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CALCULATION OF RADAR CROSS SECTION
AIRCRAFT GEOMETRY METHODS

H. C. HEATH

NORTHROP CORPORATION
AIRCRAFT DIVISION

TECHNICAL REPORT AFAL-TR-73-9
SEPTEMBER 1973

AIR FORCE AVIONICS LABORATORY

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NOTICE

CALCULATION OF RADAR CROSS SECTION

AIRCRAFT GEOMETRY METHODS

H. C. HEATH

FOREWORD

This document reports a portion of the work performed by Northrop Corporation, Aircraft Division, Hawthorne, California under USAF Contract F 33615-70-C-1820 "Calculation of Radar Cross Section." The work was sponsored by the Electromagnetic Division, Air Force Avionics Laboratory, under Project 7633 with Dr. Charles H. Krueger, AFAL/WRP as technical monitor.

The research was carried out by personnel of the Electronics Systems Research Technology Group at Northrop, with S. Stanley Locus as Principal Investigator. Other members of the group who contributed to the study were Dr. K. M. Mitzner, Messers. J. R. Coleman and F. K. Oshiro.

This document has been assigned NOR 72-401 by Northrop for internal control purposes and was submitted in September 1973.

This technical report has been reviewed and is approved for publication.

JOHN B. STURGES, JR.
Lt. Colonel, USAF
Chief, Electronic Warfare Division

ABSTRACT

Geometry description methods have been developed to transform design data of an aircraft configuration into a computer model. The computer model is used as a basis for radar cross section calculations covering the frequency range of 500 to 20,000 MHz. Accomplishments include: development of a quadric surface fitting program, complete analysis of quadric surfaces including intersections with lines, location of scattering centers, calculation of Gaussian curvature and surface zoning.

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SECTION I

INTRODUCTION AND SUMMARY

The calculation of the Radar Cross Section (RCS) of any target requires the capability of defining the geometry of the configuration being analyzed. Because the geometry description is not only essential, but is also the main link in the man-machine interface, it is important to understand how it is applied.

All geometry description techniques were developed with two goals in mind. The first goal was to minimize the time required to transform the geometry from drawings into a computer model. This was accomplished by developing a program to fit a general quadric surface through a set of design data and compute the rotation matrix and translation terms that transform the surface into a standard form. The coefficients of the surface and the rotation and translation operators are punched into data cards. The data cards have the form of a FORTRAN declaration statement and can be introduced directly into FORTRAN programs.

The second goal was to minimize the amount of computer time required to perform the analysis. This was accomplished by performing all calculations using the standard form of the surface equation (in a local quadric system) and transforming the results to the aircraft reference system. This transformation results in a significant reduction in computer time because the general equation of the surface contains ten coefficients and the standard form has four.

The manufacturing of the vehicle requires uniformity in the duplication of models and hence, exact duplication of all tooling equipment. To meet this requirement the design data is specified by a series of loft lines which are curves defining the contours of the surface. Typically, the loft lines lie in planes parallel or perpendicular to a set of reference axes. The design group specifies discrete points on the various surfaces of the configuration. The lofting group is then required to fit loft lines through the design data. Logarithmic functions are seldom used to fit the data because they are difficult to define. Equations of third degree are used occasionally but equations of higher degree are rarely employed because they require more input data. On the other hand, conic sections are easily defined, can be applied to a wide variety of shapes, give a streamlined shape, and the joins between loft lines are easily matched. Loft lines are explained in detail in References 1 and 2.

Quadric surfaces form the basis of aircraft geometry description because the majority of design data is of second degree and all operations involving quadrics can be derived analytically. The aspect of analytical development is important with respect to computer time because closed form calculations generally are much faster then iterative methods.

A literature search revealed that most books discussing quadric surfaces only covered certain aspects, but none presented a thorough outlined analysis of quadrics. This report contains a complete analysis of quadric surfaces and includes computer programs to perform the analysis. All techniques discussed in this report have been incorporated into the general high frequency radar scattering (GENSCAT) computer program.

SECTION II

LINES

2.1 Equation of a Line

The equation of a line can assume many forms. The principal application of a line in the GENSCAT program is to find its intersection with a surface. The most convenient way to express the equation of a line for this application is in parametric form.

$$
\begin{aligned}
x &= \lambda_1 d + x_0 \\
y &= \lambda_2 d + y_0 \\
z &= \lambda_3 d + z_0
\end{aligned}
\qquad (2\text{-}1)
$$

Where x_0, y_0, z_0 are the coordinates at the starting point of the line, the parameter d is the (positive) distance from P_0 (x_0, y_0, z_0) to P (x, y, z), and λ_1, λ_2, λ_3 are the direction cosines of the line (See Figure 1). The direction of the line can be expressed by the unit vector

$$\hat{\lambda} = \hat{i}\,\lambda_1 + \hat{j}\,\lambda_2 + \hat{k}\,\lambda_3. \qquad (2\text{-}2)$$

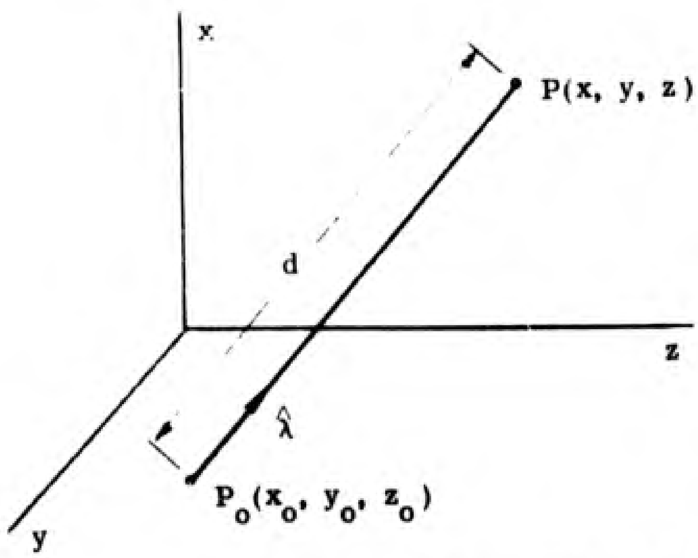


FIGURE 1. GEOMETRY OF A LINE

SECTION III

PLANES

3.1 Equation of a Plane

Of the many forms that the equation of a plane can assume, the most easily visualized is the normal form.

$$c_1 x + c_2 y + c_3 z = c_4 \qquad (3\text{-}1)$$

Where c_1, c_2, c_3 are the direction cosines of a line that is normal to the plane and passes through the origin of the coordinate system. The direction of the line is measured from the origin to the plane. The coefficient c_4 is the (positive) distance of the normal line from the origin to the plane (See Figure 2).

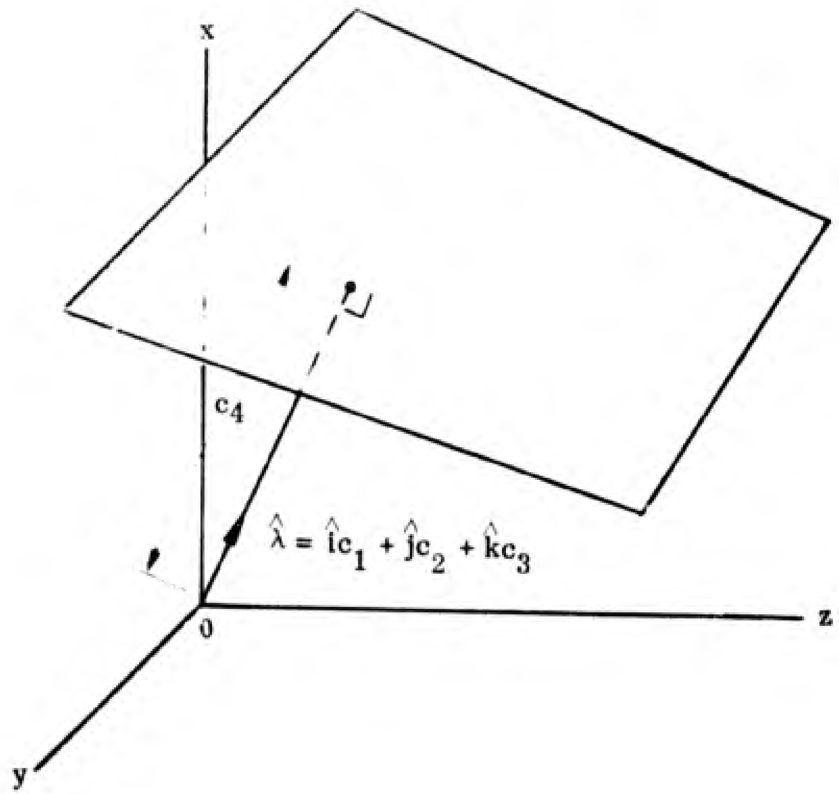


FIGURE 2. GEOMETRY OF A PLANE

3.2 Intersection of a Line with a Plane

Given the coordinates of the starting point, P_0 (x_0, y_0, z_0), and the direction cosines, λ_1, λ_2, λ_3, of a line, solve for the parameter d by substituting (2-1) into (3-1).

$$d = \frac{c_4 - (c_1 x_0 + c_2 y_0 + c_3 z_0)}{c_1 \lambda_1 + c_2 \lambda_2 + c_3 \lambda_3} \qquad (3\text{-}2)$$

To determine the coordinates at the point of intersection, substitute d into (2-1).

The unit normal at the point of intersection is

$$\hat{n} = s_n (\hat{i} c_1 + \hat{j} c_2 + \hat{k} c_3), \qquad (3\text{-}3)$$

where s_n is a sign that indicates the direction of the surface normal.

$s_n = 1$ if the surface normal is required to point away from the origin of the coordinate system.

$s_n = -1$ if the surface normal is required to point toward the origin of the coordinate system.

Note that the quantity s_n is simply an operator used to distinguish one side of the surface from the other.

SECTION IV

QUADRIC SURFACES

4.1 Discussion

The vehicle design data used to build a computer model is given with respect to a fixed aircraft or reference coordinate system. The general equation of a surface with respect to the reference system (4-3) contains ten coefficients. The general equation can be reduced to a standard equation (4-5) or (4-6), containing four coefficients, by a rotation and translation of axes. The standard equation defines the surface with respect to a local quadric system. Because of a reduction in computer time, the GENSCAT program performs all calculations in the local quadric system and transforms the results to the reference system. All transformation operators are contained in the punched output of the surface fit program.

4.2 General Procedure for Finding the Intersection of a Line With a Quadric Surface

Given the coordinates of the starting point P_0 (x_0, y_0, z_0) and the direction cosines λ_1, λ_2, λ_3 of a line, solve for the parameter d by substituting (2-1) into the equation of the surface. The result is a quadratic equation where d is the unknown.

$$A\,d^2 + B\,d + C = 0 \qquad (4\text{-}1)$$

The determination of the coefficients A, B, and C will be explained in Sections 4.3 and 4.4. The selection of the correct root of d in (4-1) is straightforward because d is the distance from the starting point P_0 (x_0, y_0, z_0) to the point of intersection P (x, y, z). This means that d must be positive and greater than zero. Because we are dealing with rays that illuminate a surface, the magnitude of d must be a minimum (if there are two positive roots, the greater distance will correspond to an intersection in the shadow region).

The correct root in Figure 3 is therefore d_1. Note that d = 0 is the starting point and is not a correct root.

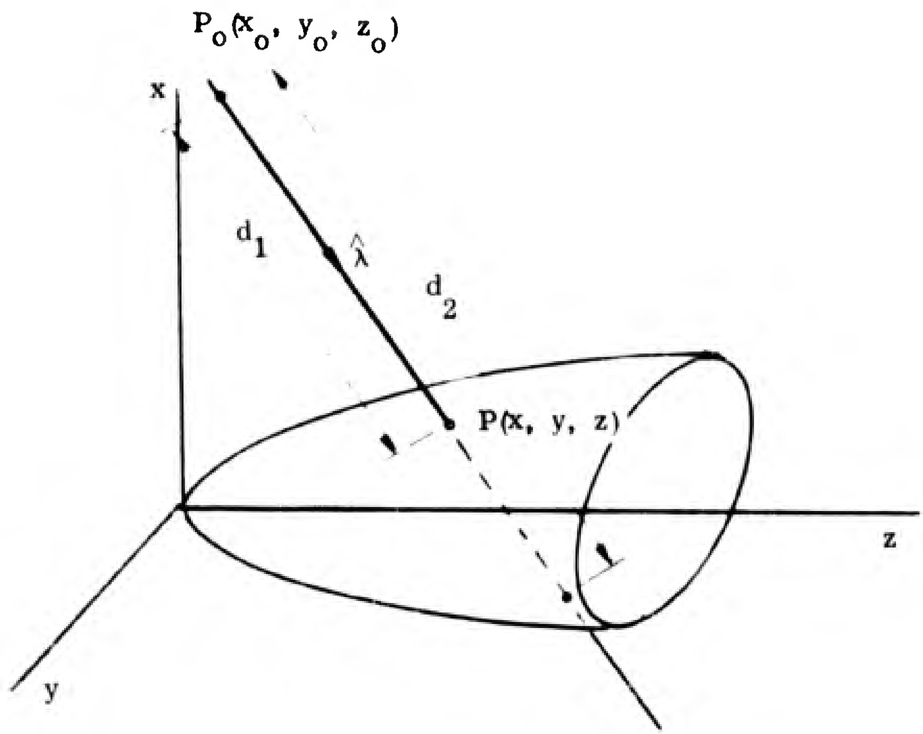


FIGURE 3. INTERSECTION OF A LINE WITH A QUADRIC SURFACE

The unit normal at the point of intersection is given by

$$\hat{n} = s_n \left[\frac{\nabla F\ (x,\ y,\ z)}{|\nabla F\ (x,\ y,\ z)|} \right], \tag{4-2}$$

where ∇F (x, y, z) is the gradient of the surface and s_n is a sign that indicates the direction of the surface normal.

$s_n = 1$ if the surface normal points away from the surface

$s_n = -1$ if the surface normal points into the surface

4.3 General Quadric Surface

The general equation of a quadric surface contains ten real coefficients and is defined by

$$F(x, y, z) = a_1 x^2 + a_2 y^2 + a_3 z^2 + 2a_4 y z + 2a_5 x z + 2a_6 x y + 2a_7 x + 2a_8 y + 2a_9 z + a_{10} = 0. \quad (4\text{-}3)$$

If the fixed or aircraft coordinate system coincides with the local quadric coordinate system, (4-3) reduces to the standard form (See Section 4.4). The second degree cross terms will be zero if there is no rotation between the two coordinate systems and the first degree terms will be zero if there is no translation between the two coordinate systems. (The elliptic and hyperbolic paraboloids are exceptions because they will contain one of the linear terms.)

4.3.1 Intersection of a Line With a General Quadric Surface

When (2-1) is substituted into (4-3), the result is a quadratic equation in d. The coefficients corresponding to (4-1) are

$$A = a_1 \lambda_1^2 + a_2 \lambda_2^2 + a_3 \lambda_3^2 + 2(a_4 \lambda_2 \lambda_3 + a_5 \lambda_1 \lambda_3 + a_6 \lambda_1 \lambda_2)$$

$$B = 2\left(\frac{\partial F}{\partial x_0}\lambda_1 + \frac{\partial F}{\partial y_0}\lambda_2 + \frac{\partial F}{\partial z_0}\lambda_3\right) = \left[\nabla F(x_0, y_0, z_0)\right] \cdot \hat{\lambda} \quad (4\text{-}4)$$

$$C = F(x_0, y_0, z_0).$$

4.4 Standard Quadric Surface

The standard equation of a quadric surface is expressed by one of two forms. If the surface is a central quadric (ellipsoid, hyperboloid of one or two sheets, elliptic cone), the surface is defined by

$$F_1(x, y, z) = b_1 x^2 + b_2 y^2 + b_3 z^2 + b_4 = 0. \tag{4-5}$$

If the surface is a noncentral quadric (elliptic or hyperbolic paraboloid, elliptic or hyperbolic cylinder), the surface is defined by

$$F_2(x, y, z) = c_1 x^2 + c_2 y^2 + 2 c_3 z + c_4 = 0. \tag{4-6}$$

The general equation (4-3) of any quadric surface can be reduced to either (4-5) or (4-6) by performing a proper rotation and translation of axes (See Appendix II).

4.4.1 Intersection of a Line With a Standard Central Quadric Surface

When (2-1) is substituted into (4-5), the result is a quadratic equation in d. The coefficients for the quadratic given in (4-1) are

$$A = b_1 \lambda_1^2 + b_2 \lambda_2^2 + b_3 \lambda_3^2$$

$$B = 2\left(\frac{\partial F_1}{\partial x_0}\lambda_1 + \frac{\partial F_1}{\partial y_0}\lambda_2 + \frac{\partial F_1}{\partial z_0}\lambda_3\right) = \left[\nabla F_1(x_0, y_0, z_0)\right] \cdot \hat{\lambda} \tag{4-7}$$

$$C = F_1(x_0, y_0, z_0).$$

4.4.2 Intersection of a Line With a Standard Noncentral Quadric Surface

When (2-1) is substituted into (4-6), the coefficients for the quadratic given in (4-1) are

$$A = c_1 \lambda_1^2 + c_2 \lambda_2^2$$

$$B = 2 (c_1 \lambda_1 x_0 + c_2 \lambda_2 y_0 + c_3 \lambda_3) \qquad (4\text{-}8)$$

$$C = F_2 (x_0, y_0, z_0).$$

SECTION V

QUADRIC SURFACE SCATTERING CENTERS

5.1 Discussion

A scattering center (specular point) exists on a surface if there is a point on the surface whose normal is coincident with the incident field unit vector. From this definition it can be seen that the scattering center location changes with incident field direction and if a surface is not exposed, it cannot contain a scattering center. To simplify the calculations, only the standard form of a quadric surface is used in the scattering center analysis. The scattering center cannot be used for elliptic cones and elliptic or hyperbolic cylinders because the scattering center location (when one does exist) becomes a line instead of a point.

The condition that the surface normal (or gradient) be coincident with the incident field unit vector can be stated mathematically as

$$\hat{\lambda} \times \left| \nabla F(x, y, z) \right| = 0. \tag{5-1}$$

If the direction of the incident field unit vector, $\hat{\lambda}$, and the surface gradient, $\nabla F(x, y, z)$, are coincident, it can be seen that each of the components of the cross product are zero. This leads to the following set of equations.

$$\begin{aligned} \lambda_2 \frac{\partial F}{\partial z} - \lambda_3 \frac{\partial F}{\partial y} &= 0 \\ \lambda_3 \frac{\partial F}{\partial x} - \lambda_1 \frac{\partial F}{\partial z} &= 0 \\ \lambda_1 \frac{\partial F}{\partial y} - \lambda_2 \frac{\partial F}{\partial x} &= 0 \end{aligned} \tag{5-2}$$

5.2 Central Quadric Surface Scattering Centers

The definition of a scattering center location restricts this derivation to ellipsoids and hyperboloids of one or two sheets. When (5-2) operates on (4-5), the result is

$$\lambda_2 b_3 z - \lambda_3 b_2 y = 0 \tag{5-3}$$

$$\lambda_3 b_1 x - \lambda_1 b_3 z = 0 \tag{5-4}$$

$$\lambda_1 b_2 y - \lambda_2 b_1 x = 0. \tag{5-5}$$

If $|\lambda_1| \geq |\lambda_2|$ and $|\lambda_1| \geq |\lambda_3|$; define i=3, j=2, k=1 and solve (5-4) for z and (5-5) for y.

$$z = \frac{\lambda_3 b_1}{\lambda_1 b_3} x \tag{5-6}$$

$$y = \frac{\lambda_2 b_1}{\lambda_1 b_2} x \tag{5-7}$$

If $|\lambda_2| \geq |\lambda_1|$ and $|\lambda_2| \geq |\lambda_3|$; define i=3, j=1, k=2 and solve (5-3) for z and (5-5) for x.

$$z = \frac{\lambda_3 b_2}{\lambda_2 b_3} y \tag{5-8}$$

$$x = \frac{\lambda_1 b_2}{\lambda_2 b_1} y \tag{5-9}$$

If $|\lambda_3| \geq |\lambda_1|$ and $|\lambda_3| \geq |\lambda_2|$; define i=2, j=1, k=3 and solve (5-3) for y and (5-4) for x.

$$y = \frac{\lambda_2 b_3}{\lambda_3 b_2} z \tag{5-10}$$

$$x = \frac{\lambda_1 b_3}{\lambda_3 b_1} z \tag{5-11}$$

Using the definitions for variable subscripts described in Appendix IV, (5-6), (5-8) and (5-10) can be written as

$$x_i = \frac{\lambda_i b_k}{\lambda_k b_i} x_k, \tag{5-12}$$

(5-7), (5-9), and (5-11) as

$$x_j = \frac{\lambda_j a_k}{\lambda_k a_j} x_k, \tag{5-13}$$

and (4-5) written as

$$F_1 (x_i, x_j, x_k) = b_i x_i^2 + b_j x_j^2 + b_k x_k^2 + b_4 = 0. \tag{5-14}$$

Substituting (5-12) and (5-13) into (5-14)

$$\left[b_i \left(\frac{\lambda_i b_k}{\lambda_k b_i}\right)^2 + b_j \left(\frac{\lambda_j b_k}{\lambda_k b_j}\right)^2 + b_k \right] x_k^2 + b_4 = 0 \tag{5-15}$$

The condition that a scattering center can exist only on an illuminated surface can be stated mathematically by

$$\hat{n} \cdot \nabla F_1 (x_i, x_j, x_k) = -1 \tag{5-16}$$

Substitute each x_k into (5-12) and (5-13) to find the corresponding x_i and x_j. The set of coordinates that satisfy (5-16) is the location of the scattering center.

5.3 Noncentral Quadric Surface Scattering Centers

The definition of a scattering center location restricts this derivation to elliptic and hyperbolic paraboloids. When (5-2) operates on (4-6), the result is

$$\lambda_2 c_3 - \lambda_3 c_2 y = 0 \tag{5-17}$$

$$\lambda_3 c_1 x - \lambda_1 c_3 = 0 \tag{5-18}$$

$$\lambda_1 c_2 y - \lambda_2 c_1 x = 0. \tag{5-19}$$

If $\lambda_3 = 0$, the scattering center is located at the vertex ($x = 0$, $y = 0$, $z = 0$) of the surface.

If $\lambda_3 \neq 0$, solve (5-17) and (5-18) for y and x,

$$y = \frac{\lambda_2 c_3}{\lambda_3 c_2} \tag{5-20}$$

$$x = \frac{\lambda_1 c_3}{\lambda_3 c_1} \tag{5-21}$$

and (4-6) for z.

$$z = -\frac{1}{2c_3} (c_4 + c_1 x^2 + c_2 y^2) \tag{5-22}$$

5.4 Gaussian Curvature

Associated with each scattering center location is a Gaussian curvature, K_g, which is defined as the reciprocal of the product of the two principal radii of curvature, $R_1 R_2$, at the scattering center. From Reference 3

$$\frac{1}{K_g} = R_1 R_2 = \frac{(F_x^2 + F_y^2 + F_z^2)^2}{\Delta} \tag{5-23}$$

is obtained, where

$$\Delta = - \begin{vmatrix} F_{xx} & F_{yx} & F_{zx} & F_x \\ F_{xy} & F_{yy} & F_{zy} & F_y \\ F_{xz} & F_{yz} & F_{zz} & F_z \\ F_x & F_y & F_z & 0 \end{vmatrix} \tag{5-24}$$

Here F_x is the first derivative of the function with respect to x, F_{xx} is the second derivative with respect to x and so on.

5.4.1 Gaussian Curvature of a Standard Central Quadric

Expanding (5-24) for a central quadric surface yields the equation

$$\Delta_1 = \frac{\Delta}{16} = b_1 b_2 b_3 (b_1 x^2 + b_2 y^2 + b_3 z^2). \tag{5-25}$$

The Gaussian curvature can then be expressed by

$$(R_1 R_2)_1 = \frac{(b_1^2 x^2 + b_2^2 y^2 + b_3^2 z^2)^2}{\Delta_1} \qquad (5\text{-}26)$$

with x, y, z being the coordinates at the scattering center.

5.4.2 Gaussian Curvature of a Standard Noncentral Quadric

Expansion of (5-24) for a noncentral quadric gives

$$\Delta_2 = \frac{\Delta}{16} = c_1 c_2 c_3^2 \qquad (5\text{-}27)$$

The Gaussian curvature can then be expressed by

$$(R_1 R_2)_2 = \frac{(c_1^2 x_2 + c_2^2 y^2 + c_3^2)^2}{\Delta_2} \qquad (5\text{-}28)$$

with x and y being the coordinates at the scattering center.

SECTION VI

QUADRIC SURFACE ZONING

6.1 Discussion

In the course of an RCS calculation, the need to subdivide a surface into zones sometimes arises. An example would be performing a summation of $2\, \underline{n} \times \underline{H}$ over a surface. A computer program that zones the surface of a standard quadric is included in the GENSCAT program package. The surface must be given in standard form because this is by far the simplest (if not the only) way to zone the surface.

The zoning procedure is developed as a function of the surface coefficients, the limits of the surface over which the zoning applies, and the parameter ΔS which is the desired incremental length of one side of a zone. The zoning program does not generate zones of constant area, but each zone will have an area that is nominally $(\Delta S)^2$. If a zone area of four square wavelengths is desired, ΔS would be specified as two wavelengths.

The zoning program will compute the area of each zone and the coordinates and surface normal at the center of each zone.

6.2 Central Quadric Surfaces

The equation of a central quadric in standard form is

$$a_1 x^2 + a_2 y^2 + a_3 z^2 + a_4 = 0. \tag{6-1}$$

For zoning purposes, it is more convenient to express (6-1) in "true" standard form.

If $a_4 \neq 0$, dividing (6-1) by $-a_4$ yields

$$b_1 x^2 + b_2 y^2 + b_3 z^2 = 1, \tag{6-2}$$

where

$$b_1 = -\frac{a_1}{a_4}, \quad b_2 = -\frac{a_2}{a_4}, \quad b_3 = -\frac{a_3}{a_4}.$$

If $a_4 = 0$, (6-1) can be written as

$$b_1 x^2 + b_2 y^2 + b_3 z^2 = 0, \tag{6-3}$$

where

$$b_1 = a_1, \quad b_2 = a_2, \quad b_3 = a_3 .$$

Using the variable subscripts discussed in Appendix IV, (6-2) and (6-3) are expressed as

$$b_i x_i^2 + b_j x_j^2 + b_k x_k^2 = b_4 . \tag{6-4}$$

where

$b_4 = 1$ for an ellipsoid and hyperboloid of one or two sheets

$b_4 = 0$ for an elliptic cone.

The following definitions of k are used to determine which variable will be assigned to the longitudinal axis (k defines the longitudinal axis).

A. Ellipsoid

If $b_1 < b_2$ and $b_1 < b_3$, define $k=1$.
If $b_2 < b_1$ and $b_2 < b_3$, define $k=2$.
If $b_3 \leq b_1$ and $b_3 \leq b_2$, define $k=3$.

B. Hyperboloid of one sheet

If $b_1 < 0$, define $k=1$.
If $b_2 < 0$, define $k=2$.
If $b_3 < 0$, define $k=3$.

C. Hyperboloid of two sheets

If $b_1 > 0$, define $k=1$.
If $b_2 > 0$, define $k=2$.
If $b_3 > 0$, define $k=3$.

D. Elliptic cone

If $b_1 < 0$, define $k=1$.
If $b_2 < 0$, define $k=2$.
If $b_3 < 0$, define $k=3$.

The following tests assign to the subscript, i, the variable (one of the remaining two) whose coefficient has the greatest magnitude. This step is performed to insure maximum computational accuracy as this is the variable whose coefficient most frequently appears in a denominator.

If $k=1$, define $I=2$, $J=3$.

If $k=2$, define $I=3$, $J=1$.

If $k=3$, define $I=1$, $J=2$.

then,

If $|b_I| > |b_J|$, define $i=I$, $j=J$.

If $|b_J| > |b_I|$, define $i=J$, $j=I$.

6.3 Central Quadric Surface Zoning

The surface is zoned by first taking planar cuts in the longitudinal direction along the x_k axis and parallel to the $x_i - x_j$ plane and then angular planar cuts rotated about the x_k axis and normal to the $x_i - x_j$ plane. All zoning occurs between the longitudinal limits, L_l and L_u, and angular limits, θ_l and θ_u, which are defined by program inputs. The zoning of a typical surface is shown in Figure 4.

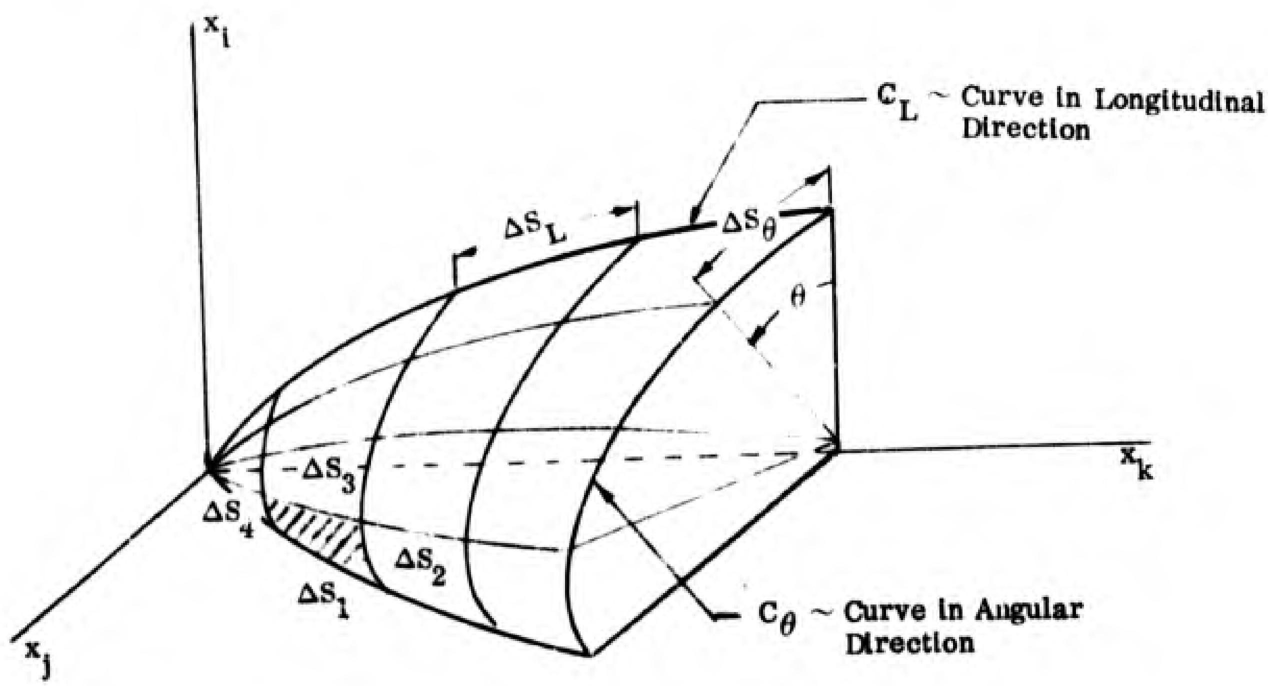


FIGURE 4. ZONING OF A TYPICAL SURFACE

The general philosophy used in determining the spacing of the cutting planes is to intersect the surface with a reference plane and numerically integrate to find the trace (arc length) of the resulting curve. For the longitudinal direction the reference plane is the $x_j = 0$ plane and for the angular direction it is either the $x_k = L_1$ or $x_k = L_2$ plane, depending on which has the greatest arc length (this decision is made by the program).

The numerical integration results in a set of data points representing arc distance as a function of x_k for the longitudinal direction and θ for the angular direction. The spacing of the cutting planes is obtained by curve fitting the coordinate (x_k or θ) as a function of arc distance and then interpolating in steps of ΔS (See Figure 6).

6.3.1 Longitudinal Cutting Planes

The curve C_L is defined as that portion of the trace (the intersection of the surface with the plane $x_j = 0$) that lies between the longitudinal limits L_1 and L_u, and whose x_i coordinate is positive. To obtain the functional relationship between the length of C_L and the x_k axis, divide the x_k axis into 50 equal increments,

$$\Delta x_k = \frac{L_u - L_1}{50} \qquad (6\text{-}5)$$

set $x_j = 0$ in (6-4) and solve for x_i as a function of x_k.

$$x_i(x_k) = \left(\frac{b_4 - b_k x_k^2}{b_i} \right)^{1/2} \qquad (6\text{-}6)$$

The length of C_L is then

$$S_L = \sum_{m=1}^{50} (\Delta S_L)_m = \sum_{m=1}^{50} \left[(\Delta x_i)_m^2 + (\Delta x_k)_m^2 \right]^{1/2} \qquad (6\text{-}7)$$

where

$$(\Delta x_i)_m = x_i \left| (m)\, \Delta x_k + L_1 \right| - x_i \left| (m-1) \Delta x_k + L_1 \right| \qquad (6\text{-}8)$$

The brackets indicate the argument for the function given in (6-6).

A table containing arc distance as a function of x_k is generated by (6-7) by incrementing m.

$$m = 1;\ x_k = L_l + \Delta x_k,\ (S_L)_1 = (\Delta S_L)_1$$
$$m = 2;\ x_k = L_l + 2\ \Delta x_k,\ (S_L)_2 = (\Delta S_L)_1 + (\Delta S_L)_2$$
$$m = 3;\ x_k = L_l + 3\ \Delta x_k,\ (S_L)_3 = (\Delta S_L)_1 + (\Delta S_L)_2 + (\Delta S_L)_3$$

. . .

. . .

$m = 50;$

This procedure is illustrated for five increments in Figure 5.

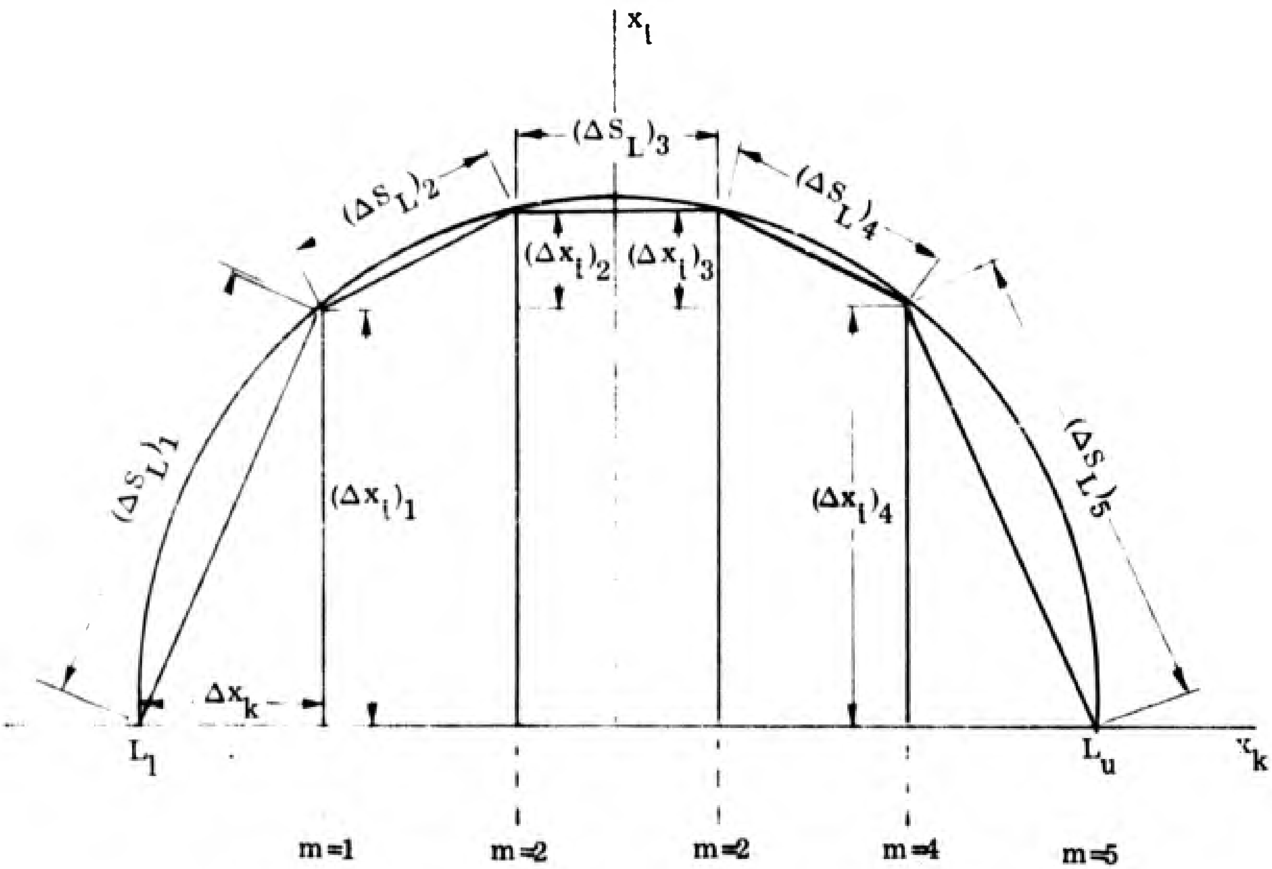


FIGURE 5. NUMERICAL INTEGRATION OF ARC LENGTH

6.3.2 Angular Cutting Planes

The curve C_θ is defined as that portion of the trace (the intersection of the surface with the plane $x_i = L_\theta$) that lies between the angular limits θ_1 and θ_u. The quantity L_θ is defined as

$L_\theta = L_l$ if arc length at $L_l \geq$ arc length at L_u

$L_\theta = L_u$ if arc length at $L_u >$ arc length at L_l

Now let

$$x_i = R\cos\theta$$

$$x_j = R\sin\theta$$

then (6-4) can be written as

$$b_i R^2 \cos^2\theta + b_j R^2 \sin^2\theta + b_k L_\theta^2 = b_4. \qquad (6\text{-}9)$$

Solve (6-9) for R

$$R = \left[\frac{b_4 - b_k L_\theta^2}{b_i \cos^2\theta + b_j \sin^2\theta} \right]^{1/2} \qquad (6\text{-}10)$$

and then differentiate (6-10) with respect to θ.

$$\frac{dR}{d\theta} = \frac{R(b_i - b_j)\sin\theta\cos\theta}{b_i \cos^2\theta + b_j \sin^2\theta} \qquad (6\text{-}11)$$

The length of curve C_θ is then given by

$$S_\theta = \int_{\theta_1}^{\theta_u} \left[R^2 + \left(\frac{dR}{d\theta}\right)^2 \right]^{1/2} d\theta . \qquad (6\text{-}12)$$

To generate the table for the functional relationship between the length of C_θ and θ, divide θ into 50 equal angles.

$$\Delta\theta = \frac{\theta_u - \theta_1}{50} \qquad (6\text{-}13)$$

The length of C_θ is then

$$S_\theta = \sum_{m=1}^{50} (\Delta S_\theta)_m = \sum_{m=1}^{50} \left[R_m^2 + \left(\frac{dR}{d\theta}\right)_m^2 \right]^{1/2} \qquad (6\text{-}14)$$

where $(\frac{dR}{d\theta})_m$ is evaluated at $\theta_m = \theta_1 + m\,\Delta\theta$.

6.3.3 Spacing of Cutting Planes

The parameter ΔS is an input to the zoning program and is used as a guide in determining the size of a zone. To insure that the entire length of C_L and C_θ are included in the zoning, the given value of ΔS is adjusted for each direction (longitudinal and angular).

$$N_L = \frac{S_L}{\Delta S} , \quad N_\theta = \frac{S_\theta}{\Delta S}$$

where N_L and N_θ are integers greater than zero. The adjusted longitudinal incremental distance is then

$$\Delta S'_L = \frac{S_L}{N_L} \qquad (6\text{-}15)$$

and the adjusted angular incremental distance is

$$\Delta S'_\theta = \frac{S_\theta}{N_\theta} . \qquad (6\text{-}16)$$

The locations of the cutting planes are found by curve fitting the tables described in Sections 6.3.1 and 6.3.2 using arc length as the independent variable (See Figure 6). The longitudinal spacing is found by using steps of $\Delta S'_L$ as the independent variable and interpolating for the corresponding x_k. The angular spacing is found by using steps of $\Delta S'_\theta$ as the independent variable and interpolating for the corresponding θ.

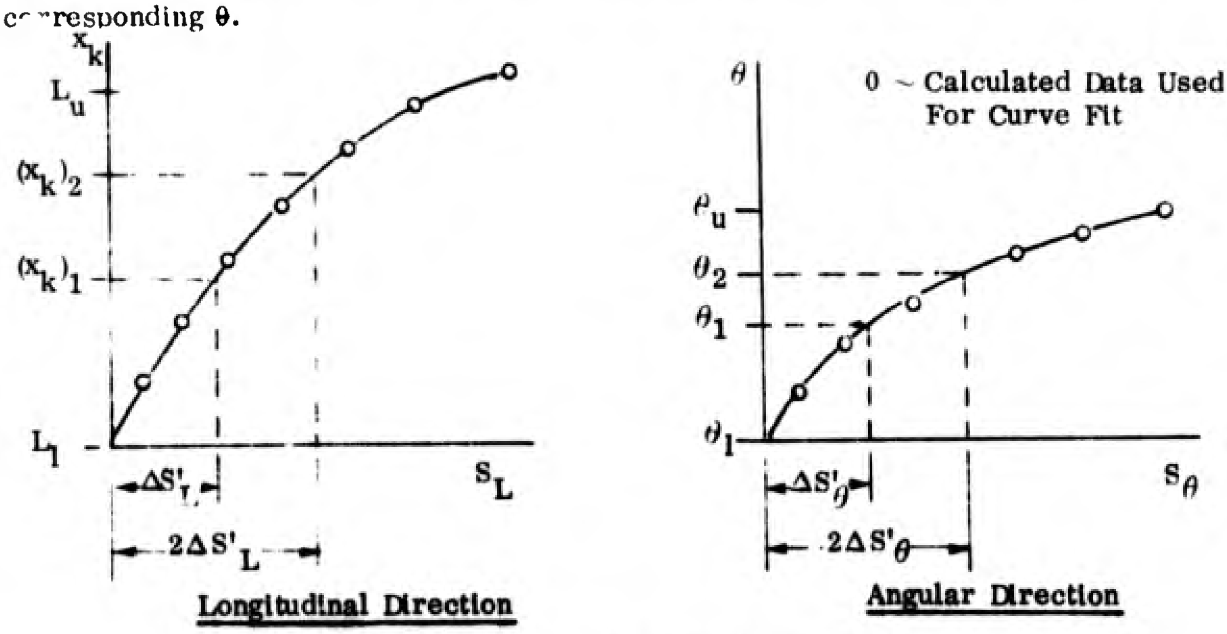


FIGURE 6. SPACING OF CUTTING PLANES

6.3.4 Surface Area of a Zone

The surface area of a zone is the area of the surface between two adjacent longitudinal and two adjacent angular cuts. In general, each zone will have a different area but the cutting planes are spaced so that each zone area is nominally $(\Delta S)^2$. To insure maximum accuracy in RCS calculations, the zoning program stores each zone area.

To calculate the zone area, consider a typical zone shown in Figure 7.

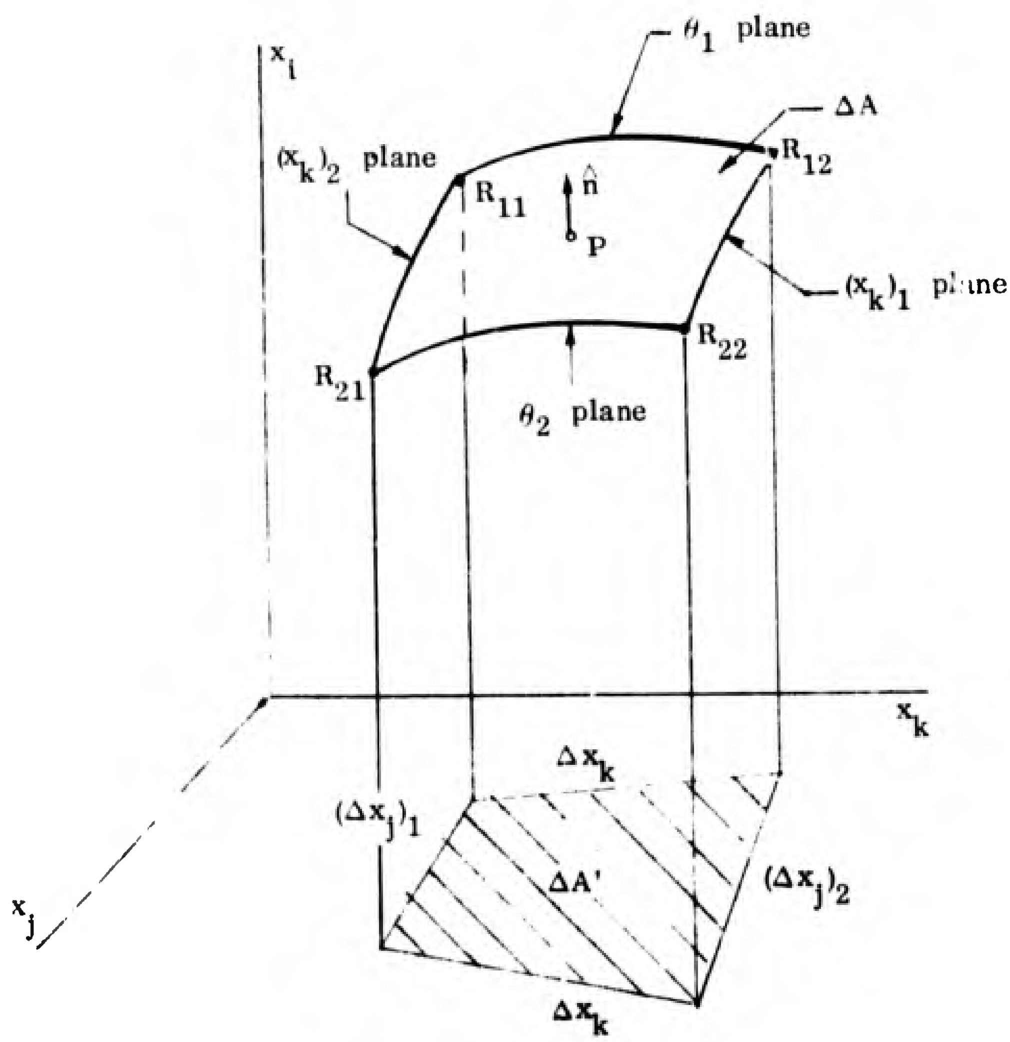


FIGURE 7. TYPICAL SURFACE ZONE

The projection of the surface area, ΔA, onto the x_j-x_k plane is a trapezoid whose area is given by

$$\Delta A' = \frac{1}{2}\left[(\Delta x_j)_1 + (\Delta x_j)_2\right]\Delta x_k \, . \qquad (6\text{-}17)$$

In some cases the projected area will be a triangle (for example, at the tips of an ellipsoid). When this occurs, either $(\Delta x_j)_1$ or $(\Delta x_j)_2$ will be zero and (6-17) reduces to the area of a triangle.

$$\cos\gamma = \frac{1}{\left[1 + \left(\frac{\partial x_i}{\partial x_j}\right)^2 + \left(\frac{\partial x_i}{\partial x_k}\right)^2\right]^{1/2}} \, . \qquad (6\text{-}18)$$

The derivatives are evaluated at P. The relationship between the surface area ΔA and the projected area $\Delta A'$ is

$$\Delta A' = \Delta A \cos\gamma$$

or

$$\Delta A = \frac{\Delta A'}{\cos\gamma} \, . \qquad (6\text{-}19)$$

Substituting (6-17) and (6-18) into (6-19)

$$\Delta A = \frac{1}{2}\left[1 + \left(\frac{\partial x_i}{\partial x_j}\right)^2 + \left(\frac{\partial x_i}{\partial x_k}\right)^2\right]^{1/2}\left[(\Delta x_j)_1 + (\Delta x_j)_2\right] x_k \qquad (6\text{-}20)$$

where

$$\frac{\partial x_i}{\partial x_j} = -\frac{b_j}{b_i}\,\frac{x_j}{x_i} \qquad (6\text{-}21)$$

$$\frac{\partial x_i}{\partial x_k} = -\frac{b_k}{b_i}\,\frac{x_k}{x_i} \qquad (6\text{-}22)$$

$$(\Delta x_j)_1 = R_{21} \sin \theta_2 - R_{11} \sin \theta_1 \qquad (6\text{-}23)$$

$$(\Delta x_j)_2 = R_{22} \sin \theta_2 - R_{12} \sin \theta_1 \qquad (6\text{-}24)$$

Equations (6-21) and (6-22) are obtained by differentiating (6-4) and evaluating the derivatives at P, which is located at the center of the zone. The coordinates at the center of the zone are

$$(x_i)_p = R \cos (\theta_p) \qquad (6\text{-}25)$$

$$(x_j)_p = R \sin (\theta_p) \qquad (6\text{-}26)$$

$$(x_k)_p = \frac{1}{2} \left[(x_k)_1 + (x_k)_2 \right] \qquad (6\text{-}27)$$

where

$$R = \left[\frac{b_4 - b_k (x_k)^2}{b_i \cos^2 \theta_p + b_j \sin^2 \theta_p} \right]^{1/2} \quad \text{and} \qquad (6\text{-}28)$$

$$\theta_p = \frac{1}{2} (\theta_1 + \theta_2)$$

The $R_{m,n}$ in (6-23) and (6-24) are the radii at the corners of the zone. R_{11} is the radius at the corner where cutting plane θ_1 intersects cutting plane $(x_k)_1$ and so on for R_{12}, R_{21} and R_{22}. The magnitude of each radius is found by substituting the corresponding x_k and θ into (6-28).

The surface normal of each zone is obtained from (6-4) and is evaluated at P.

$$\hat{n} = \frac{\hat{i}\, b_1 (x_1)_p + \hat{j}\, b_2 (x_2)_p + \hat{k}\, b_3 (x_3)_p}{\left\{ \left| b_1 (x_1)_p \right|^2 + \left| b_2 (x_2)_p \right|^2 + \left| b_3 (x_3)_p \right|^2 \right\}^{1/2}} \tag{6-29}$$

The zoning program requires the surface to be expressed in standard form so all derivations were developed in the local quadric system. The RCS calculations are made in a fixed reference coordinate system. The outputs of the zoning program are the zone area, the coordinates at the center of each zone, and the direction cosines of the surface normal at the center of each zone. The coordinates and normal components of the zone must therefore be transformed from the local quadric system into the fixed reference system.

$$|\underline{x}'| = |M|\ |\underline{x}| + |\underline{x}_o| \tag{6-30}$$

$$|\hat{n}'| = |M|\ |\hat{n}| \tag{6-31}$$

The rotation matrix $[M]$ and the translation vector $\underline{x}_o$ are included with the inputs to the zoning program. The transformations are explained in detail in Appendix II.

6.4 Noncentral Quadric Surfaces

The equation of a noncentral quadric in standard form is

$$a_1 x^2 + a_2 y^2 + 2a_3 z + a_4 = 0 \, . \tag{6-32}$$

For zoning purposes, it is more convenient to express (6-32) in "true" standard form. If $a_4 \neq 0$, dividing (6-32) by $-a_4$ yields

$$b_1 x^2 + b_2 y^2 + b_3 z = 1 \, , \tag{6-33}$$

where

$$b_1 = \frac{a_1}{a_4} \, , \quad b_2 = -\frac{a_2}{a_4} \, , \quad b_3 = -\frac{2a_3}{a_4}$$

If $a_4 = 0$, (6-32) is written as

$$b_1 x^2 + b_2 y^2 + b_3 z = 0 , \tag{6-34}$$

where

$$b_1 = a_1 , \quad b_2 = a_2 , \quad b_3 = 2a_3 .$$

Using the variable subscripts defined in Appendix IV, (6-33) and (6-34) can be expressed as

$$b_i x_i^2 + b_j x_j^2 + b_k x_k = b_4 . \tag{6-35}$$

where

$b_4 = 0$ for an elliptic or hyperbolic paraboloid

$b_4 = 1$ for an elliptic or hyperbolic cylinder

The following definitions of k are used to determine which variable will be assigned to the longitudinal axis (k defines the longitudinal axis).

A. Elliptic paraboloid

Define $k = 3$

B. Hyperbolic paraboloid

If $b_1 < 0$, define $k = 1$

If $b_2 < 0$, define $k = 2$

C. Elliptic cylinder

Define $k = 3$

D. Hyperbolic cylinder

If $b_1 < 0$, $k = 2$

If $b_2 < 0$, $k = 1$

then

if $k = 1$, define $i = 2$, $j = 3$

if $k = 2$, define $i = 3$, $j = 1$

if $k = 3$, define $i = 1$, $j = 2$

The development of the zoning of a noncentral quadric is identical to that of the central quadric with three exceptions.

1. Equation (6-6) is replaced by

$$x_i(x_k) = \left(\frac{b_4 - b_k x_k}{b_i} \right)^{1/2} \tag{6-36}$$

2. Equations (6-10) and (6-28) are replaced by

$$\eta = \left[\frac{b_4 - b_k x_k}{b_i \cos^2\theta + b_j \sin^2\theta} \right]^{1/2} \tag{6-37}$$

3. Equation (6-30) is replaced by

$$\hat{n} = \frac{\hat{i}\, 2b_1 (x_1)_p + \hat{j}\, 2b_2 (x_2)_p + \hat{k}\, b_3}{\left\{ \left[2b_1 (x_1)_p\right]^2 + \left[2b_2 (x_2)_p\right]^2 + b_3^{\,2} \right\}^{1/2}} \tag{6-38}$$

SECTION VII

APPLICATION OF QUADRICS TO AIRCRAFT SURFACES

7.1 Discussion

The primary interface between the user and the GENSCAT Program is the geometry description of the vehicle. To transform the geometry of an aircraft configuration into a computer model, the aircraft is divided into major structural components (fuselage, duct interior, nacelle, canopy, etc.). The GENSCAT Program contains user defined subroutines that correspond to each of the major structural components and it is these subroutines that are the main communication link between the user and GENSCAT. The user defined subroutines can be viewed as a means of inputing geometry data.

The breakup of the aircraft into major structural components is done for two reasons. It is easier for the user to "assemble" the aircraft by considering each component separately than to become involved with the entire aircraft at one time. Secondly, this enables GENSCAT to compute and display the RCS of each component in addition to the total RCS of the entire aircraft. This is useful because the user can determine which of the components are major contributors to the RCS.

This report is concerned with defining the geometry of major structural components by use of quadric surfaces. The user defined subroutines will be explained in a later report.

7.2 General Background Information

Before discussing the actual fitting of a surface, it is important that the user be made aware of the procedures involved in building a computer model. The procedures are broken down into the following three basic steps which are listed in order of occurrence.

7.2.1 Design Data Procurement

The first step in defining a new configuration is the procurement of design data. The most desirable form is one which gives a table of surface coordinates at specified control (or loft line) points for a series of fuselage stations (See Figures 9, 10 and 11). These data are generally obtained from the design or lofting departments. This step can become quite time consuming and should be initiated as soon as possible.

7.2.2 Extraction of Coordinate Data

Once the aircraft design data is obtained, the next step is to extract the surface coordinate data. The surface coordinate data (x, y, z) are the coordinates of the surface with respect to the fixed aircraft reference system (water line, buttock line and fuselage station). Experience has shown that 6 to 20 fuselage stations and 6 to 12 control points are usually sufficient to define a typical surface. If a surface has symmetry, be sure to include the symmetry when selecting surface coordinate data.

7.2.3 Surface Fit Data

After the coordinate data is selected, it is punched into data cards which are inputed to the surface fit program (See Appendix III).

7.3 Example

The above steps are best illustrated by use of an example. Given the design data, the information necessary to define the configuration includes:

A. Basic Diagrams (Figure 8) - Useful in visualizing the aircraft and in locating major structures.

B. Definition of Control Points (Figure 9) - shows location of tabulated control points for each fuselage station.

C. Tabulated Control Points (Figures 10 and 11) - gives the surface coordinates at the control points at various fuselage stations.

The information contained in Figures 10 and 11 is punched into data cards which in turn are used as inputs to the surface fit program. Appendix III contains a listing of the example data cards and gives the results of the surface fitting program.

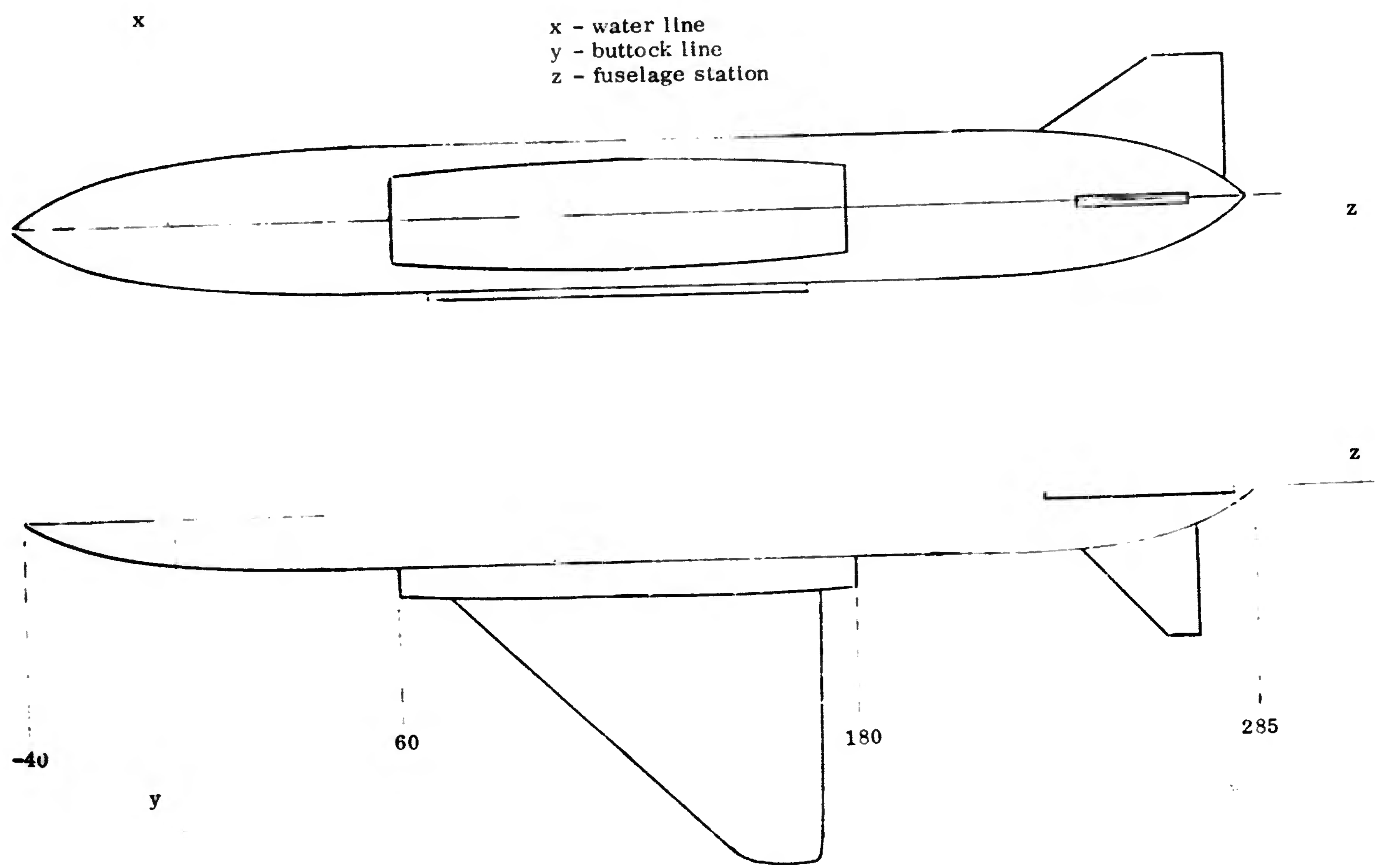
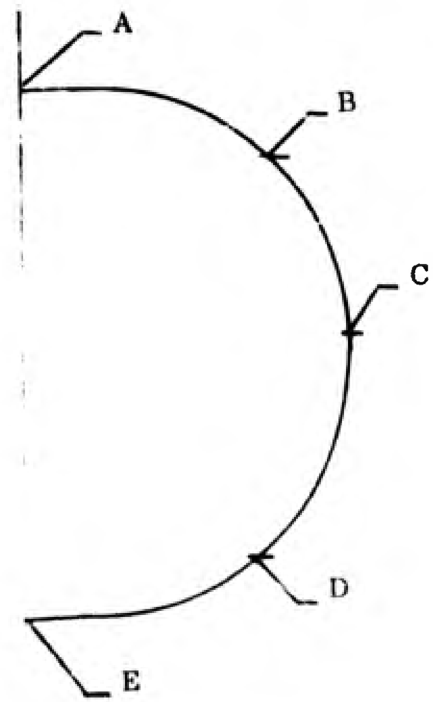
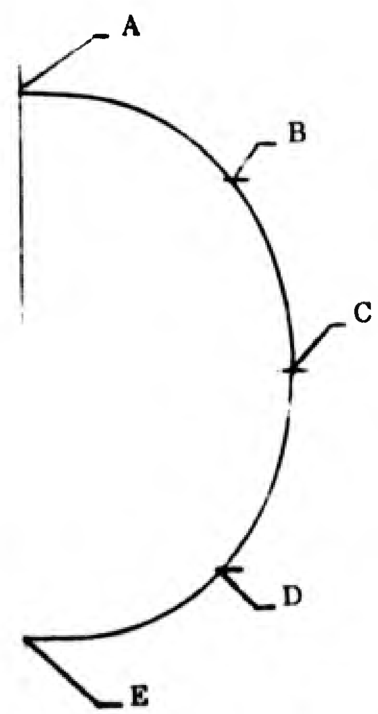


Figure 8. BASIC DIAGRAM



TYPICAL SECTION - FUSELAGE

F.S. -40 to F.S. 285



TYPICAL SECTION - NACELLE

F.S. 60 to F.S. 180

Figure 9. DEFINITION OF CONTROL POINTS

	A	B		C		D		E
z	x	x	y	x	y	x	y	x
-40	0	0	0	0	0	0	0	0
-30	8.72	5.23	5.23	↓	6.54	-5.23	5.23	-8.72
-20	12.00	7.20	7.20		9.00	-7.20	7.20	-12.00
-10	14.28	8.57	8.57		10.71	-8.57	8.57	-14.28
0	16.00	9.60	9.60		12.00	-9.60	9.60	-16.00
10	17.32	10.39	10.39		12.99	-10.39	10.39	-17.32
20	18.33	11.00	11.00		13.75	-11.00	11.00	-18.33
30	19.08	11.45	11.45		14.31	-11.45	11.45	-19.08
40	19.60	11.76	11.76		14.70	-11.76	11.76	-19.60
50	19.90	11.94	11.94		14.93	-14.94	11.94	-19.90
60	20.00	12.00	12.00		15.00	-12.00	12.00	-20.00
↓	↓	↓	↓		↓	↓	↓	↓
210	20.00	12.00	12.00		15.00	-12.00	12.00	-20.00
220	19.82	11.89	11.89		14.87	-11.89	11.89	-19.82
230	19.28	11.57	11.57		14.46	-11.57	11.57	-19.28
240	18.33	11.00	11.00		13.75	-11.00	11.00	-18.33
250	16.92	10.15	10.15		12.69	-10.15	10.15	-16.92
260	14.91	8.94	8.94		11.18	- 8.94	8.94	-14.91
270	12.00	7.20	7.20		9.00	- 7.20	7.20	-12.00
280	7.18	4.31	4.31	↓	5.39	- 4.31	4.31	- 7.18
285	0	0	0	0	0	0	0	0

TABULATED CONTROL POINTS FOR FUSELAGE

F.S. -40 TO F.S. 285

Figure 10. FUSELAGE COORDINATES

	A	B		C		D		E
z	x	x	y	x	y	x	y	x
60	12.00	6.66	21.66	0	23.00	-6.66	21.66	-12.00
70	12.99	7.21	22.21		23.66	-7.21	22.21	-12.99
80	13.75	7.63	22.63		24.17	-7.63	22.63	-13.75
90	14.31	7.94	22.94		24.54	-7.94	22.94	-14.31
100	14.70	8.15	23.15		24.80	-8.15	23.15	-14.70
110	14.93	8.28	23.28		24.95	-8.28	23.28	-14.93
120	15.00	8.32	23.32		25.00	-8.32	23.32	-15.00
130	14.93	8.28	23.28		24.95	-8.28	23.28	-14.93
140	14.70	8.15	23.15		24.80	-8.15	23.15	-14.70
150	14.31	7.94	22.94		24.54	-7.94	22.94	-14.31
160	13.75	7.63	22.63		24.17	-7.63	22.63	-13.75
170	12.99	7.21	22.21		23.66	-7.21	22.21	-12.99
180	12.00	6.66	21.66	0	23.00	-6.66	21.66	-12.00

TABULATED CONTROL POINTS FOR NACELLE

F. S. 60 TO F.S 180

Figure 11. NACELLE COORDINATES

APPENDIX I

CLASSIFICATION OF QUADRIC SURFACES

The general equation of a quadric surface is

$$F(x, y, z) = a_1 x^2 + a_2 y^2 + a_3 z^2 + 2 a_4 y z + 2 a_5 x z + 2 a_6 x y + 2 a_7 x + 2 a_8 y + 2 a_9 z + a_{10} = 0. \quad \text{(I-1)}$$

Equation (I-1) can also be written as

$$F(x, y, z) = (a_1 x + a_6 y + a_5 z + a_7) x + (a_6 x + a_2 y + a_4 z + a_8) y + (a_5 x + a_4 y + a_3 z + a_9) z + (a_7 x + a_8 y + a_9 z + a_{10}) = 0 \quad \text{(I-2)}$$

For any surface represented by (I-1) or (I-2), the four quantities

$$A = \begin{vmatrix} a_1 & a_6 & a_5 & a_7 \\ a_6 & a_2 & a_4 & a_8 \\ a_5 & a_4 & a_3 & a_9 \\ a_7 & a_8 & a_9 & a_{10} \end{vmatrix} \quad \text{(I-3)}$$

$$D = \begin{vmatrix} a_1 & a_6 & a_5 \\ a_6 & a_2 & a_4 \\ a_5 & a_4 & a_3 \end{vmatrix} \quad \text{(I-4)}$$

$$I = a_1 + a_2 + a_3 \quad \text{(I-5)}$$

$$J = \begin{vmatrix} a_1 & a_6 \\ a_6 & a_2 \end{vmatrix} + \begin{vmatrix} a_2 & a_4 \\ a_4 & a_3 \end{vmatrix} + \begin{vmatrix} a_3 & a_5 \\ a_5 & a_1 \end{vmatrix} \quad \text{(I-6)}$$

and the sign of the quantity

$$A' = A_1 + A_2 + A_3 + A_{10} \qquad \text{(I-7)}$$

are invariant with respect to translation and rotation and are used to classify a quadric surface. The brackets indicate determinants and A_i is the cofactor of a_i in the determinant A. By examining the values of A and D, the following facts about the surface can immediately be deduced.

$A \neq 0$. The surface is a proper quadric (ellipsoids, hyperboloids and paraboloids).

$A = 0$. The surface is an improper or degenerate quadric (cones and cylinders).

$D \neq 0$. The surface is a central quadric (ellipsoids, hyperboloids of one or two sheets, and elliptic cones).

$D = 0$. The surface is a non-central quadric (paraboloids and cylinders).

By performing the proper rotation and translation operations on (I-1), the surface can be transformed into a standard form. The transformation of a general quadric into a standard quadric is discussed in Appendix II. To simplify the classification of quadric surfaces, all examples in this section will be given in standard form.

The type of surface represented by (I-1) can assume one of 17 basic forms, depending on the values of the coefficients. Of these 17 basic forms, three are imaginary second degree surfaces

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1 \qquad \text{(imaginary ellipsoid)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0 \qquad \text{(imaginary cone)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1 \qquad \text{(imaginary elliptic cone)}$$

five involve either real or imaginary planes,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \qquad \text{(real intersecting planes)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \qquad \text{(imaginary intersecting planes)}$$

$$x^2 = a^2 \qquad \text{(real parallel planes)}$$

$$x^2 = -a^2 \qquad \text{(imaginary parallel planes)}$$

$$x^2 = 0 \qquad \text{(coincident planes),}$$

and one is a parabolic cylinder

$$x^2 + 2az = 0 \ .$$

For a more detailed description of the above surfaces see References 3 and 4. None of these surfaces will be used for geometry description.

The eight remaining real surfaces are used for geometry description. Each of these eight surfaces is assigned an identification number by the surface fitting program described in Appendix III. This identification has the name KODE and is included in the following classification of surfaces.

1. Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$A < 0$ $\quad D \gtrless 0$ $\quad$ DI and J both > 0 $\quad$ KODE $= 1$

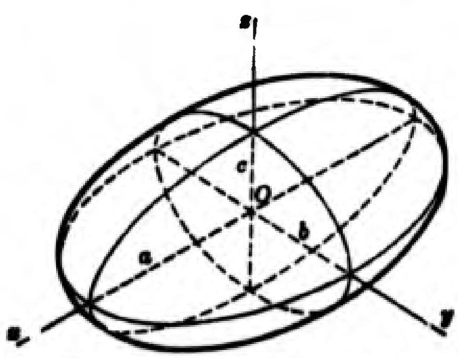


FIGURE 12. ELLIPSOID

2. Hyperboloid of one sheet.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$A > 0$ $D \neq 0$ DI and J not both > 0 KODE = 2

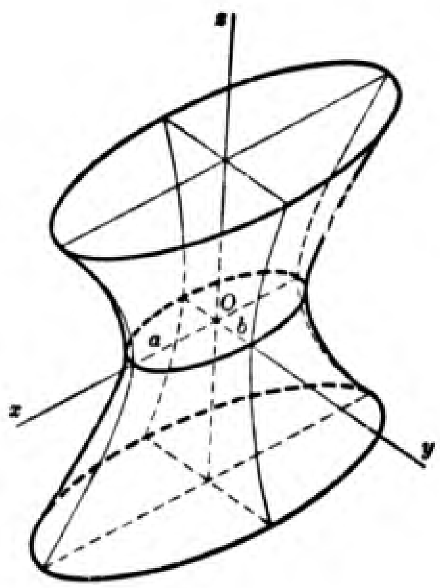


FIGURE 13. HYPERBOLOID OF ONE SHEET

3. Hyperboloid of two sheets.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

$A < 0$ $D \neq 0$ DI and J not both > 0 KODE = 3

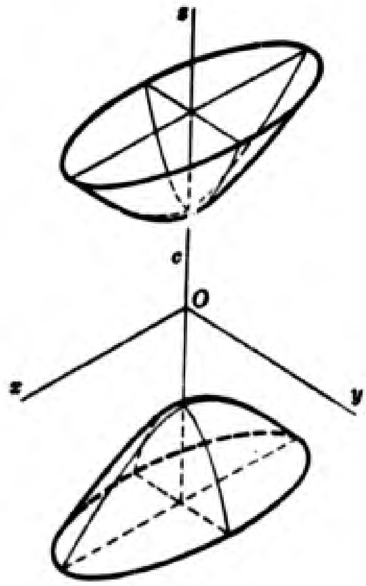


FIGURE 14. HYPERBOLOID OF TWO SHEETS

4. Elliptic cone.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

A = 0 D ≠ 0 A' ≠ 0 DI and J not both > 0 KODE = 4

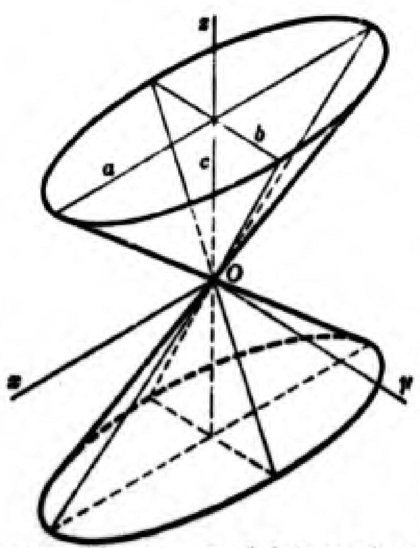


FIGURE 15. ELLIPTIC CONE

5. Elliptic paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + 2z = 0$$

A < 0 D = 0 J > 0 KODE = 5

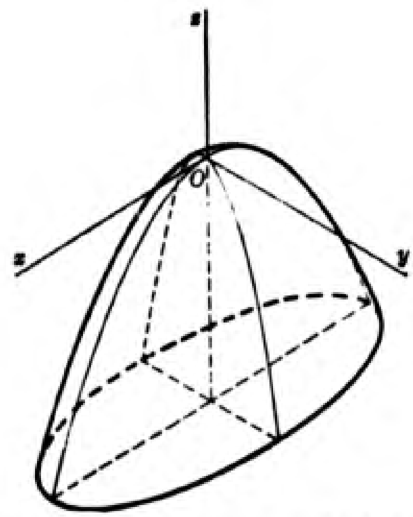


FIGURE 16. ELLIPTIC PARABOLOID

6. Hyperbolic paraboloid.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + 2z = 0$$

A > 0 D = 0 J < 0 KODE = 6

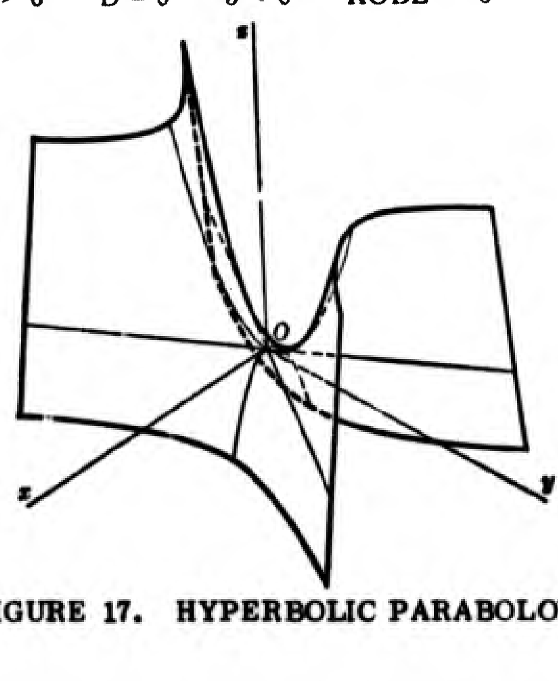


FIGURE 17. HYPERBOLIC PARABOLOID

7. Elliptic cylinder.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

A = 0 D = 0 J > 0

A' ≠ 0 A'K and J not both > 0 KODE = 7

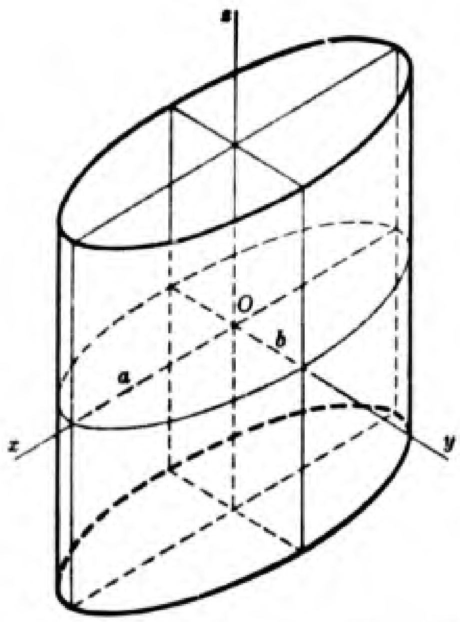


FIGURE 18. ELLIPTIC CYLINDER

8. Hyperbolic cylinder.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

A = 0 D = 0 J < 0 A' ≠ 0 KODE = 8

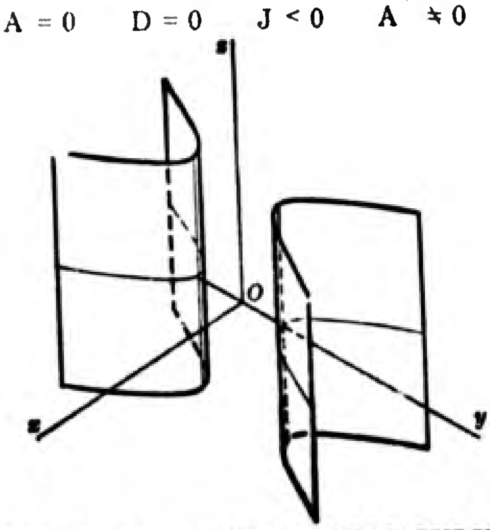


FIGURE 19. HYPERBOLIC CYLINDER

APPENDIX II

TRANSFORMATION OF A GENERAL QUADRIC INTO STANDARD FORM

All quadric surfaces defined by (II-1) can be reduced to a standard form by translation and rotation to a new coordinate system.

$$F(x, y, z) = a_1 x^2 + a_2 y^2 + a_3 z^2 + 2 a_4 yz + 2 a_5 xz$$

$$+ 2 a_6 xy + 2 a_7 x + 2 a_8 y + 2 a_9 z + a_{10} = 0 \qquad \text{(II-1)}$$

This new coordinate system is called the local quadric system. The quantities A, D, I, and J defined in Appendix I are invariant with respect to translation and rotation and are used to classify quadric surfaces. They will also be used in determining the transformation operators.

2.1 Principal Planes and Axes

The intersection of a line with a quadric surface (See Sections 4.2 and 4.2.1) results in a quadratic equation whose unknown, d, is the distance from a starting point (on the line) to the surface.

$$A d^2 + B d + C = 0 \qquad \text{(II-2)}$$

Any line with the direction, $\lambda_1, \lambda_2, \lambda_3$, for which $A \neq 0$, determines a chord of (II-1) which has a real midpoint. Let this unknown midpoint, $P_0 (x_0, y_0, z_0)$, be the starting point on the line. Then, when the two roots of (II-2) are real, their sum is zero because they are equal in magnitude and opposite in sign. A necessary and sufficient condition for the sum of the two roots of a quadratic equation to vanish is that the coefficients of the linear term vanish. The expression for the linear term is obtained from (4-4) in Section 4.3.1.

$$B = \left[\Delta F (x_0, y_0, z_0) \right] \cdot \hat{\lambda} \qquad \text{(II-3)}$$

Let (II-3) operate on (II-1) and set the result to zero.

$$(a_1 \lambda_1 + a_6 \lambda_2 + a_5 \lambda_3) x_o + (a_6 \lambda_1 + a_2 \lambda_2 + a_4 \lambda_3) y_o + (a_5 \lambda_1 + a_4 \lambda_2 + a_3 \lambda_3) z_o + (a_7 \lambda_1 + a_8 \lambda_2 + a_9 \lambda_3) = 0 \tag{II-4}$$

The locus of midpoints of chords with direction cosines $\lambda_1, \lambda_2, \lambda_3$ is the plane defined by (II-4). Note that any set of direction cosines will generate a family of chords as long as $A \neq 0$. A diametral plane is defined as a plane that bisects a family of parallel chords. Therefore, (II-4) is the equation of a diametral plane.

A principal plane is defined as a diametral plane that is perpendicular to the chords it bisects. Every quadric surface has at least two mutually perpendicular principal planes. A central quadric has at least three mutually perpendicular principal planes.

The intersection of two principal planes is an axis of symmetry or principal axis. Every quadric surface has at least one principal axis. If more than one principal axis exists, there is at least one other axis perpendicular to each of them. A central quadric has at least three mutually perpendicular principal axes.

2.2 Center of a Central Quadric Surface

For (II-3) to vanish for any given direction $\lambda_1, \lambda_2, \lambda_3$ the following condition must be met.

$$\nabla F (x_o, y_o, z_o) = 0 \tag{II-5}$$

For (II-5) to always be true, each of its components must be zero.

$$\begin{aligned} a_1 x_o + a_6 y_o + a_5 z_o + a_7 &= 0 \\ a_6 x_o + a_2 y_o + a_4 z_o + a_8 &= 0 \\ a_5 x_o + a_4 y_o + a_3 z_o + a_9 &= 0 \end{aligned} \tag{II-6}$$

For a central quadric ($D \neq 0$), the solution of (II-6) is unique and yields the center, P_o (x_o, y_o, z_o), of the surface.

$$x_o = -\frac{1}{D} \begin{vmatrix} a_7 & a_6 & a_5 \\ a_8 & a_2 & a_4 \\ a_9 & a_4 & a_3 \end{vmatrix} \qquad \text{(II-7)}$$

$$y_o = -\frac{1}{D} \begin{vmatrix} a_1 & a_7 & a_5 \\ a_6 & a_8 & a_4 \\ a_5 & a_9 & a_3 \end{vmatrix} \qquad \text{(II-8)}$$

$$z_o = -\frac{1}{D} \begin{vmatrix} a_1 & a_6 & a_7 \\ a_6 & a_2 & a_8 \\ a_5 & a_4 & a_9 \end{vmatrix} \qquad \text{(II-9)}$$

Where the brackets indicate determinants.

2.3 Rotation Matrix

A necessary and sufficient condition that the principal plane associated with the direction $\lambda_1, \lambda_2, \lambda_3$ be perpendicular to this direction is that the coefficients in (II-4) be proportional to λ_1, λ_2, and λ_3.

$$\begin{aligned} a_1\lambda_1 + a_6\lambda_2 + a_5\lambda_3 &= k\lambda_1 \\ a_6\lambda_1 + a_2\lambda_2 + a_4\lambda_3 &= k\lambda_2 \\ a_5\lambda_1 + a_4\lambda_2 + a_3\lambda_3 &= k\lambda_3 \end{aligned} \qquad \text{(II-10)}$$

Where the unknowns are k (a real non-zero number) and the direction cosines λ_1, λ_2, and λ_3. Equation (II-10) can be expressed in matrix form by

$$\begin{bmatrix} a_1 - k & a_6 & a_5 \\ a_6 & a_2 - k & a_4 \\ a_5 & a_4 & a_3 - k \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(II-11)}$$

A non-trivial solution of (II-11) exists only if

$$\begin{vmatrix} a_1 - k & a_6 & a_5 \\ a_6 & a_2 - k & a_4 \\ a_5 & a_4 & a_3 - k \end{vmatrix} = 0 \ . \qquad \text{(II-12)}$$

The characteristic equation is found by expanding the determinant in (II-12). The expansion can be expressed in terms of the quadric invariants.

$$k^3 - I k^2 + J k - D = 0 \qquad \text{(II-13)}$$

The solution of (II-13) results in three roots, k_1, k_2, and k_3, where

$$k_1 \geq k_2 \geq k_3 \ . \qquad \text{(II-14)}$$

If one of the roots of k is zero, there is no principal plane associated with that root. This occurs only for noncentral quadrics. If two of the roots are equal, the surface is a surface of revolution.

Associated with each root or eigenvalue, k_i, of (II-13) is an eigenvecter $\underline{V}_i$. The directions of the normals to the principal planes are directed along the eigenvectors. The direction numbers for each eigenvector are found by substituting each k_i into (II-11) and solving for $\underline{V}_i$ $(\lambda_1, \lambda_2, \lambda_3)$.

The elements of the rotation matrix are found by normalizing the eigenvectors

$$\hat{v}_i = \frac{\underline{V}_i}{|\underline{V}_i|} \qquad \text{(II-15)}$$

and using them as columns of the rotation matrix |R|

$$|R| = |\hat{v}_1 \ \hat{v}_2 \ \hat{v}_3| = \begin{bmatrix} v_{1x} & v_{2x} & v_{3x} \\ v_{1y} & v_{2y} & v_{3y} \\ v_{1z} & v_{2z} & v_{3z} \end{bmatrix} \qquad \text{(II-16)}$$

If there are only two eigenvalues (which is the case for a noncentral quadric), the third eigenvector is found by taking the vector cross product of the first two eigenvectors.

$$\underline{V}_3 = \underline{V}_1 \times \underline{V}_2 \qquad \text{(II-17)}$$

2.4 Vertex of Noncentral Quadric Surface

The elliptic and hyperbolic paraboloids have only two principal planes and hence only one axis of symmetry. The vertex of the surface is defined as the point where the axis of symmetry meets the surface. Before this problem can be solved, some preliminary groundwork must be laid.

The transformation from the local quadric system (primed) to the fixed reference system (unprimed) is

$$|\underline{x}| = |R|\,|\underline{x}'| + |\underline{x}_0| \qquad \text{(II-18)}$$

where |R| is the rotation matrix and $\underline{x}_0$ the translation vector. The inverse transformation is

$$|\underline{x}'| = |R|^{-1}\,|\underline{x} - \underline{x}_0|. \qquad \text{(II-19)}$$

Expand (II-19) for the x' component.

$$x' = R_{11} (x - x_o) + R_{21} (y - y_o) + R_{31} (z - z_o) \qquad \text{(II-20)}$$

In the last section it was shown that R_{11}, R_{21} and R_{31} are the components of the eigenvector associated with k_1 and that the normals of the principal planes are directed along the eigenvectors. Equation (II-20) can then be written as

$$x' = v_{1x} (x - x_o) + v_{1y} (y - y_o) + v_{1z} (z - z_o)$$

or

$$x' = v_{1x} x + v_{1y} y + v_{13} z - (v_{1x} x_o + v_{1y} y_o + v_{13} z_o). \qquad \text{(II-21)}$$

Equation (II-21) can be recognized as the equation of a plane expressed in normal form (See Section III), where the normal distance from the origin is $v_{1x} x_0 + v_{1y} y_0 + v_{1z} z_0$. Closer inspection of (II-21) reveals that it is actually the equation of a principal plane defined in normal form. Thus it can be stated that x' equals the normal equation of the principal plane for the eigenvalue k_1. The same can be said about y' and k_2.

The paraboloids in this section can be expressed in standard form (the local quadric system) as

$$c_1 x'^2 + c_2 y'^2 + 2 z' = 0 . \qquad \text{(II-22)}$$

From page 79 of Reference 4 is obtained the relationships

$$c_1 = k_1 \left(- \frac{k_1 k_2}{A} \right)^{1/2} \qquad \text{(II-23)}$$

$$c_2 = k_2 \left(- \frac{k_1 k_2}{A} \right)^{1/2} \qquad \text{(II-24)}$$

where A is one of the quadric invariants. Substituting (II-23) and (II-24) into (II-22)

$$k_1 \left(\frac{-k_1 k_2}{A} \right)^{1/2} x'^2 + k_2 \left(- \frac{k_1 k_2}{A} \right)^{1/2} y'^2 + 2 z' = 0$$

or

$$k_1 x'^2 + k_2 y'^2 + 2\left(\frac{-A}{k_1 k_2}\right)^{1/2} z' = 0 \qquad \text{(II-25)}$$

By equating ' to the normal equation of the principal plane associated with k_1, it can be stated that

$$x' = d_1 x + d_2 y + d_3 z + d_4 \qquad \text{(II-26)}$$

and for k_2

$$y' = e_1 x + e_2 y + e_3 z + e_4 . \qquad \text{(II-27)}$$

The d_n and e_n coefficients are found by substituting the direction cosines of the appropriate eigenvectors into (II-4).

Because there are only two principal planes, the third relationship

$$z' = f_1 x + f_2 y + f_3 z + f_4 \qquad \text{(II-28)}$$

contains an unknown coefficient, f_4. The direction of the third rotation vector is known from (II-17) and the coefficients f_1, f_2 and f_3 are the components of this vector. The vertex is located at the intersection point of the two principal planes and the plane defined by the right hand side of (II-28). Thus, if the coefficient f_4 of (II-28) is known, the coordinates of the vertex can be found.

To find f_4, substitute (II-26), (II-27) and (II-28) into (II-25).

$$k_1 (d_1 x + d_2 y + d_3 z + d_4)^2 + k_2 (e_1 x + e_2 y + e_3 z + e_4)^2$$

$$+ 2 \left[\left(\frac{-A}{k_1 k_2}\right)^{1/2} (f_1 x + f_2 y + f_3 z) \right] = 0 \qquad \text{(II-29)}$$

Expanding (II-29) and collecting terms.

$$(k_1 d_1^2 + k_2 e_1^2) x^2 + (k_2 d_2^2 + k_2 e_2^2) y^2 + (k_1 d_3^2 + k_2 e_3^2) z^2$$

$$+ 2 (k_1 d_2 d_3 + k_2 e_2 e_3) yz + 2 (k_1 d_1 d_3 + k_2 e_1 e_3) xz$$

$$+ 2 (k_1 d_1 d_2 + k_2 e_1 e_2) xy + 2 (k_1 d_1 d_4 + k_2 e_1 e_4) x \qquad \text{(II-30)}$$

$$+ 2 (k_1 d_2 d_4 + k_2 e_2 e_4) y + 2 (k_1 d_3 d_4 + k_2 e_3 e_4) z$$

$$+ k_1 d_4^2 + k_2 e_4^2 + 2 \left(\frac{-A}{k_1 k_2}\right)^{1/2} (f_1 x + f_2 y + f_3 z + f_4) = 0$$

Note that (II-30) expresses the surface in the fixed reference system in terms of the principal planes. Equations (II-30) and II-1) are equivalent and the unknown coefficient, f_4, is found by equating the linear and constant terms of (II-30) to those of (II-1).

$$k_1 d_1 d_4 + k_2 e_1 e_4 + N f_1 = a_7 \quad \text{(II-31)}$$

$$k_1 d_2 d_4 + k_2 e_2 e_4 + N f_2 = a_8 \quad \text{(II-32)}$$

$$k_1 d_3 d_4 + k_2 e_3 e_4 + N f_3 = a_9 \quad \text{(II-33)}$$

$$k_1 d_4^2 + k_2 e_4^2 + N f_4 = a_{10} \quad \text{(II-34)}$$

where

$$N = \pm \left(\frac{-A}{k_1 k_2}\right)^{1/2} \quad \text{(II-35)}$$

The choice of sign in (II-35) is determined from either (II-31), (II-32) or (II-33). Our friend, f_4, is then found by solving (II-34).

$$f_4 = \left| a_{10} - (k_1 d_4^2 + k_2 e_4^2) \right| / N \quad \text{(II-36)}$$

The coordinates (x_o, y_o, z_o) of the vertex are found by solving the following set of equations.

$$\begin{aligned} d_1 x_o + d_2 y_o + d_3 z_o + d_4 &= 0 \\ e_1 x_o + e_2 y_o + e_3 z_o + e_4 &= 0 \\ f_1 x_o + f_2 z_o + f_3 z_o + f_4 &= 0 \end{aligned} \quad \text{(II-37)}$$

2.5 Line of centers for Noncentral Quadric Surface

The elliptic and hyperbolic cylinders have only two principal planes and one axis of symmetry. The intersection of the two principal planes is the axis of symmetry or line of centers. The translation reference coordinates can be any point that lies on the line of centers.

The principal plane associated with k_1 is

$$d_1 x_o + d_2 y_o + d_3 z_o + d_4 = 0 \tag{II-38}$$

and the principal plane associated with k_2 is

$$e_1 x_o + e_2 y_o + e_3 z_o + e_4 = 0 . \tag{II-39}$$

Now simply choose a value for z_o and solve (II-38) and (II-39) for x_o, and y_o. A good choice for z_o is

$$z_o = \frac{z_u - z_L}{2}$$

where z_u and z_L are the values for the z coordinate of the upper and lower limits of the surface.

APPENDIX III

PROGRAM TO SURFACE FIT AND ANALYZE A GENERAL QUADRIC SURFACE

A surface fit program was developed to aid the user in defining aircraft surfaces. GENSCAT requires that a quadric surface be defined by four coefficients (standard form), a code number to identify the surface (see Appendix I), the translation vector and rotation matrix (see Appendix II). The surface fit program applies a least squares fit of a general quadric to a set of input coordinate data; performs the analysis described in Appendices I and II; compares the input coordinates with those of the fitted surface; and punches cards containing the four standard quadric coefficients, code number, translation vector and rotation matrix. The punched output has the form of a FORTRAN data declaration statement and may be inserted directly into a FORTRAN program.

The surface data inputs (x, y, z) are the coordinates of the surface with respect to a fixed reference system (for aircraft this is usually the water line, buttock line, and fuselage station). At least nine input points must be used to fit the surface. Typically, the input coordinate data is obtained from tables and/or drawings of surface cross section cuts taken at various fuselage stations. Experience has shown that 6 to 20 fuselage stations (cross section cuts) is ample. For a given fuselage station, it is usually sufficient to consider about 12 points for each full (360°) cross section. If a surface has symmetry (most will), be sure to include the symmetry when selecting data points. The number of input data points is limited only by program dimension statements. Typical aircraft surfaces are defined by between 30 and 200 points, so the dimension of 500 in the program should be ample.

There are times when a surface (or major aircraft structure) is too large or has too many inflections to be defined by a single surface. The surface is then broken down into two or more sub-surfaces and each sub-surface is fitted individually. To insure that no large gaps occur at the boundaries of the sub-surfaces, the surface fit data must include points that extend into adjacent surfaces. This is illustrated in Figure 20. This overlapping method does not guarantee that adjacent surfaces will meet exactly at their boundary. When overlapping is used, care must be taken to insure that points at or near the boundary match up as closely as possible.

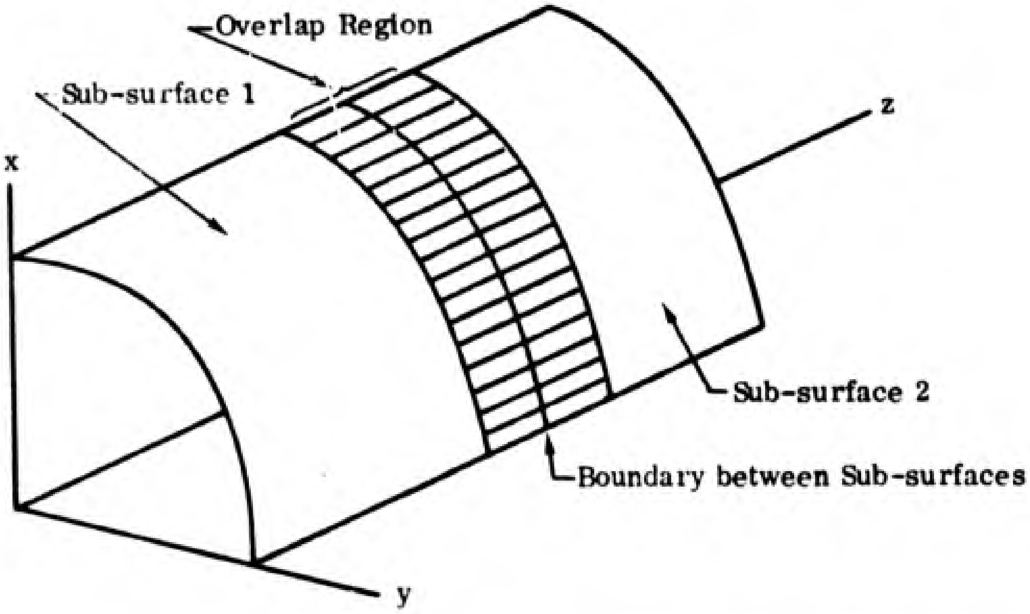


FIGURE 20. OVERLAP OF SUB-SURFACES

In addition to punching surface fit cards, the program also compares the input coordinates with their corresponding location on the fitted surface and shows the deviation of x and y for a given z. This output is used as a guide in determining the goodness of fit. If less than 10% of the input coordinates have a maximum deviation of about 8%, the surface is presently considered acceptable. If the fit is not acceptable, the following steps are followed.

1. Check for errors in coordinate data cards.
2. If the surface has symmetry, include the symmetry in the coordinate data cards. For example; if the surface is symmetric in x, be sure to include ± x coordinates for a given y; if the surface is symmetric in y, be sure to include ± y coordinates for a given x.
3. If the deviation is too large over a portion of the surface, weight that region by adding more data cards (this is possible because of the least squares fit).
4. If the above steps fail, the surface must be subdivided into two or more sub-surfaces.

3.1 Sample Problem

In Section VII an example configuration was introduced to illustrate the type of design data required for the generation of input coordinate data. The example will now be used to demonstrate the surface fit program.

Assume that the design data has been located (Figures 8, 9, 10 and 11) and the user is required to fit the fuselage and nacelle (outer duct surface). Inspection of the fuselage design data (Figure 10) reveals that it consists of three distinct surfaces. The first surface is from F.S. -40 to F.S. 60, the second from F.S. 60 to F.S. 210 and the third from F.S. 210 to F.S. 285. The nacelle design data (Figure 11) appears to be one surface.

A set of coordinate data is punched for each of the three fuselage surfaces and the nacelle surface. Each set of data is terminated by a 9999 card. The input data format is shown in Figure 21 and a listing of the input data cards is given in Section 3.2 of this Appendix.

The surface fit output is given in Section 3.3 of this Appendix. The output consists of

1. Identification title.
2. Results of surface fit in general form.
3. The quadric invariants discussed in Appendix I.
4. If surface is a central quadric
   a. Type of quadric
   b. Coordinates of the center (translation vector)
   c. Eigenvalues
   d. Kode identification
5. If surface is a noncentral quadric
   a. Type of quadric
   b. Eigenvalues
   c. Kode identification
   d. Direction of diametral planes
   e. Vertex (translation vector)
6. The rotation matrix
7. The determinate of the rotation matrix. The value must be +1 for a proper rotation.

8. Surface fit results transformed into standard form.

9. General form obtained by applying inverse transformation to standard form. The coefficients must be the same as those obtained from the original fit (line 2). This is a double check of the transformation operators.

10. Comparison data.

    XI, YI, ZI - input coordinate data.

    XC, YC - location of input data on fitted surface for a given ZI.

    DX, DY - (XI - XC) and (YI - YC).

11. Punched surface fit data containing

    a. Identification title

    b. Kode identification and coefficients of standard surface.

    c. Translation vector

    d. Rotation matrix

DATA SHEET FOR QUADRIC SURFACE FITTING PROGRAM

DATA PUNCH INDICATOR = ▨ DATA ID = ▨ CARD PUNCH UNIT = ▨	CARD1
	CARD2
X= ▨ Y= ▨ Z= ▨	CARD3
	⋮
	CARD N
9999.	

CARD 1 ~ COLS. 1-72 (18A4) ALPHA-NUMERIC DATA TO IDENTIFY THE DATA SET

CARD 2 ~ COL. 22 (I1) 0 - NO DATA CARDS PUNCHED
1 - PUNCH DATA CARDS

COL. 35 (A1) AN ALPHA NUMERIC CHARACTER.
THIS IDENTIFICATION NUMBER IS PUNCHED INTO THE OUTPUT DATA CARDS.
EACH SURFACE MUST HAVE ITS OWN ID NUMBER.

COLS. 57-58 (I2) DEFINES COMPUTER SYSTEMS PUNCH UNIT.

CARD 3 ~ COLS. 3-15 (E 13.6) X COORDINATE
COLS. 18-30 (E 13.6) Y COORDINATE
COLS. 33-45 (E 13.6) Z COORDINATE
THIS CARD IS REPEATED FOR EACH SURFACE POINT.

CARD N ~ COLS. 3-15 (E 13.6) 9999. INDICATES END OF DATA SET.

FIGURE 17. DATA SHEET

5.2 Sample Input Data

```
CHECKOUT AIRCRAFT  FUSELAGE  F.S. -40 TO F.S. 60
DATA PUNCH INDICATOR=1.   DATA ID=A.    CARD PUNCH UNIT= 6.
X=0.0           Y=0.0           Z=-40.0
X=-8.72         Y=0.0           Z=-30.0
X= 8.72         Y=0.0           Z=-30.0
X= 5.23         Y= 5.25         Z=-30.0
X=-5.23         Y= 5.25         Z=-30.0
X=-5.23         Y=-5.25         Z=-30.0
X= 5.23         Y=-5.25         Z=-30.0
X=0.0           Y= 6.54         Z=-30.0
X=0.0           Y=-6.54         Z=-30.0
X=-12.0         Y=0.0           Z=-20.0
X= 12.0         Y=0.0           Z=-20.0
X= 7.20         Y= 7.20         Z=-20.0
X=-7.20         Y= 7.20         Z=-20.0
X=-7.20         Y=-7.20         Z=-20.0
X= 7.20         Y=-7.20         Z=-20.0
X=0.0           Y= 9.00         Z=-20.0
X=0.0           Y=-9.00         Z=-20.0
X=-14.28        Y=0.0           Z=-10.0
X= 14.28        Y=0.0           Z=-10.0
X= 8.57         Y= 8.57         Z=-10.0
X=-8.57         Y= 8.57         Z=-10.0
X=-8.57         Y=-8.57         Z=-10.0
X= 8.57         Y=-8.57         Z=-10.0
X=0.0           Y= 10.71        Z=-10.0
X=0.0           Y=-10.71        Z=-10.0
X=-16.00        Y=0.0           Z=0.0
X= 16.00        Y=0.0           Z=0.0
X= 9.60         Y= 9.60         Z=0.0
X=-9.60         Y= 9.60         Z=0.0
X=-9.60         Y=-9.60         Z=0.0
X= 9.60         Y=-9.60         Z=0.0
X=0.0           Y= 12.00        Z=0.0
X=0.0           Y=-12.00        Z=0.0
X=-17.32        Y=0.0           Z=10.0
X= 17.32        Y=0.0           Z=10.0
X= 10.39        Y= 10.39        Z=10.0
X=-10.39        Y= 10.39        Z=10.0
X=-10.39        Y=-10.39        Z=10.0
X= 10.39        Y=-10.39        Z=10.0
```

X=0.0	Y= 12.99	Z=10.0
X=0.0	Y=-12.99	Z=10.0
X= 18.33	Y=0.0	Z=20.0
X=-18.33	Y=0.0	Z=20.0
X= 11.00	Y= 11.00	Z=20.0
X=-11.00	Y= 11.00	Z=20.0
X=-11.00	Y=-11.00	Z=20.0
X= 11.00	Y=-11.00	Z=20.0
X=0.0	Y= 1[illegible].75	Z=20.0
X=0.0	Y=-[illegible].75	Z=20.0
X= 19.08	Y=0.0	Z=30.0
X=-19.08	Y=0.0	Z=30.0
X= 11.45	Y= 11.45	Z=30.0
X=-11.45	Y= 11.45	Z=30.0
X=-11.45	Y=-11.45	Z=30.0
X= 11.45	Y=-11.45	Z=30.0
X=0.0	Y= 14.31	Z=30.0
X=0.0	Y=-14.31	Z=30.0
X= 19.60	Y=0.0	Z=40.0
X=-19.60	Y=0.0	Z=40.0
X= 11.76	Y= 11.76	Z=40.0
X=-11.76	Y= 11.76	Z=40.0
X=-11.76	Y=-11.76	Z=40.0
X= 11.76	Y=-11.76	Z=40.0
X=0.0	Y= 14.70	Z=40.0
X=0.0	Y=-14.70	Z=40.0
X= 19.90	Y=0.0	Z=50.0
X=-19.90	Y=0.0	Z=50.0
X= 11.94	Y= 11.94	Z=50.0
X=-11.94	Y= 11.94	Z=50.0
X=-11.94	Y=-11.94	Z=50.0
X= 11.94	Y=-11.94	Z=50.0
X=0.0	Y= 14.93	Z=50.0
X=0.0	Y=-14.93	Z=50.0
X=-20.0	Y=0.0	Z=60.0
X= 20.0	Y=0.0	Z=60.0
X= 12.00	Y= 12.00	Z=60.0
X=-12.00	Y= 12.00	Z=60.0
X=-12.00	Y=-12.00	Z=60.0
X= 12.00	Y=-12.00	Z=60.0
X=0.0	Y= 15.00	Z=60.0
X=0.0	Y=-15.00	Z=60.0
9999.		

```
CHECKOUT AIRCRAFT  FUSELAGE  F.S. 60 TO F.S. 210
DATA PUNCH INDICATOR=1.   DATA ID=8.    CARD PUNCH UNIT= 6.
X= 20.0          Y=0.0          Z=60.0
X=-20.0          Y=0.0          Z=60.0
X= 12.00         Y= 12.00       Z=60.0
X=-12.00         Y= 12.00       Z=60.0
X=-12.00         Y=-12.00       Z=60.0
X= 12.00         Y=-12.00       Z=60.0
X=0.0            Y= 15.00       Z=60.0
X=0.0            Y=-15.00       Z=60.0
X= 20.0          Y=0.0          Z=150.0
X=-20.0          Y=0.0          Z=150.0
X= 12.00         Y= 12.00       Z=150.0
X=-12.00         Y= 12.00       Z=150.0
X=-12.00         Y=-12.00       Z=150.0
X= 12.00         Y=-12.00       Z=150.0
X=0.0            Y= 15.00       Z=150.0
X=0.0            Y=-15.00       Z=150.0
X= 20.00         Y=0.0          Z=210.0
X=-20.00         Y=0.0          Z=210.0
X= 12.00         Y= 12.00       Z=210.0
X=-12.00         Y= 12.00       Z=210.0
X=-12.00         Y=-12.00       Z=210.0
X= 12.00         Y=-12.00       Z=210.0
X=0.0            Y= 15.00       Z=210.0
X=0.0            Y=-15.00       Z=210.0
  9999.
```

```
CHECKOUT AIRCRAFT FUSELAGE  F.S. 210 TO F.S. 285
DATA PUNCH INDICATOR=1.   DATA ID=C.    CARD PUNCH UNIT= 6.
X= 20.00          Y=0.0            Z=210.0
X=-20.00          Y=0.0            Z=210.0
X= 12.00          Y= 12.00         Z=210.0
X=-12.00          Y= 12.00         Z=210.0
X=-12.00          Y=-12.00         Z=210.0
X= 12.00          Y=-12.00         Z=210.0
X= 0.0            Y= 15.00         Z=210.0
X= 0.0            Y=-15.00         Z=210.0
X= 19.82          Y=0.0            Z=220.0
X=-19.82          Y=0.0            Z=220.0
X= 11.89          Y= 11.89         Z=220.0
X=-11.89          Y= 11.89         Z=220.0
X=-11.89          Y=-11.89         Z=220.0
X= 11.89          Y=-11.89         Z=220.0
X=0.0             Y= 14.87         Z=220.0
X=0.0             Y=-14.87         Z=220.0
X= 19.28          Y=0.0            Z=230.0
X=-19.28          Y=0.0            Z=230.0
X= 11.57          Y= 11.57         Z=230.0
X=-11.57          Y= 11.57         Z=230.0
X=-11.57          Y=-11.57         Z=230.0
X= 11.57          Y=-11.57         Z=230.0
X=0.0             Y= 14.46         Z=230.0
X=0.0             Y=-14.46         Z=230.0
X= 18.33          Y=0.0            Z=240.0
X=-18.33          Y=0.0            Z=240.0
X= 11.00          Y= 11.00         Z=240.0
X=-11.00          Y= 11.00         Z=240.0
X=-11.00          Y=-11.00         Z=240.0
X= 11.00          Y=-11.00         Z=240.0
X=0.0             Y= 13.75         Z=240.0
X=0.0             Y=-13.75         Z=240.0
X= 16.92          Y=0.0            Z=250.0
X=-16.92          Y=0.0            Z=250.0
X= 10.15          Y= 10.15         Z=250.0
X=-10.15          Y= 10.15         Z=250.0
X=-10.15          Y=-10.15         Z=250.0
X= 10.15          Y=-10.15         Z=250.0
X=0.0             Y= 12.69         Z=250.0
```

X=0.0	Y=-12.69	Z=250.0
X= 14.91	Y=0.0	Z=260.0
X=-14.91	Y=0.0	Z=260.0
X= 8.94	Y= 8.94	Z=260.0
X=-8.94	Y= 8.94	Z=260.0
X=-8.94	Y=-8.94	Z=260.0
X= 8.94	Y=-8.94	Z=260.0
X=0.0	Y= 11.18	Z=260.0
X=0.0	Y=-11.18	Z=260.0
X= 12.00	Y=0.0	Z=270.0
X=-12.00	Y=0.0	Z=270.0
X= 7.20	Y= 7.20	Z=270.0
X=-7.20	Y= 7.20	Z=270.0
X=-7.20	Y=-7.20	Z=270.0
X= 7.20	Y=-7.20	Z=270.0
X=0.0	Y= 9.00	Z=270.0
X=0.0	Y=-9.00	Z=270.0
X= 7.18	Y=0.0	Z=280.0
X=-7.18	Y=0.0	Z=280.0
X= 4.31	Y= 4.31	Z=280.0
X=-4.31	Y= 4.31	Z=280.0
X=-4.31	Y=-4.31	Z=280.0
X= 4.31	Y=-4.31	Z=280.0
X=0.0	Y= 5.39	Z=280.0
X=0.0	Y=-5.39	Z=280.0
X=0.0	Y=0.0	Z=285.0
9999.		

CHECKOUT AIRCRAFT NACELLE F.S. 60 TO F.S. 180

DATA PUNCH INDICATOR=1. DATA ID=D. CARD PUNCH UNIT= 6.

X= 12.00	Y=0.0	Z=60.0
X=-12.00	Y=0.0	Z=60.0
X= 6.66	Y= 21.66	Z=60.0
X=-6.66	Y= 21.66	Z=60.0
X=0.0	Y=23.00	Z=60.0
X= 12.99	Y=0.0	Z=70.0
X=-12.99	Y=0.0	Z=70.0
X= 7.21	Y=22.21	Z=70.0
X=-7.21	Y=22.21	Z=70.0
X=0.0	Y=23.66	Z=70.0
X= 13.75	Y=0.0	Z=80.0
X=-13.75	Y=0.0	Z=80.0
X= 7.63	Y=22.63	Z=80.0
X=-7.63	Y=22.63	Z=80.0
X=0.0	Y=24.17	Z=80.0
X= 14.31	Y=0.0	Z=90.0
X=-14.31	Y=0.0	Z=90.0
X= 7.94	Y=22.94	Z=90.0
X=-7.94	Y=22.94	Z=90.0
X=0.0	Y=24.54	Z=90.0
X= 14.70	Y=0.0	Z=100.0
X=-14.70	Y=0.0	Z=100.0
X= 8.15	Y=23.15	Z=100.0
X=-8.15	Y=23.15	Z=100.0
X=0.0	Y=24.80	Z=100.0
X= 14.93	Y=0.0	Z=110.0
X=-14.93	Y=0.0	Z=110.0
X= 8.28	Y=23.28	Z=110.0
X=-8.28	Y=23.28	Z=110.0
X=0.0	Y=24.95	Z=110.0
X= 15.00	Y=0.0	Z=120.0
X=-15.00	Y=0.0	Z=120.0
X= 8.32	Y=23.32	Z=120.0
X=-8.32	Y=23.32	Z=120.0
X=0.0	Y=25.00	Z=120.0
X= 14.93	Y=0.0	Z=130.0
X=-14.93	Y=0.0	Z=130.0
X= 8.28	Y=23.28	Z=130.0
X=-8.28	Y=23.28	Z=130.0

```
X=0.0          Y=24.95     Z=130.0
X= 14.7C       Y=0.0       Z=140.0
X=-14.7C       Y=0.0       Z=140.0
X= 8.15        Y=23.15     Z=140.0
X=-8.15        Y=23.15     Z=140.0
X=0.0          Y=24.80     Z=140.0
X= 14.31       Y=0.0       Z=150.0
X=-14.31       Y=0.0       Z=150.0
X= 7.94        Y=22.94     Z=150.0
X=-7.94        Y=22.94     Z=150.0
X=0.0          Y=24.54     Z=150.0
X= 13.75       Y=0.0       Z=160.0
X=-13.75       Y=0.0       Z=160.0
X= 7.63        Y=22.63     Z=160.0
X=-7.63        Y=22.63     Z=160.0
X=0.0          Y=24.17     Z=160.0
X= 12.99       Y=0.0       Z=170.0
X=-12.99       Y=0.0       Z=170.0
X= 7.21        Y=22.21     Z=170.0
X=-7.21        Y=22.21     Z=170.0
X=0.0          Y=23.66     Z=170.0
X= 12.0C       Y=0.0       Z=180.0
X=-12.00       Y=0.0       Z=180.0
X= 6.66        Y=21.66     Z=180.0
X=-6.66        Y=21.66     Z=180.0
X=0.0          Y=23.00     Z=180.0
   9999.
```

3.3. Sample Output Data

```
CHECKOUT AIRCRAFT  FUSELAGE  F.S. -40 TO F.S. 60

COEFFS. FOR SURFACE FIT
C1..C5   1.000000E+00  1.777497E+00  3.996930E-02  3.926511E-17  0.
C6.C10   0.            0.           -9.059765E-16 -2.399488E+00 -2.560075E+02



QUADRIC INVARIANTS   A= -2.842214E+01   D=  7.104531E-02   I=  2.817466E+00
                     J=  1.888511E+00   A PRIME= -4.993935E+02

CENTER OF CENTRAL QUADRIC
              X0=  0.             Y0= -8.164493E-16   Z0=  6.003326E+01

*****ELLIPSOID*****
WITH EIGENVALUES K1=  1.777E+00  K2=  1.000E+00  K3=  3.997E-02
KODE= 1

THE ROTATION MATRIX IS
   1.000000E+00    0.               0.
   0.              1.000000E+00    -2.259827E-17
   0.              2.259827E-17     1.000000E+00

DET=    1.000000E+00

COEFFS. FOR TRANSFORMED QUADRIC SURFACE
   1.0000E+00     1.7775E+00     3.9969E-02    -2.4835E-26     0.
   0.             0.            -2.7216E-29    -2.8422E-14    -4.0006E+02

COEFFS. FOR INVERSE TRANSFORMATION
   1.0000E+00     1.7775E+00     3.9969E-02     3.9265E-17     0.
   0.             0.            -9.0598E-16    -2.3995E+00    -2.5601E+02
```

CHECKOUT AIRCRAFT FUSELAGE F.S. -40 TO F.S. 60

ZI	XI	XC	DX	YI	YC	DY
-30.000	-8.720	-8.722	1.5736E-03	0.000	-.000	2.1983E-19
-30.000	8.720	8.722	-1.5736E-03	0.000	-.000	2.1983E-19
-30.000	5.230	5.220	9.5758E-03	5.250	5.240	9.6124E-03
-30.000	-5.230	-5.220	-9.5758E-03	5.250	5.240	9.6124E-03
-30.000	-5.230	-5.220	-9.5758E-03	-5.250	-5.240	-9.6124E-03
-30.000	5.230	5.220	9.5758E-03	-5.250	-5.240	-9.6124E-03
-30.000	0.000	0.000	0.	6.540	6.542	-1.6972E-03
-30.000	0.000	0.000	0.	-6.540	-6.542	1.6972E-03
-20.000	-12.000	-12.002	1.6769E-03	0.000	-.000	1.3865E-19
-20.000	12.000	12.002	-1.6769E-03	0.000	-.000	1.3865E-19
-20.000	7.200	7.201	-1.3704E-03	7.200	7.201	-1.3704E-03
-20.000	-7.200	-7.201	1.3704E-03	7.200	7.201	-1.3704E-03
-20.000	-7.200	-7.201	1.3704E-03	-7.200	-7.201	1.3704E-03
-20.000	7.200	7.201	-1.3704E-03	-7.200	-7.201	1.3704E-03
-20.000	0.000	0.000	0.	9.000	9.002	-1.9691E-03
-20.000	0.000	0.000	0.	-9.000	-9.002	1.9691E-03
-10.000	-14.280	-14.284	3.5846E-03	0.000	-.000	1.9233E-19
-10.000	14.280	14.284	-3.5846E-03	0.000	-.000	1.9233E-19
-10.000	8.570	8.571	-5.8425E-04	8.570	8.571	-5.8425E-04
-10.000	-8.570	-8.571	5.8425E-04	8.570	8.571	-5.8425E-04
-10.000	-8.570	-8.571	5.8425E-04	-8.570	-8.571	5.8425E-04
-10.000	8.570	8.571	-5.8425E-04	-8.570	-8.571	5.8425E-04
-10.000	0.000	0.000	0.	10.710	10.714	-3.5351E-03
-10.000	0.000	0.000	0.	-10.710	-10.714	3.5351E-03
0.000	-16.000	-16.000	2.3349E-04	0.000	-.000	7.8822E-21
0.000	16.000	16.000	-2.3349E-04	0.000	-.000	7.8822E-21
0.000	9.600	9.601	-6.2566E-04	9.600	9.601	-6.2566E-04
0.000	-9.600	-9.601	6.2566E-04	9.600	9.601	-6.2566E-04
0.000	-9.600	-9.601	6.2566E-04	-9.600	-9.601	6.2566E-04
0.000	9.600	9.601	-6.2566E-04	-9.600	-9.601	6.2566E-04
0.000	0.000	0.000	0.	12.000	12.001	-1.1235E-03
0.000	0.000	0.000	0.	-12.000	-12.001	1.1235E-03
10.000	-17.320	-17.321	5.1653E-04	0.000	-.000	9.3700E-21
10.000	17.320	17.321	-5.1653E-04	0.000	-.000	9.3700E-21
10.000	10.390	10.393	-2.8355E-03	10.390	10.393	-2.8355E-03
10.000	-10.390	-10.393	2.8355E-03	10.390	10.393	-2.8355E-03
10.000	-10.390	-10.393	2.8355E-03	-10.390	-10.393	2.8355E-03
10.000	10.390	10.393	-2.8355E-03	-10.390	-10.393	2.8355E-03
10.000	0.000	0.000	0.	12.990	12.991	-1.4141E-03

CHECKOUT AIRCRAFT FUSFLAGE F.S. -40 TO F.S. 60

ZI	XI	XC	DX	YI	YC	DY
10.000	0.000	0.000	0.	-12.990	-12.991	1.4141E-03
20.000	18.330	18.330	-2.8242E-04	0.000	-.000	1.3589E-21
20.000	-18.330	-18.330	2.8242E-04	0.000	-.000	1.3589E-21
20.000	11.000	10.999	1.2743E-03	11.000	10.999	1.2743E-03
20.000	-11.000	-10.999	-1.2743E-03	11.000	10.999	1.2743E-03
20.000	-11.000	-10.999	-1.2743E-03	-11.000	-10.999	-1.2743E-03
20.000	11.000	10.999	1.2743E-03	-11.000	-10.999	-1.2743E-03
20.000	0.000	0.000	0.	13.750	13.749	1.2017E-03
20.000	0.000	0.000	0.	-13.750	-13.749	-1.2017E-03
30.000	19.080	19.079	1.1019E-03	0.000	-.000	7.9545E-21
30.000	-19.080	-19.079	-1.1019E-03	0.000	-.000	7.9545E-21
30.000	11.450	11.448	2.0821E-03	11.450	11.448	2.0821E-03
30.000	-11.450	-11.448	-2.0821E-03	11.450	11.448	2.0821E-03
30.000	-11.450	-11.448	-2.0821E-03	-11.450	-11.448	-2.0821E-03
30.000	11.450	11.448	2.0821E-03	-11.450	-11.448	-2.0821E-03
30.000	0.000	0.000	0.	14.310	14.310	-3.0451E-04
30.000	0.000	0.000	0.	-14.310	-14.310	3.0451E-04
40.000	19.600	19.596	3.6842E-03	0.000	-.000	6.8369E-20
40.000	-19.600	-19.596	-3.6842E-03	0.000	-.000	6.8369E-20
40.000	11.760	11.758	1.6158E-03	11.760	11.758	1.6158E-03
40.000	-11.760	-11.758	-1.6158E-03	11.760	11.758	1.6158E-03
40.000	-11.760	-11.758	-1.6158E-03	-11.760	-11.758	-1.6158E-03
40.000	11.760	11.758	1.6158E-03	-11.760	-11.758	-1.6158E-03
40.000	0.000	0.000	0.	14.700	14.698	1.6015E-03
40.000	0.000	0.000	0.	-14.700	-14.698	-1.6015E-03
50.000	19.900	19.901	-5.7726E-04	0.000	.000	-1.7106E-20
50.000	-19.900	-19.901	5.7726E-04	0.000	.000	-1.7106E-20
50.000	11.940	11.941	-9.5028E-04	11.940	11.941	-9.5028E-04
50.000	-11.940	-11.941	9.5028E-04	11.940	11.941	-9.5028E-04
50.000	-11.940	-11.941	9.5028E-04	-11.940	-11.941	9.5028E-04
50.000	11.940	11.941	-9.5028E-04	-11.940	-11.941	9.5028E-04
50.000	0.000	0.000	0.	14.930	14.927	3.3875E-03
50.000	0.000	0.000	0.	-14.930	-14.927	-3.3875E-03
60.000	-20.000	-20.001	1.4123E-03	0.000	.000	-5.7602E-20
60.000	20.000	20.001	-1.4123E-03	0.000	.000	-5.7602E-20
60.000	12.000	12.001	-1.4544E-03	12.000	12.001	-1.4544E-03
60.000	-12.000	-12.001	1.4544E-03	12.000	12.001	-1.4544E-03
60.000	-12.000	-12.001	1.4544E-03	-12.000	-12.001	1.4544E-03
60.000	12.000	12.001	-1.4544E-03	-12.000	-12.001	1.4544E-03
60.000	0.000	0.000	0.	15.000	15.002	-2.2448E-03
60.000	0.000	0.000	0.	-15.000	-15.002	2.2448E-03

```
C
C.....CHECKOUT AIRCRAFT  FUSELAGE  F.S. -40 TO F.S. 60
      DATA KA,CA/1,     1.0000000E+00,     1.7774968E+00,
     1                  3.9969301E-02,    -4.0005654E+02/
      DATA TA/     0.              ,  -8.1644935E-16,    6.0033263E+01/
      DATA RA/     1.0000000E+00,     0.              ,  0.              ,
     1             0.              ,   1.0000000E+00,   -2.2598266E-17,
     2             0.              ,   2.2598266E-17,    1.0000000E+00/
```

```
CHECKOUT AIRCRAFT  FUSELAGE  F.S. 60 TO F.S. 210

COEFFS. FOR SURFACE FIT
C1..C5   1.000000E+00  1.777778E+00 -7.636428E-15  0.             0.
C6.C10   0.            0.            0.            1.023182E-12 -4.000000E+02



QUADRIC INVARIANTS   A=  5.430349E-12    D= -1.357587E-14    I=  2.777778E+00
                     J=  1.777778E+00    A PRIME= -7.111111E+02

THE SURFACE IS NOT A CENTRAL QUADRIC

*****ELLIPTIC CYLINDER*****
WITH EIGENVALUES K1=  1.778E+00  K2=  1.000E+00  K3=  1.075E-09
KODE= 7

DIAMETRAL PLANES
   0.                1.000000E+00    0.
   1.000000E+00      0.              0.

VERTEX OF NON-CENTRAL QUADRIC
              X0=  0.                Y0= -8.164493E-16   Z0=  0.

THE ROTATION MATRIX IS
   0.                1.000000E+00    0.
   1.000000E+00      0.              0.
   0.                0.             -1.000000E+00

DET=    1.000000E+00

COEFFS. FOR TRANSFORMED QUADRIC SURFACE
   1.7778E+00    1.0000E+00    -7.6364E-15    0.             0.
   0.           -1.4515E-15     0.           -1.0232E-12    -4.0000E+02

COEFFS. FOR INVERSE TRANSFORMATION
   1.0000E+00    1.7778E+00    -7.6364E-15    0.             0.
   0.            0.             0.            1.0232E-12    -4.0000E+02
```

CHECKOUT AIRCRAFT FUSELAGE F.S. 60 TO F.S. 210

ZI	XI	XC	DX	YI	YC	DY
60.000	20.000	20.000	6.8212E-13	0.000	-.000	2.8399E-29
60.000	-20.000	-20.000	-6.8212E-13	0.000	-.000	2.8399E-29
60.000	12.000	12.000	2.8422E-13	12.000	12.000	3.4106E-13
60.000	-12.000	-12.000	-2.8422F-13	12.000	12.000	3.4106E-13
60.000	-12.000	-12.000	-2.2737E-13	-12.000	-12.000	-2.8422E-13
60.000	12.000	12.000	2.2737E-13	-12.000	-12.000	-2.8422E-13
60.000	0.000	0.000	0.	15.000	15.000	2.8422E-13
60.000	0.000	0.000	0.	-15.000	-15.000	-2.2737E-13
150.000	20.000	20.000	5.2296E-12	0.000	-.000	2.1457E-28
150.000	-20.000	-20.000	-5.2296E-12	0.000	-.000	2.1457E-28
150.000	12.000	12.000	3.0695E-12	12.000	12.000	3.1264E-12
150.000	-12.000	-12.000	-3.0695E-12	12.000	12.000	3.1264E-12
150.000	-12.000	-12.000	-3.0127E-12	-12.000	-12.000	-3.0695E-12
150.000	12.000	12.000	3.0127E-12	-12.000	-12.000	-3.0695E-12
150.000	0.000	0.000	0.	15.000	15.000	3.7517E-12
150.000	0.000	0.000	0.	-15.000	-15.000	-3.6948E-12
210.000	20.000	20.000	8.2991E-12	0.000	-.000	3.3763E-28
210.000	-20.000	-20.000	-8.2991E-12	0.000	-.000	3.3763E-28
210.000	12.000	12.000	4.9454E-12	12.000	12.000	5.0022E-12
210.000	-12.000	-12.000	-4.9454E-12	12.000	12.000	5.0022E-12
210.000	-12.000	-12.000	-4.7748E-12	-12.000	-12.000	-4.8317E-12
210.000	12.000	12.000	4.7748E-12	-12.000	-12.000	-4.8317E-12
210.000	0.000	0.000	0.	15.000	15.000	6.0254E-12
210.000	0.000	0.000	0.	-15.000	-15.000	-5.9686E-12

```
C
C.....CHECKOUT AIRCRAFT  FUSELAGE  F.S. 60 TO F.S. 210
      DATA KB,CB/7,    1.7777778E+00,    1.0000000E+00,
     1                -1.0231815E-12,   -4.0000000E+02/
      DATA TB/    0.            ,   -8.1644935E-16,    0.            /
      DATA RB/    0.            ,    1.0000000E+00,    0.            ,
     1            1.0000000E+00,    0.            ,    0.            ,
     2            0.            ,    0.            ,   -1.0000000E+00/
```

```
CHECKOUT AIRCRAFT FUSELAGE   F.S. 210 TO F.S. 285

COEFFS. FOR SURFACE FIT
C1..C5   1.000000E+00   1.777555E+00   7.113967E-02 -1.009988E-13   0.
C6.C10   0.             0.             2.383110E-11 -1.494027E+01   2.737664E+03



QUADRIC INVARIANTS    A= -5.058082E+01    D=  1.264547E-01    I=  2.848694E+00
                      J=  1.975149E+00    A PRIME=  4.787437E+03

CENTER OF CENTRAL QUADRIC
              X0=  0.                  Y0= -1.473944E-12    Z0=  2.100132E+02

*****ELLIPSOID*****
WITH EIGENVALUES K1=  1.778E+00   K2=  1.000E+00   K3=  7.114E-02
KODE= 1

THE ROTATION MATRIX IS
   1.000000E+00    0.                 0.
   0.              1.000000E+00       5.918770E-14
   0.             -5.918770E-14       1.000000E+00

DET=     1.000000E+00

COEFFS. FOR TRANSFORMED QUADRIC SURFACE
   1.0000E+00      1.7776E+00      7.1140E-02      6.6240E-23      0.
   0.              0.              1.0017E-25     -1.7053E-13     -3.9999E+02

COEFFS. FOR INVERSE TRANSFORMATION
   1.0000E+00      1.7776E+00      7.1140E-02     -1.0100E-13      0.
   0.              0.              2.3831E-11     -1.4940E+01      2.7377E+03
```

CHECKOUT AIRCRAFT FUSELAGE F.S. 210 TO F.S. 285

ZI	XI	XC	DX	YI	YC	DY
210.000	20.000	20.000	2.0766E-04	0.000	-.000	1.5312E-17
210.000	-20.000	-20.000	-2.0766E-04	0.000	-.000	1.5312E-17
210.000	12.000	12.000	-3.5721E-04	12.000	12.000	-3.5721E-04
210.000	-12.000	-12.000	3.5721E-04	12.000	12.000	-3.5721E-04
210.000	-12.000	-12.000	3.5721E-04	-12.000	-12.000	3.5721E-04
210.000	12.000	12.000	-3.5721E-04	-12.000	-12.000	3.5721E-04
210.000	0.000	0.000	0.	15.000	15.001	-7.8532E-04
210.000	0.000	0.000	0.	-15.000	-15.001	7.8532E-04
220.000	19.820	19.822	-1.6189E-03	0.000	.000	-7.2112E-17
220.000	-19.820	-19.822	1.6189E-03	0.000	.000	-7.2112E-17
220.000	11.890	11.893	-3.4489E-03	11.890	11.893	-3.4489E-03
220.000	-11.890	-11.893	3.4489E-03	11.890	11.893	-3.4489E-03
220.000	-11.890	-11.893	3.4489E-03	-11.890	-11.893	3.4489E-03
220.000	11.890	11.893	-3.4489E-03	-11.890	-11.893	3.4489E-03
220.000	0.000	0.000	0.	14.870	14.867	2.8531E-03
220.000	0.000	0.000	0.	-14.870	-14.867	-2.8531E-03
230.000	19.280	19.276	3.7575E-03	0.000	-.000	5.6708E-17
230.000	-19.280	-19.276	-3.7575E-03	0.000	-.000	5.6708E-17
230.000	11.570	11.566	3.7901E-03	11.570	11.566	3.7901E-03
230.000	-11.570	-11.566	-3.7901E-03	11.570	11.566	3.7901E-03
230.000	-11.570	-11.566	-3.7901E-03	-11.570	-11.566	-3.7901E-03
230.000	11.570	11.566	3.7901E-03	-11.570	-11.566	-3.7901E-03
230.000	0.000	0.000	0.	14.460	14.458	1.9111E-03
230.000	0.000	0.000	0.	-14.460	-14.458	-1.9111E-03
240.000	18.330	18.331	-9.1750E-04	0.000	-.000	1.5061E-17
240.000	-18.330	-18.331	9.1750E-04	0.000	-.000	1.5061E-17
240.000	11.000	10.999	1.0079E-03	11.000	10.999	1.0079E-03
240.000	-11.000	-10.999	-1.0079E-03	11.000	10.999	1.0079E-03
240.000	-11.000	-10.999	-1.0079E-03	-11.000	-10.999	-1.0079E-03
240.000	11.000	10.999	1.0079E-03	-11.000	-10.999	-1.0079E-03
240.000	0.000	0.000	0.	13.750	13.749	9.4933E-04
240.000	0.000	0.000	0.	-13.750	-13.749	-9.4933E-04
250.000	16.920	16.919	1.2644E-03	0.000	.000	-6.6716E-17
250.000	-16.920	-16.919	-1.2644E-03	0.000	.000	-6.6716E-17
250.000	10.150	10.152	-1.6490E-03	10.150	10.152	-1.6490E-03
250.000	-10.150	-10.152	1.6490E-03	10.150	10.152	-1.6490E-03
250.000	-10.150	-10.152	1.6490E-03	-10.150	-10.152	1.6490E-03
250.000	10.150	10.152	-1.6490E-03	-10.150	-10.152	1.6490E-03
250.000	0.000	0.000	0.	12.690	12.690	1.5219E-04
250.000	0.000	0.000	0.	-12.690	-12.690	-1.5219E-04

CHECKOUT AIRCRAFT FUSELAGE F.S. 210 TO F.S. 285

ZI	XI	XC	DX	YI	YC	DY
260.000	14.910	14.908	2.3926E-03	0.000	.000	-2.3824E-16
260.000	-14.910	-14.908	-2.3926E-03	0.000	.000	-2.3824E-16
260.000	8.940	8.945	-4.9236E-03	8.940	8.945	-4.9236E-03
260.000	-8.940	-8.945	4.9236E-03	8.940	8.945	-4.9236E-03
260.000	-8.940	-8.945	4.9236E-03	-8.940	-8.945	4.9236E-03
260.000	8.940	8.945	-4.9236E-03	-8.940	-8.945	4.9236E-03
260.000	0.000	0.000	0.	11.180	11.181	-1.4070E-03
260.000	0.000	0.000	0.	-11.180	-11.181	1.4070E-03
270.000	12.000	12.000	-8.2046E-05	0.000	-.000	1.4195E-17
270.000	-12.000	-12.000	8.2046E-05	0.000	-.000	1.4195E-17
270.000	7.200	7.200	-3.3832E-04	7.200	7.200	-3.3832E-04
270.000	-7.200	-7.200	3.3832E-04	7.200	7.200	-3.3832E-04
270.000	-7.200	-7.200	3.3832E-04	-7.200	-7.200	3.3832E-04
270.000	7.200	7.200	-3.3832E-04	-7.200	-7.200	3.3832E-04
270.000	0.000	0.000	0.	9.000	9.001	-6.2618E-04
270.000	0.000	0.000	0.	-9.000	-9.001	6.2618E-04
280.000	7.180	7.179	9.1619E-04	0.000	.000	-3.4050E-16
280.000	-7.180	-7.179	-9.1619E-04	0.000	.000	-3.4050E-16
280.000	4.310	4.308	2.3768E-03	4.310	4.308	2.3768E-03
280.000	-4.310	-4.308	-2.3768E-03	4.310	4.308	2.3768E-03
280.000	-4.310	-4.308	-2.3768E-03	-4.310	-4.308	-2.3768E-03
280.000	4.310	4.308	2.3768E-03	-4.310	-4.308	-2.3768E-03
280.000	0.000	0.000	0.	5.390	5.385	5.3493E-03
280.000	0.000	0.000	0.	-5.390	-5.385	-5.3493E-03

```
C
C.....CHECKOUT AIRCRAFT FUSELAGE  F.S. 210 TO F.S. 285
      DATA KC,CC/1,     1.0000000E+00,    1.7775547E+00,
     1                  7.1139672E-02,   -3.9999171E+02/
      DATA TC/     0.            ,  -1.4739442E-12,    2.1001325E+02/
      DATA RC/     1.0000000E+00,    0.            ,    0.            ,
     1             0.            ,    1.0000000E+00,    5.9187698E-14,
     2             0.            ,   -5.9187698E-14,    1.0000000E+00/
```

```
CHECKOUT AIRCRAFT NACELLE  F.S. 60 TO F.S. 180

COEFFS. FOR SURFACE FIT
C1..C5  1.000000E+00  1.332633E+00  2.272122E-02  4.815592E-14  0.
C6.C10  0.            0.            -1.218090E+01 -2.726546E+00  1.018284E+02



QUADRIC INVARIANTS     A= -1.019485E+01    D=  3.027905E-02    I=  2.355354E+00
                       J=  1.385633E+00    A PRIME= -2.795939E+01

CENTER OF CENTRAL QUADRIC
              X0=  0.              Y0=  9.140475E+00   Z0=  1.200000E+02

*****ELLIPSOID*****
WITH EIGENVALUES K1=  1.333E+00  K2=  1.000E+00  K3=  2.272E-02
KODE= 1

THE ROTATION MATRIX IS
   1.000000E+00      0.                0.
   0.                1.000000E+00     -3.676272E-14
   0.                3.676272E-14      1.000000E+00

DET=     1.000000E+00

COEFFS. FOR TRANSFORMED QUADRIC SURFACE
   1.0000E+00     1.3326E+00     2.2721E-02    -1.7187E-23     0.
   0.             0.            -1.5708E-13    -5.6843E-14    -3.3670E+02

COEFFS. FOR INVERSE TRANSFORMATION
   1.0000E+00     1.3326E+00     2.2721E-02     4.8156E-14     0.
   0.             0.            -1.2181E+01    -2.7265E+00     1.0183E+02
```

CHECKOUT AIRCRAFT NACELLE F.S. 60 TO F.S. 180

ZI	XI	XC	DX	YI	YC	DY
60.000	12.000	11.990	1.0325E-02	0.000	.008	-7.8649E-03
60.000	-12.000	-11.990	-1.0325E-02	0.000	.008	-7.8649E-03
60.000	6.660	6.682	-2.1918E-02	21.660	21.701	-4.1201E-02
60.000	-6.660	-6.682	2.1918E-02	21.660	21.701	-4.1201E-02
60.000	0.000	0.000	0.	23.000	22.971	2.9288E-02
70.000	12.990	12.986	4.3136E-03	0.000	.003	-3.0353E-03
70.000	-12.990	-12.986	-4.3136E-03	0.000	.003	-3.0353E-03
70.000	7.210	7.214	-3.5944E-03	22.210	22.217	-6.5156E-03
70.000	-7.210	-7.214	3.5944E-03	22.210	22.217	-6.5156E-03
70.000	0.000	0.000	0.	23.660	23.633	2.7103E-02
80.000	13.750	13.749	1.3569E-03	0.000	.001	-9.0199E-04
80.000	-13.750	-13.749	-1.3569E-03	0.000	.001	-9.0199E-04
80.000	7.630	7.625	4.6967E-03	22.630	22.622	8.3035E-03
80.000	-7.630	-7.625	-4.6967E-03	22.630	22.622	8.3035E-03
80.000	0.000	0.000	0.	24.170	24.153	1.7024E-02
90.000	14.310	14.313	-2.9867E-03	0.000	-.002	1.9077E-03
90.000	-14.310	-14.313	2.9867E-03	0.000	-.002	1.9077E-03
90.000	7.940	7.933	7.0894E-03	22.940	22.928	1.2321E-02
90.000	-7.940	-7.933	-7.0894E-03	22.940	22.928	1.2321E-02
90.000	0.000	0.000	0.	24.540	24.545	-5.3474E-03
100.000	14.700	14.704	-4.0104E-03	0.000	-.002	2.4936E-03
100.000	-14.700	-14.704	4.0104E-03	0.000	-.002	2.4936E-03
100.000	8.150	8.145	4.5516E-03	23.150	23.142	7.8239E-03
100.000	-8.150	-8.145	-4.5516E-03	23.150	23.142	7.8239E-03
100.000	0.000	0.000	0.	24.800	24.820	-1.9602E-02
110.000	14.930	14.934	-4.0228E-03	0.000	-.002	2.4628E-03
110.000	-14.930	-14.934	4.0228E-03	0.000	-.002	2.4628E-03
110.000	8.280	8.273	6.9531E-03	23.280	23.268	1.1874E-02
110.000	-8.280	-8.273	-6.9531E-03	23.280	23.268	1.1874E-02
110.000	0.000	0.000	0.	24.950	24.982	-3.1876E-02
120.000	15.000	15.008	-7.9623E-03	0.000	-.005	4.8519E-03
120.000	-15.000	-15.008	7.9623E-03	0.000	-.005	4.8519E-03
120.000	8.320	8.314	5.7245E-03	23.320	23.310	9.7561E-03
120.000	-8.320	-8.314	-5.7245E-03	23.320	23.310	9.7561E-03
120.000	0.000	0.000	0.	25.000	25.036	-3.5599E-02
130.000	14.930	14.934	-4.0228E-03	0.000	-.002	2.4628E-03
130.000	-14.930	-14.934	4.0228E-03	0.000	-.002	2.4628E-03
130.000	8.280	8.273	6.9531E-03	23.280	23.268	1.1874E-02
130.000	-8.280	-8.273	-6.9531E-03	23.280	23.268	1.1874E-02
130.000	0.000	0.000	0.	24.950	24.982	-3.1876E-02

CHECKOUT AIRCRAFT NACELLE F.S. 60 TO F.S. 180

ZI	XI	XC	DX	YI	YC	DY
140.000	14.700	14.704	-4.0104E-03	0.000	-.002	2.4936E-03
140.000	-14.700	-14.704	4.0104E-03	0.000	-.002	2.4936E-03
140.000	8.150	8.145	4.5516E-03	23.150	23.142	7.8239E-03
140.000	-8.150	-8.145	-4.5516E-03	23.150	23.142	7.8239E-03
140.000	0.000	0.000	0.	24.800	24.820	-1.9602E-02
150.000	14.310	14.313	-2.9867E-03	0.000	-.002	1.9077E-03
150.000	-14.310	-14.313	2.9867E-03	0.000	-.002	1.9077E-03
150.000	7.940	7.933	7.0894E-03	22.940	22.928	1.2321E-02
150.000	-7.940	-7.933	-7.0894E-03	22.940	22.928	1.2321E-02
150.000	0.000	0.000	0.	24.540	24.545	-5.3474E-03
160.000	13.750	13.749	1.3569E-03	0.000	.001	-9.0199E-04
160.000	-13.750	-13.749	-1.3569E-03	0.000	.001	-9.0199E-04
160.000	7.630	7.625	4.6967E-03	22.630	22.622	8.3035E-03
160.000	-7.630	-7.625	-4.6967E-03	22.630	22.622	8.3035E-03
160.000	0.000	0.000	0.	24.170	24.153	1.7024E-02
170.000	12.990	12.986	4.3136E-03	0.000	.003	-3.0353E-03
170.000	-12.990	-12.986	-4.3136E-03	0.000	.003	-3.0353E-03
170.000	7.210	7.214	-3.5944E-03	22.210	22.217	-6.5156E-03
170.000	-7.210	-7.214	3.5944E-03	22.210	22.217	-6.5156E-03
170.000	0.000	0.000	0.	23.660	23.633	2.7103E-02
180.000	12.000	11.990	1.0325E-02	0.000	.008	-7.8649E-03
180.000	-12.000	-11.990	-1.0325E-02	0.000	.008	-7.8649E-03
180.000	6.660	6.682	-2.1918E-02	21.660	21.701	-4.1201E-02
180.000	-6.660	-6.682	2.1918E-02	21.660	21.701	-4.1201E-02
180.000	0.000	0.000	0.	23.000	22.971	2.9288E-02

```
C
C.....CHECKOUT AIRCRAFT NACELLE  F.S. 60 TO F.S. 180
      DATA KD,CD/1,    1.0000000E+00,    1.3326331E+00,
     1                 2.2721220E-02,   -3.3669638E+02/
      DATA TD/      0.            ,    9.1404752E+00,    1.2000000E+02/
      DATA RD/      1.0000000E+00,    0.            ,    0.            ,
     1              0.            ,    1.0000000E+00,   -3.6762719E-14,
     2              0.            ,    3.6762719E-14,    1.0000000E+00/
```

3.4 Program Listings

This section contains listings of all routines used in the quadric surface fit program. The four surfaces used in the sample problem consumed 2.8 seconds of CDC 6600 computer time.

```
C     PROGRAM TO SURFACE FIT A GENERAL SECOND DEGREE SURFACE
C
C     INPUT DATA IS DEFINED AS
C                     HEAD.....JOB IDENTIFICATION (ALPA-NUMERIC)
C                     IPUNCH=0.....DO NOT PUNCH DATA CARDS
C                     IPUNCH=1.....PUNCH DATA CARDS
C                     ID.....IDENTIFICATION PUNCHED INTO DATA CARDS
C                     IPT.....OUTPUT UNIT USED TO PUNCH DATA CARDS
C                     X,Y,Z.....COORDINATES OF INPUT DATA
C
      DIMENSION X(500),Y(500),Z(500),C(10),HEAD(18),P1(3)
      DIMENSION DP1(3),DPO(3),ZREF(70),CP(10),RI(3,3)
      DIMENSION PO(3),R(3,3),PT(3),CL(4)
  999 READ (5,400) (HEAD(I),I=1,18)
  400 FORMAT (18A4)
      READ (5,501) IPUNCH,ID,IPT
  501 FORMAT (21X,I1,12X,A1,21X,I2)
      I=1
   99 CONTINUE
      READ (5,420) X(I),Y(I),Z(I)
  420 FORMAT (3(2X,E13.6))
      IF (X(I).GT. 1000.0) GO TO 10
      I=I+1
      GO TO 99
   10 CONTINUE
      NPTS=I-1
      CALL LSF2D (X,Y,Z,NPTS,C)
      WRITE (6,450) HEAD
  450 FORMAT (1H1,18A4/)
      WRITE (6,600)
  600 FORMAT (24H COEFFS. FOR SURFACE FIT)
      WRITE (6,460) (C(I),I=1,10)
  460 FORMAT (7H C1..C5   ,     5E14.6/7H C6.C10   ,    5E14.6///)
      CALL QDPAR (C,PT,R,KODE)
      CALL DET3X3 (R,DET)
      IF (DET.GT. 0.0) GO TO 371
      DO 372 I=1,3
  372 P1(I)=R(I,1)
      DO 373 I=1,3
      R(I,1)=R(I,2)
  373 R(I,2)=P1(I)
      CALL DET3X3 (R,DET)
  371 CONTINUE
      WRITE (6,642)
  642 FORMAT (23H0THE ROTATION MATRIX IS)
      DO 32 J=1,3
      WRITE (6,620) (R(J,K),K=1,3)
  620 FORMAT (3E16.6)
   32 CONTINUE
      WRITE (6,611) DET
  611 FORMAT (5H0DET=E16.6)
      CALL ROTRAN (C,R,PT,CP,2)
```

```
      WRITE (6,643)
643   FORMAT (40HOCOEFFS. FOR TRANSFORMED QUADRIC SURFACE)
      WRITE (6,610) (CP(J),J=1,10)
610   FORMAT (5E14.4)
      DO 33 I=1,3
      DO 33 J=1,3
33    RI(I,J)=R(J,I)
      CALL ROTRAN (CP,RI,PT,C,1)
      WRITE (6,645)
645   FORMAT (35HOCOEFFS. FOR INVERSE TRANSFORMATION)
      WRITE (6,610) (C(J),J=1,10)
      WRITE (6,453) HEAD
453   FORMAT (1H1,18A4//5X,2HZI, 9X,2HXI, 9X,2HXC,6X,2HDX,
     1   14X,2HYI, 9X,2HYC,6X,2HDY)
C
C     COMPARE THE FITTED WITH THE INPUT DATA
C
      CL(1)=CP(1)
      CL(2)=CP(2)
      CL(3)=CP(3)
      IF (KODE.GT. 4) CL(3)=CP(9)
      CL(4)=CP(10)
      IC=0
      DO 205 I=1,NPTS
      PO(1)=X(I)
      PO(2)=Y(I)
      PO(3)=Z(I)
      IC=IC+1
      IF (IC.LE. 40) GO TO 203
      WRITE (6,453) HEAD
      IC=1
203   CALL COMPAR (CL,PT,R,PO,KODE)
205   CONTINUE
      WRITE (6,1122)
1122  FORMAT (1H1)
      IF (IPUNCH.EQ. 0) GO TO 999
C
C     PUNCH OUT DATA CARDS
C
      IF (IPT.NE. 6) WRITE (IPT,701) HEAD
701   FORMAT (1HC/6HC.....18A4)
      IF (IPT.EQ. 6) WRITE (IPT,601) HEAD
601   FORMAT (2H C/7H C.....18A4)
      WRITE (IPT,700) ID,ID,KODE,CL
700   FORMAT (6X,6HDATA K,A1,2H,C,A1,1H/,I1,1H,E17.7,1H,E17.7,1H,/
     1   5X,1H1,13X,E17.7,1H,E17.7,1H/)
      WRITE (IPT,710) ID,PT
710   FORMAT (6X,6HDATA T,A1,1H/,E17.7,1H,E17.7,1H,E17.7,1H/)
      WRITE (IPT,720) ID,R(1,1),R(1,2),R(1,3)
720   FORMAT (6X,6HDATA R,A1,1H/,E17.7,1H,E17.7,1H,E17.7,1H,)
      WRITE (IPT,730) R(2,1),R(2,2),R(2,3)
730   FORMAT (5X,1H1,8X,E17.7,1H,E17.7,1H,E17.7,1H,)
      WRITE (IPT,740) R(3,1),R(3,2),R(3,3)
740   FORMAT (5X,1H2,8X,E17.7,1H,E17.7,1H,E17.7,1H/)
C
      WRITE (6,1122)
      GO TO 999
      END
```

```
      SUBROUTINE COMPAR (C,PT,R,PFI,KODE)
      DIMENSION C(4),PT(3),R(3,3),PFI(3),PO(3),DPO(3),PFC(3),P1(3),
     1   DP1(3)
C
      CALL FIXLOC (R,PT,PFI,PO)
      DEN=SQRT(PO(1)**2+PO(2)**2)
      IF (ABS(DEN).LE. 1.0E-6) GO TO 90
      TMP=1.0/DEN
      DO 10 I=1,2
   10 DPO(I)=PO(I)*TMP
      DPO(3)=0.0
      PO(1)=0.0
      PO(2)=0.0
      CALL QUADS (C,PO,DPO,KODE,1.0,P1,DP1,INT)
      IF (INT.EQ. 2) GO TO 90
      CALL LOCFIX (R,PT,P1,PFC)
      DX=PFI(1)-PFC(1)
      DY=PFI(2)-PFC(2)
      WRITE (6,600) PFI(3),PFI(1),PFC(1),DX,PFI(2),PFC(2),DY
  600 FORMAT (1X,F9.3,2X,F9.3,2X,F9.3,2X,E11.4,2X,F9.3,2X,
     1   F9.3,2X,E11.4)
   90 RETURN
      END
```

```
      SUBROUTINE CROUT(A,B,C,N)
C
C     SOLVE A SET OF LINEAR EQUATIONS USING CROUT REDUCTION METHOD
C
      DIMENSION A(10,11),B(10,11),C(10)
C     A IS GIVEN MATRIX,B IS AUXILIARY MATRIX
      NN=N+1
      DO 1 J=1,N
      DO 1 K=1,NN
 1    B(J,K)=A(J,K)
      DO 2 J=2,NN
 2    B(1,J)=A(1,J)/B(1,1)
      DO 5 J=2,N
      K=J-1
      DO 5 JJ=J,N
      DO 4 L=1,K
      B(JJ,J)=B(JJ,J)-B(L,J)*B(JJ,L)
 4    B(J,JJ+1)=B(J,JJ+1)-B(L,JJ+1)*B(J,L)
 5    B(J,JJ+1)=B(J,JJ+1)/B(J,J)
      DO 6 J=1,N
 6    C(J)=B(J,NN)
      DO 7 J=2,N
      K=NN-J
      M=J-1
      DO 7 L=1,M
      NNL=NN-L
 7    C(K)=C(K)-B(K,NNL)*C(NNL)
      RETURN
      END
```

```
      SUBROUTINE CUBIC (A,B,C,D,RT)
C
C     EQ. IN THE FORM    A*Y**3+B*Y**2+C*Y+D=0.0
C     IF A IS ZERO THE PROGRAM WILL SOLVE A QUADRATIC
C     OF THE FORM B*Y**2+C*Y+D=0.0
C
      DIMENSION RT(3)
      IF (A)10,50,10
   50 TERM=C*C-4.0*B*D
      IF (TERM)51,52,52
   51 RT(1)=-12345678.0
      GO TO 32
   52 RCOT=SQRT(TERM)
      CON=2.0*B
      RT(1)=(-C+ROOT)/CON
      RT(2)=(-C-ROOT)/CON
      RT(3)=-12345678.0
      GO TO 70
   10 P=B/A
      Q=C/A
      R=D/A
      CON=P/3.0
      AA=(3.0*Q-P*P)/3.0
      BB=(P*(2.0*P*P-9.0*Q)+27.0*R)/27.0
      SAVEA=(AA*AA*AA)/27.0
      TERM=.25*BB*BB+SAVEA
      HALFB=-BB*.5
      IF (TERM+.00001)20,30,30
   30 ROOT=SQRT(ABS(TERM))
      TERMA=(ABS(HALFB+ROOT))**.33333333
      TERMB=(ABS(HALFB-ROOT))**.33333333
      TERMA=SIGN(TERMA,HALFB+ROOT)
      TERMB=SIGN(TERMB,HALFB-ROOT)
      RT(1)=TERMA+TERMB
   40 RT(2)=(-RT(1)+(TERMA-TERMB)*1.732051)*.5-CON
      RT(3)=(-RT(1)-(TERMA-TERMB)*1.732051)*.5-CON
      RT(1)=RT(1)-CON
      IF (TERM-.00001)70,70,32
   32 RT(2)=-12345678.0
      RT(3)=-12345678.0
      GO TO 70
   20 COSPHI=HALFB/(SQRT(-SAVEA))
      PHI=ACOS(COSPHI)
   43 ROOT=2.0*SQRT(-AA/3.0)
      ANGLE=PHI/3.0
      DO 44 I=1,3
      XI=I-1
   44 RT(I)=ROOT*COS(ANGLE+XI*2.0943951)-CON
   70 RETURN
      END
```

```
      SUBROUTINE DET3X3 (A,DET)
C
C     FIND THE DETERMINANT FOR A 3X3 MATRIX
C
      DIMENSION A(3,3)
      T1=A(1,1)*(A(2,2)*A(3,3)-A(2,3)*A(3,2))
      T2=A(1,2)*(A(2,3)*A(3,1)-A(2,1)*A(3,3))
      T3=A(1,3)*(A(2,1)*A(3,2)-A(2,2)*A(3,1))
      DET=T1+T2+T3
      RETURN
      END
```

```
      SUBROUTINE DET4X4 (A,DET)
C
C     FIND THE DETERMINANT FOR A 4X4 MATRIX
C
      DIMENSION A(4,4),B(3,3)
      DO 71 I=1,3
      DO 71 J=1,3
      I1=I+1
      J1=J+1
   71 B(I,J)=A(I1,J1)
      CALL DET3X3 (B,D1)
      DO 72 I=1,3
      DO 72 J=1,3
      I1=I+1
      J1=J+1
      IF (J.EQ.1) J1=1
   72 B(I,J)=A(I1,J1)
      CALL DET3X3 (B,D2)
      DO 73 I=1,3
      DO 73 J=1,3
      I1=I+1
      J1=J
      IF (J.EQ.3) J1=4
   73 B(I,J)=A(I1,J1)
      CALL DET3X3 (B,D3)
      DO 74 I=1,3
      DO 74 J=1,3
      I1=I+1
   74 B(I,J)=A(I1,J)
      CALL DET3X3 (B,D4)
      DET =A(1,1)*D1-A(1,2)*D2+A(1,3)*D3-A(1,4)*D4
      RETURN
      END
```

```
      SUBROUTINE DIAMET (C,EVEC,B)
C
C     COMPUTE THE DIAMETRAL PLANE OF A QUADRIC SURFACE
C
      DIMENSION C(10),EVEC(3),B(4)
      B(1)=C(1)*EVEC(1)+C(6)*EVEC(2)+C(5)*EVEC(3)
      B(2)=C(6)*EVEC(1)+C(2)*EVEC(2)+C(4)*EVEC(3)
      B(3)=C(5)*EVEC(1)+C(4)*EVEC(2)+C(3)*EVEC(3)
      B(4)=C(7)*EVEC(1)+C(8)*EVEC(2)+C(9)*EVEC(3)
      TMP=1.0/SQRT(B(1)*B(1)+B(2)*B(2)+B(3)*B(3))
      DO 10 I=1,4
   10 B(I)=TMP*B(I)
      RETURN
      END
```

```
      SUBROUTINE DIRCOS (A,B,C)
C
C     COMPUTE DIRECTION COSINES BY NORMALIZING THE
C             PARTIAL DERIVATIVES.
C
      D=SQRT(A*A+B*B+C*C)
      A=A/D
      B=B/D
      C=C/D
      RETURN
      END
```

```
      FUNCTION DOT(A,B)
         DIMENSION A(3),B(3)
      DOT=A(1)*B(1)+A(2)*B(2)+A(3)*B(3)
      RETURN
      END
```

```
      SUBROUTINE EIGVEC (B,EVAL,EVEC)
C
C     FIND THE EIGENVECTOR FOR A GIVEN EIGENVALUE
C
      DIMENSION B(3,3),EVEC(3),CE(3,3)
      DIMENSION A(3,4)
      DO 10 I=1,3
      DO 10 J=1,3
      CE(I,J)=B(I,J)
      IF (I.EQ.J) CE(I,J)=CE(I,J)-EVAL
   10 CONTINUE
      CALL HOM3X3 (CE,EVEC)
      RETURN
      END
```

```
      SUBROUTINE FIXLCC (R,PT,X,XP)
C
C     TRANSFORM COORDINATES FROM THE FIXED AIRCRAFT SYSTEM(X)
C        INTO THE LOCAL QUADRIC SYSTEM(XP)
C
C     INPUTS.....R.....ROTATION MATRIX
C                PT....TRANSLATION VECTOR
C                X.....POSITION VECTOR IN FIXED AIRCRAFT SYSTEM
C     OUTPUT.....XP....POSITION VECTOR IN LOCAL QUADRIC SYSTEM
C     IN MATRIX NOTATION.....(X)=((R))(XP)+(PT)
C                     OR   (XP)=INVERSE((R))(X-PT)
C
      DIMENSION R(3,3),PT(3),XP(3),X(3),T(3)
      DO 10 I=1,3
 10   T(I)=X(I)-PT(I)
      DO 20 I=1,3
 20   XP(I)=R(1,I)*T(1)+R(2,I)*T(2)+R(3,I)*T(3)
      RETURN
      END
```

```
      SUBROUTINE HOM2X2 (C,EVEC)
C
C     SOLVE A 2X2 HOMOGENEOUS EQUATION WITH A SYMMETRIC COEFF. MATRIX
C
      DIMENSION C(2,2),EVEC(3)
      DET=C(1,1)*C(2,2)-C(1,2)*C(2,2)
      IF (ABS(DET).GT. 1.0E-5) GO TO 60
      IF (ABS(C(1,1)).LT. 1.0E-6.AND.ABS(C(1,2)).LT. 1.0E-6) GO TO 50
      JJ=2
      KK=3
      IF (ABS(C(1,1)).GT.ABS(C(1,2))) GO TO 10
      JJ=3
      KK=2
 10   EVEC(JJ)=-C(1,KK-1)/C(1,JJ-1)
      EVEC(KK)=1.0
      RETURN
 50   EVEC(2)=1.0
      EVEC(3)=0.0
      RETURN
 60   DO 61 I=2,3
 61   EVEC(I)=0.0
      RETURN
      END
```

```
      SUBROUTINE HOM3X3 (CE,EVEC)
C
C     SOLVE A 3X3 HOMOGENEOUS EQUATION WITH A SYMMETRIC COEFF. MATRIX
C
      DIMENSION CE(3,3),EVEC(3),C(2,2),T(2,2)
      ACY=1.0E-6
      CALL DET3X3 (CE,DET)
      IF (ABS(DET).GT. 1.0E-3) GO TO 90
      IF (ABS(CE(1,1)).LT.ACY.AND.ABS(CE(1,2)).LT.ACY
     1   .AND.ABS(CE(1,3)).LT.ACY) GO TO 80
      VMAX=-9999.0
      DO 20 I=1,3
      TMP=ABS(CE(I,1))
      IF (TMP.LE.VMAX) GO TO 20
      II=I
      VMAX=TMP
   20 CONTINUE
      CALL INDICE (II,JJ,KK)
      TMP=1.0/CE(1,II)
      DO 30 I=1,2
      IP1=I+1
      C(I, 1)=CE(IP1,JJ)-TMP*CE(IP1,II)*CE(1,JJ)
      C(I, 2)=CE(IP1,KK)-TMP*CE(IP1,II)*CE(1,KK)
      T(I,1)=ABS(C(I,1))
   30 T(I,2)=ABS(C(I,2))
      IF (T(1,1).GT.ACY.OR .T(1,2).GT.ACY) GO TO 41
      IF (T(2,1).GT.ACY.OR .T(2,2).GT.ACY) GO TO 42
      EVEC(JJ)=1.0
      EVEC(KK)=1.0
      GO TO 40
   42 IF (T(2,1).GE.T(2,2)) GO TO 47
      EVEC(KK)=-C(2,1)/C(2,2)
      EVEC(JJ)=1.0
      GO TO 40
   47 EVEC(JJ)=-C(2,2)/C(2,1)
      EVEC(KK)=1.0
      GO TO 40
   41 IF (T(1,1).GE.T(1,2)) GO TO 46
      EVEC(KK)=-C(1,1)/C(1,2)
      EVEC(JJ)=1.0
      GO TO 40
   46 EVEC(JJ)=-C(1,2)/C(1,1)
      EVEC(KK)=1.0
   40 EVEC(II)=(-1.0/CE(1,II))*(CE(1,JJ)*EVEC(JJ)+CE(1,KK)*EVEC(KK))
      RETURN
   80 DO 81 I=1,2
      IP1=I+1
      DO 81 J=1,2
      JP1=J+1
   81 C(I,J)=CE(IP1,JP1)
      EVEC(1)=1.0
      CALL HOM2X2 (C,EVEC)
      RETURN
   90 DO 91 I=1,3
   91 EVEC(I)=0.0
      WRITE (6,633)
  633 FORMAT (59HOTHE MATRIX IS NOT SINGULAR....THE ONLY SOLUTION IS TRI
     1VIAL)
      RETURN
      END
```

```
      SUBROUTINE INDICE (K,I,J)
C
C     FROM THE GIVEN SUBSCRIPT K ASSIGN SUBSCRIPTS TO THE OTHER
C        TWO VARIBLES
C
      GO TO (1,2,3),K
 1    I=2
      J=3
      RETURN
 2    I=3
      J=1
      RETURN
 3    I=1
      J=2
      RETURN
      END
```

```
      SUBROUTINE LOCFIX (R,PT,XP,X)
C
C     TRANSFORM COORDINATES FROM THE LOCAL QUADRIC SYSTEM(XP)
C        INTO THE FIXED AIRCRAFT SYSTEM(X)
C
C     INPUTS.....R.....ROTATION MATRIX
C                PT....TRANSLATION VECTOR
C                XP....POSITION VECTOR IN LOCAL QUADRIC SYSTEM
C     OUTPUT.....X.....POSITION VECTOR IN FIXED AIRCRAFT SYSTEM
C     IN MATRIX NOTATION.....(X)=((R))(XP)+(PT)
C                      OR   (XP)=INVERSE((R))(X-PT)
C
      DIMENSION R(3,3),PT(3),XP(3),X(3)
      DO 10 I=1,3
 10   X(I)=R(I,1)*XP(1)+R(I,2)*XP(2)+R(I,3)*XP(3)+PT(I)
      RETURN
      END
```

```
      SUBROUTINE LSF2D (X,Y,Z,NPTS,C)
C
C     FIT A GENERAL QUADRIC SURFACE THRU THE INPUT DATA
C        USING A LEAST SQUARES SURFACE FIT
C
      DIMENSION SUM(10,11),B(10,11),T(11),C(10),CT(10)
      DIMENSION X(500),Y(500),Z(500)
      DO 10 I=1,10
      DO 10 J=1,11
   10 SUM(I,J)=0.0
      DO 20 I=1,NPTS
      T(1)=Y(I)*Y(I)
      T(2)=Z(I)*Z(I)
      T(3)=2.0*Y(I)*Z(I)
      T(4)=2.0*Z(I)*X(I)
      T(5)=2.0*X(I)*Y(I)
      T(6)=2.0*X(I)
      T(7)=2.0*Y(I)
      T(8)=2.0*Z(I)
      T( 9)=1.0
      T(10)=-X(I)*X(I)
      DO 20 J=1,9
      DO 20 K=1,10
      SUM(J,K)=SUM(J,K)+T(J)*T(K)
   20 CONTINUE
      DO 30 J=1,9
      DO 30 K=1,10
      IF (J.EQ.K.AND.SUM(J,K).EQ. 0.0) SUM(J,K)=1.0E-6
   30 CONTINUE
      CALL CROUT (SUM,B,CT,9)
      C(1)=1.0
      DO 71 I=1,9
      K=I+1
   71 C(K)=CT(I)
      RETURN
      END
```

```
      SUBROUTINE QDPAR (C,PT,R,KODE)
C
C     ANALYZE A GENERAL QUADRIC AND FIND THE TRANSLATION TERMS
C        AND THE ROTATION MATRIX TO TRANSFORM THE
C        GENERAL EQUATION INTO STANDARD FORM
C
      DIMENSION C(10),A(4,4),B(3,3),RT(3),EVEC(3),EVAL(3),R(3,3),PT(3)
      ACY=1.0E-5
      A(1,1)=C(1)
      A(1,2)=C(6)
      A(1,3)=C(5)
      A(1,4)=C(7)
      A(2,1)=A(1,2)
      A(2,2)=C(2)
      A(2,3)=C(4)
      A(2,4)=C(8)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      A(3,3)=C(3)
      A(3,4)=C(9)
      A(4,1)=A(1,4)
      A(4,2)=A(2,4)
      A(4,3)=A(3,4)
      A(4,4)=C(10)
      CALL DET4X4 (A,QA)
      QI=A(1,1)+A(2,2)+A(3,3)
      QJ=A(1,1)*A(2,2)-A(1,2)*A(2,1)+A(2,2)*A(3,3)
     1   -A(2,3)*A(3,2)+A(3,3)*A(1,1)-A(3,1)*A(1,3)
      DO 10 I=1,3
      DO 10 J=1,3
   10 B(I,J)=A(I,J)
      CALL DET3X3 (B,QD)
      WRITE (6,600) QA,QD,QI
  600 FORMAT (21HOQUADRIC INVARIANTS  3H A=  E14.6,3X,2HD=E14.6,3X,
     1   2HI=E14.6)
      DO 5 I=1,3
      J=I+1
      B(I,1)=A(J,2)
      B(I,2)=A(J,3)
    5 B(I,3)=A(J,4)
      CALL DET3X3 (B,A11)
      DO 6 I=1,3
      J=I
      IF (I.GT.1) J=I+1
      B(I,1)=A(J,1)
      B(I,2)=A(J,3)
    6 B(I,3)=A(J,4)
      CALL DET3X3 (B,A22)
      DO 7 I=1,3
      J=I
      IF (I.GT.2) J=I+1
      B(I,1)=A(J,1)
      B(I,2)=A(J,2)
    7 B(I,3)=A(J,4)
      CALL DET3X3 (B,A33)
      DO 8 I=1,3
      DO 8 J=1,3
```

```
 8     B(I,J)=A(I,J)
       CALL DET3X3 (B,A44)
       AP=A11+A22+A33+A44
       WRITE (6,601) QJ,AP
 601   FORMAT (21X,3H J=E14.6,3X,8HA PRIME=E14.6)
       IF (ABS(QD).LT. 1.0E-6) GO TO 50
C
C      COMPUTE COORDS. OF THE CENTER OF A CENTRAL QUADRIC
C
       TEMP=-1.0/QD
       DO 11 I=1,3
       B(I,1)=A(I,4)
       B(I,2)=A(I,2)
 11    B(I,3)=A(I,3)
       CALL DET3X3 (B,DET)
       PT(1)=TEMP*DET
       DO 12 I=1,3
       B(I,1)=A(I,1)
       B(I,2)=A(I,4)
 12    B(I,3)=A(I,3)
       CALL DET3X3 (B,DET)
       PT(2)=TEMP*DET
       DO 13 I=1,3
       B(I,1)=A(I,1)
       B(I,2)=A(I,2)
 13    B(I,3)=A(I,4)
       CALL DET3X3 (B,DET)
       PT(3)=TEMP*DET
       WRITE (6,610) PT
 610   FORMAT (26HOCENTER OF CENTRAL QUADRIC /15X,3HXO=E14.6,3X,
     1   3HYO=E14.6,3X,3HZO=E14.6)
       GO TO 20
 50    WRITE (6,620)
 620   FORMAT (37HOTHE SURFACE IS NOT A CENTRAL QUADRIC)
 20    CONTINUE
       CALL CUBIC (1.0,-QI,QJ,-QD,RT)
       DO 82 I=1,3
       DO 82 J=1,3
 82    B(I,J)=A(I,J)
       EVAL(1)=AMAX1(RT(1),RT(2),RT(3))
       DO 90 I=1,3
       II=I
       IF (EVAL(1).EQ. RT(I)) GO TO 91
 90    CONTINUE
 91    CALL INDICE (II,JJ,KK)
       EVAL(2)=AMAX1(RT(JJ),RT(KK))
       EVAL(3)=AMIN1(RT(JJ),RT(KK))
       CALL TYPE (QA,QD,QI,QJ, AP,EVAL,KODE)
       IF (KODE.EQ. 0) RETURN
       NEVAL=3
       IF (ABS(EVAL(1)-EVAL(2)).LE.ACY) GO TO 31
       IF (ABS(EVAL(2)-EVAL(3)).LE.ACY) GO TO 34
       IF (ABS(EVAL(1)-EVAL(3)).LE.ACY) GO TO 35
       GO TO 39
 31    IF (ABS(EVAL(1)-EVAL(3)).LE.ACY) GO TO 32
       NEVAL=2
       EVAL(2)=EVAL(3)
```

```
      GO TO 39
 32   NEVAL=1
      GO TO 39
 35   NEVAL=2
      GO TO 39
 34   NEVAL=2
 39   CONTINUE
      ITP=0
      DO 61 I=1,NEVAL
      IF (ABS(EVAL(I)).LT.ACY) GO TO 61
      ITP=ITP+1
      EVAL(ITP)=EVAL(I)
 61   CONTINUE
      NEVAL=ITP
      IF (NEVAL.GT. 1) GO TO 73
      DO 74 I=1,3
      DO 74 J=1,3
      R(I,J)=0.0
      IF (I.EQ.J) R(I,J)=1.0
 74   CONTINUE
      GO TO 44
 73   DO 81 I=1,NEVAL
      CALL EIGVEC (B,EVAL(I),EVEC)
      TMP=     SQRT(EVEC(1)*EVEC(1)+EVEC(2)*EVEC(2)+EVEC(3)*EVEC(3))
      IF (TMP.EQ. 0.0) TMP=1.0
      TMP=1.0/TMP
      DO 83 J=1,3
 83   R(J,I)=TMP*EVEC(J)
 81   CONTINUE
      IF (NEVAL.EQ. 3) GO TO 44
      EVEC(1)=R(2,1)*R(3,2)-R(3,1)*R(2,2)
      EVEC(2)=R(3,1)*R(1,2)-R(1,1)*R(3,2)
      EVEC(3)=R(1,1)*R(2,2)-R(2,1)*R(1,2)
      TMP=     SQRT(EVEC(1)*EVEC(1)+EVEC(2)*EVEC(2)+EVEC(3)*EVEC(3))
      IF (TMP.EQ. 0.0) TMP=1.0
      TMP=1.0/TMP
      DO 84 J=1,3
 84   R(J,3)=TMP*EVEC(J)
 44   CONTINUE
      IF (ABS(QD).GT. 1.0E-6) RETURN
      CALL VERTEX (C,EVAL(1),EVAL(2),R,QA,KODE,PT)
      WRITE (6,631) PT
631   FORMAT (30HOVERTEX OF NON-CENTRAL QUADRIC/15X,3HXO=E14.6,
     1   3X,3HYO=E14.6,3X,3HZO=E14.6)
      RETURN
      END
```

```
      SUBROUTINE QUADS (C,PO,DPO,KODE,SN,P1,DP1,INT)
C
C     FIND THE INTERSECTION OF A LINE WITH A QUADRIC
C        SURFACE (GIVEN IN STANDARD FORM)
C
      DIMENSION C(4),PO(3),DPO(3),P1(3),DP1(3),TT(3)
      DATA ACUR/1.0E-6/
      INT=2
      IF (KODE.GT. 4) GO TO 99
C
C     CENTRAL QUADRIC SURFACE
C
      CC=0.0
      DO 10 I=1,3
      CC=CC+C(I)*PO(I)*PO(I)
   10 TT(I)=C(I)*DPO(I)
      AA=DOT(TT,DPO)
      BB=2.0*DOT(TT,PO)
      CC=CC+C(4)
      GO TO 15
C
C     NONCENTRAL QUADRIC SURFACE
C
   99 AA=C(1)*DPO(1)**2+C(2)*DPO(2)**2
      BB=2.0*(C(1)*DPO(1)*PO(1)+C(2)*DPO(2)*PO(2)+C(3)*DPO(3))
      CC=C(1)*PO(1)**2+C(2)*PO(2)**2+2.0*C(3)*PO(3)+C(4)
C
   15 IF (ABS(AA)-ACUR) 20,20,30
   20 IF (ABS(BB).LE.ACUR) RETURN
      RT=-CC/BB
      GO TO 50
   30 ROOT=BB*BB-4.0*AA*CC
      IF (ROOT.LT. 0.0) RETURN
      ROOT=SQRT(ROOT)
      TMP=AA+AA
      D1=(-BB+ROOT)/TMP
      D2=(-BB-ROOT)/TMP
      IF (D1.LT. ACUR) GO TO 31
      IF (D2.LT. ACUR) GO TO 32
      RT=AMIN1(D1,D2)
      GO TO 50
   31 IF (D2.LT. ACUR) RETURN
      RT=D2
      GO TO 50
   32 RT=D1
   50 DO 60 I=1,3
      P1(I)=PO(I)+DPO(I)*RT
   60 DP1(I)=SN*C(I)*P1(I)
      IF (KODE.GT. 4) DP1(3)=SN*C(3)
      CALL DIRCOS (DP1(1),DP1(2),DP1(3))
      INT=1
      RETURN
      END
```

```
      SUBROUTINE ROTRAN (A,G,PT,B,IND)
C
C     ROTATE AND TRANSLATE A GIVEN QUADRIC SURFACE WITH COEFFS. A
C        INTO ANOTHER COORD. SYSTEM WITH COEFFS. B
C     A.....COEFFS. OF GIVEN SURFACE
C     G.....ROTATION MATRIX
C     PT.....TRANSLATION TERMS
C     B.....COEFFS. OF SURFACE IN NEW COORD. SYSTEM
C     IND=1.....X,Y,Z PRIME=F(X,Y,Z)
C     IND=2.....X,Y,Z=F(X,Y,Z PRIME)
C
      DIMENSION A(10),G(3,3),B(10),T(3,3),PT(3)
      IF (IND.NE. 1) GO TO 5
      XO=-(PT(1)*G(1,1)+PT(2)*G(1,2)+PT(3)*G(1,3))
      YO=-(PT(1)*G(2,1)+PT(2)*G(2,2)+PT(3)*G(2,3))
      ZO=-(PT(1)*G(3,1)+PT(2)*G(3,2)+PT(3)*G(3,3))
      GO TO 6
    5 XO=PT(1)
      YO=PT(2)
      ZO=PT(3)
    6 CONTINUE
      DO 10 K=1,3
      L=4
      IF (K.EQ.1) L=5
      M=6
      IF (K.EQ.3) M=5
      M1=M-3
      L1=L-3
      T(K,1)=A(K)*G(K,1)+A(L)*G(M1,1)+A(M)*G(L1,1)
      T(K,2)=A(K)*G(K,2)+A(L)*G(M1,2)+A(M)*G(L1,2)
      T(K,3)=A(K)*G(K,3)+A(L)*G(M1,3)+A(M)*G(L1,3)
   10 CONTINUE
      DO 20 K=1 3
      B(K)=G(1,K)*T(1,K)+G(2,K)*T(2,K)+G(3,K)*T(3,K)
      L=K+1
      IF (K.EQ.3) L=1
      M=K-1
      IF (K.EQ.1) M=3
      B(K+3)=G(1,L)*T(1,M)+G(2,L)*T(2,M)+G(3,L)*T(3,M)
      B(K+6)= XO*T(1,K)+YO*T(2,K)+ZO*T(3,K)+A(7)*G(1,K)+A(8)*G(2,K)
     1   +A(9)*G(3,K)
   20 CONTINUE
      B(10)=A(1)*XO*XO+A(2)*YO*YO+A(3)*ZO*ZO+2.0*(XO*(A(6)*YO
     1   +A(5)*ZO+A(7))+YO*(A(4)*ZO+A(8))+A(9)*ZO)+A(10)
      RETURN
      END
```

```
      SUBROUTINE SIMEQ (A,NE,XE)
C
C     SOLVE A NXN SYSTEM SIMULTANEOUS EQUATIONS
C
      DIMENSION A(3,4),V(4,3),C(4),T(4),XE(3)
      K=NE
      L=K+1
      IF (ABS(A(1,1)).LT. 1.0E-6) RETURN
      AONE1=A(1,1)
      X=1.0/AONE1
      DO 13 I=1,K
   13 V(1,I)=-A(1,I+1)*X
      N=1
  100 LAST=N
      K=K-1
      N=N+1
      DO 105 I=1,LAST
  105 T(I)=V(I,1)
      D=-A(N,N)
      DO 106 I=1,LAST
  106 D=D-A(N,I)*T(I)
      IF (ABS(D).LT. 1.0E-6) RETURN
      D=1./D
      DO 111 I=1,K
      M=N+I
      C(I)=A(N,M)
      DO 110 J=1,LAST
  110 C(I)=C(I)+A(N,J)*V(J,I+1)
  111 C(I)=C(I)*D
      DO 116 J=1,K
      DO 115 I=1,LAST
  115 V(I,J)=C(J)*T(I)+V(I,J+1)
  116 V(N,J)=C(J)
      IF (K.GT. 1) GO TO 100
      DO 125 I=1,N
  125 XE(I)=V(I,1)
      RETURN
      END
```

```
      SUBROUTINE TYPE (QA,QD,QI,QJ,AP,EVAL,KODE)
C
C     IDENTIFY THE QUADRIC SURFACE USING THE QUADRIC INVARIANTS
C
      DIMENSION EVAL(3)
      ACY=1.0E-6
      IF (ABS(QA).LT.ACY) GO TO 200
      IF (QA.GT. 0.0) GO TO 50
      IF (ABS(QD).GT.ACY) GO TO 20
      IF (QJ.LT. 0.0) GO TO 133
      WRITE (6,605)
  605 FORMAT (30H0*****ELLIPTIC PARABOLOID*****)
      KODE=5
      GO TO 99
   20 IF ((QD*QI).GT. 0.0.AND.QJ.GT. 0.0) GO TO 21
      WRITE (6,603)
  603 FORMAT (36H0*****HYPERBOLOID OF TWO SHEETS*****)
      KODE=3
      GO TO 99
   21 CONTINUE
      WRITE (6,601)
  601 FORMAT (20H0*****ELLIPSOID*****)
      KODE=1
      GO TO 99
   50 IF (ABS(QD).GT.ACY) GO TO 23
      IF (QJ.GT. 0.0) GO TO 133
      WRITE (6,606)
  606 FORMAT (32H0*****HYPERBOLIC PARABOLOID*****)
      KODE=6
      GO TO 99
   23 IF ((QD*QI).GT. 0.0.AND.QJ.GT. 0.0) GO TO 133
      WRITE (6,602)
  602 FORMAT (35H0*****HYPERBOLOID OF ONE SHEET*****)
      KODE=2
      GO TO 99
  200 IF (ABS(AP).LT.ACY) GO TO 133
      IF ((AP*QI).GT. 0.0.AND.QJ.GT. 0.0) GO TO 133
      IF (ABS(QD).LT. ACY) GO TO 210
      IF ((QD*QI).GT. 0.0.AND.QJ.GT. 0.0) GO TO 133
      WRITE (6,604)
  604 FORMAT (24H0*****ELLIPTIC CONE*****)
      KODE=4
      GO TO 99
  210 IF (ABS(QJ).LT.ACY) GO TO 133
      IF (QJ.GT. 0.0) GO TO 220
      WRITE (6,608)
  608 FORMAT (30H0*****HYPERBOLIC CYLINDER*****)
      KODE=8
      GO TO 99
  220 CONTINUE
      WRITE (6,607)
  607 FORMAT (28H0*****ELLIPTIC CYLINDER*****)
      KODE=7
      GO TO 99
  133 WRITE (6,699)
  699 FORMAT (46H0THE SURFACE IS EITHER IMAGINARY AND/OR PLANAR)
```

```
      KODE=0
99    WRITE (6,633) (EVAL(I),I=1,3)
633   FORMAT (21H WITH EIGENVALUES K1=E11.3,2X,3HK2=E11.3,2X,
     1    3HK3=E11.3)
      WRITE (6,678) KODE
678   FORMAT (6H KODE=I2)
      RETURN
      END
```

```
      SUBROUTINE VERTEX (C,EVAL1,EVAL2,R,QA,KODE,XE)
C
C     FIND THE VERTEX OF A NON-CENTRAL QUADRIC SURFACE
C
      DIMENSION C(10),R(3,3),D1(4),D2(4),D3(4),A(3,4),XE(3)
      IF (KODE.LT. 5) RETURN
      CALL DIAMET (C,R(1,1),D1)
      CALL DIAMET (C,R(1,2),D2)
      IF (KODE.GT. 6) GO TO 50
      TMP=SQRT(ABS(QA/(EVAL1*EVAL2)))
      DO 10 I=1,3
      T1=EVAL1*D1(I)*D1(4)+EVAL2*D2(I)*D2(4)
      T2=TMP*R(I,3)
      T3=C(I+6)
      IF (T1.NE. 0.0) GO TO 11
   10 CONTINUE
   11 SN=2.0
      IF (ABS(T1+T2-T3).GT. 1.0E-5) SN=-2.0
      D3(4)=(C(10)-EVAL1*D1(4)*D1(4)-EVAL2*D2(4)*D2(4))/(SN*TMP)
      DO 20 I=1,3
   20 D3(I)=R(I,3)
      DO 30 J=1,4
      A(1,J)=D1(J)
      A(2,J)=D2(J)
   30 A(3,J)=D3(J)
      WRITE (6,600)
  600 FORMAT (17HODIAMETRAL PLANES)
      DO 40 I=1,3
      WRITE (6,610) (A(I,J),J=1,4)
  610 FORMAT (4E16.6)
   40 CONTINUE
      CALL SIMEQ (A,3,XE)
      RETURN
C
C     FIND THE LINE OF CENTERS
C
   50 K=0
      DO 60 J=1,4
      IF (J.EQ. 3) GO TO 60
      K=K+1
      A(1,K)=D1(J)
      A(2,K)=D2(J)
   60 CONTINUE
      WRITE (6,600)
      DO 61 I=1,2
      WRITE (6,610) (A(I,J),J=1,3)
   61 CONTINUE
      CALL SIMEQ (A,2,XE)
      XE(3)=0.0
      RETURN
      END
```

APPENDIX IV

DEFINITION OF VARIABLE SUBSCRIPTS

Some of the mathematical derivations in this report are simplified by using variable subscripts to identify the coordinate variables. Define the coordinate variable as

$$x_1 = x\,, \quad x_2 = y\,, \quad x_3 = z\,. \qquad \text{(IV-1)}$$

Now let (IV-1) take on variable subscripts such that

$$(x_i = x\,, \quad x_j = z\,, \quad x_k = z); \text{ if } i = 1,\ j = 2,\ k = 3$$

$$(x_i = y\,, \quad x_j = z\,, \quad x_k = x); \text{ if } i = 2,\ j = 3,\ k = 1$$

$$(x_i = z\,, \quad x_j = x\,, \quad x_k = y); \text{ if } i = 3,\ j = 1,\ k = 2$$

The restrictions placed on the variable subscripts are that they must be integers and

$$\begin{aligned} 1 &\le i \le 3 \\ 1 &\le j \le 3 \\ 1 &\le k \le 3 \\ i &\ne j \ne k \end{aligned} \qquad \text{(IV-2)}$$

Once the coordinate variables are identified, their corresponding coefficients are also assigned variable subscripts. For example, the standard equation of a quadric surface

$$c_1 x^2 + c_2 y^2 + c_3 z^2 + c_4 = 0 \qquad \text{(IV-3)}$$

can be rewritten as

$$c_i x_i^2 + c_j x_j^2 + c_k x_k^2 + c_4 = 0 \qquad \text{(IV-4)}$$

The variable subscript concept is incorporated into the programming of the equations because it reduces the number of program statements needed to represent the problem.

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13. ABSTRACT

Geometry description methods have been developed to transform design data of an aircraft configuration into a computer model. The computer model is used as a basis for radar cross section calculations covering the frequency range of 500 to 20,000 MHz. Accomplishments include: development of a quadric surface fitting program, complete analysis of quadric surfaces including intersections with lines, location of scattering centers, calculation of Gaussian curvature and surface zoning.

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