A REDUCED-ORDER MODEL FOR EVALUATING THE DYNAMIC RESPONSE OF MULTILAYER PLATES TO IMPULSIVE LOADS

Weiran Jiang, Alyssa Bennett, Nickolas Vlahopoulos,
University of Michigan

Matthew Castanier, Ravi Thyagarajan,
US Army TARDEC

SAE INTERNATIONAL

UNIVERSITY OF MICHIGAN

TARDEC

Outline

- **Motivation**
- **Technical Approach**
- **Reduced Order Model (ROM)**
- **Validation**
  - by Spectrum Finite Element Analysis (SFEA)
  - by Numerical Analysis in Nastran
- **Dynamic Response Index (DRI) and the Screening Metric**
- **Design Optimization**

Motivation

Research Goal:

Design a light weight occupant-centric vehicle structures with high levels of protection against underbody explosive events.

- A design philosophy that improve vehicle survivability as well as force protection by mitigating soldier injury due to underbody IED & mine blast is necessary.
- Size and weight would also be paramount factors for Army vehicles in order for faster transport, greater fuel conservation, higher payload and higher mobility.
- Develop innovative multilayer materials or structures to optimize the dynamic performance as a mechanism to absorb and spread energy from an impulsive load.

Technical Approach

- Reduced order modeling of multi-layered plates for rapid evaluation of the dynamic characteristics for a large numbers of alternative configurations.

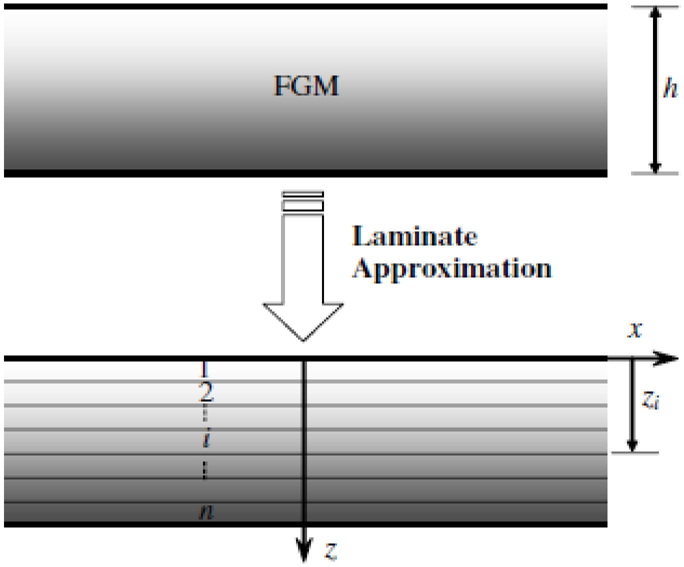
- Using Dynamic Response Index (DRI) for injury as a metric in underbody explosive events.

- Developing a meaningful screening process using the reduced order models.

- Optimizing the structural weight and levels of protection of the multilayer plates with a good combination of materials.

Reduced Order Model (ROM)

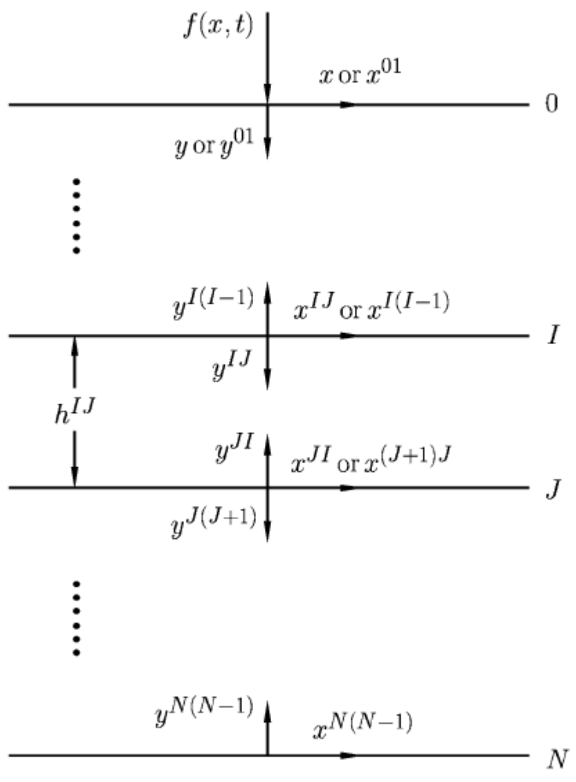
Based on the concepts of Functional Graded Materials (FGM), the Reverberation Matrix Method (RMM) is employed to build a reduce order model (ROM) to analyze the frequency response of an infinite multilayered medium under an external excitation.



The material properties of each sublayer, including modulus of elasticity (E_i), Poisson ratio (ν_i), mass density (ρ_i), loss factor (η_i), and the thickness (h_i) of sublayer will be assigned.

Reduced Order Model (ROM)

1. Define Local Coordinates at Interfaces and Boundaries



- Given a solid medium with N sublayers which has (N-1) interfaces and 2 boundaries.
- Define a coupled local coordinate systems (x^{IJ}, y^{IJ}) and $\left(x^{I(I-1)}, y^{I(I-1)}\right)$ at each interface – 4 DOFs
- Define a regular local coordinates at top boundary (x^{01}, y^{01}) and bottom boundary $\left(x^{N(N-1)}, y^{N(N-1)}\right)$ - 2DOFs
- Define p-wave potential $\varphi\,(x, y, t)$ and s-wave potential $\psi(x, y, t)$ in space and time domain for each local coordinate.

Reduced Order Model (ROM)

2. Double Fourier Transform and Elastodynamics Equations

- Wave potentials $\varphi\ (x, y, t)$ and $\psi\ (x, y, t)$ satisfy the wave equations:

$$\nabla^2\varphi = \frac{1}{{c_p}^2}\frac{\partial^2\varphi}{\partial t^2} \quad \textit{and} \quad \nabla^2\psi = \frac{1}{{c_s}^2}\frac{\partial^2\psi}{\partial t^2}$$

Where $c_p = \sqrt{\frac{\lambda+2\mu}{\rho}}$: (P-wave speed) and $c_s = \sqrt{\frac{\mu}{\rho}}$: (SV-wave speed)

- Apply double Fourier Transform on the wave equations:

$$\frac{d^2\tilde{\varphi}}{dy^2} + \alpha^2\tilde{\varphi} = 0 \ ; \frac{d^2\tilde{\psi}}{dy^2} + \beta^2\tilde{\psi} = 0$$

Where $\alpha = \sqrt{\frac{\omega^2}{{c_p}^2} - k^2}$: (p-wave number in y direction)

$\beta = \sqrt{\frac{\omega^2}{{c_s}^2} - k^2}$: (s-wave number in y direction)

And $\tilde{\varphi} = \tilde{\varphi}(k, y, \omega)$, $\tilde{\psi} = \tilde{\psi}(k, y, \omega)$ are now the potentials in the frequency (ω) and wave number (k) domain.

Reduced Order Model (ROM)

2. Double Fourier Transform and Elastodynamics Equations

- The solution of the wave equations can be expressed as:

$$\tilde{\varphi}(k,y,\omega) = \tilde{a}_p(k,\omega)e^{-i\alpha y} + \tilde{d}_p(k,\omega)e^{i\alpha y}$$
$$\tilde{\psi}(k,y,\omega) = \tilde{a}_s(k,\omega)e^{-i\beta y} + \tilde{d}_s(k,\omega)e^{i\beta y}$$

Where $\tilde{a}_p, \tilde{a}_s$ are the unknown arrival wave amplitudes;

$\tilde{d}_p, \tilde{d}_s$ are the unknown departure wave amplitudes;

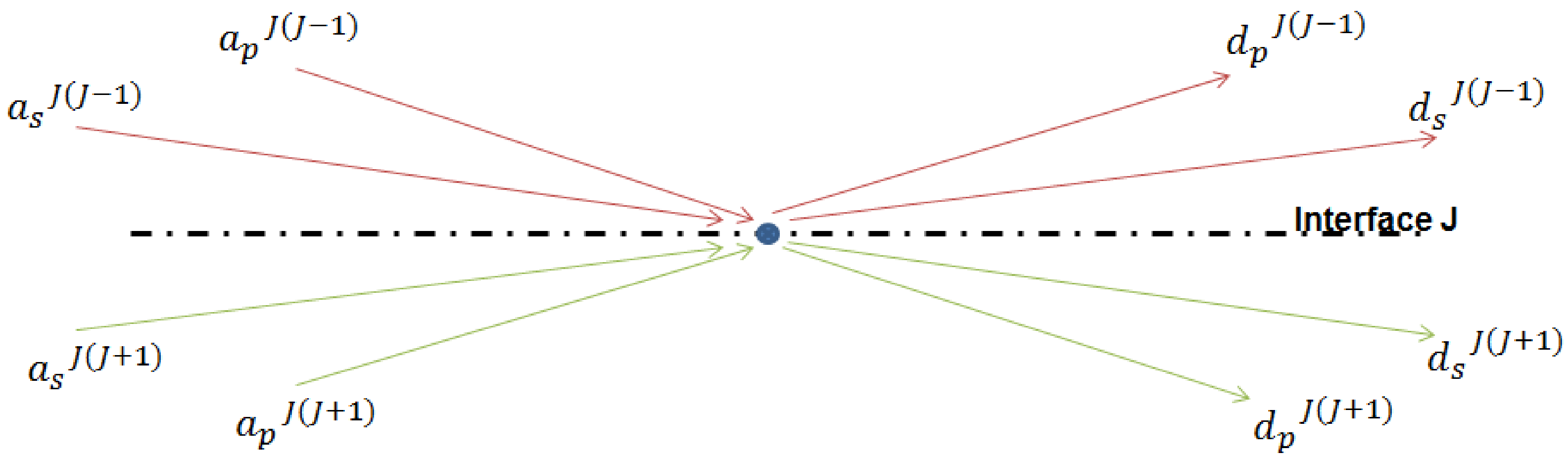
- The displacement field $u = [u_x, u_y]$ and the stress components $\sigma = [\sigma_{xy}, \sigma_{yy}]$ can be calculated by:

$$u_x = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial y}; \quad \sigma_{xy} = \mu[2\frac{\partial^2\varphi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}];$$
$$u_y = \frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial x}; \quad \sigma_{yy} = \lambda\nabla^2\varphi + 2\mu[\frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\psi}{\partial x\partial y}];$$

in each local coordinates system.

Reduced Order Model (ROM)

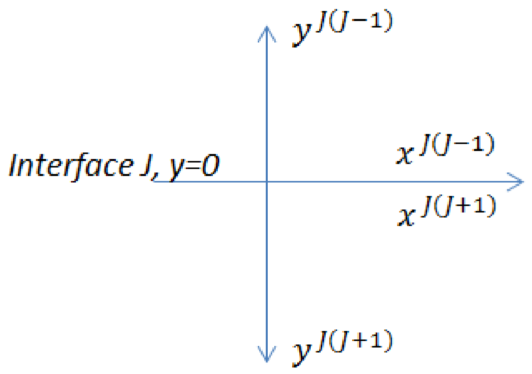
3. Scattering Matrices for Waves at Interfaces and Boundaries



- The local scattering matrix S^J at interface J is obtained by relating incident waves (arrival) to the transmitted/reflected waves (departure).
- The source wave vector s^J needs to be considered if any wave sources exist at interface J, where J = 1,2,..,N.
- a represents the "arrival waves "; d represents the "departure waves "
- p represents the "P-wave"; s represents the "SV-wave"

Reduced Order Model (ROM)

3. Scattering Matrices for Waves at Interfaces and Boundaries



- The displacements and the stresses have continuity relation at the interfaces:

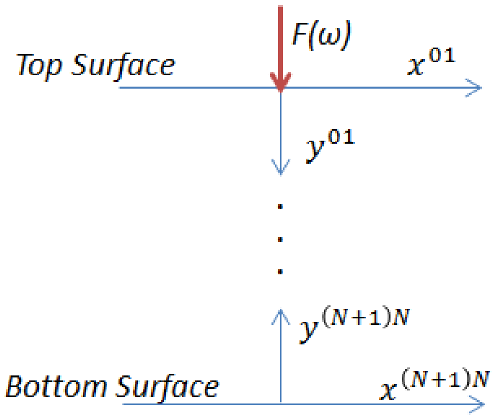
$$\tilde{u}_x{}^{J(J-1)}(k,\omega) - \tilde{u}_x{}^{J(J+1)}(k,\omega) = 0;$$
$$\tilde{u}_y{}^{J(J-1)}(k,\omega) + \tilde{u}_y{}^{J(J+1)}(k,\omega) = 0;$$
$$\tilde{\sigma}_{xy}{}^{J(J-1)}(k,\omega) + \tilde{\sigma}_{xy}{}^{J(J+1)}(k,\omega) = 0;$$
$$\tilde{\sigma}_{yy}{}^{J(J-1)}(k,\omega) - \tilde{\sigma}_{yy}{}^{J(J+1)}(k,\omega) = 0;$$

- By substituting the elastodynamics solutions, the above equations can be expressed in a linear system as: $\boldsymbol{A}^J \cdot \tilde{\boldsymbol{a}}^J + \boldsymbol{D}^J \cdot \tilde{\boldsymbol{d}}^J = \tilde{\boldsymbol{g}}^J(k,\omega) = 0$; where $\boldsymbol{A}^J$ and $\boldsymbol{D}^J$ are both 4x4 matrices which represent the material and physical properties of the adjacent layers of interface J.

- The linear system can be also written as: $\tilde{\boldsymbol{d}}^J = \boldsymbol{S}^J \cdot \tilde{\boldsymbol{a}}^J + \boldsymbol{s}^J$; where $\boldsymbol{S}^J = -(\boldsymbol{D}^J)^{-1} \cdot \boldsymbol{A}^J$, is the local scattering matrix at interface J
  $\boldsymbol{s}^J = -(\boldsymbol{D}^J)^{-1} \cdot \tilde{\boldsymbol{g}}^J = 0$, is the local source vector at interface J

Reduced Order Model (ROM)

3. Scattering Matrices for Waves at Interfaces and Boundaries



- For the top and bottom boundaries:

$$\tilde{\sigma}_{xy}{}^{01}(k,\omega) = \tilde{\sigma}_{xy}{}^{N(N+1)}(k,\omega) = \tilde{\sigma}_{yy}{}^{N(N+1)}(k,\omega) = 0;$$

$$\tilde{\sigma}_{yy}{}^{01}(k,\omega) = F(\omega);$$

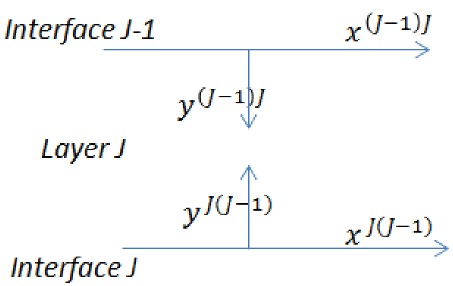
$\boldsymbol{S}^0$ and $\boldsymbol{S}^N$ now degenerate into 2x2 matrices, and the source vector $\boldsymbol{s}^0 \neq 0$ if loads apply on top surface.

- Construct a 4Nx4N global scattering matrix for entire multilayered plate:

$$\begin{pmatrix} \tilde{d}^0 \\ \tilde{d}^1 \\ \tilde{d}^2 \\ \vdots \\ \tilde{d}^{N-2} \\ \tilde{d}^{N-1} \\ \tilde{d}^N \end{pmatrix} = \begin{pmatrix} \boldsymbol{S}^0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & \boldsymbol{S}^1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{S}^2 & & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & & \boldsymbol{S}^{N-2} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \boldsymbol{S}^{N-1} & 0 \\ 0 & 0 & 0 & & 0 & 0 & \boldsymbol{S}^N \end{pmatrix} \begin{pmatrix} \tilde{a}^0 \\ \tilde{a}^1 \\ a^2 \\ \vdots \\ \tilde{a}^{N-2} \\ \tilde{a}^{N-1} \\ \tilde{a}^N \end{pmatrix} + \begin{pmatrix} \tilde{s}^0 \\ \tilde{s}^1 \\ \tilde{s}^2 \\ \vdots \\ \tilde{s}^{N-2} \\ \tilde{s}^{N-1} \\ \tilde{s}^N \end{pmatrix}$$

In a compact notation: $\tilde{\boldsymbol{d}} = \boldsymbol{S} \cdot \tilde{\boldsymbol{a}} + \tilde{\boldsymbol{s}}$

4. Phase Matrix and Reverberation Matrix



- The additional equation is supplemented by noting that waves departing from one side of the layer become the waves arriving at another side of the same layer.

$$\tilde{a}_p{}^{J(J-1)} = e^{i\alpha_J h_J} \cdot \tilde{d}_p{}^{(J-1)J};\ \tilde{a}_s{}^{J(J-1)} = -e^{i\beta_J h_J} \cdot \tilde{d}_s{}^{(J-1)J};$$

$$\tilde{d}_p{}^{J(J-1)} = e^{-i\alpha_J h_J} \cdot \tilde{a}_p{}^{(J-1)J};\ \tilde{d}_s{}^{J(J-1)} = -e^{-i\beta_J h_J} \cdot \tilde{a}_s{}^{(J-1)J}$$

Then all the "arrival waves" could be related to the "departure waves" by the global phase matrix $\boldsymbol{P}(h)$ as: $\tilde{\boldsymbol{a}} = \boldsymbol{P}(h) \cdot \tilde{\boldsymbol{d}}^*$

Local Phase Matrix:

$$\boldsymbol{P_J}(h_J) = \begin{pmatrix} e^{i\alpha_J h_J} & 0 \\ 0 & -e^{i\beta_J h_J} \end{pmatrix}$$

where the 4Nx4N Global Phase Matrix is:

$$\boldsymbol{P}(h) = \begin{pmatrix} \begin{matrix} \boldsymbol{P_1}(h_1) & 0 \\ 0 & \boldsymbol{P_1}(h_1) \end{matrix} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \begin{matrix} \boldsymbol{P_N}(h_N) & 0 \\ 0 & \boldsymbol{P_N}(h_N) \end{matrix} \end{pmatrix}$$

Reduced Order Model (ROM)

4. Phase Matrix and Reverberation Matrix

- The departure wave vectors $\tilde{d}^*$ and $\tilde{d}$ contain same elements but in different order. It can be expressed in equivalence through a global permutation matrix U.

$$\tilde{d}^* = U \cdot \tilde{d}$$

where U is a 4Nx4N block-diagonal matrix composed of N same 4x4 sub-matrix u:

$$U = \begin{pmatrix} u & 0 & 0 & 0 \\ 0 & u & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u \end{pmatrix}, \text{ where } u = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

- By combining the global scattering matrix S, global phase matrix P, and global permutation matrix U, we introduce the 4Nx4N reverberation matrix R:

$$\boldsymbol{R(k,\omega) = S \cdot P \cdot U}$$

Reduced Order Model (ROM)

5. Generalized Ray Solution and Frequency Response

- The arrival wave amplitude: $\tilde{a} = P \cdot U \cdot [I - R]^{-1} \cdot \tilde{s}$
- The departure wave amplitude: $\tilde{d} = [I - R]^{-1} \cdot \tilde{s}$

Here the matrix $[I - R]^{-1}$ relates the response of the layered medium to the excitation $\tilde{s}$ in the frequency-wave number domain.

- The dispersion relation for the layered medium is found as:

$$det[I - R(k, \omega)] = 0;$$

Based on the dispersion relation, it is possible to calculate the travelling wave number for specific frequencies.

Reduced Order Model (ROM)

5. Generalized Ray Solution and Frequency Response

- The frequency response of displacement field could be obtained by substituting $\tilde{\boldsymbol{a}}$ and $\tilde{\boldsymbol{d}}$ into the elastodynamics equation:

$$\boldsymbol{W}(k, y, \omega) = \boldsymbol{A_u} \cdot \tilde{\boldsymbol{a}} + \boldsymbol{D_u} \cdot \tilde{\boldsymbol{d}}$$
$$= (\boldsymbol{A_u} \cdot \boldsymbol{P} \cdot \boldsymbol{U} + \boldsymbol{D_u}) \cdot [\boldsymbol{I} - \boldsymbol{R}]^{-1} \cdot \tilde{\boldsymbol{s}}$$

Response of the Plate (Output) = Transfer Function * External Excitation (Input)

The matrix $\boldsymbol{A_u}$ and $\boldsymbol{D_u}$ are receiver matrix

- If we expand $[\boldsymbol{I} - \boldsymbol{R}]^{-1}$ in a form of Neumann series:

$$[\boldsymbol{I} - \boldsymbol{R}]^{-1} = \boldsymbol{I} + \boldsymbol{R} + \boldsymbol{R}^2 + \cdots + \boldsymbol{R}^M$$

Thus the response can be evaluated in a summation form, where $\boldsymbol{R}^M$ means the M times reflections and transmissions of the sources waves in the multilayered plate.

Reduced Order Model (ROM)

5. Generalized Ray Solution and Frequency Response

- By applying inverse Fourier Transform in the wave number domain, the frequency response could be represented in space-frequency domain:

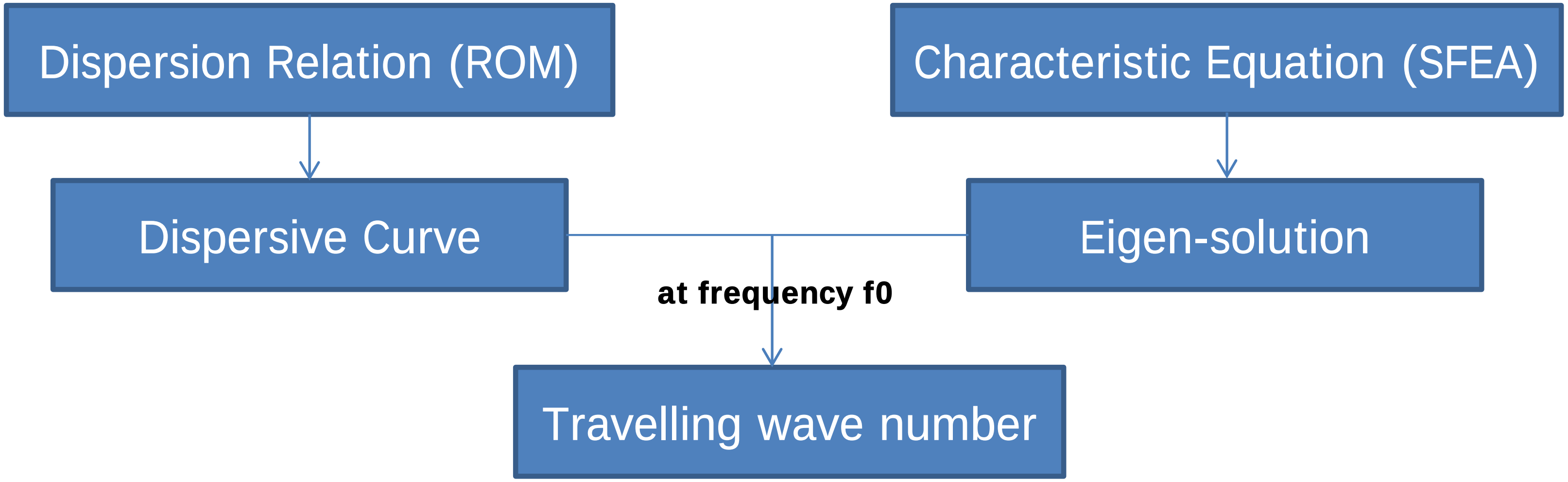
$$W(x, y, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(k, y, \omega)\, e^{ikx} dk$$

  Now the frequency response at any locations in the multilayered plate, including the top and bottom surfaces and all the interfaces, could be calculated.

- We will validate the reduced order model (ROM) by comparing with the spectral finite elements method (SFEM) and numerical analysis in Nastran in the following examples.

Validation

1. Validation by Spectral Finite Element Analysis (SFEA)



2. Validation by Numerical Analysis in Nastran

- Applying an impulsive load
- Comparing the forced response in frequency domain

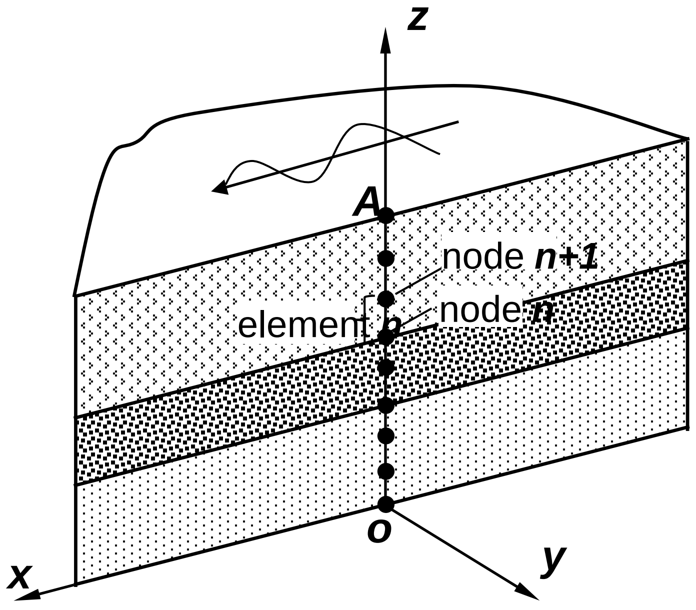
Validation by SFEA

1. Displacement Field Approximation

The displacement field within multi-layered structures is approximated as:

$$\boldsymbol{u}(x, y, z, t) = \boldsymbol{N}(z) \cdot \boldsymbol{U} e^{ikx - i\omega t}$$

where $\mathbf{u} = [u \quad v \quad w]^{\mathrm{T}}$ is the displacement vector, $\mathbf{N}(z)$ is the matrix of the element shape functions, and $\mathbf{U}$ is the nodal displacement vector.



For a linear finite element *n*, shown to the right, the displacements within the element can be expressed as:

$$\boldsymbol{u}(x, y, z, t) = \boldsymbol{N}_n(z)\boldsymbol{U}_n(k, \omega)e^{ikx - i\omega t}$$

where the shape function matrix $\boldsymbol{N}_n(z) = [(z_{n+1} - z)/h_n\boldsymbol{I} \quad (z - z_n)/h_n\boldsymbol{I}]$ and the amplitude vector $\boldsymbol{U}_n$=$[U_n \quad V_n \quad W_n \quad U_{n+1} \quad V_{n+1} \quad W_{n+1}]^T$, in which $\mathbf{I}$ is 3x3 identity matrix.

Validation by SFEA

2. SFEA Formulation

The governing differential equation for free vibration of multi-layered structures can be expressed in matrix-vector form as:

$$\rho \ddot{\boldsymbol{u}} - \boldsymbol{L}^T \cdot \boldsymbol{c} \cdot \boldsymbol{L} \cdot \boldsymbol{u} = \boldsymbol{0}$$

where ρ is the mass density, $\mathbf{c}$ is the elasticity constants matrix, and $\mathbf{L}$ is the differential operator given by

$$\boldsymbol{L}^T = \begin{bmatrix} \partial/\partial x & 0 & 0 & 0 & \partial/\partial z & \partial/\partial y \\ 0 & \partial/\partial y & 0 & \partial/\partial z & 0 & \partial/\partial x \\ 0 & 0 & \partial/\partial z & \partial/\partial y & \partial/\partial x & 0 \end{bmatrix}$$

A standard one-dimensional finite element formulation with the assumed displacement field can be applied to derive the following form of algebraic eigenvalue equations:

$$[\boldsymbol{K}(k) - \omega^2 \boldsymbol{M}]\boldsymbol{U} = \boldsymbol{0}$$

Validation by SFEA

2. SFEA Formulation

In the eigenvalue equation:

$$[\boldsymbol{K}(k) - \omega^2 \boldsymbol{M}]\boldsymbol{U} = \boldsymbol{0}$$

the global mass matrix $\mathbf{M}$ and the global stiffness matrix $\mathbf{K}(k)$ are, respectively, obtained by assembling the element mass matrix $\boldsymbol{M}_n = \int_0^{h_n} \rho_n \boldsymbol{N}_n^T \boldsymbol{N}_n dz$ and the element stiffness matrix $\boldsymbol{K}_n = \int_0^{h_n} \boldsymbol{D}^T \boldsymbol{c} \boldsymbol{D} dz$. Here,

$$\boldsymbol{D}^T = \begin{bmatrix} ik\boldsymbol{N}_n & 0 & 0 & 0 & \partial \boldsymbol{N}_n/\partial z & 0 \\ 0 & 0 & 0 & \partial \boldsymbol{N}_n/\partial z & 0 & ik\boldsymbol{N}_n \\ 0 & 0 & \partial \boldsymbol{N}_n/\partial z & 0 & ik\boldsymbol{N}_n & 0 \end{bmatrix}$$

Because the matrix $\boldsymbol{D}$ is linear in *k*, it can be shown that

$$\boldsymbol{K}(k) = \boldsymbol{K}_2 k^2 + \boldsymbol{K}_1 k + \boldsymbol{K}_0$$

Here, $\boldsymbol{K}_0$, $\boldsymbol{K}_1$, and $\boldsymbol{K}_2$ are independent of the wavenumber *k*.

Validation by SFEA

3. <u>Solution : Wavenumber and Mode Shapes</u>

Based on the eigenvalue equation, for a given frequency ω, is quadratic in k, it can be transformed to a linear eigenvalue problem of the form:

$$[\boldsymbol{A} - k\boldsymbol{B}]\boldsymbol{d} = \boldsymbol{0}$$

where $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{K}_2^{-1}(\boldsymbol{K}_0 - \omega^2 \boldsymbol{M}) & \boldsymbol{K}_2^{-1}\boldsymbol{K}_1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$ *and* $\boldsymbol{d} = \begin{Bmatrix} \boldsymbol{U} \\ k\boldsymbol{U} \end{Bmatrix}$

It is possible to seek eigenvalues *k* (i.e. wave numbers) and its corresponding eigenvectors (i.e. nodal displacement amplitude vectors) $\mathbf{U}$ at each frequency ω.

The eigenvector $\mathbf{U}_m$ gives the deformation shape (a.k.a. mode shape) for the *m*-th wave number k_m at each frequency ω. Eigenvalues with zero imaginary part and positive real part are wave numbers for propagating waves and those with non-zero imaginary part are wave numbers of evanescent waves.

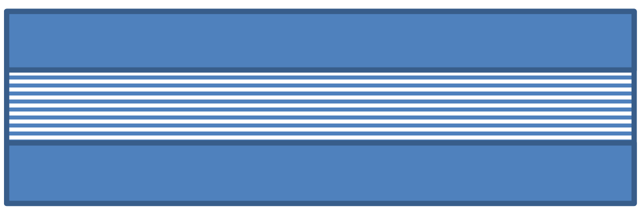
Validation by SFEA

- Calculate and compare the travelling wave numbers.
- All tests are under the frequency f = 1000Hz.

Case 1: All three layers are isotropic ($E = 200e9\ Pa, \nu = 0.3, \rho = 7800\ {}^{kg}/_{m^3}$)

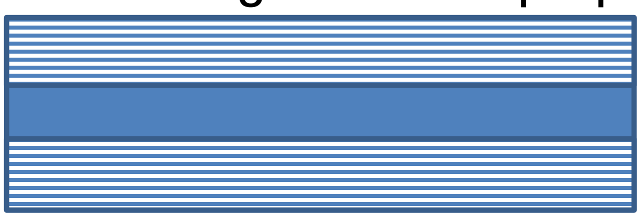
Material	RMM Code	SFEA Code
Baseline (regular steel)	20.323	20.323
Double Modulus of Elasticity	17.071	17.071
Half Modulus of Elasticity	24.204	24.204

Case 2: Change material properties in mid-layer (top & bottom unchanged)



Material	RMM Code	SFEA Code
Double E/Half E	19.991/20.525	19.991/20.525
Double ρ/Half ρ	22.118/19.215	22.118/19.215
ν=0.2/ν=0.4	20.337/20.299	20.337/20.299

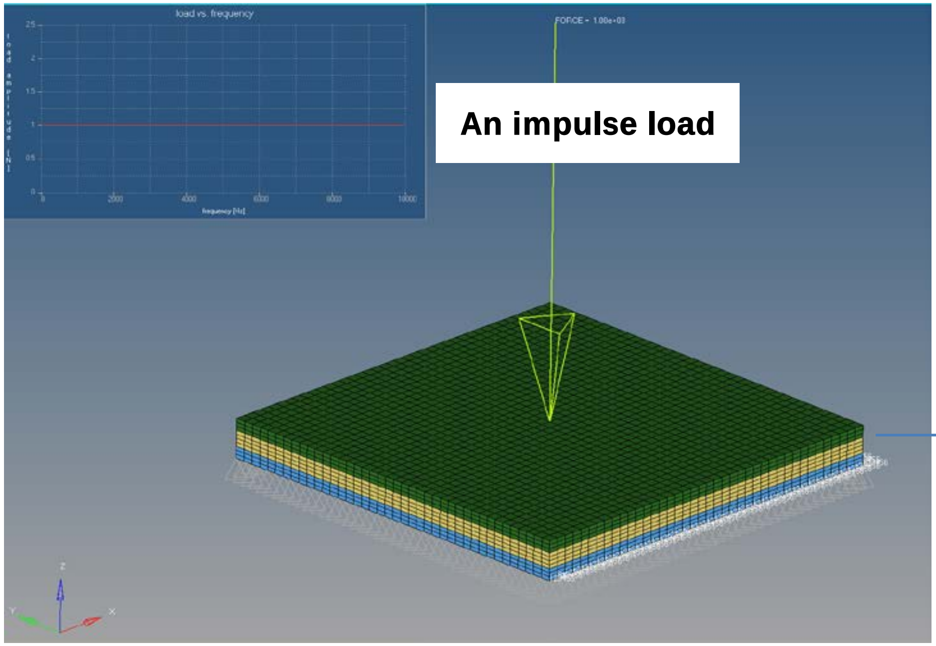
Case 3: Change material properties in top & bottom (mid-layers unchanged)



Material	RMM Code	SFEA Code
Double E/Half E	17.232/23.799	17.232/23.799
Double ρ/Half ρ	22.882/18.576	22.882/18.576
ν=0.2/ν=0.4	20.577/19.947	20.577/19.947

Validation by Nastran

A 1m*1m layered plate is created in Nastran as another reference.

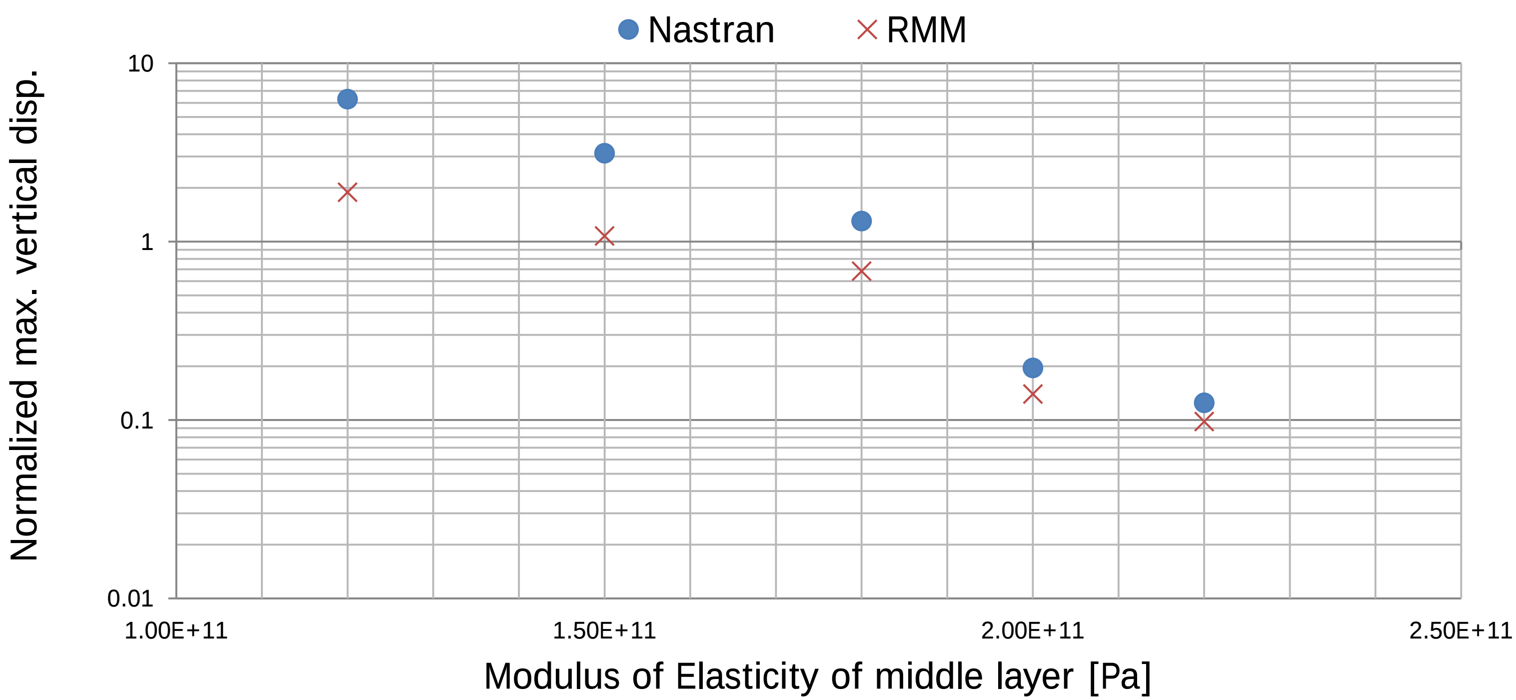


Changing the modulus of elasticity of the middle layer

- The thickness for each layer are: $h_1 = h_3 = 0.03m, h_2 = 0.04m$.
- An impulse load is applied at the center of the top surface.
- Clamped boundary condition applies on the edges of the bottom surface.

Validation by Nastran

- Capture the vertical displacement at the center of bottom surface.
- Compare maximum vertical displacement obtained from ROM/Nastran.
- All the responses are normalized based on the material properties of the first layer and the magnitude of the excitation using $u_z^* = \log(\frac{u_z * h_1 * E_1}{100 * F})$

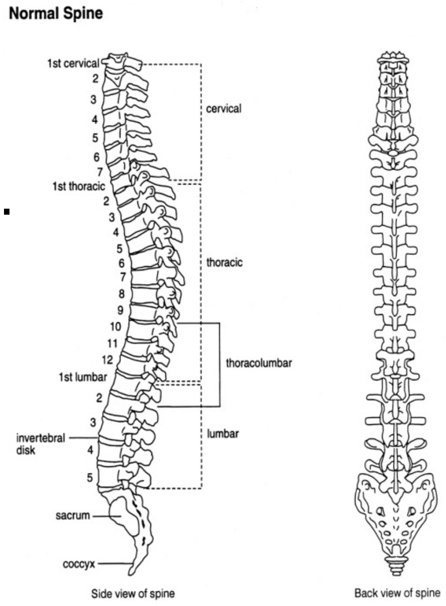


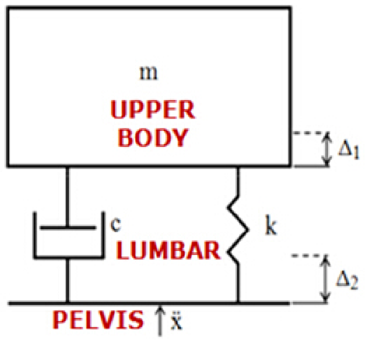
Dynamic Response Index (DRI)

- For underbody blast events, the lumbar and spinal injury would be the most threats to occupant.

- The Dynamic Response Index (DRI), which is a dimensionless number proportional to the maximum spinal compression, will be applied as standard injury metric in this research.

- The human lumbar would be simplified as a 1-DOF mass-spring-damper system.





Dynamic Response Index (DRI)

The governing equation for the simplified lumbar model is:

$$\frac{d^2\delta}{dt^2} + 2 \cdot \zeta \cdot \omega \frac{d\delta}{dt} + \omega^2 \cdot \delta = \frac{d^2z}{dt^2}$$

$\frac{d^2z}{dt^2}$ - Time-dependent shock acceleration (excitation from the hull)

ζ - Damping Ratio of human lumbar = 0.224

ω - Natural Frequency of human lumbar = 52.9 rad/s

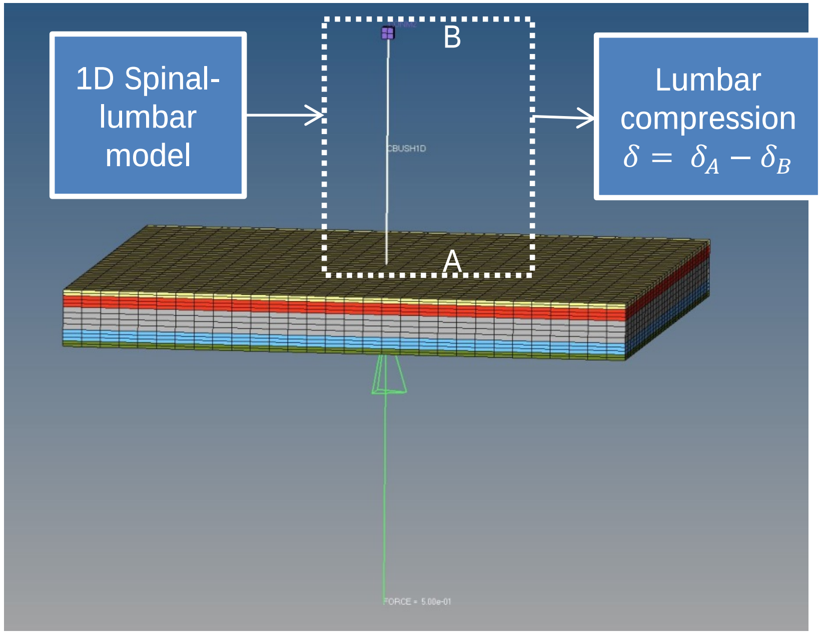
δ - Lumbar Compression

The DRI is based on the maximum lumbar compression (δ_{max}):

$$\textbf{DRI} = \frac{\omega^2}{g} \cdot \delta_{max}$$

The limiting DRI is 17.7 with a 10% chance of serious injury.

Screening Metric



The overall stiffness E_{eff} of the multilayer plates keep constant instead of intuitively changing the stiffness of one certain layer to change the dynamic response of the plate. Therefore, it yields the constraint:

$$\sum_{i=1}^{n} E_i * h_i = E_{eff} * h_{total}$$

Modulus of Elasticity	Baseline	Case 1	Case 2	Case 3
E of layer 1 (Pa)	220e9	300e9	250e9	250e9
E of layer 2 (Pa)	220e9	250e9	200e9	220e9
E of layer 3 (Pa)	220e9	150e9	225e9	205e9
E of layer 4 (Pa)	220e9	250e9	200e9	220e9
E of layer 5 (Pa)	220e9	300e9	250e9	250e9
E_{eff}	220e9	220e9	220e9	220e9

Screening Metric

The screening metric is obtained from the value of DRI $= \frac{\omega^2}{g} \cdot \delta_{max}$

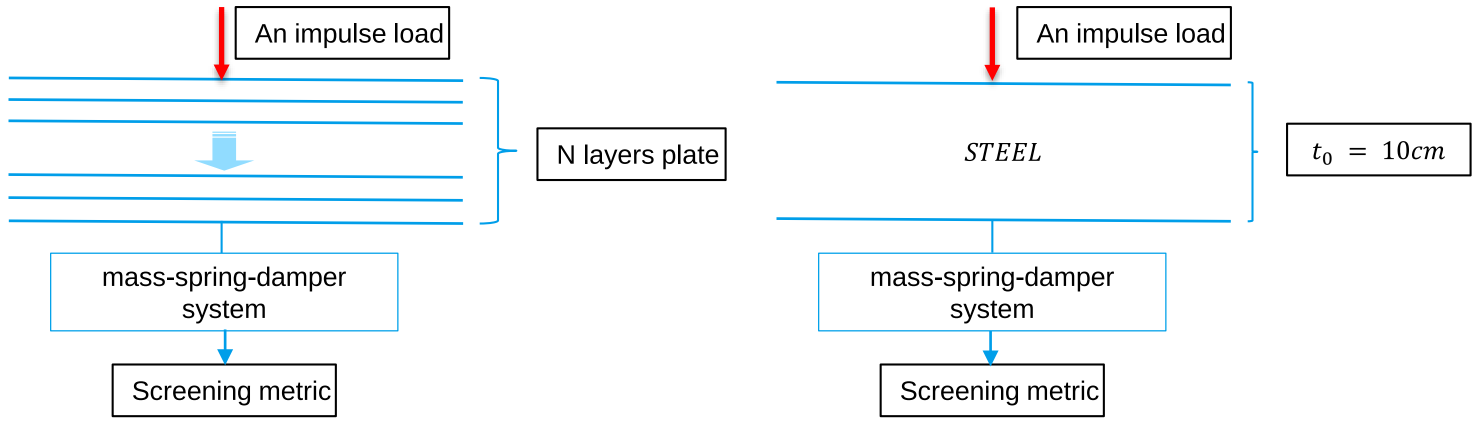
ROM	Screening Metric	Ranking	Computational Time
Baseline	100%	4	10.2s
Case 1	81.36%	1	10.5s
Case 2	94.53%	3	10.4s
Case 3	94.09%	2	10.8s

Nastran	Screening Metric	Ranking	Computational Time
Baseline	100%	4	2,025s
Case 1	87.96%	1	2,320s
Case 2	97.18%	3	2,298s
Case 3	95.33%	2	2,344s

- The screening metric match well between ROM and Nastran
- The ranking order is captured correctly
- The ROM computes about 220 times faster than Nastran

Design Optimization

1. Optimization Statement



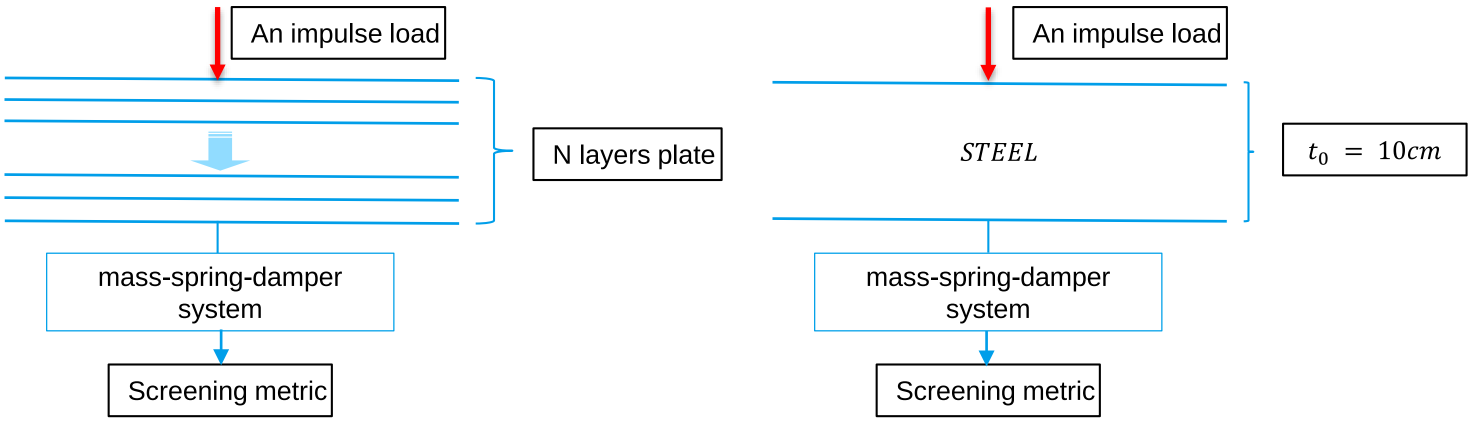
Discrete Design Variables: number of layers (N), material selection (s_j)

Continuous Design Variables: sublayer thickness (t_i)

Objective Function: $\boldsymbol{min\ (Screening\ metric)}$
The screening metric is based on the largest relative displacement experienced by the spring mass-damper system

Design Optimization

1. Optimization Statement



Constraints	Comments
$1\text{cm} < t_i < 4.8\text{cm}$	Upper and lower bounds on the thickness of each sublayer
$9.8cm < \sum_i^N t_i < 10.2\text{cm}$	Total thickness changes little to the thickness of baseline
$\sum_i^N E_i * t_i > 0.7 * (E_{steel} * t_0)$	Total surface stiffness will be kept in an acceptable range to avoid reducing the plate rigidity
$\sum_i^N \rho_i * t_i < (\rho_{steel} * t_0)$	Total surface density should be less than the baseline in order to avoid an increase in the structural mass

Design Optimization

1. Optimization Statement

- Optimization Solver: NSGA2 – type of solver of the DS Toolkit
- Generation size: 10
- Population size in each generation: 200
- Material Selection:

# (s_j)	Materials Selection	Modulus of Elasticity (GPa)	Density $\left({}^{kg}/_{m^3}\right)$	Poisson Ratio
s_1	RHA Steel	200	7823	0.30
s_2	6061 Aluminum Alloy	73.7	2704	0.34
s_3	SiC	445	3200	0.16
s_4	OFHC Copper	129	8940	0.35
s_5	Ti-6Al-4V Alloy	114	4430	0.34

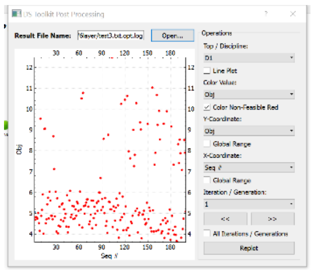
The DS Toolkit is used for conducting the optimization analysis

Design Optimization

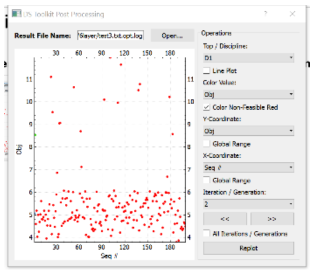
2. Optimization Results

Convergence increases when the number of generation increases.

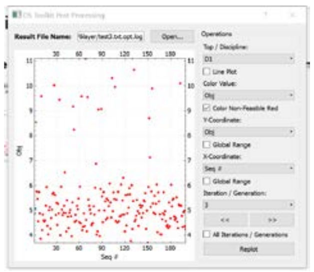
(The results of the 6 layers configuration are shown as an example)



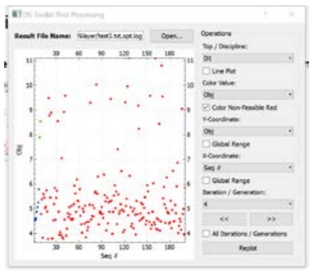
1 generation



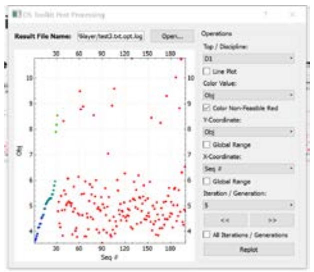
2 generation



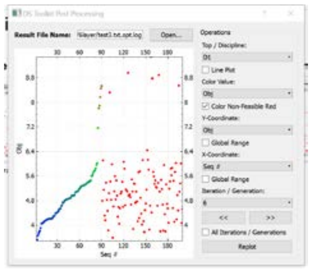
3 generation



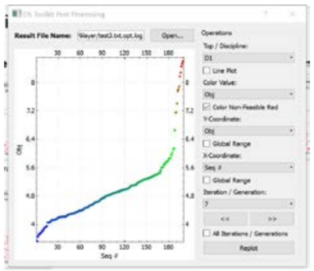
4 generation



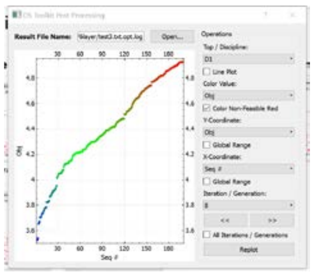
5 generation



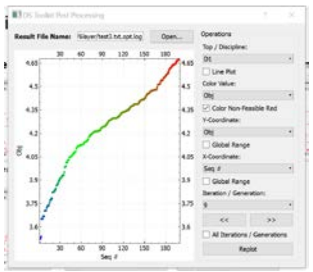
6 generation



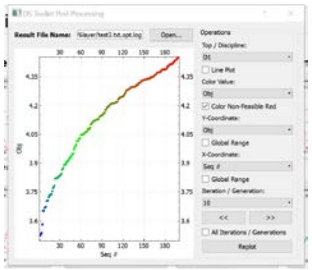
7 generation



8 generation



9 generation

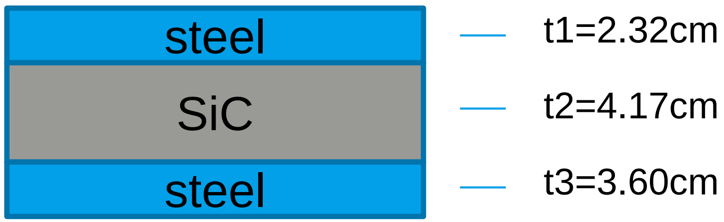


10 generation

Design Optimization

2. Optimization Results

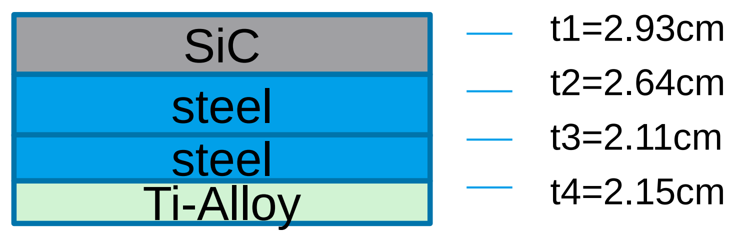
- **3 layers optimization (N=3)**



Total thickness = 10.1cm

Density/unit area = 597.31 kg/m

- **4 layers optimization (N=4)**



Total thickness = 9.83cm

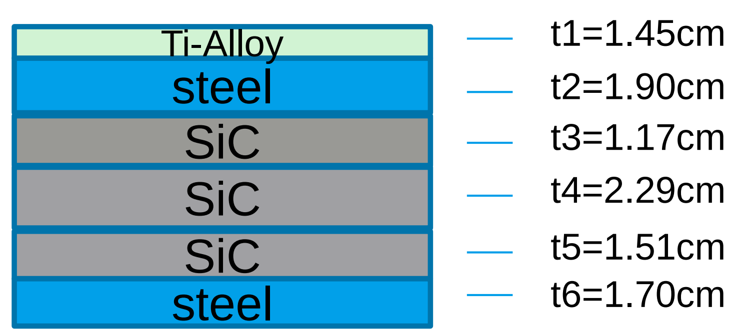
Density/unit area = 560.64 kg/m

- **5 layers optimization (N=5)**

Total thickness = 10.1cm

Density/unit area = 520.69 kg/m

- **6 layers optimization (N=6)**



Total thickness = 10.0cm

Density/unit area = 505.27 kg/m

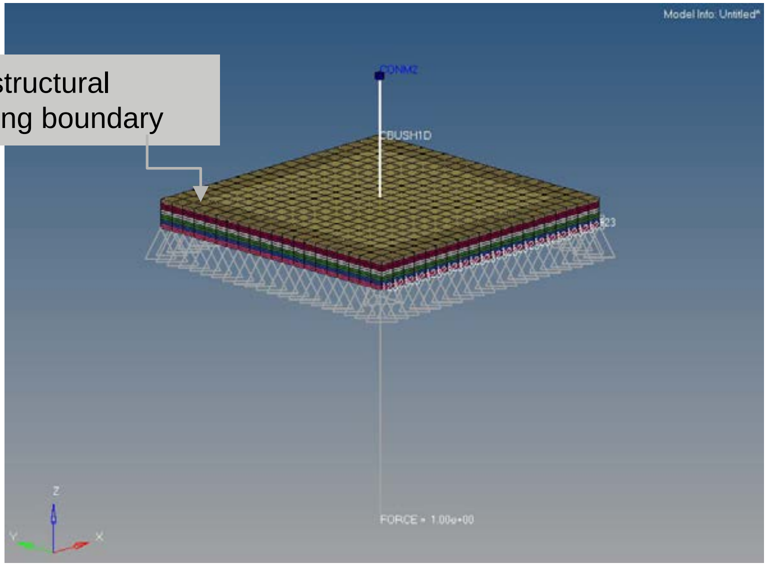
Design Optimization

3. Nastran Validation

NASTRAN is used to analyzed the optimal configuration that was identified when a different number of layers is considered. The relative ranking of the optimal configurations that correspond to the different number of layers are evaluated and compared with the ROM.

In the NASTRAN analysis:

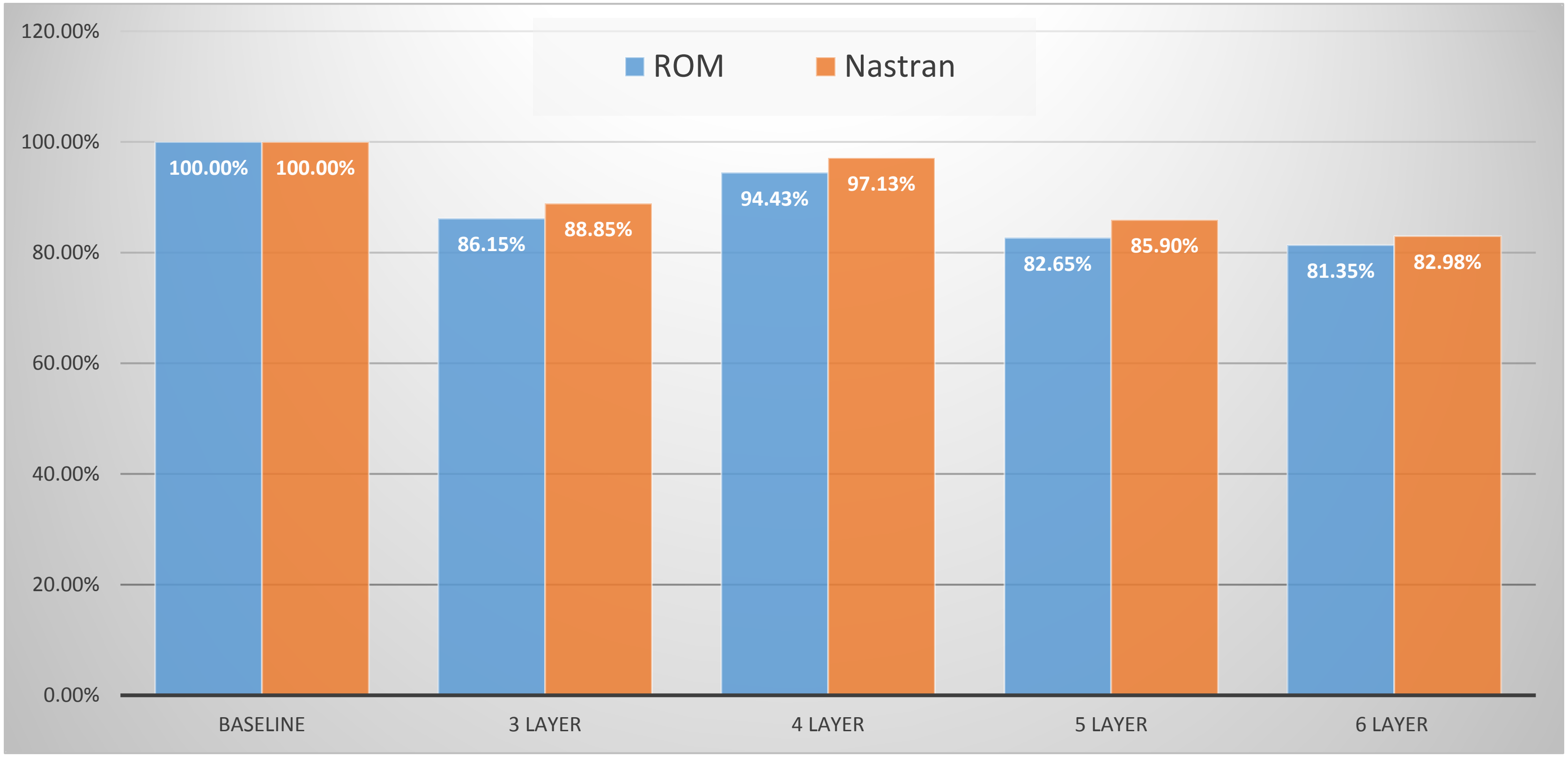
- Using 1m by 1m finite plate
- Applying the load at the center of the lower surface
- Applying simply-supported boundary condition at all edges of the lower surface
- A high structural damping boundary was added to the plate to reduce the reflected waves from the edges of the plate in the NASTRAN analysis



Design Optimization

4. Comparison Results

The screening metric is based on the value of the relative displacement experienced by the single degree of freedom system:



Q & A

Thank you!