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AFOSR ltr, 15 Feb 1972

von KARMAN INSTITUTE
FOR FLUID DYNAMICS

TECHNICAL NOTE 78

SUPERSONIC GAS - SOLID PARTICLE FLOW
IN AN AXISYMMETRIC NOZZLE
BY THE METHOD OF CHARACTERISTICS

by

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AUGUST 1971

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LIST OF SYMBOLS

C_d	particle drag coefficient
C_p	particle specific heat
C	gas specific heat
D	particle diameter
m	density per unit volume of a particle
$\dot{m}_p$	mass flow rate of particles
M	Mach number
P	pressure
Pr	Prandtl number
R	gas constant
Re	Reynolds number
T	temperature
u	x-direction velocity component
v	y-direction velocity component
W	speed
x	axial coordinate along nozzle axis
y	radial coordinate measured from nozzle axis
α	Mach angle
γ	isentropic exponent
θ	streamline angle with respect to nozzle axis
μ	viscosity
ρ	density per unit volume

Subscripts

p	particle property
x	partial derivative with respect to x
y	partial derivative with respect to y

ABSTRACT

A study was conducted to numerically treat a mixture composed of a gas and solid particles in a supersonic, axisymmetric nozzle. The governing equations are a set of eight first order, quasi-linear, partial differential equations. Seven of these equations are of the hyperbolic type when the flow is supersonic (based on the frozen speed of sound in the gas) and can be solved by the method of characteristics. The eighth equation (the particle continuity equation) is rewritten as an integral equation to be solved. The resulting seven compatibility equations and the seven characteristic equations (only four are distinct; the t. gas Mach lines and the gas and particle streamlines) are solved by the modified Euler predictor- corrector algorithm. These equations were programmed for an IBM 1130 computer. A sample nozzle calculation is given and compared with the one-dimensional calculations. These results indicate that the program is working correctly.

I. INTRODUCTION

The purpose of the present investigation was to numerically treat a mixture of a gas and solid particles in an axisymmetric, supersonic nozzle. The study was primarily intended for industrial particle micronization processes. In this process solid particles are fluidized and entrained by a high pressure air flow. The mixture is then expanded through a converging-diverging nozzle. The solid particles are accelerated during this expansion and impact a target downstream of the nozzle exit plane. If the particles have sufficient momentum they are broken upon impact. This process is repeated until the desired particle sizes are obtained.

The interest in the present study was not so much to determine two-dimensional particle velocities but rather to be able to investigate non-uniform particle distributions in each nozzle plane. One of the inherent restrictions in one-dimensional gas solid - particle analyses is that the particles are uniformly distributed in each nozzle plane. However, experimental studies definitely show that most of the particles are concentrated near the nozzle center line. To treat the problem of nonuniform particle distribution it is necessary to formulate the problem in two dimensions.

Riethmuller[1] has done an extensive literature survey on gas-solid particle flow analyses and experiments. Therefore, only articles that are directly pertinent to this analysis are referenced in this report.

II. GOVERNING EQUATIONS

The supersonic motion of most compressible fluids encountered in nozzle expansion flow can be accurately described by the governing equations of an inviscid fluid. The basic assumptions which constitute such a gas dynamic model are : (1) continuum, (2) steady, (3) inviscid, (4) adiabatic, and (5) the gas composition is frozen. The resulting partial differential equations can be treated by the well known technique of the method of characteristics. For axisymmetric or two-dimensional nozzles this method transforms the partial differential equations into a system of differential equations which are valid along certain characteristic directions in the flow field.

The transformed equations are much simpler to solve and this fact led Kliegel[2] to attempt a similar analysis for a mixture of a gas and solid particles. The major assumption necessary for such a gas-solid particle analysis is that the particles behave as a continuum. Clearly, this assumption is physically unrealistic. That it yields good engineering results is another question. Kliegel has shown that the results are in good agreement with observation for rocket engine applications for moderate loading ratios and small particle sizes. Similar comparisons will be required for particle micronization processes to determine the applicability of this assumption. However, it would appear that, even for particle micronization processes this assumption should give useful engineering results for moderate loading ratios of small particles.

The equations governing the steady axisymmetric flow of a mixture of a gas and solid particles are derived by Hoffman and Thompson[3]. The assumptions necessary for these derivations are : (1) the gas is inviscid except for its interactions with the solid particles, (2) the mixture mass and mixture energy of the system are constant, (3) the volume occupied by the solid particles is negligible, (4) the thermal motion of the solid particles is negligible, (5) the solid particles do not interact, (6) the drag and heat transfer characteristics of an actual particle shape and the size distribution of particles can be represented by spherical particles of a single size, (7) the internal temperature of each solid particle is uniform, (8) energy exchange between the gas and the solid particles occurs by convection only, (9) the only force acting on the solid particles is the viscous drag forces, (10) no mass transfer between the gas and the solid particles, and (11) no phase change.

Based on these assumptions, the following equations govern the gas-solid particle flow :

$$\rho u_x + \rho v_y + u\rho_x + v\rho_y + \rho v/y = 0 \qquad \text{(II-1)}$$

$$\rho u u_x + \rho v u_y + P_x + A\rho_p(u - u_p) = 0 \qquad \text{(II-2)}$$

$$\rho u v_x + \rho v v_y + P_y + A\rho_p(v - v_p) = 0 \qquad \text{(II-3)}$$

$$uP_x + vP_y - a^2 u\rho_x - a^2 v\rho_y - AB\rho_p = 0 \qquad \text{(II-4)}$$

$$\rho_p u_{px} + \rho_p v_{py} + u_p \rho_{px} + v_p \rho_{py} + \rho_p v_p / y = 0 \qquad \text{(II-5)}$$

$$\rho_p \left[u_p u_{px} + v_p u_{py} - A(u - u_p) \right] = 0 \qquad \text{(II-6)}$$

$$\rho_p \left[u_p v_{px} + v_p v_{py} - A(v - v_p) \right] = 0 \qquad \text{(II-7)}$$

$$\rho_p \left[u_p T_{p_x} + v_p T_{p_y} + \frac{2}{3} AC\,(T_p - T) \right] = 0 \qquad \text{(II-8)}$$

where

$$A = \frac{3}{4}\,\frac{C_D\,\rho(u - u_p)}{\rho_p D} \qquad \text{(II-9)}$$

$$C = \frac{12\,k\,N_u}{\mu\,C_D\,Re} \qquad \text{(II-10)}$$

$$B = (\gamma - 1) \left[\frac{2}{3} C_p (T_p - T) + (u - u_p)^2 + (v - v_p)^2\right] \qquad \text{(II-11)}$$

The drag coefficient and Nusselt number for a particle are calculated from equations given by Neilson and Gilchrist[7]

$$C_D = 28\ Re^{-0.85} + 0.48 \qquad \text{(II-12)}$$

$$N_u = 2.0 + 0.6\ Re^{0.5}\ Pr^{0.333} \qquad \text{(II-13)}$$

where the Reynolds number for a particle is defined as follows :

$$Re = \frac{\rho D (W - W_p)}{\mu} \qquad \text{(II-14)}$$

III. APPLICATION OF THE METHOD OF CHARACTERISTICS

In this section, the techniques of the method of characteristics will be employed to obtain the characteristic and compatibility equations for the flow field variables.

The flow field governing equations, Equations (II-1) through (II-8) can be written in the following general form :

$$L_i = a_{ij} z_{jx} + b_{ij} z_{jy} + c_i = 0 \qquad (i,j = 1,\ldots, 8) \qquad \text{(III-1)}$$

where z represents the eight dependent variables u, v, p, ρ, u_p, v_p, ρ_p and T_p and the x and y subscripts denote partial differentiation. These eight equations can be combined to form a single differential operator by employing arbitrary multipliers and summing. Thus

$$L = \sigma_i L_i = 0 \qquad (i = 1,\ldots, 8) \qquad \text{(III-2)}$$

where σ_i are the arbitrary multipliers. In this section, the convention of indicating a summation by repeated indices will be used. The partial differential equation, Equation (III-2), can be rewritten in the form of an ordinary differential equation under certain conditions. Thus

$$(a_{ij}\sigma_i) dz_j + c_i \sigma_i dx = 0 \qquad (i,j = 1,\ldots, 8) \qquad \text{(III-3)}$$

if and only if the following equations are valid :

$$a_{ij}\sigma_i \, dy/dx = b_{ij}\sigma_i \qquad (i,j = 1,\ldots, 8) \qquad \text{(III-4)}$$

Equations (III-4) are eight independent equations for dy/dx. Equations (III-4) are used to determine the unknown multipliers σ_i which can then be substituted into Equation (III-3) to yield the compatibility equations. Rearranging Equations (III-4) to solve for σ_i yields the following equations :

$$\sigma_i(a_{ij}\, dy/dx - b_{ij}) = 0 \qquad (i,j = 1,\ldots,8) \quad \text{(III-5)}$$

If this system of equations, Equations (III-5), is to have a solution other than the trivial, i.e., all $\sigma_i = 0$, then the determinant of the coefficients of σ_i must be zero. Thus,

$$[\; a_{ij}\, dy/dx - b_{ij} \;] = 0 \qquad (i,j = 1,\ldots,8) \quad \text{(III-6)}$$

Now, dy/dx can be obtained from Equation (III-6). The resulting dy/dx can then be substituted into Equations (III-4) to determine relationships in terms of the multipliers σ_i. With σ_i known, the final form of the compatibility equations, Equations (III-3), is obtained.

The governing equations for the flow field are repeated below for convenience.

$$\rho u_x + \rho v_y + u\rho_x + v\rho_y + \rho v/y = 0 \quad \text{(III-7)}$$

$$\rho u u_x + \rho v u_y + P_x + A\rho_p(u - u_p) = 0 \quad \text{(III-8)}$$

$$\rho u v_x + \rho v v_y + P_y + A\rho_p(v - v_p) = 0 \quad \text{(III-9)}$$

$$uP_x + vP_y - a^2 u\rho_x - a^2 v\rho_y - AB\rho_p = 0 \quad \text{(III-10)}$$

$$\rho_p u_{p_x} + \rho_p v_{p_y} + u_p \rho_{p_x} + v_p \rho_{p_y} + \rho_p v_p/y = 0 \quad \text{(III-11)}$$

$$\rho_p \left[u_p u_{p_x} + v_p u_{p_y} - A(u - u_p) \right] = 0 \quad \text{(III-12)}$$

$$\rho_p \left[u_p v_{p_x} + v_p v_{p_y} - A(v - v_p) \right] = 0 \quad \text{(III-13)}$$

$$\rho_p \left[c u_p T_{p_x} + c v_p T_{p_y} + \frac{2}{3} AC(T_p - T) \right] = 0 \quad \text{(III-14)}$$

The determinant, Equation (III-6) can now be written as follows :

$$\begin{vmatrix} \rho\, dy/dx & -\rho & 0 & S_1 & 0 & 0 & 0 & 0 \\ \rho S_1 & 0 & dy/dx & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho S_1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_1 & -a^2 S_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_p\, dy/dx & -\rho_p & 0 & S_2 \\ 0 & 0 & 0 & 0 & \rho_p S_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_p S_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_p c S_2 & 0 \end{vmatrix} = 0 \qquad \text{(III-15)}$$

where $S_1 = (u\, dy/dx - v)$ and $S_2 = (u_p\, dy/dx - v_p)$.
Since the upper right quarter of the above determinant is filled with zeros, the expansion of the determinant reduces to

$$[\ G\ ] \quad x \quad [\ P\ ] = 0 \qquad \text{(III-16)}$$

where G represents the upper left quarter of the determinant and P represents the lower right quarter of the determinant as shown below :

$$\left|\begin{array}{c:c} \begin{array}{c}\text{Gas} \\ \text{Properties} \\ [G]\end{array} & \text{Zeros} \\ \hdashline \text{Zeros} & \begin{array}{c}\text{Particles} \\ \text{Properties} \\ [P]\end{array} \end{array}\right| = 0$$

Equation (III-16) can be zero by either G or P being zero. Setting these two determinants to zero yields the following expressions :

$$[G] = (u\,dy/dx - v)^2 \left[(dy/dx)^2 \left[u^2 - a^2 \right] - 2\,uv\,dy/dx + (v^2 - a^2)\right] = 0 \qquad \text{(III-17)}$$

and

$$[P] = (u_p\,dy/dx - v_p)^4 = 0 \qquad \text{(III-18)}$$

Therefore, Equation (III-16) becomes :

$$(u_p\,dy/dx - v_p)^4 (u\,dy/dx - v)^2 \left[(dy/dx)^2 \left[u^2 - a^2 \right] - 2\,uv\,dy/dx + (v^2 - a^2) \right] = 0 \qquad \text{(III-19)}$$

The characteristic curves are found by solving for dy/dx which, for this system of equations, are

$$dy/dx = v/u \qquad \text{(III-20)}$$

$$dy/dx = \frac{uv \pm a^2 \sqrt{M^2 - 1}}{u^2 - a^2} \qquad \text{(III-21)}$$

$$dy/dx = v_p/u_p \qquad \text{(III-22)}$$

In terms of the Mach angle α , the gas flow angle θ and the particle flow angle θ_p, Equations (III-20), (III-21) and (III-22) can be rewritten, respectively, as

$$dy/dx = \tan\theta \qquad \text{(III-23)}$$

$$dy/dx = \tan(\theta \pm \alpha) \qquad \text{(III-24)}$$

$$dy/dx = \tan\theta_p \qquad \text{(III-25)}$$

Thus, the characteristic curves are the gas streamlines appearing twice, the Mach lines each appearing once and the particle streamlines appearing four times. Equations (III-21) are real only when M > 1, whereas, Equations (III-20) and (III-22) are always real. Therefore, the system of governing equations, Equations (III-7) through (III-14) is hyperbolic when the flow field is supersonic. Of these eight characteristic curves, four are distinct; the two Mach lines, the gas streamlines and the particle streamlines.

In order to determine the compatibility equations, the unknown multiplier σ_i must be determined. These multipliers are determined by simultaneously solving Equations (III-4), using Equations (III-20) through (III-22) for the slope of each characteristic curve.

Substituting the coefficients a_{ij} and b_{ij} from Equations (III-7) through (III-14) into Equations (III-4) yields the following results :

$$\sigma_1 dy/dx + (u dy/dx - v)\sigma_2 = 0 \qquad \text{(III-26)}$$

$$-\sigma_1 + (u dy/dx - v)\sigma_3 = 0 \qquad \text{(III-27)}$$

$$\sigma_2 dy/dx - \sigma_3 + (u dy/dx - v)\sigma_4 = 0 \qquad \text{(III-28)}$$

$$(u dy/dx - v)\ (\sigma_1 - a^2\sigma_4) = 0 \qquad \text{(III-29)}$$

$$\sigma_5 dy/dx + (u_p dy/dx - v_p)\sigma_6 = 0 \qquad \text{(III-30)}$$

$$-\sigma_5 + (u_p dy/dx - v_p)\sigma_7 = 0 \qquad \text{(III-31)}$$

$$(u_p dy/dx - v_p)\sigma_8 = 0 \qquad \text{(III-32)}$$

$$(u_p dy/dx - v_p)\sigma_5 = 0 \qquad \text{(III-33)}$$

The form of the general compatibility equation, Equation (III-3) is :

$$[\,\rho\sigma_1 + \rho u\sigma_2\,]\,du + [\,\rho u\sigma_3\,]\,dv + [\,\sigma_2 + u\sigma_4\,]\,dp$$

$$+ [\,u\sigma_1 - a^2 u\sigma_4\,]\,d\rho + [\,\rho_p\sigma_5 + \rho_p u_p\sigma_6\,]\,du_p$$

$$+ [\,\rho_p u_p\sigma_7\,]\,dv_p + [\,c\rho_p u_p\sigma_8\,]\,dT_p$$

$$+ [\,u_p\sigma_5\,]\,d\rho_p + [\,\rho v\sigma_1/y + A\rho_p(u - u_p)\sigma_2$$

$$+ A\rho_p(v - v_p)\sigma_3 - AB\rho_p\sigma_4 + \rho_p v_p\sigma_5/y - A\rho_p(u - u_p)\sigma_6$$

$$- A\rho_p(v - v_p)\sigma_7 + \tfrac{2}{3}\rho_p\,AC(T_p - T)\sigma_8\,]\,dx = 0 \qquad \text{(III-34)}$$

For the gas streamline characteristic curve, $(u dy/dx - v) = 0$. Thus, by substituting this relationship into Equations (III-26) through (III-33), the following results are obtained :

$$\sigma_1 = 0$$

$$\sigma_2 = u\sigma_3/v \qquad \text{(III-35)}$$

$$\sigma_5 = \sigma_5 = \sigma_7 = \sigma_8 = 0$$

Substituting Equation (III-35) into Equation (III-34) and regrouping terms into coefficients of the two arbitrary multipliers, σ_3 and σ_4, yields the following result :

$$[\,\rho u du + \rho v dv + dp + A\rho_p(u - u_p)dx + A\rho_p(v - v_p)dy\,]\,\frac{u}{v}\sigma_3$$

$$+ [\,u dp - a^2 u d\rho - AB\rho_p dx\,]\,\sigma_4 = 0 \qquad \text{(III-36)}$$

Since the multipliers σ_3 and σ_4 are arbitrary, their coefficients must equal zero. Equating the coefficients of these multipliers to zero yields the following compatibility equations which are valid along gas streamlines :

$$\rho u du + \rho v dv + dp + A\rho_p(u - u_p)dx + A\rho_p(v - v_p)dy = 0 \qquad \text{(III-37)}$$

$$u dp - a^2 u d\rho - AB\rho_p dx = 0 \qquad \text{(III-38)}$$

Noting that

$$u du + v dv = W dW$$

Equation (III-37) can be rewritten as

$$\rho W dW + dp + A\rho_p \left[(u - u_p)dx + (v - v_p)dy \right] = 0 \qquad \text{(III-39)}$$

Equations (III-38) and (III-39) are valid only along gas streamlines, i.e., along curves defined by

$$dy/dx = \tan\theta \qquad \text{(III-40)}$$

Along the Mach lines, the slope dy/dx is determined from either Equations (III-21) or (III-24) and $(u\,dy/dx - v) \neq 0$. Therefore, by combining Equations (III-26) through (III-33) with Equation (III-31), the relationships for the multipliers σ_i for determining the compatibility equations which are valid along the Mach lines are obtained and are as follows :

$$\sigma_1 = a^2\sigma_4$$

$$\sigma_2 = \frac{-\,a^2\sigma_4\,dy/dx}{(u\,dy/dx - v)}$$

$$\sigma_3 = \left[\frac{(u\,dy/dx - v)^2 - a^2(dy/dx)^2}{(u\,dy/dx - v)} \right] \sigma_4$$

$$\sigma_5 = \sigma_6 = \sigma_7 = \sigma_8 = 0 \qquad \text{(III-41)}$$

Since no relationship is obtained for σ_4, this multiplier is arbitrary. The compatibility equations valid along the Mach lines are obtained by substituting Equations (III-41) into Equation (III-34), the general compatibility equation, and equating the coefficient of σ_4 to zero, which yields

$$(-vdu + udv) \pm \frac{\sqrt{M^2-1}}{\rho}dp + \frac{v}{y}(udy - vdx)$$

$$- A(\rho_p/\rho)(u - u_p)dy + A(\rho_p/\rho)(v - v_p)dx$$

$$- AB(\rho_p/\rho a^2)(udy - vdx) = 0 \qquad \text{(III-42)}$$

Equation (III-42) can be rewritten in terms of θ and α as

$$d\theta \pm(\cot\alpha/\rho W^2)dp \pm \left[\frac{\sin\theta}{y} - \frac{A\rho_p}{\rho W}(1 + B/a^2)\right] \times$$

$$\left(\frac{\sin\alpha}{\cos(\theta \pm \alpha)}\right)dx + A(\rho_p/\rho W^2)\left[u_p dy - v_p dx\right] = 0$$

(III-43)

where the gas Mach lines are given by

$$dy/dx = \tan(\theta \pm \alpha) \qquad \text{(III-44)}$$

In Equations (III-43) and (III-44), the upper signs refer to left-running Mach lines and the lower signs refer to right-running Mach lines.

For the particle streamline characteristic curve, $(u_p dy/dx - v_p) = 0$. Thus, by substituting this relationship into Equations (III-26) through (III-33), the following results are obtained :

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0 \qquad \text{(III-45)}$$

Substituting Equation (III-45) into Equation (III-34) and regrouping terms into coefficients of the three arbitrary multipliers, σ_6, σ_7 and σ_8 yields the following result :

$$\left[u_p du_p - A(u - u_p)dx \right]\sigma_6 + \left[u_p dv_p - A(v - v_p)dx \right]\sigma_7$$

$$+ \left[cu_p dT_p + \frac{2}{3} AC(Tp - T)dx \right]\sigma_8 = 0 \qquad \text{(III-46)}$$

Again, the multipliers σ_6, σ_7 and σ_8 are arbitrary. Therefore, their coefficients must be zero. Equating the coefficients of these three multipliers to zero yields the following compatibility equations which are valid along the particle streamlines :

$$u_p du_p - A(u - u_p)dx = 0 \qquad \text{(III-47)}$$

$$u_p dv_p - A(v - v_p)dx = 0 \qquad \text{(III-48)}$$

$$cu_p dT_p + \frac{2}{3} AC(T_p - T)dx = 0 \qquad \text{(III-49)}$$

where the particle streamline is given by

$$dy/dx = v_p/u_p \qquad \text{(III-50)}$$

It should be pointed out that only seven compatibility equations are obtained in this analysis, i.e., no equation is obtained for solving for the particle density, ρ_p. This can be explained by noting that σ_5 is zero for all characteristic curves. Therefore, the fifth governing equation, Equation (III-11), the particle continuity equation does not enter the analysis as treated by the method of characteristics. If Equation (III-11) is removed from the set of governing equations, seven characteristic curves and seven compatibility equations are obtained, as should be. In order to have a complete set of equations it is necessary to include Equation (III-11) or some equivalent equation in order to have a relationship for ρ_p. In the present analysis an integral equivalent equation for particle continuity is employed. The equation used is as follows :

$$\dot{m}_p = 2\pi \int_0^y y\rho_p(u_p dy/dx - v_p)dx \qquad \text{(III-51)}$$

Equation (III-51) is used to calculate an average particle density at each point in the flow field.

It should be reemphasized that treating the particles as a continuum is an approximation in the present two-dimensional analysis just as in previous one-dimensional analyses.

In summary, the governing partial differential equations have been transformed to a system of differential equations valid only along certain characteristic curves. These are :

Gas Streamlines

$$dy/dx = \tan\theta \qquad \text{(III-52)}$$

$$\rho W dW + dp + A\rho_p \left[(u - u_p)dx + (v - v_p)dy \right] = 0 \qquad \text{(III-53)}$$

$$u dp - a^2 u d\rho - AB\rho_p dx = 0 \qquad \text{(III-54)}$$

Gas Mach Lines

$$dy/dx = \tan(\theta \pm \alpha) \qquad \text{(III-55)}$$

$$d\theta \pm (\cot\alpha/\rho W^2)dp \pm \left[\frac{\sin\theta}{y} - \frac{A\rho_p}{\rho W}(1 + B/a^2) \right] \frac{\sin\alpha}{\cos(\theta \pm \alpha)} dx$$

$$+ A(\rho_p/\rho W^2) \left[u_p dy - v_p dx \right] = 0 \qquad \text{(III-56)}$$

Particle Streamlines

$$dy/dx = \tan \theta p \qquad \text{(III-57)}$$

$$u_p du_p - A(u - u_p)dx = 0 \qquad \text{(III-58)}$$

$$u_p dv_p - A(v - v_p)dx = 0 \qquad \text{(III-59)}$$

$$u_p c dT_p + \frac{2}{3} AC(T_p - T)dx = 0 \qquad \text{(III-60)}$$

and the particle continuity equation

$$\dot{m}_p = 2\pi \int_0^y y\rho_p(u_p dy/dx - v_p)dx \qquad \text{(III-61)}$$

IV. NUMERICAL SOLUTION TECHNIQUE

The formulation of the problem of treating a supersonic flow field consisting of a mixture of a gas and solid particles represents only one part of the present investigation. In order to make use of the equations derived in the previous section, it is necessary to translate the equations of motion of Section III into a numerical algorithm. This section outlines the solution procedure which was programmed for the IBM 1130 computer at the von Karman Institute.

As was discussed in the previous section, the gas Mach lines (two of the characteristic curves) are real only when the gas Mach number (ratio of the gas speed and the frozen speed of sound) is greater than one. Therefore, the present scheme is only applicable in regions of the flow which are supersonic. This means that in order to calculate the flow in a nozzle by the method of characteristics one must know the flow properties in the region downstream of, but near, the gas sonic line. In order to obtain the necessary initial conditions for a gas-solid particle flow the complete subsonic and transonic flow field must be solved. Since this problem has not been solved to date, it is necessary to use approximate methods to obtain the gas and solid particle properties from which to start the solution procedure. It is clearly evident that much more work must be done in the subsonic and transonic flow regions before the present supersonic analysis can be effectively utilized, i.e., to get realistic results one must specify realistic initial conditions.

The numerical algorithm does not depend on the initial conditions. Therefore, as better methods for predicting the initial conditions become available, they can be input into the present analysis directly as input data.

The numerical algorithm used is a modified Euler, predictor-corrector technique where averaged coefficients are used. The transformed equations, Equations (III-52) through (III-61) are forward differenced. To illustrate the technique, Figure 1 presents an interior mesh point grid. The four distinct characteristic curves are shown with the point numbering scheme as used in the computer program. Point 3 is the point where the solution is sought. Points 1 and 2 are previously obtained solution points or given points on the initial value line. The prop ies at points 4 and 5 are known (by interpolation) once the location of each point is determined. The solution procedure is as follows :

(1) Predict the x and y direction of Point 3 by solving the two equations, Equations (III-55). In finite difference form this yields :

$$x_3 = (y_2 - y_1 - x_2H_{23} + x_1S_{13})/(S_{13} - H_{23})$$

$$y_3 = y_1 + S_{13}(x_3 - x_1)$$

where

$$H_{23} = \frac{1}{2} [\tan(\theta_2 + \alpha_2) + \tan(\theta_3 + \alpha_3)]$$

$$S_{13} = \frac{1}{2} [\tan(\theta_1 - \alpha_1) + \tan(\theta_3 + \alpha_3)]$$

it should be noted that the second term within the brackets in the above two equations are set equal to the first term for the predictor.

(2) Solve for the gas flow angle and the pressure by simultaneously solving the two equations, Equations (III-56).

(3) Calculate the position of Points 4 and 5 by the particle streamline equation, Equation (III-57) and the gas streamline equation, Equation (III-52), respectively.

(4) Interpolate for the properties at points 4 and 5.

(5) Calculate the gas velocity and the gas density at Point 3 from Equations (III-53) and (III-54).

(6) Calculate the particle velocity, particle flow angle, particle temperature, and particle density from Equations (III-58) through (III-61).

(7) Now that all properties are known (predicted values), evaluate all equation coefficients, average the coefficients, and repeat steps (1) through (6).

This algorithm is second order accurate. To iterate again accomplishes nothing. If the accuracy is not sufficient change the grid spacing by increasing the number of points on the initial value line and redo the calculations.

The same technique is used on the wall boundary except that on the wall the solution point is obtained by the intersection of the left characteristic and the wall contour. Since the gas flow angle is the wall angle, the flow angle is known. This reduces the unknowns by one, which is necessary since the right characteristic compatibility equation cannot be used.

The same procedure exists at the centerline. Here the gas flow angle is zero (known, a priori), and the left characteristic compatibility equation is not used.

To check the program out, a sample case was executed for which previous one-dimensional results were available. The initial value line properties were taken from the one-dimensional results. The results were then compared at the nozzle exit, 9 cm. downstream of the initial value line. The following table presents the results as obtained from the two programs. The loading ratio was 1.5, i.e., the particle flow rate was 1.5 times the gas flow rate. Particle diameter was 500 microns.

	Present Analysis 2 - d	1 - d
Gas Velocity	469.0 m/sec	467.6 m/sec
Particle Velocity	213.2 m/sec	214.7 m/sec
Gas Temperature	159.7 °K	159.7°K
Particle Temperature	295.7 °K	291.9 °K
Gas Mach number	1.850	1.845
Pressure	1.86×10^{6} n/m^2	1.88×10^{6} n/m^2

The run time for the two-dimensional calculation on the IBM 1130 was approximately 9 minutes.

The above comparison indicates that the program is working correctly and that it can be used effectively for particle-gas problems.

V. CONCLUSIONS AND RECOMMENDATIONS

The results of the comparison of the present two-dimensional, supersonic analysis with the one-dimensional analysis indicate that the program is working correctly and that it can be a useful tool for particle micronization processes. The present analysis has the advantage that non-uniform particle distributions can be treated if the necessary initial data are available. The program has been written in modular form so that any required or desired program changes are relatively simple.

Further studies are needed in two areas. These are : (1) a working subsonic and transonic analysis to take advantage of the potential applications of this analysis and (2) much more effort is required to obtain governing equations that are more physically realistic. The use of the particle continuum assumption, obviously, has restrictions. Experimental measurements are necessary to determine the region where this assumption breaks down. It should be emphasized that one-dimensional analyses suffer from the same problem and, therefore, if for particle micronization processes the continuum assumption does not provide engineering results neither approach can be used to predict particle behaviour in the nozzle.

REFERENCES

1. Riethmuller, M., VKI Report, 1971 (in preparation).

2. Kliegel, J.R., Ninth Symposium on Combustion; Cornell University, 1962.

3. Hoffman, J.D. and Thompson, H.D., AIAA Paper 66-538, June 1966.

4. Neilson, J.H. and Gilchrist, A., J.F.M., Vol. 33, Part 1, p. 137.

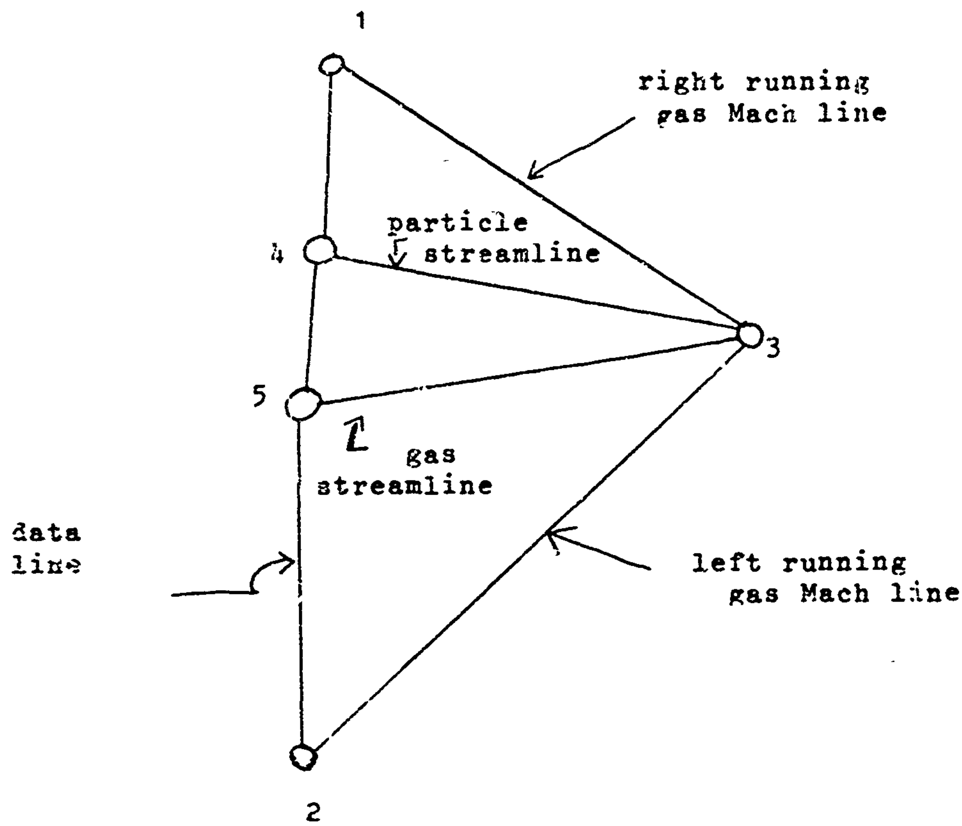

Figure 1 Gas-solid particle characteristic mesh

APPENDICES

APPENDIX A

PROGRAM DESCRIPTION

The computer program was written in the FORTRAN IV language for the IBM 1130 computer at the von Karman Institute. The program consists of the MAIN program and four subroutines. These subroutines are designated FLØWP, MØCP, MØCB and CØEFP. The structure of the program is illustrated in Figure A-1. A brief description of the program is as follows :

MAIN - This program calls the flow field logic program or CALLS EXIT as determined by whether SWITCH 10 on the console is "on" or "off".

FLØWP - This subroutine controls the logic in calculating the flow field in an axisymmetric nozzle. All input data and all output options are accomplished by this subroutine. Subroutine FLØWP calls MØCP for the solution at an interior mesh point or MØCB for the solution at the wall boundary or the centerline boundary. Two arrays store the solution along a back left characteristic and the left characteristic being solved.

MØCP - This subroutine uses the modified Euler predictor corrector technique with averaged coefficients to solve for an interior mesh point. Subroutine CØEFP is called to evaluate all the necessary coefficients used in the finite difference equations.

MØCB - This subroutine is nearly identical to the above subroutine except the solution point is on the boundary.

CØEFP - This subroutine evaluates the coefficients of the finite difference equations and is called from both MØCP and MØCB.

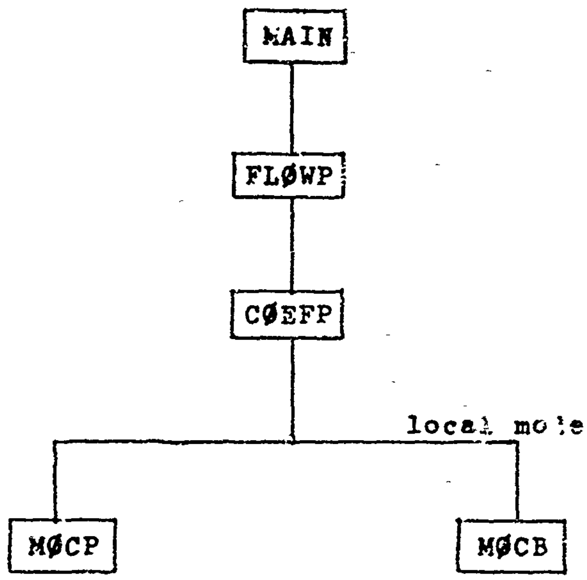

Figure A-1 Program Structure

APPENDIX B

PROGRAM INPUT

The input data for the program has been kept to a minimum to keep the program simple. The following data are required for program execution :

CARD 1 (FORMAT 3F10.0)

1-10	GAMMA, ratio of the specific heats for the gas (air, γ=1.4)
11-20	RGAS, the gas constant (air, R=29.27)
21-30	PR, the gas Prandtl number (air, Pr=0.7)

CARD 2 (FORMAT 4F10.0)

1-10	DIAM, particle diameter (meters)
11-20	RHOP, particle density (kg/m^3)
21-30	CPART, particle specific heat
31-40	PFLRT, particle flow rate (kg/sec)

CARD 3 (FORMAT I2,8X,I2,8X,I2,8X,I1)

1-2	NPTSL, the number of points for which data is specified on the initial value line (MAX=5)
11-12	IMAX, the maximum number of left characteristics to be calculated
21-22	IOUT, the number of the left characteristic where print out begins. If all characteristics are desired IOUT=0.
31	NU, not used

The following two cards are repeated for each of the NPTSL points. Two cards are required per point. The first point input is the wall point, the last point is the centerline point.

CARD 4 (FORMAT 7F10.0)

1-10	x-location of the point (meters)
11-20	y-location of the point (meters)
21-30	Gas Mach number
31-40	Gas velocity (m/sec)
41-50	Gas flow angle (degrees)
51-60	pressure (n/m^2)
61-70	gas density (kg/m^3 * g_c)

CARD 5 (FORMAT 5F10.0)

1-10	particle velocity (m/sec)
11-20	particle flow angle (degrees)
21-30	particle density (kg/m^3)
31-40	particle temperature (°K)
41-50	particle stream function (0.0 for centerline, 1.0 at limiting streamline).

APPENDIX C

PROGRAM LISTING

This appendix contains the complete program listing. All subroutines are stored on disc 6. For execution, it is only necessary to compile the MAIN program. The two subroutines MØCP and MØCB are stored in the LOCAL mode.

```
C              MAIN PROGRAM
C
C              COMMON STATEMENTS
C
      COMMON NU,CONST,THETW,PFLRT,PI,X3A,Y3A,E3A,V3A,T3A,P3A,R3A,VELP3,
     1TP3A,RP3A,TEP3A,PSI3A
      COMMON GAMMA,GC,RGAS,DIAM,RHOP,PR,CPART
C
   10 CONTINUE
      PAUSE
      CALL DATSW (10,K10)
      GO TO (20,30), K10
   20 CALL FLOWP
      GO TO 10
C
   30 CALL EXIT
C
      END
```

```
      SUBROUTINE FLOWP
C
C
C           THIS PROGRAM CONTROLS THE LOGIC NECESSARY TO CALCULATE
C           THE FLOW FIELD IN A SUPERSONIC AXISYMMETRIC (NU=1) OR
C           TWO-DIMENSIONAL (NU=0) NOZZLE BY THE METHOD OF CHARACTERIS
C
C           THE ARRAY SLV IS USED TO STORE THE INIT.AL-VALUE LINE.
C           THE FIRST SUBSCRIPT IS THE POINT NUMBER, THE SECOND DENOTE
C           THE FLUID PROPERTY DEFINED BY THE FOLLOWING
C
C                1     X POSITION
C                2     Y POSITION
C                3     GAS MACH NUMBER
C                4     GAS VELOCITY
C                5     GAS FLOW ANGLE
C                6     GAS PRESSURE
C                7     GAS DENSITY
C                8     PARTICLE VELOCITY
C                9     PARTICLE FLOW ANGLE
C                10    PARTICLE DENSITY
C                11    PARTICLE TEMPERATURE
C                12    PARTICLE STREAM FUNCTION
C
C           THE SAME SCHEME IS USED FOR THE TWO CHARACTERISTIC ARRAYS
C           CHARA AND CHARL. . TWO ARRAYS ARE NEEDED SINCE IT IS NECESS
C           TO SAVE ONE LEFT CHARACTERISTIC WHILE THE NEXT ONE IS BEIN
C           CALCULATED.
C
C       ** NOTE **
C           THE INPUT PARAMETER NPTSL GOVERNS THE SIZE OF THE THREE
C           ARRAYS.  THESE MUST BE SLV(NPTSL,12), CHARA(2*NPTSL,12)
C           AND CHARL(2*NPTSL,12).
C
C
C
C           TYPE, DIMENSION AND COMMON STATEMENTS
C
      REAL METER
      DIMENSION SLV(5,12),CHARA(10,12),CHARL(10,12)
      COMMON NU,CONST,THETW,PFLRT,PI,X3A,Y3A,E3A,V3A,T3A,P3A,R3A,VP3,
     1TP3A,RP3A,TEP3A,PSI3A
      COMMON GAMMA,GC,RGAS,DIAM,RHOP,PR,CPART
C
C           SET PROGRAM CONSTANTS AND INIATILIZE COUNTERS
C
      GC=9.81
```

```
      PI=3.1415926
      RAD=180.0/PI
      LINE=9
      METER=100.0
      ICL=1
      INDEX=1
      I=0
      J=0
      M1=1
C
C             READ INPUT GAS DATA
C
      READ (2,1001) GAMMA,RGAS,PR
C
C             READ INPUT PARTICLE DATA
C
      READ (2,1001) DIAM,RHOP,CPART,PFLRT
C
C             READ INPUT CONTROL FLAGS
C
      READ (2,1002) NPTSL,IMAX,IOUT,NU
C
C             READ INITIAL-VALUE LINE POINTS AND FLOW PROPERTIES
C
      DO 10 K=1,NPTSL
      READ (2,1003) (SLV(K,IP),IP=1,12)
      SLV(K,5)=SLV(K,5)/RAD
      SLV(K,9)=SLV(K,9)/RAD
   10 CONTINUE
C
C             STORE FIRST POINT OF INITIAL-VALUE LINE IN AUXILLIARY ARRA
C
      DO 20 K=1,12
      CHARA(1,K)=SLV(1,K)
   20 CONTINUE
      THETW=SLV(1,5)
      CONST=SQRT(GAMMA*RGAS*GC)
      WRITE (1,1006)
      GO TO 210
C
C             INCREMENT COUNTER
C
   30 I=I+1
      GO TO (40,50), INDEX
C
C             M1 IS THE NUMBER OF POINTS ON THE LEFT CHARACTERISTIC
C             BEING CALCULATED
```

```
C
   40 J=I+1
      MM=J+1
      GO TO 60
C
   50 J=NPTSL+NPTSL-1
      MM=J
C
C             THE COUNTER KK DETERMINES WHETHER THE LEFT CHARACTERISTIC
C             BEING CALCULATED STARTS FROM THE INITIAL-VALUE LINE OR FRO
C             THE CENTERLINE
C
   60 KK=I+1-NPTSL
C
C             LOOP FOR CALCULATING THE PROPERTIES ALONG A LEFT CHARACTER
C
      DO 200 L=1,J
      K=L+1
      M=L+1
      N=L-1
      IF(N) 70,70,120
   70 CONTINUE
      IF(KK) 90,80,110
   80 INDEX=2
   90 JP=I+1
      DO 100 IP=1,12
      CHARA(L,IP)=SLV(JP,IP)
  100 CONTINUE
      GO TO 120
C
C             CALCULATE FLOW PROPERTIES ON THE CENTERLINE
C
  110 ICL=2
      VELP1=CHARL(2,8)
      VELP2=CHARL(1,8)
      TP1=CHARL(2,9)
      TP2=CHARL(1,9)
      RP1=CHARL(2,10)
      RP2=CHARL(1,10)
      TEP1=CHARL(2,11)
      TEP2=CHARL(1,11)
      PSI1=CHARL(2,12)
      PSI2=CHARL(1,12)
      CALL MOCR(2,CHARL(2,1),CHARL(1,1),CHARL(2,2),CHARL(1,2),CHARL(2,3)
     1,CHARL(1,3),CHARL(2,4),CHARL(1,4),CHARL(2,5),CHARL(1,5),CHARL(2,6)
     2,CHARL(1,6),CHARL(2,7),CHARL(1,7),VELP1,VELP2,TP1,TP2,RP1,RP2,
     3TEP1,TEP2,PSI1,PSI2)
```

```
      M=N-1
      GO TO 180
C
  120 CONTINUE
      GO TO (130,140), ICL
  130 K=L
      N=L
      GO TO 150
C
  140 M=N-1
  150 VELP1=CHARL(K,8)
      VELP2=CHARA(N,8)
      TP1=CHARL(K,9)
      TP2=CHARA(N,9)
      RP1=CHARL(K,10)
      RP2=CHARA(N,10)
      TEP1=CHARL(K,11)
      TEP2=CHARA(N,11)
      PSI1=CHARL(K,12)
      PSI2=CHARA(N,12)
      MZ=L-J
      IF(MZ) 160,170,170
C
C                CALCULATE FLOW PROPERTIES AT AN INTERIOR MESH POINT
C
  160 CALL MOCP (CHARL(K,1),CHARA(N,1),CHARL(K,2),CHARA(N,2),CHARL(K,3
     1),CHARA(N,3),CHARL(K,4),CHARA(N,4),CHARL(K,5),CHARA(N,5),CHARL(K,6
     2),CHARA(N,6),CHARL(K,7),CHARA(N,7),VELP1,VELP2,TP1,TP2,RP1,RP2,
     3TEP1,TEP2,PSI1,PSI2)
      GO TO 180
C
C
C                CALCULATE FLOW PROPERTIES ON THE NOZZLE WALL
C
  170 CALL MOCP (1,CHARL(K,1),CHARA(N,1),CHARL(K,2),CHARA(N,2),CHARL(K,3
     1),CHARA(N,3),CHARL(K,4),CHARA(N,4),CHARL(K,5),CHARA(N,5),CHARL(K,6
     2),CHARA(N,6),CHARL(K,7),CHARA(N,7),VELP1,VELP2,TP1,TP2,RP1,RP2,
     3TEP1,TEP2,PSI1,PSI2)
C
C                STORE THE SOLUTION OBTAINED FROM SUBROUTINE MOC IN
C                THE AUXILLIARY ARRAY
C
  180 CHARA(N,1)=X3A
      CHARA(N,2)=Y3A
      CHARA(N,3)=E3A
      CHARA(N,4)=V3A
      CHARA(N,5)=T3A
```

```
      CHARA(M,6)=P3A
      CHARA(M,7)=R3A
      CHARA(M,8)=VELP3
      CHARA(M,9)=TP3A
      CHARA(M,10)=RP3A
      CHARA(M,11)=TEP3A
      CHARA(M,12)=PSI3A
  200 CONTINUE
  210 CONTINUE
C
C             MOVE DATA FROM AUXILLIARY ARRAY TO THE LEFT
C             CHARACTERISTIC ARRAY
C
      DO 240 K=1,MM
      DO 220 LL=1,12
      CHARL(K,LL)=CHARA(K,LL)
  220 CONTINUE
      IF(I-IOUT) 240,230,230
C
C             PRINT OUT SOLUTION ALONG THE LEFT CHARACTERISTIC
C
  230 CONTINUE
      IF(LINE-62) 234,232,232
  232 WRITE (1,1008)
      WRITE (1,1006)
      LINE=9
  234 XCM=CHARL(K,1)*METER
      YCM=CHARL(K,2)*METER
      TDEG=CHARL(K,5)*RAD
      RKG=CHARL(K,7)/GC
      PRES=CHARL(K,6)/(GC*METER*METER)
      TEMP=CHARL(K,6)/(RGAS*CHARL(K,7))
      TPDEG=CHARL(K,9)*RAD
      WRITE (1,1004) XCM,YCM,CHARL(K,3),CHARL(K,4),TDEG,PRES,RKG
     1,TEMP
      WRITE (1,1007) CHARL(K,8),TPDEG,CHARL(K,10),CHARL(K,11)
      LINE=LINE+2
  240 CONTINUE
      IF(I-IOUT) 260,250,250
  250 WRITE (1,1005)
      LINE=LINE+3
C
C             STORE WALL POINT IN THE LEFT CHARACTERISTIC ARRAY
C
  260 MN=MM+1
      DO 270 LL=1,12
      CHARL(MN,LL)=CHARA(MM,LL)
```

```
  270 CONTINUE
      IF(I-IMAX) 30,30,280
  280 RETURN
C
C               FORMAT STATEMENTS
C
 1001 FORMAT (4F10.0)
 1002 FORMAT (I2,8X,I2,8X,I2,8X,I1)
 1003 FORMAT (7F10.0/5F10.0)
 1004 FORMAT (3(F10.5,1X),2(F10.4,1X),F11.4,3X,F10.4,1X,F10.2,8X,'GAS')
 1005 FORMAT (//)
 1006 FORMAT (26X,'MACH',20X,'FLOW'/6X,'X',10X,'Y',7X,'NUMBER',4X,'VELOC
     1ITY',6X,'ANGLE',3X,'PRESSURE',5X,'DENSITY',3X,'TEMPERATURE'//4X,
     2'(CM)',7X,'(CM)',17X,'(M/SEC)',6X,'(DEG)',4X,'(KG/SQ CM)',3X,
     3'(KG/CU M)',4X,'(DEG K)'//)
 1007 FORMAT (33X,2(F10.4,1X),14X,F10.4,1X,F10.2,6X,'PARTICLE')
 1008 FORMAT (////)
C
      END
```

```
      SUBROUTINE COEFP (Y,E,V,T,P,RHO,VELP,TEP,RHOPP,TP,S,H,Q,APU,APV,
     1ARU,ARV,CE,G,F,FP,GP,BP,C,DP)
C
C        THIS SUBROUTINE CALCULATES THE COEFFICIENTS AT EACH POINT
C        WHICH ARE USED IN SOLVING THE DIFFERENTIAL EQUATIONS FOR THE
C        METHOD OF CHARACTERISTICS SOLUTION TECHNIQUE
C
C
C        TYPE, DIMENSION AND COMMON STATEMENTS
C
      REAL KGAS,MU,NU1
C
      COMMON NU,CONST,THETH,FFLRT,PI,X3A,Y3A,E3A,V3A,T3A,P3A,R3A,VELP3,
     1TP3A,RP3A,TEP3A,PSI3A
      COMMON GAMMA,G0,R,DIAM,RHOP,PR,CPART
C
      SOS=V/E
      D=1.0/SQRT(E**2-1.0)
      A=ATAN(D)
      STMA=SIN(T-A)
      STPA=SIN(T+A)
      CTMA=COS(T-A)
      CTPA=COS(T+A)
      S=STMA/CTMA
      H=STPA/CTPA
      Q=COS(A)/(SIN(A)*RHO/G0*V**2)
      UP=VELP*COS(TP)
      VP=VELP*SIN(TP)
      DU=V*COS(T)-UP
      DV=V*SIN(T)-VP
      TEMP=P/(R*RHO)
      MU=0.0000143*SQRT(TEMP)/(1.0+105.0/TEMP)
      KGAS=1000.0*MU
      REY=RHO*DIAM*ABS(V-VELP)/(MU*G0)
      CX=0.48+28.0/(REY**0.85)
      NU1=2.0+0.6*SQRT(REY)*PR**0.333
      CP=12.0*KGAS*NU1/(MU*CX*REY)
      B=(GAMMA-1.0)*(2.0*CP*(TEP-TEMP)/3.0+DU*DU+DV*DV)
      AP=0.75*CX*RHO*ABS(V-VELP)/(DIAM*RHOP*G0)
      APU=AP*DU/UP
      APV=AP*DV/UP
      ARU=AP*RHOPP*DU
      ARV=AP*RHOPP*DV
      CE=2.0*AP*CP*(TEMP-TEP)/(3.0*UP*CPART)
      F=-AP*G0*RHOPP/(RHO*V*E*STPA)*(1.0+B/(SOS*SOS))
      G=-AP*G0*RHOPP/(RHO*V*E*STMA)*(1.0+B/(SOS*SOS))
      IF(Y-0.0000001) 20,20,10
   10 STY=SIN(T)/Y
      F=F+STY/(E*STPA)
      G=G+STY/(E*STMA)
   20 FP=-AP*RHOPP*G0*VELP/(V*V*RHO)*(COS(TP)*H-SIN(TP))
      GP=-AP*RHOPP*G0*VELP/(V*V*RHO)*(COS(TP)*S-SIN(TP))
      BP=RHO*V/G0
      C=SOS*SOS/G0
      DP=AP*B*RHOPP/(C*V*COS(T))
      RETURN
      END
```

```
      SUBROUTINE MOCP (X1,X2,Y1,Y2,E1,E2,V1,V2,T1,T2,P1,P2,R1,R2,
     1VELP1,VELP2,TP1,TP2,RP1,RP2,TEP1,TEP2,PSI1,PSI2)
C
C
C          THIS SUBROUTINE IS A METHOD OF CHARACTERISTICS SUBPROGRAM
C          USED TO CALCULATE THE FLOW PROPERTIES AT AN INTERIOR
C          MESH POINT. .
C
C          THE GASDYNAMIC MODEL IS ROTATIONAL AND A PERFECT GAS IS
C          ASSUMED. , THIS SUBROUTINE USES THE MODIFIED-EULER
C          PREDICTOR-CORRECTOR TECHNIQUE, WHERE AVERAGED
C          COEFFICIENTS ARE USED. .
C
C          POINTS 1 AND 3 ARE ON THE RIGHT CHARACTERISTIC, POINTS
C          2 AND 3 ARE ON THE LEFT CHARACTERISTIC, POINTS 5 AND 3 ARE
C          ON THE GAS STREAMLINE, POINTS 4 AND 3 ARE ON THE PARTICLE
C          STREAMLINE WHERE POINT 3 IS THE DESIRED SOLUTION POINT. .
C
C
C          TYPE, DIMENSION AND COMMON STATEMENTS
C
      COMMON NU,CONST,THETW,PFLRT,PI,X3A,Y3A,E3A,V3A,T3A,P3A,R3A,VELP3,
     1TP3A,RP3A,TEP3A,PSI3A
      COMMON G,G0,R,DIAM,RHOP,PR,CPART
      I=-1
      ICL=1
C
C          CALCULATE THE COEFFICIENTS AT POINT 1
C
      CALL COEF  (Y1,E1,V1,T1,P1,R1,VELP1,TEP1,RP1,TP1,S1,H,Q1,APU,APV,
     1ARU,ARV,  G1,F,FP,GP1,B,C,D)
C
C          AVERAGE THE COEFFICIENTS FOR THE PREDICTOR
C
      S13=S1
      Q13=Q1
      G13=G1
      GP13=GP1
C
C          CALCULATE THE COEFFICIENTS AT POINT 2
C
      CALL COEFP (Y2,E2,V2,T2,P2,R2,VELP2,TEP2,RP2,TP2,S,H2,Q2,APU,APV,
     1ARU,ARV,CE,G2,F2 FP2,GP,B,C,D)
      IF(Y2-0.0000001) 70,70,80
C
C          CENTERLINE APPROXIMATION
C
```

```
   70 ICL=2
C
C            AVERAGE COEFFICIENTS FOR THE PREDICTOR
C
   80 H23=H2
      Q23=Q2
      F23=F2
      FP23=FP2
C
   90 I=I+1
C
C            INTERIOR MESH POINT SOLUTION
C
      X3A=(Y2-Y1-X2*H23+X1*S13)/(S13-H23)
      Y3A=Y1+S13*(X3A-X1)
      GO TO (150,140), ICL
C
C            SOLUTION WHEN POINT 2 IS ON THE CENTERLINE
C
  140 P3A=(-2.0*T1+Q23*P2+2.0*Q13*P1-2.0*G13*(Y3A-Y1)-2.0*GP13*(X3A-X1)
     1-F23*(Y3A-Y2)+FP23*(X3A-X2))/(2.0*Q13+Q23)
      T3A=T1+Q13*(P3A-P1)+G13*(Y3A-Y1)+GP13*(X3A-X1)
      F2=F2+T3A/Y3A
      ICL=1
      GO TO 210
C
  150 P3A=(Q23*P2+Q13*P1+T2-T1-F23*(Y3A-Y2)+FP23*(X3A-X2)-G13*(Y3A-Y1)-
     1GP13*(X3A-X1))/(Q23+Q13)
      T3A=T2-Q23*(P3A-P2)-F23*(Y3A-Y2)+FP23*(X3A-X2)
C
C
C            CALCULATE THE POSITION OF POINT 5 FROM THE GAS
C            STREAMLINE CHARACTERISTIC EQUATION.
C
  210 IXY=1
      DX12=X1-X2
      IF(ABS(DX12)-0.0001) 230,240,240
  230 DXDY=(X2-X1)/(Y2-Y1)
      DX12=Y1-Y2
      IXY=2
      GO TO 250
C
  240 DYDX=(Y1-Y2)/DX12
  250 CONTINUE
      IF(I) 260,260,270
  260 T5=T3A
  270 TT35=(SIN(T3A)/COS(T3A)+SIN(T5)/COS(T5))*0.5
```

```
      GO TO (280,290), IXY
  280 X5=(Y3A-Y2+X2*DYDX-X3A*TT35)/(DYDX-TT35)
      Y5=Y3A+TT35*(X5-X3A)
      DX=X5-X2
      GO TO 300
C
  290 Y5=(Y3A+TT35*(X1-X3A)-TT35*DXDY*Y1)/(1.0-TT35*DXDY)
      X5=X1+(Y5-Y1)*DXDY
      DX=Y5-Y2
C
C              CALCULATE THE DERIVATIVES OF THE PROPERTIES BETWEEN
C              POINTS 1 AND 2. .
C
  300 SM12=(E1-E2)/DX12
      SV12=(V1-V2)/DX12
      ST12=(T1-T2)/DX12
      SP12=(P1-P2)/DX12
      SR12=(R1-R2)/DX12
      SPV12=(VELP1-VELP2)/DX12
      STP12=(TP1-TP2)/DX12
      SRP12=(RP1-RP2)/DX12
      STE12=(TEP1-TEP2)/DX12
C
C              LINEARLY INTERPOLATE FOR THE PROPERTIES AT POINT 5
C
      E5=E2+SM12*DX
      V5=V2+SV12*DX
      T5=T2+ST12*DX
      P5=P2+SP12*DX
      R5=R2+SR12*DX
      VELP5=VELP2+SPV12*DX
      TP5=TP2+STP12*DX
      RP5=RP2+SRP12*DX
      TEP5=TEP2+STE12*DX
      IF(I) 302,302,304
C
C              CALCULATE THE POSITION OF POINT 4 FROM THE PARTICLE
C              STREAMLINE CHARACTERISTIC EQUATION. .
C
  302 TP3A=TP5
      TP4=TP5
  304 TT34=(SIN(TP3A)/COS(TP3A)+SIN(TP4)/COS(TP4))*0.5
      GO TO (306,307), IXY
  306 X4=(Y3A-Y2+X2*DYDX-X3A*TT34)/(DYDX-TT34)
      Y4=Y3A+TT34*(X4-X3A)
      DXX=X4-X2
      GO TO 308
```

```
C
  307 Y4=(Y3A+TT34*(X1-X3A)-TT34*DXDY*Y1)/(1.0-TT34*DXDY)
      X4=X1+(Y4-Y1)*DXDY
      DXX=Y4-Y2
C
C              LINEARLY INTERPOLATE FOR THE PROPERTIES AT POINT 4
C
  308 E4=E2+SM12*DXX
      V4=V2+SV12*DXX
      T4=T2+ST12*DXX
      P4=P2+SP12*DXX
      R4=R2+SR12*DXX
      VELP4=VELP2+SPV12*DXX
      TP4=TP2+STP12*DXX
      RP4=RP2+SRP12*DXX
      TEP4=TEP2+STE12*DXX
      PSI4=PSI2+PI/PFLRT*(0.5*(RP4*VELP4*COS(TP4)+RP2*VELP2*COS(TP2))
     1*(Y4**2-Y2**2)-(Y4*RP4*VELP4*SIN(TP4)+Y2*RP2*VELP2*SIN(TP2))*(X4
     2-X2))
C
C              CALCULATE THE COEFFICIENTS AT POINT 5
C
      CALL COEFP (Y5,E5,V5,T5,P5,R5,VELP5,TEP5,RP5,TP5,S,H,Q,APU,APV,
     1ARU5,ARV5,CE,G2,F,FP,GP,B5,C5,D5)
C
C              CALCULATE THE COEFFICIENTS AT POINT 4
C
      CALL COEFP (Y4,E4,V4,T4,P4,R4,VELP4,TEP4,RP4,TP4,S,H,Q,APU4,APV4,
     1ARU,ARV,CE4,G2,F,FP,GP,B,C,D)
      IF(I) 320,320,330
C
C              AVERAGE COEFFICIENTS FOR THE PREDICTOR
C
  320 C35=C5
      B35=B5
      D35=D5
      ARU35=ARU5
      ARV35=ARV5
      APU43=APU4
      APV43=APV4
      CE43=CE4
      GO TO 340
C
C              AVERAGE COEFFICIENTS FOR THE CORRECTOR
C
  330 B35=(B3+B5)*0.5
      C35=(C3+C5)*0.5
```

```
      D35=0.5*(D3+D5)
      ARU35=0.5*(ARU3+ARU5)
      ARV35=0.5*(ARV3+ARV5)
      APU43=0.5*(APU4+APU3)
      APV43=0.5*(APV4+APV3)
      CE43=0.5*(CE4+CE3)
C
C              CALCULATE THE VELOCITY AND DENSITY FROM THE STREAMLINE
C              COMPATIBILITY EQUATIONS
C
  340 V3A=V5+(P5-P3A-ARU35*(X3A-X5)-ARV35*(Y3A-Y5))/B35
      R3A=R5+(P3A-P5)/C35-D35*(X3A-X5)
      TEMP=P3A/(R*R3A)
      SOS=CONST*SQRT(TEMP)
      E3A=V3A/SOS
      UP3=VELP4*COS(TP4)+APU43*(X3A-X4)
      VP3=VELP4*SIN(TP4)+APV43*(X3A-X4)
      VELP3=SQRT(UP3**2+VP3**2)
      YDOT3=VP3/UP3
      TP3A=ATAN(YDOT3)
      TEP3A=TEP4+CE43*(X3A-X4)
      PSI3A=PSI4
      RP3A=(PFLRT*(PSI3A-PSI2)/PI-RP2*VELP2*COS(TP2)*0.5*(Y3A**2-Y2**2)
     1+(X3A-X2)*Y2*RP2*VELP2*SIN(TP2))/(0.5*UP3*(Y3A**2-Y2**2)-Y3A*VELP3
     2*SIN(TP3A)*(X3A-X2))
      IF(I) 350,350,410
C
C              CALCULATE THE COEFFICIENTS AT POINT 3
C
  350 CALL COEFP (Y3A,E3A,V3A,T3A,P3A,R3A,VELP3,TEP3A,RP3A,TP3A,S3,H3,
     1Q3,APU3,APV3,ARU3,ARV3,CE3,G3,F3,FP3,GP3,B3,C3,D3)
C
C              AVERAGE COEFFICIENTS FOR THE CORRECTOR
C
      H23=(H2+H3)*0.5
      Q23=(Q2+Q3)*0.5
      F23=(F2+F3)*0.5
      S13=(S1+S3)*0.5
      Q13=(Q1+Q3)*0.5
      G13=(G1+G3)*0.5
      FP23=0.5*(FP2+FP3)
      GP13=0.5*(GP1+GP3)
      GO TO 90
C
  410 RETURN
      END
```

```
      SUBROUTINE MOCB (IB,X1,X2,Y1,Y2,E1,E2,V1,V2,T1,T2,P1,P2,R1,R2,
     1VELP1,VELP2,TP1,TP2,RP1,RP2,TEP1,TEP2,PSI1,PSI2)
      COMMON NU,CONST,THETW,PELRT,PI,X3A,Y3A,E3A,V3A,T3A,P3A,R3A,VELP3,
     1TP3A,RP3A,TEP3A,PSI3A
      COMMON G,GC,R,DIAM,RHOP,PR,CPART
      I=-1
      GO TO (10,20), IB
C
C              REDEFINE DATA TRANSFERRED THROUGH CALL STATEMENT FOR
C              WALL POINT SOLUTION
C
   10 X5=X1
      Y5=Y1
      E5=E1
      V5=V1
      T5=T1
      P5=P1
      R5=R1
      VELP5=VELP1
      TP5=TP1
      RP5=RP1
      TEP5=TEP1
      PSI5=PSI1
      TANTW=SIN(THETW)/COS(THETW)
      X4=X1
      Y4=Y1
      E4=E1
      V4=V1
      T4=T1
      P4=P1
      R4=R1
      VELP4=VELP1
      TP4=TP1
      RP4=RP1
      TEP4=TEP1
      PSI4=PSI1
      GO TO 40
C
C              REDEFINE DATA TRANSFERRED THROUGH CALL STATEMENT FOR
C              CENTERLINE SOLUTION
C
   20 X5=X2
      Y5=Y2
      E5=E2
      V5=V2
      T5=T2
      P5=P2
```

```
      R5=R2
      VELP5=VELP2
      TP5=TP2
      RP5=RP2
      TEP5=TEP2
      PSI5=PSI2
      X4=X2
      Y4=Y2
      E4=E2
      V4=V2
      T4=T2
      P4=P2
      R4=R2
      VELP4=VELP2
      TP4=TP2
      RP4=RP2
      TEP4=TEP2
      PSI4=PSI2
      CALL COEFP (Y1,E1,V1,T1,P1,R1,VELP1,TEP1,RP1,TP1,S1,H,Q1,APU,APV,
     1ARU,ARV,CE,G1,F,FP,GP1,B,C,D)
      S13=S1
      Q13=Q1
      G13=G1
      GP13=GP1
      GO TO (40,90), IB
   40 CALL COEFP (Y2,E2,V2,T2,P2,R2,VELP2,TEP2,RP2,TP2,S,H2,Q2,APU,APV,
     1ARU,ARV,CE,G2,F2,FP2,GP,B,C,D)
      H23=H2
      Q23=Q2
      F23=F2
      FP23=FP2
   90 I=I+1
      GO TO (100,180), IB
C
C              WALL POINT SOLUTION
C
  100 X3A=(Y5-Y2+H23*X2-X5*TANTW)/(H23-TANTW)
      Y3A=Y2+H23*(X3A-X2)
      T3A=THETW
      P3A=(Q23*P2+T2-T3A-F23*(Y3A-Y2)+FP23*(X3A-X2))/Q23
      GO TO 210
C
C
C              CENTERLINE SOLUTION
C
  180 X3A=X1-Y1/S13
      Y3A=0.0
```

```
      T3A=0.0
      P3A=(Q13*P1-T1+G13*Y1-GP13*(X3A-X1))/Q13
  210 CONTINUE
      IF(I) 310,310,330
  310 CALL COEFP (Y5,E5,V5,T5,P5,R5,VELP5,TEP5,RP5,TP5,S,H,Q,APU,APV,
     1ARU5,ARV5,CE,G2,F,FP,GP,B5,C5,D5)
C
C              CALCULATE THE COEFFICIENTS AT POINT 4
C
      CALL COEFP (Y4,E4,V4,T4,P4,R4,VELP4,TEP4,RP4,TP4,S,H,Q,APU4,APV4,
     1ARU,ARV,CE4,G2,F,FP,GP,B,C,D)
      C35=C5
      B35=B5
      D35=D5
      ARU35=ARU5
      ARV35=ARV5
      APU43=APU4
      APV43=APV4
      CE43=CE4
      GO TO 340
  330 B35=(B3+B5)*0.5
      C35=(C3+C5)*0.5
      D35=0.5*(D3+D5)
      ARU35=0.5*(ARU3+ARU5)
      ARV35=0.5*(ARV3+ARV5)
      APU43=0.5*(APU4+APU3)
      APV43=0.5*(APV4+APV3)
      CE43=0.5*(CE4+CE3)
  340 V3A=V5+(P5-P3A-ARU35*(X3A-X5)-ARV35*(Y3A-Y5))/B35
      R3A=R5+(P3A-P5)/C35-D35*(X3A-X5)
      TEMP=P3A/(R*R3A)
      SOS=CONST*SQRT(TEMP)
      E3A=V3A/SOS
      UP3=VELP4*COS(TP4)+APU43*(X3A-X4)
      VP3=VELP4*SIN(TP4)+APV43*(X3A-X4)
      VELP3=SQRT(UP3**2+VP3**2)
      YDOT3=VP3/UP3
      TP3A=ATAN(YDOT3)
      TEP3A=TEP4+CE43*(X3A-X4)
      PSI3A=PSI4
      GO TO (342,344), IB
  342 RP3A=(PELRT*(PSI3A-PSI2)/PI-RP2*VELP2*COS(TP2)*0.5*(Y3A**2-Y2**2)
     1+(X3A-X2)*Y2*RP2*VELP2*SIN(TP2))/(0.5*UP3*(Y3A**2-Y2**2)-Y3A*VELP3
     2*SIN(TP3A)*(X3A-X2))
      GO TO 346
C
  344 RP3A=(PELRT*PSI1/PI-RP1*VELP1*COS(TP1)*0.5*Y1**2+(X1-X3A)*Y1   1*
```

```
   1VELP1*SIN(TP1))/(0.5*UP3*Y1**2)
 346 CONTINUE
     IF(I) 350,350,410
 350 CALL COEFP (Y3A,E3A,V3A,T3A,P3A,R3A,VELP3,TEP3A,RP3A,TP3A,S3,H3,
    1Q3,APU3,APV3,ARU3,ARV3,CE3,G3,F3,FP3,GP3,B3,C3,D3)
     GO TO (390,370), IB
C
C              CENTERLINE APPROXIMATION
C
 370 STY=SIN(T1)/Y1
     D=1.0/SQRT(E3A**2-1.0)
     A=ATAN(D)
     STMA=SIN(T3A-A)
     G3=G3+STY/(E3A*STMA)
     GO TO 400
C
 390 H23=(H2+H3)*0.5
     Q23=(Q2+Q3)*0.5
     F23=(F2+F3)*0.5
     FP23=0.5*(FP2+FP3)
     GO TO (90,400), IB
 400 S13=(S1+S3)*0.5
     Q13=(Q1+Q3)*0.5
     G13=(G1+G3)*0.5
     GP13=0.5*(GP1+GP3)
     GO TO 90
 410 RETURN
     END
```

```
C            SAMPLE DATA CASE
1.4          29.27      0.7
0.0005       2000.0     500.0      0.8972
 3           35         0          1
0.0          0.005485   1.0736     330.31     15.0        1296100.0 187.9731
123.76       15.0       76.7112    298.05     1.0
0.00002      0.0027425  1.0736     330.31     7.5         1296100.0 187.9731
123.76       7.5        76.71128   298.05     0.25
0.00004      0.0        1.0736     330.31     0.0         1296100.0 187.9731
123.76       0.0        76.7112    298.05     0.0
```