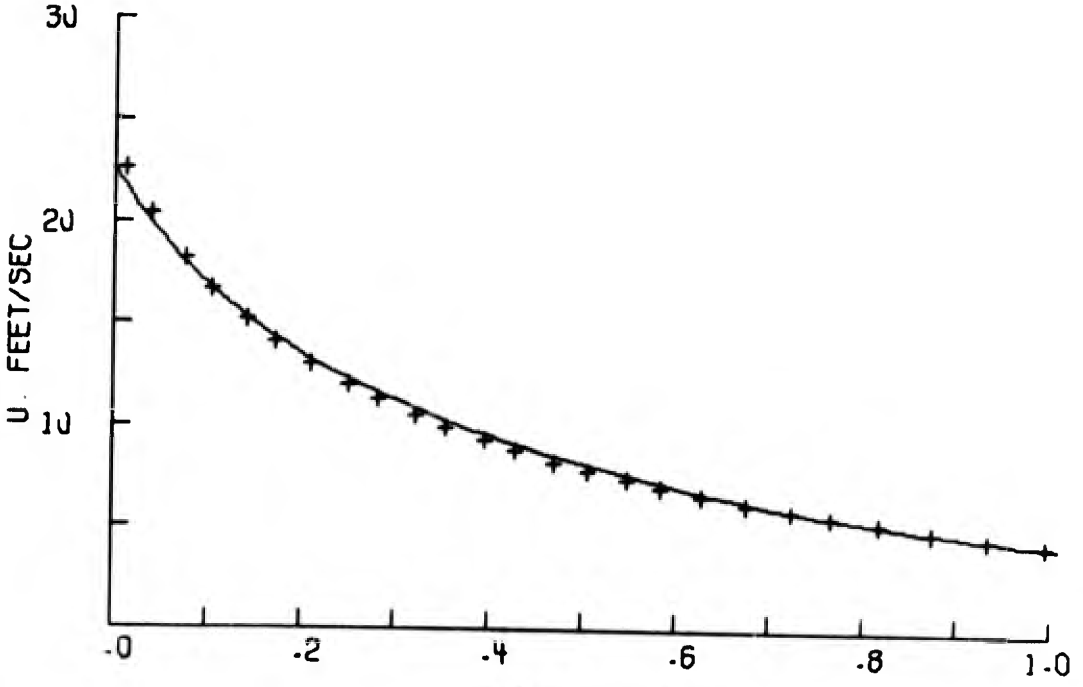
DENSITY =	3.510000E+00
SHOCK VELOCITY =	8.615810E+02
SOUND VELOCITY =	2.156211E+03
ZETA =	4.289990E-01

FITTING ERRORS

E_1 =	9.997031E-01	E_2 =	5.936263E-04
E_3 =	1.779350E-04	E_4 =	1.333923E-02
E_6 =	9.990248E-01	E_6 =	1.949446E-03
E_7 =	6.795347E-02	E_8 =	2.606789E-01

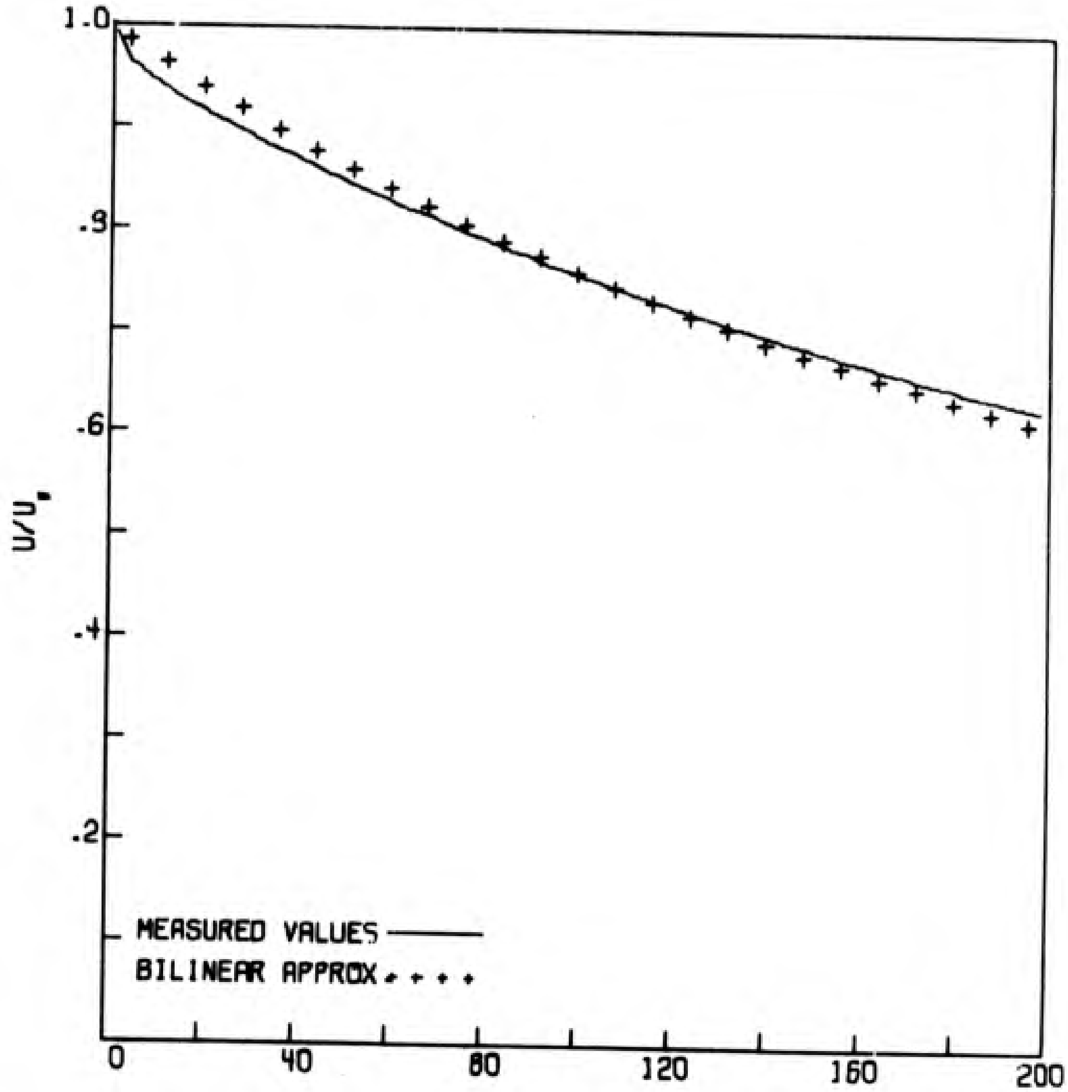
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	10.466	10.386
25	10.075	9.826
50	9.664	9.305
75	9.267	8.818
100	8.882	8.360



SURFACE PARTICLE VELOCITY

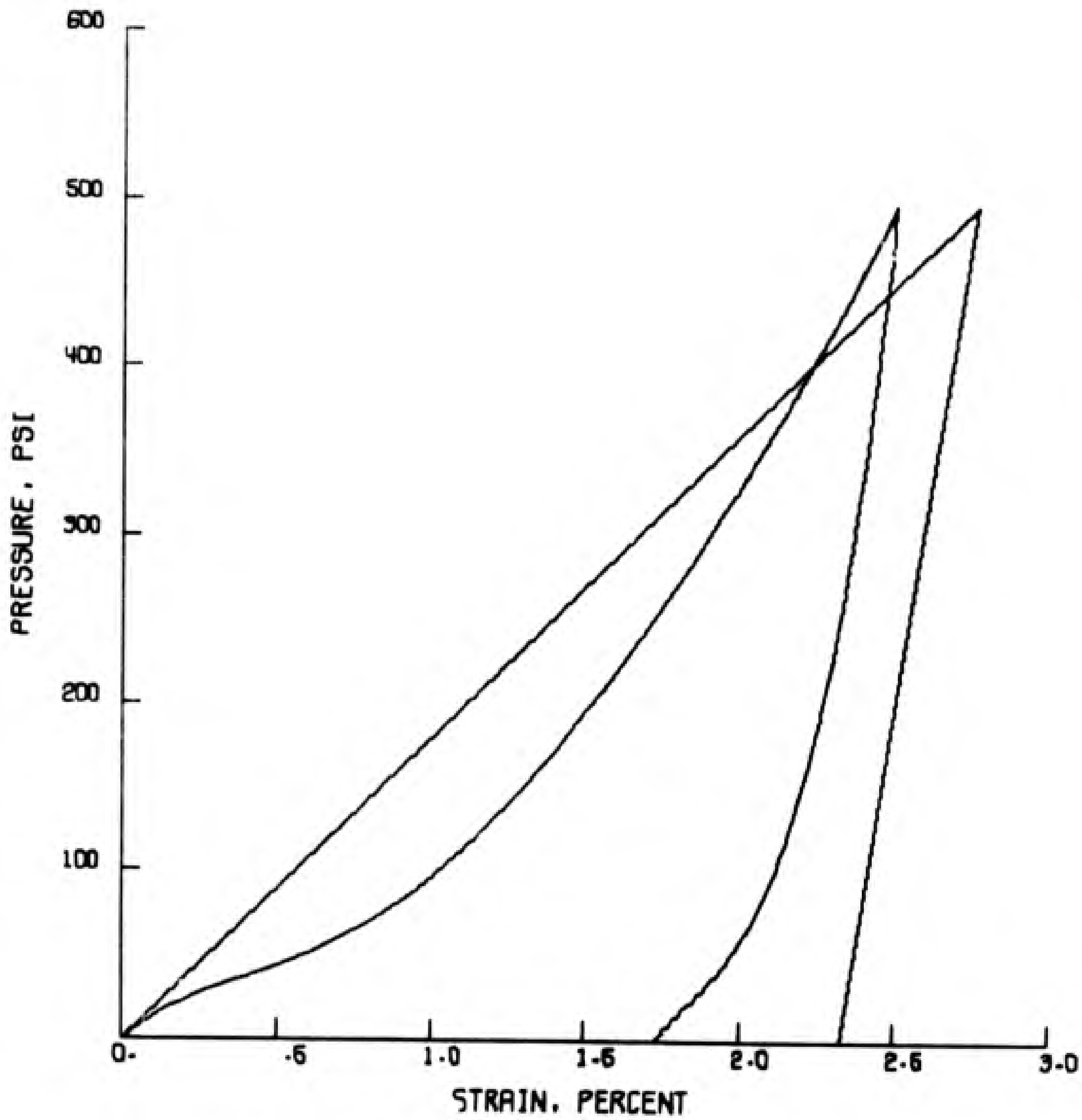
PROBLEM 1F NOV 13. 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 100 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.304202E-01 SEC.
NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 1F NOV 13. 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =1.809409E+04
M2(PSI) =1.133253E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04
HUGONIOT NONLINEARITY FACTOR = 2.745624E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161070E-01

BEST BILINEAR MODEl

PROBLEM 2F NOV 13. 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

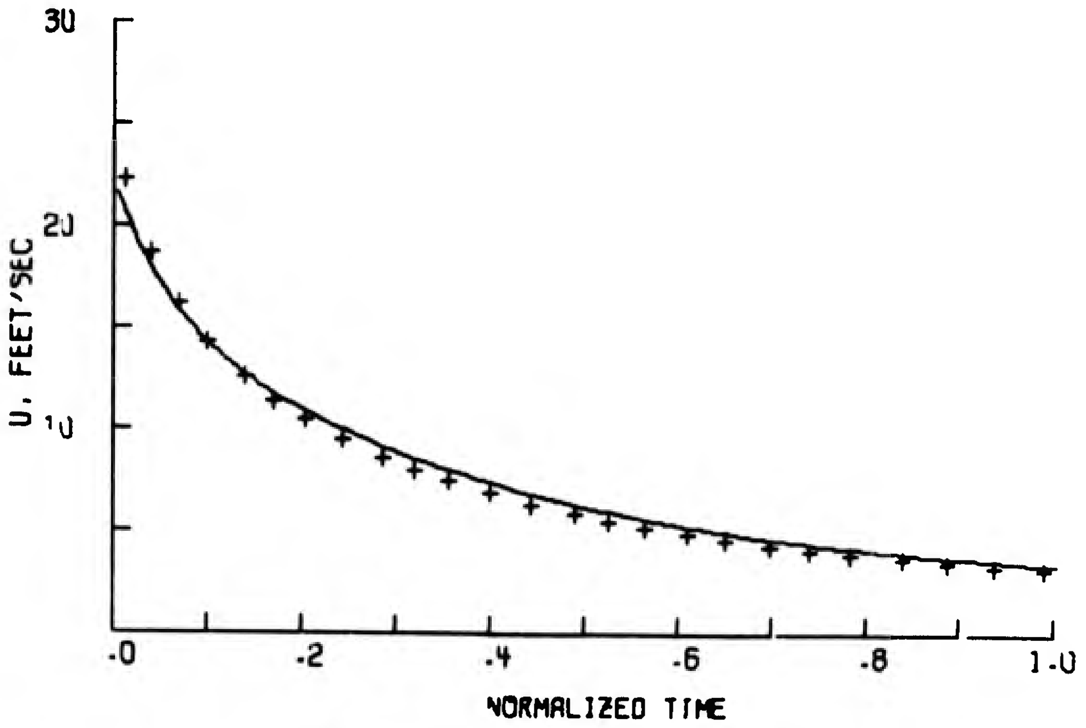
DENSITY =	3.510000E+00
SHOCK VELOCITY =	8.166776E+02
SOUND VELOCITY =	1.547937E+03
ZETA =	3.092510E-01

FITTING ERRORS

E_1 =	9.998072E-01	E_2 =	3.856390E-04
E_3 =	1.249478E-04	E_4 =	1.117801E-02
E_5 =	9.982100E-01	E_6 =	3.576812E-03
E_7 =	2.443322E-01	E_8 =	4.942998E-01

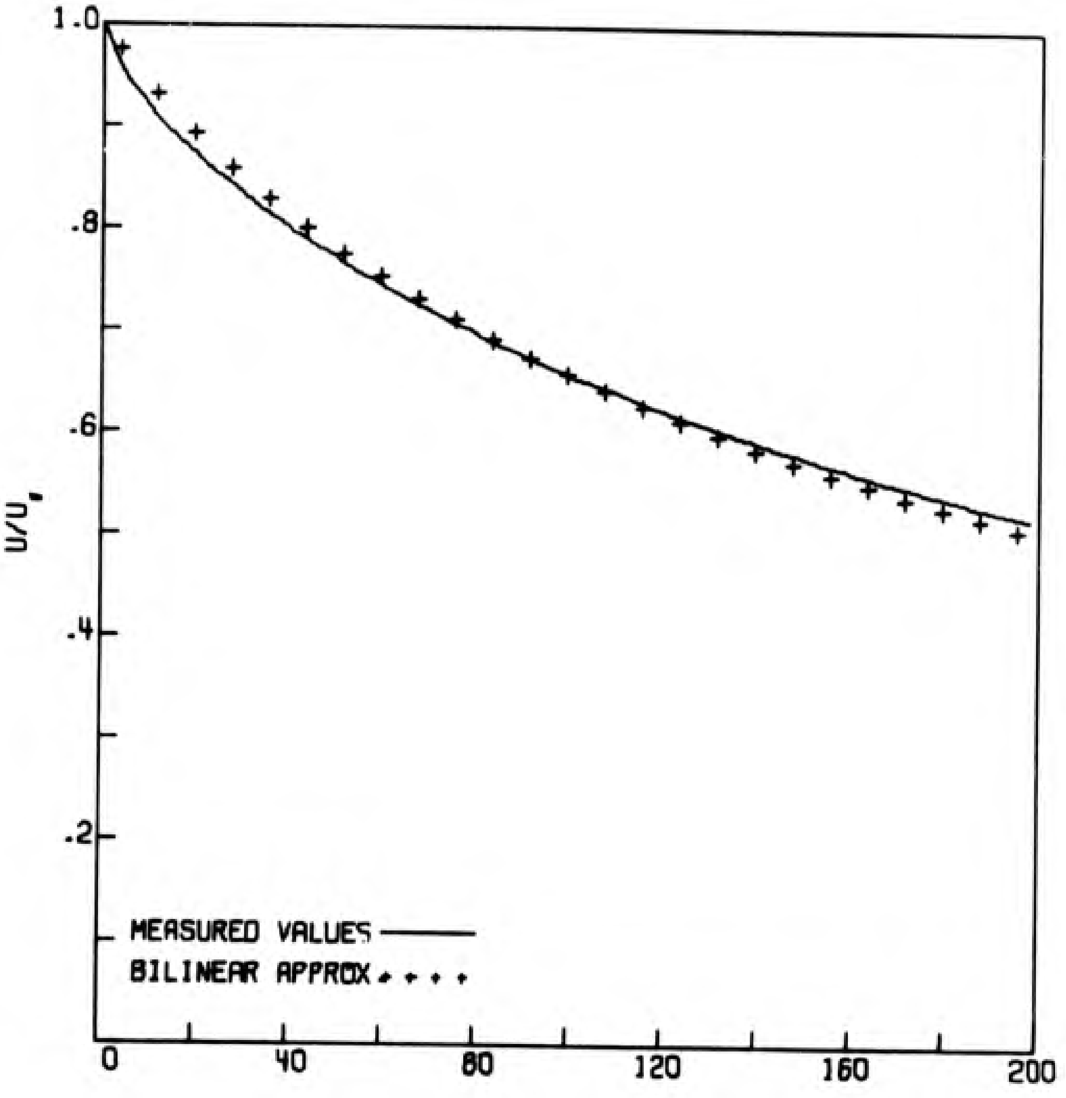
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	8.499	8.238
25	8.119	7.719
50	7.734	7.259
75	7.370	6.841
100	7.024	6.458



9 SURFACE PARTICLE VELOCITY

PROBLEM 2F NOV 13. 1969

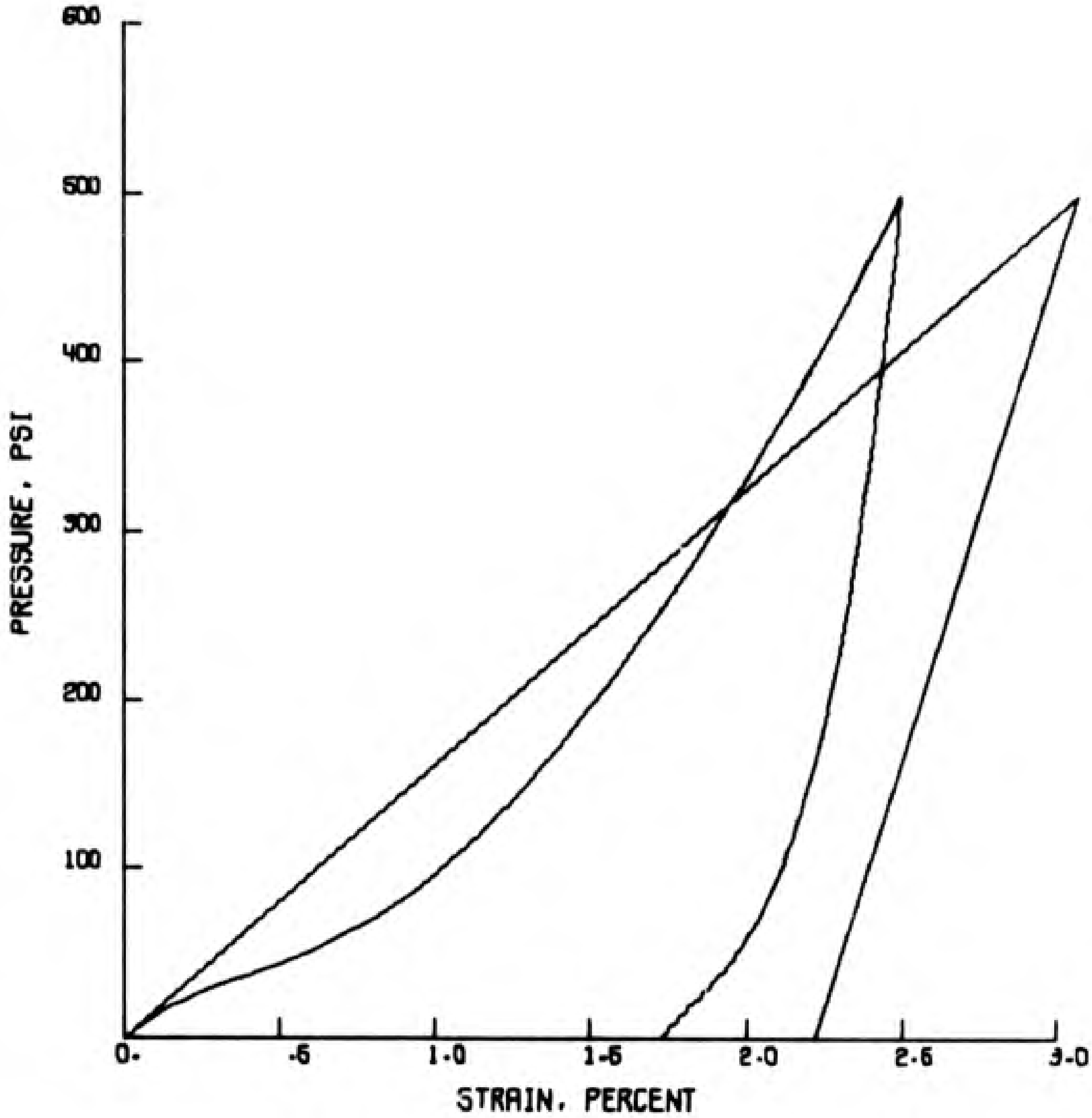


DEPTH. FEET

PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 200 PSI BRODE FORM PMAX=500 PSI TO=1.09
HALF LOAD TIME = 7.710905E-02 SEC.
NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 2F NOV 13. 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =1.625721E+04
M2(PSI) =5.840516E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04
HUGONIOT NONLINEARITY FACTOR = 2.745624E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161070E-01

BEST BILINEAR MOD

PROBLEM 3F NOV 13. 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

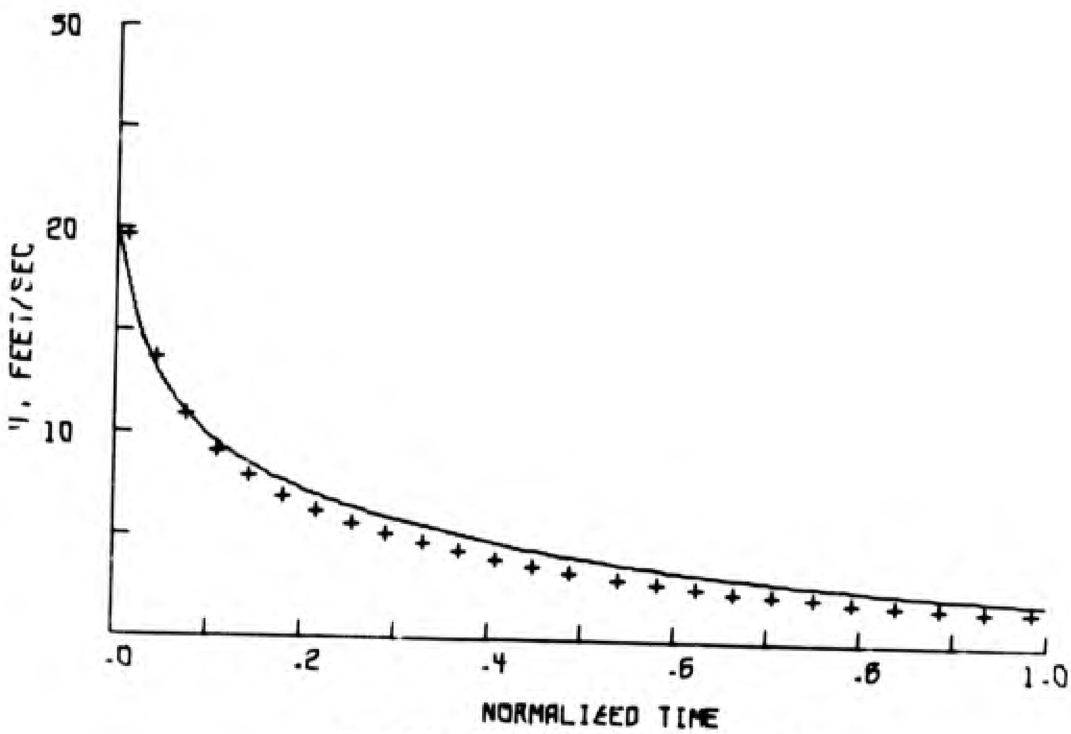
DENSITY =	3.510000E+00
SHOCK VELOCITY =	7.260303E+02
SOUND VELOCITY =	1.051057E+03
ZETA =	1.826965E-01

FITTING ERRORS

E_1 =	9.995552E-01	E_2 =	8.893764E-04
E_3 =	8.250761E-05	E_4 =	9.083370E-05
E_5 =	9.958233E-01	E_6 =	8.335903E-05
E_7 =	5.142269E-01	E_8 =	7.170962E-01

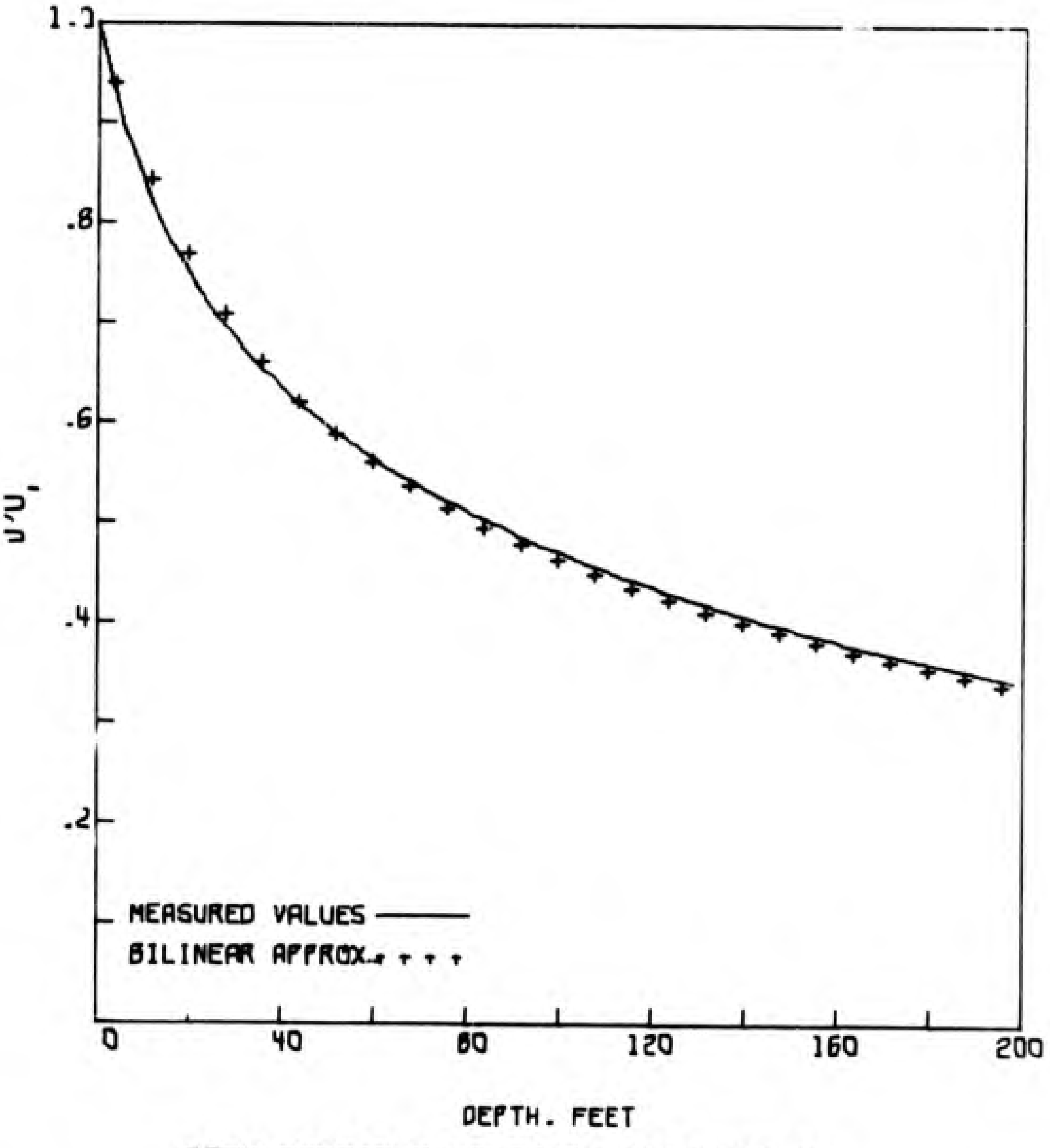
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	5.758	5.333
25	5.414	4.895
50	5.094	4.557
75	4.803	4.272
100	4.532	4.019



SURFACE PARTICLE VELOCITY

PROBLEM 3F NOV 13. 196J



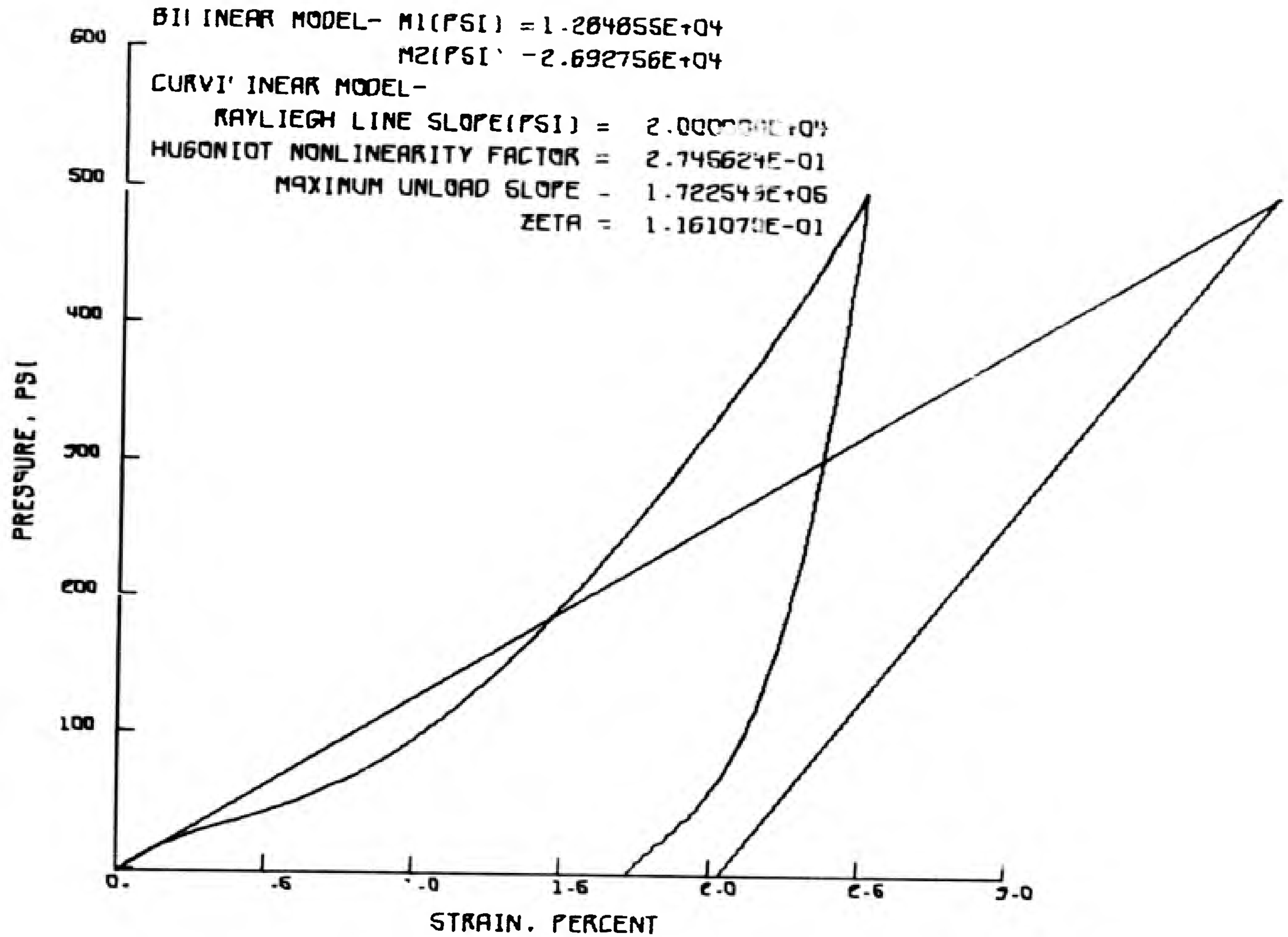
PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 500 PSI BRODE FORM PMAX=500 PSI TD=1 09

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.522903E-02

PROBLEM 3F NOV 13. 1969



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 4F NOV 13. 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

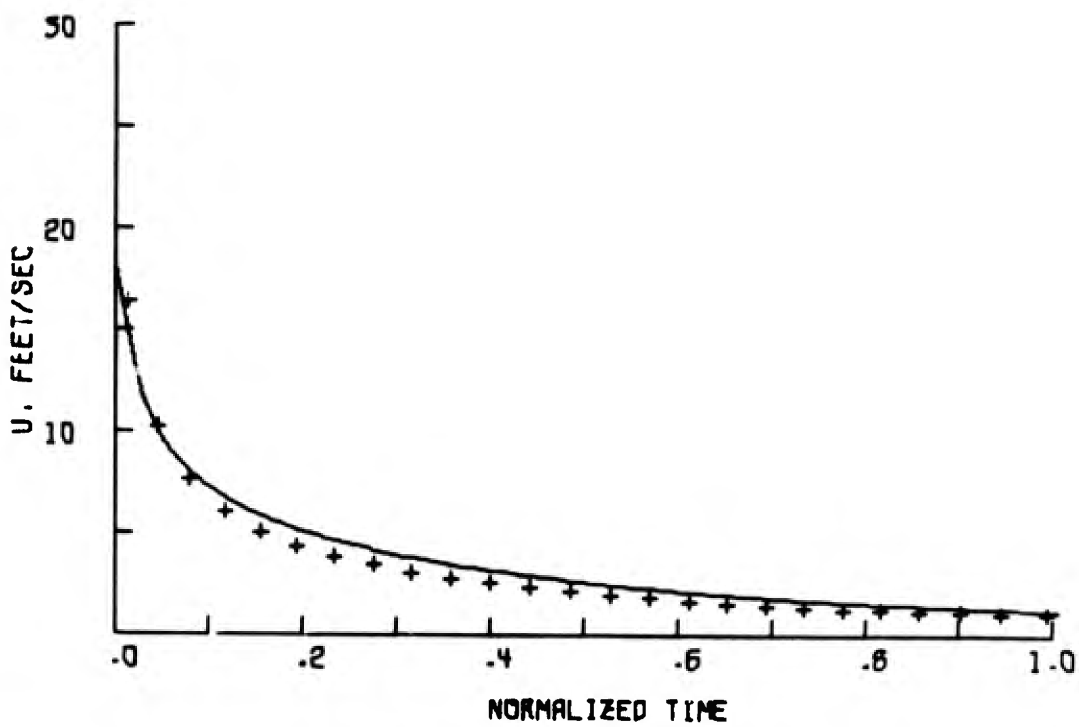
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	6.882596E+02
	SOUND VELOCITY =	9.310386E+02
	ZETA =	1.499285E-01

FITTING ERRORS

E_1 =	9.986112E-01	E_2 =	2.775645E-03
E_3 =	9.746421E-06	E_4 =	9.871890E-03
E_5 =	9.936053E-01	E_6 =	1.235154E-02
E_7 =	3.399502E-01	E_8 =	5.830624E-01

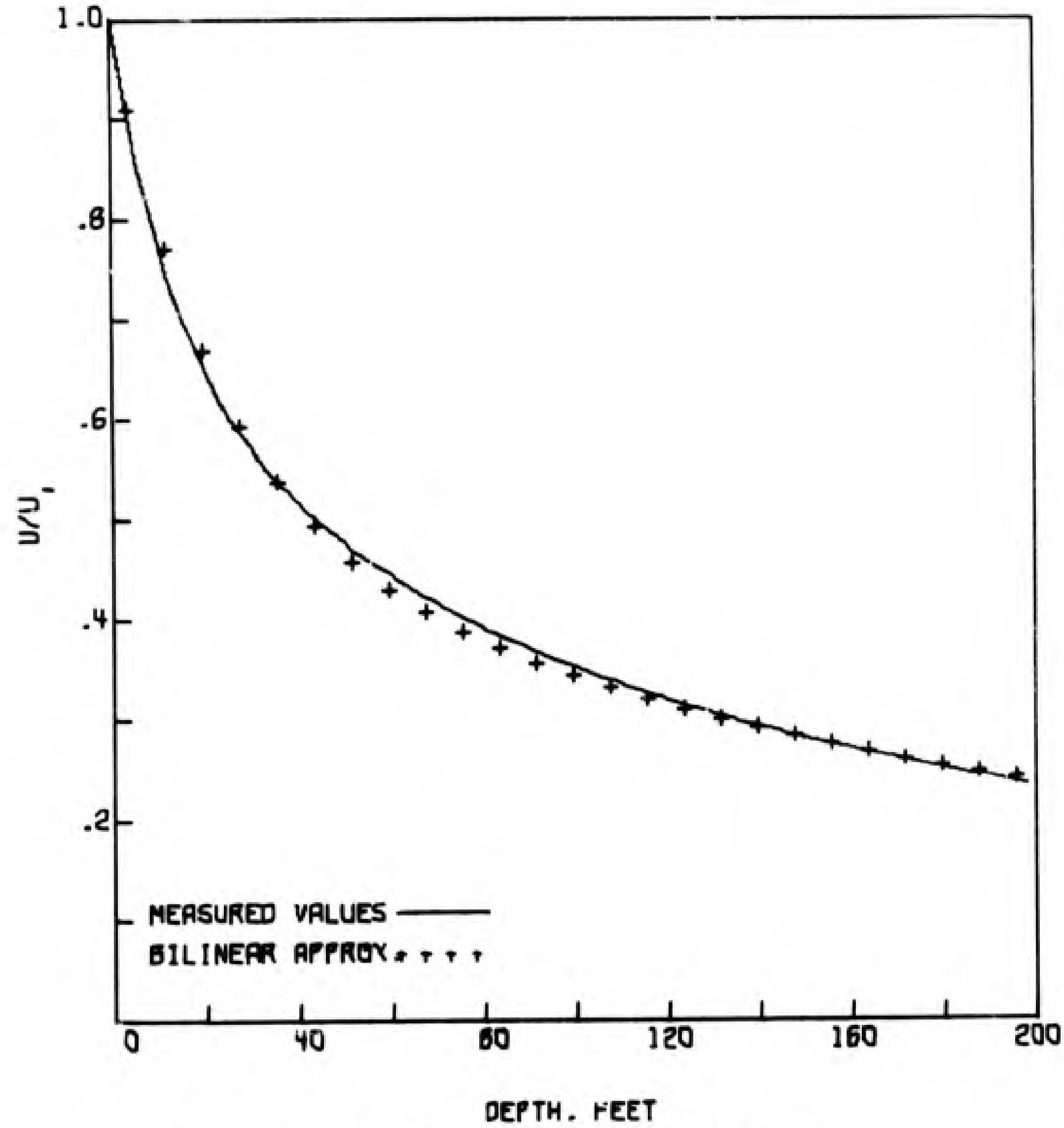
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	4.017	3.700
25	3.704	3.306
50	3.431	3.040
75	3.192	2.828
100	2.974	2.645



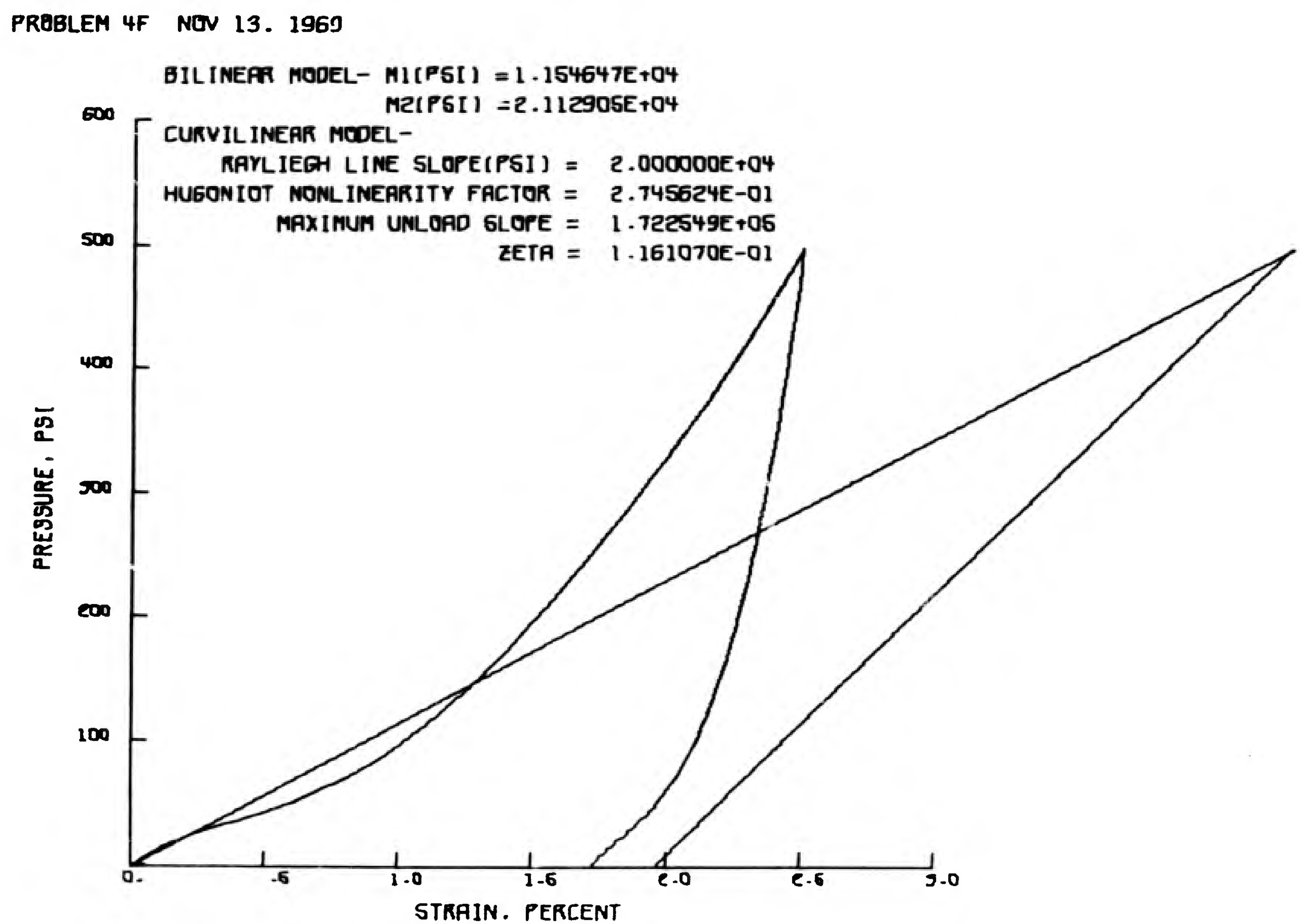
SURFACE PARTICLE VELOCITY

PROBLEM 4F NOV 13. 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 1000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 1.285354E-02 SEC.
NORMALIZED HALF LOAD TIME = 1.179224E-02



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM SF NOV 13. 1969

NUMBER OF DATA POINTS. N= 99 . M= 99

MATERIAL PROPERTIES

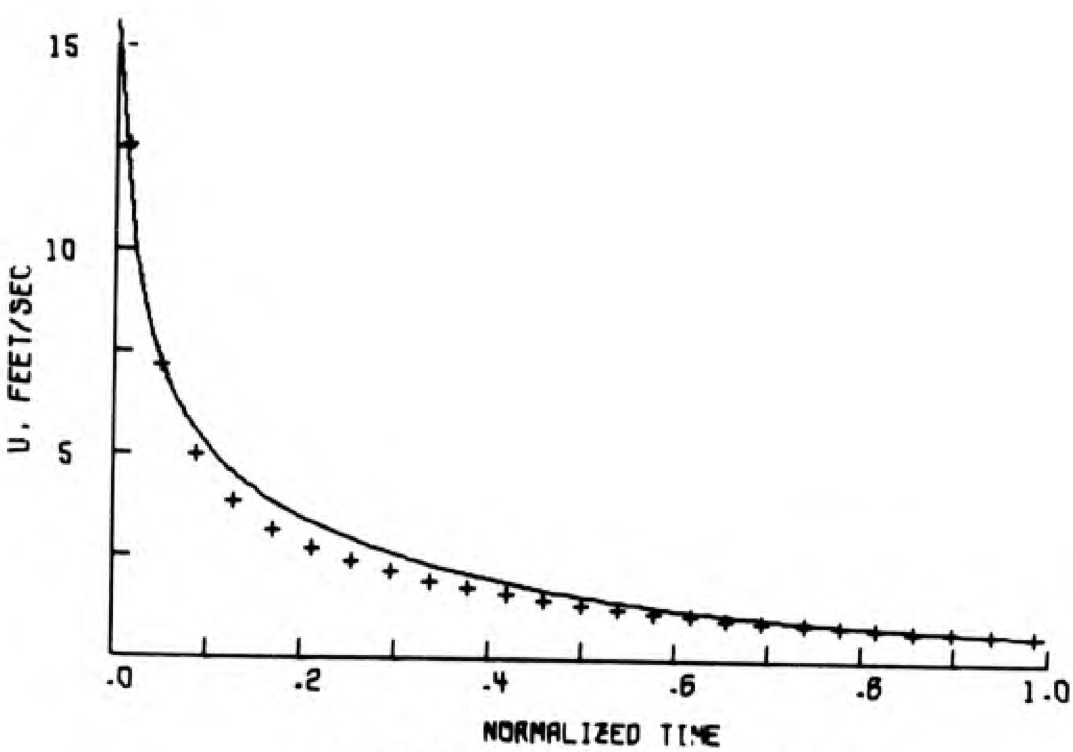
DENSITY	=	3.510000E+00
SHOCK VELOCITY	=	6.569932E+02
SOUND VELOCITY	=	8.510704E+02
ZETA	=	1.286930E-01

FITTING ERRORS

E_1 =	5.965826E-01	E_2 =	6.823209E-05
E_3 =	1.876269E-04	E_4 =	1.369770E-02
E_5 =	9.920037E-01	E_6 =	1.592859E-02
E_7 =	1.566919E-01	E_8 =	3.960958E-01

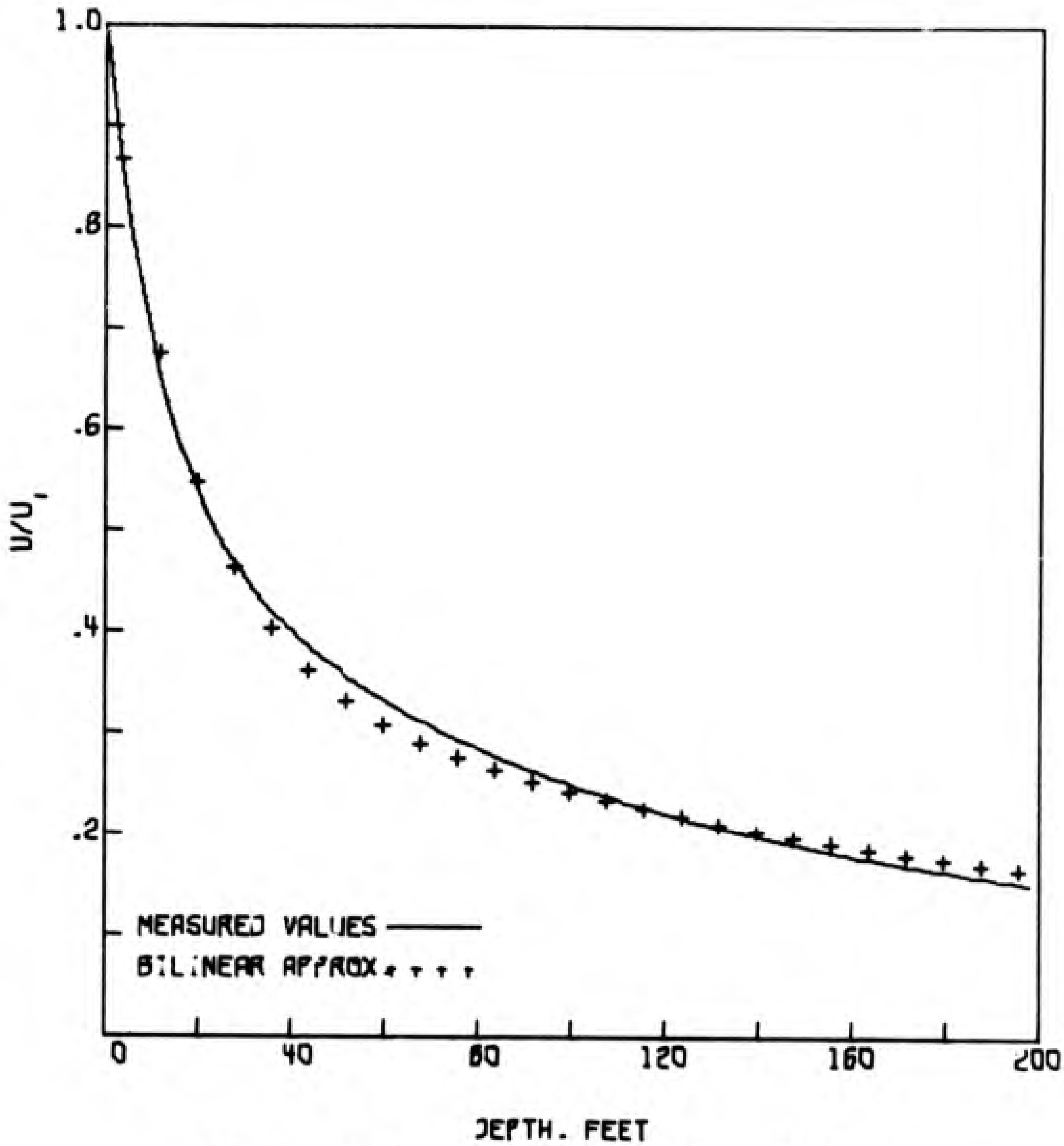
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	2.666	2.541
25	2.391	2.187
50	2.171	1.988
75	1.985	1.839
100	1.822	1.712



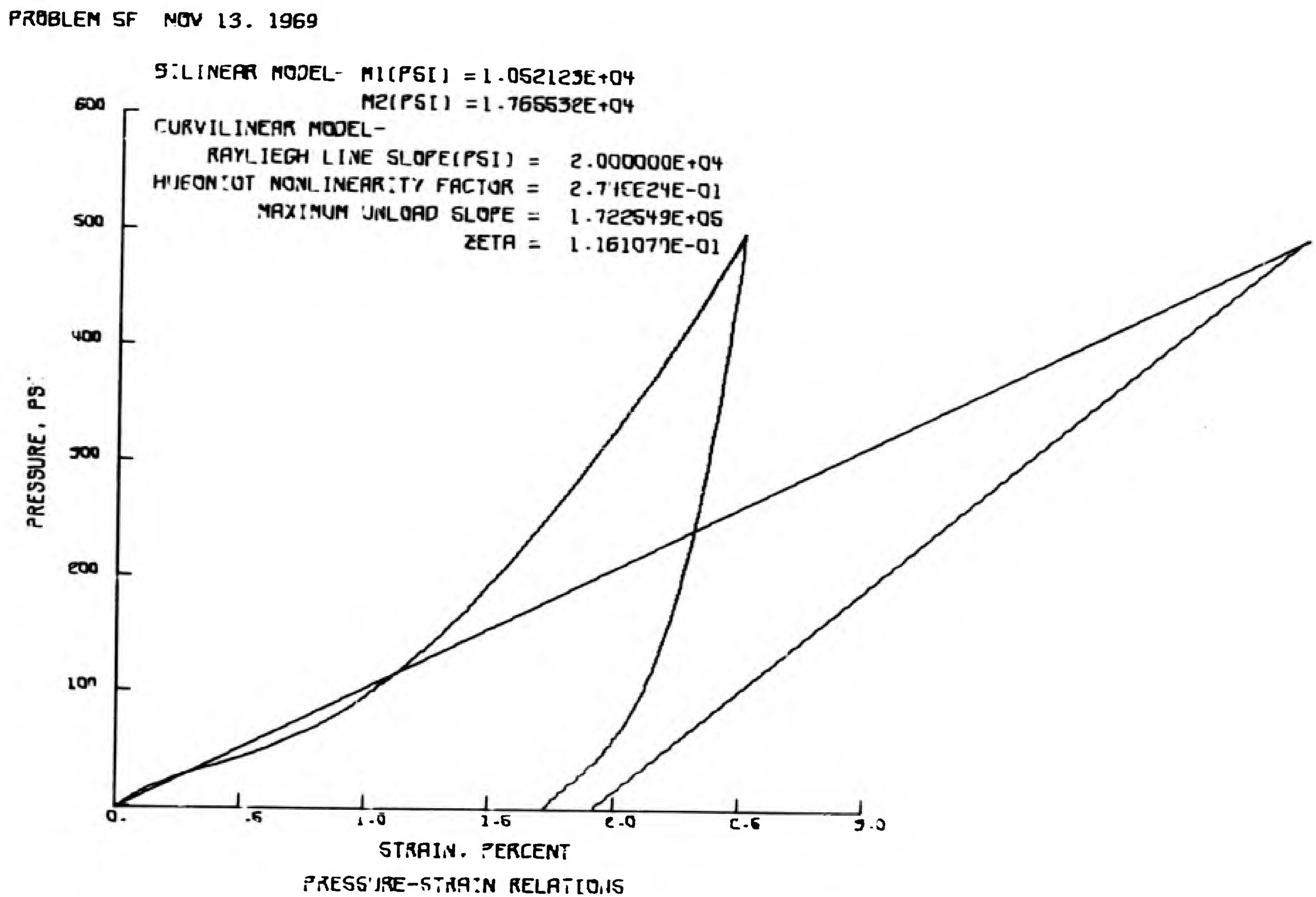
SURFACE PARTICLE VELOCITY

PROBLEM 5F NOV 13. 1969



PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE HAS 2000 PSI BRODE FORM PMAX=500 PSI TD=1.09
HALF LOAD TIME = 3.531115E-03 SEC.
NORMALIZED HALF LOAD TIME = 3.239555E-03


PROBLEM SF NOV 13. 1969
BILINEAR MODEL- M1(PSI) = 1.052123E+04
M2(PSI) = 1.766532E+04
CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04
HUGONIOT NONLINEARITY FACTOR = 2.71EE24E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161077E-01
600
500
400
300
200
100
PRESSURE, PSI
0.
.5
1.0
1.5
2.0
2.5
3.0
STRAIN, PERCENT
PRESSURE-STRAIN RELATIONS

BEST BILINEAR MOD

PROBLEM 1G -- 19 NOV 1969

NUMBER OF DATA POINTS. N=117 . M= 99

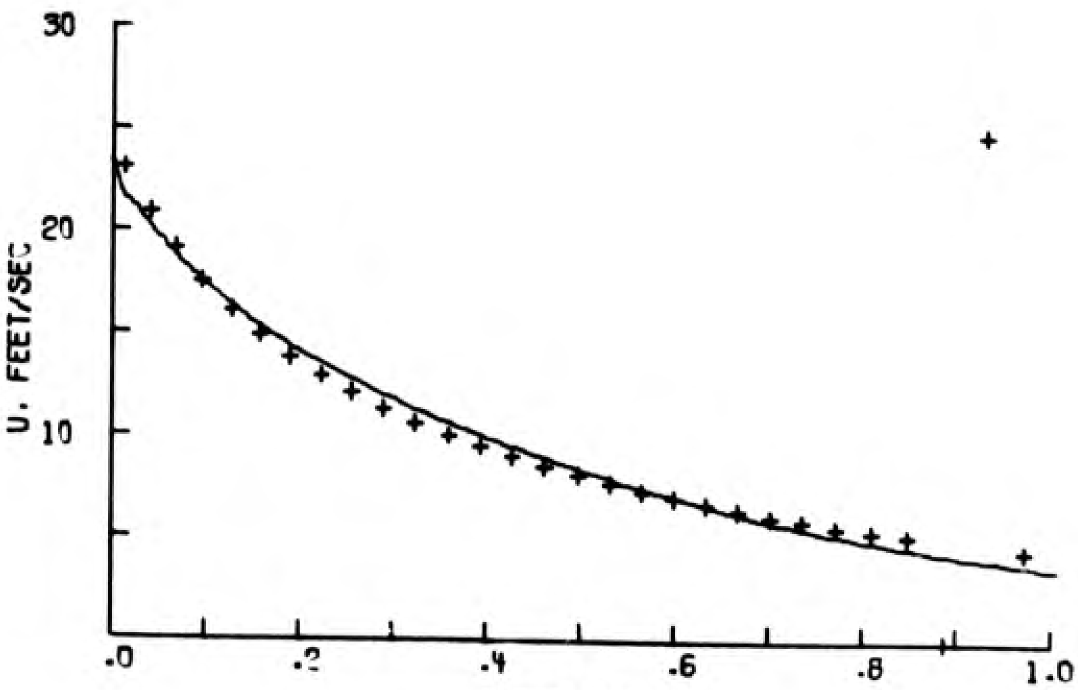
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	8.434395E+02
	SOUND VELOCITY =	2.082727E+03
	ZETA =	4.235192E-01

FITTING ERRORS

E_1 =	5.278241E+00	E_2 =	5.222080E+00
E_3 =	5.166472E+00	E_4 =	5.111408E+00
E_5 =	9.953180E-01	E_6 =	9.342041E-03
E_7 =	2.574269E-01	E_8 =	5.073725E-01

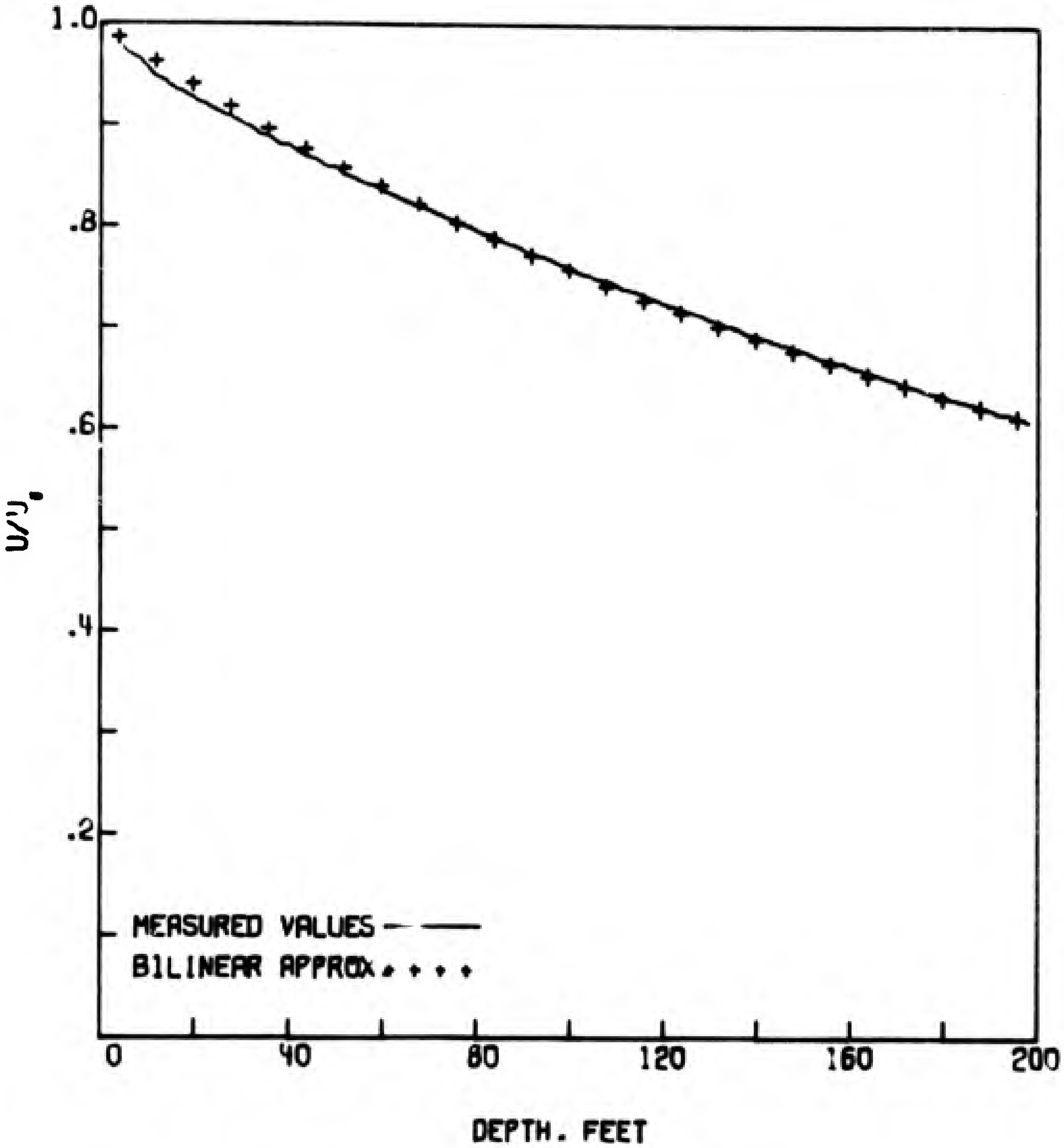
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	3.582	1.943
25	3.057	1.441
50	2.491	1.070
75	1.930	.775
100	1.374	.528



SURFACE PARTICLE VELOCITY

PROBLEM 1G -- 19 NOV 1969



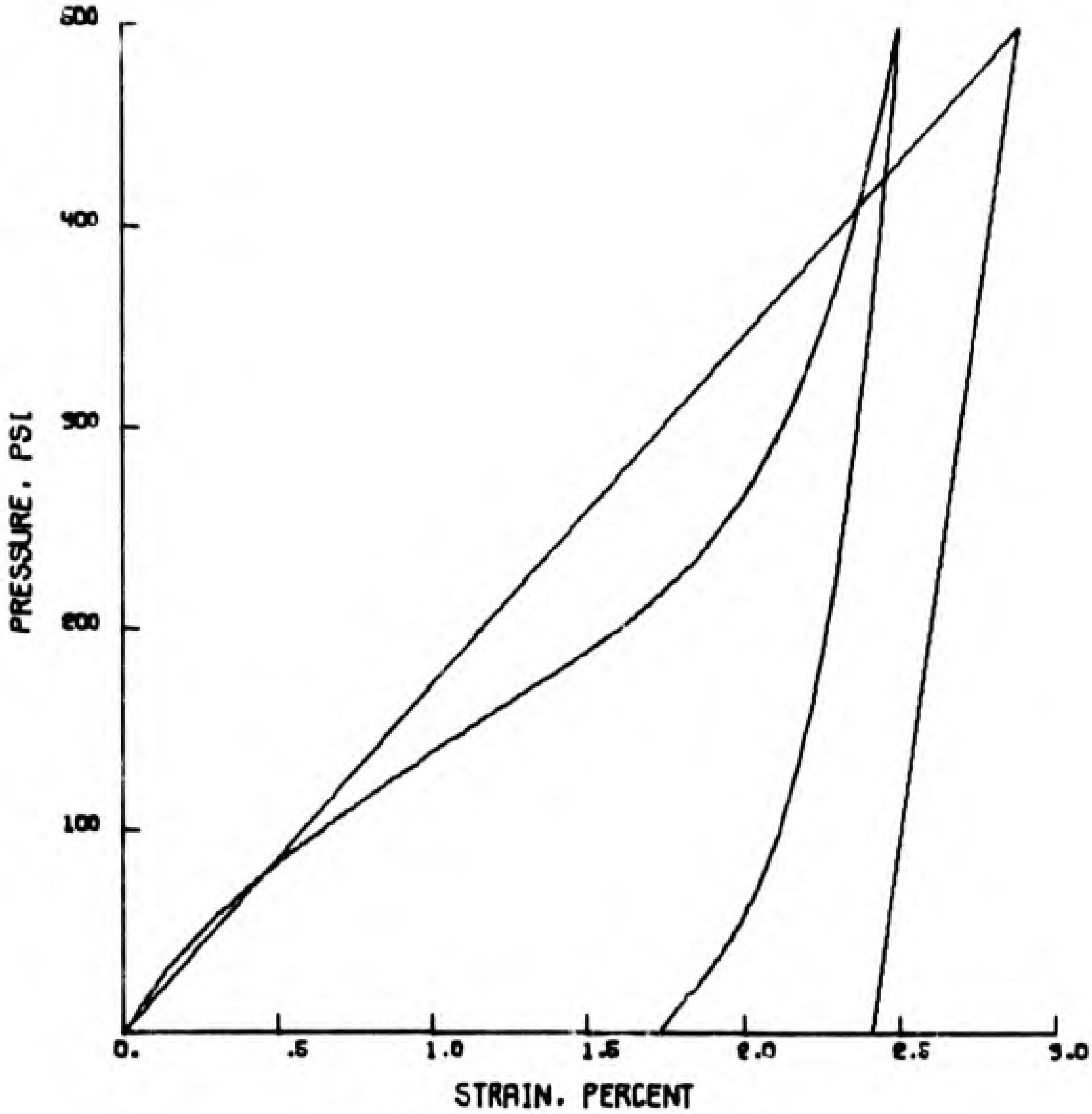
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 100 PSI BRODE WAVE - PMAX=500. TD=1.09

HALF LOAD TIME = 1.304202E-01 SEC.

NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 1G -- 19 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 1.734014E+04
M2(PSI) = 1.057327E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04
HUGONIOT NONLINEARITY FACTOR = 2.745190E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161070E-01

BEST BILINEAR MODEL

PROBLEM 2C -- 19 NOV 1963

NUMBER OF DATA POINTS. N=119 . M= 99

MATERIAL PROPERTIES

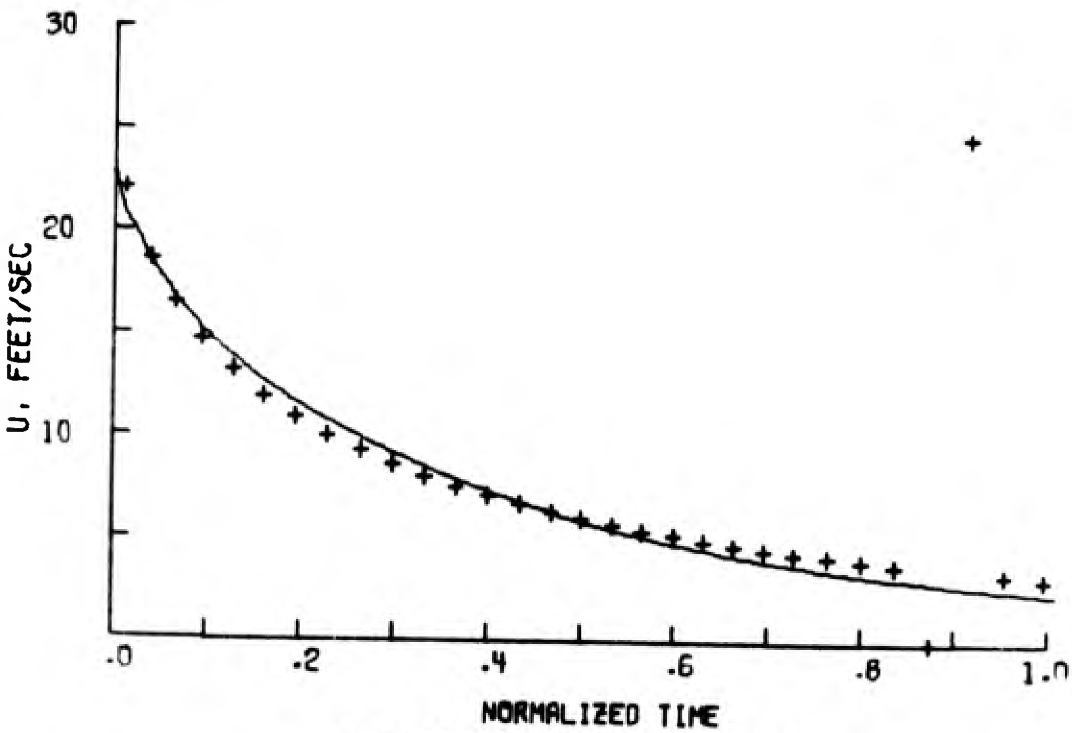
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	8.340154E+02
	SOUND VELOCITY =	1.933022E+03
	ZETA =	3.971780E-01

FITTING ERRORS

E_1 =	3.915847E+00	E_2 =	3.875391E+00
E_3 =	3.835344E+00	E_4 =	3.795695E+00
E_5 =	9.951443E-01	E_6 =	9.687796E-03
E_7 =	3.161300E-01	E_8 =	5.622543E-01

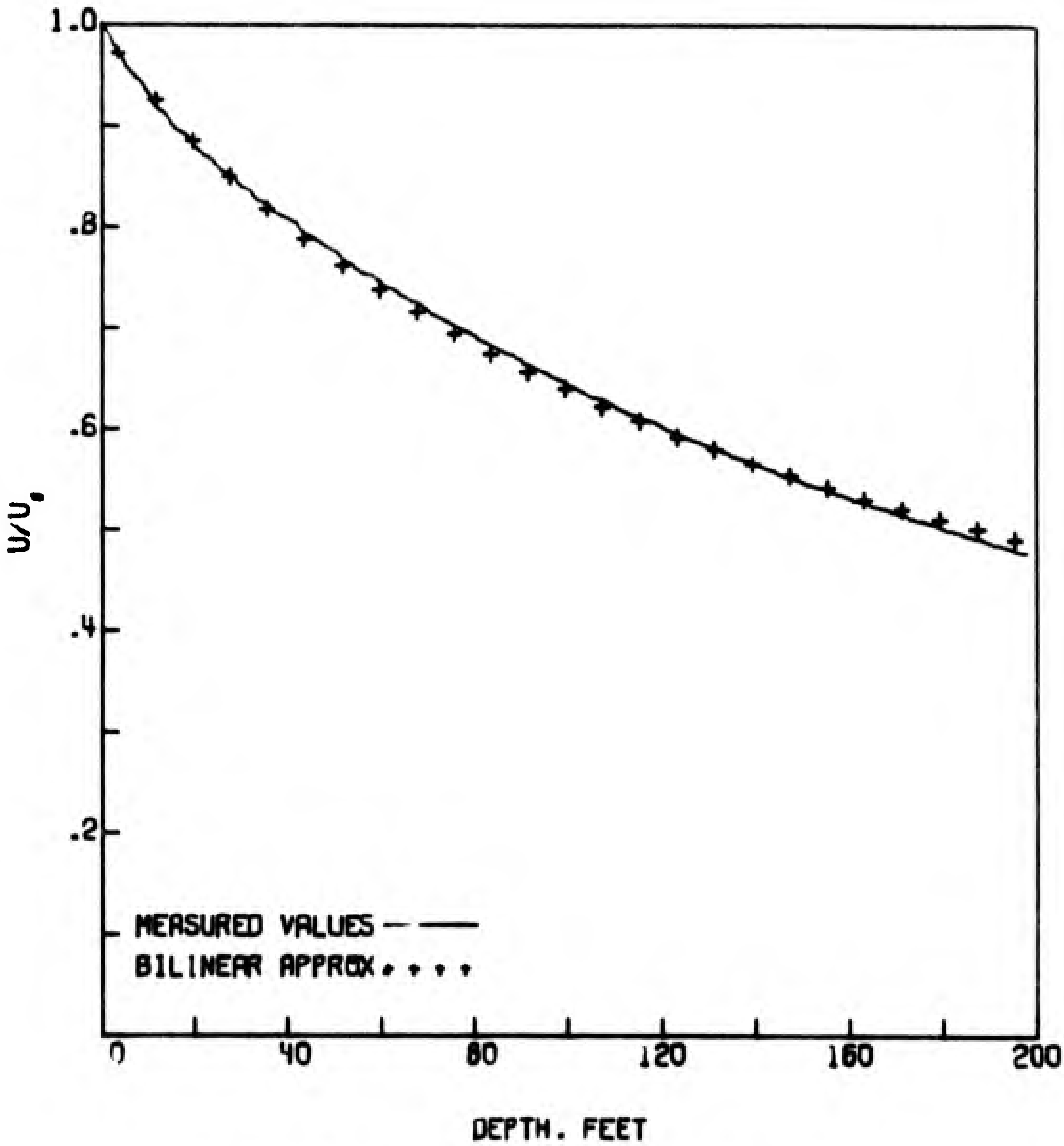
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	3.159	1.539
25	2.657	1.095
50	2.122	.800
75	1.599	.572
100	1.089	.385



SURFACE PARTICLE VELOCITY

PROBLEM 2G -- 19 NOV 1969



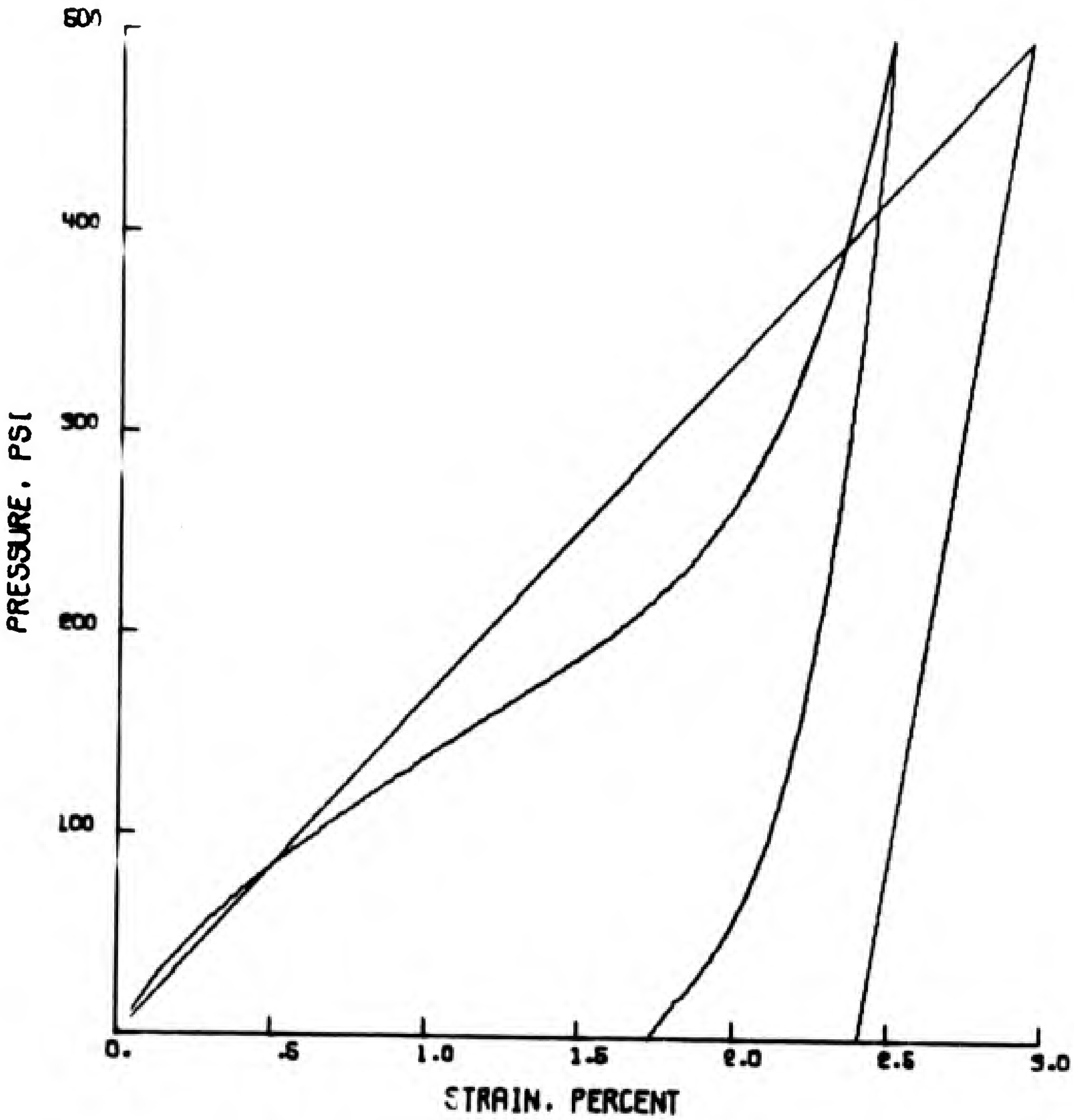
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE - PMAX=500. TD=1.09

HALF LOAD TIME = 1.373652E-01 SEC.

NORMALIZED HALF LOAD TIME = 1.260231E-01

PROBLEM 2G -- 19 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 1.695460E+04

M2(PSI) = 9.107897E+04

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04

HUGONIOT NONLINEARITY FACTOR = 2.745190E-01

MAXIMUM UNLOAD SLOPE = 1.722549E+05

ZETA = 1.161070E-01

BEST BILINEAR MODEL

PROBLEM 3G -- 19 NOV 1963

NUMBER OF DATA POINTS. N=122 . M= 99

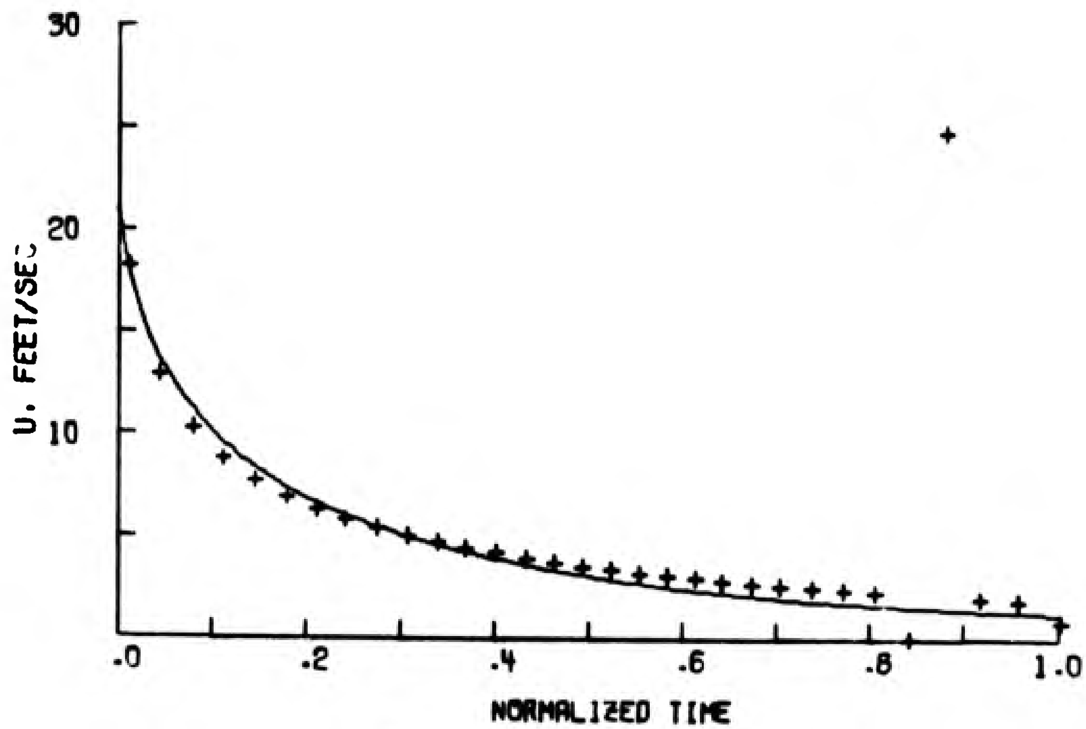
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	8.395332E+02
	SOUND VELOCITY =	1.987875E+03
	ZETA =	4.061464E-01

FITTING ERRORS

E_1 =	2.376252E+00	E_2 =	2.352100E+00
E_3 =	2.328170E+00	E_4 =	2.304459E+00
E_5 =	9.968598E-01	E_6 =	6.270467E-03
E_7 =	3.037376E-01	E_8 =	5.511240E-01

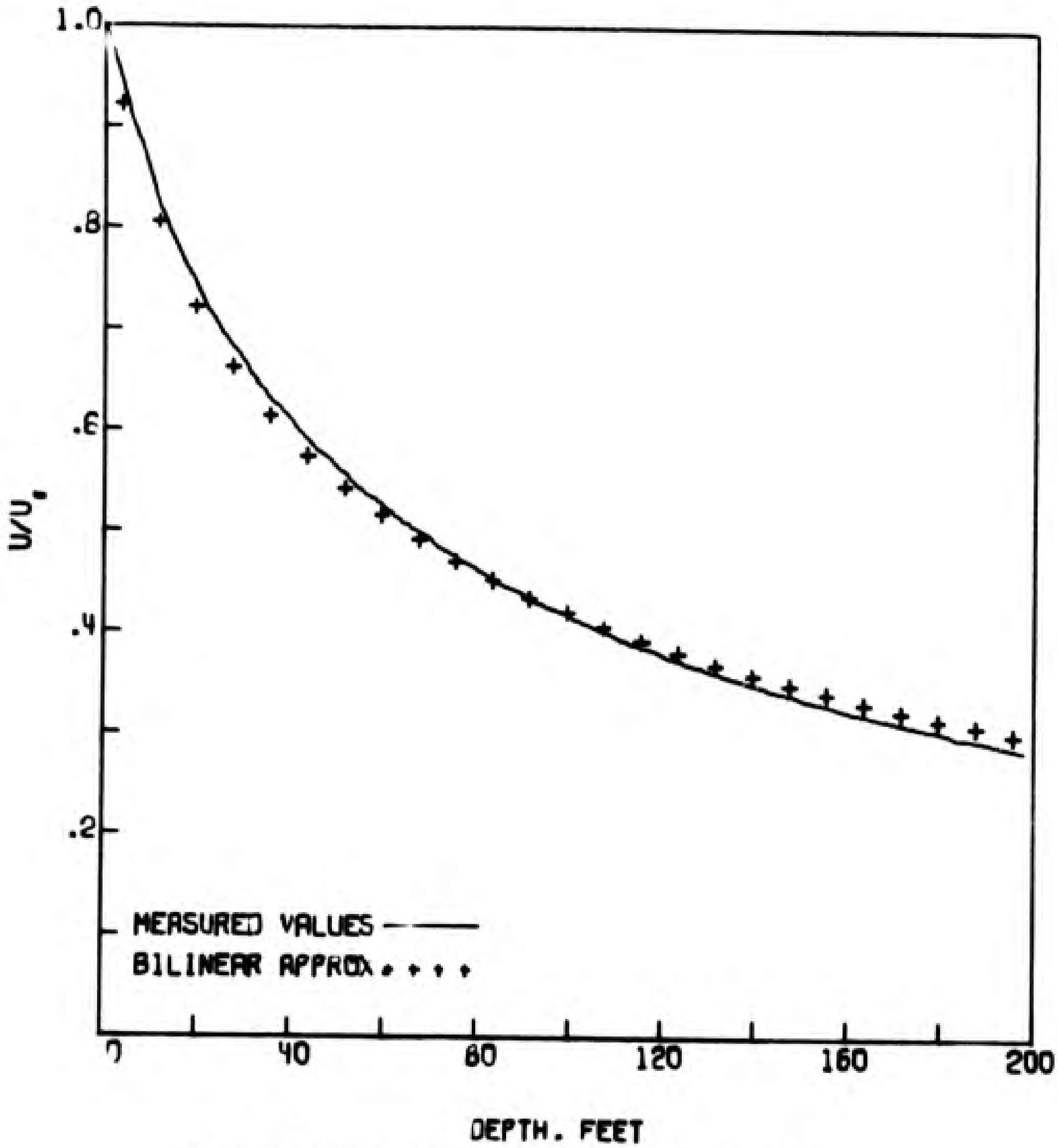
FINAL DISPLACEMENTS. FEET

DEPTH	MEASURED	COMPUTED
0	2.286	.977
25	1.834	.652
50	1.389	.471
75	.992	.336
100	.642	.226



SURFACE PARTICLE VELOCITY

PROBLEM 3G -- 19 NOV 1969



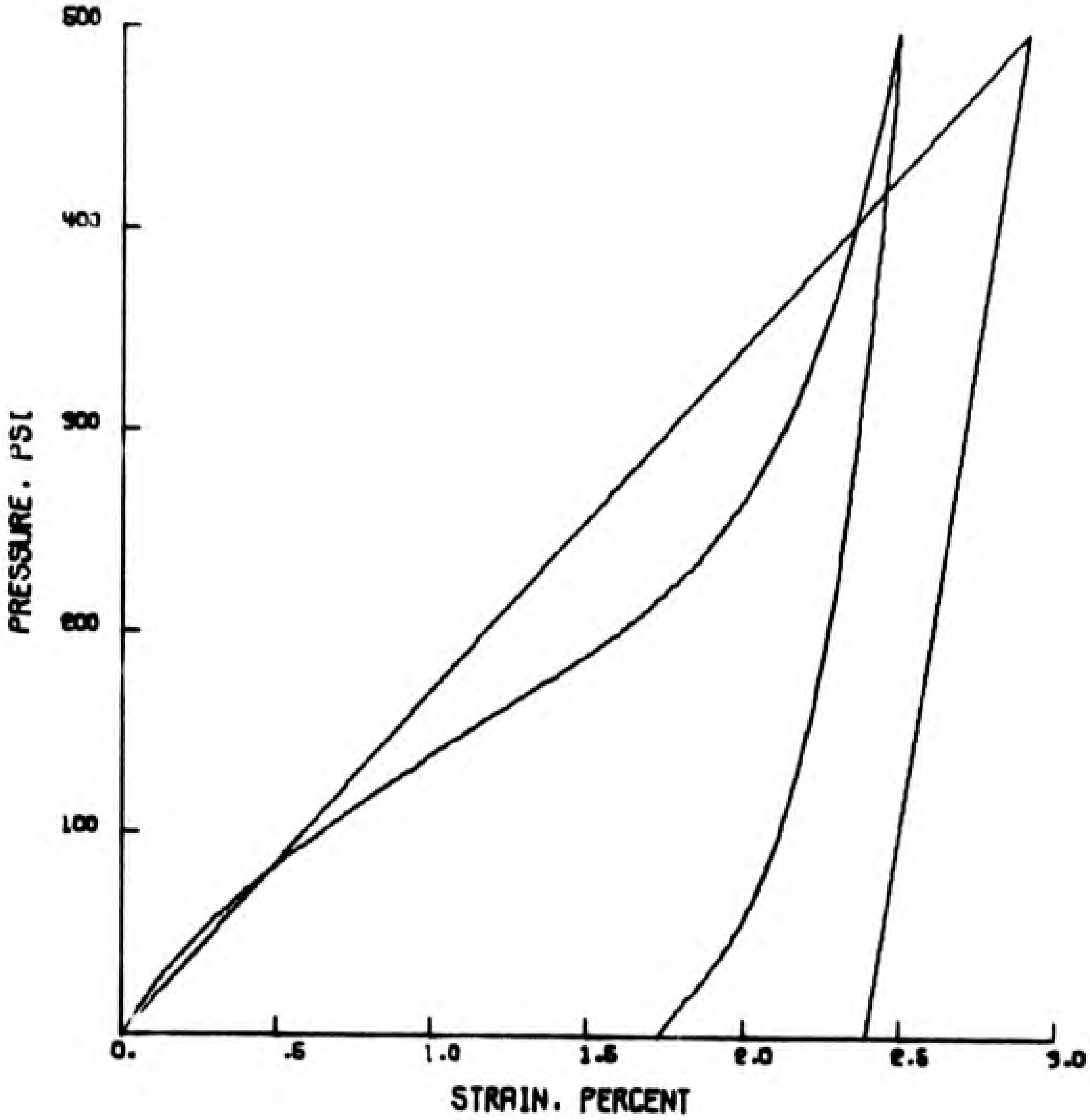
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE - PMAX=500. TD=1.09

HALF LOAD TIME = 9.545150E-01 SEC.

NORMALIZED HALF LOAD TIME = 8.757018E-01

PROBLEM 3G -- 19 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 1.717989E+04
M2(PSI) = 9.632138E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000001E+04
HUGONIOT NONLINEARITY FACTOR = 2.745190E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161070E-01

BEST BILINEAR MODEL

PROBLEM 4G -- 19 NOV 1969

NUMBER OF DATA POINTS. N=125 . M= 99

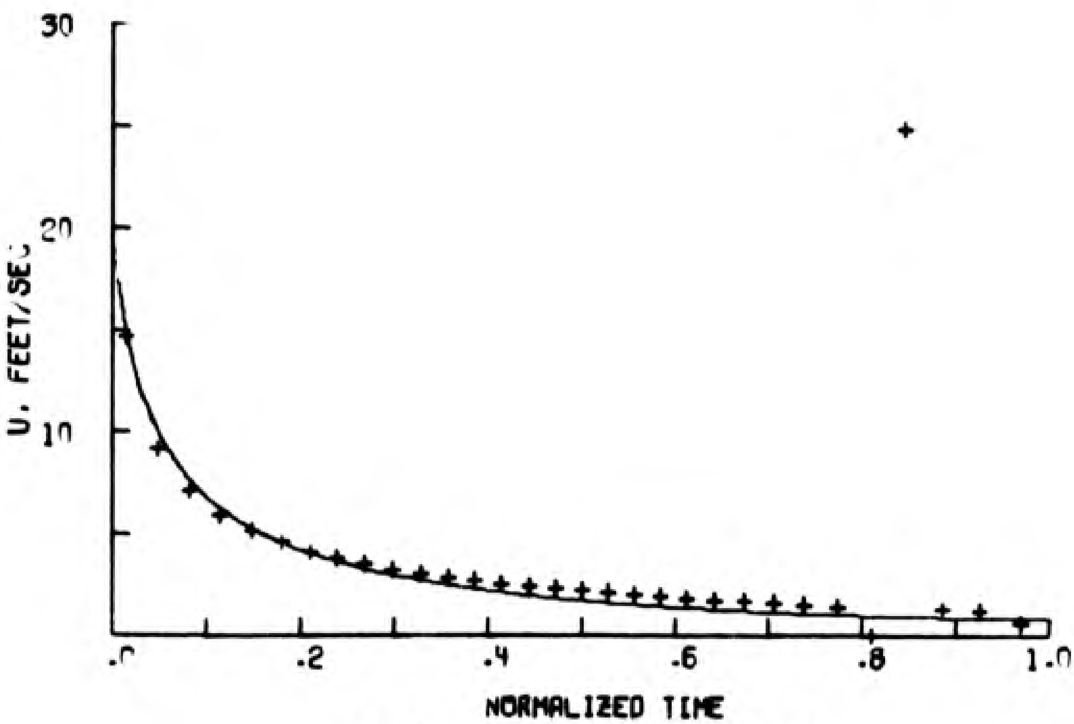
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	8.689983E+02
	SOUND VELOCITY =	2.336275E+03
	ZETA =	4.577696E-01

FITTING ERRORS

E_1 =	1.515609E+00	E_2 =	1.499577E+00
E_3 =	1.483698E+00	E_4 =	1.467968E+00
E_5 =	9.988579E-01	E_6 =	2.282965E-03
E_7 =	2.098780E-01	E_8 =	4.581245E-01

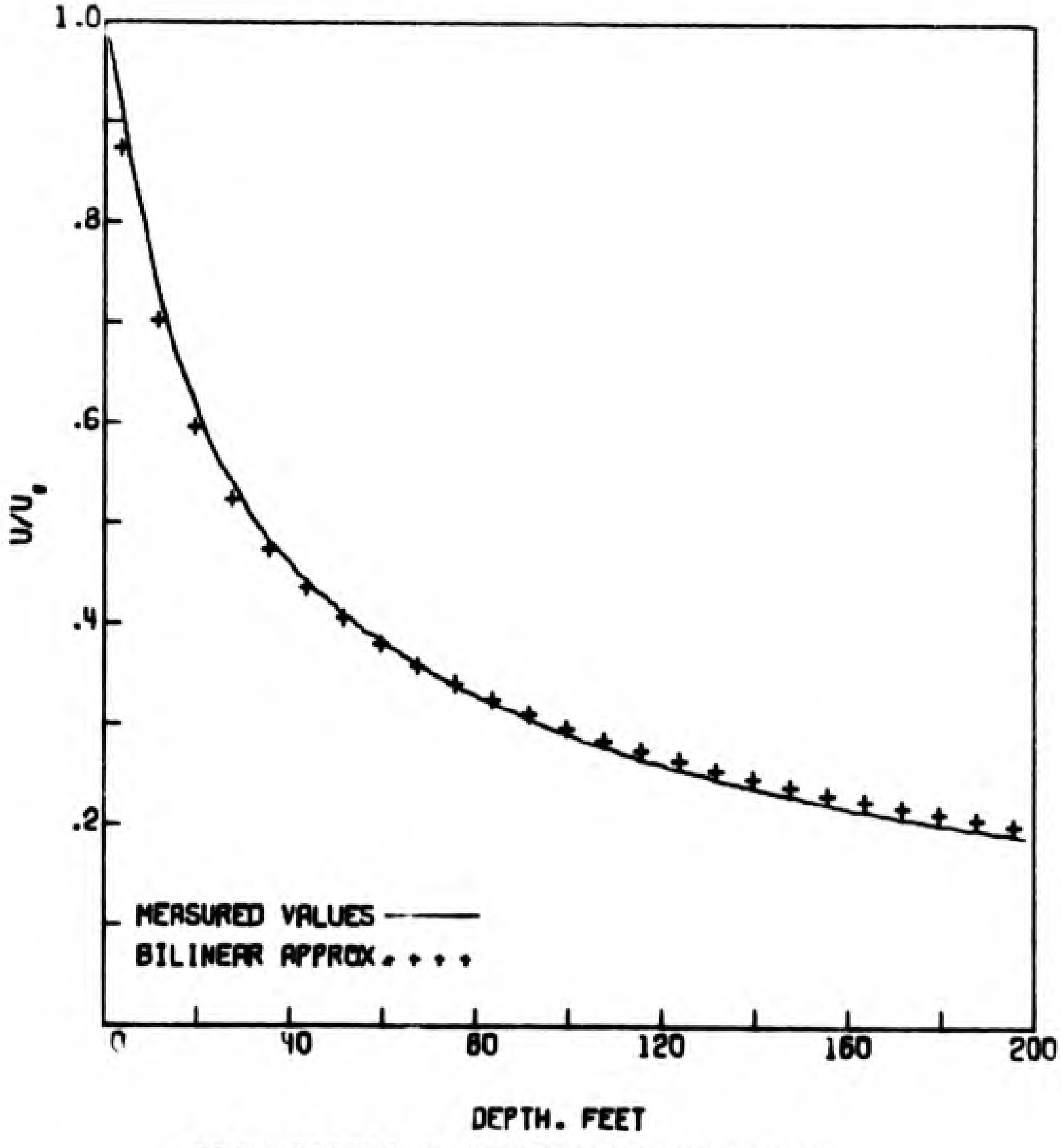
FINAL DISPLACEMENTS, FEET

DEPTH	MEASURED	COMPUTED
0	1.677	.657
25	1.279	.410
50	.945	.292
75	.678	.208
100	.454	.142



SURFACE PARTICLE VELOCITY

PROBLEM 4G -- 19 NOV 1969



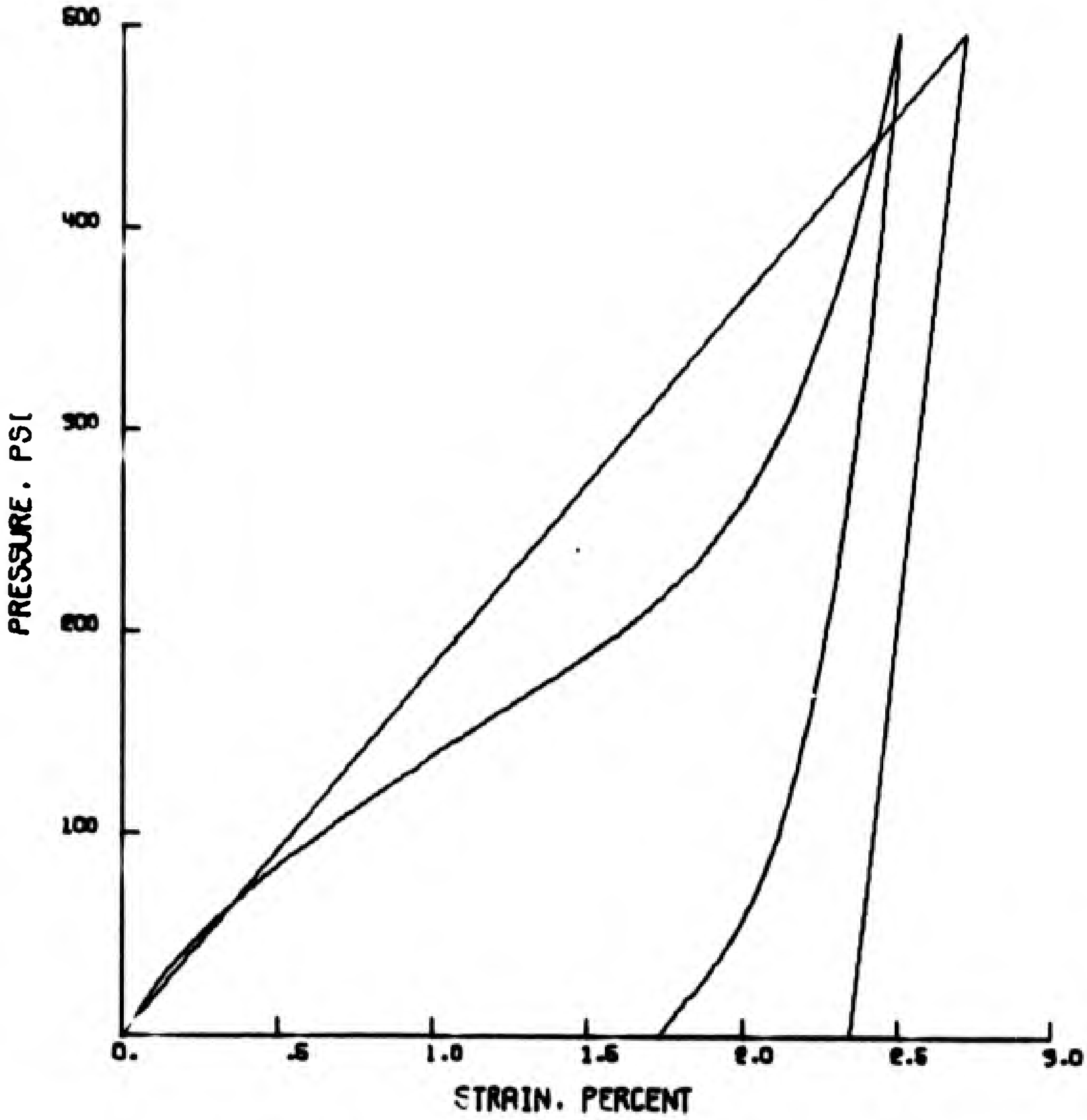
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE - PMAX=500. TD=1.09

HALF LOAD TIME = 4.704830E+00 SEC.

NORMALIZED HALF LOAD TIME = 4.316357E+00

PROBLEM 4G -- 19 NOV 1963



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 1.840698E+04

M2(PSI) = 1.330432E+05

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04

HUGONIOT NONLINEARITY FACTOR = 2.745190E-01

MAXIMUM UNLOAD SLOPE = 1.722649E+05

ZETA = 1.161070E-01

BEST BILINEAR MODEL

PROBLEM SG -- 19 NOV 1969

NUMBER OF DATA POINTS. N=128 . M= 99

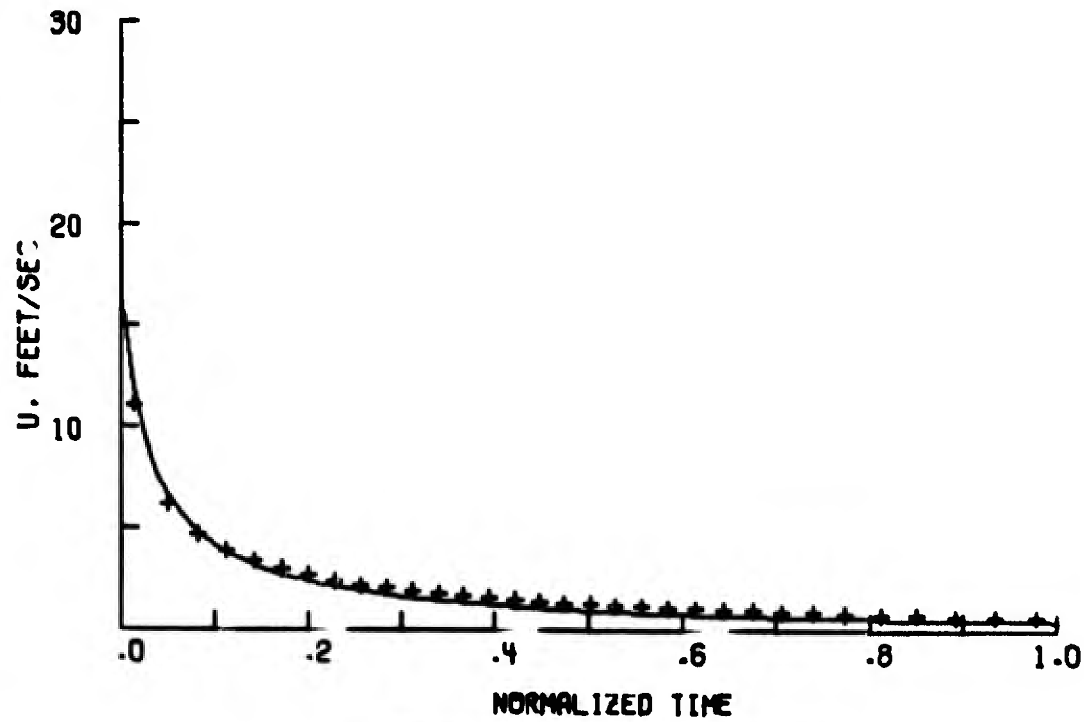
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	8.839971E+02
	SOUND VELOCITY =	2.188652E+03
	ZETA =	4.246025E-01

FITTING ERRORS

E_1 =	9.994636E-01	E_2 =	1.072420E-03
E_3 =	1.246447E-04	E_4 =	1.116444E-02
E_5 =	9.990F60E-01	E_6 =	1.867049E-03
E_7 =	1.128786E-01	E_8 =	3.359741E-01

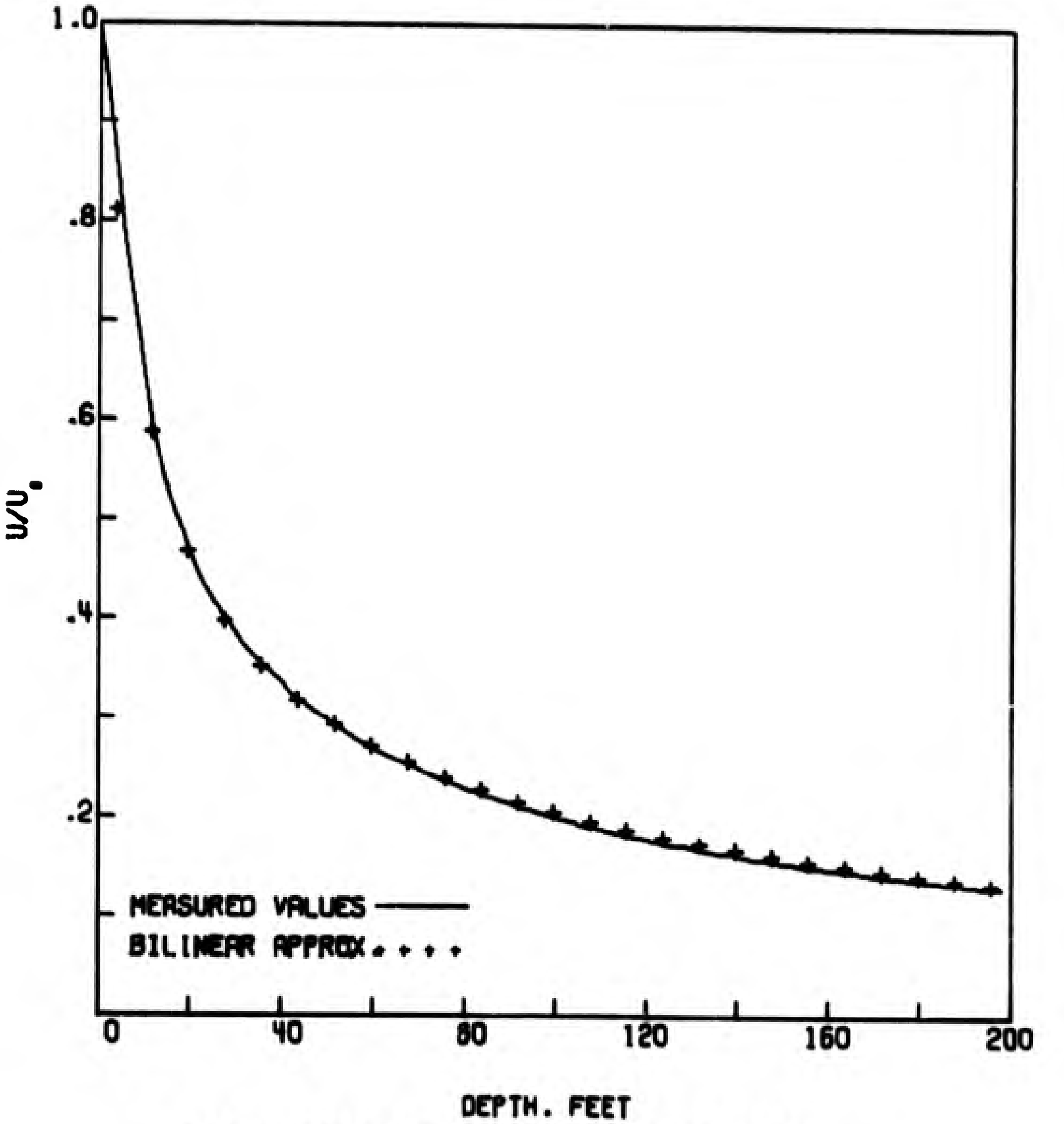
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.158	1.162
25	.838	.816
50	.620	.617
75	.454	.461
100	.318	.329



SURFACE PARTICLE VELOCITY

PROBLEM 5G -- 19 NOV 1969



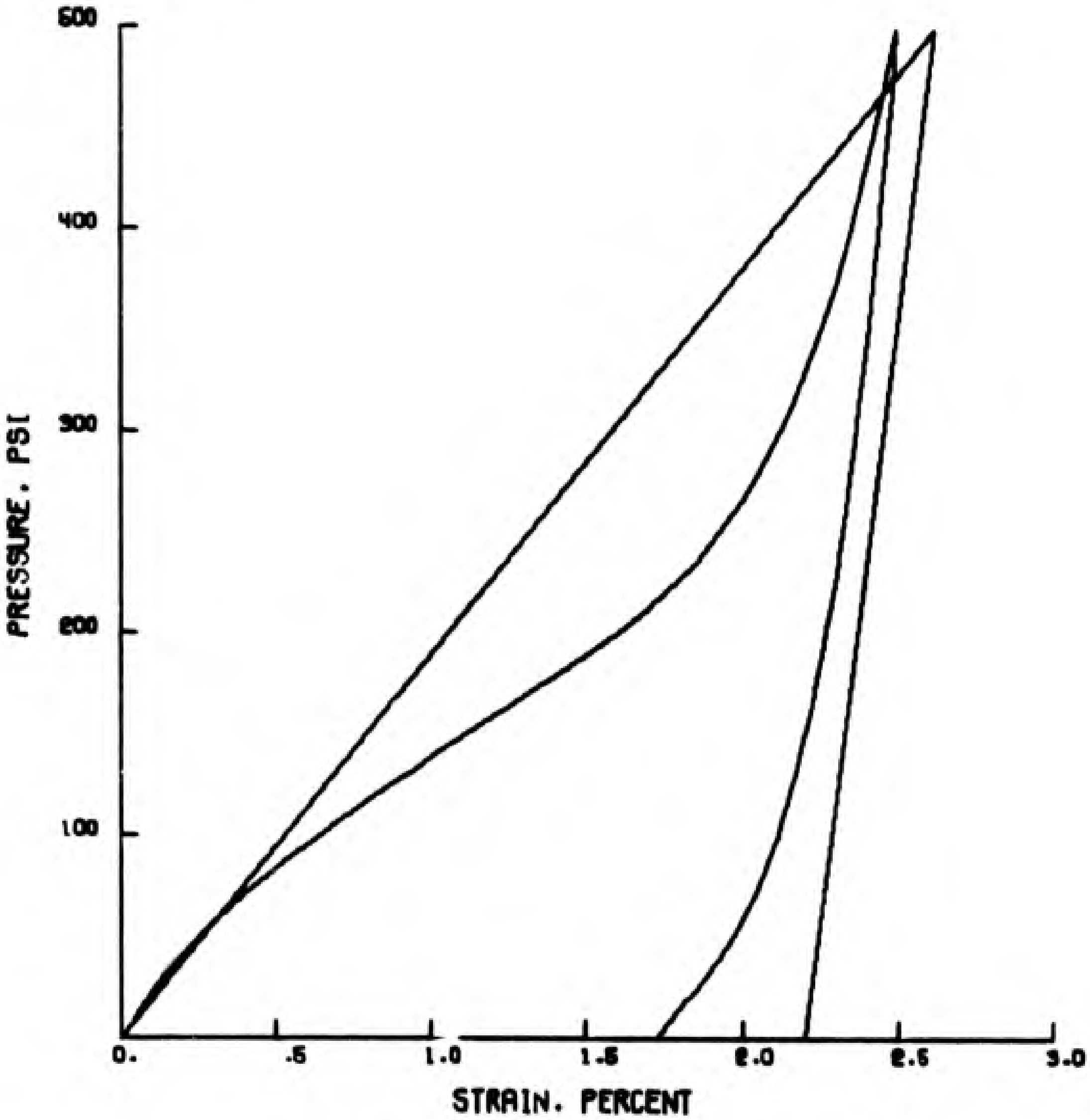
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE - PMAX=500. TD=1.09

HALF LOAD TIME = 3.531115E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM 5G -- 19 NOV 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =1.904?86E+04
M2(PSI) =1.167610E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04
HUGONIOT NONLINEARITY FACTOR = 2.745190E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161070E-01

BEST BILINEAR MODE

PROBLEM 7C -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. N= 89 . M= 99

MATERIAL PROPERTIES

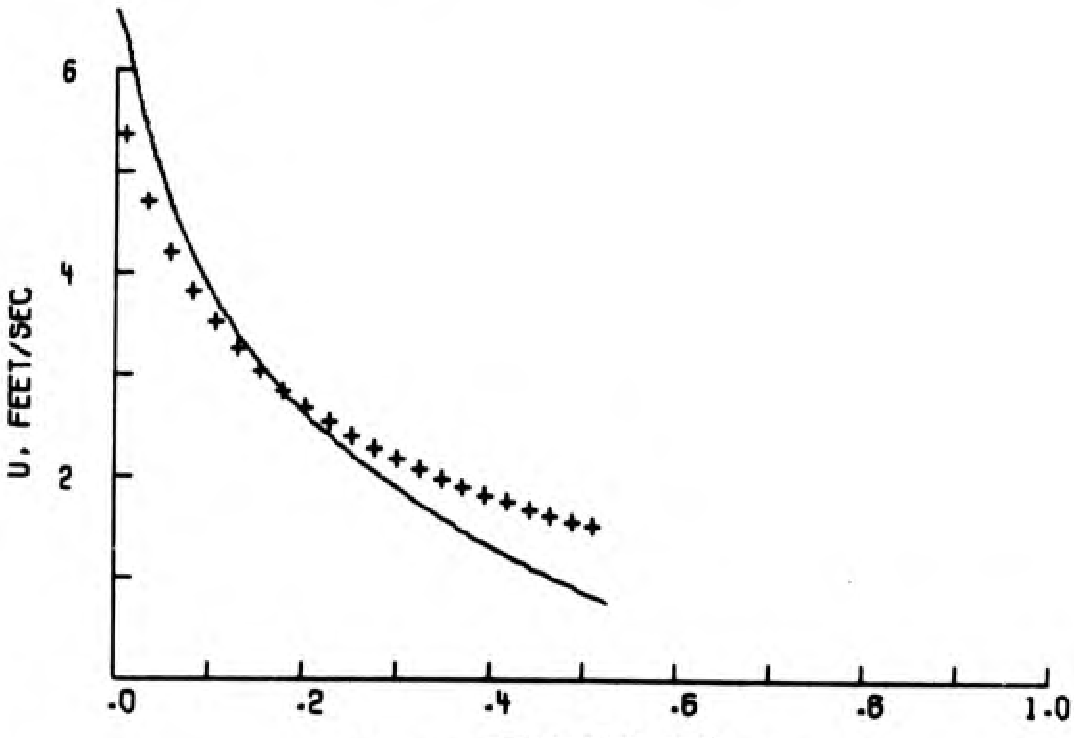
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.406883E+03
SOUND VELOCITY =	2.414007E+04
ZETA =	8.898590E-01

FITTING ERRORS

E_1 =	9.994409E-01	E_2 =	1.117935E-03
E_3 =	1.811030E-05	E_4 =	4.266620E-03
E_5 =	9.995985E-01	E_6 =	8.029066E-04
E_7 =	2.138982E-01	E_8 =	4.624913E-01

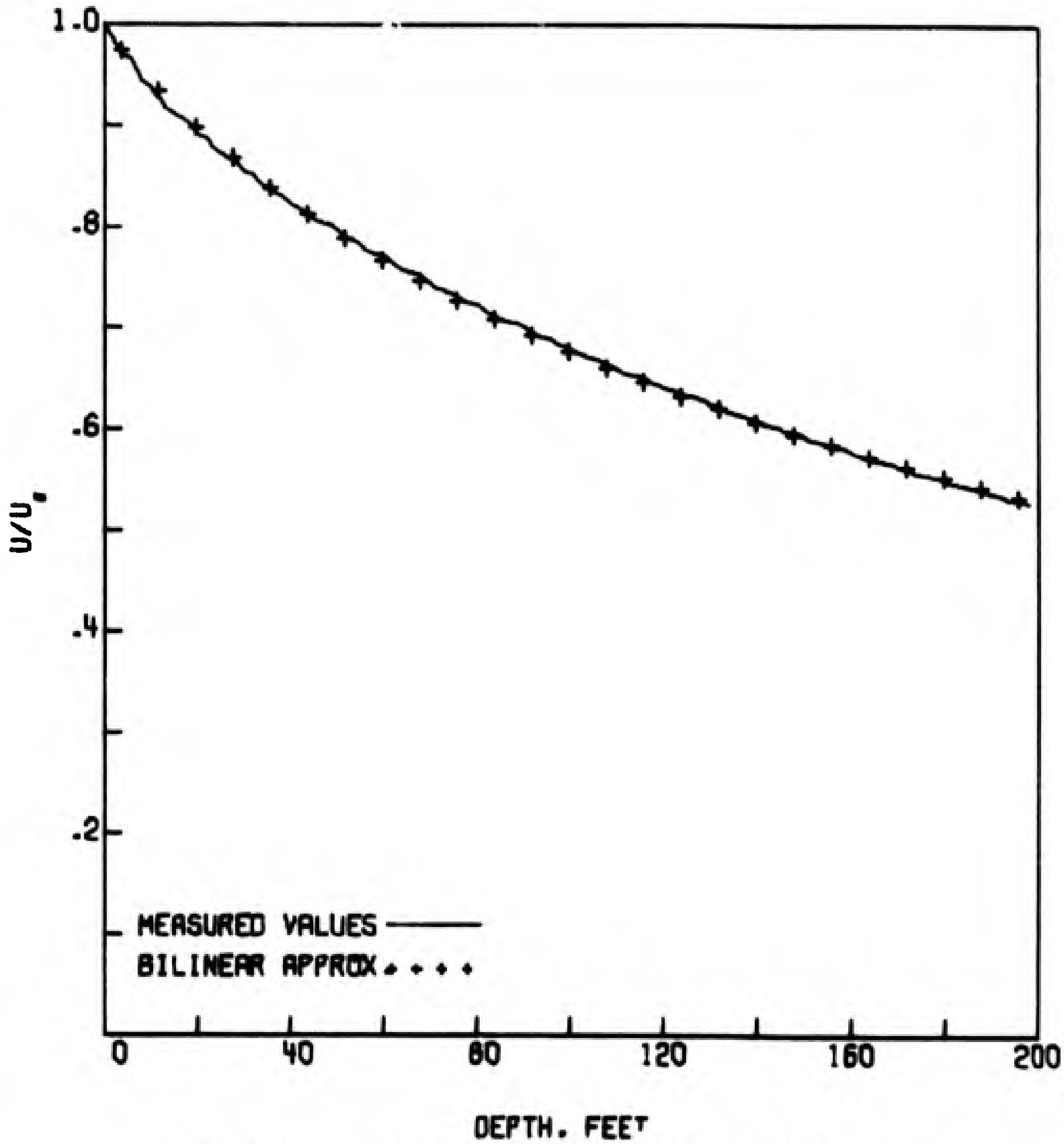
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.800	.746
25	.716	.649
50	.626	.563
75	.536	.484
100	.446	.412



SURFACE PARTICLE VELOCITY

PROBLEM 7C -- 17 FEB 1970 $.



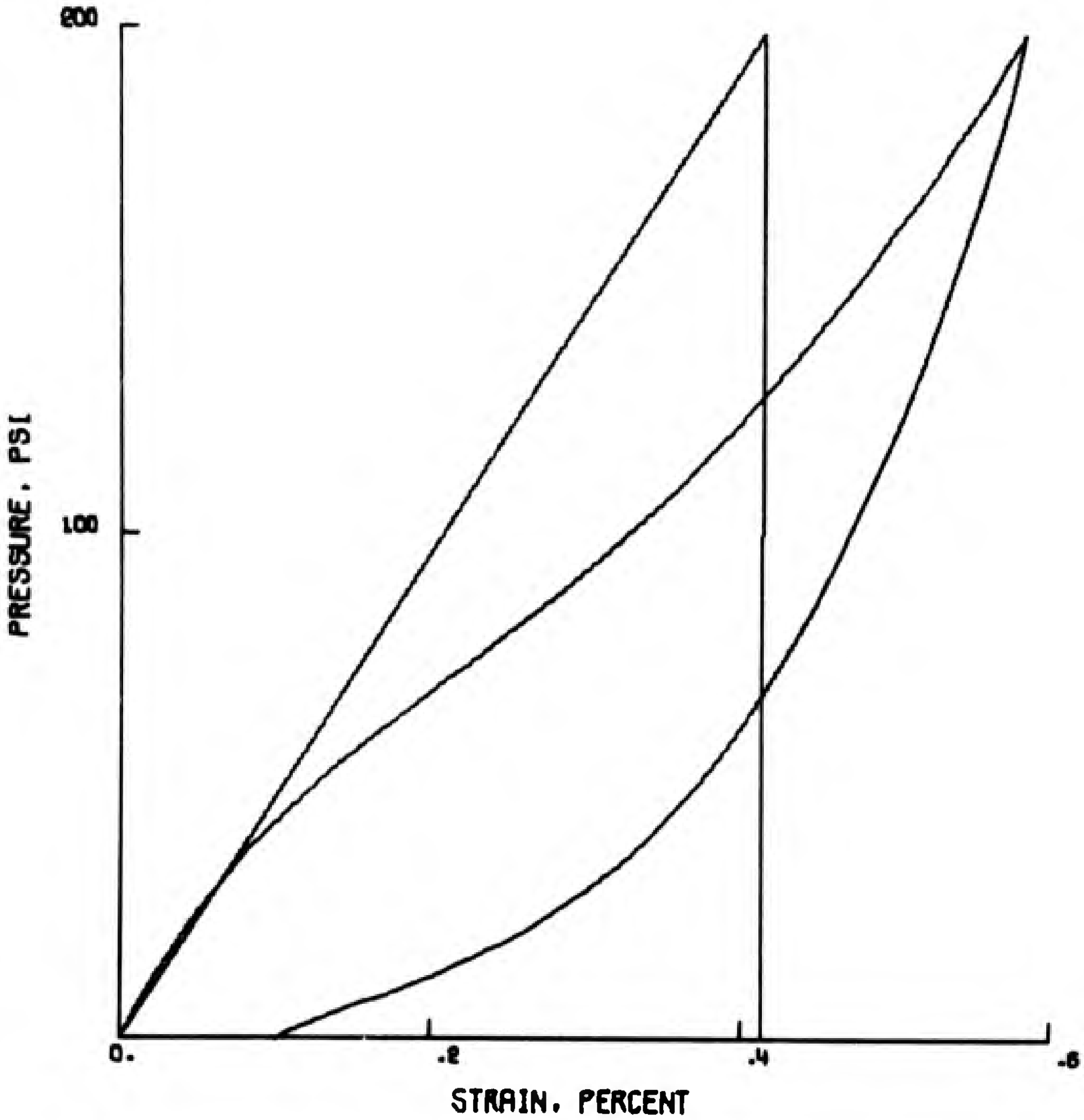
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 7C -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.62'+595E+04
M2(PSI) =1.420436E+07

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 3.432298E+04
HUGONIOT NONLINEARITY FACTOR = 4.106091E-02
MAXIMUM UNLOAD SLOPE = 1.105115E+06
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 8C -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. N= 82 . M= 99

MATERIAL PROPERTIES

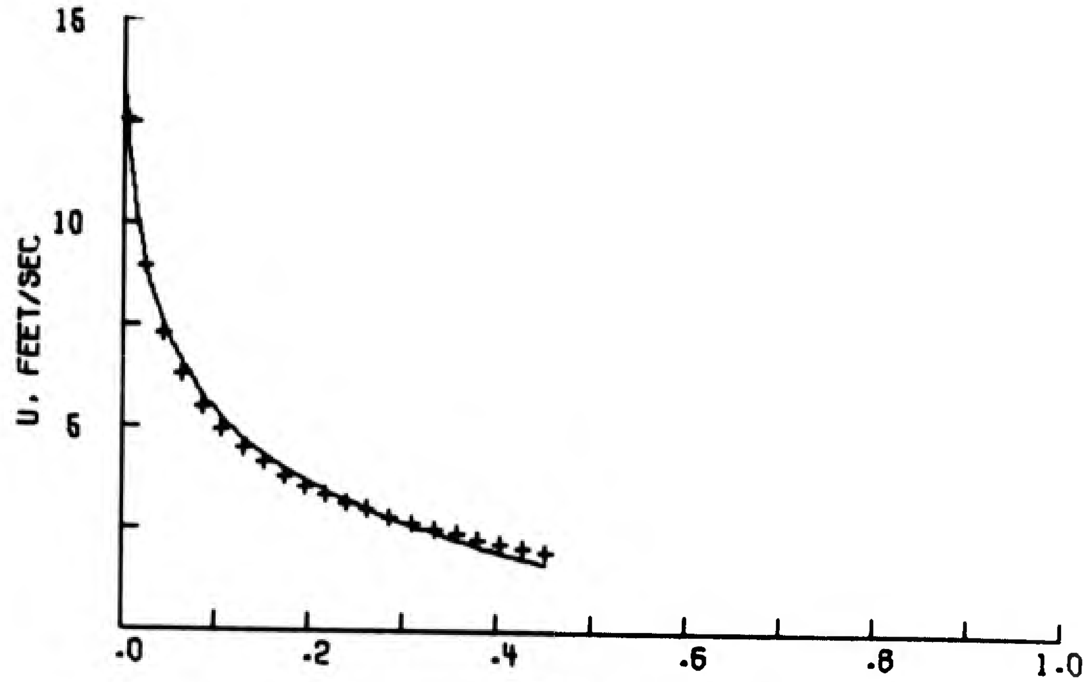
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.367490E+03
SOUND VELOCITY =	1.998965E+03
ZETA =	1.875785E-01

FITTING ERRORS

E_1 =	9.989516E-01	E_2 =	2.095984E-03
E_3 =	4.397233E-05	E_4 =	6.631164E-03
E_6 =	9.964426E-01	E_8 =	7.102426E-03
E_7 =	4.709174E-02	E_9 =	2.170063E-01

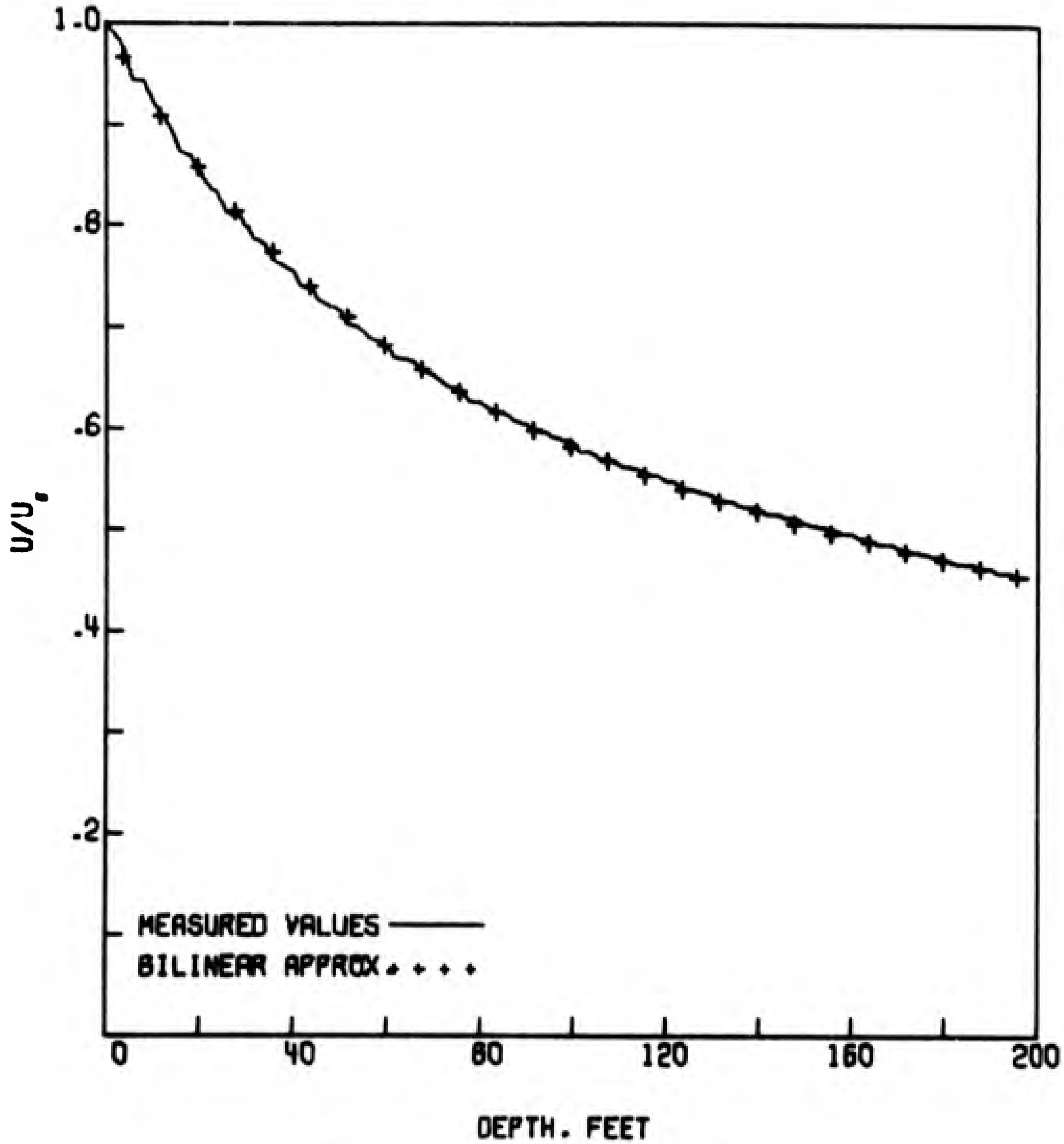
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.264	1.246
25	1.163	1.094
50	1.028	.961
75	.900	.840
100	.772	.727



SURFACE PARTICLE VELOCITY

PROBLEM 8C -- 17 FEB 1970 $.



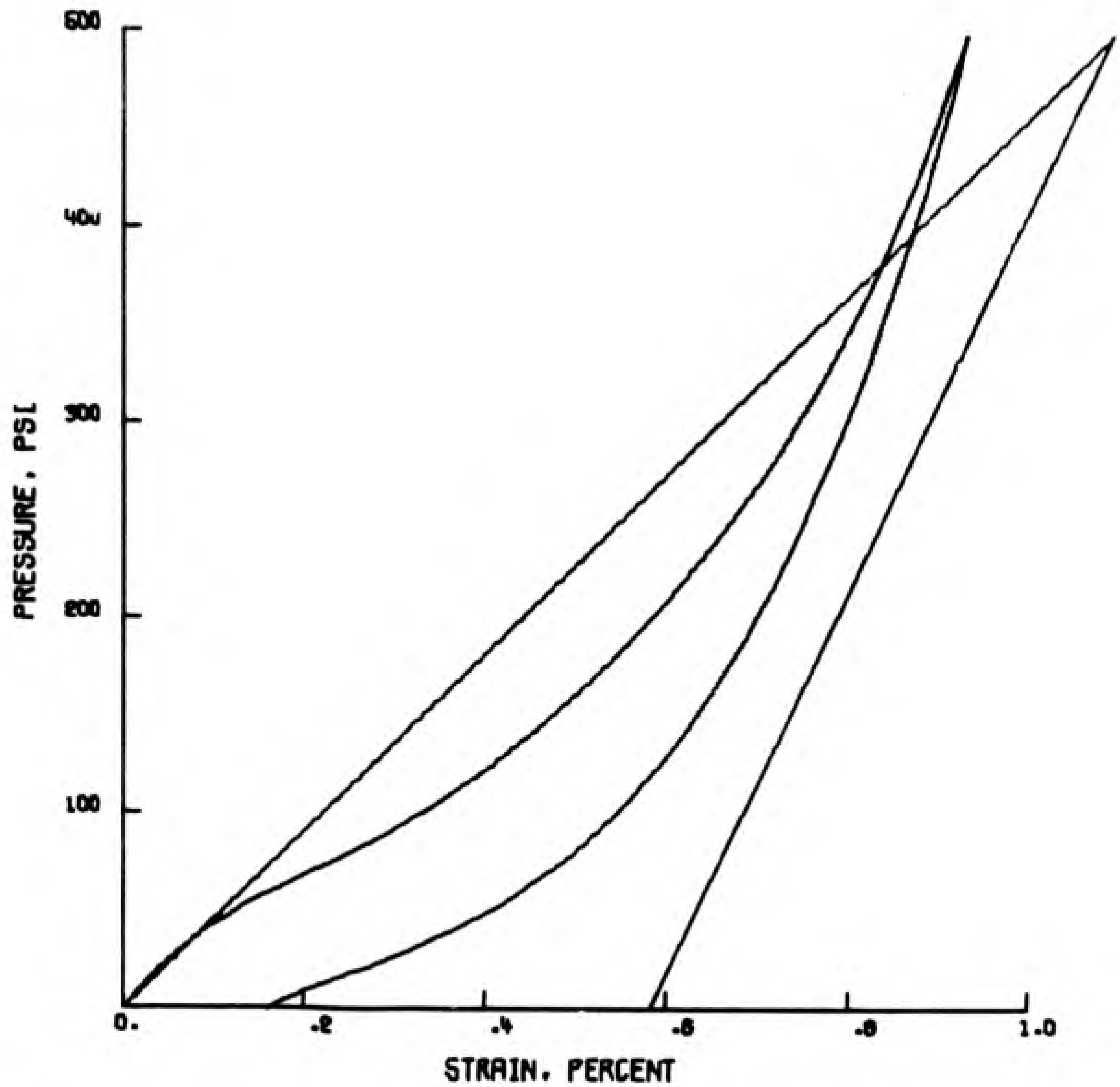
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.631964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 8C -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.558197E+04
M2(PSI) =9.739910E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 3.105830E-01

BEST BILINEAR MODEL

PROBLEM 9C -- 17 FEB 1970

NUMBER OF DATA POINTS. N= 69 . M= 99

MATERIAL PROPERTIES

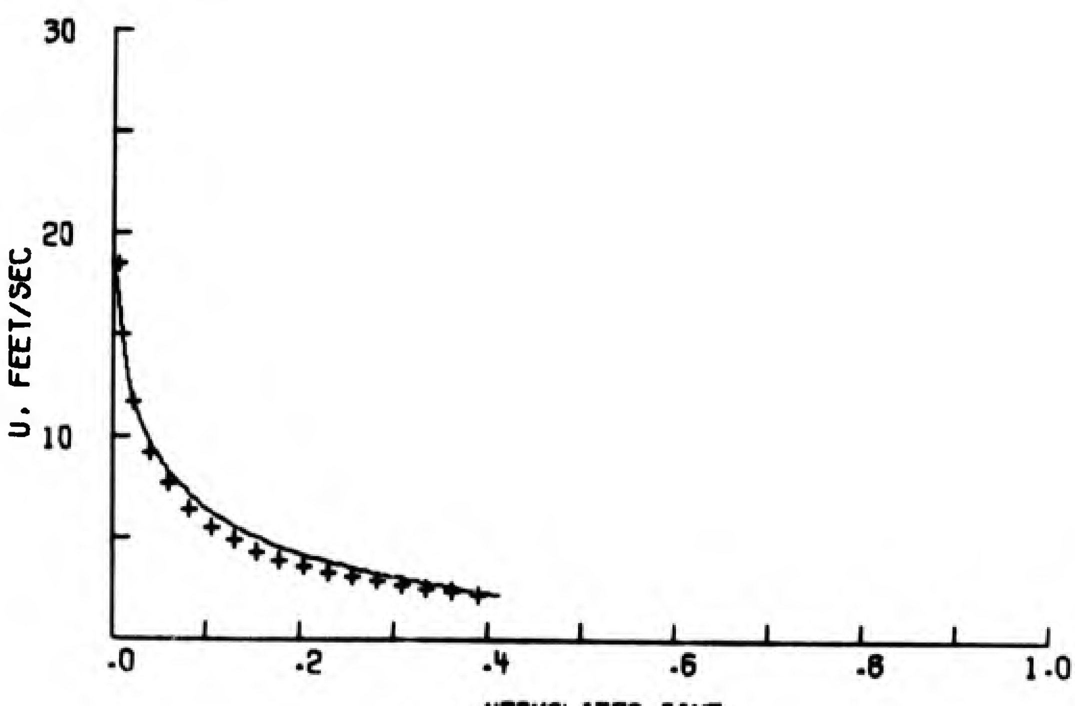
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.580583E+03
	SOUND VELOCITY =	2.054468E+03
	ZETA =	1.303655E-01

FITTING ERRORS

E_1 =	9.976822E-01	E_2 =	4.630293E-03
E_3 =	1.319095E-04	E_4 =	1.148518E-02
E_5 =	9.935535E-01	E_6 =	1.285149E-02
E_7 =	4.375344E-01	E_8 =	6.614636E-01

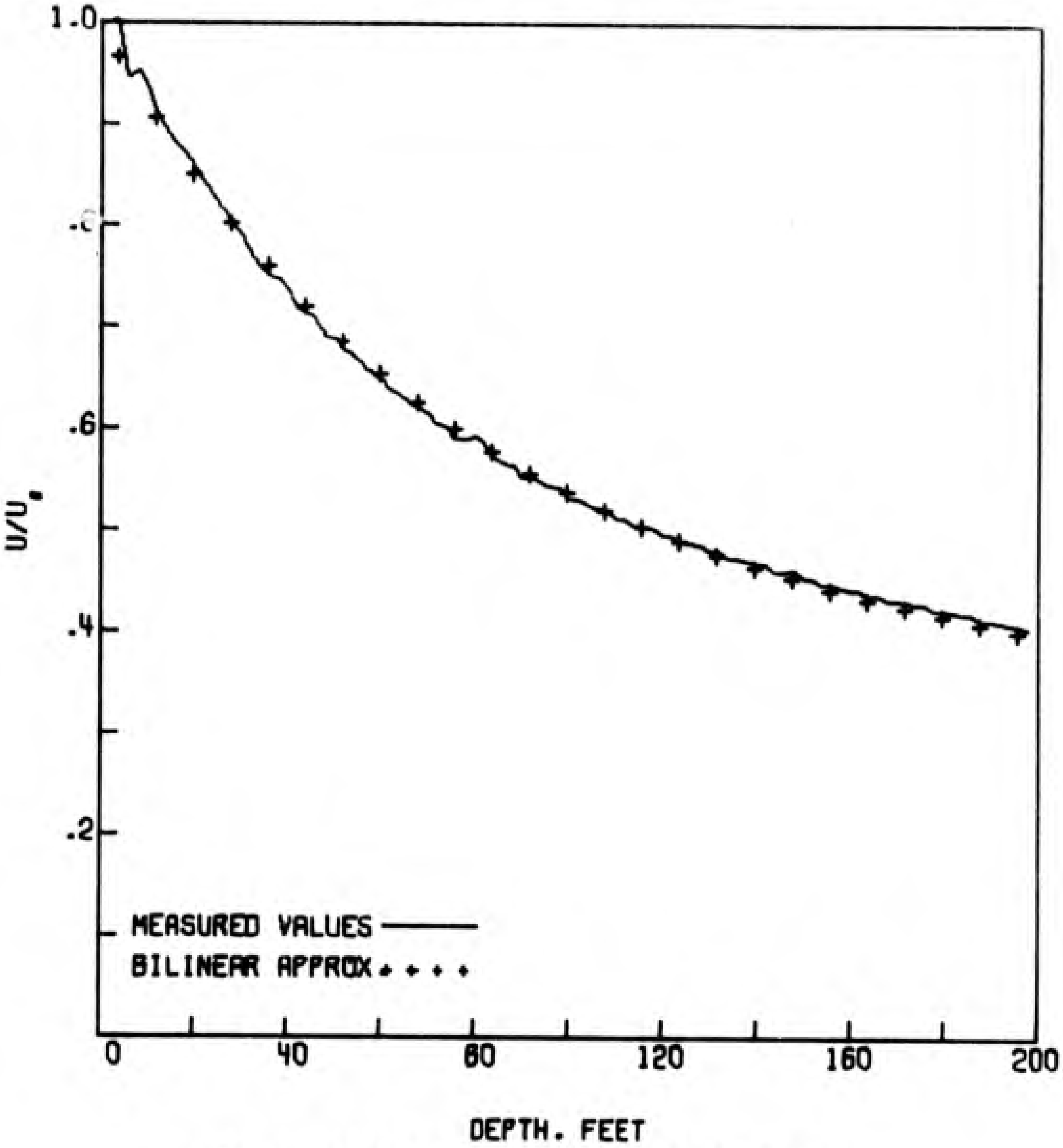
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.641	1.587
25	1.534	1.414
50	1.403	1.264
75	1.263	1.129
100	1.116	1.005



SURFACE PARTICLE VELOCITY

PROBLEM 9C -- 17 FEB 1970 $.



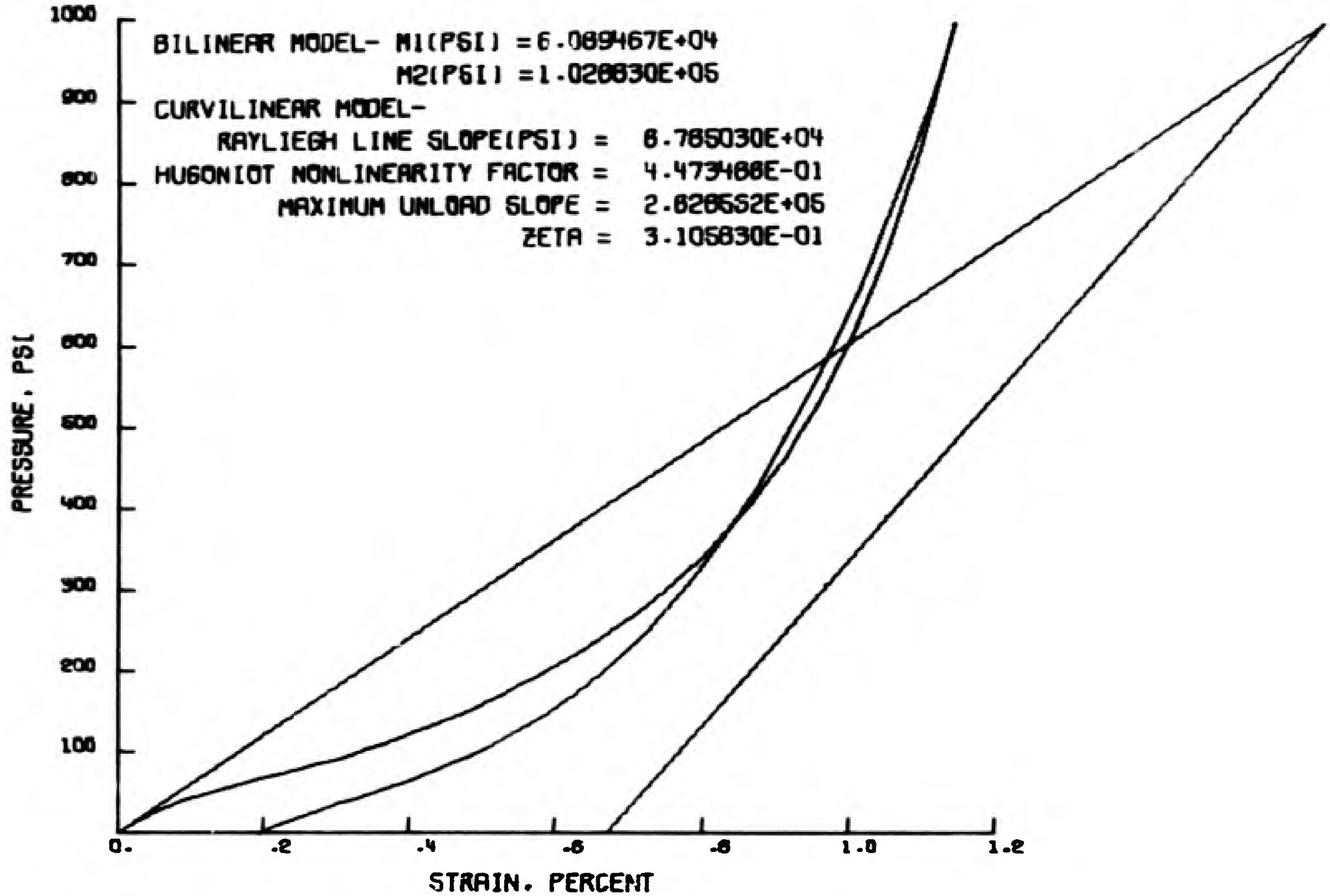
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 9C -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 10C -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. N= 57 . M= 99

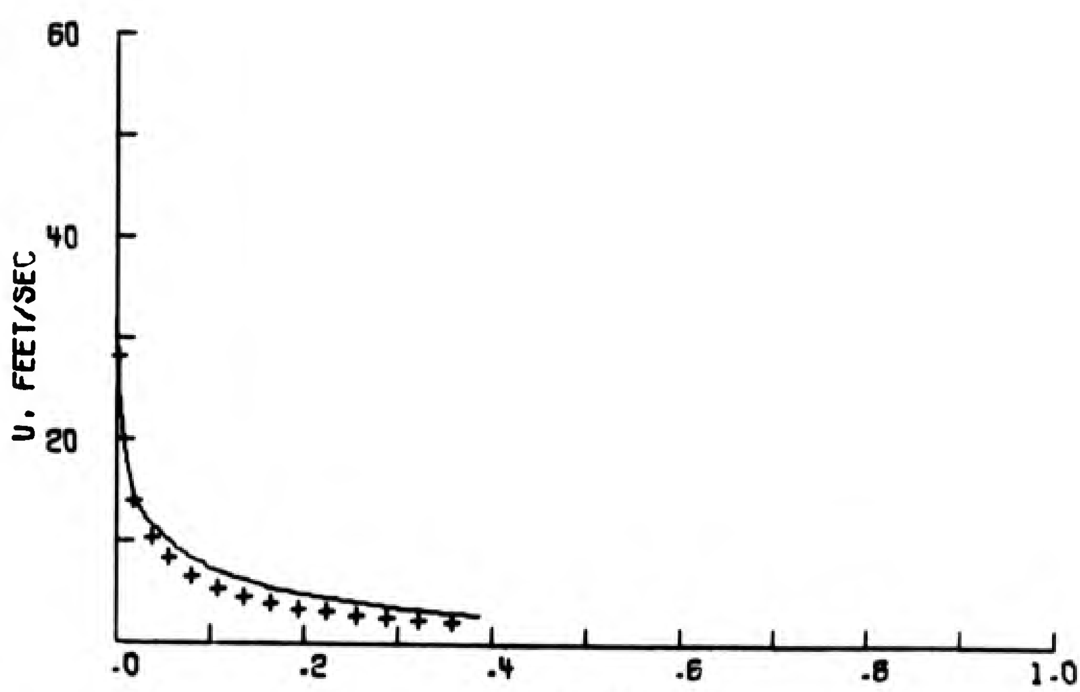
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	2.038103E+03
	SOUND VELOCITY =	2.510669E+03
	ZETA =	1.097406E-01

FITTING ERRORS

E_1 =	9.968034E-01	E_2 =	6.382958E-03
E_3 =	2.302106E 04	E_4 =	1.517269E-02
E_5 =	9.939617E-01	E_6 =	1.204023E-02
E_7 =	2.407342E+00	E_8 =	1.551561E+00

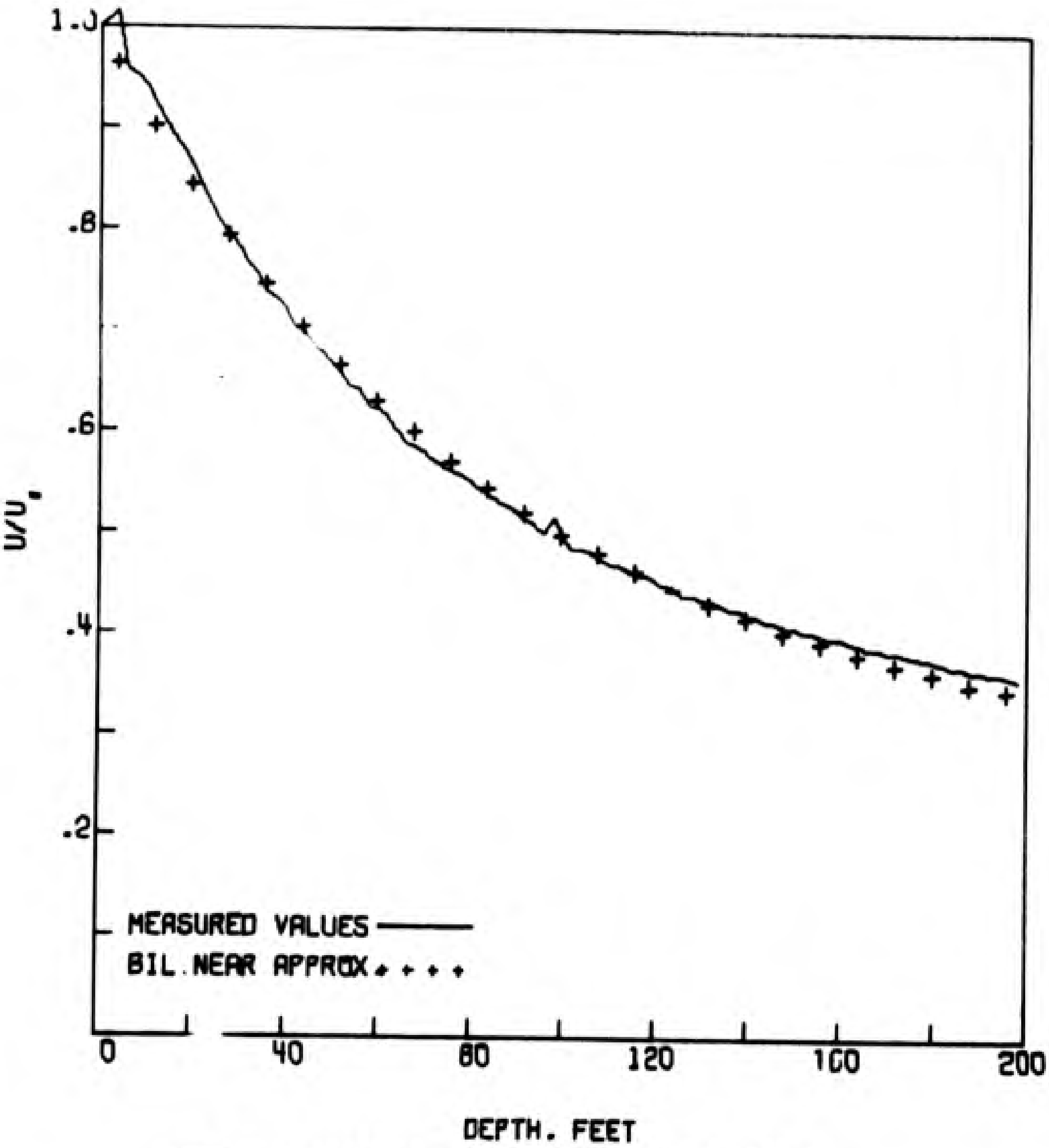
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.012	1.798
25	1.318	1.622
50	1.737	1.473
75	1.660	1.343
100	1.509	1.227



SURFACE PARTICLE VELOCITY

PROBLEM 10C -- 17 FEB 1970 $.



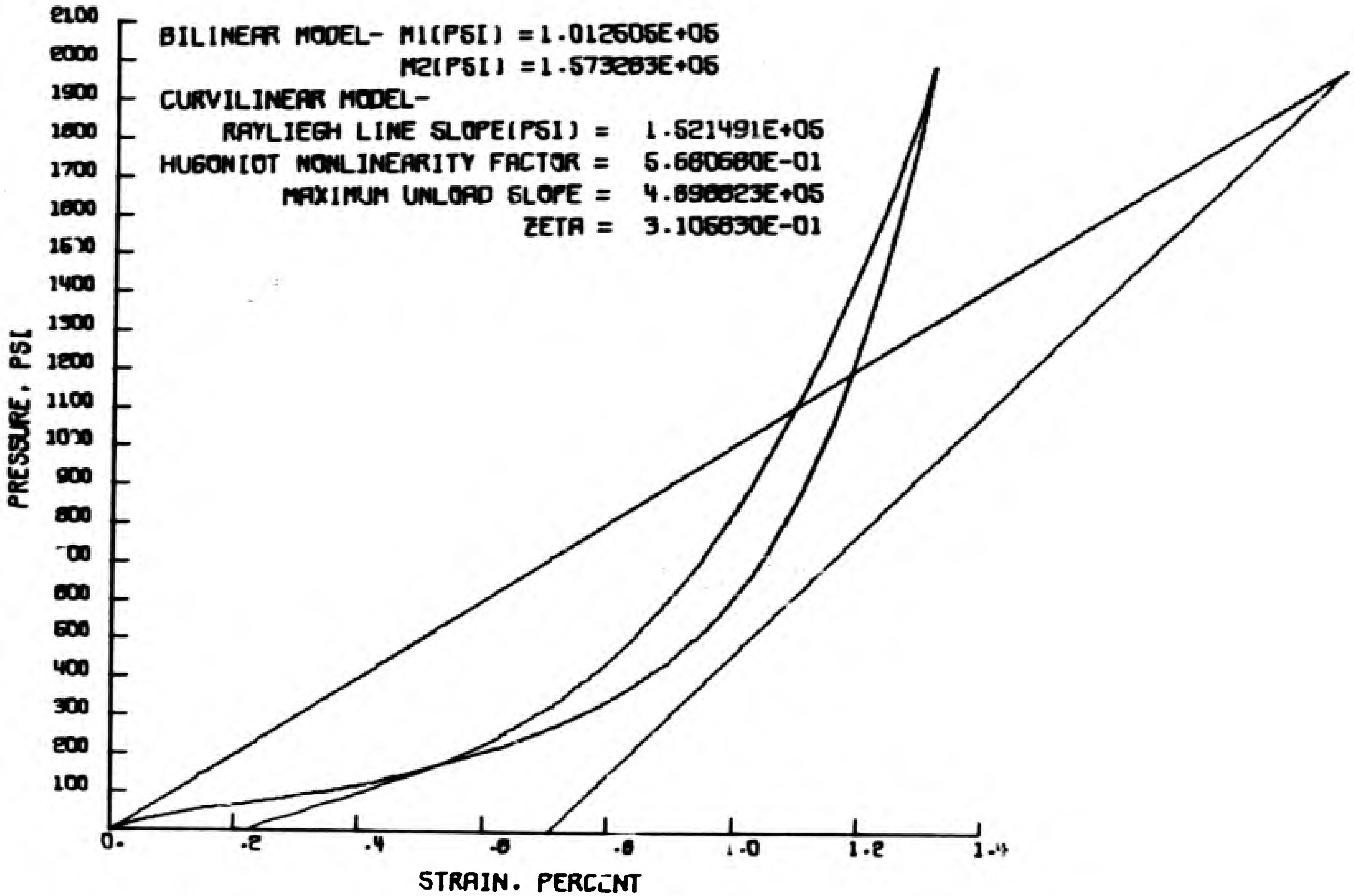
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM 10C -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 70 -- 16 DEC 1969

NUMBER OF DATA POINTS. N= 86 . M= 99

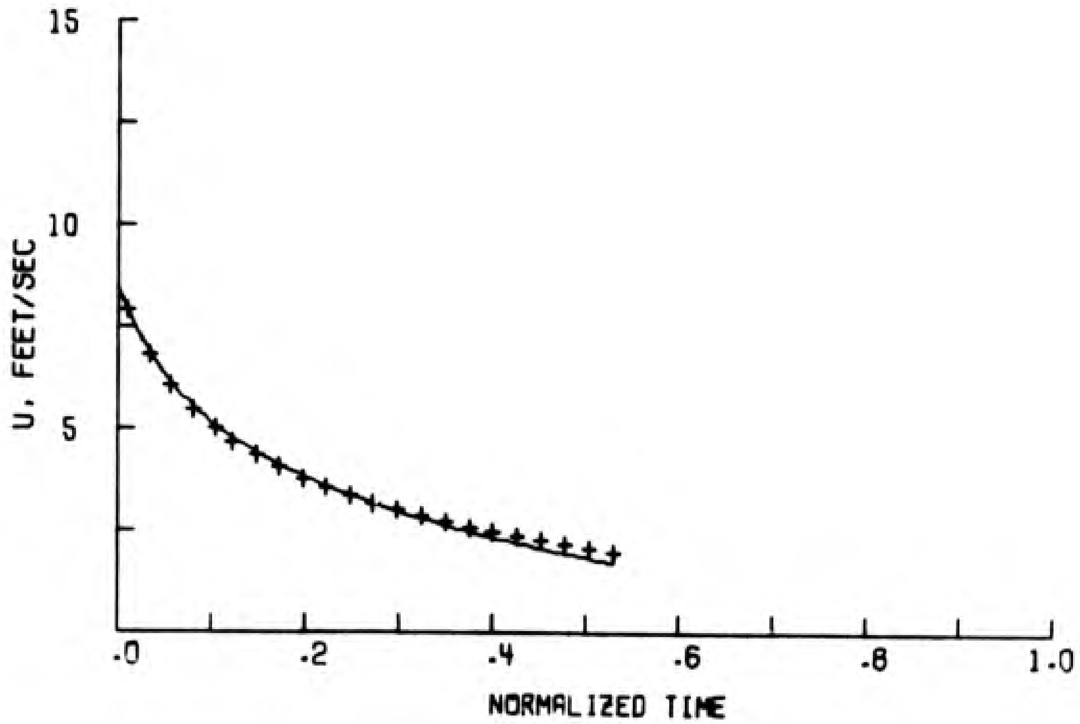
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	9.414798E+02
	SOUND VELOCITY =	1.998032E+03
	ZETA =	3.594312E-01

FITTING ERRORS

E_1 =	9.996830E-01	E_2 =	6.338156E-04
E_3 =	4.548794E-05	E_4 =	6.744474E-03
E_5 =	9.991165E-01	E_6 =	1.766120E-03
E_7 =	1.561318E-02	E_8 =	1.249527E-01

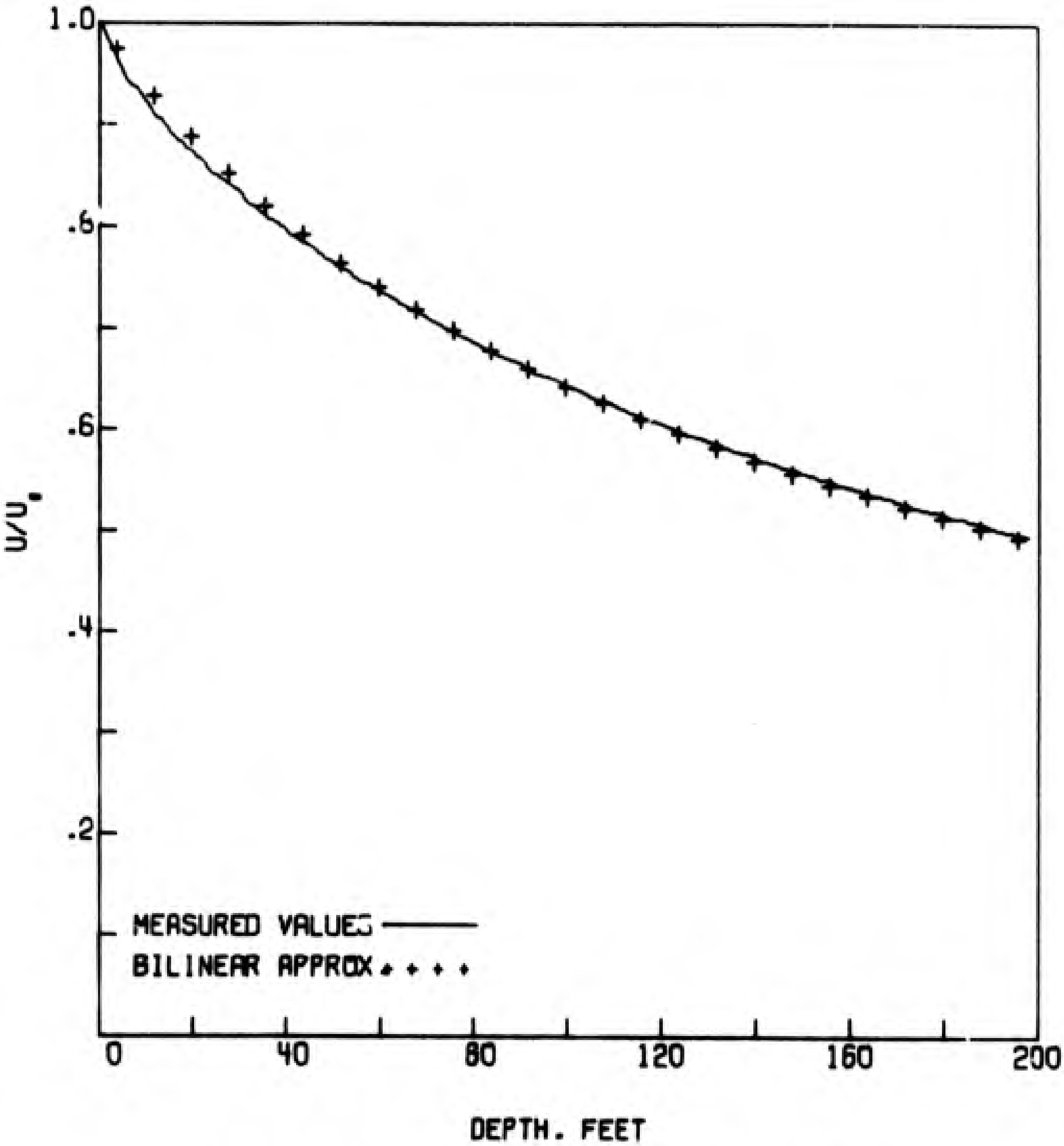
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.066	1.065
25	.911	.885
50	.750	.724
75	.595	.576
100	.445	.438



SURFACE PARTICLE VELOCITY

PROBLEM 7D -- 16 DEC 1969



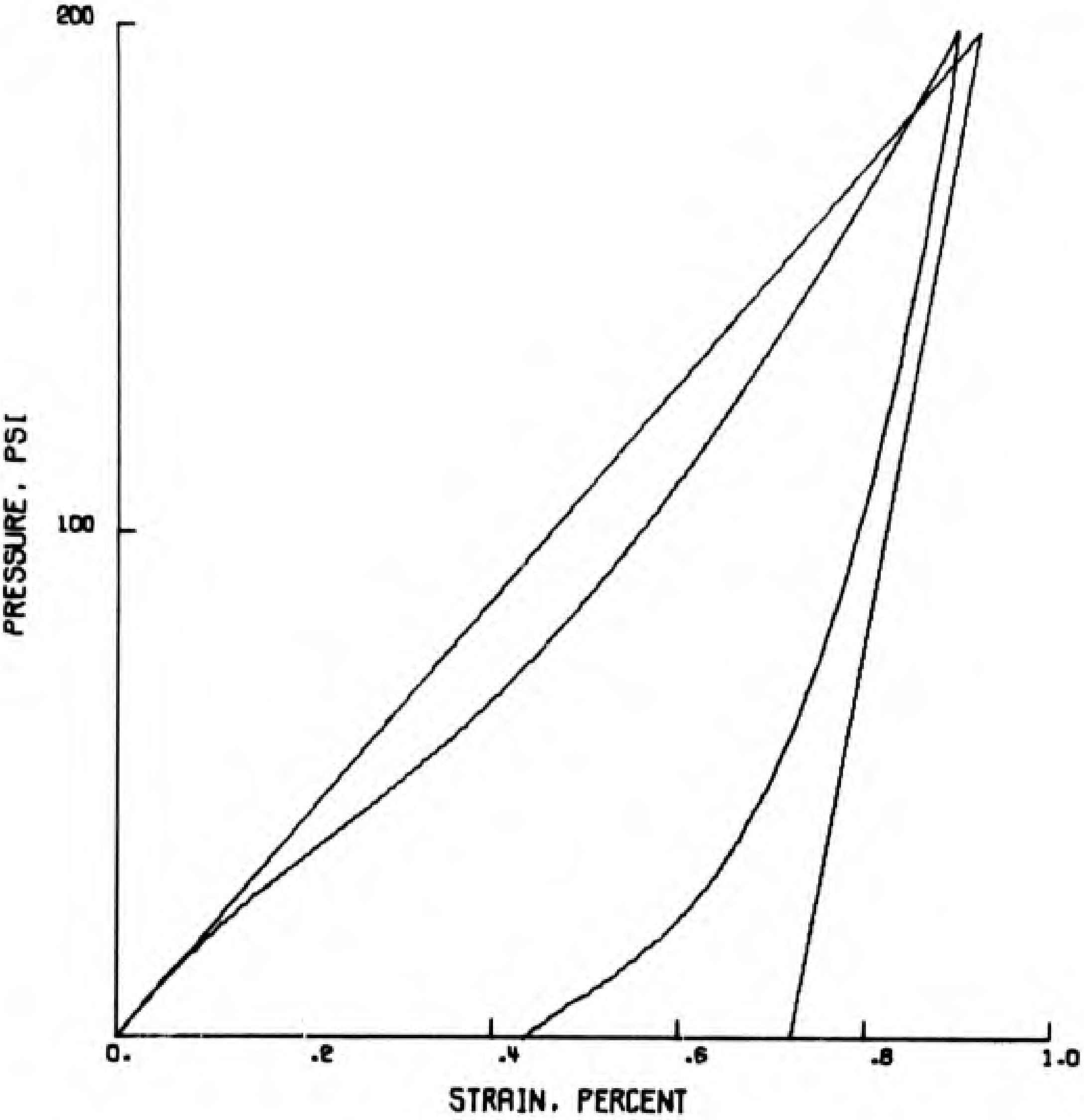
PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 7D -- 16 DEC 1969



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =2.160561E+04
M2(PSI) =9.730823E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.218672E+04
HUGONIOT NONLINEARITY FACTOR = 1.387057E-01
MAXIMUM UNLOAD SLOPE = 1.146531E+05
ZETA = 1.935118E-01

BEST BILINEAR MODEL

PROBLEM 8D -- 16 DEC 1969

NUMBER OF DATA POINTS. N= 82 . M= 99

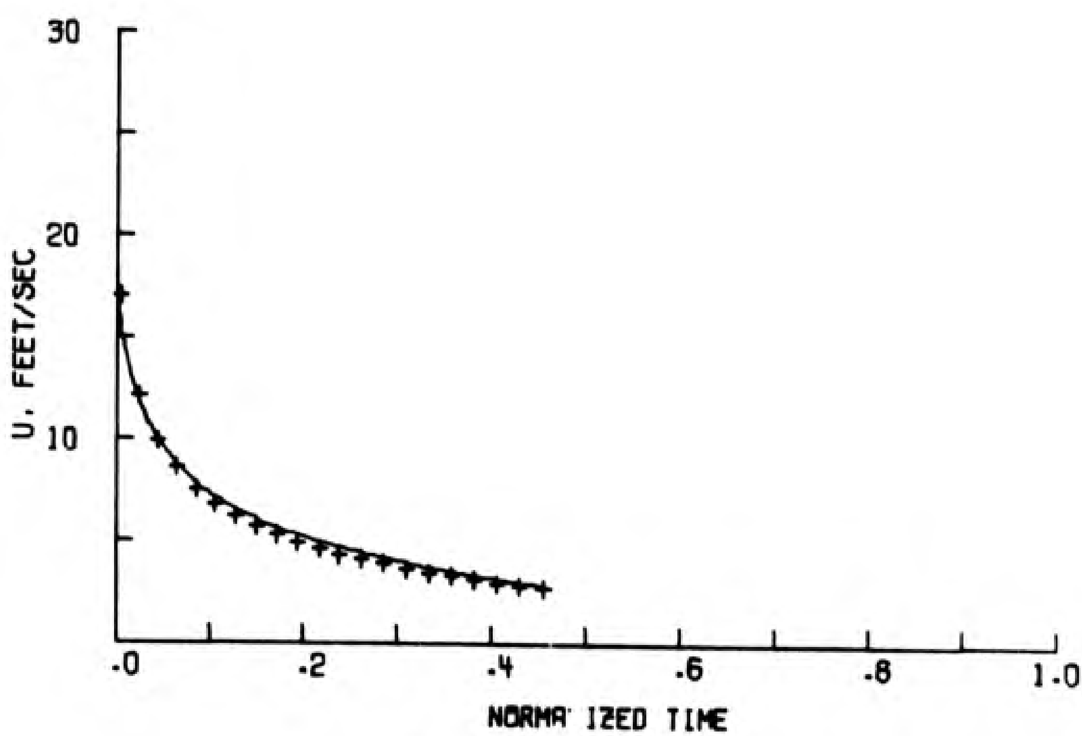
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.013448E+03
	SOUND VELOCITY =	1.540663E+03
	ZETA =	2.064183E-01

FITTING ERRORS

E_1 =	9.993938E-01	E_2 =	1.212036E-03
E_3 =	7.782300E-05	E_4 =	8.821735E-03
E_5 =	9.979969E-01	E_6 =	4.002128E-03
E_7 =	1.382808E-01	E_8 =	3.718613E-01

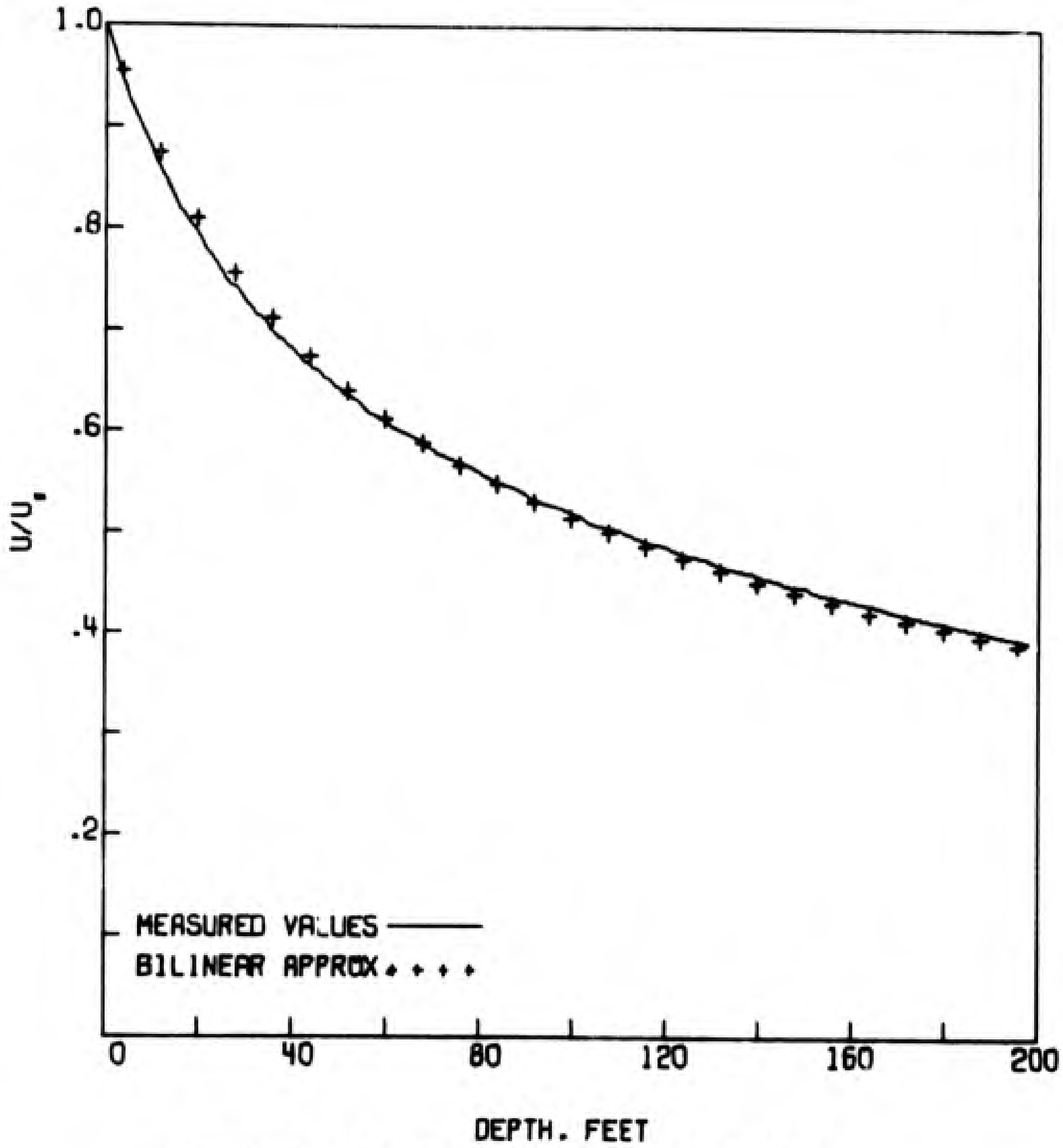
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.693	1.702
25	1.461	1.421
50	1.225	1.185
75	1.000	.972
100	.784	.773



SURFACE PARTICLE VELOCITY

PROBLEM 8D -- 16 DEC 1969



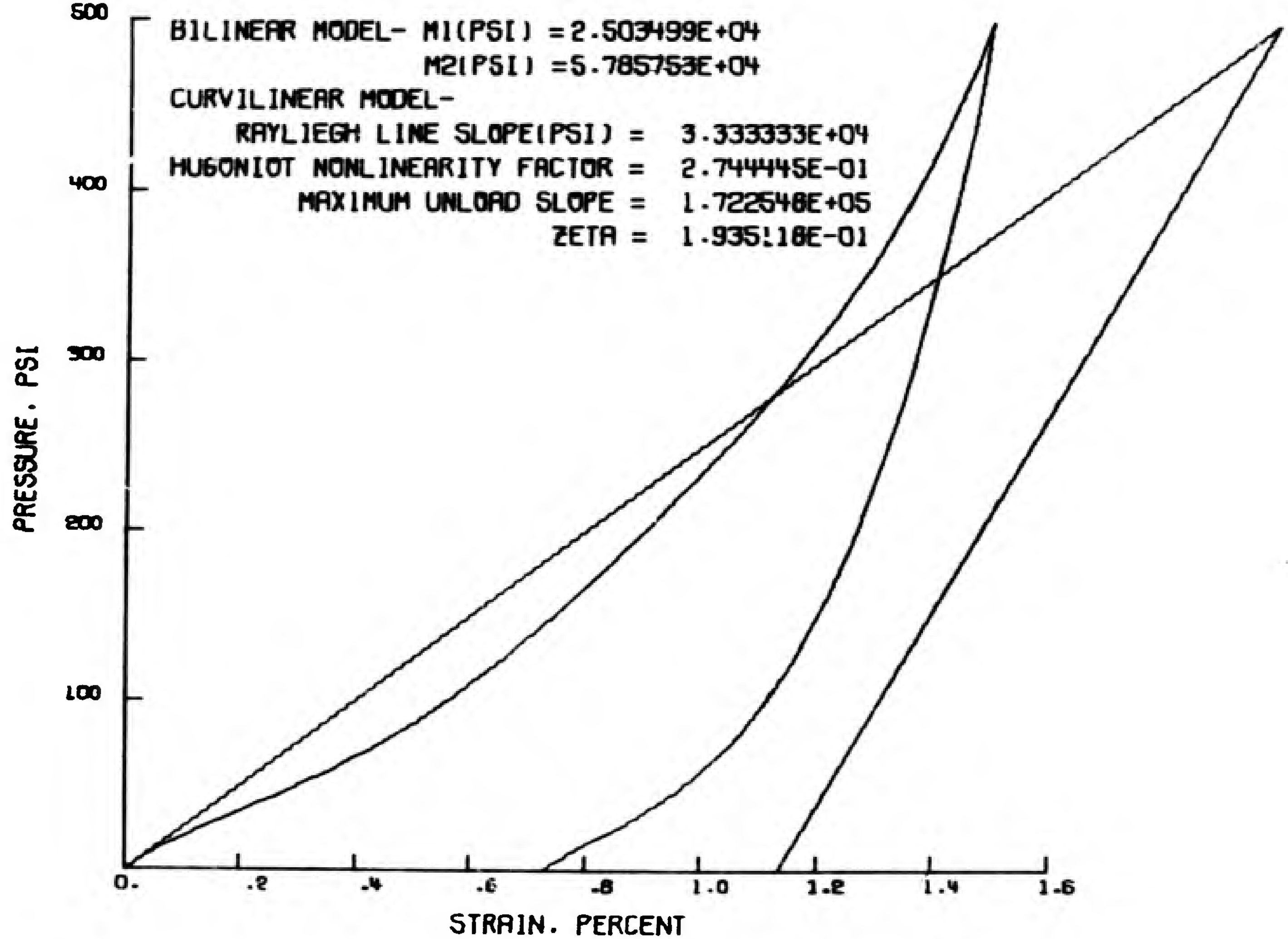
PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 9.545150E-01 SEC.

NORMALIZED HA F LOAD TIME = 8.757018E-01

PROBLEM 8D -- 16 DEC 1969



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 9D -- 16 DEC 1969

NUMBER OF DATA POINTS. N= 69 . M= 99

MATERIAL PROPERTIES

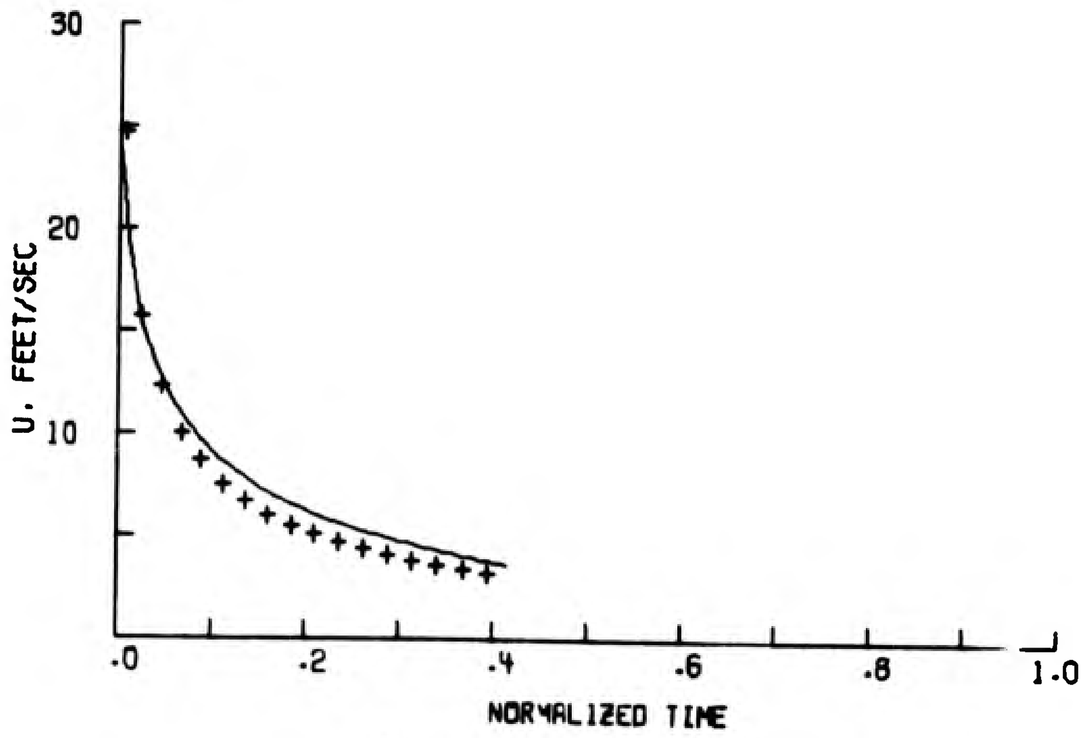
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.159796E+03
SOUND VELOCITY =	1.583515E+03
ZETA =	1.544551E-01

FITTING ERRORS

E_1 =	9 989699E-01	E_2 =	2.059046E-03
E_3 =	1.045317E-04	E_4 =	1.022408E-02
E_5 =	9.923332E-01	E_6 =	1.527486E-02
E_7 =	1.390913E+00	E_8 =	1.179370E+00

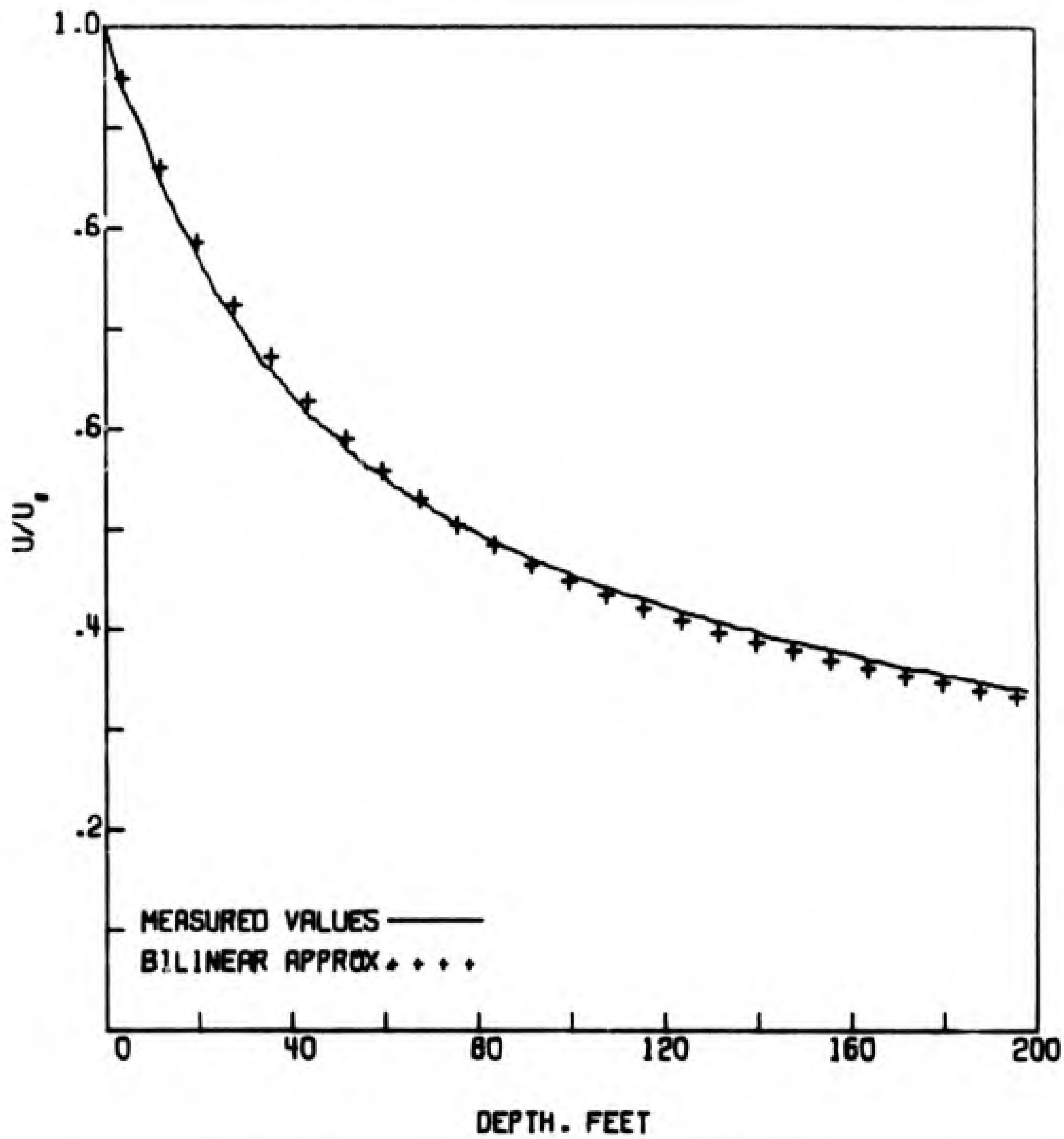
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.271	2.218
25	2.007	1.876
50	1.722	1.598
75	1.444	1.355
100	1.173	1.133



SURFACE PARTICLE VELOCITY

PROBLEM 9D - 16 DEC 1969



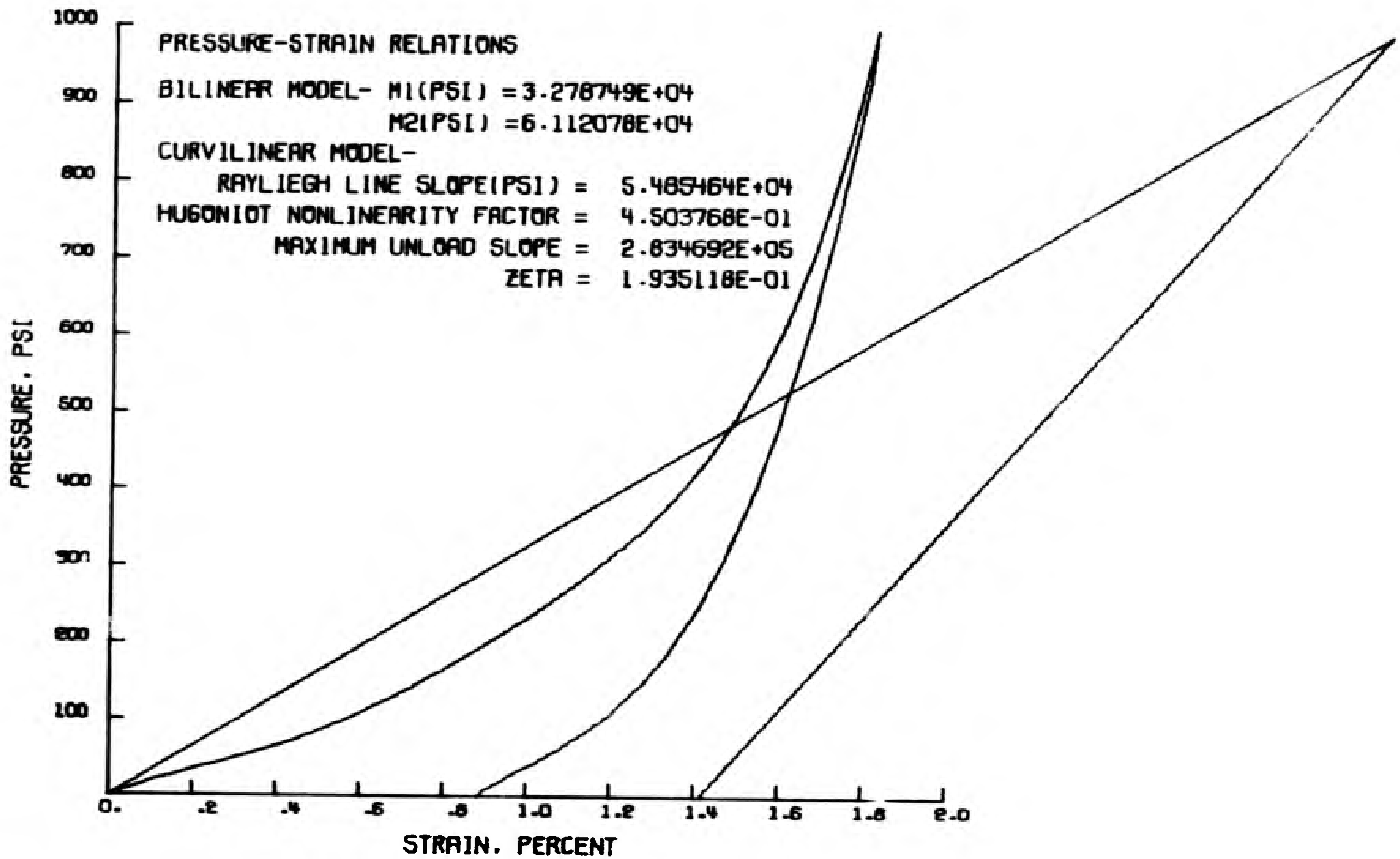
PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 4.207066E+00 SEC.

NORMALIZED HALF LOAD TIME = 3.505888E+00

PROBLEM 9D -- 16 DEC 1969



BEST BILINEAR MODEL

PROBLEM 10D -- 16 DEC 1969

NUMBER OF DATA POINTS. N= 56 . M= 99

MATERIAL PROPERTIES

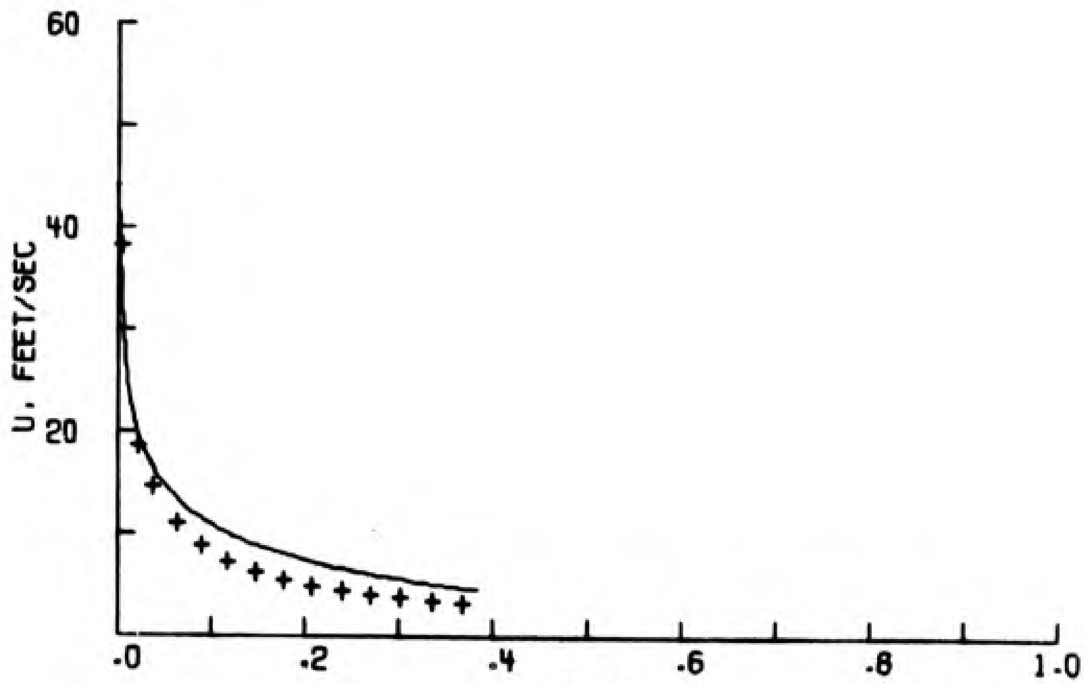
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.453719E+03
	SOUND VELOCITY =	1.877652E+03
	ZETA =	1.272550E-01

FITTING ERRORS

E_1 =	9.978011E-01	E_2 =	4.393035E-03
E_3 =	2.972847E-04	E_4 =	1.724195E-02
E_5 =	9.932535E-01	E_6 =	1.344739E-02
E_7 =	6.308164E+00	E_8 =	2.511606E+00

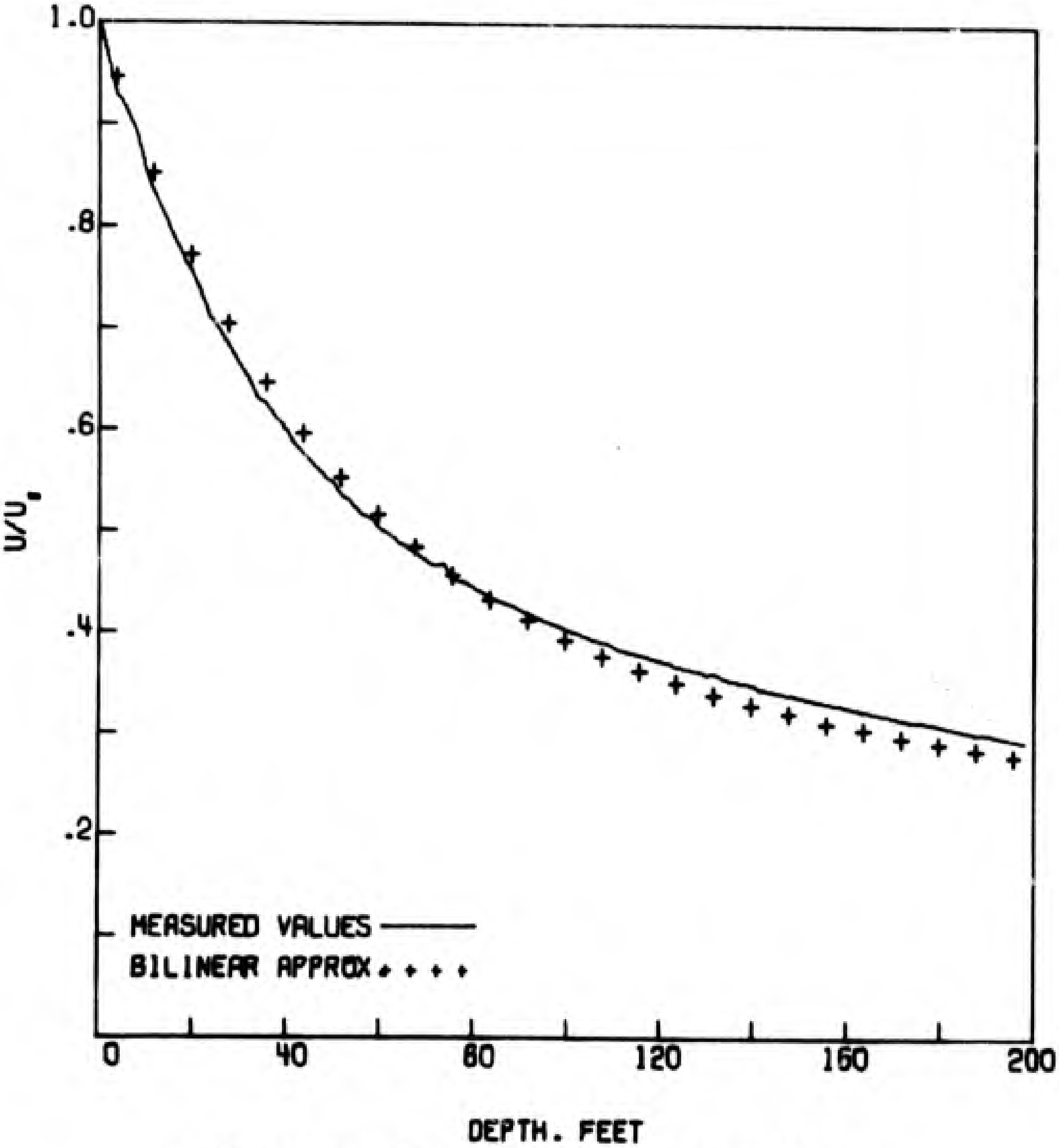
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.874	2.579
25	2.596	2.215
50	2.288	1.930
75	1.977	1.692
100	1.666	1.482



SURFACE PARTICLE VELOCITY

PROBLEM 100 -- 16 DEC 1969

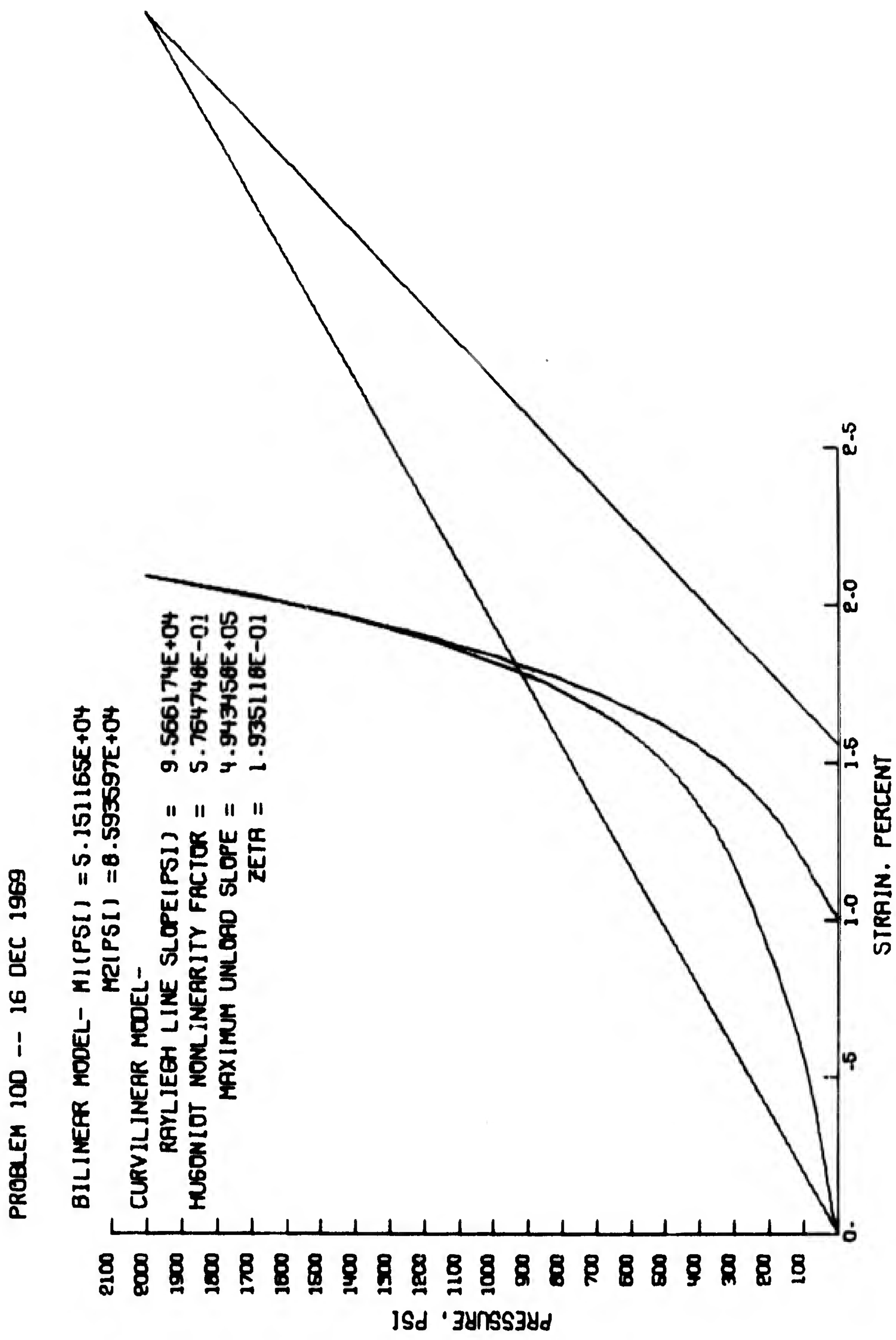


PEAK PARTICLE VELOCITY ATTENUATION

INPUT OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 1.383998E+01 SEC.

NORMALIZED HALF LOAD TIME = 1.072867E+01



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM BE -- 2 JAN 1970

NUMBER OF DATA POINTS. N= 84 . M= 99

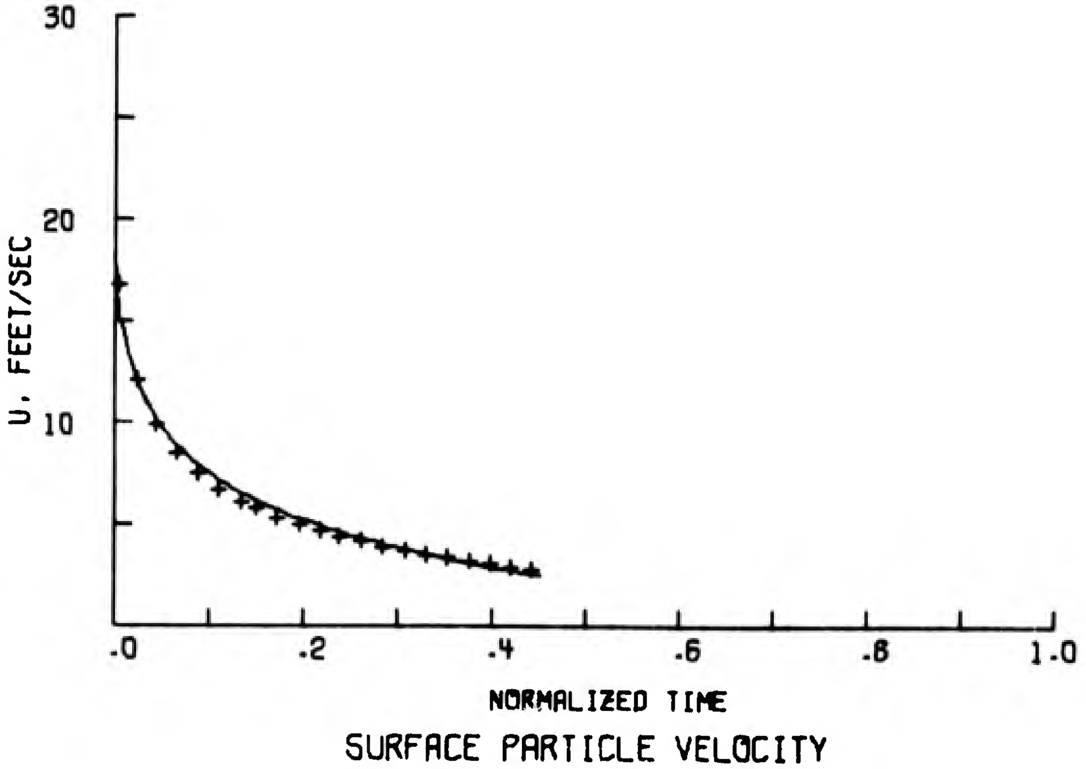
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.031202E+03
	SOUND VELOCITY =	1.701410E+03
	ZETA =	2.452629E-01

FITTING ERRORS

E_1 =	9.994335E-01	E_2 =	1.132584E-03
E_3 =	5.024486E-05	E_4 =	7.088361E-03
E_5 =	9.958324E-01	E_6 =	8.317805E-03
E_7 =	1.029820E-01	E_8 =	3.209081E-01

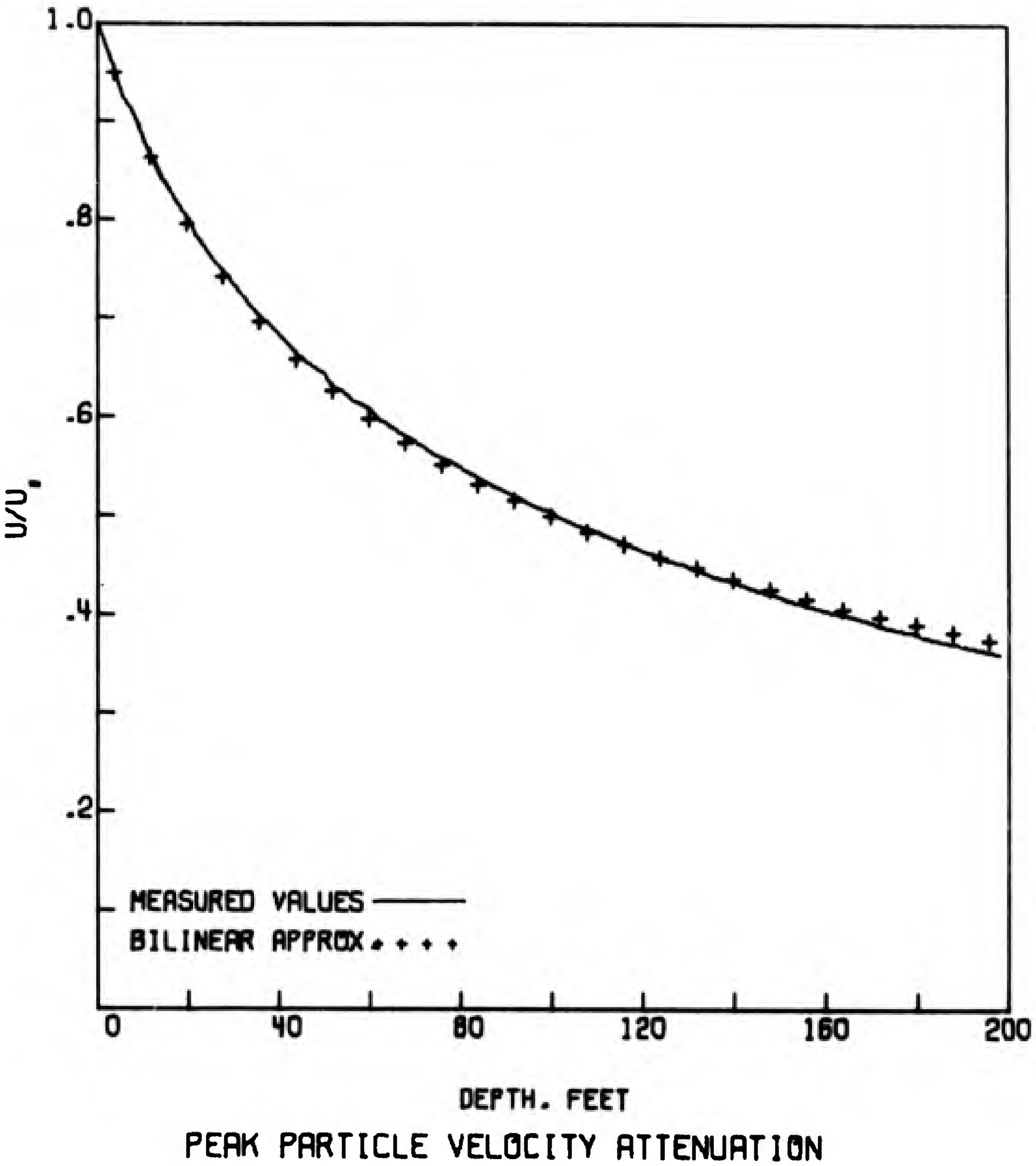
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.717	1.709
25	1.481	1.418
50	1.235	1.177
75	.998	.964
100	.771	.766



SURFACE PARTICLE VELOCITY

PROBLEM 8E -- 2 JAN 1970



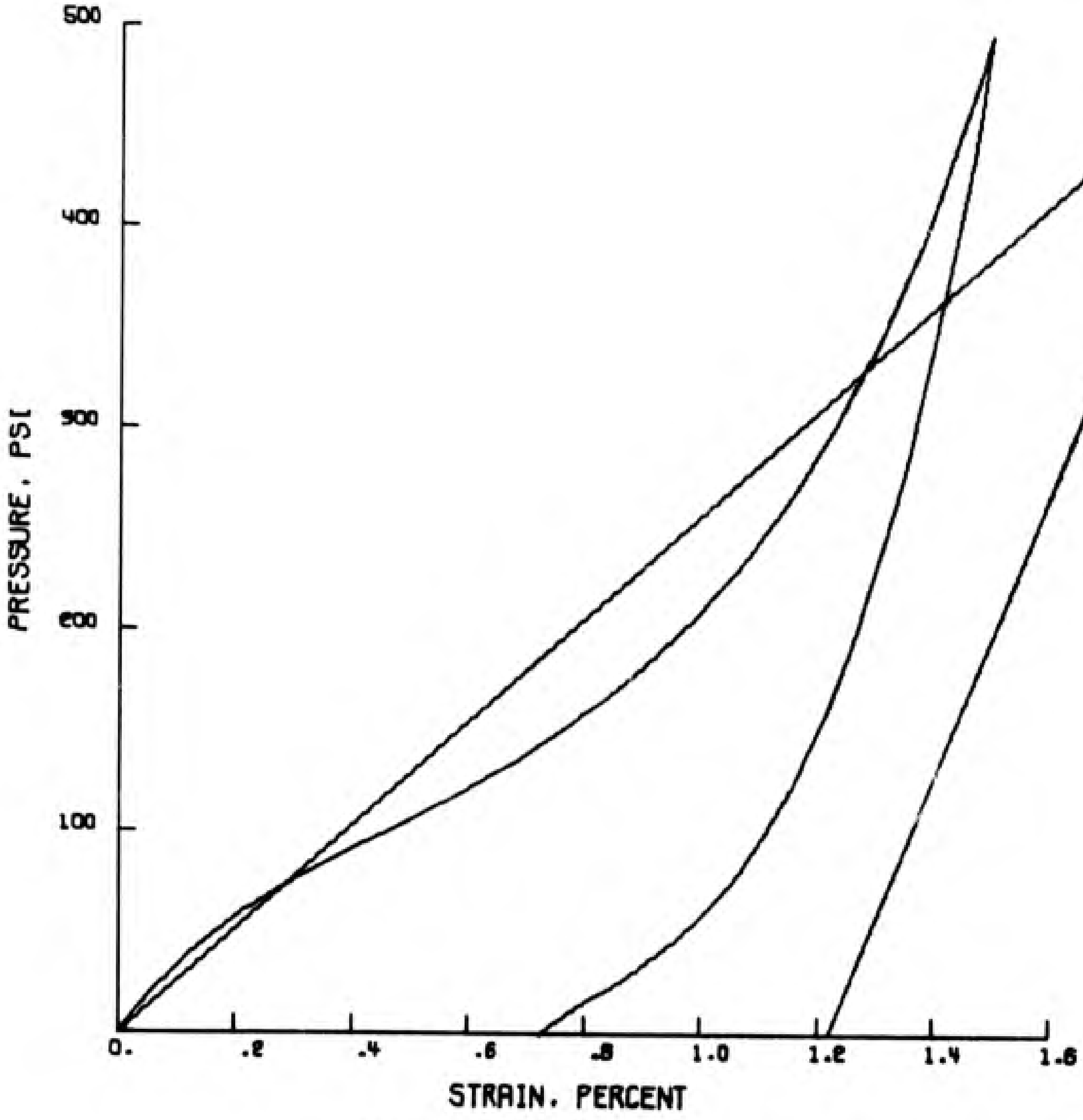
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 8E -- 2 JAN 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =2.591981E+04
M2(PSI) =7.056064E+04

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 3.333333E+04
HUGONIOT NONLINEARITY FACTOR = 2.745143E-01
MAXIMUM UNLOAD SLOPE = 1.722548E+05
ZETA = 1.935118E-01

BEST BILINEAR MODEL

PROBLEM 9E -- 2 JAN 1970

NUMBER OF DATA POINTS. N= 69 . M= 99

MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.145513E+03
	SOUND VELOCITY =	1.532943E+03
	ZETA =	1.446467E-01

FITTING ERRORS

E_1 = 9.989535E-01 E_2 = 2.091975E-03

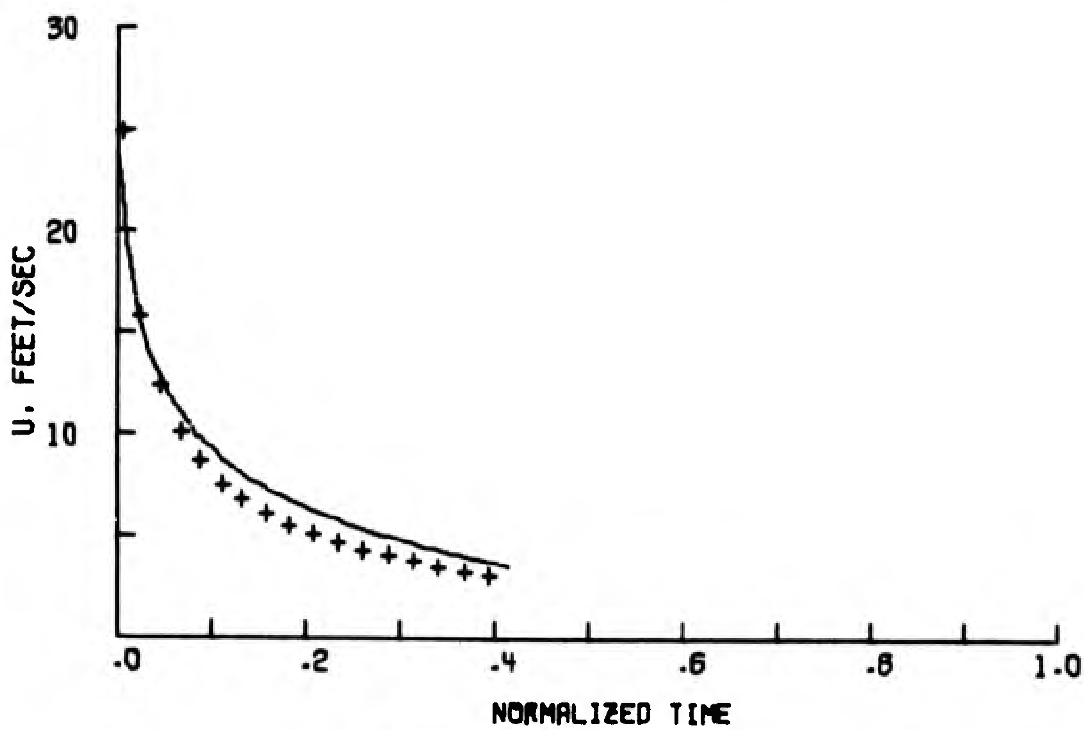
E_3 = 7.980968E-05 E_4 = 8.933626E-03

E_5 = 9.903464E-01 E_6 = 1.921403E-02

E_7 = 1.664775E+00 E_8 = 1.290262E+00

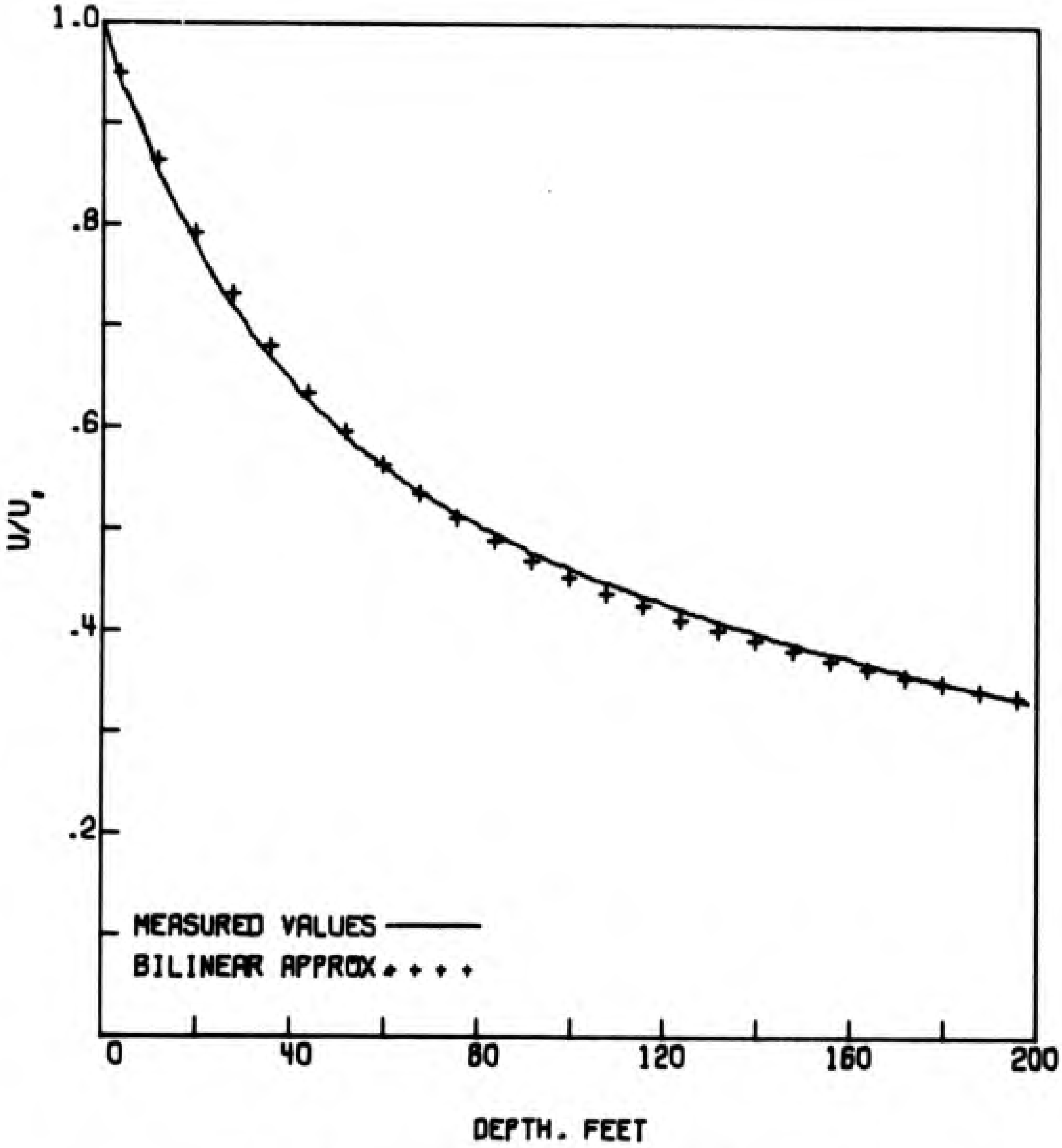
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.287	2.224
25	2.023	1.885
50	1.738	1.607
75	1.456	1.364
100	1.178	1.139



SURFACE PARTICLE VELOCITY

PROBLEM 9E -- 2 JAN 1970



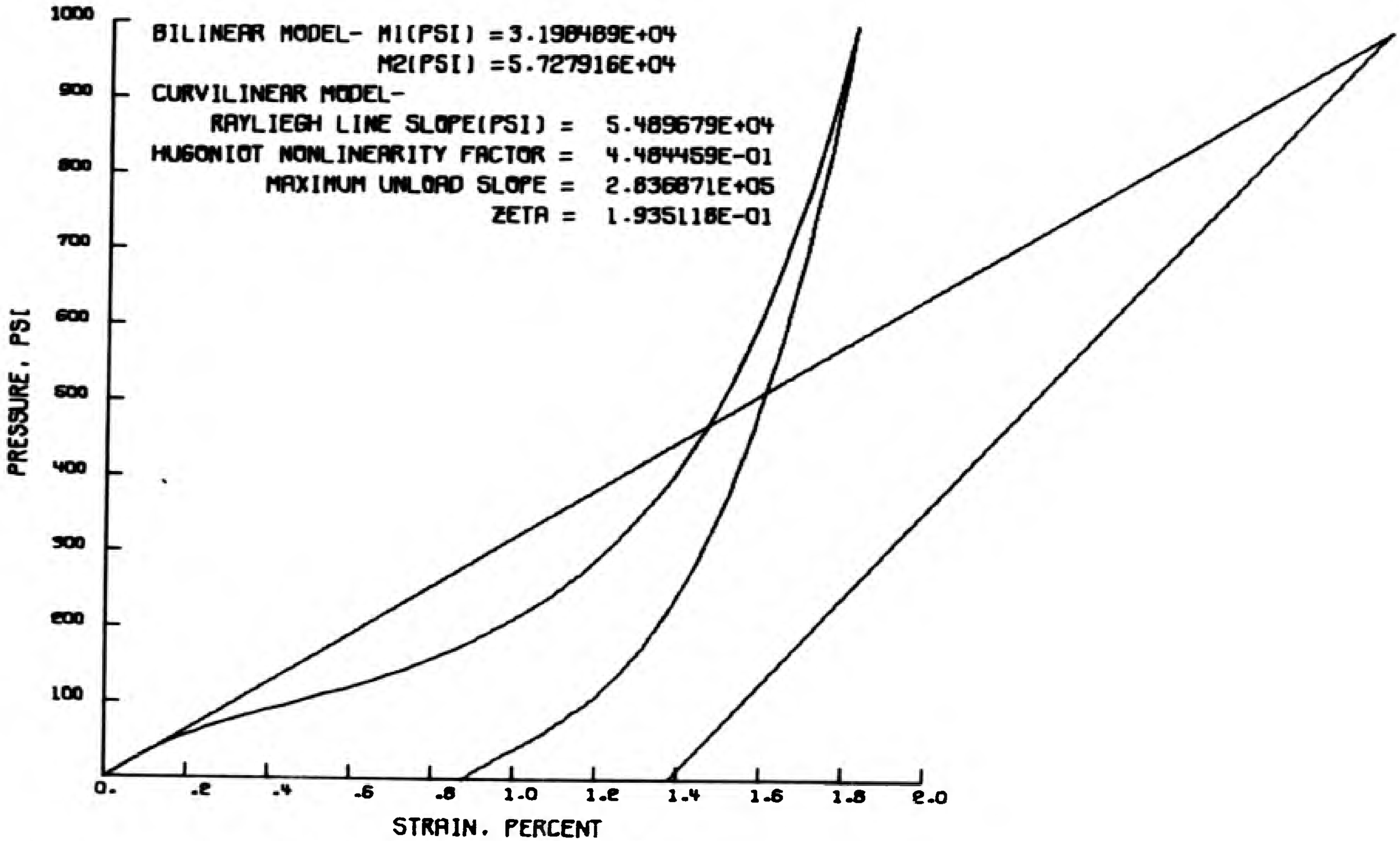
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 9E -- 2 JAN 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 10E -- 2 JAN 1970

NUMBER OF DATA POINTS. N= 54 . M= 99

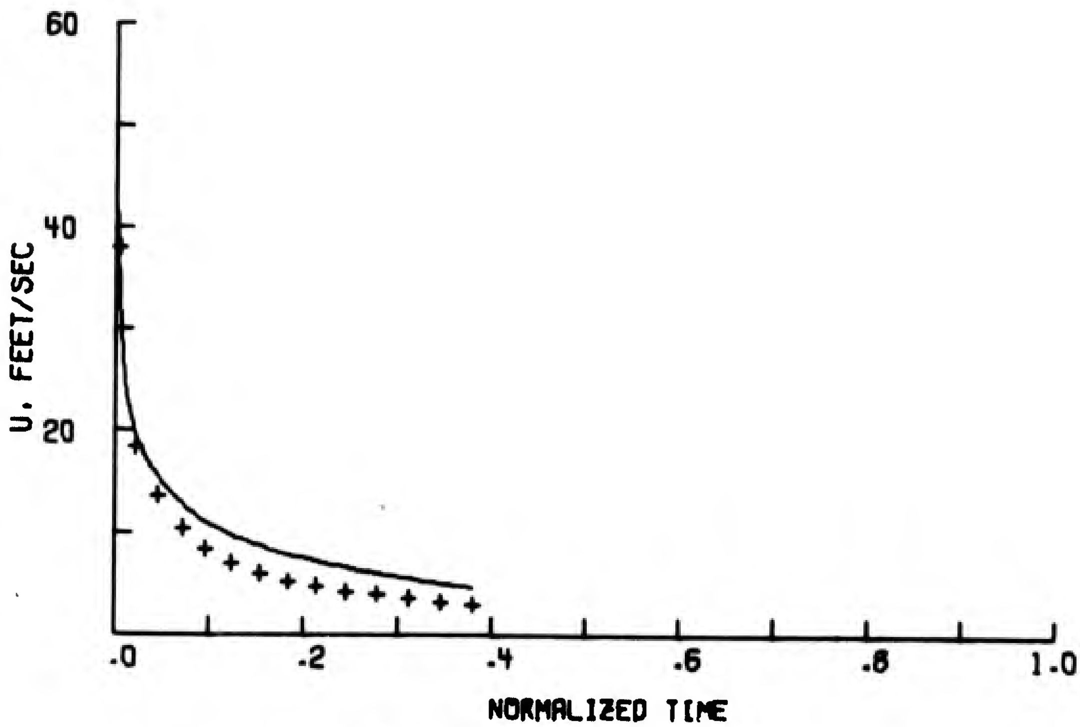
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.441768E+03
	SOUND VELOCITY =	1.837309E+03
	ZETA =	1.206256E-01

FITTING ERRORS

E_1 =	9.973951E-01	E_2 =	5.202958E-03
E_3 =	4.010558E-04	E_4 =	2.002638E-02
E_5 =	9.938848E-01	E_6 =	1.219306E-02
E_7 =	6.926474E+00	E_8 =	2.631819E+00

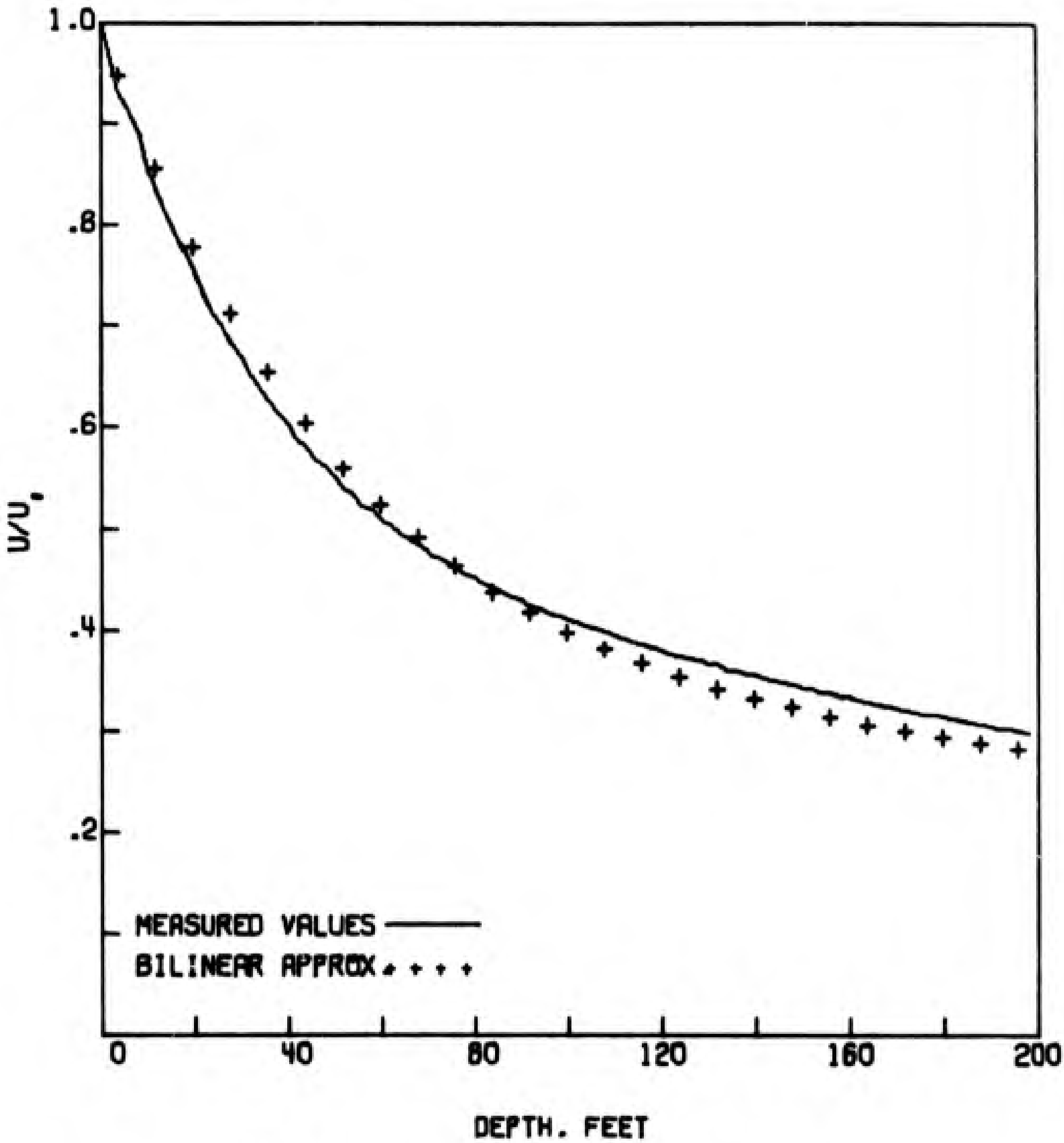
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.881	2.579
25	2.600	2.220
50	2.292	1.937
75	1.982	1.699
100	1.672	1.488



SURFACE PARTICLE VELOCITY

PROBLEM 10E -- 2 JAN 1970



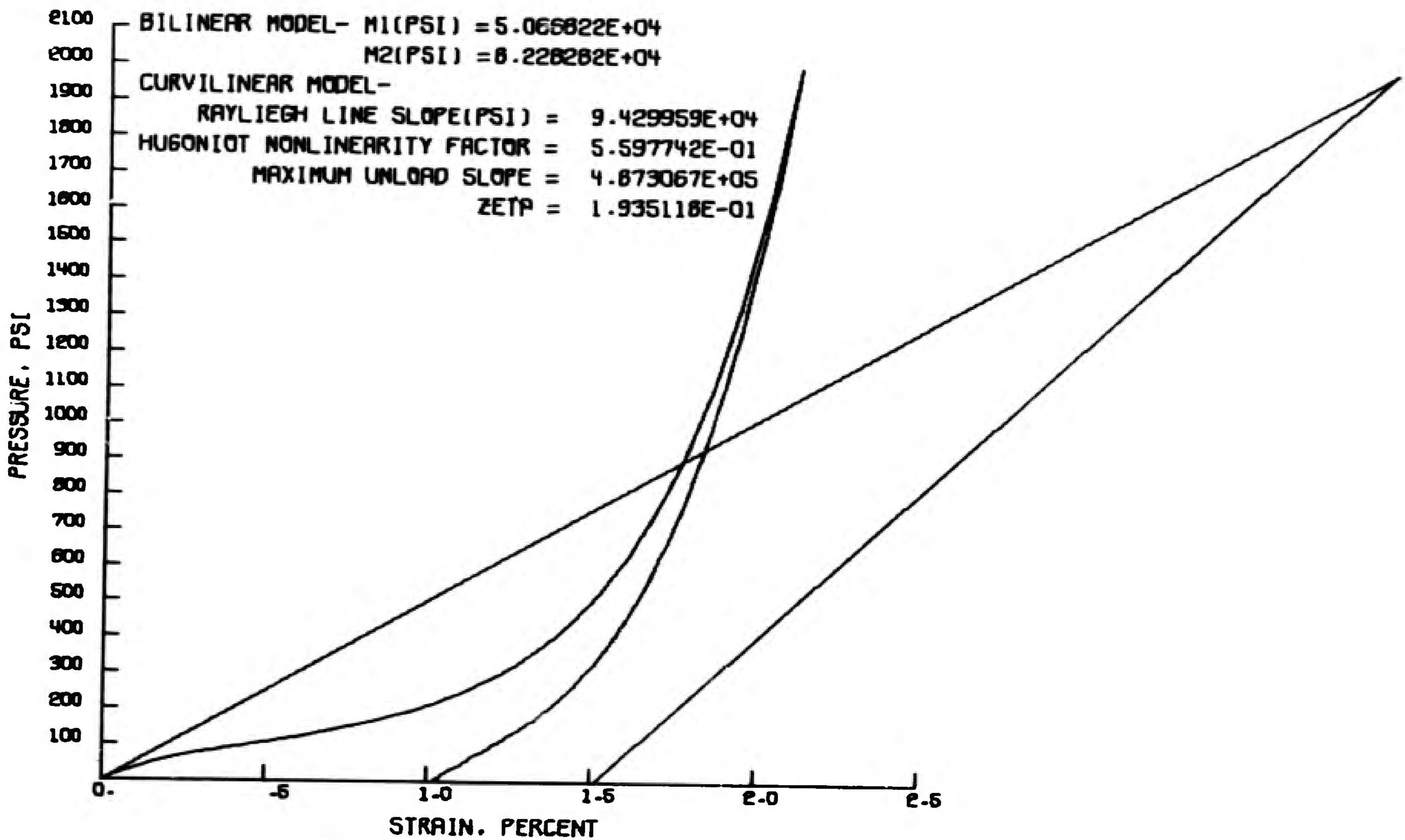
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239556E-03

PROBLEM 10E -- 2 JAN 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 7F -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. N= 81 . M= 99

MATERIAL PROPERTIES

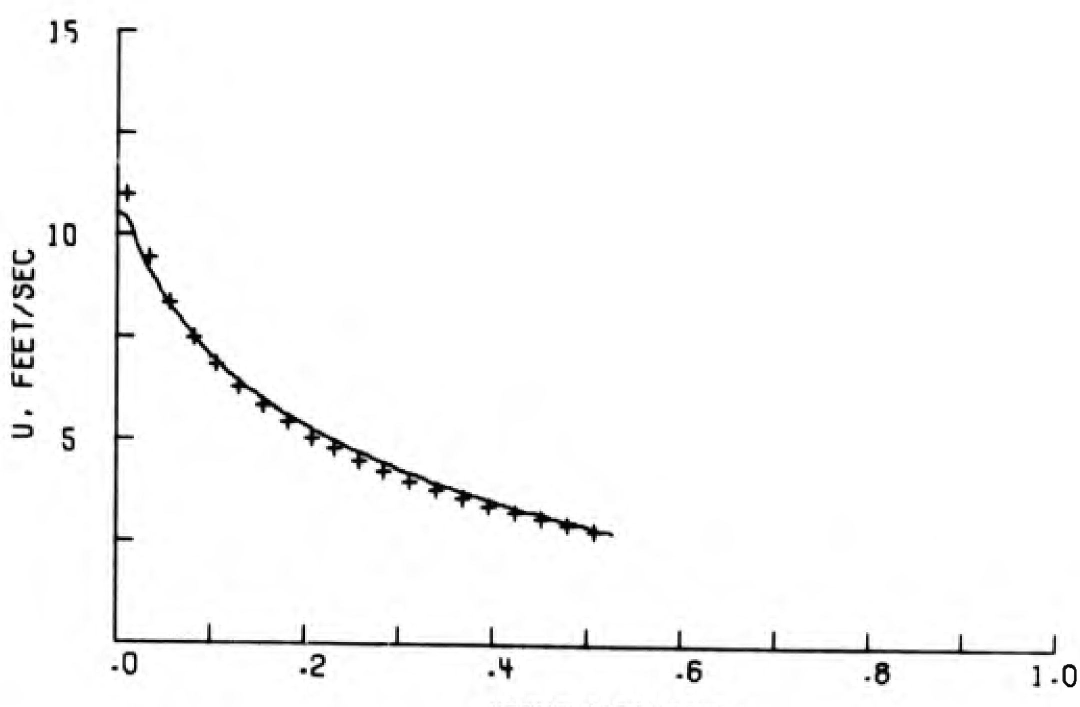
DENSITY =	3.510000E+00
SHOCK VELOCITY =	6.749238E+02
SOUND VELOCITY =	1.342317E+03
ZETA =	3.308445E-01

FITTING ERRORS

E_1 =	9.997285E-01	E_2 =	5.428772E-04
E_3 =	1.477666E-05	E_4 =	3.844042E-03
E_5 =	9.981189E-01	E_6 =	3.758571E-03
E_7 =	4.728071E-02	E_8 =	2.174413E-01

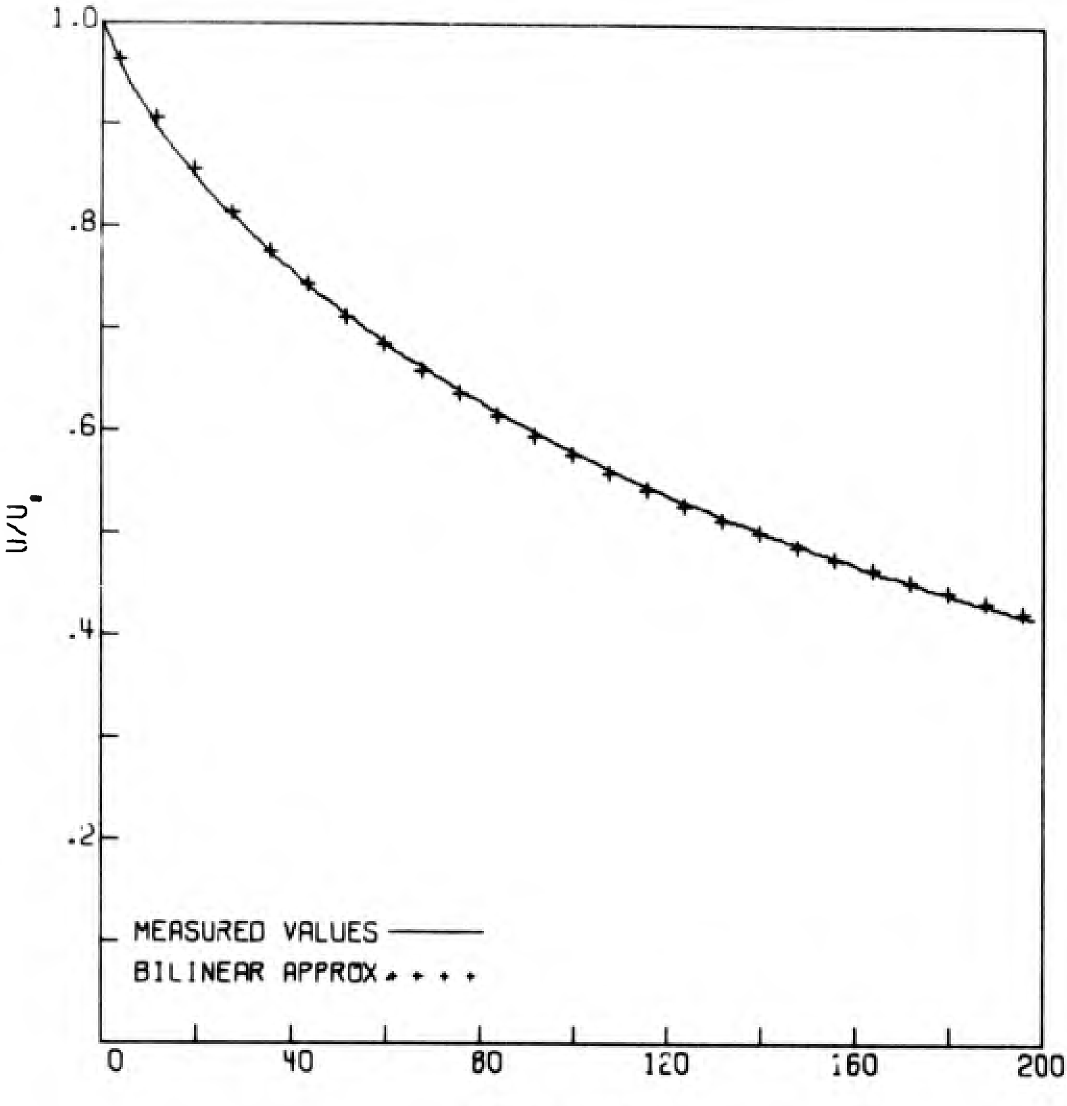
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.443	1.474
25	1.155	1.140
50	.863	.846
75	.587	.579
100	.325	.330



SURFACE PARTICLE VELOCITY

PROBLEM 7F -- 17 FEB 1970 $.



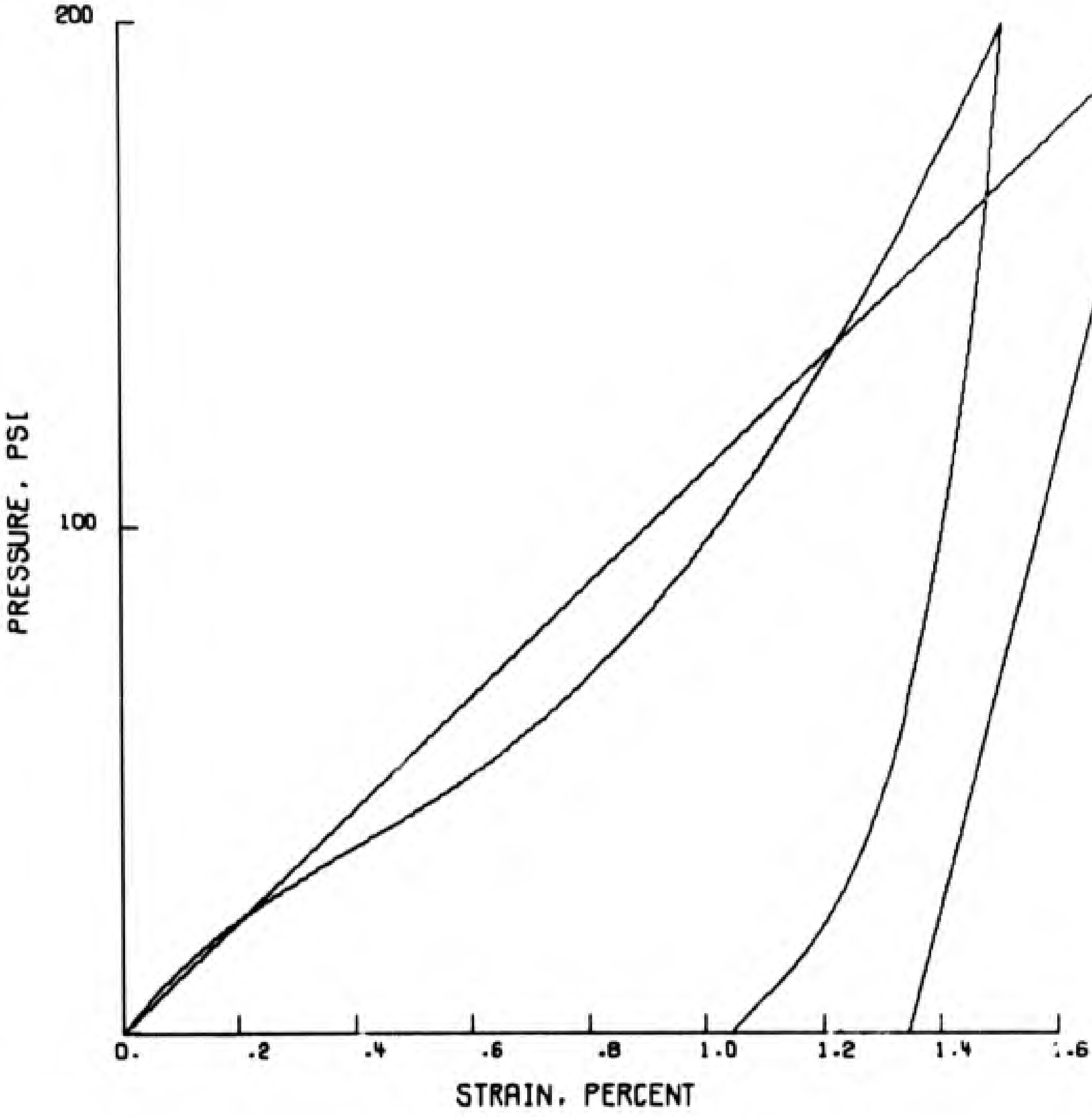
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 7F -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =1.110335E+04

M2(PSI) =4.391922E+04

CURVILINEAR MODEL-

RAYLIEGH LINE SLOPE(PSI) = 1.318913E+04

HUGONIOT NONLINEARITY FACTOR = 2.053498E-01

MAXIMUM UNLOAD SLOPE = 1.135946E+05

ZETA = 1.161070E-01

BEST BILINEAR MODEL

PROBLEM 8F -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. N= 79 . M= 99

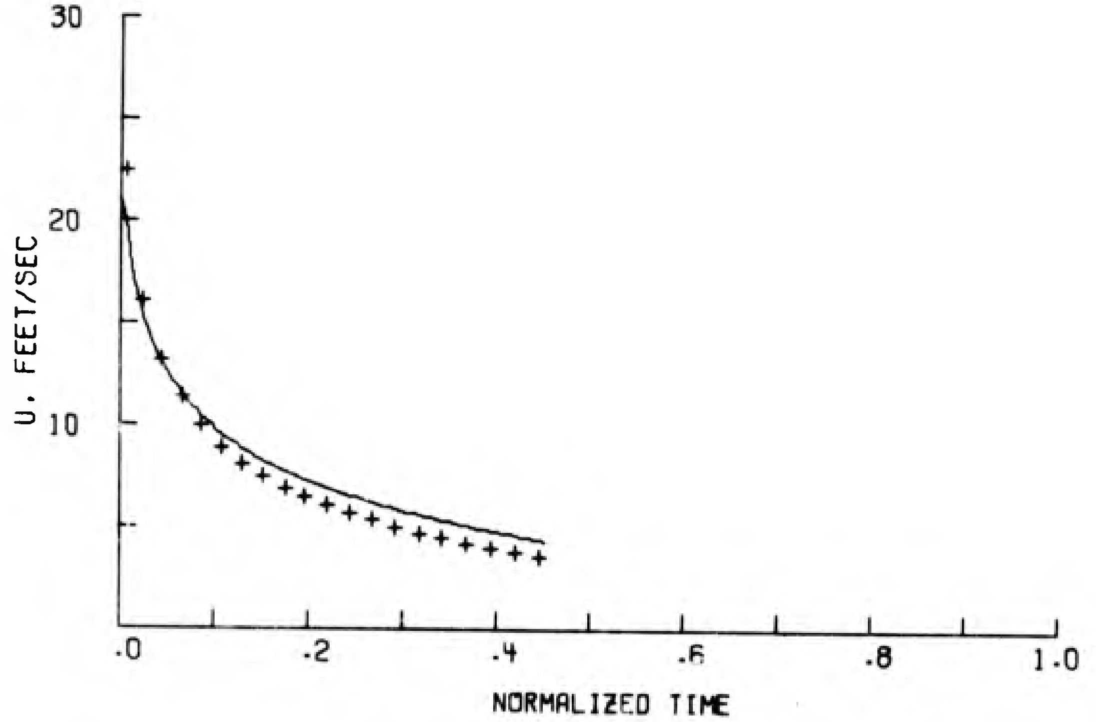
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	7.583318E+02
	SOUND VELOCITY =	1.129969E+03
	ZETA =	1.968105E-01

FITTING ERRORS

E_1 =	9.995818E-01	E_2 =	8.361596E-04
E_3 =	7.746988E-05	E_4 =	8.801698E-03
E_5 =	9.959546E-01	E_6 =	8.074339E-03
E_7 =	9.727575E-01	E_8 =	9.862847E-01

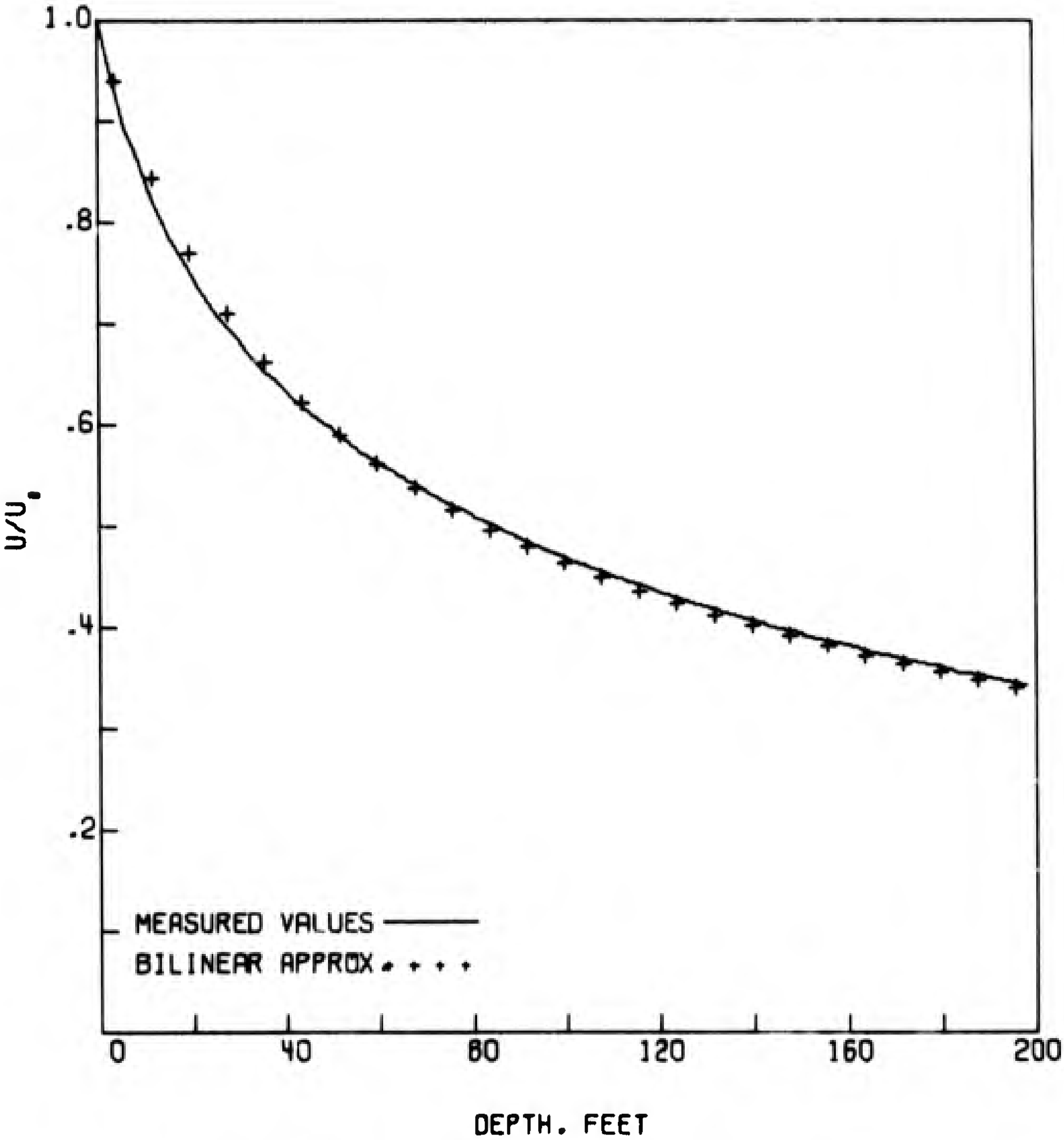
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.242	2.260
25	1.822	1.784
50	1.417	1.390
75	1.040	1.034
100	.682	.695



SURFACE PARTICLE VELOCITY

PROBLEM 8F -- 17 FEB 1970 $.



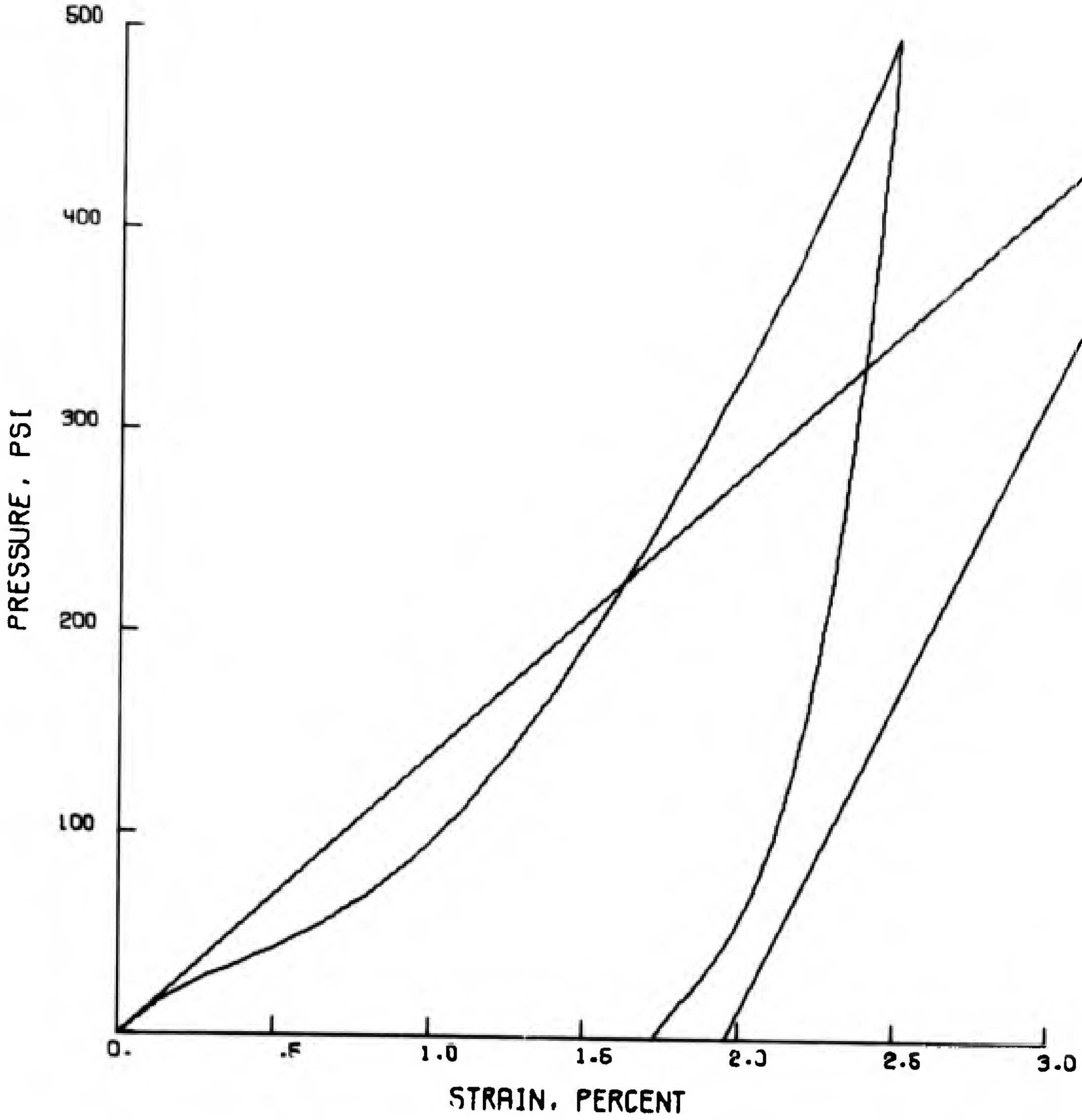
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 8F -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =1.401726E+04
M2(PSI) =3.112275E+04
CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 2.000000E+04
HUGONIOT NONLINEARITY FACTOR = 2.745624E-01
MAXIMUM UNLOAD SLOPE = 1.722549E+05
ZETA = 1.161070E-01

BEST BILINEAR MODEL

PROBLEM 9F -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. N= 68 . M= 99

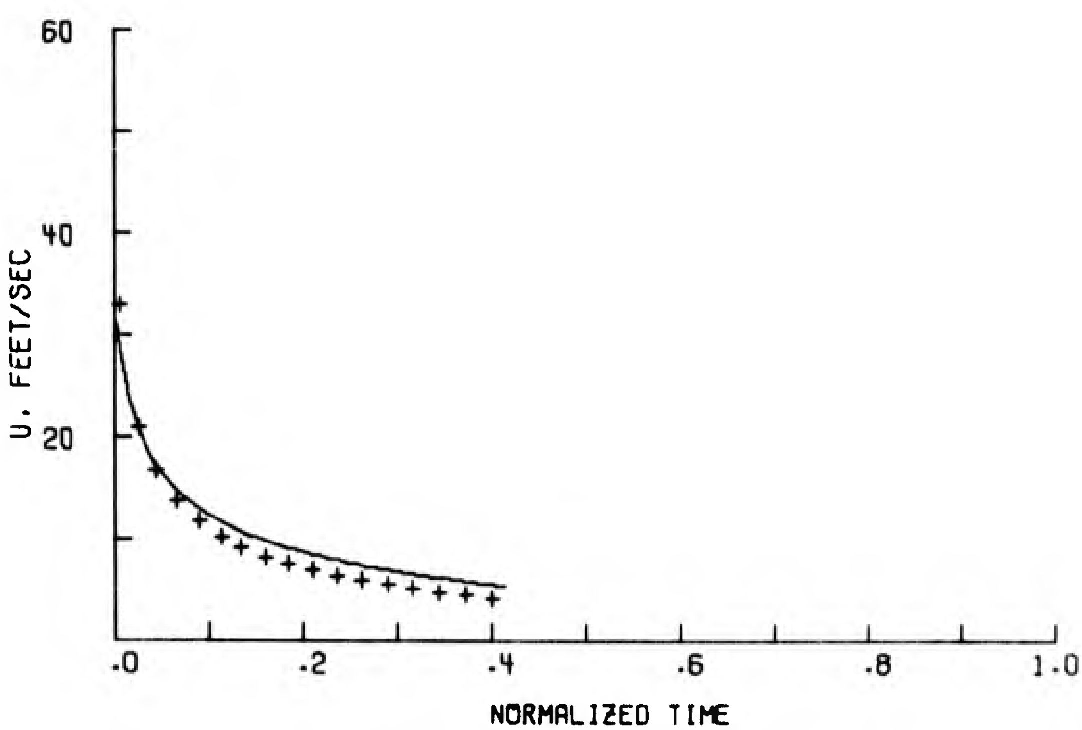
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	8.577271E+02
	SOUND VELOCITY =	1.197651E+03
	ZETA =	1.653825E-01

FITTING ERRORS

E_1 =	9.989576E-01	E_2 =	2.083749E-03
E_3 =	1.489631E-04	E_4 =	1.220504E-02
E_5 =	9.941750E-01	E_6 =	1.161611E-02
E_7 =	3.041167E+00	E_8 =	1.743894E+00

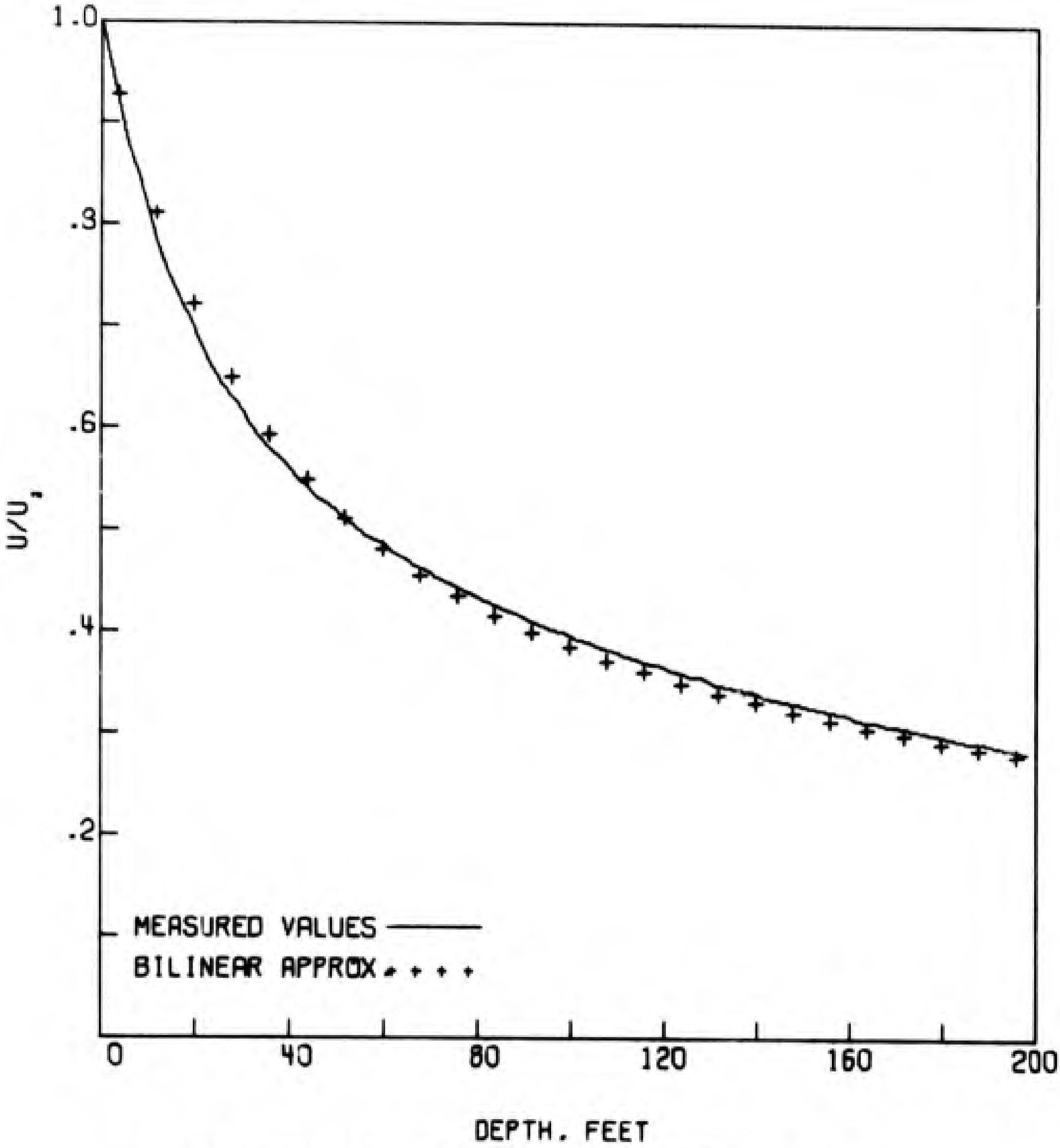
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	3.062	3.029
25	2.524	2.407
50	2.007	1.928
75	1.535	1.511
100	1.091	1.121



SURFACE PARTICLE VELOCITY

PROBLEM 9F -- 17 FEB 1970 $.



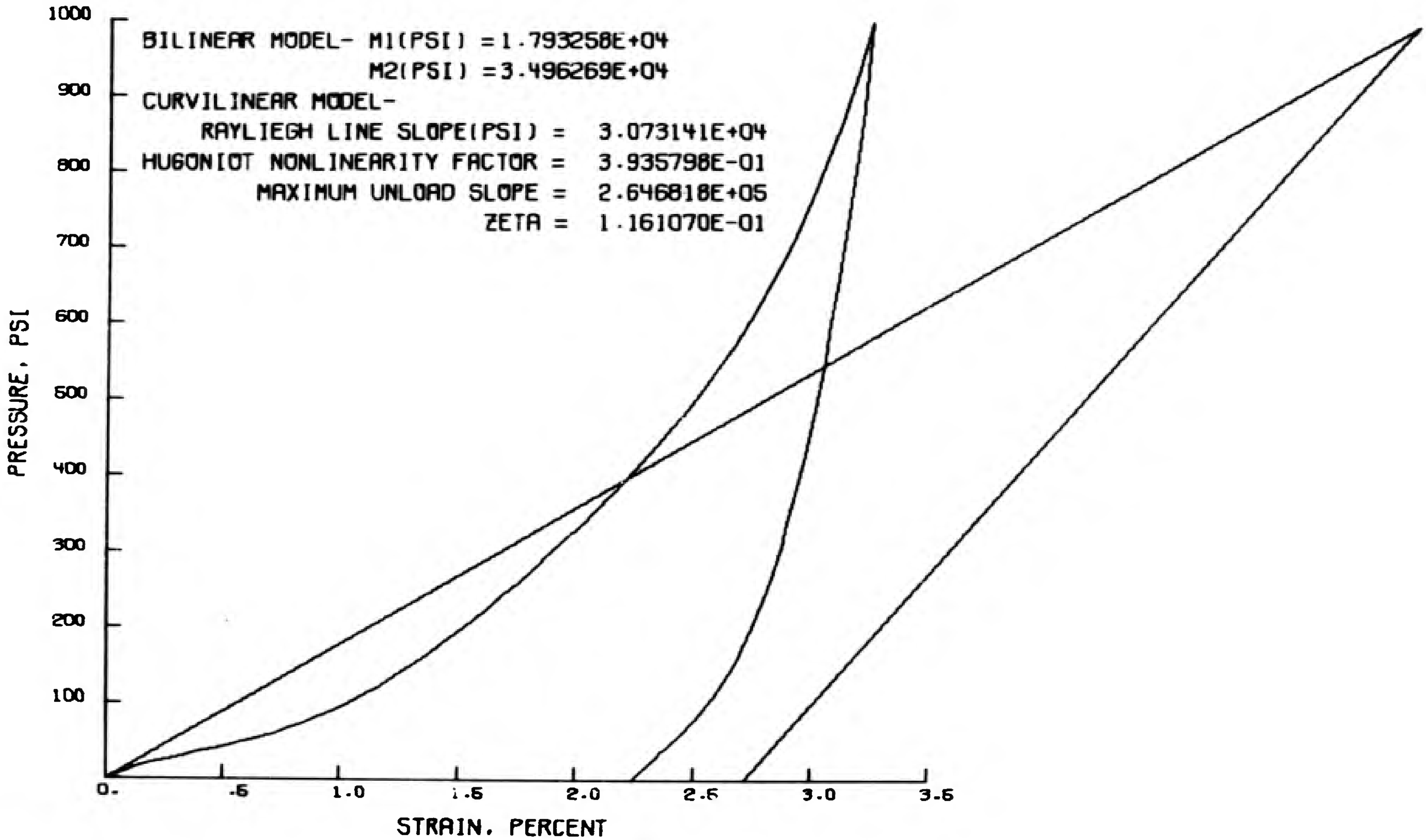
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 9F -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 10F -- 17 FEB 1970 $.

NUMBER OF DATA POINTS. = 54 . M= 99

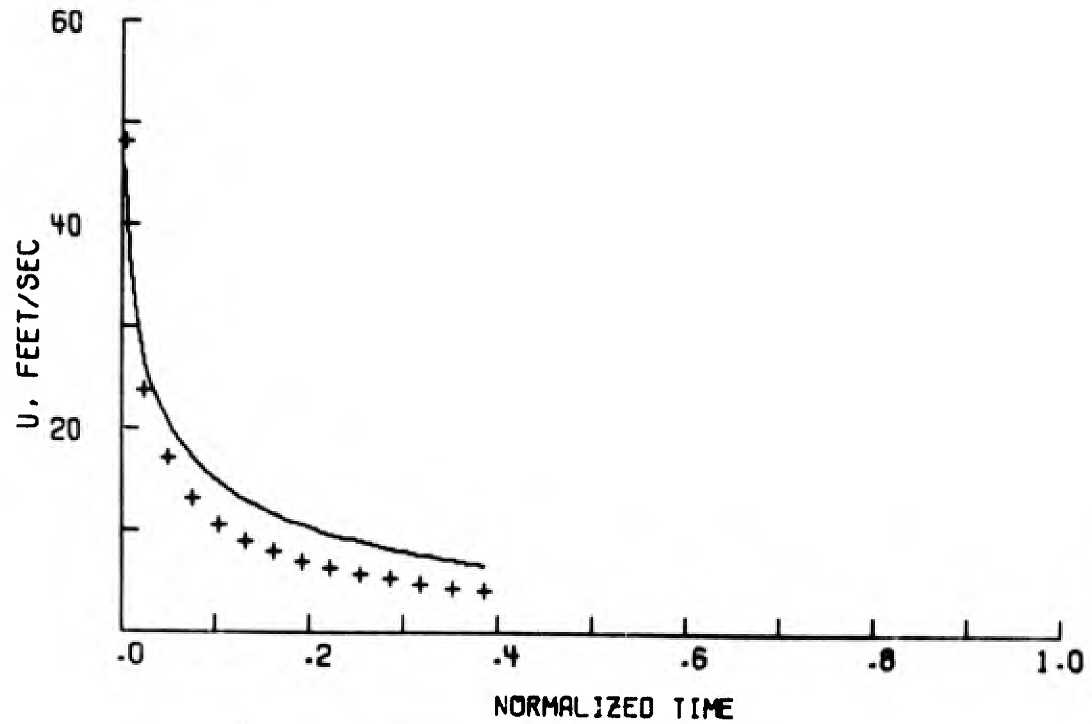
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.131898E+03
	SOUND VELOCITY =	1.551965E+03
	ZETA =	1.565160E-01

FITTING ERRORS

E_1 =	9.982563E-01	E_2 =	3.484393E-03
E_3 =	2.781559E-04	E_4 =	1.667801E-02
E_5 =	9.720128E-01	E_6 =	5.519105E-02
E_7 =	1.991715E+01	E_8 =	4.462864E+00

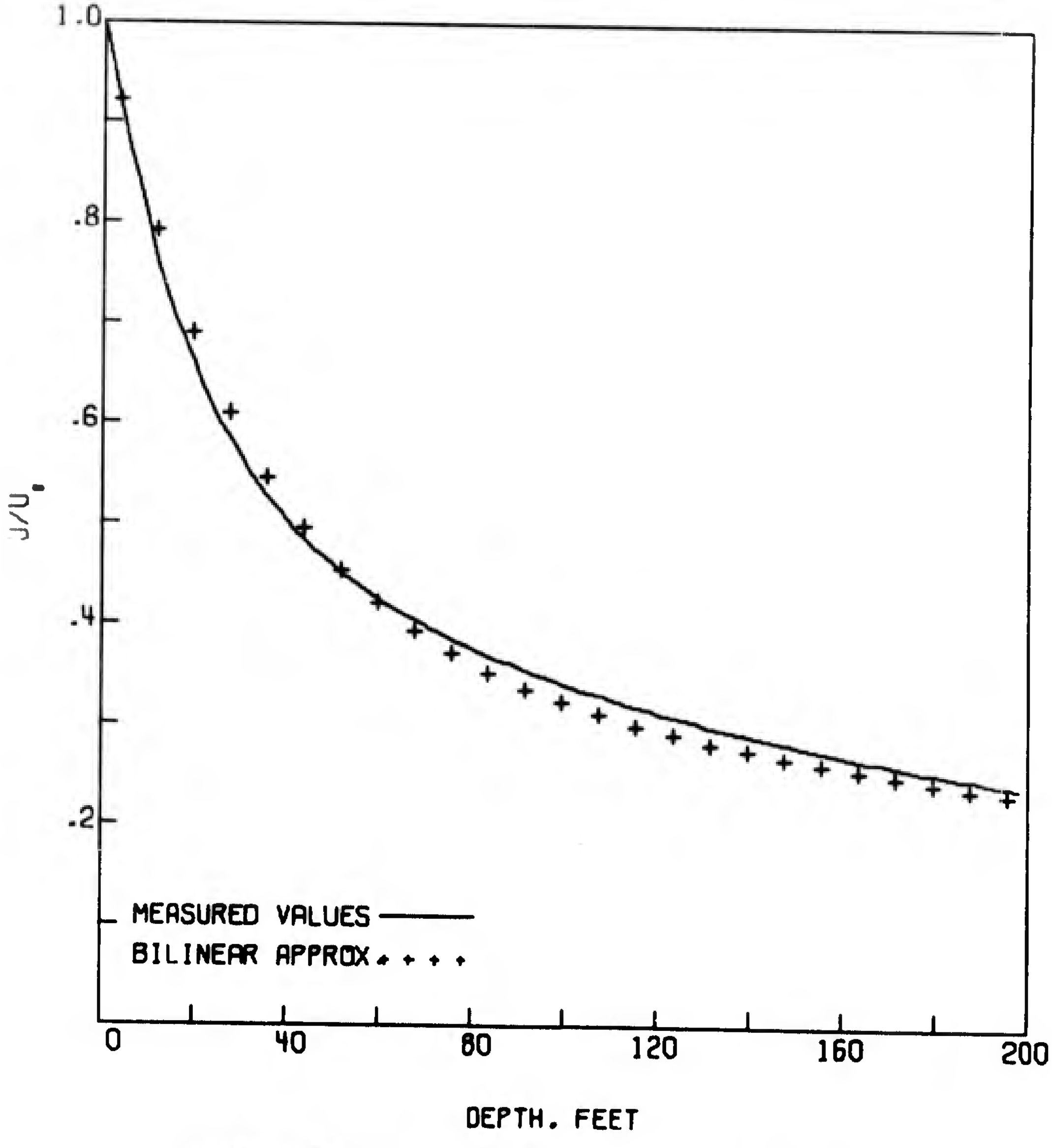
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	3.978	3.423
25	3.355	2.778
50	2.736	2.316
75	2.163	1.941
100	1.624	1.607



SURFACE PARTICLE VELOCITY

PROBLEM 10F -- 17 FEB 1970 $.



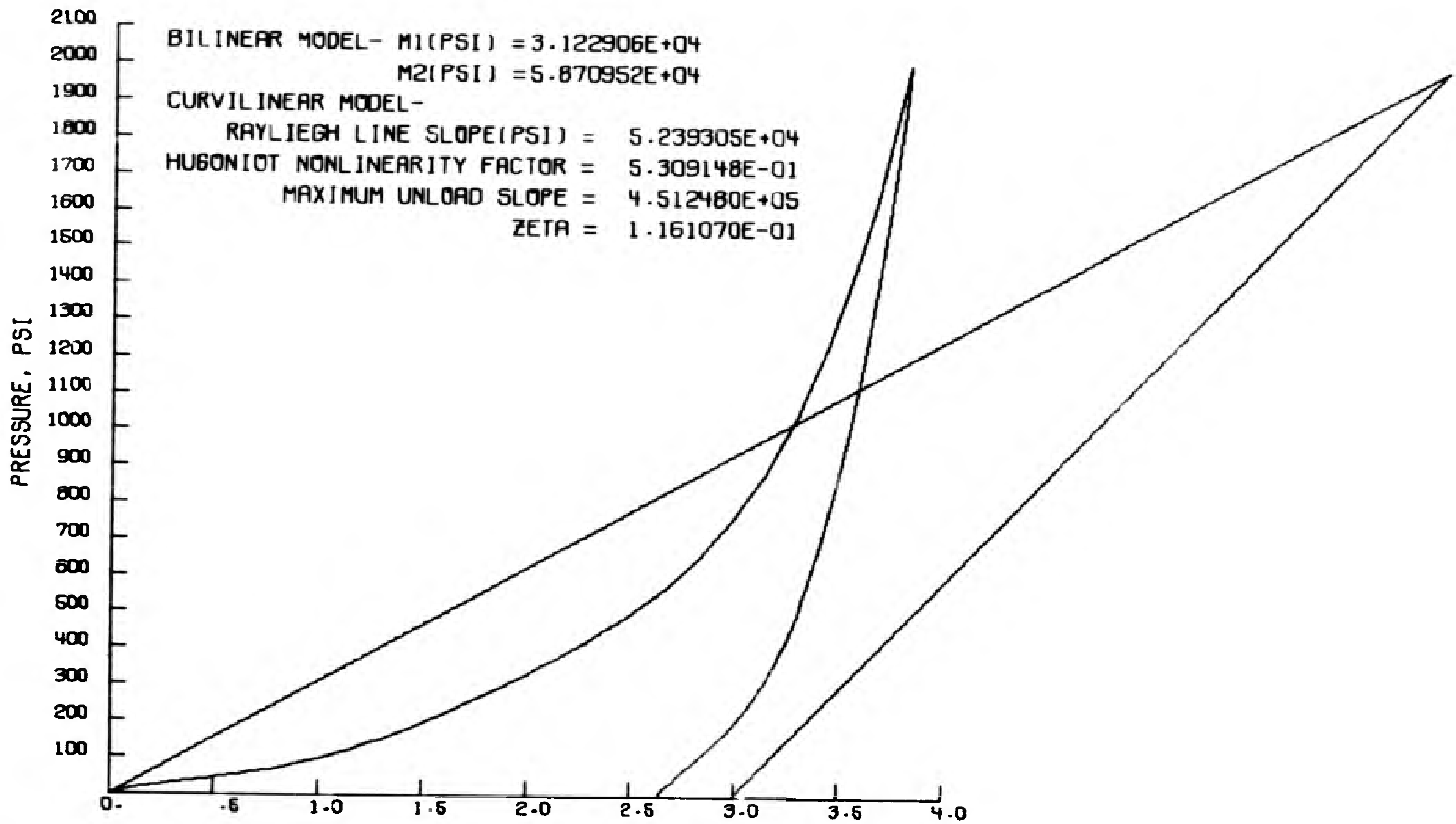
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM 10F -- 17 FEB 1970 $.



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 6H -- 23 APRIL 1970

NUMBER OF DATA POINTS. N=162 . M= 99

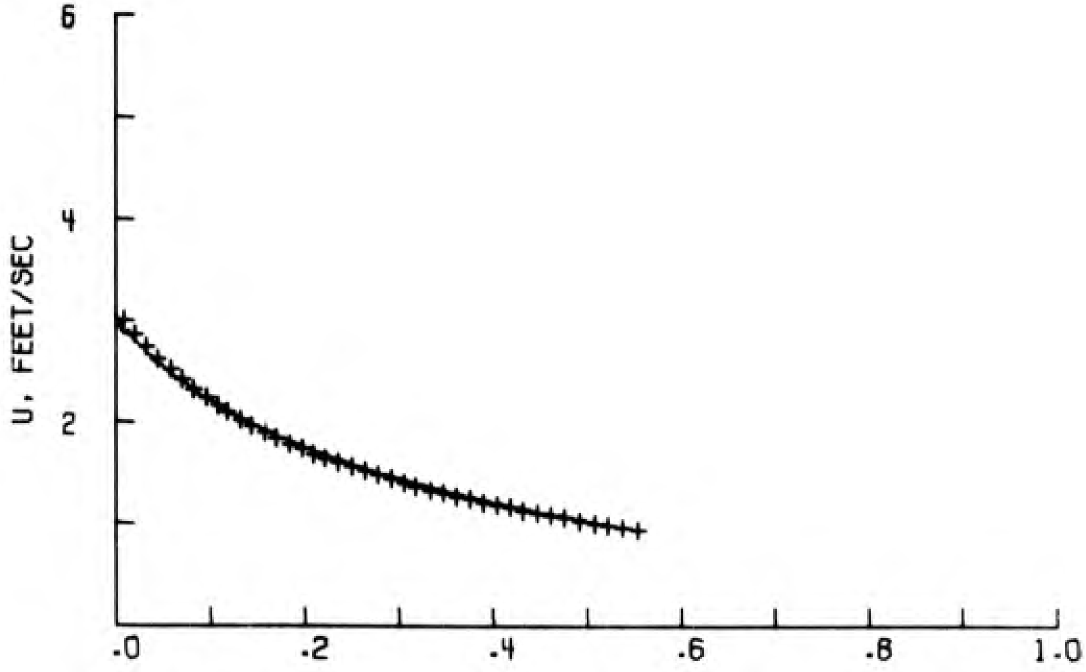
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.310215E+03
	SOUND VELOCITY =	2.953939E+03
	ZETA =	3.854750E-01

FITTING ERRORS

E_1 =	9.987558E-01	E_2 =	2.486812E-03
E_3 =	3.168588E-05	E_4 =	5.629021E-03
E_5 =	9.994195E-01	E_6 =	1.160618E-03
E_7 =	8.779511E-04	E_8 =	2.963024E-02

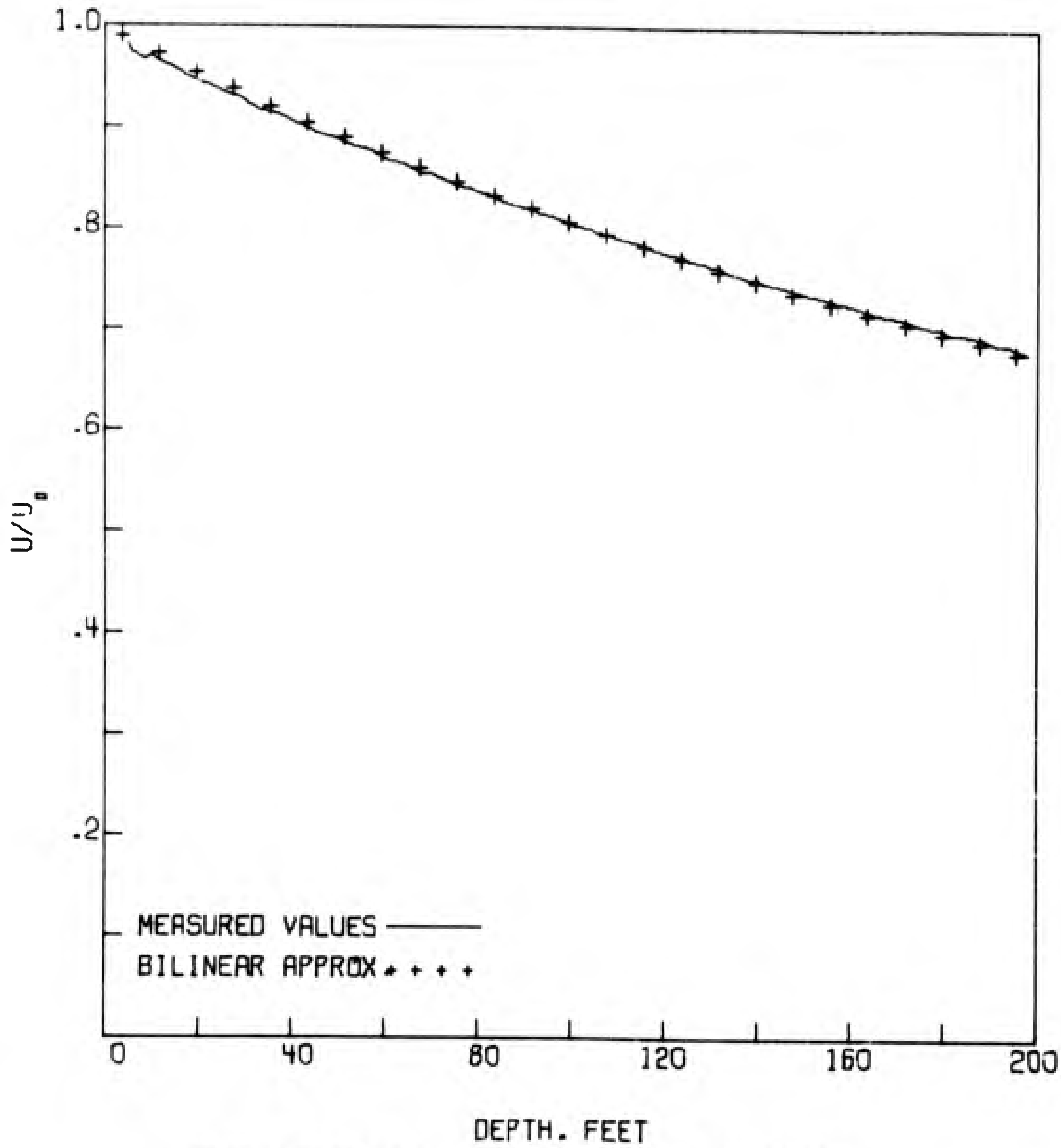
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.441	.446
25	.397	.396
50	.351	.348
75	.306	.301
100	.261	.257



SURFACE PARTICLE VELOCITY

PROBLEM 6H -- 23 APRIL 1973



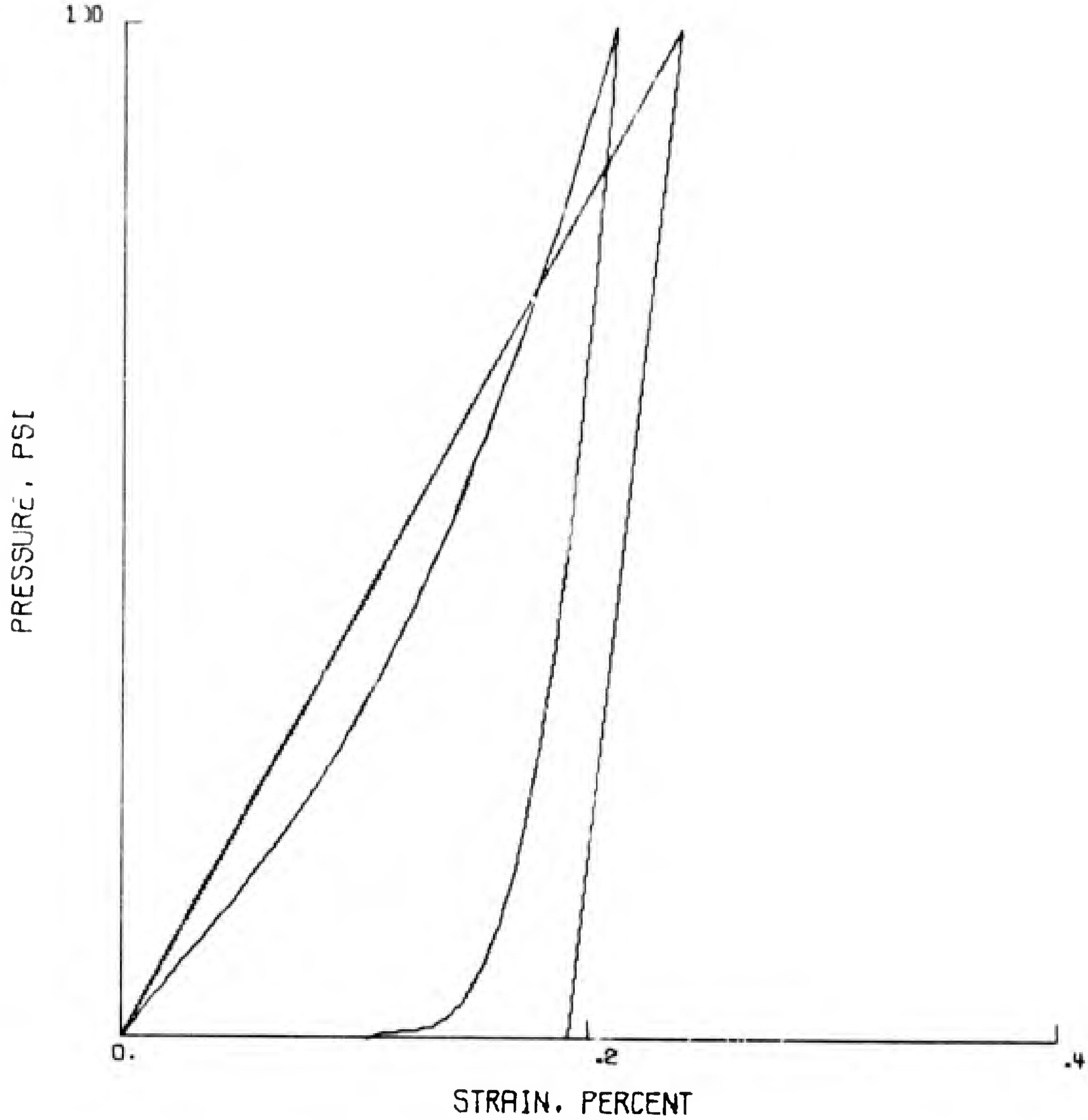
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 100 PSI BRODE WAVE

HALF LOAD TIME = 1.076864E-01 SEC.

NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 64 -- 23 APRIL 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 4.184364E+04
M2(PSI) = 2.126903E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 4.738438E+04
HUGONIOT NONLINEARITY FACTOR = 2.120586E-01
MAXIMUM UNLOAD SLOPE = 3.222138E+05
ZETA = 1.470588E-01

BEST BILINEAR MODEL

PROBLEM 7H -- 23 APRIL 1970

NUMBER OF DATA POINTS. N=155 . M= 99

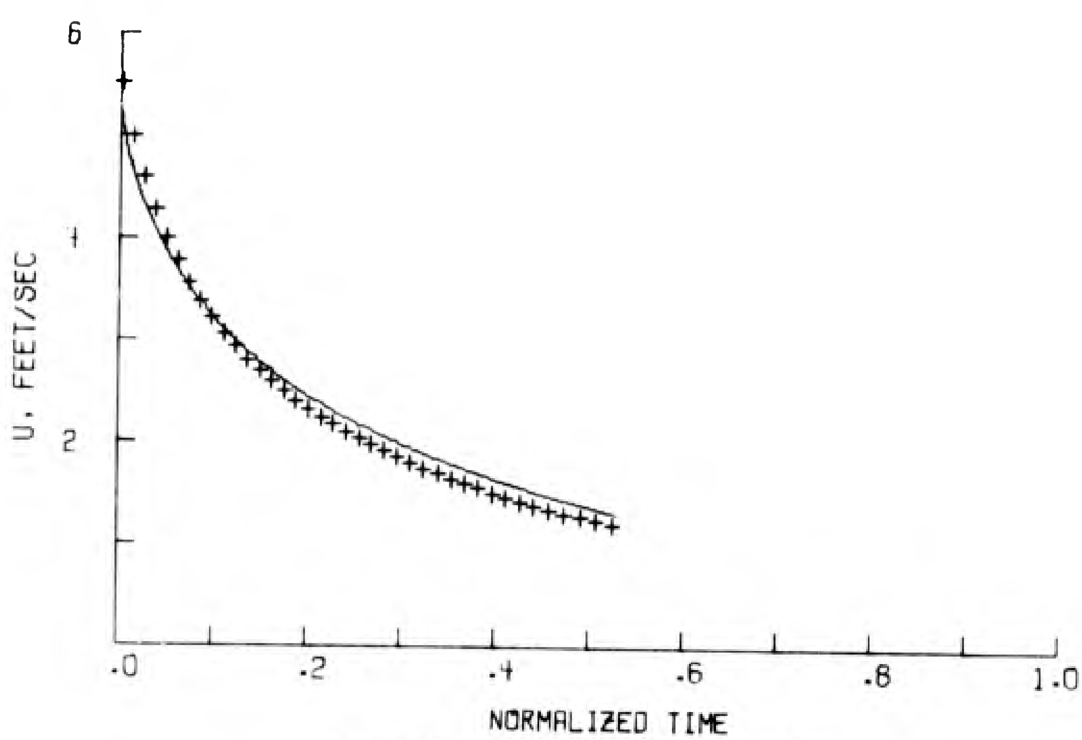
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.428263E+03
	SOUND VELOCITY =	2.381439E+03
	ZETA =	2.501970E-01

FITTING ERRORS

E_1 =	9.992735E-01	E_2 =	1.452553E-03
E_3 =	1.762504E-05	E_4 =	4.198219E-03
E_5 =	9.986020E-01	E_6 =	2.794133E-03
E_7 =	2.308888E-02	E_8 =	1.519503E-01

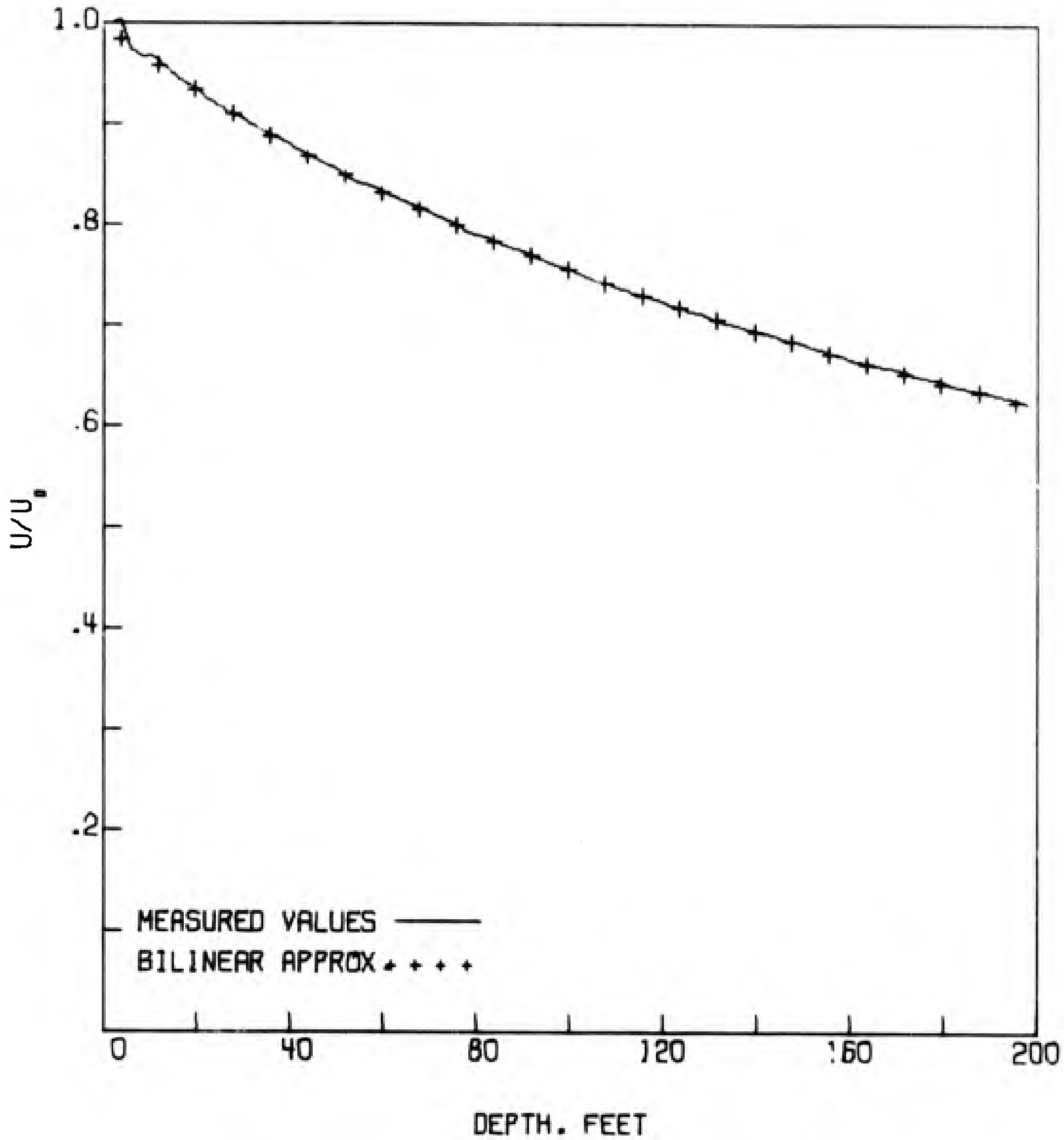
DISPLACEMENT AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.665	.677
25	.604	.606
50	.540	.540
75	.478	.477
100	.418	.416



SURFACE PARTICLE VELOCITY

PROBLEM 7H -- 25 APRIL 1970



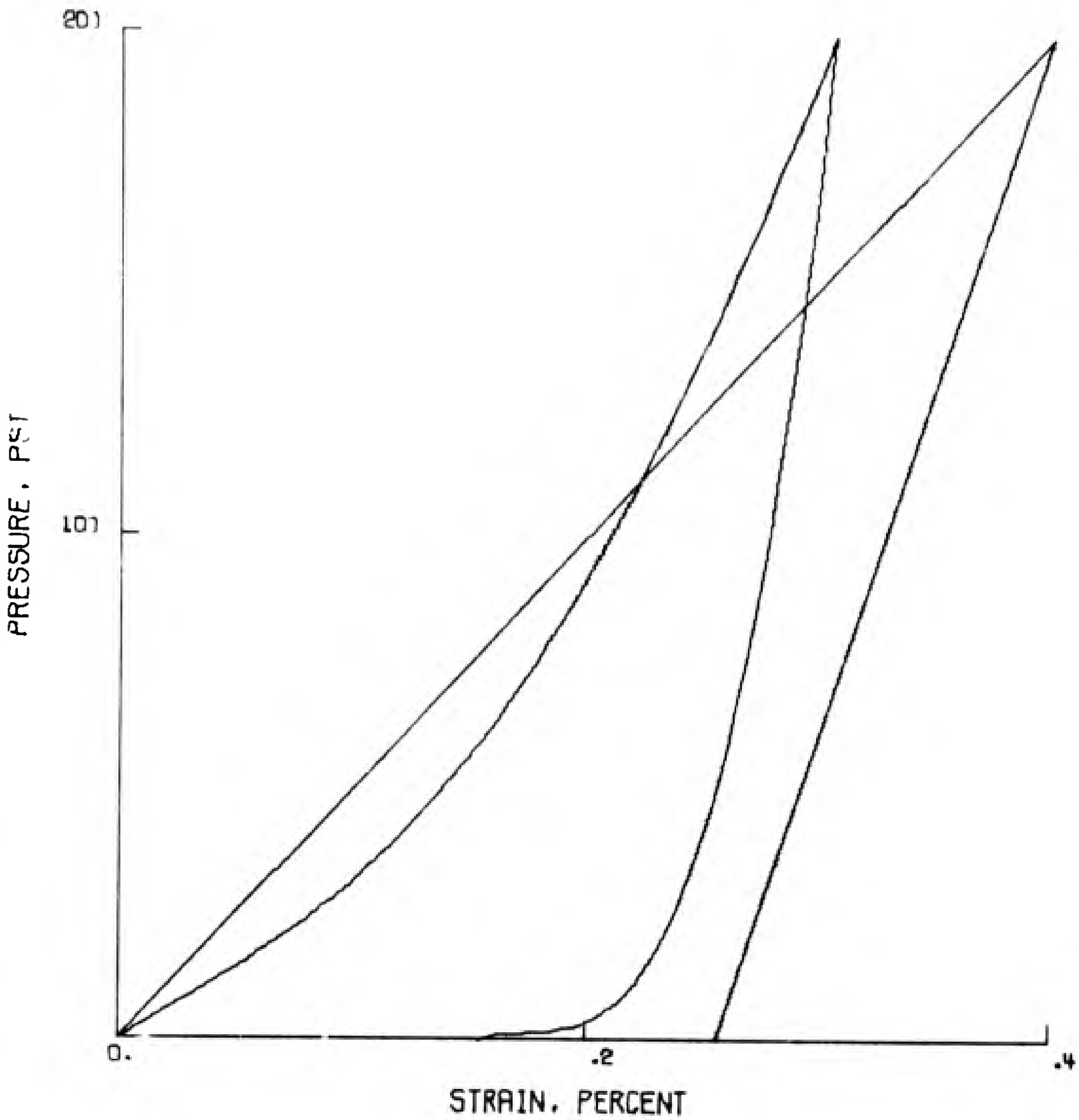
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 74 -- 23 APRIL 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.972343E+04

M2(PSI) =1.382368E+05

CURVILINEAR MODEL -

RAYLIEGH LINE SLOPE(PSI) = 6.497937E+04

HUGONIOT NONLINEARITY FACTOR = 2.677390E-01

MAXIMUM UNLOAD SLOPE = 4.418597E+05

ZETA = 1.470588E-01

BEST BILINEAR MODE'

PROBLEM 8H -- 23 APRIL 1970

NUMBER OF DATA POINTS. N=153 . M= 39

MATERIAL PROPERTIES

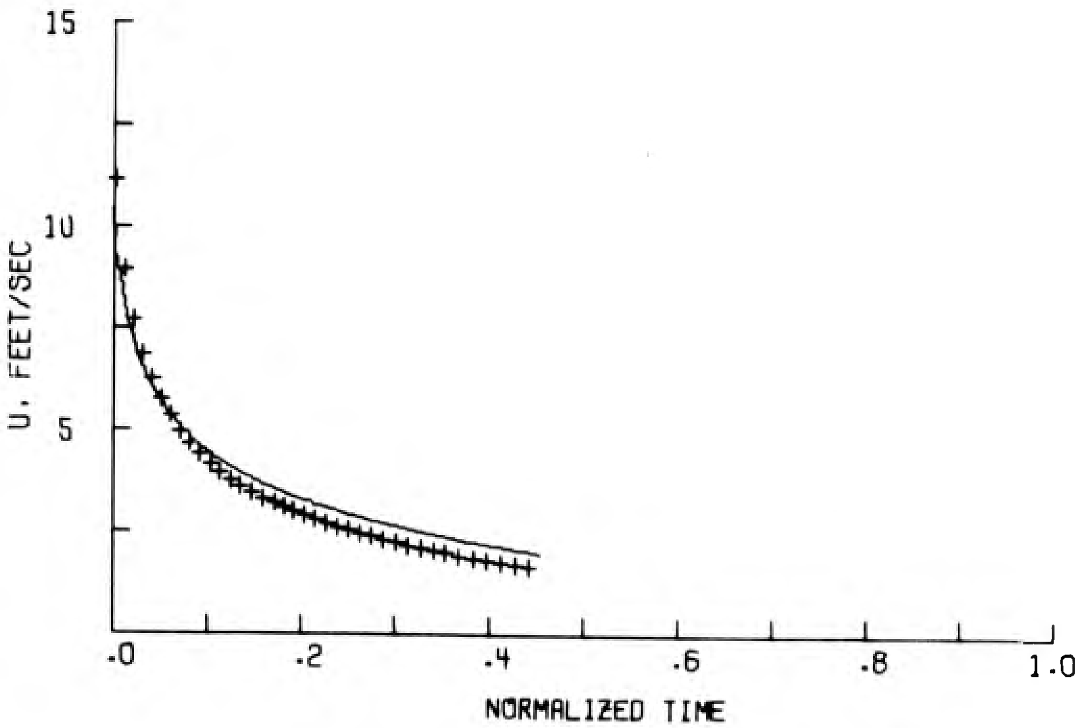
DENSITY	=	3.510000E+00
SHOCK VELOCITY	=	1.637644E+03
SOUND VELOCITY	=	2.358042E+03
ZETA	=	1.802940E-01

FITTING ERRORS

E_1 =	9.990512E-01	E_2 =	1.896722E-03
E_3 =	5.011073E-05	E_4 =	7.078893E-03
E_5 =	9.976064E-01	E_6 =	4.781397E-03
E_7 =	1.826509E-01	E_8 =	4.273768E-01

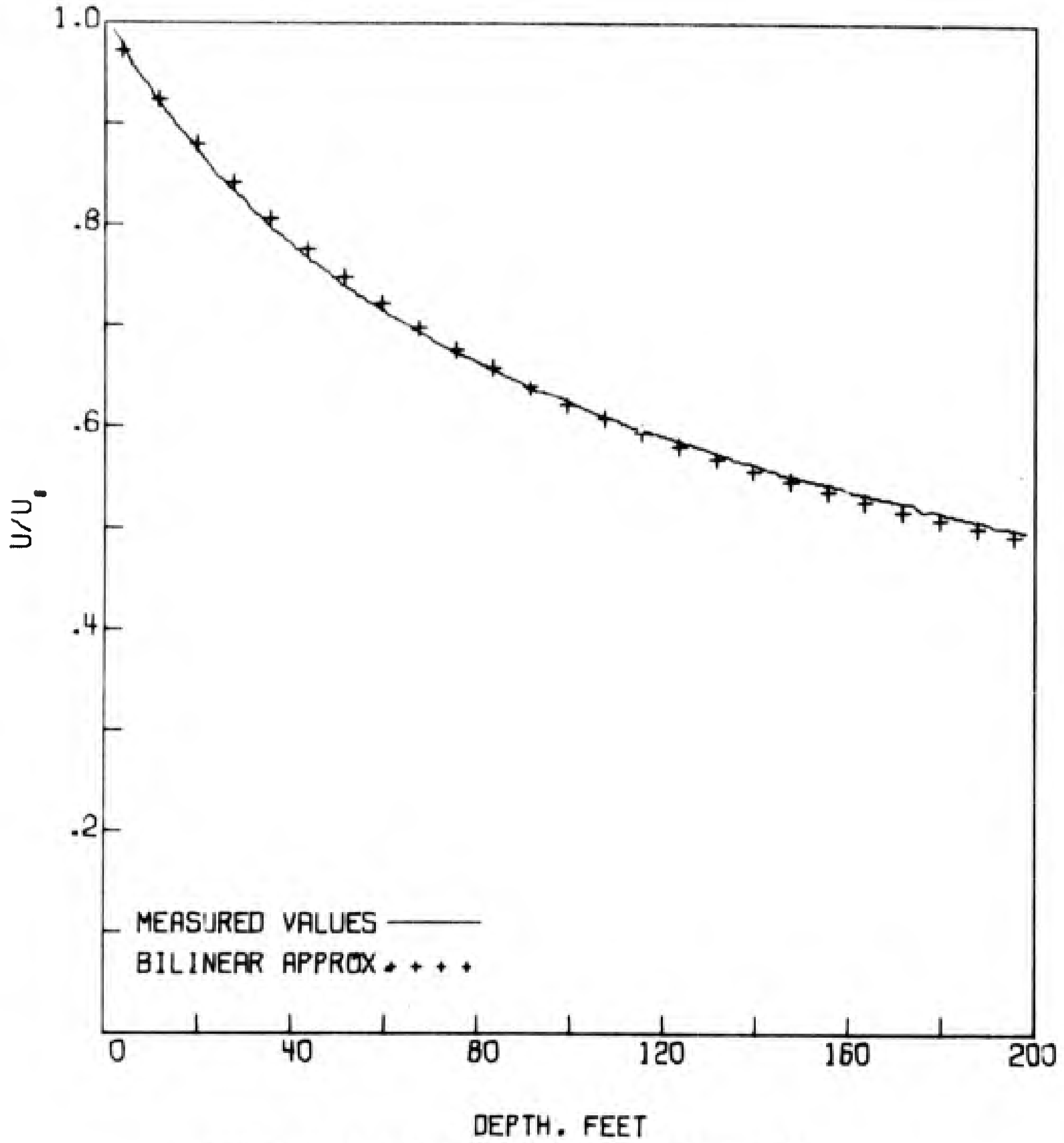
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.029	1.035
25	.939	.930
50	.847	.836
75	.761	.750
100	.678	.669



SURFACE PARTICLE VELOCITY

PROBLEM 84 -- 25 APRIL 1970

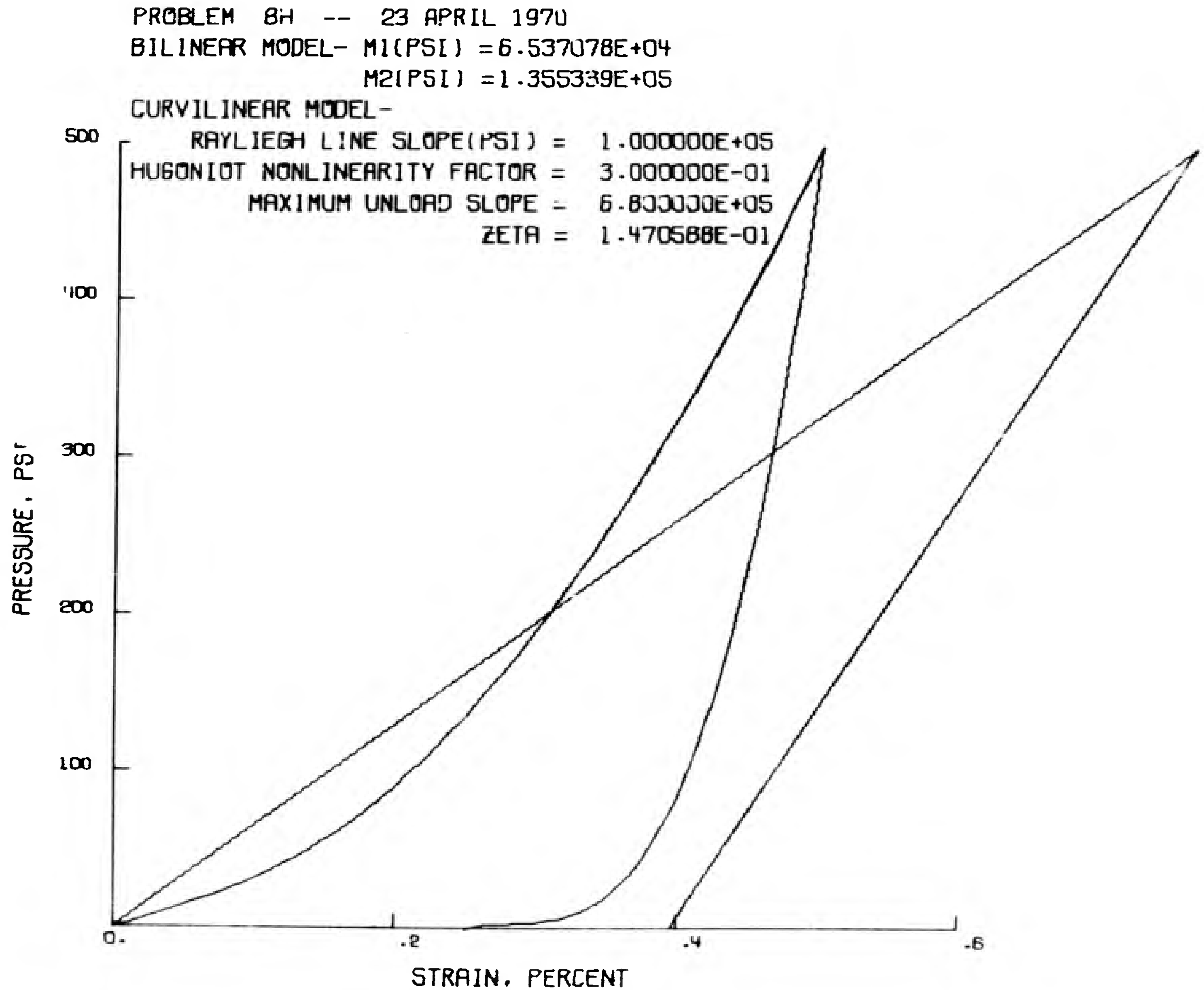


PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 9H -- 23 APRIL 1970

NUMBER OF DATA POINTS. N=123 . M= 93

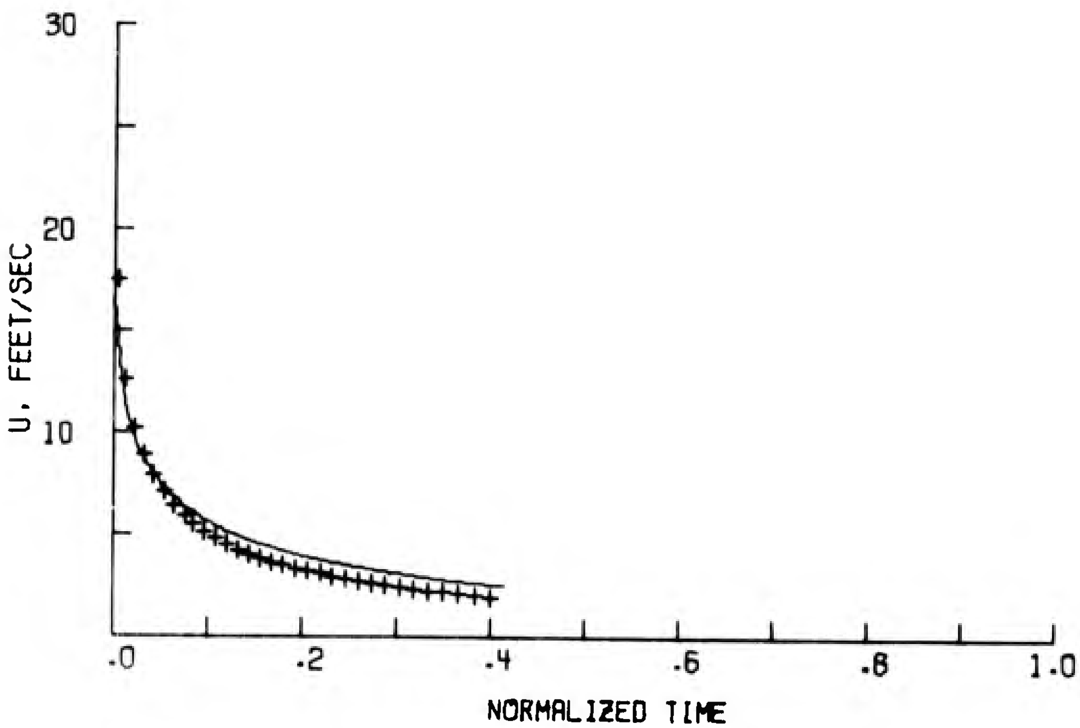
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.875949E+03
	SOUND VELOCITY =	2.599350E+03
	ZETA =	1.616430E-01

FITTING ERRORS

E_1 =	9.979896E-01	E_2 =	4.016792E-03
E_3 =	1.357584E-04	E_4 =	1.165154E-02
E_5 =	9.967443E-01	E_6 =	6.500855E-03
E_7 =	5.522623E-01	E_8 =	[illegible].431435E-01

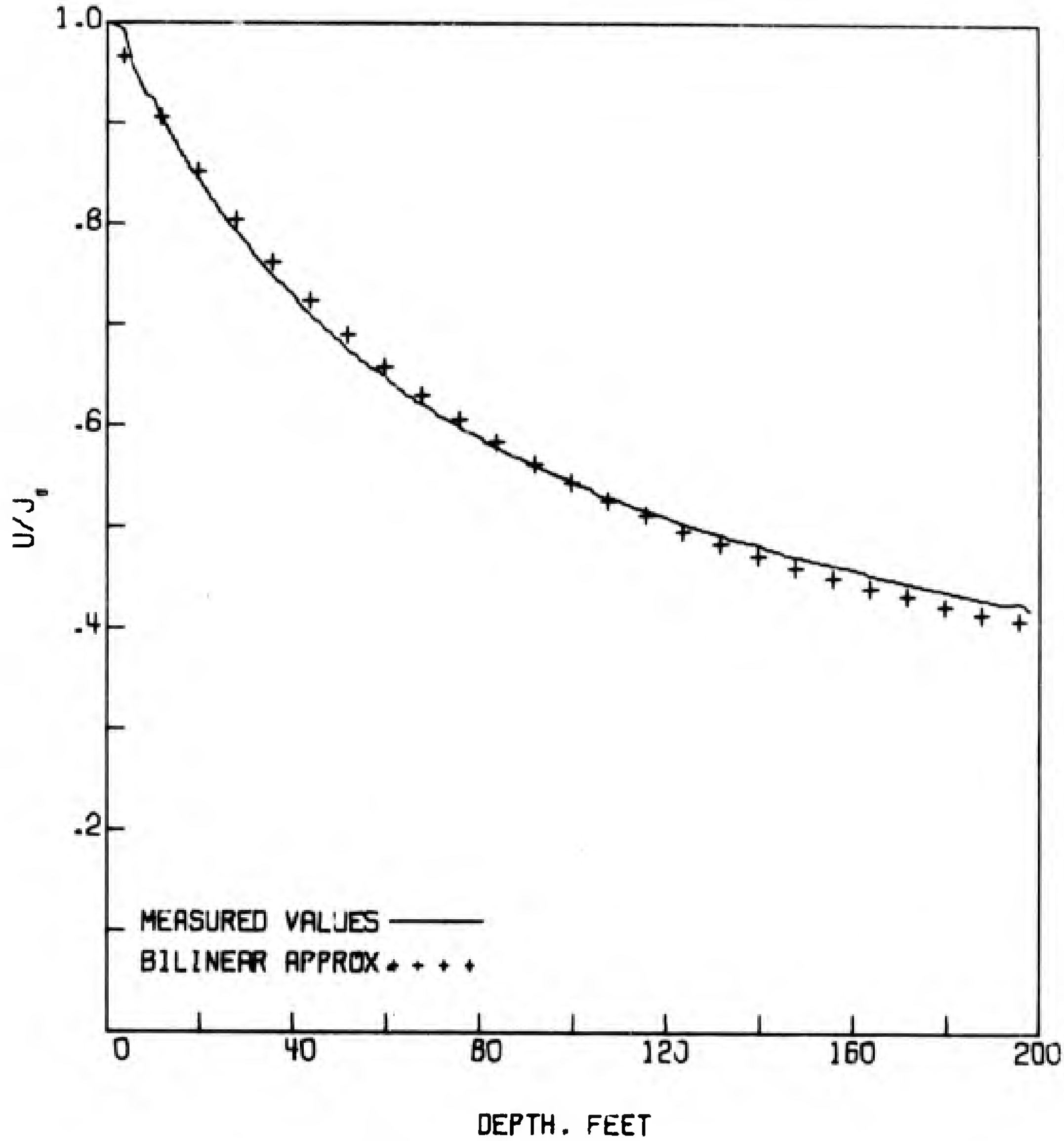
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.400	1.380
25	1.285	1.240
50	1.168	1.120
75	1.058	1.013
100	.954	.916



SURFACE PARTICLE VELOCITY

PROBLEM 94 -- 23 APRIL 1970

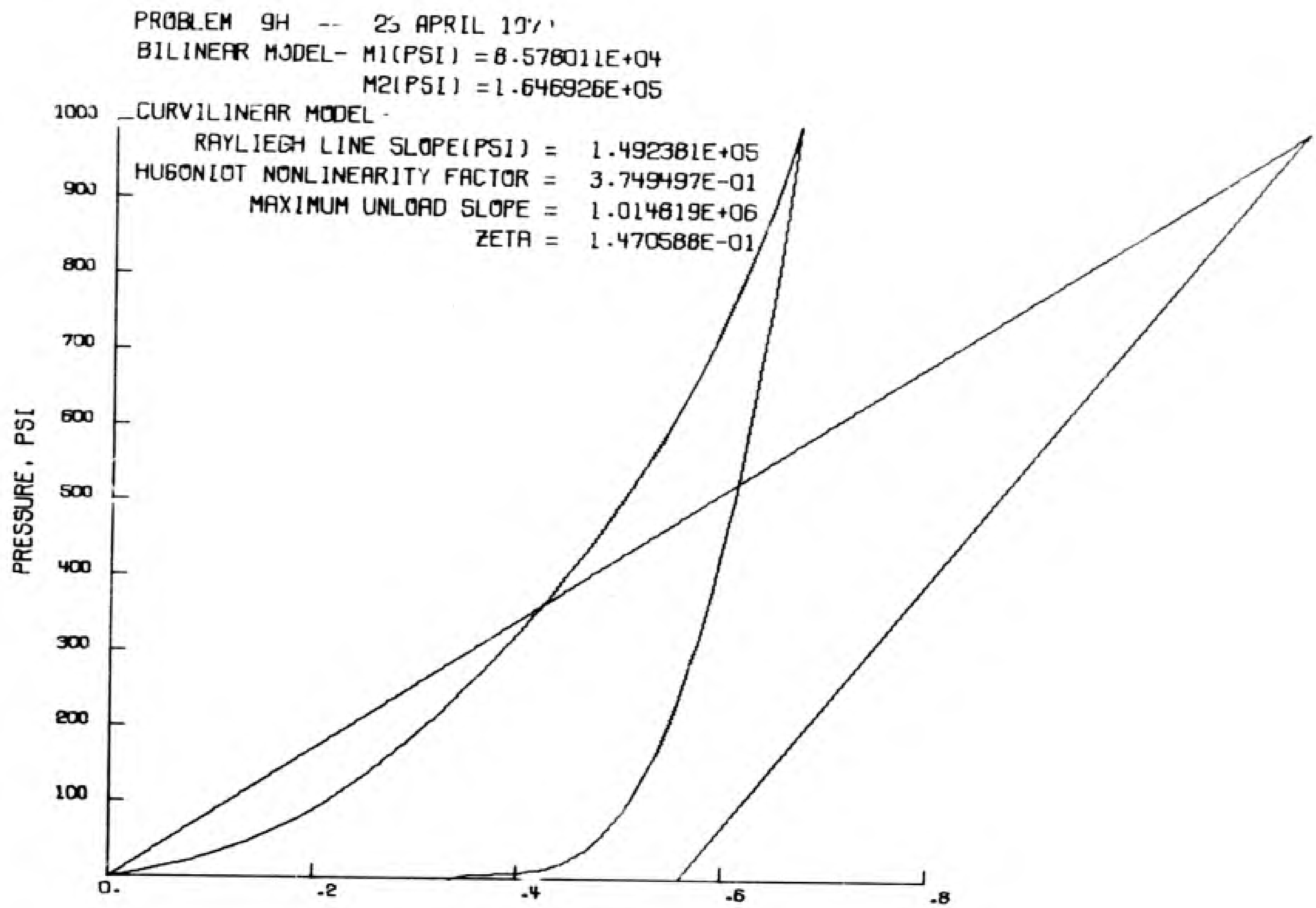


PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 104 -- 23 APRIL 1970

NUMBER OF DATA POINTS. N=101 . M= 99

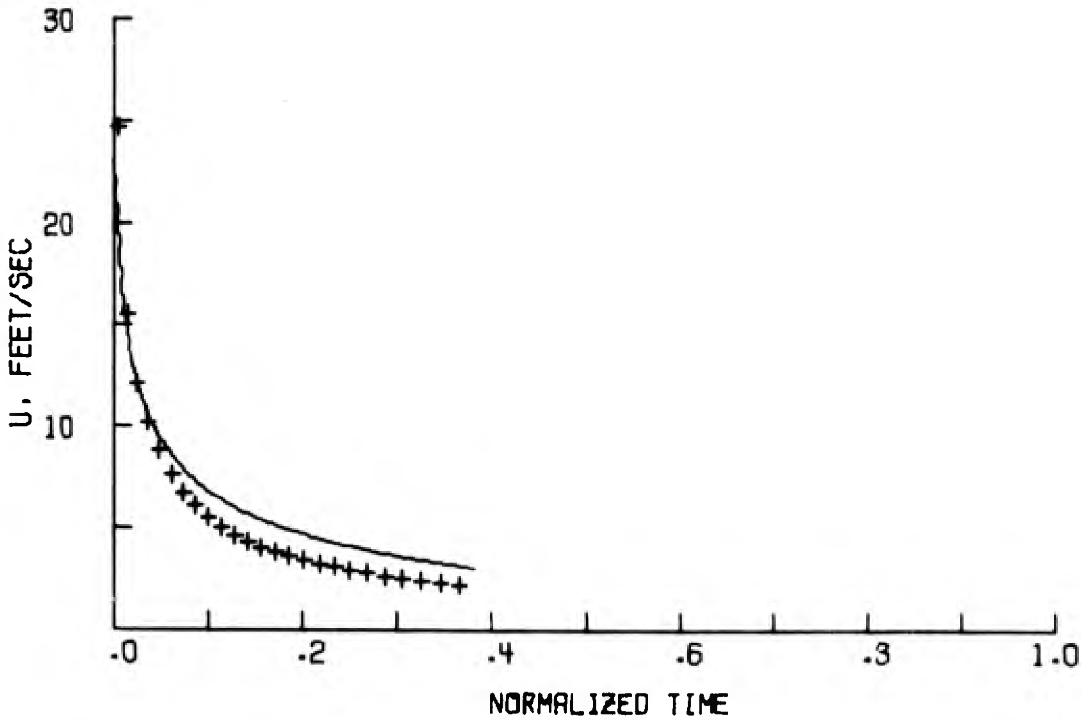
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	2.197920E+03
	SOUND VELOCITY =	2.896686E+03
	BETA =	1.371580E-01

FITTING ERRORS

E_1 =	9.963549E-01	E_2 =	7.276916E-03
E_3 =	1.609635E-04	E_4 =	1.615437E-02
E_5 =	9.929348E-01	E_6 =	1.408058E-02
E_7 =	1.894341E+00	E_8 =	1.376351E+00

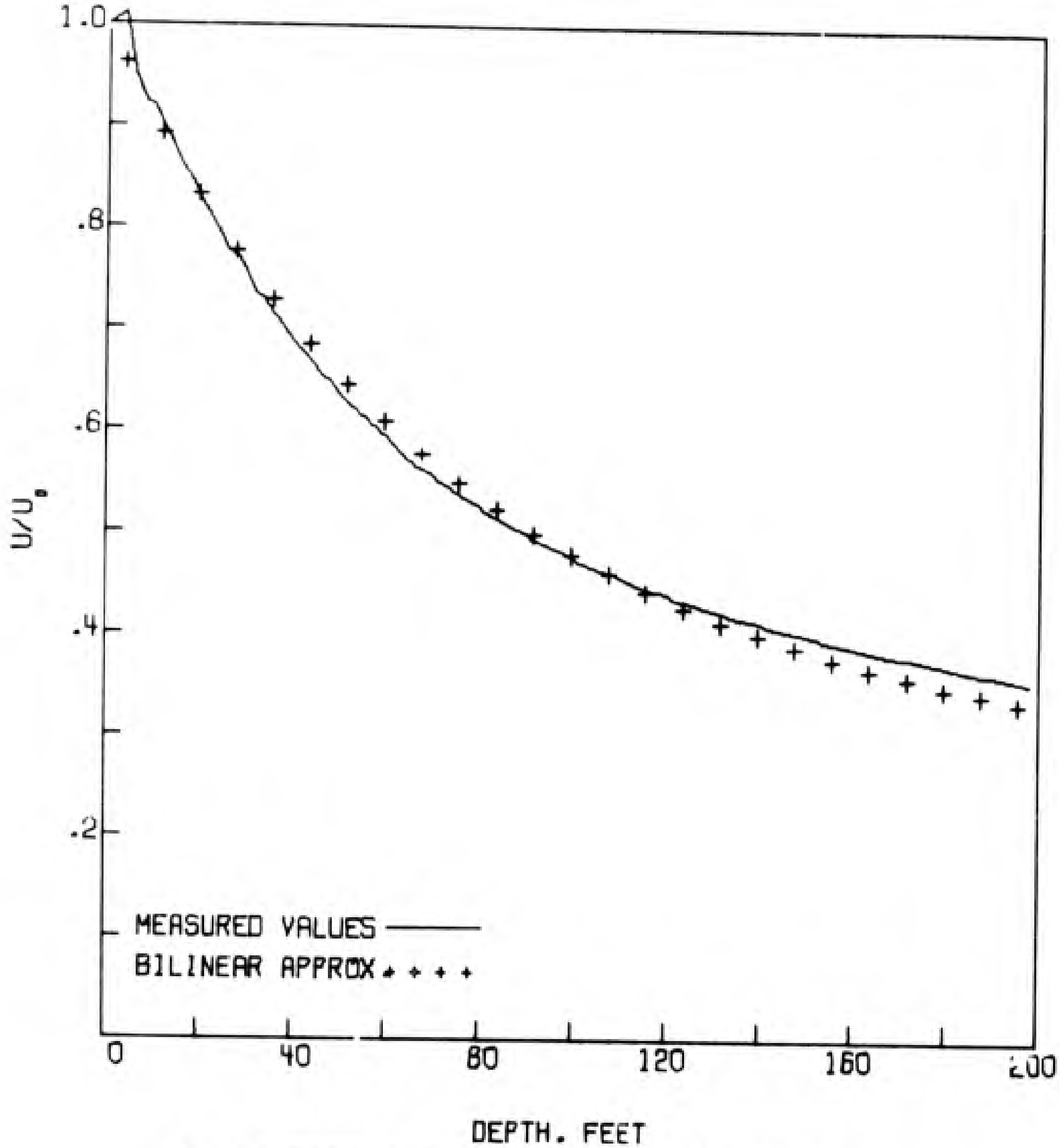
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.821	1.726
25	1.685	1.552
50	1.546	1.408
75	1.414	1.284
100	1.289	1.175



SURFACE PARTICLE VELOCITY

PROBLEM 10H -- 23 APRIL 1973

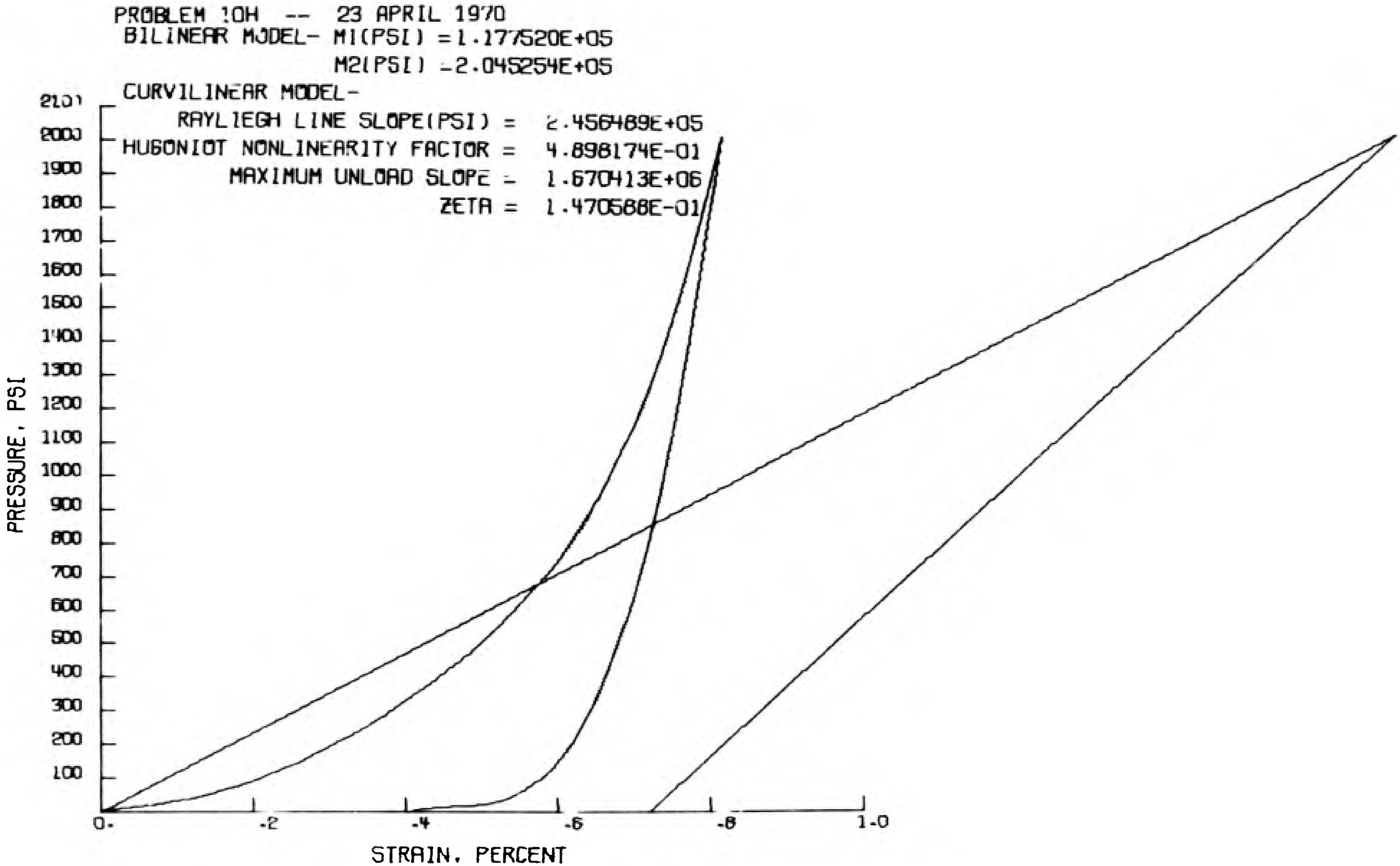


PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 7J -- 23 APRIL 1970

NUMBER OF DATA POINTS. N= 56 . M= 99

MATERIAL PROPERTIES

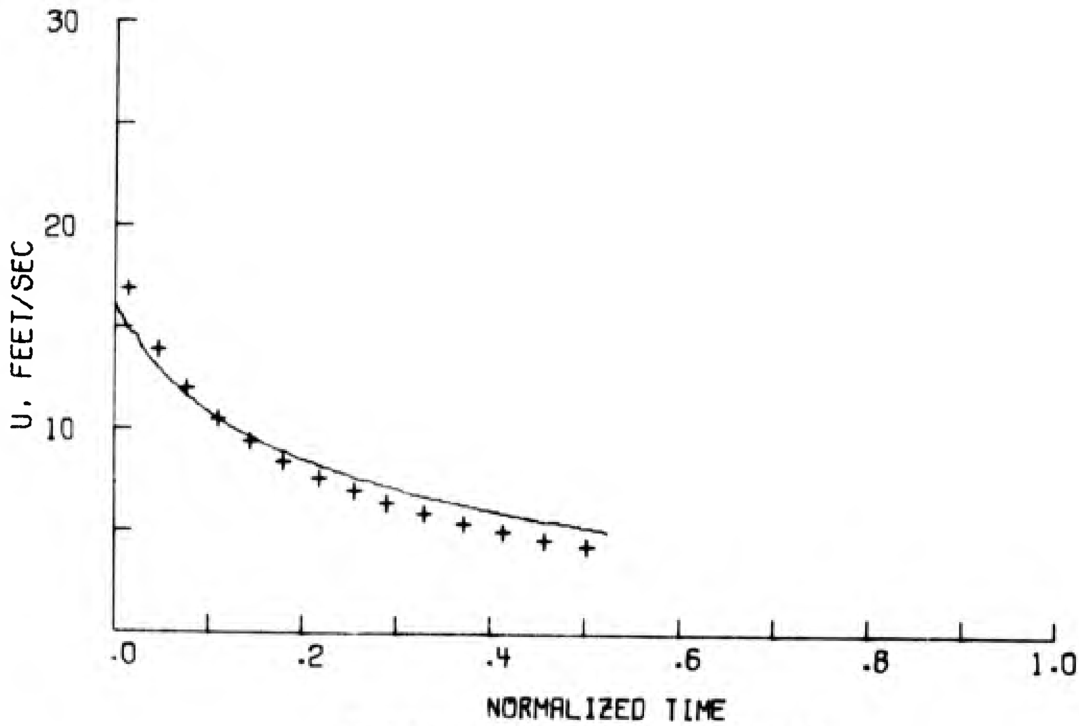
DENSITY	=	3.510000E+00
SHOCK VELOCITY	=	4.243641E+02
SOUND VELOCITY	=	7.474220E+02
ZETA	=	2.756970E-01

FITTING ERRORS

E_1 =	9.211962E-01	E_2 =	1.513976E-01
E_3 =	5.957471E-03	E_4 =	7.718465E-02
E_5 =	9.985677E-01	E_6 =	2.862642E-03
E_7 =	6.926704E-01	E_8 =	8.322682E-01

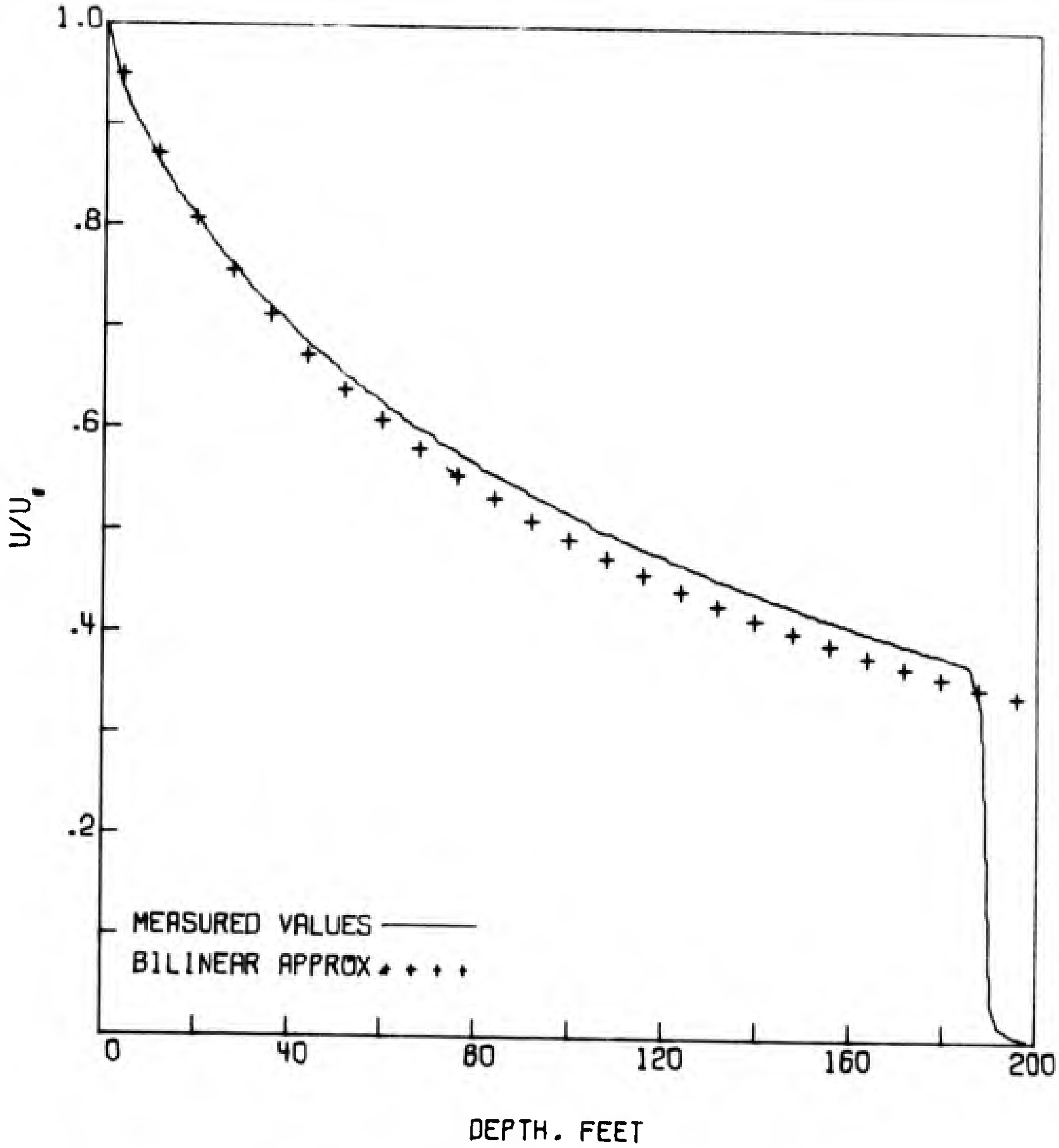
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.193	2.301
25	1.543	1.529
50	.886	.860
75	.261	.239
100	.000	0.



SURFACE PARTICLE VELOCITY

PROBLEM 7J -- 23 APRIL 1970



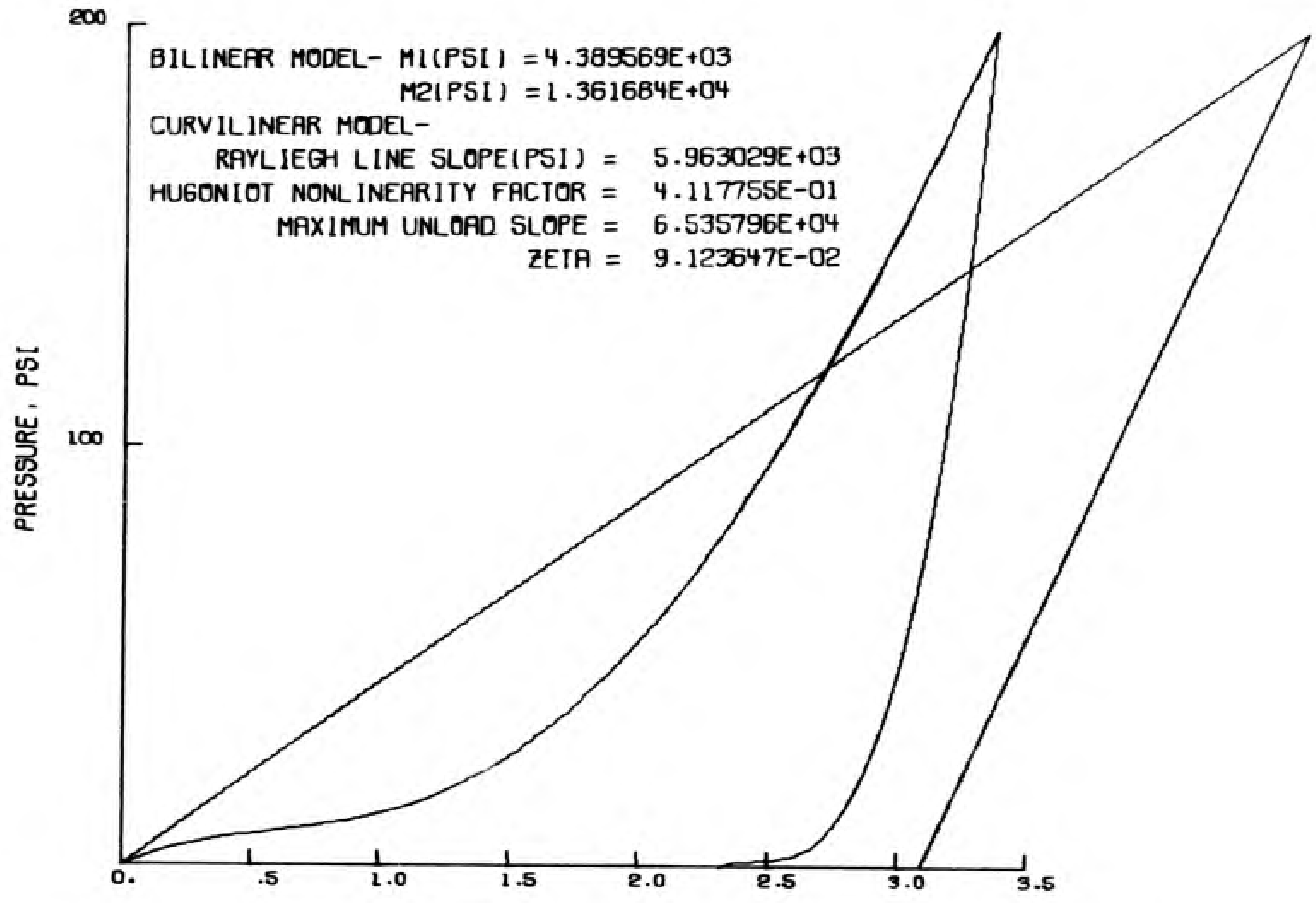
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 7J -- 23 APRIL 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 8J -- 23 APRIL 1970

NUMBER OF DATA POINTS. N= 57 . M= 99

MATERIAL PROPERTIES

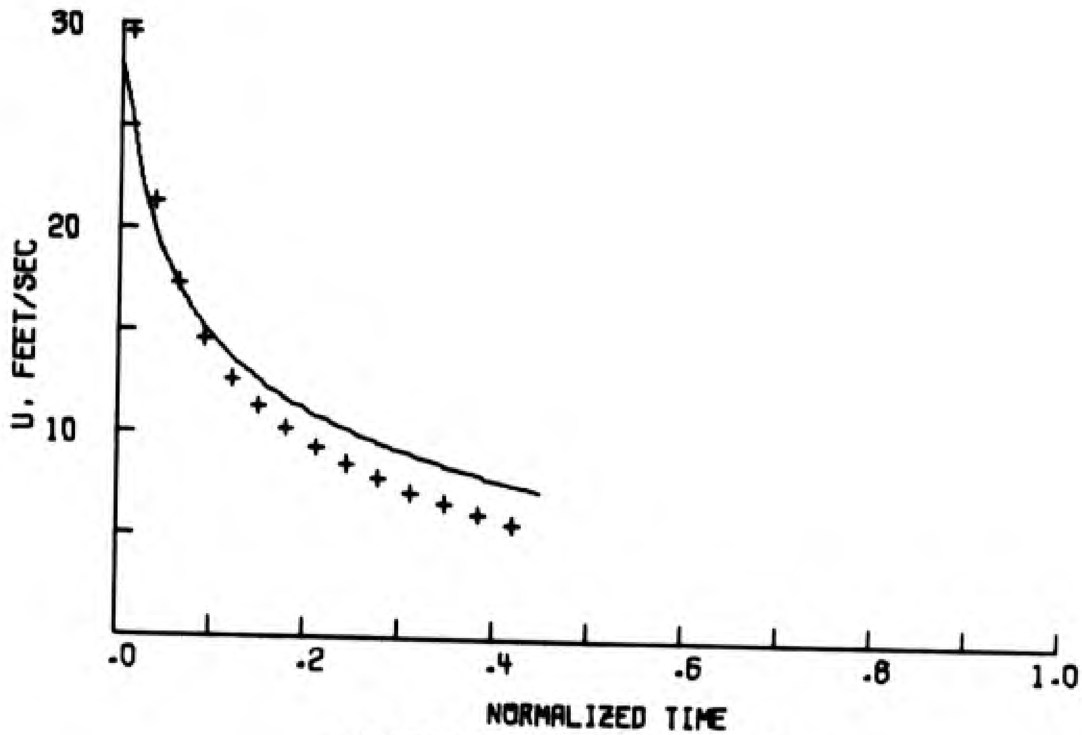
DENSITY	=	3.510000E+00
SHOCK VELOCITY	=	4.836168E+02
SOUND VELOCITY	=	6.576140E+02
ZETA	=	1.524645E-01

FITTING ERRORS

E_1 =	9.996340E-01	E_2 =	7.318878E-04
E_3 =	8.611315E-05	E_4 =	9.279717E-03
E_5 =	9.979035E-01	E_6 =	4.188524E-03
E_7 =	3.371199E+00	E_8 =	1.836082E+00

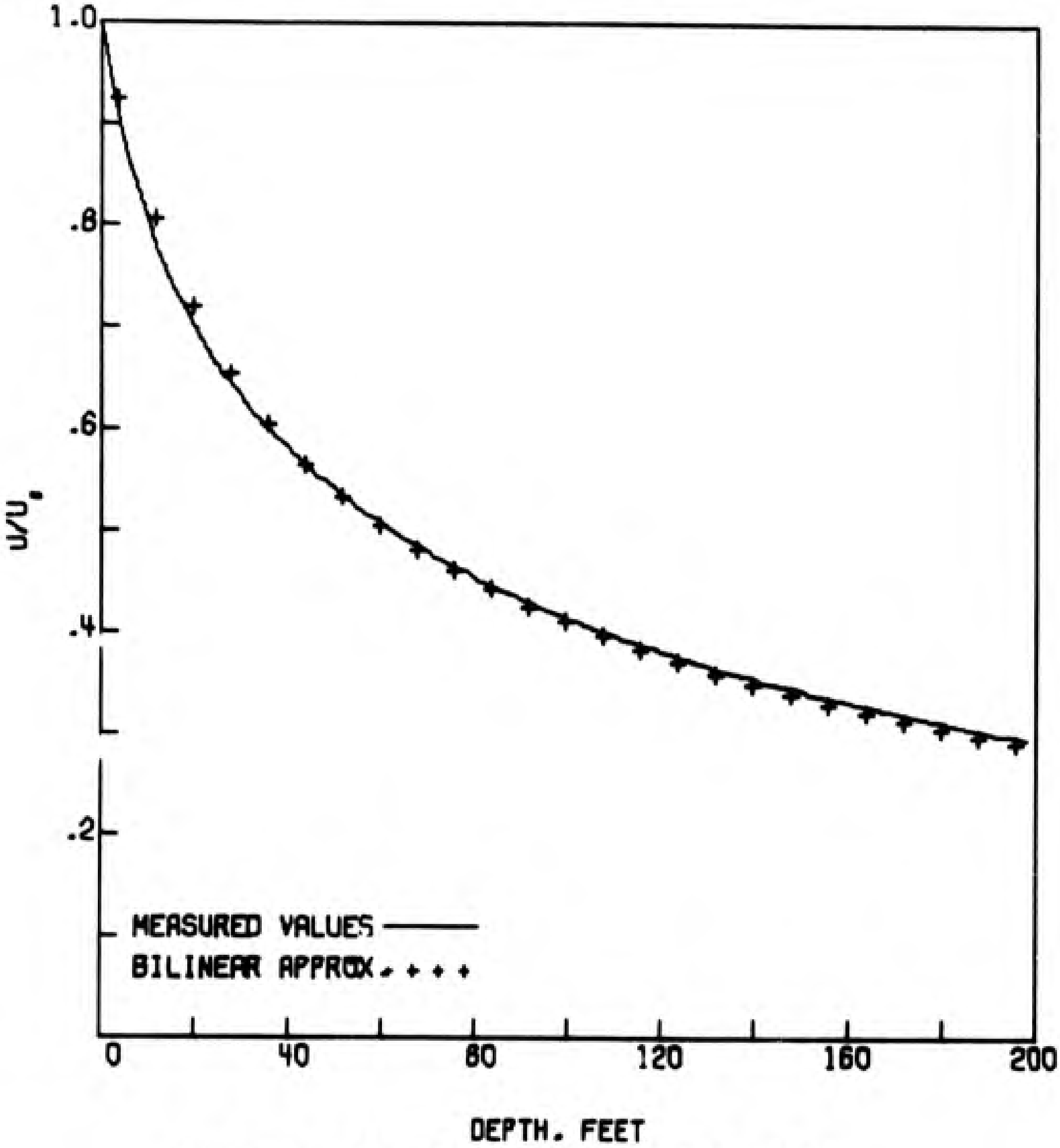
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	3.280	3.428
25	2.410	2.421
50	1.572	1.573
75	.793	.753
100	.052	0.



SURFACE PARTICLE VELOCITY

PROBLEM 8J -- 23 APRIL 1970

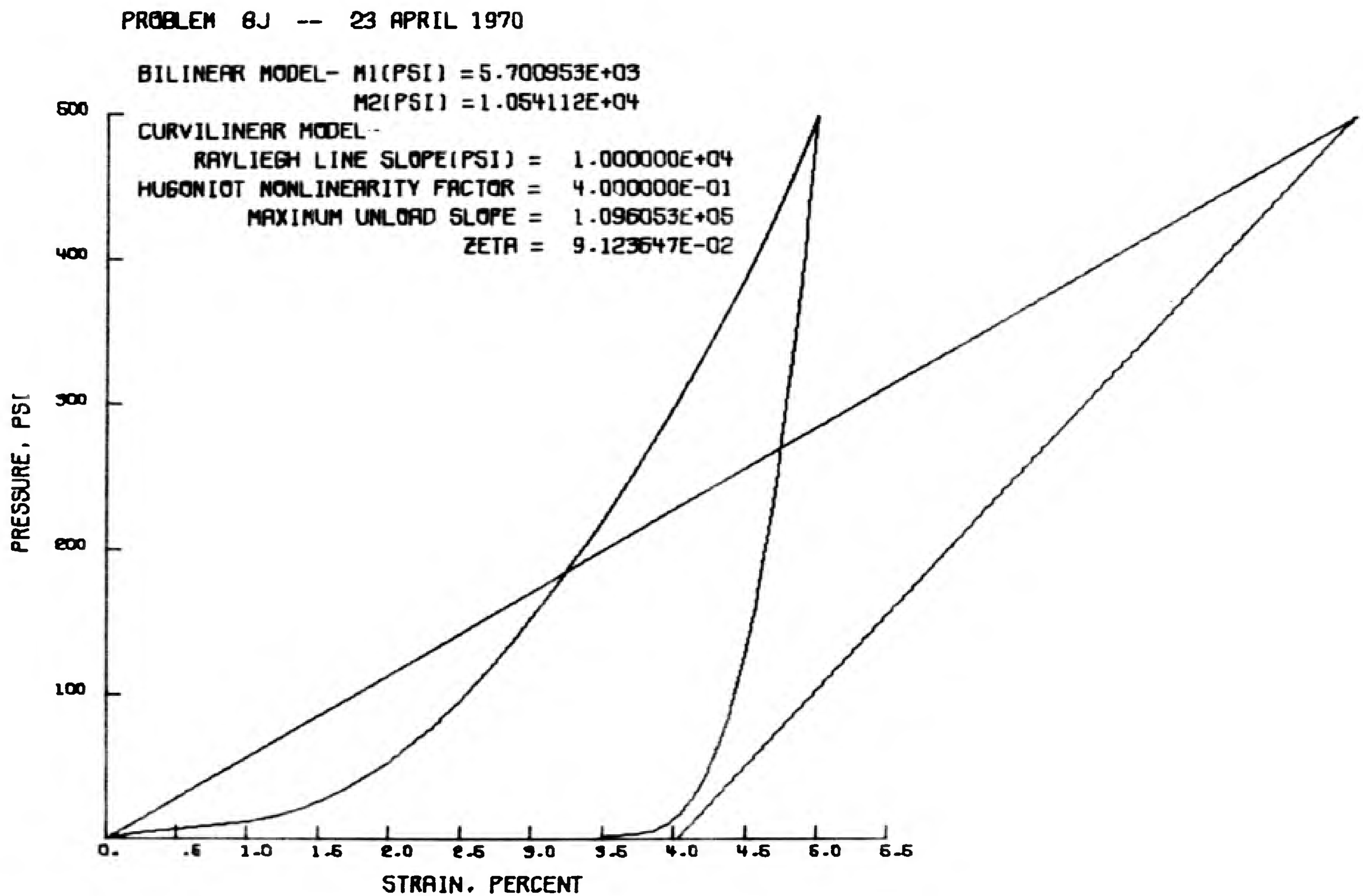


PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODE.

PROBLEM 9J -- 23 APRIL 1970

NUMBER OF DATA POINTS. N= 51 . M= 99

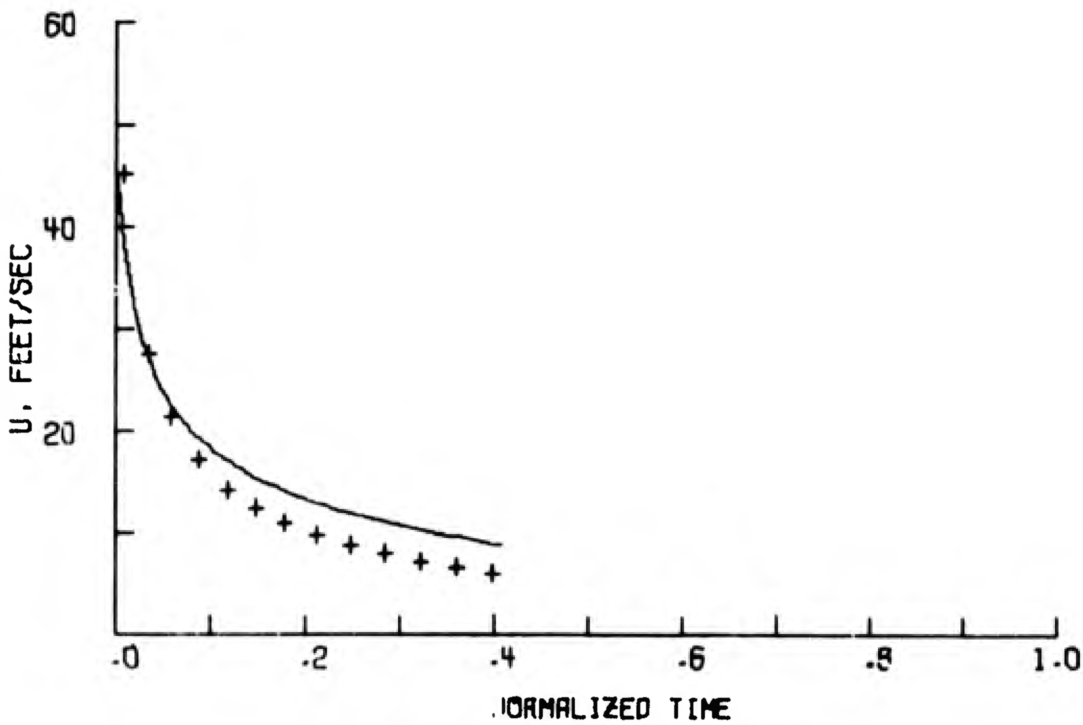
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	5.781802E+02
	SOUND VELOCITY =	7.622416E+02
	ZETA =	1.373160E-01

FITTING ERRORS

E_1 =	9.987377E-01	E_2 =	2.522935E-03
E_3 =	1.501747E-04	E_4 =	1.225458E-02
E_5 =	9.953670E-01	E_6 =	9.244625E-03
E_7 =	1.045288E+01	E_8 =	3.233091E+00

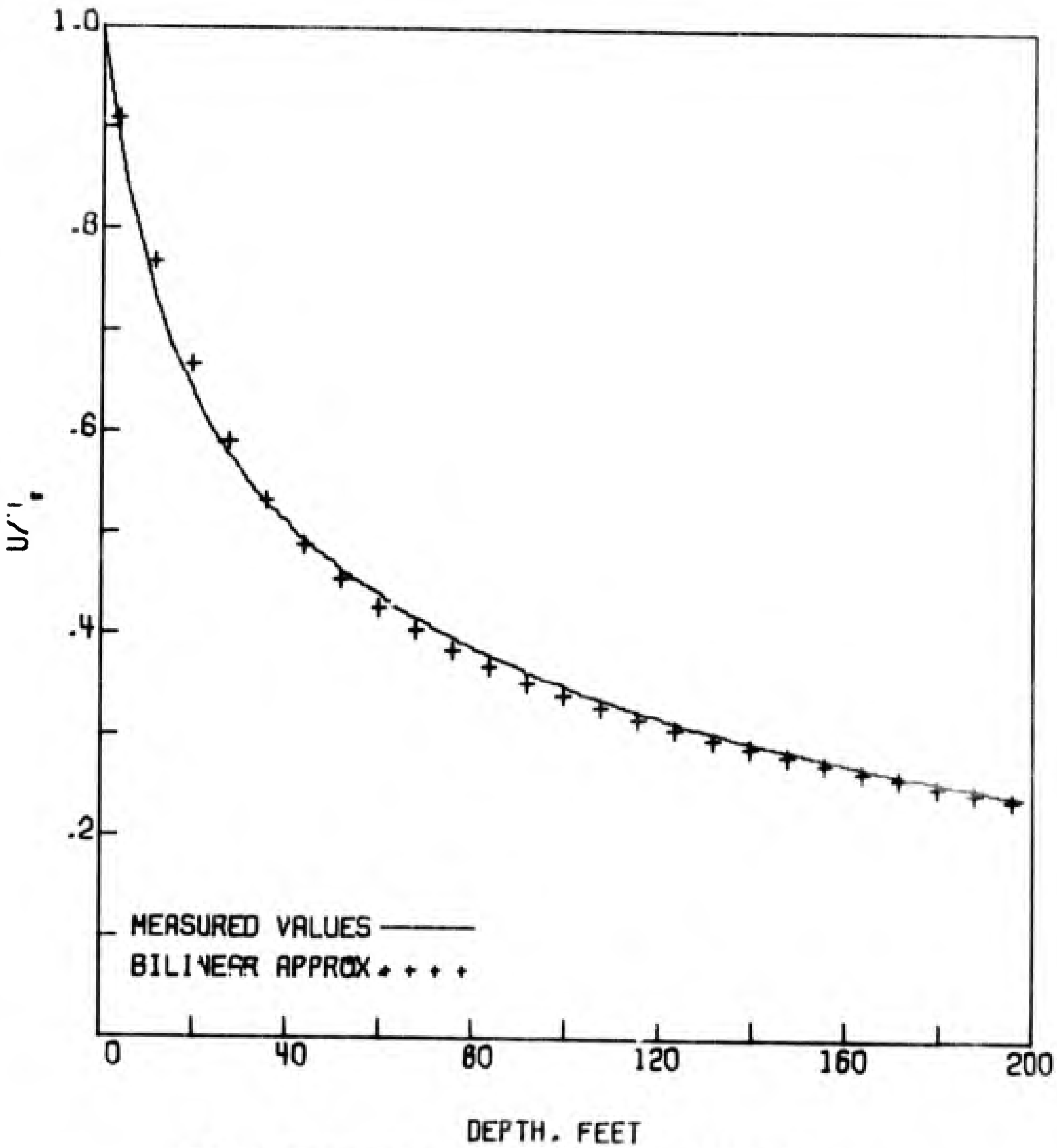
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	4.395	4.373
25	3.321	3.188
50	2.305	2.276
75	1.375	1.439
100	.499	.582



SURFACE PARTICLE VELOCITY

PROBLEM 9J -- 23 APRIL 1970



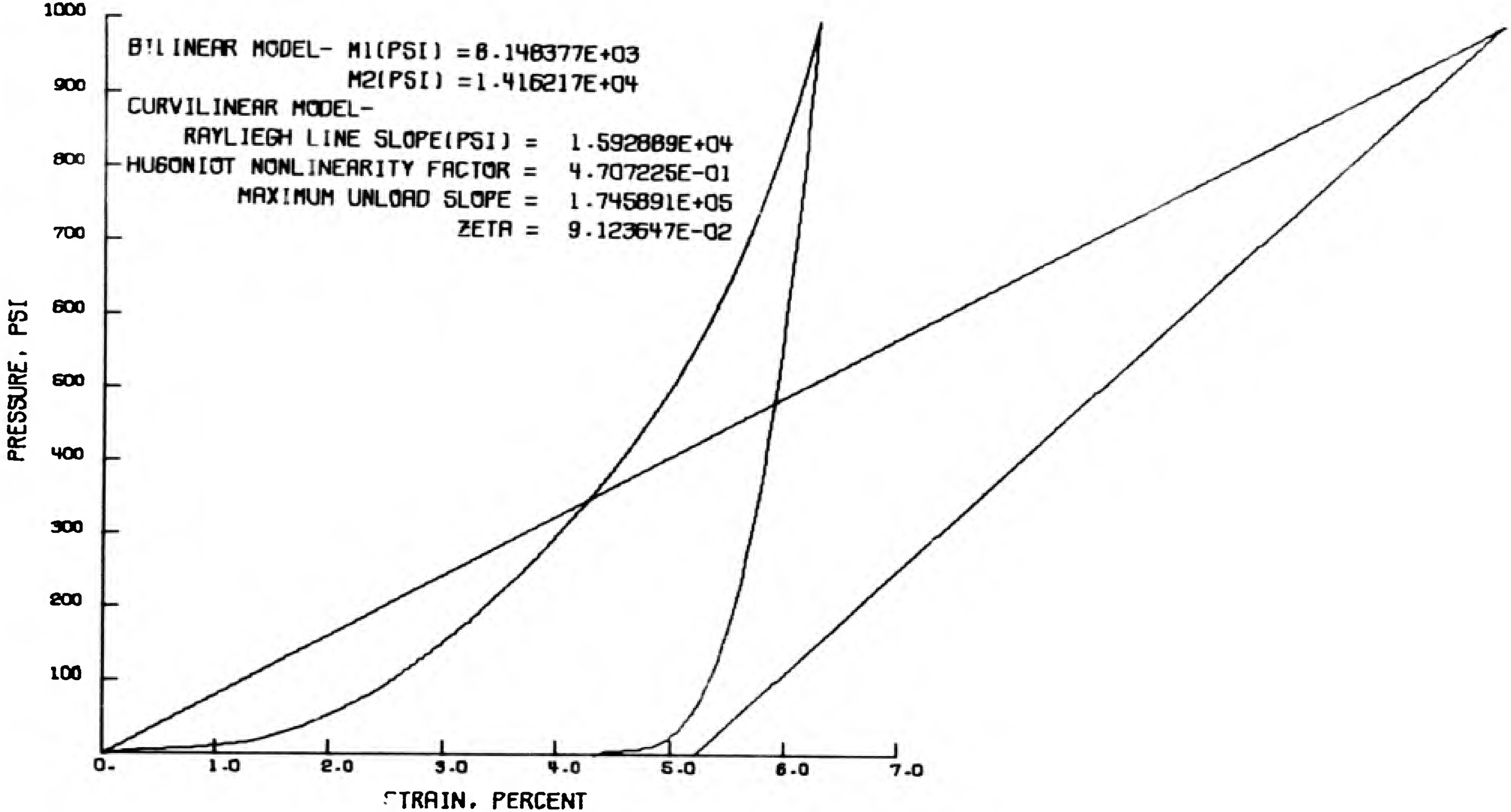
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 9J -- 23 APRIL 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 10J -- 23 APRIL 1970

NUMBER OF DATA POINTS. N= 40 . M= 99

MATERIAL PROPERTIES

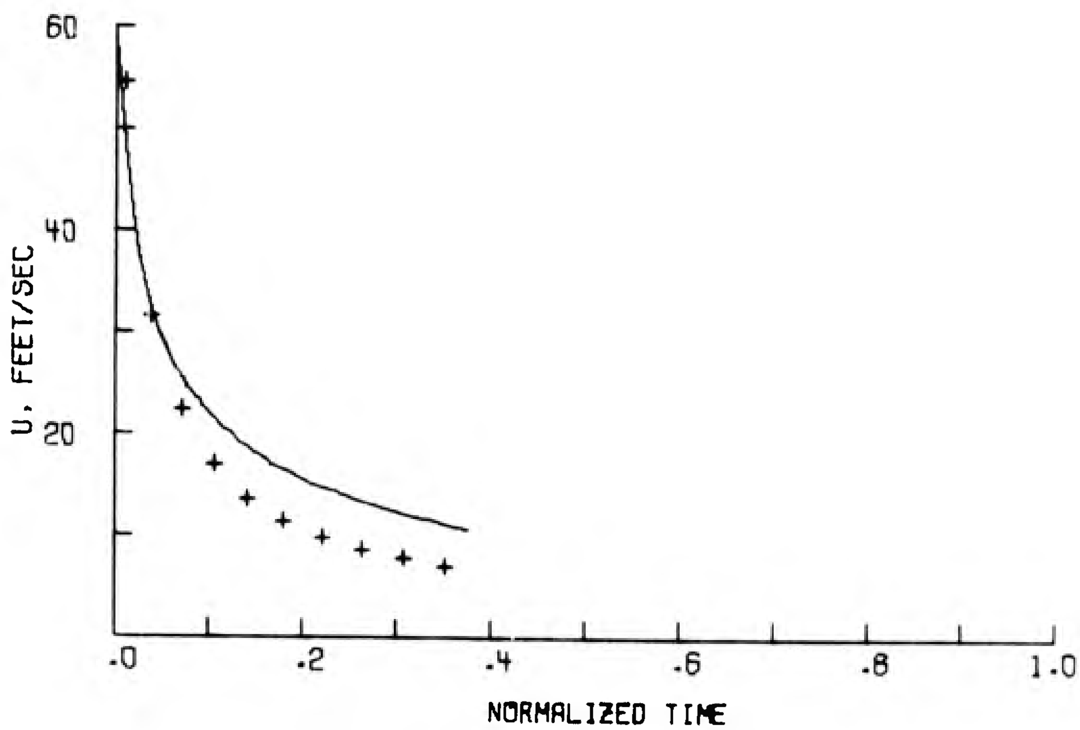
DENSITY =	3.510000E+00
SHOCK VELOCITY =	6.564554E+02
SOUND VELOCITY =	8.116939E+02
ZETA =	1.057375E-01

FITTING ERRORS

E_1 =	9.965170E-01	E_2 =	6.953816E-03
E_3 =	4.450874E-04	E_4 =	2.109709E-02
E_5 =	9.927103E-01	E_6 =	1.452629E-02
E_7 =	2.550450E+01	E_8 =	5.050198E+00

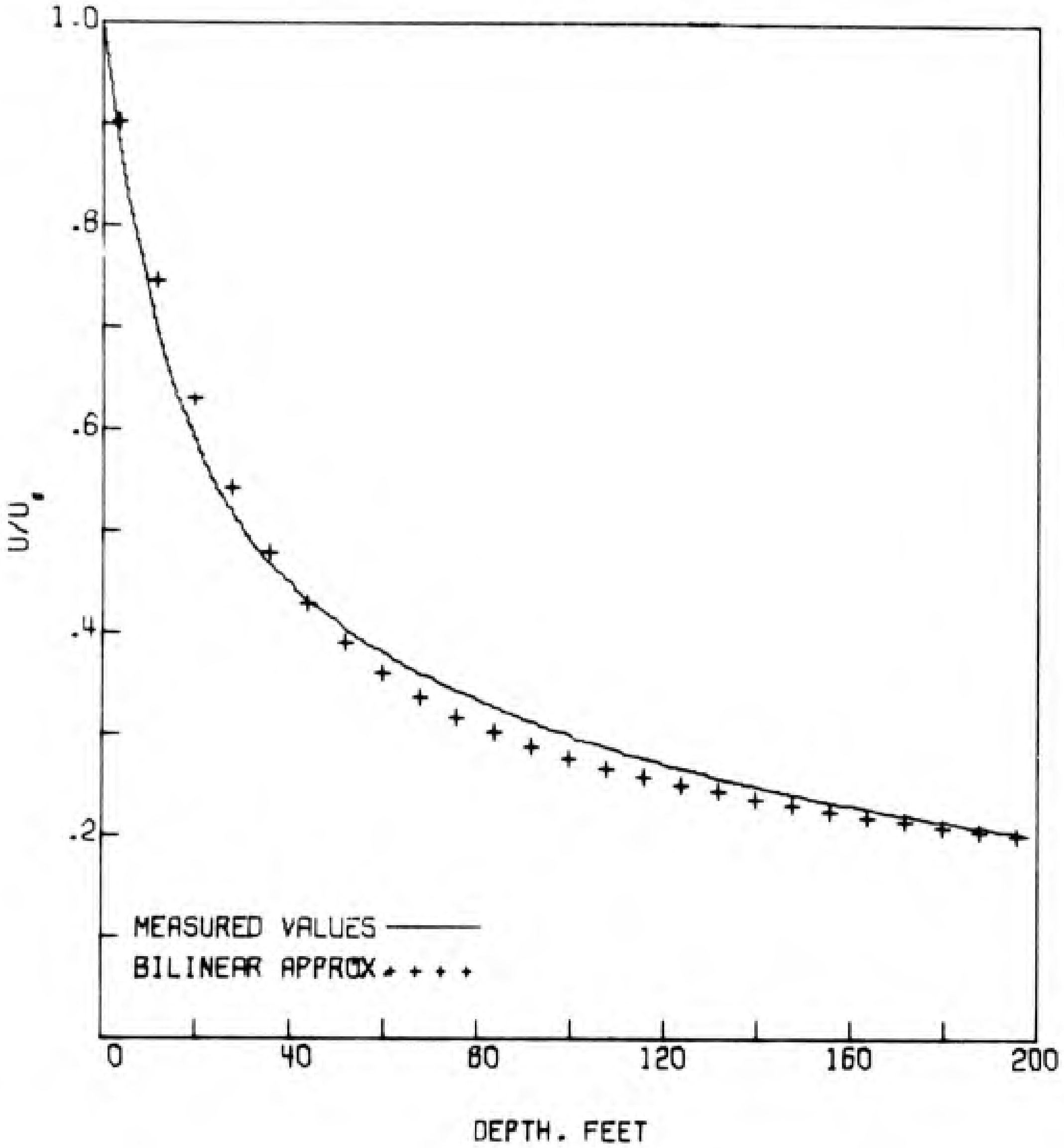
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	5.658	5.551
25	4.408	4.124
50	3.215	3.105
75	2.120	2.207
100	1.093	1.299



SURFACE PARTICLE VELOCITY

PROBLEM 10J -- 23 APRIL 1970



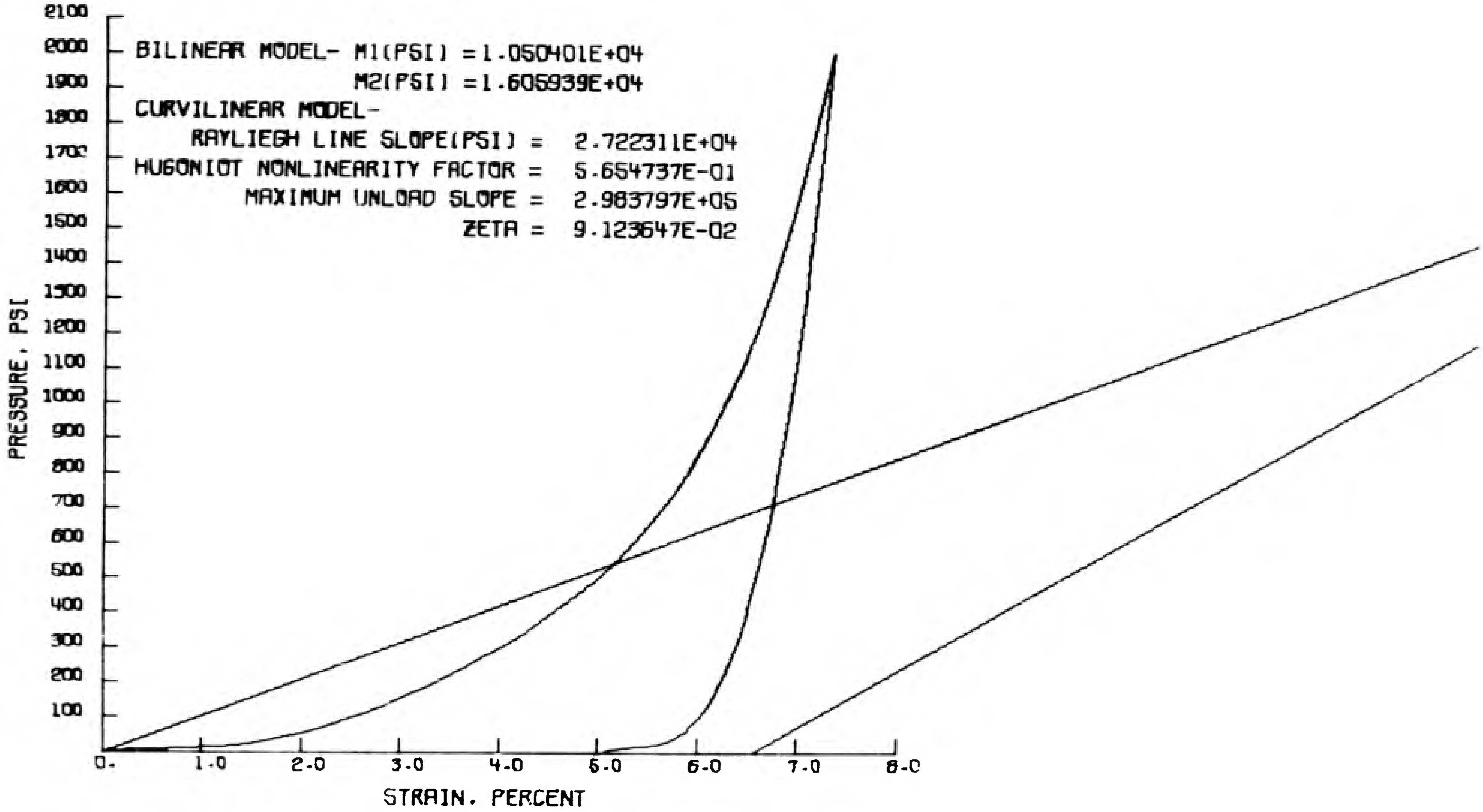
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM 10J -- 23 APRIL 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 13C -- 6 APRIL 1970

NUMBER OF DATA POINTS. N=105 . M= 99

MATERIAL PROPERTIES

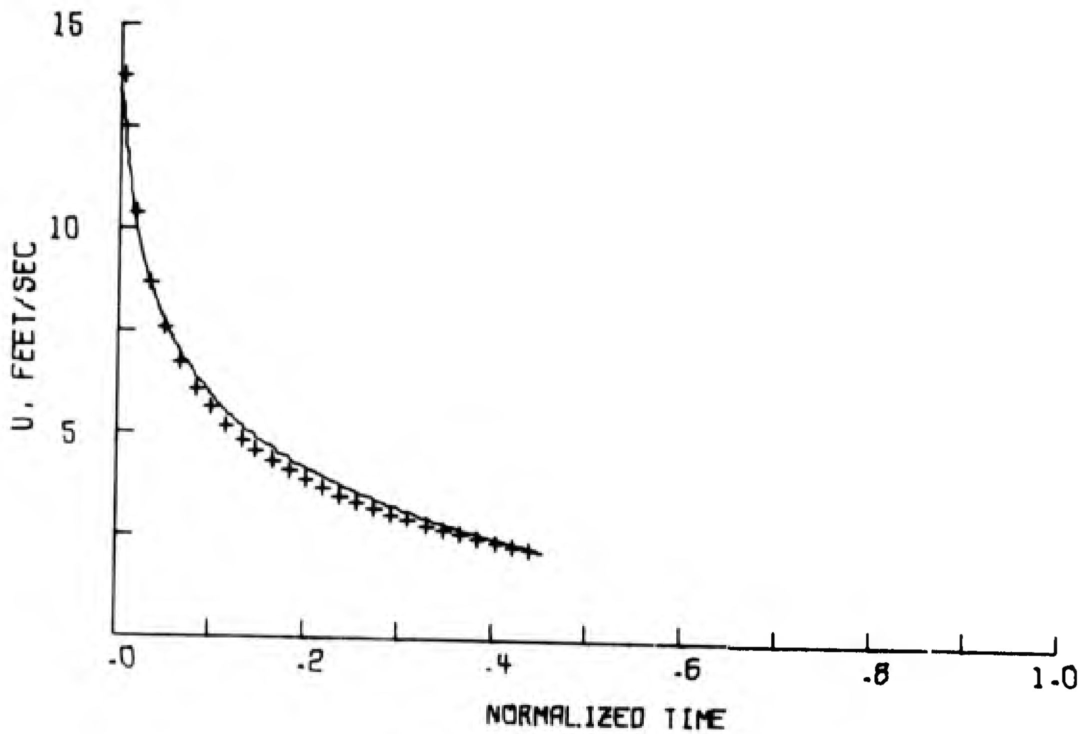
DENSITY = 3.510000E+00
SHOCK VELOCITY = 1.288919E+03
SOUND VELOCITY = 2.055151E+03
ZETA = 2.291315E-01

FITTING ERRORS

E_1 = 9.994910E-01 E_2 = 1.017770E-03
E_3 = 2.277771E-05 E_4 = 4.772600E-03
E_5 = 9.961261E-01 E_6 = 7.732720E-03
E_7 = 9.448421E-02 E_8 = 3.073828E-01

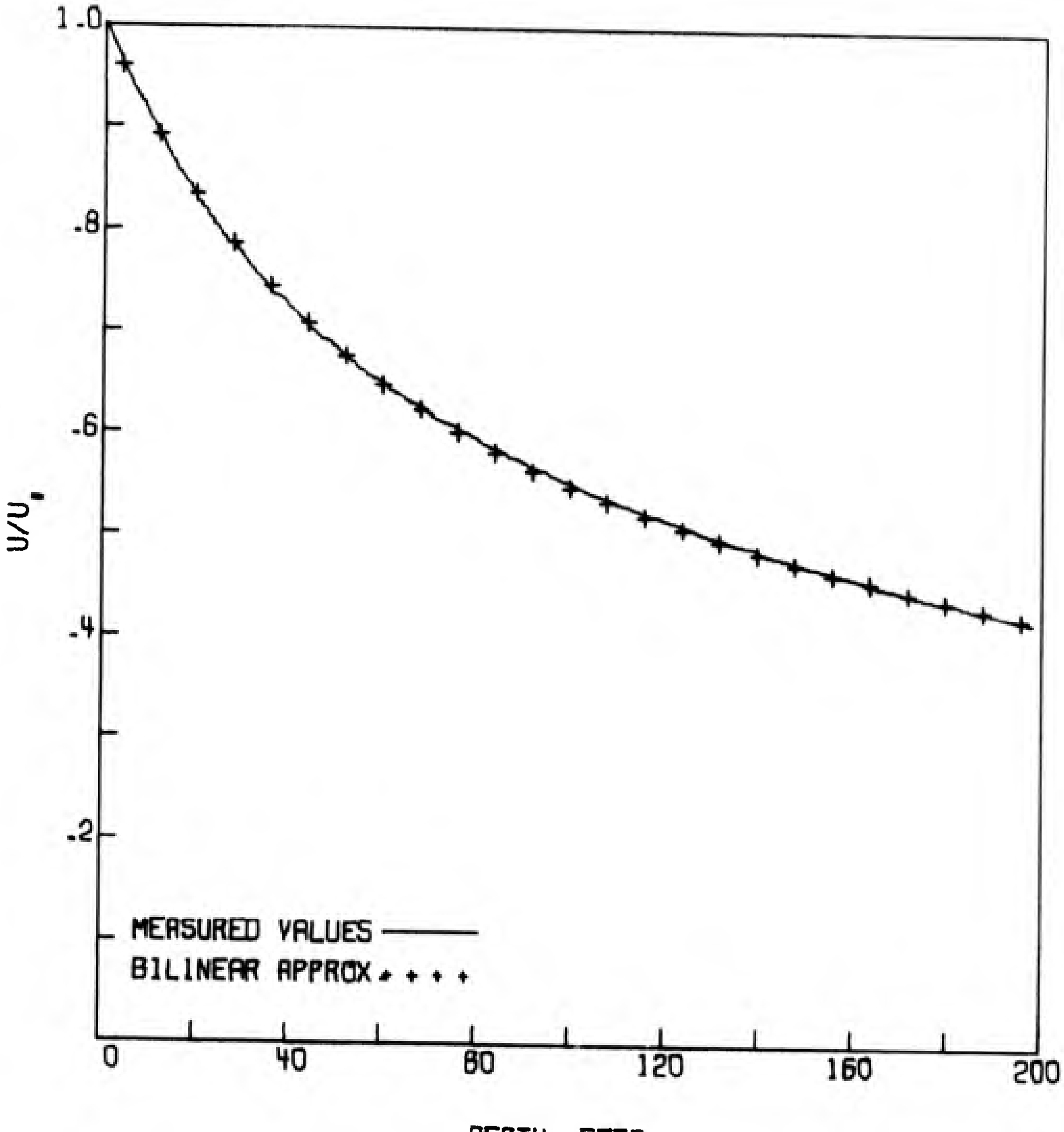
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.361	1.356
25	1.212	1.171
50	1.056	1.013
75	.906	.871
100	.761	.739



SURFACE PARTICLE VELOCITY

PROBLEM 13C -- 6 APRIL 1970



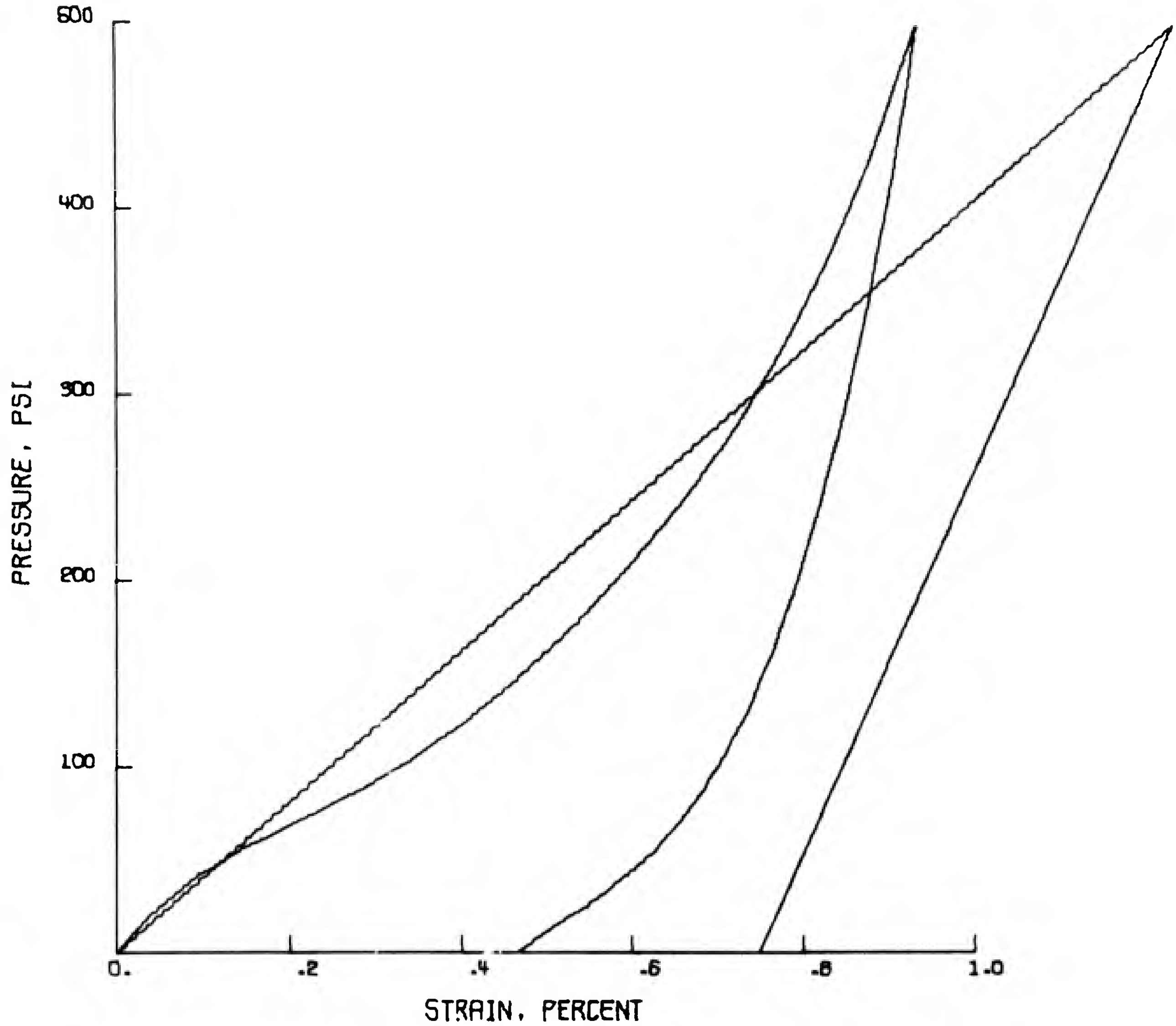
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 13C -- 6 APRIL 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.049448E+04
M2(PSI) =1.029513E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 5.349940E+04
HUGONIOT NONLINEARITY FACTOR = 2.745101E-01
MAXIMUM UNLOAD SLOPE = 2.858695E+05
ZETA = 1.871462E-01

BEST BILINEAR MODEL

PROBLEM 14C -- 6 APRIL 1970

NUMBER OF DATA POINTS. N= 88 . M= 99

MATERIAL PROPERTIES

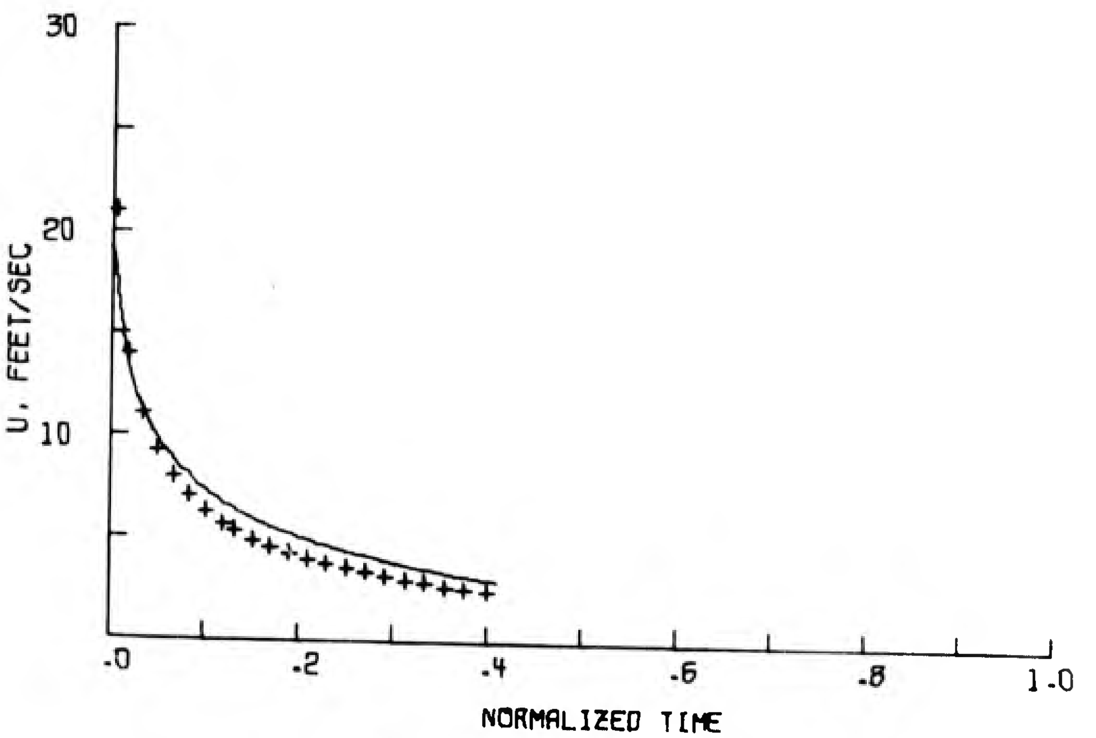
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.452081E+03
	SOUND VELOCITY =	1.972608E+03
	ZETA =	1.519925E-01

FITTING ERRORS

E_1 =	9.987045E-01	E_2 =	2.589275E-03
E_3 =	7.615599E-05	E_4 =	8.726740E-03
E_5 =	9.911574E-01	E_6 =	1.760693E-02
E_7 =	1.038517E+00	E_8 =	1.019076E+00

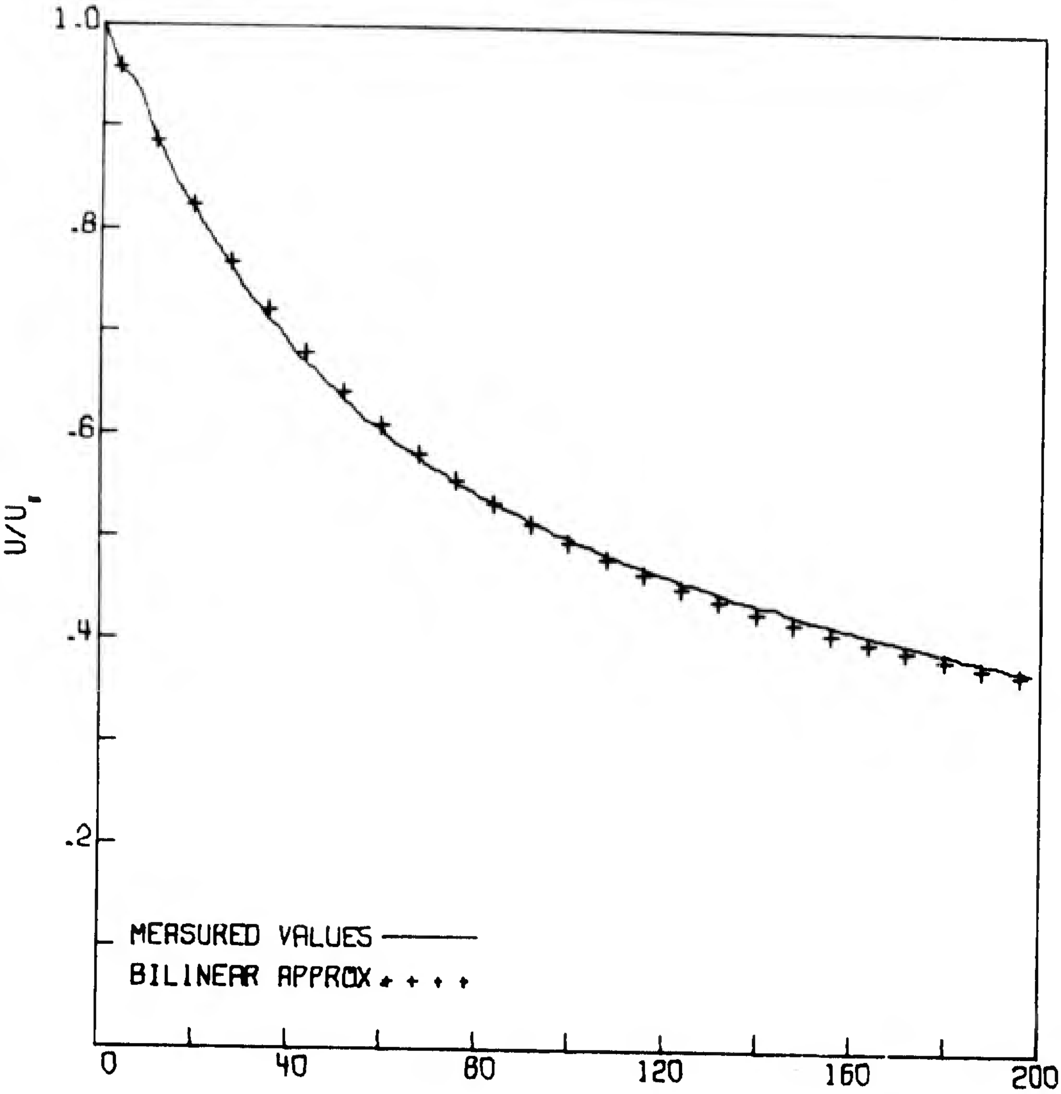
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.819	1.767
25	1.650	1.546
50	1.468	1.360
75	1.288	1.196
100	1.112	1.047



SURFACE PARTICLE VELOCITY

PROBLEM 14C -- 6 APRIL 1970



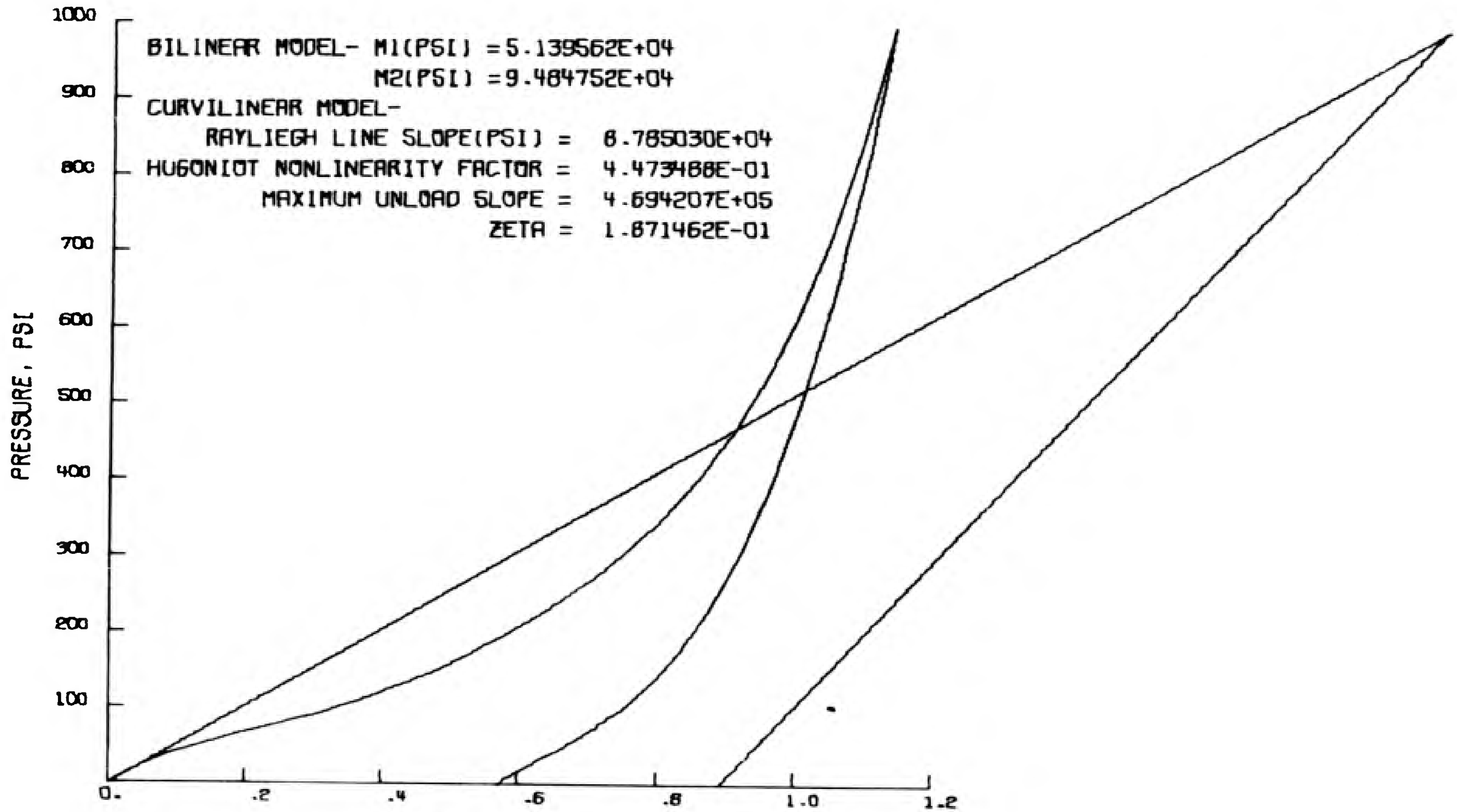
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 14C -- 6 APRIL 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 15C -- 6 APRIL 1970

NUMBER OF DATA POINTS. N= 70 . M= 99

MATERIAL PROPERTIES

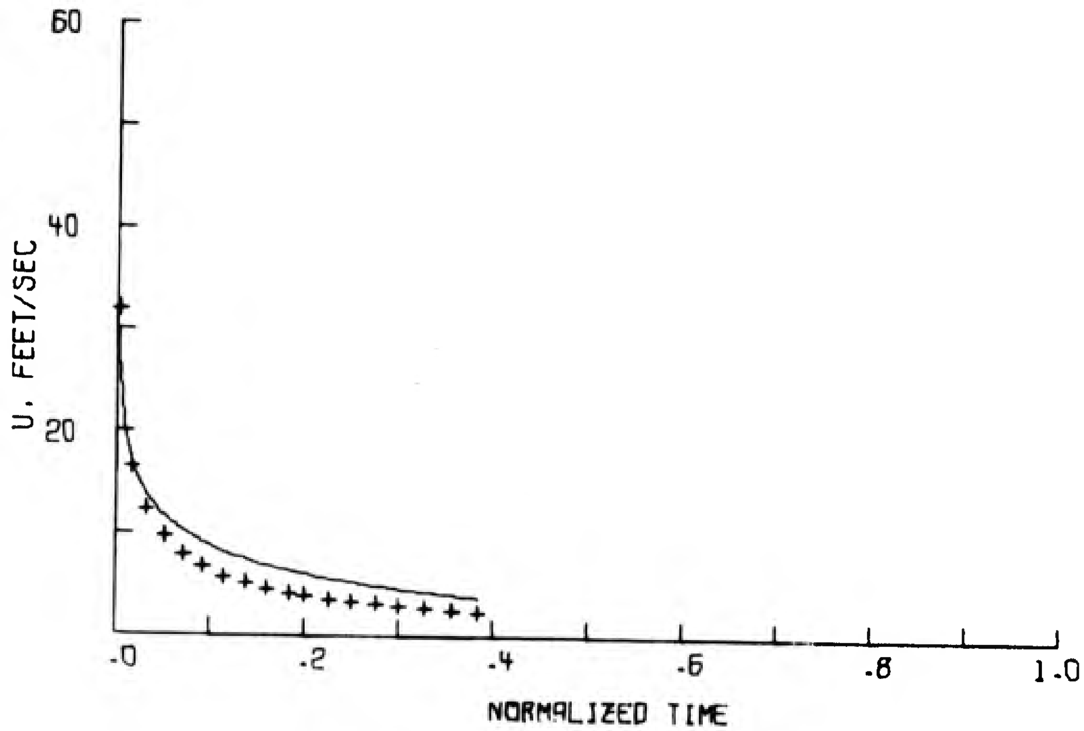
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.854296E+03
	SOUND VELOCITY =	2.403437E+03
	ZETA =	1.289750E-01

FITTING ERRORS

E_1 =	9.971398E-01	E_2 =	5.712141E-03
E_3 =	2.410239E-04	E_4 =	1.552494E-02
E_5 =	9.888789E-01	E_6 =	2.211858E-02
E_7 =	4.956790E+00	E_8 =	2.226385E+00

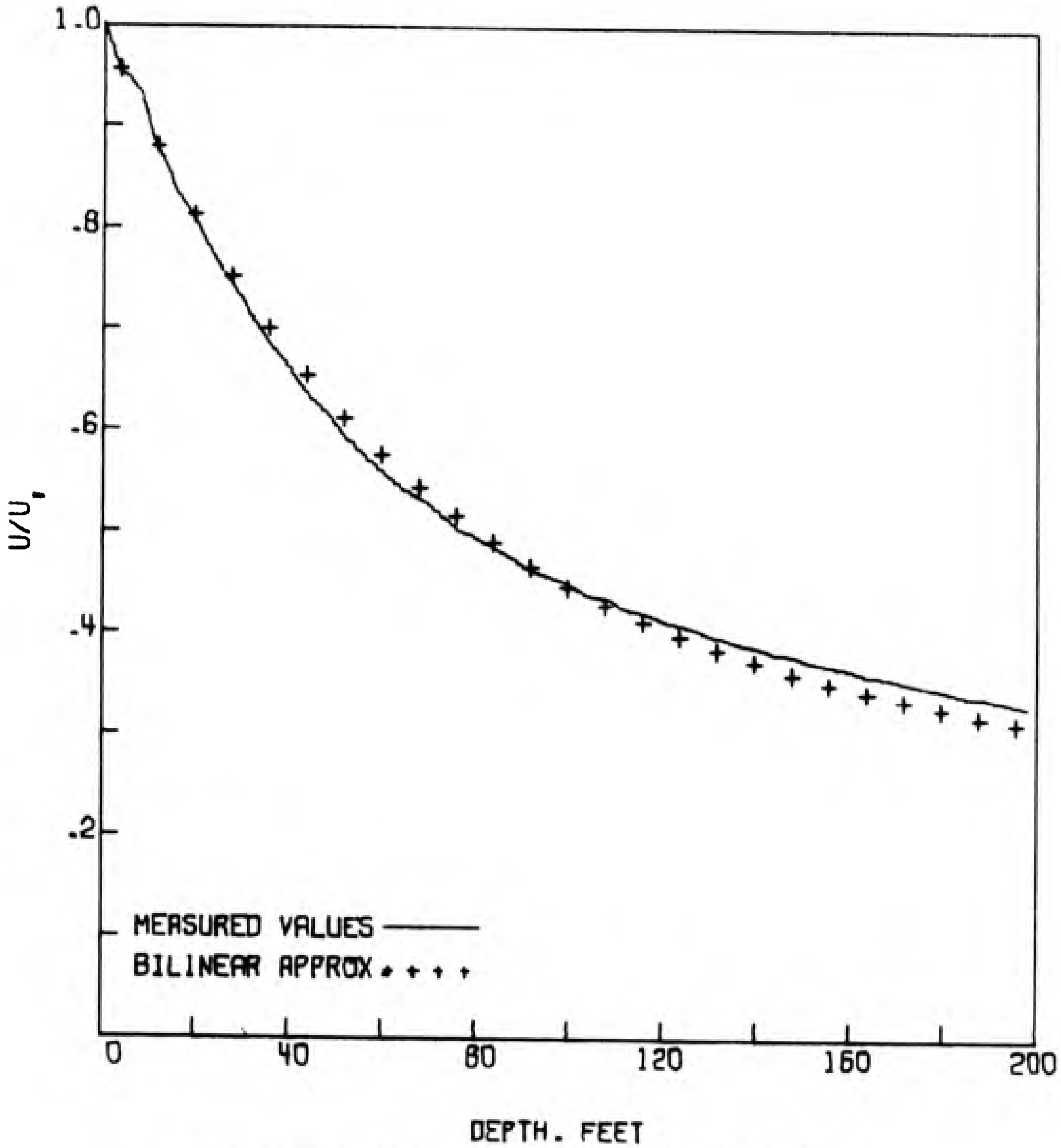
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.295	2.026
25	2.116	1.794
50	1.921	1.605
75	1.724	1.445
100	1.527	1.304



SURFACE PARTICLE VELOCITY

PROBLEM 15C -- 6 APRIL 1970



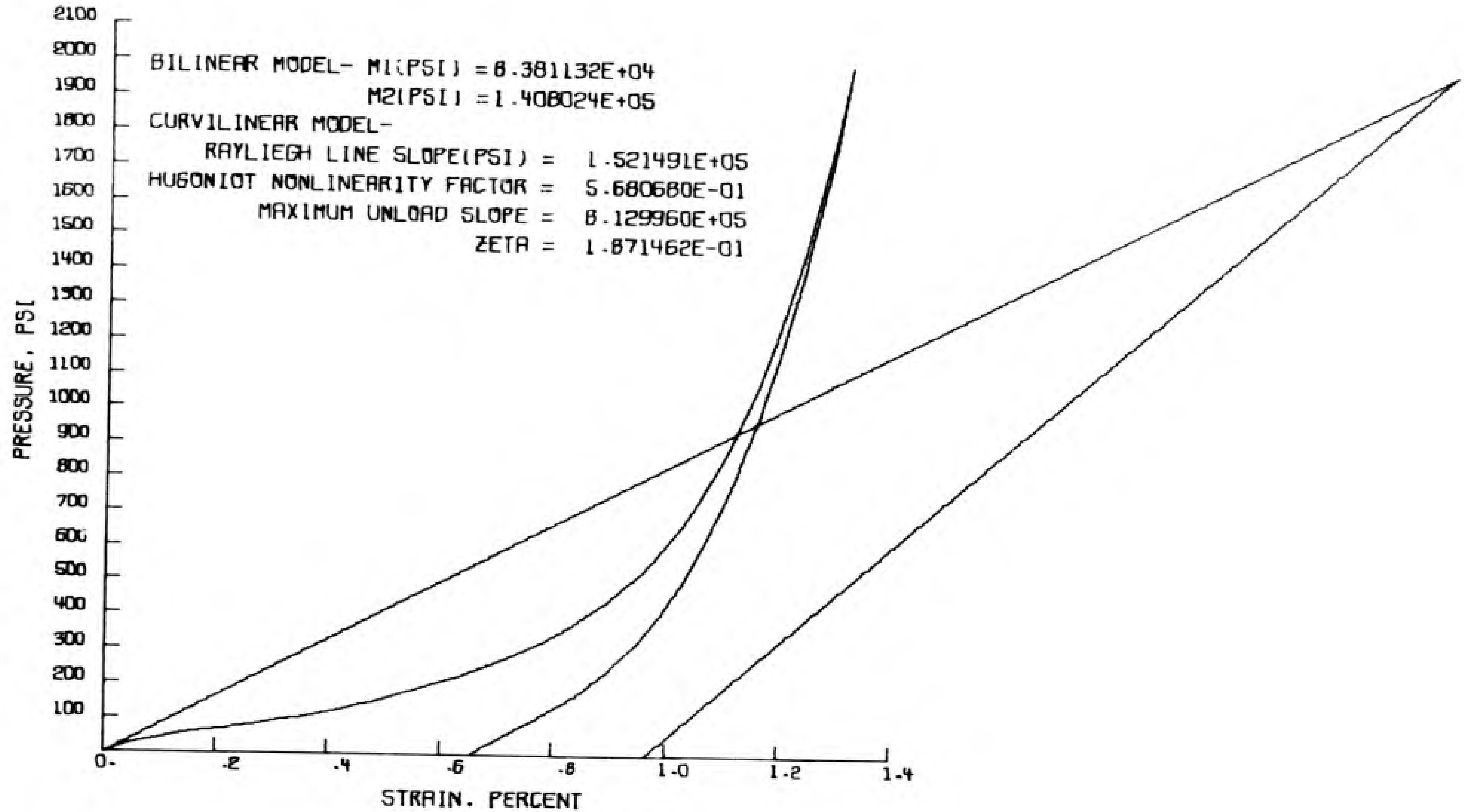
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM 15C -- 6 APRIL 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 11H -- 1 MAY 1970

NUMBER OF DATA POINTS. N=125 . M= 99

MATERIAL PROPERTIES

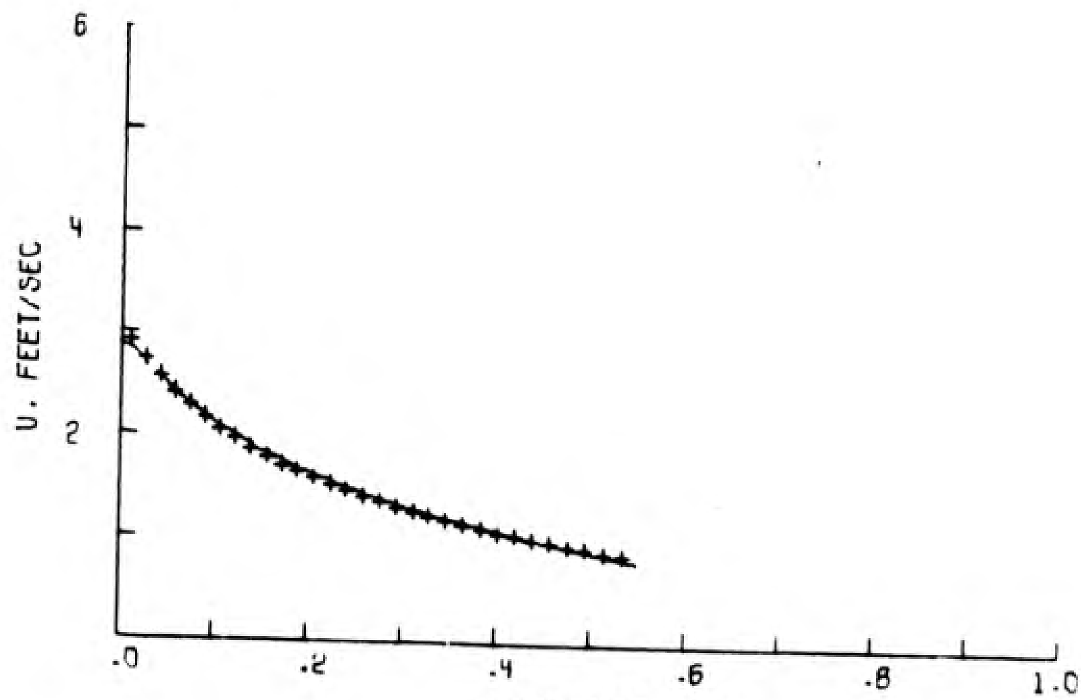
DENSITY =	3.510000E+00
SHOCK VELOCITY =	1.329457E+03
SOUND VELOCITY =	2.326237E+03
ZETA =	2.726650E-01

FITTING ERRORS

E_1 =	3.979431E-01	E_2 =	4.109513E-03
E_3 =	2.914219E-05	E_4 =	5.398351E-03
E_5 =	9.990203E-01	E_6 =	1.958383E-03
E_7 =	6.434448E-04	E_8 =	2.536621E-02

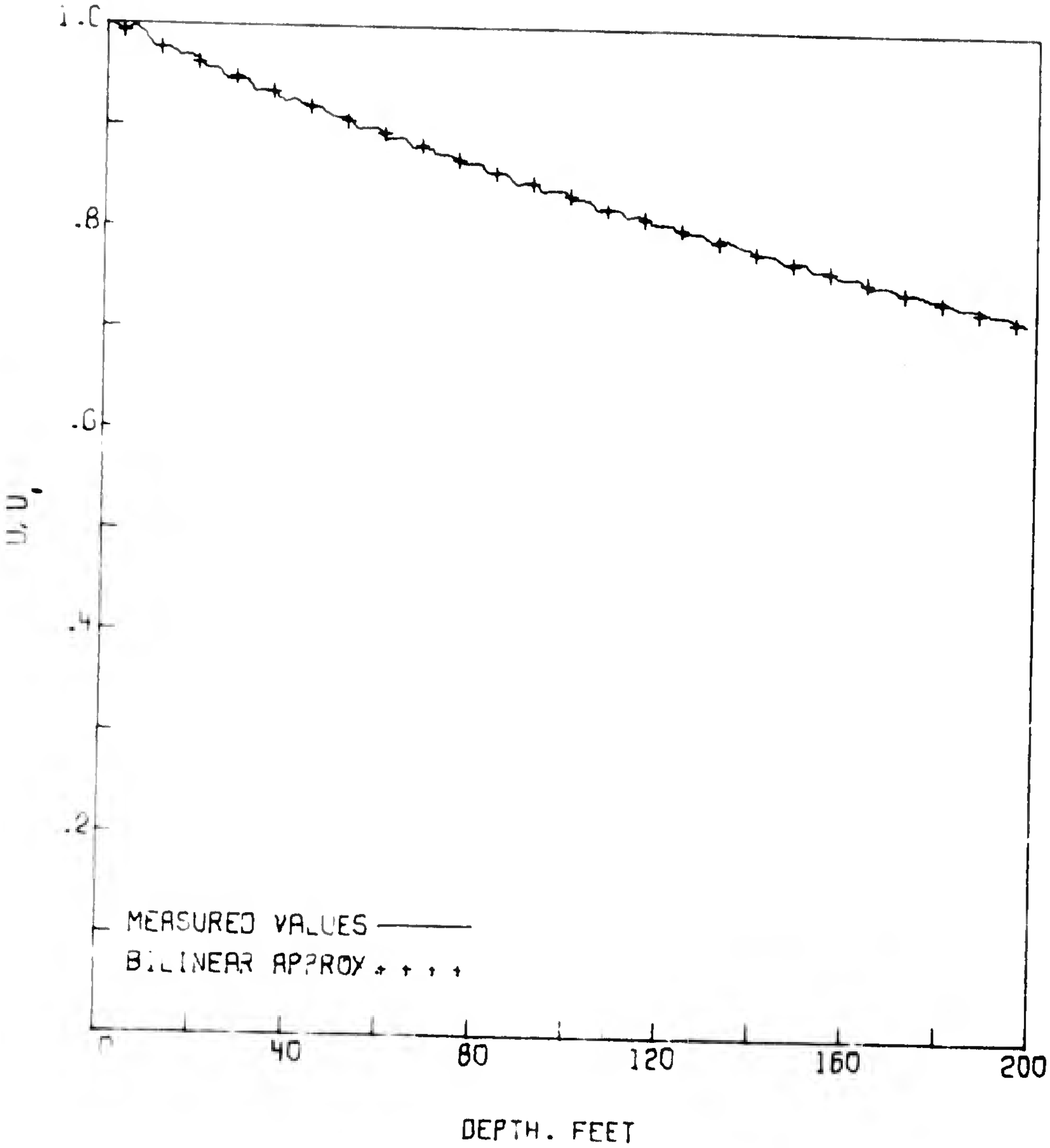
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.426	.427
25	.387	.383
50	.345	.340
75	.303	.298
100	.262	.257



SURFACE PARTICLE VELOCITY

PROBLEM 11H -- 1 MAY 1970



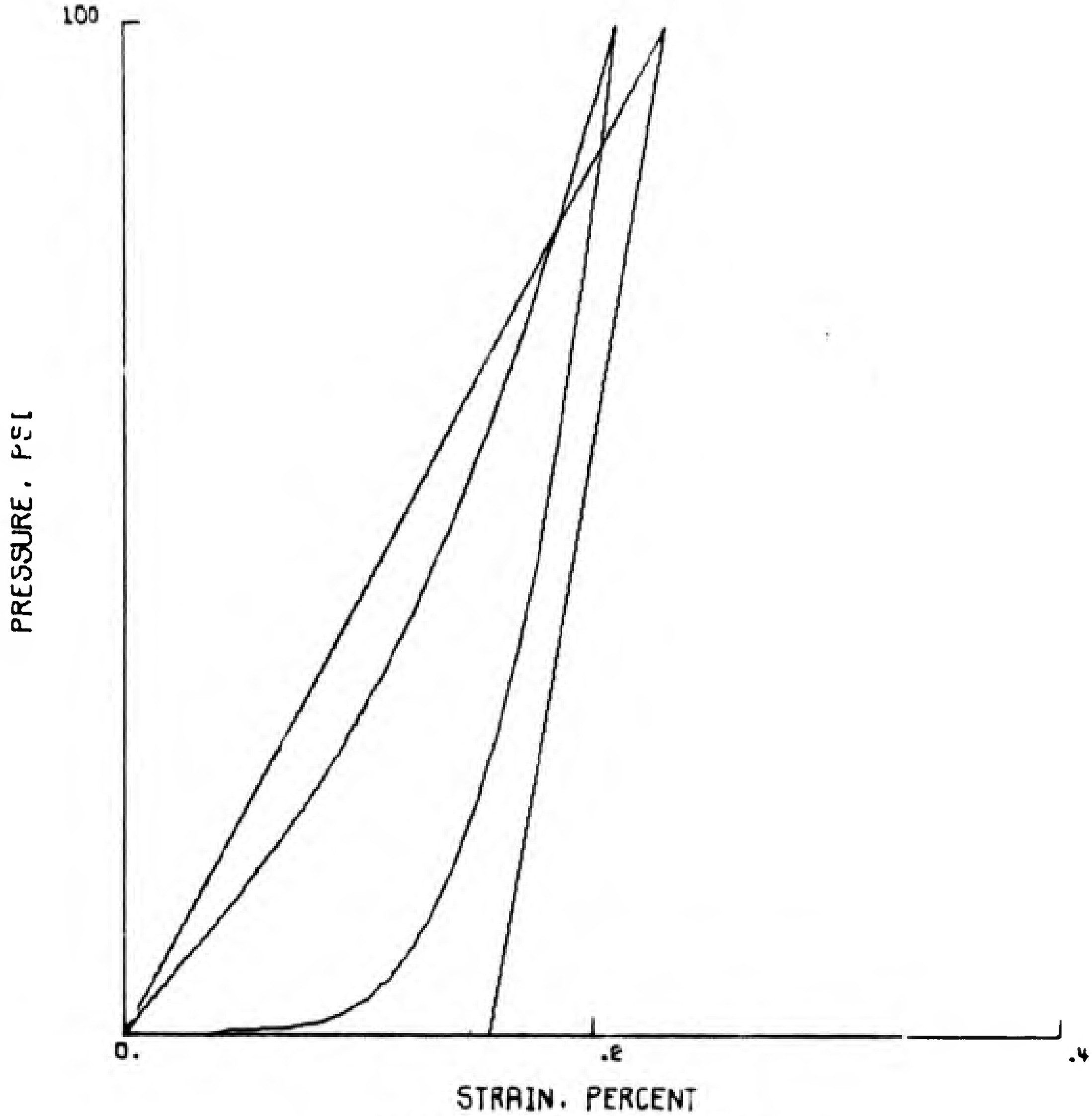
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 100 PSI BRODE WAVE

HALF LOAD TIME = 1.076864E-01 SEC.

NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 11H -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 4.308176E+04
M2(PSI) = 1.319024E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 4.738438E+04
HUGONIOT NONLINEARITY FACTOR = 2.120586E-01
MAXIMUM UNLOAD SLOPE = 1.941047E+05
ZETA = 2.441176E-01

BEST BILINEAR MODEL

PROBLEM 12H -- 1 MAY 1970

NUMBER OF DATA POINTS. N=122 . M= 99

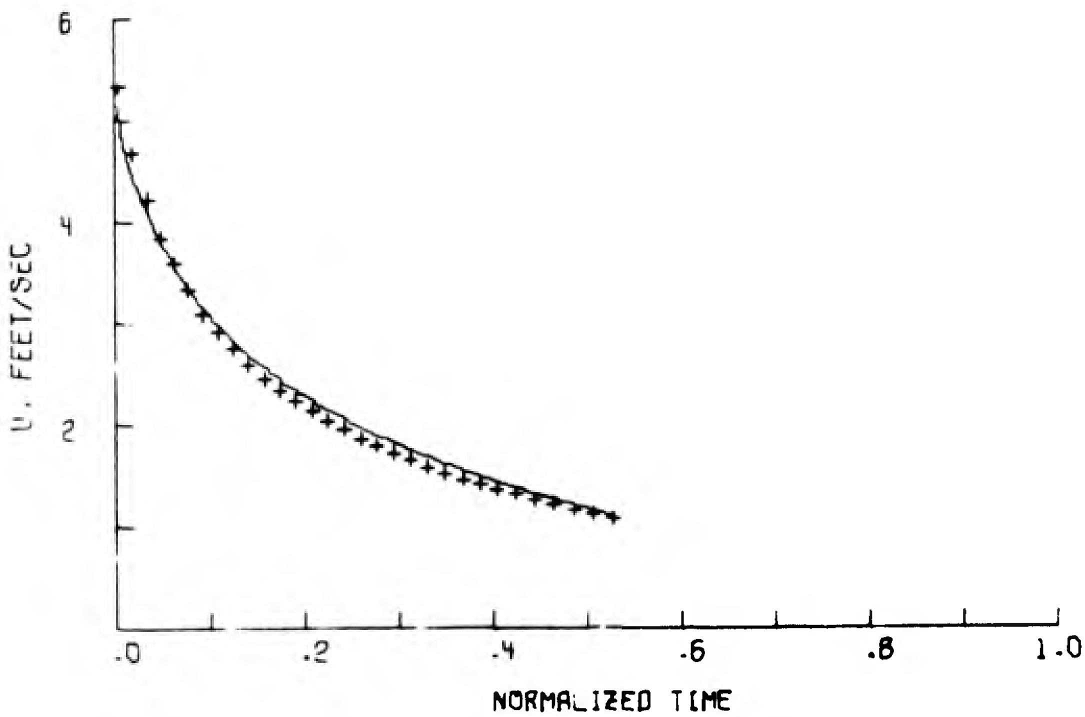
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.465692E+03
	SOUND VELOCITY =	2.178224E+03
	ZETA =	1.955400E-01

FITTING ERRORS

E_1 =	9.987874E-01	E_2 =	2.423712E-03
E_3 =	3.755049E-05	E_4 =	6.127846E-03
E_5 =	9.983171E-01	E_6 =	3.362892E-03
E_7 =	9.975170E-03	E_8 =	9.986576E-02

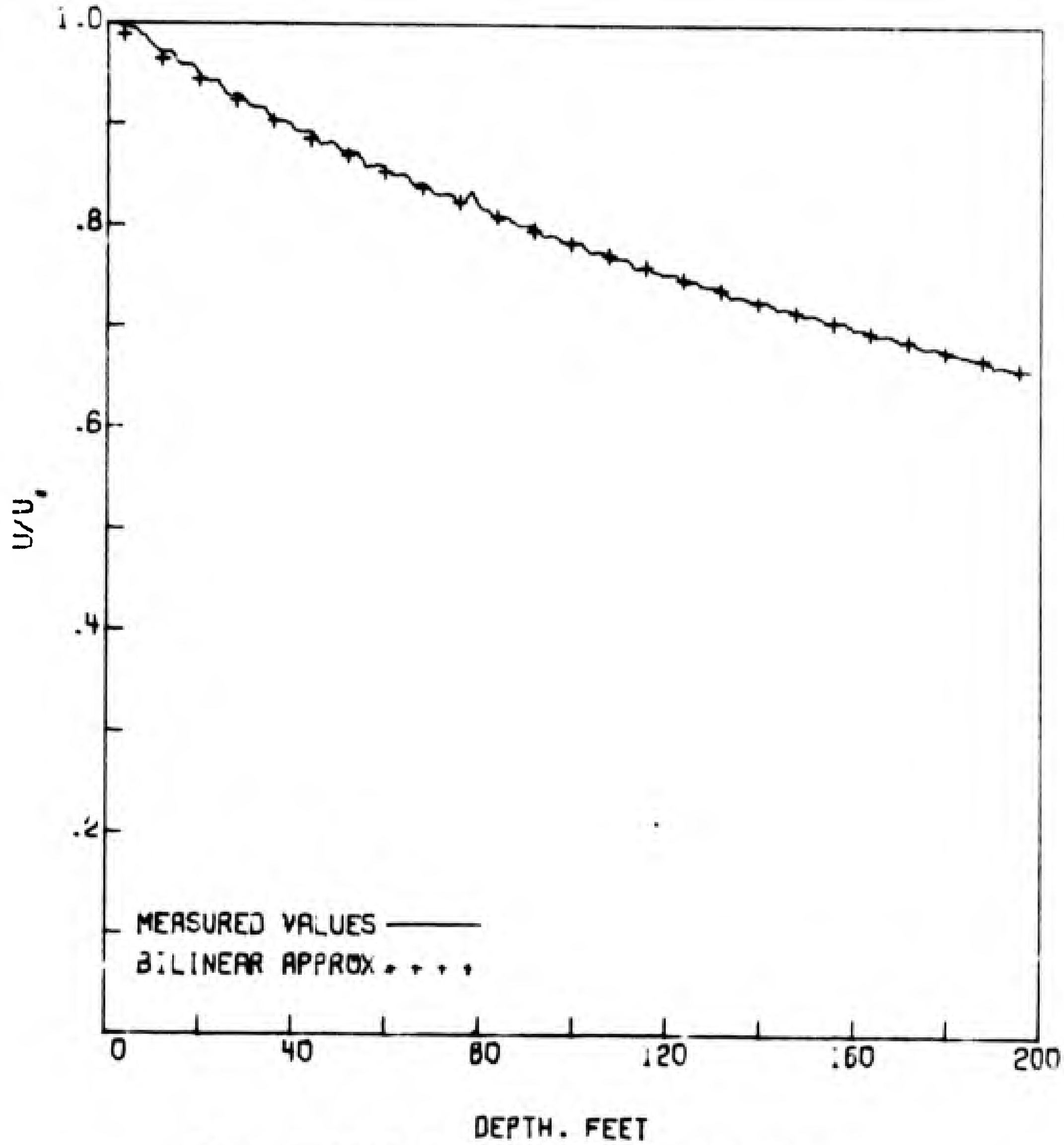
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.636	.642
25	.534	.581
50	.527	.523
75	.472	.466
100	.416	.411



SURFACE PARTICLE VELOCITY

PROBLEM 12H -- 1 MAY 1970



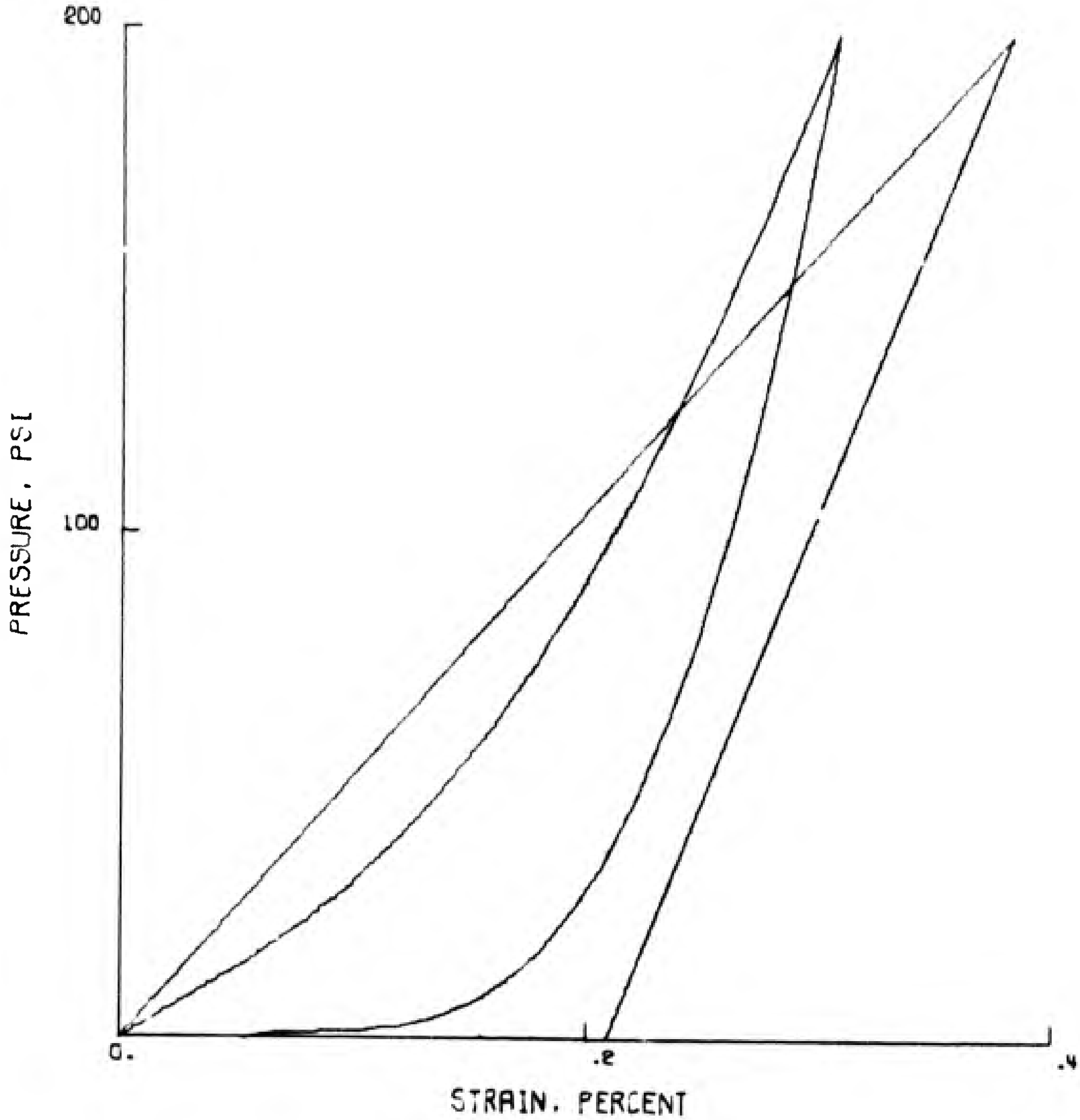
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 12H -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 5.236309E+04
M2(PSI) = 1.156510E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 6.497937E+04
HUGONIOT NONLINEARITY FACTOR = 2.677390E-01
MAXIMUM UNLOAD SLOPE = 2.661805E+05
ZETA = 2.441176E-01

BEST BILINEAR MODEL

PROBLEM 13H -- 1 MAY 1970

NUMBER OF DATA POINTS. N=119 . M= 99

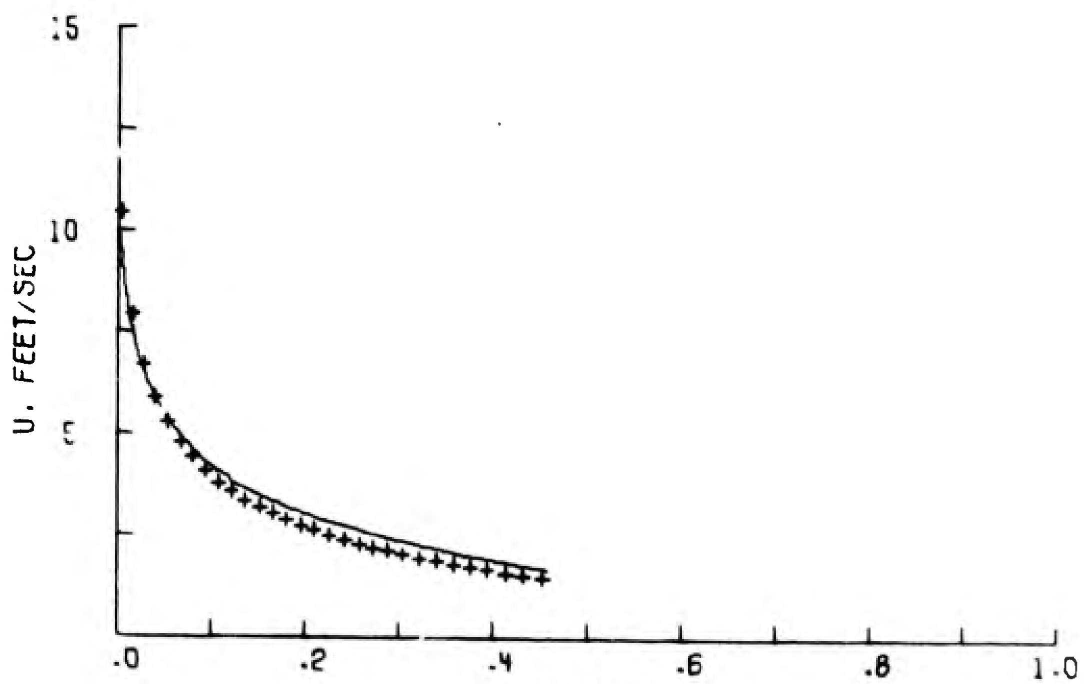
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.699751E+03
	SOUND VELOCITY =	2.305950E+03
	ZETA =	1.513340E-01

FITTING ERRORS

E_1 =	9.982450E-01	E_2 =	3.006849E-03
E_3 =	6.850892E-05	E_4 =	3.277012E-03
E_5 =	9.978885E-01	E_6 =	4.218493E-03
E_7 =	1.005914E-01	E_8 =	3.171615E-01

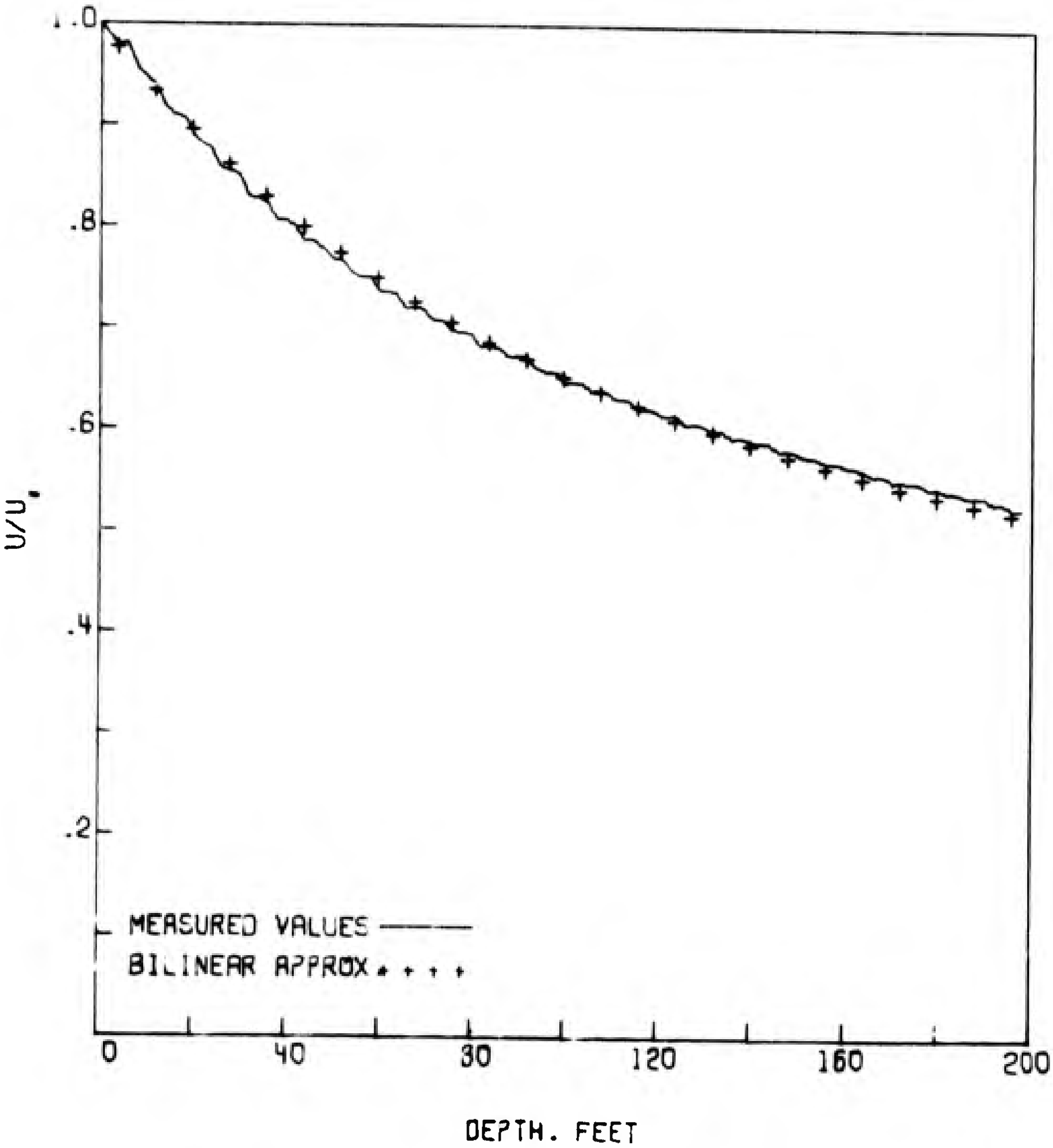
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.974	.974
25	.899	.885
50	.821	.803
75	.745	.726
100	.671	.654



SURFACE PARTICLE VELOCITY

PROBLEM 13H -- 1 MAY 1970



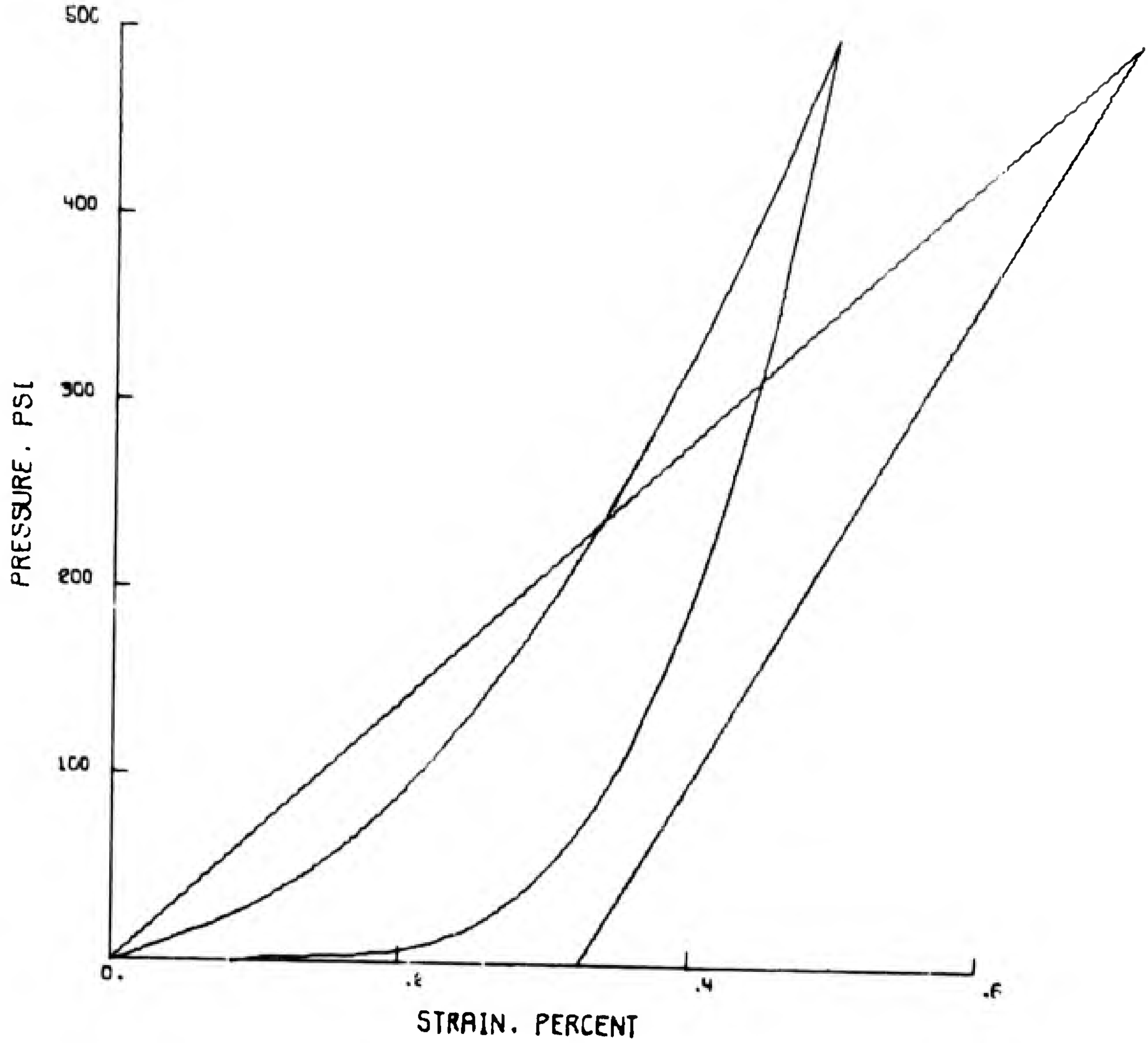
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 13H -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) = 7.042312E+04
M2(PSI) = 1.296117E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 1.000000E+05
HUGONIOT NONLINEARITY FACTOR = 3.000000E-01
MAXIMUM UNLOAD SLOPE = 4.096386E+05
ZETA = 2.441176E-01

BEST BILINEAR MODEL

PROBLEM 14H -- 1 MAY 1970

NUMBER OF DATA POINTS. N=101 . M= 99

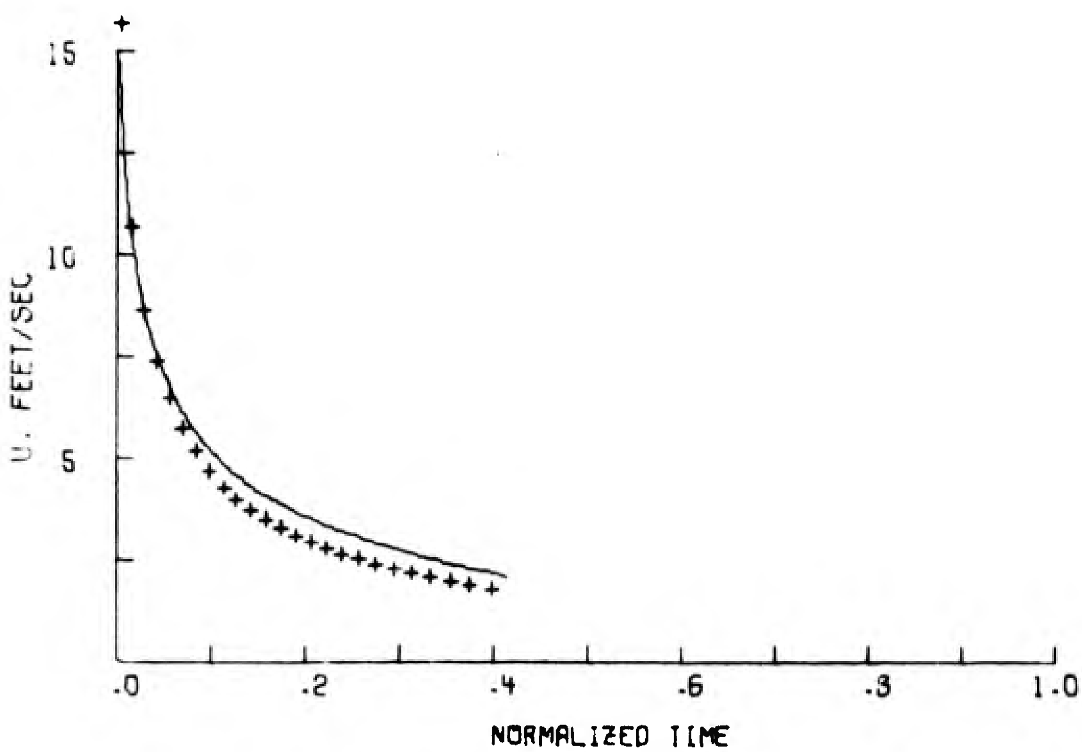
MATERIAL PROPERTIES	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	1.979245E+03
	SOUND VELOCITY =	2.635961E+03
	ZETA =	1.422940E-01

FITTING ERRORS

E_1 =	9.372461E-01	E_2 =	5.500204E-03
E_3 =	1.452848E-04	E_4 =	1.205342E-02
E_5 =	3.358625E-01	E_6 =	8.257974E-03
E_7 =	3.575361E-01	E_8 =	5.979433E-01

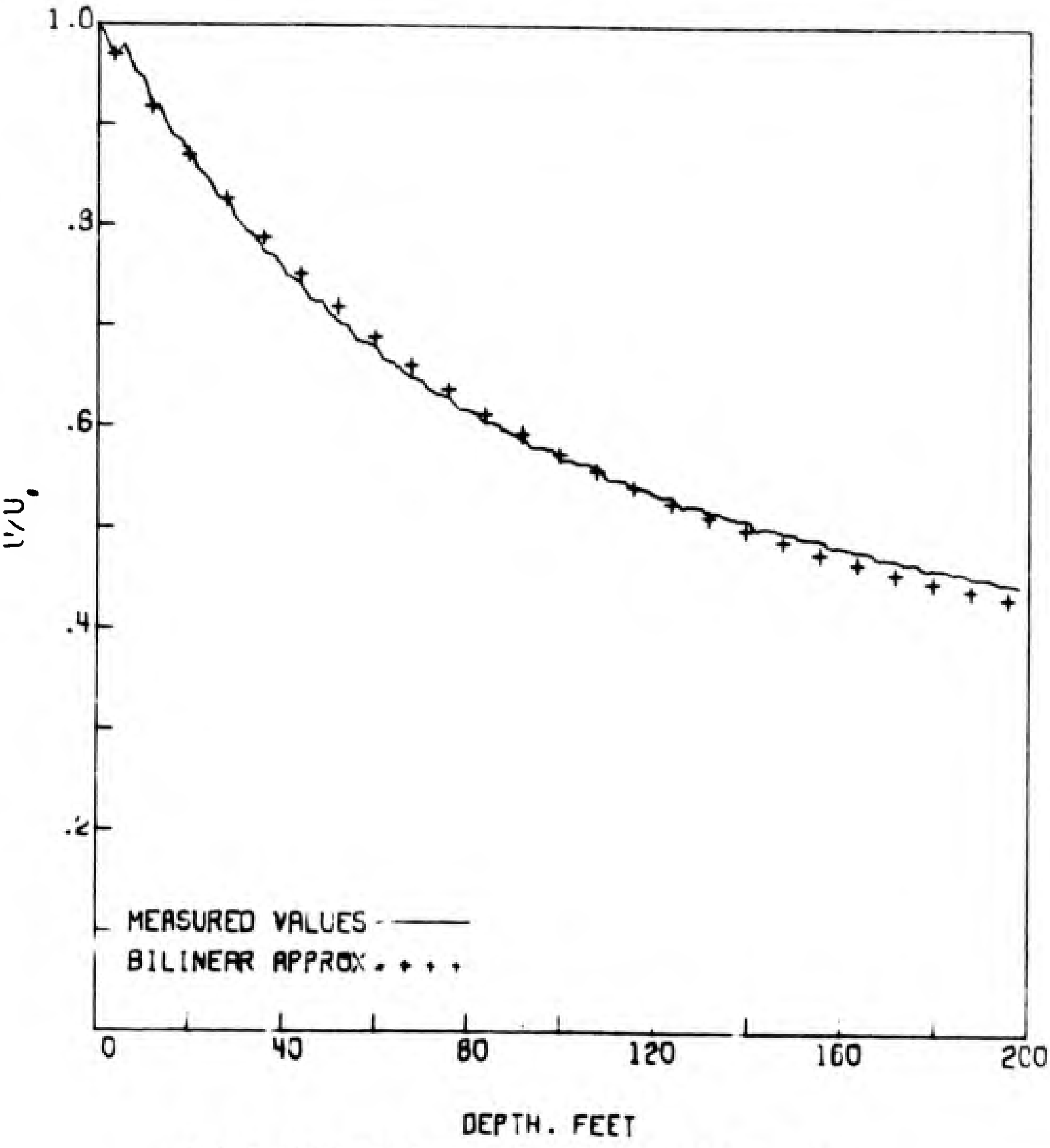
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.309	1.284
25	1.218	1.166
50	1.122	1.063
75	1.029	.969
100	.939	.884



SURFACE PARTICLE VELOCITY

PROBLEM 14H -- 1 MAY 1970

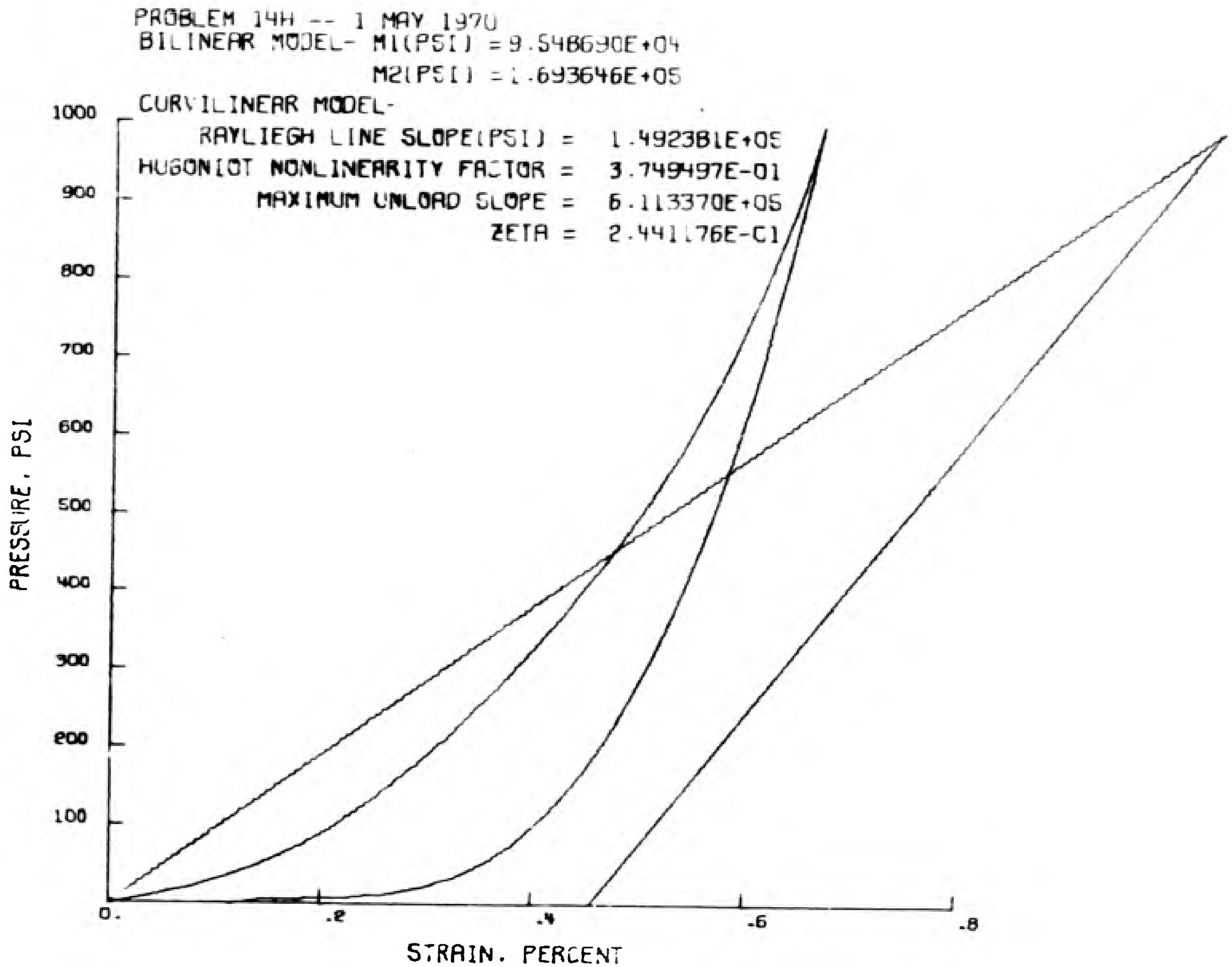


PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 12J -- 1 MAY 1970

NUMBER OF DATA POINTS. N= 69 . M= 99

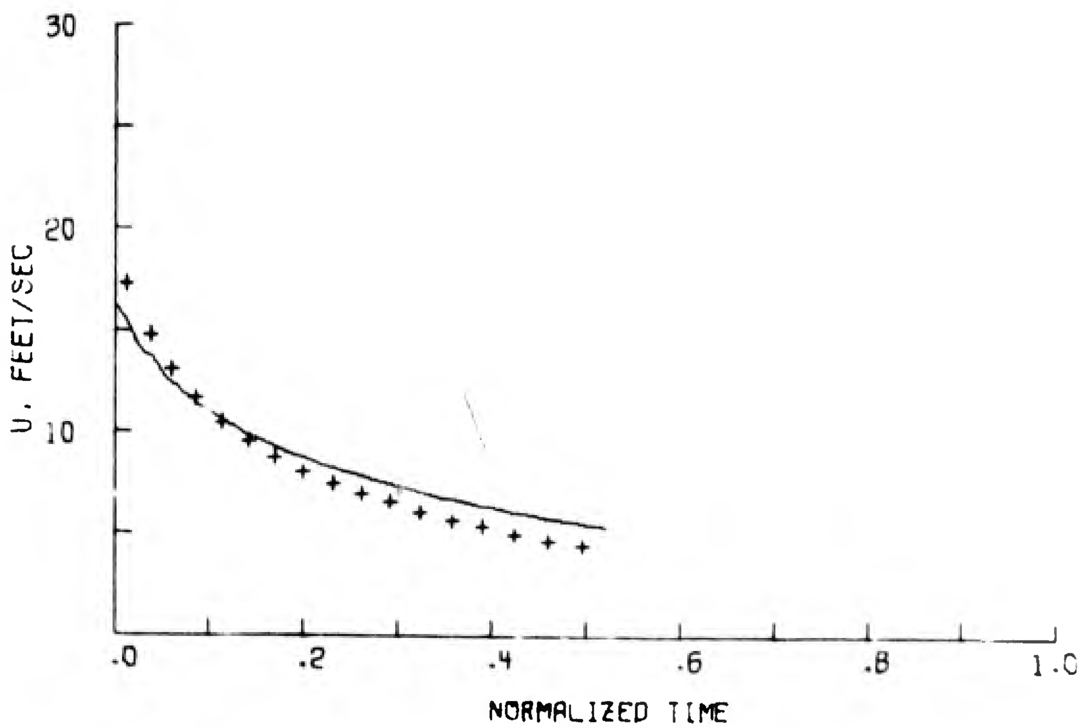
MATERIAL PROPERTIES		
	DENSITY =	3.510000E+00
	SHOCK VELOCITY =	4.213099E+02
	SOUND VELOCITY =	7.834291E+02
	ZETA =	3.005790E-01

FITTING ERRORS

E_1 = 9.178274E-01 E_2 = 1.575328E-01
E_3 = 6.708251E-03 E_4 = 3.190391E-02
E_5 = 9.987548E-01 E_6 = 2.488919E-03
E_7 = 7.908220E-01 E_8 = 8.892818E-01

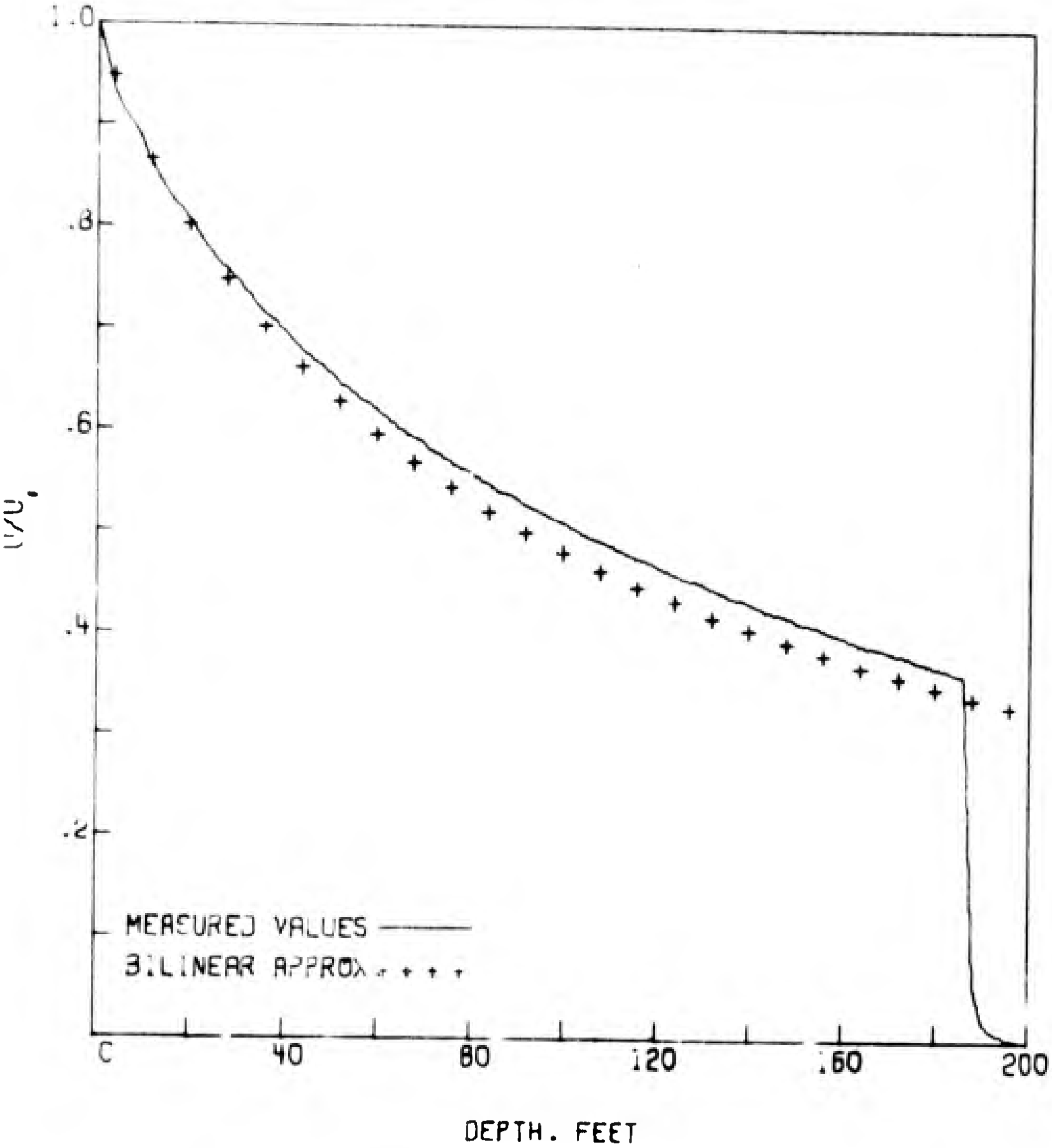
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	2.222	2.339
25	1.549	1.536
50	.878	.851
75	.249	.224
100	.000	0.



SURFACE PARTICLE VELOCITY

PROBLEM 12J -- 1 MAY 1970



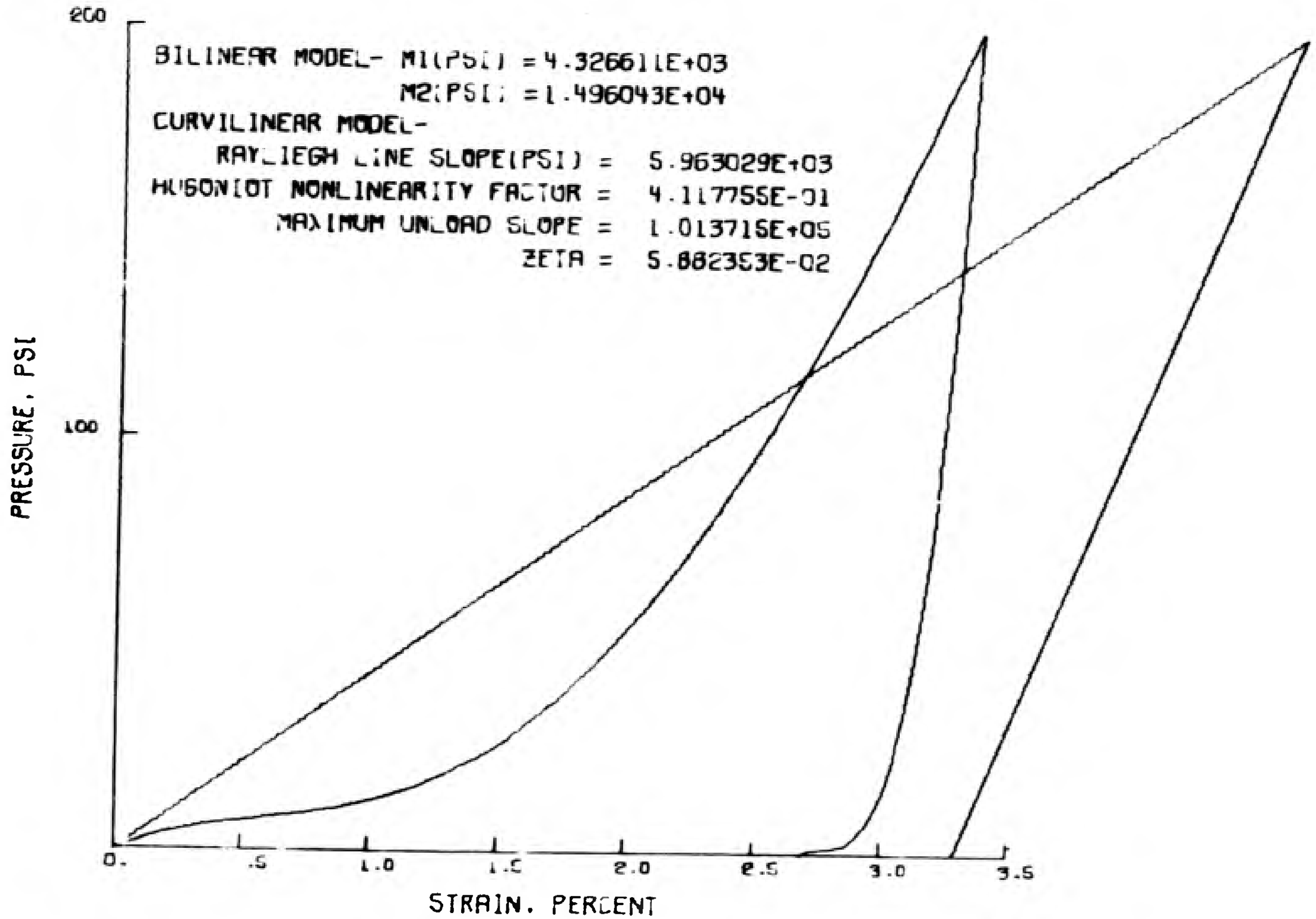
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 12J -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 13.0 -- 1 MAY 1970

NUMBER OF DATA POINTS. N= 72 . M= 39

MATERIAL PROPERTIES

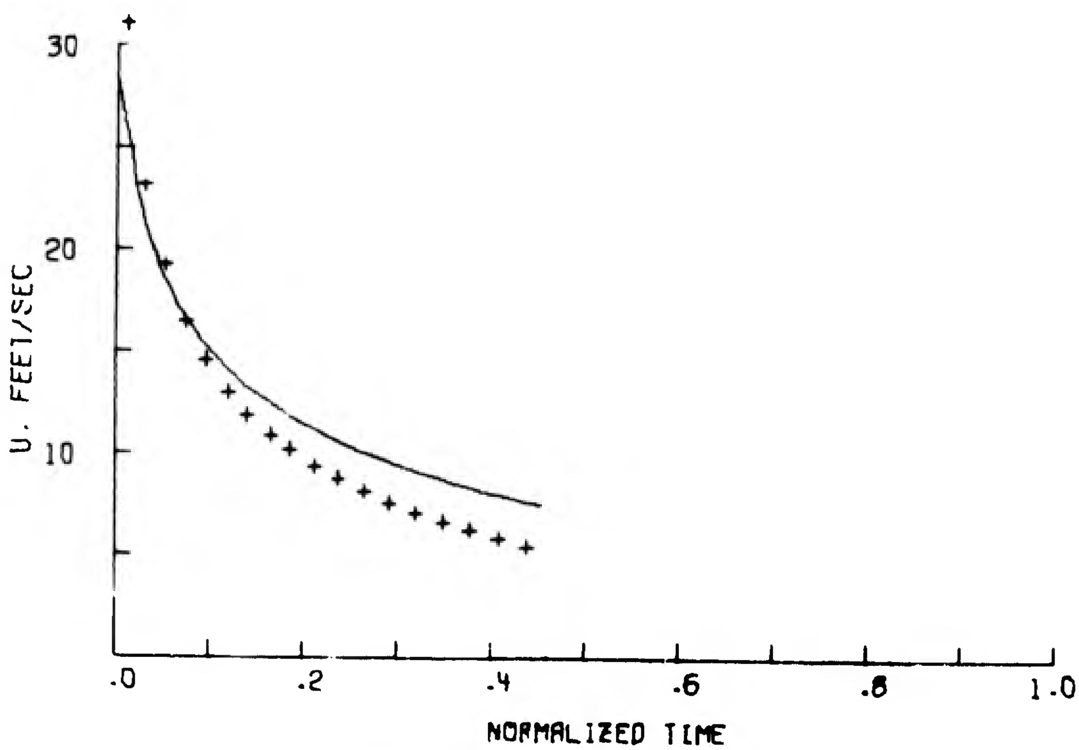
DENSITY =	3.510000E+00
SHOCK VELOCITY =	4.780395E+02
SOUND VELOCITY =	6.567104E+02
ZETA =	1.574540E-01

FITTING ERRORS

E_1 =	9.996706E-01	E_2 =	6.586383E-04
E_3 =	7.866082E-05	E_4 =	8.868530E-03
E_5 =	9.973965E-01	E_6 =	5.200276E-03
E_7 =	3.942725E+00	E_8 =	1.985630E+00

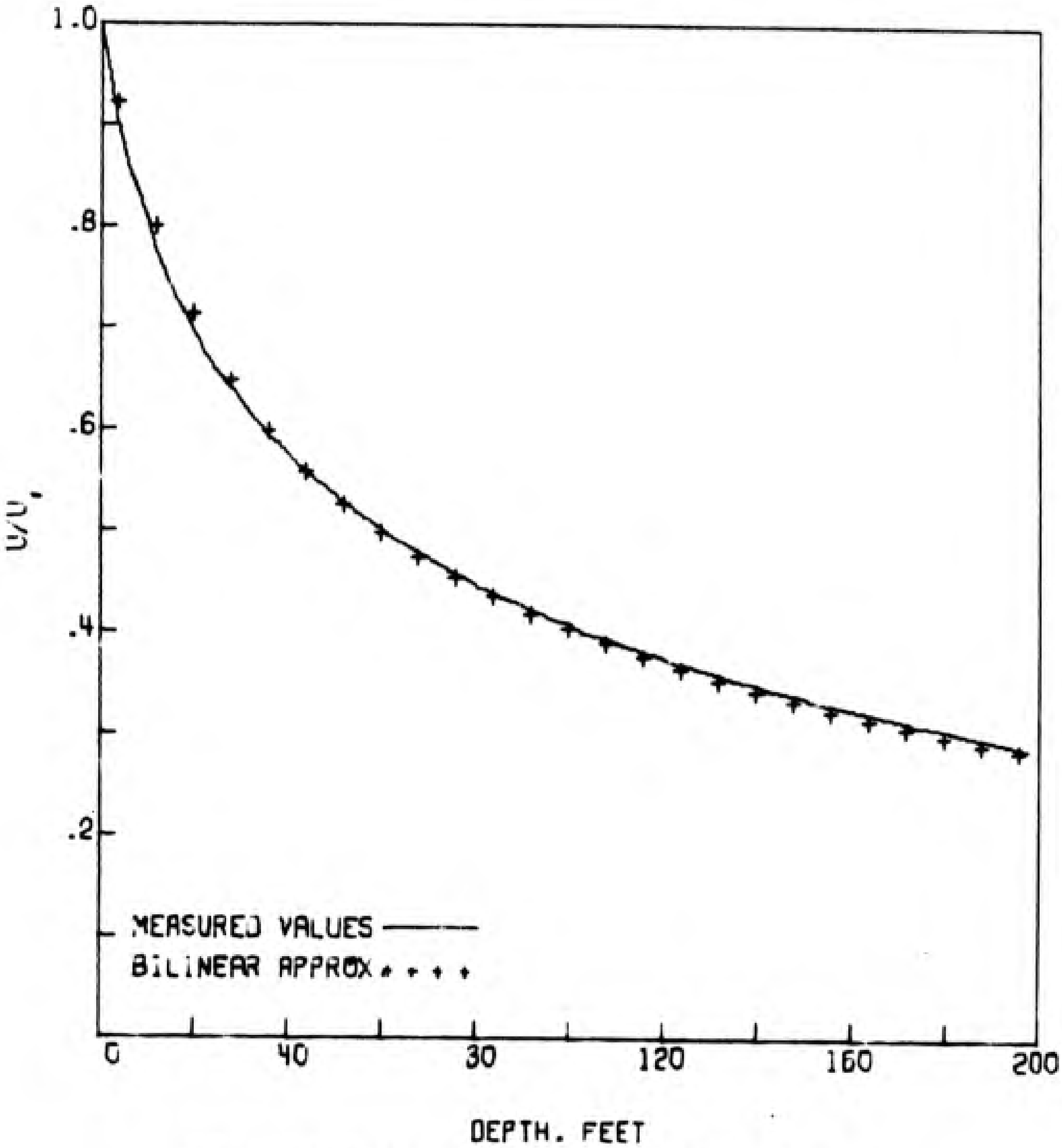
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	3.328	3.483
25	2.422	2.441
50	1.565	1.569
75	.777	.728
100	.036	0.



SURFACE PARTICLE VELOCITY

PROBLEM 13J -- 1 MAY 1970



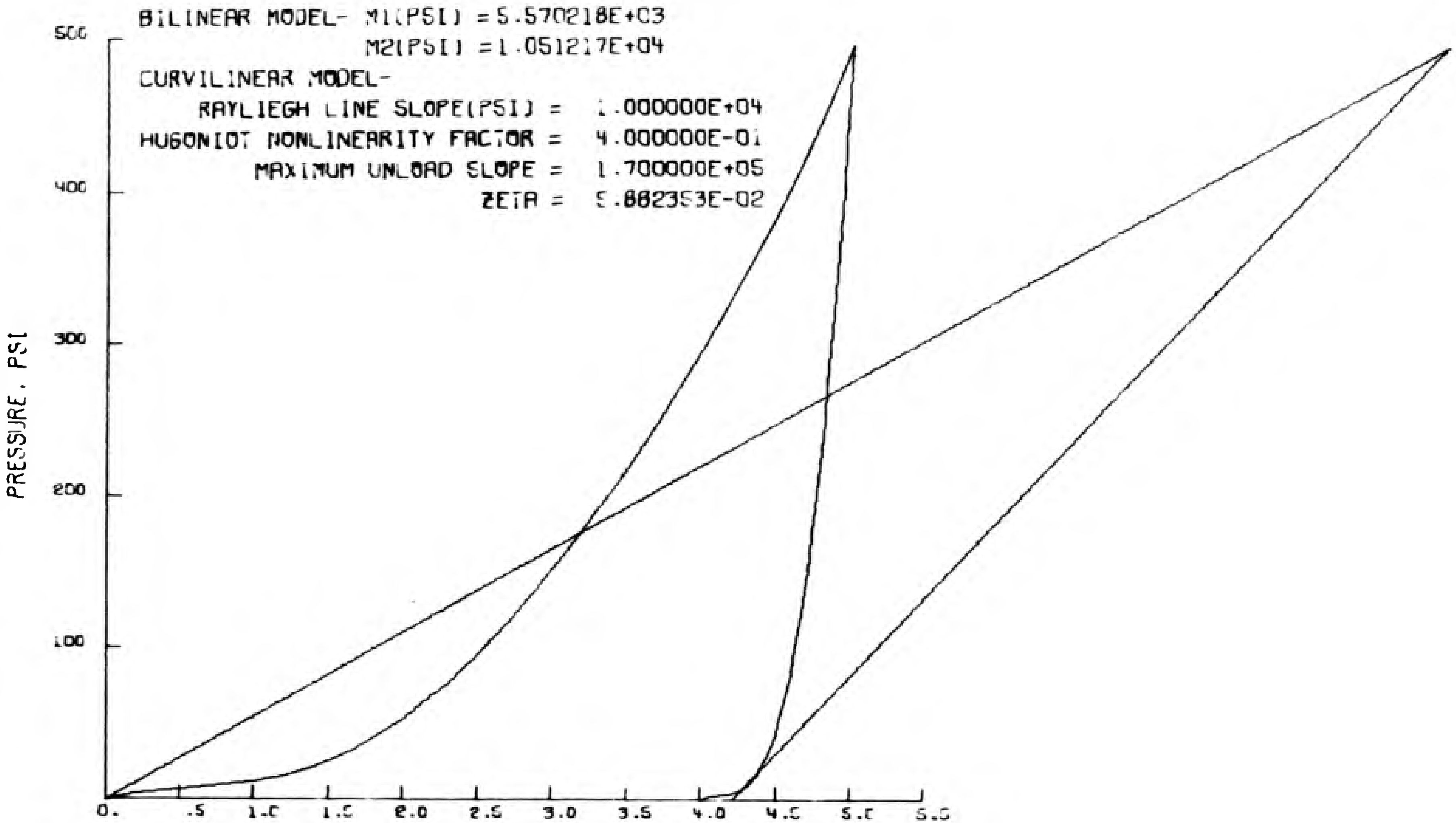
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 13_ -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 14J -- 1 MAY 1970

NUMBER OF DATA POINTS. N= 62 . M= 99

MATERIAL PROPERTIES

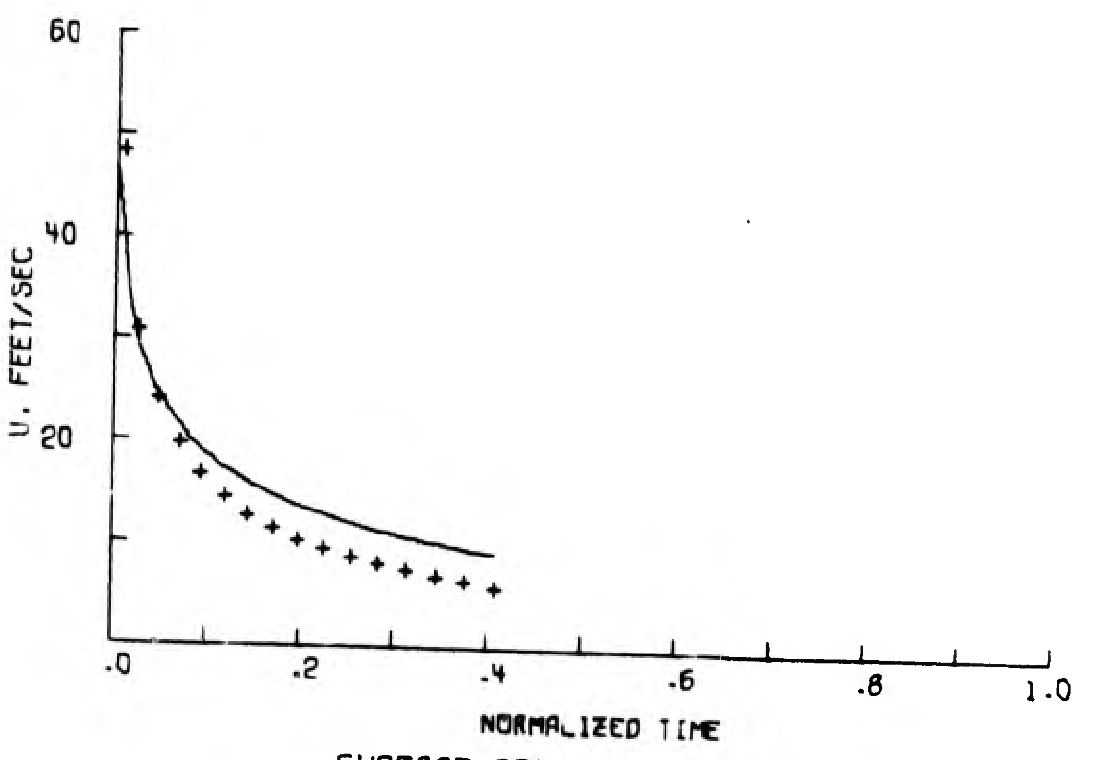
DENSITY	=	3.510000E+00
SHOCK VELOCITY	=	5.668278E+02
SOUND VELOCITY	=	7.514385E+02
ZETA	=	1.400405E-01

FITTING ERRORS

E_1 =	9.988251E-01	E_2 =	2.348429E-03
E_3 =	1.399906E-04	E_4 =	1.183176E-02
E_5 =	9.358492E-01	E_6 =	8.284316E-03
E_7 =	1.115583E+01	E_8 =	3.340034E+00

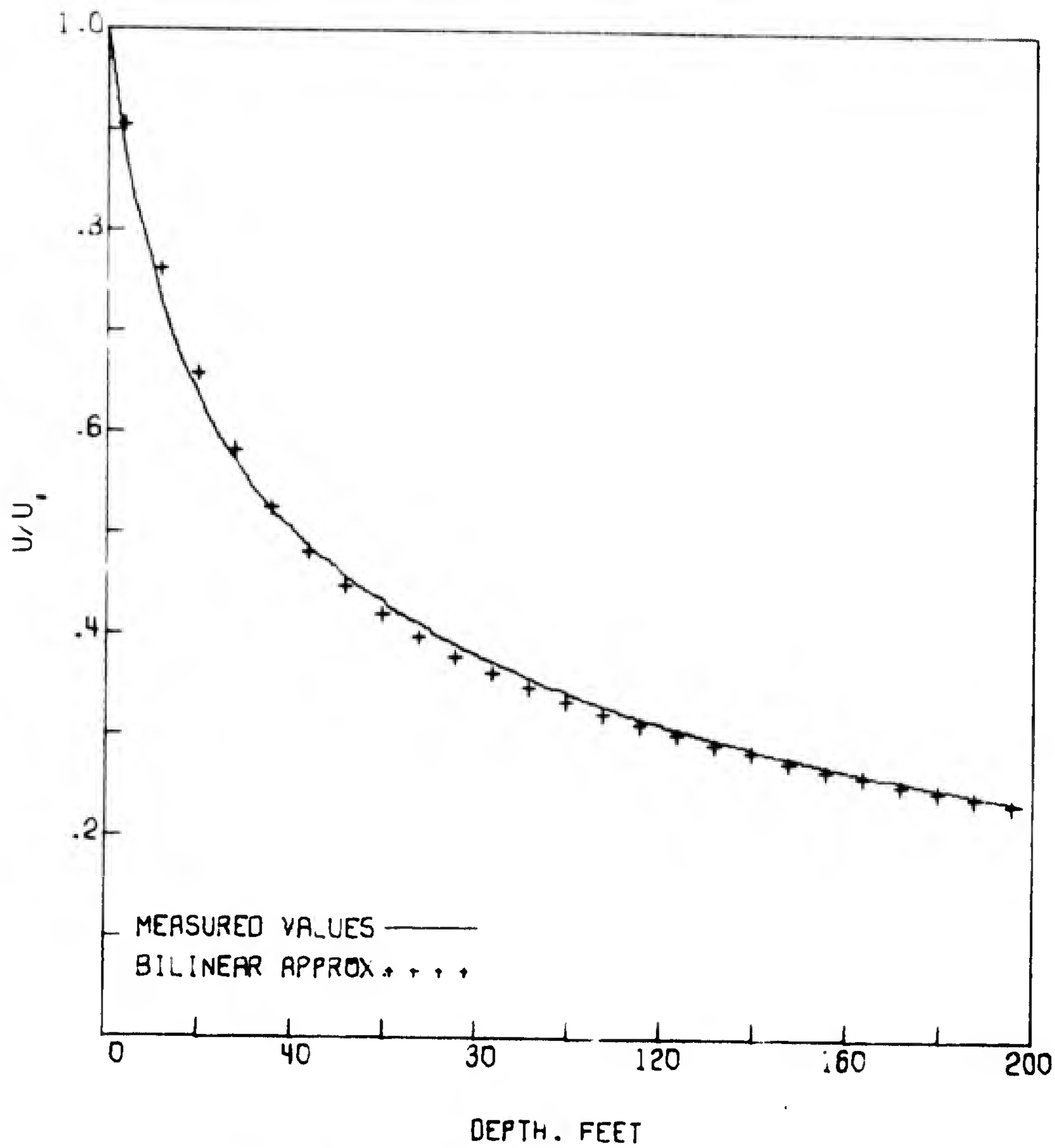
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	4.470	4.473
25	3.313	3.233
50	2.298	2.285
75	1.353	1.414
100	.475	.518



SURFACE PARTICLE VELOCITY

PROBLEM 14J -- 1 MAY 1970



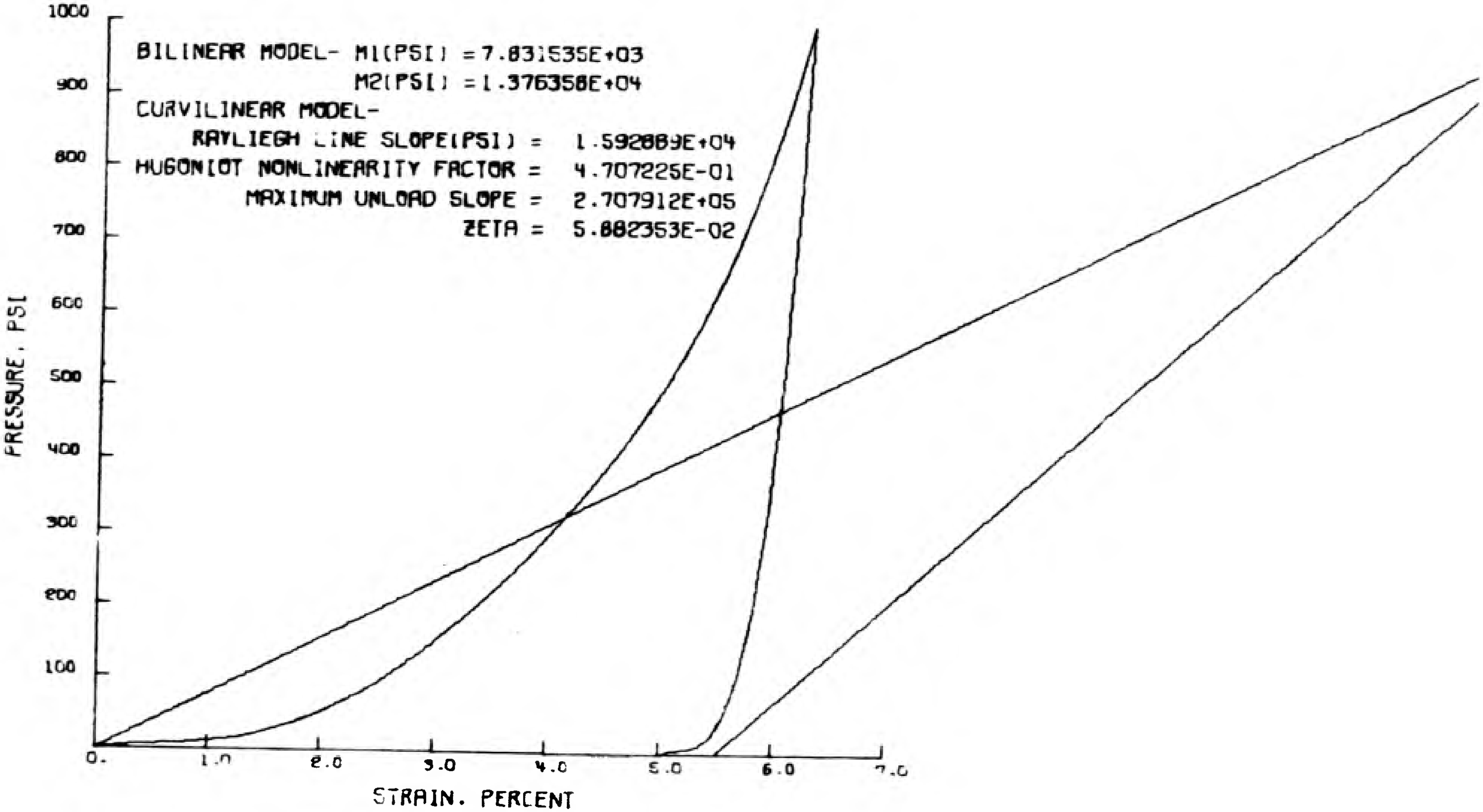
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02

PROBLEM 14J -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 15J -- 1 MAY 1970

NUMBER OF DATA POINTS. N= 51 . M= 99

MATERIAL PROPERTIES

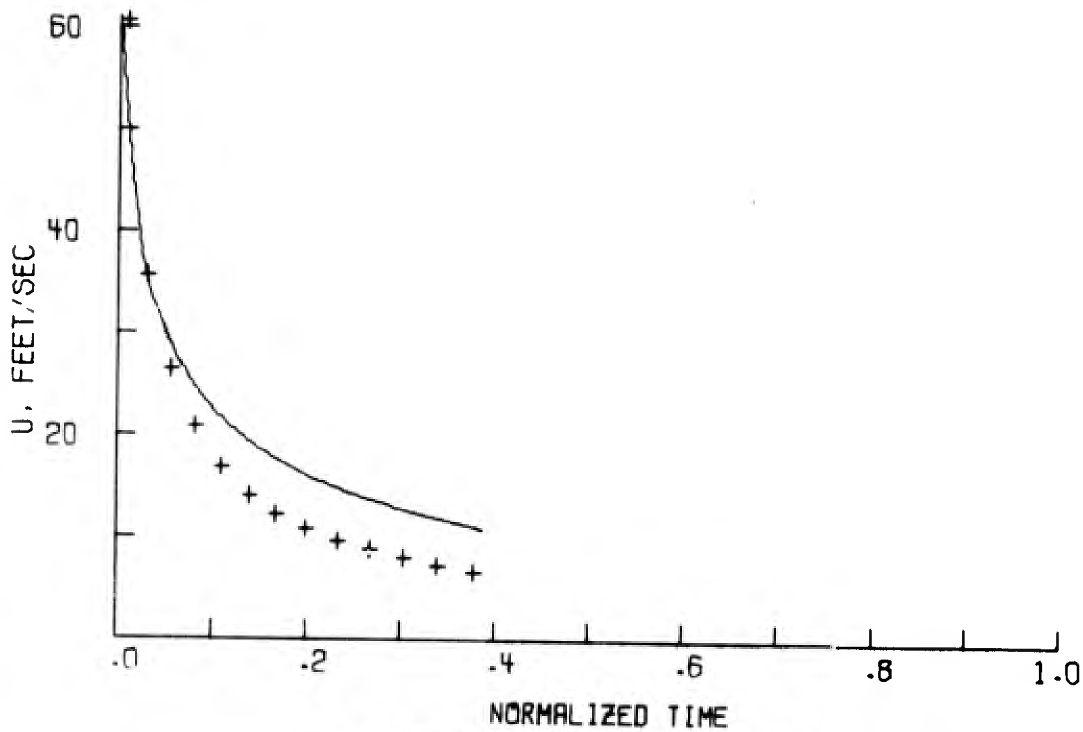
DENSITY =	3.510000E+00
SHOCK VELOCITY =	6.492500E+02
SOUND VELOCITY =	9.082870E+02
ZETA =	1.091135E-01

FITTING ERRORS

E_1 =	9.966068E-01	E_2 =	6.774951E-03
E_3 =	4.138198E-04	E_4 =	2.034256E-02
E_5 =	9.914361E-01	E_6 =	1.705454E-02
E_7 =	2.878307E+01	E_8 =	5.364986E+00

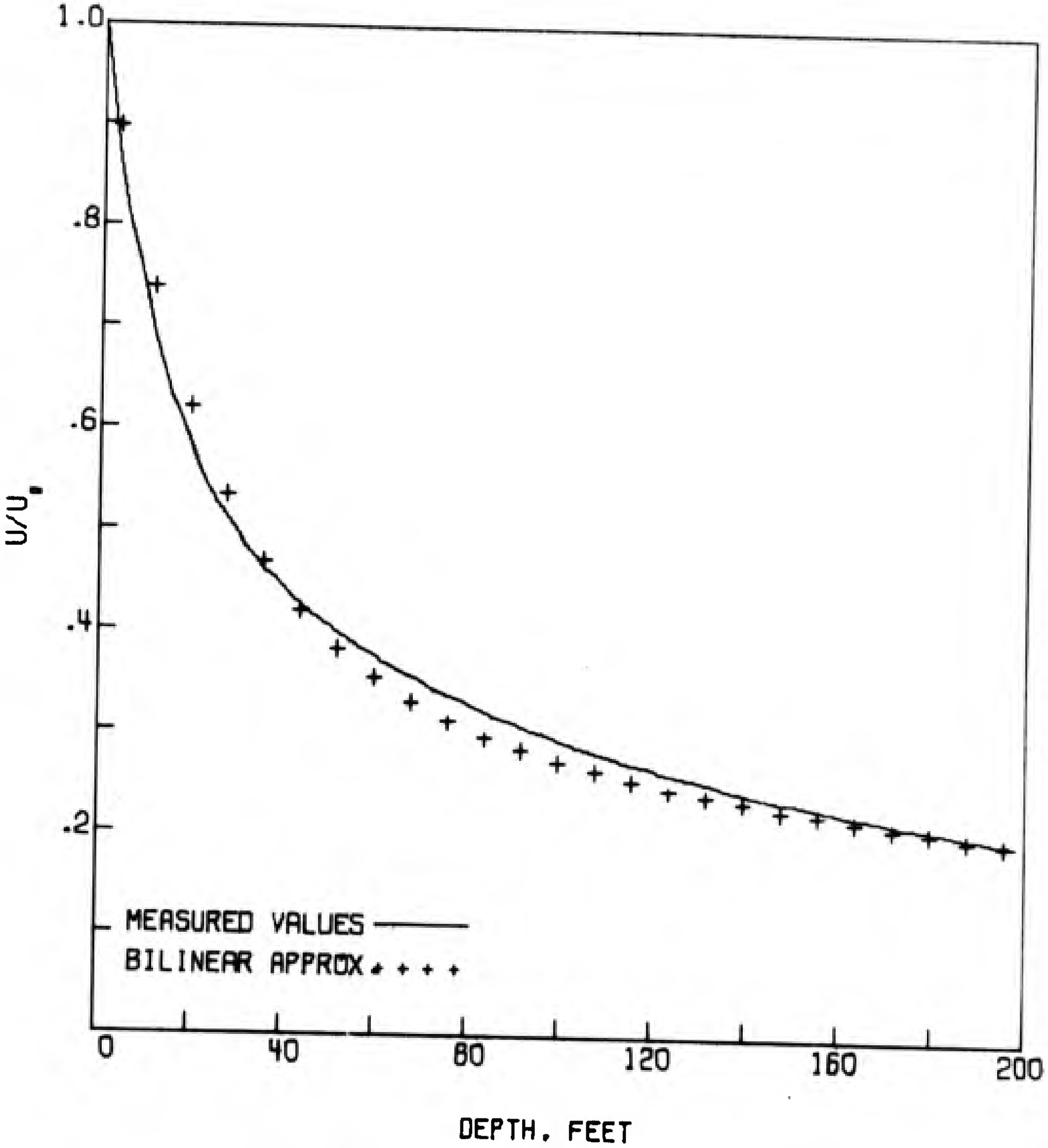
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	5.769	5.639
25	4.446	4.162
50	3.210	3.116
75	2.092	2.197
100	1.058	1.265



SURFACE PARTICLE VELOCITY

PROBLEM 15J -- 1 MAY 1970



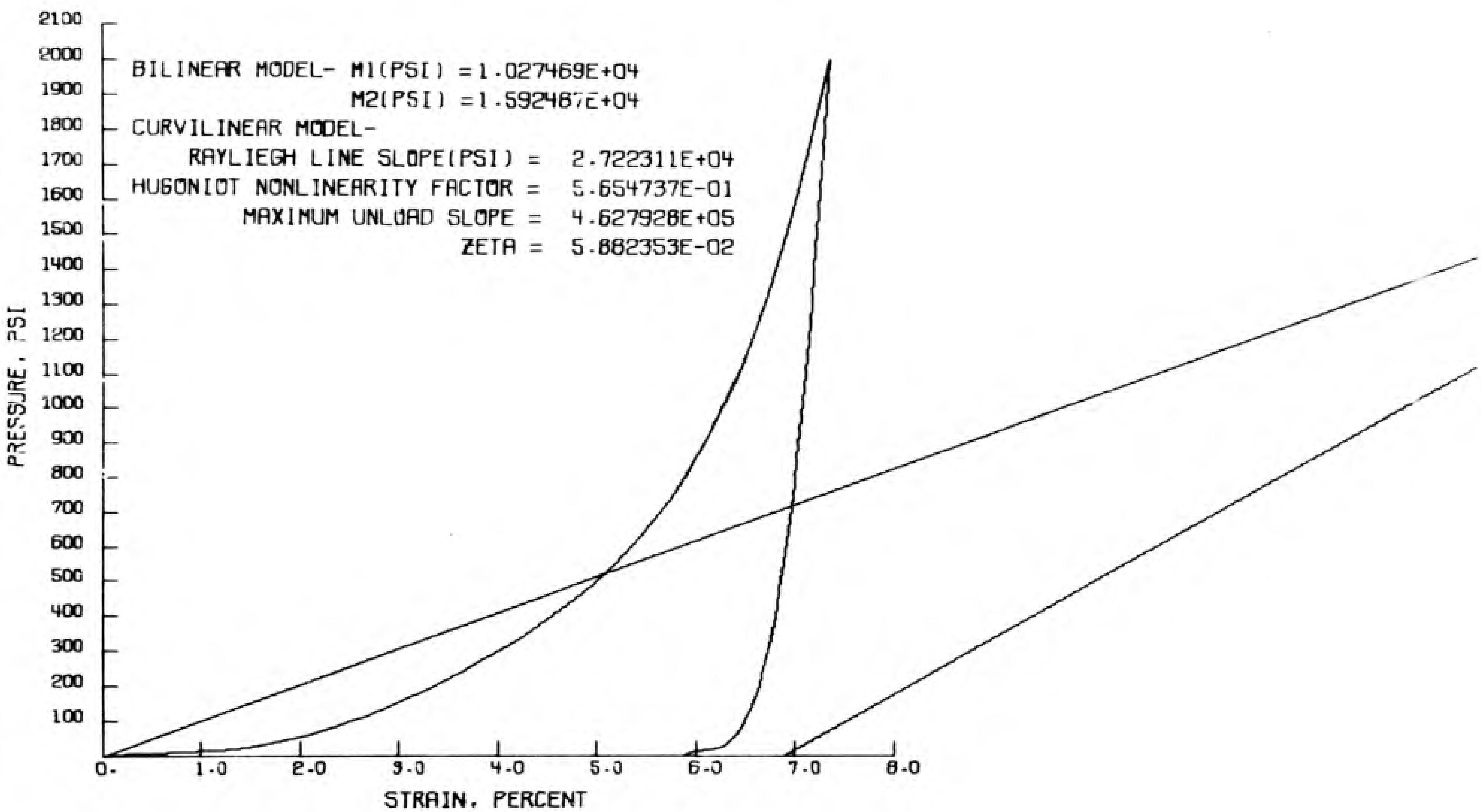
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 2000 PSI BRODE WAVE

HALF LOAD TIME = 4.179026E-03 SEC.

NORMALIZED HALF LOAD TIME = 3.239555E-03

PROBLEM 15J -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BEST BILINEAR MODEL

PROBLEM 11H -- 1 MAY 1973

NUMBER OF DATA POINTS. N=107 . M= 99

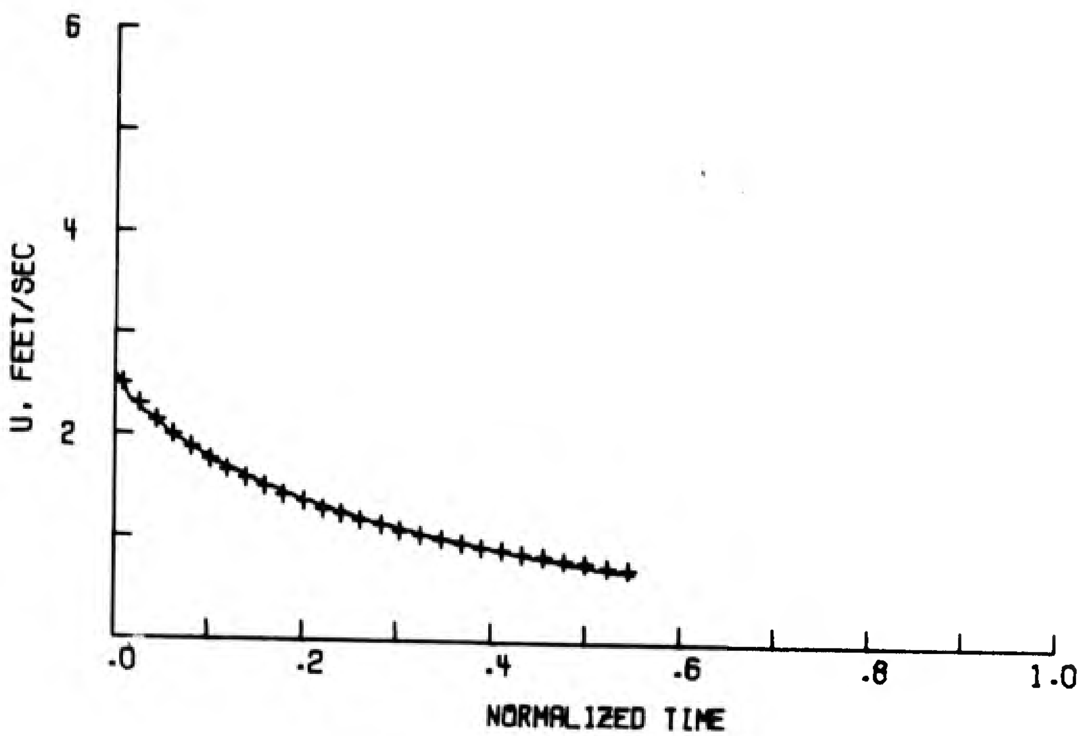
MATERIAL PROPERTIES		
	DENSITY =	5.000000E+00
	SHOCK VELOCITY =	1.110462E+03
	SOUND VELOCITY =	1.929650E+03
	ZETA =	2.694595E-01

FITTING ERRORS

E_1 =	9.982372E-01	E_2 =	3.522581E-03
E_3 =	3.284248E-05	E_4 =	5.730836E-03
E_5 =	9.992262E-01	E_6 =	1.546977E-03
E_7 =	3.943292E-04	E_8 =	1.985772E-02

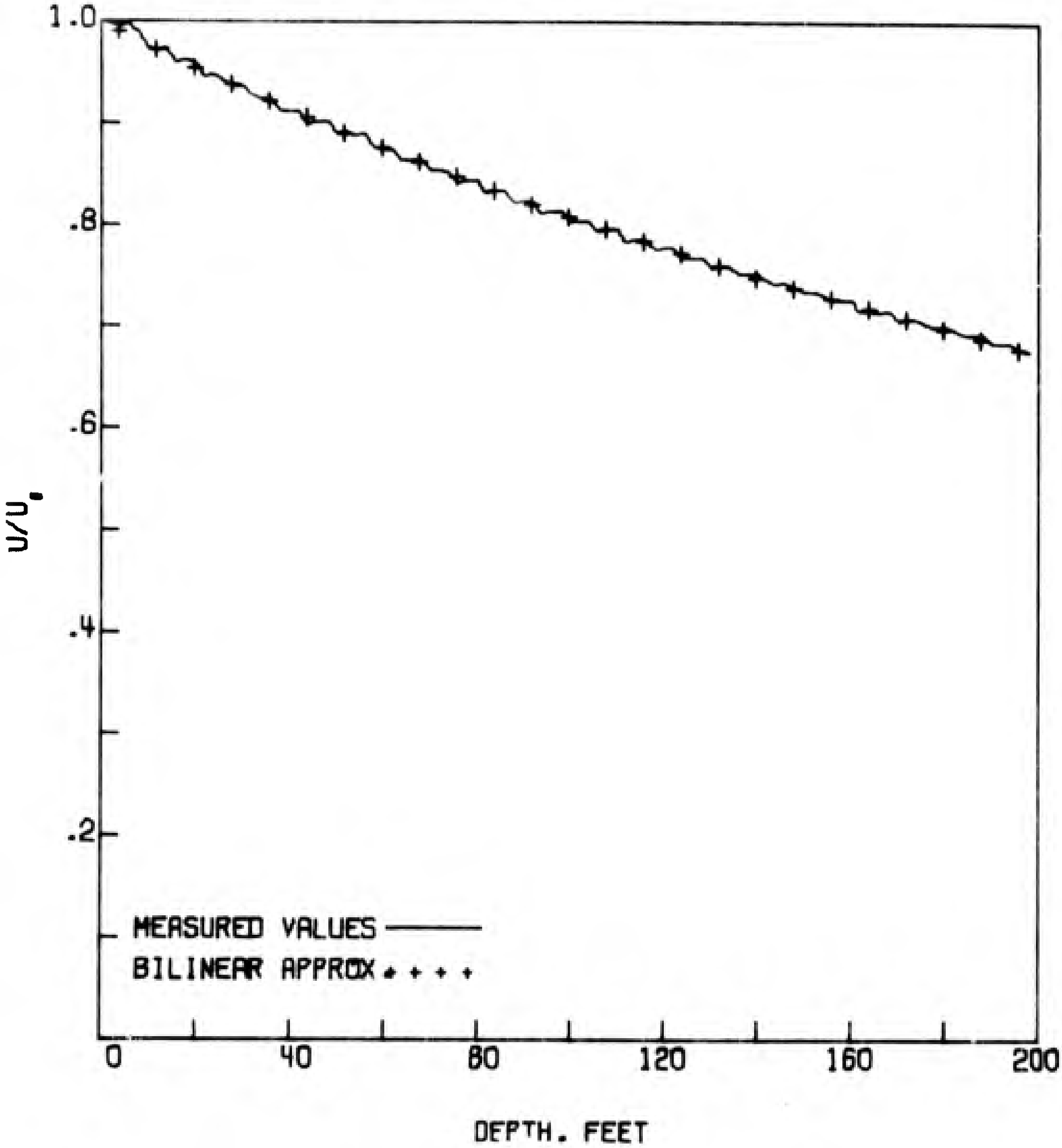
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.356	.359
25	.318	.314
50	.276	.272
75	.234	.230
100	.193	.189



SURFACE PARTICLE VELOCITY

PROBLEM 11H -- 1 MAY 1970



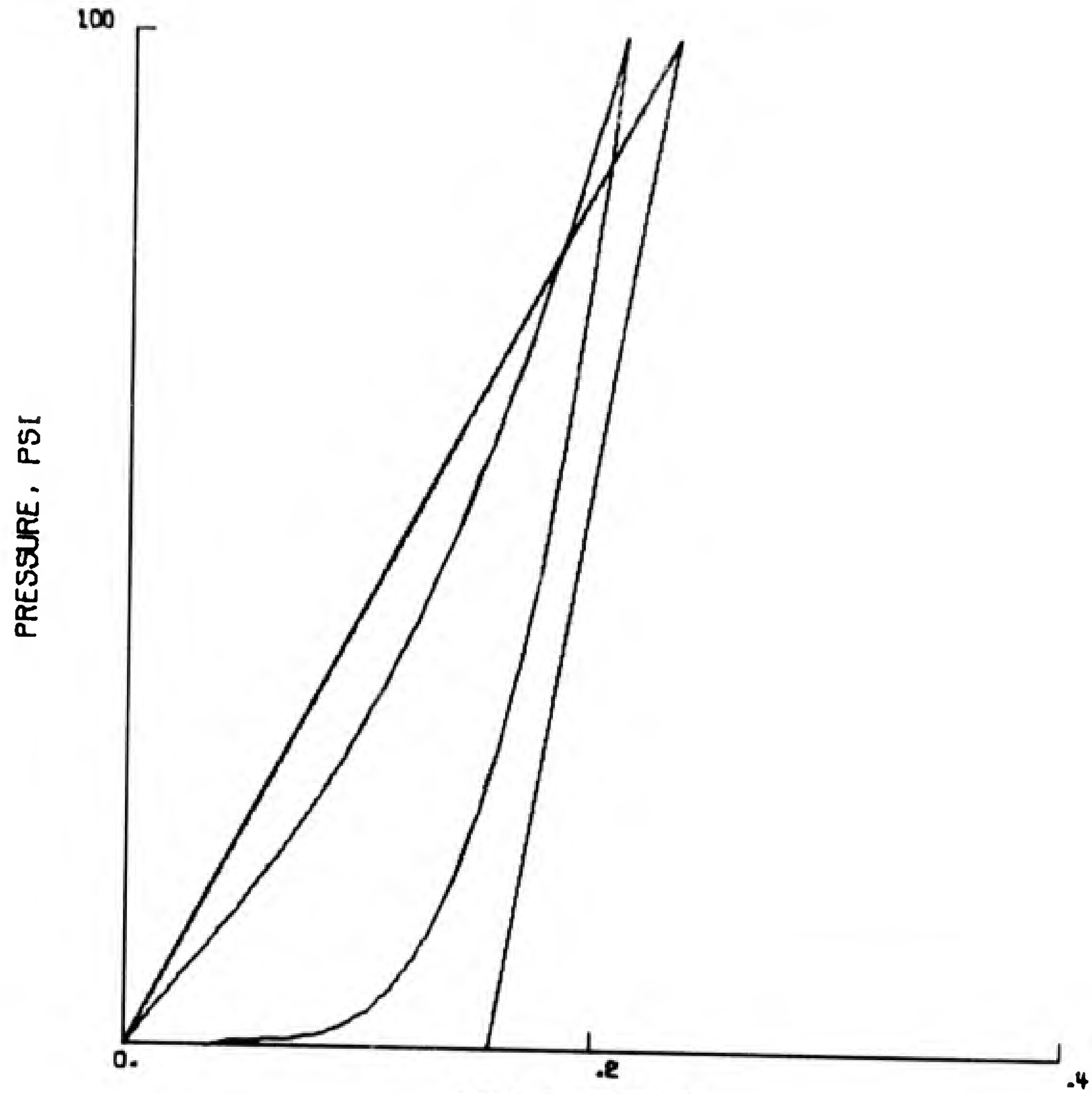
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 100 PSI BRODE WAVE

HALF LOAD TIME = 1.076864E-01 SEC.

NORMALIZED HALF LOAD TIME = 1.196516E-01

PROBLEM 11H -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =4.281691E+04
M2(PSI) =1.292898E+05
CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 4.738438E+04
HUGONIOT NONLINEARITY FACTOR = 2.120586E-01
MAXIMUM UNLOAD SLOPE = 1.941047E+05
ZETA = 2.441176E-01

BEST BILINEAR MODEL

PROBLEM 12H -- 1 MAY 1970

NUMBER OF DATA POINTS. N=103 . M= 99

MATERIAL PROPERTIES		
	DENSITY =	5.000000E+00
	SHOCK VELOCITY =	1.225203E+03
	SOUND VELOCITY =	1.814922E+03
	ZETA =	1.939785E-01

FITTING ERRORS

E_1 = 9.988471E-01 E_2 = 2.304385E-03

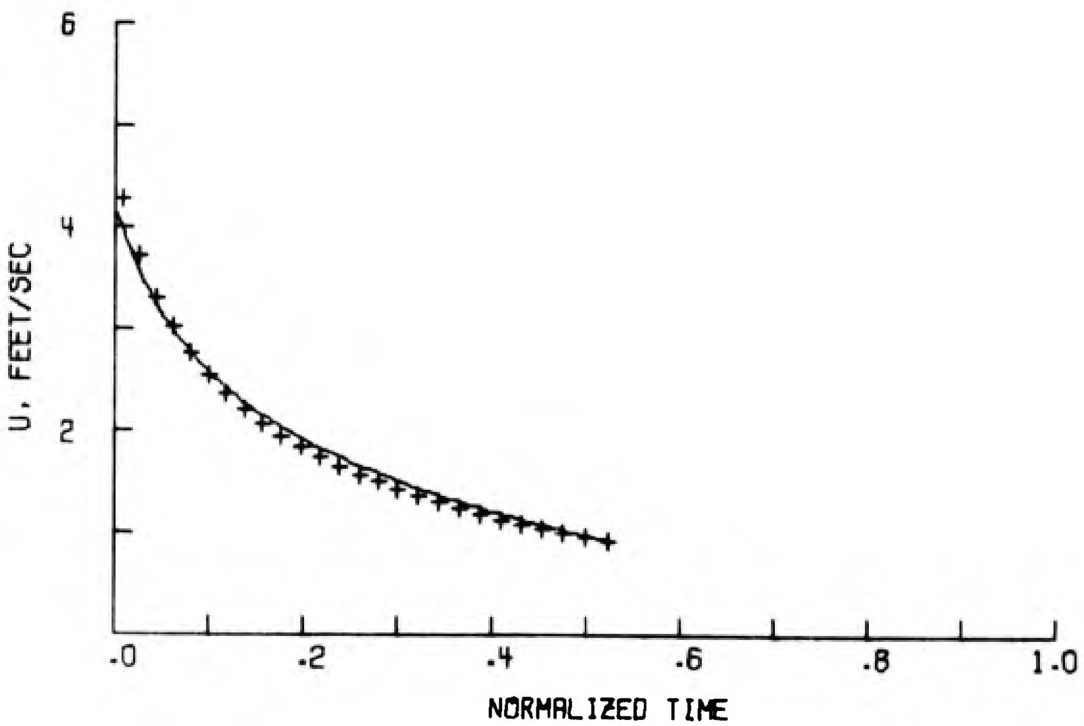
E_3 = 4.424875E-05 E_4 = 6.651973E-03

E_5 = 9.980840E-01 E_6 = 3.828344E-03

E_7 = 7.119933E-03 E_8 = 8.437970E-02

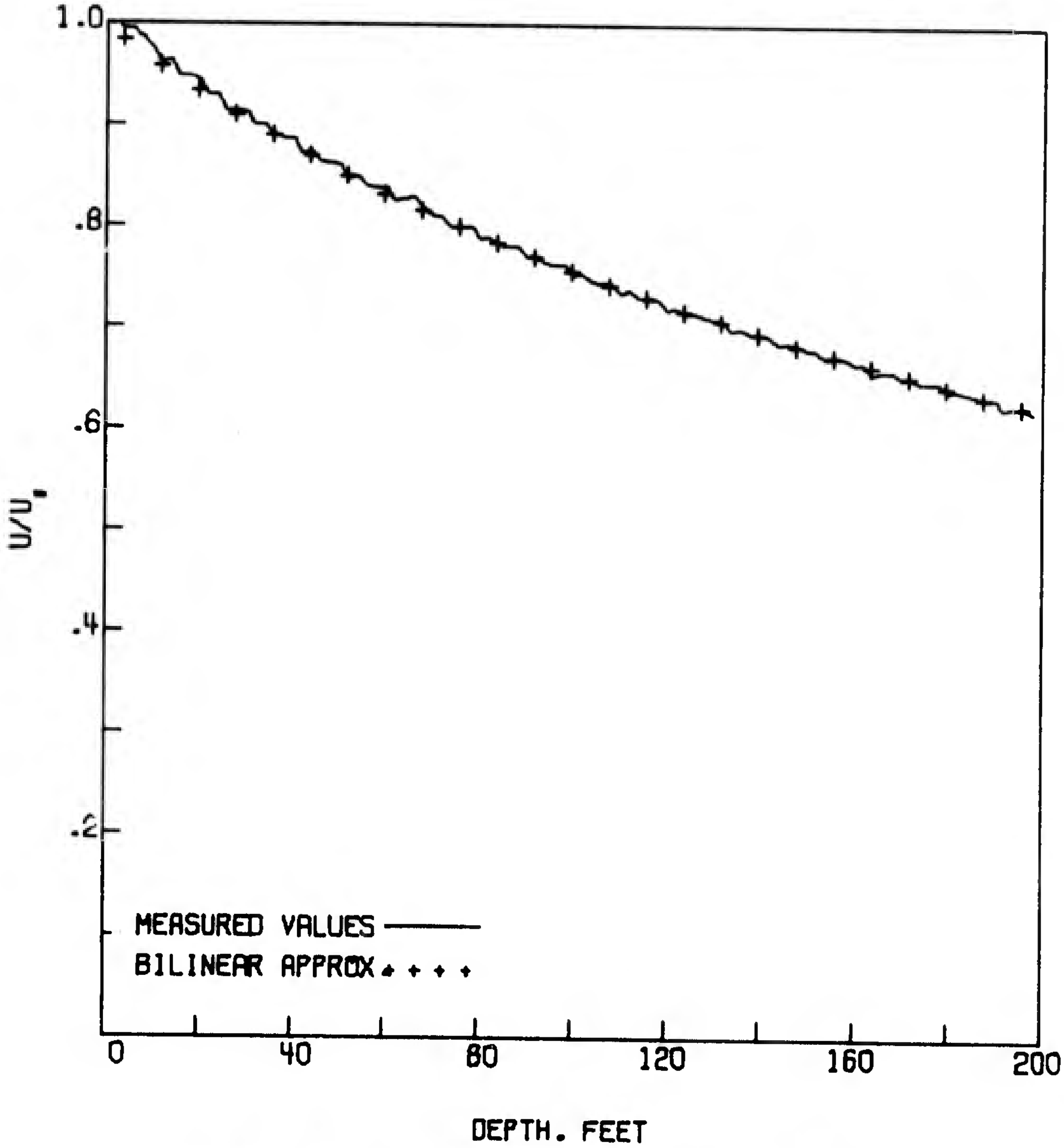
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.532	.539
25	.480	.478
50	.424	.420
75	.368	.364
100	.313	.310



SURFACE PARTICLE VELOCITY

PROBLEM 12H -- 1 MAY 1970



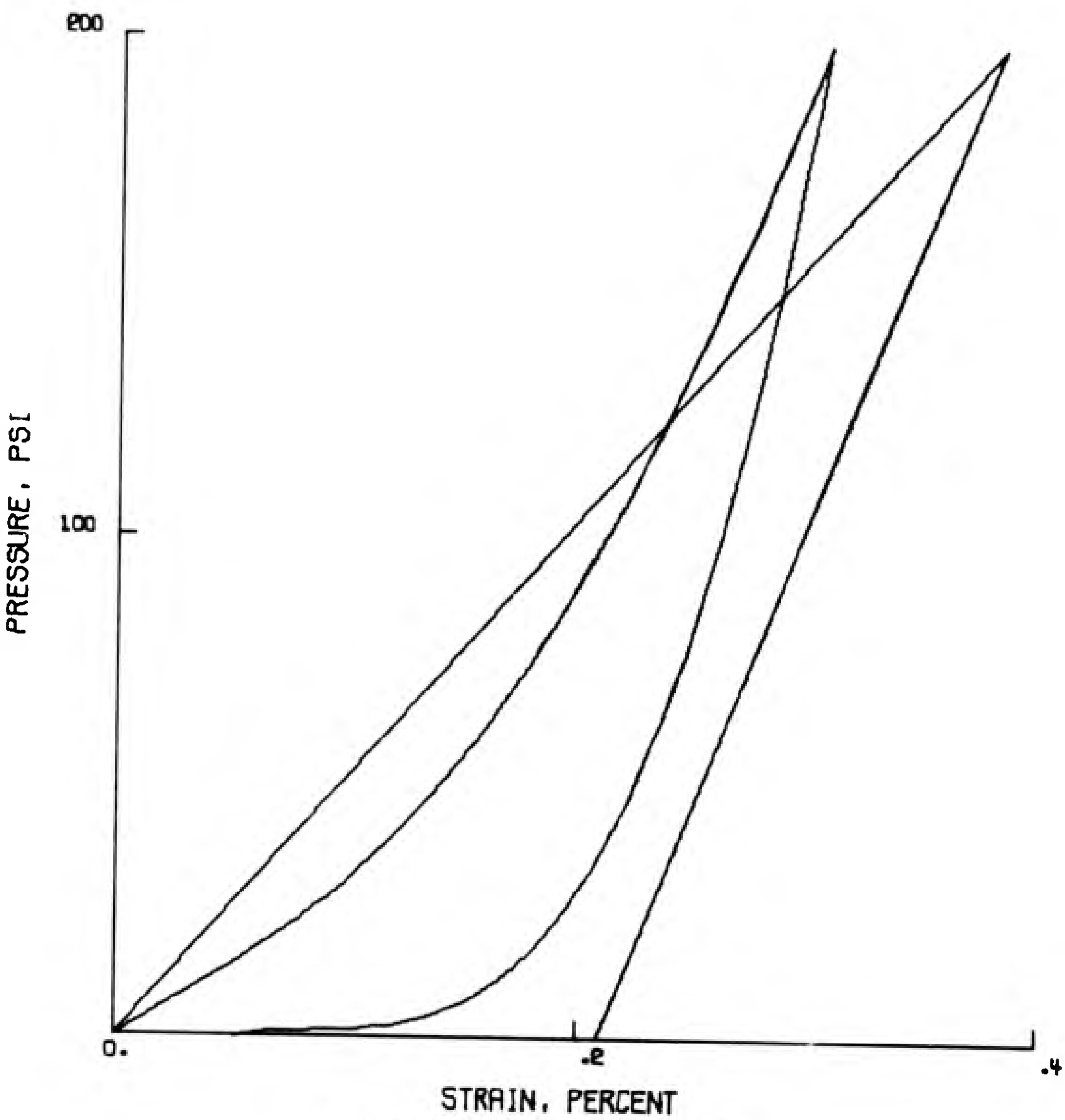
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 200 PSI BRODE WAVE

HALF LOAD TIME = 6.649772E-02 SEC.

NORMALIZED HALF LOAD TIME = 7.074225E-02

PROBLEM 12H -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =5.212230E+04
M2(PSI) =1.143730E+05
CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 6.497937E+04
HUGONIOT NONLINEARITY FACTOR = 2.677390E-01
MAXIMUM UNLOAD SLOPE = 2.661805E+05
ZETA = 2.441176E-01

BEST BILINEAR MODEL

PROBLEM 13H -- 1 MAY 1970

NUMBER OF DATA POINTS. N=101 . M= 99

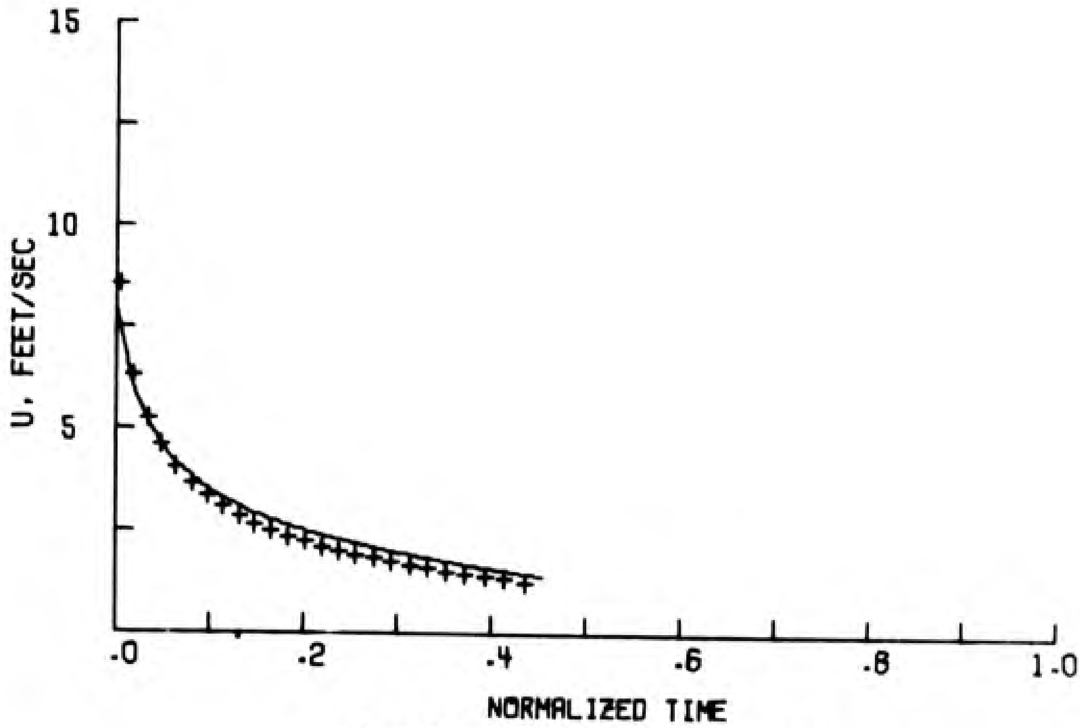
MATERIAL PROPERTIES		
	DENSITY =	5.000000E+00
	SHOCK VELOCITY =	1.431168E+03
	SOUND VELOCITY =	1.932827E+03
	ZETA =	1.491260E-01

FITTING ERRORS

E_1 =	9.984514E-01	E_2 =	3.094739E-03
E_3 =	6.956552E-05	E_4 =	8.340595E-03
E_5 =	9.959436E-01	E_6 =	8.096252E-03
E_7 =	9.228900E-02	E_8 =	3.037910E-01

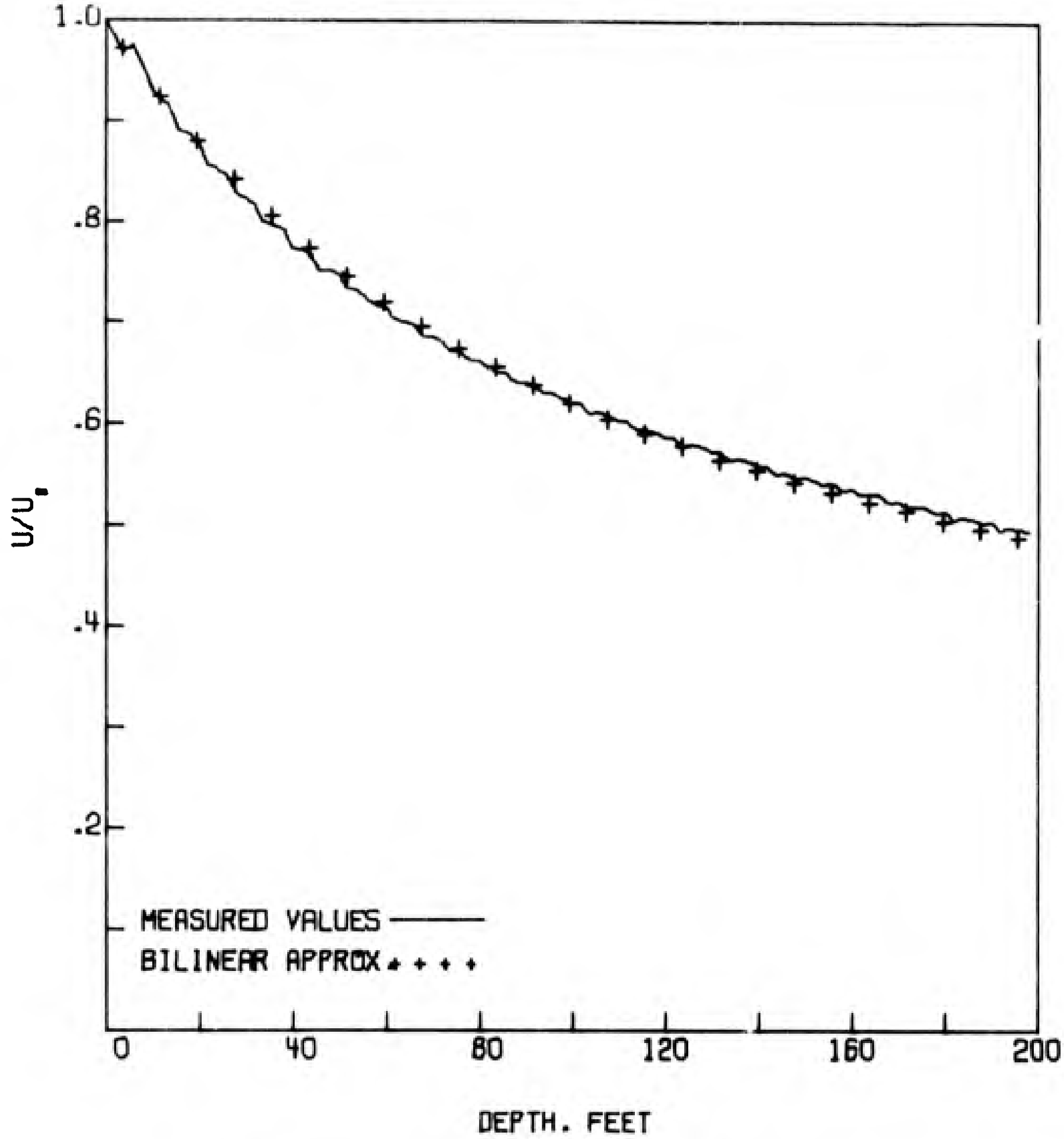
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	.815	.811
25	.741	.723
50	.663	.644
75	.588	.571
100	.514	.502



SURFACE PARTICLE VELOCITY

PROBLEM 13H -- 1 MAY 1970



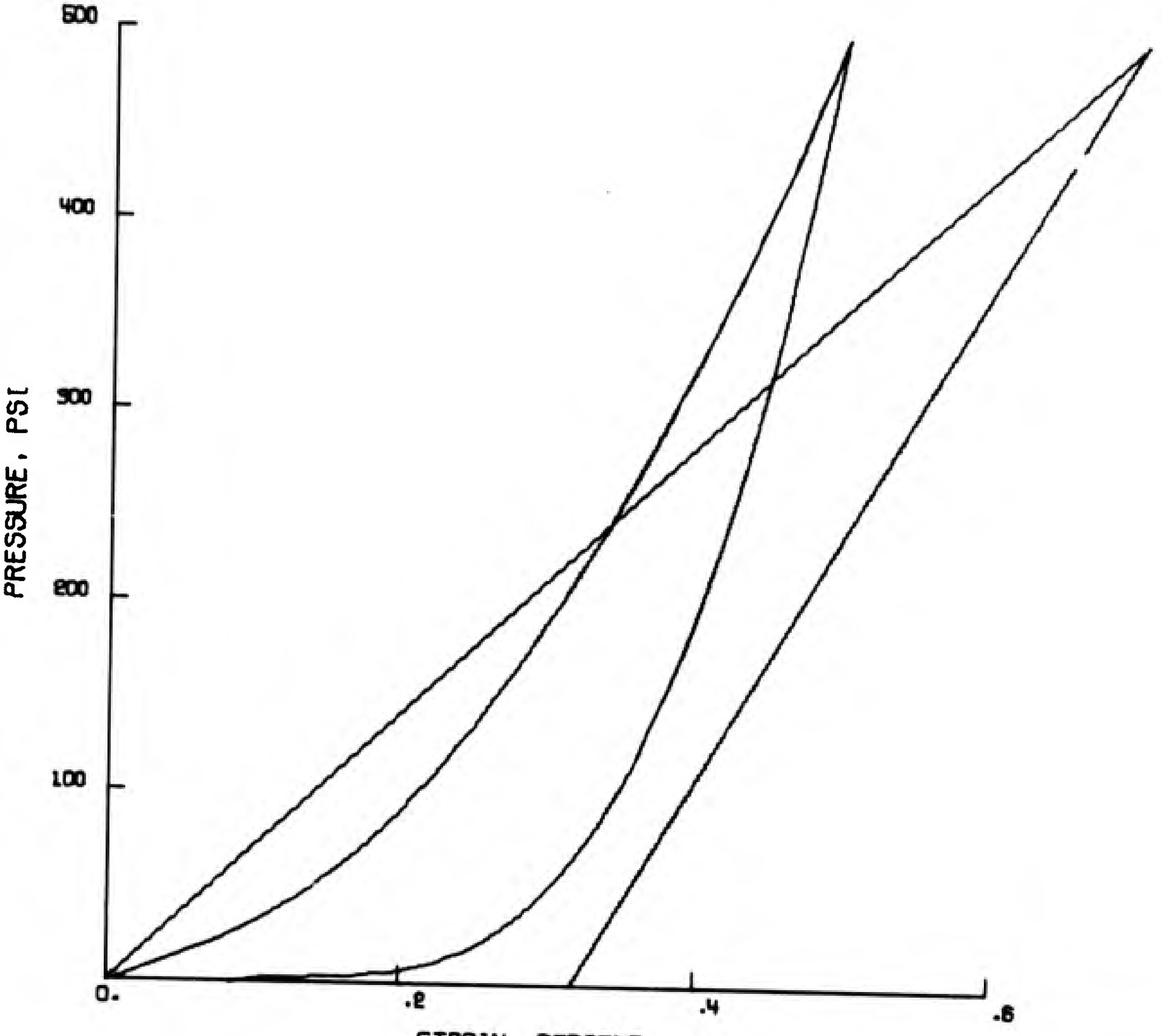
PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 500 PSI BRODE WAVE

HALF LOAD TIME = 2.531964E-02 SEC.

NORMALIZED HALF LOAD TIME = 2.322903E-02

PROBLEM 13H -- 1 MAY 1970



PRESSURE-STRAIN RELATIONS

BILINEAR MODEL- M1(PSI) =7.111952E+04
M2(PSI) =1.297160E+05

CURVILINEAR MODEL-
RAYLIEGH LINE SLOPE(PSI) = 1.000000E+05
HUGONIOT NONLINEARITY FACTOR = 3.000000E-01
MAXIMUM UNLOAD SLOPE = 4.096386E+05
ZETA = 2.441176E-01

BEST BILINEAR MODEL

PROBLEM 14H -- 1 MAY 1970

NUMBER OF DATA POINTS. N= 87 . M= 99

MATERIAL PROPERTIES

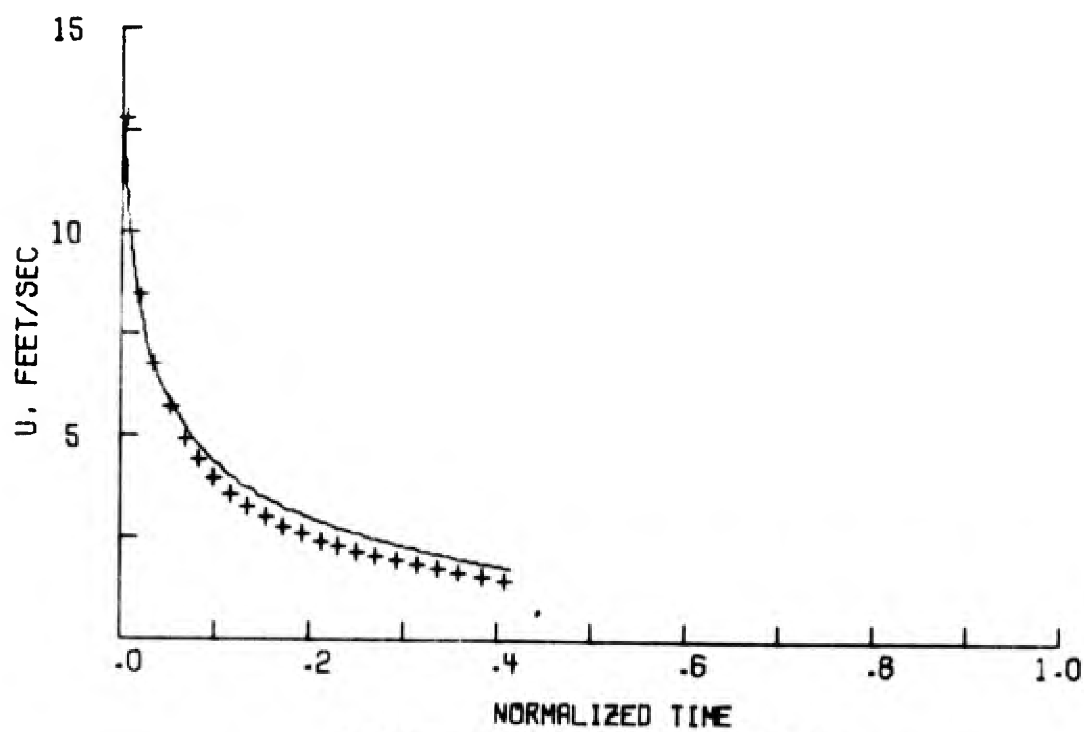
DENSITY =	5.000000E+00
SHOCK VELOCITY =	1.635817E+03
SOUND VELOCITY =	2.146784E+03
ZETA =	1.350835E-01

FITTING ERRORS

E_1 =	9.971475E-01	E_2 =	5.696769E-03
E_3 =	1.724121E-04	E_4 =	1.313058E-02
E_5 =	9.970414E-01	E_6 =	5.908375E-03
E_7 =	2.282940E-01	E_8 =	4.778012E-01

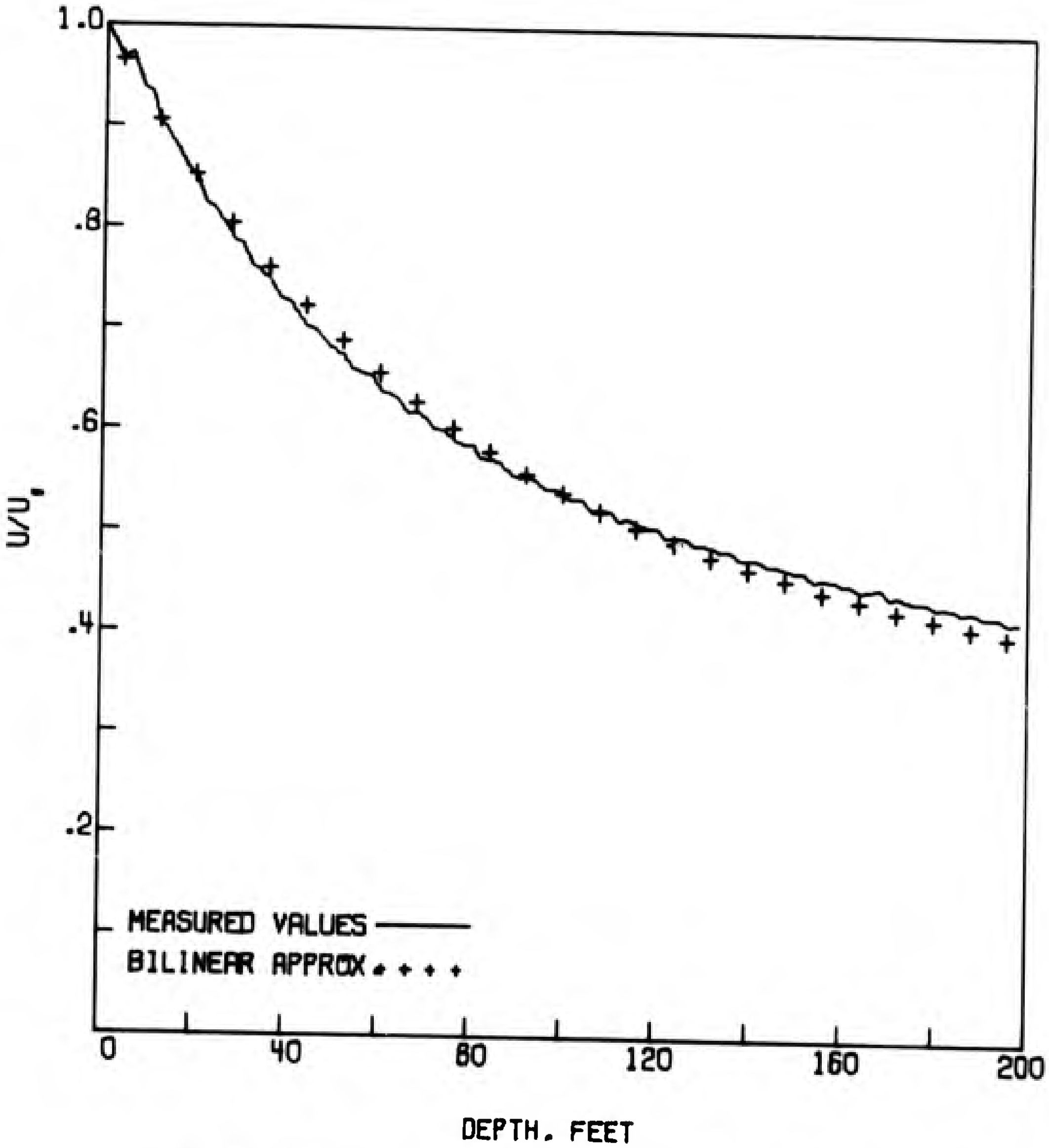
DISPLACEMENTS AFTER 200 MSEC. IN FEET

DEPTH	MEASURED	COMPUTED
0	1.095	1.082
25	1.005	.966
50	.910	.866
75	.818	.776
100	.729	.693



SURFACE PARTICLE VELOCITY

PROBLEM 14H -- 1 MAY 1970

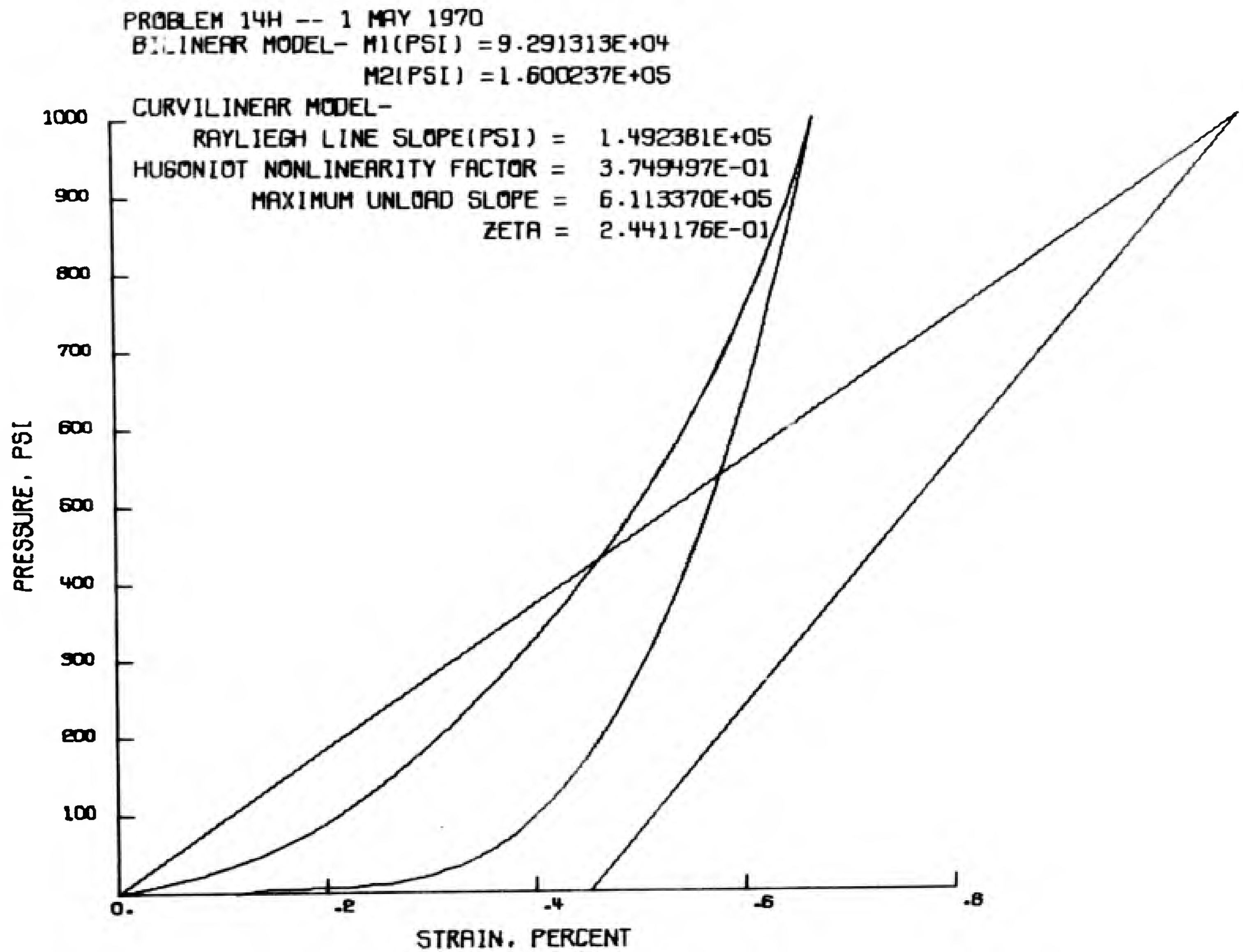


PEAK PARTICLE VELOCITY ATTENUATION

OVERPRESSURE IS 1000 PSI BRODE WAVE

HALF LOAD TIME = 1.415069E-02 SEC.

NORMALIZED HALF LOAD TIME = 1.179224E-02



PRESSURE-STRAIN RELATIONS

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