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CONVEX RELAXATION FOR HARD PROBLEM IN DATA MINING AND SENSOR LOCALIZATION

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During the three-year period of the grant, the PI's discovered new, faster and more robust optimization solvers and also new methodologies for using optimization to find hidden structure in large data sets. The results led to journal publications and conference talks. This report provides some of the highlights of the results.

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Final Report to AFOSR: CONVEX RELAXATION FOR HARD PROBLEM IN DATA MINING AND SENSOR LOCALIZATION

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During the three-year period of the grant, the PI's discovered new, faster and more robust optimization solvers and also new methodologies for using optimization to find hidden structure in large data sets. The results led to journal publications and conference talks. This report provides some of the highlights of the results.

In the realm of solvers, we discovered novel ways to combine two classical first-order methods for convex optimization, namely, accelerated gradient and conjugate gradient. Conjugate gradient, due to Hestenes and Stiefel (1952) is the optimal first-order method for solving quadratic minimization problems. Nesterov (1983) introduced the accelerated gradient method, which is optimal, though in a weaker sense, for strongly convex problems. The two algorithms seem similar, and yet their analyses are complete different. We obtained several results with former student S. Karimi [26, 27, 28] showing how to unify the two algorithms. The point of this unification is a new algorithm that is both optimal for quadratic problems and just as fast or faster than accelerated gradient for other classes of problems. With former postdoc D. Drusvyatskiy and G. Lin [8], we showed that classic alternating projection, one of the original first-order methods, can successfully solve the ill-posed semidefinite programming problems if used properly. This is immediately applicable to the class of ill-posed semidefinite instances that often arise in data mining problems.

In the area of applications of convex optimization to recovering hidden former student B. Ames [22] we developed a new algorithm for clustering problems based on semidefinite programming. The new algorithm is the best possible under the hypothesis of hardness of planted clique. With Drusvyatskiy, Krislock and Voronin [11], we developed a convex relaxation of the noisy sensor localization problem that resolves problems with inexactness in earlier methods based on facial reduction. The two PI's together, together with Drusvyatskiy [2], we developed a theory of faces of the unit ball of sums of norms; this result has immediate implications for several algorithms proposed in the literature to simultaneously reveal sparse and low-rank structure.

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