

Global Network Connectivity Assessment via Local Data Exchange for Underwater Acoustic Sensor Networks

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1 Introduction

Underwater sensor networks consist of a number of fixed or mobile sensors, deployed in the underwater environment. The sensors are capable of sending/receiving data using acoustic, optical, or radio-frequency communication channels [1], [2], [3]. A typical objective for such networks is to perform data aggregation for applications as diverse as environmental monitoring, underwater exploration, disaster prevention, climate reporting, and mine detection [1], [4], [5].

The acoustic communication channel is the most typical physical layer technology used in underwater sensors. Unlike the communication channels used in the terrestrial sensor networks, there are several sources of uncertainty which influence the communication between underwater nodes such as underwater currents, temperature fluctuations, multi-path fading, ambient noise, and sound speed profile [6], [7], [8]. Furthermore, these sources of uncertainty vary over time and space in an unpredictable manner, and hence one can use random graphs to model the underwater sensor networks more effectively [9]. This results in a time-varying network structure with the possibility of edge addition/deletion. Also, the node deletion can occur as a probable scenario for underwater applications due to limited sensor battery life [10].

Distributed computation over networks has attracted considerable attention during the last decade due to its efficiency in many cooperative control applications [11]. Consensus protocols are among the fully distributed computation techniques that are applicable to data aggregation tasks in sensor networks providing robustness to node failures. In order to achieve adequate data aggregation over a sensor network, connectivity preservation of the graph representing the network can be a sufficient and/or necessary condition [12], [13]. Thus, it is important to define a proper connectivity measure and develop an efficient algorithm to monitor, and if possible control, the network connectivity at all times. Although there exists various schemes for connectivity assessment in the literature [14], [15], [16], the corresponding algorithms are not distributed.

An adaptive algorithm for structure estimation of wireless underwater networks is devised in [9]. However, the approach is constrained merely to the case of complete expected graphs which imposes some constraints on the network communication. This limitation is relaxed to certain degree in the present publication by extending the results to the case where the expected graph is connected but is not complete.

In this publication, a distributed procedure is developed to estimate the connectivity degree of an expected communication graph representing a sensor network [17]. The sensors communicate based on a periodic broadcast schedule in such a way that data interference is avoided. An algorithm is employed by each sensor to upgrade its estimate of the expected communication graph. Moreover, an estimation method is proposed for every sensor to estimate the probability matrix of the underlying random network using the beliefs of its

neighbors. Then, weighted vertex connectivity is introduced as a novel metric of connectivity which extends the notion of vertex connectivity to random graphs. The simulation results confirm the efficacy of the proposed strategy.

The remainder of the publication is organized as follows. In Section 2, the problem is formulated and some relevant definitions are given. A distributed method to estimate the expected communication graph and the probability matrix of the network is introduced in Section 3. The weighted vertex connectivity degree is proposed in Section 4. The simulation results are provided in Section 5, and the concluding remarks are made in Section 6.

2 Problem formulation

The problem of distributed connectivity assessment for an underwater acoustic sensor network in a data aggregation application is investigated in this publication. Because of the random nature of the underwater communication channels, some notions about random graphs are borrowed from [12], [13], which are presented next.

Definition 1 Let $G = (V, E)$ denote a random graph composed of a set of n vertices V and a set of directed edges E . Let also the matrix $P = [p_{ij}]$ represent the existence probabilities for all possible edges, where $p_{ij} \in [0, 1]$ is the probability of the existence of the edge $(j, i) \in E$. Denote the adjacency matrix of the network by $A = [a_{ij}]$, and note that $(j, i) \in E$ if and only if $a_{ij} \neq 0$, where a_{ij} is defined by:

$$a_{ij} = \begin{cases} 1, & \text{with probability } p_{ij}, \\ 0, & \text{with probability } 1 - p_{ij}. \end{cases} \quad (1)$$

Definition 2 Define $\hat{G} = \mathbb{E}(G)$ as the expected graph of a random graph $G = (V, E)$, where $\mathbb{E}(\cdot)$ represents the expectation operator. The set of vertices of the expected graph $\hat{G}$ is denoted by $\hat{V}$ which is the same as the vertex set of G , and the set of its edges is denoted by $\hat{E}$. Furthermore, the weighted adjacency matrix of $\hat{G}$ is defined as $\hat{A} = [\hat{a}_{ij}]$, where $\hat{a}_{ij} = p_{ij}$ for all $i, j \in \hat{V}$. Moreover, $(j, i) \in \hat{E}$ if and only if $p_{ij} \neq 0$.

The sensor network considered in this work is composed of underwater sensors, which use acoustic waves for broadcast-based information exchange. In order to avoid transmission collisions and data interference, it is assumed that only one successful broadcast can occur at a time. In other words, a time slot is assigned to each sensor in a periodic manner for broadcasting its data. The *communication graph* of the underwater sensor network composed of n sensors is specified by a random digraph $G = (V, E)$, where its vertex and

edge sets are given by:

$$V = \{1, 2, \dots, n\}, \quad (2a)$$

$$E = \{(i, j) \mid i, j \in V, a_{ji} = 1\}. \quad (2b)$$

Also, $\hat{G} = (\hat{V}, \hat{E})$ denotes the *expected communication graph* of the underwater sensor network, where $\hat{V} = V$ and its edge set is characterized by:

$$\hat{E} = \{(i, j) \mid i, j \in \hat{V}, p_{ji} \neq 0\}. \quad (3)$$

From the results given in [12], a sufficient condition to reach asymptotic almost sure consensus in a network represented by a random graph G is that the expected graph $\hat{G}$ is strongly connected. Therefore, data aggregation is achieved successfully in an underwater sensor network as long as the expected communication graph remains strongly connected. Before introducing appropriate measures for the connectivity assessment of the underwater sensor network, it is required to estimate the topology of the expected communication graph and its underlying probability matrix in a distributed manner.

3 Distributed estimation of expected graph topology and probability matrix

3.1 Topology estimation

In the underwater sensor network under study, only one sensor is allowed to broadcast its data at any time instant in order to avoid data interference. A *broadcast cycle* of length T ($T \in \mathbb{R}_{>0}$) is considered, from which a time slot is dedicated to each sensor to broadcast its data. Denote the k -th broadcast cycle by $B(k) = [(k-1)T, kT)$, where $k \in \mathbb{N}$. Let $B^i(k) = [(k-1)T + \delta_i, (k-1)T + \delta_i + \Delta)$ be the k -th *broadcast interval* for sensor i ($i \in V$), where Δ ($\Delta \leq \frac{T}{n}$) is the length of each time slot, and $\delta_i \in [0, T - (n-i+1)\Delta)$ indicates the beginning of broadcasting for the sensor i in the initial broadcast cycle $B(1) = [0, T)$. The acoustic propagation time from any node to its neighboring nodes is taken into account by appropriately selecting Δ . The broadcast intervals $B^i(k)$'s constitute a family of disjoint subsets of $B(k)$ for every $k \in \mathbb{N}$. This implies that $|\delta_i - \delta_j| \geq \Delta$ for all pairs of distinct nodes $i, j \in V$.

Each node broadcasts its estimate of the global expected communication graph $\hat{G}$ during its designated broadcast interval. Moreover, the estimate of $\hat{G}$ is updated by each node before its broadcasting starts. The update procedure for the i -th node is performed using its previous estimate and the information it has received from the other nodes since its last broadcast. This time interval is referred to as the k -th *receive interval* for node i and is denoted by $R^i(k)$. Note that $R^i(k) = [(k-2)T + \delta_i + \Delta, (k-1)T + \delta_i)$ for all $k \geq 2$. The initial receive interval for node i is defined as $R^i(1) = [0, \delta_i)$ for any $i \in V$. A simple

example is given in Fig. 1 to illustrate the partitioning of the time axis for a network of three sensors during two broadcast cycles.

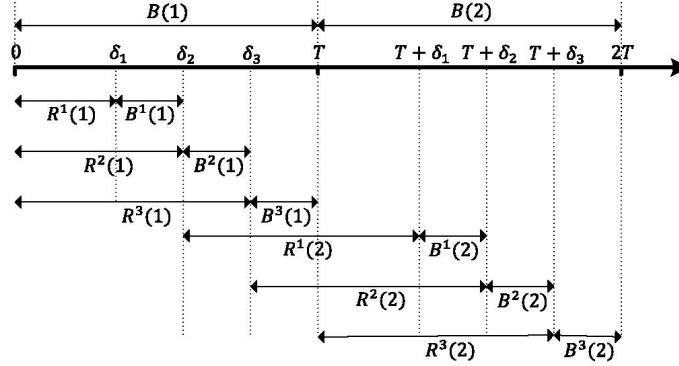


Figure 1: An example of periodic broadcast in a network of three sensors.

3.2 Probability matrix estimation

Let $X(k)$ represent a random variable with a Bernoulli distribution at discrete time instant $k \in \mathbb{N}$ such that $X(k) \in \{0, 1\}$. Assume that $p_0(k)$ and $p_1(k)$ denote the probabilities of two complementary events $X = 0$ and $X = 1$ at discrete time instant k , respectively. Then, the random variable $X(k)$ is described as:

$$X(k) = \begin{cases} 1, & \text{with probability } p_1(k), \\ 0, & \text{with probability } p_0(k), \end{cases} \quad (4)$$

where $0 \leq p_0(k), p_1(k) \leq 1$ and $p_0(k) + p_1(k) = 1$. Define $\hat{X}(k)$ as the estimation of $X(k)$ such that:

$$\hat{X}(k) = \begin{cases} 1, & \text{with probability } \hat{p}_1(k), \\ 0, & \text{with probability } \hat{p}_0(k). \end{cases} \quad (5)$$

The objective is to design an estimation procedure such that the expected values of $\hat{p}_0(k)$ and $\hat{p}_1(k)$ converge asymptotically to the real values of $p_0(k)$ and $p_1(k)$, respectively. In other words, $\mathbb{E}[\hat{p}_i(k)] \rightarrow p_i(k)$ as $k \rightarrow \infty$ for $i \in \{0, 1\}$. The update procedure of the proposed estimation scheme is given by:

$$\hat{p}_i(k+1) = \begin{cases} (1 - \alpha)\hat{p}_i(k) + \alpha, & \text{if } X(k) = i, \\ (1 - \alpha)\hat{p}_i(k), & \text{if } X(k) \neq i, \end{cases} \quad (6)$$

for $i \in \{0, 1\}$, where $\alpha \in (0, 1)$ denotes the learning rate of the estimation method. Let the probability vector representing two complementary events in a stationary Bernoulli distribution be denoted by $P = [p_0 \ p_1]^T$. It is desired to find the variance of the estimate of p_0 and p_1 given in (6), as time increases.

Theorem 1 Consider an unknown stationary Bernoulli distribution given by (4) and apply the procedure (6) to estimate the probability vector P . Then, $\mathbb{E}[\hat{P}(k)]$ converges to P as $k \rightarrow \infty$ in an asymptotic manner.

Theorem 2 Consider an unknown stationary Bernoulli distribution $X(k)$ and apply the procedure (6) to identify $X(k)$. Then, the variance of the estimated Bernoulli distribution as $k \rightarrow \infty$ is given by:

$$\text{Var}[\hat{p}_i(\infty)] = \frac{\alpha}{2 - \alpha} p_i(1 - p_i), \quad (7)$$

for $i \in \{0, 1\}$, where $\alpha \in (0, 1)$ denotes the learning rate of the update procedure.

Consider a stationary Bernoulli distribution with the estimated probabilities obtained by using (6), and let (7) denote the variance of the estimated probabilities as $k \rightarrow \infty$. It can be inferred that as $\alpha \rightarrow 0$, the variance tends to zero and results in more accurate estimation. On the other hand, the resulted estimation is deteriorated as $\alpha \rightarrow 1$ because of the increasing variance. Moreover, it can be shown that the learning rate $\alpha \in (0, 1)$ is directly proportional to the convergence rate of $\mathbb{E}[\hat{p}_i(k)]$ to p_i for $i \in \{0, 1\}$. Therefore, there is a trade-off between the convergence rate and the estimation accuracy in the choice of an appropriate learning rate α .

Remark 1 The results presented above correspond to stationary Bernoulli distributions. One can extend the results to the case of non-stationary Bernoulli distributions and derive an upper bound on the estimation error in terms of the sampling interval and the rate of change of the probabilities $p_0(k)$ and $p_1(k)$.

4 Weighted vertex connectivity measure

The objective of this section is to find a global metric to evaluate the connectivity degree of the expected communication graph $\hat{G}$ of a sensor network. It is known that the strong connectivity of $\hat{G}$ is a sufficient condition for asymptotic almost sure convergence to consensus over a network characterized by a random graph. Consider a group of sensors represented as the node set of a random digraph $G = (V, E)$, where the existence probability of every directed edge in E is assumed to have a Bernoulli distribution. Let the random variables describing the existence probability of all edges be independent and identically distributed.

To illustrate the importance of a connectivity measure in a random graph, consider the consensus problem for two random networks G_1 and G_2 composed of three sensors as shown in Fig. 2.

Let the purpose of information exchange between the nodes in each graph be to reach an agreement upon a certain quantity of interest. The simulation results, as given in Fig. 3,

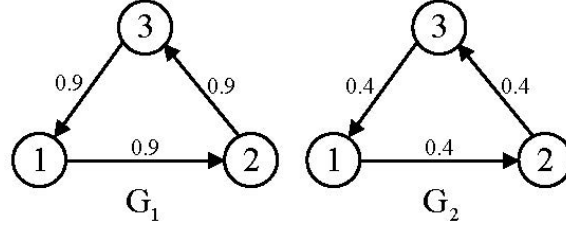


Figure 2: Two directed graphs with the same topology but different probabilities for the existence of edges.

demonstrate faster convergence for the network associated with the random graph G_1 which has a stronger connectivity due to higher probability of the existence of its edges.

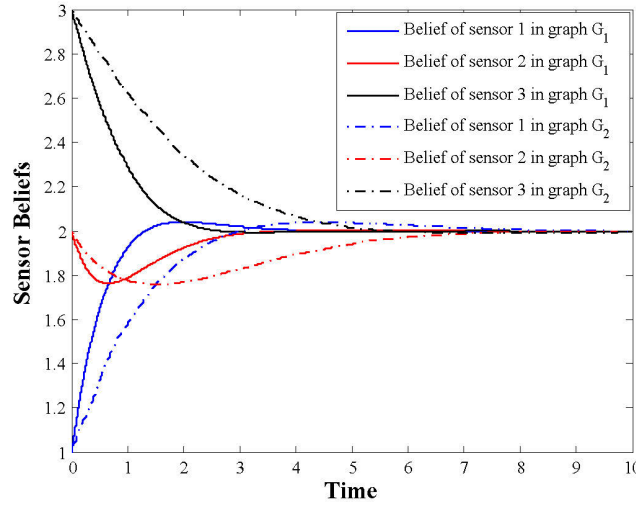


Figure 3: Consensus convergence of the sensor beliefs for the graphs G_1 and G_2 in Fig. 2.

The concept of *vertex connectivity* (VC) has been used as a measure to evaluate the global connectivity of digraphs [18]. The VC degree of G is defined as the minimum number of vertices that need to be removed such that G is no longer strongly connected. Let $G = (V, E)$ represent a digraph with node set V and edge set E . Then the VC degree of G , denoted by q , is defined as:

$$q = \min_{i,j \in V, i \neq j} q(i, j), \quad (8)$$

where,

$$q(i, j) = \begin{cases} N(i, j), & \text{if } (i, j) \notin E, \\ |V| - 1, & \text{if } (i, j) \in E, \end{cases} \quad (9)$$

and $N(i, j)$ denotes the maximum number of vertex-disjoint directed paths connecting i to j in G .

Several polynomial-time algorithms are provided in the literature to find the VC degree of a graph [19], [20], [21]. The general idea behind these algorithms is that the minimum number of vertices whose removal disconnects any pair of non-adjacent vertices is equal to the maximum number of mutually vertex-disjoint directed paths between them (see Menger's Theorem [18]).

However, this measure does not account for the probability matrix P and this calls for a more accurate measure of connectivity capturing the probabilistic nature of the environment. The *weighted vertex connectivity* (WVC) measure extends the notion of the VC degree to the more general case of weighted graphs. This nonnegative measure is positive for a strongly connected graph, and a larger value of this measure represents “stronger” connectivity. To clarify this new concept, the multiplicative path weight is defined, which is based on the mutual independence of the random variables used for describing the probabilistic nature of the network's edges.

Definition 3 *Let the ordered set of distinct nodes $\{i_0, i_1, \dots, i_m\}$ denote a directed path from i_0 to i_m in the expected communication graph $\hat{G}$. The multiplicative path weight of this directed path, denoted by $PW(i_0, i_m)$, is defined as follows:*

$$PW(i_0, i_m) = \prod_{k=1}^m p_{i_k i_{k-1}}, \quad (10)$$

where p_{ij} is the (i, j) element of the probability matrix P . Furthermore, the length of the above path is given by:

$$PL(i_0, i_m) = m. \quad (11)$$

Since every element of the matrix P represents the probability of the existence of the corresponding edge in the expected communication graph $\hat{G}$, the multiplicative path weight can be interpreted as the probability of the existence of a given path, as long as the probabilities of the existence of different edges of the path are mutually independent. In order to extend the notion of vertex connectivity degree to random graphs, it is first required to define the local WVC measure for any pair of distinct nodes $i, j \in \hat{V}$ in the expected communication graph $\hat{G} = (\hat{V}, \hat{E})$. This local measure, denoted by $\hat{q}(i, j)$, is defined as the maximum value of the summation of the multiplicative weights of all vertex-disjoint directed paths from i to j which belong to $\hat{G}$. In other words, $\hat{q}(i, j)$ represents the maximum of the summation of the existence probability of the vertex-disjoint paths from i to j . Let $\Upsilon(i, j)$ contain all possible directed paths from i to j in $\hat{G}$. Then:

$$\hat{q}(i, j) = \begin{cases} \hat{N}(i, j), & \text{if } (i, j) \notin \hat{E}, \\ \max((|\hat{V}| - 1)p_{ij}, \hat{N}(i, j)), & \text{if } (i, j) \in \hat{E}, \end{cases} \quad (12)$$

where,

$$\hat{N}(i, j) = \max_{\Xi(i, j) \subseteq \Upsilon(i, j)} \sum_{k=1}^{|\Xi(i, j)|} PW_k(i, j), \quad (13)$$

and $\Xi(i, j)$ represents a set of directed paths from i to j which are mutually vertex-disjoint. Then, the WVC metric, denoted by $\hat{q}$, is introduced as a global connectivity measure of the expected communication graph $\hat{G}$ as follows:

$$\hat{q} = \min_{i, j \in \hat{V}, i \neq j} \hat{q}(i, j). \quad (14)$$

The WVC measure $\hat{q}$ can be considered as an extension of the VC degree q for weighted graphs, where an edge weight in the present problem denotes the existence probability of that edge, and varies between zero and one. Furthermore, the WVC measure $\hat{q}$ is a non-negative real value ($\hat{q} \in \mathbb{R}_{\geq 0}$), and is more sensitive to changes in the network compared to the VC degree q which is a nonnegative integer ($q \in \mathbb{Z}_{\geq 0}$). Note that For two different expected communication graphs $\hat{G}_1$ and $\hat{G}_2$ with $q_1 > q_2$, it is possible that $\hat{q}_1 < \hat{q}_2$, depending on the probability matrices P_1 and P_2 . Note also that since the random nature of the environment is captured by the WVC measure, it is more suitable for the networks which are represented by random graphs.

4.1 Illustrative examples to compare the connectivity measures

Example 1: As the first example, consider the expected communication graph $\hat{G}$ shown in Fig. 4, where the elements of the probability matrix are shown as weights on all edges. Using the definition of VC degree and on noting that $\hat{G}$ is strongly connected, one obtains $q = 1$. Based on the definition of WVC measure, $\hat{q}(a, c) = \hat{q}(c, a) = 0.72$, which results in $\hat{q} = 0.72$.

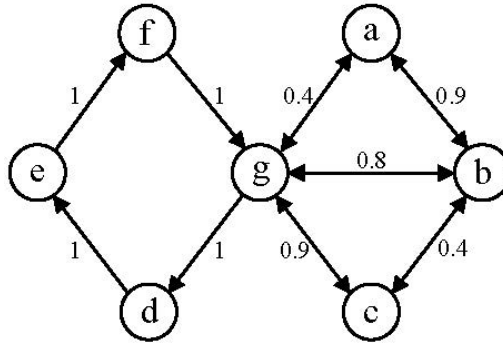


Figure 4: Expected communication graph of Example 1

Example 2: In the second example, the expected communication graph $\hat{G}$ shown in Fig. 5 is considered. This graph is similar to the one used in Example 1 with the difference that

the values of edge weights $p_{ag}, p_{ga}, p_{bc}, p_{cb}$ have been changed from 0.4 to 0.3 in this example. Like the previous example, the edge weights shown in this figure represent the existence probability of the corresponding edges. Since the graph is strongly connected and there exists at least one directed path between any pair of its distinct nodes, $\hat{G}$ is 1-vertex connected ($q = 1$) similar to Example 1. Unlike Example 1, the minimum local WVC measure is given by $\hat{q}(a, c) = \hat{q}(c, a) = 0.648$, which leads to $\hat{q} = 0.648$.

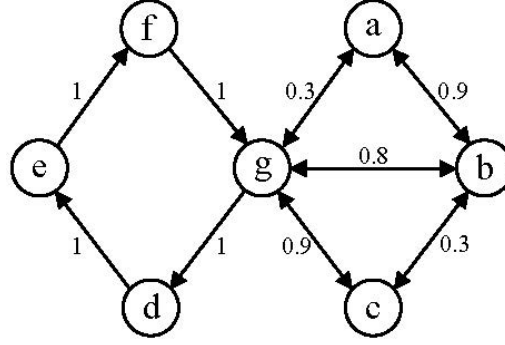


Figure 5: Expected communication graph of Example 2

Remark 2 The results developed in this work can be used to improve the performance of an underwater acoustic sensor network by identifying the relative contribution of each link in the connectivity of the overall network. For example, consider an underwater sensor network represented by the random graph given in Fig. 6. While there are two edges of

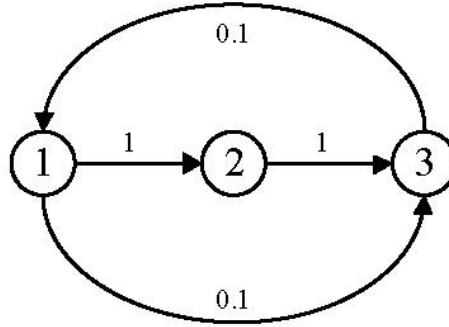


Figure 6: Expected communication graph of Remark 2

the same probability 0.1 in this graph, it results from the proposed connectivity measure that the edge from node 3 to node 1 plays a more important role than the edge from node 1 to node 3 in the connectivity of this graph because there exists another path from node 1 to node 3 with maximum probability of existence while the edge from node 3 to node 1 represents the only path from node 3 to node 1.

5 Simulation results

Consider a network of six underwater acoustic sensors with the periodic broadcast schedule described in Section 3. Assume that the existence probability of the edges in the communication digraph G is described by a time-varying matrix $P(k)$ given below:

$$P(k) = \begin{bmatrix} 0 & 0.5 & 0 & 0.8 & 0.7 & 0 \\ 0.6 & 0 & 0.6 & 0 & 0.5 & 0.9 \\ 0.7 & 0.6 & 0 & 0.9 & 0 & 0 \\ 0 & 0.7 & 0.4 & 0 & 0.9 & 0.7 \\ 0 & 0.7 & 0 & 0.9 & 0 & 0.7 + 0.3 \sin \omega k \\ 0.8 & 0 & 0.3 & 0.9 & 0 & 0 \end{bmatrix}, \quad (15)$$

where $\omega = 0.005$, and $k \in \mathbb{N}$ denotes the k -th broadcast cycle (note that the above matrix is only used in simulations, and is considered as an unknown matrix in all algorithms). The expected communication graph $\hat{G}$, which is constructed based on the random graph G with the edge weight matrix $P(k)$, has the following adjacency matrix:

$$\hat{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}. \quad (16)$$

The expected communication graph $\hat{G}$ induced by $\hat{A}$ is depicted in Fig. 7.

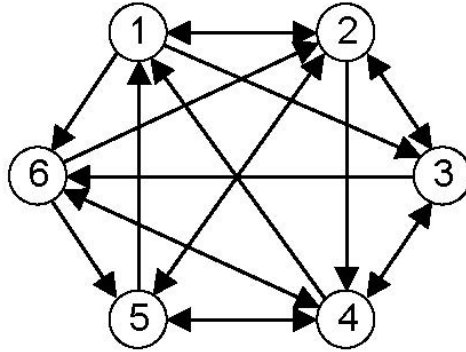


Figure 7: Expected communication graph $\hat{G}$.

Let the length of each broadcast cycle be $T = 6$ sec, and the length of each broadcast interval be $\Delta = 1$ sec. Let also the broadcast order of the i -th sensor be specified by $\delta_i = i - 1$ for $i \in \{1, 2, \dots, 6\}$. Using $\alpha = 0.01$ as the learning rate, the estimated network size

perceived by each node versus the number of broadcast intervals is shown in Fig. 8. On the other hand, Figs. 9 and 10 present estimates of the edge-set size and the VC degree, respectively, by each node. Fig. 8 demonstrates that all nodes identify the network size after 3 broadcast cycles, while Figs. 9 and 10 show that all nodes precisely estimate the edge-set size and VC degree after 10 broadcast cycles (note that each broadcast cycle consists of six broadcast intervals).

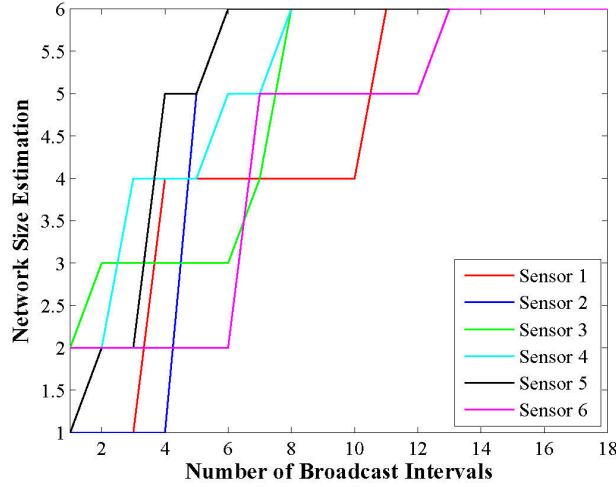


Figure 8: Network size estimation perceived by every sensor versus the number of broadcast intervals.

In order to verify the effectiveness of the probability estimation method and the proposed WVC measure, the corresponding algorithms are simulated for a sequence of 1000 broadcast cycles. The results of the probability estimation procedure to identify the constant element p_{15} and the time-varying element p_{56} are demonstrated in Figs. 11 and 12, respectively. Fig. 11 shows the efficacy of the estimation procedure for a stationary Bernoulli distribution, while Fig. 12 shows the performance of the proposed method in estimating a non-stationary Bernoulli distribution. In Fig. 13, the connectivity measures $\hat{q}$ from the viewpoint of sensor 1 is demonstrated.

6 Summary

Connectivity assessment of an underwater sensor network using distributed algorithms is investigated in this publication. The sensors are assumed to broadcast their information in a periodic manner using acoustic modems, and the underlying communication graph is modeled by a random graph due to unpredictable conditions of the underwater environment. An update procedure is used by sensors to identify the structure of the expected communication graph at first. Also, a novel estimation procedure is introduced to estimate the

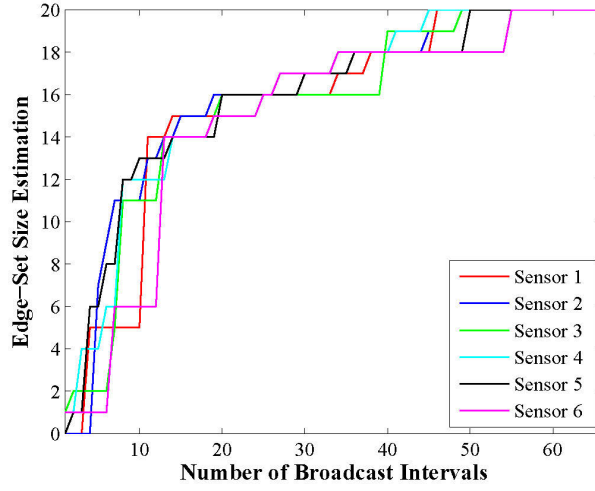


Figure 9: Edge-set size estimation perceived by every sensor versus the number of broadcast intervals.

probability matrix in a distributed manner by each sensor. Then, the weighted vertex connectivity is proposed as a novel measure to evaluate the connectivity of the network which, in fact, extends the vertex connectivity metric to the case of random graphs, capturing the probabilistic nature of the underwater sensor networks. The main challenge for the future work is to come up with computationally efficient algorithms to assess the connectivity of a random network. Simulation results demonstrate the effectiveness of the algorithms developed in this publication.

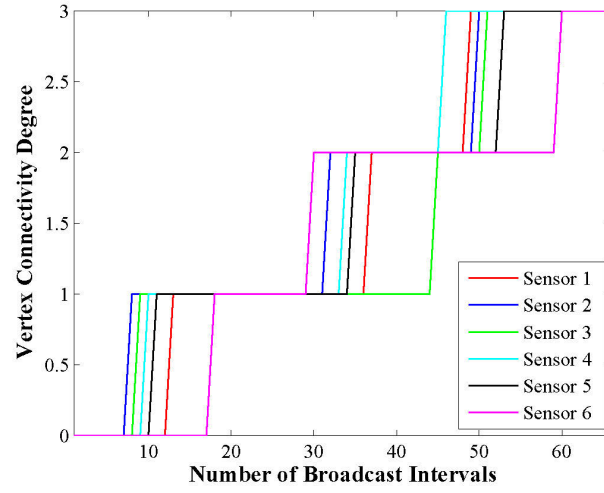


Figure 10: VC degree perceived by every sensor versus the number of broadcast intervals.

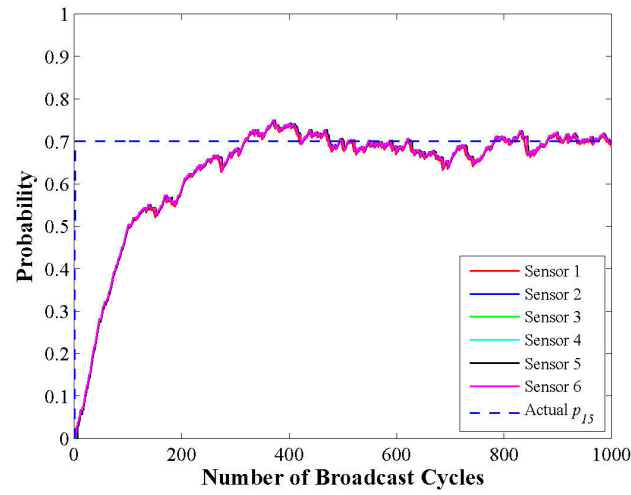


Figure 11: Estimated and actual probability p_{I5} perceived by every sensor versus the number of broadcast cycles.

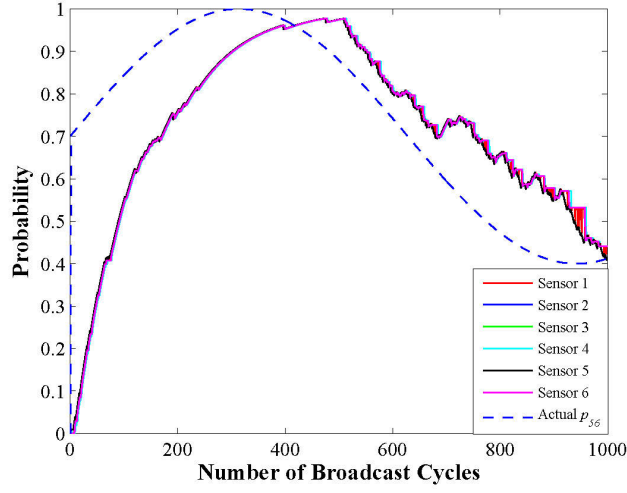


Figure 12: Estimated and actual probability p_{56} perceived by every sensor versus the number of broadcast cycles.

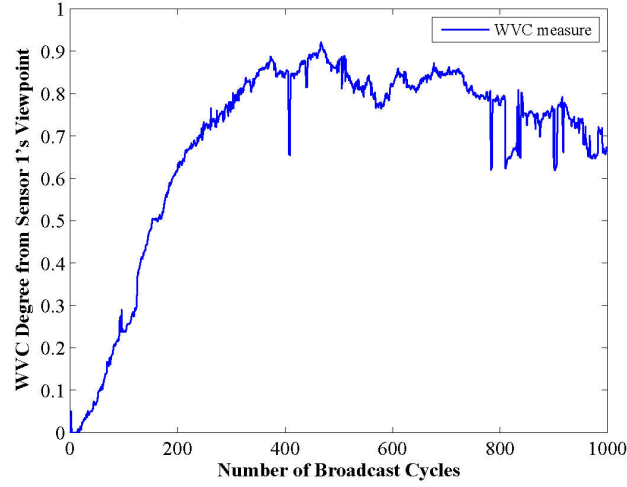


Figure 13: WVC metric $\hat{q}$ from the viewpoint of sensor 1.

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