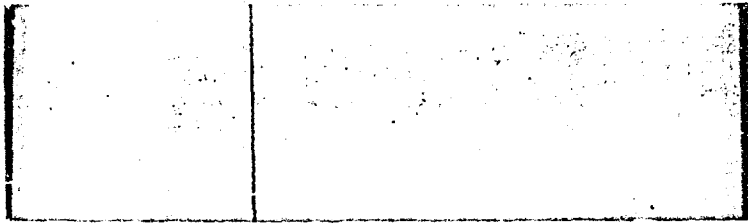


UNCLASSIFIED

AD NUMBER
AD903970
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; 13 Oct 1972. Other requests shall be referred to Commanding Officer, U.S. Army, Frankford Arsenal, Attn: SMUFA-L3300, Philadelphia, PA 19137.
AUTHORITY
FA D/A ltr, 9 Oct 1974

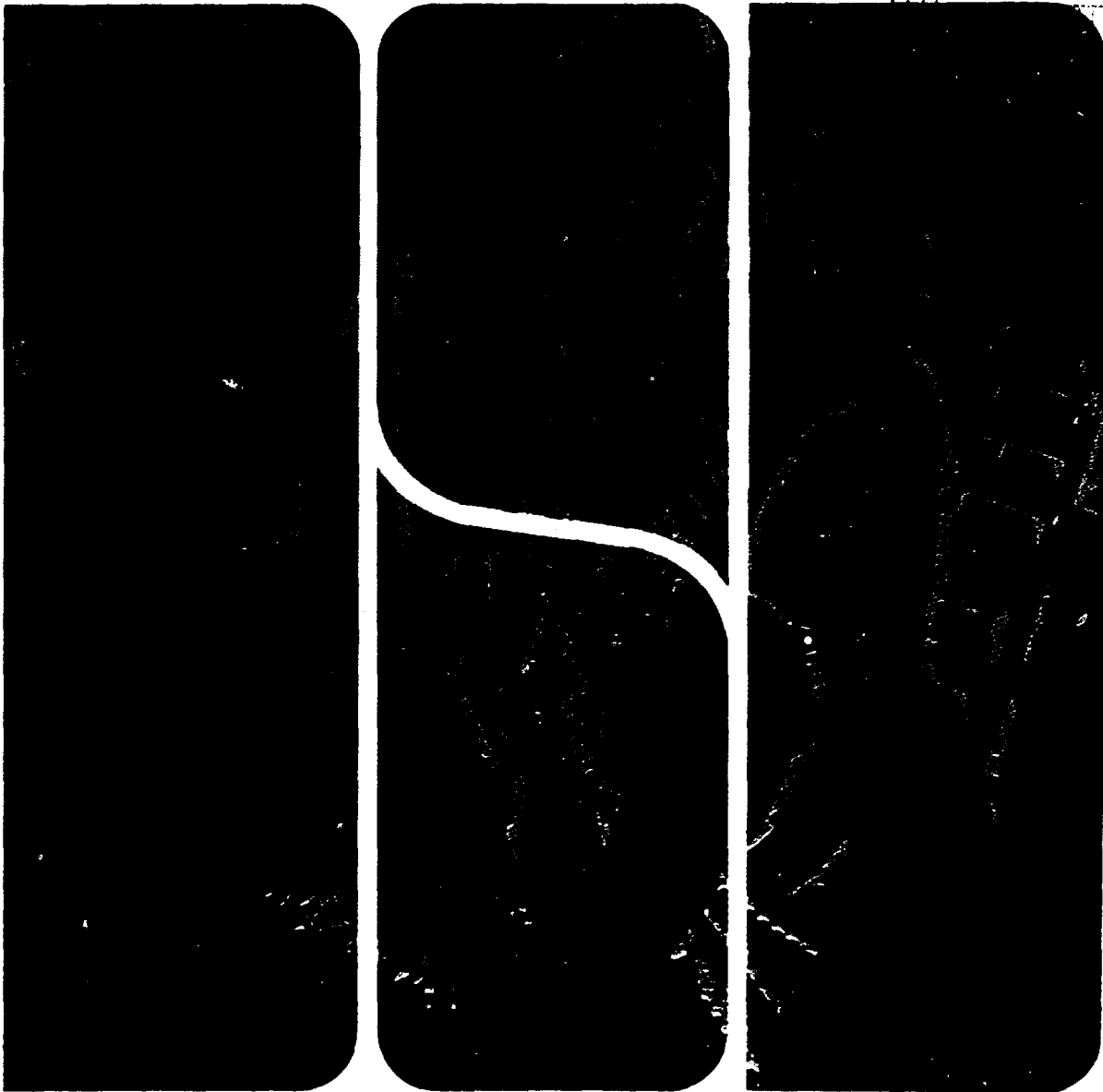
THIS PAGE IS UNCLASSIFIED

AD 903970 L



DATA SYSTEMS

DDC  
RECEIVED  
OCT 13 1972  
RECEIVED



FINAL REPORT  
SUPPLEMENT TO PARAMETRIC STUDY OF  
ADVANCED FORWARD AREA AIR DEFENSE  
WEAPON SYSTEM  
(AFAADS)  
VOLUME II, SIMULATION MODEL

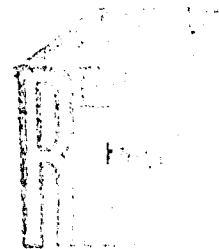
15 September 1972

Submitted to:

U. S. Army, Frankford Arsenal /SMUFA-L3300  
Bridge and Tacony Streets  
Philadelphia, Pennsylvania 19137

Prepared Under Contract: DAAG05-70-C-0328

Prepared by:  
Data Systems Division  
Litton Systems, Inc.  
8000 Woodley Avenue  
Van Nuys, California 91409



Distribution limited to U.S. Gov't. agencies only;  
Test and Evaluation; 13 OCT 1972 Other requests  
for this document must be referred to \_\_\_\_\_

## ACKNOWLEDGEMENTS

Litton is happy to acknowledge the excellent guidance provided on this contract by Mr. Walter Ryba, contract technical supervisor, and the substantive con-

tributions made by Mr. Stanley Goodman in the simulation development.

"PRECEDING PAGE BLANK-NOT FILMED."

## FOREWORD

This report describes the research effort of the Data Systems Division of Litton Industries, Inc. under Supplemental Agreement 2 to Contract DAAG05-70-C-0328, with the U.S. Army, Frankford Arsenal. The objective was to provide additional analytic and simulation effort in support of the parametric analysis of predicted fire air defense weapon systems.

The report is presented in three volumes. Volume I, Analysis, by Herbert K. Weiss, reports the analysis effort and the simulation results. Volume II, Simulation Model, by Martin P. Ginsberg, describes the Litton Air Defense

Simulation, designed by Mr. Ginsberg. The results of the simulation are included in Volume I. Volume III, Effectiveness, by Herbert K. Weiss, reports on methods of evaluating overall system effectiveness.

In the present report, frequent reference is made to the Final Report on the original contract. The previous report, titled Final Report, A Parametric Study of Advanced Forward Area Air Defense Weapon System (AFAADS) (two volumes), dated 2 October 1970, Revised Edition 1971, is referred to throughout this report as the "AFAADS-I Report."

## TABLE OF CONTENTS

Section		Page
1	INTRODUCTION	1-1
	1.1 CAPABILITY	1-1
	1.2 EXPERIENCE	1-1
	1.3 MODEL ELEMENTS	1-3
	1.3.1 Aircraft	1-3
	1.3.2 Weapon System	1-5
	1.3.3 Report Generator	1-6
	1.4 STRUCTURE OF THE REPORT BODY	1-6
2	SIMULATION CONTROL	2-1
3	EVENTS	3-1
	3.1 SENSOR DATA ACQUISITION EVENT	3-1
	3.1.1 Sensor Lag	3-1
	3.1.2 Regenerated Rates	3-2
	3.1.3 Sensor Noise	3-2
	3.1.4 Systematic Errors (Bias)	3-3
	3.1.5 Sensed Position	3-3
	3.1.6 Smoothed Values	3-3
	3.1.7 Regenerative Tracking	3-3
	3.1.8 Event Scheduling	3-3
	3.2 SENSOR FAILURE EVENTS	3-3
	3.3 PREDICTION EVENT	3-4
	3.4 GUN MOTION AND FIRING EVENT	3-6
	3.5 REPORT GENERATOR EVENT	3-7
	3.5.1 Single Replicate Reports	3-8
	3.5.2 Across Replicate Summary Report	3-11
4	UTILITY PACKAGE	4-1
	4.1 AIRCRAFT POSITION MODULE	4-1
	4.2 BALLISTICS MODULE	4-2
	4.3 TIME OF FLIGHT MODULE	4-2
	4.4 PREDICTION ALGORITHM MODULE	4-2
	4.4.1 Quadratic Prediction	4-3
	4.4.2 Simple Polar Prediction	4-3
	4.4.3 Defense of a Known Point	4-4
	4.5 RANDOM NUMBER GENERATOR	4-4
	4.6 COORDINATE TRANSFORMATION	4-5
	4.7 FIRE DOCTRINE MODULE	4-5
	4.8 VULNERABILITY MODULE	4-5
	4.8.1 The J Matrix	4-6
	4.8.2 The Covariance Matrix S	4-6
	4.8.3 The $M_a$ Vector	4-7

## LIST OF ILLUSTRATIONS

Figure		Page
1-1	Top Level Organization of the AFAADS Computer Model	1-2
1-2	Information Flow Through the AFAADS Gun Model	1-4
2-1	Logic of the Control Process	2-2
3-1	Bias Consideration Region	3-5
3-2	Straight-Line Bias Situation	3-6
3-3	Turn to Attack Bias Situation ( $S < L$ )	3-6
3-4	No Bias Situation ( $S > L$ )	3-6
3-5	Geometry of a Turn Followed by a Linear Segment	3-6
3-6	Logic Flow Diagram of the Production Event	3-8
3-7	Top Level Logic of the Report Generator	3-9
3-8	Computations For a Single Replicate	3-10
3-9	Miss Distance Report	3-11
3-10	Miss Distance Detail Report	3-12
3-11	Single Shot Kill Probability Detail Report	3-13
3-12	Single Shot Kill Probability Report	3-14
3-13	Burst Kill Probability Detail Report	3-14
3-14	Burst Kill Probability Report	3-14
3-15	Miss Distance Summary Report	3-14
3-16	Kill Seconds Over the Course Report	3-15
3-17	Single Shot Kill Probability Summary Report	3-15
3-18	Kill Probability Over the Course Report	3-15
4-1	Geometric Representation of the Point-to-Point Linear Interpolation	4-4

## LIST OF TABLES

Tables		Page
I-1	Major Model Elements	1-5
III-1	Regenerated Coordinate Computation	3-4
III-2	Reports Generated	3-11

## SECTION 1 INTRODUCTION

The AFAADS gun model provides the capability of estimating the performance of proposed (or existing) predicted fire weapon systems against attack aircraft in one-to-one combat situations.

### 1.1 CAPABILITY

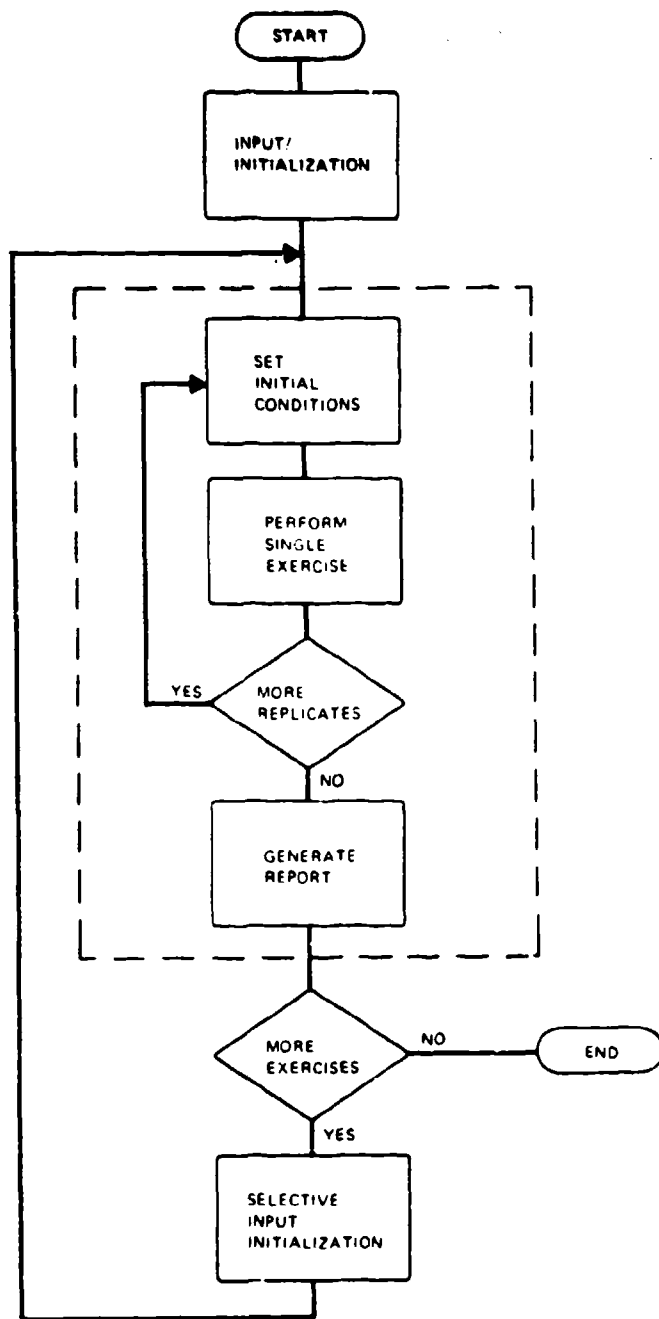
The model is not specific to one gun or aircraft system but has the ability to model a variety of systems. Each major subsystem of the weapon is represented. The particular attributes or method of operation of each subsystem for a computer experiment is determined by parameter values in the model data set. In its present configuration the model includes the facilities to:

- a. Model realistic aircraft paths.
- b. Model sensor behavior including the effects of:
  - (1) Sensor lag.
  - (2) Regenerative tracking.
  - (3) Aircraft image in the tracker plane.
  - (4) Sensor noise due to scintillation.
  - (5) Radar wave length.
  - (6) Sampling rate.
- c. Model the data smoother for:
  - (1) Position.
  - (2) Velocity.
  - (3) Acceleration.
- d. Model the following prediction methods:
  - (1) Linear.
  - (2) Linear plus energy corrections.
  - (3) Linear plus acceleration corrections.
  - (4) Linear plus thresholded acceleration corrections.
  - (5) Defense of a known point combined with any of the above.
  - (6) Simple polar.
- e. Model the ballistics of any projectile with trajectory characteristics suitable to representation as a Siacci table of no more than 300 velocity points.
- f. Automatically use the selected ballistics and prediction algorithm in the time of flight, shell position, and effectiveness computations.
- g. Model gun motion including the effects of:
  - (1) Servo lag.
  - (2) Regenerative tracking.
- h. Include the effects of up to 10 different dispersion patterns in the kill probability.
- i. Compute the single-shot kill probability for equally spaced time points over the aircraft path.
- j. Compute the burst-kill probability for each burst over the aircraft path for up to 10 fire doctrines on each execution.
- k. For each fire doctrine and dispersion combination compute the probability of a kill.
- l. Produce detail reports of the performance of any subsystem.
- m. Produce summary reports of each execution.
- n. Automatically replicate and produce summary statistics across replicates of:
  - (1) The distribution function of the miss-distances observed over the course.
  - (2) The distribution function of the single-shot kill probabilities over the course for each dispersion pattern input.
  - (3) The kill-sec (integral of single-shot kill probability) over the course for each dispersion pattern input.
  - (4) The distribution function of burst kill probabilities for each dispersion pattern, fire doctrine combination input.
  - (5) The kill probability over the course for each fire doctrine and dispersion set averaged over the set of replicates.

### 1.2 EXPERIENCE

Figure 1-1 is a top level block diagram of the model organization. The model was intentionally structured for use as a study tool. The area enclosed in dashed lines is where replication and statistics gathering over samples using the same data values and different random number streams are accomplished. The outer loop which encloses this area is used for the selective replacement of parameter values. This structure enables the completion of an experiment in one computer run. For example, if it is desired to construct a footprint, the inner loop would compute statistics over replicas for one path and the outer loop would be used to change the minimum crossing distance of the path.

The AFAADS model has been used to conduct computer experiments to measure the variability of system performance due to changes in aircraft performance, system implementation, and system operating policy. Specific experiments which have been conducted and reported include effects of:



20871 182

Figure 1-1. Top Level Organization of the AFAADS Computer Model

- Varying aircraft path including:
  - Varying velocity on linear paths.
  - Varying crossing range on linear paths.
  - Introducing jinking on linear paths.
  - Introducing 'break-away' maneuvers on linear and jinking paths.
- Varying the sensor subsystem with respect to:
  - The wave length of radar.
  - Using manual tracking.
  - Using rate-aided manual tracking.
  - Introducing regeneration.
  - Introducing angular bias.
  - Interruption of sensor data.
- Varying the smoothing algorithms with respect to:
  - Smoothing period.
  - Number of points.
- Varying the prediction algorithm including:
  - Linear prediction.
  - Quadratic prediction.
  - Energy correction.
  - Defense of a known point.
- Varying the projectile-to-projectile position deviation due to:
  - Angular dispersion.
  - Velocity dispersion.
- Varying the ballistics including:
  - Muzzle velocity.
  - Caliber.
- Varying the fire doctrine including:
  - Continuous vs. interrupted fire.
  - Rate of fire.
  - Open fire range.
  - Burst duration.

In addition to the studies listed above the model has also been used to generate 'footprints' for 20, 25, 37 and 50 mm gun systems against aircraft executing both unaccelerated and jinking fly-by courses.

The simulation model is programmed in FORTRAN IV and is currently executing on IBM 370/165 and CDC 6400 computer systems. Execution times are highly dependent on the particular experiment being conducted, however, the model is fairly fast. For exam-

ple, to execute a fly-by path from acquisition at 3600 meters to crossover for 10 fire doctrines and 5 dispersion patterns requires less than 1 CPU second per replicate on the IBM 370/165.

### 1.3 MODEL ELEMENTS

The exercise portion of the model is structured to maintain the 'real world' information pattern of the gun system being simulated. This is accomplished by restricting the access of each simulated gun component to only that information which would be available in the 'real world.'

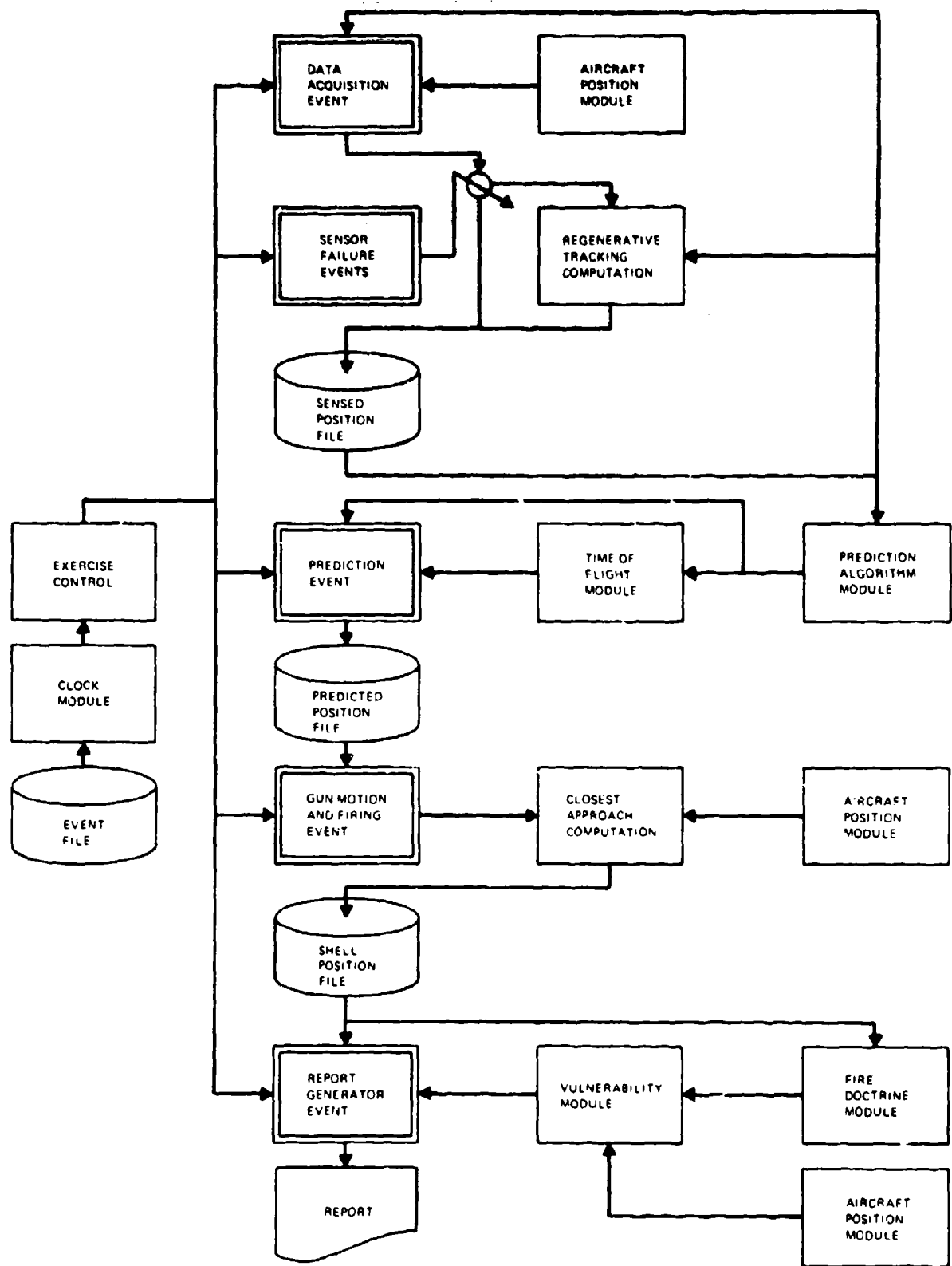
The manner in which information is acquired, generated, and transmitted through the system is presented in Figure 1-2. The primary information flow is as follows: the aircraft position as a function of time enters the system via the Data Acquisition Event, which adds the effects of the sensor system and fills the Sensed Position File; the Prediction Event uses the sensed data, the time of flight module, and the prediction algorithm module to fill the Predicted Position File with aim points; the Gun Motion and Fire Event accesses the Predicted Position File and fills the Shell Position File as if the gun fired continuously; the Report Generator Event accesses the Shell Position File and the Aircraft Position Module, applies the fire doctrines called for, and produces reports. The preceding information flow pattern demonstrates an important feature of the model. Note that, as in the 'real world', any action which the gun system takes (or can take) is ultimately based on information acquired through its sensing system.

Table 1-1 lists the major elements of the computer model. Each is described in greater detail in the following paragraphs and specific formulation and equations for the elements are presented in the main body of the text.

#### 1.3.1 Aircraft

The aircraft is described by its flight path, its size, and its shape. The flight path is specified as an initial position, velocity, and heading, and a series of path segments or legs. The legs are described by specifying their horizontal and vertical projections. Horizontal projections may be segments of lines (accelerated or unaccelerated), segments of circles, or segments of spirals. Vertical projections may be segments of lines or segments of parabolas. The bank angle of the aircraft is determined by the radial acceleration of the segment in question.

Aircraft size and shape are considered in the data acquisition phase and in the vulnerability phase. In the data acquisition phase the aircraft is represented as three perpendicular vectors: The fuselage length vector, along the line of flight; the wing length vector, perpendicular to the fuselage length vector and rotated from the horizontal plane by an angle equal to the



20871 183

Figure 1-2. Information Flow Through the AFAADS Gun Model

Table I-1. Major Model Elements

I	AIRCRAFT
	Flight Path
	Size and Shape
II	WEAPON SYSTEM
	Sensor
	Smoother
	Predictor
	Controller
	Gun
	Projectiles
III	REPORT GENERATOR
	Aircraft Vulnerability
	Fire Doctrine
	Measures of Performance for:
	Sensor   Smoother
	Predictor
	Controller
	Weapon System

20871-532

bank angle; and the aircraft height vector, in a direction perpendicular to the first two vectors. The lengths of these vectors are combined with the aircraft-sensor geometry in determining the standard deviations of the sensor error in azimuth and elevation.

Aircraft size and shape for the vulnerability phase are entered separately. The aircraft is represented as two perpendicular ellipsoids: one for the wing and one for the fuselage. Each ellipsoid is specified by its three semi-major axes: length, width, and height. Vulnerability of the aircraft then is determined by combining the vulnerability of the fuselage, the wing, and the fuselage-wing intersection.

### 1.3.2 Weapon System

The weapon system sensor model uses the actual aircraft position as a departure point and adjusts the values to reflect aircraft-sensor geometry, tracker drive elements, and stochastic error due to 'glint'.

The actual position is obtained in cartesian coordinates and transformed to polar coordinates. The position and velocity of the aircraft are used to determine the size of the aircraft in the sight plane. These sizes are combined with azimuth and elevation rates to determine the size of the error distributions in azimuth and elevation, and the current amount of error correlation across time. The history of tracker motion is used to determine the amount of tracker lag and past sensed

values are used to determine how much regeneration is present (if specified). Random numbers are used to determine current values for azimuth, elevation, and range errors due to glint. The adjusted values are transformed to cartesian coordinates and stored as sensed position.

The weapon system data smoother operates on sensed data. The particular filters to use are determined by input data. Three filters are required: Position, velocity and acceleration. The sensed position file maintains the ten most recent samples at all times. Thus, any filter which can be represented by ten or fewer coefficients to operate on equally spaced data values can be used.

The weapon system predictor operates on smoothed data. There are several algorithms available with the specific one used on a given experiment determined by the input data. Algorithms currently available are:

- a. **LINEAR.** Straight linear extrapolation of current smoothed position based on current smoothed velocity and time of flight.
- b. **QUADRATIC.** Quadratic extrapolation of current smoothed position based on current smoothed position, velocity and radial acceleration.
- c. **QUADRATIC THRESHOLD.** Uses linear extrapolation unless the radial acceleration exceeds a specified threshold. Automatically adds an acceleration correction to the linear prediction whenever the threshold is exceeded.
- d. **SIMPLE POLAR.** Linear extrapolation of aircraft position in polar coordinates based on measured position and rates.
- e. **DEFENSE OF A KNOWN POINT.** Assumes aircraft will turn and attack a known point if current heading and velocity allow such a maneuver. Used in conjunction with one of the above algorithms.
- f. **CONSTANT ENERGY.** In conjunction with the above algorithm a velocity correction based on velocity in the vertical plane can be used. It is based on a thresholding mechanism similar to the automatic linear-quadratic switch.

Aim errors due to the gun subsystem are of two types: lag between aim order and actual arrival at the aim point, and bias due to imperfections in the system alignment and/or adjustment. Lag is computed using a history of the commands given to the gun system. Regeneration may be specified and when present is also based on the command history. Provision has been made for the specification of systematic errors in azimuth and/or elevation.

Projectile trajectories are computed using a set of input tables. Any family of trajectories which can be specified using tables (of less than 500 points each) expressing time of flight and range as a function of remaining velocity may be used. Provision has been made for the specification of a bias of the muzzle velocity.

### 1.3.3 Report Generator

The report generator portion of the model is much more than an output routine. For economy of computer time certain computations are made in the report generator, specifically, the effects of both round-to-round dispersion and fire policy, as well as aircraft vulnerability, are computed in the report generator. This arrangement allows the generation of reports of up to 10 fire policies and 10 sets of dispersion parameters using the results of one model execution.

During the exercise portion of the model, the gun is 'fired' continuously. In the Report Generator, fire doctrines are used to determine when the gun system would fire, how many rounds and the rate of fire. Current implementation requires that the following data be specified:

- a. Amount of ammunition on the mount
- b. Number of rounds per burst
- c. Open fire time
- d. Duration of a burst
- e. Dead time (time between bursts) and
- f. Reload time.

The appropriate miss distances and the round-to-round dispersion parameters are combined with the aircraft size in the VULNERABILITY module to produce kill probabilities.

Measures of performance may be generated for subsystem or the gun system. They may be at the detail level for each replicate or at the summary level over a set of replicates.

### 1.4 STRUCTURE OF THE REPORT BODY

In the following sections, the model capability is described in greater detail. To facilitate the discussion, the model is viewed in three sets of functionally related program modules.

The first set of modules is used to control the simulation process. This set is composed of the main program and one subroutine which maintains a schedule of events and provides for central time accounting.

The second set of modules is composed of the Gun System Functional Event Routines. This set of routines models the various components of the gun system under study. There are five event routines:

- a. Data Acquisition Event.
- b. Sensor Failure Event.
- c. Prediction Event
- d. Gun Motion and Firing Event.
- e. Report Generation Event.

The third set of modules is the utility package. These modules are used by the event routines or each other and may not be invoked by the system scheduler. Included in this set are:

- a. Aircraft positioner.
- b. Time of flight.
- c. Prediction algorithm.
- d. Random number generator.
- e. Coordinate transformation package.
- f. Fire Doctrine Module.
- g. Vulnerability Computation Module.

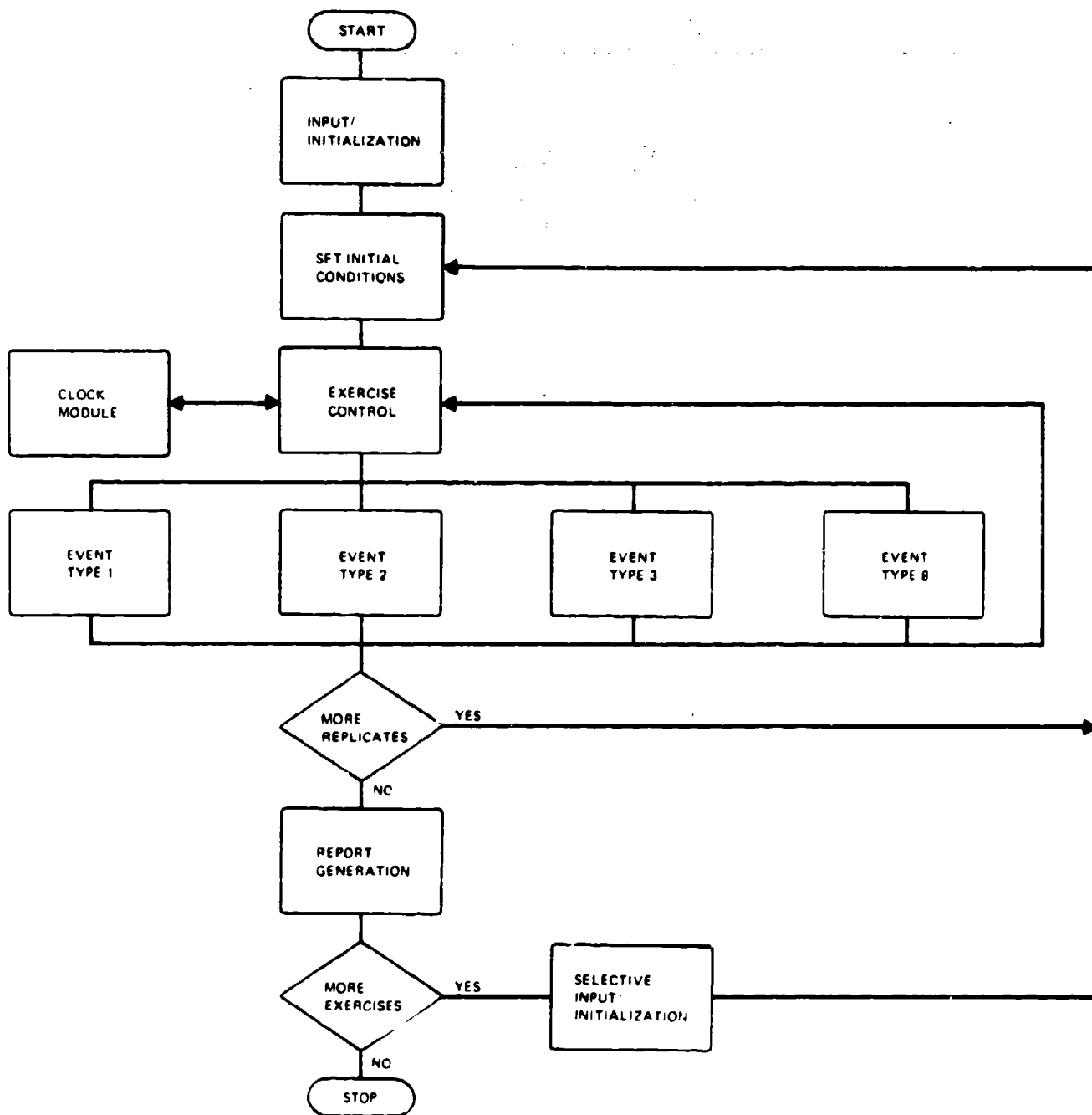
## SECTION 2 SIMULATION CONTROL

The simulation model represents the combat situation under study as a series of events centered around the antiaircraft gun system. Event sequencing and initialization are performed by the master control program and the clock module. The initialization phase consists of reading input parameters, performing and storing the results of once-only computations, and scheduling the first (a sensor data acquisition event) and last (a report event) events to be processed. Initialization tasks are accomplished by special modules which are called by the master control program prior to entering the event sequencing mode.

The logic of the control process is presented in Figure 2-1. The particular order of events, which will occur, is determined by the gun system and the aircraft path under study. There are, however, some principles worth noting: for example, the first event to occur will always be Event Type 1, and it will continue to occur at its specified period (input during initialization phase) until the end simulation event. With the exceptions of the end simulation event and the sensor inter-

ruption events, no other events will occur unless they are explicitly scheduled by Event Type 1 or some other module, either directly or indirectly scheduled by an occurrence of Event Type 1. The occurrence of Event Type 2 indicates the end of the current sequence of simulation events.

One of two actions are initiated after the occurrence of Event Type 2. If there are more replications to be performed; the event file is cleared, all flags are turned off, the initial Event Type 1 and end simulation events are scheduled, and the simulation process continues. If no more replications are required, the summary statistics over replications are printed, and the model enters the selective input phase. During the selective input phase any data set may be replaced and the model restarted. The selective input phase is very useful for making sensitivity studies since a series of exercises can be made specifying the full data set once and only the data which changes from case to case thereafter. This enables the completion of an extended computer experiment in one computer run.



20871 184

Figure 2-1. Logic of the Control Process

## SECTION 3 EVENTS

### 3.1 SENSOR DATA ACQUISITION EVENT

In the real world all gun system functions, which required a knowledge of aircraft position or rate parameters, acquire the information either directly or indirectly from the gun sensor. In the computer model the sensor function is modeled by the Sensor Data Acquisition Event. On each entry, this event routine stores current values of sensed aircraft position in the SENSED DATA File. The gun-system's data smoother is also modeled in this routine and its outputs, smoothed position and rate values, are also placed in the data file.

In the following discussion, upper case letters will be used for true values, primed upper case letters for sensed values, underscored upper case letters for smoothed values of position, and dot notation to indicate rates and accelerations. The time to which values pertain will be indicated by subscript expressed relative to current time. Thus, if A is a position coordinate:

- $A_0$  = Current true value.
- $A'_0$  = Current sensed value.
- $A_0$  = Current smoothed value.
- $A_0$  = Current smoothed rate value.
- $A'(t)$  = Sensed value at current time minus  $\Delta t$ .

The six dimensional transformations from cartesian to polar, and from polar to cartesian coordinates, will be represented by the operators; 'CTOP' and 'PTOC.' Thus:

$$(D, \theta, \phi, \dot{D}, \dot{\theta}, \dot{\phi}) = \text{CTOP} [X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}] \quad (3.1)$$

and

$$(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) = \text{PTOC} [D, \theta, \phi, \dot{D}, \dot{\theta}, \dot{\phi}] \quad (3.2)$$

Standard superscript arrow notation ( $\vec{A}$ ) will be used to designate vectors. Thus, the coordinate transformations become:

$$\vec{P} = \text{CTOP} [\vec{C}] \quad (3.3)$$

and

$$\vec{C} = \text{PTOC} [\vec{P}] \quad (3.4)$$

On each entry to the event routine, the sequence of computations is as follows. The true aircraft position and rates are obtained by invoking the aircraft position module. These values which are returned in cartesian coordinates are transformed to polar coordinates, which is the natural coordinate system of the sensor systems. From the sensor-aircraft geometry, the size of the aircraft as 'seen' in azimuth, elevation, and range are computed and saved. Based on the aircraft size and rates in the sight plane, values are computed for the standard deviation of glint noise in azimuth and elevation and for the sample-to-sample correlation. Current aircraft position is estimated based on the last values of smoothed position and velocity. A check is made to determine the state of the sensor. Any values which are in an interrupted state are replaced by the values computed from the smoothed rates and further adjustments to these values are not made. Values in an up state are adjusted for lag, regeneration, and the presence of sensor noise.

The resulting sensed position in polar coordinates is transformed to cartesian coordinates and stored in the SENSED DATA File. The most recent 'n' values stored in the SENSED DATA File are used to compute current values for smoothed position, velocity, and acceleration. These values are also stored in the SENSED DATA File.

The dimensions of the aircraft in the sight plane are obtained by a rotation and projection onto the sight plane of a set of three orthogonal vectors representing the aircraft length, height, and width. First, a rotation to orient the set in space such that the length is parallel to the aircraft velocity vector and the width is rotated to an angle equal to the bank angle is performed. Then these rotated vectors are projected onto the sight plane which is perpendicular to the range vector and the maximum value of azimuth, and elevation projections are determined and saved.

#### 3.1.1 Sensor Lag

The computation for sensor lag is based on actual aircraft rates in polar coordinates, since these are the cause of the phenomenon in the 'real world.' Lag is computed in the azimuth and elevation coordinates using the following operation:

$$L_0 = \frac{1}{K_v} \dot{P}_0 + \frac{1}{K_a} \frac{\dot{P}_0 \cdot \dot{P}_0 \Delta}{\Delta} \quad (3.5)$$

This is the usual two term servo lag approximation

with acceleration being estimated using two rate measurements.

### 3.1.2 Regenerated Rates

The computation of rates for regenerative sensor tracking is based on data acquired by the sensor. This is again in agreement with the information flow in the 'real world' situation. Estimates of current position are computed from current values of the data smoother using linear extrapolation:

$$\begin{aligned} X_0^* &= \underline{X} \cdot \Delta + \dot{\underline{X}} \cdot \Delta \cdot \Delta \\ Y_0^* &= \underline{Y} \cdot \Delta + \dot{\underline{Y}} \cdot \Delta \cdot \Delta \\ Z_0^* &= \underline{Z} \cdot \Delta + \dot{\underline{Z}} \cdot \Delta \cdot \Delta \end{aligned} \quad (3.6)$$

Ground range,  $S$ , and ground range rate,  $\dot{S}$ , are estimated as follows:

$$S = (X_0^{*2} + Y_0^{*2})^{1/2} \quad (3.7)$$

$$\dot{S} = (X \cdot \dot{\underline{X}} \cdot \Delta + Y \cdot \dot{\underline{Y}} \cdot \Delta) / S \quad (3.8)$$

Polar position and rate are obtained as follows:

$$\vec{P}_0^* = \text{CTOP} (X_0^*, Y_0^*, Z_0^*, \dot{\underline{X}} \cdot \Delta, \dot{\underline{Y}} \cdot \Delta, \dot{\underline{Z}} \cdot \Delta) \quad (3.9)$$

Angular accelerations are computed from the above values by:

$$\ddot{\theta}^* = -2.0 \dot{\theta}^* \dot{S} / S \quad (3.10)$$

$$\ddot{\theta}^* = -2.0 (D \cdot \dot{D} \cdot \dot{\theta}^* + S \dot{\theta}^{*2} Z^*) / D^2 \quad (3.11)$$

Finally, the amount of regeneration provided is given by:

$$R_\theta = \frac{\dot{\theta}}{K_v} + \frac{\ddot{\theta}}{K_a} \quad (3.12)$$

$$R_\phi = \frac{\dot{\phi}}{K_v} + \frac{\ddot{\phi}}{K_a} \quad (3.13)$$

### 3.1.3 Sensor Noise

The value for sensor noise in each of the coordinates is computed using the formula:

$$X_j = \alpha X_{j-1} + \beta \sigma_N \quad (3.14)$$

where:

- $X_j$  - is the current value of sensor noise in the coordinate
- $X_{j-1}$  - is the last value of sensor noise in the coordinate
- $N$  - is a sample from a population which is normally distributed with mean zero and variance one
- $\alpha$   
 $\beta$  - are coefficients to obtain the correct serial correlation
- $\sigma$  - is the standard deviation of the noise distribution in the coordinate.

For the range coordinate,  $\alpha$ ,  $\beta$ , and  $\sigma$  are static over a computer run. The value of  $\sigma$  is an input parameter and  $\alpha$  and  $\beta$  are computed as follows:

$$\alpha = e^{-\Delta/\tau} \quad (3.15)$$

$$\beta = (1 - \alpha^2)^{1/2} \quad (3.16)$$

where:

$\Delta$  is the sample interval

$\tau$  is the autocorrelation of the sample.

For the angular coordinates,  $\alpha$ ,  $\beta$ , and  $\sigma$  are dynamically computed using parameters which describe the sensor and the sensor-target geometry. They are defined by the following equations:

$$\sigma = 1/2 \left\{ 0.7 \left( \frac{1 + 500 \frac{\Omega L}{B \lambda}}{1 + 1000 \frac{\Omega L}{B \lambda}} \right)^2 + 0.3 \left( \frac{1 + 5 \frac{\Omega L}{B \lambda}}{1 + 10 \frac{\Omega L}{B \lambda}} \right)^2 \right\}^{1/2} \quad (3.17)$$

$$\begin{aligned} \alpha &= e^{-B \cdot 10 \cdot \frac{\Omega L}{B \lambda}} & \text{if } 10 \frac{\Omega L}{B \lambda} < 1 \\ &= e^{-B \Delta} & \text{if } 10 \frac{\Omega L}{B \lambda} > 1 \end{aligned} \quad (3.18)$$

where:

$$\beta = [1 - \alpha^2]^{1/2} \quad (3.19)$$

$\Omega$  is the angular rate in the coordinate and is composed of the rates due to aircraft path and wind

$L$  is the length of the aircraft in the coordinate

$B$  is the bandwidth of the servo

$\lambda$  is the wave length of the radar

$\Delta$  is the sample interval.

There are provisions in the model for the case when there is an interaction between lag and noise. When this condition exists, the equation becomes:

$$X_j = \alpha X_{j-1} + [\beta\sigma + \mu(L+R)] N \quad (3.20)$$

where the new term  $\mu(L+R)N$  is used to increase the standard deviation of the noise sample by a fraction,  $\mu$ , of the existing lag (the sum of lag and regeneration) at sensing time.

### 3.1.4 Systematic Errors (Bias)

Systematic errors in any combination of range, azimuth, and/or elevation may be included. The value of bias in each of these coordinates is part of the input data set and remain constant over a computer run.

### 3.1.5 Sensed Position

The sensed position stored for a given time is given by the equation:

$$\vec{P}_0' = \vec{P}_0 + \vec{L}_0 + \vec{R}_0 + \vec{N}_0 + \vec{B} \quad (3.21)$$

where:

P indicates polar coordinates

L is lag

R is regeneration

N is noise

B is bias

In addition, the values are stored in the position file in cartesian form. That is:

$$\vec{C}_0' = \text{PTOC}(\vec{P}_0') \text{ is saved for use in computing smoothed values.} \quad (3.22)$$

### 3.1.6 Smoothed Values

The tracker computes and stores values for current smoothed position, velocity, and acceleration. These values are computed using the 'n' most recent values of sensed position. In equation form:

$$\vec{C}_0 = \sum_{j=0}^{n-1} a_j \vec{C}'_{j\Delta} \quad (3.23)$$

$$\vec{C}_0 = \sum_{j=0}^{n-1} b_j \vec{C}'_{j\Delta} \quad (3.24)$$

$$\vec{C}_0 = \sum_{j=0}^{n-1} c_j \vec{C}'_{j\Delta} \quad (3.25)$$

### 3.1.7 Regenerative Tracking

The Sensor Data Acquisition event has the capability of generating 'sensed' positions which are based on extrapolations of smoothed data values. The event automatically uses this method of providing sensed data whenever data flow from the sensor is interrupted. The interruption can be due to a sensor failure in range, in angles, or in both range and angle; or due to terrain masking. Sensor failures are scheduled in the input phase and when they occur, the appropriate parameters to interrupt the data flow from the sensor are set. Terrain Masking is specified as part of the aircraft path. In the case of terrain masking, the aircraft position routine returns an indicator to the Data Acquisition Event whenever the aircraft is masked. When this indicator is on, the regenerated coordinates are used instead of sensed values.

Table III-1 presents the method used to obtain 'sensed' values in the presence of other than normal sensor operation. The equations show how the 'sensed' position is computed. Whenever one or more coordinates are still operable, lag and noise are added to them as in normal operations; however, no adjustments are made to regenerated coordinates.

### 3.1.8 Event Scheduling

The DATA ACQUISITION EVENT also performs the initial scheduling for the PREDICTION EVENT and the GUN FIRING EVENT. On the first occurrence of the DATA ACQUISITION EVENT with the aircraft in a visible state, both the PREDICTION and GUN EVENTS will be scheduled. The initial time for each of these events will be the time of the DATA ACQUISITION EVENT plus the system acquisition time, TACQ, which is an input variable.

### 3.2 SENSOR FAILURE EVENTS

SENSOR FAILURE is perhaps an error in naming. There are four events which are processed by this event routine. They are:

- a. Range Interruption.
- b. Range Resumption.

Table III-1. Regenerated Coordinate Computation

State	Computation
Range Failure	$\bar{C} = \text{PROC} (D^0, \theta_0, \psi_0, \dot{D}^0, \dot{\theta}_0, \dot{\psi}_0)$
Angle Failure	$\bar{C} = \text{PTOC} (D_0, \theta^0, \psi^0, D_0, \dot{\theta}^0, \dot{\psi}^0)$
Range and Angle Failure	$\bar{C} = \bar{C}^0$
Terrain Masking	$\bar{C} = \bar{C}^0$
$\bar{C}^0 = (X^0, Y^0, Z^0, \dot{X}^0, \dot{Y}^0, \dot{Z}^0) = (X + \Delta \cdot \dot{X}, Y + \Delta \cdot \dot{Y}, Z + \Delta \cdot \dot{Z}, \dot{X}, \dot{Y}, \dot{Z})$ $\bar{P}^0 = (D^0, \theta^0, \psi^0, \dot{D}^0, \dot{\theta}^0, \dot{\psi}^0) = \text{CTOP} (\bar{C}^0)$	

20871-533A

c. Angle Interruption.

d. Angle Resumption.

Each is scheduled during the initialization phase and may occur periodically if the period is less than the time duration of the exercise. The total extent of the action taken by the event when it occurs is to set or reset an appropriate variable to represent the current state of the sensor. The variable is used in the DATA ACQUISITION EVENT to determine the state of the sensor and perform appropriate computations.

### 3.3 PREDICTION EVENT

The Prediction Event models that portion of the real gun system in which sensor information is extrapolated into the future in order to compute lead angles for gun pointing. This event routine can operate in two modes. The first, standard mode, makes no restraining assumptions on the aircraft trajectory. The second, defense of a known point, assumes that a known point is the object of attack and biases predictions according to that assumption.

The sequence of computations which occur when an entry is made to this event routine are as follows: First, the Time of Flight Module is invoked which, with the aid of the Prediction Algorithm Module, computes a time of flight. The returned value for time of flight is combined with current time and the Prediction Algorithm Module is invoked directly to obtain the re-

quired predicted position. At this point, if standard prediction has been specified, a branch is executed to the area which stores data in the PREDICTED POSITION File. If defense of a known point has been specified, the process continues.

As mentioned above, when defense of a known point is specified, predictions are biased. The bias may take two forms based on either the assumption that the aircraft will turn toward the defended point, or the assumption that the aircraft will follow a straight line from its present position to the defended point. Consideration of the geometry of the situation reveals that neither of these biases are appropriate under certain conditions. For this reason, even when bias is specified, a particular prediction may or may not be biased. The determining factor is the position and velocity of the aircraft relative to the defended point. If the defended point is in front of the aircraft, a bias is considered. 'In front of' should be understood as an angle of less than 90 degrees between the aircraft horizontal velocity vector and the horizontal projection of the vector from the aircraft to the defended point. Since the dot product of two vectors is positive, if and only if the absolute value of the angle between the vector involved is less than 90 degrees, the decision rule becomes:

- Consider bias if:  $V \cdot R > 0$ .
- Do not consider bias if:  $V \cdot R < 0$ .

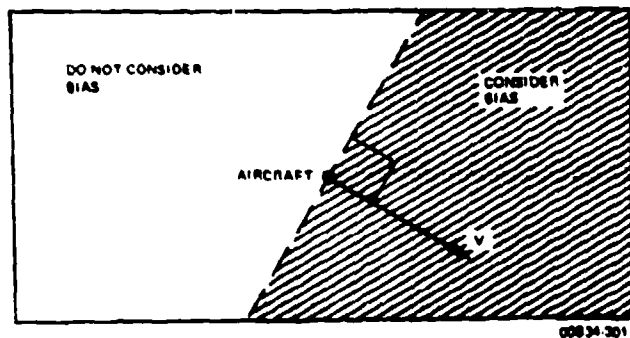


Figure 3-1. Bias Consideration Region

Figure 3-1 displays the geometry of the decision rule. If the defended point lies within the shaded area bias is considered.

Whenever the defended point lies within the bias consideration area of the aircraft, the geometry is further examined to determine if a bias can be used, and if so, which of the two biases is appropriate. Three situations are possible; 1) the aircraft heading is such that a projection of its current velocity vector would pass within a specified (during initialization) distance ( $R_c$ ) of the defended point, 2) a standard turn of specified (during initialization) acceleration followed by a straight line flight path would cause the aircraft to pass over the target, or 3) neither of the above.

The first situation, namely the straight line bias, is the easiest to explain (see Figure 3-2). The heading angle change  $\theta$  is computed as follows:

$$\theta = \tan^{-1} \left( \frac{R_x V}{R \cdot V} \cdot 1/2 \delta t \right) \quad (3.26)$$

The closest approach to the defended point is approximated by the arc:

$$S = R\theta \quad (3.27)$$

Linear bias is used if  $S < R_c$ .

The linear-bias computations are accomplished in the Time of Flight Module and the Prediction Algorithm Module. If a situation is such that linear bias is appropriate, an indicator switch is set and point to point interpolation will be used in the Prediction Algorithm Module.

If the situation is such that linear bias is not appropriate, a check is made to see if a turn followed by a straight-line path can bring the aircraft over the target. This check is made as follows: The radius of turn is computed using the following equation:

$$\delta = V^2/ng \quad (3.28)$$

where:

$V$  = the aircraft velocity

$ng$  = the standard turn acceleration ( $n$  is input)

The heading and turn radius are used to find the center of curvature of the turn and the distance ( $L$ ) from this point to the defended point. Whenever the turn radius is less than the distance from the center of curvature, a turn is possible (Figure 3-3). If no turn is possible, we have situation three; that is, no bias can be used (Figure 3-4). Under such circumstances a branch to the data storage area is executed and standard values for prediction are used.

If a turn followed by a linear segment is to be used, the process continues. Figure 3-5 presents the detailed geometry of the turn. The new heading ( $\theta$ ) and the time to turn ( $T$ ) are computed using the following formulas:

$$R_p = \sqrt{R_o^2 - 2\delta R_o \sin H} \quad (3.29)$$

$$A = \tan^{-1} \left\{ \frac{(R_o \cdot \delta \sin H) \cdot R_p \delta \cos H}{R_p (R_o \cdot \delta \sin H) + \delta^2 \cos H} \right\} \quad (3.30)$$

$$\Delta\theta = A + H \quad (3.31)$$

$$\dot{\theta} = \frac{V}{S} \quad (3.32)$$

$$T_T = \theta/\dot{\theta} \quad (3.33)$$

$$\theta = \theta_o + \Delta\theta \quad (3.34)$$

The time of the flight to the end point of the turn is compared to the time to turn. If the time of flight exceeds the time to turn, the biased prediction is used, otherwise a branch to the storage area is made and a standard prediction results.

When turn bias is included, values for the velocity in the  $x$  and  $y$  direction are computed. The prediction in these directions is a linear extrapolation along the line from the end of the turn to the defended point. In this case, the  $Z$  coordinate is unbiased.

For all cases, the last action is a branch to the data storage area where the predicted position is transformed to polar coordinates and stored along with the

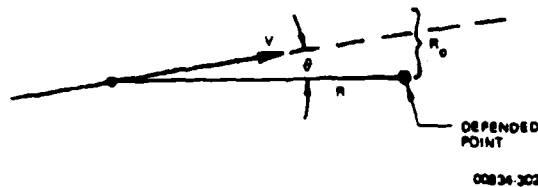


Figure 3-2. Straight-Line Bias Situation

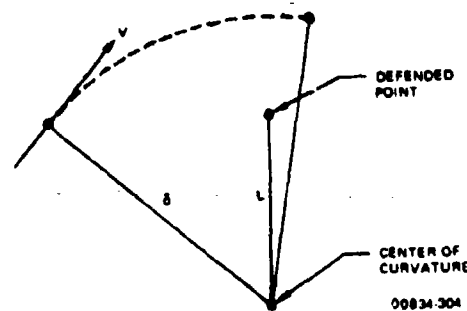


Figure 3-4. No Bias Situation ( $S > L$ )

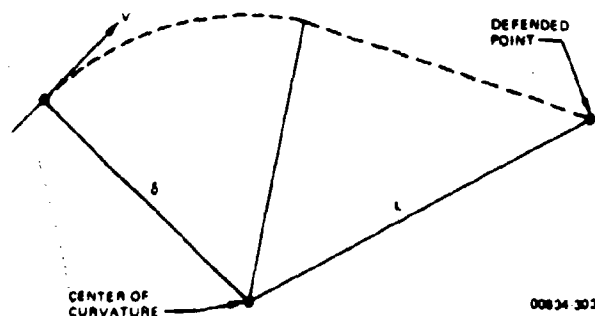


Figure 3-3. Turn to Attack Bias Situation ( $S < L$ )

estimated time of closest approach as the most recent entry in the PREDICTED POSITION File.

Figure 3-6 summarizes the logical structure of the Prediction Event.

### 3.4 GUN MOTION AND FIRING EVENT

The function of this event is to compute the closest approach of a shell fired at current clock time given; the history of predicted positions, the ballistic tables, and the actual aircraft flight path. Gun servo lag and/or servo regeneration can be included. When included, lag and/or regeneration characteristics are determined by input parameter specification.

In the following discussion, upper case letters are used to designate predicted values, subscripts designate the time to which the values pertain, primed upper case letters are used to indicate actual gun pointing angles, and the letters L and R are used to designate Lag and Regeneration Standard dot notation is used to indicate rates and accelerations. In this notation, the azimuth and elevation become:

$$\theta'_0 = \theta_0 + L_0 + R_0 \quad (3.35)$$

$$\phi'_0 = \phi_0 + L_0 + R_0 \quad (3.36)$$

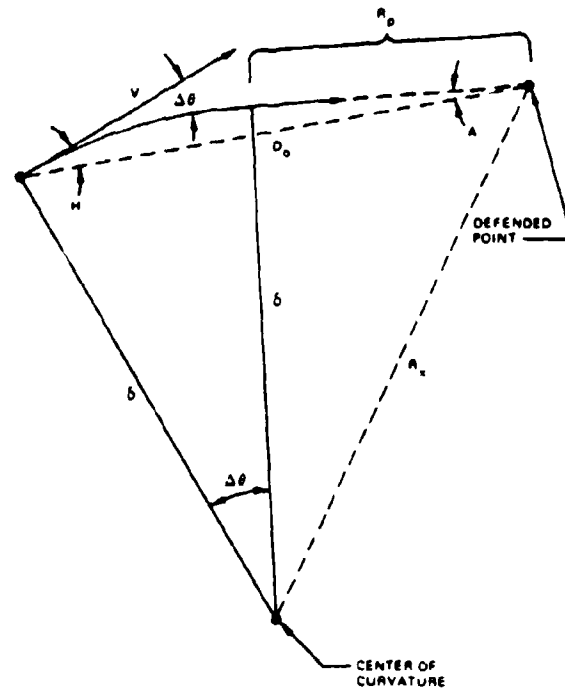


Figure 3-5. Geometry of a Turn Followed by a Linear Segment.

The lag computation is based on the four latest predicted positions. These four values are used to estimate current rate and acceleration values for the coordinate in question, which is in turn used in the expression:

$$L_A = \frac{\dot{A}}{K_V} + \frac{\ddot{A}}{K_B} \quad (3.37)$$

When the estimates are substituted into the above expression the lag equations become:

$$L_A = \left\{ \frac{21}{20K_V \Delta_t} + \frac{1}{2K_A \Delta_t^2} \right\} A_0 + \left\{ \frac{-13}{20K_V \Delta_t} + \frac{-1}{2K_A \Delta_t^2} \right\} A_{.1\Delta_t} + \left\{ \frac{-17}{20K_V \Delta_t} + \frac{-1}{2K_A \Delta_t^2} \right\} A_{.2\Delta_t} + \left\{ \frac{9}{20K_V \Delta_t} + \frac{1}{2K_A \Delta_t^2} \right\} A_{.3\Delta_t} \quad (3.38)$$

Values for regeneration are computed as follows: The current predicted position is transformed to cartesian coordinates. These are combined with current values for smoothed cartesian rates and transformed back to polar coordinates. The resulting polar rates are used to estimated polar acceleration using the following equations:

$$S = \sqrt{X_p^2 + Y_p^2} \quad (3.39)$$

$$\dot{S} = (X_p \dot{X}_p + Y_p \dot{Y}_p) / S \quad (3.40)$$

$$\ddot{\theta} = -2.0 \dot{S} \dot{\theta} / D \quad (3.41)$$

$$\ddot{\phi} = \frac{-2.0 D \dot{D} \dot{\phi} - S \dot{S} Z_p}{D^2} \quad (3.42)$$

These values are used in the standard two-term lag estimate yielding:

$$R_\theta = \left\{ \frac{\dot{\theta}}{K_V} + \frac{\ddot{\theta}}{K_A} \right\} \quad (3.43)$$

$$R_\phi = \left\{ \frac{\dot{\phi}}{K_V} + \frac{\ddot{\phi}}{K_A} \right\} \quad (3.44)$$

The projectile position, at current time plus time of flight (estimated time of closest approach) is given by: the predicted range ( $D_0$ ), the adjusted azimuth ( $\theta'$ ), and the adjusted elevation ( $\phi_0$ ). The projectile velocity is computed as the vector sum of the remaining velocity corresponding to the time of flight and a drop component. The remaining velocity is considered to be in the radial direction and the drop in the vertical

direction. The magnitude of the drop vector is computed as  $-gt$ .

Projectile position and velocity are transformed into cartesian coordinates for the computation of the actual closest approach values.

The time of closest approach is computed by correcting the estimated time of closest approach which is computed by the predictor. The correction is made by assuming: that the aircraft and projectile are moving at constant rates for the time period necessary for the projectile to get from either the predicted position to position of closest approach, or in the case when the closest approach occurs earlier than the predicted time, the time necessary to travel from the closest approach to the predicted position. Once this assumption is made, the time from predicted position to closest approach can be computed as:

$$t_c = |\bar{R} \times \bar{V}| / \bar{V} \cdot \bar{V} \quad (3.45)$$

where:

$\bar{R}$  = position of aircraft relative to the projectile at predicted impact.

$\bar{V}$  = velocity of the aircraft relative to the projectile at the predicted impact.

The time of closest approach is then given by:

$$t = t_p + t_c \quad (3.46)$$

The position of the shell at closest approach is given by:

$$\bar{R}_s = \bar{R}_p + \bar{V}_s t_c \quad (3.47)$$

If the error components are desired, they are computed as:

$$\bar{E} = \bar{R} + \bar{V} t_c \quad (3.48)$$

With  $R$ ,  $V$ , and  $t_c$  defined as above.

The time of closest approach, the position and velocity of the shell at closest approach, the position and velocity of the aircraft relative to the shell at closest approach, and the radial acceleration of the aircraft at closest approach are stored in the SHELL POSITION FILE.

### 3.5 REPORT GENERATOR EVENT

The function of the REPORT GENERATOR EVENT is to compute, tabulate, and print measures of performance. The particular performance measures reported, category boundary points for statistical outputs, and some system operating parameters are input in the initialization portion of this event. After initial-

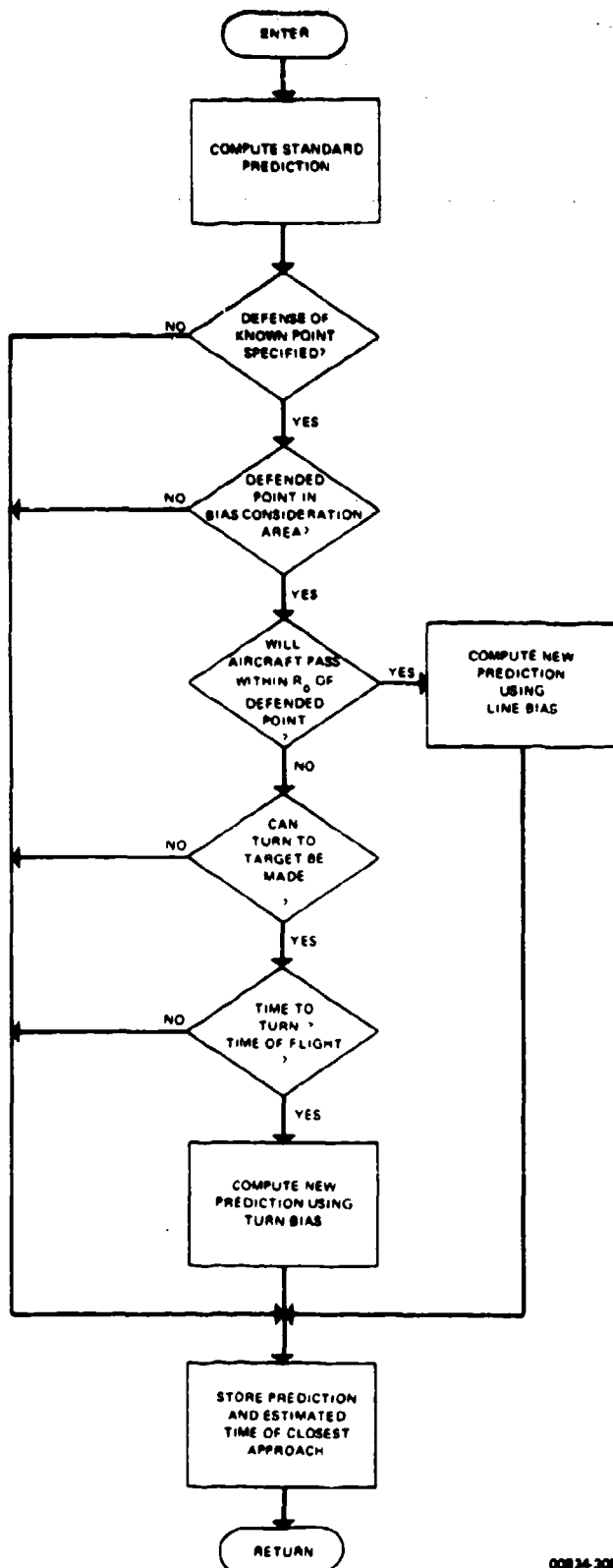


Figure 3-8. Logic Flow Diagram of the Prediction Event

ization, the event occurs once for each replicate of an experiment. On this entry, requested outputs are generated, printed and tabulated for an across replicate summary. On the last replicate of an experiment, the event also produces a report of across replicate statistics. Figure 3-7 presents the logical relationships of the above functions.

A rather unusual feature of the REPORT GENERATOR is the inclusion of the affects of dispersion, fire doctrine, and aircraft vulnerable volume computations in the report generating loops. This arrangement facilitates the direct comparison (using a constant aircraft path and stochastic tracking error sequence) of different dispersion and fire doctrine parameters. The presence of a 'nest' of two loops, one for dispersion and one for fire doctrine, allows the generation of reports for up to 100 combinations of dispersion parameters with fire doctrine parameters.

In the following sections, the content and method of computation of each report are presented. Reference is made to the following utility modules which perform major support functions:

- Aircraft Position Routine
- Fire Doctrine Routine and
- Vulnerability Routine

### 3.5.1 Single Replicate Reports

Measures of performance generated fall into four categories:

- a. Miss Distance Measures.
- b. Single Shot Probabilities.
- c. Burst Probabilities.
- d. Course Probabilities.

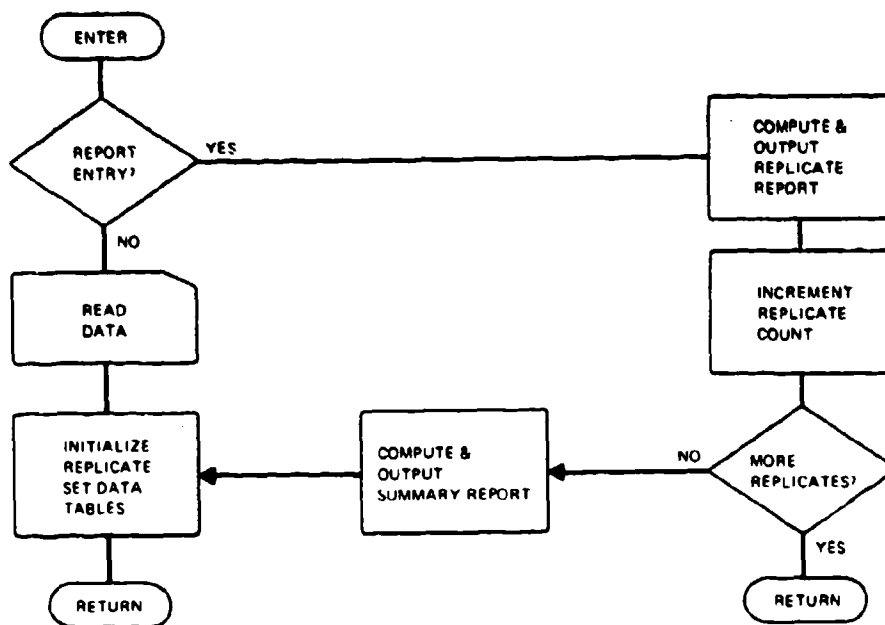
Data for a summary report is always generated for each category. In addition, detailed reports may be requested in any combination for the first three categories.

Figure 3-8 depicts the logical structure of the single replicate portion of the REPORT GENERATOR. Table III-2 presents the number of reports which can be generated for one replicate. These numbers reflect the loop structure of the REPORT GENERATOR.

#### 3.5.1.1 Miss Distance Reports

Miss distance data is computed using values stored in the SHELL POSITION FILE and accesses to the Aircraft Position Module. Values are computed without considering dispersion or fire doctrine. It is assumed that the gun fires continuously and that rounds fired at the specified point in time have zero dispersion and bias. When a detailed report is requested, one line of output for each sample time in the SHELL POSITION FILE is generated. Position and miss distance

00834-308



20871 185

Figure 3-7. Top Level Logic of the Report Generator

values are for the point of closest approach. The following data is presented:

- a. Time of fire (sec).
- b. Time of flight (sec).
- c. Range to aircraft (m).
- d. Azimuth of aircraft (degrees).
- e. Elevation of aircraft (degrees).
- f. Radial acceleration of aircraft (g).
- g. Range error (m).
- h. Azimuth error (m).
- i. Elevation error (m).
- j. Error along line of flight (m).\*
- k. Error perpendicular to line of flight (m).\*
- l. RMS error (m).\*

NOTE: \*In the plane normal to the relative velocity vector at closest approach.

A summary miss distance report is always generated and always tabulated for reporting in the across replicate report. The summary report consists of a partition of the samples computed on the basis of RMS error. Up to ten partition points are specified and the number

of samples which do not exceed the partition point of each category is tabulated and printed.

Figures 3-9 and 3-10 present samples of the miss distance report formats.

### 3.5.1.2 Single Shot Kill Probability Reports

Single shot kill probability computations are based on the data in the SHELL POSITION file, the dispersion parameter, and the vulnerable volume of the aircraft. The actual computation of the single shot kill probabilities are accomplished in the vulnerability module. The module is invoked once for each entry in the SHELL POSITION FILE (for which the aircraft is in range) and returns single shot hit probabilities for the: fuselage ( $P_f$ ), the wing ( $P_w$ ), and the fuselage-wing intersection ( $P_i$ ). These three values are combined using input parameters to form the single shot kill probability for the aircraft ( $P_a$ ). The most usual combination used was:

$$P_a = P_f + P_w - P_i \quad (3.49)$$

Values of  $P_a$  are tabulated for summary output and for the across replicate report.

When a detailed report is requested, one line of information for each entry in the SHELL POSITION FILE is generated. Each line contains the following information:

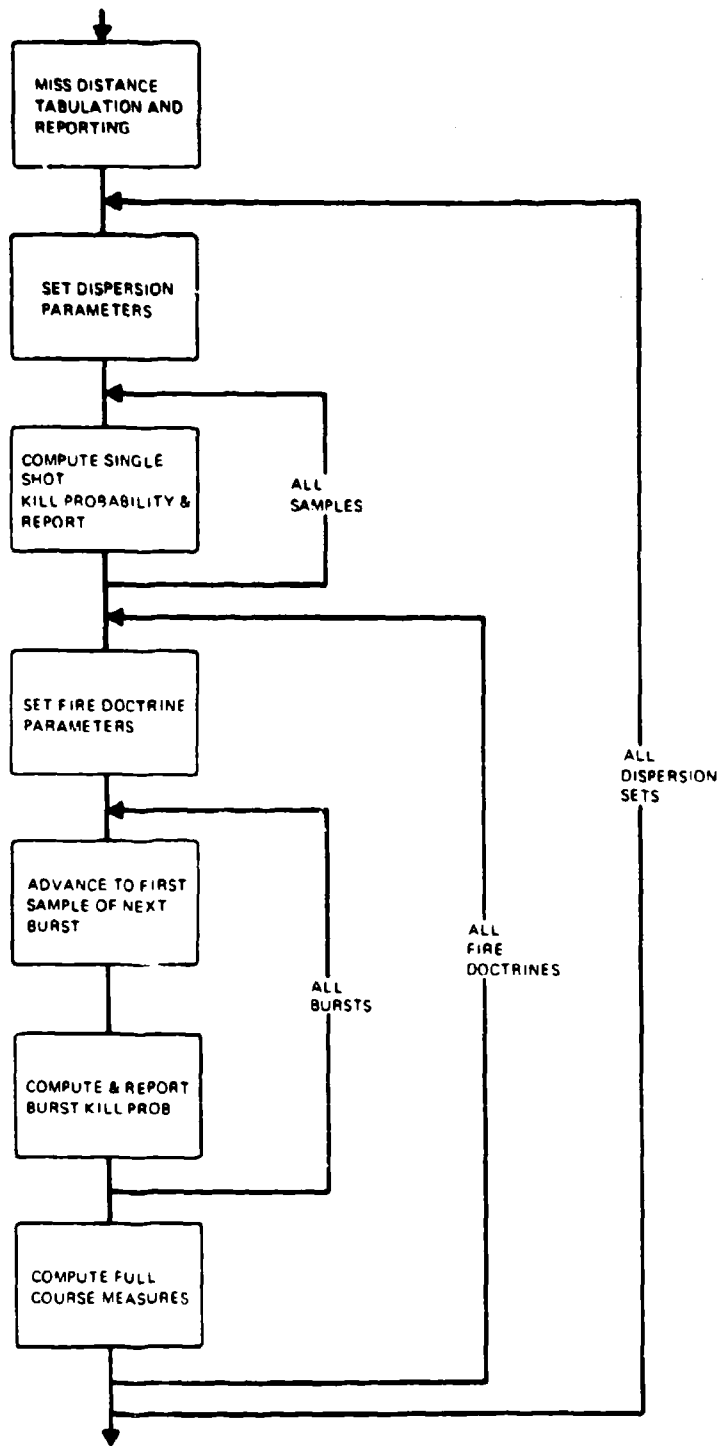


Figure 3-8. Computations For a Single Replicate

20871-186

Table III-2. Reports Generated

Report	Number of Reports
Miss Distance	1
Single Shot Kill Probability	n
Burst Kill Probability	n · m
Probability Over the Course	n · m

n = number of dispersion parameter sets ≤ 10  
m = number of fire doctrine parameter sets ≤ 10

20871-534

- Time of fire.
- Probability of hitting the fuselage.
- Probability of hitting the wings.
- Probability of hitting the fuselage-wing intersection.
- Single shot kill probability.

The summary report consists of a partition of the samples on the basis of the single shot kill probability. Up to ten partition points are specified in the input data. The number of samples which do not exceed the partition point for each category are tabulated and printed.

Figures 3-11 and 3-12 present samples of the single shot output reports.

### 3.5.1.3 Burst Reports

Burst measures are computed using the single shot kill probabilities generated in the single shot area and the fire doctrine parameters. The FIRE DOCTRINE Module is accessed to determine the time of fire of the first shot of a burst, the duration of the burst, and the number of rounds (n) in the burst. This data is used to access the table of single shot kill probabilities and compute an average value over the burst (p). The probability that a burst kills the aircraft (P<sub>B</sub>) is computed using the following equation:

$$P_B = 1.0 - e^{-n\bar{p}} \quad (3.50)$$

When requested, a detail report is generated. The detail report contains one line for each burst which can

MISS DISTANCE										
NUMBER OF SAMPLES WITH MISS DISTANCE LESS THAN										
1.00000	2.00000	3.00000	4.00000	5.00000	6.00000	8.00000	10.00000	15.00000	20.00000	
0	2	8	19	32	44	68	89	113	121	

Figure 3-9. Miss Distance Report

20871-304

be fired according to the fire doctrine. Each line contains the following:

- Time of fire.
- Average single shot kill probability.
- Probability that the burst kills the aircraft.

A summary burst kill report is always generated and always tabulated for across replicate reporting. The summary consists of a partition of the sample based on the computed values of burst kill probability. The sample is partitioned using up to ten input category points.

Figures 3-13 and 3-14 present examples of the burst kill probability reports.

### 3.5.1.4 Over the Course Measures

Two measures are generated over the entire course. They are the integral of the single shot kill probability over time which is independent of fire doctrine and the kill probability over the course, which is the kill probability resulting from combining the effects of all bursts of a fire doctrine.

The integral of the single shot kill probability is computed as follows:

$$\int P_s dt = \sum P_s \Delta t \quad (3.51)$$

The kill probability over the course is computed as follows:

$$P_c = 1 - e^{-n \sum P_s} \quad (3.52)$$

where:

- n = number of rounds per burst
- P<sub>s</sub> = average single shot kill probability over a burst, and the sum is over all bursts of a fire doctrine.

### 3.5.2 Across Replicate Summary Report

The across replicate summary report is made up of the summary data of each single replicate categories of measures and the kill probability over the course averaged over the set of replicates.

Samples of these reports are presented in Figures 3-15 through 3-18.

Copy available to DDC does not permit fully legible reproduction

TIME OF FIRE	TIME OF FLIGHT (M)	RANGE (M)	AZIMUTH (DEG)	ELEVATION (DEG)	RADIAL ACCEL. (G)	RANGE ERROR (M)	AZIMUTH ERROR (M)	ELEVATION ERROR (M)	MU (M)	MV (M)	RMS (M)
6.7	1.233	1647.657	-4.654	16.650	-0.200	2.216	5.729	1.316	-5.983	1.931	6.287
6.8	1.232	1547.159	-4.555	16.650	-0.200	2.177	5.754	1.322	-5.956	1.927	6.298
6.9	1.228	1644.264	-1.305	16.656	-0.200	1.162	3.025	0.307	-3.194	0.639	3.237
7.0	1.228	1644.264	-1.305	16.656	-0.200	1.133	3.037	0.310	-3.196	0.634	3.258
7.1	1.028	1644.620	2.139	16.690	-0.200	0.974	2.541	-0.031	-2.711	0.254	2.723
7.2	1.029	1644.623	2.145	16.689	-0.200	0.506	2.442	-0.056	-2.597	0.210	2.606
7.3	1.031	1644.475	5.609	16.626	-0.200	-0.344	-1.125	1.075	-1.282	0.947	1.593
7.4	1.032	1648.469	5.618	16.626	-0.200	-0.395	-1.276	1.040	-1.434	0.898	1.692
7.5	1.033	1648.118	9.197	16.592	-0.200	1.702	4.131	0.604	-4.380	1.991	4.512
7.6	1.034	1648.194	9.203	16.501	-0.200	1.622	4.029	0.581	-4.261	1.936	4.385
7.7	1.036	1667.549	12.718	16.321	-0.200	1.146	2.647	0.694	-2.793	1.005	2.968
7.8	1.037	1667.542	12.730	16.320	-0.200	1.024	2.442	0.650	-2.565	0.927	2.728
7.9	1.076	1693.891	16.272	16.089	-0.200	0.794	1.677	1.379	-1.597	1.527	2.237
8.0	1.077	1693.193	16.247	16.079	-0.200	0.647	1.313	1.322	-1.312	1.474	1.973
8.1	1.078	1693.193	16.247	16.079	-0.200	1.844	3.990	1.761	-4.190	2.239	4.734
8.2	1.079	1693.193	16.247	16.079	-0.200	1.049	3.673	1.694	-3.826	2.112	4.370
8.3	1.135	1626.989	23.271	15.436	-0.200	2.150	4.457	2.505	-4.659	3.031	5.559
8.4	1.136	1626.989	23.271	15.436	-0.200	1.907	4.073	2.423	-4.219	2.885	5.112
8.5	1.174	1635.628	26.679	15.066	-0.200	1.129	1.616	4.283	-1.467	4.484	4.718
8.6	1.176	1635.628	26.679	15.066	-0.200	0.852	1.078	5.169	-0.887	4.300	4.391
8.7	1.215	1638.162	29.974	14.618	-0.200	1.746	2.501	6.204	-2.328	6.514	6.917
8.8	1.216	1638.162	29.974	14.618	-0.200	1.396	1.953	6.062	-1.625	6.288	6.495
8.9	1.217	1638.162	29.974	14.618	-0.200	1.503	1.401	7.938	-1.065	8.134	8.204
9.0	1.173	1626.463	33.175	14.155	-0.200	1.107	0.849	7.767	-0.253	7.876	7.876
9.1	1.119	1606.216	36.175	13.365	-0.200	1.221	5.469	10.968	6.561	10.413	12.308
9.2	1.122	1606.216	36.175	13.365	-0.200	1.011	6.422	16.740	7.552	10.095	12.608
9.3	1.167	1616.870	39.226	13.209	-0.200	0.605	0.140	5.614	0.417	5.634	5.649
9.4	1.264	1615.150	39.642	13.206	-0.200	0.417	-0.509	5.520	0.814	5.500	5.560
9.5	1.264	1615.150	39.642	13.206	-0.200	3.422	6.248	4.525	-6.660	5.195	9.447
9.6	1.366	1664.756	41.873	12.998	-0.200	3.078	5.786	4.395	-6.119	4.997	7.900
9.7	1.537	1417.664	44.421	12.215	-0.200	1.381	2.343	2.318	-2.489	2.566	3.575
9.8	1.539	1417.664	44.421	12.215	-0.200	1.165	2.030	2.217	-2.101	2.421	3.206
9.9	1.628	1477.670	46.943	11.714	-0.200	3.777	8.379	0.033	-9.166	0.900	9.201
10.0	1.629	1477.670	46.943	11.714	-0.200	3.601	8.231	-0.015	-8.965	0.718	8.994
10.1	1.721	1547.267	49.272	11.231	-0.200	3.655	8.101	-0.328	-8.854	0.390	8.902
10.2	1.827	1545.532	49.291	11.225	-0.200	3.349	7.858	-0.414	-8.572	0.254	8.576
10.3	1.915	1625.653	51.623	10.170	-0.200	0.284	-0.379	2.455	0.420	2.469	2.505
10.4	1.917	1625.653	51.623	10.170	-0.200	0.053	-0.746	2.298	0.881	2.267	2.432
10.5	1.280	1527.162	48.807	11.325	-0.200	8.728	-24.945	10.960	27.070	8.979	28.520
10.6	1.143	1527.168	48.829	11.324	-0.200	8.666	-25.524	10.763	27.561	8.803	28.932
10.7	1.132	1527.168	48.829	11.324	-0.200	2.085	4.181	2.463	-6.658	2.810	5.444
10.8	1.133	1527.168	48.829	11.324	-0.200	1.894	4.099	2.357	-6.332	2.676	5.092
10.9	1.157	1647.017	52.676	10.495	-0.200	9.466	28.848	-5.529	-30.895	3.656	31.110
11.0	1.208	1647.145	52.676	10.494	-0.200	9.527	29.745	-5.571	-30.647	3.777	30.879
11.1	1.431	1701.236	54.124	10.157	-0.200	1.056	0.844	4.298	-0.850	4.419	4.500
11.2	1.333	1701.674	54.195	10.154	-0.200	0.863	0.463	4.150	-0.473	4.238	4.265
11.3	1.485	1761.448	55.720	9.806	-0.200	2.971	5.844	3.824	-6.273	4.281	7.595
11.4	1.367	1761.911	55.731	9.803	-0.200	2.695	5.503	3.664	-5.867	4.075	7.144
11.5	1.631	1820.559	57.193	9.485	-0.200	3.406	5.417	6.367	-5.883	6.855	9.034

20871.305

Figure 3-10. Miss Distance Detail Report

Copy available to DDC does not  
 permit fully legible reproduction

PATH 1, SMOOTHING TIME 3.6S, PERIODIC INVISIBILITY, SENSOR FAILURE  
 SIGX= 0.300E 01 SIGZC= 0.500E 01 SIGVO= 0.100E 03 VO BIAS= 0.0

SINGLE SHOT KILL PROBABILITY

TIME OF FIRE	SINGLE SHOT P(KILL)			
	FUSELAGE	WING	INTERSECTION	AIRCRAFT
13.6	0.717E-02	0.416E-02	0.153E-02	0.980E-02
13.8	0.472E-03	0.147E-05	0.701E-10	0.549E-09
14.0	0.900E-43	0.269E-34	0.299E-34	0.897E-33
14.2	0.170E-46	0.111E-49	0.200E-48	0.199E-46
14.4	0.103E-31	0.268E-33	0.313E-33	0.102E-31
14.6	0.122E-14	0.222E-15	0.101E-15	0.134E-14
14.8	0.604E-17	0.101E-17	0.409E-18	0.665E-17
15.0	0.927E-12	0.199E-12	0.940E-13	0.103E-11
15.2	0.263E-06	0.922E-07	0.436E-07	0.311E-06
15.4	0.937E-08	0.277E-09	0.127E-08	0.108E-07
15.6	0.103E-07	0.332E-08	0.140E-08	0.123E-07
15.8	0.503E-13	0.192E-13	0.493E-14	0.646E-13
15.0	0.305E-14	0.130E-14	0.293E-15	0.406E-14
16.2	0.349E-07	0.126E-07	0.503E-08	0.423E-07
16.4	0.124E-05	0.484E-06	0.205E-06	0.152E-05
16.6	0.467E-06	0.179E-06	0.752E-07	0.571E-06
16.8	0.237E-06	0.106E-06	0.389E-07	0.304E-06
17.0	0.142E-07	0.650E-08	0.221E-08	0.185E-07
17.2	0.418E-08	0.166E-09	0.620E-09	0.521E-08
17.4	0.124E-08	0.584E-09	0.185E-09	0.165E-08
17.6	0.115E-06	0.468E-07	0.185E-07	0.143E-06
17.8	0.146E-04	0.618E-05	0.265E-05	0.181E-04
18.0	0.105E-03	0.507E-04	0.206E-04	0.135E-03
18.2	0.191E-03	0.904E-04	0.378E-04	0.243E-03
18.4	0.898E-06	0.442E-06	0.155E-06	0.119E-05
18.6	0.117E-10	0.543E-11	0.168E-11	0.195E-10
18.8	0.589E-14	0.586E-14	0.772E-15	0.120E-13
19.0	0.927E-21	0.206E-20	0.100E-21	0.287E-20
19.2	0.172E-24	0.497E-24	0.173E-25	0.552E-24
19.4	0.485E-42	0.518E-41	0.302E-43	0.563E-41
19.6	0.149E-66	0.973E-65	0.501E-68	0.983E-65
19.8	0.539E-77	0.496E-75	0.0	0.501E-75
20.0	0.309E-74	0.177E-72	0.107E-75	0.180E-72
20.2	0.307E-09	0.373E-09	0.176E-09	0.100E-08
20.4	0.759E-45	0.605E-43	0.237E-45	0.610E-43
20.6	0.434E-40	0.162E-39	0.143E-40	0.164E-38
20.8	0.316E-18	0.619E-18	0.883E-19	0.104E-17
21.0	0.102E-07	0.600E-08	0.225E-08	0.145E-07
21.2	0.294E-03	0.193E-08	0.558E-09	0.430E-08
21.4	0.164E-09	0.165E-09	0.312E-10	0.298E-09
21.6	0.427E-07	0.781E-07	0.134E-07	0.127E-06
21.8	0.254E-05	0.274E-05	0.550E-05	0.473E-05
22.0	0.149E-06	0.115E-06	0.306E-07	0.236E-06
22.2	0.127E-08	0.100E-08	0.247E-09	0.203E-08
22.4	0.525E-14	0.381E-14	0.690E-15	0.518E-14
22.6	0.135E-21	0.448E-22	0.188E-22	0.201E-21
22.8	0.145E-27	0.809E-28	0.174E-28	0.211E-27
23.0	0.247E-17	0.188E-17	0.382E-18	0.370E-17
23.2	0.137E-14	0.959E-15	0.234E-15	0.210E-14
23.4	0.194E-17	0.110E-17	0.295E-18	0.265E-17

20071-206

Figure 3-11. Single Shot Kill Probability Detail Report

Copy available to DDC does not permit fully legible reproduction

SINGLE SHOT KILL PROBABILITY									
NUMBER OF SAMPLES WITH PROBABILITY GREATER THAN									
0.7500	0.5000	0.2500	0.1000	0.0500	0.0250	0.0050	0.0040	0.0020	0.0001
0	0	0	1	2	5	21	22	15	18

Figure 3-12. Single Shot Kill Probability Report

20871-307

FIRE DOCTRINE NUMBER 1			
SIGXD=	0.300E 01	SIGZN=	0.500E 01
SIGVD=	0.100E 02	VD RIAS=	0.0
START TIME	PSS AVER	PKILL)	
1.800	0.001	0.577E-01	
5.000	0.002	0.971E-01	
8.200	0.000	0.138E-01	
11.400	0.007	0.290E 00	
14.600	0.009	0.346E 00	
17.800	0.011	0.412E 00	
21.000	0.005	0.206E 00	
24.200	0.000	0.194E-02	
27.400	0.000	0.299E-02	

20871-308

Figure 3-13. Burst Kill Probability Detail Report

FIRE DOCTRINE NUMBER 1									
SIGXD=	0.300E 01	SIGZN=	0.500E 01	SIGVD=	0.100E 02	VD RIAS=	0.0		
BURST KILL PROBABILITY									
NUMBER OF SAMPLES FOR WHICH PKILL EXCEEDS									
0.7500	0.5000	0.2500	0.1000	0.0500	0.0250	0.0050	0.0040	0.0020	0.0001
0	0	2	4	6	7	7	7	8	9

20871-309

Figure 3-14. Burst Kill Probability Report

MISS DISTANCE										
NUMBER OF SAMPLES WITH MISS DISTANCE LESS THAN										
1.00000	2.00000	3.00000	4.00000	5.00000	6.00000	8.00000	10.00000	15.00000	20.00000	
0	3	9	18	30	42	67	85	108	114	

20871-310

Figure 3-15. Miss Distance Summary Report

Copy available to DDC does not permit fully legible reproduction

Copy available to DDC does not permit fully legible reproduction

KILL-SEC FOR THE COURSE

SIZE	0.500	0.500	0.500	0.500	0.500	3.000	3.000	3.000	3.000	3.000
SIZE	0.500	0.500	0.500	0.500	0.500	3.000	3.000	3.000	3.000	3.000
SIZE	0.0	0.000	10.000	15.000	30.000	0.0	0.000	10.000	15.000	30.000
VARIAS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KILL-SEC	0.111	0.112	0.109	0.094	0.065	0.184	0.177	0.164	0.147	0.102

20871-311

Figure 3-16. Kill Seconds Over the Course Report

SINGLE SHOT KILL PROBABILITY

DISP TYPE NUMBER	NUMBER OF SAMPLES WITH PROBABILITY GREATER THAN									
	0.7500	0.5000	0.2500	0.1000	0.0500	0.0250	0.0125	0.00625	0.003125	0.0015625
1	0	0	1	3	4	7	11	11	13	20
2	0	0	1	3	5	7	12	13	15	22
3	0	0	0	3	6	8	13	14	16	25
4	0	0	0	3	6	8	13	14	16	24
5	0	0	0	1	5	8	13	14	16	27
6	0	0	0	0	7	27	71	77	93	117
7	0	0	0	0	6	26	72	79	95	119
8	0	0	0	0	2	24	74	81	98	120
9	0	0	0	0	0	18	74	81	99	122
10	0	0	0	0	0	3	67	74	93	125

20871-312

Figure 3-17. Single Shot Kill Probability Summary Report

KILL PROBABILITY OVER THE COURSE

DISPERICA	0.5000	0.5000	0.5000	0.5000	0.5000	3.0000	3.0000	3.0000	3.0000	3.0000
SIZE	0.5000	0.5000	0.5000	0.5000	0.5000	3.0000	3.0000	3.0000	3.0000	3.0000
SIZE	0.0	0.0000	10.0000	15.0000	30.0000	0.0	0.0000	10.0000	15.0000	30.0000
VARIAS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

File: D:\1\1194

1	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.03/0.02	0.03/0.02	0.03/0.02	0.03/0.02	0.03/0.01
2	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.13/0.07	0.13/0.06	0.13/0.06	0.12/0.04	0.10/0.03
3	0.00/0.00	0.00/0.00	0.00/0.00	0.00/0.00	0.01/0.00	0.28/0.11	0.28/0.11	0.27/0.10	0.26/0.09	0.25/0.07
4	0.00/0.00	0.01/0.00	0.01/0.01	0.01/0.01	0.01/0.01	0.51/0.14	0.51/0.17	0.49/0.15	0.47/0.12	0.45/0.09
5	0.01/0.01	0.11/0.02	0.11/0.04	0.11/0.04	0.10/0.04	0.65/0.17	0.64/0.16	0.62/0.15	0.58/0.14	0.45/0.11
6	0.25/0.06	0.27/0.07	0.20/0.06	0.28/0.05	0.24/0.02	0.72/0.23	0.71/0.22	0.70/0.21	0.67/0.19	0.56/0.16
7	0.35/0.09	0.38/0.09	0.41/0.09	0.41/0.09	0.37/0.07	0.77/0.19	0.77/0.19	0.76/0.19	0.74/0.18	0.63/0.13
8	0.35/0.09	0.38/0.09	0.41/0.09	0.41/0.09	0.40/0.09	0.83/0.17	0.82/0.16	0.81/0.16	0.78/0.15	0.67/0.13
9	0.44/0.10	0.46/0.10	0.46/0.10	0.45/0.10	0.39/0.09	0.83/0.12	0.82/0.12	0.79/0.12	0.75/0.12	0.61/0.11
10	0.31/0.03	0.34/0.03	0.34/0.03	0.34/0.03	0.19/0.02	0.58/0.22	0.56/0.21	0.52/0.19	0.47/0.17	0.34/0.13

Figure 3-18. Kill Probability Over the Course Report

20871-313

Copy available to DDC does not permit fully legible reproduction

## SECTION 4 UTILITY PACKAGE

### 4.1 AIRCRAFT POSITION MODULE

The Aircraft Position Routine provides the position and velocity of the aircraft in cartesian coordinates centered at the gun for the simulation time specified in the invoking command.

Paths are represented as sequences of continuous path segments proceeding from an initial point of known position and velocity. Each path segment is either a segment of a line, a segment of a circle, or a segment of a logarithmic spiral. All path segments are in the horizontal plane coupled with motion of constant acceleration (possibly zero) in the vertical direction. Smooth positions and velocities are assured by using the end conditions of one segment as the initial conditions of its successor. The end point of a segment is determined relative to its initial point with the 'duration' of the segment being specified by: length for a line segment, and degrees of turn for both circular and spiral segments. The only position and velocity specification made is for the initial point. Further path description is in the form of the following statements:

- a. Line 'X' meters in length, tangential acceleration 'A<sub>t</sub>' and vertical acceleration 'A<sub>z</sub>'
- b. Circular turn 'φ' degrees right or left radial acceleration 'A<sub>r</sub>' and vertical acceleration 'A<sub>z</sub>'
- c. Spiral turn 'φ' degrees right or left, radial acceleration 'A<sub>r</sub>', tangential acceleration 'A<sub>t</sub>', and vertical acceleration 'A<sub>z</sub>'

For linear segments position and velocity are determined according to the equations:

$$X(t) = X_0 + (V_0 t + \frac{1}{2} A_T t^2) \cos \theta \quad (4.1)$$

$$Y(t) = Y_0 + (V_0 t + \frac{1}{2} A_T t^2) \sin \theta \quad (4.2)$$

$$Z(t) = Z_0 + V_{Z_0} t + \frac{1}{2} A_Z t^2 \quad (4.3)$$

$$V_X(t) = -(V_0 + A_T t) \cos \theta \quad (4.4)$$

$$V_Y(t) = (V_0 + A_T t) \sin \theta \quad (4.5)$$

$$V_Z(t) = V_{Z_0} + A_Z t \quad (4.6)$$

where:  $\theta$  = aircraft heading

For circular segments, position and velocity are determined according to the equations:

$$R = V_0^2 / A_R \quad (4.7)$$

$$\dot{\theta} = V_0 / R \quad (4.8)$$

$$\theta(t) = \theta_0 + \dot{\theta} t \quad (4.9)$$

$$X(t) = X_0 + R [\cos \theta_0 - \cos \theta(t)] \quad (4.10)$$

$$Y(t) = Y_0 + R [\sin \theta_0 + \sin \theta(t)] \quad (4.11)$$

$$Z(t) = Z_0 + V_{Z_0} t + \frac{1}{2} A_Z t^2 \quad (4.12)$$

$$V_X(t) = -V \sin \theta(t) \quad (4.13)$$

$$V_Y(t) = V \cos \theta(t) \quad (4.14)$$

$$V_Z(t) = V_{Z_0} + A_Z t \quad (4.15)$$

For spiral segments position and velocity are determined according to the equations:

$$\lambda = A_T / A_R \quad (4.16)$$

$$V(t) = V_0 + A_T t \quad (4.17)$$

$$R = V_0^2 / A_R \quad (4.18)$$

$$\theta(t) = \theta_0 - R \log_e [V^2(t) / 2\lambda V_0^2] \quad (4.19)$$

$$X(t) = X_0 - \frac{V^2(t)}{A_R (1 + 4\lambda^2)} \left\{ 2\lambda \left[ \sin \theta_0 - \frac{V^2(t)}{V_0^2} \sin \theta(t) \right] - \left[ \cos \theta_0 - \frac{V^2(t)}{V_0^2} \cos \theta(t) \right] \right\} \quad (4.20)$$

$$Y(t) = Y_0 + \frac{V^2(t)}{A_R (1 + 4\lambda^2)} \left\{ 2\lambda \left[ \frac{V^2(t)}{V_0^2} \cos \theta(t) - \cos \theta_0 \right] - \left[ \sin \theta_0 - \frac{V^2(t)}{V_0^2} \sin \theta_0 \right] \right\} \quad (4.21)$$

$$Z(t) = Z_0 + V_{Z_0}t + \frac{1}{2}A_z t^2 \quad (4.22)$$

$$V_X(t) = -V(t) \sin [\theta(t)] \quad (4.23)$$

$$V_Y(t) = V(t) \cos [\theta(t)] \quad (4.24)$$

$$V_Z(t) = V_{Z_0} + A_z t \quad (4.25)$$

#### 4.2 BALLISTICS MODULE

The function of the BALLISTICS MODULE is to provide shell ballistics information. The three entry points used during the execution phase are as follows:

- PRANGE*. Provides range as a function of time of flight.
- PVELOC*. Provides remaining velocity as a function of time of flight.
- TRANGE*. Provides time of flight as a function of range.

During the initialization phase, a table of remaining velocity, range, and time of flight is input. The table may be ragged and need not start at the desired muzzle velocity. Values of muzzle velocity and a ballistic coefficient or a point on the trajectory are also input. The input tables are adjusted to fit the input data and interpolation is carried out to produce equally spaced tables.

The tables produced are as follows:

- Range as a function of Time of Flight.
- Remaining Velocity as a function of Time of Flight.
- Time of Flight as a function of Range.

During the execution phase, any values required are obtained by a table look-up and linear interpolation in the appropriate table. Note that the ballistic information is isolated in this module and, therefore, any desired ballistics could be used with the model. Modification could be limited to performing any necessary initialization in the initialization entry, thus providing the three required functions by using the appropriate names and calling sequences.

#### 4.3 TIME OF FLIGHT MODULE

The Time of Flight Module computes the time required for a projectile to travel from the gun to a future aircraft position. The complexity of the problem is increased, because the predicted position is dependent upon the time of flight. Closed-form solutions of the problem are possible only under some rather restrictive assumptions. For this reason numerical methods have been employed. The time of flight computa-

tion is based on the fact that, at the time of predicted impact, the range to the aircraft ( $R_a$ ) and the range to the projectile ( $R_p$ ) must be equal. In essence then, the Time of Flight Module numerically solves the equation:

$$f(t) = R_a(t) - R_p(t) = 0 \quad (4.26)$$

for the smallest value of  $t$ .

The values of aircraft range are provided by the Prediction Algorithm Module and are dependent upon the values currently stored in the SENSED DATA File as well as the particular options for prediction being simulated. Values for  $R_a(t)$  are obtained by invoking the 'range as a function of time' entry of the ballistics package.

A start value for the solution is obtained by assuming a projectile which does not deaccelerate, and an aircraft which is in unaccelerated motion:

$$T_{start} = R_a(0) \left[ \frac{R_a(0)}{\sqrt{R_a^2(0) + v_m^2 - v_a^2(0)}} \right] / (v_m^2 - v_a^2(0)) \quad (4.27)$$

where:

$v_m$  = muzzle velocity of gun.

$v_a$  = current aircraft velocity.

$R_a$  = current range rate of the aircraft.

An initial computation of the function  $f(t) = R_a(t) - R_p(t)$  is made using the start value. The process is continued with each successive computation being made at a new time, which is the current value incremented by a function of the start value.

Once a sign change has been obtained, a combination of the secant method and interval bisection is employed until the desired accuracy is obtained. In general, no more than five computations are required to derive the required accuracy.

#### 4.4 PREDICTION ALGORITHM MODULE

The function of the Prediction Algorithm Module is to extrapolate current available aircraft position and velocity data to a specified future time point, thereby, producing an estimate of aircraft position for the specified time. The module has the capability of performing the extrapolation by a number of methods, each of which is somewhat flexible in that its specific behavior is determined by method parameters. The specific method, employed during a simulation run, is

determined by the designation of the general method type and its required parameters during the initialization phase of the run. The position data on which the module operates are all derived from the output of the sensor module. These data are accessed via the communications pool (specifically the SENSED DATA File).

Current module capabilities include the following three general methods:

- a. Quadratic (includes linear by parameter specification).
- b. Simple polar.
- c. Defense of a known point (in combination with quadratic).

#### 4.4.1 Quadratic Prediction

The quadratic prediction is made, using smoothed position (X, Y, Z), velocity (X, Y, Z), and acceleration (X, Y, Z) values computed and stored in the SENSED POSITION File by the TRACK Module. The prediction is computed as a linear estimate, corrected for accelerations in the horizontal plane, and changes in total aircraft speed induced by changes in aircraft altitude. The basic estimate for a time increment  $\tau$  is given by:

$$X' = \underline{X} + \underline{\dot{X}}\tau \quad (4.28)$$

$$Y' = \underline{Y} + \underline{\dot{Y}}\tau \quad (4.29)$$

$$Z' = \underline{Z} + \underline{\dot{Z}}\tau \quad (4.30)$$

The correction for acceleration in the horizontal plane is made whenever the absolute value of the estimated heading rate  $\theta$  of the aircraft exceeds the value of an input threshold parameter. The specification of an excessively large threshold value leads to suppression of the acceleration correction and linear prediction results. The heading rate is estimated by the equation:

$$\dot{\theta} = (\ddot{X}\dot{Y} - \ddot{Y}\dot{X})/(\dot{X}^2 + \dot{Y}^2) \quad (4.31)$$

When the correction is included, the resultant predictions for X and Y are as follows:

$$F = \tau^2 + a\tau + b \quad (4.32)$$

$$X' = \underline{X} + \underline{\dot{X}}\tau - 1/2 F \dot{\theta} \underline{\dot{Y}} \quad (4.33)$$

$$Y' = \underline{Y} + \underline{\dot{Y}}\tau - 1/2 F \dot{\theta} \underline{\dot{X}} \quad (4.34)$$

where:

$$a = 2.0 T_{v/a}$$

$$b = 2.0 (T_{v/a} \cdot T_{p/v} + T_{p/a}^2)$$

$T_{p/v}$  = Time lag of position due to constant velocity

$T_{p/a}^2$  = Time lag coefficient of position due to constant acceleration

$T_{v/a}$  = Time lag of velocity due to constant acceleration

The correction for change in aircraft speed, due to a change in altitude, is made whenever the absolute value of the aircraft velocity, in the vertical direction, exceeds the value of an input threshold parameter, as in acceleration, the correction can be suppressed by specification of an excessive value for that parameter. When the correction is included, the resultant predictions are as follows:

$$X' = \underline{X} + \underline{\dot{X}}\tau - 1/2 F [\dot{\theta} \underline{\dot{Y}} + g(\underline{\dot{X}}\underline{\dot{Z}}/V^2)] \quad (4.35)$$

$$Y' = \underline{Y} + \underline{\dot{Y}}\tau + 1/2 F [\dot{\theta} \underline{\dot{X}} - g(\underline{\dot{X}}\underline{\dot{Z}}/V^2)] \quad (4.36)$$

$$Z' = \underline{Z} + \underline{\dot{Z}}\tau + 1/2 F [g(\underline{\dot{Z}}^2/V^2)] \quad (4.37)$$

where:

F is defined as above

g is acceleration due to gravity

V is the aircraft velocity.

#### 4.4.2 Simple Polar Prediction

The simple polar prediction is made by estimating the range rate R, the azimuth rate A, and the elevation rate E. Assuming constant rates, and using linear extrapolation, polar coordinate rates are estimated by transforming the cartesian coordinates of the last two entries in the SENSED POSITION File and computing first-divided differences. Thus:

$$\dot{R} = (R_t - R_{t-\Delta t})/\Delta t \quad (4.38)$$

$$\dot{A} = (A_t - A_{t-\Delta t})/\Delta t \quad (4.39)$$

$$\dot{E} = (E_t - E_{t-\Delta t})/\Delta t \quad (4.40)$$

Estimated range, azimuth, and elevation are given by the following equations:

$$R' = R_t + \dot{R}\tau \quad (4.41)$$

$$A' = A_t + \dot{A}\tau \quad (4.42)$$

$$E' = E_t + \dot{E}\tau \quad (4.43)$$

The required cartesian estimates are obtained by transforming  $R'$ ,  $A'$ , and  $E'$ .

#### 4.4.3 Defense of a Known Point

The majority of the logic and computation for the defense of a known point option are in the Prediction Event Module. However, for time of flight computations, use is made of the Prediction Algorithm Module. During such entries to the module, one of two modes of prediction are possible: Quadratic (discussed previously) or point-to-point linear interpolation. An indicator switch, set in the Prediction Event Module, controls which method is used on any given entry to the module. The following pertains to point-to-point linear interpolation, used for a portion of the attack, of a known point option.

Figure 4-1 presents the geometry for the point-to-point linear interpolation. The point Q represents the last sensed aircraft position and P the point being defended. The prediction method assumes that the aircraft will travel along the line connecting the points, at rates which are consistent in magnitude to the current smoothed aircraft speed. Distance along the line is computed using the following equations:

$$\Delta X = X_p - X_Q \quad (4.44)$$

$$\Delta Y = Y_p - Y_Q \quad (4.45)$$

$$\Delta Z = Z_p - Z_Q \quad (4.46)$$

$$D = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} \quad (4.47)$$

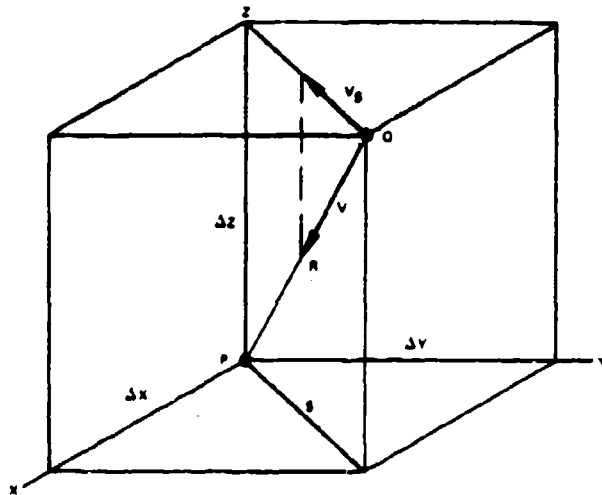
$$L(1) = V_t + \frac{1}{28} \left( 12 + 1T_s + \frac{T_s^2}{6} \right) \frac{\Delta Z}{D} \quad (4.48)$$

The desired estimate is then obtained by interpolation:

$$X' = X_Q + \Delta X \cdot L(1)/D \quad (4.49)$$

$$Y' = Y_Q + \Delta Y \cdot L(1)/D \quad (4.50)$$

$$Z' = Z_Q + \Delta Z \cdot L(1)/D \quad (4.51)$$



00824 401

Figure 4-1. Geometric Representation of the Point-to-Point Linear Interpolation

#### 4.5 RANDOM NUMBER GENERATOR

The function of the Random Number Generator is to produce samples of random variables from either a population uniformly distributed on the interval [0,1], or from a population which is normally distributed with mean zero and variance one. The uniform random variables are generated by using an additive congruous process with 31 seeds. This method was based on work, which was done at MIT's Lincoln Laboratory in 1958, by B.F. Green, J.E. Smith, and L. Klem. The sequence of samples is produced by using the following equations:

$$X_j = (X_{j-1} + X_{j-n}) \text{ Mod } 2^r \quad j \geq n \quad (4.52)$$

$$R_{j-n+1} = X_j / 2^r \quad (4.53)$$

where:

$$n = 31$$

$$r = 29$$

$$X_1 = \text{an input seed variable}$$

$$X_2 - X_{31} = \text{prestored seed values}$$

$$R_i = \text{the required uniformly distributed random numbers.}$$

Normally-distributed variables are computed from the uniformly-distributed random variables, using the well known Central Limit Theorem. The exact equation employed is as follows:

$$N = \sum_{i=1}^{12} R_i - 6.0 \quad (4.54)$$

where:

N = the required sample

R = uniformly distributed random variables

This module has the ability to return any number of normal or uniform numbers on any given entry. The sequence of samples generated is a function of the first seed value, which is an input variable.

#### 4.6 COORDINATE TRANSFORMATION

There are two coordinate transformation programs. One converts polar coordinates to cartesian coordinates and the other converts cartesian coordinates to polar coordinates. Both programs convert the three position coordinates, plus their respective rates. The transformations are not standard, due to the fact that the azimuth angle is measured counter-clockwise from the positive y-axis. This system was used in an attempt to remain as compatible as possible with systems employed in the University of Michigan Gun Model.

The cartesian to polar transformation uses Equations 4.55 through 4.61. The equations are computed in the order shown and earlier results are used in later computation in order to minimize computer time.

$$R = \sqrt{X^2 + Y^2 + Z^2} \quad (4.55)$$

$$S = \sqrt{X^2 + Y^2} \quad (4.56)$$

$$\theta = \tan^{-1}(-X/Y) \quad (4.57)$$

$$\phi = \tan^{-1}(Z/S) \quad (4.58)$$

$$\dot{R} = (X\dot{X} + Y\dot{Y} + Z\dot{Z})/R \quad (4.59)$$

$$\dot{\theta} = (X\dot{Y} - \dot{X}Y)/S^2 \quad (4.60)$$

$$\dot{\phi} = (Z - Z\dot{R}/R)/S \quad (4.61)$$

The polar to cartesian conversion uses the following equations:

$$S = R \cos \phi \quad (4.62)$$

$$Z = R \sin \phi \quad (4.63)$$

$$X = -S \sin \theta \quad (4.64)$$

$$Y = S \cos \theta \quad (4.65)$$

$$T = \dot{R}/R \cdot Z\dot{\phi}/S \quad (4.66)$$

$$\dot{X} = X\dot{\theta} - Y\dot{\theta} \quad (4.67)$$

$$\dot{Y} = X\dot{\theta} + Y\dot{\theta} \quad (4.68)$$

$$\dot{Z} = Z\dot{R}/R + S\dot{\phi} \quad (4.69)$$

#### 4.7 FIRE DOCTRINE MODULE

The function of the Fire Doctrine Module is to provide a FIRE - NO FIRE decision for a given point in time. When a FIRE decision is made, Fire Doctrine Module provides values for, burst duration, rate of fire, and number of rounds in the burst.

The current configuration works in conjunction with the report generator, which is responsible for maintaining the count of rounds available for firing and not initiating the Fire Doctrine Module, unless there are rounds on the mount. The following parameters are input during initialization:

- a. *Open fire time.* First time at which a burst may be fired.
- b. *Round per burst.* Number of rounds in one burst.
- c. *Burst duration.* Period of time over which burst will be fired.
- d. *Dead time.* Minimum time between firing bursts.
- e. *Load time.* Time required to reload the mount.
- f. *Total rounds.* Number of rounds on the mount.

The Fire Doctrine Module logic and REPORT GENERATOR EVENT logic work together to assure that bursts are fired in a manner consistent with the input parameters. If the number of rounds in the mount is greater than the rounds in the burst, the logic spaces the bursts by the dead time. When ammunition is exhausted, the logic prevents the gun from firing, until sufficient time has elapsed, to accomplish reloading the mount.

#### 4.8 VULNERABILITY MODULE

The function of the Vulnerability Module is to operate on the data in the SHELL POSITION FILE and produce values for the single shot hit probability for the fuselage, wing, and fuselage-wing intersection.

The equation used to compute the single shot hit probability is as follows:

$$P_{ss} = \frac{|J|^{1/2}}{|J+S|^{1/2}} e^{-1/2 M_a^T [J+S]^{-1} M_a} \quad (4.70)$$

where:

- [J] is the inverse of the matrix [V].
- [V] describing the diffuse ellipsoid representing the aircraft segment.
- [S] is the covariance matrix of the round to round dispersion of the shots.
- M<sub>a</sub> is the vector of displacement of the aim point (point of closest approach) from the center of the target.

The following paragraphs expand the above definition in terms of aircraft and projectile parameters.

#### 4.8.1 The J Matrix

The J matrix is the inverse of the V matrix, which represents the projection of the ellipsoid, representing the fuselage, the wing, or the fuselage-wing intersection onto a plane normal to the relative velocity vector of the shell with respect to the aircraft. The V matrix is obtained by: 1) rotating the aircraft (actually the appropriate matrix representing the ellipsoid of vulnerable volume corresponding to the aircraft component in question) to an orientation in space corresponding to the true position, heading, and bank angle of the aircraft at the time in question; 2) project this rotated ellipsoid onto the plan normal to the relative velocity vector of the shell with respect to the aircraft at the point where the ellipse is of maximum extent in the U direction to obtain the required V matrix. The above process in equation form is as follows:

Define:

$$M = \begin{bmatrix} M_{11} & M_{21} & M_{31} \\ M_{21} & M_{22} & M_{32} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} = R^T H R \quad (4.71)$$

where:

$$H = \begin{bmatrix} \frac{2}{a} & 0 & 0 \\ 0 & \frac{2}{b} & 0 \\ 0 & 0 & \frac{2}{c} \end{bmatrix} \text{ is the aircraft ellipsoid} \quad (4.72)$$

$$R = B^T \phi^T \theta^T A_R [e_R + \pi/2] \quad (4.73)$$

- B = Rotation Matrix for Bank angle.
- $\theta$  = Rotation Matrix for Climb angle.
- $\phi$  = Rotation Matrix for Heading angle.
- A<sub>R</sub> = Rotation Matrix for Relative azimuth.
- e<sub>R</sub> = Rotation Matrix for Relative elevation.

Then:

$$V = \begin{bmatrix} M_{11} \cdot \frac{M_{31}M_{31}}{M_{33}} & M_{21} \cdot \frac{M_{31}M_{32}}{M_{33}} \\ M_{21} \cdot \frac{M_{32}M_{31}}{M_{33}} & M_{22} \cdot \frac{M_{32}M_{32}}{M_{33}} \end{bmatrix} \quad (4.74)$$

$$J = V^{-1}$$

#### 4.8.2 The Covariance Matrix S

The covariance matrix [S] is computed as the sum of the matrix  $[\sigma_a^2]$  of angular dispersion and the matrix  $[\sigma_m^2]$  or muzzle velocity dispersion.

The angular dispersion matrix  $[\sigma_a^2]$  is obtained by rotation of the angular dispersion matrix, in gun coordinates by an elevation angle, e, and an azimuth angle, A, and then back through a relative azimuth angle, A<sub>R</sub>, and an augmented elevation angle (e + π/2). Only the upper right hand four elements are retained in equation form:

$$[\sigma_r^2] = C [\sigma_g^2] C^T \quad (4.75)$$

$$C^T = C^T A A_r [e_r + \pi/2] \quad (4.76)$$

The muzzle velocity dispersion matrix  $[\sigma_m^2]$  is computed by treating the error as an error along the line of flight, due to an early or late arrival of the shell, and applying a rotation to obtain the appropriate values in the plane normal to the shell trajectory near the aircraft. The error is estimated using a 3/2 law for ballistics and the rotation, R, is the same as R used in the definition of the J Matrix.

In equation form:

$$\sigma_e^2 = \left(\frac{\sigma_{v_0}}{V_0}\right)^2 \left(\frac{V}{V_p}\right)^2 D_p^2 \quad (4.77)$$

where:

- $\sigma_{v_0}$  = muzzle velocity dispersion
- $V_0$  = muzzle velocity
- $V$  = aircraft velocity
- $V_p$  = projectile velocity
- $D_p$  = range of projectile

$$[\sigma_m^2] = R^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} R \quad (4.78)$$

#### 4.8.3 The M<sub>r</sub> Vector

The M<sub>r</sub> vector is the displacement from the point of closest approach to the target center. The vector is composed of the error due to prediction and the error

due to any systematic difference between the muzzle velocity used for prediction and the actual muzzle velocity.

$$M_a = M_p + M_B \quad (4.79)$$

where:

- $M_p$  = displacement due to prediction error
- $M_B$  = displacement due to muzzle velocity bias.

$M_p$  is obtained by rotation of the prediction errors from the gun based system to the projectile based system and retaining only the U and V values.

$$M_p = [X Y Z] [A_r] [E_r + \pi/2] \quad (4.80)$$

$M_B$  is produced by computing the error in gun coordinates and rotating it to projectile coordinates. The error computation is an error due to early or late arrival assuming a 3/2 ballistics law while the projectile is in the neighborhood of the target. In equation form:

$$B_v = \left(\frac{V_a}{V_p}\right) \left(\frac{\Delta V_0}{V_0}\right) D_p \quad (4.81)$$

$$M_B = [0 B_v 0] R \quad (4.82)$$

where:

- $V_a$  = aircraft velocity
- $V_p$  = projectile velocity
- $\Delta V_0$  = muzzle velocity bias
- $V_0$  = muzzle velocity
- $D_p$  = range of the projectile
- $R$  = the rotation matrix defined in Section 3.8.1.