A Cascaded Self-Similar Rat-Race Hybrid Coupler Architecture and its Compact Ka-band Implementation

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Abstract: *We present a cascaded rat-race coupler architecture that performs broadband in-phase and differential signal generation while maintaining a very compact passive structure size. A proof-of-concept 24-42 GHz 2nd order cascaded rat-race coupler is implemented in the Globalfoundries 45nm CMOS SOI process. Based on 3D full-wave electromagnetic (EM) modeling and simulations, the cascaded rat-race design achieves a less-than 5° phase error, less-than 1dB magnitude mismatch, an average insertion loss of only 2.5dB, and better-than -10dB input matching across the entire Ka-band.*

Keywords: hybrid coupler; rat-race; ring coupler; size reduction.

Introduction

Silicon device scaling has led to the continuous performance improvement of active devices. However, monolithic passive structures do not directly benefit from such device scaling, where their performance and form-factor are mostly determined by their intrinsic circuit topologies. As a result, in many RF/mm-wave integrated circuits, on-chip passive components often consume large and expensive chip real-estate and limit the system-level performance, including bandwidth, gain, and energy-efficiency. These many challenges are positioning passive networks as major technology differentiator in RF/mm-wave circuit and system designs.

180° hybrid couplers are a fundamental passive building block and are used in numerous RF/mm-wave systems for radar and wireless communications. Although a Marchand balun covers a large bandwidth, it is inherently a 3-port network and cannot be used as a true 4-port coupler in applications such as antenna array beamforming. Among 180° hybrids, the rat-race coupler is a popular solution due to its compatibility with coplanar fabrication technologies [1]. However, a conventional rat-race coupler requires multiple λ/4 transmission lines (t-lines), making its on-chip designs very costly even for RF/mm-wave bands. Reported miniaturized rat-race couplers have extended bandwidth but at the expense of severely degraded insertion loss, amplitude/phase imbalance, or complex 3D fabrication processes [2], [3].

In this paper, we propose a cascaded 180° rat-race coupler topology that uses narrow-band rat-race couplers as building blocks to form a high-order self-similar 180° coupler network and achieve a substantial bandwidth expansion and improved phase/amplitude balancing with low insertion loss [4]. A 2^{nd}-order proof-of-concept cascaded 180° rat-race coupler is built based on previously reported folded inductor based rat-race coupler [5], and achieves a less-than 1dB magnitude mismatch, less-than 5° phase mismatch, and less than 2.9dB insertion loss from 24 to 42 GHz over the entire Ka-band.

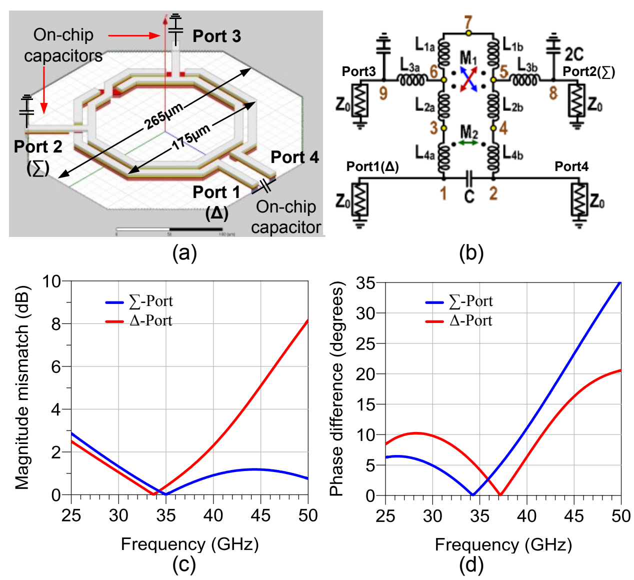

Figure 1. (a) Folded-inductor rat-race 3D EM model, (b) equivalent schematic, and 3D EM simulation results of (c) magnitude mismatch and (d) phase error.

Folded Inductor Based Rat-Race Coupler

Conventionally, rat-race couplers are synthesized using λ/4 and 3λ/4 t-lines. To shrink the coupler size, lumped elements are employed to replace the t-line sections with their Π or T lumped equivalent circuits [6]. In our previous research, the rat-race coupler size is decreased even further by combining the three inductors needed into one folded inductor geometry, thus creating a rat-race coupler with only one inductor footprint [5]. Figure 1a shows the 3D EM model of the rat-race coupler utilized in this paper where port 1 and port 2 are the difference and sum ports, respectively. Figure 1b shows the equivalent schematic for the folded inductor based rat-race coupler, which differs from the classical six element lumped model (three inductors and three capacitors) due to the magnetic coupling among

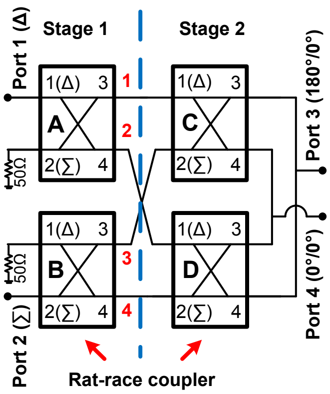

Figure 2. Folded-inductor rat-race equivalent schematic.

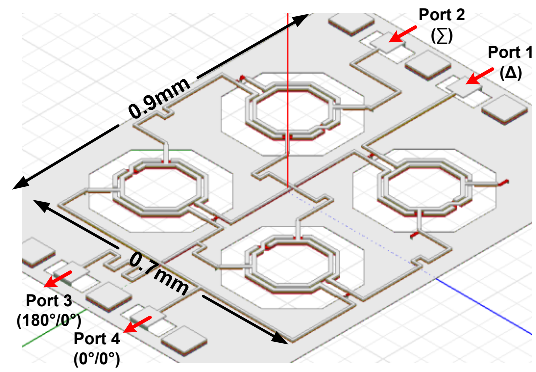

Figure 3. 3-D EM model of proposed 2^{nd} order rat-race coupler using folded-inductor rat-race coupler.

adjacent traces in the folded structure. Figure 1c and 1d show the 3D EM simulation results of the magnitude mismatch and phase error for the single rat-race coupler.

Cascaded Rat-Race Coupler Network

Figure 2 shows the circuit block diagram of a 2^{nd}-order design of our proposed cascaded rat-race coupler, comprising four unit 180° couplers together forming a true 4-port coupler network. Moreover, our proposed design is a self-similar structure, and it can be extended to it higher-order (n^{th}-order) implementations with a total of $2^{2\times n}$ individual rat-races (n = 1 for a 2^{nd}-order design).

For this 2^{nd}-order design, the rat-race coupler network can be divided into two stages, where each stage consists of two narrow-band rat-race couplers (Figure 2). For the stage 1, each individual coupler has one of its input ports terminated with a 50Ω resistor. To obtain the correct magnitude and phase difference at the output, port 3 and port 4 of the individual rat-race couplers at the stage 2 are combined as shown in Figure 2. This cascaded structures greatly suppresses the magnitude/phase mismatches across a broader bandwidth due to the high-pass or low-pass response of different signal paths [5], which cancels the magnitude and phase errors when combined appropriately. It is important to note that the phase and magnitude error cancelling effects are correlated, thus this cancelling effect is limited in most cases by the magnitude mismatch across frequencies of each rat-race.

To analyze the magnitude and phase mismatch reduction due to our proposed cascaded rat-race coupler structure, we can compare the mismatch of a single rat-race with that of a cascaded rat-race coupler. In our analysis we use the subscripts '*s*' and '*c*' to differentiate between the S-parameters of a single and cascaded rat-race, respectively. We also assume that the four unit rat-race couplers are identical designs with the same magnitude and phase mismatches at corresponding ports.

We will first consider the magnitude and phase mismatch of a single rat-race. Its magnitude and phase mismatch can be calculated using the 4-port S-parameters in (1)-(4). Additionally, we can define the Common-Mode

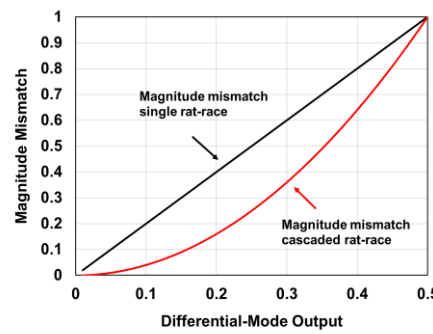

Figure 4. Calculated mismatch of the Δ and Σ ports as a function of the Differential-Mode output of a single rat-race.

(CM) and Differential-Mode (DM) outputs of a single stage rat-race for the Δ-Port and Σ-Port using (5)-(8), as

$$|\Delta\text{-Port}_s|_{mismatch} = ||S_{31}| - |S_{41}|| \quad (1)$$

$$|\Sigma\text{-Port}_s|_{mismatch} = ||S_{32}| - |S_{42}|| \quad (2)$$

$$\angle\Delta\text{-Port}_{s_{mismatch}} = |\angle(S_{31}) - \angle(S_{41})| \quad (3)$$

$$\angle\Sigma\text{-Port}_{s_{mismatch}} = |\angle(S_{32}) - \angle(S_{42})| \quad (4)$$

$$CM_{\Delta_s} = \frac{1}{2}(|S_{41}| + |S_{31}|) \quad (5)$$

$$DM_{\Delta_s} = \frac{1}{2}(|S_{41}| - |S_{31}|) \quad (6)$$

$$CM_{\Sigma_s} = \frac{1}{2}(|S_{42}| + |S_{32}|) \quad (7)$$

$$DM_{\Sigma_s} = \frac{1}{2}(|S_{42}| - |S_{32}|). \quad (8)$$

Furthermore, for a single stage rat-race, equations (5)-(8) can be used to define the 4-port S-parameters as expressed in (9)-(12), as

$$S_{41} = (CM_{\Delta_s} + DM_{\Delta_s})e^{j\pi} \quad (9)$$

$$S_{31} = (CM_{\Delta_s} - DM_{\Delta_s})e^{j0} \quad (10)$$

$$S_{42} = (CM_{\Sigma_s} + DM_{\Sigma_s})e^{j\pi} \quad (11)$$

$$S_{32} = (CM_{\Sigma_s} - DM_{\Sigma_s})e^{j\pi}. \quad (12)$$

Next, we will analyze the magnitude and phase mismatches in the proposed cascaded rat-race for its 2^{nd} order implementation. Considering the losses through each

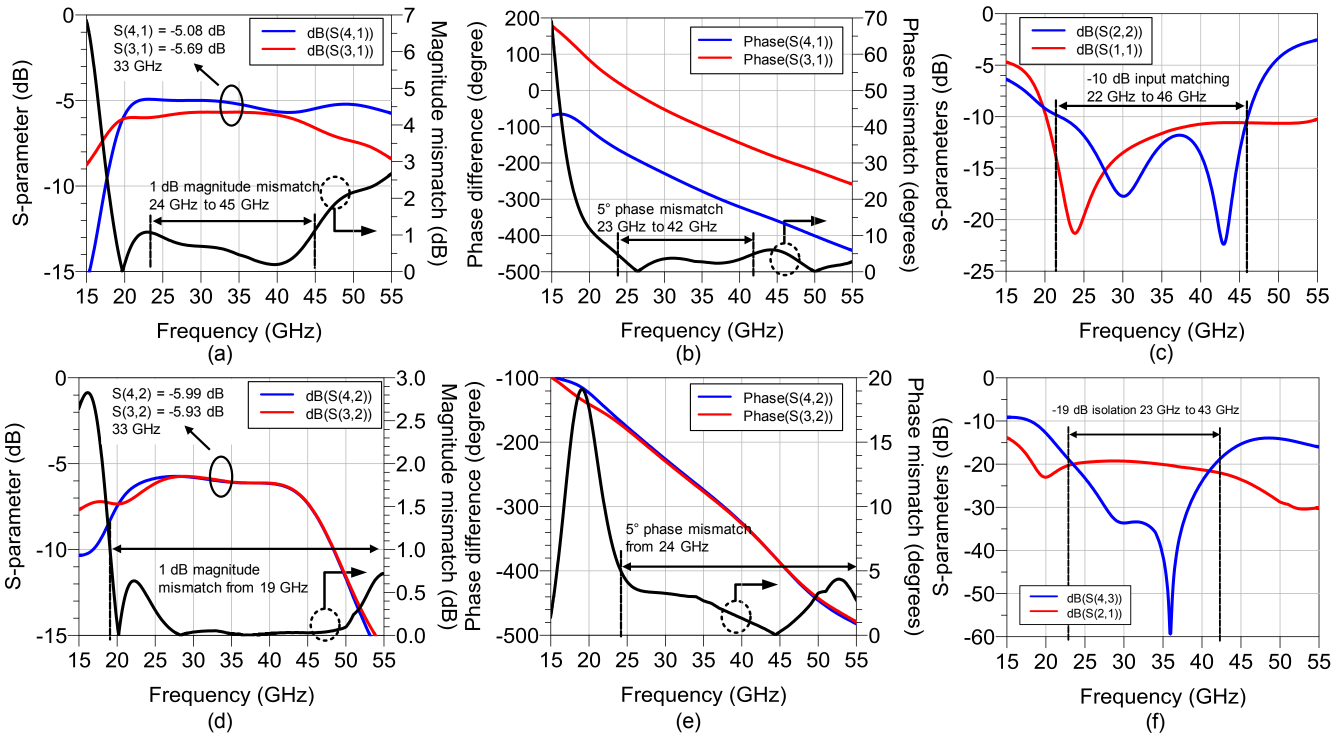

Figure 5. 3D EM simulation results of a 2nd order cascade rat-race coupler. (a) Magnitude response and (b) phase response of S(3,1) and S(4,1). (c) Return loss for port 2 and port4. (d) Magnitude and (e) phase response of S(3,2) and S(4,2). (f) Isolation between port 1 and 2 and port 3 and 4.

signal path (Figure 2), we can first find the magnitude mismatch of the cascaded rat-race. For the magnitude mismatch analysis, we will assume that the phase mismatch of each individual rat-race is very small for simplicity. The resulting first order analysis will aid in understanding the bandwidth extension and mismatch reduction attained by our cascaded structure. For the 2nd order cascaded rat-race, the magnitude mismatch is given by (13)-(14), which can also be expressed using the Common-Mode and Differential-Mode output equations (9)-(12) of the single rat-race, as expressed in equations (13)-(14), as

$$\begin{aligned}&|\Delta\text{-Port}_c|_{\text{mismatch}}\\&=|(|S_{31}|\times|S_{41}|+|S_{41}|\times|S_{31}|)\\&-(|S_{31}|\times|S_{31}|+|S_{41}|\times|S_{41}|)|\\&=|[2\times(CM_{\Delta_s}-DM_{\Delta_s})\times(CM_{\Delta_s}+DM_{\Delta_s})]\\&-[(CM_{\Delta_s}-DM_{\Delta_s})^2\times(CM_{\Delta_s}+DM_{\Delta_s})^2]|\\&=(2\times DM_{\Delta_s})^2\end{aligned}\tag{13}$$

$$\begin{aligned}&|\Sigma\text{-Port}_c|_{\text{mismatch}}\\&=(|S_{32}|\times|S_{42}|+|S_{42}|\times|S_{32}|)\\&-(|S_{32}|\times|S_{32}|+|S_{42}|\times|S_{42}|)\\&=|[2\times(CM_{\Sigma_s}-DM_{\Sigma_s})\times(CM_{\Sigma_s}+DM_{\Sigma_s})]\\&-[(CM_{\Sigma_s}-DM_{\Sigma_s})^2\times(CM_{\Sigma_s}+DM_{\Sigma_s})^2]|\\&=(2\times DM_{\Sigma_s})^2.\end{aligned}\tag{14}$$

Equations (13) and (14) demonstrate that the magnitude mismatch of a 2nd-order cascaded rat-race coupler will be equal to the square of the magnitude mismatch of a single rat-race. Since the magnitude mismatch of a single rat-race is always a quantity with its magnitude less than 1, there will be a magnitude mismatch reduction by employing the cascaded structure (Figure 4).

Phase mismatch suppression in hybrid couplers are of particular interest, since phase mismatches can degrade the signal integrity along the signal path and performance in various beam-former/beam-steering systems. Similar to the amplitude mismatch analysis, one can also analyze the phase mismatch of the proposed cascaded rat-race coupler using the S-parameters of a single rat-race. This will aid in comparing the phase mismatch improvements offered by the cascaded structure. We begin our analysis by assuming that the magnitude mismatch at the output ports of the single rat-race is very small. For the Δ-Port of a single rat-race, we can define the phase mismatch using equation (15) as 2δ and the arithmetic average of the output phases as ε, as shown in (16). Based on (15) and (16), the phase of each signal path for the single and cascaded rat-race are given as

$$\delta=\frac{1}{2}(\angle S_{41}-\angle S_{31})\tag{15}$$

$$\varepsilon=\frac{1}{2}(\angle S_{41}+\angle S_{31})\tag{16}$$

$$\angle S_{41}=\varepsilon+\delta\tag{17}$$

$$\angle S_{31}=\varepsilon-\delta\tag{18}$$

$$\angle(S_{31}\times S_{41})=(\varepsilon+\delta)+(\varepsilon-\delta)=2\varepsilon\tag{19}$$

$$\angle(S_{41}\times S_{31})=(\varepsilon-\delta)+(\varepsilon+\delta)=2\varepsilon\tag{20}$$

$$\angle(S_{31}\times S_{31})=(\varepsilon-\delta)+(\varepsilon-\delta)=2(\varepsilon-\delta)\tag{21}$$

$$\angle(S_{41} \times S_{41}) = (\varepsilon + \delta) + (\varepsilon + \delta) = 2(\varepsilon + \delta). \quad (22)$$

Next, we will investigate the phase mismatch using (19)-(22) to compute the error through each signal path. As shown in Figure 2, the input signal at the Δ-Port passes through two single stage rat-races in the case of a 2-stage design, and then the outputs of the last stage are combined to obtained the desired phase and magnitude. The phase mismatch at each of the outputs of the cascaded rat-race can be found using (23)-(24) as shown below

$$(\angle S_{41})_{mismatch} \text{ (cascaded rat-race)} = tan^{-1}\left[\frac{\sin(2\varepsilon-2\epsilon))}{1+\cos(2\varepsilon-\varepsilon)}\right] = 0 \quad (23)$$

$$(\angle S_{31})_{mismatch} \text{ (cascaded rat-race)} = tan^{-1}\left[\frac{\sin[2(\varepsilon-\delta)-2(\epsilon+\delta)]}{1+\cos[2(\varepsilon-\delta)-(\varepsilon+\delta)]}\right] = -2\delta. \quad (24)$$

It is important to note that S_{41} and S_{31} in (23)-(24) represent the S-parameters of the 2nd order cascaded rat-race coupler. Then, we can compute the total phase mismatch between the outputs of ports 3 and 4 using (20)-(24). The total phase mismatch is computed by subtraction of the output phases of port 3 and 4 as

$$(\angle S_{41} - \angle S_{31})(cacaded\ rat-race) = [2(\varepsilon + \delta) - 2\delta] - [2\varepsilon - 0] = 0^{o}. \quad (25)$$

This first order analysis shows the phase mismatch cancelling effect offered by cascading multiple rat-race couplers. The same analysis can be performed on the Σ-Port to yield the same result obtained in (25). Note that the above equation (25) means that the phase mismatch of the unit rat-race couplers will cancel each other and lead to ideally zero phase error for our proposed cascaded rat-race coupler as long as the magnitude mismatch is small and the unit rat-race couplers have the same magnitude and phase mismatches at corresponding ports. In summary, the above derivations show that cascading more stages can achieve further magnitude/phase mismatch suppression and bandwidth extension, however, at the cost of additional insertion loss.

Simulation Results

A proof-of-concept 24-42 GHz 2nd order cascaded rat-race coupler is implemented in the Globalfoundries 45nm CMOS SOI process. The simulation results of a 2nd order cascade rat-race coupler are based on a 3-D electromagnetic (EM) model in HFSS shown in Figure 3. In this design, the input ports are taken to be ports 1 and 2. The magnitude and phase response of port 1 shows a less than 1dB magnitude mismatch from 24 to 45 GHz and a phase mismatch of less than 5° from 23 to 42 GHz (Fig 5a and 5b). For port 2, the magnitude mismatch is less than 0.5dB from 23 to 53 GHz, and the phase mismatch is less than 5° beyond 24 GHz (Fig. 5d and 5e). In addition, the return loss of port 1 and port 2 is better than -10dB throughout the frequency range of interest (Fig. 5c). Additionally, the insertion loss between input and output ports is between 2dB to 2.9dB, making this design very symmetric. Figure 5f shows that the isolation between the input ports and out ports is better than -19dB from 23 to 43 GHz. These simulation results demonstrate that this cascaded rat-race design is a desirable candidate for implementing Ka-band beam-formers (26 to 40 GHz) or other radar/communication systems.

Conclusion

A new cascaded rat-race coupler network topology is proposed to achieve coupler bandwidth extension and amplitude/phase mismatch suppression. As a proof-of-concept demonstration, a 24-42 GHz fully integrated 2nd-order cascaded rat-race coupler is presented using four identical unit rat-race couplers. The cascaded design cancels the amplitude/phase mismatches of the unit rat-race couplers through each signal path and greatly extents the operation bandwidth. Based on 3D full-wave EM modeling and simulations, our cascaded design achieves a less than 1dB magnitude mismatch, a less than 5° phase mismatch, while providing excellent input/output matching and better than -19dB isolation, throughout the entire Ka-band frequency band. The cascaded rat-race coupler design including the pads occupies only 0.9mm by 0.7mm in the Globalfoundries 45nm SOI CMOS process.

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