



NRL/MR/5674--18-9792

# A Noninvasive Optical Technique for Characterizing the Normal Acoustic Modes of a Water Tank

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May 7, 2018

# REPORT DOCUMENTATION PAGE

*Form Approved*  
*OMB No. 0704-0188*

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<b>1. REPORT DATE (DD-MM-YYYY)</b> 07-05-2018			<b>2. REPORT TYPE</b> Memorandum Report		<b>3. DATES COVERED (From - To)</b> August 2017 - March 2018	
<b>4. TITLE AND SUBTITLE</b>  A Noninvasive Optical Technique For Characterizing The Normal Acoustic Modes Of A Water Tank					<b>5a. CONTRACT NUMBER</b>	
					<b>5b. GRANT NUMBER</b>	
					<b>5c. PROGRAM ELEMENT NUMBER</b>	
<b>6. AUTHOR(S)</b>  Sean Krupa, <sup>1</sup> Matthew Pittman, <sup>1</sup> Brandon Redding and Allen Davis					<b>5d. PROJECT NUMBER</b>	
					<b>5e. TASK NUMBER</b>	
					<b>5f. WORK UNIT NUMBER</b> 56-6A94-08	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  U.S. Naval Research Laboratory 4555 Overlook Avenue, SW Washington, DC 20375-5320					<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  NRL/MR/5674--18-9792	
<b>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  U.S. Naval Research Laboratory 4555 Overlook Avenue, SW Washington, DC 20375-5320					<b>10. SPONSOR / MONITOR'S ACRONYM(S)</b>  NRL	
<b>11. SPONSOR / MONITOR'S REPORT NUMBER(S)</b>						
<b>12. DISTRIBUTION / AVAILABILITY STATEMENT</b>  DISTRIBUTION STATEMENT A: Approved for public release. Distribution is unlimited.						
<b>13. SUPPLEMENTARY NOTES</b>  <sup>1</sup> Keyw Corporation, Hanover, MD 21076						
<b>14. ABSTRACT</b>  In order to characterize and develop underwater acoustic sensors, researchers are often forced to choose between using portable calibrators (restricted size), dedicated acoustic testing facilities (costly, restricted access), or using water tanks with unknown acoustic properties. Additionally, analytic methods often fall short in predicting the acoustic properties of tanks with unknown geometries, material properties, and boundary conditions. To address these limitations, we propose and demonstrate an optical technique to efficiently and noninvasively calibrate the normal modes of an arbitrary water tank with partially reverberant boundary conditions. The technique relies on measuring the integrated optical phase change introduced by the acoustic induced change in the refractive index of water. In this work, we use this approach to find the fundamental mode of a large oval water tank at two water depths and validate the optical measurements using conventional hydrophones. By enabling the rapid and non-invasive characterization of the normal acoustic modes of an arbitrary water tank, this technique enables a wide range of custom designed water tanks to be used for underwater acoustic experiments.						
<b>15. SUBJECT TERMS</b>						
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>  SAR	<b>18. NUMBER OF PAGES</b>  15	<b>19a. NAME OF RESPONSIBLE PERSON</b> Sean Krupa	
<b>a. REPORT</b> Unclassified Unlimited	<b>b. ABSTRACT</b> Unclassified Unlimited	<b>c. THIS PAGE</b> Unclassified Unlimited			<b>19b. TELEPHONE NUMBER (include area code)</b> 202-767-2193	

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# A Noninvasive Optical Technique For Characterizing The Normal Acoustic Modes Of A Water Tank

## I. Introduction

Indoor water tanks are a necessary piece of laboratory equipment for developing and characterizing acoustic devices such as hydrophones<sup>1,2</sup> and underwater sources<sup>3</sup>. Typically, the calibration of commercial hydrophones is performed with portable calibrators such as the G40<sup>4</sup> or at specialized facilities which are designed and optimized for this purpose<sup>5,6</sup>. However, practical limitations such as the size of the G40 calibrator and cost or availability of specialized facilities often force researchers with unique requirements to pursue testing in self-procured water tanks.

Home-built water tanks, often repurposed from their original intended application, are convenient but exhibit unknown acoustic properties. Moreover, different applications may require the generation or detection of acoustic fields with distinct properties. For example, the characterization of line arrays of hydrophones requires the generation of a uniform acoustic field in a water tank large enough to hold the array. While non-anechoic tanks support resonant modes<sup>7,8</sup> which could be used for applications such as line array characterization, predicting the frequency of the resonant modes in a tank with complex geometry and unknown boundary conditions is non-trivial. The challenge in accurately modeling the behavior of an arbitrary water tank leads most researchers to resort to experimental testing. Nonetheless, experimental characterization of a water tank with unknown acoustic properties can be tedious as it often requires scanning a hydrophone over the entire tank in step sizes which are a fraction of the wavelength of interest<sup>7,9,10</sup>. This process can be expedited by employing many hydrophones in parallel<sup>7</sup> at the cost of increasing experimental complexity and potentially changing the acoustic properties of the tank itself.

In this paper, we present an optical technique to quickly and non-invasively characterize the resonant modes of an arbitrary water tank with unknown boundary conditions. As an example, we show that the proposed optical technique can rapidly find the fundamental mode of an oval shaped water tank with unknown boundary conditions and validate this technique using conventional hydrophones. Finally, we discuss how this technique could be extended to identify higher frequency modes in arbitrary water tanks.

The paper is structured as follows: Section II described the analytic model used to guide the experiments, Section III describes the optical technique including details of the background, experimental setup, and results; Section IV presents the acoustic testing performed to validate the optical characterization technique; Section V summarizes the work.

## II. Analytic Predictions

To guide the experiments performed in this paper, an analytic model was used to estimate the location of the tank's normal modes. Although the actual geometry and boundary conditions will differ, we approximated the tank as a perfect rectangular cavity with rigid walls. The cavity, with dimensions:  $L_x$ ,  $L_y$ , and  $L_z$ , will have normal modes given by Eq. 3<sup>11</sup>:

$$f_N = \sqrt{f_{N_x}^2 + f_{N_y}^2 + f_{N_z}^2}. \quad (3)$$

In this model, the individual frequencies for each dimension are given by Eq. 4<sup>11</sup>:

$$f_{N_i} = \frac{N_i v}{2L_i} \quad (4)$$

where  $N_i$  is an integer corresponding to the  $N^{th}$  order mode in the  $i$ -axis and  $v$  is the speed of sound. For all measurements presented in this work, the temperature dependent speed of sound in water was found to be 1480 m/s based on Medwin<sup>12</sup>.

From this analytic model, we were able to estimate the approximate frequency of the normal modes for varying water depths. These estimates were then used to determine the range of frequencies we scanned to find the actual resonant modes.

## III. Optical Technique For Finding Normal Acoustic Modes

### *Background*

The fundamental resonant mode of a water tank can be particularly useful for applications such as characterizing line arrays of hydrophones or other extended sensors, since the amplitude and phase of the mode are relatively uniform through the tank. In addition, exciting a resonant mode is relatively insensitive to the position of the acoustic source and can result in an acoustic field which remains stable over a long period of time (e.g. hours). The relatively uniform pressure and phase of the fundamental mode also enables us to design an optical interferometer which will experience the most phase change when this mode is excited. This approach relies on measuring the acoustic induced change in the refractive index of water, which can be expressed as:

$$\Delta n(x, t) = (n_{H_2O} - 1) \frac{P(x, t)}{K_o} \quad (1)$$

where  $n_{H_2O}$  is the refractive index of water,  $P$  is the differential pressure, and  $K_o$  is the bulk elasticity of water (2.2 GPa)<sup>13</sup>. For an acoustic field at a frequency of  $f_{ac}$ , the pressure will vary as

a function of position and time as  $P(x, t) = A(x) \sin(2\pi f_{ac}t + \Phi(x))$ , where  $A$  and  $\Phi$  are the position dependent amplitude and phase of the acoustic field. This acoustic induced change in the refractive index will introduce a time-varying change in the phase of an optical beam passing through the water. For a beam travelling along the  $x$ -axis of a water tank, the integrated optical phase change,  $\Delta\varphi$ , can then be expressed as:

$$\Delta\varphi(t) = \int 2\Delta n(x, t) \frac{2\pi}{\lambda_{opt}} dx \quad (2)$$

where  $\lambda_{opt}$  is the optical wavelength, and the factor of 2 accounts for passing twice through the water tank. Measuring this integrated optical phase change provides a means to identify the resonant modes of a water tank. That is, if we introduce an acoustic source, we expect that the acoustic field excited within the tank will be strongest when the source frequency aligns with a resonant mode of the tank. In addition, the integrated optical phase is most sensitive to acoustic fields in which the acoustic phase is uniform along the optical path, whereas the optical phase change introduced by acoustic fields in which  $\Phi(x)$  varies will average toward zero. Thus, to find the fundamental mode, we can align the optical beam to integrate along the length of the cavity where the phase of the acoustic field is expected to be relatively uniform. If we are interested in finding a higher order mode, we could align the beam along the expected position of one of the nodes of the desired acoustic mode. In addition, this optical technique could be used to efficiently estimate the quality factor of the resonant acoustic mode.

### *Experimental Setup*

To experimentally demonstrate this approach, we characterized the resonant modes of an oval-shaped fiber glass water tank (Water Management Technologies OVAL-2010). The OVAL-2010 has a ~1600-gallon volume and measured interior dimensions of  $x = 5.84$  m,  $y = 1.52$  m and  $z \leq 0.92$  m. Photographs of the water tank are shown in Fig. 1. The unknown reflectivity of the fiberglass walls combined with the irregular geometry make the acoustic properties difficult to predict analytically.

An interferometer was constructed to measure the integrated phase along the length of the tank (the  $x$ -axis shown in Fig. 1). A 20 mW continuous wave (CW) laser (Coherent Prometheus) operating at a wavelength of 532 nm was divided into a reference beam and an interrogation beam using a polarizing beam splitter (PBS). The interrogation beam was directed along the length of the water tank, passing through two 12" windows mounted at opposite ends of the water tank before being reflected by a mirror. A quarter-wave plate inserted between the mirror and the far window served to rotate the polarization, rejecting undesired specular reflections from the windows. Meanwhile, the reference arm consisted of a fiber-optic delay line which was used to match the optical path of the interrogation beam and introduce a pair of fiber-coupled acousto-optic modulators (AA Optoelectronics MT80-G40). The reference arm was then coupled back to free space and recombined with the reflected interrogation beam on a polarizing beam splitter before being focused on a high speed Si photodetector (Thorlabs PDA10A). The AOMs introduced frequency shifts of +80.125 MHz and -80 MHz, resulting in a net frequency shift on the reference arm of 125 kHz with respect to the interrogation arm. This intermediate frequency was used to extract the time varying phase of the interrogation beam.

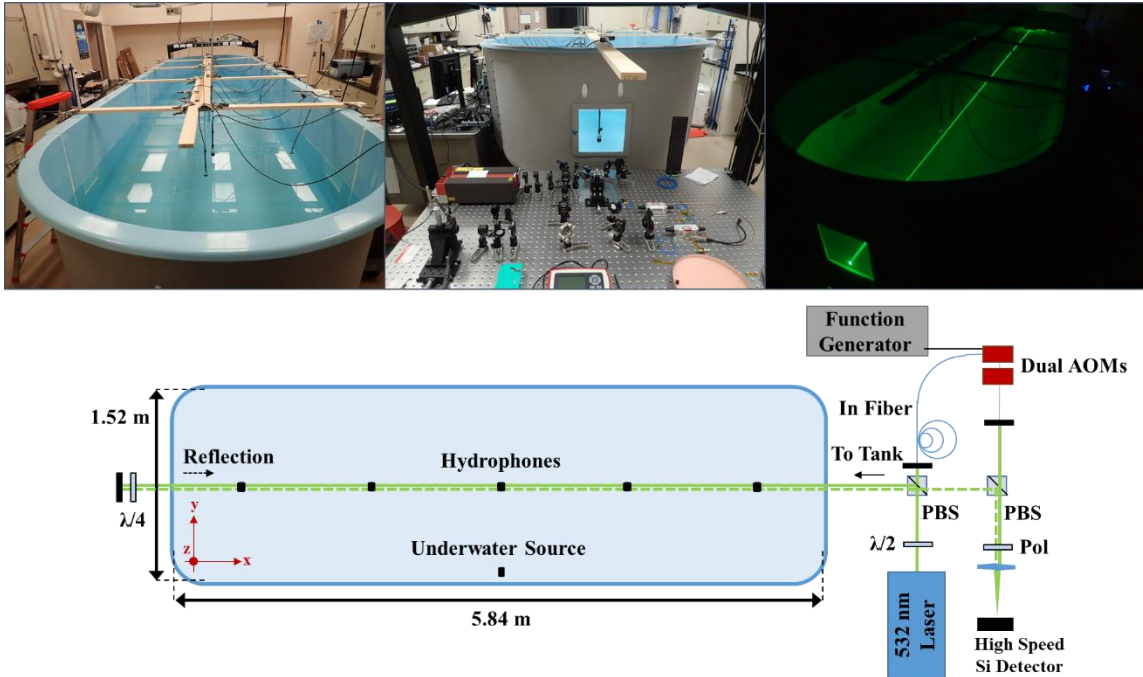


FIG. 1. OVAL-2010 tank and hydrophone support structure shown from various angles with a schematic representation of the optical setup.

In order to find the fundamental mode of the water tank, the acoustic-induced phase change in the optical interrogation beam was recorded while an underwater acoustic source introduced narrow-band tones at frequencies ranging from 1032 Hz to 1600 Hz in 8 Hz step sizes. The acoustic source (Aquarian Audio Products H1a) was positioned at the center of the tank along the  $x$  and  $z$  axes and 10 cm from the wall along the  $y$  axis as shown in Fig. 1. At each frequency, the acoustic source was allowed to stabilize for 0.5 seconds before recording 1 s of data using the optical interrogator. The optical interference signal was digitized at 1 MHz using a National Instruments DAQ board (USB-6356) and demodulated computationally to provide the time varying phase of the interrogation beam. The power spectral density was then calculated, providing the RMS phase change at the frequency of the acoustic source. A typical power spectral density plot is shown in Fig. 2 where the acoustic tone at 1080 Hz is clearly visible. In this case, the tone introduced an RMS phase change of 12 mrad. Note that at each frequency, the RMS phase change was measured with a signal to noise ratio  $>20$  dB.

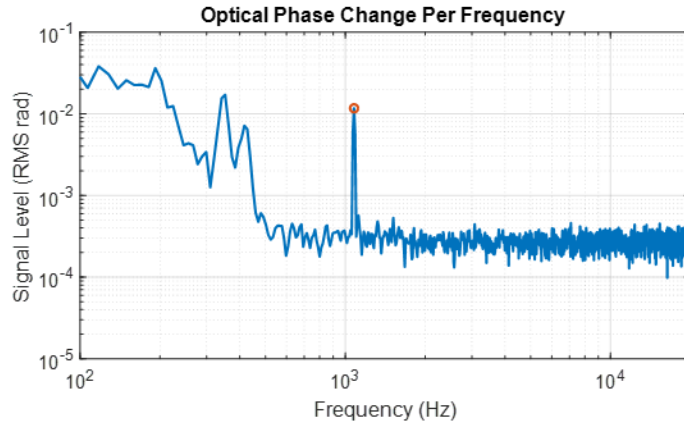


FIG. 2. Power spectral density from a 1080 Hz tone. Lower frequency (<500 Hz) noise appears from ambient sources in the laboratory.

To ensure the fidelity of the optical measurement, the phase change resulting from the movement of the tank windows themselves was measured separately and found to be 20 dB smaller than the phase change observed when the beam passed through the tank. In addition, repeated measurements over long time scales (hours) showed the optical measurement to be very stable, with a standard deviation in the measured phase of 0.84%. Lastly, because the tank used in this work has one axis ( $x$ ) which is much longer than the other two, the lowest frequency higher order modes will exhibit multiple nodes along this dimension, motivating the alignment of the laser to probe this dimension.

### *Results*

The acoustic induced optical phase change was recorded as a function of frequency at two different tank depths: 0.75 meters and 0.55 meters, as shown in Figure 3. The scans revealed a peak optical phase change occurring at 1064 Hz for the 0.75-meter depth and at 1376 Hz for the 0.55-meter depth. Based on our discussion above, we expect that these frequencies correspond to the fundamental modes of the water tank at these two depths. Using the optical frequency scan, we can also estimate the cavity  $Q$ , which we found to be 22.1 and 22.5 for the 0.55 m depth and 0.75 m depths, respectively. Note that the predicted curves shown on the plot will be discussed below.

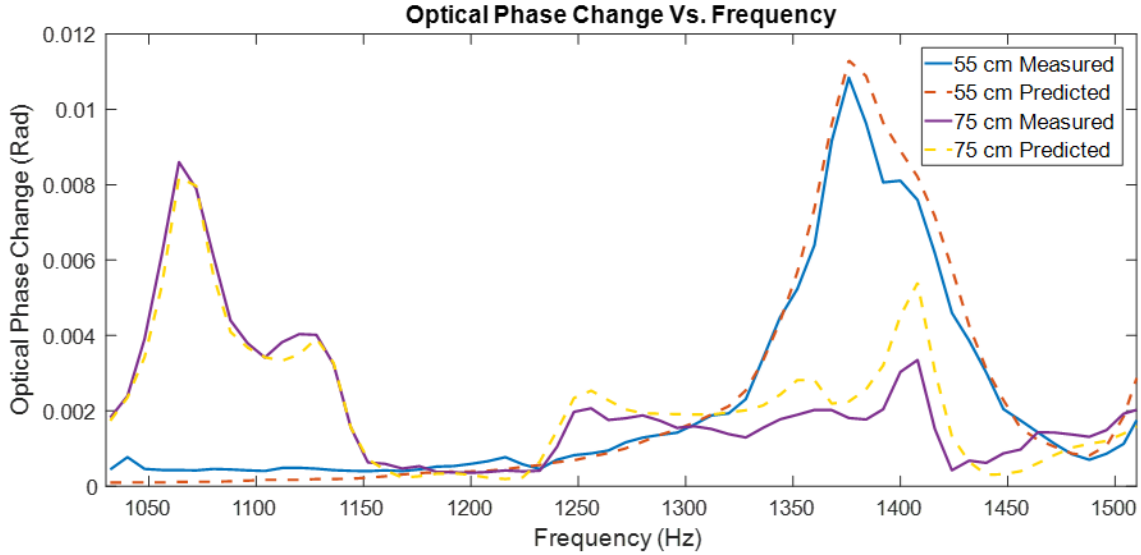


FIG. 3. Scan of optical phase change versus frequency done at 75 cm and 55 cm water depths. Each depth has a peak (1064 Hz and 1376 Hz) corresponding to the tank’s fundamental acoustic mode. The measured optical data is compared to expected results generated from acoustic hydrophone measurements.

#### IV. Acoustic Verification of Normal Modes

##### *Experimental Setup*

To confirm that the maximum observed optical phase change did in fact correspond to the frequency of the fundamental acoustic mode, we performed a thorough acoustic characterization of the water tank using five Aquarian Audio Products H1a hydrophones. The recorded RMS pressure and phase from these hydrophones was recorded and digitized via a National Instruments DAQ board (USB-6356). Using the wooden support structure shown in Figure 1, the five hydrophones were lowered to the approximate depth of the laser beam and then scanned along the  $x$ -axis giving a high resolution measurement of the pressure and acoustic phase inside the tank. Each H1a hydrophone used in this work was calibrated using a scientific grade hydrophone (Brüel and Kjær C174). The C174 hydrophone was calibrated at the USRD Bugg Spring<sup>1</sup> facility prior to this work. Similar to the process for collecting optical data, the underwater source was allowed to stabilize for 0.5 seconds at each frequency before recording 1 s of data from the hydrophones using a National Instruments DAQ board (USB-6356). To reduce the scan time, the five hydrophones were moved together such that each hydrophone sampled  $\sim 1/5$  of the tank and data from the five hydrophones was stitched together to provide a continuous measurement of the pressure field along the length of the water tank. Even so, scanning the five hydrophones took  $\sim 2$  hours whereas the optical measurement found the fundamental mode in  $\sim 5$  minutes.

## Results

To confirm the location of the tank's fundamental mode and find the location of the tank's higher order modes, the five hydrophones were stepped along the  $x$ -direction in 10 cm intervals while the underwater source was driven over a range of frequencies. Figure 4 shows the pressure and phase as a function of position along the tank (0.75 m depth) for frequencies ranging from 1032 Hz to 1304 Hz in 8 Hz steps.

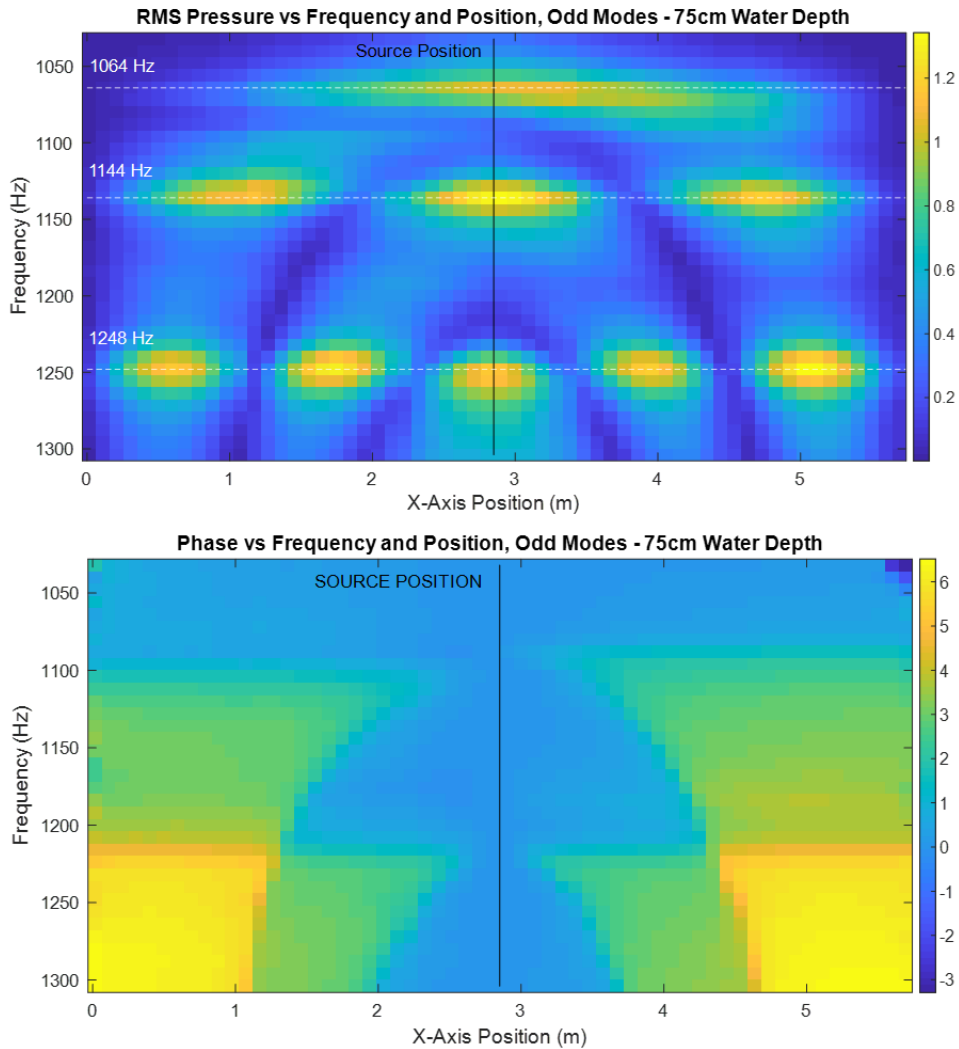


FIG. 4. Acoustic pressure and phase measurements from a five-hydrophone array stepped along the tank's  $x$ -axis in 10 cm increments at a range of frequencies (1032 Hz to 1304 Hz). The solid black line represents the physical location of the single underwater acoustic source.

The first three odd order modes are clearly visible in Figure 4 with locations of 1064 Hz, 1136 Hz, and 1248 Hz respectively. This measurement confirmed the location of the fundamental mode

predicted by the optical technique at the 0.75 m water depth. Using the pressure and phase values obtained from the high resolution hydrophone scan, we also calculated the expected optical phase change at each frequency using Eqns. 1 and 2. The expected optical phase change calculated from this hydrophone data is plotted along with the results of the optical experiment in Figure 3 for both water depths, showing excellent agreement.

Figure 5 shows the normalized pressure and phase of the 1<sup>st</sup> and 3<sup>rd</sup> order modes taken from the high resolution hydrophone scan. The 1<sup>st</sup> order mode shows a single node with relatively uniform phase while the 3<sup>rd</sup> order mode shows three amplitude maxima with  $\pi$  phase shifts between the neighboring nodes. Note that the phase of the fundamental mode at the 1064 Hz has a standard deviation of 0.19 rad. Moreover, most of this variation occurs near the boundaries of the tank where the amplitude approaches zero. If we consider only the center 2 m of the tank, the standard deviation of the phase is reduced to 0.07 rad, sufficiently uniform to characterize hydrophone line arrays.

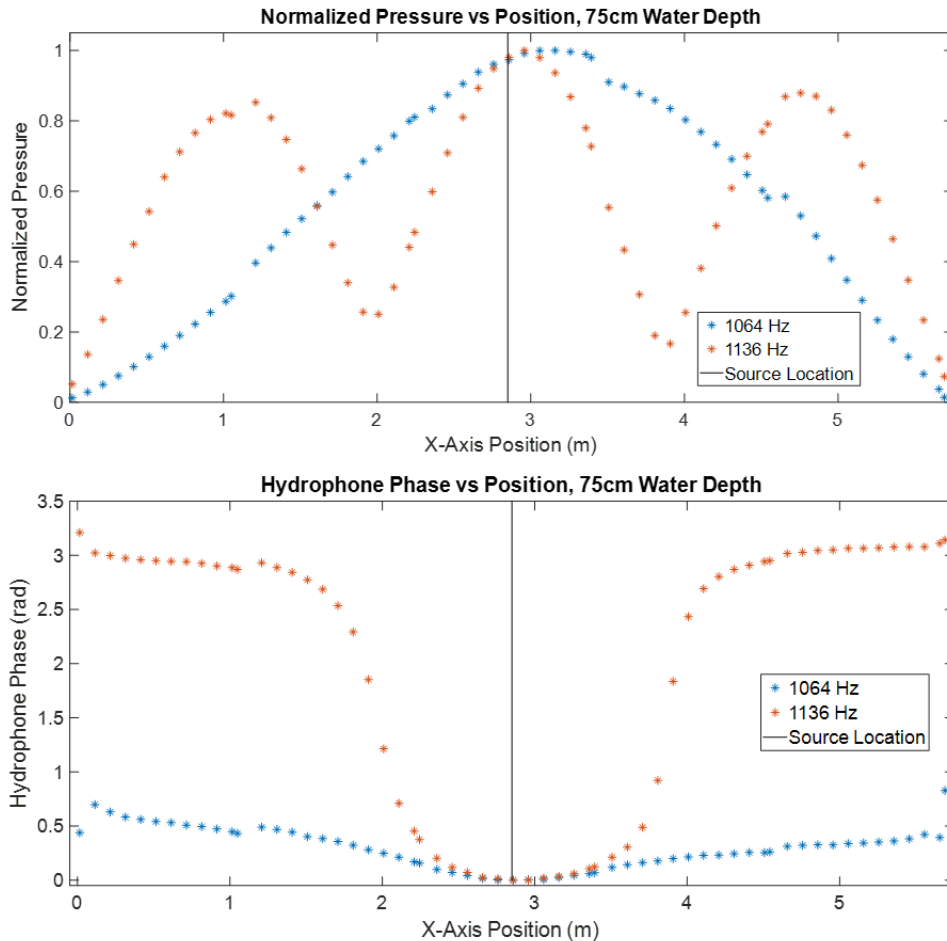


FIG. 5. Acoustic pressure (normalized) and phase along the tank's x-axis at 1064 Hz and 1136 Hz. The solid black line represents the physical location of the single underwater acoustic source.

The small discontinuities seen in both the pressure and phase plots appear when data from one hydrophone is stitched to another. The calibration process minimizes these discontinuities but does not eliminate them completely. The frequency and position scan was repeated for the 0.55 m water depth confirming the optical prediction that the fundamental mode for that depth was at 1376 Hz.

In addition to scanning along the  $x$ -axis, we measured the spatial profile of the fundamental mode along the  $y$  and  $z$  axes for a 0.75 m water depth. In this case, the frequency of the acoustic source was fixed at 1064 Hz, while a single hydrophone scanned along the  $y$  and  $z$  axes. The results of this scan are displayed in Figure 6. The pressure field along the  $y$  and  $z$  axes again exhibit a sinusoidal profile consistent with the fundamental mode.

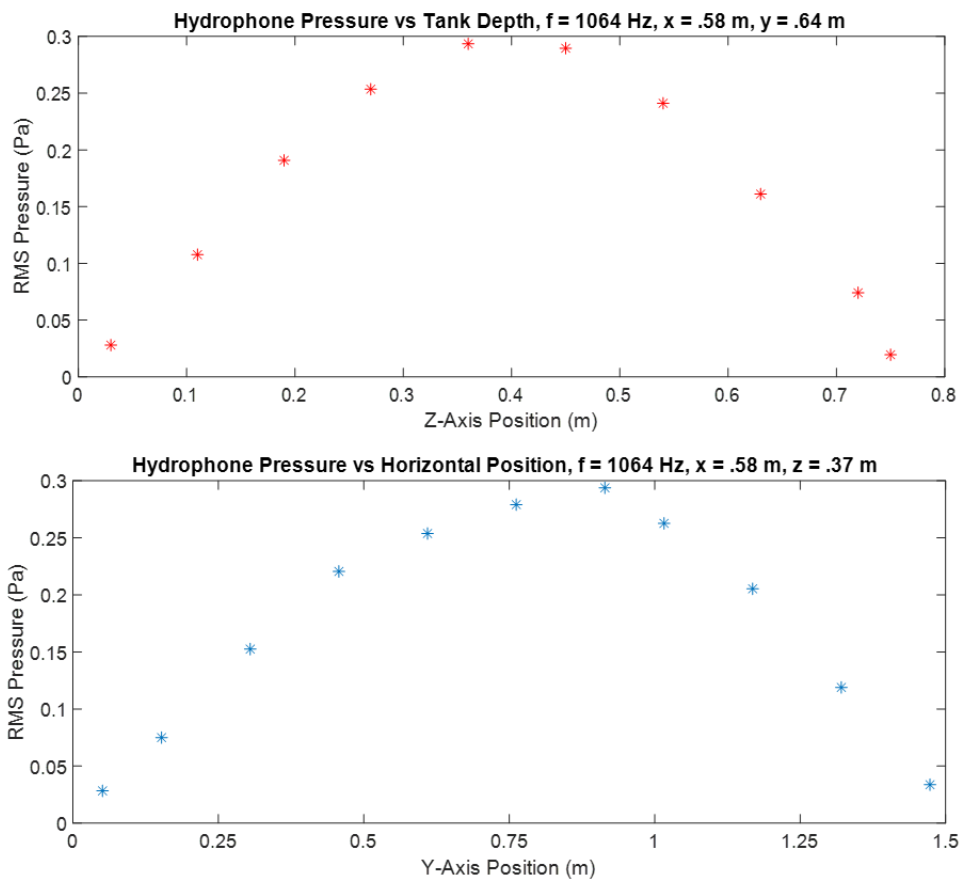


FIG. 6. Acoustic pressure profiles along the tank’s  $z$ -axis and  $y$ -axis at 1064 Hz, the fundamental mode for a 75 cm water depth.

In the measurements presented above, we only observed the odd modes of the water tank. This can be explained by the position of the acoustic source, which was centered along the  $x$ -axis and thus did not efficiently couple energy into the even modes of the tank which exhibit amplitude minima at the center of the tank. To confirm this explanation, we sought to excite the even modes of the water tank by positioning two acoustic sources at  $x = 0.25L_x = 1.45$  m and  $x = 0.75L_x = 4.35$  m

(corresponding to the expected peaks of the nodes of the second order mode). We then repeated the hydrophone scan over the frequency range of 1050 Hz to 1400 Hz. Also note that the signals from the two acoustic sources were  $180^\circ$  out of phase. As shown in figure 7, this approach successfully excited the 2<sup>nd</sup> and 6<sup>th</sup> order modes. Note that we still did not excite the 4<sup>th</sup> order mode since the updated source locations correspond to minima in the amplitude of the 4<sup>th</sup> mode.

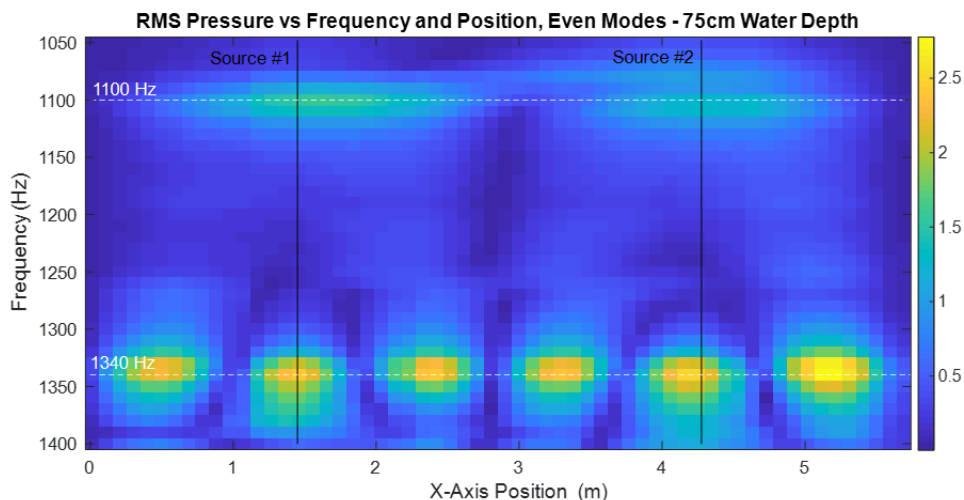


FIG. 7. Acoustic pressure measurements from a five-hydrophone array stepped along the tank’s x-axis in 10 cm increments. The solid black lines represent the physical location of two underwater sources which are out of phase by  $180^\circ$ .

While the optical interrogation beam was aligned to select for the fundamental mode in this work, we could use the same technique to search for a higher order mode by changing the orientation of the laser beam and location of the underwater source(s). For example, to optically search for the second order mode, we could send the “reference” beam along the y-axis of the tank at the expected position of the first node (at  $x = 0.25L_x$ ) and the “interrogation” beam along the y-axis at the position of the second node (at  $x = 0.75L_x$ ). The relative phase between these beams would be largest for the second order mode since the two nodes are out of phase, whereas the fundamental mode would not influence the measured optical phase since the acoustic induced phase change in both arms would be the same.

Finally, Figure 8 shows a summary of the measured normal mode locations compared with the predictions of the analytic model outlined in Section II. While the model was sufficiently accurate to guide our experimental search, the experimental measurements were required to identify the actual resonant frequencies. These discrepancies illustrate the limitations of simple analytic models when applied to tanks with unknown boundary conditions and irregular geometries.

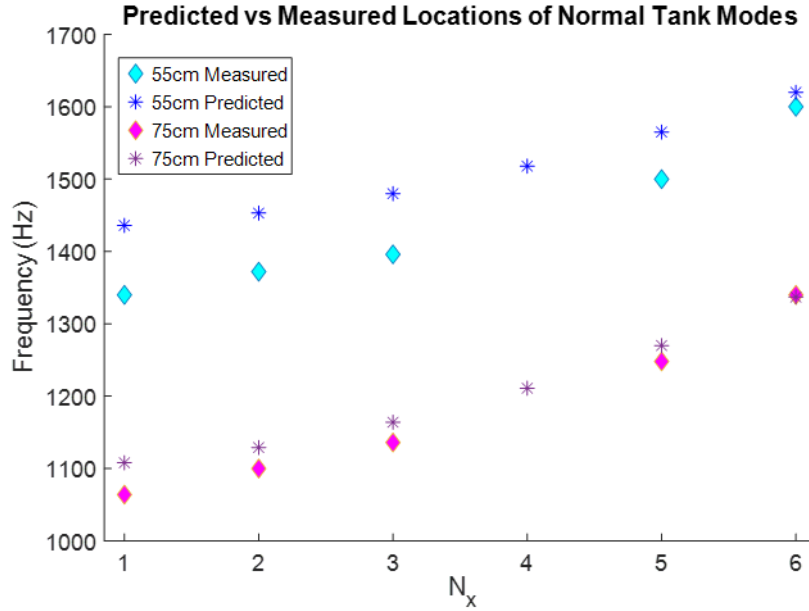


FIG. 8. Predicted and measured locations of the tank’s normal modes along the x-axis. The two underwater source configurations used for acoustic characterization will not efficiently excite the 4<sup>th</sup> order mode.

## V. Summary

This paper presents an optical technique to quickly and noninvasively identify the normal acoustic modes of an arbitrary water tank. By directing one arm of an optical interferometer through a water tank, pressure induced changes in the index of refraction result in a measurable optical phase change. By aligning the optical interrogation beam to the long axis of the tank, the optical phase change is maximized for the fundamental acoustic mode. Using this optical technique, the fundamental mode of a large oval water tank was identified and characterized at two different water depths. A thorough characterization of the tank’s acoustic properties was performed using an array of hydrophones, verifying the location of the fundamental mode identified with the optical technique. Higher order modes could be identified by changing the laser orientation to align with the nodes of a desired acoustic mode. This technique could enable the use of a wide range of custom built water tanks to be used for underwater acoustic experiments.

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