AIMdyn, Inc.
Engineering Consulting & Software Development

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Contract: **N00014-18-P-2004** **Date: 04/30/2018**
Project Title: Analysis and Prediction of Sea Ice Evolution using Koopman Mode Decomposition Techniques
Subject: Monthly Progress Report
Period of Performance: April 4, 2018 – April 30, 2018

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AIMdyn, Inc. respectfully submits Progress Report 1 for contract N00014-18-P-2004**.** Please direct any Technical questions on this report to the undersigned.
V/r

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BUSINESS STATUS REPORT

(1) Resource Status:
Resourced to plan.

(2) Contributions by AIMdyn Inc:
See technical report.

(3) Resource status VS Original schedule:
On plan to original scope of work. Effort is fixed priced.

PROGRAM FINANCIAL STATUS

Work Breakdown	Cumulative to Date	At Completion
CLIN#0001	$18,687	$112,125

Structure or Task Element	Planned Expend	Actual Expend	% Budget Compl	At Compl	Latest Revised Estimate	Remark Remark
CLIN#0001	$18,687	$18,687	16.7%	100 %	N/A	N/A

Subtotal: $18,687
Management Reserve: N/A
Or Unallocated Resources: N/A
TOTAL: $18,687

TECHNICAL STATUS REPORT

Abstract

The program goal is analysis of sea ice dynamical behavior using Koopman Mode Decomposition (KMD) techniques. The work in the program's first month consisted of improvements to data processing code, inclusion of additional arctic sea ice concentration data, and identification of additional data sets of interest. We used recently developed improved KMD algorithms and windowing over 5, 10, 15, 20, and 30 year time periods to capture trends with shorter time scales that are less apparent in the full 39 year data set. The expected annual variation in sea ice concentration was clearly visible in the resulting Koopman spectra as well as a potentially novel slowly decaying mode that indicates a possible change in dynamical behavior beginning in the 1990's. Future work will include analysis of southern hemisphere sea ice concentration data for comparison, analysis of the spatial modes corresponding to eigenvalues of interest, and initial analysis of sea ice thickness data.

Accomplishments

Summary

Initial work was done to build on the basic analysis presented in the program proposal, including use of the newest algorithms for Koopman Mode Decomposition (KMD) and varying the time window of the KMD processing to better show trends over periods shorter than entire year range of the data set. The resulting Koopman eigenvalues and mode spectra included features that were both expected (annual cycle) and novel (consistent existence of slow-decay or growing mode from the 1990's onward).

Introduction

Work in the first month consisted of updating the analysis work done for the proposal by using the latest KMD algorithms and expanding the analysis to better capture behavior over time scales shorter than the 39 year period of the sea ice concentration data set.

Methods

The analysis method was based on an improved version of the Matlab code used for sea ice concentration and the most recent versions of the various KMD algorithm implementations developed by AIMdyn. The data set used was the monthly arctic sea ice concentration from the NSIDC Sea Ice Index. A wider year range of data was used in the present work than was available for the analysis in the proposal, because the data for all of 2017 is now available, and we interpolated over the three missing months of data in the 1980's to allow use of all data back to 1979, giving 39 full years of data (1979 to 2017). In brief, the analysis method was to convert the image data files showing the average monthly concentration to numerical arrays, removing the pixels corresponding to land areas, and reshaping the remaining sea pixels into a 1-D array for each month. The arrays for each month were then combined into a 2-D data matrix for performing KMD analysis using multiple algorithms (Arnoldi, DMD RRR, iDMD, Arnoldi Vandermonde Cauchy RRD, and Arnoldi Vandermonde LDU) and the resulting Koopman eigenvalues and mode spectra were examined.

To capture trends in the data that vary over relatively short time scales, the above analysis was also performed on windowed data sets. The windowing consisted of performing KMD on subsets of the sea ice concentration data covering time periods of 5, 10, 15, 20, and 30 years (e.g. the five year windows consisted of 1979-1983, 1980-1984, . . . , 2013-2017). Again the Koopman eigenvalues and mode spectra for each time period were examined.

Additional work involved identifying other promising data sets provided by NSIDC and other sources to examine. The criteria for appropriate data sets were the availability of data with at least relatively uniform temporal and spatial sampling, and measured over a time scale of approximately a decade or longer.

Results and Discussion

A general result observed upon performing KMD processing on the sea ice concentration data using the KMD algorithms listed above was that all of the algorithms gave identical or effectively identical outputs. This suggests that the sea ice concentration data dynamical behavior is "well behaved" in some sense, presumably that the resulting condition number is sufficiently small that any of the various approximations of the Koopman decomposition are valid in this case. This is a desirable result, as it supports the conclusion that the KMD results obtained here are physically meaningful and not numerical artifacts.

Specific observations from the windowed processing were:

1. The identification based on eigenvalue plots of the existence of growing or slowly decaying modes appear only intermittently before the 1990's and are mostly consistently present from the 1990's onward. These were most visible in the five year window case from also visible in the ten year case (see the eigenvalues for the five year case in Figure 1).

2. The existence of a consistent annual (i.e. period one year) signal as expected, and generally consistent spectral results from year to year (see the spectrograph for the five year case in Figure 2).

The most promising additional data set to examine, based on the criteria defined above, is the sea ice thickness data from CryoSat-2 satellite measurements. This data covers a shorter time period than the Sea Ice Index data (from October 2010 to the present), but it is expected to be complementary as another data set to compare with the trends observed in the post 1990's sea ice concentration data.

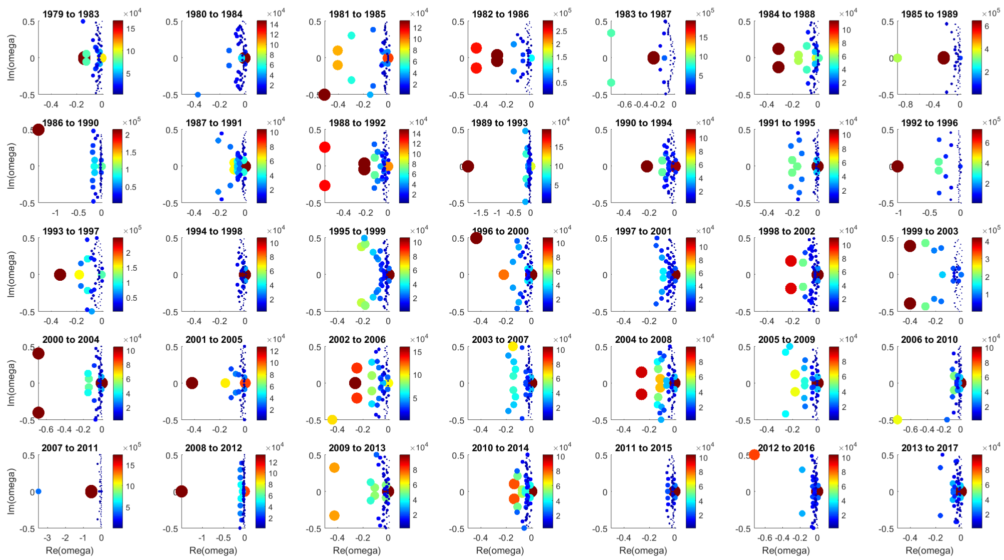

Figure 1: Koopman eigenvalues for 5 year time windows beginning in 1979. Each circle corresponds to the complex eigenvalue associated with a Koopman mode, where the size and color of the circle are both representations of the L2 norm of the corresponding mode, the position along the horizontal axis shows the growth or decay constant of the mode (e.g. the more negative the value the faster the mode decays), and the position along the vertical axis shows the oscillatory frequency of the mode. The time scale is in months, so the two circles generally visible at approximately ± 0.08 on the vertical axis are the expected annual variation of the sea ice concentration. The novel result apparent is the mostly consistent presence of a large norm mode near the origin beginning in the 1990-1994 window. This slowly decaying or growing mode suggests a change in sea ice dynamics beginning in the 1990's.

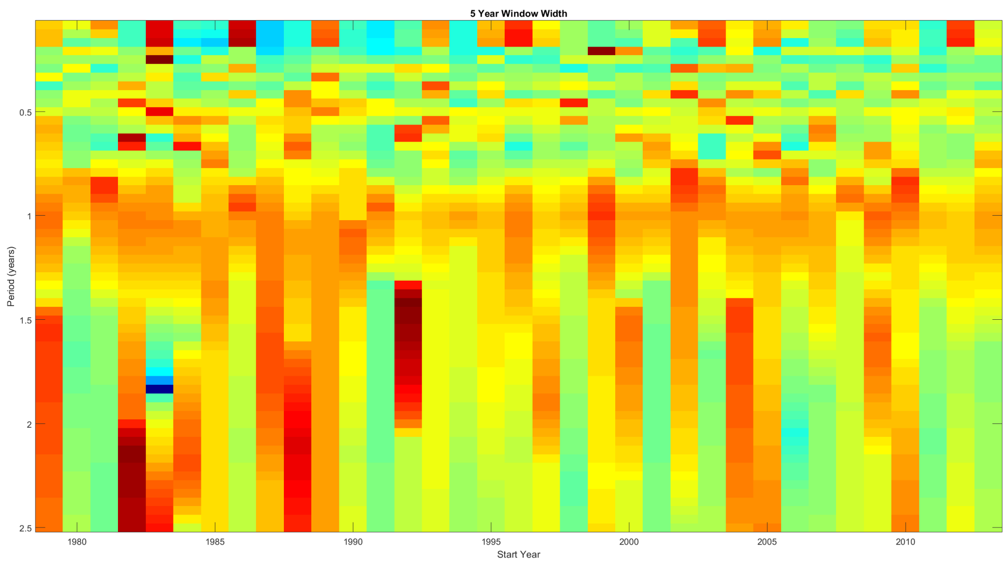

Figure 2: Spectrograph of the Koopman mode spectra for 5 year time windows. The period (in years) is shown here instead of frequency, because the multi-year dynamics are of greater interest than month-scale behavior. The expected annual cyclical behavior is clearly visible in each time window, and harmonics at two years are also apparent in many cases.

Conclusions

The results obtained in the first month are consistent with expectations and have shown potentially novel new results, giving us confidence in the suitability of KMD for the study of sea ice dynamics. These results have pointed the way to future work, specifically:

1. Repeat the previously described analysis for antarctic sea ice concentration data, to compare with the results obtained for the arctic.

2. Examine the spatial modes corresponding to modes of interest identified in the eigenvalue plots to identify the geographic significance of the growth/decay behavior observed.

3. Begin analysis of the sea ice thickness data set and any other data sets of interest.

Personnel Supported

Dr. Maria Fonoberova, Dr. Igor Mezic, Dr. James Hogg