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How Many Grid Points are Required for Time Accurate Simulations?

Scheme Selection and Scale-Discriminant Stabilization

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APS DSD 2015
Boston, MA

UCLA
ENERGY & PROPULSION RESEARCH LABORATORY

Motivation

- computational efficiency
  - coarser grids
  - larger time steps

spatial concerns:

- how well are gradients captured? (resolution requirement)

spatial/temporal concerns:

- dispersion and dissipation error
  - better characteristics for a broader range of wavenumbers

scheme stabilization concerns:

- balancing accuracy with stability
  - artificial dissipation, filtering

Spectral Representation

<u>Fourier basis:</u>

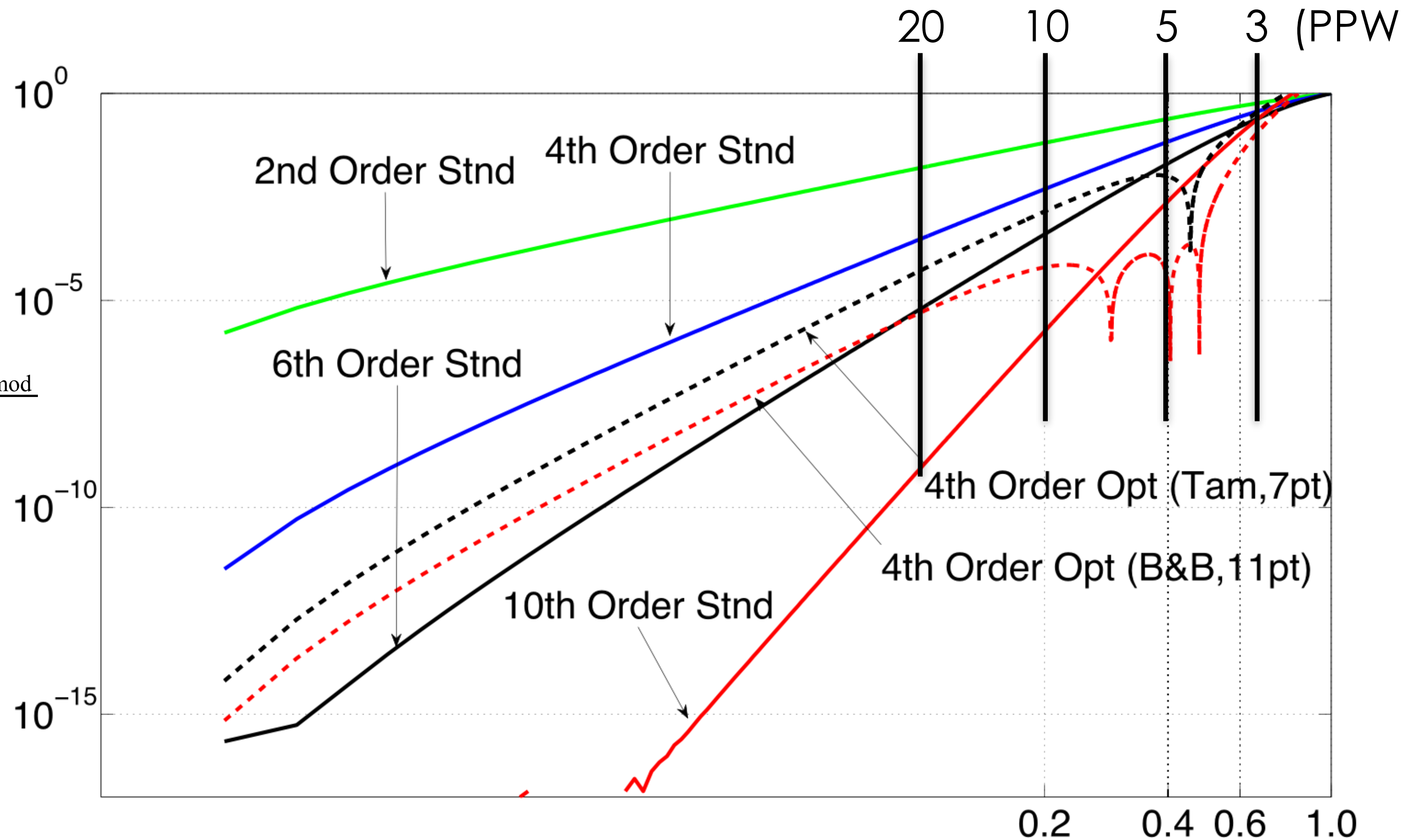
$$
\begin{aligned}
u_j &= \sum_k \hat{u}(k) e^{ikx_j} \\
u_{j+1} &= \sum_k \hat{u}(k) e^{ikx_j} e^{ik\Delta x} \quad \text{with} \quad \theta = (k\Delta x) \in [-\pi, \pi]
\end{aligned}
$$

$$
\delta_x u \rightarrow a_o u'_j + \sum_{l=1} a_l \left(u'_{j+l} - u'_{j-l} \right) = \frac{1}{\Delta x} \left[b_o u_j + \sum_{r=1} b_r \left(u_{j+r} - u_{j-r} \right) \right]
$$

$$
Z_{conv,spec} = i \cdot k \qquad \rightarrow Z_{conv}(\theta) = \underbrace{b_o + i \cdot \left[\frac{2 \sum_{r=1}^{R} b_r \sin(r\theta)}{1 + 2 \sum_{l=1}^{L} a_l \sin(l\theta)} \right]}_{\text{modified wavenumber}}
$$

Modified Wavenumber

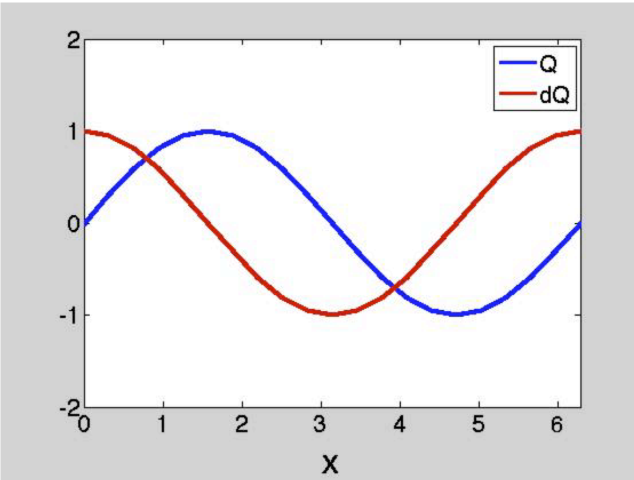
$$\text{error}_k = \frac{k - k_{\text{mod}}}{k}$$



goal: proper representation of derivative

Gradient Capture vs. Resolution: Single Mode

Solution/Derivative:

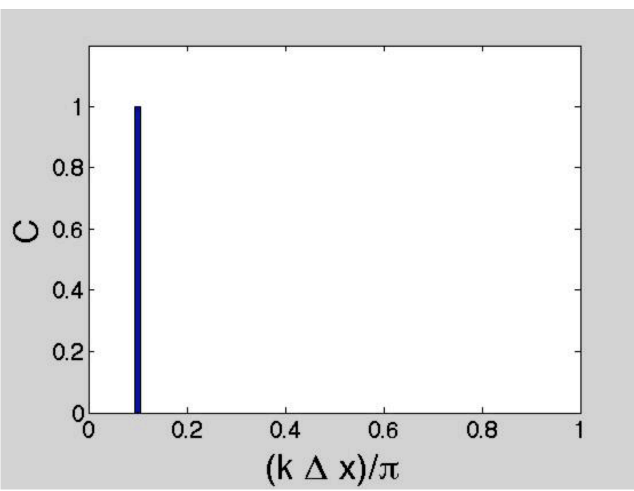


$$f(x) = \sin(x) \quad \text{with } x \in [0, 2\pi]$$

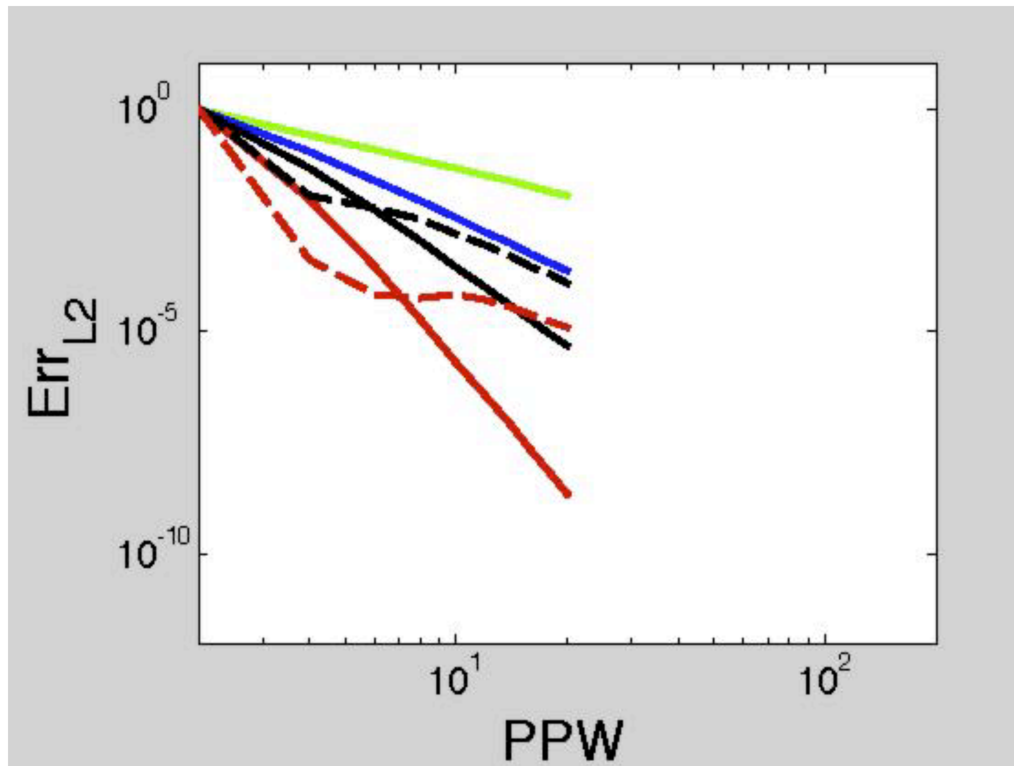
$$\frac{df}{dx} = \cos(x)$$

__ CD02
__ CD04
__ CD06
-- CD04 (Tam,7pt)
__ CD10
-- CD04 (B&B,11pt)

FFT:

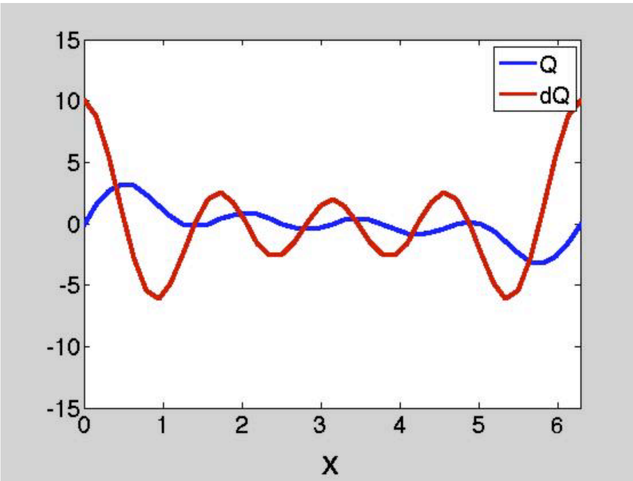


Convergence:



Gradient Capture vs. Resolution: Multiple Modes

Solution/Derivative:



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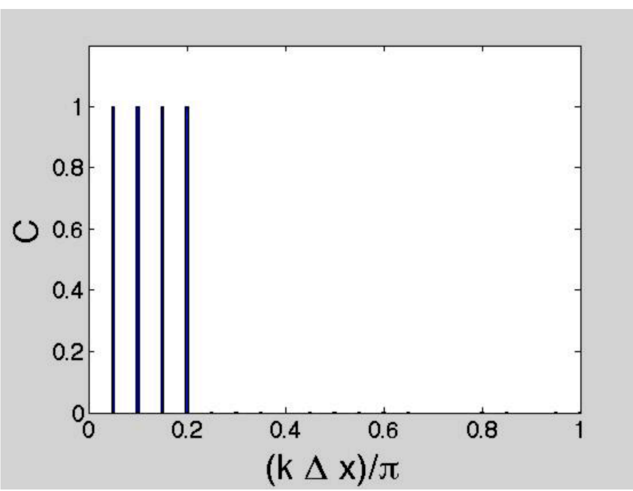
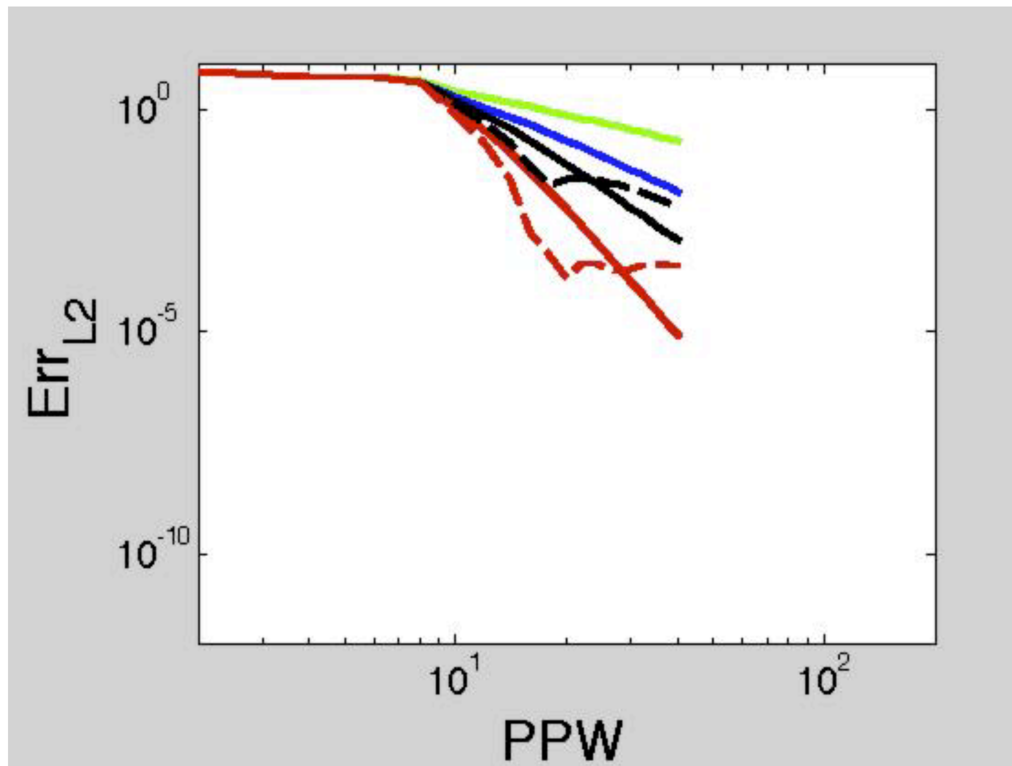


$$f(x) = \sum_{m=1}^{4} \sin(mx) \quad \text{with } x \in [0, 2\pi]$$

$$\frac{df}{dx} = \sum_{m=1}^{4} m\cos(mx)$$

__ CD02
__ CD04
__ CD06
-- CD04 (Tam,7pt)
__ CD10
-- CD04 (B&B,11pt)

Convergence:

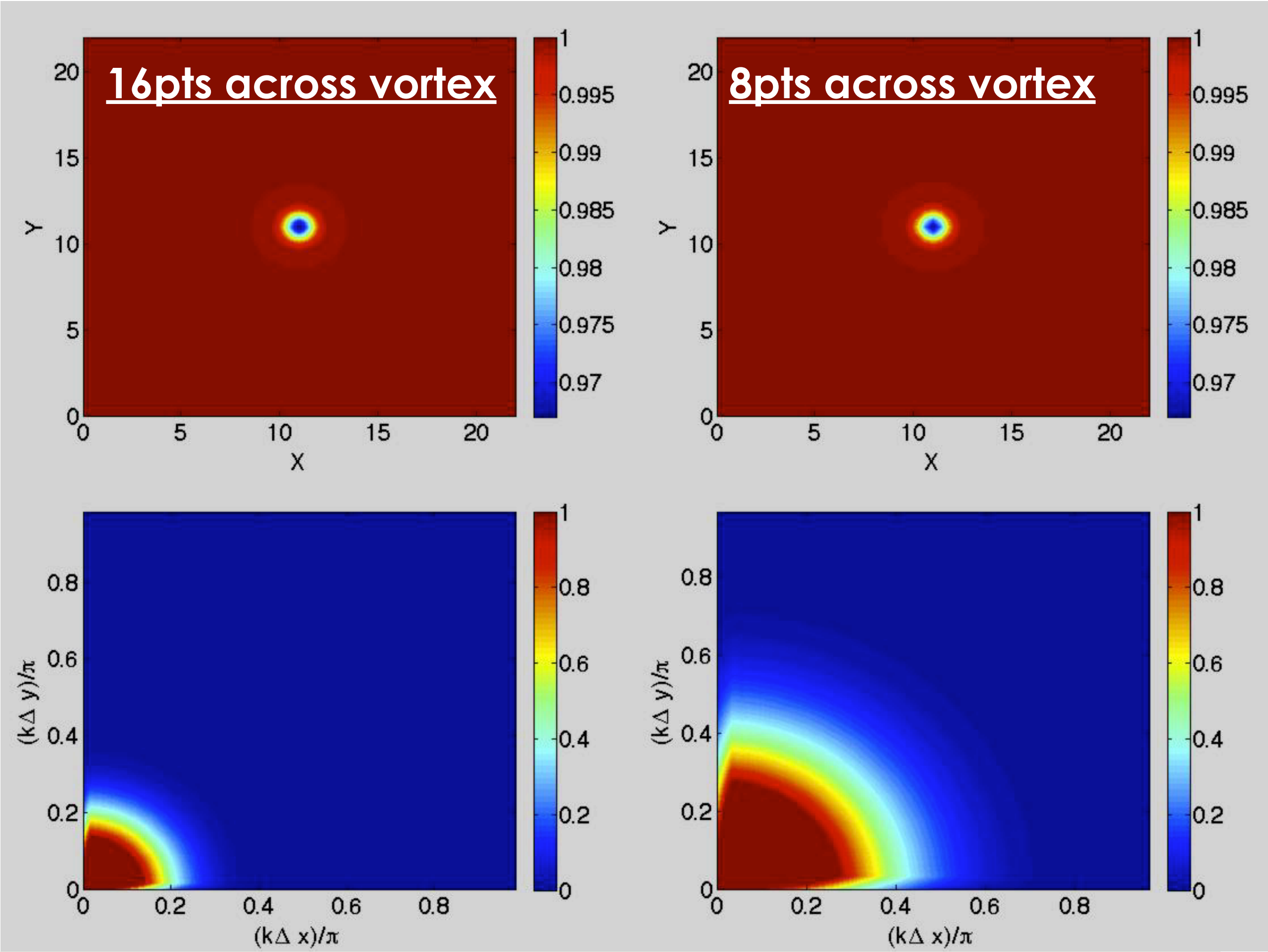


Isentropic Vortex: no Stabilization

$$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$$

$$M_\infty = 0.5$$

<u>CD04 (11pt B&B)</u>



need to stabilize accumulation of high frequency error

Stabilization: Artificial Dissipation and Filtering

Artificial Dissipation:

$$\frac{\partial Q}{\partial t} = -\frac{\partial E}{\partial x} + \sum_m (-1)^{m-1} (\Delta x)^{2m-1} \varepsilon_{2m} |\lambda_{u+c}| \frac{\partial^{2m} Q}{\partial x^{2m}}$$

damping strongly dependent on base scheme
(couples with temporal scheme)

Filtering:

$$\frac{\partial Q^*}{\partial t} = -\frac{\partial E}{\partial x}$$

$$\left[1 + \sum_m^M (-1)^{m+1} \varepsilon_{IF,2m} \left(\frac{\Delta x}{2}\right)^{2m} \frac{\partial^{2m}}{\partial x^{2m}}\right] Q = \left[1 + \sum_n^N (-1)^{n+1} \varepsilon_{EF,2n} \left(\frac{\Delta x}{2}\right)^{2n} \frac{\partial^{2n}}{\partial x^{2n}}\right] Q^*$$

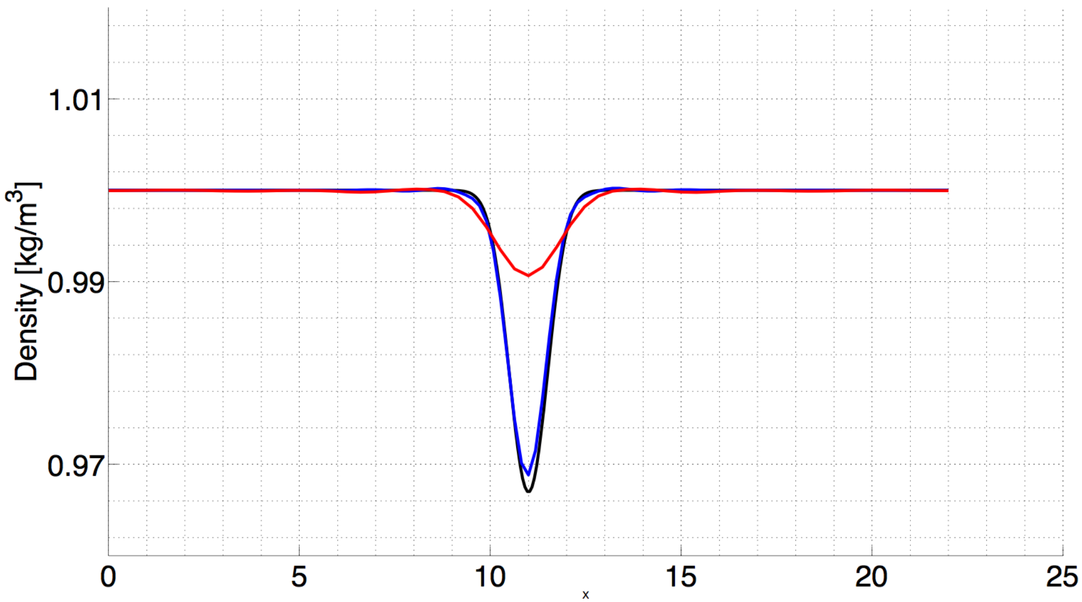
adds consistent amount of damping to base scheme
(decoupled from temporal integration)

Isentropic Vortex: Traditional Stabilization

EF10(1.0)

$$Q = \left[1 + \left(\frac{\Delta x}{2} \right)^{10} \frac{\partial^{10}}{\partial x^{10}} \right] Q^*$$

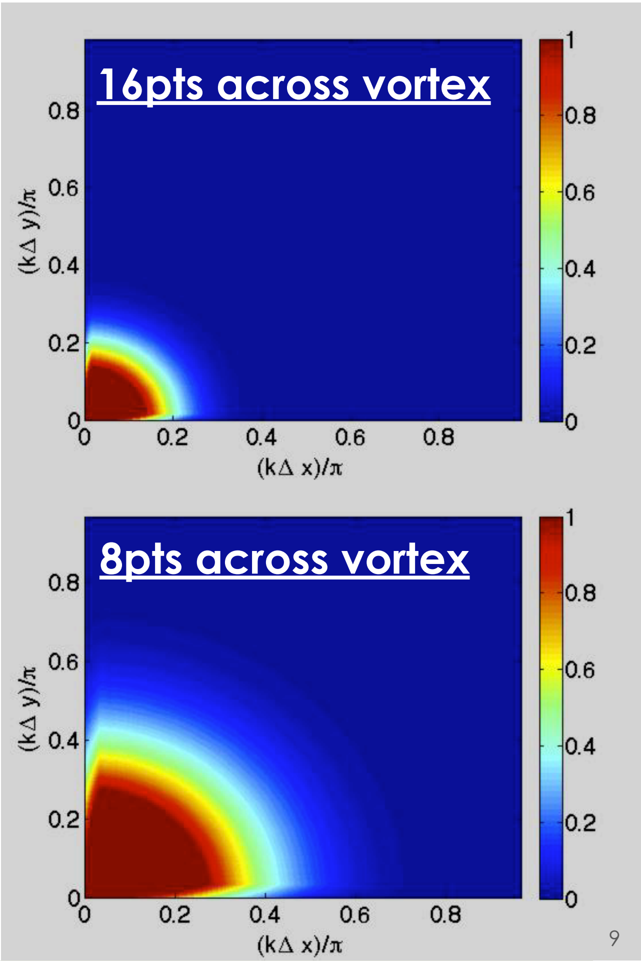
$$CFL_{u,1D} \approx 0.01 \qquad \alpha = 1, \phi = 1$$



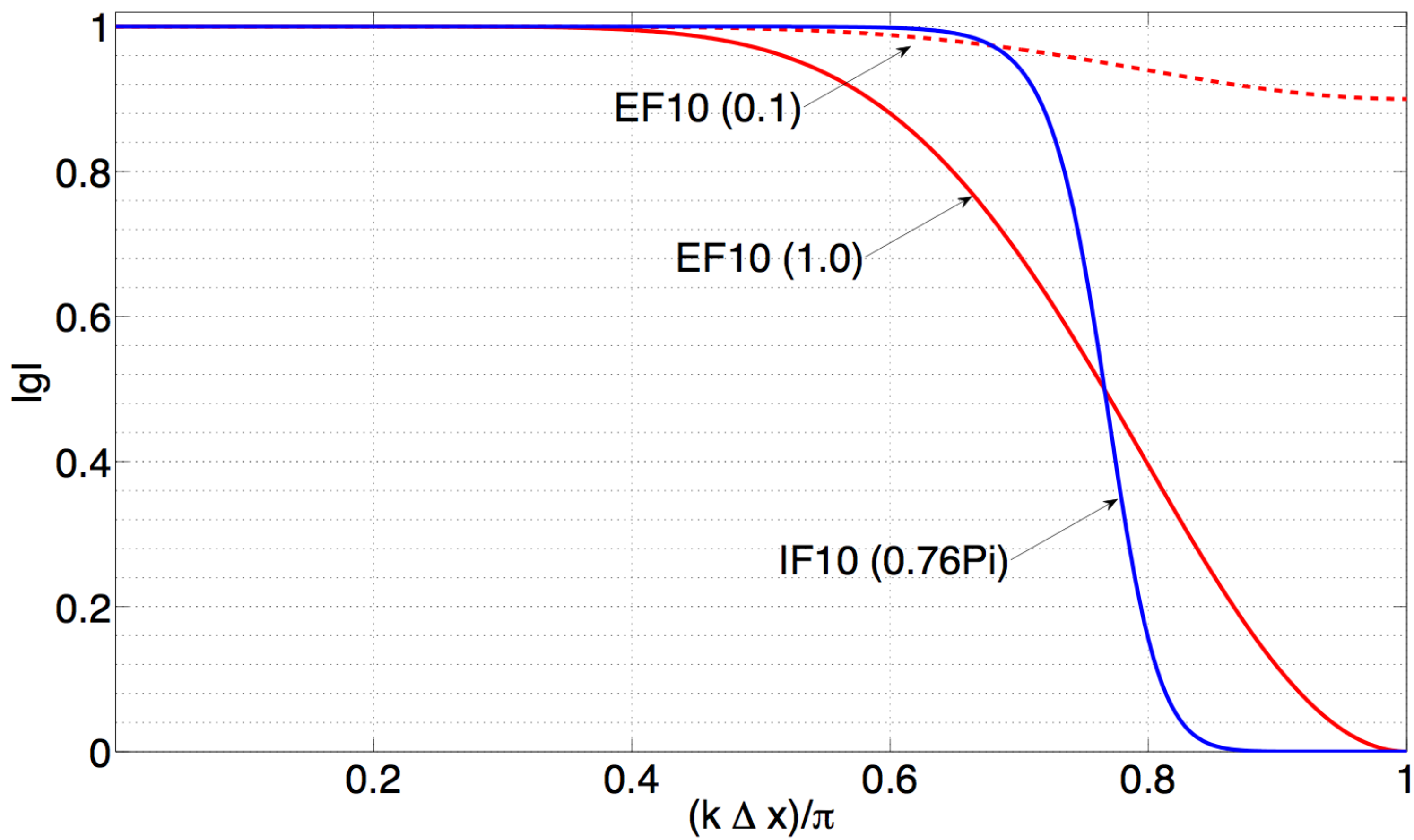
distance traveled: 500 widths

effectiveness of stabilization strategy dependent on spectral content

CD04 (11pt B&B)



Damping Characteristics: Growth Factor

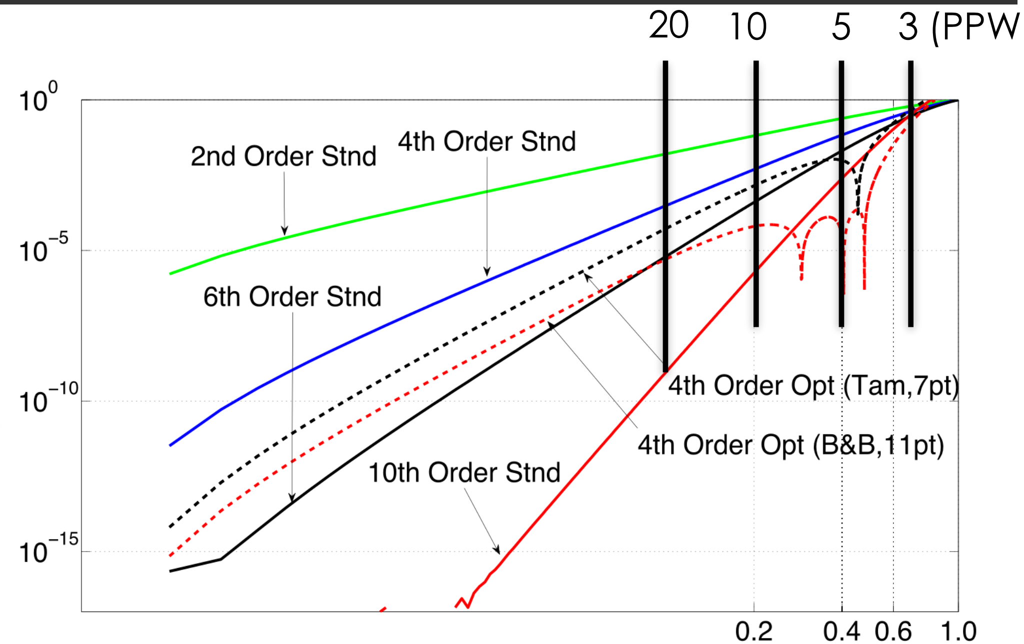


- need scale-discriminant, tunable formulations
- strong preservation of resolvable modes

Scale-Discriminant Stabilization

goal:
preserve accurately resolved frequencies and remove error-prone content

scale-discriminant dissipation



$$\text{error}_k = \frac{k - k_{\text{mod}}}{k} = 0.01$$

FD stencil	$(k\Delta x)_{cutoff}/\pi$	~PPW
CD02	***0.08***	***25***
CD04	0.24	9
CD06	***0.35***	***6***
CD10	0.46	5
CD04 (7pt Tam)	***0.48***	***5***
CD04 (11pt B&B)	0.55	4

for the isentropic vortex…

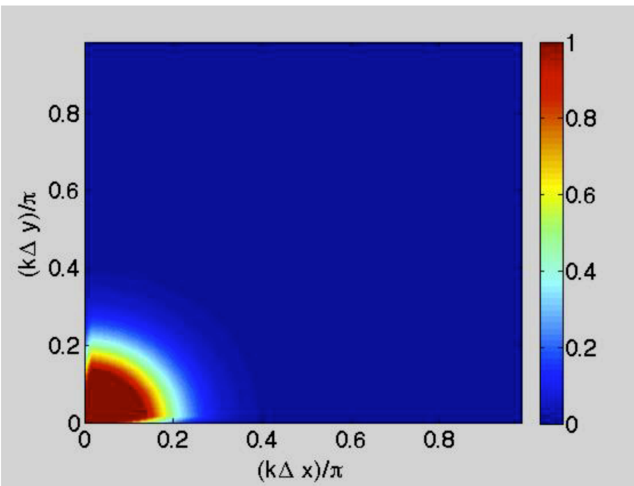
- maintain coherence of vortex
- preserve vortex intensity

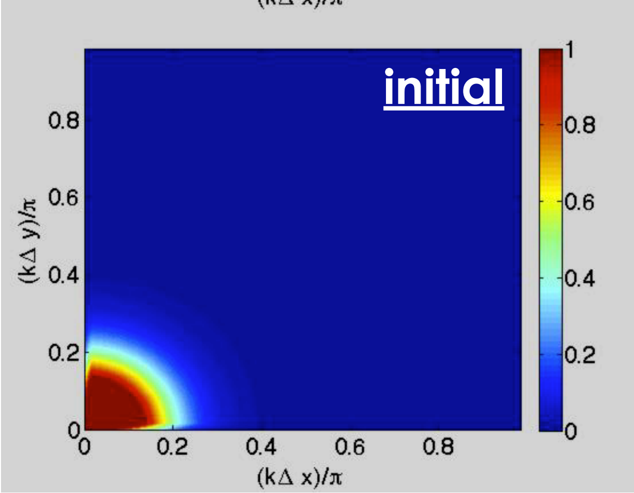
Isentropic Vortex: Scale-Discriminant Stabilization

$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

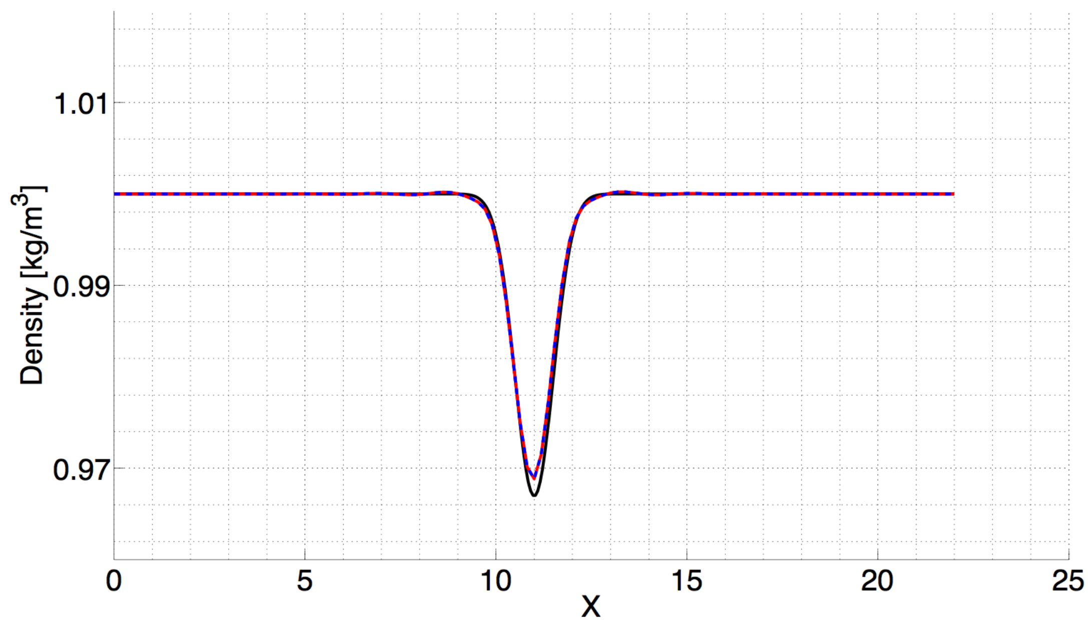
CD04 (11pt B&B) + IF10 (2/3Pi)

__ ref
__ IF10 (2/3Pi)
__ EF10





16 points across vortex



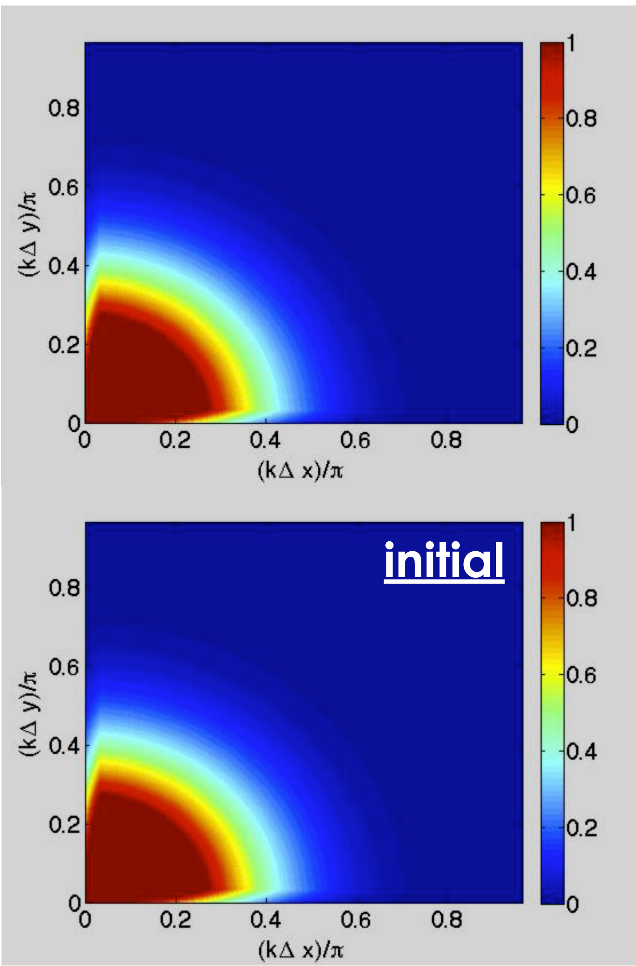
distance traveled: 500 widths

- traditional dissipation is sufficient when spectrum is well resolved

Isentropic Vortex: Scale-Discriminant Stabilization

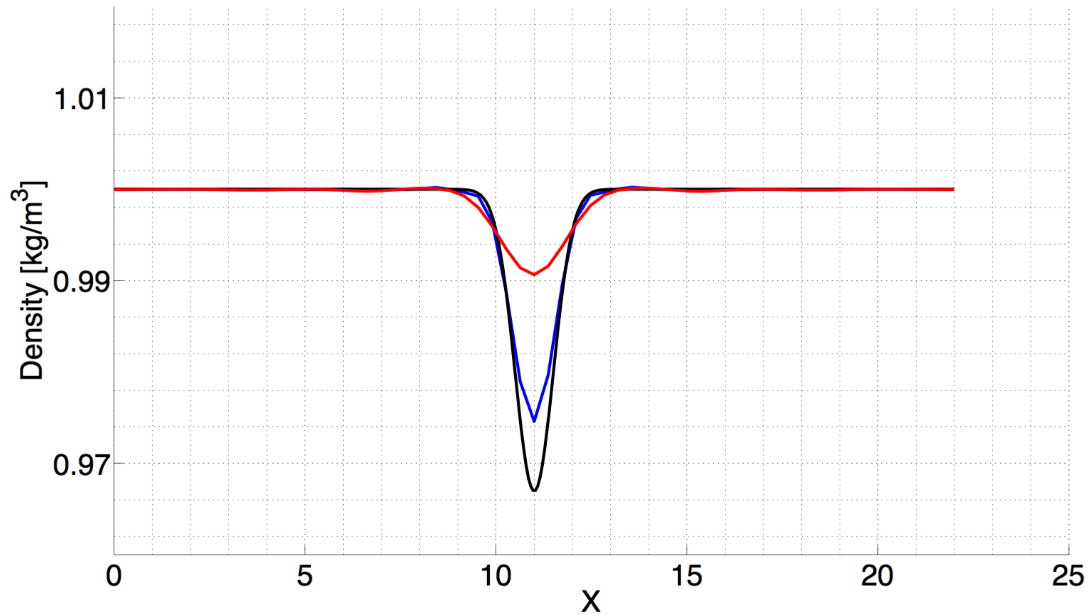
$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

<u>CD04 (11pt B&B) + IF10 (2/3Pi)</u>



__ ref
__ IF10 (2/3Pi)
__ EF10

<u>8 points across vortex</u>



distance traveled: 500widths

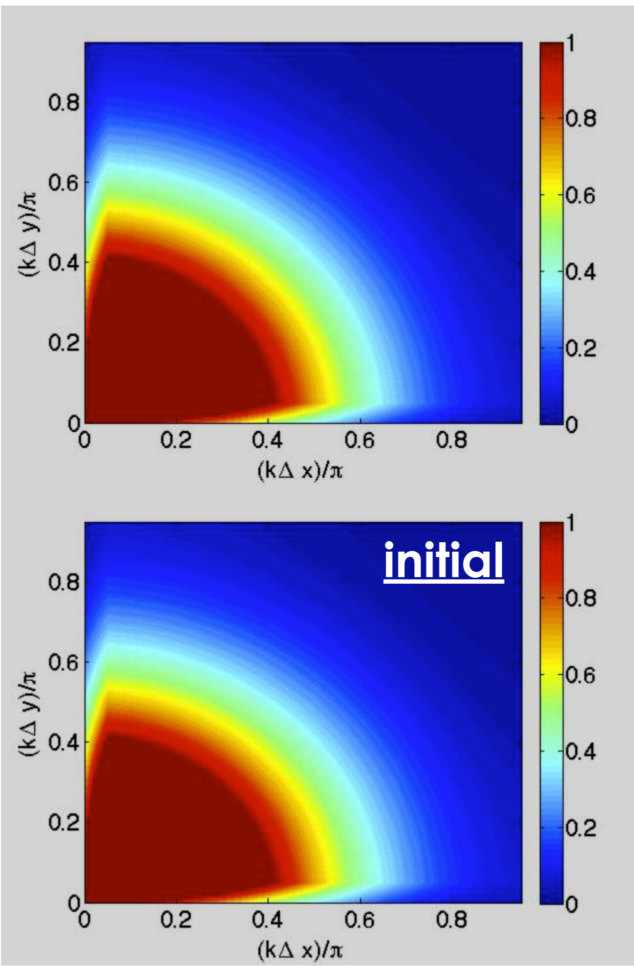
- scale-discriminant dissipation preserves structure
- robustness requires tuning to scheme resolvability
- efficacy limited by dissipation scheme

Isentropic Vortex: Scale-Discriminant Stabilization

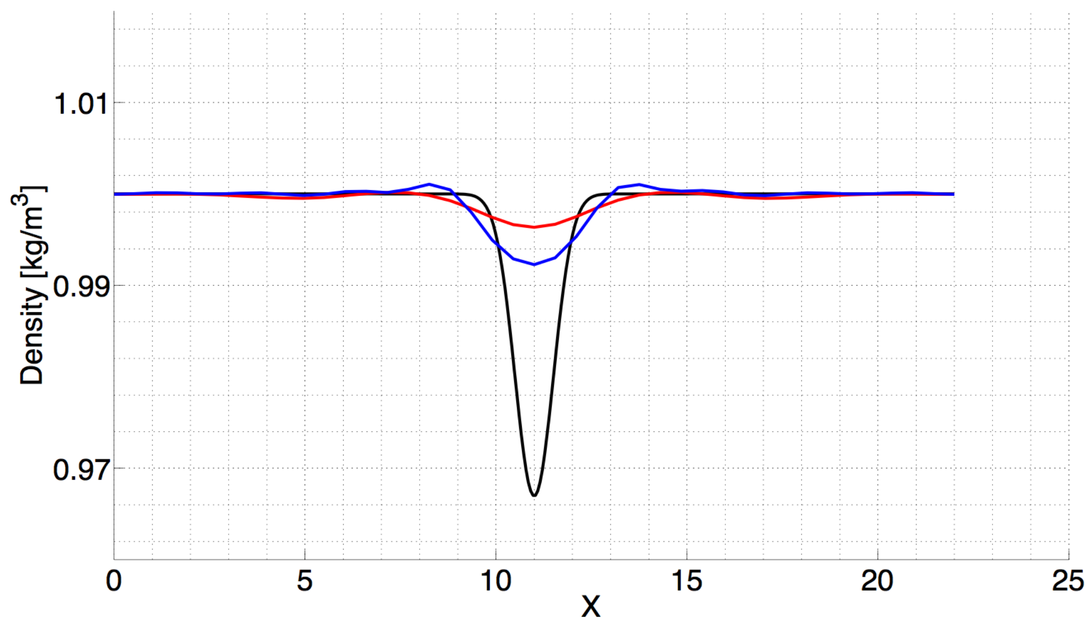
$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

CD04 (11pt B&B) + IF48 (2/3Pi)

__ **ref**
__ **IF48 (2/3Pi)**
__ **EF10**



4 points across vortex



distance traveled: 500widths

- scale-discriminant dissipation preserves structure
- robustness requires tuning to scheme resolvability
- efficacy limited by dissipation scheme

Conclusions/Going Forward

- non-linear instabilities instigated by high frequency error (i.e.: dispersion, aliasing)
- remove error with respect to overall scheme resolvability – a robust strategy
- scale-discriminant dissipation provides stability while minimizing dissipation error

going forward:

- non-linear stability (residual filtering, skew-symmetric forms etc…)
- incorporating temporal error
- applications to explicit LES