

REPORT DOCUMENTATION PAGE			Form Approved OMB NO. 0704-0188		
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1. REPORT DATE (DD-MM-YYYY) 23-10-2017		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 1-Oct-2013 - 31-Jul-2017	
4. TITLE AND SUBTITLE Final Report: Double Tunneling-Injection Quantum Dot Lasers for High-Speed Operation			5a. CONTRACT NUMBER W911NF-13-1-0445		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER 611102		
6. AUTHORS			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES Virginia Polytechnic Institute & State Univ North End Center, Suite 4200 300 Turner Street, NW Blacksburg, VA 24061 -0001			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSOR/MONITOR'S ACRONYM(S) ARO		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) 64568-EL.25		
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Levon Asryan
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 540-231-7033

RPPR Final Report

as of 26-Oct-2017

Agency Code:

Proposal Number: 64568EL

Agreement Number: W911NF-13-1-0445

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DUNS Number: 003137015

EIN: 546001805

Report Date: 31-Oct-2017

Date Received: 23-Oct-2017

Final Report for Period Beginning 01-Oct-2013 and Ending 31-Jul-2017

Title: Double Tunneling-Injection Quantum Dot Lasers for High-Speed Operation

Begin Performance Period: 01-Oct-2013

End Performance Period: 31-Jul-2017

Report Term: 0-Other

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Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 1

STEM Participants: 2

Major Goals: The objective of this project was to explore the potential of a novel type of semiconductor laser – double tunneling-injection (DTI) quantum dot (QD) laser – for high-speed operation. To accomplish this objective, a comprehensive theory of such lasers should have been developed and the major factors affecting their modulation bandwidth should have been thoroughly studied.

The following particular major goals were set for this project:

- 1) To calculate the upper limit for the modulation bandwidth of DTI QD lasers.
- 2) To study the effect of noninstantaneous pumping of QDs on the modulation bandwidth of DTI QD lasers.
- 3) To study the effect of out-tunneling leakage from QDs on the modulation bandwidth of DTI QD lasers.

Accomplishments: 1) The upper limit for the modulation bandwidth of double tunneling-injection (DTI) quantum dot (QD) lasers has been calculated. The optimum cavity length, surface density of QDs, and dc injection current density, maximizing the modulation bandwidth, have been shown to exist. The higher the dc injection current density, the smaller should be the optimum values of the cavity length and the surface density of QDs. While the maximum bandwidth has been shown to be the same in DTI and conventional QD lasers and unaffected by the differential gain, the optimum dc current density, being inversely proportional to the differential gain, is lower in a DTI laser.

2) The modulation bandwidth of DTI QD lasers has been studied taking into account noninstantaneous pumping of QDs. In this novel type of semiconductor laser, carriers are first captured from the bulk waveguide region into two-dimensional regions (quantum wells – QWs) and then they tunnel from the QWs into zero-dimensional regions (QDs). The two processes are noninstantaneous and thus could delay the delivery of carriers to the QDs. The modulation bandwidth of DTI QD lasers has been calculated as a function of two characteristic times (capture time from the waveguide region into the QW and tunneling time from the QW into the QD-ensemble) and shown to increase as either of these times is reduced. In the specific structure considered, which lases near 1.55 micrometer, the capture and tunneling times of 1 and 0.1 ps, respectively, have been shown to characterize fast capture and tunneling processes — as the capture and tunneling times are brought below 1 and 0.1 ps, the bandwidth remains almost unchanged and close to its upper limit.

3) Effect of out-tunneling leakage of carriers from QDs on modulation bandwidth of DTI QD lasers has been

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studied. In this type of laser, as an alternative to conventional pumping, electrons and holes are injected into QDs by tunneling from two separate quantum wells (QWs) located on the opposite sides of the QD layer and separated from the latter by thin barrier layers. While fast tunneling-injection into QDs should be maintained to ensure high modulation bandwidth, it may seem that undesirable out-tunneling leakage from QDs (tunneling of electrons to the hole-injecting QW and holes to the electron-injecting QW) should be kept as slow as possible (ideally, totally suppressed) to maximize the modulation bandwidth. However, it has been shown that, due to zero-dimensional nature of QDs, the modulation bandwidth is practically unaffected by out-tunneling leakage from them – it depends only slightly on the out-tunneling time. Provided that tunneling-injection is fast, high bandwidth can be attained in DTI QD lasers no matter the out-tunneling time.

4) A theory of operating characteristics of QD lasers has been developed in the presence of excited states in QDs. Three possible situations for lasing have been considered: (i) ground-state lasing only; (ii) ground-state lasing at first and then the onset of also excited-state lasing with increasing injection current; (iii) excited-state lasing only. The following characteristics have been studied: occupancies of the ground-state and excited-state in QDs, free carrier density in the optical confinement layer, threshold currents for ground- and excited-state lasing, densities of photons emitted via ground and excited-state stimulated transitions, output power, internal and external differential quantum efficiencies. Under the conditions of ground-state lasing only, the output power saturates with injection current. Under the conditions of both ground- and excited-state lasing, the output power of ground-state lasing remains pinned above the excited-state lasing threshold while the power of excited-state lasing increases. There is a kink in the light-current curve at the excited-state lasing threshold. The case of excited-state lasing only is qualitatively similar to that for single-state QDs—the role of ground-state transitions is simply reduced to increasing the threshold current.

5) The light-current characteristic (LCC) of a QD laser has been calculated under the conditions of both direct and indirect capture of carriers from the optical confinement layer into the lasing ground state in QDs. The direct capture has been shown to be a dominant process determining the ground-state LCC. Only when direct capture is slow, the role of indirect capture (capture into the QD excited state and subsequent intradot relaxation to the ground state) becomes important.

6) A comprehensive analytical model for the operating characteristics of semiconductor lasers with a low-dimensional active region has been developed. Particular emphasis has been given to the effect of capture delay of both electrons and holes from a bulk optical confinement layer into a quantum-confined active region and an extended set of rate equations is used. A closed-form expression for the internal quantum efficiency has been derived as an explicit function of the injection current and parameters of a laser structure. Due to either electron or hole capture delay, the internal efficiency decreases with increasing injection current above the lasing threshold thus causing sublinearity of the light-current characteristic of a laser.

7) An extended model for the operating characteristics of semiconductor quantum well (QW) lasers has been developed. The model exploits the condition of global electroneutrality, which includes the charge carriers both in the two-dimensional (2D) active region (QW) and bulk waveguide region (optical confinement layer – OCL). The charge of each sign in the OCL has been shown to be significantly larger than that in the QW. As a result of this, (i) the global electroneutrality condition reduces to the condition of electroneutrality in the OCL and (ii) the local electroneutrality in the QW can be strongly violated, i.e., the 2D electron and hole densities in the QW can significantly differ from each other.

8) A simple method for the determination of the capture velocity of charge carriers from a three-dimensional (3D) region (waveguide region) into a 2D region (quantum well) has been proposed. The method is based on measurement of the threshold current density and internal differential quantum efficiency in a semiconductor laser structure. The method also allows determining the 2D carrier density in a quantum well, which is otherwise not easy to measure in a multilayer laser structure.

9) A comprehensive rate equations model for semiconductor quantum dot solar cells (QDSCs) has been developed. The model is based on the continuity equations with a proper account for quantum dots (QDs). A general analytical expression for the total current density has been obtained and the current-voltage characteristic has been studied for several specific situations. The degradation in the open circuit voltage of the QDSC has been shown to be due to strong spontaneous radiative recombination in QDs. Due to small absorption coefficient of the QD ensemble, the improvement in the short circuit current density is negligible if only one QD layer is used. If spontaneous radiative recombination would be suppressed in QDs, a QDSC with multiple QD layers would have significantly higher short circuit current density and power conversion efficiency than its conventional counterpart.

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The effects of photoexcitation of carriers from discrete-energy states in QDs to continuum-energy states have been discussed. An extended model, which includes excited states in QDs, has also been introduced.

Training Opportunities: Nothing to Report

Results Dissemination: Results of the work obtained during the period of performance of this project were disseminated at the following international conferences:

[1] L. V. Asryan, "Modulation bandwidth of a double tunneling-injection quantum dot laser: The upper limit and limiting factors," 2017 IEEE Photonics Society Summer Topicals Meeting Series, San Juan, Puerto Rico, July 10-12, 2017.

[2] S. Kar and L. V. Asryan, "Effect of out-tunneling leakage on modulation response of double tunneling-injection quantum dot lasers," SPIE Photonics West, San Francisco, CA, USA, Jan. 28 – Feb. 2, 2017.

[3] Z. N. Sokolova, I. S. Tarasov, and L. V. Asryan, "Theoretical model for the operating characteristics of semiconductor quantum well lasers," 13th International Workshop on Nonlinear Optics and Excitation Kinetics in Semiconductors, Dortmund, Germany, Oct. 9-13, 2016.

[4] L. V. Asryan, "Modulation response of double tunneling-injection quantum dot lasers," 17th International Conference "Laser Optics 2016", June 27 – July 1, 2016, St. Petersburg, Russia.

[5] L. V. Asryan, "Effect of pumping delay on modulation response of double tunneling-injection quantum dot lasers," SPIE Photonics West, San Francisco, CA, USA, Feb. 13-18, 2016.

[6] L. V. Asryan, "Double tunneling-injection quantum dot laser: Temperature-stable and high-power operation," International School on Nanophotonics and Photovoltaics, Cefalù, Sicily, Italy, Sept. 15-23, 2015.

[7] L. V. Asryan, "Dynamic characteristics of double tunneling-injection quantum dot lasers," SPIE Photonics West, San Francisco, CA, USA, Feb. 7-12, 2015.

[8] L. V. Asryan, "Double tunneling-injection quantum dot lasers: A novel type of semiconductor lasers," SPIE-funded lecture (SPIE Visiting Lecturer Program), St. Petersburg Academic University, Russia, Apr. 25, 2014.

Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI

Participant: Levon Asryan

Person Months Worked: 3.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

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Person Months Worked: 3.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

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National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Saurav Kar

Person Months Worked: 15.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

ARTICLES:

Publication Type: Journal Article

Peer Reviewed: Y

Publication Status: 1-Published

Journal: Journal of Applied Physics

Publication Identifier Type: DOI

Publication Identifier: 10.1063/1.4868472

Volume: 115

Issue: 10

First Page #: 0

Date Submitted:

Date Published:

Publication Location:

Article Title: Direct and indirect capture of carriers into the lasing ground state and the light-current characteristic of quantum dot lasers

Authors:

Keywords: semiconductor lasers, quantum dot lasers, light-current characteristic, capture of carriers

Abstract: We calculate the light-current characteristic (LCC) of a quantum dot (QD) laser under the conditions of both direct and indirect capture of carriers from the optical confinement layer into the lasing ground state in QDs. We show that direct capture is a dominant process determining the ground-state LCC. Only when direct capture is slow, the role of indirect capture (capture into the QD excited state and subsequent intradot relaxation to the ground state) becomes important.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support:

Publication Type: Journal Article

Peer Reviewed: Y

Publication Status: 1-Published

Journal: Journal of Applied Physics

Publication Identifier Type: DOI

Publication Identifier: 10.1063/1.4861408

Volume: 115

Issue: 2

First Page #: 0

Date Submitted:

Date Published:

Publication Location:

Article Title: Optical power of semiconductor lasers with a low-dimensional active region

Authors:

Keywords: semiconductor lasers, quantum well lasers, quantum dot lasers, internal quantum efficiency

Abstract: A comprehensive analytical model for the operating characteristics of semiconductor lasers with a low-dimensional active region is developed. Particular emphasis is given to the effect of capture delay of both electrons and holes from a bulk optical confinement region into a quantum-confined active region and an extended set of rate equations is used. We derive a closed-form expression for the internal quantum efficiency as an explicit function of the injection current and parameters of a laser structure. Due to either electron or hole capture delay, the internal efficiency decreases with increasing injection current above the lasing threshold thus causing sublinearity of the light-current characteristic of a laser.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support:

RPPR Final Report as of 26-Oct-2017

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published

Journal: Proc. 16th International Conf. "Laser Optics 2014"

Publication Identifier Type: Publication Identifier:

Volume: 16 Issue: 1 First Page #: 1

Date Submitted: Date Published:

Publication Location:

Article Title: Intradot relaxation and modulation bandwidth in quantum dot lasers

Authors:

Keywords: quantum dot lasers; modulation bandwidth

Abstract: Excited-to-ground-state relaxation delay in quantum dots is shown to strongly limit the ground-state modulation bandwidth of a quantum dot laser. The longest tolerable intradot relaxation time exists, beyond which the direct modulation of the laser optical output becomes impossible.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support:

Publication Type: Journal Article Peer Reviewed: **Publication Status:** 1-Published

Journal:

Publication Identifier Type: Publication Identifier:

Volume: 0 Issue: 0 First Page #: 0

Date Submitted: Date Published:

Publication Location:

Article Title: Modulation bandwidth of a double tunneling-injection quantum dot laser: The best-case scenario

Authors:

Keywords: semiconductor lasers, quantum dot lasers, tunneling-injection lasers, modulation response, modulation bandwidth

Abstract: The modulation bandwidth of a double tunneling-injection (DTI) quantum dot (QD) laser is studied in the best-case scenario of fast pumping of QDs and no carrier leakage from QDs. The maximum modulation bandwidth and the optimum values of the cavity length and dc component of the injection current density, at which it is obtained, are calculated. While the maximum bandwidth is shown to be the same in DTI and conventional QD lasers and unaffected by the differential gain, it is more easily attainable in a DTI laser since the optimum dc current density, being inversely proportional to the differential gain, is lower there.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support:

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published

Journal: Journal of Physics: Conference Series

Publication Identifier Type: DOI Publication Identifier: 10.1088/1742-6596/740/1/012002

Volume: 740 Issue: First Page #: 012002

Date Submitted: 10/22/17 12:00AM Date Published: 9/1/16 4:00AM

Publication Location: United Kingdom

Article Title: Theory of operating characteristics of a semiconductor quantum well laser: Inclusion of global electroneutrality in the structure

Authors: Z N Sokolova, N A Pikhtin, I S Tarasov, L V Asryan

Keywords: semiconductor lasers, quantum well lasers

Abstract: A model for calculating the operating characteristics of semiconductor quantum well (QW) lasers is presented. The model exploits the condition of global electroneutrality, which includes the charge carriers both in the two-dimensional (2D) active region (QW) and bulk waveguide region (optical confinement layer – OCL). The charge of each sign in the OCL is shown to be significantly larger than that in the QW. As a result of this, (i) the global electroneutrality condition reduces to the condition of electroneutrality in the OCL and (ii) the local electroneutrality in the QW can be strongly violated, i.e., the 2D electron and hole densities in the QW can significantly differ from each other.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support: Y

RPPR Final Report
as of 26-Oct-2017

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published

Journal: Semiconductors

Publication Identifier Type: DOI

Publication Identifier: 10.1134/S1063782617070326

Volume: 51

Issue: 7

First Page #: 959

Date Submitted: 10/22/17 12:00AM

Date Published: 7/1/17 4:00AM

Publication Location:

Article Title: Increase in the internal optical loss with increasing pump current and the output power of quantum well lasers

Authors: Z. N. Sokolova, D. A. Veselov, N. A. Pikhtin, I. S. Tarasov, L. V. Asryan

Keywords: semiconductor lasers, quantum well lasers, internal optical loss

Abstract: The operating characteristics of semiconductor quantum-well lasers, calculated with consideration for an increase in the internal optical loss in the waveguide region with increasing pump current, are presented. The condition of global electroneutrality in the structure is used. This condition consists in that the total charge of electrons in the 2D active region (quantum well) and bulk waveguide region (optical confinement layer) is equal to the total charge of holes in these two regions. Good agreement between the calculated and experimentally determined light-current characteristics is obtained.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support: Y

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 4-Under Review

Journal: Journal of Lightwave Technologies

Publication Identifier Type:

Publication Identifier:

Volume:

Issue:

First Page #:

Date Submitted: 10/22/17 12:00AM

Date Published:

Publication Location:

Article Title: Two-valued characteristics in semiconductor

Authors: Z.N. Sokolova, N.A. Pikhtin, L.V. Asryan

Keywords: quantum well laser, semiconductor laser

Abstract: Due to internal optical absorption loss, which varies with carrier density in the optical confinement region, the operating characteristics can be two-valued in semiconductor quantum well lasers. Particularly, there can be two lasing thresholds and two branches in the light-current characteristic. While the internal differential quantum efficiency for the first (conventional) branch of the light-current characteristic is less than one and decreases with increasing pump current, that for the second (unconventional) branch is greater than one and nonmonotonous.

Distribution Statement: 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support: Y

RPPR Final Report as of 26-Oct-2017

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 4-Under Review
Journal: Journal of Applied Physics
Publication Identifier Type: **Publication Identifier:**
Volume: **Issue:** **First Page #:**
Date Submitted: 10/22/17 12:00AM **Date Published:**
Publication Location: United States of America
Article Title: Modulation bandwidth of double tunneling-injection quantum dot lasers: Effect of out-tunneling leakage
Authors: S. Kar, L.V. Asryan
Keywords: quantum dot laser, semiconductor laser
Abstract: Effect of out-tunneling leakage of carriers from quantum dots (QDs) on modulation bandwidth of double tunneling-injection (DTI) QD lasers is studied. In this type of laser, as an alternative to conventional pumping, electrons and holes are injected into QDs by tunneling from two separate quantum wells (QWs) located on the opposite sides of the QD layer and separated from the latter by thin barrier layers. While fast tunneling-injection into QDs should be maintained to ensure high modulation bandwidth, it may seem that undesirable out-tunneling leakage from QDs should be kept as slow as possible to maximize the modulation bandwidth. However, it is shown in this work that, due to zero-dimensional nature of QDs, the bandwidth is practically unaffected by out-tunneling leakage from them – it depends only slightly on the out-tunneling time. Provided that tunneling-injection is fast, high bandwidth can be attained in DTI QD lasers no matter the out-tunneling time.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: Y

CONFERENCE PAPERS:

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 17th International Conference "Laser Optics 2016"
Date Received: 21-Aug-2016 **Conference Date:** 27-Jun-2016 **Date Published:** 27-Jun-2016
Conference Location: St. Petersburg, Russia
Paper Title: Modulation response of double tunneling-injection quantum dot lasers
Authors: Levon V. Asryan
Acknowledged Federal Support: Y

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 13th International Workshop on Nonlinear Optics and Excitation Kinetics in Semiconductors
Date Received: 20-Dec-2016 **Conference Date:** 09-Oct-2016 **Date Published:** 13-Oct-2016
Conference Location: Dortmund, Germany
Paper Title: Theoretical model for the operating characteristics of semiconductor quantum well lasers
Authors: Z. N. Sokolova, I. S. Tarasov, L. V. Asryan
Acknowledged Federal Support: Y

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2017 IEEE Photonics Society Summer Topicals Meeting Series
Date Received: 22-Oct-2017 **Conference Date:** 11-Jul-2017 **Date Published:** 13-Jul-2017
Conference Location: San Juan, Puerto Rico
Paper Title: Modulation bandwidth of a double tunneling-injection quantum dot laser: The upper limit and limiting factors
Authors: L. V. Asryan
Acknowledged Federal Support: Y

DISSERTATIONS:

RPPR Final Report
as of 26-Oct-2017

Publication Type: Thesis or Dissertation

Institution: Virginia Polytechnic Institute and State University

Date Received: 22-Oct-2017

Completion Date: 8/4/17 2:16AM

Title: Effect of out-tunneling leakage and electron-hole asymmetry on modulation response of semiconductor double tunneling-injection quantum dot lasers

Authors: Saurav Kar

Acknowledged Federal Support: Y

Modulation bandwidth of a double tunnelling-injection quantum dot laser

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Received 27 November 2014, revised 4 February 2015

Accepted for publication 6 February 2015

Published 3 March 2015



Abstract

The modulation response of a double tunnelling-injection (DTI) quantum dot (QD) laser is studied. Closed-form expressions are obtained for the dynamic characteristics of the laser and the upper limit for the modulation bandwidth is estimated. The optimum cavity length, surface density of QDs, and dc injection current density, maximizing the modulation bandwidth, are shown to exist. The higher the dc injection current density, the smaller should be the optimum values of the cavity length and the surface density of QDs. While the maximum bandwidth is shown to be the same in DTI and conventional QD lasers and unaffected by the differential gain, the optimum dc current density, being inversely proportional to the differential gain, is lower in a DTI laser.

Keywords: semiconductor lasers, quantum dot lasers, modulation response, tunneling-injection lasers

(Some figures may appear in colour only in the online journal)

1. Introduction

As an alternative to conventional pumping, tunnelling-injection of carriers into a low-dimensional active region of diode lasers was considered. In [1–9], single tunnelling-injection was proposed, which was tunnelling-injection of carriers of one type only (electrons) while the carriers of the opposite sign (holes) were injected conventionally, i.e., first injected into the optical confinement layer (OCL) and then captured into the active region. In [1], single tunnelling-injection was utilized into a two-dimensional (2D) active region (quantum well, QW), and in [2–9] into a zero-dimensional (0D) active region (quantum dots, QDs). Double tunnelling-injection (DTI), i.e., tunnelling-injection of both electrons and holes into QDs was proposed in [10, 11] and further studied theoretically in [12–14]. There has also been experimental work on double tunnelling-injection into QDs [15–18]. More tunnelling-injection schemes were considered, such as injection of excitons into quantum dashes [19, 20] or into QDs via nanobridges [21].

A study of static characteristics showed that DTI QD lasers possess a potential for low-threshold, temperature-stable [10, 11], and high-power operation [12–14].

In this work, dynamic characteristics of DTI QD lasers are studied. To reveal their potential for high-speed direct modulation of optical output by pump current, the upper limit for the modulation bandwidth is estimated here assuming fast carrier injection into QDs and no carrier leakage from QDs.

2. Theoretical model

The energy band diagram of a DTI QD laser is shown in figure 1. Electrons and holes are first injected into the left- and right-hand sides of the OCL, respectively, and then captured into the corresponding quantum wells (QWs). QDs are pumped by tunnelling of electrons from the left-hand-side QW and holes from the right-hand-side QW. More details on DTI QD laser structures were thoroughly discussed in [10–14].

In the general case, eleven rate equations describe the dynamics in a DTI QD laser—one equation for photons and ten equations for carriers (see [13]). The ten equations are those for electrons and holes for each of the following five regions of the structure (figure 1): left-hand side of the OCL, left-hand-side QW, QDs, right-hand-side QW, and right-hand side of the OCL.

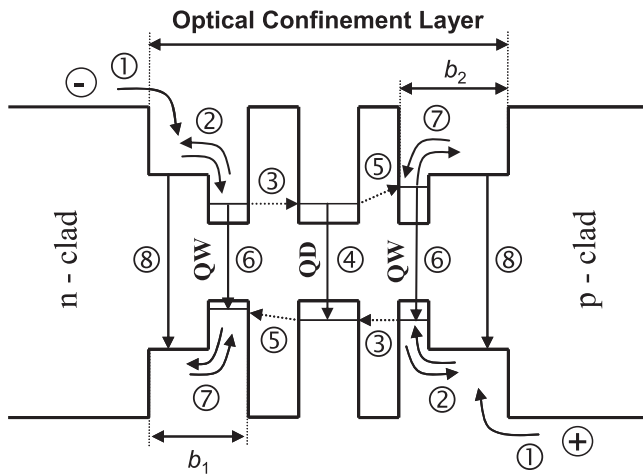


Figure 1. Energy band diagram of a double tunnelling-injection QD laser and the main processes: ① injection from the cladding layers to the OCL, ② majority carrier capture from the OCL into the QW and thermal escape from the QW to the OCL, ③ tunnelling-injection from the QW into a QD, ④ spontaneous and stimulated recombination in a QD, ⑤ out-tunnelling from a QD into the ‘foreign’ QW, ⑥ spontaneous recombination in the QWs, ⑦ minority carrier thermal escape from the QW to the OCL and capture from the OCL into the QW, and ⑧ spontaneous recombination in the OCL. Reproduced with permission from figure 1 of [12]. Copyright [2008], AIP Publishing LLC.

To estimate the upper limit for the modulation bandwidth of the laser, the following assumptions are made here:

- (1) There is no out-tunnelling leakage of carriers from QDs (processes 5 in figure 1), i.e., no minority carrier leakage through the junction—electrons will not leak to the right-hand-side QW and holes to the left-hand-side QW. This assumption ensures that there will be no minority carriers on either side of the structure—no electrons will be in the right-hand-side QW and right-hand side of the OCL and, correspondingly, no rate equations will be needed for electrons for those regions. Similarly, no holes will be in the left-hand-side QW and left-hand side of the OCL and, correspondingly, no rate equations will be needed for holes for those regions. Thus the number of rate equations is reduced from eleven to seven—one equation for photons, three equations for electrons (in the left-hand-side QW, in the left-hand side of the OCL, and in QDs), and three equations for holes (in the right-hand-side QW, in the right-hand side of the OCL, and in QDs).
- (2) In [22] and [23], static characteristics of conventional QD and QW lasers were studied taking into account the electron–hole asymmetry. In particular, a parameter of electron–hole asymmetry was introduced in [22], which effectively accounts for the differences between the electron and hole energy levels in a QD, conduction and valence band offsets, and doping levels of the n- and p-cladding layers. Dynamic characteristics of both conventional and DTI QD lasers also deserve a separate study with a proper inclusion of electron–hole asymmetry. Here, however, as the first step, an assumption of

electron–hole symmetry is used, which is commonly made in the literature—see, e.g., [24–33]. With this assumption, the three rate equations for holes become identical to the three rate equations for electrons and hence the number of rate equations is further reduced from seven to four—these are equations (A1)–(A4) of the appendix.

- (3) The majority carrier exchange between the OCL and QW is fast on each side of the structure, i.e., the electron exchange between the left-hand side of the OCL and the left-hand-side QW (i.e., capture from the OCL into the QW and escape from the QW to the OCL, processes 2 in figure 1) is fast and so is the hole exchange between the right-hand side of the OCL and the right-hand-side QW. As a result, thermal equilibrium establishes between the time-varying bulk and 2D majority-carrier populations on each side of the structure (see equation (A14) below).
- (4) Tunnelling-injection of majority carriers is fast, i.e., the electron tunnelling from the left-hand-side QW into QDs is fast and so is the hole tunnelling from the right-hand-side QW into QDs (processes 3 in figure 1). As a result, thermal equilibrium establishes between the time-varying 2D majority-carrier population in each QW and 0D population of carriers of the same type in QDs (see equation (A13)). Hence, assumptions 3 and 4 allow expressing the time-varying 3D and 2D carrier densities in the OCL and QW, respectively, in terms of the carrier level occupancy in QDs. This further reduces the number of rate equations from four to two—one for total carrier density (equation (1) below) and the other for photon density (equation (2)).
- (5) As discussed above, assumption 1 of no out-tunnelling leakage from QDs ensures that there are no minority carriers outside QDs. As a result, there is no parasitic recombination outside QDs—neither nonradiative monomolecular recombination, nor radiative bimolecular recombination, nor nonradiative Auger recombination. These parasitic recombination channels outside QDs will only become nonvanishing if out-tunnelling leakage from QDs occurs.
- (6) High quality laser structures without surface states in QDs are assumed here. The presence of such states may lead to nonradiative monomolecular recombination in QDs and can be accounted for by including the rate of this recombination into the total rate of non-stimulated recombination $R_{\text{non-stim}}$, which enters into equation (1) below.
- (7) QDs without excited states are considered here. Similarly to recombination via surface states, recombination via excited states in QDs can be accounted for by including the rate of this process into $R_{\text{non-stim}}$.

Noninstantaneous carrier exchange between the OCL and QWs and noninstantaneous tunnelling-injection from the QWs into QDs will lead to delay in carrier delivery to QDs. As a result, the modulation bandwidth of the laser will be decreased. The electron–hole asymmetry will also adversely

affect the modulation response of the laser—even in the case of fast injection of carriers of one type into QDs, a delay in delivery of carriers of the opposite sign will still limit the bandwidth. The extent to which the modulation bandwidth is impacted by these factors will be addressed in separate studies.

In the appendix, the following set of two rate equations is obtained using the above assumptions:

$$\frac{\partial n}{\partial t} = \frac{j}{eb} - R_{\text{non-stim}} - R_{\text{stim}}, \quad (1)$$

$$\frac{\partial n_{\text{ph}}}{\partial t} = R_{\text{stim}} - R_{\text{loss}}, \quad (2)$$

where j is the injection current density, e is the electron charge, b is the OCL thickness, n is the total electron density, and n_{ph} is the photon density—both n and n_{ph} are per unit volume of the OCL.

The rates (in units of $\text{cm}^{-3} \text{s}^{-1}$) of the processes entering into (1) and (2) are as follows: $R_{\text{non-stim}}$ is the non-stimulated recombination rate, R_{stim} is the stimulated recombination rate (which is the rate of stimulated emission of photons), and R_{loss} is the photon loss rate.

The total electron density is given by

$$n = \frac{b_1}{b} n_1 \frac{f_n}{1 - f_n} + \frac{N_c^{2D}}{b} \frac{f_n}{1 - f_n} + 2 \frac{N_S}{b} f_n, \quad (3)$$

where b_1 is the thickness of the left-hand side of the OCL (the separation between the n-cladding layer and the left-hand-side barrier—figure 1), f_n is the electron level occupancy in QDs, N_S is the surface density of QDs, and N_c^{2D} is the 2D effective density of states in the conduction band in the QW. The quantity n_1 is

$$n_1 = N_c^{3D} \exp\left(-\frac{\Delta E_c - \epsilon_n^{\text{QW}}}{T}\right), \quad (4)$$

where N_c^{3D} is the three-dimensional (3D) effective density of states in the conduction band in the OCL, ΔE_c is the conduction band offset between the OCL and the QW, ϵ_n^{QW} is the energy of the electron-subband edge in the QW (which is in resonance with the electron energy level in QDs in figure 1), and T is the temperature (in units of energy).

The first, second, and third terms in (3) are the contributions to the total electron density from the left-hand side of the OCL, left-hand-side QW, and QDs, respectively.

The rate of non-stimulated recombination is given by

$$R_{\text{non-stim}} = \frac{N_S}{b} \frac{f_n^2}{\tau_{\text{QD}}}, \quad (5)$$

where τ_{QD} is the spontaneous radiative lifetime in QDs.

The stimulated recombination rate is

$$R_{\text{stim}} = v_g g n_{\text{ph}}, \quad (6)$$

where v_g is the group velocity of light and

$$g = g^{\text{max}} (2f_n - 1) \quad (7)$$

is the modal gain of the laser with g^{max} being its maximum value [34].

The photon loss rate is

$$R_{\text{loss}} = v_g \beta n_{\text{ph}}, \quad (8)$$

where $\beta = (1/L) \ln(1/R)$ is the mirror loss coefficient, L is the cavity length, and R is the mirror reflectivity.

In contrast to conventional QD lasers [32], where non-stimulated recombination occurs in and outside QDs, there is no bipolar (i.e., simultaneously electron and hole) population and hence no parasitic recombination outside QDs in a DTI laser (processes 6 and 8) in the case of no out-tunnelling from QDs (assumption 1). This is why the spontaneous recombination rate in QDs only enters into equation (5). There is, however, monopolar population outside QDs on each side of a DTI structure—electrons in the left-hand side (see equation (3)) and holes in the right-hand side. Hence, similarly to conventional QD lasers [32], the carrier population outside QDs (although monopolar here) will considerably reduce the differential gain of a DTI QD laser—see equation (19) below.

3. Dynamic characteristics

Using the small-signal analysis, the injection current density in (1) is presented as the sum of the dc and small ac components,

$$j = j_0 + (\delta j_m) \exp(i\omega t). \quad (9)$$

The solutions of (1) and (2) for n and n_{ph} (or, equivalently, for f_n and n_{ph}) are accordingly found as the sums of the dc and small ac components. The amplitude $\delta n_{\text{ph-m}}(\omega)$ of the ac component of the photon density is then used to derive the following expression for the modulation response function:

$$H(\omega) = \left| \frac{\delta n_{\text{ph-m}}(\omega)}{\delta n_{\text{ph-m}}(0)} \right|^2 = \frac{\omega_0^4}{(\omega^2 - \omega_0^2)^2 + 4\Gamma_{\text{dec}}^2 \omega^2}, \quad (10)$$

where

$$\omega_0 = \sqrt{v_g G^{\text{dif}} n_{\text{ph},0} \frac{1}{\tau_{\text{ph}}}} \quad (11)$$

and the decay rate of relaxation oscillations is given by

$$\Gamma_{\text{dec}} = \frac{1}{2} \left(\frac{1}{\tau_{\text{non-stim}}^{\text{dif}}} + v_g G^{\text{dif}} n_{\text{ph},0} \right). \quad (12)$$

The angular frequency of relaxation oscillations is expressed in terms of Γ_{dec} and ω_0 as follows:

$$\Omega_{\text{osc}} = \sqrt{\omega_0^2 - \Gamma_{\text{dec}}^2}. \quad (13)$$

The dc component of the photon density entering into (11) and (12) is given by

$$\begin{aligned} n_{\text{ph},0} &= \frac{1}{eb} (j_0 - j_{\text{th}}) \tau_{\text{ph}} \\ &= \frac{1}{eb} \left(j_0 - eN_S \frac{f_{n0}^2}{\tau_{\text{QD}}} \right) \tau_{\text{ph}}, \end{aligned} \quad (14)$$

where j_{th} is the threshold current density,

$$j_{\text{th}} = eN_S \frac{f_{n0}^2}{\tau_{\text{QD}}}, \quad (15)$$

and f_{n0} is the dc component of the electron level occupancy in QDs,

$$\begin{aligned} f_{n0} &= \frac{1}{2} \left(1 + \frac{\beta}{g^{\text{max}}} \right) \\ &= \frac{1}{2} \left(1 + \frac{L^{\text{min}}}{L} \right) \\ &= \frac{1}{2} \left(1 + \frac{N_S^{\text{min}}}{N_S} \right). \end{aligned} \quad (16)$$

In (16), L^{min} is the shortest cavity length and N_S^{min} is the minimum surface density of QDs, at which the lasing can occur [34, 35]. The expression for L^{min} is

$$L^{\text{min}} = \frac{1}{g^{\text{max}}} \ln \frac{1}{R}. \quad (17)$$

The photon lifetime in the cavity is

$$\tau_{\text{ph}} = \frac{1}{v_g \beta} = \frac{L}{v_g} \frac{1}{\ln(1/R)}. \quad (18)$$

The *effective differential gain* G^{dif} entering into (11) and (12) presents the derivative of the modal gain with respect to the *total carrier density*. With (7) and (3), the following expression is obtained for G^{dif} :

$$\begin{aligned} G^{\text{dif}} &= \frac{\partial g_0}{\partial n_0} \\ &= \frac{2g^{\text{max}}}{2\frac{N_S}{b} + \frac{b_1 n_1 + N_c^{2D}}{b} \frac{1}{(1-f_{n0})^2}} \\ &= \frac{1}{1 + \frac{b_1 n_1 + N_c^{2D}}{2N_S} \frac{1}{(1-f_{n0})^2}} G_{\text{act}}^{\text{dif}} \ll G_{\text{act}}^{\text{dif}}, \end{aligned} \quad (19)$$

where

$$G_{\text{act}}^{\text{dif}} = \frac{\partial g_0}{\partial n_{\text{act},0}} = \frac{b}{N_S} g^{\text{max}} \quad (20)$$

is the differential gain of the active region and $n_{\text{act},0}$ is given by the last term in (3). As in (14) and (16), ‘0’ in the subscripts of the corresponding quantities in (19)–(21) denotes their dc (steady-state) values.

The reciprocal of the *effective differential non-stimulated recombination time* $\tau_{\text{non-stim}}^{\text{dif}}$ entering into (12) is determined as the derivative of $R_{\text{non-stim}}$ with respect to the *total carrier density*. With (5) and (3),

$$\begin{aligned} \frac{1}{\tau_{\text{non-stim}}^{\text{dif}}} &= \frac{\partial R_{\text{non-stim},0}}{\partial n_0} \\ &= \frac{2\frac{N_S}{b} \frac{f_{n0}}{\tau_{\text{QD}}}}{2\frac{N_S}{b} + \frac{b_1 n_1 + N_c^{2D}}{b} \frac{1}{(1-f_{n0})^2}} \\ &= \frac{1}{1 + \frac{b_1 n_1 + N_c^{2D}}{2N_S} \frac{1}{(1-f_{n0})^2}} \\ &\times \frac{1}{\tau_{\text{non-stim,act}}^{\text{dif}}} \ll \frac{1}{\tau_{\text{non-stim,act}}^{\text{dif}}}, \end{aligned} \quad (21)$$

where

$$\frac{1}{\tau_{\text{non-stim,act}}^{\text{dif}}} = \frac{f_{n0}}{\tau_{\text{QD}}} \quad (22)$$

is the reciprocal differential non-stimulated recombination time in the active region.

The expression for the modulation bandwidth (defined as the –3dB-bandwidth) is

$$\omega_{-3\text{dB}} = \left\{ \left(\omega_0^2 - 2\Gamma_{\text{dec}}^2 \right) + \left[\left(\omega_0^2 - 2\Gamma_{\text{dec}}^2 \right)^2 + (r-1)\omega_0^4 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}, \quad (23)$$

where $r = 10^{0.3} \approx 1.995$.

As seen from equations (10)–(13) and (23), the dynamic characteristics of the laser ($H(\omega)$, ω_0 , Γ_{dec} , Ω_{osc} , and $\omega_{-3\text{dB}}$) are controlled by the *effective* values of the differential gain and non-stimulated recombination time rather than by those for the active region. While G^{dif} in a DTI QD laser is higher than that in a conventional QD laser, it is still significantly lower than $G_{\text{act}}^{\text{dif}}$. For a specific DTI laser and a reference conventional laser considered here, both GaInAsP-based and emitting at $T = 300$ K near $1.55 \mu\text{m}$, at the cavity length $L = 1.139$ mm (which corresponds to the mirror loss $\beta = 10 \text{ cm}^{-1}$), G^{dif} values are $1.61 \times 10^{-16} \text{ cm}^2$ and $1.27 \times 10^{-16} \text{ cm}^2$, respectively, and $G_{\text{act}}^{\text{dif}} = 1.36 \times 10^{-14} \text{ cm}^2$. Hence, the presence of even monopolar carrier population outside QDs (electrons on the left- and holes on the right-hand side) significantly reduces the differential gain in a DTI laser.

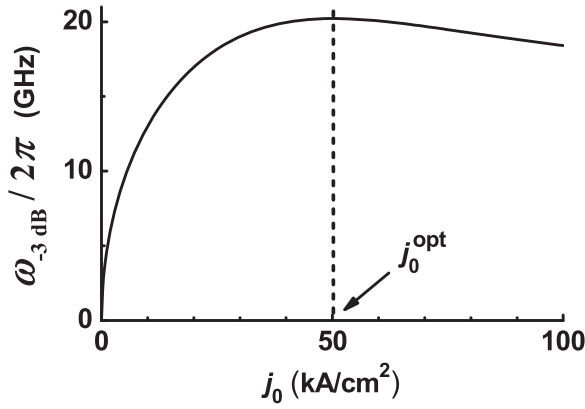


Figure 2. Modulation bandwidth $\omega_{-3\text{dB}}/2\pi$ versus dc component of the injection current density. A GaInAsP structure emitting at $T=300$ K near $1.55 \mu\text{m}$ is considered [12, 13]. The cavity length $L=1.139$ mm (the mirror loss $\beta=10 \text{ cm}^{-1}$) and the surface density of QDs $N_S=6.11 \times 10^{10} \text{ cm}^{-2}$. The vertical dashed line marks j_0^{opt} .

4. Optimum dc current density

With increasing dc component j_0 of the injection current density above the threshold current density, $\omega_{-3\text{dB}}$ increases from zero, approaches its maximum, and then decreases (figure 2). The optimum value of j_0 , at which $\omega_{-3\text{dB}}$ is maximum, is given by

$$j_0^{\text{opt}} = j_{\text{th}} + \frac{2eb}{v_g G^{\text{dif}} \tau_{\text{ph}}^2} \left(\frac{1}{2} - \frac{1}{2} \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}} + \frac{1}{2} \sqrt{1 - 2 \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}}} \right), \quad (24)$$

and the maximum value of $\omega_{-3\text{dB}}$ is

$$\omega_{-3\text{dB}}^{\text{max}} = \sqrt[4]{r-1} \frac{\sqrt{2}}{\tau_{\text{ph}}} \times \sqrt{\frac{1}{2} - \frac{1}{2} \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}} + \frac{1}{2} \sqrt{1 - 2 \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}}}}. \quad (25)$$

For the entire range of values of the cavity length, i.e., for $L > L^{\text{min}}$ [see (17) for L^{min}] and up to very high L , the photon lifetime in the cavity remains much shorter than the differential non-stimulated recombination time (figure 3(a)),

$$\tau_{\text{ph}} \ll \tau_{\text{non-stim}}^{\text{dif}}. \quad (26)$$

With inequality (26) and taking into account that j_{th} is small compared to the second term in (24), equations (24) and (25) simplify,

$$j_0^{\text{opt}} = \frac{2eb}{v_g G^{\text{dif}} \tau_{\text{ph}}}, \quad (27)$$

$$\omega_{-3\text{dB}}^{\text{max}} = \sqrt[4]{r-1} \frac{\sqrt{2}}{\tau_{\text{ph}}} \approx \frac{\sqrt{2}}{\tau_{\text{ph}}} = \sqrt{2} \frac{v_g}{L} \ln\left(\frac{1}{R}\right). \quad (28)$$

As seen from (27) and (28), while the maximum modulation bandwidth does not depend on the differential gain

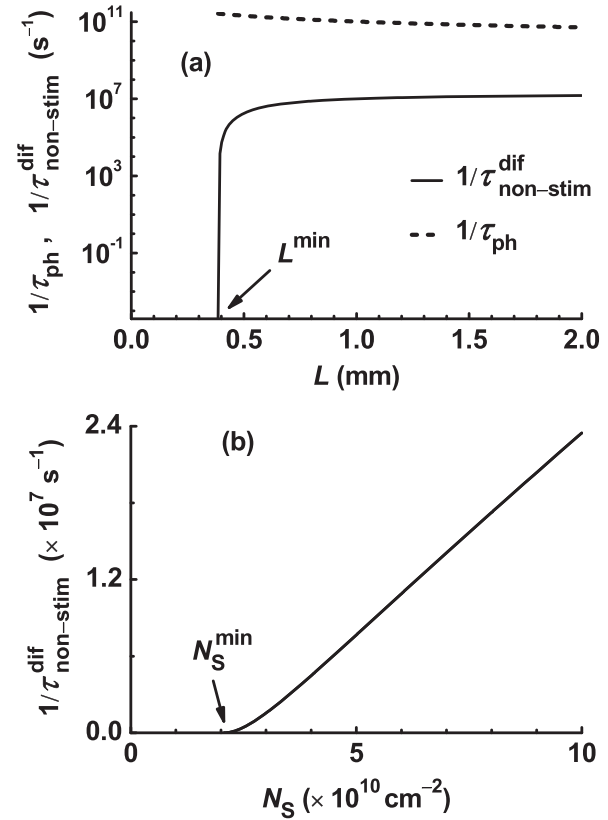


Figure 3. Reciprocal effective differential non-stimulated recombination time versus cavity length (a) and surface density of QDs (b). In (a), $N_S=6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $L=1.139$ mm. The dashed curve in (a) is the reciprocal photon lifetime in the cavity.

and is controlled by merely τ_{ph} , the optimum dc current density is strongly controlled not only by τ_{ph} but by G^{dif} as well—the higher G^{dif} , the lower j_0^{opt} . Since G^{dif} in a DTI QD laser is higher than that in a conventional QD laser, j_0^{opt} is lower in a DTI laser. For a DTI laser and a reference conventional laser considered here, at the cavity length $L=1.139$ mm, j_0^{opt} values are 50 and 64 kA cm^{-2} , respectively. The maximum modulation bandwidth, being controlled by the photon lifetime only, i.e., by the cavity length, is however the same ($\omega_{-3\text{dB}}^{\text{max}}/2\pi=20 \text{ GHz}$).

5. Optimum cavity length and surface density of QDs

As $L \rightarrow L^{\text{min}}$ or $N_S \rightarrow N_S^{\text{min}}$, the steady-state level occupancy $f_{n0} \rightarrow 1$ (see (16)). As this happens, both $1/\tau_{\text{non-stim}}^{\text{dif}}$ and G^{dif} go to zero—see figures 3 and 4, respectively. With increasing L above L^{min} and N_S above N_S^{min} , both $1/\tau_{\text{non-stim}}^{\text{dif}}$ and G^{dif} increase monotonously and will finally saturate as $L \rightarrow \infty$ or $N_S \rightarrow \infty$ ($f_{n0} \rightarrow 1/2$).

As with the dependence on j_0 , the dependences of the modulation bandwidth on L and N_S are also nonmonotonous. At $L=L^{\text{min}}$ or $N_S=N_S^{\text{min}}$, $\omega_{-3\text{dB}}=0$. With increasing L above L^{min} or N_S above N_S^{min} , $\omega_{-3\text{dB}}$ increases, approaches its maximum at a certain optimum cavity length L^{opt} or surface density of QDs N_S^{opt} and then decreases (figure 5).

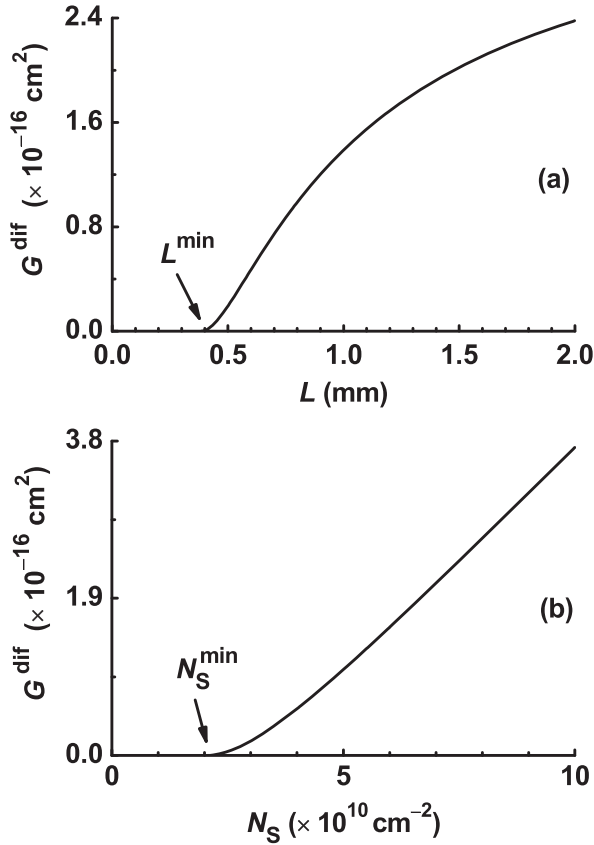


Figure 4. Effective differential gain versus cavity length (a) and surface density of QDs (b). In (a), $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $L = 1.139$ mm.

The optimum cavity length and surface density of QDs decrease with increasing dc component of the injection current density (figure 6). At high j_0 , when

$$\frac{j_0}{e(b_1 n_1 + N_c^{2D})} \gg v_g g^{\text{max}}, \quad (29)$$

the optimum cavity length and surface density of QDs are very close to L^{min} and N_S^{min} , respectively, and given by

$$\frac{L^{\text{opt}}}{L^{\text{min}}} = \frac{N_S^{\text{opt}}}{N_S^{\text{min}}} = 1 + 2\sqrt{\frac{2}{3}} \sqrt[4]{r} \sqrt{\frac{v_g g^{\text{max}}}{j_0}} \sqrt{\frac{e(b_1 n_1 + N_c^{2D})}{j_0}}. \quad (30)$$

Since L^{opt} is close to L^{min} , $\omega_{-3\text{dB}}$ in this case is close to $\omega_{-3\text{dB}}^{\text{max}}$ value at L^{min} , which is the upper limit for the modulation bandwidth,

$$\omega_{-3\text{dB}}^{\text{highest}} = \sqrt{2} \frac{v_g}{L^{\text{min}}} \ln\left(\frac{1}{R}\right) = \frac{\sqrt{2}}{\tau_{\text{ph}}^{\text{min}}} = \sqrt{2} v_g g^{\text{max}}, \quad (31)$$

where

$$\tau_{\text{ph}}^{\text{min}} = \frac{1}{v_g g^{\text{max}}} = \frac{L^{\text{min}}}{v_g} \frac{1}{\ln(1/R)} \quad (32)$$

is the photon lifetime in the shortest cavity, in which the lasing can occur. As in conventional lasers [32], the upper

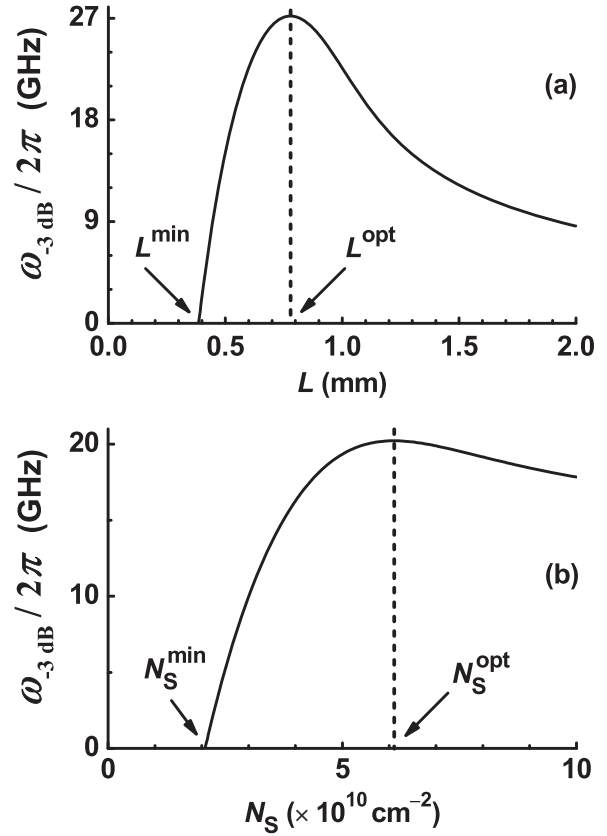


Figure 5. Modulation bandwidth $\omega_{-3\text{dB}}/2\pi$ versus cavity length (a) and surface density of QDs (b). In (a), $j_0 = 100 \text{ kA cm}^{-2}$ and $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $j_0 = 50 \text{ kA cm}^{-2}$ and $L = 1.139$ mm. The vertical dashed line marks L^{opt} in (a) and N_S^{opt} in (b).

limit for the modulation bandwidth in a DTI laser is controlled by the maximum modal gain rather than the differential gain.

With increasing L , both G^{dif} and τ_{ph} increase. Hence, as seen from (27), j_0^{opt} decreases. Taking into account that the first term in the denominator of (19) is small compared to the second term, G^{dif} can be written as follows:

$$\begin{aligned} G^{\text{dif}} &= \frac{2g^{\text{max}}}{b_1 n_1 + N_c^{2D}} (1 - f_{n0})^2 \\ &= \frac{1}{2} \frac{g^{\text{max}}}{b_1 n_1 + N_c^{2D}} \left(1 - \frac{L^{\text{min}}}{L}\right)^2. \end{aligned} \quad (33)$$

Using (33) in (27) gives

$$j_0^{\text{opt}} = \frac{4e(b_1 n_1 + N_c^{2D})}{\tau_{\text{ph}}^{\text{min}}} \frac{1}{\left(\frac{L}{L^{\text{min}}} - 1\right)^2}. \quad (34)$$

Hence, to reduce the optimum dc pump current density, the cavity should be made longer. As a sacrifice, the maximum modulation bandwidth will also decrease (see (28)). At a cavity length $L = 1.139$ mm, $j_0^{\text{opt}} = 50 \text{ kA cm}^{-2}$ and

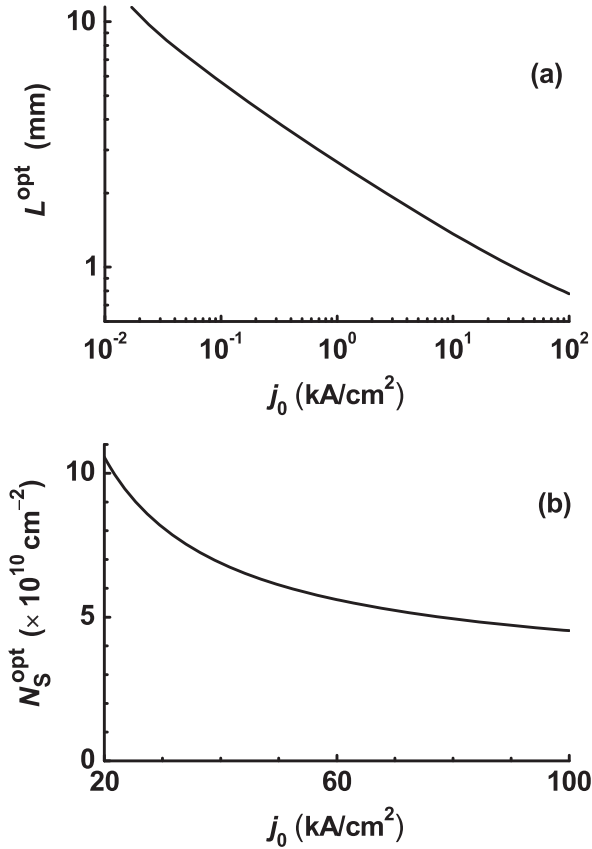


Figure 6. Optimum cavity length (a) and surface density of QDs (b) versus dc component of the injection current density. In (a), $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $L = 1.139 \text{ mm}$.

$\omega_{-3 \text{ dB}}^{\text{max}}/2\pi = 20 \text{ GHz}$. This value of j_0^{opt} is lower than that in a conventional QD laser (64 kA cm^{-2}) at the same cavity length.

6. Conclusions

The modulation response of a DTI QD laser has been studied. Closed-form expressions have been obtained for the dynamic characteristics of the laser and the upper limit for the modulation bandwidth has been estimated. The optimum cavity length, surface density of QDs, and dc injection current density, maximizing the modulation bandwidth, have been shown to exist. The higher the dc injection current density, the smaller should be the optimum values of the cavity length and the surface density of QDs. While the maximum bandwidth has been shown to be the same in DTI and conventional QD lasers and unaffected by the differential gain, the optimum dc current density, being inversely proportional to the differential gain, is lower in a DTI laser.

Acknowledgments

This work was supported by the US Army Research Office (Grant No. W911NF-13-1-0445).

Appendix. Derivation of rate equations (1) and (2) and expression (3) for the total carrier density for the case of fast pumping of QDs and no carrier leakage from QDs

As mentioned in section 2, eleven rate equations describe the dynamics in a DTI QD laser in the general case (see [13]). With assumption 1 of section 2 of no out-tunnelling leakage of carriers from QDs and assumption 2 of electron-hole symmetry, the general set reduces to the following set of four rate equations:

for free electrons in the left-hand side of the OCL,

$$b_1 \frac{\partial n_L}{\partial t} = \frac{j}{e} + \frac{n_{\text{QW}}^L}{\tau_{n,\text{esc}}^L} - v_{n,\text{capt}}^L n_L, \quad (\text{A1})$$

for electrons in the electron-injecting (left-hand-side) QW,

$$\begin{aligned} \frac{\partial n_{\text{QW}}^L}{\partial t} = & v_{n,\text{capt}}^L n_L - \frac{n_{\text{QW}}^L}{\tau_{n,\text{esc}}^L} - w_{n,\text{tunn}}^L N_S \\ & \times (1 - f_n) n_{\text{QW}}^L + w_{n,\text{tunn}}^L n_1^{\text{L,QW}} N_S f_n, \end{aligned} \quad (\text{A2})$$

for electrons confined in QDs,

$$\begin{aligned} 2N_S \frac{\partial f_n}{\partial t} = & w_{n,\text{tunn}}^L N_S (1 - f_n) n_{\text{QW}}^L \\ & - w_{n,\text{tunn}}^L n_1^{\text{L,QW}} N_S f_n - N_S \frac{f_n^2}{\tau_{\text{QD}}} \\ & - v_g g^{\text{max}} (2f_n - 1) \frac{N}{S}, \end{aligned} \quad (\text{A3})$$

and for photons,

$$\frac{\partial N}{\partial t} = v_g g^{\text{max}} (2f_n - 1) N - v_g \beta N. \quad (\text{A4})$$

In equations (A1)–(A4), n_L is the free-electron density per unit volume of the left-hand side of the OCL, n_{QW}^L is the 2D electron density in the left-hand-side QW, f_n is the electron-level occupancy in QDs, $S = WL$ is the cross-section of the junction, W is the lateral size of the device, and N is the number of photons in the lasing mode. The other parameters in (A1)–(A4) not listed here were introduced in section 2.

In (A1) and (A2), $v_{n,\text{capt}}^L$ is the capture velocity of electrons from the OCL into the QW (measured in units of cm s^{-1}) and $\tau_{n,\text{esc}}^L$ is the thermal escape time of electrons from the QW to the OCL. These quantities are related to each other (see [13]). For undoped OCL and QW,

$$\tau_{n,\text{esc}}^L = \frac{1}{v_{n,\text{capt}}^L} \frac{N_c^{2D}}{n_1}, \quad (\text{A5})$$

where n_1 is given by (4).

Using the capture velocity, the capture time into the QW can be also introduced,

$$\tau_{n,\text{capt}}^L = \frac{b_1}{v_{n,\text{capt}}^L}. \quad (\text{A6})$$

Note that the escape and capture times, while both being expressed in terms of the capture velocity, are distinct.

The quantity $w_{n,\text{tunn}}^L$ in (A2) and (A3) is the tunnelling coefficient for electrons between the QW and the QD-ensemble (measured in units of $\text{cm}^2 \text{s}^{-1}$).

The quantity $n_1^{L,QW}$ entering into the electron tunnelling flux from the QD-ensemble to the QW in (A2) and (A3) is measured in units of cm^{-2} . In the case of undoped QW and resonance between the QD energy level and the lowest sub-band edge in the QW (figure 1), it is given as [13]

$$n_1^{L,QW} = N_c^{2D}. \quad (\text{A7})$$

Using $w_{n,\text{tunn}}^L$, the tunnelling time from the QW into unoccupied QD-ensemble (when $f_n = 0$) can be introduced,

$$\tau_{n,\text{tunn},0}^{QW \rightarrow QDs} = \frac{1}{w_{n,\text{tunn}}^L N_S}. \quad (\text{A8})$$

Adding up equations (A1), (A2), and (A3) and dividing both sides of the obtained equation by the OCL thickness b , the following rate equation is obtained for the total electron density *per unit volume of the OCL*:

$$\frac{dn}{dt} = \frac{j}{eb} - \frac{N_S}{b} \frac{f_n^2}{\tau_{QD}} - v_g g^{\max} (2f_n - 1) n_{ph}, \quad (\text{A9})$$

where $n_{ph} = N/(Sb)$ is the photon density (also per unit volume of the OCL).

The total electron density is given as

$$n = \frac{b_1}{b} n_L + \frac{n_{QW}^L}{b} + 2 \frac{N_S}{b} f_n. \quad (\text{A10})$$

While equation (A9) always holds true, the total electron density n entering into it includes three quantities (n_L , n_{QW}^L , and f_n), which should be found from the solution of the set of rate equations. However, as shown below, in the case of fast electron exchange between the OCL and the QW and fast tunnelling-injection from the QW into QDs (assumptions 3 and 4 of section 2), any two of these three quantities are expressed in terms of the third quantity. Below, for definiteness, n_L and n_{QW}^L are expressed in terms of f_n .

From equation (A3),

$$\begin{aligned} n_{QW}^L &= n_1^{L,QW} \frac{f_n}{1 - f_n} + \frac{1}{w_{n,\text{tunn}}^L N_S (1 - f_n)} \\ &\times \left[N_S \frac{f_n^2}{\tau_{QD}} + v_g g^{\max} (2f_n - 1) \frac{N}{S} + 2N_S \frac{df_n}{dt} \right] \\ &= n_1^{L,QW} \frac{f_n}{1 - f_n} + \tau_{n,\text{tunn},0}^{QW \rightarrow QDs} \frac{1}{1 - f_n} \\ &\times \left[N_S \frac{f_n^2}{\tau_{QD}} + v_g g^{\max} (2f_n - 1) \frac{N}{S} + 2N_S \frac{df_n}{dt} \right]. \quad (\text{A11}) \end{aligned}$$

Equation (A8) was used in (A11).

From equation (A1),

$$\begin{aligned} n_L &= \frac{1}{v_{n,\text{capt}}^L \tau_{n,\text{esc}}^L} n_{QW}^L + \frac{1}{v_{n,\text{capt}}^L} \left(\frac{j}{e} - b_1 \frac{dn_L}{dt} \right) \\ &= \frac{n_1}{N_c^{2D}} n_{QW}^L + \frac{1}{v_{n,\text{capt}}^L} \left(\frac{j}{e} - b_1 \frac{dn_L}{dt} \right) \\ &= \frac{n_1}{N_c^{2D}} n_{QW}^L + \tau_{n,\text{capt}}^L \left(\frac{j}{eb_1} - \frac{dn_L}{dt} \right). \quad (\text{A12}) \end{aligned}$$

Equations (A5) and (A6) were used in (A12).

Assuming fast tunnelling-injection of electrons from the left-hand-side QW into QDs ($w_{n,\text{tunn}}^L \rightarrow \infty$ or, equivalently, $\tau_{n,\text{tunn},0}^{QW \rightarrow QDs} \rightarrow 0$), the following relation is immediately obtained from (A11) between the 2D-electron density in the left-hand-side QW and the electron level occupancy in QDs:

$$n_{QW}^L = n_1^{L,QW} \frac{f_n}{1 - f_n} = N_c^{2D} \frac{f_n}{1 - f_n}. \quad (\text{A13})$$

Equation (A7) was used in (A13).

Assuming fast electron exchange between the left-hand side of the OCL and the left-hand-side QW ($v_{n,\text{capt}}^L \rightarrow \infty$, or, equivalently, $\tau_{n,\text{capt}}^L \rightarrow 0$), the following relation is obtained from (A12) between the free-electron density in the left-hand side of the OCL, the 2D-electron density in the left-hand-side QW, and the electron level occupancy in QDs:

$$n_L = \frac{n_1}{N_c^{2D}} n_{QW}^L = n_1 \frac{f_n}{1 - f_n}. \quad (\text{A14})$$

Using (A13) and (A14) in (A10), equation (3) for the total electron density is obtained. As seen from equation (3), a single quantity f_n defines now the total electron density n . Thus, the rate equation (1) is obtained, which is equation (A9) wherein the total electron density is given by (3). This rate equation now substitutes for three equations (A1)–(A3). Hence, the number of rate equations is finally reduced to two—these are the rate equations for the total electron density (equation (1)) and for the photon density (equation (2) is simply the other form of writing equation (A4)).

Assumptions 3 and 4 of fast capture from the OCL into the QW and tunnelling-injection from the QW into QDs are justified if the rates of these processes are much higher than the rates of all the other processes and the time derivatives entering into equations (A1)–(A3). In particular, for the case of time-harmonic modulation of the output power by injection current considered here, the capture time from the OCL into the QW and the tunnelling time from the injector-QW into QDs should be much shorter than the period of modulating current,

$$\tau_{n,\text{capt}}^L, \tau_{n,\text{tunn},0}^{QW \rightarrow QDs} \ll \frac{2\pi}{\omega}. \quad (\text{A15})$$

For example, at a very high modulation frequency $\omega/2\pi = 30$ GHz, these times should be much shorter than 33 ps.

It should be emphasized that, while (A13) and (A14) present the equilibrium relationships between n_L , n_{QW}^L , and f_n (if f_n would be given by the Fermi–Dirac distribution

function, equilibrium expressions would be obtained for n_L and n_{QW}^L), these relationships hold true for time-varying quantities n_L , n_{QW}^L , and f_n in the case of fast capture and tunnelling-injection.

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Dynamic characteristics of double tunneling-injection quantum dot lasers

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ABSTRACT

Dynamic characteristics of double tunneling-injection (DTI) quantum dot (QD) lasers are studied. To reveal the potential of such lasers for high-speed direct modulation of their optical output by pump current, fast carrier injection into QDs and no carrier leakage from QDs are assumed. The small-signal analysis of rate equations is applied. The modulation bandwidth is calculated as a function of the dc component of the injection current density and parameters of the laser structure.

Keywords: semiconductor lasers, quantum dot lasers, modulation response, tunneling-injecting lasers

1. INTRODUCTION

As an alternative to conventional pumping, tunneling-injection of carriers into a low-dimensional active region of diode lasers was considered. In [1]-[9], single-tunneling injection was proposed, which was tunneling-injection of carriers of one type only (electrons) while the carriers of the opposite sign (holes) were injected conventionally, i.e., first injected into the optical confinement layer (OCL) and then captured into the active region. In [1], single tunneling-injection was utilized into a two-dimensional (2D) active region (quantum well – QW), and in [2]-[9] – into a zero-dimensional (0D) active region (quantum dots – QDs). Double tunneling-injection, i.e., tunneling-injection of both electrons and holes into QDs was proposed in [10]-[12] and further studied theoretically in [13]-[15]. There has also been experimental work on double tunneling-injection into QDs [16]-[19]. More tunneling-injection schemes were considered, such as injection of excitons into quantum dashes [20, 21] or into QDs via nanobridges [22].

A study of static characteristics [10]-[15] showed that DTI QD lasers possess a potential for low-threshold, temperature-stable, and high-power operation.

In this work, dynamic characteristics of DTI QD lasers are studied. To reveal their potential for high-speed direct modulation of optical output by pump current, the best-case scenario of fast carrier injection into QDs and no carrier leakage from QDs is considered here.

2. DETAILS OF ANALYTICAL MODEL

The energy band diagram of a DTI QD laser is shown in Fig. 1. Electrons and holes are first injected into the left- and right-hand sides of the OCL, respectively, and then captured into the corresponding quantum wells (QWs). QDs are pumped by tunneling of electrons from the left-hand-side QW and holes from the right-hand-side QW. More details on DTI QD laser structures are thoroughly discussed in [10]-[15].

In the general case, eleven rate equations should be used to adequately describe a DTI QD laser – one equation for photons and ten equations for carriers (see [10]). The ten equations are those for electrons and holes for each of the following five regions of the structure (Fig. 1): left-hand side of the OCL, left-hand-side QW, QDs, right-hand-side QW, and right-hand side of the OCL. To estimate the highest possible modulation bandwidth of a DTI QD laser, the following assumptions are made here, which significantly simplify the consideration and allow to obtain closed-form expressions for the dynamic characteristics of the laser:

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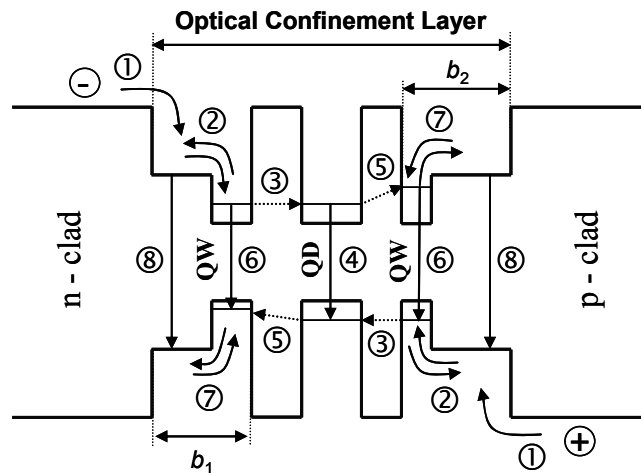


Fig. 1. Energy band diagram of a double tunneling-injection QD laser and the main processes: ① injection from the cladding layers to the OCL, ② majority carrier capture from the OCL into the QW and thermal escape from the QW to the OCL, ③ tunneling-injection from the QW into a QD, ④ spontaneous and stimulated recombination in a QD, ⑤ out-tunneling from a QD into the 'foreign' QW, ⑥ spontaneous recombination in the QWs, ⑦ minority carrier thermal escape from the QW to the OCL and capture from the OCL into the QW, and ⑧ spontaneous recombination in the OCL. Reproduced with permission from Fig. 1 of Ref. [13]. Copyright [2008], AIP Publishing LLC.

In the general case, eleven rate equations should be used to adequately describe a DTI QD laser – one equation for photons and ten equations for carriers (see [10]). The ten equations are those for electrons and holes for each of the following five regions of the structure (Fig. 1): left-hand side of the OCL, left-hand-side QW, QDs, right-hand-side QW, and right-hand side of the OCL. To estimate the highest possible modulation bandwidth of a DTI QD laser, the following assumptions are made here, which significantly simplify the consideration and allow to obtain closed-form expressions for the dynamic characteristics of the laser:

- 1) There is no out-tunneling leakage of carriers from QDs, i.e., no electron leakage to the right-hand-side QW and no hole leakage to the left-hand-side QW (processes #5 in Fig. 1).
- 2) The electron exchange between the left-hand side of the OCL and the left-hand-side QW (i.e., capture from the OCL into the QW and escape from the QW to the OCL) is instantaneous and so is the electron tunneling from the left-hand-side QW into QDs; the same for the hole exchange between the right-hand side of the OCL and the right-hand-side QW and for the hole tunneling from the right-hand-side QW into QDs.
- 3) The electron-hole asymmetry is not taken into account.

Assumption #1 ensures that no electrons will be in the right-hand-side QW and right-hand side of the OCL and, correspondingly, no rate equations will be needed for electrons for those regions. Similarly, no holes will be in the left-hand-side QW and left-hand side of the OCL and, correspondingly, no rate equations will be needed for holes for those regions. Thus the number of rate equations is reduced from eleven to seven. Assumption #2 allows to easily express the free-electron density in the left-hand side of the OCL and the 2D-electron density in the left-hand-side QW in terms of the electron level occupancy in QDs [see eq. (3) below] and, similarly, the hole densities in the right-hand side of the OCL and the right-hand-side QW in terms of the hole level occupancy in QDs. As a result, the number of rate equations is further reduced to three – these are the rate equations for the total electron and hole densities and for photons. And finally, if assumption #3 is also made, the rate equations for the total electron and hole densities become identical and hence only two rate equations need to be used. These equations are

$$\frac{\partial n}{\partial t} = \frac{j}{eb} - R_{\text{non-stim}} - R_{\text{stim}}, \quad (1)$$

$$\frac{\partial n_{\text{ph}}}{\partial t} = R_{\text{stim}} - R_{\text{loss}}, \quad (2)$$

where j is the injection current density, e is the electron charge, b is the OCL thickness, n is the total electron density, and n_{ph} is the photon density – both n and n_{ph} are per unit volume of the OCL.

The following expression is obtained for the total electron density:

$$n = \frac{b_1 n_1}{b} \frac{f_n}{1 - f_n} + \frac{N_c^{2D}}{b} \frac{f_n}{1 - f_n} + 2 \frac{N_s}{b} f_n, \quad (3)$$

where b_1 is the separation between the n-cladding layer and the left-hand-side barrier (Fig. 1), f_n is the electron level occupancy in QDs, N_s is the surface density of QDs, and N_c^{2D} is the 2D effective density of states in the conduction band in the QW. The quantity n_1 is

$$n_1 = N_c^{3D} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right), \quad (4)$$

where N_c^{3D} is the three-dimensional (3D) effective density of states in the conduction band in the OCL, ΔE_c is the conduction band offset between the OCL and the QW, $\varepsilon_n^{\text{QW}}$ is the energy of the electron-subband edge in the QW (which is in resonance with the electron energy level in QDs in Fig. 1), and T is the temperature (in units of energy).

The first, second, and third terms in (3) are the contributions to the total electron density from the left-hand side of the OCL, left-hand-side QW, and QDs, respectively.

The rate of non-stimulated recombination is given by

$$R_{\text{non-stim}} = \frac{N_s}{b} \frac{f_n^2}{\tau_{\text{QD}}}, \quad (5)$$

where τ_{QD} is the spontaneous radiative lifetime in QDs.

The stimulated recombination rate (which is the rate of stimulated emission of photons) is

$$R_{\text{stim}} = v_g g n_{\text{ph}}, \quad (6)$$

where v_g is the group velocity of light and

$$g = g^{\text{max}} (2f_n - 1) \quad (7)$$

is the modal gain of the laser with g^{max} being its maximum value [23, 24].

The photon loss rate is

$$R_{\text{loss}} = v_g \beta n_{\text{ph}} \quad (8)$$

where $\beta = (1/L) \ln(1/R)$ is the mirror loss coefficient, L is the cavity length, and R is the mirror reflectivity.

In contrast to conventional QD lasers [25], where non-stimulated recombination occurs in and outside QDs, there is no bipolar (i.e., simultaneously electron and hole) population and hence no parasitic recombination outside QDs in a DTI laser in the case of no out-tunneling from QDs (assumption #1). This is why the spontaneous recombination rate in QDs only enters into (5). There is, however, monopolar population outside QDs on each side of a DTI structure – electrons in the left-hand side [see (3)] and holes in the right-hand side. Hence, similarly to conventional QD lasers [25], the carrier population outside QDs (although monopolar here) will considerably reduce the differential gain of a DTI QD laser – see eq. (19) below.

3. DYNAMIC CHARACTERISTICS

Using the small-signal analysis, the injection current density in (1) is presented as the sum of the dc and small ac components,

$$j = j_0 + (\delta j_m) \exp(i\omega t) . \quad (9)$$

The solutions of (1) and (2) for n and n_{ph} (or, equivalently, for f_n and n_{ph}) are accordingly found as the sums of the dc and small ac components. The amplitude $\delta n_{ph-m}(\omega)$ of the ac component of the photon density is then used to derive the following expression for the modulation response function:

$$H(\omega) = \left| \frac{\delta n_{ph-m}(\omega)}{\delta n_{ph-m}(0)} \right|^2 = \frac{\omega_0^4}{(\omega^2 - \omega_0^2)^2 + 4\Gamma_{dec}^2 \omega^2} , \quad (10)$$

where

$$\omega_0 = \sqrt{v_g G^{dif} n_{ph,0} \frac{1}{\tau_{ph}}} \quad (11)$$

and the decay rate of relaxation oscillations is given by

$$\Gamma_{dec} = \frac{1}{2} \left(\frac{1}{\tau_{non-stim}^{dif}} + v_g G^{dif} n_{ph,0} \right) . \quad (12)$$

The angular frequency of relaxation oscillations is expressed in terms of Γ_{dec} and ω_0 as follows:

$$\Omega_{osc} = \sqrt{\omega_0^2 - \Gamma_{dec}^2} . \quad (13)$$

The dc component of the photon density entering into (11) and (12) is given by

$$n_{ph,0} = \frac{1}{eb} (j_0 - j_{th}) \tau_{ph} = \frac{1}{eb} \left(j_0 - eN_s \frac{f_{n0}^2}{\tau_{QD}} \right) \tau_{ph} , \quad (14)$$

where j_{th} is the threshold current density,

$$j_{th} = eN_s \frac{f_{n0}^2}{\tau_{QD}} , \quad (15)$$

and f_{n0} is the dc component of the electron level occupancy in QDs,

$$f_{n0} = \frac{1}{2} \left(1 + \frac{\beta}{g_{max}} \right) = \frac{1}{2} \left(1 + \frac{L^{min}}{L} \right) = \frac{1}{2} \left(1 + \frac{N_s^{min}}{N_s} \right) . \quad (16)$$

In (16), L^{min} is the shortest cavity length and N_s^{min} is the minimum surface density of QDs, at which the lasing can occur [23, 24]. The expression for L^{min} is

$$L^{min} = \frac{1}{g_{max}} \ln \frac{1}{R} . \quad (17)$$

The photon lifetime in the cavity is

$$\tau_{ph} = \frac{1}{v_g \beta} = \frac{L}{v_g} \frac{1}{\ln(1/R)} . \quad (18)$$

The effective differential gain G^{dif} entering into (11) and (12) presents the derivative of the modal gain with respect to the total carrier density. With (7) and (3), the following expression is obtained for G^{dif} :

$$G^{\text{dif}} = \frac{\partial g_0}{\partial n_0} = \frac{2g^{\text{max}}}{2\frac{N_s}{b} + \frac{b_1 n_1 + N_c^{2D}}{b} \frac{1}{(1-f_{n0})^2}} = \frac{1}{1 + \frac{b_1 n_1 + N_c^{2D}}{2N_s} \frac{1}{(1-f_{n0})^2}} G_{\text{act}}^{\text{dif}} \ll G_{\text{act}}^{\text{dif}}, \quad (19)$$

where

$$G_{\text{act}}^{\text{dif}} = \frac{\partial g_0}{\partial n_{\text{act},0}} = \frac{b}{N_s} g^{\text{max}} \quad (20)$$

is the differential gain of the active region and $n_{\text{act},0}$ is given by the last term in (3). As in (14) and (16), “0” in the subscripts of the corresponding quantities in (19)-(21) denotes their dc (steady-state) values.

The reciprocal of the effective differential non-stimulated recombination time $\tau_{\text{non-stim}}^{\text{dif}}$ entering into (12) is determined as the derivative of $R_{\text{non-stim}}$ with respect to the total carrier density. With (5) and (3), we have

$$\frac{1}{\tau_{\text{non-stim}}^{\text{dif}}} = \frac{\partial R_{\text{non-stim},0}}{\partial n_0} = \frac{2\frac{N_s}{b} \frac{f_{n0}}{\tau_{\text{QD}}}}{2\frac{N_s}{b} + \frac{b_1 n_1 + N_c^{2D}}{b} \frac{1}{(1-f_{n0})^2}} = \frac{1}{1 + \frac{b_1 n_1 + N_c^{2D}}{2N_s} \frac{1}{(1-f_{n0})^2}} \frac{1}{\tau_{\text{non-stim,act}}^{\text{dif}}} \ll \frac{1}{\tau_{\text{non-stim,act}}^{\text{dif}}}, \quad (21)$$

where

$$\frac{1}{\tau_{\text{non-stim,act}}^{\text{dif}}} = \frac{f_{n0}}{\tau_{\text{QD}}} \quad (22)$$

is the reciprocal differential non-stimulated recombination time in the active region.

The expression for the modulation bandwidth (defined as the -3dB-bandwidth) is

$$\omega_{-3\text{ dB}} = \sqrt{(\omega_0^2 - 2\Gamma_{\text{dec}}^2) + \sqrt{(\omega_0^2 - 2\Gamma_{\text{dec}}^2)^2 + (r-1)\omega_0^4}}, \quad (23)$$

where $r = 10^{0.3} \approx 1.995$.

As seen from eqs. (10)-(13) and (23), the dynamic characteristics of the laser [$H(\omega)$, ω_0 , Γ_{dec} , Ω_{osc} , and $\omega_{-3\text{ dB}}$] are controlled by the effective values of the differential gain and non-stimulated recombination time rather than by those for the active region. While G^{dif} in a DTI QD laser is higher than that in a conventional QD laser, it is still significantly lower than $G_{\text{act}}^{\text{dif}}$. For a specific DTI laser and a reference conventional laser considered here, both GaInAsP-based and emitting at $T = 300$ K near $1.55 \mu\text{m}$, at the cavity length $L = 1.139$ mm (which corresponds to the mirror loss $\beta = 10 \text{ cm}^{-1}$), G^{dif} values are $1.61 \times 10^{-16} \text{ cm}^2$ and $1.27 \times 10^{-16} \text{ cm}^2$, respectively, and $G_{\text{act}}^{\text{dif}} = 1.36 \times 10^{-14} \text{ cm}^2$. Hence the presence of even monopolar carrier population outside QDs (electrons on the left- and holes on the right-hand side) significantly reduces the differential gain in a DTI laser.

4. OPTIMUM DC CURRENT DENSITY

With increasing dc component j_0 of the injection current density above the threshold current density, $\omega_{-3\text{ dB}}$ increases from zero, approaches its maximum, and then decreases (Fig. 2). The optimum value of j_0 , at which $\omega_{-3\text{ dB}}$ is maximum, is given by

$$j_0^{\text{opt}} = j_{\text{th}} + \frac{2eb}{v_g G^{\text{dif}} \tau_{\text{ph}}^2} \left(\frac{1}{2} - \frac{1}{2} \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}} + \frac{1}{2} \sqrt{1 - 2 \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}}} \right), \quad (24)$$

and the maximum value of $\omega_{-3\text{ dB}}$ is

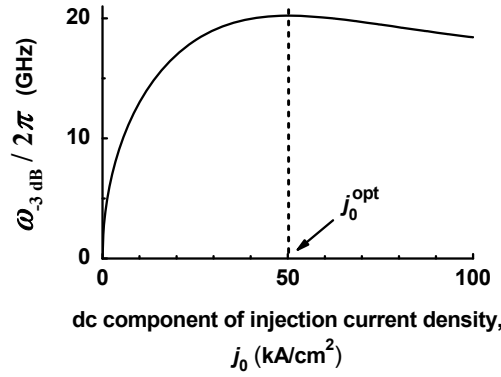


Fig. 2. Modulation bandwidth $\omega_{-3\text{ dB}}/2\pi$ vs. dc component of the injection current density. A GaInAsP structure emitting at $T = 300$ K near $1.55\text{ }\mu\text{m}$ is considered [13, 14]. The cavity length $L = 1.139$ mm (the mirror loss $\beta = 10\text{ cm}^{-1}$) and the surface density of QDs $N_s = 6.11 \times 10^{10}\text{ cm}^{-2}$. The vertical dashed line marks j_0^{opt} .

$$\omega_{-3\text{ dB}}^{\text{max}} = \sqrt[4]{r-1} \frac{\sqrt{2}}{\tau_{\text{ph}}} \sqrt{\frac{1}{2} - \frac{1}{2} \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}} + \frac{1}{2} \sqrt{1 - 2 \frac{\tau_{\text{ph}}}{\tau_{\text{non-stim}}^{\text{dif}}}}} . \quad (25)$$

For the entire range of values of the cavity length, i.e., for $L > L^{\text{min}}$ [see (17) for L^{min}] and up to very high L , the photon lifetime in the cavity remains much shorter than the differential non-stimulated recombination time [Fig. 3(a)],

$$\tau_{\text{ph}} \ll \tau_{\text{non-stim}}^{\text{dif}} . \quad (26)$$

With inequality (26) and taking into account that j_{th} is small compared to the second term in (24), eqs. (24) and (25) simplify,

$$j_0^{\text{opt}} = \frac{2eb}{v_g G^{\text{dif}} \tau_{\text{ph}}^2} , \quad (27)$$

$$\omega_{-3\text{ dB}}^{\text{max}} = \sqrt[4]{r-1} \frac{\sqrt{2}}{\tau_{\text{ph}}} \approx \frac{\sqrt{2}}{\tau_{\text{ph}}} = \sqrt{2} \frac{v_g}{L} \ln \left(\frac{1}{R} \right) . \quad (28)$$

As seen from (27) and (28), while the maximum modulation bandwidth does not depend on the differential gain and is controlled by merely τ_{ph} , the optimum dc current density is strongly controlled not only by τ_{ph} but by G^{dif} as well – the higher G^{dif} , the lower j_0^{opt} . Since G^{dif} in a DTI QD laser is higher than that in a conventional QD laser, j_0^{opt} is lower in a DTI laser. For a DTI laser and a reference conventional laser considered here, at the cavity length $L = 1.139$ mm, j_0^{opt} values are 50 and 64 kA/cm², respectively. The maximum modulation bandwidth, being controlled by the photon lifetime only, i.e., by the cavity length, is however the same ($\omega_{-3\text{ dB}}^{\text{max}}/2\pi = 20$ GHz).

5. OPTIMUM CAVITY LENGTH AND SURFACE DENSITY OF QDS

As $L \rightarrow L^{\text{min}}$ or $N_s \rightarrow N_s^{\text{min}}$, the steady-state level occupancy $f_{n0} \rightarrow 1$ [see (16)]. As this happens, both $1/\tau_{\text{non-stim}}^{\text{dif}}$ and G^{dif} go to zero – see Figs. 3 and 4, respectively. With increasing L above L^{min} and N_s above N_s^{min} , both $1/\tau_{\text{non-stim}}^{\text{dif}}$ and G^{dif} increase monotonously and will finally saturate as $L \rightarrow \infty$ or $N_s \rightarrow \infty$ ($f_{n0} \rightarrow 1/2$).

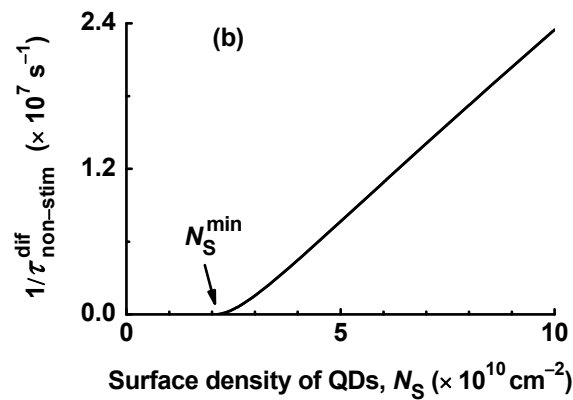
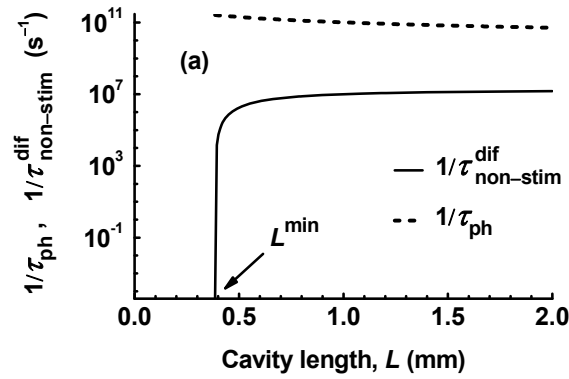


Fig. 3. Reciprocal effective differential non-stimulated recombination time vs. cavity length (a) and surface density of QDs (b). In (a), $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $L = 1.139 \text{ mm}$. The dashed curve in (a) is the reciprocal photon lifetime in the cavity.

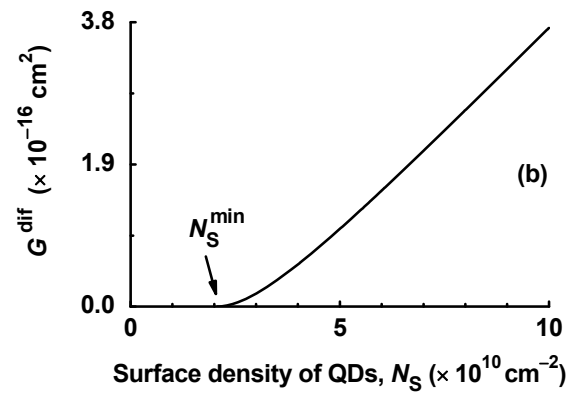
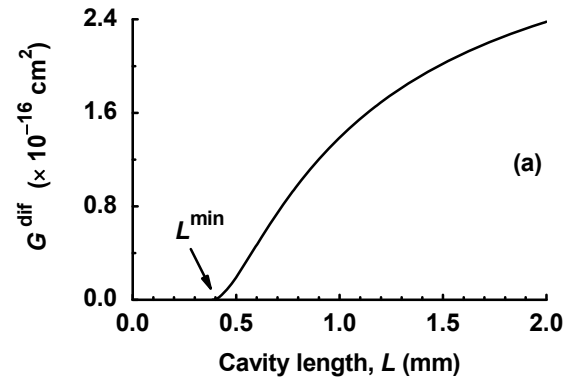


Fig. 4. Effective differential gain vs. cavity length (a) and surface density of QDs (b). In (a), $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $L = 1.139 \text{ mm}$.

As with the dependence on j_0 , the dependences of the modulation bandwidth on L and N_S are also nonmonotonous. At $L = L^{\min}$ or $N_S = N_S^{\min}$, $\omega_{-3 \text{ dB}} = 0$. With increasing L above $L^{\min}$ or N_S above $N_S^{\min}$, $\omega_{-3 \text{ dB}}$ increases, approaches its maximum at a certain optimum cavity length L^{opt} or surface density of QDs N_S^{opt} and then decreases (Fig. 5).

The optimum cavity length and surface density of QDs decrease with increasing dc component of the injection current density (Fig. 6). At high j_0 , when

$$\frac{j_0}{e(b_1 n_1 + N_c^{2D})} \gg v_g g^{\max}, \quad (29)$$

the optimum cavity length and surface density of QDs are very close to $L^{\min}$ and $N_S^{\min}$, respectively, and given by

$$\frac{L^{\text{opt}}}{L^{\min}} = \frac{N_S^{\text{opt}}}{N_S^{\min}} = 1 + 2 \sqrt{\frac{2}{3}} \sqrt[4]{r} \sqrt{\frac{v_g g^{\max}}{j_0 / (e(b_1 n_1 + N_c^{2D}))}}. \quad (30)$$

Since L^{opt} is close to $L^{\min}$, $\omega_{-3 \text{ dB}}$ in this case is close to $\omega_{-3 \text{ dB}}^{\max}$ value at $L^{\min}$, which is the upper limit for the modulation bandwidth,

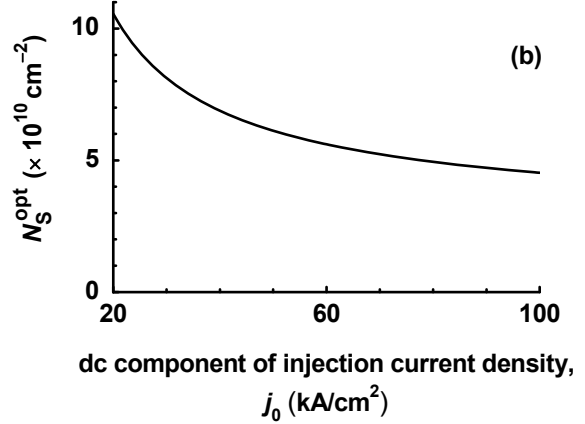
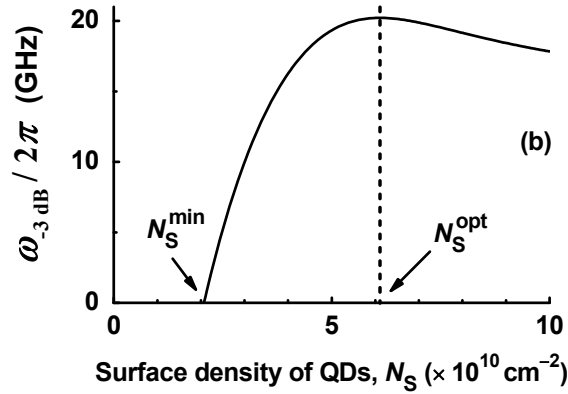
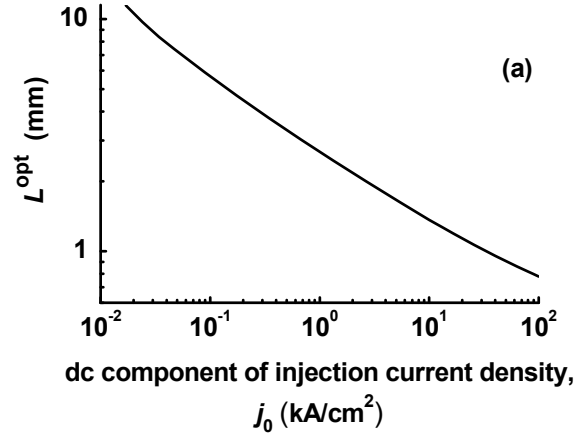
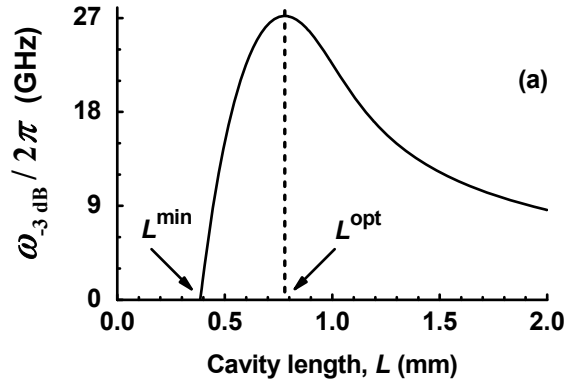


Fig. 5. Modulation bandwidth $\omega_{-3 \text{ dB}}/2\pi$ vs. cavity length (a) and surface density of QDs (b). In (a), $j_0 = 100 \text{ kA/cm}^2$ and $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $j_0 = 50 \text{ kA/cm}^2$ and $L = 1.139 \text{ mm}$. The vertical dashed line marks L^{opt} in (a) and N_S^{opt} in (b).

Fig. 6. Optimum cavity length (a) and surface density of QDs (b) vs. dc component of the injection current density. In (a), $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. In (b), $L = 1.139 \text{ mm}$.

$$\omega_{-3 \text{ dB}}^{\text{highest}} = \sqrt{2} \frac{v_g}{L_{\min}} \ln \left(\frac{1}{R} \right) = \frac{\sqrt{2}}{\tau_{\text{ph}}^{\min}} = \sqrt{2} v_g g^{\max}, \quad (31)$$

where

$$\tau_{\text{ph}}^{\min} = \frac{1}{v_g g^{\max}} = \frac{L_{\min}}{v_g} \frac{1}{\ln(1/R)} \quad (32)$$

is the photon lifetime in the shortest cavity, in which the lasing can occur. As in conventional lasers [25], the upper limit for the modulation bandwidth in a DTI laser is controlled by the maximum modal gain rather than the differential gain.

With increasing L , both G^{dif} and τ_{ph} increase. Hence, as seen from (27), j_0^{opt} decreases. However, $\omega_{-3\text{ dB}}^{\text{max}}$ decreases as well [see (28)]. Taking into account that the first term in the denominator of (19) is small compared to the second term, G^{dif} can be written as follows:

$$G^{\text{dif}} = \frac{2g^{\text{max}}}{b_1 n_1 + N_c^{2\text{D}}} (1 - f_{n0})^2 = \frac{1}{2} \frac{g^{\text{max}}}{b_1 n_1 + N_c^{2\text{D}}} \left(1 - \frac{L^{\text{min}}}{L}\right)^2. \quad (33)$$

With (33), we have from (27)

$$j_0^{\text{opt}} = \frac{4e(b_1 n_1 + N_c^{2\text{D}})}{\tau_{\text{ph}}^{\text{min}}} \frac{1}{\left(\frac{L}{L^{\text{min}}} - 1\right)^2}. \quad (34)$$

Hence to reduce the optimum dc pump current density, the cavity should be made longer. As a sacrifice, the maximum modulation bandwidth will also decrease. Even in a long-cavity device ($L = 1.139$ mm), j_0^{opt} is high (50 kA/cm²) although it is lower than in a conventional laser (64 kA/cm²). At this cavity length, the maximum bandwidth ($\omega_{-3\text{ dB}}^{\text{max}}/2\pi = 20$ GHz), while being considerably lower than its upper limit given by (31), is still high, at least higher than any data reported so far for single QD-layer lasers.

6. CONCLUSIONS

The modulation response of a DTI QD laser has been studied in the best-case scenario of fast pumping of QDs and no carrier leakage from QDs. Closed-form expressions have been obtained for the dynamic characteristics of the laser. The optimum cavity length, surface density of QDs, and dc injection current density, maximizing the modulation bandwidth, have been shown to exist. The higher the dc injection current density, the smaller should be the optimum values of the cavity length and the surface density of QDs. While the maximum bandwidth has been shown to be the same in DTI and conventional QD lasers and unaffected by the differential gain, it is more easily attainable in a DTI laser since the optimum dc current density, being inversely proportional to the differential gain, is lower there.

ACKNOWLEDGEMENTS

This work was supported by the U.S. Army Research Office (Grant No. W911NF-13-1-0445).

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Proceedings

2016 International Conference Laser Optics

St. Petersburg, Russia
June 27 - July 1, 2016

Proceedings

**2016 International Conference
Laser Optics.**

St. Petersburg, Russia

June 27 - July 1, 2016

ISBN: 978-1-4673-9735-3

IEEE Catalog number: CFP1636X-CDR

2016 International Conference Laser Optics.

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IEEE Catalog number: CFP1636X-CDR
ISBN: 978-1-4673-9735-3

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Modulation Response of Double Tunneling-Injection Quantum Dot Lasers

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Abstract—The upper limit for the modulation bandwidth in a double tunneling-injection (DTI) quantum dot (QD) laser is discussed. While the maximum bandwidth is the same in DTI and conventional QD lasers, the optimum dc current, at which it is obtained, is lower in a DTI laser. Different factors limiting the modulation bandwidth in a DTI QD laser are also discussed.

Keywords—quantum dot lasers; quantum well lasers; semiconductor lasers; modulation bandwidth; tunneling-injection

As an alternative to conventional pumping [1], double tunnelling-injection (DTI), i.e., tunnelling-injection of both electrons and holes into quantum dots (QDs), was proposed in [2]–[4] and further studied in [5]–[8] (Fig. 1).

A study of static characteristics showed that DTI QD lasers possess a potential for low-threshold, temperature-stable [2–4, 8], and high-power operation [5]–[7].

Here, dynamic characteristics of DTI QD lasers are discussed. To understand the potential of such lasers for high-speed direct modulation of their optical output by pump current, the upper limit for the modulation bandwidth is first analyzed [9, 10].

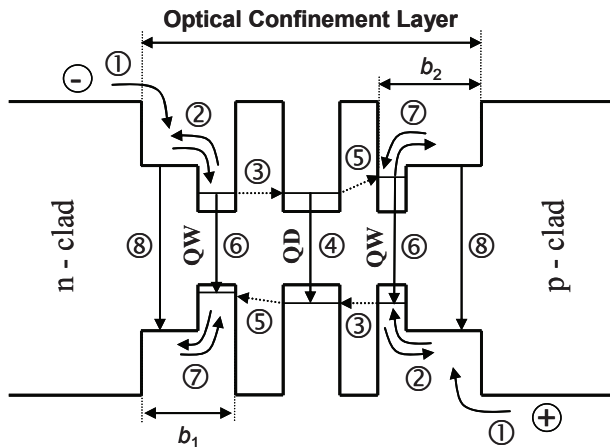


Fig. 1. Energy band diagram of a DTI QD laser and the main processes: ① injection from the cladding layers to the optical confinement layer (OCL), ② majority carrier capture from the OCL into the quantum well (QW) and thermal escape from the QW to the OCL, ③ tunnelling-injection from the QW into a QD, ④ spontaneous and stimulated recombination in a QD, ⑤ out-tunnelling from a QD into the ‘foreign’ QW, ⑥ spontaneous recombination in the QWs, ⑦ minority carrier thermal escape from the QW to the OCL and capture from the OCL into the QW, and ⑧ spontaneous recombination in the OCL. Reproduced with permission from figure 1 of Ref. [5]. Copyright [2008], AIP Publishing LLC.

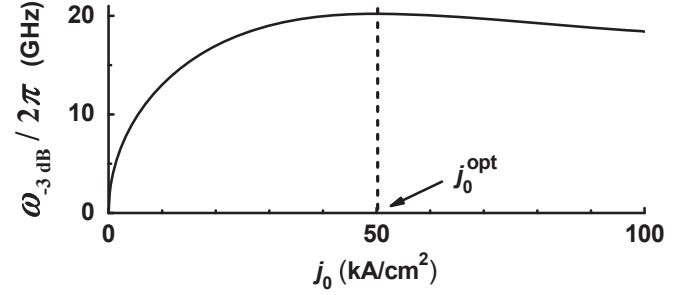


Fig. 2. Modulation bandwidth $\omega_{-3 \text{ dB}}/2\pi$ vs. dc component of the injection current density. A GaInAsP structure emitting at $T = 300$ K near $1.55 \mu\text{m}$ is considered [9, 10]. The cavity length $L = 1.139$ mm (the mirror loss $\beta = 10 \text{ cm}^{-1}$) and the surface density of QDs $N_s = 6.11 \times 10^{10} \text{ cm}^{-2}$. The vertical dashed line marks the optimum dc current density maximizing the bandwidth.

Fig. 2 shows the small-signal modulation bandwidth (defined as the -3 dB bandwidth) as a function of the dc component of the injection current density. As seen from the figure, $\omega_{-3 \text{ dB}}/2\pi$ has a maximum at a certain optimum value of the dc current density. This optimum value, maximizing the bandwidth, is lower in a DTI QD laser as compared to conventional QD lasers [11].

As shown in [9, 10], there also exist the optimum cavity length and surface density of QDs, maximizing the modulation bandwidth in a DTI QD laser.

To estimate the upper limit for the bandwidth, fast carrier injection into QDs was assumed in [9, 10]. Here, the effects of different factors (such as noninstantaneous carrier injection into QDs) on the modulation bandwidth in a DTI QD laser are also discussed.

This work was supported by the US Army Research Office (Grant No. W911NF-13-1-0445).

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Optics Letters

Effect of pumping delay on the modulation bandwidth in double tunneling-injection quantum dot lasers

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Received 19 October 2016; revised 6 November 2016; accepted 29 November 2016; posted 2 December 2016 (Doc. ID 279019); published 22 December 2016

The modulation bandwidth of double tunneling-injection (DTI) quantum dot (QD) lasers is studied, taking into account noninstantaneous pumping of QDs. In this advanced type of semiconductor lasers, carriers are first captured from the bulk waveguide region into two-dimensional regions (quantum wells [QWs]); then they tunnel from the QWs into zero-dimensional regions (QDs). The two processes are non-instantaneous and, thus, could delay the delivery of the carriers to the QDs. Here, the modulation bandwidth of DTI QD lasers is calculated as a function of two characteristic times (the capture time from the waveguide region into the QW and the tunneling time from the QW into the QD ensemble) and is shown to increase as either of these times is reduced. The capture and tunneling times of 1 and 0.1 ps, respectively, are shown to characterize fast capture and tunneling processes; as the capture and tunneling times are brought below 1 and 0.1 ps, the bandwidth remains almost unchanged and close to its upper limit. © 2016 Optical Society of America

OCIS codes: (250.0250) Optoelectronics; (250.5960) Semiconductor lasers; (250.5590) Quantum-well, -wire and -dot devices.

<https://doi.org/10.1364/OL.42.000097>

Several tunneling schemes were considered for pumping low-dimensional active regions in semiconductor lasers [1–19]. In double tunneling-injection (DTI) quantum dot (QD) lasers, electrons and holes are injected into QDs via tunneling from two separate quantum wells (QWs) located on the opposite sides of the QD layer and separated from the latter by thin barrier layers [9–12]; see (Fig. 1). Such a pumping mechanism should ideally suppress electron-hole recombination outside QDs, which is a major cause of (1) the temperature dependence of the threshold current [20] and (2) the sublinearity of the light-current characteristic in conventional (not tunneling-injection) QD lasers [21].

The carrier delivery to the QDs in DTI lasers is a two-step process. The electrons and holes are first captured from the bulk waveguide region (optical confinement layer [OCL]) into the respective QWs and then they tunnel from the QWs into

the QDs. In this Letter, the modulation response of DTI QD lasers is studied, taking into account the fact that both the capture and tunneling processes are noninstantaneous. The modulation bandwidth is calculated as a function of two characteristic times: the capture time from a three-dimensional (3D) region (OCL) into a two-dimensional (2D) region (QW), and the tunneling time from a 2D region into a QD ensemble.

To focus on the effects of capture and tunneling delays, it is assumed here that there is no out-tunneling of carriers from QDs into the “foreign” (not injecting) QWs (process ⑤ in Fig. 1), i.e., no electron leakage into the right-hand side QW and no hole leakage into the left-hand side QW in Fig. 1. Hence, there are no electrons on the hole-injecting side of the structure and no holes on the electron-injecting side, the right- and left-hand sides, respectively, in Fig. 1. There is also no carrier leakage from the QDs to the wetting layer [22,23] in this model. With these assumptions, the following set of rate equations is used here: for free majority carriers in the injecting side of the OCL (for definiteness, for electrons in the left-hand side),

$$b_1 \frac{\partial n_{\text{OCL}}^L}{\partial t} = \frac{j}{e} + \frac{n_{\text{QW}}^L}{\tau_{n,\text{esc}}^L} - v_{n,\text{capt}}^L n_{\text{OCL}}^L, \quad (1)$$

for 2D majority carriers in the injecting QW,

$$\begin{aligned} \frac{\partial n_{\text{QW}}^L}{\partial t} = & v_{n,\text{capt}}^L n_{\text{OCL}}^L - \frac{n_{\text{QW}}^L}{\tau_{n,\text{esc}}^L} - w_{n,\text{tunn}}^L N_S (1 - f_n) n_{\text{QW}}^L \\ & + w_{n,\text{tunn}}^L n_1^{L,\text{QW}} N_S f_n, \end{aligned} \quad (2)$$

for carriers confined in QDs,

$$\begin{aligned} 2N_S \frac{\partial f_n}{\partial t} = & w_{n,\text{tunn}}^L N_S (1 - f_n) n_{\text{QW}}^L - w_{n,\text{tunn}}^L n_1^{L,\text{QW}} N_S f_n \\ & - N_S \frac{f_n^2}{\tau_{\text{QD}}} - c_g g^{\text{max}} (2f_n - 1) n_{\text{ph}}, \end{aligned} \quad (3)$$

and for photons,

$$\frac{\partial n_{\text{ph}}}{\partial t} = c_g g^{\text{max}} (2f_n - 1) n_{\text{ph}} - c_g \beta n_{\text{ph}}. \quad (4)$$

In Eqs. (1)–(4), n_{OCL}^L is the free majority carrier density (per unit volume of the injecting side of the OCL), n_{QW}^L is the 2D majority carrier density in the injecting QW, f_n is the confined

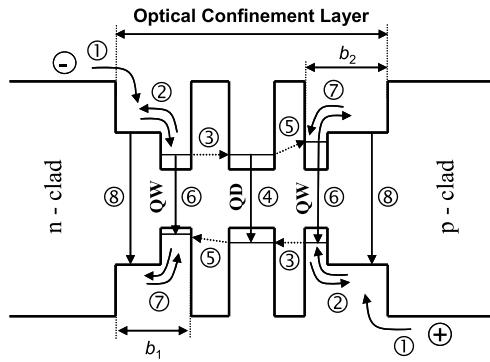


Fig. 1. Energy band diagram of a DTI QD laser and the main processes: ① injection from the cladding layers to the OCL, ② majority carrier capture from the OCL into the QW and thermal escape from the QW to the OCL, ③ tunneling injection from the QW into a QD, ④ spontaneous and stimulated recombination in a QD, ⑤ out-tunneling from a QD into the “foreign” QW, ⑥ spontaneous recombination in the QWs, ⑦ minority carrier thermal escape from the QW to the OCL and capture from the OCL into the QW, and ⑧ spontaneous recombination in the OCL. Reproduced from Fig. 1 of [11], with the permission of AIP Publishing.

carrier level occupancy in QDs, n_{ph} is the number of photons in the lasing mode (*per unit area of the junction*), j is the injection current density, e is the electron charge, b_1 is the thickness of the injecting side of the OCL (Fig. 1), N_S is the surface density of QDs, τ_{QD} is the spontaneous radiative recombination time in QDs, c_g is the group velocity of light, $g^{\max}$ is the maximum value of the modal gain of the laser, $\beta = (1/L) \ln(1/R)$ is the mirror loss coefficient, L is the cavity length, and R is the mirror reflectivity.

In the case of an undoped QW, the quantity n_1^{LQW} in Eqs. (2) and (3) is equal to the 2D effective density of states in the conduction band in the QW [12]:

$$n_1^{LQW} = N_c^{2D}. \quad (5)$$

Two important parameters entering into Eqs. (1)–(3) are the velocity $v_{n,capt}^L$ of the carrier capture from a bulk region (OCL) into a 2D region (QW) and the coefficient $w_{n,tunn}^L$ (in units of cm^2/s) of majority carrier tunneling between the injecting QW and QDs.

In Eqs. (1) and (2), $\tau_{n,esc}^L$ is the thermal escape time of the carriers from the QW to the OCL. For undoped OCL and QW, $\tau_{n,esc}^L$ is expressed in terms of $v_{n,capt}^L$ as follows [12]:

$$\tau_{n,esc}^L = \frac{1}{v_{n,capt}^L} \frac{N_c^{2D}}{n_1}, \quad (6)$$

with the quantity n_1 being given as

$$n_1 = N_c^{3D} \exp\left(-\frac{\Delta E_c - \epsilon_n^{QW}}{T}\right), \quad (7)$$

where N_c^{3D} is the 3D effective density of states in the OCL, ΔE_c is the band offset between the OCL and the QW, ϵ_n^{QW} is the energy of the carrier-subband edge in the QW, and T is the temperature (in units of energy).

With $v_{n,capt}^L$ and $w_{n,tunn}^L$, the following two characteristic times can be introduced: the carrier capture time from a 3D (OCL) into a 2D region (QW):

$$\tau_{n,capt}^L = \frac{b_1}{v_{n,capt}^L}, \quad (8)$$

and the carrier tunneling time from a 2D region (QW) into an unoccupied (when $f_n = 0$) QD ensemble:

$$\tau_{n,tunn,0}^{QW \rightarrow QDs} = \frac{1}{w_{n,tunn}^L N_S}. \quad (9)$$

The tunneling coefficient $w_{n,tunn}^L$ and time $\tau_{n,tunn,0}^{QW \rightarrow QDs}$ depend on the compositions and widths of the QW and barrier layer, as well as on the composition, shape, and size of the QDs. For disk-shaped QDs, the tunneling rate was discussed in [24].

The tunneling time also depends on N_S . Actually, tunneling injection should inherently select the QDs of the “preferred” size and, hence, lead to the effective narrowing of inhomogeneous linewidth [9]. This was observed in [7].

The small-signal response of the laser is considered here and, hence, the injection current density is presented as

$$j = j_0 + (\delta j_m) \exp(i\omega t), \quad (10)$$

where j_0 is the dc component, δj_m is the small amplitude of the time-harmonic ac component, and ω is the modulating frequency.

The solutions of Eqs. (1)–(4) are correspondingly sought for as

$$n_{OCL}^L = n_{OCL,0}^L + (\delta n_{OCL-m}^L) \exp(i\omega t), \quad (11)$$

$$n_{QW}^L = n_{QW,0}^L + (\delta n_{QW-m}^L) \exp(i\omega t), \quad (12)$$

$$f_n = f_{n0} + (\delta f_{n-m}) \exp(i\omega t), \quad (13)$$

$$n_{ph} = n_{ph,0} + (\delta n_{ph-m}) \exp(i\omega t), \quad (14)$$

where $n_{OCL,0}^L$, $n_{QW,0}^L$, f_{n0} , and $n_{ph,0}$ are the dc components, and δn_{OCL-m}^L , δn_{QW-m}^L , δf_{n-m} , and δn_{ph-m} are the small amplitudes of the time-harmonic ac components.

The dc components are found from the solutions of the rate Eqs. (1)–(4) at the steady state ($\partial/\partial t = 0$) and are given as

$$n_{OCL,0}^L = n_1 \frac{f_{n0}}{1 - f_{n0}} + \left[\frac{1}{v_{n,capt}^L} + \frac{n_1}{N_c^{2D}} \frac{1}{w_{n,tunn}^L N_S (1 - f_{n0})} \right] \frac{j_0}{e}, \quad (15)$$

$$n_{QW,0}^L = n_1^{LQW} \frac{f_{n0}}{1 - f_{n0}} + \frac{j_0}{e w_{n,tunn}^L N_S (1 - f_{n0})}, \quad (16)$$

$$f_{n0} = \frac{1}{2} \left(1 + \frac{\beta}{g^{\max}} \right), \quad (17)$$

$$n_{ph,0} = \tau_{ph} \frac{j_0 - j_{th}}{e}, \quad (18)$$

where the threshold current density and the photon lifetime in the cavity are introduced as follows:

$$j_{th} = e N_S \frac{f_{n0}^2}{\tau_{QD}}, \quad (19)$$

$$\tau_{\text{ph}} = \frac{1}{c_g \beta} = \frac{L}{c_g \ln(1/R)}. \quad (20)$$

Using Eqs. (11)–(14) in Eqs. (1)–(4), a set of linear algebraic equations is obtained for the small ac amplitudes as functions of the modulating frequency ω . The solution of this set yields, in particular, the ratio $\delta n_{\text{ph}-m}(\omega)/\delta n_{\text{ph}-m}(0)$:

$$\frac{\delta n_{\text{ph}-m}(\omega)}{\delta n_{\text{ph}-m}(0)} = \frac{A_0}{(\omega^4 - A_2 \omega^2 + A_0) - i(A_3 \omega^3 - A_1 \omega)}, \quad (21)$$

and then the modulation response function

$$H(\omega) = \left| \frac{\delta n_{\text{ph}-m}(\omega)}{\delta n_{\text{ph}-m}(0)} \right|^2 = \frac{A_0^2}{(\omega^4 - A_2 \omega^2 + A_0)^2 + (A_3 \omega^3 - A_1 \omega)^2}, \quad (22)$$

where the coefficients A_0 , A_1 , A_2 , and A_3 are given by

$$A_0 = \frac{v_{n,\text{capt}}^L}{b_1} w_{n,\text{tunn}}^L N_S (1 - f_{n0}) c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S} \frac{1}{\tau_{\text{ph}}}, \quad (23)$$

$$A_1 = \frac{v_{n,\text{capt}}^L}{b_1} w_{n,\text{tunn}}^L N_S (1 - f_{n0}) \left(\frac{f_{n0}}{\tau_{\text{QD}}} + c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S} \right) + \left[\frac{v_{n,\text{capt}}^L}{b_1} + \frac{1}{\tau_{n,\text{esc}}^L} + w_{n,\text{tunn}}^L N_S (1 - f_{n0}) \right] c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S} \frac{1}{\tau_{\text{ph}}}, \quad (24)$$

$$A_2 = w_{n,\text{tunn}}^L N_S (1 - f_{n0}) \left(\frac{v_{n,\text{capt}}^L}{b_1} + \frac{f_{n0}}{\tau_{\text{QD}}} + c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S} \right) + \left(\frac{v_{n,\text{capt}}^L}{b_1} + \frac{1}{\tau_{n,\text{esc}}^L} \right) \left[\frac{1}{2} w_{n,\text{tunn}}^L (n_{\text{QW},0}^L + n_1^{L,\text{QW}}) + \frac{f_{n0}}{\tau_{\text{QD}}} + c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S} \right] + c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S} \frac{1}{\tau_{\text{ph}}}, \quad (25)$$

$$A_3 = \frac{v_{n,\text{capt}}^L}{b_1} + \frac{1}{\tau_{n,\text{esc}}^L} + w_{n,\text{tunn}}^L \left[N_S (1 - f_{n0}) + \frac{1}{2} (n_{\text{QW},0}^L + n_1^{L,\text{QW}}) \right] + \frac{f_{n0}}{\tau_{\text{QD}}} + c_g g^{\text{max}} \frac{n_{\text{ph},0}}{N_S}. \quad (26)$$

In Eqs. (23)–(26), $n_{\text{QW},0}^L$, f_{n0} , and $n_{\text{ph},0}$ are given by Eqs. (16), (17), and (18). Hence, the coefficients A_0 , A_1 , A_2 , and A_3 are functions of the dc component j_0 of the injection current density and parameters of the laser structure. A_1 , A_2 , and A_3 are positive; A_0 is positive above the lasing threshold and zero at the lasing threshold.

The modulation bandwidth is defined as the -3 dB-bandwidth [the frequency at which $H(\omega)$ has fallen to half its dc ($\omega = 0$) value]:

$$10 \log_{10} H(\omega_{-3\text{dB}}) = -3. \quad (27)$$

With Eq. (22), the following quartic equation for the square $\omega_{-3\text{dB}}^2$ of the modulation bandwidth is obtained:

$$\omega_{-3\text{dB}}^8 + (A_3^2 - 2A_2)\omega_{-3\text{dB}}^6 + (A_2^2 + 2A_0 - 2A_1A_3)\omega_{-3\text{dB}}^4 + (A_1^2 - 2A_0A_2)\omega_{-3\text{dB}}^2 - (r - 1)A_0^2 = 0, \quad (28)$$

where the numerical parameter $r = 10^{0.3} \approx 1.995$. From the solution of Eq. (28), $\omega_{-3\text{dB}}$ is found.

The bandwidth is controlled by the slowest of the processes of the carrier delivery to QDs. Thus, if capture is slow (i.e., the capture velocity $v_{n,\text{capt}}^L$ is low or, equivalently [see Eq. (8)], the capture time $\tau_{n,\text{capt}}^L$ is long), $\omega_{-3\text{dB}}$ is low and inversely proportional to $\tau_{n,\text{capt}}^L$:

$$\omega_{-3\text{dB}} \propto v_{n,\text{capt}}^L \propto \frac{1}{\tau_{n,\text{capt}}^L}. \quad (29)$$

If tunneling is slow (i.e., the tunneling coefficient $w_{n,\text{tunn}}^L$ is small or, equivalently [see Eq. (9)], the tunneling time $\tau_{n,\text{tunn},0}^{\text{QW} \rightarrow \text{QDs}}$ is long), $\omega_{-3\text{dB}}$ is low and inversely proportional to $\tau_{n,\text{tunn},0}^{\text{QW} \rightarrow \text{QDs}}$:

$$\omega_{-3\text{dB}} \propto w_{n,\text{tunn}}^L \propto \frac{1}{\tau_{n,\text{tunn},0}^{\text{QW} \rightarrow \text{QDs}}}. \quad (30)$$

If the capture and tunneling processes are fast, the bandwidth approaches its upper limit (see below). In this case, the peak value of $\omega_{-3\text{dB}}$ as a function of the dc component j_0 of the injection current density will be given by

$$\omega_{-3\text{dB}}^{\text{max}} = \sqrt[4]{r - 1} \frac{\sqrt{2}}{\tau_{\text{ph}}} \approx \frac{\sqrt{2}}{\tau_{\text{ph}}}. \quad (31)$$

Below, the dependences of the modulation bandwidth on the dc component of the current density (Fig. 2), cavity length (Fig. 3), and surface density of QDs (Fig. 4) are discussed for a GaInAsP structure lasing at $T = 300$ K near $1.55 \mu\text{m}$ [25]. Each of these dependences is calculated for several values of (a) the capture time into the QW and (b) the tunneling time into the QD ensemble.

As seen from Figs. 2(a), 3(a), and 4(a), there is a considerable increase in $\omega_{-3\text{dB}}$ with reducing the capture time from both 100 to 10 ps and then 10 to 1 ps. On the further reduction of the capture time, $\omega_{-3\text{dB}}$ remains almost unchanged and close to its upper limit calculated in [25]. This is also clearly seen from Fig. 5(a), in which the bandwidth is plotted against the capture time. Hence, for the capture into the QW to be

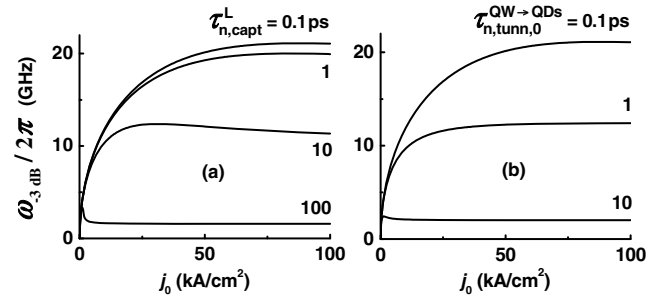


Fig. 2. Modulation bandwidth $\omega_{-3\text{dB}}/2\pi$ versus dc component of the injection current density at different values of (a) the capture time into the QW and (b) the tunneling time into the QD ensemble. A GaInAsP structure lasing at $T = 300$ K near $1.55 \mu\text{m}$ is considered [25]. The cavity length $L = 1$ mm; the surface density of QDs $N_S = 6 \times 10^{10} \text{ cm}^{-2}$. In Figs. 2–5(a), to focus on the effect of the capture delay, fast tunneling ($\tau_{n,\text{tunn},0}^{\text{QW} \rightarrow \text{QDs}} = 0.1$ ps) is assumed. In Figs. 2–5(b), to focus on the effect of tunneling delay, fast capture ($\tau_{n,\text{capt}}^L = 0.1$ ps) is assumed.

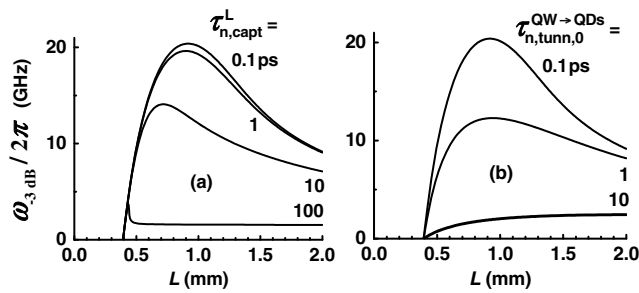


Fig. 3. Modulation bandwidth $\omega_{-3\text{dB}}/2\pi$ versus cavity length at different values of (a) the capture time into the QW and (b) the tunneling time into the QD ensemble. The dc component of the injection current density $j_0 = 50 \text{ kA/cm}^2$; $N_S = 6 \times 10^{10} \text{ cm}^{-2}$.

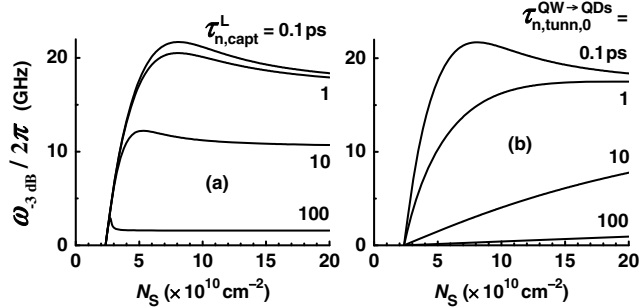


Fig. 4. Modulation bandwidth $\omega_{-3\text{dB}}/2\pi$ versus surface density of QDs at different values of (a) the capture time into the QW and (b) the tunneling time into the QD ensemble. $j_0 = 50 \text{ kA/cm}^2$; $L = 1 \text{ mm}$.

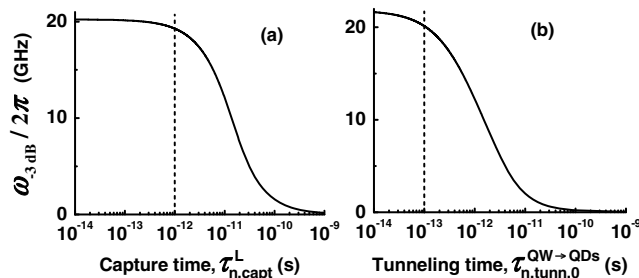


Fig. 5. Modulation bandwidth $\omega_{-3\text{dB}}/2\pi$ versus (a) capture time into the QW and (b) tunneling time into the QD ensemble. $j_0 = 50 \text{ kA/cm}^2$, $L = 1 \text{ mm}$, and $N_S = 6 \times 10^{10} \text{ cm}^{-2}$. The vertical dashed lines mark the values of the capture and tunneling times [1 and 0.1 ps in (a) and (b), respectively], below which the capture and tunneling processes can be considered as fast.

considered as a fast process in the specific structure considered here, the capture time should be shorter than 1 ps.

While the capture time of 1 ps describes a fast capture into the QW, the tunneling into the QD ensemble cannot

be considered as a fast process at the tunneling time of 1 ps. As seen in Figs. 2(b), 3(b), and 4(b), the modulation bandwidth continues to increase with reducing tunneling time beyond 1 ps and down to 0.1 ps. For the tunneling time below 0.1 ps, the modulation bandwidth is almost constant and close to its upper limit [25]. This is also seen in Fig. 5(b), showing $\omega_{-3\text{dB}}$ against the tunneling time into the QD ensemble. Therefore, for the tunneling into the QD ensemble to be considered as a fast process in a structure considered here, the tunneling time should be shorter than 0.1 ps.

In conclusion, the modulation bandwidth of DTI QD lasers has been shown to increase as either the capture time from the waveguide region into the QW or the tunneling time from the QW into the QD ensemble is reduced.

Funding. U.S. Army Research Office (ARO) (W911NF-13-1-0445).

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Modulation Bandwidth of a Double Tunneling-Injection Quantum Dot Laser: The Upper Limit and Limiting Factors

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Abstract—The modulation response in double tunnelling-injection quantum dot lasers is analyzed. The highest possible modulation bandwidth and the effect of noninstantaneous pumping of quantum dots on the bandwidth are discussed.

Keywords—quantum dot lasers; quantum well lasers; semiconductor lasers; modulation bandwidth; tunnelling-injection

As an alternative to conventional pumping [1], double tunnelling-injection (DTI), i.e., tunnelling-injection of both electrons and holes into quantum dots (QDs), was proposed in [2]–[4] and further studied in [5]–[8] (Fig. 1). Such a pumping mechanism should ideally suppress electron-hole recombination outside QDs, which is a major cause of (i) temperature-dependence of the threshold current [9] and (ii) sublinearity of the light-current characteristic in conventional (not tunneling-injection) QD lasers [10].

A study of static characteristics showed that DTI QD lasers possess a potential for low-threshold, temperature-stable [2–4, 8], and high-power operation [5]–[7].

Here, dynamic characteristics of DTI QD lasers are discussed. To understand the potential of such lasers for high-speed direct modulation of their optical output by pump current, the upper limit for the modulation bandwidth is first analyzed [11, 12].

The carrier delivery to the QDs in DTI lasers is a two-step process. The electrons and holes are first captured from the bulk waveguide region (optical confinement layer [OCL]) into the respective quantum wells (QWs) and then they tunnel from the QWs into the QDs. To estimate the upper limit for the bandwidth, fast carrier capture from the OCL into the QWs and fast tunneling injection of carriers from the QWs into the QDs were assumed in [11, 12].

Fig. 2 shows the small-signal modulation bandwidth (defined as the -3 dB bandwidth) as a function of the dc component of the injection current density. A GaInAsP structure emitting at $T = 300$ K near 1.55 μm is considered [11, 12]. As seen from the figure, $\omega_{-3\text{ dB}}/2\pi$ has a maximum at a certain optimum value of the dc current density. This

optimum value, maximizing the bandwidth, is lower in a DTI QD laser as compared to that in conventional QD lasers [13].

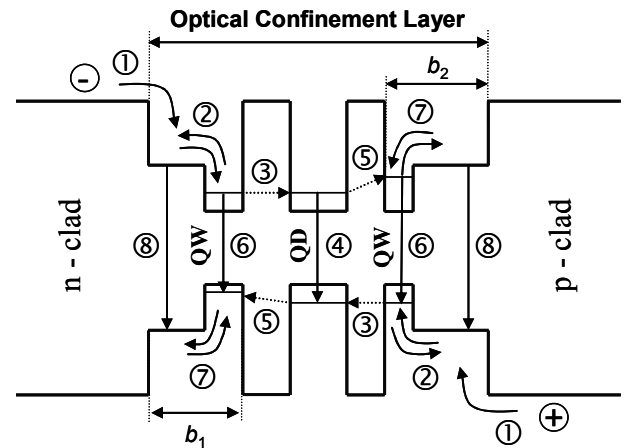


Fig. 1. Energy band diagram of a DTI QD laser and the main processes: ① injection from the cladding layers to the optical confinement layer (OCL), ② majority carrier capture from the OCL into the quantum well (QW) and thermal escape from the QW to the OCL, ③ tunnelling-injection from the QW into a QD, ④ spontaneous and stimulated recombination in a QD, ⑤ out-tunnelling from a QD into the ‘foreign’ QW, ⑥ spontaneous recombination in the QWs, ⑦ minority carrier thermal escape from the QW to the OCL and capture from the OCL into the QW, and ⑧ spontaneous recombination in the OCL. Reproduced with permission from Fig. 1 of Ref. [5]. Copyright [2008], AIP Publishing LLC.

As shown in [11, 12], there also exist the optimum cavity length and surface density of QDs, maximizing the modulation bandwidth in a DTI QD laser.

In [14], the effects of noninstantaneous capture of carrier from the OCL into the QWs and noninstantaneous tunneling-injection from the QWs into the QDs were studied. The modulation bandwidth was calculated as a function of two characteristic times – the capture time from a three-dimensional (3D) region (OCL) into a two-dimensional (2D)

region (QW) and the tunneling time from a 2D region into a QD-ensemble.

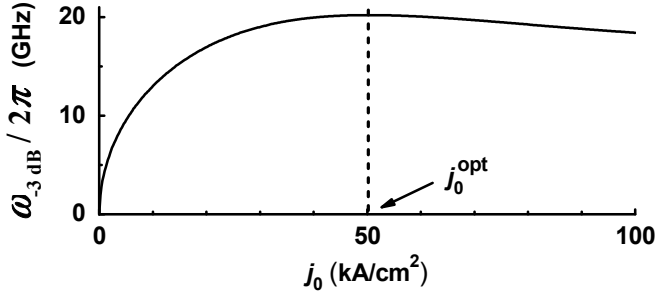


Fig. 2. Modulation bandwidth $\omega_{-3\text{ dB}}/2\pi$ vs. dc component of the injection current density. A GaInAsP structure emitting at $T = 300\text{ K}$ near $1.55\text{ }\mu\text{m}$ is considered [11, 12]. The cavity length $L = 1.139\text{ mm}$ (the mirror loss $\beta = 10\text{ cm}^{-1}$) and the surface density of QDs $N_S = 6.11 \times 10^{10}\text{ cm}^{-2}$. The vertical dashed line marks the optimum dc current density maximizing the bandwidth.

Fig. 3 shows the modulation bandwidth against the capture time. As seen from the figure, there is a considerable increase in $\omega_{-3\text{ dB}}$ both with reducing the capture time from 100 to 10 ps and then from 10 to 1 ps. On the further reduction of the capture time, $\omega_{-3\text{ dB}}$ remains almost unchanged and close to its upper limit. Hence, for the capture into the QW to be considered as a fast process in the specific structure considered here, the capture time should be shorter than 1 ps.

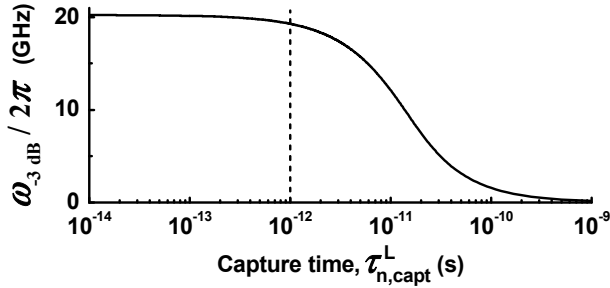


Fig. 3. Modulation bandwidth $\omega_{-3\text{ dB}}/2\pi$ vs. capture time into the QW. $j_0 = 50\text{ kA/cm}^2$, $L = 1.139\text{ mm}$, and $N_S = 6.11 \times 10^{10}\text{ cm}^{-2}$. The vertical dashed line marks the value of the capture time (1 ps), below which the capture process can be considered as fast.

Fig. 4 shows the modulation bandwidth against the tunneling time. While the capture time of 1 ps describes a fast capture into the QW, tunneling into the QD-ensemble cannot be considered as a fast process at the tunneling time of 1 ps. As seen from Fig. 4, the modulation bandwidth continues to increase with reducing tunneling time beyond 1 ps and down to 0.1 ps. For the tunneling time below 0.1 ps, the modulation bandwidth is almost constant and close to its upper limit.

Therefore, for the tunneling into the QD-ensemble to be considered as a fast process in a structure considered here, the tunneling time should be shorter than 0.1 ps.

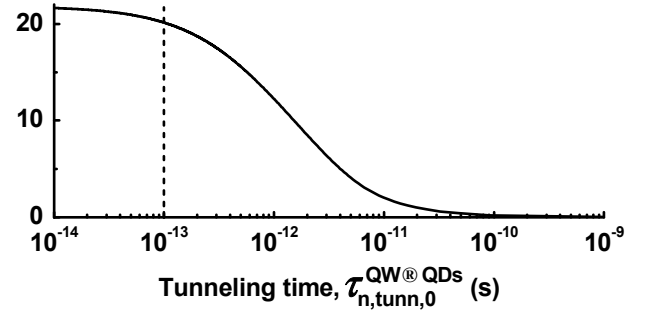


Fig. 4. Modulation bandwidth $\omega_{-3\text{ dB}}/2\pi$ vs. tunneling time into the QD-ensemble. $j_0 = 50\text{ kA/cm}^2$, $L = 1.139\text{ mm}$, and $N_S = 6.11 \times 10^{10}\text{ cm}^{-2}$. The vertical dashed line marks the value of the tunneling time (0.1 p), below which the tunneling process can be considered as fast.

This work was supported by the US Army Research Office (Grant No. W911NF-13-1-0445).

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Modulation bandwidth of double tunneling-injection quantum dot lasers: Effect of out-tunneling leakage

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Effect of out-tunneling leakage of carriers from quantum dots (QDs) on modulation bandwidth of semiconductor double tunneling-injection (DTI) QD lasers is studied. In this type of laser, as an alternative to conventional pumping, electrons and holes are injected into QDs by tunneling from two separate quantum wells (QWs) located on the opposite sides of the QD layer and separated from the latter by thin barrier layers. While fast tunneling-injection into QDs should be maintained to ensure high modulation bandwidth, it may seem that undesirable out-tunneling leakage from QDs (tunneling of electrons to the hole-injecting QW and holes to the electron-injecting QW) should be kept as slow as possible (ideally, totally suppressed) to maximize the modulation bandwidth. However, it is shown in this work that, due to zero-dimensional nature of QDs, the modulation bandwidth is practically unaffected by out-tunneling leakage from them – it depends only slightly on the out-tunneling time. Provided that tunneling-injection is fast, high bandwidth can be attained in DTI QD lasers no matter the out-tunneling time.

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I. Introduction

In conventional semiconductor lasers with a quantum-confined active region, there is always bipolar (i.e., both electron and hole) population and hence electron-hole recombination outside the active region [1, 2]. This recombination negatively affects the laser operating characteristics. In particular, the threshold current is increased [3] and more temperature-sensitive (the characteristic temperature is reduced) [4]. Besides, since the rate of this recombination increases superlinearly with increasing injection current above the lasing threshold, the efficiency of stimulated recombination decreases and hence the light-current characteristic (LCC) becomes sublinear [5].

In [6]-[8], to eliminate the parasitic electron-hole recombination outside quantum dots (QDs), double tunneling-injection (DTI), i.e., tunneling injection of both electrons and holes, was proposed as a mechanism for pumping the QDs. In [9]-[12], the concept of double tunneling-injection was applied to AlGaAs-GaAs-InGaAs-InAs QD heterostructure lasers. To improve the QD laser characteristics, other tunneling-injection approaches have also been proposed and realized, such as tunneling injection of only electrons into QDs [13]-[21].

The energy band diagram of a DTI QD laser is shown in Fig. 1. As in conventional QD lasers, electrons and holes are injected from n- and p-cladding layers, respectively, into the optical confinement layer (OCL). In contrast to conventional QD lasers, electrons and holes do not occupy the entire OCL in DTI QD lasers; each type of carriers is contained in its own part of the OCL – the one adjacent to the cladding layer injecting them. This is accomplished by means of the use of two quantum wells (QWs) and barrier layers (one on each side of the QD layer). Each type of carriers is captured from the corresponding part of the OCL into the adjacent QW. The only way for the further transport of electrons and holes is tunneling through the corresponding thin barrier layers into the QDs. Ideally, the wells and barriers should be designed so that electrons will not out-tunnel into the hole-injecting QW and holes into the electron-injecting QW [6]-[8]. In such an ideal situation, bipolar population will be maintained in QDs only. Outside the QDs, i.e., in the OCL and QWs, there will be monopolar population – electron population on the electron-injecting side of the structure and hole population on the hole-injecting side. As a result of this, there will be no electron-hole recombination outside QDs. The total suppression of parasitic recombination will result in a very low and virtually temperature-insensitive threshold current [6]-[8] and a linear LCC.

In actual structures, however, out-tunneling of carriers into the foreign QWs will not be totally suppressed. In the presence of out-tunneling, electrons injected into QDs will leak into the hole-injecting QW and further escape to the hole-injecting side of the OCL. Similarly, holes injected into QDs will leak into the electron-injecting QW and then escape to the electron-injecting side of the OCL. There will be, therefore, certain fractions of minority carriers on each side of the structure – electrons on the hole-injecting side and holes on the electron-injecting side. As a result of this, parasitic recombination will still occur outside QDs in DTI lasers. In [22]-[24], the effect of out-tunneling leakage on static characteristics of DTI QD lasers was studied.

In this work, to explore the potential of DTI QD lasers for high-frequency direct modulation of the optical output by injection current, we study the effect of out-tunneling leakage on their dynamic characteristics. The modulation bandwidth is calculated as a function of the characteristic time of out-tunneling leakage from QDs. It is shown that, due to zero-dimensional (0-D) nature of QDs, the net out-tunneling current from them remains limited with increasing out-tunneling time. As a result of this, the modulation bandwidth is practically unaffected by out-tunneling leakage and can be made high (provided that tunneling-injection is fast) no matter the out-tunneling time.

II. Theoretical model

We use the following set of rate equations derived in Appendixes A and B:

$$\frac{\partial}{\partial t} \left[\left(\frac{b_1 n_1}{N_c^{2D}} + 1 \right) N_c^{2D} \frac{f_n}{1-f_n} + 2N_s f_n \right] = \frac{j}{e} - \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_n}{1-f_n} F_{QW} - N_s \frac{f_n^2}{\tau_{QD}} - [w_{\text{out-tunn}} F_1^{\text{QW}} N_s f_n - w_{\text{out-tunn}} N_s (1-f_n) F_{QW}] - c_g g^{\text{max}} (2f_n - 1) n_{\text{ph}} \quad (1)$$

$$\frac{\partial}{\partial t} \left[\left(\frac{b_1 n_1}{N_c^{2D}} + 1 \right) F_{QW} \right] = [w_{\text{out-tunn}} F_1^{\text{QW}} N_s f_n - w_{\text{out-tunn}} N_s (1-f_n) F_{QW}] - \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_n}{1-f_n} F_{QW} \quad (2)$$

$$\frac{\partial n_{\text{ph}}}{\partial t} = c_g g^{\text{max}} (2f_n - 1) n_{\text{ph}} - c_g \beta n_{\text{ph}} \quad (3)$$

The unknowns in eqs. (1)-(3) are the carrier level occupancy (occupation probability) f_n in QDs, the minority carrier density F_{QW} in the foreign QW (i.e., the electron density in the hole-injecting QW and the hole density in the electron-injecting QW), and the two-dimensional (2-D) density of photons n_{ph} in the lasing mode (number of photons per unit area of the junction).

The parameters in eqs. (1)-(3) are the injection current density j , the electron charge e , the thickness of one side of the OCL b_1 [the separation between the cladding layer and the barrier – Fig. 1], the spontaneous radiative recombination constants B and B_{2D} for the bulk (OCL) and 2-D regions (QWs) measured in units of cm^3/s and cm^2/s , respectively, the surface density of QDs N_s , the spontaneous radiative lifetime in QDs τ_{QD} , the group velocity of light c_g , the maximum value of the modal gain g^{max} [3], the mirror loss $\beta = (1/L)\ln(1/R)$, the cavity length L , and the facet reflectivity R .

The quantities N_c^{2D} , n_1 , and F_1^{QW} entering into (1) and (2) are given by eqs. (A13), (A14), and (B16) in Appendixes A and B.

The parameter $w_{\text{out-tunn}}$ in (1) and (2) is the out-tunneling coefficient, i.e., the coefficient of tunneling (measured in units of cm^2/s) between QDs and the foreign QW. The out-tunneling time from a QD to the foreign QW is expressed in terms of $w_{\text{out-tunn}}$ as follows:

$$\tau_{\text{out-tunn}}^{\text{QD} \rightarrow \text{QW}} = \frac{1}{w_{\text{out-tunn}} F_1^{\text{QW}}} \quad (4)$$

We apply the small-signal analysis to rate equations (1)-(3) and hence consider the injection current density in (1) in the form of

$$j = j_0 + (\delta j_m) \exp(i\omega t), \quad (5)$$

where ω is the modulation frequency, j_0 is the dc component and δj_m is the small amplitude of the time-harmonic ac component.

We correspondingly look for f_n , F_{QW} , and n_{ph} in (1)-(3) in the form of

$$F_{QW} = F_{QW,0} + (\delta F_{QW-m}) \exp(i\omega t), \quad (6)$$

$$f_n = f_{n,0} + (\delta f_{n-m}) \exp(i\omega t), \quad (7)$$

$$n_{\text{ph}} = n_{\text{ph},0} + (\delta n_{\text{ph-m}}) \exp(i\omega t), \quad (8)$$

where the dc components $f_{n,0}$, $F_{QW,0}$, and $n_{\text{ph},0}$ are the solutions of rate equations (1)-(3) at the steady-state, which are given by

$$f_{n,0} = \frac{1}{2} \left(1 + \frac{\beta}{g^{\max}} \right), \quad (9)$$

$$F_{\text{QW},0} = \frac{w_{\text{out-tunn}} F_1^{\text{QW}} N_S f_{n,0}}{w_{\text{out-tunn}} N_S (1 - f_{n,0}) + \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_{n,0}}{1 - f_{n,0}}}, \quad (10)$$

$$n_{\text{ph},0} = \tau_{\text{ph}} \frac{j_0 - j_{\text{th}}}{e}. \quad (11)$$

In (11), the threshold current density and the photon lifetime in the cavity are given by

$$j_{\text{th}} = e N_S \frac{f_{n,0}^2}{\tau_{\text{QD}}} + 2e \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_{n,0}}{1 - f_{n,0}} F_{\text{QW},0}, \quad (12)$$

$$\tau_{\text{ph}} = \frac{1}{c_g \beta}. \quad (13)$$

The first term in the right-hand side of (12) is the spontaneous radiative recombination current density in QDs. The second term is the parasitic recombination current density outside QDs (i.e., in the OCL and QWs).

Using (5)-(8) in (1)-(3), we obtain the following set of linear algebraic equations in the modulation frequency-dependent small amplitudes δf_{n-m} , $\delta F_{\text{QW}-m}$, and $\delta n_{\text{ph}-m}$:

$$\begin{pmatrix} i\omega Y_1 + D_1 & C_{12} & C_{13} \\ C_{21} & i\omega Y_2 + D_2 & 0 \\ C_{31} & 0 & i\omega \end{pmatrix} \begin{pmatrix} \delta f_{n-m} \\ \delta F_{\text{QW}-m} \\ \delta n_{\text{ph}-m} \end{pmatrix} = \begin{pmatrix} \delta j_m / e \\ 0 \\ 0 \end{pmatrix}, \quad (14)$$

where the coefficients (all independent of the modulation frequency ω) are given by

$$Y_1 = \left(\frac{b_1 n_1}{N_c^{2D}} + 1 \right) N_c^{2D} \frac{1}{(1 - f_{n,0})^2} + 2N_S, \quad (15)$$

$$Y_2 = \frac{b_1 n_1}{N_c^{2D}} + 1, \quad (16)$$

$$D_1 = \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{1}{(1 - f_{n,0})^2} F_{\text{QW},0} + 2N_S \frac{f_{n,0}}{\tau_{\text{QD}}}, \quad (17)$$

$$+ w_{\text{out-tunn}} N_S (F_1^{\text{QW}} + F_{\text{QW},0}) + 2c_g g^{\max} n_{\text{ph},0}$$

$$D_2 = \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_{n,0}}{1 - f_{n,0}} + w_{\text{out-tunn}} N_S (1 - f_{n,0}), \quad (18)$$

$$C_{12} = \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_{n,0}}{1 - f_{n,0}} - w_{\text{out-tunn}} N_S (1 - f_{n,0}), \quad (19)$$

$$C_{13} = \frac{1}{\tau_{\text{ph}}}, \quad (20)$$

$$C_{21} = \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{1}{(1 - f_{n,0})^2} F_{\text{QW},0} - w_{\text{out-tunn}} N_S (F_1^{\text{QW}} + F_{\text{QW},0}), \quad (21)$$

$$C_{31} = -2c_g g^{\max} n_{\text{ph},0}. \quad (22)$$

From the solution of (14), we find the ratio $\delta n_{\text{ph-m}}(i\omega)/\delta n_{\text{ph-m}}(0)$,

$$\frac{\delta n_{\text{ph-m}}(i\omega)}{\delta n_{\text{ph-m}}(0)} = \frac{C_{13}C_{31}D_2 + i\omega Y_2C_{13}C_{31}}{[C_{13}C_{31}D_2 + \omega^2(Y_1D_2 + Y_2D_1)] + i\omega[(Y_2C_{13}C_{31} + C_{12}C_{21} - D_1D_2) + \omega^2 Y_1Y_2]}, \quad (23)$$

and then the modulation response function $H(\omega)$,

$$H(\omega) = \left| \frac{\delta n_{\text{ph-m}}(\omega)}{\delta n_{\text{ph-m}}(0)} \right|^2 = \frac{C_{13}^2C_{31}^2D_2^2 + \omega^2 Y_2^2C_{13}^2C_{31}^2}{[C_{13}C_{31}D_{22} + \omega^2(Y_1D_2 + Y_2D_1)]^2 + \omega^2[(Y_2C_{13}C_{31} + C_{12}C_{21} - D_1D_2) + \omega^2 Y_1Y_2]^2}. \quad (24)$$

The modulation bandwidth $\omega_{-3\text{ dB}}$ is defined as the -3 dB bandwidth [25]-[27],

$$10\log_{10} H(\omega_{-3\text{ dB}}) = -3. \quad (25)$$

Using (24) in (25), we obtain the following cubic equation for the square $\omega_{-3\text{ dB}}^2$ of the bandwidth:

$$\omega_{-3\text{ dB}}^6 + E_4\omega_{-3\text{ dB}}^4 + E_2\omega_{-3\text{ dB}}^2 - E_0 = 0, \quad (26)$$

where the coefficients E are expressed in terms of the coefficients C , D , and Y as follows:

$$E_0 = \frac{1}{Y_1^2Y_2^2}(r-1)C_{13}^2C_{31}^2D_2^2, \quad (27)$$

$$E_2 = \frac{1}{Y_1^2Y_2^2}[(C_{12}C_{21} - D_1D_2)^2 - (r-1)Y_2^2C_{13}^2C_{31}^2 + 2C_{13}C_{31}(Y_2C_{12}C_{21} + Y_1D_2^2)], \quad (28)$$

$$E_4 = \frac{1}{Y_1^2Y_2^2}[Y_1^2D_2^2 + Y_2^2D_1^2 + 2Y_1Y_2(Y_2C_{13}C_{31} + C_{12}C_{21})], \quad (29)$$

where $r = 10^{0.3} \approx 1.995$.

III. Discussion

We apply the above analysis and calculate the modulation bandwidth for a GaInAsP laser structure emitting near $1.55\text{ }\mu\text{m}$ [28]-[31]. The surface density of QDs $N_S = 6.11 \times 10^{10}\text{ cm}^{-2}$. The Fabry-Perot cavity length $L = 1.139\text{ mm}$; at this cavity length and assuming as-cleaved facets (the mirror reflectivity $R = 0.32$), the mirror loss $\beta = 10\text{ cm}^{-1}$.

Figure 2 shows the modulation bandwidth $\omega_{-3\text{ dB}}$ against the out-tunneling time from a QD to the foreign QW $\tau_{\text{out-tunn}}^{\text{QD} \rightarrow \text{QW}}$ (the bottom axis). In the figure, the out-tunneling time is varied [see eq. (4)] via varying the out-tunneling coefficient $w_{\text{out-tunn}}$ (shown in the top axis). As seen from the figure, the modulation bandwidth is practically unaffected by out-tunneling from QDs – there is only a slight dip in the curve for $\omega_{-3\text{ dB}}$. In order to gain insight into this, we analyze the net out-tunneling current density, which is the difference between the current density of out-tunneling leakage from QDs to the foreign QW and the current density of backward tunneling from the foreign QW into QDs [see (2)],

$$e w_{\text{out-tunn}} F_1^{\text{QW}} N_S f_n - e w_{\text{out-tunn}} N_S (1 - f_n) F_{\text{QW}}. \quad (30)$$

At the steady-state, we have from (2),

$$e w_{\text{out-tunn}} F_1^{\text{QW}} N_S f_{n,0} - e w_{\text{out-tunn}} N_S (1 - f_{n,0}) F_{\text{QW},0} = e \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_{n,0}}{1 - f_{n,0}} F_{\text{QW},0}. \quad (31)$$

For the sum of the net out-tunneling current densities of electrons and holes, we have

$$j_{\text{out-tunn},0}^{\text{net}} = 2e \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} \frac{f_{n,0}}{1-f_{n,0}} F_{\text{QW},0}. \quad (32)$$

Naturally, $j_{\text{out-tunn},0}^{\text{net}}$ is equal to the parasitic recombination current density outside QDs [the second term in the right-hand side of (12)] since the latter is totally due to out-tunneling from QDs.

As seen from (32) and (10), $j_{\text{out-tunn},0}^{\text{net}}$ depends on $w_{\text{out-tunn}}$ via such dependence of the 2-D carrier density $F_{\text{QW},0}$ in the foreign QW. However, as seen from (10) and Fig. 3, while $F_{\text{QW},0}$ starts to increase from zero with increasing $w_{\text{out-tunn}}$, it remains saturated as $w_{\text{out-tunn}} \rightarrow \infty$, i.e., as out-tunneling becomes instantaneous ($\tau_{\text{out-tunn}}^{\text{QD} \rightarrow \text{QW}} \rightarrow \infty$). For such a case of instantaneous exchange between QDs and the foreign QW, we have from (10),

$$F_{\text{QW},0} \Big|_{w_{\text{out-tunn}} \rightarrow \infty} = F_1^{\text{QW}} \frac{f_{n,0}}{1-f_{n,0}}, \quad (33)$$

which is the same expression as eqs. (B1) and (B2) in Appendix B relating the 2-D carrier density in the injecting QW and the level occupancy in QDs for the case of instantaneous tunneling-injection.

Due to 0-D nature of QDs, the QD population remains limited – the level occupancy $f_{n,0} < 1$. Hence the highest 2-D carrier density in the foreign QW, which is given by (33), is finite even in the worst case scenario of instantaneous out-tunneling from QDs. So remains the net out-tunneling current density, which is shown in Fig. 4 (the solid curve) against the out-tunneling coefficient (the top axis) and the out-tunneling time (the bottom axis). With (32) and (33), we have for the saturation value of $j_{\text{out-tunn},0}^{\text{net}}$,

$$j_{\text{out-tunn},0}^{\text{net}} \Big|_{w_{\text{out-tunn}} \rightarrow \infty} = 2e \left[\frac{b_1 B n_1^2}{(N_c^{2D})^2} + B_{2D} \right] N_c^{2D} F_1^{\text{QW}} \left(\frac{f_{n,0}}{1-f_{n,0}} \right)^2. \quad (34)$$

With eq. (B16) of Appendix B for F_1^{QW} , we can rewrite (34) as follows:

$$j_{\text{out-tunn},0}^{\text{net}} \Big|_{w_{\text{out-tunn}} \rightarrow \infty} = 2e \left[b_1 B n_1^2 + B_{2D} (N_c^{2D})^2 \right] \left(\frac{f_{n,0}}{1-f_{n,0}} \right)^2 \exp\left(-\frac{\Delta}{T}\right). \quad (35)$$

Hence, while the out-tunneling current density from QDs to the foreign QW [the first term in (30) and the dashed line in Fig. 4] increases with increasing $w_{\text{out-tunn}}$ and becomes high, this increase is compensated for by the increase of the backward tunneling current density from the foreign QW into QDs [the second term in (30) and the dotted curve in Fig. 4] so that their difference, presenting the net out-tunneling current density (the solid curve in Fig. 4), remains saturated and low with increasing $w_{\text{out-tunn}}$.

Figure 5 also shows the modulation bandwidth against the out-tunneling time. In contrast to Fig. 2, $\tau_{\text{out-tunn}}^{\text{QD} \rightarrow \text{QW}}$ is now varied by varying the offset Δ between the energies of the subband edge in the foreign QW and the level in a QD [see eqs. (4) and (B16) and Fig. 6]. As seen from (4) and (B16), $\tau_{\text{out-tunn}}^{\text{QD} \rightarrow \text{QW}}$ decreases (i.e., out-tunneling occurs faster) exponentially with decreasing Δ . Hence, it would be intuitive that the modulation bandwidth should decrease considerably with decreasing Δ . However, as seen from Fig. 5, the decrease in $\omega_{-3 \text{ dB}}$ is actually small; $\omega_{-3 \text{ dB}}$ decreases by 18% only with reducing Δ from $4T$ (high offset between the foreign QW subband edge and the QD level) to 0 (resonance between the foreign QW subband edge and the QD level), i.e., reducing $\tau_{\text{out-tunn}}^{\text{QD} \rightarrow \text{QW}}$ by a factor of $\exp(4) \approx 55$.

IV. Conclusions

We studied the effect of out-tunneling leakage of carriers from QDs on dynamic characteristics of DTI QD lasers. We calculated the modulation bandwidth as a function of the characteristic time of out-tunneling leakage from QDs. The out-tunneling time is varied by varying the out-tunneling coefficient and the offset energy between the subband edge in the foreign QW and the level in a QD. We showed that, due to 0-D nature of QDs, the 2-D carrier densities in the foreign QWs and the net out-tunneling current from QDs to the foreign QWs remain limited with decreasing out-tunneling time, i.e., making out-tunneling faster. As a result of this, the modulation bandwidth depends only slightly on the out-tunneling time. Hence, in contrast to tunneling-injection of carriers into QDs, which strongly limits the modulation bandwidth and should be made fast enough [31] to attain high bandwidth, out-tunneling leakage has little effect on modulation bandwidth in DTI QD lasers. Provided that tunneling-injection is fast, suppressing out-tunneling leakage should not be too significant in practical DTI QD lasers.

Acknowledgments

This work was supported by the U.S. Army Research Office (Grant No. W911NF-13-1-0445).

Appendix A: General rate equations

In the general case, the operating characteristics of DTI QD lasers (see Fig. 1 for the energy band diagram) are described by the following set of rate equations:

for electrons and holes in the left-hand side of the OCL,

$$b_1 \frac{\partial n_L}{\partial t} = \frac{j}{e} + \frac{n_{QW}^L}{\tau_{n,esc}^L} - v_{n,capt}^L n_L - b_1 B n_L p_L, \quad (A1)$$

$$b_1 \frac{\partial p_L}{\partial t} = \frac{p_{QW}^L}{\tau_{p,esc}^L} - v_{p,capt}^L p_L - b_1 B n_L p_L, \quad (A2)$$

for holes and electrons in the right-hand side of the OCL,

$$b_2 \frac{\partial p_R}{\partial t} = \frac{j}{e} + \frac{p_{QW}^R}{\tau_{p,esc}^R} - v_{p,capt}^R p_R - b_2 B n_R p_R, \quad (A3)$$

$$b_2 \frac{\partial n_R}{\partial t} = \frac{n_{QW}^R}{\tau_{n,esc}^R} - v_{n,capt}^R n_R - b_2 B n_R p_R, \quad (A4)$$

for electrons and holes in the electron-injecting (left-hand-side) QW,

$$\frac{\partial n_{QW}^L}{\partial t} = v_{n,capt}^L n_L - \frac{n_{QW}^L}{\tau_{n,esc}^L} - w_{n,tunn}^L N_S (1 - f_n) n_{QW}^L + w_{n,tunn}^L n_1^{L,QW} N_S f_n - B_{2D} n_{QW}^L p_{QW}^L, \quad (A5)$$

$$\frac{\partial p_{QW}^L}{\partial t} = v_{p,capt}^L p_L - \frac{p_{QW}^L}{\tau_{p,esc}^L} - w_{p,tunn}^L N_S (1 - f_p) p_{QW}^L + w_{p,tunn}^L p_1^{L,QW} N_S f_p - B_{2D} n_{QW}^L p_{QW}^L, \quad (A6)$$

for holes and electrons in the hole-injecting (right-hand-side) QW,

$$\frac{\partial p_{QW}^R}{\partial t} = v_{p,capt}^R p_R - \frac{p_{QW}^R}{\tau_{p,esc}^R} - w_{p,tunn}^R N_S (1 - f_p) p_{QW}^R + w_{p,tunn}^R p_1^{R,QW} N_S f_p - B_{2D} n_{QW}^R p_{QW}^R, \quad (A7)$$

$$\frac{\partial n_{\text{QW}}^R}{\partial t} = v_{\text{n,capt}}^R n_R - \frac{n_{\text{QW}}^R}{\tau_{\text{n,esc}}^R} - w_{\text{n,tunn}}^R N_S (1 - f_n) n_{\text{QW}}^R + w_{\text{n,tunn}}^R n_1^{R,\text{QW}} N_S f_n - B_{2\text{D}} n_{\text{QW}}^R p_{\text{QW}}^R, \quad (\text{A8})$$

for electrons and holes confined in QDs,

$$2N_S \frac{\partial f_n}{\partial t} = w_{\text{n,tunn}}^L N_S (1 - f_n) n_{\text{QW}}^L - w_{\text{n,tunn}}^L n_1^{L,\text{QW}} N_S f_n + w_{\text{n,tunn}}^R N_S (1 - f_n) n_{\text{QW}}^R - w_{\text{n,tunn}}^R n_1^{R,\text{QW}} N_S f_n - N_S \frac{f_n f_p}{\tau_{\text{QD}}} - v_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}}, \quad (\text{A9})$$

$$2N_S \frac{\partial f_p}{\partial t} = w_{\text{p,tunn}}^R N_S (1 - f_p) p_{\text{QW}}^R - w_{\text{p,tunn}}^R p_1^{R,\text{QW}} N_S f_p + w_{\text{p,tunn}}^L N_S (1 - f_p) p_{\text{QW}}^L - w_{\text{p,tunn}}^L p_1^{L,\text{QW}} N_S f_p - N_S \frac{f_n f_p}{\tau_{\text{QD}}} - c_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}}, \quad (\text{A10})$$

and for photons,

$$\frac{\partial n_{\text{ph}}}{\partial t} = c_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}} - c_g \beta n_{\text{ph}}. \quad (\text{A11})$$

The quantities entering into eqs. (A1)–(A11), which were not introduced earlier, are the thickness b_1 (b_2) of the left- (right-) hand side of the OCL [the separation between the n - (p -) cladding layer and the left- (right-) hand-side barrier – see Fig. 1], the free-electron and -hole densities n_L (n_R) and p_L (p_R) in the left- (right-) hand side of the OCL, the 2-D electron and hole densities n_{QW}^L (n_{QW}^R) and p_{QW}^L (p_{QW}^R) in the left- (right-) hand-side QW (Fig. 1), and the electron- and hole-level occupancies $f_{\text{n,p}}$ in QDs.

The thermal escape times of electrons and holes from the QWs to the OCL, $\tau_{\text{n,p,esc}}^{L,R}$, and the capture velocities from the OCL to the QWs, $v_{\text{n,p,capt}}^{L,R}$, are related to each other [23]. For undoped OCL and QW, the relation is

$$\tau_{\text{n,esc}} = \frac{1}{v_{\text{n,capt}}} \frac{N_{\text{c}}^{2\text{D}}}{n_1}, \quad \tau_{\text{p,esc}} = \frac{1}{v_{\text{p,capt}}} \frac{N_{\text{v}}^{2\text{D}}}{p_1}, \quad (\text{A12})$$

where

$$N_{\text{c,v}}^{2\text{D}} = \frac{m_{\text{c,v}}^{\text{QW}} T}{\pi \hbar^2} \quad (\text{A13})$$

are the 2-D effective densities of states in the conduction and valence bands in the QWs, $m_{\text{c,v}}^{\text{QW}}$ are the electron and hole effective masses in the QWs, and the temperature T is measured in units of energy.

The quantities n_1 and p_1 are

$$n_1 = N_{\text{c}}^{3\text{D}} \exp\left(-\frac{\Delta E_{\text{c}} - \varepsilon_{\text{n}}^{\text{QW}}}{T}\right), \quad p_1 = N_{\text{v}}^{3\text{D}} \exp\left(-\frac{\Delta E_{\text{v}} - \varepsilon_{\text{p}}^{\text{QW}}}{T}\right), \quad (\text{A14})$$

where

$$N_{\text{c,v}}^{3\text{D}} = 2 \left(\frac{m_{\text{c,v}}^{\text{OCL}} T}{2\pi \hbar^2} \right)^{3/2} \quad (\text{A15})$$

are the three-dimensional (3-D) effective densities of states in the conduction and valence bands in the OCL, $m_{c,v}^{\text{OCL}}$ are the electron and hole effective masses in the OCL, $\Delta E_{c,v}$ are the conduction and valence band offsets between the OCL and the QW, and $\varepsilon_{n,p}^{\text{QW}}$ are the energies of the electron- and hole-subband edges in the QW.

The coefficients $w_{n,p,\text{tunn}}^{L,R}$ (measured in units of cm^2/s) describe the electron and hole tunneling between the QD ensemble and the QWs.

The quantities $n_1^{L,R,\text{QW}}$ and $p_1^{L,R,\text{QW}}$ entering into the electron and hole tunneling fluxes from the QD ensemble to the QWs [see (A5)–(A10)] are measured in units of cm^{-2} . In the general case, they are given by expression (B3) of [23]. In the case of undoped QW and if $\varepsilon_n^{\text{QW}} \geq \varepsilon_n^{\text{QD}}$ and $\varepsilon_p^{\text{QW}} \geq \varepsilon_p^{\text{QD}}$, where $\varepsilon_{n,p}^{\text{QD}}$ are energies of the electron and hole levels in a QD, the expressions for n_1^{QW} and p_1^{QW} are

$$n_1^{\text{QW}} = N_c^{2D} \exp\left(-\frac{\varepsilon_n^{\text{QW}} - \varepsilon_n^{\text{QD}}}{T}\right), \quad p_1^{\text{QW}} = N_v^{2D} \exp\left(-\frac{\varepsilon_p^{\text{QW}} - \varepsilon_p^{\text{QD}}}{T}\right). \quad (\text{A16})$$

The flux of electron (and similarly hole) tunneling from a QD ensemble to a QW can be written as

$$w_{n,\text{tunn}} n_1^{\text{QW}} N_S f_n = N_S \frac{f_n}{\tau_{n,\text{tunn}}^{\text{QD} \rightarrow \text{QW}}}, \quad (\text{A17})$$

where

$$\tau_{n,\text{tunn}}^{\text{QD} \rightarrow \text{QW}} = \frac{1}{w_{n,\text{tunn}} n_1^{\text{QW}}} \quad (\text{A18})$$

can be viewed as the tunneling time from a QD to a QW.

The flux of electron tunneling from a QW to a QD ensemble can be written as

$$w_{n,\text{tunn}} N_S (1 - f_n) n_{\text{QW}} = \frac{n_{\text{QW}}}{\tau_{n,\text{tunn}}^{\text{QW} \rightarrow \text{QDs}}}, \quad (\text{A19})$$

where

$$\tau_{n,\text{tunn}}^{\text{QW} \rightarrow \text{QDs}} = \frac{\tau_{n,\text{tunn},0}^{\text{QW} \rightarrow \text{QDs}}}{1 - f_n} \quad (\text{A20})$$

can be considered as the tunneling time from a QW to a QD ensemble, and

$$\tau_{n,\text{tunn},0}^{\text{QW} \rightarrow \text{QDs}} = \frac{1}{w_{n,\text{tunn}} N_S} \quad (\text{A21})$$

can be correspondingly considered as the tunneling time into an unoccupied QD ensemble (when $f_n = 0$).

Adding up eqs. (A1), (A5), and (A9), we obtain

$$\begin{aligned} \frac{\partial}{\partial t} (b_1 n_L + n_{\text{QW}}^L + 2N_S f_n) = & \frac{j}{e} - b_1 B n_L p_L - B_{2D} n_{\text{QW}}^L p_{\text{QW}}^L - N_S \frac{f_n f_p}{\tau_{\text{QD}}} - v_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}} \\ & - [w_{n,\text{tunn}}^R n_1^{R,\text{QW}} N_S f_n - w_{n,\text{tunn}}^R N_S (1 - f_n) n_{\text{QW}}^R] \end{aligned} \quad (\text{A22})$$

Adding up eqs. (A2) and (A6), we obtain

$$\frac{\partial}{\partial t} (b_1 p_L + p_{\text{QW}}^L) = [w_{p,\text{tunn}}^L p_1^{L,\text{QW}} N_S f_p - w_{p,\text{tunn}}^L N_S (1 - f_p) p_{\text{QW}}^L] - b_1 B n_L p_L - B_{2D} n_{\text{QW}}^L p_{\text{QW}}^L. \quad (\text{A23})$$

Adding up eqs. (A3), (A7), and (A10), we obtain

$$\frac{\partial}{\partial t}(b_2 p_R + p_{QW}^R + 2N_S f_p) = \frac{j}{e} - b_2 B n_R p_R - B_{2D} n_{QW}^R p_{QW}^R - N_S \frac{f_n f_p}{\tau_{QD}} - v_g g^{\max}(f_n + f_p - 1)n_{ph} - [w_{p, \text{tunn}}^L p_1^{L, QW} N_S f_p - w_{p, \text{tunn}}^L N_S (1 - f_p) p_{QW}^L] \quad (\text{A24})$$

Adding up eqs. (A4) and (A8), we obtain

$$\frac{\partial}{\partial t}(b_2 n_R + n_{QW}^R) = [w_{n, \text{tunn}}^R n_1^{R, QW} N_S f_n - w_{n, \text{tunn}}^R N_S (1 - f_n) n_{QW}^R] - b_2 B n_R p_R - B_{2D} n_{QW}^R p_{QW}^R \quad (\text{A25})$$

Adding up eqs. (A1), (A4), (A5), (A8), and (A9), we obtain

$$\frac{\partial}{\partial t}(b_1 n_L + n_{QW}^L + 2N_S f_n + b_2 n_R + n_{QW}^R) = \frac{j}{e} - b_1 B n_L p_L - B_{2D} n_{QW}^L p_{QW}^L - b_2 B n_R p_R - B_{2D} n_{QW}^R p_{QW}^R - N_S \frac{f_n f_p}{\tau_{QD}} - c_g g^{\max}(f_n + f_p - 1)n_{ph} \quad (\text{A26})$$

Adding up eqs. (A2), (A3), (A6), (A7), and (A10), we obtain

$$\frac{\partial}{\partial t}(b_2 p_R + p_{QW}^R + 2N_S f_p + b_1 p_L + p_{QW}^L) = \frac{j}{e} - b_1 B n_L p_L - B_{2D} n_{QW}^L p_{QW}^L - b_2 B n_R p_R - B_{2D} n_{QW}^R p_{QW}^R - N_S \frac{f_n f_p}{\tau_{QD}} - c_g g^{\max}(f_n + f_p - 1)n_{ph} \quad (\text{A27})$$

As seen from (A26) and (A27),

$$\frac{\partial}{\partial t}(b_1 n_L + n_{QW}^L + 2N_S f_n + b_2 n_R + n_{QW}^R) = \frac{\partial}{\partial t}(b_2 p_R + p_{QW}^R + 2N_S f_p + b_1 p_L + p_{QW}^L), \quad (\text{A28})$$

which gives

$$b_1 n_L + n_{QW}^L + 2N_S f_n + b_2 n_R + n_{QW}^R = b_2 p_R + p_{QW}^R + 2N_S f_p + b_1 p_L + p_{QW}^L. \quad (\text{A29})$$

Eq. (A29) is the condition of global charge neutrality in the structure.

Appendix B: Derivation of rate equations (1)-(3)

In this work, to focus on the effect of out-tunneling leakage from QDs, we make the following assumptions:

1) Carrier exchange between the OCL and QW on each side of the structure (processes ② and ⑦ in Fig. 1) is instantaneous ($v_{n,p, \text{capt}}^{L,R} \rightarrow \infty$ and hence $\tau_{n,p, \text{esc}}^{L,R} \rightarrow 0$).

2) Tunneling-injection into QDs (processes ③ in Fig. 1) is instantaneous ($w_{n, \text{tunn}}^L \rightarrow \infty$ and hence $\tau_{n, \text{tunn}, 0}^{L, QW \rightarrow \text{QDs}} \rightarrow 0$; similarly, $w_{p, \text{tunn}}^R \rightarrow \infty$ and hence $\tau_{p, \text{tunn}, 0}^{R, QW \rightarrow \text{QDs}} \rightarrow 0$).

Out-tunneling leakage from QDs (processes ⑤ in Fig. 1) is characterized by the coefficients $w_{n, \text{tunn}}^R$ and $w_{p, \text{tunn}}^L$, or, equivalently, the out-tunneling times $\tau_{n, \text{tunn}}^{\text{QD} \rightarrow R, QW}$ and $\tau_{p, \text{tunn}}^{\text{QD} \rightarrow L, QW}$ to the “foreign” QWs, i.e., the tunneling times of electrons from a QD to the hole-injecting QW [see eq. (A18)] and holes from a QD to the electron-injecting QW. We will calculate the modulation bandwidth as a function of these parameters.

With the assumption of instantaneous tunneling-injection into QDs, we have from (A5) and (A7),

$$n_{QW}^L = n_1^{L, QW} \frac{f_n}{1 - f_n} = N_c^{2D} \frac{f_n}{1 - f_n}, \quad (\text{B1})$$

$$p_{\text{QW}}^R = p_1^{R, \text{QW}} \frac{f_p}{1-f_p} = N_v^{2D} \frac{f_p}{1-f_p}. \quad (\text{B2})$$

In (B1) and (B2), eqs. (A16) were used assuming $\varepsilon_n^{L, \text{QW}} = \varepsilon_n^{\text{QD}}$ and $\varepsilon_p^{R, \text{QW}} = \varepsilon_p^{\text{QD}}$.

With the assumption of instantaneous carrier exchange between the OCL and QW on each side of the structure and using (A12), we have from (A1)-(A4),

$$n_L = \frac{n_1}{N_c^{2D}} n_{\text{QW}}^L = n_1 \frac{f_n}{1-f_n}, \quad (\text{B3})$$

$$p_L = \frac{p_1}{N_v^{2D}} p_{\text{QW}}^L, \quad (\text{B4})$$

$$p_R = \frac{p_1}{N_v^{2D}} p_{\text{QW}}^R = p_1 \frac{f_p}{1-f_p}, \quad (\text{B5})$$

$$n_R = \frac{n_1}{N_c^{2D}} n_{\text{QW}}^R. \quad (\text{B6})$$

We used eq. (B1) in (B3) and eq. (B2) in (B5).

Using (B1)-(B6) in (A22), (A23), (A25), and (A29), we obtain

$$\frac{\partial}{\partial t} \left[\left(\frac{b_1 n_1}{N_c^{2D}} + 1 \right) N_c^{2D} \frac{f_n}{1-f_n} + 2N_s f_n \right] = \frac{j}{e} - \left(\frac{b_1 B n_1 p_1}{N_c^{2D} N_v^{2D}} + B_{2D} \right) N_c^{2D} \frac{f_n}{1-f_n} p_{\text{QW}}^L \quad (\text{B7})$$

$$- N_s \frac{f_n f_p}{\tau_{\text{QD}}} - c_g g^{\text{max}} (f_n + f_p - 1) n_{\text{ph}} - [w_{n, \text{tunn}}^R n_1^{R, \text{QW}} N_s f_n - w_{n, \text{tunn}}^R N_s (1-f_n) n_{\text{QW}}^R]$$

$$\frac{\partial}{\partial t} \left[\left(\frac{b_1 p_1}{N_v^{2D}} + 1 \right) p_{\text{QW}}^L \right] = [w_{p, \text{tunn}}^L p_1^{L, \text{QW}} N_s f_p - w_{p, \text{tunn}}^L N_s (1-f_p) p_{\text{QW}}^L] \quad (\text{B8})$$

$$- \left(\frac{b_1 B n_1 p_1}{N_c^{2D} N_v^{2D}} + B_{2D} \right) N_c^{2D} \frac{f_n}{1-f_n} p_{\text{QW}}^L$$

$$\frac{\partial}{\partial t} \left[\left(\frac{b_2 n_1}{N_c^{2D}} + 1 \right) n_{\text{QW}}^R \right] = [w_{n, \text{tunn}}^R n_1^{R, \text{QW}} N_s f_n - w_{n, \text{tunn}}^R N_s (1-f_n) n_{\text{QW}}^R] \quad (\text{B9})$$

$$- \left(\frac{b_2 B n_1 p_1}{N_c^{2D} N_v^{2D}} + B_{2D} \right) N_v^{2D} \frac{f_p}{1-f_p} n_{\text{QW}}^R$$

$$\left(\frac{b_1 n_1}{N_c^{2D}} + 1 \right) N_c^{2D} \frac{f_n}{1-f_n} + 2N_s f_n + \left(\frac{b_2 n_1}{N_c^{2D}} + 1 \right) n_{\text{QW}}^R = \left(\frac{b_2 p_1}{N_v^{2D}} + 1 \right) N_v^{2D} \frac{f_p}{1-f_p} + 2N_s f_p + \left(\frac{b_1 p_1}{N_v^{2D}} + 1 \right) p_{\text{QW}}^L. \quad (\text{B10})$$

Using eq. (B10), we can express n_{QW}^R in terms of p_{QW}^L, f_p , and f_n as follows:

$$n_{\text{QW}}^R = \frac{1}{\frac{b_2 n_1}{N_c^{2D}} + 1} \left\{ \left(\frac{b_1 p_1}{N_v^{2D}} + 1 \right) p_{\text{QW}}^L + \left[\left(\frac{b_2 p_1}{N_v^{2D}} + 1 \right) N_v^{2D} \frac{f_p}{1-f_p} + 2N_s f_p \right] - \left[\left(\frac{b_1 n_1}{N_c^{2D}} + 1 \right) N_c^{2D} \frac{f_n}{1-f_n} + 2N_s f_n \right] \right\}. \quad (\text{B11})$$

Eqs. (B7)-(B9) [wherein n_{QW}^R is given by (B11)] and eq. (A11) present four equations for finding four unknowns $p_{\text{QW}}^L, f_p, f_n$, and n_{ph} .

We can still further simplify our consideration assuming electron-hole symmetry. In such a case, in particular,

$$w_{n, \text{tunn}}^R = w_{p, \text{tunn}}^L, \quad (\text{B12})$$

$$n_1^{R, \text{QW}} = p_1^{L, \text{QW}}, \quad (\text{B13})$$

$$f_n = f_p, \quad (\text{B14})$$

$$n_{\text{QW}}^R = p_{\text{QW}}^L. \quad (\text{B15})$$

Let us denote the out-tunneling coefficients $w_{n, \text{tunn}}^R = w_{p, \text{tunn}}^L$ by $w_{\text{out-tunn}}$, the minority carrier densities $n_{\text{QW}}^R = p_{\text{QW}}^L$ in the foreign QWs by F_{QW} , and the parameters $n_1^{R, \text{QW}} = p_1^{L, \text{QW}}$ by F_1^{QW} . The parameter F_1^{QW} can be written as [see (A16)]

$$F_1^{\text{QW}} = N_c^{2\text{D}} \exp\left(-\frac{\Delta}{T}\right), \quad (\text{B16})$$

where $\Delta = \varepsilon_n^{\text{QW}} - \varepsilon_n^{\text{QD}}$ is the offset energy between the subband edge in the foreign QW and the level in a QD (Fig. 6).

Equations (B8) and (B9) will now coincide and we will have from (B7), (B8), and (A11) the set (1)-(3) of three rate equations for finding f_n, F_{QW} , and n_{ph} .

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Figure captions

Fig. 1. Energy band diagram of a DTI QD laser and the main processes: ① injection from the cladding layers to the OCL, ② majority carrier capture from the OCL into the QW and thermal escape from the QW to the OCL, ③ tunneling-injection from the QW into a QD, ④ spontaneous and stimulated recombination in a QD, ⑤ out-tunneling from a QD into the ‘foreign’ QW, ⑥ spontaneous recombination in the QWs, ⑦ minority carrier thermal escape from the QW to the OCL and capture from the OCL into the QW, and ⑧ spontaneous recombination in the OCL. Reproduced from Fig. 1 of Ref. [22], with the permission of AIP Publishing.

Fig. 2. Modulation bandwidth $\omega_{-3\text{ dB}}/2\pi$ vs. out-tunneling time (the bottom axis) and out-tunneling coefficient (the top axis). A GaInAsP heterostructure emitting near $1.55\text{ }\mu\text{m}$ at $T = 300\text{ K}$ is considered [28]-[31]. In Figs. 2-5, the cavity length $L = 1.139\text{ mm}$ (the mirror loss $\beta = 10\text{ cm}^{-1}$) and the surface density of QDs $N_s = 6.11 \times 10^{10}\text{ cm}^{-2}$. In Figs. 2-4, the offset energy between the carrier subband edge in the foreign QW and the level in a QD is $\Delta = T$.

Fig. 3. 2-D carrier density in the foreign QW vs. out-tunneling time (the bottom axis) and out-tunneling coefficient (the top axis).

Fig. 4. Out-tunneling current density from QDs to the foreign QWs (the dashed line), current density of backward tunneling from the foreign QWs to QDs (the dotted curve), and net out-tunneling current density (the solid curve) vs. out-tunneling time (the bottom axis) and out-tunneling coefficient (the top axis).

Fig. 5. Modulation bandwidth $\omega_{-3\text{ dB}}/2\pi$ vs. out-tunneling time (the bottom axis) and normalized offset energy between the carrier subband edge in the foreign QW and the level in a QD (the top axis). The out-tunneling coefficient is $w_{\text{out-tunn}} = 6.14\text{ cm}^2/\text{s}$.

Fig. 6. Conduction band diagram in the right-hand side of the structure. Δ is the offset energy between the electron subband edge in the foreign (hole-injecting) QW and the electron level in a QD.

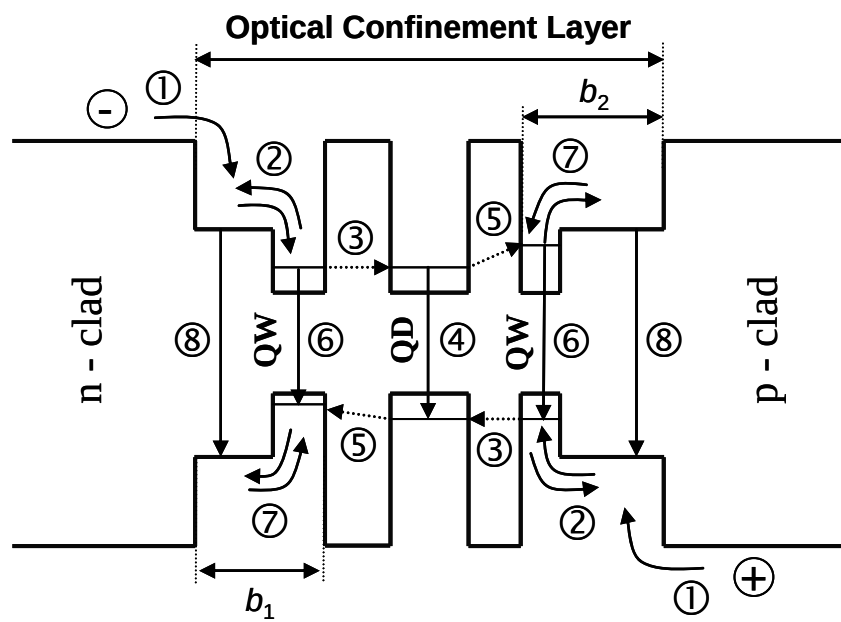


Fig. 1

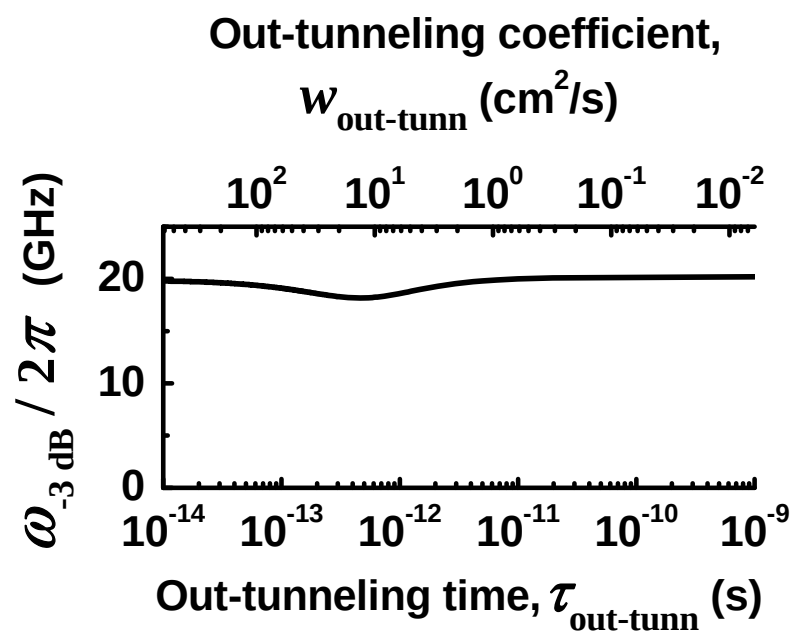


Fig. 2

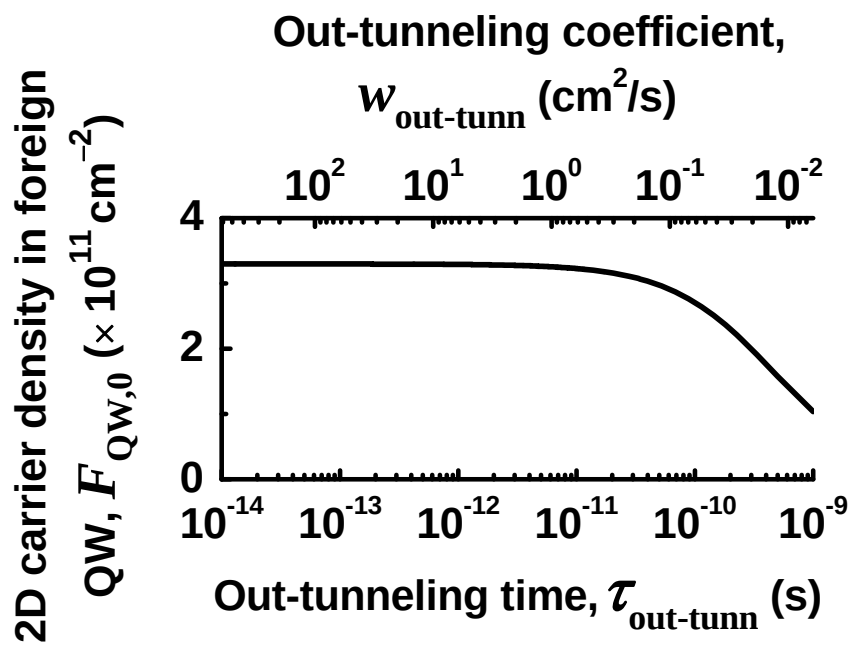


Fig. 3

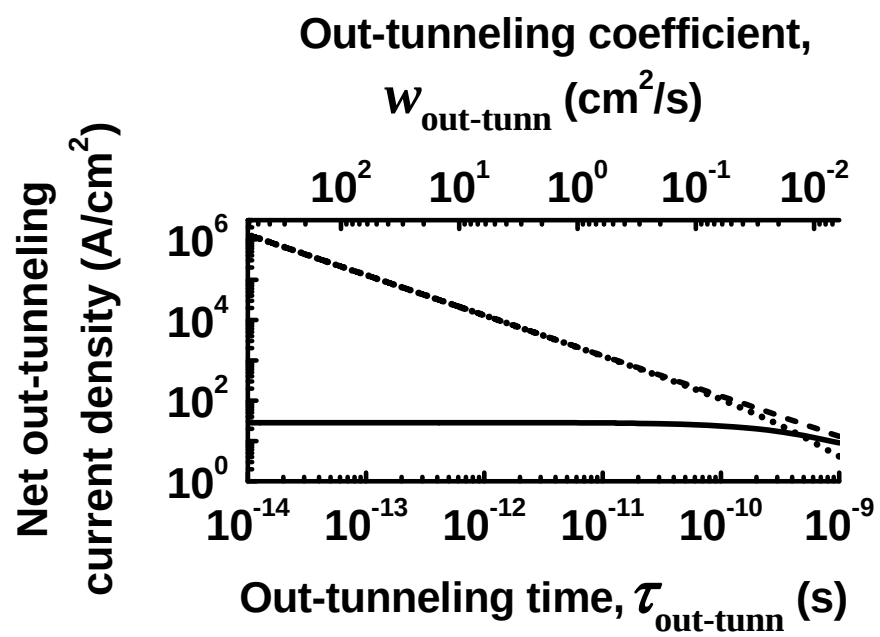


Fig. 4

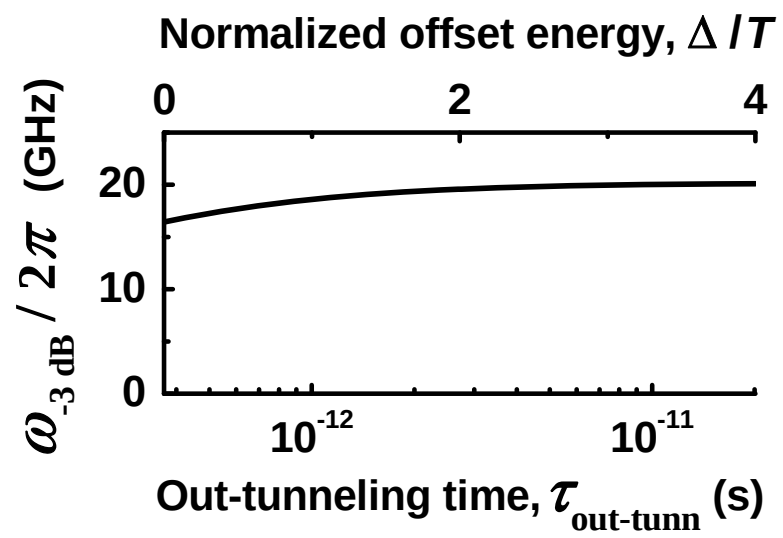


Fig. 5

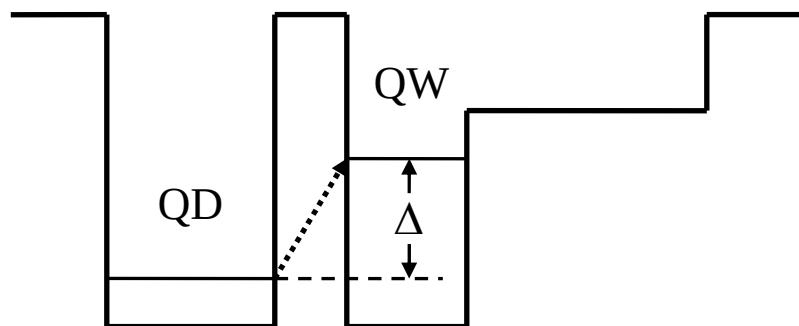


Fig. 6

Direct and indirect capture of carriers into the lasing ground state and the light-current characteristic of quantum dot lasers

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(Received 18 January 2014; accepted 3 March 2014; published online 13 March 2014)

We calculate the light-current characteristic (LCC) of a quantum dot (QD) laser under the conditions of both direct and indirect capture of carriers from the optical confinement layer into the lasing ground state in QDs. We show that direct capture is a dominant process determining the ground-state LCC. Only when direct capture is slow, the role of indirect capture (capture into the QD excited state and subsequent intradot relaxation to the ground state) becomes important. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4868472>]

I. INTRODUCTION

Excited states in quantum dots (QDs) strongly affect the operation of QD lasers.^{1–18} In Refs. 19–21, the light-current characteristic (LCC) of semiconductor lasers (the optical power as a function of the pump current) was calculated considering only direct capture of carriers from the optical confinement layer (OCL) into the lasing state in a quantum-confined active region. In Ref. 9, the LCC of a QD laser was studied assuming only an indirect (excited-state-mediated) mechanism of capture into the QD ground state. In this work, we calculate the LCC of a QD laser under realistic conditions of both direct and indirect capture into the lasing ground state in QDs. We focus here on the effects of carrier exchange between a bulk OCL and zero-dimensional QDs and intradot relaxation. We show that, while indirect capture (capture into the QD excited state and subsequent intradot relaxation to the ground state) plays a part in determining the ground-state LCC, the role of direct capture is dominant.

II. RATE EQUATIONS MODEL

We use the following set of steady-state rate equations: for free carriers in the OCL ($\partial n_{\text{OCL}}/\partial t = 0$),

$$\begin{aligned} \frac{j}{eb} - \sigma_{n2} v_n \frac{N_S}{b} (1 - f_{n2}) n_{\text{OCL}} + \sigma_{n2} v_n n_2 \frac{N_S}{b} f_{n2} \\ - \sigma_{n1} v_n \frac{N_S}{b} (1 - f_{n1}) n_{\text{OCL}} + \sigma_{n1} v_n n_1 \frac{N_S}{b} f_{n1} - B n_{\text{OCL}}^2 = 0, \end{aligned} \quad (1)$$

for carriers confined in the QD excited state [$2(N_S/b) \partial f_{n2}/\partial t = 0$],

$$\begin{aligned} \sigma_{n2} v_n \frac{N_S}{b} (1 - f_{n2}) n_{\text{OCL}} - \sigma_{n2} v_n n_2 \frac{N_S}{b} f_{n2} \\ + \frac{N_S}{b} \frac{f_{n1}(1 - f_{n2})}{\tau_{12}} - \frac{N_S}{b} \frac{f_{n2}(1 - f_{n1})}{\tau_{21}} - \frac{N_S}{b} \frac{f_{n2}^2}{\tau_{\text{QD2}}} = 0, \end{aligned} \quad (2)$$

for carriers confined in the QD ground state [$2(N_S/b) \partial f_{n1}/\partial t = 0$],

$$\begin{aligned} \sigma_{n1} v_n \frac{N_S}{b} (1 - f_{n1}) n_{\text{OCL}} - \sigma_{n1} v_n n_1 \frac{N_S}{b} f_{n1} + \frac{N_S}{b} \frac{f_{n2}(1 - f_{n1})}{\tau_{21}} \\ - \frac{N_S}{b} \frac{f_{n1}(1 - f_{n2})}{\tau_{12}} - \frac{N_S}{b} \frac{f_{n1}^2}{\tau_{\text{QD1}}} - v_g g_1^{\text{max}} (2f_{n1} - 1) \frac{N_1}{Sb} = 0, \end{aligned} \quad (3)$$

and for photons ($\partial N_1/\partial t = 0$),

$$v_g g_1^{\text{max}} (2f_{n1} - 1) N_1 - v_g \beta N_1 = 0, \quad (4)$$

where n_{OCL} is the free-carrier density in the OCL, f_{n1} and f_{n2} are the occupancies of the ground and excited state in a QD, and N_1 is the number of photons in the ground-state lasing mode.

The parameters in Eqs. (1)–(4) are as follows: j is the injection current density, e is the electron charge, b is the OCL thickness, v_n is the free-carrier thermal velocity in the OCL, N_S is the surface density of QDs, B is the spontaneous radiative recombination constant for the OCL, τ_{QD1} and τ_{QD2} are the spontaneous radiative recombination lifetimes via the QD ground and excited state, $S = WL$ is the cross section of the junction, W is the lateral size of the device, and L is the cavity length.

To adequately describe the carrier exchange between the OCL and QDs, we use the cross-sections σ_{n1} and σ_{n2} of carrier capture from the OCL into the QD ground and excited state.

The quantities n_1 and n_2 entering into the rates of thermal escape of carriers from the QD ground and excited state to the OCL [see Eqs. (1)–(3)] are given by $n_1 = N_c^{3D} \exp(-E_{n1}/T)$ and $n_2 = N_c^{3D} \exp(-E_{n2}/T)$, where E_{n1} and E_{n2} are the energies of carrier excitation from the QD ground and excited state to the OCL (Fig. 1), and T is the temperature (in units of energy).

In Eqs. (2) and (3), τ_{12} is the upward (ground-to-excited state) transition time and τ_{21} is the excited-to-ground state relaxation time in a QD. The detailed balance condition yields the following relationship between these intradot times:

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$$\tau_{12} = \tau_{21} \exp\left(\frac{E_{n1} - E_{n2}}{T}\right). \quad (5)$$

In Eqs. (3) and (4), v_g is the group velocity of light, $g_1^{\max}$ is the maximum modal gain of ground-state lasing,²² $\beta = (1/L)\ln(1/R)$ is the mirror loss, and R is the facet reflectivity.

We use quadratic (in free-carrier density n_{OCL} and confined-carrier level occupancies f_{n1} and f_{n2}) rates [the last terms in the left-hand sides of Eqs. (1) and (2) and the fifth term in the left-hand side of Eq. (3)] for spontaneous radiative recombination in the OCL and QDs, since this process is bimolecular.²²

We focus here on the effects of carrier capture into QDs and intradot relaxation on the lower- (ground-) state lasing, and, for this reason, we do not consider the stimulated emission via the upper (excited) state in QDs. To rule out the stimulated emission via the excited state in QDs, we set the maximum modal gain via excited-state transitions lower than the mirror loss β . Correspondingly, our model does not incorporate the onset of excited-state lasing with increasing pump current, simultaneous two (or even more) state lasing, or quenching of ground-state lasing—see, e.g., Refs. 1–8 and Refs. 10–18 for discussion of these and other interesting phenomena occurring due to the presence of excited states in QDs.

Our model does not also include the electron-hole asymmetry. Consideration of the electron-hole asymmetry would involve at least seven rate equations instead of four equations (1)–(4).

Solving the set of rate equations (1)–(4) and finding the number of photons N_1 in the lasing mode will give us the output optical power of the laser,

$$P = \hbar\omega v_g \beta N_1 = \hbar\omega \frac{N_1}{\tau_{\text{ph}}}, \quad (6)$$

where $\hbar\omega$ is the energy of photons emitted via the ground-state transitions and we introduced the photon lifetime in the cavity,

$$\tau_{\text{ph}} = \frac{1}{v_g \beta}. \quad (7)$$

III. SOLUTIONS OF RATE EQUATIONS

As seen from Eq. (4), for the number of photons to be nonzero ($N_1 \neq 0$), the condition of equality of the gain to the loss should hold

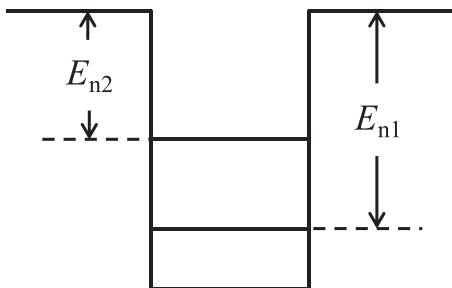


FIG. 1. Carrier excitation energies E_{n1} and E_{n2} from the QD ground and excited state to the OCL. For the QD laser structures considered in this paper, $E_{n1} = 49$ meV and $E_{n2} = 23$ meV.

$$g_1^{\max}(2f_{n1} - 1) = \beta, \quad (8)$$

which immediately yields the following expression for the ground-state level occupancy in QDs:

$$f_{n1} = \frac{1}{2} \left(1 + \frac{\beta}{g_1^{\max}} \right). \quad (9)$$

As seen from Eq. (9), f_{n1} does not depend on j , i.e., is pinned at its threshold value. In contrast to f_{n1} , as shown below [Figs. 2(a) and 2(b)], the excited-state level occupancy

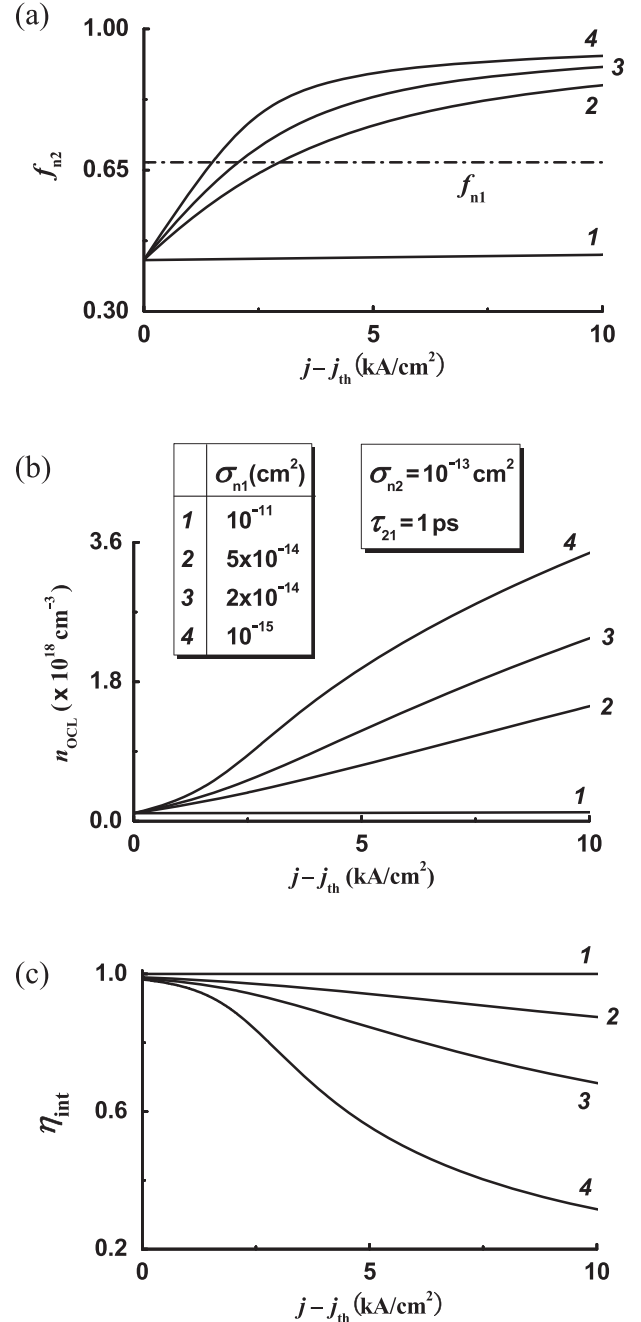


FIG. 2. Excited-state level occupancy in QDs (a), free-carrier density in the OCL (b), and internal differential quantum efficiency (c) vs. excess of the injection current density over the threshold current density for different values of the capture cross-section σ_{n1} into the QD ground state. The horizontal dashed-dotted line in (a) shows the ground-state level occupancy. The values of parameters σ_{n1} , σ_{n2} , and τ_{21} are indicated in the inset to (b).

in QDs, f_{n2} , and the free-carrier density in the OCL, n_{OCL} , increase with j above the lasing threshold.

We use the following approach to solving the rate equations: we express n_{OCL} and the number of photons N_1 in terms of f_{n2} , and then obtain a single equation for finding f_{n2} [Eq. (13) below].

From Eq. (2), the free-carrier density in the OCL can be written as follows in terms of f_{n2} :

$$n_{\text{OCL}}(f_{n2}) = n_2 \frac{f_{n2}}{1 - f_{n2}} + \frac{1}{\sigma_{n2} v_n} \frac{1}{1 - f_{n2}} \times \left[\frac{f_{n2}(1 - f_{n1})}{\tau_{21}} - \frac{f_{n1}(1 - f_{n2})}{\tau_{12}} + \frac{f_{n2}^2}{\tau_{\text{QD2}}} \right]. \quad (10)$$

From Eq. (3), we have for the number of photons

$$N_1(f_{n2}) = \left[\tau_{\text{ph}} N_S S \sigma_{n1} v_n (1 - f_{n1}) n_{\text{OCL}}(f_{n2}) - \sigma_{n1} v_n n_1 f_{n1} + \frac{f_{n2}(1 - f_{n1})}{\tau_{21}} - \frac{f_{n1}(1 - f_{n2})}{\tau_{12}} - \frac{f_{n1}^2}{\tau_{\text{QD1}}} \right]. \quad (11)$$

An expression for N_1 equivalent to Eq. (11) can also be obtained by adding Eqs. (2) and (3)

$$N_1(f_{n2}) = \tau_{\text{ph}} N_S S \left\{ [\sigma_{n1} v_n (1 - f_{n1}) n_{\text{OCL}}(f_{n2}) - \sigma_{n1} v_n n_1 f_{n1}] + [\sigma_{n2} v_n (1 - f_{n2}) n_{\text{OCL}}(f_{n2}) - \sigma_{n2} v_n n_2 f_{n2}] - \frac{f_{n1}^2}{\tau_{\text{QD1}}} - \frac{f_{n2}^2}{\tau_{\text{QD2}}} \right\}. \quad (12)$$

Adding equations (1), (2), and (3), we can naturally express the injection current density as a sum of the current densities of spontaneous recombination in the OCL and in QDs (the first three terms in the right-hand side below) and stimulated recombination in QDs (the last term)

$$j = ebB n_{\text{OCL}}^2(f_{n2}) + eN_S \frac{f_{n1}^2}{\tau_{\text{QD1}}} + eN_S \frac{f_{n2}^2}{\tau_{\text{QD2}}} + \frac{e}{S} \frac{N_1(f_{n2})}{\tau_{\text{ph}}}. \quad (13)$$

Equation (13) [wherein $n_{\text{OCL}}(f_{n2})$ and $N_1(f_{n2})$ are given by Eqs. (10) and (11), respectively] presents the injection current density j as an explicit function of the excited-state level occupancy in QDs f_{n2} . We need to find the inverse function, i.e., f_{n2} versus j . In terms of f_{n2} , Eq. (13) can be presented as a quartic equation. Solving this equation, we obtain $f_{n2}(j)$; then, using Eqs. (10) and (11), we calculate $n_{\text{OCL}}(j)$ and $N_1(j)$, and finally $P(j)$ [see Eq. (6)].

Equation (13) can also be written as

$$j = j_{\text{th}} + \Delta j_{\text{spon}}(j) + j_{\text{stim}}(j), \quad (14)$$

where

$$j_{\text{th}} = eN_S \frac{f_{n1}^2}{\tau_{\text{QD1}}} + eN_S \frac{f_{n2,\text{th}}^2}{\tau_{\text{QD2}}} + ebB n_{\text{OCL,th}}^2 \quad (15)$$

is the threshold current density (see Appendix for the expressions for the free-carrier density in the OCL $n_{\text{OCL,th}}$ and the

excited-state level occupancy in QDs $f_{n2,\text{th}}$ at the lasing threshold),

$$\Delta j_{\text{spon}}(j) = \frac{eN_S}{\tau_{\text{QD2}}} [f_{n2}^2(j) - f_{n2,\text{th}}^2] + ebB [n_{\text{OCL}}^2(j) - n_{\text{OCL,th}}^2] \quad (16)$$

is the raise in the spontaneous radiative recombination current density above the lasing threshold, and

$$j_{\text{stim}}(j) = \frac{e}{S} \frac{N_1(j)}{\tau_{\text{ph}}} \quad (17)$$

is the stimulated recombination current density.

For the internal differential quantum efficiency (efficiency of stimulated recombination),²³ we have

$$\eta_{\text{int}}(j) = \frac{j_{\text{stim}}(j)}{j - j_{\text{th}}} = 1 - \frac{\Delta j_{\text{spon}}(j)}{j - j_{\text{th}}} = 1 - \frac{\frac{eN_S}{\tau_{\text{QD2}}} [f_{n2}^2(j) - f_{n2,\text{th}}^2] + ebB [n_{\text{OCL}}^2(j) - n_{\text{OCL,th}}^2]}{j - j_{\text{th}}}. \quad (18)$$

IV. DISCUSSION

For our calculations, we use a GaInAsP heterostructure operating at room-temperature.^{24,25} The lasing wavelength is near 1.55 μm . The maximum gain for the ground-state transitions $g_1^{\text{max}} = 29.52 \text{ cm}^{-1}$, which corresponds to 10% QD-size fluctuations and surface density of QDs $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$. We assume the mirror loss $\beta = 10 \text{ cm}^{-1}$ (for as-cleaved facet reflectivity $R = 0.32$, this corresponds to the cavity length $L = 1.139 \text{ mm}$). The OCL thickness $b = 0.28 \mu\text{m}$ and the lateral size $W = 2 \mu\text{m}$.

Figures 2(a) and 2(b) show the excited-state level occupancy in QDs and the free-carrier density in the OCL against excess of the injection current density over the threshold current density. As seen from the figures, carriers accumulate in non-lasing states in the OCL and QDs above the lasing threshold—both f_{n2} and n_{OCL} raise with j . The accumulation is due to delay in carrier delivery (capture and intradot relaxation) from these non-lasing states into the lasing state in QDs. When the capture into the QD ground state is very fast (curve 1 in the figures), f_{n2} and n_{OCL} are practically constant.

Figure 2(c) shows the internal differential quantum efficiency against excess injection current density for different values of the capture cross-section σ_{n1} into the QD ground state. As seen from the figure, η_{int} decreases with increasing j . As clear from Eq. (18), this decrease is related to a raise Δj_{spon} in the spontaneous radiative recombination current density above the lasing threshold caused in turn by a raise in f_{n2} and n_{OCL} . With making faster the direct capture (increasing σ_{n1}), fewer carriers are accumulated in non-lasing states thus reducing Δj_{spon} and enhancing η_{int} . At very fast direct capture [curve 1 in Fig. 2(c)], η_{int} is almost unchanged with increasing j .

The output optical power vs. excess of the injection current density j over the threshold current density j_{th} is

shown in Fig. 3 for wide ranges of parameters σ_{n1} , σ_{n2} , and τ_{21} . In Fig. 3(a), σ_{n1} is different for different curves, while σ_{n2} and τ_{21} are fixed. In Fig. 3(b), σ_{n2} is different for different curves, while σ_{n1} and τ_{21} are fixed. In Fig. 3(c), τ_{21} is

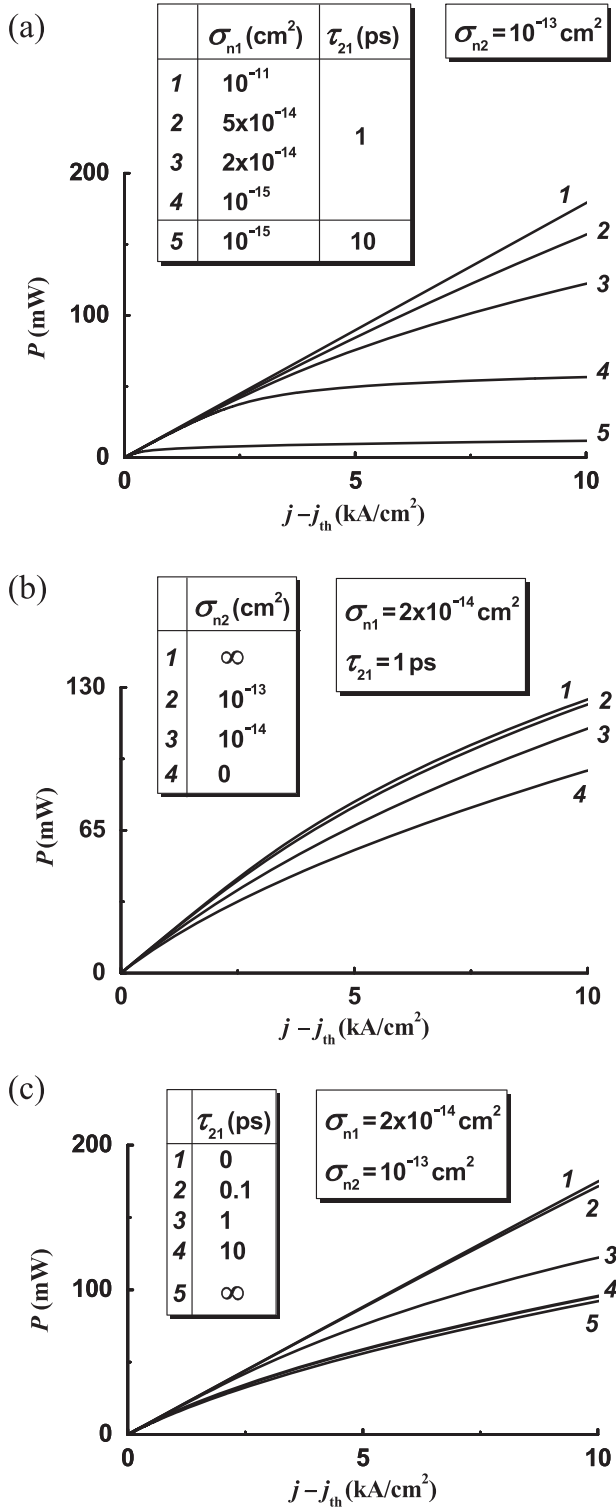


FIG. 3. Light-current characteristic of a QD laser. The abscissa shows the excess of the injection current density over the threshold current density—that is why all the curves start from the origin. In (a), σ_{n1} is different for different curves, while σ_{n2} and τ_{21} are fixed. In (b), σ_{n2} is different for different curves, while σ_{n1} and τ_{21} are fixed. In (c), τ_{21} is different for different curves, while σ_{n1} and σ_{n2} are fixed. The values of parameters σ_{n1} , σ_{n2} , and τ_{21} are indicated in the corresponding insets.

different for different curves, while σ_{n1} and σ_{n2} are fixed. With increasing either σ_{n1} or σ_{n2} and with decreasing τ_{21} , the LCC becomes more linear and the output power becomes higher at the same value of $j - j_{th}$. As seen from Fig. 3(a), the effect of direct capture (capture into the QD ground state) on the LCC is strong. The role of indirect capture (capture into the QD excited state and subsequent intradot relaxation), as seen from Figs. 3(b) and 3(c), becomes important only when direct capture is slow. In order to show a noticeable change in the LCC in Figs. 3(b) and 3(c), we used a small σ_{n1} and varied σ_{n2} and τ_{21} within entire ranges of their possible values [σ_{n2} —from infinity (instantaneous capture into the excited state) to zero (no capture into the excited state), τ_{21} —from zero (instantaneous intradot relaxation) to infinity (no intradot relaxation)].

V. CONCLUSIONS

We have calculated the LCC of a QD laser under the conditions of both direct and indirect capture of carriers from the OCL into the lasing ground state in QDs. We have shown that direct capture is a dominant process determining the ground-state LCC. Only when direct capture is slow, the role of indirect capture (capture into the excited state and subsequent intradot relaxation to the ground state) becomes important.

ACKNOWLEDGMENTS

This work was supported by the U.S. Army Research Office (Grant No. W911NF-13-1-0445).

APPENDIX: EXCITED-STATE LEVEL OCCUPANCY IN QDS AND FREE-CARRIER DENSITY IN THE OCL AT THE LASING THRESHOLD

At the lasing threshold, the number of photons $N_1 = 0$. The free-carrier density in the OCL at the lasing threshold can be obtained from either of equations (10), (11), or (12). Correspondingly, we have the following three equivalent expressions for $n_{OCL,th}$:

$$n_{OCL,th} = n_2 \frac{f_{n2,th}}{1 - f_{n2,th}} + \frac{1}{\sigma_{n2} v_n} \frac{1}{1 - f_{n2,th}} \times \left[\frac{f_{n2,th}(1 - f_{n1})}{\tau_{21}} - \frac{f_{n1}(1 - f_{n2,th})}{\tau_{12}} + \frac{f_{n2,th}^2}{\tau_{QD2}} \right], \quad (A1)$$

$$n_{OCL,th} = n_1 \frac{f_{n1}}{1 - f_{n1}} + \frac{1}{\sigma_{n1} v_n (1 - f_{n1})} \times \left[\frac{f_{n1}(1 - f_{n2,th})}{\tau_{12}} - \frac{f_{n2,th}(1 - f_{n1})}{\tau_{21}} + \frac{f_{n1}^2}{\tau_{QD1}} \right], \quad (A2)$$

$$n_{OCL,th} = \frac{\sigma_{n1} v_n n_1 f_{n1} + \sigma_{n2} v_n n_2 f_{n2,th} + \frac{f_{n1}^2}{\tau_{QD1}} + \frac{f_{n2,th}^2}{\tau_{QD2}}}{\sigma_{n1} v_n (1 - f_{n1}) + \sigma_{n2} v_n (1 - f_{n2,th})}. \quad (A3)$$

Equalizing (A1) and (A2), we obtain the following quadratic equation in the excited-state level occupancy $f_{n2,th}$ in QDs at the lasing threshold:

$$\left[\left(\frac{f_{n1}}{1-f_{n1}} \frac{1}{\tau_{12}} + \frac{1}{\tau_{21}} \right) \sigma_{n2} - \frac{1}{\tau_{QD2}} \sigma_{n1} \right] f_{n2,th}^2 - \left[\sigma_{n1} \sigma_{n2} v_n \left(n_1 \frac{f_{n1}}{1-f_{n1}} + n_2 \right) + \left(\frac{f_{n1}^2}{1-f_{n1}} \frac{1}{\tau_{QD1}} + \frac{2f_{n1}}{1-f_{n1}} \frac{1}{\tau_{12}} + \frac{1}{\tau_{21}} \right) \sigma_{n2} + \left(\frac{f_{n1}}{\tau_{12}} + \frac{1-f_{n1}}{\tau_{21}} \right) \sigma_{n1} \right] f_{n2,th} + \left[\sigma_{n1} \sigma_{n2} v_n n_1 \frac{f_{n1}}{1-f_{n1}} + \left(\frac{f_{n1}^2}{1-f_{n1}} \frac{1}{\tau_{QD1}} + \frac{f_{n1}}{1-f_{n1}} \frac{1}{\tau_{12}} \right) \sigma_{n2} + \frac{f_{n1}}{\tau_{12}} \sigma_{n1} \right] = 0. \quad (A4)$$

Only one of the roots of Eq. (A4) is real, positive, and smaller than unity, and hence is physical.

With the solution of Eq. (A4) for $f_{n2,th}$ and either of expressions (A1), (A2), or (A3) for $n_{OCL,th}$, we calculate the threshold current density [see Eq. (15)].

If the carrier capture/escape into/from a QD is excited-state-mediated only (i.e., $\sigma_{n1}=0$), the expression for $f_{n2,th}$ considerably simplifies

$$f_{n2,th} = \frac{1}{(1-f_{n1}) \frac{\tau_{12}}{\tau_{21}} + f_{n1}} \left(f_{n1} + \frac{\tau_{12}}{\tau_{QD1}} f_{n1}^2 \right). \quad (A5)$$

In this case, Eq. (A3) for $n_{OCL,th}$ becomes

$$n_{OCL,th} = n_2 \frac{f_{n2,th}}{1-f_{n2,th}} + \frac{1}{\sigma_{n2} v_n} \frac{1}{1-f_{n2,th}} \left(\frac{f_{n1}^2}{\tau_{QD1}} + \frac{f_{n2,th}^2}{\tau_{QD2}} \right). \quad (A6)$$

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Output power of a quantum dot laser: Effects of excited states

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(Received 17 August 2015; accepted 26 October 2015; published online 11 November 2015)

A theory of operating characteristics of quantum dot (QD) lasers is discussed in the presence of excited states in QDs. We consider three possible situations for lasing: (i) ground-state lasing only; (ii) ground-state lasing at first and then the onset of also excited-state lasing with increasing injection current; (iii) excited-state lasing only. The following characteristics are studied: occupancies of the ground-state and excited-state in QDs, free carrier density in the optical confinement layer, threshold currents for ground- and excited-state lasing, densities of photons emitted via ground- and excited-state stimulated transitions, output power, internal and external differential quantum efficiencies. Under the conditions of ground-state lasing only, the output power saturates with injection current. Under the conditions of both ground- and excited-state lasing, the output power of ground-state lasing remains pinned above the excited-state lasing threshold while the power of excited-state lasing increases. There is a kink in the light-current curve at the excited-state lasing threshold. The case of excited-state lasing only is qualitatively similar to that for single-state QDs—the role of ground-state transitions is simply reduced to increasing the threshold current. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4935296>]

I. INTRODUCTION AND THEORETICAL MODEL

Excited states of carriers confined in semiconductor quantum dots (QDs) significantly affect the operating characteristics of injection lasers based on them (see, e.g., Refs. 1–21). In this paper, we develop a theory of output optical power of QD lasers in the presence of such states. The details of our model are discussed in the following text.

(I) To describe the actual situation of indirect injection of carriers into QDs, our model includes the bulk optical confinement layer (OCL) and processes therein—the carriers are first injected from the cladding layers into the OCL and then captured into QDs (Fig. 1).

(II) The carrier capture from the OCL into QDs is noninstantaneous—this presents one of the key components of our model. To describe the capture into a QD, we use the capture cross-section. As discussed in Ref. 22, no capture time into a single QD can be properly introduced; instead, using the capture cross-section, the capture time into the entire ensemble of QDs can be introduced that thus depends on the surface density of QDs.

(III) The spontaneous radiative recombination rate outside QDs (i.e., in the OCL) is quadratic in the carrier density n_{OCL} there; nonradiative Auger recombination (which rate is cubic in n_{OCL}) can also be easily included into our model. It is the superlinearity of recombination rate outside QDs, which, combined with noninstantaneous capture into QDs and intradot relaxation, causes (i) saturation of output power of ground-state lasing and (ii) sublinearity of output power of excited-state lasing with increasing injection current. Hence we do not assume monomolecular (linear in n_{OCL}) recombination rate

outside QDs, i.e., constant recombination time outside QDs, which does not depend on n_{OCL} . Monomolecular recombination outside QDs could be a factor only in the presence of high concentration of recombination centers there, which should be avoided in laser-quality structures. More importantly, even if such recombination is present, it will be first dominated by spontaneous radiative recombination and then Auger recombination with increasing injection current, i.e., with increasing n_{OCL} . Monomolecular recombination outside QDs can cause neither saturation of output power of ground-state lasing nor sublinearity of power of excited-state lasing, both of which are important derivations from our model.

(IV) To mainly focus on the effects of excited-states, we assume that the carrier capture into and escape from the QD ground-state occur via the QD excited-state (Fig. 1). For the case of direct capture from the OCL into single-state QDs, the optical power was calculated in Refs. 22 and 23.

Depending on the parameters of the structure, there can be three possible situations for lasing. We consider them separately in the following text.

II. GROUND-STATE LASING ONLY: HIGH GAIN FOR GROUND-STATE TRANSITIONS AND LOW GAIN FOR EXCITED-STATE TRANSITIONS

If the maximum modal gain for ground-state transitions is higher than the mirror loss (the strict criterion will be formulated in the following text) and the maximum gain for excited-state transitions is lower than the mirror loss, the lasing will always occur via ground-state transitions.

A. Rate equations

Our model is based on the following set of rate equations:

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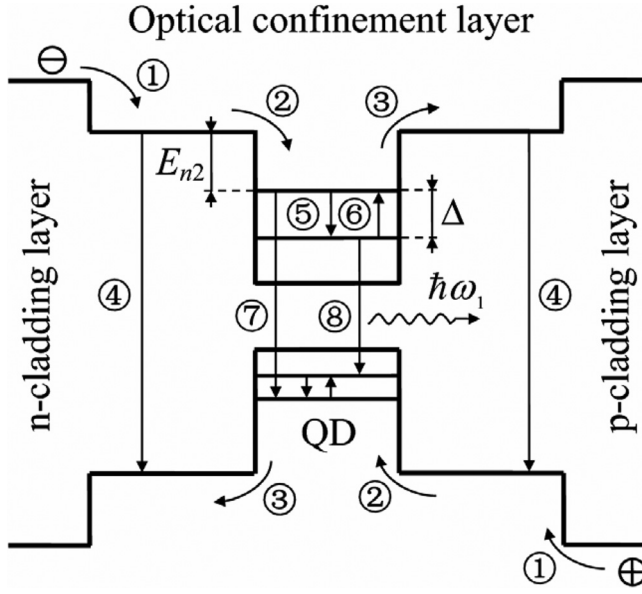


FIG. 1. Energy band diagram of a QD laser (the layers are not drawn to scale). The main processes (shown by arrows) are as follows: ① carrier injection from the cladding layers to the OCL, ② carrier capture from the OCL into the QD excited-state, ③ carrier escape from the QD excited-state to the OCL, ④ spontaneous radiative recombination in the OCL, ⑤ downward transition in QDs (intradot relaxation), ⑥ upward transition in QDs, ⑦ spontaneous and stimulated radiative recombinations via the excited-state in QDs, and ⑧ spontaneous and stimulated radiative recombinations via the ground-state in QDs.

for free carriers in the OCL

$$\frac{\partial n_{\text{OCL}}}{\partial t} = \sigma_2 v_n n_2 \frac{N_S}{b} f_2 - \sigma_2 v_n n_{\text{OCL}} \frac{N_S}{b} (1 - f_2) - B n_{\text{OCL}}^2 + \frac{j}{eb}, \quad (1)$$

for carriers confined in the excited state in QDs

$$\frac{\partial}{\partial t} \left(2 \frac{N_S}{b} f_2 \right) = \sigma_2 v_n \frac{N_S}{b} (1 - f_2) n_{\text{OCL}} - \sigma_2 v_n n_2 \frac{N_S}{b} f_2 + \frac{N_S f_1 (1 - f_2)}{b \tau_{12}} - \frac{N_S f_2 (1 - f_1)}{b \tau_{21}} - \frac{N_S f_2^2}{b \tau_{\text{QD2}}}, \quad (2)$$

for carriers confined in the ground state in QDs

$$\frac{\partial}{\partial t} \left(2 \frac{N_S}{b} f_1 \right) = \frac{N_S f_2 (1 - f_1)}{b \tau_{21}} - \frac{N_S f_1 (1 - f_2)}{b \tau_{12}} - \frac{N_S f_1^2}{b \tau_{\text{QD1}}} - v_{g1} g_1^{\text{max}} (2f_1 - 1) n_{\text{ph1}}, \quad (3)$$

and for photons

$$\frac{\partial n_{\text{ph1}}}{\partial t} = v_{g1} g_1^{\text{max}} (2f_1 - 1) n_{\text{ph1}} - v_{g1} \beta_1 n_{\text{ph1}}. \quad (4)$$

The physical quantities and terms entering into Eqs. (1)–(4) are presented in Tables I and II, respectively. We assume electron–hole symmetry in our model—that is why n_{OCL}^2 , f_2^2 , f_1^2 , and $(2f_1 - 1)$ enter into Eqs. (1)–(4) instead of $n_{\text{OCL}} p_{\text{OCL}}$, $f_{n2} f_{p2}$, $f_{n1} f_{p1}$, and $(f_{n1} + f_{p1} - 1)$, respectively.

Continuous-wave operation is considered here and hence steady-state rate equations ($\partial/\partial t = 0$) are used.

Equation (3) can be written as follows at the steady state:

$$\frac{N_S f_2 (1 - f_1)}{b \tau_{21}} - \frac{N_S f_1 (1 - f_2)}{b \tau_{12}} = \frac{N_S f_1^2}{b \tau_{\text{QD1}}} + v_{g1} g_1^{\text{max}} (2f_1 - 1) n_{\text{ph1}}, \quad (5)$$

TABLE I. Physical quantities entering into the rate equations (1)–(4).

f_1	Occupancy of the ground-state in QDs
f_2	Occupancy of the excited-state in QDs
τ_{QD1}	Ground-state spontaneous radiative lifetime in QDs
τ_{QD2}	Excited-state spontaneous radiative lifetime in QDs
τ_{21}	Transition time from the excited- to ground-state in QDs (intradot relaxation time)
τ_{12}	Transition time from the ground- to excited-state in QDs
v_{g1}	Group velocity of photons emitted via ground-state transitions in QDs
N_S	Surface density of QDs
$S = WL$	Cross-section of the junction (QD layer area)
W	Lateral size of the device (QD layer width)
L	Cavity length
g_1^{max}	Maximum modal gain for ground-state transitions in QDs
n_{ph1}	Density of photons (per unit OCL volume) emitted via ground-state transitions in QDs
σ_2	Cross-section of carrier capture from the OCL into the QD excited-state
v_n	Free carrier thermal velocity in the OCL
n_{OCL}	Free carrier density in the OCL
b	OCL thickness
B	Radiative recombination constant for the OCL
j	Injection current density
$\beta_1 = (1/L) \ln(1/R_1)$	Mirror loss coefficient for ground-state lasing
R_1	Facet reflectivity at the energy of ground-state transitions
$n_2 = N_c^{\text{OCL}} \exp(-E_{n2}/T)$	Effective density of states in the OCL
$N_c^{\text{OCL}} = 2(m_c^{\text{OCL}} T / 2\pi \hbar^2)^{3/2}$	Carrier excitation energy from the QD excited-state to the OCL
E_{n2}	Effective mass in the OCL
m_c^{OCL}	Temperature measured in units of energy
T	Planck's constant
$\hbar$	

TABLE II. Rates of the processes entering into Equations (1)–(4).

$\frac{N_S f_2 (1 - f_1)}{b \tau_{21}}$	Downward transitions in QDs: Intradot relaxation
$\frac{N_S f_1 (1 - f_2)}{b \tau_{12}}$	Upward transitions in QDs
$\frac{N_S f_1^2}{b \tau_{QD1}}$	Spontaneous radiative recombination via the ground-state in QDs
$v_{g1} g_1^{\max} (2f_1 - 1) n_{ph1}$	Stimulated radiative recombination (stimulated emission of photons) via the ground-state in QDs
$\sigma_2 v_n \frac{N_S}{b} (1 - f_2) n_{OCL}$	Capture from the OCL into the excited-state in QDs
$\sigma_2 v_n n_2 \frac{N_S}{b} f_2$	Escape from the excited-state in QDs to the OCL
$\frac{N_S f_2^2}{b \tau_{QD2}}$	Spontaneous radiative recombination via the excited-state in QDs
$B n_{OCL}^2$	Spontaneous radiative recombination in the OCL
$\frac{j}{eb}$	Carrier injection to the OCL
$v_{g1} \beta_1 n_{ph1}$	Mirror loss of photons

which is simply the condition of equality of the net downward transitions rate in QDs (the left-hand side) to the net recombination rate via the ground-state in QDs (the right-hand side).

B. Solutions of rate equations: Level occupancies in QDs, free-carrier density in the OCL, photon density, and output power

Because $n_{ph1} \neq 0$, it follows immediately from Eq. (4) that the ground-state occupancy is pinned at its threshold value

$$f_1 = \frac{1}{2} \left(1 + \frac{\beta_1}{g_1^{\max}} \right). \quad (6)$$

From Eq. (3), the excited-state occupancy can be expressed in terms of the photon density n_{ph1}

$$f_2 = \frac{1}{f_1 + (1 - f_1) \frac{\tau_{12}}{\tau_{21}}} \left[f_1 + \tau_{12} \left(\frac{f_1^2}{\tau_{QD1}} + \frac{b}{N_S} \frac{n_{ph1}}{\tau_{ph1}} \right) \right], \quad (7)$$

where we introduced the lifetime in the cavity for photons emitted via ground-state transitions

$$\tau_{ph1} = \frac{1}{v_{g1} \beta_1} = \frac{1}{v_{g1} g_1^{\max} (2f_1 - 1)}. \quad (8)$$

Using the detailed balance condition, the upward-to-downward transition time ratio in QDs entering into Eq. (7) can be written as

$$\frac{\tau_{12}}{\tau_{21}} = \exp\left(\frac{\Delta}{T}\right), \quad (9)$$

where Δ is the separation between the energies of the excited and ground states in QDs (Fig. 1).

From Eq. (2), the free carrier density in the OCL is

$$n_{OCL} = n_2 \frac{f_2}{1 - f_2} + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_2} \left[\frac{f_2 (1 - f_1)}{\tau_{21}} - \frac{f_1 (1 - f_2)}{\tau_{12}} + \frac{f_2^2}{\tau_{QD2}} \right], \quad (10)$$

or, taking into account Eqs. (5) and (6)

$$n_{OCL} = n_2 \frac{f_2}{1 - f_2} + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_2} \left(\frac{f_1^2}{\tau_{QD1}} + \frac{f_2^2}{\tau_{QD2}} + \frac{b}{N_S} \frac{n_{ph1}}{\tau_{ph1}} \right). \quad (11)$$

From Eqs. (1) and (2), the injection current density is

$$j = eb B n_{OCL}^2 + e N_S \left[\frac{f_2 (1 - f_1)}{\tau_{21}} - \frac{f_1 (1 - f_2)}{\tau_{12}} + \frac{f_2^2}{\tau_{QD2}} \right], \quad (12)$$

or, taking into account Eqs. (5) and (6)

$$j = eb B n_{OCL}^2 + e N_S \frac{f_1^2}{\tau_{QD1}} + e N_S \frac{f_2^2}{\tau_{QD2}} + eb \frac{n_{ph1}}{\tau_{ph1}}. \quad (13)$$

Equation (13) simply states that the injection current goes into spontaneous recombination (via the OCL states and ground and excited states in QDs—the first, second, and third terms in the right-hand side, respectively) and stimulated recombination via the ground-state in QDs (the last term).

In Eq. (13), f_2 and n_{OCL} are functions of n_{ph1} —see Eqs. (7) and (11). Using Eqs. (7) and (11) in (13), we obtain an expression for the injection current density as an explicit function $j(n_{ph1})$ of the photon density. Our task is to calculate the inverse function, i.e., $n_{ph1}(j)$, and then the output power versus j . This can be done and a closed-form expression can be obtained from the solution of a quartic equation. This expression is, however, rather cumbersome and, for this reason, we use a different procedure to plot the functional dependences here. As shown in the preceding text, the quantities f_2 , n_{OCL} , and j are expressed as explicit functions of n_{ph1} . Hence, we first consider n_{ph1} as a variable, change it throughout the entire range of its possible values (from 0 to $n_{ph1}^{\max}$ —see following text), calculate and plot f_2 , n_{OCL} , and j versus n_{ph1} . The dependence of n_{ph1} on j is then simply obtained by switching between the abscissa and ordinate. Thus the light-current characteristic (LCC), i.e., the output optical power versus the injection current density, is calculated

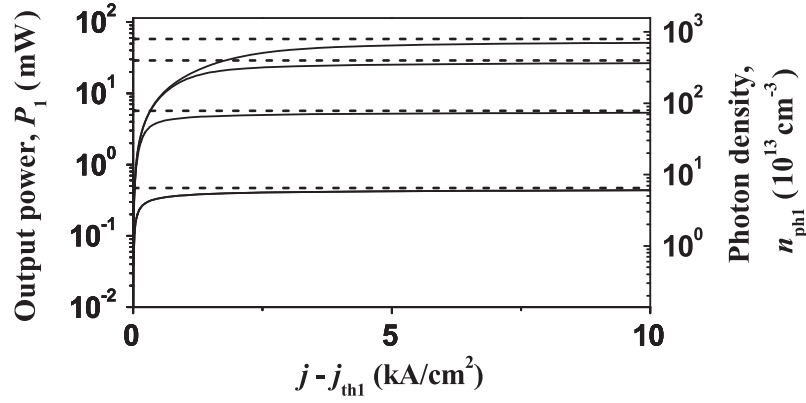


FIG. 2. Output power P_1 (left axis) and photon density n_{ph1} (right axis) of ground-state lasing against excess injection current density. Both P_1 and n_{ph1} go to zero at the lasing threshold ($j = j_{th1}$); the lowest shown values of P_1 and n_{ph1} are nonvanishing because the log-scale is used for the vertical axes. The horizontal short-dashed lines show $P_1^{\max}$ and $n_{ph1}^{\max}$ [Eqs. (21) and (20)]. A GaInAsP/InP heterostructure lasing near $1.55 \mu\text{m}$ is considered here for illustration.^{34,35} In Figs. 2 and 4, the parameters are as follows: $N_S = 6.11 \times 10^{10} \text{ cm}^{-2}$, $\tau_{QD1} = 0.71 \text{ ns}$, $\tau_{QD2} = 2.31 \text{ ns}$, $g_1^{\max} = 29.52 \text{ cm}^{-1}$, $g_2^{\max} = 7.92 \text{ cm}^{-1}$, $\sigma_2 = 10^{-13} \text{ cm}^2$, $L = 0.114 \text{ cm}$, and $T = 300 \text{ K}$; 5% QD-size fluctuations are assumed. The mirror reflectivity at the ground- and excited-state transition energies is put the same, $R_1 = R_2 = 0.32$ (as-cleaved facets at both ends), and hence the mirror loss $\beta_1 = \beta_2 = 10 \text{ cm}^{-1}$. The values of τ_{21} for different curves (from bottom to top) are 100, 10, 2, and 1 ps. The corresponding values of j_{th1} are 21.20, 13.60, 13.11, and 13.05 A/cm^2 . Because the preceding values of τ_{21} are much smaller than $\tau_{21}^{\max} = 521 \text{ ps}$ [Eq. (22)], $P_1^{\max}$ is inversely proportional to τ_{21} [Eq. (25)].

$$P_1(j) = \hbar\omega_1 v_{g1} \beta_1 n_{ph1}(j) S b = \hbar\omega_1 \frac{n_{ph1}(j)}{\tau_{ph1}} S b, \quad (14)$$

where $\hbar\omega_1$ is the energy of photons emitted via ground-state transitions (Fig. 1).

The dependences of f_2 and n_{OCL} on j are also easily obtained from those on n_{ph1} by converting the variable on the x axis from n_{ph1} into $j(n_{ph1})$.

Fig. 2 shows n_{ph1} and P_1 versus the excess of the injection current density j over the threshold current density j_{th1} for ground-state lasing [see Eq. (19) for j_{th1} in the following text]. The dependences are plotted for different values of the excited-to-ground-state relaxation time τ_{21} . Experimental and calculated values of τ_{21} taken from Refs. 18, 20, 21, and 24–31 are presented in Table III.

The LCC was shown to be sublinear in the case of direct capture of carriers from the OCL into single-state QDs.^{22,23} The sublinearity is due to (i) noninstantaneous capture from

the OCL into QDs and (ii) recombination in the OCL, the rate of which is superlinear in carrier density (quadratic or cubic for spontaneous radiative or nonradiative Auger recombination). In the case under study here, as seen from Fig. 2, the output power is more severely impacted—it becomes saturated with increasing j . The point is that the capture into the QD lasing state is now a two-step process. In addition to capture delay from the OCL to the QD excited-state, there is now excited-to-ground-state relaxation delay. It is the relaxation delay, which now controls the carrier supply to the lasing state and strongly limits the output power by causing its saturation at high injection currents.

1. Characteristics at the lasing threshold

Putting $n_{ph1} = 0$ into Eqs. (7), (10), and (12), the values of f_2 and n_{OCL} at the ground-state lasing threshold and the threshold current density are obtained

$$f_{2,th1} = \frac{1}{f_1 + (1-f_1)\frac{\tau_{12}}{\tau_{21}}} \left(f_1 + \frac{\tau_{12}}{\tau_{QD1}} f_1^2 \right), \quad (15)$$

$$n_{OCL,th1} = n_2 \frac{f_{2,th1}}{1-f_{2,th1}} + \frac{1}{\sigma_2 v_n} \frac{1}{1-f_{2,th1}} \times \left[\frac{f_{2,th1}(1-f_1)}{\tau_{21}} - \frac{f_1(1-f_{2,th1})}{\tau_{12}} + \frac{f_{2,th1}^2}{\tau_{QD2}} \right], \quad (16)$$

$$j_{th1} = ebBn_{OCL,th1}^2 + eN_S \left[\frac{f_{2,th1}(1-f_1)}{\tau_{21}} - \frac{f_1(1-f_{2,th1})}{\tau_{12}} + \frac{f_{2,th1}^2}{\tau_{QD2}} \right]. \quad (17)$$

Taking into account Eq. (15), we can write $n_{OCL,th1}$ and j_{th1} as

$$n_{OCL,th1} = n_2 \frac{f_{2,th1}}{1-f_{2,th1}} + \frac{1}{\sigma_2 v_n} \frac{1}{1-f_{2,th1}} \left(\frac{f_1^2}{\tau_{QD1}} + \frac{f_{2,th1}^2}{\tau_{QD2}} \right), \quad (18)$$

TABLE III. Reported values of the intradot relaxation time τ_{21} .

τ_{21} (ps)	Temperature (K)	Energy separation between the excited and ground states in QDs, Δ (meV)	Material system	Source
7	Room temperature (RT)		InAs/In _{0.15} Ga _{0.85} As/ GaAs	18 and 20
5.6	10	60	In(Ga)As/GaAs	21
1, 570	12	11	In _{0.1} Ga _{0.9} As/GaAs/InP	24
19		<2	GaAs/Al _{0.3} Ga _{0.7} As	25
2.7–17	12	11	In _{0.1} Ga _{0.9} As/GaAs/InP	26
150	RT	50	GaAs-AlGaAs	27
30–50	RT		In _{0.4} Ga _{0.6} As/GaAs	28
0.6–6	2	40	InP/Ga _{0.5} In _{0.5} P	29
30, 100			InGaAs/GaAs	30
>30	Liquid helium		InAs/GaAs	31

$$j_{\text{th1}} = ebBn_{\text{OCL,th1}}^2 + eN_S \frac{f_1^2}{\tau_{\text{QD1}}} + eN_S \frac{f_{2,\text{th1}}^2}{\tau_{\text{QD2}}}. \quad (19)$$

The first, second, and third terms in the right-hand side of Eq. (19) are the threshold values of the current densities of spontaneous radiative recombination via the OCL states, and ground and excited states in QDs, respectively.

2. Maximum output power and necessary condition for ground-state lasing

It is seen from Eq. (7) that, as $f_2 \rightarrow 1$ (the level occupancy cannot exceed unity) with increasing injection current, the photon density n_{ph1} remains finite and tends to its maximum (saturation) value (Fig. 2). Putting $f_2 = 1$ in Eq. (7) gives

$$n_{\text{ph1}}^{\text{max}} = \tau_{\text{ph1}} \left(\frac{1-f_1}{\tau_{21}} - \frac{f_1^2}{\tau_{\text{QD1}}} \right) \frac{N_S}{b}. \quad (20)$$

For the maximum (saturation) value of the output power of ground-state lasing, we have

$$P_1^{\text{max}} = \hbar\omega_1 \frac{n_{\text{ph1}}^{\text{max}}}{\tau_{\text{ph1}}} Sb = \hbar\omega_1 \left(\frac{1-f_1}{\tau_{21}} - \frac{f_1^2}{\tau_{\text{QD1}}} \right) N_S S. \quad (21)$$

The output power P_1 approaches its saturation value P_1^{max} according to $(1 - \text{const}/\sqrt{j}) P_1^{\text{max}}$ —see Eq. (A6) in Appendix A for the asymptotic expression for the LCC at high injection currents.

As seen from Eq. (21), the maximum power P_1^{max} of ground-state lasing is a decreasing function of the intradot relaxation time τ_{21} . At a certain value of τ_{21} given by

$$\tau_{21}^{\text{max}} = \frac{1-f_1}{f_1^2} \tau_{\text{QD1}}, \quad (22)$$

P_1^{max} vanishes. Hence the condition for lasing can be formulated as

$$\tau_{21} < \tau_{21}^{\text{max}}. \quad (23)$$

As can be seen from Eqs. (15) and (22), if $\tau_{21} = \tau_{21}^{\text{max}}$, the excited-state occupancy is unity already at the ground-state lasing threshold ($f_{2,\text{th1}} = 1$); hence $n_{\text{OCL,th1}}$ and the threshold current density j_{th1} of ground-state lasing [see Eqs. (18) and (19)] become infinitely high.

τ_{21}^{max} presents the longest, cut-off value of the intradot relaxation time—no ground-state lasing is possible if $\tau_{21} > \tau_{21}^{\text{max}}$. This cut-off time depends on the ground-state occupancy f_1 . As seen from Eq. (6), f_1 can range from 1/2 (infinitely long cavity, i.e., no mirror loss, $\beta_1 = 0$) to 1 (shortest cavity, $\beta_1 = g_1^{\text{max}}$). As seen from (22), τ_{21}^{max} decreases from $2\tau_{\text{QD1}}$ to 0 as f_1 varies from 1/2 to 1. Hence

$$\tau_{21}^{\text{abs,max}} = 2\tau_{\text{QD1}} \quad (24)$$

presents the absolute upper limit for τ_{21} . If $\tau_{21} > \tau_{21}^{\text{abs,max}}$, the lasing is unattainable even in a structure with no mirror loss.

The fact that $\tau_{21}^{\text{max}} = 0$ in the device with the shortest cavity means that no lasing is possible in such a device even if intradot relaxation is instantaneous.

With Eq. (22), the maximum value of the stimulated recombination rate per QD, $\frac{b}{N_S} \frac{n_{\text{ph1}}^{\text{max}}}{\tau_{\text{ph1}}} = \frac{1}{N_S} \frac{P_1^{\text{max}}}{\hbar\omega_1}$, normalized to the ground-state spontaneous recombination rate per QD, f_1^2/τ_{QD1} , can be written as the following universal decaying function of the normalized relaxation time $\tau_{21}/\tau_{21}^{\text{max}}$ (Fig. 3):

$$\frac{\frac{b}{N_S} \frac{n_{\text{ph1}}^{\text{max}}}{\tau_{\text{ph1}}}}{\frac{f_1^2}{\tau_{\text{QD1}}}} = \frac{\frac{1}{N_S} \frac{P_1^{\text{max}}}{\hbar\omega_1}}{\frac{f_1^2}{\tau_{\text{QD1}}}} = \frac{1}{\frac{\tau_{21}}{\tau_{21}^{\text{max}}} - 1}. \quad (25)$$

When $\tau_{21}/\tau_{21}^{\text{max}} \ll 1$, the normalized P_1^{max} is inversely proportional to $\tau_{21}/\tau_{21}^{\text{max}}$ (the linear portion of the curve in Fig. 3 in log-scale; see also the horizontal dotted lines in Fig. 2).

Inequality Eq. (23), which is the necessary condition for ground-state lasing, is written in terms of allowed values of τ_{21} at a given f_1 , i.e., given g_1^{max} and β_1 . This condition can be rewritten in terms of allowed values of g_1^{max} at given τ_{21} and β_1 . Thus we obtain

$$g_1^{\text{max}} > \frac{\beta_1}{\frac{1}{\frac{1}{4} + \sqrt{\frac{1}{16} + \frac{1}{4} \frac{\tau_{21}}{\tau_{\text{QD1}}}}} - 1}. \quad (26)$$

For the entire range of allowed values of τ_{21} , the right-hand side of Eq. (26) is larger than or equal to β_1 . Indeed the denominator of the expression in the right-hand side is less than or equal to unity—it varies from 1 to 0 as τ_{21} varies from 0 to $\tau_{21}^{\text{abs,max}} = 2\tau_{\text{QD1}}$.

Hence, when the carrier capture into the QD ground-state is excited-state-mediated, a simple excess of the maximum gain g_1^{max} over the mirror loss β_1 is not sufficient for ground-state lasing to occur. A stronger condition [inequality Eq. (26)] should be satisfied. Only if intradot relaxation is instantaneous, Eq. (26) reduces to the condition $g_1^{\text{max}} > \beta_1$. The longer is τ_{21} , the higher should be g_1^{max} . For Eq. (26) to hold in the case of the slowest intradot relaxation ($\tau_{21} = \tau_{21}^{\text{abs,max}} = 2\tau_{\text{QD1}}$), the maximum gain for ground-state transitions g_1^{max} should be infinitely high or the cavity infinitely long ($\beta_1 = 0$).

3. Internal and external differential quantum efficiencies

Above the lasing threshold ($j \geq j_{\text{th1}}$), the internal differential quantum efficiency of a semiconductor laser is defined

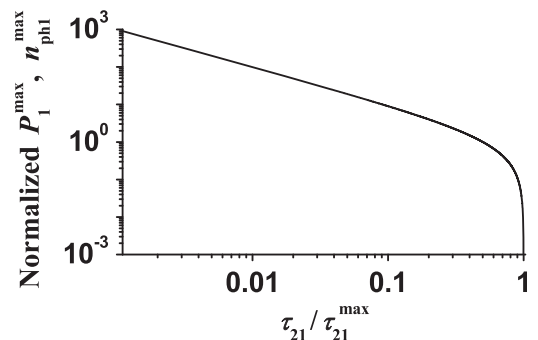


FIG. 3. Normalized maximum output power and photon density [Eq. (25)] of ground-state lasing as a universal function of normalized intradot relaxation time.

as the fraction of the excess of the injection current over the threshold current that results in stimulated emission

$$\eta_{\text{int1}} = \frac{j_{\text{stim1}}}{j - j_{\text{th1}}}, \quad (27)$$

where

$$j_{\text{stim1}} = eb \frac{n_{\text{ph1}}}{\tau_{\text{ph1}}} \quad (28)$$

is the current density of stimulated recombination via the ground-state in QDs.

With Eqs. (13) and (19) for j and j_{th1} , Eq. (27) becomes

$$\eta_{\text{int1}} = \frac{1}{1 + \frac{\tau_{12}}{\tau_{\text{QD2}}} \frac{f_2 + f_{2,\text{th1}}}{f_1 + (1 - f_1)} + \frac{B(n_{\text{OCL}}^2 - n_{\text{OCL,th1}}^2)}{\frac{n_{\text{ph1}}}{\tau_{\text{ph1}}}}}. \quad (29)$$

In Eq. (29), Eqs. (7) and (15) for f_2 and $f_{2,\text{th1}}$ were used.

The external differential quantum efficiency is defined as

$$\eta_{\text{ext1}} = \frac{1}{e} \frac{\partial P_1}{\partial I} = eb \frac{1}{\tau_{\text{ph1}}} \frac{\partial n_{\text{ph1}}}{\partial j} = \frac{\partial j_{\text{stim1}}}{\partial j}, \quad (30)$$

where $I = Sj$ is the injection current. With Eqs. (7), (10), and (11), we have for η_{ext1}

$$\eta_{\text{ext1}} = \left\{ 1 + 2 \frac{\tau_{12}}{\tau_{\text{QD2}}} \frac{f_2}{f_1 + (1 - f_1)} + 2\tau_{12} \frac{b}{N_s} B n_{\text{OCL}} \right. \\ \times \frac{1}{1 - f_2} \frac{1}{f_1 + (1 - f_1)} \frac{\tau_{12}}{\tau_{21}} \\ \times \left[n_{\text{OCL}} + n_2 + \frac{1}{\sigma_2 V_n} \left(\frac{1 - f_1}{\tau_{21}} + \frac{f_1}{\tau_{12}} + \frac{2f_2}{\tau_{\text{QD2}}} \right) \right] \Bigg\}^{-1}. \quad (31)$$

In Eqs. (29) and (31), f_2 , n_{OCL} , and n_{ph1} are functions of the injection current density j .

Even at the lasing threshold, the internal and external efficiencies (being equal to each other) are less than unity (Fig. 4).

At $j > j_{\text{th1}}$, η_{ext1} is smaller than η_{int1} (Fig. 4). Both efficiencies decrease rapidly with j (Fig. 4). The asymptotic expressions for them at high j are derived in Appendix A— η_{int1} and η_{ext1} decay as $1/j$ and $1/j^{3/2}$, respectively [Eqs. (A8) and (A9)].

The shorter the intradot relaxation time τ_{21} , the higher η_{int1} and η_{ext1} (Fig. 4). The limiting case of instantaneous relaxation ($\tau_{21} = 0$) is considered in Appendix B.

We considered in Section II the situation when the lasing occurs via ground-state transitions only. This means that with increasing pump current, the lasing condition will never be satisfied for excited-state transitions. A sufficient condition for this is the inequality

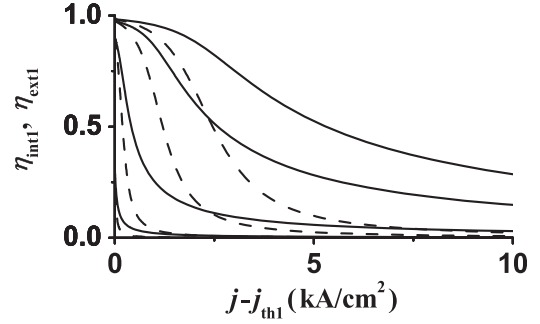


FIG. 4. Internal and external differential quantum efficiencies (solid and dashed curves, respectively) against excess injection current density. The values of τ_{21} for different curves (from bottom to top) are 100, 10, 2, and 1 ps.

$$g_2^{\text{max}} < \beta_2, \quad (32)$$

where g_2^{max} and β_2 are the maximum modal gain and mirror loss for excited-state transitions. For calculations in this section, we used 29.52 and 7.92 cm^{-1} for g_1^{max} and g_2^{max} and the same value (10 cm^{-1}) for β_1 and β_2 . Other parameters are presented in the caption to Fig. 2.

III. GROUND- AND EXCITED-STATE LASING: HIGH GAIN FOR BOTH GROUND- AND EXCITED-STATE TRANSITIONS

The condition Eq. (32) of low gain for excited-state transitions is not always satisfied. When the inequality reverse to Eq. (32) holds

$$g_2^{\text{max}} > \beta_2, \quad (33)$$

excited-state lasing occurs above a certain injection current (threshold current for excited-state lasing). Depending on the maximum gain g_1^{max} for ground-state transitions, two situations are possible. If g_1^{max} is high enough [the criterion is formulated in the following text—see Eq. (40)], ground-state lasing turns on first with increasing pump current and then so does excited-state lasing. Such a situation is considered in this section; for calculations here, we use $g_1^{\text{max}} = 29.52 \text{ cm}^{-1}$ and $g_2^{\text{max}} = 12 \text{ cm}^{-1}$; other parameters are presented in the caption to Fig. 5. If g_1^{max} is low, lasing will occur via excited-state transitions only. Such a situation is considered in Section IV.

A. Above ground-state lasing threshold and below excited-state lasing threshold: Ground-state lasing only

All the equations and analysis of Section II apply in this case of $j_{\text{th1}} < j < j_{\text{th2}}$, where j_{th1} is the threshold current density for ground-state lasing given by Eq. (19), and j_{th2} is the threshold current density for excited-state lasing given by Eq. (46) in the following text. Particularly, with increasing pump current density above j_{th1} , the photon density and output power of ground-state lasing increase from zero, and the excited-state occupancy increases from its value $f_{2,\text{th1}}$ [given by Eq. (15)]; f_2 and n_{ph1} are related by Eq. (7). The increase in n_{ph1} and f_2 continues up to the onset of excited-state lasing (Fig. 5).

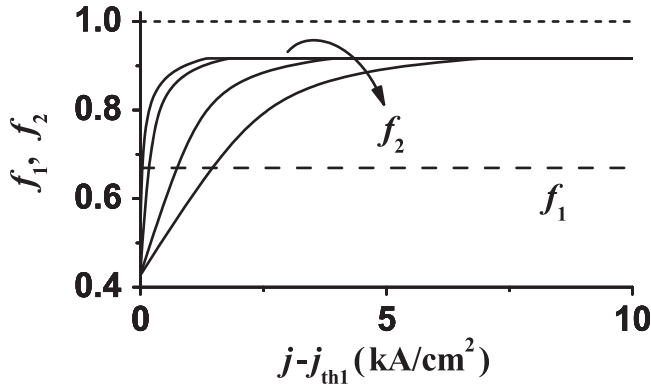


FIG. 5. Ground- (horizontal dashed line) and excited-state (solid curves) occupancies in QDs against excess injection current density. The horizontal short-dashed line indicates the maximum possible value (unity) for the level occupancies. In Figs. 5–12, the parameters are as follows (those not listed here are the same as in the caption to Fig. 2): $\tau_{QD2} = 1.53$ ns and $g_2^{\max} = 12$ cm⁻¹. The values of τ_{21} for different curves (from left to right) are 100, 10, 2, and 1 ps. The corresponding values of j_{th1} are 22.00, 13.60, 13.11, and 13.05 A/cm², and the values of j_{th2} are 1.34×10^3 , 1.60×10^3 , 2.71×10^3 , and 4.10×10^3 A/cm². The pinning of f_2 occurs at $j = j_{th2}$.

B. Above excited-state lasing threshold: Simultaneous ground- and excited-state lasing

The set of the rate Eqs. (1)–(4) should now be added by the rate equation for photons emitted via excited-state stimulated transitions. This equation is similar to Eq. (4)

$$v_{g2} g_2^{\max} (2f_2 - 1) n_{ph2} - v_{g2} \beta_2 n_{ph2} = 0, \quad (34)$$

where v_{g2} and n_{ph2} are the group velocity and density of photons emitted via excited-state stimulated transitions.

At and above j_{th2} (i.e., when $n_{ph2} \neq 0$), Eq. (34) reads as the condition of equality of the gain for excited-state transitions to the mirror loss (condition for excited-state lasing), from which we have

$$f_2 = f_{2,th2} = \frac{1}{2} \left(1 + \frac{\beta_2}{g_2^{\max}} \right). \quad (35)$$

Similarly to the ground-state occupancy above j_{th1} , the excited-state occupancy above j_{th2} is also pinned (Fig. 5).

1. Pinning of output power and necessary condition for ground-state lasing

Because n_{ph1} is related to f_2 [see Eq. (7), which is just another way of writing the rate Eq. (3)], it follows immediately

from pinning of f_2 that n_{ph1} (and hence P_1 —see Fig. 6) is also pinned above the excited-state lasing threshold. The pinning value of n_{ph1} is found from Eq. (7) by putting $f_2 = f_{2,th2}$ there

$$n_{ph1}^{\text{pin}} = \tau_{ph1} \left\{ \frac{f_{2,th2} \left[f_1 + (1 - f_1) \frac{\tau_{12}}{\tau_{21}} \right] - f_1}{\tau_{12}} - \frac{f_1^2}{\tau_{QD1}} \right\} \frac{N_S}{b}. \quad (36)$$

For the pinning value of P_1 , we thus have

$$P_1^{\text{pin}} = \hbar \omega_1 \frac{n_{ph1}^{\text{pin}}}{\tau_{ph1}} S b = \hbar \omega_1 \left\{ \frac{f_{2,th2} \left[f_1 + (1 - f_1) \frac{\tau_{12}}{\tau_{21}} \right] - f_1}{\tau_{12}} - \frac{f_1^2}{\tau_{QD1}} \right\} N_S S. \quad (37)$$

Compare Eqs. (36) and (37) with Eqs. (20) and (21) for $n_{ph1}^{\max}$ and $P_1^{\max}$. Because $f_{2,th2} < 1$, the pinning value of the output power of ground-state lasing P_1^{pin} is lower than the saturation value $P_1^{\max}$. If $f_{2,th2} \rightarrow 1$, $P_1^{\text{pin}} \rightarrow P_1^{\max}$.

For ground-state lasing to occur, P_1^{pin} must be positive. At a certain value of τ_{21} given by

$$\tau_{21}^{\max'} = \frac{f_{2,th2} - f_1 \left[f_{2,th2} + (1 - f_{2,th2}) \frac{\tau_{21}}{\tau_{12}} \right]}{f_1^2} \tau_{QD1}, \quad (38)$$

P_1^{pin} vanishes. Hence the condition for ground-state lasing can now be formulated as

$$\tau_{21} < \tau_{21}^{\max'}. \quad (39)$$

$\tau_{21}^{\max'}$ is the cut-off value of τ_{21} in the case when excited-state lasing is also possible—there can be no ground-state lasing if $\tau_{21} > \tau_{21}^{\max'}$ [compare Eq. (38) with Eq. (22) for $\tau_{21}^{\max}$]. Because $f_{2,th2} < 1$, this cut-off value (shown by the vertical dotted line in Fig. 7) is shorter than $\tau_{21}^{\max}$. If $f_{2,th2} \rightarrow 1$, $\tau_{21}^{\max'} \rightarrow \tau_{21}^{\max}$.

Inequality Eq. (39) presents the necessary condition for ground-state lasing in terms of allowed values of τ_{21} at given f_1 and $f_{2,th2}$, i.e., given β_1 , $g_1^{\max}$, β_2 , and $g_2^{\max}$. It can be rewritten in terms of allowed values of $g_1^{\max}$ at given τ_{21} as follows:

$$g_1^{\max} > \frac{\beta_1}{f_{2,th2} \left[\frac{1}{4} \left[f_{2,th2} + (1 - f_{2,th2}) \frac{\tau_{21}}{\tau_{12}} \right] + \sqrt{\frac{1}{16} \left[f_{2,th2} + (1 - f_{2,th2}) \frac{\tau_{21}}{\tau_{12}} \right]^2 + \frac{1}{4} \frac{\tau_{21}}{\tau_{QD1}} f_{2,th2}} \right] - 1}, \quad (40)$$

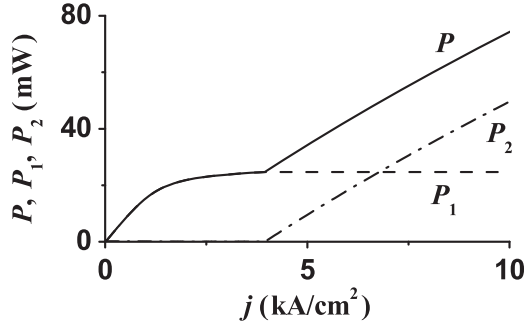


FIG. 6. Total power (solid curve) and powers of ground- (dashed curve) and excited-state (dash-dotted curve) lasing against injection current density. The kink in the curve for the total power occurs at $j = j_{th2}$. $\tau_{21} = 2$ ps.

where f_1 and $f_{2,th2}$ are given by Eqs. (6) and (35), correspondingly. If $f_{2,th2} \rightarrow 1$, inequality Eq. (40) reduces to Eq. (26).

As in the case of only ground-state lasing (Section II), the longer is τ_{21} , the higher should be $g_1^{\max}$. At $\tau_{21} = 0$, Eq. (40) reduces to a simple condition $g_1^{\max} > \beta_1$. At $\tau_{21} = \tau_{21}^{\max'}$, the denominator in the right-hand side of Eq. (40) becomes equal to 0, and hence $g_1^{\max}$ should be infinitely high to have ground-state lasing.

In terms of the normalized relaxation time $\tau_{21}/\tau_{21}^{\max'}$, the pinning value of the ground-state stimulated recombination rate per QD, $\frac{b}{N_S} \frac{n_{ph1}^{\text{pin}}}{\tau_{ph1}} = \frac{1}{SN_S} \frac{P_1^{\text{pin}}}{\hbar\omega_1}$, normalized to the ground-state spontaneous recombination rate per QD, f_1^2/τ_{QD1} , is given by the same function as the normalized maximum value of the ground-state stimulated recombination rate per QD in terms of $\tau_{21}/\tau_{21}^{\max'}$ [Section II, Eq. (25) and Fig. 3].

2. Free-carrier density in the OCL, photon density, and output power of excited-state lasing

For $j \geq j_{th2}$, instead of Eq. (2), the following modified rate equation should be used, which includes the rate of stimulated radiative recombination transitions via the QD excited-state (the last term in the left-hand side in the following text):

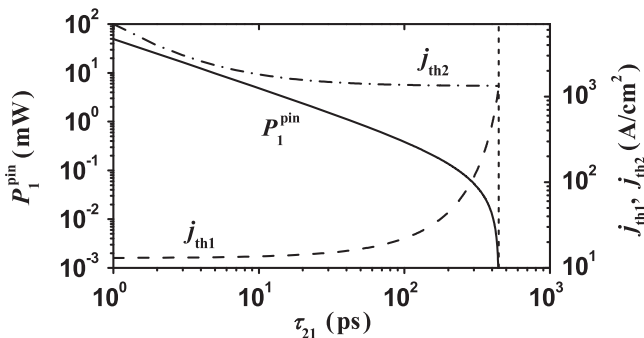


FIG. 7. Output power of ground-state lasing at which excited-state lasing starts (solid curve, left axis) and threshold current densities for ground- and excited-state lasing (dashed and dash-dotted curves, right axis) against intra-dot relaxation time. The vertical short-dashed line marks $\tau_{21}^{\max'} = 445$ ps at which the ground-state lasing becomes unattainable.

$$\begin{aligned} & \sigma_2 v_n \frac{N_S}{b} (1 - f_{2,th2}) n_{OCL} - \sigma_2 v_n n_2 \frac{N_S}{b} f_{2,th2} \\ & + \frac{N_S f_1 (1 - f_{2,th2})}{b \tau_{12}} - \frac{N_S f_{2,th2} (1 - f_1)}{b \tau_{21}} \\ & - \frac{N_S f_{2,th2}^2}{b \tau_{QD2}} - v_{g2} g_2^{\max} (2f_{2,th2} - 1) n_{ph2} = 0. \end{aligned} \quad (41)$$

Using Eqs. (35) and (36), we have from Eq. (41) for the free carrier density in the OCL

$$\begin{aligned} n_{OCL} = n_2 \frac{f_{2,th2}}{1 - f_{2,th2}} + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_{2,th2}} \\ \times \left(\frac{f_1^2}{\tau_{QD1}} + \frac{f_{2,th2}^2}{\tau_{QD2}} + \frac{b}{N_S} \frac{n_{ph1}^{\text{pin}}}{\tau_{ph1}} + \frac{b}{N_S} \frac{n_{ph2}}{\tau_{ph2}} \right), \end{aligned} \quad (42)$$

where we introduced the lifetime in the cavity for photons emitted via excited-state transitions

$$\tau_{ph2} = \frac{1}{v_{g2} \beta_2} = \frac{1}{v_{g2} g_2^{\max} (2f_{2,th2} - 1)}. \quad (43)$$

Using Eq. (41), we have from Eq. (1) for the injection current density

$$j = ebBn_{OCL}^2 + eN_S \frac{f_1^2}{\tau_{QD1}} + eN_S \frac{f_{2,th2}^2}{\tau_{QD2}} + eb \frac{n_{ph1}^{\text{pin}}}{\tau_{ph1}} + eb \frac{n_{ph2}}{\tau_{ph2}}, \quad (44)$$

where n_{OCL} is given by Eq. (42).

At the excited-state lasing threshold ($n_{ph2} = 0$), we have from Eqs. (42) and (44)

$$\begin{aligned} n_{OCL,th2} = n_2 \frac{f_{2,th2}}{1 - f_{2,th2}} \\ + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_{2,th2}} \left(\frac{f_1^2}{\tau_{QD1}} + \frac{f_{2,th2}^2}{\tau_{QD2}} + \frac{b}{N_S} \frac{n_{ph1}^{\text{pin}}}{\tau_{ph1}} \right), \end{aligned} \quad (45)$$

$$j_{th2} = ebBn_{OCL,th2}^2 + eN_S \frac{f_1^2}{\tau_{QD1}} + eN_S \frac{f_{2,th2}^2}{\tau_{QD2}} + eb \frac{n_{ph1}^{\text{pin}}}{\tau_{ph1}}. \quad (46)$$

Compare Eq. (46) for j_{th2} with Eq. (19) for j_{th1} . Because excited-state lasing follows ground-state lasing, this explains why the current density of stimulated recombination via the ground-state in QDs enters as a component [the last term in the right-hand side of Eq. (46)] into the expression for the threshold current density j_{th2} for excited-state lasing.

As seen from Eq. (35), $f_{2,th2}$ is unaffected by τ_{21} (Fig. 5). Hence, as it follows from Eq. (36), n_{ph1}^{pin} decreases with increasing τ_{21} ; consequently, $n_{OCL,th2}$ and j_{th2} decrease [see Eqs. (45) and (46)]. Conversely, $n_{OCL,th1}$ and j_{th1} increase with τ_{21} . The opposite tendencies in j_{th1} and j_{th2} are easily understood—increasing τ_{21} , i.e., delaying the excited-to-ground-state relaxation, hinders the ground-state lasing (hence j_{th1} increases) while making for excited-state lasing (that is why j_{th2} decreases). Hence as τ_{21} increases, j_{th1} and j_{th2} approach each other (Fig. 7). At $\tau_{21} = \tau_{21}^{\max'}$ [see Eq. (38) for $\tau_{21}^{\max'}$], n_{ph1}^{pin} becomes zero and $f_{2,th2}$, $n_{OCL,th2}$, and j_{th2} become equal to

$f_{2,\text{th}1}$, $n_{\text{OCL},\text{th}1}$, and $j_{\text{th}1}$, respectively (Fig. 7). In contrast to the case of ground-state lasing only (wherein $j_{\text{th}1} \rightarrow \infty$ as $\tau_{21} \rightarrow \tau_{21}^{\text{max}}$), here $j_{\text{th}1}$ remains finite at $\tau_{21} = \tau_{21}^{\text{max}}$ (Fig. 7). As $\tau_{21} \rightarrow 0$, $n_{\text{ph}1}^{\text{pin}} \rightarrow \infty$ [Eq. (36)], $n_{\text{OCL},\text{th}2} \rightarrow \infty$ [Eq. (45)], and $j_{\text{th}2} \rightarrow \infty$ [Eq. (46) and Fig. 7], i.e., the excited-state lasing becomes unattainable—there can be ground-state lasing only with $n_{\text{OCL},\text{th}1}$ and $j_{\text{th}1}$ being given by Eqs. (B2) and (B3) (wherein $n_{\text{ph}1}$ is put zero).

The current density of stimulated recombination via the excited-state in QDs

$$j_{\text{stim}2} = eb \frac{n_{\text{ph}2}}{\tau_{\text{ph}2}} \quad (47)$$

is calculated in Appendix C. With Eq. (C5) for $j_{\text{stim}2}$, the output power of excited-state lasing as a function of the injection current density is

$$\begin{aligned} P_2 &= \hbar\omega_2 v_{g2} \beta_2 n_{\text{ph}2} S b = \frac{\hbar\omega_2}{e} j_{\text{stim}2} S \\ &= \frac{\hbar\omega_2}{e} \frac{1}{\frac{1}{2} + \frac{j_{\text{th}2}^{\text{OCL}}}{j_{\text{capt},\text{th}2}} + \sqrt{\left(\frac{1}{2} + \frac{j_{\text{th}2}^{\text{OCL}}}{j_{\text{capt},\text{th}2}}\right)^2 + \frac{j_{\text{th}2}^{\text{OCL}}}{j_{\text{capt},\text{th}2}} \frac{j - j_{\text{th}2}}{j_{\text{capt},\text{th}2}}}} \\ &\quad \times (j - j_{\text{th}2}) S, \end{aligned} \quad (48)$$

where

$$j_{\text{capt},\text{th}2} = e\sigma_2 v_n n_{\text{OCL},\text{th}2} (1 - f_{2,\text{th}2}) N_S \quad (49)$$

is the current density of carrier capture from the OCL into the excited-state in QDs at $j = j_{\text{th}2}$, and

$$j_{\text{th}2}^{\text{OCL}} = ebBn_{\text{OCL},\text{th}2}^2 \quad (50)$$

is the spontaneous radiative recombination current density in the OCL at $j = j_{\text{th}2}$.

Because the energies of photons emitted via the ground- and excited-state transitions are not the same, the total optical power $P = P_1^{\text{pin}} + P_2$ is not directly proportional to the total photon density $n_{\text{ph}} = n_{\text{ph}1}^{\text{pin}} + n_{\text{ph}2}$ at $j > j_{\text{th}2}$.

Eq. (44) can be rewritten as

$$j_{\text{stim}1}^{\text{pin}} + j_{\text{stim}2} = j - \left(ebBn_{\text{OCL}}^2 + eN_S \frac{f_1^2}{\tau_{\text{QD}1}} + eN_S \frac{f_{2,\text{th}2}^2}{\tau_{\text{QD}2}} \right), \quad (51)$$

where

$$j_{\text{stim}1}^{\text{pin}} = ebv_{g1} \beta_1 n_{\text{ph}1}^{\text{pin}} = eb \frac{n_{\text{ph}1}^{\text{pin}}}{\tau_{\text{ph}1}} \quad (52)$$

is the current density of stimulated recombination via the ground-state in QDs above the excited-state lasing threshold with $n_{\text{ph}1}^{\text{pin}}$ being given by Eq. (36). Because $f_{2,\text{th}2}$ is unaffected by τ_{21} [see Eq. (35)], as it immediately follows from the rate Eq. (1), so is n_{OCL} ; this is also seen from Fig. 8—the curves for n_{OCL} calculated at different values of τ_{21} merge together above $j_{\text{th}2}$. Hence none of the quantities n_{OCL} , f_1 , and $f_{2,\text{th}2}$ in

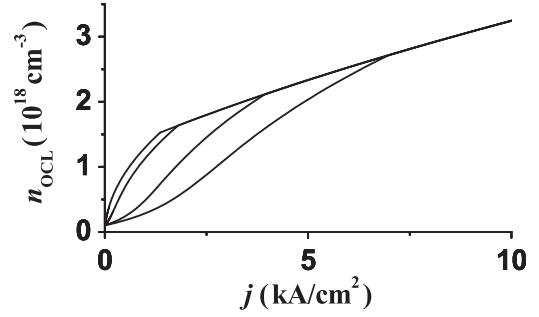


FIG. 8. Free carrier density in the OCL against injection current density. The values of τ_{21} for different curves (from left to right) are 100, 10, and 1 ps. The kinks in the curves occur at $j = j_{\text{th}2}$.

the right-hand side of Eq. (51) depends on τ_{21} . Therefore the spontaneous recombination current densities [via the OCL states and ground and excited states in QDs—the three terms in the brackets in the right-hand-side of Eq. (51)] do not depend on τ_{21} at $j \geq j_{\text{th}2}$. Thus the total stimulated recombination current density is also independent of τ_{21} though both $j_{\text{stim}1}^{\text{pin}}$ and $j_{\text{stim}2}$ depend on τ_{21} on their own.

Hence, as seen from Eq. (51), the sum of the photon densities of ground- and excited-state lasing, each weighted by its own reciprocal lifetime in the cavity, does not depend on τ_{21} at $j \geq j_{\text{th}2}$. The following expression is obtained for this sum:

$$\begin{aligned} \frac{n_{\text{ph}1}^{\text{pin}}}{\tau_{\text{ph}1}} + \frac{n_{\text{ph}2}}{\tau_{\text{ph}2}} &= \frac{1}{eb} \left(j_{\text{stim}1}^{\text{pin}} + j_{\text{stim}2} \right) \\ &= \frac{1}{eb} \left\{ \frac{j_{\text{capt},\text{th}2}^2}{j_{\text{th}2}^{\text{OCL}}} \left[\sqrt{\frac{1}{4} + \frac{j_{\text{th}2}^{\text{OCL}}}{j_{\text{capt},\text{th}2}^2} (j_{\text{esc}2}^{\text{OCL}} + j)} - \frac{1}{2} \right] \right. \\ &\quad \left. - j_{\text{esc}2}^{\text{OCL}} - eN_S \frac{f_1^2}{\tau_{\text{QD}1}} - eN_S \frac{f_{2,\text{th}2}^2}{\tau_{\text{QD}2}} \right\} = \text{const}(\tau_{21}), \end{aligned} \quad (53)$$

where

$$j_{\text{esc}2}^{\text{OCL}} = e\sigma_2 v_n n_{2,\text{th}2} N_S \quad (54)$$

is the current density of carrier escape from the excited-state in QDs to the OCL.

If $\tau_{\text{ph}1} = \tau_{\text{ph}2}$, the total photon density $n_{\text{ph}1}^{\text{pin}} + n_{\text{ph}2}$ is independent of τ_{21} [Fig. 9(a)]. Because, as mentioned in the preceding text, the total power P is not directly proportional to the total photon density, there is a slight dependence of P on τ_{21} even in this case [Fig. 9(b)].

The kinks in the curves in Fig. 9 occur at $j = j_{\text{th}2}$. As seen from the figure, with increasing τ_{21} , the contribution of ground-state lasing to the total output power decreases. For $\tau_{21} = 100$ ps, this contribution is negligible. This is also seen from Fig. 10 showing the ratio of $n_{\text{ph}2}$ to $n_{\text{ph}1}$ at $j \geq j_{\text{th}2}$.

For $\tau_{21} = 2$ ps, Fig. 6 shows the total power P and the powers of ground- and excited-state lasing P_1 and P_2 , respectively.

We should emphasize here that other factors, which are not included into our theoretical model, may also affect the LCC of a laser in the presence of excited states in QDs. Thus, experimental LCCs both with (see, e.g., Refs. 1, 8, 12, 16, 17, 19, and 20)

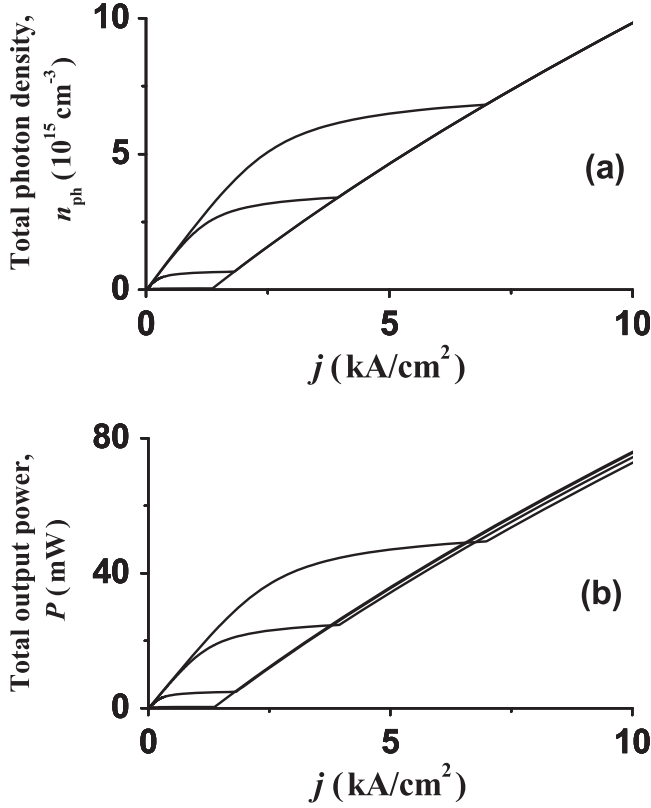


FIG. 9. Total photon density (a) and output power (b) of both ground- and excited-state lasing against injection current density. The kinks in the curves occur at $j = j_{th2}$ and show n_{ph1}^{pin} and P_1^{pin} [see Eqs. (36) and (37)]. The values of τ_{21} for different curves (from bottom to top) are 100, 10, 2, and 1 ps.

and without^{4,5,7} kinks were reported. Furthermore, even in the presence of a kink, the dependence of the ground-state power on the pump current is not always the same. It remains constant above the excited-state lasing threshold in some structures;^{8,19} in others, the ground-state power approaches its maximum and then rollover occurs.^{1,4,7,12,16,17}

3. Internal and external differential quantum efficiencies

For $j_{th1} \leq j < j_{th2}$, the internal quantum efficiency can be introduced for ground-state lasing only [see Eq. (29)]. Above j_{th2} , the internal efficiency can be introduced for both ground- and excited-state lasing.

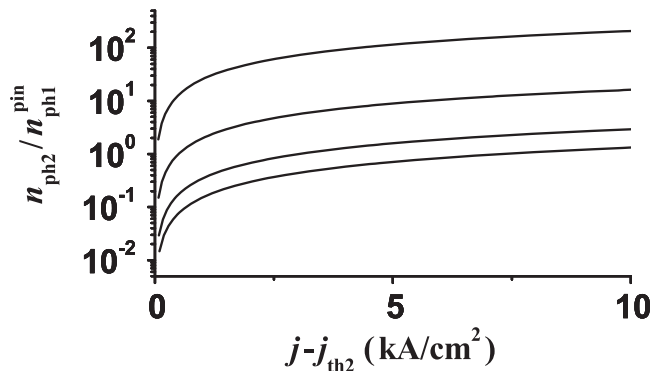


FIG. 10. n_{ph2}/n_{ph1}^{pin} ratio against excess of the injection current density over the threshold current density for excited-state lasing. The values of τ_{21} for different curves (from the top down) are 100, 10, 2, and 1 ps.

For $j \geq j_{th2}$, the current density of stimulated recombination via the ground-state in QDs is pinned [see Eq. (52)]. Hence the internal efficiency for ground-state lasing simply decreases as the reciprocal of $j - j_{th1}$

$$\eta_{int1} = \frac{j_{stim1}^{pin}}{j - j_{th1}}. \quad (55)$$

Above j_{th2} , the internal efficiency for excited-state lasing is

$$\eta_{int2} = \frac{j_{stim2}}{j - j_{th2}}. \quad (56)$$

With (C5) for $j_{stim2}(j)$, the following expression is obtained for η_{int2} :

$$\eta_{int2} = \frac{1}{\frac{1}{2} + \frac{j_{th2}^{OCL}}{j_{capt,th2}} + \sqrt{\left(\frac{1}{2} + \frac{j_{th2}^{OCL}}{j_{capt,th2}}\right)^2 + \frac{j_{th2}^{OCL}}{j_{capt,th2}} \frac{j - j_{th2}}{j_{capt,th2}}}}, \quad (57)$$

which is similar to Eq. (31) of Ref. 22.

Fig. 11 shows the internal differential quantum efficiency against injection current density.

Above j_{th2} , P_1 is pinned at P_1^{pin} . Hence, the external differential quantum efficiency is given by the derivative of only P_2 with respect to j

$$\eta_{ext2} = \frac{1}{e} \frac{1}{\hbar\omega_2 S} \frac{\partial P_2}{\partial j} = \frac{\partial j_{stim2}}{\partial j}. \quad (58)$$

With Eq. (48), we have

$$\begin{aligned} \eta_{ext2} &= \frac{1}{\sqrt{\left(1 + 2 \frac{j_{th2}^{OCL}}{j_{capt,th2}}\right)^2 + 4 \frac{j_{th2}^{OCL}}{j_{capt,th2}} \frac{j - j_{th2}}{j_{capt,th2}}}}} \\ &= \frac{1}{\sqrt{1 + 4 \frac{j_{th2}^{OCL}}{j_{capt,th2}^2} (j_{esc2}^{OCL} + j)}}. \end{aligned} \quad (59)$$

Neither j_{esc2}^{OCL} nor the ratio $j_{th2}^{OCL}/j_{capt,th2}^2$ [see Eqs. (49) and (50)] depends on τ_{21} . Hence η_{ext2} does not depend on τ_{21} for $j \geq j_{th2}$; this is also seen from Fig. 12—the curves for η_{ext2} calculated at different values of τ_{21} merge together above j_{th2} .

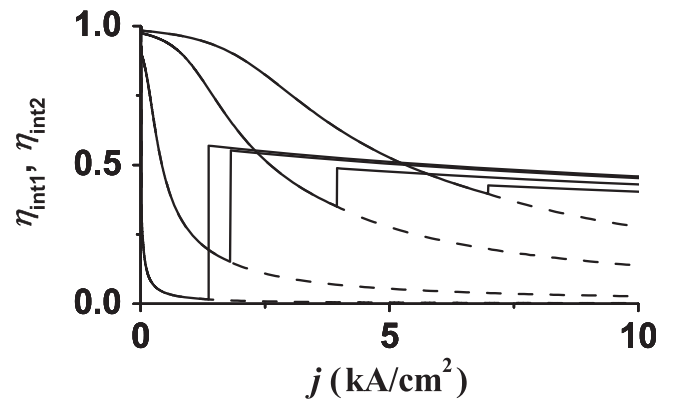


FIG. 11. Internal differential quantum efficiency against injection current density. The values of τ_{21} for different curves (from left to right) are 100, 10, 2, and 1 ps. The solid curves show η_{int1} for $j \leq j_{th2}$ and η_{int2} for $j > j_{th2}$. The dashed curves show η_{int1} for $j \geq j_{th2}$.

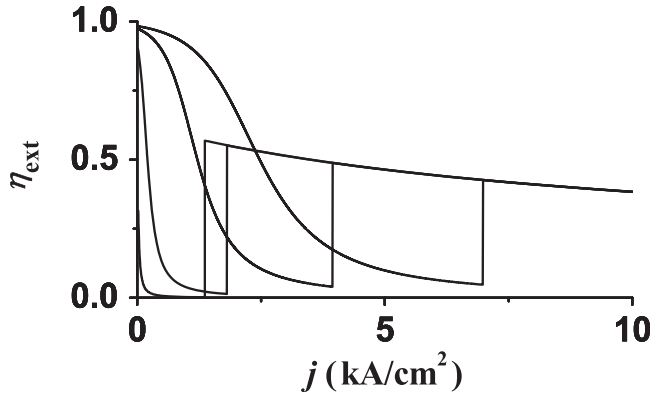


FIG. 12. External differential quantum efficiency against injection current density. The values of τ_{21} for different curves (from left to right) are 100, 10, 2, and 1 ps. The steps in the curves occur at $j = j_{th2}$.

At $j = j_{th2}$, we have from Eqs. (57) and (59)

$$\eta_{int,th2} = \eta_{ext,th2} = \frac{1}{1 + 2 \frac{j_{th2}^{OCL}}{j_{capt,th2}}}. \quad (60)$$

In the absence of piezoelectric effects (which is not always the case—see, e.g., Refs. 32 and 33), transitions from excited electron- to excited hole-states are degenerate in pyramidal (with square base), cubic, and cylindrical QDs; hence, the maximum gain for excited-state transitions $g_2^{\max}$ could be higher than that for ground-state transitions $g_1^{\max}$. If such would be the case, the findings of Section III should be modified as follows. If the mirror reflectivity at the ground- and excited-state transition energies is the same ($R_1 = R_2$ and hence $\beta_1 = \beta_2$) and both $g_1^{\max}$ and $g_2^{\max}$ are higher than the mirror loss, excited-state lasing will turn on first with increasing pump current and then so will do ground-state lasing. If $R_1 \neq R_2$ and $g_1^{\max}$ and $g_2^{\max}$ are higher than the mirror

losses β_1 and β_2 , respectively, whether lasing will first occur via ground- or excited-state transitions will be determined by the lower of the two threshold currents. If $j_{th1} < j_{th2}$, Section III remains unchanged. If $j_{th2} < j_{th1}$, Section III will still apply if the terms “ground” and “excited” and all the related quantities are interchanged throughout the text and expressions.

IV. EXCITED-STATE LASING ONLY: LOW GAIN FOR GROUND-STATE TRANSITIONS AND HIGH GAIN FOR EXCITED-STATE TRANSITIONS

In Section III, we considered the situation when inequality Eq. (40) is satisfied. If the reverse condition holds, there will be excited-state lasing only. Qualitatively, this situation is similar to that for single-state QDs considered in Refs. 22 and 23.

Dropping out the stimulated recombination term in Eq. (3), the rate equation for the QD ground-state occupancy reads as

$$\frac{f_2(1-f_1)}{\tau_{21}} - \frac{f_1(1-f_2)}{\tau_{12}} - \frac{f_1^2}{\tau_{QD1}} = 0. \quad (61)$$

The rate equations for the QD excited-state occupancy, free-carrier density in the OCL, and photon density are given by Eqs. (41), (1), and (34), respectively.

A. Solutions of rate equations

At and above the excited-state lasing threshold, the excited-state occupancy f_2 is pinned at its threshold value $f_{2,th2}$ given by Eq. (35). From Eq. (61) (which is a quadratic equation in f_1), we find that the ground-state occupancy f_1 is also pinned at the following value:

$$f_{1,th2} = \frac{f_{2,th2}}{\frac{1}{2} \left[f_{2,th2} + \frac{\tau_{21}}{\tau_{12}} (1 - f_{2,th2}) \right] + \sqrt{\frac{1}{4} \left[f_{2,th2} + \frac{\tau_{21}}{\tau_{12}} (1 - f_{2,th2}) \right]^2 + \frac{\tau_{21}}{\tau_{QD1}} f_{2,th2}}}. \quad (62)$$

With Eq. (61), we have from Eq. (41)

$$n_{OCL} = n_2 \frac{f_{2,th2}}{1 - f_{2,th2}} + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_{2,th2}} \left(\frac{f_{1,th2}^2}{\tau_{QD1}} + \frac{f_{2,th2}^2}{\tau_{QD2}} + \frac{b}{N_S} \frac{n_{ph2}}{\tau_{ph2}} \right). \quad (63)$$

With Eqs. (41) and (61), we have from Eq. (1)

$$j = ebBn_{OCL}^2 + eN_S \frac{f_{1,th2}^2}{\tau_{QD1}} + eN_S \frac{f_{2,th2}^2}{\tau_{QD2}} + eb \frac{n_{ph2}}{\tau_{ph2}}, \quad (64)$$

where n_{OCL} is given by Eq. (63).

At the excited-state lasing threshold (when $n_{ph2} = 0$), we have from Eqs. (63) and (64)

$$n_{OCL,th2} = n_2 \frac{f_{2,th2}}{1 - f_{2,th2}} + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_{2,th2}} \left(\frac{f_{1,th2}^2}{\tau_{QD1}} + \frac{f_{2,th2}^2}{\tau_{QD2}} \right), \quad (65)$$

$$j_{th2} = ebBn_{OCL,th2}^2 + eN_S \frac{f_{1,th2}^2}{\tau_{QD1}} + eN_S \frac{f_{2,th2}^2}{\tau_{QD2}}. \quad (66)$$

Eqs. (63)–(66) could also be immediately obtained from Eqs. (42) and (44)–(46) by putting there $n_{ph1}^{\text{pin}} = 0$ and $f_1 = f_{1,th2}$.

With Eqs. (47) and (49) for j_{stim2} and $j_{capt,th2}$, Eqs. (63) and (64) can be rewritten in the form of Eqs. (C2) and (C3) (see Appendix C). Consequently, the stimulated recombination current density j_{stim2} , photon density n_{ph2} , output power P_2 , internal and external efficiencies η_{int2} and η_{ext2} will be given as functions of the injection current density by Eqs. (C5), (48), (57), and (59).

V. CONCLUSIONS

We developed a theory of operating characteristics of QD lasers in the presence of excited states in QDs. We considered three possible situations for lasing.

Under the conditions of ground-state lasing only (high gain for ground-state transitions and low gain for excited-state transitions), the output power asymptotically approaches its maximum (saturation) value with increasing injection current. A simple universal expression is obtained for the normalized maximum power as a function of the normalized intradot relaxation time.

Under the conditions of both ground- and excited-state lasing, the output power of ground-state lasing remains pinned above the excited-state lasing threshold while the power of excited-state lasing increases. At the excited-state lasing threshold, a kink appears in the LCC. Above the excited-state lasing threshold, the free carrier density in the OCL, current of total stimulated recombination via the ground and excited states in QDs, and external differential quantum efficiency become independent of the intradot relaxation time. As in the case of ground-state lasing only, there exists a cut-off value of the intradot relaxation time, at which the output power of ground-state lasing vanishes. With increasing relaxation time, the threshold current for ground-state lasing increases while that for excited-state lasing decreases. At the cut-off value of the relaxation time, the threshold currents for ground- and excited-state lasing become the same.

The case of excited-state lasing only (low gain for ground-state transitions and high gain for excited-state transitions) is qualitatively similar to that for single-state QDs. Above the lasing threshold (which is the excited-state lasing threshold in this case), the role of ground-state transitions is simply reduced to adding an extra component (current of spontaneous recombination via the ground-state in QDs) to the threshold current. The free-carrier density in the OCL is also increased due to spontaneous recombination via the ground-state in QDs.

ACKNOWLEDGMENTS

This work was supported by the U.S. Army Research Office (Grant No. W911NF-13-1-0445). Y.W. also acknowledges the Fundamental Research Funds for the Central Universities (Grant No. 2015QNA10) for support of this work.

APPENDIX A: GROUND-STATE LASING ONLY: LCC AT HIGH INJECTION CURRENTS

With Eqs. (8) and (22) for τ_{ph1} and $\tau_{21}^{\max}$, we have from Eq. (25)

$$\frac{b}{N_S} \frac{1}{\tau_{ph1}} = \frac{1}{\tau_{21}} - \frac{1}{\tau_{21}^{\max}} (1 - f_1). \quad (A1)$$

With Eqs. (A1) and (22), Eq. (7) can be written as

$$1 - f_2 = \frac{(1 - f_1) \left(1 - \frac{\tau_{21}}{\tau_{21}^{\max}}\right) \left(1 - \frac{n_{ph1}}{n_{ph1}^{\max}}\right)}{1 - f_1 + \frac{\tau_{21}}{\tau_{12}} f_1}. \quad (A2)$$

Using Eq. (A2) for $1 - f_2$ in the denominators in Eq. (10), we have

$$n_{OCL} = \left\{ n_2 f_2 + \frac{1}{\sigma_2 V_n} \left[\frac{f_2 (1 - f_1)}{\tau_{21}} - \frac{f_1 (1 - f_2)}{\tau_{12}} + \frac{f_2^2}{\tau_{QD2}} \right] \right\} \times \frac{1 - f_1 + \frac{\tau_{21}}{\tau_{12}} f_1}{(1 - f_1) \left(1 - \frac{\tau_{21}}{\tau_{21}^{\max}}\right)} \frac{1}{1 - \frac{n_{ph1}}{n_{ph1}^{\max}}}. \quad (A3)$$

As n_{ph1} tends to $n_{ph1}^{\max}$ (i.e., as $f_2 \rightarrow 1$), the free carrier density n_{OCL} in the OCL tends to infinity. The asymptotic expression for n_{OCL} is apparent from Eq. (A3)

$$n_{OCL} = \left[n_2 + \frac{1}{\sigma_2 V_n} \left(\frac{1 - f_1}{\tau_{21}} + \frac{1}{\tau_{QD2}} \right) \right] \times \frac{1 - f_1 + \frac{\tau_{21}}{\tau_{12}} f_1}{(1 - f_1) \left(1 - \frac{\tau_{21}}{\tau_{21}^{\max}}\right)} \frac{1}{1 - \frac{n_{ph1}}{n_{ph1}^{\max}}} \propto \frac{1}{1 - \frac{n_{ph1}}{n_{ph1}^{\max}}}. \quad (A4)$$

At high n_{OCL} , the first term in the right-hand side of Eq. (12) becomes dominant. Thus Eq. (12) becomes

$$j = eb n_{OCL}^2 \propto \frac{1}{\left(1 - \frac{n_{ph1}}{n_{ph1}^{\max}}\right)^2}. \quad (A5)$$

With Eqs. (A4) and (A5), we obtain the following asymptotic expression for the LCC at high j :

$$\frac{P_1}{P_1^{\max}} = \frac{n_{ph1}}{n_{ph1}^{\max}} = 1 - \frac{1 - f_1 + \frac{\tau_{21}}{\tau_{12}} f_1}{(1 - f_1) \left(1 - \frac{\tau_{21}}{\tau_{21}^{\max}}\right)} \times \left[n_2 + \frac{1}{\sigma_2 V_n} \left(\frac{1}{\tau_{QD2}} + \frac{1 - f_1}{\tau_{21}} \right) \right] \sqrt{\frac{ebB}{j}} = 1 - \frac{\text{const}}{\sqrt{j}}. \quad (A6)$$

The asymptotic expression for the free carrier density in the OCL versus j is apparent from Eq. (A5)

$$n_{OCL} = \sqrt{\frac{j}{ebB}}. \quad (A7)$$

At high current density (when $n_{ph1} \rightarrow n_{ph1}^{\max}$), the asymptotic expression for η_{int1} is easily obtained from Eq. (27)

$$\eta_{int1} = \frac{j_{stim1}^{\max}}{j} = \frac{eb n_{ph1}^{\max}}{j}, \quad (A8)$$

where we introduced the saturation value $j_{\text{stim1}}^{\text{max}}$ of the current density of stimulated recombination via the ground-state in QDs.

With Eq. (30) and the asymptotic expression Eq. (A6) for P_1 , we have for the asymptotic expression for η_{ext1} at high j

$$\eta_{\text{ext1}} = \frac{1}{2} \frac{1 - f_1 + \frac{\tau_{21}}{\tau_{12}} f_1}{(1 - f_1) \left(1 - \frac{\tau_{21}}{\tau_{21}^{\text{max}}}\right)} \times \left[n_2 + \frac{1}{\sigma_2 v_n} \left(\frac{1}{\tau_{\text{QD2}}} + \frac{1 - f_1}{\tau_{21}} \right) \right] j_{\text{stim1}}^{\text{max}} \times \frac{\sqrt{ebB}}{j^{3/2}} \propto \frac{1}{j^{3/2}}. \quad (\text{A9})$$

APPENDIX B: GROUND-STATE LASING ONLY: INSTANTANEOUS INTRADOT RELAXATION

In the limiting case of instantaneous intradot relaxation ($\tau_{21} = 0$), Eqs. (7), (11), and (13) become

$$f_2 = \frac{f_1}{f_1 + (1 - f_1) \exp\left(\frac{\Delta}{T}\right)}, \quad (\text{B1})$$

$$n_{\text{OCL}} = n_2 \frac{f_1}{1 - f_1} \exp\left(-\frac{\Delta}{T}\right) + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_1} \times \exp\left(-\frac{\Delta}{T}\right) \left[f_1 + (1 - f_1) \exp\left(\frac{\Delta}{T}\right) \right] \times \left(f_1^2 \left\{ \frac{1}{\tau_{\text{QD1}}} + \frac{1}{\tau_{\text{QD2}}} \frac{1}{\left[f_1 + (1 - f_1) \exp\left(\frac{\Delta}{T}\right) \right]^2} \right\} + \frac{b}{N_S} \frac{n_{\text{ph1}}}{\tau_{\text{ph1}}} \right), \quad (\text{B2})$$

$$j = ebB n_{\text{OCL}}^2 + eN_S f_1^2 \times \left\{ \frac{1}{\tau_{\text{QD1}}} + \frac{1}{\tau_{\text{QD2}}} \frac{1}{\left[f_1 + (1 - f_1) \exp\left(\frac{\Delta}{T}\right) \right]^2} \right\} + eb \frac{n_{\text{ph1}}}{\tau_{\text{ph1}}}. \quad (\text{B3})$$

In Eqs. (B1)–(B3), Eq. (9) was used.

APPENDIX C: GROUND- AND EXCITED-STATE LASING: CURRENT DENSITY OF STIMULATED RECOMBINATION VIA THE EXCITED-STATE IN QDS

With Eq. (45), we can write Eq. (42) for the free-carrier density in the OCL as

$$n_{\text{OCL}} = n_{\text{OCL,th2}} + \frac{1}{\sigma_2 v_n} \frac{1}{1 - f_{2,\text{th2}}} \frac{b}{N_S} \frac{n_{\text{ph2}}}{\tau_{\text{ph2}}}, \quad (\text{C1})$$

or, using Eqs. (47) and (49), as

$$n_{\text{OCL}} = n_{\text{OCL,th2}} \left(1 + \frac{j_{\text{stim2}}}{j_{\text{capt,th2}}} \right). \quad (\text{C2})$$

With Eqs. (46) and (47), we can rewrite Eq. (44) as

$$j = j_{\text{th2}} + ebB(n_{\text{OCL}}^2 - n_{\text{OCL,th2}}^2) + j_{\text{stim2}}. \quad (\text{C3})$$

Substituting n_{OCL} from Eq. (C2) into Eq. (C3), we have the following quadratic equation in j_{stim2} :

$$\frac{j - j_{\text{th2}}}{j_{\text{th2}}^{\text{OCL}}} = \left(1 + \frac{j_{\text{stim2}}}{j_{\text{capt,th2}}} \right)^2 - 1 + \frac{j_{\text{stim2}}}{j_{\text{th2}}^{\text{OCL}}}. \quad (\text{C4})$$

The solution of this equation is

$$j_{\text{stim2}} = j_{\text{capt,th2}} \times \left[\sqrt{\left(1 + \frac{j_{\text{capt,th2}}}{2 j_{\text{th2}}^{\text{OCL}}} \right)^2 + \frac{j - j_{\text{th2}}}{j_{\text{th2}}^{\text{OCL}}} - \left(1 + \frac{j_{\text{capt,th2}}}{2 j_{\text{th2}}^{\text{OCL}}} \right)} \right]. \quad (\text{C5})$$

With Eq. (C5), we have for the free-carrier density from Eq. (C2)

$$n_{\text{OCL}} = n_{\text{OCL,th2}} \left[\sqrt{\left(1 + \frac{j_{\text{capt,th2}}}{2 j_{\text{th2}}^{\text{OCL}}} \right)^2 + \frac{j - j_{\text{th2}}}{j_{\text{th2}}^{\text{OCL}}} - \frac{j_{\text{capt,th2}}}{2 j_{\text{th2}}^{\text{OCL}}}} \right]. \quad (\text{C6})$$

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Optical power of semiconductor lasers with a low-dimensional active region

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(Received 28 November 2013; accepted 20 December 2013; published online 13 January 2014)

A comprehensive analytical model for the operating characteristics of semiconductor lasers with a low-dimensional active region is developed. Particular emphasis is given to the effect of capture delay of both electrons and holes from a bulk optical confinement region into a quantum-confined active region and an extended set of rate equations is used. We derive a closed-form expression for the internal quantum efficiency as an explicit function of the injection current and parameters of a laser structure. Due to either electron or hole capture delay, the internal efficiency decreases with increasing injection current above the lasing threshold thus causing sublinearity of the light-current characteristic of a laser. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4861408>]

I. INTRODUCTION

A number of processes are involved in the operation of semiconductor lasers with a quantum-confined active region.^{1,2} Among them is the carrier capture from a bulk reservoir region [optical confinement layer (OCL)] into a low-dimensional active region.^{3–16} Analytical models can provide insights into these processes and be used to optimize the laser structures to achieve desired operating characteristics. To develop analytical approaches, certain assumptions and simplifications should be made. Reduction of the number of rate equations describing the carrier dynamics in a laser structure is one of such simplifications. Particularly, the use of rate equations for carriers of only one type can considerably simplify the analysis. This is commonly done in the literature—see, e.g., Refs. 14 and 15, in which just three rate equations were exploited (one for free carriers in the OCL, one for carriers confined in the active region, and one for photons) to study the laser power characteristics.

In this paper, we analyze the effect of both electron and hole capture delay from the OCL into a low-dimensional active region on the internal quantum efficiency η_{int} and output optical power of semiconductor lasers. To describe such a general situation, we use rate equations both for electrons and holes. Remarkably, while the number of equations is now five (two for free electrons and holes in the OCL, two for electrons and holes confined in the active region, and one for photons), we are able to derive a closed-form expression for η_{int} as an explicit function of the injection current, electron and hole capture velocities, and parameters of the laser structure.

II. THEORETICAL MODEL

While our model is general and applies to semiconductor lasers with a quantum-confined active region of any type [quantum wells (QWs), quantum wires, or quantum dots], we use here a QW laser to illustrate the model. We study the

steady-state characteristics of the laser and, correspondingly, use the following set of steady-state rate equations:

for free electrons in the OCL [$b(\partial n^{\text{OCL}}/\partial t) = 0$],

$$\frac{j}{e} + N_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{\text{n,esc}}} - N_{\text{QW}} v_{\text{n,capt},0} (1 - f_{\text{n}}) n^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0, \quad (1)$$

for free holes in the OCL [$b(\partial p^{\text{OCL}}/\partial t) = 0$],

$$\frac{j}{e} + N_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{\text{p,esc}}} - N_{\text{QW}} v_{\text{p,capt},0} (1 - f_{\text{p}}) p^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0, \quad (2)$$

for electrons confined in the QWs ($\partial n^{\text{QW}}/\partial t = 0$),

$$v_{\text{n,capt},0} (1 - f_{\text{n}}) n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{\text{n,esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - v_{\text{g}} g^{\text{max}} (f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} = 0, \quad (3)$$

for holes confined in the QWs ($\partial p^{\text{QW}}/\partial t = 0$),

$$v_{\text{p,capt},0} (1 - f_{\text{p}}) p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{\text{p,esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - v_{\text{g}} g^{\text{max}} (f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} = 0, \quad (4)$$

and for photons ($\partial N/\partial t = 0$),

$$v_{\text{g}} N_{\text{QW}} g^{\text{max}} (f_{\text{n}} + f_{\text{p}} - 1) N - v_{\text{g}} (\beta + \alpha_{\text{int}}) N = 0. \quad (5)$$

We first list the quantities to be found from the solution of Eqs. (1)–(5): n^{OCL} and p^{OCL} are the free-electron and -hole densities in the OCL, n^{QW} and p^{QW} are the two-dimensional (2D) densities of electrons and holes confined in each of the QWs, and N is the number of photons in the lasing mode. f_{n} is the occupancy of the lower edge of the electron

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quantum-confinement sub-band in the QW and f_p is the occupancy of the upper edge of the hole quantum-confinement sub-band in the QW. The occupancies f_n and f_p are not independent quantities themselves—they are expressed in terms of the 2D-carrier densities n^{QW} and p^{QW} as follows:^{17,18}

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_c^{2\text{D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_v^{2\text{D}}}\right), \quad (6)$$

where $N_{c,v}^{2\text{D}} = m_{c,v}^{\text{QW}} T / (\pi \hbar^2)$ are the 2D effective densities of states in the conduction and valence bands in the QW, $m_{c,v}^{\text{QW}}$ are the electron and hole effective masses in the QW, and the temperature T is measured in units of energy.

We now list the parameters entering into Eqs. (1)–(5): b is the thickness of the OCL, j is the injection current density, e is the electron charge, N_{QW} is the number of QWs, $B_{3\text{D}}$ and $B_{2\text{D}}$ are the spontaneous radiative recombination constants for the bulk (OCL) and 2D regions (QWs) measured in units of cm^3/s and cm^2/s , respectively, v_g is the group velocity of light, g^{max} ($f_n + f_p - 1$) is the modal gain provided by each of

the QWs, g^{max} is the maximum value of the modal gain in each QW, $S = WL$ is the cross-section of the junction, W is the lateral size of the device, L is the cavity length, $\beta = (1/L)\ln(1/R)$ is the mirror loss, R is the facet reflectivity, and α_{int} is the internal optical loss.

We denoted the thermal escape times of electrons and holes from a QW to the OCL by $\tau_{n,p,\text{esc}}$ and the capture velocities from the OCL into an empty (at $f_{n,p} = 0$) QW by $v_{n,p,\text{capt},0}$. It is the capture velocity (measured in cm/s), which is a proper parameter describing the carrier capture into a QW.^{6–11,14–16} The thermal escape time and the capture velocity are related to each other (see Ref. 16 for the general expression relating $\tau_{n,p,\text{esc}}$ and $v_{n,p,\text{capt},0}$). For undoped OCL and QW, the relation is

$$\tau_{n,\text{esc}} = \frac{1}{v_{n,\text{capt},0}(1-f_n)} \frac{N_c^{2\text{D}}}{n_1}, \quad \tau_{p,\text{esc}} = \frac{1}{v_{p,\text{capt},0}(1-f_p)} \frac{N_v^{2\text{D}}}{p_1}. \quad (7)$$

In Eq. (7), the quantities n_1 and p_1 are

$$n_1 = N_c^{3\text{D}} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right), \quad p_1 = N_v^{3\text{D}} \exp\left(-\frac{\Delta E_v - \varepsilon_p^{\text{QW}}}{T}\right), \quad (8)$$

where $N_{c,v}^{3\text{D}} = 2[m_{c,v}^{\text{OCL}} T / (2\pi \hbar^2)]^{3/2}$ are the 3D effective densities of states in the conduction and valence bands in the OCL, $m_{c,v}^{\text{OCL}}$ are the electron and hole effective masses in the OCL, $\Delta E_{c,v}$ are the conduction and valence band offsets between the OCL and the QW, and $\varepsilon_{n,p}^{\text{QW}}$ are the energies of the electron- and hole-sub-band edges in the QW.

Solving the set of rate equations (1)–(5) and, particularly, finding the number of photons N as a function of the injection current density j will allow us to calculate the light-current characteristic (LCC) of the laser (the output optical power P versus j),

$$P(j) = \hbar \omega v_g \beta N(j), \quad (9)$$

where $\hbar \omega$ is the photon energy.

Introducing the current density of stimulated recombination in the laser active region (i.e., in the QWs) and the internal differential quantum efficiency,¹⁹

$$j_{\text{stim}} = e N_{\text{QW}} v_g g^{\text{max}} (f_n + f_p - 1) \frac{N}{S} = e v_g (\beta + \alpha_{\text{int}}) \frac{N}{S}, \quad (10)$$

$$\eta_{\text{int}} = \frac{j_{\text{stim}}}{j - j_{\text{th}}}, \quad (11)$$

where j_{th} is the threshold current density [see Eq. (A14)], we can write Eq. (9) as follows:

$$P(j) = \frac{\hbar \omega}{e} S j_{\text{stim}}(j) \frac{\beta}{\beta + \alpha_{\text{int}}} = \frac{\hbar \omega}{e} S (j - j_{\text{th}}) \eta_{\text{int}}(j) \frac{\beta}{\beta + \alpha_{\text{int}}}. \quad (12)$$

When writing the second equation in Eq. (10), we used the lasing condition (5).

III. DISCUSSION

In Appendix, the set of rate equations (1)–(5) is solved. The following closed-form expression is derived for η_{int} , which applies to QW lasers, quantum wire lasers, and quantum dot lasers:

$$\eta_{\text{int}}(j) = \left\{ \frac{1}{2} + \frac{1}{2} \left(\frac{1}{j_{\text{capt},n,\text{th}}} + \frac{1}{j_{\text{capt},p,\text{th}}} \right) j_{\text{spon},\text{th}}^{\text{OCL}} + \sqrt{\left[\frac{1}{2} + \frac{1}{2} \left(\frac{1}{j_{\text{capt},n,\text{th}}} + \frac{1}{j_{\text{capt},p,\text{th}}} \right) j_{\text{spon},\text{th}}^{\text{OCL}} \right]^2 + \frac{j_{\text{spon},\text{th}}^{\text{OCL}}}{j_{\text{capt},n,\text{th}} j_{\text{capt},p,\text{th}}}} (j - j_{\text{th}}) \right\}^{-1}, \quad (13)$$

where $j_{\text{spn,th}}^{\text{OCL}}$ is the spontaneous recombination current density in the OCL at the lasing threshold [see Eq. (A15)] and $j_{\text{capt,n,th}}$, $j_{\text{capt,p,th}}$ are the current densities of electron and hole

capture from the OCL into a quantum-confined active region at the lasing threshold.

Equation (13) can be rewritten as follows:

$$\eta_{\text{int}}(j) = \left[\frac{1}{2} + \frac{j_{\text{spn,th}}^{\text{OCL}}}{\langle j_{\text{capt,th}} \rangle_{\text{harmon}}} + \sqrt{\left(\frac{1}{2} + \frac{j_{\text{spn,th}}^{\text{OCL}}}{\langle j_{\text{capt,th}} \rangle_{\text{harmon}}} \right)^2 + \frac{j_{\text{spn,th}}^{\text{OCL}}}{\langle j_{\text{capt,th}} \rangle_{\text{geom}}} \frac{j - j_{\text{th}}}{\langle j_{\text{capt,th}} \rangle_{\text{geom}}}} \right]^{-1}, \quad (14)$$

where we introduced the harmonic and geometric means of the current densities of electron and hole capture at the lasing threshold,

$$\frac{1}{\langle j_{\text{capt,th}} \rangle_{\text{harmon}}} = \frac{1}{2} \left(\frac{1}{j_{\text{capt,n,th}}} + \frac{1}{j_{\text{capt,p,th}}} \right), \quad (15)$$

$$\langle j_{\text{capt,th}} \rangle_{\text{geom}} = \sqrt{j_{\text{capt,n,th}} j_{\text{capt,p,th}}}. \quad (16)$$

If the electron and hole parameters (capture velocities, effective masses, conduction and valence band offsets) were the same, the rate equations (2) and (4) would become identical to the rate equations (1) and (3), respectively, and our “five rate equations” model would naturally reduce to the

“three rate equations” model of Refs. 14 and 15. The internal differential quantum efficiency would be given by Eq. (14) in which both $\langle j_{\text{capt,th}} \rangle_{\text{harmon}}$ and $\langle j_{\text{capt,th}} \rangle_{\text{geom}}$ would simply turn into the capture current density at the lasing threshold, which would be the same for electrons and holes. Hence, we see that, in the expression for η_{int} in the “five rate equations model,” the asymmetry between the electron and hole capture is effectively taken into account by the harmonic and geometric means of the current densities of electron and hole capture at the lasing threshold. In case of significant asymmetry, these mean quantities $\langle j_{\text{capt,th}} \rangle_{\text{harmon}}$ and $\langle j_{\text{capt,th}} \rangle_{\text{geom}}$ differ considerably from either $j_{\text{capt,n,th}}$ and $j_{\text{capt,p,th}}$.

For QW lasers, using Eqs. (A25) and (A26) for $j_{\text{capt,n,th}}$ and $j_{\text{capt,p,th}}$, we can rewrite Eq. (13) as follows:

$$\eta_{\text{int}}(j) = \left(\frac{1}{2} + \frac{1}{2} \frac{bB_{3D}}{N_{\text{QW}}} \left[\frac{p_{\text{th}}^{\text{OCL}}}{v_{\text{capt,n,0}}(1-f_n)} + \frac{n_{\text{th}}^{\text{OCL}}}{v_{\text{capt,p,0}}(1-f_p)} \right] + \sqrt{\left\{ \frac{1}{2} + \frac{1}{2} \frac{bB_{3D}}{N_{\text{QW}}} \left[\frac{p_{\text{th}}^{\text{OCL}}}{v_{\text{capt,n,0}}(1-f_n)} + \frac{n_{\text{th}}^{\text{OCL}}}{v_{\text{capt,p,0}}(1-f_p)} \right] \right\}^2 + \frac{bB_{3D}}{eN_{\text{QW}}^2 v_{\text{capt,n,0}} v_{\text{capt,p,0}}(1-f_n)(1-f_p)} (j - j_{\text{th}})} \right)^{-1}, \quad (17)$$

where $n_{\text{th}}^{\text{OCL}}$ and $p_{\text{th}}^{\text{OCL}}$ are the threshold values of n^{OCL} and p^{OCL} , which are given by Eqs. (A12) and (A13), respectively.

Using Eq. (17), we calculate the LCC of the laser [Eq. (12)], the free-electron and -hole densities and spontaneous recombination current density in the OCL [Eqs. (A27), (A28), and (A2)], the stimulated recombination current density [from Eq. (11)], and the number of photons in the lasing mode [from Eq. (10)].

For illustration of our theoretical model, we use an InGaAs/GaAs/AlGaAs heterostructure laser with a broadened waveguide.²⁰ The QW material is $\text{In}_{0.28}\text{Ga}_{0.72}\text{As}$ and the QW thickness is 80 Å. The lasing wavelength is 1.044 μm. The materials of the waveguide region (OCL) and cladding layers are GaAs and $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$, respectively. The thickness of the OCL $b = 1.7$ μm. The other parameters of the structure are: $L = 1.5$ mm, $R = 0.32$, $\beta = 7.6 \text{ cm}^{-1}$, $\alpha_{\text{int}} = 1 \text{ cm}^{-1}$, and $T = 300$ K. We consider structures with a single QW (Figs. 1–3) and multiple QWs (Figs. 4 and 5).

Figure 1 shows the free-electron (a) and -hole (b) densities in the OCL against injection current density j above

the lasing threshold for several values of the electron and hole capture velocities $v_{\text{n,p,capt,0}}$. Both n^{OCL} and p^{OCL} increase considerably with increasing j . As seen from Eqs. (A27) and (A28) [or Eqs. (A9) and (A10)], this increase is due to the fact that the electron and hole capture from the OCL into the QW is not instantaneous—in the case of instantaneous capture ($v_{\text{n,p,capt,0}} \rightarrow \infty$ and $j_{\text{capt,n,p,th}} \rightarrow \infty$), n^{OCL} and p^{OCL} would remain pinned at their threshold values $n_{\text{th}}^{\text{OCL}}$ and $p_{\text{th}}^{\text{OCL}}$. The comparison of curve 1 with curve 3 and curve 2 with curve 4 in Fig. 1(a) [and also curve 1 with curve 2 and curve 3 with curve 4 in Fig. 1(b)] shows that, at a given j , the higher is the capture velocity for carriers of one type (at a fixed capture velocity for carriers of the opposite type), the lower is the density of carriers of that type—naturally, the faster is the carrier capture into a low-dimensional active region, the fewer carriers are accumulated in a bulk reservoir region (OCL). At the same time, and notably, the density of carriers of the opposite type becomes higher—compare curve 1 with curve 2 and curve 3 with curve 4 in Fig. 1(a) [and also curve 1 with curve 3 and curve 2 with curve 4 in

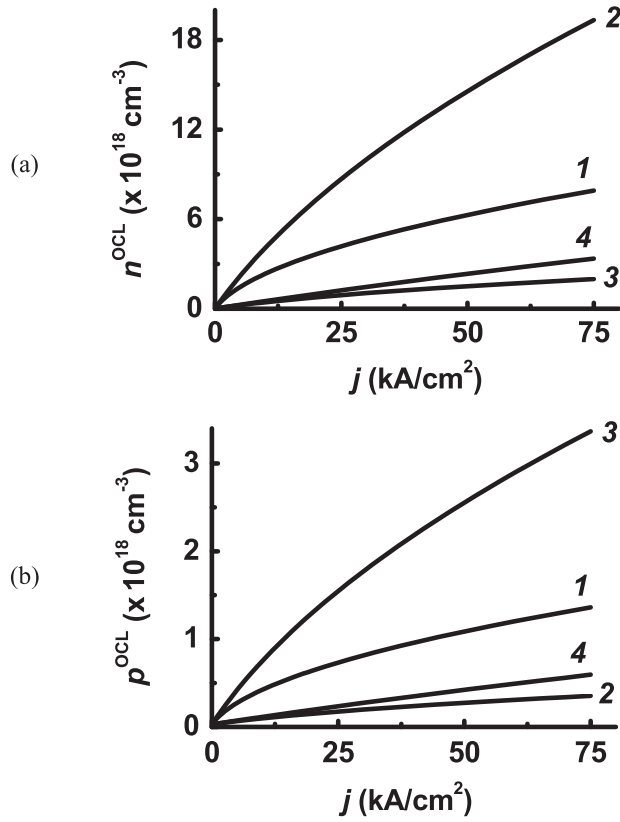


FIG. 1. Free-electron (a) and -hole (b) densities in the OCL vs. injection current density above the lasing threshold. Figs. 1–3 are for laser structures with a single QW. In Figs. 1–3, curves 1–4 correspond to the following values of the electron and hole capture velocities: 1— $v_{n,\text{capt},0} = 10^5$ cm/s, $v_{p,\text{capt},0} = 10^5$ cm/s; 2— $v_{n,\text{capt},0} = 10^5$ cm/s, $v_{p,\text{capt},0} = 10^6$ cm/s; 3— $v_{n,\text{capt},0} = 10^6$ cm/s, $v_{p,\text{capt},0} = 10^5$ cm/s; 4— $v_{n,\text{capt},0} = 10^6$ cm/s, $v_{p,\text{capt},0} = 10^6$ cm/s. The threshold current density values for curves 1–4 are: 1— $j_{\text{th}} = 82.9$ A/cm²; 2— $j_{\text{th}} = 81.4$ A/cm²; 3— $j_{\text{th}} = 76.1$ A/cm²; 4— $j_{\text{th}} = 75.7$ A/cm².

Fig. 1(b)]. These tendencies can be easily understood from Eqs. (A27) and (A28). For the sake of definiteness, let us consider increasing $v_{n,\text{capt},0}$ at a fixed $v_{p,\text{capt},0}$. Both η_{int} [see Fig. 3(a)] and $j_{\text{capt},n,\text{th}}$ increase with $v_{n,\text{capt},0}$ but $j_{\text{capt},n,\text{th}}$ increases faster, since it is directly proportional to $v_{n,\text{capt},0}$ [see Eq. (A25)]. Hence, the ratio $\eta_{\text{int}}/j_{\text{capt},n,\text{th}}$ in Eq. (A27) decreases and so does n^{OCL} [if $v_{n,\text{capt},0}$ is not very low, the change in n^{OCL} and j_{th} with $v_{n,\text{capt},0}$ can be neglected in Eq. (A27)—see the j_{th} values in the caption to Fig. 1]. In contrast to $\eta_{\text{int}}/j_{\text{capt},n,\text{th}}$, since $j_{\text{capt},p,\text{th}}$ is fixed, the ratio $\eta_{\text{int}}/j_{\text{capt},p,\text{th}}$ in Eq. (A28) increases with $v_{n,\text{capt},0}$ and so does p^{OCL} .

Figure 2 shows the current densities of spontaneous recombination in the OCL (a) and stimulated recombination in the QW (b) against injection current density j for several values of $v_{n,p,\text{capt},0}$. Both $j_{\text{spon}}^{\text{OCL}}$ and j_{stim} increase with j . Since the spontaneous radiative recombination is bimolecular [$j_{\text{spon}}^{\text{OCL}}$ is proportional to the product of n^{OCL} and p^{OCL} —see Eq. (A2)], $j_{\text{spon}}^{\text{OCL}}$ increases superlinearly [Fig. 2(a)]. As seen from Eq. (A1), the sum $j_{\text{spon}}^{\text{OCL}} + j_{\text{stim}}^{\text{QW}}$ is equal to $j - j_{\text{spon}}^{\text{QW}}$, i.e., is linear in j (as discussed in Appendix, the current density $j_{\text{spon}}^{\text{QW}}$ of spontaneous recombination in the QW is pinned at its threshold value and does not change with j). Hence, it follows that j_{stim} should be sublinear in j [Fig. 2(b)]. The faster either the electron or hole capture into the QW, the lower is $j_{\text{spon}}^{\text{OCL}}$ and,

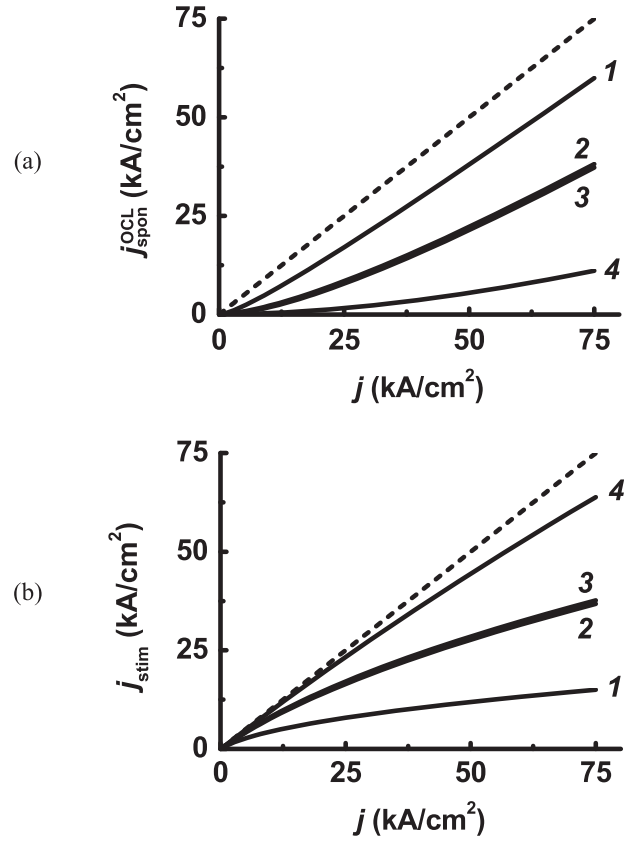


FIG. 2. Current densities of spontaneous recombination in the OCL (a) and stimulated recombination in the QW (b) vs. injection current density. The dashed lines show the injection current density itself.

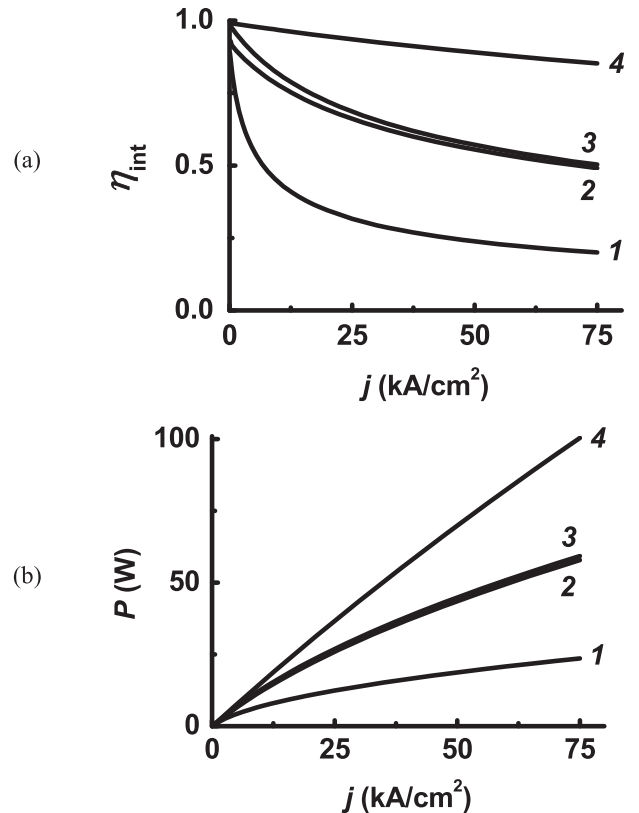


FIG. 3. Internal differential quantum efficiency (a) and output optical power (b) vs. injection current density.

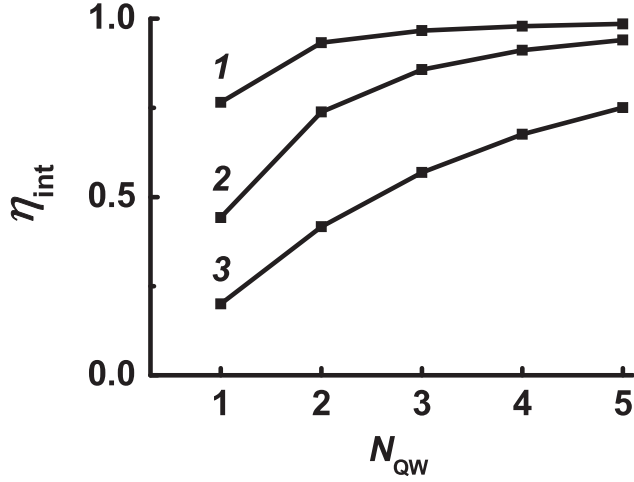


FIG. 4. Internal quantum efficiency vs. number of QWs for different values of the injection current density: 1— $j = 1 \text{ kA/cm}^2$; 2— $j = 10 \text{ kA/cm}^2$; 3— $j = 75 \text{ kA/cm}^2$. In Figs. 4 and 5, $v_{n,\text{capt},0} = 10^5 \text{ cm/s}$ and $v_{p,\text{capt},0} = 10^5 \text{ cm/s}$.

consequently, the higher is j_{stim} . In the ideal case of instantaneous capture of both electrons and holes, $j_{\text{spon}}^{\text{OCL}}$ would remain pinned at its threshold value $j_{\text{spon,th}}^{\text{OCL}}$, hence j_{stim} would be equal to $j - j_{\text{spon}}^{\text{QW}} - j_{\text{spon,th}}^{\text{OCL}}$, i.e., to $j - j_{\text{th}}$ [see Eq. (A14)], and the internal differential quantum efficiency [see Eq. (11)] would be equal to unity.

Figure 3 shows the internal quantum efficiency (a) and the output optical power (b) of the laser against injection current density for several values of $v_{n,\text{p,capt},0}$. As it follows from Eq. (11) and seen from Fig. 3(a), η_{int} decreases with j —since j_{stim} increases sublinearly with j [Fig. 2(b)], the ratio of j_{stim} to $j - j_{\text{th}}$ defining η_{int} is a decreasing function of j . As is clear from our discussion, the decrease of η_{int} is due to electron and hole capture delay into the QW and bimolecular character of spontaneous recombination in the OCL. Due to sublinearity of j_{stim} , so is the LCC of the laser [Fig. 3(b)]. Except for the case of fast capture of both electrons and holes into the QW [curves 4 in Figs. 3(a) and 3(b)], the drop of η_{int} with j and the sublinearity of the LCC are significant.

For the case of slow capture of both electrons and holes into individual QWs, Fig. 4 shows the internal quantum

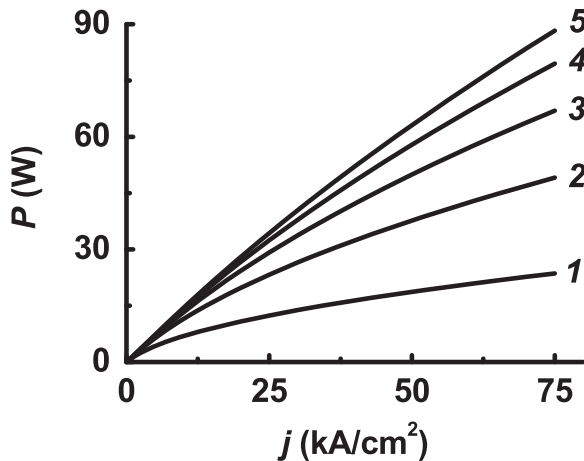


FIG. 5. Light-current characteristics of laser structures with multiple QWs: 1— $N_{\text{QW}} = 1$; 2— $N_{\text{QW}} = 2$; 3— $N_{\text{QW}} = 3$; 4— $N_{\text{QW}} = 4$; 5— $N_{\text{QW}} = 5$.

efficiency against number of QWs and Fig. 5 shows the LCC of laser structures with multiple QWs.

As seen from Eq. (17) and Fig. 4, η_{int} becomes higher with increasing number of QWs. This is because both the electron and hole capture current densities are directly proportional to N_{QW} [see Eqs. (A16) and (A17)] and hence increasing N_{QW} acts similarly to increasing both the electron and hole capture velocities $v_{n,\text{p,capt},0}$ into individual QWs.

Due to increase in η_{int} , the output optical power increases and the LCC becomes considerably more linear with increasing number of QWs (Fig. 5). Hence, if the carrier capture into individual QWs is not fast, the use of multiple QWs can effectively enhance the internal quantum efficiency and linearity of the LCC of the laser.

IV. CONCLUSIONS

We have developed a comprehensive analytical model for the operating characteristics of semiconductor lasers with a low-dimensional active region. Particular emphasis has been given to the effect of capture delay of both electrons and holes from a bulk OCL into a quantum-confined active region and an extended set of rate equations have been used. We have derived closed-form expressions for the electron and hole densities, current densities of spontaneous and stimulated recombination, internal quantum efficiency, and output optical power as explicit functions of the injection current and parameters of a laser structure. Due to either electron or hole capture delay, the internal efficiency decreases with increasing injection current above the lasing threshold thus causing sublinearity of the LCC of a laser. As an illustration of our model, we have considered QW lasers.

ACKNOWLEDGMENTS

L.V.A. acknowledges the U.S. Army Research Office (Grant No. W911NF-13-1-0445) for support of this work.

APPENDIX: DERIVATION OF SOLUTIONS TO RATE EQUATIONS

We derive here closed-form expressions for the solutions of the set of rate equations (1)–(5).

Using Eq. (3) or (4) in, respectively, Eq. (1) or (2), we can present the injection current density as a sum of the current densities of spontaneous recombination in the OCL and QWs and stimulated recombination in the QWs,

$$j = j_{\text{spon}}^{\text{OCL}} + j_{\text{spon}}^{\text{QW}} + j_{\text{stim}}, \quad (\text{A1})$$

where

$$j_{\text{spon}}^{\text{OCL}} = ebB_{3\text{D}}n^{\text{OCL}}p^{\text{OCL}}, \quad (\text{A2})$$

$$j_{\text{spon}}^{\text{QW}} = eN_{\text{QW}}B_{2\text{D}}n^{\text{QW}}p^{\text{QW}}, \quad (\text{A3})$$

and j_{stim} is given by Eq. (10).

Assuming that (i) the internal optical loss α_{int} does not depend on the free-carrier densities n^{OCL} and p^{OCL} in the OCL and (ii) charge neutrality holds in each QW ($n^{\text{QW}} = p^{\text{QW}}$) at the lasing threshold, we obtain from the lasing condition (5) that the electron and hole densities in the QWs do not depend on the injection current density and hence remain equal to each other also above the lasing threshold,

$$n^{\text{QW}} = p^{\text{QW}} = \text{const}(j). \quad (\text{A4})$$

In view of pinning the 2D-carrier densities, the spontaneous recombination current density in the QWs is also pinned above the lasing threshold,

$$j_{\text{spon}}^{\text{QW}} = eN_{\text{QW}}B_{2\text{D}}(n^{\text{QW}})^2 = \text{const}(j). \quad (\text{A5})$$

With Eqs. (6) and (A4), we have from Eq. (5) the following equation for finding n^{QW} :

$$N_{\text{QW}}g^{\text{max}} \left[1 - \exp\left(-\frac{n^{\text{QW}}}{N_{\text{c}}^{2\text{D}}}\right) - \exp\left(-\frac{m_{\text{c}}^{\text{QW}} n^{\text{QW}}}{m_{\text{v}}^{\text{QW}} N_{\text{c}}^{2\text{D}}}\right) \right] = \beta + \alpha_{\text{int}}. \quad (\text{A6})$$

With Eqs. (3) and (4), we can express n^{OCL} and p^{OCL} in terms of the 2D-carrier density in the QWs n^{QW} and the photon number N as follows:

$$n^{\text{OCL}} = \frac{1}{v_{\text{n,capt},0}(1-f_{\text{n}})} \times \left[\frac{n^{\text{QW}}}{\tau_{\text{n,esc}}} + B_{2\text{D}}(n^{\text{QW}})^2 + v_{\text{g}}g^{\text{max}}(f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} \right], \quad (\text{A7})$$

$$p^{\text{OCL}} = \frac{1}{v_{\text{p,capt},0}(1-f_{\text{p}})} \times \left[\frac{n^{\text{QW}}}{\tau_{\text{p,esc}}} + B_{2\text{D}}(n^{\text{QW}})^2 + v_{\text{g}}g^{\text{max}}(f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} \right], \quad (\text{A8})$$

or, using Eq. (7),

$$n^{\text{OCL}} = n_1 \frac{n^{\text{QW}}}{N_{\text{c}}^{2\text{D}}} + \frac{1}{v_{\text{n,capt},0}(1-f_{\text{n}})} \times \left[B_{2\text{D}}(n^{\text{QW}})^2 + v_{\text{g}}g^{\text{max}}(f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} \right], \quad (\text{A9})$$

$$p^{\text{OCL}} = p_1 \frac{n^{\text{QW}}}{N_{\text{v}}^{2\text{D}}} + \frac{1}{v_{\text{p,capt},0}(1-f_{\text{p}})} \times \left[B_{2\text{D}}(n^{\text{QW}})^2 + v_{\text{g}}g^{\text{max}}(f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} \right]. \quad (\text{A10})$$

As seen from Eqs. (A9) and (A10), the following linear relationship between n^{OCL} and p^{OCL} holds:

$$\begin{aligned} & \left(n^{\text{OCL}} - n_1 \frac{n^{\text{QW}}}{N_{\text{c}}^{2\text{D}}} \right) v_{\text{n,capt},0}(1-f_{\text{n}}) \\ &= \left(p^{\text{OCL}} - p_1 \frac{n^{\text{QW}}}{N_{\text{v}}^{2\text{D}}} \right) v_{\text{p,capt},0}(1-f_{\text{p}}). \end{aligned} \quad (\text{A11})$$

The threshold values of n^{OCL} and p^{OCL} are obtained by putting the number of photons $N=0$ in Eqs. (A9) and (A10),

$$n_{\text{th}}^{\text{OCL}} = n_1 \frac{n^{\text{QW}}}{N_{\text{c}}^{2\text{D}}} + \frac{1}{v_{\text{n,capt},0}(1-f_{\text{n}})} B_{2\text{D}}(n^{\text{QW}})^2, \quad (\text{A12})$$

$$p_{\text{th}}^{\text{OCL}} = p_1 \frac{n^{\text{QW}}}{N_{\text{v}}^{2\text{D}}} + \frac{1}{v_{\text{p,capt},0}(1-f_{\text{p}})} B_{2\text{D}}(n^{\text{QW}})^2. \quad (\text{A13})$$

The threshold current density is

$$j_{\text{th}} = j_{\text{spon,th}}^{\text{OCL}} + j_{\text{spon}}^{\text{QW}}, \quad (\text{A14})$$

where the spontaneous recombination current density in the OCL at the lasing threshold is

$$j_{\text{spon,th}}^{\text{OCL}} = ebB_{3\text{D}}n_{\text{th}}^{\text{OCL}}p_{\text{th}}^{\text{OCL}}. \quad (\text{A15})$$

Let us introduce the current densities of electron and hole capture from the OCL into the QWs and escape from the QWs to the OCL,

$$j_{\text{capt,n}} = eN_{\text{QW}}v_{\text{n,capt}}n^{\text{OCL}} = eN_{\text{QW}}v_{\text{n,capt},0}(1-f_{\text{n}})n^{\text{OCL}}, \quad (\text{A16})$$

$$j_{\text{capt,p}} = eN_{\text{QW}}v_{\text{p,capt}}p^{\text{OCL}} = eN_{\text{QW}}v_{\text{p,capt},0}(1-f_{\text{p}})p^{\text{OCL}}, \quad (\text{A17})$$

$$j_{\text{esc,n}} = eN_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{\text{n,esc}}} = \text{const}(j), \quad (\text{A18})$$

$$j_{\text{esc,p}} = eN_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{\text{p,esc}}} = \text{const}(j). \quad (\text{A19})$$

In Eqs. (A16) and (A17), $v_{\text{n,capt}}$ and $v_{\text{p,capt}}$ are the capture velocities, which take into account the QW filling by electrons and holes and are related to the capture velocities $v_{\text{n,capt},0}$ and $v_{\text{p,capt},0}$ into an unoccupied QW as follows:

$$v_{\text{n,capt}} = v_{\text{n,capt},0}(1-f_{\text{n}}), \quad v_{\text{p,capt}} = v_{\text{p,capt},0}(1-f_{\text{p}}). \quad (\text{A20})$$

Since the 2D-carrier densities are pinned [see Eq. (A4)], the escape current densities [Eqs. (A18) and (A19)] are also pinned above the lasing threshold.

From Eqs. (3) and (4), we have

$$j_{\text{capt,n}} = j_{\text{esc,n}} + j_{\text{spon}}^{\text{QW}} + j_{\text{stim}}, \quad (\text{A21})$$

$$j_{\text{capt,p}} = j_{\text{esc,p}} + j_{\text{spon}}^{\text{QW}} + j_{\text{stim}}. \quad (\text{A22})$$

Since $j_{\text{esc},n}$, $j_{\text{esc},p}$, and $j_{\text{spon}}^{\text{QW}}$ do not change with increasing j above the lasing threshold, we can write Eqs. (A21) and (A22) in the form

$$j_{\text{capt},n}(j) = j_{\text{capt},n,\text{th}} + j_{\text{stim}}(j), \quad (\text{A23})$$

$$j_{\text{capt},p}(j) = j_{\text{capt},p,\text{th}} + j_{\text{stim}}(j), \quad (\text{A24})$$

where the capture current densities at the lasing threshold are

$$j_{\text{capt},n,\text{th}} = eN_{\text{QW}}V_{n,\text{capt}}n_{\text{th}}^{\text{OCL}} = j_{\text{esc},n} + j_{\text{spon}}^{\text{QW}}, \quad (\text{A25})$$

$$j_{\text{capt},p,\text{th}} = eN_{\text{QW}}V_{p,\text{capt}}p_{\text{th}}^{\text{OCL}} = j_{\text{esc},p} + j_{\text{spon}}^{\text{QW}}. \quad (\text{A26})$$

Using Eqs. (A16), (A17) and Eqs. (A23)–(A26), we can write the free carrier densities as follows:

$$\begin{aligned} n^{\text{OCL}}(j) &= \frac{j_{\text{capt},n}(j)}{eN_{\text{QW}}V_{n,\text{capt}}} = n_{\text{th}}^{\text{OCL}} \frac{j_{\text{capt},n}(j)}{j_{\text{capt},n,\text{th}}} \\ &= n_{\text{th}}^{\text{OCL}} \left[1 + \frac{j_{\text{stim}}(j)}{j_{\text{capt},n,\text{th}}} \right] = n_{\text{th}}^{\text{OCL}} \left[1 + \frac{j - j_{\text{th}}}{j_{\text{capt},n,\text{th}}} \eta_{\text{int}}(j) \right], \end{aligned} \quad (\text{A27})$$

$$\begin{aligned} p^{\text{OCL}}(j) &= \frac{j_{\text{capt},p}(j)}{eN_{\text{QW}}V_{p,\text{capt}}} = p_{\text{th}}^{\text{OCL}} \frac{j_{\text{capt},p}(j)}{j_{\text{capt},p,\text{th}}} \\ &= p_{\text{th}}^{\text{OCL}} \left[1 + \frac{j_{\text{stim}}(j)}{j_{\text{capt},p,\text{th}}} \right] = p_{\text{th}}^{\text{OCL}} \left[1 + \frac{j - j_{\text{th}}}{j_{\text{capt},p,\text{th}}} \eta_{\text{int}}(j) \right]. \end{aligned} \quad (\text{A28})$$

We used Eq. (11) in the last equations in Eqs. (A27) and (A28).

Using Eq. (A14), we can write Eq. (A1) in the form

$$j - j_{\text{th}} = [j_{\text{spon}}^{\text{OCL}}(j) - j_{\text{spon},\text{th}}^{\text{OCL}}] + j_{\text{stim}}(j) \quad (\text{A29})$$

or

$$\frac{j - j_{\text{th}}}{j_{\text{spon},\text{th}}^{\text{OCL}}} = \left[\frac{n^{\text{OCL}}(j)}{n_{\text{th}}^{\text{OCL}}} \frac{p^{\text{OCL}}(j)}{p_{\text{th}}^{\text{OCL}}} - 1 \right] + \frac{j_{\text{stim}}(j)}{j_{\text{spon},\text{th}}^{\text{OCL}}}. \quad (\text{A30})$$

Using Eqs. (A27), (A28), and (11) in Eq. (A30), we obtain the following quadratic equation in the internal differential quantum efficiency $\eta_{\text{int}}(j)$:

$$\begin{aligned} &\frac{j_{\text{spon},\text{th}}^{\text{OCL}}}{j_{\text{capt},n,\text{th}}j_{\text{capt},p,\text{th}}} (j - j_{\text{th}}) \eta_{\text{int}}^2(j) \\ &+ \left[1 + \left(\frac{1}{j_{\text{capt},n,\text{th}}} + \frac{1}{j_{\text{capt},p,\text{th}}} \right) j_{\text{spon},\text{th}}^{\text{OCL}} \right] \eta_{\text{int}}(j) - 1 = 0. \end{aligned} \quad (\text{A31})$$

The solution of Eq. (A31) is given by Eq. (13). With Eq. (13), we easily calculate the LCC of the laser [Eq. (12)], the free carrier densities in the OCL [Eqs. (A27) and (A28)], the spontaneous recombination current density in the OCL [Eq. (A2)], the stimulated recombination current density [from Eq. (11)], and the number of photons in the lasing mode [from Eq. (10)].

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Calculation of output characteristics of semiconductor quantum-well lasers with account for both electrons and holes

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Abstract. Using an extended theoretical model, which includes the rate equations for both electrons and holes, we have studied the output characteristics of semiconductor quantum-well lasers. We have found non-trivial dependences of electron and hole concentrations in the waveguide region of the laser on the capture velocities of both types of carriers from the waveguide region into the quantum well. We have obtained the dependences of the internal differential quantum efficiency and optical output power of the laser on the capture velocities of electrons and holes. An increase in the capture velocities has been shown to result in suppression of parasitic recombination in the waveguide region and therefore in a substantial increase in the quantum efficiency and output power.

Keywords: semiconductor laser, quantum well, capture velocity of electrons and holes.

1. Introduction

It was theoretically demonstrated in [1–4] that with increasing injection current the internal differential quantum efficiency (efficiency of stimulated emission) η_{int} of semiconductor lasers with a quantum-confined active region decreases, resulting in a sublinear behaviour of the light–current (L – I) characteristic of such lasers. One of the reasons for η_{int} reduction and L – I sublinearity is the noninstantaneousness of charge carrier capture from the waveguide region (optical confinement layer, OCL) into the active region. With the growth of the pump current above the lasing threshold, the carriers are accumulated in the waveguide region due to the delay in their capture. An increase in the concentration of carriers in the waveguide region, in turn, causes an increase in the intensity of parasitic spontaneous recombination, experimentally observed in the early 1990s in InGaAsP/InP [5] and AlGaAs/GaAs [6] laser diodes.

The need to account for an increase in the carrier concentration in the waveguide region in order to calculate the internal quantum efficiency of the laser was discussed in monograph [7], where the difficulty of assessing the degree of influence of an increasing carrier concentration on the reduction of the internal quantum efficiency was emphasised.

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Received 24 March 2014

Kvantovaya Elektronika 44 (9) 801–805 (2014)

Translated by I.A. Ulitkin

Theoretical studies (including Refs [1–4]) on the output characteristics of injection lasers, for simplicity, make use of the rate equations for only one type of charge carriers. An analytical model for calculating the characteristics of semiconductor lasers with a low-dimensional active region, taking into account the dynamics of both electrons and holes and, therefore, including the rate equations for both types of charge carriers, has been developed in Ref. [8]. In particular, the authors derived a universal expression for the internal differential quantum efficiency in the form of an explicit function of the pump current density and the laser structure parameters [see below expression (10)]. The theoretical model from paper [8] can be applied to both quantum-well (QW) lasers and to lasers with a lower-dimensional active region (i.e., quantum-wire and quantum-dot lasers).

In this paper, we use the theoretical model from paper [8] to study the dependences of the output characteristics of single-quantum-well semiconductor lasers on the capture velocity of electrons and holes. Calculations have been carried out for an InGaAs/GaAs/AlGaAs separate-confinement laser heterostructure.

2. Description of the analytical model

We present here the steady-state rate equations of [8] for the case of a single quantum well in the active region of the laser.

The equation for free electrons in a bulk waveguide region of thickness b [$b(\partial n^{\text{OCL}}/\partial t) = 0$] reads:

$$\frac{j}{e} + \frac{n^{\text{QW}}}{\tau_{n,\text{esc}}} - v_{n,\text{capt},0}(1 - f_n)n^{\text{OCL}} - bB_{3\text{D}}n^{\text{OCL}}p^{\text{OCL}} = 0. \quad (1)$$

The first two terms in the left-hand side of (1) describe the influx of the electrons in the waveguide region due to the injection current and escape from the quantum well, and the last two terms – outflow of the electrons from the waveguide region as a result of their capture in the QW and spontaneous radiative recombination in the waveguide region.

The equation for free holes in the waveguide region [$b(\partial p^{\text{OCL}}/\partial t) = 0$] is similar to equation (1):

$$\frac{j}{e} + \frac{p^{\text{QW}}}{\tau_{p,\text{esc}}} - v_{p,\text{capt},0}(1 - f_p)p^{\text{OCL}} - bB_{3\text{D}}n^{\text{OCL}}p^{\text{OCL}} = 0. \quad (2)$$

The equation for the electrons in the QW ($\partial n^{\text{QW}}/\partial t = 0$) reads:

$$v_{n,\text{capt},0}(1 - f_n)n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{n,\text{esc}}} - B_{2\text{D}}n^{\text{QW}}p^{\text{QW}} - v_g g^{\text{max}}(f_n + f_p - 1) \frac{N}{S} = 0. \quad (3)$$

The first term in the left-hand side of (3) describes the influx of the electrons in the quantum well due to the capture from the waveguide region, and the other three terms – their outflow from the QW as a result of escape into the waveguide region, as well as spontaneous and stimulated recombination in the QW.

The equation for the holes in the QW ($\partial p^{\text{QW}}/\partial t = 0$) is similar to equation (3):

$$v_{p,\text{capt},0}(1-f_p)p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{p,\text{esc}}} - B_{2\text{D}}n^{\text{QW}}p^{\text{QW}} - v_g g^{\text{max}}(f_n + f_p - 1)\frac{N}{S} = 0. \quad (4)$$

For the number N of photons in a laser mode ($\partial N/\partial t = 0$)

$$v_g g^{\text{max}}(f_n + f_p - 1)N - v_g(\beta + \alpha_{\text{int}})N = 0. \quad (5)$$

The first term in the left-hand side of (5) describes the ‘influx’ of the photons due to stimulated emission, and the second term – their ‘escape’ through the cavity mirrors and as a result of internal optical losses.

In equations (1)–(5) the following notations are used: n^{OCL} and p^{OCL} are the volume concentrations of free electrons and holes in the waveguide region; n^{QW} and p^{QW} are the two-dimensional concentrations of electrons and holes in the QW; N is the number of photons in the cavity; f_n and f_p are the filling factors (occupancies) of the states corresponding to the lower and upper edges of the quantum-confinement sub-bands of electrons and holes in the QW, respectively; j is the pump current density; e is the electron charge; $\tau_{n,\text{esc}}$ and $\tau_{p,\text{esc}}$ are the escape times of electrons and holes from the QW into the waveguide region; $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ are the capture velocities of electrons and holes in an unfilled (at $f_n = 0$ and $f_p = 0$) single quantum well (cm s^{-1}); $B_{3\text{D}}$ and $B_{2\text{D}}$ are the coefficients of spontaneous radiative recombination in a three-dimensional waveguide region and in a two-dimensional QW ($\text{cm}^3 \text{s}^{-1}$ and $\text{cm}^2 \text{s}^{-1}$, respectively); analytical expressions for the $B_{2\text{D}}$ and $B_{3\text{D}}$ are given in Ref. [9]; v_g is the group velocity of light; g^{max} is the maximum modal gain in the QW; $S = WL$ is the stripe-contact area; W is the stripe-contact width; L is the length of the Fabry–Perot cavity; $\beta = (1/L)\ln(1/R)$ is the cavity mirror loss; R is the mirror reflectivity; and α_{int} is the internal optical loss in the laser structure.

The filling factors f_n and f_p are related to two-dimensional concentrations n^{QW} and p^{QW} as follows [10]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_{\text{c},\text{v}}^{\text{2D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_{\text{v},\text{c}}^{\text{2D}}}\right), \quad (6)$$

where $N_{\text{c},\text{v}}^{\text{2D}}$ are the two-dimensional effective densities of states in the conduction band and the valence band in the QW.

The key parameters describing the capture of electrons and holes from the bulk waveguide region into a two-dimensional QW are the capture velocities of electrons and holes into the unfilled QW. The current densities of electron and hole capture in the QW are expressed through $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ as follows [1–4, 8]:

$$j_{\text{capt},n} = ev_{n,\text{capt},0}(1-f_n)n^{\text{OCL}}, \quad (7)$$

$$j_{\text{capt},p} = ev_{p,\text{capt},0}(1-f_p)p^{\text{OCL}}.$$

Factors $1-f_n$ and $1-f_p$ in (7) take into account the filling of the quantum-confinement sub-bands in the conduction band and the valence band of the QW by charge carriers. The capture velocities of electrons and holes are analogous to the surface recombination velocities. It should be noted that the number of papers devoted to the capture velocity in the QW is small [1–4, 8, 11–19].

The output optical power P of the semiconductor laser as a function of the pump current density j is given by the expression:

$$P(j) = \frac{\hbar\omega}{e} S(j - j_{\text{th}})\eta_{\text{int}}(j) \frac{\beta}{\beta + \alpha_{\text{int}}}, \quad (8)$$

where $\hbar\omega$ is the photon energy; and j_{th} is the threshold current density.

The internal differential quantum efficiency η_{int} , appearing in (8), is defined as the ratio of the current density of stimulated recombination, $j_{\text{stim}}(j)$, to the excess of the injection current density j over the threshold density j_{th} :

$$\eta_{\text{int}}(j) = \frac{j_{\text{stim}}(j)}{j - j_{\text{th}}}. \quad (9)$$

In [8] a set of steady-state rate equations (1)–(5) was solved and the following expression for η_{int} was derived:

$$\eta_{\text{int}}(j) = \left\{ \frac{1}{2} + \frac{j_{\text{spon},\text{th}}^{\text{OCL}}}{\langle j_{\text{capt},\text{th}} \rangle_{\text{harmon}}} + \left[\left(\frac{1}{2} + \frac{j_{\text{spon},\text{th}}^{\text{OCL}}}{\langle j_{\text{capt},\text{th}} \rangle_{\text{harmon}}} \right)^2 + \frac{j_{\text{spon},\text{th}}^{\text{OCL}}}{\langle j_{\text{capt},\text{th}} \rangle_{\text{geom}}} \frac{j - j_{\text{th}}}{\langle j_{\text{capt},\text{th}} \rangle_{\text{geom}}} \right]^{1/2} \right\}^{-1}, \quad (10)$$

where $\langle j_{\text{capt},\text{th}} \rangle_{\text{harmon}}$ and $\langle j_{\text{capt},\text{th}} \rangle_{\text{geom}}$ are, respectively, the harmonic mean and the geometric mean of the current densities of electron and hole capture in the QW at the lasing threshold,

$$\frac{1}{\langle j_{\text{capt},\text{th}} \rangle_{\text{harmon}}} = \frac{1}{2} \left(\frac{1}{j_{\text{capt},n,\text{th}}} + \frac{1}{j_{\text{capt},p,\text{th}}} \right), \quad (11)$$

$$\langle j_{\text{capt},\text{th}} \rangle_{\text{geom}} = \sqrt{j_{\text{capt},n,\text{th}} j_{\text{capt},p,\text{th}}}. \quad (12)$$

Expression (10) includes the threshold current density component $j_{\text{spon},\text{th}}^{\text{OCL}}$ related to the parasitic spontaneous radiative recombination in the waveguide region:

$$j_{\text{spon},\text{th}}^{\text{OCL}} = ebB_{3\text{D}}n_{\text{th}}^{\text{OCL}}p_{\text{th}}^{\text{OCL}}, \quad (13)$$

where $n_{\text{th}}^{\text{OCL}}$ and $p_{\text{th}}^{\text{OCL}}$ are the concentrations of free electrons and holes in the waveguide region at the lasing threshold.

In this paper we have studied in detail the dependences of the concentrations of electrons and holes in the waveguide region, as well as the dependences of the internal differential quantum efficiency and output optical power of the laser on the capture velocities $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ in the QW.

3. Dependences of laser characteristics on capture velocities of electrons and holes in the QW

Below we present the results of calculations for a laser structure with an extended waveguide studied experimentally in papers [20–22]. An InGaAs/GaAs/AlGaAs structure is considered, the active region of which contains one strained

$\text{In}_{0.28}\text{Ga}_{0.72}\text{As}$ QW of thickness 80 Å. The material for such a wide ($b = 1.7 \mu\text{m}$) waveguide was GaAs, and the emitters were fabricated from $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$. The laser wavelength was 1.044 μm , the length of the Fabry–Perot was $L = 1.5 \text{ mm}$, the width of the stripe contact was $W = 100 \mu\text{m}$, the mirror reflectivity was $R = 0.32$, the mirror loss was $\beta = 7.6 \text{ cm}^{-1}$, the internal optical loss was $\alpha_{\text{int}} = 1 \text{ cm}^{-1}$ and the temperature was $T = 300 \text{ K}$. The maximum modal gain in the QW amounted to $g^{\text{max}} = 49.1 \text{ cm}^{-1}$.

The concentrations of free electrons and holes in the bulk waveguide region as a function of the capture velocities of electrons and holes, $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$, at various pump current densities are presented in Figs 1 and 2. One can see that the concentrations of both electrons and holes in the waveguide region increase with increasing pump current. From Fig. 1a it follows that when $v_{n,\text{capt},0}$ increases, the concentration of electrons in waveguide region decreases. The higher the pump current density, the stronger this dependence. However, when $v_{p,\text{capt},0}$ increases, the concentration of electrons in the waveguide region increases (Fig. 1b). Similar dependences hold for holes in Fig. 2.

An increase in the electron concentration in the waveguide region with increasing capture velocity of holes can be

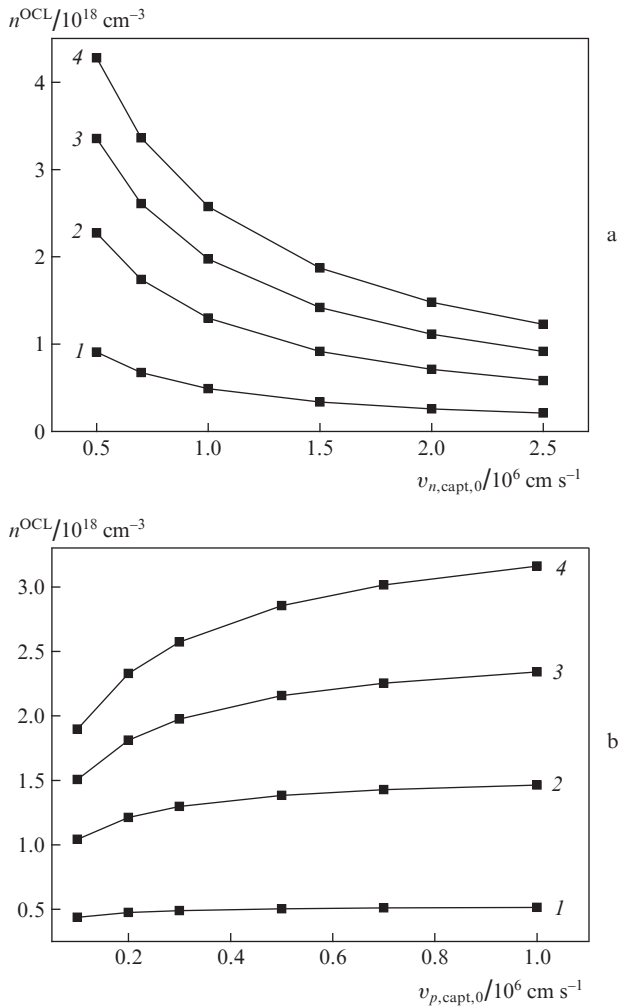


Figure 1. Concentrations of free electrons in the waveguide region as functions of (a) $v_{n,\text{capt},0}$ and (b) of $v_{p,\text{capt},0}$ at $v_{p,\text{capt},0} = 3 \times 10^5 \text{ cm s}^{-1}$ and $v_{n,\text{capt},0} = 1 \times 10^6 \text{ cm s}^{-1}$, respectively; the pump current density is $j = (1) 10$, (2) 30, (3) 50 and (4) 70 kA cm^{-2} .

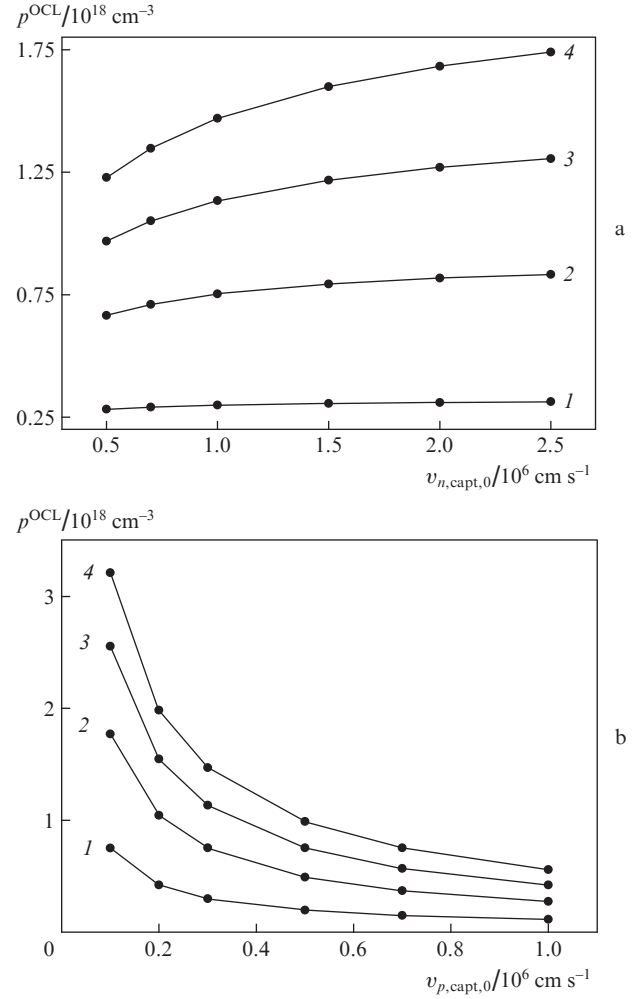


Figure 2. Concentrations of free holes in the waveguide region as functions of (a) $v_{n,\text{capt},0}$ and (b) of $v_{p,\text{capt},0}$ at $v_{p,\text{capt},0} = 3 \times 10^5 \text{ cm s}^{-1}$ and $v_{n,\text{capt},0} = 1 \times 10^6 \text{ cm s}^{-1}$, respectively; $j = (1) 10$, (2) 30, (3) 50 and (4) 70 kA cm^{-2} .

explained as follows. When we consider the capture of both electrons and holes, the electron concentration in the waveguide region depends on the capture velocity of both electrons and holes:

$$n^{\text{OCL}}(j, v_{n,\text{capt},0}, v_{p,\text{capt},0}) = n_{\text{th}}^{\text{OCL}} + \frac{j_{\text{stim}}(j, v_{n,\text{capt},0}, v_{p,\text{capt},0})}{e v_{n,\text{capt},0} (1 - f_n)}. \quad (14)$$

With increasing capture velocity of holes the current density j_{stim} of stimulated recombination in the QW is increased (see Fig. 3b), which, as follows from (14), should lead to an increase in the concentration of electrons in the waveguide region.

Comparison of curves in Figs 1a and 2a shows that with increasing capture velocity of electrons at a constant capture velocity of holes, the electron concentration in the waveguide region decreases faster than the hole concentration increases. Similarly, from Figs 1b and 2b follows that with increasing capture velocity of holes at a constant capture velocity of electrons, the hole concentration decreases more rapidly than the electron concentration increases.

The dependences of the current density of stimulated recombination, j_{stim} , in the QW on capture velocities of electrons and holes are shown in Fig. 3.

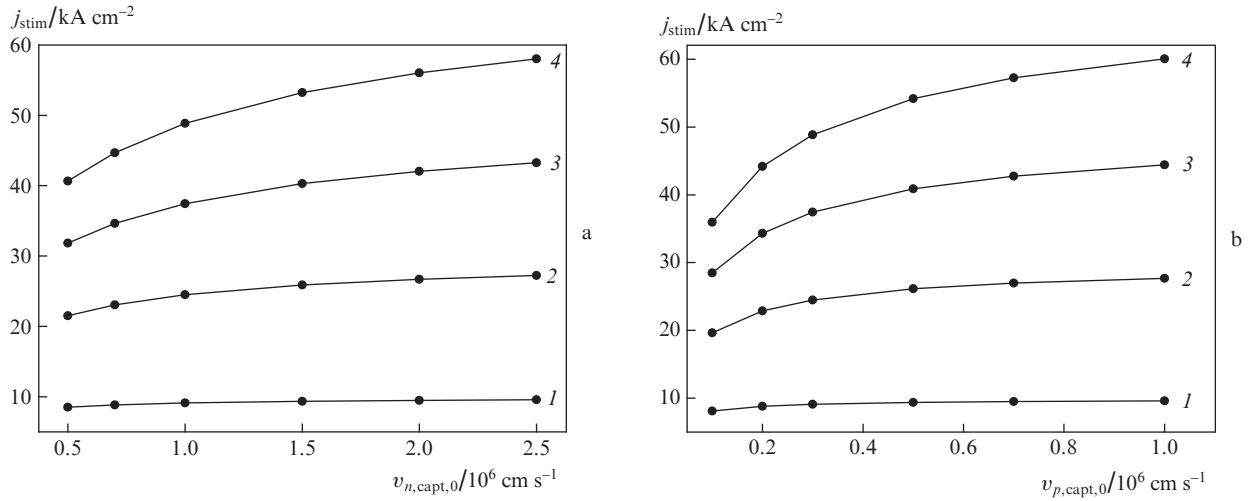


Figure 3. Current densities of stimulated radiative recombination in the QW as functions of (a) $v_{n,capt,0}$ and (b) of $v_{p,capt,0}$ at $v_{p,capt,0} = 3 \times 10^5\ cm\ s^{-1}$ and $v_{n,capt,0} = 1 \times 10^6\ cm\ s^{-1}$, respectively; $j = (1) 10, (2) 30, (3) 50$ and $(4) 70\ kA\ cm^{-2}$.

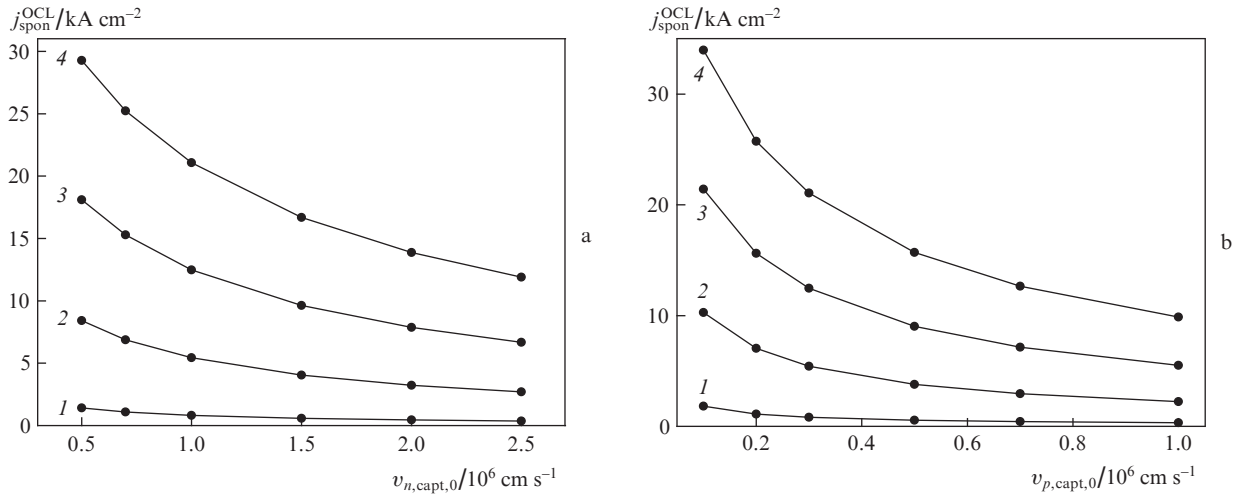


Figure 4. Current densities of parasitic spontaneous radiative recombination in the waveguide region as functions of (a) $v_{n,capt,0}$ and (b) of $v_{p,capt,0}$ at $v_{p,capt,0} = 3 \times 10^5\ cm\ s^{-1}$ and $v_{n,capt,0} = 1 \times 10^6\ cm\ s^{-1}$, respectively; $j = (1) 10, (2) 30, (3) 50$ and $(4) 70\ kA\ cm^{-2}$.

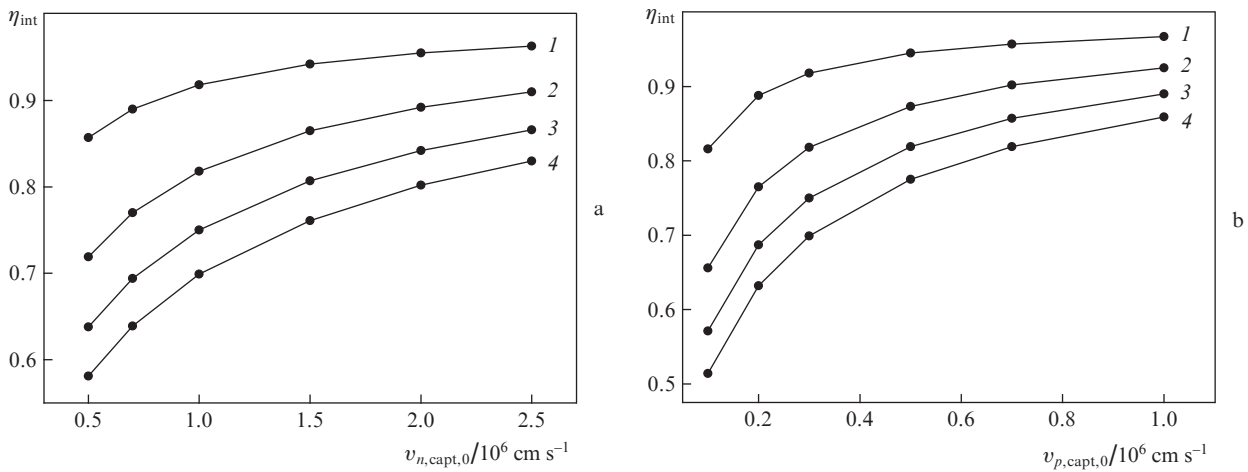


Figure 5. Internal differential quantum efficiency as a function of (a) $v_{n,capt,0}$ and (b) of $v_{p,capt,0}$ at $v_{p,capt,0} = 3 \times 10^5\ cm\ s^{-1}$ and $v_{n,capt,0} = 1 \times 10^6\ cm\ s^{-1}$, respectively; $j = (1) 10, (2) 30, (3) 50$ and $(4) 70\ kA\ cm^{-2}$.

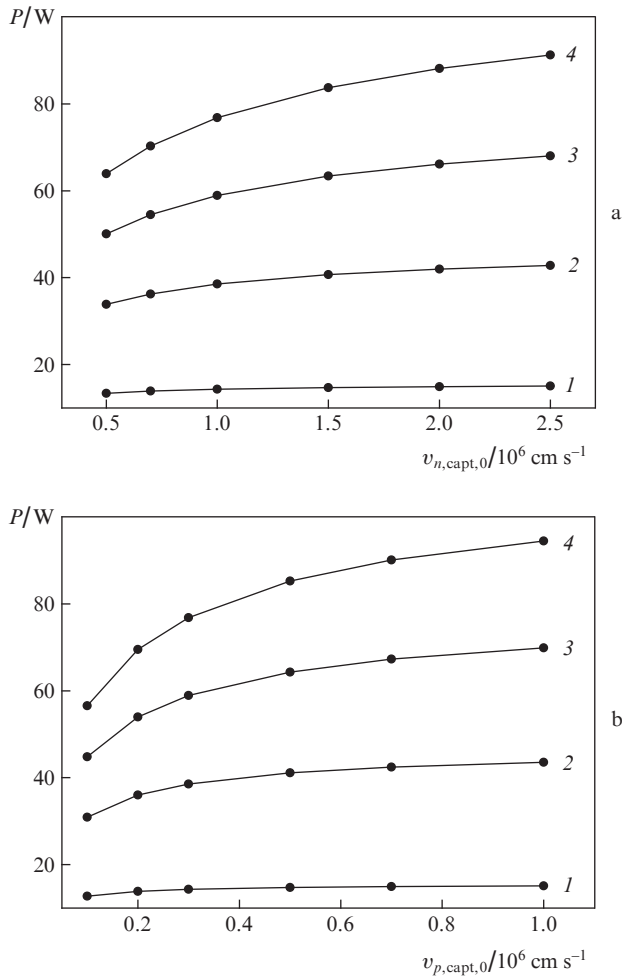


Figure 6. Output optical power of the laser as a function of (a) $v_{n,capt,0}$ and (b) of $v_{p,capt,0}$ at $v_{p,capt,0} = 3 \times 10^5 \text{ cm s}^{-1}$ and $v_{n,capt,0} = 1 \times 10^6 \text{ cm s}^{-1}$, respectively; $j = (1) 10, (2) 30, (3) 50$ and $(4) 70 \text{ kA cm}^{-2}$.

The dependences of the current density of spontaneous radiative recombination in the waveguide region, $j_{\text{spont}}^{\text{OCL}}$, on capture velocities of electrons and holes are shown in Fig. 4. Spontaneous radiative recombination in the waveguide region is a ‘parasitic’ process, which reduces the internal differential quantum efficiency. One can see from Fig. 4 that $j_{\text{spont}}^{\text{OCL}}$ decreases with increasing capture velocities.

The internal differential quantum efficiency and output optical power of the laser are shown in Figs 5 and 6 as a function of capture velocities of electrons and holes in the QW at different injection current densities. One can see that η_{int} and P significantly increase with increasing capture velocities.

4. Conclusions

We have studied the output characteristics of quantum-well diode lasers. We have used a theoretical model, which includes the rate equations for both electrons and holes. We have found non-trivial dependences of concentrations n^{OCL} and p^{OCL} in the waveguide regions of the laser on capture velocities of both types of carriers from the waveguide region to the QW. We have obtained the dependences of the internal differential quantum efficiency and optical output power of the lasers on capture velocities of electrons and holes. It has been shown that increasing capture velocities results in suppression of

parasitic recombination in the waveguide region and hence in a substantial increase in η_{int} and P .

Acknowledgements. The study was carried out by I.S.T. and Z.N.S. under the research plan of the A.F. Ioffe Physical-Technical Institute, Russian Academy of Sciences. L.V.A. thanks the U.S. Army Research Office for support of the present study (Grant No. W911NF-13-1-0445).

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PHYSICS OF SEMICONDUCTOR DEVICES

Comparative Analysis of the Effects of Electron and Hole Capture on the Power Characteristics of a Semiconductor Quantum-Well Laser

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Submitted April 21, 2015; accepted for publication May 5, 2015

Abstract—The operating characteristics of a semiconductor quantum-well laser calculated using three models are compared. These models are (i) a model not taking into account differences between the electron and hole parameters and using the electron parameters for both types of charge carriers; (ii) a model, which does not take into account differences between the electron and hole parameters and uses the hole parameters for both types of charge carriers; and (iii) a model taking into account the asymmetry between the electron and hole parameters. It is shown that, at the same velocity of electron and hole capture into an unoccupied quantum well, the laser characteristics, obtained using the three models, differ considerably. These differences are due to a difference between the filling of the electron and hole subbands in a quantum well. The electron subband is more occupied than the hole subband. As a result, at the same velocities of electron and hole capture into an empty quantum well, the effective electron-capture velocity is lower than the effective hole-capture velocity. Specifically, it is shown that for the laser structure studied the hole-capture velocity of 5×10^5 cm/s into an empty quantum well and the corresponding electron-capture velocity of 3×10^6 cm/s into an empty quantum well describe the rapid capture of these carriers, at which the light–current characteristic of the laser remains virtually linear up to high pump-current densities. However, an electron-capture velocity of 5×10^5 cm/s and a corresponding hole-capture velocity of 8.4×10^4 cm/s describe the slow capture of these carriers, causing significant sublinearity in the light–current characteristic.

DOI: 10.1134/S1063782615110202

1. INTRODUCTION

An important factor, which affects the operating characteristics of semiconductor lasers with a nanoscale active region, is the noninstantaneous capture of charge carriers from the waveguide region of the separate-confinement laser structure into the nanoscale active region. The series of theoretical works [1–6] is devoted to the effect of charge-carrier capture on the characteristics of quantum-well (QW) lasers. The power characteristics of the lasers were studied under the assumption of symmetry between the electron and hole parameters [1–4]. An expanded analytical model, which takes into account the asymmetry between the electron and hole parameters, was developed in [5, 6]. In particular, the capture of electrons and holes into the nanoscale active region of the laser was separately taken into account in this model.

In the present work we compare the characteristics of a semiconductor quantum-well laser, calculated using three models: (i) a model, which does not take into account differences between the electron and hole

parameters and uses the electron parameters for both types of charge carriers (model 1); (ii) a model, which does not take into account differences between the electron and hole parameters and uses the hole parameters for both types of charges carriers (model 2); and (iii) a model, which takes into account the asymmetry between the electron and hole parameters (model 3).

2. DESCRIPTION OF THE LASER STRUCTURE

Calculations of the light–current characteristic (LCC) and internal differential quantum efficiency were performed for a separate-confinement laser heterostructure with a broad waveguide region and active region made up of one strained QW. The charge carriers are delivered to the QW in two steps, namely, injection from emitters into the waveguide region (optical-confinement layer (OCL)) and subsequent capture from the waveguide region into the QW.

In our calculations the key parameters are the capture velocity of electrons $v_{n,capt,0}$ and holes $v_{p,capt,0}$ from the bulk waveguide region into the unoccupied two-dimensional region (quantum well). The capture velocity into the empty QW is the QW characteristic and depends on its width and depth, i.e., on the materials of the QW and its surrounding layers.

Due to the finiteness of the capture velocities of electrons and holes, carriers are accumulated in the waveguide region as the pump current increases above the laser generation threshold [1–6]. As it was for the first time experimentally shown in [7], when the density of carriers in the waveguide region increases, parasitic spontaneous radiative recombination intensifies in this region. This leads to a decrease in the internal differential quantum efficiency of stimulated radiation from the QW with increasing pump current and, along with an increase in the internal optical loss is one of the main reasons for the LCC sublinearity in high-power semiconductor lasers.

In this work calculations were performed for an InGaAs/GaAs/AlGaAs laser structure, which was fabricated and experimentally studied in [8]. The active region of the structure contains one strained $\text{In}_{0.28}\text{Ga}_{0.72}\text{As}$ QW with a thickness of 80 Å. The material of the broad (1.7 μm) waveguide region is GaAs, and the material of the emitters is $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$. The effective masses of electrons and holes in the QW were calculated in accordance with [9]: $m_c^{\text{QW}} = 0.058m_0$ and $m_v^{\text{QW}} = 0.362m_0$, where m_0 is the free electron mass. In the studied structure the QW is tensile-strained; the subband splitting of heavy and light holes is about ~100 meV. Therefore, we neglect radiative processes with the participation of light holes in the QW. The emission wavelength of the laser is 1.044 μm.

In the calculations the following laser parameters were used: the length of the Fabry-Perot resonator was $L = 1.5$ mm, the width of the strip contact was $W = 100$ μm, the reflection coefficient of the mirrors were $R = 0.32$, the mirror loss were $\beta = 7.6 \text{ cm}^{-1}$, and the temperature was $T = 300\text{K}$. In this work (as in [1–6]) the internal optical loss (α_{int}) was considered constant, i.e., independent of the pump current, and assumed equal to 1 cm^{-1} . The maximum modal gain coefficient, calculated using the Kane model, was $g_{\text{max}} = 49.1 \text{ cm}^{-1}$.

3. LASER CHARACTERISTICS

As was already mentioned, the laser characteristics were calculated using three models: (i) the model, which does not take into account differences between the electron and hole parameters and uses the electron parameters for both types of charge carriers (model 1); (ii) the model, which does not take into account differences between the electron and hole parameters and uses the hole parameters for both types of charges

carriers (model 2); and (iii) the model, which takes into account the asymmetry between the electron and hole parameters (model 3). Calculations within the framework of model 1 were performed according to formulas in [3], using the following expression for the threshold electron density in the waveguide region:

$$n_{\text{th}}^{\text{OCL}} = n_1 \frac{n^{\text{QW}}}{N_c^{2\text{D}}} + \frac{1}{v_{n,capt,0}} \exp\left(\frac{n^{\text{QW}}}{N_c^{2\text{D}}}\right) B_{2\text{D}} (n^{\text{QW}})^2. \quad (1)$$

The value n_1 is specified by the expression:

$$n_1 = N_c^{3\text{D}} \exp\left(-\frac{\Delta E_c - \epsilon_n^{\text{QW}}}{T}\right), \quad (2)$$

in which ΔE_c is the QW depth for electrons, ϵ_n^{QW} is the energy of the edge of the size-quantization subband of electrons in the QW, T is the temperature in energy units; $v_{n,capt,0}$ is the velocity of electron capture into the empty single QW, n^{QW} is the density of electrons in the QW (n^{QW} is considered constant, i.e., independent of the pump current above the generation threshold), and $B_{2\text{D}}$ is the coefficient of spontaneous radiative recombination in a two-dimensional QW [10] (for the studied experimental structure $B_{2\text{D}} = 2.5 \times 10^{-4} \text{ cm}^2/\text{s}$).

The bulk effective densities of electron and hole states in the waveguide region ($N_{c,v}^{3\text{D}}$) and two-dimensional effective densities of states in the QW ($N_{c,v}^{2\text{D}}$) are specified by the following expressions:

$$N_{c,v}^{3\text{D}} = 2 \left(\frac{m_{c,v}^{\text{OCL}} T}{2\pi\hbar^2} \right)^{3/2}, \quad (3)$$

$$N_{c,v}^{2\text{D}} = \frac{m_{c,v}^{\text{QW}} T}{\pi\hbar^2}, \quad (4)$$

where the effective mass of electrons in the waveguide region $m_c^{\text{OCL}}/m_0 = 0.07$ and the effective mass of holes $m_v^{\text{OCL}}/m_0 = 0.35$.

In this work calculations of the laser characteristics using model 2 were performed for the first time; those using model 3 were based on formulas in [5, 6]. The calculation results of the laser characteristics using all three models are represented in Figs. 1–4. In all three models the velocities of electron and hole capture into the empty QW were considered identical: $v_{n,capt,0} = v_{p,capt,0} = 5 \times 10^5 \text{ cm/s}$.

Figure 1 shows the dependence of the internal differential quantum efficiency of the laser on the pump current density. It can be seen from the figure that the values of the internal differential quantum efficiency (η_{int}), obtained using three models, greatly differ. The same is true for other laser characteristics, i.e., the output optical power P (Fig. 2), the current density of spontaneous radiative recombination in the waveguide

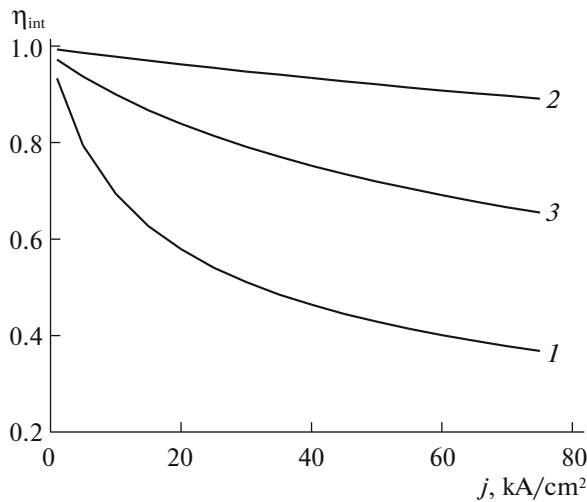


Fig. 1. Dependence of the internal differential quantum efficiency of the laser on the pump-current density: (1, 2, 3) calculations using models 1, 2, and 3. The capture velocities of electrons and holes into the empty quantum well are equal to 5×10^5 cm/s.

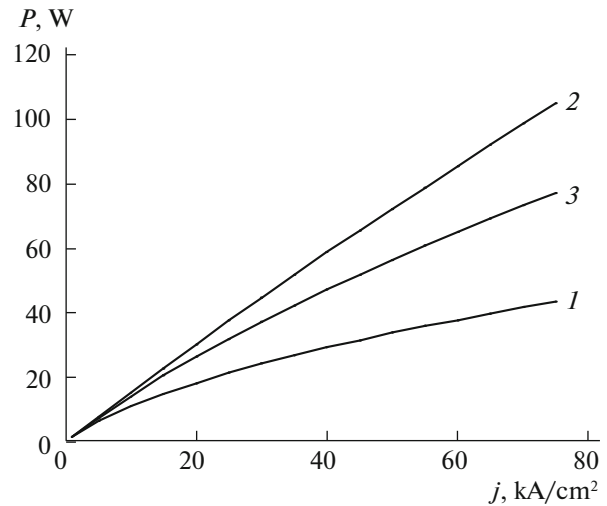


Fig. 2. Light-current characteristic of the laser: (1, 2, 3) calculations using models 1, 2, and 3. The capture velocities of electrons and holes into the empty quantum well are equal to 5×10^5 cm/s.

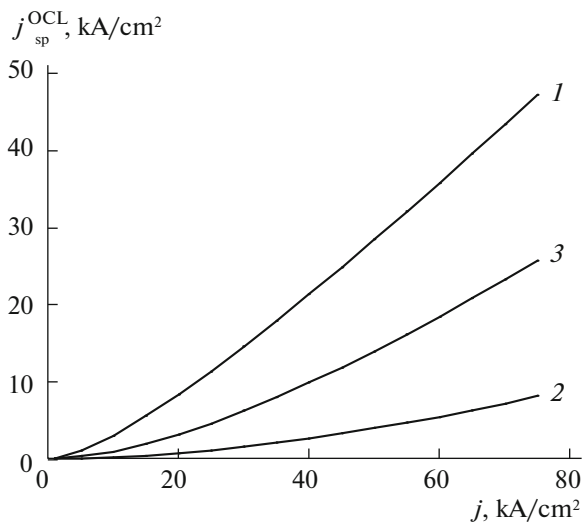


Fig. 3. Dependence of the current density of spontaneous recombination in the waveguide region on the pump-current density: (1, 2, 3) calculations using models 1, 2, and 3. The capture velocities of electrons and holes into the empty quantum well are equal to 5×10^5 cm/s.

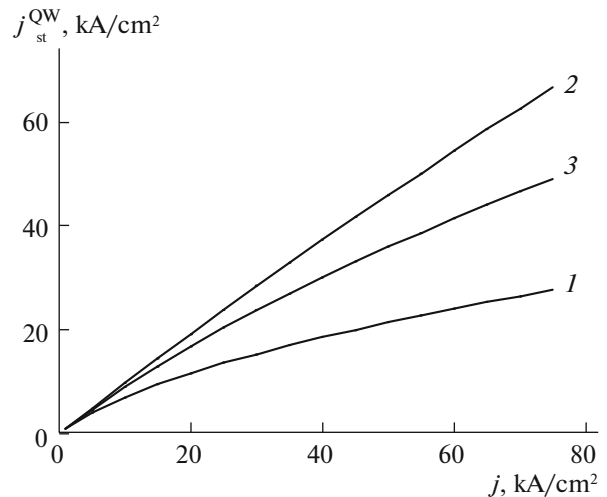


Fig. 4. Dependence of the current density of stimulated recombination in the quantum well on the pump-current density: (1, 2, 3) calculations using models 1, 2, and 3. The capture velocities of electrons and holes into the empty quantum well are equal to 5×10^5 cm/s.

region $j_{\text{sp}}^{\text{OCL}}$ (Fig. 3), and the current density of stimulated recombination in the QW $j_{\text{st}}^{\text{QW}}$ (Fig. 4).

The highest internal quantum efficiency, which slightly changes with increasing pump current density j (up to 75 kA/cm^2), is given by model 2, which uses the parameters of holes for both types of charge carriers (Fig. 1, curve 2), and the lowest is given by model 1, which uses the parameters of electrons for both types

of charge carriers (Fig. 1, curve 1). These results mean that the selected capture velocity in the QW (5×10^5 cm/s) in the studied structure is rather high, if the hole parameters are used for both types of charge carriers. In this case, the capture process does not delay the delivery of carriers to the quantum well and does not limit stimulated recombination in it (Fig. 4, curve 2). In addition, the LCC remains virtually linear (Fig. 2, curve 2). The same capture velocity (5×10^5 cm/s) is low if the electron parameters are used for

both types of carriers. In this case, the capture process slows down the delivery of carriers to the quantum well. Consequently, the growth of stimulated recombination in the QW is significantly limited (Fig. 4, curve 1) while spontaneous recombination in the waveguide region considerably increases (Fig. 3, curve 1). As a result, the internal quantum efficiency quickly drops with increasing pump current density (Fig. 1, curve 1), and the LCC is sublinear (Fig. 2, curve 1).

As was expected, the laser characteristics, obtained in model 3, which takes into account the asymmetry between the electron and hole parameters (curves 3 in Figs. 1–4), occupy the intermediate position between the calculation results, obtained using models 1 and 2.

The differences between the characteristics, obtained in models 1 and 2 using the same velocity of capture into the unoccupied quantum well, are due to the difference in the subband filling for electrons and holes. According to formula (4), due to the fact that the effective mass of holes is larger than the effective mass of electrons, the inequality $N_v^{2D} > N_c^{2D}$ holds true. This means that with consideration for the electrical neutrality condition in the QW used by us ($n^{QW} = p^{QW}$) the electron subband filling factor [11, 12]:

$$f_n = 1 - \exp\left(-\frac{n^{QW}}{N_c^{2D}}\right) \quad (5)$$

is larger than the hole subband filling factor:

$$f_p = 1 - \exp\left(-\frac{p^{QW}}{N_v^{2D}}\right). \quad (6)$$

In the considered laser structure $f_n = 0.881$ and $f_p = 0.294$. Thus, the electron subband is filled to a greater extent (i.e., less empty) than the hole subband. Therefore, at the same velocities of electron and hole capture into the empty QW ($v_{n,capt,0} = v_{p,capt,0}$), the effective velocity of electron capture into the QW defined as [5]:

$$v_{n,capt} = v_{n,capt,0}(1 - f_n), \quad (7)$$

is smaller than the effective rate of hole capture in the QW:

$$v_{p,capt} = v_{p,capt,0}(1 - f_p). \quad (8)$$

Analysis using the formulas of [5] and the calculations, performed in this work, showed that the laser characteristics, obtained using models 1 and 2, coincide, if the effective capture velocities of electrons and holes are equal, i.e., when $v_{n,capt} = v_{p,capt}$. Thus, for coincidence of the characteristics of the laser structure, obtained using models 1 and 2, the capture rate of electrons into the empty QW ($v_{n,capt,0}$) should be higher by a factor of $(1 - f_p)/(1 - f_n)$ than the velocity of hole capture into the empty QW ($v_{p,capt,0}$). Naturally, use of thus chosen values $v_{n,capt,0}$ and $v_{p,capt,0}$ in model 3 will give laser characteristics that are identical

to those, obtained in models 1 and 2. Thus, e.g., for coincidence of the characteristics at $v_{p,capt,0} = 5 \times 10^5$ cm/s, $v_{n,capt,0}$ should be equal to 3×10^6 cm/s; at $v_{p,capt,0} = 5 \times 10^5$ cm/s, $v_{p,capt,0}$ should be equal to 8.4×10^4 cm/s.

Thus, one can see from our analysis that for the considered laser structure capture velocities of electrons of 3×10^6 cm/s and holes of 5×10^5 cm/s into the empty QW describe the rapid capture, and the LCC remains linear at these velocities. Capture velocities of electrons of 5×10^5 cm/s and holes of 8.4×10^4 cm/s describe slow capture, resulting in substantial nonlinearity of the LCC.

4. CONCLUSIONS

The characteristics of a semiconductor laser, calculated using three models, are compared. These three models are (i) a model, which does not take into account differences between the electron and hole parameters and uses the electron parameters for both types of charge carriers (model 1); (ii) a model, which does not take into account differences between the electron and hole parameters and uses the hole parameters for both types of charge carriers (model 2); and (iii) a model, which takes into account the asymmetry between the electron and hole parameters (model 3). In the calculations the internal optical loss (α_{int}) was considered constant and independent of the pump current. It is shown that at the same velocities of electron and hole capture into the empty QW ($v_{n,capt,0} = v_{p,capt,0}$) the laser characteristics (LCC, internal quantum efficiency, the current of stimulated recombination in the QW, the current of spontaneous recombination in the waveguide region), obtained using the three models, differ considerably. These differences are the consequence of a difference in the subband filling by electrons and holes in the conduction band and valence band in the quantum well. The electron subband is filled to a greater extent than the hole subband, and owing to this fact at the same capture velocities into the empty QW the effective electron-capture velocity is smaller than the effective hole-capture velocity.

It is shown in particular that for the studied laser structure the velocity of capture in the empty QW for holes $v_{p,capt,0} = 5 \times 10^5$ cm/s and the corresponding velocity of electron capture $v_{n,capt,0} = 3 \times 10^6$ cm/s describe the fast capture of these carriers. In this case, the LCC remains virtually linear up to high pump-current velocities. The capture velocity of electrons $v_{n,capt,0} = 5 \times 10^5$ cm/s and the corresponding capture velocity of holes $v_{p,capt,0} = 8.4 \times 10^4$ cm/s describe the slow capture of these carriers, at which the LCC is substantially sublinear.

ACKNOWLEDGMENTS

This work was supported by the state program of the Ioffe Physical–Technical Institute. L.V. Asryan is thankful to the U.S. Army Research Office for supporting this work, grant no. W911NF-13-1-0445.

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Translated by N. Pakhomova

Method for determination of capture velocity of charge carriers into quantum well in semiconductor laser

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A simple method for the determination of the capture velocity of charge carriers from a three-dimensional (3D) region (waveguide region) into a 2D region (quantum well) is proposed. The method is based on measurement of the threshold current density and internal differential quantum efficiency in a semiconductor laser structure. The method also allows determining the 2D carrier density in a quantum well, which is otherwise not easy to measure in a multilayer laser structure.

Introduction: Increasing the output optical power is one of the most important tasks for developers of semiconductor lasers with a low-dimensional active region. To achieve high powers, the light-current characteristic (LCC) of a laser should remain close to linear at high levels of pumping. However, saturation of the LCC occurs at high injection currents in separate-confinement double-heterostructure lasers with a low-dimensional active region. Saturation of the LCC can be due to different factors. There have been a number of studies devoted to this issue (see e.g. [1–12]). As theoretically shown in [7–12], the non-instantaneous capture of carriers from a waveguide region into a low-dimensional active region is among the major factors affecting the operating characteristics of semiconductor lasers. In the case of quantum well (QW) lasers, the main parameter in [7–12] describing the carrier capture from a bulk waveguide region into a two-dimensional (2D) active region (QW) is the capture velocity $v_{\text{capt},0}$ (in units of cm/s). The capture velocity is a characteristic of a QW and thus depends on its depth (i.e. on the material compositions of a QW and the surrounding barrier layers) and width. The capture velocity has been theoretically estimated in [13–18]. However, to the best of our knowledge, the capture velocity into a QW has not yet been determined experimentally.

In this Letter, a simple method for determination of the capture velocity of charge carriers into a QW is proposed, which is based on

- (i) measurement of the threshold current density (j_{th}) and internal differential quantum efficiency at the lasing threshold ($\eta_{\text{int,th}}$), and
- (ii) use of the theoretical model of [7–12].

Experimental laser structure: An experimental laser structure with a single strained InGaAs QW and a broadened GaAs waveguide is studied (Fig. 1). The structure was grown by metal-organic chemical vapour deposition [19]. Using standard post-growth operations [20], a mesastripe-construction laser (with the stripe width of 100 μm and Fabry-Pérot cavity length of 2.5 mm) was fabricated from this structure.

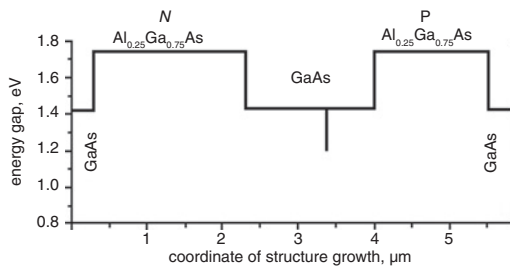


Fig. 1 Schematic energy band diagram of experimental laser structure

From the emission spectrum, the lasing wavelength λ (the wavelength at the spectrum peak) was determined (Table 1). The LCC was measured in the continuous-wave regime from which the threshold current density j_{th} and the internal differential quantum efficiency $\eta_{\text{int,th}}$ at the lasing threshold were determined [21] (Table 1). These experimental values of j_{th} and $\eta_{\text{int,th}}$ were then used to calculate the capture velocity of charge carriers into a QW.

Determination of capture velocity of charge carriers into QW: Having the QW width and lasing wavelength λ and using [22], the composition of the strained InGaAs alloy of the QW was found, as well as the electron

and hole effective masses and the conduction and valence band offsets ΔE_c and ΔE_v at the QW-waveguide heteroboundary. Using these quantities, the energies of the edges of quantum-confinement subbands in the conduction and valence bands in the QW were calculated.

Table 1: Experimental parameters of laser structure

Waveguide	GaAs
Strained QW	In _{0.309} Ga _{0.691} As
QW width	75 Å
λ	1.063 μm
j_{th}	117 A/cm ²
$\eta_{\text{int,th}}$	0.93

The velocity $v_{\text{capt},0}$ of the carrier capture into a single unoccupied QW can be found from the following set of equations for the threshold current density j_{th} and the internal differential quantum efficiency $\eta_{\text{int,th}}$ at the lasing threshold [8, 9]

$$j_{\text{th}} = j_{\text{th}}^{\text{QW}} + j_{\text{th}}^{\text{OCL}} \quad (1)$$

$$\eta_{\text{int,th}} = \left(1 + 2 \frac{j_{\text{th}}^{\text{OCL}}}{j_{\text{capt,th}}}\right)^{-1} \quad (2)$$

In (1), the threshold current density components associated with the spontaneous radiative recombination in the QW and the waveguide [i.e. the optical confinement layer (OCL)] are given by

$$j_{\text{th}}^{\text{QW}} = eB_{2D}(n^{\text{QW}})^2 \quad (3)$$

$$j_{\text{th}}^{\text{OCL}} = ebB_{3D}(n_{\text{th}}^{\text{OCL}})^2 \quad (4)$$

where e is the electron charge, b is the thickness of the OCL, which is equal to 1.7 μm in our structure, and B_{2D} and B_{3D} are the spontaneous radiative recombination constants for the 2D and 3D bulk regions (QW and OCL, respectively) calculated according to the expressions of [23] and [24]. For the experimental structure studied here, $B_{2D} = 2.45 \times 10^{-4} \text{ cm}^2/\text{s}$ and $B_{3D} = 2.04 \times 10^{-10} \text{ cm}^3/\text{s}$. n^{QW} and $n_{\text{th}}^{\text{OCL}}$ are the carrier densities in the QW and the OCL at the lasing threshold.

The bulk carrier density in the OCL at the lasing threshold $n_{\text{th}}^{\text{OCL}}$ is as follows expressed in terms of the 2D carrier density in the QW n^{QW} [11]:

$$n_{\text{th}}^{\text{OCL}} = n_1 \frac{n^{\text{QW}}}{N_c^{2D}} + \frac{1}{v_{\text{capt},0}} \exp\left(\frac{n^{\text{QW}}}{N_c^{2D}}\right) B_{2D}(n^{\text{QW}})^2 \quad (5)$$

with the quantity n_1 being given by

$$n_1 = N_c^{3D} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right) \quad (6)$$

where $\varepsilon_n^{\text{QW}}$ is the energy of the lower edge of the carrier quantum-confinement subband in the QW and T is the temperature in units of energy. In our experimental structure, the QW depth for carriers is $\Delta E_c = 182.3 \text{ meV}$ and the carrier activation energy from the QW is $\Delta E_c - \varepsilon_n^{\text{QW}} = 135.3 \text{ meV}$.

The bulk effective density of states in the OCL and the 2D effective density of states in the QW are given by

$$N_c^{3D} = 2 \left(\frac{m_c^{\text{OCL}} T}{2\pi\hbar^2} \right)^{3/2} \quad (7)$$

$$N_c^{2D} = \frac{m_c^{\text{QW}} T}{\pi\hbar^2} \quad (8)$$

In (2), the current density of the carrier capture from the waveguide (OCL) into the QW at the lasing threshold is [8, 9]

$$j_{\text{capt,th}} = ev_{\text{capt},0} \exp\left(-\frac{n^{\text{QW}}}{N_c^{2D}}\right) n_{\text{th}}^{\text{OCL}} \quad (9)$$

With the values of j_{th} and $\eta_{\text{int,th}}$ obtained from the experiment (see Table 1), (1) and (2) present a set of two equations for the determination of two unknown quantities – the velocity of carrier capture into a single unoccupied QW $v_{\text{capt},0}$ and the carrier density in the QW n^{QW} . For our experimental structure, the solution of this set yields the following values: $v_{\text{capt},0} = 4.3 \times 10^5 \text{ cm/s}$ and $n^{\text{QW}} = 1.69 \times 10^{12} \text{ cm}^{-2}$. It should

be noted that the above value of $v_{\text{capt},0}$ is in good agreement with the theoretical estimates of [13–18].

Conclusion: A simple method for the determination of the capture velocity of charge carriers from a 3D region (waveguide region) into a 2D region (QW) has been proposed. The method is based on measurement of the threshold current density and internal differential quantum efficiency in a semiconductor laser structure. The method also allows determining the 2D carrier density in a QW, which is otherwise not easy to measure in a multilayer laser structure. The ease of the method is particularly due to the fact that both the threshold current density and internal differential quantum efficiency are basic laser characteristics that are conveniently determined for experimental structures. The use of experimental values of these characteristics in the set of (1) and (2) allows calculating the capture velocity of carriers into a QW $v_{\text{capt},0}$ as well as the carrier density in a QW n^{QW} . For the experimental structure studied in this work, $v_{\text{capt},0} = 4.3 \times 10^5$ cm/s and $n^{\text{QW}} = 1.69 \times 10^{12}$ cm⁻².

Acknowledgments: This study was performed under the research plan of the A.F. Ioffe Physical-Technical Institute. L.V. Asryan thanks the US Army Research Office for support of this work (grant no. W911NF-13-1-0445).

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17 February 2015
doi: 10.1049/el.2015.0605

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PHYSICS OF SEMICONDUCTOR DEVICES

Dependence of the Electron Capture Velocity on the Quantum-Well Depth in Semiconductor Lasers

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Submitted November 3, 2015; accepted for publication November 13, 2015

Abstract—Three laser structures with a single strained InGaAs quantum well of different depths and a GaAs or Al_{0.1}Ga_{0.9}As waveguide region are grown by metal-organic chemical vapor deposition. Using the experimentally determined values of the threshold current density and internal differential quantum efficiency, the velocity of electron capture into the quantum well is calculated for each of these structures. It is found that the capture velocity into deep quantum wells is significantly lower than into shallow ones.

DOI: 10.1134/S1063782616050225

1. INTRODUCTION

The injection of charge carriers into quantum wells (QWs) in double-heterostructure semiconductor lasers with a QW in the active region (Fig. 1) occurs via a two-step, indirect process. First, charge carriers are injected from the cladding layers, acting as carrier emitters, into the bulk waveguide region, or the optical-confinement layer (OCL). Next, carriers occupying the OCL are captured into the QW. Carrier capture is characterized by a finite velocity [1, 2]. The capture velocity has a significant effect on the characteristics of semiconductor lasers. In particular, a decrease in the capture velocity can lead to a considerable reduction in the output optical power of the laser [3].

In a series of theoretical papers [3–7], we investigated the impact of charge-carrier capture on the characteristics of heterostructure lasers. The main parameter of this theory is the charge-carrier capture velocity $v_{\text{capt},0}$ for an empty QW. This parameter characterizes the QW and depends on its effective depth, i.e., on the energy of charge-carrier activation from the QW to the waveguide region. Thus, the capture velocity depends on both the QW width and the materials forming the QW and the waveguide regions.

Previously, we suggested a method for determining the velocity of electron capture into a single QW using experimentally obtained characteristics of the laser under study and theoretical calculations [8]. The required experimental parameters are the threshold current density j_{th} and the internal differential quantum efficiency $\eta_{\text{int},\text{th}}$ of the laser at the threshold, and the calculations are based on the model developed in [5].

Here, we use this method to experimentally investigate the dependence of the electron capture velocity on the effective depth of the QW in different laser structures.

2. EXPERIMENTAL CHARACTERISTICS OF THE LASER STRUCTURES

We investigated three laser structures of similar design in which the active region was formed by a sin-

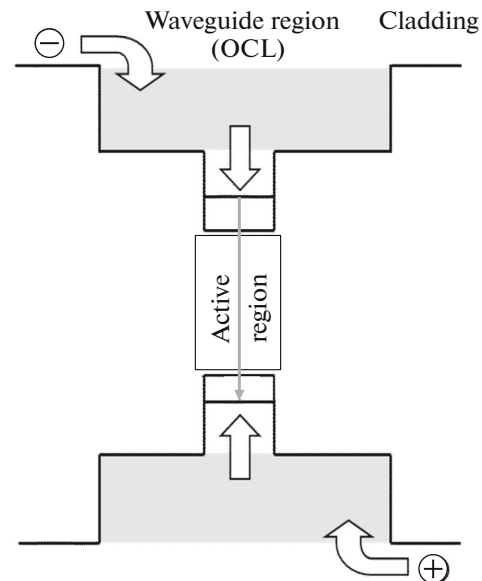


Fig. 1. Schematic layout of a laser with a quantum-confined active region.

Table 1. Technological and measured parameters of the laser structures

Structure number	Composition of the waveguide region	Composition of the QW	QW thickness, Å	λ , μm	j_{th} , A/cm ²	$\eta_{\text{int, th}}$, %
1	GaAs	In _{0.309} Ga _{0.691} As	75	1.063	117	93
2	Al _{0.1} Ga _{0.9} As	In _{0.327} Ga _{0.673} As	60	1.053	120	91
3	Al _{0.1} Ga _{0.9} As	In _{0.302} Ga _{0.698} As	90	1.066	101	93.5

Table 2. Calculated parameters of the laser structures

Structure number	ΔE_c , meV	ε_{n1} , meV	$\Delta E_c - \varepsilon_{n1}$, meV	$B_{2D} \times 10^{-4}$, cm ² /s	$B_{3D} \times 10^{-10}$, cm ³ /s
1	182.3	47	135.3	2.45	2.04
2	256.5	69.1	187.4	2.49	3.03
3	242.5	40	202.5	2.47	3.03

gle strained InGaAs-alloy QW with different alloy compositions (Table 1). All three structures had a broadened 1.7- μm -wide waveguide region. The waveguide was composed of GaAs in one of the structures and solid solution Al_{0.1}Ga_{0.9}As (Table 1) in the other two. The energy-band diagram of these structures is shown schematically in Fig. 2.

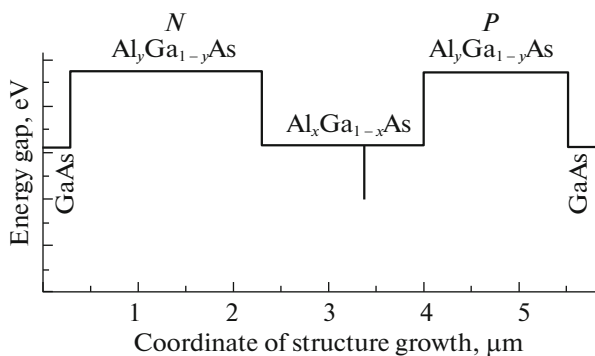
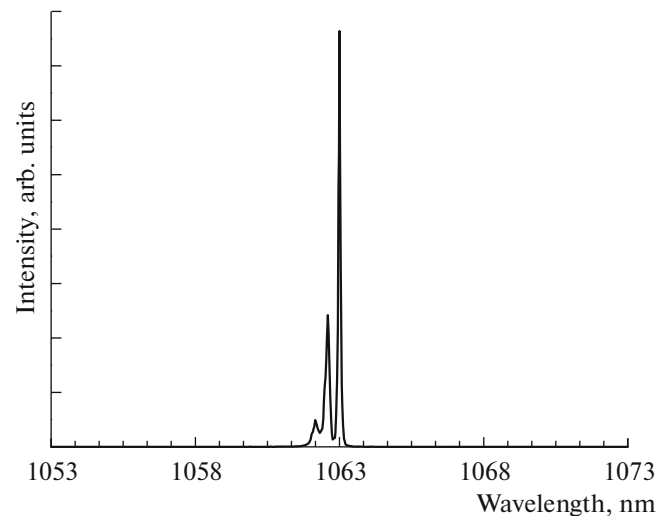
The structures were grown by metal-organic chemical vapor deposition (MOCVD) [9]. Then, mesa stripe lasers were fabricated by conventional post-growth operations [10]. All lasers were shaped as a 100- μm -wide stripe and had the same Fabry–Perot cavity length of 2.5 mm.

For all three lasers, the emission spectra near the lasing threshold were measured (see Fig. 3) and the wavelength λ corresponding to the peak emission intensity was determined (Table 1). Then, the light–current characteristics of these lasers in the continuous-wave (cw) mode were measured, from which the threshold current density j_{th} was found (see Table 1) and the internal differential quantum efficiency $\eta_{\text{int, th}}$ at the lasing threshold was determined (see Table 1) according to the known procedure [11]. Experimental

values of j_{th} and $\eta_{\text{int, th}}$ were then used to calculate the velocities of electron capture into the QWs in the three structures under study.

3. CALCULATION OF THE VELOCITY OF ELECTRON CAPTURE INTO THE QUANTUM WELLS

The experimental values of the QW thickness and the peak wavelength λ of the emission spectrum were used to determine the composition of the strained InGaAs-alloy layer in the active region (Table 1). The calculations were carried out within the framework of the model described in [12]. On the basis of this model, we also determined the electron effective masses in the strained alloy layers and the conduction-band offsets ΔE_c at the heterointerface between the strained QW and the waveguide region (Table 2). Relying on these values, we calculated the energy ε_{n1} of

**Fig. 2.** Energy-band diagram of the experimental laser structures.**Fig. 3.** Laser oscillation spectrum.

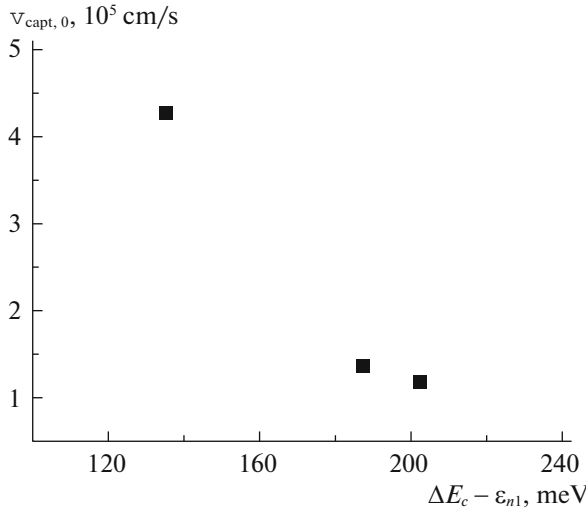


Fig. 4. Electron capture velocity versus the effective width of the QW.

the lowest quantum-confinement subband edge with respect to the conduction-band bottom and the effective QW depth $\Delta E_c - \varepsilon_{n1}$ (Table 2).

In [8], the velocity of electron capture into a single QW was determined from a system of equations for two quantities, i.e., the threshold current density j_{th} and the internal differential quantum efficiency at the lasing threshold $\eta_{\text{int, th}}$, both of which are experimentally determined (Table 1). Let us demonstrate that all quantities that enter these equations can be expressed in terms of two unknowns, i.e., the electron concentration in the QW n^{QW} at the lasing threshold and the capture velocity $v_{\text{capt},0}$ of electrons into an empty QW.

The first equation is that for the threshold current density:

$$j_{\text{th}} = eB_{2\text{D}}(n^{\text{QW}})^2 + ebB_{3\text{D}}(n_{\text{th}}^{\text{OCL}})^2. \quad (1)$$

The first and second terms in Eq. (1) are the densities of the spontaneous-recombination current in the QW and the OCL (waveguide region), respectively, at the lasing threshold. Here, e is the elementary charge; b is the OCL width, which is $1.7 \mu\text{m}$ in all of the structures under study; n^{QW} and $n_{\text{th}}^{\text{OCL}}$ are the electron concentrations in the QW and the OCL at the lasing threshold; $B_{2\text{D}}$ and $B_{3\text{D}}$ are the coefficients of spontaneous recombination in the two-dimensional QW and the three-dimensional OCL, respectively, calculated using the formulas given in [13, 14]. The values of $B_{2\text{D}}$ and $B_{3\text{D}}$ are listed in Table 2.

The electron concentration in the waveguide region at the lasing threshold can be expressed via the

electron concentration in the QW n^{QW} and the capture velocity $v_{\text{capt},0}$ as follows [5]:

$$n_{\text{th}}^{\text{OCL}} = n_1 \frac{n^{\text{QW}}}{N_c^{2\text{D}}} + \frac{1}{v_{\text{capt},0}} \exp\left(\frac{n^{\text{QW}}}{N_c^{2\text{D}}}\right) B_{2\text{D}}(n^{\text{QW}})^2. \quad (2)$$

The quantity n_1 depends exponentially on the effective depth of the QW:

$$n_1 = N_c^{3\text{D}} \exp\left(-\frac{\Delta E_c - \varepsilon_{n1}}{T}\right); \quad (3)$$

here, the temperature is expressed in energy units. The bulk effective density of states $N_c^{3\text{D}}$ in the conduction band of the waveguide region and the two-dimensional effective density of states $N_c^{2\text{D}}$ in the conduction band of the QW are given by the following expressions:

$$N_c^{3\text{D}} = 2 \left(\frac{m_c^{\text{OCL}} T}{2\pi\hbar^2} \right)^{3/2}, \quad (4)$$

$$N_c^{2\text{D}} = \frac{m_c^{\text{QW}} T}{2\pi\hbar^2}, \quad (5)$$

where m_c^{OCL} and m_c^{QW} are the electron effective masses in the OCL and the QW, respectively.

The second equation is that for the internal differential quantum efficiency at the lasing threshold [5]:

$$\eta_{\text{int, th}} = \frac{1}{1 + 2[ebB_{3\text{D}}(n_{\text{th}}^{\text{OCL}})^2 / j_{\text{capt, th}}]}. \quad (6)$$

Here, the current density of carrier capture from the waveguide region into the QW at the lasing threshold is [2, 3]

$$j_{\text{capt, th}} = ev_{\text{capt},0} \exp\left(-\frac{n^{\text{QW}}}{N_c^{2\text{D}}}\right) n_{\text{th}}^{\text{OCL}}. \quad (7)$$

According to Eqs. (6) and (7), the value of $\eta_{\text{int, th}}$ can be expressed via the same two unknown quantities n^{QW} and $v_{\text{capt},0}$ as the value of j_{th} .

Thus, with the values of j_{th} and $\eta_{\text{int, th}}$ known from experiments, the system of equations (1) and (6) contains two quantities to be found, i.e., n^{QW} and $v_{\text{capt},0}$. This system was solved numerically for each of the three experimental laser structures using the values of j_{th} and $\eta_{\text{int, th}}$ from Table 1. The calculated values of the electron capture velocities and electron concentrations in the QW are shown in Figs. 4 and 5. In all three cases, independent of the effective QW depth, the threshold concentrations of electrons in the QW differ insignificantly. However, the electron capture velocity into deep QWs is much lower than into shallow QWs. The use of $\text{Al}_{0.1}\text{Ga}_{0.9}\text{As}$ alloy instead of GaAs as the material for the waveguide region led to an increase in the effective QW depth by 50–65 meV, and this led to a reduction in the capture velocity by a factor of 3.1–3.6.

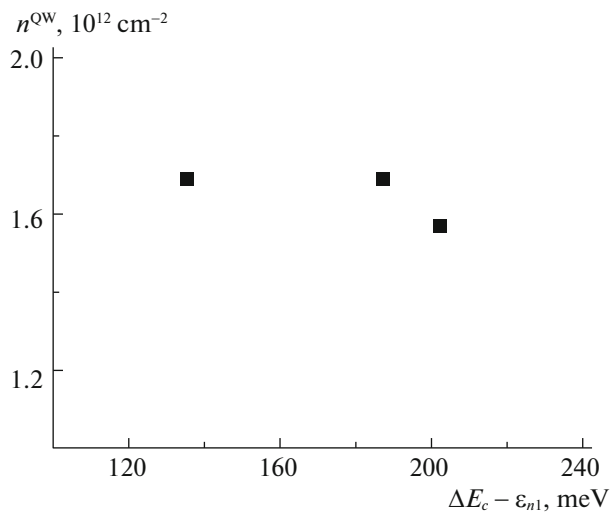


Fig. 5. Electron concentration at the lasing threshold versus the effective width of the QW.

4. CONCLUSIONS

We have investigated the dependence of the velocity of electron capture by a QW on the effective well depth in three experimental laser structures. The structures were grown by MOCVD and contained a single QW of different depths. In these structures, the threshold current density and internal differential quantum efficiency have been measured. Using the experimentally determined values of these quantities and the theoretical model developed in [5], we have calculated the electron capture velocity by the QW in each of the three structures. We have demonstrated that the capture velocity decreases considerably with an increase in the depth of the QW.

ACKNOWLEDGMENTS

This study was carried out as part of the federal program for the Ioffe Physical–Technical Institute.

L.V. Asryan acknowledges support from the US Army Research Office, grant no. W911NF-13-1-0445.

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Translated by M. Skorikov

Threshold characteristics of a semiconductor quantum-well laser: inclusion of global electroneutrality in the structure

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Abstract. Threshold characteristics of a semiconductor quantum-well (QW) laser are calculated using the global electroneutrality condition, which includes charge carriers both in the active and waveguide regions and thus presents an equality of the total charge of electrons to the total charge of holes in these two regions. It is shown that at the lasing threshold, the densities of electrons in the QW and the waveguide region are not equal to the densities of holes in these regions, i.e., the local electroneutrality condition is violated in each of the regions. Depending on the velocities of the carrier capture from the waveguide region into the QW, the electron density can be either higher or lower than the hole density (both in the QW and in the waveguide region). The charge of the carriers of each sign in the waveguide region is shown to be greater than that in the QW.

Keywords: semiconductor lasers, quantum wells, electroneutrality condition.

1. Introduction

In modern semiconductor laser structures, stimulated emission is generated in a low-dimensional active region surrounded by a wideband bulk waveguide region (optical confinement layer, OCL) [1, 2]. In such structures, electrons and holes are first injected from emitters into the OCL, and then captured by the active region [3]. For a high output power to be obtained, the threshold current of the laser structure should be low, and therefore, threshold densities of charge carriers in the active region and the OCL should be low, too.

In calculating threshold characteristics of heterostructure lasers, it is generally assumed that densities of electrons and holes in the active region are equal [4], i.e. there is a local electroneutrality. In reality, however, due to differences between electron and hole parameters (primarily between the parameters that control the capture in a nanosized active region and the position of sub-bands or size quantization levels therein), the carrier densities in the active region may be different, which means violation of the local electroneutrality condition.

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Received 15 July 2016
Kvantovaya Elektronika 46 (9) 777–781 (2016)
Translated by I.A. Ulitkin

In our paper [5] we calculated threshold characteristics of a Fabry–Perot cavity planar injection laser at different ratios between the densities of electrons and holes in a quantum well (QW).

In this paper, threshold characteristics of a semiconductor QW laser are calculated using a global electroneutrality condition, which includes charge carriers both in the active region and the OCL. This condition presents an equality of the total charge of electrons to the total charge of holes in these two regions. It is shown that at the lasing threshold, the densities of electrons in the QW and in the OCL are not equal to the densities of holes in them. Depending on the velocity of carrier capture from the OCL into the QW, the electron density can be either higher or lower than the hole density (both in the QW and OCL).

2. Theoretical model

To calculate the characteristics of the laser, we used five steady-state rate equations [6]:

for free electrons in the bulk OCL of thickness b [$b(\partial n^{\text{OCL}}/\partial t) = 0$]

$$\begin{aligned} \frac{j}{e} + N_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{\text{nesc}}} - N_{\text{QW}} v_{\text{ncapt}0} (1 - f_n) n^{\text{OCL}} \\ - b B_{3D} n^{\text{OCL}} p^{\text{OCL}} = 0; \end{aligned} \quad (1)$$

for free holes in the OCL [$b(\partial p^{\text{OCL}}/\partial t) = 0$]

$$\begin{aligned} \frac{j}{e} + N_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{\text{pesc}}} - N_{\text{QW}} v_{\text{pcapt}0} (1 - f_p) p^{\text{OCL}} \\ - b B_{3D} n^{\text{OCL}} p^{\text{OCL}} = 0; \end{aligned} \quad (2)$$

for electrons confined in the QW ($\partial n^{\text{QW}}/\partial t = 0$)

$$\begin{aligned} v_{\text{ncapt}0} (1 - f_n) n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{\text{nesc}}} - B_{2D} n^{\text{QW}} p^{\text{QW}} \\ - v_{\text{gr}} g^{\text{max}} (f_n + f_p - 1) \frac{N}{S} = 0; \end{aligned} \quad (3)$$

for holes confined in the QW ($\partial p^{\text{QW}}/\partial t = 0$)

$$v_{\text{pcapt}0} (1 - f_p) p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{\text{pesc}}} - B_{2D} n^{\text{QW}} p^{\text{QW}} -$$

$$-v_{gr}g^{\max}(f_n + f_p - 1)\frac{N}{S} = 0; \quad (4)$$

and for photons in the lasing mode ($\partial N/\partial t = 0$)

$$v_{gr}N_{QW}g^{\max}(f_n + f_p - 1)N - v_{gr}(\beta + \alpha_{int})N = 0. \quad (5)$$

Here, n^{OCL} and p^{OCL} are the densities of free electrons and holes in the OCL; n^{QW} and p^{QW} are the two-dimensional densities of electrons and holes confined in the QW; N is the number of stimulated-emission photons; and f_n and f_p are the filling factors (occupancies) of the states corresponding to the lower edge of the electron quantum-confinement sub-band and to the upper edge of the hole quantum-confinement sub-band in the QW.

The occupancies f_n and f_p are expressed in terms of two-dimensional densities of electrons and holes in the QW, n^{QW} and p^{QW} , as follows [7, 8]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_c^{\text{2D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_v^{\text{2D}}}\right), \quad (6)$$

where $N_{c,v}^{\text{2D}} = m_{c,v}^{\text{QW}} T/(\pi\hbar^2)$ are the two-dimensional effective densities of states in the conduction and valence bands in the QW; $m_{c,v}^{\text{QW}}$ are the effective masses of electrons and holes in the QW; and T is the temperature in energy units.

Equations (1)–(5) also include the following parameters: j is the injection current density; e is the electron charge; N_{QW} is the number of identical (having the same width and the same composition) QWs; $\tau_{n\text{esc}}$ and $\tau_{p\text{esc}}$ are the thermal escape times of electrons and holes from the QW into the OCL; v_{ncapt0} and v_{pcapt0} are the capture velocities of electrons and holes into an empty (at $f_n = 0$ and $f_p = 0$) single QW (cm s^{-1}); B_{3D} and B_{2D} are the coefficients of spontaneous radiative recombination in the three-dimensional (OCL) region and in the two-dimensional (QW) region ($\text{cm}^3 \text{s}^{-1}$ and $\text{cm}^2 \text{s}^{-1}$, respectively, see expressions for B_{3D} and B_{2D} in [9] and [10]); v_{gr} is the group velocity of light; $g^{\max}$ is the maximum gain in each QW; $S = WL$; W is the width of the stripe contact; L is the length of Fabry–Perot cavity; $\beta = (1/L)\ln(1/R)$ are the losses associated with the emission output from the cavity; R is the reflectance of the mirrors; and α_{int} is the internal optical loss coefficient in the laser structure.

In accordance with [6, 11], the thermal escape times of electrons and holes from the QW into the OCL have the form

$$\tau_{n\text{esc}} = \frac{1}{v_{ncapt0}(1-f_n)} \frac{N_c^{\text{2D}}}{n_1}, \quad (7)$$

$$\tau_{p\text{esc}} = \frac{1}{v_{pcapt0}(1-f_p)} \frac{N_v^{\text{2D}}}{p_1},$$

where

$$n_1 = N_c^{\text{3D}} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right); \quad (8)$$

$$p_1 = N_v^{\text{3D}} \exp\left(-\frac{\Delta E_v - \varepsilon_p^{\text{QW}}}{T}\right);$$

$N_{c,v}^{\text{3D}} = 2[m_{c,v}^{\text{OCL}} T/(\pi\hbar^2)]^{3/2}$ are the three-dimensional effective densities of states in the conduction and valence bands in

the OCL; $m_{c,v}^{\text{OCL}}$ are the effective masses of electrons and holes in the OCL; $\Delta E_{c,v}$ are the offsets of the edges of the conduction and valence bands at a heterojunction between a QW and OCL; and $\varepsilon_n^{\text{QW}}$ and $\varepsilon_p^{\text{QW}}$ are, respectively, the energies of the lower and upper edges of the electron and hole quantum-confinement sub-bands in QWs.

The capture velocities of electrons and holes (v_{ncapt0} and v_{pcapt0}) from the OCL into an empty QW are the characteristics of a QW, depend on its width and depth, i.e. on the composition of the QW material and its surrounding layers, and can vary greatly in different laser structures. In papers [12, 13], we found the electron capture velocities into a single QW in real laser structures. To do this, we used experimental laser characteristics and theoretical calculations.

The total capture velocities v_{ncapt} and v_{pcapt} are defined as follows [6]:

$$v_{ncapt} = v_{ncapt0}(1-f_n), \quad v_{pcapt} = v_{pcapt0}(1-f_p). \quad (9)$$

The global electroneutrality condition for the OCL and the QWs is written in the form:

$$e(N_{QW}n^{\text{QW}} + bn^{\text{OCL}}) = e(N_{QW}p^{\text{QW}} + bp^{\text{OCL}}). \quad (10)$$

3. Discussion of threshold characteristics of a laser

The set of equations (1)–(5), supplemented with condition (10), was solved at the lasing threshold when the number of coherent photons in the cavity can be assumed equal to zero ($N = 0$). The set was solved numerically for a InGaAs/GaAs/AlGaAs laser structure comprising a single strained QW $\text{In}_{0.28}\text{Ga}_{0.72}\text{As}$ of thickness 80 Å. The material of the waveguide region (OCL) having a width $b = 1.7 \mu\text{m}$ was GaAs; the emitter material was $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$. The lasing wavelength was equal to 1.044 μm . The length of the Fabry–Perot cavity was $L = 1.5 \text{ mm}$, the width of the stripe contact was $W = 100 \mu\text{m}$, the reflectance of the mirrors was $R = 0.32$, the mirror loss was $\beta = 7.6 \text{ cm}^{-1}$, the temperature was $T = 300 \text{ K}$, and the internal optical loss was $\alpha_{int} = 1 \text{ cm}^{-1}$ (in this paper α_{int} was considered to be independent of the injection current). The maximum modal gain in the QW was $g^{\max} = 49.1 \text{ cm}^{-1}$. The capture velocities of electrons and holes into an empty QW, v_{ncapt0} and v_{pcapt0} , were varied from 10^5 to 10^6 cm s^{-1} .

Using equation (5) with (6) taken into account, the electron density n^{QW} in the QW can be expressed through the hole density p^{QW} in the QW as follows [5]:

$$n^{\text{QW}} = N_c^{\text{2D}} \ln \left[1 - \frac{\beta + \alpha_{int}}{N_{QW}g^{\max}} - \exp\left(-\frac{p^{\text{QW}}}{N_v^{\text{2D}}}\right) \right]^{-1}. \quad (11)$$

Using expression (11) in equations (3) and (4), the densities of electrons and holes in the OCL, n^{OCL} and p^{OCL} , can also be expressed through p^{QW} . And then using expression (11) for n^{QW} and thus obtained expressions for n^{OCL} and p^{OCL} in (10), we obtain an equation for one unknown (p^{QW}). Determining p^{QW} from it, we find n^{QW} , n^{OCL} and p^{OCL} . The threshold current density is calculated according to the formula

$$j_{th} = eN_{QW}B_{2D}n^{\text{QW}}p^{\text{QW}} + ebB_{3D}n^{\text{OCL}}p^{\text{OCL}}. \quad (12)$$

Figure 1a shows the dependences of the threshold densities of electrons and holes in the QW on the capture velocity of holes into an empty QW at electron capture velocities $v_{n\text{capt}0} = 10^5$ and 10^6 cm s⁻¹. One can see that when $v_{n\text{capt}0}$ is changed by an order of magnitude (from 10^5 to 10^6 cm s⁻¹), p^{QW} and n^{QW} weakly depend on $v_{p\text{capt}0}$; at a low capture velocity of electrons ($v_{n\text{capt}0} = 10^5$ cm s⁻¹), the threshold density of holes in the active region p^{QW} [curve (1)] is higher than the threshold density of electrons n^{QW} [curve (2)]; at a high capture velocity of electrons ($v_{n\text{capt}0} = 10^6$ cm s⁻¹), the ratio between p^{QW} and n^{QW} is reversed, i.e. the density of electrons [curve (4)] in the active region is about twice the density of holes [curve (3)].

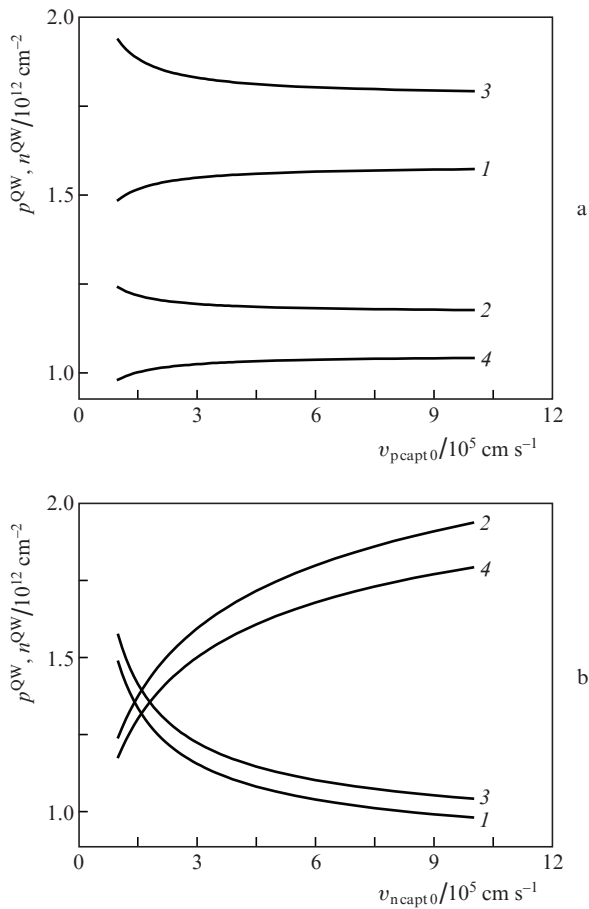


Figure 1. Dependences of the densities of (1, 3) holes and (2, 4) electrons in the QW at the lasing threshold on (a) the hole capture velocity at $v_{n\text{capt}0} = (1, 2) 10^5$ and (3, 4) 10^6 cm s⁻¹ and (b) the electron capture velocity at $v_{p\text{capt}0} = (1, 2) 10^5$ and (3, 4) 10^6 cm s⁻¹.

Figure 1b shows the dependences of the densities n^{QW} and p^{QW} on the capture velocity of electrons into an empty QW at capture velocities of holes $v_{p\text{capt}0} = 10^5$ and 10^6 cm s⁻¹. One can see that p^{QW} and n^{QW} quite strongly depend on $v_{n\text{capt}0}$; at small $v_{n\text{capt}0}$, the density of holes in the QW [curve (1)] is greater than that of electrons [curve (2)]; at large $v_{p\text{capt}0}$, the density of electrons in the QW can significantly exceed the density of holes, which coincides with the results shown in Fig. 1a.

The results obtained (Fig. 1) demonstrate that in laser structures the electron and hole densities in the active region at the lasing threshold can differ greatly, which means viola-

tion of the local electroneutrality condition in it [5]. Depending on the capture velocity, the electron density in the QW can be either higher or lower than the hole density. Note that the densities n^{QW} and p^{QW} are strongly dependent on the electron capture velocity and are weakly dependent on the hole capture velocity.

Our calculations show that at the lasing threshold when changing $v_{n\text{capt}0}$ and $v_{p\text{capt}0}$ from 10^5 to 10^6 cm s⁻¹, the occupancy of the lower edge of the electron quantum-confinement sub-band in the QW, f_n [see (6)], varies from 0.843 to 0.952, and the occupancy of the upper edge of the hole quantum-confinement sub-band in the QW, f_p [see (6)], varies from 0.223 to 0.332. Thus, holes in the QW at the lasing threshold are non-degenerate. In the studied range of the capture velocities, the quasi-Fermi level of holes is located in the forbidden zone. Electrons in the QW at the lasing threshold are strongly degenerate, i.e. the quasi-Fermi level of electrons is located in the quantum-confinement sub-band. Thus, already at the lasing threshold, unlike the hole quantum-confinement sub-band, the electron quantum-confinement sub-band in the QW is strongly filled.

Figure 2a shows the threshold densities of electrons and holes in the OCL as functions of the capture velocities of holes, $v_{p\text{capt}0}$, for two values of the electron capture velocity $v_{n\text{capt}0}$. One can see a weak dependence of the threshold densities in the OCL on the capture velocity of holes. It is also seen that at small values of $v_{n\text{capt}0}$, the densities n^{OCL} and p^{OCL} do not significantly differ, the electron density in the OCL

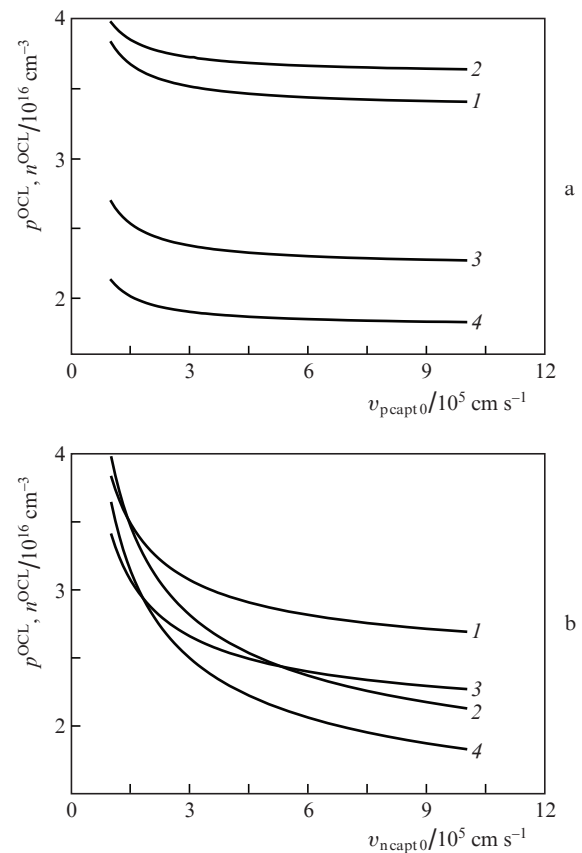


Figure 2. Dependences of the densities of (1, 3) holes and (2, 4) electrons in the OCL at the lasing threshold on (a) the hole capture velocity at $v_{n\text{capt}0} = (1, 2) 10^5$ and (3, 4) 10^6 cm s⁻¹ and (b) the electron capture velocity at $v_{p\text{capt}0} = (1, 2) 10^5$ and (3, 4) 10^6 cm s⁻¹.

being higher than the hole density. At high capture velocities of electrons ($v_{\text{n capt}0} = 10^6 \text{ cm s}^{-1}$), the hole density in the OCL exceeds the electron density.

Figure 2b shows the threshold densities of electrons and holes in the OCL as functions of the capture velocities of electrons, $v_{\text{n capt}0}$, for two values of the hole capture velocity $v_{\text{p capt}0}$. One can see that there is a strong dependence of the threshold densities in the OCL on the electron capture velocity $v_{\text{n capt}0}$. At $v_{\text{n capt}0} > 2 \times 10^5 \text{ cm s}^{-1}$, the hole density in the OCL starts to exceed the electron density.

Thus, as follows from Figure 2, with increasing capture velocities of both electrons and holes, the threshold densities of the charge carriers in the OCL decrease.

Figure 3 shows the dependence of the ratio of the electron charge in the OCL to that in the QW and the ratio of the hole charge in the OCL to that in the QW on the capture velocities of electrons and holes into the QW. It follows from Fig. 3a that these ratios are virtually independent of the capture velocity of holes. The ratio for the holes is also independent of the electron capture velocity, while the ratio for the electrons decreases by approximately three times with increasing electron capture velocity by an order of magnitude (Fig. 3b). However, the charge of both electrons and holes in the OCL is larger than that in the QW.

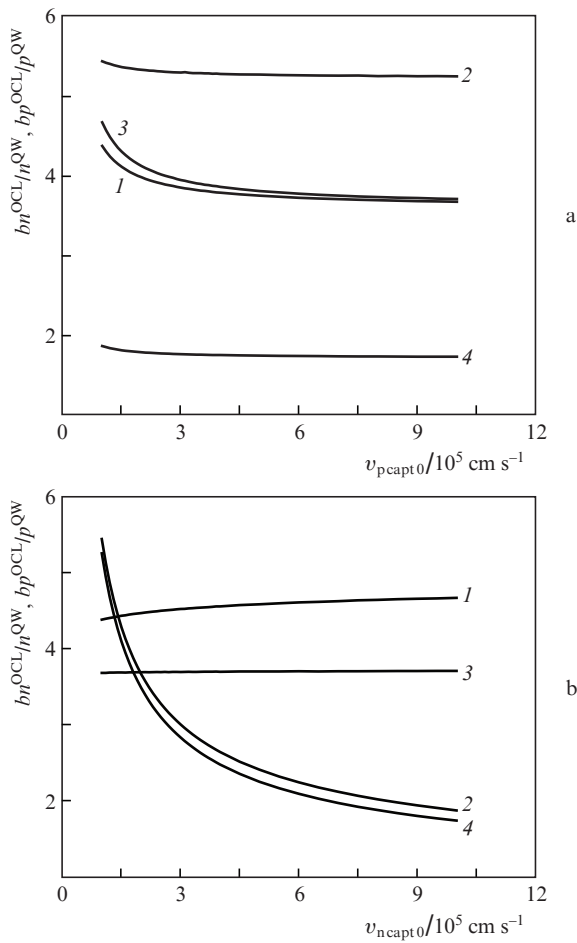


Figure 3. Ratios of (1, 3) the charge of holes in the OCL to the charge of holes in the QW and (2, 4) the charge of electrons in the OCL to the charge of electrons in the QW vs. (a) the capture velocity of holes at $v_{\text{n capt}0} = (1, 2) 10^5$ and $(3, 4) 10^6 \text{ cm s}^{-1}$ and (b) the electron capture velocity at $v_{\text{p capt}0} = (1, 2) 10^5$ and $(3, 4) 10^6 \text{ cm s}^{-1}$.

The dependences of the threshold current density on the capture velocity of holes in the QW at electron capture velocities $v_{\text{n capt}0} = 10^5$ and 10^6 cm s^{-1} are shown in Fig. 4a, and the dependences of the threshold current density on the capture velocity of electrons at hole capture velocities $v_{\text{p capt}0} = 10^5$ and 10^6 cm s^{-1} – in Fig. 4b. One can see from Fig. 4 that in the laser structure in question, the threshold current density is weakly dependent on the capture velocities of both holes and electrons.

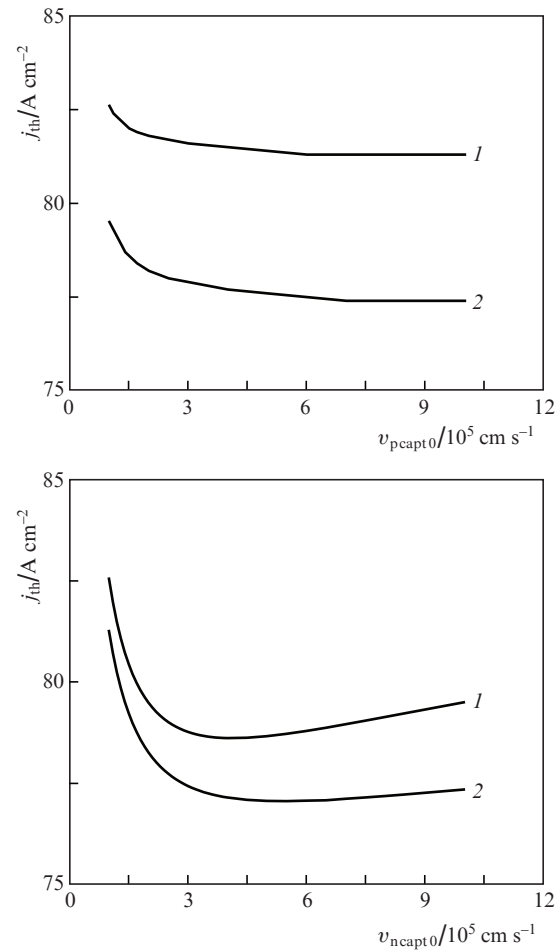


Figure 4. Dependences of the threshold current density on (a) the hole capture velocity at $v_{\text{n capt}0} = (1, 2) 10^5$ and $(3, 4) 10^6 \text{ cm s}^{-1}$ and (b) the electron capture velocity at $v_{\text{p capt}0} = (1, 2) 10^5$ and $(3, 4) 10^6 \text{ cm s}^{-1}$.

4. Conclusions

We have used the global electroneutrality condition, i.e. the condition of the equality of the total charge of electrons in the OCL and QW to the total charge of holes in the OCL and QW.

Calculations with the global electroneutrality condition taken into account have shown that at the lasing threshold in laser structures, the densities of electrons and holes in the active region are strongly different (see Fig. 1), which means a violation of the local electroneutrality condition. Depending on the capture velocity, the electron density in the QW can be either higher or lower than the hole density. It is shown that the densities n^{QW} and p^{QW} strongly depend on the electron capture velocity and weakly – on the hole capture velocity.

The densities of electrons and holes in the OCL at the lasing threshold are also not equal: depending on the capture velocity, the electron density in the OCL can be either higher or lower than the hole density. With increasing capture velocity, the densities of both types of charge carriers in the OCL are reduced; however, the charge of the carriers of each sign in the OCL is greater than that in the QW.

Thus, we have shown that at the lasing threshold, the local electroneutrality condition is violated in laser layers – in the active region and the OCL (under the condition of the global electroneutrality).

Acknowledgements. This work was carried out in accordance with the State Programme of the Ioffe Institute. L.V. Asryan thanks the US Army Research Office (Grant No. W911NF-13-1-0445) for support of this work.

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Theory of operating characteristics of a semiconductor quantum well laser: Inclusion of global electroneutrality in the structure

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Abstract

A model for calculating the operating characteristics of semiconductor quantum well (QW) lasers is presented. The model exploits the condition of global electroneutrality, which includes the charge carriers both in the two-dimensional (2D) active region (QW) and bulk waveguide region (optical confinement layer – OCL). The charge of each sign in the OCL is shown to be significantly larger than that in the QW. As a result of this, (i) the global electroneutrality condition reduces to the condition of electroneutrality in the OCL and (ii) the local electroneutrality in the QW can be strongly violated, i.e., the 2D electron and hole densities in the QW can significantly differ from each other.

1. Introduction

The stimulated emission in contemporary semiconductor lasers is generated in a low-dimensional active region, which is surrounded by a wider band gap bulk waveguide region (optical confinement layer – OCL) [1, 2]. In such structures, electrons and holes are first injected from the cladding layers into the OCL and then captured into the active region [3] (figure 1).

In [4], the operating characteristics of semiconductor QW lasers were calculated with a proper account for noninstantaneous capture of charge carriers. In calculations, we assumed the local electroneutrality in QWs, i.e., equality of the 2D electron and hole densities in QWs to each other.

In this work, the operating characteristics of a semiconductor QW laser are calculated exploiting the condition of global electroneutrality, which includes the charge carriers both in the active region (QWs) and OCL. This condition thus presents the equality of the total charge of electrons to that of holes – see equation (10) below. The charge of each sign in the OCL is shown to be significantly larger than that in the QW. As a result of this, (i) the global electroneutrality condition reduces to the condition of electroneutrality in the OCL and (ii) the local electroneutrality in the QW can be strongly violated, i.e., the 2D electron and hole densities in the QW can significantly differ from each other.

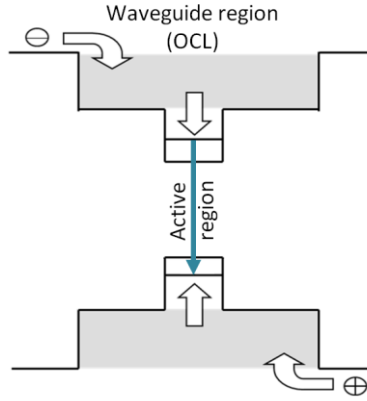


Figure 1. A schematic of a semiconductor laser with a quantum-confined active region.

2. Theoretical model

To calculate the laser characteristics, the following set of steady-state rate equations is used [4]:

for free electrons in the bulk OCL [$b(\partial n^{\text{OCL}}/\partial t) = 0$],

$$\frac{j}{e} + N_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{\text{n,esc}}} - N_{\text{QW}} v_{\text{n,capt},0} (1 - f_{\text{n}}) n^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0 \quad (1)$$

for free holes in the OCL [$b(\partial p^{\text{OCL}}/\partial t) = 0$],

$$\frac{j}{e} + N_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{\text{p,esc}}} - N_{\text{QW}} v_{\text{p,capt},0} (1 - f_{\text{p}}) p^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0 \quad (2)$$

for electrons confined in the QW ($\partial n^{\text{QW}}/\partial t = 0$),

$$v_{\text{n,capt},0} (1 - f_{\text{n}}) n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{\text{n,esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - v_{\text{g}} g^{\text{max}} (f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} = 0 \quad (3)$$

for holes confined in the QW ($\partial p^{\text{QW}}/\partial t = 0$),

$$v_{\text{p,capt},0} (1 - f_{\text{p}}) p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{\text{p,esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - v_{\text{g}} g^{\text{max}} (f_{\text{n}} + f_{\text{p}} - 1) \frac{N}{S} = 0 \quad (4)$$

and for photons in the lasing mode ($\partial N/\partial t = 0$),

$$v_{\text{g}} N_{\text{QW}} g^{\text{max}} (f_{\text{n}} + f_{\text{p}} - 1) N - v_{\text{g}} (\beta + \alpha_{\text{int}}) N = 0 \quad (5)$$

The following quantities are the unknowns in equations (1)-(5): n^{OCL} and p^{OCL} are the free electron and hole densities in the OCL, n^{QW} and p^{QW} are the 2D densities of electrons and holes confined in each of the QWs, N is the number of photons in the stimulated emission, f_{n} (f_{p}) is the occupancy of the lower (upper) edge of the electron (hole) quantum-confinement subband in the QW. The occupancies f_{n} and f_{p} are expressed in terms of the 2D electron and hole densities n^{QW} and p^{QW} as follows [7, 8]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_c^{2\text{D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_v^{2\text{D}}}\right) \quad (6)$$

where $N_{c,v}^{2\text{D}} = m_{e,h}^{\text{QW}} T / (\pi \hbar^2)$ are the 2D effective densities of states in the conduction and valence bands in the QW, $m_{e,h}^{\text{QW}}$ are the electron and hole effective masses in the QW, and the temperature T is measured in units of energy.

The following parameters enter into equations (1)-(5): j is the injection current density, e is the electron charge, N_{QW} is the number of identical (of the same material composition and width) QWs, $\tau_{n,\text{esc}}$ and $\tau_{p,\text{esc}}$ are the thermal escape times of electrons and holes from a QW to the OCL, $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ are the capture velocities of electrons and holes into an empty (at $f_n = 0$ and $f_p = 0$) QW measured in units of cm/s, b is the thickness of the OCL, $B_{3\text{D}}$ and $B_{2\text{D}}$ are the spontaneous radiative recombination coefficients for the bulk (OCL) and 2D (QW) regions measured in units of cm^3/s and cm^2/s , respectively – see [5] and [6] for the expressions for $B_{3\text{D}}$ and $B_{2\text{D}}$; v_g is the group velocity of light, g^{max} is the maximum gain in each QW, $S = WL$ is the cross-section of the junction, W is the lateral size of the device, L is the Fabry-Pérot cavity length, $\beta = (1/L)\ln(1/R)$ is the mirror loss, R is the facet reflectivity, and α_{int} is the coefficient of internal optical loss in the structure.

The thermal escape times of electrons and holes from the QW into the OCL are [4, 9]:

$$\tau_{n,\text{esc}} = \frac{1}{v_{n,\text{capt},0}(1-f_n)} \frac{N_c^{2\text{D}}}{n_1}, \quad \tau_{p,\text{esc}} = \frac{1}{v_{p,\text{capt},0}(1-f_p)} \frac{N_v^{2\text{D}}}{p_1} \quad (7)$$

Where

$$n_1 = N_c^{3\text{D}} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right), \quad p_1 = N_v^{3\text{D}} \exp\left(-\frac{\Delta E_v - \varepsilon_p^{\text{QW}}}{T}\right) \quad (8)$$

and $N_{c,v}^{3\text{D}} = 2[m_{c,v}^{\text{OCL}} T / (2\pi \hbar^2)]^{3/2}$ are the 3D effective densities of states in the conduction and valence bands in the OCL, $m_{c,v}^{\text{OCL}}$ are the electron and hole effective masses in the OCL, $\Delta E_{c,v}$ are the conduction and valence band offsets at the heterointerface of the QW and OCL, and $\varepsilon_n^{\text{QW}}$ ($\varepsilon_p^{\text{QW}}$) is the energy of the lower (upper) edge of the electron (hole) subband in the QW.

The velocities $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ of electron and hole capture from the OCL into an empty QW are the characteristics of the QW; they thus depend on the QW width and depth, i.e., on the material compositions of the QW and surrounding layers. The capture velocity can be significantly different in different laser structures. In [10], we determined the value of the electron capture velocity in the laser structure similar to the one used here as an example for our calculations (see below).

The total capture velocities $v_{n,\text{capt}}$ and $v_{p,\text{capt}}$ are defined as [4]

$$v_{n,\text{capt}} = v_{n,\text{capt},0}(1-f_n), \quad v_{p,\text{capt}} = v_{p,\text{capt},0}(1-f_p). \quad (9)$$

The condition of global electroneutrality for the OCL and QW is written as follows:

$$e(N_{\text{QW}} n^{\text{QW}} + b n^{\text{OCL}}) = e(N_{\text{QW}} p^{\text{QW}} + b p^{\text{OCL}}) \quad (10)$$

The set of equations (1)-(5), added by equation (10), was solved numerically for a InGaAs/GaAs/AlGaAs laser structure containing a single strained $\text{In}_{0.28}\text{Ga}_{0.72}\text{As}$ QW of 80 Å width. The

material of the OCL of thickness $b = 1.7 \mu\text{m}$ was GaAs, the material of the cladding layers was $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$. The lasing wavelength was $1.044 \mu\text{m}$. The Fabry-Pérot cavity length $L = 1.5 \text{ mm}$, the stripe width $W = 100 \mu\text{m}$, the mirror reflectivity $R = 0.32$, the mirror loss $\beta = 7.6 \text{ cm}^{-1}$, the temperature $T = 300 \text{ K}$, the coefficient of internal optical loss $\alpha_{\text{int}} = 1 \text{ cm}^{-1}$ (in this work, α_{int} is assumed to be independent of the injection current). The maximum modal gain in the QW $g^{\text{max}} = 49.1 \text{ cm}^{-1}$. The following values were used for the electron and hole capture velocities [10]: $v_{n,\text{capt},0} = 4 \times 10^5 \text{ cm/s}$ and $v_{p,\text{capt},0} = 4 \times 10^5 \text{ cm/s}$.

3. Discussion of laser characteristics

The following characteristics were calculated versus the injection current density j (up to $j = 75 \text{ kA/cm}^2$): i) electron and hole densities in the QW, n^{QW} and p^{QW} (figure 2), ii) electron and hole densities in the OCL, n^{OCL} and p^{OCL} (figure 4), iii) stimulated recombination current density in the QW, $j_{\text{stim}}^{\text{QW}}$, and spontaneous recombination current density in the OCL, $j_{\text{spon}}^{\text{OCL}}$ (figure 8), and iv) output optical power (figure 9).

The above listed characteristics, calculated using the global electroneutrality condition (10), were compared with those calculated using the condition of local electroneutrality in the QW

$$e n^{\text{QW}} = e p^{\text{QW}}. \quad (11)$$

As seen from figure 2, the electron and hole densities in the QW, calculated using the global electroneutrality condition (10), differ significantly.

However, the product $n^{\text{QW}} p^{\text{QW}}$, calculated with the use of the condition (10), remains virtually constant with increasing injection current (figure 3).

The electron and hole densities in the OCL are shown in figure 4.

As seen from figure 5, the charge of each sign in the OCL is significantly larger than that in the QW,

$$e b n^{\text{OCL}} \gg e N_{\text{QW}} n^{\text{QW}}, \quad e b p^{\text{OCL}} \gg e N_{\text{QW}} p^{\text{QW}} \quad (12)$$

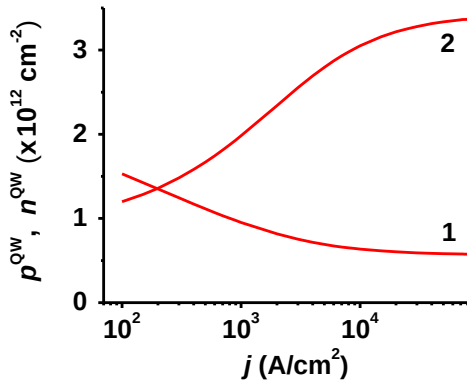


Figure 2. Electron (1) and hole (2) densities in the QW against injection current density calculated using the global electroneutrality condition.

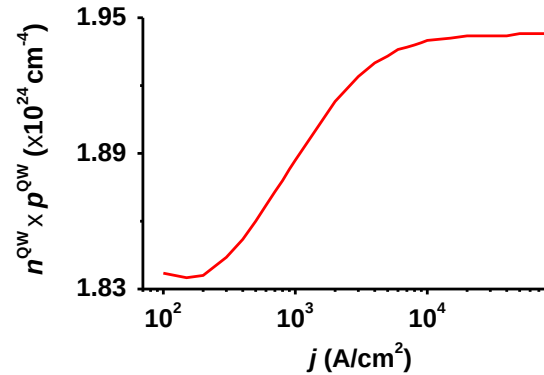


Figure 3. Product of electron and hole densities in the QW against injection current density calculated using the global electroneutrality condition.

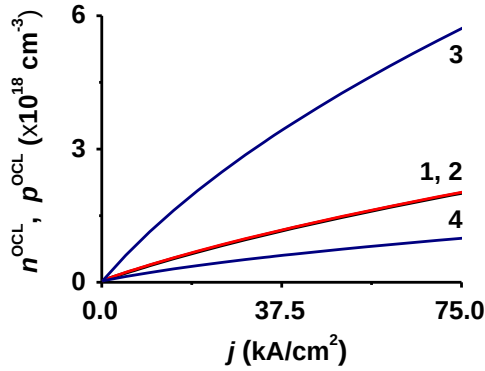


Figure 4. Electron (1, 3) and hole (2, 4) densities in the OCL against injection current density: (1, 2) - using the global electroneutrality condition; (3, 4) - using the local electroneutrality condition in the QW.

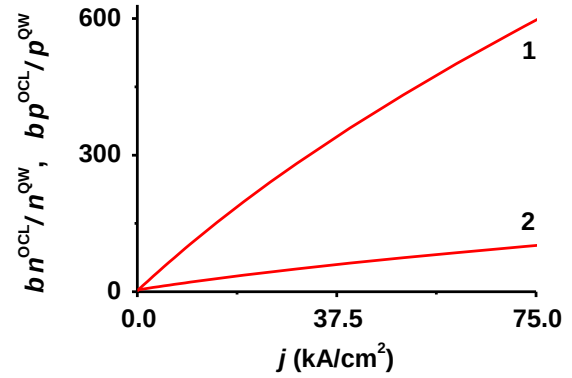


Figure 5. Ratio of the electron charge in the OCL to that in the QW (curve 1) and ratio of the hole charge in the OCL to that in the QW (curve 2) against injection current density calculated using the global electroneutrality condition.

In view of inequalities (12), the condition (10) of global electroneutrality reduces to the condition of electroneutrality in the OCL, i.e., i.e., to the equality of the free electron and hole densities in the OCL (figure 4, curves 1 and 2),

$$n^{\text{OCL}} = p^{\text{OCL}} \quad (13)$$

In contrast, the electron and hole densities in the OCL, calculated using the condition (11) of local electroneutrality in the QW, differ considerably (figure 4, curves 3 and 4).

Using in (13) expressions (A27) and (A28) of [4] for n^{OCL} and p^{OCL} and considering high injection current densities, the following relationship can be derived:

$$v_{n,\text{capt},0}(1 - f_n) = v_{p,\text{capt},0}(1 - f_p) \quad (14)$$

which is the equality of the total capture velocity of electrons to that of holes at high injection currents.

It follows from (14) that, at equal electron and hole capture velocities into an empty QW ($v_{n,\text{capt},0} = v_{p,\text{capt},0}$) and no matter what is their common value, $f_n = f_p$; in the laser structure considered here, $f_n = f_p = 0.588$ (figure 6). If, however, $v_{n,\text{capt},0} \neq v_{p,\text{capt},0}$, then, as seen from (14), the level occupancies f_n and f_p tend to different constant values with increasing injection current (figure 7).

The spontaneous radiative recombination current density in the OCL calculated using the expression

$$j_{\text{spon}}^{\text{OCL}} = ebB_{3D}n^{\text{OCL}}p^{\text{OCL}} \quad (15)$$

is shown in figure 8 as a function of the injection current density (curves 1 and 2). The spontaneous recombination in the OCL is a parasitic processes, which reduces the quantum efficiency and output optical power of the laser. As seen from the figure, $j_{\text{spon}}^{\text{OCL}}$, calculated using the global electroneutrality condition (curve 1), is lower than $j_{\text{spon}}^{\text{OCL}}$, calculated using the local electroneutrality condition in the QW (curve 2).

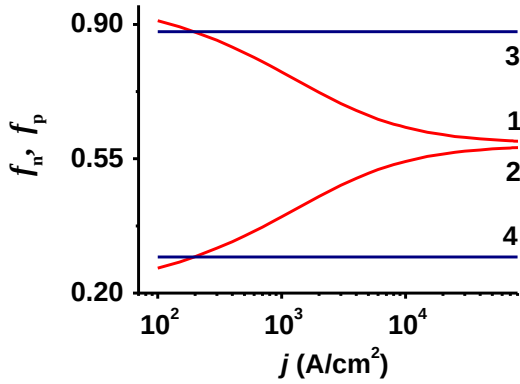


Figure 6. Electron (1, 3) and hole (2, 4) level occupancies in the QW against injection current density: (1, 2) - using the global electroneutrality condition; (3, 4) - using the local electroneutrality condition in the QW. The electron and hole capture velocities into an empty QW $v_{n,capt,0} = v_{p,capt,0} = 4 \times 10^5$ cm/s.

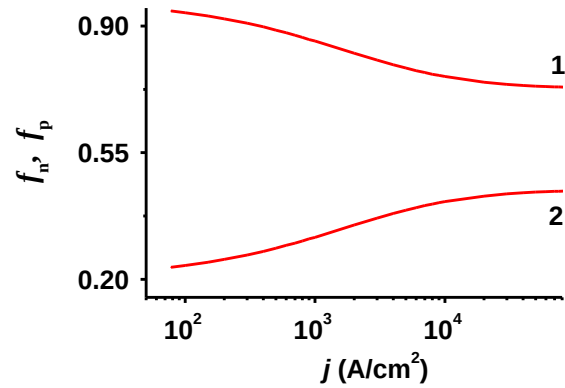


Figure 7. Electron (1) and hole (2) level occupancies in the QW against injection current density calculated using the global electroneutrality condition. The electron and hole capture velocities into an empty QW $v_{n,capt,0} = 10^6$ cm/s and $v_{p,capt,0} = 5 \times 10^5$ cm/s.

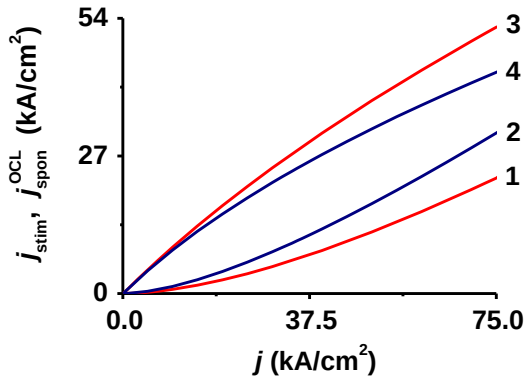


Figure 8. Spontaneous recombination current density in the OCL (1, 2) and stimulated recombination current density in the QW (3, 4) against injection current density: (1, 3) - using the global electroneutrality condition, (2, 4) - using the local electroneutrality condition in the QW.

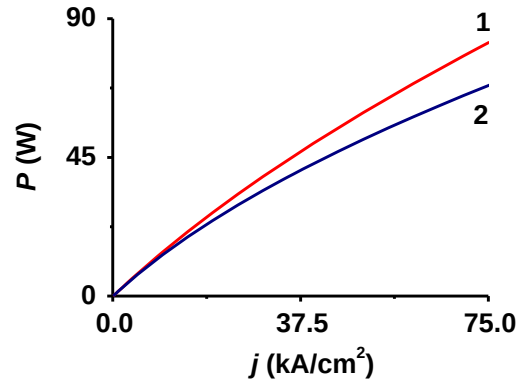


Figure 9. Optical power of the laser against injection current density (light-current characteristic): (1) - using the global electroneutrality condition, (2) - using the local electroneutrality condition in the QW.

The current density of stimulated radiative recombination in the QW,

$$j_{stim} = e v_g (\beta + \alpha_{int}) \frac{N}{S} \quad (16)$$

is also shown in figure 8. The current density of stimulated recombination, calculated using the global electroneutrality condition (curve 3), is higher than that, calculated using the local electroneutrality condition in the QW (curve 4).

The output optical power P against injection current density,

$$P(j) = \frac{\hbar\omega}{e} S j_{\text{stim}}(j) \frac{\beta}{\beta + \alpha_{\text{int}}} \quad (17)$$

is shown in figure 9. The calculation using the global electroneutrality condition (curve 1) gives higher values of the power as compared to that, which uses the local electroneutrality condition in the QW (curve 2).

4. Conclusions

A model for calculating the operating characteristics of semiconductor QW lasers has been presented. The model exploits the condition of global electroneutrality, which includes the charge carriers both in the 2D active region (QW) and bulk waveguide region (OCL). The charge of each sign in the OCL has been shown to be significantly larger than that in the QW. As a result of this, (i) the global electroneutrality condition reduces to the condition of electroneutrality in the OCL and (ii) the local electroneutrality in the QW can be strongly violated, i.e., the 2D electron and hole densities in the QW can significantly differ from each other. The product of the 2D electron and hole densities in the QW, however, remains virtually constant with increasing injection current.

The use of the global electroneutrality condition gives higher values of the power than the use of the local electroneutrality condition in the QW.

Acknowledgments

This work was supported by the federal program of the Ioffe Institute. Z. N. Sokolova is grateful for the hospitality of MEPhI during her visit supported by the Competitiveness Program of National Research Nuclear University MEPhI. L. V. Asryan acknowledges the U.S. Army Research Office (Grant No. W911NF-13-1-0445) for support of this work.

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13th International Workshop on Nonlinear Optics and Excitation Kinetics in Semiconductors

Dortmund, 9. - 13. October, 2016

Theoretical model for the operating characteristics of semiconductor quantum well lasers

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A comprehensive theoretical model for the operating characteristics of semiconductor lasers with a low-dimensional active region is developed. A particular emphasis is given to the effect of capture delay of both electrons and holes from a bulk optical confinement layer (OCL) into a quantum-confined active region and to a proper account of the condition of global electroneutrality in the laser structure.

Increasing the output optical power has been one of the most important tasks for developers of semiconductor lasers with a low-dimensional active region [1, 2]. To achieve high powers, the light-current characteristic (LCC) of a laser should remain close to linear at high levels of pumping. However, saturation of the LCC occurs at high injection currents in separate-confinement double-heterostructure lasers with a low-dimensional active region. The saturation can be due to different factors. Among them is the carrier capture from a bulk reservoir region (OCL) into a low-dimensional active region [3].

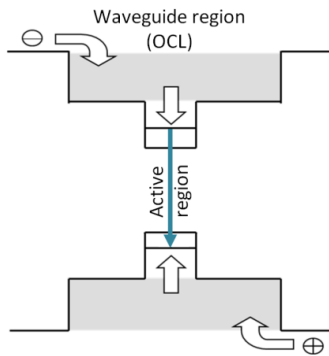


Figure 1: A schematic of a semiconductor laser with a quantum-confined active region.

Analytical model can provide insights into the processes controlling the laser performance and be used to optimize the laser structures to achieve desired operating characteristics. In this presentation, we analyze the effect of both electron and hole capture delay from the OCL into a low-dimensional active region on the operating characteristics of semiconductor lasers. We take into account the condition of global electroneutrality in the laser structure, which includes the charge carriers both in the active region [quantum wells – (QWs)] and OCL. The threshold characteristics of the laser, as well as characteristics at high pumping levels, are calculated. At the lasing threshold, the charge of each sign, both in the OCL and QW, has been shown to differ from each

other; hence the electroneutrality in these layers is violated. At high pumping levels, it turns out that the charge of each sign in the OCL is significantly larger than that in the QW. As a result of this, (i) the global electroneutrality condition reduces to the condition of electroneutrality in the OCL and (ii) the local electroneutrality in the QW can be strongly violated, i.e., the 2D electron and hole densities in the QW can significantly differ from each other. The product of the 2D electron and hole densities in the QW, however, remains virtually constant with increasing injection current.

Z.N.S. and I.S.T. acknowledge the support of the Federal Target Program “R&D in High Priority Directions of the Science and Technology Complex of Russia in 2014–2020” subsidy agreement no. 14.607.21.0048, identifier RFMEFI60714X0048. L.V.A. acknowledges the US Army Research Office (Grant No. W911NF-13-1-0445) for support of this work.

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PHYSICS OF SEMICONDUCTOR DEVICES

Increase in the Internal Optical Loss with Increasing Pump Current and the Output Power of Quantum Well Lasers

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Submitted January 19, 2017; accepted for publication January 25, 2017

Abstract—The operating characteristics of semiconductor quantum-well lasers, calculated with consideration for an increase in the internal optical loss in the waveguide region with increasing pump current, are presented. The condition of global electroneutrality in the structure is used. This condition consists in that the total charge of electrons in the 2D active region (quantum well) and bulk waveguide region (optical confinement layer) is equal to the total charge of holes in these two regions. Good agreement between the calculated and experimentally determined light–current characteristics is obtained.

DOI: 10.1134/S1063782617070326

1. INTRODUCTION

Raising the optical power of infrared planar semiconductor lasers with a nanoscale active region still remains a topical task despite achievements made in this field. One of the reasons why the power is limited is an increase in the internal optical loss in a laser structure with increasing pump current. This problem has been studied before. In particular, the influence exerted by the increase in the internal optical loss with increasing pump current on the operating characteristics of lasers with quantum-confined active regions was theoretically studied in [1–6]. We have recently carried out experimental studies of this problem in laser structures with an emission wavelength of $\sim 1 \mu\text{m}$ [7–10]. In [9], we developed a special procedure for measuring the absorption coefficient in the layers of a semiconductor laser under pulsed current pumping.

In modern semiconductor lasers, the low-dimensional active region is embedded in a wider gap bulk material, i.e., the waveguide region (optical confinement region, OCL). Electrons and holes are first injected from emitters into the waveguide region, then they are captured at a finite velocity into the active region [11]. Thus, the capture of charge carriers from the waveguide region of a laser into its active region is not an instantaneous process.

Since the use of bulk (3D) active regions, when calculating the characteristics of semiconductor lasers, it has been a common practice to assume that the concentrations of electrons and holes in the active region are equal to each other [12], i.e., there is local electro-

neutrality in the active region. The local electroneutrality condition may cease to be valid for quantum-confined active regions, so that the charge-carrier concentrations in the active region may greatly differ [13]. Thus, a more substantiated way to calculate the characteristics of semiconductor lasers is by using the global electroneutrality condition, which consists in that the total charge of electrons in the active and waveguide regions is equal to the total charge of holes in these two regions [14, 15].

In this study, we examine the influence exerted by the rise in internal optical loss with increasing pump current on the characteristics of semiconductor lasers with quantum wells (QWs). We use the global electroneutrality condition and take into account that the capture of charge carriers into QWs is not instantaneous. In [1–3], we used the local electroneutrality condition when constructing the analytical model, and in [4–6], the finiteness of the velocity of charge-carrier capture into wells was disregarded.

2. THEORETICAL MODEL

We used the following five steady-state rate equations to calculate the laser characteristics [16].

$$\begin{aligned} \frac{j}{e} + N_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{n,\text{esc}}} - N_{\text{QW}} v_{n,\text{capt},0} (1 - f_n) n^{\text{OCL}} \\ - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0, \end{aligned} \quad (1)$$

[†] Deceased.

for free electrons in a bulk waveguide region with the thickness b [$b(\partial n^{\text{OCL}}/\partial t) = 0$],

$$\frac{j}{e} + N_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{p,\text{esc}}} - N_{\text{QW}} v_{p,\text{capt},0} (1 - f_p) p^{\text{OCL}} - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0, \quad (2)$$

for free holes in a bulk waveguide region [$b(\partial p^{\text{OCL}}/\partial t) = 0$],

$$v_{n,\text{capt},0} (1 - f_n) n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{n,\text{esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - c_g g^{\text{max}} (f_n + f_p - 1) \frac{N}{S} = 0, \quad (3)$$

for electrons localized in QWs ($\partial n^{\text{QW}}/\partial t = 0$),

$$v_{p,\text{capt},0} (1 - f_p) p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{p,\text{esc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} - c_g g^{\text{max}} (f_n + f_p - 1) \frac{N}{S} = 0, \quad (4)$$

for holes localized in QWs ($\partial p^{\text{QW}}/\partial t = 0$), and

$$c_g N_{\text{QW}} g^{\text{max}} (f_n + f_p - 1) N - c_g (\beta + \alpha_{\text{int}}) N = 0, \quad (5)$$

for photons in the lasing mode ($\partial N/\partial t = 0$).

The following are unknowns in Eqs. (1)–(5): n^{OCL} and p^{OCL} are the 3D concentrations of free electrons and holes in the waveguide region; n^{QW} and p^{QW} are the 2D concentrations of electrons and holes in the QWs; N is the number of stimulated-emission photons; and f_n and f_p are the filling factors (occupancies) of the states corresponding to the lower edge of the electron subband and to the upper edge of the hole subband in a QW.

The occupancies f_n and f_p are related to the 2D electron and hole concentrations in the QWs, n^{QW} and p^{QW} , by [1, 2, 17]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_c^{2\text{D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_v^{2\text{D}}}\right). \quad (6)$$

Here, $N_{c,v}^{2\text{D}} = m_{e,h}^{\text{QW}} T (\pi \hbar^2)$ are the 2D effective densities of states in the conduction band and valence band in the QWs; $m_{e,h}^{\text{QW}}$ are the effective electron and hole masses in the QWs; and T is the temperature in energy units.

The following parameters enter in Eqs. (1)–(5): j is the injection current density; e is the elementary charge; N_{QW} is the number of identical QWs, i.e., those having the same width and the same composition; $\tau_{n,\text{esc}}$ and $\tau_{p,\text{esc}}$ are the thermal excitation times of electrons and holes from the QWs into the waveguide region; $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$ are the capture velocities of electrons and holes into an empty (at $f_n = 0$ and $f_p = 0$) single QW, measured in cm/s; $B_{3\text{D}}$ and $B_{2\text{D}}$ are the spontaneous radiative recombination coefficients

in the bulk (waveguide) region and in the 2D region (QWs), measured in cm^3/s and cm^2/s , respectively (see [18] and [19] for the expressions for $B_{3\text{D}}$ and $B_{2\text{D}}$); c_g is the group speed of light; g^{max} is the maximum modal gain in each QW; $S = WL$, where W is the stripe contact width and L is the Fabry–Perot cavity length; $\beta = (2L)^{-1} \ln(R_1 R_2)^{-1}$ is the cavity output loss; R_1 and R_2 are the reflectivities of mirrors for a laser with a Fabry–Perot cavity; and α_{int} is the internal optical loss coefficient for the laser structure.

The thermal excitation times of electrons and holes from QWs into the waveguide region are given by [16, 20]:

$$\tau_{n,\text{esc}} = \frac{1}{v_{n,\text{capt},0} (1 - f_n)} \frac{N_c^{2\text{D}}}{n_1}, \quad \tau_{p,\text{esc}} = \frac{1}{v_{p,\text{capt},0} (1 - f_p)} \frac{N_v^{2\text{D}}}{p_1}, \quad (7)$$

where

$$n_1 = N_c^{3\text{D}} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right), \quad p_1 = N_v^{3\text{D}} \exp\left(-\frac{\Delta E_v - \varepsilon_p^{\text{QW}}}{T}\right), \quad (8)$$

$N_{c,v}^{3\text{D}} = 2[m_{c,v}^{\text{OCL}} T / (2\pi \hbar^2)]^{3/2}$ are the 3D effective densities of states in the conduction band and valence band of the waveguide region; $m_{c,v}^{\text{OCL}}$ are the electron and hole effective masses in the waveguide region; $\Delta E_{c,v}$ are the band edge offsets for the conduction band and valence band at the heterointerface between the QWs and the waveguide region; and $\varepsilon_n^{\text{QW}}$ and $\varepsilon_p^{\text{QW}}$ are the energies of, respectively, the lower and upper edges of the electron and hole quantum-confinement subbands in the QWs.

Equations (1)–(4) have as parameters the electron ($v_{n,\text{capt},0}$) and hole ($v_{p,\text{capt},0}$) capture velocities from the waveguide region into an empty QW. The capture velocities are characteristics of a QW and depend on its width and depth, i.e., on the composition of the materials constituting a QW and its surrounding layers. The capture velocities vary between different laser structures. In [21, 22], the electron capture velocities into an empty single QW in real laser structures were determined. These studies used experimental laser characteristics and theoretical calculations for this purpose.

In the present study, the capture velocities of electrons and holes into an empty QW were considered the same and equal to 5×10^5 cm/s. In this case, the capture velocities into a filled QW [16],

$$v_{n,\text{capt}} = v_{n,\text{capt},0} (1 - f_n), \quad v_{p,\text{capt}} = v_{p,\text{capt},0} (1 - f_p), \quad (9)$$

are found to be different according to formula (6). It was shown in [15] that the characteristics of the lasers

Parameters of the experimental laser structure

Layer	Layer thickness, μm	Layer material	Layer doping
Substrate	100	GaAs	$N, 1 \times 10^{18} \text{ cm}^{-3}$
Emitter	1.5	$\text{Al}_{0.1}\text{Ga}_{0.9}\text{As}$	$N, 5 \times 10^{17} \text{ cm}^{-3}$
Waveguide	1.73	GaAs	$n, 5 \times 10^{16} \text{ cm}^{-3}$
Waveguide	0.2	GaAs	undoped
QW	0.009	InGaAs	undoped
Spacer layer	0.03	GaAs	undoped
QW	0.009	InGaAs	undoped
Waveguide	0.2	GaAs	undoped
Waveguide	1	GaAs	$p, 5 \times 10^{16} \text{ cm}^{-3}$
Emitter	1	$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$	$P, 1 \times 10^{18} \text{ cm}^{-3}$
Contact layer	0.3	GaAs	$P, 3 \times 10^{18} \text{ cm}^{-3}$

under study heavily depend on the electron capture velocity, and only slightly, on the hole capture velocity.

We used in our calculations the global electroneutrality condition, which consists in that the total charge of electrons in the waveguide region and in the QWs is equal to the total charge of holes in these two regions:

$$e(N_{\text{QW}}n^{\text{QW}} + bn^{\text{OCL}}) = e(N_{\text{QW}}p^{\text{QW}} + bp^{\text{OCL}}), \quad (10)$$

where b is the thickness of the waveguide region.

3. DISCUSSION OF LASER CHARACTERISTICS

The set of Eqs. (1)–(5), supplemented with condition (10), was solved numerically for a laser structure with a room-temperature emission wavelength of $1.075 \mu\text{m}$. The AlGaAs/GaAs/InGaAs laser structure was grown on a GaAs substrate (see table). The structure contains two strained $90 \text{ \AA}$ -wide $\text{In}_{0.31}\text{Ga}_{0.69}\text{As}$ QWs. The material of the waveguide with width $b = 3.2 \mu\text{m}$ is GaAs, and the material of the emitters is AlGaAs. The length of the Fabry–Perot cavity is $L = 5.1 \text{ mm}$, the width of the stripe contact is $W = 100 \mu\text{m}$, the reflectivity of both mirrors is $R = 0.05$, and the emission output loss is $\beta = 5.87 \text{ cm}^{-1}$.

The cavity parameters of the laser under examination enable an experimental study of the rise in the internal optical loss by the procedure described in [9]. The optical output power of the laser was measured (and calculated) from both cavity mirrors. As shown in [8], lasers with two antireflective mirrors have a light–current (L – I) characteristic that is closer, compared with ordinary lasers, to a linear characteristic due to the weaker influence exerted by the internal optical loss on the output power. The maximum modal gain in a single QW, $g^{\text{max}} = 36.5 \text{ cm}^{-1}$.

In the present study, we took into account the fact that the internal optical loss grows with increasing pump current. In the case of a rather wide waveguide region ($3.2 \mu\text{m}$), the electromagnetic wave propagates nearly completely within the waveguide region, with the optical loss occurring in this region. In lightly doped semiconductors (the waveguide region of our structure is undoped), the internal optical loss coefficient linearly depends on the charge-carrier concentration [12],

$$\alpha_{\text{int}} = \alpha_0 + \alpha^{\text{OCL}}, \quad \alpha^{\text{OCL}} = \alpha_n + \alpha_p, \quad (11)$$

$$\alpha_n = \Gamma^{\text{OCL}} \sigma_n n^{\text{OCL}}, \quad \alpha_p = \Gamma^{\text{OCL}} \sigma_p p^{\text{OCL}}. \quad (12)$$

Here, α^{OCL} is the internal optical loss coefficient in the waveguide region; σ_n and σ_p are the cross sections of light absorption by free electrons and holes in the waveguide region; Γ^{OCL} is the optical confinement factor in the waveguide region; and α_0 is the constant component of the internal optical loss coefficient, primarily related to the absorption of light in the emitters.

Because charge-carrier capture from the waveguide region into the QWs is not instantaneous, the concentrations of these charge carriers in the waveguide region grow with increasing injection current in the lasing mode [1–4, 14, 16]. As a result, as can be seen in (12), the internal optical loss in the waveguide region of the laser structure grows.

We used the following values for the free-charge-carrier absorption cross sections at photon energies close to the band-gap width of GaAs: $\sigma_n = 3 \times 10^{-18} \text{ cm}^2$ and $\sigma_p = 7 \times 10^{-18} \text{ cm}^2$ [12], with $\Gamma^{\text{OCL}} = 0.9843$. For the calculated and experimental values of P to coincide at a current density of $j = 1 \text{ kA/cm}^2$, α_0 was taken to be 0.35 cm^{-1} .

Having solved numerically the set of equations (1)–(5), supplemented with the global electroneutrality condition (10), we obtained the dependences of the

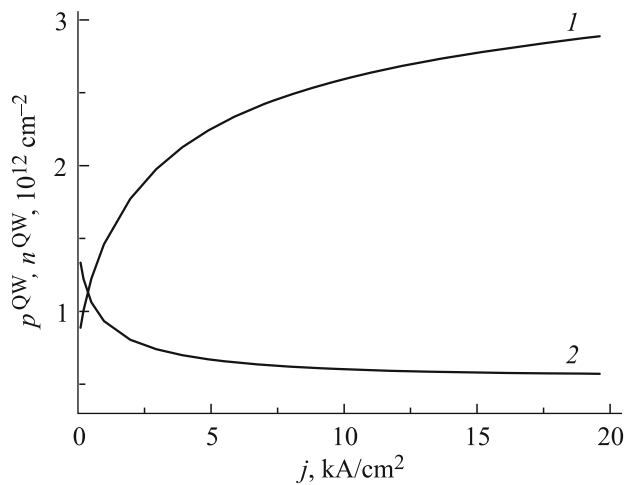


Fig. 1. (1) Hole and (2) electron concentrations in a QW vs. the pump-current density.

following quantities on the pump-current density: (a) electron and hole concentrations in the QWs, (b) electron and hole concentrations in the waveguide region, (c) internal optical loss, (d) occupancies of the electron and hole quantum-confined subbands in the QWs, and (e) output optical emission power.

The calculated dependences of the electron and hole concentrations in a QW on the pump-current density are shown in Fig. 1. It can be seen that the electron and hole concentrations greatly differ in the active region of the laser structure under consideration. At the lasing threshold the electron concentration is 1.5 times that of holes, whereas with increasing pump-current density, the hole concentration starts to exceed that of electrons. At $j = 20 \text{ kA/cm}^2$, the hole concentration is already 5 times the electron concentration. This means that there is a “deficiency” of electrons in the active region, and, consequently, there is no local electroneutrality for electrons and holes. This is manifested in the fact that the occupancy of the lower edge of the electron quantum-confinement subband in the QW decreases and that of the upper edge of the hole subband grows with increasing pump-current density (Fig. 2).

Figure 3 shows the electron and hole concentrations in the waveguide region. It can be seen that these concentrations differ from each other only slightly. Even at $j = 20 \text{ kA/cm}^2$, the electron concentration exceeds that of holes by only a factor of 1.05.

It should be noted that the charge of each sign (i.e., of both electrons and holes) in the waveguide region substantially exceeds the corresponding charge in two QWs (Fig. 4). At $j = 20 \text{ kA/cm}^2$, the charge of holes in the waveguide region is 15 times that in the QWs, and the charge of electrons in the waveguide region is 82 times that in the QWs.

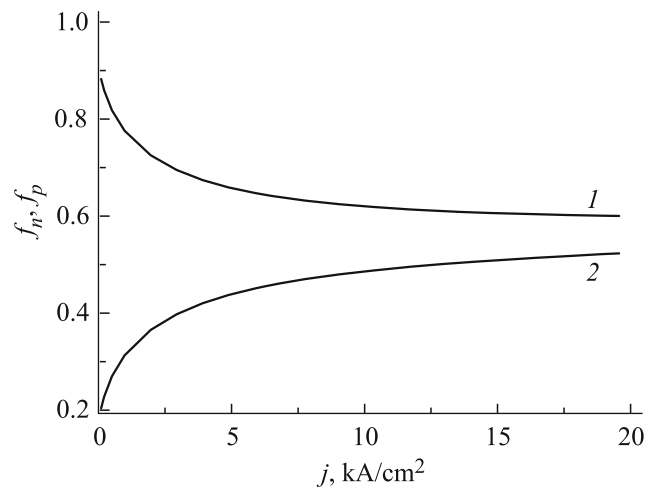


Fig. 2. Occupancy of states corresponding to (1) the lower edge of the electron quantum-confinement subband and (2) upper edge of the hole subband in the QW vs. the pump-current density.

Thus, charge carriers accumulate in the waveguide region with increasing pump-current density. According to expressions (11) and (12), this leads to a rise in the internal optical loss in the waveguide region with increasing pump-current density (Fig. 5, curve 1). Figure 5 also shows how the internal optical loss in the waveguide region, caused by the absorption of light by holes (α_p , curve 2) and electrons (α_n , curve 3) depends on the pump-current density. At the injection current densities under consideration, which correspond to the experimental range of pump currents in the laser structure under study (see table), the charge-carrier concentrations and the internal optical loss in the waveguide region grow close-to-linearly with increasing current density (Figs. 3 and 5).

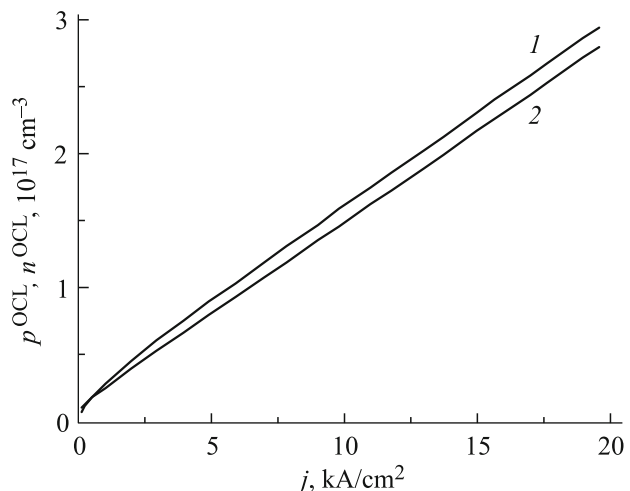


Fig. 3. (1) Electron and (2) hole concentrations in the waveguide region vs. the pump-current density.

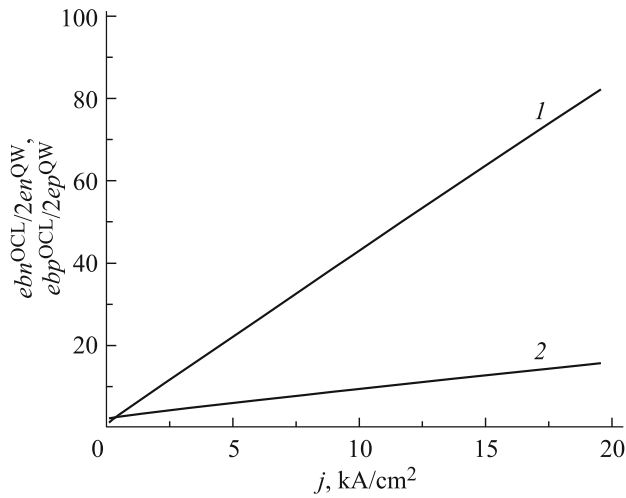


Fig. 4. (1) Ratio of the electron charge in the waveguide region to that in the QWs and (2) ratio of the hole charge in the waveguide region to that in the QWs vs. the pump-current density.

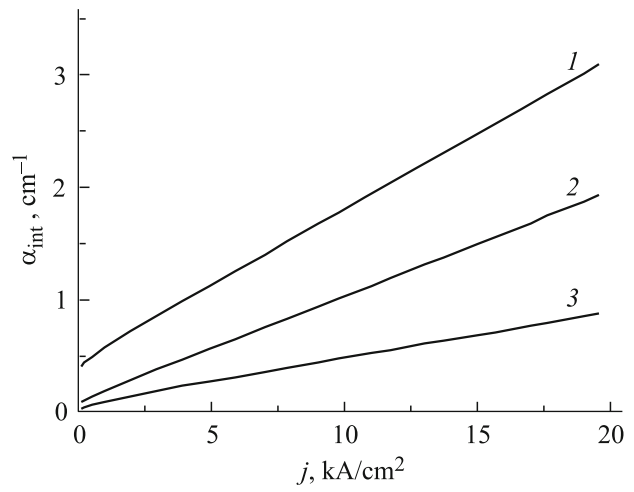


Fig. 5. (1) Internal optical loss coefficient in the waveguide region and (2, 3) "contribution" due to the absorption of light by (2) holes and (3) electrons vs. the pump-current density.

The increase in the internal optical loss results in a weaker rise in the photon concentration in the laser cavity with increasing pump-current density. This, in turn, reduces the power of the optical emission emerging from the laser:

$$P(j) = \frac{\hbar\omega}{e} S j_{\text{stim}}(j) \frac{\beta}{\beta + \alpha_{\text{int}}}, \quad (13)$$

where

$$j_{\text{stim}} = e c_g (\beta + \alpha_{\text{int}}) \frac{N}{S} \quad (14)$$

is the current density of the stimulated radiative recombination in two QWs [16]. Thus, the rise in the internal optical loss with increasing pump current limits the rise in the output emission of the laser.

The light–current ($L-I$) characteristics of the laser are shown in Fig. 6. The solid curves represent the $L-I$ characteristics calculated with consideration for the rise in the internal optical loss in the waveguide region of the laser (curve 1) and at a constant internal optical loss $\alpha_{\text{int}} = 0.5 \text{ cm}^{-1}$ (curve 2). At $j = 20 \text{ kA/cm}^2$, the values of power for curves 1 and 2 differ by a factor of 1.4. The points in Fig. 6 show the experimental values of the $L-I$ characteristics measured in the present study for the AlGaAs/GaAs/InGaAs laser structure (see table). It can be seen in the figure that there is a very good agreement between the experimental $L-I$ characteristic and that calculated with consideration for the rise in the optical loss with increasing pump current. It should be noted that, for the calculated power to coincide with the experimental value at $j = 1 \text{ kA/cm}^2$, the constant component α_0 of the internal

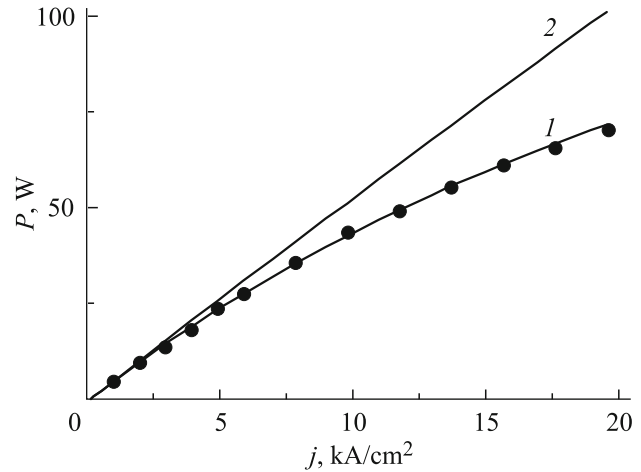


Fig. 6. Optical power P of the laser vs. the pump-current density ($L-I$ characteristic). The solid curves represent (1) calculation with consideration for the dependence of the internal optical loss on the pump current and (2) that for $\alpha_{\text{int}} = \text{const}$. The points are experimental data.

optical loss coefficient [see expression (11)] was taken to be 0.35 cm^{-1} in our calculations.

4. CONCLUSIONS

The results of calculations of the operating characteristics of a semiconductor laser based on an AlGaAs/GaAs/InGaAs structure with two quantum wells (QWs) are presented. The rise in the internal optical loss in the waveguide region with increasing pump-current density in the lasing mode was taken into account (Fig. 5). The global electroneutrality condition in the structure [expression (10)] was used.

This condition consists in that the total charge of electrons in the waveguide region and two QWs is equal to the total charge of holes in these regions.

A good agreement was obtained between the experimentally determined and calculated light–current (L – I) characteristics (Fig. 6). The coincidence of the calculated and experimental L – I characteristics confirms that the rise in the internal optical loss in the waveguide region of the laser structure with increasing pump current should be taken into account.

To make less pronounced the leveling-off of the L – I characteristic it is necessary to reduce the internal optical loss in the waveguide region of the laser, for which purpose the charge-carrier concentration in this region should be decreased. For example, laser structures with multiple QWs can be used. Lower concentrations in the waveguide region can also be attained by using structures with an optimized width of the waveguide region [18] and structures with a higher charge-carrier capture velocity from the waveguide region into QWs [15].

ACKNOWLEDGMENTS

The study was carried out under the State program of the Ioffe Physical–Technical Institute. L.V. Asryan is grateful to U.S. Army Research Office, grant no. W911NF-13-1-0445, for the support of this study.

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Translated by M. Tagirdzhanov

Two-Valued Characteristics in Semiconductor Quantum Well Lasers

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Abstract— Due to internal optical absorption loss, which varies with carrier density in the optical confinement region, the operating characteristics can be two-valued in semiconductor quantum well lasers. Particularly, there can be two lasing thresholds and two branches in the light-current characteristic. While the internal differential quantum efficiency for the first (conventional) branch of the light-current characteristic is less than one and decreases with increasing pump current, that for the second (unconventional) branch is greater than one and nonmonotonous.

Index Terms—Quantum well laser, semiconductor laser.

I. INTRODUCTION

INTERNAL optical loss is among the key factors affecting the operating characteristics of semiconductor lasers [1]–[15]. An interesting peculiarity in such lasers was theoretically studied in [16]–[20] — under certain conditions and due to internal loss, which varies with carrier density in the optical confinement layer (OCL), the second lasing threshold can exist in addition to the first (conventional) threshold. Above the second threshold, the laser characteristics become degenerated, namely, two-valued. In particular, the second, ‘unconventional’, branch appears in the dependence of the output optical power on the pump current (light-current characteristic – LCC) in addition to the conventional (first) branch. With increasing pump current and approaching a certain value (which is the maximum operating current), the two branches of the LCC merge together. At the merging point, the optical power is non-vanishing; immediately beyond this point, the lasing quenches. Possibility of existing of such two-valued, lobe-shape, LCC, presents considerable interest from the viewpoints of both fundamental physics and practical applications of semiconductor lasers with a quantum-confined active region.

While a general approach was developed in [16]–[20], which applies to the effects of carrier-density-dependent internal loss

in semiconductor lasers with a quantum-confined active region of any dimensionality, detailed calculations were performed there for quantum dot lasers only. In this work, we study two-valued characteristics for the particular case of semiconductor quantum well (QW) lasers. In contrast to [16]–[20], wherein local charge neutrality was assumed to hold separately in the quantum-confined active region and in the OCL, we use here the condition of global charge neutrality [21]–[23] in the laser structure.

II. THEORETICAL MODEL

Our theoretical model for the operating characteristics of QW lasers is based on a set of five rate equations in terms of the following unknowns: the three-dimensional (3D) densities of free electrons and holes in the OCL, n^{OCL} and p^{OCL} , the two-dimensional (2D) densities of electrons and holes confined in each QW, n^{QW} and p^{QW} , and the number of photons N in the lasing mode.

The condition of global charge neutrality presents the equality of the total electron charge in the OCL and QWs to that of holes [21],

$$e(N_{\text{QW}} n^{\text{QW}} + b n^{\text{OCL}}) = e(N_{\text{QW}} p^{\text{QW}} + b p^{\text{OCL}}), \quad (1)$$

where N_{QW} is the number of QWs, b is the thickness of the OCL, and e is the electron charge.

We use a set of rate equations presented in [21, 24]. In [21, 24], constant internal optical loss was assumed. A critical element of the theoretical model in this paper is the carrier-density-dependent internal loss in the OCL. We present the internal optical absorption loss coefficient as

$$\alpha_{\text{int}}(j) = \alpha_0 + \sigma_{\text{n,int}} n^{\text{OCL}}(j) + \sigma_{\text{p,int}} p^{\text{OCL}}(j), \quad (2)$$

where α_0 is the constant component, $\sigma_{\text{n,int}}$ and $\sigma_{\text{p,int}}$ are the cross sections of internal loss due to optical absorption by free electrons and holes in the OCL, and j is the injection current density. The internal optical loss in the QWs, which depends on the 2D carrier densities there, is included into the constant component α_0 . This is because the 2D carrier densities in the QWs are considerably lower than the 3D carrier densities in the OCL (see below). Free carrier absorption loss in the cladding layers and intervalence band absorption loss [25] are also included into the constant component α_0 .

Due to the fact that carrier capture from the OCL into the QWs is noninstantaneous, the free carrier densities in the OCL,

Manuscript received . This work was supported by the Federal program of the Ioffe Physical-Technical Institute and the U.S. Army Research Office under Grant No. W911NF-13-1-0445.

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n^{OCL} and p^{OCL} , are not pinned after the onset of lasing – they depend on the injection current j [26]. Hence, as seen from (2), the internal loss coefficient α_{int} is also injection-current-dependent (see also [27]). The capture of electrons and holes from the OCL into an empty (unoccupied) QW is described by the capture velocities (measured in units of cm/s) $v_{n,\text{capt},0}$ and $v_{p,\text{capt},0}$. The capture velocities, being dependent on the size and material composition of the QW and surrounding layers, can considerably vary in different laser structures. In [28, 29], a combined experimental-theoretical method for determination of the capture velocities was described and the capture velocities were determined for a specific QW laser structure emitting at 1.063 μm .

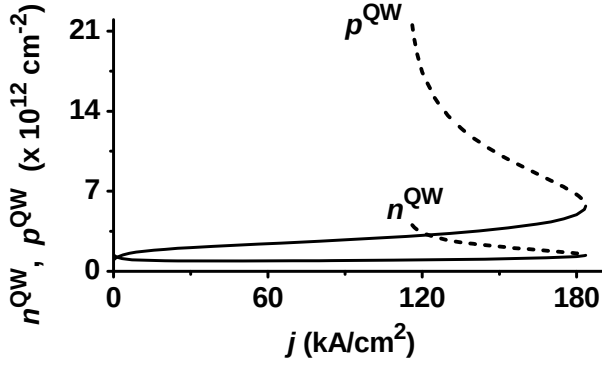


Fig. 1. 2D electron and hole densities in the QW against injection current density. In Figs. 1-8, the first (conventional) branch is shown by the solid curve and the second (unconventional) branch by the dashed curve. In Figs. 1-5, and 8, the OCL thickness $b = 0.8 \mu\text{m}$.

III. DISCUSSION OF LASER CHARACTERISTICS

We consider here the laser structure emitting at 1.01 μm and containing a single 50 $\text{\AA}$ -wide strained InGaAs QW; the materials of the OCL and cladding layers are GaAs and AlGaAs, respectively. The following parameters are used in the calculations: the Fabry-Perot cavity length is $L = 0.15 \text{ cm}$; the electron and hole capture velocities into an empty QW are $v_{n,\text{capt},0} = 2 \times 10^6 \text{ cm/s}$ and $v_{p,\text{capt},0} = 10^6 \text{ cm/s}$, respectively; the internal loss cross sections due to optical absorption by electrons and holes in the OCL are $\sigma_{n,\text{int}} = 3 \times 10^{-18} \text{ cm}^2$ and $\sigma_{p,\text{int}} = 10^{-17} \text{ cm}^2$; the doping concentrations in the n- and p-cladding layers are $5 \times 10^{17} \text{ cm}^{-3}$ and $3.5 \times 10^{18} \text{ cm}^{-3}$; the constant component of the internal loss coefficient is $\alpha_0 = 2 \text{ cm}^{-1}$. The OCL thickness $b = 0.8 \mu\text{m}$ is used in the calculations. The LCC and the internal differential quantum efficiency are also calculated for $b = 0.5$ and $0.2 \mu\text{m}$.

Solving the set of rate equations at the steady-state, we obtain the electron and hole densities in the QW and OCL and the number of photons N in the lasing mode as functions of the injection current density j . We then calculate the output optical power [30, 31],

$$P(j) = \hbar \omega c_g \beta N(j), \quad (3)$$

where ω is the angular frequency of the lasing emission, c_g is the group velocity of light, $\beta = (1/L) \ln(1/R)$ is the mirror loss coefficient, and R is the mirror reflectivity.

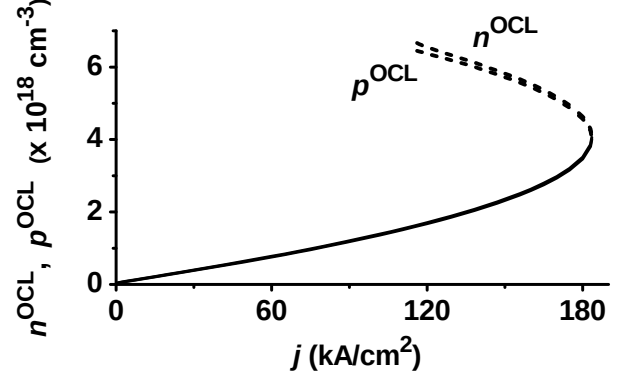


Fig. 2. 3D electron and hole densities in the OCL against injection current density.

We find that under certain conditions and due to internal optical loss, which varies with electron and hole densities in the OCL, the second lasing threshold can exist in addition to the first (conventional) threshold. Above the second threshold, the set of rate equations has two solutions, i.e., all the laser characteristics become degenerated, namely, two-valued. In particular, the second, unconventional, branch appears in the dependence of the output optical power on the pump current (LCC) in addition to the conventional branch. With increasing pump current and approaching a certain value (which is the maximum operating current), the two branches of the LCC merge together. At the merging point, the optical power is non-vanishing; immediately beyond this point, the lasing quenches.

In [19], assuming that local charge neutrality holds separately in the quantum-confined active region and in the OCL, closed-form solutions were obtained from the steady-state rate equations and the following analytical criterion for the emergence of the second branch of the LCC was derived:

$$\frac{\sigma_{\text{int}} v_{\text{capt},0}}{2b B^{\text{OCL}} g^{\text{max}}} > 1, \quad (4)$$

where B^{OCL} is the spontaneous radiative recombination constant for the bulk (OCL) region measured in units of cm^3/s (see [32] for the expression for B^{OCL}) and g^{max} is the maximum value of the modal gain in the active region. In the specific laser structure considered here, $g^{\text{max}} = 89.7 \text{ cm}^{-1}$ [33].

In this work, we use a general approach, which is based on exploiting the global charge neutrality condition in the laser structure [see eq. (1) above]. With this condition, no closed-form solutions of the set of rate equations can be obtained and no analytical criterion for the emergence of the

second branch of the LCC can be derived. However, the findings of [19] expressed by inequality (4) hold true also in the general case of global charge neutrality in the laser structure: the internal loss cross-section and the capture velocity should be high enough and/or the OCL thickness and the maximum modal gain should be low for the second branch of the LCC to appear.

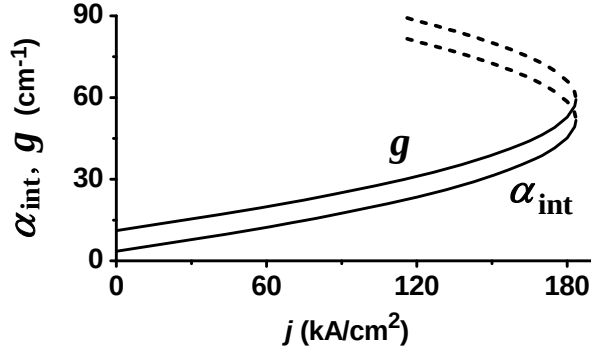


Fig. 3. Gain and internal optical absorption loss against injection current density.

Figure 1 shows the 2D electron and hole densities in the QW against the injection current density j . As seen from the figure, the character of the dependence of the carrier density on j in the second (unconventional) branch is opposite to that in the first (conventional) branch. While the electron and hole densities increase with j in the first branch, they decrease in the second branch. Also, as seen from the figure, the decrease of the hole density with j in the second branch is substantial. In both branches (except for the initial portions in the first branch near the first lasing threshold), the hole density in the QW is considerably higher than the electron density. At the second lasing threshold, the hole-to-electron carrier density ratio is 3.1 in the second branch and 5.3 in the second. At the maximum operating current (beyond which the lasing quenches in both branches), the hole-to-electron carrier density ratio is 4.1. Hence, as seen from Fig. 1 and the above numbers, there is no local charge neutrality in the QW.

In contrast to the 2D electron and hole densities in the QW, the free electron and hole densities in the OCL are close to each other in both the first and second branches for the entire range of injection current values (Fig. 2). This is because the density of each type of carriers in the OCL is considerably higher than that in the QWs, i.e.,

$$bn^{\text{OCL}} \gg N_{\text{QW}} n^{\text{QW}}, \quad bp^{\text{OCL}} \gg N_{\text{QW}} p^{\text{QW}}. \quad (5)$$

Hence, to maintain the global charge neutrality condition (1), n^{OCL} and p^{OCL} should be close to each other. Similarly to the QW, while the carrier densities in the OCL increase with j in the first branch, they decrease in the second branch. The relative variation is, however, greater in the first branch – the carrier densities increase by a factor of 120 on going from the

first lasing threshold to the lasing quenching point; the carrier densities in the second branch decrease by a factor of 1.6 only on going from the second lasing threshold to the lasing quenching point. As seen from Fig. 2, above the second lasing threshold, the free carrier densities in the OCL (especially in the second branch) are significantly high.

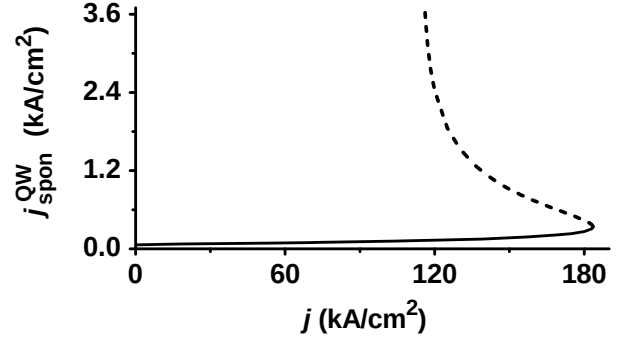


Fig. 4. Spontaneous radiative recombination current density in the QW against injection current density.

With the calculated dependences of the carrier densities in the OCL on j , we can easily obtain such dependences of the internal loss [see eq. (2)] and modal gain (both shown in Fig. 3). For the modal gain of a QW laser, we use [24]

$$g(j) = N_{\text{QW}} g^{\text{max}} [f_n(j) + f_p(j) - 1], \quad (6)$$

where f_n and f_p are the occupancies of the electron and hole subband edges in the QW, which are related to the 2D-carrier densities in the QW as follows [16, 17, 34]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_{\text{c}}^{\text{2D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_{\text{v}}^{\text{2D}}}\right), \quad (7)$$

where $N_{\text{c,v}}^{\text{2D}} = m_{\text{c,v}}^{\text{QW}} T / (\pi \hbar^2)$ are the 2D effective densities of states in the conduction and valence bands in the QW, $m_{\text{c,v}}^{\text{QW}}$ are the electron and hole effective masses in the QW, and the temperature T is measured in units of energy.

The modal gain and the internal loss are related to each other by the lasing condition,

$$g(j) = \alpha_{\text{int}}(j) + \beta. \quad (8)$$

As seen from (8) and Fig. 3, since the mirror loss β does not depend on the injection current, the dependences of g and α_{int} reproduce each other – the curve for g is obtained from that for α_{int} by simply shifting up the latter along the vertical axis by the value equal to the mirror loss β , in the laser structure considered here, $\beta = 7.6 \text{ cm}^{-1}$. Since the dependences of n^{OCL} and p^{OCL} on j have two branches, $g(j)$ and $\alpha_{\text{int}}(j)$ also consist of two branches, which correspond to two solutions of the steady-state set of rate equations.

As seen from Fig. 3, both the internal loss and gain change

substantially with j . In the first branch, α_{int} and g increase from 3.5 to 51.9 cm^{-1} and from 11.1 to 59.5 cm^{-1} , respectively, on going from the first lasing threshold (63 A/cm^2) to the lasing quenching point ($j_{\text{max}} = 183.5 \text{ kA}/\text{cm}^2$). In the second branch, α_{int} and g decrease from 81.6 to 51.9 cm^{-1} and from 89.2 to 59.5 cm^{-1} , respectively, on going from the second lasing threshold (116 kA/cm^2) to the lasing quenching point ($j_{\text{max}} = 183.5 \text{ kA}/\text{cm}^2$). At the lasing quenching point, both branches of α_{int} merge together (at this point, $\alpha_{\text{int}} = 51.9 \text{ cm}^{-1}$) and so do the two branches of g (at this point, $g = 59.5 \text{ cm}^{-1}$).

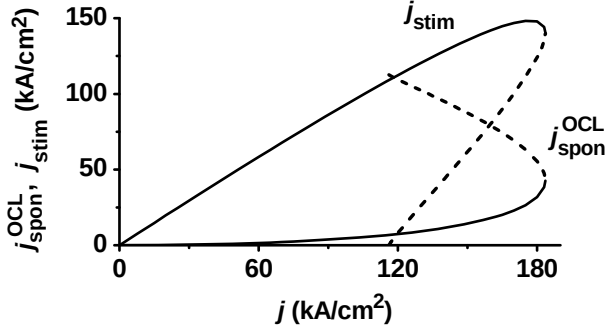


Fig. 5. Spontaneous and stimulated radiative recombination current densities in the QW against injection current density.

Figure 4 shows the current density of spontaneous radiative recombination in the QWs,

$$j_{\text{spon}}^{\text{QW}}(j) = eN_{\text{QW}}B^{\text{QW}}n^{\text{QW}}(j)p^{\text{QW}}(j), \quad (9)$$

against the injection current density. In (9), B^{QW} is the spontaneous radiative recombination constant for the 2D region (QW), which is measured in units of cm^2/s – see [32] for the expression for B^{QW} . As seen from Fig. 4, $j_{\text{spon}}^{\text{QW}}$ drops considerably in the second branch (from 3.6 to 0.35 kA/cm^2 , i.e., by more than an order of magnitude) on going from the second lasing threshold to the lasing quenching point. However, even at the second lasing threshold, $j_{\text{spon}}^{\text{QW}}$ is much lower than the current density of spontaneous radiative recombination in the OCL [see eq. (10) and Fig. 5 below]. This is due to the fact that the carrier densities in the QW are considerably lower than those in the OCL – see (5).

The current density of spontaneous radiative recombination in the OCL,

$$j_{\text{spon}}^{\text{OCL}}(j) = ebB^{\text{OCL}}n^{\text{OCL}}(j)p^{\text{OCL}}(j), \quad (10)$$

is shown in Fig. 5 against the injection current density. As seen from the figure, $j_{\text{spon}}^{\text{OCL}}$ is very high in the second branch – it presents a considerable fraction of the total injection current density.

Figure 5 also shows the current density of stimulated radiative recombination in the QWs,

$$j_{\text{stim}}(j) = \frac{1}{S} e c_g g(j) N(j) = \frac{1}{S} e c_g [\alpha_{\text{int}}(j) + \beta] N(j), \quad (11)$$

against the injection current density. In (11), $S = WL$ is the cross section of the junction and W is the laser stripe width. As seen from the figure and as expected, in each of two branches, j_{stim} is zero at the corresponding lasing threshold. In the first branch, j_{stim} increases with j and approaches its maximum value; as j further increases and approaches the maximum operating current density, j_{stim} decreases to a non-vanishing value (140 kA/cm^2) – on further increase of j , the lasing quenches. In the second branch, j_{stim} increases monotonously with j . At $j = j_{\text{max}}$, the two branches of j_{stim} merge together.

Figure 6 shows the output optical power of the laser against the injection current density (LCC) for the OCL thicknesses $b = 0.2, 0.5$, and $0.8 \text{ }\mu\text{m}$; the corresponding values of the maximum modal gain g^{max} are 125, 115.2, and 89.7 cm^{-1} . As seen from the figure, due to the fact that the set of rate equations has two solutions in the presence of carrier-density-dependent internal loss in the OCL, the LCC has two branches; these two branches actually form a lobe-shape LCC. The first (conventional) branch of the LCC is nonmonotonous and the second (unconventional) branch is monotonous.

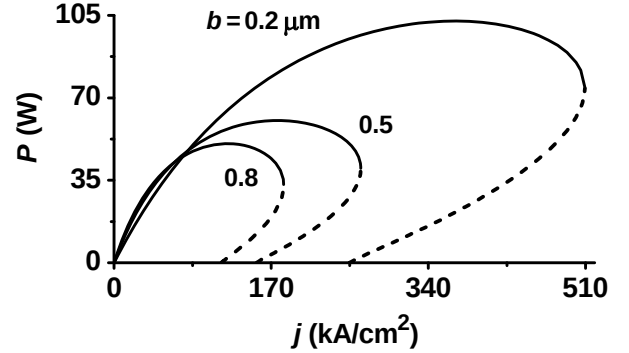


Fig. 6. Light-current characteristics for the structures with the OCL thicknesses $b = 0.2, 0.5$, and $0.8 \text{ }\mu\text{m}$.

As seen from Fig. 6, making the OCL thinner increases the maximum optical power, the second lasing threshold, and the maximum operating current at which the lasing quenches.

Figure 7 shows the internal differential quantum efficiency for the first and second branches of the LCC against the injection current density for three values of the OCL thickness. The internal efficiency (efficiency of stimulated recombination) is defined as [24, 26]

$$\eta_{\text{int},1,2}(j) = \frac{j_{\text{stim},1,2}(j)}{j - j_{\text{th},1,2}}, \quad (12)$$

where “1” and “2” in the subscripts denote the first and second branches of the LCC and $j_{\text{th},1,2}$ are the first and second threshold current densities.

While the two branches of j_{stim} and of the LCC merge

together at the maximum operating current density (Figs. 5 and 6), i.e., $j_{\text{stim},1}(j_m) = j_{\text{stim},2}(j_m)$, the curves for the internal efficiency for the two branches do not merge, i.e., $\eta_{\text{int},1}(j_m) \neq \eta_{\text{int},2}(j_m)$ (see Fig. 7). This is because the denominators in (12) for the two branches are not the same; namely, since $j_{\text{th},2} > j_{\text{th},1}$, the denominator for the second branch is smaller than that for the first branch, $j - j_{\text{th},2} < j - j_{\text{th},1}$. Hence, at $j = j_m$, when $j_{\text{stim},1}(j_m) = j_{\text{stim},2}(j_m)$, the internal efficiency for the second branch is higher than that for the first branch, $\eta_{\text{int},2}(j_m) > \eta_{\text{int},1}(j_m)$ (Fig. 7).

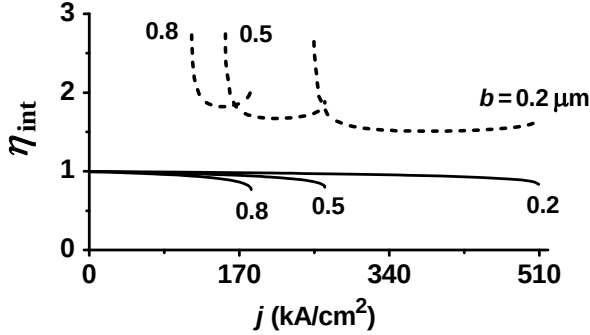


Fig. 7. Internal differential quantum efficiency against injection current density for the structures with the OCL thicknesses $b = 0.2, 0.5$, and $0.8 \mu\text{m}$.

Moreover, as seen from Fig. 7, while the internal efficiency for the first branch is less than one and decreases with increasing injection current, the internal efficiency for the second branch is greater than one and nonmonotonous. To explain these dependences, let us present the injection and threshold current densities as follows:

$$j = j_{\text{spon}} + j_{\text{stim}} = j_{\text{spon}}^{\text{QW}} + j_{\text{spon}}^{\text{OCL}} + j_{\text{stim}}, \quad (13)$$

$$j_{\text{th}} = j_{\text{spon,th}} = j_{\text{spon,th}}^{\text{QW}} + j_{\text{spon,th}}^{\text{OCL}}. \quad (14)$$

With (12)-(14), we can rewrite the expressions for η_{int} for the first and second branches as follows:

$$\eta_{\text{int},1} = \frac{j_{\text{stim},1}}{j_{\text{stim},1} + j_{\text{spon},1} - j_{\text{spon,th},1}} = \frac{1}{1 + \frac{j_{\text{spon},1} - j_{\text{spon,th},1}}{j_{\text{stim},1}}} < 1, \quad (15)$$

$$\eta_{\text{int},2} = \frac{j_{\text{stim},2}}{j_{\text{stim},2} + j_{\text{spon},2} - j_{\text{spon,th},2}} = \frac{1}{1 + \frac{j_{\text{spon},2} - j_{\text{spon,th},2}}{j_{\text{stim},2}}} = \frac{1}{1 - \frac{j_{\text{spon,th},2} - j_{\text{spon},2}}{j_{\text{stim},2}}} > 1. \quad (16)$$

As seen from Fig. 8, the total spontaneous radiative recombination current density (the sum of spontaneous radiative recombination current densities in the QWs and OCL) in the first branch (the solid curve) increases with increasing j , i.e., at any j , it is higher than at the first lasing threshold, $j_{\text{spon},1} > j_{\text{spon,th},1}$ – hence, as seen from (15), the internal efficiency for the first branch is less than one (Fig. 7). The total spontaneous

radiative recombination current density in the second branch (the dashed curve) decreases with increasing j , i.e., at any j , it is lower than at the second lasing threshold, $j_{\text{spon},2} < j_{\text{spon,th},2}$ – hence, as seen from (16), the internal efficiency for the second branch is greater than one (Fig. 7).

Since the sum of the total spontaneous radiative recombination current density and the stimulated recombination current density is equal to the total injection current density [see (13)], the superlinearity of $j_{\text{spon},1}(j)$ (the solid curve in Fig. 8) leads to sublinearity of $j_{\text{stim},1}(j)$ (the solid curve for j_{stim} in Fig. 5); the sublinearity of $j_{\text{spon},2}(j)$ (the dashed curve in Fig. 8) leads to superlinearity of $j_{\text{stim},2}(j)$ (the dashed curve for j_{stim} in Fig. 5).

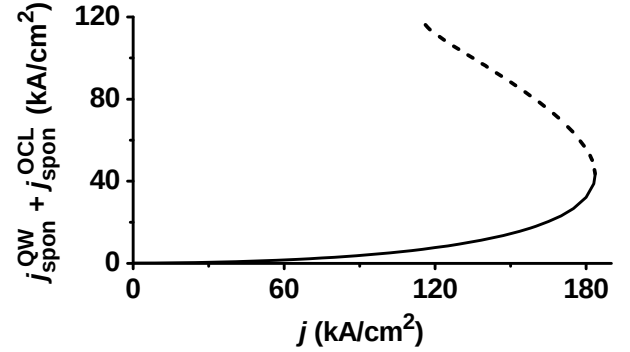


Fig. 8. Total spontaneous radiative recombination current density, $j_{\text{spon}}^{\text{QW}} + j_{\text{spon}}^{\text{OCL}}$, against injection current density. In view of (5), $j_{\text{spon}}^{\text{QW}} + j_{\text{spon}}^{\text{OCL}}$ is very close to the spontaneous radiative recombination current density in the OCL, $j_{\text{spon}}^{\text{OCL}}$ (shown in Fig. 5).

IV. CONCLUSION

We discussed the operating characteristics of semiconductor QW lasers in the presence of internal optical absorption loss, which varies with electron and hole densities in the OCL. Due to such loss, the second lasing threshold can emerge in addition to the conventional (first) threshold. Above the second threshold, the laser characteristics are two-valued. In particular, the second, unconventional, branch appears in the dependence of the output optical power on the pump current (light-current characteristic – LCC) in addition to the conventional branch. With increasing pump current and approaching a certain value (which is the maximum operating current), the two branches of the LCC merge together. At the merging point, the optical power is non-vanishing; immediately beyond this point, the lasing quenches. The characteristics of the laser are different in two branches. Thus, while the internal differential quantum efficiency for the first branch of the LCC is less than one and decreases with increasing pump current, that for the second branch is greater than one and nonmonotonous. The dependences of other characteristics on the injection current are also different in two branches – e.g., while the electron and hole densities in the OCL increase with increasing current in the first

branch, those in the second branch decrease.

The fact that the LCC can be two-valued (lobe-shaped) presents considerable interest from the viewpoints of both fundamental physics and practical applications of semiconductor lasers with a quantum-confined active region. For instance, switching between the two branches of the LCC (assuming that this is feasible) would provide an opportunity for controllable change of the internal quantum efficiency of the laser.

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Theory of photovoltaic characteristics of semiconductor quantum dot solar cells

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(Received 15 April 2016; accepted 3 August 2016; published online 23 August 2016)

We develop a comprehensive rate equations model for semiconductor quantum dot solar cells (QDSCs). The model is based on the continuity equations with a proper account for quantum dots (QDs). A general analytical expression for the total current density is obtained, and the current-voltage characteristic is studied for several specific situations. The degradation in the open circuit voltage of the QDSC is shown to be due to strong spontaneous radiative recombination in QDs. Due to small absorption coefficient of the QD ensemble, the improvement in the short circuit current density is negligible if only one QD layer is used. If spontaneous radiative recombination would be suppressed in QDs, a QDSC with multiple QD layers would have significantly higher short circuit current density and power conversion efficiency than its conventional counterpart. The effects of photoexcitation of carriers from discrete-energy states in QDs to continuum-energy states are discussed. An extended model, which includes excited states in QDs, is also introduced. Published by AIP Publishing. [<http://dx.doi.org/10.1063/1.4961046>]

I. INTRODUCTION

Due to quantum confinement of charge carriers (electrons and holes) in all three dimensions, quantum dots (QDs) have drawn much attention for their potential applications in optoelectronic devices such as solar cells (SCs).^{1–13} Changing the sizes of QDs and, hence, the position of confined carrier energy levels can easily alter the optical properties of QD-based devices. When QDs are used in a multi-junction SC, the current due to photon absorption by QDs can be tuned to achieve the current-match condition leading to an increase in the power conversion efficiency.⁶ QDs were also proposed to provide an intermediate band (IB) in an intermediate band solar cell (IBSC).¹

It was shown earlier that the power conversion efficiency of an ideal IBSC^{14,15} could exceed the Shockley-Queisser (SQ) limit of a single bandgap SC.¹⁶ The idea is that, besides the optical transition from the valence band (VB) to the conduction band (CB), by introducing an IB into the bandgap of the host material, an additional IB-mediated two-step two-photon-absorption process (VB $\rightarrow$ IB and the subsequent IB $\rightarrow$ CB transitions) becomes possible. The otherwise wasted sub-bandgap energy photons can now be absorbed via IB and, hence, both the short circuit current and the overall power conversion efficiency should be increased.

In a semiconductor quantum dot solar cell (QDSC), in which one or multiple layers with QDs are embedded in a conventional semiconductor SC, the QD confined energy level⁷ or the miniband¹ can act as an IB. The miniband is formed when multiple layers with QDs are grown within a larger bandgap material matrix and the distance between

neighboring layers or the height of the potential energy barrier is small enough for tunneling to occur between the layers. Making high performance QDSC turned, however, to be a difficult task. The problem is that, although the short circuit current can be increased due to absorption of sub-bandgap energy photons, the open circuit voltage is decreased. The increase in the short circuit current can hardly compensate for the decrease in open circuit voltage and the overall effect is a reduction of the power conversion efficiency.

Due to the fact that zero-dimensional QDs are incorporated in the bulk material and due to the presence of multiple processes in QDSCs, certain simplifications were made to provide theoretical insights into the operating characteristics of QDSCs.^{17–22} For instance, the confined electron energy level in a QD or the miniband were treated as the IB. The VB offset was assumed to be zero. While the absorption coefficient of a QD ensemble is energy-dependent (with a peak at the photon energy corresponding to the optical transition in a mean-sized QD), the IB was characterized by two constant absorption coefficients—one for a range of sub-bandgap energy photons corresponding to the VB $\rightarrow$ IB transitions and the other for the IB $\rightarrow$ CB optical transitions. QDs were also assumed to be evenly distributed in the active region. The carrier dynamics between QDs and the host material and the spontaneous radiative recombination within QDs were also not properly included into the existing models. Most of the present QDSC theories consider InAs/GaAs QDSCs, in which InAs QDs are grown in the GaAs matrix and which are relatively easy to fabricate. The problem is, however, that the energy levels of carriers confined in QDs correspond to the GaAs bandgap. The VB and CB offset energies are not zero, they are actually large, which is far

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from an ideal situation.⁹ As a result, transitions between the electron- and hole-ground states in QDs and carrier exchange (via both thermal and non-thermal processes) between the confined states in QDs and the continuous energy states in the bulk will occur. Hence, two constant absorption coefficients cannot properly describe the presence of QDs in a QDSC.

In this paper, we develop a rate equations model for semiconductor QDSCs based on such a model for QD lasers.^{23,24} Unlike lasers, which operate by current injection, a QDSC is a power source and hence the photocarrier generation term should now enter into the rate equations. We use the Dirac delta function to describe spatial discreteness of a QD layer.²⁴ The carrier thermal exchange between QDs and the host material is characterized by capture cross-sections. Photon absorptions both via the QD ground-state transitions and photoexcitation are studied. We show that inferior performance of QDSCs as compared to their conventional counterpart is due to weak photon absorption in QDs and weak photoexcitation of carriers from QDs as well as to strong spontaneous radiative recombination in QDs.

II. THEORETICAL MODEL

Our model is based on the following set of steady-state rate equations that describe the carrier generation, recombination, and transport processes in a QDSC as shown in Fig. 1:

For free carriers in the bulk i-region

$$\sigma_n v_n n_1 N_S f_n \delta(x - x_0) - \sigma_n v_n N_S (1 - f_n) n(x) \delta(x - x_0) + g_{\text{non-th},n} \delta(x - x_0) + g_{\text{bulk}} - R_{\text{bulk}} + \frac{1}{q} \frac{dj_n}{dx} = 0, \quad (1)$$

$$\sigma_p v_p p_1 N_S f_p \delta(x - x_0) - \sigma_p v_p N_S (1 - f_p) p(x) \delta(x - x_0) + g_{\text{non-th},p} \delta(x - x_0) + g_{\text{bulk}} - R_{\text{bulk}} - \frac{1}{q} \frac{dj_p}{dx} = 0, \quad (2)$$

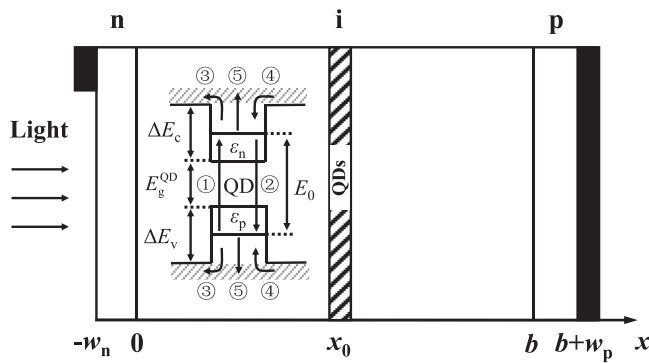


FIG. 1. Simplified structure and schematic energy band diagram (the inset) of a typical QDSC (not drawn to scale). The layer with QDs is located at the position x_0 in the i-region. The thickness of the n-, i-, and p-region is w_n , b , and w_p , respectively. The front and back contacts are shown in black. The main processes (shown by arrows) associated with QDs are as follows: ① photocarrier generation by absorption of photons, ② radiative recombination of carriers, ③ thermal escape of carriers from QDs to the bulk, ④ carrier capture from the bulk into QDs, and ⑤ photoexcitation or tunneling of carriers confined in QDs. For our calculations, a typical InAs/GaAs QDSC is used; $w_n = 300$ nm, $b = 1000$ nm, $w_p = 200$ nm, $x_0 = b/2$, and the surface recombination velocities for the front and back contacts $S_p = S_n = 10^4$ cm/s. The doping levels in the n- and p-region are 2×10^{16} and 4×10^{17} cm⁻³, respectively.

for carriers confined in QDs

$$\sigma_n v_n N_S (1 - f_n) n(x) \delta(x - x_0) - \sigma_n v_n n_1 N_S f_n \delta(x - x_0) - g_{\text{non-th},n} \delta(x - x_0) + g_{\text{QD}} \delta(x - x_0) - R_{\text{QD}} \delta(x - x_0) = 0, \quad (3)$$

$$\sigma_p v_p N_S (1 - f_p) p(x) \delta(x - x_0) - \sigma_p v_p p_1 N_S f_p \delta(x - x_0) - g_{\text{non-th},p} \delta(x - x_0) + g_{\text{QD}} \delta(x - x_0) - R_{\text{QD}} \delta(x - x_0) = 0, \quad (4)$$

where $n(x)$ and $p(x)$ are the free electron and hole densities in the i-region, respectively, $\sigma_{n,p}$ are the cross-sections of carrier capture into a QD, $v_{n,p}$ are the carrier thermal velocities in the bulk material, N_S is the surface density of QDs (i.e., the number of QDs per unit area of the QD layer), the quantities $n_1 = N_c \exp[-(\Delta E_c - \epsilon_n)/T]$ and $p_1 = N_v \exp[-(\Delta E_v - \epsilon_p)/T]$ characterize the carrier thermal escape from a QD to the bulk, $N_{c,v}$ are the effective densities of states in the bulk, $\Delta E_{c,v}$ are the energy band offsets, $\epsilon_{n,p}$ are the confined energies, T is the temperature (in units of energy), $f_{n,p} = \{1 + \exp[(\epsilon_{n,p} - \mu_{n,p})/T]\}^{-1}$ are the confined carrier level occupancies in QDs, and $\mu_{n,p}$ are the quasi-Fermi levels. Both $\mu_{n,p}$ and $\epsilon_{n,p}$ are measured from the respective band edges in the QD material (see the inset in Fig. 1 for $\epsilon_{n,p}$; the positions of $\mu_{n,p}$ will be determined later). g_{bulk} and R_{bulk} are the photocarrier generation and radiative recombination rates in the i-region, respectively. q is the elementary charge, j_n and j_p are the electron and hole current densities in the i-region, respectively.

Carriers confined in QDs are involved in the following processes: generation due to absorption of photons with a rate g_{QD} (process ① in Fig. 1), radiative recombination with a rate R_{QD} (process ② in Fig. 1), thermal escape to the bulk (process ③ in Fig. 1) or non-thermal escape to the bulk (photoexcitation or tunneling; process ⑤ in Fig. 1). The first terms in the left-hand side of Eqs. (1) and (2) are the rates of electron and hole thermal escape from QDs to the bulk, respectively. $g_{\text{non-th},n,p}$ are the rates of non-thermal escape. Once electrons (holes) escape to the bulk, they can drift across the i-region to the n- (p-) region due to the built-in electric field. Free electrons and holes can also be captured back from the bulk into QDs (process ④ in Fig. 1); this process is described by the second terms in the left-hand side of Eqs. (1) and (2).

The QD layer occupies only a very small fraction of the i-region, and hence, we use the Dirac delta function $\delta(x - x_0)$ (where x_0 is the location of the QD layer) in the rate equations to describe the QD-related local events.

III. RESULTS AND DISCUSSIONS

A. General expression for the total current density

The free carrier densities $n(x)$ and $p(x)$ can be discontinuous at the QD layer $x = x_0$. Defining the integration of the product of the Dirac delta-function and a discontinuous function as follows:

$$\int_{x_0-0}^{x_0+0} \delta(x-x_0)n(x)dx = \lim_{\Delta \rightarrow 0} \left\{ \frac{1}{\Delta\sqrt{2\pi}} \int_{x_0-0}^{x_0+0} \exp\left[-\frac{(x-x_0)^2}{2\Delta^2}\right] n(x)dx \right\}, \quad (5)$$

and integrating (3) and (4) over an infinitesimal interval around $x = x_0$, we obtain

$$\sigma_n v_n N_S (1-f_n) \frac{n(x_0-0) + n(x_0+0)}{2} - \sigma_n v_n n_1 N_S f_n - g_{\text{non-th,n}} + g_{\text{QD}} - R_{\text{QD}} = 0, \quad (6)$$

$$\sigma_p v_p N_S (1-f_p) \frac{p(x_0-0) + p(x_0+0)}{2} - \sigma_p v_p p_1 N_S f_p - g_{\text{non-th,p}} + g_{\text{QD}} - R_{\text{QD}} = 0. \quad (7)$$

From (6) and (7), we have for the average values of the free electron and hole densities at the QD layer

$$\frac{n(x_0-0) + n(x_0+0)}{2} = n_1 \frac{f_n}{1-f_n} + \frac{1}{\sigma_n v_n N_S (1-f_n)} \times (R_{\text{QD}} - g_{\text{QD}} + g_{\text{non-th,n}}), \quad (8)$$

$$\begin{aligned} & \frac{p(x_0-0) + p(x_0+0)}{2} \\ &= p_1 \frac{f_p}{1-f_p} + \frac{1}{\sigma_p v_p N_S (1-f_p)} (R_{\text{QD}} - g_{\text{QD}} + g_{\text{non-th,p}}). \end{aligned} \quad (9)$$

Instead of (1) and (2), equations

$$g_{\text{bulk}} - R_{\text{bulk}} + \frac{1}{q} \frac{dj_n}{dx} = 0 \quad (10)$$

and

$$g_{\text{bulk}} - R_{\text{bulk}} - \frac{1}{q} \frac{dj_p}{dx} = 0 \quad (11)$$

can be equivalently used separately for the left- and right-hand sides (with respect to the QD layer) of the i-region (i.e., the regions $0 \leq x \leq x_0 - 0$ and $x_0 + 0 \leq x \leq b$, respectively) provided Eqs. (10) and (11) are supplemented with the following boundary conditions at the QD layer:

$$j_n|_{x=x_0+0} - j_n|_{x=x_0-0} = q(R_{\text{QD}} - g_{\text{QD}}), \quad (12)$$

$$j_p|_{x=x_0+0} - j_p|_{x=x_0-0} = q(g_{\text{QD}} - R_{\text{QD}}). \quad (13)$$

These boundary conditions are obtained by integrating Eqs. (1)–(4) over an infinitesimal interval around x_0 .

It should be noted that, although the total current density $j = j_n(x) + j_p(x)$ is constant in position, the electron and hole current densities may be discontinuous at the QD layer. As can be seen from Eqs. (12) and (13), the discontinuity is due to the fact that the generation and spontaneous radiative recombination rates in QDs may not equal to each other. This discontinuity is a feature related to the use of the δ -function to describe the QD layer in our model.

As seen from Eqs. (6) and (7), the net escape rate from QDs to the i-region is the same for electrons and holes [and equals to $(g_{\text{QD}} - R_{\text{QD}})$] even if their individual capture and escape rates are different.

At the front and back contacts, the current is completely due to the majority carriers so that

$$j = j_n(-w_n) = j_p(b + w_p). \quad (14)$$

It follows from (14) that

$$j_p(-w_n) = j_n(b + w_p) = 0. \quad (15)$$

Integrating Eq. (10) from $-w_n$ to $b + w_p$ and using Eq. (12), we obtain a general expression for the total current density in a QDSC

$$\begin{aligned} j = q \int_{-w_n}^{x_0-0} (g_{\text{bulk}} - R_{\text{bulk}}) dx \\ + q \int_{x_0+0}^{b+w_p} (g_{\text{bulk}} - R_{\text{bulk}}) dx + q(g_{\text{QD}} - R_{\text{QD}}). \end{aligned} \quad (16)$$

Eq. (16) simply states that the total current density is due to the net generation of photocarriers by the entire device.

We did not consider here the nonradiative recombination loss of carriers. However, this process can be easily incorporated into our model by simply rewriting R_{bulk} and R_{QD} to include both radiative and nonradiative recombinations in Eq. (16).

A straightforward approach to obtain the j - V characteristic of the QDSC would be to solve the carrier transport equations and Poisson's equation supplemented by the corresponding boundary conditions. Having thus obtained $n(x)$ and $p(x)$ and the spatial distribution of the electric field, the total current density could be calculated. In the i-region, however, we would have a set of coupled integro-differential equations [Eqs. (10) and (11)]. At the QD layer, there would be a step in the electric field equal to $N_S(f_p - f_n)/\epsilon_0\epsilon_r$, where ϵ_0 is the permittivity of free space and ϵ_r is the relative permittivity (dielectric constant). This set of integro-differential equations could only be solved numerically. Our goal here is, however, to derive analytical expressions for the j - V characteristic. To do this, we use the approach which is described in Ref. 25.

Since the total current density is constant in position [$j = j_p(x) + j_n(x) = \text{const}(x)$], it can be calculated by adding the electron and hole current densities at any position x . Let us choose $x = 0$ for doing this. At $x = 0$, $j_p(0)$ can be easily obtained by solving the ambipolar transport equation for minority holes, and $j_n(0)$ can be obtained by integrating Eq. (10) from 0 to b at which position electrons can be treated as minority carrier. With Eq. (12), the total current density is given by

$$\begin{aligned} j = j_p(0) + q \int_0^{x_0-0} (g_{\text{bulk}} - R_{\text{bulk}}) dx + q(g_{\text{QD}} - R_{\text{QD}}) \\ + q \int_{x_0+0}^b (g_{\text{bulk}} - R_{\text{bulk}}) dx + j_n(b). \end{aligned} \quad (17)$$

In (17), the photocarrier generation rate for the bulk i-region is given by

$$g_{\text{bulk}}(x) = \int_0^{\lambda_{\text{bulk}}} F \alpha_{\text{bulk}} \exp[-\alpha_{\text{bulk}}(x + w_n)] d\lambda, \quad (18)$$

where λ_{bulk} is the longest photon wavelength for optical transitions in the bulk. The absorption coefficient $\alpha_{\text{bulk}}(\lambda)$ for the i-region material is a function of the wavelength λ ,²⁶ the photon flux $F(\lambda)$ of the incident sunlight at a certain wavelength λ is given by

$$F(\lambda) = 2\pi c \left(\frac{r_{\text{sun}}}{d_{\text{se}}} \right)^2 \frac{1}{\lambda^4} \frac{1}{\exp(hc/\lambda T_{\text{sun}}) - 1}, \quad (19)$$

where c is the speed of light in vacuum, r_{sun} is the radius of the sun, d_{se} is the mean distance between the sun and earth, h is Planck's constant, T_{sun} is set to be 5762 K.²⁷

So far, one layer with QDs was only considered. If multiple layers are used (with Z_L being their number), we will have Z_L pairs of rate equations for confined carriers. Integrating Eq. (10), we will obtain for the total current density

$$j = j_p(0) + q \int (g_{\text{bulk}} - R_{\text{bulk}}) dx + q \sum_{i=1}^{Z_L} (g_{\text{QD},i} - R_{\text{QD},i}) + j_n(b), \quad (20)$$

where the integration is taken over the interval $[0, b]$ with an effective thickness that does not include the total thickness of QD layers.

When multiple layers with QDs are used, tunneling can occur between neighboring QD layers (do not confuse with the tunneling escape of confined carriers to the i-region), which is ignored in Eq. (20). It has been experimentally demonstrated that such tunneling is not required for better performance.⁷ If this process is, however, present, the rate equations for confined carriers in each QD layer should be revised to include the in- and out-tunneling rates of carriers. As a consequence, the boundary conditions at each QD layer should also contain the in- and out-tunneling terms. Following a procedure similar to that for deriving Eq. (17), we can obtain the expression for the total current density, which has exactly the same form as Eq. (20). This is reasonable since the total current density should be given by the net generation of photocarriers. See Appendix A for the case of tunneling between neighboring QD layers.

It should be noted that Eqs. (16) and (20) for the total current density are general. They are derived from the continuity equations with the boundary conditions at the QD layer.

B. Assumptions and results

To simplify the calculations, g_{bulk} can be approximated by an exponential function^{3,28}

$$g_{\text{bulk}}(x) = \alpha_{\text{bulk}} \phi_0 \exp[-\alpha_{\text{bulk}}(x + w_n)], \quad (21)$$

where α_{bulk} and ϕ_0 are numerical constants.

To obtain an analytical expression for j , we make the following assumptions, which are often used in the analysis of p-n junctions:²⁸

- (i) In the n- (p-) type region (i.e., in the regions $-w_n \leq x \leq 0$ and $b \leq x \leq b + w_p$, respectively), holes (electrons) are minority carriers even under illumination.
- (ii) n- and p-type regions are quasi-neutral so that the electric fields are zero there.
- (iii) The front and back contacts can be characterized by the surface recombination velocities S_p and S_n , respectively.
- (iv) The space charge regions are neglected.

Using above assumptions and Eq. (21), analytical expressions for $j_p(0)$ and $j_n(b)$ can be easily obtained by solving the ambipolar transport equations with the following boundary conditions:

For minority holes

$$p(0) = p_{\text{no}} \exp(V/T), \quad (22a)$$

$$\left. \frac{dp}{dx} \right|_{x=-w_n} = \frac{S_p}{D_p} [p(-w_n) - p_{\text{no}}], \quad (22b)$$

and for minority electrons

$$n(b) = n_{\text{po}} \exp(V/T), \quad (23a)$$

$$\left. \frac{dn}{dx} \right|_{x=b+w_p} = -\frac{S_n}{D_n} [n(b + w_p) - n_{\text{po}}], \quad (23b)$$

where V is the output voltage (in units of energy), and p_{no} and n_{po} are the hole and electron densities at thermal equilibrium in n- and p-regions, respectively. $D_{n,p}$ are the diffusion coefficients for electrons and holes, respectively.

In the bulk i-region, the radiative recombination process is bimolecular and the electric field is present; therefore, it is impossible to calculate R_{bulk} analytically. Besides, the electric field may not be constant—it is determined not only by carrier densities and doping levels but also by the net charge in the QD layer.²⁹ Therefore, to obtain an analytical expression for the current density due to radiative recombination $j_{\text{rec}}^{\text{bulk}}$, we need to use one more assumption:³⁰

- (v) In the i-region, the drops of the quasi-Fermi levels $E_{\text{Fn},p}$ for electrons and holes are so small that we approximately have

$$E_{\text{Fn}} - E_{\text{Fp}} = V. \quad (24)$$

Eq. (24) is equivalent to the assumption that electrons and holes have high mobilities so that they quickly drift to the n- and p-regions, respectively.¹⁹

With (24), we have for the recombination current density in i-region

$$j_{\text{rec}}^{\text{bulk}} = q \int_0^b R_{\text{bulk}} dx = q b B n_i^2 \left[\exp\left(\frac{V}{T}\right) - 1 \right], \quad (25)$$

where B is the radiative recombination constant for the bulk.

The current density associated with absorption of photons via ground-state transitions in QDs is given by

$$j_{\text{ph}}^{\text{QD}} = qg_{\text{QD}} = j_{\text{ph}}^{\text{QD-max}}(1 - f_n - f_p), \quad (26)$$

where the photocarrier generation rate by the QD ensemble is

$$g_{\text{QD}} = \left| \int_{\lambda_{\text{bulk}}}^{+\infty} \sigma_{\text{QD}} N_S F d\lambda \right| (1 - f_n - f_p). \quad (27)$$

The maximum value of $j_{\text{ph}}^{\text{QD}}$ (obtained when $f_n = f_p = 0$) is

$$j_{\text{ph}}^{\text{QD-max}} = q \left| \int_{\lambda_{\text{bulk}}}^{+\infty} \sigma_{\text{QD}} N_S F d\lambda \right|. \quad (28)$$

In (28), $\sigma_{\text{QD}}(\lambda) = \bar{a}^3 \alpha_{\text{QD}}(\lambda)$ is the cross-section (in units of cm^2) of photon absorption by a QD, $\bar{a}$ is the mean size of QDs, and $\alpha_{\text{QD}}(\lambda)$ is the absorption coefficient per QD (in units of cm^{-1}) averaged over the inhomogeneously broadened ensemble of QDs^{31,32}

$$\alpha_{\text{QD}}(\lambda) = \frac{8}{3} \pi^2 \frac{\alpha_{\text{fine}}}{\sqrt{\epsilon_r}} P^2 \frac{1}{E(\lambda)} I_{\text{overlap}} d \left\langle \frac{1}{V_{\text{QD}}} \delta[E(\lambda) - E_0] \right\rangle, \quad (29)$$

where α_{fine} is the fine-structure constant, V_{QD} is the volume of a QD, P is Kane's parameter,³³ I_{overlap} is the overlap integral of the electron and hole wave functions in a QD, d is the degeneracy of the optical transition in a QD, $E(\lambda) = hc/\lambda$ is the incident photon energy, $\langle \dots \rangle$ means averaging over the inhomogeneously broadened ensemble of QDs, the delta-function of energy in $\langle \dots \rangle$ originates from the density of states in a QD, $E_0 = E_g^{\text{QD}} + \bar{\epsilon}_n + \bar{\epsilon}_p$ is the energy of the optical transition in a QD, and E_g^{QD} is the band-gap energy of the QD material.

The absorption coefficient α_{QD} has a peak at the energy of the optical transition in a mean-sized QD $\bar{E}_0 = E_g^{\text{QD}} + \bar{\epsilon}_n + \bar{\epsilon}_p = hc/\bar{\lambda}_0$, where $\bar{\epsilon}_n$ and $\bar{\epsilon}_p$ are the electron and hole quantized energies in a mean-sized QD. Since $\alpha_{\text{QD}}(\lambda)$ is very low for λ far beyond $\bar{\lambda}_0$, we can safely use $+\infty$ as the upper limit for the integral in Eq. (27).

The current density associated with spontaneous radiative recombination of electrons and holes in QDs is

$$j_{\text{rec}}^{\text{QD}} = qR_{\text{QD}} = j_{\text{rec}}^{\text{QD-max}} f_n f_p, \quad (30)$$

where

$$j_{\text{rec}}^{\text{QD-max}} = q \frac{N_S}{\tau_{\text{QD}}} \quad (31)$$

is the maximum value (when $f_n = f_p = 1$) of $j_{\text{rec}}^{\text{QD}}$, R_{QD} is the radiative recombination rate in QDs

$$R_{\text{QD}} = N_S \frac{f_n f_p}{\tau_{\text{QD}}}, \quad (32)$$

and τ_{QD} is the lifetime of spontaneous radiative recombination of an electron and a hole in a QD^{31,32}

$$\tau_{\text{QD}} = \left[\frac{8}{3} \alpha_{\text{fine}} \sqrt{\epsilon_r} \frac{\bar{E}_0}{\hbar} \left(\frac{P}{\hbar c} \right)^2 I_{\text{overlap}} \right]^{-1}. \quad (33)$$

The ratio of $j_{\text{ph}}^{\text{QD}}$ to $j_{\text{rec}}^{\text{QD}}$ is

$$\frac{j_{\text{ph}}^{\text{QD}}}{j_{\text{rec}}^{\text{QD}}} = \frac{g_{\text{QD}}}{R_{\text{QD}}} = \frac{j_{\text{ph}}^{\text{QD-max}}}{j_{\text{rec}}^{\text{QD-max}}} \left\{ \exp \left[\frac{(\bar{\epsilon}_n + \bar{\epsilon}_p) - (\mu_n + \mu_p)}{T} \right] - 1 \right\}. \quad (34)$$

As seen from (34), to have $j_{\text{ph}}^{\text{QD}}$ positive, the condition $\bar{\epsilon}_n + \bar{\epsilon}_p > \mu_n + \mu_p$ should hold. Note that the opposite of this inequality is the population inversion condition in a laser diode. This is reasonable since lasers are power-consumption devices and population inversion is achieved by injection of carriers, while a solar cell is a power source and hence there should be more carriers in lower energy states to have more absorption than recombination.

Using Eqs. (21), (25), (26), and (30), the total current density is obtained

$$j = j_{\text{ph}} - j_s \left[\exp \left(\frac{V}{T} \right) - 1 \right] + j_{\text{QD}}, \quad (35)$$

where

$$j_{\text{ph}} = q\phi_0 \frac{\beta_2^2 e^{-\beta_2}}{\beta_2^2 - \beta_1^2} \left(\frac{1 + \beta_1 \beta_3 / \beta_2}{\beta_3 \sinh \beta_1 + \cosh \beta_1} e^{\beta_2} - \frac{\beta_1 \beta_3 \cosh \beta_1 + \sinh \beta_1}{\beta_2 \beta_3 \sinh \beta_1 + \cosh \beta_1} - 1 \right) + q\phi_0 \frac{\beta_6^2 e^{-\beta_4}}{\beta_6^2 - \beta_5^2} \left(\frac{\beta_5 \beta_7 / \beta_6 - 1}{\beta_7 \sinh \beta_5 + \cosh \beta_5} e^{-\beta_6} - \frac{\beta_5 \beta_7 \cosh \beta_5 + \sinh \beta_5}{\beta_6 \beta_7 \sinh \beta_5 + \cosh \beta_5} + 1 \right) + q\phi_0 (e^{-\beta_2} - e^{-\beta_4}), \quad (36)$$

$$j_s = \frac{qD_p p_{\text{no}}}{L_p} \frac{\beta_3 \cosh \beta_1 + \sinh \beta_1}{\beta_3 \sinh \beta_1 + \cosh \beta_1} + \frac{qD_n n_{\text{po}}}{L_n} \frac{\beta_7 \cosh \beta_5 + \sinh \beta_5}{\beta_7 \sinh \beta_5 + \cosh \beta_5} + qbBn_i^2, \quad (37)$$

$$j_{\text{QD}} = j_{\text{ph}}^{\text{QD}} - j_{\text{rec}}^{\text{QD}}, \quad (38)$$

$L_{n,p}$ are the diffusion lengths for minority electrons and holes, respectively. Parameters β_{1-7} (defined in Table I) are determined by the cell structure and optical properties of cell materials.²⁵

With Eqs. (6) and (7), j_{QD} can also be written as

$$j_{\text{QD}} = q \left\{ g_{\text{non-th,n}} + \sigma_n v_n N_S (1 - f_n) \left[n_1 \frac{f_n}{1 - f_n} - \frac{n(x_0 - 0) + n(x_0 + 0)}{2} \right] \right\} \\ = q \left\{ g_{\text{non-th,p}} + \sigma_p v_p N_S (1 - f_p) \left[p_1 \frac{f_p}{1 - f_p} - \frac{p(x_0 - 0) + p(x_0 + 0)}{2} \right] \right\}. \quad (39)$$

Since $j_{\text{QD}}(V)$ can be voltage-dependent through such dependences of $f_{n,p}(V)$, for the short circuit current density and the open circuit voltage, we have

$$j_{\text{sc}} = j_{\text{ph}} + j_{\text{QD}}(0), \quad (40)$$

$$V_{\text{oc}} = T \ln \left[\frac{j_{\text{ph}} + j_{\text{QD}}(V_{\text{oc}})}{j_s} + 1 \right]. \quad (41)$$

It is worth noting that, if j_{QD} is voltage-dependent, the total current density cannot simply be written in the form of $j_{\text{sc}} - j_s [\exp(V/T) - 1]$.

C. Discussion

There have been experimental reports showing that the position of the QD layer can play an important role in determining the cell performance.^{34,35} Due to the position-dependence of $f_{n,p}$ and g_{QD} , the quantities R_{QD} and j_{QD} are also functions of position. Hence, our model can be used to study the effects of the QD layer position. We will, however, assume here that the QD layer is located in the middle of the i-region, which is the most common situation. We leave the effects of the QD layer position for a separate study.

Using our model, numerical calculations would allow us to obtain the spatial profiles of the electric field and carrier densities, as well as the energy band diagram. We are, however, able to derive here an analytical expression for the j - V characteristic [Eq. (35)] without involving extensive numerical calculations.

The primary focus of our study is the factors limiting the QDSC performance. For this purpose, we will separately study the QDSC in the following three cases.

1. Quasi-equilibrium between QDs and the bulk i-region

If we assume that the carrier exchange between QDs and the i-region is instantaneous (i.e., the carrier capture

cross-sections $\sigma_{n,p} = \infty$ and the tunneling escape of confined carriers to the i-region is instantaneous³⁶), the quasi-equilibrium will be established, the output voltage will be given as $V = E_g^{\text{QD}} + \mu_n + \mu_p$, and Eq. (34) can be rewritten as

$$\frac{j_{\text{ph}}^{\text{QD}}}{j_{\text{rec}}^{\text{QD}}} = \frac{j_{\text{ph}}^{\text{QD-max}}}{j_{\text{rec}}^{\text{QD-max}}} \left[\exp\left(\frac{\bar{E}_0 - V}{T}\right) - 1 \right]. \quad (42)$$

Clearly, to have a positive $j_{\text{ph}}^{\text{QD}}$, the output voltage should be lower than $\bar{E}_0/q$.

Depending on the doping levels, conduction and valence band offsets, thickness of the i-region, QD mean size, and location of the QD layer, we will have the following two situations [(i) and (ii) below] in which expressions for $f_{n,p}$ can be obtained.²⁹

- (i) The QD layer is charge neutral. Then, we have

$$f_n = f_p = f_{\text{CN}}(V) = \left[1 + \exp\left(\frac{\bar{E}_0 - V}{2T}\right) \right]^{-1}. \quad (43)$$

Since $\bar{E}_0 \gg T$, at short circuit $f_{\text{CN}} \approx 0$, $j_{\text{QD}}(0) \approx j_{\text{ph}}^{\text{QD-max}}$, and j_{sc} becomes

$$j_{\text{sc}} = j_{\text{ph}} + j_{\text{ph}}^{\text{QD-max}}. \quad (44)$$

V_{oc} is calculated numerically from Eq. (41).

Fig. 2 shows $j_{\text{ph}}^{\text{QD}}$, $j_{\text{rec}}^{\text{QD}}$, and j_{QD} as functions of the output voltage. With increasing V , both $j_{\text{ph}}^{\text{QD}}$ and j_{QD} decrease (from $j_{\text{ph}}^{\text{QD-max}} = 3.9 \mu\text{A}/\text{cm}^2$) and become zero as the voltage equals to 0.6 and 0.99 V [when $V = \bar{E}_0$ —see Eq. (42)], respectively. $j_{\text{rec}}^{\text{QD}}$ increases with V and approaches $j_{\text{rec}}^{\text{QD-max}}$ ($= 32.8 \text{ A}/\text{cm}^2$) as V becomes much larger than $\bar{E}_0$. Since $j_{\text{rec}}^{\text{QD-max}} \gg j_{\text{ph}}^{\text{QD-max}}$, as V approaches $\bar{E}_0$, $j_{\text{rec}}^{\text{QD}}$ becomes larger than $j_{\text{ph}}^{\text{QD}}$ and thus j_{QD} becomes negative. In our calculations, the most commonly studied InAs/GaAs QDSC is considered, and typical values for materials properties are used.³⁷

Fig. 3 shows the j - V characteristic and the output electrical power densities (mW/cm^2) for both the conventional SC and QDSC under different scenarios. The solid curves represent the conventional SC and the dashed-dotted curves represent the QDSC under quasi-equilibrium. The current density $j(V)$ for the conventional SC is calculated using Eq. (35) and neglecting the last term j_{QD} therein. As seen from Eq. (44), j_{sc} for the QDSC is increased compared to that for the conventional SC. However, the increase is practically negligible. This is because of a very small

TABLE I. Expressions for parameters entering into Eqs. (36) and (37).

Parameter	Expression
β_1	w_n/L_p
β_2	$w_n \alpha_{\text{bulk}}$
β_3	$L_p S_p / D_p$
β_4	$(b + w_n) \alpha_{\text{bulk}}$
β_5	w_p / L_n
β_6	$w_p \alpha_{\text{bulk}}$
β_7	$L_n S_n / D_n$

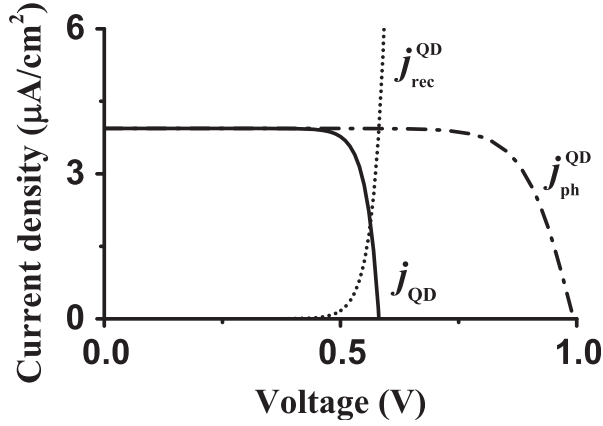


FIG. 2. Voltage dependences of current densities for the processes associated with QDs in a QDSC. The current densities due to photon absorption (j_{ph}^{QD}), spontaneous radiative recombination (j_{rec}^{QD}), and $j_{QD} = j_{ph}^{QD} - j_{rec}^{QD}$ are shown by the dashed-dotted, dotted, and solid curve, respectively. For the QDSC, the following two assumptions are made: (i) there is quasi-equilibrium between QDs and the bulk i-region and (ii) the QD layer is charge neutral (i.e., $f_n = f_p$). At 0.58 V, $j_{QD} = 0$, and at $\bar{E}_0/q = 0.99V$, $j_{ph}^{QD} = 0$ [see Eqs. (26) and (43)]. The mean size of QDs is 9 nm, an infinitely deep potential well for confined carriers is assumed, the root mean square of QD-size fluctuations $\delta = 0.05$, the overlap integral of the confined electron and hole wave functions $I_{overlap} = 1$, the spontaneous radiative lifetime in a QD $\tau_{QD} = 0.49$ ns, and the QD surface density (i.e., the number of QDs per unit area of the QD layer) $N_S = 10^{11} \text{ cm}^{-2}$.

QD absorption coefficient as can be seen from Eq. (28). The major contribution to j_{sc} is still from the optical absorption by the bulk, not by QDs. On the other hand, V_{oc} for the QDSC is much lower than that for the conventional SC. The reason for voltage degradation is in a very strong spontaneous radiative recombination in QDs. Since $f_{n,p}$ increase with V , j_{ph}^{QD} decreases; however, j_{rec}^{QD} increases with V . Since $j_{rec}^{QD} \gg j_{ph}^{QD}$, j_{QD} is already negative at $V = V_{oc}$ (see Fig. 2) and, therefore, the open circuit voltage for the QDSC is lower than its bulk counterpart. Our findings on a negligible increase in j_{sc} and a significant decrease in V_{oc} in a QDSC are consistent with experimental results.^{3,7,8,19}

If multiple layers with QDs are used, it might seem that $j_{ph}^{QD_{max}}$ could be multiplied by the number of QD layers Z_L to calculate j_{sc} and V_{oc} . However, since $f_{n,p}$ depend on the position, each layer will now have its own values of $f_{n,p}$. Therefore, $j_{ph}^{QD_{max}}$ should be calculated separately for each layer and the j - V characteristic should be derived according to Eq. (20).

- (ii) Charge neutrality is violated in the QD layer, i.e., $f_n \neq f_p$. Analytical expressions for $f_{n,p}$ for this situation have already been derived.²⁹ It can be easily proved that, in this situation

$$f_n f_p \leq f_{CN}^2. \quad (45)$$

A smaller value of the product $f_n f_p$ means a weaker recombination in QDs [see Eq. (32)]. Hence, violation of charge neutrality in QDs is beneficial for better performance of QDSCs. Since the i-region of the QDSC structure considered here is considerably thicker than the QD layer, we did not take into account the effect

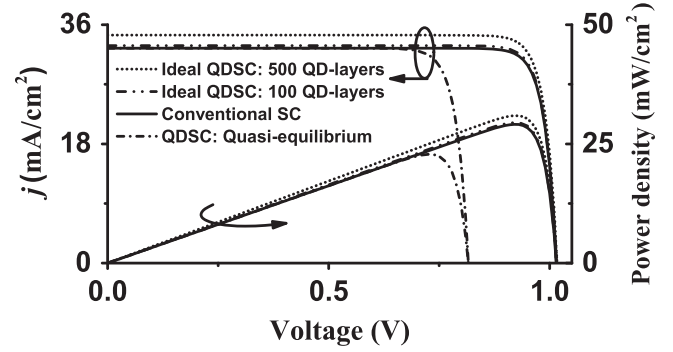


FIG. 3. Current density j and output electrical power density as a function of the output voltage for the conventional SC (solid curves) and QDSC under different scenarios. The dashed-dotted curves for the QDSC are calculated assuming (i) quasi-equilibrium between QDs and the bulk and (ii) charge neutrality in the QD layer. The dotted and dashed-dotted-dotted curve represent the QDSC under the best-case scenario with the number of QD layers $Z_L = 500$ and 100, respectively. V_{oc} for the QDSC under quasi-equilibrium is 0.81 V, while $V_{oc} = 1.01$ V is practically the same for the conventional SC and QDSCs under the best-case scenario. The power conversion efficiencies are 16.9%, 21.6%, 21.9%, and 23.0% for the curves from bottom up, respectively.

of violation of charge neutrality in QDs. However, in QDSCs structures with a thinner i-region, the effect of violation of charge neutrality in QDs on the j - V characteristic may be stronger.

2. Best-case scenario: $f_n = f_p = 0$ for the entire range of V

In a QDSC, both photocarrier generation and recombination processes will occur in QDs. To have better performance, it is desirable to enhance the generation process while suppressing the carrier recombination process. This can be done by simultaneously decreasing f_n and f_p [see Eqs. (26) and (30)]. Ideally, we should have $j_{QD} = j_{ph}^{QD} = j_{ph}^{QD_{max}}$ and $j_{rec}^{QD} = 0$, i.e., $f_n = f_p = 0$ for the entire range of V (i.e., from 0 to V_{oc} , the inequality $\bar{e}_{n,p} \gg \mu_{n,p}$ is always satisfied). In such a case, there will only be photocarrier generation and no radiative recombination in QDs ($R_{QD} = 0$); QDs will act as photocarrier generation centers and their inclusion into a conventional SC will lead to better performance—both the short circuit current density and open circuit voltage will be improved according to Eqs. (40) and (41), respectively. If Z_L layers with QDs are used and assuming the best-case scenario ($f_n = f_p = 0$) in each layer, j_{sc} and V_{oc} will be given by

$$j_{sc} = j_{ph} + Z_L j_{ph}^{QD_{max}}, \quad (46)$$

$$V_{oc} = T \ln \left(\frac{j_{ph} + Z_L j_{ph}^{QD_{max}}}{j_s} + 1 \right). \quad (47)$$

In this case, we obtain an analytical expression for the power conversion efficiency

$$\eta_{best} = \frac{V_{opt} \{j_{sc} - j_s [\exp(V_{opt}/T) - 1]\}}{\int_0^{+\infty} F(\lambda) hc / \lambda d\lambda}, \quad (48)$$

where the optimum voltage V_{opt} , at which the output power density is maximum, is found from the solution of the equation

$$\left(1 + \frac{V_{\text{opt}}}{T}\right) \exp\left(\frac{V_{\text{opt}}}{T}\right) = \frac{j_{\text{sc}} + j_s}{j_s}. \quad (49)$$

For this ideal case, the dotted and dashed-dotted-dotted curves in Fig. 3 are plotted for the QDSCs with 500 and 100 QD layers, respectively. Compared with the conventional SC (the solid curve) and non-ideal QDSC (the dashed-dotted curves), j_{sc} is increased due to absorption of sub-bandgap energy photons and elimination of radiative recombination at short circuit [see Eqs. (40) and (46)]. However, in view of a very small absorption coefficient [see Eq. (28)], the improvement in j_{sc} for the ideal QDSC with 100 QD layers is still quite small. Only with the use of large number of QD layers ($Z_L = 500$ for the top dotted curve), a large increase in j_{sc} is obtained. Compared with the non-ideal QDSC (the dashed-dotted curve), a significant recovery in V_{oc} is obtained in the ideal QDSCs. The improvement in V_{oc} is due to elimination of strong spontaneous radiative recombination in QDs. Due to a logarithmic dependence on $j_{\text{ph}}^{\text{QD-max}}$ (with $j_{\text{ph}}^{\text{QD-max}}$ being very small) and Z_L [see Eq. (47)], V_{oc} for the ideal QDSC is practically the same as that for the conventional SC. This confirms again that it is the strong spontaneous radiative recombination of carriers in QDs, which is responsible for the open circuit voltage degradation in a QDSC.

In Fig. 3, the output electrical power densities (in units of mW/cm^2) are also shown. The power conversion efficiencies are 16.9%, 21.6%, 21.9%, and 23.0% for the QDSC under quasi-equilibrium, conventional SC, and ideal QDSCs with $Z_L = 100$ and 500, respectively. As seen from the figure, a multi-layer QDSC will have a higher efficiency than its conventional counterpart if spontaneous radiative recombination of carriers in QDs is suppressed.

3. Photoexcitation of confined carriers

Carriers that are confined in QDs can absorb photons and thus be excited to continuum-energy states in the bulk. This process of photoexcitation is another possible mechanism for carriers to escape from QDs (process ⑤ in Fig. 1), especially under concentrated sunlight. The electron and hole photoexcitation coefficients (in units of cm^{-1}) are given by^{31,38}

$$\beta_{\text{ex},n,p}(\lambda) = 64\sqrt{2}\pi^3 \frac{1}{a} \frac{\alpha_{\text{fine}}}{\sqrt{\epsilon_r}} \frac{\Delta E_{c,v}}{hc/\lambda} \left(\frac{\hbar^2}{m_{c,v} a^2 hc/\lambda} \right)^{5/2} \times [f_{n,p}(\bar{e}_{n,p}) - f_{n,p}(\bar{e}_{n,p} + hc/\lambda)], \quad (50)$$

where $m_{c,v}$ are the electron and hole effective masses in the QD material.

Depending on the mechanism by which photons are provided (incident sunlight or spontaneous radiative recombination in QDs), we will have the following two cases:

- (i) If photons are provided by the incident sunlight, the photoexcitation rates are given as

$$g_{\text{ex},n,p} = \int_{\lambda_{\text{bulk}}}^{\lambda_{c,v}} N_S \sigma_{\text{ex},n,p} F(\lambda) d\lambda, \quad (51)$$

where $\lambda_{c,v}$ are the longest wavelengths of photons, which correspond to the excitation energies $\Delta E_{c,v} - \bar{e}_{n,p}$, and $\sigma_{\text{ex},n,p}(\lambda) = \bar{a}^3 \beta_{\text{ex},n,p}(\lambda)$ are the cross-sections (in units of cm^2) for the photoexcitation processes.

Fig. 4(a) shows the photoexcitation rates as functions of $f_n(\bar{e}_n)$ at different values of the conduction band offset ΔE_c . If $f_n = 0$ or 1, $g_{\text{ex},n} = 0$ since $\beta_{\text{ex},n}$ becomes zero as is seen from Eq. (50). Depending on the values of $\Delta E_c - \bar{e}_n$, there can be two different types for the dependence of $g_{\text{ex},n}$ on f_n . When $\Delta E_c - \bar{e}_n$ is small, $g_{\text{ex},n}$ increases with increasing f_n , approaches its maximum value, and then decreases down to zero as f_n becomes unity. With increasing ΔE_c , the value of f_n , at which $g_{\text{ex},n}$ is maximum, increases and the maximum value of $g_{\text{ex},n}$ decreases. When $\Delta E_c - \bar{e}_n$ is large, $g_{\text{ex},n}$ increases monotonically with f_n for $0 \leq f_n < 1$. The difference between the dependences is due to the following: when $\Delta E_c - \bar{e}_n$ is small and comparable with T , as $f_n(\bar{e}_n)$ approaches unity, $f_n(\bar{e}_n + hc/\lambda_c)$ becomes comparable with $f_n(\bar{e}_n)$; however, when $\Delta E_c - \bar{e}_n$ is much larger than T , $f_n(\bar{e}_n)$ is always much larger than $f_n(\bar{e}_n + hc/\lambda)$ for $0 \leq \lambda \leq \lambda_c$.

Although $\beta_{\text{ex},n} \propto \Delta E_c$ [see (50)], a smaller ΔE_c also means a longer λ_c in Eq. (51). Besides, since $\beta_{\text{ex},n}/[f_n(\bar{e}_n) - f_n(\bar{e}_n + hc/\lambda)]$ increases as $\lambda^{7/2}$ with increasing λ [see (50)], absorption of longer wavelength photons is stronger. Therefore, the maximum value of $g_{\text{ex},n}$ is achieved when $\Delta E_c = \bar{e}_n$ (i.e., when λ_c is longest), as is shown by the top curve in Fig. 4(a).

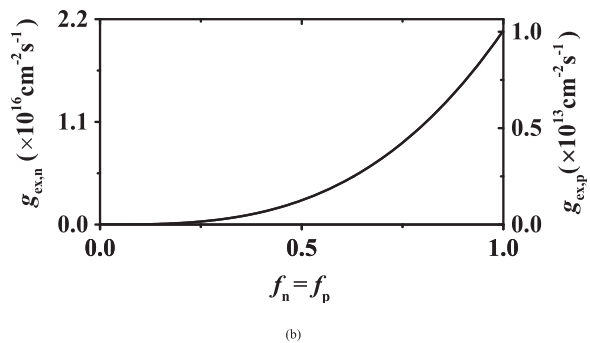
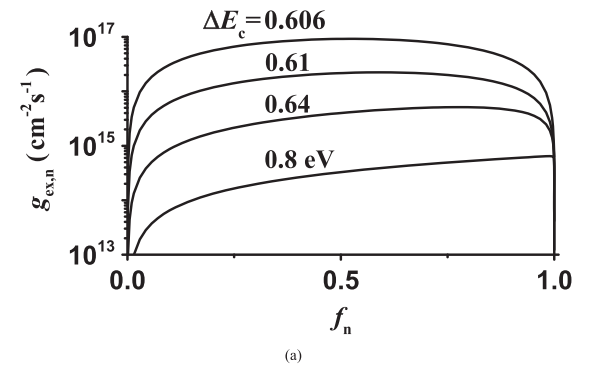


FIG. 4. Photoexcitation rate versus QD level occupancy for the cases when absorbed photons are provided by (a) the incident sunlight and (b) the spontaneous radiative recombination process in QDs. $\bar{e}_{n,p}$ are 0.606 and 0.034 eV, respectively. In (a), the curves for $g_{\text{ex},n}$ versus f_n are plotted at different conduction band offsets ΔE_c . In (b), $f_n = f_p$ is assumed and $\Delta E_{c,v}$ are 0.64 and 0.426 eV, respectively.

The dependences of $g_{\text{ex},p}$ on f_p are similar to those of $g_{\text{ex},n}$ on f_n . However, since the hole effective mass is much larger than the electron effective mass and since $\beta_{\text{ex},n,p}$ are inversely proportional to $m_{c,v}^{5/2}$, $g_{\text{ex},p}$ is several orders of magnitude smaller than $g_{\text{ex},n}$.

- (ii) If photons are provided by spontaneous radiative recombination in QDs, the expressions for $g_{\text{ex},n,p}$ are

$$g_{\text{ex},n,p} = N_S \bar{a}^3 \beta_{\text{ex},n,p}(\bar{\lambda}_0) R_{\text{QD}}. \quad (52)$$

Since the transition energy $hc/\bar{\lambda}_0 \gg T$, then $f_{n,p}(\bar{\epsilon}_{n,p}) \gg f_{n,p}(\bar{\epsilon}_{n,p} + hc/\bar{\lambda}_0)$ and $\beta_{\text{ex},n,p}(\bar{\lambda}_0) \propto f_{n,p}$. Hence, $g_{\text{ex},n,p} \propto f_{n,p}^2 f_{p,n}$ depends on both f_n and f_p . The ratio of the electron photoexcitation rate to that for holes is

$$\frac{g_{\text{ex},n}}{g_{\text{ex},p}} = \frac{\Delta E_c}{\Delta E_v} \left(\frac{m_v}{m_c} \right)^{5/2} \frac{f_n}{f_p}. \quad (53)$$

It is clear that when $f_n = f_p$, $g_{\text{ex},n}$ will be much larger than $g_{\text{ex},p}$ since $m_v \gg m_c$.

Fig. 4(b) shows the dependences of $g_{\text{ex},n,p}$ on the level occupancy for $f_n = f_p$. The maximum values (obtained when $f_n = f_p = 1$) of the photoexcitation rates are 2.1×10^{16} and $1.0 \times 10^{13} \text{ cm}^{-2} \text{ s}^{-1}$ for $g_{\text{ex},n,p}$, respectively.

To study the effects of photoexcitation, let us assume that this process is the dominant mechanism for both electron and hole escape from QDs. Then from Eqs. (6) and (7), we have

$$g_{\text{ex},n} + R_{\text{QD}} - g_{\text{QD}} = 0, \quad (54)$$

$$g_{\text{ex},p} + R_{\text{QD}} - g_{\text{QD}} = 0. \quad (55)$$

As seen from (54) and (55), the photoexcitation rates are the same for electrons and holes

$$g_{\text{ex},n} = g_{\text{ex},p}. \quad (56)$$

If the net escape of confined carriers is due to the photoexcitation process, j_{QD} becomes

$$j_{\text{QD}} = q g_{\text{ex},n} = q g_{\text{ex},p}. \quad (57)$$

For both cases discussed above, $f_{n,p}$ and thus $g_{\text{ex},n,p}$ and j_{QD} are found from the numerical solution of Eqs. (54) or (55). In contrast with the case of quasi-equilibrium, in which $f_{n,p}$ depend on the voltage [see Eq. (43)], $f_{n,p}$ for photoexcited carriers are voltage-independent. With Eq. (57), we immediately find from Eq. (41) that the open circuit voltage V_{oc} is larger than that for the conventional SC.

The maximum values of the photoexcitation rates are, however, several orders of magnitude smaller than the radiative recombination rate in QDs. For a typical InAs/GaAs QDSC considered here, the maximum values of R_{QD} (obtained when $f_n = f_p = 1$), g_{QD} (obtained when $f_n = f_p = 0$), and $g_{\text{ex},n}$ [see Fig. 4(a)] are on the order of 10^{20} , 10^{13} , and $10^{17} \text{ cm}^{-2} \text{ s}^{-1}$, respectively. The point is that, in order to satisfy Eqs. (54) and (55), $g_{\text{ex},n,p}$ cannot be larger than g_{QD} and even if $g_{\text{ex},n,p} = g_{\text{QD}}$, the maximum possible improvements in j_{sc} (on the order of 10^{-3} mA/cm^2) and V_{oc} are negligible.

This means that practically no improvements in j_{sc} and V_{oc} will be brought about by the photoexcitation processes, and, as a result, the j - V curve of the QDSC will be virtually the same as that of the SC without QDs (as shown by the solid curve in Fig. 3) if photoexcitation is the dominant mechanism for escape of confined carriers from QDs.

Hence, the fact of negligible improvement in the j - V characteristic is not only due to a very small photoexcitation coefficient but also to a large spontaneous radiative recombination rate and a very small optical absorption coefficient of QDs. To take advantages of photoexcitation, it is necessary (i) to increase the photon absorption by QDs and photoexcitation coefficients and (ii) to reduce the spontaneous radiative recombination rate R_{QD} .

To reduce R_{QD} , type-II energy band alignment between QDs and the host material can be used. In such structures, only one type of carriers is confined in QDs while the other is delocalized; as a result, electrons and holes are spatially separated so that the overlap integral of the electron and hole wave functions I_{overlap} will be reduced, thus leading to an increase in the carrier lifetime [see Eq. (33)] and a reduction in R_{QD} . There are both experimental and theoretical studies confirming that, due to such spatial separation of carriers, τ_{QD} is increased from nano- to microsecond range.^{39,40} It should be noted that I_{overlap} enters into both the photocarrier generation and radiative recombination rates. Reducing I_{overlap} will reduce R_{QD} ; however, the QD absorption rate will also be reduced to the same extent [see Eq. (29)]—hence I_{overlap} cancels in Eq. (34). Nevertheless, if concentrated sunlight is used, the reduction in absorption can be compensated and if, at the same time, R_{QD} remains reduced (in fact, while the absorption should ideally be proportional to the sunlight concentration [see Eq. (27)], R_{QD} has a maximum [see Eq. (31)]), the cell performance can be improved.

In this paper, the optical transitions only between the ground states of electrons and holes in QDs are considered. However, if the band offsets and/or QD sizes are large enough to support excited states, both $\beta_{\text{ex},n,p}$ and α_{QD} will be increased, which will result in a stronger QD photon absorption and lead to better performance. Our theoretical model can be easily revised to include the presence of excited states in QDs—see Appendix B.

IV. CONCLUSIONS

To conclude, we have developed a comprehensive theoretical model for semiconductor QD solar cells. In our model, the spatial discreteness of a QD layer is described by the Dirac delta-function and the carrier thermal exchange between QDs and the bulk i-region are characterized by the capture cross-sections. The coefficient of photon absorption (averaged over the inhomogeneously broadened ensemble of QDs) due to ground-state transitions in QDs is calculated as a function of the wavelength and QD level occupancy. The processes of carrier escape from QDs (namely, photoexcitation and tunneling) are also included in our model. The electron and hole current densities are shown to be discontinuous at the QD layer when the carrier generation and recombination rates in QDs are not equal to each other. A general

analytical expression for the total current density in the QDSC is obtained and the current-voltage characteristic is studied for several specific situations. Our theoretical model confirms that, in line with earlier experimental studies,¹³ due to much stronger (compared with the photon absorption in QDs and photoexcitation from QDs) spontaneous radiative recombination in QDs, the performance of the solar cell can hardly be improved by using QDs no matter the dominant mechanism of escape of confined carriers from QDs to the bulk—thermal escape, tunneling, or photoexcitation.

- (1) If QDs are at quasi-equilibrium with the bulk *i*-region, the expressions for the voltage-dependent $f_{n,p}$ can be obtained and the j - V characteristic can be calculated. Compared with their conventional counterparts, j_{sc} is increased while V_{oc} is decreased. Due to small absorption coefficient α_{QD} of the QD ensemble, the improvement in j_{sc} is negligible. Strong spontaneous radiative recombination in QDs leads to large degradation in V_{oc} .
- (2) In the best-case scenario, when the spontaneous radiative recombination in QDs is completely suppressed (i.e., when $f_n = f_p = 0$), both j_{sc} and the power conversion efficiency are higher than those in a conventional SC or in a non-ideal QDSC. Again, due to small value of α_{QD} , the increase in j_{sc} is negligible if only one QD layer is used. If multiple layers with QDs are used and the best-case scenario is maintained in each layer, j_{sc} will be much higher. V_{oc} in an ideal QDSC is much higher than that in a non-ideal QDSC. However, due to a logarithmic dependence on Z_L and j_{ph}^{QD-max} , V_{oc} in an ideal QDSC is practically the same as that in the conventional SC.
- (3) When photoexcitation is the dominant mechanism for escape of carriers from QDs to the bulk, $f_{n,p}$ become voltage-independent and both j_{sc} and V_{oc} are larger than those for the conventional SC. However, due to small values of α_{QD} and $\beta_{ex,n,p}$ and as well as a large value of R_{QD} , the improvements in j_{sc} and V_{oc} are negligible—the maximum possible improvement in j_{sc} is on the order of 10^{-3} mA/cm². As a result, the j - V curve is practically the same as that for the conventional SC.

ACKNOWLEDGMENTS

Y.W. acknowledges the support of the Fundamental Research Funds for the Central Universities (Grant No. 2015QNA10). L.V.A. acknowledges the U.S. Army Research Office (Grant No. W911NF-13-1-0445) for support of this work.

APPENDIX A: TUNNELING BETWEEN NEIGHBORING QD LAYERS

If multiple layers with QDs are used, tunneling can occur between neighboring QD layers. As shown in Fig. 5, carriers confined in a given QD layer can tunnel out to the neighboring layers located on its left- and/or right-hand sides. Carriers can also tunnel into a given QD layer from its neighboring layers. Hence, the rate equations for carriers confined in each QD layer should be revised to include the in- and out-tunneling terms. Denote the number of QD layers as Z_L . We have then the following rate equations:

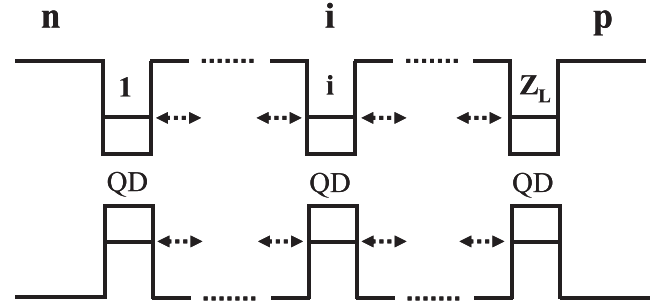


FIG. 5. Tunneling (shown by arrows) between neighboring QD layers in a QDSC. Out- and in-tunneling of carriers occur between a QD layer and its neighboring layers on the left- and/or right-hand sides.

For electrons confined in the 1st QD layer (counted from the left)

$$\sigma_n v_n N_S (1 - f_{n,1}) n - \sigma_n v_n n_1 N_S f_{n,1} + g_{QD,1} - R_{QD,1} - g_{non-th,1} + R_{in-tunn,1}^R - R_{out-tunn,1}^R = 0, \quad (A1)$$

for electrons confined in the i -th layer ($2 \leq i \leq Z_L - 1$)

$$\sigma_n v_n N_S (1 - f_{n,i}) n - \sigma_n v_n n_1 N_S f_{n,i} + g_{QD,i} - R_{QD,i} - g_{non-th,i} + R_{in-tunn,i}^R - R_{out-tunn,i}^R + R_{in-tunn,i}^L - R_{out-tunn,i}^L = 0, \quad (A2)$$

and for electrons confined in the Z_L -th layer

$$\sigma_n v_n N_S (1 - f_{n,Z_L}) n - \sigma_n v_n n_1 N_S f_{n,Z_L} + g_{QD,Z_L} - R_{QD,Z_L} - g_{non-th,Z_L} + R_{in-tunn,Z_L}^L - R_{out-tunn,Z_L}^L = 0, \quad (A3)$$

where $R_{in-tunn}$ and $R_{out-tunn}$ are the in- and out-tunneling rates of confined electrons, respectively. Superscripts “L” and “R” denote the tunneling exchange rates between a given QD layer and its left- and right-hand side neighbor, respectively. For the sake of simplicity, the Dirac delta-function $\delta(x-x_i)$ (x_i is the location of the i -th layer) is dropped in each term in Eqs. (A1)–(A3). Similar equations can also be written for the confined holes.

As is clear from Eqs. (A1)–(A3), the following relations hold:

$$R_{in-tunn,i}^R = R_{out-tunn,i+1}^L, \quad (A4)$$

$$R_{out-tunn,i}^R = R_{in-tunn,i+1}^L. \quad (A5)$$

Similarly to Eq. (12), we will have the following boundary conditions at the layers:

$$j_n|_{x=x_1+0} - j_n|_{x=x_1-0} = q(R_{QD,1} - g_{QD,1} + R_{out-tunn,1}^R - R_{in-tunn,1}^R), \quad (A6)$$

$$j_n|_{x=x_i+0} - j_n|_{x=x_i-0} = q(R_{QD,i} - g_{QD,i} + R_{out-tunn,i}^R - R_{in-tunn,i}^R + R_{out-tunn,i}^L - R_{in-tunn,i}^L), \quad (A7)$$

$$j_n|_{x=x_{Z_L}+0} - j_n|_{x=x_{Z_L}-0} = q(R_{QD,Z_L} - g_{QD,Z_L} + R_{out-tunn,Z_L}^L - R_{in-tunn,Z_L}^L). \quad (A8)$$

Integrating Eq. (10) and using Eqs. (A4)–(A8), we obtain for the total current density

$$j = j_p(0) + q \int (g_{bulk} - R_{bulk}) dx + q \sum_{i=1}^{Z_L} (g_{QD,i} - R_{QD,i}) + j_n(b). \quad (A9)$$

Although tunneling affects the j - V characteristic through $f_{n,p}$, Eq. (A9) has exactly the same form as that for the case of no tunneling between neighboring QD layers [Eq. (20)].

APPENDIX B: QDSC IN THE PRESENCE OF EXCITED STATES IN QDS

Consider now a situation when excited states (ES) are present in QDs. Let us assume that the carrier exchange between QDs and the i -region is ES-mediated only. At the steady-state, we can write the following rate equations for electrons:

For excited state electrons

$$0 = \sigma_n v_n N_S (1 - f_{n2}) n(x) \delta(x - x_0) - \sigma_n v_n N_S f_{n2} n_2 \delta(x - x_0) + g_{QD2} \delta(x - x_0) - R_{QD2} \delta(x - x_0) + g_{QD,1 \rightarrow 2} \delta(x - x_0) + N_S \frac{f_{n1}(1 - f_{n2})}{\tau_{12}} \delta(x - x_0) - N_S \frac{f_{n2}(1 - f_{n1})}{\tau_{21}} \delta(x - x_0), \quad (B1)$$

for ground-state (GS) electrons

$$0 = g_{QD1} \delta(x - x_0) - R_{QD1} \delta(x - x_0) - g_{QD,1 \rightarrow 2} \delta(x - x_0) - N_S \frac{f_{n1}(1 - f_{n2})}{\tau_{12}} \delta(x - x_0) + N_S \frac{f_{n2}(1 - f_{n1})}{\tau_{21}} \delta(x - x_0), \quad (B2)$$

and for free electrons in the i -region

$$0 = \sigma_n v_n n_2 N_S f_{n2} \delta(x - x_0) - \sigma_n v_n N_S (1 - f_{n2}) n(x) \delta(x - x_0) - R_{bulk} + g_{bulk} + \frac{1}{q} \frac{dj_n}{dx}. \quad (B3)$$

Similar equations can also be written for holes. “1” and “2” in the subscripts represent the GS and ES, respectively. $g_{QD,1 \rightarrow 2}$ is the transition rate of carriers from ground to excited state due to the absorption of photons. τ_{12} is the ground-to-excited state transition time, and τ_{21} is the excited-to-ground state relaxation time in a QD.

Integrating Eq. (B3) over an infinitesimal interval around x_0 and using Eqs. (B1) and (B2), we have

$$\begin{aligned} & \frac{1}{q} (j_n|_{x=x_0+0} - j_n|_{x=x_0-0}) \\ &= \sigma_n v_n N_S (1 - f_{n2}) \frac{n(x_0 + 0) + n(x_0 - 0)}{2} - \sigma_n v_n n_2 N_S f_{n2} \\ &= (R_{QD1} + R_{QD2}) - (g_{QD1} + g_{QD2}). \end{aligned} \quad (B4)$$

The expression for the total current density will then be

$$j = q \int (g_{bulk} - R_{bulk}) dx + q(g_{QD1} + g_{QD2}) - q(R_{QD1} + R_{QD2}), \quad (B5)$$

which is similar to Eq. (16). The total current density is again due to the net photocarrier generation by the entire device.

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