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# An Examination of the JWL Equation of State

by Steven B Segletes

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14. ABSTRACT The JWL equation of state, ubiquitous in the field of explosive–metal interactions, is examined from the perspective of thermodynamic foundation and compatibility. Because there is no founding document for the JWL equation of state, there exist different interpretations of the equation, which is the source of substantial confusion. In starting with a general entropy-based equation, we show what assumptions are necessary to simplify the equation of state to the JWL equation forms commonly found in the literature.					
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## 1. Introduction

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The JWL equation of state (EOS) (named for Jones, Wilkins, and Lee) is ubiquitous in the community that deals with explosive–metal interactions. However, it did not derive from a single source document and therefore different evolutions of JWL, derivative from its amorphous origin, are generally still labeled as “JWL”, so that there is no authoritative standard of comparison, but rather a multitude of similar JWL forms<sup>1–7</sup> (see the Appendix). The situation is further complicated by the fact that the JWL “model” has a large, empirical component to it, so the number of discrepancies between alternate implementations, while real, cannot be deemed as theoretically incorrect, merely as “different”.

This short report draws upon the commonalities present across the spectrum of JWL specifications and endeavors to study the JWL EOS in terms of the underlying thermodynamic relations that must apply to any EOS form. In so doing, we can distinguish the EOS from a mere JWL isentrope. Further, we may ascertain whether the EOS specification is a subset of Grüneisen theory or not.

## 2. The JWL Equation of State Form

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The most commonly found variety of the JWL EOS can be expressed as a family of isentropes of the form

$$p(S, V) = Ae^{-R_1V} + Be^{-R_2V} + C^*(S)V^{-(\omega+1)} \quad , \quad (1)$$

where  $p$  is the pressure,  $S$  the entropy per unit initial volume ( $s/v_0$ ), and  $V$  is the volume relative to the undetonated state ( $v/v_0$ ). The parameters  $A$ ,  $B$ ,  $R_1$ , and  $R_2$  are constant fitting parameters associated with the model, while the parameter  $\omega$  is an assumed-constant material parameter, which, we later see, can characterize the Grüneisen function. Finally, the function  $C^*(S)$  is dependent only upon the entropy  $S$ , such that individual isentropes may be obtained by holding the value of  $C^*$  constant at particularly selected values. (That  $C^*$  must be a nontrivial function of  $S$  is ensured by the fact that, in removing  $S$  as an independent variable,  $p(V)$  is not an EOS, but merely a pressure–volume relationship.)

The JWL EOS form is more commonly presented as  $p(E, V)$ , where  $E$  is the current internal energy per unit initial volume ( $e/v_0$ ). To (eventually) show its derivation from Eq. 1, we rely on the fact that pressure and energy along an isentrope are

related by the relation

$$p = -(\partial E / \partial V)_S \quad , \quad (2)$$

where the  $(\partial f / \partial x)_y$  nomenclature implies a derivative taken upon  $f(x, y)$  with respect to  $x$ , while holding  $y$  constant. Substituting Eq. 2 into Eq. 1 to eliminate  $p$  provides the relationship

$$-(\partial E / \partial V)_S = Ae^{-R_1 V} + Be^{-R_2 V} + C^*(S) V^{-(\omega+1)} \quad . \quad (3)$$

This equation may be integrated with respect to  $V$ , holding the entropy constant:

$$E(S, V) = \frac{A}{R_1} e^{-R_1 V} + \frac{B}{R_2} e^{-R_2 V} + \frac{C^*(S)}{\omega} V^{-\omega} + E^*(S) \quad , \quad (4)$$

where  $E^*$  is the integration function, which in this case is independent of  $V$ , the integration variable. In Eq. 4, the internal energy  $E$  is expressed in terms of its natural\* thermodynamic variables,  $S$  and  $V$ .

Now, let us consider a particular reference isentrope of interest—say, one that goes through a particular thermodynamic state point associated with the entropy level of  $S_{\text{ref}}$ . Denoting the constants  $C^*(S_{\text{ref}})$  as  $C$  and, likewise,  $E^*(S_{\text{ref}})$  as  $E_0$ , the volume-dependent pressure and energy along this reference isentrope (substituting into Eqs. 1 and 4) are given, respectively, by

$$p_{\text{ref}}(V) = Ae^{-R_1 V} + Be^{-R_2 V} + CV^{-(\omega+1)} \quad (5)$$

and

$$E_{\text{ref}}(V) = \frac{A}{R_1} e^{-R_1 V} + \frac{B}{R_2} e^{-R_2 V} + \frac{C}{\omega} V^{-\omega} + E_0 \quad . \quad (6)$$

The form of Eq. 5 is often seen in the literature as describing the JWL isentrope, where  $S_{\text{ref}}$  is taken at the value associated with the Chapman–Jouguet (CJ) point. Note, though, that the actual value of entropy  $S_{\text{ref}}$  need not be known to exercise Eq. 5; rather, only a fit to parameter  $C$  is needed.

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\*Entropy and volume are the “natural” variables by which to express internal energy because the internal energy may be expressed as an exact differential in terms of them, namely,  $dE = TdS - PdV$ .

### 3. From Isentrope to $p = p(E, V)$ EOS

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An important thermodynamic parameter associated with the EOS, essential for use in high-pressure simulations, is the Grüneisen function,  $\Gamma$ . In its most general manifestation, it is a state function, so that  $\Gamma = \Gamma(S, V)$ . However, as employed in simulations for explosive–metal interactions, it is almost universally treated in a constrained manner, as  $\Gamma = \Gamma(V)$ . This restricted case conforms to what is known as the “Grüneisen assumption” and materials obeying it are called “Grüneisen materials”. In the general case,  $\Gamma$  is thermodynamically defined as

$$\Gamma(S, V) \equiv V \frac{(\partial p / \partial S)_V}{(\partial E / \partial S)_V} . \quad (7)$$

However, when the Grüneisen assumption applies, it is often more convenient to eliminate entropy altogether and express Eq. 7 as

$$\Gamma(V) \equiv V(\partial p / \partial E)_V . \quad (8)$$

For Grüneisen formulations in which a dependence on  $S$  is eliminated from  $\Gamma$ , it follows that the EOS may be expressed in the incomplete\* form,  $p = p(E, V)$ . This incomplete form may also be preferred because experimental shock data are obtained as  $p$  and  $E$  in terms of  $V$ .

For cases in which  $\Gamma$  can be successfully characterized as  $\Gamma = \Gamma(V)$ , the  $p = p(E, V)$  EOS of the Grüneisen variety may always be reformulated as

$$p - p_{\text{ref}}(V) = \frac{\Gamma(V)}{V} (E - E_{\text{ref}}(V)) , \quad (9)$$

where the reference function is a curve (*i.e.*, path) in thermodynamic space, so chosen because material behavior along it is known, either analytically or experimentally. Reference functions typically employed include an isentrope, isotherm, or Hugoniot.

For the JWL case, one can combine Eqs. 1 and 4 to eliminate  $C^*$ , to obtain so-

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\*An incomplete EOS arises when it is not expressed in terms of the natural thermodynamic variables. It is called incomplete because there are some thermodynamic variables that are unknowable from the incomplete form. For example, when  $p = p(E, V)$  is the sole source of EOS information, temperature is not knowable without the additional specification of, for example, a specific heat formulation.

something that has a passing resemblance to a Grüneisen EOS, in which  $p = p(E, V)$ :

$$p = A \left( 1 - \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left( 1 - \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \frac{\omega}{V} (E - E^*) \quad . \quad (10)$$

Note, however, that while this may look like an EOS in the Grüneisen form, that is not the case, since the term  $E^*$  is a function of entropy,  $S$ . Rather, to cast the JWL equation in Grüneisen form, we must pursue Eq. 9, where the JWL reference path is the CJ isentrope, given by Eqs. 5 and 6.

Applying this reference path to Eq. 9 gives a generalized Grüneisen form of the JWL EOS, in which  $p = p(E, V)$ , where  $\Gamma = \Gamma(V)$  is assumed:

$$p = A \left( 1 - \frac{\Gamma}{R_1 V} \right) e^{-R_1 V} + B \left( 1 - \frac{\Gamma}{R_2 V} \right) e^{-R_2 V} + C \left( 1 - \frac{\Gamma}{\omega} \right) V^{-(\omega+1)} + \frac{\Gamma}{V} (E - E_0). \quad (11)$$

This, however, is not the JWL form one finds in the literature. We must look into Eq. 7 more closely to see what conditions are necessary to ensure that  $\Gamma = \Gamma(V)$ .

### 3.1 A Constant Grüneisen Form of JWL

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To better understand the functional nature of  $\Gamma$  that might arise naturally from the JWL form, substitute Eqs. 1 and 4 into Eq. 7 to obtain

$$\Gamma(S, V) = \frac{\frac{dC^*}{dS} V^{-\omega}}{\frac{dC^*}{dS} \frac{V^{-\omega}}{\omega} + \frac{dE^*}{dS}} \quad \equiv \quad \frac{\omega}{1 + \omega V^\omega \frac{dE^*}{dC^*}} \quad . \quad (12)$$

From Eq. 12, we immediately see that, for the JWL case, the nature of  $\Gamma$  will strongly depend upon the functional relationships describing  $E^*(S)$  and  $C^*(S)$ .

If we assume an additional thermodynamic constraint, namely, that the function  $E^*(S)$  is not a function of entropy, but, in fact everywhere constant, such that  $E^*(S) \equiv E_0$ , then it follows that  $dE^*/dS = 0$ . When substituted into Eq. 12, the Grüneisen function reduces to a constant,

$$\Gamma = \omega \quad . \quad (13)$$

Therefore, under this constant  $E^*$  assumption, the JWL EOS will satisfy the Grüneisen EOS assumption (*i.e.*, that  $(\partial p / \partial E)_V$ , and thus  $\Gamma$ , is a function of volume  $V$  alone) with the Grüneisen function reducing to a constant parameter  $\omega$ .

Under this constraining simplification, both Eqs. 10 and 11 reduce to

$$p(E, V) = A \left( 1 - \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left( 1 - \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \frac{\omega}{V} (E - E_0) \quad . \quad (14)$$

This is the form most often seen in the literature for the JWL EOS (though often seen with the  $E_0$  term tacitly taken as zero).

### **3.2 A Volume-Dependent Grüneisen Form of the JWL EOS**

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To obtain Eq. 14, one was required to assume that the integration function  $E^*(S)$  was equivalent to a mere constant,  $E_0$ . Consider the more general case when that assumption is not made.

If  $E^*$  were, instead, to be found solely linear in  $C^*$ , such that

$$E^*(S) = E_0 + F(C^*(S) - C) \quad , \quad (15)$$

then  $dE^*/dC^* = F$ , a constant. In that case, Eq. 12 may be reevaluated as

$$\Gamma \equiv V(\partial p / \partial E)_V = \frac{\omega}{1 + \omega F V \omega} \quad . \quad (16)$$

Note that this form, too, satisfies the Grüneisen constraint that  $\Gamma = \Gamma(V)$ . However, in this case,  $\Gamma$  is no longer a constant, but actually varies with compression.

We are not proposing this particular form as preferable in any way, but developed it to show that the JWL form of Eq. 1 does not preclude more sophisticated Grüneisen functional forms. In such cases, however, Eq. 11 (and not Eq. 14) must be employed as the governing  $p(E, V)$  EOS form.

## 4. Conclusions

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In this report, the connection between the JWL isentrope and its associated EOS was investigated. Starting with a thermodynamically unrestricted  $p = p(S, V)$  family of isentropes, it was shown how the incomplete  $p = p(E, V)$  EOS form could be derived for the JWL form. The associated Grüneisen function was derived, first in general terms and then according to the assumptions necessary to reformulate the EOS into the form commonly associated with JWL in the literature. It was shown how more general JWL EOS formulations are possible, permitting  $\Gamma = \Gamma(V)$  forms in which  $\Gamma$  varies with compression.

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## **Appendix. JWL Confusion**

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The description of the JWL equation of state (EOS) in the literature is a confusing array of contradictions. The fact that there is apparently no master document published by Jones, Wilkins, and Lee setting down the equation and its theoretical underpinnings has no doubt set the stage for many reinterpretations. This state of affairs is further encouraged by the fact that JWL can and has been expressed both as a full EOS as well as a single expansion isentrope.

### A.1 Dobratz

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The Lawrence Livermore National Laboratory (LLNL) Handbook<sup>1</sup>, in presenting the isentropic JWL form equal to Eq. 5, refers to it as “the” isentrope, without indicating which one it is (presumably the one passing through the Chapman–Jouguet [CJ] point). When presenting the Grüneisen EOS form of JWL equation as

$$P = A \left( 1 - \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left( 1 - \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \frac{\omega E}{V} , \quad (\text{A-1})$$

the only energy term used is  $E$ , which it identifies as the “detonation energy per unit volume”, which one would presumably interpret as a constant. Yet, on the following page of the handbook, the tabulated data provide a column labeled  $E_0$  (not  $E$ ). The current analysis, given by Eq. 14, would indicate that the term should be the state-dependent internal energy modified by a constant,  $(E - E_0)$ . Further, for consistency, the term  $E$  should be defined as energy per unit *initial* volume and not (as it seems to be) per unit (*current*) volume.

### A.2 Lu and Kennedy

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Lu and Kennedy<sup>2</sup> provide a form, which appears to be a hybrid between the isentrope and EOS forms:

$$P = A e^{-R_1 V} + B e^{-R_2 V} + \frac{\omega C_v T}{V} . \quad (\text{A-2})$$

While the term  $C_v T \equiv E$  expresses the internal energy in thermodynamic terms

<sup>1</sup>Dobratz BM, Crawford PC. LLNL explosives handbook: properties of chemical explosives and explosive simulants. Livermore (CA): Lawrence Livermore National Laboratory; 1985 Jan. Report No.: UCRL-52997 (ADA272275).

<sup>2</sup>Lu JP, Kennedy DL. Modelling of PBX-115 using kinetic CHEETAH and the DYNA codes. Edinburgh (Australia): Defence Science and Technology Organization; 2003 Sep. Report No.: DSTO-TR-1496 (ADA421656).

under the assumption of a constant specific heat, it is not clear where such a form derives, since the exponential terms are of a form associated with the isentrope, whereas the temperature term is clearly not associated with the isentrope.

### A.3 Price and Ghee

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When discussing ideal detonation, Price and Ghee<sup>3</sup> employ the form of Eq. 5, citing it as an expansion isentrope. Additionally, they define detonation energy in several ways, noting the definitions are not quite identical:

$$\Delta H_d^0 = -\left\{\sum n \cdot \Delta H_f^0(\text{products}) - \sum n \cdot \Delta H_f^0(\text{reactants})\right\} \quad (\text{A-3})$$

and

$$E_{\text{det}} = W = \int_{V_{CJ}}^V P dV - E_c, \text{ where } E_c = \frac{1}{2}(P_{CJ})(1 - V_{CJ}) \quad . \quad (\text{A-4})$$

According to Price and Ghee, “the detonation energy from the AUTODYN models is calculated with

$$E_{\text{det}} = E_0 - E_{INT} \quad , \quad (\text{A-5})$$

where  $E_0$  is the initial explosive internal energy of the explosive and  $E_{INT}$  is the internal energy calculated by the code.”

### A.4 Larcher

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Larcher<sup>4</sup> provides a Grüneisen form of JWL identical in form to that of Dobratz, which he cites as

$$P_{EOS} = A \left(1 - \frac{\omega}{R_1 V}\right) e^{-R_1 V} + B \left(1 - \frac{\omega}{R_2 V}\right) e^{-R_2 V} + \omega \frac{E}{V} \quad . \quad (\text{A-6})$$

However, unlike Dobratz, Larcher identifies the term  $E$  as the current internal energy per initial volume. He notes that at large expansion, the form reduces to that of an ideal gas:

$$P_{EOS} = \omega \frac{E}{V} \quad . \quad (\text{A-7})$$

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<sup>3</sup>Price MA, Ghee AH. Modeling for detonation and energy release from peroxides and non-ideal improvised explosives. Central European Journal of Energetic Materials. 2009;6(3–4):239–254.

<sup>4</sup>Larcher M. Simulation of the effects of an air blast wave. Brussels (Belgium): European Commission Joint Research Centre; The Institute for the Protection and Security of the Citizen; 2007. Report No.: PUBSY JRC41337.

Then, after clearly implying that  $E$  is a state-dependent internal energy (how else could one refer to it in Eq. A-7?), he provides a table in which the “current internal energy” is specified with particular values. Once again, this would seem to introduce confusion on the distinction between the  $E$  and the  $E_0$  terms referred to in Eq. 14.

## A.5 LS-DYNA

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The LS-DYNA Keyword manual<sup>5</sup> provides several EOS forms that employ a JWL-like structure. The EOS Form 2 (JWL) reflects the equation given by Dobratz, with a  $\omega E/V$  energy term. Again, like Dobratz, input of energy is specified with a term  $E_0$ , which is not otherwise identified.

The EOS Form 7 (Ignition and Growth of Reaction in High Explosive [HE]) describes a JWL form more complex (and different) than other JWL use cases cited. No more will be said of it.

EOS Form 10 (Propellant) employs a form much like Lu and Kennedy<sup>2</sup>:

$$p = Ae^{-R_1V} + Be^{-R_2V} + \frac{\omega C_v T}{V-d}, \quad (\text{A-8})$$

which defies understanding in terms of the derivations of Section 2. The exponential terms are reminiscent of a JWL isentrope, whereas the  $C_v T$  term is an internal energy term that one would associate with an EOS.

## A.6 Wikipedia

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Wikipedia<sup>6</sup> has a page concerning various EOS forms. They present a section on JWL:

$$p = A \cdot \left(1 - \frac{\omega}{R_1 \cdot V}\right) \cdot \exp(-R_1 \cdot V) + B \cdot \left(1 - \frac{\omega}{R_2 \cdot V}\right) \cdot \exp(-R_2 \cdot V) + \frac{\omega \cdot e_0}{V}, \quad (\text{A-9})$$

where the undefined term given as  $e_0$  is explained only through tabulations of its singular value for different explosives. Like other instances of JWL description, it leaves an impression that the energy term is a constant, rather than a state term.

<sup>5</sup>LS-DYNA keyword user manual, volume 1 (version 971). Livermore (CA): Livermore Software Technology Corporation; 2007 May [accessed 2018 Jun 4]. [http://lstdc.com/pdf/lstdyna\\_971\\_manual\\_k.pdf](http://lstdc.com/pdf/lstdyna_971_manual_k.pdf).

<sup>6</sup>Equation of state. Wikipedia: The Free Encyclopedia; 2015 Mar 5 [accessed 2015 Mar 12]. [http://en.wikipedia.org/wiki/Equation\\_of\\_state#Jones-Wilkins-Lee\\_equation\\_of\\_state\\_for\\_explosives\\_.28JWL-equation.29](http://en.wikipedia.org/wiki/Equation_of_state#Jones-Wilkins-Lee_equation_of_state_for_explosives_.28JWL-equation.29).

## A.7 Baudin and Serradeill

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Baudin and Serradeill<sup>7</sup> purport to review the foundations of the JWL EOS. The earliest reference they are able to locate is a 1968 paper by Lee, Hornig, and Kury (UCRL-50422). The bulk of their explanation is in trying to justify the empirical selection of the exponential decay terms in the JWL. They end up with the isentropic form of Eq. 5 and note that the values of the parameters are chosen to correspond to the isentrope that traverses the CJ point.

They then put forth a version of Eq. 9, and like the present effort, indicate that the reference isentrope should be substituted into it. However, they never derive a form for the energy along the isentrope, along the lines of Eq. 4, that would validate the conversion from an isentropic description to a Mie–Grüneisen EOS.

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<sup>7</sup>Baudin G, Serradeill R. Review of Jones-Wilkins-Lee equation of state. In: *New Models and Hydrocodes for Shock Wave Processes in Condensed Matter*; Vol. 10; 2010 May 24–28; Paris, France. EDP Sciences; 2010. p. 00021·p.1–00021·p.4.

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B BREECH  
RDRL WML C  
K MCNESBY  
B ROOS  
RDRL WML E  
P WEINACHT  
RDRL WML F  
J CONDON  
RDRL WML H  
T EHLERS  
E KENNEDY  
L MAGNESS  
C MEYER  
B SORENSEN  
R SUMMERS  
RDRL WMM B  
B LOVE  
G GAZONAS

RDRL WMM C  
R JENSEN  
RDRL WMM D  
S WALSH  
RDRL WMM E  
P PATEL  
RDRL WMP  
D LYON  
J HOGGE  
T VONG  
RDRL WMP A  
P BERNING  
J CAZAMIAS  
M COPPINGER  
J FLENIKEN  
M J GRAHAM  
W C UHLIG  
C WOLFE  
RDRL WMP B  
C HOPPEL  
S SATAPATHY  
RDRL WMP C  
T BJERKE  
M FERMEN-COKER  
R BECKER  
S R BILYK  
D CASEM  
J CLAYTON  
M GREENFIELD  
B LEAVY  
S SEGLETES  
A SOKOLOW  
A TONGE  
C WILLIAMS  
RDRL WMP D  
J RUNYEON  
A BARD  
N BRUCHEY  
R DONEY  
R FREY  
S HALSEY  
M KEELE  
D KLEPONIS  
H W MEYER  
R MUDD  
F MURPHY  
D PETTY  
C RANDOW  
S SCHRAML  
B SCOTT  
K STOFFEL  
G VUNNI  
V WAGONER

M ZELLNER  
RDRL WMP E  
P SWOBODA  
P BARTKOWSKI  
S BARTUS  
M BURKINS  
D GALLARDY  
D HACKBARTH  
D HORNBAKER  
E HORWATH  
J HOUSKAMP  
E KLIER  
C KRAUTHAUSER  
M LOVE  
K MCNAB  
D SHOWALTER  
RDRL WMP F  
N GNIAZDOWSKI  
C CUMMINS  
E FIORAVANTE  
D FOX  
R GUPTA  
S HUG  
RDRL WMP G  
R EHLERS  
S KUKUCK  
C PECORA  
J STEWART