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MS-ABMDA-1683

Technical Report

COMPUTER PROGRAM FOR SIZING AND PERFORMANCE ANALYSIS OF SOLID PROPELLANT ROCKET MOTORS

November 1972

TELEDYNE
BROWN ENGINEERING

Research Park • Huntsville, Alabama 35807

TELEDYNE
BROWN ENGINEERING

RESEARCH PARK

HUNTSVILLE, ALABAMA 35807

(205) 536-4455 TWX (810) 726-2103

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In accordance with Sequence No. G003 of the DD Form 1423 for the subject contract, Teledyne Brown Engineering is submitting seven (7) copies of its Technical Report MS-ABMDA-1683, "Computer Program for Sizing and Performance Analysis of Solid Propellant Rocket Motors". This report was verbally approved. Distribution is being made per the DD Form 1423.

Sincerely,

BROWN ENGINEERING COMPANY, INC.

Martha H. Teenor

Martha H. Teenor
Contract Manager

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TECHNICAL REPORT
MS-ABMDA-1683

COMPUTER PROGRAM FOR SIZING AND PERFORMANCE ANALYSIS OF SOLID PROPELLANT ROCKET MOTORS

By

J. L. Thurman

November 1972

Prepared For

U.S. ARMY ADVANCED BALLISTIC MISSILE DEFENSE AGENCY
DEPARTMENT OF THE ARMY
HUNTSVILLE, ALABAMA

Contract No. DAHC60-69-C-0037

Prepared By

MILITARY SYSTEMS
TELEDYNE BROWN ENGINEERING
HUNTSVILLE, ALABAMA

ABSTRACT

A FORTRAN digital computer program was developed which is generally applicable for preliminary design tradeoff analyses of solid-propellant rocket motors for a variety of applications. The program computes rocket motor length, propellant and inert component weights, mass fraction, burn time, specific impulse, ideal stage burnout velocity, and other performance parameters as a function of input motor diameter, chamber pressure, thrust, payload mass, propellant ballistic properties, propellant web fraction, port-to-throat area ratio, and other required parameters. Either conical or contoured nozzle exit geometries may be analyzed. Options are available for considering a head-end propellant web and inert residual slivers.

Approved:

G. R. Guinn, Ph.D.
Manager
Missile Requirements and
Technology Branch

Approved:

S. M. Gilbert, Ph. D.
Manager
System Synthesis Department

Approved:

H. A. Matheny
Deputy
Program Management

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LIST OF SYMBOLS

Symbol	Definition
A_b	Average burning surface area, in^2
A_{b_c}	Average burning surface area within cylindrical length, in^2
A_{b_h}	Combined average burning surface area of forward and aft head-end propellant, in^2
A_{bo}	Thrust-to-weight ratio at burnout
A_{e_i}	Nozzle exit area (inside), in^2
A_f	Cross sectional area corresponding to outside diameter of propellant, in^2
A_{ign}	Thrust-to-weight ratio at ignition
A_p	Initial port area, in^2
A_{pt}	Initial port-to-throat area ratio
A_s	Cross sectional area of residual sliver (active or inert), in^2
A_t	Nozzle throat area, in^2
A_w	Initial cross sectional area of propellant web, in^2
a	Propellant burning rate coefficient
C_1	Constant; 0.0163 for upper stage; 0.0055 for remaining stages
C_D	Mass discharge coefficient, lbm/lbf sec
C_f	Thrust coefficient

LIST OF SYMBOLS (Continued)

Symbol	Definition
c_i	Specific heat of insulation, Btu/lbm-°R
c_s	Specific heat of structural material, Btu/lbm-°R
D_{c_i}	Motor case inside diameter, in.
D_{e_i}	Nozzle exit inside diameter, in.
D_{e_m}	Maximum nozzle exit outside diameter, in.
D_{e_o}	Nozzle exit outside diameter, in.
D_f	Outside diameter of propellant charge, in.
D_m	Motor case outside diameter, in.
D_{h_i}	Inside major axis of ellipsoidal forward head, in.
E_c	Young's modulus of motor case material, psi
E_{int}	Young's modulus of interstage structure, psi
E_s	Young's modulus of nozzle throat structural shell, psi
E_t	Young's modulus of throat insert material, psi
F	Required thrust (input), lbf
F_p	Fraction of propellant burned at web burn-through
F_{sm}	Motor case design safety factor
F_{sn}	Nozzle design safety factor
F_w	Propellant web fraction ($2\tau_w/D_f$)
g_c	Conversion constant, 32.174 lbm ft/lbf sec^2

LIST OF SYMBOLS (Continued)

Symbol	Definition
g_{max_i}	Maximum longitudinal acceleration of stage i, g's
$g_{max_{i-1}}$	Maximum longitudinal acceleration of stage i-1, g's
Isp_d	Delivered specific impulse, lbf sec/lbm
k_t	Ratio of nozzle throat radius of curvature to throat radius
L_{ah}	Length of aft head, in.
L_c	Rocket motor cylindrical length, in.
L_{cont}/L_{con}	Ratio of contoured nozzle length to conical nozzle length
L_{fh}	Length of forward head, in.
L_{h_i}	Inside semiminor axis of ellipsoidal forward head, in.
L_{h_o}	Outside semiminor axis of ellipsoidal forward head, in.
L_{int}	Interstage clearance between forward head of stage i and nozzle exit plane of stage (i + 1), in.
L_m	Rocket motor length, in.
L_n	Nozzle length, in.
$L_{n_{i+1}}$	Nozzle length of stage (i + 1), in.
$\dot{m}$	Mass flowrate, lbm/sec
n	Propellant burning rate exponent
P_{amb}	Ambient pressure, psia

LIST OF SYMBOLS (Continued)

Symbol	Definition
P_b	Average propellant burning perimeter, in.
P_c	Average chamber pressure, psia
P_D	Design chamber pressure for structural analysis, psia
P_{D_x}	Local design pressure, psia
P_e	Nozzle exit pressure, psia
R	Specific gas constant, ft lbf/lbm°R
r_{at}	Nozzle inside radius at exit cone tangent point (see Figure 3-2), in.
r_b	Propellant burning rate, in/sec
r_c	Throat insert radius of curvature, in.
r_e	Inside radius at nozzle exit plane, in.
r_{es}	Inside radius of nozzle structure at aft end of throat insert, in.
r_{op}	Radius of motor case aft head opening, in.
r_{os}	Inside radius of nozzle structure at throat, in.
r_{ot}	Inside radius at nozzle entrance cone tangent point, in.
r_t	Nozzle throat radius, in.
T_c	Propellant flame temperature, °R
t	Insulation exposure time, sec

LIST OF SYMBOLS (Continued)

Symbol	Definition
t_b	Elapsed time between ignition and web burn-through, sec
$V_{p_{cw}}$	Volume of propellant within cylindrical length which is burned prior to web burnthrough, in^3
V_{p_h}	Propellant volume in forward and aft heads, in^3
$V_{p_{sc}}$	Total volume of propellant charge in cylindrical chamber (including slivers), in^3
V_{p_w}	Total volume of propellant which is burned prior to web burnthrough, in^3
V_s	Volume of residual slivers, in^3
W_{ab}	Weight of motor case aft head attachment boss, lbm
W_{ahi}	Weight of aft head insulation, lbm
W_{ahs}	Weight of aft head membrane structure, lbm
W_{as}	Weight of aft head attachment skirt, lbm
W_{bo}	Stage burnout weight, lbm
$W_{c\ell}$	Liner weight in cylindrical chamber, lbm
W_{cs}	Weight of cylindrical shell, lbm
W_{ei}	Weight of nozzle entrance insulation, lbm
W_{es}	Weight of structural material in nozzle entrance cone, lbm
W_{fhs}	Weight of forward head membrane structure, lbm

LIST OF SYMBOLS (Continued)

Symbol	Definition
W_{fs}	Weight of forward head attachment skirt, lbm
W_{hi}	Weight of forward head insulation, lbm
W_{ib}	Weight of ignitor boss, lbm
W_{ign}	Ignitor weight, lbm
W_{int}	Weight of interstage structure, lbm
W_{IP}	Total weight of inert parts, lbm
W_n	Total nozzle weight, lbm
W_{nb}	Weight of nozzle attachment boss (nozzle portion), lbm
W_o	Gross stage weight, lbm
W_p	Propellant weight, lbm
W_{PL}	Stage payload weight, lbm
W_t	Weight of nozzle throat insert, lbm
W_{ti}	Weight of nozzle throat insulation, lbm
W_{ts}	Weight of throat structural shell, lbm
W_{xi}	Weight of nozzle exit cone insulation, lbm
W_{xs}	Weight of nozzle exit cone structure, lbm
α	Nozzle exit cone half-angle, radians
α_e	Contoured nozzle half-angle at exit plane, radians

LIST OF SYMBOLS (Continued)

Symbol	Definition
α_i	Thermal diffusivity of nozzle insulation, in^2/sec
α_o	Contoured nozzle half-angle immediately downstream of throat, radians
β	Forward and aft head ellipse ratio
γ	Specific heat ratio of combustion gas
ΔT	Predicted temperature rise of protected structural material, °R
ΔV_I	Ideal velocity increment, ft/sec
ϵ	Nozzle expansion ratio
ϵ_e	Equivalent conical nozzle expansion ratio for contoured nozzle analysis
η_D	Mass discharge correction factor for nonideal flow
η_v	Exit velocity correction factor for nonideal flow
λ	Rocket motor mass fraction
λ_n	Nozzle divergence loss factor
μ	Stage mass fraction
μ_s	Poisson's ratio of throat structural shell
μ_t	Poisson's ratio of throat insert material
ϕ	Nozzle entrance cone half angle, radians
ψ	$[1 - (1/\beta^2)]$
ρ_ℓ	Liner density, lbm/in^3

LIST OF SYMBOLS (Continued)

Symbol	Definition
ρ_i	Insulation density, lbm/in^3
ρ_m	Motor case density, lbm/in^3
ρ_p	Propellant density, lbm/in^3
ρ_s	Density of structural material, lbm/in^3
ρ_t	Density of throat insert, lbm/in^3
σ_m	Yield strength of motor case, psi
σ_n	Yield strength of nozzle structural material, psi
τ_e	Wall thickness of nozzle entrance cone, in.
τ_{ℓ_c}	Liner thickness of cylindrical chamber, in.
τ_h	Forward head membrane thickness, in.
τ_i	Insulation thickness, in.
τ_{ie}	Insulation thickness in nozzle entrance cone, in.
τ_{i_x}	Insulation thickness at nozzle exit plane, in.
τ_m	Motor case wall thickness, in.
τ_{mn}	Minimum wall thickness due to fabrication constraints, in.
τ_{n_e}	Nozzle wall thickness at exit plane, in.
τ_s	Structural material thickness, in.
τ_{ts}	Thickness of throat structural shell, in.

LIST OF SYMBOLS (Concluded)

Symbol	Definition
τ_w	Propellant web thickness, in.
τ_{xs}	Thickness of nozzle structure at aft end of throat insert, in.
θ	Time constant $(\tau_i^2 / \pi^2 \alpha_i)$, sec

1. INTRODUCTION

Preliminary sizing analyses of aerospace vehicles which utilize solid propellant rocket motors usually require rather extensive trade-off analyses to ensure the near-optimal selection of pertinent motor design parameters. For multistage vehicles, the motor design analyses usually follow energy management studies (e.g., Ref. 1) which apportion propellant masses optimally among the constituent stages to achieve prescribed vehicle performance characteristics; e.g., to minimize gross weight for a required velocity increment or to maximize velocity for a fixed gross weight. The objective of this investigation was to develop a digital computer program which would permit rapid, accurate, preliminary design tradeoff analyses of solid propellant rocket motor concepts.

2. PROGRAM CAPABILITIES

The program described in this report computes rocket motor length, propellant and inert component weights, mass fraction, burn time, specific impulse, stage burnout velocity, and other performance parameters as a function of input motor diameter, chamber pressure, thrust, payload mass, propellant ballistic properties, propellant web fraction, port-to-throat area ratio, residual sliver fraction, and other required parameters.

The generic representation of a selected propellant grain configuration by its web fraction, F_W, and residual sliver fraction, F_p, and a prescribed port-to-throat ratio, Apt, permits evaluation of the applicability of many candidate grain designs without requiring detailed surface-web calculations. This feature simplifies input preparation and enhances program flexibility for its intended use in conceptual design tradeoff analyses. Realistic values of F_W and F_p may be derived from known geometric characteristics of previously configured grain cross sections (Refs. 2 and 3). All star, wagonwheel, cylindrical port, and slotted tube grain configurations wherein the propellant perforations traverse the entire cylindrical motor length are ideally suited for analysis using this technique. Tapered grain configurations and those which employ radial slots, or longitudinal slots which traverse only a fraction of the cylindrical motor length, may be analyzed provided a length-averaged propellant web fraction is entered. Ellipsoidal head-end propellant webs and inert propellant slivers may be included in the design analysis simply through the proper selection of input code words, as described later.

Dimensions and/or weights of inert components, including the motor case, head closures, nozzle, attachment skirts, interstage structure, liner, insulation, and igniter, are computed using proven theoretical and empirical relationships in conjunction with input material physical properties. Either conical or contoured nozzle designs may be analyzed. The nozzle expansion ratio is sized to provide as near optimum expansion as possible while maintaining the nozzle exit diameter less than the input maximum allowable value. (Optimum expansion occurs when the nozzle exit pressure is equivalent to the input ambient pressure.) The validity of the computer program was confirmed through the analytical reproduction of various rocket motor designs which are documented in Reference 4.

3. DESIGN ANALYSIS

3.1 INTERNAL BALLISTICS

The internal ballistics analysis described in this section is based on the assumption of constant mass flow rate throughout motor operation. The appropriate gas dynamic relationships for combustion gas flow within the motor port and nozzle are based on the assumption of one-dimensional flow of a perfect gas. Most of the applicable thermodynamic relationships may be found in standard propulsion textbooks; for example, Reference 5. Following program initialization and the reading of appropriate input parameters, the rocket motor design analysis proceeds as described in the following paragraphs.

The required thickness of the motor case wall is sized using the well-known hoop stress relationship

$$\tau_m = \frac{P_c D_m F_{sm}}{2\sigma_m} . \tag{3-1}$$

The independent variables P_c, D_m, and F_{sm} are program inputs. The yield strength of the motor case, σ_m, and most of the other required inert component physical properties discussed later are specified within the program in well-defined DATA statements, as described in Section 5.2. This method of specifying material properties provides flexibility for analyzing various candidate materials (at the expense of program recompilation for each DATA statement alteration) while reducing the number of inputs required for normal design analyses. The case wall thickness computed from Equation 3-1 is compared with a specified minimum wall thickness based on fabrication limitations, and the larger value is employed in further calculations.

Following an initial estimation of the required cylindrical case liner thickness, τ_{ℓ_c}, the outer diameter of the propellant charge is computed from

$$D_f = D_m - 2\,(\tau_m + \tau_{\ell_c}) \quad . \tag{3-2}$$

The final design value of liner thickness is determined through iteration using an empirical relationship which is described later.

Equation 3-2 begins an iteration wherein the nozzle expansion ratio, cylindrical case liner thickness, and propellant geometrical characteristics are established. An attempt is first made to size the nozzle for optimum expansion such that the nozzle exit pressure is equivalent to the input ambient pressure. If this is not feasible within the constraints of the input maximum nozzle exit diameter, D_{e_m}, and the thrust-dictated throat area requirement, the nozzle design which most nearly approaches optimum expansion within these constraints is established. In the latter situation, the nozzle exit pressure is always greater than ambient; i. e., the nozzle is underexpanded. As a first trial estimate, the nozzle exit pressure, P_e, is set equal to the input ambient pressure P_{amb}. The corresponding nozzle expansion ratio, ϵ, is then determined from the relationship

$$\epsilon = \left[\left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{P_e}{P_c}\right)^{\frac{1}{\gamma}} \left\{\frac{\gamma+1}{\gamma-1}\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{0.5}\right]^{-1} \quad . \tag{3-3}$$

The thrust coefficient, C_f, is computed from

$$C_f = \lambda_n \eta_D \eta_v \left\{ \frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{0.5} \tag{3-4}$$

$$+ \frac{(P_e - P_{amb})\epsilon}{P_c} \; .$$

The divergence loss factor, λ_n, is defined as

$$\lambda_n = \frac{1}{2}(1 + \cos\alpha) \; . \tag{3-5}$$

For analysis of contoured nozzles using this program, an effective exit half-angle of 17.5 degrees should be input, as described later. The nozzle discharge correction factor, η_D, is usually greater than unity, as discussed in Reference 6, but may range from 0.98 to 1.15. (A value of 1.02 is typical.) The velocity correction factor, η_v, ranges between 0.85 and 0.98, with an average near 0.92 (Ref. 6).

The required nozzle throat area is established by

$$A_t = F/(P_c C_f) \; . \tag{3-6}$$

The corresponding nozzle exit area, A_{ei}, is computed from

$$A_{ei} = A_t \epsilon \; . \tag{3-7}$$

The required nozzle wall thickness at the exit plane (τ_{n_e}) is sized from the hoop stress relation

$$\tau_{n_e} = \frac{P_e \; D_{ei} \; F_{sn}}{2\sigma_n} \quad . \tag{3-8}$$

The variable F_{sn} is a program input, and σ_n is specified within the program by a DATA statement. Again, the computed wall thickness from Equation 3-8 is compared with a specified minimum thickness based on fabrication constraints, and the larger value is selected for further calculations. The outside diameter at the nozzle exit plane is then computed from

$$D_{e_o} = D_{ei} + 2(\tau_{ne} + \tau_{ix}) \tag{3-9}$$

where τ_{ix}, the insulation thickness at the nozzle exit plane, is computed using procedures described later in this section (Equation 37). If the computed outside exit diameter is less than the input maximum value, the design analysis continues with the computed value. Moreover, if this test is satisfied on the first iteration, with $P_e = P_{amb}$, the nozzle is designed for optimum expansion. If, however, the computed outside exit diameter exceeds the input maximum value, the exit pressure estimate is increased slightly and the analysis returns to Equation 3-3. The iteration continues until a satisfactory exit diameter and corresponding exit pressure and expansion ratio are obtained.

Following the establishment of the nozzle throat and exit areas, the internal ballistics analysis and propellant geometrical characterization proceed. The mass discharge coefficient is computed from

$$C_D = \eta_D \left[\frac{g_c \gamma}{R T_c} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \right]^{0.5} . \qquad (3\text{-}10)$$

The independent variables γ, R, T_c are program inputs. The average mass flow rate, $\dot{m}$, discharged by the nozzle is computed from

$$\dot{m} = C_D P_c A_t . \qquad (3\text{-}11)$$

This discharged mass flowrate must equal the rate of mass generation at the propellant burning surface, which is determined from

$$\dot{m} = r_b A_b \rho_p \qquad (3\text{-}12)$$

where the average propellant burning rate is governed, for nonerosive flow conditions, by the well-known relation

$$r_b = a P_c^n . \qquad (3\text{-}13)$$

Equating the mass generation rate (Equation 3-11) to the mass discharge rate (Equation 3-10) yields, after rearrangement, the following relationship for computing the required average burning surface area:

$$A_b = \frac{C_D P_c A_t}{r_b \rho_p} . \qquad (3\text{-}14)$$

The initial motor port area is established by the input port-to-throat area ratio and the previously computed throat area

$$A_p = A_{pt} A_t \quad . \tag{3-15}$$

The initial port-to-throat area ratio, A_{pt}, must be input as unity or greater to assure a nonerosive flow condition inside the motor port. For tapered grains, A_{pt} is the length-averaged value. The cross sectional area, A_w, of the propellant web at ignition is computed from the geometric relationship

$$A_w = (A_f - A_p) F_p \tag{3-16}$$

where F_p is the fraction of propellant burned prior to web burnthrough. Values of F_p usually range from 0.9 to 1.0 (Refs. 2 and 3). The corresponding area, A_s, of residual slivers (active or inert) remaining after web burnthrough is established by the expression

$$A_s = A_f - A_w - A_p \quad . \tag{3-17}$$

The propellant web thickness is determined from

$$\tau_w = \frac{D_f F_w}{2} \tag{3-18}$$

As described previously, F_w represents the input propellant web fraction. Values of F_w typically range from 0.2 to 0.5 for star or wagonwheel designs and from 0.28 to 0.9 for slotted-tube designs. Star and wagonwheel configurations are usually employed with slow or medium burning rate propellants where relatively small burn times and large thrust levels are required. Slotted tube configurations yield large motor mass fractions; they are usually employed with relatively slow burning propellants for long burn time, low thrust applications or with fast burning propellants for short burn time, high thrust applications.

The motor burn time is computed from

$$t_b = \frac{\tau_w}{r_b} \quad . \tag{3-19}$$

At this point in the analysis, the required liner thickness adjacent to the cylindrical case wall, τ_{ℓ_c}, may be recalculated more accurately using the proven empirical expression developed by Thiokol Chemical Corporation (Ref. 7).

$$\tau_{\ell_c} = 0.02\, D_m^{0.5} (t_b + 1)^{0.1} (1 - F_p)^{0.25} \quad . \tag{3-20}$$

The liner thickness computed from Equation 3-20 is compared with the previously estimated value used in Equation 3-2. If the computed liner thickness differs by more than a specified amount from the previous estimate, the analysis returns to Equation 3-2, with the computed value used as the new estimate. This iteration continues until the computed and estimated values converge.

The next step in the analysis is the determination of the required propellant volume and corresponding motor length. Before motor length may be computed, the amount of propellant must be evaluated which may be loaded into the available forward and aft head-end volumes, if head-end webs are desirable. (The computer program is formulated such that the analysis of head-end webs is optional.) For cases in which a forward head-end web analysis is requested, a fractional ellipsoidal propellant volume, conforming to the head closure shape, is considered. A cylindrical perforation is allowed for protrusion of the ignitor through the head-end web. The analysis of the propellant in the propellant volume in the aft head is similar to that for the forward head, except the cross-sectional area of the center perforation is equivalent to the port area.

Following evaluation of the combined forward and aft head-end propellant volume, V_{p_h}, and corresponding average burning surface, A_{b_h}, if applicable, the required average burning surface, A_{b_c}, within the cylindrical motor length is computed from

$$A_{b_c} = A_b - A_{b_h} \quad . \tag{3-21}$$

The required total volume of the propellant charge, $V_{p_{sc}}$, including residual slivers (active or inert), in the cylindrical portion may now be computed from the geometrical relationship

$$V_{p_{sc}} = \frac{\tau_w A_{b_c}}{F_p} \quad . \tag{3-22}$$

Likewise, the volume of propellant, $V_{p_{cw}}$, within the cylindrical length which is burned before web burnthrough is determined from

$$V_{p_{cw}} = V_{p_{sc}} F_p \quad . \tag{3-23}$$

The total volume of propellant, V_{p_w}, which is burned before web burnthrough is determined by the sum

$$V_{p_w} = V_{p_{cw}} + V_{p_h} \tag{3-24}$$

and the volume of residual slivers, V_s, is computed from the geometrical relationship

$$V_s = V_{p_{sc}} (1 - F_p) \; . \tag{3-25}$$

Because of the simple propellant geometry selected for the head-end web, residual slivers there will be negligible.

The required cylindrical length of the motor, L_c, may now be computed from

$$L_c = V_{p_{cw}} / A_w \; . \tag{3-26}$$

The propellant and residual sliver weights are evaluated by multiplying the previously computed volumes by their respective densities. For active slivers, the propellant density is used, whereas for inert slivers a density of 0.02 lbm/in^3, which is typical of conventional phenolic/microballoon filler material is used.

3.2 DESIGN OF INERT COMPONENTS

Sufficient information is now available to permit computation of weights and dimensions of the remaining constituent inert components of the rocket motor. These calculations are performed within the computer program by Subroutines INERT, NOZZ, and INSUL. In the interest of brevity, only the final formulations of the pertinent working equations will be presented herein. The reader is referred to the cited references for the appropriate derivations. The nomenclature for the type of motor case considered herein is defined in Figure 3-1. Also, the nomenclature which applies to the conical and contoured nozzle analyses is defined in Figures 3-2 and 3-3, respectively.

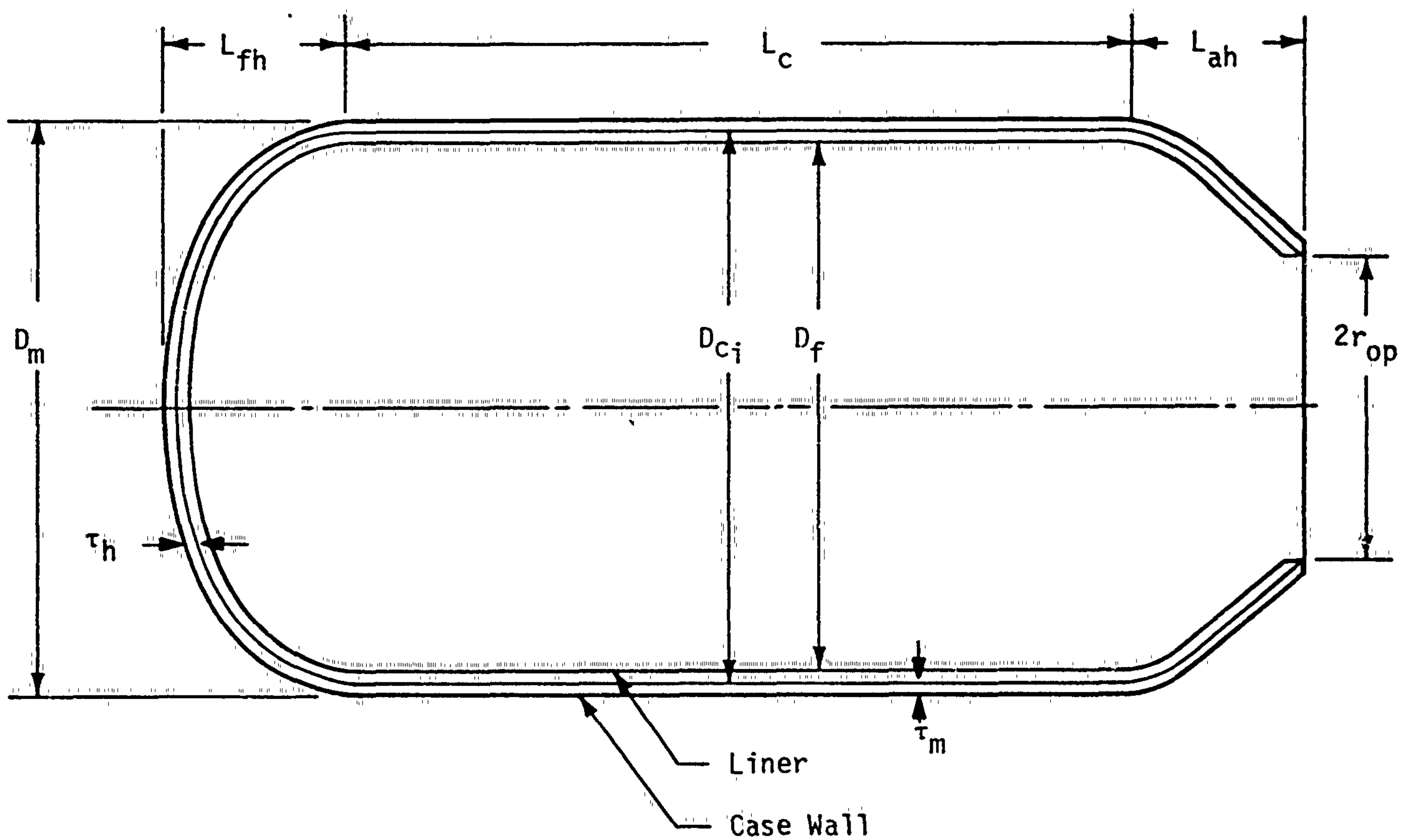


FIGURE 3-1. MOTOR CASE NOMENCLATURE

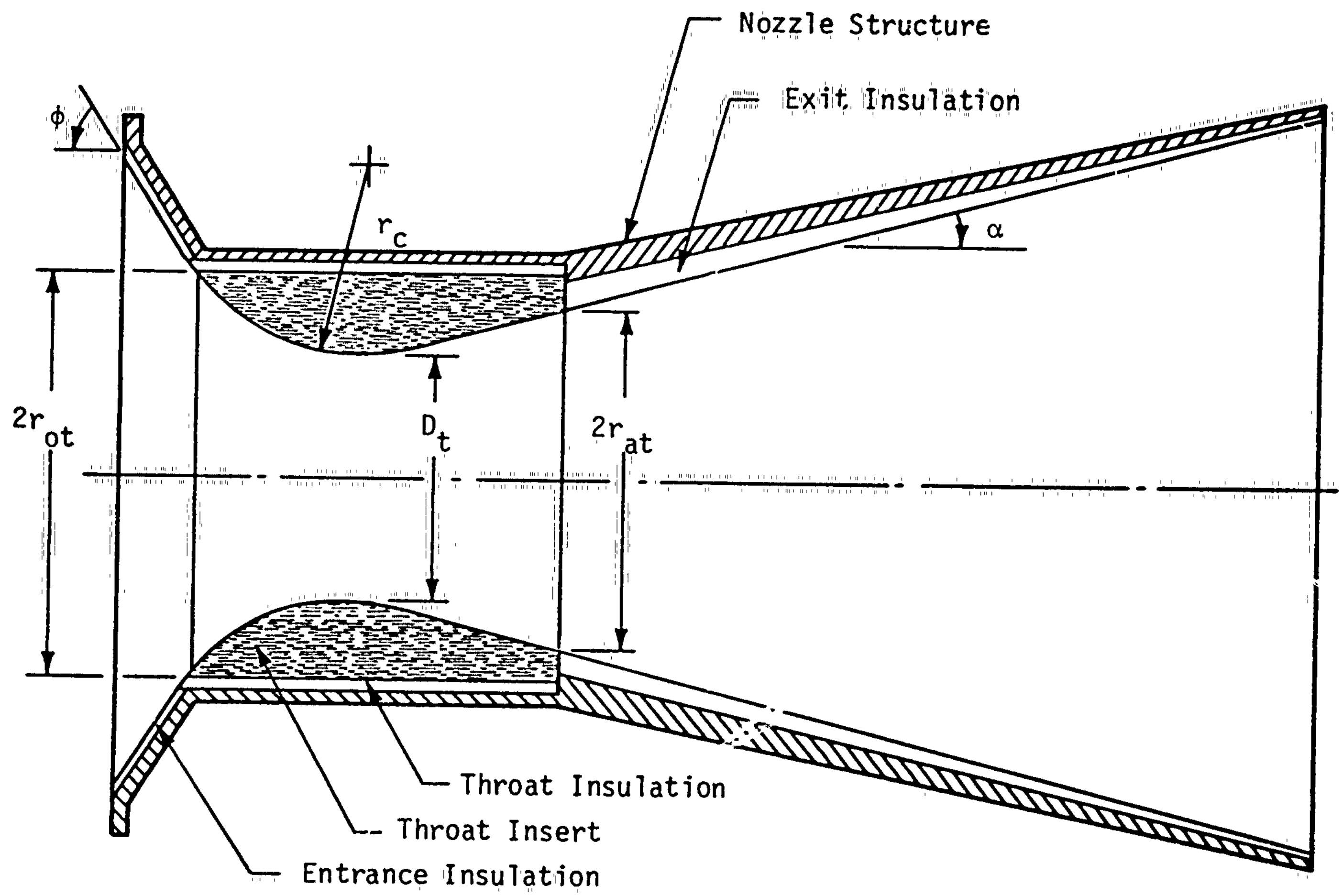


FIGURE 3-2. CONICAL NOZZLE NOMENCLATURE

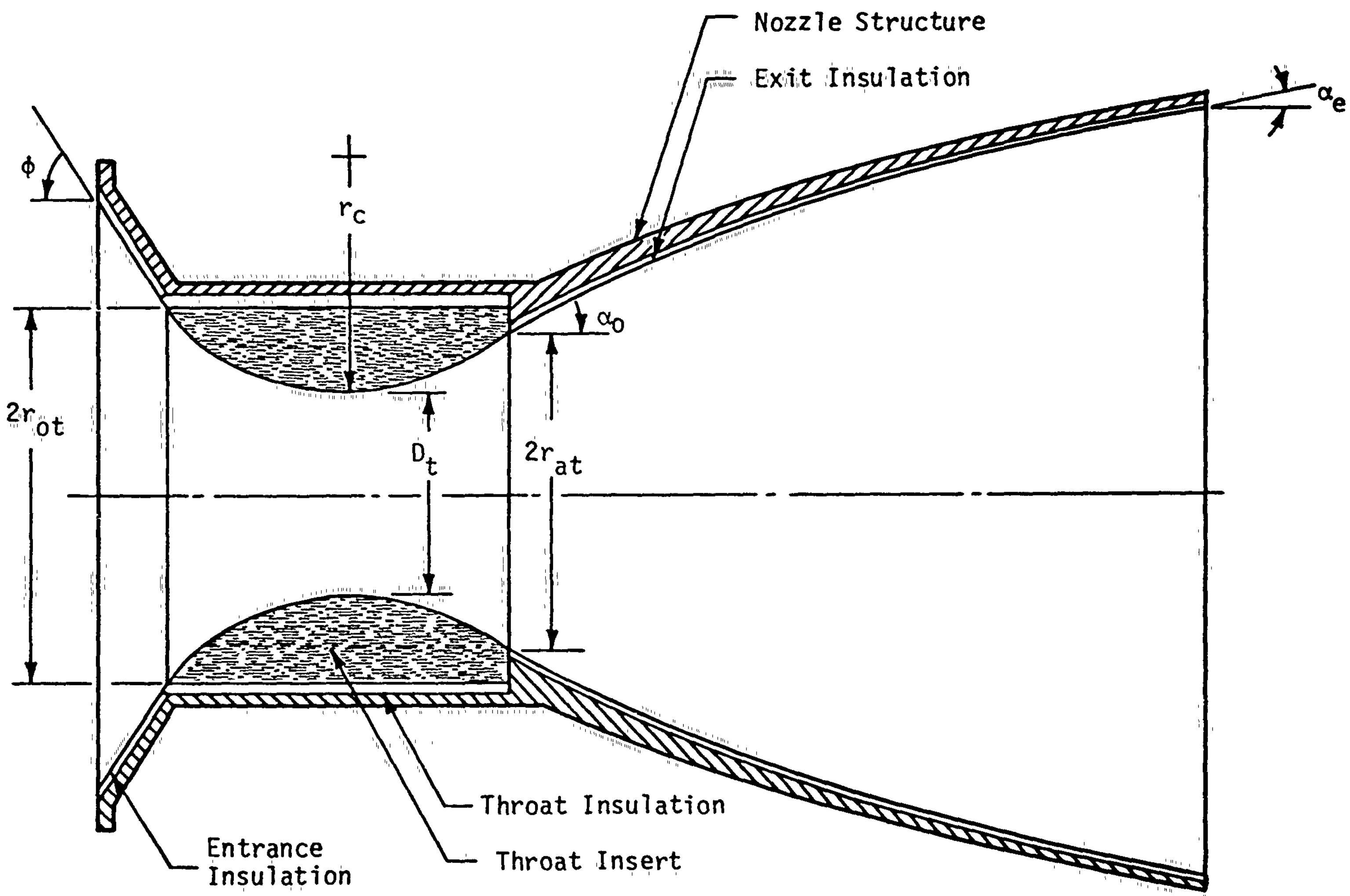


FIGURE 3-3. CONTOURED NOZZLE NOMENCLATURE

3.2.1 Forward Head Membrane

The required thichness, τ_h, of the forward head membrane is sized by the resultant of the meridional and tangential stresses as discussed in Reference 8. Thus,

$$\tau_h = \frac{0.177\ P_D\ D_m^2}{L_{h_o}\ \sigma_m} \qquad (3\text{-}27)$$

The design chamber pressure, P_D, is taken as 1.5 times the average chamber pressure. The value of Lh_o is dictated by the input motor diameter and the head-end ellipse ratio, β, which typically ranges from 1.0 to 2.0. The values of the inside major axis, D_{h_i}, and semi-minor axis, L_{h_i}, are determined from the input motor diameter, ellipse ratio, and membrane thickness. The weight of the forward head membrane structure is then computed from

$$W_{fhs} = 0.167\ \rho_m \pi\ (L_{h_o}\ D_m^2 - L_{h_i}\ D_{h_i}^2) \quad . \qquad (3\text{-}28)$$

3.2.2 Forward Head Insulation

The weight of the forward head insulation is computed from the following empirical relationship (Ref. 9):

$$W_{hi} = 1.93\ (10^{-10})\ D_m^2 \times \left(0.7854 + \frac{0.3925}{\beta^2\ \psi}\ \ln \frac{1+\psi}{1-\psi}\right) P_D^{0.8}\ t_b\ C_D^{-1.7} \quad . \qquad (3\text{-}29)$$

where

$$\psi = \left(1 - \frac{1}{\beta^2}\right)^{1/2} \qquad (3\text{-}29a)$$

3.2.3 Ignitor Boss

The weight of the ignitor boss, W_{ib}, on the forward head is estimated from the empirical expression (Ref. 9):

$$W_{ib} = 1.13 \, P_D \, D_m \, \beta \, A_t \left(\frac{\rho_m}{\sigma_m} \right) \tag{3-30}$$

3.2.4 Ignitor

The weight of the ignitor, W_{ign}, is estimated from the following empirical relationship which was derived through correlation of reported ignitor weights (Ref. 4) for previously designed and fabricated rocket motors:

$$W_{ign} = 10 + \exp \left[0.9 \, (10^{-4}) \, A_b \right] . \tag{3-31}$$

Equation 3-31 was found to produce an excellent fit of reported ignitor weight data for rocket motors with an average burning surface area ranging between 3500 and 44000 in².

3.2.5 Cylindrical Case Structure

The weight of the cylindrical portion of the motor case structure, W_{cs}, is computed from the relationship

$$W_{cs} = \frac{\pi}{4} \rho_m \, L_c \left(D_m^{\,2} - D_{c_i}^{\,2} \right) \tag{3-32}$$

3.2.6 Cylindrical Case Liner

The weight of the liner adjacent to the cylindrical motor case, $W_{c\ell}$, is determined from

$$W_{c\ell} = \pi D_{c_i} L_c \tau_{\ell_c} \rho_\ell \tag{3-33}$$

For the computer program described herein, a typical liner density, ρ_ℓ, of 0.06 lbm/in³ is employed

3.2.7 Aft Head Membrane

The weight of the aft head membrane structure, W_{ahs}, is estimated from the following empirical relationship (Ref. 9):

$$W_{ahs} = 0.25 \, P_D \, D_m \, \beta \left(\frac{\rho_m}{\sigma_m} \right)$$

$$\times \left\{ D_m^2 \left[0.7854 + \frac{0.3925}{\beta^2 \psi} \, \ell n \, \frac{1 + \psi}{1 + \psi} \right] - 2A_t \right\} \tag{3-34}$$

3.2.8 Aft Head Insulation

The weight of the aft head insulation, W_{ahi}, is scaled as 1.54 times that of the previously computed forward head insulation as recommended by Reference 9.

3.2.9 Aft Head Nozzle Attachment

The weight of the aft head nozzle attachment boss, W_{ab}, is estimated using the following relationship (Ref. 9):

$$W_{ab} = 10.26\ P_D\ D_m^{\ 2}\ \beta \left(\frac{\rho_m}{\sigma_m} \right) . \qquad (3\text{-}35)$$

3.2.10 Nozzle

Either conical or contoured nozzle designs may be analyzed using this program. Conical nozzles of the type illustrated in Figure 3-2 are designed using nondocumented procedures developed by Mr. A. R. Maykut of the U.S. Army Missile Command (MICOM) at Redstone Arsenal. The validity of these procedures has been proven at Redstone Arsenal through numerous design analyses and reproductions of previous operational nozzle designs. Contoured nozzle length and weight, when applicable, are scaled from a corresponding design of a conical nozzle with equivalent throat area and fixed exit cone half-angle of 17.5 degrees using procedures described in Reference 10. A more detailed description of the contoured nozzle scaling procedure is presented later.

Discussing first the conical nozzle analysis, the entrance wall thickness, τ_e, of the nozzle at the aft head attachment radius is computed from

$$\tau_e = \frac{r_{op}\ P_D}{\sigma_n\ \cos\phi} \qquad (3\text{-}36)$$

The parameter r_{op} usually is governed by the size of the aft head opening which is required for insertion and extraction of the propellant mandrel. For this computer program, the nozzle entrance cone half-angle, ϕ, is fixed at 60 degrees.

The thickness computed by Equation 3-36 is compared with a specified minimum value which is dictated by fabrication constraints, and the larger of the two values is used in further calculations.

The next step in the nozzle analysis is the determination of the required insulation thickness at the nozzle entrance. The procedure which is employed in sizing the insulation thickness is described in Reference 5. A thermally thin structural wall is assumed to be protected from the combustion gases by a refractory insulation. The temperature of the exposed insulation surface is assumed to be equivalent to that of the adjacent combustion gases, T_c, and the temperature rise of the protected structure during motor operation is assumed to be much smaller than T_c. The predicted temperature rise of the structural wall during time t is computed from the general expression

$$\Delta T = \frac{c_i \rho_i \tau_i}{c_s \rho_s \tau_s} T_c f\left(\frac{t}{\theta}\right) \tag{3-37}$$

where θ is the time constant $(\tau_i^2/\pi^2 \alpha_i)$. The function $f(t/\theta)$ is reported in Reference 5 as

$$f\left(\frac{t}{\theta}\right) = \frac{t}{\pi^2\theta} - \frac{1}{6} + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left[-n^2 \frac{t}{\theta}\right] \tag{3-38}$$

Equation 3-37 cannot be solved directly for τ_i because of the implicit relationship of τ_i with the series function $f(t/\theta)$. Therefore, an iterative solution is accomplished in which Equation 3-37 is solved repetitively to compute values of predicted temperature rise for progressively improved estimates of τ_i. The least value of τ_i for which the predicted structural temperature rise is maintained below a specified limit of 10°R is selected for design application.

The first 20 terms are utilized in the series evaluation of $f(t/\theta)$. Ablation of the insulation is not accounted for in the determination of the required insulation thickness. Therefore, the selected thickness should be conservative. An approximate, but nonconservative, means of accounting for ablation would be to substitute the insulation ablation temperature for T_c in Equation 3-37.

The weight of insulation in the nozzle entrance cone, W_{ei}, is computed from

$$W_{ei} = \frac{\pi \rho_i}{\sin \phi} \left(r_{op} + \tau_{ie} \cos \phi + r_{ot} \right) \left(r_{op} - r_{ot} \right) \tau_{ie} \quad . \tag{3-39}$$

The weight of the nozzle entrance cone structural material, W_{es}, is computed from

$$W_{es} = \frac{\pi \rho_s}{\sin \phi} \left[r_{op} + \left(\tau_{ie} + \frac{\tau_e}{2} \right) \cos \phi + r_{ot} \right] \left(r_{op} - r_{ot} \right) \tau_e \quad . \tag{3-40}$$

The weight of the nozzle throat insulation is computed from the relation

$$W_{ti} = \pi \rho_i \; \tau_{ie} \left[r_{ot} + \tau_{ie} \left(\cos \phi + \cos \alpha \right) + r_{at} \right] \left(2k_t \, r_t \right) \sin \left(\frac{\phi + \alpha}{2} \right) . \tag{3-41}$$

The thickness of the throat structural shell, τ_{ts}, is sized from the following expression which accounts for the reduction in pressure loading of the structural shell due to the load carried by the throat insert material:

$$\tau_{ts} = \frac{r_{os}^2}{r_{ot}} \; \frac{P_D}{\sigma_n} \; \frac{E_t}{E_s} \left(\frac{1 - \mu_s}{1 - \mu_t} \right) \tag{3-42}$$

The design pressure P_D in Equation 3-42 should rigorously be based upon a gas pressure whose value lies between the expected values at the throat and in the combustion chamber, since the inner surface of the throat insert is exposed to pressures in this range. However, for design conservatism, the value of P_D used in this program is computed as 1.5 times the chamber pressure. The corresponding weight of the throat structural shell, W_{ts}, is determined from

$$W_{ts} = \pi \tau_{ts} \rho_s \left[r_{ot} + \tau_{ie} (\cos\phi + \cos\alpha) + r_{at} + \frac{\tau_{ts}}{2} (\cos\phi + \cos\alpha) \right]$$

$$\times \quad 2 k_t r_t \sin\left(\frac{\phi + \alpha}{2}\right) . \qquad (3\text{-}43)$$

From geometrical considerations, the weight of the throat insert, W_t, is established as

$$W_t = \pi \rho_t k_t^2 r_t^3 \left[\phi + \alpha - \sin(\phi + \alpha) \right]$$

$$\times \left\{ \left(1 + k_t\right) - \left[\frac{0.667 k_t \sin^3 \left(\frac{\phi+\alpha}{2}\right)}{\phi + \alpha - 0.5 \sin(\phi + \alpha)} \right] \cos\left(\frac{\phi + \alpha}{2}\right) \right\} . \qquad (3\text{-}44)$$

The thickness of the exit cone structure at the downstream end of the nozzle throat insert, τ_{xs}, is sized from

$$\tau_{xs} = \frac{P_{D_x} r_{es}}{\sigma_n \cos\alpha} . \qquad (3\text{-}45)$$

The design pressure P_{D_x} is defined as 1.5 times the local pressure in the nozzle exit cone. The structural thickness computed from Equation 3-45 is compared with a specified minimum value which is dictated by fabrication constraints, and the larger of the two values is selected for design application. The corresponding weight of the exit cone structure, W_{xs}, is computed from

$$W_{xs} = \frac{\pi}{3} \rho_s \operatorname{ctn}\alpha \left(r_e - r_{at}\right) \Bigg\{ \left[r_{at} + \left(\tau_{ie} + \tau_{xs}\right)\cos\alpha\right]^2 +$$

$$+ \left[r_{at} + \left(\tau_{ie} + \tau_{xs}\right)\cos\alpha\right]\left(r_e + \tau_{ie} + \tau_{mn}\right) + \left(r_e + \tau_{ie} + \tau_{mn}\right)^2$$

$$- \left(r_{at} + \tau_{ie}\cos\alpha\right)^2 - \left(r_{at} + \tau_{ie}\cos\alpha\right)\left(r_e + \tau_{ie}\right) - \left(r_e + \tau_{ie}\right)^2 \Bigg\} . \tag{3-46}$$

The weight of the nozzle exit cone insulation, W_{xi}, is determined from

$$W_{xi} = \pi \tau_{ie} \rho_i \operatorname{ctn}\alpha \left(r_{at} + \tau_{ie} + r_e\right)\left(r_e - r_{at}\right) . \tag{3-47}$$

The weight of the attachment boss, W_{nb}, which is an integral part of the nozzle structure, is estimated from

$$W_{nb} = 18\pi r_{op} \tau_e \rho_s . \tag{3-48}$$

From the previously computed nozzle component weights, the total nozzle weight, W_n, is computed as the sum

$$W_n = W_{ei} + W_{es} + W_{ti} + W_{ts} + W_t + W_{xs} + W_{xi} + W_{nb} . \tag{3-49}$$

The nozzle length, L_n, for conical entrance and exit geometries, may be computed from

$$L_n = \overbrace{\left(r_{op} - r_{ot}\right)\operatorname{ctn}\phi}^{\text{Entrance}} + \overbrace{r_t k_t (\sin\phi + \sin\alpha)}^{\text{Throat}} + \overbrace{\left(r_e - r_{at}\right)\operatorname{ctn}\alpha}^{\text{Exit}} . \tag{3-50}$$

Equations 3-36 through 3-50 were derived for nozzles with conical entrance and exit geometries.

However, most modern-day rocket motors which are designed for flight operation employ nonconical contours which are optimized for improved performance and reduced length. Because of the difficulty associated with detailed design of a contoured nozzle, the simplified empirical procedure described in Reference 10 for approximating contoured nozzle weight and length from a corresponding conical nozzle design was employed in this program.

To calculate contoured nozzle weight using the previously described approximate method, the following assumptions are made:

- The weights of conical and contoured nozzles are identical from the forward attachment boss to an expansion ratio downstream of the throat of 3.0 for nozzles of equal throat area. (Nozzle shape within this region is relatively independent of downstream shape.)

- The nozzle weights downstream of an exit expansion ratio of 3.0 are identical for contoured and conical nozzles of equal surface area.

The two types of nozzles are assumed to have identical throat areas, chamber presures, burning durations, and propellant flame temperature. The family of contoured nozzles for which this procedure applies is described by O. J. Demuth in Reference 11.

To determine the conical nozzle surface area which is equivalent to that of a contoured nozzle having the same throat area, an equivalent conical nozzle expansion ratio, ϵ_e, is derived from previously computed contoured nozzle data. This equivalent expansion ratio is defined as that of a conical nozzle with a 17.5-degree exit cone half-angle which has the same surface area as a particular contoured nozzle of the same throat area. Contoured nozzle weights are then computed from the previously described conical nozzle equations (Equations 3-36 through 3-50) when the applied value of the inside radius of the throat exit plane, r_e, is based on the equivalent expansion ratio, ϵ_e, and the input nozzle exit cone half-angle is taken as 17.5 degrees. The curve of ϵ_e as a function of ϵ (Ref. 10), which is incorporated into this computer program, is presented in Figure 3-4. Figure 3-4 applies for a Demuth-type contoured nozzle with an initial divergence half-angle (immediately downstream of the throat) of 32.5 degrees and an expansion ratio at the point of parallel flow of 30. A review of previously designed contoured nozzles (Ref. 4) revealed that this particular exit shape is generally applicable for relatively large tactical rocket motors. Of course, flight nozzles are truncated at expansion ratios slightly less than that required for parallel flow with little performance penalty. Additionally, Figure 3-4 is based on a ratio of combustion gas specific heats of 1.2 and a ratio of throat radius of curvature to throat radius of 1.2.

Direct calculation of the length of a contoured nozzle is again quite difficult. To simplify the calculations in this program, contoured nozzle length is related empirically to a 17.5-degree conical nozzle of the same throat area and expansion ratio. In determining contoured nozzle length, the length from the throat to the exit plane is first calculated as for a 17.5-degree conical nozzle. This length is then multiplied by an empirically derived factor, L_{cont}/L_{con}, for the appropriate contoured nozzle expansion ratio. The curve of L_{cont}/L_{con} as a function of ϵ (Ref. 10), which is employed in the computer program described herein, is presented in Figure 3-5. The curve shown in Figure 3-5 is for the same Demuth-type nozzle shape as discussed previously for nozzle weight calculations. The corrected length between the throat and the nozzle exit plane is then added to the entrance length, between the forward attachment boss and the throat, to determine the total contoured nozzle length.

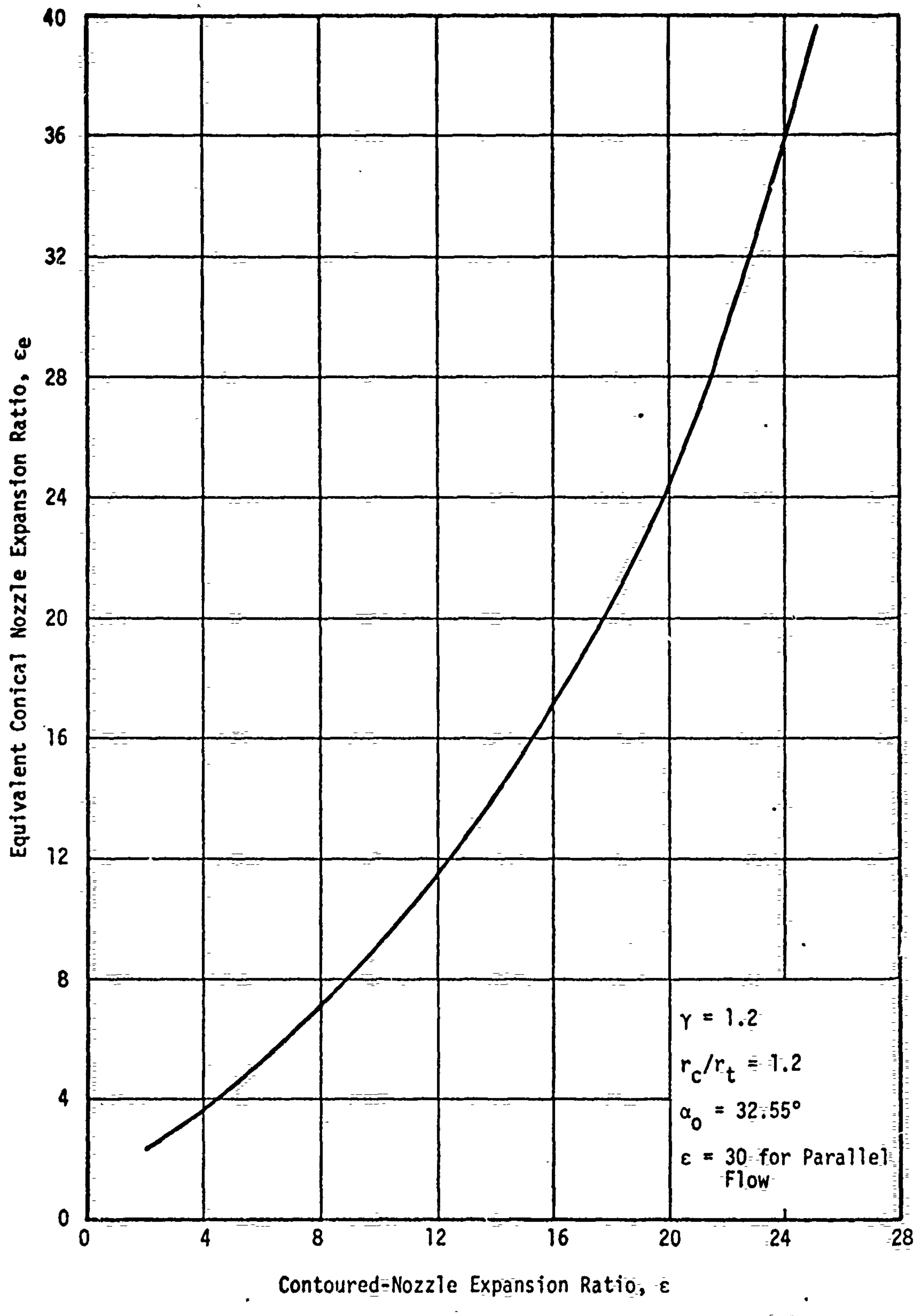


FIGURE 3-4. EQUIVALENT CONICAL EXPANSION RATIO AS A FUNCTION OF CONTOURED NOZZLE EXPANSION RATIO

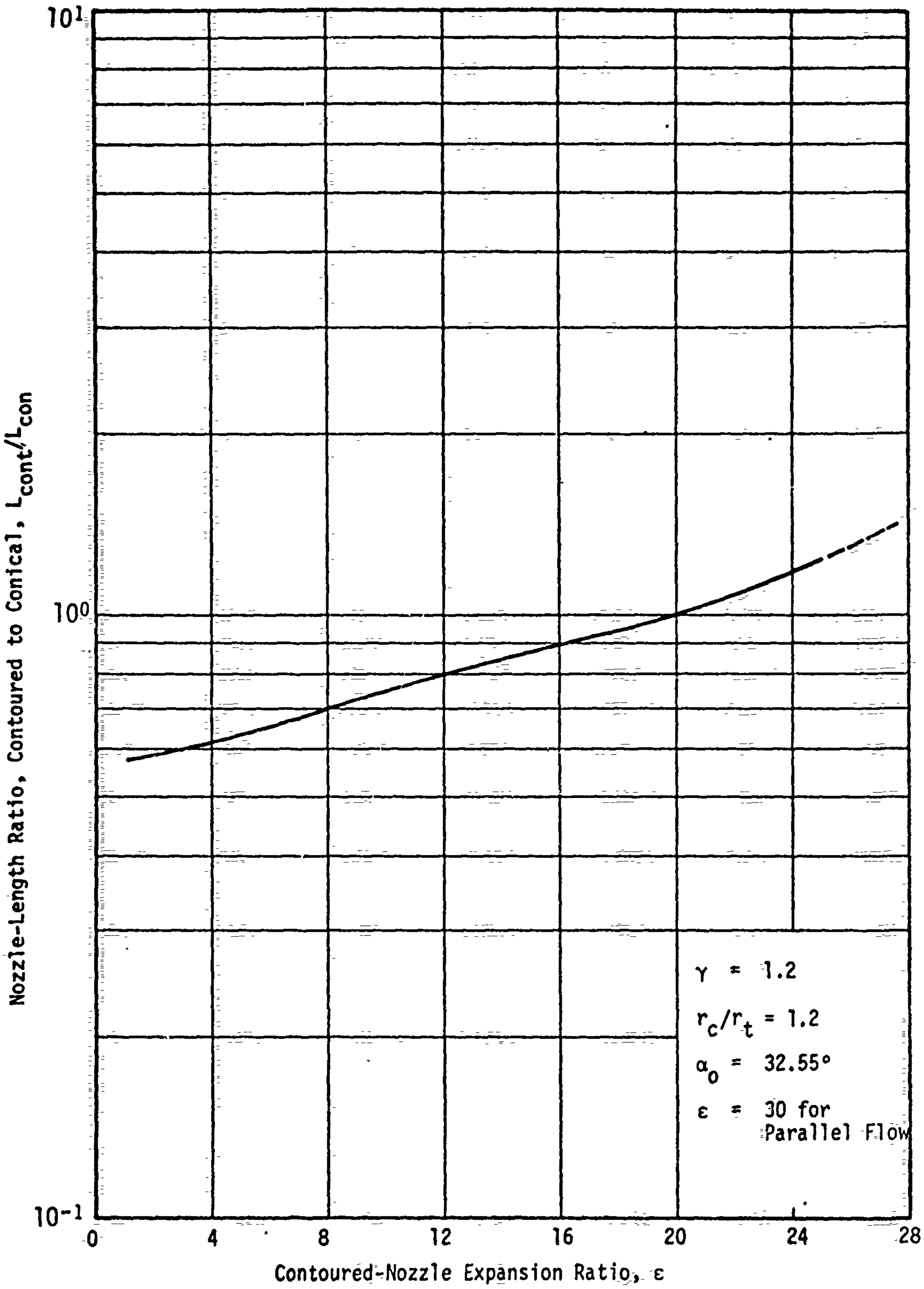


FIGURE 3-5. L_{cont}/L_{con} AS A FUNCTION OF CONTOURED NOZZLE EXPANSION RATIO

The previously described procedure could be extended for analyzing other Demuth-type exit shapes, with different values of α_i and ϵ for parallel flow, by incorporating additional curves of ϵ_e and L_{cont}/L_{con} as a function of ϵ from Reference 10.

3.2.11 Motor Attachment Skirts

The weight of the forward attachment skirt, W_{fs}, is estimated from the following relationship recommended by Reference 9:

$$W_{fs} = C_1 D_m^2 \left\{ \frac{g_{max_i} W_{PL}}{E_c} \left[\frac{0.215 \left(L_c + D_m/\beta \right)}{D_m} + 1 \right] \right\}^{0.5} \quad (3\text{-}51)$$

The maximum longitudinal acceleration of the stage under consideration, g_{max_i}, is assumed to occur at stage burnout:

$$g_{max_i} = F/W_{bo} \quad . \quad (3\text{-}52)$$

Now, at this stage of the analysis, W_{bo} is unknown, since its value depends upon the undetermined weights of the attachment skirts and interstage structure. However, these weights are usually relatively small in comparison with the remaining inert component weights. Therefore, an iterative solution is accomplished in which g_{max_i} is first estimated using a value of W_{bo} which includes only the previously computed weights of the motor case, nozzle, ignitor, liner, insulation, and the stage payload. This value of g_{max_i} is utilized in the calculation of the attachment skirt and interstage weights. The value of g_{max_i} is then recalculated using the estimated skirt and interstage weights included in W_{bo}. This iteration is continued until the computed values of W_{bo} (or g_{max_i}) on successive iterations are equivalent within a specified tolerance. At this point, the correct values of the attachment skirt and interstage weights are considered to be established.

The weight of the aft attachment skirt, W_{as}, is estimated from

$$W_{as} = 0.0055\, D_m^2 \left\{ \frac{g_{max_{i-1}}\, W_o}{E_c} \right. \tag{3-53}$$

$$\left. \times \left[\frac{0.215 \left(L_c + \frac{D_m}{\beta} \right)}{D_m} + 1 \right] \right\}^{0.5} .$$

In the evaluation of W_{as}, the value of $g_{max_{i-1}}$ must be estimated from a separate analysis of stage i-1, and W_o is estimated as part of the previously described iteration on W_{bo}.

3.2.12 Interstage Structure

The interstage structural weight, W_{int}, is estimated, when required, from the following relationship which is recommended by Reference 9:

$$W_{int} = 0.155\, D_m \left(\frac{D_m}{\beta} + L_{n_{i+1}} + L_{int} \right) \tag{3-54}$$

$$\times \left\{ \frac{g_{max_i}\, W_{PL}}{E_{int}} \left[\frac{0.215 \left(L_c + \frac{D_m}{\beta} \right)}{D_m} + 1 \right] \right\}^{0.5}$$

The evaluation of g_{max_i} and W_{int} is included in the previously described iteration on W_{bo}.

3.2.13 Rocket Motor Length

Rocket motor length between the nozzle exit plane and the forward head, L_m, is computed by summing the constituent lengths.

$$L_m = L_{fh} + L_c + L_{ah} + L_n \quad . \tag{3-55}$$

The length of the forward head, L_{fh}, is determined from

$$L_{fh} = D_m/2\beta \tag{3-56}$$

and the length of the aft head, L_{ah}, is computed from

$$L_{ah} = \frac{D_m}{2\beta} \left[1 - \left(\frac{2r_{op}}{D_m}\right)^2\right]^{0.5} \quad . \tag{3-57}$$

3.3 ROCKET MOTOR PERFORMANCE CHARACTERIZATION

Sufficient preliminary design information has now been computed from the previously described analysis to permit prediction of rocket motor performance. After setting the sum of the previously computed inert component weights equal to W_{IP}, the gross stage weight, W_o, is computed from

$$W_o = W_{IP} + W_{PL} + W_p \quad . \tag{3-58}$$

Stage mass fraction, μ, is defined as

$$\mu = \frac{W_p}{W_o - W_{PL}} \quad . \tag{3-59}$$

Rocket motor mass fraction, λ, is defined as

$$\lambda = \frac{W_p}{W_o - W_{int} - W_{PL}} \quad . \tag{3-60}$$

The delivered specific impulse, I_{sp_d}, is computed from

$$I_{sp_d} = \frac{C_f}{C_D} \quad . \tag{3-61}$$

The predicted ideal velocity increment produced by the stage, neglecting drag and gravity losses, is computed from

$$\Delta V_I = I_{sp_d} \, g_c \, \ln \frac{W_o}{W_{bo}} \quad . \tag{3-62}$$

Thrust-to-weight ratios at ignition, A_{ign}, and burnout, A_{bo}, are computed from Equations 3-63 and 3-64, respectively:

$$A_{ign} = F/W_o \tag{3-63}$$

$$A_{bo} = F/W_{bo} \quad . \tag{3-64}$$

This completes the description of the rocket motor design analysis as it is programmed for the digital computer. The program is formulated in such a manner that parametric tradeoff analyses of pertinent design variables may be readily accomplished by stacking input cases back-to-back. Moreover, an automatic variable adjusting procedure, which would determine the required value of a specific independent variables (e.g., motor diameter, chamber pressure, propellant web fraction, etc.) to produce a desired motor performance or design characteristic (e.g., mass fraction, motor length, or velocity increment) may be easily incorporated into the program if desired.

4. PROGRAM APPLICATION

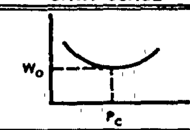
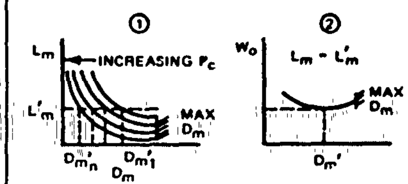
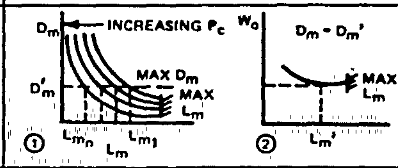
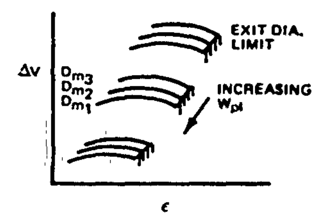
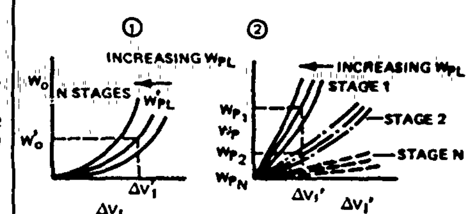
4.1 METHODOLOGY

The computer program described herein may be utilized to generate both point and optimized preliminary designs of solid propellant rocket motors through the proper selection and manipulation of appropriate input variables. Typical design problems and corresponding methods for using the program to solve these problems are presented in Table 4-1. The computer program contains no automated variable optimization scheme. Therefore, the optimization of pertinent design variables must be accomplished using "brute force" techniques wherein graphs are plotted of computed performance and/or weight characteristics which result from manual perturbation of the variables of interest. Optimum values of design variables which yield either maximum performance or minimum weight characteristics are then selected from these graphs. For example, Problem 1 of Table 4-1 illustrates the technique which would be employed to establish the optimum chamber pressure for a minimum weight design, assuming all other input variables are fixed.

Problem 2a of Table 4-1 illustrates how the designer would determine the optimum motor diameter and corresponding chamber pressure which would be required to provide a given motor length. Similarly, Problem 2b illustrates the technique which would be used to determine the optimum motor length and corresponding chamber pressure which would yield a given motor diameter. The latter two examples are frequently encountered in the design of tactical and strategic missiles which must be contained within fixed envelope dimensions.

Problem 3 illustrates how the program would be employed to evaluate the variation of stage impulsive velocity with motor diameter, expansion ratio and payload weight.

TABLE 4-1. PROGRAM APPLICATION TO TYPICAL DESIGN PROBLEMS

PROB. NO.	DESIGN OBJECTIVE	FIXED PARAMETERS	REQUIRED COMPUTER PROGRAM MANIPULATION	DATA USAGE	PROBLEM SOLUTION
1	Find optimum chamber pressure for minimum stage weight.	Motor diameter, grain configuration, propellant type, thrust, port-to-throat ratio, payload weight	Process several stacked cases with varying input values of P_c, plot computed W_o versus P_c.		Select optimum P_c from plot, rerun point design case with selected P_c.
2a	Find optimum motor diameter, chamber pressure to provide a given motor length, L_m', within maximum diameter constraint.	Maximum allowable motor length, grain configuration, propellant type, thrust, port-to-throat ratio, payload weight	Process several stacked cases with varying D_m; for each D_m, run several input values of P_c. Plot L_m versus D_m for selected values of P_c to find values of D_m' which yield L_m'. Plot W_o versus D_m to find D_m', P_c for minimum W_o and required length.		1. From 1 select values of D_m' which yield L_m'; plot 2. 2. From 2 select optimum D_m'.
2b	Find optimum motor length, chamber pressure to provide a given motor diameter, D_m', within maximum length constraint	Maximum allowable motor diameter, others same as 2a	Same as 2a, except plot D_m versus computed L_m for each input P_c to find values of L_m' which yield D_m'. Plot W_o versus L_m' to find L_m', P_c for minimum W_o and required diameter.		1. From 1 select values of L_m' which yield D_m'; plot 2. 2. From 2 select optimum L_m'.
3	Examine variation of ideal impulsive velocity with motor diameter, expansion ratio, payload weight.	Chamber pressure, thrust, grain configuration, propellant type, port-to-throat ratio	For each selected payload weight, W_{PL}, run several stacked cases with varying D_m; for each D_m, input several values of exit diameter, D_{em}. Plot computed ΔV versus computed expansion ratio for selected motor diameters, payload weights.		Select optimum D_m and corresponding optimum ϵ for each W_{PL}.
4	For a multistage missile, determine optimum number of stages and propellant weight distribution as a function of payload weight and total impulsive velocity requirements. For selected optimized vehicle concept, determine optimum diameter, chamber pressure for each stage within envelope constraints.	Payload weight range, total impulsive velocity range, grain configuration, propellant type, port-to-throat ratio, maximum allowable diameter, length	1. Using Missile Optimization Program (Appendix B), establish variation of missile gross weight, optimum propellant weight distribution with payload, ideal ΔV requirement for N stages. (Assuming stage mass fraction, I_{sp} values) plot 1 and 2. 2. For selected $\Delta V_I'$, W_{PL}', pick propellant weights W_{P1}, W_{P2}...W_{PN} from 2. Compute total impulse, corresponding thrust requirement for each stage. 3. Perform optimization of diameter (or length) as in 2a, 2b, and of expansion ratio as in 3. 4. Repeat with other numbers of stages, select minimum weight design.		Select optimum number of stages, optimum propellant weight distribution, optimum stage diameters, chamber pressures.

Problem 4 illustrates how the sizing program described herein may be used in conjunction with a stage optimization routine to develop a near-optimum preliminary design of the propulsion components of a multistage vehicle. A typical stage optimization routine, which was computerized by Mr. J. H. Dobkins of Teledyne Brown Engineering, is described in Appendix B. Results from this routine, which is entitled the Missile Optimization Program (MOP), have been demonstrated to agree closely with those from other more elaborate analyses. Since MOP requires ideal impulsive velocity as input, the designer must estimate velocity losses which will result from drag and gravitational effects. Additionally, input values of mass fraction and specific impulse for each stage must be specified. Minimized gross vehicle weight and optimized propellant weight distributions for a prescribed number of stages are then computed using MOP for the ranges of payload weights and ideal impulsive velocities of interest. For selected point values of ideal impulsive velocity and payload weight, the designer computes corresponding values of required total impulse and thrust (for a prescribed burn time) for each stage. The sizing program may then be utilized to establish optimum diameters (or lengths), chamber pressures and expansion ratios for all of the stages as discussed previously.

Following the preliminary design of a single or multistage vehicle, the evaluation of its predicted flight performance is usually desired. The trajectory analysis which is required to evaluate flight performance is beyond the scope of this investigation. However, vehicle design characteristics, e.g., weights, mass flow rates, thrust levels, envelope dimensions, which are desired from the sizing program may be input into a trajectory simulation computer program to predict flight performance. The trajectory simulation program described in Reference 12 is typical of many applicable programs which are available.

4.2 SAMPLE PROBLEM

To demonstrate the validity of the computer program, a sample problem was processed which consisted of an analytical reproduction of the Thiokol TX-354-3 (Castor II-A) rocket motor design described in Reference 13. This motor was chosen for discussion herein because of the ready availablity of unclassified documentation describing its design and performance characteristics in detail. Other rocket motor designs described in Reference 4, e.g., Spartan, were also analyzed successfully using this program, but the classified nature of their design characteristics precluded their discussion herein.

The cross-sectional arrangement of the propellant grain in the TX-354-3 rocket motor is illustrated in Figure 4-1. The grain design is basically a cylindrical port type which is segmented by two radial slots, as shown in Figure 4-1. A propellant web fraction, F_W, of 0.76 was deduced from the published geometric characteristics of the propellant charge. The TX-354-3 nozzle design (Figure 4-2) utilizes a conical expansion geometry, a graphite throat insert, and requires no entrance structure. The nozzle is designed for operation at near-vaccum conditions. AISI 4130 steel is utilized as the structural material for the rocket motor chamber and nozzle body.

The required input parameters corresponding to the TX-354-3 rocket motor were translated into a set of motor design and performance characteristics using the computer program described herein. The computer printout of the input and output parameters of the sample problem analysis is presented in Figure 4-3. For comparison, published values (Ref. 13) of pertinent design and performance parameters of the TX-354-3 motor are included in parentheses adjacent to the corresponding computer values. Comparison of the published and computed

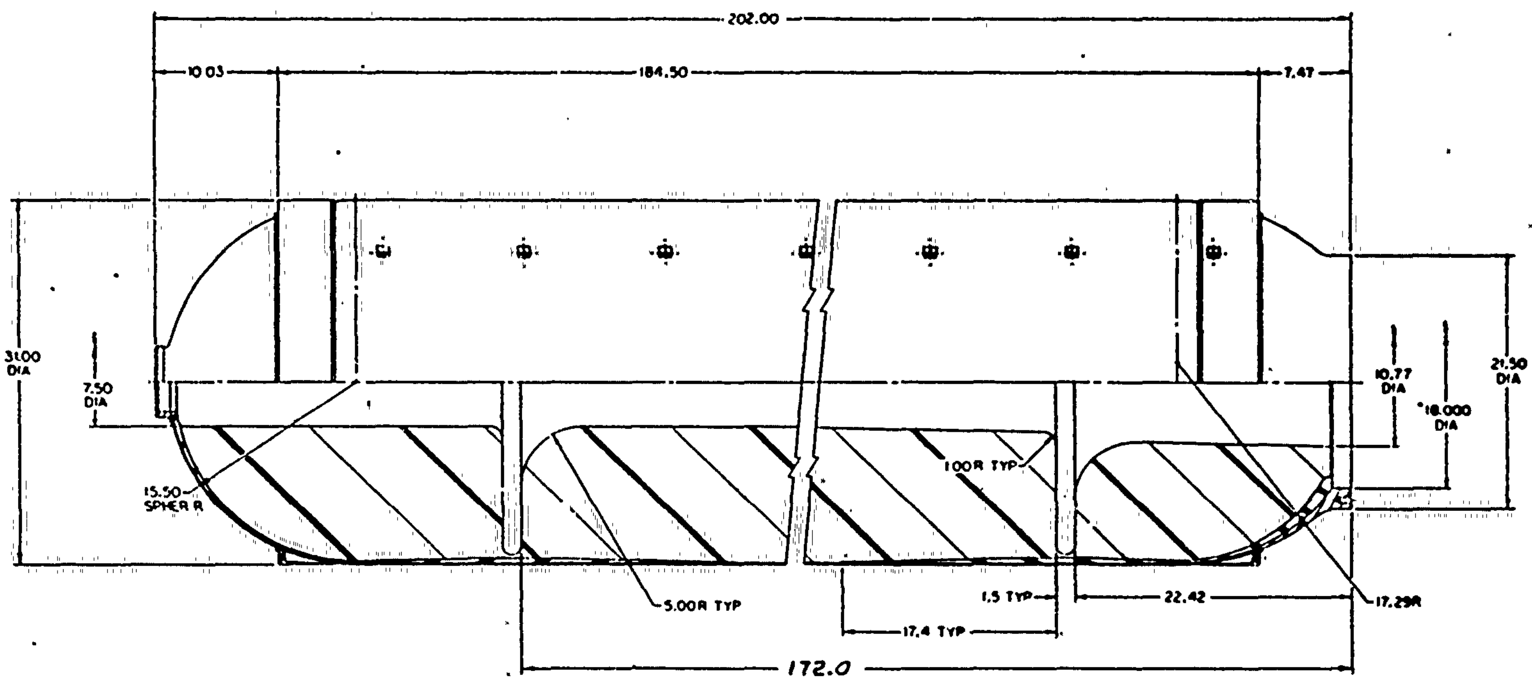


FIGURE 4-1. GENERAL ARRANGEMENT OF LOADED TX-354-3 ROCKET MOTOR CHAMBER

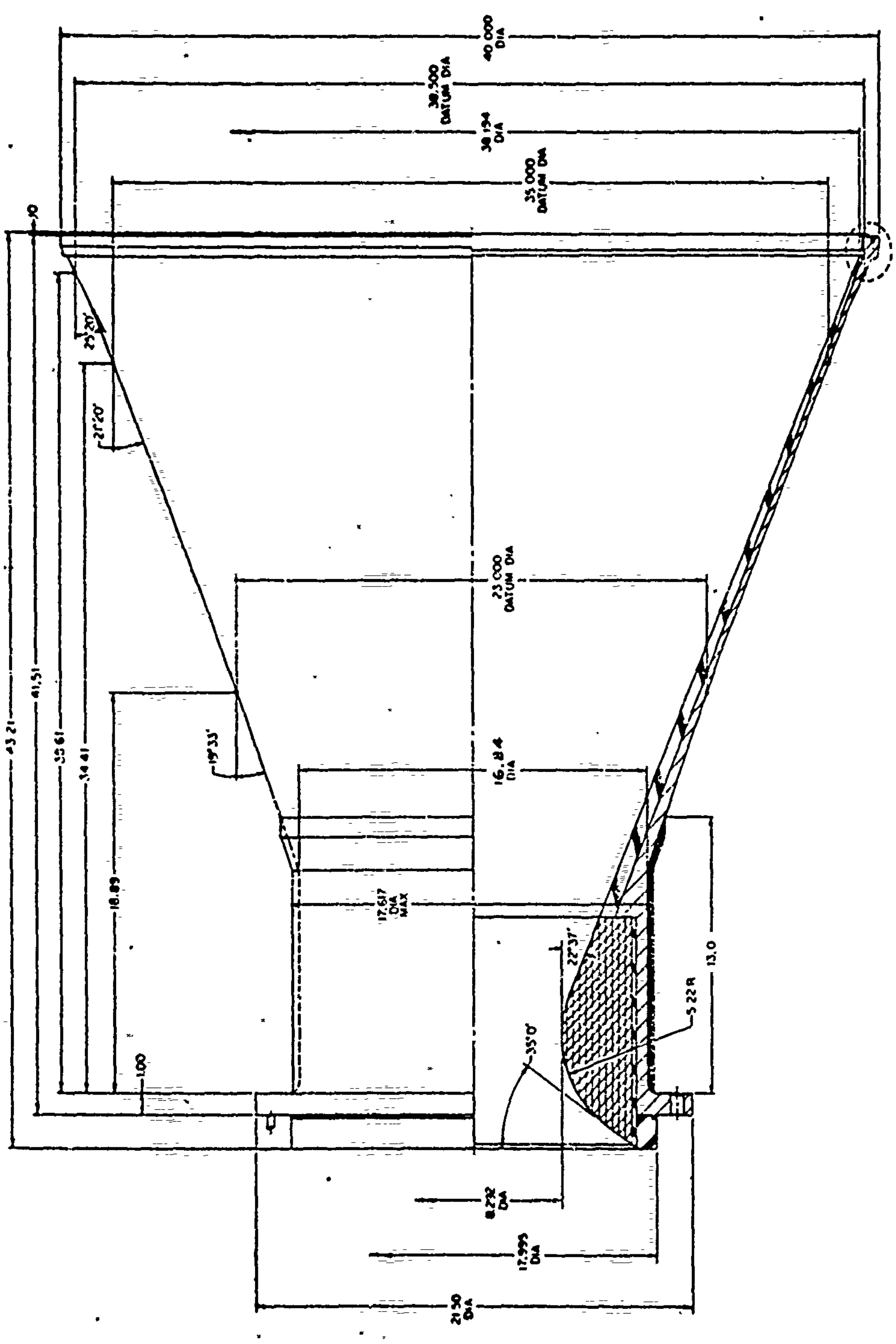


FIGURE 4-2. TX-354-3 ROCKET NOZZLE ASSEMBLY

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                        ROCKET MOTOR PRELIMINARY DESIGN PROGRAM

                      * * * * * PROGRAM INPUT DATA * * * * *

DM, MOTOR O.D.(IN.) . . . . . . . =     31.0000     DEM,    NOZZLE EXIT O.D. (IN) . . =      40.0000
PC, CHAMBER PRESSURE      (PSI)  =    635.0000     ALF,NOZZLE EXIT HALF-ANGLE(DEG) . =      22.6300
F, REQ. THRUST (LBF)    . . . .  =  51763.3330     FW,PROPELLANT WEB FRACTION . . . . =        .7500
WPL, PAYLOAD MASS   (LBM)  . . . =   3510.0000     WEX,MASS OF EXTRAS(LBM) . . . . .  =       0.0000
RHOP, PROPELLANT DENSITY (LBM/IN3)=      .0647     ATB,BURN RATE COEFF.. . . . . . .  =        .0530
AN,BURN RATE EXPONENT . . . . . . =      .2740     GAM, RATIO OF SPECIFIC HEATS  . .  =       1.1630
TC, GAS CHAMBER TEMP. (DEG R). .  =   5542.30      AMW, MOL. WT. OF EXH. GAS(LBM/MOLE)=      29.0000
APT, PORT-TO-THROAT AREA RATIO .  =     1.2700     PAM, ATM. PRESSURE(PSIA) . . . .  =        .5000
SFM, CASE DESIGN SAFETY FACTOR  . =     1.50       SFN,NOZZLE DESIGN SAFETY FACTOR. . =        1.50
ETAD, DISCHARGE CORRECTION FACTOR =     1.0000     ETAV, VELOCITY CORRECTION FACTOR   =        .9700
FP,FRACT. OF PROP. BURNE. TO WEB  =     1.0000
ZLN, NOZZ. LGTH. OF NEXT STAGE(IN)=     3.0000     ZLI,INTERSTAGE CLEARANCE(IN) . . . =       0.0000
CODE,STAGE DESIGNATION  . . . . . =     2.0000     FLAG(0.-CYL,1.-FWD,2.,FWD+AFT,3.-AFT)       2.0000
NOZ(0.-CONICA. NOZZ..1.0-CONTOUR) =     0.0000     ISLIV(SET.GT. 0. FOR INERT SLIVERS)=       0.0000

                      * * * * * ROCKET DESIGN SUMMARY * * * * *

NOZZLE THROAT I.D. (IN.)  . . . . =   8.445927 (8.41)    NOZZLE  EXIT I.D. (IN.)  . . . . . . =   38.849859 (*)
NOZZLE EXIT O.D.(IN.) . . . . . . =  39.333970 (40.0)    NOZZLE EXIT WALL THICK (IN.) . . . . =     .050000 (*)
NOZZLE THROAT AREA (IN2) . . . .     56.025236 (55.6)    NOZZLE EXIT AREA (SQ IN.)  . . . . . = 1185.409495 (*)
NOZZLE AREA RATIO (AE/AT) . . . . =  21.159469 (20.95)   NOZZLE EXIT MACH NUMBER  . . . . . . =    3.631016 (3.63)
NOZZLE CONE ANGLE (DEG) . . . . . =      27.53 (**)      NOZZLE EXIT PRESSURE (PSI) . . . . . =    3.446013 (3.47)

MOTOR CHAMBER I.D. (IN.)  . . . . =  30.502220 (30.72)   MOTOR CHAMBER O.D. (IN.) . . . . . . =   31.000000 (**)
MOTOR WALL THICK (IN.)  . . . . . =    .099890 (0.11)    MOTOR XSECT AREA (SQ IN.)  . . . . . =  742.266582 (*)
FORWARD HEAD THICK (IN) . . . . . =    .076937 (*)

PORT AREA (SQ IN.)  . . . . . . . =  71.152126 (*)       WEB XSECT AREA (SQ IN.)  . . . . . . =  671.114556 (*)
PORT-TO-THROAT AREA . . . . . . . =   1.270000 (**)      WEB THICKNESS (IN.)  . . . . . . . . =   11.682344 (11.6)
BURNING AREA (SQ IN.) . . . . . . = 10313.391 (10,930)   CYLINDRICAL MOTOR LENGTH (IN.) . . . =  172.514374 (170)
LENGTH OF FWD. HEAD (IN)  . . . . =  14.093919 (*)       LENGTH OF AFT HEAD (IN)  . . . . . . =   13.685996 (*)
NOZZLE LENGTH (IN)  . . . . . . . =  42.319375 (43.2)    MOTOR LGTH(NOZ EXIT TO FWD HEAD,IN)=   242.611252 (245)

MASS OF NOZZLE ASSY (LBM) . . . . = 170.510721 (539)     MASS OF CYL.(TUBE+LINER) (LBM) . . . =  498.742842 (720-Chamber Ass'y)
MASS OF FWD HEAD ASSY. (LBM). . . =  47.961359 (*)       MASS  OF AFT HEAD ASSY. (LBM). . . . =   61.640951
MASS OF EXTRAS (LBM)  . . . . . . =   0.000000 (*)       MASS  OF IGNITER (LBM) . . . . . . . =   12.675147 (14)
MASS OF FWD. SKIRT (LBM)  . . . . =   9.929359 (*)       MASS OF AFT SKIRT (LBM)  . . . . . . =    9.929359 (*)
MASS OF INTERSTAGE STRUCTURE (LBM)=   0.000000 (N/A)     TOTAL MASS OF INERT PARTS (LBM)  . . =  811.369757 (1523)

TOT PROPELLANT MASS,WP  (LBM) . . = 8263.757966 (8,220)
MASS OF PROP. IN FWD. HEAD (LBM)  =  380.331764 (*)      MASS  OF PROP. IN AFT  HEAD (LBM)  =    392.634231 (*)
MASS OF PROP. BURNED TO WEB(LBM)  = 8263.757966 (8,220)  MASS OF SLIVERS(PROP OR INERT)(LBM)=      0.000000 (0.0)

GROSS MASS AT STAGE IGN.,WO (LBM) = 12585.1277 (*)       MASS OF PAYLOAD,WPL (LBM)  . . . . . =   3510.0000 (*)
GROSS MASS AT STAGE BURNOUT (LBM) =  4321.3698 (*)

                      * * * * * ROCKET PERFORMANCE SUMMARY * * * * *

PROPELLANT BURNING RATE (IN./SEC)  . . . . . .        .3113  (0.31)
MASS DISCHARGE COEFFICIENT (LBM/LBF-SEC) . .          .00516 (0.0060)
MASS FLOW RATE (LBM/SEC) . . . . . . . . . . .       220.32  (210.4)
C*, CHARACTERISTIC VELOCITY(FPS) . . . . . . .      5227.0   (5211)
THRUST(LBF)  . . . . . . . . . . . . . . . . .     61760.00  (**)
SPECIFIC IMPULSE (LBF-SEC/LBM) . . . . . . . .       280.71  (281.9)
THRUST/WEIGHT AT IGNITION (G)  . . . . . . . .         4.91  (*)
THRUST/WEIGHT AT BURNOUT (G) . . . . . . . . .        14.29  (*)
CF,THRUST COEFF.   . . . . . . . . . . . . . .     1.727836  (1.74)
IDEAL VEL. INCR. (FPS)   . . . . . . . . . . .      9654.19  (*)
BURNING TIME      (SEC)  . . . . . . . . . . .    37.559713  (37.0)
STAGE MASS FRACT. (WP/(WO-WPL)). . . . . . . .       .91059  (0.84)
```

Legend: (*) - No Comparable Data Available
(**) - Input Value
(N/A) - Not Applicable

NOTE: Numbers in parentheses represent published values for TX-354-3 rocket motor (Reference 13)

FIGURE 4-3. SAMPLE PROBLEM SOLUTION

values reveals certain discrepancies which must be discussed. For example, the computer nozzle weight of 170.5 lbm is much less than the published value of 539 lbm. This large discrepancy is difficult to explain without a knowledge of the details of the original structural analysis from which the nozzle was designed. However, it may be speculated that bending stresses from predicted lateral flight loads could have dictated larger required structural thicknesses than those which resulted from the pure hoop stress equations employed in this computer program. (The credibility of the nozzle design procedures employed herein was demonstrated through the almost exact analytical reproduction of the existing Spartan booster nozzle.) The only other conceivable explanation for this discrepancy is that relatively large design safety factors ($\gg$ 1.5) could have been employed in the original design analysis of the TX-354-3 nozzle.

The only other significant discrepancies between published and computed motor design parameters are in total motor length, total inert parts weight, and mass fraction. The discrepancy between the published motor length of 245 inches and the computed value of 242.6 inches results primarily from the neglect in the computer program of the reported lengths associated with the pyrogen and nozzle attachment bosses (Figure 4-1). The discrepancy between the published total inert parts weight of 1523 lbm and the computed value of 811.4 lbm results primarily from the nozzle weight deficit (368.5 lbm), the chamber weight deficit (92 lbm), and the weight deficit caused by neglect of the additional insulation weight required by the two radial slots in the propellant. Additionally, the discrepancy between the published mass fraction of 0.84 and the computed value of 0.91 results directly from the previously described discrepancy in total inert parts weight.

Additional test cases, covering a variety of motor configurations, should be processed in the future to clearly establish the accuracy and general applicability of the computer program.

5. PROGRAM OPERATING INSTRUCTIONS

The previously described design equations were coded into a FORTRAN computer program consisting of approximately 530 cards. Multiple cases may be submitted simultaneously by stacking the input cards back-to-back. Only the input parameters which change need be added to complete each new case. The remaining input parameters always revert to the values used in the preceding case.

5.1 INPUT DATA

All program input variables are read into the array A(i), i = 1, 27. A card-by-card description of the inputs, including symbol definition and format, is presented below.

- Card 1, Format (I2)
  - ▲ Code word KTR
  - ▲ KTR is the number of input variables (one per card) to be read following this card. At the end of the last case, input KTR as 99 to end the job.
- Cards 2 through 28, Format (I2, E15.8)
  - ▲ These 27 cards contain the 27 input variables which are read into the array A(i). On each of the 27 cards, the proper index, i, is punched into columns 1 and 2, and the corresponding element A(i) is punched into columns 3 through 17, using the format given above.
  - ▲ The 27 inputs, A(i), i = 1, 27, and the equivalent program variables are defined in Table 5-1.
- Card 29, Format (I2)
  - ▲ Code word KTR
  - ▲ This is the same code word as described above. If another case is to follow, KTR is the number of input elements A(i) to be changed from the values used in the preceding case. If no more cases follow, KTR should be punched as 99.

TABLE 5-1. INPUT VARIABLE DEFINITION
FOR CARDS 2 THROUGH 28

ELEMENT	EQUIVALENT PROGRAM VARIABLE	DEFINITION	UNITS
A(1)	DM	Rocket motor outside diameter	in.
A(2)	PC	Chamber pressure	psia
A(3)	F	Required thrust	F
A(4)	FW	Propellant web fraction (Ratio of web thickness to propellant outside radius)	--
A(5)	APT	Port-to-throat area ratio	--
A(6)	PAM	Ambient pressure	psia
A(7)	TC	Propellant flame temperature	°R
A(8)	RHOP	Propellant density	lbm/in^3
A(9)	GAM	Combustion gas ratio of specific heats	--
A(10)	ATB	Pressure coefficient in propellant burning rate law	--
A(11)	AMW	Molecular weight of combustion gas	lbm/mole
A(12)	AN	Pressure exponent in propellant burning rate law	--
A(13)	ETAD	Mass discharge correction factor for nonideal flow	--
A(14)	ETAV	Exit velocity correction factor for nonideal flow	--
A(15)	ALF	Nozzle exit cone half-angle (for contoured nozzle analysis, input effective value of ALF of 17.5 degrees)	deg

TABLE 5-1. INPUT VARIABLE DEFINITION
FOR CARDS 2 THROUGH 28 (Continued)

ELEMENT	EQUIVALENT PROGRAM VARIABLE	DEFINITION	UNITS
A(16)	DEM	Maximum permissible nozzle exit plane diameter	in.
A(17)	FP	Fraction of propellant burned before web burn through	--
A(18)	SFM	Motor case design safety factor	--
A(19)	SFN	Nozzle design safety factor	--
A(20)	WPL	Stage payload weight (includes terminal payload and upper stage weight, where applicable)	lbm
A(21)	WEX	Miscellaneous weights to be added to stage weight (e.g., thrust vector control hardware)	lbm
A(22)	ZLN	Nozzle length of next higher stage	in.
A(23)	ZLI	Interstage clearance between forward head of stage being designed and nozzle exit plane of next higher stage	in.
A(24)	CODE	Code designating which stage is being designed (CODE = 2.0 for terminal stage of multiple vehicle or for single-stage vehicle; CODE = 1.0 for remaining stages)	--
A(25)	FLAG	Set FLAG = 0.0 when head end web(s) are <u>not</u> present; FLAG = 1.0 forward head web only; FLAG = 2.0 for forward and aft head webs; FLAG = 3.0 for aft head web only.	--

TABLE 5-1. INPUT VARIABLE DEFINITION FOR CARDS 2 THROUGH 28 (Concluded)

ELEMENT	EQUIVALENT PROGRAM VARIABLE	DEFINITION	UNITS
A(26)	NOZ	Set NOZ = 0.0 for conical nozzle analysis; NOZ = 1.0 for contoured nozzle analysis	--
A(27)	ISLIV	For cases where FP < 1.0, set ISLIV = 0.0 for active slivers; ISLIV > 0.0 for inert slivers	--

In preparation of the program inputs, FORTRAN coding sheets are usually completed to serve as a guide in punching the appropriate values onto input cards. Example input coding sheets, including input variable definitions and typical ranges and/or recommended values of the inputs, are presented in Figure 5-1.

FIGURE 5-1. SAMPLE INPUT CODING SHEET

SAMPLE INPUTS* (columns 1–20)	PROGRAM VARIABLES**	TYPICAL RANGES OR RECOMMENDED VALUES
27	KTR	Number of inputs to follow or 99 to end job
1+.31 E+02	DM	Constrained by envelope limitations
2+.638 E+03	PC	300 to 2000 psia
3+.61760 E+05	F	Dictated by total impulse, burn time requirements
4+.76 E+00	FW	Star, wagonwheel: 0.2 to 0.5 slotted tube: 0.3 to 0.9
5+.127 E+01	APT	1.1 to 3.5
6+.5 E+00	PAM	0.0 to 14.7 psia
7+.6542 E+04	TC	4000 to 6600 °R
8+.0647 E+00	RHOP	0.053 to 0.07 lbm/in^3
9+.116 E+01	GAM	1.12 to 1.2
10+.053 E+00	ATB	0.002 to 0.1
11+.29 E+02	AMW	25 to 29
12+.274 E+00	AN	0.1 to 0.9
13+.1 E+01	ETAD	0.98 to 1.15 (1.02 typ.)
14+.97 E+00	ETAV	0.85 to 0.98 (0.92 typ.)
15+.2263 E+02	ALF	10 to 25 deg. (15 typ.)
16+.40 E+02	DEM	Constrained by envelope, expansion ratio limitations
17+.1 E+00	FP	0.9 to 1.0
18+.15 E+01	SFM	1.5
19+.15 E+01	SFN	1.5
20+.3510 E+04	WPL	Dictated by mission objectives
21+.0 E+00	WEX	Additional miscellaneous weight (Fins, TVC Hardware, etc.)
22+.0 E+00	ZLN	Zero for single or upper stages; Appropriate value for lower stages.
23+.0 E+00	ZLI	3 - 10 in.
24+.2 E+01	CODE	**
25+.2 E+01	FLAG	**
26+.0 E+00	NOZ	**
27+.0 E+00	ISLIV	**
99	KTR	End of job

* Values listed are inputs for Sample Problem (Figure 4-3)
** Refer to page 5-1 and Table 5-1 for Definitions

5.2 STORED DATA

Most of the material physical properties which are required in the motor design are defined through the use of FORTRAN DATA statements within the main segment of the program. This procedure was adopted to reduce the number of inputs which would be required in parametric analyses once the motor case, insulation, and nozzle materials have been selected for a particular rocket motor design. Program flexibility is not impaired significantly, because various structural materials may be readily analyzed by simply modifying the appropriate property values in the previously mentioned DATA statements before program compilation. These property values are then employed by the program until further changes in the DATA statements are made.

The program variables which represent the various physical properties are well defined in the main program listing in the Appendix by several COMMENT cards. Program variable definitions and corresponding typical property values are presented in Table 5-2. The values shown in Table 5-2 are those incorporated in the program deck which is listed in the Appendix.

TABLE 5-2. TYPICAL MATERIAL PHYSICAL PROPERTIES AND EQUIVALENT PROGRAM VARIABLES

	PROGRAM VARIABLE	DEFINITION	TYPICAL VALUE
Motor[1] Case	RHOM	Motor case density	0.283 lbm/in^3
	SIGM	Allowable tensile stress	150,000 psi
	EC	Young's modulus of case	30 (10^6) psi
	EI	Young's modulus of interstage structure	30 (10^6) psi
	BETA	Forward and aft head ellipse ratio	1.0 - 1.6
Nozzle[1] Structure	RHOS	Density	0.283 lbm/in^3
	SIGN	Allowable tensile stress	150,000 psi
	ESUBS	Young's modulus	30 (10^6) psi
	MUS	Poisson's ratio	0.3
	CSUB2	Specific heat	0.11 Btu/lbm-°R
Nozzle[2] Insulation	RHOI	Density	0.062 lbm/in^3
	KSUB1	Thermal conductivity	0.5 (10^{-5}) Btu/in.-sec-°R
	CSUB1	Specific heat	0.21 Btu/lbm-°R
Throat[3] Insert	RHOT	Density	0.062 lbm/in^3
	ESUBT	Young's Modulus	1.7 (10^6) psi
	MUT	Poisson's ratio	0.12
	SIGT	Allowable tensile stress	3,355 psi
Insert[4] Slivers	RHOSL	Density	0.02 lbm/in^3

NOTES:

[1] Typical values for 4130 steel.

[2] Typical values for FM 5048 silica/phenolic.

[3] Typical values for ATJ graphite.

[4] Typical value for phenolic microballoon.

6. REFERENCES

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APPENDIX A

FORTRAN LISTING OF SIZING PROGRAM

A complete listing of the FORTRAN statements is furnished in this appendix as a reference for making any desired changes to the source deck of the solid rocket motor sizing program.

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     ROCKET MOTOR PRELIMINARY DESIGN PROGRAM -J L THURMAN
C     PROG AS-803
      REAL NOZ,LC,ME,MDOT,LAM ,ISP ,MU,LFM,LAM,LN,LM,MUT,MUS,KSUB1,ISLIV
      DIMENSION A(50),EXP(10),CFVCON(10)
      EQUIVALENCE (A(1),DM),(A(2),PC),(A(3),F ),(A(4),FW),(A(5),APT),
     1(A(6),FAM),(A(7),TC),(A(8),RHOF),(A(9),GAM),(A(10),ATB),
     2(A(11),AMM),(A(12),AN),(A(13),ETAO),(A(14),ETAV),(A(15),ALF),
     3(A(16),DEM),(A(17),FP),(A(18),SFM),(A(19),SFN),(A(20),WPL),(A(21),
     4WEX),(A(22),ZLN),(A(23),ZLI),(A(24),CODE),(A(25),FLAG),(A(26),NOZ)
     5,(A(27),ISLIV)
C
C     MOTOR CASE PROPERTIES(4340STEEL)
C     RHOM-DENSITY,SIGM-YEILD STR.,EC-YOUNGS MOD.,BETA-HEAD ELLIPSE RAT.
C     EI-YOUNGS MOD. OF INTERSTAGE
      DATA RHOM/.283/,SIGM/1.50E+05/,EC/3.0E+07/,BETA/1.1/,EI/3.0E+07/
C
C     NOZZLE STRUCTURE PROPERTIES(4340 STEEL)
C     RHOS-DENSITY,SIGN-YEILC STR.,ESUBS-YOUNGS MOD.,MUS-POISSON RATIO
C     CSUB2-SPECIFIC HEAT
      DATA RHOS/.283/,SIGN/1.50E+05/,ESUBS/3.0E+07/,MUS/0.3/,CSUB2/0.11/
C
C     NOZZLE INSULATION PROPERTIES(SILICA/PHENOLIC FM-5248)
C     RHOI-DENSITY,KSUB1-CONDUCTIVITY(B/IN SEC F),CSUB1-SPECIFIC HEAT
      DATA RHOI/0.062/,KSUB1/0.5E-05/,CSUB1/0.21/
C
C NOZZLE THROAT INSERT PROPERTIES (SILICA/PHENOLIC FM-5248)
C     RHOT-DENSITY,ESUBT-YOUNGS MOD.,MUT-POISSON RATIO,SIGT-YEILD STR.
      DATA RHOT/0.062/,ESUBT/1.7E+06/,MUT/.12/,SIGT/3355./
C
C     INERT SLIVER DENSITY (ZEUS PHENOLIC MICROBALLOON)
      DATA RHOSL/0.024/
C     DIGN=DIA. OF IGNITOR OPENING,IN.
      DATA DIGN/4./
      DATA EXP/1.0,2.,3.,4.,6.,8.,10.,14.,20.,28./
      DATA CFVCON/1.325,1.355,1.455,1.518,1.6,1.66,1.7,1.755,1.81,1.855/
      KIN=5
      KOUT=6
      DO 1 I=1,50
    1 A(I)=0
    2 READ(KIN,101) KTR
  101 FORMAT(1012)
      IF(KTR-99)4,3,3
    3 CALL EXIT
    4 READ(KIN,102) (I,A(I),KK=1,KTR)
  102 FORMAT(I2,E15.8)
C     SET CONSTANTS
      KODE=CODE
      KFLAG=FLAG
      KNOZ=NOZ
      KSLIV=ISLIV
      RHOSV=RHOSL
      GC=32.174
      PI=3.14159
      GAM1=2./(GAM+1.)
```

```
      GAM2=2.*GAM**2/(GAM-1.)
      GAM3=(GAM+1.)/(GAM-1.)
      GAM4=GAM1**GAM3
      GAM5=GAM/(GAM-1.)
      GAM6=1./GAM5
      GAM7=((GAM+1.)/2.)**(1./(GAM-1.))
      R=1545./AMW
      CD=ETAD*(GC*GAM*GAM4/(R*TC))**.5
      CSTAR=GC/CD
      TAUMN=0.05
      TAULM=0.03
      LAM=0.5*(1.+COS(ALF/57.3))
      WRITE(KOUT,105)
      WRITE(KOUT,106)
      WRITE(KOUT,107)DM,DEM,FC,ALF   , F ,FW, WPL, WEX
      WRITE(KOUT,108)RHOP,ATB,AN,GAM,TC,AMW
      WRITE(KOUT,109)APT,PAM
      WRITE(KOUT,110)             SFM,SFN,ETAD,ETAV,FP
      WRITE(KOUT,111)ZLN,ZLI,CODE,FLAG,NOZ,ISLIV
      RP=ATB*PC**AN
      TAUM=PC*DM*SFM/(2.*SIGP)
      IF(TAUM-TAUMN)6,7,7
    6 TAUM=TAUMN
    7 DCI=DM-2.*TAUM
      TAULI=TAULM
    5 DF=DCI-2.*TAULI
      AF=PI/4.*DF**2
      TAUW=DF*FW/2.
      TB=TAUW/RB
      TAULC=0.02*DM**.5*(TB+1.)**0.1*(1.-FP)**0.25
      IF(TAULC-TAULM)21,22,22
   21 TAULC=TAULM
   22 IF((ABS(TAULC-TAULI)/TAULI)-0.1)24,24,23
   23 TAULI=TAULC
      GO TO 5
   24 PE=PAM
   20 EPS=1./(GAM7*(PE/PC)**(1./GAM)*(GAM3*(1.-(PE/PC)**GAM6))**0.5)
      PEPC=PE/PC
      IF(KNOZ)30,30,31
   30 CF=LAM*ETAD*ETAV*(GAM2*GAM4*(1.-(PE/PC)**GAM6))**.5+
     1(PE-PAM )/PC*EPS
      GO TO 32
   31 CALL ITERP(EPS,EXP,CFVCON,10,4,CFVCT,NERR)
      CF=ETAV*ETAD*(CFVCT-(PEPC*EPS))+(PE-PAM)/PC*EPS
   32 AT=F/(PC*CF)
      AEI=AT*EPS
      DEI=(4./PI*AEI)**0.5
      TAUNE=PE*DEI*SFN/SIGN/2.
      IF(TAUNE-TAUMN)10,11,11
   10 TAUNE=TAUMN
   11 E=1./EPS
      CALL MACH(GAM,1.2,1.0,E,ME,KOUT)
      TE=TC/(1.+(GAM-1.)/2.*ME**2)
      CALL INSUL(TAUNE,TI,TD,SMTS,CSUB1,KSUB1,RHOI,CSUB2,TE,DELTT)
      DEO=DEI+2.*(TAUNE+TI)
```

```
      IF(DEO-DEM)13,13,12
   12 PE=PE*1.01
      GO TO 23
   13 AEO=PI/4.*DEO**2
      MDOT=CD*PC*AT
      AB=MDOT/(RB*RHOP)
      AP=AT*APT
      AW=(AF-AP)*FP
      AS=(AF-AP)*(1.-FP)
      ROP=DF/2.-TAUW
      LFH=DM/(2.*BETA)
      LAH=LFH*(1.-(ROP/BETA/LFH)**2)**0.5
C     FWD AND/OR AFT HEAD WEBS CONSIDERED WHEN KFLAG.GT.0
C     IF KFLAG =0.$NO FWD OR AFT HEAD WEBS+=1.$FWD HEAD WEB ONLY+=2.$FWD
C     AND AFT HEAD WEBS+=3.$AFT HEAD WEB ONLY
      IF(KFLAG)14,14,15
   14 AFHAV=0.
  114 AAHAV=0.
      VPAH=0.
      GO TO 18
   15 GO TO(201,201,200),KFLAG
  200 AFHAV=0.
      GO TO 202
C     FWD HEAD WEB ANALYSIS
  201 IF(DIGN-DF)17,16,16
   16 WRITE(KOUT,940)
  940 FORMAT(73H0DIA. OF IGNITER CAVITY .GT. GRAIN O.D. -HEAD END PROPEL
     1LANT NEGLECTED    )
      AFHAV=0.
      GO TO 214
   17 AIGN=PI/4.*DIGN**2
      ASEM=DF/2.-TAUW/2.
      BSEM=DF/(2.*BETA)-TAUW/2.
      ECC=(ASEM**2-BSEM**2)**.5/ASEM
      AFHAV=PI*(ASEM**2+BSEM**2/(2.*ECC)*ALOG((1.+ECC)/(1.-ECC)))-AIGN
  214 IF(KFLAG-1)114,114,202
C     AFT HEAD WEB ANALYSIS
  202 AA=DF/(2.*BETA)
      VOLA=PI*BETA**2*(AA**2*LAH-LAH**3/3.)-PI*ROP**2*LAH
      VOLAB=PI*LAH*((DF/2.)**2-ROP**2)
      VFAC=VOLA/VOLAB
      VPA=AW*LAH
      VPAH=VPA*VFAC
      AAHAV=VPA*FP/TAUW*VFAC
   18 VPFH=AFHAV*TAUW
      VPH=VPFH+VPAH
      WPFH=VPFH*RHOP
      WPAH=VPAH*RHOP
      WPH=VPH*RHOP
      ABC=AB-AFHAV-AAHAV
      VPSC=TAUW*ABC/FP
      VPCH=VPSC*FP
      LC=VPCH/AW
      VP=VPCH+VPH
      [illegible]SC*(1.-FP)
```

```
      IF(KSLIV)121,121,122
  121 RHOSV=RHOP
      VP=VPSC+VPH
      GO TO 225
  122 VP=VPH
  225 WS=VS*RHOSV
      WP=VP*RHOP
      WPW=VPW*RHOP
      ISP=CF/CD
      DT=(4./PI*AT)**0.5
C     CALC WT OF INERT PARTS
      CALL INERT(PC,SFM,SFN,BETA,DM,SIGM,RHCM,TB,CD,AT,LC,AP,DCI,TAULC,
     1DEI,DT,ALF,DEO,DTO,WPL,F,KODE,EC,EI,ZLN,ZLI,TAUMN,PI,EPS,WP,WIP,
     2WN,WC,WH,WIGN,TAUH,AB,WAFT,WFS,WAS,WINT,ROP,     SIGN,RHOT,MUT,
     3ESUBT,SIGT,MUS,    RHOS,ESUBS,CSUB2,RHOI,KSUB1,CSUB1,LN,TC,WEX,KNOZ
     4                                                                  )
      IF(KSLIV)125,125,124
  124 WIP=WIP+WS
  125 WBO=WIF+ WPL
      WO=WBO+ WP
      MU= WP/(WO- WPL)
      DELV=ISP *GC*ALOG(WO/WBO)
      AIG=F/WO
      ABO=F/WBO
      LM=LFM+LC+LN+LAM
      WRITE(KOUT,950)
      WRITE(KOUT,952)DT,DEI,CEO,TAUNE
      WRITE(KOUT,953)AT,AEI,EPS,ME,ALF,PE
      WRITE(KOUT,954)DCI,DM,TAUM,AF,TAUH
      WRITE(KOUT,956)AP,AW,APT,TAUN,AB,LC
      WRITE(KOUT,957)LFH,LAH,LN,LM
      WRITE(KOUT,958)WN,WC,WH,WAFT,WEX,WIGN
      WRITE(KOUT,961)WFS,WAS,WINT,WIF
      WRITE(KOUT,960)WP,WPFH,WPAH,WPW,WS
      WRITE(KOUT,959)WO ,WPL,WBO
      WRITE(KOUT,970)
      WRITE(KOUT,971)RB,CO,MDOT,CSTAR
      WRITE(KOUT,972)F,ISP ,AIG,ABO
      WRITE(KOUT,973) CF,DELV,TB,MU
  105 FORMAT(1H1, 43X ,41HROCKET MOTOR PRELIMINARY DESIGN PROGRAM      )
  106 FORMAT(1H0,30X30H* * * * * PROGRAM INPUT DATA * * * * */)
  107 FORMAT(36H DM,  MOTOR O.D.(IN.) . . . . . . . =F12.4,12X36HDEM,
     1NOZZLE EXIT O.D. (IN) . .  =F12.4/36H PC,  CHAMBER PRESSURE      (
     2PSI)  =F12.4,12X36HALF,NOZZLE EXIT HALF-ANGLE(DEG) .  =F12.4/36H F
     3, REQ. THRUST (LBF)      . . . .  =F12.4,12X36HFW,PROPELLANT WEB FR
     4ACTION . . . . =F12.4/36H WPL, PAYLOAD MASS   (LBM)  . . .  =F12.4
     5,12X,36HWEX,MASS OF EXTRAS(LBM) . . . . .  =F12.4)
  108 FORMAT(36H RHOP, PROPELLANT DENSITY (LBM/IN3)=F12.4,12X,36HAT9,BUR
     1N RATE COEFF.. . . . . . . .  =F12.4/36H AN,BURN RATE EXPONENT . . .
     2 . . . =F12.4,12X,36HGAM, RATIO OF SPECIFIC HEATS  . .  =F12.4/
     3         36H TC, GAS CHAMBER TEMP. (DEG R). .  =F12.2,12X,36HAMW, MO
     4L. WT. OF EXH. GAS(LBM/MOLE)=F12.4)
  109 FORMAT(36H APT, PORT-TO-THROAT AREA RATIO .  =F12.4,12X,
     2                     36HPAM,  ATM. PRESSURE(PSIA) . . . .  =F12.4)
  110 FORMAT(                                                 36H SFM, CA
```

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      1SE DESIGN SAFETY FACTOR  . = F12.2,12X,
      236HSFN,NOZZLE DESIGN SAFETY FACTOR. . =F12.2                /36H
      3 ETAD, DISCHARGE CORRECTION FACTOR =F12.4,12X,36HETAV, VELOCITY CO
      4RRECTION FACTOR   =F12.4/36H FP,FRACT. OF PROP. BURNED TO WEB  =F1
      52.4)
  111 FORMAT(36H ZLN, NOZZ. LGTH. OF NEXT STAGE(IN)=F12.4,12X
      136HZLI,INTERSTAGE CLEARANCE(IN) . . . = F12.4 /
      236H CODE,STAGE DESIGNATION  . . . . . =F12.4,12X,
      337HFLAG(0.-CYL,1.-FWD,2.,FWD+AFT,3.-AFT)F11.4/
      436H NOZ(0.-CONICAL NOZZ.,1.0-CONTOUR) = F12.4,12X
      536HISLIV(SET.GT. 0. FOR INERT SLIVERS)= F12.4)
  950  FORMAT (1H0,  29X42H * * * * * ROCKET DESIGN SUMMARY * * * * * /)
  952  FORMAT (1H , 35HNOZZLE THROAT I.D. (IN.)  . . . . = F12.6, 12X
      336HNOZZLE  EXIT I.D. (IN.)  . . . . . = F12.6/
      436H NOZZLE EXIT O.D.(IN.) . . . . . . = F12.6,  12X
      636HNOZZLE EXIT WALL THICK (IN.) . . . = F12.6 )
  953 FORMAT(36H NOZZLE THROAT AREA (IN2) . . . .   F12.6,12X
      836HNOZZLE EXIT AREA (SQ IN.)  . . . . = F12.6 /
      936H NOZZLE AREA RATIO (AE/AT) . . . . = F12.6, 12X
      136HNOZZLE EXIT MACH NUMBER  . . . . . = F12.6 /
      236H NOZZLE CONE ANGLE (DEG) . . . . . = F12.2,  12X
      336HNOZZLE EXIT PRESSURE (PSI) . . . . = F12.6)
  954  FORMAT ( 36HMOTOR CHAMBER I.D. (IN.)  . . . . = F12.6, 12X
      236HMOTOR CHAMBER O.D. (IN.) . . . . . = F12.6 /
      336H MOTOR WALL THICK (IN.)  . . . . . = F12.6,  12X
      436HMOTOR XSECT AREA (SQ IN.)  . . . . = F12.6 /
      536H FORWARD HEAD THICK (IN) . . . . . = F12.6/)
  957 FORMAT(36H LENGTH OF FWD. HEAD (IN)  . . . . = F12.6,12X
      136HLENGTH OF AFT HEAD (IN)  . . . . . = F12.6/
      236H NOZZLE LENGTH (IN)  . . . . . . . =F12.6,12X
      336HMOTOR LGTH(NOZ EXIT TO FWD HEAD,IN)= F12.6)
  956  FORMAT (36H PORT AREA (SQ IN.)  . . . . . . . = F12.6, 12X
      236HWEB XSECT AREA (SQ IN.)  . . . . . = F12.6 /
      336H PORT-TO-THROAT AREA . . . . . . . = F12.6, 12X
      436HWEB THICKNESS (IN.)  . . . . . . . = F12.6 /
      536H BURNING AREA (SQ IN.) . . . . . . = F12.3, 12X
      636HCYLINDRICAL MOTOR LENGTH (IN.) . . = F12.6 )
  958  FORMAT (36H0MASS OF NOZZLE ASSY (LBM) . . . . = F12.6, 12X
      136HMASS OF CYL.(TUBE+LINER) (LBM) . . = F12.6 /
      236H MASS OF FWD HEAD ASSY. (LBM). . . = F12.6, 12X
      336HMASS  OF AFT HEAD ASSY.  (LBM) . . = F12.6/
      436H MASS OF EXTRAS (LBM)  . . . . . . = F12.6,  12X
      536HMASS  OF IGNITER (LBM) . . . . . . = F12.6)
  961 FORMAT(36H MASS OF FWD. SKIRT (LBM)   . . . . = F12.6,12X
      136HMASS OF AFT SKIRT (LBM)  . . . . . = F12.6 /
      236H MASS OF INTERSTAGE STRUCTURE (LBM)= F12.6,12X
      3         36HTOTAL MASS OF INERT PARTS (LBM)  . = F12.6/ )
  960 FORMAT(36H TOT PROPELLANT MASS,WP  (LBM) . . = F12.6/
      136H MASS OF PROP. IN FWD. HEAD (LBM)   = F12.6,12X
      236HMASS  OF PROP. IN AFT  HEAD (LBM)   = F12.6/
      336H MASS OF PROP. BURNED TO WEB(LBM)   =F12.6,12X
      436HMASS OF SLIVERS(PROP OR INERT)(LBM)=F12.6/)
  959 FORMAT(36H GROSS MASS AT STAGE IGN.,WO (LBM) = F12.4,12X
      136HMASS OF PAYLOAD,WPL (LBM)   . . . . = F12.4 /
      236H GROSS MASS AT STAGE BURNOUT (LBM) = F12.4     )
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  970  FORMAT(1H0,26X45H* * * * * ROCKET PERFORMANCE SUMMARY * * * * */)
  971  FORMAT(45H PROPELLANT BURNING RATE (IN./SEC)  . . . . . .  F15.5/     4
      145H MASS DISCHARGE COEFFICIENT (LBM/LBF-SEC) . .  F15.5 /
      245H MASS FLOW RATE (LBM/SEC) . . . . . . . . . .  F15.2 /
      345H C*, CHARACTERISTIC VELOCITY(FPS) . . . . . .  F15.1 )
  972  FORMAT(45H THRUST(LBF)  . . . . . . . . . . . . . . . . . .  F15.2 /
      145H SPECIFIC IMPULSE (LBF-SEC/LBM) . . . . . . .  F15.2 /
      245H THRUST/WEIGHT AT IGNITION (G)  . . . . . . .  F15.2 /
      345H THRUST/WEIGHT AT BURNOUT (G) . . . . . . . .  F15.2 )
  973 FORMAT(45H ... CEFF.  . . . . . . . . . . . . . . . . F15.6/
      345H ... VEL. INC... (FPS)  . . . . . . . . . . .  F15.2 /
      4 ... BURNING TIME ... . . . . . . . . . . . . . .  F15.5 /
      545H STA... ...(WO/WB...)). . . . . . . . .  F15.5       )
       GO ...
       E...
```

```
      SUBROUTINE INERT(PC,SFM,SFN,BETA,DM,SIGM,RHOM,TB,CD,AT,LC,AP,DCI,
     1TAULC,DEI,DT,ALF,DEO,DTO,WPL,F,KODE,EC,EI,ZLN,ZLI,TAUMN,PI,EPS,WP,
     2WIP,WN,WC,WH,WIGN,TAUH,AB,WAFT,WFS,WAS,WINT,ROP,    SIGN,RHOT,MUT,
     3ESUBT,SIGT,MUS,    RHOS,ESUBS,CSUB2,RHOI,KSUB1,CSUB1,LN,TC,WEX,KNOZ
     4                                                                   )
      REAL LC,LHO,LHI,MUT,MUS,LN,KSUB1
      RHOL=0.06
      WINT=0.0
      PDN=PC*SFN
C
C     FORWARD HEAD
C     MEMBRANE
      PDM=PC*SFM
      LHO=DM/(2.*BETA)
      TAUH=0.707*PDM*DM**2/(4.*LHO*SIGM)
      IF(TAUH-TAUMN)1,2,2
    1 TAUH=TAUMN
    2 DM=DM-2.*TAUH
      LHI=DM/(2.*BETA)
      VH=0.667*PI*(LHO*DM**2/4.-LHI*DM**2/4.)
      WHS=VH*RHOM
C     INSULATION
      FACT=(1.-1./BETA**2)**0.5
      PART1=ALOG((1.+FACT)/(1.-FACT))
      PART2=PDM**0.8*TB*(1./CD)**1.7
      WHI=1.93E-10*DM**2*(0.7854+0.393/(BETA**2*FACT)*PART1)*PART2
C     IGNITER BOSS
      WHIB=1.132*PDM*DM*BETA*AT*RHOM/SIGM
C     IGNITER
      WIGN=10.*EXP(0.9E-04*AB)
C
C     CYLINDER
C     STRUCTURE
      WCS=RHOM*PI/4.*(DM**2-DCI**2)*LC
C     LINER
      WCL=PI*DCI*LC*TAULC*RHOL
C
C     AFT HEAD
C     MEMBRANE
      PART3=0.25*PDM*DM*BETA*RHOM/SIGM
      WAFTS=PART3*(DM**2*(0.7854+0.393/(BETA**2*FACT)*PART1)-2.*AT)
C     INSULATION
      WAFTI=WHI*2.96/1.93
C     BOSS
      WAFTB=10.26*PDM*DM**2*BETA*RHOM/SIGM
C
C     NOZZLE
      CALL NOZZ(WN,LN,ROP,PDN,DT,DEO,SIGN,RHOS,TAUMN,RHOT,MUT,ESUBT,
     1ESUBS,MUS,ALF,SIGT,TB,TC,CSUB1,KSUB1,RHOI,CSUB2,DELTT,DEI,KNOZ)
C
C     INERT WEIGHT SUBTOTAL
C     FORWARD HEAD ASSEMBLY
      WH=WHS+WHI+WHIB
C     CYLINDER
      WC=WCS+WCL
```

```
C     AFT HEAD ASSEMBLY
      WAFT=WAFTS+WAFTI+WAFTB
C     SUBTOTAL
      WSUB=WH+WC+WAFT+WN+WIGN
C
C     CALC. OF VEHICLE MASS AT START OF BURNING OF THIS STAGE AND MAX.
C     LONG. ACCELERATION
C     INITIAL ESTIMATE
      WOEST=WPL+WP+WSUB
  100 WBEST=WOEST-WP
      GMAX=F/WBEST
C
C     FORWARD AND AFT SKIRTS AND INTERSTAGE MASSES
      IF(KODE-2)9,10,10
    9 C1=0.055
      GO TO 11
   10 C1=0.163
   11 WFS=C1*DM**2*(GMAX*WPL/EC*(0.215*(LC+DM/BETA)/DM+1.))**0.5
      WAS=WFS
      IF(KODE-2)12,13,13
   12 WINT=0.155*DM*(DM/BETA+ZLN+ZLI)*(GMAX*WPL/EI*(0.215*(LC+DM/BETA)/
     1DM+1.0))**0.5
   13 WO=WSUB+WFS+WAS+WINT+WPL+WP
      TEST=ABS((WO-WOEST)/WOEST)
      IF(TEST-0.005)15,15,14
   14 WOEST=WO
      GO TO 100
C     SUM INERT PARTS MASSES
   15 WIP=WSUB+WFS+WAS+WINT+WEX
      RETURN
      END
```

```
      SUBROUTINE NOZZ  (WN,LN,ROP,PDN,DT,DEQ,SIGN,RHOS,TAUHN,RHOT,MUT,
     1ESUBT,ESUBS,MUS,ALF,SIGT,TB,TC,CSUB1,KSUB1,RHOI,CSUB2,DELTT,DEI,
     2KNOZ)
      REAL KSUBT,MUS,MUT,LN,KSUB1
      DIMENSION FACL(10), EX(10),EPE(10),ECONT(10)
      DATA  EX/2.,4.,6.,8.,10.,12.,14.,16.,20.,24/
      DATA FACL/.59,.61,.65,.70,.75,.80,.84,.89,1.0,1.2/
      DATA ECONT/2.,4.,6.,8.,10.,12.,14.,18.,22.,25./
      DATA EPE/2.4,3.7,5.3,7.1,9.1,11.4,14.1,20.5,29.6,39.2/
      DATA PI/3.14159/,ONE/1.0/,TWO/2.0/,THREE/3.0/,RAD/57.2957795/    NOZO 220
      DATA KSUBT/1.30 /,BETAE/60.0/
      FAC=1.0
      TMIN=TAUHN
      ALFR=ALF/RAD                                                     NOZO 24
      BETAR=BETAE/PAD
      RSUBT=DT/2.
      RSUBE=DEI/2.
      EPS=(RSUBE/RSUBT)**2
      IF(KNOZ) 101,101,100
  100 CALL ITERP(EPS, EX,FACL,10,4,FAC,NERR)
      CALL ITERP(EPS,ECONT,EPE,10,4,EPEQ,NERR)
      AE=EPEQ*PI*RSUBT**2
      RSUBE=(AE/PI) **0.5
      ALFR=17.5/RAD
  101 CTF=COS(ALFR)/SIN(ALFR)
      TSUBE=ROP*PDN/SIGN/COS(BETAR)
      IF(TSUBE-TAUHN)1,2,2
    1 TSUBE=TAUHN
    2 CALL INSUL(TSUBE,TIN,TB,RHOS,CSUB1,KSUB1,RHOI,CSUB2,TC,DELTT)
      ROT   =RSUBT+KSUBT*RSUBT*(ONE-COS( BETAR))                       NOZO 320
      RAT   =RSUBT+KSUBT*RSUBT*(ONE-COS(ALFR  ))                       NOZO 330
      IF(ROP-ROT)888,888,889
  888 WEI=0.
      WES=0.
      HSUBS=0.
      GO TO 890
  889 HSUBS =SQRT((ROP   -ROT)**2)/SIN( BETAR)                         NOZO 340
      WEI   =PI*(ROP    +TIN*COS( BETAR)+ROT)*HSUBS*TIN*RHOI           NOZO 350
      WES   =PI*(ROP    +(TIN+TSUBE/TWO)*COS( BETAR)+ROT)*HSUBS*TSUBE* NOZO 360
     1       RHOS                                                      NOZO 370
  890 WTI   =PI*(ROT+TIN*(COS( BETAR)+COS(ALFR  ))+RAT)*TWO*KSUBT*RSUBT*NOZO 380
     1       SIN(( BETAR+ALFR  )/TWO)*TIN*RHOI                         NOZO 390
      ROS   =ROT+TIN*COS( BETAR)                                       NOZO 400
      TTS   =ROS**2/ROT*PDN  /SIGT  *ESUBT/ESUBS*(ONE-MUS   )/         NOZO 410
     1       (ONE-MUT   )                                              NOZO 420
      WTS   =PI*(ROT+TIN*(COS( BETAR)+COS(ALFR  ))+RAT+TTS   *COS( BETARNOZO 430
     1       )/TWO+TTS    *COS(ALFR  )/TWO)*TWO*KSUBT*RSUBT*SIN(( BETAR+ NOZO 440
     2       ALFR  )/TWO)*TTS   *RHOS                                  NOZO 450
      WT    =PI*KSUBT**2*RSUBT**3*( BETAR+ALFR  -SIN( BETAR+ALFR  ))*( NOZO 460
     1       (ONE+KSUBT)-((TWO*KSUBT/THREE*SIN(( BETAR+ALFR  )/TWO)**3)/NOZO 470
     2       ( BETAR+ALFR  -(SIN( BETAR+ALFR  )/TWO)))*COS(( BETAR-     NOZO 480
     3       ALFR  )/TWO))*RHOT                                        NOZO 490
      RES   =RAT+TIN*COS( BETAR)                                       NOZO 500
      TXS=PDN*RES/SIGN/COS(ALFR)
      IF(TXS-TAUHN)3,4,4
```

```
    3 TXS=TAUMN
    4 WXS   =RHOS*PI/THREE*CTF        *(RSUBE-RAT)*((RAT+(TIN+TXS   )*  NOZO 550
     1        COS(ALFR  ))**2+(RAT+(TIN+TXS   )*COS(ALFR  ))*(RSUBE+TIN+ NOZO 560
     2        TMIN)+(RSUBE+TIN+TMIN)**2-(RAT+TIN*COS(ALFR  ))**2-(RAT+   NOZO 570
     3        TIN*COS(ALFR  ))*(RSUBE+TIN)-(RSUBE+TIN)**2)               NOZO 580
      WXI   =PI*TIN*RHOI*CTF         *(RAT+TIN+RSUBE)*(RSUBE-RAT)        NOZO 590
      WB    =10.0*PI*ROP   *TSUBE*RHOS                                   NOZO 600
      WN=WES+WEI+WTI+WTS+WT+WXS+WXI+WB
      IF(ROP-ROT)892,892,891
  892 ENTL=0.
      GO TO 893
  891 ENTL=(ROP-ROT)*COS(BETAR)/SIN(BETAR)
  893 TLENT=RSUBT*KSUBT*SIN(BETAR)
      TLEX=RSUBT*KSUBT*SIN(ALFR)
      EXITL=(RSUBE-RAT)*CTF
      CORRL=(TLEX+EXITL)*FAC
      LN=ENTL+TLENT+CORRL
      RETURN                                                             NOZO 640
      END                                                                NOZO 650
```

```
      SUBROUTINE INSUL (THEM,THICK,TB,RHOS,CSUB1,KSUB1,RHOI,CSUB2,TC,
     1DELTT)
      REAL KSUB1                                                        INSL 110
      DATA PI/3.14159/,MONE/-1/,SIXTH/0.1666667/,TWO/2.0/               INSL 120
      DELTT=10.
      THICK = (0.008*CSUB1/CSUB2*RHOI/RHOS*TC   /THEM/DELTT *(TB   *PI/ INSL 140
     1        CSUB1*PI/RHOI*KSUB1)**3.91414)**0.14644                   INSL 150
      SMIN=0.
      SMAX=THICK*5.
    4 TBTAU =PI*PI/CSUB1*KSUB1/RHOI*TB   /THICK/THICK
      SUM=0.0                                                           INSL 170
      DO 5 N=1,20                                                       INSL 180
      FN=N
      EPOW  =FN*FN*TBTAU                                                INSL 200
      IF(EPOW-10.)5,5,51
    5 SUM=SUM+(FLOAT(MONE**(N+1))/FN/FN/EXP(EPOW  ))
   51 CONTINUE                                                          INSL 240
      FTTAU =TBTAU /PI/PI-SIXTH+TWO*SUM/PI/PI                           INSL 250
      TCDEL =CSUB1*RHOI/CSUB2*THICK/RHOS*TC   /THEM*FTTAU               INSL 260
      TEST=ABS ((TCDEL-DELTT)/DELTT)
      IF(TEST-0.01)6,6,1
    1 IF(TCDEL-DELTT)2,6,3
    2 SMAX=THICK
      COR=(SMAX-SMIN)*0.5
      THICK=THICK-COR
      GO TO 4
    3 SMIN=THICK
      COR=(SMAX-SMIN)*0.5
      THICK=THICK+COR
      GO TO 4
    6 RETURN                                                            INSL 310
      END                                                               INSL 320
```

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SUBROUTINE  MACH                                  76/6 FTN V2.0  CY=43     OPT=1  01/29/73  14.39.55        PAGE      1

            SUBROUTINE MACH(G,ZN,ZT,AP,AMOK,KOUT)
      C     ONE DIMENSIONAL NOZZLE ANALYSIS  JL THURMAN
         13 FORMAT(25H  MACH COULD NOT CONVERGE/4H AP=E15.8,6H XM=E15.8,5H XNA
           1=E15.8)
            GAMM=G-1.
            GAMA=GAMM/2.
            GAMAA=G/GAMM
            GAMB=G+1.
            GAMBB=GAMB/2.
            GAMEX=GAMBB/GAMM
        116 IF(ZN-ZT)25,12,25
         25 XM=AP
            GO TO 115
         26 XM=1.2
        115 ICTR=0
          6 ALIP=(1.+GAMA*XM**2)/GAMBB
          1 XNA=XM-(AP*ALIP**GAMEX)
            IF(ABS (XNA)-.0001)118,118,2
          2 IF(XM-1.)3,12,4
          3 XM=XM-XNA
            GO TO 5
          4 XM=XM+AP*XNA
          5 ICTR=ICTR+1
            IF(ICTR-999)6,6,800
        118 CONTINUE
         11 AMOK=XM
            GO TO 21
         12 AMOK=1.
            GO TO 21
        800 WRITE(KOUT,13)AP,XM,XNA
         21 RETURN
            END
```

```
SUBROUTINE  ITERP                                  7600 FTN V2.0  CY=40    OPT=1  01/29/73  10.18.31        PAGE     1

      SUBROUTINE ITERP (ARG,XTAB,YTAB,NX,NPTS,ANS,NERR)
C
C                    1. ARG  THE INDEPENDENT VARIABLE FOR THE DESIRED  INTR  20
C                            UNKNOWN.
C                    2. XTAB TABLE OF INDEPENDENT VALUES. MUST BE IN   INTR  40
C                            INCREASING ORDER.
C                    3. YTAB TABLE OF DEPENDENT VALUES.
C                    4. NX   NUMBER OF POINTS IN XTAB.
C                    5. NPTS NUMBER OF POINTS USED IN THE INTERPOLATINGINTR  80
C                            FORMULA. (NPTS-1) IS THE DEGREE OF THE    INTR  90
C                            INTERPOLATION FORMULA USED.
C                    6. ANS  THE DEPENDENT VALUE CORRESPONDING TO THE  INTR 110
C                            VALUE OF ARG.
C                    7. NERR WILL BE SET UNEQUAL ZERO IF ARG IS NOT ON INTR 130
C                            XTAB. ROUTINE WILL EXTRAPOLATE.
C                            NERR = 1, ARG .LT. XTAB(1)
C                            NERR = 2, ARG .GT. XTAB(NX)
      DIMENSION XTAB(NX),YTAB(NX)
      NERR = 0
      NP = NPTS
      IF(NX-NP)1,2,2
    1 NP=NX
    2 NS = (NX+169)/26
      IH = NP/2
      I = 1
      IF (XTAB(I) - ARG)30,20,10
   10 IH = I
   12 NERR = 01
      GO TO 70
   13 NERR = 2
      GO TO 70
   20 ANS = YTAB(I)
      GO TO 999
   30 I = NX
      IF (XTAB(I) - ARG)13,20,50
   50 L = IH + 1
      IS = NS+L
      IF(NX-IS)58,58,52
   52 DO 54   I=IS,NX,NS
      IF(XTAB(I)-ARG)54,20,56
   54 L = I
      GO TO 58
   56 L = I - NS
   58 DO 60   I=L,NX
      IF (XTAB(I) - ARG)60,20,70
   60 CONTINUE
   70 K = I - IH
      N = K + NP - 1
      ANS = 0.0
      IF (N - NX)90,90,80
   80 N = NX
      K = NX-NP+1
   90 DO 120  J=K,N
      P = 1.0
      DO 110  I=K,N

SUBROUTINE  ITERP                                  7600 FTN V2.0  CY=40    OPT=1  01/29/73  10.18.31        PAGE     2

      IF (I-J)100,110,100
  100 P = P * (ARG - XTAB(I)) / (XTAB(J) - XTAB(I))
  110 CONTINUE
  120 ANS = ANS + YTAB(J) * P
  999 RETURN
      END
```

APPENDIX B

DESCRIPTION OF MISSILE OPTIMIZATION PROGRAM (MOP)

B.1 ANALYSIS

The following analysis* is based primarily on the work of Weisbord (Ref. 1) with modifications to simplify automation and notational changes to fit current usage.

The ideal impulsive velocity, ΔV_I, achievable for any rocket with constant specific impulse for each stage is given in Equation 1.

$$\Delta V = C_1 \, \ell n \frac{W_{o_1}}{W_{bo_1}} + C_2 \, \ell n \frac{W_{o_2}}{W_{bo_2}} \quad . \; . \; . \; . \tag{1}$$

where

W_{o_i} - gross start weight for the ith stage, lbm

W_{bo_i} - burnout gross weight for the ith stage, lbm

C_1 - $g_c Isp$, ft/sec

For any given mission, ΔV_I can be estimated from Equation 2.

$$\Delta V_I = \Delta V_{act} + \int_0^{t_b} g \sin \theta \, dt + \Delta V_D \tag{2}$$

* The Missile Optimization Program was developed by Mr. J. H. Dobkins of Teledyne Brown Engineering.

where

ΔV_{act} - actual impulsive velocity, ft/sec

θ - angle of velocity vector above local horizon, rad

ΔV_D - drag losses, ft/sec

t - time, sec

t_b - burn time, sec

For large missiles drag losses usually amount to less than 5 percent and are not considered important for preliminary analyses. However, for smaller, high-velocity vehicles this factor becomes more important and must be estimated.

The gravity loss term is zero for horizontal launch and is (gt) for a vertical launch. Thus it is strongly trajectory dependent and some approximation between the limits must be made.

Each stage consists of motors, propellant tanks, miscellaneous hardware, guidance, etc. The mass fraction (XMF_i) and structural fraction (XMS_i) of stage i are defined in Equations 3 and 4.

$$XMF_i = W_{p_i}/(W_{o_i} - W_{PL_i}) \tag{3}$$

$$XMS_i = W_{s_i}/(W_{o_i} - W_{PL_i}) \tag{4}$$

where

W_{p_i} - weight of propellant, lbm

W_{s_i} - weight of stage - empty, lbm

W_{PL_i} - weight of stage payload (upper stages plus payload), lbm.

Mass fraction data can be estimated from past experience. Structural fraction is calculated within the program from the mass fraction and thus need not be input.

The burnout weight of any stage may be written in the form of Equation 5.

$$W_{bo_i} = W_{o_{i+1}} + XMS_i \, (W_{o_i} - W_{o_{i+1}})$$

$$= XMS_i \, W_{o_i} + (1 - XMS_i) \, W_{o_{i+1}} \qquad (5)$$

and Equation 1 becomes

$$\Delta V_I = \sum_{i=1}^{N} C_i \, \ell n \, \frac{W_{o_i}}{XMS_i \, W_{o_i} + (1 - XMS_i) \, W_{o_{i+1}}} \qquad (6a)$$

or

$$0 = \sum_{i=1}^{N} C_i \, \ell n \left[\frac{W_{o_i}}{XMS_i \, W_{o_i} + (1 - XMS_i) \, W_{o_{i+1}}}\right] - \Delta V_I \qquad (6b)$$

The problem requires that W_{o_i} be minimized for a given ΔV_I. Therefore, the W_{o_i} become variables and the maximum or minimum take-off weight is achieved when

$$\frac{\partial W_{o_1}}{\partial W_{o_i}} = 0_{i=2,\, N} \qquad (7)$$

Since it is difficult to explicitly write these differentials, note that Equation 6 is of the form

$$f(W_{o_i}) = 0 \tag{8}$$

and the partials can be written

$$\frac{\partial W_{o_1}}{\partial W_{o_i}} = - \frac{\partial f / \partial W_{o_i}}{\partial f / \partial W_{o_1}} \tag{9}$$

from the chain rule for differentiation. Equation 9 implies that if Equation 7 is to be satisfied then

$$\frac{\partial f}{\partial W_{o_i}} = 0 = \frac{\partial W_{o_1}}{\partial W_{o_i}} \tag{10}$$

must also be satisfied. The process yields for i = 2, 3

$$0 = \frac{\partial f}{\partial W_{o_2}} = - \frac{C_1 \ (1 - XMS_1)}{XMS_1 \ W_{o_1} + (1 - XMS_1) \ W_{o_2}} + \frac{C_2}{W_{o_2}} - \frac{C_2 \ XMS_2}{XMS_2 \ W_{o_2} + (1 - XMS_2) \ W_{o_3}}$$

$$0 = \frac{\partial f}{\partial W_{o_3}} = - \frac{C_2 \ (1 - XMS_2)}{XMS_2 \ W_{o_2} + (1 - XMS_2)} + \frac{C_3}{W_{o_3}} - \frac{C_3 \ XMS_3}{XMS_3 \ W_{o_3} + (1 - XMS_3) \ W_{o_4}} \tag{11}$$

This can be algebraically manipulated to become

$$\frac{W_{o_3}}{W_{o_2}} = \frac{C_1 \ XMS_2 \ (1 - XMS_1)}{(1 - XMS_1)(1 - XMS_2)(C_2 - C_1) + C_2 \ XMS_1 \ (W_{o_1}/W_{o_2})(1 - XMS_2)}$$

$$\frac{W_{o_4}}{W_{o_3}} = \frac{C_2 \ XMS_3 (1 - XMS_2)}{(1 - XMS_2)(1 - XMS_3)(C_3 - C_2) + C_3 \ XMS_2 (W_{o_2}/W_{o_3})(1 - XMS_3)} \tag{12}$$

and can be generally stated as

$$\frac{W_{o_{i+1}}}{W_{o_i}} = \frac{C_{i-1}\, XMS_i\, [1 - XMS\ (i-1)]}{(1 - XMS_{i-1})(1 - XMS_i)(C_i - C_{i-1}) + C_i\, XMS_{i-1}\, (W_{o_{i-1}}/W_{o_i})(1 - XMS_i)} \qquad (13)$$

$$i = 2, N$$

Note that difficulties will occur when i = 1. An iterative process with an estimated W_{o_2}/W_{o_1} ratio is required until the required ΔV_I is available.

It has not been shown in the previous discussion that the resulting data will represent a minimum weight vehicle (rather than maximum). A simple test is to try any other weight ratio between stages that produces the same performance and compare the resulting total vehicle weight, W_{o_1}. Other weight ratios invariably produce greater values of W_{o_2}, thus proving that stage weight ratios determined by the previously described procedures represent a minimum W_{o_1}.

The terminal payload weight is finally used to calculate the W_{o_i} for the terminal stage. The vehicle is then specified totally from the following equations.

$$W_{st_i} = W_{o_{i+1}}/(W_{o_{i+1}}/W_{o_i}) - W_{o_{i+1}} \qquad (14)$$

$$W_{p_i} = W_{st_i}\ XMF_i \qquad (15)$$

$$W_{s_i} = W_{st_i} - W_{p_i} \qquad (16)$$

$$W_{bo_i} = W_{o_i} + W_{s_i} \qquad (17)$$

$$\Delta V_{I_i} = C_i\ \ell n\ W_{o_i}/(W_{o_i} - W_{p_i}) \qquad (18)$$

where W_{st_i} is the total weight of stage i.

B.2 PROGRAM APPLICATION

The Missile Optimization Program (MOP) will determine the optimum weight distribution between stages for up to nine stages. If for some reason not enough stages were input, the system will assume extra stages with parameters equal to the top stage until it is possible to build the system. Conversely, when the number of stages is too great the last stage is ignored until the system is feasible. This does not mean however, that the number of stages has been optimized.

The input data required is given in Table B-1. Provision has been made to study a spectrum of velocities and payloads and obtain vehicles optimized for each combination. Appendix C contains a FORTRAN listing of the program.

TABLE B-1. INPUT DESCRIPTION FOR MOP

INPUT CARD TYPE	VARIABLE	FORMAT	DEFINITION
1	DV	F10.2	Ideal Velocity, ft/sec
	WPL	F10.2	Payload, lbm
	N	I2	Number of stages, 9 max.
	LAST	I2	Last case 0 No 1 Yes
2 (N cards)	XISP(I)	F10.2	Specific Impulse, sec
	XMF(I)	F10.2	Mass Fraction
3	VST	F10.2	Velocity Step Desired
	WST	F10.2	Weight Step Desired
	JVS	I2	Number of Velocity Steps Desired
	JWST	I2	Number of Weight Steps Desired

APPENDIX C

FORTRAN LISTING OF MISSILE OPTIMIZATION PROGRAM (MOP)

A complete listing of the FORTRAN statements is furnished in this appendix as a reference for making any desired changes to the source deck of the missile optimization program (MOP).

```
PROGRAM    MAIN           DEMO CENTERS        CDC 6400 FTN V3.0-P251 OPT=1  10/18/72  14.58.19.       PAGE    1

          PROGRAM MAIN(INPUT,OUTPUT,TAPE2=INPUT,TAPE3=OUTPUT)
          DIMENSION XISP(9),XMF(9),C(9),XMS(9),R(9)
   98     FORMAT(1H1)
  100     READ (2,1) DV,WPL,N,LAST,(XISP(I),XMF(I),I=1,N)
          READ (2,5) VST,WST,JVS,JWST
    1     FORMAT ( 2F10.2,2I2/(2F10.2))
    5     FORMAT ( 2F10.2,2I2)
          WRITE(3,98)
          DDV = DV
          NN = N
          WPP= WPL
          DO 10 I = 1,N
          C (I) = 32.1658*XISP(I)
   10     XMS (I) = 1.00000-XMF(I)
          DO 90 J = 1,JVS
          DV = DDV + (J-1)*VST
          N = NN
          DO 89 JJ=1,JWST
          WPL = WPP + (JJ-1)*WST
          K = 0
          G = 0.0
   11     R (1) = 0.01
          DR = -0.1999
   20     DO 30 I = 2,N
   30     R(I) =C(I-1)*XMS(I)*(1.-XMS(I-1))/((1.-XMS(I-1))*(1.-XMS(I))*(C(I)
         1-C(I-1)) + C(I)*XMS(I-1)*(1.-XMS(I))/R(I-1))
   35     DO 40 I = 1,N
   40     G = G + C(I)*ALOG(1./(XMS(I) + (1.-XMS(I))*R(I)))
          DER = DV - G
          IF (ABS(DER)-0.1) 60,60,50
   50     IF (K) 51,51,52
   51     K = 1
          R (1) = R(1) - DR
          ER = DER
          G = 0.0
          GO TO 20
   52     IF (DER) 54,53,53
   53     IF (ER) 58,55,55
   55     IF (DER-ER) 59,58,58
   54     IF (ER) 56,56,58
   56     IF (DER-ER) 58,59,59
   58     DR = -0.5*DR
   59     R (1) = R(1) - DR
          ER = DER
          G = 0.0
          GO TO 20
   60     IF (R(1) - 1.) 62,63,63
   62     IF (R(1)) 64,64,61
   61     CONTINUE
          GO TO 65
   63     N = N-1
          GO TO 11
   64     N = N+1
          C(N) = C(N-1)
          XMS(N) = XMS(N-1)
```

```
      XMF(N) = XMF(N-1)
      XISP(N) = XISP(N-1)
      GO TO 11
55    WRITE (3,4) DV,WPL,(I,XMF(I),XISP(I),I=1,N)
      WRITE (3,2)
2     FORMAT (/////6X,70HSTAGE     DV       WT PROP     WT STAGE    TOT STG
     1WT     IGN WT       BO WT/7X,69HNO.    FT/SEC      LBS.          LBS.
     2      LBS.          LBS)           LBS.)
4     FORMAT (///,5X,17H IDEAL VELOCITY =,F10.2,7H FT/SEC//6X,13HPAYLOAD
     1     =,F10.2,4H LBS//5X,33HSTAGE     MASS FRACTION     ISP-SEC/(8X,
     2I2,9X,F6.4,7X,F7.2))
      DO 70 I = 1,N
      M = N + 1 -I
      WT = WPL/R(M) -WPL
      WP = WT*XMF(M)
      WS = WT -WP
      WBO = WPL + WS
      WPL = WPL + WT
      DLV = C(M)*ALOG (WPL/(WPL-WP))
70    WRITE (3,3) M,DLV,WP,WS,WT,WPL,WBO
3     FORMAT (7X,I2,3X,F7.2,5F12.2)
89    CONTINUE
99    CONTINUE
      IF (LAST) 80,100,80
80    CALL EXIT
      END
```