



**A STUDY OF QUASI-STATIC AND
DYNAMIC ANALYSES OF A HEXAKIS
ICOSAHEDRON FRAME FOR USE IN A
VACUUM LIGHTER-THAN-AIR VEHICLE**

THESIS

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AFIT-ENY-MS-17-M-291

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VEHICLE

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Abstract

This research discusses the design, modal analysis, and dynamic analysis of a hexakis icosahedron frame for use in a vacuum lighter than air vehicle (VLTAV). The work was carried out in two parts. In the first part, an experiment was conducted to verify that the method of analysis and assumed member connections was correct. The second part consisted of a finite element analysis of the nonlinear dynamic behavior present within the frame with an attached membrane, including possible chaos. For the first part, a 0.1778 m diameter hexakis icosahedron frame was additively manufactured using VeroWhitePlus plastic. Previous research indicated that the global modulus of elasticity for the frame was 50% of that listed in the manufacturer's specification sheet so an *equivalent* modulus of 1.1 GPa was initially assumed for the frame. A modal experiment using a laser vibrometer was performed in order to determine the eigenvalues of the structure for comparison to the modal analysis results of a finite element (FE) model of the frame assuming a free-free boundary condition. The FE model predicted the first two frequency modes within 1% of the experimental results. Because the FE model accurately predicted the dynamic properties of the frame, the modeling technique in which the assumed member connections were verified, led to confidence of modeling. Once the method of analysis was confirmed, a nonlinear dynamic study of the frame and the attached skin response to different loading conditions using the Beryllium material previously incorporated was then carried out. Thus, the comparison could be made between the hexakis icosahedron results and previous work performed for an icosahedron. The frame was analyzed under both a symmetric and an unsymmetric boundary condition with three different load rates, 4.05 MPa/s, 5.06 MPa/s, and 8.11 MPa/s. The results of analyses exhibited chaotic

behavior at load rates at or above 4.5 MPa/s for symmetric boundary conditions and at load rates of 4.0 MPa/s for unsymmetric boundary conditions. For the six different load cases, the hexakis icosahedron frame exhibited less chaotic behavior in terms of the Lyapunov exponent than did the icosahedron frame except for load case 1 where the boundary condition was symmetric and the load rate was 5.06 MPa/s. The Mises stresses of the frame were analyzed for the six load cases in order to determine the impact of boundary conditions, load rates, and load magnitudes. Load magnitudes of 15% sea level pressure or higher exceeded the yield stress limits of the frame, and loads of 40% sea level pressure exceeded the material ultimate stress limits. Unlike the icosahedron, the frame-skin model of the hexakis icosahedron still exhibited nonlinear dynamic behavior due to the frame's construction. An analysis of the inertia and stiffness properties of the beams for each model was conducted and revealed that the icosahedron beams had inertia values 6.5 times higher than the hexakis icosahedron and stiffness values 2.5 times higher. This lack of stiffness at the four-beam vertices of the hexakis icosahedron allowed a *snapback* behavior to occur and nonlinear dynamic behavior to dominate.

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Jordan W. Snyder

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List of Acronyms

BC	boundary condition
FE	finite element
FEA	finite element analysis
FEM	Finite Element Model
FFT	fast fourier transform
FRF	frequency response function
LTAV	Lighter-Than-Air Vehicle
N-R	Newton-Raphson
PSD	power spectral density
VLTA	Vacuum Lighter-Than-Air Vehicle
W/B	weight-to-buoyancy

A STUDY OF QUASI-STATIC AND DYNAMIC ANALYSES OF A HEXAKIS ICOSAHEDRON FRAME FOR USE IN A VACUUM LIGHTER-THAN-AIR VEHICLE

I. Introduction

1.1 Chapter Overview

The development of Lighter-Than-Air Vehicles (LTAVs) using heated air or lighter than air gases such as hydrogen or helium marked a significant achievement in the history of human flight; however, multiple design disadvantages such as their size and maneuverability limit their usefulness. One design idea that has the potential to greatly improve conventional balloons is to evacuate the air within the balloon structure itself (thereby creating a vacuum) in order to generate positive buoyancy for the LTAV. This chapter will investigate the background of LTAVs, the motivation for a Vacuum Lighter-Than-Air Vehicle (VLTAV) design, the objectives of the research, and the general methodology used to meet the objectives.

1.2 Background

The idea of achieving flight using an LTAV has existed for thousands of years. The first recorded use of LTAVs was found in China, where they were used as unmanned military signaling lanterns beginning in 220 A.D [14]. It was not until 1783 that the Montgolfier brothers successfully achieved manned flight in one of the most common types of LTAV - a hot air balloon [15]. Hot air balloons, like all LTAVs, achieve buoyancy by displacing a volume of air weighing more than the combined weight of

the skin, internal structure and cargo. Figure 1.1 shows a schematic of the opposing gravitational and buoyancy forces.

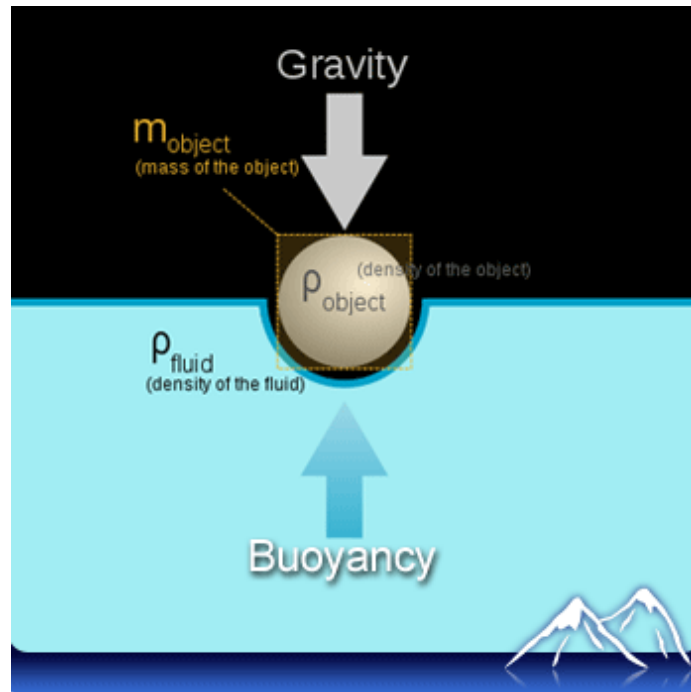


Figure 1.1. Buoyancy schematic [1]

Heating the inside of the balloon decreases the density of the air it contains, making it lighter than the surrounding air. If the balloon is sufficiently large, the weight of the displaced volume of air will be greater than the combined weight of the balloon and the lower density air contained within the balloon, and positive buoyancy will be reached.

Buoyancy is typically characterized using a weight/buoyancy (W/B) ratio. A vehicle with a W/B greater than one will sink, a W/B less than one will rise, and a W/B equal to one will be neutrally buoyant. All LTAVs are designed with a W/B less than one; however, there is a limit to how far they can rise. As altitude increases, air density decreases so W/B will steadily rise until it is greater than one. The altitude at which weight equals buoyancy is the maximum operating altitude for the vehicle.

Other LTAVs are filled with lighter-than-air gases such as helium or hydrogen;

however, this presents other challenges. Gases like hydrogen are extremely flammable and present a significant safety concern. Helium, while fire retardant, is heavier than hydrogen, so a greater volume of it is needed to create positive buoyancy. In both cases, large stores of additional gas would have to be carried to allow the LTAV to change altitude. Thus, LTAVs using lighter-than-air gases must be very large.

In 1663, a Jesuit priest named Francesco Lana de Terzi developed another idea for an LTAV - one which used an internal vacuum to create positive buoyancy rather than heated air or a lighter-than-air gas. As shown in figure 1.2, de Terzi designed an airship that used four copper spheres from which the air was evacuated to produce the buoyant force. Although his design could have worked from a purely mathematical perspective, in that the weight of his design was less than the weight of air displaced, the thin copper spheres would not have been able to withstand the atmospheric pressure induced by air evacuation [16] [17]. In order for such a design to succeed, an internal supporting structure would have to be included.

One modified VLTAV design proposed by A. Akhmeteli and A. Gavrilin in their 2005 patent incorporated layered shells around a spherical outline providing rigidity to the structure as well as allowing the frame to achieve positive buoyancy [16]. Their design was comprised of two thin layers sandwiched around a honeycomb matrix core. The honeycomb material would provide the necessary stiffness for the structure, and the two thin layers around it would encase it from the ambient air, allowing the internal air to be evacuated.

Other VLTAV internal frame designs were proposed by T. Metlen during his research at the Air Force Institute of Technology (AFIT) in 2013 [18]. His preliminary research focused on both icosahedron and hexakis icosahedron-shaped frames. The icosahedron frame consisted of 20 equilateral triangles and 12 vertices while the hexakis icosahedron frame consisted of 120 identical triangles and 62 vertices. These

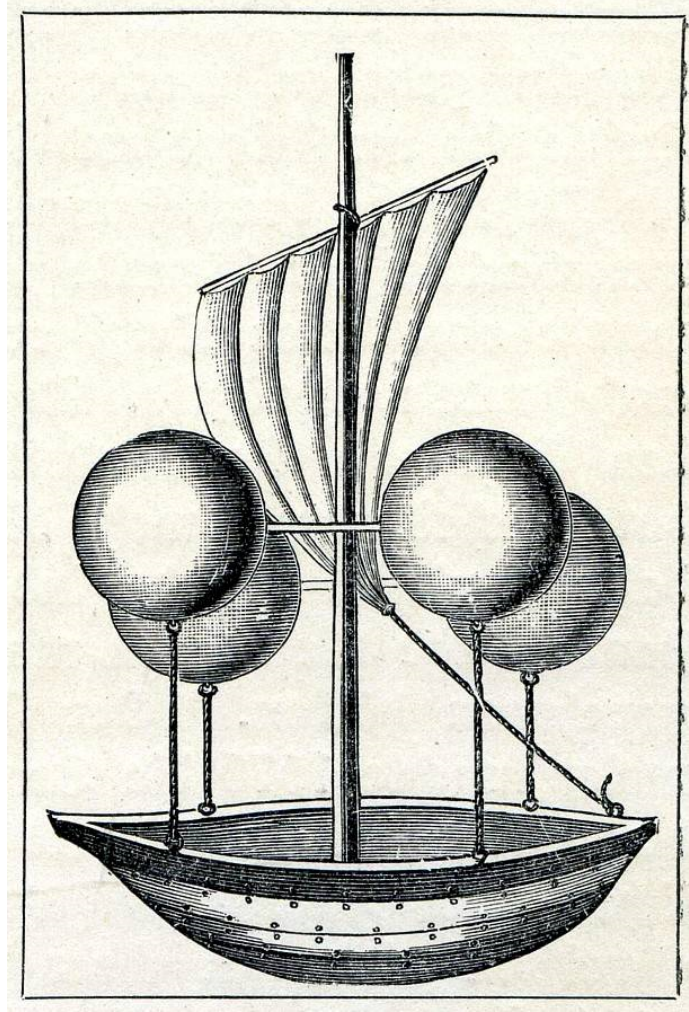
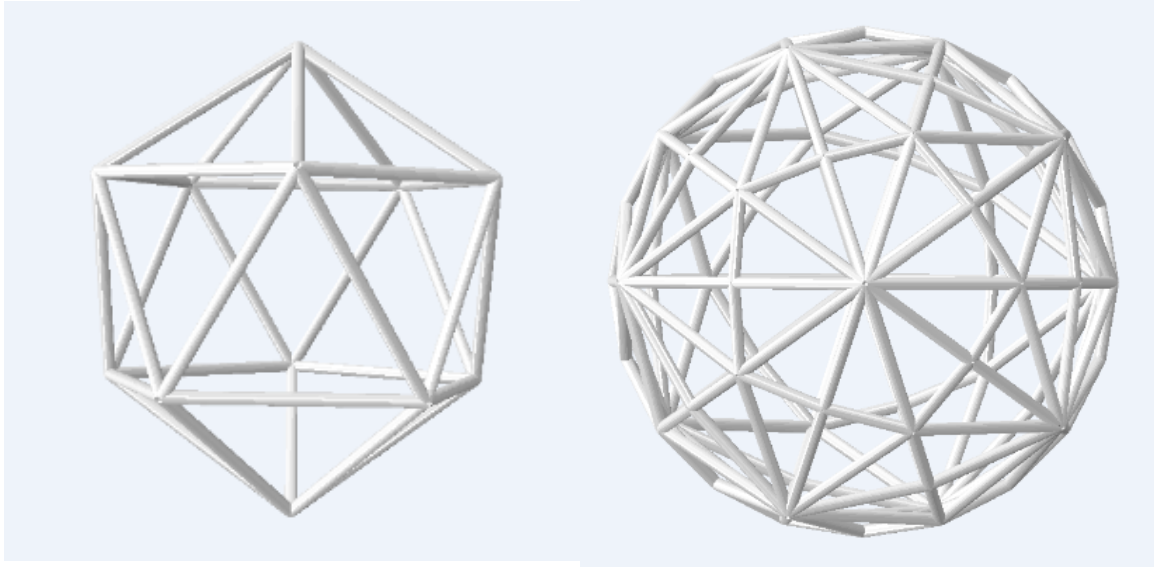


Figure 1.2. Francesco Lana de Terzi's vacuum airship design [2]

frames were to be encased with a thin, membrane-like skin to complete the VLTAV design. R. Adorno Rodriguez, M. AlGhofaily, and L. Just continued Metlen's research on the icosahedron frame from 2013 to 2015 and analyzed it under static and dynamic loading conditions as well as its frequency mode shapes [5] [8] [19]. Figure 1.3 shows a side-by-side comparison of the icosahedron and hexakis icosahedron.



(a) Icosahedron

(b) Hexakis Icosahedron

Figure 1.3. Icosahedron and Hexakis Icosahedron Comparison

1.3 Motivation

VLTAVs could prove to be a far more useful airship design than current lighter-than-air vehicles. As one LTAV researcher, David Noel, states, “the greatest advantage of the vacuum balloon lies in the way it avoids one of the major limitations of conventional balloons. They leak.” LTAVs using lighter than air gases, such as helium, must carry reserves of the gas on board in order to compensate for the volume of gas that escapes the balloon during flight. This added weight reduces the amount of payload the balloon can carry, and because gas is constantly leaking at a certain rate, the balloon cannot stay in the air indefinitely. VLTAVs, on the other hand, are not limited by this issue; they are only limited by their power source, likely either a battery pack or a solar panel array covering the upper skin of the airship. Even with small leaks, a VLTAV could stay aloft indefinitely as it could simply pump out the air that leaks into the structure. Additionally, it could climb by pumping air out of the balloon and descend by allowing some air to leak back into the structure. Because it

would not need to carry additional stores of gas on board, the theoretical payload is much larger than that of a conventional LTAV balloon. The primary weight limitation is the weight of the internal supporting structure [20].

A VLTAV design presents multiple civilian and military applications. In his paper, Noel suggests a variety of civilian applications for a VLTAV design. The first, and most apparent, application is transport. Because a VLTAV could rise and fall easily by simply controlling the amount of air present within the structure, the airship could take advantage of the varying wind directions at different altitudes to carry either people or cargo. A more creative application included in his paper is an array of tethered VLTAVs that could support a cable-car system or an electromagnetic launching frame for space vehicles. Many such balloons positioned inside clouds could collect and harness the potential energy of the water, and larger balloons positioned above the clouds could be used to harvest solar energy [20]. The primary military-specific application for such a design is surveillance. A small, vacuum balloon would be ideal for maneuvering in a tight, urban environment for use in special operations as it would only require a battery pack to operate. Special operators in the field would not need to carry a lifting gas with them, and the vacuum balloon design would allow them to get closer to the urban environment than traditional micro air vehicles (MAVs).

1.4 Objective

Recent research has shown that there are two primary ways to develop a VLTAV. One method proposed by A. Akhmeteli and A. Gavrilin consists of a spherical structure with a thick, sandwiched wall designed to resist buckling [16]. Research conducted at AFIT has focused on using an internal supporting structure encased by a thin membrane-like skin to minimize weight and maximize structural strength.

This thesis focuses on one such structure, a hexakis icosahedron, and analyzes its suitability for such an application with regards to its dynamic response. Specifically, the following objectives are pursued:

- Construct a 0.1778 m (7 inches) diameter model hexakis icosahedron using additive manufacturing techniques;
- Determine natural frequencies and mode shapes of the hexakis icosahedron frame using Finite Element Analysis (FEA);
- Validate the dynamic Finite Element Model (FEM) of the hexakis icosahedron frame by conducting experimental modal analysis considering the first and second frequencies using a laser vibrometer;
- Analyze the response of the hexakis icosahedron with an attached skin under dynamic loading scenarios at two different boundary conditions to assess chaotic behavior; and
- Compare results for a 0.3048 m (12 inches) diameter icosahedron and hexakis icosahedron with a weight-to-buoyancy (W/B) of 0.9 assuming material properties of Beryllium;

1.5 Methodology

A 0.1778 m diameter model of the hexakis icosahedron frame was constructed using additive manufacturing techniques using VeroWhitePlus material. The natural frequencies of the frame were determined using a laser vibrometer and compared to the results calculated using the Abaqus FEA model. These results were used both to validate the dynamic FEM of the frame as well as provide additional modal response characteristics of an additively manufactured hexakis icosahedron.

A computer-aided design (CAD) model of the 0.3048 m hexakis icosahedron frame was created using Solidworks software and imported into the Abaqus FEA solver. Analyses were conducted within Abaqus to analyze the dynamic instability characteristics and chaotic behavior of the frame under various loading scenarios and boundary conditions assuming non-linear effects. Abaqus software was also used to investigate the mode shapes and natural frequencies of the proposed frame assuming a free-free boundary condition. Two different models were constructed in Abaqus: one with the standalone frame and one with a thin membrane-like skin attached to the frame to provide a more realistic model of the VLTAV design. The mode shapes and natural frequencies of the two models were compared to better understand the interaction of the frame and skin components as well as note the frequencies likely to cause failure due to harmonic resonance.

1.6 Overview

- Chapter I: Thesis background, motivation, objective, and methodology overview
- Chapter II: Theory overview and previous research on the icosahedron and hexakis icosahedron frames
- Chapter III: Detailed methodology, including FEA model development and analysis techniques
- Chapter IV: FEM analysis and experimental results
- Chapter V: Results summary and conclusion, including future research recommendations

II. Background

2.1 Chapter Overview

A static loading assumption is very common for most structural analyses; however, static loading conditions can only be assumed if the load is applied very slowly. As the rate of loading is increased, dynamic analysis becomes much more critical as well as an examination into chaotic behavior. Previous research conducted at AFIT by Metlen, Adorno-Rodriguez, Just, AlGhofaily, and Cranston analyzed the static and dynamic responses of the icosahedron frame and is used as a template for this thesis in examining the dynamic response of a hexakis icosahedron frame [5] [8] [18] [19] [9].

This chapter will summarize the previous research on both the icosahedron and hexakis icosahedron-shaped VLTAVs as well as explain the theory needed to solve this particular structural dynamics problem. Specifically, dynamic FEA methods, chaotic behavior, and experimental modal analysis theories will be discussed.

2.2 Theory of Finite Element Analyses and Dynamic Behavior

Due to the complexity of the hexakis icosahedron frame, the dynamic response was calculated using FEA, a numerical method for approximating solutions to field problems such as heat transfer and stress analysis. FEA works by splitting the structure into small pieces called elements. The elements are connected at nodes, and the arrangement of elements is called a mesh. This mesh is discretized by a system of equations which solves for the desired unknown variables at the nodes of the structure. The problem must be classified, modeled, and discretized to approximate a solution using FEM [21].

The first step is to classify the model. This involves answering several questions regarding the overall structure such as whether the problem is time-dependent, linear

or nonlinear, and static or dynamic [21]. After classification is complete, the problem must be modeled.

In this step, a geometric, computerized replica of the structure is created in the FEA solver. Care must be taken to design the model with sufficient detail that the behavior can be accurately approximated but not too detailed such that the solution is overly complex and runtime is excessive. Once boundary conditions, loads and solver options are specified, the geometric model becomes a mathematical model which must be discretized [21].

Discretization is the final step in the FEA process. The mathematical model is split into a mesh of finite elements which are solved in piecewise fashion. Errors can stem from either the modeling or discretization steps. These can be reduced by improving the geometric details of the model and by increasing the mesh density along the elements [21].

2.2.1 Nonlinear Systems

An important question to answer during the first step of the FEA process is whether the structure should be analyzed linearly or nonlinearly. Equation 2.1 shows the general structure stiffness equation for FEA systems.

$$[K]\{D\} = \{R\} \tag{2.1}$$

where:

$[K]$ = Stiffness matrix

$\{D\}$ = Displacement vector

$\{R\}$ = Load vector

For linear systems, both the stiffness matrix, $[K]$, and the load vector, $\{R\}$, are known as they are assumed to be independent of the structure's response to the

loading condition. For nonlinear systems, however, both the stiffness matrix and at times, the load vector, are functions of the displacement vector, $\{D\}$. In other words, the stiffness matrix changes as the structure displaces. This makes the solution much more complex as additional information is required. FEA solvers utilize an iterative process to solve for $\{D\}$, $[K]$, and $\{R\}$ such that the product $[K]\{D\}$ and $\{R\}$ are in equilibrium [22].

One method of solving nonlinear systems is the Newton-Raphson (N-R) technique. In mathematics, this method is simply called Newton's method and provides a way of approximating the root of a polynomial through iteration. A simplified example is a pressure applied to a nonlinear spring which can be represented by equation 2.2 where k , u , and P represent the stiffness coefficient, displacement, and pressure load, respectively [21].

$$ku = P \quad (2.2)$$

An initial displacement of $u_0 = 0$ is assumed, and a pressure, P_1 is applied. The goal is to approximate the final displacement, u_1 . The initial stiffness coefficient is labeled as k_0 . Because P_1 is an initial load, the load increment is equal to the load itself for this first increment. Thus the first increment is represented by equation 2.3 where $u_A = \Delta u$ where u_A represents the initial estimate of the desired displacement result, u_1 .

$$k_0 u_A = \Delta P_1 \quad (2.3)$$

After this initial step, the error is represented by equation 2.4 where ku_A represents

the spring force resisting the applied pressure, P_1 .

$$e_A = P_1 - ku_A \quad (2.4)$$

In order to reach a more accurate result, a second iteration is calculated beginning at point A and moving to point B. The second increment equation is represented by equation 2.5.

$$k_A \Delta u = e_A \quad (2.5)$$

This provides a more accurate solution to the displacement u_1 . Equation 2.6 shows the updated error equation after the second increment where $u_B = u_A + \Delta u$.

$$e_B = P_1 - ku_B \quad (2.6)$$

After multiple similar displacement increments, the load increment, ΔP_2 can be added and the iteration process begins again. By increasing the number of increments as well as reducing the increment size, a more accurate solution will be reached [21].

2.2.2 Modal Analysis using FEM

Finite Element Modeling (FEM) programs perform modal analysis by solving the equations of motion as outlined in equation 2.7 for one-dimensional systems [23].

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} = \{F(t)\} \quad (2.7)$$

where:

$[C]$ = Damping matrix

$\{D\}, \{\dot{D}\}, \{\ddot{D}\}$ = Nodal position, velocity, and acceleration vectors

$[K]$ = Stiffness matrix

$[M]$ = Mass matrix

$\{F(t)\}$ = Applied forcing function (equal to zero for a free-free boundary condition)

Frequency analyses on the structure can be determined by assuming the damping matrix, $[C]$ as well as the forcing function, is equal to zero and subsequently solving the matrix equation. The solution will yield two critical dynamic properties - eigenvalues and eigenvectors. Eigenvalues represent the natural frequencies of a given system, and eigenvectors describe the shape of the response when subjected to the given frequency. Eigenvalues for an undamped system are calculated according to equation 2.8a. Reconstructing the general equation of motion in terms of eigenvalues and eigenvectors then yields equation 2.8b [24]. These quantities represent a unique, characteristic property of the system and are critical to the research in this thesis as they can be compared to the results of experimental modal analysis to determine the material properties of the beams in the structure.

$$\omega_n = \frac{\frac{EI}{\rho A} n^4 \pi^4}{L^4} \quad (2.8a)$$

$$(-\omega_n^2 [M] + [K]) \{\phi_n\} = \{0\} \quad (2.8b)$$

where:

ω_n = Eigenvalue (natural frequency)

E = Elastic modulus

I = Area moment of inertia

ρ = Material density

A = Beam cross-sectional area

n = Eigenvalue number

L = Beam length

ϕ_n = Eigenvector (mode shape)

FEA programs use a technique called direct integration to solve such problems computationally. Direct integration calculates response history by incrementally integrating with respect to time using a time step, Δt . Equations 2.9a and 2.9b show the equations of motion for the n th time step, used to calculate the structural displacements.

$$[M]\{\ddot{D}\}_n + [C]\{\dot{D}\}_n + \{R^{int}\}_n = \{R^{ext}\}_n \quad (2.9a)$$

$$[M]\{\ddot{D}\}_n + [C]\{\dot{D}\}_n + [K]\{D\}_n = \{R^{ext}\}_n \quad (2.9b)$$

where:

$\{R^{int}\}$ = Internal force vector

$\{R^{ext}\}$ = Externally applied loads

Both forms of this equation of motion can be used; however, equation 2.9a is better applied for nonlinear systems as it avoids the changing $[K]$ matrix. FEA solvers that utilize direct integration use finite difference approximations to estimate the time derivatives for $\{D\}$, $\{\dot{D}\}$, and $\{\ddot{D}\}$. Algorithms to calculate response history

can be either explicit or implicit. Explicit algorithms use the general expression shown in equation 2.10. They are generally best used in solving wave propagation problems; however, they are also useful in solving structural dynamics problems that have convergence issues in implicit solvers.

$$\{D\}_{n+1} = f(\{D\}_n, \{\dot{D}\}_n, \{\ddot{D}\}_n, \{D\}_{n-1}, \dots) \quad (2.10)$$

For explicit algorithms, the solution accuracy is dependent on the size of the time increment. Explicit solvers typically require a very small Δt in order to yield a stable solution. According to Cook et al., "...a Δt adequate for stability is likely to guarantee accuracy" [21]. Although explicit algorithms are effective at producing a stable and, thus, accurate solution, the small size of the time increment means the solution will take a long time to converge.

Implicit algorithms use the expression shown in equation 2.11 and are typically used in solving structural dynamics problems.

$$\{D\}_{n+1} = f(\{\dot{D}\}_{n+1}, \{\ddot{D}\}_{n+1}, \{D\}_n, \{\dot{D}\}_n, \{\ddot{D}\}_n, \dots) \quad (2.11)$$

Implicit algorithms have a slight advantage over explicit algorithms. Most implicit methods used are unconditionally stable. In other words, the solver will converge on a solution regardless of the time increment, Δt , used. Although the accuracy will be greatly reduced when large time steps are used, a stable solution will be produced, and the time step can be adjusted for accuracy improvement [21]. Because the problem addressed in this thesis is related to structural dynamics, implicit direct integration will be utilized.

Modal analysis using FEA does have one disadvantage. One of the key underlying assumptions is that the system is linear; however, this is not always true. Many

structural systems may be slightly nonlinear due to material nonlinearity, large displacements, or boundary conditions that are allowed to change over time [22]. As previously stated, the research conducted by Adorno-Rodriguez, Just, and Cranston demonstrated certain nonlinearities may exist in the icosahedron frame as exhibited by its chaotic behavior under certain loads and boundary conditions [5] [8] [9]. Dynamic analyses on the hexakis icosahedron frame assume nonlinear behavior exists, so modal analysis was conducted separately from other dynamic conditions in order to approximate the eigenvalues and eigenvectors of the structure. In other words, for the purposes of calculating the dynamic properties of the frame, nonlinearities were assumed to be minor.

2.2.3 Experimental Modal Analysis

Experimental modal testing is invaluable not only for quickly and accurately measuring data on the dynamic properties of a given system, but also for validating a dynamic Finite Element Model. Through experimentation, the eigenvalues and eigenvectors of a structure can be extracted and compared to the frequency results of a computerized model [22]. Errors in the model can be mitigated by examining the initial assumptions, including material properties, constraints, loads, and boundary conditions. If the results from the model match the experimental results, the engineer can gain confidence in the applicability of the model to various dynamic loading scenarios. Three key components are critical to experimental modal analysis - fast fourier transforms (FFTs), frequency response functions (FRFs), and power spectral density (PSD) functions [3].

One of the most common techniques used to derive the response of a system to an arbitrary excitation is an FFT. The arbitrary input function is represented by a

Fourier series as shown in equations 2.12a through 2.12d by $f(t)$ [24].

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad (2.12a)$$

$$\omega_0 = \frac{2\pi}{T} \quad (2.12b)$$

$$a_n = \frac{2}{T} \int_0^T (f(t) \cos n\omega_0 t) dt \quad (2.12c)$$

$$b_n = \frac{2}{T} \int_0^T (f(t) \sin n\omega_0 t) dt \quad (2.12d)$$

where:

ω_0 = Fundamental frequency

T = Period of arbitrary input function

n = Integers

t = Time

Once the input function is represented by a periodic Fourier Expansion, the signal can be converted from the time domain to the frequency domain using a Fourier transform represented by equations 2.13a through 2.13c [23] [24].

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (2.13a)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad (2.13b)$$

$$G(\omega) = \frac{X(\omega)}{F(\omega)} \quad (2.13c)$$

where:

$F(\omega)$ = Fourier transform of $f(t)$ as a function of frequency ω

$x(t)$ = System response as a function of time

$X(\omega)$ = Fourier transform of $x(t)$ as a function of frequency ω

$G(\omega)$ = Frequency Response Function (FRF)

The result of these calculations is the FRF, which is the ratio of the output response to the input excitation force. Due to the FFT equations, the FRF contains both real and imaginary components that describe the magnitude and the phase of the response. Figure 2.1 provides an example of the plots of the time and frequency response functions related by the FFT for a simple, unrestricted beam.

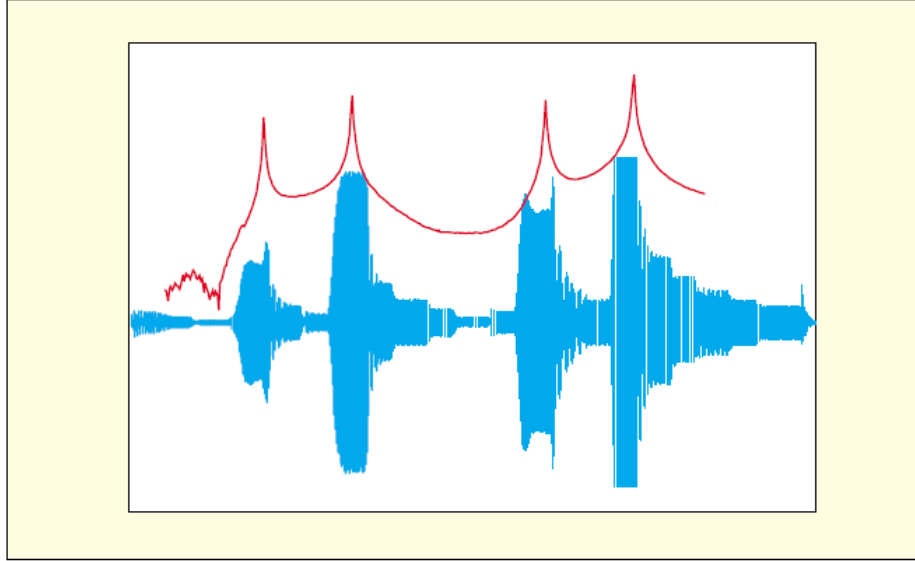


Figure 2.1. Time (blue) and frequency (red) response functions plotted as response magnitude (in dB) vs frequency (in Hz) [3]

The eigenvalues (resonances) are represented by the peaks of the FRF plot (as shown in red in figure 2.1 and are separated by anti-resonances. If multiple FRFs are collected along the length of a beam, information regarding the mode shapes of the structure at different resonances can be extracted. Figure 2.2 illustrates the first four

mode shapes of a sample beam where modes 1 and 3 are bending modes, and modes 2 and 4 are torsional modes.

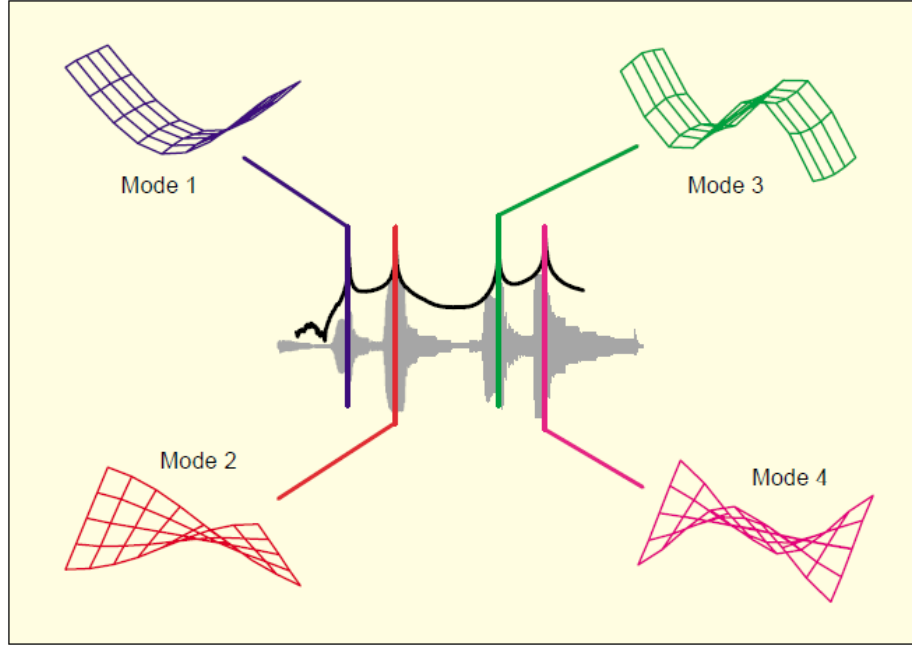


Figure 2.2. FRF mode shapes at the first four eigenvalues [3]

One other method of displaying dynamic information is plotting the Power Spectral Density (PSD). This technique has a slight advantage over FRFs in that only the output response as a function of time is needed rather than both the output response and input force. PSD functions rely on the principle of the autocorrelation function which relates the value of a particular variable at one point in time to the value of the same variable one time interval, τ , later. Equation 2.14a shows the autocorrelation function as a function of the representative sample function, $f(t)$. The PSD function is then calculated by a Fourier transform as shown in equation 2.14b [24].

$$R_f(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t)f(t + \tau)dt \quad (2.14a)$$

$$S_f(\omega) = \int_{-\infty}^{\infty} R_f(\tau)e^{-i\omega\tau}d\tau \quad (2.14b)$$

where:

$R_f(\tau)$ = Autocorrelation as a function of τ

$S_f(\omega)$ = PSD as a function of frequency, ω

τ = Time step

T = Signal period

FRFs and PSD functions provide abundant experimental information regarding the dynamics of structural systems, and both techniques will be utilized to validate the accuracy of the computerized hexakis icosahedron model. Additionally, this information will be useful in assessing nonlinear dynamic behavior. Previous research on the icosahedron frame studied a *snapback* behavior, discussed later in this chapter, which was determined to be a nonlinear dynamic event attributed to chaotic behavior. A similar dynamic analysis on the hexakis icosahedron model will be conducted to determine whether similar chaotic behavior exists under the same loading conditions.

2.3 Chaotic Behavior

The field of chaotic dynamics is what S. Rasband describes as “Arguably the most broad based revolution in the worldview of science in the twentieth century...” He goes on to justify his claim by stating “...chaotic dynamics is a consequence of mathematics itself and hence appears in a broad range of physical systems” [25]. Typically, the word “chaos” is defined as a state of total confusion or disorder. A common misconception is that a chaotic system is simply one where measurements vary unpredictably or randomly; however, chaotic dynamics refers to deterministic development in the response of a system. In other words, the “state of a system depends on the previous state in a rigidly determined way” [25]. Chaotic systems are highly dependent on the initial conditions specified [7]. For non-chaotic systems,

error generally grows linearly with time; however, for chaotic systems, error grows exponentially with time so the state of a system is only known for very short periods of time [6]. In order to measure the amount of chaos present in a given dynamic system, displacement plots, Power Spectral Density (PSD) functions, phase-space diagrams, and Lyapunov exponents will be analyzed for the hexakis icosahedron frame. To illustrate the difference between chaotic and non-chaotic systems, two examples using pendulums are presented.

2.3.1 Simple Examples of Chaotic Behavior

A simple example of a non-chaotic dynamic system is a single pendulum. A single pendulum system without any losses, such as friction or air resistance, will oscillate indefinitely. This is because all of the potential energy at the top of its swing will be converted to kinetic energy at the bottom. All of the potential energy will be restored to the system back at the top of its swing. The left image in figure 2.3 shows the motion of the pendulum as well as its phase-space diagram where the pendulum's position and velocity are plotted on x and y-axes, respectively. The motion and phase-space diagram of this pendulum will remain the same regardless of the initial conditions. Another type of single pendulum is one in which losses such as friction are included. Due to the losses in the system, kinetic energy will slowly be reduced and the pendulum's motion and associated phase-space diagram will resemble the right image in figure 2.3.

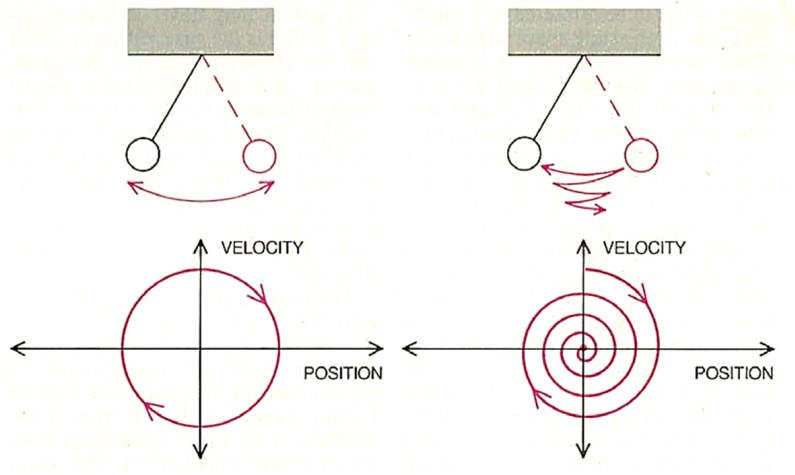


Figure 2.3. Single pendulum motion and associated phase-space diagrams, without friction (left) and with friction (right) [4]

As can be seen in the phase-space diagram, the pendulum's motion slowly converges on the origin of the plot. This phenomenon will be discussed later in this chapter and is known as an attractor [4].

A simple example of a chaotic dynamic system is a double pendulum. Unlike the non-chaotic single pendulum, the double pendulum system exhibits tremendously different behavior depending on the initial conditions. In figure 2.4, the images on the left show the different initial conditions while the images on the right show traces of the two pendulums. Notice that the traces show significantly different responses based on the different initial conditions. As the two double pendulums in figure 2.4 show, although the initial conditions differ by only a few degrees, the responses of the two dynamic systems are completely different. Because of the system's extreme sensitivity to the initial conditions, the system is categorized as chaotic [6].

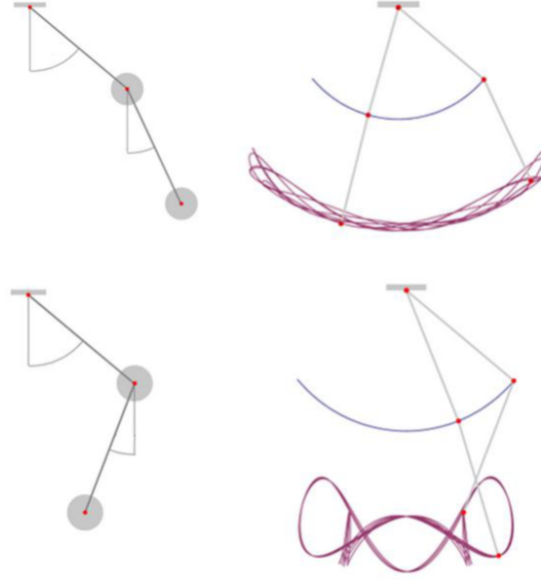


Figure 2.4. Motion of two double pendulums with two slightly different initial conditions [5]

2.3.2 PSD Functions

PSD plots are another tool useful for chaotic system analysis. According to Baker, the power spectrum is a “representation of the relative abundance of different frequencies in a given time series” [6]. The power spectrum plots reveal the harmonic frequencies for a system by using logarithmic plots which accentuate the components with low power levels. Figures 2.5 and 2.6 compare the time series and PSD plots for the same system constrained by different initial conditions. This serves to demonstrate the importance of initial conditions in exhibiting the chaotic behavior of a system as well as prove the usefulness of PSD plots in determining the state of chaotic systems [6] [7].

Figure 2.5 shows the time series and PSD plots for a system exhibiting periodic motion. The time series plot shown in the top of figure 2.5 demonstrates the periodic oscillation of the system. The bottom diagram shows the corresponding PSD plot for the system where the eigenvalues are represented by the peaks.

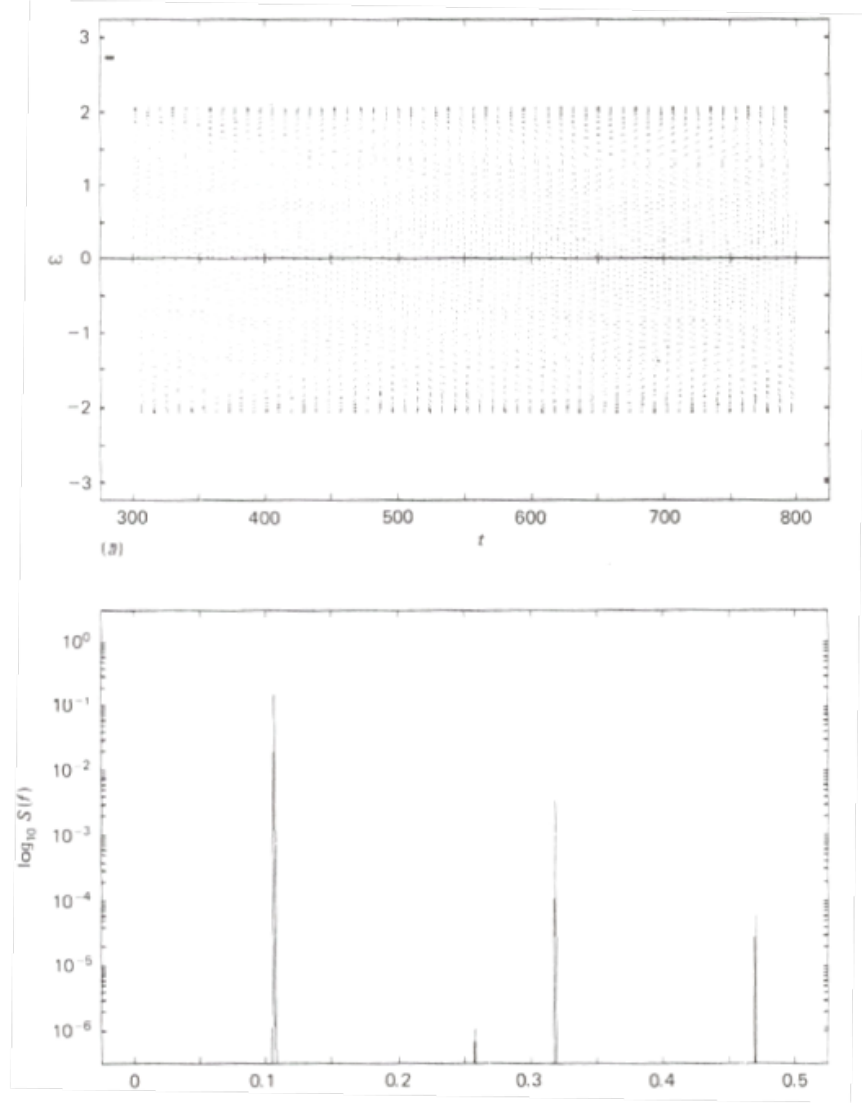


Figure 2.5. Time series (upper) and PSD (lower) plots for periodic motion [6]

Figure 2.6 shows the time series and PSD plots for the same system under different initial conditions. The response represents a chaotic state. The time series plot shown in the top of the figure is non-periodic and irregular, and the corresponding PSD plot does not have clearly distinct peaks as in figure 2.5. Chaotic PSD plots exhibit most power at lower frequencies as can be seen in figure 2.6. Although not as simple as non-chaotic systems, PSD plots can still be used to determine the state of chaotic systems.

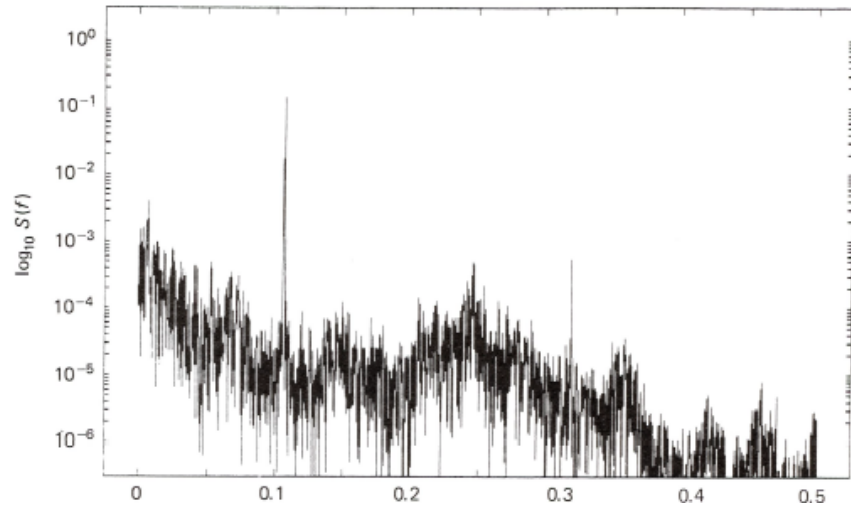
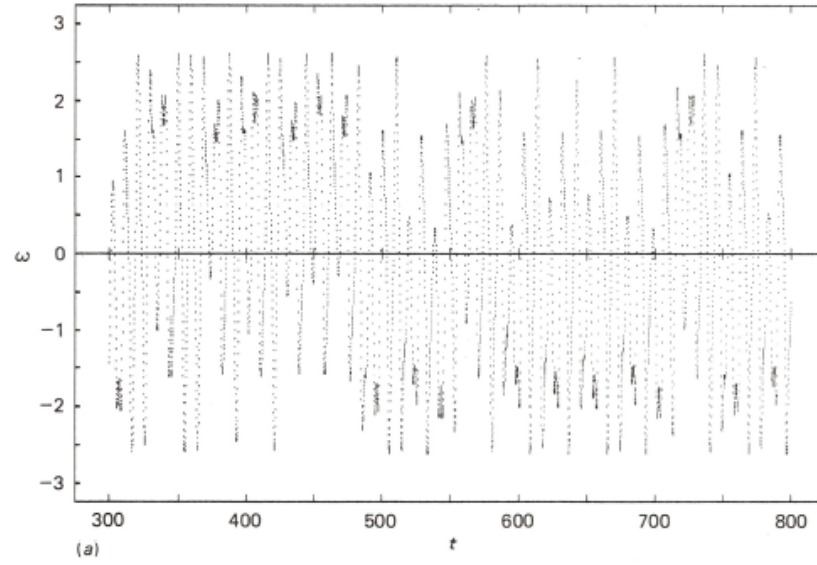


Figure 2.6. Time series (upper) and PSD (lower) plots for chaotic system [6]

2.3.3 Phase-Space Diagrams

Another method for analyzing chaotic systems involves phase-space diagrams. According to Baker, “The phase space of a dynamical system is a mathematical space with orthogonal coordinate directions representing each of the variables needed to specify the instantaneous state of the system” [6]. To best illustrate this, the example of the single pendulum from section 2.3.1 will be used. The equation of motion for the single pendulum can be written as $\frac{d^2\theta}{dt^2} + \theta = 0$ and $\omega = \frac{d\theta}{dt}$ where θ is the angular displacement of the pendulum and ω is the angular velocity. Solving the general equation of motion yields the following two equations: $\theta = a_i \cos t$ and $\omega = a_i \sin t$ where a_i represents the amplitudes of the motion. Combining these two equations yields the *phase-space* of the system represented by the two variables θ and ω . Thus, the phase-space diagram of a system in static equilibrium will appear as a single point, and one that is dynamically stable with periodic motion will appear as a single, closed orbit. Assuming there are no losses such as friction or air resistance in the trajectory, the energy of the system is fixed and the orbit will remain constant. This is illustrated in the difference between the two phase space diagrams shown in figure 2.3. For dissipative systems, the phase-space plot converges on a single point at the origin as the motion decays. This point is called the *attractor* [7].

Attractors provide critical information for dynamic analyses as they portray the long term behavior of the systems regardless of the initial conditions [4] [7]. Dissipative systems typically converge on either a single point or a closed curve as shown in figure 2.7; however, for systems with higher dimensionality, the system can converge on a torus shape as well [11]. These types of attractors represent non-chaotic systems.

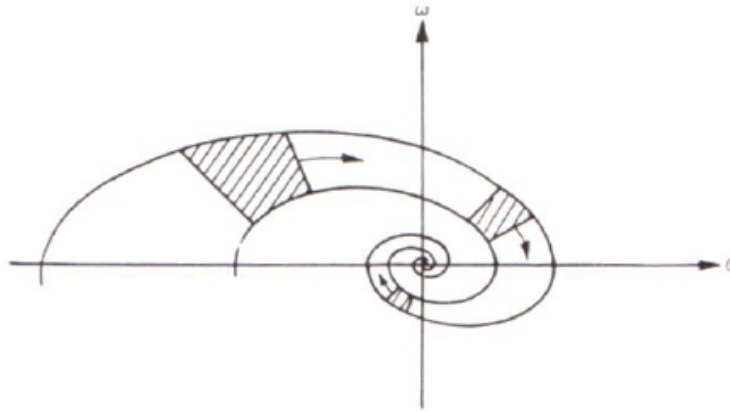


Figure 2.7. Phase-space diagram of a non-chaotic pendulum system converging on an attractor [6]

Chaotic systems can also converge on an attractor; however, its shape will be much more geometrically intricate as shown in figure 2.8. Although attractors and phase-space diagrams do not provide exact details on the dynamic state of the system in the form of eigenvalues and eigenvectors, they do provide another method to differentiate between chaotic and non-chaotic systems.

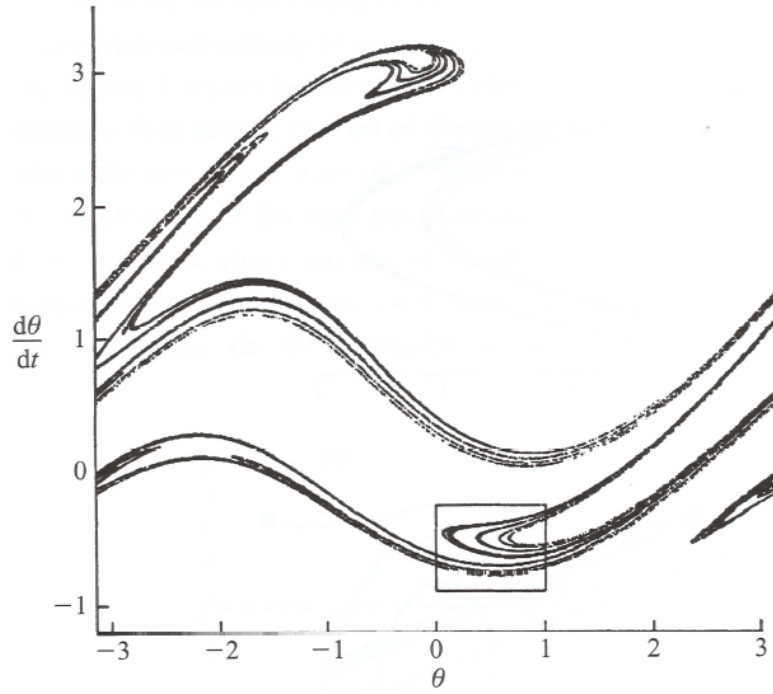


Figure 2.8. Phase-space diagram of a chaotic system converging on an attractor [7]

2.3.4 Lyapunov Exponents

As stated in section 2.3.1, one characteristic of chaotic systems is their sensitivity to initial conditions such as boundary conditions, load magnitudes, or load rates. Lyapunov exponents provide a measure of this sensitivity by evaluating a logistic map of two slightly different initial conditions, x and $x + \epsilon$, and estimating their divergence from each other. Equation 2.15 below is used to characterize the system's divergence after n iterations [6].

$$\epsilon(n) \approx \epsilon e^{\lambda n} \quad (2.15)$$

where:

λ = Lyapunov exponent

ϵ = Initial condition

n = Number of increments

Lyapunov exponents are very similar to eigenvalues; however, they do have some key differences. According to Sprott, “Whereas eigenvalues are usually calculated at a point in state space, such as an equilibrium point, Lyapunov exponents are usually geometrically *averaged* along the orbit or trajectory” [11]. An n dimension system will have n eigenvalues and n Lyapunov exponents. More differences between Lyapunov exponents and eigenvalues are listed in table 2.1 below.

Table 2.1. Eigenvalues and Lyapunov Exponents Compared [11]

Eigenvalue	Lyapunov exponent
<i>Local</i> quantity	<i>Global</i> quantity
<i>Constant</i> value	<i>Average</i> value
<i>Complex</i> number	<i>Real</i> number
<i>Not</i> usually orthogonal	Mutually <i>orthogonal</i>

For a one-dimensional logistic map of the system, the global Lyapunov exponent is calculated using equation 2.16 below for N increments. The equation for the one-dimensional map is represented by $f(X_n)$ [11].

$$\lambda = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} \ln|f'(X_n)| \quad (2.16)$$

A Lyapunov exponent less than or equal to zero ($\lambda \leq 0$) indicates a non-chaotic system that is dissipating towards an attractor. A positive Lyapunov exponent ($\lambda > 0$) indicates a chaotic system which is diverging at a rate equal to the Lyapunov exponent. The higher the value of the exponent, the more chaotic the system [7].

Although these equations are useful when the one-dimensional logistic map is known, they do not allow the calculation of Lyapunov exponents from experimental

data. In their paper, Wolf et al. provide a method for calculating the Lyapunov exponent from time series data. Equation 2.17 was used to create a MATLAB algorithm to experimentally determine the magnitude of the exponent. This code, written by Wolf et al. and modified by Just, can be found in Appendix A [5] [26].

$$\lambda_1 = \frac{1}{t_N - t_0} \sum_{k=1}^n \log_2 \frac{L'_p(t_k)}{L_p(t_{k-1})} \quad (2.17)$$

where:

t_0 = Initial time

t_N = Time at increment N

N = Total number of increments

$L_p(t_{k-1})$ = Distance between two points on the trajectory

$L'_p(t_k)$ = Evolved distance between two points at a later time step

2.4 Previous Research on VLTAV Structures

Creating a Vacuum Lighter-Than-Air Vehicle (VLTAV) using an internal frame covered with a membrane-like skin is a fairly new concept that has been explored to a limited extent, though the idea of a vacuum airship is centuries old. Five previous AFIT students have conducted extensive research on one internal frame option, an icosahedron, and have analyzed it under both static and dynamic loading conditions. The concept originated from Metlen's research into structural options for VLTAVs. His thesis largely discussed the feasibility of using an icosahedron frame, pictured in figure 1.3a, but also introduced the hexakis icosahedron, pictured in figure 1.3b, as a possible frame configuration [18].

As mentioned in chapter 1.2, a sphere is the ideal shape for a VLTAV as it max-

imizes the ratio of internal volume to surface area. For this reason, Metlen focused his research on geodesic structures that could function as internal supporting frames. Through mathematical theory, Metlen concluded that creating a VLTAV using an icosahedron frame was possible using certain materials and postulated that a hexakis icosahedron frame would also work [18].

Adorno-Rodriguez continued Metlen's work by conducting static analyses on the icosahedron frame, including an investigation into the optimal materials, beam sizing, and skin thickness [8]. His work was later used by Cranston to develop the beam sizing and skin thickness equations for the hexakis icosahedron frame later discussed in this section [9]. Adorno-Rodriguez began by creating equation 2.18 which allows calculation of the weight-to-buoyancy ratio (W/B) for a given structure including atmospheric conditions and internal volume reduction caused by structural deformation. This equation was invaluable for optimizing the frame design at different atmospheric conditions.

$$\frac{W}{B} = \frac{V_s \rho_s + V_f \rho_f + (V_i - V_r) \rho_{air,i}}{(V_i - V_r) \rho_{air,o}} \quad (2.18)$$

where:

V_s = Volume of membrane-like skin

V_f = Volume of internal frame

V_i = Initial internal volume

V_r = Volume reduction due to structural deformation

ρ_s = Density of skin

ρ_f = Density of frame

$\rho_{air,i}$ = Density of internal air

$\rho_{air,o}$ = Density of ambient air

Adorno-Rodriguez also carried out extensive analyses on the optimal materials to use in constructing the VLTAV that would minimize weight and maximize resiliency to atmospheric loading conditions. He chose seven material combinations from three different materials for the frame and skin, and plotted the applied pressure vs. maximum Von Mises stress for each model option as shown in figures 2.9 and 2.10.

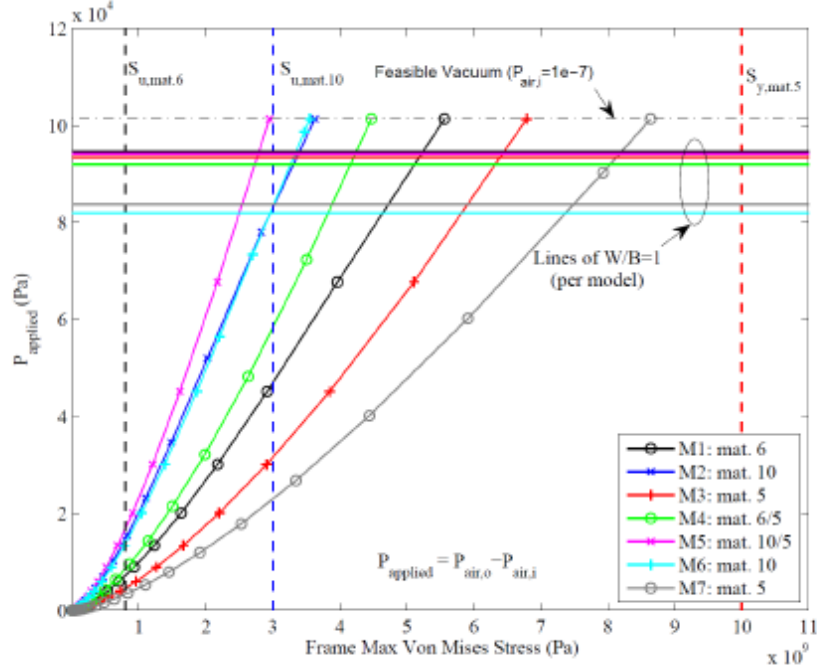


Figure 2.9. Applied pressure versus Von Mises Stress for different frame materials [8]

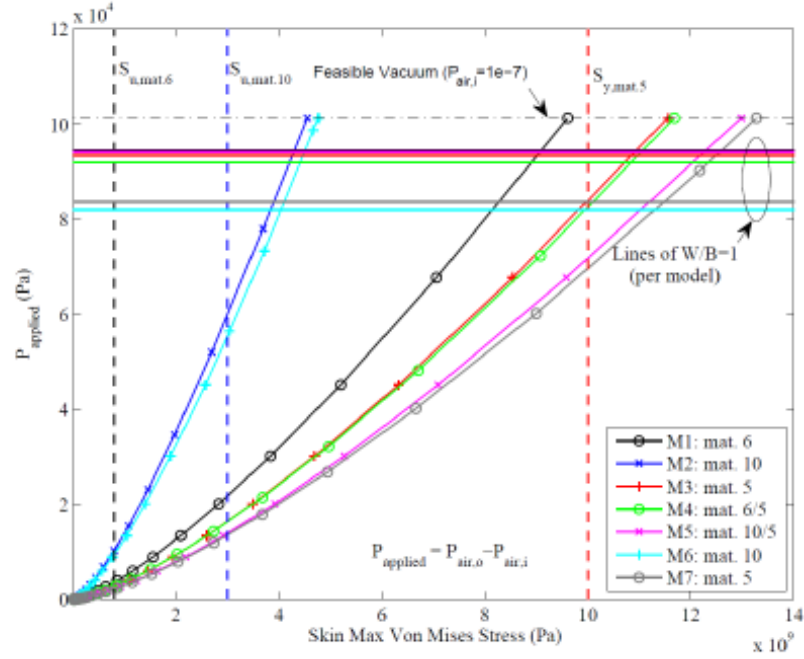
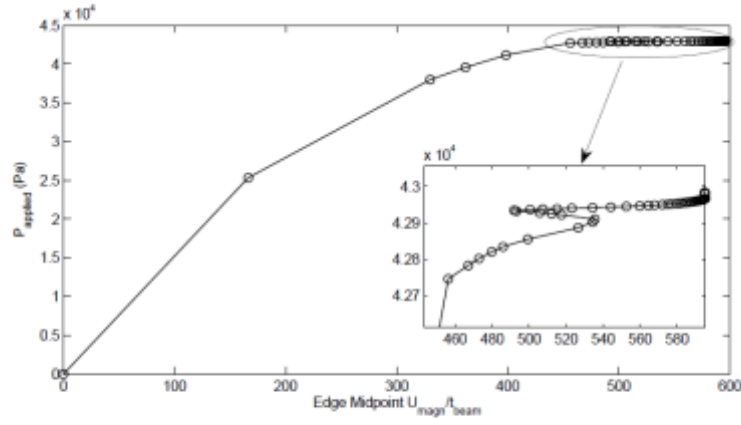


Figure 2.10. Applied pressure versus Von Mises Stress for different skin materials [8]

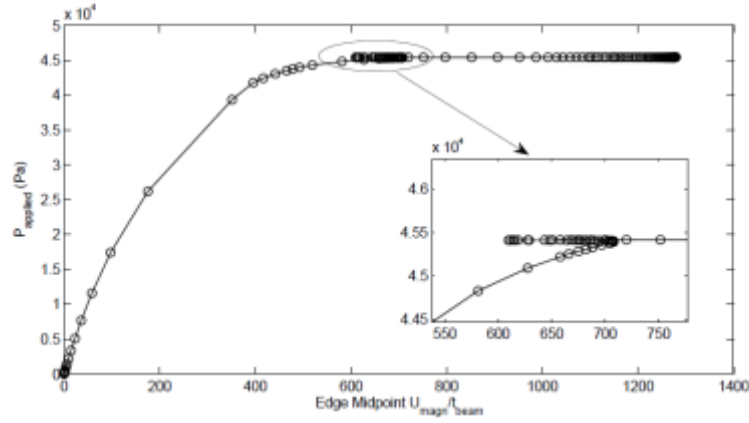
In these graphs, horizontal lines represent lines of positive buoyancy for each material combination, and dashed vertical lines represent the yield strength of each material. From these graphs, Adorno-Rodriguez determined that models M3 and M7 best resisted the loading and allowed positive buoyancy. Both of these models were composed of Nanocyl NC7000 Thin multi-wall carbon nanotubes (CNTs) for both the frame and the skin. Although CNTs seem best suited for a VLTAV application, they are still being researched and are not yet commercially available. Because of this, beryllium was chosen as the primary material for the remainder of the current work as it is commercially available and its properties are well known.

Adorno-Rodriguez conducted static analyses on the icosahedron under three different boundary conditions (BCs) imposed at the top and bottom nodes of the frame. BC1 restricted all degrees of freedom (DOFs) at the bottom node and left the top node free. BC2 restricted all DOFs at the bottom node and also restrained translational motion in the 1 and 2 directions. BC3 was the only symmetric BC, which

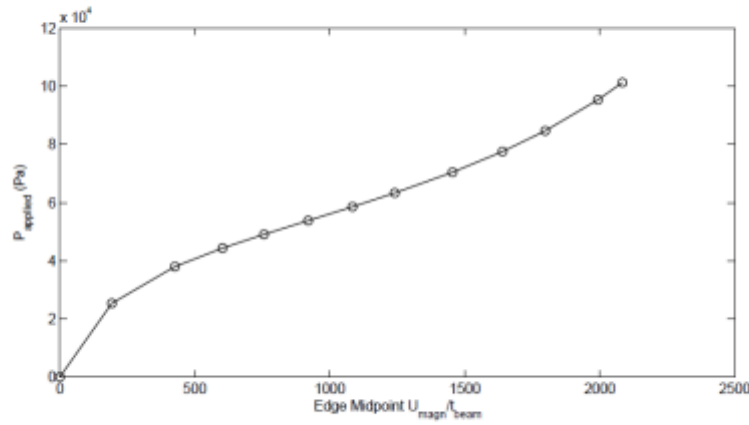
analyzed and restricted translational motion in the 1 and 2 directions for both the top and bottom nodes. By plotting the pressure load vs. displacement curves, evaluated at the midpoint of a beam for each BC as shown in figure 2.11, he was able to observe a *snapback* behavior for BCs 1 and 2 at 45% sea level loading conditions. The top two graphs in figure 2.11 show the *snapback* behavior occurring near 45% of sea level pressure. The plot of BC3, the bottom image, exhibits no *snapback* behavior even with 100% sea level pressure applied.



(a) Boundary Condition 1



(b) Boundary Condition 2



(c) Boundary Condition 3

Figure 2.11. Applied pressure versus displacement plots for two unsymmetric boundary conditions exhibiting *snapback* behavior (a and b) and one symmetric boundary condition (c) [8]

Adorno-Rodriguez compared this *snapback* behavior to the buckling of a thin shell where "an instantaneous reversal of geometry occurs, but the structure retains a load-bearing capacity" [5]. Just then theorized that the *snapback* was due to nonlinear dynamic behavior present within the frame [5]. To validate this theory, he analyzed the icosahedron frame quasi-statically and dynamically to characterize the chaotic behavior of the structure.

Just modeled the load rate for each BC using a ramp input to determine which BCs would exhibit *snapback* behavior. Figure 2.12 below shows an example of the different load rates used in his quasi-static analyses.

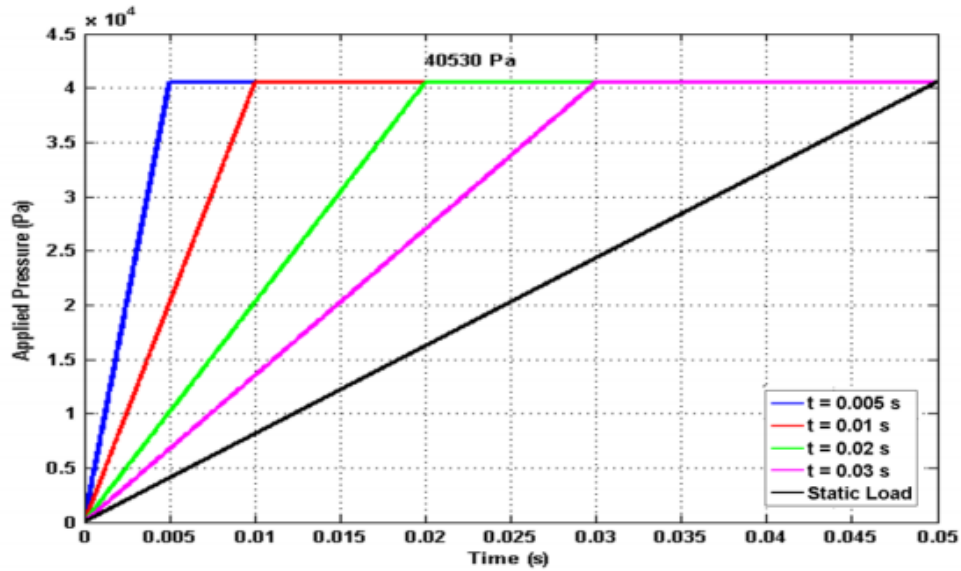


Figure 2.12. Load rates modeled as ramp inputs for a 40% sea level loading condition [5]

Although only BCs 1 and 2 exhibited *snapback* behavior in Adorno-Rodriguez's static analyses, Just's research demonstrated that the *snapback* behavior would be created in all three BCs depending on the load rate applied. The icosahedron VL-TAV modeled with both the frame and the skin, however, did not exhibit any chaotic behavior [5]. The techniques used to characterize the chaotic behavior of the icosahedron were previously discussed in section 2.3, and Just's findings will be compared to

the dynamic analysis results of the hexakis icosahedron frame in Chapter IV of this thesis.

Research by Cranston provided detailed information on the model development of the hexakis icosahedron frame [9]. A hexakis icosahedron is a another geodesic sphere with 120 faces, 180 edges, and 62 vertices as shown in figure 1.3b. It is a nearly spherical frame composed of 120 identical right triangles. To create a model of the frame, the vertex locations were calculated using a MATLAB program. C-Equations shown in table 2.2 were used to find the 62 vertices as shown in figure 2.13. The only input required to solve these equations is the desired radius of the frame, labeled as r_{hex} . The MATLAB code used to generate the vertex locations can be found in Appendix A.

Table 2.2. Hexakis Icosahedron Vertex Coordinate C-Equations [9]

$C_0 = \frac{r_{hex} \sqrt{\frac{5}{2}(85-31\sqrt{5})}}{22}$
$C_1 = \frac{r_{hex} \sqrt{\frac{5}{2}(25-11\sqrt{5})}}{3}$
$C_2 = \frac{r_{hex} \sqrt{5(25-2\sqrt{5})}}{22}$
$C_3 = r_{hex} \sqrt{\frac{1}{10}(5 - \sqrt{5})}$
$C_4 = \frac{5r_{hex}}{3\sqrt{5+2\sqrt{5}}}$
$C_5 = \frac{r_{hex} \sqrt{\frac{5}{2}(65+19\sqrt{5})}}{22}$
$C_6 = r_{hex} \sqrt{\frac{1}{10}(5 + \sqrt{5})}$
$C_7 = \frac{r_{hex} \sqrt{\frac{5}{2}(5-\sqrt{5})}}{3}$
$C_8 = \frac{r_{hex} \sqrt{5(25-2\sqrt{5})}}{11}$

Hexakis Icosahedron Vertex Locations		
V0 = (0.0, 0.0, C8)	V20 = (-C3, 0.0, C6)	V41 = (C5, -C0, -C2)
V1 = (0.0, 0.0, -C8)	V21 = (-C3, 0.0, -C6)	V42 = (-C5, C0, C2)
V2 = (C8, 0.0, 0.0)	V22 = (C6, C3, 0.0)	V43 = (-C5, C0, -C2)
V3 = (-C8, 0.0, 0.0)	V23 = (C6, -C3, 0.0)	V44 = (-C5, -C0, C2)
V4 = (0.0, C8, 0.0)	V24 = (-C6, C3, 0.0)	V45 = (-C5, -C0, -C2)
V5 = (0.0, -C8, 0.0)	V25 = (-C6, -C3, 0.0)	V46 = (C2, C5, C0)
V6 = (0.0, C1, C7)	V26 = (0.0, C6, C3)	V47 = (C2, C5, -C0)
V7 = (0.0, C1, -C7)	V27 = (0.0, C6, -C3)	V48 = (C2, -C5, C0)
V8 = (0.0, -C1, C7)	V28 = (0.0, -C6, C3)	V49 = (C2, -C5, -C0)
V9 = (0.0, -C1, -C7)	V29 = (0.0, -C6, -C3)	V50 = (-C2, C5, C0)
V10 = (C7, 0.0, C1)	V30 = (C0, C2, C5)	V51 = (-C2, C5, -C0)
V11 = (C7, 0.0, -C1)	V31 = (C0, C2, -C5)	V52 = (-C2, -C5, C0)
V12 = (-C7, 0.0, C1)	V32 = (C0, -C2, C5)	V53 = (-C2, -C5, -C0)
V13 = (-C7, 0.0, -C1)	V33 = (C0, -C2, -C5)	V54 = (C4, C4, C4)
V14 = (C1, C7, 0.0)	V34 = (-C0, C2, C5)	V55 = (C4, C4, -C4)
V15 = (C1, -C7, 0.0)	V35 = (-C0, C2, -C5)	V56 = (C4, -C4, C4)
V16 = (-C1, C7, 0.0)	V36 = (-C0, -C2, C5)	V57 = (C4, -C4, -C4)
V17 = (-C1, -C7, 0.0)	V37 = (-C0, -C2, -C5)	V58 = (-C4, C4, C4)
V18 = (C3, 0.0, C6)	V38 = (C5, C0, C2)	V59 = (-C4, C4, -C4)
V19 = (C3, 0.0, -C6)	V39 = (C5, C0, -C2)	V60 = (-C4, -C4, C4)
	V40 = (C5, -C0, C2)	V61 = (-C4, -C4, -C4)

Figure 2.13. Hexakis Icosahedron Vertex Coordinate Locations Based on C-Equation Calculations [9]

Once the vertex locations were known, the lengths of the sides of the triangles could be calculated using equations 2.19a through 2.19c. The lengths, l_{se} , l_{me} , and l_{le} , corresponded to the short, medium, and long edge lengths of the triangle, respectively.

$$l_{se} = r_{hex} \frac{5}{11} \sqrt{49 - \frac{65\sqrt{5}}{3}} \quad (2.19a)$$

$$l_{me} = r_{hex} \frac{1}{11} \sqrt{81 - 21\sqrt{5}} \quad (2.19b)$$

$$l_e = r_{hex} 2\sqrt{\frac{7}{3} - \sqrt{5}} \quad (2.19c)$$

The surface area and internal volume contained by the hexakis icosahedron frame were calculated using equations 2.20a through 2.20c. These values were then substituted into equation 2.18 to solve for the beam cross-section.

$$p_{ht} = \frac{l_{se} + l_{me} + l_e}{2} \quad (2.20a)$$

$$A_{HI} = 120\sqrt{p_{ht}(p_{ht} - l_{se})(p_{ht} - l_{me})(p_{ht} - l_e)} \quad (2.20b)$$

$$V_{HI} = \frac{100r_{hex}^3}{3\sqrt{53 + \frac{118}{\sqrt{5}}}} \quad (2.20c)$$

where:

p_{ht} = Average beam length within structure

A_{HI} = Surface area of hexakis icosahedron (HI)

V_{HI} = Internal volume of hexakis icosahedron (HI)

Figure 2.14 shows a sketch of the proposed beam cross-section where r_{beam} represents the radius of the beam and t_{beam} represents the thickness of the beam.

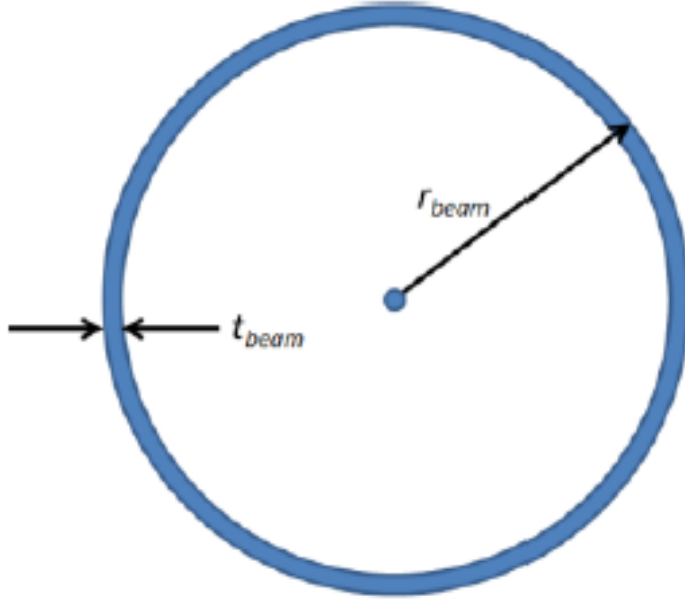


Figure 2.14. Beam cross-section sketch [5]

Equations 2.21a through 2.21c were used to calculate the beam radius, r_{beam} , and skin thickness, t_{skin} , for the hexakis icosahedron frame given a desired weight-to-buoyancy (W/B) for both the frame and the skin, indicated as $\frac{W}{B_f}$ and $\frac{W}{B_s}$ respectively [9].

$$r_{beam} = r_{icos} \sqrt{\frac{\frac{W}{B_f} \rho_a V_{HI}}{((\pi 60(2c - c^2))(l_{se} + l_{me} + l_{le})) \rho_f}} \quad (2.21a)$$

$$c = \frac{t_{beam}}{r_{beam}} \quad (2.21b)$$

$$t_{skin} = \frac{\rho_a \frac{W}{B_s} V_{HI}}{A_{HI} \rho_s} \quad (2.21c)$$

The equations provided by Cranston were critical in creating a Finite Element Model (FEM) of the hexakis icosahedron frame for future dynamic analyses as well as manufacturing a physical model of the frame using additive manufacturing techniques.

Quasi-static and dynamic analyses on the hexakis icosahedron frame were conducted very similarly to Just's analyses on the icosahedron.

2.5 Summary

Previous research by Metlen, Adorno-Rodriguez, Just, and Cranston studied two different geodesic structures for use in a VLTAV. Metlen analyzed the icosahedron and hexakis icosahedron mathematically to determine whether such structures would be suitable for this application. Adorno-Rodriguez continued Metlen's study of the icosahedron frame and performed static analyses using different boundary conditions. A *snaphack* phenomenon was exhibited at two unsymmetric BCs, suggesting the need for dynamic analyses on the frame and an examination into its chaotic behavior. This research was conducted by Just using the techniques and theory outlined above. In the current work, the same methods were used to characterize the dynamic behavior of a hexakis icosahedron. Chapter III will discuss the model development and methodology used to analyze the dynamics and chaos of the VLTAV system.

III. Research Methodology

3.1 Chapter Overview

Quasi-static and dynamic analyses of the hexakis icosahedron frame require an accurate FEM to converge to a solution. Just modified Adorno-Rodriguez's static FEM in Abaqus to analyze the icosahedron under quasi-static and dynamic loading conditions [5]. For this research, the model of the icosahedron was re-created for further analysis and compared to a model of the hexakis icosahedron. A comprehensive discussion of the finite element analysis (FEA) methods, modeling techniques, and analysis process for the hexakis icosahedron frame are included in this chapter as well as the general strategy for comparing the icosahedron and hexakis icosahedron results.

The hexakis icosahedron model was developed using a combination of MATLAB, Solidworks, and Abaqus software and was used to study the behavior of the frame under quasi-static loading conditions. Additionally, the model was used to estimate the dynamic properties of the frame in the form of eigenvalues and eigenvectors. An additively manufactured 0.1778 m diameter hexakis icosahedron frame, with beam radii of 0.0012 m was created using VeroWhitePlus plastic for experimental analysis. The natural frequencies of the frame were obtained using a laser vibrometer and compared to the eigenvalues estimated by the Abaqus model in order to validate the dynamic analysis technique.

3.2 Hexakis Icosahedron Design

The process of creating the hexakis icosahedron frame and importing it into Abaqus began with calculating the vertex coordinates using MATLAB software. The equations outlined at the end of Chapter II were programmed into MATLAB, and

the list of vertex coordinates were subsequently output to a “.txt” file as a matrix of x, y, and z-coordinates. A separate MATLAB code was created to calculate the beam radius and thickness as well as skin thickness. Additionally, this code provided estimates of the frame volume and surface area. The input parameters and output dimensions for the hexakis icosahedron frame are outlined in table 3.1. The output dimensions for the icosahedron frame for the same input values are listed in table 3.2.

Table 3.1. Hexakis Icosahedron Dimensions

	Dimension	Units
Frame Diameter	1.0 (0.3048)	<i>ft (m)</i>
W/B frame	0.5	<i>unitless</i>
W/B skin	0.4	<i>unitless</i>
W/B total	0.9	<i>unitless</i>
Air Density	0.0749 (1.225)	<i>lb/ft³ (kg/m³)</i>
Beam cross-section radius	3.133e-03 (9.5497e-04)	<i>ft (m)</i>
Beam cross-section thickness	1.567e-04 (4.7749e-05)	<i>ft (m)</i>
Skin thickness	4.0079e-05 (1.2216e-05)	<i>ft (m)</i>

Table 3.2. Icosahedron Dimensions

	Dimension	Units
Beam cross-section radius	4.996e-3 (1.523e-03)	<i>ft (m)</i>
Beam cross-section thickness	2.498e-04 (7.614e-05)	<i>ft (m)</i>
Skin thickness	3.6627e-05 (1.1164e-05)	<i>ft (m)</i>

As stated in Chapter II, Just conducted his FEM analyses assuming the material properties of beryllium which are summarized in table 3.3. For simplicity, both the frame and the skin were assumed to be constructed from beryllium.

Table 3.3. Beryllium Material Properties [12]

	Dimension	Units
Density	115.12 (1844)	<i>lb/ft³ (kg/m³)</i>
Modulus of Elasticity	6.33 (303)	<i>lb/ft² (GPa)</i>
Poisson’s Ratio	0.18	<i>unitless</i>
Yield Stress, σ_y	62.9 (434)	<i>ksi (MPa)</i>
Ultimate Stress, σ_u	116 (800)	<i>ksi (MPa)</i>

The “.txt” file containing all of the vertex coordinates of the hexakis icosahedron was then input into SolidWorks, a 3D Computer-Aided Design (CAD) software, using a macro, which generated a 3D point cloud from which the frame could be sketched by connecting the vertices with lines [27]. As shown in figure 3.1, this created a baseline model of the frame that could be modified to be additively manufactured or imported into a finite element (FE) solver.

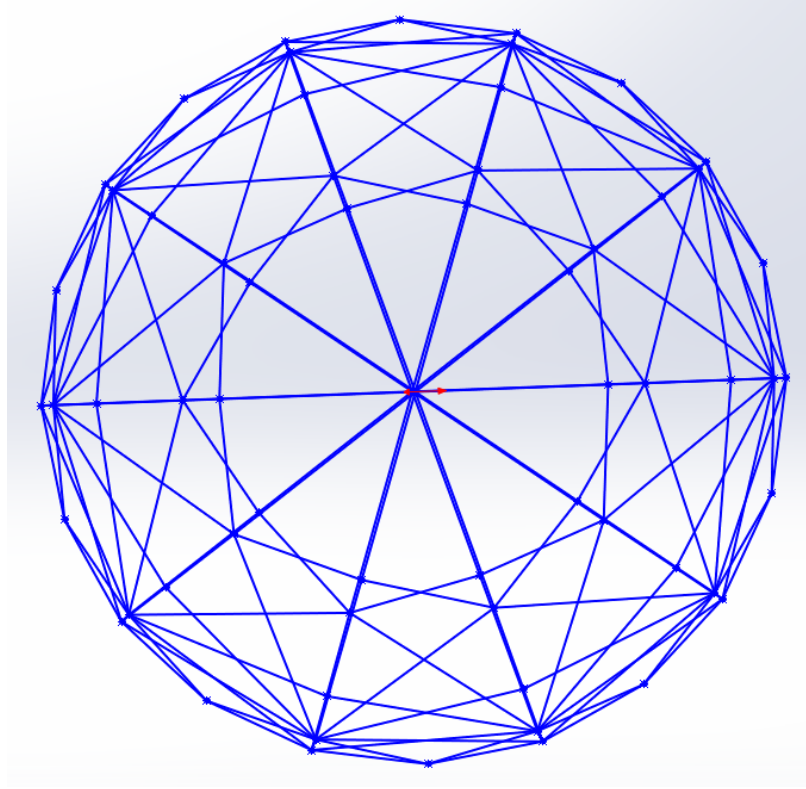


Figure 3.1. Baseline SolidWorks hexakis icosahedron frame model

The Abaqus Unified FEA product suite was used for all FE modeling in this thesis and was the primary focus of this research. The baseline SolidWorks model was saved as a “.stp” file, imported into Abaqus as a wire frame and scaled as desired. Material properties and beam profiles were assigned based on the MATLAB calculations with the desired weight-to-buoyancy (W/B) ratios as shown in figure 3.2.

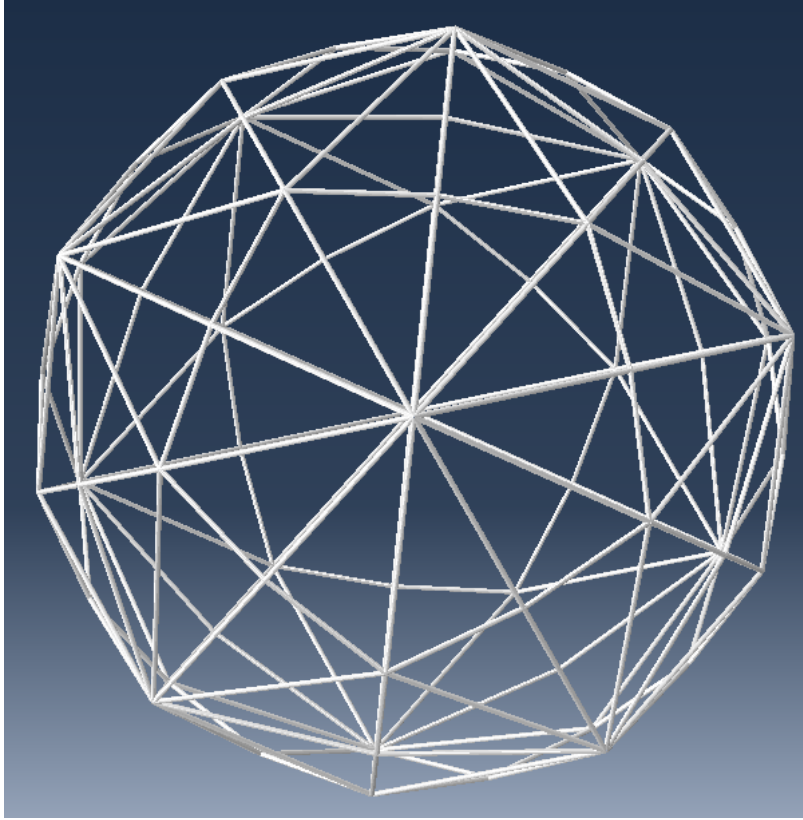


Figure 3.2. Abaqus hexakis icosahedron model with beam profiles assigned

Abaqus's standard implicit solver was used for the FEA analysis. Because non-linear behavior was expected under certain loading conditions as evidenced in Just's icosahedron research, nonlinear geometry was included in the implicit solver. Just used two different fixed time increment sizes depending on whether or not the loading condition was above or below the frame's snapping load (a load rate of approximately 8.1 MPa/s). If the load rate was below 8.1 MPa/s, the time increment size was fixed at 1.0E-05s. If the load rate was above 8.1 MPa/s, the time increment size was fixed at 1.0E-06s [5].

When only the behavior of the frame was being investigated rather than a model of both the frame and skin, a reference point technique was used to apply the atmospheric pressure load as a concentrated force. This technique was first analyzed by Adorno-Rodriguez in his research where he showed that applying the pressure load as

a concentrated force using a coupling constraint yielded accurate results [8]. The centers of mass of the individual triangular faces were calculated in Abaqus, and their coordinates were identified as reference points. Once all 120 reference points were created, a coupling constraint within Abaqus was used to couple the motion of the triangular faces to the motion of the individual reference points. Each triangle was identified as a circumferential surface, and the reference points were identified as the constraint control points. The continuum distributing coupling type was selected to constrain in all six degrees of freedom. According to the Abaqus manual, “the continuum coupling method couples the translation and rotation of the reference node to the average translation of the coupling nodes. The constraint distributes the forces and moments at the reference node as a coupling nodes force distribution only. No moments are distributed at the coupling nodes” [28]. Figure 3.3 shows the continuum coupling constraint applied to a single triangular face of the hexakis icosahedron frame.

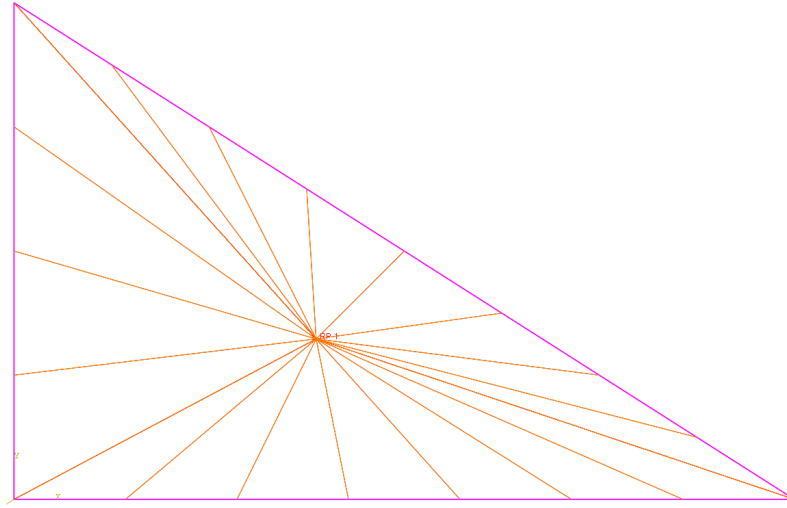


Figure 3.3. Coupling constraint applied to a triangular face of the frame

The pressure load was then applied as a concentrated force at the reference point of each triangular face. In order to most accurately model the pressure load, the

concentrated force was set to follow nodal rotation as a follower force. This concept is best illustrated in figure 3.4.

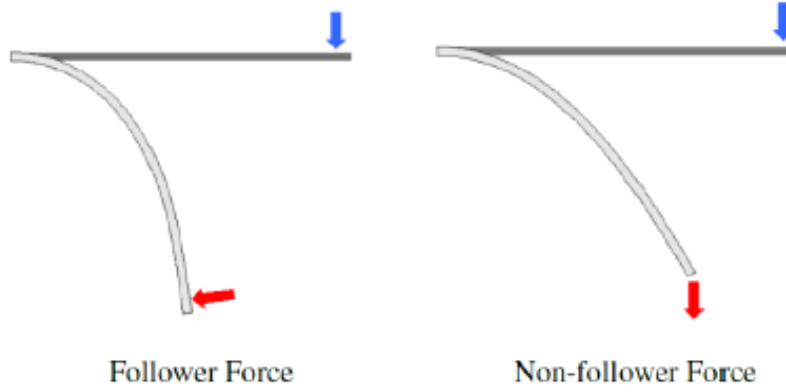


Figure 3.4. Illustration of follower and non-follower forces [5]

To approximate the magnitude of this force, atmospheric pressure in Pascals was multiplied by the surface area of each triangle. This equated to approximately 210.3 Newtons for the 0.3048 m diameter hexakis icosahedron frame and 1126.61 Newtons for the icosahedron frame. The loads were applied using a spherical coordinate system where the center of the frame was identified as the origin. The boundary conditions were applied to the top and bottom nodes of the frame as shown in figure 3.5.

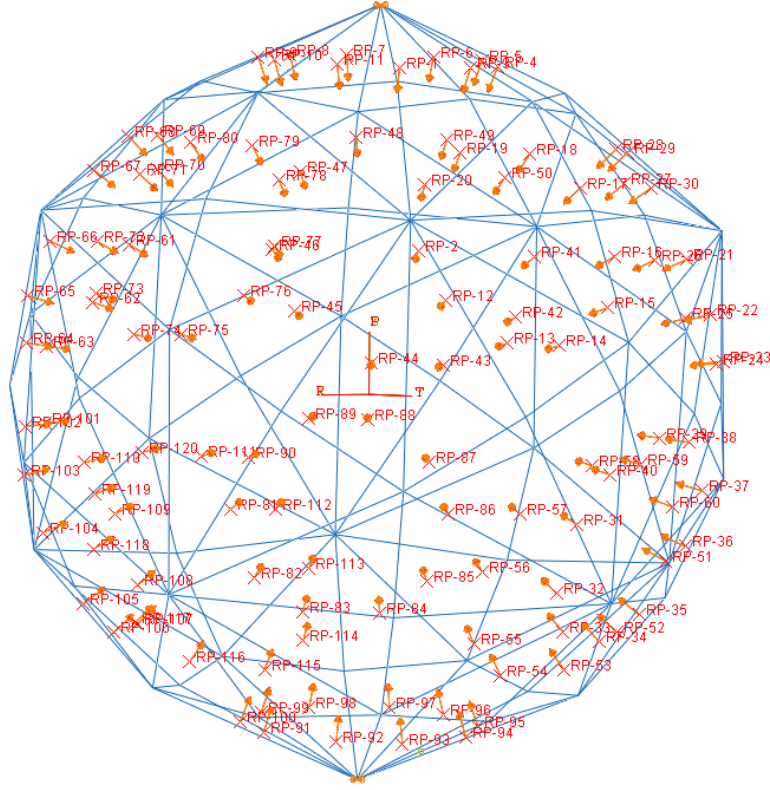


Figure 3.5. Loads applied to the reference points (in yellow) and boundary conditions applied at the top and bottom nodes (in orange)

Though briefly explained earlier in chapter II, the boundary conditions are summarized in table 3.4. BCs 1 and 2 are unsymmetric boundary conditions while BC3 is symmetric.

Table 3.4. Boundary Conditions

	Bottom Node	Top Node
BC1	$U1 = U2 = U3 = UR1 = UR2 = UR3 = 0$	<i>Free</i>
BC2	$U1 = U2 = U3 = UR1 = UR2 = UR3 = 0$	$U1 = U2 = 0$
BC3	$U1 = U2 = 0$	$U1 = U2 = 0$

Because the frame was analyzed under quasi-static loading conditions, the loads were applied using amplitudes created in Abaqus. The amplitude frequency and magnitude corresponded to the ramp duration and percent sea level pressure for each load case. Just evaluated eight different load cases in his research (six for the

standalone frame model and two for the frame with skin model). These load cases are summarized in table 3.5.

Table 3.5. Load Cases [5]

Load Case	Sea Level (%)	Ramp Duration	Load Rate	Boundary Condition
1	10	0.002s	5.0663 <i>MPa/s</i>	BC3 (Frame)
2	20	0.005s	4.053 <i>MPa/s</i>	BC3 (Frame)
3	40	0.005s	8.106 <i>MPa/s</i>	BC3 (Frame)
4	10	0.002s	5.0663 <i>MPa/s</i>	BC2 (Frame)
5	20	0.005s	4.053 <i>MPa/s</i>	BC2 (Frame)
6	40	0.005s	8.106 <i>MPa/s</i>	BC2 (Frame)
7	40	0.005s	8.106 <i>MPa/s</i>	BC3 (Frame-skin)
8	60	0.005s	12.159 <i>MPa/s</i>	BC3 (Frame-skin)

The icosahedron and hexakis icosahedron frames were modeled using B32 elements in Abaqus as shown in figure 3.6. Adorno-Rodriguez conducted a convergence study on the icosahedron frame and determined that each beam in the model should be meshed with eight B32 elements. Abaqus B32 elements are Timoshenko beam elements that allow for transverse shear deformation using a quadratic interpolation between the nodes [28]. To maintain consistency with his and Just’s research, the same number of element divisions was used in this thesis. The icosahedron frame was discretized with 300 B32 elements, and the hexakis icosahedron frame was discretized with 1440 B32 elements.

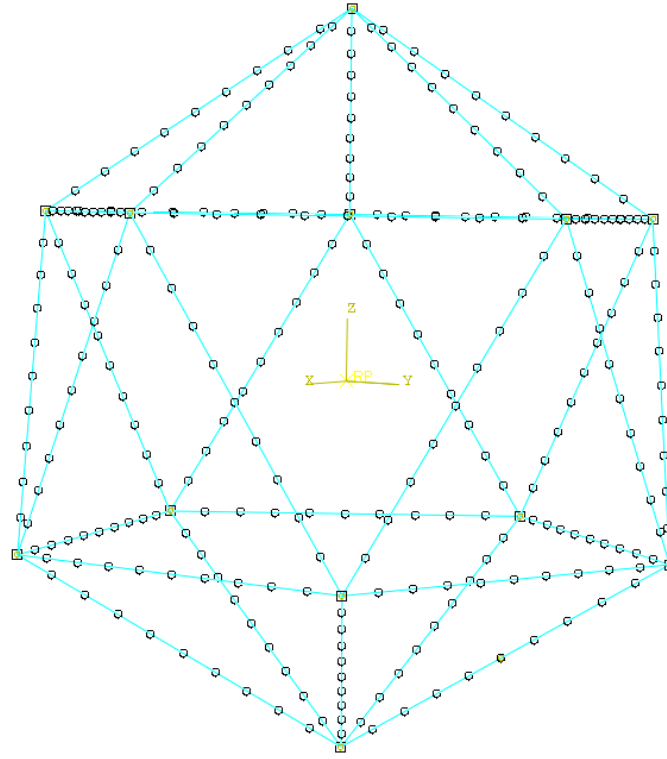
The membrane-like skin designed to cover the icosahedron and hexakis icosahedron frames was modeled using S3R elements, three-noded triangular large strain shell elements that account for transverse shear, as shown in figure 3.7. Just researched which element type should be used to discretize the skin of the VLTAV. Although Adorno-Rodriguez used M3D3 elements, reduced integration triangular elements, to discretize the skin for static analyses, Just utilized S3R elements to conduct dynamic analyses [5] [8] [28]. As he stated, “S3R elements must replace M3D3 elements in this research to calculate the eigenvalues and mode shapes because a membrane does not

possess initial stiffness when subjected to a force perpendicular to the membrane” [5]. The icosahedron skin was meshed with 2000 S3R elements, and the hexakis icosahedron skin was meshed with 5802 S3R elements.

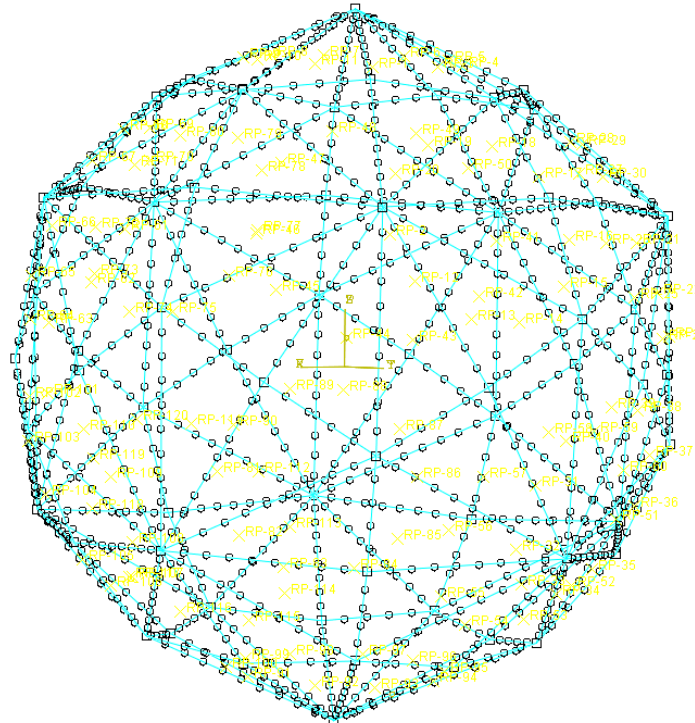
A physical model of the hexakis icosahedron frame was additively manufactured for experimental testing. The Stratasys Objet500 Connex3 3D printer was used to create the frame using Stratasys’s VeroWhitePlus RGD835 material (or simply VeroWhite). VeroWhite is a rigid, opaque photopolymer used to create strong and durable prototypes. Adorno-Rodriguez, Just, and Cranston also utilized the Objet500 3D printer in their research; however, they used Stratasys’s VeroBlue RGD840 material. VeroBlue was switched to VeroWhitePlus for this research because of its more consistent material properties and better quality. This was thought to improve the data gathered by the laser vibrometer in experimental testing. Table 3.6 compares the properties of the two materials as stated in the Stratasys specification sheet [13].

Table 3.6. VeroBlue and VeroWhitePlus Material Properties [13]

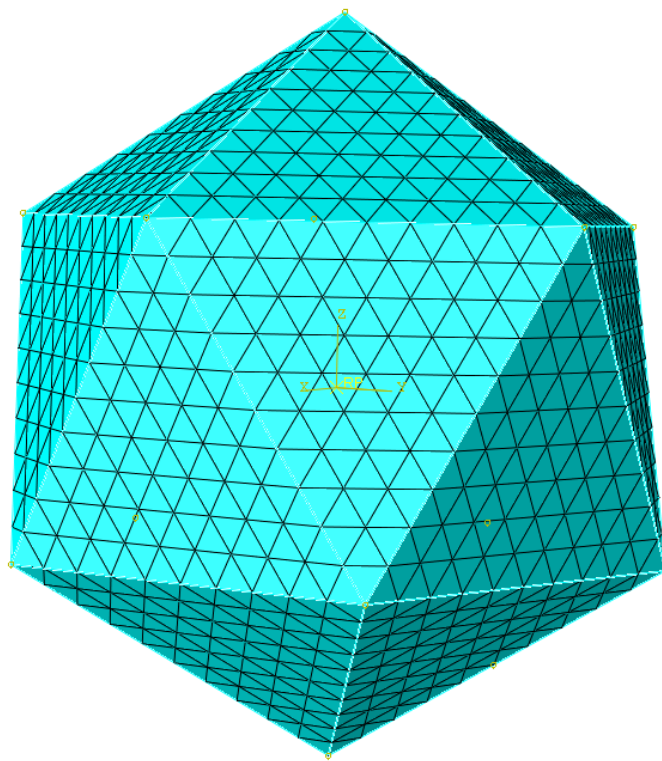
	VeroBlue	VeroWhitePlus	Units
Density	1180-1190	1170-1180	kg/m^3
Modulus of Elasticity	2-3	2-3	GPa
Tensile Strength	50-60	50-65	MPa
Poisson’s Ratio	0.35	0.35	Unitless



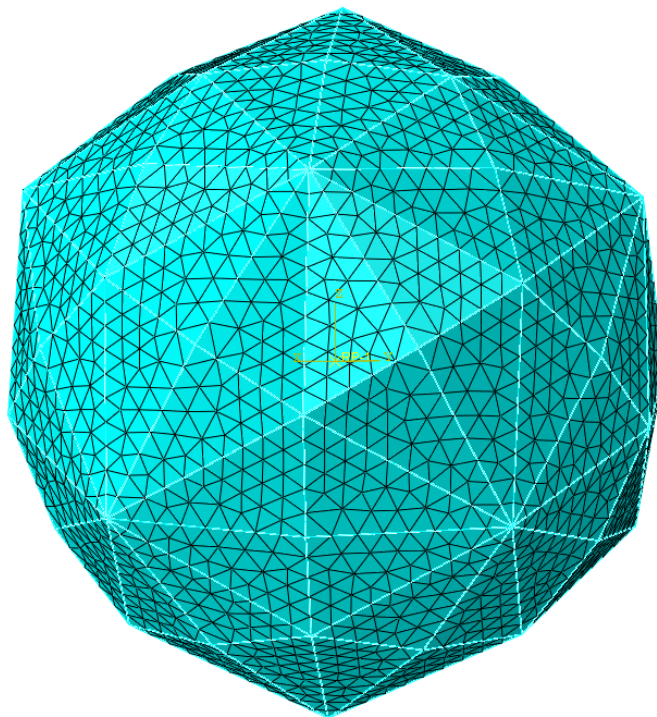
(a) Icosahedron frame meshed with 300 B32 elements



(b) Hexakis icosahedron frame meshed with 1440 B32 elements



(a) Icosahedron skin meshed with 2000 S3R elements



(b) Hexakis icosahedron skin meshed with 5802 S3R elements

Both VeroWhite and VeroBlue are photopolymers that are UV-cured using PolyJet technology. Polyjet technology allows for smooth 3D printing with complex geometries as well as much finer detail than traditional 3D printers. As Stratasys explains, “PolyJet 3D Printing works similarly to inkjet printing, but instead of jetting drops of ink onto paper, PolyJet 3D Printers jet layers of curable liquid photopolymer onto a build tray” [13]. The 3D printer software calculates the exact placement of the photopolymer and support material and UV-cures the photopolymers during the printing process. Once printing is complete, the user removes the support material from around the cured photopolymer structure using either a water jet or a solution bath. Figure 3.8 shows a 0.1778 m diameter hexakis icosahedron frame immediately after printing completion with the support material still attached. This was followed by a water jet cleaning of the supporting material from the cured photopolymer frame.

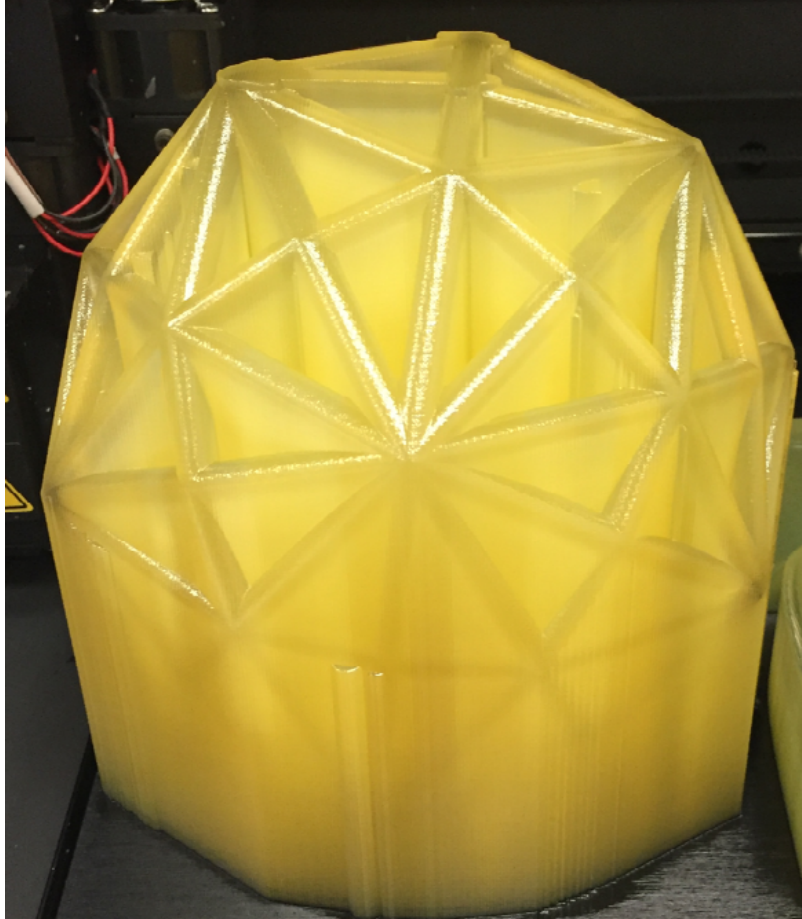


Figure 3.8. A 0.1778 m diameter hexakis icosahedron frame with supporting material after 3D printing

3.3 Experimental Test Setup

Although Polyjet technology allows for quality 3D printing of complex geometries, layering the photopolymer material along an angled beam can drastically reduce the modulus of elasticity of the structure. Cranston's research concluded that the modulus of elasticity for an icosahedron frame printed with VeroBlue was approximately 50% (1 GPa) of the 2-3 GPa stated by the Stratasys specification sheet [9]. His results were confirmed by comparing the experimental modal analysis results of the icosahedron to an FEA model. A modal analysis of the hexakis icosahedron frame was conducted in the AFIT vibrations lab to verify Cranston's research on the *global*

material properties of an additively manufactured frame as well as confirm that a correct method of analysis had been established for dynamic FE modeling.

An impact hammer was used to apply an excitation force to the structure. The impact hammer was connected to a signal conditioner which allowed for amplification as necessary. The amplified signal was then sent to the Polytec OFV-5000 vibrometer controller and data acquisition system. The Polytec PSV-400 laser scanning head was also connected to the vibrometer controller, and all other settings for both the impact hammer and laser vibrometer were programmed using version 8.7 of the Polytec software. The general test setup for the experiment is shown in figure 3.9.

A Polytec scanning laser vibrometer was used to measure the response of the hexakis icosahedron. For best results, the default data acquisition settings were modified to amplify the signal from the laser vibrometer and input hammer. The Fast Fourier Transform (FFT) was selected as the measurement mode including magnitude averaging. The default voltage range for the input hammer was changed from 10V to 316.2 mV. Based on FEA calculations, the first eigenvalue was expected to be between 300 and 600 Hz so the bandwidth was set to range from 0 to 2000 Hz in order to capture the first several frequency modes. The quality of all desired measurements was assessed using the coherence signal.

In order to isolate the response of the structure, a free-free boundary condition was applied to the frame. Because the energy was not absorbed by the boundary condition, the response due to the material properties could be isolated in the analysis. This was accomplished by suspending the hexakis icosahedron with a thread from a support frame positioned on top of a Vibraplane platform. To prevent the hexakis icosahedron from swinging when impacted by the hammer, a foam support was added on the bottom to lightly hold it in place as shown in figure 3.10

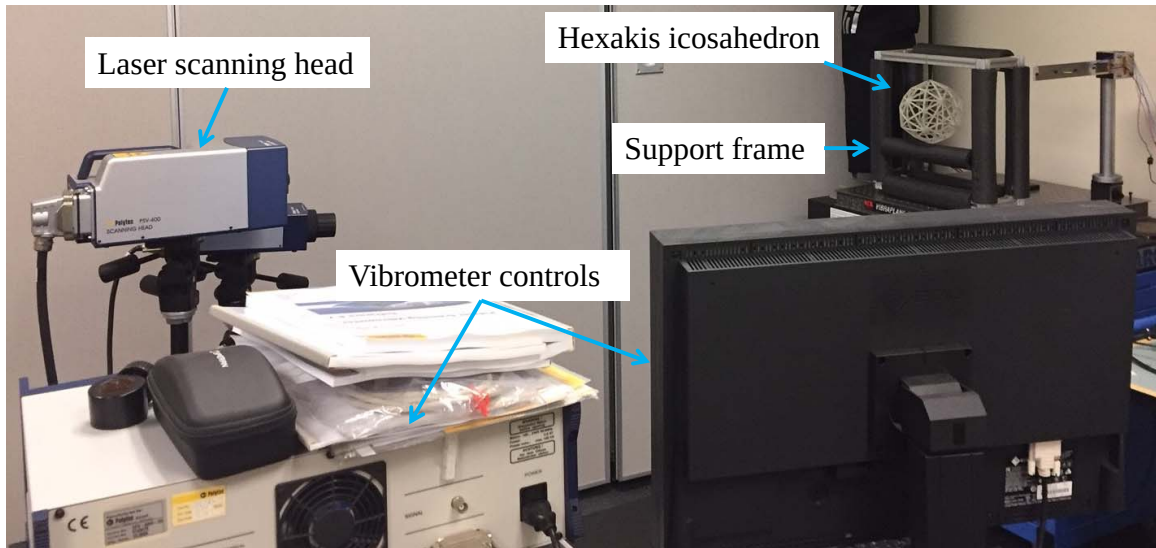


Figure 3.9. Laser vibrometer test setup in AFIT vibrations lab

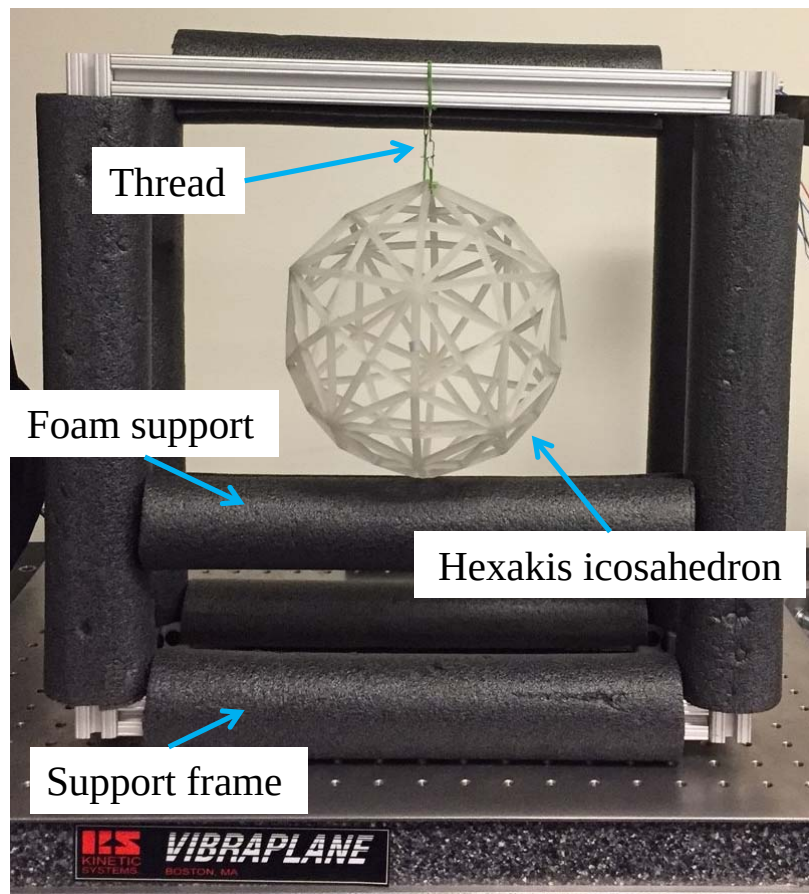


Figure 3.10. Hexakis icosahedron frame suspended from support frame in a free-free boundary condition

The hammer input was applied at a vertex near the top of the structure to minimize its tendency to swing or rotate. The frame was hit on a vertex in order to maximize the energy imparted to the whole structure as an input applied to the midpoint of a beam could absorb some of the energy and decrease the coherence of the response. Although a free-free boundary condition was being modeled, excessive movement of the structure would decrease the coherence of the signal, especially at higher frequencies. The laser response was recorded for two separate tests - one for each laser response point - in order to ensure the frequency modes were consistent across the structure. The first point was at the center of a beam closest to the vibrometer scanning head. The second point was directly below the first point at a vertex of the structure as shown in figure 3.11. Reflective tape was placed on each of the two points to enhance the laser response.

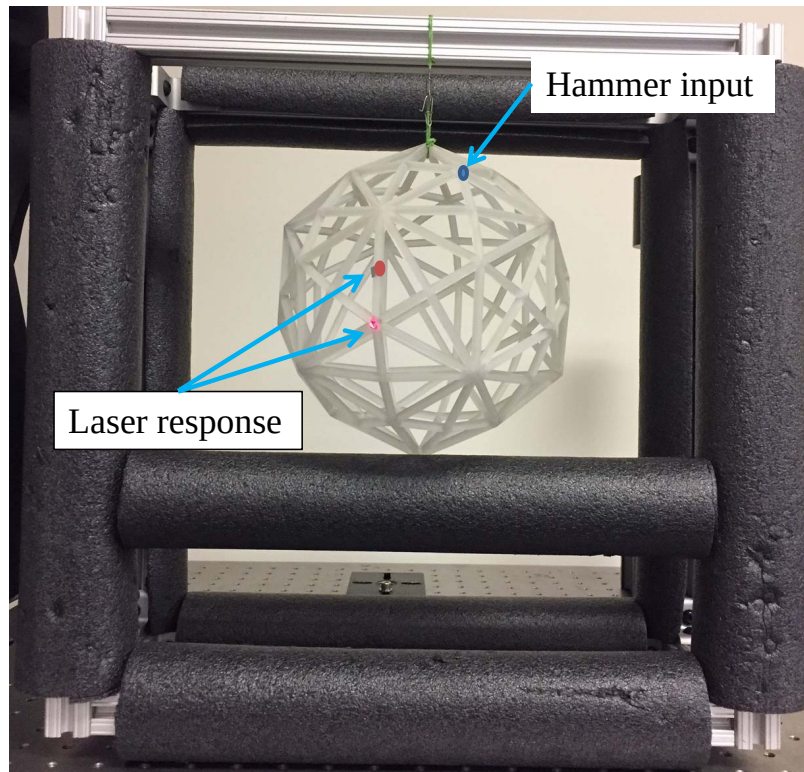


Figure 3.11. Hammer input and laser response points on hexakis icosahedron frame

3.4 Summary

This chapter presented the procedures followed to conduct quasi-static and dynamic analyses of a hexakis icosahedron frame. Quasi-static and dynamic simulations were run using Abaqus FEA software for comparison to previous results on an icosahedron frame. Six different load rates were applied to both frames using two different boundary conditions, as specified by Just and Cranston [5] [9]. The results for both frames were tested for chaotic behavior using four different plots as prescribed in Chapter II. The methodology for an experimental modal analysis using a laser vibrometer was also explained including the process for creating a SolidWorks model of a hexakis icosahedron frame for 3D printing. In the following chapter, the results from the experimental modal testing will be presented and discussed as well as the results of the quasi-static and dynamic analyses.

IV. Analysis and Results

4.1 Chapter Overview

Modeling techniques for analyzing the hexakis icosahedron frame under quasi-static and dynamic loading conditions were described at length in chapter III. In order to confirm that a correct method of analysis for the dynamic FE model had been established, experimental modal analysis results of an additively manufactured 0.1778 m diameter hexakis icosahedron frame were compared to the frequency results calculated using Abaqus software. These results will also be useful in the future for determining the viability of additively manufacturing such a frame for use in a VLTAV.

In this chapter, the results of Just's analyses on the icosahedron will be discussed and compared to the hexakis icosahedron frame. Displacement and velocity results were extracted from the models and used to determine whether nonlinear dynamic behavior was present in the system and whether it was more influenced by the load rates, load magnitudes, or boundary conditions applied. Von mises stress data was also gathered using FEA and compared to the yield stress of the beryllium material used.

4.2 Modal Analysis Results

In order to ensure a correct method of analysis had been established and the member connections were modeled accurately, the additively manufactured 0.1778 m diameter hexakis icosahedron frame was modeled in Abaqus to determine its dynamic properties and then compared to the results of an experimental modal analysis. Previous research conducted by Cranston studied how the material properties of the icosahedron frame varied due to the additive manufacturing process. Although the

manufacturer specification sheet for the material stated the modulus of elasticity was 2.0 GPa, through experimental testing, he determined that the global modulus of the frame was reduced by 50% (1 GPa) [9]. The manufacturer specifications for additively manufactured materials can, in certain layups, be significantly higher than reality due to the various angles used in the additive manufacturing technique. In order to simplify the analysis, the structure was assumed to be isotropic and a *global* (or *equivalent*) modulus of elasticity for the frame was assumed. Though a different material was used and the structure was more complex for this research, a similar approach was used in the modeling technique. The material used in this research was VeroWhitePlus, as compared to the VeroBlue material used by Cranston. Applying Cranston's conclusion, the *global* modulus of elasticity of the structure was assumed to be 50% of the modulus stated in the manufacturer specification sheet. Thus, an elastic modulus of 1.1 GPa was assumed for the analysis of the additively manufactured hexakis icosahedron frame.

The first two frequency modes were calculated using Abaqus software to later be compared to experimental data. Because Abaqus outputs a long list of local eigenvalue results, the shape of each eigenvector was visually inspected to determine when the second frequency mode first occurred. This list can be found in Appendix B. The first frequency mode occurred at 413.9 Hz and its mode shape is shown in figure 4.1. The second mode occurred at 613.28 Hz and its mode shape is shown in figure 4.2. Because the experimental setup attempted to simulate a free-free boundary condition where the frame was excited by a hammer input, only the first two frequency modes could be captured accurately so only the first two frequency modes were recorded from the FE model.

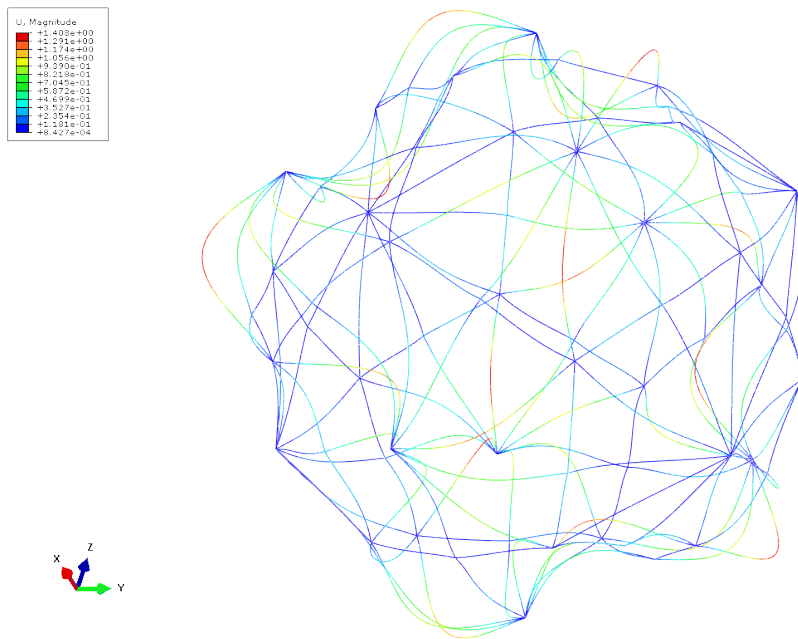


Figure 4.1. FEA first frequency mode at 413.9 Hz

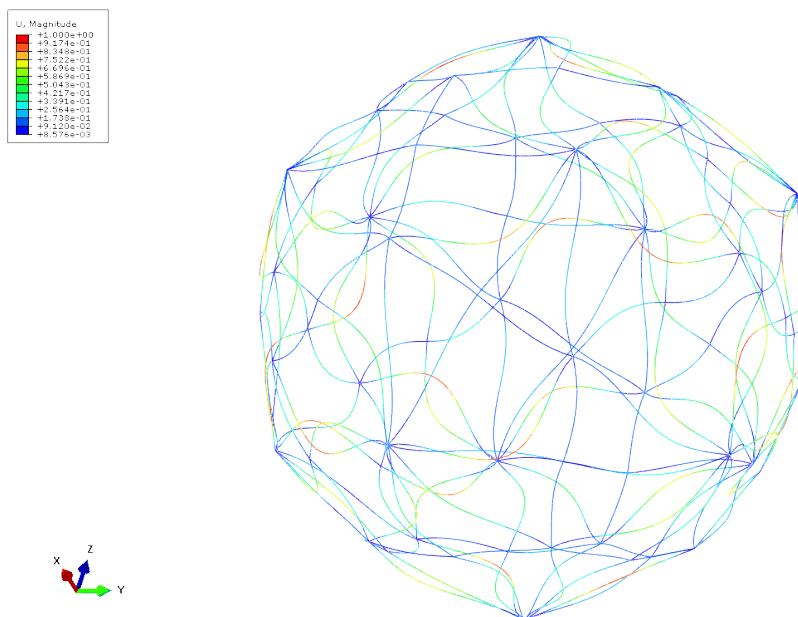


Figure 4.2. FEA second frequency mode at 613.28 Hz

Figure 4.3 shows the Frequency Response Function (FRF) recorded from the experiment with the laser vibrometer. The first mode was recorded at 414.1 Hz, and the second mode was recorded at 607.8 Hz. With each additional mode, the peaks in the FRF plot become less distinct, making it difficult to accurately approximate the frequency value. The coherence value for each mode was recorded from the coherence plot in figure 4.4. The coherence value was 0.955 for the first mode and 0.870 for the second mode. After the second mode, the coherence was too low to make an approximation for other frequency values.

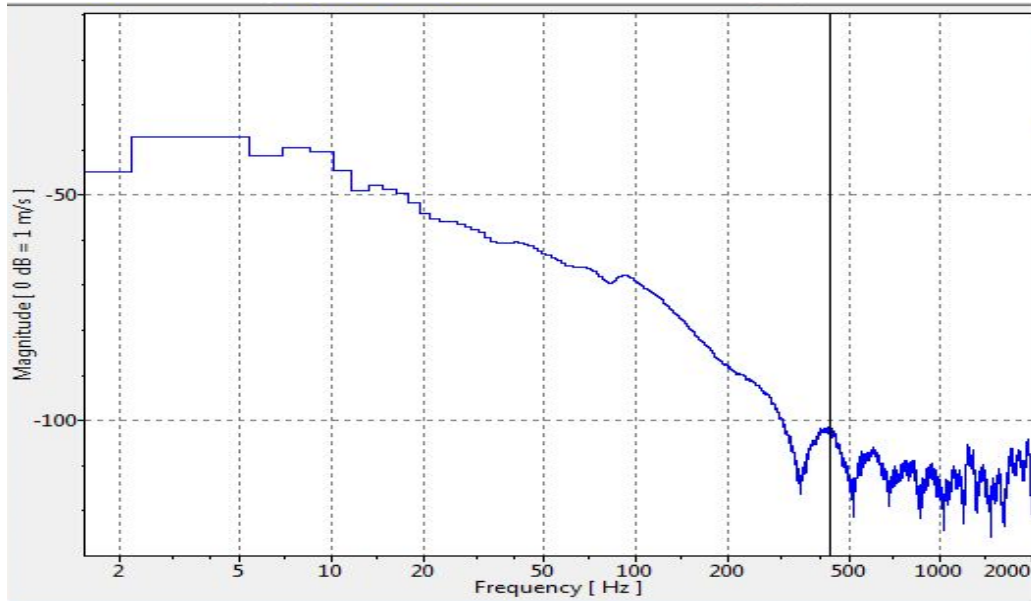


Figure 4.3. Experimental FRF with first mode at 414.1 Hz and second mode at 607.8 Hz

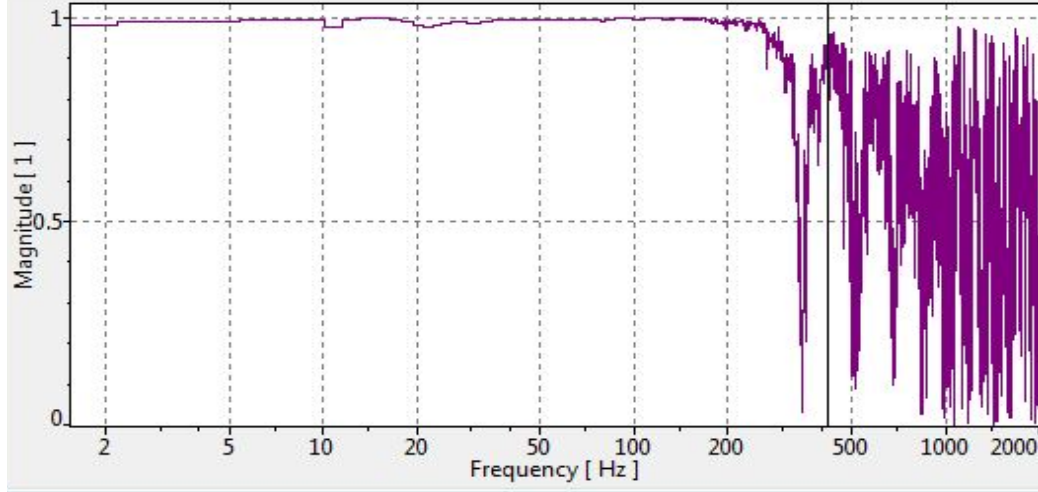


Figure 4.4. Experimental Coherence plot with a value of 0.955 at first mode and 0.870 at second mode

Table 4.1 summarizes the results of the FE model and the experimental results. The technique of applying a *global* modulus of elasticity to the frame as recommended by Cranston yielded an accurate model for the dynamic properties of the frame. The first mode matched the experimental test almost exactly with an error of only 0.0483%. The second mode was also extremely accurate with an error of 0.894%.

Table 4.1. FEA and Experimental Frequency Modes

Mode	FEA (Hz)	Experimental (Hz)	% error	Coherence
First	413.9	414.1	0.0483	0.955
Second	613.28	607.8	0.894	0.870

Because the dynamic model of the hexakis icosahedron frame was successful in matching experimental data, the properties of the model were determined to be a suitable method of analysis for further dynamic and quasi-static modeling.

4.3 Icosahedron FEA Results Summary

In order to more easily understand the quasi-static analysis results on the hexakis icosahedron as well as provide a more meaningful comparison between the icosahedron

and hexakis icosahedron, some of Just's research results will be briefly summarized in this section.

Just's quasi-static analysis of the icosahedron discovered that chaotic behavior, present at certain load rates, caused the *snapback* phenomenon observed in Adorno-Rodriguez's research [5]. As was explained in Chapter II, four different techniques were used to assess chaos - displacement plots, phase-space diagrams, Power Spectral Density (PSD) plots, and Lyapunov exponents. Figures 4.5 through 4.8 are examples of *non-chaotic* behavior. These plots were created from displacement and velocity data of the icosahedron using load case 2 (BC3 and a load rate of 4.053 MPa/s).

For a non-chaotic frame, the displacement plot should resemble figure 4.5. The plot matches the ramped load input pictured in the subplot of the figure. Maximum displacement is reached when the load is at its highest at 0.005 s, and the displacement oscillates periodically around steady state. It is worth noting that the displacements are very small for this load case, only on the order of 10^{-3}m .

The phase-space diagram is shown in figure 4.6. Once the load is fully applied, the frame reaches a steady state attractor after 0.005 s, indicated by the ellipse on the right of the plot. According to Sprott, the quicker the structure reaches steady state and the thinner the attractor, the less chaotic the system. Cases where the attractor is thicker typically represent more dissipative systems where the energy is being lost due to factors such as friction; however, they can also be indicative of nonlinear dynamic behavior [6] [11].

The non-chaotic PSD plot of the icosahedron is shown in figure 4.7. A non-chaotic PSD plot should have clearly distinct peaks that correspond to the natural frequencies of the structure as shown in the plot [6]. This is expected due to the periodic motion shown in the time series data of the displacement plot.

Finally, the Lyapunov exponent convergence plot for the non-chaotic load case is

shown in figure 4.8. As was explained in Chapter II, a Lyapunov exponent, λ , greater than zero indicates a chaotic system while an exponent less than zero indicates a non-chaotic system. As can be seen in the plot, the Lyapunov exponent converges to -0.0137 , meaning it is non-chaotic.

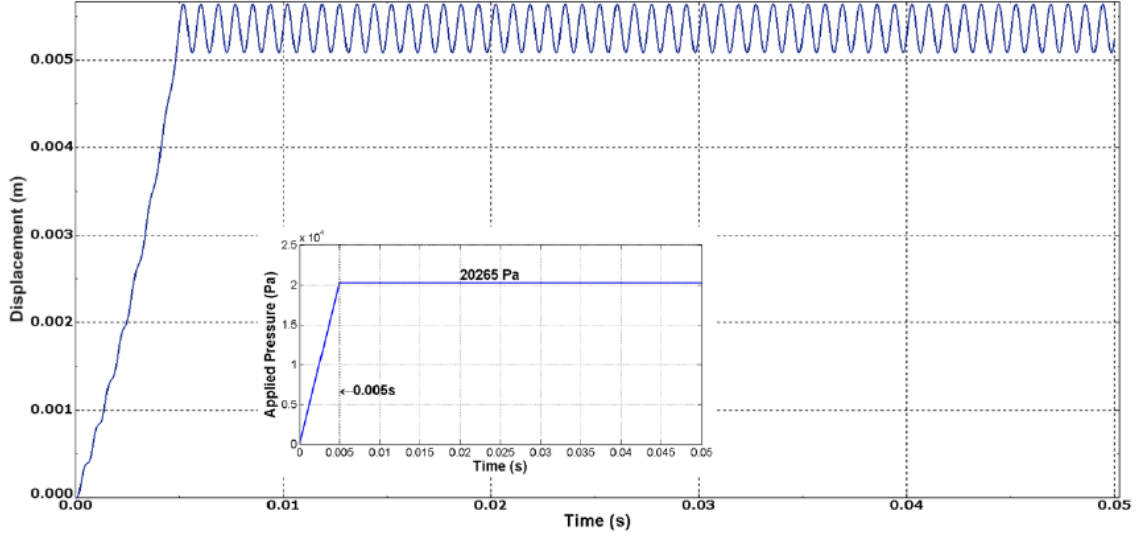


Figure 4.5. Icosahedron load case 2 displacement plot ($\lambda = -0.0137$)

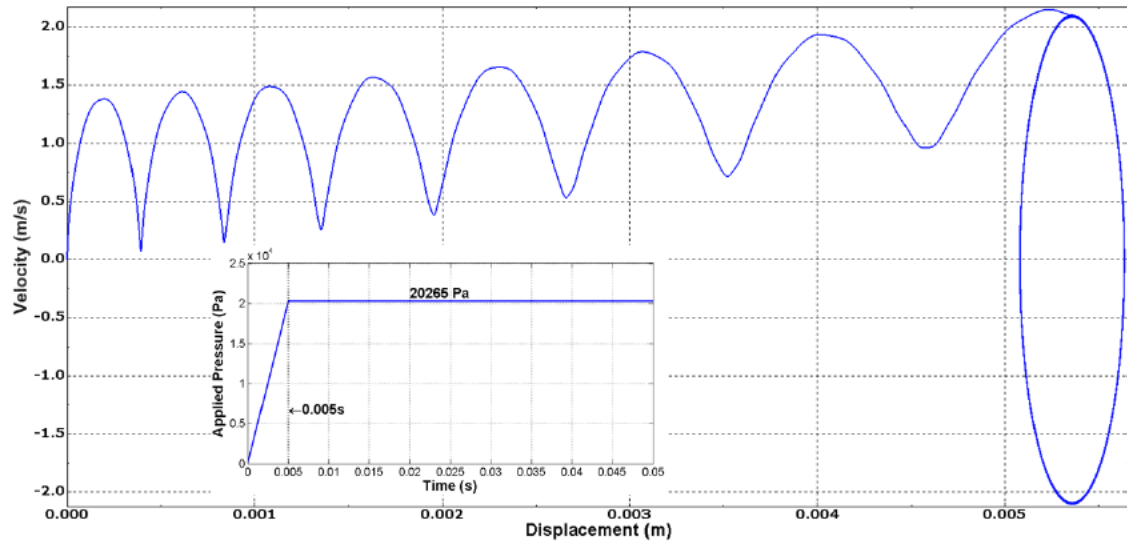


Figure 4.6. Icosahedron load case 2 phase-space diagram ($\lambda = -0.0137$)

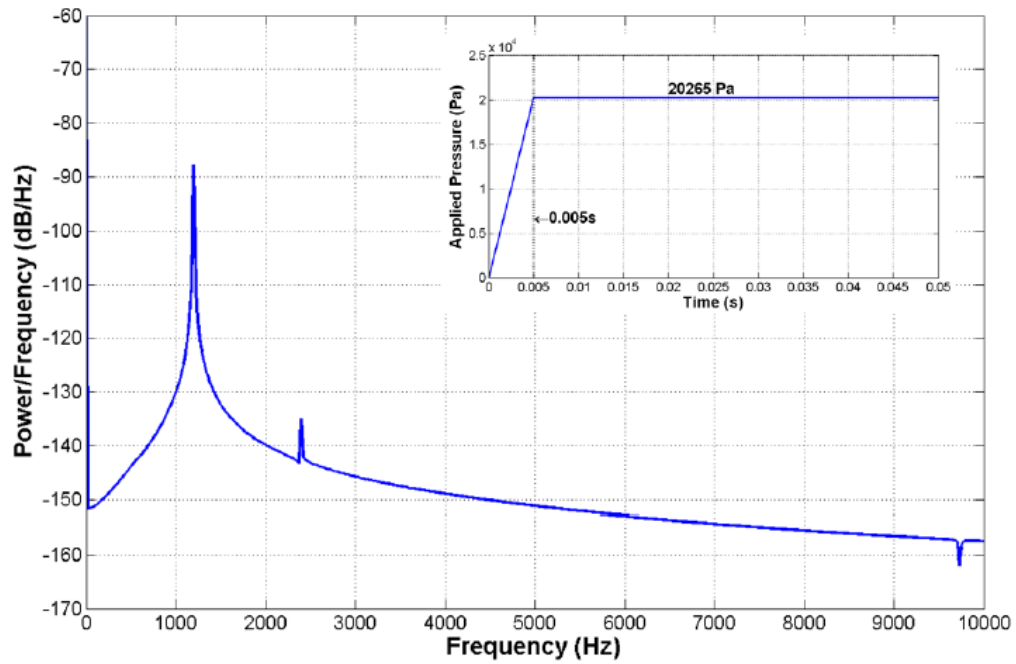


Figure 4.7. Icosahedron load case 2 PSD plot ($\lambda = -0.0137$)

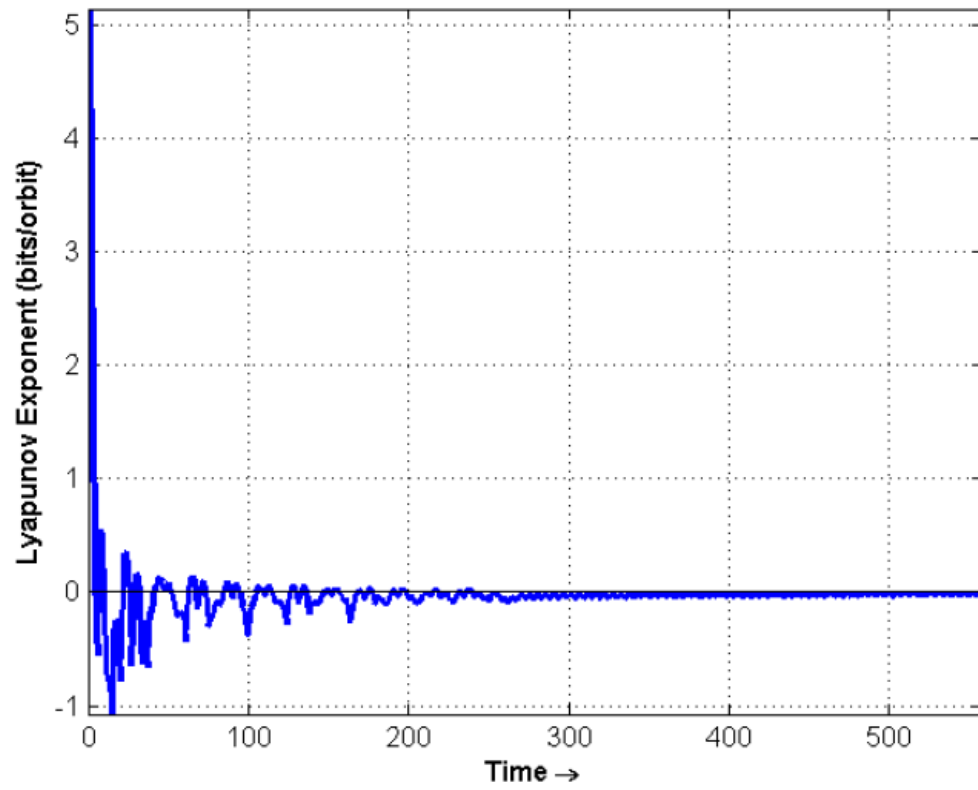


Figure 4.8. Icosahedron load case 2 Lyapunov exponent convergence plot ($\lambda = -0.0137$)

Figures 4.9 through 4.12 represent *chaotic* behavior in the icosahedron frame using load case 6 (BC2 and a load rate of 8.106 MPa/s). The displacement plot in figure 4.9 is characterized by significant nonlinear behavior. Once the load is fully applied in the first 0.005 s, the displacement plot does not oscillate periodically but rather oscillates erratically. Also of note is the significantly larger displacement values than in the non-chaotic load case of figure 4.5. The displacement oscillation has varying periods with large amplitudes.

The phase-space diagram of the chaotic system is shown in figure 4.10. Unlike the plot in figure 4.6, the chaotic phase-space diagram never reaches a clear steady state. Although it appears to be approaching an elliptical pattern after 0.05s, there is no clear attractor in the plot, thus indicating a chaotic system.

The PSD plot for the given load case is shown in figure 4.11. This PSD plot does not exhibit any distinct dominant frequencies. Although the general shape of the plot has a downward slope that is characteristic of most PSD plots, no dominant frequencies can be determined from the plot, in contrast to the non-chaotic PSD plot shown in figure 4.7. This means that one mode is not absorbing the dynamic energy more than others.

Perhaps the most convincing evidence that the system is chaotic for this load case is the Lyapunov exponent convergence plot shown in figure 4.12. The Lyapunov exponent converges to a value of approximately 19.67. This indicates a highly chaotic system and a load case that should be avoided in the design of the VLTAV. The instability of the frame at these loads is important to the effect of exhausting air from within the structure. It is believed that any possible instability in the frame may cause problems when the membrane is attached. This theory will be tested later in this chapter.

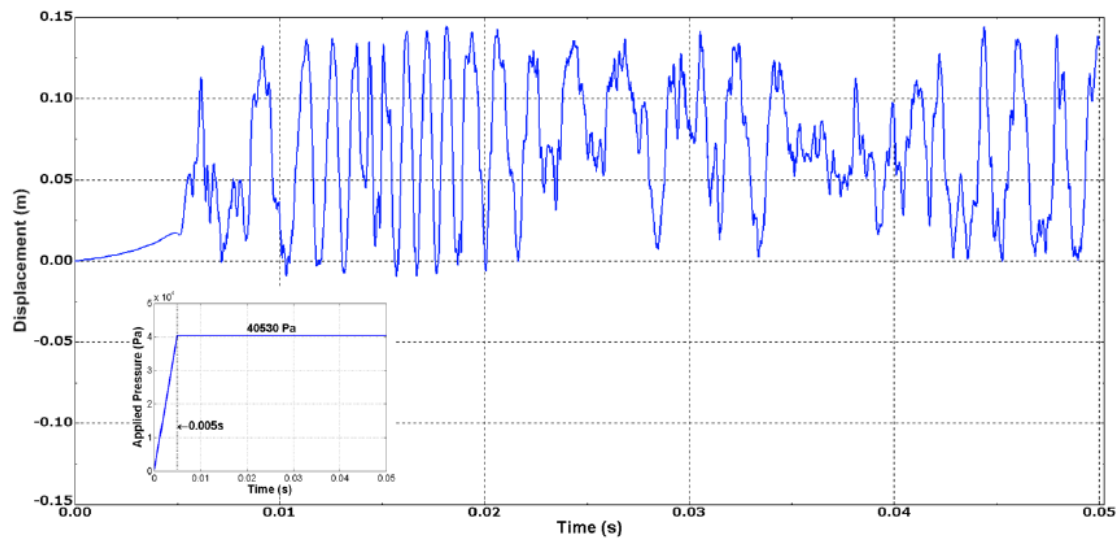


Figure 4.9. Icosahedron load case 6 displacement plot ($\lambda = 19.67$)

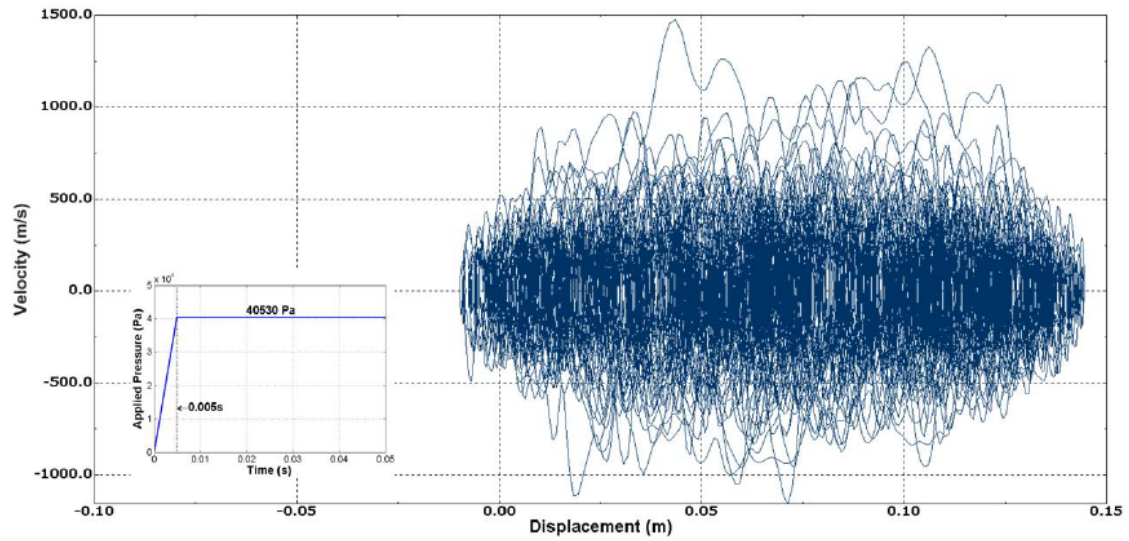


Figure 4.10. Icosahedron load case 6 phase-space diagram ($\lambda = 19.67$)

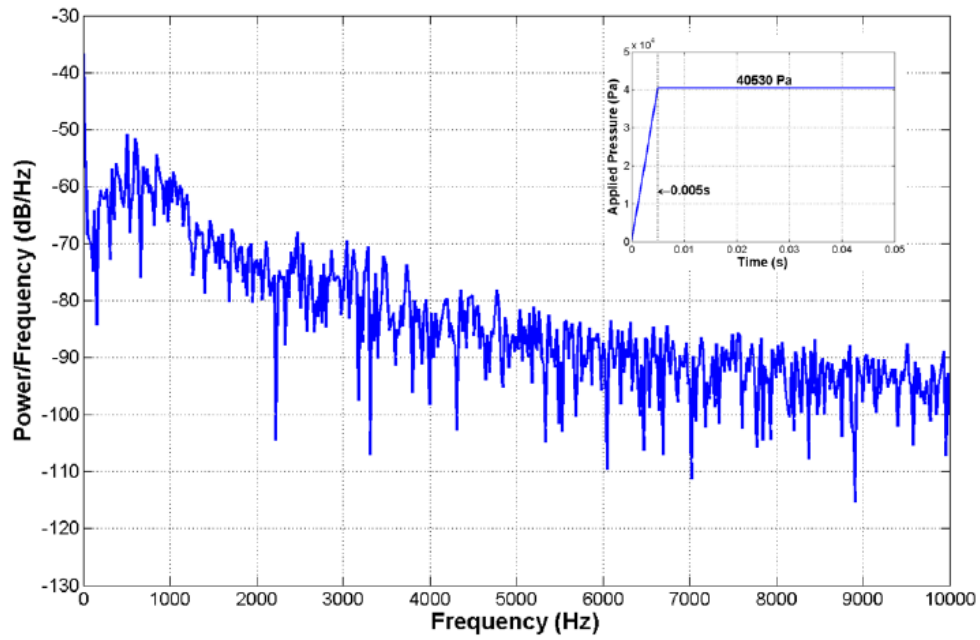


Figure 4.11. Icosahedron load case 6 PSD plot ($\lambda = 19.67$)

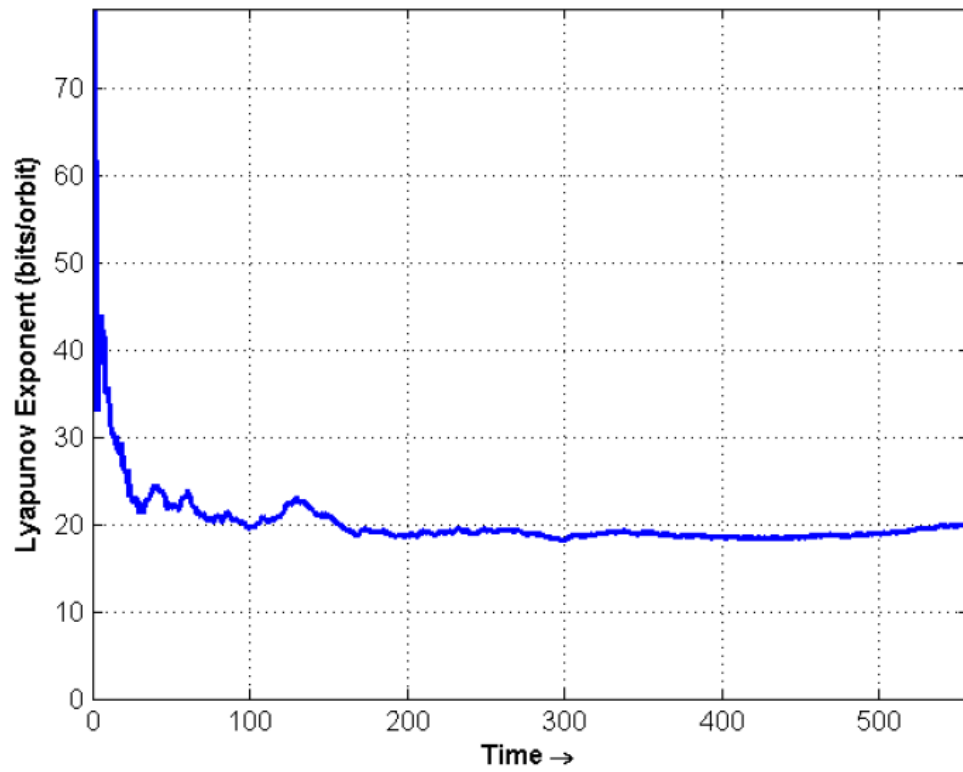


Figure 4.12. Icosahedron load case 6 Lyapunov exponent convergence plot ($\lambda = 19.67$)

The results of Just’s quasi-static analysis of the icosahedron are summarized in table 4.2. A more detailed discussion of his results in the context of a comparison with the hexakis icosahedron results will be included later in this chapter.

Table 4.2. Icosahedron Quasi-Static Analysis Results [5]

Load Case	λ	Load Rate (MPa/s)	BC
1	-0.0121	5.0663	3
2	-0.0137	4.053	3
3	3.8814	8.106	3
4	0.303	5.0663	2
5	0.371	4.053	2
6	19.67	8.106	2

4.4 Hexakis Icosahedron FEA Results

The hexakis icosahedron frame FE model was analyzed under the same six loading conditions as the icosahedron in Just’s research. Displacement and velocity data was used to evaluate the amount of chaos present in the frame at three different load rates and two boundary conditions. By modifying these parameters, the structure’s sensitivity to its initial conditions and thus, chaos, could be estimated.

For the first load case, the frame was loaded to 10% sea level pressure at a rate of 5.0663 MPa/s and restrained by the symmetric BC3. Figure 4.13 shows the deformed shape of the frame after the load has been fully applied, and figures 4.14 through 4.17 provide evidence for chaotic behavior present in the frame. At first glance, the displacement plot shown in figure 4.14 seems to exhibit perfect periodic motion once the load has been fully applied after 0.002s; however, a closer examination reveals that the amplitude and period of the plot vary with time. Variations that do not follow a particular pattern can indicate a chaotic system; however, it is difficult to gauge this based strictly on this plot. The displacement magnitudes are very small (on the order of 10^{-4}m), typically indicative of non-chaotic systems.

Figure 4.15 depicts the phase-space diagram of the frame. As evidenced by the thin elliptical ring on the right side of the plot, the frame quickly reached a steady state after the load was applied, also indicative of a non-chaotic system.

The PSD plot in figure 4.16 seems to match that of a non-chaotic system; however, minor irregularities exist that suggest otherwise. The plot shows a clear, dominant frequency near 3.5 kHz, yet irregular frequencies also appear at 1.9 kHz, 2.6 kHz, 6.6 kHz, and 10 kHz. Though seemingly insignificant, these blips can designate a chaotic system. To further investigate the effect of chaotic behavior on the PSD plot, an additional FE model of the hexakis icosahedron frame was constructed to conduct a frequency analysis on the structure. The first frequency mode from the FE model occurred at approximately 2.13 kHz while the dominant frequency in figure 4.16 shows the first mode at approximately 3.5 kHz - another indication that the load case could be chaotic.

The Lyapunov exponent convergence plot for load case 1 is shown in figure 4.17 where the exponent, λ , converges to 0.2552. Because the value of the exponent is greater than zero, it is clear the system is chaotic.

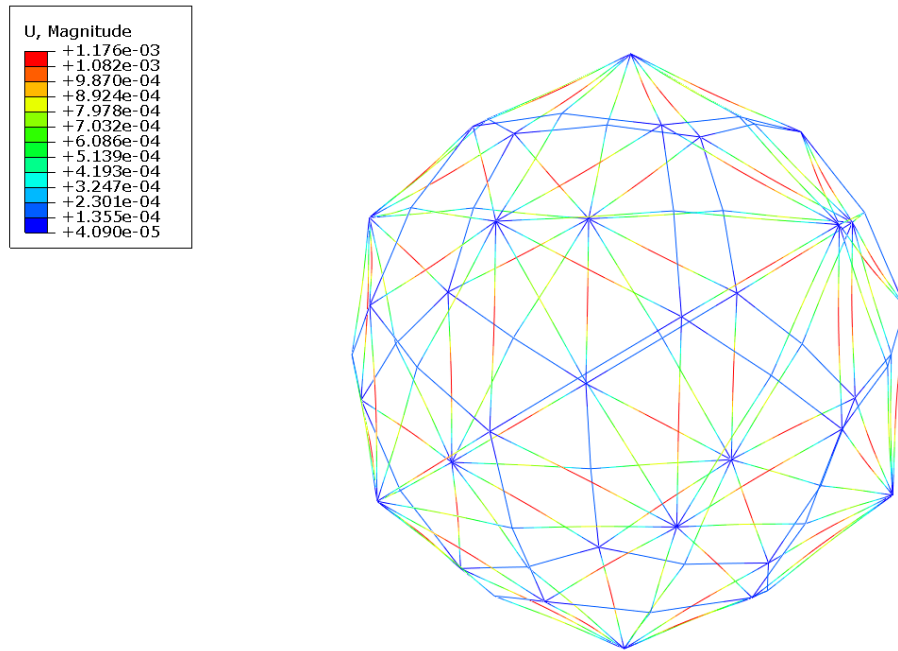


Figure 4.13. Hexakis icosahedron load case 1 deformed shape

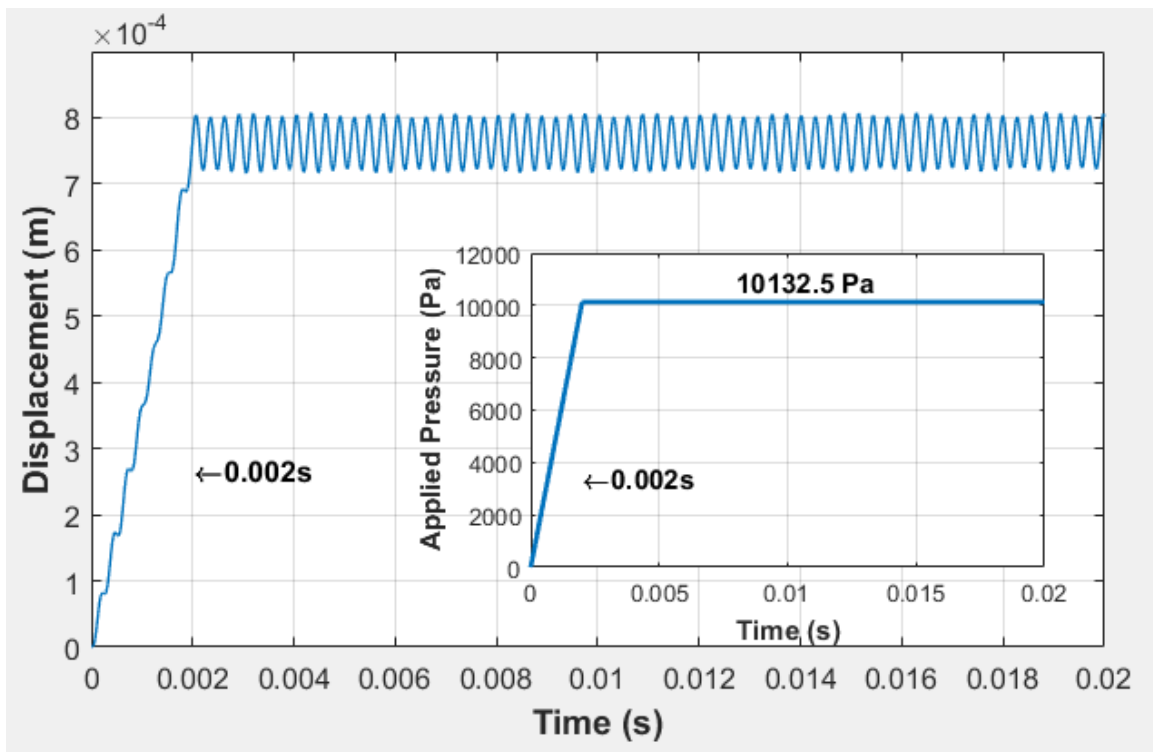


Figure 4.14. Hexakis icosahedron load case 1 displacement plot ($\lambda = 0.2552$)

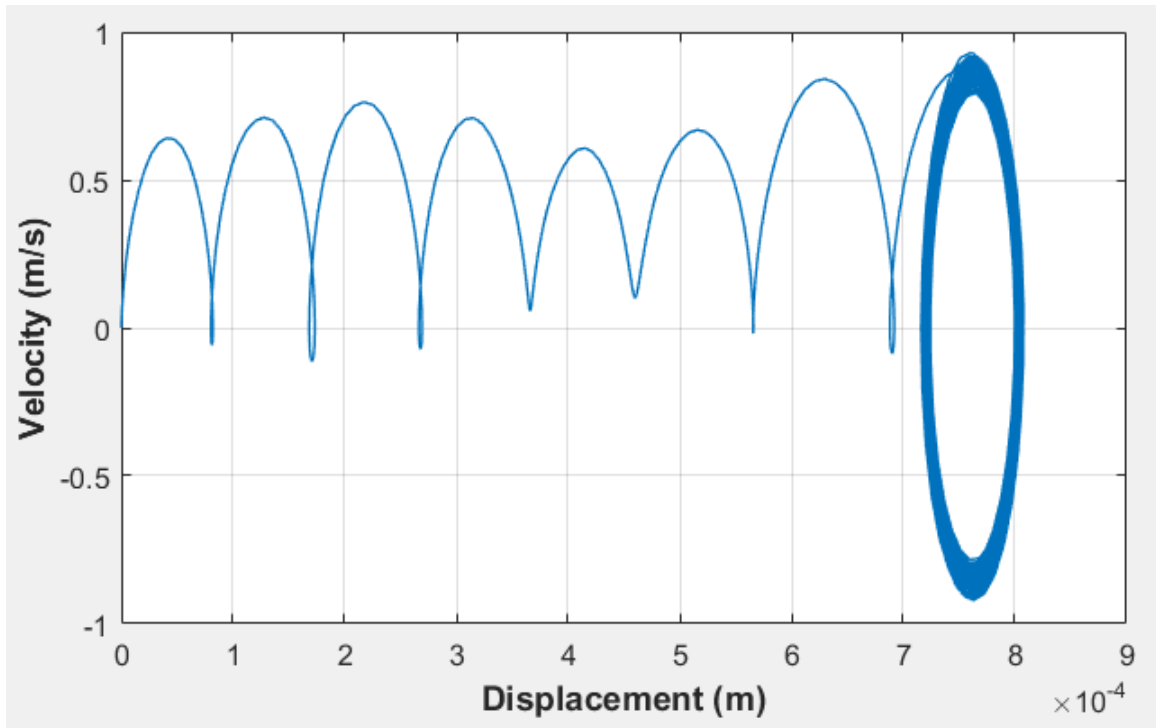


Figure 4.15. Hexakis icosahedron load case 1 phase-space diagram ($\lambda = 0.2552$)

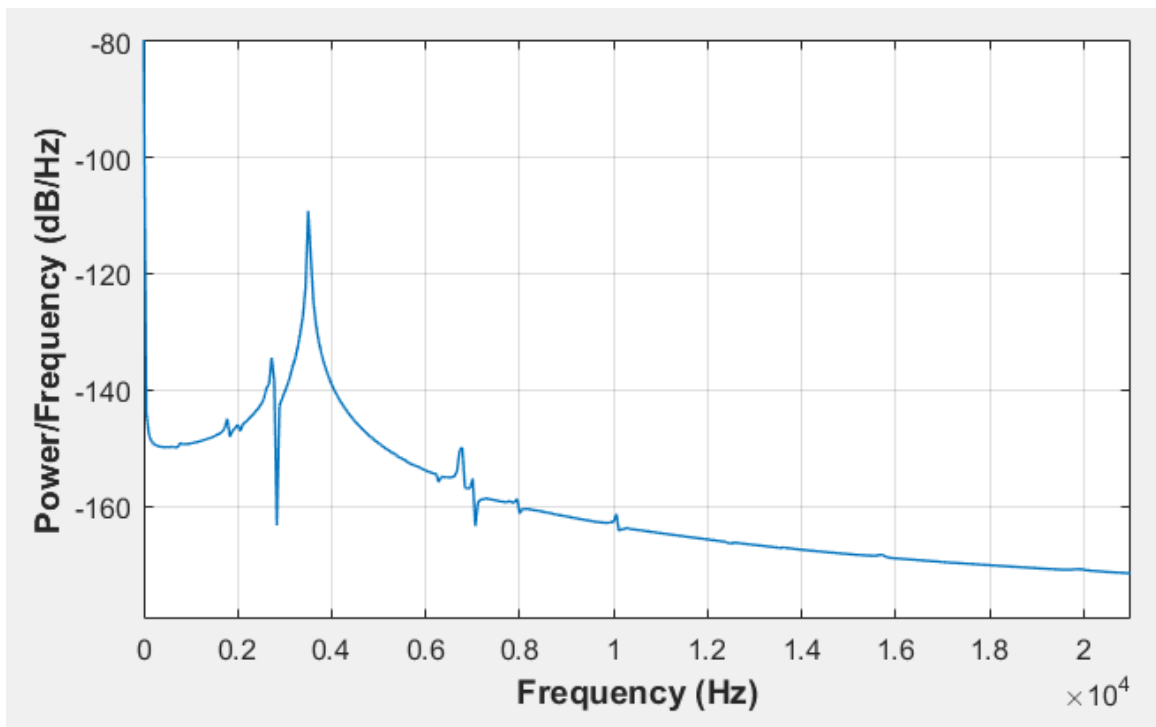


Figure 4.16. Hexakis icosahedron load case 1 PSD plot ($\lambda = 0.2552$)

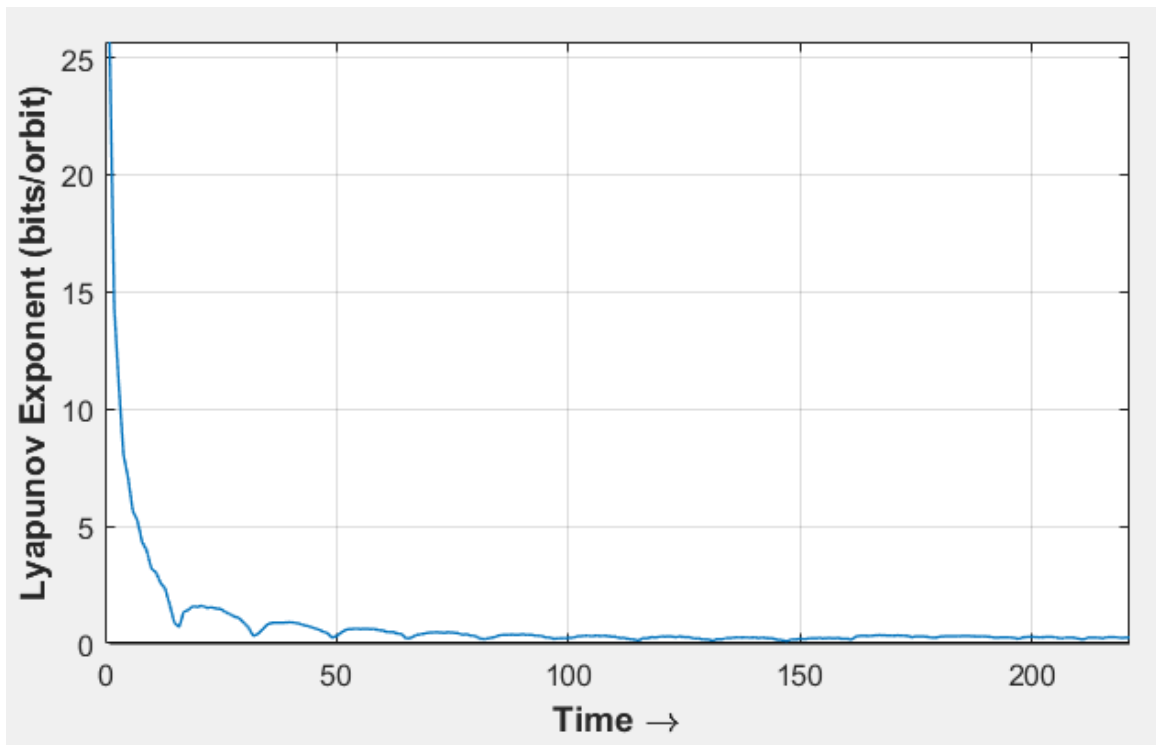


Figure 4.17. Hexakis icosahedron load case 1 Lyapunov exponent convergence plot ($\lambda = 0.2552$)

For the second load case, the frame was loaded to 20% sea level pressure at a load rate of 4.053 MPa/s and restricted with symmetric BC3. The deformed shape of the frame can be seen in figure 4.18. The load rate for this case is 1 MPa/s slower than the first load case, and figures 4.19 through 4.22 show the system has transitioned to a non-chaotic load case. As can be seen in the displacement plot in figure 4.19, once the load has been fully applied after 0.005 s, the displacement oscillates in a consistent, periodic pattern.

As with load case 1, the phase-space diagram in figure 4.20 shows the frame reach steady state as soon as the load is fully applied. The elliptical attractor is thicker than in the previous load case; however, this does not necessarily signify chaotic behavior is present. As stated by Baker, this can also be caused by energy being dissipated from the system in other forms such as friction [6].

The PSD plot in figure 4.21 is an example of a non-chaotic system. Two distinct frequencies are shown at 2.2 kHz and 3.0 kHz. Unlike the first load case, this first frequency matches the FE model with an error of only 3.3%. The ability of the PSD plot to accurately predict the frequency mode of the frame is reflective of its non-chaotic behavior.

The Lyapunov exponent plot is shown in figure 4.22 where the exponent converges to $\lambda = -0.0175$. This negative exponent value confirms that the load case is non-chaotic.

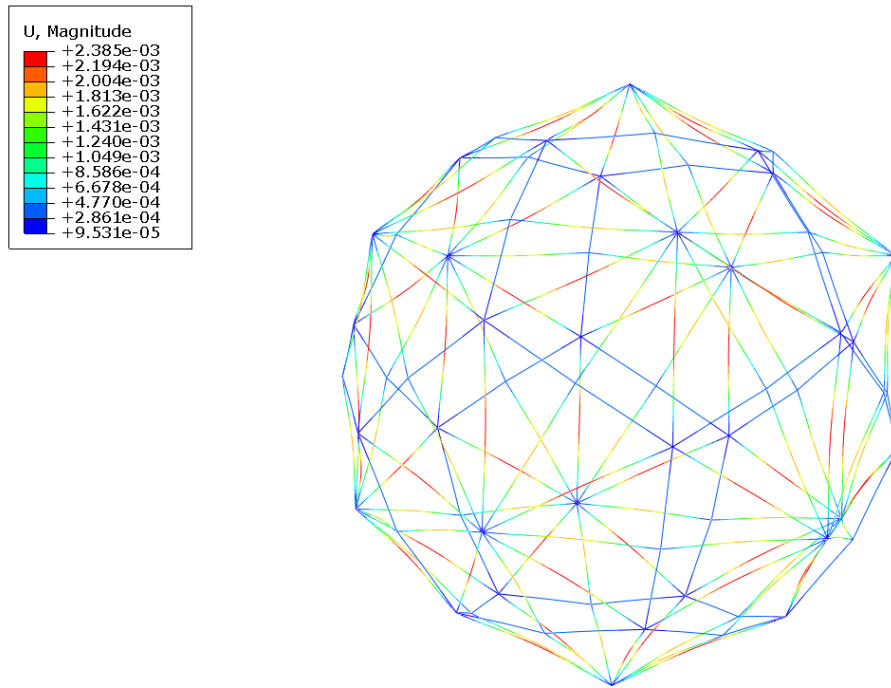


Figure 4.18. Hexakis icosahedron load case 2 deformed shape

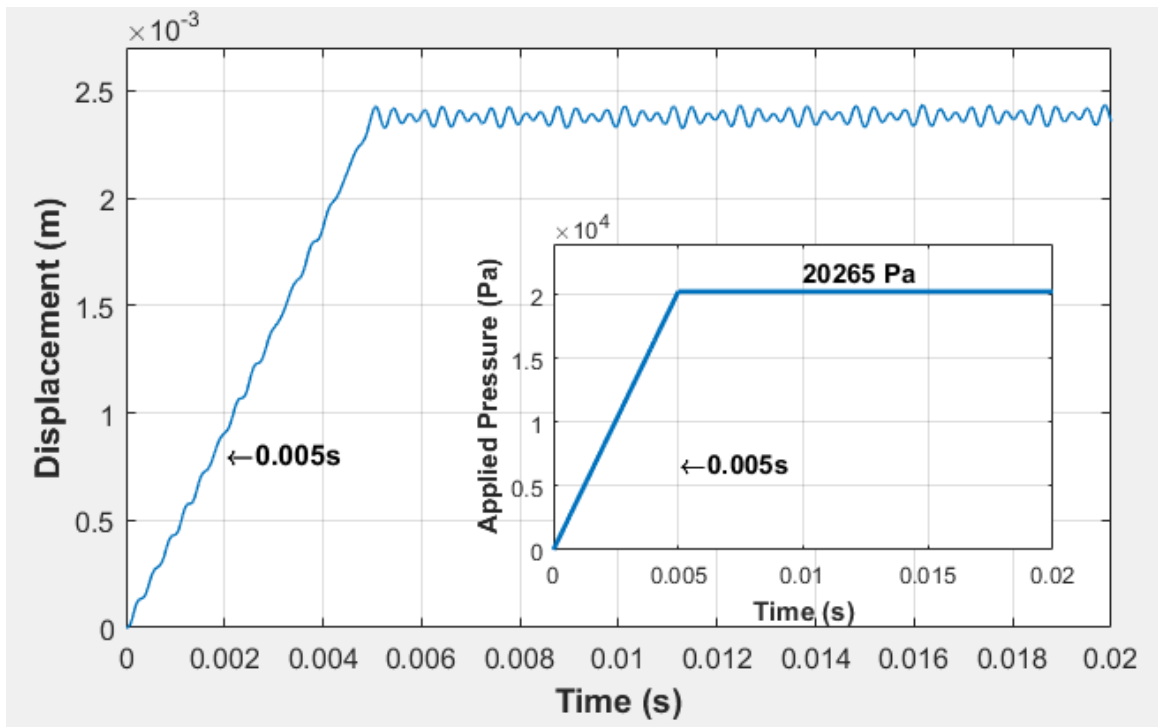


Figure 4.19. Hexakis icosahedron load case 2 displacement plot ($\lambda = -0.0175$)

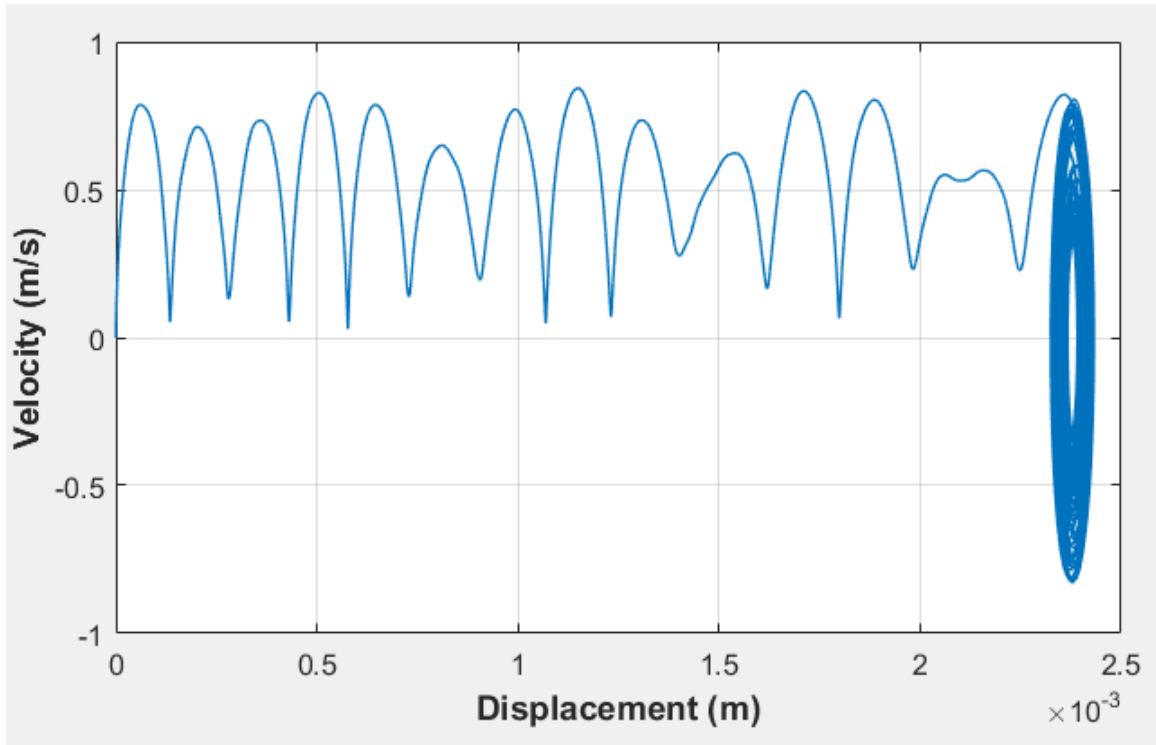


Figure 4.20. Hexakis icosahedron load case 2 phase-space diagram ($\lambda = -0.0175$)

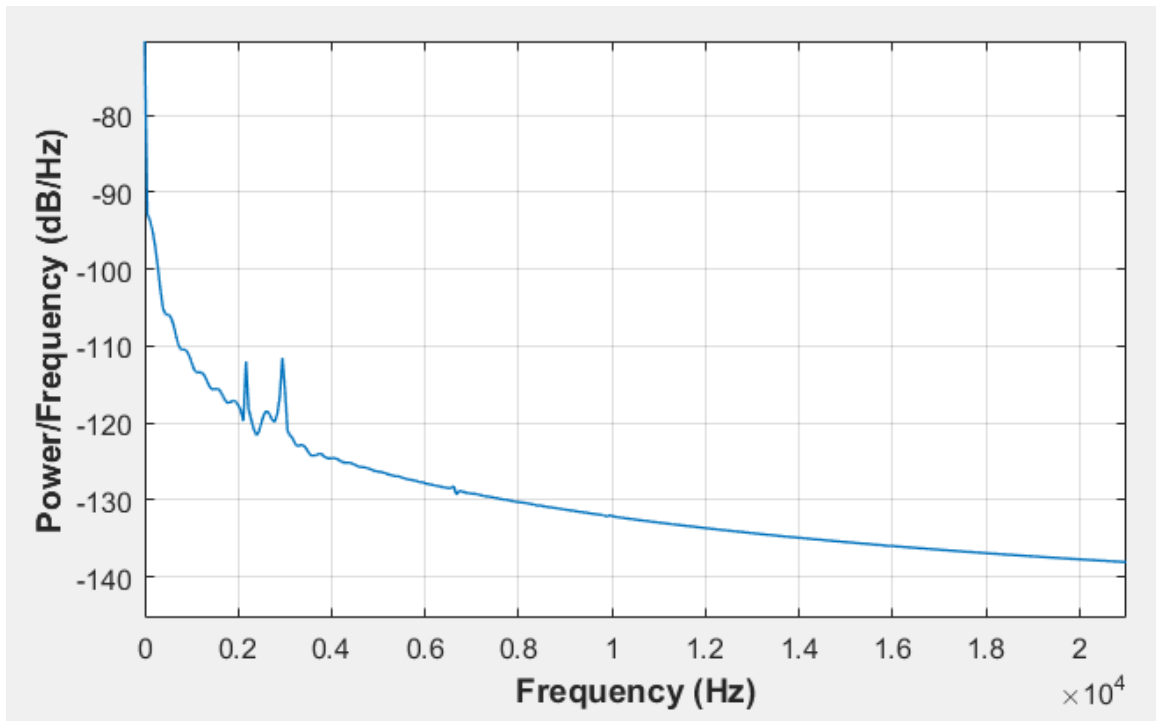


Figure 4.21. Hexakis icosahedron load case 2 PSD plot ($\lambda = -0.0175$)

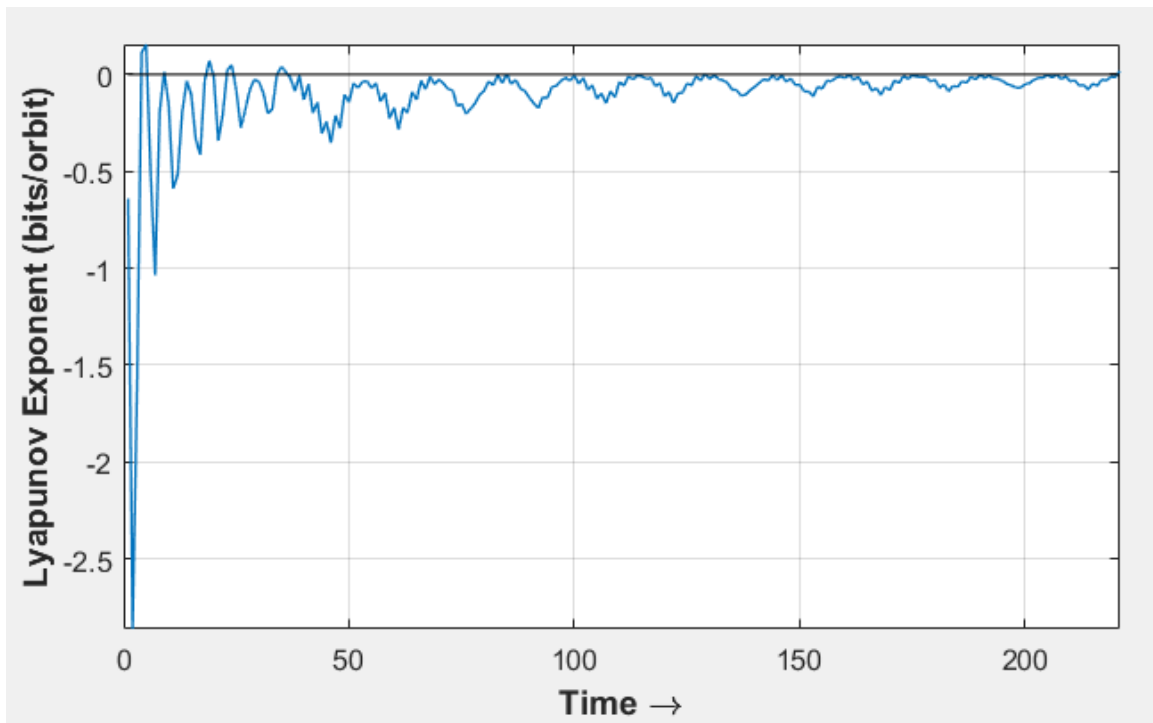


Figure 4.22. Hexakis icosahedron load case 2 Lyapunov exponent convergence plot ($\lambda = -0.0175$)

For load case 3, the frame was loaded to 40% sea level pressure at a rate of 8.106 MPa/s and restrained by boundary condition 3. The deformed shape of the frame, as shown in figure 4.23, is noticeably characterized by nonlinear behavior. The displacement plot in figure 4.24 looks very different than for load cases 1 and 2. For these load cases, the displacement plot closely resembled the load input and quickly reached a steady state. For load case 3, however, the system does not appear to reach this steady state even long after the load was fully applied. The displacement appears to grow unbounded, reaching displacement magnitudes of 0.9 m - far larger than the diameter of the frame - within the first 0.02 s. Figure 4.25 shows the displacement plot expanded to 0.048 s. The displacement continues to grow in magnitude to 20 m and still does not seem to approach a steady state.

The phase-space diagram in figure 4.26 confirms that the frame does not reach steady state even long after the load has been fully applied. The plot does not converge to an elliptical attractor as with the first two load cases.

The PSD plot in figure 4.27 provides further evidence that the load case is chaotic. Unlike non-chaotic PSD plots, figure 4.27 has no clear dominant frequency. The plot is highly erratic, and the first frequency mode calculated by the FE model is indistinguishable. The Lyapunov exponent convergence plot in figure 4.28 confirms that the load case is chaotic with a positive exponent value of $\lambda = 0.6245$.



Figure 4.23. Hexakis icosahedron load case 3 deformed shape

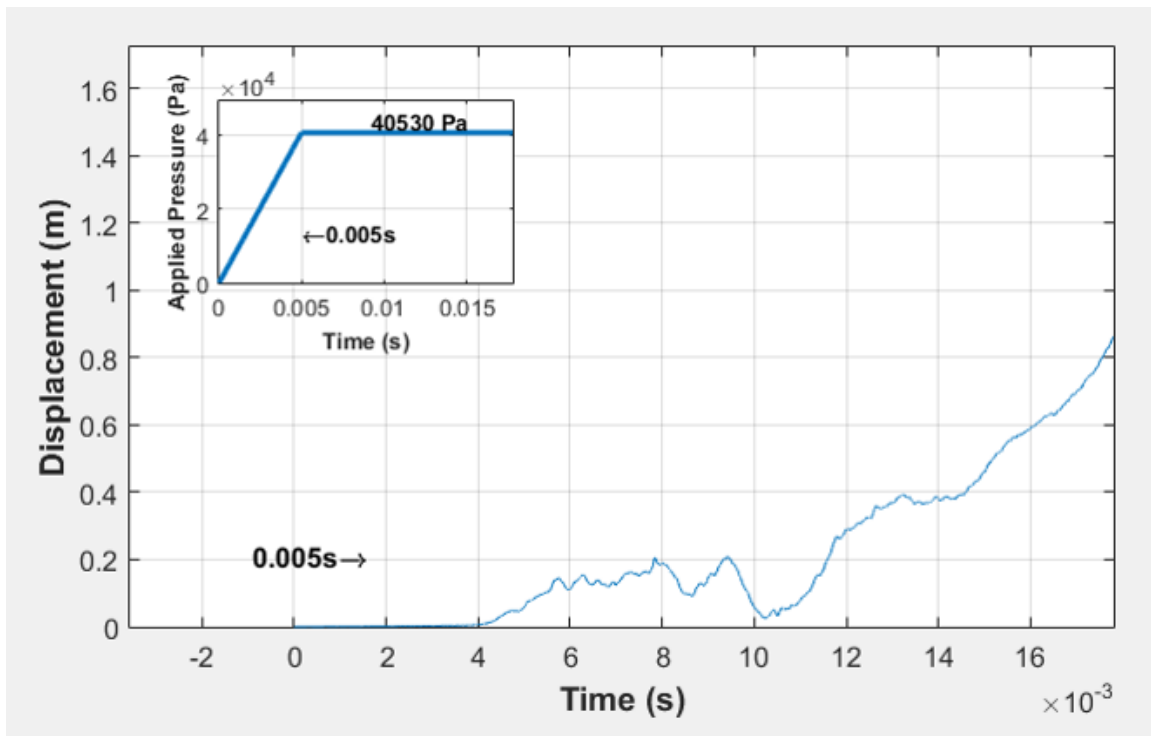


Figure 4.24. Hexakis icosahedron load case 3 displacement plot ($\lambda = 0.6245$)

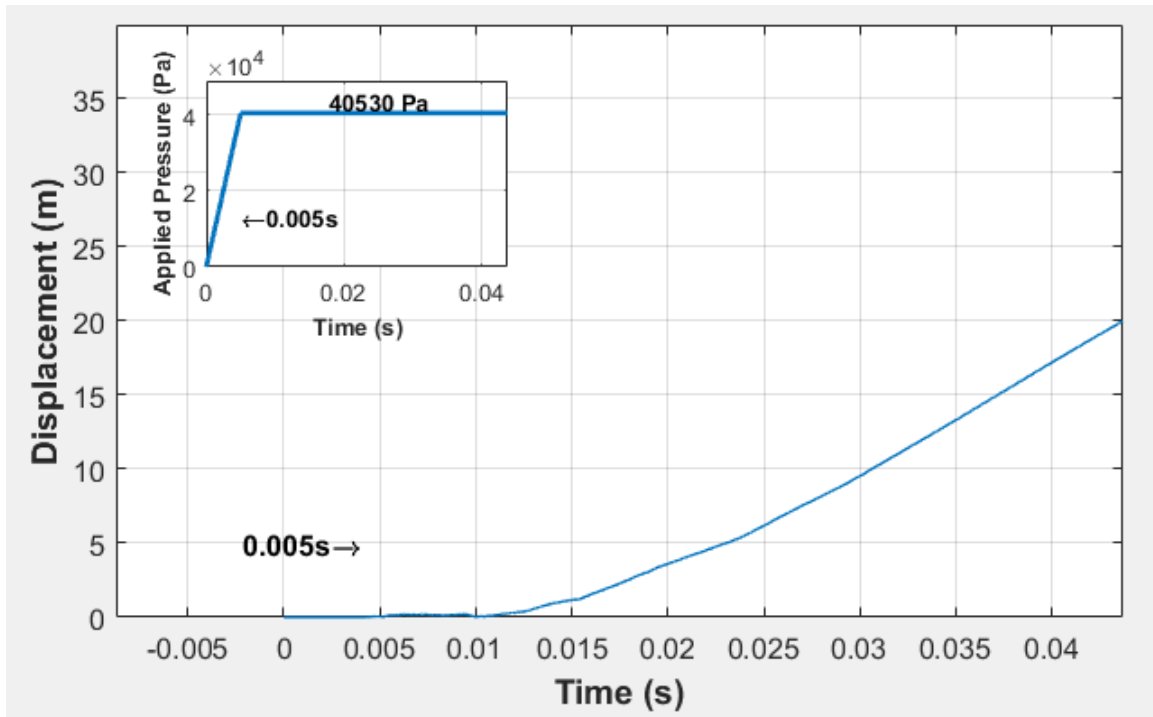


Figure 4.25. Hexakis icosahedron load case 3 displacement plot expanded to 0.048s ($\lambda = 0.6245$)

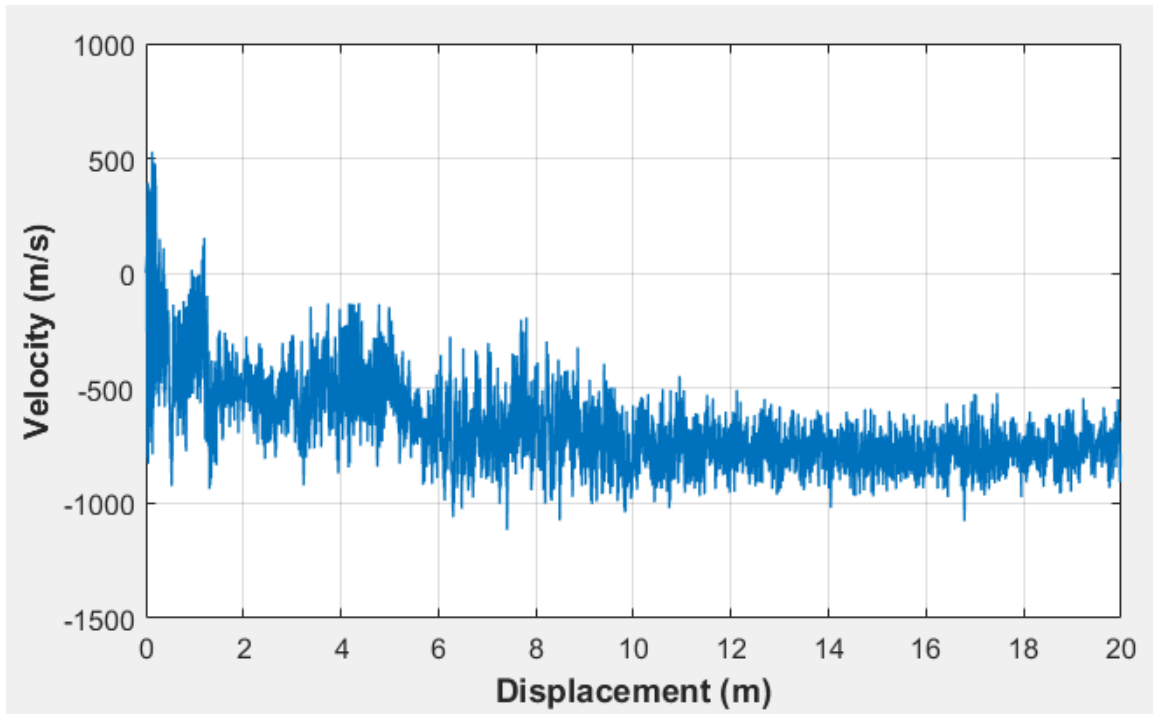


Figure 4.26. Hexakis icosahedron load case 3 phase-space diagram ($\lambda = 0.6245$)

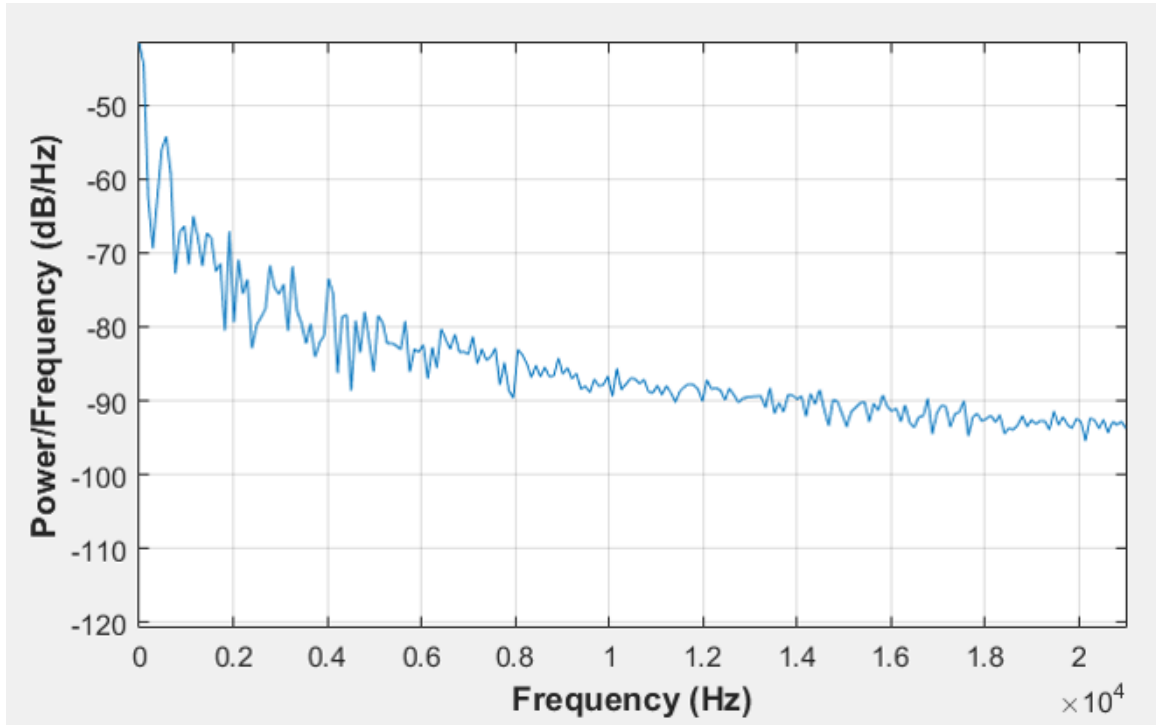


Figure 4.27. Hexakis icosahedron load case 3 PSD plot ($\lambda = 0.6245$)

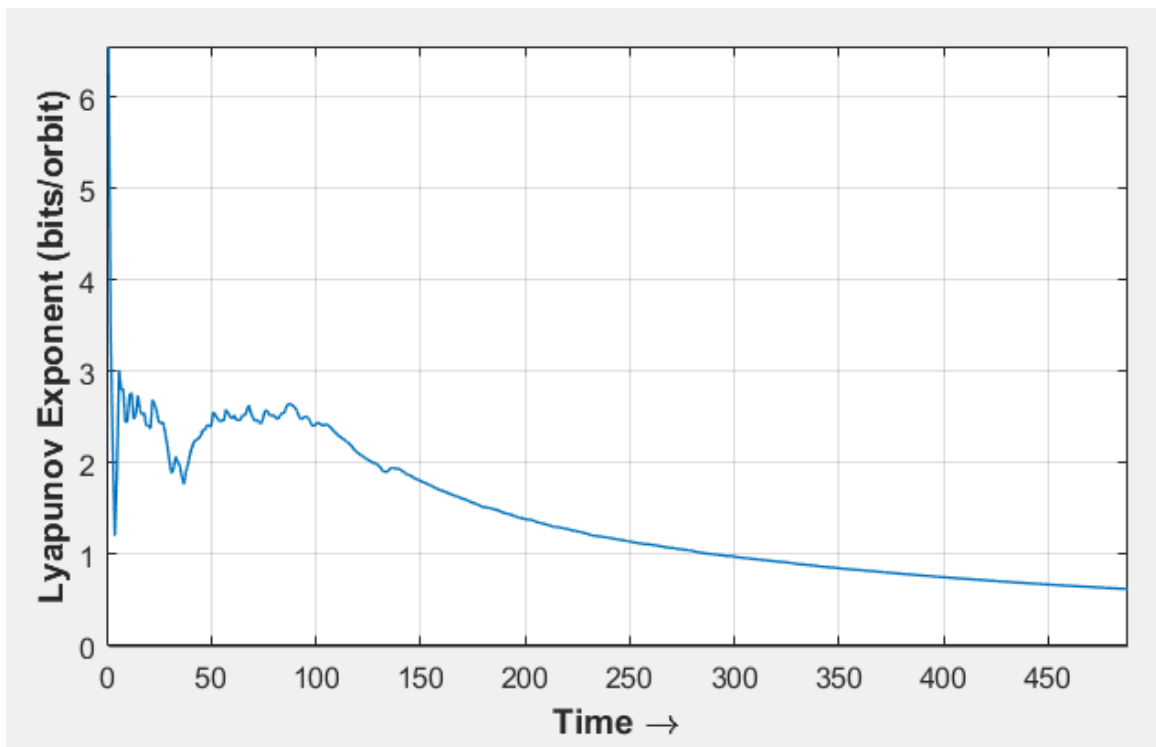


Figure 4.28. Hexakis icosahedron load case 3 Lyapunov exponent convergence plot ($\lambda = 0.6245$)

In load case 4, the frame was again loaded to 10% sea level pressure at a rate of 5.0663 MPa/s as in load case 1; however, the boundary condition was changed from the symmetric boundary condition BC3 to the unsymmetric BC2. The deformed shape of the frame after the load was fully applied is shown in figure 4.29. The results of the chaotic behavior analysis are provided in figures 4.30 through 4.33.

The displacement plot in figure 4.30 depicts displacements slightly larger than the displacements in load case 1. Also, the irregular pattern of the displacement at steady state, a trend more easily visible in this load case than load case 1, suggests chaotic behavior may be present in the system.

Chaotic behavior is also more apparent in the phase-space diagram for load case 4. As shown in figure 4.31, although the system reaches a steady state, the oval-shaped attractor is significantly thicker than previous load cases - a strong indication that chaos is present.

The PSD plot in figure 4.32 is again more visibly chaotic than in load case 1 where a symmetric boundary condition was used. No distinct frequency is clearly dominant in the plot, and the frequencies that can be distinguished do not match the calculated first mode of the frame as can be seen in the non-chaotic load case 2.

The Lyapunov exponent convergence plot in figure 4.33 shows the graph converging to a exponent value of $\lambda = 0.2341$. Although the exponent values of load cases 1 and 4 were nearly identical, chaotic behavior was more apparent in load case 4 where the boundary condition was unsymmetric. This demonstrates the frame's sensitivity to its initial conditions at load rates greater than 4.0-4.5 MPa/s, a trait that will be further proven by load case 5.

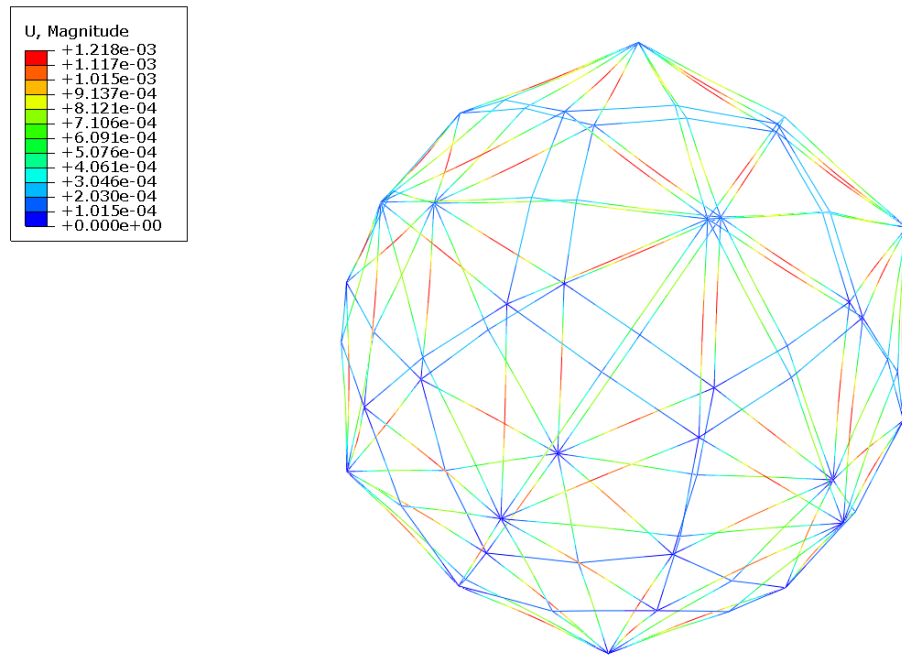


Figure 4.29. Hexakis icosahedron load case 4 deformed shape

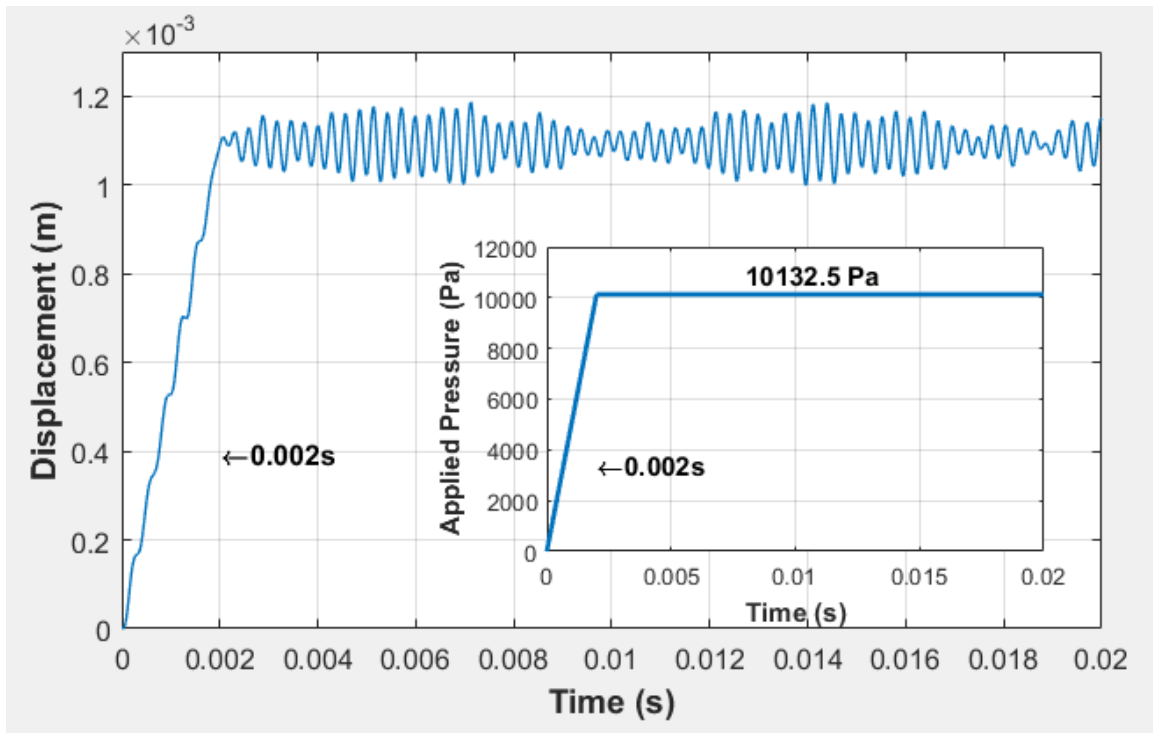


Figure 4.30. Hexakis icosahedron load case 4 displacement plot ($\lambda = 0.2341$)

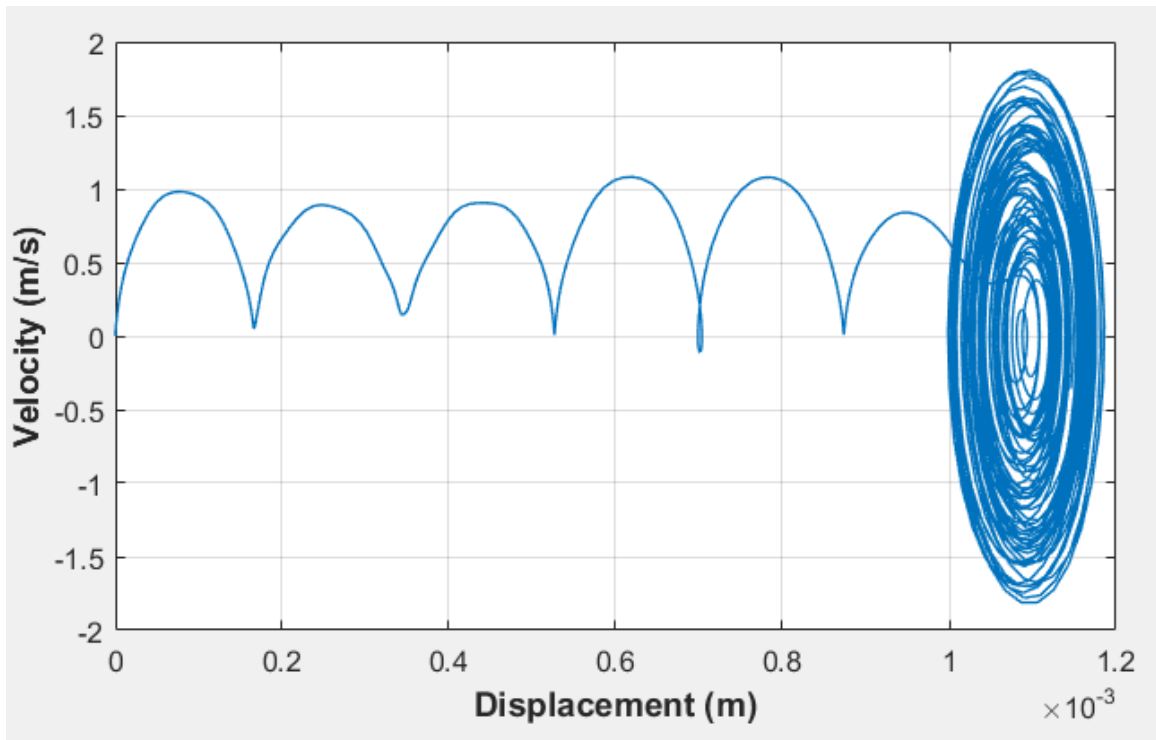


Figure 4.31. Hexakis icosahedron load case 4 phase-space diagram ($\lambda = 0.2341$)

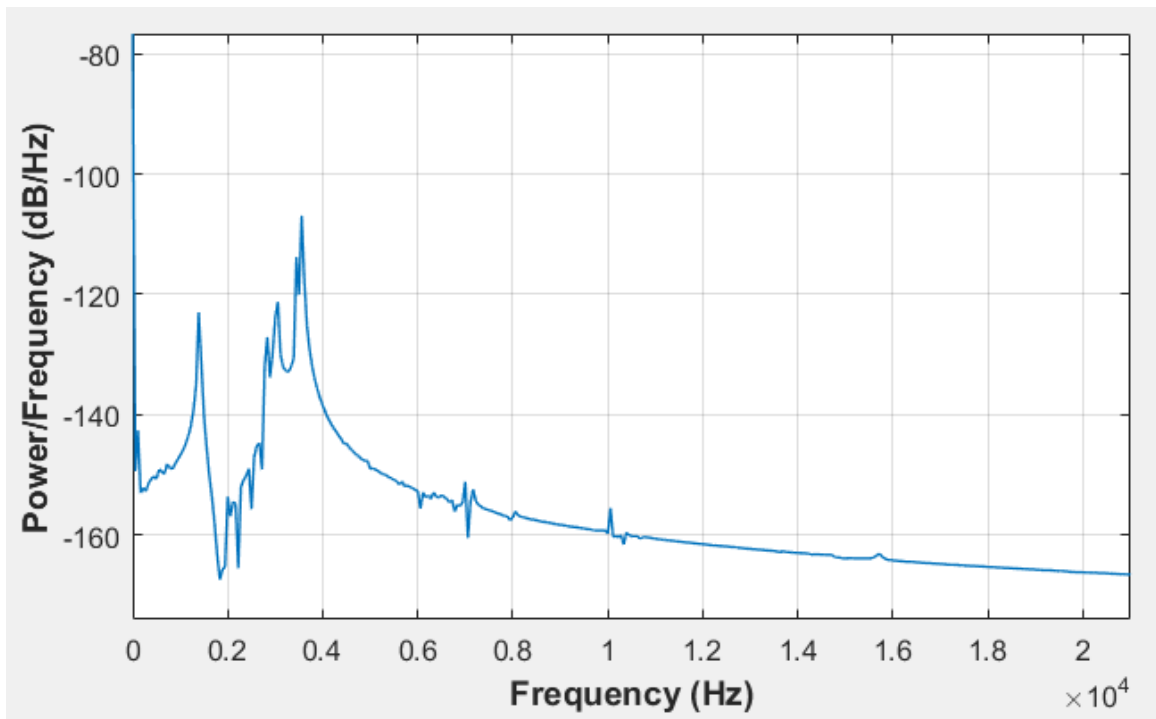


Figure 4.32. Hexakis icosahedron load case 4 PSD plot ($\lambda = 0.2341$)

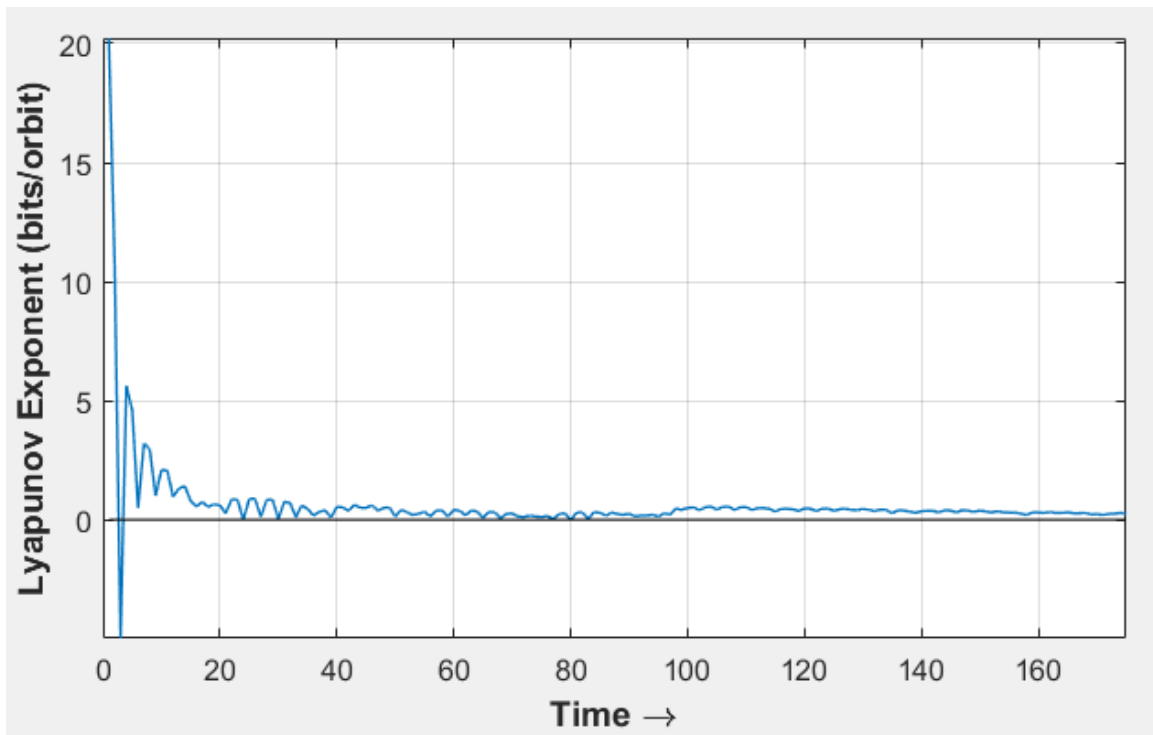


Figure 4.33. Hexakis icosahedron load case 4 Lyapunov exponent convergence plot ($\lambda = 0.2341$)

For load case 5, the load was applied to 20% sea level pressure as in load case 2; however, like load case 4, the boundary condition was changed from symmetric BC3 to unsymmetric BC2 for the analysis. The deformed shape of the frame after loading can be seen in figure 4.34.

As with load case 4, load case 5 exhibits larger displacements and a more irregular displacement pattern than load case 2. This plot is shown in figure 4.35. This leads to the conclusion that changing the boundary condition may have caused a transformation from non-chaotic to chaotic behavior.

The phase-space diagram in figure 4.36 reinforces this conclusion. The steady state attractor is significantly thicker than the attractor in figure 4.20. This is both due to the chaotic behavior developing in the system as well as the asymmetry of the boundary conditions.

The PSD plot in figure 4.37 resembles that of load case 2 in figure 4.21; however, the first frequency mode is higher than the calculated value of 2.13 kHz. From the plot, it seems as though this frequency is exhibited as a small blip; however, the dominant frequency occurs at approximately 3.1 kHz - an error of 45.5%.

Finally, the Lyapunov exponent convergence plot in figure 4.38 confirms that the frame has transitioned to a chaotic system due to the change in boundary condition with an exponent of $\lambda = 0.0143$. It is worth noting that the magnitude of the Lyapunov exponent has not increased by much; however, it shifted enough to increase from a negative to a positive value.

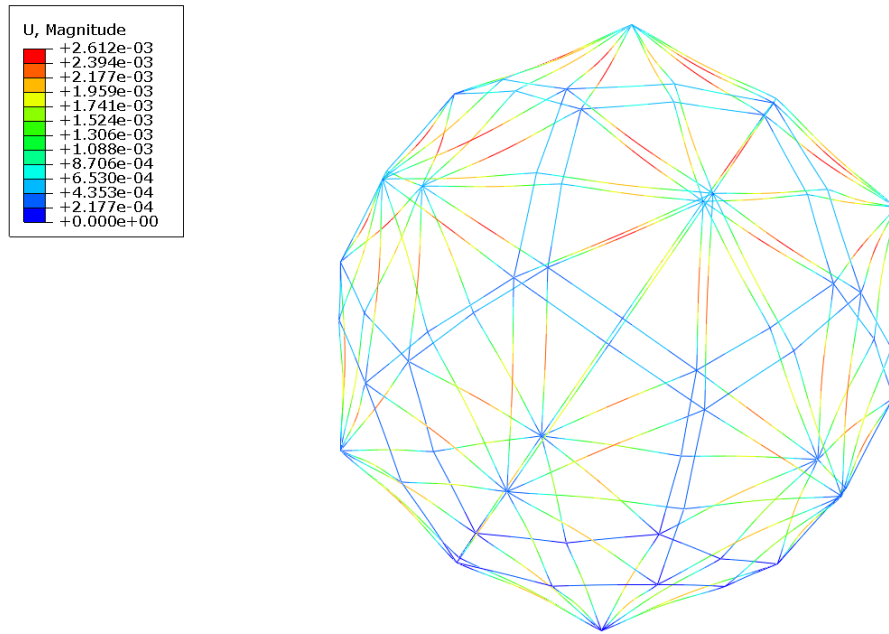


Figure 4.34. Hexakis icosahedron load case 5 deformed shape

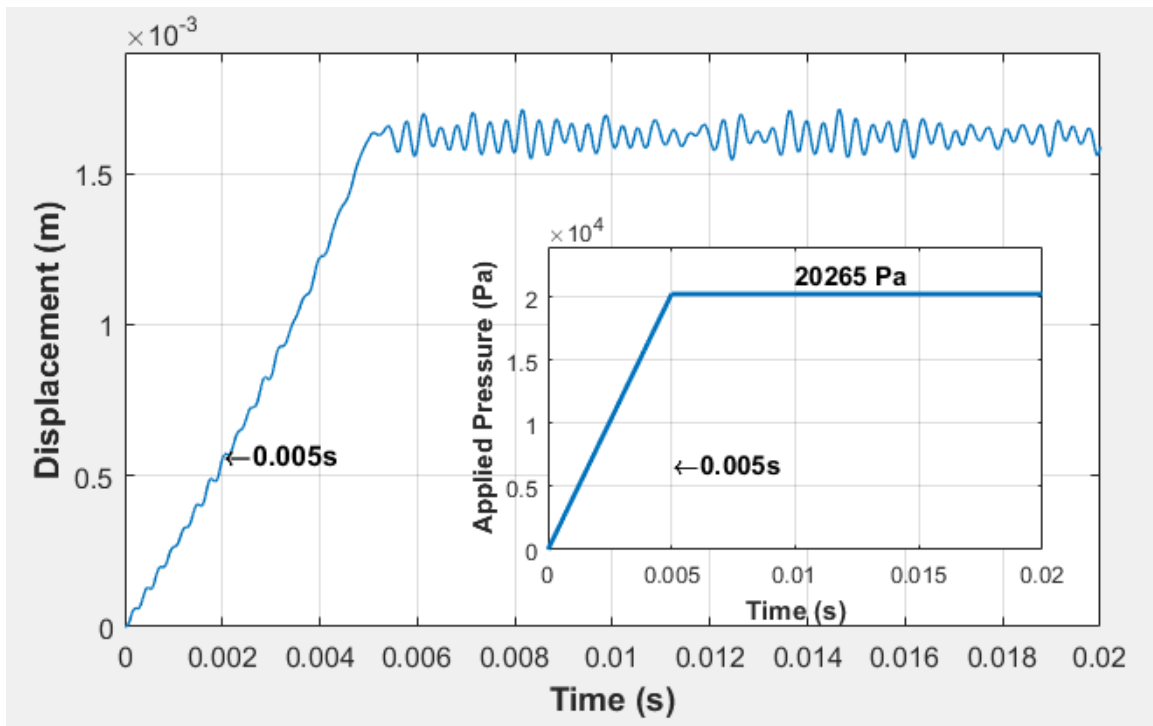


Figure 4.35. Hexakis icosahedron load case 5 displacement plot ($\lambda = 0.0143$)

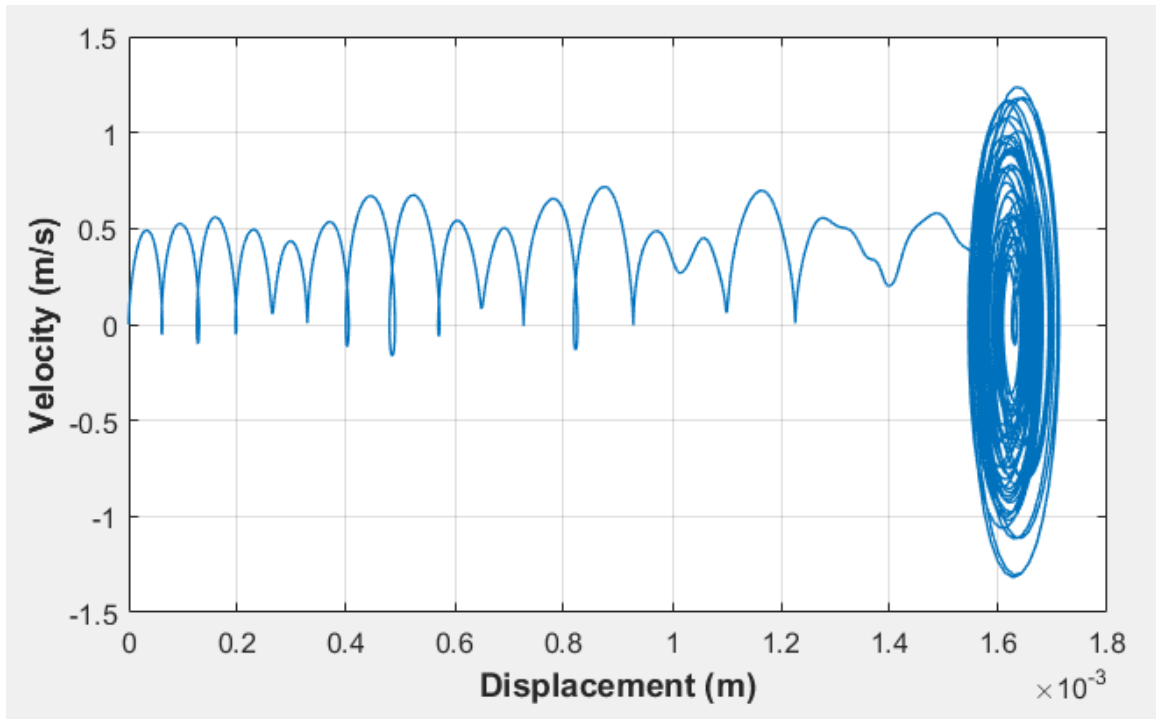


Figure 4.36. Hexakis icosahedron load case 5 phase-space diagram ($\lambda = 0.0143$)

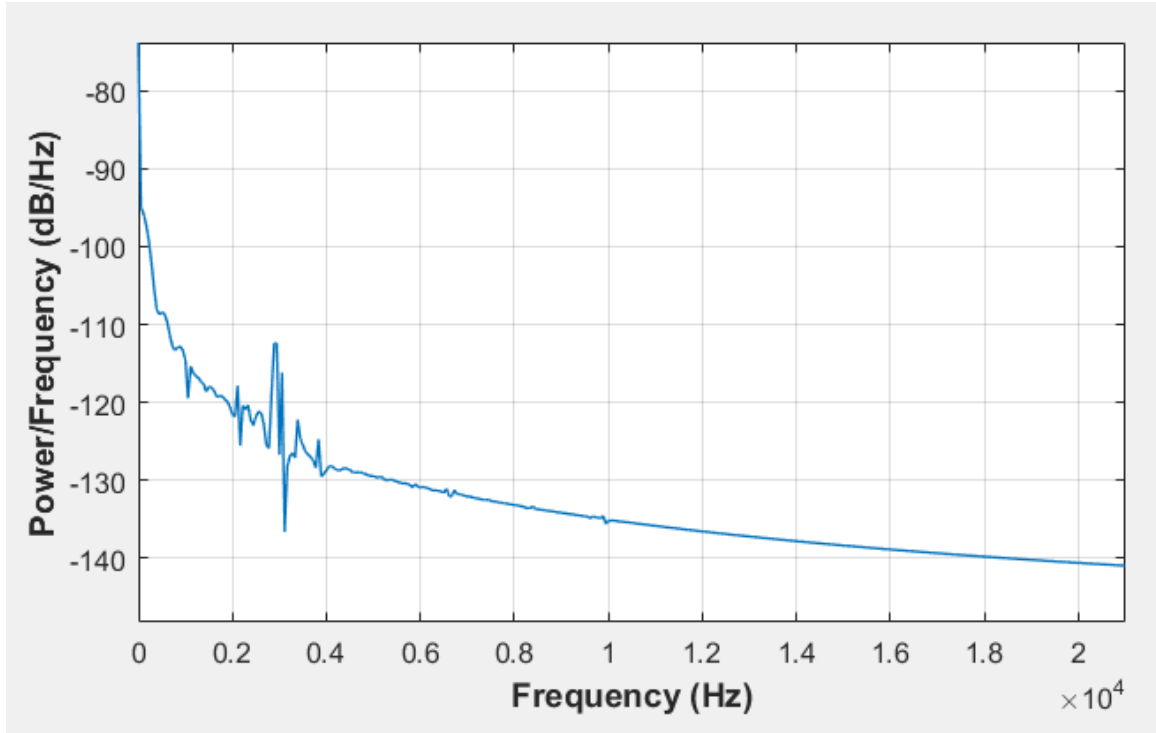


Figure 4.37. Hexakis icosahedron load case 5 PSD plot ($\lambda = 0.0143$)

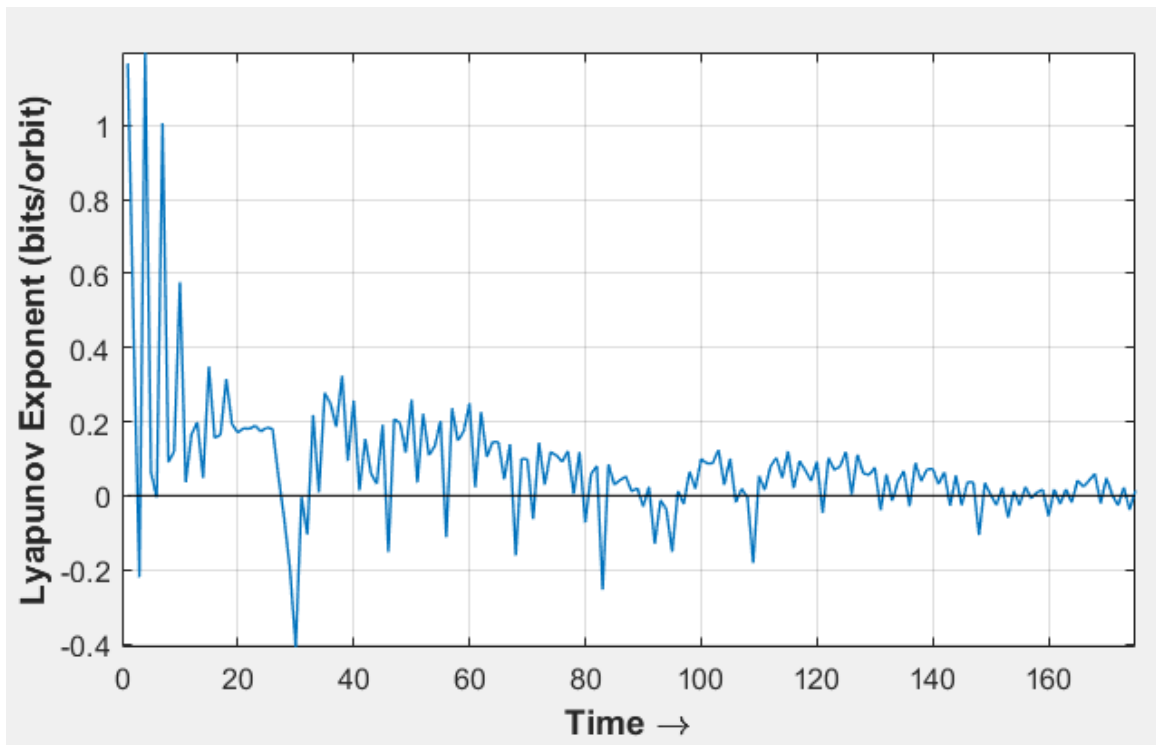


Figure 4.38. Hexakis icosahedron load case 5 Lyapunov exponent convergence plot ($\lambda = 0.0143$)

The final scenario, load case 6, applied a 40% sea level pressure load to the frame at a rate of 8.106 MPa/s. Unsymmetric boundary condition 2 was used to restrain the frame rather than boundary condition 3 as in load case 3, and the deformed shape of the frame can be seen in figure 4.39. The displacement plot in figure 4.40 resembles that of load case 3. It is also characterized by large displacement values and non-periodic behavior once the load has been applied.

The phase-space diagram in figure 4.41 proves that the frame did not reach a steady state attractor. Also of note are the high velocity values (reaching magnitudes of over 750 m/s) shown in the plot. This demonstrates highly non-linear behavior present in the system.

The PSD plot in figure 4.42 also matches that of load case 3 in figure 4.27. No clear dominant frequency is exhibited in the plot, indicating a chaotic load case. The calculated first frequency mode at 2.13 kHz is indistinguishable in the erratic behavior of the plot.

The Lyapunov exponent convergence plot in figure 4.43 confirms that the load case is chaotic with an exponent value of $\lambda = 4.767$. This higher exponent value and increased chaotic behavior was caused by simply changing the boundary condition in the loading scenario. This demonstrates the extreme dependency of the frame on its initial conditions under certain load rates. The load rate at which the system changes from non-chaotic to chaotic appears to be between 4.0 and 4.5 MPa/s depending on the boundary condition used.

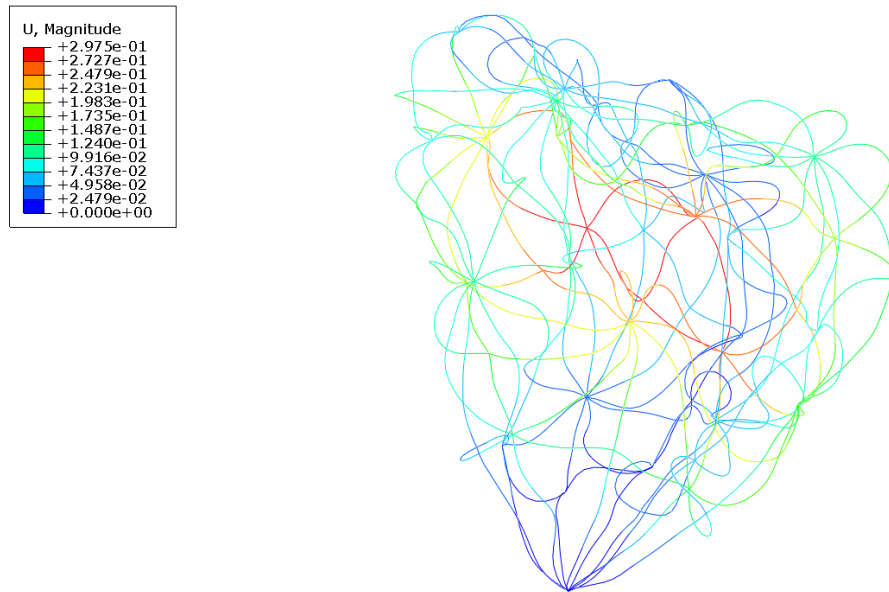


Figure 4.39. Hexakis icosahedron load case 6 deformed shape

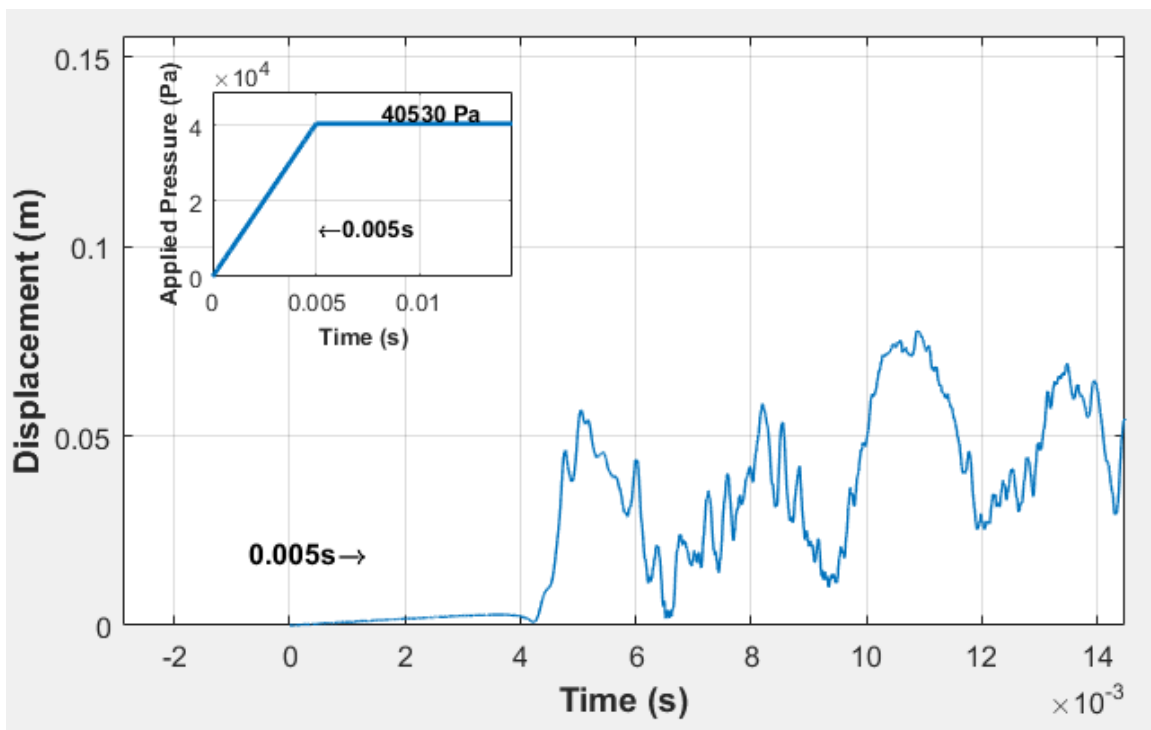


Figure 4.40. Hexakis icosahedron load case 6 displacement plot ($\lambda = 4.767$)

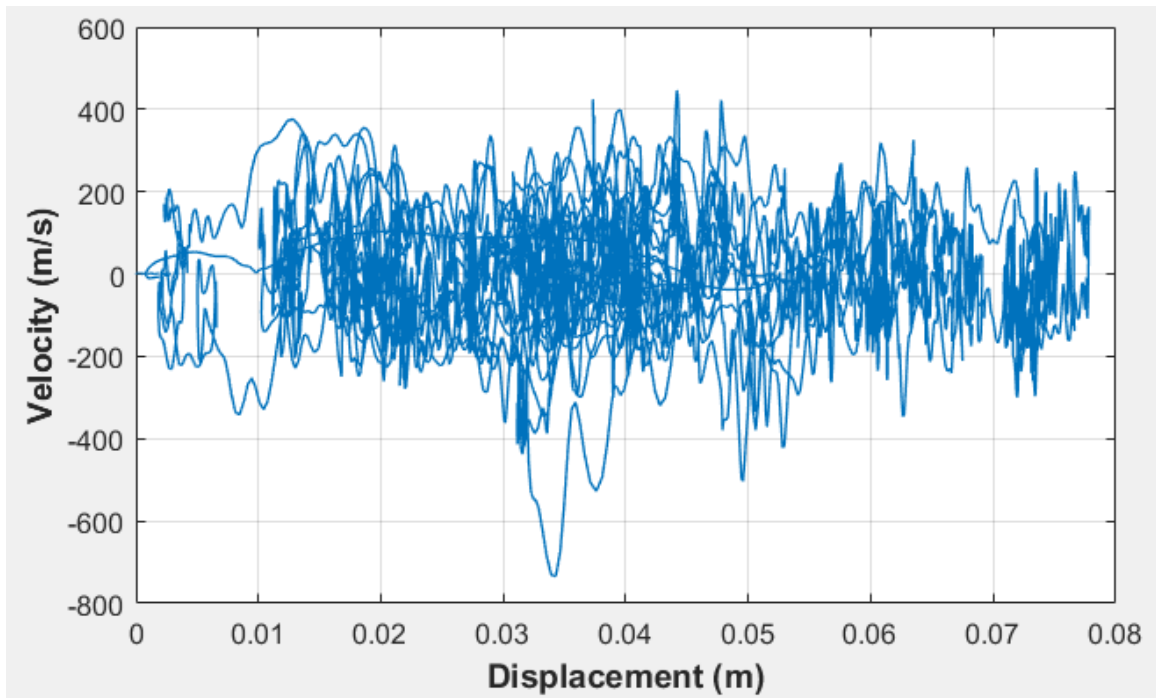


Figure 4.41. Hexakis icosahedron load case 6 phase-space diagram ($\lambda = 4.767$)

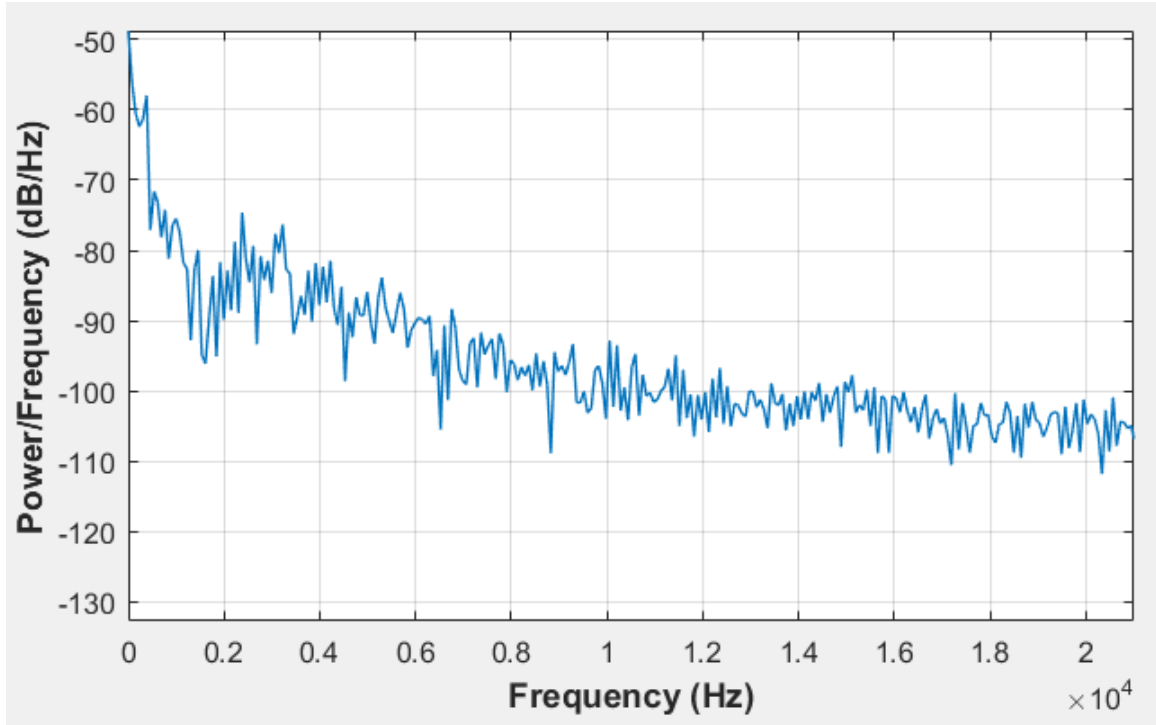


Figure 4.42. Hexakis icosahedron load case 6 PSD plot ($\lambda = 4.767$)

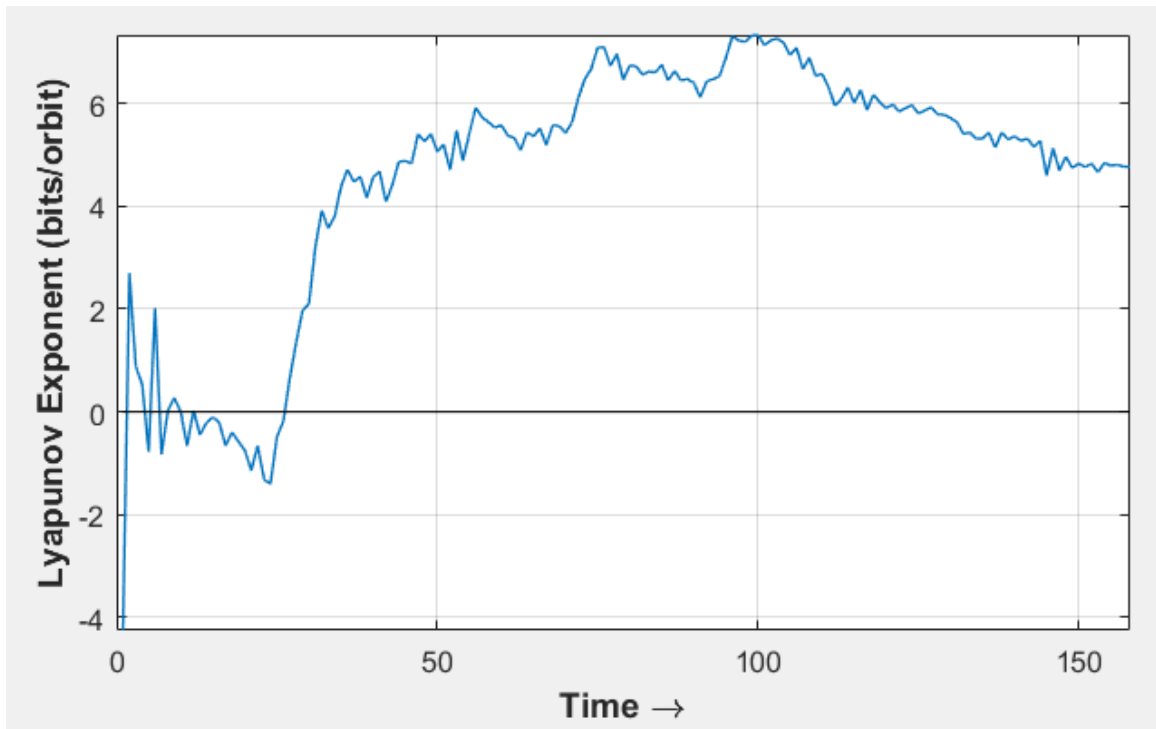


Figure 4.43. Hexakis icosahedron load case 6 Lyapunov exponent convergence plot ($\lambda = 4.767$)

The Lyapunov exponent results for the hexakis icosahedron are summarized and compared to the values for the icosahedron in table 4.3. From the Lyapunov exponents, the hexakis icosahedron exhibited less chaos than the icosahedron for all load cases except load case 1. Based on the results of the above analysis, the hexakis icosahedron is less sensitive to changing its initial conditions than the icosahedron. As can be seen in table 4.3, the icosahedron Lyapunov exponents change much more drastically than the hexakis icosahedron exponents. The data suggests that the shape of the hexakis icosahedron may be more rigid than the icosahedron and thus, able to withstand higher load rates without exhibiting chaos.

Table 4.3. Icosahedron and Hexakis Icosahedron λ Analysis Results

Load Case	Icosahedron	Hexakis	Load Rate (MPa/s)	BC
1	-0.0121	0.2552	5.0663	3
2	-0.0137	-0.0175	4.053	3
3	3.8814	0.6245	8.106	3
4	0.303	0.2341	5.0663	2
5	0.371	0.0143	4.053	2
6	19.67	4.767	8.106	2

Although analyzing the chaotic behavior of the frame under various load rates and boundary conditions is crucial, it does not account for the yield properties of the Beryllium material.

4.5 Hexakis Icosahedron Stress Analysis Results

Beryllium S-200 tubing was used in Adorno-Rodriguez's and Just's research because of its low density and known material properties; however, neither researcher did an analysis of the stresses present in the frame to evaluate the effect of chaotic behavior on the material itself. A typical stress-strain curve for Beryllium S-200 tubing is shown in figure 4.44. The yield stress is approximately 434 MPa, and the ultimate stress is approximately 800 MPa [10] [12]. Although previous research has shown

beryllium would not be an appropriate material choice for a VLTAV, this research sought to discover the effect, if any, of chaotic behavior on the Mises stresses in the frame. The six load cases previously analyzed were also used to study the structural stresses, shown in figures 4.45 through 4.50.

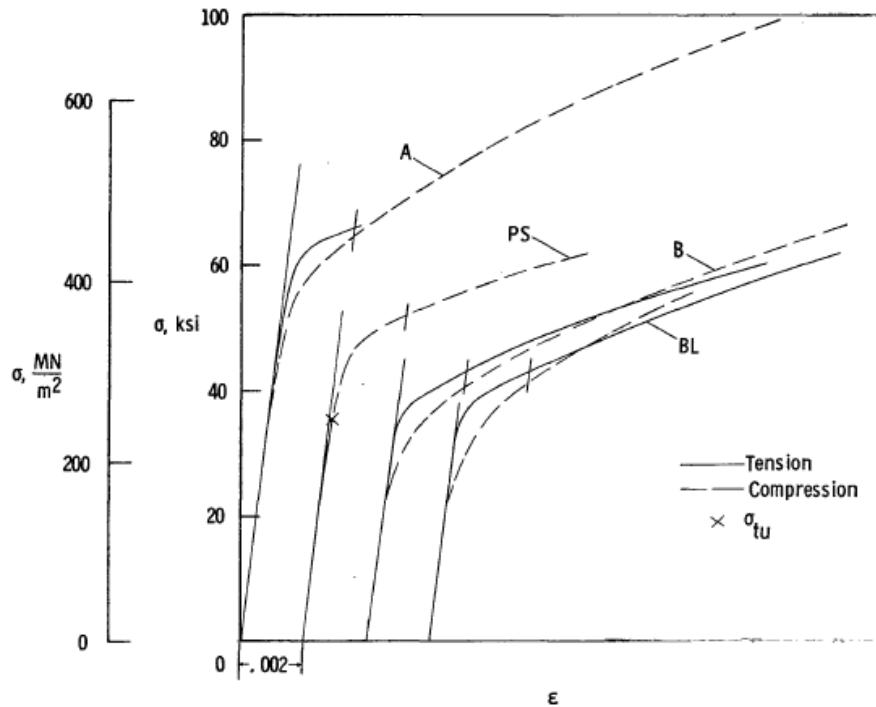


Figure 4.44. Typical stress-strain curve for Beryllium S-200 Tubing at standard temperature with $\sigma_y = 434$ MPa and $\sigma_u = 800$ MPa [10]

From figures 4.45 through 4.50, the Mises stress magnitudes are more dependent on the load magnitude than the boundary condition, applied load rate, or presence of chaotic behavior. Comparing, for example, load cases 2 and 5 in figures 4.46 and 4.49, where only the boundary condition changed, the stress barely changed. For load case 2, the maximum stress was approximately 750 MPa, and for load case 5, the maximum stress was approximately 760 MPa.

The applied load rate also seemed to have little effect on the stress plots. Comparing load cases 1 and 2 in figures 4.45 and 4.46 where the load rates were 5.0663 MPa/s

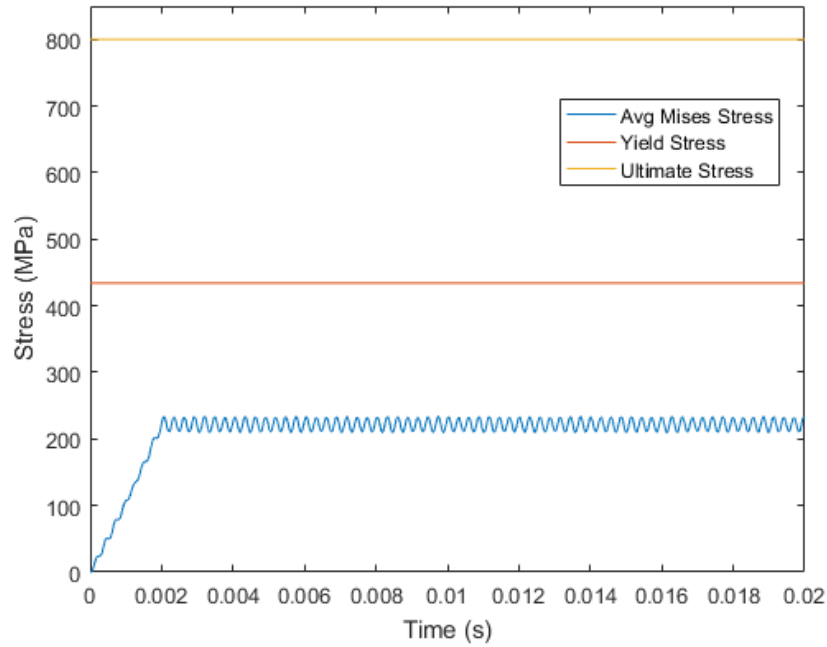


Figure 4.45. Mises stress results for load case 1

and 4.053 MPa/s respectively, the results are opposite of what may be expected. The case with the higher load rate, load case 1, had lower stress values than the case with the lower load rate, load case 2. The reason for higher stress values in load case 2 appears to be due to the higher load magnitude (20% sea level pressure for load case 2 and 10% sea level pressure for load case 1).

The presence of chaotic behavior seemed to have little impact on the stress plots. Again, comparing load cases 2 and 5 in figures 4.46 and 4.49, the frame transitions from a non-chaotic to a slightly chaotic system. The Mises stresses in the plots are nearly identical, suggesting that chaos has a minimal effect on the beam stresses.

Of the six load cases evaluated, only load cases 1 and 4 had stress values below both the yield stress and ultimate stress of the beryllium S-200 tubing. These load cases corresponded to load magnitudes of only 10% sea level pressure. When the frame was loaded to 20% sea level pressure as in load cases 2 and 5, the stress was less than the ultimate stress of the tubing, but greater than the yield stress. Load

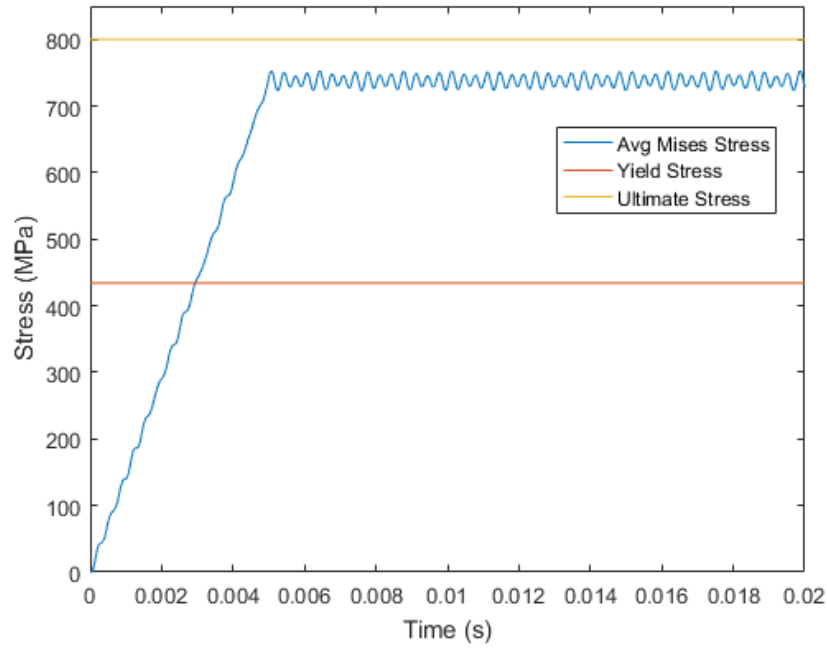


Figure 4.46. Mises stress results for load case 2

cases 3 and 6 caused significantly higher stress values as the load applied was 40% of sea level pressure. It can be seen from the plots in figures 4.47 and 4.50 that the stress gradually increases as the load is applied until it reaches the *snapping* load where the chaotic behavior begins to dominate and the stress increases dramatically.

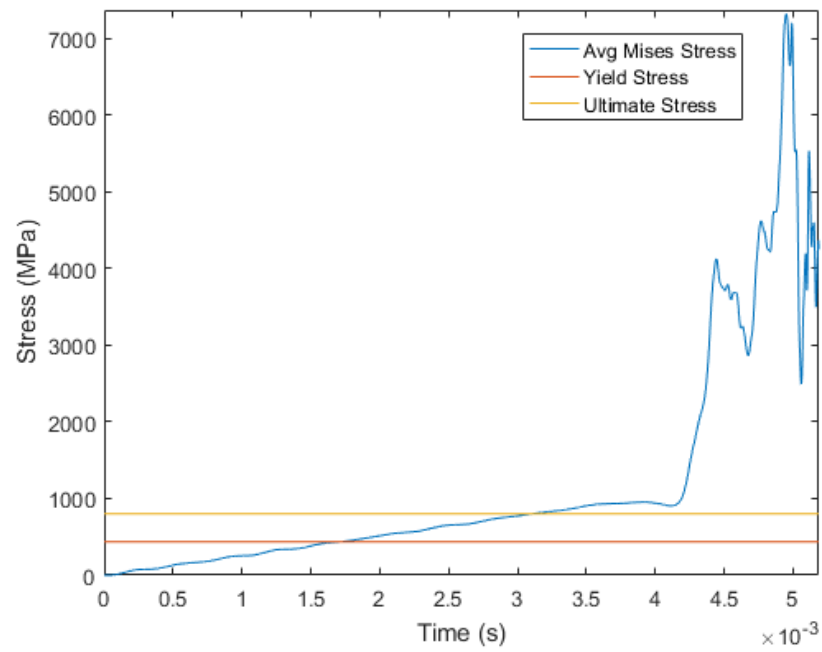


Figure 4.47. Mises stress results for load case 3

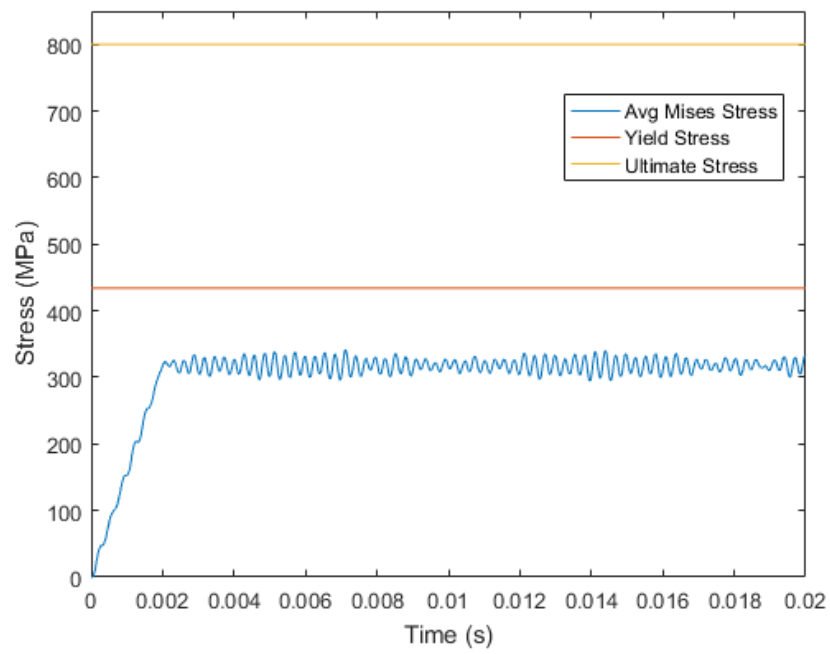


Figure 4.48. Mises stress results for load case 4

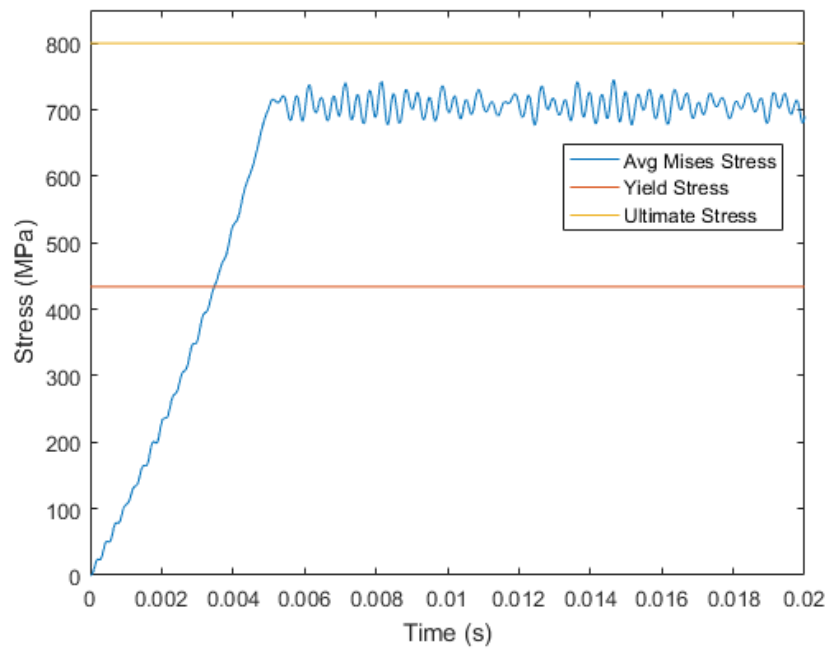


Figure 4.49. Mises stress results for load case 5

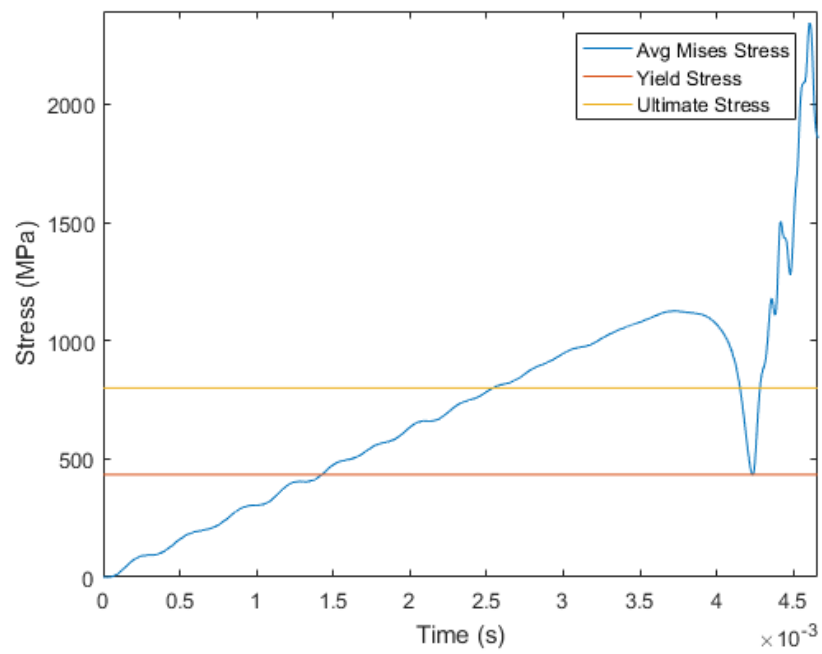


Figure 4.50. Mises stress results for load case 6

4.6 Hexakis Icosahedron Frame and Skin Model Results

In order to provide a more realistic analysis of the hexakis icosahedron shaped VLTAV, the membrane was included in the model and tied to the frame. Load cases 7 and 8, described in table 3.5, were applied to the frame-skin model for comparison to Just's results from the icosahedron.

Figure 4.51 depicts the deformed shape of the frame-skin model after load case 7 is applied. Load case 7 is equivalent to load case 3 for the standalone frame model where the load magnitude is 40% sea level pressure, the load rate is 8.106 MPa/s, and the symmetric boundary condition 3 is used. As can be seen in figure 4.51, the model deforms differently than may be expected in that the maximum displacement occurs at the center of the medium length beams rather than at the center of the individual membrane panels. The displacement plot, phase-space diagram, and PSD plot were compared to Just's icosahedron results to determine whether chaotic behavior was present even after the skin was included.

The displacement plot for load case 7 is shown in figure 4.52. As with chaotic scenarios, the plot exhibits non-periodic displacement over time. Compared to load case 3 where the standalone frame was analyzed, the displacement magnitudes for load case 7 were significantly smaller with the maximum displacement value being only 1.1 mm. The phase-space diagram in figure 4.53 shows the system is beginning to converge to a steady state after 0.02 seconds in the analysis; however, the plot is indicative of a chaotic system. The PSD plot in figure 4.54 confirms that the load case is chaotic as there is no clear dominant frequency present in the plot.

Although the skin added stiffness to the frame and reduced displacement magnitudes, the chaotic behavior of the frame was not damped out as was the case with the icosahedron in Just's research. A similar analysis of load case 8 provided a clearer picture of what was creating the nonlinear dynamic behavior in the model.

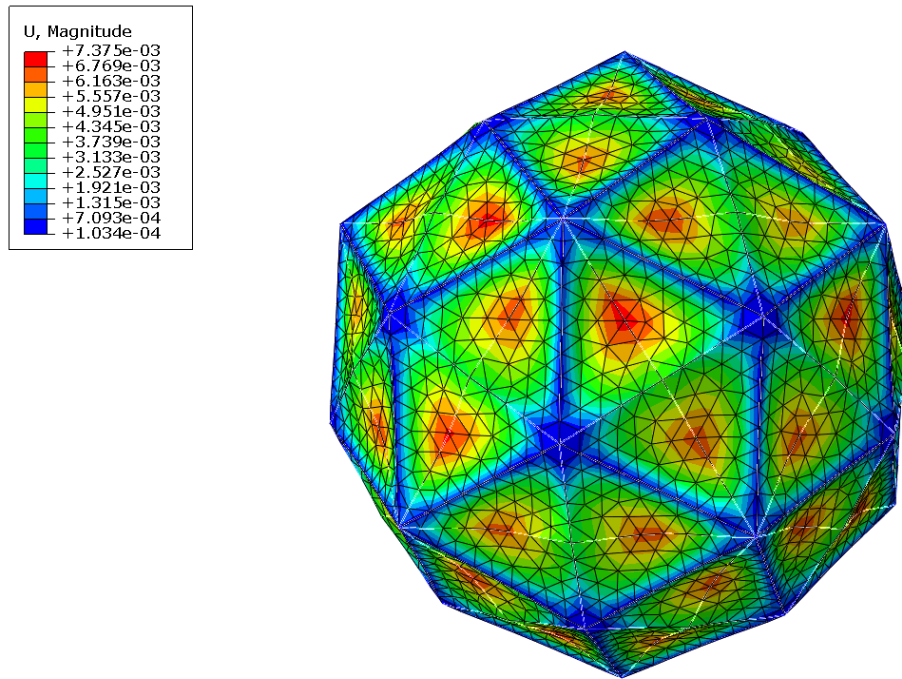


Figure 4.51. Hexakis icosahedron load case 7 deformed shape

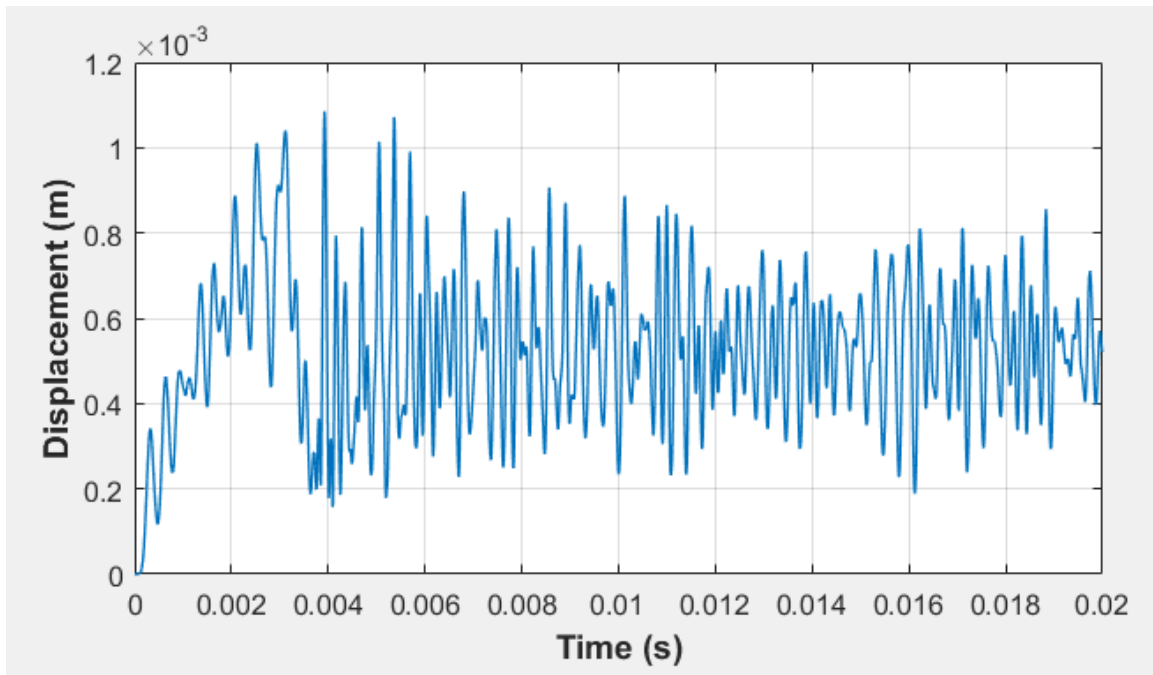


Figure 4.52. Hexakis icosahedron load case 7 displacement plot

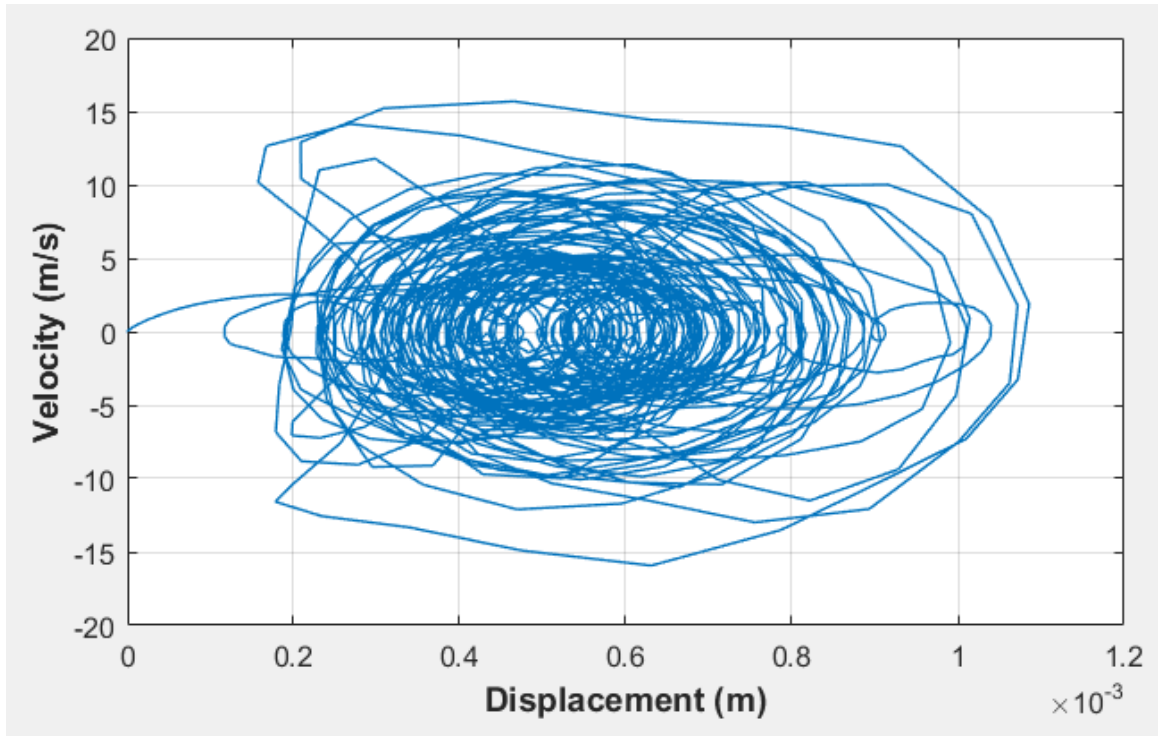


Figure 4.53. Hexakis icosahedron load case 7 phase-space diagram

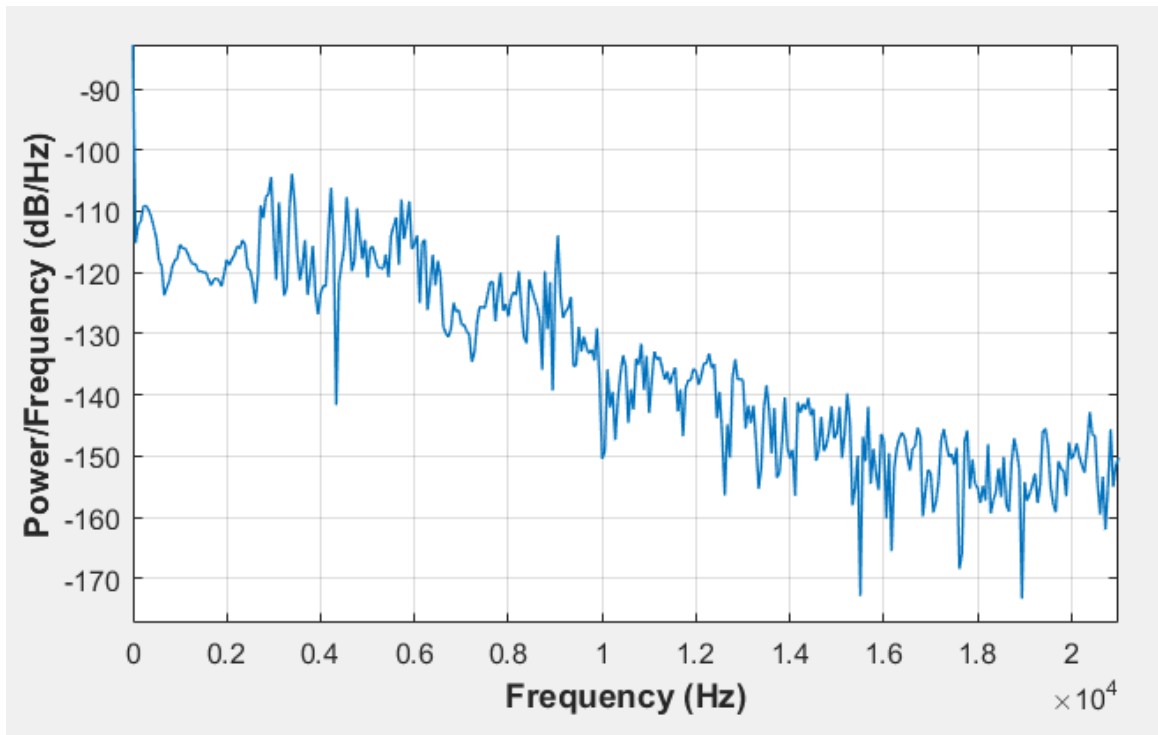


Figure 4.54. Hexakis icosahedron load case 7 PSD plot

Load case 8 corresponded to a load magnitude of 60% sea level pressure and a load rate of 12.159 MPa/s. Symmetric BC3 was applied to the structure. The deformed shape of the model is shown in figure 4.56. The points of maximum displacement shifted from the center of the medium length beams to the intersections of the medium and short beams. As with load case 7, this was a different result than initially expected. The dynamic behavior of the frame was also analyzed for this load case in order to determine the cause of this deformation result.

Snapback behavior of previous research is exhibited in this load case as well. Figure 4.55 shows the deformation of the model prior to the snapback occurring while figure 4.56 shows the deformation of the model after snapback has occurred. In the displacement plot of figure 4.57, the *snapback* phenomenon of previous research occurs at approximately 0.0025 seconds, just after the load was fully applied. Once the beam regained a load bearing capacity, the displacement oscillated non-periodically and converged to a steady state. This *snapback* is seen again in figure 4.58 on the left side of the phase-space diagram. The system's nonlinear dynamic behavior is exhibited by the thickness and shape of the attractor. The PSD plot in figure 4.59 confirms that the load case is chaotic as in load case 7 in that no dominant frequency can be determined from the plot.

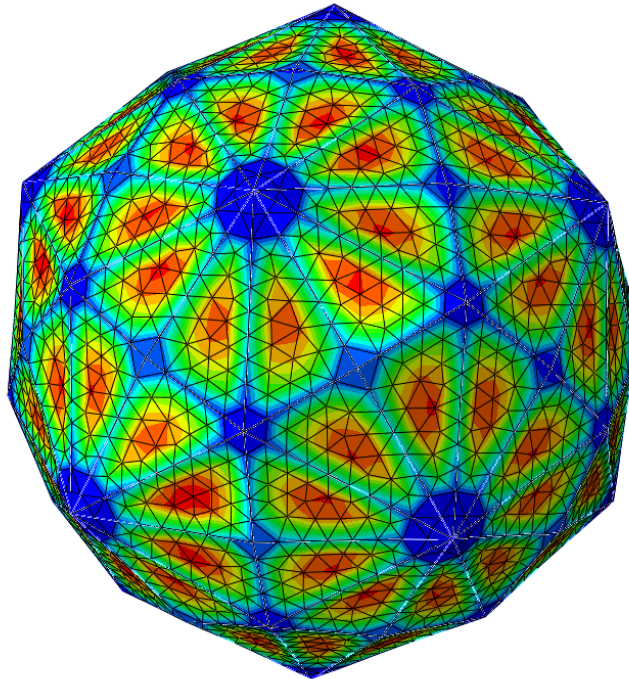


Figure 4.55. Hexakis icosahedron load case 8 deformed shape before snapping

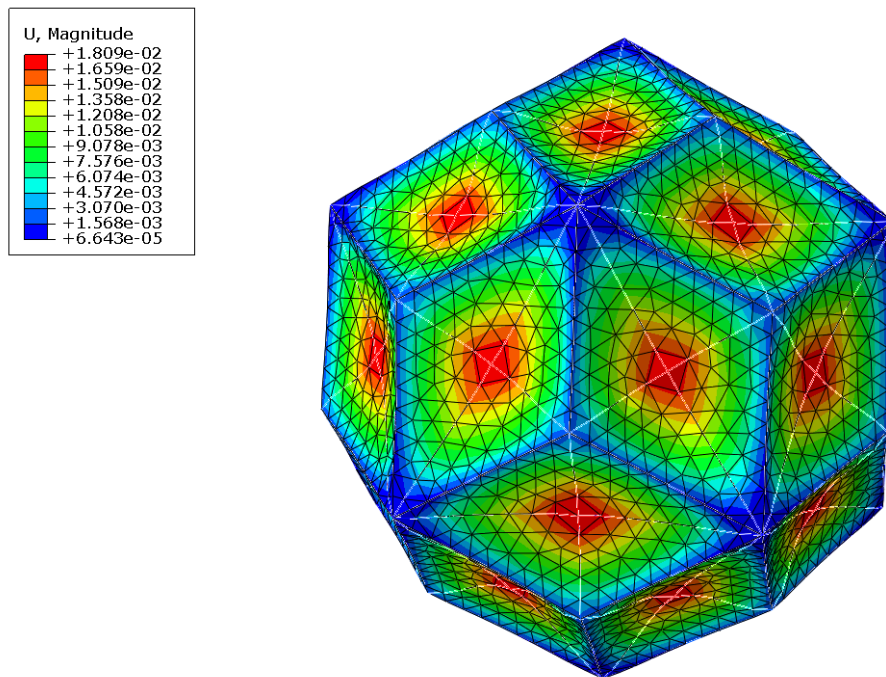


Figure 4.56. Hexakis icosahedron load case 8 deformed shape after snapping

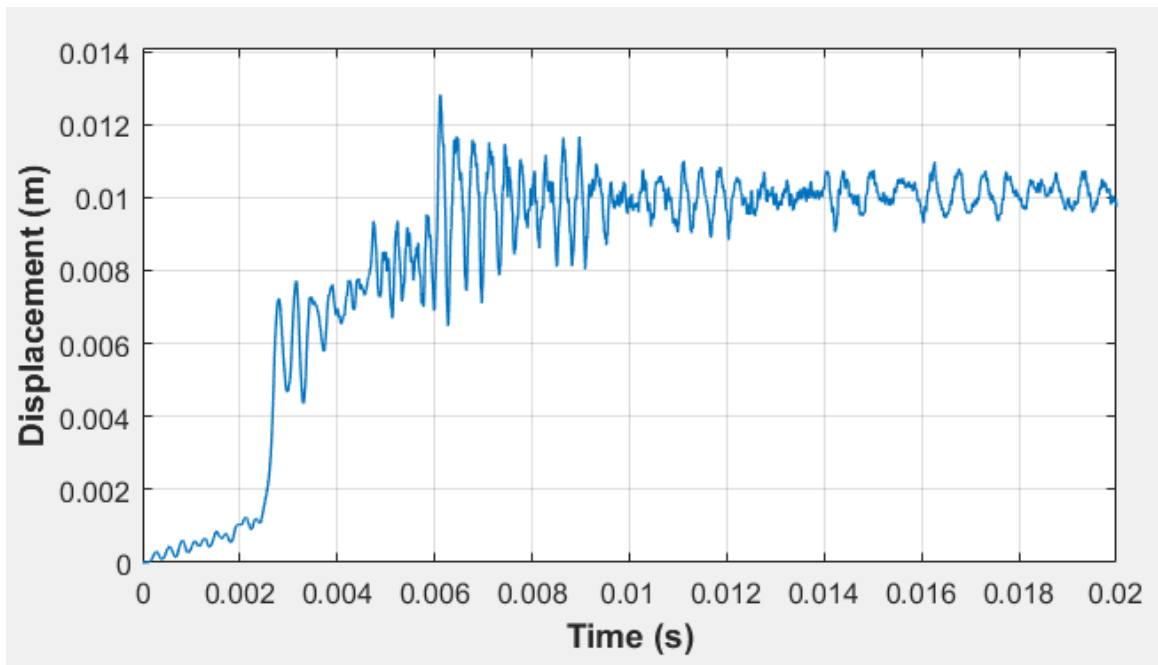


Figure 4.57. Hexakis icosahedron load case 8 displacement plot

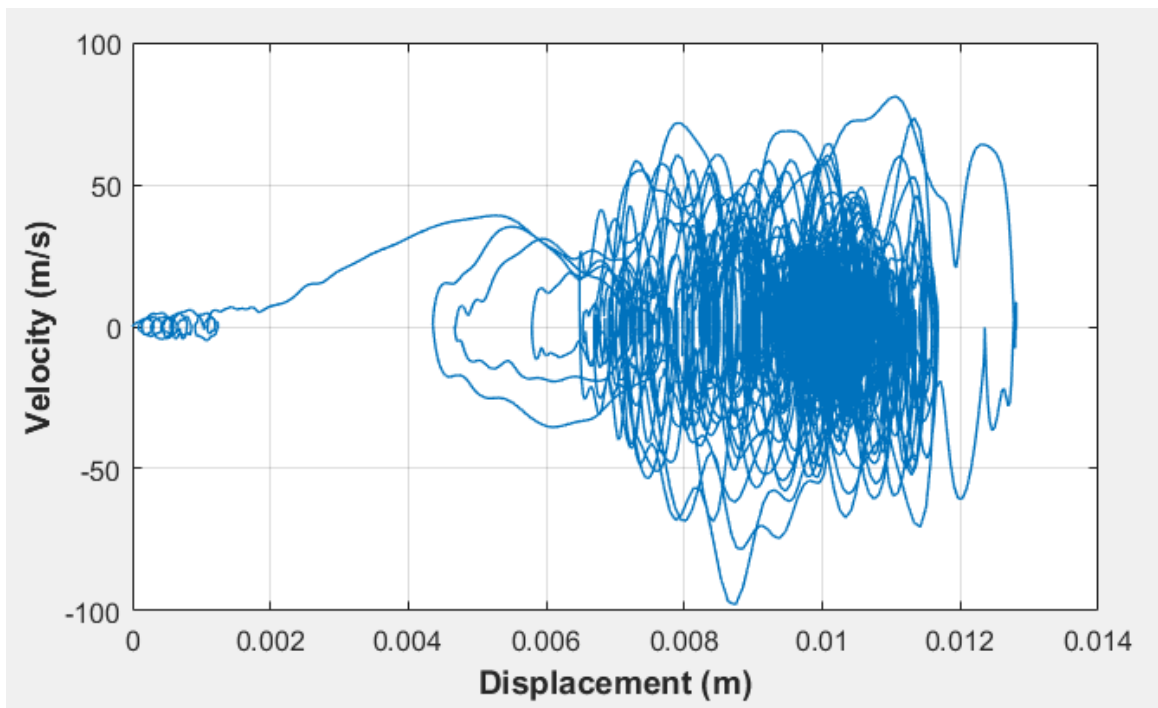


Figure 4.58. Hexakis icosahedron load case 8 phase-space diagram

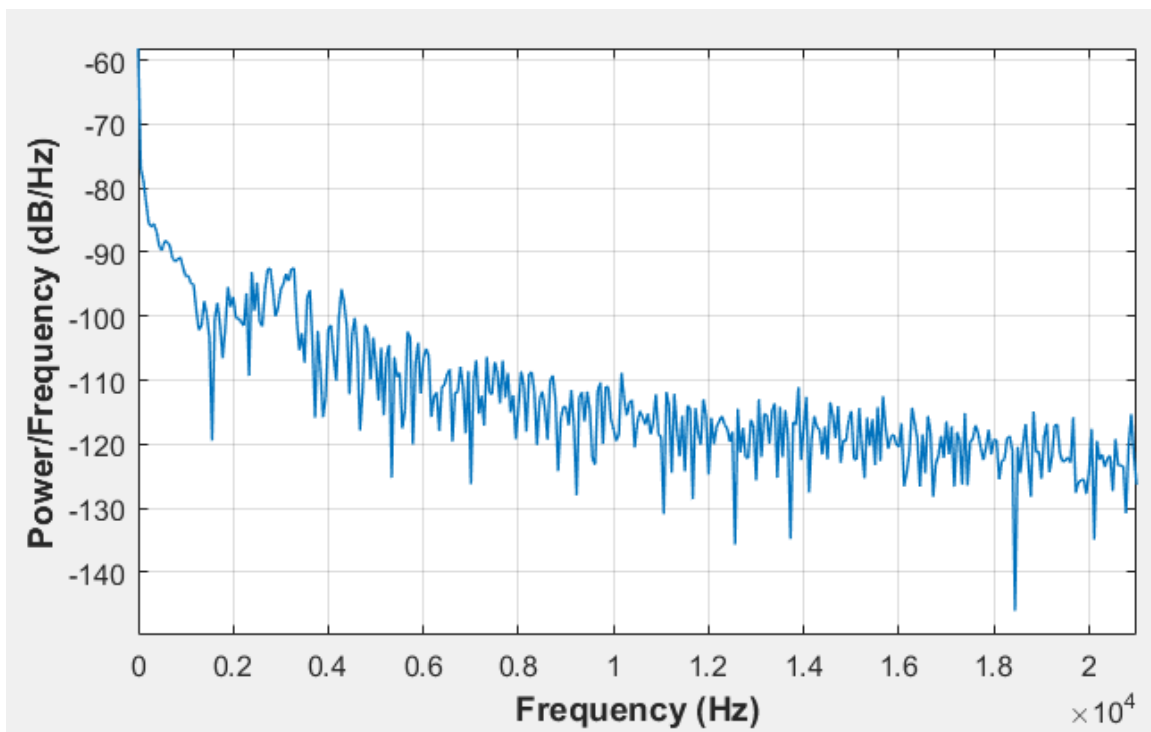


Figure 4.59. Hexakis icosahedron load case 8 PSD plot

In order to determine the reason why dynamic nonlinearities were present in the hexakis icosahedron models but not in the icosahedron models, an investigation was conducted into how each frame absorbed loads. The icosahedron consists of 12 vertices, each with five beams “welded” together. The hexakis icosahedron, however, has vertices with ten beams, six beams, and four beams intersecting and thus, each vertex absorbs loading differently. From studying the output files of the model, the vertices with four beams appeared to be the cause of the dynamic nonlinearities at certain load cases.

As a sea level pressure load is applied to any vertex of the icosahedron, the beams connected to the vertex absorb the load through a combination of compression and bending. For a four beam vertex on the hexakis icosahedron frame, however, this is not the case. These vertices lie relatively flat on the surface of the structure and thus, absorb the load almost exclusively through bending. In order to verify this theory, the inertia and stiffness of the beams were compared between the two models. Equation 4.1 was used to calculate the moment of inertia of the beams, and equation 4.2 was used to compare the stiffness of the beams [21]. The results of this comparison are shown in table 4.4.

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) \quad (4.1)$$

where:

I = Moment of Inertia

d_o = Outer beam diameter

d_i = Inner beam diameter

$$K = \frac{6EI}{L^2} \quad (4.2)$$

where:

K = Stiffness coefficient

E = Modulus of elasticity

I = Moment of inertia

L = Beam length

Table 4.4. Icosahedron and Hexakis Icosahedron Stiffness Analysis Results

Model	$I(m^4) * 10^{-13}$	$K(N)$
Icosahedron	7.8373	84.779
Hexakis	1.2116	33.697

As can be seen in table 4.4, the icosahedron beams have a moment of inertia nearly 6.5 times that of the hexakis icosahedron beams. Also, they are 2.5 times stiffer than the beams of the hexakis icosahedron. The lack of inertia and stiffness in the beams of the hexakis icosahedron could be the cause of nonlinear dynamic behavior under certain load cases.

4.7 Summary

The dynamic FE modeling technique was verified by conducting an experimental modal analysis on a 0.1778 m diameter hexakis icosahedron frame. The experimentally determined first two frequency modes of the frame matched the calculated FE model results within less than 1%. This confirmed that a correct method of analysis for dynamic and quasi-static scenarios had been established. The frame was analyzed under six different load cases to determine which load rates, magnitudes, and boundary conditions would produce a significant dynamic response in the form of chaotic behavior. This allowed for characterization of the *snapping* load described in previous research conducted by Adorno-Rodriguez and Just. The Mises stresses for each load case were calculated using FE software to determine if a relationship existed between chaotic behavior and Mises stresses. Finally, the frame-skin model was analyzed for

nonlinear dynamic behavior, and the stiffness of the beams was compared to the stiffness of beams in the icosahedron frame.

V. Conclusions and Recommendations

5.1 Chapter Overview

Chapters I through IV discussed the background, theory, methodology, and results of dynamic and quasi-static analyses of a hexakis icosahedron frame for use in a VLTAV. This chapter summarizes the key results and outcomes of the analyses and how they apply to developing a VLTAV with a hexakis icosahedron internal frame. Additionally, the significance of these analyses and recommendations for future research efforts will be discussed.

5.2 Research Conclusions

- Computational analysis for quasi-static and dynamic loading scenarios can be verified using experimental laser vibrometer testing on an additively manufactured frame assuming a free-free boundary condition. The FE model frequency results were compared to experimental data for a 0.1778 m diameter hexakis icosahedron frame and found to match within 1% for the first two frequency modes. The accuracy of the model in predicting these frequencies confirmed that a correct method of analysis had been established for dynamic modeling.
- Material properties provided by additive manufacturing material manufacturers may not be accurate for certain layups. In the case of the hexakis icosahedron, the listed modulus of elasticity was far too high due to the various angles used in the additive manufacturing technique. A *global* modulus of elasticity, much lower than that listed by the manufacturer, can be used to approximate the overall elastic modulus of the frame. The listed modulus of elasticity for Vero-Clear, the material used to create the hexakis icosahedron frame, was between

2.0 and 2.2 GPa. The approximated *global* modulus for the structure was found to be 1.1 GPa.

- From testing six different load cases with varying load magnitudes, load rates, and boundary conditions, nonlinear dynamic behavior was found to be primarily driven by the load rate. This chaos was characterized using four different techniques, and a load rate of 4.5 MPa/s at symmetric boundary condition 3 was sufficient to transition to a chaotic case. When unsymmetric boundary condition 2 was used, chaotic behavior could be seen at a load rate greater 4.0 MPa/s.
- The hexakis icosahedron exhibited less chaos, per the Lyapunov exponents, than the icosahedron frame for all load cases except case 1. Because the frame of the hexakis icosahedron has shorter beams throughout the structure, it provided more rigidity to withstand the loads. Though less chaotic, this frame was significantly more difficult to model due to its complex geometry. Thus, computational modeling was more limited due to excessive runtimes, even while using an AFRL supercomputer (10 hours or more for chaotic load cases).
- The Mises stresses in the frame are more dependent on the load magnitude applied than the boundary condition, load rate, or presence of chaotic behavior. At load magnitudes of 15% of sea level pressure or higher, the stresses in the frame exceeded the yield stress limits of the beryllium material used. Loads of 40% sea level pressure created Mises stresses between 1.7 and 6.7 GPa higher than the ultimate stress of the material.
- The nonlinear dynamic behavior seen in the frame-skin model is possibly due to a lack of inertia and stiffness in the beams of the hexakis icosahedron model. A comparison between beams in the icosahedron model and hexakis icosahedron

model revealed that the beams in the icosahedron had 6.5 times higher inertia and 2.5 times higher stiffness than the beams in the hexakis icosahedron. This made the hexakis icosahedron weakest at the nodes where only four beams intersected, causing *snapback* to occur.

5.3 Research Impact

This research studies the nonlinear dynamic response of a complex structure for use in a VLTAV. Thus, the research is significant both because of the structural analysis as well as its unique application. By comparing the frequency results of the dynamic FE model to the experimental results of an additively manufactured frame, the method of analysis was confirmed. This allows future designs and dynamic load cases for other additively manufactured frames to be tested using FE software. Also, this research demonstrated how to incorporate a *global* modulus of elasticity for an additively manufactured structure. As the additive manufacturing technique becomes more popular, this will prove invaluable in modeling other complex structures. The characterization of specific load cases and boundary conditions as chaotic in addition to evaluating the Mises stress in the individual beams helps establish an operating envelope for future hexakis icosahedron VLTAV designs. Finally, by analyzing the frame-skin model, the source of the nonlinear dynamic behavior was determined to be the lack of inertia and stiffness in the beams of the model, particularly near the four-beam vertices.

5.4 Recommendations for Future Research

- The hexakis icosahedron frame was additively manufactured using VeroClear material; however, a more homogeneous and isotropic material, such as a metal, would be better suited for laser vibrometer experimentation. The first two

frequency modes were accurately determined, but a greater hammer input is required to excite higher frequencies. To compare these to the results of the FE model, boundary conditions other than free-free should be tried as well.

- The *global* elastic modulus for the frame constructed with VeroClear was determined by studying Cranston's research with VeroBlue material and the knowledge that VeroClear was of a higher quality. Further testing on the material properties of various beams in the frame would yield more accurate values for the true elastic modulus of the frame.
- Up to this point, primarily the static and dynamic behavior of icosahedron and hexakis icosahedron frames have been studied. Only a few materials have been considered for use in either the frame or the membrane, and no method has been established to connect them in an actual design. More research is required into the finer details of the VLTAV design such as optimal motors, energy sources, propulsion methods, control systems, payloads, etc.

Appendix A. Matlab Code

```
1 function [XYZ,c,k,c_bar2] = hexakis_coordinates(r)
2 %UNTITLED Summary of this function goes here
3 % Detailed explanation goes here
4
5 %Disdyakis Triacontahedron
6 r1=.5; % desired radius in meters
7 r =r1/(3*sqrt(5*(5+2*sqrt(5)))/5);
8 C0 = 3 * (15 + sqrt(5)) / 44;
9 C1 = (5 - sqrt(5)) / 2;
10 C2 = 3 * (5 + 4 * sqrt(5)) / 22;
11 C3 = 3 * (5 + sqrt(5)) / 10;
12 C4 = sqrt(5);
13 C5 = (75 + 27 * sqrt(5)) / 44;
14 C6 = (15 + 9 * sqrt(5)) / 10;
15 C7 = (5 + sqrt(5)) / 2;
16 C8 = 3 * (5 + 4 * sqrt(5)) / 11;
17
18 V0 = [0.0, 0.0, C8];
19 V1 = [0.0, 0.0, -C8];
20 V2 = [C8, 0.0, 0.0];
21 V3 = [-C8, 0.0, 0.0];
22 V4 = [0.0, C8, 0.0];
23 V5 = [0.0, -C8, 0.0];
24 V6 = [0.0, C1, C7];
25 V7 = [0.0, C1, -C7];
26 V8 = [0.0, -C1, C7];
27 V9 = [0.0, -C1, -C7];
28 V10 = [C7, 0.0, C1];
29 V11 = [C7, 0.0, -C1];
30 V12 = [-C7, 0.0, C1];
31 V13 = [-C7, 0.0, -C1];
32 V14 = [C1, C7, 0.0];
33 V15 = [C1, -C7, 0.0];
34 V16 = [-C1, C7, 0.0];
35 V17 = [-C1, -C7, 0.0];
36 V18 = [C3, 0.0, C6];
37 V19 = [C3, 0.0, -C6];
38 V20 = [-C3, 0.0, C6];
39 V21 = [-C3, 0.0, -C6];
40 V22 = [C6, C3, 0.0];
41 V23 = [C6, -C3, 0.0];
42 V24 = [-C6, C3, 0.0];
43 V25 = [-C6, -C3, 0.0];
44 V26 = [0.0, C6, C3];
45 V27 = [0.0, C6, -C3];
46 V28 = [0.0, -C6, C3];
47 V29 = [0.0, -C6, -C3];
48 V30 = [C0, C2, C5];
```

```

49 V31 = [ C0,  C2, -C5];
50 V32 = [ C0, -C2,  C5];
51 V33 = [ C0, -C2, -C5];
52 V34 = [-C0,  C2,  C5];
53 V35 = [-C0,  C2, -C5];
54 V36 = [-C0, -C2,  C5];
55 V37 = [-C0, -C2, -C5];
56 V38 = [ C5,  C0,  C2];
57 V39 = [ C5,  C0, -C2];
58 V40 = [ C5, -C0,  C2];
59 V41 = [ C5, -C0, -C2];
60 V42 = [-C5,  C0,  C2];
61 V43 = [-C5,  C0, -C2];
62 V44 = [-C5, -C0,  C2];
63 V45 = [-C5, -C0, -C2];
64 V46 = [ C2,  C5,  C0];
65 V47 = [ C2,  C5, -C0];
66 V48 = [ C2, -C5,  C0];
67 V49 = [ C2, -C5, -C0];
68 V50 = [-C2,  C5,  C0];
69 V51 = [-C2,  C5, -C0];
70 V52 = [-C2, -C5,  C0];
71 V53 = [-C2, -C5, -C0];
72 V54 = [ C4,  C4,  C4];
73 V55 = [ C4,  C4, -C4];
74 V56 = [ C4, -C4,  C4];
75 V57 = [ C4, -C4, -C4];
76 V58 = [-C4,  C4,  C4];
77 V59 = [-C4,  C4, -C4];
78 V60 = [-C4, -C4,  C4];
79 V61 = [-C4, -C4, -C4];
80
81 for i = 0:61
82     j = i +1;
83     p(j,:) = r.*eval(['V' num2str(i)]);
84 end
85 XYZ = p;
86
87 f = [18,  0,  8 ...,
88      18,  8, 32 ...,
89      18, 32, 56 ...,
90      18, 56, 40 ...,
91      18, 40, 10 ...,
92      18, 10, 38 ...,
93      18, 38, 54 ...,
94      18, 54, 30 ...,
95      18, 30,  6 ...,
96      18,  6,  0 ...,
97      19,  1,  7 ...,
98      19,  7, 31 ...,
99      19, 31, 55 ...,
100     19, 55, 39 ...];

```

101	19,	39,	11	,
102	19,	11,	41	,
103	19,	41,	57	,
104	19,	57,	33	,
105	19,	33,	9	,
106	19,	9,	1	,
107	20,	0,	6	,
108	20,	6,	34	,
109	20,	34,	58	,
110	20,	58,	42	,
111	20,	42,	12	,
112	20,	12,	44	,
113	20,	44,	60	,
114	20,	60,	36	,
115	20,	36,	8	,
116	20,	8,	0	,
117	21,	1,	9	,
118	21,	9,	37	,
119	21,	37,	61	,
120	21,	61,	45	,
121	21,	45,	13	,
122	21,	13,	43	,
123	21,	43,	59	,
124	21,	59,	35	,
125	21,	35,	7	,
126	21,	7,	1	,
127	22,	2,	11	,
128	22,	11,	39	,
129	22,	39,	55	,
130	22,	55,	47	,
131	22,	47,	14	,
132	22,	14,	46	,
133	22,	46,	54	,
134	22,	54,	38	,
135	22,	38,	10	,
136	22,	10,	2	,
137	23,	2,	10	,
138	23,	10,	40	,
139	23,	40,	56	,
140	23,	56,	48	,
141	23,	48,	15	,
142	23,	15,	49	,
143	23,	49,	57	,
144	23,	57,	41	,
145	23,	41,	11	,
146	23,	11,	2	,
147	24,	3,	12	,
148	24,	12,	42	,
149	24,	42,	58	,
150	24,	58,	50	,
151	24,	50,	16	,
152	24,	16,	51	,

153	24,	51,	59	,
154	24,	59,	43	,
155	24,	43,	13	,
156	24,	13,	3	,
157	25,	3,	13	,
158	25,	13,	45	,
159	25,	45,	61	,
160	25,	61,	53	,
161	25,	53,	17	,
162	25,	17,	52	,
163	25,	52,	60	,
164	25,	60,	44	,
165	25,	44,	12	,
166	25,	12,	3	,
167	26,	4,	16	,
168	26,	16,	50	,
169	26,	50,	58	,
170	26,	58,	34	,
171	26,	34,	6	,
172	26,	6,	30	,
173	26,	30,	54	,
174	26,	54,	46	,
175	26,	46,	14	,
176	26,	14,	4	,
177	27,	4,	14	,
178	27,	14,	47	,
179	27,	47,	55	,
180	27,	55,	31	,
181	27,	31,	7	,
182	27,	7,	35	,
183	27,	35,	59	,
184	27,	59,	51	,
185	27,	51,	16	,
186	27,	16,	4	,
187	28,	5,	15	,
188	28,	15,	48	,
189	28,	48,	56	,
190	28,	56,	32	,
191	28,	32,	8	,
192	28,	8,	36	,
193	28,	36,	60	,
194	28,	60,	52	,
195	28,	52,	17	,
196	28,	17,	5	,
197	29,	5,	17	,
198	29,	17,	53	,
199	29,	53,	61	,
200	29,	61,	37	,
201	29,	37,	9	,
202	29,	9,	33	,
203	29,	33,	57	,
204	29,	57,	49	,

```

205     29, 49, 15 ...,
206     29, 15, 5];
207 f = f+1;
208 j = 1;
209 for i = 1:length(f)/3
210     c(j:j+2,:) = [f(j),f(j+1);f(j),f(j+2);f(j+1),f(j+2)];
211     j = j+3;
212 end
213 c = c';
214 c_bar2 = unique(c','rows');
215 %c_bar = unique(c','rows','stable');
216 j = 170;
217 for i = 1:24
218     remove(i) = j;
219     j = j+2;
220 end
221 c_bar2([1:12,14:15,17:18,20:21,23:24,26:27,29:30,32:33,35:36,38:39,41:42,44:45,47:48,ren
    = [];
222 j = 1;
223 for i = 1:120
224     k(i,:) = [f(j),f(j+1),f(j+2)];
225     j = j+3;
226 end
227 k = k';
228 end

```

```

1 % Jordan Snyder
2 % 13 July, 2016
3
4 % Thesis
5 %% Hexakis Icosahedron Beam Radius Calculation
6 clc; close all; clear all;
7
8 % Inputs: Assume a W/B total of 1 (WB_f + WB_s = 1)
9
10 r_icos=.3048/2 % radius in meters % .1778/2
11 rb_icos= .00238125; % icosahedron frame beam radius (m)
12 c=.05; % beam wall thickness to radius ratio (0<c<1)
13 tb_icos=c*rb_icos; % icosahedron frame beam wall thickness (m)
14 WB_f= .9; % W/B ratio for frame
15 WB_s= .1; % W/B ratio for skin
16 c= tb_icos/rb_icos; % c-ratio parameter
17 rho_a= 1.225; % density of air at STP conditions (kg/m^3)
18 rho_f= 1190; % density of frame (kg/m^3)
19 rho_s= 970; % density of skin (kg/m^3)
20
21 % Calculations
22
23 l_se=r_icos*(5/11)*sqrt(49-(65*sqrt(5))/3); % short edge length of ...
    triangle
24 l_me=r_icos*(1/11)*sqrt(81-21*sqrt(5)); % medium edge length of triangle

```

```

25 l_le=r_icos*2*sqrt((7/3)-sqrt(5)); % long edge length of triangle
26 p_ht=(l_se+l_me+l_le)/2;
27 A_HI=120*sqrt(p_ht*(p_ht-l_se)*(p_ht-l_me)*(p_ht-l_le)) % surface ...
    area of frame
28 V_HI=r_icos^3*100/(3*sqrt(53+(118/sqrt(5)))); % volume of frame
29
30
31 % Outputs
32
33 r_b=sqrt((WB_f*rho_a*V_HI)/((pi*60*1)*(l_se+l_me+l_le)*rho_f)) % ...
    beam radius output
34 t_s=(rho_a*WB_s*V_HI)/(A_HI*rho_s) % skin thickness output


1 % clear all; close all; clc;
2 % After reporting out xy (displacement/velocity) data from ABAQUS,
3 % need to save the data of interest as a single column and save as
4 % a text file to send into basegen
5
6 % First column is incrementation, second column is displacement ...
    (Umag), third
7 % column is velocity (V3).
8
9 fid = uigetfile('.txt');
10 % Enter which load case to run calculation for
11 load_case = str2num(fid(end-5));
12 percSL = 0.01*str2num(fid(1:2));
13 tstep=.001*str2num(fid(12));
14 rawdata = importdata(fid, ' ');
15 time_1 = rawdata(:,1);
16 disp_1 = rawdata(:,2);
17 vel_1 = rawdata(:,3);
18 % Cut off the transient response data
19 [m,~] = size(rawdata);
20 time_2 = rawdata(round(0.1*m)+2:end,1);
21 disp_2 = rawdata(round(0.1*m)+2:end,2);
22 vel_2 = rawdata(round(0.1*m)+2:end,3);
23
24 t_end = time_1(end);
25
26 figure
27 set(gcf, 'Position', get(0,'Screensize'));
28
29 % subplot(2,2,1);
30 % plotLoad(percSL,t_end);
31
32 subplot(2,2,1);
33 plotDisp(load_case,time_1,disp_1,percSL,tstep);
34
35
36 subplot(2,2,2);
37 plotPSD(time_2,disp_2);

```

```

38
39 subplot(2,2,3);
40 plotPhaseSpace(displ_1,vel_1);
41
42 subplot(2,2,4);
43 [b_s, b_o] = plotLyapunov(time_2,disp_2,load_case)

1 function [ ] = plotDisp(load_case,time,displacement,percSL,tstep)
2
3 if load_case == 3 || load_case == 6
4     plot(time,displacement,'LineWidth',1)
5     xlabel('Time (s)')
6     ylabel('Displacement (m)')
7     set(gca,'FontSize',12)
8     handxlabel1 = get(gca,'XLabel');
9     set(handxlabel1,'FontSize',14,'FontWeight','bold')
10    handylabel1 = get(gca,'ylabel');
11    set(handylabel1,'FontSize',14,'FontWeight','bold')
12    grid on;
13    axis([-0.2*time(end) time(end) roundn(min(displacement)*1.1,-4) ...
14          roundn(max(displacement)*2,-4)])
15    %hx = graph2d.constantline(time(end)*0.1, ...
16          'LineStyle',':','LineWidth',2, 'Color','k');
17    %changedependvar(hx,'x');
18    x2 = time(end)*-0.05;
19    y2 = max(displacement)/4;
20    str2 = [num2str(tstep),'s','\rightarrow'];
21    text(x2,y2,str2,'FontSize',12,'FontWeight','bold');
22    handaxes2 = axes('Position',[0.16 0.786 0.10 0.107]);
23    plotLoad(load_case,percSL,time(end),tstep)
24 else
25     plot(time,displacement,'LineWidth',1)
26     xlabel('Time (s)')
27     ylabel('Displacement (m)')
28     set(gca,'FontSize',12)
29     handxlabel1 = get(gca,'XLabel');
30     set(handxlabel1,'FontSize',14,'FontWeight','bold')
31     handylabel1 = get(gca,'ylabel');
32     set(handylabel1,'FontSize',14,'FontWeight','bold')
33     grid on;
34     axis([time(1) time(end) displacement(1) ...
35           roundn(max(displacement)*1.1,-4)])
36     %hx = graph2d.constantline(time(end)*0.1, ...
37           'LineStyle',':','LineWidth',2, 'Color','k');
38     %changedependvar(hx,'x');
39     x2 = time(end)*0.1;
40     y2 = max(displacement)/3;
41     str2 = ['\leftarrow',num2str(tstep),'s'];
42     text(x2,y2,str2,'FontSize',12,'FontWeight','bold');
43     handaxes2 = axes('Position',[0.275 0.63 0.17 0.18]);
44     plotLoad(load_case,percSL,time(end),tstep)

```

```

41 end
42
43 end

1 function [ ] = plotLoad(load_case,percSL,t_end,tstep)
2
3 if load_case == 3 || load_case == 6
4     t1 = 0:1e-5:tstep;
5     t2 = tstep:1e-5:t_end;
6     SL_pressure = 101325;
7     Slope = polyfit([0,tstep],[0,SL_pressure*percSL],1);
8     Pressure1 = t1.*Slope(1);
9     Pressure2 = SL_pressure*percSL*ones(size(t2));
10    appliedPressure = [Pressure1 Pressure2];
11    t = [t1 t2];
12    plot(t,appliedPressure,'LineWidth',2)
13    xlabel('Time (s)')
14    ylabel('Applied Pressure (Pa)')
15    set(gca,'FontSize',10)
16    handxlabel2 = get(gca,'XLabel');
17    set(handxlabel2,'FontSize',10,'FontWeight','bold');
18    handylabel2 = get(gca,'ylabel');
19    set(handylabel2,'FontSize',10,'FontWeight','bold');
20    grid on;
21    axis([t(1) t(end) appliedPressure(1) ...
22          roundn(max(appliedPressure)*1.2,3)])
23    x1 = t2(round(length(t2)/3));
24    y1 = Pressure2(1)+0.08*Pressure2(1);
25    str1 = [num2str(Pressure2(1)),' Pa'];
26    text(x1,y1,str1,'FontSize',10,'FontWeight','bold');
27    %hx = graph2d.constantline(t1(end), ...
28    'LineStyle',':','LineWidth',2, 'Color','k');
29    %changedependvar(hx,'x');
30    x2 = t1(end);
31    y2 = Pressure2(1)/3;
32    str2 = ['\leftarrow',num2str(tstep),'s'];
33    text(x2,y2,str2,'FontSize',10,'FontWeight','bold');
34 else
35     % t1 = 0:1e-5:0.1*t_end;
36     % t2 = 0.1*t_end:1e-5:t_end;
37     t1 = 0:1e-5:tstep;
38     t2 = tstep:1e-5:t_end;
39     SL_pressure = 101325;
40     Slope = polyfit([0,tstep],[0,SL_pressure*percSL],1);
41     Pressure1 = t1.*Slope(1);
42     Pressure2 = SL_pressure*percSL*ones(size(t2));
43     appliedPressure = [Pressure1 Pressure2];
44     t = [t1 t2];
45     plot(t,appliedPressure,'LineWidth',2)
46     xlabel('Time (s)')
47     ylabel('Applied Pressure (Pa)')

```

```

46     set(gca, 'FontSize', 10)
47     handxlabel2 = get(gca, 'XLabel');
48     set(handxlabel2, 'FontSize', 11, 'FontWeight', 'bold');
49     handylabel2 = get(gca, 'ylabel');
50     set(handylabel2, 'FontSize', 11, 'FontWeight', 'bold');
51     grid on;
52     axis([t(1) t(end) appliedPressure(1) ...
           roundn(max(appliedPressure)*1.2,3)])
53     x1 = t2(round(length(t2)/3));
54     y1 = Pressure2(1)+0.075*Pressure2(1);
55     str1 = [num2str(Pressure2(1)), ' Pa'];
56     text(x1,y1,str1, 'FontSize', 11, 'FontWeight', 'bold');
57     %hx = graph2d.constantline(t1(end), ...
           'LineStyle', ':', 'LineWidth', 2, 'Color', 'k');
58     %changedependvar(hx, 'x');
59     x2 = t1(end);
60     y2 = Pressure2(1)/3;
61     str2 = ['\leftarrow', num2str(tstep), 's'];
62     text(x2,y2,str2, 'FontSize', 11, 'FontWeight', 'bold');
63 end
64
65 end

```

```

1 function [ ] = plotPhaseSpace(displacement, velocity)
2
3 plot(displacement, velocity, 'LineWidth', 1)
4 xlabel('Displacement (m)')
5 ylabel('Velocity (m/s)')
6 set(gca, 'FontSize', 12)
7 handxlabel1 = get(gca, 'XLabel');
8 set(handxlabel1, 'FontSize', 14, 'FontWeight', 'bold')
9 handylabel1 = get(gca, 'ylabel');
10 set(handylabel1, 'FontSize', 14, 'FontWeight', 'bold')
11 grid on;
12
13
14 end

```

```

1 function [ ] = plotPSD(time, displacement)
2
3 dt = time(2) - time(1);
4 Fs = 1/dt;
5 x = displacement;
6
7 N = length(x);
8 xdft = fft(x);
9 xdft = xdft(1:N/2+1);
10 psdx = (1/(Fs*N)) * abs(xdft).^2;
11 psdx(2:end-1) = 2*psdx(2:end-1);
12 freq = 0:Fs/length(x):Fs/2;

```

```

13
14 try
15     plot(freq,10*log10(psd), 'LineWidth',1)
16 catch
17     xdft = xdft(1:round(N/2));
18     psd = (1/(Fs*N)) * abs(xdft).^2;
19     psd(2:end-1) = 2*psd(2:end-1);
20     freq = 0:Fs/length(x):Fs/2;
21     plot(freq,10*log10(psd), 'LineWidth',1)
22 end
23
24 set(gca, 'FontSize',12)
25 grid on
26
27 xlabel('Frequency (Hz)')
28 ylabel('Power/Frequency (dB/Hz)')
29 axis([0 21000 -inf inf])
30 handxlabel1 = get(gca, 'XLabel');
31 set(handxlabel1, 'FontSize', 14, 'FontWeight', 'bold')
32 handylabel1 = get(gca, 'YLabel');
33 set(handylabel1, 'FontSize', 14, 'FontWeight', 'bold')
34 % figureHandle = gcf;
35 % ...
36     set(findall(figureHandle,'type','text'),'fontSize',14,'fontWeight','bold')
37 end

1 function [LyaExp_b_sec,LyaExp_b_orb] = ...
2     plotLyapunov(time,displacement,load_case)
3
4 time = time(2:end);
5 disp = displacement(2:end);
6
7 % input the dominant frequency as calculated using PSD
8 domFreqs = [3500 3000 4000 3500 2900 2200 3100 3100];
9 domFreq = domFreqs(load_case); %Changes for load case
10 save Disp_Data.txt disp -ASCII
11 fname = 'Disp_Data.txt';
12
13 datcnt = length(disp);
14 taus = [8 8 80 15 15 150 15 8];
15 tau = taus(load_case); %Changes for load case
16 ndim = 3;
17 ires = 10;
18 maxbox = 6000;
19
20 db = basgen(fname, tau, ndim, ires, datcnt, maxbox);
21
22 dt = time(2)-time(1);
23 evolves = [8 8 80 10 10 80 8 8];
24 evolve = evolves(load_case); %Changes for load case

```

```

24 dismin = 0.00000001;
25 dismaxs = [0.0002 0.0002 0.02 0.0002 0.0002 0.02 0.002 0.002];
26 dismax = dismaxs(load_case); %Changes for load case
27 thmax = 30;
28
29 [out, SUM] = fet(db, dt, evolve, dismin, dismax, thmax);
30
31 % makeplot(db, out, evolve, 'NorthWest')
32
33 % [exp_bps,exp_bpo] = lyapunov_expEst(domFreq)
34
35 meanPeriod = 1/domFreq;
36 bitsPerSec_Data = importdata('fetout.txt');
37 bitsPerSec_Exp = bitsPerSec_Data(:,8);
38
39 i = 1;
40 ind = 1;
41 while i <= length(bitsPerSec_Exp)
42     if isnan(bitsPerSec_Exp(i)) ~= 1
43         BPS(ind) = bitsPerSec_Exp(i);
44         ind = ind + 1;
45     end
46     i = i + 1;
47 end
48
49 BPS = BPS';
50 bitsPerOrbit = BPS.*meanPeriod;
51 plot(bitsPerOrbit,'linewidth',1)
52 hold on; plot([1 length(BPS)],[0 0],'-k')
53 grid on;
54 set(gca,'FontSize',12)
55 axis([0 length(BPS) -inf inf])
56
57 xlabel('Time \rightarrow')
58 ylabel('Lyapunov Exponent (bits/orbit)')
59 handxlabel1 = get(gca, 'XLabel');
60 set(handxlabel1, 'FontSize', 14, 'FontWeight', 'bold')
61 handylabel1 = get(gca, 'ylabel');
62 set(handylabel1, 'FontSize', 14, 'FontWeight', 'bold')
63 % figureHandle = gcf;
64 % ...
65     set(findall(figureHandle,'type','text'),'fontSize',14,'fontWeight','bold')
66
67 LyaExp_b_sec = BPS(end);
68 LyaExp_b_orb = bitsPerOrbit(end);
69
70 end
71
72 % Jordan Snyder
73 % Thesis plots
74

```

```

4  clc; close all; clear all;
5  %% Stress vs. Time
6  fid = uigetfile('.txt');
7  % Identify load case parameters from .txt filename
8  percSL = 0.01*str2num(fid(1:2));
9  tstep=.001*str2num(fid(12));
10 rawdata = importdata(fid, ' ');
11 time2=rawdata(end,1);
12
13 % Identify input variables
14 time = rawdata(:,1);
15 Su= .8E9; % ultimate stress for Beryllium
16 Sy= .434E9; % yield stress for Beryllium
17 SMises_SP1 = rawdata(:,2);
18
19
20
21 SMises_SP3 = rawdata(:,3);
22 SMises_SP5 = rawdata(:,4);
23 SMises_SP7 = rawdata(:,5);
24 Umag = rawdata(:,6);
25 Vmag = rawdata(:,7);
26 V3 = rawdata(:,8);
27 SMises_avg=(SMises_SP1+SMises_SP3+SMises_SP5+SMises_SP7)/4;
28
29 % Create subplots
30
31 if max(rawdata(:,2))>Sy
32 subplot(2,2,1);
33 plot(time,SMises_SP1/(1E6))
34 hold on
35 plot(time,ones(size(time))*Sy/(1E6))
36 axis([0 time2 0 max(rawdata(:,2))/(1E6)+50])
37 xlabel('Time (s)')
38 ylabel('S Mises SP:1 (MPa)')
39 else
40 subplot(2,2,1);
41 plot(time,SMises_SP1/(1E6))
42 hold on
43 plot(time,ones(size(time))*Sy/(1E6))
44 axis([0 time2 0 Sy/(1E6)+50])
45 xlabel('Time (s)')
46 ylabel('S Mises SP:1 (MPa)')
47 end
48
49 if max(rawdata(:,3))>Sy
50 subplot(2,2,2);
51 plot(time,SMises_SP3/(1E6))
52 hold on
53 plot(time,ones(size(time))*Sy/(1E6))
54 axis([0 time2 0 max(rawdata(:,3))/(1E6)+50])
55 xlabel('Time (s)')

```

```

56 ylabel('S Mises SP:3 (MPa)')
57 else
58 subplot(2,2,2);
59 plot(time,SMises_SP3/(1E6))
60 hold on
61 plot(time,ones(size(time))*Sy/(1E6))
62 axis([0 time2 0 Sy/(1E6)+50])
63 xlabel('Time (s)')
64 ylabel('S Mises SP:3 (MPa)')
65 end
66
67 if max(rawdata(:,4))>Sy
68 subplot(2,2,3);
69 plot(time,SMises_SP5/(1E6))
70 hold on
71 plot(time,ones(size(time))*Sy/(1E6))
72 axis([0 time2 0 max(rawdata(:,4))/(1E6)+50])
73 xlabel('Time (s)')
74 ylabel('S Mises SP:5 (MPa)')
75 else
76 subplot(2,2,3);
77 plot(time,SMises_SP5/(1E6))
78 hold on
79 plot(time,ones(size(time))*Sy/(1E6))
80 axis([0 time2 0 Sy/(1E6)+50])
81 xlabel('Time (s)')
82 ylabel('S Mises SP:5 (MPa)')
83 end
84
85 if max(rawdata(:,5))>Sy
86 subplot(2,2,4);
87 plot(time,SMises_SP7/(1E6))
88 hold on
89 plot(time,ones(size(time))*Sy/(1E6))
90 axis([0 time2 0 max(rawdata(:,5))/(1E6)+50])
91 xlabel('Time (s)')
92 ylabel('S Mises SP:7 (MPa)')
93 else
94 subplot(2,2,4);
95 plot(time,SMises_SP7/(1E6))
96 hold on
97 plot(time,ones(size(time))*Sy/(1E6))
98 axis([0 time2 0 Sy/(1E6)+50])
99 xlabel('Time (s)')
100 ylabel('S Mises SP:7 (MPa)')
101 end
102
103 if max(SMises_avg)>Su
104 figure
105 plot(time,SMises_avg/(1E6))
106 hold on
107 plot(time,ones(size(time))*Sy/(1E6))

```

```

108 hold on
109 plot(time,ones(size(time))*Su/(1E6))
110 axis([0 time2 0 max(SMises_avg)/(1E6)+50])
111 xlabel('Time (s)')
112 ylabel('Stress (MPa)')
113 legend('Avg Mises Stress','Yield Stress','Ultimate Stress')
114 else
115 figure
116 plot(time,SMises_avg/(1E6))
117 hold on
118 plot(time,ones(size(time))*Sy/(1E6))
119 hold on
120 plot(time,ones(size(time))*Su/(1E6))
121 axis([0 time2 0 Su/(1E6)+50])
122 xlabel('Time (s)')
123 ylabel('Stress (MPa)')
124 legend('Avg Mises Stress','Yield Stress','Ultimate Stress')
125 end

```

Appendix B. Abaqus Frequency Outputs

1		
2	Mode	Frequency (Hz)
3		
4	1.	0.
5	2.	76.8918E-06
6	3.	131.28E-06
7	4.	163.029E-06
8	5.	192.106E-06
9	6.	265.659E-06
10	7.	413.927
11	8.	413.939
12	9.	414.032
13	10.	414.035
14	11.	414.048
15	12.	417.849
16	13.	417.858
17	14.	417.869
18	15.	418.145
19	16.	425.674
20	17.	425.674
21	18.	425.675
22	19.	445.072
23	20.	448.983
24	21.	449.036
25	22.	449.047
26	23.	449.055
27	24.	449.226
28	25.	449.235
29	26.	449.236
30	27.	449.237
31	28.	449.241
32	29.	449.326
33	30.	449.329
34	31.	449.344
35	32.	466.216
36	33.	466.216
37	34.	466.216
38	35.	486.029
39	36.	486.031
40	37.	486.158
41	38.	486.159
42	39.	486.16
43	40.	497.613
44	41.	497.615
45	42.	497.616
46	43.	520.884
47	44.	520.888
48	45.	520.892

49	46.	521.085
50	47.	521.087
51	48.	521.904
52	49.	521.907
53	50.	521.909
54	51.	551.967
55	52.	551.967
56	53.	551.968
57	54.	552.231
58	55.	553.556
59	56.	553.56
60	57.	553.636
61	58.	553.638
62	59.	553.638
63	60.	558.011
64	61.	558.011
65	62.	558.011
66	63.	558.11
67	64.	558.862
68	65.	558.862
69	66.	558.862
70	67.	562.542
71	68.	562.546
72	69.	562.548
73	70.	563.007
74	71.	563.009
75	72.	565.989
76	73.	565.989
77	74.	565.99
78	75.	566.739
79	76.	566.741
80	77.	566.742
81	78.	566.743
82	79.	566.746
83	80.	572.35
84	81.	572.706
85	82.	572.71
86	83.	572.711
87	84.	585.222
88	85.	585.326
89	86.	585.331
90	87.	585.332
91	88.	588.784
92	89.	588.786
93	90.	588.786
94	91.	613.282
95	92.	613.282
96	93.	613.282
97	94.	618.643
98	95.	623.42
99	96.	623.422
100	97.	623.424

101	98.	632.048
102	99.	632.573
103	100.	632.574

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