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Electric field controlled spin interference in a system with Rashba spin-orbit coupling

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There have been intense research efforts over the last years focused on understanding the Rashba spin-orbit coupling effect from the perspective of possible spintronics applications. An important component of this line of research is aimed at control and manipulation of electron's spin degrees of freedom in semiconductor quantum dot devices. A promising way to achieve this goal is to make use of the tunable Rashba effect that relies on the spin-orbit interaction in a two-dimensional electron system embedded in a host semiconducting material that lacks inversion-symmetry. This way, the Rashba spin-orbit coupling effect may potentially lead to fabrication of a new generation of spintronic devices where control of spin, thus magnetic properties, is achieved via an electric field and not a magnetic field. In this work we investigate theoretically the electron's spin interference and accumulation process in a Rashba spin-orbit coupled system consisting of a pair of two-dimensional semiconductor quantum dots connected to each other via two conducting semi-circular channels. The strength of the confinement energy on the quantum dots is tuned by gate potentials that allow "leakage" of electrons from one dot to another. While going through the conducting channels, the electrons are spin-orbit coupled to a microscopically generated electric field applied perpendicular to the two-dimensional system. We show that interference of spin wave functions of electrons travelling through the two channels gives rise to interference/conductance patterns that lead to the observation of the geometric Berry's phase. Achieving a predictable and measurable observation of Berry's phase allows one to control the spin dynamics of the electrons. It is demonstrated that this system allows use of a microscopically generated electric field to control Berry's phase, thus, enables one to tune the spin-dependent interference pattern and spintronic properties with no need for injection of spin-polarized electrons. [http://dx.doi.org/10.1063/1.4952756]

I. INTRODUCTION

Past advances in the field of nanotechnology have enabled the precise, controlled fabrication of materials at atomic and molecular scales.[1,2] In nanoscale territory, the electron's quantum mechanical nature dominates with the payoff that electronic devices built on nanoscale show remarkable properties. A novel class of confined electronic systems called semiconductor quantum dot systems has been identified as one of the very promising avenues for meeting current technological challenges.[3–5] A typical semiconductor quantum dot system consists of a finite number of electrons confined in a two-dimensional (2D) region of space.[6] The great interest in 2D semiconductor quantum dot devices stems from the fact that such systems utilize the discreteness of the electron's charge and spin in a highly tunable way, thus, they offer a possible breakthrough in device/circuit technology.[7–16] In particular, there is great interest on electron's spin effects in 2D

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semiconductor quantum dot systems since such phenomena may lead to device applications on the field of spin-based electronics (spintronics).[17,18]

Spintronics deals with the control and manipulation of electron's spin degrees of freedom in a device. There is intensive research going on in the field of spintronics since it is believed that spin-based devices represent the next generation of electronic devices. It is widely known that an external magnetic field influences the spin of the electrons. However, all current mainstream electronic applications rely on the ability to manipulate the charge of electrons using electric fields. Thus, it would be extremely desirable if we could find a way to manipulate electron's spin via electric fields, too. The spin-orbit (SO) coupling effect creates a way to manipulate electron's spin by means of an electric field. Consider a 2D system of electrons confined (for instance, at the doped/undoped interface of a hetero-structure) of a semiconductor host that lacks inversion symmetry (e.g., *GaAs*, *InAs* or *InGaAs*). The essence of the SO coupling phenomenon is that the 2D system of electrons will feel an effective in-plane magnetic field when they move in presence of a perpendicular electric field that has broken the inversion symmetry. This explains the great current interest on the Rashba effect that relies on the SO interaction.[19,20] The key outcome of this scenario is that an electric field can tune the spin of electrons via SO coupling. The SO-induced spin splitting of the electron states may lead to many interesting applications.

A detectable SO effect requires a strong electric field (as well as a semiconductor host for the electrons that satisfies a number of specific criteria). In a typical experiment, sufficiently strong electric fields are microscopically generated by "strategically" placing nano/micron-sized gate electrodes on top of the 2D system of electrons/holes. Experiments show that gate-controlled microscopically generated electric fields of this nature are quite strong resulting in measurable SO interaction effects for a variety of systems.[21–23] In this work we investigate theoretically the spin interference process in a Rashba SO coupled 2D system of electrons consisting of a pair of 2D semiconductor quantum dots connected to each other via two conducting semi-circular channels. This system allows use of a perpendicularly applied microscopically generated electric field in conjunction with a weak perpendicular magnetic field to control the Berry's phase[24] acquired by the electrons. Hence, this way one can tune the spin-dependent interference pattern and spintronic properties of the system with no need for injection of spin-polarized electrons.

II. MODEL AND THEORY

We consider the following system consisting of two laterally coupled 2D semiconductor quantum dots on a $x - y$ plane. The two quantum dots are connected to each other via two conducting channels. We may assume that each of the two conducting channels are connected at a single connection point at each of the quantum dots. For simplicity, we may assume that each of the two conducting channels is perfectly half-circular. Electrons in each of the two quantum dots (which may be considered identical) are confined by an electrostatically created potential that can be tuned to allow "leakage" of electrons from one quantum dot to another. Electrons moving through the conducting channels are SO coupled to a sufficiently strong microscopically generated electric field, $\vec{E} = (0, 0, E)$ applied perpendicular to the 2D system. In a typical experimental setup, such an electric field is created by means of small gate electrodes placed in vicinity of the system using well-established fabrication techniques.[21–23] We also assume that a static magnetic field, $\vec{B} = (0, 0, B)$ (that may be weak, not necessarily strong) is applied perpendicular to the 2D plane of motion of electrons. A schematic view of the system under consideration is given in Fig. 1. If only a magnetic field (like $\vec{B}$) is applied, such a field acts upon the electron's spin through the Zeeman effect term:

$$\hat{H}_Z = \frac{g\,\mu_B}{\hbar}\,\vec{B}\,\hat{\vec{S}}, \tag{1}$$

where g is the electron's g-factor (or the effective one for that particular semiconductor host), μ_B is the Bohr magneton of the electron (or the effective one for that particular semiconductor host), $\hbar$ is the reduced Planck's constant and $\hat{\vec{S}}$ denotes the spin operator of the electron (a fermion with spin $S = 1/2$).

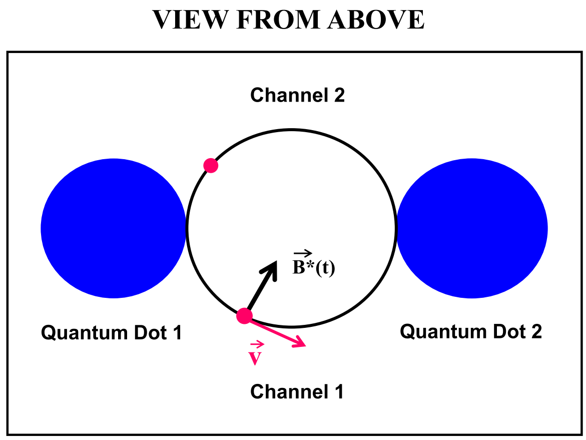

FIG. 1. View from above of the proposed 2D spintronic system. Both the microscopically generated electric field, $\vec{E}$ and the magnetic field, $\vec{B}$ are perpendicular to the 2D system (with direction pointing towards the reader) and are not drawn. The two quantum dots (Quantum Dot 1 and Quantum Dot 2) are connected to each other via two semi-circular conducting channels (Channel 1 and Channel 2). Electrons may travel from Quantum Dot 1 to Quantum Dot 2 via either Channel 1 or Channel 2. Because of SO coupling effects an electron (for instance, moving through Channel 1) with instantaneous velocity, $\vec{v}$ will sense an additional effective magnetic due the applied microscopic electric field. The resulting time-varying effective magnetic field, $\vec{B}^{*}(t)$ is in-plane and given from Eq. (2).

The essence of the SO coupling phenomenon for such a setup is that the microscopically generated electric field under consideration gives rise to an effective magnetic field which on turn acts on the electron's spin. The origin of the effect is relativistic and it can be shown that such an effective magnetic field reads:

$$\vec{B}^{*}(t) = \frac{\vec{E} \times \vec{v}}{2\, c^{2}}, \tag{2}$$

where $\vec{E}$ is the applied electric field, $\vec{v}$ is the velocity of the electron and c is the speed of light in vacuum. Note that, differently from $\vec{B}$, the effective magnetic field in Eq. (2) is not a constant quantity. For the sake of simplicity, we can assume that $\vec{B}^{*}(t)$ represents a vector with a constant magnitude but uniformly changing direction. Despite the relativistic origin of the effect, the effective magnetic field is not negligible when dealing with microscopically generated electric fields even when $|\vec{v}| \ll c$. The textbook example of the Bohr model of the Hydrogen atom shows that when the electron is at its lowest energy state (at a distance from the proton equal to one Bohr radius), the electric field felt by the electron is $E \approx 5.14 \times 10^{11}\, V/m$. The speed of an electron at that distance from the proton, calculated from the Bohr model, can be shown to be $v \approx 2.19 \times 10^{6}\, m/s$. Insertion of these values into Eq. (2) leads to an effective magnetic field of about 6.25 Tesla! Advances in nano-fabrication methods and techniques have made possible the creation of strong microscopically generated electric fields that ultimately lead to effective magnetic fields that are quite large.[21–23]

For the setup in Fig. 1, the electrons are restricted to move on a 2D $x - y$ plane subject to an electric field, $\vec{E} = E\, \vec{e}_z$ where $\vec{e}_z$ is a unit vector in the z-direction perpendicular to the 2D layer. In such a case, the Rashba SO part of the quantum Hamiltonian is written as:

$$\hat{H}_R = \alpha_R \left(\vec{e}_z \times \hat{\vec{k}} \right) \hat{\vec{\sigma}} \propto \left(\vec{E} \times \hat{\vec{v}} \right) \hat{\vec{S}}, \tag{3}$$

where α_R represents the Rashba SO coupling strength parameter (which can be estimated from experimental data), $\hat{\vec{\sigma}}$ are Pauli's spin matrices, $\left(\vec{E} \times \hat{\vec{v}} \right) \propto \vec{B}^{*}(t)$ represents an in-plane effective magnetic field acting on the electrons, $\hbar\, \hat{\vec{k}} = m_e\, \hat{\vec{v}} = \left(\hat{\vec{p}} + e\, \vec{A} \right) = \hat{\vec{\pi}}$ is the kinetic momentum operator, $\hat{\vec{v}} = \hat{\vec{\pi}}/m_e$ is the velocity operator for an electron, $-e$ $(e > 0)$ is electron's charge, m_e is electron's mass (or, more precisely, electron's effective mass for the given semiconductor host), $\vec{A}$ is the magnetic gauge vector potential and $\hat{\vec{S}} = \frac{\hbar}{2}\, \hat{\vec{\sigma}}$, as usual.

It is well-known that the Hamiltonian in Eq. (3) is the SO term of the relativistic Dirac Hamiltonian:

$$\hat{H}_{SO} = \frac{e\,\hbar}{(2\,m_e\,c)^2}\left(\vec{E}\times\hat{\vec{\pi}}\right)\hat{\vec{\sigma}}. \tag{4}$$

A comparison of Eq. (3) to Eq. (4) allows one to obtain the explicit form of α_R as:

$$\alpha_R = \frac{e\,\hbar^2\,E}{(2\,m_e\,c)^2}. \tag{5}$$

Let's now consider the transport of electrons from one 2D semiconductor quantum dot to the other via the two half-circular conducting channels. We imagine a situation where the electrons in the first quantum dot (all in the same quantum state) travel to the other quantum dot through each of the two conducting channels. We assume that roughly half of them go through either channel before being recombined. The Aharonov and Bohm (AB) effect[25] theory explains that when electrons arrive in the other quantum dot, they arrive with a phase difference. This phase difference is proportional to the magnetic flux their paths encircle once the curve is closed:

$$\Gamma = \frac{e}{\hbar}\Phi, \tag{6}$$

where Φ is the magnetic flux through the closed loop. When the two groups of electrons are recombined, the total wave function has the form:

$$\Psi = \Psi_0 + \Psi_0\,e^{i\,\Gamma}, \tag{7}$$

where Ψ_0 is the wave function of one group of electrons that went through the same channel (for instance, channel 1). In this case, the magnitude squared of the wave function can be calculated:

$$|\Psi|^2 = 4\,|\Psi_0|^2\,\cos\left(\frac{\Gamma}{2}\right). \tag{8}$$

As a result, one can easily estimate Γ by looking at the interference patterns. What has been stated so far seems to be quite generic. However, a new "tweak" in the theory comes from the presence of the "effective" in-plane magnetic field, $\vec{B}^*(t)$. As seen from Eq. (2), such an "effective" magnetic field depends on the electron's instantaneous wave vector (velocity quantum operator) and on the microscopically generated electric field. In addition to lifting the spin degeneracy, this effective magnetic field tilts the total magnetic field away from the vertical z-direction:

$$\vec{B}_{tot}(t) = \vec{B} + \vec{B}^*(t)\,. \tag{9}$$

As a result, the total magnetic field (which is perpendicular to the velocity vector of the electrons) is neither "vertical" (in the z direction), nor "horizontal" (in the $x-y$ plane). For such a scenario, we have a total magnetic field, $\vec{B}_{tot}(t)$ that precesses around uniformly with some precession angular frequency, ω (as determined by the Rashba SO effect parameters) while making a fixed angle with the z-axis. The spin of the travelling electrons may or may not follow the total magnetic field along the tilted direction. All depends on the relative strength of the precession frequency (that is tuned by the applied microscopic electric field) with respect to the frequency $\omega_0 = e\,|\vec{B}_{tot}(t)|/m_e$. That said, the important point made here is that, for the given circumstances, a tunable microscopically generated electric field can control the spin state of the electrons. In the non-adiabatic regime ($\omega \gg \omega_0$), it is expected that the system with change between spin up and spin down states. However, for slow rotations (in the adiabatic regime), the total magnetic field will guide the electrons around. This is the standard situation that leads to a new type of phase difference in the wave functions of recombining electrons called Berry's geometric phase[24] written as:

$$\gamma = i\oint\left\langle\psi_n(\vec{R})\middle|\vec{\nabla}_{\vec{R}}\psi_n(\vec{R})\right\rangle\cdot d\vec{R}\,, \tag{10}$$

where the closed line integral is taken around a closed curve in parameter space, denoted as $\vec{R}\equiv\vec{R}(t)$ (for instance, such a time-dependent parameter is the azimuthal angle of the precessing total magnetic field, namely, the rotating angle of the SO effective magnetic field), $\psi_n(\vec{R})$ is a time-dependent spin state which depends on t through $\vec{R}$ (thus, precessing angle) and $\vec{\nabla}_{\vec{R}}$ is to be interpreted as a gradient operator in parameter space. The outcome is a spatial modulation of the net spin polarization of the current, which can be controlled by means of a locally applied electric field and detected through observation of interference patterns. This suggests a feasible avenue to build spintronic devices in which electron spin states are split by the Rashba SO effect and spin currents would be detected by measuring variations of the interference/conductance patterns (that, normally, are expected to show oscillatory behavior).

Since the Berry's phase acquired by travelling electrons is varied independently of the AB phase, one should expect changes on the characteristic AB interference patterns due to the appearance of Berry's phase. As a result the expected AB oscillatory patterns of the conductance as a function of the magnetic field (due to AB interference) will be influenced by the additional Berry's phase acquired by the interfering electrons. While detailed patterns of this behavior should depend on the system/model being investigated, it is expected that changes in the oscillatory AB-induced conductance (and/or transmission probabilities) would broadly follow the patterns seen in simpler systems already studied via various methods.[26–28] Such changes would lead to the detection of Berry's phase. It is important to point out that, in this model, a microscopically generated electric field that can be tuned experimentally with a gate voltage controls Berry's phase. Detection of Berry's phase, in return, provides us information on the spin dynamics of electrons.

III. SUMMARY AND CONCLUSIONS

It has been demonstrated in the past that injection of spin-polarized electrons from/to various materials is doable.[29] However, spin manipulation in these devices is done by magnetic fields and not electric fields. Therefore, this line of research faces a major challenge which is controlled manipulation (injection, transport and detection) of electron's spin in a functional electronic device without a magnetic field. This explains the current interest on devices that would operate on principles of the Rashba SO effect where one may control the electron's spin, thus, overall magnetic properties, via an electric field. The Rashba SO effect is found to be very pronounced in certain 2D semiconductor systems of electrons where its strength can be controlled experimentally by using a conveniently placed gate voltage. Thus, these materials represent a promising avenue to build spintronic devices where control and manipulation of spin degrees of freedom in a 2D system of electrons is done by using an electric field. This is very important for the future of spintronic technologies.[30]

In this work we focused our attention on highly-tunable systems such as 2D semiconductor quantum dots in which electrons are confined in a semiconductor host that lacks inversion symmetry and, thus, may manifest a sizeable Rashba SO effect. While transporting through the conducting channels, the 2D electrons are SO coupled to a microscopically generated perpendicular electric field tuned by means of gate electrodes. A weak magnetic field which is perpendicular to the 2D system gives rise to the usual AB phase when electrons recombine after having travelled in a closed path through the device. The Rashba SO effect manifests itself as an additional in-plane (radial) time-varying effective magnetic field. The key outcome of the effect is an additional geometric Berry phase accumulated by the wave functions of the recombining electrons. Since AB and Berry phases can be varied separately, the interplay between these two phases may lead to intricate behavior. The most obvious expectation is observation of changes on the normally oscillating AB interference patterns of electrons when they recombine after having gone through the two semi-circular conducting channels. Arguably, changes of the expected AB interference patterns are to be attributed to the presence of Berry's phase. Therefore, detection of extra features on measured AB oscillation patterns lead to the observation of the geometric spin Berry's phase. Achieving a predictable observation of Berry's phase allows one to control the spin dynamics of electrons.

We also emphasized the importance of 2D semiconductor quantum dot systems in the context of novel spintronic device technologies where spin is manipulated via a microscopically generated electric field and detected via interference/conductance experiments. Specifically speaking, we investigated the spin interference and accumulation process patterns in a distinct Rashba SO coupled system consisting of a pair of 2D semiconductor quantum dots connected to each other via two semi-circular channels. Interference of electron spin wave functions when they recombine after travelling accross the two conducting channels of the device leads to the observation of the geometric Berry's phase. This model allows use of a gate-controlled microscopically generated electric field to tune the Berry's phase of electrons which in turn influences the nature of AB oscillations. This ultimately leads to observable spin-dependent interference features. Past studies of simpler systems have indicated that also amplitudes of the AB oscillations are affected by Berry's phase.[31] Thus, it is possible that something qualitatively similar may also happen in the present case enhancing the overall effect. Differently from earlier proposals such as the Datta and Das' two-dimensional elecron gas (2DEG) spin transistor,[29] in this setup, there is no need for injection of spin-polarized electrons from one quantum dot to the other. Therefore, the proposed spintronic device would operate without any further requirement regarding injection of spin-polarized electrons. We further argue that the system under consideration can represent a highly tunable and reliable spintronic device given that interference effects are expected to be generally robust and relatively insensitive to experimental fabrication details.

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