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**Linear Quantum Systems: Non-Classical States and Robust Stability**

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**AUSTRALIAN NATIONAL UNIVERSITY RESEARCH OFFICE ACTON (AUSTRALIA)**

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**Final Report**

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| <b>14. ABSTRACT</b><br>The proposed work attempted to establish a theoretical framework for the analysis and control of quantum linear systems and their interactions with non-classical quantum fields by developing control theoretic concepts exploiting special features of quantum systems. The research aimed to continue developing a new mathematical framework for robust control of quantum linear systems and extend results to quantum linear systems subject to non-classical quantum fields. The major outcomes of this project are (i) derivation of quantum filtering equations for systems non-classical input states including single photon states, (ii) determination of how linear quantum systems respond to multichannel non-classical states, (iii) linear quantum models for quantum memories and the zero dynamics principle for perfect information transfer, (iv) development of new structured uncertainty methods that ensure robust stability of quantum systems based on nominal linear models, and (v) physical realizability results for finite level quantum systems. |                                        |                                         |                                              |                                                                             |                                                                  |
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## **Final Report for AOARD Grant 12RSZ047-124075**

Linear Quantum Systems: Non-Classical  
States and Robust Stability

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**Period of Performance:** 28-Jun-2012 – 27-Jun-2015

### **Summary:**

The major outcomes of this project are (i) derivation of quantum filtering equations for systems non-classical input states including single photon states, (ii) determination of how linear quantum systems respond to multichannel non-classical states, (iii) linear quantum models for quantum memories and the zero dynamics principle for perfect information transfer, (iv) development of new structured uncertainty methods that ensure robust stability of quantum systems based on nominal linear models, and (v) physical realizability results for finite level quantum systems.

### **Introduction:**

Classical linear systems theory has a history going back some 50 years, to the birth of modern control theory with Kalman's foundational work on filtering and LQG optimal control. However, there are new imperatives arising in emerging quantum technologies that demand new concepts and tools from control theory. This has motivated recent work in developing aspects of a new quantum linear systems theory intended to provide a framework for exploiting the special features of quantum mechanics.

*Non-classical quantum states.* Gaussian distributions play a fundamental role in classical (non-quantum) linear systems theory, and underlies many well-known and fundamental results including LQG control, Kalman filtering, and spectral factorization. However, quantum states have unique features not shared by classical distributions, and it is the exploitation of such features that hold promise for new quantum technologies. Of particular interest is the potential for linear systems methods in the understanding and processing of highly non-classical quantum states, such as single photon states, and superposition states.

*Robust stability.* Models are only approximations to actual physical systems. It is therefore of basic importance to develop analysis and design methods that take uncertainty into account. Specifically, in the area of robust control for linear quantum systems, we will consider perturbed quantum linear systems described by coupling and Hamiltonian operators with components that depend on a nominal model and on uncertainty variables. This will provide a structured approach

that enables uncertainty and its impact on performance to be analyzed.

**Experiment:** N/A

## **Results and Discussion:**

### *Non-classical quantum states.*

A key accomplishment of this project is the derivation of the quantum filter when the system is driven by single photon states. A general approach to determining photon-generating models was developed to enable the filter to be derived. These results are currently being applied to develop methods for non-classical quantum state generation. Results extending the type of input states to include continuous matrix product states were also obtained.

Detailed results on how linear quantum systems respond to pulsed-Gaussian states were developed in a multichannel setting. The results involve general convolution (for photon pulses) and spectral (for Gaussian terms) input-output relationships. Aspects of these results have been extended to develop explicit analytical results for output states of finite level systems.

The research in this project made several significant contributions to the theory of quantum memories. A general infinite dimensional linear quantum models for a gradient echo memory was developed, and applied to determine how such memories store non-classical photon states. The important system-theoretic concept of zero dynamics was applied to linear quantum systems to characterize exactly how such systems can perfectly store single photon states.

In quantum optics, nonlinearities are important for quantum information processing. Modeling and simulation results were obtained for cross-phase modulation in optical cavities and in gradient echo memories.

### *Robust stability.*

Within the area of robust stability of quantum systems, a series of results was obtained in which quantum versions of the small gain approach and the Popov approach to robust stability were derived for uncertain quantum systems. These results guaranteed robust stability for the cases of both linear and nonlinear perturbations to a nominal linear quantum system. Some applications of these results were considered involving, Kerr nonlinearities, Josephson Junctions, and optical parametric amplifiers.

Also, results were obtained on approximating linear quantum systems using singular perturbation methods. In addition, results were obtained on the coherent control of linear quantum systems along with results on physical realizability for bilinear quantum systems.

In addition, results were obtained for quantum state estimation and quantum observers including the case of coherent quantum observers in which the observer is also a quantum system.

## Refereed Journal Papers

Shuangshuang Fu, André RR Carvalho, Michael R Hush, Matthew R James, *Cross-phase modulation and entanglement in a compound gradient echo memory*, Physical Review A 93(2) 023809 2016.

We present a theoretical model for a Kerr-like interaction between two registers of a compound gradient echo memory (GEM). This type of interaction is known to generate cross-phase modulation (XPM) between optical fields, an effect that is limited by the typically small values of nonlinearities in crystals. Here we show that in GEM systems the phase shift increases linearly with the interaction time and quadratically with the strength of the field. Increasing storage (interaction) times would then lead to stronger XPM effects even with fields with very low intensity. This interaction also generates two other effects: entanglement between the registers, which depends on the strength of the interaction and its spatial profile, and an interaction-induced gradient. We show that the latter produces leakage during the storage stage depending on the shape of the stored pulses, an undesirable consequence that can be minimized by carefully designing the temporal profile of the input fields.

A. R. R. Carvalho, M. R. Hush, and M. R. James, *Cavity driven by a single photon: Conditional dynamics and nonlinear phase shift*, Physical Review A, 86, 023806 , 2012.

We apply the stochastic master equations (quantum filter) derived by Gough et al. (Proc. 50th IEEE Conference on Decision and Control, 2011) to a system consisting of a cavity driven by a multimode single photon field. In particular, we analyse the conditional dynamics for the problem of cross phase modulation in a doubly resonant cavity. Through the exact integration of the stochastic equations, our results reveal features of the problem unavailable from previous models.

I.R. Petersen, V. Ugrinovskii and M.R. James, *Robust Stability of Uncertain Linear Quantum Systems*, Phil. Trans. R. Soc. A 28 November 2012 vol. 370 no. 1979 5354-5363.

This paper considers the problem of robust stability for a class of uncertain linear quantum systems subject to unknown perturbations in the system Hamiltonian. The case of a nominal linear quantum system is considered with quadratic perturbations to the system Hamiltonian. A robust stability condition is given in terms of a strict bounded real condition.

J. Gough, H. Nurdin and M.R. James, *Single photon quantum filtering using non-Markovian embeddings*, Phil. Trans. R. Soc. A 28 November 2012 vol. 370 no. 1979 5408-5421.

The purpose of this paper is to determine quantum master and filter equations for systems coupled to fields in certain non-classical continuous-mode states. Specifically, we consider two types of field states (i) single photon states, and (ii) superpositions of coherent states. The system and field are described using a

quantum stochastic unitary model. Master equations are derived from this model and are given in terms of systems of coupled equations. The output field carries information about the system, and is continuously monitored. The quantum filters are determined with the aid of an embedding into a larger non-Markovian quantum system.

D. Gwion Evans, John E. Gough, and Matthew R. James, *Non-abelian Weyl commutation relations and the series product of quantum stochastic evolutions*, Phil. Trans. R. Soc. A 28 November 2012 vol. 370 no. 1979 5437-5451.

We show that the series product, which serves as an algebraic rule for connecting state-based input–output systems, is intimately related to the Heisenberg group and the canonical commutation relations. The series product for quantum stochastic models then corresponds to a non-abelian generalization of the Weyl commutation relation. We show that the series product gives the general rule for combining the generators of quantum stochastic evolutions using a Lie–Trotter product formula.

John E. Gough, Matthew R. James, Hendra I. Nurdin, and Joshua Combes, *Quantum filtering for systems driven by fields in single-photon states or superposition of coherent states*, Phys. Rev. A 86, 043819 (2012).

We derive the stochastic master equations, that is to say, quantum filters, and master equations for an arbitrary quantum system probed by a continuous-mode bosonic input field in two types of nonclassical states. Specifically, we consider the cases where the state of the input field is a superposition or combination of (1) a continuous-mode, single-photon wave packet and vacuum, and (2) any continuous-mode coherent states.

Shi Wang, Hendra I Nurdin, Guofeng Zhang, Matthew R James, *Quantum optical realization of classical linear stochastic systems*, Automatica, 49(10), 3090-3096, 2013.

The purpose of this paper is to show how a class of classical linear stochastic systems can be physically implemented using quantum optical components. Quantum optical systems typically have much higher bandwidth than electronic devices, meaning faster response and processing times, and hence have the potential for providing better performance than classical systems. A procedure is provided for constructing the quantum optical realization.

G. Zhang and M.R. James, *On the Response of Quantum Linear Systems to Single Photon Input Fields*, IEEE Trans. Aut. Control, 58(5), 1221 - 1235, May 2013.

The purpose of this paper is to extend linear systems and signals theory to include single photon quantum signals. We provide detailed results describing how quantum linear systems respond to multichannel single photon quantum signals. In particular, we characterize the class of states (which we call photon-Gaussian states) that result when multichannel photons are input to a quantum linear system. We show that this class of quantum states is preserved by quantum linear systems. Multichannel

photon-Gaussian.

M. R. Hush, A. R. R. Carvalho, M. Hedges and M R James, *Analysis of the operation of gradient echo memories using a quantum input-output model*, New Journal of Physics, 15 (2013) 085020 (Special Issue on Quantum Memories). 10.1088/1367-2630/15/8/085020

The gradient echo memory (GEM) technique is a promising candidate for real devices due to its demonstrated performance, but to date high performance experiments can only be described numerically. In this paper we derive a model for GEM as a cascade of infinite interconnected harmonic oscillators. We take a quantum input–output approach to analyse this system, describing the read and write processes of GEM each as a linear-time-invariant process. We provide an analytical solution to the problem in terms of transfer functions which describe the memory behaviour for arbitrary inputs and operating regimes. This allows us to go beyond previous works and analyse the storage quality in the regimes of high optical depth and memory-bandwidth comparable to input bandwidth, exactly the regime of high-efficiency experiments.

J. E. Gough, M.R. James and H.I. Nurdin, *Quantum Trajectories for a Class of Continuous Matrix Product Input States*, New Journal of Physics, 16 (2014) 075008.

We introduce a new class of continuous matrix product (CMP) states and establish the stochastic master equations (quantum filters) for an arbitrary quantum system probed by a bosonic input field in this class of states. We show that this class of CMP states arise naturally as outputs of a Markovian model, and that input fields in these states lead to master and filtering (quantum trajectory) equations which are matrix-valued. Furthermore, it is shown that this class of CMP states include the (continuous-mode) single photon and time-ordered multi-photon states.

N. Yamamoto and M.R. James, *Zero-dynamics principle for perfect quantum memory in linear networks*, New Journal of Physics, 16 (2014) 073032. 10.1088/1367-2630/16/7/073032

In this paper, we study a general linear networked system that contains a tunable memory subsystem; that is, it is decoupled from an optical field for state transportation during the storage process, while it couples to the field during the writing or reading process. The input is given by a single photon state or a coherent state in a pulsed light field. We then completely and explicitly characterize the condition required on the pulse shape achieving the perfect state transfer from the light field to the memory subsystem. The key idea to obtain this result is the use of zero-dynamics principle, which in our case means that, for perfect state transfer, the output field during the writing process must be a vacuum. A useful interpretation of the result in terms of the transfer function is also given. Moreover, a four-node network composed of atomic ensembles is studied as an example, demonstrating how the input field state is transferred to the memory subsystem and what the input pulse shape to be engineered for perfect memory looks like.

Shuangshuang Fu, Guodong Shi, Alexandre Proutiere, Matthew R James, *Feedback policies for*

*measurement-based quantum state manipulation*, PRA 90(6), 062328, 2014.

In this paper, we propose feedback designs for manipulating a quantum state to a target state by performing sequential measurements. In light of Belavkin's quantum feedback control theory, for a given set of (projective or nonprojective) measurements and a given time horizon, we show that finding the measurement selection policy that maximizes the probability of successful state manipulation is an optimal control problem for a controlled Markovian process. The optimal policy is Markovian and can be solved by dynamical programming. Numerical examples indicate that making use of feedback information significantly improves the success probability compared to classical scheme without taking feedback. We also consider other objective functionals including maximizing the expected fidelity with the target state as well as minimizing the expected arrival time. The connections and differences among these objectives are also discussed.

Shi Wang and M.R. James, *Quantum Feedback Control of Linear Stochastic Systems with Feedback-loop Time Delays*, Automatica, 52, 277-282, 2015.

In a quantum feedback control loop, one is often confronted with time delays that mainly originated from the transition delay of signals, which may cause the quantum feedback control system unstable. The purpose of this paper is to deal with a problem of quantum measurement-based feedback control subject to feedback-loop time delays. A quantum version of delay-independent stability criterion as well as an upper bound on a cost function is derived based on quantum Itô rules. We propose one numerical procedure for quantum feedback controller design.

Ian R. Petersen, *Singular perturbation approximations for a class of linear quantum systems*, IEEE Transactions on Automatic Control, 58(1):193–198, 2013.

This paper considers the use of singular perturbation approximations for a class of linear quantum systems arising in the area of linear quantum optics. The paper presents results on the physical realizability properties of the approximate system arising from singular perturbation model reduction.

Luis Augusto Duffaut Espinosa, Zibo Miao, Ian R. Petersen, Valery Ugrinovskii, and Matthew R. James, *Physical realizability and preservation of commutation and anticommutation relations for n-level quantum systems*, SIAM Journal on Control and Optimization, 54(2):632–661, 2016.

The purpose of this paper is to address the problem of physical realizability for n-level quantum systems. We provide necessary and sufficient conditions for quantum stochastic differential equations that ensure the existence of physical parameters characterizing the unitary evolution required by the laws of quantum mechanics. Also, these conditions guarantee the preservation of the

commutation and anticommutation relations of the underlying algebra  $SU(n)$ .

## Refereed Conference Papers

Ian R Petersen, Valery Ugrinovskii, Matthew R James, *Robust stability of quantum systems with a nonlinear coupling operator*, 51st IEEE Conference on Decision and Control (CDC), 1078-1082, 2012.

This paper considers the problem of robust stability for a class of uncertain quantum systems subject to unknown perturbations in the system coupling operator. A general stability result is given for a class of perturbations to the system coupling operator. Then, the special case of a nominal linear quantum system is considered with non-linear perturbations to the system coupling operator. In this case, a robust stability condition is given in terms of a scaled strict bounded real condition.

Shi Wang, Hendra I Nurdin, Guofeng Zhang, Matthew R James, *Synthesis and Structure of Mixed Quantum-Classical Linear Systems*, Proc. IEEE CDC, Maui, USA, December, 2012.

The purpose of this paper is to formulate and solve a synthesis problem for a class of linear quantum equations that may describe mixed quantum-classical systems. We propose a standard model for mixed quantum-classical linear stochastic systems for the design process, which can present the internal structure of a mixed quantum-classical system. Physical realizability conditions are derived for the standard model to ensure that it can correspond to a physical system.

Matthew R. James, Ian R. Petersen and Valery Ugrinovskii, *A Popov Stability Condition for Uncertain Linear Quantum Systems*, American Control Conference, 2013.

This paper considers a Popov type approach to the problem of robust stability for a class of uncertain linear quantum systems subject to unknown perturbations in the system Hamiltonian. A general stability result is given for a general class of perturbations to the system Hamiltonian. Then, the special case of a nominal linear quantum system is considered with quadratic perturbations to the system Hamiltonian. In this case, a robust stability condition is given in terms of a frequency domain condition.

S. Wang and M.R. James, *Extended LMI Approach to Coherent Quantum LQG Control Design*, 52nd IEEE Conference on Decision and Control, 1265-1270, December, 2013.

A coherent quantum controller is itself a quantum system that is required to be physically realizable. Thus, additional non-linear and linear constraints must be imposed on the coefficients of a physically realizable quantum controller, which differs the quantum Linear Quadratic Gaussian (LQG) design from the standard LQG problem. The purpose of this paper is to propose one numerical procedure based on extended linear matrix inequality (LMI) approach and new physical realizability conditions.

Shi Wang, Matthew R James *H-Infinity control of quantum feedback control systems with time delay*, Proc. 10th Asian Control Conference (ASCC), 1-6, 2015

In a quantum feedback control loop, it is often confronted with time delays mainly originated from the transition delay of signals, which may cause quantum feedback control systems unstable. The purpose of this paper is to deal with a problem of quantum measurement-based feedback control systems subject to feedback-loop time delays. A physical model for a quantum measurement-based feedback control system with time delay is presented for the H-infinity control.

Luis Augusto Duffaut Espinosa, Zibo Miao, Ian R. Petersen, Valery Ugrinovskii, and Matthew R. James, *On the preservation of commutation and anticommutation relations of n-level quantum systems*, Proceedings of the 2013 American Control Conference, Washington, DC, June 2013.

The goal of this paper is to provide conditions under which a quantum stochastic differential equation (QSDE) preserves the commutation and anticommutation relations of the  $SU(n)$  algebra, and thus describes the evolution of an open n-level quantum system. One of the challenges in the approach lies in the handling of the so-called anomaly coefficients of  $SU(n)$ . Then, it is shown that the physical realizability conditions recently developed by the authors for open n-level quantum systems also imply preservation of commutation and anticommutation relations.

Ian R. Petersen, *Quantum Popov robust stability analysis of an optical cavity containing a saturated Kerr medium*, Proceedings of the 2013 European Control Conference, Zurich, Switzerland, July 2013.

This paper applies results on the robust stability of nonlinear quantum systems to a system consisting an optical cavity containing a saturated Kerr medium. The system is characterized by a Hamiltonian operator which contains a non-quadratic term involving a quartic function of the annihilation and creation operators. A saturated version of the Kerr nonlinearity leads to a sector bounded nonlinearity which enables a quantum small gain theorem to be applied to this system in order to analyze its stability. Also, a non-quadratic version of a quantum Popov stability criterion is presented and applied to analyze the stability of this system.

Ian R. Petersen, *Robust stability analysis of an optical parametric amplifier quantum system*, Proceedings of the 2013 Asian Control Conference, Istanbul, Turkey, July 2013.

This paper considers the problem of robust stability for a class of uncertain nonlinear quantum systems subject to unknown perturbations in the system Hamiltonian. The case of a nominal linear quantum system is considered with non-quadratic

perturbations to the system Hamiltonian. The paper extends recent results on the robust stability of nonlinear quantum systems to allow for non-quadratic perturbations to the Hamiltonian which depend on multiple parameters. A robust stability condition is given in terms of a strict bounded real condition. This result is then applied to the robust stability analysis of a nonlinear quantum system which is a model of an optical parametric amplifier.

Ian R. Petersen, *Coherent-classical estimation for quantum linear systems*. In *Australian Control Conference*, Perth, Australia, November 2013.

This paper introduces a problem of coherent-classical estimation for a class of linear quantum systems. In this problem, the estimator is a mixed quantum-classical system which produces a classical estimate of a system variable. The coherent-classical estimator may also involve coherent feedback. An example involving optical squeezers is given to illustrate the efficacy of this idea.

Ian R. Petersen, *Notes on coherent feedback control for linear quantum systems*. In *Australian Control Conference*, Perth, Australia, November 2013.

This paper considers some formulations and possible approaches to the coherent LQG and  $H_\infty$  quantum control problems. Some new results for these problems are presented in the case of annihilation operator only quantum systems showing that in this case, the optimal controllers are trivial controllers.

Ian R. Petersen, *Control and robustness for quantum linear systems*, Proceedings of the 2013 Chinese Control Conference, Xian, China, July 2013.

This paper surveys some recent results on the feedback control of quantum linear systems and the robustness properties of these systems. Quantum linear systems are a class of systems whose dynamics, which are described by the laws of quantum mechanics, take the specific form of a set of linear quantum stochastic differential equations (QSDEs). These systems can also be described in terms of a Hamiltonian operator  $H$  and a coupling operator  $L$ , which in the case of quantum linear systems have a specific quadratic and linear form respectively. Such systems commonly arise in the area of quantum optics and related disciplines. Systems whose dynamics can be described or approximated by linear QSDEs include interconnections of optical cavities, beam-splitters, phase-shifters, optical parametric amplifiers, optical squeezers, and cavity quantum electrodynamic systems.

An important approach to the feedback control of quantum linear systems involves the use of a controller which itself is a quantum linear system. This approach to quantum feedback control, referred to as coherent quantum feedback control, has the advantage that it does not destroy quantum information, is fast, and has the potential for efficient implementation. The paper discusses recent results concerning the synthesis of coherent quantum controllers such as in coherent quantum  $H_\infty$  control.

Another important issue in the design of quantum feedback controllers is the robustness of the closed loop quantum system to perturbations in the quantum plant dynamics arising from perturbations in the system Hamiltonian  $H$  and the system coupling operator  $L$ . The paper discusses recent robust stability results for such perturbed quantum systems which take the form of a quantum small gain theorem and a quantum Popov stability criterion. These results are useful when combined with quantum  $H_\infty$  control in synthesizing robust quantum feedback controllers.

Ian R. Petersen, *Robust stability of quantum systems with nonlinear dynamic uncertainties*, Proceedings of the 52nd IEEE Conference on Decision and Control, Florence, Italy, December 2013.

This paper considers the problem of robust stability for a class of uncertain nonlinear quantum systems subject to unknown perturbations in the system Hamiltonian. The nominal system is a linear quantum system defined by a linear vector of coupling operators and a quadratic Hamiltonian. This paper extends previous results on the robust stability of nonlinear quantum systems to allow for quantum systems with dynamic uncertainties. These dynamic uncertainties are required to satisfy a certain quantum stochastic integral quadratic constraint. The robust stability condition is given in terms of a strict bounded real condition. This result is applied to the robust stability analysis of an optical parametric amplifier.

I.R. Petersen, *Guaranteed non-quadratic performance for quantum systems with nonlinear uncertainties*, Proceedings of the 2014 American Control Conference, Portland OR, June 2014.

This paper presents a robust performance analysis result for a class of uncertain quantum systems containing sector bounded nonlinearities arising from perturbations to the system Hamiltonian. An LMI condition is given for calculating a guaranteed upper bound on a non-quadratic cost function. This result is illustrated with an example involving a Josephson junction in an electromagnetic cavity.

Arash Khodaparastsichani, Ian R. Petersen, and Igor G. Vladimirov, *Parameterization of stabilizing linear coherent quantum controllers*, Proceedings of the 10th ASIAN CONTROL CONFERENCE 2015, Kota Kinabalu, Malaysia, May 2015.

This paper is concerned with application of the classical Youla-Kucera parameterization to finding a set of linear coherent quantum controllers that stabilize a linear quantum plant. The plant and controller are assumed to represent open quantum harmonic oscillators modelled by linear quantum stochastic differential equations. The interconnections between the plant and the controller are assumed to be established through quantum bosonic fields. In this framework, conditions for the stabilization of a given linear quantum plant via linear coherent quantum feedback are addressed using a stable factorization approach. The class of stabilizing quantum

controllers is parameterized in the frequency domain. Also, this approach is used in order to formulate coherent quantum weighted  $H_2$  and  $H_\infty$  control problems for linear quantum systems in the frequency domain. Finally, a projected gradient descent scheme is proposed to solve the coherent quantum weighted  $H_2$  control problem.

Ian R. Petersen, *A direct coupling coherent quantum observer*, Proceedings of the 2014 IEEE Multi-conference on Systems and Control, Antibes, France, October 2014.

This paper considers the problem of constructing a direct coupling quantum observer for a closed linear quantum system. The proposed observer is shown to be able to estimate some but not all of the plant variables in a time averaged sense.

Ian R. Petersen, *A direct coupling coherent quantum observer for a single qubit finite level quantum system*, Proceedings of 2014 Australian Control Conference, Canberra, Australia, November 2014.

This paper considers the problem of constructing a direct coupling quantum observer for a single qubit finite level quantum system plant. The proposed observer is a single mode linear quantum system which is shown to be able to estimate one of the plant variables in a time averaged sense. A numerical example and simulations are included to illustrate the properties of the observer.