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# PHYSICAL FACTORS IN PRECIPITATION PROCESSES AND THEIR INFLUENCE ON THE EFFECTIVENESS OF CLOUD SEEDING

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*“An old wisdom from the Far East says: you cannot tell how the flower looks if you know only the seed. You have to know first how the bud looks like. It appears that in our age of modern technology of the conquest of space and time, we sometimes tend to forget this simple wisdom. So, for instance, when man tries to make or prevent rain or hail artificially without even knowing all the intermediate steps that Nature has provided in the evolutionary process that leads from a cloud droplet or ice crystal to a rain drop or hailstone.”*

—from the foreword to the *Proceedings of the International Conference on Cloud Physics, May 24–June 1, 1965*, by Helmut Weickmann

## 1. Introduction

The above quotation focuses on the crux of the situation with which I shall deal. From the physical standpoint, the problem with respect to the effectiveness of attempts to modify the precipitation process is that while we have a fairly clear understanding of the process in qualitative terms, adequate theories and observational data for quantitative evaluation of process rates are not available. Consequently, we cannot tell whether the natural process in a given case will proceed at optimum efficiency, or whether a particular change in the conditions will lead to an increase or a decrease in the rate.

Under these circumstances it is likely, if not inevitable, that insofar as they have any effect at all the attempts to affect precipitation by cloud seeding will in some cases increase it and in other cases decrease it, with results which are impossible to predict and difficult to detect.

In the following I shall review the status of our knowledge of the process of formation of precipitation and point out the implications of our present knowledge and the requirements for future research to enable the intelligent selection of cases and procedures to seed to produce a particular effect in optimum fashion.

## 2. Qualitative description of precipitation processes

Broadly, the processes of formation of precipitation may be divided into the *dynamic processes*, concerned with the motions of air currents which give rise to the general conditions for the formation of clouds and precipitation, and the *microphysical processes*, concerned with the growth of the individual precipitation particles from gas phase by condensation and from smaller cloud particles by collision and coalescence. There is, of course, a strong interaction between the two kinds of processes. The upward motions determine the rate of cooling due to expansion and thus control the rate at which the microphysical processes go on. The release of latent heat in condensation and the drag of the particles formed affect the buoyant forces which determine the upward motion. While the dynamic processes are prerequisite to the microphysical ones, it is convenient to discuss the processes of particle growth first, and subsequently to turn to the larger scale setting in which it occurs.

It is a fact of common experience that clouds can remain in the sky for long periods without precipitating. Since clouds consist of water particles, liquid or solid, which are heavier than air, this phenomenon requires explanation; the usual explanation is, of course, that the particles are being sustained in a current of air moving upward as fast as or faster than they are falling. Alternatively, the particles may evaporate as they fall from the base of cloud and vanish into vapor a short distance below.

Measurements show that the radii of drops in nonprecipitating liquid clouds are in the range 2 to 20 microns, with the modal radius usually between 5 and 10 microns. These drops have terminal velocities ranging from 0.05 to 5 cm/sec, so that very slight upward flow of air would be required to offset their falling. Further, it has been shown that if drops of these sizes fall out of a cloud into air with 90 per cent relative humidity they would evaporate before they go as much as one meter.

Rain drops, on the other hand, range in radii from 0.1 mm to 3 mm, with terminal velocities from 70 cm/sec to 9 m/sec. They thus can fall relative to ascending air and reach the ground before evaporating, even when the vertical air velocity is considerable and low humidities prevail below the clouds.

The key difference between cloud and precipitation is thus the particle size, and the central question in precipitation physics concerns the conditions under which the particles can grow, roughly one to ten million times in mass, from cloud size to precipitation size.

The process of condensation by itself can be shown to be much too slow to explain the rates at which precipitation forms. For instance, the development from clear air to showers in the course of a summer day may occur in a matter of an hour or less. While condensation results in very rapid growth of drops to the size of average cloud drops, say 10 microns, continued growth is progressively slower, and with the number of drops which form, there is not enough water vapor available for millimeter drops to be produced by condensation alone.

The two ways that cloud particles can grow rapidly to precipitation are (1) by collision and coalescence, and (2) by the three phase, or Bergeron process. The nature of the first process is obvious: if the cloud drops are not of uniform size the larger ones will fall relative to the smaller, and tend to overtake and capture them. After collecting one small drop the large drop becomes larger, falls faster, and is more effective in collecting others. But as we shall see, because of the tendency for the air to carry drops around each other, there are limitations on the initiation of this process.

The three phase process is based on the fact that drops remain liquid at temperatures below 0° C, and ice crystals, if they form, are much fewer in number than the liquid drops. Since the equilibrium vapor pressure over ice is lower than that over water at the same (subzero) temperature, there is a strong gradient of vapor density away from the liquid drops toward the ice crystals, so that rapid transfer of water occurs from the drops, which evaporate, to the crystals, which quickly grow large compared to the preexisting supercooled drops. The crystals fall relative to the remaining small drops and collect them. Process (2) thus may initiate process (1), and the two acting together can readily lead to the formation of precipitation sized particles in subfreezing clouds. In warm clouds which precipitate, collision and coalescence alone must be the activating process.

As a drop falls, the air ahead of it is pushed out of its way, and if a smaller drop is contained in that air it likewise will tend to be carried out of its way by the moving air. Because of its inertia the smaller drop may be struck by the large one if it is not too far from the axis of fall. The fraction of the small drops in its way which would be collected by a large drop is called the *collision efficiency*  $E$ . It is a function of the radii  $A$  and  $a$  of the large and small drops, as well as their density and the density and viscosity of the medium (air) through which they fall.

If  $Y_c$  is the limiting distance from the axis of fall of the large drop, within which the small drop must lie to be collected, the collision efficiency is

$$(1) \quad E = Y_c^2/(a + A)^2 = y_c^2/(1 + p)^2,$$

where  $y_c = Y_c/A$  and  $p = a/A$  are nondimensional values of the limiting distance and the small radius, in units of the large radius. It is convenient to show  $y_c$  as a function of  $p$  for various values of  $A$ .

Evaluations of  $y_c$  have been carried out by computing drop trajectories for various conditions [11]. It turns out, as shown in figure 1, that for large  $A$  (greater than 60  $\mu$ ) the geometric value corresponding to  $E = 1$  is approximated for most values of  $p$ ; but for relatively small  $A$ , say 20  $\mu$ ,  $E$  is at most about 0.25, and is zero for half of the range of  $p$  (for  $p < 0.3$ ;  $> 0.8$ ); and for  $A \leq 18$ ,  $E$  is zero for all values of  $p$ .

(It should be mentioned that these computed values of  $y_c$  and  $E$  involve approximations, and a recent investigation suggests that, rather than a complete cutoff for  $A \leq 18$ , the values for small  $A$  are not much smaller than for  $A = 20$ .

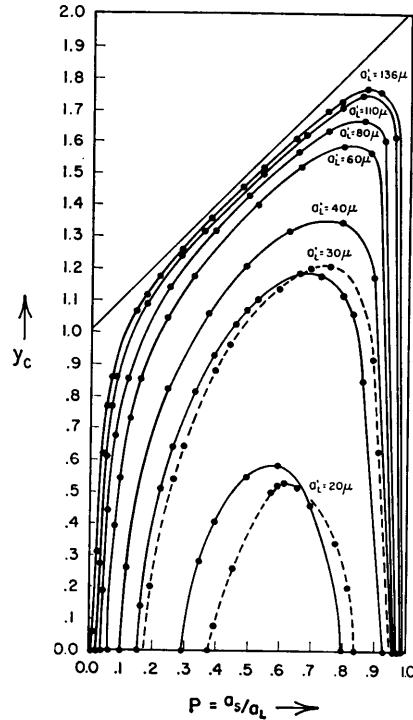


FIGURE 1

Linear collision efficiencies  $y_c$  of water drops of radius  $a_L$  falling through air, as a function of the ratio  $p$  of the radius of the small droplet  $a_s$  to the large.

If strong electric fields are present they would modify the collision efficiency, but present indications suggest that strong fields arise only after some cloud particles have become large or ice has formed in the cloud.)

This result, together with the data presented earlier on drop sizes in clouds, indicates that precipitation by the warm process should not be expected to take place from most clouds. If, as the data shows, most clouds do not contain drops of radius as large as  $20 \mu$ , the likelihood of drop growth by coalescence is very slight. It is only in those cases where the size spectrum is broadened to include a sizable number of drops up to 30 or 40 microns in radius that the coalescence process is likely to be initiated. To understand why clouds sometimes have broad drop size spectra which lead to warm rain, but frequently do not, we must look to the details of the condensation process.

### 3. Condensation and the formation of clouds

Saturation vapor pressure or one hundred per cent relative humidity is defined in terms of the vapor in equilibrium with a plane surface of pure water. In the

absence of surfaces, however, the vapor pressure can exceed the saturation value several fold before condensation will begin. In the atmosphere surfaces are always present, in the form of particles of haze or dust. These particles are predominantly in the size range  $0.005 \mu$  to  $5 \mu$ . The lower limit is due to the tendency for smaller particles to agglomerate rapidly because of Brownian motion. Particles larger than one micron will tend to settle out even though the effect of turbulence is to diffuse them upward.

Typical observed size distributions of nuclei are shown [7] in figure 2. It will be seen that the most frequent sizes are the smallest. While individual cases vary considerably Junge found that on the average the frequency of sizes varies as the inverse fourth power of the equivalent radius.

The reason why nuclei are needed for condensation to occur at reasonable humidities is that the equilibrium vapor pressure of a spherical drop is inversely proportional to its radius. For this reason condensation will occur more readily on large than on small nuclei. Condensation on the few large nuclei will keep the vapor pressure from rising to the higher value required for condensation on the small ones, and the number of cloud drops will thus be much smaller than the total number of nuclei.

Among the various particles in the atmosphere some are hygroscopic, that is, they tend to attract water even at relative humidities less than 100 per cent. These nuclei will grow into droplets before the nonhygroscopic nuclei of the same size.

Figure 3 shows the equilibrium vapor pressure of nuclei composed of various quantities of hygroscopic material as a function of their size. Each curve shows a maximum, and if this maximum vapor pressure is exceeded the drop will grow continuously, for the larger it becomes the smaller its equilibrium vapor pressure. The favored nuclei are *large hygroscopic nuclei*.

In general the number of nuclei which are effective is in the range 50 to 1000 per cubic centimeter, and this correspondingly is the number of cloud drops formed. The exact number and size distribution depends on the rate of cooling (updraft velocity) during the time when saturation is being attained and exceeded and the spectrum of nucleus sizes.

The rate of growth of a single drop of radius  $r$  growing from a nucleus of equivalent radius  $r_0$ , density  $\rho_n$ , and hygroscopicity  $\Gamma$ , in an environment with temperature  $T$  and vapor pressure  $e$  is

$$(2) \quad \frac{dr}{dt} = \frac{D}{r\rho_s R_v T} \left\{ e - e_s(T) \left[ \exp \frac{L\delta}{R_v T} \right] \left[ \exp \frac{2\sigma}{r\rho_s R_v T} \right] \left[ 1 - \frac{\Gamma\rho_n r_0^3}{\rho_s r^3} \right] \right\}.$$

Here  $D$  is the coefficient of molecular diffusion,  $R_v$  the gas constant, and  $e_s(T)$  the saturation pressure of water vapor,  $L$  is the latent heat of condensation,  $\rho_s$  the density and  $\sigma$  the surface tension of the liquid, and  $\delta = (\rho_s L r dr/dt)/kT$ , the fractional difference between the temperature of the drop and the ambient temperature, where  $k$  is the thermal conductivity.

That the rate of growth decreases with increasing size is readily demonstrated

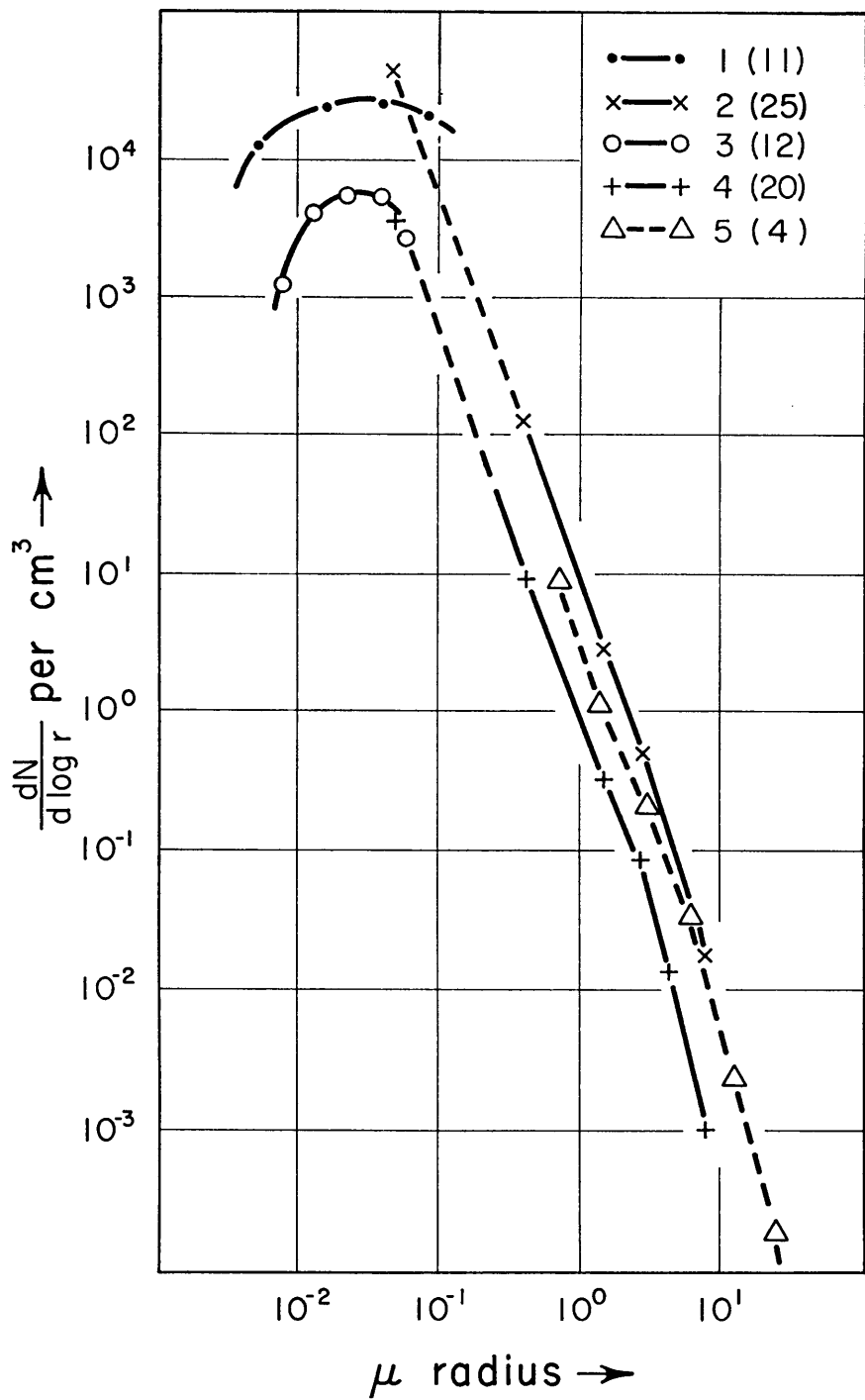


FIGURE 2

Average size distributions of natural aerosols occurring in a city (curves 1, 2 and 5) and at a mountain peak (3 and 4). The curves are composites of measurements made for the various ranges of sizes by different techniques at different times.

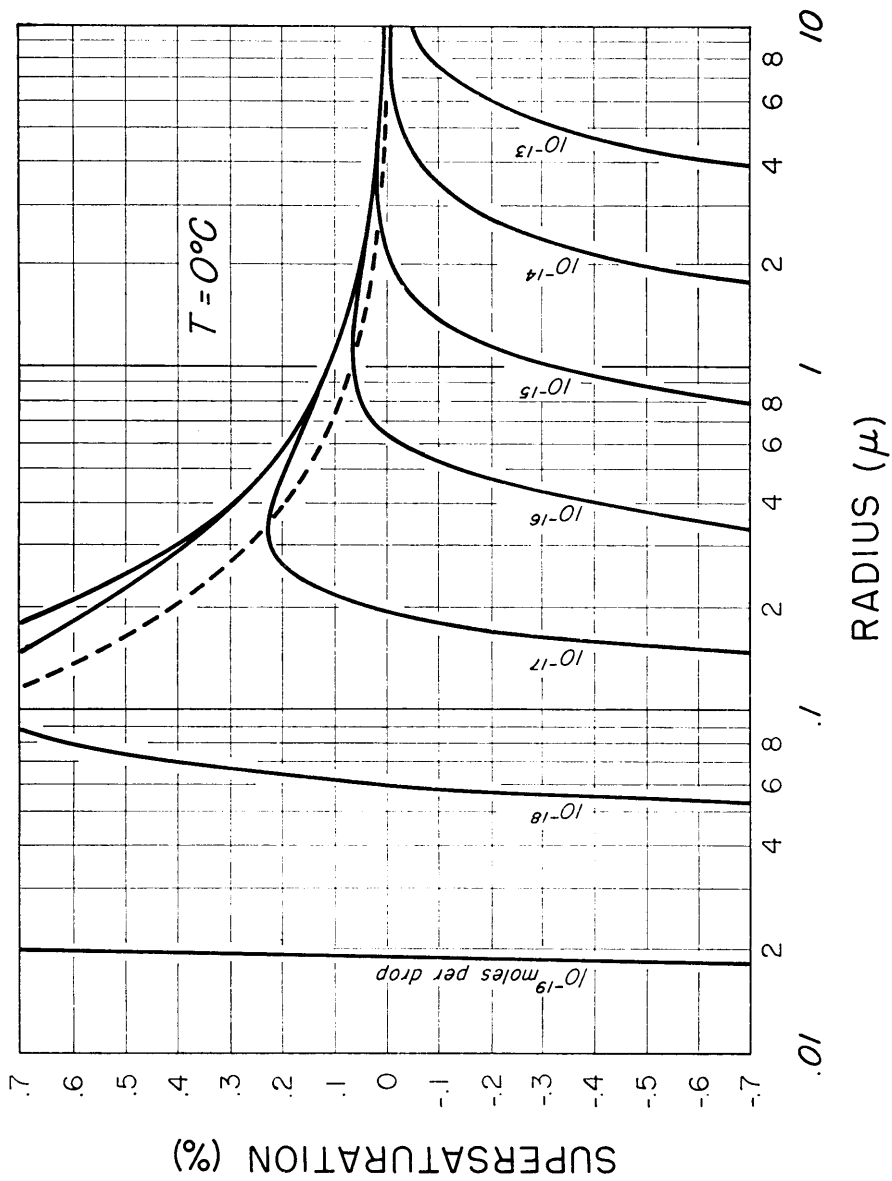


FIGURE 3  
 Equilibrium supersaturation over hygroscopic nuclei  
 of various masses.

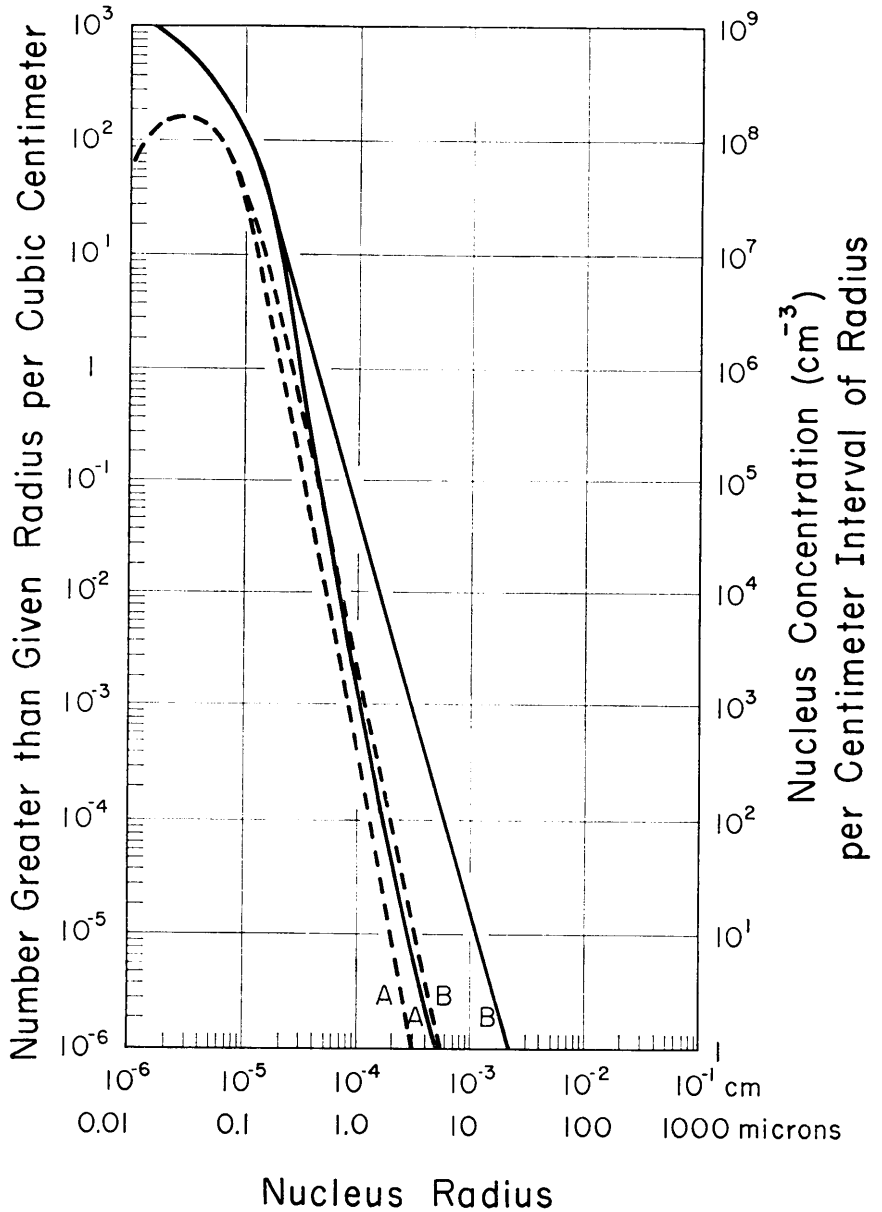


FIGURE 4

Distributions of nucleus sizes used in drop growth computations. Solid curves: cumulative distributions, referred to left hand scale of ordinates. Dashed curves, differential concentrations, referred to right hand scale.

by this equation. Table I shows values of  $dr/dt$  for various  $r_0$  and  $r$ , assuming  $T = 278K$  and  $e = 1.01e_s$ . The rates in table I cannot be applied to estimate

TABLE I  
RATE OF GROWTH OF DROPS ON A 0.1 MICRON "RADIUS" SALT NUCLEUS,  
IN A ONE PER CENT SUPERSATURATED ENVIRONMENT

$r$ (microns)	$\frac{dr}{dt}$ (microns/sec)	$r$ (microns)	$\frac{dr}{dt}$ (microns/sec)
0.13	197.22	2.51	0.34
0.25	40.17	4.64	0.16
0.46	3.27	10.00	0.075
1.00	0.77	100.00	0.007

the rate of growth of drops in a cloud, for as the drops grow the condensed vapor reduces the environmental vapor pressure at a rate which depends on the number of activated nuclei. To see the size distribution which develops in a cloud, the growth equation must be applied to specific nucleus distributions.

C. W. Chien and I [9] computed some cases of cloud growth for certain typical rates of cooling and nuclei spectra. The distributions of nucleus sizes assumed are shown in figure 4. In the type A distribution there are more than 1000 nuclei per  $\text{cm}^3$  larger than  $0.01 \mu$ , there are 115 per  $\text{cm}^3$  larger than  $0.1 \mu$ , and 1.5 per liter larger than  $1 \mu$ . In the type B distribution, which may be typical of air near the ocean, the first two numbers are almost unchanged, but there are 50 per liter larger than one micron, and 14 per  $\text{m}^3$  larger than 10 microns.

As a simulation of an active convective cloud, we used a distribution of vertical velocities based on the measurements of the Thunderstorm Project [3] as shown in figure 5. The air is assumed to start near the ground with a relative humidity of 75 per cent and type A nucleus distribution. The growth curves for nuclei of various sizes are shown in figure 6. Once the relative humidity slightly exceeds 100 per cent the drops formed on nuclei of  $0.1 \mu$  radius or larger grow rapidly, while those formed on smaller nuclei do not continue to grow. The size distributions after various time intervals are shown in figure 7. Shortly after the cloud forms the separation between the cloud drops and the inactivated nuclei show up, with the drop size mode at a radius of about 7 microns. At the end of the computation, corresponding to a rise of the air parcel to 9 km, the mode is at  $20 \mu$ , with about 70 per  $\text{cm}^3$  greater than  $16 \mu$  but only one per liter greater than about  $22 \mu$ . This very narrow drop size spectrum would not favor collision and coalescence even though the critical size for nonzero collision efficiency is slightly exceeded.

The influence of the rate of cooling was tested by computing the growth of the same nuclei at a much smaller cooling rate, corresponding to a constant vertical velocity of 17 cm/sec. The growth of the cloud drops took longer, of course, and the computation was continued only until the modal cloud drop

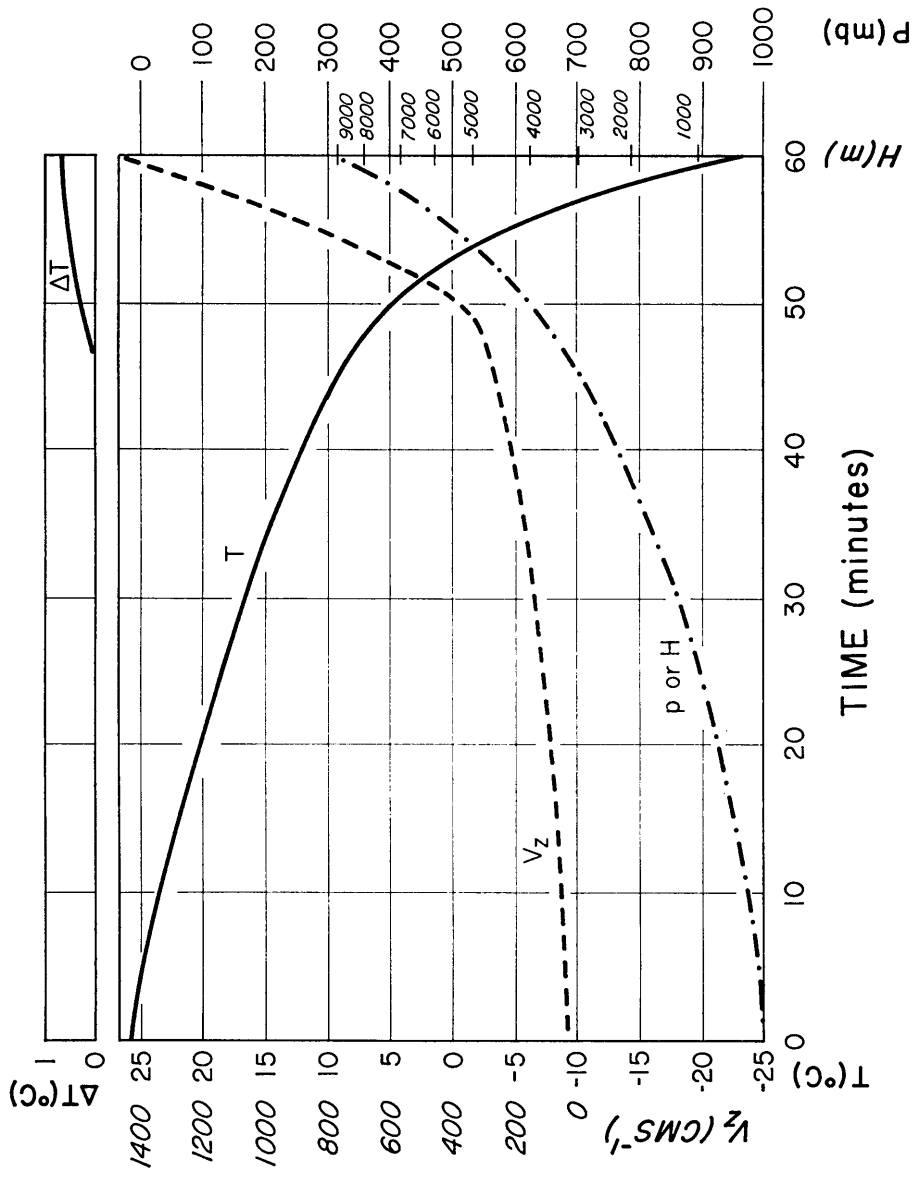


FIGURE 5

Variation of vertical velocity with time and corresponding changes of temperature, pressure and height assumed for computation of drop growth in cumulonimbus cloud.

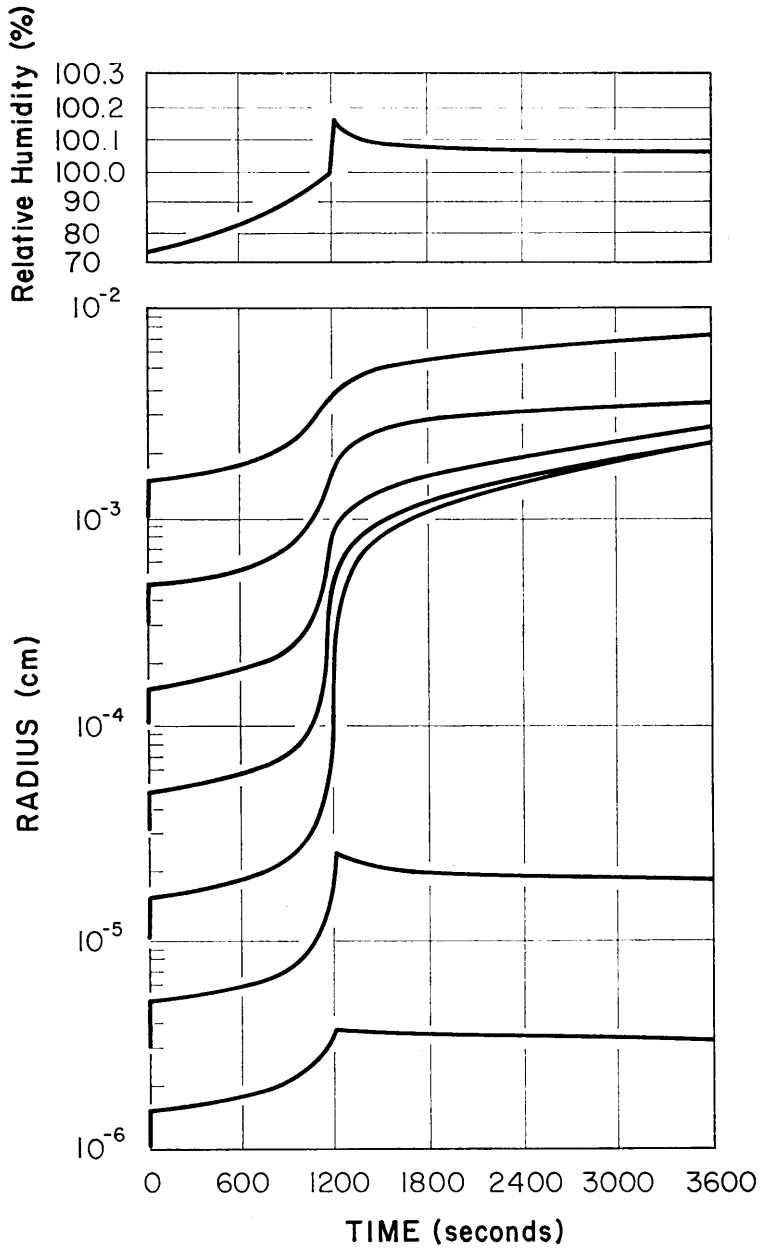
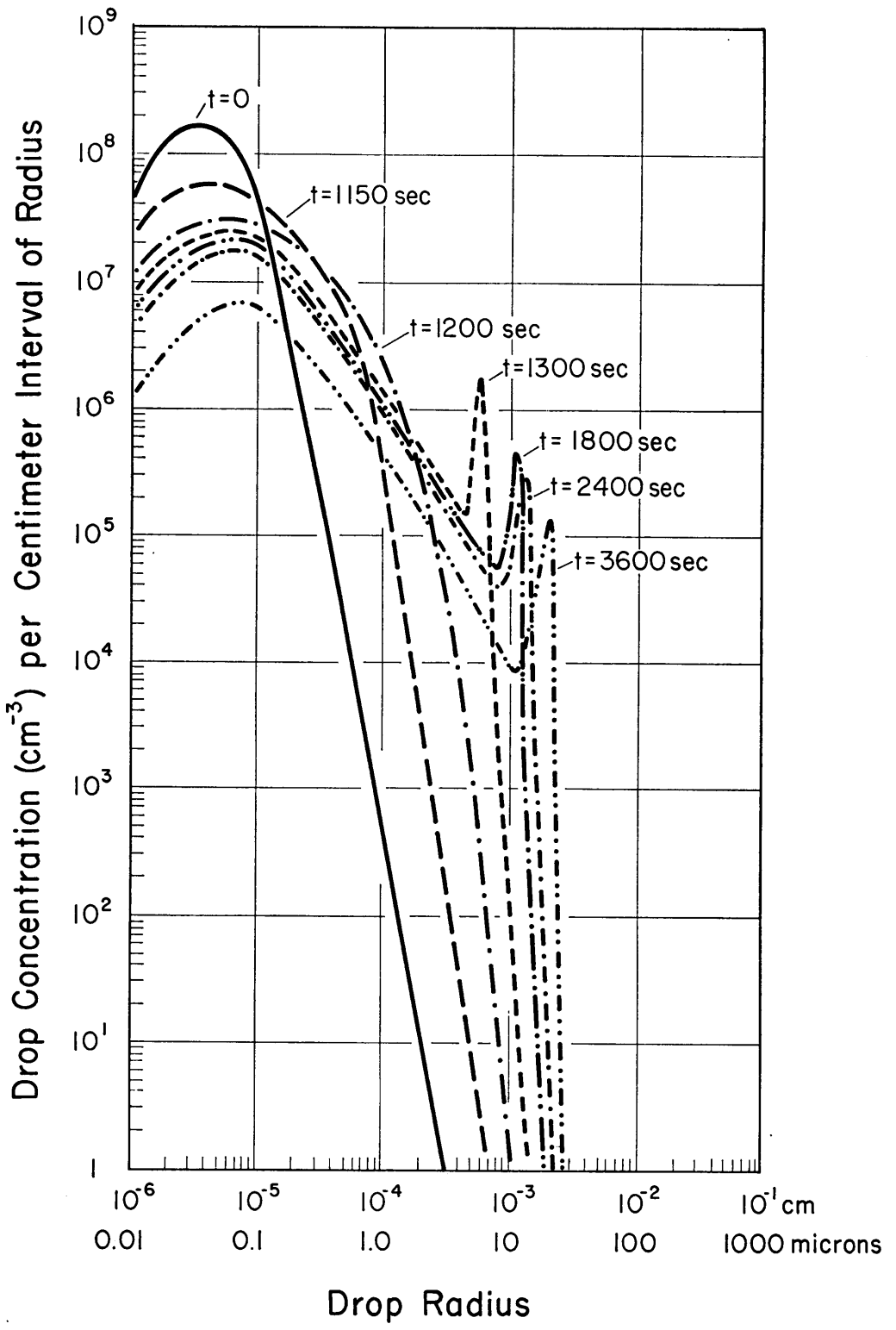


FIGURE 6

Computed growth of drops on nuclei of various sizes in a cumulonimbus cloud. The upper part of the diagram shows the variation of relative humidity in the rising air parcel.



Drop Radius

FIGURE 7

Drop size distributions after various elapsed times of rise of air in a cumulonimbus cloud.

radius reached  $12 \mu$ , but already at that time one per liter exceeded  $20 \mu$ . Thus, slower rates of cooling tend to produce a broader spectrum, with more large drops.

To see how the increased number of large nuclei in the type B distribution affect the cloud drop spectrum, the growth was computed for it with the same slow rate and duration of cooling. The modal radius of  $12 \mu$  was unaffected, but in the type B case there were 50 per liter greater than  $20 \mu$ , compared to one per liter in the type A case.

In marine air there may be many more large nuclei than assumed in the type B distribution, which would lead to even broader spectra, but in continental air sparsity of large nuclei like that in type A may frequently occur, with correspondingly narrow cloud drop spectra developing by condensation.

#### 4. Growth by collection

Since condensation on nuclei, even when due to air rising through the entire troposphere, as illustrated in the above computations, does not lead to drops large enough to fall as precipitation, we must turn to collision and coalescence for combining the many small cloud drops to form the size required.

The simplest way to treat the collection problem is to deal with the growth of a large collector drop falling through a cloud of uniform small drops sufficiently dense that the growth is essentially continuous. In this case the rate of growth of the large drop radius is

$$(3) \quad \frac{dA}{dt} = \frac{y_c^2(V-v)w_L}{4\rho_s} = \frac{Ww_L}{4\rho_s}$$

where  $W = y_c^2(V-v)$ , with  $V$  and  $v$  being the velocities of fall of the drops, and  $w_L$  is the liquid content of the cloud of small drops of density  $\rho_s$ .

If the drops form a spectrum of varied sizes, it is necessary to compute the change in drop spectrum by collisions of various sized pairs of drops. The so called kinetic coagulation equation expresses this change. Let  $n(m) dm$  be the number of drops of mass  $m$  to  $m + dm$  per unit volume. We consider drops of masses  $M$  and  $m$ , corresponding to radii  $A$  and  $a$ . The volume swept out by the  $M$  drop per second within which  $m$  drops will be collected is

$$(4) \quad q(M, m) = \pi A^2 y_c^2 (V - v) = \pi A^2 W.$$

The change in number of  $M$  drops will therefore be

$$(5) \quad \frac{\partial n(M)}{\partial t} = \frac{1}{2\rho_s} \int_0^M q(M-m, m) n(M-m) n(m) dm \\ - \frac{n(M)}{\rho_s} \int_0^\infty q(M, m) n(m) dm.$$

The first term on the right represents the rate at which  $M$  drops are formed by collision of drops of mass  $M - m$  with drops of mass  $m$ . The second term is the rate at which  $M$  drops are taken away by collision with other drops.

Even though equation (5) represents the process somewhat more realistically, we shall discuss the collection process by means of equation (3), because it illustrates the process most clearly.

In both equations for growth by accretion the expression  $W = y_c(V - v)$  occurs. Figure 8 gives  $W$  as a function of  $A$  for various values of  $a$ . It is seen that

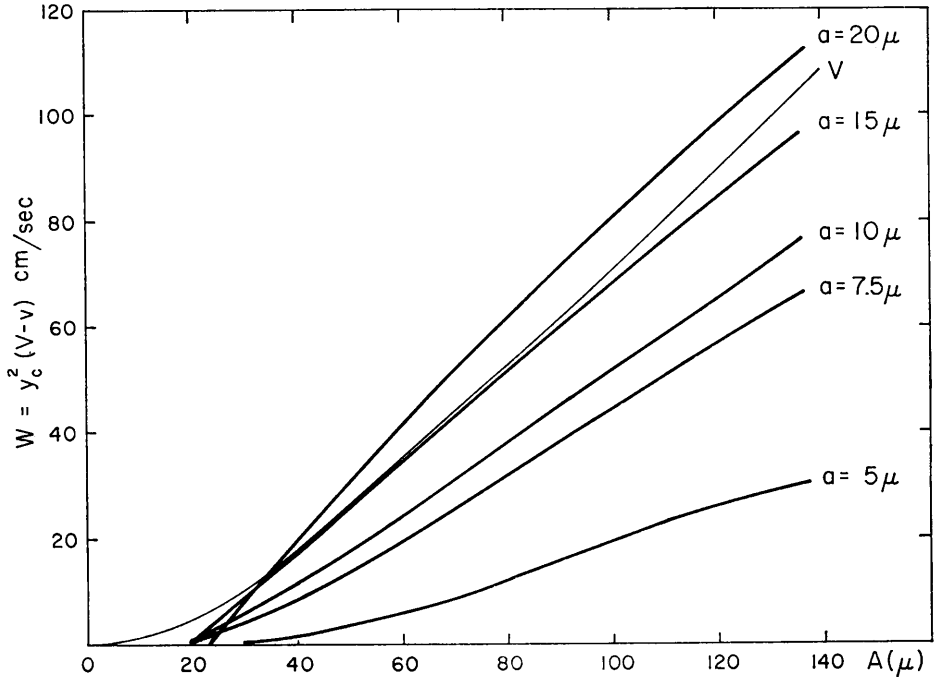


FIGURE 8

Collection coefficient  $W$  as a function of large drop radius  $A$  for clouds of droplets of radius  $a$ . The thin curve labeled  $V$  gives the terminal velocity of the large drops.

$W$  is very nearly linear in  $A$  for constant  $a$ . If we write  $W = kA - B$ , equation (2) can be integrated, yielding,

$$(6) \quad t_1 = \frac{4}{kw_L} \ln \frac{kA_1 - B}{kA_0 - B} \\ = \frac{9.2}{kw_L} (\log_{10} W_1 - \log_{10} W_0).$$

For  $a = 10 \mu$ ,  $k = 0.682 \times 10^4$ , and if  $w_L = 10^{-6}$  ( $1 \text{ g/m}^3$ ) we see that it will take 1345 seconds (22.4 minutes) for a tenfold increase in  $A$ . Table II gives the time of growth.

TABLE II  
 TIME FOR GROWTH TO 100 MICRONS BY COLLECTION OF DROPS OF  
 UNIFORM RADIUS  $a$  WITH LIQUID CONTENT  $w_L = 1 \text{ gm}^{-3}$

Initial Radius $A_0$ (microns)	$t_1$ (sec)		
	$a = 7.5$ microns ( $k = 6.1 \times 10^3$ )	$a = 10$ microns ( $k = 6.8 \times 10^3$ )	$a = 15$ microns ( $k = 8.2 \times 10^3$ )
80	190	181	132
60	493	427	334
40	1080	890	676
30	[1800]	1200	970

It is seen that for the same liquid content  $w_L$  a large drop falling through a cloud will grow more rapidly if the cloud is composed of fewer larger droplets than many smaller ones. For the same droplet size the collector drop will grow more rapidly the larger it is. The drop falling through the  $10 \mu$  droplet cloud would take almost as long in growing from 30 to 100 microns as it would from 100 microns to one millimeter in radius, assuming the linear relationship would hold to that size. Actually the linearity is fairly good to about one half millimeter, above which the terminal velocity increases much less than linearly.

These results must be considered only indicative, because of the assumption of uniform droplet size and constant  $w$  and of continuous accretion (as though the droplets were smeared throughout the volume). The assumption of uniform droplet size is acceptable at any one level, because of the narrowness of the droplet spectrum, but the size and liquid content both vary with height in the cloud, and thus with time as a large drop falls through it.

The length of time the collector drop can grow will depend on the depth of cloud through which it falls. Since initially it may be small enough to fall slower than the rising air current which is forming the cloud, the total depth of cloud through which it passes may be much larger than the geometric cloud thickness. Bowen [2] computed the duration of growth and final drop size for various updraft speeds for a collector drop starting near the base of a cloud of  $10 \mu$  radius droplets,  $w_L = 1 \text{ g/m}^3$ . Figure 9 shows the variation of drop size as a function of updraft speeds. It is seen that the accretion growth increases with updraft speed.

In considering equation (4), we see that since  $W$  varies approximately linearly with  $A$ ,  $q$  is approximately proportional to  $M$ . Golovin [5] and Scott [10] have given analytic solutions for equation (5) for certain forms of  $n(m)$  with  $q = b(m + M)$ , which is probably an adequate approximation to  $q = BM$ , the relationship corresponding to the linear variation. Scott computed the change in drop spectra with time for the case of an initial Gaussian distribution. His results show an extension of the tail of the distribution towards the large end, but not the development of a separate group of rapidly growing collector drops. On the other hand, Berry [1] carried out numerical integrations of equation (5), using the Shafrir-Neiburger collision efficiencies, and found "an explosive forma-

tion of large drops after an initial period of little noticeable change." Telford [13] and Twomey [14] examined the statistical effects of the random distribution of coalescences, and found that the fortunate few which collide sooner than the average would give rise to large drops in greater number and much sooner than

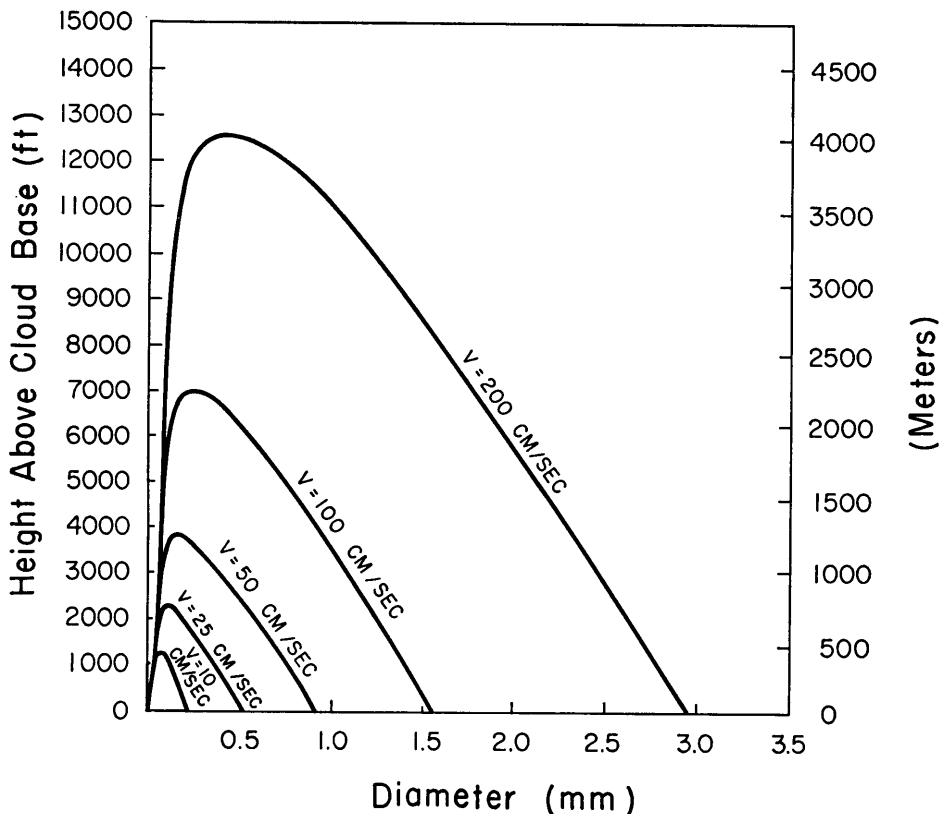


FIGURE 9

Dependence of drop growth on updraft speed.

the continuous theory indicates. Beyond suggesting that the broader the spectrum the more rapid the growth, none of these theories shed light on which factors contribute to the development of precipitation and in what way.

It is clear that the minimum requirement for growth by accretion is the presence of drops at least  $20 \mu$  in radius. (Perhaps  $25$  or  $30 \mu$  might be the lower limit in view of figure 9.) Condensation may give rise to such drops in favorable conditions, particularly if giant nuclei are present. But on many occasions these conditions appear not to be present, and we must turn to the ice crystal process for the development of the collector particles.

## 5. Growth of ice crystals in supercooled clouds

That cloud drops remain liquid at temperatures considerably below  $0^{\circ}\text{C}$  has been known for many decades. The role of nuclei for the formation of ice in atmospheric clouds, while suggested as early as 1911 by Wegener, has only recently become well understood. Thus, when Bergeron pointed out in 1933 that the coexistence of a few ice particles in a supercooled liquid cloud would lead to rapid evaporation of the liquid drops and deposition on the ice, leading to ice crystals large enough to fall, he proposed that the ice particles formed by the spontaneous freezing of some of the drops. It is now known that spontaneous freezing (homogeneous nucleation) of drops of the size occurring in clouds requires temperatures between  $-35^{\circ}\text{C}$  and  $-40^{\circ}\text{C}$ . Freezing at higher temperatures requires nuclei. The reason why supercooling occurs is that there are relatively few nuclei effective at or slightly below  $0^{\circ}\text{C}$ , and the probability of a small cloud drop containing one is very low. At progressively lower temperatures, the number of nuclei which are effective increases, but even for  $-15^{\circ}\text{C}$  naturally occurring nuclei which are effective are sparse.

Figure 10 shows some examples of the concentration in the air of natural nuclei effective at various temperatures. The number effective varies exponentially with the temperature, ranging from about one per cubic meter at  $-5^{\circ}\text{C}$  to one per cubic centimeter at  $-30^{\circ}\text{C}$ . These concentrations are orders of magnitude less than that of the condensation nuclei. For a given temperature the number varies greatly (by several orders of magnitude) from time to time and place to place. The most frequent natural nuclei are particles of clay, mostly kaolin, which begin to be active at about  $-12^{\circ}\text{C}$  and are highly effective at  $-24^{\circ}\text{C}$ .

The rapidity of the growth of ice crystals in the presence of supercooled drops is due to the fact that the equilibrium vapor pressure over ice is lower than over liquid water at the same temperature below  $0^{\circ}\text{C}$ . If the air is saturated with respect to liquid water, it will be supersaturated with respect to ice. The amount of supersaturation, proportional to the amount of supercooling, is about 10 per cent at  $-10^{\circ}\text{C}$ . Since the growth rate depends on the supersaturation, it is clear that ice crystals in a supercooled cloud will grow much faster than condensing drops, for which supersaturations rarely exceed a few tenths of a per cent. More important to the ultimate size of the crystals, however, is the fact that the ice crystal is not limited with respect to availability of water vapor, for the vapor which condenses on it is replenished from the evaporating drops, and the sparsity of crystals means there is no competition among them.

There have been several studies of the comparative rates of the ice crystal growth and drop growth by accretion. Thus Houghton [6] in one of the earliest of these found that a plane dendritic crystal growing in liquid water saturation at  $-15^{\circ}\text{C}$  would increase its mass by a factor of about  $10^4$  in 20 minutes, corresponding to equivalent drop radii increasing from 10 microns to 250 microns.

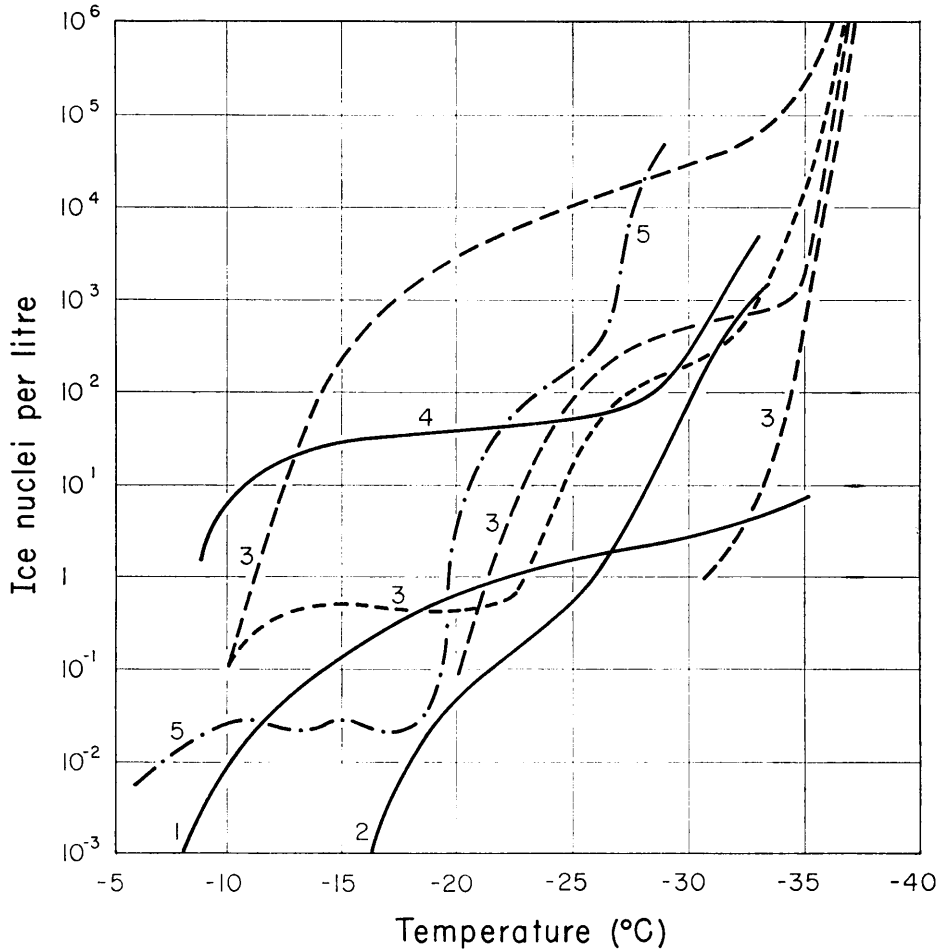


FIGURE 10

Examples [4] of ice nucleus "spectra," that is, numbers of nuclei effective at various temperatures.

This is much faster than the collection process in the early stages, and of course such a crystal would fall relative to the cloud drops and may grow by accretion as well. Ludlam [8] carried out computations which indicate that in summer cumulus congestus clouds with bases warmer than  $10^{\circ}\text{C}$  and large updrafts, drops starting sufficiently large near the base will grow faster by accretion than ice particles originating near the top, but in colder clouds the ice particles will grow faster.

The specific rate of ice crystal growth in any situation will depend on the amount of supercooling and the activity spectrum of the ice nuclei. At present ice nuclei are measured at only a few places, usually with respect to their activity

at  $-20^{\circ}\text{C}$ , but in principle they could be measured wherever it is desired. The amount of supercooling will depend on the temperature and flow of air producing the cloud and the temperature environment. These factors are determined by the larger scale structure of the atmosphere and the dynamics of its motion.

## 6. Dynamics of clouds and precipitation

Full consideration of the dynamic processes in clouds would require as much discussion as the microphysical processes. Instead we shall outline in skeleton fashion the essential facts which are known about it. The meteorological conditions for the formation of precipitation were clearly set forth by J. Bjerknes and H. Solberg in 1921. They demonstrated that any considerable amount of rain or snow requires upward motion of the air. The ascent of the air can be associated with either free convection or due to forced lifting by mountains, fronts, or in regions of general convergence of the horizontal air flow.

Free convection, which is induced by establishment of a large lapse rate of temperature by heating from below, advection of cold air at high levels, or vertical stretching of air columns, has been the subject of extensive analytic and numerical studies, but the details of its mechanism are only partially understood, and we are far short of being able to predict its nature in a particular case. One of the two principal contending theories of cumulus convection treats the phenomenon as a continuous jet or plume; the other treats it as a succession of "bubbles" or pulses of rising air. In both theories air is pulled into the rising current from the sides, and this entrainment of environmental air reduces the buoyancy and liquid content of the rising air and limits its upward acceleration and the height to which it rises.

The variation of temperature with height in convective conditions is illustrated in figure 11, which presents the averages of observations on thunderstorm days in Ohio and Florida. The soundings show that a saturated air parcel near the ground rising adiabatically without entrainment would rise almost to the tropopause. Entrainment would lead to more rapid cooling of the rising parcel, and it is to be expected that the development of the thunderstorm cells consisted of successive impulses in which an undiluted core penetrated higher and higher before being affected by dilution. The life of such a cell, as observed in the Thunderstorm Project [3] is shown in figures 12, 13, and 14.

Entrainment and drag may sufficiently limit convection to prevent precipitation from developing. Thus J. Simpson, R. Simpson, Andrews and Eaton [11] studied the situation shown in figure 15, in which the clouds reached only about 7 km, even though an undiluted saturated parcel would be buoyant to the stratosphere. They computed that entrainment would lead to a cooling rate shown by the short dashed curve, intersecting the environmental sounding curve near the cloud top and just below a stable dry layer. These cooling rates are based on the cloud remaining liquid. If the cloud turned to ice beginning at  $-4^{\circ}\text{C}$  their computation shows that the cooling would follow the dotted curve

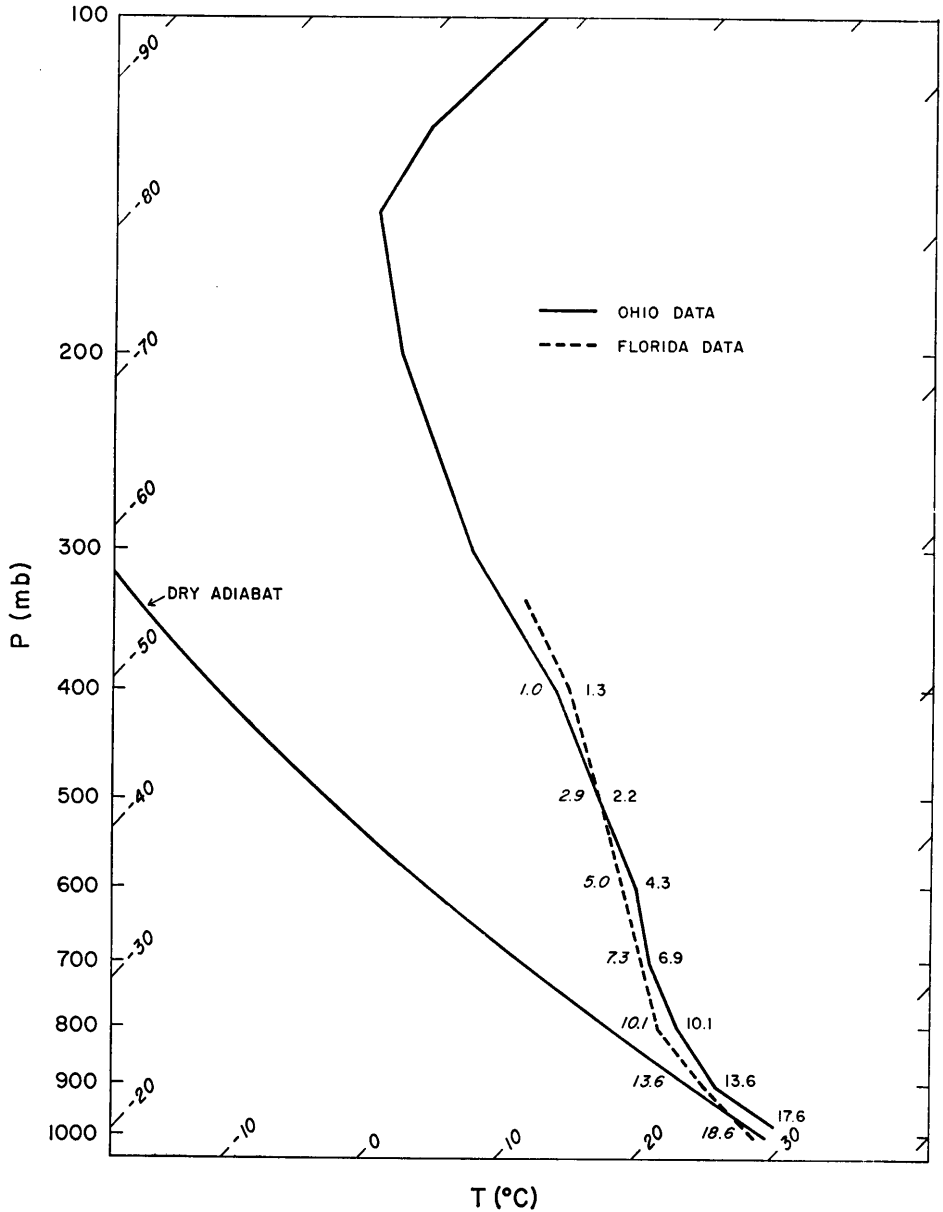


FIGURE 11

Average variation of temperature with pressure on days with thunderstorms in Ohio (solid curve) and Florida (dotted curve). The water vapor mixing ratios in parts per thousand are shown by numbers beside the curves every hundred millibars.

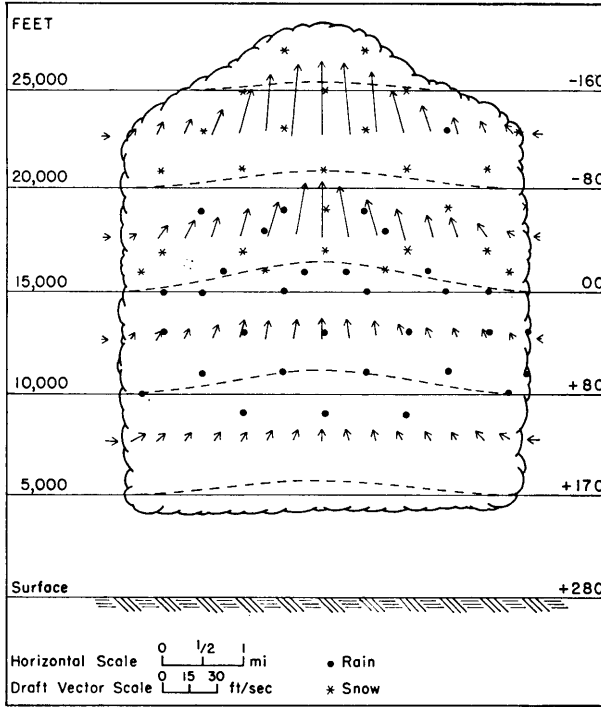


FIGURE 12

Schematic cross section through a thunderstorm cell in its cumulus or growing stage, as shown by aircraft traverses in the Thunderstorm Project. Dashed lines bulging upward are isotherms. Arrows show air motion, with speed given by scale in lower left of diagram.

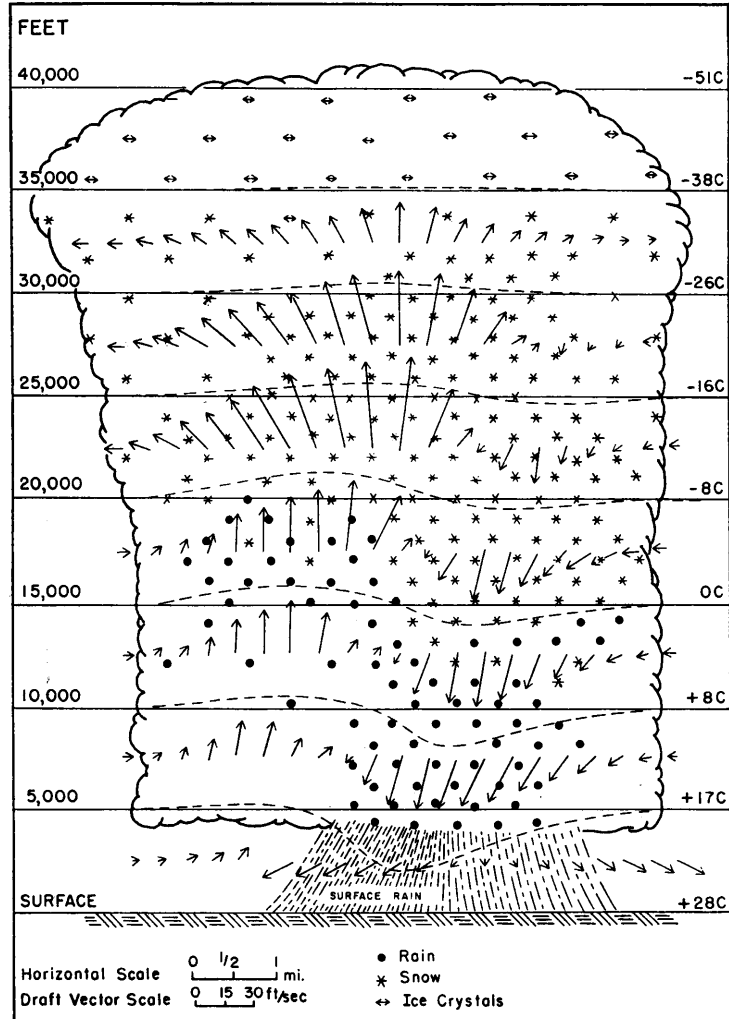


FIGURE 13  
Schematic cross section through a thunderstorm  
cell in its mature stage.

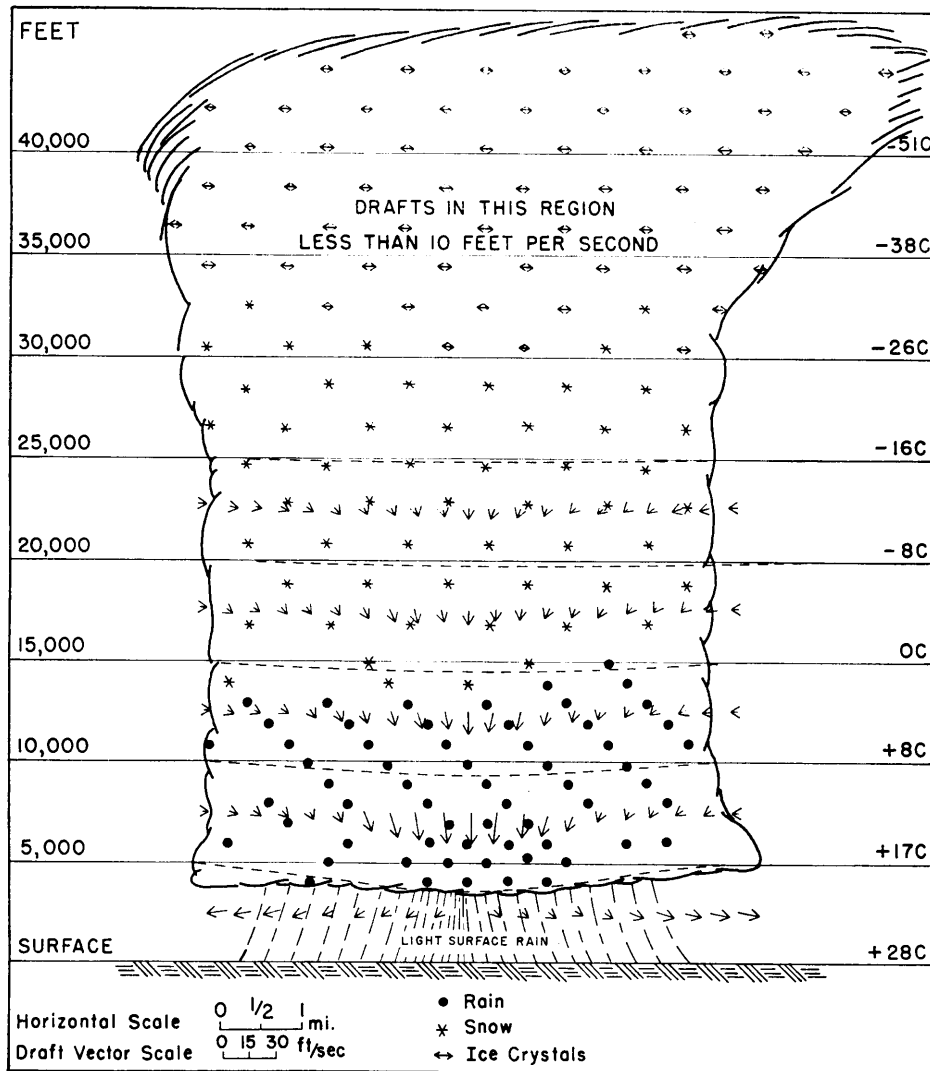


FIGURE 14

Schematic cross section through a thunderstorm cell in its dissipating stage.

and the cloudy air could rise through the stable layer. Seeding a cloud with massive quantities of silver iodide on this day was followed by growth of the cloud to great heights.

The example just discussed illustrates the potential effect of the microphysical processes on the dynamics, whether or not the seeding was responsible in this instance. While the details of the entrainment process are not sufficiently certain for confidence in the accuracy of the cooling curve, it is clear that the general

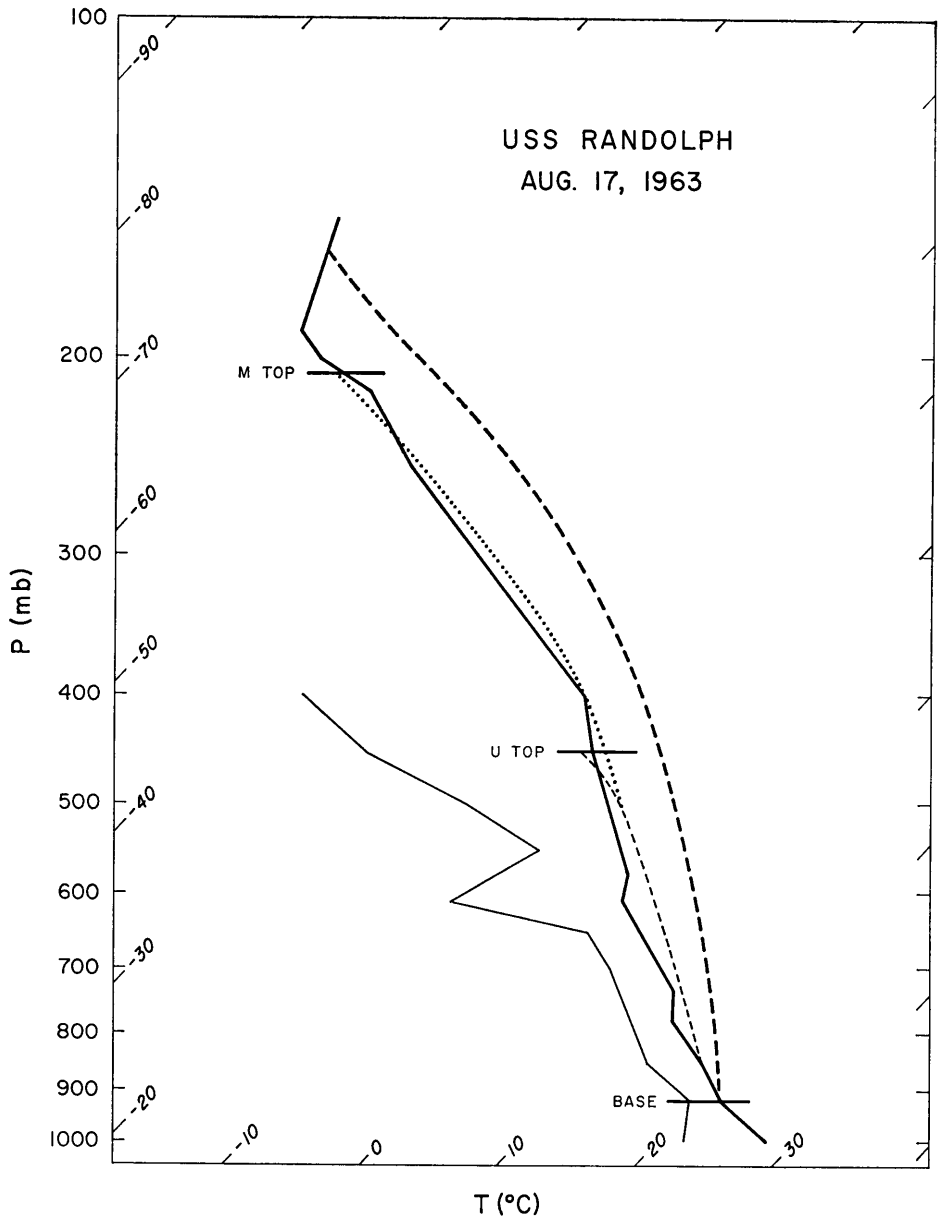


FIGURE 15

Variation of temperature with pressure over Caribbean Sea at 2030 local time August 17, 1963, shown on a tephigram. Constant pressure lines would be diagonals sloping upward to the right. As in figure 11, the dashed curve represents the variation with temperature which would be experienced by an undiluted saturated air parcel rising adiabatically from the cloud base. The observed temperature variation is shown by the solid curve, and the dew point by the thin solid curve.

character of the result is correct, and that the question whether or not the clouds would penetrate the stable layer would depend on whether ice nuclei effective at these temperatures were present as well as on the amount of entrainment.

The flow over mountains and fronts has likewise been studied considerably in recent years without complete solution. Under some circumstances the upward velocity increases with distance above the sloping surface, in others it decreases, and in still others mountain waves form. Potential instability may be released and convective vertical motions added to the forced ascent. In any given situation the exact pattern of the upward currents may be difficult to predict, and consequently the rate of condensation and precipitation will be likewise difficult to estimate.

## 7. Conclusions

From the above it is clear that the question whether precipitation will occur naturally at optimum rates cannot be answered adequately either in general or in a specific situation.

From the overall standpoint no one has yet established a coordinated theory which would enable estimation of the number and sizes of condensation nuclei and of ice nuclei effective at various temperatures which would lead to maximum precipitation for a given distribution of temperature, humidity, and vertical velocity. Added to this, for proper solution the mutual interaction of the other parameters with the vertical velocity would have to be considered. Until such a theory is arrived at, only guesses can be made concerning what direction and amount the parameters should be changed in order to produce a desired effect.

In specific cases if the natural nuclei counts, temperature, humidity, and vertical velocity are known, computations can be carried out that will indicate approximately the cloud drop distribution which will develop, whether and at what rate the liquid accretion process will result in drop growth, and at what level and rate the ice crystal process will occur. The effects of changes, that is, increases in the number of large condensation nuclei or ice nuclei or introduction of large water drops, could likewise be computed. Unfortunately, in addition to the uncertainty of the physical theory, the large amount of machine time required for the computations would delay the availability of the results beyond the time when the changes could be put to experimental test.

A way of handling the timing problem would be to carry out computations for enough combinations of parameters so that at the time a situation arises a combination approximating that which is observed could be pulled out of the file.

The need for complete observations is obvious. Without knowing the actual physical conditions one cannot intelligently vary them.

Finally, the need for additional investigation of the precipitation processes is clear. Until the objective of capability to compute accurately the rates of these

processes for given conditions is attained, the effect of varying them cannot be predicted.

The essential objective must be the ability to discriminate among the situations when a particular artificial seeding procedure will increase the precipitation rates and those in which it will decrease them. At present such a discrimination is not possible, and the probability is that seeding as carried out in practice sometimes increases and other times decreases precipitation with a net result that is usually close to zero and therefore hard to evaluate. When we can seed only the truly favorable situations, we may expect clearly demonstrable results which will not strain the ability of statistical methods to detect.

In the above I have concerned myself mainly with the problem of increase in precipitation. The applicability of what I have said to attempts to decrease it, dissipate fog and cloud, and eliminate hail is clear. In all of these, the same need to understand the physical processes and discriminate between favorable and unfavorable cases is present. It is to be hoped that more effort will go into gaining of this understanding and ability to discriminate, and less to indiscriminate seeding experiments.

Finally I should like to make clear that I recognize that field experiments in which cloud seeding is used to study the conditions under which it will produce particular effects represent one way of advancing the desired understanding. As a rule I would favor well designed experiments in which the physical parameters are carefully measured and specific hypotheses set out in advance are tested. However, incompletely instrumented experiments may be informative, and *post facto* hypotheses may be suggested and tested by the results of seeding experiments which initially did not adequately discriminate with respect to seeding opportunities. In all *post facto* studies, however, especial care must be exerted to avoid depreciating their significance by using the same experimental results to arrive at the hypothesis and to test it.

There is much to be done before we have developed efficient methods of controlling precipitation processes. We must act wisely and deploy our resources well in choosing our procedures. The objective promises so much in social and economic benefit as well as in scientific gratification that we should seek to attain it as quickly as sound scientific methods permit. To help identify these sound methods has been my purpose in this paper.

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