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## USAAVLABS TECHNICAL REPORT 70-64

# THE LAMINAR BOUNDARY LAYER ON A ROTATING BLADE OF SYMMETRICAL AIRFOIL SHAPE

By

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December 1970

**EUSTIS DIRECTORATE  
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY  
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THE LAMINAR BOUNDARY LAYER ON A ROTATING BLADE  
OF SYMMETRICAL AIRFOIL SHAPE

Final Report

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### ABSTRACT

A theoretical study has been conducted to determine the effects of rotation, inflow, and forward flight on the development of the laminar boundary layer on a helicopter blade. Particular emphasis was placed on the determination of the separation line. In order to facilitate the computation of the inviscid flow about the blade, an 11.9%-thick symmetrical Joukowski airfoil was used. The essential feature of the analysis was the scaling of the chordwise coordinate so that the separation line is invariant with span and time in the transformed coordinate system. The transformed boundary layer equations were expanded in an asymptotic series in span, and the resulting equations were solved by the method of Smith and Clutter.

The major effect of rotation is a delay in separation. The separation line delay is most pronounced near the axis of rotation. Forward flight causes an oscillation about this separation line, so that the delay is greatest in the first and fourth quadrants. The oscillations are affected by the blade angle of attack and the inflow due to lift. The phase advance between the wall shear and the free-stream velocity is in qualitative agreement with the results of Lighthill.

Rotation alone does not influence the separation line greatly. However, its combination with forward flight and inflow contribute, at least in part, to the increase in maximum lift observed on helicopter blades.

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# LIST OF SYMBOLS

In the following list, all quantities for which dimensions are not given have been nondimensionalized by the angular velocity  $\Omega$  (units of  $\text{time}^{-1}$ ) and/or the chord  $c$  (units of length).

|                             |                                                                                     |
|-----------------------------|-------------------------------------------------------------------------------------|
| $A_R$                       | the aspect ratio of the blade                                                       |
| $a_0$                       | the slope of the lift curve                                                         |
| $B$                         | a function of $\sigma$ that defines $dx/d\sigma$ , given by Equation (6)            |
| $C_t$                       | the coefficient of thrust                                                           |
| $c$                         | the chord of the airfoil; units of length                                           |
| $f, g, \bar{w}$             | stream functions for $u, v$ , and $w$                                               |
| $F_{nj}, G_{nj}$            | terms in the series for $f$ and $g$                                                 |
| $\vec{i}, \vec{j}, \vec{k}$ | unit vectors in the $X, Y, Z$ coordinate system                                     |
| $K_n, K_{nj}$               | terms in the series for $q$                                                         |
| $Q$                         | the velocity vector in the $X, Y$ , and $Z$ coordinate system; units of length/time |
| $M_t$                       | Mach number at the tip of the blade in an azimuthal angle of 90 degrees             |
| $N_b$                       | the number of blades in the rotor disk                                              |
| $q$                         | a factor in the transformation of $x$ into $\xi$                                    |
| $R$                         | radius of the rotor disk                                                            |
| $S_g$                       | denotes the upper surface of the blade by +1, the lower surface by -1               |
| $S_H$                       | the speed of forward flight; units of length/time                                   |
| $s_H$                       | the speed of forward flight                                                         |
| $s_s$                       | the resultant of induced and forward flight speeds                                  |

|                             |                                                                                    |
|-----------------------------|------------------------------------------------------------------------------------|
| $T$                         | the thrust generated by the entire rotor disk; units of force                      |
| $T_j$                       | a function of time and forward flight speed                                        |
| $t$                         | time; units of time                                                                |
| $\bar{t}$                   | time                                                                               |
| $U, V, W$                   | velocities in the $X, Y, Z$ system; units of length/time                           |
| $\bar{U}, \bar{V}, \bar{W}$ | velocities in the $X, Y, Z$ coordinate system                                      |
| $U_0$                       | the value of $U$ far from the blade; defined in Equation (3); units of length/time |
| $\bar{U}_0$                 | the value of $\bar{U}$ far from the blade                                          |
| $u, v, w$                   | velocities in the $x, y, z$ coordinate system                                      |
| $u_\delta, v_\delta$        | velocities at the edge of the boundary layer                                       |
| $u_a, u_c, u_e$             | the derivatives, with respect to $x$ , of $\phi_a, \phi_c$ , and $\phi_e$          |
| $v_a$                       | the constant part of the inflow, units of length/time                              |
| $v_a$                       | the constant part of the inflow                                                    |
| $v_i$                       | the inflow induced by the combined action of all the blades; units of length/time  |
| $v_i$                       | the inflow induced by all the blades                                               |
| $\bar{x}, \bar{y}, \bar{z}$ | the Cartesian coordinate system that rotates with the blade                        |
| $x_{NS}$                    | a value of $x_0$ chosen to be near the separation point                            |
| $x_R, z_R$                  | the Cartesian coordinate system that is fixed on the blade                         |
| $x_0, z_0$                  | the coordinates of the surface of the blade in the $x_R, z_R$ coordinate system    |

|                      |                                                                                                                     |
|----------------------|---------------------------------------------------------------------------------------------------------------------|
| $x_0$                | the distance of the axis of rotation from the leading edge, measured along the chord line                           |
| $x$                  | the distance from the leading edge of the blade, measured in the chordwise direction along the surface of the blade |
| $y$                  | the distance from the axis of rotation, measured along the span of the blade                                        |
| $z$                  | the distance from the surface of the blade, measured along a normal to the surface                                  |
| $\alpha$             | the angle between the normal to the surface of the blade and a normal to the chord line of the blade                |
| $\alpha_a$           | the aerodynamic angle of attack of the blade                                                                        |
| $\alpha_b$           | the geometric blade angle of attack; the angle between a normal to the chord line and the axis of rotation          |
| $\beta$              | the skew angle of the flow on the blade surface                                                                     |
| $\gamma$             | phase angle between the maximums of shear stress and chordwise velocity                                             |
| $\delta_x, \delta_y$ | the displacement thickness multiplied by $\sqrt{\Omega / \nu}$                                                      |
| $\theta_x, \theta_y$ | the momentum thickness multiplied by $\sqrt{\Omega / \nu}$                                                          |
| $\Delta_h$           | the spacing between $\xi_h$ and $\xi_{h-1}$                                                                         |
| $\epsilon$           | a parameter determining the maximum thickness of the Joukowski                                                      |
| $\nu$                | the kinematic viscosity; units of length <sup>2</sup> /time                                                         |
| $\rho$               | the density of the fluid; units of mass/length <sup>3</sup>                                                         |
| $\xi, \zeta, \eta$   | the transform of the $x, y, z$ coordinates                                                                          |
| $\sigma$             | a parametric variable for $x$ in the potential flow                                                                 |
| $\sigma_r$           | the solidity of the rotor disk                                                                                      |
| $\phi$               | the potential function for a unit velocity; units of length                                                         |

|                          |                                                                                        |
|--------------------------|----------------------------------------------------------------------------------------|
| $\phi$                   | the potential function for a unit velocity                                             |
| $\phi_a, \phi_c, \phi_e$ | potential functions that depend on $\omega_1$ and $\alpha_b$ , defined by Equation (5) |
| $\phi_\sigma$            | the potential for a unit velocity parallel to the chord                                |
| $\phi_\rho$              | the potential for a unit velocity normal to the chord                                  |
| $\psi$                   | time; the azimuthal angle measured as shown in Figure 1                                |
| $\Omega$                 | the rotational velocity; units of time <sup>-1</sup>                                   |
| $\Omega_1$               | the constant of proportionality for nonuniform inflow; units of time <sup>-1</sup>     |
| $\omega_1$               | the constant of proportionality for nonuniform inflow                                  |

The following variables are defined for the hover case only:

|       |                                                           |
|-------|-----------------------------------------------------------|
| $C^C$ | $2 \cos (\alpha / \alpha_b) / \bar{u}_a$                  |
| $C^S$ | $\omega_1 (x - x_I) \sin (\alpha - \alpha_b) / \bar{u}_a$ |
| $H$   | $(x - x_I) \bar{u}_a$                                     |
| $M$   | $(x - x_I) (d\bar{u}_a / dx) / \bar{u}_a$                 |
| $P$   | $(x - x_I) / \bar{u}_a$                                   |

$C_n^C, C_n^S, h_n, m_n, P_n$  terms in the series expansion for  $C_n^C, C_n^S, H, M,$  and  $P$

The following variables are defined for the forward flight case only:

|               |                                                                 |
|---------------|-----------------------------------------------------------------|
| $c_n$         | a term in the series expansion for $2 \cos (\alpha - \alpha_b)$ |
| $m_n, m_{nj}$ | functions defined in Equations (51) through (54)                |
| $P_0$         | $\xi / \mu_0^\delta$                                            |

|                                  |                                                                 |
|----------------------------------|-----------------------------------------------------------------|
| $\bar{v}$                        | the part of $v_\delta$ that does not depend on time             |
| $x_n$                            | a term in the series for $x_I$                                  |
| $\mu_n^a, \mu_n^c, \mu_n^\delta$ | terms in the series for $\bar{u}_a, \bar{u}_c$ , and $u_\delta$ |
| $\bar{v}_n$                      | a term in the series for $\bar{v}$                              |

The following symbols are used as subscripts:

|          |                                                                                 |
|----------|---------------------------------------------------------------------------------|
| c        | $K_n$ is not a coefficient of the subscripted function                          |
| I        | at the stagnation point                                                         |
| i        | induced; concerned with the induced downflow                                    |
| k        | $K_n$ is a coefficient of the subscripted function                              |
| j        | the subscripted function is to be multiplied by $T_j$ where $j = 0, 1, 2 \dots$ |
| n        | a coefficient of $\zeta^{-n}$ where $n = 0, 1, 2 \dots$                         |
| r        | neither $v_a$ nor $T_j$ is a coefficient of the subscripted function            |
| s        | at the separation point                                                         |
| $\delta$ | at the edge of the boundary layer                                               |

The following symbols are used as superscripts:

|   |                                                        |
|---|--------------------------------------------------------|
| ' | the derivative with respect to $n$                     |
| * | the asymptotic value for large values of span          |
| - | nondimensionalized with respect to $\Omega$ and/or $c$ |

In the forward flight case, terms in the series expansion for  $\zeta$  are denoted by the first numeral subscript. For example,  $p_{21}$ ,  $m_1$ , and  $G_{nj}$  are coefficients of  $\zeta^{-2}$ ,  $\zeta^{-1}$ , and  $\zeta^{-n}$ , respectively. The second numeral subscript (often denoted in general by  $j$ ) indicates that the function is a coefficient of  $T_j$ , where  $T_0 = 1$ ,  $T_1 = s_H \sin \psi$ ,  $T_2 = s_H \cos \psi$ .

The double numerical subscript (often denoted by  $nj$ ), or a single 0 subscript, also indicates that the function is independent of  $v_a$ ,  $s_H$ ,  $\zeta$ , and  $\psi$ ; the single exception is  $G_0$ , which has the components

$$G_0 = G_{00} + T_1 G_{01}$$

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## INTRODUCTION

The separation of the boundary layer near the leading edge of an airfoil in two-dimensional flow produces well defined and easily interpretable symptoms. As the aerodynamic angle of attack increases beyond a certain point, the lift of the airfoil does not increase, the drag increases more rapidly, and the pitching moment becomes large and negative. These three symptoms characterize the two-dimensional stall of an airfoil. Experimental measurements have shown that helicopter rotors do not experience this type of stall.<sup>1</sup> The lift does not decrease at the expected angle of attack,<sup>2</sup> but the drag and pitching moment are more nearly predicted by the two-dimensional situation.<sup>3,4</sup>

The helicopter rotor in forward flight (translation perpendicular to the axis of rotation) has a constantly changing angle of yaw. Spanwise flow is also generated by centrifugal and Coriolis forces. Both the angle of attack and the chordwise velocity vary periodically as the blade rotates. Additional time dependence is introduced by feathering, flapping, and lead-lag motions of the blade. Each of these effects may contribute to the peculiarities of stall on helicopter blades, but no single effect has been identified as the cause. Examinations of the lift, drag, and pitching moment in three-dimensional (yawed) steady-state flow<sup>3</sup> and in unsteady two-dimensional flows<sup>4</sup> have shown trends that suggest that yaw and time variation of the flow are important. Such investigations have outlined the effects of unsteady and three-dimensional behavior on helicopter rotors, but the cause of stall is to be found in the boundary layer. This suggests the approach of the present work: an analysis of the boundary layer on a blade that is simultaneously undergoing rotation, inflow due to lift, and forward flight. It is not yet feasible to include, in one analysis, every factor that influences the boundary layer. The present work is valuable in evaluating some of the factors that influence the boundary layer, and since separation is the most important feature of the boundary layer, the primary interest of this work will be the determination of the position of the separation line. Most experimental work has been concerned with such aerodynamic characteristics as the lift, drag, and pitching moment. There is a need for experimental measurements in the boundary layer. Except for some measurements of surface streamlines,<sup>5,6</sup> the investigation of the boundary layer has been analytical.

The consideration of rotational effects on blades began after the discovery by Sears<sup>7</sup> of a simple but powerful potential flow transformation for the flow about rotating blades. The application of this transformation to the rotating cylindrical nonlifting blade reduced the potential flow problem to an easily solved two-dimensional problem. By utilizing this potential flow solution, several solutions to the boundary layer problem on rotating blades have been found for the special case where the span is large compared to the chord. The solutions of Rott and Smith<sup>8</sup> for wedge-type flows, and of McCroskey and Yaggy<sup>9</sup> for the linearly decelerating flow, are of this type. The assumption of a large span-to-chord ratio simplifies the

boundary layer equations by uncoupling the equations which govern the chordwise and spanwise flows. The chordwise equation then becomes the equation for two-dimensional flow and may be solved by standard techniques. Once the chordwise flow is known, it is a straightforward procedure to determine the spanwise boundary layer flow. These solutions give no information on the effect of the spanwise flow on the chordwise flow, and they are not valid as separation is approached.

S. W. Liu<sup>10</sup> removed the restriction of large span-to-chord ratio by expanding the velocities in a series in the spanwise coordinate and then using a Blasius series technique to investigate the boundary layer on a cubic cylinder for several positions of the axis of rotation. Although this solution presents many interesting features of the boundary layers on rotating blades, especially at small spans, it is limited in that the equations solved have been simplified by the assumption of thin blade sections. The solutions are not valid, therefore, in the vicinity of the stagnation point. Furthermore, it does not seem reasonable that the procedure used by Liu could be extended for use on a realistic airfoil (blade) section, since it is well known that the Blasius series technique requires a large number of terms to represent the boundary layer on practical airfoil sections. McCroskey and Yaggy applied Liu's method to a flat blade in forward flight. Forward flight solutions for linearly decelerating flows at large span-to-chord ratios were also obtained. This work gives considerable insight into the simultaneous action of forward flight and rotation, as well as establishes the method for including forward flight in the analysis.

A solution for a symmetric Joukowski airfoil with forward flight and rotation was found by Young and Williams.<sup>11</sup> By utilizing a series in the spanwise coordinate similar to that of Liu, a solution that was valid for small values of span, and from the stagnation point to near separation, was found. The variation of the separation point with time and span was found for several positions of the axis of rotation, but only the zero angle of attack (no lift) case was considered. Centrifugal and Coriolis effects were found to be most important in the strong adverse pressure gradients near separation. Forward flight caused the separation line to oscillate about the hover (no forward flight) separation line. Separation was delayed most on the downstream side of the rotor disk because of the predominance of the time derivatives of the nonsteady flow. The effects of forward flight, in general, were greater than the rotational effects. By examining the order of magnitude of the terms in the boundary layer equations, Dwyer and McCroskey<sup>6</sup> confirmed this conclusion. From series solutions on a rotating flat plate in forward flight, from finite difference solutions on airfoils in hover, and from unsteady two-dimensional solutions, it was concluded that the unsteady effects and rotational effects are larger at smaller blade angles of attack. It was also suggested that the chordwise dimension should be nondimensionalized by the span.

Beyond the preceding investigations, little if anything has been done to answer pressing questions regarding the effects of blade shape, blade lift (angle of attack), forward flight velocity, and rotation on the boundary layer development and separation on a blade of practical airfoil shape. The present investigation considers these factors analytically. The boundary layer equations are expanded in an asymptotic series in span, and then in a finite series in time by utilizing the principle of superposition. The chordwise and spanwise stream functions are governed by a series of two-dimensional differential equations. These equations are solved numerically. The velocities at any point, and for any time, in the unseparated boundary layer are now calculable. The equation for the separation line may also be found. The effects of the inflow due to the lift of the rotor disk, the angle of attack of the blade, and the speed of forward flight are assessed.

## THEORY

### ASSUMPTIONS

The axis of rotation is taken to be the Z axis in the rotating coordinate system shown in Figures 1 and 2. The blade rotates about the Z axis with constant rotational speed  $\Omega$  so that there is no lead-lag motion. The blade is assumed to be straight and rigid and to have no twist or taper. This restricts the blade from any flapping motion. It is further assumed that the blade remains normal to the axis of rotation (without coning), so that the Y axis remains fixed in the blade. Both the X and Y axes rotate with the blade so as to define a plane of rotation which is normal to the axis of rotation. Translation due to forward flight results in a velocity  $S_H$  which lies in this plane of rotation. The azimuthal angle  $\psi$  is the angle between the vector  $S_H$  and the Y axis. The blade under consideration is only one of several blades that make up the rotor disk. The entire disk will induce an inflow  $V_i$ . This inflow velocity is assumed to be parallel to the axis of rotation and may be a function of the span, but not of the azimuthal angle. The inflow may be proportional to span, or it may be constant over the rotor disk. That is, it has the functional form

$$V_i = -V_a - \Omega_1 Y$$

Both  $V_a$  and  $\Omega_1$  are constants; for a lifting rotor they will be greater than or equal to zero. If  $\Omega_1$  is not zero, the potential flow is no longer irrotational.

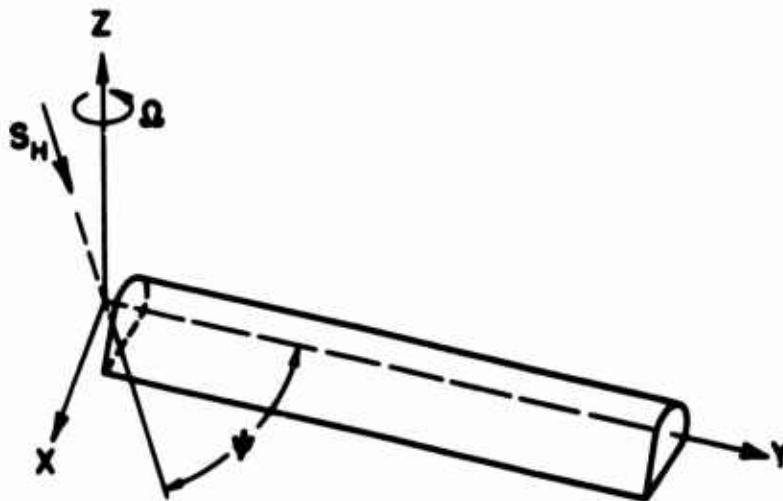


Figure 1. Rotating Coordinates.

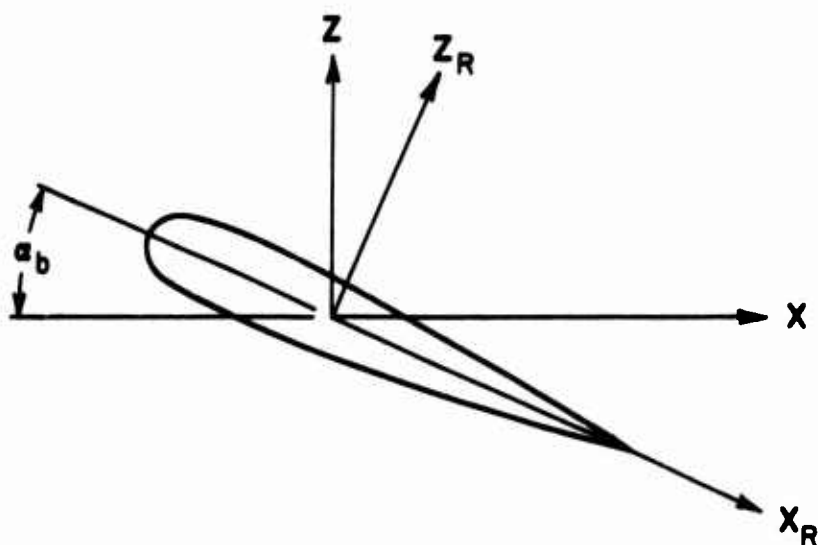


Figure 2. Blade Coordinates.

No attempt will be made to account for end effects or tip vortices. This is equivalent to assuming that the blade is infinitely long. The blade length  $R$  cannot be neglected entirely, however, since it must be used when finding a relation between the thrust of the rotor disk, the inflow, and the angle of attack of the blade (see APPENDIX I). The boundary layer solution must not be applied to the region of the blade near the tip. The reverse flow region must also be excluded from consideration. This is the region on the retreating blade where the flow is from the trailing edge to the leading edge. It is a circular region in the plane of rotation represented mathematically by

$$\Omega Y + S_H \sin \psi < 0$$

A cross section of the blade is shown in Figure 2. The geometric angle of attack  $\alpha_b$  is the angle between the chord line of the blade and the  $X$  axis. It is assumed that this geometric angle of attack does not vary along the span (the  $Y$  direction), nor does it vary with azimuthal angle. This precludes any consideration of feathering of the blade. When there is no inflow, only the forward flight velocity and the velocity due to rotation are present. Both of these velocities lie in the plane of rotation, and thus the geometric angle of attack coincides with the aerodynamic angle of attack. If inflow is present, as it must be if lift is generated, the aerodynamic angle of attack will be the angle between the chord line and the resultant of the inflow, forward flight, and rotational velocities. The component of forward flight in the chordwise direction is  $S_H \cos \psi$ . This results in an aerodynamic angle of attack which varies with azimuthal angle. The velocity due to rotation, given by  $\Omega Y$ , gives rise to a spanwise variation.

In order to define a tractable problem, the variation of the aerodynamic angle of attack due to blade twist, flapping, feathering, nonuniform inflow, and lead-lag motion has been neglected. The neglect of the spanwise variation can be assessed, and justified, from the results. The change with azimuthal angle presents more difficulty. The change of angle of attack with time is a difficult problem even in two-dimensional flow. The present method of solution can account for the time dependence introduced by forward flight very well, but this effect is usually small compared to the effects of flapping or feathering. The failure to account fully for the change of the aerodynamic angle of attack with azimuthal angle is probably the most restrictive assumption in the present analysis. The forward tilt of the rotor disk in forward flight can be easily included in the present mathematical formulation, but the results obtained would not be realistic. The effects of tilt are partially cancelled out by blade twist and by feathering or flapping. Since it is not practicable to include the cancelling effects, tilt was omitted, hence the assumption that the forward flight velocity lies in the plane of rotation.

Assumptions have been made that would not be justified if the performance of the helicopter were to be calculated. The tip vortices have a definite effect on helicopter performance, but since this is not a boundary layer effect, their neglect is justified. The flow outside the boundary layer is assumed to be incompressible and inviscid, and the circulation in this potential flow is calculated by the Kutta condition. The Kutta condition is chosen for convenience. If a more accurate determination of the circulation were desired, the present method of solution for the boundary would still be valid.

With regard to the Kutta-Joukowski trailing edge condition, one should recognize that this condition is only an approximate condition which accounts for the viscous origin of the airfoil circulation. When separation occurs on the airfoil, this approximation is poor. Unfortunately, there has not been established, as yet, an appropriate condition to replace the Kutta-Joukowski condition when the separation is extensive or when the separation line is time dependent.

The assumptions that have been made lead to equations that are solvable, and the results will contain several effects that have not been considered in the previous solutions for the boundary layer on rotating blades. It has not been possible to include all the factors that influence the development of the boundary layer, but the importance of some of the omitted factors can be better assessed from the results.

#### POTENTIAL FLOW SOLUTION

Before solving the boundary layer equations, it is necessary to furnish the boundary conditions at the edge of the boundary layer from a potential flow solution. This solution has been chosen to present the features of the potential flow that significantly influence the boundary layer. The

circulation will be calculated from the Kutta condition, but the thickness of the boundary layer will not be taken into account. As in the boundary layer, the flow is assumed incompressible. For ease of calculations, results will be found for a symmetrical Joukowski airfoil. Interest in this investigation is focused on the boundary layer itself, although the method of solution could be applied to a more precise determination of the potential flow.

The inviscid flow over a rotating cylindrical blade was found by Sears and Fogarty<sup>12</sup> for a constant inflow. In the rotating coordinate system, the velocity vector  $\vec{Q}$  must obey

$$\vec{\nabla} \cdot \vec{Q} = 0 \quad \vec{\nabla} \times \vec{Q} = -2\Omega \vec{k} - \Omega_1 \vec{i}$$

The first of these equations is an expression of conservation of mass (continuity equation) in the fluid flow; the second defines the rotation of the external flow. It is possible to write  $\vec{Q}$  in terms of a potential function  $\phi$  and a two-dimensional stream function  $\psi$ ,

$$\vec{Q} = \vec{\nabla} \phi + \vec{\nabla} \psi \times \vec{k} \quad (1)$$

The equation for the rotation of the flow is then satisfied if  $\psi$  is chosen as

$$\psi = \Omega(x^2 + y^2)/2 - \Omega_1 xz \quad (2)$$

To include the effects of forward flight,  $\phi$  is written as

$$\phi = U_0 \phi_1 + V_1 \phi_2 + Y(-\Omega X + S_H \cos \Omega t) \quad (3)$$

where  $U_0 = \Omega Y + S_H \sin \Omega t$ .

The boundary conditions on  $\phi_1$  and  $\phi_2$  are

$$\text{at } x^2 + z^2 = - \quad \phi_1 = x, \quad \phi_2 = z$$

on the surface of the airfoil:

$$\frac{\partial \phi_1}{\partial x} \tan (\alpha - \alpha_b) = \frac{\partial \phi_1}{\partial z}$$

$$\frac{\partial \phi_2}{\partial x} \tan (\alpha - \alpha_b) = \frac{\partial \phi_2}{\partial z}$$

where  $(\alpha - \alpha_b)$  is the angle between the Z axis and the normal to the surface of the blade.

The equation of continuity is satisfied if  $\phi_1$  and  $\phi_2$  are two dimensional (in X and Z), and if they satisfy

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2} = 0$$

$$\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2} = 0$$

Thus,  $\phi_1$  and  $\phi_2$  are the two-dimensional potentials for flow around the rotor blade section in a unit stream. If the rotor blade generates lift, there will be a circulation included in the potential.

Nondimensionalizing the lengths by the chord of the airfoil  $c$  and the velocities by  $\Omega c$ , the velocities become

$$\bar{u} = \bar{u}_0 \frac{\partial \phi_1}{\partial \bar{x}} + v_1 \frac{\partial \phi_2}{\partial \bar{x}}$$

$$\bar{v} = \phi_1 - 2\bar{x} + s_H \cos \bar{t} + \omega_1 (\bar{z} - \phi_2)$$

$$\bar{w} = \bar{u}_0 \frac{\partial \phi_1}{\partial \bar{z}} + v_1 \frac{\partial \phi_2}{\partial \bar{z}}$$

In order to find the velocities at the edge of the boundary layer, the velocities must be written in a coordinate system fixed on the body. The distance along the surface of the blade  $x$  is measured from the leading edge. The  $z$  dimension is measured normal to the surface. The spanwise dimension  $y$  is measured in the spanwise direction from the axis of rotation.

It is also convenient to define a Cartesian coordinate system that is fixed with respect to the blade, as shown in Figure 2. The  $X_R$  axis will lie along the chord line of the blade. The angle between the  $X$  axis and the  $X_R$  axis will be the blade angle of attack  $\alpha_b$ . The  $Z_R$  axis is perpendicular to  $X_R$ , and is at angle  $\alpha_b$  to the  $Z$  axis (the axis of rotation). The origin of the  $X_R, Z_R$  coordinate system is the point where the  $Z$  axis intersects the chord line of the blade. This intersection is at a distance  $X_0$  from the leading edge of the blade.

Finally, it is convenient to define two new potential functions in the new  $X_R, Z_R$  coordinate system. In this system,  $\phi_\sigma$  is the two-dimensional potential, on the surface, for a unit stream in the chordwise direction, and  $\phi_\rho$  is the two-dimensional potential, evaluated on the surface, for a unit stream normal to the chord. There is no circulation in  $\phi_\sigma$ , but matching the Kutta condition requires a circulation in  $\phi_\rho$ . The velocities at the edge of the boundary layer,  $u_\delta$  and  $v_\delta$ , are now given by

$$u_\delta = y\bar{u}_a + v_a \bar{u}_c + T_2 \bar{u}_e \quad (4)$$

$$v_\delta = \phi_a + T_1 - X_R(2 \cos \alpha_b + \omega_i \sin \alpha_b) + Z_R(-2 \sin \alpha_b + \omega_i \cos \alpha_b) \quad (5)$$

where  $T_0 = 1$ ,  $T_1 = S_H \cos \bar{t}$ ,  $T_2 = S_H \sin \bar{t}$

$$v_i = -v_a - \omega_i y$$

$$\bar{u}_a = \partial \phi_a / \partial x, \quad \bar{u}_c = \partial \phi_c / \partial x, \quad \bar{u}_e = \partial \phi_e / \partial x$$

$$\phi_a = (\cos \alpha_b + \omega_i \sin \alpha_b) \phi_\sigma + (\sin \alpha_b - \omega_i \cos \alpha_b) \phi_\rho$$

$$\phi_c = \phi_\sigma \sin \alpha_b - \phi_\rho \cos \alpha_b$$

$$\phi_e = \phi_\sigma \cos \alpha_b + \phi_\rho \sin \alpha_b$$

Once  $\phi_\sigma$  and  $\phi_\rho$  are found as functions of the body coordinate  $x$ , the velocities at the edge of the boundary layer can be evaluated. All velocities and lengths in the remainder of this section, and in the following sections, are nondimensional.

For ease of calculating  $\phi_\sigma$  and  $\phi_\rho$ , a symmetrical Joukowski airfoil will be used. The parameter  $\epsilon$  is related to the thickness of the airfoil; the value  $\epsilon = 0.092$  gives an 11.9%-thick airfoil. This thickness was chosen so that the airfoil would closely resemble an NACA 0012 airfoil. The NACA 0012 is widely used for helicopters. It is less blunt near the

leading edge and is flatter near the quarter chord than a Joukowski airfoil. To lessen these differences, the Joukowski airfoil was chosen to be thinner (11.9% thick as compared to the 12%-thick NACA 0012). In terms of the parameter  $\sigma$ , the potentials are

$$\phi_\sigma = (1 + \epsilon) \sigma / 2$$

$$\phi_\rho = (1 + \epsilon) [S_g \sqrt{1 - \sigma^2} - \tan^{-1} (S_g \sqrt{1 - \sigma^2} / \sigma)] / 2$$

where  $S_g = +1$  denotes the upper surface of the airfoil

$S_g = -1$  denotes the lower surface of the airfoil

The parameter  $\sigma$  is given as a function of  $x$  by

$$\frac{dx}{d\sigma} = B(\sigma) \quad , \quad x = \int_{-1}^{\sigma} B d\sigma \quad (6)$$

where  $B = \frac{dX_\sigma}{d\sigma} / \cos \alpha$

$$\tan \alpha = \frac{dZ_\sigma}{dX_\sigma}$$

$$X_\sigma = \frac{1 + \epsilon}{4} (\sigma - \epsilon) \left[ 1 + \frac{(1 - \epsilon)^2}{1 + \epsilon^2 - 2\epsilon\sigma} \right] + \frac{1 + \epsilon^2}{2} - X_0$$

$$Z_\sigma = \frac{1 + \epsilon}{4} S_g \sqrt{1 - \sigma^2} \left[ 1 + \frac{(1 - \epsilon)^2}{1 + \epsilon^2 - 2\epsilon\sigma} \right]$$

The angle  $\alpha$  is the angle between the normal to the surface and the  $Z_R$  axis.  $X_\sigma$  and  $Z_\sigma$  are the coordinates of the airfoil surface in the  $X_R, Z_R$  coordinate system.

Considerable simplification can be obtained for certain special cases. A solution is available for the rotor blade at zero angle of attack and

no induced flow ( $\alpha_b = 0$ ,  $v_i = 0$ ) in Young and Williams.<sup>11</sup> For this case, the velocities at the edge of the boundary layer are

$$u_\delta = (y + T_2) u_\sigma \quad (7)$$

$$v_\delta = \phi_\sigma + T_1 - 2X_\sigma \quad (8)$$

This, in turn, is contained within the special case of forward flight with constant induced velocity ( $\omega_i = 0$ ), for which

$$u_\delta = y u_a + v_a u_c + T_2 u_a \quad (9)$$

$$v_\delta = \phi_\sigma + T_1 - 2X_\sigma \cos \alpha_b - 2Z_\sigma \sin \alpha_b \quad (10)$$

$$\phi_a = \phi_\sigma \cos \alpha_b + \phi_\rho \sin \alpha_b$$

$$\phi_c = \phi_\sigma \sin \alpha_b - \phi_\rho \cos \alpha_b$$

In the case of hover ( $s_H = 0$ ), it is more convenient to allow the induced velocity to be proportional to the span ( $v_a = 0$ ). In this case,

$$u_\delta = y u_a \quad (11)$$

$$v_\delta = \phi_a - X_\sigma (2 \cos \alpha_b + \omega_i \sin \alpha_b) + Z_\sigma (-2 \sin \alpha_b + \omega_i \cos \alpha_b) \quad (12)$$

In order to calculate  $X_\sigma$ ,  $Z_\sigma$ ,  $\phi_\sigma$  and  $\phi_\rho$ , it is necessary to specify the airfoil shape (in all examples herein, a Joukowski airfoil) and the airfoil thickness (always 11.9%). The only purpose of finding the potential flow is to furnish the boundary conditions on the velocities  $u_\delta$  and  $v_\delta$ . These depend on the choices made for the constants  $\alpha_b$ ,  $\omega_i$  and  $X_0$ ; once these are specified, the boundary layer equations may be solved. The general theory in the next section is applicable to both the hover case and the forward flight case. In succeeding sections, advantage is taken of the simplifications in  $u_\delta$  and  $v_\delta$  that occur in the above two cases.

### BOUNDARY LAYER EQUATIONS

By making the usual boundary layer assumptions, the Navier-Stokes equations can be reduced to the simplified equations for the boundary layer on a rotating blade:<sup>9</sup>

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (13)$$

$$\begin{aligned} \frac{\partial u}{\partial \bar{t}} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - 2(v - v_\delta) \cos(\alpha - \alpha_b) - v \frac{\partial^2 u}{\partial z^2} / \Omega c^2 \\ = \frac{\partial u_\delta}{\partial \bar{t}} + u_\delta \frac{\partial u_\delta}{\partial x} + v_\delta \frac{\partial u_\delta}{\partial y} \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{\partial v}{\partial \bar{t}} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + 2(u - u_\delta) \cos(\alpha - \alpha_b) - v \frac{\partial^2 v}{\partial z^2} / \Omega c^2 \\ = \frac{\partial v_\delta}{\partial \bar{t}} + u_\delta \frac{\partial v_\delta}{\partial x} + v_\delta \frac{\partial v_\delta}{\partial y} \end{aligned} \quad (15)$$

The boundary conditions for these equations are:

$$\text{at } z = 0 \quad u = v = w = 0$$

$$\text{at } z = \infty \quad u = u_\delta, \quad v = v_\delta$$

These equations differ from the usual unsteady, three-dimensional boundary layer equations in the terms that give the Coriolis acceleration. These terms, involving the product of velocity and  $\cos(\alpha - \alpha_b)$ , appear explicitly because the equations are written in a rotating coordinate system.

As the span increases, there is little change in the spanwise flow. The potential flow in the spanwise direction  $u_\delta$  is independent of span. Time dependence is introduced into the equations by the forward flight speed, which is also independent of span. However, the chordwise velocity due to rotation is proportional to span. Thus, at large values of span, the chordwise flow is dominated entirely by the flow due to rotation, and both time dependence and the spanwise flow become negligible in the chordwise

momentum equation. It is not that the spanwise flow and time dependence have become so small, but that the chordwise flow due to rotation has become so large when the span is large, that the effects of spanwise flow and time dependence are relatively small. The underlined terms in the equations may be neglected at large values of span. Typically, the effect of spanwise flow is negligible in the chordwise equation at values of span greater than three to ten chord lengths and at time dependence from five to fifteen chord lengths. Over a significant portion of a typical helicopter rotor, the boundary layer flow is governed by the asymptotic solution at large span. The chordwise flow in such an asymptotic solution is given by the steady, two-dimensional equations, and thus separation occurs at the same point as in two-dimensional, steady-state flow. Figure 3 shows the separation line approaching the two-dimensional value at large span.

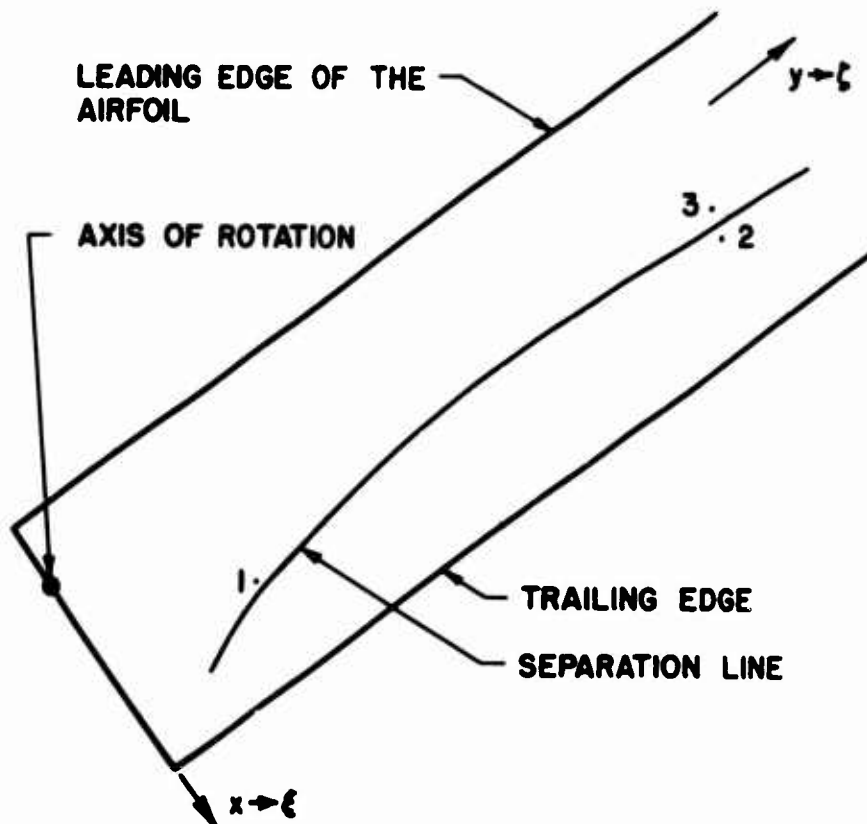


Figure 3. The Separation Line.

The solution at smaller values of span is found by expanding the stream functions in an asymptotic series in the span. This may be done most simply by allowing the first term in the series (or the first

approximation to the solution) to be the two-dimensional solution at the same chordwise position. In Figure 3, if this technique is used, the first approximation to the velocity profile at point 1 is the profile at point 2. The profile at point 2 is in the separated region. The profile at point 2 is difficult to obtain, and once it is found, it will be a poor first approximation to the profile at point 1.

A superior technique for finding a first approximation to point 1 has been devised. The  $x$  coordinate may be transformed into the  $\xi$  coordinate. The  $\xi$  coordinate is stretched as span decreases so that point 1 has the same  $\xi$  coordinate as point 3. Point 3, like point 1, is in the unseparated region. The  $\xi$  coordinate, if chosen so that the separation line always occurs at the same value of  $\xi$ , say  $\xi_s$ , will give a much better solution near separation. If time dependence is introduced by forward flight, then the  $\xi$  coordinate will expand and contract with time as well as be dependent on the span. The transform from  $x$  to  $\xi$  is accompanied by a complete coordinate transform, which is given by

$$\xi = [x - x_I(y, \bar{t})] q(y, \bar{t}) \quad , \quad \zeta = y \quad (16)$$

$$\eta = cz \sqrt{\Omega u_\delta / \nu (x - x_I)} \quad , \quad \psi = \bar{t}$$

$$u = u_\delta f'(\xi, \zeta, \eta, \psi) \quad , \quad v = g'(\xi, \zeta, \eta, \psi) \quad (17)$$

$$w = \sqrt{\nu (x - x_I) / \Omega u_\delta} \bar{w}(\xi, \zeta, \eta, \psi) / c$$

In the transformed system, the boundary conditions become

$$\text{at } \eta = 0 \quad f = f' = g = g' = \bar{w} = 0$$

$$\text{at } \eta = \infty \quad f' = 1 \quad , \quad g' = v_\delta$$

The primes denote differentiation with respect to  $\eta$ . The  $\xi, \zeta, \eta$  coordinate system has the same orientation as the  $x, y, z$  system, but the dimensions are stretched. For example, the  $\eta$  direction is the same as the  $z$  direction; it is normal to the body. The  $\eta$  coordinate, however, is stretched by the familiar Falkner-Skan factor  $\sqrt{u_\delta / \nu x}$ . This factor accounts for the dependence of the boundary layer height on  $u_\delta, \nu$ , and  $x$ . The stretching of the  $\xi$  coordinate to account for the position of the separation line is accomplished by including the factor  $q$ . If  $q$  is chosen correctly, the separation line will have the constant location  $\xi_s$  for all values of  $\zeta$  and  $\psi$ . Since the separation line is not yet known,  $q$  cannot be found immediately. Instead,  $q$  will be written in the form of a series

in  $1/\zeta$ , with the coefficients in the series still undetermined. After solving a series of differential equations, these coefficients will be determined from the condition that the separation line be invariant with  $\zeta$  and  $\psi$ .

The choice of  $x_I$  determined the origin of the  $\xi$  coordinate. It is most convenient to let  $x_I$  be the position of the stagnation point in the  $x$  coordinate system. It will usually be a negative number; it is defined as the value of  $x$  at which

$$u_\delta = 0$$

For this choice of  $x_I$ , the stagnation line will be given by  $\xi = 0$ .

The velocities have been written in terms of stream functions. The stream function for flow normal to the surface  $w$  can be eliminated through the continuity equation. The stream functions for chordwise flow ( $f$ ) and for spanwise flow ( $g$ ) are given as functions of  $\xi$ ,  $\zeta$ ,  $\eta$  and  $\psi$  by the simultaneous equations

$$\begin{aligned} & u_\delta^2 \left( -f'''' - \frac{1}{2} f f'' + \xi f' \frac{\partial f'}{\partial \xi} - \xi f'' \frac{\partial f}{\partial \xi} \right) + \xi u_\delta \frac{\partial u_\delta}{\partial \xi} \left( f'^2 - \frac{1}{2} f f'' - 1 \right) \\ & + \frac{\xi u_\delta}{q \zeta} \left[ \left( \zeta \frac{\partial g}{\partial \bar{t}} - \frac{\zeta g^2}{\xi} \frac{\partial x_I}{\partial \bar{t}} \right) \frac{\xi}{q} \frac{\partial f'}{\partial \xi} + \frac{\eta g \zeta}{2 \xi} f'' \frac{\partial x_I}{\partial \bar{t}} + \zeta \frac{\partial f'}{\partial \psi} \right. \\ & + \zeta \left( g' \frac{\partial f'}{\partial \zeta} - f'' \frac{\partial g}{\partial \zeta} \right) + \left( \frac{\zeta}{q} \frac{\partial g}{\partial y} - \frac{\zeta g}{\xi} \frac{\partial x_I}{\partial y} \right) \xi \left( g' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial g}{\partial \xi} \right) \\ & + \frac{q \zeta}{2 \xi} f'' g \frac{\partial x_I}{\partial y} \left. \right] + \frac{\xi u_e}{q} T_1 \left( f' + \frac{\eta}{2} f'' - 1 \right) + \frac{\xi u_a}{q} \left( g' f' + \frac{1}{2} f'' g - v_\delta \right) \\ & - 2 \frac{\xi}{q} (g' - v_\delta) \cos(\alpha - \alpha_b) = 0 \quad (18) \end{aligned}$$

$$\begin{aligned} & u_\delta^2 \left( -g'''' - \frac{1}{2} f g' + \xi \left( f' \frac{\partial g'}{\partial \xi} - g'' \frac{\partial f}{\partial \xi} \right) - \frac{\xi u_a}{q} - \omega_1 \frac{\xi}{q} \sin(\alpha - \alpha_b) \right. \\ & + 2 f' \frac{\xi}{q} \cos(\alpha - \alpha_b) \left. \right) + \frac{\xi}{q} u_\delta T_2 - \frac{1}{2} \xi u_\delta \frac{\partial u_\delta}{\partial \xi} f g'' \\ & + \frac{\xi u_\delta}{q \zeta} \left[ \zeta \frac{\partial g'}{\partial \psi} + \left( \frac{\zeta}{q} \frac{\partial g}{\partial \bar{t}} - \frac{\zeta g}{\xi} \frac{\partial x_I}{\partial \bar{t}} \right) \xi \frac{\partial g'}{\partial \xi} + \zeta \eta \frac{q}{\xi} \frac{\partial x_I}{\partial \bar{t}} g'' + \zeta g' \frac{\partial g'}{\partial \zeta} \right. \end{aligned}$$

$$\begin{aligned}
& + \left( \frac{\xi}{q} \frac{\partial q}{\partial y} - \frac{q\xi}{\xi} \frac{\partial x_I}{\partial y} \right) \xi \left( g' \frac{\partial g'}{\partial \xi} - g'' \frac{\partial g}{\partial \xi} \right) - \zeta g'' \frac{\partial g}{\partial \zeta} + \frac{q\xi}{2\xi} \frac{\partial x_I}{\partial y} g g'' \\
& + \frac{\xi u_a}{q} g g'' + \frac{\xi u_e}{q} n T_1 g'' = 0 \quad (19)
\end{aligned}$$

From this point on, the developments of the hover case and the forward flight case differ, but the general method is the same. The stream functions are expanded in asymptotic series in  $1/\zeta$ . Also, functions such as  $\bar{u}_a$  and  $q$  are converted to functions of  $\xi$ ,  $\zeta$ , and  $\psi$  and expanded in series. The boundary layer equations then become a series of equations. The details of these calculations are outlined in the following sections.

### HOVER CASE

In the hover case, the forward flight speed  $s_H$  is zero. It will simplify the solution considerably if the constant part of the inflow is eliminated. For the hover case, therefore, the induced velocity is taken to be proportional to span, i.e.,

$$v_i = -\omega_i y$$

This case is considerably simplified by the elimination of time dependence and by having a constant aerodynamic angle of attack  $\alpha_a$ . The geometric angle of attack of the blade  $\alpha_b$  (the angle between the chord line and the plane of rotation) is constant by assumption. The aerodynamic angle of attack is then

$$\alpha_a = \alpha_b - v_i / \bar{U}_0 = \alpha_b - \omega_i \quad (20)$$

for small values of  $\omega_i$ . This makes the position of the stagnation point  $x_I$  constant. The value of  $x_I$  is found by determining the value of  $x$  at which  $u_x$  is equal to zero. This gives a value of  $\sigma$  ( $\sigma$  is the parametric variable for  $x$ ) at the stagnation point  $\sigma_I$ .

$$\begin{aligned}
\sigma_I &= (1 - L^2) / (1 + L^2) \\
L &= -(1 + \omega_i \tan \alpha_b) / (\tan \alpha_b - \omega_i) \quad (21)
\end{aligned}$$

If  $L < 0$ , the stagnation point is on the underside of the airfoil, and  $S_g$  is taken to be -1. In body coordinates, the stagnation point is

$$x_I = \int_{-1}^{\sigma_I} B \, d\sigma$$

with  $s_g = L/|L|$  .

It is convenient to define the following functions of  $x$ :

$$C^C = \frac{2 \cos (\alpha - \alpha_b)}{u_a} \quad H = (x - x_I) u_a$$

$$C^S = \omega_1 (x - x_I) \sin (\alpha - \alpha_b) \quad , \quad P = \frac{x - x_I}{u_a}$$

$$M = \frac{x - x_I}{u_s} \frac{du_a}{dx}$$

These functions appear in the boundary layer equations. Before solving the boundary layer equations, the functions of  $x$ ,  $y$ , and  $z$  must be converted to functions of  $\xi$ ,  $\zeta$ , and  $\eta$ . Since the transform between the two coordinate systems contains the function  $q$ , this function must be chosen before solving the equations. The first step in preparing the equations for solution is to decrease the number of independent variables from three to two. The variable  $\zeta$  will be eliminated from the equations by expanding the dependent variables in infinite series in  $1/\zeta$ . The coefficients in the series, in general, will be functions of  $\xi$  and  $\eta$ . First, the stream functions will be expressed as infinite series in  $\zeta$  as follows:

$$f' = \sum_{n=0}^{\infty} F'_{2n} \zeta^{-2n} \quad (22)$$

$$g' = \sum_{n=0}^{\infty} G'_{2n} \zeta^{-2n} \quad (23)$$

where the  $F_n$  and  $G_n$  are functions of  $\xi$  and  $\eta$ . The series is infinite, so the accuracy of the solution will depend on the number of terms retained

and the rapidity of convergence of the series. In order to expand the functions  $C^C$ ,  $C^S$ ,  $H$ ,  $P$ ,  $M$ ,  $\bar{u}_a$  and  $v_\delta$  as functions of  $\zeta$ , the form of the function  $q(y)$  must be chosen. The function  $q$  will stretch the chordwise dimension  $\xi$  so that separation will always occur at the same value of  $\xi$ , say  $\xi_s$ . The criterion for separation is taken to be

$$f''|_{\eta=0} = 0 = F''_0 + F''_2/\zeta^2 + \dots$$

For the separation to be independent of  $\zeta$ , it is required for all  $n$  that

$$F''_n|_{\eta=0} = 0 \quad \text{at} \quad \xi = \xi_s \quad (24)$$

This is not a requirement that separation occur at the same point along the chord or that it be independent of the span. The spanwise dependence of the separation will be removed from the equations and expressed explicitly in the function  $q$ . The relation between the physical chordwise dimension  $x$  and the transformed, stretched dimension  $\xi$  is

$$x = \xi/q(y) + x_I$$

Thus, since  $\xi_s$  is independent of  $\zeta$ , the position of the separation line in physical dimensions is

$$x_s = \xi_s/q(y) + x_I \quad (25)$$

By intuition, and by trial and error, an appropriate form for the arbitrary function  $q$  has been found to be

$$q = 1 - K_2 y^2 + \dots \quad (26)$$

The constant  $K_2$  will be determined after the equations have been solved. The functions  $C^C$ ,  $C^S$ ,  $M$ ,  $P$ ,  $H$ ,  $\bar{u}_a$ , and  $v_\delta$  are all functions of  $(\xi/q + x_I)$ , and they can be expanded in series in the following manner:

$$C^C = \sum_{n=0}^{\infty} c_{2n}^C \zeta^{-2n} \quad H = \sum_{n=0}^{\infty} h_{2n} \zeta^{-2n} \quad (27), (28)$$

$$c^s = \sum_{n=0}^{\infty} c_{2n}^s \zeta^{-2n}, \quad M = \sum_{n=0}^{\infty} m_{2n} \zeta^{-2n} \quad (29), (30)$$

$$v_\delta = \sum_{n=0}^{\infty} v_{2n} \zeta^{-2n}, \quad a_a = \sum_{n=0}^{\infty} u_{2n}^2 \zeta^{-2n} \quad (31), (32)$$

$$P = \sum_{n=0}^{\infty} P_{2n} \zeta^{-2n} \quad (33)$$

These functions depend on the values of  $\alpha$  and  $\phi_a$  obtained from the potential flow solutions. Once the constant parameters  $\omega_1$ ,  $\alpha_b$  and  $X_0$  are chosen, the techniques of a Taylor series can be applied; for example,

$$P(\xi q + x_I) = P(\xi + x_I) + K_2 \xi \frac{dP(\xi + x_I)}{d(\xi + x_I)} / \zeta^2 + \dots$$

Now that the  $\zeta$  dependence has been expressed by an infinite series, the governing equations become an infinite set of equations in  $\xi$  and  $\eta$ . Retaining only the first two terms of the velocity series yields

$$-F_0'''' - \frac{m_0 + 1}{2} F_0 F_0'' + m_0 F_0'^2 - m_0 + \xi (F_0' \frac{\partial F_0'}{\partial \xi} - F_0'' \frac{\partial F_0}{\partial \xi}) = 0 \quad (34)$$

$$-G_0'''' - (\frac{m_0 + 1}{2} F_0 + \xi \frac{\partial F_0}{\partial \xi}) G_0'' + \xi F_0' \frac{\partial G_0'}{\partial \xi} - h_0 (1 - c_0^c F_0') - c_0^s = 0 \quad (35)$$

$$-F_2'''' - (\frac{m_0 + 1}{2} F_0 + \xi \frac{\partial F_0}{\partial \xi}) F_2'' + (2m_0 F_0' + \xi \frac{\partial F_0'}{\partial \xi}) F_2' - \frac{m_0 + 1}{2} F_0'' F_2$$

$$+ \xi (F_0' \frac{\partial F_2'}{\partial \xi} - F_0'' \frac{\partial F_2}{\partial \xi}) + m_2 (F_0'^2 - 1 - \frac{1}{2} F_0'' F_0)$$

$$+ p_0 [G_0' F_0' - v_0 + \frac{1}{2} F_0'' G_0 - c_0^c (G_0' - v_0)] = 0 \quad (36)$$

$$\begin{aligned}
- G_2'''' - \left( \frac{m_0 + 1}{2} F_0 + \xi \frac{\partial F_0}{\partial \xi} \right) G_2'' + \xi F_0' \frac{\partial G_2'}{\partial \xi} \\
- \left[ \frac{m_0 + 1}{2} F_2 + \xi \frac{\partial F_2}{\partial \xi} + \frac{m_2}{2} F_0 - \frac{p_0}{2} G_0 \right] G_0'' + \xi F_2' \frac{\partial G_0'}{\partial \xi} - h_2 (1 - c_0^c F_0') \\
+ h_0 (c_2^c F_0' + c_0^c F_2') - c_2^s = 0 \quad (37)
\end{aligned}$$

The boundary conditions are

$$\text{at } \eta = 0 \quad F_j = G_j = F_j' = G_j' = 0$$

$$\text{at } \eta = \infty \quad F_0' = 1, \quad F_j' = 0 \quad \text{for } j > 1, \quad G_j' = v_j$$

At  $\xi = 0$ , the product  $u_a F_2'$  has a finite, non-zero value. Since  $\xi = 0$  is the stagnation point of the potential flow,  $u_a = 0$  there. However, rotational effects cause a small flow in the boundary layer there, and

$$f' = u/y u_a$$

A singularity in the stream function is caused by the method of non-dimensionalization chosen. To avoid this, and to remove  $K_2$  from the equations, let

$$F_2' u_0^s = K_2 F_{2k}' + F_{2c}' \quad (38)$$

The equations for the stream functions can now be solved if the potential flow solution is known. This solution is fixed by the type of the airfoil (in this case, a symmetrical Joukowski airfoil); the maximum thickness (fixed by the parameter  $\epsilon$ ; in this case  $\epsilon = .092$  for a maximum thickness of 11.9%); the position of the axis of rotation,  $X_0$ ; the geometric angle of attack,  $\alpha_b$ ; and the induced velocity, which is proportional to  $\omega_1$ . All these are nondimensional quantities; the lengths are nondimensional with respect to the chord and the velocities are nondimensional with respect to the product of chord and angular velocity. Once the equations for  $F_{2k}$  and  $F_{2c}$  are solved,  $K_2$  is found from Equation (32), which may now be written, for  $n = 2$ , as

$$K_2 F_{2k}'' + F_{2c}'' = 0 \quad \text{at} \quad \eta = 0 \quad \text{and} \quad \xi = \xi_s \quad (39)$$

and  $\xi_s$  is the value of  $\xi$  at which

$$F_0'' = 0 \quad \text{at} \quad \eta = 0$$

The velocities in the boundary layer are

$$u = y \bar{u}_a(x) [F_0' + (K_2 F_{2k}' + F_{2c}') / u_0^a \zeta^2 + \dots] \quad (40)$$

$$v = G_0' + G_2' / \zeta^2 + \dots \quad (41)$$

Once the equations for  $F_0$ ,  $G_0$ ,  $F_{2k}$  and  $F_{2c}$  have been solved, and the value of  $K_2$  is found, the chordwise and spanwise velocities are known.

#### FORWARD FLIGHT CASE

For this case, the analysis is simplified if the induced velocity is taken as constant ( $\omega_i = 0$ ), so that

$$v_i = -v_a$$

For a lifting helicopter,  $v_a$  is a positive constant. Both the aerodynamic angle of attack and the stagnation point vary with time and position along the span. The stagnation point is found by setting  $u_s$  equal to zero. This gives

$$\sigma_I = (1 - L^2) / (1 + L^2)$$

$$L = - [(y + T_2) \cos \alpha_b + v_a \sin \alpha_b] / [(y + T_2) \sin \alpha_b - v_a \cos \alpha_b]$$

$$s_g = L / |L|$$

In body coordinates, the value of  $x$  at the stagnation point is given by

$$x_I = \int_{-1}^{\sigma_I} B \, d\sigma \quad (42)$$

where  $B$  is the function of  $\sigma$  given by Equation (6). The dependence of  $x_I$  on  $y$  and  $t$  is complex, but  $x_I$  may be expanded in a series in  $y$ , i.e.,

$$x_I = x_0 + x_1/y + x_2/y^2$$

$$x_0 = \int_{-1}^{\sigma_1^*} B d\sigma$$

$$x_1 = v_a x_{10} = -v_a 4B_I^* \cos \alpha_b \sin \alpha_b$$

$$x_2 = v_a^2 x_{20} + v_a^T x_{22} \quad x_{22} = -x_{10}$$

$$x_{20} = (\partial B / \partial \sigma)_I^* 8 \cos^2 \alpha_b \sin^2 \alpha_b + B_I^* [2 \sin^2 \alpha_b (3 \cot^2 \alpha_b - 1) - 4 \cos^2 \alpha_b] \quad (43)$$

The superscript \* denotes "the asymptotic value for large values of  $y$ ." For example, as  $y$  becomes large,  $x$  becomes  $\xi^*$ , where  $\xi^* = \xi + x_0$ .  $x_0$ ,  $x_{10}$ ,  $x_{20}$  and  $x_{22}$  are constants and are independent of  $v_a$ ,  $\psi$ , and  $\zeta$ . The equations will be made independent of  $v_a$  by a careful choice of the coefficients of the double subscripted terms. A variable with a single number as a subscript will be independent of  $\zeta$ . A double subscript or the single subscript 0 (except for  $G_0$ ) indicates independence of  $\zeta$ ,  $\psi$ ,  $s_H$ , and  $v_a$ . This removal of induced velocity is possible because  $\omega_1$  is zero and the flow at the edge of the boundary layer has been simplified to

$$\phi_e = \phi_a = \phi_\sigma \cos \alpha_b + \phi_\rho \sin \alpha_b$$

$$\phi_c = \phi_\sigma \sin \alpha_b - \phi_\rho \cos \alpha_b$$

$$u_\delta = (y + T_2) \bar{u}_a + v_a \bar{u}_c$$

$$v_\delta = T_1 + \bar{v}$$

$$\bar{v} = \phi_a - 2(x_\sigma \cos \alpha_b + z_\sigma \sin \alpha_b)$$

The functions  $\bar{u}_a$ ,  $\bar{u}_c$ ,  $\bar{v}$ , and  $\cos(\alpha - \alpha_b)$  are functions of  $x$  only. However, in the transformed coordinate system  $(\xi, \zeta, \eta, \psi)$  they are functions of  $\zeta$  and  $\psi$  (as well as  $\xi$ ), because  $x_I$  and  $q$  depend on  $\zeta$  and  $\psi$ . As in the hover case,  $q$  is chosen so that separation, given by

$$f''(\xi_s, \zeta, \eta = 0, \psi) = 0,$$

occurs at a fixed value of  $\xi$ , say  $\xi_s$ . An appropriate form for  $q$  has been found to be

$$q = 1 - K_1/\zeta - K_2/\zeta^2 \dots \quad (44)$$

$$K_1 = v_a K_{10}$$

$$K_2 = K_{20r} + v_a^2 K_{20} + K_{21} T_1 + v_a K_{22} T_2$$

The double subscripted  $K$ 's are constants.

This choice of  $q$  allows functions of  $x$  to be expanded in a Taylor series in  $1/\zeta$ :

$$\bar{v} = \sum_{n=0}^{\infty} v_n \zeta^{-n} \quad \cos(\alpha - \alpha_b) = \sum_{n=0}^{\infty} c_n \zeta^{-n} \quad (45), (46)$$

$$\bar{u}_a = \sum_{n=0}^{\infty} u_n^a \zeta^{-n} \quad \bar{u}_c = \sum_{n=0}^{\infty} u_n^c \zeta^{-n} \quad (47), (48)$$

$$u_\delta = \zeta \sum_{n=0}^{\infty} u_n^\delta \zeta^{-n} = \zeta(u_0^a + \sum_{n=1}^{\infty} (u_n^a + v_a u_{n-1}^c + T_2 u_{n-1}^a) \zeta^{-n}) \quad (49)$$

As an example, the coefficients for  $\bar{v}$  are found to be

$$v_0 = \bar{v}^*$$

$$v_1 = (x_1 + \xi K_1) (\partial \psi / \partial x)^\circ$$

$$v_2 = (x_2 + \xi K_1^2 + \xi K_2) (\partial \psi / \partial x)^\circ + (x_1 + \xi K_1)^2 (\partial^2 \psi / \partial x^2)^\circ / 2$$

The  $v_n$  are still functions of  $\psi$  and  $v_a$ , so a further expansion is necessary:

$$v_1 = v_a v_{10} = v_a (x_{10} + \xi K_{10}) (\partial \psi / \partial x)^\circ$$

$$v_2 = v_a^2 v_{20} + v_{20r} + T_1 v_{21} + v_a T_2 v_{22}$$

$$v_{20} = (x_{20} + \xi K_{10}^2 + \xi K_{20}) (\partial \psi / \partial x)^\circ + (x_{10} + \xi K_{10})^2 (\partial^2 \psi / \partial x^2)^\circ / 2$$

$$v_{20r} = \xi K_{20r} (\partial \psi / \partial x)^\circ$$

$$v_{21} = \xi K_{21} (\partial \psi / \partial x)^\circ$$

$$v_{22} = (x_{22} + \xi K_{22}) (\partial \psi / \partial x)^\circ$$

The other functions of  $x$  are handled in a similar manner. In order to express the coefficients of the stream functions in the boundary layer equations, it is convenient to define the following:

$$p_0 = \frac{\xi}{\mu_0^\delta} \quad (50)$$

$$m_0 = \frac{\xi}{\mu_0^\delta} \frac{\partial \mu_0^\delta}{\partial \xi} \quad (51)$$

$$m_1 = \frac{\partial \mu_1^\delta}{\partial \xi} - \frac{\mu_1^\delta}{\mu_0^\delta} \frac{\partial \mu_0^\delta}{\partial \xi} = v_a m_{10} \quad (52)$$

$$m_{10} = K_{10} m_{1k} + m_{10c} \quad (53)$$

$$\begin{aligned}
m_2 &= \frac{\partial u_2^\delta}{\partial \xi} - \frac{u_2^\delta}{\nu_0} \frac{\partial u_0^\delta}{\partial \xi} \\
&= m_{2k} K_{20r} + v_a^2 (m_{2k} K_{20} + m_{20c}) + T_1 m_{2k} K_{21} \\
&\quad + v_a T_2 (m_{2k} K_{22} + m_{22c}) \quad (54)
\end{aligned}$$

At this point, all the expressions in the boundary layer equations have been written in series form except for the stream functions. They are now written as

$$f = f_0(\xi, \eta) + f_1(\xi, \eta, \psi)/\zeta + f_2(\xi, \eta, \psi)/\zeta^2 + \dots \quad (55)$$

$$g = g_0(\xi, \eta, \psi) + g_1(\xi, \eta, \psi)/\zeta + \dots \quad (56)$$

$$F_1 = v_a F_{10} = v_a (x_{10} F_{1k} + F_{10c}) \quad (57)$$

$$\begin{aligned}
F_2 &= [(F_{20r} + K_{20r} F_{2k}) + (F_{20c} + K_{20} F_{2k}) v_a^2 + T_1 (F_{21c} + K_{21} F_{2k}) \\
&\quad + v_a T_2 (F_{22c} + K_{22} F_{2k})] / \nu_0^\delta \quad (58)
\end{aligned}$$

$$G_0 = G_{00} + G_{01} T_1 \quad (59)$$

$$G_1 = v_a G_{10} + v_a T_1 G_{11} + T_2 G_{12} \quad (60)$$

Substituting all the infinite series into the boundary layer equations and neglecting the terms of higher order in  $1/\zeta^2$  in the  $F$  series and  $1/\zeta$  in the  $G$  series, the equations to be solved may be found. The equations for the first terms in the stream functions are

$$-F_0'''' - \frac{m_0 + 1}{2} F_0 F_0'' + m_0 F_0'^2 - m_0 + \xi (F_0' \frac{\partial F_0'}{\partial \xi} - F_0'' \frac{\partial F_0}{\partial \xi}) = 0 \quad (61)$$

$$D^G(G_{00}) = - \xi (F'_0 c_0 - \mu_0^a) \quad (62)$$

$$D^G(G_{01}) = 0 \quad (63)$$

$$\text{where } D^G(G_0) = - G_0''' - \left(\frac{m_0 + 1}{2} F_0 + \xi \frac{\partial F_0}{\partial \xi}\right) G_0'' + \xi F_0' \frac{\partial G_0'}{\partial \xi}$$

The equations for the second terms in the series for the stream functions are

$$D_1^F(F_{1k}) = - p_{0m_{1k}} (F_0'^2 - \frac{1}{2} F_0 F_0'' - 1) \quad (64)$$

$$D_1^F(F_{1c}) = - p_{0m_{10c}} (F_0'^2 - \frac{1}{2} F_0 F_0'' - 1) \quad (65)$$

$$\begin{aligned} \text{where } D_1^F(F_1) = & - F_1''' - \left(\frac{m_0 + 1}{2} F_0 + \xi \frac{\partial F_0}{\partial \xi}\right) F_1'' \\ & + (2m_0 F_0' + \xi \frac{\partial F_0'}{\partial \xi}) F_1' - \frac{m_0 + 1}{2} F_0'' F_1' \\ & + \xi (F_0' \frac{\partial F_1'}{\partial \xi} - F_0'' \frac{\partial F_1}{\partial \xi}) \end{aligned}$$

$$\begin{aligned} D^G(G_{10}) = & \left(\frac{m_0 + 1}{2} F_{10} + \xi \frac{\partial F_{10}}{\partial \xi} + \frac{1}{2} p_{0m_{10}} F_0\right) G_{00}'' \\ & - F_{10}' \xi \frac{\partial G_{00}'}{\partial \xi} - \xi (F_{10}' c_0 + F_0' c_{10} - \mu_{10}^a) \\ & - K_{10} \xi (F_0' c_0 - \mu_0^a) \end{aligned} \quad (66)$$

$$\begin{aligned} D^G(G_{11}) = & \left(\frac{m_0 + 1}{2} F_{10} + \xi \frac{\partial F_{10}}{\partial \xi} + \frac{1}{2} p_{0m_{10}} F_0\right) G_{01}'' \\ & - F_0' \xi \frac{\partial G_{01}'}{\partial \xi} \end{aligned} \quad (67)$$

$$D^G(G_{12}) = -p_0(1 - G'_{01})$$

The operator  $D^G(G_0)$  is given above. The third term in the series for the chordwise stream function is given by

$$D_2^F(F_{2k}) = -\xi m_{2k} (F_0'^2 - \frac{1}{2} F_0'' F_0 - 1) \quad (69)$$

$$D_2^F(F_{20r}) = -\xi (F_0' F_{00}' + \frac{1}{2} G_{00} F_0'' - v_0) - p_0 c_0 (v_0 - G'_{00}) \quad (70)$$

$$\begin{aligned} D_2^F(F_{20c}) = & -\frac{\delta}{\mu_0} \left( -\frac{m_0 + 1}{2} F_{10}'' F_{10} - m_0 F_{10}'^2 + \xi F_{10}' \frac{\partial F_{10}'}{\partial \xi} - \xi F_{10}'' \frac{\partial F_{10}}{\partial \xi} \right) \\ & - \xi m_{10} (2F_0' F_{10}' - \frac{1}{2} F_{10}'' F_0 - \frac{1}{2} F_0'' F_{10}) \\ & - \xi (m_{20c} - \frac{\mu_{10}^{\delta}}{\delta} m_{10}) (F_0'^2 - \frac{1}{2} F_0' F_0'' - 1) \end{aligned} \quad (71)$$

$$\begin{aligned} D_2^F(F_{21c}) = & -\xi (F_0' - 2 + \frac{n}{2} F_0'') - p_0 c_0 (1 - G'_{01}) \\ & - \xi (\frac{1}{2} F_0'' G_{01} + F_0' G'_{01}) \end{aligned} \quad (72)$$

$$D_2^F(F_{22c}) = -\xi (m_{22c} - \frac{\mu_{12}^{\delta}}{\delta} m_{10}) (F_0'^2 - \frac{1}{2} F_0' F_0'' - 1) \quad (73)$$

$$\begin{aligned} \text{where } D_2^F(F_2) = & -F_2'''' - \left( \frac{m_0 + 1}{2} F_0' + \xi \frac{\partial F_0'}{\partial \xi} \right) F_2'' \\ & + (m_0 F_0' + \xi \frac{\partial F_0'}{\partial \xi}) F_2' + \frac{m_0 - 1}{2} F_0'' F_2 \\ & + \xi (F_0' \frac{\partial F_2'}{\partial \xi} - F_0'' \frac{\partial F_2}{\partial \xi}) \end{aligned}$$

The variable  $\psi$  has been eliminated from the equation by applying the principle of superposition. The independent variables in the equations are  $\xi$  and  $\eta$ . The equations have the boundary conditions given by

$$\text{At } \eta = 0 \quad F_{nj} = F'_{nj} = G_{nj} = G'_{nj} = 0$$

$$\text{at } \eta = \infty \quad F'_0 = 1, \quad F'_{nj} = 0 \quad \text{for } n > 0$$

$$G'_0 = v_0, \quad G'_{01} = 1, \quad G'_{nj} = v_{nj} \quad \text{for } n > 0$$

These equations may be solved sequentially, once values have been fixed for the thickness of the airfoil ( $\varepsilon$ ), the type of airfoil (symmetrical Joukowski), the geometric angle of attack ( $\alpha_p$ ), and the location of the axis of rotation ( $X_0$ ). The separation point  $\xi_s$  is the point at which

$$F''_0(\xi_s, 0) = 0$$

The values of  $K_{nj}$  are found from the equations

$$F''_{nk}(\xi_s, 0) K_{nj} + F''_{njc}(\xi_s, 0) = 0 \quad (74)$$

The equation for the separation line is given by

$$\begin{aligned} x_s = X_0 + v_a X_{10}/y + (v_a^2 X_{20} + v_a X_{22} T_2)/y^2 \\ + \xi_s / [1 - v_a K_{10}/y - (K_{20r} + v_a^2 K_{20} + T_1 K_{21} + v_a T_2 K_{22})/y^2] \end{aligned} \quad (75)$$

The velocities in the boundary layer are given by

$$u = (y \tilde{u}_a(x) + v_a \tilde{u}_c(x) + T_2 \tilde{u}_a) f'(\xi, \zeta, \eta, \psi) \quad (76)$$

$$v = g'(\xi, \zeta, \eta, \psi) \quad (77)$$

The stream functions  $f$  and  $g$  are given by Equations (61) through (72).

### METHOD OF SOLUTION

For both the hover case and the forward flight case, a series of equations for the stream functions must be solved. The dependent variables in these partial differential equations are the transformed chordwise coordinate  $\xi$  and the transformed coordinate normal to the surface of the airfoil  $\eta$ . By finite difference techniques, numerical solutions can be found from straightforward calculations. Special techniques are required to start the solution at the stagnation point, and some numerical experimentation is required to find the optimum spacing in the  $\xi$  direction. The calculations for the potential flow and the conversion from the transformed coordinates to the real coordinates involve considerable algebraic manipulation, but the computer programs, especially the one for forward flight, are simple and straightforward in logic. Both are shown in APPENDIX II.

The potential flow solution requires the solution of the differential equation

$$dx / d\sigma = B(\sigma) \quad (78)$$

where  $x = 0$  at  $\sigma = -1$ . This equation must be solved to find the value of  $x$  at the stagnation point, and then for each value of  $x$ , the corresponding value of  $\sigma$  must be found. This was accomplished by the RK1 Runge-Kutta scheme in the IBM System 360 Scientific Subroutine Package. Since  $B$  becomes infinite at  $\sigma = -1$ , the value of  $x$  must be known at a value of  $\sigma$  greater than  $-1$ . For  $(\sigma + 1) \ll 1$ ,  $B(\sigma)$  can be simplified and integrated analytically. The relation between  $x$  and  $\sigma$  near  $\sigma = -1$  is

$$x = \frac{\epsilon \sqrt{2(1 + \sigma)}}{1 + \epsilon}$$

Since the integration of the differential equation cannot begin at  $\sigma = -1$ , it begins at  $(\sigma + 1) = 10^{-20}$ .

A similar situation is encountered at the stagnation point, where several functions become indeterminate. The function  $m_0$ ,

$$m_0 = \frac{\xi}{\mu_0^a} \frac{d\mu_0^a}{d\xi}$$

is indeterminate in that both the numerator and denominator are zero at  $\xi = 0$ . It is well known that the velocity  $\mu_0^a$  is proportional to  $\xi$  at a

stagnation point, and that  $m_0$  should equal 1. To avoid exponent overflow (the computer, an IBM 360/75, is limited to exponents of about 75), the solution was started at values of  $\xi$  of  $10^{-5}$  to  $10^{-6}$ . For  $\xi = 1.2 \times 10^{-6}$ , the value  $m_0 = 1.00004$  was found, and  $F_0'' = 1.23259$ . This is close enough to the stagnation point value of  $F_0''$  of 1.2326 to ensure the desired accuracy. Varying the starting value of  $\xi$  by a factor of two produced insignificant changes in the solution.

A similar situation was encountered for  $\alpha_b = 0$ . Although negative angles of attack cause no problem, exponent overflow occurs for  $\alpha_b$  much less than  $0.005^\circ$ . A solution at  $0.005^\circ$  was compared to a solution specifically designed for only zero angle of attack, and the results of this comparison indicate that  $0.005^\circ$  is a satisfactory approximation to zero degrees. The results for the hover case for zero angle of attack are found from Reference 11, and for forward flight, the  $0.005^\circ$  approximation was used.

Since all the functions of  $x$ , such as  $\bar{u}_a$  and  $\cos(\alpha - \alpha_b)$ , are actually known as analytic functions of the parametric variation  $\sigma$ , the determination of  $\sigma$  for the desired value of  $x$  allowed the functions of  $x$ , and their derivatives, to be found. This allows the functions of  $\xi$ , such as  $\mu_2^a$  and  $c_0^c$ , to be found, and all the coefficients of the stream functions in the equations to be solved are determined. Each equation is a partial differential equation in  $\xi$  and  $\eta$ . If they are solved in the correct order, each equation will contain only one undetermined stream function. This allows the equations to be solved sequentially, and not simultaneously. The starting of the solution in the  $\xi$  direction is simplified by the form into which the  $\xi$  derivatives have been cast. Each derivative with respect to  $\xi$  is multiplied by  $\xi$ . Thus, at  $\xi = 0$  the equations become ordinary differential equations in  $\eta$ . Once a solution is found at the stagnation point ( $\xi = 0$ ), the solution proceeds in the  $\xi$  direction by the method of A. M. O. Smith.<sup>13</sup>

This method reduces each partial differential equation to an ordinary differential equation in  $\eta$  at each successive value of  $\xi$ , by expressing the  $\xi$  derivatives in terms of the values at the preceding stations by Lagrangian finite differences. When the solution is known at the stagnation point ( $\xi = \xi_1$ ),  $\xi$  is incremented and a solution at  $\xi = \xi_2$  is sought. The  $\xi$  derivatives are approximated by a two-point Lagrangian formula using values at  $\xi_1$  and  $\xi_2$ . The equations are solved in  $\eta$  at  $\xi_2$ . At the next point,  $\xi_3$ , a three-point formula is used, and at each succeeding point a four-point formula is employed. The spacing of the points required some numerical experimentation. The two-point formula is considerably less accurate than the three point, so  $\Delta_2(\Delta_h = \xi_h - \xi_{h-1})$  is small, typically  $.5 \times 10^{-3}$ . A. M. O. Smith<sup>13</sup> has given several guidelines for the selection of  $\xi$  spacing. If the value of  $\xi_h / \Delta_h$  is greater than 25, the equation for  $F_0$  may become unstable. This instability is manifested in an inability to find a solution for the ordinary differential equation in  $\eta$ . If the value of  $\Delta_h$  is too large, the  $\xi$  derivatives will be inaccurate. The accuracy can be checked in two ways. After finding a solution at point  $\xi_{h+1}$ , a central difference

differentiation using  $\xi_{h-1}$ ,  $\xi_h$ , and  $\xi_{h+1}$  can be compared to the backward formula (using  $\xi_h$ ,  $\xi_{h-1}$ ,  $\xi_{h-2}$ , ...) used in the solution. This method is not as complete a check as rerunning the entire solution with smaller values of  $\Delta_h$ . Since the accuracy of the  $\xi$  derivatives depends strongly on  $\Delta_h$ , this will reveal any inaccuracies that affect the results. From 30 to 60 stations in  $\xi$  were required for the case run, with values of  $\xi_h / \Delta_h$  ranging from 8 to 25 away from the stagnation point. Near the stagnation point,  $\Delta_h$  was varied from about 0.002 (at large angles of attack) to 0.005 (at small angles of attack). Near stagnation,  $\Delta_h$  ranged from 0.01 to 0.02.

The equation for  $F_0$  is the equation for a two-dimensional boundary layer. At the point of separation it has a singularity, and the solution cannot be carried to the separation point. Smith and Clutter<sup>13</sup> have shown that it may be accurately found by extrapolation. To approach the separation point closely enough to successfully extrapolate often required two or three attempts to find a suitable spacing. The instability in the ordinary differential equation near separation is similar to that caused by large values of  $\xi_h / \Delta_h$ . In both cases, the scheme used to integrate the equation in  $\eta$  failed to converge. The conditions under which the  $\eta$  equation diverged depended partly on the integration scheme used.

For the hover case, a Runge-Kutta scheme, attributed to Kutta<sup>14</sup>, was used to solve the ordinary differential equation in  $\eta$ . One of the boundary conditions on  $F'_n$  (or  $G'_n$ ) is given at  $\eta = \infty$ . This requires that guesses be made of a maximum value of  $\eta$  large enough to cause little error, and of the value of  $F''_n$  at the wall that will match the boundary condition on  $F'_n$  at the maximum value of  $\eta$ . Little difficulty was experienced with the linear equations, but for the nonlinear equation for  $F_0$ , and error of  $10^{-12}$  in  $F''_0$  often caused appreciable error in  $F'_0$  at the maximum value of  $\eta$ . This problem was greatly accentuated if the maximum value of  $\eta$  was so large that  $F''_0$  became much less than  $10^{-3}$ . Near separation, or if  $\xi_h / \Delta_h$  became too large, the equation would not converge. Since the computer retained only 15 significant digits, this could limit the accuracy of  $F''_0$  to less than that needed to match the boundary condition in  $F'_0$  at the maximum  $\eta$ . This divergence of the Runge-Kutta integration made it difficult to approach the separation point closely, and required constant adjustment of the maximum value of  $\eta$ .

For the forward flight case, Lew's method of accelerated successive replacement<sup>15</sup> was used to solve the equation in  $\eta$ . This method has some of the features of quasi-linearization. For each equation, an estimate of  $F'_n$  (or  $G'_n$ ) at discrete values of  $\eta$  ( $\eta_1, \eta_2 \dots \eta_{l \max}$ ) is required. The values of  $F_n$ ,  $F''_n$ , and  $F'''_n$  at  $\eta_l$  are found by finite differences from the current value of  $F'_n$  at  $\eta_{l-1}$ ,  $\eta_l$  and  $\eta_{l+1}$ . The error in the equation is used to find a new value of  $F'_n$ , and the old value is immediately replaced. This procedure continues until the change in  $F'_n$  is less than  $10^{-6}$  or  $10^{-7}$ . Both the accuracy and speed of this method depend strongly on the spacing between the values of  $\eta$ . A spacing of .1 gives sufficient accuracy (3 to 4 places). The computer time required is greater than that of the

Runge-Kutta scheme, except near separation, where they are comparable. On an IBM Model 360/75, even the Runge-Kutta scheme required 5 to 8 minutes of computer time. For  $\eta$  greater than 4 or 5, the spacing in  $\eta$  was gradually increased, but the decrease in computer time is probably not worth the effort involved. Using Lew's method, the maximum value of  $\eta$  and the values of  $\Delta_h$  are less critical, and separation can be approached more closely and easily than with the Runge-Kutta scheme. The method of Lew proved much simpler to program, but comparison with the results of the Runge-Kutta scheme were necessary to establish confidence in its accuracy.

Since neither the Runge-Kutta solution or the solution by Lew's method is clearly superior, the advantages of each are listed. First, for Lew's method:

1. Separation may be approached more easily.
2. Larger values of  $\eta$ , and smaller values of  $\Delta_h$ , may be used.
3. The computer program is simpler.

The advantages of the Runge-Kutta scheme are:

1. Less computer time is required.
2. Greater accuracy is obtainable.

Other programmers have successfully used predictor-corrector methods and the method of quasi-linearization, in conjunction with A. M. O. Smith's method.

About one-half the storage required by the program (100 to 200 thousand bytes) is occupied by a matrix with four subscripts,  $Y_{m,j,1,i}$ , that contains all the stream functions and their derivatives. The subscript  $j$  denotes the variable. In the hover case, for example,  $j = 1$  denotes  $F_0$ , 2 denotes  $G_0$ , 3 denotes  $F_{2k}$ , and so on. The number of primes is given by  $m - 1$ . Thus,  $m = 1$  denotes  $F_0$ , 2 denotes  $F_0'$ , and 3 denotes  $F_0''$ . The value of  $\eta$  is  $\eta_l$ , and the value of  $\xi$  is  $\xi_{h-i+1}$  where  $\xi_h$  is the current value of  $\xi$ . When a solution at  $\xi_h$  is sought, values at  $\xi_{h-1}$ ,  $\xi_{h-2}$ , and  $\xi_{h-3}$  are needed to find the derivatives with respect to  $\xi$ . Thus  $Y_{2,2,6,3}$  denotes  $G_0'$  evaluated at  $\eta_6$  and  $\xi_{h-2}$ . Much computer time was saved by evaluating the  $\xi$  derivatives only once at each  $\xi$  station and storing the result in a matrix  $A_{m,j,1}$ . When the value of  $\xi$  is incremented, the  $Y$  and  $A$  matrices must be revised.

Whatever method is used to solve the equations in  $\xi$  and  $\eta$ , the solution will be limited in accuracy by the truncation of the asymptotic series in span. Like any asymptotic series, the convergence of the series cannot be determined mathematically. It does appear that for values of span greater than 12 or 15 times the chord of the blade an adequate approximation is provided by the first term of the series,  $F_0$ . The spanwise series is taken to terms of the order  $1 / \zeta^2$ , which has a coefficient  $F_2$ . In the hover case,  $F_1$  is zero and  $F_2'$  is smaller than  $F_0'$ . These terms may approximate the solution for values of span as small as 1 or 2. In the

forward flight case, the magnitude of  $F_1'$  and  $F_2'$  depends on the magnitude of the inflow  $v_\infty$  and the forward flight speed  $s_H$ . Helicopters may have speeds to give values of  $s_H$  of 3 to 5, and the inflow may be in the range 0.05 to 0.2. The accuracy of the series is in doubt for values of span less than 5 to 10 and near the region of reverse flow where  $s_H \sin \psi + y < 1$ .

The spanwise series has been taken to terms of order  $1 / \zeta$ . Over most of the airfoil, the spanwise velocity is of the order of  $1 / \zeta$  times the chordwise velocity. The primary interest in this investigation lies in the chordwise, not the spanwise, flow. Determination of the spanwise flow is primarily for evaluating its effect on the chordwise flow. Since terms of order  $1 / \zeta^2$  in the spanwise flow affect the chordwise flow only to order  $1 / \zeta^3$ , the truncation of the spanwise series at  $1 / \zeta$  is consistent with truncation of the chordwise series at  $1 / \zeta^2$ .

RESULTS

HOVER CASE

The computer program in APPENDIX II was run for four conditions, and a fifth condition, for zero angle of attack, was taken from a special computer program.<sup>11</sup> All conditions were for a symmetrical Joukowski airfoil with a thickness of 11.9% and for the axis of rotation at the quarter chord ( $x_0 = 0.25$ ). For each condition, the geometric angle of attack  $\alpha_b$  and the downflow constant  $\omega_i$  must be chosen. These have not been selected arbitrarily, but have been chosen to be representative of conditions found in two specific helicopters. Values of thrust of 6000 lbf, 8000 lbf, and 10,000 lbf were given for a helicopter with a 24-foot radius rotor. For a 40-foot radius rotor, values of geometric angle of attack of zero and 6 degrees were given. The relationship of the inflow constant, geometric angle of attack, and thrust is described in APPENDIX I.

It has been found that the solution depends strongly on the aerodynamic angle of attack, which was found to be

$$\alpha_a = \alpha_b - \omega_i$$

For this reason, the conditions are identified by the aerodynamic angle of attack, and the corresponding values of  $\alpha_b$  and  $\omega_i$  are understood to be those in Table I. Table I also shows the position of the stagnation point  $x_I$ , the value of  $\xi$  at separation  $\xi_s$ , and the constant in the expression for  $q$ , i.e.,  $K_2$ .

| TABLE I. SUMMARY OF CALCULATED RESULTS FOR THE HOVER CASE |                     |            |            |           |         |         |       |          |
|-----------------------------------------------------------|---------------------|------------|------------|-----------|---------|---------|-------|----------|
| $\alpha_a$<br>(deg)                                       | $\alpha_b$<br>(deg) | $\omega_i$ | T<br>(lbf) | R<br>(ft) | $x_I$   | $\xi_s$ | $K_2$ | $x_{NS}$ |
| 0                                                         | 0                   | 0          | -          | 40        | 0       | .440    | .784  | 30.0     |
| 2.38                                                      | 6.00                | .0631      | -          | 40        | -.00721 | .300    | .450  | 24.2     |
| 3.12                                                      | 6.01                | .0504      | 6000       | 24        | -.0096  | .256    | .365  | 20.1     |
| 4.16                                                      | 7.50                | .0582      | 8000       | 24        | -.01323 | .188    | .226  | 13.3     |
| 5.21                                                      | 8.94                | .0651      | 10000      | 24        | -.01718 | .123    | .081  | 6.48     |

The percent of chord called "near separation" is used only in constructing some of the figures. Its position depends mainly on how close to separation the solution was taken. The third digit in the values for  $\xi_s$  and  $K_2$  is probably not accurate. No attempt was made to calculate the derivatives of the stream functions to more than three significant figures in order to conserve computer time. The instability of the Runge-Kutta solution near the separation point makes these values at separation less reliable than those obtained by Lew's method.

In examining the results for the hover case, it should be remembered that the  $\xi$  dimension is not a physical dimension. The  $x$  dimension is the distance along the surface of the airfoil, measured from the leading edge. The  $\xi^*$  dimension is the distance along the surface of the airfoil, measured from the stagnation point. For a given angle of attack,  $x$  and  $\xi^*$  differ only by a constant. However, for one value of  $\xi$  the distance along the surface of the airfoil will vary with span. Figure 4 shows that, for  $\alpha_a = 3.12^\circ$ , a value of  $\xi$  of 0.15 corresponds to values of  $x$  ranging from 0.2266 at  $y = 1$  to 0.1426 at  $y = 5$ . In every case, as  $y$  becomes very large, the value of  $\xi$  approaches  $\xi^*$ .

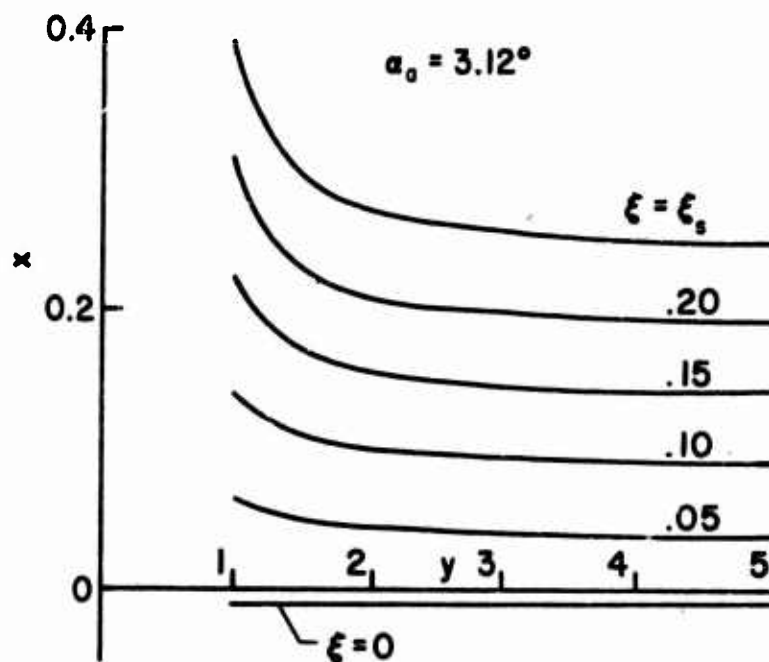


Figure 4. The  $\xi$  Coordinate.

Some of the results are more readily seen by examining the stream functions, and other results are made more apparent by considering the velocities. Both approaches will be used. A detailed examination of the

stream functions, similar in approach to that of Young and Williams,<sup>11</sup> will evaluate the various effects that occur in the boundary layer for angles of attack greater than zero. Then, for conditions applicable to the two helicopters described in APPENDIX I, the results will be discussed in terms of such physically recognizable variables as velocity profiles, skew angles, and boundary layer thicknesses.

The effect of the angle of attack on the chordwise flow can be seen in Figures 5 and 6. The function  $F_0''$  is just the result found on a two-dimensional airfoil. The function  $F_2''$  reflects the rotational effects in the chordwise flow. These rotational effects cause a spanwise flow, which in turn gives rise to the function  $F_2$  in the chordwise flow. The increase in  $F_2''$  as the angle of attack decreases is reflected in the values of  $K_2$  in Table I. At large angles of attack, the effect of rotation on the chordwise flow is much less than at small angles of attack. Experimental and analytical results with an NACA 0012 airfoil led Dwyer and McCroskey<sup>6</sup> to the same conclusion. The reason for this dependence on angle of attack can be seen by examining the spanwise flow. Positive values of spanwise velocity indicate flow away from the axis of rotation. For large values of  $\eta$ ,  $G_0'$  is influenced primarily by the spanwise velocity in the potential flow  $v_\delta$ . Figure 7 shows that  $v_\delta$  is directed toward the axis of rotation, and, over most of the airfoil, its magnitude increases along the chord. At smaller values of  $\eta$ , the spanwise velocity is influenced by rotational effects which tend to cause flow away from the axis of rotation. This flow increases along the chord and is greatest near separation. As the blade slows the fluid flow through shear stress in the boundary layer, the fluid experiences Coriolis and centrifugal accelerations. These accelerations must act on the flow for some length of time, or equivalently, for some distance along the chord, before the velocity is affected. A convenient measure of the rotational effects is  $G_0''$ , shown in Figure 8. This shows a steady buildup of the cumulative effect of rotational accelerations along the chord, even though  $v_\delta$ , which opposes this increase in  $G_0''$ , also increases in magnitude along the chord over most of the airfoil.

The spanwise flow affects the chordwise flow through the Coriolis acceleration of the fluid. This acceleration must act on the chordwise flow for some distance, or more precisely, for some time, before it affects the velocity. Figure 8 shows that the rate of increase of  $G_0''$  at the wall increases with angle of attack, but the magnitude of  $G_0''$  at the wall at separation depends more on the length of the boundary layer. The spanwise flow also experiences a Coriolis acceleration. When the spanwise flow is outward, this will delay separation. The spanwise inviscid flow is usually more important than the Coriolis acceleration. The terms in the chordwise boundary layer equation that involve the spanwise potential flow are written as  $-2v_\delta \cos(\alpha - \alpha_b) + v_\delta \bar{u}_a(x)$ . Figure 7 shows that the magnitude of  $v_\delta$  depends upon the distance along the surface of the airfoil, and over the rear portion of the airfoil,  $v_\delta$  increases along the chord. Thus, the greater the length of the boundary layer, the greater the magnitude of  $v_\delta$ , resulting in a larger effect due to the Coriolis force terms.

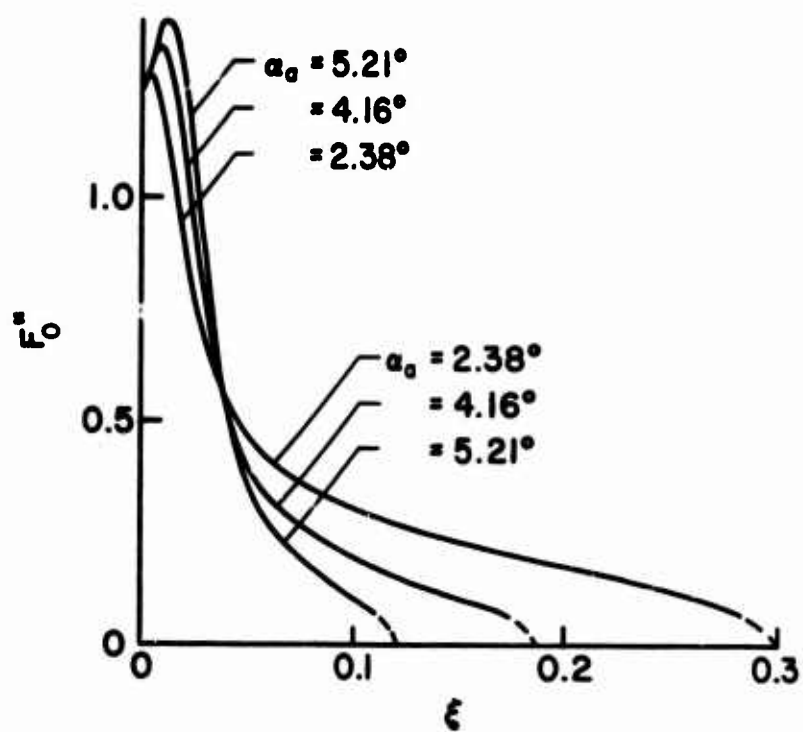


Figure 5. The Function  $F_0''$  on the Surface of the Blade.

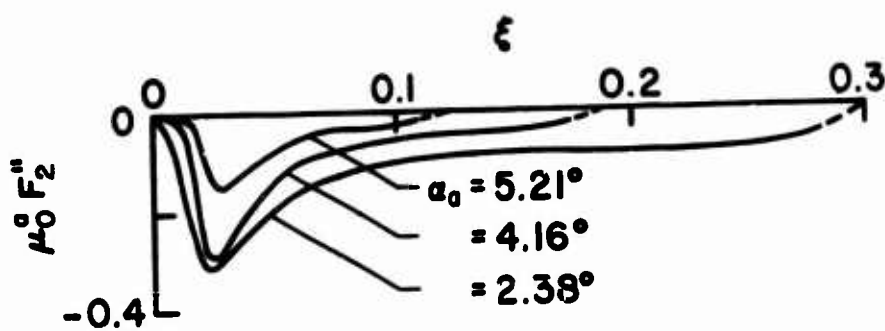


Figure 6. The Function  $\mu_0^a F_2''$  on the Surface of the Blade.

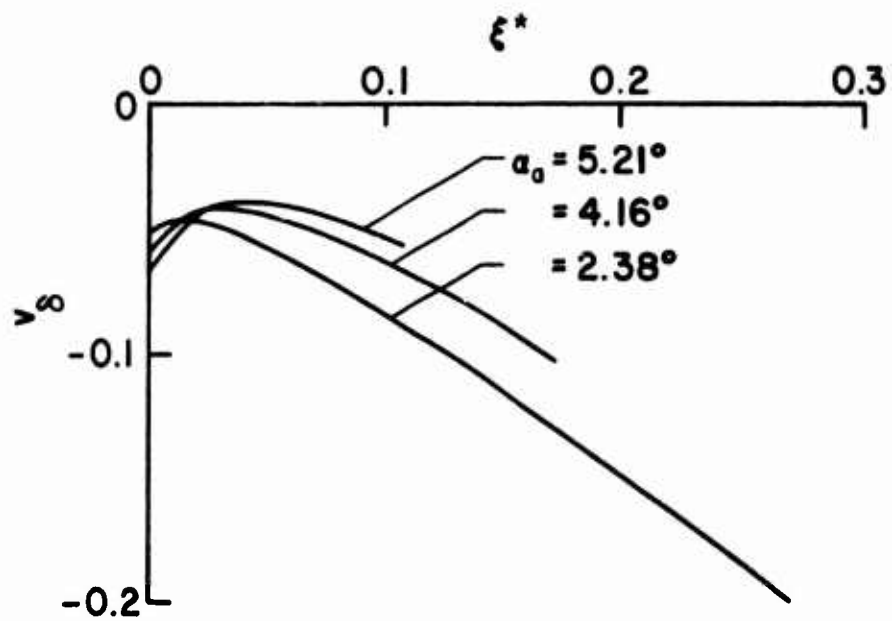


Figure 7. The Spanwise Velocity in the Potential Flow.

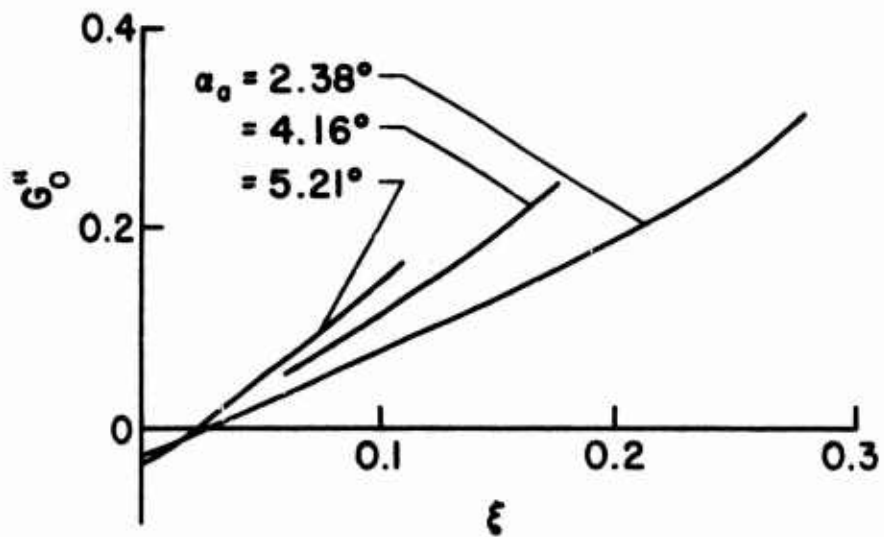


Figure 8. The Function  $G_0''$  on the Surface of the Blade.

The length of the boundary layer, or the time the fluid spends in the boundary layer, not only allows the spanwise flow to grow but increases the chordwise velocity caused by accelerations in the spanwise flow. Since only one body shape has been considered, a real distinction between time and length of the boundary layer has not been found. However, this point of view seems to provide more understanding than an emphasis on chordwise pressure gradient.

A more familiar view of the boundary layer is provided by examination of velocity profiles calculated for specific applications. Figures 9 through 14 show the chordwise velocity profiles for two helicopters. One has a 24-foot radius rotor; the other has a 40-foot radius rotor; both are described in APPENDIX I. Figures 11 through 14 show that little spanwise variation occurs for values of span greater than 30%. The spanwise variation is not a function of the length of the blade, but the percent span can easily be converted back to the number of chord lengths from the axis of rotation. For the 24-foot rotor, 30% span is about 4 chord lengths from the axis of rotation (actually,  $y = 4.113$ ). For the 40-foot rotor, 30% span is 6 chord lengths. Since such quantities as percent span, thrust in pounds force, percent chord, and forward flight speed in knots are usual and easily recognizable variables, they will be used in presenting the velocity profiles, boundary layer thicknesses, and skew angles. The notation "Near Separation" (or "NS") denotes the values of  $X_{NS}$  in Table I. Figures 10, 11, and 14 show that chordwise velocity profiles near separation have a shape like Falkner-Skan profiles near separation. Figure 13 shows that the profile is fuller at smaller values of chord. A small spanwise variation is seen at 24.4% chord, whereas none can be seen (in fact, none was seen on the original, computer-drawn graph) at 10% chord. This is related to the increase in rotational effects along the chord.

The spanwise velocity profiles, shown in Figures 15 through 20, show the characteristic S shape. The positive values of velocity (flow away from the axis of rotation) are caused by centrifugal forces in the boundary layer. The Coriolis force acting on the chordwise flow causes negative spanwise flow. The sum of the two effects is small and positive near the wall. In comparing spanwise and chordwise profiles, the magnitude of  $u_\delta$  must be considered. The factor  $\bar{u}_a$  in  $u_\delta$  is the velocity at the edge of the boundary layer on the airfoil in a two-dimensional flow where the impinging stream has unit velocity. Away from the stagnation point,  $\bar{u}_a$  is of the order of unity. The factor  $\Omega Y$  thus determines the order of magnitude of the chordwise potential flow. For comparison, the nondimensionalizing factor for the spanwise velocity is  $\Omega R$ , the value of  $\Omega Y$  at the blade tip. This arrangement is dictated by the physical situation: the chordwise velocity is nearly proportional to span and the spanwise velocity is nearly independent of it. In both spanwise and chordwise velocity profiles, there is dependence on span through the variable  $\eta$ . A chordwise variation is expressed implicitly in  $\eta$ , and in addition, both  $v_\delta$  and  $v$  near the wall increase along the chord. This is seen in Figures 18 and 19.

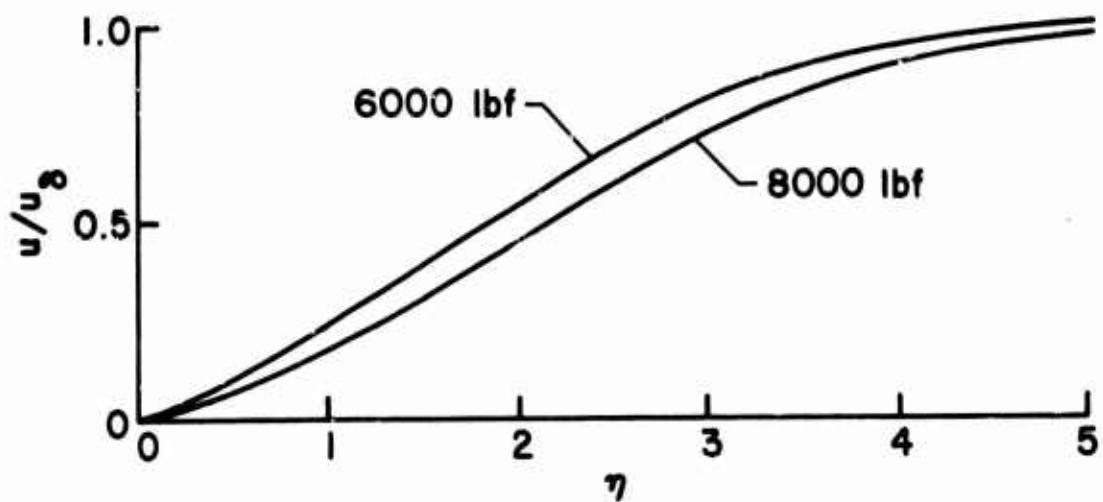


Figure 9. The Chordwise Velocity Profile for the 24-Foot Rotor (at 10% Chord and 60% Span) in Hover.

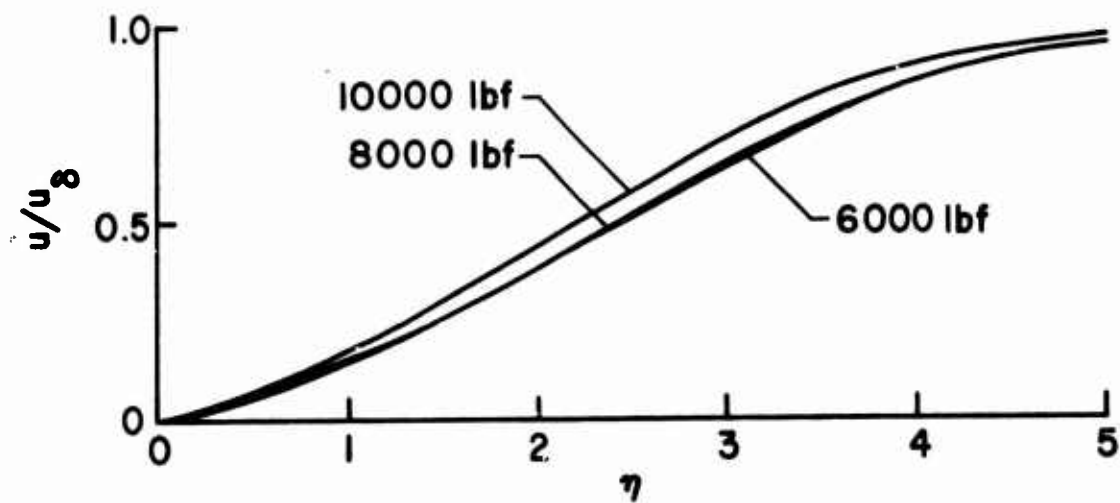


Figure 10. The Chordwise Velocity Profile for the 24-Foot Rotor (Near Separation and 60% Span) in Hover.

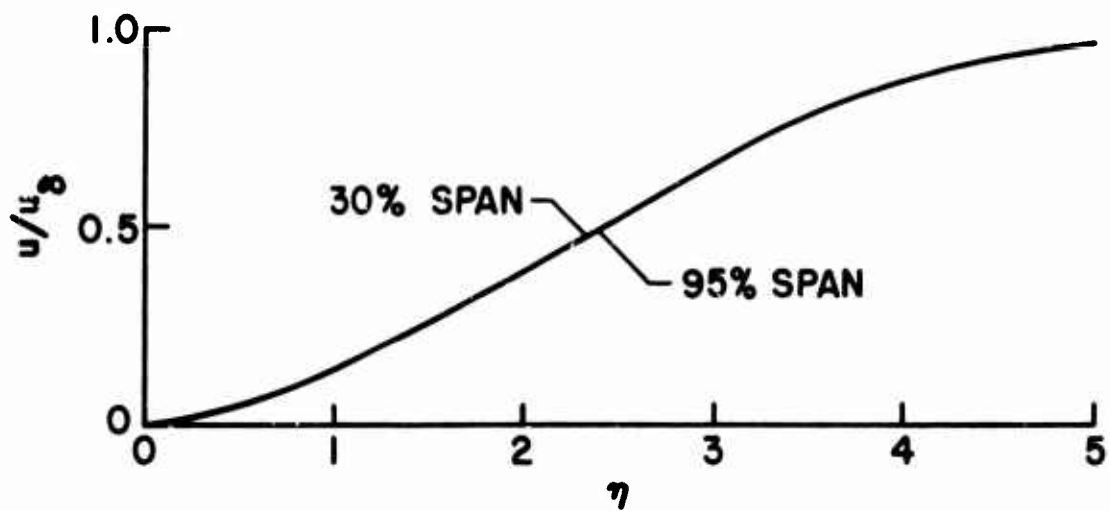


Figure 11. The Chordwise Velocity Profile for the 24-Foot Rotor (Near Separation and With 8000 lbf Thrust) in Hover.

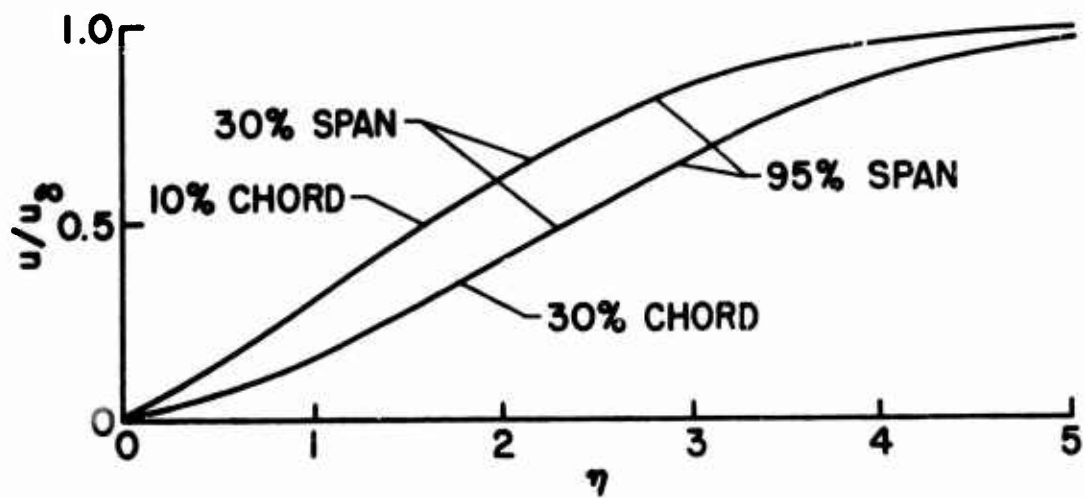


Figure 12. The Chordwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack) in Hover.

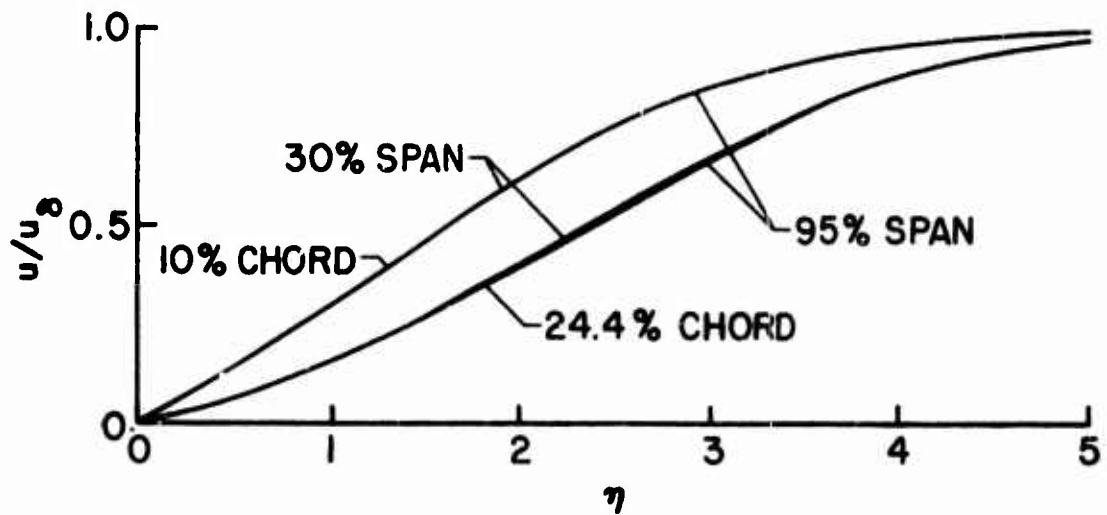


Figure 13. The Chordwise Velocity Profile for the 40-Foot Rotor (at 6 Degrees Blade Angle of Attack) in Hover.

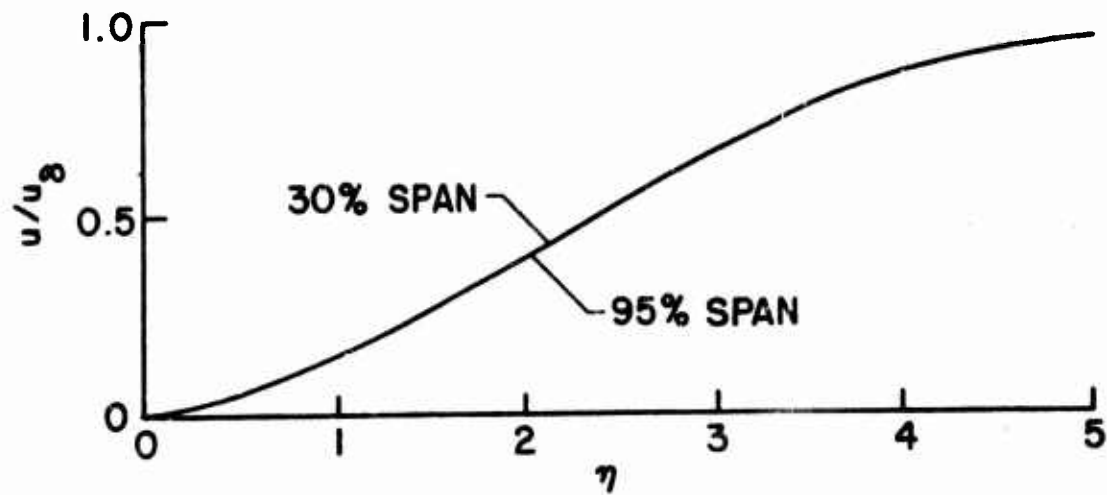


Figure 14. The Chordwise Velocity Profile for the 40-Foot Rotor (at 6 Degrees Blade Angle of Attack and 24.4% Chord) in Hover.

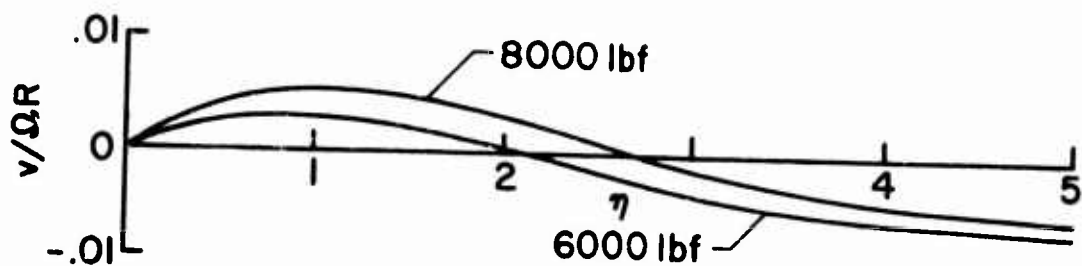


Figure 15. The Spanwise Velocity Profile for the 24-Foot Rotor (at 10% Chord and 60% Span) in Hover.

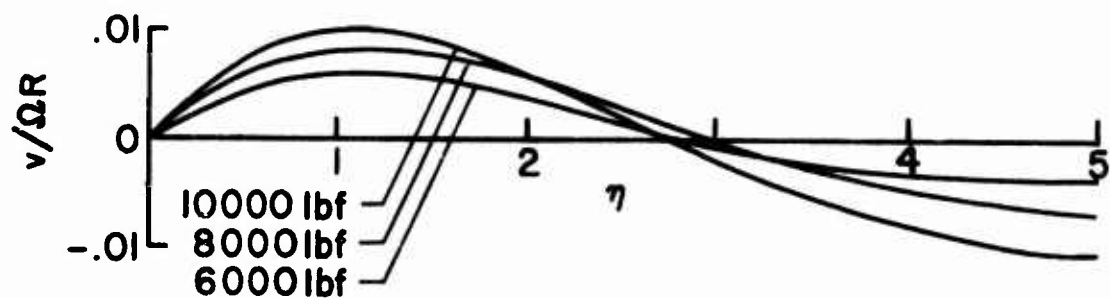


Figure 16. The Spanwise Velocity Profile for the 24-Foot Rotor (at 60% Span and Near Separation) in Hover.

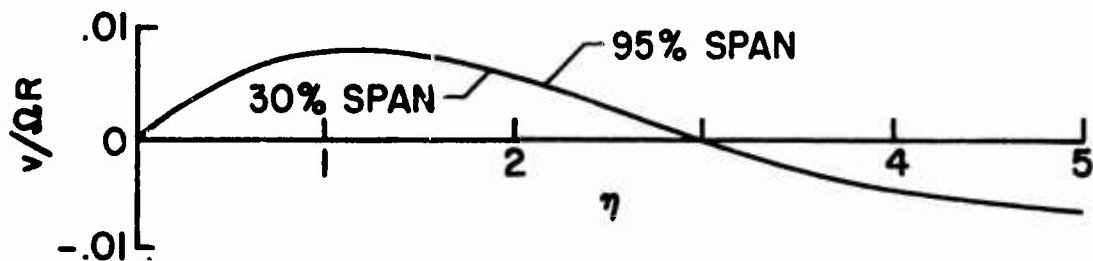


Figure 17. The Spanwise Velocity Profile for the 24-Foot Rotor (Near Separation and With 8000 lbf Thrust) in Hover.

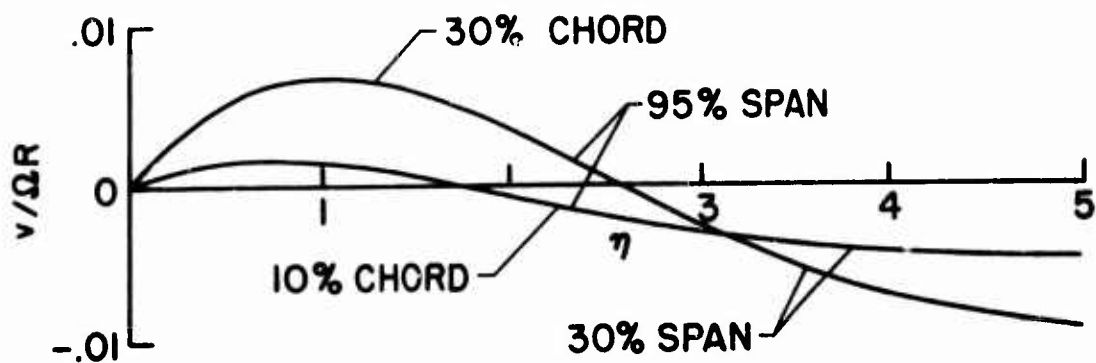


Figure 18. The Spanwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack) in Hover.

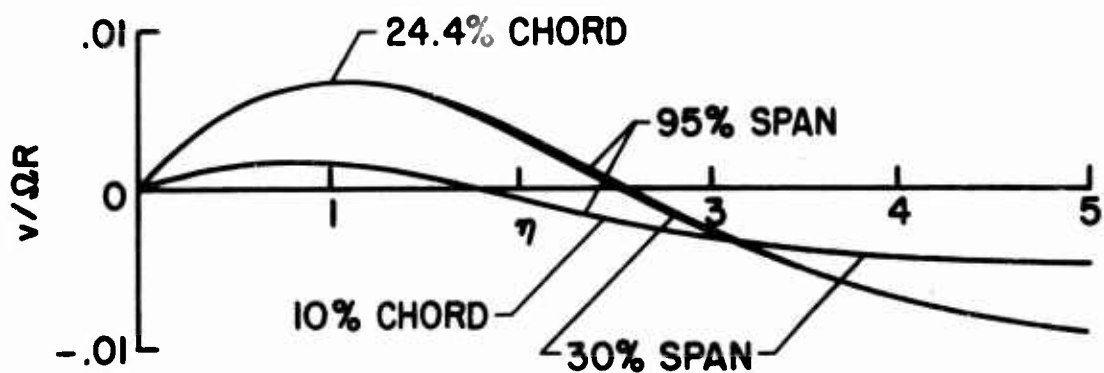


Figure 19. The Spanwise Velocity Profile for the 40-Foot Rotor (at 6 Degrees Blade Angle of Attack) in Hover.

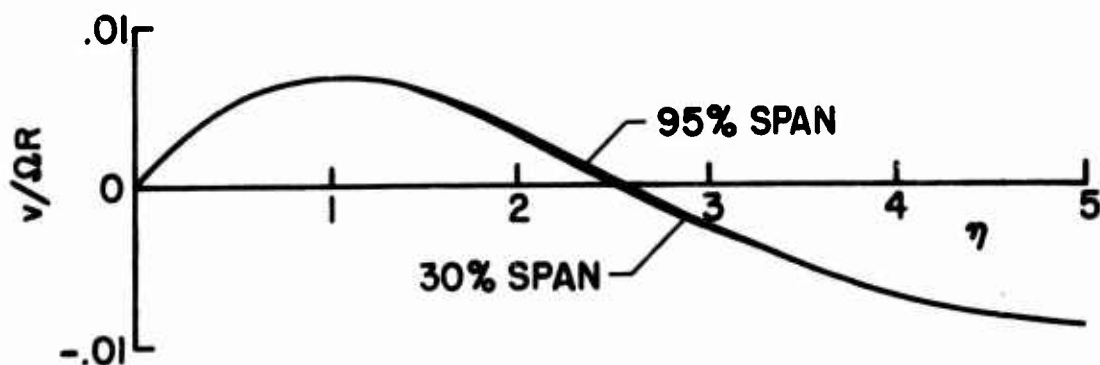


Figure 20. The Spanwise Velocity Profile for the 40-Foot Rotor (at 24.4% Chord and 6 Degrees Blade Angle of Attack) in Hover.

Figure 15 shows that an increase in thrust (a higher angle of attack) gives larger wall shear stress but smaller potential flow. Plotting velocity profiles as a function of  $\eta$  conceals some dependence on chord and span. The boundary layer displacement thicknesses  $\delta_x$  (in the chordwise direction) and  $\delta_y$  (in the spanwise direction) are nondimensionalized by a constant  $\sqrt{\Omega/\nu}$ . The equation for  $\delta_y$ , for example, is

$$\delta_y = \sqrt{(x - x_I) / u_\delta} \left[ \eta_{\max} - \int_0^{\eta_{\max}} (v / v_\delta) d\eta \right] \quad (78)$$

The term in brackets is very nearly independent of span,<sup>16</sup> but, through  $u_\delta$ , the displacement thickness has the variation shown in Figures 21 through 28. The graphs begin at the leading edge of the airfoil ( $x = 0$ ) in order to avoid the stagnation point. A true stagnation point occurs only in the chordwise potential flow. At the point where  $u_\delta = 0$ , rotational effects cause flow in the boundary layer. This was discussed in the arguments leading to Equation (38). Away from the stagnation point, the displacement thicknesses increase smoothly with chord. In some cases an upturn near separation can be detected. This is because the boundary layer thickness at separation becomes very large (in fact, the flow is not adequately described by a boundary layer there). The spanwise displacement thickness is larger than the chordwise, but the shape of the curves is similar. The spanwise graphs show a larger increase of thickness along the chord. In both cases the rate of increase along the chord increases with thrust (or angle of attack), but because of the greater length of the boundary layer, the final thickness is greater for smaller angles of attack (or lower thrust). The S shape of the spanwise velocity profiles can intuitively be seen to cause a greater displacement thickness than a chordwise profile, which has no reverse flow.

The chordwise momentum thicknesses (Figures 29 and 30) show much the same effects as the displacement thicknesses. The spanwise momentum thickness will decrease along the span and even become negative. Because of the S shape of the spanwise profile, the physical meaningfulness of the momentum thickness is lost.

To allow meaningful comparison between the two different rotors and to allow the implications of Equation (25) to be seen, the separation line in Figures 31 and 32 is plotted as a function of  $y$  ( $y$  is spanwise position/chord). The position of the separation line  $x_s$  is the distance along the surface measured from the leading edge of the airfoil. This separation line has the same criterion as in two-dimensional flow, but separation occurs in a different way. The description of separation that best fits the present case was given by Maskell<sup>17</sup> and is shown in Figure 33. The separation line is an asymptote of all the limiting streamlines (the streamlines on the surface of the blade).

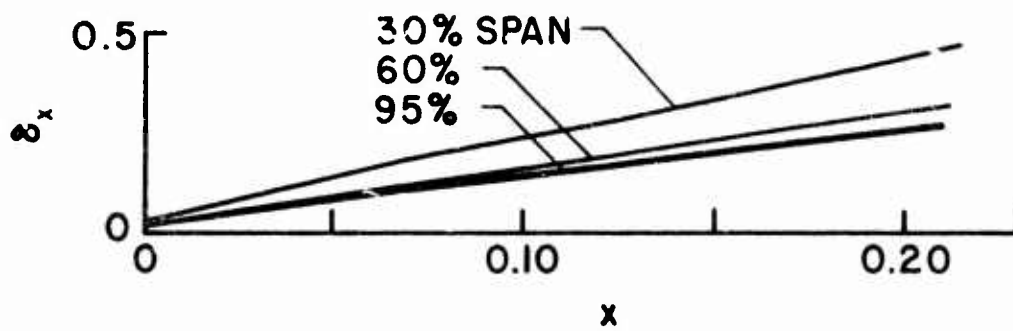


Figure 21. The Chordwise Displacement Thickness for the 24-Foot Rotor (at 6000 lbf Thrust) in Hover.

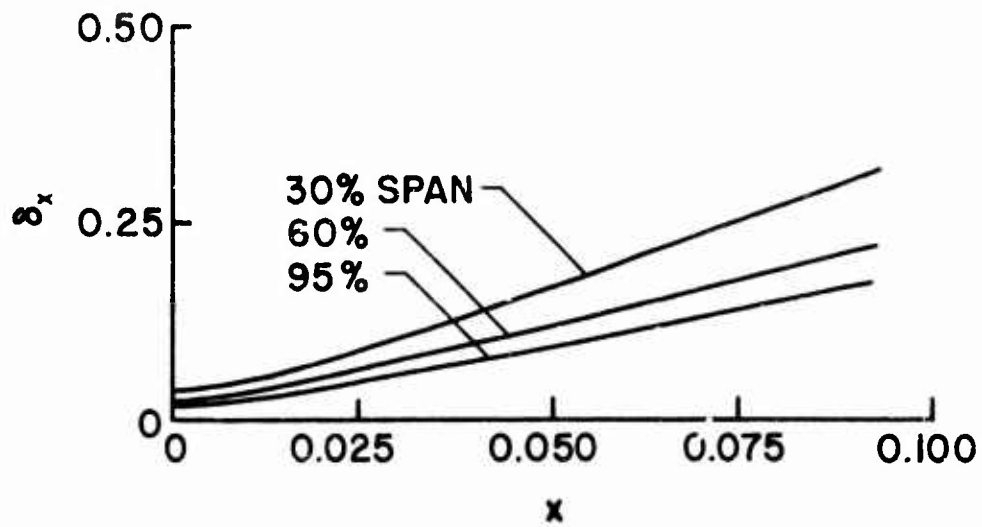


Figure 22. The Chordwise Displacement Thickness for the 24-Foot Rotor (at 10,000 lbf Thrust) in Hover.

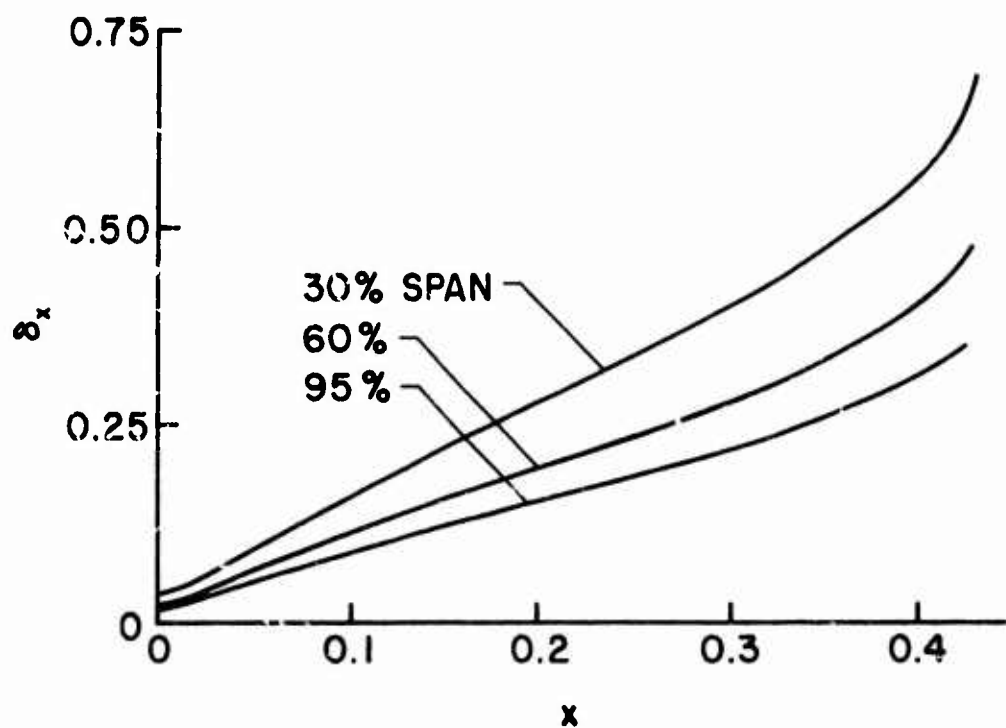


Figure 23. The Chordwise Displacement Thickness for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack) in Hover.

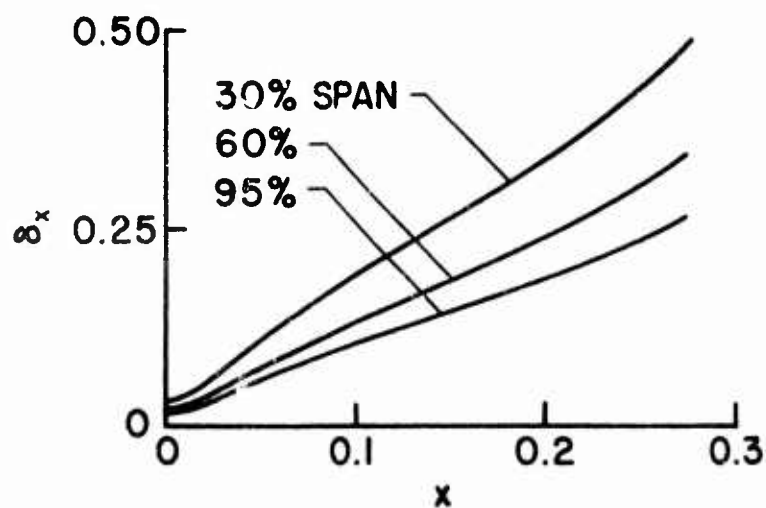


Figure 24. The Chordwise Displacement Thickness for the 40-Foot Rotor (at 6 Degrees Blade Angle of Attack) in Hover.

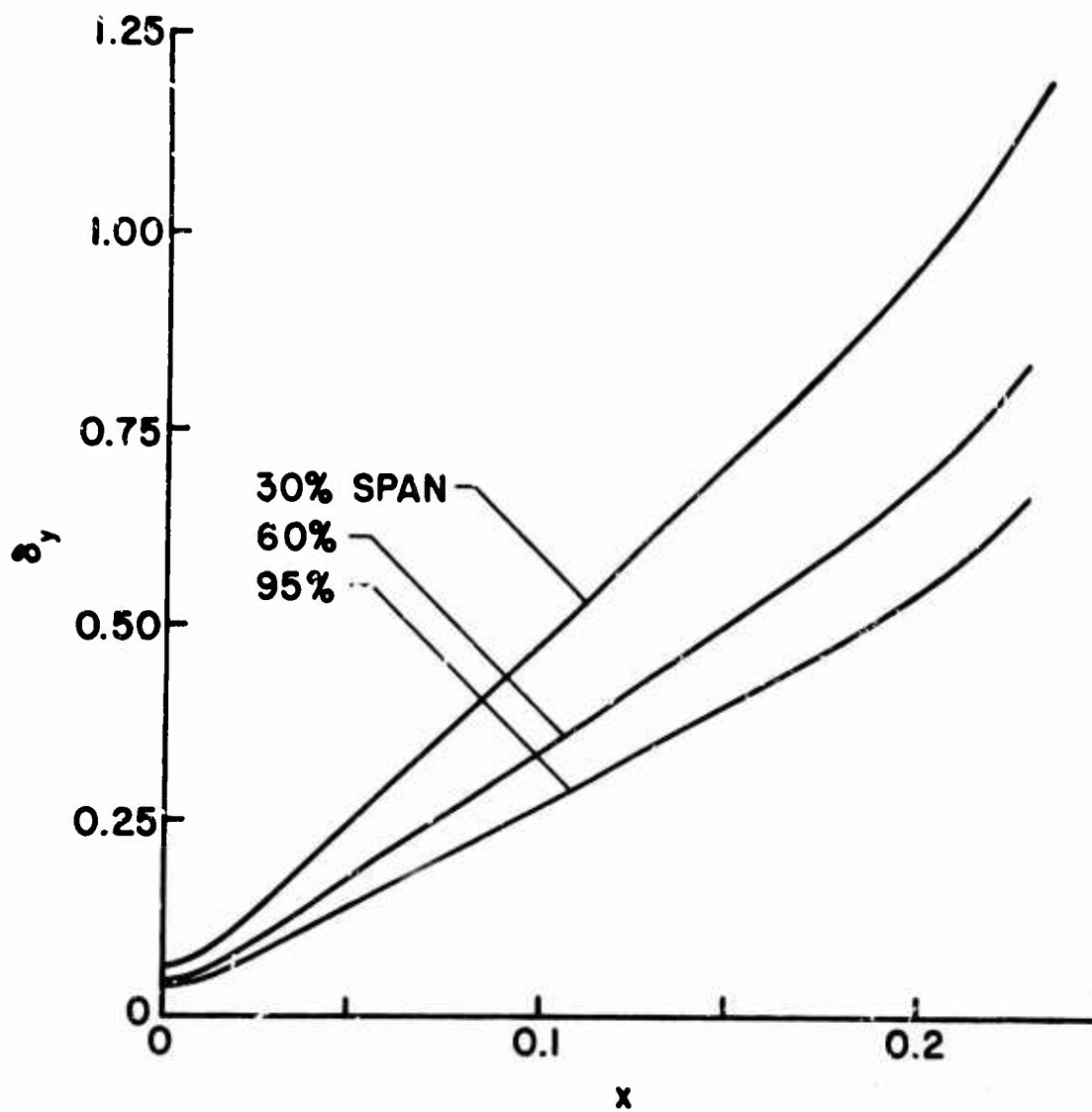


Figure 25. The Spanwise Displacement Thickness for the 24-Foot Rotor (for 6000 lbf Thrust) in Hover.

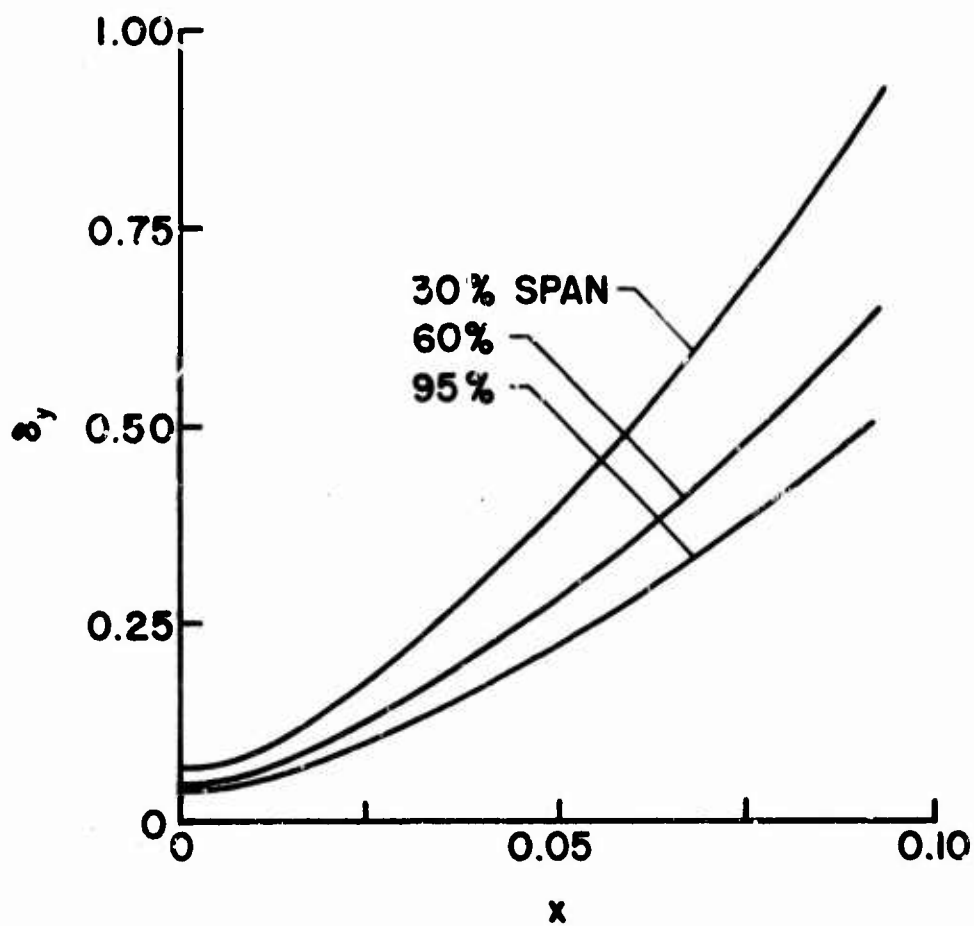


Figure 26. The Spanwise Displacement Thickness for the 24-Foot Rotor (for 10,000 lbf Thrust) in Hover.

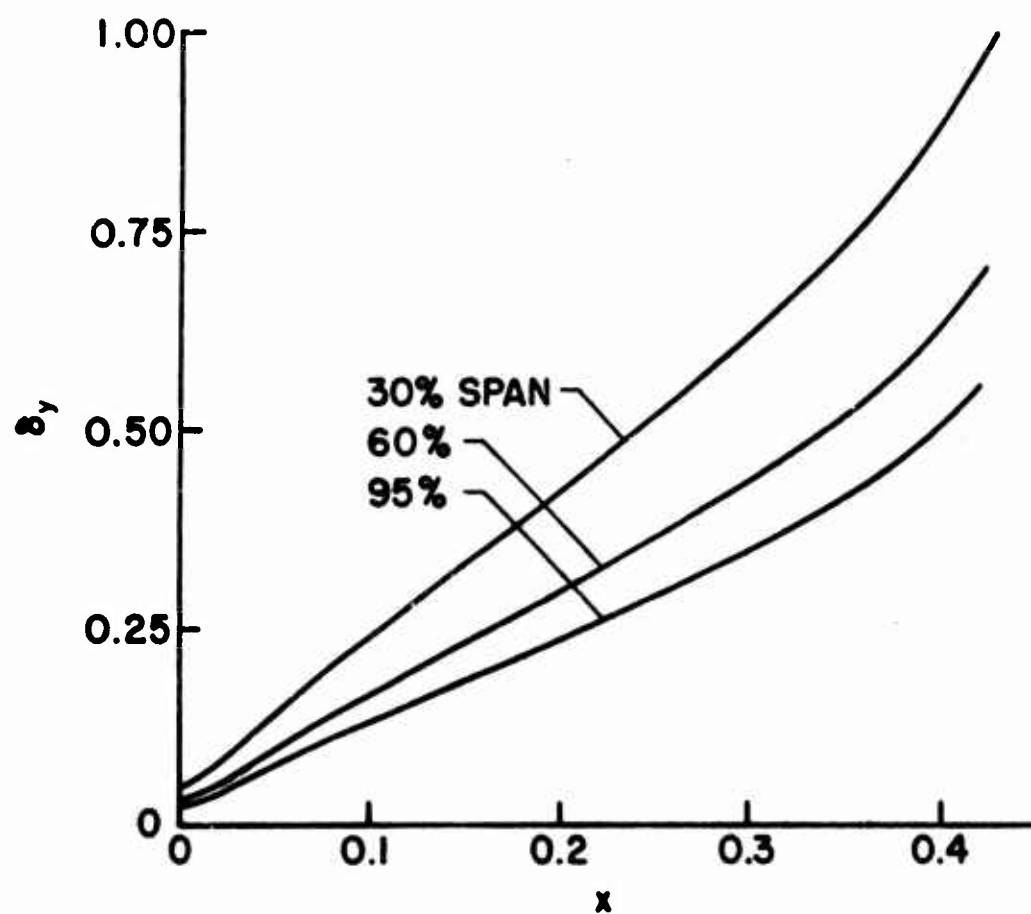


Figure 27. The Spanwise Displacement Thickness for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack) in Hover.

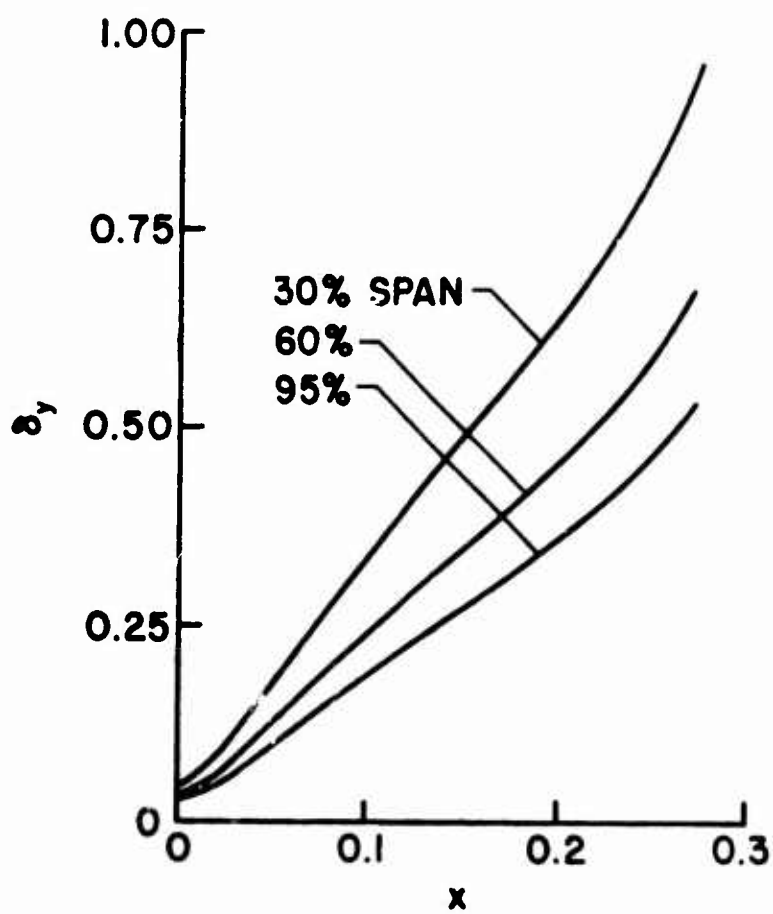


Figure 28. The Spanwise Displacement Thickness for the 40-Foot Rotor (at 6 Degrees Blade Angle of Attack) in Hover.

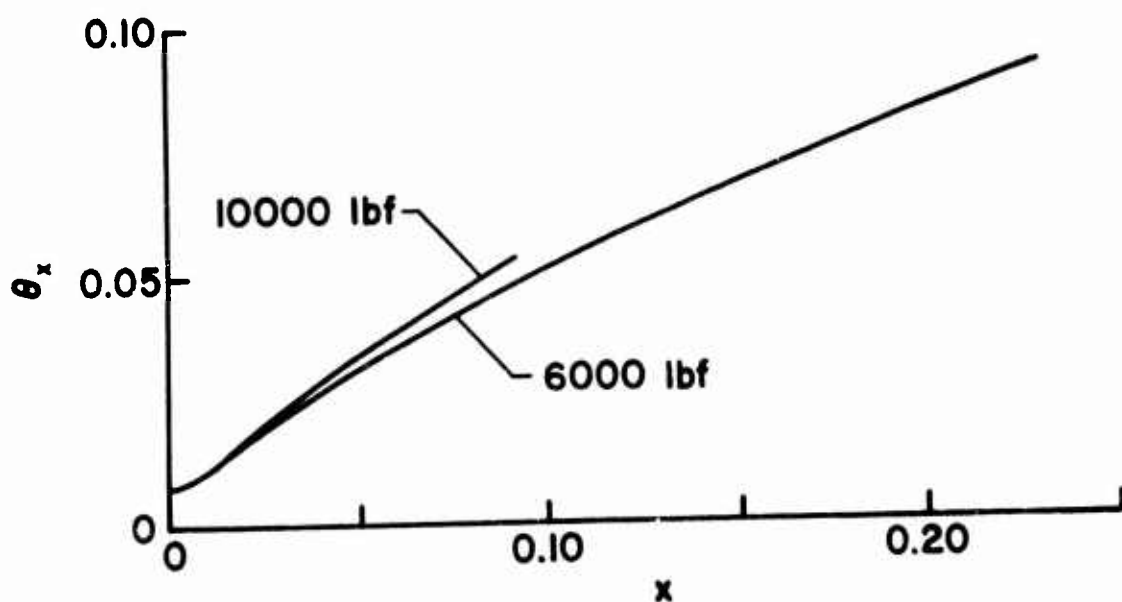


Figure 29. The Chordwise Momentum Thickness for the 24-Foot Rotor (at 60% Span) in Hover.

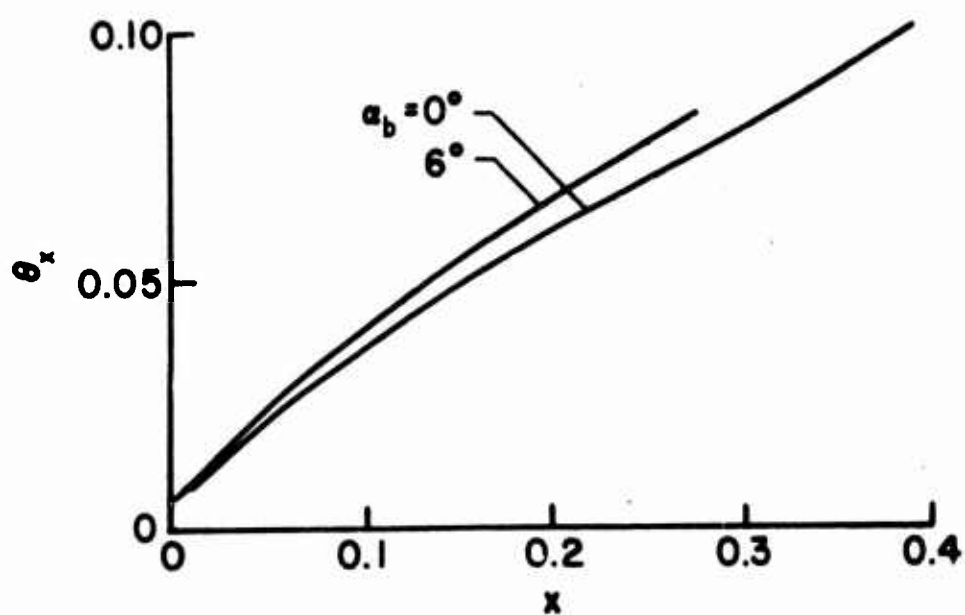


Figure 30. The Chordwise Momentum Thickness for the 40-Foot Rotor (at 60% Span) in Hover.

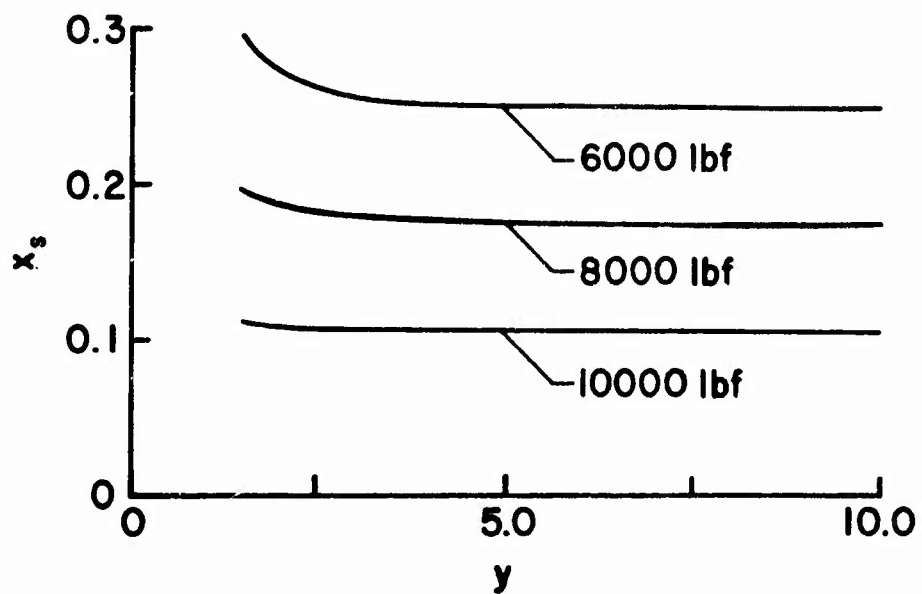


Figure 31. The Separation Line for the 24-Foot Rotor in Hover.

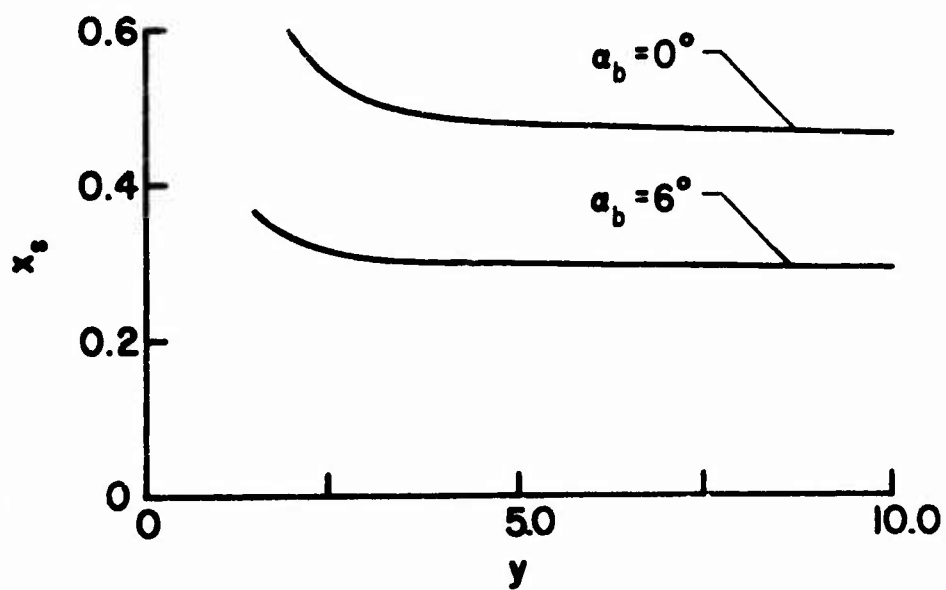


Figure 32. The Separation Line for the 40-Foot Rotor in Hover.

Because the streamline that lies on the separation line cannot carry an infinite amount of fluid (it is the asymptote for an infinite number of limiting streamlines), the limiting streamlines must leave the surface of the blade; *i.e.*, separation must occur. The skew angles (Figures 34 through 37) show how the streamline turn outward to approach the separation line. At smaller angles of attack the turn is more gradual, but in all cases an extension to separation would show the same type of approach to the separation line. The spanwise dependence can be better understood from the equation for skew angle:

$$\tan \beta = \lim_{\eta \rightarrow 0} \frac{v}{u} = \frac{g''}{u_\delta f''} \bigg|_{\eta=0} \quad (79)$$

The  $y$  dependence in  $u_\delta$  is the primary, but not the only, source of  $y$  dependence. A skew angle of  $90^\circ$  indicates outward flow parallel to the span. If the skew angle is zero, there is no spanwise flow at the surface.

The rotational effects are not large in the hover case, but much understanding of their effect can be gained. When forward flight is added to the problem, the complexity of the flow field obscures many of the basic phenomena.

#### FORWARD FLIGHT CASE

The results for the forward flight case are summarized in Table II. The aerodynamic angle of attack varies with span and time, so it may not be used to identify the conditions as in the hover case. If a graph is drawn with application to a specific helicopter, then the length of the rotor in feet  $R$  and either thrust in pounds force  $T$  or the geometric angle of attack in degrees  $\alpha_b$  will be given. All results are understood to be for an 11.9%-thick symmetrical Joukowski airfoil with the axis of rotation at the quarter-chord point. The values for  $\xi_s$  and  $K_{nj}$  are more accurate than those in the hover case. This is due to the superior stability of Lew's method, which allows a closer approach to separation.

The results will be presented in parts. First, each factor that influences the separation line will be analyzed. A series of special cases will be used to investigate the various effects. The values of inflow and forward flight speed will be chosen for illustration and do not necessarily reflect realistic values. The second method of presentation of the results uses values of inflow, thrust, and forward flight speed for the two helicopters described in APPENDIX I. Plots of velocity profiles and the separation lines are presented in terms of parameters related to helicopters.

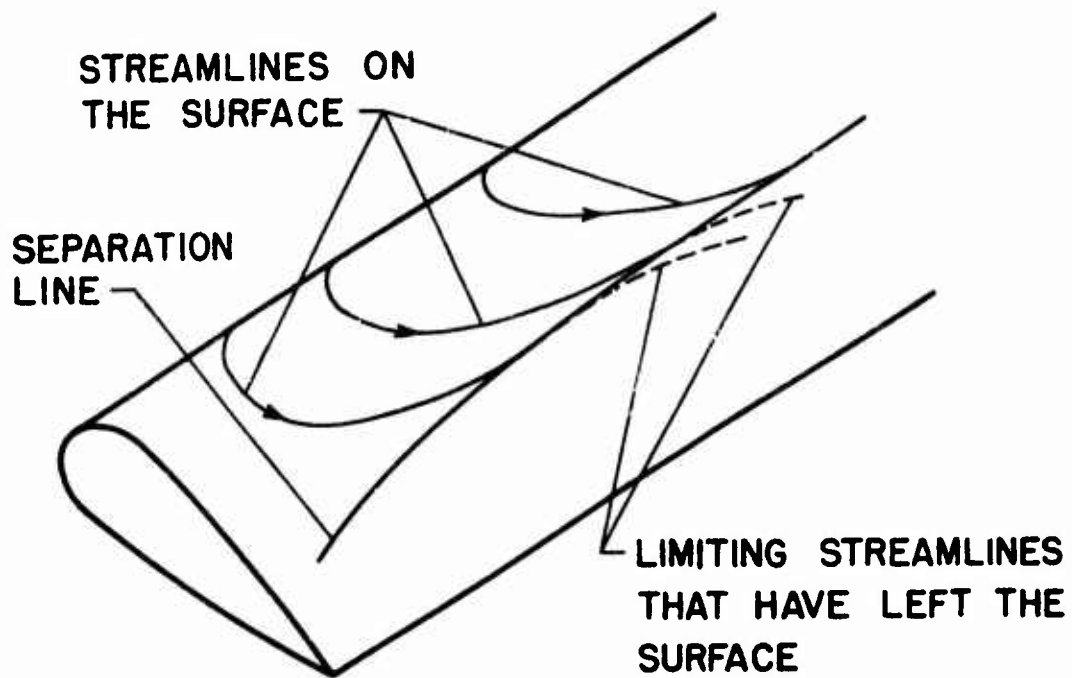


Figure 33. The Streamlines at Separation in the Hover Case.

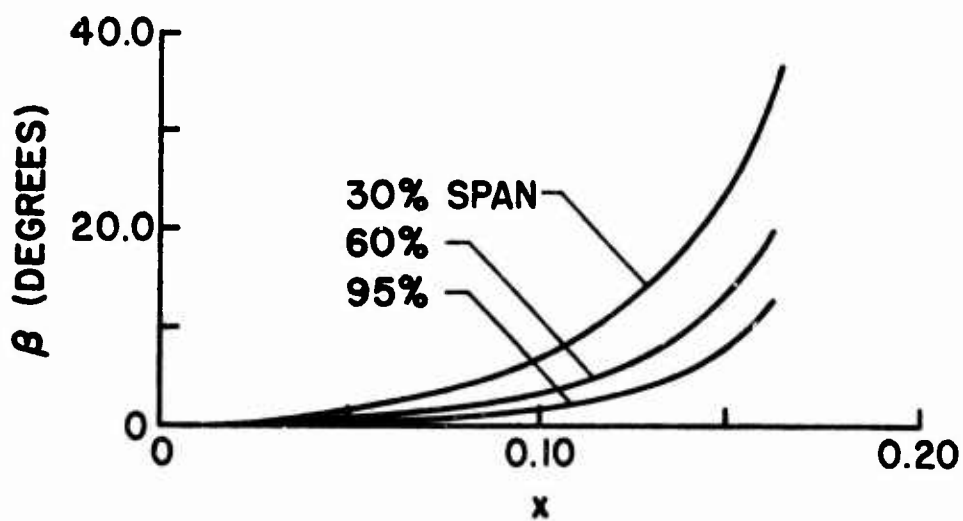


Figure 34. The Surface Skew Angle for the 24-Foot Rotor (at 8000 lbf Thrust) in Hover.

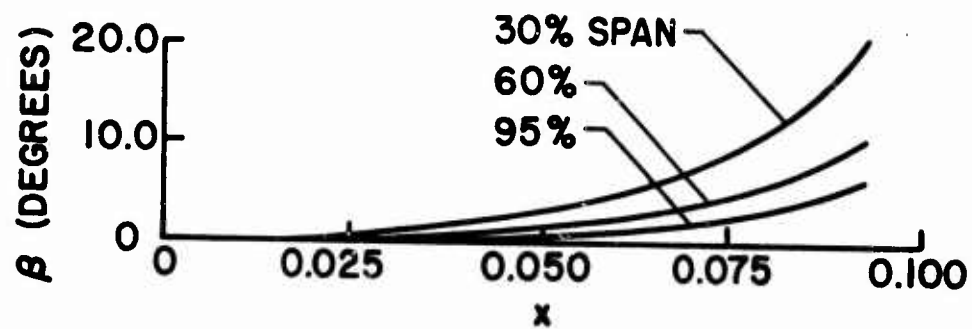


Figure 35. The Surface Skew Angle for the 24-Foot Rotor (at 10,000 lbf Thrust) in Hover.

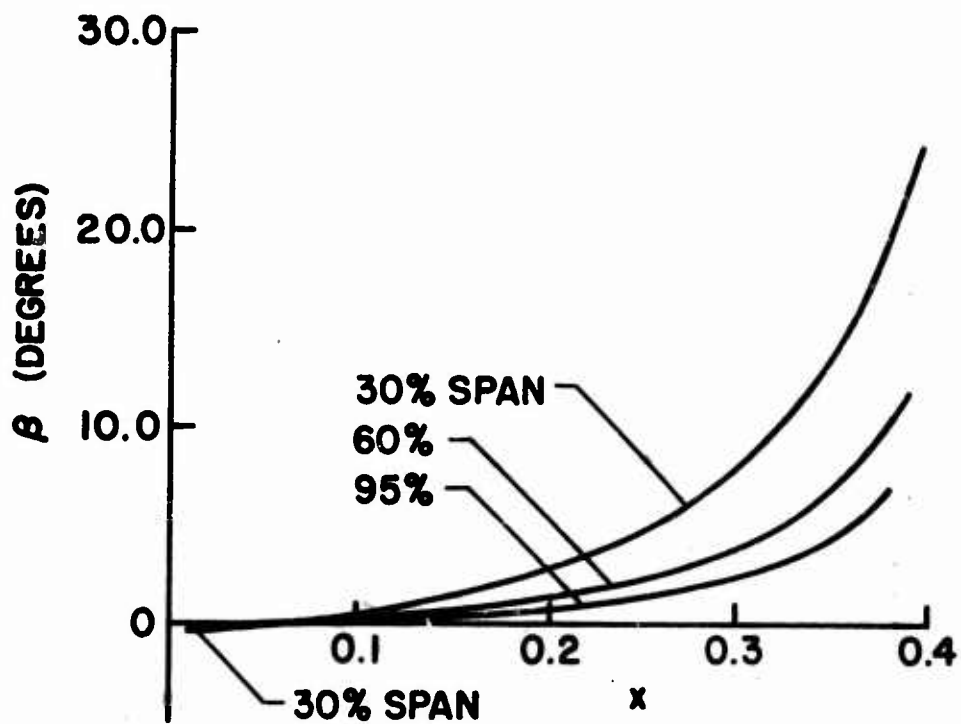


Figure 36. The Surface Skew Angle for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack) in Hover.

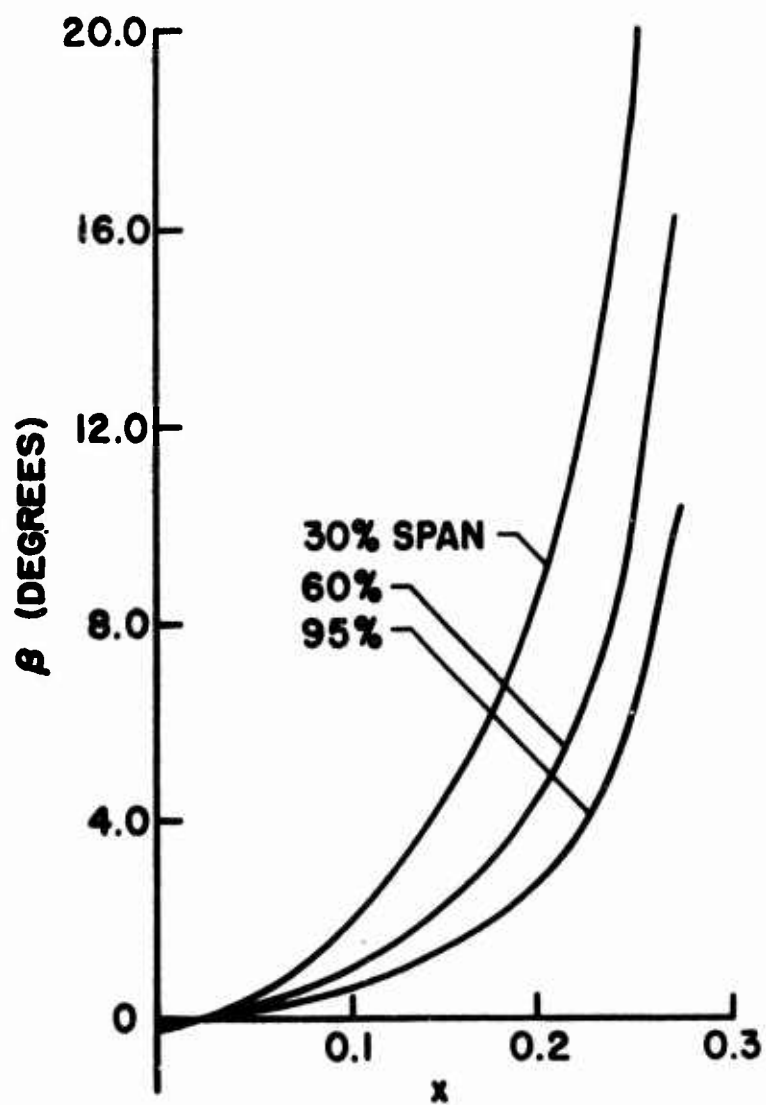


Figure 37. The Surface Skew Angle for the 40-Foot Rotor (at 6 Degrees Blade Angle of Attack) in Hover.

| TABLE II. SUMMARY OF CALCULATED RESULTS FOR THE FORWARD FLIGHT CASE |           |                        |          |          |          |         |          |           |          |          |          |
|---------------------------------------------------------------------|-----------|------------------------|----------|----------|----------|---------|----------|-----------|----------|----------|----------|
| $\alpha_b$<br>(deg)                                                 | R<br>(ft) | $X_0$                  | $X_{10}$ | $X_{20}$ | $X_{22}$ | $\xi_s$ | $K_{10}$ | $K_{2Cr}$ | $K_{2C}$ | $K_{21}$ | $K_{22}$ |
| .005                                                                | 40        | $-.161 \times 10^{-4}$ | .1685    | -.000706 | -.1685   | .440    | 7.18     | .795      | - 55.05  | .423     | - 7.19   |
| 2.0                                                                 | -         | -.00599                | .1781    | -.2667   | -.1781   | .323    | 10.4     | .485      | -122.4   | .596     | -10.46   |
| 3.349                                                               | 24        | -.01036                | .1941    | -.4084   | -.1941   | .244    | 15.01    | .329      | -252.6   | .703     | -15.05   |
| 4.0                                                                 | 40        | -.01262                | .2040    | -.4631   | -.2040   | .201    | 18.80    | .246      | -368.7   | .728     | -18.82   |
| 4.465                                                               | 24        | -.01431                | .2118    | -.4971   | -.2118   | .170    | 22.15    | .178      | -447.5   | .709     | -22.15   |
| 5.581                                                               | 24        | -.0186                 | .2325    | -.5631   | -.2325   | .1065   | 24.22    | .0482     | -86.0    | .450     | -24.13   |

### Special Case of Zero Inflow and Forward Flight Speed

The solution for forward flight is particularly well suited to the evaluation of the effects of inflow and forward flight because both  $v_a$  and  $s_H$  appear explicitly in the equations. For example, if both  $s_H$  and  $v_a$  are set equal to zero, the separation line becomes

$$x_s = x_0 + \xi_s / (1 - K_{20r} / y^2) \quad (80)$$

However, this is not a physically realistic situation. In calculating the potential flow for the rotor blade under consideration, circulation sufficient to satisfy the Kutta condition was included. Yet by setting  $v_a$  equal to zero, the net sum of the inflow of all the blades in the rotor disk was made equal to zero. This situation is interesting, however, because of its similarity to the hover case. In the two cases, the equations for the separation line and  $F_0$  have the same form. Also,  $F_{20}$  in the hover case is governed by the same equation as that for  $F_{20c}$  in the forward flight case. The two cases are not identical, however, because the spanwise flow due to  $G'_{00}$  is different.

In the forward flight case, the nonhomogeneous terms in Equation (62) for  $G'_{00}$  arise from both the pressure gradient terms and the Coriolis term. The term  $\xi \mu_0^a$  comes from the product of  $u_\delta$  and the  $x$  derivative of  $\phi_a$  ( $v_\delta$  depends on  $\phi_a$  and  $X$ ). The term  $-\xi F'_0 c_0$  comes from both the Coriolis acceleration  $2(u - u_\delta) \cos(\alpha - \alpha_b)$  and the product of  $u_\delta$  and the  $x$  derivative of  $X$ . In the hover case an additional term, arising from  $u_\delta \partial v_\delta / \partial x$ , that depends on the inflow, appears. For the forward flight case, neither inflow nor forward flight speed appear explicitly in the reduced equations of motion. The nonhomogeneous terms in Equation (70) for  $F_{20r}$  (or  $F_{20}$  in the hover case) depend on the spanwise flow. The term  $p_0 c_0 (v_0 - G'_{00})$  comes from the Coriolis acceleration of the spanwise flow due to rotation. The convective term  $v \partial u / \partial y$  contributes the term  $\xi F'_0 G'_0$ , and the part of  $w$  due to  $G'_{00}$  gives rise to  $\xi F''_0 G_{00} / 2$  through  $w \partial u / \partial z$ . The term  $\xi v_0$  comes from  $v_\delta \partial u_\delta / \partial y$ .

For both  $v_a$  and  $s_H$  equal to zero, the aerodynamic angle of attack and the geometric angle of attack are identical and constant. Although the form of the separation line in this situation is the same as in the hover case, the values of  $K_2$  and  $K_{20r}$  will not agree for the same geometric angle of attack because the inflow in the hover case will influence both the aerodynamic angle of attack (making it differ from the geometric) and the spanwise flow (through the part of  $u_\delta \partial v_\delta / \partial x$  that arises from the inflow). However, if the values of  $K_2$  are compared on the basis of aerodynamic angle of attack, it appears that separation is influenced more by the aerodynamic angle of attack than by the dependence of the inflow on the span.

### Special Case of Zero Inflow

The present solution is formally valid for  $v_a = 0$ , even if  $s_H$  is not zero. This situation is as physically unrealistic as the previous one for blade angles of attack that give rise to lift. The zero blade angle of attack case was investigated by Young and Williams.<sup>11</sup> The present investigation is in agreement except for the value of  $\xi_s$ . Improvements in the computer program that allowed separation to be approached more closely, and improvements in the extrapolation technique, give better values for  $\xi_s$  and the constants for the separation line. The separation line for  $v_a = 0$  is given by

$$x_s = x_0 + \xi_s / (1 - K_{20r} / y^2 - K_{21} T_1 / y^2) \quad (81)$$

In the present situation, the same effects operate to give the time dependence of the separation line as in Young and Williams<sup>11</sup>, but  $\alpha_b$  may be other than zero.

The spanwise flow contains an additional contribution,  $G'_{01}$ , due to the yaw angle of the blade. This is the time dependent part of the spanwise flow, and it causes a Coriolis force which affects the chordwise flow. The nonhomogeneous terms in the equation for  $F_{21c}$  (Equation (72)) come from the Coriolis acceleration of the spanwise flow due to yaw ( $-p_0 c_0 + p_0 c_0 G'_{01}$ ); from the time dependent part of  $v$  in  $v \partial u / \partial y$  ( $-\xi F'_0 G'_{01}$ ); from the time derivatives of the chordwise flow found in the terms  $\partial u / \partial t$  and  $\partial u_\delta / \partial t$  ( $-\xi F'_0$  and  $\xi$ ); from the time dependent part of the spanwise potential flow in  $v_\delta \partial u_\delta / \partial y$  ( $\xi$ ); and from the dependence of  $\eta$  on time in  $\partial u / \partial t$  ( $-\xi \eta F''_0 / 2$ ). Figure 38 shows that, as the angle of attack becomes larger than  $2^\circ$ , the magnitude of the oscillations of the separation line with azimuthal angle decreases. This is in agreement with a two-dimensional, time-dependent solution of Dwyer and McCroskey<sup>6</sup> on an NACA 0012 airfoil at  $8^\circ$  angle of attack which showed no time dependence of the separation line. The combined effects of rotation and forward flight, without inflow, seem to decrease in magnitude, but remain unchanged in character, as the angle of attack is increased.

The maximum displacement of the separation line occurs at  $\psi = 0$ . This is a  $90^\circ$  phase advance from the maximum of the velocity at the edge of the boundary layer, which occurs at  $\psi = 90^\circ$ . The time dependence of the chordwise velocity resides partly in  $u_\delta$  and partly in  $f'$ . Since separation occurs at the point where  $f''$  is zero, only the time dependence of the  $f'$  part appears in the equation for the separation line. At other points along the chord, the shear stress reflects the time dependence of both  $u_\delta$  and  $f'$ . The phase advance (the angle  $\gamma$  between the maximum of the shear stress at the wall and the maximum of  $u_\delta$ ) has been calculated by Lighthill<sup>18</sup> for two-dimensional oscillatory flow. Lighthill based  $\gamma$  on the steady-state displacement thickness and shear stress, and specifically excluded the region near separation.

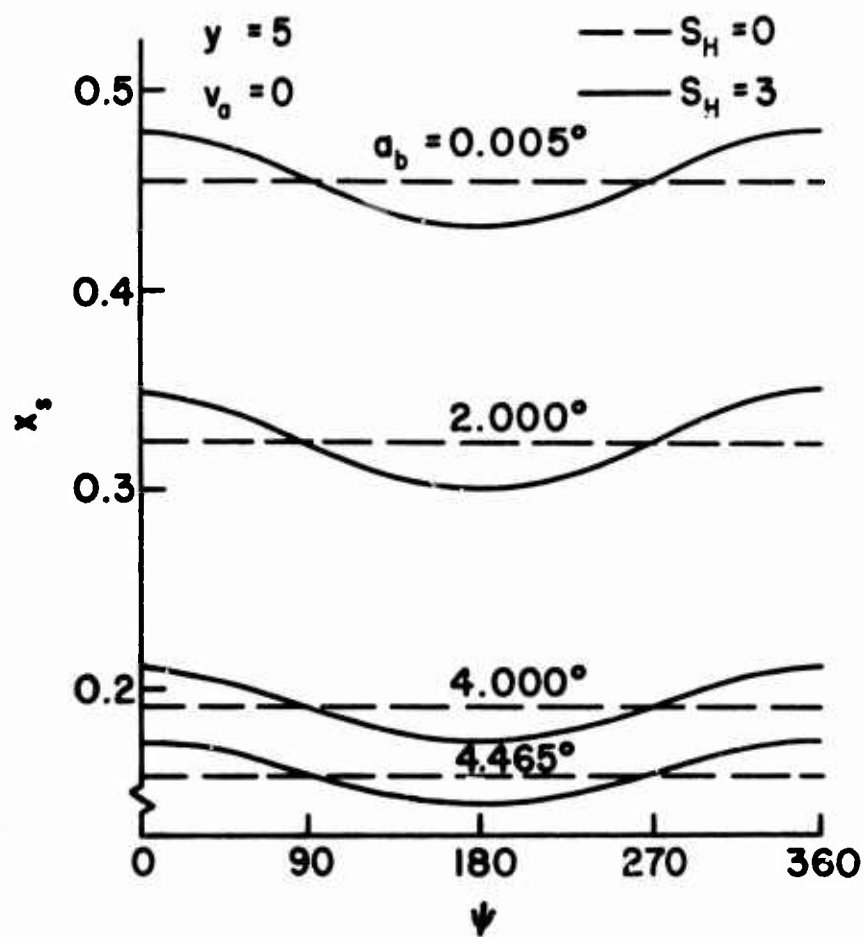


Figure 38. The Separation Line in Forward Flight for No Inflow.

Figure 39 shows that the trend of Lighthill's solution and the phase advance in the present solution are in general agreement for no inflow. A quantitative comparison is not justified because of the difference in the physical situation and the method of solution between the two cases. In the present solution, three-dimensional effects cause a phase shift in the chordwise pressure gradient, and the asymptotic series is formed quite differently from that of Lighthill.

#### Special Case of Zero Forward Flight Speed

Another special case may be isolated at this point; namely, the solution obtained by setting  $s_H$  equal to zero and retaining the inflow. This is basically a solution for hover but will differ from the previously presented hover case in that now the inflow is a constant. The constancy of the inflow causes the aerodynamic angle of attack to vary with span. The aerodynamic angle of attack is given in this case by

$$\alpha_a = \alpha_b - \tan^{-1} v_a / y \quad (82)$$

Relative to the chordwise flow, which increases with span, the downflow decreases as the span increases. This relative change is reflected in the potential flow both indirectly by a change in the position of the stagnation point and directly by a change in the importance of the potential flow function  $\bar{u}_c$  relative to  $\bar{u}_a$ . Since the inflow was proportional to span in the previously considered hover case, these effects were not present.

The transform of the  $x$  coordinate into the  $\xi$  coordinate causes the terms in the series for  $f'$  of higher order than  $F'_0$  to be larger over most of the airfoil for the hover and forward flight cases. In the forward flight case, however, the function  $F'_{10}$  will be large even if the problem had been solved in the  $x, y, z$  coordinate system. The plot of  $F'_{10}$  in Figure 40 demonstrates the behavior of the function. The large negative values behind the leading edge grow rapidly with angle of attack; for an  $\alpha_b$  of 4.465,  $F'_{10} = -29.0$  at  $\xi = .022$ . At the large values of  $F'_{10}$ , the dominant effect is the change in the pressure gradient due to the change in the position of the stagnation point along the span. For the positive values of  $F'_{10}$ , the change in the pressure gradient due to the change in the importance of  $\bar{u}_c$  relative to  $\bar{u}_a$  becomes the largest influence on  $F'_{10}$ . In addition to  $F'_{10}$ , the constant inflow gives rise to the function  $F'_{20}$ . For comparison to  $F'_{10}$ , a plot of  $F'_{20}$  is shown in Figure 41. The large magnitude of  $F'_{20}$  is partly due to the nonhomogeneous terms in the equation for  $F'_{20}$  being dependent on  $F'_{10}$ . The magnitudes of  $F'_{10}$  and  $F'_{20}$  seem large in comparison to  $F'_0$ . The proper comparison, however, should be to  $v_a F'_{10} / \zeta$  and  $v_a^2 F'_{20} / u_0^2$ .

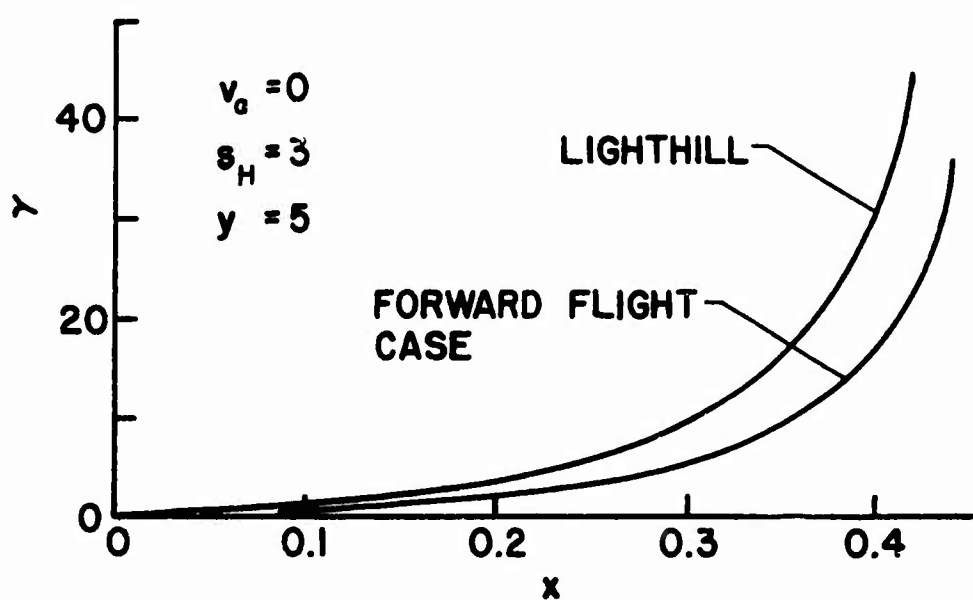


Figure 39. The Advance Angle (in Degrees) of the Shear Stress on the Surface.

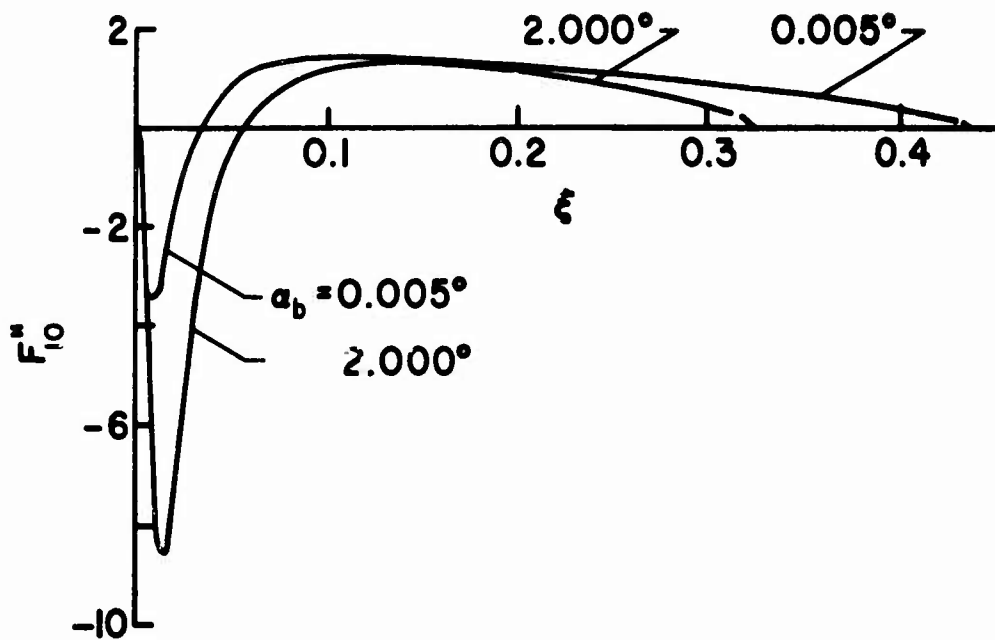


Figure 40. The Function  $F''_{10}$  on the Surface of the Blade.

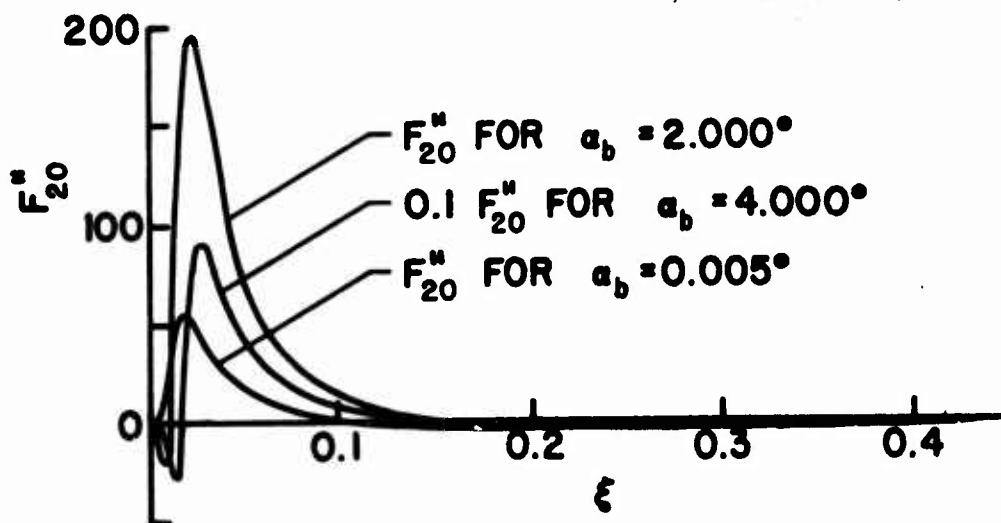


Figure 41. The Function  $F''_{20}$  on the Surface of the Blade.

Since  $v_a$  is small (typically .1), and since  $F_{10}'$  and  $F_{20}''$  are of opposite sign where they are largest, the infinite series for span may be accurate over the entire chord for values of span / chord as small as 3 or 4 for the present situation ( $s_H = 0$ ). The functions  $F_{10}$  and  $F_{20}$ , through the constants  $K_{10}$  and  $K_{20}$ , affect the separation line through

$$x_s = x_0 + v_a x_{10}/y + v_a^2 x_{20}/y^2 + \xi_s / (1 - v_a K_{10}/y - K_{20}r/y^2 - v_a^2 K_{20}/y^2) \quad (83)$$

As shown in Table II,  $K_{10}$  and  $K_{20}$  are of opposite sign. Separation is delayed by  $K_{10}$  and advanced by  $K_{20}$ , so that their effect is to tend to cancel each other. Both increase in magnitude as the angle of attack increases in such a way that the net effect on separation is delay (except for values of span so small as to cast doubt on the accuracy of the series). Figure 42 shows the separation line as a function of span for  $s_H = 0$ . The situation of large inflow at near zero angle of attack is not realistic, but it is included for completeness.

The effect of the constant inflow is to delay separation, and this delay is increased as span decreases. At least part of this effect is due to the increase in aerodynamic angle of attack as span decreases. The importance of the aerodynamic angle of attack may be assessed from Figure 43. When the forward flight speed is zero, the results for the two different cases agree very well when compared on the basis of aerodynamic angle of attack. The inflow affects the separation line almost entirely through a change in the aerodynamic angle of attack in the hover case (with inflow proportional to span) and in the forward flight case (with  $s_H = 0$ ), which has constant downflow.

#### General Case With Inflow and Forward Flight

Combining the effects of inflow and forward flight introduces one function that has not yet been considered,  $F_{22}$ . It has the value  $-v_0^2 F_{10}$  and arises from the combined action of the constant inflow and the time dependence due to forward flight. It is not present unless both are acting simultaneously. Its effect on the separation line is shown in Figure 44. As the inflow increases, the mean (with respect to time) position of the separation line is moved rearward on the airfoil. Also, the oscillations about this mean separation line increase in magnitude, and the maxima and minima occur at smaller values of azimuthal angle. For no inflow, the time dependence of the flow is dominated by time derivatives of the chordwise velocity.<sup>6,11</sup> This effect is represented by the function  $F_{21c}$ , which has the coefficient  $\cos \psi$ . As the inflow increases, the function  $F_{22c}$ , with the coefficient  $\sin \psi$ , becomes important.

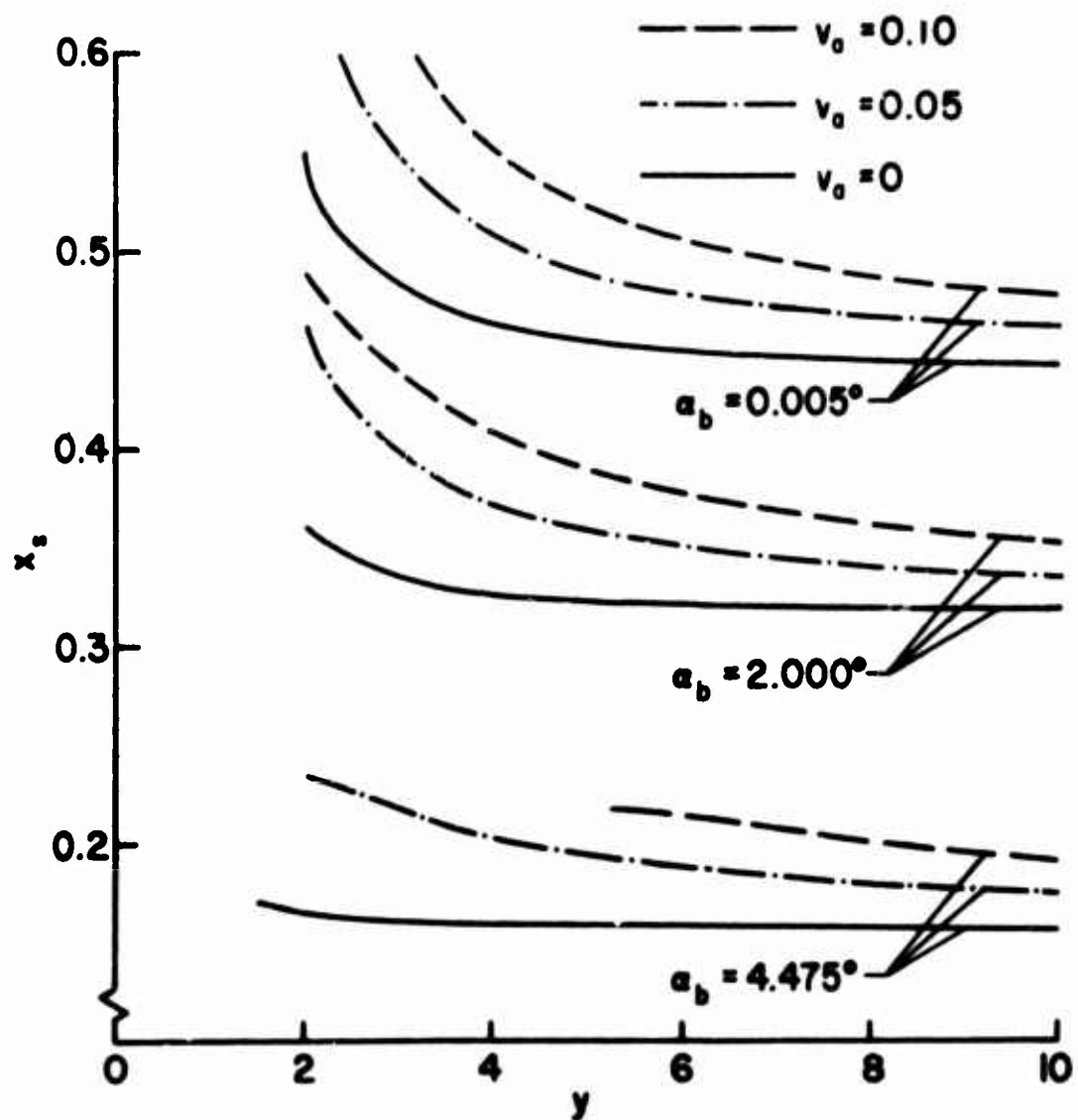


Figure 42. The Separation Line as a Function of Span and Inflow for No Forward Flight.

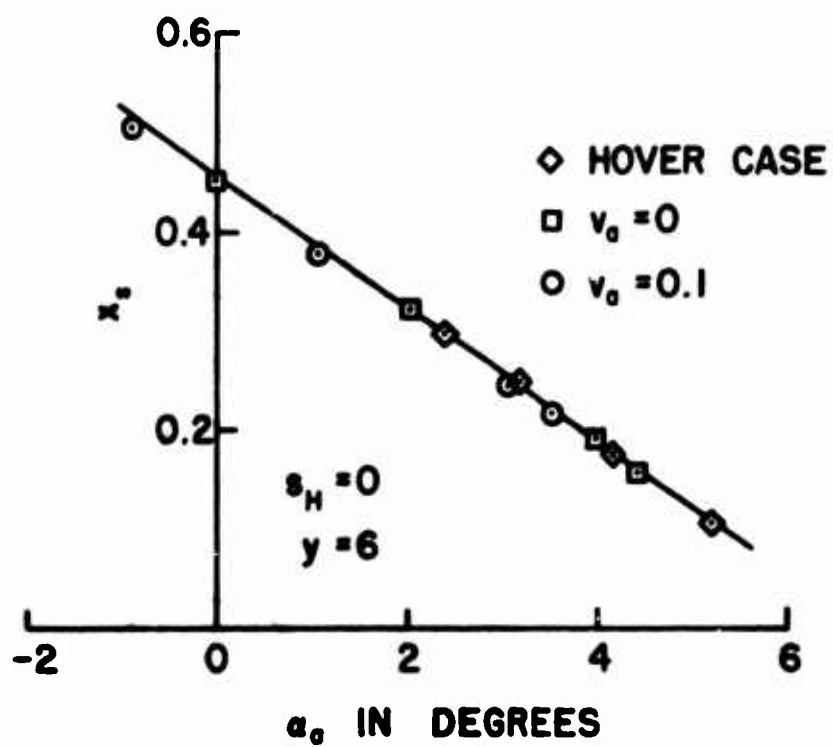


Figure 43. Separation Points With and Without Inflow for No Forward Flight.

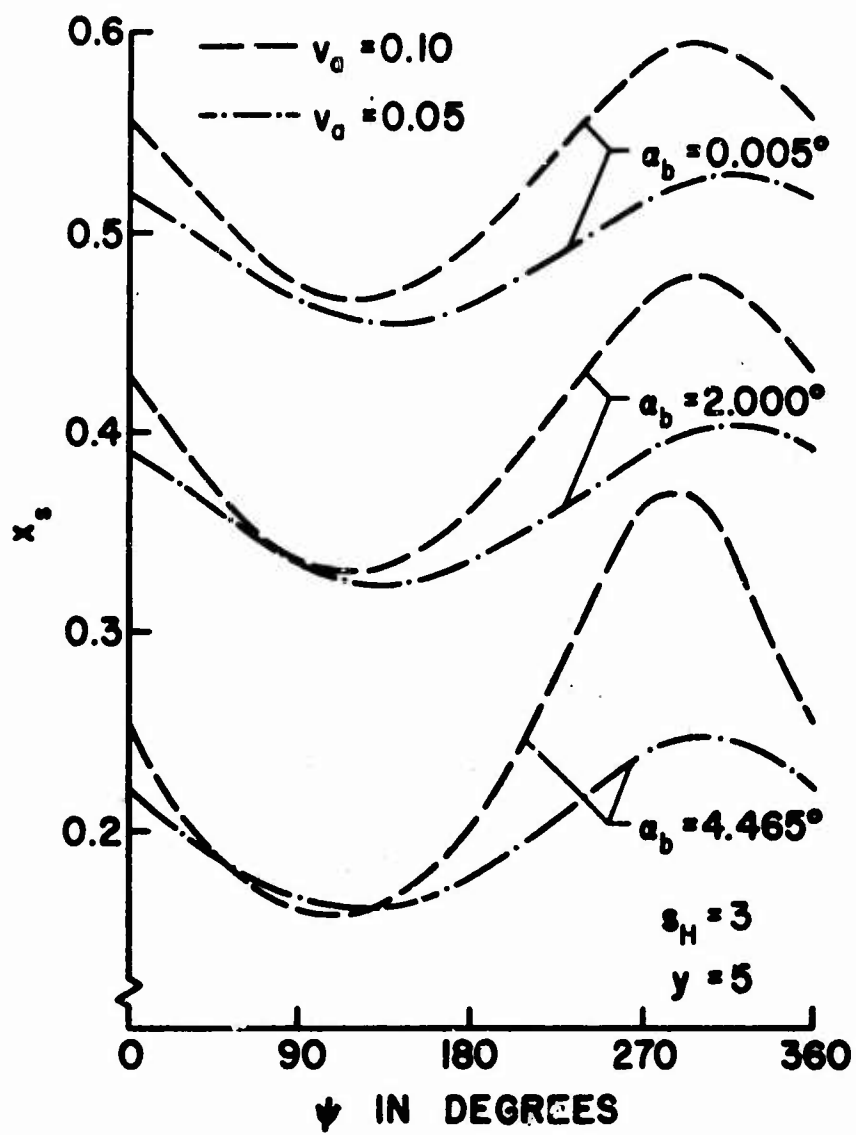


Figure 44. The Separation Line as a Function of Azimuthal Angle and Inflow.

This function increases the magnitude of the oscillations and causes the phase shift in the separation line. The forward flight speed and the inflow cause a change with time of the aerodynamic angle of attack which is given by

$$\alpha_a = \alpha_b - \tan^{-1} (v_a / (y + T_2)) \quad (84)$$

The aerodynamic angle of attack is a minimum at  $\psi = 270^\circ$ . This is a result of the simplifying assumptions imposed to obtain a tractable solution. For an actual helicopter, the blade angle of attack is maximum at  $\psi = 270^\circ$  due to cyclic pitch. As the inflow increases, the maximum of the separation line moves away from  $360^\circ$  toward  $270^\circ$ . For zero forward flight speed, the inflow did not significantly affect the relationship between separation and aerodynamic angle of attack. From Figure 45, it can be seen that this is not true in forward flight. Going from  $\psi = 0$  to  $\psi = 360^\circ$  is equivalent to traversing the closed curves in a clockwise direction. For no inflow, the aerodynamic angle of attack is constant, and the separation point is represented by a straight vertical line. This time-dependent behavior of the separation line could not be found except by considering rotation, inflow, and forward flight simultaneously. It must be ascribed to an interaction of all three.

#### General Case of Two Specific Helicopters

The values of inflow, thrust, and angle of attack have been calculated in APPENDIX I for two specific helicopters. There are, of course, other effects that are not included in the present analysis. Only special forms of time dependence, those due to forward flight, can be handled. The aerodynamic angle of attack has been calculated without taking into account blade twist or flapping. Due to these factors and others discussed in the theory, the results may not be applied directly to a helicopter rotor. For example, results are shown at various values of span. At 60% span, the assumption of no end effects is defensible. At 95% span, the results still properly represent the effects which the present work is investigating, but the end effects, which this theory cannot calculate, will have to be accounted for in some other way.

The chordwise velocity profiles are shown in Figures 46 through 55. The general shape is that of Falkner-Skan profiles. Figures 50 and 51 show that the effect of forward flight speed is small at  $\psi = 90^\circ$ . Graphs of the separation line will show the effects of forward flight speed more clearly. The dependence of the chordwise velocity profiles on azimuthal angle shown in Figures 52 and 53 will grow larger if the span is decreased or the separation point is approached. At  $45^\circ$  and  $135^\circ$ , similar values are found for the aerodynamic angle of attack. The aerodynamic angle of attack at  $225^\circ$  is about the same as at  $315^\circ$ .

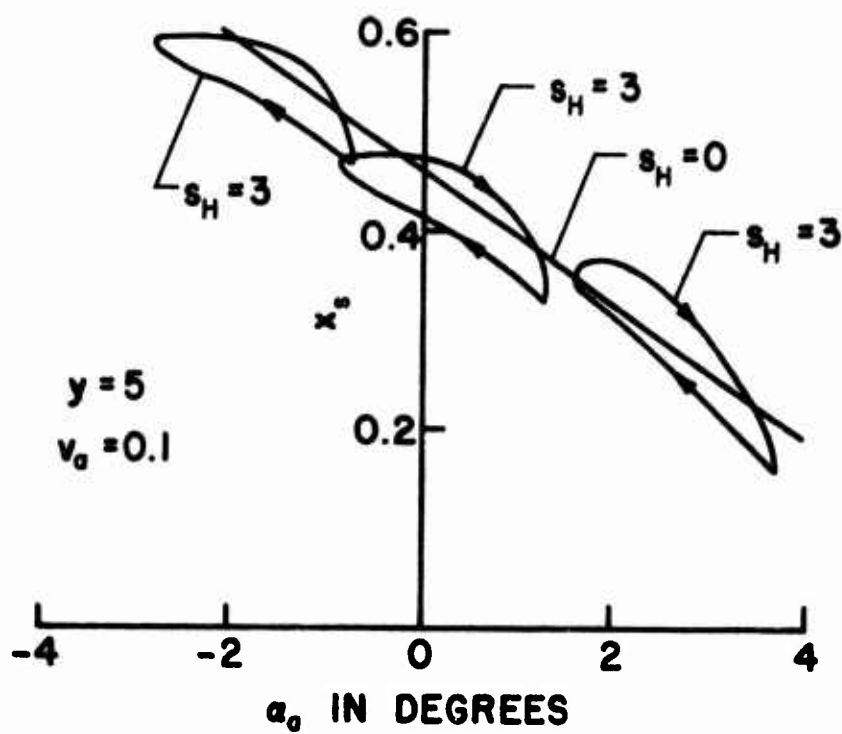


Figure 45. The Separation Line as a Function of Aerodynamic Angle of Attack in Forward Flight With Lift.

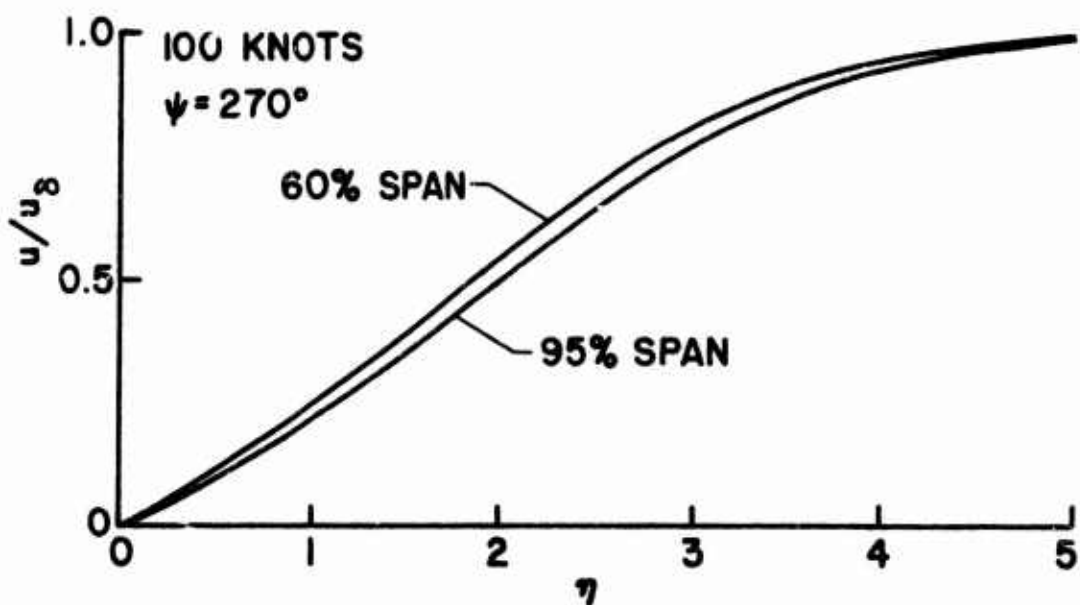


Figure 46. The Chordwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf and 10% Chord).

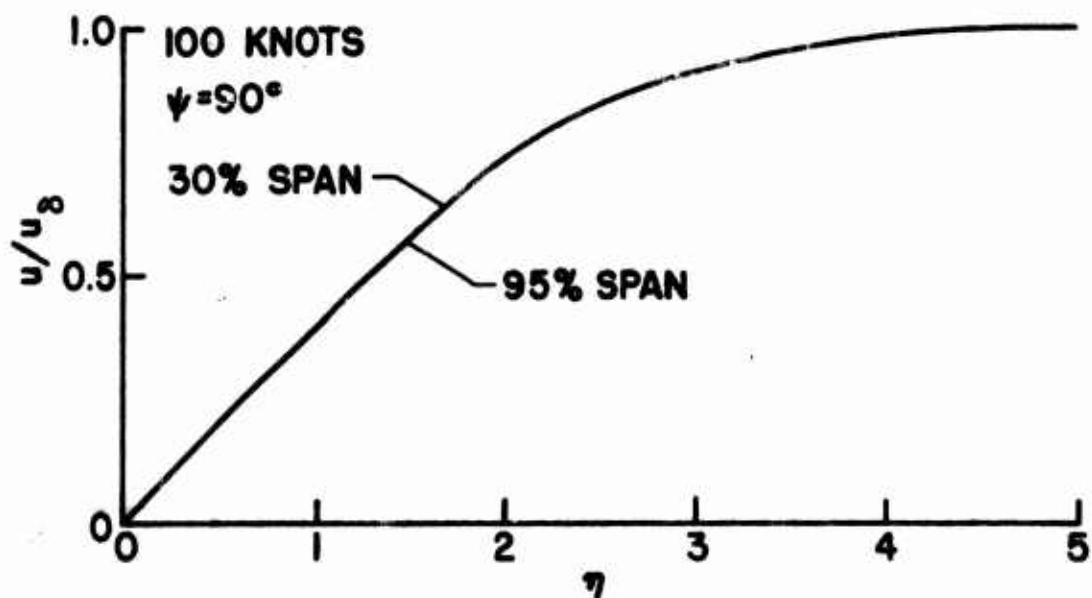


Figure 47. The Chordwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 10% Chord).

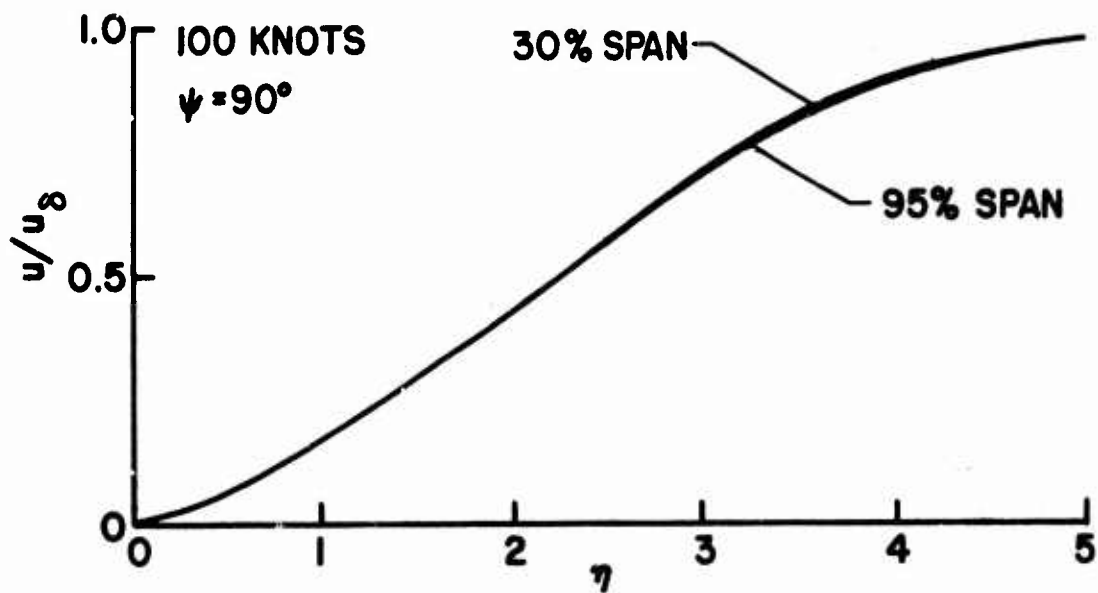


Figure 48. The Chordwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf and 12% Chord).

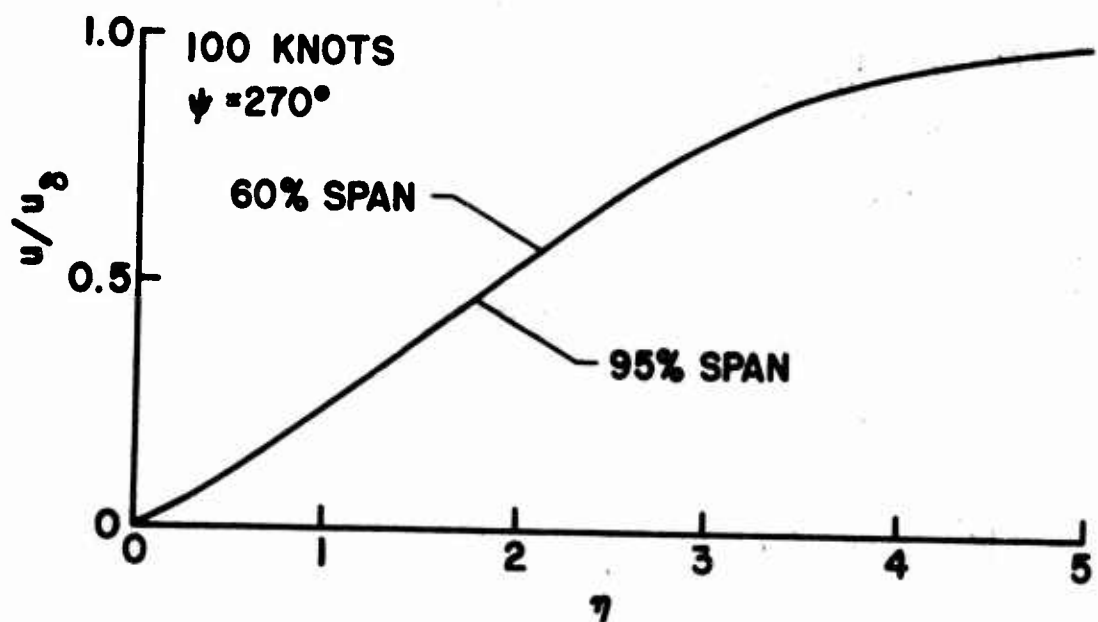


Figure 49. The Chordwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 30% Chord).

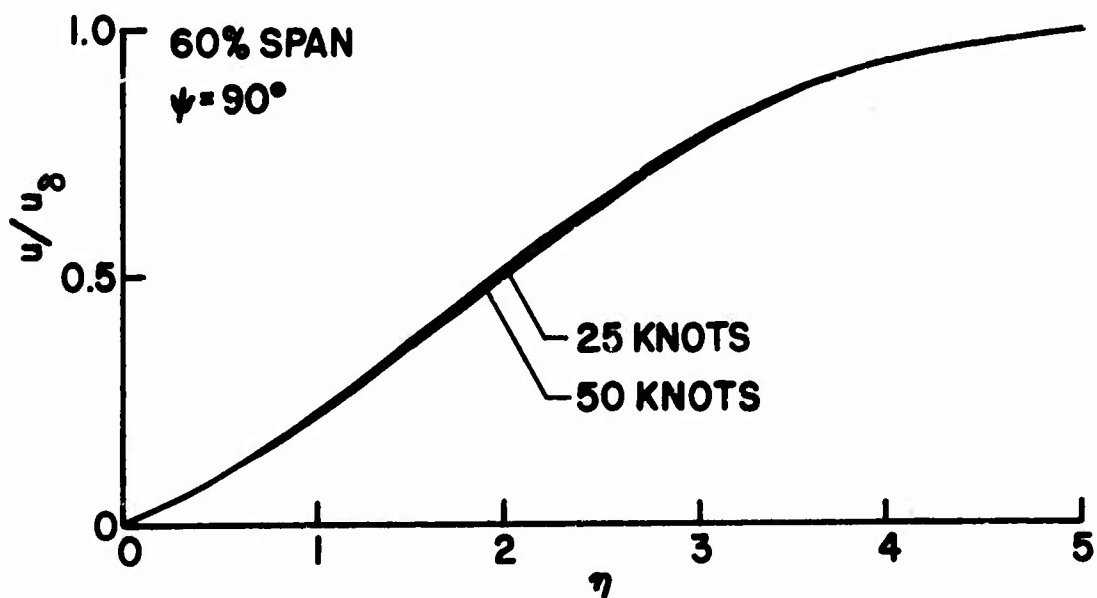


Figure 50. The Chordwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf, 60% Span, and 10% Chord).

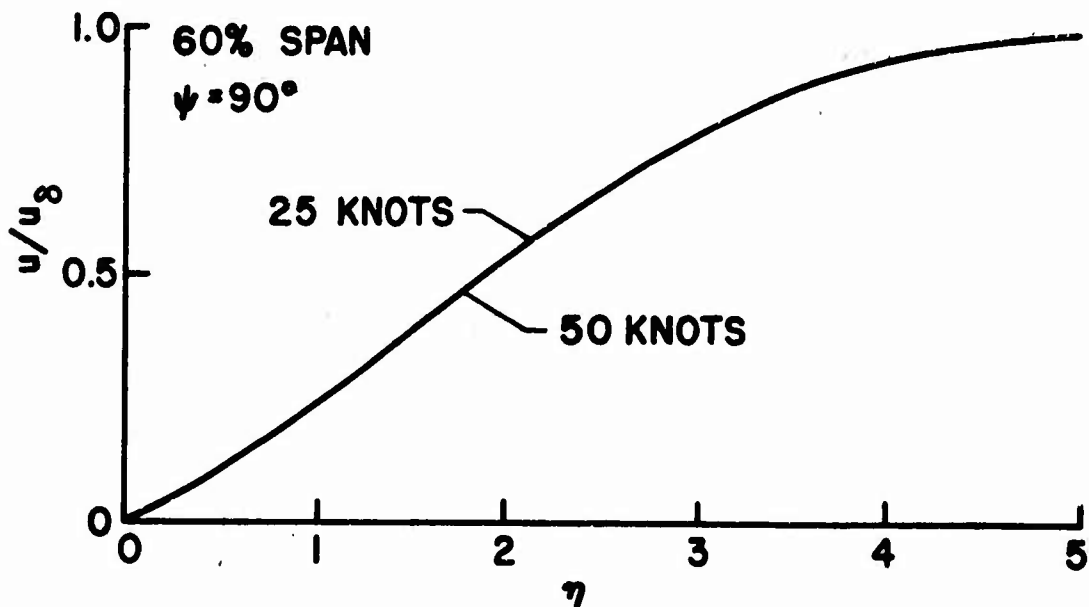


Figure 51. The Chordwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack, 60% Span, and 30% Chord).

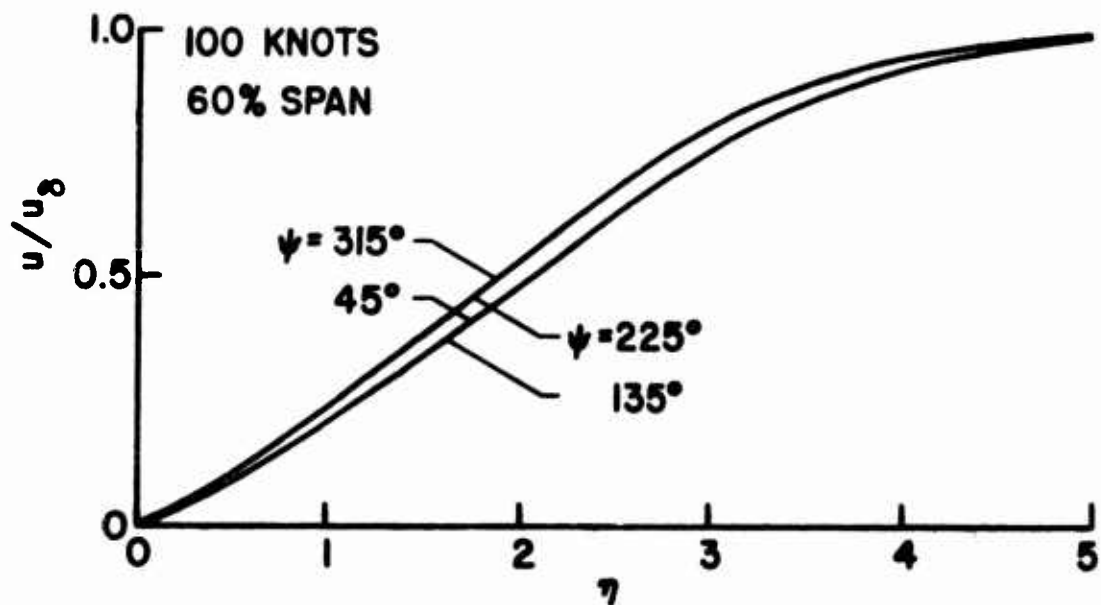


Figure 52. The Chordwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf and 10% Chord) in Forward Flight.

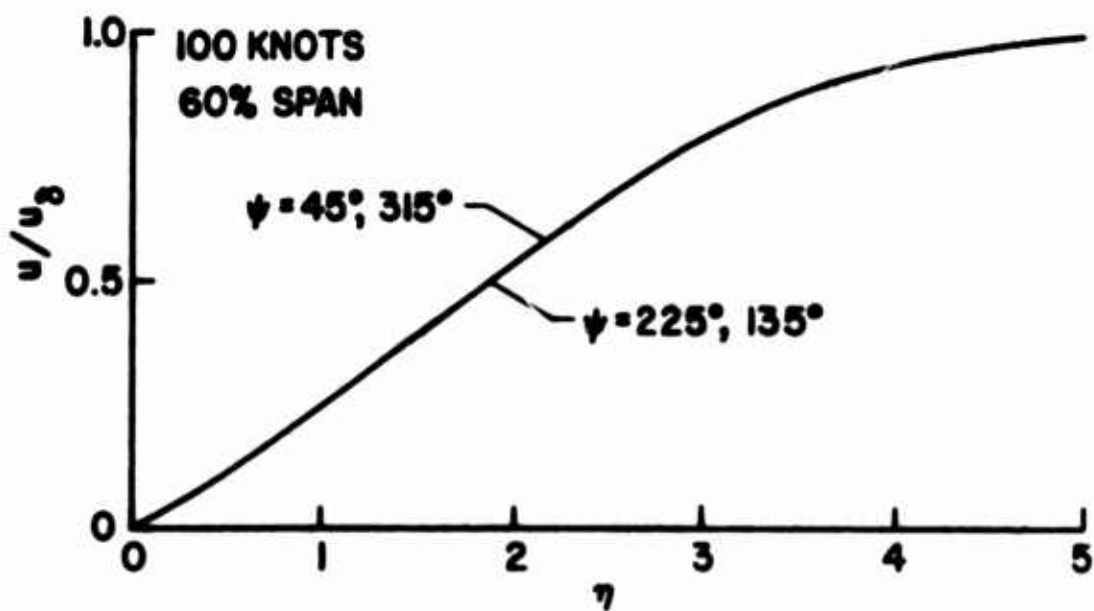


Figure 53. The Chordwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 30% Chord) in Forward Flight.

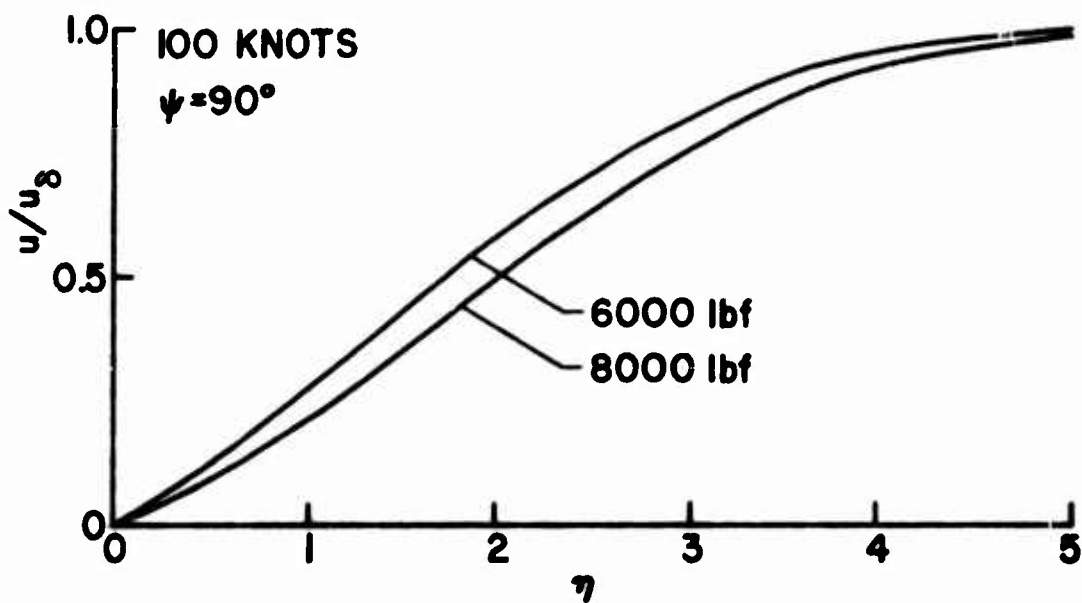


Figure 54. The Chordwise Velocity Profile for the 24-Foot Rotor (at 10% Chord and 60% Span) in Forward Flight.

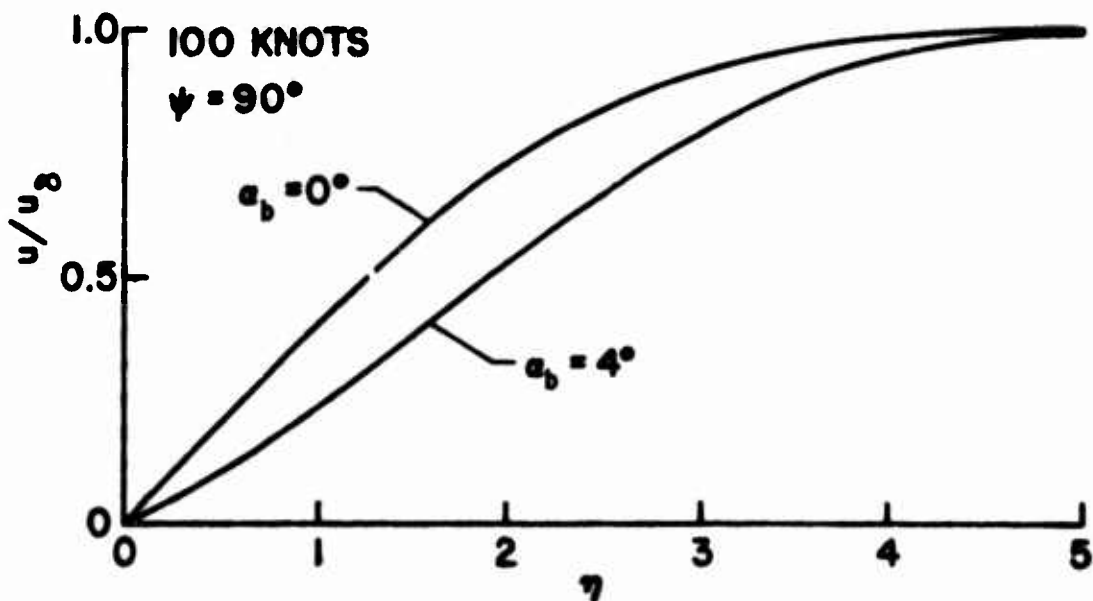


Figure 55. The Chordwise Velocity Profile for the 40-Foot Rotor (at 10% Chord and 60% Span) in Forward Flight.

The pattern of the aerodynamic angle of attack with azimuthal angle is reversed from normal helicopter operation (since flapping is not included), but the implication that time derivatives and spanwise flow are not dominant over angle of attack will apply. Figures 54 and 55 indicate that fuller profiles occur at smaller angles of attack for a given spanwise and chordwise position.

From Figures 56 through 64, the characteristics of the spanwise velocity profiles can be seen. Figures 56 through 59 show that the spanwise variation is greater at  $270^\circ$  than at  $90^\circ$ . Figure 57 does not have the characteristic S shape because the centrifugal forces have not yet caused positive values near the wall. The points  $\psi = 90^\circ$  and  $\psi = 270^\circ$  are not typical points; the spanwise flow due to yaw is zero there. Figures 60 and 61 show clearly that the flow due to forward flight dominates the flow due to rotational effects. At  $45^\circ$  and  $315^\circ$  there is outward spanwise flow, and at  $135^\circ$  and  $225^\circ$  there is inward spanwise flow. This flow is so much larger than the induced flow due to rotation that a larger scale is required. Near the wall, however, an asymmetry between the upper and lower curves can be detected. This is due to rotational effects; or more precisely, it is not due to the yaw angle. There is some dependence on forward flight at  $90^\circ$  or  $270^\circ$  due to the time derivative of spanwise velocity and due to the combination of inflow and forward flight (from Equations (67) and (68)). Figures 62 and 63 show that at  $90^\circ$ , these effects are small.

The dependence of the chordwise displacement thickness on azimuthal angle, shown in Figure 65, is much larger than that seen in the velocity profiles (Figures 52 and 53). In the velocity profiles, this time dependence (or dependence on azimuthal angle) is contained in the variable  $\eta$ , and so cannot be seen on the graph. In Figure 65, this time dependence is displayed. The large difference between the  $225^\circ$ ,  $315^\circ$  pair and the  $45^\circ$ ,  $135^\circ$  pair occurs because of the appearance of  $u_0$  in the Falkner-Skan transform from  $z$  to  $\eta$ . A similar effect occurs in Figure 66; the forward flight speed also appears in  $u_0$ . This dependence on forward flight speed will become more important at smaller values of span.

In examining the graphs of the separation line, the shape of the curves in Figures 67 and 68 should be understood. For no inflow ( $\alpha_0 = .005^\circ$ ), there is a minimum in the separation line at  $180^\circ$  azimuthal angle and a maximum at  $360^\circ$  (or  $0^\circ$ ). The separation line with inflow, shown in Figure 68, has a minimum near  $90^\circ$  and a maximum near  $270^\circ$ . The time derivatives of chordwise flow tend to cause a maximum at  $360^\circ$ . The inflow makes the aerodynamic angle of attack smallest at  $270^\circ$  (since flapping and cyclic pitch are not considered) and tends to cause a maximum in the separation line at  $270^\circ$ . The resultant maximum lies between  $270^\circ$  and  $360^\circ$  azimuthal angle. If the  $y$  (spanwise) dependence is examined near a minimum, the separation line will move forward on the blade as  $y$  decreases. Near the maximum, the opposite is true. However, the mean or time-averaged separation line will move backward as  $y$  decreases.

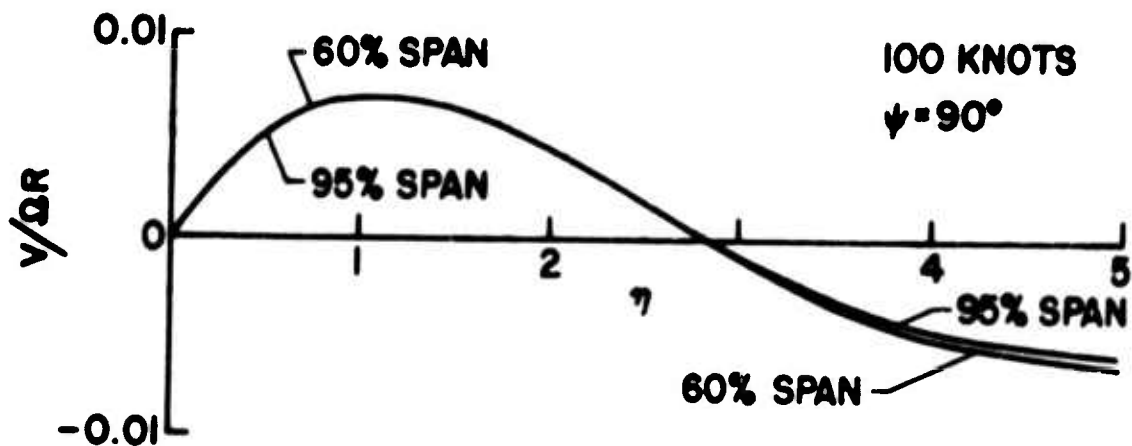


Figure 56. The Spanwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf and 12% Chord) in Forward Flight.

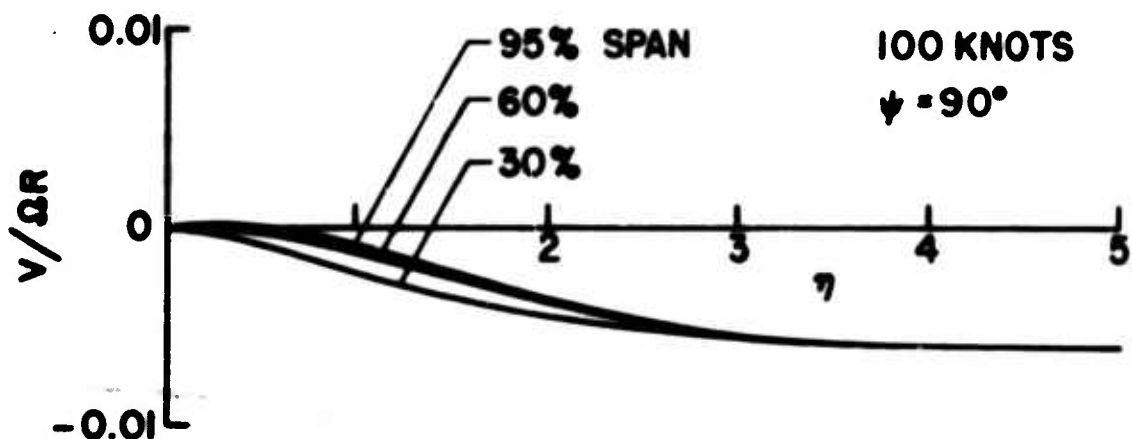


Figure 57. The Spanwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 10% Chord) in Forward Flight.

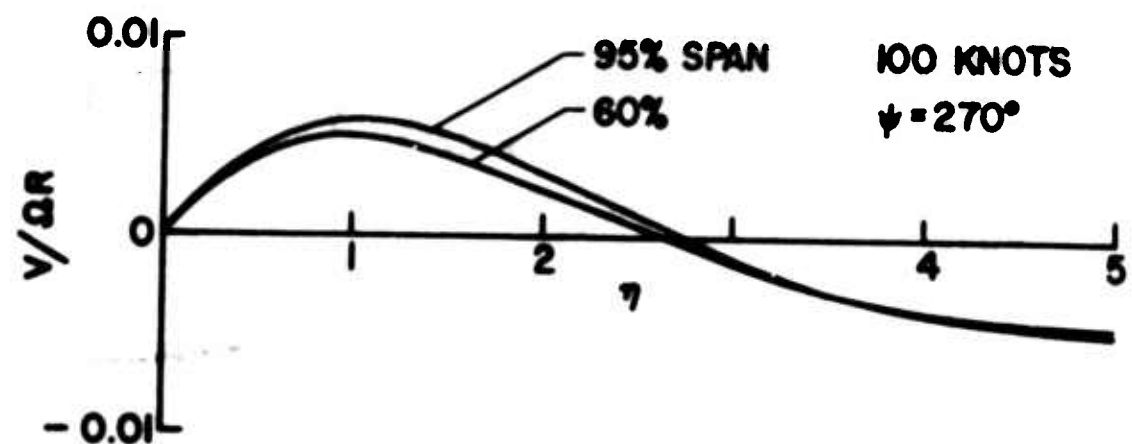


Figure 58. The Spanwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf and 10% Chord) in Forward Flight.

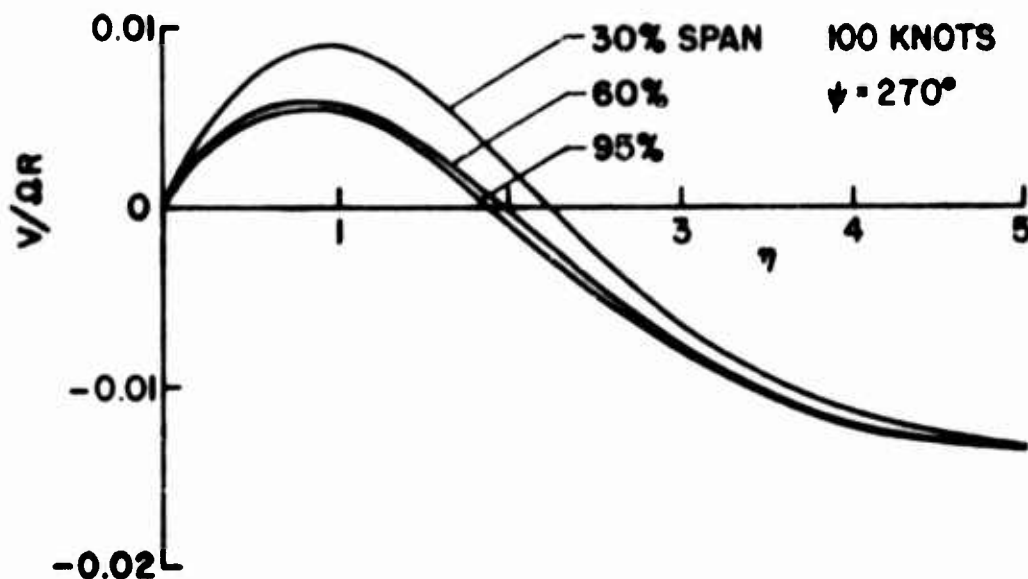


Figure 59. The Spanwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 30% Chord) in Forward Flight.

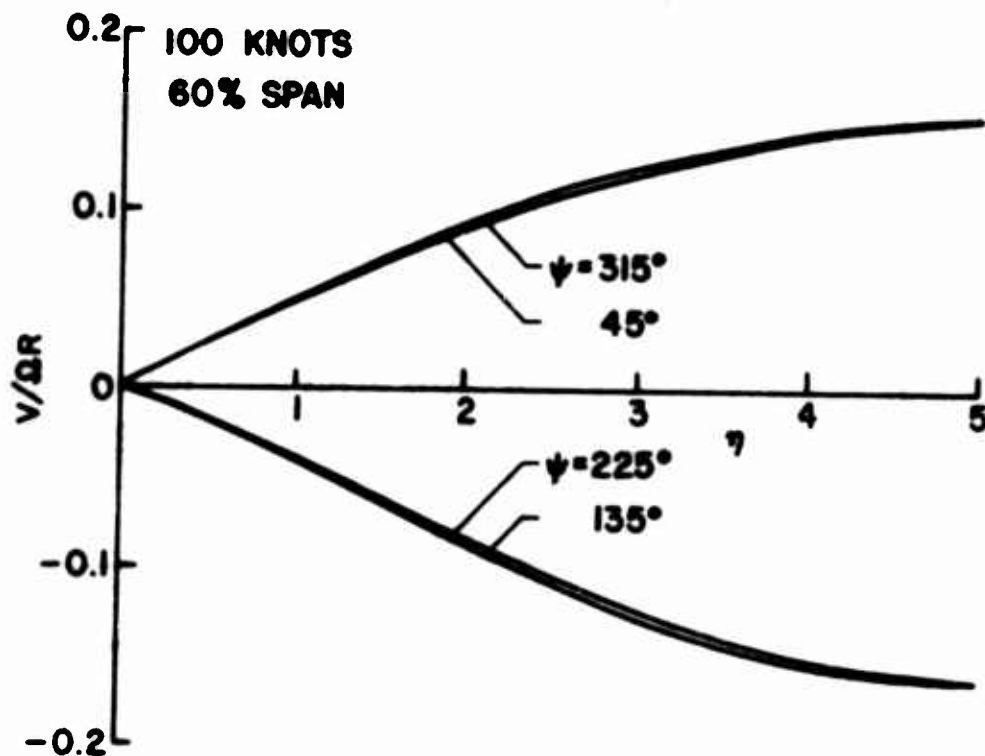


Figure 60. The Spanwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf and 10% Chord) in Forward Flight.

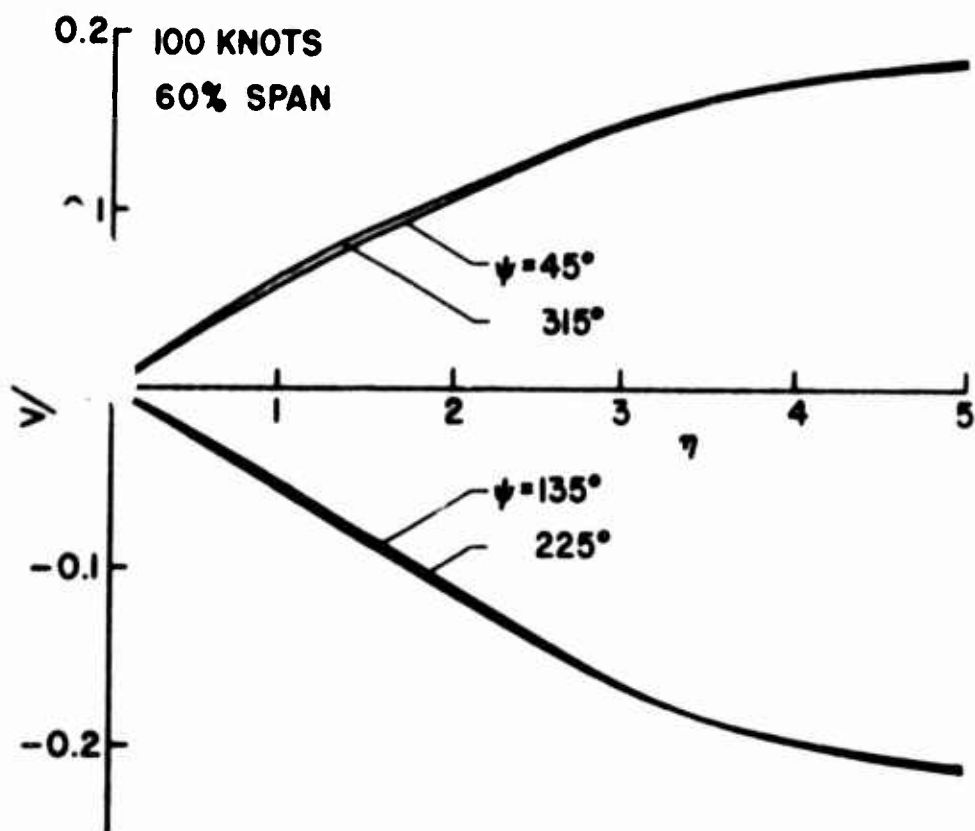


Figure 61. The Spanwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 30% Chord) in Forward Flight.

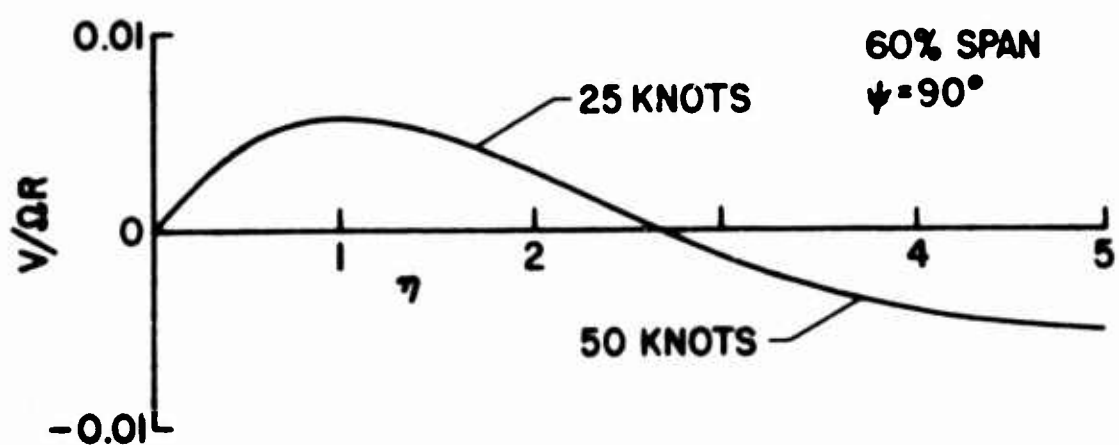


Figure 62. The Spanwise Velocity Profile for the 24-Foot Rotor (at 8000 lbf, 60% Span, and 10% Chord) in Forward Flight.

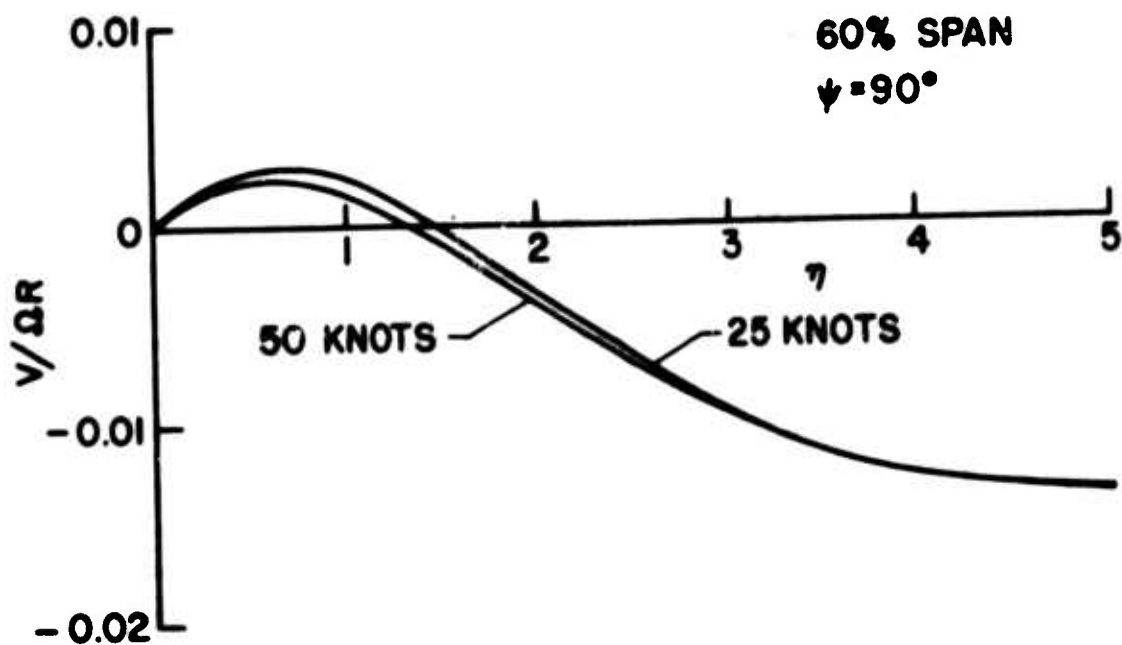


Figure 63. The Spanwise Velocity Profile for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack, 60% Span, and 30% Chord) in Forward Flight.

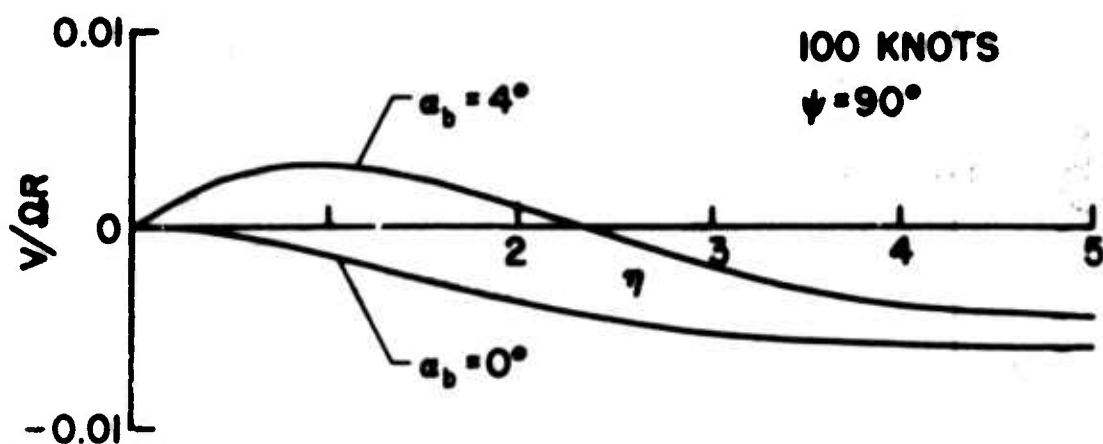


Figure 64. The Spanwise Velocity Profile for the 40-Foot Rotor (at 10% Chord and 60% Span) in Forward Flight.

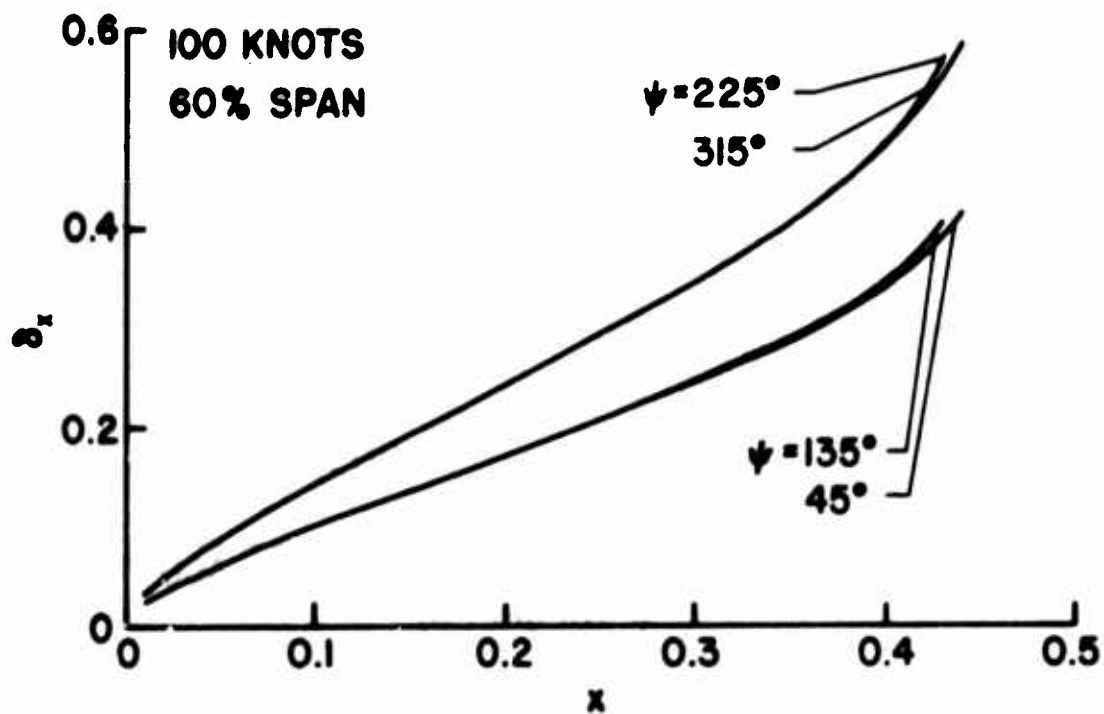


Figure 65. The Chordwise Displacement Thickness for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack and 60% Span) in Forward Flight.

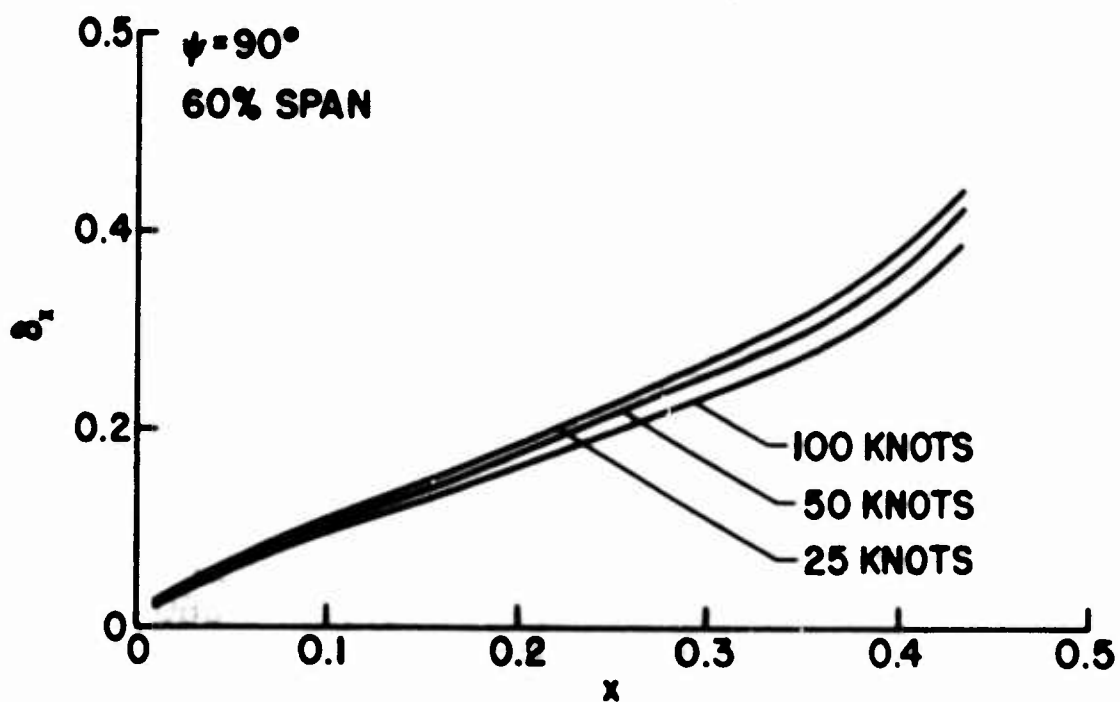


Figure 66. The Chordwise Displacement Thickness for the 40-Foot Rotor (at Zero Degrees Blade Angle of Attack) in Forward Flight.

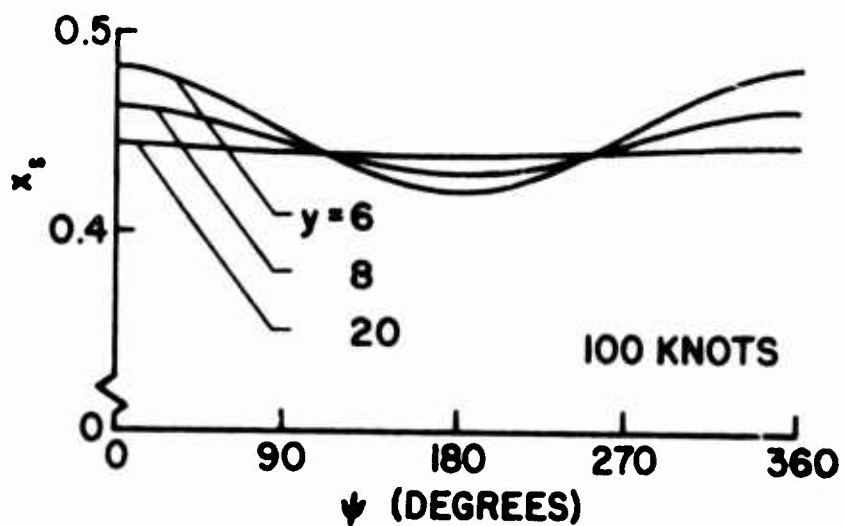


Figure 67. The Separation Line for the 40-Foot Rotor at Zero Degrees Blade Angle of Attack and 100 Knots Forward Flight Speed.

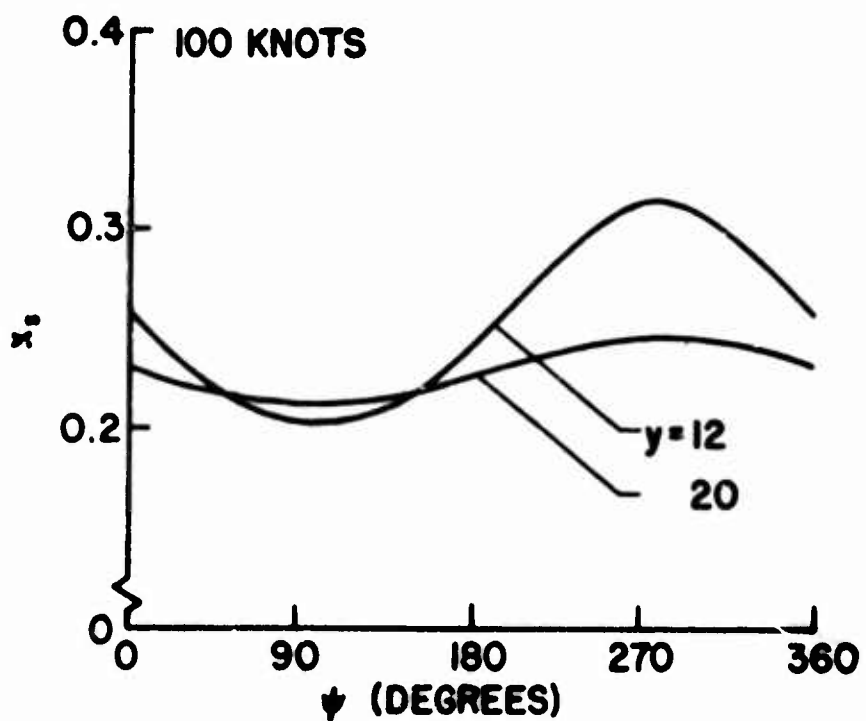


Figure 68. The Separation Line for the 40-Foot Rotor at 4 Degrees Blade Angle of Attack and 100 Knots Forward Flight Speed.

Figure 69 shows this effect more explicitly when there is no inflow. For no inflow, the curve has symmetry about the line  $\psi = 180^\circ$ , so the mean curve coincides with the line marked  $\psi = 90^\circ, 270^\circ$ . With inflow, the curves are no longer symmetric. Figures 70 and 71 show that the delay in separation at  $270^\circ$  is greater than the advance at  $90^\circ$ . Figures 72 and 73 emphasize the difference the inflow makes. For no inflow (Figure 72), the dependence on forward flight is eliminated at  $\psi = 90^\circ$ . With inflow the separation line actually advances with increased forward flight speed, as shown in Figure 73. This is misleading; Figure 74 gives a better understanding of the situation. The separation line will be advanced or delayed according to the azimuthal angle chosen. The effect of forward flight speed, span, and azimuthal angle should not be examined without consideration of Equation (75).

In Figure 75, the thrust dependence appears to contradict a previous result: the increase of angle of attack reduces the time dependent effects. However, the increase of inflow tends to increase time dependence, so there is no contradiction. The increase in thrust causes an increase in both angle of attack and inflow, and the inflow increase is the more important. A better view of the mean separation line, and its relation to separation on an airfoil in two-dimensional flow, can be found in Figures 76 and 77. The aerodynamic angle of attack variation decreases with span because only the change due to the time dependence of the chordwise component of forward flight is considered. When the angle of attack decreases, the separation line retreats toward the trailing edge with very nearly a linear dependence on angle of attack. As the angle of attack begins to increase, the response of the separation line lags. The separation line may even retreat for a time. As the angle of attack increases further, the separation line moves forward more rapidly than it moves back. It moves forward past the two-dimensional separation line but stops its advance as soon as the angle of attack ceases to increase.

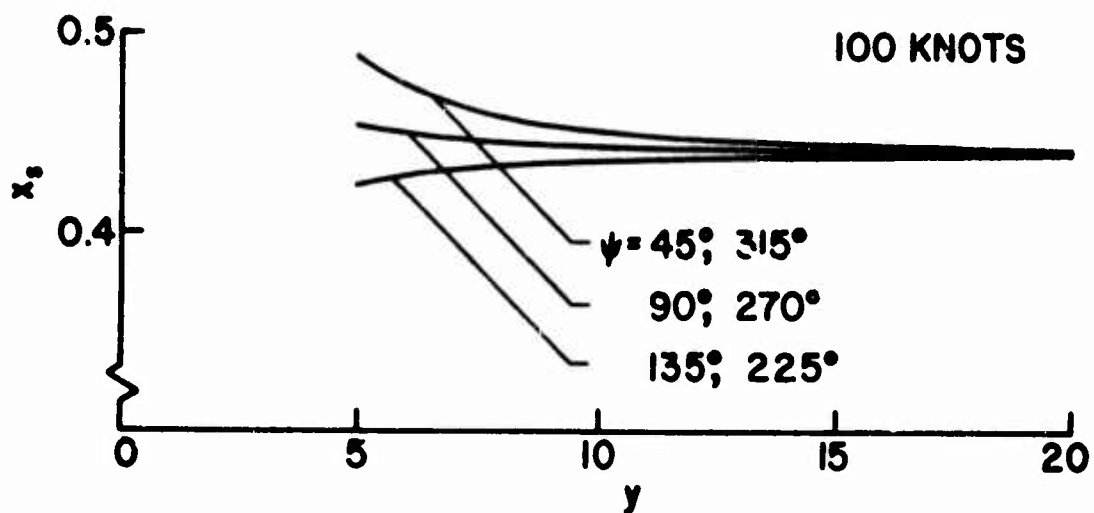


Figure 69. The Separation Line for the 40-Foot Rotor at Zero Degrees Blade Angle of Attack in Forward Flight.

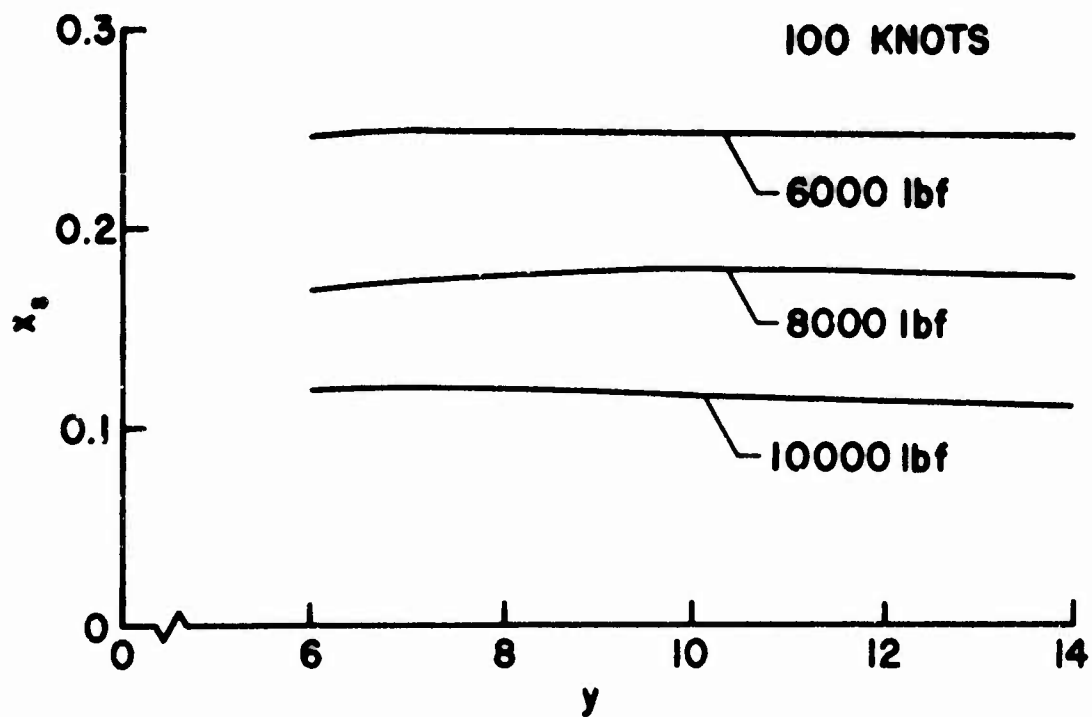


Figure 70. The Separation Line for the 24-Foot Rotor at 100 Knots and 90° Azimuthal Angle.

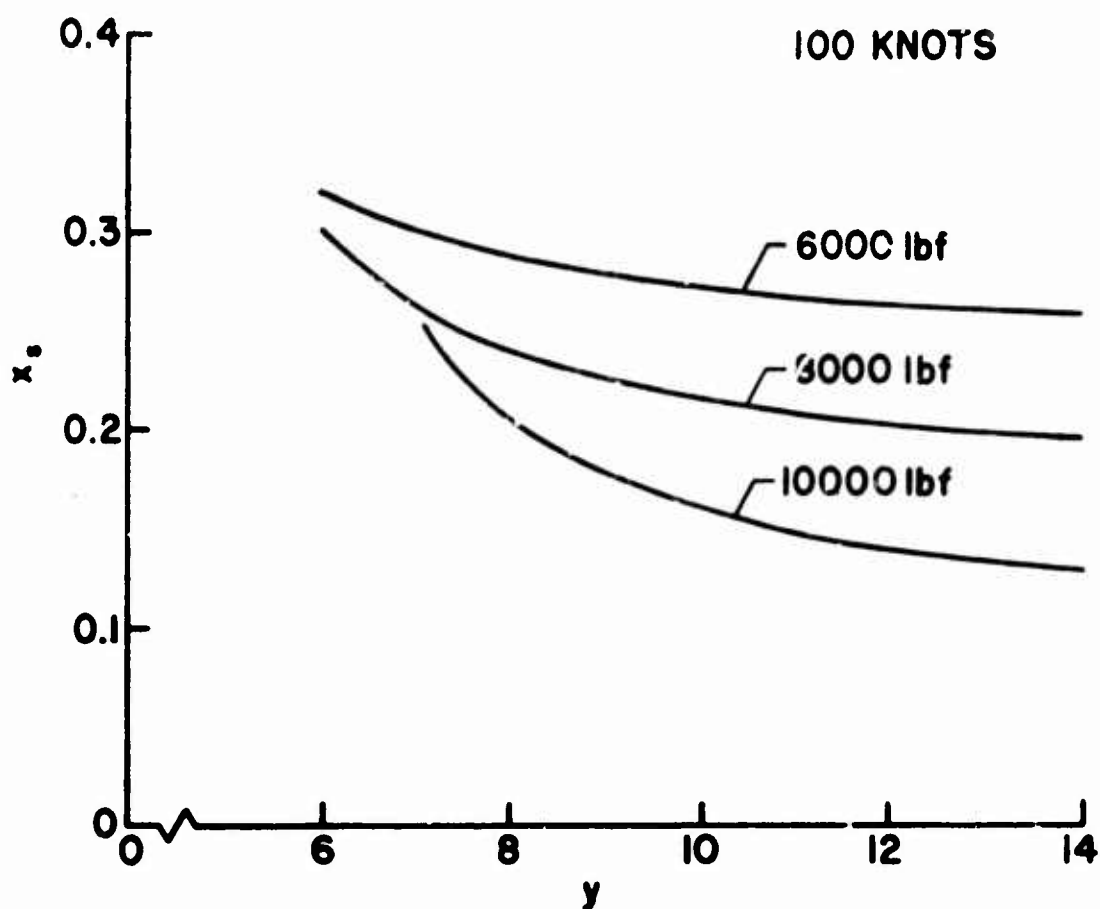


Figure 71. The Separation Line for the 24-Foot Rotor at 100 Knots and 270° Azimuthal Angle.

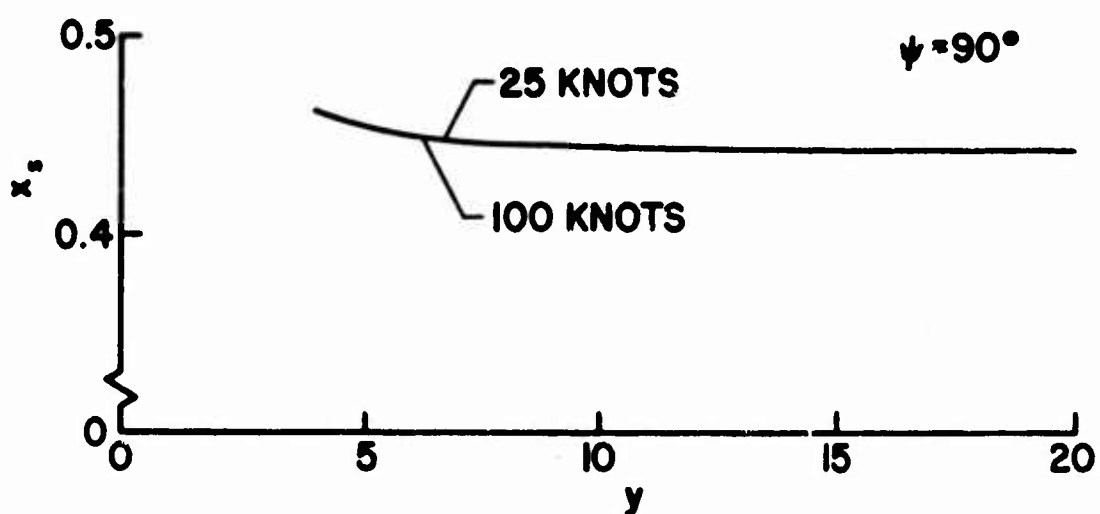


Figure 72. The Separation Line for the 40-Foot Rotor at Zero Degrees Blade Angle of Attack and 90° Azimuthal Angle.

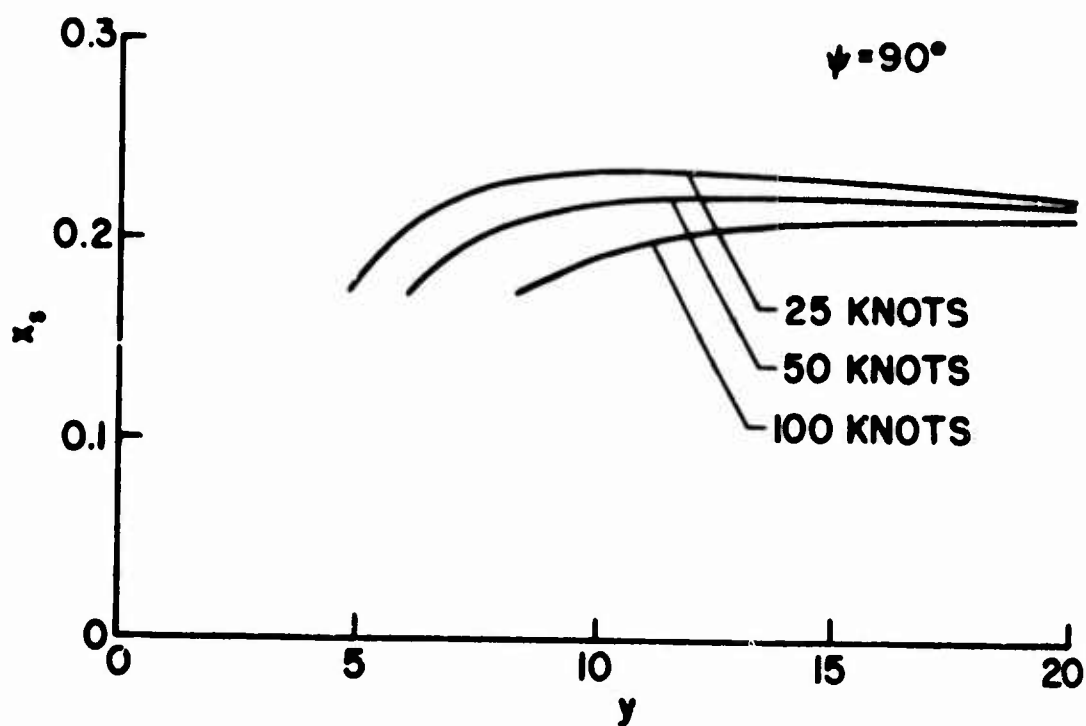


Figure 73. The Separation Line for the 40-Foot Rotor at 4 Degrees Blade Angle of Attack.

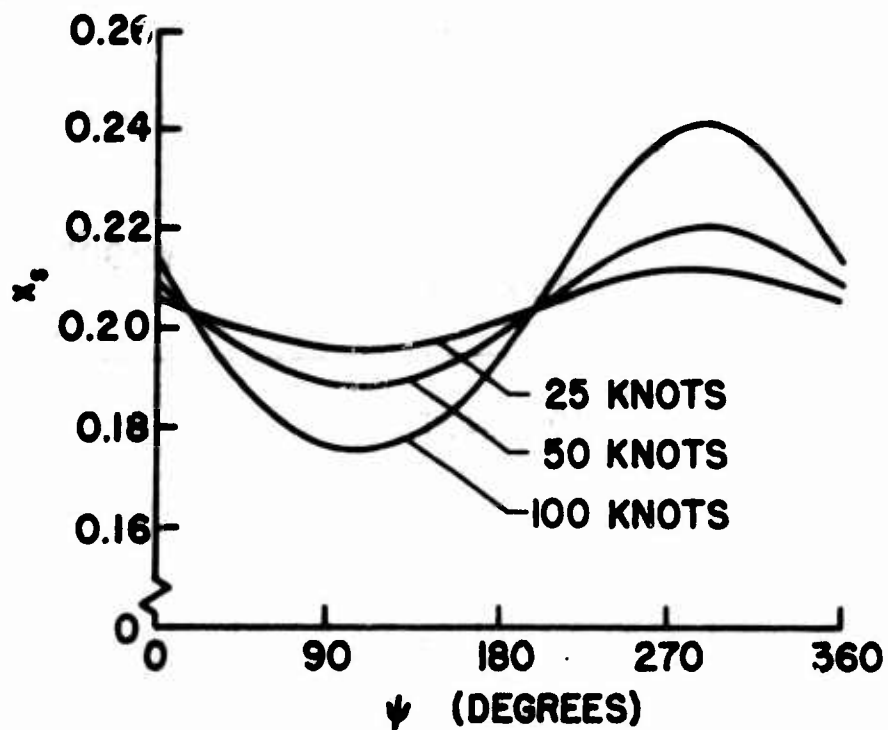


Figure 74. The Separation Line for the 24-Foot Rotor at 8000 lbf and 8 Chord Lengths From the Axis of Rotation.

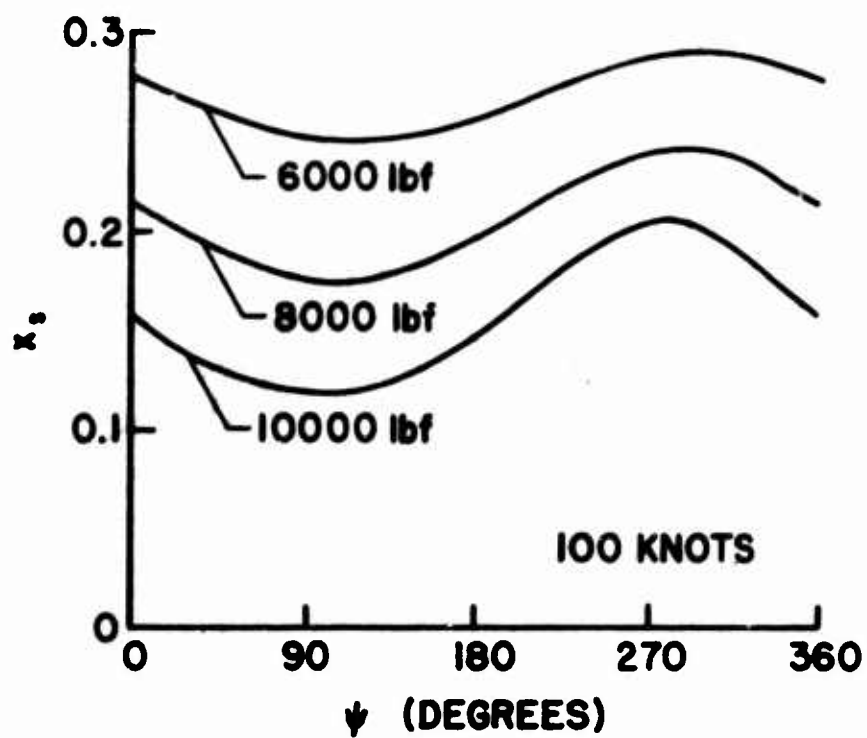


Figure 75. The Separation Line for the 24-Foot Rotor at 100 Knots and 8 Chord Lengths From the Axis of Rotation.

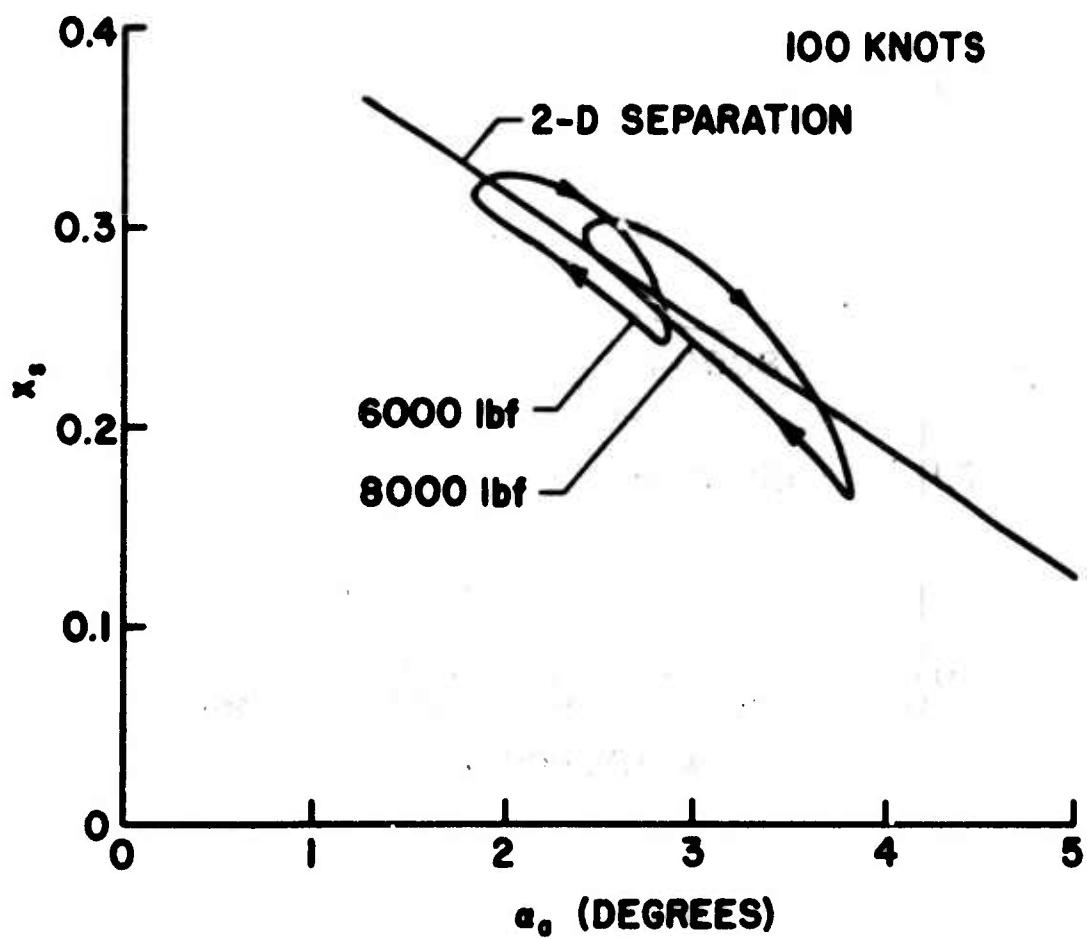


Figure 76. The Separation Line for the 24-Foot Rotor at 6 Chord Lengths From the Axis of Rotation.

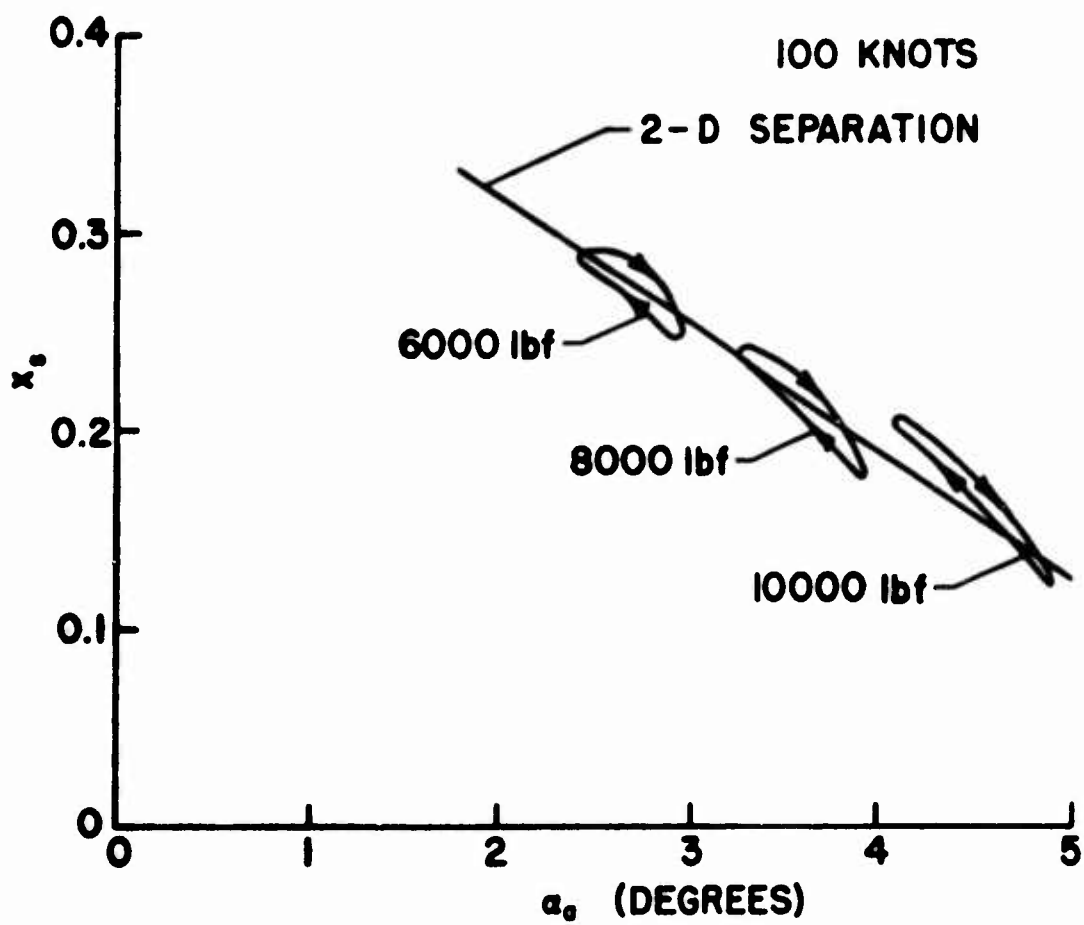


Figure 77. The Separation Line for the 40-Foot Rotor at 8 Chord Lengths From the Axis of Rotation.

## SUMMARY

A solution has been obtained for the laminar boundary layer on a rotating blade that includes the effects of lift and forward flight, but does not include tip effects, cyclic pitch, or flapping. The results are primarily applicable to a rotating blade that is part of a system of rotors. The entire rotor disk induces an inflow velocity that is proportional to span in the part of the analysis called the hover case. In the forward flight case, the induced velocity is a constant  $v_a$  and the speed of forward flight is  $s_H$ . Either  $s_H$  or  $v_a$ , or both, may be eliminated from consideration since they appear explicitly in the equations for velocity and separation. In all calculations, it is implicit that the axis of rotation is located at the quarter-chord point and that the blade is a symmetrical, 11.9-percent-thick Joukowski airfoil. The geometric blade angle of attack must be known before calculations are made. In the hover case, the constant of proportionality between the inflow and the span must also be known.

For both cases, the chordwise dimension is transformed so that in the transformed coordinates, the distance from the stagnation point to the separation point is independent of span and time. Time is more conveniently thought of as the angle through which the blade has rotated, the azimuthal angle  $\psi$ . In the transformed coordinates, the velocities are expanded in an asymptotic series in the span. The use of transformed coordinates makes it possible to approach the separation point. The first term in the series for the chordwise velocity satisfies the two-dimensional boundary layer equations at the geometric angle of attack. Higher terms in the asymptotic series are given by linear equations, allowing the time dependence to be removed by applying the principle of superposition. The accuracy of the asymptotic series cannot be easily estimated, but the hover case seems to be accurate for  $y$  greater than about 2. The forward flight case, with  $s_H = 0$ , depends strongly on the value of  $v_a$ . For  $v_a = .1$ , there may be sufficient accuracy for values of span greater than 3 or 4. For forward flight,  $y + T_2$ , as well as  $y$ , must be large. The asymptotic series is useful for relatively small deviations from the solution at large span. The most practicable criterion for accuracy is that the higher terms be small compared to the first term in the series. By this criterion, the solution well fulfills its purpose. Since most helicopter blades have aspect ratios greater than 10 or 12, the solution is able to assess the importance of each effect over most of the blade.

Many of the effects seen in the results have been described in previous work. The asymptotic solutions at large span showed that the chordwise flow approaches the two-dimensional flow over an airfoil. At smaller values of span, separation is delayed by rotational effects. Rotational effects cause an S-shaped velocity profile in the spanwise flow. The potential flow generally causes flow toward the axis of rotation, but rotational effects which become important near the surface cause outward

flow. Rotation, by itself, has little influence on the separation line except for the region close to the axis of rotation.

In forward flight, the spanwise flow is much larger than in the hover case. The rotational effects are still present, but the spanwise flow due to yaw is dominant. Through the effect of changes in the yaw angle, the chordwise flow becomes time dependent. Although the present solution cannot account for flapping or feathering, it is clear that time-dependent effects in the forward flight case are much larger than the effects of rotation in the hover case. The aerodynamic angle of attack, which changes with time, is an important influence on the chordwise flow, and the time derivative of the chordwise flow is also important. The time dependence decreases as span increases, and its effect seems to diminish as angle of attack increases, provided the inflow is held constant.

The time dependence can best be assessed by examining the displacement thicknesses or the separation line, instead of the velocity profiles. Much of the spanwise dependence and time dependence of the velocity has been accounted for by nondimensionalizing by  $u_0$  and by transforming the dimensional normal to the surface by a Falkner-Skan transformation.

The separation line oscillates due to forward flight. The maximum delay occurs at an azimuthal angle that seems to depend primarily on the aerodynamic angle of attack and secondarily on the time derivative of the chordwise flow. In hover, separation depends almost entirely on the aerodynamic angle of attack. In forward flight, even for small oscillations of the angle of attack, there is a lag in the response of the separation line to increases in the angle of attack.

The rotor blade under consideration is part of a rotor disk which generates thrust. The values of angle of attack, thrust, and inflow were chosen to be representative of two specific helicopters. As thrust increases, both angle of attack and inflow increase. In hover, separation is closely correlated to the aerodynamic angle of attack. The form or magnitude of inflow, thrust or geometric angle of attack was unimportant except for its influence on aerodynamic angle of attack. In forward flight, the situation is much more complex. At constant angle of attack, increased inflow delays separation and increases time dependence. For no inflow, time dependence decreases as angle of attack increases, but for large inflow the opposite is true. Inflow also shifts the phase angle of the separation line through its influence on the aerodynamic angle of attack. When the dependence of the separation line on thrust is considered, the magnitude of the oscillations increases as thrust increases, and for normal values of thrust, inflow is the primary influence on the phase angle of the separation line.

## CONCLUSIONS

As a result of the present study, certain conclusions can be made regarding the effects of rotation, forward flight speed and inflow on the laminar boundary layer development and laminar separation line on a rotating helicopter blade. The analysis is limited in that such effects as lead-lag, flapping, feathering, and reverse flow must be excluded at this time due to the mathematical complexity involved in including these effects. Within the framework of the present analysis, it is concluded that:

1. The technique of scaling the chordwise coordinate so that separation always occurs at the same location in the scaled coordinate offers an excellent method for studying three-dimensional time-dependent boundary layers where the separation line varies with spanwise location and time.
2. The normal delay of the separation line near the axis of rotation due to blade rotation is obtained. This effect has been previously obtained by a number of investigators and is well known. As usual, the chordwise flow asymptotically approached the two-dimensional flow over the blade at large span.
3. In the hover case, two solutions have been obtained: the normal hover case with a linear variation of inflow, and in the forward flight solution, the limiting case of zero forward flight speed in which the inflow is constant. In each case the delay in separation due to rotation is still present, but the main effect of the inflow is to change the aerodynamic angle of attack. Compared at the same span and aerodynamic angle of attack, neither the form of the inflow (constant or variable along the span) nor its magnitude has a significant effect on the separation point. As in two-dimensional flow, increasing the aerodynamic angle of attack moves separation forward on the blade.

The main effect of thrust level is to change the aerodynamic angle of attack, by altering both the geometric angle of attack and the inflow velocity. As the thrust level is increased, the geometric angle of attack, the inflow velocity, and the aerodynamic angle of attack are all increased. In general, then, increasing the thrust level moves the separation line forward on the blade. However, increasing the thrust level decreases the spanwise variation of the separation line.

4. At normal helicopter forward flight speeds, the dominant spanwise flow is the time-dependent flow due to the angle of yaw. This, together with the time-dependent chordwise flow

due to changing yaw angle, causes a time-dependent separation line oscillating about the no-forward-flight separation line. Both the average delay in separation and the oscillations are diminished as the angle of attack of the blade increases (if inflow is held constant).

The simultaneous action of inflow and forward flight speed causes the magnitude of the oscillations of the separation line to increase, for a given geometric angle of attack. When the inflow is zero, the time dependence of the separation line is dominated by the time derivative of the chordwise flow. It is most favorable to delayed separation at the extreme downstream position of the blade. This makes the maximum delay in separation occur at  $\psi = 360^\circ$ . Inflow causes the aerodynamic angle of attack to vary with time so that it is smallest at  $\psi = 270^\circ$ . The combination of inflow and forward flight shifts the maximum delay of the separation line into the fourth quadrant.

In the forward flight case, increasing the thrust level moves the separation line forward at all values of  $\psi$  and increases the magnitude of the oscillations of the separation line.

5. Increases in forward flight speed cause the magnitude of oscillations of the separation line to increase.
6. When plotted as a function of angle of attack, the separation line in forward flight, for fixed span, describes a loop (for normal thrust levels). This indicates that the oscillations of the separation line are not correlated with angle of attack alone, as in the hover case. This indicates that other nonsteady effects, in addition to the time varying angle of attack, are important in determining the separation line.
7. When the inflow is zero, the phase advance angle (the angle between the maximum of the shear stress at the wall and the maximum of the velocity at the edge of the boundary layer) agrees qualitatively with the solution of Lighthill.<sup>18</sup>

### RECOMMENDATIONS

1. The technique of scaling the chordwise coordinate so that separation always occurs at the same location in the scaled coordinate offers an effective method for attacking the more realistic problem of turbulent boundary layers on rotating blades. It is recommended, therefore, that this method be employed in a study of the turbulent boundary layer on a rotating helicopter blade.
2. Additional studies should be conducted to isolate and determine more clearly the true nature of the unsteady boundary layer effects on the separation line on a helicopter blade. A first step would be to eliminate the effects of rotation and study the nonsteady boundary layer effects alone on an airfoil blade.
3. Additional work must be done to include the effects of lead-lag, feathering, cyclic pitch, and flapping, which occur in real helicopter blade motions, into the analysis of the boundary layer on rotating blades. It is recommended that a continuing effort be made to develop techniques which will allow the incorporation of these effects into the boundary layer analysis.

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## APPENDIX I

### THRUST CALCULATIONS

In order to calculate the boundary layer development on a rotating blade, it is necessary first to determine the inviscid flow over the blade. In practice, this inviscid flow depends on such factors as the geometry of the blade (airfoil section), the geometric angle of attack of the blade, the forward flight speed, and the inflow velocity due to the blade system. In the design of a real helicopter, these factors are not independent and therefore cannot be chosen arbitrarily. The inflow, for example, depends upon the airfoil section, the geometric angle of attack, and the forward flight speed, as well as on the number of blades in the helicopter rotor  $N_b$ , the radius (span) of the blade system  $R$ , the thrust level  $T$ , the rotational speed  $\Omega$ , and the blade chord  $c$ . It is not possible, therefore, to assign values to each of these parameters independently. In order to obtain a reasonably consistent set of parameters required to determine the potential flow, calculations were made for two helicopters for which some of the above parameters were specified in the contractual arrangement for the present work. The remaining parameters were then calculated using simple momentum theory and blade element theory. The calculations required to obtain the above parameters are outlined herein.

In the present work, the potential flow calculations were made for two helicopters. The first was a helicopter with a two bladed rotor of 24-foot radius, a blade chord of 21 inches with the rotor rotational velocity of 295 revolutions per minute, and blade loadings corresponding to vehicle gross weights of 6,000, 8,000, and 10,000 pounds. The second helicopter was to have six blades of 40-foot radius and 24-inch chord, with a rotational velocity of 143.3 revolutions per minute and blade geometric angles of attack of 0 and 4 or 6 degrees. In each case the blade airfoil section was a 11.9%-thick symmetrical Joukowski airfoil and the blades were assumed to operate in an atmosphere of density 0.0765 lbm/ft<sup>3</sup> and with a speed of sound of 1117 ft/sec. These operating conditions are summarized in Table III, together with several parameters which characterize the airfoil aerodynamics.

It is also necessary, in the forward flight case, to prescribe the forward flight speed. In order to compare the two helicopters on the same basis, the speed calculations were made for both helicopters at forward flight speeds of 25, 50 and 100 knots. This corresponds to values of  $s_H$  of 0.7807, 1.5613 and 3.1226 for the 24-foot rotor and to values of 1.406, 2.812 and 5.625 for the 40-foot rotor.

These prescribed values are still not sufficient for the calculation of the potential flow over the blade section. For the helicopter with the 24-foot rotor, it will be necessary to determine the geometric angle of attack and the inflow velocity; for the helicopter with the 40-foot rotor, it will be necessary to determine the thrust level and the inflow velocity.

| TABLE III. CHARACTERISTICS OF TWO HELICOPTERS |                 |                     |                     |
|-----------------------------------------------|-----------------|---------------------|---------------------|
| Symbol                                        | Dimensions      | 24-Foot Rotor       | 40-Foot Rotor       |
| R                                             | ft              | 24                  | 40                  |
| $\Omega$                                      | RPM             | 295                 | 143.3               |
| c                                             | ft              | 1.75                | 2.00                |
| $N_b$                                         | -               | 2                   | 6                   |
| $A_R$                                         | -               | 13.71               | 20.00               |
| T                                             | lb              | 6,000               | -                   |
|                                               |                 | 8,000               | -                   |
|                                               |                 | 10,000              | -                   |
|                                               |                 | -                   | 0                   |
| $\alpha_b$                                    | deg             | -                   | 4 or 6              |
|                                               |                 | -                   | 4 or 6              |
| $\Omega c$                                    | ft/sec          | 54.06               | 30.01               |
| $\sigma_r$                                    | -               | 0.04642             | 0.09549             |
| $M_t$                                         | -               | 0.665               | 0.6884              |
| $\pi \rho \Omega^2 R^4$                       | lb <sup>f</sup> | $2.360 \times 10^6$ | $4.302 \times 10^6$ |

The calculations required to obtain these additional parameters were based on momentum theory and blade element theory. For clarity and convenience, the simplest form of each theory will be used and all variables will be written in nondimensional form. Both of these theories are available in texts on helicopter aerodynamics.<sup>19,20</sup>

The starting point for each of these theories is the thrust coefficient defined by

$$C_T = \frac{T}{\pi \rho R^2 (\Omega R)^2} \tag{85}$$

The simple or classical momentum theory relates an element of thrust  $dc_T$  to the downflow velocity  $v_i$  in an annulus of radius  $y$  and width  $dy$  by

$$d c_T = - S_s (2 v_i) \frac{2}{A_R} y dy \quad (86)$$

where  $S_s = \sqrt{s_H^2 + v_i^2}$ . The thrust coefficient is found by integration over the span and azimuthal angle:

$$c_T = \int_0^{A_R} \int_0^{2\pi} d c_T d \psi dy \quad (87)$$

For the hover case, the forward flight speed is zero, and the inflow velocity is taken herein to have the form  $v_i = -\omega_i y$ . This yields

$$c_T = \omega_i^2 \quad (88)$$

For the forward flight case, the inflow is taken herein to be constant, i.e.,  $v_i = -v_a$ . Integration of Equation (87) for this case yields

$$c_T = 2 \sqrt{s_H^2 + v_a^2} \frac{v_a}{A_R^2} \quad (89)$$

In either case, then, the momentum theory yields a relation between thrust coefficient and inflow velocity.

The simplest blade element theory calculates the lift from the slope of the two-dimensional lift curve  $a_0$ . It is assumed that the angle of attack and inflow are small so that  $\tan \alpha_b \approx \alpha_b$  and  $\tan (v_i / \bar{U}_0) \approx v_i / \bar{U}_0$ . The velocity relative to the blade in the chordwise direction consists of the inflow in the  $z$  direction and  $\bar{U}_0$  in the plane of rotation. Further  $\bar{U}_0$  contains two components: one due to the rotation and the other due to forward flight. An element of one blade of length  $dy$  then produces an element of thrust of

$$d c_t = \frac{a_0}{2\pi A_R} (\alpha_b \bar{U}_0^2 + \bar{U}_0 v_i) dy$$

where

$a_0$  is the slope of the lift curve (taken as 6.016 per radian, corresponding to an NACA 0012 airfoil)

$$\bar{U}_0 = y + s_H \sin \psi$$

For the hover case ( $s_H = 0$ ,  $v_i = -\omega_i y$ ),

$$C_t = a_0 \sigma_r (\alpha_b - \omega_i) / 6$$

where

$$\sigma_r = N_b c / \pi R$$

The number of blades in the helicopter rotor has been incorporated into the solidity of the rotor  $\sigma_r$ . For the forward flight case ( $v_i = -v_a$ ),

$$C_t = a_0 \sigma_r \left( \frac{\alpha_b}{6} + \frac{\alpha_b}{4} \left( \frac{s_H}{A_R} \right) - \frac{v_a}{4A_R} \right)$$

The blade element theory relates the thrust coefficient to the geometric angle of attack and the inflow. When the results of blade element theory are combined with the results of momentum theory, one has two relations between the thrust coefficient, the geometric angle of attack and the inflow. If any one of these is given, the other two are then uniquely determined.

The aerodynamic angle of attack is also necessary in the calculation of the potential flow. Once the geometric angle of attack is known, the aerodynamic angle of attack is easily computed from the simple geometric relationship

$$\alpha_a = \alpha_b - \arctan \frac{v_i}{\bar{U}_0}$$

These relations have been utilized to obtain the additional information necessary for the potential flow calculations in the present analysis. In the case of the 24-foot rotor system, they are used to obtain the blade geometric angle, the inflow constant, and the aerodynamic angle of attack. In the case of the 40-foot blade system, they are used to obtain

the inflow, the aerodynamic angle of attack, and the thrust. The thrust level for the 40-foot rotor system is not used directly in the calculations but is obtained as a matter of interest. The results of these calculations are presented in Table IV for the hover case and in Table V for the forward flight case. In Table IV, the aerodynamic angle of attack corresponds to the presented values of geometric angle of attack  $\alpha_p$  and inflow constant  $w_i$ . In the case of forward flight, the aerodynamic angle of attack varies with azimuthal angle and therefore is not presented.

| TABLE IV. THRUST CALCULATIONS IN HOVER |                                         |                              |                                       |                    |
|----------------------------------------|-----------------------------------------|------------------------------|---------------------------------------|--------------------|
| Rotor Span<br>(ft)                     | Aerodynamic<br>Angle of Attack<br>(deg) | Thrust<br>(lb <sup>f</sup> ) | Geometric<br>Angle of Attack<br>(deg) | Inflow<br>Constant |
| 24                                     | 3.12                                    | 6,000                        | 6.01                                  | 0.0504             |
| 24                                     | 4.16                                    | 8,000                        | 7.50                                  | 0.0582             |
| 24                                     | 5.21                                    | 10,000                       | 8.94                                  | 0.0651             |
| 40                                     | 0                                       | 0                            | 0                                     | 0                  |
| 40                                     | 2.38                                    | 17,140                       | 6.00                                  | 0.0631             |

| TABLE V. THRUST CALCULATIONS FOR A FORWARD FLIGHT SPEED OF 100 KNOTS |                              |                                       |                    |
|----------------------------------------------------------------------|------------------------------|---------------------------------------|--------------------|
| Rotor Span<br>(ft)                                                   | Thrust<br>(lb <sup>f</sup> ) | Geometric<br>Angle of Attack<br>(deg) | Inflow<br>Constant |
| 24                                                                   | 6,000                        | 3.349                                 | 0.0765             |
| 24                                                                   | 8,000                        | 4.465                                 | 0.1020             |
| 24                                                                   | 10,000                       | 5.581                                 | 0.1274             |
| 40                                                                   | 0                            | 0                                     | 0                  |
| 40                                                                   | 25,600                       | 4.000                                 | 0.2117             |

The information given in Tables IV and V, together with that given in Table III, is sufficient to make the calculations of the inviscid flow over the blade.

## APPENDIX II

### COMPUTER PROGRAMS

Separate computer programs were used for the hover case and the forward flight case. In "Method of Solution," the more important features of these programs were discussed. The programs themselves have been well annotated by comment cards, and a brief list of the correspondence between the variable names used in the report and the FORTRAN names is presented in Table VI. The listing of the programs follows. The subroutine RK1 is omitted from the forward flight program because it is shown in the hover program.

| TABLE VI. FORTRAN NOMENCLATURE         |                 |
|----------------------------------------|-----------------|
| Name Used in the Report                | FORTTRAN Name   |
| $F''_0, G''_0, G''_{01}, \text{ etc.}$ | DVDZ            |
| $F'_0, G'_0, G'_{01}, \text{ etc.}$    | VEL             |
| $m_{10}$                               | XM10            |
| $x$                                    | X               |
| $\Delta x$                             | DX              |
| $\xi$                                  | XI              |
| $\Delta \xi$                           | DXI             |
| $\alpha_b$ (degrees)                   | ALPHB           |
| $\alpha_b$ (radians)                   | AB              |
| $\epsilon$                             | EPS             |
| $\delta$                               | DELTA, DEP, DET |
| $\delta_I$                             | DEI             |
| $\sigma$                               | T               |
| $\sigma_I$                             | SI              |
| $\eta$                                 | Z               |
| $\Delta \eta$                          | DZ              |

```

C A FORTRAN PROGRAM FOR A ROTATING, HOVERING, SYMMETRIC AIRFOIL WITH
C LIFT ; INFLCW=TR*SPAN
C PROGRAM SIZE: 97880 BYTES IN FORTRAN IV-G,LEVEL1,MOD3,RELEASE15
C LESS THAN 100 PAGES ARE PRINTED
C RUN TIME (CPU) IN MINUTES=5+.4*(KMAX-34) WHERE KMAX IS THE VALUE
C OF K AT THE LAST STATION AND MACHINE IS IBM 360/75 AT TUCC
C IMPLICIT REAL*8 (A-H,O-Z)
C COMMON/BIJ/EPS,SGN,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10
C THIS DIMENSION STMT ASSUMES K<46,IBLUP<61,M<4,J<6,L<73,I<5,ITER<10
C DIMENSION X(45),DX(45),XI(45),DXI(45),BP(20),NBLUP(60),V(3,5),
1 DD(3,5),Y(3,5,72,4),ST(6),U(3,5),ERR(9),DVZ(5,9),ITERKP(5),
2 HIS(9),VINP(5),Z(72),CZ(72),A(3,5,72),S(4),THT(5),XD2(5),
C DIMENSION PCC(50),UTIL(50),VDEL(50),TK(6),E(4),UP(75),VP(75),
C D ILIM(4,4)
C EXTERNAL DSDX
C ACCURACY IS CONTROLLED BY THE VALUES OF ERRMAX,ZMAX,DXI(K),JS,
C CZ(1),IMAX
C ERRMAX SEEMS TO HAVE LITTLE EFFECT ON ACCURACY,BUT IF IT IS TOO
C SMALL THE VALUE OF THE 2ND DERIVATIVE OF THE STREAM FUNCTION WRT
C ETA (DVDZ) CANNOT BE FOUND TO SUFFICIENT ACCURACY AT THE WALL TO
C CONVERGE THE EQUATION.
C ZMAX MUST BE LARGE ENOUGH SO THAT DVDZ IS LESS THAN ABOUT .001
C AT ZMAX; IF IT IS LARGER THAN NECESSARY SPEED AND CONVERGENCE
C WILL SUFFER. THE LARGER ZMAX IS, THE MORE AN ERROR IN DVDZ AT THE
C WALL WILL AFFECT THE ERROR IN VELOCITY AT ZMAX, ESPECIALLY NEAR
C SEPARATION.
C THE EFFECT OF DXI(K) IS DESCRIBED AT LENGTH IN THE AIAA JOURNAL,
C VOL. 1, P.2062, 1963, AND IN PREVIOUS WORK BY A.M.O. SMITH
C THE ERROR IN THE RUNGE KUTTA INTEGRATION IS PROPORTIONAL TO DZ**4
C THE ERROR IN RK3 IS PROPORTIONAL TO H**4, H IS DETERMINED BY JS
C IMAX-1 MAY NOT BE LARGER THAN THE NO. OF PREVIOUS K STATIONS
C KNOWN. FOR K>=8 (K STARTS AT 5) IMAX=3 MAY GIVE SUFFICIENT
C ACCURACY FOR THE XI DERIVATIVES.
C BEGIN SECTION 1
C PARAMETERS FOR VELOCITY EQNS
C LMAXU=69
C LPAX=40
C JCIP=5
C JCIM IS THE NO. OF EQNS;IT IS THE MAXIMUM VALUE OF J
C DO 67 I=1,4
67 S(I)=0.00
C CC 1480 J=1,5
C JCIM<=5 REQUIRED
C VINP(J)=0.00
C CC 1480 P=1,3
C CC 1480 L=1,LMAXU
C CC 1480 I=1,4
1480 Y(M,J,L,I)=0.00
C VINP(1)=1.00
C IN THE LOCAL IMPLEMENTATION AT TRIANGLE UNIVERSITIES COMPUTING
C CENTER (TUCC), FILE 1 IS THE DATA CARDS AT THE END OF THIS DECK,
C FILE 2 IS THE CARD PUNCH, FILE 3 IS AN 133 SPACE/LINE PRINTER
C THE FIRST DATA CARD HAS, IN FORMAT(5F15.10) THE INITIAL ESTIMATES
C OF CVZ FOR THE 5 EQNS.
C THE 2ND DATA CARD CONTAINS, IN FORMAT(F10.7,F10.4,2F10.7,F20.16,
C F10.7) THE VALUES OF:
C 1 XO: THE POSITION OF AXIS OF ROTATION ALONG THE CHORD;XO=.25 IS
C 25% CHORD
C 2 AR: THE ASPECT RATIO; IT IS USED ONLY IN STMT 8189
C 3 ALPHB: THE GEOMETRIC ANGLE OF ATTACK OF THE BLADE IN DEGREES
C 4 TR: THE NONDIMENSIONAL INDUCED VELOCITY=TR*(SPAN/CHORD)
C IT IS OF THE ORDER OF THE SQUARE ROOT OF THE COEFFICIENT

```

```

C      OF THRUST
C      5 XIINF: THE POSITION OF THE STAGNATION POINT IN BODY COORDINATES
C              IT IS NORMALLY NEGATIVE, I.E. ON THE BOTTOM SURFACE OF
C              THE AIRFOIL. IF IT IS NOT KNOWN, READ IN A VALUE OF 14.00
C              AND THE PROGRAM WILL CALCULATE A CORRECT VALUE
C      6 RK2: THIS IS THE CONSTANT K2 IN THE EQN FOR LOWER CASE Q. IF IT
C              IS NOT KNOWN, PUT IN AN ESTIMATE (SAY .1) AND AFTER THE PGM
C              RUNS IT MAY BE FOUND BY EXTRAPOLATING  $-F2C^*/F2K^*$  TO THE
C              SEPARATION POINT XIS. XIS IS THE VALUE OF XI AT WHICH  $F0^*$ 
C              AT THE WALL (Y(3,1,2,1)) GOES TO ZERO. SINCE THE 1ST EQN
C              (FOR F0) WILL NOT CONVERGE FOR  $F0^*$  AT WALL < .06 TO .04,
C              XIS IS FOUND BY EXTRAPOLATION.
C      READ(1,1481) (DVZ(J,1), J=1,JDIM)
1481 FORMAT(5F15.10)
C      THIS WRITE STMT IS FOR CHECKOUT PURPOSES
C      WRITE(3,2001) (DVZ(J,1), J=1,JDIM)
2001 FORMAT( ' DVZ=',6D18.9 )
C      CC 1399 IS=1,5
1399 HIS(IS)=.500*(IS/3+IS/5)
C      CC 1485 M=1,3
1485 U(M,1)=0.00
C      DVZINC=.00500
C      ERRMAX=2.0-5
C      WRITE(3,152) JDIM,LMAX,ERRMAX
152 FORMAT(/' NO. EQNS=',I2,5X,'LMAX MUST BE < ',I3,5X,'MAX ERROR IN VE
C      LOCITY AT MAX ETA=',D12.5/)
C      CZST=.100
C      ZMAX=5.5
C      Z(2)=0.00
C      DZ(1)=.1
C      Z(1)=-.1
C      FIX ST2P SIZE IN ETA (Z) AND INCREMENT OF ETA (DZ)
C      DF=0
C      CC 1484 L=2,LMAXU
C      Z(L)=(L-2)*DZST
C      IF( Z(L) .LT. ZMAX ) LLL=L
1484 DZ(L)=Z(L)-Z(L-1)
C      ZMAX=Z(LLL-1)
C      CZST=DZ(LLL)
C      N=0
C      CC 954 L=LLL,LMAXU
C      CZE=2.00*DZ(L-1)-DZ(L-2)
C      D12=1.00/CZST+ZMAX-Z(L-1)-CZE/4.00
C      CZ(L)=(1.00+ZMAX**2+Z(L-1)*(Z(L-1)-2.00*ZMAX) )/D12
C      IF( DZ(L) .GT. .400) N=1
C      IF(N.EQ. 1) DZ(L)=.400
954 Z(L)=Z(L-1)+DZ(L)
C      WRITE(3,1482) (L,Z(L),DZ(L),L=1,LMAXU)
1482 FORMAT(/ ' L',6X,'Z',9X,'DZ',6X,'L',6X,'Z',9X,'DZ',6X,'L',6X,'Z',
1,9X,'DZ',6X,'L',6X,'Z',9X,'DZ',6X,'L',6X,'Z',9X,'DZ' / (I4,2F10.6,
2',',I4,2F10.6,',',I4,2F10.6,',',I4,2F10.6,',',I4,2F10.6))
C      INITIALIZATION FOR *VELOCITY PROFILES AT SELECTED CHORDWISE
C      POSITIONS* & MAY BE REMOVED IF THAT SECTION IS DELETED
C      CC 8025 IYP=1,4
C      CC 8025 IXP=1,4
8025 ILIP(IXP,IYP)=0.00
C      PCNS=50.00
C      END INITIALIZATION FOR *VELOCITY PROFILES AT SELECTED CHORDWISE
C      END PARAMETERS FOR VELOCITY EQNS
C      END OF SECTION 1
C      BEGIN SECTION 2
C      PARAMETERS FOR POTENTIAL FLOW

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C THRUST(LBF) 6000 8000 10000 .18 17140
C TR .0504 .0582 .0651 .0002057 .063115
C AB(RADIANS) .1050 .1310 .1561 .000207 .1048
C ALPHB(DEG.) 6.01 7.50 8.94 .0118379 6.00
C XIS .256 .108 .123 .460 .30
C RK2 .365 .226 .081 .898 .450
C RCTCR(Feet) 24 24 24 40
      READ(1,246) XO,AR ,ALPHB,TR,XIINF,RK2
246 FORMAT(F10.7,F10.4,2F10.7,F20.16,F10.7)
C TAB=TR*DSQRT(2.00*DSRT) IS NECESSARY TO AVOID OVERFLOW
EPS=9.20-2
C EPS DETERMINES THICKNESS OF JOUKOWSKI AIRFOIL
C XO IS POSITION ALONG THE CHORD OF THE AXIS OF ROTATION; .25 IS 25%
C CHORD
C AR IS THE ASPECT RATIO;IT IS USED ONLY IN 'VELOCITY PROFILES AT
C SELECTED CHORDWISE POSITIONS'
C ALPHB IS GEOMETRIC ANGLE OF ATTACK
C THE NONDIMENSIONALIZED INDUCED VELOCITY=TR*(SPAN/CHORD)
C XIINF IS THE CHORDWISE POSITION OF THE STAGNATION POINT. IT IS IN
C BODY COORDINATES AND IS NORMALLY A NEGATIVE NUMBER.
C RK2 IS THE CONSTANT K2 IN  $Q=1-K2/YY^{*2}$ ;YY=SPAN/CHORD
AR=ALPHB/57.29578
TAB=DTAN(AB)
WRITE(3,400) XO,AR ,ALPHB,TR,XIINF,RK2,EPS,TAB
400 FORMAT(// ' THE POSITION OF THE AXIS OF ROTATION IS',F8.4,'CHORD'//
1' ASPECT RATIO=',F8.4/' GEOMETRIC ANGLE OF ATTACK=',F12.8,'DEG.'//
2' COEFFICIENT OF INDUCED VELOCITY=TR=',F12.8/' POSITION OF THE ST
3AGNATION POINT IN BODY COORDINATES: XIINF=',D15.8/' K2=',F8.4/
4' THE THICKNESS OF THE AIRFOIL IS CONTROLLED BY THE PARAMETER EPS'
5,' ;EPS=',F12.8/' TANGENT OF AOA=',D15.8//)
FRS=DSIN(AB)
FBR=-CCCS(AB)
FAS=-FBR+TR*FBS
FAR=FBS+TR*FBR
FI=-(1.00+TR*TAB)/(TAB-TR)
SGN=1.00
DEI=2.00/(1.00+FI*FI)
C DE IS THE PARAMETER DELTA;IT APPEARS IN THE PARAMETRIC EQNS FOR
C A JOUKOWSKI AIRFOIL. AS DE VARIES FROM 0 TO 2, %CHORD GDS FROM
C 0 TO 100%
C DEI IS DE AT THE STAGNATION POINT.
SI=DEI-1.00
JS=1200
F=1.0-3/JS
CSTRT=1.0-20
XSTRT=EPS*DSQRT(2.00*DSRT)/(1.00+FI)
IF( XIINF .GT. 13.00) IBLUP=2070
IF( IBLUP .NE. 2070) GO TO 8190
DEP=1.0-3*DEI
CALL RK3(CSDX,H,XSTRT,CSTRT,3.00,DEP,XIINT,DV,IER)
H=.100/JS
CALL RK3(CSDX,H,XIINT,DEP,3.00,DEI,XIINF,DV,IER)
C CSDX CALCULATES THE DERIVATIVE OF DE (OR S,SINCE S=DE-1)
8190 CONTINUE
IF( FI .LT. 0.00 ) SGN=-1.00
C SGN =1.00 FOR THE TOP(ABOVE THE CHORD LINE) OF THE AIRFOIL;
C SGN=-1.00 FOR THE BOTTOM
IF( IBLUP .EC. 2070 ) XIINF=XIINF*SGN
IBLUP=0
WRITE(3,401) SI,XIINF,DEI,FI
401 FORMAT( ' SI,XIINF,DEI,FI ',6D18.10 ///)
CUNNY=13.00

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      B99=DSOX(DUMMY,DEI)
C      DSOX WILL CALCULATE XR (THE POSITION ALONG THE CHORD, MEASURED FROM
C      THE AXIS OF ROTATION) AND ZR (ONE-HALF THE AIRFOIL THICKNESS) AND
C      THEIR DERIVATIVES WITH RESPECT TO DE (E2 THRU B10)
      B10I=B10
      CXI(4)=1.2D-6
      DELX=2.5C-3
      XI(4)=0.00
      X(4)=XIINF
      CE=DEI
      T=SI
      ATX=0
C  END PARAMETERS FOR POTENTIAL FLOW
C  END OF SECTION 2
C  BEGIN K LOOP IN XI
C      K IS THE INDEX FOR XI ; CXI(K)=XI(K)-XI(K-1)
      DO 30 K=5,45
C      BEGIN SECTION 3
C      THE OPTIMUM XI SPACING DEPENDS ON ALPHB AND TR; THE SPACINGS USED
C      FOR VARIOUS COMBINATIONS OF THE TWO ARE GIVEN:
C      FOR ALPHB=6.01; TR=.05C4
      DXI(K)=DELX*(1+K/23+2*(K/33)+4*(K/40))
C      FOR ALPHB=.50; TR=.0582
      DXI(K)=DELX*(1+K/23+2*(K/33)+2*(K/40))
C      FOR ALPHB=8.94; TR=.0651
      CXI(K)=DELX*(1+K/23+2*(K/33))
C      FOR ALPHB=6.00; TR=.0631
      DXI(K)=DELX*(1+K/23-K/46+2*(K/33)+2*(K/40)+6*(K/44))
C      FOR ALPHB=.0118379; TR=.0002057
      DXI(K)=DELX*(2+K/13-K/26+K/21+8*(K/22)+5*(K/29))
153 XI(K)=XI(K-1)+CXI(K)
      X(K)=X(K-1)+XIINF
      DX(K)=DXI(K)
      IMAX=4
      IF(K .LT. 7) GO TO 516
      DO 1489 J=1,JDIM
1489 DVZ(J,1)=(Y(3,J,2,2)*(X(K)-X(K-2))-Y(3,J,2,3)*DX(K))/DX(K-1)
      IF(K .LE. 7) GO TO 516
      K>=8
C      S(1)=XI(K)*(1.00/(X(K)-X(K-3))+1.00/(DX(K)+DX(K-1))+1.00/DX(K))
      S(2)=-XI(K)*(X(K)-X(K-3))*(DX(K)+DX(K-1))/(DX(K-1)+DX(K-2))*
2      DX(K-1)*DX(K))
      S(3)=XI(K)*(X(K)-X(K-3))*[DX(K)/(DX(K-2)*DX(K-1)+DX(K)+DX(K-1))]
      S(4)=-XI(K)*(DX(K)+DX(K-1))*DX(K)/(DX(K-2)*(DX(K-1)+DX(K-2))*
4      (X(K)-X(K-3)))
C      S(1) IS THE COEFFICIENT OF THE VALUE AT THE POINT K-1+1 IN FORMING
C      THE DERIVATIVE XI*(DERIVATIVE WRT XI)
C      IMAX IS THE MAX VALUE OF I
      DO 1437 J=1,JDIM
1437 DVZ(J,1)=DVZ(J,1)+DX(K)*(X(K)-X(K-2))*[Y(3,J,2,3)*(X(K-1)-X(K-3))
      -Y(3,J,2,4)*DX(K-1)-Y(3,J,2,2)*DX(K-2)]/(DX(K-1)*DX(K-2))
516 IF(K=6) 518,519,520
518 XI(5)=CXI(4)
      CXI(K)=XI(K)-XI(K-1)
      IMAX=2
      GO TO 521
519 CONTINUE
      XI(6)=.5C-3
C      K=6
      S(1)=1.00
      S(2)=-1.00
      CXI(K)=XI(K)-XI(K-1)

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      IPAX=2
      CC 1490 J=1,JOIP
1490 CVZ(J,1)=Y(3,J,2,2)*(1.00-1.500*DVZINC)*.8
      GO TO 521
520 IF(K .GT. 7) GO TO 521
C      K=7
      XI(7)=CXI(7)
      CXI(K)=XI(K)-XI(K-1)
      CX(K)=DXI(K)
      X(K)=XI(K)+XIINF
      S(1)=(1.00/DX(K)+1.00/(DX(K)+DX(K-1)))*XI(K)
      S(2)=-(DX(K)+DX(K-1))/(DX(K)*DX(K-1))*XI(K)
      S(3)=DX(K)/((DX(K)+DX(K-1))*CX(K-1))
      S(3)=S(3)*XI(K)
      IPAX=3
521 CCNTINUE
      X(K)=XI(K)+XIINF
      CX(K)=DXI(K)
      SGN=1.00
      IF(X(K) .LT. 0.00) SGN=-1.00
      JS=15000*25/(K*K)+1000
      H=(X(K)-X(K-1))/JS
      H=10.00*H
      CFP=DE
C      END OF SECTION 3
C      BEGIN SECTION 4
C      BEGIN CALCULATION OF THE FUNCTIONS OF XI THAT APPEAR IN B.C. AND ECNS.
      IF(X(K)*X(K-1)) 3314,1405,3315
3314 H=.0100*H
C      THE VALUE OF DE IS FOUND BY INTEGRATING DSDX TO X(K) FROM XSTR
      CALL RK3(CSDX,H,XSTR,CSTR,X(K),27.00,XF,DE,IER)
      GO TO 320
C      THE VALUE OF DE IS FOUND BY INTEGRATING DSDX TO X(K) FROM X(K-1)
3315 CALL RK3(CSDX,H,X(K-1),CFP,X(K),27.00,XF,DE,IER)
320 CCNTINUE
      T=DE-1.00
      CUMMY=13.00*SGN
319 B99=DSDX(CUMMY,DE)
      WRITE(3,450) K,XI(K),IER,CXI(K),DE,SGN,X(K)
450 FORMAT(' K=',I3,5X,'XI(K)=' ,D19.11,10X,'IER=',I5 / ' DXI(K)=' ,
1C19.11,5X,'DE=' ,D19.11,5X,'SGN=' ,F5.2,5X,'X(K)=' ,D19.11)
      B11=2.00*(B2*B4+B3*B5)
      B12=2.00*(B4*B4+B2*B6+B5*B5+B3*B7)
      XR=(1.00+EPS)*(T-EPS)*(1.00+((1.00-EPS)**2)/B1)/4.00
      XR=XR+(1.00+EPS*EPS)/2.00-X0
      ZR=.2500*(1.00+EPS)*DSQRT(DE*(1.00-T))*(1.00-(1.00-EPS)**2/B1)*SGN
      WRITE(3,451) XR,B2,B4,B6,ZR,B3,B5,B7,B1,B10,B11,B12
451 FORMAT(' XR,B2,B4,B6,ZR,B3,B5,B7,B1,B10,B11,B12',5D16.9 /7016.1C)
      CES=DSQRT(DE)
      CMSS=DSQRT(1.00-T)
C      PS1 IS 1ST DERIVATIVE OF THE POTENTIAL, SUBSCRIPT SIGMA, WRT SIGMA
C      PR3 IS 3RD DERIVATIVE OF THE POTENTIAL, SUBSCRIPT RHC, WRT SIGMA
      PS0=(1.00+EPS)*T/2.00
      PRO=(1.00+EPS)*(SGN*DES*CMSS-DATAN(SGN*DES*CMSS/T))/2.00
      PS1=(1.00+EPS)/2.00
      PS2=0.00
      PS3=0.00
      PR1=SGN*PS1*CMSS/DES
      PR2=-SGN*PS1/(CMSS*(DES**3))
      PR3=SGN*PS1*(1.00-2.00*T)/((CMSS**3)*(DES**5))
      BP(1)=B10
      CC 317 I=2,10

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317 BP(I)=BP(I-1)*B10
C   US0 IS 1ST (0+1) DERIVATIVE OF POTENTIAL, SUBSCRIPT SIGMA, WRT XI
C   UR2 IS 3RD (2+1) DERIVATIVE OF POTENTIAL, SUBSCRIPT RHO, WRT XI
    US0=PS1/B10
    URO=PR1/B10
    US1=-.5D0*B11/BP(4)
    UR1=US1*PR1+PR2/BP(2)
    US1=US1*PS1
    US2=-.5D0*B12/BP(5)+B11*B11/BP(7)
    UR2=US2*PR1-PR2*1.5D0*B11/BP(5)+PR3/BP(3)
    US2=US2*PS1
C   PHIA IS PHI, SUBSCRIPT A. UAO IS THE FIRST DERIVATIVE OF PHIA WRT X
C   UA1 IS THE SECCND DERIVATIVE...., ETC.
    PHIA=FAS*PS0+FAR*PRO
    UAO=FAS*US0+FAR*URO
    UA1=FAS*US1+FAR*UR1
    UA2=FAS*US2+FAR*UR2
    WRITE(3,459) PRO,PR1,PR2,PR3,PS0,PS1,PS2,PS3, US0,US1,US2,URO,
    W UR1,UR2, PHIA,UA0,UA1,UA2
459 FORMAT( ' PRO,PR1,PR2,PR3/PS0,PS1,PS2,PS3 ',4D2C.11/4D2C.11,
    F ' US0,US1,US2,URO,UR1,UR2'/6D2C.11/' PHIA,UA0,UA1,UA2 ',4D20.11 )
C   ALPHA IS THE ANGLE BETWEEN THE NORMAL TO THE SURFACE OF THE
C   AIRFOIL AND THE UPWARD NORMAL TO THE CHORD LINE
    ALPHA=DATAN(B3/B2)
    IF(SGN .LT. 0.D0) ALPHA= 3.1415926536 +ALPHA
    ADEG=57.2958*ALPHA
    CC=2.D0*DCOS(ALPHA-AB)/UA0
C   VDELTA IS THE FIRST TERM IN THE EXPANSION FOR THE SPANWISE
C   POTENTIAL FLOW, WHICH= VDELTA+VDEL2/VY**2
    VDELTA=PHIA+XR*(FBR-FAS)-ZR*(FBS+FAR)
    PO=XI(K)/UA0
    HO=XI(K)*UA0
C   XMO IS M, SUBSCRIPT 0. XM2K IS M, SUBSCRIPT 2K
    XMO=PO*UA1
    RM=(XMO+1.D0)/2.D0
    XM2K=XMO*(1.D0-XMO)+XI(K)*XI(K)*UA2/UA0
    DCDXA=(B4-B2*B11/(2.D0*B10*B10))/(B1C*B1C)
    DSDXA=(B5-B3*B11/(2.D0*B1C*B1C))/(B1C*B1C)
    DCDX=DCDXA*DCOS(AB)+DSDXA*DSIN(AB)
    DSWX=DSOXA*DCOS(AB)-DCDXA*DSIN(AB)
    CS=TR*XI(K)*CSIN(ALPHA-AB)
    CS2=RK2*(CS+TR*XI(K)*XI(K)*DSWX)
    CC2=RK2*(-XMO*CC+2.D0*PO*CCDX)
    RL2=2.D0*RK2
    P2=RK2*PO*(1.D0-XMO)
    VDEL2=RK2*(HO*(1.D0-CC)+CS)
    H2=RK2*HO*(1.D0+XMO)
C   PCC IS %CHORD; PCNS IS %CHORD NEAR SEPARATION
    PCC(K)=100.D0*(XR+X0)
    UTIL(K)=UA0
    VDEL(K)=VDELTA
C   THIS WRITE STMT IS FOR CHECKOUT PURPOSES
C   WRITE(3,166) ALPHA,AB,DCDXA,DSDXA,DCDX,DSWX
C 166 FORMAT( ' ALPHA,AB,DCDXA,DSDXA,DCDX,DSWX ',2C18.1C,4D16.7 )
C   VINFI(J) IS THE B.C. ON THE VELOCITY IN THE JTH ECN.
    VINFI(5)=VDEL2
    VINFI(2)=VDELTA
    WRITE(3,471) XR,ZR,PCC(K),ADEG,CC,CC2,H0,H2,PC,P2,UAC,VDELTA,VDEL2
    W,CS,CS2, XMO,XM2K
471 FORMAT( ' THE POINT HAS COORDINATES XR=',D17.10,IX,
    1' AND ZR=',D17.10,5X, ' %CHORD(AT INFINITE SPAN)=' ,D17.10/
    2' THE ANGLE BETWEEN NORMAL TO SURFACE AND NORMAL TO CHORD=' ,D17.10

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3,'DEG',5X,'CC=',D17.10,5X,'CC2=',D17.10/ ' H0=',D17.10,5X,
4 'H2=',D17.10,5X,'P0=',D17.10,5X,'P2=',D17.10,5X,'UA0=',D17.10/
5 ' VDELTA=',D17.10,5X,'VDEL2=',D17.10,5X,' CS=',D17.10,5X,'CS2=',
6 D17.10,5X, / ' XMC=',D17.10,5X,'XM2K=',D17.10 )
ZMAX=5.4C0+11.C0*X1(K)
C ZMAX IS THE VALUE OF ETA TO WHICH THE SOLUTIONS ARE TAKEN. FOR
C GOOD ACCURACY, ZMAX SHOULD BE CHOSEN SO THAT THE 2ND DERIVATIVE
C OF THE STREAM FUNCTION WRT ETA (DVOZ) IS <=.001
C SPEED AND CONVERGENCE ARE IMPROVED IF LMAX (WHERE Z(LMAX)>=ZMAX)
C IS AS SMALL AS POSSIBLE
C END CALCULATION OF THE FUNCTIONS OF XI THAT APPEAR IN B.C. AND ECNS.
C END OF SECTION 4
1345 DO 979 L=LMAX,LMAXU
IF(Z(L) .LE. ZMAX) GO TO 979
LMAX=L
GO TO 980
979 CONTINUE
LMAX=LMAXU-1
980 IF( LMAX .GT. LMAXU-1) LMAX=LMAXU-1
WRITE(3,148) LMAX,ZMAX,Z(LMAX),H,CCDX,ALPHA
148 FORMAT( ' LMAX,ZMAX,Z(LMAX),H,CCDX,ALPHA ',15,3D14.6,2D20.12 )
WRITE(3,149) LMAX,(S(I),I=1,4)
149 FORMAT( ' LMAX=',I3,' S(1)=' ,4D20.10 )
C BEGIN SECTION 5
C BEGIN J LOOP TO CHOOSE VARIABLE
C WHEN J=1S 1 2 3 4 4 5
C STREAM FUNC IS FO GO F2K F2C G2
C M DENOTES NO. PRIMES+1
CC 1500 J=1,JDIM
IF(J .EQ. 2) LMAX=LMAX-1
IF( J .EQ. 3) LMAX=LMAX+1
C LMAX<LMAXU IS REQUIRED
DVZINC=.005D0
IF(J.EQ. 1 .AND. K .GE. 8) DVZINC=1.D-9
C ERRMAX MAY BE REDefined HERE
Y(2,J,LMAX,1)=VINP(J)
C Y(M,J,L,1) IS THE (M-1)DERIVATIVE OF THE STREAM FUNCTION OF THE
C JTH VARIABLE AT ETA=Z(L) AND XI=XI(K-I+1)
C FOR EXAMPLE, IF K CURRENTLY=12, Y(2,5,37,3) IS G2' AT ETA=Z(37) AND
C XI=XI(10)
C A(M,J,L)+S(1)*Y(M,J,L,1)=XI*(DERIVATIVE OF Y(M,J,L,1) WRT XI)
CC 1491 L=1,LMAXU
CC 1491 M=1,3
A(M,J,L)=0.D0
CC 1491 I=2,IMAX
1491 A(M,J,L)=A(M,J,L)+S(I)*Y(M,J,L,I)
NBLUP(J)=0
C
C BEGIN ITERATIONS ON B.C. ON VELOCITY AT EDGE OF B.L.
C ONCE THE RUNGE-KUTTA SCHEME HAS INTEGRATED THE JTH EQN FROM ETA=0
C TO ETA=Z(LMAX-1),THE ERROR IN VELOCITY (Y(2,J,LMAX-1,1)-VINP(J))
C AT Z(LMAX-1) IS STORED IN ERR(ITER). THE SECOND DERIVATIVE AT THE
C WALL (DVZ(J,ITER)) IS CHANGED AND ITER INCREMENTED, THIS CONTINUES
C UNTIL ITER=6 (EQN PRESUMABLY NOT CONVERGENT) OR ERR(ITER)<ERRMAX
C THE RUNGE-KUTTA SCHEME USED IS DESCRIBED IN "INTRODUCTION TO
C NUMERICAL ANALYSIS" ,F.B. HILDEBRAND,"MCGRAW-HILL,1956,PAGE237
CC 1400 ITER=1,6
IRLUP=0
WAYN=0.D0
WAYP=0.D0
ERRA=0.D0
ERRP=0.D0

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      LLH=LMAX*.9+K/5+K/12
2848 Y(3,J,2,1)=DVZ(J,ITER)
      STIRL=0.00
      LLL=LMAX-2
C
C BEGIN L LOOP IN ETA (I.E. IN Z)
      DO 1300 L=2,LLL
        ST(1)=-.12500*(DZ(L+1)**2)/((DZ(L+1)+DZ(L))*DZ(L))
        ST(2)  =-.12500*DZ(L+1)*(DZ(L)-DZ(L+1)-DZ(L+2))/((DZ(L+1)
        2 +DZ(L+2))*DZ(L))
        ST(3)  =-.12500*DZ(L+1)*(DZ(L+2)-DZ(L+1)-DZ(L))/((DZ(L+1)
        3 +DZ(L))*DZ(L+2))
        ST(4)  =-.12500*(DZ(L+1)**2)/((DZ(L+1)+DZ(L+2))*DZ(L+2))
        N=0
C BEGIN ITERATIONS FOR RUNGE KUTTA
      DO 1200 IS=2,5
        JM1=J-1
        IF( (IS-3)*(2-L) .GT. 0 .OR. IS .EQ. 4 ) GO TO 1527
        IF( (IS-3)*(IS-4) .EQ. 0 ) GO TO 176
        DO 1492 JT=1,JM1
        DO 1492 M=1,3
C      V(M,J) IS THE Y(M,J,L,1) EVALUATED AT THE VALUE OF Z REQUIRED BY
C      STEP "IS" OF THE RUNGE KUTTA SCHEME
        V(M,JT)=Y(M,JT,L+N,1)
1492  CC(M,JT)=S(1)*Y(M,JT,L+N,1)+A(M,JT,L+N)
        DO 1493 M=1,2
1493  CC(M,J)=A(M,J,L+N)
        GO TO 1527
176  DO 1593 M=1,3
        CC(M,J)=.500*(A(M,J,L) +A(M,J,L+1) )
        IF(J-1) 1692,1692,1693
1693  DO 1592 JT=1,JM1
        V(M,JT)=.500*(Y(M,JT,L,1)+Y(M,JT,L+1,1) )
        CC(M,JT)=.500*(S(1)*(Y(M,JT,L,1)+ Y(M,JT,L+1,1))+ A(M,JT,L)+
        D A(M,JT,L+1) )
        DO 1592 IST=1,4
        V(M,JT)=V(M,JT)+STIRL*ST(IST)*Y(M,JT,L-2+IST,1)
1592  CC(M,JT)=CC(M,JT)+STIRL*ST(IST)*(S(1)*Y(M,JT,L-2+IST,1)+
        D A(M,JT,L-2+IST) )
1692  CONTINUE
        DO 1593 IST=1,4
1593  CC(M,J)=CC(M,J)+STIRL*ST(IST)*A(M,J,L-2+IST)
1527  N=1
        DO 2732 M=1,3
2732  V(M,J)=Y(M,J,L,1)+HIS(IS)*U(M,IS-1)
C      U(M,IS) IS DZ(L+1)*(DERIVATIVE OF Y(M,J, ,1)) EVALUATED AT THE
C      VALUE OF ETA CALLED FOR BY IS. FOR IS=2,ETA=Z(L); FOR IS=3&4,ETA=
C      =(Z(L)+Z(L+1))/2; FOR IS=5,ETA=Z(L+1)
        U(2,IS)=DZ(L+1)*(Y(3,J,L,1)+HIS(IS)*U(3,IS-1))
        U(1,IS)=DZ(L+1)*(Y(2,J,L,1)+HIS(IS)*U(2,IS-1))
        GO TO (1,2,3,4,5),J
1  CONTINUE
C      EQN 1 J=1 Y(2,J,L,1)=F0
        U(3,IS)=XMO*V(2,J)**2-RH*V(1,J)*V(3,J)-XMO+V(2,J)*(S(1)*V(2,J)+
        1 CC(2,1))-V(3,J)*(S(1)*V(1,J)+DD(1,1))
        GO TO 1200
2  CONTINUE
C      EQN 2 J=2 Y(2,J,L,1)=G0
        U(3,IS)=-(RH*V(1,1)+CC(1,1))*V(3,J)+V(2,1)*(S(1)*V(2,J)
        2 +CC(2,2)) -HO*(1.00-CC*V(2,1))-CS
        GO TO 1200
3  CONTINUE

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C      EQN 3      J=3      Y(2,J,L,1)=F2K*
      U(3,IS)=- (RM*V(1,1)+DD(1,1))*V(3,J)+(XMO*V(2,1)+DD(2,1))*V(2,J)
      3 +(XMO-1.00)*.500*V(3,1)*V(1,J)+V(2,1)*(S(1)*V(2,J)+DD(2,J))
      3 -V(3,1)*(S(1)*V(1,J)+DD(1,J))
      3 +XM2K*UA0*(V(2,1)**2-.500*V(3,1)*V(1,1)-1.00)
      GO TO 1200
4 CONTINUE
C      EQN 4      J=4      Y(2,J,L,1)=F2C*
      U(3,IS)=- (RM*V(1,1)+DD(1,1))*V(3,J)+(XMO*V(2,1)+DD(2,1))*V(2,J)
      4 +(XMO-1.00)*.500*V(3,1)*V(1,J)+V(2,1)*(S(1)*V(2,J)+DD(2,J))
      4 -V(3,1)*(S(1)*V(1,J)+DD(1,J))
      4 +XI(K)*(V(2,2)*V(2,1)+.5*V(3,1)*V(1,2)+CC*(VDELTA-V(2,2))-VDELTA)
      GO TO 1200
5 CONTINUE
C      EQN 5      J=5      Y(2,J,L,1)=G2*
C      RK2 IS USED IN THIS EQN
      U(3,IS)=- (RM*V(1,1)+DD(1,1))*V(3,J)+V(2,1)*(S(1)*V(2,J)+DD(2,J))
      5 +RK2*(-((1.00-XMO)*.500*V(1,3)/UA0+P0*DD(1,3)/XI(K)+V(1,1)*XM2K*
      5 .500)*V(3,2)+P0*V(2,3)*DD(2,2)/XI(K)-(1.00-CC*V(2,1))*H2/RK2
      5 +H0*V(2,1)*CC2/RK2 +XI(K)*CC*V(2,3)-CS2/RK2 )
      5 -((1.00-XMO)*.500*V(1,4)/UA0+P0*DD(1,4)/XI(K)+.5*P0*V(1,2))*V(3,2)
      5 +P0*V(2,4)*DD(2,2)/XI(K)+XI(K)*CC*V(2,4)
1200 U(3,IS)=U(3,IS)*CZ(L+1)
C END      ITERATIONS FOR RUNGE KUTTA
      DO 1201 M=1,3
C      THE VALUES AT Z(L+1) ARE EVALUATED
1201 Y(M,J,L+1,1)=Y(M,J,L,1)+(U(M,2)+2.00*(U(M,3)+U(M,4))+U(M,5))/6.00
C      IF THE VELOCITY IS TOO LARGE OR SMALL, THE RUNGE-KUTTA INTEGRATION
C      IS STOPPED AND THE VALUE OF DVZ(J,ITER) IS ADJUSTED AND THE
C      PROGRAM RETURNS TO STMT 2848. AFTER 50 ADJUSTMENTS (IBLUP=50), THE
C      PROGRAM WILL CALL EXIT (I.E. STOP) AT STMT 1405
      IF( Y(2,1,L+1,1) .LT. 0 .OR. Y(2,1,L+1,1) .GT. 1.8 ) GO TO 1299
      IF( DABS(Y(2,J,L+1,1)) .LT. 1.06 ) GO TO 1300
1299 IBLUP= IBLUP+1
      NBLUP(J)=NBLUP(J)+1
      IF( IBLUP .GT. 40 )
        IWRITE(3,2849) IBLUP,J,L,ITER,DVZ(J,ITER),Y(2,J,L+1,1),Y(3,J,L+1,1)
2849 FORMAT( 14,' TH BLOWUP: J=',I3,' L=',I3,' ITER=', 14,' DVZ=',
        F20.13, ' VEL=',D15.6, ' DVDZ=',D15.6)
      IWAY=Y(2,J,L+1,1)/DABS(Y(2,J,L+1,1))
      IF( IWAY .GT. 0 ) WAYP=DVZ(J,ITER)
      IF( IWAY .LT. 0 ) WAYN=DVZ(J,ITER)
      IF( IWAY .GT. 0 .AND. L .GT. LLM ) ERRP=Y(2,J,LLM,1)-VINP(J)
      IF( IWAY .LT. 0 .AND. L .GT. LLM ) ERRN=Y(2,J,LLP,1)-VINP(J)
      IF( WAYN*WAYP )=2219,1218,2219
2219 DVZ(J,ITER)=(WAYN+WAYP)*.500
      IF( ERRP*ERRN .NE. 0.00 ) DVZ(J,ITER)=(WAYN*ERRP-WAYP*ERRN)/(ERRP
      0 -ERRN)
      GO TO 2220
1218 DVZ(J,ITER)=DVZ(J,ITER)*(1.00-IBLUP*IWAY*.100 *DVZ(J,ITER)/
      1 DABS(DVZ(J,ITER)) )
      IF( WAYN*WAYP+(IBLUP/10-1).NE.0 .AND. DABS(DVZ(J,ITER)).LT. 1.0-10)
      1 DVZ(J,ITER)=DVZ(J,ITER)-IWAY*.001
2220 IF( IBLUP .GT. 50 ) GO TO 1405
      GO TO 2848
1300 STIRL=1.00
C END      L LOOP IN ETA (I.E. IN Z)
      ERR(ITER)=Y(2,J,LLL+1,1) -VINP(J)
C      THE ERROR IN MATCHING THE B.C. ON VELOCITY AT THE EDGE OF THE
C      BOUNDARY LAYER ( ERR(ITER) ) IS USED TO FIND A BETTER VALUE FOR
C      THE SECOND DERIVATIVE OF THE STREAM FUNCTION AT THE WALL (DVZ(J,IT
C      (DVZ(J,ITER)) UNTIL ERR(ITER)<ERRMAX

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      IF(ITER .GT. 3 )
C      THIS WRITE STMT IS FOR CHECKOUT PURPOSES
      1WRITE(3,965) K,J,ITER,L,IS,DVZ(J,ITER),ERR(ITER)
      965 FORMAT( ' K,J,ITER,L,IS,DVZ,ERR',5I5,3D15.6)
      IF(DABS(ERR(ITER)) .LT. ERRMAX) GO TO 1401
      IF(ITER-2) 81,82,83
C      ERRMAX& DVZINC CAN BE FUNCTIONS OF J
      81, CVZ(J,2)=CVZ(J,1)*(1.D0-DVZINC)
      GO TO 1400
      82 DVZ(J,3)=(ERR(2)*DVZ(J,1)-ERR(1)*DVZ(J,2))/(ERR(2)-ERR(1))
      GO TO 1400
      83 C12=(DVZ(J,ITER-1)-DVZ(J,ITER-2))/(ERR(ITER-1)-ERR(ITER-2))
      C23=(DVZ(J,ITER) -DVZ(J,ITER-1))/(ERR(ITER) -ERR(ITER-1))
      D123=(D23-D12)/(ERR(ITER)-ERR(ITER-2))
      DVZ(J,ITER+1)=DVZ(J,ITER)-ERR(ITER)*(D23-ERR(ITER-1)*D123)
      1400 CONTINUE
C END   ITERATIONS ON B.C. ON VELOCITY AT EDGE OF B.L.
      1405 WRITE(3,1403) J
      1403 FORMAT(' EQN NO.',I2,' FAILS TO CONVERGE BECAUSE' )
      IF(1BLUP .GT. 50) WRITE(3,8186) ITER,Z(L)
      8186 FORMAT(' FOR ITER=',I2,', THE VELOCITY BECAME TOO LARGE OR TOO',
      1'SMALL FOR ETA=',D15.6 / ' THE VELOCITIES ARE CHECKED FOR SIZE',
      2'AFTER STMT 1201')
      IF( ITER .GE. 6) WRITE(3,8187) (ERR(I),DVZ(J,I),I,I=1,6)
      8187 FORMAT(' THE B.C. AT Z(LMAX) WERE NOT MATCHED AFTER 6 ITERATIONS,'
      1'F'CN THE SECOND DERIVATIVE OF STREAM FUNCTION AT WALL (DVDZ) '//
      2' THE ERROR IN MATCHING B.C. DVDZ AT THE WALL ITERATION'/
      3 ( 8X,D14.7,8X,D22.15,5X,I2) )
      CALL EXIT
      1401 CONTINUE
      ITERKP(J)=ITER
      CC 2324 L=LMAX,LMAXU
C      THE STREAM FUNCTIONS (Y(1,J,L,1)), "VELOCITIES"(Y(2,J,L,1)), AND
C      "SHEAR RATE", (Y(3,J,L,1)) ARE CALCULATED UP TO Z(LMAXU) IN CASE
C      LMAX INCREASES AT THE NEXT XI(K) STATION
      Y(1,J,L,1)=Y(1,J,LLL+1,1)+VINP(J)*(Z(L)-Z(LLL+1))
      Y(3,J,L,1)=Y(3,J,LLL+1,1)
      2324 Y(2,J,L,1)=VINP(J)
      1500 CONTINUE
C END   J LOOP TO CHOOSE VARIABLE
C      END OF SECTION 5
C      BEGIN SECTION 6
      DO 157 J=1,JDIM
      157 IF(DABS(Y(3,J,LLL+1,1)) .GT. 1.D-3) N=27
      WRITE(3,8183) (J,J=1,JDIM)
      8183 FORMAT(/' CONVERGENCE ACHIEVED FOR J= ',5X,6(I3,11X))
      WRITE(3,8184) (NBLUP(J),J=1,JDIM)
      8184 FORMAT( ' NO. ITERATIONS TO REACH Z(LMAX)',5X,6(I3,11X))
      WRITE(3,8185) (ITERKP(J),J=1,JDIM)
      8185 FORMAT( ' NO. ITERATIONS TO MATCH B.C. ',5X,6(I3,11X))
      IF(N .EQ. 27) WRITE(3,158) (Y(3,J,LLL+1,1),J=1,JDIM)
      158 FORMAT( ' ZMAX TOO SMALL; DVDZ(J) AT ZMAX=', 6D14.5)
      WRITE(3,144)
      144 FORMAT(/ ' Z L ' , T25,'F0',T43,'G0', T61,'F2K',
      1 F T79,'F2C', T97,'G2' )
      JM1=LMAX/4
      CC 861 L=2,LMAX
C      VELOCITY OR VEL IS ACTUALLY THE FIRST DERIVATIVE WRT ETA OF THE
C      STREAM FUNCTION OR Y(2,J,L,1). THE 2ND DERIVATIVE IS CALLED DVDZ
C      CP DVZ (WHEN EVALUATED AT THE WALL)
      IF((L/2)*2.NE.L .AND. L.GT.10 .AND. L.LT.LMAX-2) GO TO 861
      WRITE(3,142) Z(L),L, (Y(2,J,L,1),J=1,JDIM)

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      IF(L.GT. 10 .AND. L.LT. LMAX-2 .AND. L.NE. (L/JM1)*JM1)GOTO 861
      WRITE(3,143) (Y(3,J,L,1),J=1,JDIM)
861  CONTINUE
      WRITE(3,161) (Y(1,J,LMAX,1),J=1,JDIM)
161  FORMAT( ' STREAM FUNCTION ' ,6D18.9 )
      WRITE(3,147) (VINFI(J),J=1,JDIM)
147  FORMAT(' VELOCITY AT ZMAX' ,6D18.9)
142  FORMAT( F8.4,I4,' VEL ', 6D18.9 )
143  FORMAT( 13X,'CVDZ ',6D18.9)
C    END OF SECTION 6
C    BEGIN SECTION 7
C    CHECK OF X DERIVATIVES          AF
C    THIS SECTION MAY BE REMOVED IF DESIRED
      IF(K.LT. 7) GO TO 7139
      CK=CX(K)
      CP=DX(K-1)
      L=LMAXU-3
      CC 7138 J=1,JDIM
      A(3,J,5)=-DK*Y(2,J,10,3)/((DK+DP)*DP)+(DK-DP)*Y(2,J,10,2)/(CK+DP)
5    +DP*Y(2,J,10,1)/((DK+DP)*DK)
      A(3,J,5)=A(3,J,5)*XI(K-1)
7138 CCNTINUE
C    THIS IS THE VALUE OF XI*(DERIVATIVE OF Y(2,J,10,2) WRT XI) USED IN
C    CALCULATIONS AT K-1
      WRITE(3,7140) (XD2(J) ,J=1,JDIM)
C    THIS IS THE VALUE OF XI*(DERIVATIVE OF Y(2,J,10,2) WRT XI ) AT
C    XI=XI(K-1) FOUND FROM VALUES AT K,K-1,&K-2. THE DIFFERENCE IN
C    THESE TWO IS A MEASURE OF THE ERROR. REDUCTION OF THE XI SPACING
C    WILL REDUCE ERROR BUT ALSO SLOW CONVERGENCE.
      WRITE(3,7140) (A(3,J,5),J=1,JDIM)
7140 FORMAT( ' XI DERIVATIVES ' ,6D18.9)
7139 CC 7136 J=1,JDIM
7136 XD2(J) =A(2,J,10)+S(1)*Y(2,J,10,1)
C    THIS SECTION MAY BE REMOVED IF DESIRED
C END CHECK OF X DERIVATIVES          AFTER 1428
C    END OF SECTION 7
C    BEGIN SECTION 8
C    DISPLACEMENT THICKNESS, SHAPE FACTOR & SKEW ANGLE      AFTER 143
C    THIS SECTION MAY BE REMOVED IF DESIRED
      IF(K.LT. 8 .OR. JDIM.LT. 5 ) GO TO 8090
      DO 8040 IYP=1,4
      YP=(.300*IYP-.2500*(IYP/4))*AR
      XP=XI(K)/(1.00-RK2/YP**2)+XIINF
      E(1)=(XP-X(K-1))*(XP-X(K-2))/( (X(K)-X(K-1))*(X(K) -X(K-2)) )
      E(2)=(XP-X(K) )*(XP-X(K-2))/( (X(K-1)-X(K))*(X(K-1)-X(K-2)) )
      E(3)=(XP-X(K) )*(XP-X(K-1))/( (X(K-2)-X(K))*(X(K-2)-X(K-1)) )
      PCCP=0.00
      UTILP=0.00
      VDELP=0.00
      DVAR=UAO*YP*YP
      DO 8020 I=1,3
      PCCP=PCCP+E(I)*PCC(K-I+1)
      UTILP=UTILP+E(I)*UTIL(K-I+1)
8020 VDELP=VDELP+E(I)*VDEL(K-I+1)
      VDELS=VDELTA+VDEL2/YP**2
C    RK2 IS USED IN THIS EQN
      TK(3)=Y(1,1,LLL,1)+(RK2*Y(1,3,LLL,1)+Y(1,4,LLL,1))/DVAR
      TK(4)=Y(1,2,LLL,1)+Y(1,5,LLL,1)/(YP*YP)
      TK(1)=Z(LLL)-TK(3)
      TK(2)=Z(LLL)-TK(4)/VDELP
      UP(2)=0.00
      VP(2)=0.00

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      CC 8030 L=3,LLL
C      RK2 IS USED IN THIS EQN
      UP(L)=(Y(2,1,L,1)+(RK2*Y(2,3,L,1)+Y(2,4,L,1))/DVAR)**2
      VP(L)=(Y(2,2,L,1)+Y(2,5,L,1))/(YP*YP)**2
8030 VP(L)=VP(L)/VDELS
      CC 8031 L=4,LLL,2
      TK(3)=TK(3)-CZ(L)*(UP(L-2)+4.00*UP(L-1)+UP(L))/3.00
      TK(4)=TK(4)-CZ(L)*(VP(L-2)+4.00*VP(L-1)+VP(L))/3.00
      LST=L
      IF(CZ(L+2).NE.CZ(3)) GO TO 8033
8031 CONTINUE
8032 DC 8032 L=LST,LLL
      TK(3)=TK(3)-CZ(L)*(UP(L)+UP(L-1))*50C
8032 TK(4)=TK(4)-CZ(L)*(VP(L)+VP(L-1))*50C
      DVAR=DSQRT(DABS((XP-KIINF)/(YP+UTILP)))
      CC 8016 I=1,4
8016 TK(I)=TK(I)*DVAR
      TK(3)=TK(1)/TK(3)
      TK(4)=TK(2)*VDELP/TK(4)
C      RK2 IS USED IN THIS EQN
      DVAR=YP+UTILP*(Y(3,1,2,1)+(RK2*Y(3,3,2,1)+Y(3,4,2,1))/(LAG*YP*YP))
C      / (Y(3,2,2,1)+Y(3,5,2,1))/(YP*YP))
      DVAR=1.00/DVAR
      BETA=DATAN(DVAR)*57.2957800
      WRITE(3,8015)
8015 FORMAT(8X,'X',RX,' X DISPLACEMENT ', ' Y DISPLACEMENT ',
1' X SHAPE FACTOR ', ' Y SHAPE FACTOR ', ' SKEW ANGLE ',
2' ASPECT RATIO ', ' SPAN/CHORD ' / 19X, ' THICKNESS ',
3' THICKNESS ', T86,' IN DEGREES' )
      WRITE(3,8014) XP,(TK(I),I=1,4),BETA,AR,YP
8014 FORMAT(5D17.10,2C16.9,D15.8)
8040 CONTINUE
C      THIS SECTION MAY BE REMOVED IF DESIRED
C      END DISPLACEMENT THICKNESS, SHAPE FACTOR & SKEW ANGLE AFTER 143
C      END OF SECTION 8
      IF(JDIM.LT.4) GO TO 1428
      RK2A=-Y(3,4,2,1)/ Y(3,3,2,1)
      WRITE(3,146) RK2A
146 FORMAT( ' -F2C*/F2K*=',D2C.9 )
1428 CONTINUE
C      BEGIN SECTION 9
C      VELOCITY PROFILES AT SELECTED CIRCUMFERENCE POSITIONS
C      THIS SECTION MAY BE REMOVED IF DESIRED
      IF(Y(3,1,2,1).LT..0885) PCNS=PCC(K) -1.1
      CC 8070 IXP=1,4
      GO TO (8071,8072,8073),IXP
8071 PCCP=60.00
      GO TO 8074
8072 PCCP=10.00+20.00*(IXP-2)
      GO TO 8074
8073 PCCP=PCNS
8074 IF(PCCP.GT.PCC(K).OR.PCCP.LT.PCC(K-2)) GO TO PCC(L)
      E(1)=(PCCP-PCC(K-1))*(PCCP-PCC(K-2))/((PCC(K)-PCC(K-1))*(PCC(K)-
1PCC(K-2)))
      E(2)=(PCCP-PCC(K))*(PCCP-PCC(K-2))/((PCC(K-1)-PCC(K))*(PCC(K-1)-
2PCC(K-2)))
      E(3)=(PCCP-PCC(K))*(PCCP-PCC(K-1))/((PCC(K-2)-PCC(K))*(PCC(K-2)-
3PCC(K-1)))
      XP=E(1)*X(K)+E(2)*X(K-1)+E(3)*X(K-2)
C      IF XP BE FOUND FOR EACH PCCP, ABOVE XP&E NEEDED ONLY FOR PCNS
      CC 8084 IYP=1,4
      YP=(.300*IYP-.2500*(IYP/4))*AR

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      IF(ILIM(IXP,IYP) .GT. 0) GO TO 8084
C     RK2 IS USED IN THIS EQN
      XIP=( XP-XIINF)*(1.00-RK2/YP**2)
      IF( XI(K) .LT. XIP ) GO TO 8084
      ILIM(IXP,IYP)=1.00
      E(1)=(XIP-XI(K-1))*(XIP-XI(K-2))/((XI(K)-XI(K-1))*(XI(K)-XI(K-2)))
      E(2)=(XIP-XI(K))*(XIP-XI(K-2))/((XI(K-1)-XI(K))*(XI(K-1)-XI(K-2)))
      E(3)=(XIP-XI(K))*(XIP-XI(K-1))/((XI(K-2)-XI(K))*(XI(K-2)-XI(K-1)))
      OVAR=YP*YP*(E(1)*UTIL(K)+E(2)*UTIL(K-1)+E(3)*UTIL(K-2) )
      DO 8083,L=2,LMAX
      UP(L)=0.00
      VP(L)=0.00
      DO 8083 I=1,3
C     RK2 IS USED IN THIS EQN
      UP(L)=UP(L)+E(1)*(Y(2,1,L,1)+(RK2*Y(2,3,L,1)+Y(2,4,L,1))/DVAR)
8083 VP(L)=VP(L)+E(1)*(Y(2,2,L,1)+Y(2,5,L,1)/(YP*YP))
      WRITE(3,8091) PCCP,YP,XP,XIP
8091 FORMAT(//' %CHORD=',F10.6,5X,'SPAN/CHORD=',F12.8,5X'X=',F15.10,5X,
F %XI=',F15.10 )
      WRITE(3,8093) (UP(L),L=2,LMAX)
8093 FORMAT(' CHORDWISE VELOCITIES' /( 1X,10F8.4))
      WRITE(3,8092) (VP(L),L=2,LMAX)
8092 FORMAT(' SPANWISE VELOCITIES' /( 1X,10F8.4))
8084 CONTINUE
8080 CONTINUE
C     THIS SECTION MAY BE REMOVED IF DESIRED
C END VELOCITY PROFILES AT SELECTED CHORDWISE POSITIONS
C END OF SECTION 9
8090 CONTINUE
      WRITE(3,872) K
872 FORMAT(' END OF K=',I5 ///)
C     UPDATE MATRIX FOR NEW VALUE OF K
      DO 1429 IT=1,3
      DO 1429 L=1,LMAXU
      DO 1429 J=1,JDIM
      DO 1429 M=1,3
1429 Y(M,J,L,5-IT)=Y(M,J,L,4-IT)
      30 CONTINUE
C END K LOOP IN XI
C
      CALL EXIT
      END
      REAL FUNCTION OSDX*8(DUMMY,DE)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BIJ/EPS,SGN,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10
      IF(SGN) 85,85,75
85 IF( DUMMY .GT. 0.00) SGN=1.00
75 S=DE-1.00
      CMS2=DE*(1.00-S)
      IF(DABS(CMS2) .LT. 1.0-50) WRITE(3,401) DE,SGN,CMS2,DUMMY
      IF(DABS(CMS2) .LT. 1.0-50) CMS2=1.0-50
      BA=(1.00-EPS)**2
      BB=1.00-EPS*EPS
      BC=1.00+EPS
      B1=1.00+EPS*EPS-2.00*EPS*S
      B2=BC*(1.00+BA*BB/(B1*B1))/4.00
      B3=SGN*BC*(-S*(1.00-BA/B1)-CMS2*2.00*EPS*BA/B1**2)/(4.00+
3 DSGRT(CMS2))
      IF( DABS(DUMMY*SGN-13.00) .GT. 1.0-4) GO TO 18
      BD=EPS*BC*BA/(2.00*B1*DSGRT(CMS2))
      B4=BC*BA*BB*EPS/B1**3
      BE=B1*B1

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      B5=SGN*BD*(-1.00/(BA*DE)+2.00*S/B1-4.00*EPS*OMS2/BE)
      B6=6.00*EPS*B4/H1
      BF=1.00/(CE*DE*BA)+2.00/B1+12. *EPS*S/BE-16.00*EPS*EPS*OMS2/B1**3
      B7=B5*(2.00*FFS/B1+S/OMS2)+SGN*BD*BF
401  FORMAT( ' R2,B3,DE,SGN,OMS2,DUMMY ',6D16.7 )
      R10=DSQRT(B2*B2+B3*B3)*SGN
C    IF(DABS(DUMY).LT.1.D-6)WRITE(3,401)R2,B3,DE,SGN,OMS2,DUMMY
      DSOX=1.00/B10
      RETURN
      END
      SUBROUTINE RK3(FUN,HI,XI,YI,XF,YF,ANSX,ANSY,IER)
C    RK3 IS THE DOUBLE PRECISION VERSION OF THE PROGRAM RK1 DOCUMENTED
C    IN THE IBM MANUAL H20-0205-3 "SYSTEM/360 SCIENTIFIC SUBROUTINE
C    PACKAGE-PROGRAMMERS MANUAL" PAGE 331
      DOUBLE PRECISION HI,XI,YI,XF,YF,ANSX,ANSY,H,XN,YN,HNEW,XN1,YN1,
1      XX,YY,XNEW,YNEW,H2,T1,T2,T3,T4,FUN
      IEP = 0
      IF(XF-XI) 11,11,12
11     ANSX=XI
      ANSY=YI
      RETURN
C    TEST INTERVAL VALUE
12     H=HI
      IF(HI) 16,14,20
14     IER=1
      ANSX=XI
      ANSY=0.0
      RETURN
16     H=-HI
C    SET XN=INITIAL X,YN=INITIAL Y
20     XN=XI
      YN=YI
C    INTEGRATE ONE TIME STEP
      HNEW=H
      JUMP=1
      GO TO 170
25     XN1=XX
      YN1=YY
C    COMPARE XN1 (=X(N+1)) TO X FINAL AND BRANCH ACCORDINGLY
      IF(XN1-XF)50,30,40
C    XN1=XF, RETURN (XF,YN1) AS ANSWER
30     ANSX=XF
      ANSY=YN1
      GO TO 160
C    XN1 GREATER THAN XF, SET NEW STEP SIZE AND INTEGRATE ONE STEP
C    RETURN RESULTS OF INTEGRATION AS ANSWER
40     HNEW=XF-XN
      JUMP=2
      GO TO 170
45     ANSX=XX
      ANSY=YY
      GO TO 160
C    XN1 LESS THAN X FINAL, CHECK IF (YN,YN1) SPAN Y FINAL
50     IF((YN1-YF)*(YF-YN))60,70,110
C    YN1 AND YN DO NOT SPAN YF. SET (XN,YN) AS (XN1,YN1) AND REPEAT
60     YN=YN1
      XN=XN1
      GO TO 170
C    EITHER YN OR YN1 =YF. CHECK WHICH AND SET PROPER (X,Y) AS ANSWER
70     IF(YN1-YF)80,100,80
80     ANSY=YN
      ANSX=XN

```

```

RK1  580
RK1  590
RK1  705
RK1  710
RK1  720
RK1  730
RK1  740
RK1  760
RK1  780
RK1  790
RK1  800
PK1  810
RK1  820
RK1  830
RK1  840
RK1  860
RK1  880
RK1  890
RK1  910
RK1  930
RK1  940
RK1  950
RK1  960
RK1  970
RK1  990
RK1 1010
RK1 1030
RK1 1050
RK1 1060
RK1 1070
RK1 1090
RK1 1100
RK1 1120
RK1 1130
RK1 1140
RK1 1150
RK1 1160
RK1 1170
RK1 1190
RK1 1220
RK1 1240
RK1 1260
RK1 1270
RK1 1280
RK1 1300
RK1 1320
RK1 1330
RK1 1340

```

|                                                              |          |
|--------------------------------------------------------------|----------|
| GC TO 160                                                    | RK1 1350 |
| 100 ANSY=YN1                                                 | RK1 1360 |
| ANSX=XN1                                                     | RK1 1370 |
| GC TO 160                                                    | RK1 1380 |
| C YN AND YN1 SPAN YF. TRY TO FIND X VALUE ASSOCIATED WITH YF | RK1 1400 |
| 110 GO 140 I=1,10                                            | RK1 1420 |
| C INTERPOLATE TO FIND NEW TIME STEP AND INTEGRATE ONE STEP   | RK1 1440 |
| C TRY TEN INTERPOLATIONS AT MOST                             | RK1 1450 |
| HNEW=((YF-YN)/(YN1-YN))*(XN1-XN)                             | RK1 1470 |
| JUMP=3                                                       | RK1 1480 |
| GC TO 170                                                    | RK1 1490 |
| 115 XNEW=XX                                                  | RK1 1500 |
| YNEW=YY                                                      | RK1 1510 |
| C COMPARE COMPUTED Y VALUE WITH YF AND BRANCH                | RK1 1530 |
| IF(YNEW-YF)120,150,130                                       | RK1 1550 |
| C ADVANCE, YF IS BETWEEN YNEW AND YN1                        | RK1 1570 |
| 120 YN=YNEW                                                  | RK1 1590 |
| XN=XNEW                                                      | RK1 1600 |
| GC TO 140                                                    | RK1 1610 |
| C ADVANCE, YF IS BETWEEN YN AND YNEW                         | RK1 1630 |
| 130 YN1=YNEW                                                 | RK1 1650 |
| XN1=XNEW                                                     | RK1 1660 |
| 140 CONTINUE                                                 | RK1 1670 |
| C RETURN (XNEW,YF) AS ANSWER                                 | RK1 1690 |
| 150 ANSX=XNEW                                                | RK1 1710 |
| ANSY=YF                                                      | RK1 1720 |
| 160 RETURN                                                   | RK1 1730 |
| 170 H2=HNEW/2.0                                              | RK1 1750 |
| T1=HNEW*FUN(XN,YN)                                           | RK1 1760 |
| T2=HNEW*FUN(XN+H2,YN+T1/2.0)                                 | RK1 1770 |
| T3=HNEW*FUN(XN+H2,YN+T2/2.0)                                 | RK1 1780 |
| T4=HNEW*FUN(XN+HNEW,YN+T3)                                   | RK1 1790 |
| YY=YN+(T1+2.0*T2+2.0*T3+T4)/6.0                              | RK1 1800 |
| XX=XN+HNEW                                                   | R-1 1810 |
| GC TO (25,45,115), JUMP                                      |          |
| END                                                          | RK1 1840 |

```

C
C
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      A FORTRAN PROGRAM FOR A ROTOR BLADE IN FORWARD FLIGHT WITH LIFT
C      THE PROGRAM OCCUPIES LESS THAN 200,000 BYTES IN FORTRAN IV-G,
C      LEVEL 1, MOD 3
      COMMON/BIJ/EP,SGN,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10
      DIMENSION X(80),DX(80),XI(80),DXI(80),BP(4),Y(3,12,75,4),
1 A(2,12,75), S(4), ST(4), ERROR(12),BC(12),BCL(12),LMAXR(12),
2 CMEGA(12),ITERK(12), Z(75),CZ(75),DN(75),EN(75),
3 SDZ(75),EY(75),CUMV(75),DT(75), EYK(450),EBC(12),RHSF(75)
4 , XDC(12),XDP(12),PCE(12),RKJ(12),IKERR(80)
      DIMENSION SHV(20),PSIV(20),VAV(20),ARV(20),PCCV(20),SHT(20),
D PSIT(20),VAT(20),ART(20), RHS(12,75),DT2(75),PCC(80),UAPK(80),
D UCPK(80),E(3)
V, ILIM(20,4),RNUK(80),TK(6),UP(75),VP(75)
C      THIS DIMENSION STATEMENT IS FOR: K<=80, M<=3, J<=12, L<=75, I<=4, NGT&NGVEL<61
C      EXTERNAL DSDX
C      SYMMETRIC, LIFTING ROTOR UNDERGOING FORWARD FLIGHT
      JDIM=5
C      JDIM IS THE NO. OF EQUATIONS TO BE SOLVED; IT IS THE MAXIMUM
C      VALUE OF J
C      IF JDIM=5 THE PGM ASSUMES THAT K10 IS TO BE FOUND
C      IF JDIM=9 THE PGM ASSUMES THAT K20R,K20,K21,&K22 ARE TO BE FOUND
C      AND THE VALUE OF K10 (RK10) THAT WAS READ IN IS CORRECT
C      IF JDIM>9 THE PGM ASSUMES THAT K10,K20R,K20,K21 & K22 ARE KNOWN
C      AND HAVE BEEN READ IN CORRECTLY
C
C      SECTION 1: INPUT VALUES NEEDED FOR THE POTENTIAL FLOW SOLUTION
C      READ(1,XXXX) DENOTES THE DATA CARDS AT THE END OF THE DECK;
C      WRITE(2,XXXX) DENOTES THE CARD PUNCH; WRITE(3,XXXX) DENOTES A
C      133 SPACE/LINE PRINTER
C      PARAMETERS FOR POTENTIAL FLOW
C      EPS(EPSILON) OF .092 GIVES AN 11.9% THICK AIRFOIL
C      EPS=9.2D-2
C      LESS THAN 160 PAGES ARE NORMALLY PRINTED.
C      ON IBM MODEL 360/75 A CPU TIME OF 9 MINUTES IS ALWAYS ADEQUATE.
C      IF NEITHER K10 NOR THE POSITION OF THE STAGNATION POINT (XIINF) IS
C      KNOWN, SET JDIM=5 AND PUT XIINF=14. ON THE FIRST DATA CARD.
C      SINCE THE VALUE OF K10 HAS BEEN FOUND ( BY EXTRAPOLATING
C      FLOC*/F1K*WALL TO SEPARATION VALUE OF XI, XIS), THEN JDIM SHOULD
C      BE SET TO 9 AND THE PROGRAM RERUN. VALUES OF K20R,K20,K21, AND
C      K22 ARE FOUND BY EXTRAPOLATING THE VALUES OF -DVC1/F2K* TO XIS.
C      SINCE THE CORRECT VALUES OF THE K2'S HAVE BEEN ENTERED ON THE
C      SECOND DATA CARD, SET JDIM TO 12 AND RERUN THE PROGRAM. THE
C      SUCCEEDING DATA CARDS WILL BE USED TO SPECIFY THE NUMBER OF
C      VELOCITY PROFILES      DISPLACEMENT THICKNESSES, AND MOMENTUM
C      THICKNESS DESIRED.
C      SUMMARY
C      1ST DATA CARD:FORMAT(F10.6,F10.7,F20.16,3F10.7) XC,ALPHA,XIINF,
C      XIS,RK10
C      SUCCEEDING DATA CARDS ARE NOT READ IF JDIM<10
C      2ND DATA CARD:FORMAT(8F10.5) RK20R,RK20,RK21,RK22
C      3RD DATA CARD:FORMAT(I5) NGVEL ( NO. VEL. PROFILES TO BE FOUND)
C      NEXT NGVEL CARDS:FORMAT(8F10.5) SH,PSI,VA,AR,PCC
C      (NGVEL+4)TH CARD:FORMAT(I5) NGT (NO. THICKNESSES TO BE FOUND)
C      NEXT NGT CARDS: FORMAT(8F10.5) SH,PSI,VA,AR
C      XC IS THE POSITION OF THE AXIS OF ROTATION ALONG THE CHORD.
C      ALPHA IS THE BLADE ANGLE OF ATTACK IN DEGREES
C      XIINF IS THE POSITION OF THE STAGNATION POINT AT LARGE SPAN. IF
C      IT IS UNKNOWN, SET IT EQUAL TO 14. AND THE PROGRAM WILL CALCULATE IT

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C      RK10 IS THE CONSTANT K10 IN THE EQUATION FOR Q. IF IT IS UNKNOWN,
C      SET JOIN=5 AND PUT SOME VALUE, SAY 10., ON THE DATA CARD.
C      XIS IS THE VALUE OF XIS AT SEPARATION. IT IS FOUND BY EXTRAPOLATING
C      FOR (Y(3,1),L,1) OR DVDZ AT Z=0 FOR J=1)
      READ(1,482) XO,ALPHB,XIINF,XIS,RK10
482  FORMAT(F10.6,F10.7,F20.16,3F10.7)
      AB=ALPHB/57.29578
      TAB=DTAN(AB)
      SAB=DSIN(AB)
      CAB=DCOS(AB)
      WRITE(3,400) XO,ALPHB,XIINF,XIS,RK10,F/S
400  FORMAT(/' INPUT VALUES '/' AXIS OF ROTATION IS AT',F7.4, 'CHORD'/
1  ' ANGLE OF ATTACK IS',F9.6, 'DEGREES', ' STAGNATION POINT AT INFINI
2TE SPAN IS',D20.14/ ' ESTIMATED SEPARATION POINT IS',F12.7/
3  ' RK10=',F10.6/ ' PARAMETER EPS FOR MAX. THICKNESS OF AIRFOIL=',
4  F12.7/ )
C
C  SECTION 2: INPUT NEEDED IN SECTIONS 15 AND 16
      IF(JOIN.LT. 10) GO TO 4412
      READ(1,4404) RK20R,RK20,RK21,RK22
      WRITE(3,4039) RK20R,RK20,RK21,RK22
4039  FORMAT(/' IN ORDER TO CALCULATE VELOCITIES: RK2CR=',F9.4,5X,
F 'RK2O=',F9.4,5X,'RK21=',F9.4,5X,'RK22=',F9.4//)
      READ(1,7056) NGVEL
7056  FORMAT(I5)
      WRITE(3,4400) NGVEL
4400  FORMAT(15, ' NO. CONDS. TO CALCULATE VELOCITY' )
      IF(NGVEL) 4401,4401,4402
4402  WRITE(3,4405)
4405  FORMAT(/' VELOCITY PROFILES WILL BE FOUND FOR:' /
1  ' SPEED RATIO   AZIMUTH ANGLE   INDUCED VEL   ASPECT RATIO
28CHORD')
      DO 4403 NG=1,NGVEL
      DO 4760 IYP=1,4
4760  ILIM(NG,IYP)=0
4404  FORMAT(8F10.5)
      READ(1,4404) SHV(NG),PSIV(NG),VAV(NG),ARV(NG),PCCV(NG)
C      SH IS A SPEED OF FORWARD FLIGHT (LIKE ALL VELOCITIES, IT IS
C      NON-DIMENSIONALIZED WRT CHORD TIMES ROTATIONAL VELOCITY)
C      PSI IS THE AZIMUTHAL ANGLE IN DEGREES
C      VA IS THE DOWNFLOW
C      AR IS THE ASPECT RATIO OF THE BLADE. POINTS WILL BE TAKEN AT 30°,
C      60°, 90°, AND 95% OF SPAN.
C      PCC IS THE PER CENT CHORD
C      FOR EACH SET OF CONDITIONS (SH,PSI,VA,AR,&PCC) BOTH THE CHORDWISE
C      AND SPANWISE VELOCITY PROFILES WILL BE CALCULATED.
4403  WRITE(3,4406) SHV(NG),PSIV(NG),VAV(NG),ARV(NG),PCCV(NG)
4406  FORMAT(5F15.5)
4401  READ(1,7056) NGT
      WRITE(3,4410) NGT
4410  FORMAT(15, 'STATIONS TO CALCULATE THICKNESSES ')
      IF(NGT) 4412,4412,4411
4411  WRITE(3,4413)
4413  FORMAT(/' THICKNESSES WILL BE FOUND FOR:' / ' SPEED RATIO
1  ' AZIMUTH ANGLE   INDUCED VEL   ASPECT RATIO ')
      DO 4414 NG=1,NGT
      READ(1,4414) SHT(NG),PSIT(NG),VAT(NG),ART(NG)
C      FOR EACH SET OF CONDITIONS (SH,PSI,VA,AR) THE D.L. THICKNESSES
C      WILL BE CALCULATED FOR K>0
4414  WRITE(3,4406) SHT(NG),PSIT(NG),VAT(NG),ART(NG)
4412  CONTINUE
C

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C SECTION 3: SOLUTION FOR THE STAGNATION POINT
  WRITE(3,7048) AB,TAB,CAB,SAB
7048 FORMAT(/' AB=',D20.13,'RADIANS; TAN(AB)='D20.12,'; COS(AB)='
      8 D20.13,'; SIN(AB)='D20.13/')
  SGN=1.00
  DEI=2.00*SAB*SAB
C DEI IS THE VALUE OF DELTA AT THE STAGNATION POINT. DELTA IS 1-SIGMA.
  SI=DEI-1.00
  MST=.5D-3
  CET=.121212771515D-6
  XIINT=.41493299383D-4
  H=.5D-4*MST
C H IS THE SPACING IN RK3 SUBROUTINE
  CSTRT=1.0-20
  XSTRT=EPS*DSQR(2.00*DSTRT)/(1.00+EPS)
C SINCE DSDX BLOWS UP AT(DELTA=0,X=0); RK3 BEGINS INTEGRATING AT
C DELTA=DSTRT,X=XSTRT
  DEP=1.0-5*DEI
C RK3 INTEGRATES FIRST FROM DSTRT TO DEP. THEN H IS INCREASED AND
C INTEGRATION GOES FROM DEP TO DEI. IF THERE IS AN ABNORMAL
C CONDITION IN RK3 (IER .NE. 0) SEE THE WRITE UP ON THE RK1 SUB-
C ROUTINE IN THE IBM 360 SCIENTIFIC SUBROUTINE PACKAGE.
C SGN=-1.00 DENOTES THE UNDERSIDE OF THE AIRFOIL; +1 IS UPPER SIDE
  J=0
  IF(XIINF .GT. 13.00) J=2070
  IF(J .NE. 2070) GO TO 397
  CALL RK3(DSDX,H,XSTRT,DSTRT,3.00,DEP,XIINT,DEI,IER)
  WRITE(3,435) XIINT,DET,XSTRT,DSTRT,IER,H
435 FORMAT(/' RK3 FINDS X='D19.12,2X,'AT DELTA='D19.12/5X,'BY INTEGR
  LATING FROM X='D19.12,2X,'AND DELTA='D19.12/' THE NUMBER OF ABNCR
  2PAL CONDITIONS ENCOUNTERED BY RK3 IS',I3,8X,'THE SPACING IN X IS',
  3 D15.6 /)
  H=.01D0*MST
  CALL RK3(DSDX,H,XIINT,DET,3.00,DEI,XIINF,DV,IER)
  IF(TAB .GT. 0.00) SGN=-1.00
  XIINF=XIINF*SGN
C XIINF (ALSO CALLED CH10) IS THE VALUE OF X AT THE STAGNATION
C POINT AT Y-INFINITY. NORMALLY IT IS A NEGATIVE NUMBER
  WRITE(3,435) XIINF,DV,XIINT,CET,IER,H
397 IF(TAB .GT. 0.00) SGN=-1.00
  WRITE(3,401) XIINF,DEI
  401 FORMAT(/' XIINF='D22.15,' IS THE VALUE OF X, AND DEI='D22.14,
  F' IS THE VALUE OF DELTA, AT THE STAGNATION POINT')
  IF(JDIM .EQ. 5) WRITE(2,246) X0,ALPHA,XIINF
246 FORMAT(F10.6,F10.7,F20.16,30X,'EXTRASTAFF' )
  CV=SGN*13.00
  B99=DSDX(CV,DEI)
C DSDX WILL CALCULATE XR AND ZR AND THEIR DERIVATIVES WRT DELTA (OR
C SIGMA)
  B11=2.00*(B2+B4+B3+B5)
  WRITE(3,436) B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11
  436 FORMAT(' B1,B2,B3,B4,B5/B6,B7,B8,B9,B10,B11',501E.10/6D18.10)
  CH10=XIINF
  CH11=-4.00*B10*CAB*SAB
  CH120=(B11/B10)*4.00*(CAB*SAB)**2+2.00*B10*( CAB**2-SAB**2)
  CH122=-CH110
  WRITE(3,434)CH10,CH110,CH120,CH122
434 FORMAT(/' THE COEFFICIENTS IN THE SERIES FOR STAGNATION POINT'/
  1' CH10='D15.8,5X'CH110='D15.8,5X,'CH120='D15.8,5X,'CH122='
  2D15.8 /)
C END PARAMETERS FOR POTENTIAL FLOW
C

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C SECTION 4: PARAMETERS AND FUNCTIONS OF Z FOR LEW'S METHOD
C PARAMETERS FOR SOLUTION BY LEW'S METHOD
  CELX=Z,5C-3
C THE INCREMENT OF XI IS PROPORTIONAL TO DELX. FOR REASONS PEYCN0
C RECALL THE STAGNATION POINT (XI=0,X=-XIINF) IS TAKEN AT K=4, WHERE
C K IS THE SUBSCRIPT FOR XI.
  XI(4)=0.00
  X(4)=XIINF
  CE=CEI
  T=SI
C DELTA (CE) AND SIGMA (T) ARE SET TO STAGNATION POINT VALUES.
  ERRCR=5C-6
  CMEGAR=1.7D0
C ERROR AND OMEGA ARE THE PARAMETERS THAT H.G. LEW CALLS EPSILON
C AND OMEGA. ERROR AND OMEGA ARE PROPORTIONAL TO ERRCR & CMEGAR.
C SEE STMT 528
  LMAXU=75
C L IS THE SUBSCRIPT FOR Z (ETA); FOR EACH J AND K, A VALUE OF LMAX
C WILL BE FOUND; L WILL BE <=LMAX AND LMAX WILL BE <=LMAXU. SEE THE
C DIMENSION STMT.
  KMAX=70
C K WILL BE <=KMAX. IF K=KMAX BEFORE FOM REACHES .05 OR .04, THEN
C EITHER DXI(K) MUST BE INCREASED (SEE DELX) OR KMAX INCREASED (SEE
C DIMENSION STMT).
  LST=55
  ITERM=450
C ITERM IS MAXIMUM NO. OF ITERATIONS BEFORE LEW'S METHOD IS
C DECLARED NON-CONVERGENT. COMPARE IT TO VALUES OF ITER (SEE
C WRITE(3,474))
  CZST=.1D0
C CALCULATE Z(L) & RELATED FUNCTIONS
  DO 521 L=1,LST
    CZ(L)=CZST
  521 Z(L)=CZST*(L-1)
  DO 522 L=LST,LMAXU
    CMISSION OF THIS DO LOOP AND SETTING LST=LMAXU WILL MAKE CZ(L)
    CONSTANT
    CZ(L)=CZST*(2.0D0*(Z(L-1)-Z(LST))+CZST))
    IF( CZ(L) .GT. .4C0) CZ(L)=.4C0
  522 Z(L)=Z(L-1)+CZ(L)
  WRITE(3,1482) (L,Z(L),CZ(L),L=1,LMAXU)
1482 FORMAT(// ' L',6X,'Z',9X,'CZ',6X,'L',6X,'Z',9X,'CZ',6X,'L',6X,'Z',
1,9X,'CZ',6X,'L',6X,'Z',9X,'CZ',6X,'L',6X,'Z',9X,'CZ',6X,'L',6X,'Z',
2,' ',14,2F10.6,' ',14,2F10.6,' ',14,2F10.6,' ',14,2F10.6))
  LPM1=LMAXU-1
  DO 523 L=1,LPM1
    CN,EN AND SDZ ARE USED RIGHT AFTER "DO 130C" TO FIND THE STREAM
    FUNCTION (FO,GO,ETC), THE SECOND DERIVATIVE OF THE STREAM FUNCTION
    (FOM,GOM,ETC), AND THE THIRD DERIVATIVE (FOM',ETC).
    CN(L)=1.00/CZ(L+1)-1.00/CZ(L)
    EN(L)=1.00/CZ(L+1)+1.00/CZ(L)
  523 SDZ(L)=2.00/(CZ(L)+CZ(L+1))
  WRITE(3,440) CZST,LST,LMAXU,CMEGAR,ERRCR,DELX,JOIN
440 FORMAT(// ' THE ETA SPACING IS',F6.4,' UP TO L=',I3,'; IT GRADUALLY
1 INCREASES UP TO THE MAXIMUM OF L=',I3// ' THE SPEED OF CONVERGENCE
2 IN ETA DEPENDS ON OMEGAR=',F8.4//
3 ' SUCCESSIVE ITERATIONS MUST MATCH VELOCITY WITHIN AN ERROR THAT',
4 ' DEPENDS ON ERRCR=',E12.5// ' THE X SPACING DEPENDS ON DELX=',
5 F12.8// ' THE NO. OF EQUATIONS IS',I3// ' BEGIN SOLUTION'/// )
C END CALCULATE Z(L) & RELATED FUNCTIONS
C
C SECTION 5: STARTING VALUES

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C      FIX STARTING VALUES
C      J=1 DENOTES F0,FC,FC* OR FC**;J=2 IS G03,G03*,ETC;J=3 IS G01;
C      J=6 IS F20C;J=7 IS F20C;8 IS F21C;9 IS F22C;10 IS G1C;11 IS G11;
C      12 IS G12;IF JDIM<5 THEN J=4 IS F1K AND J=5 IS F10C ; IF JDIM>6
C      THEN J=4 IS F10 (F10=F10C+K1C*F1K) AND J=5 IS F2K.
C      CC 525 J=1,JDIM
C      PC(J)=0.00
C      PC(J) IS THE VALUE OF THE J*TH VEL (VEL IS FC*,G03*,G01*,ETC AND
C      CVDZ IS FC*,G01*,ETC) AT ZMAX (AT ETA=INFINITY)
C      EDC(J)=0.00
C      LMAXR(J)=0
C      CC 525 L=1,LMAXU
C      RHS(J,L)=0.00
C      A(1,J,L)=0.00
C      A(2,J,L)=0.00
C      CC 525 P=1,3
C      CC 525 I=1,4
525 Y(M,J,L,1)=0.00
C      CC 526 I=1,4
C      ST(I)=0.00
526 S(I)=0.00
C      THESE MATRICES (EDC,RHS,A,Y,S,AND ST) WILL BE REDEFINED AS NEEDED
C      RC(1)=1.00
C      RC(3)=1.00
C      PC(2) AND UC(4) WILL BE REDEFINED LATER.
C      CC 527 L=1,LMAXU
C      THIS IS THE INITIAL ESTIMATE FOR THE VELOCITY PROFILES.
C      Y(2,1,L,1)=1.00-CEXP(-7(L))
C      CC 546 J=2,JDIM
546 Y(2,J,L,1)=Y(2,1,L,1)
527 CONTINUE
C      Y(3,1,1,2)=1.200
C      END FIX STARTING VALUES
C      END PARAMETERS FOR SOLUTION BY LEWIS METHOD
C      BEGIN K LOOP IN XI
C      K IS THE SUBSCRIPT FOR XI,PXI,X AND CX
C      CC 30 K=5,KMAX
C      CX(K)=DELX*(1+K/23+2*(K/33)+2*(K/40))
C
C      SECTION 6: VALUES OF XI AND X AT K*TH CHORDWISE STATION
C      THESE VALUES FOR THE INCREMENTS OF XI (DXI) HAVE PROVED BOTH
C      ACCURATE AND REASONABLE FAST. IF DIFFERENT VALUES OF ALPHB ARE
C      NEEDED, DXI(K) NEAR SEPARATION CAN BE ADJUSTED SO THAT FC* (LVCZ
C      FOR J=1) IS < ABOUT .05 AT THE FINAL VALUE OF XI(K).
C      IF(DABS(ALPHB) .LT. 2.0-2) DXI(K)=DELX*(2+K/13-K/27+K/21
C      +4*(K/22)-3*(K/38))
C      IF(ALPHB .GE. 1.00) DXI(K)=DELX*(1.500+.500*(K/23)+2*(K/27)
C      +4*(K/31))
C      IF(DABS(ALPHB-2.00) .LT. 2.0-2) DXI(K)=DELX*(1+.600*(K/19-K/38-K/
C      2.57)+.800*(K/29-K/58)+1.000*(K/39)+2*(K/50)-K/62)
C      IF(DABS(ALPHB-3.34900) .LT. 2.0-2) DXI(K)=.800*DELX*(1+(K/27-K/54)
C      +.500*(K/37)+.500*(K/44+K/51))
C      IF(DABS(ALPHB-4.09000) .LT. 2.0-2) DXI(K)=.800*DELX*(1+(K/27-K/54)
C      +.500*(K/37)+.500*(K/44+K/51))
C      IF(DABS(ALPHB-4.46500) .LT. 2.0-2) DXI(K)=.800*DELX*(1+(K/27-K/54)
C      +.500*(K/37)+.500*(K/44))
C      IF(DABS(ALPHB-5.58100) .LT. 2.0-2) DXI(K)=.800*DELX*(1+(K/27-K/54)
C      +.500*(K/37)+.500)
C      IF(K .GE. 8) GO TO 513
C      IF(K-6) 502,503,504
C      K=5
502 XI(5)=.50-3*DELX

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      IMAX=2
      DXI(5)=XI(5)
      GO TO 520
503 XI(6)=2000*CELX
      K=6
      S(1)=1.00
      S(2)=-1.00
      CXI(K)=XI(K)-XI(K-1)
      GO TO 520
504 XI(7)=DXI(7)
      K=7
      S(1)=(1.00/DXI(K)+1.00/(CXI(K-1)+CXI(K)))*XI(K)
      S(2)=-XI(K)*(DXI(K)+DXI(K-1))/(CXI(K)+DXI(K-1))
      S(3)=XI(K)*DXI(K)/((DXI(K)+DXI(K-1))*DXI(K-1))
      XI*(DERIVATIVE OF Y(M,J,L,1) WRT XI)= SUM OF S(I)*Y(M,J,L,1) FOR
      I=1 TO IMAX
      IMAX=3
      DXI(K)=XI(K)-XI(K-1)
      GO TO 512
513 XI(K)=XI(K-1)+DXI(K)
      K=8
      S(1)=(1.00/(XI(K)-XI(K-3))+1.00/(XI(K)-XI(K-2))+1.00/DXI(K))*XI(K)
      S(2)=-XI(K)*(XI(K)-XI(K-3))*(XI(K)-XI(K-2))/(DXI(K)+DXI(K-1))*
      2 (XI(K-1)-XI(K-3)))
      S(3)=XI(K)*(XI(K)-XI(K-3))*DXI(K)/(CXI(K-2)*(XI(K)-XI(K-2))*
      3 DXI(K-1))
      S(4)=-XI(K)*(XI(K)-XI(K-2))*CXI(K)/(DXI(K-2)*(XI(K-1)-XI(K-3))*
      4 (XI(K)-XI(K-3)))
      IF ALL DXI ARE EQUAL; S1=11/6, S2=-3, S3=1.5, S4=-1/3
      IMAX=4
512 CCNTINUE
520 X(K)=XI(K)+XIINF
      K=7
      DX(K)=DXI(K)
C
C SECTION 7: FUNCTIONS OF XI FOUND FROM POTENTIAL FLOW
C CALCULATIONS FOR POTENTIAL FLOW
      SGN=1.00
      IF(X(K) .LT. 0.00) SGN=-1.00
      H=.100*HST
      IF(CABS(X(K)) .LT. 1.0-3) H=.0500*HST
      IF(X(K) .GT. 0.00 .AND. X(K) .LT. 3.0-2) H=.0300*HST
      IF(X(K) .GT. .0600) H=.200*HST
      IF(H .GT. 1.0-2*CXI(K)) H=1.0-2*CXI(K)
      CEP=DE
      IF(X(K)*X(K-1)) 3314,1405,3315
3314 H=1.0-2*HST
      IF(X(K) .LT. XIINT) GO TO 1405
      CALL RK3(CSDX,H,XIINT,CEP,X(K),27.DC,XF,DE,IER)
      WRITE(3,435) XF,DE,XIINT,CEP,IER,H
      GO TO 320
3315 CALL RK3(CSDX,H,X(K-1),CEP,X(K),27.DC,XF,DE,IER)
C RK3 INTEGRATES CSDX TO FIND THE VALUE OF SIGMA(NEW CALLED T) AT
C X(K).
320 CCNTINUE
      T=DE-1.00
      CUMPY=13.00*SGN
319 P99=CSDX(CUMPY,DE)
      WRITE(3,450) K,XI(K),IER,CXI(K),DE,SGN,X(K)
450 FCMPAT(' K=',13,5X,'XI(K)=' ,C19.11,10X,'IER=',15 / 1 DXI(K)=' ,
      1C19.11,5X,'DE=',C19.11,5X,'SGN=',F5.2,5X,'X(K)=' ,D19.11)
C P2=DERIV. OF XR WRT SIGMA (CR DELTA, WHERE DELTA=1+SIGMA);

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C      H3=DERIV. OF ZH WRT SIGMA; B4=DERIV. OF R2 WRT SIGMA; B5=DERIV. OF
C      B3 WRT SIGMA; AND SO ON UP TO B9=DERIV. OF H7 WRT SIGMA.
C      B10= DERIV. OF X WRT SIGMA =SGN*DSQRT(B2*B2+B3*B3);
C      B11= DERIV. OF B10**2 WRT SIGMA; B12=DERIV. B11; B13= DERIV. B12
C      B20 AND B21 ARE DEFINED FOR CONVENIENCE, AS ARE B6,B7,ETC.
      B11=2.00*(B2*B4+B3*B5)
      B12=2.00*(B4*B4+B2*B6+B5*B5+B3*B7)
      B13=2.00*(3.00*(B4*B6+B5*B7)+B2*B8+B3*B9)
      B20=(B11/B10)/(B10*B10)
      B21=((B12/B10)/(B10**4)
      XR=(1.00+EPS)*(T-EPS)*(1.00+((1.00-EPS)**2)/B1)/4.00
      XR=XR+(1.00+EPS*EPS)/2.00-X0
      ZR=.2500*(1.00+EPS)*DSQRT(DE*(1.00-T))*(1.00-(1.00-EPS)**2/B1)*SGN
C      XR IS CAPITAL X(SUBSCRIPT R); ZR IS CAP Z(SUB R)
      WRITE(3,479) B1,B10,B11,B12,B13
479  FORMAT(' B1,B10,B11,B12,B13 ',5D22.14 )
      DES=DSQRT(DE)
      CMSS=DSQRT(1.00-T)
C      PSC IS PHI(SUBSCRIPT SIGMA); PRO IS PHI(SUBSCRIPT RHO);
C      PS1 IS THE FIRST DERIV. OF PSC WRT SIGMA; PR2 IS 2ND DERIV. OF PRO
C      AND SO ON.
      PS0=(1.00+EPS)*T/2.00
      PRO=(1.00+EPS)*(SGN*DES*CMSS-CATAN(SGN*DES*CMSS/T))/2.00
      PS1=(1.00+EPS)/2.00
      PS2=0.00
      PS3=0.00
      PR1=SGN*PS1*CMSS/DES
      PR2=-SGN*PS1/(CMSS*(DES**3))
      PR3=SGN*PS1*(1.00-2.00*T)/((CMSS**3)*(DES**5))
      PR4=-SGN*PS1*3.00*(1.00-2.00*(1.00-T)*T)/((CMSS**5)*(DES**7))
      BP(1)=P10
      CC 317 I=2,4
317  BP(1)=PP(1-1)*B10
C      USC IS U(SUPERSCRIPT TILDA,SUBSCRIPT SIGMA); URC IS U(SUPERSCRIPT
C      TILDA,SUB RHO); US1 IS FIRST DERIV. OF USO WRT X, ETC.
      USO=PS1/B10
      UPO=PR1/B10
      US1=-.500*B11/BP(4)
      UR1=US1*PR1+PR2/BP(2)
      US1=US1*PS1
      US2=B20*B20/B10-.500*B21
      UR2=US2*PR1-PR2*1.500*B20/BP(2)+PR3/BP(3)
      US2=US2*PS1
      US3= B20*(3.2500*B21-3.5000*B20*B20/B10)-.500*B13/(BP(3)**2)
      UR3= US3*PR1+(4.7500*B20*B20/B10-2.00*B21)*PR2/B10
      US3=-3.00*B20*PR1/BP(3)+PR4/BP(4)
      US3=US3*PS1
C      PHIA IS PHI(SUB A) ; UAC IS U(SUPER TILDA,SUB A) ; NAL IS FIRST
C      DERIV. OF UAC WRT X, AND SO ON.
      PHIA=CAB*PS0+SAB*PRO
      UAO= CAB*US0+SAB*UR0
      UA1= CAB*US1+SAB*UR1
      UA2= CAB*US2+SAB*UR2
      UA3= CAB*US3+SAB*UR3
      PHIC=SAB*PS0-CAB*PRO
      UCO= SAB*US0-CAB*UR0
      UC1= SAB*US1-CAB*UR1
      UC2= SAB*US2-CAB*UR2
      UC3= SAB*US3-CAB*UR3
C      UAPK,UCPK,PCC, AND RNOK STORE VALUES FOR LATER USE TO INTERPOLATE
C      FOR VELOCITY PROFILES AND THICKNESSES
      PCC(K)=(XR+X0)*1.02

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      UAPK(K)=UAO
      UCPK(K)=UCO
      WRITE(3,490)
490  FORMAT(' PSO,PS1...PS4 |PRO,PR1...PR4 |PSC,USC...US3 |PRO,URO'
      F,'...UR3 |PHIA,UA0...UA3 |PHIC,UC0...UC3 |XR,B2,B4,B6,B8 |'
      F,'ZR,B3,B5,B7,B9' )
      WRITE(3,492) PSO,PRO,PSO,PRO,PHIA,PHIC,XR,ZR, PS1,PR1,USO,URO,UAO,
      IUCO,B2,B3,PS2,PR2,US1,UR1,UA1,UC1,B4,B5, PS2,PR3,US2,UR2,UA2,UC2,
      2B6,B7, PS2,PR4,US3,UR3,UA3,UC3,B8,B9
492  FORMAT( ' ', 8C16.8 )
      ALPHA=DATAN(B3/B2)
      IF(SGN .LT. 0.00 ) ALPHA= 3.1415926536 +ALPHA
      ADEG=57.2958*ALPHA
      PO=XI(K)/UAO
      XMO=PO*UA1
C     XMO IS M(SUB O);XMIK IS M(SUB 1K); RNUO IS NU(SUB O)
      RM=(XMO+1.00)/2.00
C     RM,RN,RPS ARE DEFINED FOR CONVENIENCE
      RN=(XMO-1.00)/2.00
      RPS=RM+S(1)
      RNUO=PHIA-2.00*(XR*CAR+ZR*SAB)
      RNUK(K)=RNUO
      CO=2.00*(CAB*B2+SAB*B3)/B10
C     PARAMETERS NEEDED FOR F10
      XMIK=XI(K)*UA2+UA1*(1.00-XMO)
      XPIOC=CHI10*UA2+UC1-(CHI10*UA1+UCC)*UA1/UAC
      WRITE(3,430) XMO,PO,CO,RM,ALPHA,H,ADEG
430  FORMAT( ' XMO,PO,CO,RM,ALPHA,H,ADEG ',7D14.6)
      WRITE(3,431) XMO,XMIK,XPIOC,RNUC
431  FORMAT( ' XMO,XMIK,XPIOC,RNUC ',6D15.7)
C     END PARAMETERS FOR F1
      IF(JDIM .LE. 5) GO TO 8632
C     PARAMETERS NEEDED FOR G1
      DCDXA=(B4-B2*B11/(2.00*B10*B10))/(B10*B1C)
      CSCXA=(B5-B3*B11/(2.00*B1C*B1C))/(B1C*B1C)
      CCCX=CCCX*CCOS(AB)+DCDX*DSIN(AB)
C     RK10 MUST BE DEFINED HERE
      XPI0=XPIK*RK10+XPIOC
      CPK=CHI10+XI(K)*RK10
      RNUA10=CPK*UA1
C     RNUA10 IS MU(SUPER A,SUB 10), UD10 IS U(SUPER DELTA,SUB 10)
      RNU10=CPK*(UA0-CO)
      C10=CPK*2.00*CCCX
      UC10=CPK*UA1+UC0
      UD12=UAC
C     PARAMETERS NEEDED FOR F2
      XP2K=XPIK
      XP22C=(CPK+CHI22)*(UA2-UA1*UA1/UA0)+RK1C*UA1
      XP20C=(CHI20+XI(K)*RK10**2)*(UA2-UA1*UA1/UA0)+(UA3-UA1*UA2/UAC)*
      X .500*CPK*CPK +CPK*(UC2+RK10*UA2-UA1*UC1/UAC)+RK10*(UC1+RK1C*UA1)
      WRITE(3,432) XMO,XP10,XP20C,XP22C,RK1C,CPK
432  FORMAT( ' XMO,XP10,XP20C,XP22C,RK1C,CPK ',6D16.8)
      WRITE(3,433) C10,RNU10,UD10,CCDX
433  FORMAT( ' C10,RNU10,UD10,CCDX ',5D17.9)
C  J=1 2 3 4 5 6 7 8 9 10 11 12
C  FC G00 G01 F10 F2K F2CR F2CC F21C F22C G10 G11 G12
      PC(10)=RNU10
C     END PARAMETERS FOR G1 & F2
8632 CONTINUE
C END CALCULATIONS FOR POTENTIAL FLOW
C
C SECTION 8: PARAMETERS FOR LEW'S METHOD AND A GUESS AT NEW VELOCITY

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C          PROFILE BY EXTRAPOLATION
C      FIX BC, PARAMETERS, AND VELOCITIES IN NEW X STATION
      BC(2)=RNUO
      CC 528 J=1,JDIM
      LMAXT=LMAXU+(8.00+1/J)*XI(K)/XIS-10/J-10
      IF(JDIM .GT. 5 .AND. J .GT. 3) LMAXT=LMAXT-1
      IF(LMAXT .GT. LMAXR(J)) LMAXR(J)=LMAXT
      IF(CABS(BC(J)) .GT. 0.00) LMAXR(J)=LMAXR(J)+CABS(BC(J))/1.0-3)
C      EBC IS DVDZ( 2ND DERIV. OF STREAM FUNC. WRT ETA) AT Z(LMAXR(J))
C      EACH STREAM FUNCTION IS INTEGRATED TO ITS OWN MAX VALUE OF ETA,
      WHICH IS GIVEN BY Z(LMAXR(J))
      IF(LMAXR(J) .GT. LMAXU-1) LMAXR(J)=LMAXU-1
      CMEGA(J)=OMEGA +.02DC*(J-1)
528 ERROR(J)=ERRCRR+( 1.0-1 +CABS(BC(J))+ .5C-2*Y(2,J,15,1)**2+J/5)
      IF(Y(3,1,1,2) .LT. .12CC) ERROR(1)=ERROR(1)+2.0C
C      NEW VELOCITIES BY INTERPOLATION
      IF(K=8) 543,543,542
542 ST(2)=(XI(K)-XI(K-2))*(XI(K)-XI(K-3))/(DXI(K-1)*(XI(K-1)-XI(K-3)))
      ST(3)=-DXI(K)*(XI(K)-XI(K-3))/(CXI(K-1)*CXI(K-2))
      ST(4)=CXI(K)*(XI(K)-XI(K-2))/(XI(K-1)-XI(K-3))*CXI(K-2))
      CC 544 J=1,JDIM
      CC 544 L=2,LMAXU
544 Y(2,J,L,1)=ST(2)*Y(2,J,L,2)+ST(3)*Y(2,J,L,3)+ST(4)*Y(2,J,L,4)
543 CONTINUE
      WRITE(3,481) (S(I),I=1,4),(ST(I),I=2,4)
481 FORMAT(' S(I)=',4F15.10,1CX,' ST(I)=',3F15.10)
C END NEW VELOCITIES BY INTERPOLATION
C END FIX BC, PARAMETERS, AND VELOCITIES IN NEW X STATION
C      CHOOSE VARIABLE TO BE SOLVED FOR          DO 1500 J
      CC 1500 J=1,JDIM

C SECTION 9: FIX UP ESTIMATED PROFILE TO MATCH BOUNDARY CONDITIONS
      LMAX=LMAXR(J)
      LPM=LMAX-1
C      CORRECT VELOCITIES TO MATCH BC & CALCULATE A'S
C      IF BC(J) CHANGES WITH X (OR XI), THEN A NEW PROFILE FOR VELOCITY
C      (ACTUALLY DERIV. OF STREAM FUNC. WRT ETA) MUST BE FOUND. STMT 549
C      IS NECESSARY TO BE SURE TO AVOID DIVISION BY ZERO
      IF(CABS(BC(J)-Y(2,J,LMAX,1)) -1.0-14) 547,547,549
549 IF(CABS(Y(2,J,LMAX,1)) -1.0-10) 551,545,545
545 BCO=BC(J)/Y(2,J,LMAX,1)
      CC 531 L=2,LMAX
531 Y(2,J,L,1)=Y(2,J,L,1)*BCO
      GC TO 547
551 BCO=(BC(J)-Y(2,J,LMAX,1))/Z(LMAX)
      CC 548 L=2,LMAX
548 Y(2,J,L,1)=Y(2,J,L,1)+BCO*Z(L)
547 CC 532 M=1,2
      CC 532 L=1,LMAXU
      A(M,J,L)=0.00
      CC 532 I=2,IPAX
532 A(P,J,L)=A(M,J,L)+S(I)*Y(P,J,L,1)
C      AT THIS POINT, A(M,J,L)=SUM OF S(I)*Y(M,J,L,1) FROM I=2 TO IPAX,
C      OR XI=DERIV. OF Y(M,J,L,1) WRT XI MINUS S(I)*Y(M,J,L,1). AS SOON
C      AS Y(M,J,L,1) IS KNOWN, IT WILL BE ACCED IN SO A CAN BE USED FOR
C      SUBSEQUENT VALUES OF J TO REPRESENT ALL THE DERIVATIVE
C END CORRECT VELOCITIES TO MATCH BC & CALCULATE A'S
C
C SECTION 10: SOLUTION OF CE'S BY LEW'S METHOD
C      BEGIN ITERATION ON LEW'S SUCCESSIVE REPLACEMENTS          DO 1400 ITER
      CC 1400 ITER=1,ITERM
C      BEGIN MARCHING IN Z DIRECTION          DO 1300 L

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DO 1300 L=2,LMM1
C   DEFINE VEL,DVDZ & STREAM FUNCTION
   YT=Y(2,J,L,1)
   Y(1,J,L,1)=Y(1,J,L-1,1)+DZ(L)*(Y(2,J,L-1,1)+YT)/2.00
   Y(3,J,L,1)=(Y(2,J,L+1,1)/DZ(L+1)-DN(L)*YT-Y(2,J,L-1,1)/DZ(L))*500
   YP=SDZ(L)*(Y(2,J,L+1,1)/DZ(L+1)-EN(L)*YT+Y(2,J,L-1,1)/DZ(L))
C   YP IS 2ND DERIVATIVE OF VELOCITY
C   YP IS USED INSTEAD OF Y(4,J,L,1) TO DECREASE STORAGE.
C   J=1 2 3 4 5
C   FO GOO 801 F1K F10C
C   GC TO (1,2,3,4,5,6,6,6,6,10,10,10),J
C   THIS STMT CHOOSES THE EQUATION TO BE SOLVED. IF TIME IS NOT
C   IMPORTANT A FUNCTION SUBROUTINE TO CALCULATE FN AND FNN (THE
C   DERIV. OF FN WRT YT) COULD BE USEC. THE 6,6,6,6 OCCURS BECAUSE
C   FOUR EQNS ARE IDENTICAL EXCEPT FOR RHS. RHS CONTAINS NO YT SC IT
C   CAN BE CALCULATED JUST ONCE AND SAVE TIME(SEE DC LOOP FOR STMT
C   NO. 8220)
C   YT=FO*
1  FN=-YP-Y(3,J,L,1)*(RMS      *Y(1,J,L,1)+A(1,J,L) )+YT*((XMC+S(1))
   1 *YT+A(2,J,L) )-XMO
   FNN= EN(L)*SDZ(L)+.500*DN(L)*(RMS      *Y(1,J,L,1)+A(1,J,L))
   1-Y(3,J,L,1)*RMS      *.500*DZ(L)+YT*2.00*(XMC+S(1))+A(2,J,L)
   GC TO 117
C   YT=GOO*
2  FN=-YP-DUMV(L)*Y(3,J,L,1)+Y(2,1,L,1)*(S(1)*YT+A(2,J,L) )
   2 *XI(K)*(CO*Y(2,1,L,1)-UA0)
   FNN=EN(L)*SDZ(L)+DUMV(L)*.500*DN(L)+Y(2,1,L,1)*S(1)
   GC TO 117
C   YT=GO1*
3  FN=-YP-DUMV(L)*Y(3,J,L,1)+Y(2,1,L,1)*(S(1)*YT+A(2,J,L) )
   FNN=EN(L)*SDZ(L)+DUMV(L)*.500*DN(L)+Y(2,1,L,1)*S(1)
   GC TO 117
4  IF(JDIM-5) 8720,8720,8721
C   YT=F1K*   J=4   JDIM<=5
8720 FN=-YP-DUMV(L)*Y(3,J,L,1)+(DT(L)+XMC*Y(2,1,L,1))*YT+
   1 Y(2,1,L,1)*A(2,J,L)-Y(3,1,L,1)*(RMS      *Y(1,J,L,1)+A(1,J,L))
   2 +PO*RHSF(L)*XMIK
   GC TO 8722
C   YT=F10*   J=4   JDIM>=6
8721 FN=-YP-DUMV(L)*Y(3,J,L,1)+(DT(L)+XMC*Y(2,1,L,1))*YT+
   1 Y(2,1,L,1)*A(2,J,L)-Y(3,1,L,1)*(RMS      *Y(1,J,L,1)+A(1,J,L))
   2 +PO*RHSF(L)*XMI0
8722 FNN=SDZ(L)*EN(L)+DUMV(L)*DN(L)*.500+ CT(L)+XMC*Y(2,1,L,1)
   1 -Y(3,1,L,1)*RMS      *CZ(L)*.500
   GC TO 117
5  IF(JDIM-5) 8726,8726,8724
C   YT=F10C*   J=5   JDIM<=5
8726 FN=-YP-DUMV(L)*Y(3,J,L,1)+(CT(L)+XMO*Y(2,1,L,1))*YT+
   1 Y(2,1,L,1)*A(2,J,L)-Y(3,1,L,1)*(RMS      *Y(1,J,L,1)+A(1,J,L))
   2 +PO*RHSF(L)*XMI0C
   FNN=SDZ(L)*EN(L)+DUMV(L)*DN(L)*.500+ DT(L)+XMC*Y(2,1,L,1)
   1 -Y(3,1,L,1)*RPS      *CZ(L)*.500
   GC TO 117
C   YT=F2K*   J=5   JDIM>=6
8724 FN=-YP-DUMV(L)*Y(3,J,L,1)+CT(L)*YT+DT2(L)*Y(1,J,L,1)+Y(2,1,L,1)*
   1 A(2,J,L)-Y(3,1,L,1)*A(1,J,L)
   5 *XI(K)*XM2K*RHSF(L)
   FNN=SDZ(L)*EN(L)+DUMV(L)*DN(L)*.500+DT(I)+DT2(L)*CZ(L)*.500
   GC TO 117
C   YT=F20K*   J=6
6  FN=-YP-DUMV(L)*Y(3,J,L,1)+DT(L)*YT+DT2(L)*Y(1,J,L,1)+Y(2,1,L,1)*
   1 A(2,J,L)-Y(3,1,L,1)*A(1,J,L) +RHS(J,L)

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FNN=SDZ(L)*EN(L)+DUMV(L)*CN(L)*.5CO+DT(L)+DT2(L)*DZ(L)*.5D0
GC TO 117
C      YT=F20C' J=7
C      YT=F21C' J=8
C      YT=F22C' J=9
10 FN=-YP-DUMV(L)*Y(3,J,L,1)+Y(2,1,L,1)*(S(1)*YT+A(2,J,L))+RHS(J,L)
' FNN=SDZ(L)*EN(L)+DUMV(L)*CN(L)*.5CO+Y(2,1,L,1)*S(1)
C      YT=G11' J=11
C      YT=G12' J=12
117 EY(L)=-OMEGA(J)*FN/FNN
C      CALCULATE NEW VALUES FOR Y(2,J,L,1)
      Y(2,J,L,1)=YT+EY(L)
1300 CONTINUE
C END BEGIN MARCHING IN Z DIRECTION DO 13CC L
      CC 536 L=2,LPM1
      IF(DABS(EY(L))-ERROR(J)) 536,535,535
C      THE CHANGE IN Y(2,J,L,1) SINCE THE LAST PASS THRU THE L LOOP IS
C      COMPARED TO ERROR (LEW CALLS IT EPSILON)
536 CONTINUE
      GO TO 1401
535 CONTINUE
      EYK(ITER)=EY(10)
C      IT IS NOT NECESSARY TO STORE A VALUE OF EY IF EYK IS REMOVED FROM
C      WRITE(3,462) IN SECTION 11.
1400 CONTINUE
C END BEGIN ITERATION ON LEW'S SUCCESSIVE REPLACEMENTS DO 14CC ITER
C
C SECTION 11: TIDY UP Y AND A MATRICES
      WRITE(3,462) J
      WRITE(3,462) J,K,ITER,LMAX,(EYK(I),I=1,ITER)
462 FORMAT(' ITER FAILS ',4I5/(6C18.9))
      GC TO 1405
1401 ITERK(J)=ITER
C      LEW'S METHOD HAS NOW CONVERGED.
463 FORMAT(' J,ITER=', 2I5/(10C13.5))
C      THE VALUE OF Y(3,J,1,1) (I.E. DVOZ AT Z=0) IS CALCULATED. THIS
C      FORMULA IS GOOD ONLY IF DZ(5)=DZ(4)=...DZ(2)
      Y(3,J,1,1)=( -25.D0*Y(2,J,1,1)+48.D0*Y(2,J,2,1)-36.D0*Y(2,J,3,1)
      +16.D0*Y(2,J,4,1)-3.C0*Y(2,J,5,1) )/(12.D0*DZ(3) )
      CC 537 L=LMAX,LMAXU
      Y(2,J,L,1)=HC(J)
C      THIS DO LOOP EXTENDS THE SOLUTION FROM LMAX TO LMAXU
      Y(1,J,L,1)=Y(1,J,LPM1,1)+(Z(L)-Z(LMM1))*BC(J)
      Y(3,J,L,1)=Y(3,J,LMM1,1)*DEXP((Z(LMM1)-Z(L))/DZST)
537 CONTINUE
      CC 538 L=1,LMAXU
      DO 538 M=1,2
C      THE ADDITIONAL TERM FOR A (SEE SEC 9) IS ADDED IN
538 A(M,J,L)=A(M,J,L)+S(1)*Y(M,J,L,1)
C END DEFINE VEL,CVDZ & STREAM FUNCTION
C
C SECTION 12: FUNCTIONS OF ETA USED IN DE
      IF(JDIM .LE. 5 .OR. J .NE. 5) GO TO 633
      CC 6220 L=1,LMAXU
C      TERMS THAT INVOLVE FUNCTIONS OF Z (ETA) BUT DO NOT CONTAIN YT
C      WHEN LEW'S METHOD IS APPLIED, ARE BEST CALCULATED HERE AND STORED.
C      THIS AVOIDS DOING THE CALCULATION ITERK(J) TIMES. RHS(J,L) IS NOT
C      USED FOR THE FIRST 5 EQUATIONS (GIVEN BY STMTS 1 TO 5). DUMV,RHSF,
C      DT7 AND CT ARE DEFINED FOR CONVENIENCE.
626 RHS(6,L)=X1(K)*(Y(2,1,L,1)*Y(2,2,L,1)-RNLC+.5DC*Y(3,1,L,1)
      +Y(1,2,L,1) ) +PO*CO*(RNUL-Y(2,2,L,1)).
627 RHS(7,L)=UAO*(-RM*Y(1,4,L,1)*Y(3,4,L,1)+XMO*Y(2,4,L,1))*2

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7 +Y(2,4,L,1)*A(2,4,L,1)-Y(3,4,L,1)*A(1,4,L,1))
7 +XI(K)*XM10*(2.00*Y(2,1,L,1)*Y(2,4,L,1)-.500*Y(3,1,L,1)*
7 Y(1,4,L,1)-.500*Y(1,1,L,1)*Y(3,4,L,1) ) + XI(K)*(XM20C-XM10*UD10
7 /UA0) * RHSF(L)
628 RHS(8,L)=XI(K)*(Y(2,1,L,1)-2.00+.500*Y(3,1,L,1)*(Y(1,3,L,1)+Z(L))+
8 Y(2,1,L,1)*Y(2,3,L,1) ) +P0*CO*(1.00-Y(2,3,L,1) )
629 RHS(9,L)=XI(K)*(XM22C-XM1C*UC12/UA0)*RHSF(L)
IF( JDIM .LT. 10 ) GO TO #220
RHS(10,L)=-(RM*Y(1,4,L,1)+A(1,4,L,1)+.500*P0*XM10*Y(1,1,L,1))*
C Y(3,2,L,1) +Y(2,4,L,1)*A(2,2,L,1) +XI(K)*( CC*Y(2,4,L,1)-RMUA10
C +Y(2,1,L,1)*C10+RK10*(Y(2,1,L,1)*CO-UA0))
RHS(11,L)=-(RM*Y(1,4,L,1)+A(1,4,L,1)+.500*P0*XM10*Y(1,1,L,1))*
1 Y(3,3,L,1) +Y(2,4,L,1)*A(2,3,L,1)
RHS(12,L)=PU*(1.00-Y(2,3,L,1))
8220 CCNTINUE
633 CCNTINUE
IF(J-1) 539,539,1500
539 CC 540 L=1,LMAXU
CUMV(L)=RM*Y(1,1,L,1)+A(1,1,L,1)
RHSF(L)=Y(2,1,L,1)*2-.500*Y(3,1,L,1)*Y(1,1,L,1)-1.00
C12(L)= (RN-S(1))*Y(3,1,L,1)
540 CT(L)=(XMO +S(1))*Y(2,1,L,1)+A(2,1,L,1)
1500 CONTINUE
C END CHOOSE VARIABLE TO BE SOLVED FOR DO 1500 J
C
C SECTION 13: WRITE UP OF RESULTS
WRITE(3,4628)
4628 FORMAT(1X, T22,'J=1',T35,'J=2',T46,'J=3',T57,'J=4',T66,'J=5',T76,
8 'J=6',T86,'J=7',T94,'J=8',T103,'J=9',T113,'J=10',T121,'J=11',T128
8 'J=12')
IF(JDIM .LE. 5) WRITE(3,4640)
4640 FORMAT( ' Z L ', T23,'F0',T35,'G00',T46,'G01',T57,
1 'F1K',T67,'F10C' )
IF(JDIM .GT. 5) WRITE(3,464)
464 FORMAT( ' Z L ', T23,'F0',T35,'G00',T46,'G01',T57,
1 'F10',T66,'F2K', T76,'F2CR',T86,'F2CC',T94,'F21C',T103,'F22C',T113
2, 'G10',T121,'G11',T128,'G12' )
465 FORMAT(1X,F7.4,I4,'VEL ', F12.8,2F11.7,2F10.6,F9.6,F9.3, F9.5,
5 F9.4,F9.5,F8.4,F9.5 )
466 FORMAT(10X, ' DVCZ ', F12.8,2F11.7,2F10.6,F9.6,F9.3, F9.6,
5 F9.5,F9.6,F8.4,F9.5)
467 FORMAT(' B.C. AT Z=INF. ', F12.8,2F11.7,2F10.7,5F9.6,2F8.5 )
468 FORMAT(' MAX ERROR IN VEL', F12.8,2F11.7,2F10.7,5F9.6,2F8.5 )
469 FORMAT(' OMEGA-FINAL VAL', F12.8,2F11.7,2F10.7,5F9.6,2F8.5 )
470 FORMAT(' STREAM FUNG LMAXU', F12.8,2F11.7,F10.6,F10.7,5F9.4,2F8.3)
473 FORMAT(' MAX VALUE OF L ' T7.2I11,2I10,5I9,2I8 )
474 FORMAT(' NO OF ITERATIONS', I5,2I11,2I10,5I9,2I8 )
475 FORMAT(' DVCZ TOC B1G2LMX', F12.8,2F11.7,2F10.7,5F9.6,2F8.5 )
NTEST=0
LMIN=LMAXU
LPAX=1
DO 552 J=1,JDIM
IF(LMAXR(J) .LT. LMIN) LMIN =LMAXR(J)
IF(LMAXR(J) .GT. LMAX) LMAX=LMAXR(J)
C IF DVCZ IS NOT SMALL (LESS THAN SAY 1.0-3) AT Z(LMAXR(J)) THEN
C THE B.C. AT EDGE OF BOUNDARY LAYER IS NOT CORRECTLY SATISFIED.
C THIS MEANS THAT Z(LMAXR(J)) ( THE MAX. VALUE OF ETA FOR EQN. NO.J)
C IS NOT LARGE ENOUGH. LMAXR(J) SHOULD BE INCREASED.
C THIS DO LOOP CHECKS DVCZ AT Z(LMAXR(J)) . LMIN AND LMAX ARE JUST
C USED FOR WRITE(3,465) AND WRITE(3,466).
IF(DABS(Y(3,J,LMAXR(J),1))-1.E-3) 600,600,601
601 ERC(J)=Y(3,J,LMAXR(J),1)

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NTEST=1
GC TO 552
600 EBC(J)=0.00
552 CONTINUE
IF(LMAX .GT. LMAXU) GC TO 1405
JM1=LMAX/5
DO 861 L=1,LMAX
IF((L/2)*2.EQ. L .AND. L .GT. 8 .AND. L .LT. LMAX-2) GC TO 861
WRITE(3,465) Z(L),L,(Y(2,J,L,1),J=1,JDIM)
IF(L.GT. 10 .AND. L.LT.LMIN-2 .AND. L.NE.(L/JM1)*JM1) GC TO 861
WRITE(3,466) (Y(3,J,L,1),J=1,JDIM)
861 CONTINUE
WRITE(3,467) ( PC(J),J=1,JDIM)
WRITE(3,468) ( ERROR(J),J=1,JDIM)
WRITE(3,469) ( CMFGA(J),J=1,JDIM)
WRITE(3,473) ( LMAXR(J),J=1,JDIM)
WRITE(3,474) ( ITERK(J),J=1,JDIM)
WRITE(3,470) (Y(1,J,LMAXU,1),J=1,JDIM)
IF(NTEST .GT. 0) WRITE(3,475) (EBC(J),J=1,JDIM)
C END WRITE STMTS
C
C SECTION 14: CHECK OF CHORDWISE DERIVATIVES (OPTIONAL)
C THIS SECTION CAN BE OMITTED IF DESIRED. THIS CHECK CONSIDERABLY
C OVERESTIMATES THE ERROR, ESPECIALLY IF THE FUNCTION IS DECREASING
C IN MAGNITUDE.
CP=DX(K-1)
CK=DX(K)
IERRXD=0
IF(K .LT. 9) GC TO 7140
DO 7138 J=1,JDIM
XDC(J)=X1(K-1)*(-DK*Y(2,J,10,3)/((CK+CP)*CP)+(CK-CP)*Y(2,J,10,2)
1/(CK*CP)+CP*Y(2,J,10,1)/((CK+CP)*CK))
C XDC IS A CENTRAL DIFFERENCE AT THE (K-1) POINT. YDP IS THE BACK-
C WARD DIFFERENCE USED AT (K-1)
PCE(J)=XDC(J)-XCP(J)
IF(DABS(PCE(J))-1.D-2-1.D-3*Y(2,15,J,1)**2) 7138,7139,7137
7137 IF(DABS(PCE(J)/XDC(J)).GT. 5.D-2) IERRXD=J
7138 CONTINUE
IKERR(K-1)=IERRXD
7140 DO 7141 J=1,JDIM
7141 XDP(J)=A(2,J,10)
IF(IERRXD .GT. 0) WRITE(3,476)(PCE(J),J=1,JDIM)
476 FORMAT(' ERROR X DERIV. ' ,F12.7,2F11.6,2F10.5,5F9.4,2F8.3 )
IF(IERRXD .GT. 0) WRITE(3,477)(XDC(J),J=1,JDIM)
477 FORMAT(' VELOC. X DERIV. ' ,F12.7,2F11.6,2F10.5,5F9.4,2F8.3 )
C END CHECK OF X DERIVATIVES AFTER WRITE STATEMENTS
C
C SECTION 15: B.L. THICKNESSES (OPTIONAL, SEE SECTIONS 2 AND 16)
C BEGIN CALCULATION OF THICKNESSES
C THIS SECTION CAN BE OMITTED IF DESIRED. FOR GIVEN VALUES OF
C FORWARD FLIGHT SPEED (SHT), AZIMUTHAL ANGLE (PSIT), (COWFLONG VAT)
C SPAN/CHORD (YP, FCORD FROM THE ASPECT RATIO, ART) AND FOR THE
C CURRENT VALUE OF XT (XT(K)) THE VALUE OF X (XP) IS CALCULATED. YY
C INTERPOLATION, UAD (UAP), UCC(UCP), PER CENT CHORD (PCCP), AND
C AL(RNUP) ARE FOUND AT XP. DISPLACENT AND MOMENTUM THICKNESS ARE
C CALCULATED AND WRITTEN.
IF(K .LT. 8 .OR. JDIM .LT. 10 .OR. NGT .EQ. 0) GO TO 5050
DO 5040 NG=1,NGT
T1 =SHT(NG)*DCOS(PSIT(NG)/57.2957800)
T2 =SHT(NG)*DSIN(PSIT(NG)/57.2957800)
VA2=VAT(NG)**2
WRITE(3,5036) PSIT(NG),VAT(NG),SHT(NG),ART(NG)

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5036 FORMAT(/ ' AZIMUTH ANGLE=',F6.1,5X,' INDICED VEL=',F9.6,5X,' SPEED R
1ATIC=',F9.5,5X,' ASPECT RATIO=',F7.3 /
2 ' SPAN/CHORD',4X,' X',4X,' 3CHORD ', ' X DIS'MENT THK ', ' Y DIS'
3PENT THK ', ' X MOM THK ', ' Y MOM THK ',
4 ' XI ', ' Q ' )
CC 5040 IYP=1.4
YP=(.300+IYP-.2500*(IYP/4))*ART(NG)
IF(YP+T2 .LT. 0) GO TO 5038
QP=1.00-VAT(NG)*RK10/YP-(RK20R+RK20*VA2+RK21+T1+RK22+T2*VAT(NG))/
C(YP*YP)
IF(QP .LT. .300 .OR. QP.GT. 2.00) GO TO 5038
XIIP= CH10+VAT(NG)*(CH110+(VAT(NG)*CH120+T2*CH122)/YP)/YP
XP=XI(K)/CP + XIIP
CALL COEX(E,X(K),X(K-1),X(K-2),XP)
C SUBROUTINE COEX JUST CALCULATES INTERPOLATION CONSTANTS FOR 3
C POINT LANGRANGIAN INTERPLATION.
PCCP=0.00
UAP=0.00
UCP=0.00
RNUP=0.00
CC 5020 I=1,3
PCCP=PCCP+E(I)*PCC(K-I+1)
UAP=UAP+E(I)*UAPK(K)
UCP=UCP+E(I)*UCPK(K)
5020 RNUP=RNUP+E(I)*RNUK(K)
RNUP=RNUP+T1
CVAR= (XP-XIIP)/(IYP+T2)*UAP+VAT(NG)*UCP
IF(DVAR) 4740,4740,4741
4740 WRITE(3,4742) YP,XP,QP,UAP,UCP,CVAR
C IF Q<.3 OR>2, THEN SERIES IN SPAN IS PROBABLY INACCURATE.
4742 FORMAT(1X,2F10.6,' DVAR<0; YP+T2 TOO SMALL QP=',F10.6,' LAP=1,
F10.6,' UCP=',F10.6,' CVAR=',C11.4 )
GO TO 5040
4741 CVAR=DSQRT(DVAR)
CON0=1.00/(UAP*YP*YP)
CON1=VAT(NG)/YP
CON2=(VAT(NG)*2)*CON0
CON3=T1*CON0
CON4=VAT(NG)*T2*CON0
CON5=VAT(NG)*T1/YP
CC 4750 L=1,LMAXU
UP(L)=Y(2,1,L,1)+CON1*Y(2,4,L,1)+CON0*(Y(2,6,L,1)+RK20R*Y(2,5,L,1)
1) +CON2*(Y(2,7,L,1)+RK20*Y(2,5,L,1))+ CON3*(Y(2,8,L,1)+RK21*
2 Y(2,5,L,1))+ CON4*(Y(2,9,L,1)+RK22*Y(2,5,L,1))
VP(L)=Y(2,2,L,1)+T1*Y(2,3,L,1)+CON1*Y(2,10,L,1)+CON5*Y(2,11,L,1)
V +T2*Y(2,12,L,1)/YP
UP(L)=UP(L)+UP(L)
4750 VP(L)=VP(L)*VP(L)/RNUP
L=LMAXU
TK(3)=Y(1,1,L,1)+CON1*Y(1,4,L,1)+CON0*(Y(1,6,L,1)+RK20R*Y(1,5,L,1)
1) +CON2*(Y(1,7,L,1)+RK20*Y(1,5,L,1))+CON3*(Y(1,8,L,1)+RK21*
2 Y(1,5,L,1))+ CON4*(Y(1,9,L,1)+RK22*Y(1,5,L,1))
TK(4)=Y(1,2,L,1)+T1*Y(1,3,L,1)+CON1*Y(1,10,L,1)+CON5*Y(1,11,L,1)
V +T2*Y(1,12,L,1)/YP
TK(1)=DVAR*(Z(LMAXU)-TK(3))
TK(2)=DVAR*(Z(LMAXU)-TK(4))/RNUP
CC 4751 L=3,LMAXU,2
TK(3)=TK(3)-CZ(L)*(UP(L-2)+4.00*UP(L-1)+UP(L))/3.00
TK(4)=TK(4)-CZ(L)*(VP(L-2)+4.00*VP(L-1)+VP(L))/3.00
LST=L+1
IF(CZ(L+2) .NE. CZ(3)) GOTO 4755
4751 CONTINUE

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4755 DO 4756 L=LST,LMAXU
      TK(3)=TK(3)-CZ(L)*(UP(L)+UP(L-1))*5DC
4756 TK(4)=TK(4)-CZ(L)*(VP(L)+VP(L-1))*5DC
      TK(3)=DVAR*TK(3)
      TK(4)=TK(4)*CVAR/RNUP
      WRITE(3,5035) YP,XP,PCCP,(TK(I),I=1,4),XIIP,CP
5035 FORMAT(1X,3F10.6,4F16.1C,2F10.7)
      GO TO 5040
5038 WRITE(3,5034) YP,T2,CP,PSIT(NG)
5034 FORMAT(' EITHER YP=',F10.6,' IS < T2=',F10.6,6X'OR QP=',F10.6,
      F' IS <.3 OR > 2.',5X,'PSI=',F7.2,'%CHORD=',F6.2,'SPEED RATIO=',
      3 F7.3/)
5040 CONTINUE
5050 CONTINUE
      END CALCULATION OF THICKNESSES

      SECTION 16: VELOCITY PROFILES(OPTIONAL,SEE SECTIONS 2 AND 15)
      THIS SECTION CAN BE OMITTED IF DESIRED. VELOCITY PROFILES ARE
      FOUND AT A GIVEN PER CENT CHORD (WHICH GIVES A VALUE OF X). THUS
      XI MUST BE FOUND (XIP). SINCE FO',600',ETC ARE KNOWN ONLY AT THE
      POINTS XI(K),THEY MUST BE INTERPOLATED TO XIP.
      BEGIN CALCULATION OF VELOCITY PROFILES
      IF(NGVEL.EQ. 0 .OR. JDIR.LT. 10 .OR. K.LT. 8) GO TO 5080
      DO 5076 NG=1,NGVEL
      IF(PCCV(NG).GT. 3.14159) GO TO 5076
      CALL COEX(E,PCC(K),PCC(K-1),PCC(K-2),PCCV(NG))
      XP=E(1)*X(K)+E(2)*X(K-1)+E(3)*X(K-2)
      T1=SHV(NG)*CCOS(PSIV(NG)/57.29578)
      T2=SHV(NG)*CSIN(PSIV(NG)/57.29578)
      VA=VAV(NG)
      VA2=VA*VA
      DO 5075 IYP=1,4
      YP=(.300+IYP-.250*(IYP/4))*ARV(NG)
      IF(ILIN(NG,IYP).GT. 0) GO TO 5075
      IF(IYP+T2.LT. 0) GO TO 5073
      CP=1.00-VA*RK10/YP-(RK20+RK20*VA2+RK21*T1+RK22*VA*T2)/(YP*YP)
      IF(CP.LT. .300 .OR. CP.GT. 2) GO TO 5073
      XIIP=CHI0+VA*(CHI10+(VA*CHI12C+T2*CHI22)/YP)/YP
      XIP=QP*(XP-XIIP)
      IF(XIP.GT. XI(K)) GO TO 5075
      ILIN(NG,IYP)=K
      CALL COEX(E,XI(K),XI(K-1),XI(K-2),XIP)
      DVAR=YP*YP*(E(1)*UAPK(K)+E(2)*UAPK(K-1)+E(3)*UAPK(K-2))
      DO 5070 L=1,LMAXU
      UP(L)=0.00
      VP(L)=0.00
      DO 5069 I=1,3
      UP(L)=UP(L)+E(I)*( Y(2,1,L,I)+VA*Y(2,4,L,I)/YP+( Y(2,6,L,I)+
      1 RK20*Y(2,5,L,I)+VA2*(Y(2,7,L,I)+RK2C*Y(2,5,L,I)) +T1*(Y(2,8,L,I)
      2 + RK21*Y(2,5,L,I)+VA*T2*(Y(2,9,L,I)+RK22*Y(2,5,L,I)) )/DVAR )
5069 VP(L)=VP(L)+E(I)*( Y(2,2,L,I)+T1*Y(2,3,L,I)+( VA*(Y(2,10,L,I)+
      1 T1*Y(2,11,L,I))+T2*Y(2,12,L,I) )/YP )
5070 VP(L)=VP(L)/ARV(NG)
      IF(IYP.EQ.1) GO TO 5060
      WRITE(3,5062) PSIV(NG),PCCV(NG),XP,SHV(NG),VAV(NG),ARV(NG)
5062 FORMAT('/' AZIMUTH ANGLE=',F6.1,4X,'CHORD=',F6.5,4X,'X=',F12.8,
      1 4X,'SPEED RATIO=',F9.5,4X,'INDUCED VEL',F7.3,4X,'ASPECT RATIO',
      2 F7.3 )
5060 WRITE(3,5059) YP
      WRITE(3,5064) (L,Z(L),UP(L),L=1,LMAXU)
5064 FORMAT(1X,12,F7.3,F8.4,4X,12,F7.3,F8.4,4X,12,F7.3,F8.4,4X,12,
      1 F7.3,F8.4,4X,12,F7.3,F8.4 )

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WRITE(3,5065) CP,XIIP,XIP,DVAR
5065 FORMAT( ' QP=',015.5,'XIIP,XIP,DVAR=',3D15.6 )
WRITE(3,5066)
5066 FORMAT(/ 1X,51' L    Z    V/R*OMEGA  ' )
WRITE(3,5064) (L,Z(L),VP(L),L=1,LMAXU)
GO TO 5075
5073 WRITE(3,5034) YP,T2,QP,PSIV(NG),PCCV(NG),SHV(NG)
5075 CONTINUE
5059 FCMPAT(/' VELOCITY PROFILES FOR SPAN/CHORD=',F8.5/1X,
1 5(' L    Z    U/DELTA  ' ))
5076 CONTINUE
5080 CONTINUE
END CALCULATION OF VELOCITY PROFILES

SECTION 17: VALUES OF XIS AND K'S
IF(JDIM=5) 566,567,568
FO*2WALL IS EXTRAPOLATED TO ZERO TO FIND XIS. THEN K10 IS THE
VALUE OF -F10C/F1K*2WALL EXTRAPOLATED TO XIS.
567 RK10TE =-Y(3,5,1,1)/Y(3,4,1,1)
WRITE(3,489) RK10TE,Y(3,1,1,1)
489 FORMAT(' -F10C*/F1K*2WALL= ',D20.12,10X,' FO*2WALL=',D20.10)
568 CONTINUE
IF(K .LT. 15) GO TO 568
IF(JDIM=9) 568,569,568
569 CC 570 J=6,9
570 RKJ(J)=-Y(3,J,1,1)/Y(3,5,1,1)
RKJ(6)...RKJ(9) ARE EXTRAPOLATED TO XIS TO FIND K2CR,K20,K21,K22.
WRITE(3,571) RKJ(6),RKJ(8),RKJ(7),RKJ(9)
571 FCMPAT(' -DVC2WALL/F2K* ',52X,D13.6,5X,D13.6 /80X,D13.6,5X,
1 D13.6)
568 CONTINUE

SECTION 18: UPDATE VELOCITIES
UPDATE VELOCITIES FOR NEW X STATION
THE VALUE OF K WILL BE INCREMENTED SOON. SINCE I=1 DENOTES THE
CURRENT VALUE OF K, Y(M,J,L,1) MUST BE UPDATED. THESE DO LOOPS
SET Y(M,J,L,IMAX)=Y(M,J,L,IMAX-1)...Y(M,J,L,2)=Y(M,J,L,1). AFTER
K IS INCREMENTED (BY DO 3C K=1,KMAX) Y(M,J,L,1) BECOMES THE
VARIABLE TO BE SOLVED FOR.
DO 1429 IT=1,3
CC 1429 L=1,LMAXU
CC 1429 J=1,JDIM
CC 1429 M=1,3
1429 Y(M,J,L,5-IT) =Y(M,J,L,4-IT)
: END UPDATE VELOCITIES FOR NEW X STATION
8723 CONTINUE
WRITE(3,872) K,PCC(K)
872 FORMAT(' END OF K=',I5,10X,' PER CENT CHORD AT INFINITE SPAN=',
F12.8,'%'//)
GC TO 30

SECTION 19: DIAGNOSTIC WRITE STMTS AND TERMINATION
THESE WRITE STMTS GIVE SOME INFORMATION ON WHY THE PGM STOPPED.
1405 IF(X(K) .LT. XIINT.AND.X(K)*X(K-1).LT.0.DO)WRITE(3,480) X(K)
480 FCMPAT(/' AT STMT 3314 X(K) (=,E20.12,' ) IS < XIINT/' CHANGE
1DXI(K) ' /)
IF(LMAX .GT. LMAXU) WRITE(3,704C) LMAX
704C FORMAT(/' AT STMT 552, LMAXU<LMAX, WHICH=',I5 )
IF(ITER .GE. ITERM) WRITE(3,7041)J,LMAX,ITER,(Y(2,J,L,1),L=1,LMAX)
7041 FCMPAT(/' SUCCESSIVE REPLACEMENTS DIVERGE FOR J=',I3/' LMAX=',
1 I3/' ITER=',I4/' Y(2,J,L,1)='/( 1X,10D13.5))
IF(ITER .GE. ITERM .AND. K .GT. 15 .AND. Y(3,1,1,1) .LT. .09D0)

```

```

WRITE(3,7044) V(3,1,1,1),XI(K)
7044 FORMAT(/' FOM AT Z=0 IS',D19.7,'; LEM'S METHOD WILL NOT CONVERGE
1FOR FOM < .03 OR .04' // ' IF THE CURRENT VALUE OF XI ('D19.7,
2') SEEMS LIKELY TO GIVE FOM TOO SMALL, THIS IS PROBABLY A NORMAL
3TERMINATION ' )
THE FINAL VALUE OF XI SHOULD BE CHOSEN (BY TRIAL AND ERROR) TO
GIVE FOM BETWEEN .06 AND .04
IF(X(K)*X(K-1) .EQ. 0.00) WRITE(3,7042) X(K)
7042 FCRPAT(/' AT STMT 3314 X(K)='D20.10/ ' CHANGE DXI(K) ' //)
KP=K-2
DO 7051 I=9,KM
7051 IF(IKERR(I).GT.0 .AND. KP .GT.9) WRITE(3,7052) I,IKERR(I)
7052 FCRPAT(' X DERIVATIVES MAY BE INACCURATE AT K=',I3,' FOR J=',I3)
9996 WRITE(3,7043) K
7043 FCRPAT(/' EXIT CALLED AT STMT 1405 FOR K=',I5)
IF(J .EQ. 1) CALL EXIT
JDIM=J-1
IF( IITER .EQ. ITERM ) GO TO 1401
CALL EXIT
30 CONTINUE
END K LOOP IN XI
WRITE(3,7055)
7055 FCRPAT(' PGM STOPS BECAUSE KMAX IS TOO SMALL ' )
CALL EXIT
END

```

```

SUBROUTINE COEX(E,A,B,C,C)
IMPLICIT REAL*8 (A-H,O-Z)
COEX PROVIDES COEFFICIENTS FOR 3 POINT LAGRANGIAN INTERPOLATION.
IT IS USED IN SECTIONS 15 AND 16.
DIMENSION E(3)
E(1)=(D-B)*(C-C)/( (A-B)*(A-C) )
E(2)=(D-A)*(D-C)/( (B-A)*(B-C) )
E(3)=(D-A)*(D-B)/( (C-A)*(C-B) )
F(D)=E(1)*F(A)+E(2)*F(B)+E(3)*F(C), WHERE F IS SOME FUNCTION
RETURN
END

```

```

REAL FUNCTION DSDX(B,DUMMY,DE)
IMPLICIT REAL*8 (A-H,O-Z)
DSDX CALCULATES THE VALUES OF B1....B10 NEEDED TO FIND THE
POTENTIAL FLOW AND TO RELATE X AND SIGMA.
COMMON/B10/EP5,SGN,B1,B2,B3,B4,B5,B6,B7,B8,B9,B10
IF(SGN) 85,85,75
85 IF(DUMMY) 75,75,74
74 SGN=1.00
WRITE(3,401) DE,SGN,SGN,DUMMY
75 S=DE-1.00
CPS2=DE*(1.00-S)
IF(CABS(CMS2) .LT. 1.0-50) WRITE(3,401) DE,SGN,CPS2,DUMMY
IF(DABS(CMS2) .LT. 1.0-50) CPS2=1.0-50
BA=(1.00-EP5)*2
BB=1.00-EP5*EP5
BC=1.00+EP5

```

```

B1=1.00+EPS*EPS-2.00*EPS*S
B2=BC*(1.00+BA*BB/(B1*B1))/4.00
B3=SGN*BC*(-5*(1.00-BA/B1)-OMS2+2.00*EPS*BA/B1**2)/(4.00*
3.0SQRT(OMS2))
IF( DABS(DUMMY*SGN-13.00) .GT. 1.0-4) GO TO 18
C0=EPS*BC*BA/(2.00*B1*SQRT(OMS2))
P4=BC*BA*BB*EPS/B1**3
DE=B1*B1
B5=SGN*BD*(-1.00/(BA*DE)+2.00*S/B1-4.00*EPS*OMS2/DE)
B6=6.00*EPS*B4/B1
B8=8.00*EPS*B6/B1
B9=2.00*EPS/B1+S/OMS2
BF=1.00/(DE*DE*BA)+2.00/B1+12. *EPS*S/DE-16.00*EPS*EPS*OMS2/B1**3
B7=B5*BG+SGN*BC*BF
B9=BG*B7+(4.00*EPS*EPS/DE+(1.00+2.*S/OMS2)/OMS2)*P5+SGN*BD*BF*BG
9 +SGN*BD*(-2.00/(BA*DE**3)+16.00*EPS/DE+80.00*EPS*EPS*S/(DE*B1)
9 -96.00*(EPS**3)*OMS2/(DE*DE) )
401 FCRPAT( ' B2,B3,DE,SGN,OMS2,DUMMY ',6D16.7 )
18 B10=DSQRT(B2*B2+B3*B3)*SGN
IF(DABS(DUMMY).LT.1.0-6)WRITE(3,401)B2,B3,DE,SGN,OMS2,DUMMY
OSDX=1.00/B10
RETURN
END

```

| DATA CARDS                          |          |                     |        |       |                  |
|-------------------------------------|----------|---------------------|--------|-------|------------------|
| SAMPLE DATA CARDS FOR ALPHA = .005  |          |                     |        |       |                  |
| .25                                 | 0.005    | -.0000161284596007  | .44C   | 7.1E  | EXTRASTAFF       |
| .795                                | -55.05   | .423                | -7.19  |       | RK2 FOR .005 DEG |
| 10                                  |          |                     |        |       |                  |
| 5.625                               | 90.      | 0.                  | 20.    | 10.   | .005             |
| 5.625                               | 90.      | 0.                  | 20.    | 30.   | .005             |
| 5.625                               | 270.     | 0.                  | 20.    | 10.   | .005             |
| 5.625                               | 270.     | 0.                  | 20.    | 30.   | .005             |
| 2.812                               | 90.      | 0.                  | 20.    | 30.   | .005             |
| 1.406                               | 90.      | 0.                  | 20.    | 30.   | .005             |
| 5.625                               | 45.      | 0.                  | 20.    | 30.   | .005             |
| 5.625                               | 135.     | 0.                  | 20.    | 30.   | .005             |
| 5.625                               | 225.     | 0.                  | 20.    | 30.   | .005             |
| 5.625                               | 315.     | 0.                  | 20.    | 30.   | .005             |
| 7                                   |          |                     |        |       |                  |
| 5.625                               | 90.      | 0.                  | 20.    |       | .005             |
| 2.812                               | 90.      | 0.                  | 20.    |       | .005             |
| 1.406                               | 90.      | 0.                  | 20.    |       | .005             |
| 5.625                               | 45.      | 0.                  | 20.    |       | .005             |
| 5.625                               | 135.     | 0.                  | 20.    |       | .005             |
| 5.625                               | 225.     | 0.                  | 20.    |       | .005             |
| 5.625                               | 315.     | 0.                  | 20.    |       | .005             |
| SAMPLE DATA CARDS FOR ALPHA = 2.    |          |                     |        |       |                  |
| 0.250000                            | 2.000000 | -0.0059944141423248 | .323   | 10.4  | EXTRASTAFF       |
| .485                                | -122.4   | .596                | -10.46 |       | RK2 FOR 2 DEG    |
| SAMPLE DATA CARDS FOR ALPHA = 3.345 |          |                     |        |       |                  |
| 0.250000                            | 3.349000 | -0.0103629678220279 | .244   | 15.01 | EXTRASTAFF       |
| .329                                | -252.6   | .703                | -15.05 | 6K    |                  |
| 1                                   |          |                     |        |       |                  |
| 3.1226                              | 90.      | .07650              | 13.71  | 10.   | 6K               |
| 1                                   |          |                     |        |       |                  |
| 3.1226                              | 90.      | .07650              | 13.71  |       | 6K               |

|                                     |          |                     |        |       |       |                    |
|-------------------------------------|----------|---------------------|--------|-------|-------|--------------------|
| SAMPLE DATA CARDS FOR ALPHB = 4.    |          |                     |        |       |       |                    |
| 0.250000                            | 4.000000 | -0.0126237115182189 | .201   | 18.8  |       | EXTRASTAFF         |
| .246                                | -968.7   | .728                | -18.82 |       |       | RK2 FOR 4 DEG      |
| 1                                   |          |                     |        |       |       |                    |
| 7.625                               | 90.      | .2117               | 20.    | 10.   | 4 DEG |                    |
| 0                                   |          |                     |        |       |       |                    |
| SAMPLE DATA CARDS FOR ALPHB = 4.465 |          |                     |        |       |       |                    |
| .25                                 | 4.465    | -.01431093492       | .170   | 22.15 |       | EXTRASTAFF         |
| .170                                | -447.5   | .709                | -22.15 |       |       | RK2 FOR 4.465 DEG  |
| 10                                  |          |                     |        |       |       |                    |
| 3.1226                              | 90.      | .10198              | 13.71  | 10.   | 8K    |                    |
| 3.1226                              | 90.      | .10198              | 13.71  | 12.   | 8K    |                    |
| 3.1226                              | 270.     | .10198              | 13.71  | 10.   | 8K    |                    |
| 3.1226                              | 270.     | .10198              | 13.71  | 12.   | 8K    |                    |
| 1.5613                              | 90.      | .10198              | 13.71  | 10.   | 8K    |                    |
| .7807                               | 90.      | .10198              | 13.71  | 10.   | 8K    |                    |
| 3.1226                              | 45.      | .10198              | 13.71  | 10.   | 8K    |                    |
| 3.1226                              | 135.     | .10198              | 13.71  | 10.   | 8K    |                    |
| 3.1226                              | 225.     | .10198              | 13.71  | 10.   | 8K    |                    |
| 3.1226                              | 315.     | .10198              | 13.71  | 10.   | 8K    |                    |
| 7                                   |          |                     |        |       |       |                    |
| 3.1226                              | 90.      | .10198              | 13.71  |       | 8K    |                    |
| 1.5613                              | 90.      | .10198              | 13.71  |       | 8K    |                    |
| .7807                               | 90.      | .10198              | 13.71  |       | 8K    |                    |
| 3.1226                              | 45.      | .10198              | 13.71  |       | 8K    |                    |
| 3.1226                              | 135.     | .10198              | 13.71  |       | 8K    |                    |
| 3.1226                              | 225.     | .10198              | 13.71  |       | 8K    |                    |
| 3.1226                              | 315.     | .10198              | 13.71  |       | 8K    |                    |
| SAMPLE DATA CARDS FOR ALPHB = 5.581 |          |                     |        |       |       |                    |
| 0.250000                            | 5.581000 | -0.0186346963551100 | .106   | 24.22 |       | EXTRASTAFF         |
| .048                                | -86.     | .450                | -24.13 |       |       | RK2 FOR 5.581 DEG. |
| 1                                   |          |                     |        |       |       |                    |
| 3.1226                              | 90.      | .12743              | 13.71  | 10.   | 10K   |                    |
| 1                                   |          |                     |        |       |       |                    |
| 3.1226                              | 90.      | .12743              | 13.71  |       | 10K   |                    |

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| 13. ABSTRACT<br>A theoretical study has been conducted to determine the effects of rotation, inflow, and forward flight on the development of the laminar boundary layer on a helicopter blade. Particular emphasis was placed on the determination of the separation line. In order to facilitate the computation of the inviscid flow about the blade, an 11.9%-thick symmetrical Joukowski airfoil was used. The essential feature of the analysis was the scaling of the chordwise coordinate so that the separation line is invariant with span and time in the transformed coordinate system. The transformed boundary layer equations were expanded in an asymptotic series in span, and the resulting equations were solved by the method of Smith and Clutter. The major effect of rotation is a delay in separation. The separation line delay is most pronounced near the axis of rotation. Forward flight causes an oscillation about this separation line, so that the delay is greatest in the first and fourth quadrants. The oscillations are affected by the blade angle of attack and the inflow due to lift. The phase advance between the wall shear and the free-stream velocity is in qualitative agreement with the results of Lighthill. Rotation alone does not influence the separation line greatly. However, its combination with forward flight and inflow contribute, at least in part, to the increase in maximum lift observed on helicopter blades. |                               |                                                                                                                           |  |

DD FORM 1473

REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

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