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Subj: Grant: N00014-15-1-2233
"APL-UW Deep Water Propagation 2015-2017: Philippine Sea Data Analysis"

Encl: (1) Final Performance/Technical Report with SF298

Enclosed please find the Final Performance/Technical Report for the project titled "APL-UW Deep Water Propagation 2015-2017: Philippine Sea Data Analysis."

Cc: ONR Seattle - Robert Rice
DTIC
Office of Sponsored Programs – UW Closeout
APL-UW Grants and Contracts Closeout Coordinator

**APL-UW Deep Water Propagation 2015-2017:
Philippine Sea Data Analysis**

Final Report

Award N00014-15-1-2233

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Abstract

This is the final report for Award N00014-15-1-2233. Analysis of low-frequency long-range acoustic signals acquired during the Philippine Sea 2010 experiment (Philsea10) was concluded. The pulse spreads for the 200 Hz and 300 Hz signals were found to be negligible. This was supported by time-independent Monte Carlo parabolic equation simulations. The autospectrum of normalized intensity had an excellent match to that of a time-dependent Monte Carlo simulation, but the timescales of fades and glints could not be replicated. The PhilSea10 datasets had considerably more short fades and glints with timescales less than several minutes. The North Pacific ambient noise collection program terminated, and trends for the final seven systems were estimated. Trends for all systems were compared for the 19-year duration of the experiment. Systems in the northeast Pacific demonstrated a decrease in ambient noise; systems along the Aleutian chain have either no significant trend or a slight increasing trend; systems in the central Pacific Ocean provided contradictory characterizations, having both increasing and decreasing trends. The DOS computer at Kauai died, and was replaced by a Linux system. At the end of the grant, it was determined that the Kauai cable had suffered a break in the shallow near-shore region. Additional contractual authority for some of this research was provided by Award Numbers N00014-14-P-1098 and N00014-17-P-7015.

Objectives

Complete the analysis of low-frequency long-range acoustic signals acquired during the Philippine Sea 2010 experiment (PhilSea10.) Support the North Pacific Ambient Noise Laboratory (NPANL) for studies of low-frequency ambient noise in the northern Pacific Ocean. Maintain the infrastructure supporting the Kauai electro-acoustic projector.

Summary of Results

Philippine Sea Analysis Primary activities on this grant involved analyzing a combination of at-sea measurements, theoretical modeling and computational simulations to understand statistical features of the 510 km low-frequency acoustic signals transmitted by APL-UW during the Philippine Sea 2010 experiment.

North Pacific Ambient Noise Laboratory Primary activities on this grant involved

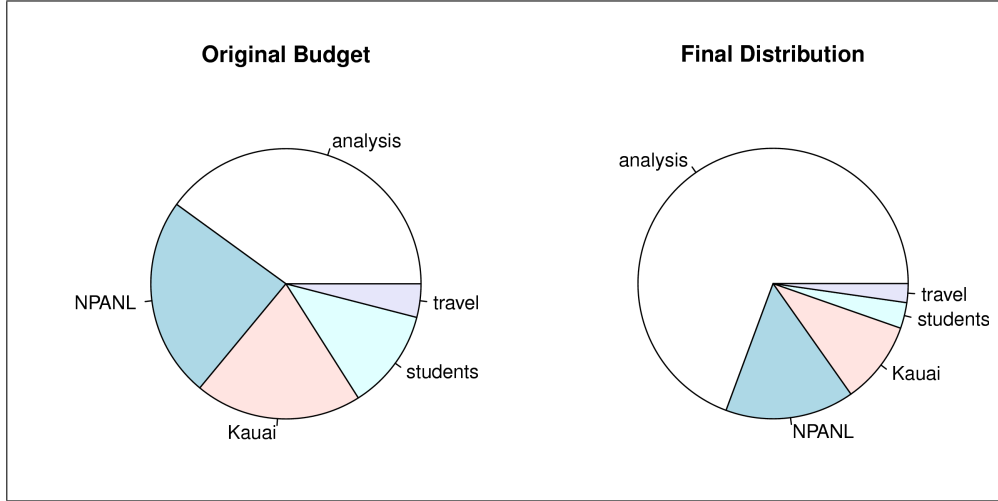


Figure 1: Original grant funding breakdown.

curating all the ambient noise data acquired via the North Pacific Ambient Noise Laboratory programs in preparation for long-term storage, analyzing the datasets for long-term trends, and publishing the findings. Secondary activities involved collaboration with researchers on topics involving oceanic ambient noise.

Kauai Source Maintenance This task initially involved occasional monitoring of the APL-UW system at Kauai. At the end of this grant, hardware anomalies prompted significant investigations.

The original budget breakdown for this grant is shown in Fig. 1. The primary objective was to spend most of the effort analyzing Philippine Sea 2010 data. This was our first major experience working with the data from the Scripps “distributed vertical line array” (DVLA), which itself was an instrument embodying some new technologies. NPANL operational support was expected to be negligible; the principal portion of the NPANL budget was targeted for preparing the datasets for long-term storage, analyzing the datasets, and publishing findings. We envisioned using a combination of undergraduate and graduate students to assist. Kauai system maintenance was expected to involve only minimal travel to Kauai, as most diagnostics could be accomplished from Seattle. Since all engineers previously associated with this system have left APL-UW, one month was budgeted to transition the system to new engineers in APL-UW’s Ocean Engineering (APL-OE) Dept.

The final cost breakdown is also shown in Fig. 1. PhilSea10 analysis required far more

time than expected. This task included some additional analysis efforts related to long-range low-frequency sound (but not directly related to the Philippine Sea), as noted below. The additional effort on this task was offset by reductions in NPANL and Kauai support tasks. The NPANL task of curating, analyzing and publishing the 19+ year archive did not require the expected effort, and secondary ambient noise collaborations were minimal because the ambient noise research community was largely inactive. The Kauai system was not fully transitioned to APL-OE because of a lack of personnel availability. Student support was used for analysis of PhilSea10 data, but, due to classification concerns, students were not used with NPANL activities. This decreased the expected cost of student support.

Almost all travel costs supported NPANL (travel to Dam Neck, VA, and the Pentagon for briefings) and Kauai system maintenance (PMRF on Kauai.) Information technology support costs (e.g. administering our workstations and our computer cluster) were handled by the PI because APL-IT support staff do not support compute clusters in any way, nor hardware upgrades on grant workstations.

Journal papers written by the PI (as first author) under this grant are appended.

Detailed Results

Philippine Sea Analysis

The primary grant task involved analysis of data collected on two experiments in the Philippine Sea (PhilSea09 in 2009 and PhilSea10 in 2010.)

The PhilSea09 experiment provided the data for the doctoral research of Dr. Andrew White (APL-UW). The PI conducted additional analysis of the Philsea09 results under this grant.

The PhilSea10 experiment involved transmissions over 54+h over a range of roughly 500 km. APL-UW transmitted the signals: the acoustic receptions (and associated receiver details) are provided by Worcester (Scripps Institution of Oceanography), and the environmental measurements at the receiver by Colosi (Naval Postgraduate School). The transmitter was the the “multiport” transducer which supported two simultaneous signals with carriers at 200 and 300 Hz. Each multiport signal had its own m-sequence law, allowing complete separation of the two signals (hereafter denoted “red” and “violet”, respectively) in code space.

As an ONR Postdoctoral Fellow at APL-UW, Dr. Andrew White conducted corresponding Monte Carlo parabolic equation simulations under ONR grant N00014-14-1-0218, which were reduced and analyzed under this grant for comparison to PhilSea10 observations and theoretical predictions.

- The PI compared predictions of log-amplitude variance from the Munk-Zachariasen theory[Mun76] to observed PhilSea09 measurements[Whi13] and associated Monte Carlo parabolic equation (MCPE) simulations[Whi13]. The latter two statistics were published by White *et al.*[Whi13]. The PI and undergraduate student Mumm also calculated the log-amplitude variance as predicted by CAFI, yielding a three-way comparison using controlled environmental inputs. This resulted in a paper published in the *J. Acoust. Soc. Am.*[3]. [Appended.]
- The PI computed the spread of pulses observed during PhilSea10 for both the “red” and “violet” signals radiated simultaneously by the APL-UW multiport transmitter. Roughly 9000 pulses were transmitted for both signals over 54 hrs. The PI compared the observed pulse spread against spread predictions obtained from the program “Computations of Acoustic Fluctuations from Internal Waves” (CAFI [CAFI].) Additionally, Dr. Andrew White conducted MCPE simulations using NSPE 5.4 of the PhilSea10 scenario at 200 and 300 Hz, and undergraduate student Mumm extracted pulses from the simulated timefronts. This resulted in a dataset of 240 independent realizations for either 200 or 300 Hz for all corresponding sections of the timefront. The PI compiled these results and published the findings in the *J. Acoust. Soc. Am.*[5]. [Appended.]
- The PI developed a new MCPE code that integrated the Henyey-Reynolds internal wave engine[Hen13] and RAM version 1.5 (upgraded to 1.6.)[RAM] into a multiprocessing application targeted for the APL-UW **bluewater** compute cluster. The RAM code, which solves the parabolic equation at a single frequency, was wrapped in “single frequency” C++ class. A “multi-frequency” C++ class was developed that distributed single-frequency calculations to all available cores on a compute host using **openMP**. Each multi-frequency computation is one simulation. A master/servant architecture was developed using **openMPI** to distribute multi-frequency computations to all cluster nodes in an efficient manner. Multi-frequency computations could be either time-independent or time-evolving. This application was the workhorse for all time-evolving simulations performed under this grant.
- An effort was made to reduce the computation time of time-evolving Monte Carlo PE simulations. The code that interfaces the output of the internal wave engine to the single-frequency PE solver adjusts the x, y sampling grid from the internal wave

engine schema to the schema required by the parabolic equation. The PI designed a Fourier-based 2D resampler that runs much faster than the previous algorithm (2-D interpolation.) For a typical PhilSea10 simulation grid, the speed-up went from about 6 hours to about 20 minutes.

- Intensity fluctuations can degrade the performance of long-range signaling systems. Fluctuations below the mean are called “fades” and those above the mean are called “glints”. The PhilSea10 experiment was unique in that the signal interrogation occurred on a dense, regular time increment, thus permitting measurement of the time-scales of fades and glints. The PI analyzed the red and violet pulses extracted from the PhilSea10 data to compile actual fade and glint statistics. Since there are no theoretical expressions for these statistics, the PI adopted the approach of comparing the observed statistics against those generated using a time-evolving MCPE. These findings have been written up as a paper for the *J. Acoust. Soc. Am.* and submitted to co-authors for review.[8] [Draft appended.]

Secondary efforts related to long-range acoustic propagation were:

- The PI worked with Dr. Tarun Chandrayadula of the India Institute of Technology Madras, and Prof Kathleen Wage of George Mason University on analysis and interpretation of long range low-frequency data collected during the 2004 LOAPEX experiment. The effort involves acoustic tomography using the low-frequency (62-75 Hz) APL-UW signal and normal modes, over ranges from 250 to 500 km, with support from APL-UW seaglider environmental measurements. Multiple extensive revisions of the manuscript occurred. This manuscript is currently in co-author review[2].
- I collaborated with Dr. Fannie Shabangu, University of Pretoria, S. Africa, and Dr. Kathleen Stafford, APL-UW, to calculate detection ranges for baleen whales off the southwest coast of S. Africa. This modeling augmented estimation of whale stocks by Shabangu, Stafford and collaborators based on acoustic records of low-frequency (15-20 Hz) baleen vocalizations collected on two multi-season moorings in the Benguela current.

The quantity of interest was the probability of detection at a receiver of a call originating from a marine mammal located at all points near the receiver. An alternate schema of multiple ranges along multiple bearings was used for “all points near the receiver”, with the result rendered as a polar plot.

Modeling made use of reciprocity: the field at the receiver from a field point is the same as that at the field point if radiated from the receiver position. Thus, the calculations

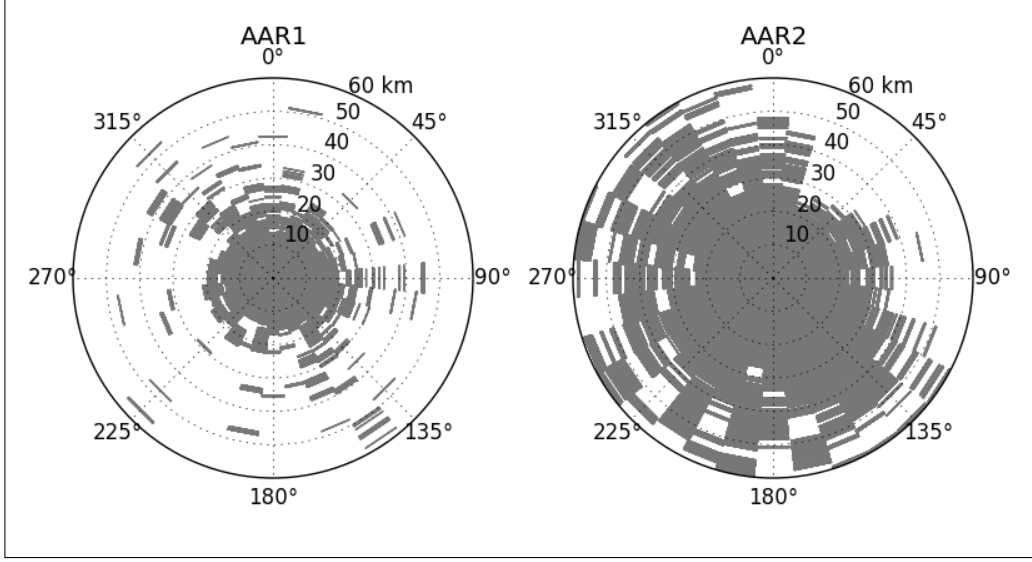


Figure 2: Example detection plot for Spring, mooring AAR1 (left) and mooring AAR2 (right). A “detection” is defined by an $\text{SNR} > 0$ and is indicated by a grey patch. Modeling done for Shabangu *et al.*

considered a substitute field radiated out from the receiver position. The field was represented by $N \times 2\text{D}$ calculations. Computation of detection fields used BELLHOP for acoustic propagation modeling, ambient noise at the receiver (supplied by Stafford), baleen vocalization source level drawn from the literature, and a simple detection statistic based on SNR.

Each BELLHOP run required bathymetry and sound speed profiles along a given radial. A maximum range of 60 km was deemed sufficient based on preliminary modeling. Sound speed profiles were derived from the 2013 version of World Ocean Atlas (WOA2013)[WOA13]. WOA2013 provides temperature and salinity profiles on a regular grid ($1^\circ \times 1^\circ$) over most of the world’s oceans. These profiles were converted to sound speed profiles using TEOS-10[TEOS10]. The bathymetry was extracted from the Smith-Sandwell database[Smi11], version 11.1. This database contains ocean depths at grid points on a world-wide grid. The resolution ($1' \times 1'$) is sufficient to detail seamounts and escarpments.

The PI automated these calculations using python, bash scripting, R, octave and C++. (BELLHOP was coded in Fortran.) Plots of detection fields were made for two moorings for every season. These figures were included in the paper written by Shanbangu *et al.* and submitted to the *J. Mar. Systems* [7]. An example figure is shown in Fig. 2.

- The PI began collaboration with Prof. Mike Ritzwoller and doctoral student Mengyu Wu, University of Colorado (CU). They are interested in investigating the predictability of signals at the DVLA in PhilSea10 using a variety of ocean models and a variety of acoustic propagation approaches, including modes and rays. The PI supplied raw PhilSea10 DVLA multiport receptions, typical APL-UW transmitter drive signal waveforms, full documentation and associated notes, and advice to CU.
- The PI continued collaboration with Dr. Andrew White, BAE Systems, Inc., involving the predictability of APL-UW signals at the DVLA during PhilSea10 using a variety of ocean models.

North Pacific Ambient Noise Laboratory

- All the classified ATOC/NPAL data were copied from their original optical media (CDs and DVDs) onto new DVDs. The shelf life of standard commercial grade organic optical media is about 3 - 5 years. The NPANL archive — which of course uses COTS optical media — dates back to 1994. All the data were recovered and transferred to new classified (SECRET) media, which were properly marked and transferred to APL Security. All original media (1231 CDs and DVDs) were released to APL Security and scheduled for destruction. In addition, 136 copies of original unclassified media were also eliminated in order to reduce unnecessary data storage.
- The PI updated all system calibrations and corrections and re-analyzed all NPANL datasets for long-term trends. Findings from seven of these datasets had never been published before. The findings for all NPANL receivers for the duration of the experiment were compiled into a classified manuscript. Permission was sought and obtained from N2N6F24 to publish the classified manuscript in an appropriate restricted journal. The paper was submitted to and published by the *J. Underwater Acoust.*[6] A summary figure of the observed trends on all systems is shown in Fig. 3.
- The PI briefed both CUS and N2N6F24 on the NPANL findings.
- The PI conferred with CUS to determine if there was a way forward to continue measurements of ambient noise in the N. Pacific. A restrictive approach was identified. This will be pursued in the follow-on grant.
- The PI collaborated with Ms. Lisa Pflug of NAVOCEANO for a comparison of in-situ North Pacific ambient noise levels and levels predicted by the latest NAVOCEANO noise model.

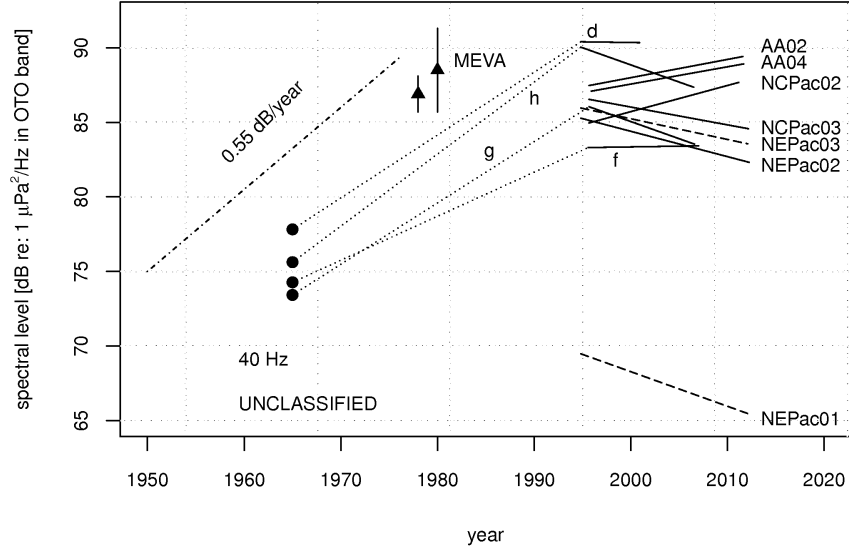


Figure 3: Trends for all datasets in the NPANL experiment.

- The PI worked with Dr. Melania Guerra, APL-UW Postdoctoral Fellow (K. Stafford, advisor), on the interpretation of ambient noise measured on AURAL moorings in the Arctic. Issues involving electrical noise and noise induced by signal processing artifacts were identified and resolved. This work resulted in a presentation at the 172nd Meeting of the Acoustical Society of America, Honolulu, HI, November 2016. [D].
- The PI worked with Ms. Dara Farrell, UW Mechanical Engineering doctoral student (advisor, P. Dahl) on the interpretation of ambient noise measured during TREX. This effort resulted in a paper [Far17].

Kauai Source Maintenance

The Kauai source was interrogated in November 2014 and found to be healthy. In March 2015, the remote DOS computer was determined to be dead. Two Linux systems, based on the successful design deployed in the Philippine Sea experiments, were prepared and tested at APL. The DOS system was replaced with one of the Linux systems (the other is an in-house back-up) in later 2015. Mr. Ben Brand from APL-OE accompanied, with the intent to transition the system to Brand and APL-OE.

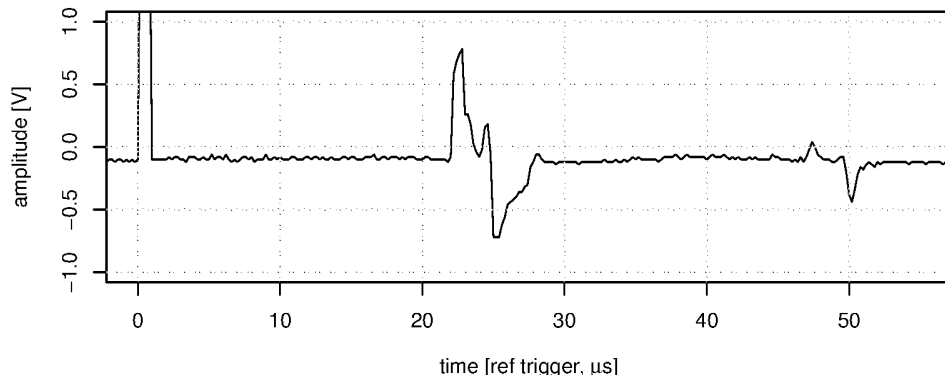


Figure 4: Reflectometer trace. Measured at the cable lugs at the T/R board, in APL-UW rack in the Calibration Lab, using the APL-UW reflectometer pulser and Fluke scopemeter.

The Kauai source was monitored for months and appeared to be in good health. However, in early 2017, the system diagnostics indicated a significant electrical fault. We examined the diagnostic data available and devised some follow-on measurements. However, it was deemed too risky to the system to make these measurements remotely from Seattle.

The PI went to Kauai in December 2017 to make further on-site measurements. APL-OE support personnel were not available. The electrical problem was not in the APL-UW rack, but further downstream. There seemed to be a significant electrical short. With the help of PMRF engineer Mr. Shayan Sohbatazadehlanbar, a reflectometer was used to check the cable. There appeared to be a cable short at a distance of about 5 km from the APL-UW equipment. (See Fig. 4.) As the cable runs underground from the Calibration Lab (where the APL-UW rack is located) for approximately 2000 m, this cable fault is located about 1 mile offshore. The PI conferred with Mr. Mike Dick, PMRF Range Instrumentation Dept. head regarding options.

Further resolution of this situation will continue under the PI's follow-on grant.

Auxiliary Activities

- IQOE Working Group on Standardization.
- Technical paper reviewer for the *J. Acoustic. Soc. Am.* (2), and *Science Rep. (Nature)*.

Bibliography

- [CAFI] <http://oalib.hlsresearch.com/Other/#IW>, last accessed 2013.08.20.
- [Hen13] FS Henyey and SA Reynolds, “A numerical simulator of ocean internal waves for long range acoustics”, Technical Memo TM 1-13, Applied Physics Laboratory, University of Washington, 2013.
- [Mun76] WH Munk and F Zachariasen, “Sound propagation through a fluctuating stratified ocean: Theory and observation”, *J. Acoust. Soc. Am.* **59**, pp 818–838 (1976).
- [Whi13] AW White, RK Andrew, JA Mercer, PF Worcester, MA Dzieciuch, and JA Colosi, “Wavefront intensity statistics for 284-Hz broadband transmissions to 107-km range in the Philippine Sea: observations and modeling I”, *J. Acoust. Soc. Am.* **134**(4), pp 3347-3358, (2013).
- [RAM] This effort used a modified version of RAM version 1.6. RAM is described in *User’s Guide to RAM Versions 1.0 and 1.0p*, written by Michael D. Collins. The original code, written by Michael D. Collins, was downloaded years ago from the NRL website, but the original website is no longer active. The code and the user’s guide can be found at <http://oalib.hlsresearch.com/PE/RAM>: last accessed 2016.12.26
- [WOA13] World Ocean Atlas products provide temperature and salinity. The latest versions can be obtained from <http://www.nodc.noaa.gov/OC5/woa13>, although the 2013 version appears the most recent. I converted these files to netCDF.
- [TEOS10] <http://www.teos-10.org>.
- [Smi11] I used the Smith-Sandwell bathymetry, version 11.1. The latest version can be obtained from http://topex.ucsd.edu/marine_topo. However, the format is not very friendly. I have converted earlier versions (specifically 11.1) into netCDF format.
- [Far17] Dara M. Farrell, David R. Dall’Osto, Peter H. Dahl, “The Background Noise Environment During the 2013 Target and REverberation eXperiment”, *IEEE Journal of Oceanic Engineering*, **42**(4), 1088-1093 (2017).

Publications and Manuscripts in Preparation

The following is a list of publications or attempted publications from this grant involving the Principal Investigator. Published first-author papers are appended and noted as such (when possible.)

- [1] Andrew A. Ganse, **RK Andrew**, Frank S. Henyey, James A. Mercer Peter F. Worcester, Matthew A. Dzieciuch, “Deep fades during the low frequency Philippine Sea experiment without apparent destructive interference”, [submitted to co-authors, 2015.]
- [2] TK Chandrayadula, KE Wage, PF Worcester, MA Dzieciuch, JA Mercer and **RK Andrew**, “Mode tomography using signals measured during the Long-range Ocean Acoustic Propagation EXperiment (LOAPEX)”, [submitted to co-authors]
- [3] **RK Andrew**, Andrew W. White, James A. Mercer, Peter F. Worcester, Matthew A. Dzieciuch and John A. Colosi, “A test of deep water Rytov theory at 284 Hz and 107 km in the Philippine Sea”, *J. Acoust. Soc. Am.* **138**(4), pp 2015-2023 (2015). [Published, refereed.][Appended.]
- [4] AW White, FS Henyey, **RK Andrew**, JA Mercer, PF Worcester, MA Dzieciuch, JA Colosi, “Internal tides and deep diel fades in acoustic intensity”, *J. Acoust. Soc. Am.*, **140**(5), pp 3952-3962 (2016). [Published, refereed.]
- [5] **RK Andrew**, A. Ganse, AW White, JA Mercer, MA Dzieciuch, PF Worcester and JA Colosi, “Low-frequency Pulse Propagation over 510 km in the Philippine Sea: A Comparison of Observed and Theoretical Pulse Spreading”, *J. Acoust. Soc. Am.* **140**, pp 216-228, (2016), [published, refereed][Appended.]
- [6] **RK Andrew**, BM Howe and JA Mercer, “Decadal Trends in Low-Frequency Ambient Ocean Noise for Seven Sites in the North Pacific Ocean”, *J. Underwater Acoust.* **66**(4), pp 1-27, October 2016. [published, refereed] *Classified, not appended.*
- [7] Fannie W. Shabangu, Ken P. Findlay, Dawit Yemane, Kathleen M. Stafford, Marcel van den Berg, Bradley Blows, **Rex K. Andrew**, “Seasonal occurrence and diel calling behaviour of Antarctic blue whales and fin whales in relation to environmental conditions off the west coast of South Africa” [submitted to *J. Marine Sys.*].
- [8] **RK Andrew**, A. Ganse, AW White, JA Mercer, MA Dzieciuch, PF Worcester and JA Colosi, “Timescales of Low-Frequency Flat Fading over 510 km in the Philippine Sea”, [submitted to the *J. Acoust. Soc. Am.*][Appended.]

Presentations

- [A] **RK Andrew**, “Results of Long-Term Low-Frequency Ambient Noise Measurements in the North Pacific”, presented to N2N6F24, the Pentagon, Arlington, VA, 23 September 2016.
- [B] **RK Andrew**, “Results of Long-Term Low-Frequency Ambient Noise Measurements in the North Pacific”, presented to CUS, Dam Neck, VA, 19 September 2016.
- [C] **RK Andrew**, A Ganse, AW White, “Pulse Spreads in the PhilSea10 Experiment” , Acoustics Dept. Applied Physics Laboratory, Spring 2016.
- [D] M Guerra, KM Stafford, **RK Andrew**, “Evaluating underwater ambient noise levels across a changing Arctic environment”, 172nd Meeting of the Acoustical Society of America, Honolulu, HI, November 2016.
- [E] **RK Andrew**, “Low-frequency ambient noise trends over (almost) 2 decades in the northern Pacific Ocean”, Acoustics Dept. Applied Physics Laboratory, Spring 2017.

Attached papers.

A test of deep water Rytov theory at 284 Hz and 107 km in the Philippine Sea
Rex k. Andrew, Andrew W. White, James A. Mercer, Peter F. Worcester, Matthew A.
Dzieciuch and John A. Colosi

Journal of the Acoustic Society of America **138**(4), pp 2015-2023 (2015).

A test of deep water Rytov theory at 284 Hz and 107 km in the Philippine Sea

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Predictions of log-amplitude variance are compared against sample log-amplitude variances reported by White, Andrew, Mercer, Worcester, Dzieciuch, and Colosi [J. Acoust. Soc. Am. **134**, 3347–3358 (2013)] for measurements acquired during the 2009 Philippine Sea experiment and associated Monte Carlo computations. The predictions here utilize the theory of Munk and Zachariasen [J. Acoust. Soc. Am. **59**, 818–838 (1976)]. The scattering mechanism is the Garrett–Munk internal wave spectrum scaled by metrics based on measured environmental profiles. The transmitter was at 1000 m depth and the receivers at nominal range 107 km and depths 600–1600 m. The signal was a broadband *m*-sequence centered at 284 Hz. Four classes of propagation paths are examined: the first class has a single upper turning point at about 60 m depth; the second and third classes each have two upper turning points at roughly 250 m; the fourth class has three upper turning points at about 450 m. Log-amplitude variance for all paths is predicted to be 0.04–0.09, well within the regime of validity of either Born or Rytov scattering. The predictions are roughly consistent with the measured and Monte Carlo log-amplitude variances, although biased slightly low. Paths turning in the extreme upper ocean (near the mixed layer) seem to incorporate additional scattering mechanisms not included in the original theory.

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[TFD]

Pages: 2015–2023

I. INTRODUCTION

In a previous paper, White *et al.*¹ reported on acoustical fluctuations that were observed in broadband signals centered at 284 Hz and transmitted for over 60 h during a 2009 experiment in the Philippine Sea. The low-latitude sound speed profile and the range of 107 km supported several distinct wholly refracted paths having one, two, or three upper turning points. Paths confined to and below the main thermocline had small log-amplitude variances of 0.04–0.09. The propagation regime suggests that a theory advanced by Munk and Zachariasen² (hereafter, referred to as MZ), which itself is an extension of an underlying theory that assumes weak fluctuations, might accurately model the observations. This paper shows that MZ calculations of log-amplitude variance are roughly consistent within observation error with the actual log-amplitude variance for these paths, although biased slightly low. A path turning very near the surface had twice as much log-amplitude variance and appears to be in a different propagation and/or scattering regime.

The underlying theory belongs to Rytov.³ When the index of refraction inhomogeneities along a propagation

path are weak and smoothly varying in space, Rytov *et al.*³ found an approximate solution to the fundamental scattering equations called the method of smooth perturbations. Conditions for validity of this approximation occur in a number of practical cases, and Rytov theory is used today in characterizing scattering in, for example, terrestrial microwave links and infrared and optical paths.⁴

Adaptation of Rytov theory to underwater acoustics, however, has not been straightforward. In contexts where the propagation path is more or less straight, Rytov theory has proven accurate enough. These configurations generally involve short ranges and high frequencies, e.g., probing turbulent scales of mixing in a coastal underwater channel⁵ over 670 m at 67 kHz, or using ultrasound in a fluid test cell⁶ over 30 cm at 100 kHz.

For longer ranges, the depth dependence of the sound speed causes the propagation path to refract upward, or downward, or both. In order to accommodate these additional propagation features, Munk and Zachariasen applied Rytov theory locally, invoking the local depth and angle of the unperturbed ray (i.e., the ray theoretic solution in an ocean having only a non-stochastic background sound-speed profile.) Statistics were computed by integrating local scattering contributions along the unperturbed ray path from transmitter to receiver.

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Predictions from MZ theory (and others) were compared to observations made during several experiments conducted in the 1970s. In the first set of experiments, called here the Cobb Seamount experiments,^{7,8} Ewart⁷ used signals from 2 to 8 kHz over a range of 17.2 km with lower paths characterized by upward turning depths of about 975–1500 m. Desaubies⁹ showed that MZ theory incorporating an appropriately parameterized Garrett–Munk (GM) internal wave spectrum yielded reasonably accurate measures of log-amplitude variance at 4 kHz, but not at 8 kHz, and showed significant discrepancies explaining the log-amplitude auto-spectra for both frequencies.

In a second experiment, Worcester¹⁰ analyzed signals at 2250 Hz transmitted over 25 km via both an upper and lower path. The lower path turned upward at a depth of 1471 m, and the upper path turned downward at 185 m. The analysis showed that fluctuations along both paths were not inconsistent with MZ theory using a version of the GM spectrum. Comparisons of intensity auto- and cross-correlation functions were, however, ambiguous because the experiment duration spanned only 15 h.

It is important to note that in both of these experiments, the acoustic paths can be characterized by eigenrays having shallow launch angles $|\theta_0| \leq 4^\circ$ (with respect to horizontal). We will return to this issue in Sec. VI.

Following these experiments, the focus of deep water research shifted for more than a decade to much longer ranges and much lower frequencies. Rytov theory, in general, and the deep water adaptation of MZ theory in particular, were not expected to be valid in these configurations. Recently, however, Colosi *et al.*¹¹ demonstrated that MZ predictions had reasonable agreement with several signal statistics for an 87 km path at 75 Hz at a mid-latitude site in the North Pacific. This regime is longer in range than the earlier Ewart and Worcester experiments, but at a much lower frequency.

White *et al.*¹ describe a scattering experiment in the Philippine Sea with a similar range—107 km—but a signal frequency (284 Hz) nearly two octaves higher. Corresponding *a posteriori* theoretical Monte Carlo calculations were remarkably accurate for most of the reception configurations. (The calculations are *a posteriori* in the sense that the calculations are made after the experiment using oceanographic measurements obtained during the experiment.) This paper provides associated *a posteriori* MZ theory predictions, completing a three-way comparison between at-sea measurements, Monte Carlo simulations, and theoretical predictions. MZ theory represents an analytical approach to predicting acoustic field statistics, and can provide greater physical insight to mechanisms of randomization than can Monte Carlo simulations. In addition, the MZ calculations are much faster to obtain compared to Monte Carlo results. [In the case presented here, a single MZ calculation required about 1 min, and a Monte Carlo parabolic equation (MCPE) simulation with 226 realizations required several weeks.] This effort therefore serves to further delineate the regions where MZ theory makes reasonably accurate predictions and where the theory develops some discrepancies.

This report is organized as follows. Section II outlines the structure of the calculations used here, including a Fresnel tube computation that is an alternative to that used in Colosi *et al.*¹¹ Section III summarizes the key features of the experiment. Section IV summarizes the theoretical Monte Carlo calculations reported by White *et al.*¹ Section V provides the comparison between the observed acoustical log-amplitude variance and the corresponding *a posteriori* MZ predictions. The theoretical expressions require several environmental profiles and a parameter—the scattering strength—which must be determined from oceanographic measurements. Provenance and construction of these are described in this section. Some of these are common and some are not. Estimation of the scattering strength parameter is not well-known: Sec. V also describes this calculation and, importantly, the error in the estimate. A discussion of the apparent regimes of validity of MZ theory and directions for further investigations are provided in Sec. VI.

II. MUNK-ZACHARIASEN THEORY

There are three key constructs in MZ theory: the eigenray, the associated Fresnel tube, and the space-time spectrum of sound-speed anomalies. Eigenrays have been thoroughly treated in the literature and will not be reviewed here. The theory is, in principle, agnostic with respect to the exact form of the sound-speed anomaly field, but here, as in most previous applications to the open ocean, the sound-speed anomalies are assumed to be due to fluid displacements caused by internal waves, with the space-time internal wave displacement field given by the GM model. This derivation is briefly summarized in Sec. II A. Construction of the Fresnel tube is described in Sec. II B, and Sec. II C describes the numerical calculation required for this study.

MZ theory writes the field $\psi = \psi_0 \exp(\chi + i\phi)$, where ψ_0 is the field in the absence of sound-speed anomalies, and χ is the (zero-mean) log-amplitude fluctuation and ϕ is the zero-mean phase fluctuation. Write $X = \chi + i\phi$. Then

$$\text{var}(\chi) = \langle \chi^2 \rangle = \frac{1}{2} \left(\langle |X|^2 \rangle + \text{Re} \{ \langle X^2 \rangle \} \right). \quad (1)$$

Using MZ Eq. (66) for $\langle |X|^2 \rangle$ and MZ Eq. (74) for $\langle X^2 \rangle$ [note that this equation has a sign error; see MZ Eq. (75)] one can obtain

$$\langle \chi^2 \rangle = \frac{2}{\pi} k_0^2 N_0 b \int_0^R dx \sum_{j=1}^{\infty} \frac{1}{j} \int_{\omega_L}^{\omega} d\omega \frac{S_{\mu\mu}(\omega, j)}{\sqrt{\omega^2 - \omega_L^2}} \frac{\sin^2 \gamma}{\cos^2 \theta}. \quad (2)$$

Quantities θ , ω_L , and γ are all functions of the unperturbed eigenray path depth $z_{\text{ray}}(x)$, which in turn is parameterized by range x from 0 to R . Wave number $k_0 = 2\pi f/c_0$, where f is the acoustical frequency in Hertz and c_0 is a reference sound speed. The product $N_0 b$ incorporates the Brunt–Väisälä frequency scale N_0 and a depth scale b , and is $\int N(z) dz$ where $N(z)$ is the Brunt–Väisälä or buoyancy frequency profile; however, N_0 and b always appear together, and therefore $N_0 b$ will always be treated as a single

quantity.¹² Angle θ is the path inclination relative to the horizontal. The term $\omega_L^2 = f_C^2 + N(z)^2 \tan^2 \theta$, where f_C is the local Coriolis parameter (here, in radians/s). $S_{\mu\mu}(\omega, j)$ is the spectrum (described more fully in Sec. II A) of fractional sound-speed anomalies μ ($\equiv \delta c/c$) as a function of internal wave frequency ω and mode number j . The term γ is given in MZ by

$$\gamma = \frac{\pi}{4} \frac{j^2}{(N_0 b)^2} \left((\omega^2 - \omega_L^2) R_{Fy}^2 + N(z)^2 R_{Fz}^2 \right), \quad (3)$$

where $R_{Fy}^2 = 2\pi x(R-x)/k_0 R$ is the horizontal Fresnel radius, and R_{Fz}^2 is the vertical Fresnel radius. Since the horizontal correlation length of the sound-speed anomalies is much greater than the vertical correlation length, the first term is generally much smaller than the second and is neglected. This leaves

$$\gamma \approx \frac{1}{4\pi} \left(\frac{\pi j N(z)}{N_0 b} \right)^2 R_{Fz}^2 = \pi \left(\frac{j N(z) R_{Fz}}{2 N_0 b} \right)^2. \quad (4)$$

A. Spectrum of sound-speed anomalies

The spectrum of fractional sound-speed anomalies $S_{\mu\mu}(\omega, j)$ due to internal wave displacements requires a mapping $\mu = T\zeta$ between fractional sound-speed anomaly μ ($= \delta c/c$) and vertical fluid displacement ζ . The standard transformation is given by MZ as

$$\mathcal{T}_c = (1/\bar{c})(\partial_z \bar{c}_P). \quad (5)$$

Here and below, the term $\bar{c}$ denotes the depth-dependent background sound-speed profile; explicit depth dependence will be omitted. The potential sound-speed gradient is

$$\partial_z \bar{c}_P = \partial_z \bar{c} - \partial_z c_A. \quad (6)$$

Here, c_A is the adiabatic sound speed, such that $-\bar{c}^{-1} \partial_z c_A \equiv \gamma_A = 1.14 \times 10^{-5} \text{ m}^{-1}$ [Flatté *et al.*,¹³ Eq. (1.1.10)].

Given this mapping, the spectrum of fractional sound-speed anomalies is

$$S_{\mu\mu}(\omega, j) = \mathcal{T}_c^2 S_{\zeta\zeta}(\omega, j). \quad (7)$$

The most mature version¹⁴ of the GM spectrum of vertical fluid displacements can be written as¹²

$$S_{\zeta\zeta}(\omega, j) = b E_0 \frac{N_0 b}{N(z)} \frac{\omega^2 - f_C^2}{\omega^2} H(j) B(\omega). \quad (8)$$

Here $H(j) = H_0^{-1}(j^2 + j_*^2)^{-1}$ is the modal spectrum and H_0 is such that $\sum_0^\infty H(j) = 1$, and $B(\omega)$ is

$$B(\omega) = \frac{2f_C}{\pi \omega} \frac{1}{\sqrt{\omega^2 - f_C^2}} \quad (9)$$

and is normalized so that $\int_{f_C}^{N(z)} B(\omega) d\omega \approx 1$ for $N(z)/f_C \gg 1$. The product $b E_0$ incorporates the energy scale E_0 and the depth scale b ; however, these two terms always also appear

together, and like $N_0 b$ will be considered a single quantity.¹² Combining Eqs. (7)–(9) yields

$$S_{\mu\mu}(\omega, j) = \mathcal{T}_c^2 b E_0 \frac{N_0 b}{N(z)} H(j) D(\omega), \quad (10)$$

where the function $D(\omega)$ is

$$D(\omega) = \frac{2f_C}{\pi} \frac{\sqrt{\omega^2 - f_C^2}}{\omega^3}. \quad (11)$$

The normalization of $B(\omega)$ provides $\int_{f_C}^{N(z)} D(\omega) d\omega \approx \frac{1}{2}$ for $N(z)/f_C \gg 1$.

B. Fresnel tubes

The line of sight in ocean acoustics is the eigenray, which is generally not straight over any typical range. Let the eigenray be the parametric curve $z_{\text{ray}}(x)$; there is no loss of generality in restricting the path to the two-dimensional system with range x and depth z . The transmitter is at point $\mathcal{S} = (0, z_{\text{ray}}(0))$ and the receiver at point $\mathcal{R} = (R, z_{\text{ray}}(R))$. The Fresnel tube is defined as that volume around the eigenray whose boundary points $\mathcal{B}$ satisfy

$$\frac{1}{2} T = |[\tau(\mathcal{B}, \mathcal{S}) + \tau(\mathcal{B}, \mathcal{R}) - \tau(\mathcal{R}, \mathcal{S})]|, \quad (12)$$

where $\tau(\mathcal{B}, \mathcal{A})$ is the travel time from point $\mathcal{A}$ to point $\mathcal{B}$. The eigenray does not formally have an associated frequency, but the Fresnel concept is frequency-dependent, and therefore T is the period of the propagating field, assumed here to be monochromatic. Equation (12) says that the Fresnel tube boundary is defined when the difference in travel time between the travel time along the eigenray and the travel time along an alternate nearby path to and from a boundary point is $\pm$ half a wave period. (Some authors use $\pm$ quarter wave period.¹⁵)

Flatté *et al.*¹³ give a prescription for computing the boundary of the Fresnel tube. An alternate approach is taken here, utilizing dynamic ray tracing theory.¹⁶ In addition to producing Fresnel tubes for simple propagation paths such as those defined here, dynamic ray tracing can handle more complex situations such as range-dependent media and reflections from interfaces. The approach uses a local coordinate system in which one axis (say, the third axis, $\hat{\mathbf{q}}_3$) is aligned along the local tangent to the ray. The first axis, $\hat{\mathbf{q}}_1$, is then located in the (x, z) plane (i.e., in the vertical plane), and the second axis, $\hat{\mathbf{q}}_2$, is directed out-of-plane.

The derivation of the Fresnel tube is thoroughly described in the dynamic ray tracing literature¹⁷; only key points are mentioned here. In the paraxial approximation, one obtains

$$\frac{1}{2} T = \frac{1}{2} \mathbf{u}^T [\mathbf{M}(\mathcal{O}, \mathcal{S}) + \mathbf{M}(\mathcal{O}, \mathcal{R})] \mathbf{u}, \quad (13)$$

where $\mathbf{u} = [u_1 \hat{\mathbf{q}}_1 + u_2 \hat{\mathbf{q}}_2]$. $\mathbf{M}$ is a 2×2 propagator matrix. Equation (13) is solved for (u_1, u_2) at every point $\mathcal{O}$ on the eigenray. Due to the anisotropic nature of the sound-speed

anomaly inhomogeneities, the influence of u_1 dominates that of u_2 , and therefore Eq. (13) can be simplified to

$$\frac{1}{2}T = \frac{1}{2}u_1^2([\mathbf{M}(\mathcal{O}, \mathcal{S})]_{1,1} + [\mathbf{M}(\mathcal{O}, \mathcal{R})]_{1,1}), \quad (14)$$

which only requires the first element of the propagator matrix. The boundary of the Fresnel tube (under the paraxial approximation) in the $\hat{\mathbf{q}}_1$ direction is then given by

$$u_1^{-1} = \sqrt{f} \sqrt{[\mathbf{M}(\mathcal{O}, \mathcal{S})]_{1,1} + [\mathbf{M}(\mathcal{O}, \mathcal{R})]_{1,1}} \quad (15)$$

for an acoustical frequency f .

For a cylindrically symmetric ocean model, the propagator matrix is particularly simple and is completely described in this case by Porter and Bucker.¹⁸ The computations are implemented in the propagation code BELLHOP. In practice, there are two complications. BELLHOP uses an adaptive step integrator and therefore solutions for the propagator elements and, hence, solutions u_1 do not occur on a regular x -axis grid. Second, the boundary point $\mathcal{B}$ for each solution $u_1(x)$ does not occur on a regular x -axis grid either, because the local coordinate system is generally rotated relative to the local Cartesian (x, y, z) system. Both problems are easily circumvented via interpolation.

An example tube calculation (for one of the paths described later) is shown in Fig. 1.

C. Numerical details

Equation (2) is the general MZ expression. For the specific case involving the GM spectrum,¹⁴ $S_{\mu\mu}(\omega, j)$ is separable and the integration over ω can be done analytically. The numerical evaluation of Eq. (2) proceeds by writing

$$\langle \chi^2 \rangle = A \sum_{j=1}^J j^{-1} H(j) I_j, \quad (16)$$

where the summation upper limit has been truncated at $J, A = (4/\pi^2) b E_0 (N_0 b)^2 f_C k_0^2$ and I_j is the one-dimensional integral

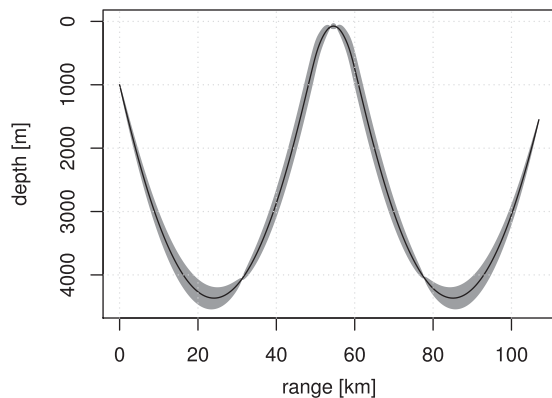


FIG. 1. Fresnel tube (vertical slice), 284 Hz, Philippine Sea 2009 experiment. The eigenray is the solid black line, the tube is represented by the shaded region.

$$I_j = \int_0^R dx Q_j(x) = R \int_0^1 d\alpha Q_j(\alpha R), \quad (17)$$

with integrand

$$Q_j(x) = Q(x; j) = \frac{T_c^2 \sin^2 \gamma}{N \cos^2 \theta} W. \quad (18)$$

All terms are implicit functions of range via a dependence on $z_{\text{ray}}(x)$. The term $W(x)$ is

$$W(x) = \int_{\omega_L}^{N(z)} \frac{d\omega \sqrt{\omega^2 - f_C^2}}{\omega^3 \sqrt{\omega^2 - \omega_L^2}}. \quad (19)$$

This integral has branch points at $|\omega| = \omega_L$ (which are also singular points.) Generally, $\omega_L > f_C$ except for $\theta = 0$. Using the branch cut $[-\omega_L, \omega_L]$, a closed form solution was found for $\theta \neq 0$ by Colosi *et al.*¹¹ An equivalent expression for $\theta \neq 0$ is

$$W(x) = \frac{\sqrt{1 - f_C^2/N(z)^2} \sqrt{1 - \omega_L^2/N(z)^2}}{2\omega_L^2} + \frac{1}{2\omega_L^2} \left(\frac{\omega_L}{f_C} - \frac{f_C}{\omega_L} \right) \coth^{-1} \left(\frac{\omega_L \sqrt{1 - f_C^2/N(z)^2}}{f_C \sqrt{1 - \omega_L^2/N(z)^2}} \right) \quad (20)$$

and at $\theta = 0$ the solution follows directly from Eq. (19) and is

$$W(x) = [1 - f_C^2/N(z)^2]/2f_C^2.$$

Romberg quadrature is used for the numerical estimation of the integrals of Eq. (17).

III. EXPERIMENTAL MEASUREMENTS

The at-sea data were collected during the 2009 Philippine Sea experiment (PhilSea09). Worcester *et al.*¹⁹ has given an overview of the entire Philippine Sea effort. White *et al.*¹ provided a comprehensive description of this experiment, so only a summary will be provided here.

An electro-acoustical transmitter was suspended at 22°16.95'N, 126°19.77'E to a depth of 1000 m from the R/V *Melville*. The transducer has sharp resonances at about 210 Hz and 320 Hz. The transmitted signal was a 1023-bit m -sequence²⁰ with a carrier frequency of 284 Hz and 2 cycles per bit. The transducer drive signal was equalized to mitigate the distortion induced by the doubly peaked system transfer function, yielding a reasonable facsimile of an m -sequence with a 130 Hz bandwidth and pulse resolution of 7.7 ms. The root-mean-square (RMS) source level was roughly 183 dB re 1 μ Pa at 1 m.

The receiver was located at 21°21.90'N, 126°01.02'E and involved a 5 km mooring suspended from a subsurface float at a range of 107 km and a bearing of 198° from the transmitter. The mooring included two autonomous subarrays of hydrophones, each consisting of 30 hydrophones and

one controller.²¹ The upper subarray had an inner interelement spacing of 25 m for 25 hydrophones and of 75 m for two hydrophones above and three hydrophones below the inner hydrophones. It spanned the deep sound channel (centered around about 1040 m). Wavefield statistics from the lower subarray were not reported by White *et al.*¹ and therefore the lower subarray will not be described further. The mooring was also outfitted with Sea-Bird SBE37-SMP pumped conductivity-temperature-pressure microCAT conductivity-temperature-depth (CTD) recorders,²² Sea-Bird SBE39 temperature recorders,²³ and a long-baseline acoustical navigation system.

The *m*-sequences were transmitted beginning roughly at 10:00 UTC on yearday 117 and continued for 60 h. Each *m*-sequence was 7.2 s in length. Sequences were transmitted continuously in a schedule of 10 min on, 5 min off, 10 min on, 5 min off, 15 min on, and off for the remainder of each hour. Data records heavily contaminated by in-water signals from unidentifiable sources were removed.

After pulse-compressing the raw data, White *et al.*¹ mapped eigenray path identifiers to individual branches of the acoustical time front. Subsections of four branches could be resolved, each involving roughly a dozen receiver depths. In every receiver time series where a particular branch could be isolated, a short time segment containing the complex pulse waveform was extracted for that branch. This yielded sample sets over the duration of the experiment of $\sim 15\,700$ pulses for every depth and branch combination, excluding combinations where branches overlapped or crossed. Each pulse was transformed into the Fourier domain. The data record for each available sensor depth and time front branch combination then consisted of the time series of squared magnitudes (i.e., the intensity) of the Fourier coefficient corresponding to the carrier frequency. White *et al.*¹ do not give the sample variances of these records themselves, but rather the span of the corresponding 95% confidence interval; see their paper for a discussion of how this confidence interval was estimated. The log-amplitude variance spans used here are taken directly from their Fig. 8 (where they give RMS log-intensity in decibels.)

White *et al.*¹ analyzed four time front branch subsections. Eigenray paths for each subsection were similar: (a) three turning points initially launched downward, denoted ID-3; (b) four turning points launched upward (ID+4); (c) four turning points launched downward (ID-4); (d) five turning points launched upward (ID+5). Representative paths are shown in Fig. 2.

IV. MONTE CARLO CALCULATIONS

The commonly accepted standard for wave-theoretic solutions in the paraxial approximation for weakly range-dependent channels is the parabolic equation, particularly for long-range, deep-water ocean acoustic scenarios where bathymetric interaction is minimal or absent, and only fields propagating near horizontal are important. This approach integrates well with the Monte Carlo method for simulating the distribution of wavefield statistics. White *et al.*¹ used this MCPE approach in an *a posteriori* prediction of

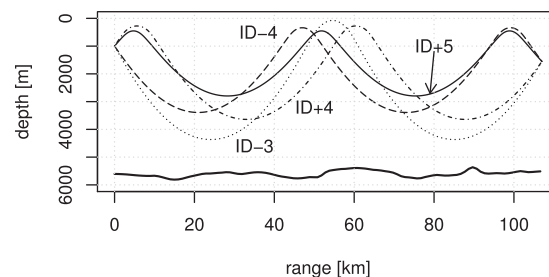


FIG. 2. Example eigenrays. The transmitter was at depth of 1000 m. Paths are shown to sensor HM028 at a nominal depth of 1550 m at range 107 km. The thick solid line at the bottom is the Smith-Sandwell bathymetry (Ref. 24). Note that paths ID+4 and ID-4 have very similar characteristics—numbers of upper and lower turns, upper turning depth—but path ID-3 is quite different.

log-amplitude variances. Details on the parabolic equation computation are given by White *et al.*¹ Each Monte Carlo realization built an acoustic channel on the background sound speed $\bar{c}(z)$ by adding a random range-depth realization of sound-speed anomalies constructed using the Henryey-Reynolds algorithm.²⁵ White *et al.*¹ set the strength of the sound-speed anomalies by analyzing the original receiver mooring microCAT CTD data to obtain a GM strength of ≈ 1.6 . (More on this in Sec. V.) This value is similar to the value of ≈ 1.4 found by Colosi *et al.*²⁶ Since the MCPE method is computationally intensive, only 226 realizations were obtained.

V. ANALYSIS

In an *a posteriori* prediction, there are several environmental quantities that must be estimated and/or constructed from measurements made during the experiment prior to the primary calculation. The background sound speed profile $\bar{c}(z)$, Brunt-Väisälä frequency profile $N(z)$, and $\partial_z \bar{c}_p$ profile were the same profiles used by White *et al.*¹ in their MCPE simulations.

Estimation of the product bE_0 , required in Eq. (10), is less straightforward. Colosi *et al.*²⁶ provide a thorough discussion of the random oceanographic fields during PhilSea09, but do not provide an explicit estimate of bE_0 . Alternatively, White *et al.*¹ describe the estimation of the GM strength coefficient in terms of a ratio $\hat{r}$ of observed integrated spectral energy $\hat{s}$ to integrated spectral energy s_{GM1} based on the standard GM79 model at unity strength. Under the paradigm that $s_{\text{GM1}} = \frac{1}{2}(bE_{\text{GM1}})(N_0b)$ [using their Eq. (1) and our notation], one can define

$$\hat{r} = \widehat{bE_0} / bE_{\text{GM1}}. \quad (21)$$

As with bE_0 , the product bE_{GM1} is considered a single variable. Under the standard model, one has²⁵

$$\frac{1}{2}(bE_{\text{GM1}})(N_0b) = 0.277 \text{ m}^2\text{rads/sec} \quad (22)$$

and hence $bE_{\text{GM1}} = 0.056 \text{ m}$ for the value of N_0b in effect here. White *et al.*¹ describe depth-averaged lower and upper bounds on $\hat{r}$, derived by examining one month records of

sound-speed time-series as measured by the 12 pumped microCATs on the receiver mooring. A lower bound of 1.6 is derived by ignoring tidal peaks in the spectra, under the assumption that tidal processes have insignificant effects on intensity fluctuations. This assumption is commonly made because tidal effects are considered to be large vertical scale perturbations, and large scale perturbations affect primarily the travel time. The intensity, however, is affected mainly by the small scale perturbations. Using their Fig. 8, the apparent standard error on this lower bound is 0.12 (assuming 12 independent measurements.) From Eq. (21), this implies $\widehat{bE_0} = 0.089 \pm 0.007$ m, ignoring tidal energy. An upper bound of 3.33 is also given, which includes tidal energy. The lower bound is used by White *et al.*¹ in reporting their MCPE calculations (Sec. IV). We address both bounds in the discussion below.

With this value of $\widehat{bE_0}$ and the other parameters shown in Table I, the log-amplitude variance equation (2) can now be computed and compared to the actual measurements. The integrals of Eq. (17) were estimated numerically with Romberg quadrature. Romberg quadrature is an adaptive algorithm whose result will be interpreted below as a random variable with a uniform distribution of $U_{[I_* - \delta, I_* + \delta]}$, where the estimated integral (i.e., at a given mode number j) is I_* and $\delta = \epsilon I_*$. Here, ϵ is the relative estimated error, which is preset in the algorithm to 1 part in 10^6 . The variance of a random variable with distribution $U_{[a,b]}$ is $(b-a)^2/12$; hence, the RMS error is $\delta/\sqrt{3}$. This value is considered the standard error. In order to compare to the 95% confidence interval error bars used by White *et al.*,¹ each integration was assigned an error of twice this RMS value. Errors for the entire calculation (i.e., over all modes) used a quadrature sum of the individual errors. The final error was obtained by the quadrature sum of the total quadrature estimation errors and the error for $\widehat{bE_0}$. In all cases, the error on $\widehat{bE_0}$ dominated.

The ranges of MZ log-amplitude variance for each class of path are shown in Table II. Also given are the corresponding values for the scintillation index and the RMS log-intensity. The full comparison of at-sea measurements, Monte Carlo calculations, and MZ predictions versus depth for all four path classes is shown in Fig. 3.

TABLE I. Parameters used in the calculation. The term $N_0 b = \int N(z) dz$ and is computed to be roughly 9.98 rads-m/s. (The integral is over the entire water column.)

Parameter	Symbol	Value
Latitude		21.5°
Buoyancy frequency	$N(z)$	See text
	$N_0 b$	See below
Coriolis parameter	f_c	5.33×10^{-5} rad/s
Modal bandwidth	j_*	3
Maximum modes	J	120
Reference sound speed	c_0	1480 m/s
Background sound speed	$\bar{c}(z)$	See text
Acoustical frequency	f	284 Hz
Strength	$\widehat{bE_0}$	0.089 ± 0.007 m

TABLE II. Comparison of typical log-amplitude and log-intensity fluctuation statistics from the MZ computations. The scintillation index (SI) is approximately $4 \text{ var } \chi$ for $\text{var } \chi \ll 1$. The RMS log-intensity (RMS I) is given in units of decibels, following Colosi *et al.* (Ref. 11); these are the same units as those used in Fig. 3.

Path	$\text{var } \chi$	SI	RMS I
ID-3	0.038–0.043	0.15–0.17	1.7–1.8
ID+4	0.067–0.074	0.27–0.30	2.3–2.4
ID-4	0.067–0.076	0.27–0.30	2.3–2.4
ID+5	0.080–0.090	0.32–0.36	2.5–2.6

VI. DISCUSSION

These comparisons indicate that *a posteriori* MZ theory is largely consistent with at-sea measurements and MCPE calculations of log-amplitude variance at the 5% level of significance for paths ID+4, ID-4, and ID+5 at 284 Hz over 107 km. This conclusion is reinforced by the power of the three measurement campaigns:

- The duration (60 h) and pulse repetition rate (500/h) guaranteed a formidable at-sea dataset; assuming an intensity correlation time of 5 m and a 50% gap loss, White *et al.*¹ still obtained roughly 360 independent samples.
- The MCPE calculations used 226 independent samples; while more would be preferable, this is commensurate with the actual data.
- The numerical estimation of integrals equation (17) continually refines its estimate of the integral until it reaches the pre-defined error bound 1 part in 10^6 .

The three-way comparison reported here is remarkably consistent. Even though the actual measurements were mediated by a mixture of oceanographic processes and the MCPE predictions were driven solely by GM-style internal waves, the correspondence between the actual measurements and the MCPE predictions is excellent. The correspondence between the MCPE and MZ predictions is also very good, which should follow because both calculations used the same scattering model. The results also suggest that MCPE calculations are more accurate predictions of the measurements than MZ predictions (using the scattering strength lower bound), with MZ slightly underpredicting the log-amplitude variance. More stringent error bounds would be needed to firmly support this conclusion.

The results for paths ID+4 and ID-4 are similar, which should be expected since the two paths themselves have similar characteristics. Agreement between observed data and either MZ predictions or MCPE calculations is not as good for path ID-3, but this is not unexpected. Figure 2 shows that this path has an upper turning depth of roughly 60 m. This near-surface region experiences considerable oceanographic variability (mixed layer dynamics, convective turbulence, etc.) not included in the GM model. This variability can cause a variety of effects not represented by the simplistic eigenray solution shown in Fig. 2, such as a surface duct or even surface reflections. White *et al.*¹ consider possible hypotheses in more comprehensive detail; here, it should suffice to note that inspection of the intensity record for path

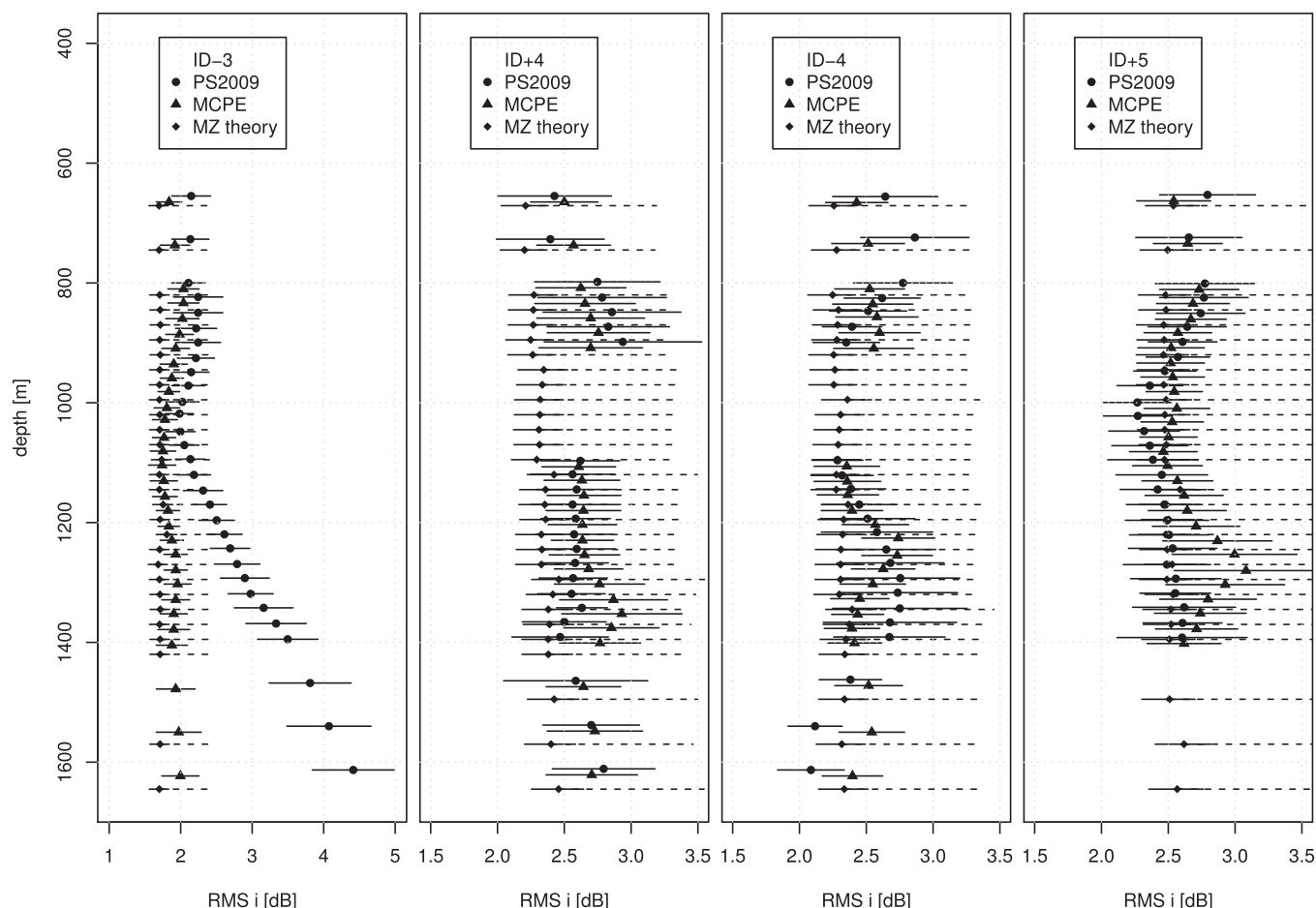


FIG. 3. Comparison of log-intensity variances (presented as RMS log-intensity i) for the Philippine Sea 2009 experiment, 107 km, 284 Hz. Panels left to right correspond to ray path geometries ID-3, ID+4, ID-4, and ID+5. Error bar spans for at-sea measurements (PS2009) and Monte Carlo estimates (MCPE) are taken from White *et al.* (Ref. 1) and are two standard errors. The at-sea and Monte Carlo error span center points are merely the span midpoints and are only provided for reference; they are not given *per se* by White *et al.* See the text for a discussion of the solid MZ prediction error bars. The dashed MZ error bars reflect the lower (left endpoint) and upper (right endpoint) bounds given by White *et al.* MCPE results offset downward by 10 m, MZ results by 20 m for clarity. Note that the left panel has a different abscissa scale to accommodate the at-sea measurements below about 1000 m. Since $\text{var } i = 4 \text{ var } \chi$, $\text{RMS } i = (10/\ln 10)\sqrt{\text{var } i} = (20/\ln 10)\sqrt{\text{var } \chi}$ in decibels.

ID-3 (their Fig. 4) seems to indicate that the intensity fluctuations were modulated by a strong non-GM diurnal process. MZ theory incorporating the GM model has no mechanism to include this.

White *et al.*¹ allowed for internal wave strength to be between GM strength parameter 1.6 and 3.33, largely because it is not well understood how tidal spectral energy, a significant component in the PhilSea09 environment, influences acoustic signal fluctuations. Tidally forced sound-speed fluctuations are considerably unidirectional and have no representation in the GM model, which assumes horizontally isotropic internal waves. MZ predictions using GM strengths [1.6,3.33] span all measurements for paths ID+4, ID-4, and ID+5: this level of uncertainty supports only the broadest of conclusions on the model. White *et al.* speculated that the lower bound (vice the upper bound) was a better estimate of scattering strength for intensity fluctuations because it provided a better spectral fit at high frequencies, away from the strong tidal peaks. Indeed, the results presented here can be used in a rudimentary inverse problem: the MZ predictions suggest that the scattering strength is at or perhaps slightly higher than the lower bound provided by

White *et al.*¹ The scattering strength upper bound is clearly not appropriate. These acoustic results can therefore be used to reduce the uncertainty in the ocean model.

In a more general sense, questions surrounding the validity of the Rytov approximation have had a durable longevity. This issue has been debated in the western literature long after Pisareva showed in the Russian literature that the issue depended on a wave parameter D , and that when this wave parameter was small, the Rytov approximation was valid when the log-amplitude variance was less than unity.^{27,28} The wave parameter was $2\lambda R/\pi L^2$, where λ is the acoustic wavelength, R is the propagation distance, and L is a correlation length scale (e.g., in a spherical isotropic medium.) For PhilSea09, the wave parameter is $\mathcal{O}(10^{-2})$ using order-of-magnitude estimates. This suggests that this experiment involved only the Fresnel scattering regime and, therefore, only the bound on log-amplitude variance is relevant. Since the observed log-amplitude variance is less than unity, the Rytov approximation would be valid.

Scattering in the deep ocean is much more complex than the scenarios assumed in the early Russian literature due to the refraction of unperturbed paths and the anisotropy of the

sound speed inhomogeneities and, therefore, the wave parameter used by Pisareva should be inappropriate. Correlation length scales in the horizontal and vertical differ considerably. Flatté–Dashen theory,¹³ which develops moment expressions starting with the path integral formalism, was built specifically to accommodate the additional complexities of the ocean scenario. In Flatté–Dashen theory, the scattering regime is categorized by the product $\Lambda\Phi^2$ involving a diffraction parameter Λ and a strength parameter Φ . Under Flatté–Dashen theory, the Rytov approximation is valid when $\Lambda\Phi^2 < 1$. (See, e.g., Fig. 8.6 in Flatté *et al.*¹³) For the PhilSea09 experiment, calculations from CAFI^{29,30} (which implements Flatté–Dashen expressions) show that $\Lambda\Phi^2$ ranges from 0.3 to 0.6 using the parameter values of Table I and an effective GM strength of 1.6 for the four rays considered here. Hence, under Flatté–Dashen theory, the Rytov approximation should be valid. (As an aside, the Pisareva condition is therefore not misleading.)

Even though Flatté–Dashen theory is a more complete theory¹¹ and supercedes MZ theory, the literature^{31,32} indicates that MZ theory and Flatté–Dashen theory are the same when $\Lambda\Phi^2 < 1$, at least to first order in the Rytov series. Flatté–Dashen theory dictates that the log-amplitude variance in the geometrical-optics approximation is given approximately by $\frac{1}{4}\Lambda\Phi^2$ (Flatté *et al.*¹³, Eq. 8.3.2). Table III shows the depth-averaged values of log-amplitude variance predictions from MZ theory and from Flatté–Dashen theory (implemented via CAFI) for the four eigenrays studied here. These are compared to MCPE calculations, which here are considered the benchmark result. This comparison shows that MZ theory slightly underpredicts the MCPE result. While it is not the purpose of this paper to compare Flatté–Dashen theory or CAFI against MCPE calculations, Table III shows that CAFI overpredicts the MCPE result, and that the CAFI result for $\frac{1}{4}\Lambda\Phi^2$ is almost twice the MZ result within the observed error. [This is, however, consistent with comments offered by Flatté *et al.*:¹³ the log-amplitude variance in the geometric region is $\mathcal{O}(\frac{1}{4}\Lambda\Phi^2)$, an assertion which proves true here.] The value of MZ theory is therefore that it provides Rytov-based expressions for the log-amplitude variance and, by extension, the scintillation index, which, within the limits of its validity, appear to be more accurate than the CAFI-based Flatté–Dashen theory results.

Calculations of $\Lambda\Phi^2$ for the Worcester experiment¹⁰ at 2250 Hz and the Cobb Seamount experiments¹³ at 4 and

8 kHz dictate that the Rytov approximation (hence, MZ theory) remains valid for these scenarios. This is consistent with the lower frequency Worcester measurements,¹⁰ but not the higher frequency Ewart measurements.^{8,9} Both of these experiments analyzed near-horizontal propagation paths. Thus, measurements covering a sparse set of scenarios from high angle, low frequency, hundred kilometer range (Colosi *et al.*,¹¹ White *et al.*¹) to low angle, mid frequency, ten kilometer range may have delineated the boundaries of valid Rytov and MZ theory application. In an effort to probe the sensitivity of the phase and log-amplitude variance and spectral predictions of MZ theory versus frequency, range, and propagation path steepness, Tombul conducted an unpublished numerical study to compare MZ predictions against direct MCPE simulation.³³ Individual eigenrays were simulated using beams with launch angles of 0°, 5°, 10°, and 14°. Ranges varied from 50 to 200 km. Frequencies were 75, 200, and 400 Hz. The medium profiles were analytic models scaled for good correspondence with typical oceanographic data. For shallow launch angles, he found good agreement for log-amplitude variance. For steeper launch angles, he observed discrepancies. The MZ results failed completely at 400 Hz and 14° ray angles. At 200 Hz and 10°, which compares closely with these data, he found that MZ underpredicted MCPE results to the extent that he concluded that MZ theory was no longer valid for these and higher frequencies at this angle. Tombul’s results are generally consistent with the results presented here: Fig. 3 and Table III show that MZ calculations are underpredicting the MCPE results. Tombul’s conclusion may be too severe; MZ theory is largely consistent with measured and MCPE results, but it does appear that a higher order correction is required to eliminate the prediction bias. Simulations were not conducted at the frequencies used by Worcester¹⁰ and Ewart.^{7,8}

The transition region from Rytov/MZ theory to the more comprehensive Flatté–Dashen theory seems to be bounded by a couple of experiments and simulations performed over the past several decades, but several gaps remain in these boundaries. In order to eliminate these gaps, MCPE simulations should be pushed to higher frequencies, and careful ocean experiments pursued at low mid-frequencies for steeper propagation paths.

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¹A. W. White, R. K. Andrew, J. A. Mercer, P. F. Worcester, M. A. Dzieciuch, and J. A. Colosi, “Wavefront intensity statistics for 284-hz broadband transmissions to 107-km range in the Philippine Sea: Observations and modeling,” *J. Acoustic. Soc. Am.* **134**, 3347–3358 (2013).

TABLE III. Comparison of Rytov-based predictions of log-amplitude variance from MZ and Flatté–Dashen theory against MCPE calculations for the four eigenrays studied here. The MCPE, MZ, and $\frac{1}{4}\Lambda\Phi^2$ columns are the depth-averaged values. The final row provides the eigenray-averaged ratios of MZ predictions against the MCPE calculations and the CAFI predictions against the MCPE calculations.

Ray ID	MCPE	MZ	MZ/MCPE	$\frac{1}{4}\Lambda\Phi^2$	$\frac{1}{4}\Lambda\Phi^2$ /MCPE
−3	0.0473	0.0389	0.82	0.075	1.59
+4	0.0961	0.0705	0.73	0.133	1.38
−4	0.0832	0.0725	0.87	0.148	1.78
+5	0.0940	0.0830	0.88	0.113	1.20
		Average	0.83	Average	1.49

- ²W. H. Munk and F. Zachariassen, "Sound propagation through a fluctuating stratified ocean: Theory and observation," *J. Acoust. Soc. Am.* **59**, 818–838 (1976).
- ³S. M. Rytov, Y. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics 4, Wave Propagation Through Random Media* (Springer, Berlin Germany, 1989), pp. 49–50.
- ⁴A. D. Wheelon, *Electromagnetic Scintillation 2, Weak Scattering* (Cambridge University Press, Cambridge, UK, 2003), pp. 1–9.
- ⁵D. Di Iorio and D. M. Farmer, "Path-averaged turbulent dissipation measurements using high frequency acoustical scintillation analysis," *J. Acoust. Soc. Am.* **96**, 1056–1069 (1994).
- ⁶T. A. Andreeva and W. W. Durgin, "Experimental investigation of ultrasound propagation in turbulent, diffractive media," *J. Acoust. Soc. Am.* **115**, 1532–1536 (2004).
- ⁷T. E. Ewart, "Acoustic fluctuations in the open ocean—A measurement using a fixed refracted path," *J. Acoust. Soc. Am.* **60**, 46–59 (1976).
- ⁸T. E. Ewart and S. A. Reynolds, "The mid-ocean acoustic transmission experiment—MATE," *J. Acoust. Soc. Am.* **75**, 785–802 (1984).
- ⁹Y. J. F. Desaubies, "On the scattering of sound by internal waves in the ocean," *J. Acoust. Soc. Am.* **64**, 1460–1469 (1978).
- ¹⁰P. F. Worcester, "Reciprocal acoustic transmission in a midocean environment: Fluctuations," *J. Acoust. Soc. Am.* **66**, 1173–1181 (1979).
- ¹¹J. A. Colosi, J. Xu, P. F. Worcester, M. A. Dzieciuch, B. M. Howe, and J. A. Mercer, "Temporal and vertical scales of acoustic fluctuations for 75-Hz, broadband transmissions to 87-km range in the eastern North Pacific Ocean," *J. Acoust. Soc. Am.* **126**, 1069–1083 (2009).
- ¹²F. S. Henyey, D. Rouseff, J. M. Grochocinski, S. A. Reynolds, K. L. Williams, and T. E. Ewart, "Effects of internal waves and turbulence on a horizontal aperture sonar," *J. Ocean. Eng.* **22**, 270–280 (1997).
- ¹³S. M. Flatté, R. Dashen, W. H. Munk, K. Watson, and F. Zachariassen, *Sound Transmission through a Fluctuating Ocean* (Cambridge University Press, Cambridge, UK, 1979), pp. 6–7.
- ¹⁴W. Munk, "Internal waves and small-scale processes," in *The Evolution of Physical Oceanography*, edited by C. Wunsch and B. Warren (MIT Press, Cambridge, MA, 1981), pp. 264–291.
- ¹⁵I. I. Rypina and M. G. Brown, "On the width of a ray," *J. Acoust. Soc. Am.* **122**, 1440–1448 (2007).
- ¹⁶V. Červený, *Seismic Ray Theory* (Cambridge University Press, Cambridge, UK, 2001), pp. 102–235.
- ¹⁷V. Červený and J. E. P. Soares, "Fresnel volume ray tracing," *Geophysics* **57**, 902–915 (1992).
- ¹⁸M. B. Porter and H. P. Buckner, "Gaussian beam tracing for computing ocean acoustic fields," *J. Acoust. Soc. Am.* **82**, 1349–1359 (1987).
- ¹⁹P. F. Worcester, M. A. Dzieciuch, J. A. Mercer, R. K. Andrew, B. D. Dushaw, A. B. Baggeroer, K. D. Heaney, G. D. D'Spain, J. A. Colosi, R. A. Stephen, J. N. Kemp, B. M. Howe, L. J. V. Uffelen, and K. E. Wage, "The North Pacific Acoustic Laboratory deep-water acoustic propagation experiments in the Philippine Sea," *J. Acoust. Soc. Am.* **134**, 3359–3375 (2013).
- ²⁰W. H. Munk, P. F. Worcester, and C. Wunsch, *Ocean Acoustic Tomography* (Cambridge University Press, Cambridge, UK, 1995), pp. 190–195.
- ²¹P. F. Worcester, S. Carey, M. A. Dzieciuch, L. L. Green, D. Horwitt, J. C. Lemire, and M. Norenberg, "Distributed vertical line array (DVLA) acoustic receiver," in *Proc. 3rd International Conf. on Underwater Acoustic Measurements*, edited by J. S. Papadakis and L. Bjørnø (Foundation for Research and Technology Hellas, Nafplion, Greece, 2009), pp. 113–118.
- ²²Sea-Bird Electronics, Inc., "MicroCAT C-T (P optional) recorder SBE 37-SMP," Technical Report, Sea-Bird Electronics, Inc., 13431 NE 20th Street, Bellevue WA (2012), available at http://www.seabird.com/products/spec_sheets/37smpdata.htm (Last viewed 6/11/2012).
- ²³Sea-Bird Electronics, Inc., "Temperature and pressure recorder SBE 39," Technical Report, Sea-Bird Electronics, Inc., 13431 NE 20th Street, Bellevue WA (2012), available at http://www.seabird.com/products/spec_sheets/39data.htm (Last viewed 6/11/2012).
- ²⁴W. H. F. Smith and D. Sandwell, "Global seafloor topography from satellite altimetry and ship depth soundings," *Science* **277**, 1956–1962 (1997).
- ²⁵F. S. Henyey and S. A. Reynolds, "A numerical simulator of ocean internal waves for long-range acoustics," Technical Report No. APL-UW TM 1-13, Applied Physics Laboratory, University of Washington, Seattle, WA (2013).
- ²⁶J. A. Colosi, L. J. van Uffelen, B. D. Cornuelle, M. A. Dzieciuch, P. F. Worcester, B. D. Dushaw, and S. R. Ramp, "Observations of sound-speed fluctuations in the western Philippine Sea in the spring of 2009," *J. Acoust. Soc. Am.* **134**, 3185–3200 (2013).
- ²⁷V. V. Pisareva, "O granitsakh primenimosti metoda 'plavnykh' ozmushcheniĭ v zadache o rasprostraneniĭ izlucheniya cherez sredy neodnorodnostyami" ("Limits of the applicability of the method of 'smooth' perturbations in the problem of radiation propagation through a medium containing inhomogeneities"), *Akust. Zh.* **6**, 87–91 (1960).
- ²⁸A. D. Wheelon, *Electromagnetic Scintillation 2, Weak Scattering* (Cambridge University Press, Cambridge, UK, 2003), pp. 23–30.
- ²⁹S. M. Flatté and G. Rovner, "User's guide to CAFI" (1998), available at http://oalib.hlsresearch.com/Other/cafi/CAFI_Guide.pdf (Last viewed 3/29/2015).
- ³⁰S. M. Flatté and G. Rovner, "Calculations of internal-wave-induced fluctuations in oceanacoustic propagation," *J. Acoust. Soc. Am.* **108**, 526–534 (2000).
- ³¹R. Dashen, "Path integrals for waves in random media," *J. Math. Phys.* **20**, 894–920 (1979).
- ³²D. R. Palmer, "An introduction to the application of Feynman path integrals to sound propagation in the ocean," Naval Research Laboratory Report No. 8148, Naval Electronic Systems Command, Washington, D.C. 20360 (1978).
- ³³S. Tombul, "A numerical study of the validity regimes of weak fluctuation theory for ocean acoustic propagation through random internal wave sound speed fields," Master's thesis, Naval Postgraduate School, Monterey, CA, 2007.

**Low-frequency Pulse Propagation over 510 km in the Philippine Sea: A
Comparison of Observed and Theoretical Pulse Spreading**

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Low-frequency pulse propagation over 510 km in the Philippine Sea: A comparison of observed and theoretical pulse spreading

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Observations of the spread of wander-corrected averaged pulses propagated over 510 km for 54 h in the Philippine Sea are compared to Monte Carlo predictions using a parabolic equation and path-integral predictions. Two simultaneous *m*-sequence signals are used, one centered at 200 Hz, the other at 300 Hz; both have a bandwidth of 50 Hz. The internal wave field is estimated at slightly less than unity Garrett-Munk strength. The observed spreads in all the early ray-like arrivals are very small, <1 ms (for pulse widths of 17 and 14 ms), which are on the order of the sampling period. Monte Carlo predictions show similar very small spreads. Pulse spread is one consequence of scattering, which is assumed to occur primarily at upper ocean depths where scattering processes are strongest and upward propagating rays refract downward. If scattering effects in early ray-like arrivals accumulate with increasing upper turning points, spread might show a similar dependence. Real and simulation results show no such dependence. Path-integral theory prediction of spread is accurate for the earliest ray-like arrivals, but appears to be increasingly biased high for later ray-like arrivals, which have more upper turning points. © 2016 Acoustical Society of America.

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I. INTRODUCTION

Time-domain statistics of low-frequency long-range sonar signals are important for underwater acoustic communication and navigation, but the current theory that predicts these statistics is not as broadly verified as the theory for frequency-domain statistics. This study presents observations of the spread of wander-corrected averaged pulses propagated over 510 km for 54 h in the Philippine Sea, and compares the measured spread against simulations from Monte Carlo predictions using a parabolic equation (PE), and against path-integral predictions. Two simultaneous *m*-sequence signals are used, one centered at 200 Hz, the other at 300 Hz; both have a bandwidth of 50 Hz. Pulses are isolated in the early ray-like part of the timefront, and collected on several dozen hydrophones on a water-column-spanning mooring. The observed spreads are very small, on the order of the sample rate period. Monte Carlo predictions show similar very small spreads. Path-integral theory prediction of spread is accurate for 17 and 18 total turning points (8 to 9

upper turning points), contrary to results reported for much longer ranges. However, path-integral predictions appear to be increasingly biased high with increasing numbers of turning points.

In 1999, Colosi *et al.*¹ noted that pulse spread predictions at 75 Hz over 3250 km in the Acoustic Thermometry of Ocean Climate (ATOC) Engineering Test were nearly two orders of magnitude too large and Colosi *et al.*² concluded that wide-band (i.e., pulse) propagation predictions remain an unsolved problem in long-range ocean acoustics. Further efforts bore out that assessment. In 2000, Flatté and Rovner³ suggested that the pulse-spread prediction from Flatté-Dashen theory did not seem to be well-understood in long-range low-frequency regimes. Finally, in 2003, Flatté and Vera conducted a comparison of Flatté-Dashen theory to Monte Carlo parabolic equation (MCPE) simulations at 250 Hz and 1000 km, and offered evidence that Flatté-Dashen theory over-predicted the MCPE pulse spread by an order of magnitude.⁴

More recently, the desire to send communication and navigation information over long ranges in the ocean has increased the importance of utilizing time-domain waveforms. Communication theory has a long history in understanding and developing fading multipath channels. The confluence of

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communication theory concepts with underwater acoustical propagation appears now to be the logical evolution of stochastic wave propagation research in underwater acoustics.

The analysis presented here involves a three-way comparison between measurements made in the Philippine Sea, extensive MCPE simulations, and theoretical calculations. The theoretical context is provided in Sec. II. Section III describes the background for the three datasets. Each dataset consists of depth- and path-indexed pulse spreads and associated statistics. First, the actual experiment is described, followed by the data processing used on the at-sea data necessary to calculate the spreads. Second, the MCPE simulations are described, including the input parameters, which were tuned to mimic at-sea conditions as closely as possible within the constraints of the internal wave model. Third, the input parameters and the physical interpretation of the calculated measures are described for corresponding theoretical calculations, which implement Flatté-Dashen theory. The main results are given in Sec. IV and compare pulse spreads observed in the Philippine Sea data, estimated from the MCPE simulations and calculated from Flatté-Dashen theory. Discussion of these results is given in Sec. V.

II. PULSE SIGNAL MODEL

Following Worcester *et al.*,⁵ if $p(t)$ is the pulse at a receiver due to a transmitted pulse,

$$\langle |p(t)|^2 \rangle = \iint d\omega_1 d\omega_2 \tilde{h}^*(\omega_2) \tilde{h}(\omega_1) \Gamma(\omega_1, \omega_2) e^{i(\omega_1 - \omega_2)t}, \quad (1)$$

where $\tilde{h}(\omega)$ is the Fourier transform of a transmitted pulse $h(t)$, $\Gamma(\omega_1, \omega_2)$ is the mutual coherence function (MCF) and the notation $\langle \cdot \rangle$ throughout denotes an ensemble average. The MCF is a Fourier domain function, which plays a central role in the analysis of pulse propagation experiments. In the acoustics literature, it has been called the MCF of frequency,^{6,7} the two-frequency MCF,^{8,9} and the bichromatic MCF.¹⁰ Traditionally, derivations for it are sought first, and then predictions on the time-domain dual, which is an ensemble-averaged pulse, are computed using a Fourier or Fourier-like synthesis. Dashen *et al.*⁶ derived expressions for the MCF for the stochastic ocean propagation problem, using a path-integral treatment. Flatté and Stoughton¹¹ provided an alternate expression using Floquet theorems to exploit the periodicity inherent in vertically cycling eigenray paths. More recently, Kalugin *et al.*¹² used local transformations of the PE, a technique originally introduced by Mazar and Felsen.¹³

Flatté-Dashen theory assumes that $\Gamma(\omega_1, \omega_2)$ is described by the model

$$\Gamma(\bar{\omega}, \Delta\omega) = e^{-(\Delta\omega^2/2\bar{\omega}^2)\Phi^2} Q(\Delta\omega) e^{-i\Delta\omega\tau_1}, \quad (2)$$

at least for small $\Delta\omega$. Here, $\Delta\omega = (1/2)(\omega_2 - \omega_1)$, $\bar{\omega} = (1/2)(\omega_2 + \omega_1)$, Φ^2 is the phase variance (of a single frequency phasor) due to wander, and τ_1 represents a mean bias or delay in arrival time. The Q function represents the additional decoherence when more than one path reaches the receiver.

In this study, the probe signal bandwidth was narrower than the bandwidth of the MCF and, therefore, the direct frequency-domain approach of Havelock *et al.*⁸ for estimating the MCF is not practicable. Instead, this analysis examines a time-domain measure—the pulse spread—of the observed and simulated data.

III. METHOD

A. Philippine Sea data

1. The 510 km pulsed propagation experiment

The at-sea observations were obtained during an experiment conducted 5–29 May 2010 in the Philippine Sea. This effort was part of a larger multi-cruise, multi-institution “PhilSea10” experiment¹⁴ to investigate the deterministic and stochastic properties of long-range deep ocean sound propagation in a region of energetic oceanographic processes. The assets for this experiment are shown in Fig. 1. A transmitter package was suspended from the R/V *Roger Revelle* at location “SS500” at 19°00.00’N, 130°12.00’E to a depth of 998 m. The acoustic data were collected by an autonomous vertical line array (VLA) receiver deployed to the seafloor at nominally 21°21.7418’N, 126°0.7867’E, at a bearing of 302° and a range of 509 783 m from the transmitter. Typical eigenrays are shown in Fig. 2: depth excess along the propagation path allowed a significant number of non-boundary interacting paths from transmitter to receiver. The ranges of upper turning point depths for the paths with unique, resolvable pulses are shown in Table I.

The transmitter package has been described by Andrew¹⁶ and White *et al.*¹⁷ Briefly, the package consisted of the experimental double-ported doubly resonant electro-acoustic MP200/TR1446 transducer (designed by ImageAcoustics Inc., Cohasset, MA and manufactured by Massa Products, Corp, Hingham, MA), a matching auto-transformer/tuner manufactured by Coiltron, Inc. (Tigard, OR), a tracking pinger, auxiliary sensors, a monitor hydrophone attached to the deployment cable 20 m above the MP200/TR1446, and a SeaBattery (Deepsea Power and Light, San Diego, CA) for sensor electronics power. The transmitter was synchronized to the global positioning system (GPS) for precision drive waveform delivery to the power amplifier. A long baseline acoustical navigation array was established at SS500 to track the three-dimensional (3-D) location of the transmitter package. Additionally, the R/V *Roger Revelle* used dynamic positioning driven by a high-precision C-Nav (Lafayette, LA) GPS to

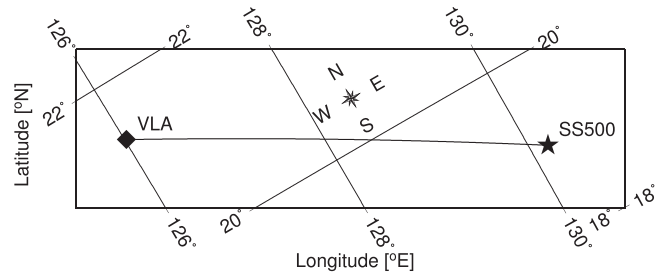


FIG. 1. Geographic configuration in the deep Philippine Sea for the pulse propagation experiment.

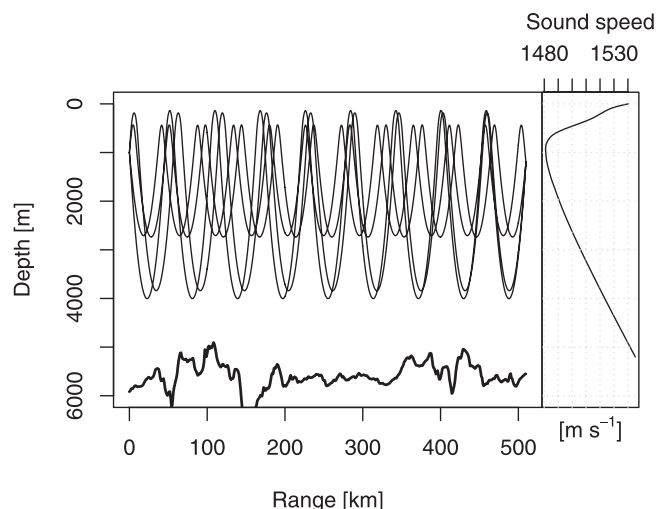


FIG. 2. (Left) Four representative eigenrays. The transmitter was at depth 998 m. Paths are shown to a typical receiver depth of 1200 m. Eigenrays launched upward are ray ID +18 and +22; eigenrays launched downward are -17 and -22. The thick solid line is bathymetry from SRTM30_PLUS (Ref. 15). (Right) The range-averaged sound speed profile is shown. This was also the sound speed profile used in the MCPE and CAFI calculations.

maintain station at the transmit location. Measurements indicated that the package did not deviate from the target transmit location by more than a couple of meters throughout the experiment.

The VLA concept has been described by Worcester *et al.*¹⁸ and the PhilSea10 configuration by Worcester *et al.*¹⁴ The instrument was designed to capture as much of the full-water-column acoustic field in deep water as possible. The PhilSea10 mooring included 5 autonomous subarrays of hydrophone modules for a combined total of 150 hydrophone modules. Each subarray had one controller. Each hydrophone module was an intelligent sensor package containing the transducer, signal conditioning and digitizing circuitry, a microcontroller, a local clock, flash memory, and a battery. The modules were commanded by their associated controller using inductive modems communicating over the mooring wire itself. The number of hydrophone modules and the inter-element spacing varied within and between subarrays. Nominal configurations are shown in Table II. The inter-element spacing throughout the middle of the deep sound channel was designed to resolve low-frequency low-order modes, while the shallower and deeper sensor spacings were larger to achieve (nearly) full water column coverage. The mooring was also outfitted with Sea-Bird SBE37-SMP pumped conductivity-temperature-pressure (CTD) microCAT recorders (Sea-Bird Electronics, Bellevue, WA),¹⁹ Sea-Bird SBE39 temperature recorders,²⁰ and a long baseline acoustical navigation system, which navigated the controllers on the

TABLE I. Upper turning point depth spans in meters for those paths which yielded unique, resolvable pulses in the timefront. Eigenray paths computed using the sound speed profile in Fig. 2 and the computer code ray (Ref. 35).

Ray ID	+18	+19	+20	+21	+22
Depths	139–195	221–242	308–335	354–364	422–432
–17	–18	–19	–20	–21	Ray ID
106–150	183–218	294–311	335–351	409–420	Depths

TABLE II. Hydrophone module subarray population, inter-element spacing, and nominal depths for the five VLA subarrays.

Subarray	Depths (m)	Spacing (m)	Number of modules
1	180–540	40	10
	580–1180	20	31
2	1200–2120	20	47
	2160–2200	40	2
3	2280–3240	60	17
4	3360–4320	60	17
5	4383–5381	40	26

mooring once per hour. The hourly position solutions were considered accurate to within a couple of meters. Positions between hours used linear interpolation between adjacent hours: positions of hydrophone modules used linear interpolation between adjacent controller positions.

2. Environmental section

From May 15 to May 21, the R/V *Roger Revelle* obtained a CTD section along the nominal acoustic propagation path between SS500 and the VLA. Casts were made every 10 km. Most casts were made to a depth of about 1500 m, starting from the VLA. Every fifth cast was to within several hundred meters of the bottom, roughly 5000–6000 m, depending on location. This section involved 51 casts. The difference between the derived sound speed and the corresponding World Ocean Atlas (WOA) 2009 May climatology^{21–23} in the upper ocean is shown in Fig. 3. There was a significant warm-core eddy down to roughly 300 m toward SS500, but colder than usual water closer to the VLA. These are typical time-varying mesoscale features of the Philippine Sea not captured by the standard oceanographic climatologies.

3. The at-sea pulsed signals

The MP200/TR1446 transducer has sharp resonances at about 210 Hz and 320 Hz. In the PhilSea09 Pilot Study/

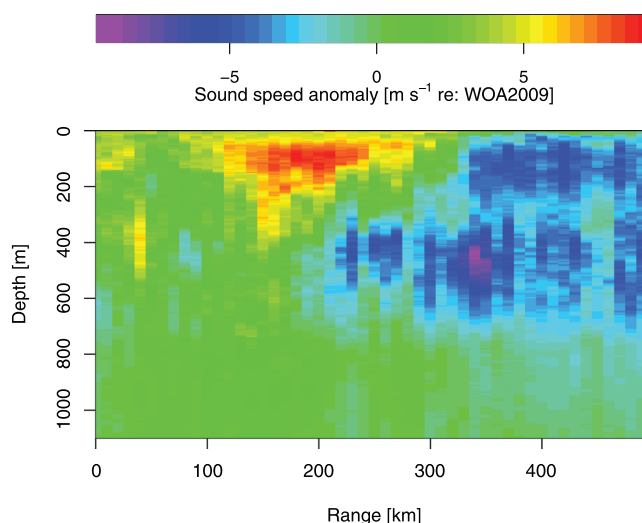


FIG. 3. Section sound speed difference from the WOA2009 climatology. The reference sound speed was computed from May WOA2009 temperature and salinity, and converted to sound speed using TEOS-10 (Ref. 23). Range is section distance from SS500.

Engineering Test,¹⁷ the transducer was treated as a device with a single passband from below 210 Hz to above 320 Hz, and a 130-Hz-wide drive signal was centered between the two resonances. In the PhilSea10 experiment, the transducer was treated as two separate devices with passbands around each resonance. The drive signal was a superposition of two *m*-sequences²⁴ with carriers of 200 and 300 Hz. Based on an optical analogy, the 200 Hz (longer wavelength) signal will be denoted the “red” signal, and the 300 Hz (shorter wavelength) signal will be denoted the “violet” signal. Parameters for these signals are shown in Table III: the parameters were selected so that each *m*-sequence was 20.46 s in duration in order to superpose exactly.

Transmissions began 2010 May 8 at 16:48:10 UTC (Universal Coordinated Time) and continued for 54 h. Each transmission involved a preamble and guard sequence, followed by 161 end-to-end, identical, interrogation sequences, for a total of 8694 interrogations. A transmission gap occurred at the end of each 161 sequence block to allow the transmitter to resynchronize data capture files, rest the power amplifier, and provide an ensionification-free time window at the VLA for mooring navigation. The source level, measured in the water with the monitor hydrophone, was roughly 191 dB re: 1 μ Pa at 1 m, corrected to broadside.

The sharp resonances of the transducer induced considerable unacceptable distortion to the radiated signal spectra and, therefore, equalization was required to obtain sharp pulses after pulse-compression. The equalization filters were designed using the actual radiated waveforms as measured on the monitor hydrophone. The Fourier characteristics of the filters and their effects are shown in Figs. 4 and 5. The filters greatly reduce the trailing “afterglow” and deliver clean, narrow pulses. These were not perfectly triangular pulses, the perfect theoretical shape, but were slightly rounded due to the finite bandwidth of the transducer. In addition, poor signal-to-noise ratio in the signal sidelobes prevented high-fidelity equalization of the higher frequencies, further rounding the equalized time-domain pulse.

4. Data processing and reduction

The data records from each hydrophone were first processed to yield pulse-compressed data frames. Processing involved (a)

TABLE III. MP200/TR1446 dual *m*-sequence signal parameters. The drive signal was the superposition of both red and violet signals. Neither the red nor violet signal was equalized prior to transmission.

Parameter	Red	Violet
Carrier	200 Hz	300 Hz
Law	2033	3471
Sequence length	1023	1023
Cycles/digit	4	6
Digit length	20.00 ms	20.00 ms
Sequence length	20.46 s	20.46 s
Phase mod angle	88.209°	88.209°
Bandwidth	50 Hz	50 Hz
Sequences	161/hr	161/hr
Total sequences	8694	8694

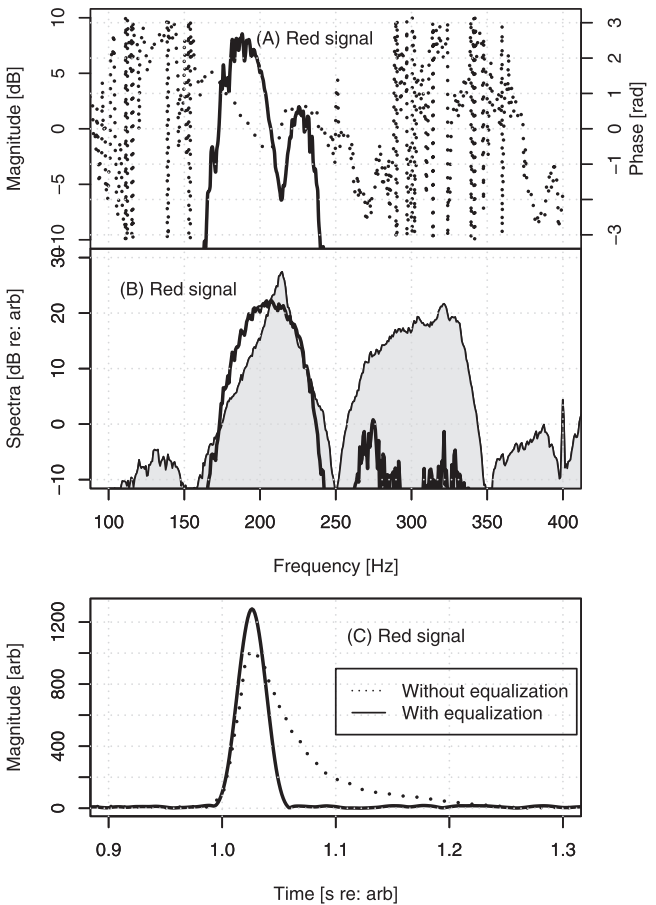


FIG. 4. PhilSea10 “red” signal features. (Top) The Fourier domain magnitude (black line) and phase (dashed line) of the red equalization filter; (middle) the autospectra over 30 s of the radiated signal (grey-filled) and after application of the red signal filter (black line); (bottom) the pulse-compressed pulse before and after equalization.

correction (via a depth-, module-, and frequency-invariant transfer function) to in-water units of μ Pa, (b) data compression or expansion via a chirp-Z transform to counter any local hydrophone module timebase dilation, (c) forward-backward bandpass filtering with second-order Butterworth filters with red (violet) bandwidth of 175 Hz (220 Hz) to attenuate out-of-band noise, (d) filtering (in the frequency domain) by the appropriate post-equalizer filter function, (e) resampling to a standard *m*-sequence frequency of four times the carrier frequency, and (f) pulse-compression. Additionally, the hydrophone module records were time-shifted to account for timebase drift between the Maxim DS4026 temperature compensated crystal oscillator (TCXO) module clock (Maxim Integrated, San Jose, CA) and the controller Q-Tech (Culver City, CA) micro-computer compensated crystal oscillator (MCXO) QT2002 clock, and between the QT2002 and the controller Symmetricom X72 (San Jose, CA) rubidium frequency standard, which had been disciplined to UTC via a GPS link prior to deployment. Finally, each frame was time-shifted to the surveyed center VLA position, based on the 3-D navigation solution for the mooring at each frame time. No timebase corrections were made for transmitter position error because deviations of the transmitter from its design target location were deemed too small to be significant.

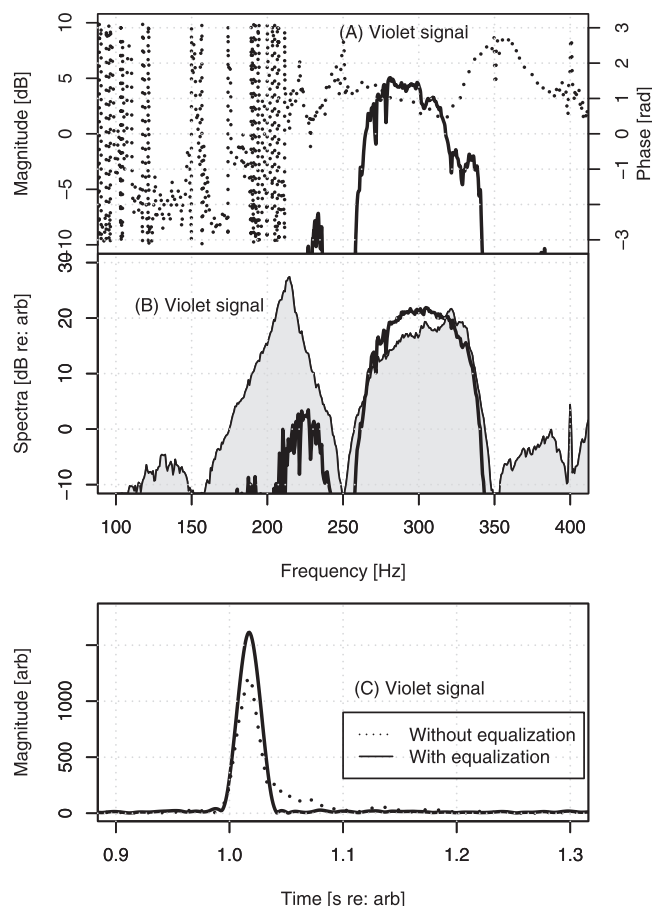


FIG. 5. PhilSea10 “violet” signal features. (Top) The Fourier domain magnitude (black line) and phase (dashed line) of the violet equalization filter; (middle) the autosepectra over 30 s of the radiated signal (grey-filled) and after application of the violet signal filter (black line); (bottom) the pulse-compressed pulse before and after equalization.

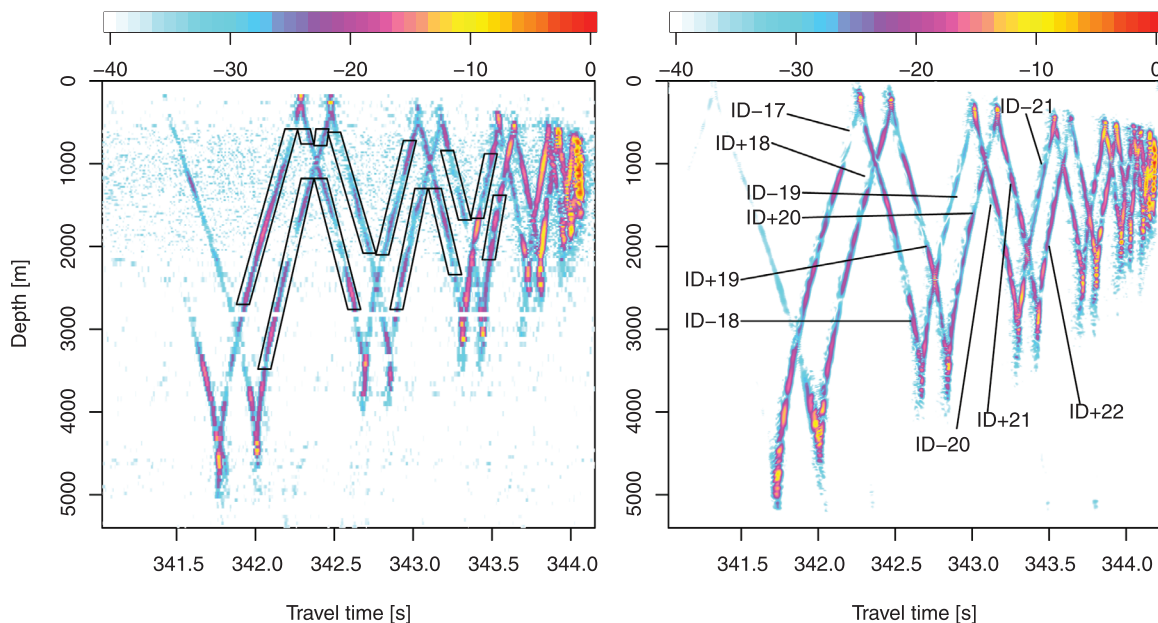


FIG. 6. Example timefronts. (Left) Entire (149 hydrophones) observed timefront after post-equalization and pulse compression, red signal, m -sequence transmitted 2010/2005/08 16:48:10. Polygons show roughly the timefront regions extracted for analysis. The violet signal timefronts had a similar appearance; the polygons for the violet signal pulses were nearly identical to those of the red signal timefront. (Right) Emulated MCPE timefront realization for the 300 Hz simulation (corresponding to the violet signal). The 200 Hz timefront emulations were similar. The original solution was produced on a 1 m depth grid; the MCPE timefronts were resampled to a 10 m grid for plotting. The branches are labeled with their corresponding ray identifiers. Each timefront has been normalized by its largest value. The color scales shown on top are decibels.

The composition of each frame for all hydrophones yields the observed timefront: the unparalleled size of the VLA provides nearly complete full-water-column resolution of the timefront. An example timefront using the red signal is shown in Fig. 6. The timefront consists of early, ray-like arrivals and a complicated unresolved finale. A 10-h record of red and violet pulses for ray ID-17 on hydrophone module 57 is shown in Fig. 7: the records are somewhat identical, but the fades and twinkles are slightly different. It should be noted that both signals travel through the same path to the receiver; any loss of correlation is therefore directly indicative of a decrease in the MCF between (roughly) 200 and 300 Hz. A more complete frequency domain analysis of the MCF spanning the red and violet signals will be pursued in a future paper.

The noise level associated with each frame was required. The ambient noise at the receivers varied throughout the experiment due to nearby shipping, local weather conditions and other interfering in-water signals. Rather than estimate the actual ambient noise levels, a representative level was determined from the final pulse-compressed frames. The frames were time shifted so that the finale appeared approximately in the middle of each 20.46 s frame. The early arrivals appear prior to this time. After this time, the data dropped to about 10 s of more-or-less stationary complex noise of constant character, which we consider to be the background noise process in the context of the pulse-compressed waveforms. The final 4 s of this complex noise were extracted and processed into autospectral estimates via the Welch method using 1024-pt blocks with 50% overlap. The lowest Fourier coefficient was then used to calculate a representative background level.

The first stage of data reduction followed that of White *et al.*¹⁷ two-dimensional regions of the timefront containing

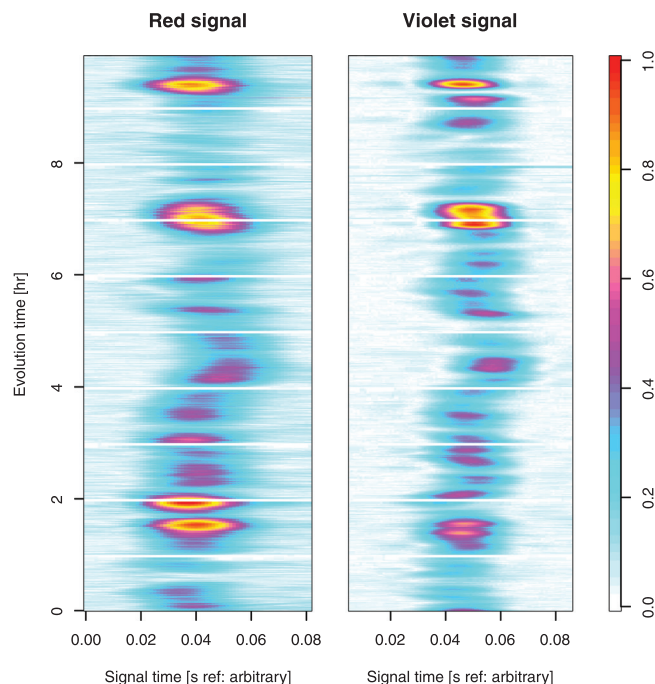


FIG. 7. Pulse-by-pulse magnitude records after timing correction, post-equalization, and pulse compression for 10h starting (at the transmitter) at 2010/05/08 16:48:10. Gaps occur hourly for VLA navigation. (Left) Red signal. (Right) Violet signal. Ray ID –17. Both signals were in the water at the same time and were separated in post-processing in code space. Data are normalized by the maximum value. The color scale is linear in amplitude.

distinct, non-overlapping sections of branches were identified. These are shown for the red signal in Fig. 6. The (complex) waveforms for each region for all associated hydrophone records were extracted. These waveforms formed the sample pulse population. The timefront branches were easily identified with downward-launched eigenray identifiers –17, –18, –19, –20, –21, and upward-launched identifiers +18, +19, +20, +21, and +22. Waveforms were only extracted where there was an unambiguous branch: regions at the cusps and at the branch crossings were therefore excluded in order to avoid introducing the additional complexity of interpreting and decomposing interfering pulses. The consistency of the polygon boundaries along the branches made it feasible to identify and extract the waveform containing the pulse, even if the pulse itself was deeply faded and buried in noise. Such deeply faded and barely recognizable pulses (if recognizable at all) were generally down-weighted in subsequent averaging for being too corrupted by noise.

In the second stage of data reduction, the ensemble-averaged pulse was estimated. The ensemble-averaged pulse is represented by an arithmetic average of the observed data. Although each data frame is synchronized to GPS and synthetically time-shifted to eliminate arrival time offsets due to differences from the nominal transmitter-receiver distance, the arrival times of the observed pulses still wander forward and backward due to random ocean propagation influences. Correcting for this ocean-induced wander isolates the Q function and permits an unambiguous test of this factor. Following Colosi *et al.*,¹ each waveform was therefore wander-corrected with an alignment algorithm. This was

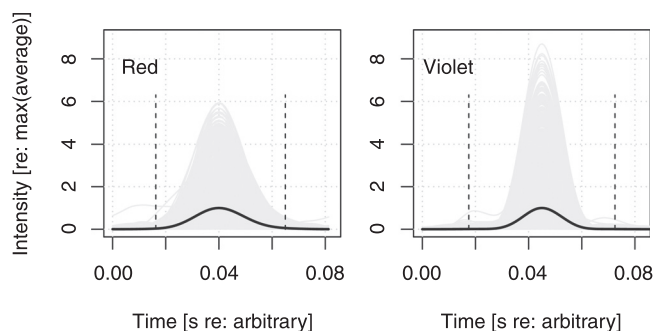


FIG. 8. Wander-corrected averaged pulses from HM S/N 57 ID –17. The individual wander-corrected sample pulses are shown in grey. There are 8692 red sample pulses and 8694 violet sample pulses. The averaged pulses are shown by black curves. All pulses are normalized by the maximum value of the associated averaged pulse. Vertical dashed lines indicate the boundary outside of which the waveforms are smoothly attenuated with cosine tapers to zero during the alignment step. (Left) Red signal. (Right) Violet signal.

straightforward because the majority of strong pulses observed in the data were unimodal and not seriously misshapen. (This is unlike the much longer range data of Colosi *et al.*¹)

Arithmetically averaging the resulting sample of wander-corrected waveforms still retained an inconsistency. During fades, particularly deep fades, the pulse is easily obscured by the background noise structure. If the observed signal is $s(t) = p(t) + n(t)$, where $p(t)$ and $n(t)$ are complex pulse and noise processes, then the background noise structure in $|s(t)|^2$ is all-positive, and does not average to zero. It was therefore desirable to down-weight the contribution of those waveforms representing a faded pulse. Including this weighting, the estimated wander-corrected pulse is $[\sum_i w_i |s_i(t)|^2] / (\sum_i w_i)$, where w_i is the weighting on the i th pulse and is a function of the noise level in the i th frame. As an example, Fig. 8 shows this calculation for the timefront branch ID–17 for hydrophone module S/N (serial number) 57, the population of observed pulses, and the associated averaged pulse over the entire experiment for both the red and violet signals. Averaged pulses were calculated for all receiver depth and timefront branch combinations for which unique and distinct pulses could be extracted. Pulses could be resolved on almost 100 hydrophones on as many as 10 separate branches. The total number of pulses processed was roughly 4.9×10^6 for either the red or violet signals.

Presentations of the observed wander-corrected pulses versus the transmitted pulses for both signals are shown in Fig. 9. The comparisons are cataloged by MP200/TR1446 signal color and eigenray ID. All the pulses for a given eigenray ID—i.e., for a single timefront branch—are normalized and overlaid. The transmitted pulse is also normalized and overlaid. The averaged pulses are very closely the same shape as the transmitted pulse. The red signal averaged pulses seem to have a slightly elevated trailing tail. The violet averaged pulses are fit very well by the transmitted pulse down to a local minimum near the bottom; further out, the pulses have some low-level structure. This is characteristic of all comparisons except for ID–17 and ID+19. The spread

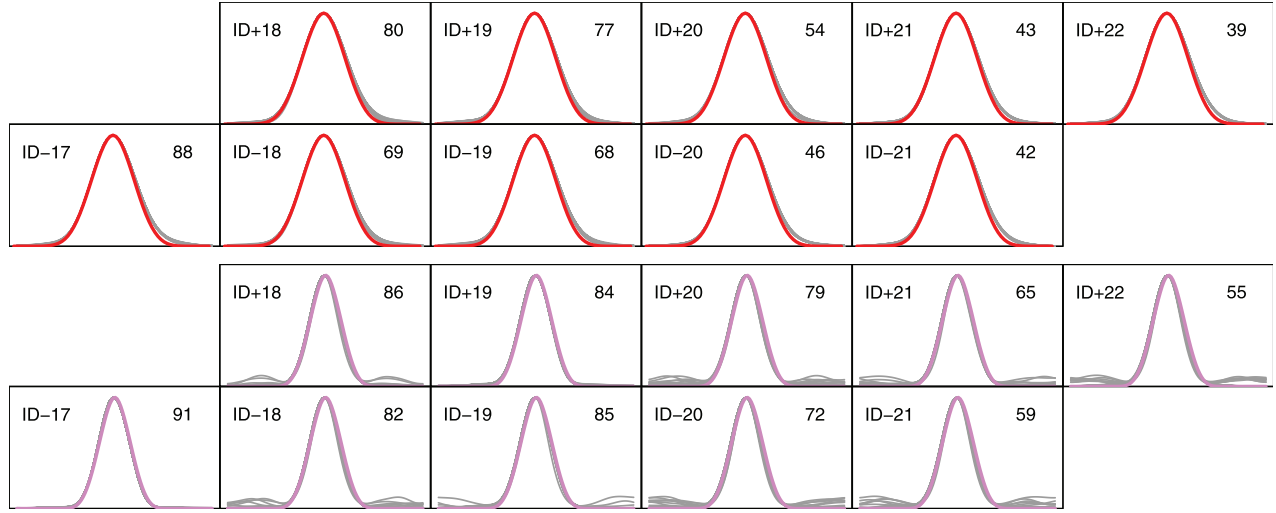


FIG. 9. Shape-by-shape comparison of MP200/TR1446 pulses as measured on the transmitter package monitor hydrophone, and the weighted-average pulses observed on the receiver. The timefront branch identifier is given in each panel in the upper left corner. The pulses are all normalized to unity maximum and drawn in grey: the number of pulses in each panel is given in the upper right corner of each panel. The transmitted pulse has also been normalized to unity and has been time-shifted by eye in order to align roughly with the pulses. (Top) “Red” signal, 66 points at 800 samples/s, spanning 82.5 ms. (Bottom) “Violet” signal, 110 points at 1200 samples/s, spanning 91.7 ms.

in the averaged pulses versus the transmitted pulses is very small and is difficult to gauge by eye.

B. Monte Carlo PE calculations

Standard wave-theoretic solutions in the paraxial approximation for the typical weakly range-dependent channels encountered in mid- to low-latitude long-range deep water ocean acoustic scenarios (where bathymetric interaction can be neglected, and only fields propagating near horizontal are important) are defined by the PE. The PE solver was then embedded in a Monte Carlo algorithm to simulate the distribution of the wave field statistics. Each Monte Carlo realization builds an acoustic channel on the background sound speed field by adding a random realization of sound-speed anomalies. Following Colosi and Brown,²⁵ the anomalies are defined as

$$\delta c(x, z) = \frac{\partial \bar{c}_P}{\partial z} \zeta(x, z) \quad (3)$$

where x is range, z is depth, $\bar{c}_P$ is a background potential sound speed, and $\zeta(x, z)$ is a random vertical fluid displacement field due to internal waves. The theory for devising an anomaly field from an ocean with range-dependent background temperature and salinity fields is not well-established. Therefore, following White *et al.*,¹⁷ the problem is simplified by considering the slowly varying background $\bar{c}(x, z)$ to be a range-independent function $\bar{c}(z)$. In this ansatz, the internal wave field can be defined with the Garrett-Munk GM79 model.²⁶

The key environmental factors required for the simulation of the $\zeta(x, z)$ field under the GM79 model are described in Sec. III B 1. Generation of the randomized sound speed field is described in Sec. III B 2. The full Monte Carlo calculation is briefly reviewed in Sec. III B 3. The data processing

applied to the simulated MCPE data is covered finally in Sec. III B 4.

1. Vertical displacement field

The vertical fluid displacement field was constructed using the Henyey-Reynolds algorithm.²⁷ In addition to the background sound speed profile $\bar{c}(z)$, the Henyey-Reynolds algorithm requires the Brunt-Väisälä buoyancy frequency profile $N(z)$, the potential sound-speed gradient $\partial_z \bar{c}_P$, and the strength product $bE_{GM;\Sigma}$ for a Garrett-Munk field with strength Σ . (The product $bE_{GM;\Sigma}$ always appears as a single term, and will be treated below as such.) The profiles were all derived by smoothing and averaging the CTD casts, described in Sec. III A 2, made along the section. The buoyancy frequency and potential sound-speed gradient profiles are shown in Fig. 10. The term $bE_{GM;\Sigma}$ is a key scalar parameter whose estimation is not as straightforward. This is given briefly next.

White *et al.*¹⁷ describe the estimation of the Garrett-Munk strength from month-long records of sound speed measured on the VLA mooring during the PhilSea09 Pilot Study/Engineering Test. A similar procedure was applied to the VLA mooring microCAT CTD records for the period encompassing this pulse propagation experiment. The procedure involves the ratio $\hat{r}$ of observed weighted integrated spectral energy $\hat{s}_{GM;\Sigma}$ of the inferred vertical fluid displacement at strength Σ to the weighted integrated spectral energy $s_{GM;1}$ of vertical fluid displacement based on the unity strength GM79 model. [The hat notation ($\hat{\cdot}$) represents an estimated value.] The weighting is the Brunt-Väisälä frequency profile $N(z)$; the product of the profile $N(z)$ and the vertical fluid displacement spectrum is depth invariant. Under the paradigm that this depth-invariant quantity $s_{GM;\Sigma} = (1/2)(bE_{GM;\Sigma})(N_0 b)$ one has

$$\widehat{bE}_{GM;\Sigma} = \hat{r}(bE_{GM;1}). \quad (4)$$

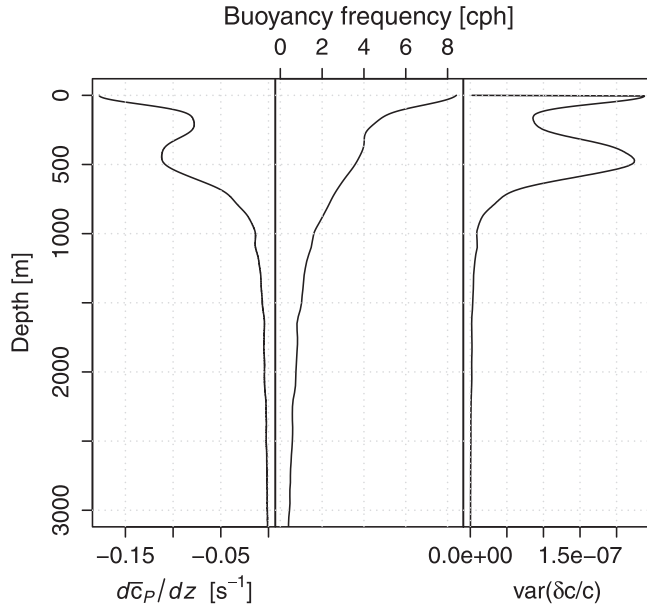


FIG. 10. Profiles for the internal wave simulations. (Left) The non-adiabatic sound-speed gradient, used to convert the vertical fluid displacement to sound-speed anomaly; (center) the Brunt-Väisälä buoyancy frequency profile; (right) the profile of the variance of the fractional sound-speed anomalies. The first two profiles were used in the MCPE simulations, the latter two profiles in the CAFI calculations. Only the top 3000 m of the water column are shown.

Here, the product $N_0 b$ is the integral of $N(z)$ over the water column, and will be treated as a single term. Under the standard model,²⁷

$$\frac{1}{2} (bE_{GM:1})(N_0 b) = 0.277 \text{ m}^2 \text{ rad s}^{-1} \quad (5)$$

for a unity strength Garrett-Munk internal wave field and, hence, $bE_{GM:1} = 0.055 \text{ m}$ for the value of $N_0 b = 10.13 \text{ m rad s}^{-1}$ computed from the CTD data. This defines the unity strength energy level. The ratio $\hat{r}$ was estimated to be 0.93 ± 0.078 (for 13 CTDs) and, therefore, $\widehat{bE}_{GM:\Sigma} = 0.051 \pm 0.004 \text{ m}$.

The Henyey-Reynolds algorithm²⁷ uses this value of $\widehat{bE}_{GM:\Sigma}$ and the other parameters shown in Table IV to generate realizations of the $\zeta(x, z)$ field.

TABLE IV. Sound speed perturbation field parameters used in the MCPE calculations. The term $n_0 B = \int n(z) dz$ and is computed to be roughly $10.129 \text{ rad m s}^{-1}$. (The integral is over the entire water column.) The value of f_* was taken from Colosi *et al.* (Ref. 36).

Parameter	Symbol	Value
Latitude		21.3624°
Buoyancy frequency	$N(z)$	See Fig. 10
	$N_0 b$	See caption
Inertial frequency	f_C	$5.2980 \times 10^{-5} \text{ rad s}^{-1}$
Modal bandwidth	j_*	6
Maximum modes	J	200
Reference sound-speed	c_0	1481.81 m s^{-1}
Background sound speed	$\bar{c}(z)$	See Fig. 2
Signal carrier frequencies	f	200 or 300 Hz
Strength	$\widehat{bE}_{GM:\Sigma}$	0.051 m
GM strength		0.93

2. Random sound speed field

Equation (3) and the profile $\partial_z \bar{c}_P$ are used to calculate the sound speed anomaly field. This anomaly field is added to the background sound speed profile $\bar{c}(z)$ to generate the final realizations of the range-depth sound speed field $c(x, z)$.

3. MCPE calculation

The PE calculations were carried out using the split-step Padé algorithm in the Navy Standard Parabolic Equation (NSPE).²⁸ Use of the NSPE in the analysis of the PhilSea09 experiment was outlined in White *et al.*:¹⁷ application here was similar. Computation time scales with the number of Padé coefficients and, therefore, convergence tests were conducted on the number of Padé coefficients: six coefficients were found to give reasonable convergence. White *et al.*¹⁷ found that a range step of 25 m also provided reasonable convergence, and that was used here as well. Each NSPE computation produced an acoustic field solution $\psi(z, r = R; f)$ at the range R of the VLA for frequency f . A full-ocean depth grid with 1 m spacing was used. Solutions were generated on a grid of frequencies, weighted with a spectral shaping curve, and then inverse Fourier transformed to synthesize the time-domain timefront. The spectral shaping followed that used by White *et al.*¹⁷ The spectral shaping for the red simulation signal was designed to fit the dominant region of the equalized monitor hydrophone red signal spectrum. It was found that a Gaussian function fit the spectrum well, giving the parametric shaping $S(f) = \exp(-(f - f_o)^2 / (2\sigma^2))$. $S(f)$ was required to be $1/e$ at $f_{\pm} = f_o \pm B/2$, where f_o is the carrier frequency, $B = f_o/N_Q$ and N_Q is the number of cycles per digit. Hence, $\sigma = f_o / (2N_Q \sqrt{2})$. For the red signal ($f_o = 200 \text{ Hz}$), this located the -3 dB points of the amplitude spectrum at roughly 180 and 220 Hz. A similar procedure was followed for the simulations corresponding to the violet signal ($f_o = 300 \text{ Hz}$). Gaussian shaping in the Fourier domain results in Gaussian-shaped pulses in the time domain.

As an example, a violet signal timefront is shown in Fig. 6: the red signal timefront was similar.

Even though the split-step Padé algorithm in the NSPE is a wide-angle algorithm, appreciable pulse distortion occurs even for the shallow, long-range eigenrays used in this study. This distortion broadens the received pulse relative to the transmitted pulse. Figure 11 shows the widths of pulses in a red signal calculation using a sound speed field consisting solely of the range-independent background profile, i.e., without perturbations from internal waves. The measured widths are greater than the transmitted widths in the earlier, steeper arrivals, with widths converging toward the width of the transmitted pulse as the eigenray launch angles constrict toward the horizontal. These widths are mostly branch dependent, with a weaker dependence on depth. These broadened pulse widths, and not the original transmitted pulse widths, were used in subsequent spread calculations. Similar branch/depth dependent widths for a violet signal calculation (not shown) were used for the violet signal spreads.

Since the MCPE method is computationally intensive, only 240 realizations were obtained for each the 200 Hz and 300 Hz signals.

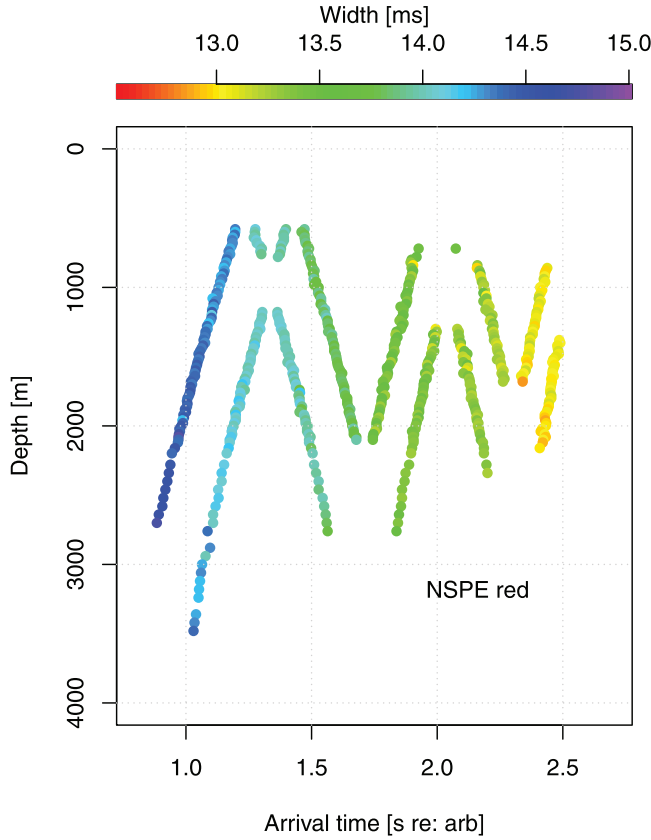


FIG. 11. Pulse widths measured using the NSPE, no scattering, 200 Hz carrier (“red signal”). Widths only computed for branch/depth combinations used in the MCPE simulations.

4. Simulation data processing

The MCPE timefronts were basebanded prior to data reduction. The first stage of data reduction selected uniquely identifiable sections of each timefront, corresponding to the at-sea ray paths ID–17, +18, –18, +19, –19, +20, –20, +21, –21, and +22. These sections were similar to those used for the at-sea timefronts. Complex waveforms were extracted for those depths that corresponded to the nominal hydrophone depths on the VLA. These waveforms formed the sample pulse population for the simulated data. In the second stage of data reduction, the pulses were similarly wander-corrected, squared, and averaged. Since there is formally no additive ambient noise in the simulation data, the weighted average utilized unity weighting on all pulses. Roughly 144 000 pulses were processed for either the red or violet MCPE simulations. Example averaged pulses are shown in Fig. 12.

C. CAFI calculations

The averaged pulse is often written as the Fourier transform of $\Gamma(\omega_1, \omega_2)$ as a function of $\Delta\omega$:

$$\langle I(t) \rangle \propto \int d\Delta\omega \Gamma(\Delta\omega) e^{i\Delta\omega t}. \quad (6)$$

This result can be derived, following Uscinski,²⁹ by using the change of variables $\Delta\omega = (1/2)(\omega_2 - \omega_1)$ and $\bar{\omega} = (1/2)(\omega_2 + \omega_1)$ in Eq. (1). Thus,

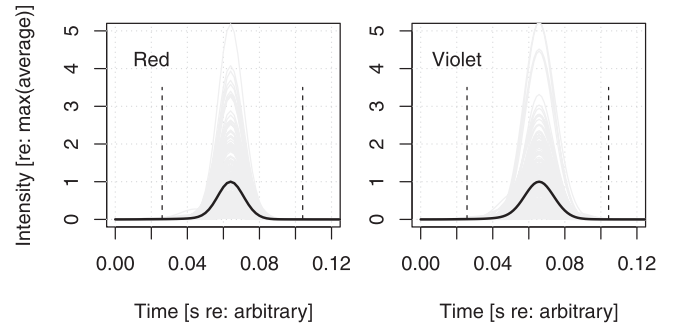


FIG. 12. MCPE ensemble averaged intensity pulses from depth $z = 1360$ m, corresponding to the nominal depth of VLA hydrophone S/N 057. Timefront branch corresponding to ID–17, 240 pulses. The pulses are normalized by the maximum value of the averaged pulse. Vertical dashed lines indicate the boundary outside of which the waveforms are smoothly attenuated to zero during the coarse alignment step. (Left) Red simulations. (Right) Violet simulations.

$$\langle |p(t)|^2 \rangle \propto \int \int d\bar{\omega} d\Delta\omega \tilde{H}(\bar{\omega}, \Delta\omega) \Gamma(\bar{\omega}, \Delta\omega) e^{i2\Delta\omega t}, \quad (7)$$

where

$$\tilde{H}(\bar{\omega}, \Delta\omega) = \tilde{h}^*(\bar{\omega} + \Delta\omega) \tilde{h}(\bar{\omega} - \Delta\omega). \quad (8)$$

Under the assumptions that $\Gamma(\bar{\omega}, \Delta\omega)$ does not vary significantly with $\bar{\omega}$ and therefore can be written approximately as $\Gamma(\Delta\omega)$, and that $\tilde{H}(\bar{\omega}, \Delta\omega)$ is approximately constant [say equal to $\tilde{H}_0(\bar{\omega})$] over the essential region of support of $\Gamma(\Delta\omega)$, the averaged pulse becomes

$$\langle |p(t)|^2 \rangle \approx \int d\bar{\omega} \tilde{H}_0(\bar{\omega}) \int d\Delta\omega \Gamma(\Delta\omega) e^{i\Delta\omega t}, \quad (9)$$

from which the form Eq. (6) can be obtained.

In order to interpret the Flatté-Dashen expression Eq. (2) in the context of the wander-corrected averaged pulses estimated in Secs. III A 4 and III B 4, one observes that wander-correction eliminates the variability represented by Φ .² Furthermore, referencing the averaged pulse to an arbitrary origin renders unnecessary any considerations of a mean delay. Under these conditions,

$$\Gamma(\bar{\omega}, \Delta\omega) \approx Q(\Delta\omega). \quad (10)$$

Flatté-Dashen theory uses path-integral techniques to evaluate the Q function.^{6,7} The resulting expression is complicated in the general case; Dashen *et al.*⁶ offers a simpler parametric approximation

$$Q(\Delta\omega) \approx e^{-(1/2)(\tau_0 \Delta\omega)^2}. \quad (11)$$

The code “calculations of fluctuations in internal waves” (CAFI)^{3,30} computes τ_0 .

Under approximation Eq. (10), the equation for $\langle |p(t)|^2 \rangle$ can be rewritten as

$$\begin{aligned} \langle |p(t)|^2 \rangle &\propto \int d\Delta\omega \left[\int d\bar{\omega} \tilde{H}(\bar{\omega}, \Delta\omega) \right] Q(\Delta\omega) e^{i\Delta\omega t} \\ &= \mathcal{F}^{-1} \{ \mathcal{F} \{ H(t) \} \mathcal{F} \{ q(t) \} \}, \end{aligned}$$

TABLE V. Input parameters for CAFI calculations.

Parameter	Value
Carrier f_o	200 or 300 Hz
Adiabatic gradient γ_A	$1.119 \times 10^{-5} \text{ m}^{-1}$
Extrapolated buoyancy frequency N_0	3 cph
Integrated buoyancy frequency $N_0 b$	5812 m cph
Modal bandwidth j_*	6.0
Reference displacement ζ_0	7.01 m

where $q(t) \rightleftharpoons Q(\Delta\omega)$ are Fourier duals and $H(t) \rightleftharpoons \int d\bar{\omega} \bar{H}(\bar{\omega}, \Delta\omega)$ are Fourier duals and $\mathcal{F}\{\cdot\}(\mathcal{F}^{-1}\{\cdot\})$ is a forward (inverse) Fourier transform. $H(t)$ is just $|h(t)|^2$. Using the Fourier duality of convolution and multiplication, the expression for $\langle |p(t)|^2 \rangle$ becomes

$$\langle |p(t)|^2 \rangle \sim |h(t)|^2 * q(t). \quad (12)$$

Thus, for wander-corrected data, there is a simple convolution interpretation to the Fourier synthesis of the ensemble averaged pulse. For reference, the Fourier transform of Eq. (11) is

$$q(t) \sim e^{-(1/2)t^2/\tau_0^2}, \quad (13)$$

and is also a Gaussian function.

The input profiles for CAFI are shown in Fig. 10. The input parameters are given in Table V. Typical scattering regime values for these calculations are $\Phi \approx 4$, $\Lambda\Phi \approx 0.8$, $\Lambda \approx 0.2$, and $\Lambda\Phi^2 \approx 3.4$ at 200 Hz. Values for 300 Hz are similar. These scattering regime values classify the regime as partially saturated.

IV. SPREAD RESULTS

Pulse spread will be defined as a width measure of the pulse minus the same measure of the transmitted pulse. That is, the spread of an arbitrary pulse waveform $p(t)$ based on a

reference pulse $p_0(t)$ will be defined as $\mathcal{S}(p; p_0) \equiv \mathcal{W}(p) - \mathcal{W}(p_0)$ where the width of a pulse p is $\mathcal{W}(p)$. In all cases presented here, Gaussian functions were fitted to the data, and the corresponding width metric derived from the estimated dispersion parameter. For a function $G(t) = e^{-(t/a)^2}$, one has $\mathcal{W}_G(G) = \sqrt{2}a$.

After equalization, the red and violet pulses measured on the monitor hydrophone still retained a rounded central peak, and fit very well throughout the central peak with a Gaussian function. A parametric fit gave the width $\mathcal{W}_G$ for the squared equalized transmitted red (violet) pulse of 17.38 ms (14.04 ms).

Comparisons of spreads for red and violet wave fields are shown in Figs. 13 and 14, respectively.

Since the at-sea records are serially correlated, the error in each pulse width estimator was estimated using the block bootstrap³¹ with a block size of 50 samples resampled 50 times. The estimated errors were not sensitive to greater block sizes. The MCPE records, however, are independent and, therefore, the error in each averaged pulse width estimator was estimated using the standard bootstrap with 50 resamplings. Two standard error bars are shown for both the at-sea data and the MCPE data in Figs. 13 and 14: many of the error bars are too small to see in these figures.

Figures 13 and 14 are the main results of this paper and show that the wander-corrected averaged pulse spread incurred over this 510 km section was very small, much less than the pulse width. In most cases, the spread is even less than the final frame sample rate period, which is 1.25 ms (833 μ s) for the red (violet) signal. The MCPE results conform very well to the at-sea measurements. There does not seem to be a dependence on the sign of the initial ray path angle. There is a trend for CAFI calculations in which the error is small for the earlier paths but the over-prediction seems to grow with the number of turning points. This suggests a dependence on initial ray path angle that does not appear in the real data or the simulation data.

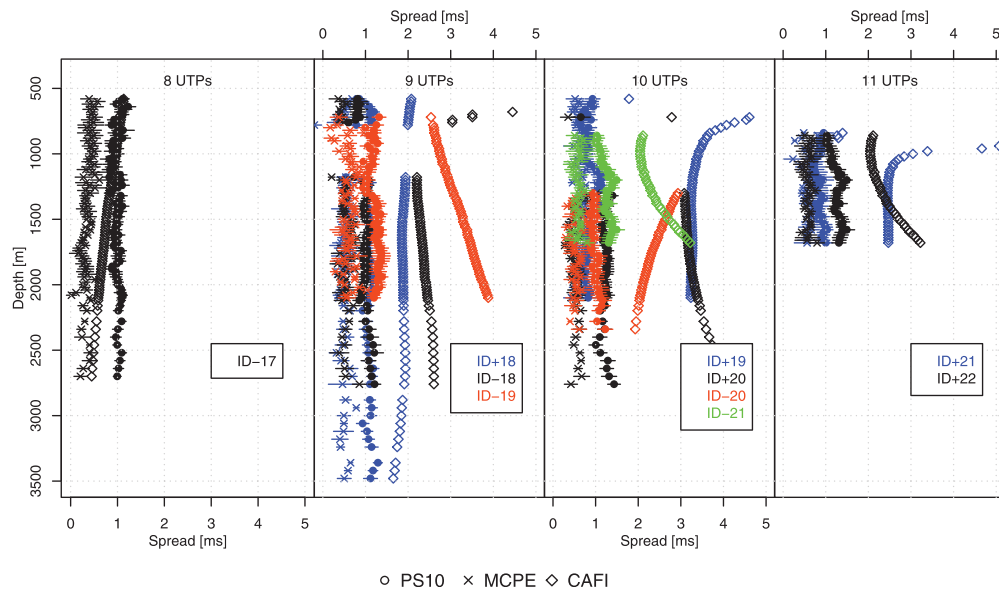


FIG. 13. Pulse spreads for the “red” signal in the PhilSea10 experiment. Panels grouped by total number of upper turning points (UTPs) along the path. Path identifiers are shown in the inset. Symbols denoting observed measurements, MCPE predictions, and CAFI calculations shown at the bottom.

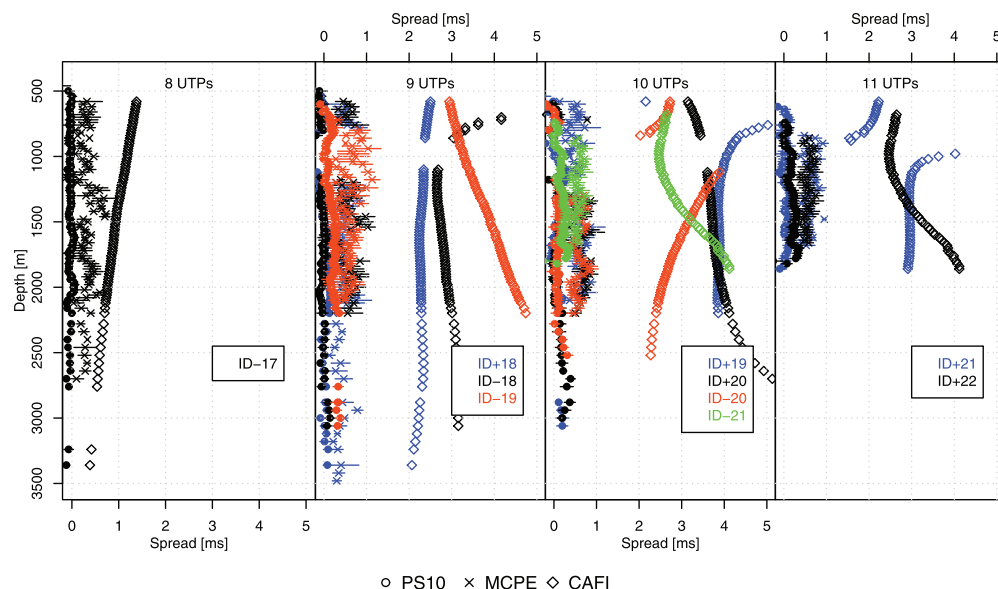


FIG. 14. Pulse spreads for the “violet” signal in the PhilSea10 experiment. Panels grouped by total number of upper turning points (UTPs) along the path. Path identifiers are shown in the inset. Symbols denoting observed measurements, MCPE predictions, and CAFI calculations are shown at the bottom.

V. DISCUSSION

Estimates of spread are clearly affected very strongly by the choice of a reference width. In the MCPE calculation, which is a controlled numerical experiment, the reference width was determined by turning off the internal wave perturbations. This cannot be done in the real world. For this real-world application, the field measured near the transducer was used as a surrogate for the unknown, early, ray-like, unscattered fields at 510 km. The validity of this choice relies on the angular/frequency dependence of the radiated acoustic field, the validity of the ray approximation, and the fundamental linearity of the wave equation.

The influence of scattering is generally assumed to grow with increasing frequency and, therefore, one might expect the spread to be larger for the violet signal than for the red signal. The results presented here suggest otherwise. The discrepancy may be due to inaccuracies in equalization. The equalizer filter, after all, is designed to sharpen the pulse. The performance of the two filters was not identical. The violet filter seemed to achieve a narrower pulse in the monitor hydrophone data than did the red filter. Figures 4 and 5 show that the radiated red signal spectrum required heavier modification than did the radiated violet spectrum to achieve the desired spectral shape. Ambient noise at the lower frequencies is also generally slightly greater than at higher frequencies, making red signal equalization slightly more problematic. Nevertheless, in both cases, the same filter was applied to the receiver data and the monitor data.

This may not have been the best form of equalization: the field measured at the monitor hydrophone, while possessing excellent signal-to-noise power ratio, was located in the end-fire direction of the MP200/TR1446, while the field observed at long range was radiated into a cone within roughly $\pm 12^\circ$ of broadside. Full dual resonance double doublet modeling of the transducer acoustic response, however, does not suggest that this configuration can account for

excessive sharpening of the violet receiver fields or excessive broadening of the red receiver fields. In fact, modeling suggests that end-fire/broadside variations might affect the red and violet signals similarly.

Unfortunately, we do not have MP200/TR1446 test data for both the red and violet signals in a broadside configuration to compare to the end-fire monitor hydrophone measurements and/or the transducer model, or for use in the construction of a broadside equalization filter. The degree to which the equalizer filters are over-sharpening the violet signal (or under-sharpening the red signal) is unknown without further system calibration.

Regardless, the at-sea results show very little pulse spreading at 510 km, a result corroborated by the MCPE simulations, in which no equalizer filters were involved. The CAFI calculation trends are consistent with the observations of Colosi *et al.*¹ that the pulse spread predictions were much larger—by as much as an order or two—for experiments at longer ranges (and hence many more turning points.) The over-prediction measured here, for up to 22 turning points, is about a factor of 3 for the red signal, but about a factor of 10 for the violet signal. These are the main results of this study.

Propagation through a randomizing medium is known to produce a trailing tail in the averaged pulse.²⁹ In the oceanic case, where there exists a sound channel, Dashen *et al.*⁶ argued that a precursor could also exist. After equalization and pulse-compression, leading or trailing sidelobes outside the triangle are not evident in the transmitted signal (Figs. 4 and 5). From Fig. 9, it can be seen that there is a slight trailing edge for the red signal averaged pulses. There appears to be no such feature for the violet signal averaged pulses. No precursors are evident. However, corresponding figures for the MCPE averaged pulses (not shown) do display precursors (but no trailing edges), confirming the Dashen assertion, at least for controlled simulation data.

Several authors^{32,33} have reported an additional feature in simulations in the averaged timefront: a low-level

background or plateau or pedestal extending far from prominent pulses. From Figs. 8 and 12, the PhilSea10 observations have almost all of the pulse amplitude concentrated within twice $\mathcal{W}_G$ from the mode of a given pulse. The low-level background, however, is described as extending much further, i.e., many times $\mathcal{W}_G$, from the mode of a given pulse. Beron-Vera *et al.*³² observed a “cloud” of very faint (i.e., very low amplitude) arrivals extending far from the prominent main arrivals. Hegewisch³³ also noted this feature in her simulations and called it a “fluctuation front.” It should be noted that these observations were made using simulated data, in which there is no additive ambient noise, only numerical noise. It is not clear that this front or cloud is strong enough to be seen in the weak signal-to-noise ratio situations encountered in actual long-range measurements.

If this front or cloud is some kind of scattering leakage from an individual pulse, then the convolutional kernel operator in Eq. (12) is responsible for a much greater morphological change than currently believed and, indeed, the very definition of pulse width would need serious revision. Such a feature cannot be a consequence of the convolution of a Gaussian kernel with the radiated magnitude-squared pulse, as indicated in Eq. (12). While it is true that a Gaussian kernel has infinite support, and that convolution against any finite support waveform will result in infinite support, the Gaussian tails fall off too fast to resemble the features reported by Beron-Vera *et al.*³² and Hegewisch.³³ Additionally, it has been noted by Dashen *et al.*⁶ that the quadratic exponential dependence of the MCF is only valid for small frequency differences: the tails of $\Gamma(\Delta\omega)$ are not well described by current theory, and it is these tails that would contribute most strongly to an extended low-level feature. Dashen *et al.*⁶ has indicated that the quadratic exponential model of Eq. (11) may be improved, particularly in the tails, by a higher order expansion in the underlying path-integral treatment, and this improvement may explain a broader low-level averaged pulse pedestal or background.

The MCF was not estimated directly in this study because the signal bandwidths used in this experiment were narrower than the coherent bandwidth of the channel. MCPE simulations should be used at these and greater ranges in order to image the actual MCF. MCPE simulations can implement signal spectra with unrealizably large bandwidth, and also do not suffer from additive ambient noise. Direct estimation of the MCF would permit a careful comparison of the relationship of the averaged pulse and MCF.

The critiques of path-integral theory by Flatté,³⁴ Colosi *et al.*,² Colosi *et al.*,¹ and Flatté and Rovner³ are not warranted at 510 km for the earliest arriving pulses: Flatté-Dashen theory as implemented in CAFI characterizes wander-corrected broadband behavior well, at least with regard to the average pulse spread, for eight upper turning points. Those critiques gain significance, however, with increasing numbers of turning points: CAFI calculations become increasingly inaccurate and appear to over-predict the pulse spread as the number of turning points increase. Low-frequency experiments should be conducted at ranges from 500 km to more than 1 Mm to determine if Flatté-Dashen theory indeed has an increasing discrepancy with increasing turning points.

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- ¹J. A. Colosi, E. K. Scheer, S. M. Flatté, B. D. Cornuelle, M. A. Dzieciuch, W. H. Munk, P. F. Worcester, B. M. Howe, J. A. Mercer, R. C. Spindel, K. Metzger, T. G. Birdsall, and A. B. Baggeroer, “Comparisons of measured and predicted acoustic fluctuations for a 3250-km propagation experiment in the eastern North Pacific Ocean,” *J. Acoust. Soc. Am.* **105**, 3202–3218 (1999).
- ²J. A. Colosi, A. B. Baggeroer, T. G. Birdsall, C. Clark, B. D. Cornuelle, D. Costa, B. D. Dushaw, M. A. Dzieciuch, A. M. G. Forbes, B. M. Howe, D. Menemenlis, J. A. Mercer, K. Metzger, W. Munk, R. C. Spindel, P. F. Worcester, and C. Wunsch, “A review of recent results on ocean acoustic wave propagation in random media: Basin scales,” *J. Ocean. Eng.* **24**, 138–155 (1999).
- ³S. M. Flatté and G. Rovner, “Calculations of internal-wave-induced fluctuations in ocean-acoustic propagation,” *J. Acoust. Soc. Am.* **108**, 526–534 (2000).
- ⁴S. M. Flatté and M. D. Vera, “Comparison between ocean-acoustic fluctuations in parabolic-equation simulations and estimates from integral approximations,” *J. Acoust. Soc. Am.* **114**, 697–706 (2003).
- ⁵P. F. Worcester, G. O. Williams, and S. M. Flatté, “Fluctuations of resolved acoustic multipaths at short range in the ocean,” *J. Acoust. Soc. Am.* **70**, 825–840 (1981).
- ⁶R. Dashen, S. M. Flatté, and S. A. Reynolds, “Path-integral treatment of acoustic mutual coherence functions for rays in a sound channel,” *J. Acoust. Soc. Am.* **77**, 1716–1722 (1985).
- ⁷S. A. Reynolds, S. M. Flatté, R. Dashen, B. Buehler, and P. Maciejewski, “AFAR measurements of acoustic mutual coherence functions of time and frequency,” *J. Acoust. Soc. Am.* **77**, 1723–1731 (1985).
- ⁸D. I. Havelock, M. R. Stinson, and G. A. Daigle, “Measurements of the two-frequency mutual coherence function for sound propagation through a turbulent atmosphere,” *J. Acoust. Soc. Am.* **104**, 91–99 (1998).
- ⁹J. H. Targ, L. K. Wang, C. C. Yang, and S. T. McDaniel, “Arrival time and pulse width of acoustic pulses in a turbulent ocean,” *J. Acoust. Soc. Am.* **84**, 1802–1807 (1988).
- ¹⁰S. Frankenthal, “The mutual coherence function in a scattering channel—A two-scale solution,” *J. Acoust. Soc. Am.* **85**, 104–113 (1989).
- ¹¹S. M. Flatté and R. B. Stoughton, “Predictions of internal-wave effects on ocean acoustic coherence, travel-time variance and intensity moments for very long-range propagation,” *J. Acoust. Soc. Am.* **84**, 1414–1424 (1988).
- ¹²A. Kalugin, A. Bronshtein, and R. Mazar, “Propagation of the two-frequency coherence function in an inhomogeneous background medium,” *Waves Random Media* **14**, 389–409 (2004).
- ¹³R. Mazar and L. B. Felsen, “High-frequency coherence functions propagated along ray paths in the inhomogeneous background of a weakly random medium: I-formalism and evaluation of the second moment,” *J. Acoust. Soc. Am.* **81**, 925–937 (1987).
- ¹⁴P. F. Worcester, M. A. Dzieciuch, J. A. Mercer, R. K. Andrew, B. D. Dushaw, A. B. Baggeroer, K. D. Heaney, G. D. D’Spain, J. A. Colosi, R. A. Stephen, J. N. Kemp, B. M. Howe, L. J. V. Uffelen, and K. E. Wage, “The North Pacific Acoustic Laboratory deep-water acoustic propagation experiments in the Philippine Sea,” *J. Acoust. Soc. Am.* **134**, 3359–3375 (2013).
- ¹⁵D. T. Sandwell, R. D. Müller, W. H. F. Smith, E. Garcia, and R. Francis, “New global marine gravity model from CryoSat-2 and Jason-1 reveals buried tectonic structure,” *Science* **346**, 65–67 (2014).

- ¹⁶R. K. Andrew, "The APL/UW Multiport Acoustic Projector System," Technical Report APL-UW TR-0902, Applied Physics Laboratory, University of Washington, Seattle, WA (2009).
- ¹⁷A. W. White, R. K. Andrew, J. A. Mercer, P. F. Worcester, M. A. Dzieciuch, and J. A. Colosi, "Wavefront intensity statistics for 284-Hz broadband transmissions to 107-km range in the Philippine Sea: Observations and modeling," *J. Acoust. Soc. Am.* **134**, 3347–3358 (2013).
- ¹⁸P. F. Worcester, S. Carey, M. A. Dzieciuch, L. L. Green, D. Horwitt, J. C. Lemire, and M. Norenberg, "Distributed vertical line array (DVLA) acoustic receiver," in *Proceedings of the 3rd International Conference on Underwater Acoustic Measurements*, edited by J. S. Papadakis and L. Bjørnø (Foundation for Research and Technology, Hellas, Nafplion, Greece, 2009), pp. 113–118.
- ¹⁹Sea-Bird Electronics, Inc., "MicroCAT C-T (P optional) Recorder SBE 37-SMP," Technical Report, Sea-Bird Electronics, Inc., 13431 NE 20th Street, Bellevue, WA (2012), available at http://www.seabird.com/products/spec_sheets/37smpdata.htm (Last viewed 6/11/2012).
- ²⁰Sea-Bird Electronics, Inc., "Temperature & Pressure Recorder SBE 39," Technical Report, Sea-Bird Electronics, Inc., 13431 NE 20th Street, Bellevue, WA (2012), available at http://www.seabird.com/products/spec_sheets/39data.htm (Last viewed 6/11/2012).
- ²¹R. A. Locarnini, A. V. Mishonov, J. I. Antonov, T. P. Boyer, H. E. Garcia, O. K. Baranova, M. M. Zweng, and D. R. Johnson, *World Ocean Atlas 2009, Volume 1: Temperature*, edited by S. Levitus NOAA Atlas NESDIS 68 (U.S. Government Printing Office, Washington, DC, 2010).
- ²²J. I. Antonov, D. Seidov, T. P. Boyer, R. A. Locarnini, A. V. Mishonov, H. E. Garcia, O. K. Baranova, M. M. Zweng, and D. R. Johnson, *World Ocean Atlas 2009, Volume 2: Salinity*, edited by S. Levitus, NOAA Atlas NESDIS 69 (U.S. Government Printing Office, Washington, DC, 2010).
- ²³"Thermodynamic Equation of Seawater-2010 (TEOS-10)," available at <http://www.teos-10.org> (Last viewed 3/29/2015).
- ²⁴W. H. Munk, P. F. Worcester, and C. Wunsch, *Ocean Acoustic Tomography* (Cambridge University Press, Cambridge, UK, 1995), pp. 190–195.
- ²⁵J. A. Colosi and M. G. Brown, "Efficient numerical simulation of stochastic internal-wave-induced sound-speed perturbation fields," *J. Acoust. Soc. Am.* **103**, 2232–2235 (1998).
- ²⁶W. Munk, "Internal waves and small-scale processes," in *The Evolution of Physical Oceanography*, edited by C. Wunsch and B. Warren (MIT Press, Cambridge, MA) (1981), pp. 264–291.
- ²⁷F. S. Henyey and S. A. Reynolds, "A numerical simulator of ocean internal waves for long-range acoustics," Technical Memorandum APL-UW TM 1-13, Applied Physics Laboratory, University of Washington, Seattle, WA (2013).
- ²⁸Navy Standard Parabolic Equation (NSPE), version 5.3, part of the Oceanographic and Atmospheric Master Library, Naval Oceanographic Office. For more information, write to Walt Moskal, moskalw@navo.navy.mil or Naval Oceanographic Office customer support at CSO.navo.fct@navy.mil. (Last viewed 9/5/2015).
- ²⁹B. J. Uscinski, *The Elements of Wave Propagation in Random Media* (McGraw-Hill, New York, 1977), pp. 37–50.
- ³⁰S. M. Flatté and G. Rovner, "User's guide to CAFI," available at http://oalib.hlsresearch.com/Other/cafi/CAFI_Guide.pdf (Last viewed 3/29/2015).
- ³¹A. C. Davison and D. V. Hinkley, *Bootstrap Methods and their Applications* (Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press, Cambridge, UK, 1997), Chap. 8, pp. 385–436.
- ³²F. J. Beron-Vera, M. G. Brown, J. A. Colosi, S. Tomsovic, A. L. Virovlyansky, M. A. Wolfson, and G. M. Zaslavsky, "Ray dynamics in a long-range acoustic propagation experiment," *J. Acoust. Soc. Am.* **114**, 1226–1242 (2003).
- ³³K. Hegewisch, "A random matrix approach to long range acoustic propagation in the ocean," Ph.D. thesis, Washington State University, Pullman, WA, 2010.
- ³⁴S. M. Flatté, "Wave propagation through random media: Contributions from ocean acoustics," *Proc. IEEE* **71**, 1267–1294 (1983).
- ³⁵J. B. Bowlin, J. L. Spiesberger, T. F. Duda, and L. F. Freitag, "Ocean Acoustical Ray-Tracing Software RAY," Technical Report WHOI-93-10, Woods Hole Oceanographic Institution, Woods Hole, MA (1992).
- ³⁶J. A. Colosi, L. J. van Uffelen, B. D. Cornuelle, M. A. Dzieciuch, P. F. Worcester, B. D. Dushaw, and S. R. Ramp, "Observations of sound-speed fluctuations in the western Philippine Sea in the Spring of 2009," *J. Acoust. Soc. Am.* **134**, 3185–3200 (2013).

**Decadal Trends in Low-Frequency Ambient Ocean Noise for Seven Sites in the
North Pacific Ocean**

RK Andrew, BM Howe and JA Mercer.

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Timescales of Low-Frequency Flat Fading over 510 km in the Philippine Sea

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Timescales of Low-Frequency Flat Fading over 510 km in the Philippine Sea

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(Dated: 4 June 2018)

Flat fading for 54 hours of m -sequence signals at 200 and 300 Hz over 510 km in the Philippine Sea showed deep fades into the ambient noise (signal-to-noise ratio ≈ 11 to 13 dB). A significant proportion of the fades had time-scale durations less than a few minutes. Short time-scale duration glints were also evident, with stronger glints associated with 10 to 40 minute time-scales. Neither process had significant time-scales beyond about one hour. The datasets are compared to Monte Carlo parabolic equation simulations using Garrett-Munk internal waves evolved over 64+ hours. Both simulation distributions suggested far fewer short time-scale events and non-negligible time-scales beyond one hour. The deepest fades and strongest glints were associated with time-scales near 30 to 40 minutes. All datasets show frequency-dependent fade and glint rates for events with time-scales from a few minutes to one hour, with the 300 Hz signal having deeper fades and stronger glints. Observed and simulated normalized intensity autospectra match very well up to about 4 cph, but the observed data have greater high-frequency energy. Scintillation indexes for the sea data (1.49-1.93) were much higher than for the simulation data (0.78-1.01). Additional non-Garrett-Munk processes appear to be affecting the intensity distribution.

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I. INTRODUCTION

“Fadeouts, or scintillations, are the rule rather than the exception” (Munk *et al.*, 1995a)

Flat fading for 54 hours of m -sequence signals at 200 and 300 Hz over 510 km in the Philippine Sea showed deep fades into the ambient noise (signal-to-noise ratio ≈ 11 to 13 dB). A significant proportion of the fades had short time-scale durations, less than a few minutes. A prevalence of short time-scale duration glints was also evident, with stronger glints associated with 10 to 40 minute time-scales. Neither process had significant time-scales beyond about one hour. The datasets are compared to Monte Carlo parabolic equation (MCPE) simulations using Garrett-Munk (GM) internal waves evolved over 64+ hours. Both MCPE distributions suggested

far fewer short time-scale events and non-negligible time-scales beyond one hour. The deepest fades and strongest glints were associated with time-scales near 30 to 40 minutes. All datasets show frequency-dependent fade and glint rates for events with time-scales from a few minutes to one hour, with the 300 Hz signal having deeper fades and stronger glints. Observed and MCPE normalized intensity autospectra match very well up to about 4 cph, but the observed data have much greater high-frequency energy. Scintillation indexes for the sea data (1.49-1.93) were much higher than for the MCPE data (0.78-1.01). Additional non-Garrett-Munk processes appear to be affecting the intensity distribution.

The statistical characteristics of signal fades — particularly deep fades — are important parameters in the design of communication systems. These characteristics could be derived from a mature theory of the distribution of the propagating field in the presence of randomness. Theory in long range, low frequency, underwater acoustics has not yet reached this point. Characterizations of these processes must still be derived from observations.

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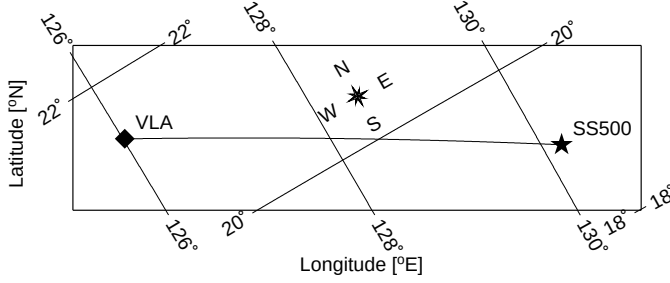


FIG. 1. Assets involved in the experiment. The transmitter was located at SS500, and the vertical array denoted VLA. Depth along the section was almost everywhere greater than 5000 m.

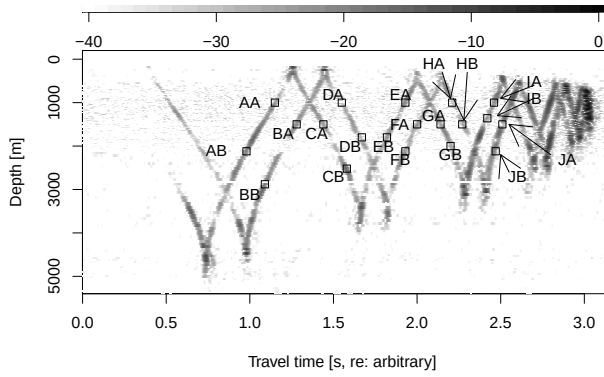


FIG. 2. Entire (149 hydrophones) observed timefront after post-equalization and pulse compression, red signal, m -sequence transmitted 2010/05/08 16:48:10. The apertures selected for further analysis are shown with corresponding labels. The timefront has been normalized by its largest value. The scale shown on top is decibels. Branches are identified as follows. AA,AB: ID-17. BA,BB: ID+18. CA,CB: ID-18. DA,DB: ID+19. EA,EB: ID-19. FA,FB: ID+20. GA,GB: ID-20. HA,HB: ID+21. IA,IB: ID-21. JA,JB: ID+22.

transformed into the Fourier domain, and the squared magnitude of the zeroth Fourier coefficient was used as “intensity plus noise”. An associated surrogate “noise level” (Andrew *et al.*, 2016) was also calculated. The ratio of the former to the latter is indicative of the intensity “signal to noise ratio” (SNR). Two red and violet examples from the entire dataset, shown in Fig. 3 are representative. The average of the red (violet) SNR was 11.6 dB (13.2 dB). The violet SNR is generally slightly better due to a decrease in ambient noise level at 300 Hz versus 200 Hz.

B. Monte Carlo Parabolic Equation Calculations

The Monte Carlo parabolic equation (MCPE) solution uses multiple split-step Padé range-adaptive method (“RAM”) (RAM) single frequency PE solvers and an in-

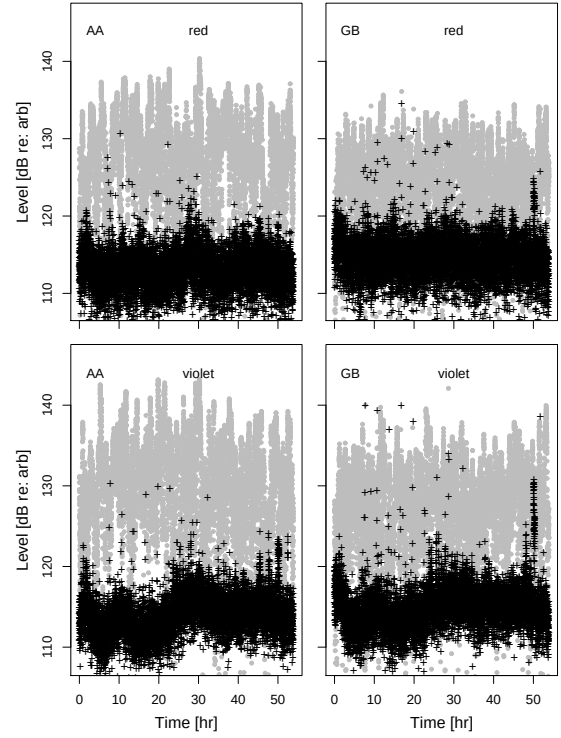


FIG. 3. Signal-to-noise examples from apertures AA and GB, red and violet signals. Horizontal axis is evolution time in hours, vertical axis is power in decibels against an arbitrary reference. Intensity-plus-noise is grey-shaded, and an estimate of the background noise is black.

verse Fourier transform to synthesize the complex time-domain Green function at the receiver range for each realization of the acoustic channel. The baseband timefront was computed using a Gaussian spectral weighting with a 50 Hz bandwidth for either the red or violet signals (White *et al.*, 2013). Realizations simulated a random ocean channel according to the Henyey-Reynolds algorithm (Henyey and Reynolds, 2013). The simulation parameters were similar to those described in Andrew *et al.* (Andrew *et al.*, 2016), with the following exceptions. First, the simulations were evolved with a time step of 200 s with the random number generator seed held constant. Second, the total number of ocean modes J_{MAX} was considerably increased. The upper limit J_{MAX} on the number of contributory internal wave modes can be estimated from a scaling argument that places an upper limit of unity on the inverse Richardson number (Munk, 1981)

$$\text{Ri}^{-1} = \frac{3}{2} \pi^2 E_0 \frac{N}{N_0} \sum_{j=1}^{J_{MAX}} j^2 H(j) \approx 1. \quad (1)$$

Here, $N(z)$ is the Brunt-Väisälä (buoyancy) frequency, N_0 is a scaling parameter, $H(j)$ is the modal weighting and E_0 represents the internal wave field energy. Approximating $\sum j^2 H(j)$ as $2\pi^{-1} j_* J_{MAX}$ for $\pi j_* \gg 1$, where j_*

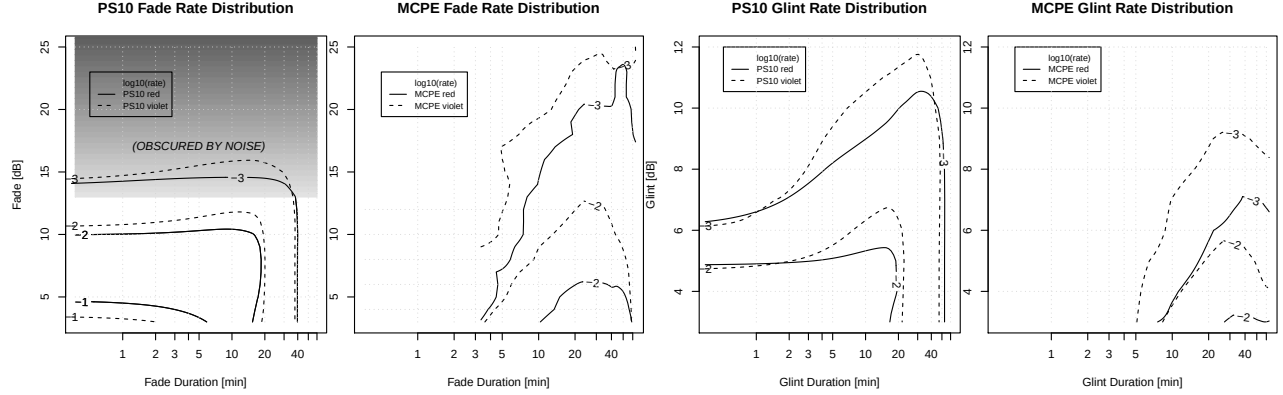


FIG. 4. Fade and glint rate statistics for the PhilSea10 and MCPE datasets, red and violet signals. The quantity shown is the base 10 logarithm of the average number of observed fades per hour, averaged additionally over all 20 apertures. Left to right: fade rate for PhilSea10 red signal, fade rate for MCPE red signal, glint rate for PhilSea10 violet signal, glint rate for MCPE violet signal. Fade rates for deep fades in the PhilSea10 data were unmeasurable due to noise contamination.

B. Glints

Glint processes are not obscured by additive noise in the PhilSea10 datasets, and the observed joint distributions are readily comparable to the MCPE distributions. Glint rate joint distributions are shown in Fig. 4 for both red and violet signals.

The PhilSea10 glint rate distributions appear frequency-independent for short-duration events, but a moderate frequency dependence for events with timescales from 3 minutes to 1 hour. Long-duration events are more likely to attain higher peaks than short-duration events. As was the case for the PhilSea10 fades, the distributions reveal a significant proportion of short-timescale events (e.g., order 1 minute.) Most of the glints have durations within the first 60 minutes.

The MCPE glint rate distribution shows a distinct frequency dependence for events with durations from 5 minutes to at least 1 hour. The maximum glint levels are not as strong as those observed in the PhilSea10 data. Unlike the PhilSea10 dataset, a considerable proportion of events have durations beyond 60 minutes, and far fewer have durations within the first 3 minutes.

As was the case for the MCPE fade distributions, the MCPE glint distributions have no comparable results for durations less than 200 s because the simulation time step (of 200 s) prevented higher time resolution observations. As was also the case for MCPE fades, auxiliary 1-hour simulations (not shown) for the red and violet signals using 20 s time steps showed fields that evolved smoothly, with no sign of bursty events on sub-200 s timescales.

The largest glint for the PhilSea10 red (violet) field was 15.7 (18.7) dB. The largest glint for the MCPE red (violet) field was 10.6 (12.4) dB.

IV. INTENSITY AUTOSPECTRA

Intensity fades and glints are temporal statistics, and should be closely linked to the distribution of intensity and to the intensity autospectrum. The PhilSea10 experiment provided nearly two inertial periods of almost continuous sampling at a very high rate, allowing comprehensive assessment of the intensity autospectrum. The time-evolving MCPE simulations generated more than 64 hours of simulation, and provide similar datasets for comparisons. For comparison purposes, the autospectrum of the intensity I normalized by the mean intensity $\bar{I}$ is more useful than the autospectrum of intensity (Ewart *et al.*, 1983). The variance of the normalized intensity $\tilde{I} = I/\bar{I}$ is the scintillation index. Section IV A therefore describes the intensity distributions and scintillation indexes for the datasets. The estimation of the autospectra is described in section IV B. Finally, representative autospectrum comparisons are given in section IV C.

A. Intensity Distributions and Scintillation Indexes

The intensity distribution is valuable because it details the depth of fades (at the origin) and the strength of glints (in the tail.) Direct estimates of the intensity distributions are shown in Appendix A. Unfortunately, the PhilSea10 data have distributions at the origin obscured by noise. Alternately, sample size in the tail is limited, and therefore Q-Q plots show tail deviations from, for example, an exponential distribution to be inconclusive. The two most important regions of the empirical distribution are either unavailable or too poorly sampled.

One hopes that careful simulation could provide additional insight. This would be particularly true at the near-origin region of the distributions because the simulations have no additive noise. The red MCPE dataset has histograms which appear to be trending toward zero

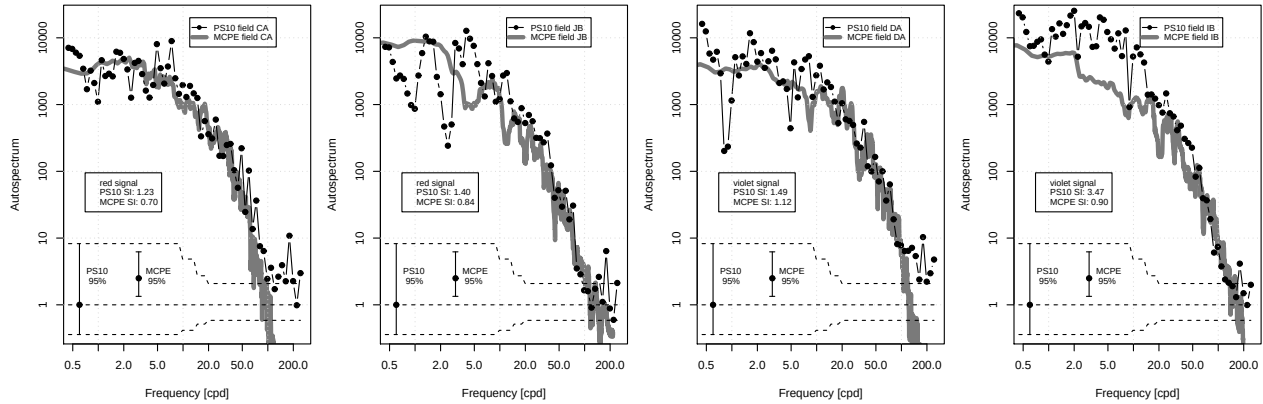


FIG. 6. Normalized intensity autospectra comparisons for PhilSea10 (PS10) and MCPE datasets. The left two figures (apertures CA and JB) are selected examples from the red signal. The right two figures (apertures DA and IB) are selected examples from the violet signal. The upper and lower dashed lines give the bootstrapped 95% confidence interval about the central dashed line for the PhilSea10 autospectra. This confidence interval is frequency-dependent due to the variable number of degrees of freedom in the estimator. The 95% confidence intervals for the MCPE autospectra are not frequency-dependent, and are shown as the single vertical line. The estimated scintillation index is given for each autospectrum; shown are the autospectra normalized by the observed scintillation index.

tra, this is not inconsistent with observations. First, the record duration (54 hours) is not long enough to provide high resolution at tidal frequencies. Second, Zhao(Zhao, 2014) shows that the K_1 and O_1 diurnal tides, which radiate from the Luzon Strait into the Philippine Sea, have wavelengths of 200 – 450 km; over 510 km, these could interact in ways that might obscure an autospectral signature. Additionally, the M_1 semi-diurnal tide, the strongest internal tide in the region, shows evidence of break-up after it passes through the Kuroshio, and may not present a distinctive autospectral signature.

A primary objective to this experiment was to understand the role of energetic ocean processes on acoustic scattering. These results indicate that the standard model involved in simulation needs further augmentation to account accurately for the observations.

APPENDIX A: DISTRIBUTION OF INTENSITY

According to the literature, the intensity has an exponential distribution in the fully saturated regime, otherwise a log-normal distribution. The difference between these two distributions occurs at/near the origin, where the log-normal distribution is zero but the exponential distribution has a positive maximum, and throughout the upper tail, where the two distributions generally have different shapes. To investigate these two characteristics, intensity histograms and Q-Q plots are shown.

Wide-span histograms provide the overall shape of the distribution, but do not resolve the complexity near the origin, for which a near-origin detailed perspective is required. Representative histograms are shown in Fig. 7 for both signals. Both the PhilSea10 and the MCPE

distributions are shown. PhilSea10 intensities with estimated SNR less than 10 dB have been redacted. In addition, theoretical curves are shown for exponential and log-normal distributions. The intensity timeseries has been normalized by the sample mean intensity, and therefore there are no free parameters for the exponential curves. The maximum-likelihood estimates of the mean and standard deviation of the logarithm of the normalized intensity timeseries are used for the log-normal parameters.

The PhilSea10 wide-span histograms in Fig. 7 show that the red signal tail conforms well to the exponential distribution, but the violet signal tail has extra weight, more like a log-normal distribution. The upper tails of the MCPE distributions are poorly defined due to inadequate sampling, and little can be said about them.

Near-origin region details are also shown in Fig. 7, again for both signals. The noise clearly obscures the shape of the PhilSea10 distributions and nothing can be inferred regarding these distributions. The MCPE distributions show a definitive decrease towards the origin for the red signal, but a maximum at the origin for the violet signal.

Representative Q-Q plots of the normalized observed intensity (signal intensity plus noise) are shown in Fig. 8. Crude 95% confidence interval bounds are also shown. These bounds are pointwise intervals about the true distribution; serial correlation in the data is moderated by assuming one new degree of freedom approximately every 5 minutes. The diagonal line in each figure represents the null hypothesis of an exponential distribution. In general, the curves deviate above the exponential distribution in the upper tail, but in most cases, the deviation from the exponential distribution is not significant.

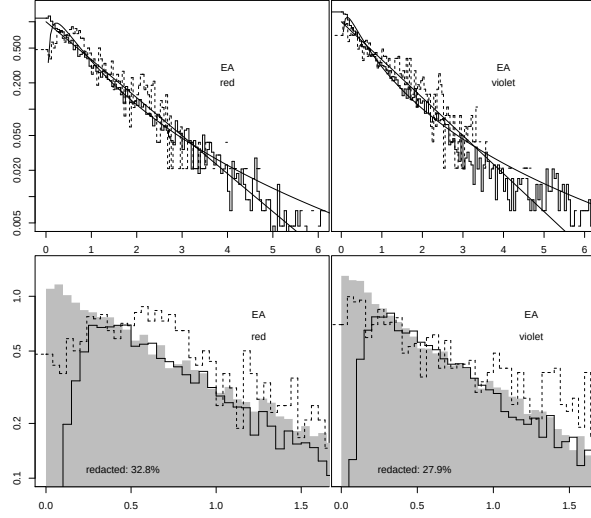


FIG. 7. Histograms for aperture EA for the “red” and “violet” signals, Top two figures: wide-span histograms. Bottom two figures: origin details. Solid line: PhilSea10, dashed line: MCPE simulation. Left axes denote density, bottom axes intensity normalized by the sample intensity. Corresponding exponential and log-normal densities are shown as smooth curves in the top two figures. The exponential model is a straight line in this representation. The grey-shaded regions in the bottom figures are the PhilSea10 histograms without redaction of low SNR samples.

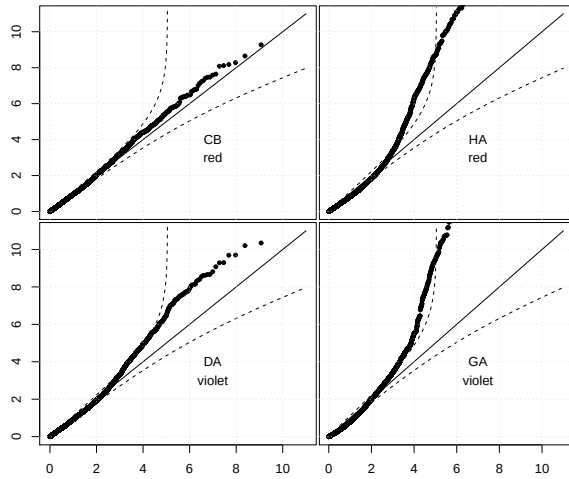


FIG. 8. Q-Q plots for the “red” (top two figures) and “violet” (bottom two figures) signals, PhilSea10. Both axes are normalized (observed) intensity. The diagonal line represents an exponential distribution. The dashed lines estimate the 95% confidence interval bounds assuming independent samples approximately every 5 minutes. Left two figures display Q-Q curves well within the confidence intervals. Almost all Q-Q plots had this character. Right two figure display the Q-Q curves that deviated (at some point) the most outside the intervals.

APPENDIX B: ESTIMATION OF SCINTILLATION INDEX WITH ADDITIVE NOISE

Estimation of the scintillation index of a process in the presence of additive noise requires a correction. Consider an intensity time series $I(t)$ observed with a noise process $n(t)$. Let both I and n be non-negative random variates. The observed data

$$d = I + n. \quad (\text{B1})$$

Assume that the intensity and noise processes are independent, so that

$$E\{In\} = E\{I\}E\{n\} = \mu_I\mu_n. \quad (\text{B2})$$

Sample statistics of the mean μ_d of d and the variance $\text{var}(d)$ of d are available. Additionally, an estimate of the mean noise μ_n is also available.

The scintillation index of the observed data is

$$\text{SI}(d) = \text{var}(d)/\mu_d^2; \quad (\text{B3})$$

the desired statistic is the scintillation index of the intensity,

$$\text{SI}(I) = \text{var}(I)/\mu_I^2; \quad (\text{B4})$$

From Eq. B1 using Eq. B2,

$$\text{SI}(d) = \frac{\text{var}(d)}{\mu_d^2} \quad (\text{B5})$$

$$= \frac{\text{var}(I) + \text{var}(n)}{\mu_I^2 + 2\mu_I\mu_n + \mu_n^2} \quad (\text{B6})$$

$$= \frac{\text{SI}(I) + \mu_I^{-2}\text{var}(n)}{1 + 2\mu_n/\mu_I + \mu_n^2/\mu_I^2}. \quad (\text{B7})$$

Thus,

$$\text{SI}(I) = \text{SI}(d) \left[1 + \frac{2\mu_n}{\mu_d - \mu_n} + \frac{\mu_n^2}{(\mu_d - \mu_n)^2} \right] - \frac{\text{var}(n)}{(\mu_d - \mu_n)^2} \quad (\text{B8})$$

Now write

$$r^{-1} = \mu_d/\mu_n. \quad (\text{B9})$$

This is like a “signal plus noise” to “noise” ratio. When the noise goes to zero, this ratio goes to infinity, (or r goes to zero) and $\text{SI}(d) \rightarrow \text{SI}(I)$. Using Eq. B9,

$$\text{SI}(I) = \frac{\text{SI}(d)}{(1-r)^2} - \frac{r^2\text{var}(n)}{\mu_n^2(1-r)^2}. \quad (\text{B10})$$

Eq. B10 is used for the PhilSea10 datasets.

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- Andrew, R. K., Ganse, A., White, A. W., Mercer, J. A., Worcester, P. F., Dzieciuch, M. A., and Colosi, J. A. (2016). "Low-frequency pulse propagation over 510 km in the philippine sea: A comparison of observed and theoretical pulse spreading," *J. Acoustic. Soc. Am.* **140**(1), 216–228.
- Bronez, T. P. (1985). "Nonparametric spectral estimation of irregularly-sampled multidimensional random processes," Ph.D. thesis, Arizona State University.
- Bronez, T. P. (1988). "Spectral Estimation of Irregularly Sampled Multi-dimensional Processes by Generalized Prolate Spheroidal Sequences," *IEEE Trans. ASSP* **36**(12), 1862–1873.
- Colosi, J. A. (2016). *Sound Propagation through the Stochastic Ocean*, Chap. 3, 124 (Cambridge University Press, Cambridge, UK).
- Desaubies, Y. J. F. (1978). "On the scattering of sound by internal waves in the ocean," *J. Acoust. Soc. Am.* **64**(5), 1460–1469.
- Ewart, T. E., Macaskill, C., and Uscinski, B. J. (1983). "Intensity Fluctuations. Part II. Comparison with the Cobb Experiment," *J. Acoust. Soc. Am.* **74**(5), 1484–1499.
- Forsyth, P. A., Vogan, E., Hansen, D. R., and Hines, C. O. (1957). "The principles of JANET- a meteor-burst communication system," *Proc. IRE* **45**(12), 1642–1657.
- Henyey, F. S., Mercer, J. A., Andrew, R. K., and White, A. W. (2014). "Smaller-scale internal waves and spice fluctuations on acoustic transmission paths from conductivity-temperature-depth profiles" Technical Memorandum APL-UW TM 1-14, Applied Physics Laboratory, University of Washington, Seattle.
- Henyey, F. S., and Reynolds, S. A. (2013). "A numerical simulator of ocean internal waves for long-range acoustics" Technical Memorandum APL-UW TM 1-13, Applied Physics Laboratory, University of Washington, Seattle.
- Kuciewicz, J., Dunmire, B., and Bailey, M. R. (2012). "Improved detection of kidney stone twinkling using autoregressive signal processing method," *J. Urology* **187**(4), e689–e690.
- Kvicera, M., Pechac, P., Grabner, M., Kvicera, V., Valtr, P., and Martellucci, A. (2015). "Experimental study on terrestrial links enhancement at 11 and 38 GHz," *IEEE T. Ant. Prop* **63**(7), 3179–3186.
- Magaldi, A., Mateu, M., Bech, J., and Lorente, J. (2016). "A long term (1999 - 2008) study of radar anomalous propagation conditions in the Western Mediterranean," *Atmos. Res.* **169**(A), 73–85.
- Maynard, L. A. (1961). "Propagation of meteor burst signals during the polar disturbance of November 1216, 1960," *Can. J. Phys.* **39**(4), 628–630.
- Munk, W. (1981). "Internal waves and small-scale processes," in *The Evolution of Physical Oceanography*, edited by C. Wunsch and B. Warren (MIT Press, Cambridge MA), pp. 264–291.
- Munk, W. H., Worcester, P. F., and Wunsch, C. (1995a). *Ocean Acoustic Tomography*, 147 (Cambridge University Press, Cambridge UK).
- Munk, W. H., Worcester, P. F., and Wunsch, C. (1995b). *Ocean Acoustic Tomography*, 190–195 (Cambridge University Press, Cambridge UK).
- Proakis, J. G. (1983). *Digital Communications*, 454 – 543 (McGraw-Hill, Inc.).
- RAM. This effort used a modified version of RAM version 1.6. RAM is described in *User's Guide to RAM Versions 1.0 and 1.0p*, written by Michael D. Collins The original code, written by Michael D. Collins, was downloaded years ago from the NRL website, but the original website is no longer active. The code and the user's guide can be found at <http://oalib.hlsresearch.com/PE/RAM>: last accessed 2016.12.26.
- White, A. W., Andrew, R. K., Mercer, J. A., Worcester, P. F., Dzieciuch, M. A., and Colosi, J. A. (2013). "Wavefront intensity statistics for 284-hz broadband transmissions to 107-km range in the Philippine Sea: observations and modeling," *J. Acoustic. Soc. Am.* **134**, 3347–3358.
- White, A. W., Henyey, F. S., Andrew, R. K., Mercer, J. A., Worcester, P. F., Dzieciuch, M. A., and Colosi, J. A. (2016). "Internal tides and deep diel fades in acoustic intensity," *J. Acoustic. Soc. Am.* **140**(5), 3952–3962.
- Worcester, P. F., Carey, S., Dzieciuch, M. A., Green, L. L., Horwitt, D., Lemire, J. C., and Norenberg, M. (2009). "Distributed vertical line array (DVLA) acoustic receiver," in *Proc. 3rd International Conf. on Underwater Acoustic Measurements*, edited by J. S. Papadakis and L. Bjørnø, Foundation for Research and Technology Hellas, Nafplion Greece, pp. 113–118.
- Worcester, P. F., Dzieciuch, M. A., Mercer, J. A., Andrew, R. K., Dushaw, B. D., Baggeroer, A. B., Heaney, K. D., D'Spain, G. D., Colosi, J. A., Stephen, R. A., Kemp, J. N., Howe, B. M., Uffelen, L. J. V., and Wage, K. E. (2013). "The North Pacific Acoustic Laboratory deep-water acoustic propagation experiments in the Philippine Sea," *J. Acoustic. Soc. Am.* **134**, 3359–3375.
- Zhao, Z. (2014). "Internal tide radiation from the Luzon Strait," *J. Geophy. Res. Oceans* **119**, 5434–5448.

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