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Ion Acoustic Turbulence, Anomalous Transport, and System Dynamics in Hall Effect Thrusters

Robert Martin[1], Jonathan Tran[2]

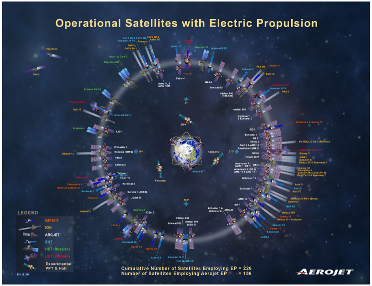
[1]Air Force Research Laboratory,
[2]ERC Inc.,
Edwards Air Force Base, CA USA

IPAM Mathematics of Turbulence Retreat, June 2017
UCLA Lake Arrowhead Conference Center, CA

Electric Propulsion

EP-Devices:

- EP Improves Thrust/Mass (Isp)
- Ion/Elec.-Thermal/HET Flying
- FRC/MPD/Electrospray/etc. in Dev.
- Space notoriously Risk Adverse (i.e. Tech must be "Proven")



Aerojet Overview of EP Satellites (3/08)

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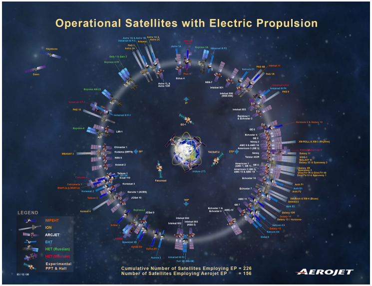
- Plasma still Threat Thruster/Plume
- Improved Life Estimates Needed
- Space not Replicated in Ground Test

Models needed to Bridge Gap



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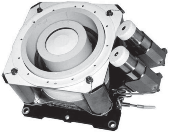

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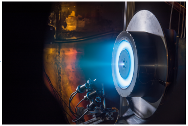

Fakel SPT-100 NASA Hermes Thruster

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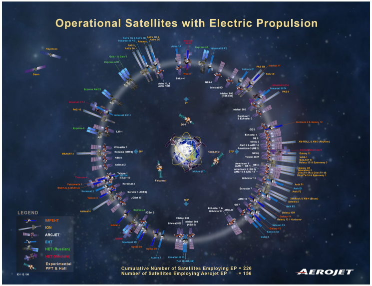

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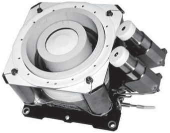
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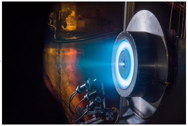
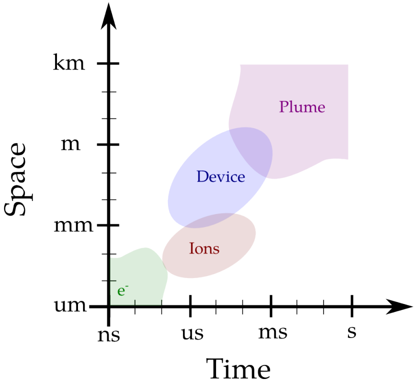


Fakel SPT-100 NASA Hermes Thruster

Inherently Multiscale:

- $\mathcal{O}(10^9)$-Space and Time
- Naive 3D-Spatial Scales $\rightarrow$ Cost^3



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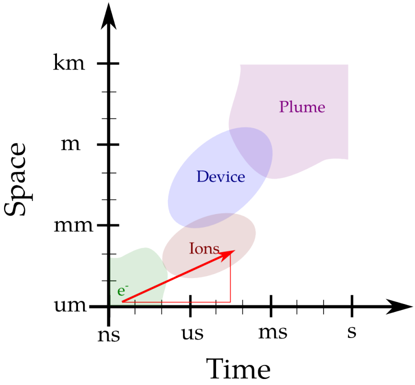

$m_i/m_e \approx \mathcal{O}(10^{3.5})$-$\mathcal{O}(10^{5.5})$

Time and Space Scales

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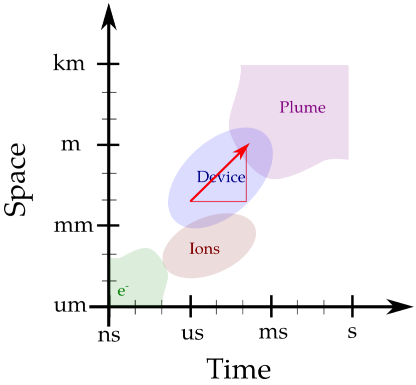
- Isp & Device Scale $\rightarrow$ Transit Time



Isp=1000 Line

Time and Space Scales

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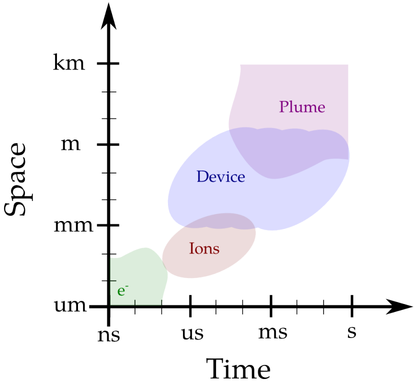
- Pulsed: Spread Right at Scale & Isp



$\mathcal{O}(\mathrm{GHz})$

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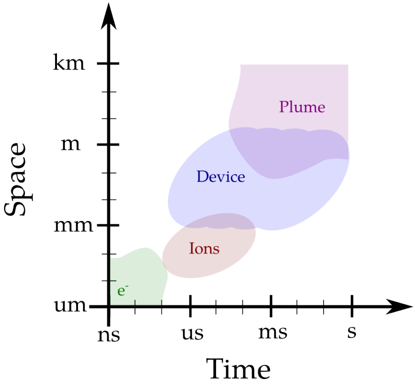
- Spacecraft Lifetime Performance?



$\mathcal{O}$(GHz)$\rightarrow$ $\mathcal{O}$(Years!)

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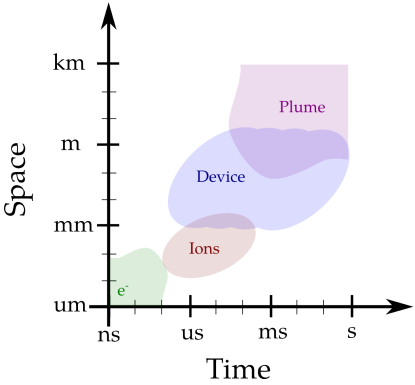
Modeling Must Exploit Scale Separation



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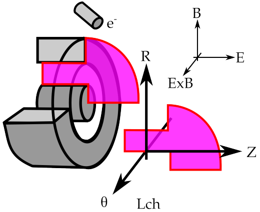
Hall Effect Thrusters (HET):

- Traditionally Modeled in R-Z
- Named for Hall Current in θ
- Uses Quasi-1D Electron Fluid Solve
- Ohm's Law $\rightarrow$ No e^--momentum

RZ-Plane Simulation



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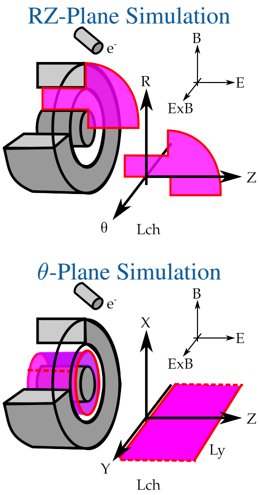

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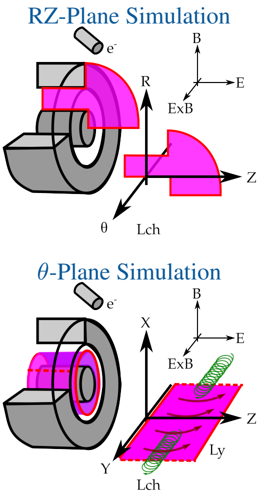
- Unmagnetized Ions
- Results in Hall Current (Namesake)



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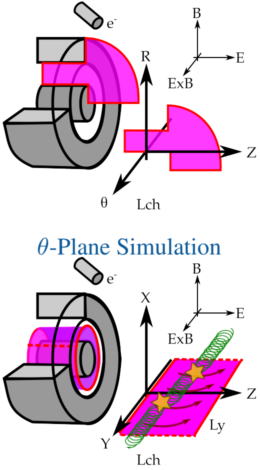

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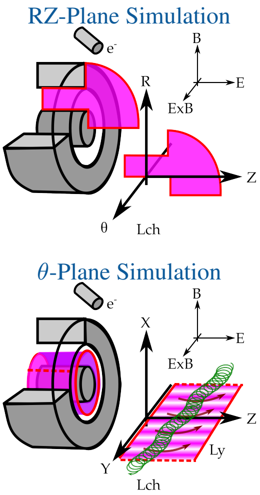
- Ion-Acoustic Waves: Collisionless Mobility?



Model Sensitive to Mobility:

- Classical Mobility Insufficient in Near Plume

From LaFleur, Phys Plasmas 23, 053503 (2016)

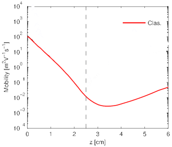


FIG. 3. Electron cross-field mobility as a function of axial position within the PPS®1350 thruster. The solid black line is an empirical mobility which is needed in fluid simulations in order to get agreement with experiment,[16] the red line is the mobility based on classical diffusion across a magnetic field, while the open blue triangles show the mobility due to the saturated instability-enhanced electron-ion friction force. The vertical dashed line indicates the thruster exit plane.

(Thruster Simulation will not "Light")

Model Sensitive to Mobility:

- Classical Mobility Insufficient in Near Plume
- Enhanced Emperical “Bohm” (1/B) Mobility
- Also Needs Coefficient by “Zone” (i.e. Anode/Channel/Plume Regions)

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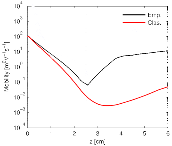

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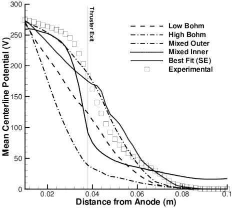
Impact of Mobility on Discharge Profile



(Koo, PhD Dissertation)

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- Behavior Sensitive to Plume Coefficient

Critical if Operating Near Mode Change

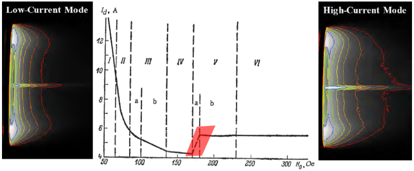

Fig. 1 Discharge current as function of magnetic field with constant discharge voltage showing operational regimes defined by Tilinin [2]

Sekerak, Gallimore, Brown, Hofer, and Polk, JPP 2016

Brown, Lobbia, and Blakely, JPC 2014

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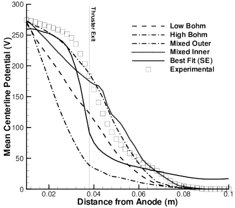

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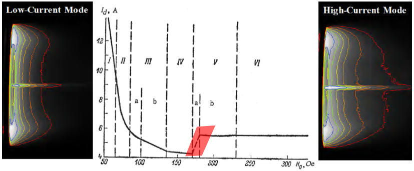
Need Physics Based Model

Critical if Operating Near Mode Change



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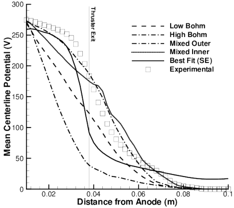

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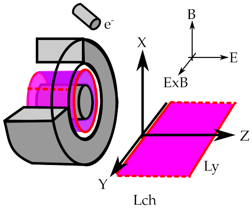
Current Driven Instability:

- HET Ion-Acoustic Instability
  -Driven by j_θ Hall Current
  -Anomalous e-Transport? (LaFleur)
  -Focus of 1D/2D Full-PIC Studies



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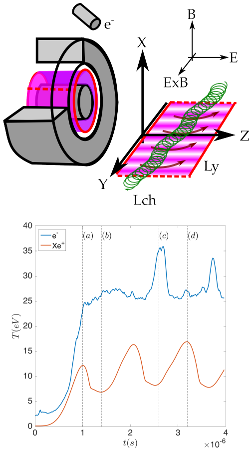
  Initial Exponential Growth Saturates



Instability and Transport

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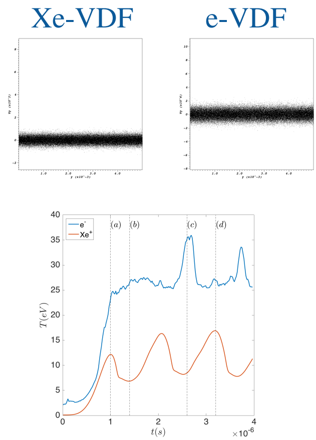

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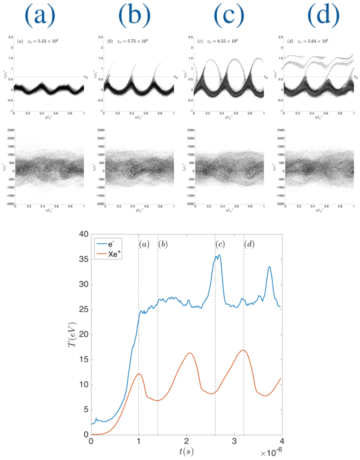

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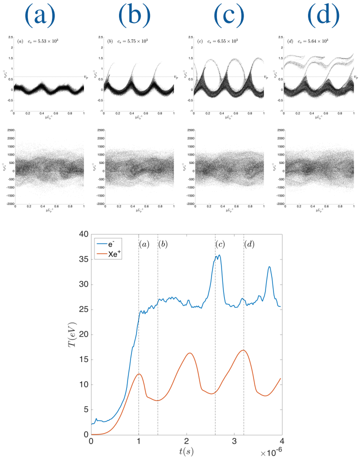

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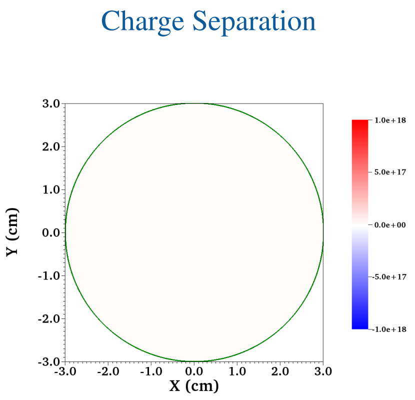

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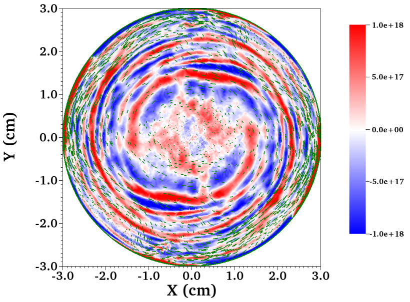
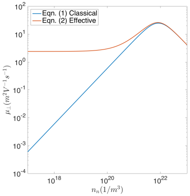
- $j_\theta \rightarrow$ FRC Spiral Charge Separation?
- Onset of Plasma Turbulence?

Charge Separation



Impact of Ion-Acoustic Instability:

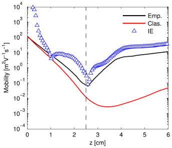
- Instability → Extra Mobility



Impact of Ion-Acoustic Instability:

- Instability → Extra Mobility
- LaFleur Results Promising

From LaFleur, Phys Plasmas 23, 053503 (2016)



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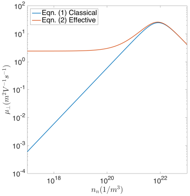
(A Posteriori Mobility)



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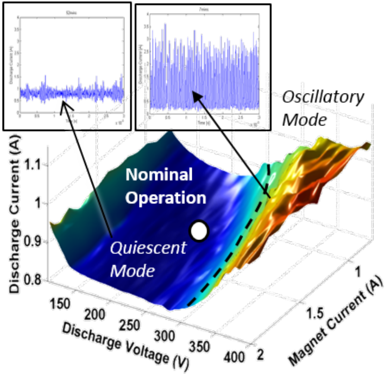
- Need High-Dim Validation

Experimental Discharge Current Oscillations



I-V-B Plot of Hall Thruster Operation

Brown, EP TEMPEST Program Review, 2015

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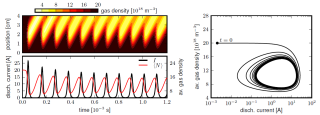
- Need High-Dim Validation
- Model via Lagged Lotka-Volterra?



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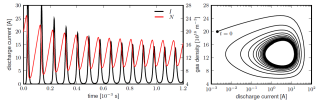

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Barral & Peradzyński, IEPC-2009-070

Impact of Ion-Acoustic Instability:

- Instability → Extra Mobility
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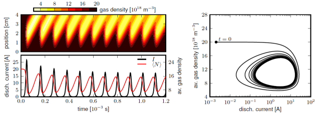
- Model via Lagged Lotka-Volterra?
- Model Captures a Bifurcation



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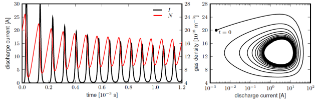

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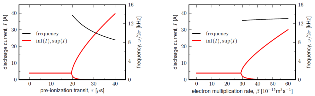

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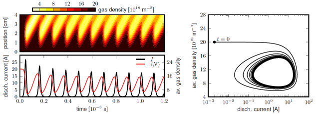
How can we Compare
Model, Experiment, and Simulation?



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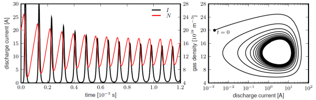

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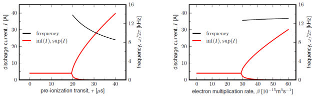

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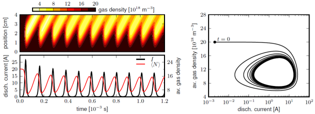
With only I(t) Accessible Experimentally?



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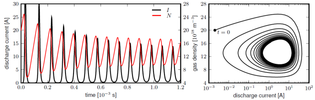

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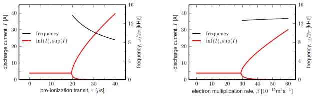

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State Space Reconstruction:
Time Series and Dynamic Systems
A supplemental simulation and animation for
"Detecting Causality in Complex Ecosystems"
George Sugihara, Robert May, Hao Ye, Chih-hao Hsieh,
Ethan Deyle, Mike Fogarty, and Stephan Munch
animation by: Peter Sugihara, Hao Ye, and George Sugihara
© August 2012

Convergent Cross Mapping I.

Same Could be Done for Hall Thruster Simulation

Same Could be Done for Hall Thruster Simulation
But Experiment has Limited Access?

State Space Reconstruction:
Takens' Theorem and Shadow Manifolds
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Shadow Maifold from only I(t)?

Convergent Cross Mapping II.

Shadow Maifold from only I(t)?

Useful Tuning Model/Simulation Parameters?


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Convergent Cross Mapping III.

Useful Detecting Causal Relationships.

Convergent Cross Mapping III.

Useful Detecting Causal Relationships.

Potential Tool for Self-Similarity/Causality in Turbulence?

Turbulence from Dynamic System

DMD → Tubulence:

- Low Re Vortex Shedding

Von Karman Vortex Street
http://commons.wikimedia.org/wiki/File:Karman_Vortex_Street_Off_Cylinder.ogv

DMD → Tubulence:

- Low Re Vortex Shedding
- Sparse Dynamic System via DMD

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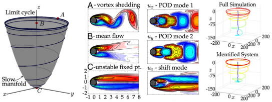

Fig. 2. Example of high-dimensional dynamical system from fluid dynamics. The vortex shedding past a cylinder is a prototypical example that is used for flow control, with relevance to many applications, including drag reduction behind vehicles. The vortex shedding is the result of a Hopf bifurcation. However, because the Navier–Stokes equations have quadratic nonlinearity, it is necessary to use a mean-field model with a separation of timescales, where a fast mean-field deformation is slave to the slow vortex shedding dynamics. The parabolic slow manifold is shown (*Left*), with the unstable fixed point (*C*), mean flow (*B*), and vortex shedding (*A*). A POD basis and shift mode are used to reduce the dimension of the problem (*Middle Right*). The identified dynamics closely match the true trajectory in POD coordinates, and most importantly, they capture the quadratic nonlinearity and timescales associated with the mean-field model.

Brunton, Proctor, and Kutz, 3932-3937, PNAS, 4/12/16, v.113, no. 15

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- What if Flow is Non-Sparse? (Turbulent)

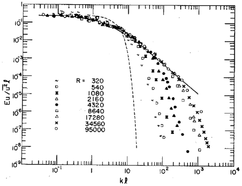

FIG. 6. Dependence of the frequency spectra $E_u/\overline{u^2}\,l$ on Reynolds number in the center of the wake.

Uberoi and Freymuth, Phys. Fluids 12, 1359 (1969).

DMD → Tubulence:

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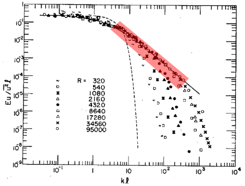
- Universal if Agnostic to Large and Small k



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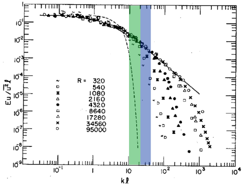
  (Filter Local Space/Time: Gabor? Wavelets?)



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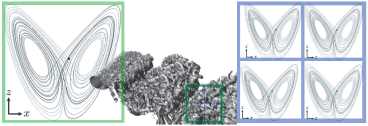

Piomelli, Phil. Trans. Royal Soc., A 372. 2014.
https://commons.wikimedia.org/wiki/File:A_Trajectory_Through_Phase_Space_in_a_Lorenz_Attractor.gif

Turbulence from Dynamic System

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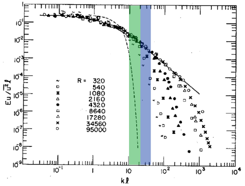

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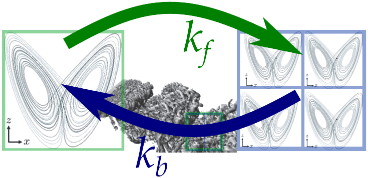

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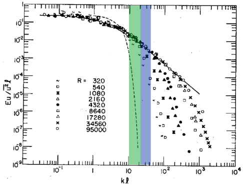
- CCM → K_j in Mori-Zwanzig LES?



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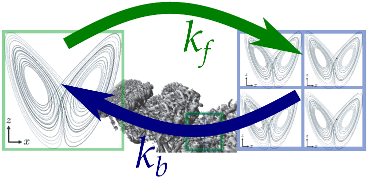

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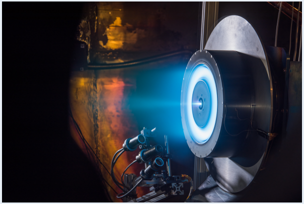
$$w_j^{(0)} = \mathcal{P} \int_o^t K_j(\overline{u}(t-s), s)ds$$

Parish and Duraisamy, AIAA Expo, 6/16

Thank You



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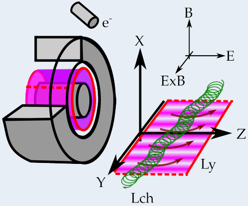

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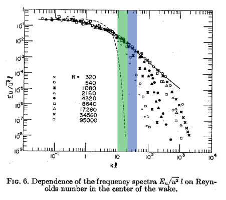

Work Supported through AFOSR Task 17RQCOR465 (PM: Birkan)

Questions?