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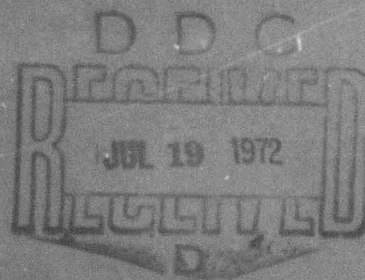
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# INCREASING SALVO KILL PROBABILITY THROUGH AIM POINT PATTERNING

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22448

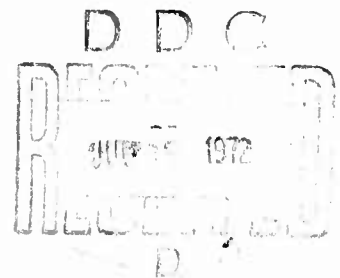
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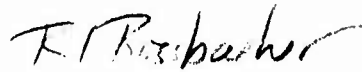
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FOREWORD

The work covered in this Technical Report was performed in the Mathematical Statistics and Systems Simulation Branch (KCM), Operations Research Division, Warfare Analysis Department. The date of completion was 16 May 1972.

The authors wish to acknowledge with appreciation the useful suggestions of Dr. Milton P. Jarnagin and the assistance of Mr. David Wolper who performed the computer programming for the report. This report was reviewed by Mr. Carl M. Hynden, Jr.

Released by:



R. I. ROSSBACHER, Head  
Warfare Analysis Department

## ABSTRACT

An expression is developed for the probability of at least one hit on a target in a salvo of  $N$  rounds under the following assumptions: (1) the target is circular of radius  $a$ ; (2) the rounds are aimed at a pattern of points about the target center; (3) the aiming error, common to all rounds in the salvo, is governed by a circular normal distribution with variance  $\sigma_1^2$ ; (4) the round-to-round or random errors, separate and independent for each round in the salvo, are governed by a circular normal distribution with variance  $\sigma_2^2$ ; and (5) there is independence between the aiming error and the random errors. Restricting the pattern of aim points to equidistant points on a circle centered at the target center, the expression is used to obtain pattern radii which yield maximum probability for a variety of values of the parameters  $\bar{a} = a/\sigma_2$ ,  $\bar{\sigma}_1 = \sigma_1/\sigma_2$ , and  $N$ . A comparison of these maximum probabilities with those obtained by aiming all rounds at the target center shows that a substantial increase in probability can be achieved through aim point patterning.

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## I. INTRODUCTION

The concept of aim point patterning is not new; it has been recognized for some time as a potential source of salvo kill probability increase. To fully understand the concept it will be instructive to first consider the case where patterning is not employed. Suppose one has a target of radius "a" with center at the origin of the Cartesian coordinate system. Suppose further that a salvo of N shots or rounds is aimed at the target center and that the error in the mean impact point of the salvo from the target center, represented by the vector (u,v), is distributed according to a circular normal distribution with variance  $\sigma_1^2$ . Also assume that the errors in the impact points of the N shots or rounds from (u,v), represented by the vectors  $(z_i, w_i)$ , are distributed separately and independently according to a circular normal distribution with variance  $\sigma_2^2$ . The former error is sometimes referred to as the "aiming" or "bias" error and is common to all rounds in the salvo. For a particular salvo, it results in a definite bias in some direction and of some magnitude, but the direction and magnitude of this error vary from salvo to salvo. For example, if a salvo is fired at map coordinates from a naval vessel, this error could be due to navigational and/or geodetic inaccuracies. The latter error, assumed independent of the first, is sometimes referred to as the "round-to-round" or "random" error as it varies from round to round within the salvo. For example, it could be due to ballistic characteristics peculiar to individual rounds.

A typical salvo configuration is shown in Figure 1 for N=4 where the circular dot represents the mean impact point (drawn from a circular normal distribution with mean vector (0,0) and variance  $\sigma_1^2$ ) and the crosses represent the individual round impact points (drawn independently from a circular normal distribution with mean vector (u,v) and variance  $\sigma_2^2$ ).

One measure of weapons systems performance is the probability that at least one round in the salvo hits the target, i.e., falls inside the circle of radius "a." With the assumption that a target hit represents a target kill (damage to specified degree), the probability that at least one round in the salvo falls inside the target radius is sometimes referred to as the salvo kill probability. This is the term which will be applied to such probabilities throughout this report.

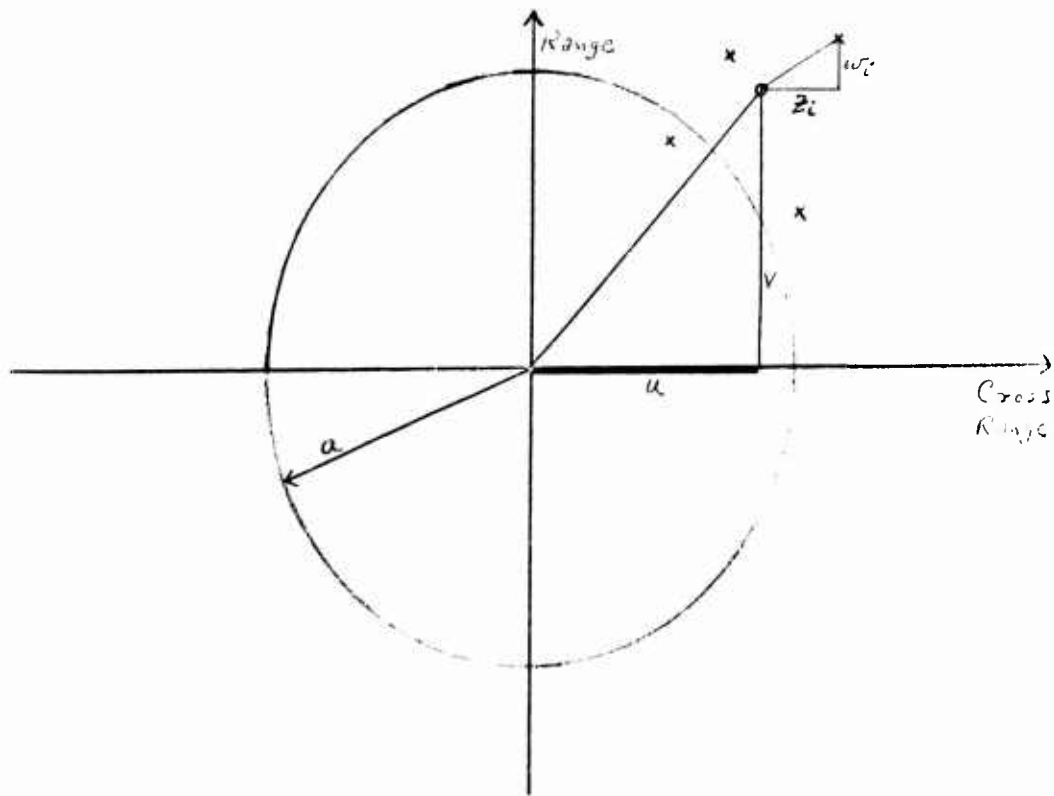


Figure 1.

Under the assumptions stated above, the formulation for salvo kill probability has been documented by Jarnagin (reference 2) and is shown in (1) below as

$$A(a, \sigma_1, \sigma_2, N) = 1 - \frac{1}{\sigma_1^2} \int_0^{\infty} [1 - P(a/\sigma_2, r/\sigma_2)]^N e^{-r^2/(2\sigma_1^2)} r dr \quad (1)$$

where  $P(R, d)$  represents the well known circular coverage function which is the integral of a circular normal distribution of unit standard deviation over a circle of radius  $R$  offset from the origin by a distance  $d$ . Such probabilities were referred to as axisymmetric kill probabilities (AKP in brief) by Jarnagin due to the circular symmetry about the origin. An extensive table of such probabilities has been documented by Thomas (reference 3) as a function of  $T = \sigma_1/\sigma_2$ ,  $R = a/\sigma_2$ , and  $N$ .

With this brief discussion in mind, consider now the case where all rounds in the salvo are not necessarily aimed at the target center but are rather aimed at a pattern of points  $(h_i, k_i)$ ,  $i=1, \dots, N$ , about the target center. The assumptions concerning the distribution of errors are the same as before, but the errors are now distributed about different points. For example, the aiming error  $(u, v)$  is still assumed circular normal with variance  $\sigma_1^2$ , but this error is now distributed about the aim points  $(h_i, k_i)$  vice the target center or origin. Also, the random or round-to-round errors  $(z_i, w_i)$  are still assumed to be separately and independently distributed according to a circular normal distribution with variance  $\sigma_2^2$  but these errors are now distributed about  $(h_i+u, k_i+v)$  vice  $(u, v)$ .

A typical salvo configuration is shown in Figure 2 for  $N=4$ . In this figure the points  $(h_i, k_i)$  represent the four aim points,  $(u, v)$  the common aiming error (circular normal about the aim points with variance  $\sigma_1^2$ ),  $(z_i, w_i)$  the separate and independent random errors (circular normal about  $(h_i+u, k_i+v)$  with variance  $\sigma_2^2$ ), and the crosses represent the actual hit points.

The problem is to (1) develop an expression for the probability of at least one hit (salvo kill probability) under the above assumptions, (2) determine a pattern of aim points which maximizes this probability, and (3) compare this maximum probability with that obtained in the axisymmetric cases where all rounds are aimed at the target center. Problem (1) is solved in this report and problem (2) is solved for the restricted case where the pattern of aim points is equidistant on a circle centered at the target center. The comparison, problem (3), shows that a substantial increase in salvo kill probability can be obtained by the careful patterning of aim points.

## II. THEORETICAL DEVELOPMENT

An expression is first needed for the salvo kill probability under the assumptions in the last section where aim point patterning is employed. To this end, consider defining the discrete random variable  $z$  to be the number of times the target receives at least one hit in a salvo of size  $N$ . Clearly,  $z$  can take on only the values 0 or 1, and the conditional density of  $z$ , given an aiming error  $(u, v)$ , can be expressed as

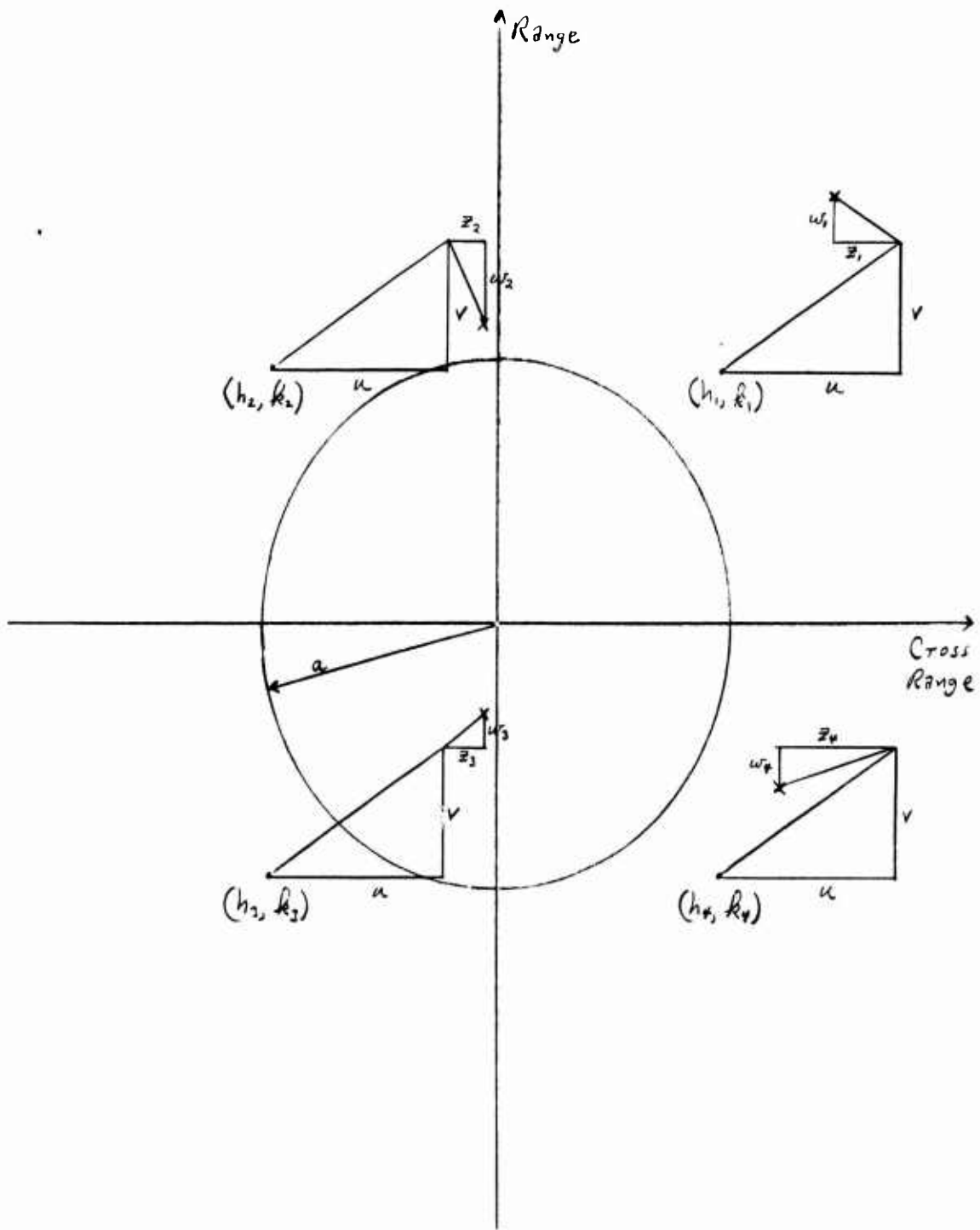


Figure 2.

$$f(z|u,v) = \left\{ 1 - \prod_{i=1}^N \left[ 1 - P\left( \frac{a}{\sigma_2}, \frac{\sqrt{(h_i+u)^2 + (k_i+v)^2}}{\sigma_2} \right) \right] \right\}^z .$$

$$\left\{ \prod_{i=1}^N \left[ 1 - P\left( \frac{a}{\sigma_2}, \frac{\sqrt{(h_i+u)^2 + (k_i+v)^2}}{\sigma_2} \right) \right] \right\}^{1-z} \quad (2)$$

for  $z=0,1$ ;  $-\infty < u < \infty$ ;  $-\infty < v < \infty$ . The rationale for equation (2) can be explained by first considering the expression for  $z=1$  which represents the conditional probability of at least one hit. Within this expression,  $P(\cdot, \cdot)$  represents the circular coverage function which in this case is the integral of a circular normal distribution with variance  $\sigma_2^2$  over a circle of radius "a," offset from the origin

by a distance  $\sqrt{(h_i+u)^2 + (k_i+v)^2}$ . This, of course, is the probability that the  $i$ th round, aimed at the  $i$ th pattern point, hits the

target, given an aiming error  $(u,v)$ . The product  $\prod_{i=1}^N [1-P(\cdot, \cdot)]$  then

represents the conditional probability that all  $N$  rounds miss the target and one minus this expression is the conditional probability that at least one round hits the target. Similar reasoning leads to a validation of (2) for  $z=0$  which is simply the conditional probability that all  $N$  rounds miss the target.

The joint density of all three variables, say  $g(z,u,v)$ , can now be obtained by multiplying  $f(z|u,v)$  by the joint density of  $(u,v)$ , say  $h(u,v)$ . By assumption

$$h(u,v) = \frac{1}{2\pi\sigma_1^2} e^{-(u^2+v^2)/(2\sigma_1^2)}, \quad -\infty < u, v < \infty \quad (3)$$

so that

$$g(z,u,v) = f(z|u,v) h(u,v) \quad (4)$$

for  $z=0, 1, -\infty < u < \infty, -\infty < v < \infty$  where  $f(z|u,v)$  and  $h(u,v)$  are given by expressions (2) and (3), respectively. The marginal or unconditional density of  $z$ , say  $p(z)$ , can now be found by integrating  $g(z,u,v)$  over the entire range of both  $u$  and  $v$ , that is

$$p(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(z,u,v) du dv \quad (5)$$

However, since only the probability of at least one hit,  $p(1)$ , is of interest, it is found to be

$$p(1) = 1 -$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_1^2} e^{-\frac{(u^2+v^2)}{(2\sigma_1^2)}} \prod_{i=1}^N \left[ 1 - P\left(\frac{a}{\sigma_2}, \frac{\sqrt{(h_i+u)^2 + (k_i+v)^2}}{\sigma_2}\right) \right] du dv \quad (6)$$

where, as aforementioned,  $P$  represents the circular coverage function. One notes, of course, that if  $h_i=k_i=0, i=1, \dots, N$ , then equation (6) degenerates to a form which is equivalent to equation (1), that is, the axisymmetric form.

Unfortunately, the double integral in expression (6) cannot be expressed in closed form. However, after finding finite limits of integration which will result in negligible error, it can be evaluated numerically for any specified values of  $a, \sigma_1, \sigma_2, N$ , and  $(h_i, k_i), i=1, \dots, N$ . It may be well to mention that the circular coverage function,  $P$ , in the integral in (6) is a double integral itself so that the evaluation of (6) numerically actually involves quadruple integration. This task is eased by use of an efficient algorithm for computing  $P$  which has been developed by DiDonato and Jarnagin (reference 1).

Let us now consider the problem of finding suitable finite limits of integration. Let  $E$  represent the error in integrating over a circle of radius  $c\sigma_1$ , centered at the origin. With  $R$  representing the neglected region of integration,

$$E = \frac{1}{2\pi\sigma_1^2} \iint_R e^{-(u^2+v^2)/(2\sigma_1^2)} \prod_{i=1}^N [1-P(\cdot, \cdot)] du dv$$

$$\leq \frac{1}{2\pi\sigma_1^2} \iint_R e^{-(u^2+v^2)/(2\sigma_1^2)} du dv \quad (7)$$

since  $\prod_{i=1}^N [1-P(\cdot, \cdot)]$  is less than or equal to one for all  $(u,v)$ .

The right hand side of (7) can now be expressed in polar coordinates as

$$\frac{1}{2\pi\sigma_1^2} \int_0^{2\pi} \int_{c\sigma_1}^{\infty} e^{-r^2/(2\sigma_1^2)} r dr d\theta$$

so that

$$E \leq \frac{1}{\sigma_1^2} \int_{c\sigma_1}^{\infty} e^{-r^2/(2\sigma_1^2)} r dr = e^{-c^2/2} \quad (8)$$

For example, if the integration is performed over a circle of radius  $4.5 \sigma_1$ , the error in neglecting the region outside the circle would be less than .000040. The results in this report are based on integration over the square  $-4.5\sigma_1 < u < 4.5\sigma_1$ ,  $-4.5\sigma_1 < v < 4.5\sigma_1$  which results in even smaller errors.

Consider now the parameters which must be specified in order to evaluate  $p(1)$ . One sees that

$$p(1) = B(a, \sigma_1, \sigma_2, N, (h_i, k_i), i=1, \dots, N) \quad (9)$$

that is,  $p(1)$  is a function of  $N+4$  parameters. This number of parameters can be reduced by one by normalization. That is, if in  $p(1)$ , one makes the transformation from  $(u, v)$  to  $(y_1, y_2)$  where  $y_1 = u/\sigma_2$  and  $y_2 = v/\sigma_2$ , one finds that  $p(1)$  is a function of  $N+3$  normalized parameters (normalized with respect to  $\sigma_2$ ) as shown below:

$$p(1) = B(a/\sigma_2, \sigma_1/\sigma_2, 1, N, (h_i/\sigma_2, k_i/\sigma_2), i=1, \dots, N) \quad (10)$$

This, of course, is highly desirable as it not only reduces the number of parameters but also places the remaining parameters on nondimensional scales. However, in order to solve problem (2) by numerical methods, one has to take this a step further and restrict the  $N$  pattern points to lie in some geometrical pattern. For example, if one restricts the pattern points to lie at equidistant intervals on a circle of radius  $PR$  (pattern radius) with center at the target center, then the entire set of  $N$  points can be specified with the parameters  $PR$  and  $N$ . This is easily seen since in this case

$$h_i = PR \cos (2\pi i/N)$$

$$k_i = PR \sin (2\pi i/N)$$

so that both  $h_i$  and  $k_i$  are functions of  $i$ , designating the  $i$ th point and  $N$ , the number of rounds in the salvo. With this pattern restriction,  $p(1)$  is a function of only four parameters:

$$\bar{a} = a/\sigma_2$$

$$\bar{\sigma}_1 = \sigma_1/\sigma_2$$

$$N = \text{salvo size}$$

$$\bar{PR} = \text{pattern radius}/\sigma_2$$

(11)

For this restricted pattern, one can now solve problem (2) by evaluating  $p(1)$  numerically for varying  $\overline{PR}$  at selected values of  $\overline{a}$ ,  $\overline{\sigma_1}$  and  $N$ . The results are shown in the next section.

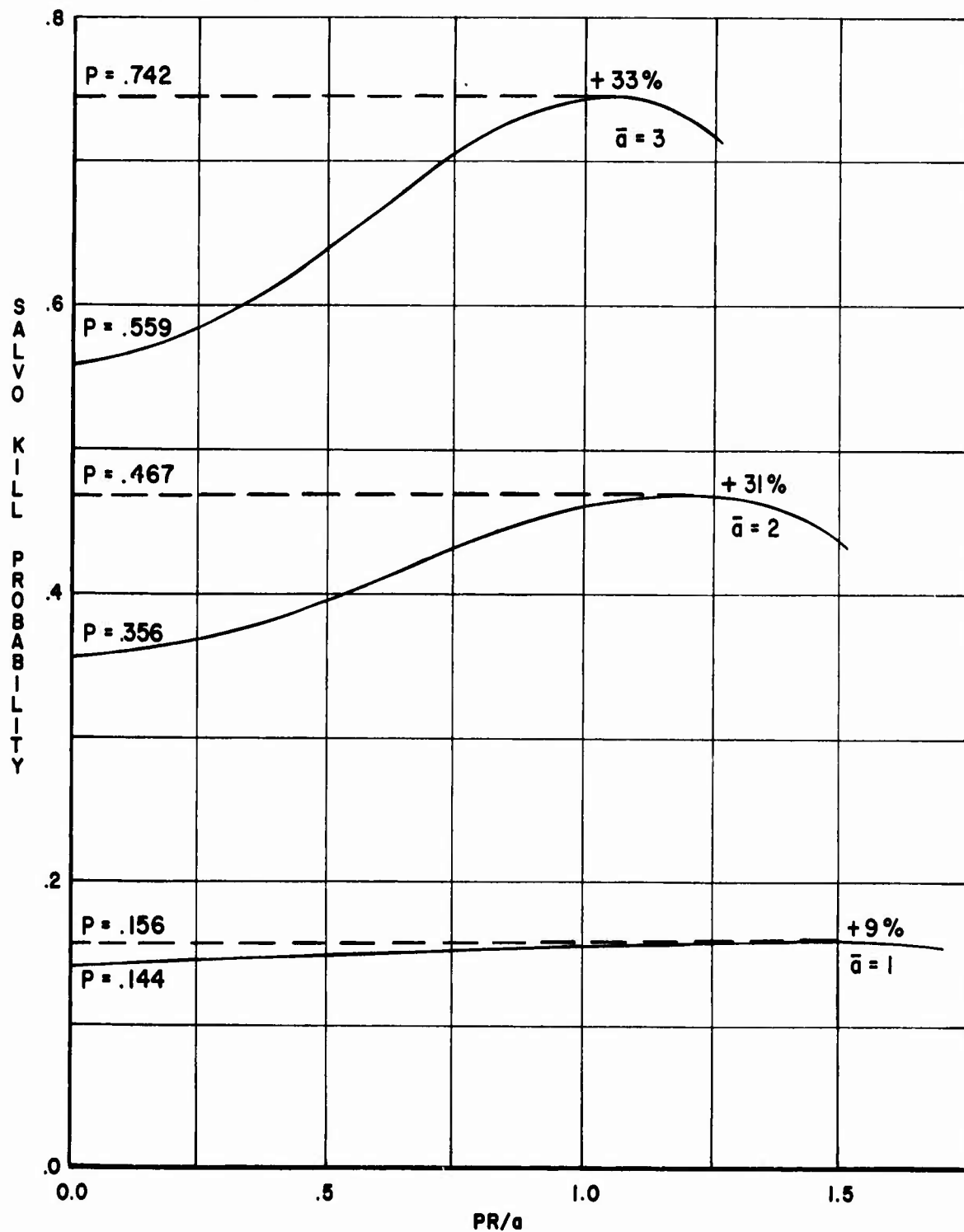
### III. OPTIMUM CIRCULAR PATTERN RADII

Before presenting general results, it will be instructive to consider a specific example. Consider a salvo of size  $N=4$  with  $\overline{\sigma_1} = 3$ , i.e., with an aiming standard deviation three times the random standard deviation. Also consider the three normalized target radii  $\overline{a} = 1, 2, 3$ . For these values of  $N$ ,  $\overline{\sigma_1}$ , and  $\overline{a}$ ,  $p(1)$  was evaluated for  $PR/a = 0(.05)c$  with  $c = 1.75, 1.50$ , and  $1.25$  for  $\overline{a} = 1, 2$ , and  $3$  respectively using the bivariate analog to the parabolic integration rule. The integration tolerance was set to provide the accuracy of three significant digits. The results are shown in Figure 3 which deserves some explanation. First, the ordinate represents the salvo kill probability or  $p(1)$ , which is plotted as a function of  $PR/a$ , the ratio of pattern radius to target radius. Note that  $PR/a = 0$  corresponds to the axisymmetric case, that is, the case where all rounds are aimed at the target center. The curve for  $\overline{a} = a/\sigma_2 = 3$  shows that a pattern radius  $PR = 1.05a$  yields a maximum  $p(1)$  equal to .742. Had all rounds been aimed at the target center, the salvo kill probability would be .559 so that by aiming the rounds equidistantly on a circle of radius slightly larger than the target radius, one can obtain a 33% increase in probability. For  $\overline{a} = 2$ , the maximum  $p(1)$  occurs at  $PR = 1.2a$  and affords a 31% increase in probability (.467 vice .356). Similarly, for  $\overline{a} = 1$ , the maximum  $p(1)$  occurs at  $1.5a$  and affords a 9% increase (.156 vice .144).

These increases in salvo kill probability cannot be fully appreciated until one examines the salvo size necessary to achieve these increased probabilities when aiming all rounds at the target center. Consider the top curve in Figure 3, the one for  $N=4$ ,  $\overline{\sigma_1} = 3$ ,  $\overline{a} = 3$ . As aforementioned,  $PR = 0$  (all rounds aimed at the target center) yields a salvo kill probability of .559 while aiming the four rounds at equidistant intervals on a circle of radius  $PR = 1.05a$  yields a salvo kill probability of .742, an increase of 33%. Now consider the salvo size necessary to achieve this .742 probability without patterning, i.e., with all rounds aimed at the target center. The results are shown in Figure 4 where it is seen that the salvo kill probability increases quite slowly with  $N$ , the salvo size. One sees that a salvo of size 36 is required to obtain a probability as large as .742. In this respect, one sees a decrease of 89% in salvo size through patterning to achieve a probability of at least .742.

FIGURE 3

$N = 4$   $\bar{\sigma}_1 = 3$



Case N = 4,  $\bar{\sigma}_1 = 3$ ,  $\bar{x} = 3$

PR = 0                      PR = 1.05a (optimal)

p(1) = .559                p(1) = .742

Increase in salvo kill probability through patterning = 33%.

Salvo size necessary to achieve p(1) = .742 without patterning:

<u>N</u>	<u>p(1) PR = 0</u>	<u>N</u>	<u>p(1) PR = 0</u>
4	.559	30	.732
8	.631	32	.736
12	.667	34	.740
20	.705	36	.743

Decrease in salvo size through patterning = 89%.

Figure 4.

This latter comparison may appear as a scheme to make a 33% increase in probability appear better than it really is. However, a brief reflection to the practical aspects of the problem reveals that this is not the case. From a practical standpoint, a minimum salvo kill probability is usually specified and then the number of rounds necessary to achieve it is determined. Hence, a percentage decrease in salvo size through patterning is a more meaningful comparison than is a percentage increase in salvo kill probability. Even so, for ease in computation and display of results, latter comparisons are based on a percentage increase in probability vice a percentage decrease in salvo size. The reader can make the latter comparisons by using the aforementioned tables by Thomas (reference 3) for the axisymmetric case.

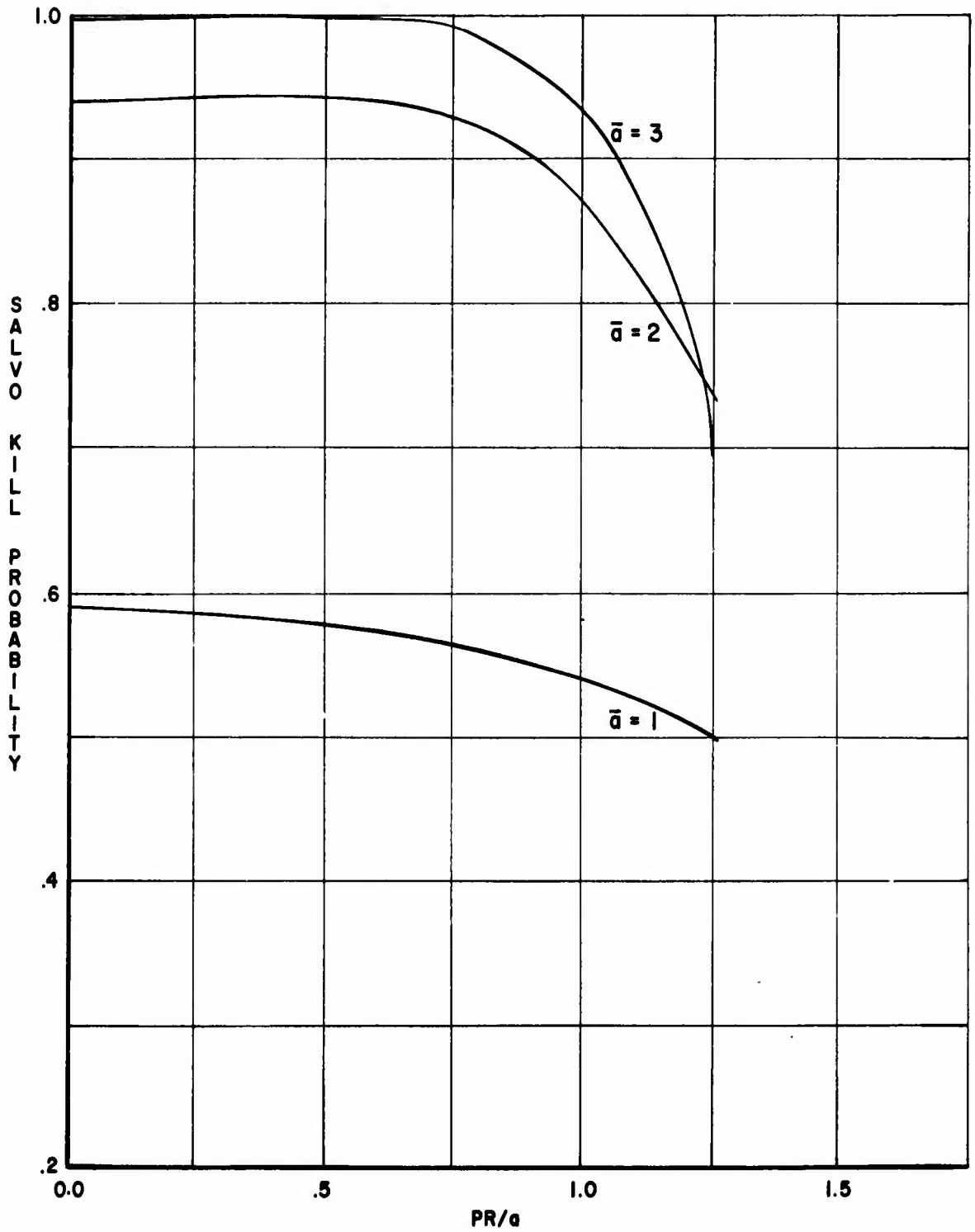
The results above show that a substantial savings in rounds can be achieved through proper aim point patterning. However, it cannot be overemphasized that patterning cannot be employed indiscriminately; one must know the pattern radius which affords the maximum probability and this pattern radius could very well be zero. To illustrate this, consider the example with  $N = 4$ ,  $\overline{\sigma}_1 = 1$ , and  $\overline{a} = 1, 2, 3$ . This is the same as the last example except that here the two standard deviations are equal while in the last example, the aiming standard deviation was three times the random standard deviation. Figure 5 shows plots of salvo kill probability as a function of  $PR/a$  as in Figure 3 for each  $\overline{a}$ . Here, however, one sees that the pattern radius which yields maximum probability is zero for the target of radius  $\overline{a} = 1$ . One also sees that if an arbitrary pattern of radius  $PR > .75a$  were used for targets  $\overline{a} = 2$  and  $\overline{a} = 3$ , a substantial decrease in probability would be experienced.

As an aid to improve weapons systems effectiveness, optimal pattern radii have been developed for  $\overline{a} = .5, 1, 2, 3, 5$ ;  $N = 4, 8, 12$ ;  $0 \leq \overline{\sigma}_1 \leq 5$ . The results are shown in Figures 6 thru 10 where the optimal pattern radii (expressed in units of "a") are shown as a function of  $\overline{\sigma}_1$  for each  $N$  and each  $\overline{a}$ . These curves were obtained by the brute force technique of evaluating  $p(1)$  for each  $N$ ,  $\overline{\sigma}_1$ , and  $\overline{a}$  at incremental values of  $PR/a$  (increments of .05) until the maximum probability was reached. These optimal pattern radii were then plotted and smoothly connected as a function of  $\overline{\sigma}_1$ .

As an example of employing these curves, consider the following. Suppose one is employing a weapon system with  $\sigma_1 = 40$  feet and  $\sigma_2 = 10$  feet against a circular target of radius 20 feet. What are the optimal pattern radii for  $N = 4, 8$ , and 12? For this case,  $\overline{a} = 20/10 = 2$  so Figure 8 is appropriate. One now reads the ordinate from these curves for  $\overline{\sigma}_1 = 40/10 = 4$  and obtains the following results:

FIGURE 5

$N = 4$     $\bar{\sigma}_1 = 1$



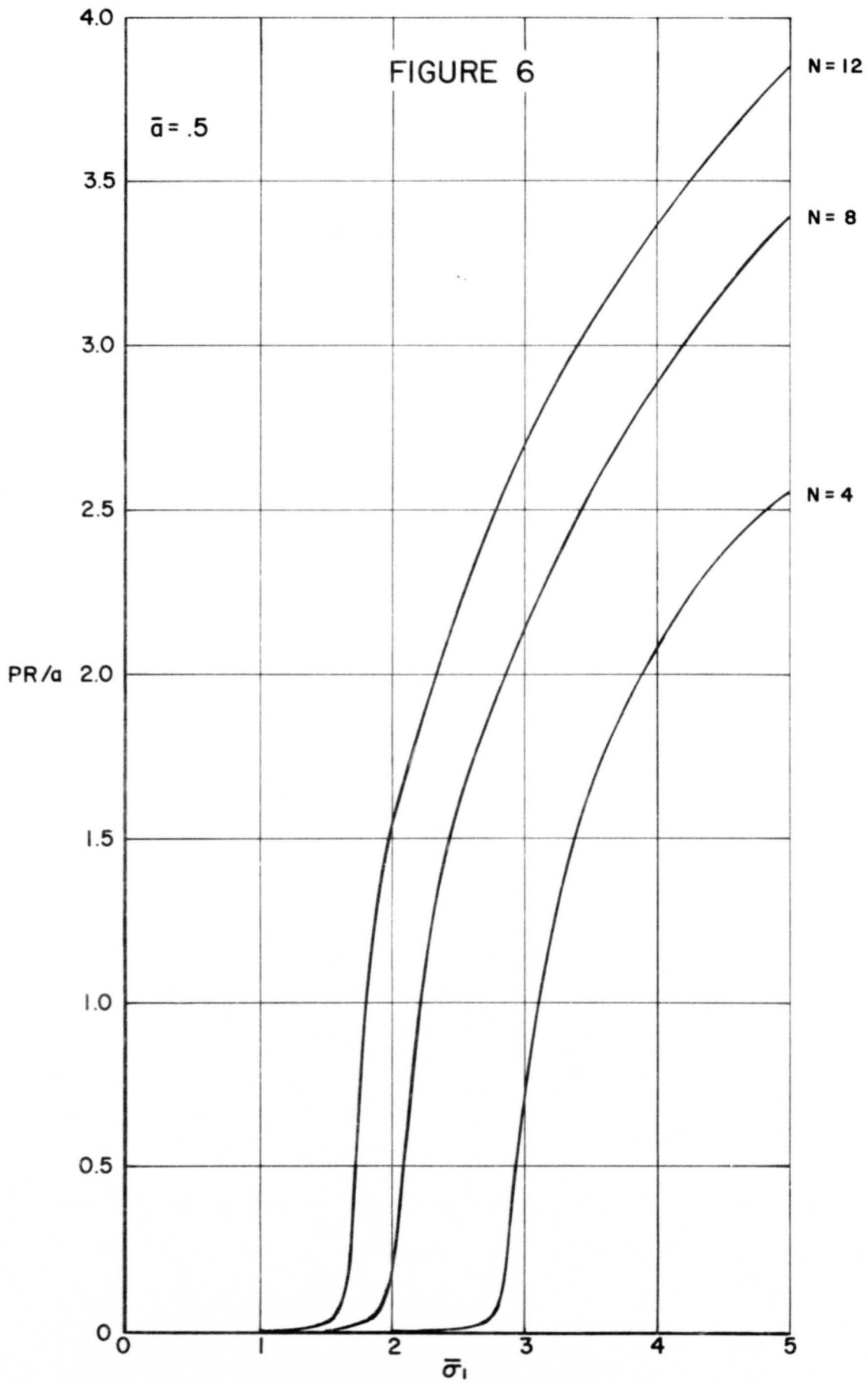


FIGURE 7

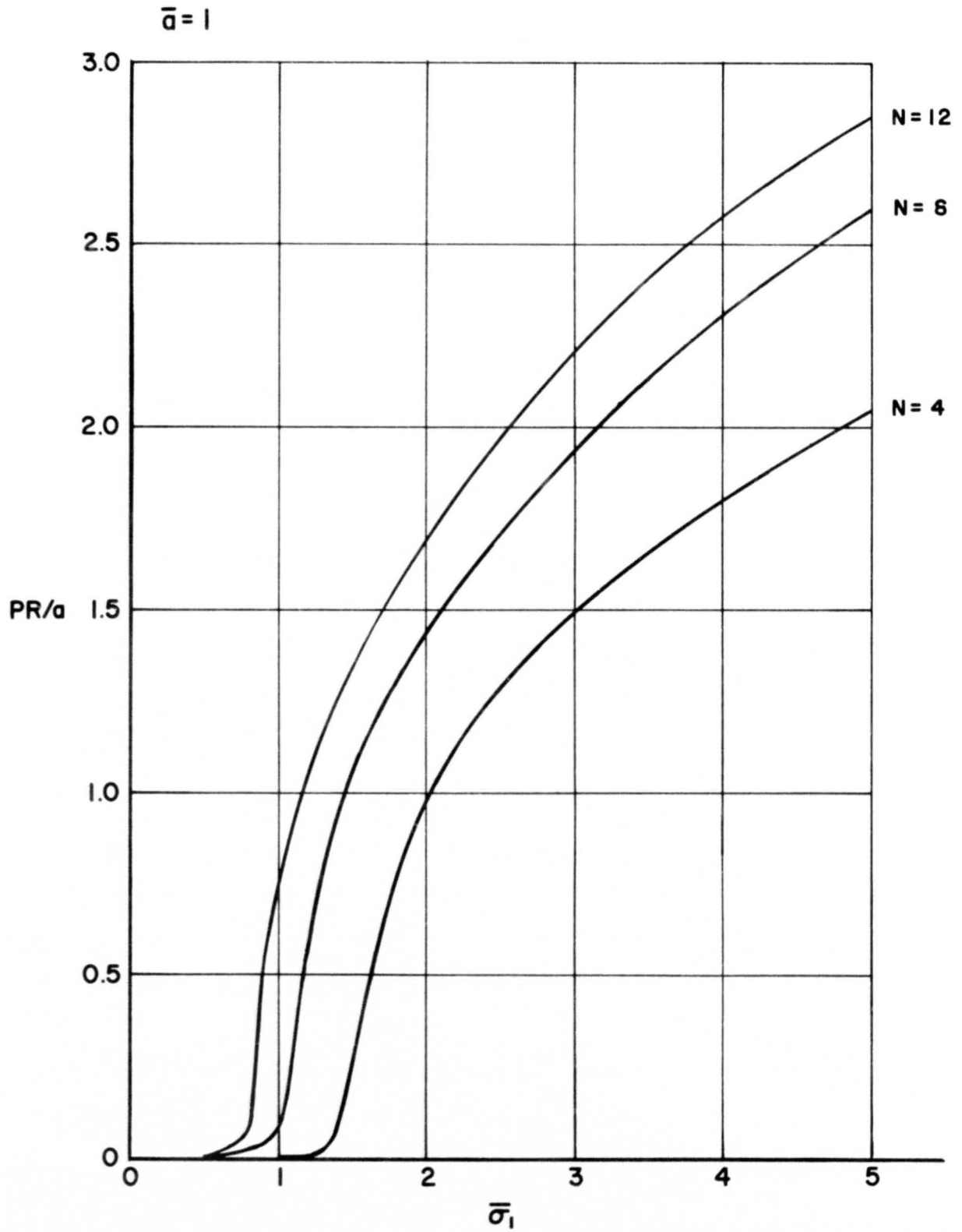


FIGURE 8

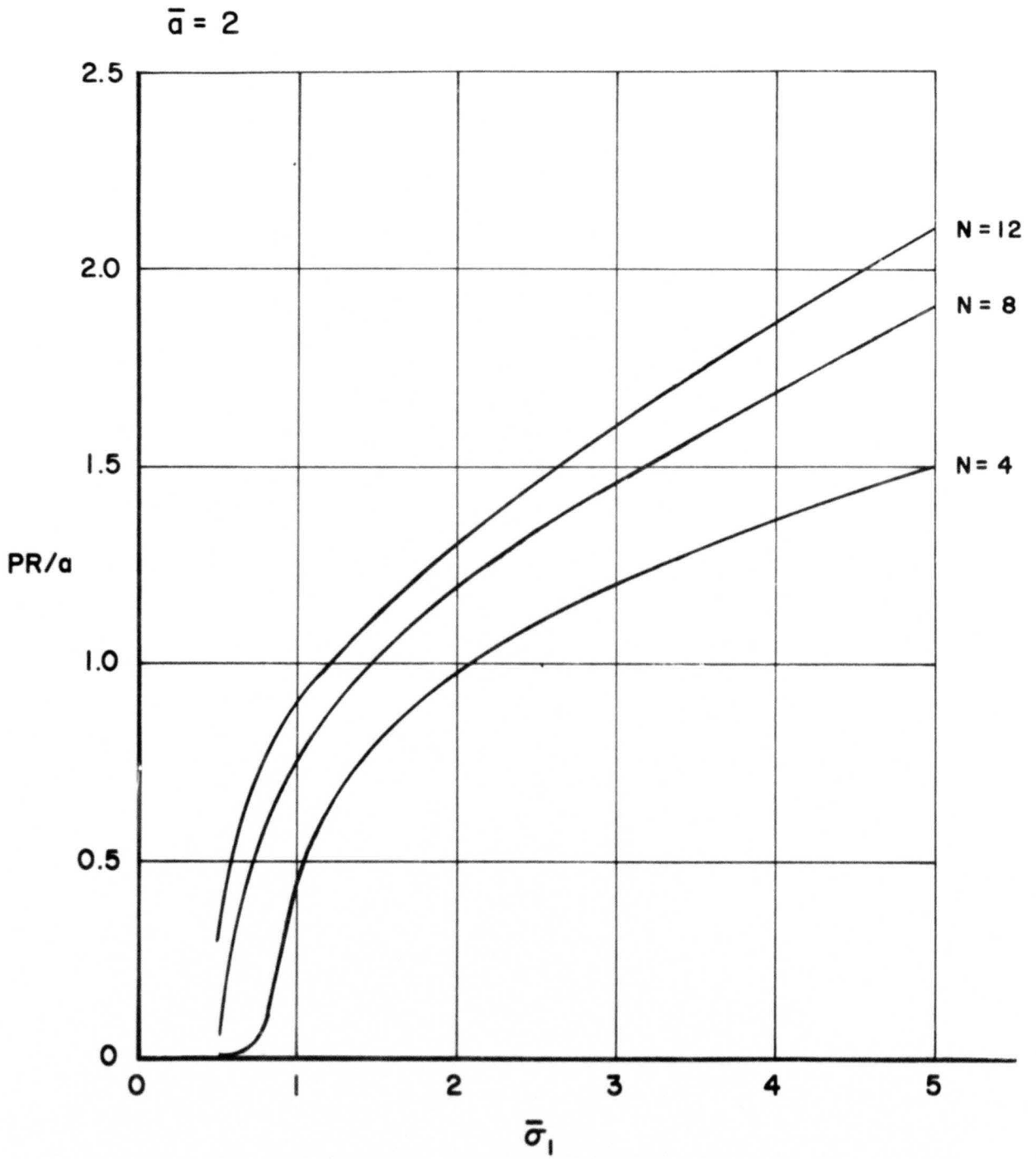


FIGURE 9

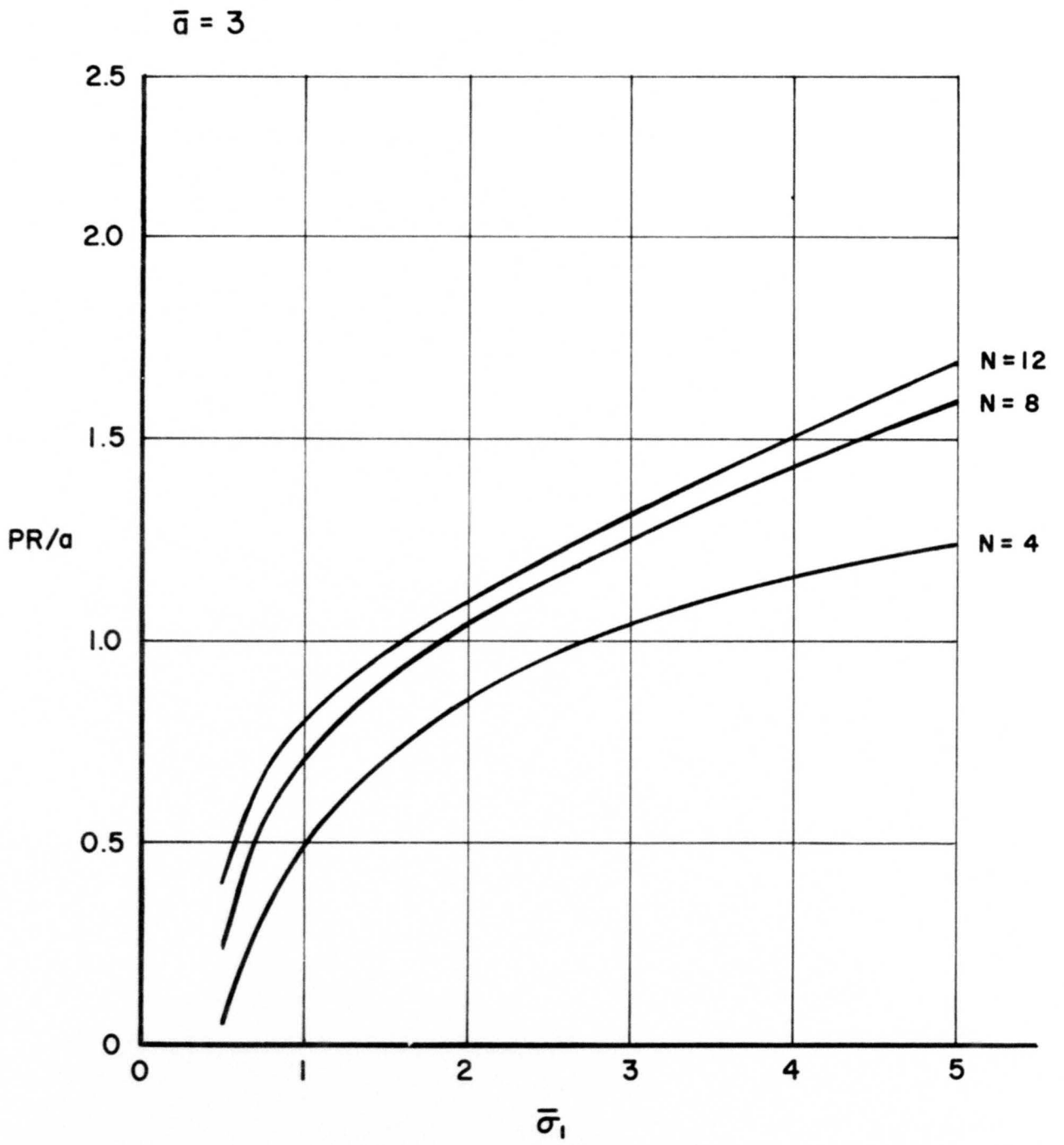
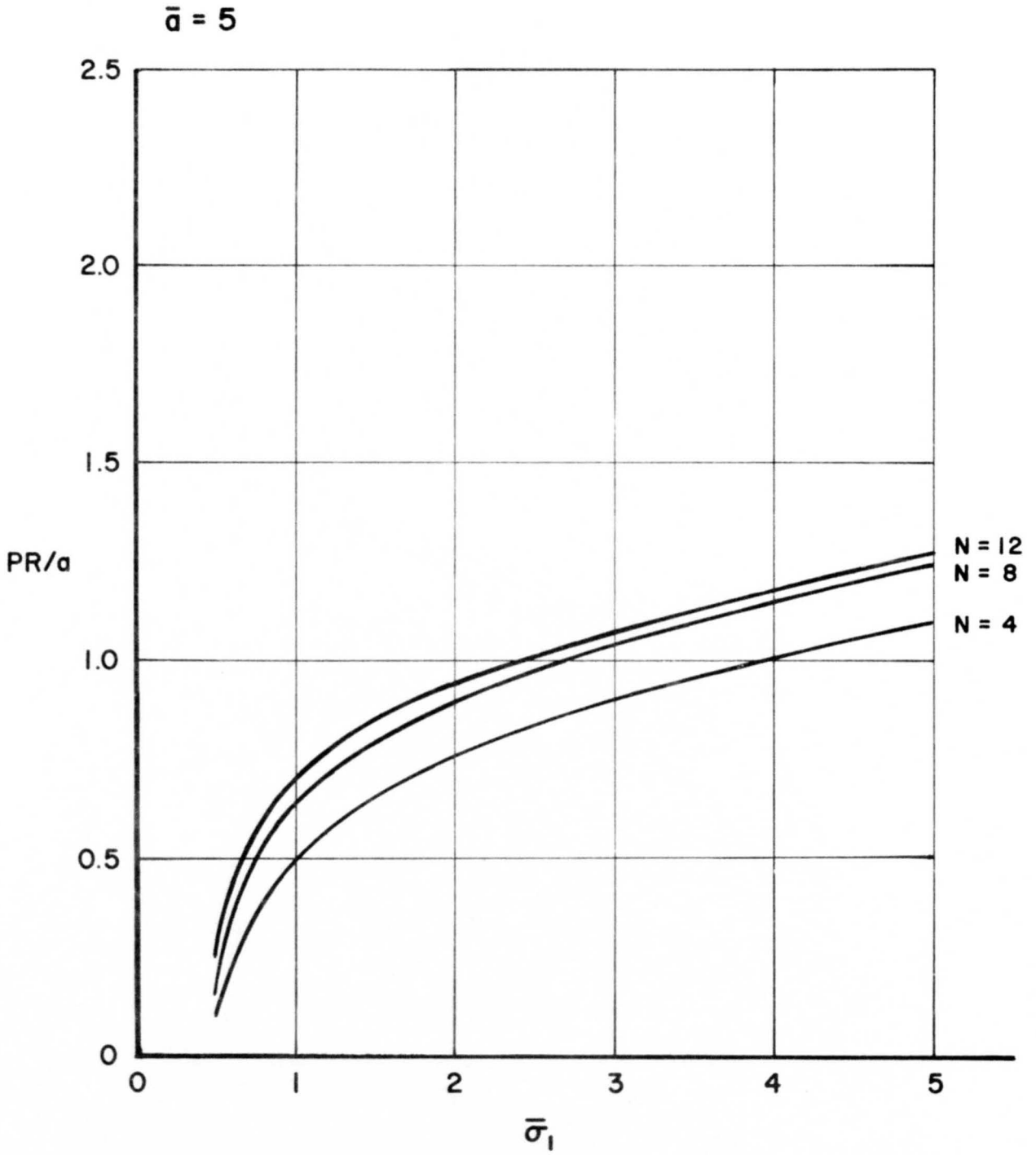


FIGURE 10



<u>N</u>	<u>Optimal Pattern Radius</u>
4	1.36a = 27.2 feet
8	1.68a = 33.6 feet
12	1.86a = 37.2 feet

Hence for a salvo of size  $N = 4$ , the maximum kill probability (with respect to circular patterns) can be obtained by aiming the rounds equidistantly on a circle of radius 27.2 feet centered at the target center. For  $N = 8$ , the optimal pattern radius is 33.6 feet, etc.

The above leads to the question of what increases are afforded by using these optimal pattern radii. This can be answered by referring to Figures 11a thru 15c which show the salvo kill probabilities obtained by using the optimal pattern radius and by using the target center as the aim point for all rounds in the salvo. In these figures, both salvo kill probabilities are plotted as a function of  $\bar{\sigma}_1$  for  $\bar{a} = .5, 1, 2, 3, 5$  and  $N = 4, 8, 12$ . For the case at hand,  $\bar{a} = 2$  so one refers to Figure 13. Reading the ordinate in Figure 13a ( $N=4$ ) for  $\bar{\sigma}_1 = 4$ , one finds the corresponding probabilities to be .32 and .22 for the patterned and centered aim points, respectively. From Figure 13b ( $N=8$ ), one finds these probabilities to be .48 and .28, and from Figure 13c ( $N=12$ ), he finds these to be .56 and .31. These are set out below along with the corresponding percentage increases.

<u>N</u>	<u>Pattern Probability</u>	<u>Axisymmetric Probability</u>	<u>Percent Increase</u>
4	.32	.22	45
8	.48	.28	71
12	.56	.31	81

Here one sees that he can achieve a higher kill probability with  $N=4$  rounds by aiming at equidistant intervals on a circle of radius 27.2 feet (centered at the target center) than he can achieve by aiming  $N=12$  rounds at the center of the target! In fact, to achieve a kill probability at least as great as .32 without patterning, one would have to use  $N=16$  rounds (.3303 from reference 3). This represents a decrease of 75% in salvo size through aim point patterning.

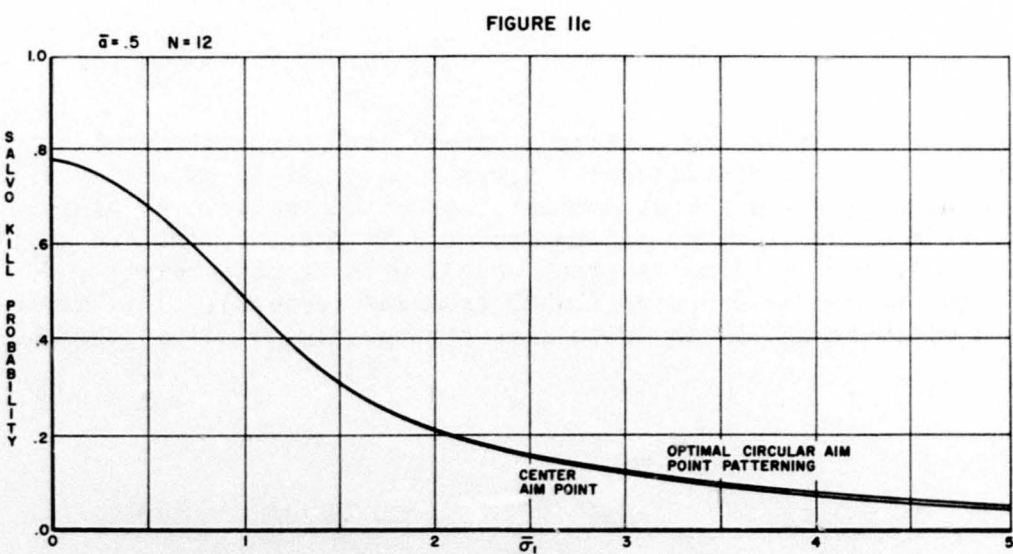
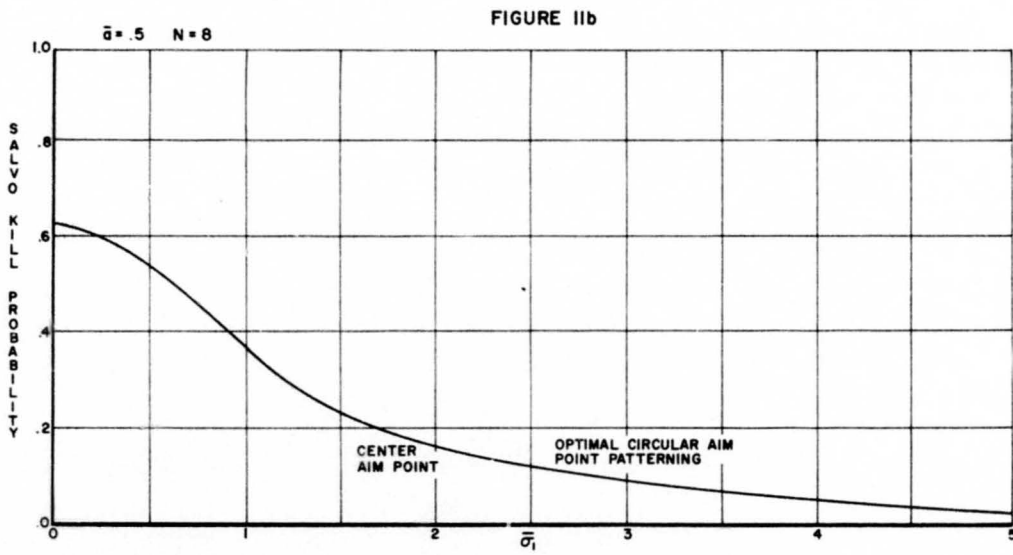
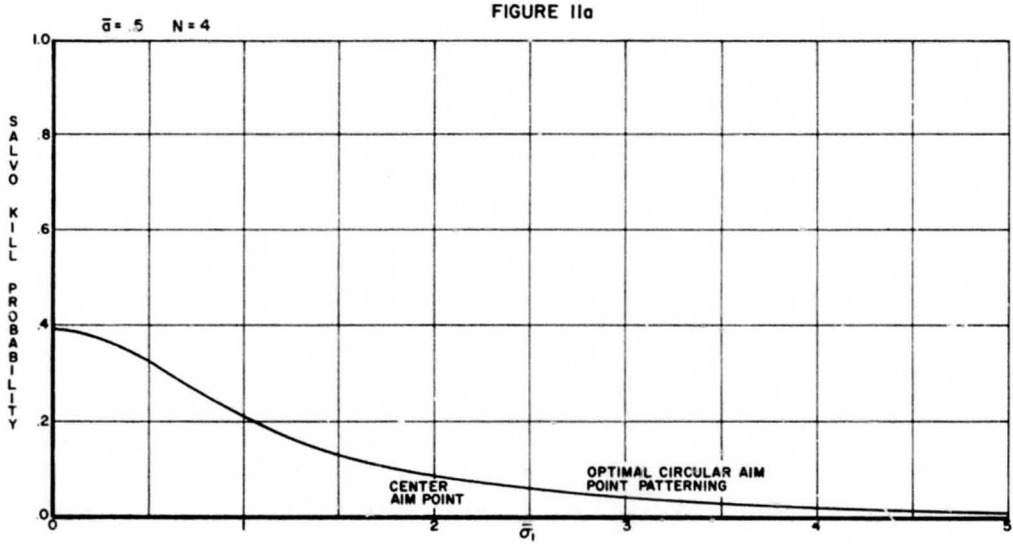


FIGURE 12a

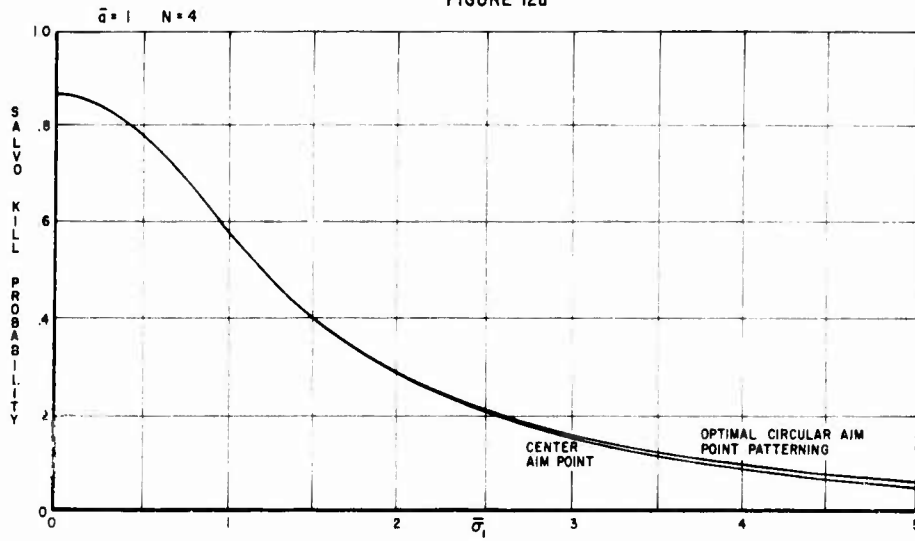


FIGURE 12b

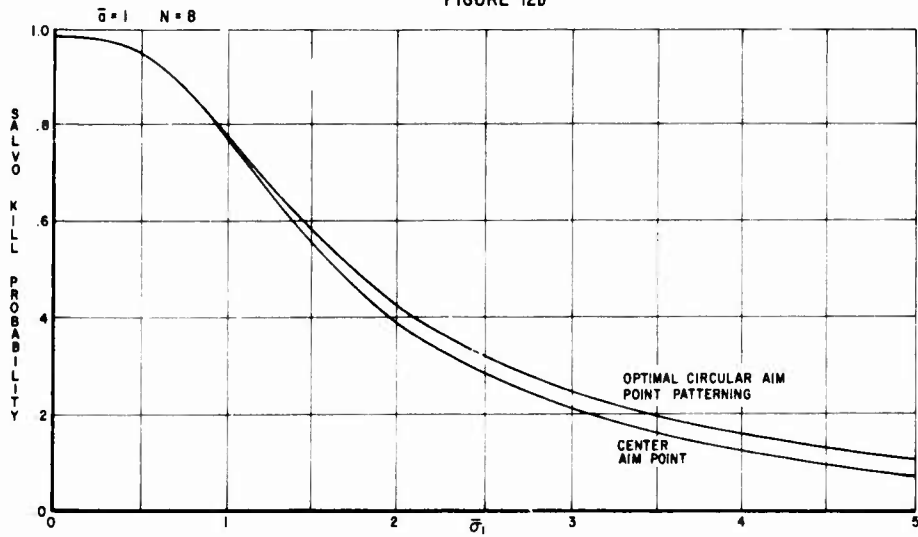


FIGURE 12c

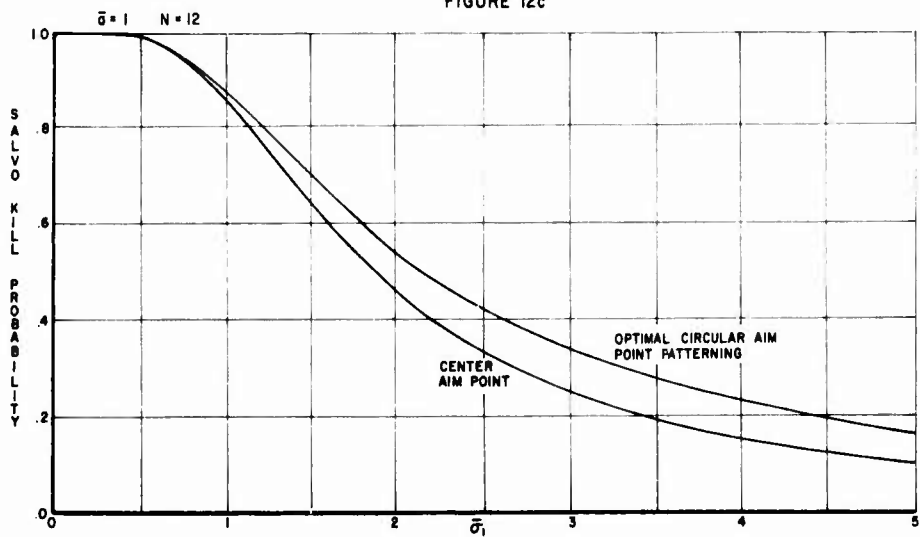


FIGURE 13a

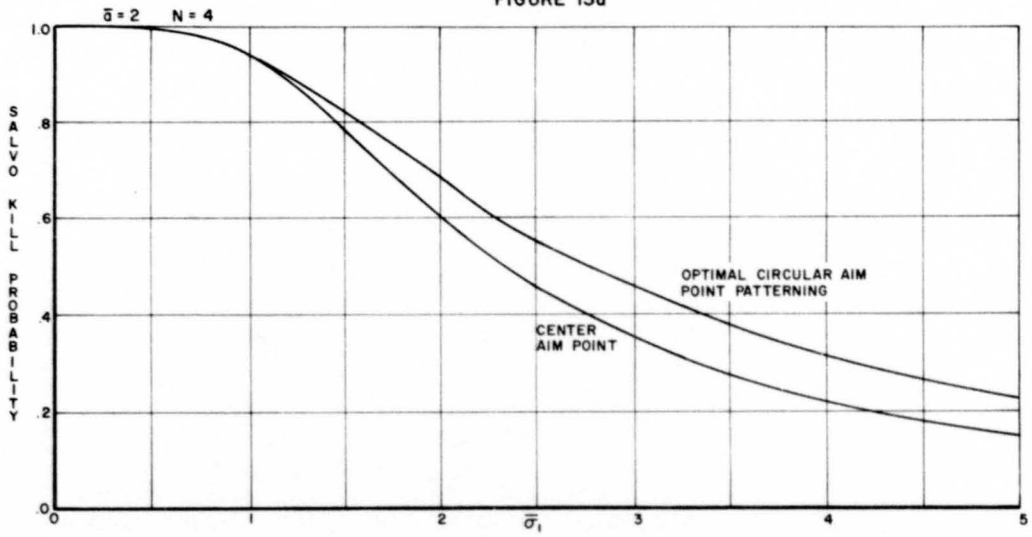


FIGURE 13b

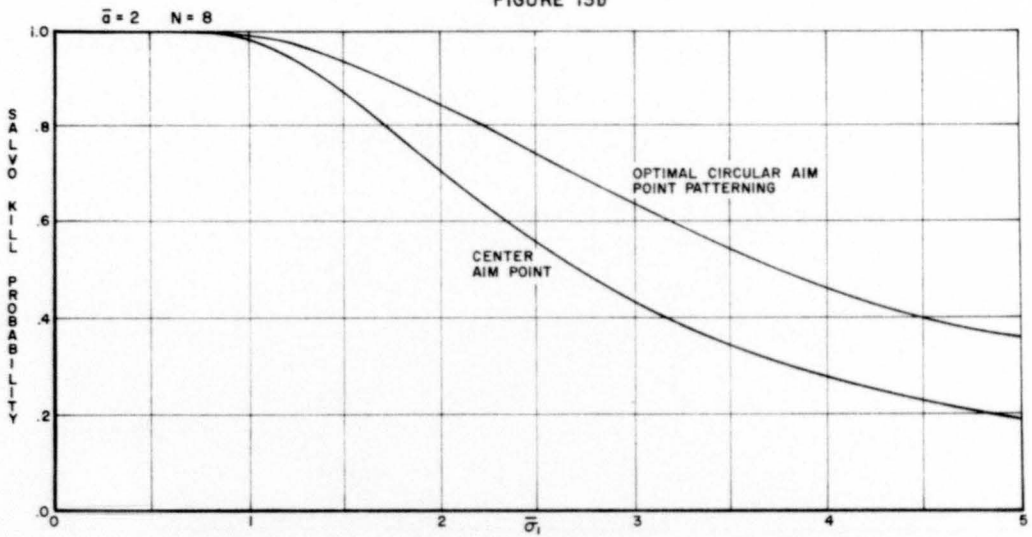
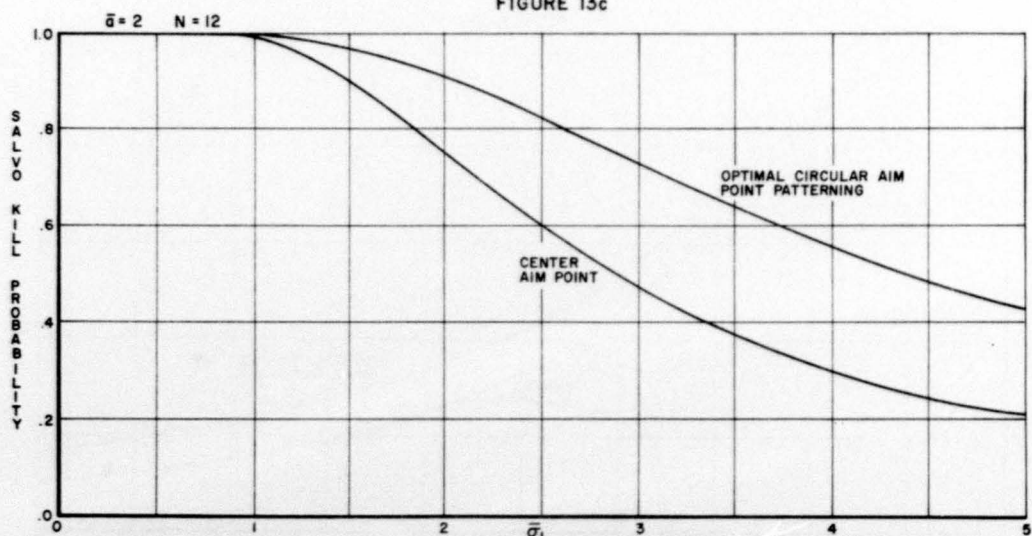


FIGURE 13c



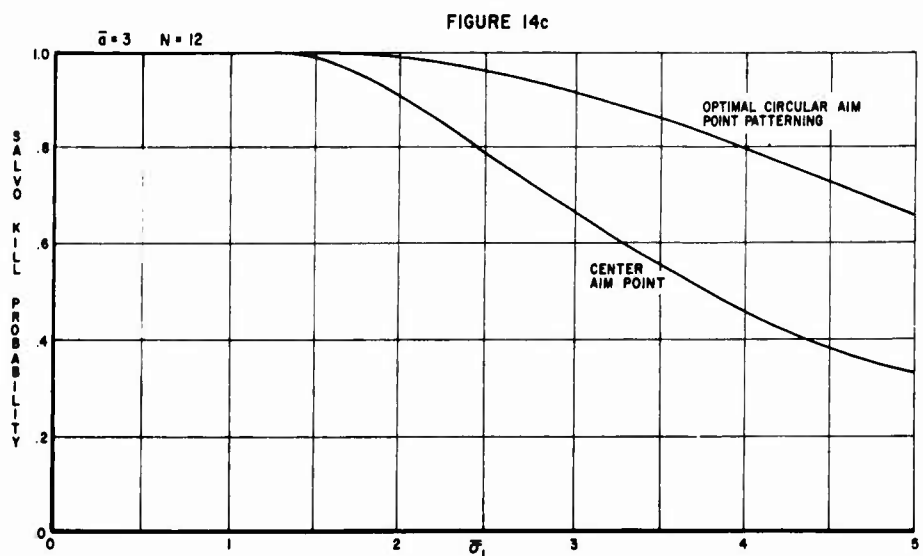
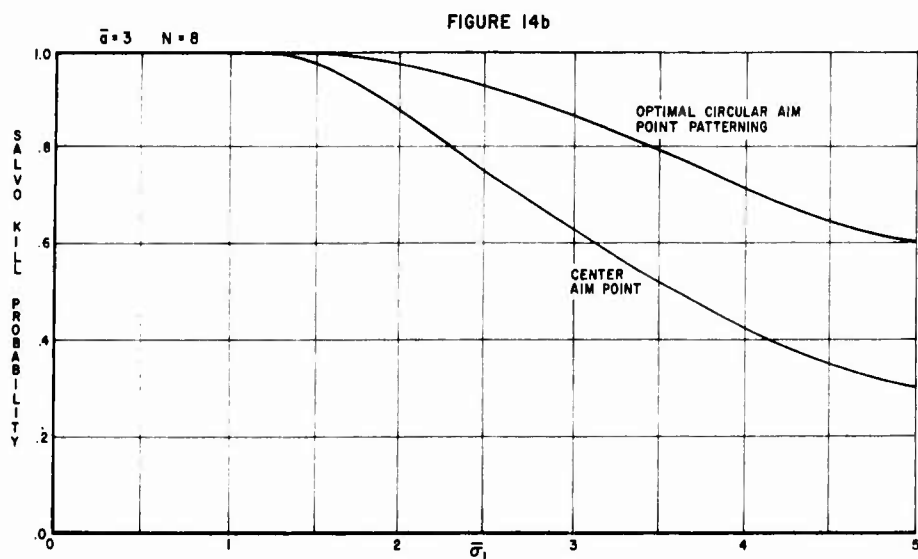
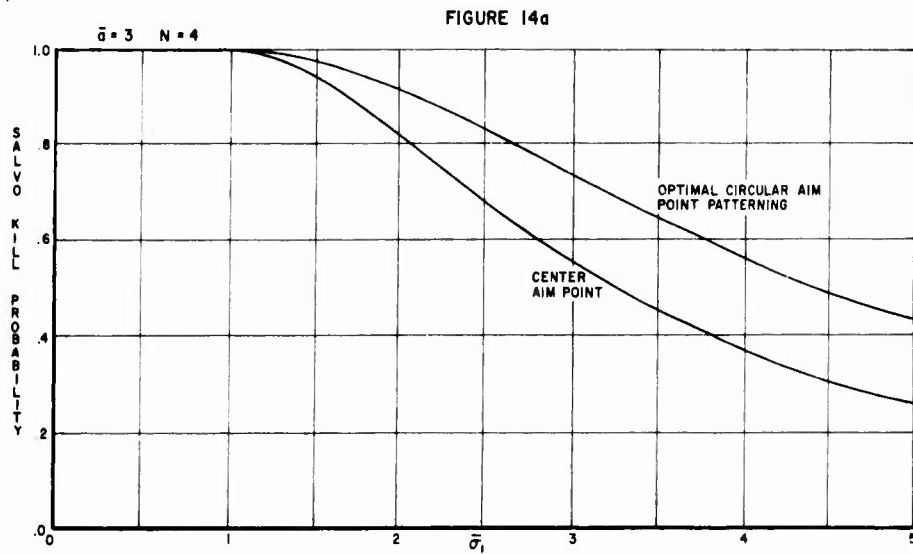


FIGURE 15a

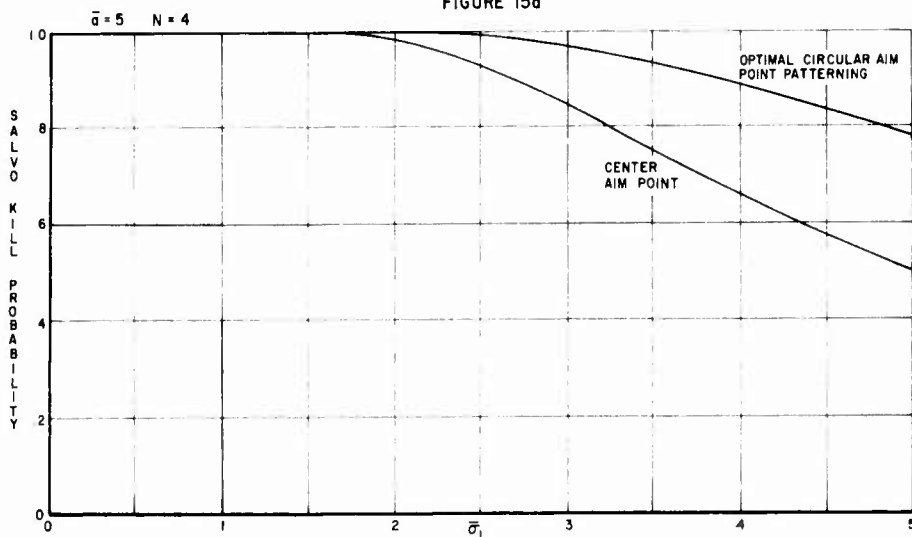


FIGURE 15b

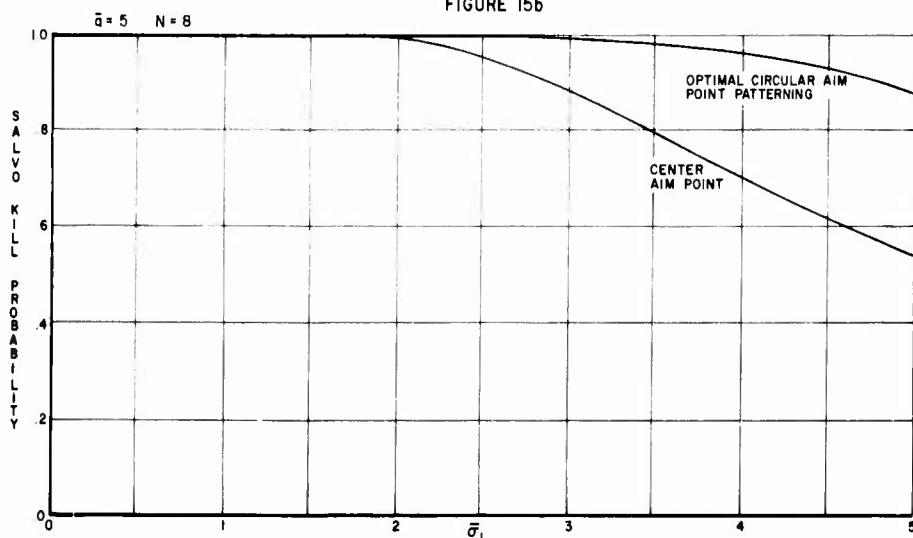
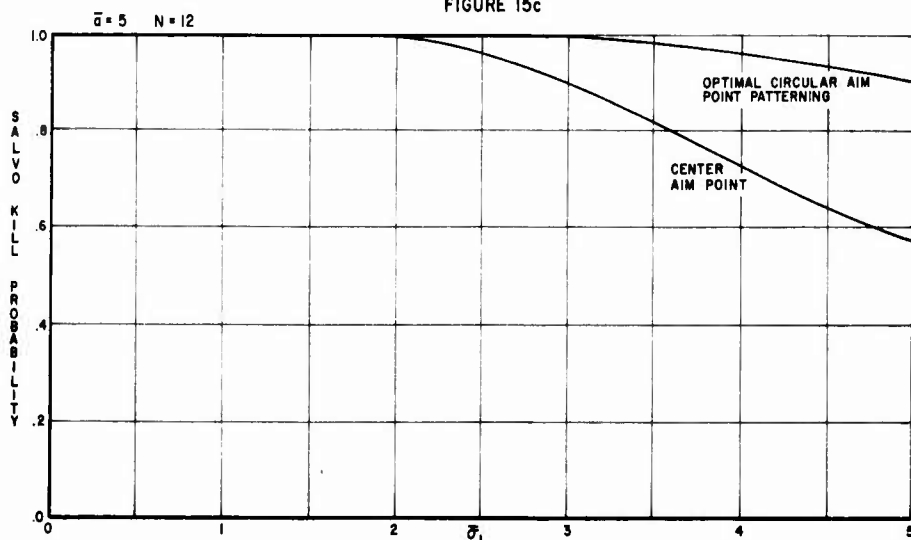


FIGURE 15c



#### IV. CONCLUSIONS

The results of this report show that an impressive savings in round requirements can be achieved through aim point patterning. However, one should be sure, before attempting to implement the procedure, that the assumptions in the Introduction have been met. Recapitulating, these assumptions are (1) the target is circular, (2) the aiming error (common to all rounds in the salvo) follows a circular normal distribution, (3) the separate random errors follow a circular normal distribution, and (4) there is independence among random errors and between the random errors and the aiming error. It is well to note that if one is using a spotter to adjust the aiming point onto the target center and/or if one is firing from a known location at a target with a known location, the assumption of a variable aiming error is probably not tenable. In the latter situation, one could experience a fixed aiming error or bias which is not covered in this report.

The assumption of circular normality also deserves comment. While there is some justification in the assumption that weapons systems errors are normal or near normal, there is no guarantee that they will be circular normal. In fact, most weapons systems experience errors which are elliptical rather than circular. Needless to say, unless the degree of eccentricity is near zero, the results in this report will apply only approximately and the degree of approximation is unknown. The elliptical extension to the problem is being given consideration and may be covered in a subsequent report if sufficient interest is shown.

The aim point patterns in this report are restricted to equidistant points on a circle about the target center and the optimal patterns developed are optimal only in the sense of selecting pattern radii which yield maximum probability. The class of patterns was restricted to the circular case because (1) it reduced the number of parameters to the point where the problem was tractable and (2) this class seemed most reasonable in light of a circular target and circular normal errors. While there is no claim that the optimal circular patterns developed are overall optimal, the authors of this report have strong reason to believe this to be the case. This belief is based on the results of empirical studies involving other patterns including square and rectangular lattices as well as concentric circles. In the large number of cases examined, none of these provided salvo kill probabilities as high as the single circular pattern.

One final point deserves attention. It is well known, in problems of this nature, that one way to increase the salvo kill probability while aiming all rounds at the target center is to increase the round-to-round dispersion. Since it may be argued that this procedure would be better than the patterning concept discussed in this report, a comment on the matter is in order. First, from a practical standpoint, it appears to these authors that it is easier to aim at different points about a target than it is to build a system which provides for controlled round-to-round dispersion. Second, our empirical investigations have shown that while one can increase the salvo kill probability by increasing the random dispersion, he can increase it more by optimal aim point patterning. This is illustrated in the following example for  $\sigma_1 = 40$ ,  $\sigma_2 = 10$ ,  $a = 20$ , and  $N = 4, 8, 12$ . The results are set forth for each value of  $N$ .

<u>N</u>	<u>Optimal <math>\sigma_2</math></u>	<u>Probability</u>	<u>Probability with <math>\sigma_2 = 10</math></u>	<u>Percent Increase by Optimizing <math>\sigma_2</math></u>
4	22	.2742	.2244	22
8	26	.4158	.2796	49
12	30	.5145	.3098	66

Here we see percent increases of 22, 49, and 66 as compared with 45, 71, and 81 through aim point patterning. Hence, even if a system could be built with controlled random dispersion, one could not attain kill probabilities as large as he could by simply employing aim point patterning.

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<p>An expression is developed for the probability of at least one hit on a target in a salvo of N rounds under the following assumptions: (1) the target is circular of radius a; (2) the rounds are aimed at a pattern of points about the target center; (3) the aiming error, common to all rounds in the salvo, is governed by a circular normal distribution with variance <math>\sigma_1^2</math>; (4) the round-to-round or random errors, separate and independent for each round in the salvo, are governed by a circular normal distribution with variance <math>\sigma_2^2</math>; and (5) there is independence between the aiming error and the random errors. Restricting the pattern of aim points to equidistant points on a circle centered at the target center, the expression is used to obtain pattern radii which yield maximum probability for a variety of values of the parameters <math>\bar{a} = a/\sigma_2</math>, <math>\bar{\sigma}_1 = \sigma_1/\sigma_2</math>, and N. A comparison of these maximum probabilities with those obtained by aiming all rounds at the target center shows that a substantial increase in probability can be achieved through aim point patterning.</p>			

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