



**EFFECTS OF POSITION UNCERTAINTY ON
PASSIVE MULTISTATIC SYNTHETIC
APERTURE RADAR RESOLUTION
MEASURES**

THESIS

Duy K. Nguyen, Capt, USAF
AFIT-ENG-MS-18-M-050

**DEPARTMENT OF THE AIR FORCE
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RESOLUTION MEASURES

THESIS

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Capt, USAF

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Abstract

Given numerous exciting potentials including substantial cost savings, passive multistatic synthetic aperture radar (PMSAR) has been gaining more attention in the radar community. However, PMSAR research efforts such as emitters of opportunity selection thus far have not considered radar position uncertainty. Seeking to fill this knowledge gap, we run Monte Carlo trials to simulate transmitter position uncertainty and examine the effects on the multistatic resolution of radar sets of different geometries, center frequencies and bandwidths. By monitoring the 3dB contour of the multistatic point spread function, we find that the multistatic resolution worsens on average and the data spread grows with increasing levels of radar position uncertainty; the extent of the effects depends on the bistatic pair with predominantly larger position uncertainty. We observe a robust correlation between the effects of position uncertainty and the bistatic angles; and the extent of the effects is determined by the bistatic pair with predominantly larger bistatic angle. Furthermore, we find that the same amount of position uncertainty has less effects on transmitters with high frequencies and large bandwidths; however, including these transmitters does not guarantee better multistatic resolution, especially in the presence of significant levels of position uncertainty. The findings and methods in this work can be immediately applied to enhance the objective functions for the emitters of opportunity selection problem, producing more realistic rankings that account for non-ideal conditions.

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To my beloved family.

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Duy K. Nguyen

Table of Contents

	Page
Abstract	iv
Acknowledgements	vi
List of Figures	x
List of Tables	xvi
I. Introduction	1
1.1 General Issue	1
1.2 Problem Statement	1
1.3 Research Objectives	2
1.4 Methodology	2
1.5 Assumptions and Implications	3
1.6 Expected Contributions	4
1.7 Thesis Overview	5
II. SAR Imaging Essentials	6
2.1 Chapter Overview	6
2.2 SAR Imaging Essentials	6
2.2.1 Bistatic SAR	6
2.2.2 Passive Multistatic SAR	8
2.2.3 Differential Range	9
2.2.4 Ideal Phase History	11
2.2.5 Point Spread Function	12
2.3 Multistatic Resolution Measures	13
2.4 Effects of Position Measurement Errors	16
2.5 Effects of Velocity and Acceleration Measurement Errors	18
III. Background Research and Methodology	21
3.1 Chapter Overview	21
3.2 Relevant Research	21
3.2.1 Effects of Motion Measurement Errors (MME)	21
3.2.2 Resolution Ellipse	22
3.3 Methodology	24
3.3.1 Simulating Position Measurement Errors	24
3.3.2 Simulating Velocity Measurement Errors	26
3.3.3 Simulating Acceleration Measurement Errors	29
3.3.4 Monte Carlo Trials	31

	Page
3.3.5	Computing the Multistatic Resolution Ellipse (MRE) Measures 32
3.3.6	Computing the Intergrated Sidelobe Ratio (ISLR) of the Multistatic Point Spread Function 35
3.3.7	Data Visualization 36
IV.	Results and Analysis 38
4.1	Chapter Overview 38
4.2	Scenario 1: Effects of Position Error Variance 38
4.2.1	Simulation Setup 38
4.2.2	Results 40
4.2.3	Key Findings 51
4.3	Scenario 2: Effects of Position Error Variance of Individual Bistatic Pairs 51
4.3.1	Simulation Setup 51
4.3.2	Results 52
4.3.3	Key Findings 58
4.4	Scenarios 3a, 3b and 3c: Effects of Transmitter Position Uncertainty and the Radar Geometry 59
4.4.1	Simulation Setup 59
4.4.2	Result Analysis 62
4.4.3	Key Findings 76
4.5	Scenario 4a: Effects of Velocity Measurement Errors 76
4.5.1	Simulation Setup 77
4.5.2	Results 78
4.5.3	Key Findings 84
4.6	Scenario 4b: Effects of Acceleration Measurement Errors 84
4.6.1	Simulation Setup 84
4.6.2	Results 85
4.6.3	Key Findings 92
4.7	Scenario 5: Effects of Transmitter Position Uncertainty and the Radar Center Frequency 92
4.7.1	Simulation Setup 92
4.7.2	Results 93
4.7.3	Key Findings 99
4.8	Scenario 6: Effects of Transmitter Position Uncertainty and the Radar Bandwidth 100
4.8.1	Simulation Setup 100
4.8.2	Results 101
4.8.3	Key Findings 107
4.9	Scenario 7: Selection of Emitters of Opportunity with Position Uncertainty 108
4.9.1	Simulation Setup 109

	Page
4.9.2 Results.....	111
4.9.3 Key Findings	114
4.10 Additional Thoughts on the ISLR	114
V. Conclusions and Recommendations	117
5.1 Key Conclusions	117
5.2 Significance of the Research.....	119
5.3 Recommendations for Future Studies.....	120
Bibliography	121

List of Figures

Figure		Page
1	Top view of an example bistatic SAR configuration. Either the transmitter or receiver has to be mobile, at the minimum, in order to resolve targets in the cross-range direction (along the y-axis).....	7
2	Top view of an example passive multistatic SAR system. The system consists of two bistatic pairs: Rx-Tx1 and Rx-Tx2.	8
3	Range versus differential range in the bistatic context. Since the actual slant range is often much greater than the scene size, differential range is used instead for mathematical convenience.	10
4	Example of an error-free bistatic point spread function (PSF). With limited phase history support, the isotropic point scatterer appears as a two-dimensional sinc function rather than a delta function.	13
5	3dB contour of a bistatic point spread function (PSF).	14
6	3dB contour of a multistatic point spread function (PSF) combined coherently. The grating lobes are visible inside the contour (blue).	15
7	3dB contour of a multistatic point spread function (PSF) combined using the GLRT method.	16
8	Examples of a bistatic point spread function (PSF) without and with transmitter position measurement errors. In the presence of position measurement errors, the PSF is spatially shifted from the scene center.	26
9	Examples of PSFs without and with a velocity measurement error. The PSF is shifted and blurred out by the linear and quadratic components of the differential range error.	28
10	Examples of PSFs without and with an acceleration measurement error. The quadratic and cubic components of the differential range error induce a blurring effect on the image.	30

Figure		Page
11	Illustration of semi-major axis, semi-minor axis, tilt angle, effective multistatic resolution area (EMRA), linear eccentricity and eccentricity.	32
12	Examples of MRE measures without and with transmitter position uncertainty.	35
13	Radar geometry in Scenarios 1 and 2.	39
14	Scenario 1 multistatic resolution ellipse (MRE) measures data with different position error variance values.	41
15	Scenario 1 MRE measures' distributions with 3 m^2 position error variance.	43
16	Convergence check results for EMRA data set with 3 m^2 position error variance in Scenario 1.	45
17	Amount of data spread of each MRE measure in Scenario 1.	47
18	Scenario 1 integrated sidelobe ratio (ISLR) data for different position error variance values.	49
19	Behaviors of the average multistatic resolution ellipse (MRE) in the presence of transmitter position uncertainty in Scenario 1. The MREs are color-coded based on position error variance (PEV), as indicated on the color bar. The MRE grows in area and eccentricity as PEV increases.	50
20	Scenario 2 MRE data with different Tx2's position error variances.	53
21	Amount of data spread of each MRE measure in Scenario 2.	56
22	Scenario 2 ISLR data for different Tx2 position error variances.	57

Figure		Page
23	Behaviors of the average MRE in the presence of transmitter position uncertainty in Scenario 2. The MREs are color-coded based on the PEV of Tx2, as indicated on the color bar. Similar to Scenerio 1, The average MRE in Scenario 2 also grows in area and eccentricity as the PEV of Tx2 increases.	58
24	Radar geometry in Scenario 3a. With Tx1 fixed at 0° azimuth, Tx2's azimuth is increased by 18° after each set of 8000 MC trials.	60
25	Radar geometry in Scenario 3b. With Tx1 fixed at 85° azimuth, Tx2's azimuth is increased by 18° after each set of 8000 MC trials.	61
26	Radar geometry in Scenario 3c. After each set of 8000 MC trials, the azimuth of Tx1 and that of Tx2 are incremented by 18 degrees such that they are equal but have opposite signs.	62
27	EMRA distribution and data spread in Scenarios 3a-3c, compared side by side.	64
28	Semi-major axis distribution and data spread in Scenarios 3a-3c, compared side by side.	66
29	Linear eccentricity distribution and data spread in Scenarios 3a-3c, compared side by side.	68
30	Eccentricity distribution and data spread in Scenarios 3a-3c, compared side by side.	70
31	Tilt distribution and data spread in Scenarios 3a-3c, compared side by side.	72
32	ISLR distribution and data spread in Scenarios 3a-3c, compared side by side.	74
33	Behaviors of the MRE, on average, in the presence of transmitter position uncertainty in Scenarios 3a-3c. The MREs are color-coded based on Tx2's azimuth, as indicated on the color bars.	75
34	Radar geometry in Scenarios 4a and 4b.	77

Figure		Page
35	The MREs resulted from the 51 deterministic trials. The MRE, which is color-coded according the amount of velocity measurement error, grows then reverts back to the baseline MRE as the cross-range velocity error increases in either direction.	79
36	Behaviors of MRE measures as Tx2's velocity error in the cross-range direction $\tilde{v}_y$ is varied through $[-4, 4]$ m/s. The bistatic resolution ellipse (BRE), which is the 3dB contour of the PSF of bistatic pair Rx-Tx1, serves as the baseline.	81
37	ISLR values resulted from different velocity errors from -4 m/s to 4 m/s in the cross-range direction.	82
38	Example of MRE beyond velocity measurement error limits. The shifting and blurring effects cause the MRE to become the bistatic resolution ellipse of Rx-Tx1 pair.	83
39	The MREs resulted from the 51 deterministic trials. The MRE, which is color-coded according the amount of acceleration measurement error, grows then reverts back to the baseline MRE as the cross-range acceleration error increases in either direction.	86
40	Behaviors of MRE measures as the acceleration error in the cross-range direction is varied through $[-0.8, 0.8]$ m/s ² . The BRE is the 3dB contour of the PSF of bistatic pair Rx-Tx1.	88
41	ISLR values resulted from different Tx2 acceleration error values from -0.8 m/s ² to 0.8 m/s ² in the cross-range direction.	89
42	Example of MRE beyond acceleration measurement error limits. The combined blurring effects due to the quadratic and cubic components of the differential range error in (19) cause the mainlobe of Rx-Tx2 to fall quickly. As a result, the MRE becomes the bistatic resolution ellipse of Rx-Tx1.	91
43	The reference MREs in Scenario 5. The MREs are color-coded based on the center frequency, as indicated on the color bar on the right. As the center frequency increases, the MRE decreases in area and eccentricity.	93

Figure		Page
44	Scenario 5 MRE measures data of different center frequencies.	94
45	Amount of data spread of each MRE measure in Scenario 5.	96
46	ISLR data at different center frequencies.	97
47	± 1 standard deviation of the difference between ISLR samples and the reference value as the center frequency is varied through [600, 2000] MHz range.	98
48	Average MRE under the effects of 5 m ² position error variance, compared with the reference MRE at different transmitter center frequencies. The MREs are color-coded based on the center frequency, as indicated on the color bars.	99
49	The reference MREs in Scenario 6. The MREs are color-coded based on the bandwidth, as indicated on the color bar. As the bandwidths of both transmitters are varied incrementally from 6 MHz to 20 MHz, the MRE reduces in size and generally retains its shape and orientation.	101
50	Scenario 6 MRE measures data with different transmitter bandwidths.	102
51	Amount of data spread of each MRE measure in Scenario 6.	104
52	ISLR data for different transmitter bandwidths.	105
53	± 1 standard deviation of the difference between ISLR samples and the reference value as the bandwidths of both transmitters are varied through [6,20] MHz range.	106
54	Average MRE under the effects of 5 m ² position error variance, compared with the reference MRE of different transmitter bandwidths.	107
55	Five emitters of opportunity are diverse in geometry.	110
56	The MRE of each multistatic set in Scenario 7 when there is no position uncertainty.	111

Figure		Page
57	Average MREs of 10 emitter sets compared with the corresponding reference MREs.	112
58	Comparisons of EMRA data of 10 emitter sets.	113
59	Comparisons of eccentricity data of 10 emitter sets.	113
60	Examples of MRE derived using the GLRT and non-coherent summing methods with large transmitter position error variance. In both cases, the MRE contains some pixels below the 3dB threshold.	115
61	Alternate view of the multistatic PSF examples seen in Figures 60a and 60b. In both cases, the pixels below the 3dB threshold (red) appear inside the MRE.....	116

List of Tables

Table		Page
1	Notations used in (9)-(13)	19
2	Number of Monte Carlo (MC) trials for each variance to maintain 95% confidence level.	31
3	Geometry of Scenarios 1 and 2.	40
4	Geometry of Scenarios 3a and 3b.	60
5	Geometry of Scenarios 4a and 4b.	78
6	Characteristics of transmitters of opportunity in Scenario 7	108

EFFECTS OF POSITION UNCERTAINTY
ON PASSIVE MULTISTATIC SYNTHETIC APERTURE RADAR
RESOLUTION MEASURES

I. Introduction

1.1 General Issue

Passive multistatic synthetic aperture radar (PMSAR) is increasingly gaining more traction due to its various advantages, the most important of which is arguably the capability to exploit emitters of opportunity. Many of these emitters are envisioned to be of commercial sources, broadcasting orthogonal frequency division multiplexing (OFDM) signals. The logical next step is to devise a means to predict the achievable resolution performance for this promising new class of imaging systems. Previous works including [1–3] have attempted to predict the system’s performance using the multistatic resolution ellipse (MRE), the 3dB contour of the mainlobe of the multistatic point spread function (PSF), as the primary resolution metric. However, the efforts thus far have assumed perfect knowledge of the transmitters’ positions, ignoring the effects of transmitter position uncertainty. Seeking to fill this knowledge gap, we attempt to characterize the effects of transmitter position uncertainty on the PMSAR image resolution; parts of this work have been selected for publication in [4].

1.2 Problem Statement

Efforts to predict PMSAR resolution performance thus far have not taken into account radar position uncertainty caused by factors including (but are not limited

to) position, velocity, and acceleration measurement errors. Little is known about the effects of position uncertainty on the resolution performance of a PMSAR system. These effects must be understood in order to advance areas such as PMSAR selection of emitters of opportunity and flight path planning to next level. Using the multistatic resolution ellipse as the primary metric, we run Monte Carlo trials in different scenarios to characterize the effects of the transmitter's position uncertainty on the resolution performance of PMSAR systems.

1.3 Research Objectives

The following are the primary goals of our work:

- Extensively characterize the effects of transmitter position uncertainty caused by position measurement error on various radar configurations of different geometries, bandwidths, and center frequencies
- Investigate the preliminary effects of transmitter velocity and acceleration measurement errors on the image resolution.

1.4 Methodology

Based on the work described in [5], we first establish a set of metrics derived from the multistatic point spread function (PSF) to monitor the effects of transmitter position uncertainty. These metrics include six multistatic resolution ellipse (MRE) measures: semi-major axis, semi-minor axis, linear eccentricity, eccentricity, tilt angle and the effective multistatic resolution area (EMRA). The last measure is simply the area of the MRE, which is defined as the 3dB contour of the mainlobe of the multistatic point spread function. In addition to the MRE measures, we also collect and analyze the integrated sidelobe ratio (ISLR). All of the metrics along with the

PSF will be explained in detail in Chapter II and Chapter III.

To simulate the position uncertainty, we run Monte Carlo (MC) trials. In each random trial, leveraging the results in [6, 7], we corrupt the bistatic phase history with Cartesian position errors drawn from a zero-mean Gaussian distribution. By changing the variance of the Gaussian distribution, we effectively control the amount of position uncertainty as we run different simulated scenarios. In each MC trial, we compute the bistatic PSF using a filtered-back projection routine to ensure a high degree of precision. The multistatic PSF is determined with the generalized likelihood ratio test (GLRT), and the MRE measures, as well as the ISLR, are calculated in the manners described in [5].

In the first stage of the research, the investigation is focused on the effects of the transmitter position uncertainty due to position measurement errors only. In the subsequent stages, we extend the investigation to include effects of velocity and acceleration measurement errors. Finally, we look at how the uncertainty affects the radar’s resolution performance at different frequencies and bandwidths.

1.5 Assumptions and Implications

Leveraging the mathematical characterization of the effects of radar motion measurement errors described in [6], we corrupt the multistatic image with random errors drawn from a zero-mean Gaussian distribution. The variance of the distribution is used as the means to control the level of position uncertainty introduced into the system. Our main underlying assumptions include:

- The receiver’s position is precisely known for each pulse; the range error $\tilde{r}$ is due to the transmitter position uncertainty only. This assumption allows us to simplify the simulation and enables us to isolate the effects of interest.
- The transmitters, which operate in frequencies and bandwidths typical of many

existing commercial broadcast waveforms, do not interfere with each other. The receiver is assumed capable of receiving and processing echoed signals originating from the transmitters separately. These assumptions enable us to reduce the complexity of the simulation.

- Signals originating from different transmitters have equal power levels, allowing to us to focus on the effects on parameters of interest including the radar geometry, frequency and bandwidth. Furthermore, the transmitter power is assumed constant for the entire bandwidth. As a result, the observations we make in this work may differ once signal power levels are taken into account.
- The simulation starts after the matched filter step in the signal processing chain. The echoes are assumed perfectly recovered, and the matched filter is assumed ideal. Consequently, factors that contribute to the overall position uncertainty arising prior to this step are not considered in our study.

1.6 Expected Contributions

Insights drawn from the data and the methodology developed in this work may be used to:

- improve the understanding of how the PMSAR image quality will be affected in the presence of position uncertainty limited to position, velocity and acceleration measurement errors
- enhance the understanding of the extents to which position uncertainty affects radar systems with different geometries, frequencies and bandwidths
- update or devise a more realistic objective function in the emitter selection optimization problem that takes into account position uncertainty

- aid in the design of adaptive flight path planning algorithms for PMSAR.

1.7 Thesis Overview

The rest of the thesis is organized as follows: Chapter II introduces the reader to passive multistatic SAR systems and equips the reader with some basic background information to facilitate the upcoming data analysis and discussions. The first half of Chapter III informs the reader of past and current relevant works, while the latter half of the chapter explains the methodology we use to achieve our research objectives. Chapter IV shows the reader how we set up each simulation scenario, presents the results, and walks the reader through our observations of the results. Finally, all key findings are summarized in Chapter V, followed by our recommendations for future works.

II. SAR Imaging Essentials

2.1 Chapter Overview

To facilitate the upcoming discussions, this chapter is aimed at equipping the reader with basic SAR imaging concepts including bistatic SAR imaging, passive multistatic SAR (PMSAR) imaging, differential range, ideal phase history, and the point spread function (PSF). Next, a description of multistatic resolution ellipse (MRE) measures, based on [5], is provided. Finally, the chapter presents a mathematical characterization of the effects of position, velocity and acceleration measurement errors, based on [6–8].

2.2 SAR Imaging Essentials

2.2.1 Bistatic SAR.

A bistatic SAR system consists of a transmitter and a receiver at separate locations [8]. The signal, emanating from the transmitter, impinges upon the target and gets reflected back to the receiver, which collects and processes it. Figure 1 provides an illustration of a simplified bistatic radar system.

The return signal is the combination of echos of all targets in the scene. The range information of the targets can be determined by match-filtering the return signal with the reference waveform to determine the amount of delay t , whereby the target's range can be computed as $R = t \times c$, where c is the speed of light. In the bistatic context, the range R is the total distance the signal has to travel from the transmitter to the target and from the target to the receiver, that is

$$R = R_{tm} + R_{rm} \tag{1}$$

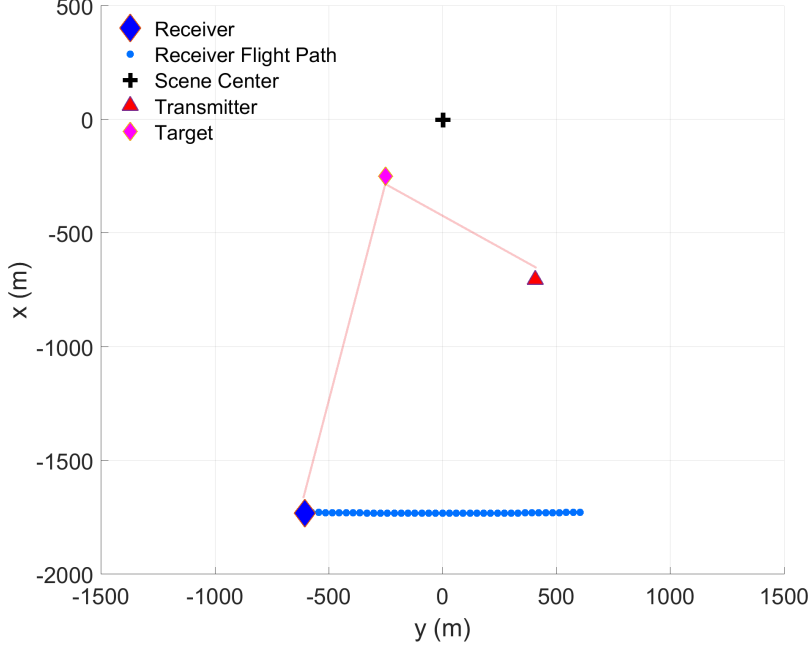


Figure 1. Top view of an example bistatic SAR configuration. Either the transmitter or receiver has to be mobile, at the minimum, in order to resolve targets in the cross-range direction (along the y-axis).

where R_{tm} represents the distance from the transmitter to the m^{th} target and R_{rm} denotes the distance between the m^{th} target and the receiver. In the monostatic context, in which the transmitter and the receiver are co-located, R represents the round-trip distance the signal travels. Consequently, monostatic synthetic aperture radar (SAR) can be regarded as a special bistatic configuration.

The problem is that the returned signal with a particular delay t consists of contributions from all targets on the isorange contour [9]. The bistatic isorange contour is the ellipse whose foci are the locations of the receiver and the transmitter [10]. All targets on the same isorange contour have the same bistatic range. A single pulse does not give enough information to resolve targets having the same bistatic range; as a result, forming an image of the scene based on the targets' reflectivity values is not possible.

To resolve targets in the cross-range direction (along the flight path), multiple

return pulses from different collection angles are needed. Match-filtering each return pulse gives a range profile, which is a complex function of range. After all received signal pulses are processed into range profiles, an image can be formed using techniques such as back-projection [9]. In back-projection, each image pixel is formed by combining contributions from all range profiles that contain the pixel's range.

2.2.2 Passive Multistatic SAR.

A passive multistatic SAR imaging system consists of multiple bistatic pairs with different geometries to take advantage of aspect diversity [11]. Figure 2 provides an illustration of a simplified PMSAR system consisting of two stationary transmitters and a mobile receiver.

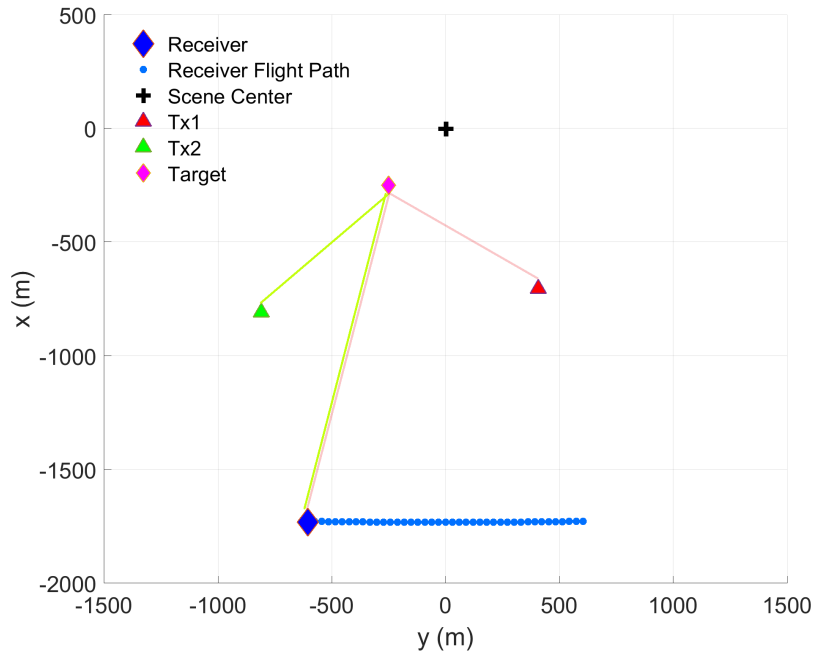


Figure 2. Top view of an example passive multistatic SAR system. The system consists of two bistatic pairs: Rx-Tx1 and Rx-Tx2.

The PMSAR system in Figure 2 is comprised of two separate bistatic pairs: Receiver-Transmitter 1 and Receiver-Transmitter 2. PMSAR systems are considered passive because the receivers, usually mobile but not necessarily, do not actively

radiate energy; they collect and process echoed signals originating from emitters of opportunity, which comprise the transmitting components of the system. The radar configuration used throughout this work consists of a single receiver and two transmitters working in spotlight mode, in which the receiver's antenna is aimed at the scene center for the entire aperture (collection flight path).

During operation, return pulses originating from each transmitter during the collection flight path (the aperture) are received, processed and stored separately. A bistatic SAR image is formed for each receiver-transmitter pair. The multistatic image is subsequently constructed by combining the individual bistatic images using a coherent or non-coherent technique. The latter technique produces each multistatic image pixel by combining the magnitude of the corresponding pixels from the bistatic images, whereby disregarding the phase information of the bistatic pixels [5]. In this work, the bistatic images are combined using the generalized likelihood ratio test (GLRT), a non-coherent technique that takes the maximum value pixel across the bistatic image stack [12].

2.2.3 Differential Range.

In SAR image applications, the scene size is much smaller than the slant range from the receiver to the scene center. For mathematical convenience, differential range is used in place of true range, as illustrated in Figure 3.

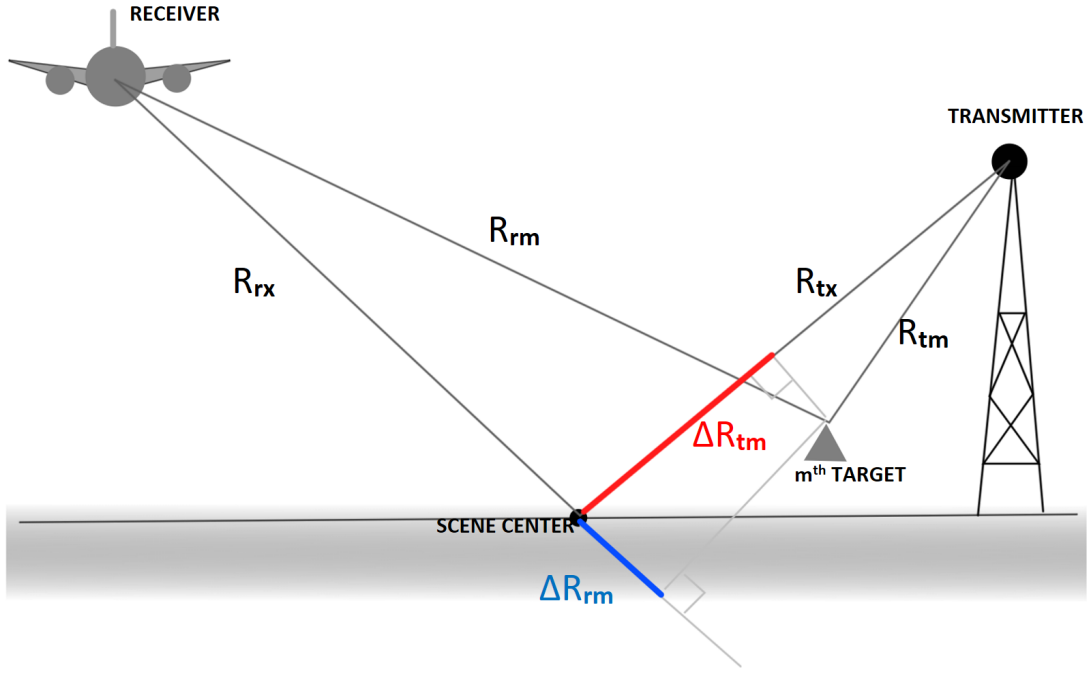


Figure 3. Range versus differential range in the bistatic context. Since the actual slant range is often much greater than the scene size, differential range is used instead for mathematical convenience.

In Figure 3, R_{rx} and R_{tx} denote the receiver's range and the transmitter's range relative to the scene center, respectively; R_{rm} and R_{tm} denote the receiver's range and the transmitter's range to the m^{th} target, respectively; lastly ΔR_{rm} denotes the receiver-to-target differential range and ΔR_{tm} denotes the transmitter-to-target differential range, referenced off of the scene center. Note that in SAR imaging, the scene center is the center of reference for the entire system. Targets at the scene center have differential range of zero. Targets further than the scene center have positive differential range while targets closer than the scene center have negative differential range.

2.2.4 Ideal Phase History.

The phase history consists of samples of the scene reflectivity in the Fourier domain, resulted from either deramp processing or match-filtering the return signals with a reference signal [9]. Based on [6–8], the ideal phase history at slow time¹ τ and frequency f of a simplified scene comprised of point scatterers is mathematically represented as

$$S(f, \tau) = \sum_m A_m e^{j\phi_m(f, \tau)} + w(f, \tau) \quad (2)$$

where A_m is the reflectivity coefficient of the m^{th} scatterer while $w(f, \tau)$ represents the receiver's circular white Gaussian noise (WGN). Since our analysis is centered around the point spread function (PSF), the image of a hypothetical scene comprised of a single isotropic point target located at the scene center, we can ignore the WGN term and simplify (2) to

$$S(f, \tau) = A_0 e^{j\phi_0(f, \tau)} \quad (3)$$

where the subscript zero refers to the single isotropic point scatterer at the scene center. Note that the phase history is primarily dependent on the phase ϕ_0 , which is in turn a function of f and differential range $\Delta R(\tau)$:

$$\phi_0(f, \tau) = -2\pi f \frac{\Delta R(\tau)}{c} \quad (4)$$

where c denotes the speed of light in vacuum and $\Delta R(\tau)$ represents the total transmitter-

¹In radar, slow time refers to time intervals between successive received pulses, as opposed to fast time, which refers to the time the signals spend traversing the scene [13].

target-receiver differential range; that is

$$\Delta R(\tau) = \Delta R_t(\tau) + \Delta R_r(\tau) \quad (5)$$

where $\Delta R_t(\tau)$ represents the transmitter-to-target differential range and $\Delta R_r(\tau)$ denotes the target-to-receiver differential range. Note that subscript m is dropped in (5) since we are dealing with a single isotropic point scatterer at the scene center. Furthermore, given our target is at the scene center, both $\Delta R_t(\tau)$ and $\Delta R_r(\tau)$ are zero for all slow time τ when there is no radar position uncertainty; consequently, the phase history is constant at any frequency f and slow time τ . Numerically, the ideal phase history can be represented as a two-dimensional array whose every element is equal to A_0 . Using image construction techniques such as polar formatting algorithm (PFA) or filtered back-projection (FBP), the ideal phase history can be transformed into an image, the point spread function.

2.2.5 Point Spread Function.

In predicting the performance of a SAR system, we form and analyze its point spread function (PSF). The PSF of the system can be thought of as the impulse response of the radar system [14], i.e., it is the image the system would produce if the scene consists of a single isotropic point target at the scene center. The shape and size of the mainlobe and the sidelobes of the PSF serve as indicators of the system's resolution performance. In deriving the PSF for each bistatic pair, we replace all the targets in the hypothetical scene with a single isotropic target and form an image using the bistatic SAR imaging process. The multistatic PSF may be formed by using either a coherent or non-coherent technique. With the multistatic PSF formed, one can proceed to evaluate the resolution performance of the multistatic SAR system. An example of a bistatic PSF can be seen in Figure 4.

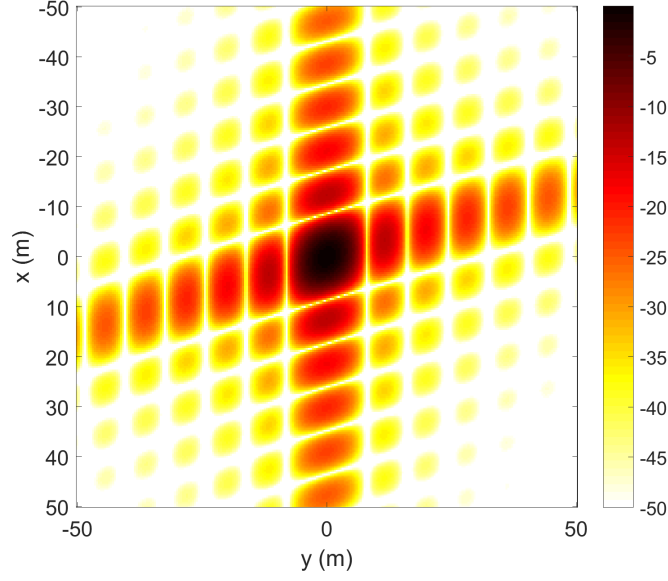


Figure 4. Example of an error-free bistatic point spread function (PSF). With limited phase history support, the isotropic point scatterer appears as a two-dimensional sinc function rather than a delta function.

Note that, in Figure 4, the single isotropic point takes on the shape of a two-dimensional sinc function rather than a delta function. This phenomenon is due to the fact that we have a limited phase history support. In other words, the spatial frequency domain samples are windowed. The larger the phase history support, the more the PSF resembles a delta function.

2.3 Multistatic Resolution Measures

The resolution ellipse is defined in [2, 15] as the 3dB contour of the bistatic PSF's mainlobe, as illustrated in Figure 5.

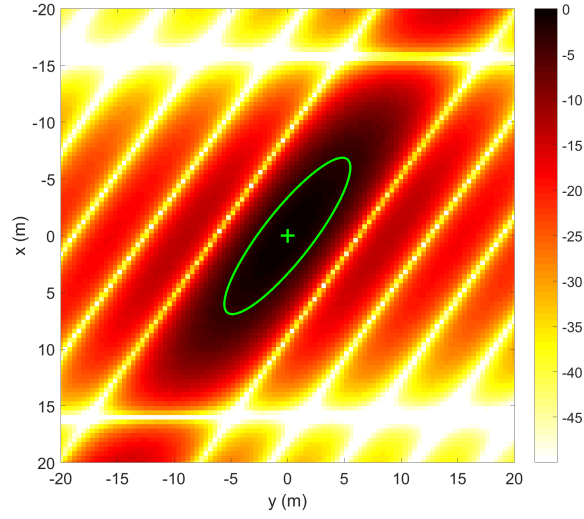


Figure 5. 3dB contour of a bistatic point spread function (PSF).

Previous works defined multistatic resolution as the intersection or union of bistatic resolution ellipses [1]. However, bistatic pairs are not necessarily equal contributors. The difference in power levels among the transmitter-receiver pairs affects the shape of the multistatic PSF. Furthermore, coherent combination of bistatic pairs can result in grating lobes, as illustrated in Figure 6.

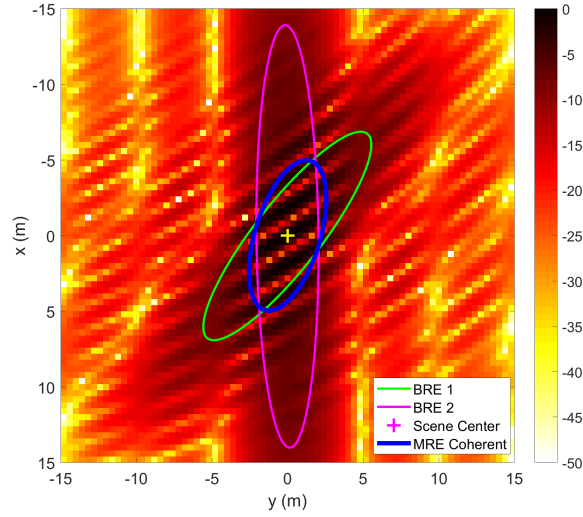


Figure 6. 3dB contour of a multistatic point spread function (PSF) combined coherently. The grating lobes are visible inside the contour (blue).

In Figure 6, although the coherently combined MRE (blue) is smaller than the individual bistatic resolution ellipses (BRE), the grating lobes are visible inside the multistatic 3dB contour. To avoid grating lobes, we combine individual bistatic PSFs using the generalized likelihood ratio test (GLRT). The GLRT is a non-coherent method, which compares the absolute reflectivity values of corresponding pixels from the individual bistatic PSFs and takes the largest value for the multistatic PSF, pixel by pixel [12]. An example of a GLRT-derived multistatic PSF can be seen in Figure 7.

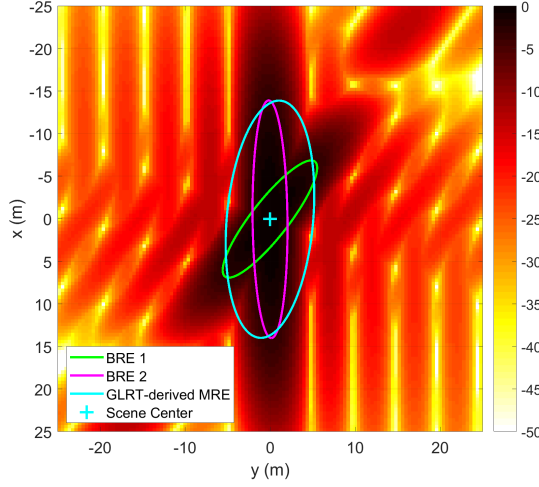


Figure 7. 3dB contour of a multistatic point spread function (PSF) combined using the GLRT method.

Note that in Figure 7, the grating lobes are completely absent at the expense of a larger resolution ellipse. In addition to the area of the MRE, which is termed the effective multistatic resolution area (EMRA) in [2, 5], we determine other measures of the MRE, including the semi-major and semi-minor axis lengths, linear eccentricity, eccentricity, and the ellipse tilt angle. The axis lengths and orientation provide additional directionality interpretation of resolution (akin to range and cross-range resolution), not included in the single scalar of the EMRA. Examples of all six MRE measures and the methodology we use to determine them are given in Section 3.3.

2.4 Effects of Position Measurement Errors

As discussed previously in Section 2.2, the ideal phase history of a bistatic point spread function is constant for all frequency f and slow time τ , since the differential range $\Delta R(\tau)$ is zero at every point along the radar's flight path. However, in the presence of radar position uncertainty, the position errors result in two additional terms $\Delta \tilde{R}_t(\tau)$ and $\Delta \tilde{R}_r(\tau)$ which respectively denote the transmitter-to-target and target-to-receiver differential range errors, referenced to the scene center. The erred

phase history can be represented as

$$S(f, \tau) = A_0 e^{-j2\pi f \frac{\Delta \tilde{R}_t(\tau) + \Delta \tilde{R}_r(\tau)}{c}}. \quad (6)$$

Without loss of generality, we assume the receiver's position is perfectly known for all slow time τ ; (6) is thus reduced to

$$S(f, \tau) = A_0 e^{-j2\pi f \frac{\Delta \tilde{R}_t(\tau)}{c}}. \quad (7)$$

Factors that can contribute to the transmitter differential range error $\Delta \tilde{R}_t(\tau)$ in (7) include (but are not limited to) transmitter position, velocity, and acceleration measurement errors. With the transmitter stationary, the transmitter-to-target differential range error $\Delta \tilde{R}_t(\tau)$ is constant across all slow time τ , resulting in a constant phase error term. According to [6, 7], a constant phase error term causes the bistatic PSF to be spatially shifted. Similar results are described in Jakowatz's treatment of the linear phase error for the monostatic case in [9]. Furthermore, in cases where the transmitter is mobile, there could be radar velocity and/or acceleration errors, which induce a linear, a quadratic and/or a cubic phase error term. Similar to the constant phase error term, the linear phase error term causes the bistatic PSF to shift while the quadratic and cubic phase error terms have blurring effects on the image [6, 7].

2.5 Effects of Velocity and Acceleration Measurement Errors

According to [6, 8], the position errors due to the transmitter's position, velocity and acceleration measurement errors can be written as

$$\vec{r}(\tau) = \vec{p} + \vec{v}\tau + \vec{a}\frac{\tau^2}{2} \quad (8)$$

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} \tilde{p}_x \\ \tilde{p}_y \\ \tilde{p}_z \end{bmatrix} + \begin{bmatrix} \tilde{v}_x \\ \tilde{v}_y \\ \tilde{v}_z \end{bmatrix} \tau + \begin{bmatrix} \tilde{a}_x \\ \tilde{a}_y \\ \tilde{a}_z \end{bmatrix} \frac{\tau^2}{2} \quad (9)$$

where $\vec{r} = [\tilde{x}, \tilde{y}, \tilde{z}]^T$ represents the transmitter's position errors in Cartesian coordinates, $\vec{p} = [\tilde{p}_x, \tilde{p}_y, \tilde{p}_z]^T$ denotes the transmitter's position measurement errors at the aperture's² midpoint while $\vec{v} = [\tilde{v}_x, \tilde{v}_y, \tilde{v}_z]^T$ and $\vec{a} = [\tilde{a}_x, \tilde{a}_y, \tilde{a}_z]^T$ symbolize the transmitter's velocity and acceleration measurement errors, respectively. In this work, these errors are considered fixed for the entire aperture. To facilitate the upcoming discussion, notations used in (9)-(13) are listed in Table 1. Since we assume perfect knowledge of the receiver's position, all quantities denoted with the tilde refer to errors on the transmitter side.

²In this work, the terms receiver's azimuth extent, aperture, and collection flight path are used interchangeably.

Table 1. Notations used in (9)-(13)

Notation	Parameter
τ	Slow time
$\vec{r} = [\tilde{x}, \tilde{y}, \tilde{z}]^T$	Transmitter's position errors in Cartesian coordinates
$\vec{p} = [\tilde{p}_x, \tilde{p}_y, \tilde{p}_z]^T$	position measurement errors
$\vec{v} = [\tilde{v}_x, \tilde{v}_y, \tilde{v}_z]^T$	velocity measurement errors
$\vec{a} = [\tilde{a}_x, \tilde{a}_y, \tilde{a}_z]^T$	acceleration measurement errors
$\Delta R(\tau)$	Transmitter-target-receiver differential range
$R_{tm}(\tau)$	Transmitter's true range to the m^{th} target
$R_{rm}(\tau)$	Receiver's true range to the m^{th} target
$R_{tx}(\tau)$	Transmitter's true range to scene center
$R_{rx}(\tau)$	Receiver's true range to the scene center
R_t	Transmitter's observed range at the aperture's midpoint
x_t, y_t, z_t	Transmitter's observed coordinates at the aperture's midpoint
v_{xt}, v_{yt}, v_{zt}	Transmitter's velocity in the x, y, and z directions
$\Delta \tilde{R}(\tau)$	Total differential range error

The total differential range $\Delta R(\tau)$ for the m^{th} point scatterer when there is no motion measurement error is determined as

$$\Delta R(\tau) = R_{tm}(\tau) + R_{rm}(\tau) - R_{tx}(\tau) - R_{rx}(\tau), \quad (10)$$

where $R_{tm}(\tau)$ denotes the true distance between the transmitter and the m^{th} target while $R_{rm}(\tau)$ denotes the true distance from the receiver to the m^{th} target. Furthermore, $R_{tx}(\tau)$ and $R_{rx}(\tau)$ in (10) represent the transmitter's true range and the receiver's true range to the scene center, respectively. Since our target is an isotropic point scatterer at the scene center, the right side of (10) is reduced to zero, independent of the slow time τ . In other words, the true differential range is always zero for a single isotropic point target at the scene center.

Assuming far-field³ operations, the differential range error $\Delta \tilde{R}(\tau)$ resulted from

³When the scene size is much smaller than the distance from its center to the radar, the operating conditions are considered far-field.

the transmitter's position errors $\vec{r} = [\tilde{x}, \tilde{y}, \tilde{z}]^T$ can be approximated as [6, 8]

$$\Delta\tilde{R}(\tau) \approx -\tilde{x}(\tau)\frac{x_t + v_{xt}\tau}{R_t} - \tilde{y}(\tau)\frac{y_t + v_{yt}(\tau)}{R_t} - \tilde{z}(\tau)\frac{z_t + v_{zt}(\tau)}{R_t} \quad (11)$$

where $\tilde{x}(\tau)$, $\tilde{y}(\tau)$, and $\tilde{z}(\tau)$ represent the position errors in Cartesian coordinates at slow time τ . Applying the position error definition in (9) to (11) gives

$$\begin{aligned} \Delta\tilde{R}(\tau) \approx & -(\tilde{p}_x + \tilde{v}_x\tau + \frac{1}{2}\tilde{a}_x\tau^2)(\frac{x_t}{R_t} + \frac{v_{xt}\tau}{R_t}) \\ & -(\tilde{p}_y + \tilde{v}_y\tau + \frac{1}{2}\tilde{a}_y\tau^2)(\frac{y_t}{R_t} + \frac{v_{yt}\tau}{R_t}) \\ & -(\tilde{p}_z + \tilde{v}_z\tau + \frac{1}{2}\tilde{a}_z\tau^2)(\frac{z_t}{R_t} + \frac{v_{zt}\tau}{R_t}). \end{aligned} \quad (12)$$

Distributing the terms in (12) and regrouping them according to the order of the slow time τ result in

$$\begin{aligned} \Delta\tilde{R}(\tau) \approx & \left[-\tilde{p}_x\frac{x_t}{R_t} - \tilde{p}_y\frac{y_t}{R_t} - \tilde{p}_z\frac{z_t}{R_t} \right] \\ & + \left[-\tilde{v}_x\frac{x_t}{R_t} - \tilde{v}_y\frac{y_t}{R_t} - \tilde{v}_z\frac{z_t}{R_t} - \tilde{p}_x\frac{v_{xt}}{R_t} - \tilde{p}_y\frac{v_{yt}}{R_t} - \tilde{p}_z\frac{v_{zt}}{R_t} \right] \tau \\ & + \left[-\frac{1}{2}\tilde{a}_x\frac{x_t}{R_t} - \frac{1}{2}\tilde{a}_y\frac{y_t}{R_t} - \frac{1}{2}\tilde{a}_z\frac{z_t}{R_t} - \tilde{v}_x\frac{v_{xt}}{R_t} - \tilde{v}_y\frac{v_{yt}}{R_t} - \tilde{v}_z\frac{v_{zt}}{R_t} \right] \tau^2 \\ & + \left[-\frac{1}{2}\tilde{a}_x\frac{v_{xt}}{R_t} - \frac{1}{2}\tilde{a}_y\frac{v_{yt}}{R_t} - \frac{1}{2}\tilde{a}_z\frac{v_{zt}}{R_t} \right] \tau^3 \end{aligned} \quad (13)$$

where x_t , y_t , and z_t represent the observed Cartesian coordinates of the transmitter at the aperture's midpoint. Equation (13) serves as the basis for our simulation of velocity and acceleration measurement errors, as explained in Section 3.3.

III. Background Research and Methodology

3.1 Chapter Overview

In this chapter, we discuss and summarize the insights gained during our literature research. These works not only inform us on the state of the art, they also help us identify areas where we can make contributions and enable us to carry out this research. The second half of the chapter is dedicated to explaining the methodology we use to achieve our research objectives.

3.2 Relevant Research

The works reviewed are organized around two main topics: the effects of radar motion measurement errors on bistatic SAR images and the metrics used to evaluate the resolution performance of a PMSAR system.

3.2.1 Effects of Motion Measurement Errors (MME).

The effects of radar motion measurement errors (MME) on the image resolution in the bistatic configuration receive a thorough treatment in [6–8]. In [6], the author states that the position information of a point scatterer is embedded in the observed phase history (samples in the Fourier domain). Under far-field assumptions, the author demonstrates that the point scatterer’s coordinates can be mathematically derived from the observed phase history and that the process is reversible. This finding plays a key role in evaluating the effects of radar position uncertainty on the image in this research. The sources of transmitter (and receiver) position uncertainty include (but are not limited to) position, velocity, and acceleration measurement errors. Excluding other factors, these measurement errors all contribute to the differential

range error, which can have up to four components: constant, linear, quadratic, and cubic (see (13)).

Using the finding that the coordinates of a point scatterer are embedded in the phase history, the author in [6] is able to prove that each pixel of the image is spatially shifted in the presence of position measurement errors. Thus, position measurement errors contribute to the constant and linear components of the differential range error. The quadratic and cubic components, which arise from the transmitter velocity and/or acceleration measurement errors, both have blurring effects on the image.

More recently, the effects of motion measurement errors in the bistatic context get a fresh look in [16]. The authors in [16] propose a generalized method to characterize these effects on bistatic SAR images. The key idea in the approach proposed in [16] is to compare the compensating phase of the measured geometry and that of the actual geometry for each pixel center in the image [16].

Building on the results in [6] and [16], we extend the study of motion measurement errors' effects to cover the multistatic case. The findings in [6] serve as an enabler for our analysis, especially the investigation of the effects of velocity and acceleration measurement errors on the multistatic PSF, as explained in later sections in this chapter.

3.2.2 Resolution Ellipse.

The concept of resolution ellipses was described in detail in [1]. According to the authors in [1], the range and cross-range resolution measures are not adequate to precisely describe the target resolving capability of a bistatic SAR system or a PMSAR system. Instead, they argue, the resolution performance of such a system should be described in terms of the resolution ellipse, the 3dB contour drawn on the mainlobe of the system's PSF¹. The major axis of the ellipse represents the minimum

¹PSF is the image of a hypothetical scene consisting of a single isotropic point target.

separation required between two closely-spaced targets in the worst case. Conversely, the minor axis of the ellipse represents the minimum separation required between two closely-spaced targets in the best case.

In an effort to predict the resolution performance in the passive multistatic and passive inverse SAR contexts, the author in [3] also uses the 3dB mainlobe contour as the basis for his proposed methods for combining outputs from multiple bistatic pairs. The authors of [2, 5, 15] also advocate using ellipses to define the resolution performance of a PMSAR system instead of range and cross-range resolutions; these measures, the authors reason, are no longer well-defined as the bistatic images are coherently or non-coherently combined to form a multistatic image. In setting up the emitters of opportunity selection problem described in [2, 5, 15], the authors introduce the concept of effective multistatic resolution area (EMRA) and include it as one of the objective functions. The EMRA is defined as the area of the 3dB contour of the multistatic PSF mainlobe. The multistatic PSF is formed by coherently or non-coherently combining the individual bistatic PSFs, pixel by pixel.

Using EMRA as an objective function, the authors in [2, 5, 15] proceed to rank various multistatic radar sets, assuming perfect knowledge of the emitters' positions. The effects of position uncertainty, which is highly possible in an operational scenario, might influence the EMRA-based emitter rankings. This is one of our primary motivations looking into this subject.

In this work, we make an extensive use of this multistatic resolution ellipse concept and will refer to it as the MRE from this point forward. However, instead of focusing on EMRA, the primary MRE resolution metric proposed in [2, 5, 15], we also look at other MRE measures including the ellipse semi-major axis, semi-minor axis, linear eccentricity, eccentricity, and tilt angle, equipping ourselves with a set of metrics for a comprehensive analysis. Beside the MRE measures, we also determine the

integrated sidelobe ratio (ISLR) of the multistatic PSF for each set of data and use it as an alternate resolution performance metric. We will revisit and explain the MRE measures and the ISLR in Section 3.3.

3.3 Methodology

3.3.1 Simulating Position Measurement Errors.

Based on the discussion in Section 2.4, we simulate the transmitter position measurement errors by multiplying the ideal bistatic phase history with the phase term $e^{-j2\pi f \frac{\Delta\tilde{R}(\tau)}{c}}$. The differential range error $\Delta\tilde{R}(\tau)$ can be approximated using (13). To isolate the effects of interest, we make the transmitter stationary and consider the position measurement errors only. As a result, the approximated differential range error $\Delta\tilde{R}(\tau)$ in (13) is simplified to

$$\Delta\tilde{R} \approx -\tilde{p}_x \frac{x_t}{R_t} - \tilde{p}_y \frac{y_t}{R_t} - \tilde{p}_z \frac{z_t}{R_t}. \quad (14)$$

where $\tilde{p}_x$, $\tilde{p}_y$, and $\tilde{p}_z$ represent the transmitter position measurement errors in each Cartesian coordinate; x_t , y_t and z_t denote the transmitter's observed mid-aperture coordinates; and R_t represents the transmitter's observed mid-aperture range to the scene center. Note that in (14), the approximated differential range error does not depend on the slow time τ .

Instead of using the far-field approximation in (14), we determine the differential range error $\Delta\tilde{R}$ directly as

$$\Delta\tilde{R} = R_{tx} - R_t, \quad (15)$$

leveraging the fact that the transmitter is stationary and that only position measurement errors are considered. Note that in (15), R_{tx} represents the transmitter's true range to the scene center and R_t denotes the transmitter's observed mid-aperture range to the scene center. Using (15) to compute the differential range error directly allows us to avoid approximation errors and ensure a high degree of precision in our simulation.

To simulate transmitter position uncertainty due to position measurement errors, we randomize the differential range error $\Delta\tilde{R}$ by drawing random numbers from a zero-mean Gaussian distribution with variance σ^2 , forming position error vector $[\tilde{x}, \tilde{y}, \tilde{z}]^T \sim N([0, 0, 0]^T, \sigma^2 I_3)$, where I_3 is a 3×3 identity matrix. Note that $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$, which represent the position errors in Cartesian coordinates, are assumed to be independent and identically distributed Gaussian random variables. The random position errors $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are then added to the true coordinates of the transmitter, resulting in the observed (erred) coordinates x_t , y_t , and z_t of the transmitter. The transmitter's observed mid-aperture range R_t is found by taking the Euclidean distance of $[x_t, y_t, z_t]^T$. We then compute differential range error $\Delta\tilde{R}$ using (15). With $\Delta\tilde{R}$ determined, we proceed to corrupt the ideal phase history with a constant phase error term, resulting in

$$S(f, \tau) = A_0 e^{-j2\pi f \frac{\Delta\tilde{R}}{c}}. \quad (16)$$

Figure 8 shows examples of a bistatic PSF before and after being corrupted with random position measurement errors drawn from a zero-mean Gaussian distribution with 20 m^2 variance.

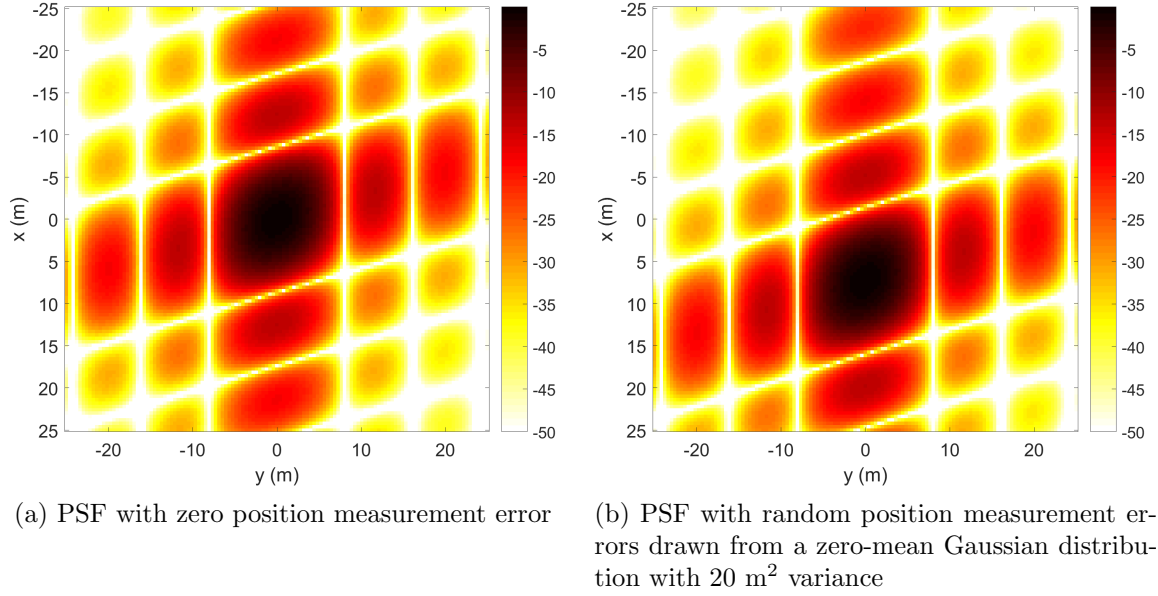


Figure 8. Examples of a bistatic PSF without and with transmitter position measurement errors. In the presence of position measurement errors, the PSF is spatially shifted from the scene center.

Compared to the error-free PSF in Figure 8a, the PSF in Figure 8b is spatially shifted from the scene center, being corrupted with 20 m² position error variance. By changing the variance of the Gaussian distribution, we control the amount of transmitter position uncertainty in our simulation.

3.3.2 Simulating Velocity Measurement Errors.

In simulating the effects of velocity measurement errors on the bistatic PSF, we make the transmitter mobile with a constant true velocity $\vec{v}_t = [v_{xt}, v_{yt}, v_{zt}]^T$. The transmitter has a velocity measurement error $\vec{\tilde{v}} = [\tilde{v}_x, \tilde{v}_y, \tilde{v}_z]^T$, which is constant for the entire aperture. To isolate of the effects of velocity measurement errors, we exclude position and acceleration measurement errors; the differential range error Equation (13) is simplified to

$$\begin{aligned}\Delta\tilde{R}(\tau) \approx & \left[-\tilde{v}_x \frac{x_t}{R_t} - \tilde{v}_y \frac{y_t}{R_t} - \tilde{v}_z \frac{z_t}{R_t} \right] \tau \\ & + \left[-\tilde{v}_x \frac{v_{xt}}{R_t} - \tilde{v}_y \frac{v_{yt}}{R_t} - \tilde{v}_z \frac{v_{zt}}{R_t} \right] \tau^2\end{aligned}\tag{17}$$

where x_t , y_t , and z_t represent the observed Cartesian coordinates of the transmitter at the aperture's midpoint while R_t denotes the transmitter's observed mid-aperture range to the scene center. Note that in (17), the constant and cubic components of $\Delta\tilde{R}(\tau)$ are no longer present with the errors limited to the transmitter velocity measurement errors.

With the errors consisting of transmitter velocity measurement errors only, the differential range error ends up with a linear component and a quadratic component (see (17)). The former causes a spatial shift of the bistatic PSF while the latter has a blurring effect on the PSF [6, 8]. To illustrate, Figure 9 shows a bistatic PSF without and with a velocity measurement error. The PSF mainlobe in Figure 9b is not only shifted to the right but also blurred out, compared to that in Figure 9a.

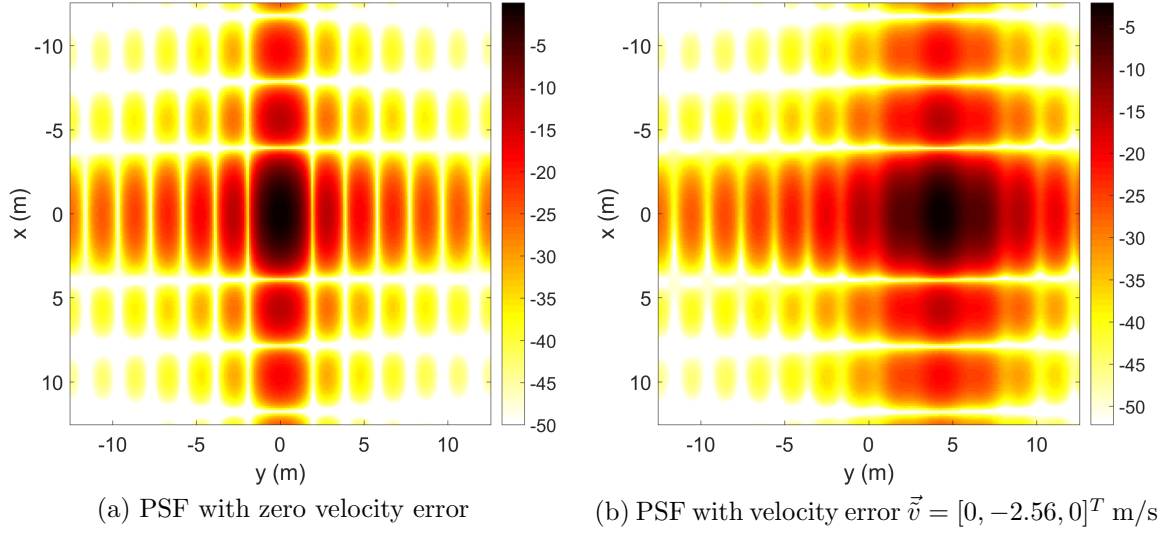


Figure 9. Examples of PSFs without and with a velocity measurement error. The PSF is shifted and blurred out by the linear and quadratic components of the differential range error.

To simulate the effects of velocity measurement errors, we first compute the position error in each Cartesian coordinate axis $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ for each slow time τ , using (9), and add the results to the true coordinates of the transmitter. We then take the medians of the transmitter's erred coordinates in each direction and assign the results to the transmitter's observed mid-aperture coordinates x_t , y_t , and z_t . The transmitter's observed mid-aperture range R_t is simply the Euclidean distance of $[x_t, y_t, z_t]^T$. Using (17), we approximate the differential range error $\Delta\tilde{R}(\tau)$ for each slow time τ . With $\Delta\tilde{R}(\tau)$ determined, we proceed to corrupt the bistatic ideal phase history, resulting in

$$S(f, \tau) = A_0 e^{-j2\pi f \frac{\Delta\tilde{R}(\tau)}{c}}. \quad (18)$$

Equation (18) represents the bistatic phase history embedded with the velocity measurement error $\vec{v}$. Once the velocity measurement error is encoded in the phase history, we construct the bistatic PSF using the filtered back-projection routine.

3.3.3 Simulating Acceleration Measurement Errors.

To simulate the effects of acceleration measurement errors on the bistatic PSF, we set the transmitter in motion with constant true velocity $\vec{v}_t = [v_{xt}, v_{yt}, v_{zt}]^T$. The transmitter has an acceleration measurement error $\vec{a} = [\tilde{a}_x, \tilde{a}_y, \tilde{a}_z]^T$, which is constant for the entire receiver's aperture. To isolate the effects of acceleration measurement errors, we exclude position and velocity measurement errors. The resulting approximation for $\Delta\tilde{R}(\tau)$ (Equation (13)) is

$$\begin{aligned} \Delta\tilde{R}(\tau) \approx & \left[-\frac{1}{2}\tilde{a}_x \frac{x_t}{R_t} - \frac{1}{2}\tilde{a}_y \frac{y_t}{R_t} - \frac{1}{2}\tilde{a}_z \frac{z_t}{R_t} \right] \tau^2 \\ & + \left[-\frac{1}{2}\tilde{a}_x \frac{v_{xt}}{R_t} - \frac{1}{2}\tilde{a}_y \frac{v_{yt}}{R_t} - \frac{1}{2}\tilde{a}_z \frac{v_{zt}}{R_t} \right] \tau^3. \end{aligned} \quad (19)$$

Equation (19) shows that, under the effects of constant acceleration measurement errors, the differential range error $\Delta\tilde{R}(\tau)$ only has the quadratic and cubic components, both of which are expected to cause the image to blur [6, 8]. For example, Figure 10 depicts a bistatic PSF without and with an acceleration error. Compared to the error-free PSF in Figure 10a, a blurring effect is dominant on the erred bistatic PSF in Figure 10b.

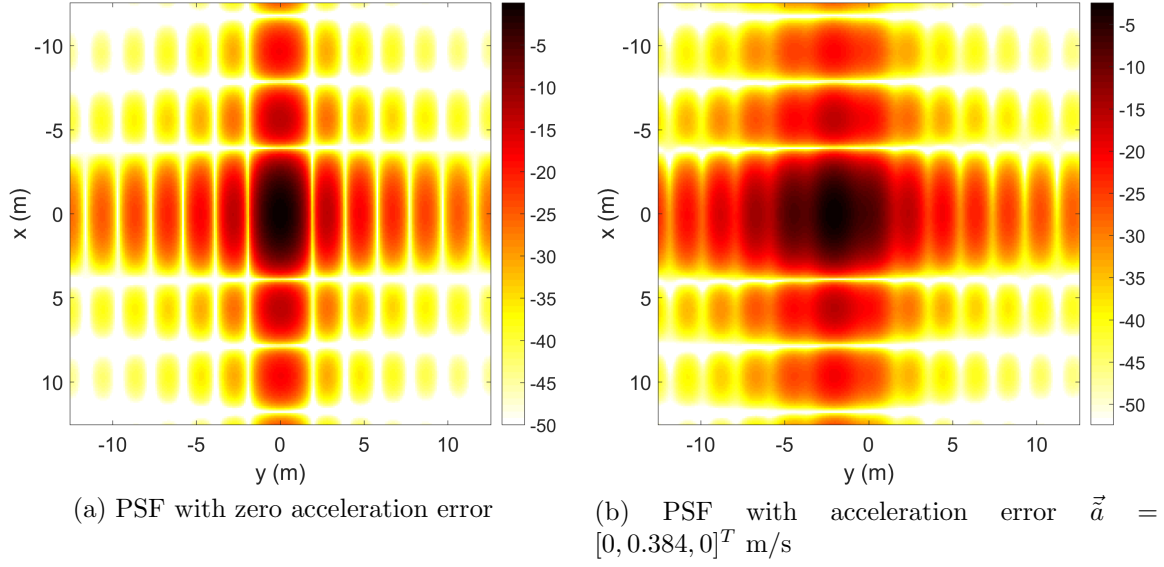


Figure 10. Examples of PSFs without and with an acceleration measurement error. The quadratic and cubic components of the differential range error induce a blurring effect on the image.

In simulate the effects of acceleration measurement errors, we first find the position error in each Cartesian coordinate axis $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ for each slow time τ , using (9), and add the results to the true coordinates of the transmitter. We then take the medians of transmitter's erred coordinates in each direction and assign the results to the transmitter's observed mid-aperture coordinates x_t , y_t , and z_t . The transmitter's observed mid-aperture range R_t is simply the Euclidean distance of $[x_t, y_t, z_t]^T$ to the scene center. In the next step, we approximate the differential range error $\Delta\tilde{R}(\tau)$ for each slow time τ using (19). The resulting differential range error $\Delta\tilde{R}(\tau)$ is then applied in (18) to embed the acceleration measurement error into the ideal phase history of the bistatic pair. Using the corrupted phase history, we proceed to construct the bistatic PSF using the back-projection routine.

3.3.4 Monte Carlo Trials.

To make sure that we run a sufficient number of trials, for each variance listed in Table 2, the number of trials n is determined using

$$\epsilon_\alpha = \frac{\sigma_X}{\sqrt{n}} Q^{-1} \left(\frac{\alpha}{2} \right) \quad (20)$$

where ϵ_α denotes the amount the sample mean is allowed to deviate from the true mean; σ_X denotes the design standard deviation; α represents the level of significance desired and n denotes the minimum number of trials to achieve α [17]. In this work, we set α and ϵ_α equal to 0.05 and 0.1 m, respectively, to ensure that the mean position error for each Cartesian coordinate axis stays within the ± 0.1 m interval with 95% confidence level. Using (20), we determine the minimum number of trials n for each variance listed in Table 2 and round it up to the next thousand.

Table 2. Number of MC trials for each variance to maintain 95% confidence level.

Variance (m ²)	n trials	Variance (m ²)	n trials
1	1000	8	4000
2	1000	9	4000
3	2000	10	4000
4	2000	20	8000
5	2000	30	12000
6	3000	40	16000
7	3000		

In all Monte Carlo (MC) trials, the multistatic radar set consists of a receiver and two transmitters, resulting in two bistatic pairs: Rx-Tx1 and Rx-Tx2. In each Monte Carlo (MC) trial, random position errors $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ are generated for Tx1. The differential range error $\Delta\tilde{R}$ is computed according to (15) and introduced into the phase history of Rx-Tx1 in form of the phase term $e^{-j2\pi\frac{f}{c}\Delta\tilde{R}}$. Using the corrupted phase history, we proceed to generate the bistatic PSF of Rx-Tx1 using a filtered back-projection routine. The process is repeated to generate the bistatic PSF for

Rx-Tx2. The multistatic PSF is determined using the GLRT method, whereby the individual bistatic PSFs' reflectivity magnitudes are compared and the larger of the two is chosen for each multistatic PSF pixel [12]. With the multistatic PSF generated, the MRE measures are determined using the methodology described in Section 3.3.5. This completes the trial and the next trial begins with a new set of random position errors until n trials have been ran.

3.3.5 Computing the Multistatic Resolution Ellipse (MRE) Measures.

An illustration of all six MRE measures can be seen in Figure 11.

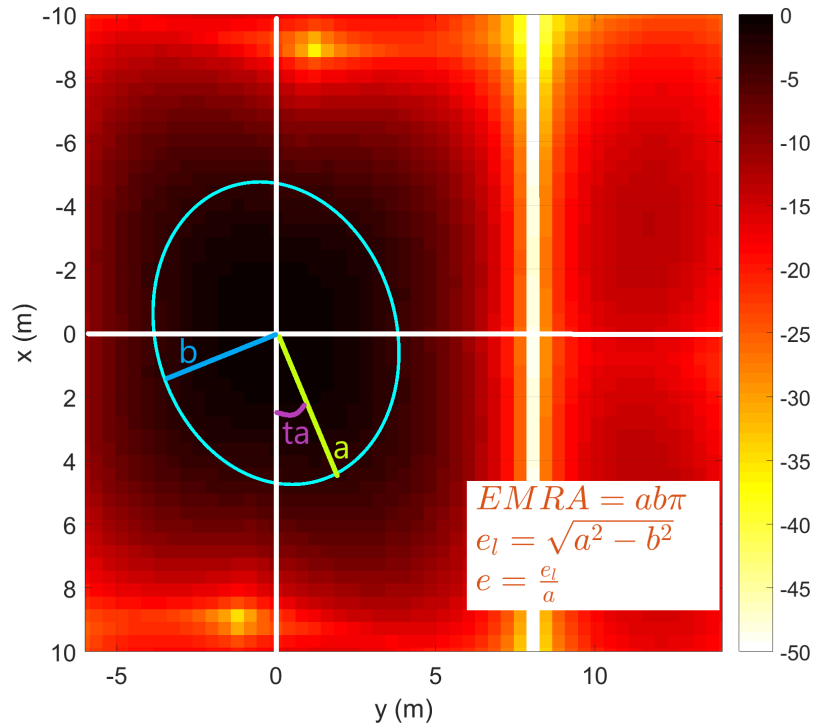


Figure 11. Illustration of semi-major axis, semi-minor axis, tilt angle, EMRA, linear eccentricity and eccentricity.

Note that in Figure 11, the EMRA, linear eccentricity e_l and eccentricity e are derived from the semi-major axis a , semi-minor axis b and the ellipse tilt angle, which are approximated using the singular value decomposition (SVD) process. Specifically, for each MC trial, we determine the MRE measures using the following steps (based

on [5]):

1. Normalize the multistatic PSF by dividing the reflectivity value of each pixel by the maximum value of the image
2. Perform a logical operation to extract the indices of the pixels whose reflectivity magnitudes are equal to or greater than 3dB. Next, use these indices to obtain the Cartesian coordinates (in meters) of these pixels. This step results in a 2-column matrix, denoted as $\mathbf{P}$, representing the x and y coordinates of pixels inside the 3dB contour.
3. Perform a SVD on $\mathbf{P}$ from the previous step, resulting in $\mathbf{U}$, $\mathbf{S}$, and $\mathbf{V}$ matrices. Mathematically, the SVD operation is expressed as

$$\mathbf{P} = \mathbf{U}\mathbf{S}\mathbf{V} \quad (21)$$

where $\mathbf{U}$ is a matrix whose columns represent the eigenvectors of $\mathbf{P}\mathbf{P}^T$, $\mathbf{S}$ is a square matrix whose diagonal entries are the eigenvalues of $\mathbf{P}^T\mathbf{P}$, and $\mathbf{V}$ is a matrix whose columns are the eigenvectors of $\mathbf{P}^T\mathbf{P}$ [18].

4. The semi-major axis, denoted as a , is determined as

$$a = \mathbf{S}_{(1,1)} \max(\mathbf{U}_{(:,1)}) \quad (22)$$

where $\mathbf{S}_{(1,1)}$ represents the first entry on the diagonal of $\mathbf{S}$ and $\max(\mathbf{U}_{(:,1)})$ denotes the largest element of the first column of $\mathbf{U}$.

5. The semi-minor axis, denoted as b , is determined as

$$b = \mathbf{S}_{(2,2)} \max(\mathbf{U}_{(:,2)}) \quad (23)$$

where $\mathbf{S}_{(2,2)}$ represents the second entry on the diagonal of $\mathbf{S}$ and $\max(\mathbf{U}_{(:,2)})$ denotes the largest element of the second column of $\mathbf{U}$.

6. The tilt angle ta (relative to the positive x axis, which points toward the bottom of the page) is determined as

$$ta = \text{atan} \left(\frac{V_{1,1}}{V_{2,1}} \right). \quad (24)$$

7. The MRE's linear eccentricity e_l is determined as

$$e_l = \sqrt{a^2 - b^2}. \quad (25)$$

8. The MRE's eccentricity e is determined as

$$e = \frac{e_l}{a}. \quad (26)$$

9. Finally, EMRA represents the area of the resolution ellipse and is determined as

$$EMRA = ab\pi. \quad (27)$$

Sample values of all six MRE measures are given in Figure 12.

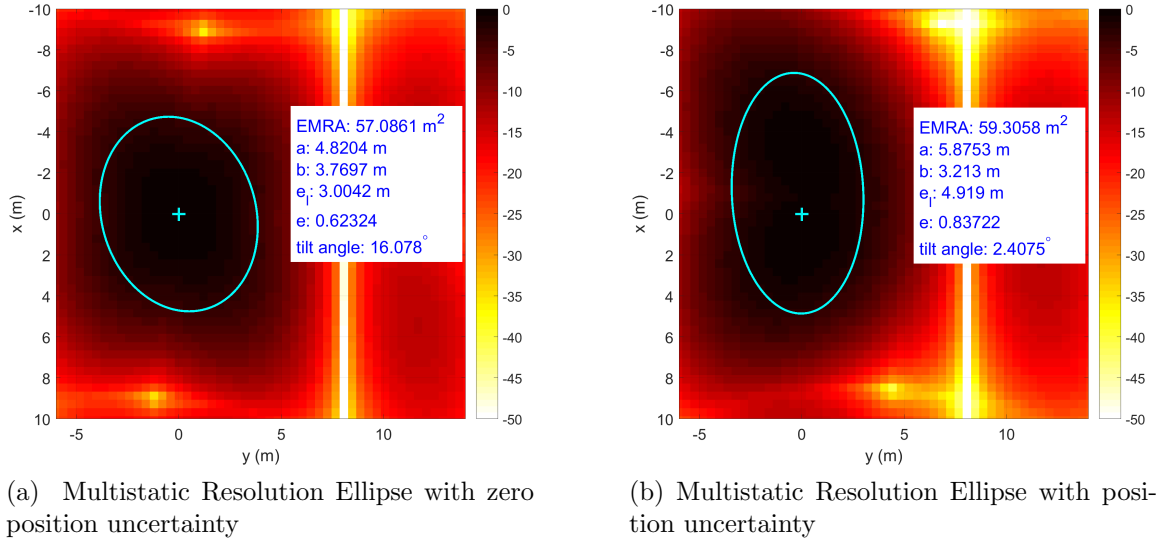


Figure 12. Examples of MRE measures without and with transmitter position uncertainty.

Comparing Figure 12a and Figure 12b, we observe a change in all six MRE measures as a result of introducing position measurement errors into the system. We collect data on all six MRE measures for a thorough analysis. Since some of these measures are strongly related, some similarity in data behaviors is expected. In addition to the aforementioned MRE measures, we also use the integrated sidelobe ratio (ISLR) as a metric since our analysis is based on the PSF.

3.3.6 Computing the Intergrated Sidelobe Ratio (ISLR) of the Multistatic Point Spread Function.

The ISLR is defined in [13] as the ratio of the energy of the sidelobes of the PSF and that of the mainlobe. Since it is a ratio, it is unitless. In this work, the ISLR of the multistatic PSF is determined based on [5], [14] and [19]. With the image size fixed, the ISLR is defined as the ratio of the total reflectivity magnitude of pixels below the 3dB threshold and that of pixels above the 3dB threshold. Symbolically,

the ISLR is expressed as

$$ISLR = \frac{\sum_{x,y: 20 \log_{10} \hat{g}(x,y) < -3dB} \hat{g}(x,y)}{\sum_{x,y: 20 \log_{10} \hat{g}(x,y) \geq -3dB} \hat{g}(x,y)}, \quad (28)$$

where $\hat{g}(x, y)$ represents the normalized PSF and (x, y) denotes the x and y coordinates of the pixel. Specifically, the ISLR is determined for each MC trial using the following steps:

- After forming the multistatic PSF using the GLRT method, normalize the image by dividing each pixel's value by the largest reflectivity magnitude
- Using 3dB threshold, sum up the reflectivity magnitudes of all pixels above the threshold, resulting in the denominator part of the ISLR
- Sum up the reflectivity magnitudes of all pixels below the threshold, resulting in the numerator part of the ISLR
- Compute the ISLR by dividing the numerator by the denominator.

A desirable bistatic PSF has a tall narrow mainlobe and shallow sidelobes; most of the reflectivity is concentrated inside the 3dB contour. As a result, the smaller the ISLR the better the image resolution.

3.3.7 Data Visualization.

To fully examine the behaviors of the MRE and ISLR measures, we use different data visualization methods:

- Box plots such as those in Figure 14 are useful in comparing the amount of data spread and the sample mean among the different position error variances. The blue box in each box plot represents the inter-quartile range containing 50% of the data. The upper whisker and the lower whisker represent data above

the 3rd quartile and below the 1st quartile, respectively. The lengths of the whiskers are limited to 5 times of the inter-quartile range. The red line on the box represents the median. Additionally, the red dot represents the data set's mean value while the green dot denotes the reference value, which is determined with zero position uncertainty.

- We use histograms such as those shown in Figure 15 as a means to visualize how each data set is distributed. The histogram in each subplot in Figure 15 represents the estimated probability density function (PDF) of the data set. The red vertical line represents the data set's mean value while the green vertical line represents the reference value.
- Additionally, we use data spread charts such as those shown in Figure 21 to compare the amounts of data spread among the data sets. The green dot in Figure 21 represents the difference between the sample mean and the reference value. This parameter will be referred to as the sample mean - reference value gap at various points. For each data set, we find the difference between the samples and the corresponding reference value then take the standard deviation of the results. In Figure 21, the $\pm$ standard deviations are depicted with a red and an orange dot; the length of the vertical line segment connecting the red and orange dots indicates the amount of data spread and thus the extent of the effects of transmitter position uncertainty. We use this metric as a means to compare various random data sets.

IV. Results and Analysis

4.1 Chapter Overview

This chapter covers the simulation setup and our result analysis of ten scenarios. The chapter is divided into sections, each of which, except for Section 4.4 and 4.10, is dedicated to covering a scenario. Each section begins with a statement of objectives for the scenario, followed by an explanation of how we set up the simulation. We then present a thorough analysis of the results. Key findings are summarized at the end of each section.

4.2 Scenario 1: Effects of Position Error Variance

In Scenario 1, we examine the effects of position uncertainty on the multistatic image resolution. The effects are observed via the MRE measures and the ISLR. In Scenario 1, the transmitter position uncertainty is due to position measurement errors only.

4.2.1 Simulation Setup.

In Scenario 1, the radar consists of two transmitters and a single receiver (see Figure 13 for a depiction of the radar geometry). The radar geometry is listed in detail in Table 3.

In Scenario 1, both transmitters are stationary and emit a typical commercial broadcast orthogonal frequency division multiplexing (OFDM) waveform. Without loss of generality, the receiver's position is assumed precisely known for each pulse; all position errors occur at the transmitter end. For simplicity, Tx1 and Tx2 are assumed identical in terms of operating frequencies and bandwidths but not interfering with each other. The single receiver is assumed capable of receiving and processing the

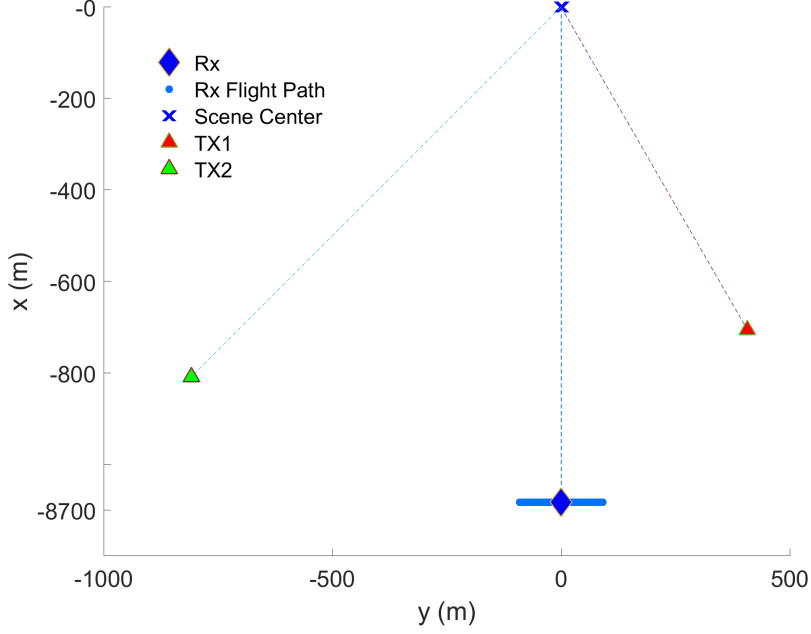


Figure 13. Radar geometry in Scenarios 1 and 2.

echoed signals originating from the two transmitters separately. The simulation starts immediately after the matched filter block in the signal processing chain. The return signals are assumed perfectly recovered and the matched filter is assumed ideal.

We corrupt the ideal bistatic phase history of each bistatic pair with random position errors in all three Cartesian coordinate axes, denoted as $\tilde{x}_t$, $\tilde{y}_t$ and $\tilde{z}_t$, drawn from a zero-mean Gaussian distribution. Using 13 position error variance values, ranging from 1 to 10, 20, 30 and 40 m², we run 13 sets of Monte Carlo (MC) trials. The number of MC trials, denoted as n , for each set of trials is dependent on the position error variance (PEV) used to generate random position errors for both transmitters. The number of MC trials for each of the 13 sets of trials in Scenario 1 are listed in Table 2. The reference value for each MRE measure and that of the ISLR in the upcoming box plots and data spread plots are determined with zero transmitter position uncertainty.

Table 3. Geometry of Scenarios 1 and 2.

		Scenario 1	Scenario 2
Tx1	Range (m)	820	820
	Azimuth ($^{\circ}$)	30	30
	Elevation ($^{\circ}$)	7	7
	x-coordinate (m)	705	705
	y-coordinate (m)	407	407
	z-coordinate (m)	100	100
	variance (m^2)	$\in [1, 40]$	1
Tx2	Range (m)	1147	1147
	Azimuth ($^{\circ}$)	-45	-45
	Elevation ($^{\circ}$)	5	5
	x-coordinate (m)	808	808
	y-coordinate (m)	808	808
	z-coordinate (m)	100	100
	variance (m^2)	$\in [1, 40]$	$\in [1, 40]$
Rx	Range (m)	10000	10000
	Azimuth ($^{\circ}$)	$[-0.6, 0.6]$	$[-0.6, 0.6]$
	Elevation ($^{\circ}$)	30	30

4.2.2 Results.

The data distribution of all six MRE measures in each set of trials can be seen in the box plots in Figure 14. Note that the upper and lower whisker are set to extend up to five times the inner quartile range (the length of the box part of the box plot). In other words, the whiskers can be lengthened as much as five times the inner quartile range to cover most of the data; data points beyond this range are considered outliers and marked individually with red plus signs, which appear on several upcoming box plots.

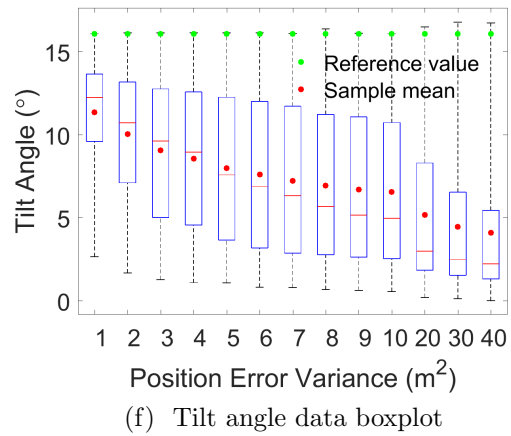
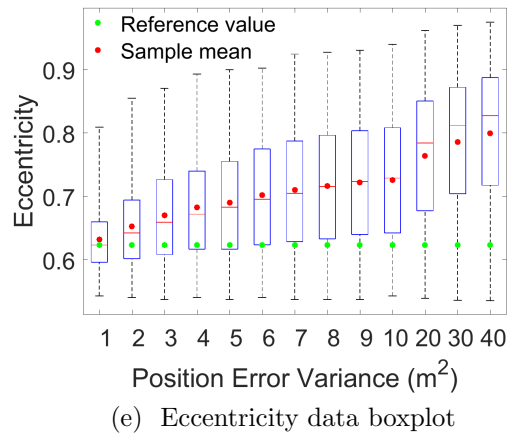
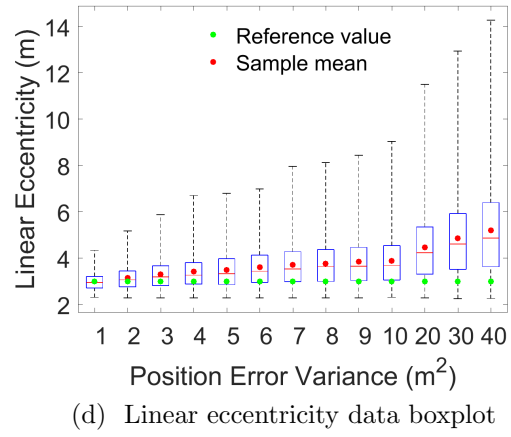
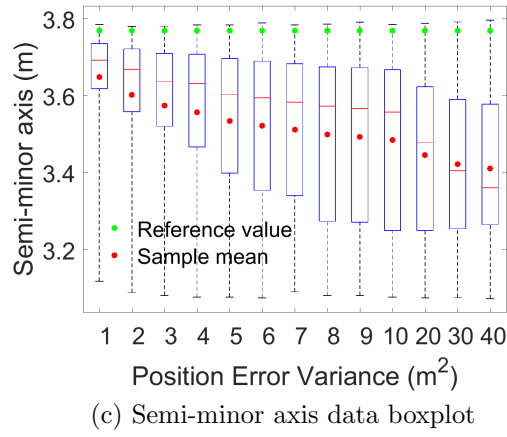
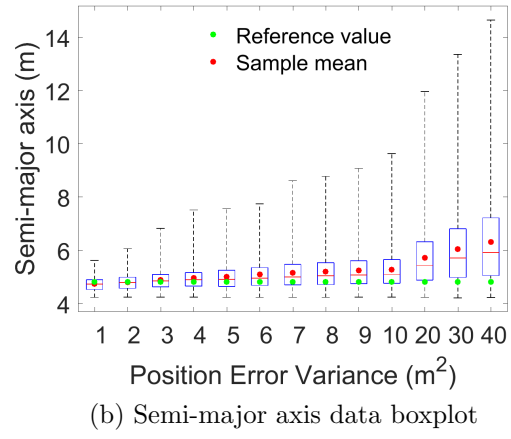
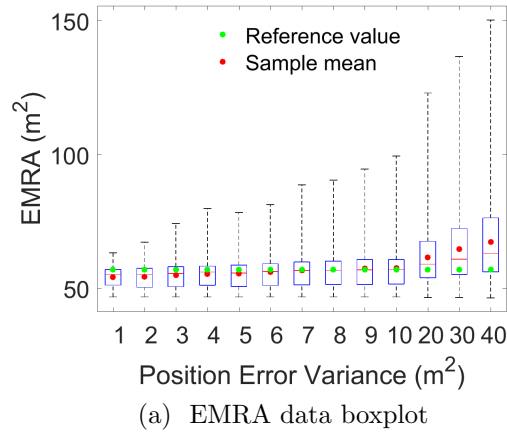
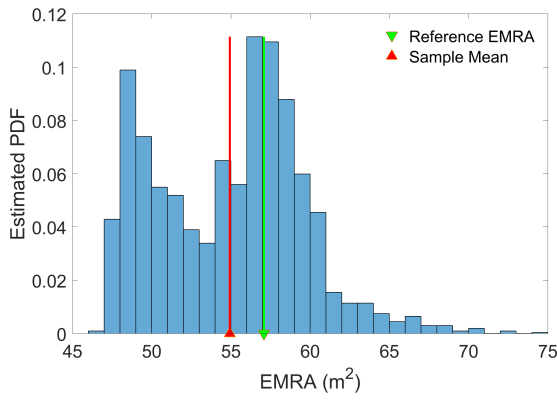


Figure 14. Scenario 1 MRE measures data with different position error variance values.

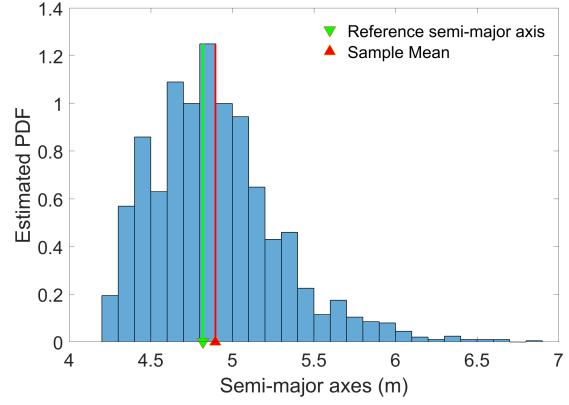
In Figures 14a, 14b, and 14d, the EMRA, semi-major axis, and linear eccentricity data spread over a larger range as the position error variance (PEV) increases, telling by the overall lengths of the box plots. Furthermore, the EMRA, semi-major axis, linear eccentricity sample means (red dots) increase with growing PEV, compared to the reference values (green dots). While the eccentricity box plots in Figure 14e only show moderate growth in range as the PEV increases, the eccentricity sample mean (red dot) aggressively grows away from the reference value (green dot). The semi-minor axis box plots in Figure 14c do not vary by much in overall range, but the sample mean (red dot) decreases away from the reference semi-minor axis (green dot), as the PEV increases. The sample observations apply to the tilt angle box plots in Figure 14f.

Overall, the trends observed in Figure 14 suggest that, as the transmitter position uncertainty increases, the MRE grows larger, more eccentric and less tilted on average. Furthermore, we observe more data spread on the MRE measures as the position uncertainty grows.

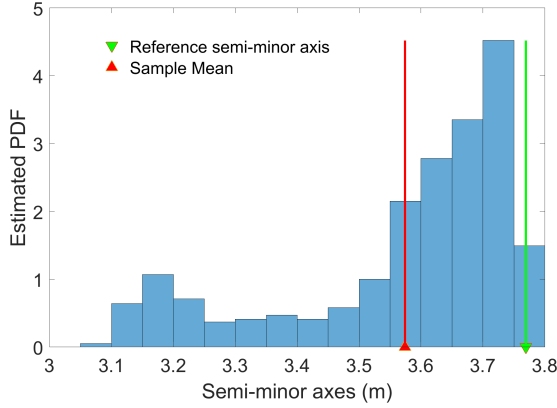
The estimated PDF of all six MRE measures in one of 13 sets of trials are shown in Figure 15. In this set of trials, we use 3 m^2 variance to generate random errors for both transmitters.



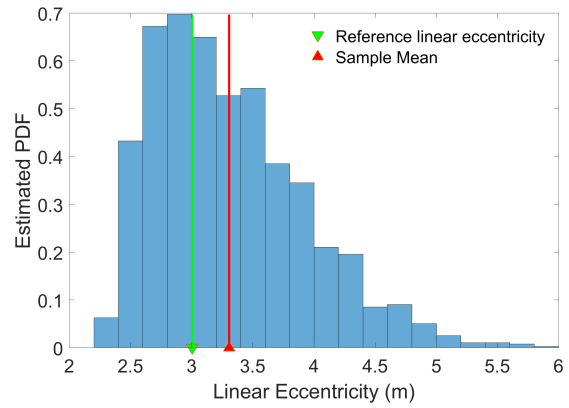
(a) EMRA data distribution



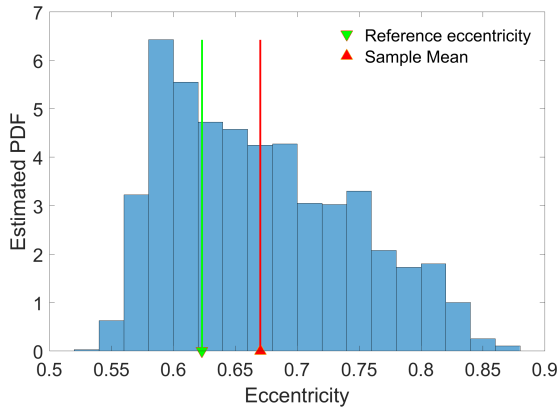
(b) Semi-major axis data distribution



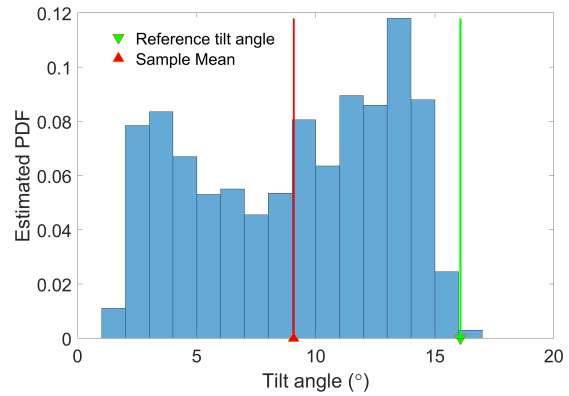
(c) Semi-minor axis data distribution



(d) Linear eccentricity data distribution



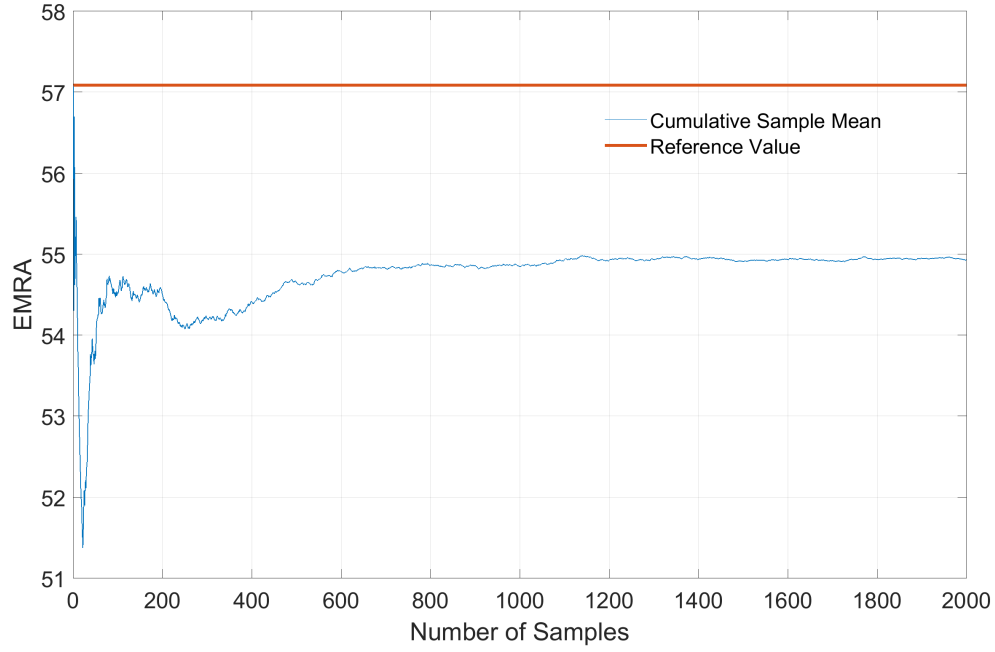
(e) Eccentricity data distribution



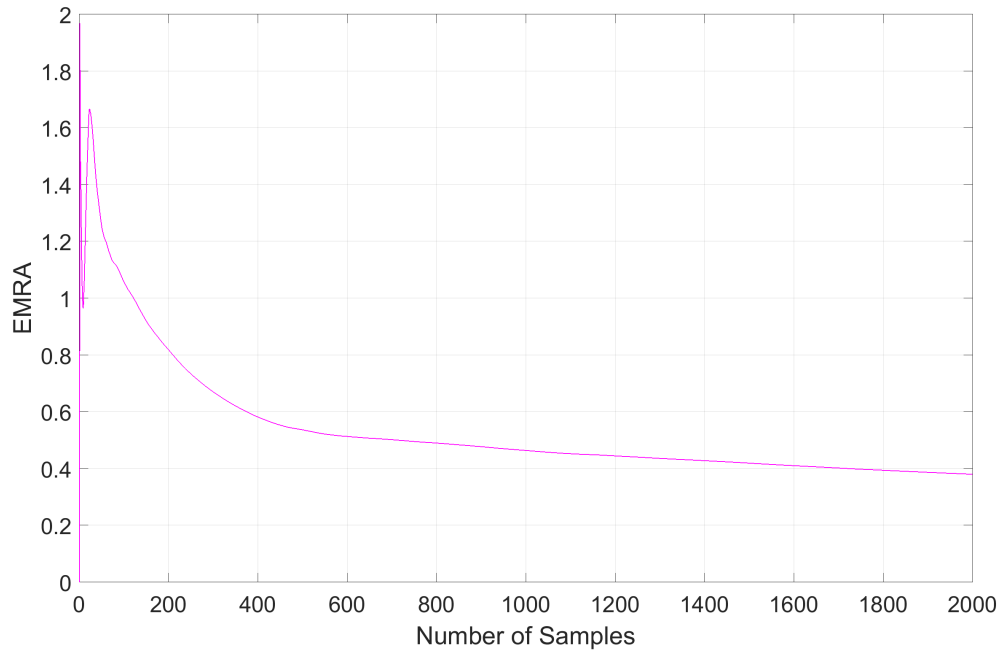
(f) Tilt angle data distribution

Figure 15. Scenario 1 MRE measures' distributions with 3 m² position error variance.

Note that in the estimated PDF plots in Figure 15, the sample mean does not agree with the reference value. In fact, it can be seen in the box plots in Figure 14 that the sample mean grows away from the reference value (determined with no transmitter position uncertainty) as the position error variance increases. However, a convergence check shows that, for each variance, the sample mean approaches a stable value while the variance of the sample mean approaches zero as the number of samples increases (see Figures 16a and 16b). These observations agree with the central limit theorem's provisions [17], indicating that the number of data points taken are statistically sufficient. We perform a similar convergence check for each MRE measure data set and observe that the convergence conditions are met in all.



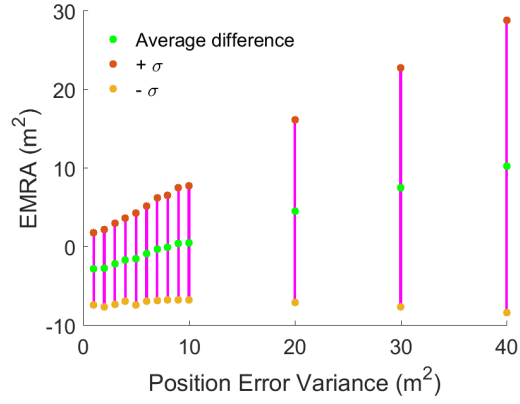
(a) EMRA sample mean converging to a stable value as number of samples increases.



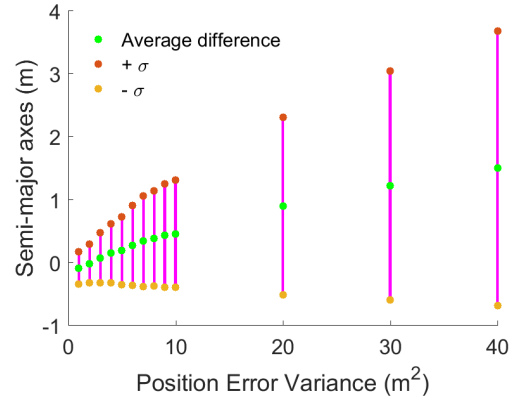
(b) The standard deviation of EMRA sample mean decreases toward zero as the number of samples increases.

Figure 16. Convergence check results for EMRA data set with 3 m² position error variance in Scenario 1.

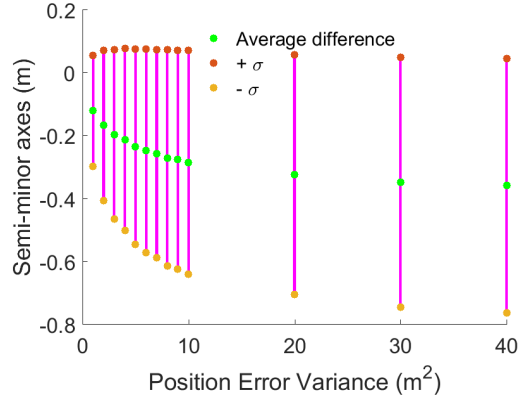
In order to characterize the data spread of MRE measures, we use data spread plots like those in Figure 17. In each subplot, the green dots represent the difference between the sample mean and the corresponding reference value. For each MRE measure, the red and orange dots represent the $\pm$ standard deviation of the difference between the samples and the corresponding reference value. This parameter is referred to as the sample mean-reference value gap at various points. The length of each vertical line segment indicates the amount of data spread corresponding to each position error variance.



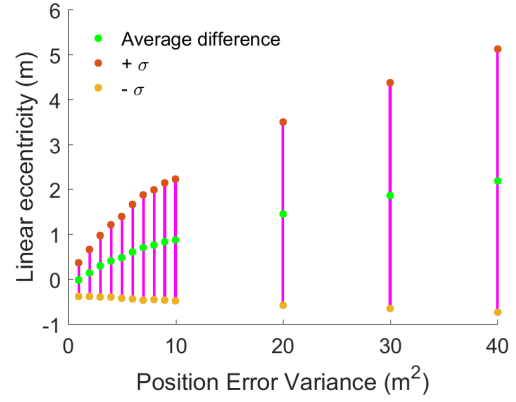
(a) ± 1 standard deviation of the difference between the EMRA samples and the reference value



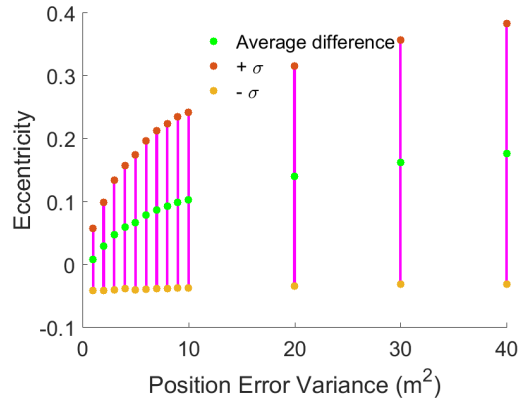
(b) ± 1 standard deviation of the difference between the semi-major axis samples and the reference value



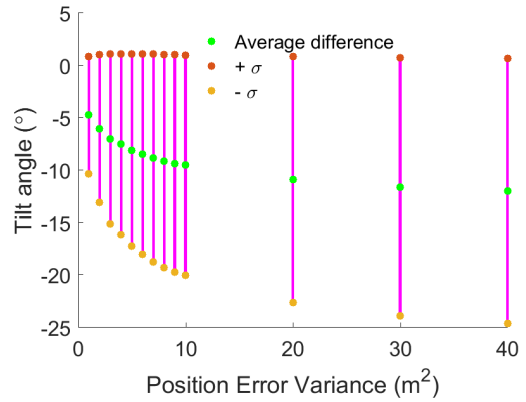
(c) ± 1 standard deviation of the difference between the semi-minor axis samples and the reference value



(d) ± 1 standard deviation of the difference between the linear eccentricity samples and the reference value



(e) ± 1 standard deviation of the difference between the eccentricity samples and the reference value

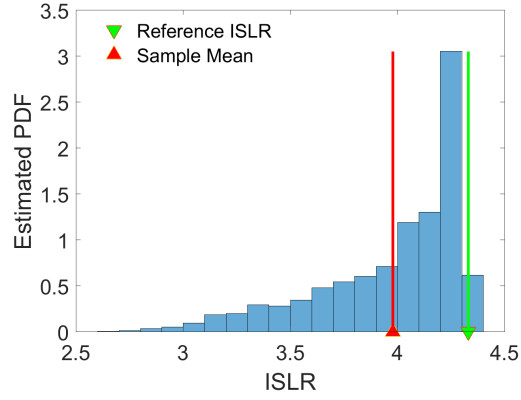


(f) ± 1 standard deviation of the difference between the tilt angle samples and the reference value

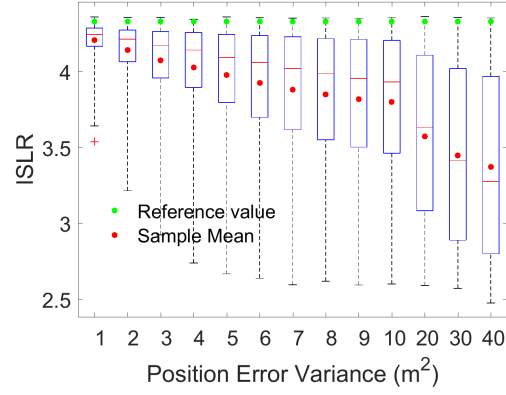
Figure 17. Amount of data spread of each MRE measure in Scenario 1.

Figure 17a shows that as the position error variance (PEV) grows, the sample mean-reference value gap increases. The EMRA spread also increases with growing PEV as indicated by the lengthening vertical line segment. The same observations apply to semi-major axis, linear eccentricity and eccentricity data as seen in Figures 17b, 17d and 17e, respectively. The semi-minor axis and the tilt angle data (Figures 17c and 17f) show a slightly different trend: the sample mean-reference value gap and the $\pm$ standard deviation also grow with increasing PEV but at a slowing rate. These observations suggest that as the PEV increases, the MRE grows larger, more elongated and less tilted on average; furthermore, we observe more data spread on the MRE measures.

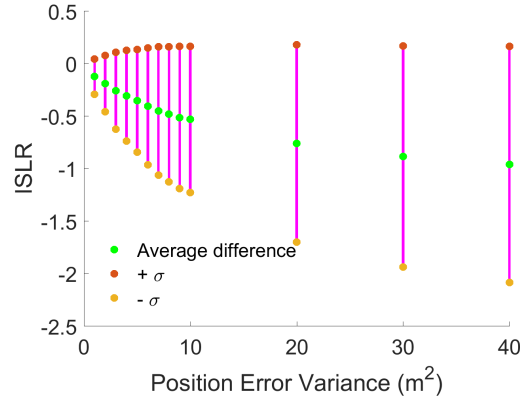
Traditionally, the ISLR of a PSF represents the how much reflectivity of pixels making up the sidelobes compared to that of the pixels making up the mainlobe [13, 14]. Consequently, a lower ISLR indicates better image resolution. Figure 18a depicts the distribution of the ISLR data generated with 5 m² variance. Note that the sample mean is lower than the reference ISLR (determined with zero position uncertainty). The box plots in Figure 18b show that the sample mean continues to decrease further as the PEV grows. Additionally, the data spread plots in Figure 18c indicate that the ISLR's data spread increases as the PEV grows.



(a) ISLR distribution for 5 m² position error variance. The sample mean is smaller than the reference value (with zero-uncertainty).



(b) ISLR data for different position error variances. The sample mean continues to decrease from the reference value as the PEV increases.



(c) ± 1 standard deviation of the difference between the ISLR samples and the reference value. The ISLR data spread increases with growing position error variance.

Figure 18. Scenario 1 ISLR data for different position error variance values.

Comparing the MRE data in Figure 17 and the ISLR data in Figure 18c, we observe opposite patterns. Compared to the reference values, the MRE measures including the EMRA, semi-major axis, eccentricity, and linear eccentricity, on average, grow with increasing PEV while the ISLR sample mean decreases with increasing PEV. On the other hand, both the MRE measures and the ISLR tend to spread out more with increasing PEV.

Finally, Figure 19 depicts the averaged MREs in 13 sets of trials; the MREs are color-coded based on PEV, as indicated on the color bar in the figure. Figure 19 confirms that, on average, the MRE grows larger, more eccentric and less tilted, compared with the reference MRE (highlighted in green), as transmitter position uncertainty increases.

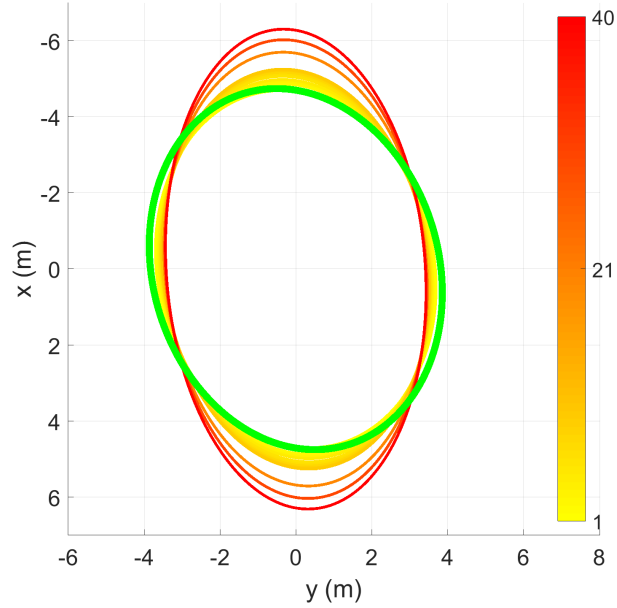


Figure 19. Behaviors of the average multistatic resolution ellipse (MRE) in the presence of transmitter position uncertainty in Scenario 1. The MREs are color-coded based on PEV, as indicated on the color bar. The MRE grows in area and eccentricity as PEV increases.

4.2.3 Key Findings.

- In the presence of transmitter position uncertainty, the MRE measures and the ISLR, on average, are different from the reference values. The gap between the sample mean and the reference value is dependent on the amount of transmitter position uncertainty.
- As the transmitter position uncertainty increases, the MRE, on average, grows in size and eccentricity, compared to the error-free MRE.
- As the transmitter position uncertainty grows, we observe increasing amounts of data spread on the MRE measures and the ISLR.

4.3 Scenario 2: Effects of Position Error Variance of Individual Bistatic Pairs

In Scenario 2, we examine the effects of position error variance of individual transmitters on the multistatic resolution, using MRE measures and the ISLR as metrics. Unlike Scenario 1, the transmitters in Scenario 2 are affected by different amounts of position uncertainty.

4.3.1 Simulation Setup.

The radar geometry of Scenario 2 is identical to that of Scenario 1 (see Figure 13 and Table 3 for more details). Similar to Scenario 1, Scenario 2 also consists of 13 sets of trials. For all trial sets, the position error variance (PEV) of Rx-Tx1 pair is maintained at 1 m^2 . The PEV of Rx-Tx2 pair is stepped through 13 values, from 1 to 10, 20, 30 and 40 m^2 . The number of MC trials n for each set of trials is determined by the PEV used to generate random errors for Rx-Tx2 pair (see Table 2).

4.3.2 Results.

By fixing Tx1's position error variance (PEV) at 1 m^2 and letting Tx2's PEV grow in the same manner as Scenario 1, we observe the same phenomena in Figure 20 as we did in Scenario 1.

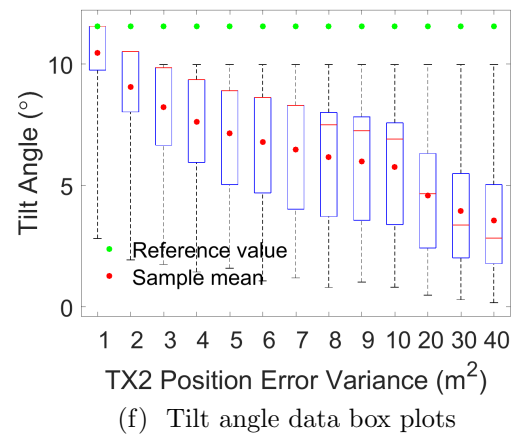
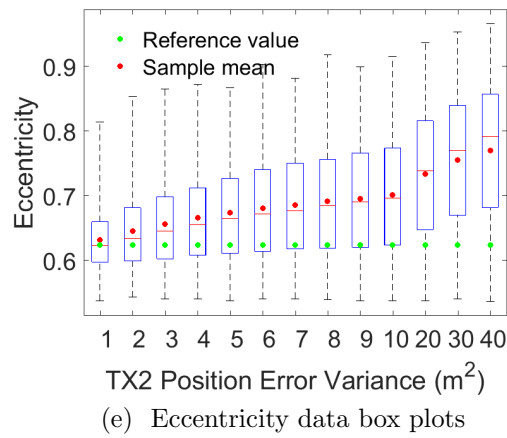
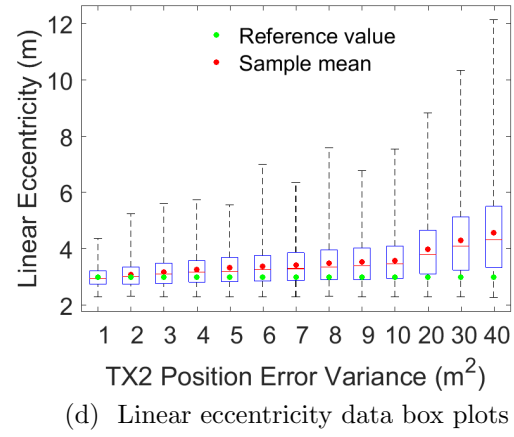
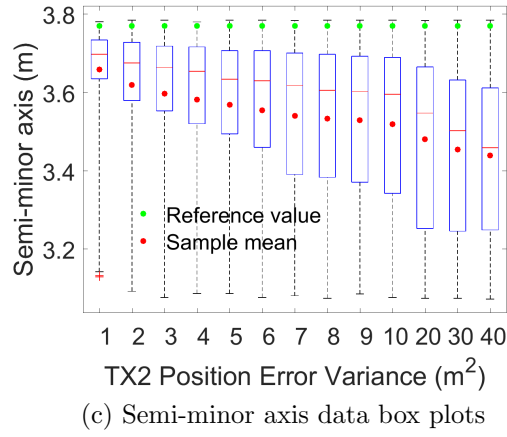
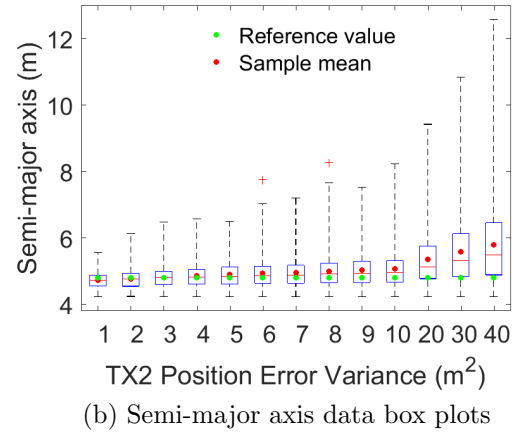
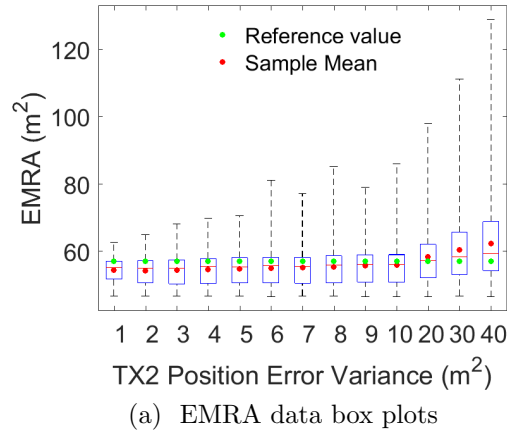


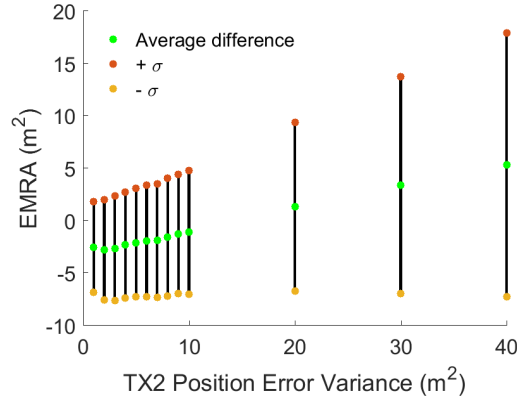
Figure 20. Scenario 2 MRE data with different Tx2's position error variances.

In Figures 20a, 20b, and 20d, the EMRA, semi-major axis, and linear eccentricity sample mean increases with growing Tx2's position error variance (PEV). More significant, the data spread for each of the three measures also increases as Tx2's PEV increases. Figure 20e shows the eccentricity sample mean increases with larger Tx2's PEVs while the eccentricity data range only grows moderately. The semi-minor axis and tilt angle box plots in Figures 20c and 20f, respectively, show the sample mean decrease with increasing Tx2's PEV. These observations suggest that the MRE measures and thus, the multistatic resolution, are strongly dependent on the bistatic pair with predominantly larger position uncertainty.

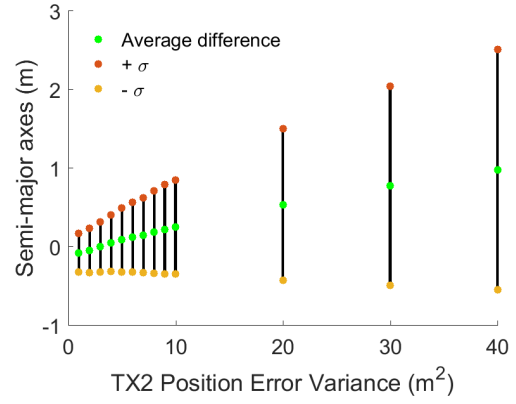
As in Scenario 1, we attempt to characterize the data spread of MRE measures as Tx2's position error variance (PEV) increases in plots like those showed in Figure 21. Figure 21a shows that, as Tx2's PEV grows, the sample mean-reference value gap (the green dot) increases. Again, the sample mean-reference value gap refers to the difference between the sample mean and the reference value (determined with zero uncertainty) for each measure. More important, the $\pm$ standard deviation of the difference between EMRA data points and the reference value also grows with increasing Tx2's PEV, telling by the lengthening vertical line segments. The same observations apply to semi-major axis, linear eccentricity, and eccentricity in Figures 21b, 21d, and 21e, respectively. Figure 21c reveals that the semi-minor axis sample mean-reference value gap and the semi-minor axis data spread ($\pm$ standard deviation) grow at a slower rate with increasing Tx2's PEV. The same observations apply to the tilt angle data in Figure 21f. These observations suggest that as Tx2's PEV increases, the data spread observed on the MRE measures also increases; however, on average, the MRE grows larger, more eccentric and less tilted.

As in Scenario 1, for each Scenario 2 MC trial, the ISLR is determined from the same multistatic PSF used to approximate the MRE measures. The ISLR data for

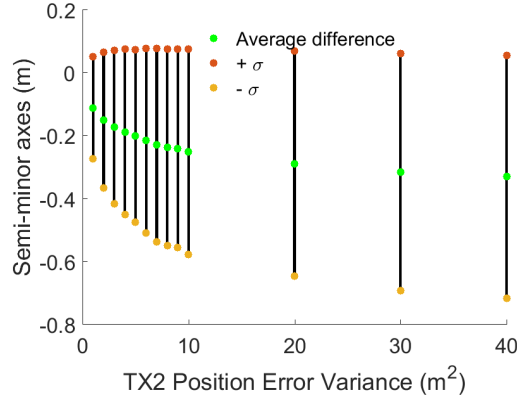
all 13 sets of trials are shown in Figure 22. Similar to Figure 18, Figure 22 shows that the ISLR sample mean for each PEV value is lower than the reference value and the data spread increases with growing PEV.



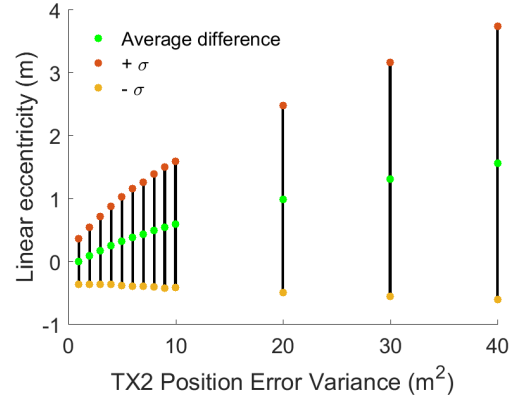
(a) ± 1 standard deviation of the difference between the EMRA samples and the reference value



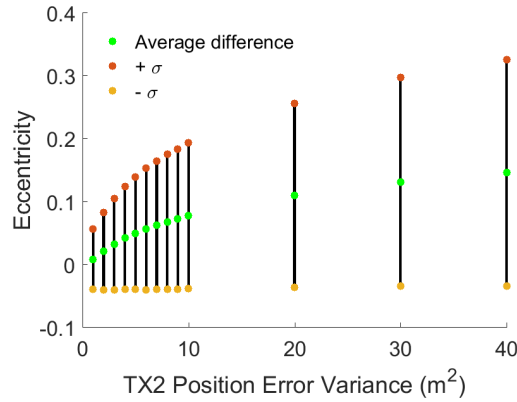
(b) ± 1 standard deviation of the difference between the semi-major axis samples and the reference value



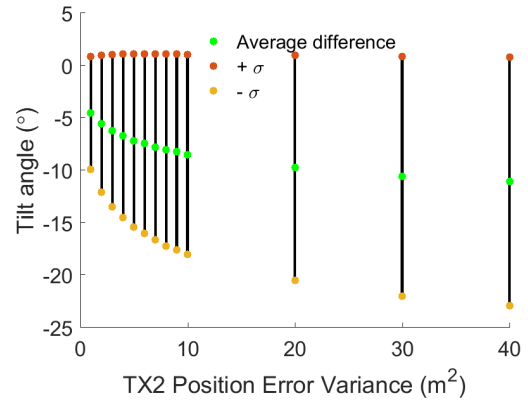
(c) ± 1 standard deviation of the difference between the semi-minor axis samples and the reference value



(d) ± 1 standard deviation of the difference between the linear eccentricity samples and the reference value

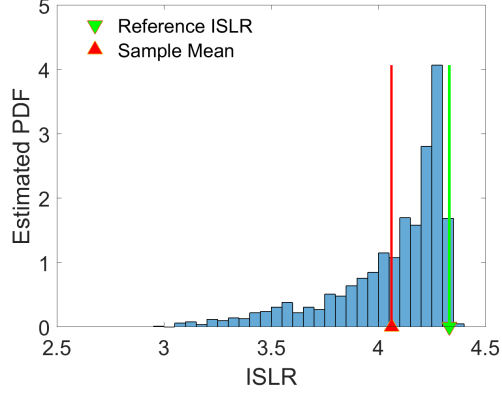


(e) ± 1 standard deviation of the difference between the eccentricity samples and the reference value

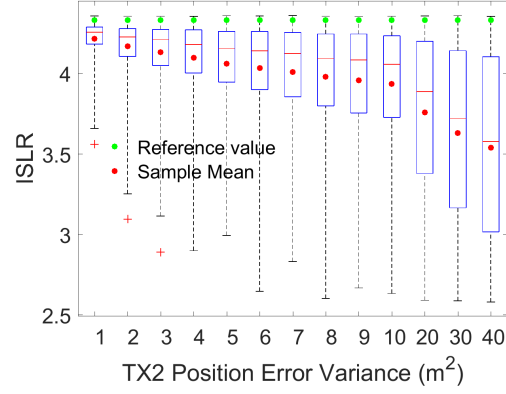


(f) ± 1 standard deviation of the difference between the tilt angle samples and the reference value

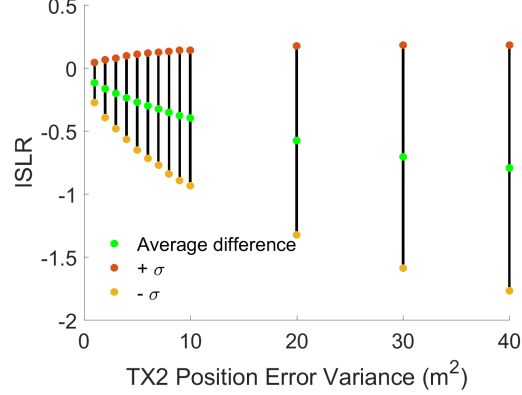
Figure 21. Amount of data spread of each MRE measure in Scenario 2.



(a) ISLR distribution for 5 m² Tx2 position error variance. The sample mean is smaller than the reference value (with zero-uncertainty).



(b) ISLR data for different Tx2 position error variances. The sample mean continues to decrease from the reference value as the PEV increases.



(c) ± 1 standard deviation of the difference between the ISLR samples and the reference value. The ISLR data spread increases with growing Tx2 position error variance.

Figure 22. Scenario 2 ISLR data for different Tx2 position error variances.

Finally, the above observations are further confirmed in Figure 23, which shows the average MRE increases in size and eccentricity, compared to the error-free MRE (highlighted in green), as Tx2's PEV grows and becomes more dominant. In Figure 23, Tx2's PEV is indicated on the color bar.

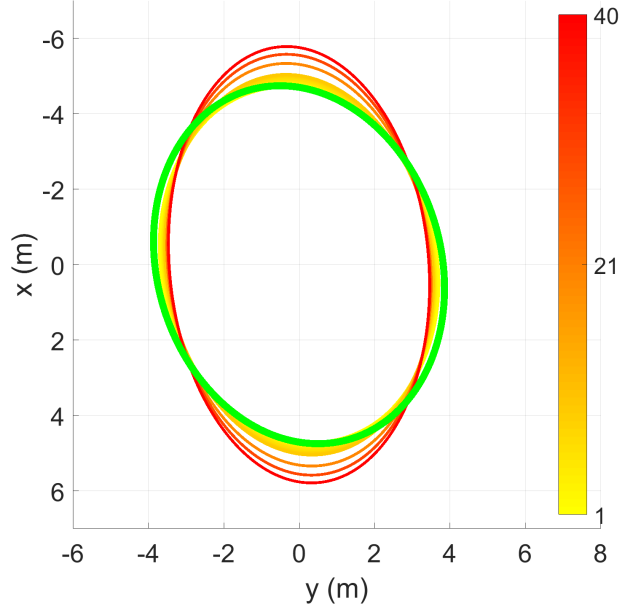


Figure 23. Behaviors of the average MRE in the presence of transmitter position uncertainty in Scenario 2. The MREs are color-coded based on the PEV of Tx2, as indicated on the color bar. Similar to Scenerio 1, The average MRE in Scenario 2 also grows in area and eccentricity as the PEV of Tx2 increases.

4.3.3 Key Findings.

- The position error variance of individual bistatic pairs contribute to the overall behaviors of the MRE. As the PEV increases, the MRE grows in area and eccentricity. More important, we observe larger amounts of data spread on all measures with larger PEV.
- The MRE measures and thus the multistatic resolution are strongly dependent on the bistatic pair with predominantly larger position uncertainty. The larger

the difference in levels of position uncertainty between the individual bistatic pairs, the stronger the dependency and the more obvious the effects.

4.4 Scenarios 3a, 3b and 3c: Effects of Transmitter Position Uncertainty and the Radar Geometry

In Scenarios 3a-3c, we look into how transmitter position uncertainty affects the multistatic resolution in different radar geometries. The transmitter position uncertainty in Scenarios 3a-3c is due to position measurement errors only since the transmitters are stationary. Henceforth, the term bistatic angle, which is defined as the angle formed by the transmitter and the receiver with the vertex being the target's location, will come up often [8]. Since our target is located at the scene center and the receiver's azimuth extent is centered at 0° , a larger azimuth angle invariably means a larger bistatic angle.

4.4.1 Simulation Setup.

In Scenario 3a, we change the azimuth of Tx2 11 times, from -90° to 90° in 18° increments while fixing Tx1's azimuth at 0° , resulting in 11 sets of MC trials. For each Tx2's azimuth, we run 8000 MC trials, using 20 m^2 position error variance to generate random position errors for both bistatic pairs, Rx-Tx1 and Rx-Tx2. The radar geometry in Scenario 3a is illustrated in Figure 24 and listed in Table 4.

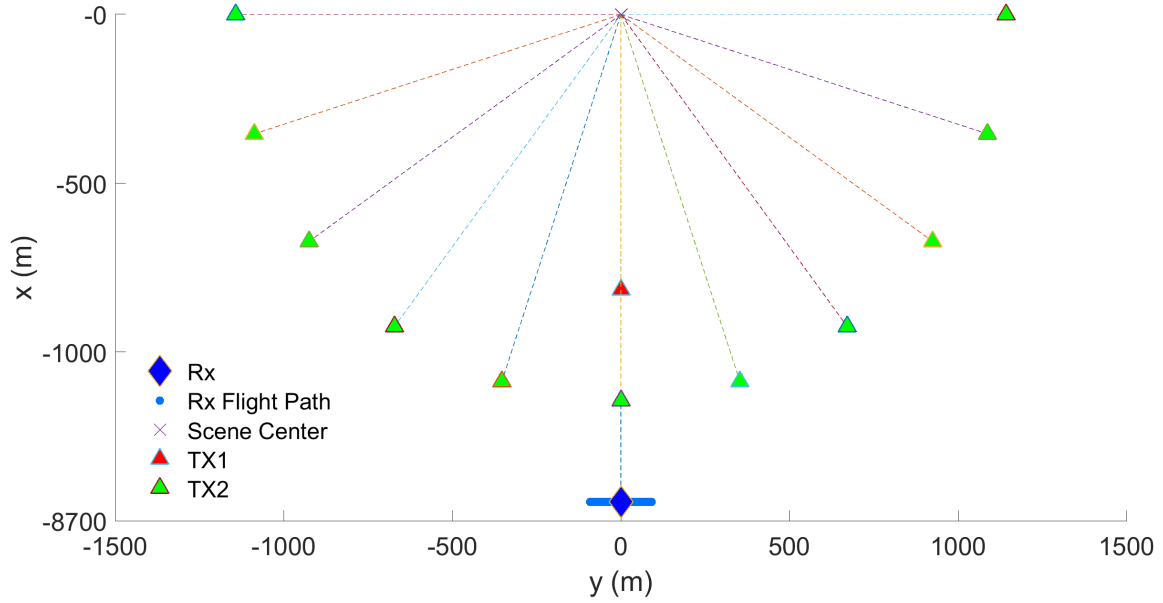


Figure 24. Radar geometry in Scenario 3a. With Tx1 fixed at 0° azimuth, Tx2's azimuth is increased by 18° after each set of 8000 MC trials.

Table 4. Geometry of Scenarios 3a and 3b.

		Scenario 3a	Scenario 3b	Scenario 3c
Tx1	Range (m)	820	820	820
	Azimuth ($^\circ$)	0	85	$\in [-90, 90]$
	Elevation ($^\circ$)	7	7	7
	x-coordinate (m)	814	71	$\in [0, 814]$
	y-coordinate (m)	0	811	$\in [-814, 814]$
	z-coordinate (m)	100	100	100
	variance (m^2)	20	20	20
Tx2	Range (m)	1147	1147	1147
	Azimuth ($^\circ$)	$\in [-90, 90]$	$\in [-90, 90]$	$\in [-90, 90]$
	Elevation ($^\circ$)	5	5	5
	x-coordinate (m)	$\in [0, 1143]$	$\in [0, 1143]$	$\in [0, 1143]$
	y-coordinate (m)	$\in [-1143, 1143]$	$\in [-1143, 1143]$	$\in [-1143, 1143]$
	z-coordinate (m)	100	100	100
	variance (m^2)	20	20	20
Rx	Range (m)	10000	10000	10000
	Azimuth ($^\circ$)	$[-0.75, 0.75]$	$[-0.75, 0.75]$	$[-0.75, 0.75]$
	Elevation ($^\circ$)	30	30	30

In Scenario 3b, we fix Tx1 at 85° azimuth and change Tx2's azimuth 11 times, from -90° to 90° in 18° increments, resulting in 11 sets of MC trials. For each set of trials, we run 8000 MC trials, which is the minimum number of trials required for 20 m^2 position error variance, as listed in Table 2. The reference value for each MRE measure and the ISLR are determined with zero transmitter position uncertainty. The radar geometry in Scenario 3b is depicted in Figure 25 and listed in Table 4.

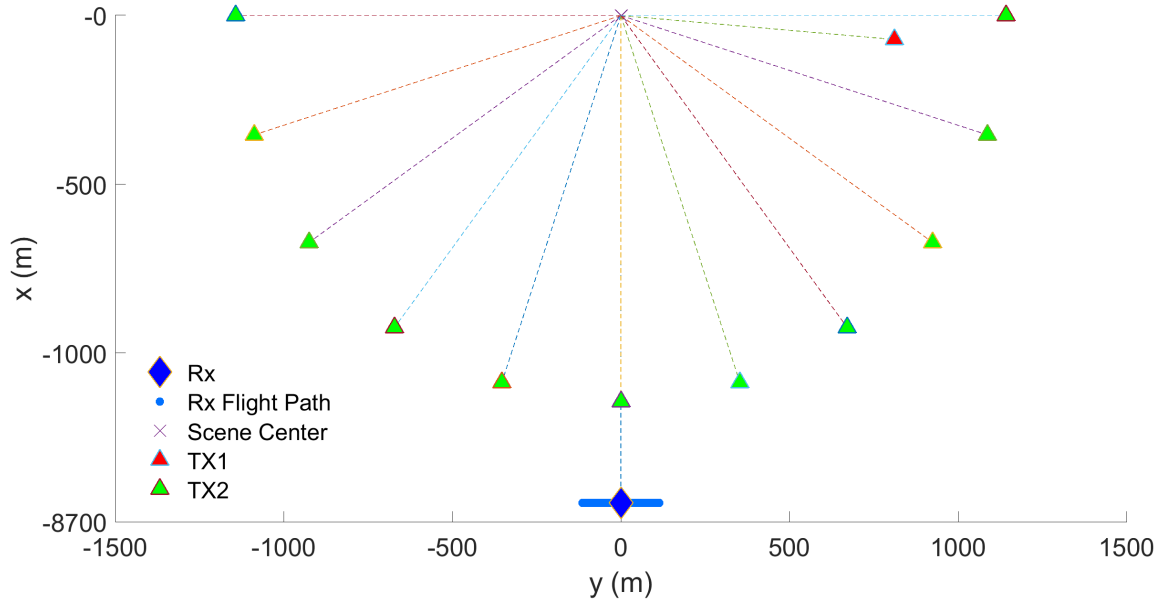


Figure 25. Radar geometry in Scenario 3b. With Tx1 fixed at 85° azimuth, Tx2's azimuth is increased by 18° after each set of 8000 MC trials.

Lastly, in Scenario 3c, we initially place Tx1 at 90° azimuth and Tx2 at -90° azimuth. After each set of 8000 MC trials, we decrease Tx1's azimuth by 18 degrees and increase Tx2's azimuth by the same amount, resulting in 11 sets of MC trials. The radar geometry in Scenario 3c is depicted in Figure 26 and listed in detail in Table 4.

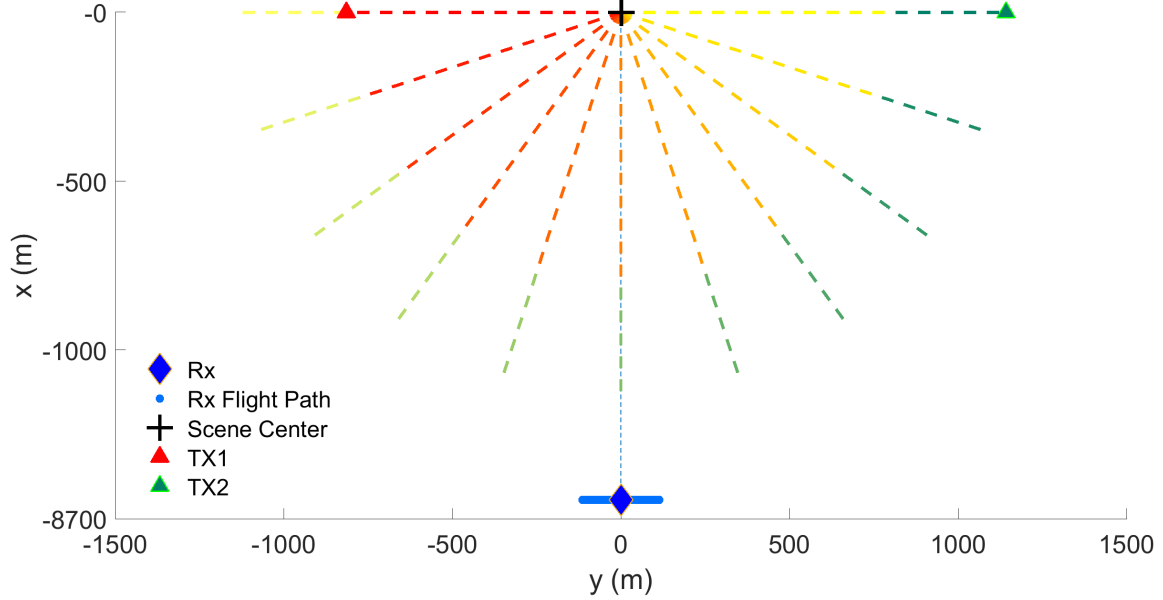


Figure 26. Radar geometry in Scenario 3c. After each set of 8000 MC trials, the azimuth of Tx1 and that of Tx2 are incremented by 18 degrees such that they are equal but have opposite signs.

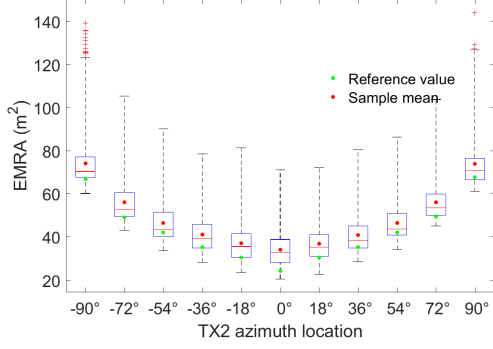
In each MC trial, the ISLR and all six MRE measures are determined. The reference value for each measure in the upcoming box plots and data spread plots is determined with zero transmitter position uncertainty.

4.4.2 Result Analysis.

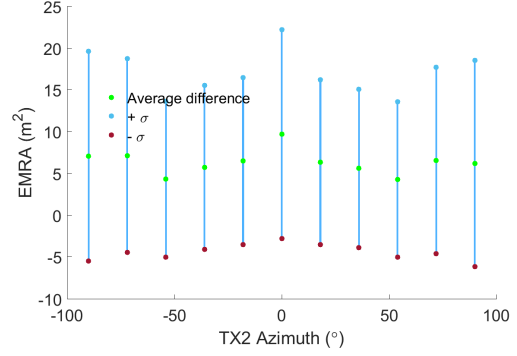
For ease of data analysis, we present the data of each measure from the three scenarios side by side as we make comparisons and look for trends. We are able to identify patterns in the ISLR data and the data of five MRE measures including EMRA, semi-major axis, linear eccentricity, eccentricity and tilt angle. No consistent trend is discernible in the semi-minor axis data.

In Figures 27a and 27b, with Tx1 fixed at 0° azimuth, a symmetry is readily observable, hinting at a relationship between the effects of transmitter position uncertainty and the bistatic angles. However, the EMRA sample mean-reference value gap does not change by much. The same position error variance appears to cause

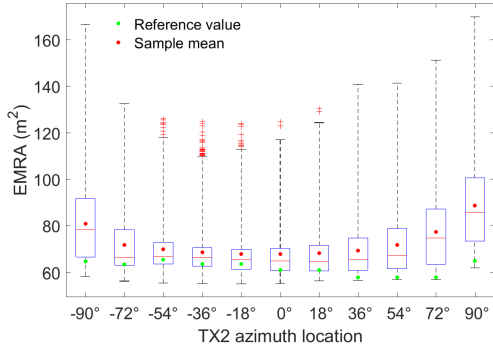
slightly more spread in the EMRA data with larger Tx2's azimuths.



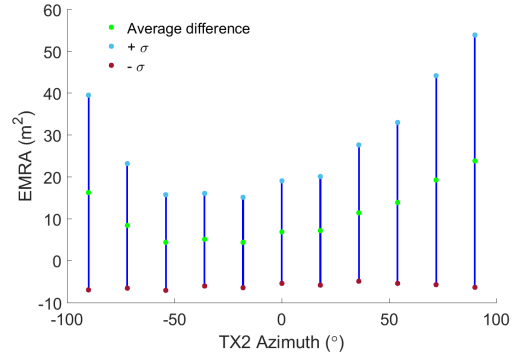
(a) EMRA data distribution in Scenario 3a



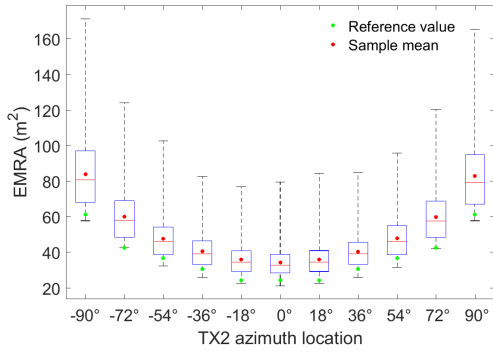
(b) EMRA data spread in Scenario 3a



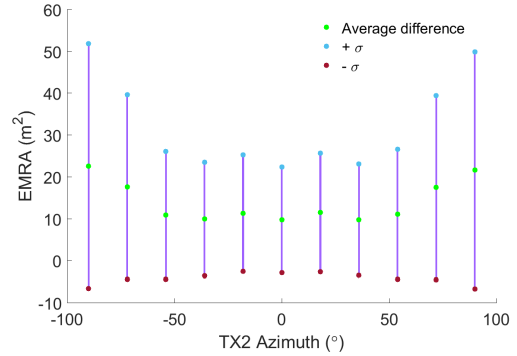
(c) EMRA data distribution in Scenario 3b



(d) EMRA data spread in Scenario 3b



(e) EMRA data distribution in Scenario 3c

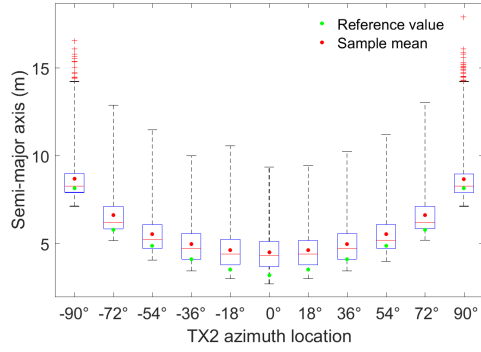


(f) EMRA data spread in Scenario 3c

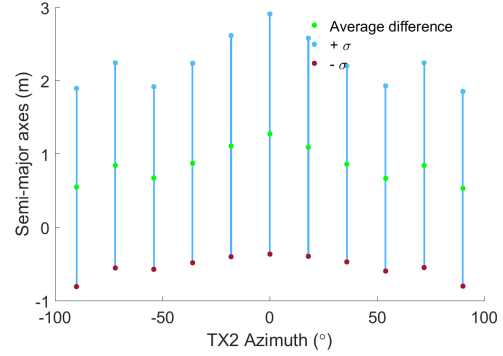
Figure 27. EMRA distribution and data spread in Scenarios 3a-3c, compared side by side.

With Tx1 fixed at 85° , Figures 27c and 27d show much more variations in EMRA data spread as Tx2 is moved through different azimuths. In addition, the EMRA sample mean-reference value variations are much more pronounced, with larger gaps observed at larger Tx2's azimuths. In Scenario 3c, where Tx1's azimuth and Tx2's azimuth are equal but have opposite signs, the more the transmitters are moved away from the 0° , the larger the EMRA sample mean-reference value gap; the same observation applies to the EMRA spread. The above observations suggest that the same amount of transmitter position uncertainty has more effects on the EMRA of multistatic sets with larger bistatic angles. Furthermore, the EMRA's behaviors in the presence of position uncertainty are controlled by the transmitter with predominantly larger bistatic angle.

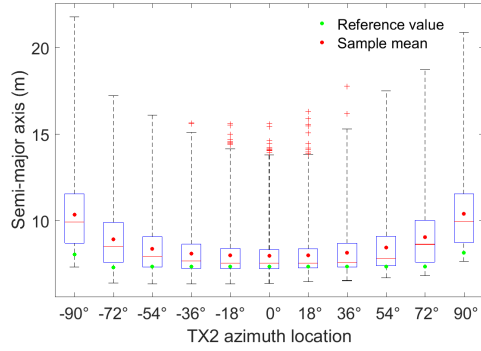
With Tx1 fixed at 0° azimuth, not much variation is observed in the semi-major axis data in either Figure 28a or Figure 28b. However, when Tx1 is fixed at 85° azimuth, larger semi-major axis sample mean-reference value gap and more data spread are observed in Figures 28c and 28d, respectively. The large bistatic angle of Rx-Tx1 pair apparently causes more spread in the semi-major axis data in the presence of the same amount of transmitter position uncertainty.



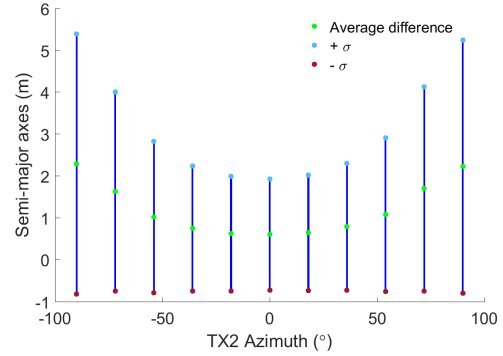
(a) Semi-major axis data distribution in Scenario 3a



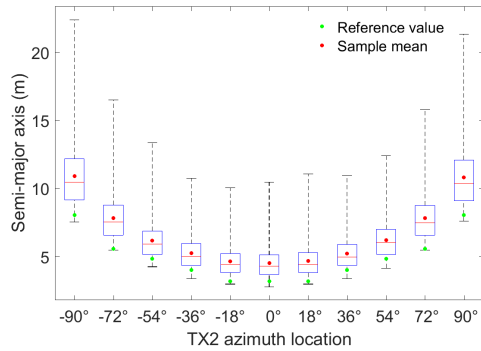
(b) Semi-major axis data spread in Scenario 3a



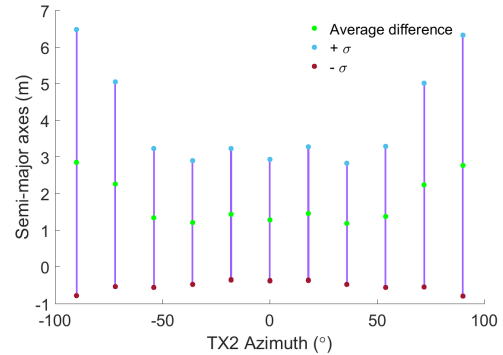
(c) Semi-major axis data distribution in Scenario 3b



(d) Semi-major axis data spread in Scenario 3b



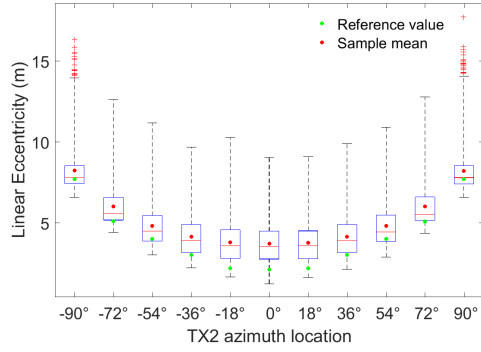
(e) Semi-major axis data distribution in Scenario 3c



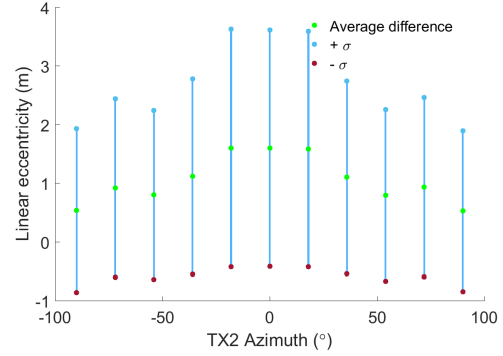
(f) Semi-major axis data spread in Scenario 3c

Figure 28. Semi-major axis distribution and data spread in Scenarios 3a-3c, compared side by side.

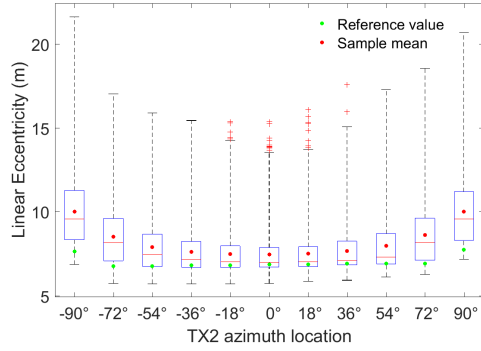
The relationship between the bistatic angle and the transmitter position uncertainty's effects are further confirmed in Figures 28e and 28f. With Tx1 and Tx2 placed at azimuths opposite to each other, more spread and larger semi-major axis sample mean-reference value difference are observed at larger azimuth angles. The same patterns can be seen in linear eccentricity data box plots and spread plots in Figure 29.



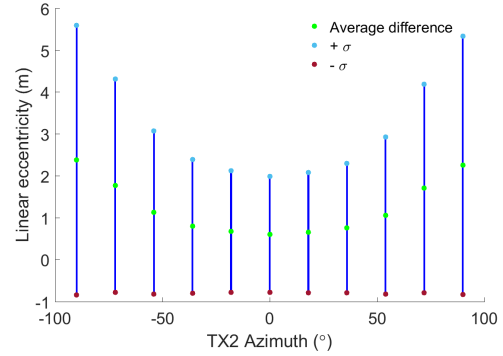
(a) Linear eccentricity data distribution in Scenario 3a



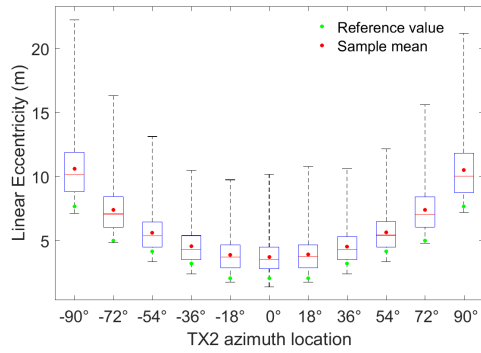
(b) Linear eccentricity data spread in Scenario 3a



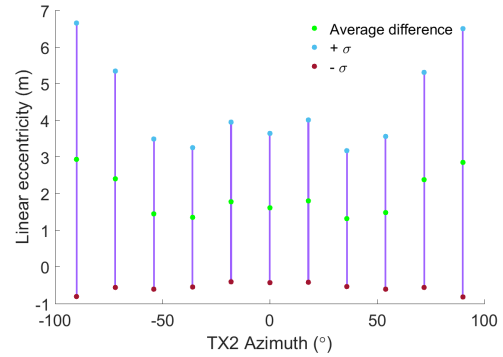
(c) Linear eccentricity data distribution in Scenario 3b



(d) Linear eccentricity data spread in Scenario 3b



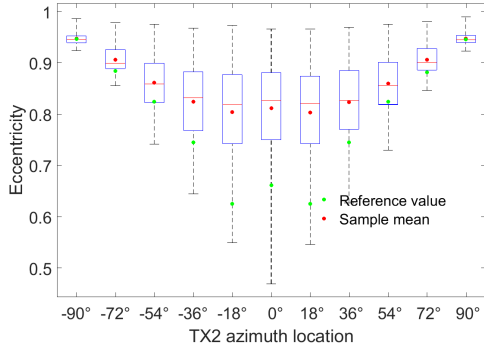
(e) Linear eccentricity data distribution in Scenario 3c



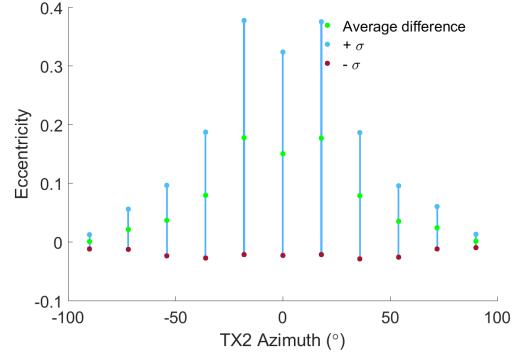
(f) Linear eccentricity data spread in Scenario 3c

Figure 29. Linear eccentricity distribution and data spread in Scenarios 3a-3c, compared side by side.

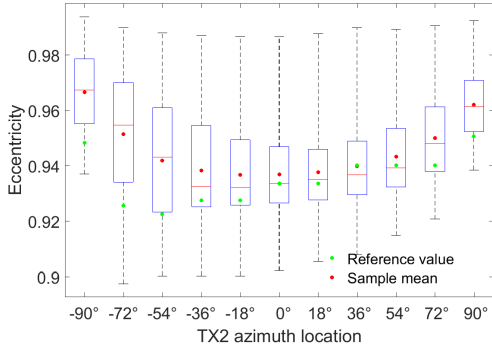
While the same position error variance causes more spread in EMRA, semi-major axis and linear eccentricity data at larger azimuths, the pattern is opposite in the eccentricity data shown in Figure 30. With Tx1's azimuth fixed at 0° , Figure 30a shows wider data range and larger eccentricity sample mean-reference value gap at smaller Tx2's azimuths. Alternatively, the trend can be observed in the data spread plots in Figure 30b. The same patterns can be discerned from Scenario 3c eccentricity box plots (Figure 30e) and data spread plots (Figure 30f). With the transmitters placed at opposite azimuths, the eccentricity data spread and the sample mean-reference value gap decrease with larger azimuth angles. We do not see much variation in eccentricity in either Figure 30c or Figure 30d, telling by the eccentricity scale in the figures. This observation suggests that the larger the bistatic angle the more likely that the MRE becomes more eccentric, for the same amount of transmitter position uncertainty.



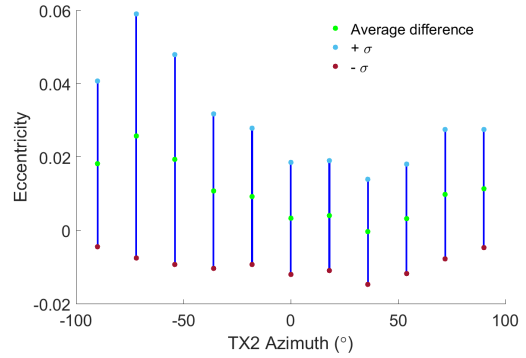
(a) Eccentricity data distribution in Scenario 3a



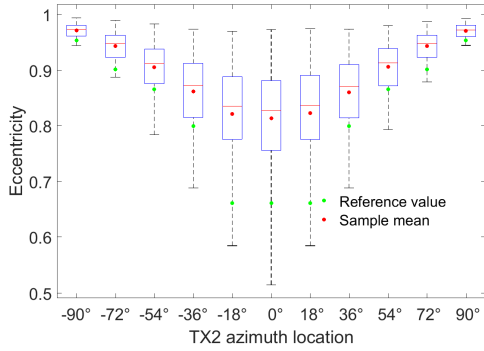
(b) Eccentricity data spread in Scenario 3a



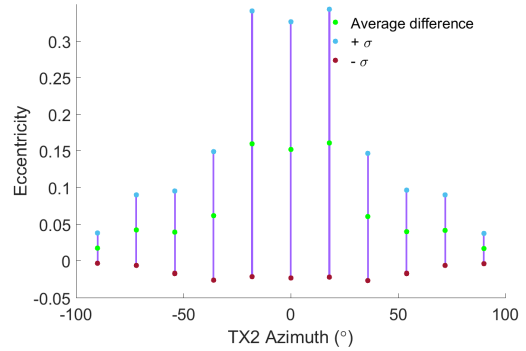
(c) Eccentricity data distribution in Scenario 3b



(d) Eccentricity data spread in Scenario 3b



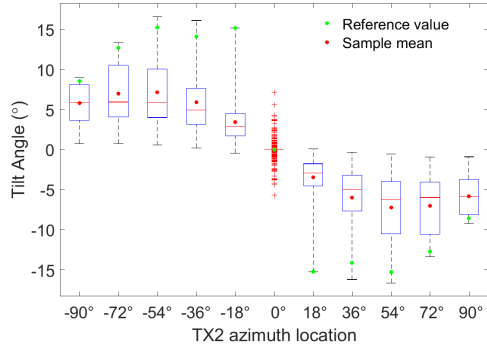
(e) Eccentricity data distribution in Scenario 3c



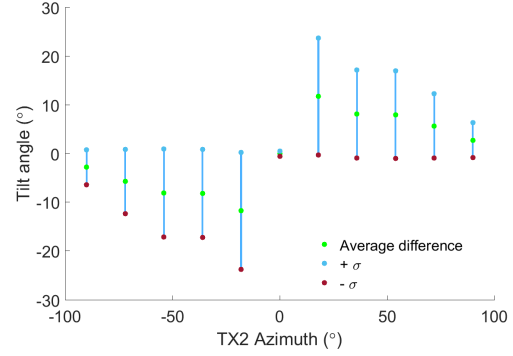
(f) Eccentricity data spread in Scenario 3c

Figure 30. Eccentricity distribution and data spread in Scenarios 3a-3c, compared side by side.

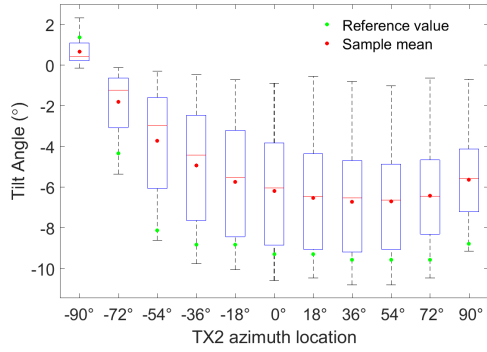
The tilt angle box plots from Scenario 3a exhibit a pattern resembling a cosine function, as seen in Figure 31a. The same pattern can be seen in Figure 31b which shows the tilt angle data spread reaches its maximum when Tx2 is at -45° and 45° azimuths while Tx1 is fixed at 0° .



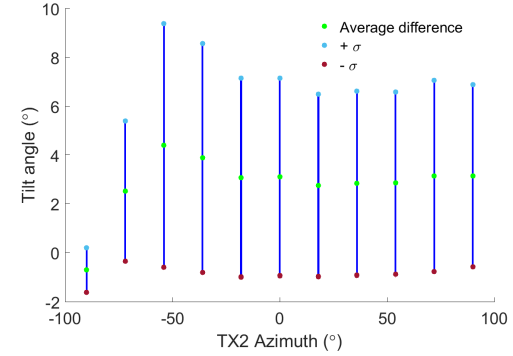
(a) Tilt angle data distribution in Scenario 3a



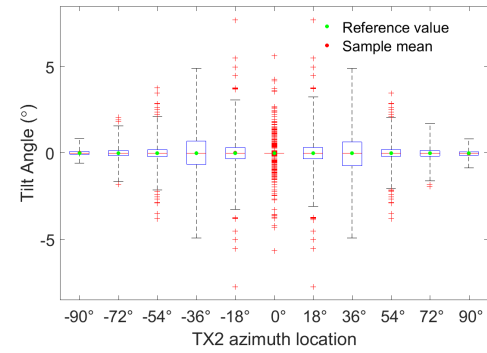
(b) Tilt angle data spread in Scenario 3a



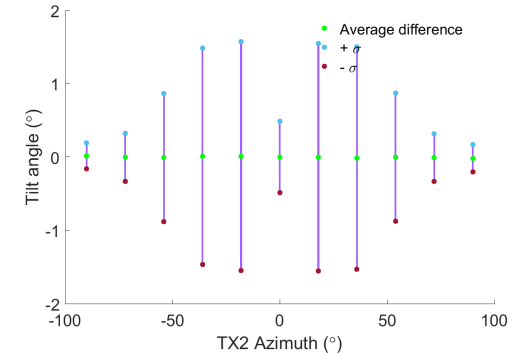
(c) Tilt angle data distribution in Scenario 3b



(d) Tilt angle data spread in Scenario 3b



(e) Tilt angle data distribution in Scenario 3c

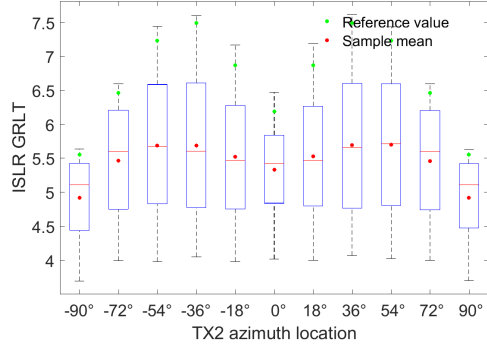


(f) Tilt angle data spread in Scenario 3c

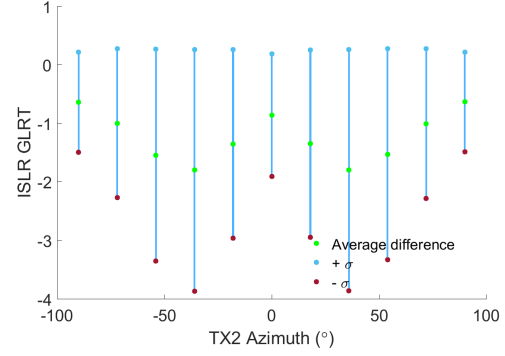
Figure 31. Tilt distribution and data spread in Scenarios 3a-3c, compared side by side.

When the two transmitters are placed at opposite azimuths, as in Scenario 3c, the data spread also maxes out at -45° and 45° , as seen in Figures 31e and 31f. With Tx1 fixed at 85° , a relatively large azimuth, the MRE tilt angle in Scenario 3b is distributed over mostly negative angle ranges. We assess that the tilt angle data in Scenario 3b is dominated by the predominantly larger azimuth of Tx1, based on the tilt angle box plots for Tx2 azimuths near 90° in Figure 31a, which shows the tilt angle occupy negative angle ranges.

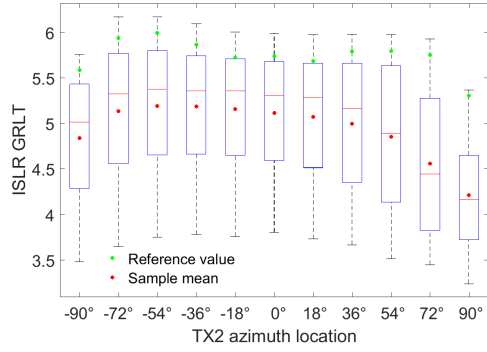
Compared to the tilt angle data, the ISLR data from Scenarios 3a-3c exhibits similar behaviors, as seen in Figure 32. In Scenario 3a, when Tx1 is fixed at 0° , the ISLR data range reaches its maximum when Tx2 is at -45° and 45° azimuths and is smallest when Tx2 is at 0° , -90° and 90° azimuths, telling by the lengths of the box plots in Figure 32a. Additionally, Figure 32b shows that the ISLR data spread and sample mean-reference value gap also reach the maxima and minima at similar Tx2's azimuths. The same observations apply to Scenario 3c ISLR data in Figures 32e and 32f, suggesting a strong correlation between the bistatic angles and the effects of transmitter position uncertainty on the ISLR of the multistatic image. Like the tilt angle data in Scenario 3b, when Tx1 is fixed at 85° , the ISLR data distribution across all Tx2 azimuths is strongly influenced by the predominantly larger bistatic angle of Rx-Tx1 pair. A close look at Figure 32c reveals that the ISLR box plots in Scenario 3b resemble to those in Scenario 3a (Figure 32a) when Tx2 is at 72° and 90° azimuths. This observation further confirms that the effects of transmitter position uncertainty are controlled by the transmitter with predominantly larger bistatic angle.



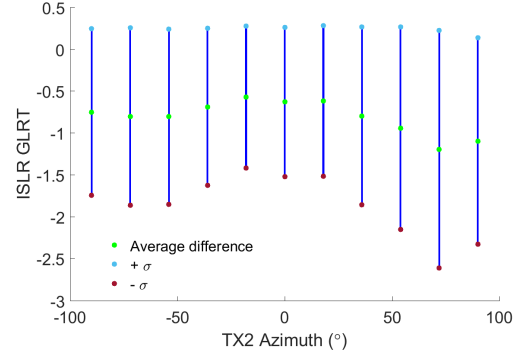
(a) ISLR data distribution in Scenario 3a



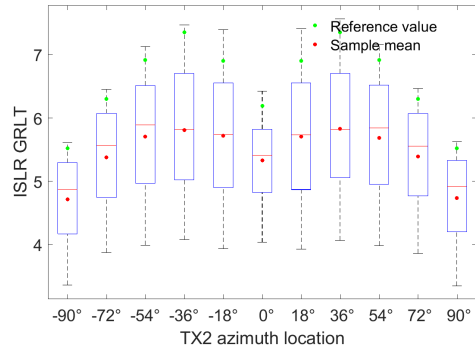
(b) ISLR data spread in Scenario 3a



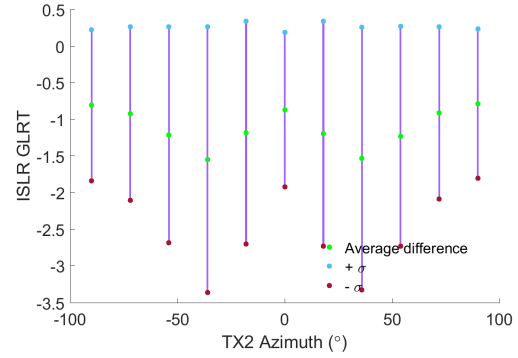
(c) ISLR data distribution in Scenario 3b



(d) ISLR data spread in Scenario 3b



(e) ISLR data distribution in Scenario 3c



(f) ISLR data spread in Scenario 3c

Figure 32. ISLR distribution and data spread in Scenarios 3a-3c, compared side by side.

Finally, the behaviors of the MRE on average in Scenarios 3a-3c are depicted in Figure 33. In Figures 33a-33c, the MREs are color-coded based on the azimuth angle of Tx2, as indicated on the color bars. The baseline MREs (highlighted in green) in Figures 33a and 33b are determined with Tx2's azimuth equal to 0° . In Figure 33c, the baseline MRE (highlighted in green) is determined with Tx1's and Tx2's azimuths both equal to 0° .

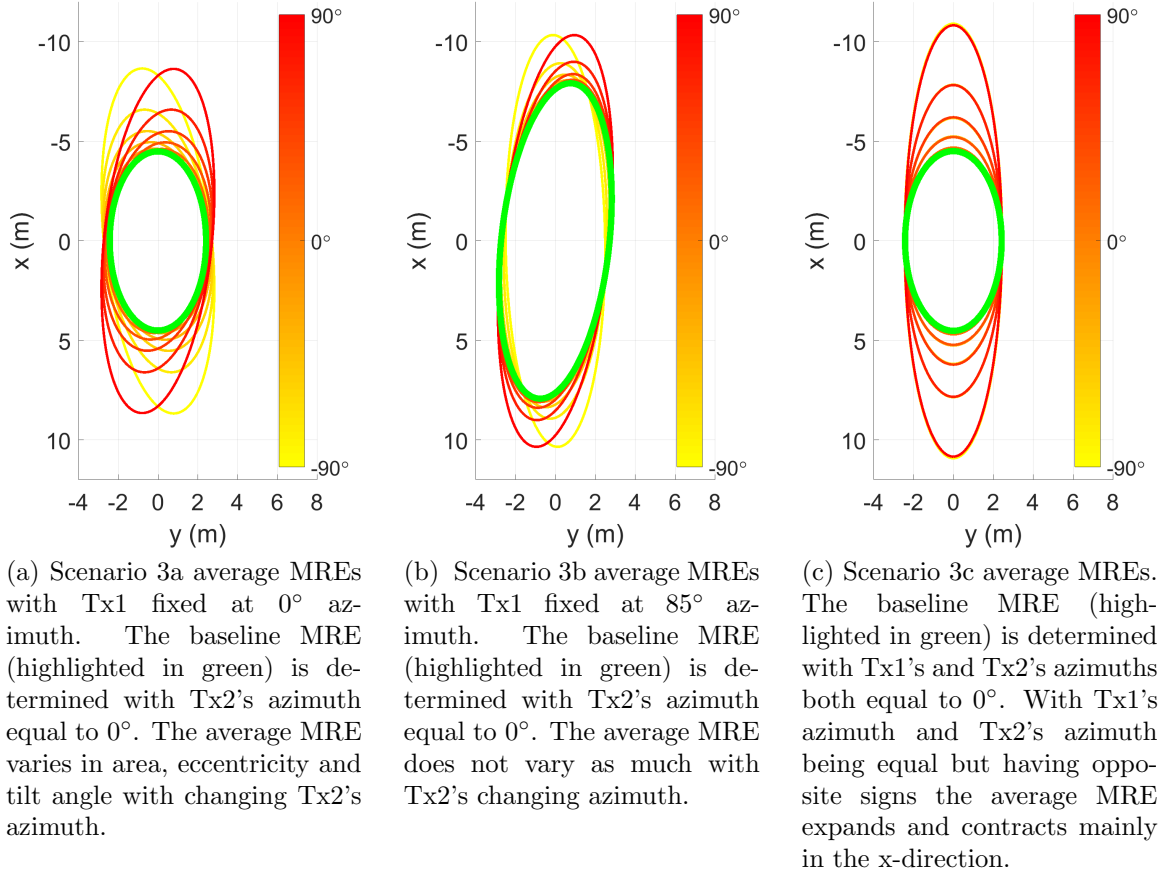


Figure 33. Behaviors of the MRE, on average, in the presence of transmitter position uncertainty in Scenarios 3a-3c. The MREs are color-coded based on Tx2's azimuth, as indicated on the color bars.

Figure 33a shows that when Tx1 is fixed at 0° , the average MRE's area, eccentricity and tilt angle are dependent on Tx2's azimuth. However, when Tx1 is fixed at 85° , a predominantly larger azimuth, less variation is seen on the average MRE, as shown

in Figure 33b. Finally, with the transmitters placed at opposite azimuths, Figure 33c shows that the average MRE expands in the x-direction (range direction) in addition to having zero tilt angle.

4.4.3 Key Findings.

- On average, the MRE's size (EMRA), shape (eccentricity) and orientation (tilt angle) are dependent on the bistatic angle of the individual pairs, with or without transmitter uncertainty.
- For the same amount of transmitter position uncertainty, the larger the bistatic angle, the more data spread observed on the MRE measures including EMRA, semi-major axis, and linear eccentricity.
- For the same amount of transmitter position uncertainty, the eccentricity data spread decreases with larger bistatic angles, meaning the MRE is highly likely to be quite elongated.
- Judging by the MRE measures, the extent to which the transmitter uncertainty affects the multistatic resolution is dependent on the transmitter with larger bistatic angle. The larger the difference in bistatic angle, the stronger the dependency and the more obvious the effects.

4.5 Scenario 4a: Effects of Velocity Measurement Errors

In Scenario 4a, we seek to investigate the effects of velocity measurement errors on the MRE and the ISLR of the multistatic PSF. As explained in Section 2.5, transmitter position errors may include contributions from position, velocity and acceleration measurement errors. For simplicity, the position errors in this scenario include contributions due to the transmitter's velocity measurement errors only.

4.5.1 Simulation Setup.

The radar geometry in Scenario 4a is illustrated in Figure 34 and Table 5. The receiver's position is assumed known at all time. Tx1 is stationary and its position is also assumed precisely known. Tx2 flies a linear path from the left to the right of the page with constant velocity $\vec{v}_t = [0, 35, 0]^T$ m/s. Additionally, Tx2 has a velocity measurement error $\tilde{v}_y$ in the y-direction; in each trial, $\tilde{v}_y$ is constant for the entire receiver's aperture. We incrementally vary the velocity measurement error $\tilde{v}_y$ through $[-4, 4]$ m/s, resulting in 51 different velocity measurement errors, one for each deterministic trial. In contrast to the scenarios covered previously, we do not run MC trials for Scenario 4a since our objective is limited to preliminary examination of the effects of velocity measurement errors on the multistatic PSF.

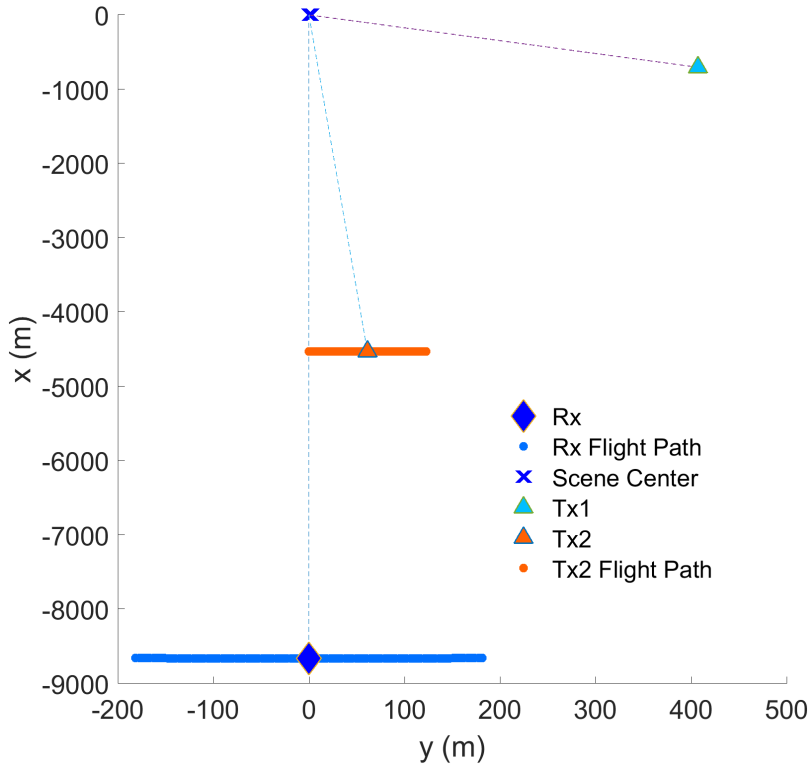


Figure 34. Radar geometry in Scenarios 4a and 4b.

In Scenario 4a, the baseline MRE measures and the baseline ISLR are determined

Table 5. Geometry of Scenarios 4a and 4b.

		Scenario 4a	Scenario 4b
Tx1	Range (m)	820	820
	Azimuth ($^{\circ}$)	30	30
	Elevation ($^{\circ}$)	7	7
	x-coordinate (m)	705	705
	y-coordinate (m)	407	407
	z-coordinate (m)	100	100
	variance (m^2)	N/A	N/A
Tx2	Range (m)	1147	5000
	Azimuth ($^{\circ}$)	$\in [-0, 1.55]$	$[0, 1.55]$
	Elevation ($^{\circ}$)	25	25
	x-coordinate (m)	4531.5	4531.5
	y-coordinate (m)	$\in [0, 112.5]$	$[0, 122.5]$
	z-coordinate (m)	2113.1	2113.1
	Velocity errors $\tilde{v}_y$ (m/s)	$[-4:4]$	0
	Acceleration errors $\tilde{a}_y$ (m/s^2)	0	$[-0.8:0.8]$
Rx	Range (m)	10000	10000
	Azimuth ($^{\circ}$)	$[-1.5, 1.5]$	$[-1.5, 1.5]$
	Elevation ($^{\circ}$)	30	30

based on the PSF of the bistatic pair Rx-Tx1, using the same methodology described in Sections 3.3.5 and 3.3.6.

4.5.2 Results.

The MRE derived in each of the 51 deterministic trials is depicted in Figure 35. To display the MRE resulted from each velocity measurement error, we break up the MREs into four groups and display them in Figures 35a-35d. Note that each MRE in Figures 35a-35d is color-coded according the amount of velocity measurement error, as indicated on the color bars. Figure 35e shows the baseline MRE, which is determined based on the PSF of bistatic pair Rx-Tx1. The baseline MRE will be referred to as the bistatic resolution ellipse (BRE) at various points in the upcoming discussion.

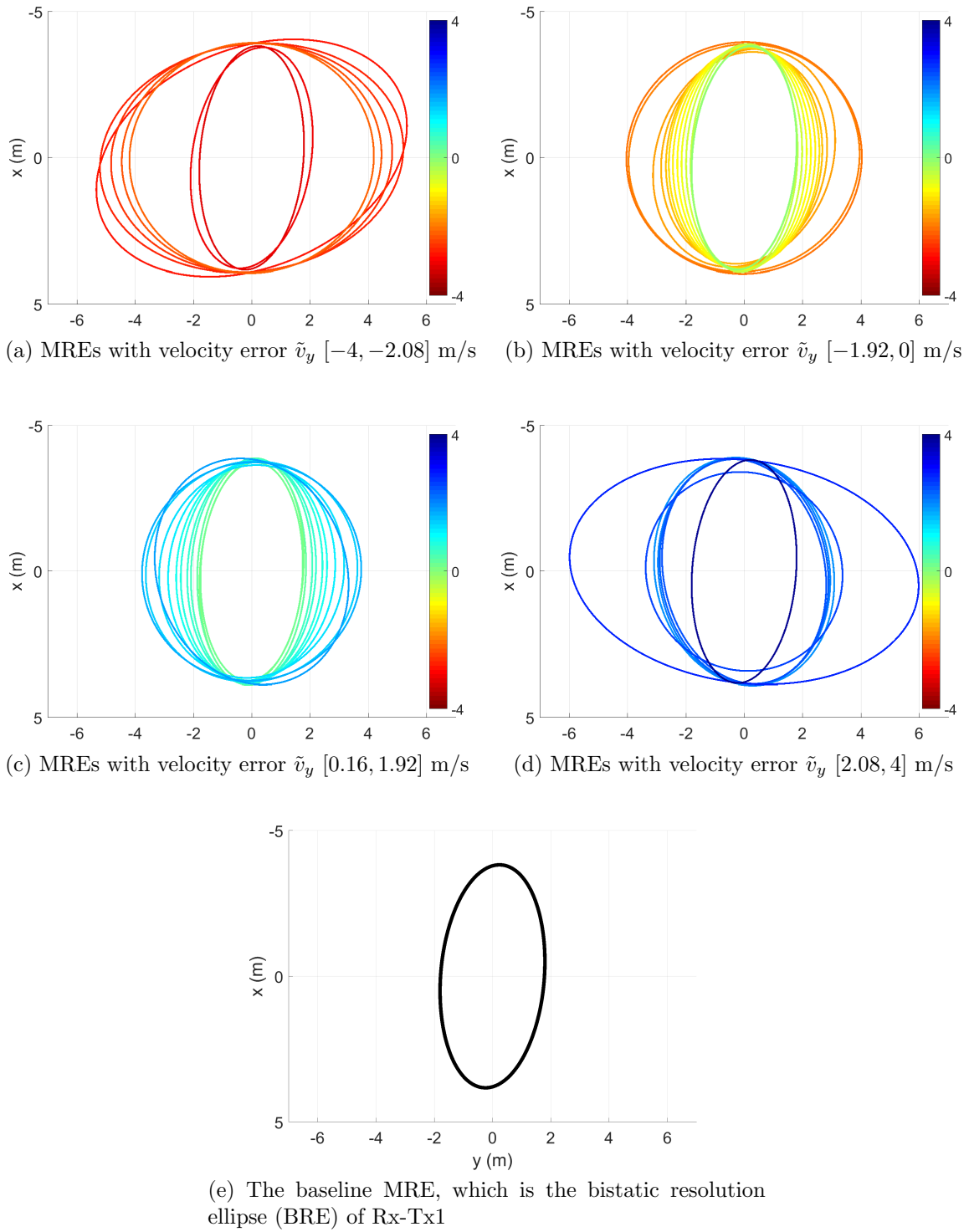


Figure 35. The MREs resulted from the 51 deterministic trials. The MRE, which is color-coded according the amount of velocity measurement error, grows then reverts back to the baseline MRE as the cross-range velocity error increases in either direction.

From Figure 35, we note that as Tx2's velocity measurement error $\tilde{v}_y$ is incrementally varied from -4 m/s to 4 m/s, the MRE starts out having the same size and shape as the baseline MRE, which is the error-free bistatic resolution ellipse (BRE) of Rx-Tx1 pair. The MRE then elongates in the y-direction (cross-range direction) and shrinks back to the BRE's size as the error approaches zero (Figures 35a and 35b). As the velocity measurement error of Tx2 becomes more positive, the MRE once again elongates in the y-direction (Figure 35c) just to shrink back to the BRE's size eventually (Figure 35d).

Examining each MRE measure's scatter plot in Figure 36, we observe that there is a clear breakdown point on each side of the x-axis, beyond which the data suddenly reverts back to the baseline value and remains there. For this particular radar geometry, the breakdown points for Tx2's velocity measurement error are -3.04 m/s and 3.04 m/s on the negative and positive end of the velocity measurement error spectrum, respectively. Examining the ISLR scatter plot (Figure 37), we do not observe the same phenomenon.

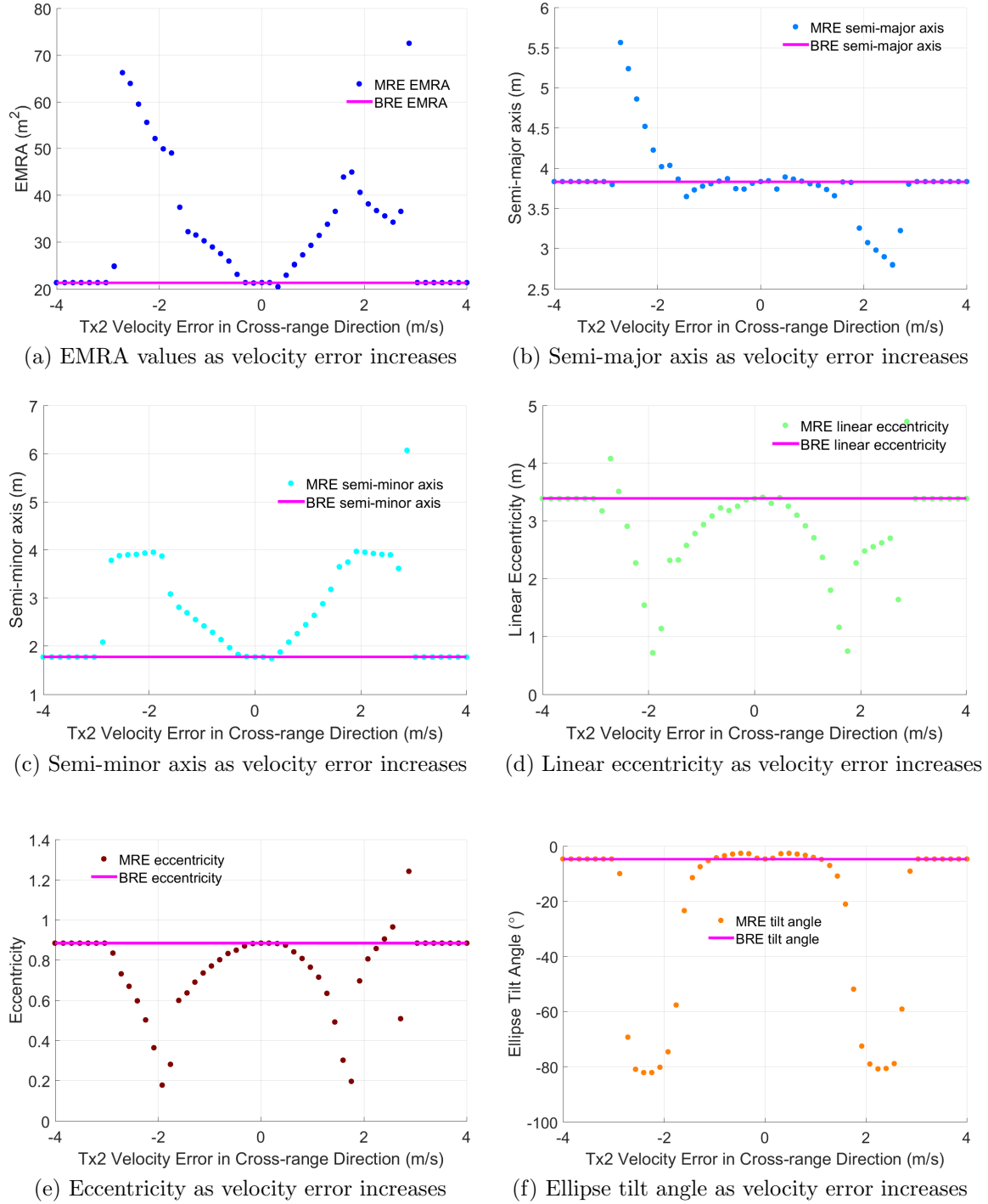


Figure 36. Behaviors of MRE measures as Tx2's velocity error in the cross-range direction $\tilde{v}_y$ is varied through $[-4, 4]$ m/s. The BRE, which is the 3dB contour of the PSF of bistatic pair Rx-Tx1, serves as the baseline.

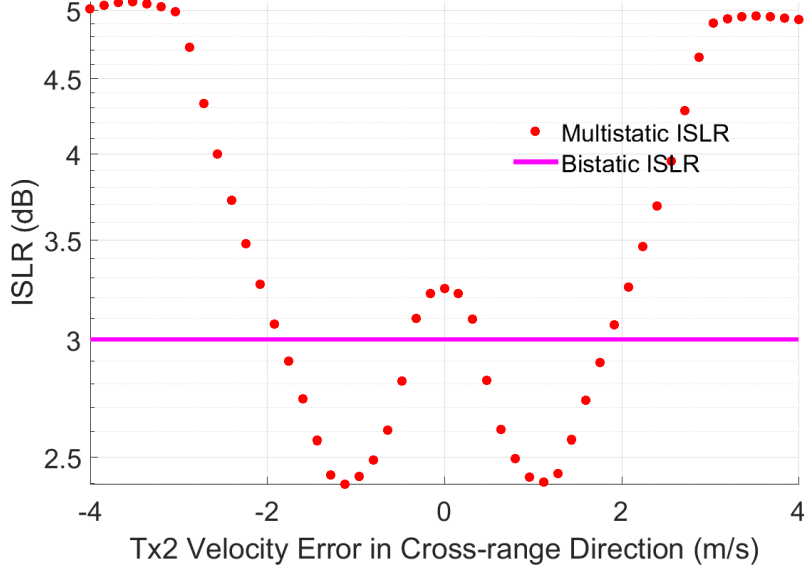
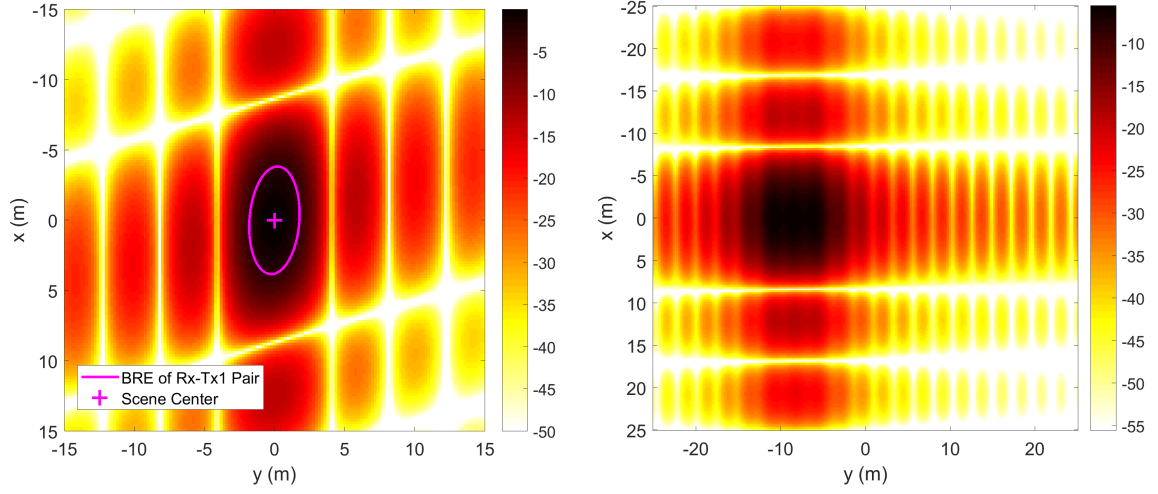


Figure 37. ISLR values resulted from different velocity errors from -4 m/s to 4 m/s in the cross-range direction.

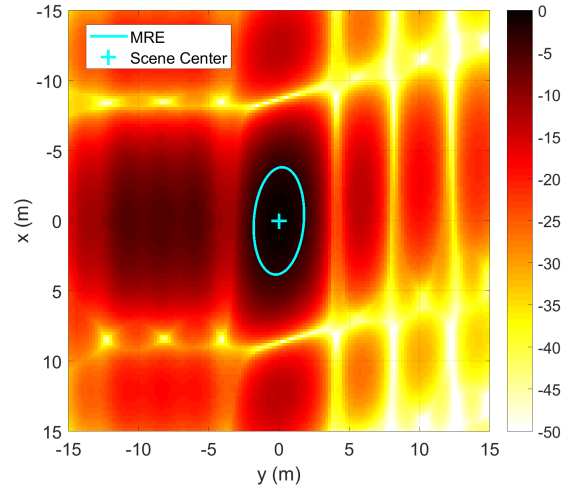
The lack of breakdown points indicates the ISLR is still able to capture the effects of the velocity measurement errors, at least for range of velocity errors tested.

Overall, the analysis of Figures 35 to 37 indicates that the MRE can function as a multistatic resolution metric for up to a finite amount of transmitter velocity measurement error. We are highly confident that at the velocity measurement error limits, the mainlobe of the bistatic PSF of Rx-Tx2 pair drops below the 3dB threshold, due to the shifting and blurring effects induced by the velocity measurement error. By design, the algorithm we use to obtain the MRE disregards all of Rx-Tx2's mainlobe pixels. The resulting MRE consists of solely pixels from Rx-Tx1 pair, causing all MRE measures to approach the corresponding BRE values of Rx-Tx1 pair, as illustrated in Figure 38.



(a) The bistatic resolution ellipse (BRE) of Rx-Tx1, subjected to no position uncertainty

(b) The point spread function (PSF) of Rx-Tx2. Subjected to a large velocity measurement error, the PSF blurs out and shifts to left.



(c) The MRE under the effects of a large velocity measurement error

Figure 38. Example of MRE beyond velocity measurement error limits. The shifting and blurring effects cause the MRE to become the bistatic resolution ellipse of Rx-Tx1 pair.

Figure 38c depicts a multistatic PSF corrupted by a velocity measurement error of 4 meters per second in the y-direction, which is well beyond the limits observed in Figure 36. The MRE in Figure 38c resembles the bistatic resolution ellipse of Rx-Tx1 pair in Figure 38a.

4.5.3 Key Findings.

- The MRE can function as a multistatic resolution metric for only up to a finite amount of transmitter velocity measurement error due to the amplitude decrease associated with image blur.
- Within these limits, the MRE grows in area and elongates in the cross-range direction (y-direction) as the magnitude of the transmitter's velocity error increases.
- The ISLR is still able to record the effects of velocity measurement errors beyond the limits seen in MRE's data, at least for the range of errors tested.

4.6 Scenario 4b: Effects of Acceleration Measurement Errors

Scenario 4b is aimed at examining the effects of acceleration measurement errors on image quality metrics including the MRE measures and the ISLR. In Scenario 4b, the position errors are solely due to transmitter acceleration measurement errors, allowing us to isolate the effects of interest. Consequently, according to (19), the differential range error consists of a quadratic component and a cubic component, both of which have blurring effects on the image [6, 8].

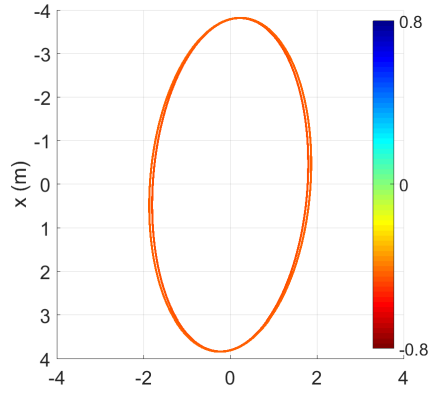
4.6.1 Simulation Setup.

The radar geometry is similar to that of Scenario 4a (see Figure 34). Again, Tx1 is stationary and its position is assumed precisely known. Tx2 flies a linear path

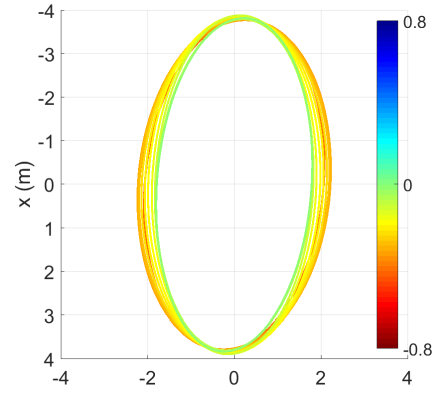
from the left to the right of the page with constant velocity $\vec{v}_t = [0, 35, 0]^T$ m/s. We replace velocity measurement error $\tilde{v}_y$ with acceleration error $\tilde{a}_y$ in the y-direction; in each trial, $\tilde{a}_y$ is constant for the entire receiver's aperture. We incrementally vary $\tilde{a}_y$ through $[-0.8, 0.8]$ m/s², resulting in 51 different acceleration measurement errors, one for each deterministic trial. Similar to Scenario 4a, we do not run MC trials for Scenario 4b since our objective is limited to preliminary examination of the effects of acceleration measurement errors on the multistatic PSF.

4.6.2 Results.

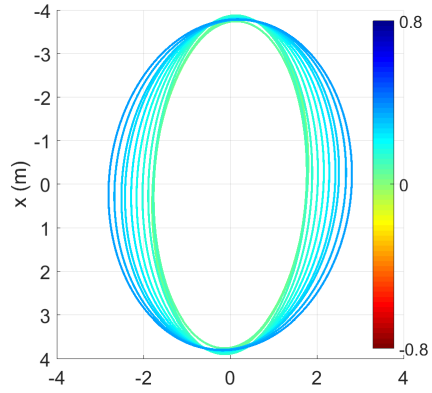
Figures 39a-39d offer an intuitive visualization of the behaviors of the MRE as the acceleration measurement error $\tilde{a}_y$ is varied incrementally through $[-0.8, 0.8]^T$ m/s². Again, to make sure that the MRE resulted from each acceleration measurement error is clearly visible, we break up the MREs into four groups and display them in Figures 39a-39d.



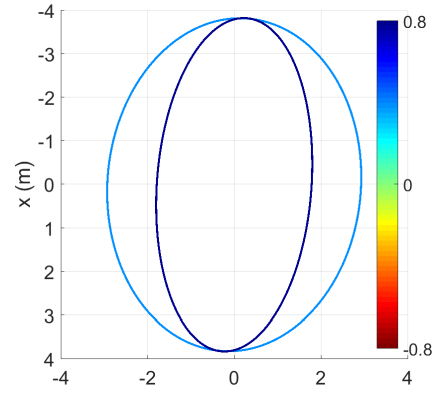
(a) MREs with acceleration error $\tilde{a}_y$
 $[-0.8, -0.416]$ m/s²



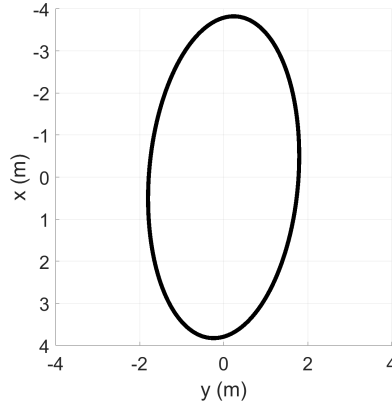
(b) MREs with acceleration error $\tilde{a}_y$
 $[-0.384, 0]$ m/s²



(c) MREs with acceleration error $\tilde{a}_y$
 $[0.032, 0.384]$ m/s²



(d) MREs with acceleration error $\tilde{a}_y$
 $[0.416, 0.8]$ m/s²



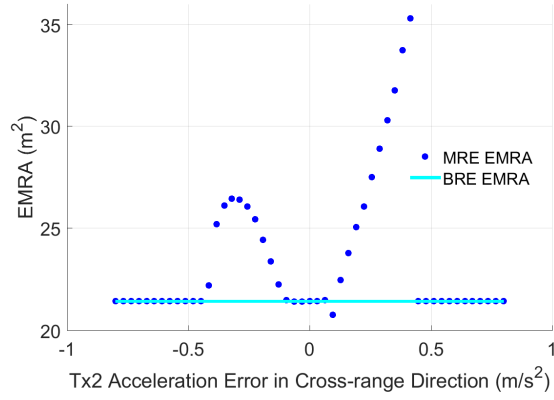
(e) The baseline MRE, which is the bistatic resolution
ellipse (BRE) of Rx-Tx1

Figure 39. The MREs resulted from the 51 deterministic trials. The MRE, which is color-coded according the amount of acceleration measurement error, grows then reverts back to the baseline MRE as the cross-range acceleration error increases in either direction.

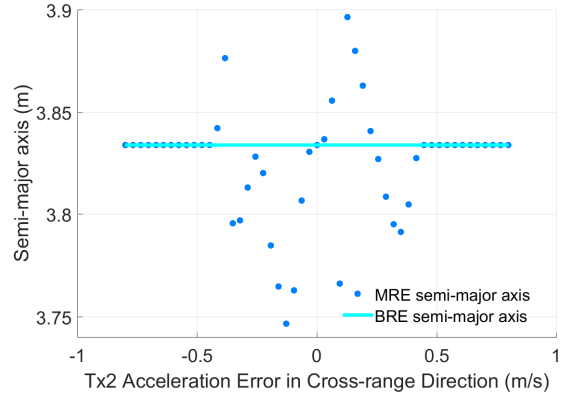
Note that each MRE in Figures 39a-39d is color-coded according the amount of acceleration measurement error, as indicated on the color bars. Figure 39e shows the baseline MRE, which is determined based on the PSF of bistatic pair Rx-Tx1. The terms baseline MRE and bistatic resolution ellipse (BRE) are used interchangeably at various points in the upcoming discussion.

In Figures 39a and 39b, we observe that as the Tx2's acceleration error $\tilde{a}_y$ is incrementally varied from -0.8 m/s^2 to 0 m/s^2 , the MRE starts out having the same size and shape as the baseline MRE (see Figure 39e), which is determined using the PSF of bistatic pair Rx-Tx1. The MRE then expands in the y-direction (cross-range) and eventually shrinks back to the BRE. As $\tilde{a}_y$ increases in the positive direction, the MRE once again expands then contracts to the baseline MRE when $\tilde{a}_y$ reaches 0.8 m/s^2 .

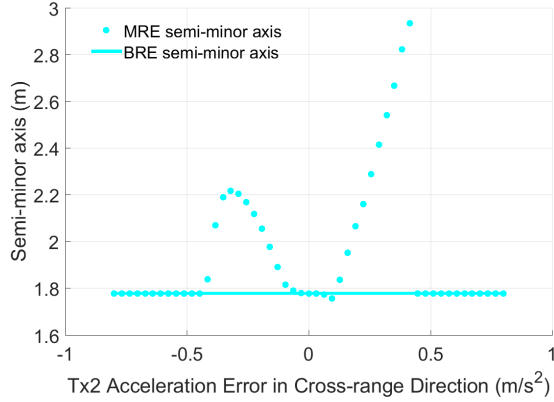
As acceleration measurement error $\tilde{a}_y$ is incrementally varied from -0.8 m/s^2 to 0.8 m/s^2 , the MRE measures are determined and captured in the scatter plots in Figure 40.



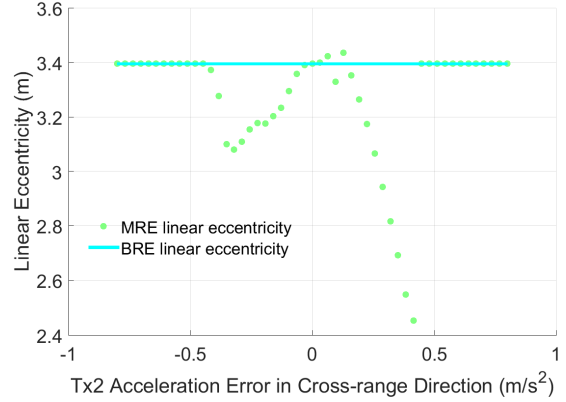
(a) EMRA values as acceleration error increases



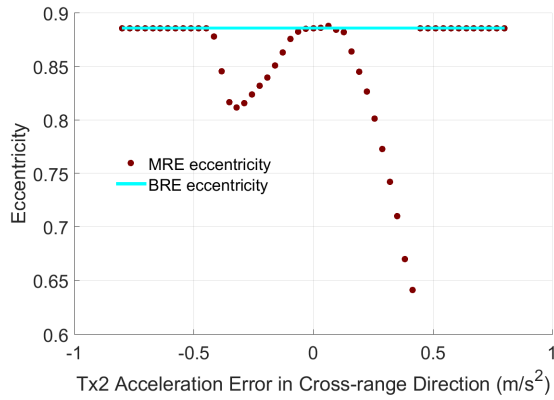
(b) Semi-major axis as acceleration error increases



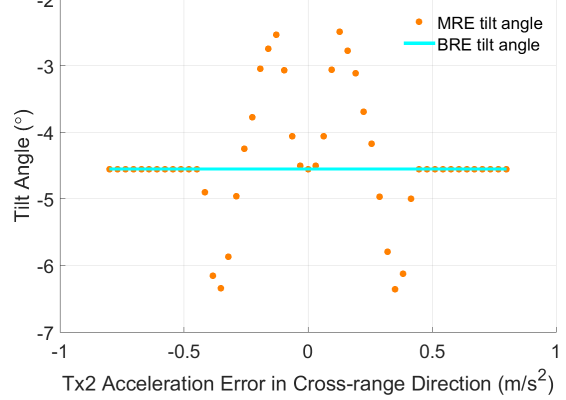
(c) Semi-minor axis as acceleration error increases



(d) Linear eccentricity as acceleration error increases



(e) Eccentricity as acceleration error increases



(f) Ellipse tilt angle as acceleration error increases

Figure 40. Behaviors of MRE measures as the acceleration error in the cross-range direction is varied through $[-0.8, 0.8] \text{ m/s}^2$. The BRE is the 3dB contour of the PSF of bistatic pair Rx-Tx1.

Examining the scatter plots of the MRE measures individually (Figure 40), we observe that the MRE data behave normally until the acceleration error reaches the breakdown points. In this particular configuration, the MRE's behaviors are normal up to -0.448 m/s^2 and 0.448 m/s^2 on the negative and positive end of the acceleration error spectrum, respectively. Beyond these error values, the scatter plots in Figure 40 show that the MRE measures revert back to the baseline values, i.e. those determined based on the PSF of Rx-Tx1 pair. However, when examining the ISLR data in Figure 41, we do not see such clear cut limits.

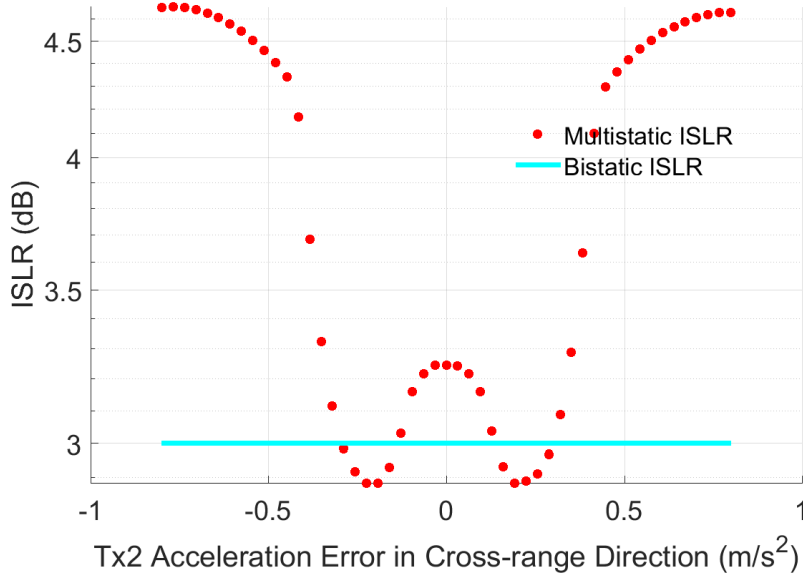


Figure 41. ISLR values resulted from different Tx2 acceleration error values from -0.8 m/s^2 to 0.8 m/s^2 in the cross-range direction.

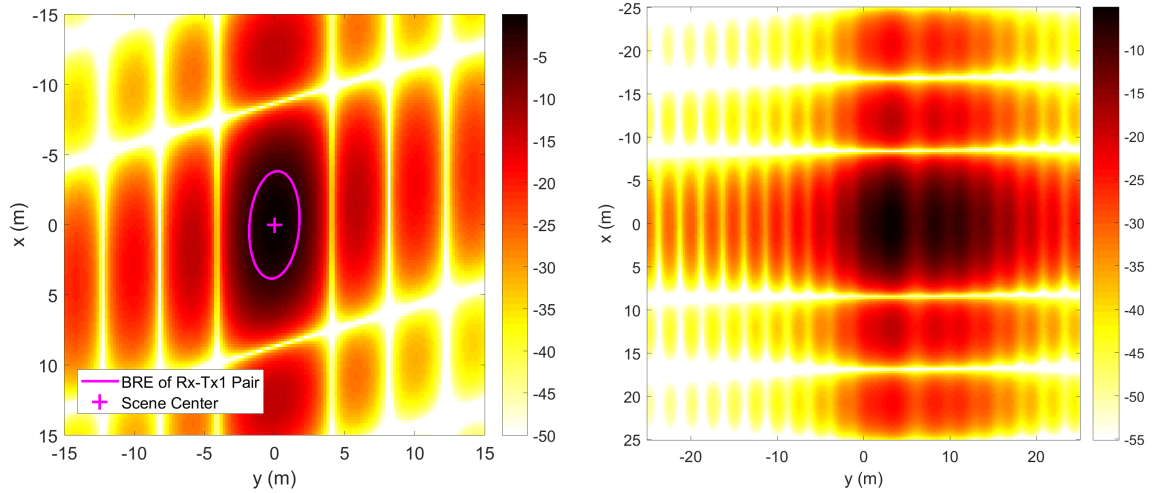
Again, we see that the ISLR is still able to capture the effects of the acceleration measurement errors when the MRE measures have started to fail. Beyond the breakdown points, the fall of the PSF mainlobe of Rx-Tx2 pair causes its sidelobes to rise, resulting in a higher ISLR, as confirmed by the ISLR scatter plot in Figure 41.

We assess that the phenomena observed on the MRE in Figures 39 and 40 are possibly due to the same cause as that in Scenario 4a: as the error grows beyond the breakdown points, the PSF mainlobe of Rx-Tx2 falls below the 3dB threshold,

due to the combined blurring effects of the quadratic and cubic components of the differential range error (see (19)). As a result, the MRE now consists of contributions from the bistatic PSF of Rx-Tx1 only, as illustrated in Figure 42.

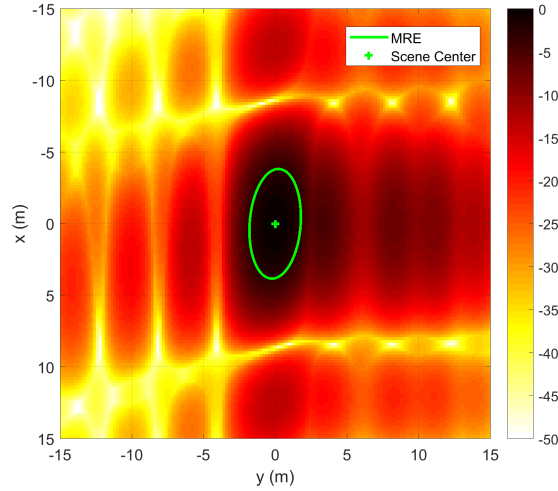
The multistatic image in Figure 42c is corrupted with acceleration error $\tilde{a}_y = -0.8$ m/s², which is well beyond the acceleration error limits observed in Figures 39 and 40. Under severe blurring effects, the PSF mainlobe of Rx-Tx2 falls below the 3dB threshold, causing the MRE to become the BRE of Rx-Tx1, which is subjected to no position uncertainty.

Additionally, compared to Scenario 4a, the MRE in Scenario 4b does not expand as much before the breakdown points. The phenomenon is likely due to the fact that acceleration measurement errors result in only blurring effects. In addition to blurring effects, velocity measurement errors also cause the image to shift, resulting in more expansion of the MRE as the error grows, as observed in Scenario 4a.



(a) The bistatic resolution ellipse (BRE) of Rx-Tx1, subjected to no position uncertainty

(b) The point spread function (PSF) of Rx-Tx2. Subjected to a large acceleration measurement error, the mainlobe drops below 3dB threshold.



(c) The MRE under the effects of a large acceleration measurement error

Figure 42. Example of MRE beyond acceleration measurement error limits. The combined blurring effects due to the quadratic and cubic components of the differential range error in (19) cause the mainlobe of Rx-Tx2 to fall quickly. As a result, the MRE becomes the bistatic resolution ellipse of Rx-Tx1.

4.6.3 Key Findings.

- The MRE can function as a multistatic resolution metric for up to a certain amount of transmitter acceleration measurement error. Within this error range, as the acceleration measurement error increases, the MRE grows in area, expanding in the cross-range direction.
- Due to the combined blurring effects of the quadratic and cubic components of the differential range error, acceleration measurement errors cause a rapid degradation of the multistatic resolution.
- The ISLR is still capable of capturing the effects of acceleration errors, at least in the range of errors tested.

4.7 Scenario 5: Effects of Transmitter Position Uncertainty and the Radar Center Frequency

In Scenario 5, we examine the effects of transmitter position uncertainty on the multistatic image resolution at different radar center frequencies.

4.7.1 Simulation Setup.

In Scenario 5, we vary the center frequency of both transmitters through [600, 2000] MHz range in 200 MHz increments, resulting in 8 sets of MC trials. For each transmitter center frequency, we run 5000 MC trials and apply the same position error variance of 5 m^2 to generate zero-mean Gaussian position errors for both transmitters. Figure 43 shows the MRE generated with different transmitter center frequencies when there is no position uncertainty.

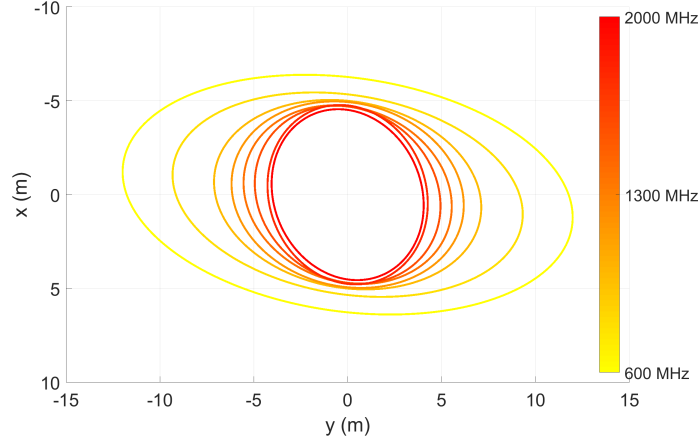
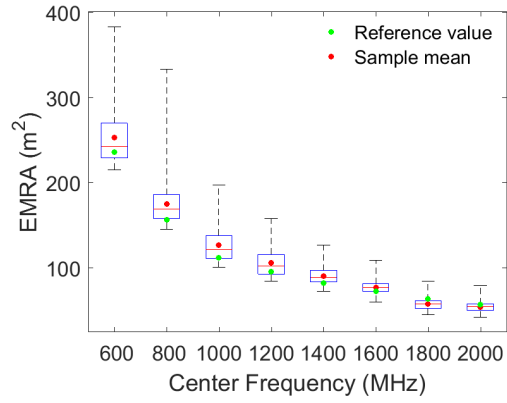


Figure 43. The reference MREs in Scenario 5. The MREs are color-coded based on the center frequency, as indicated on the color bar on the right. As the center frequency increases, the MRE decreases in area and eccentricity.

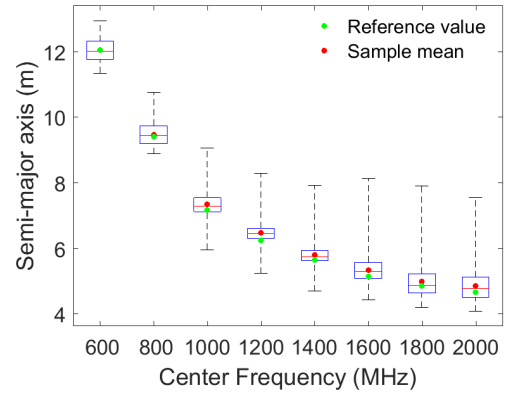
Note that in Figure 43 the MREs are color-coded based on the center frequency, as indicated by the color bar on the right. As the center frequencies of both transmitters increase from 600 MHz to 2000 MHz, the MRE becomes smaller and less eccentric, suggesting an improvement in resolution at higher frequencies. The MREs in Figure 43 will serve as the reference for the upcoming data analysis.

4.7.2 Results.

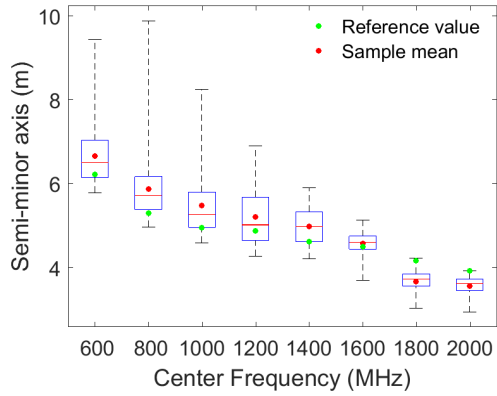
The data distribution of each MRE measure at different transmitter center frequencies can be observed on the box plots in Figure 44.



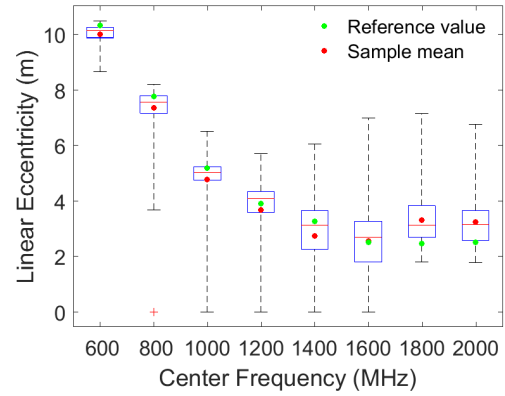
(a) EMRA data boxplot



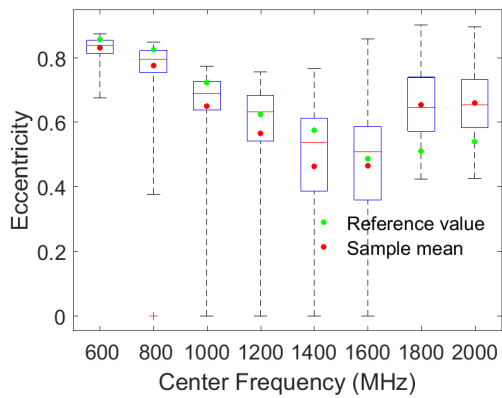
(b) Semi-major axis data boxplot



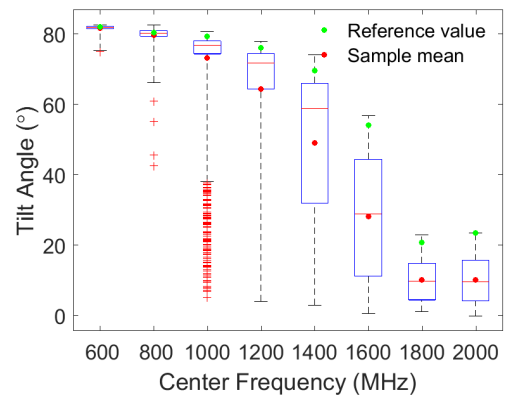
(c) Semi-minor axis data boxplot



(d) Linear eccentricity data boxplot



(e) Eccentricity data boxplot

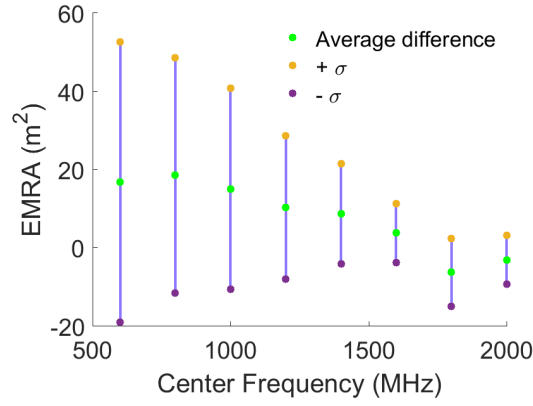


(f) Tilt angle data boxplot

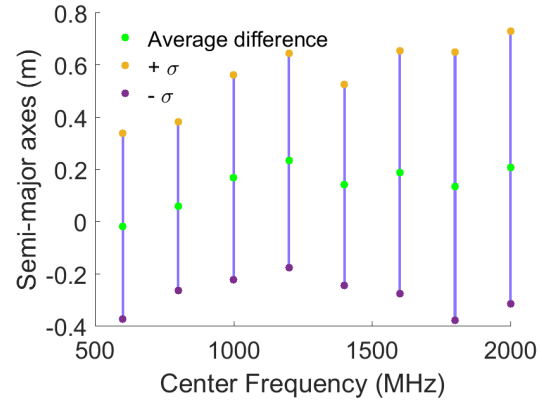
Figure 44. Scenario 5 MRE measures data of different center frequencies.

In Figure 44a, we observe that as the center frequency increases, on average, the EMRA decreases, suggesting an improvement in resolution at higher frequencies. The semi-major axis in Figure 44b and semi-minor axis in Figure 44c both generally shorten as the center frequency increases. The box plots in Figures 44d and 44e suggests the MRE, on average, becomes less eccentric at higher transmitter center frequencies. The trend in tilt angle data is not so straight forward in Figure 44f, with the tilt angle data distributed over the shortest range when the center frequency is at 600 MHz; the tilt angle data spread maxes out at around 1400 MHz before starting to fall off.

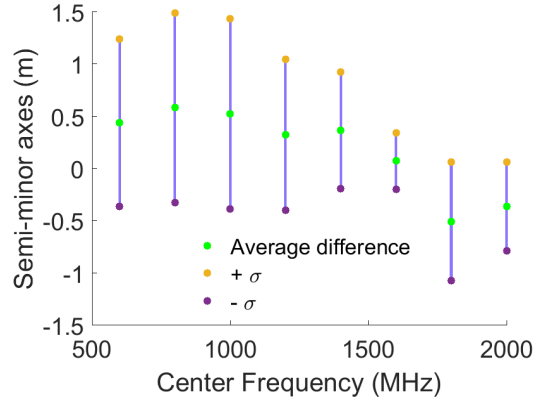
When analyzing the amount of data spread of the MRE measures (Figure 45), we observe that the same amount of transmitter position uncertainty has less effects at higher frequencies, telling by the data spread and the EMRA sample mean-reference value gap in Figure 45a. The same observation applies to the semi-minor axis (Figure 45c), while the opposite trend is true for the semi-major axis (Figure 45b). In Figures 45d and 45e, the linear eccentricity and the eccentricity experience the most effects at 1000 MHz center frequency; no direct relationship can be drawn between the data spread and the center frequency, however. The ellipse tilt angle experiences the least effects (least data spread observed) at 600 MHz and the most effects at 1400 MHz.



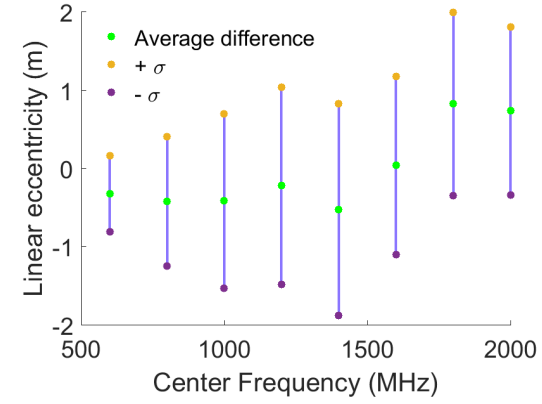
(a) ± 1 standard deviation of the difference between the EMRA samples and the reference value



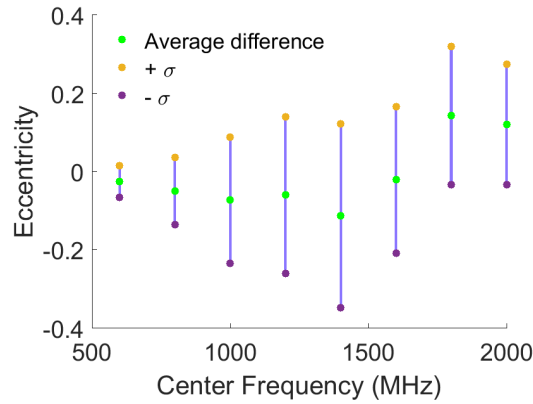
(b) ± 1 standard deviation of the difference between the semi-major axis samples and the reference value



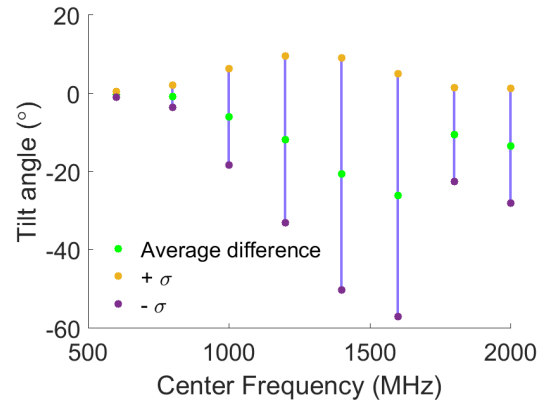
(c) ± 1 standard deviation of the difference between the semi-minor axis samples and the reference value



(d) ± 1 standard deviation of the difference between the linear eccentricity samples and the reference value



(e) ± 1 standard deviation of the difference between the eccentricity samples and the reference value



(f) ± 1 standard deviation of the difference between the tilt angle samples and the reference value

Figure 45. Amount of data spread of each MRE measure in Scenario 5.

Regarding the ISLR, the distribution of the ISLR data at different transmitter center frequencies is depicted in Figure 46.

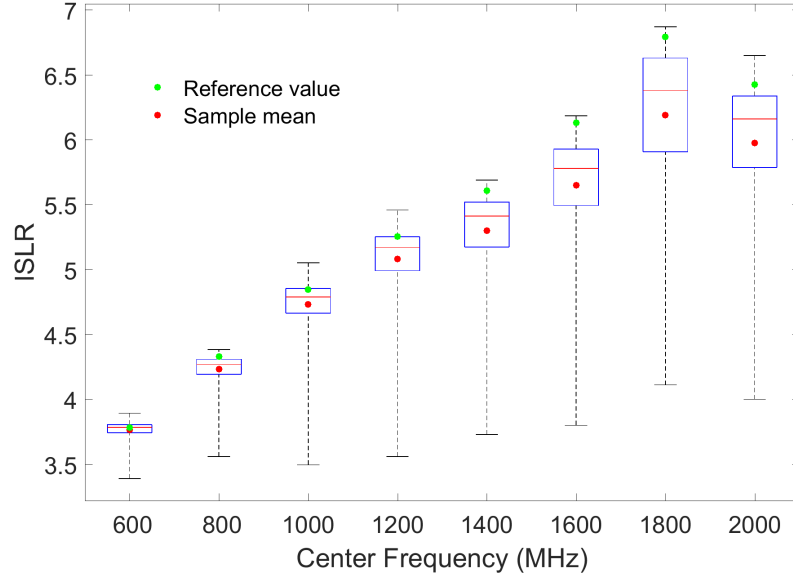


Figure 46. ISLR data at different center frequencies.

In Figure 46, we observe that, on average, the ISLR increases with the center frequencies. Additionally, we observe more ISLR data spread with increasing center frequency, as indicated by the expanding $\pm$ standard deviation (purple line segments) and widening ISLR sample mean-reference value (green dots) in Figure 47.

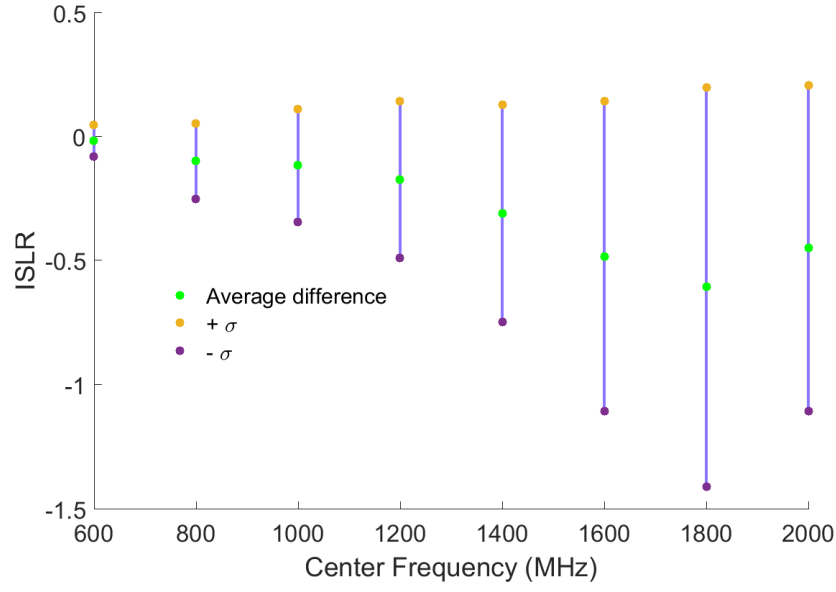


Figure 47. ± 1 standard deviation of the difference between ISLR samples and the reference value as the center frequency is varied through [600, 2000] MHz range.

Finally, the average MREs at different center frequencies are placed side by side with the reference MREs in Figure 48 for comparisons. Again, each MRE depicted in Figure 48 is color coded based on the center frequency (refer to the color bars).

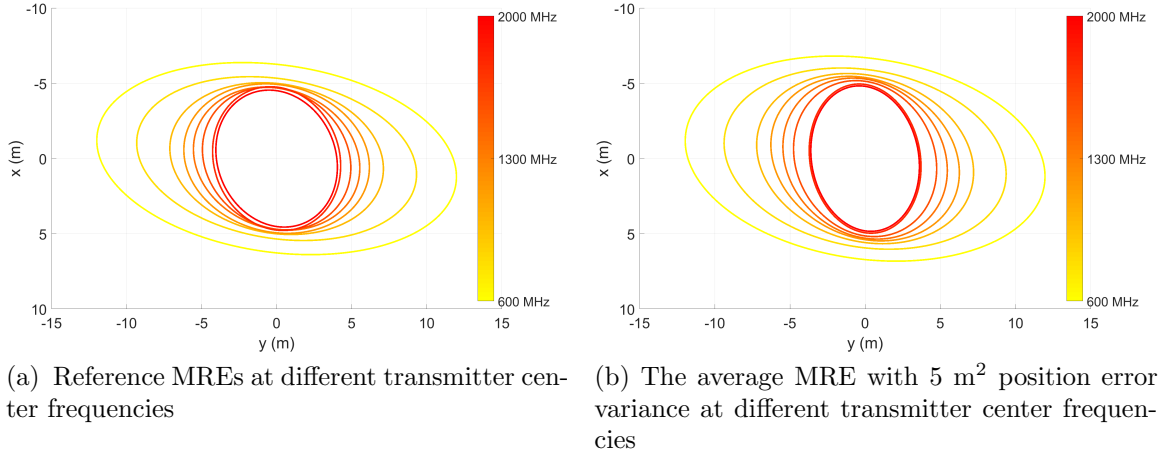


Figure 48. Average MRE under the effects of 5 m^2 position error variance, compared with the reference MRE at different transmitter center frequencies. The MREs are color-coded based on the center frequency, as indicated on the color bars.

Figure 48b indicates a strong agreement between the average MREs and the error-free MREs in Figure 48a. As a result, we assess that as the radar's center frequency increases, the MRE on average becomes smaller and less eccentric with decreasing tilt angle; furthermore, the higher the center frequency the more stable the EMRA.

4.7.3 Key Findings.

- As the center frequencies of individual bistatic pairs increase, the MRE on average becomes smaller and less eccentric with decreasing tilt angle. This phenomenon is observable with or without transmitter position uncertainty.
- Judging by the EMRA, the transmitter position uncertainty effects are more prominent (resulting the most data spread) at lower frequencies.
- Using ISLR as the metric, we observe that multistatic sets of high frequency experience the effects more than sets of low frequency.

4.8 Scenario 6: Effects of Transmitter Position Uncertainty and the Radar Bandwidth

In Scenario 6, we investigate the effects of transmitter position uncertainty on the image resolution of multistatic radar sets with different bandwidths.

4.8.1 Simulation Setup.

The radar geometry in Scenario 6 is similar to that in Scenario 1 (see Figure 13 and Table 3). We vary the bandwidths of both transmitters through $[6, 20]$ MHz range in 2 MHz increments, resulting in eight sets of MC trials. For each bandwidth, we run 5000 MC trials. During each MC trial, random errors are generated for both transmitters using 5 m^2 variance. To ensure the effects are caused by the changing bandwidth only, we select a sub-aperture for each bistatic pair so that it produces more or less equal range and cross-range image resolution. The sizes of the sub-apertures vary according to the transmitter bandwidth, with the smallest being $[-0.6^\circ, 0.6^\circ]$ and the largest being $[-1.5^\circ, 1.5^\circ]$, which is the azimuth extent of the receiver. The center frequencies of both transmitters are kept at 800 MHz throughout eight sets of trials.

To establish the baseline, for each transmitter bandwidth, we generate an error-free MRE, which is depicted in Figure 49. Note that the MREs in Figure 49 are color-coded based on the bandwidth, as indicated on the color bar.

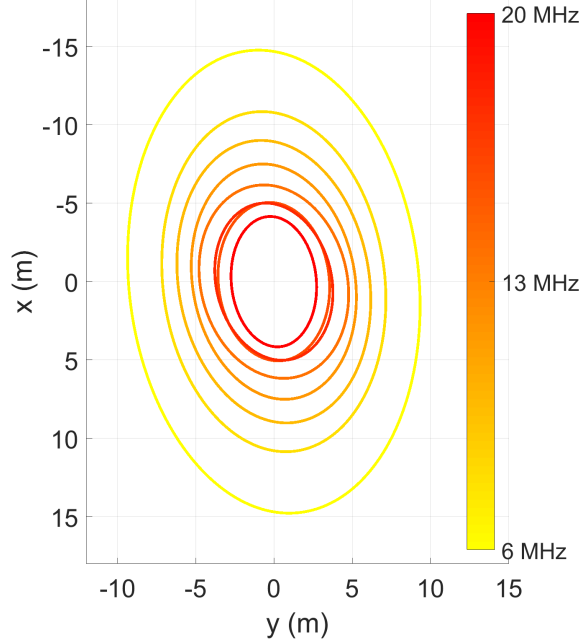
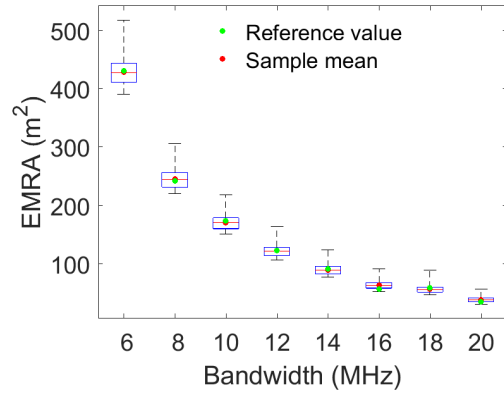


Figure 49. The reference MREs in Scenario 6. The MREs are color-coded based on the bandwidth, as indicated on the color bar. As the bandwidths of both transmitters are varied incrementally from 6 MHz to 20 MHz, the MRE reduces in size and generally retains its shape and orientation.

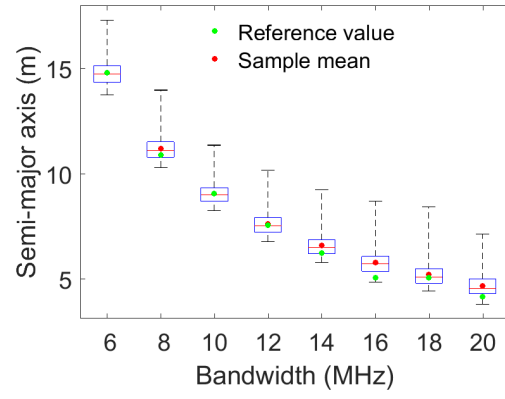
Figure 49 shows that without transmitter position uncertainty, the error-free MRE reduces in area, as indicated by the MRE shrinking. This behavior is expected because the range resolution is dependent on the radar's bandwidth and the cross-range resolution is dependent on the receiver's aperture size [8,9,13]. The MREs in Figure 49 serve as the reference for the upcoming analysis.

4.8.2 Results.

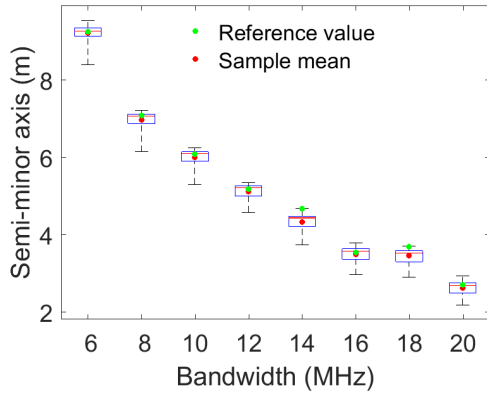
The data distribution of each MRE measure from the random trials is displayed in Figure 50.



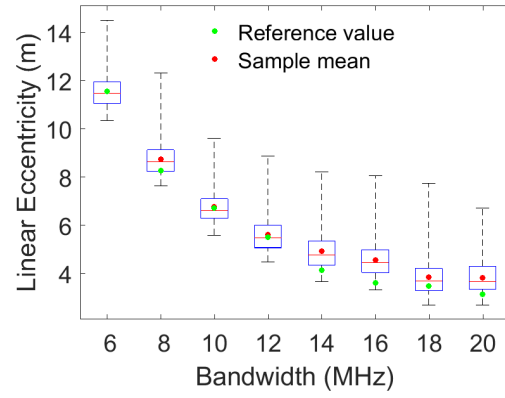
(a) EMRA data boxplot



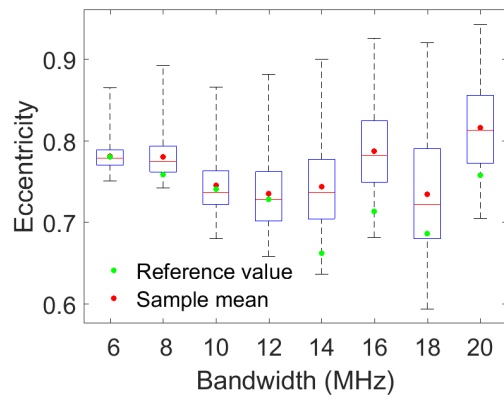
(b) Semi-major axis data boxplot



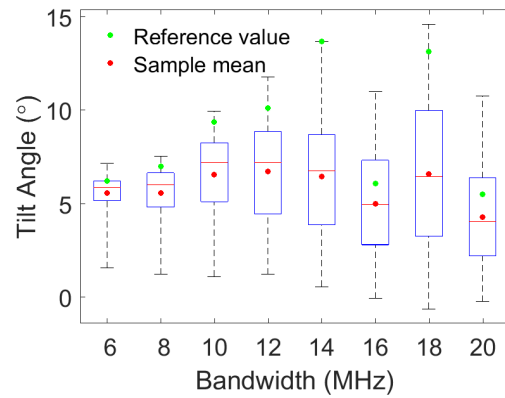
(c) Semi-minor axis data boxplot



(d) Linear eccentricity data boxplot



(e) Eccentricity data boxplot

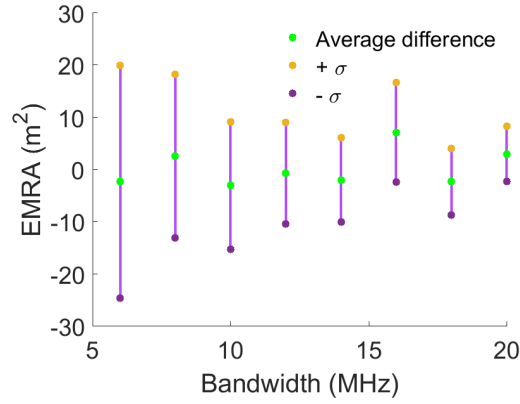


(f) Tilt angle data boxplot

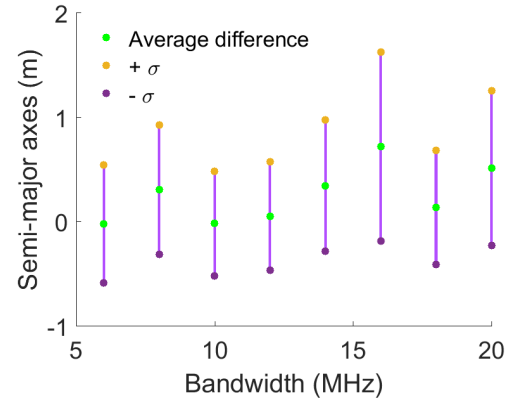
Figure 50. Scenario 6 MRE measures data with different transmitter bandwidths.

In Figure 50a, we observe that, on average, the EMRA reduces with increasing bandwidth, suggesting an improved resolution with larger bandwidths. The semi-major axis, semi-minor axis, and linear eccentricity box plots in Figures 50b-50d all indicate an improvement in resolution as the bandwidth is increased. Meanwhile, Figure 50e shows no clear trend on the average eccentricity; the same observation applies to the average tilt angle in Figure 50f.

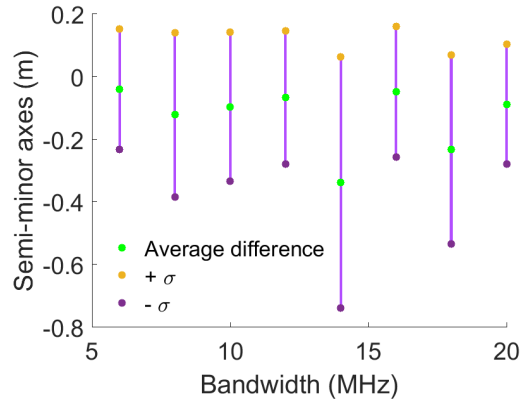
As far as the data spread is concerned, telling by the lengths of the box plots in Figure 50, EMRA data spread decreases with increasing bandwidth (Figure 50a). On the other hand, eccentricity and tilt angle data spread out more for larger transmitter bandwidths, as seen in Figures 50e and 50f. The data spread of all six MRE measures can also be observed on the data spread plots in Figure 51.



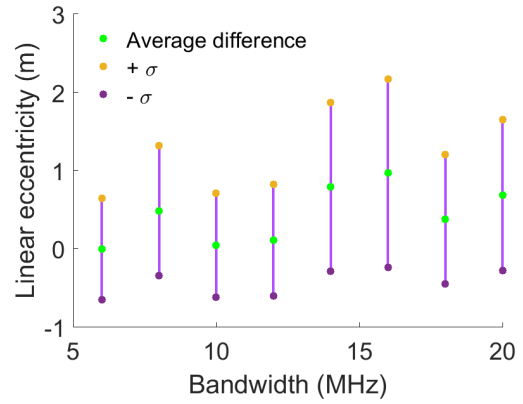
(a) ± 1 standard deviation of the difference between the EMRA samples and the reference value



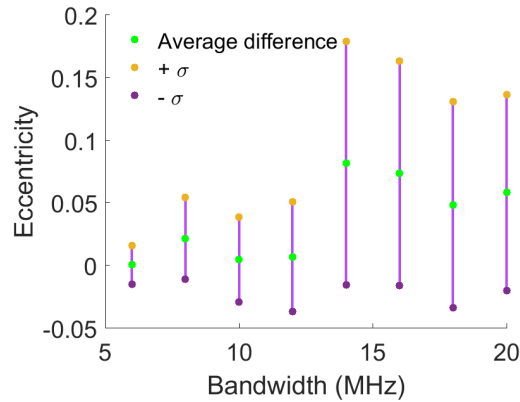
(b) ± 1 standard deviation of the difference between the semi-major axis samples and the reference value



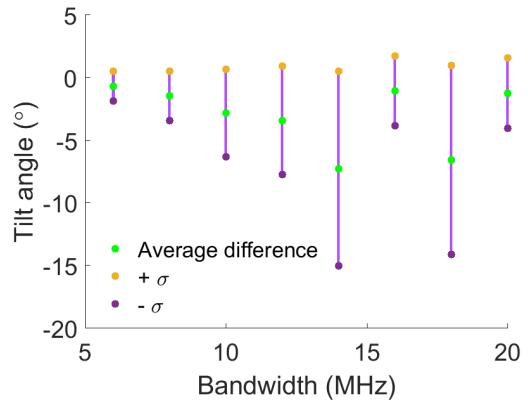
(c) ± 1 standard deviation of the difference between the semi-minor axis samples and the reference value



(d) ± 1 standard deviation of the difference between the linear eccentricity samples and the reference value



(e) ± 1 standard deviation of the difference between the eccentricity samples and the reference value



(f) ± 1 standard deviation of the difference between the tilt angle samples and the reference value

Figure 51. Amount of data spread of each MRE measure in Scenario 6.

We observe lower levels of data spread in the EMRA data in Figure 51a as the bandwidth grows. Meanwhile, the trend is reversed in the eccentricity data: the eccentricity data spread increases as the bandwidth grows; the same trend is observable in the ISLR data in Figures 52 and 53.

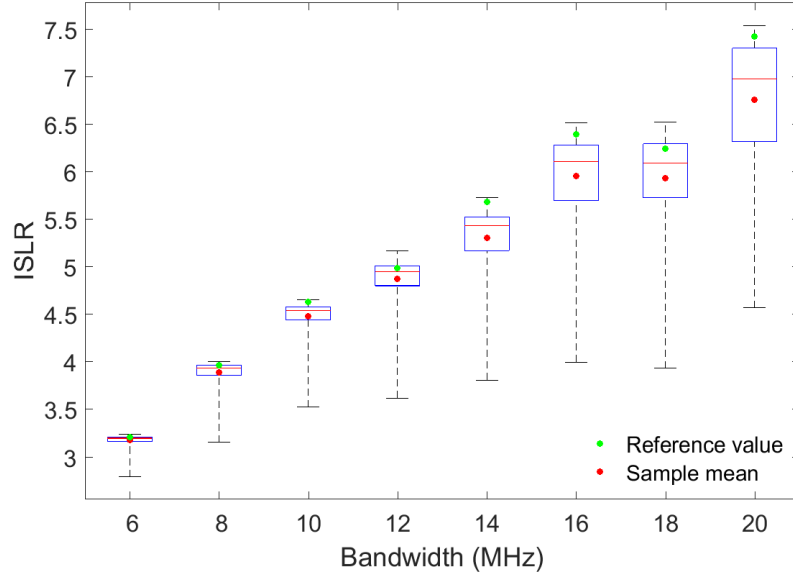


Figure 52. ISLR data for different transmitter bandwidths.

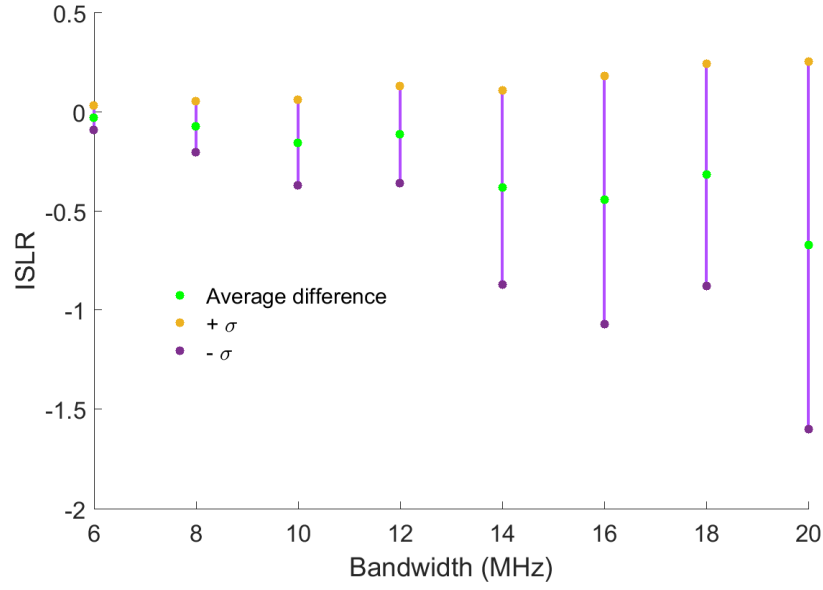
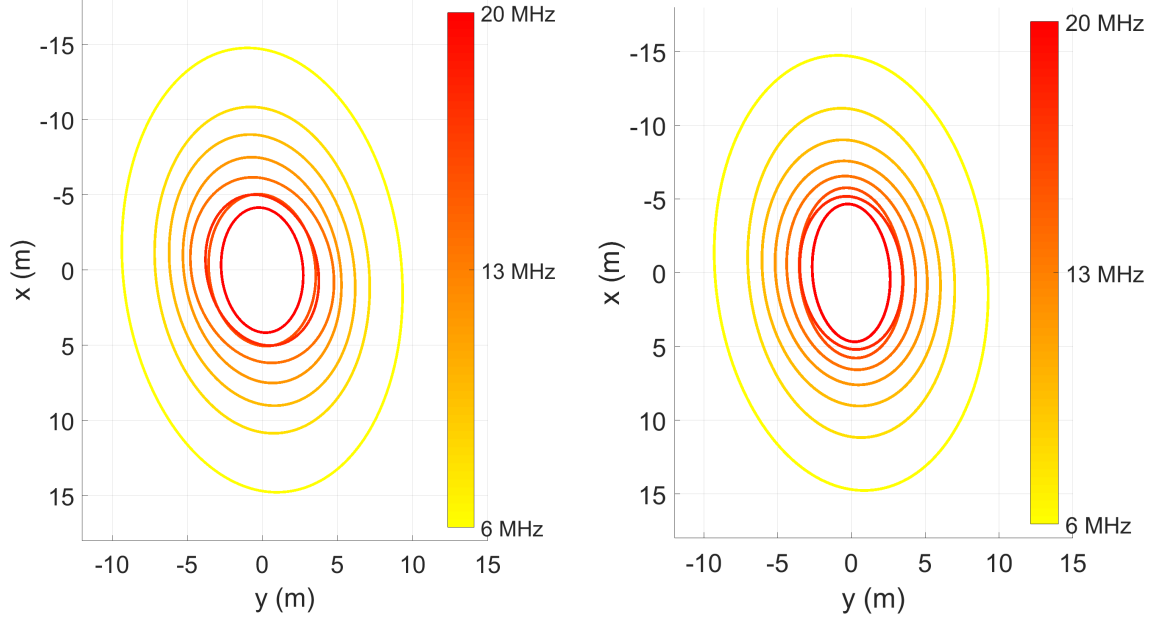


Figure 53. ± 1 standard deviation of the difference between ISLR samples and the reference value as the bandwidths of both transmitters are varied through [6,20] MHz range.

Figure 52 shows an increasing ISLR sample mean as the bandwidth grows, while Figure 53 reveals an expanding $\pm$ standard deviation interval (purple vertical lines) in addition to the widening ISLR sample mean-reference value gap (green dots).

Finally, the mean MRE for each bandwidth is placed side by side with the corresponding reference MRE for comparison purposes in Figure 54.



(a) Reference MREs of different transmitter bandwidths (b) The average MRE with 5 m^2 position error variance of different transmitter bandwidths

Figure 54. Average MRE under the effects of 5 m^2 position error variance, compared with the reference MRE of different transmitter bandwidths.

From 54b, the average MREs closely resemble the reference MREs in Figure 54a, allowing us to assess, with high confidence, that the MRE on average improves with large bandwidths. In the presence of position uncertainty, having larger bandwidths yields more stable EMRA at the expense of more eccentricity data spread.

4.8.3 Key Findings.

- With or without transmitter position uncertainty, the MRE improves on average, as the bandwidth increases.
- For the same amount of transmitter position uncertainty, multistatic sets with smaller bandwidths are affected more than sets with larger bandwidths in terms of EMRA.
- For the same amount of transmitter position uncertainty, multistatic sets with

smaller bandwidths are less affected than sets with larger bandwidths in terms of eccentricity.

4.9 Scenario 7: Selection of Emitters of Opportunity with Position Uncertainty

In the last scenario, we sum up our findings with a simplified emitters of opportunity selection problem. Henceforth, the terms transmitter and emitter are used interchangeably. In Scenario 7, there are 5 transmitters broadcasting in frequencies and bandwidths common to digital communication waveforms. The radar geometry and characteristics are summarized in Table 6.

Table 6. Characteristics of transmitters of opportunity in Scenario 7

	Tx1	Tx2	Tx3	Tx4	Tx5
Range (m)	1000	1200	700	800	900
Azimuth ($^{\circ}$)	-15	0	40	-70	85
Elevation ($^{\circ}$)	2.3	1.9	3.3	2.9	2.5
x-coordinate (m)	965	1199	535	273	78
y-coordinate (m)	-259	0	449	-750	895
z-coordinate (m)	40	40	40	40	40
Variance (m^2)	1	2	5	10	20
Frequency (MHz)	500	698	1880	1945	3460
Bandwidth (MHz)	6	6	5	5	10

For simplicity, we limit the multistatic configuration to one receiver and two transmitters, resulting in ten possible multistatic radar sets or emitter sets. We assume that the transmit power of each emitter is constant over its entire bandwidth and all five emitters have equivalent power levels. However, the emitters will differ in terms of geometry, frequency, bandwidth, and the levels of position uncertainty. Furthermore, all transmitters are stationary and the receiver's position is assumed perfectly known. As a result, the position uncertainty is due to transmitter position measurement errors only. The geometry and the position uncertainty for each transmitter

are arbitrarily selected. The frequencies are selected consistent with [5] to introduce some diversity into the scenario. Lastly, although the receiver’s azimuth extends from -0.8° to 0.8° , we use a sub-aperture within the receiver’s azimuth extent to form the PSF for each bistatic pair so that the bistatic range and cross-range resolutions are more or less equal.

Here, we carve out a much smaller and simpler emitters of opportunity selection problem, compared to that in [5]. The difference here is we take into account the position uncertainty. With the results obtained in this scenario, one can proceed to rank the multistatic sets using the MRE measures individually or in combinations. Finding an optimized way to combine the rankings to produce the optimal multistatic pair is beyond the scope of this study. Our goal is here to point out different ways the transmitter position uncertainty can affect the emitter rankings.

4.9.1 Simulation Setup.

The positions of the five emitters of opportunity in Scenario 7 are depicted in Figure 55.

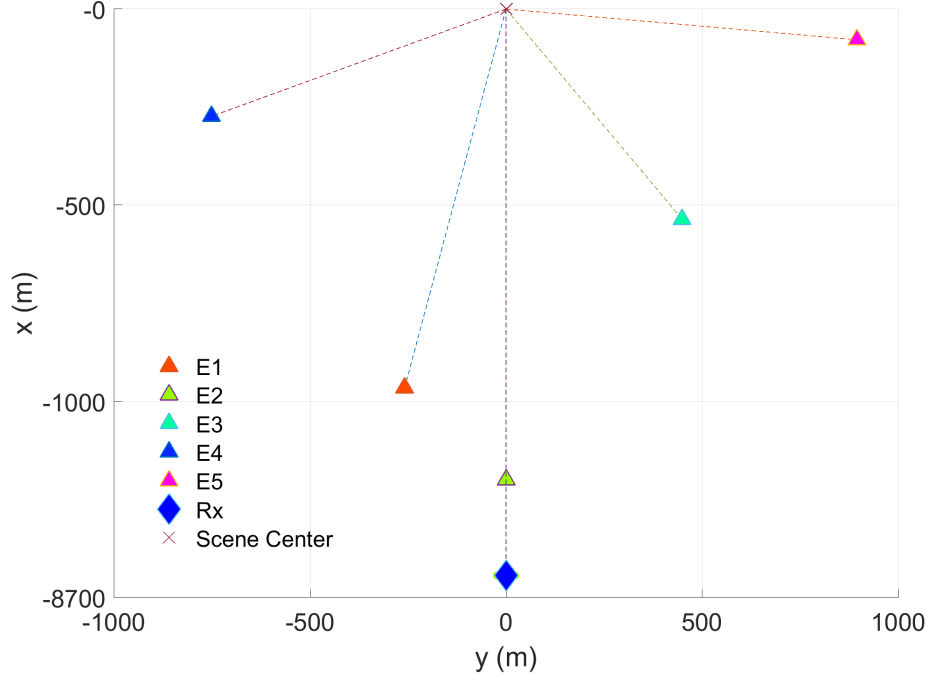


Figure 55. Five emitters of opportunity are diverse in geometry.

For each combination of emitters, the number of MC trials is determined based on the larger position error variance of the two emitters. Without any position uncertainty, the error-free MRE of each multistatic set is depicted in Figure 56. The legend numbers denote the various emitter pairings for multistatic imaging with the receiver.

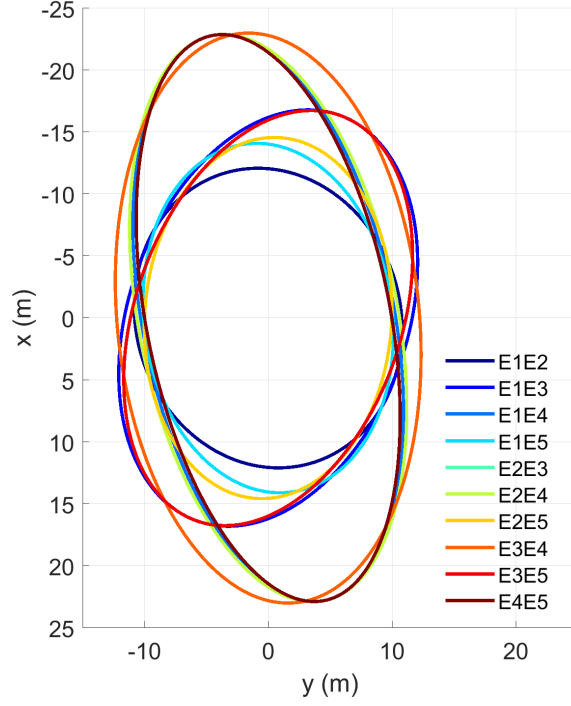
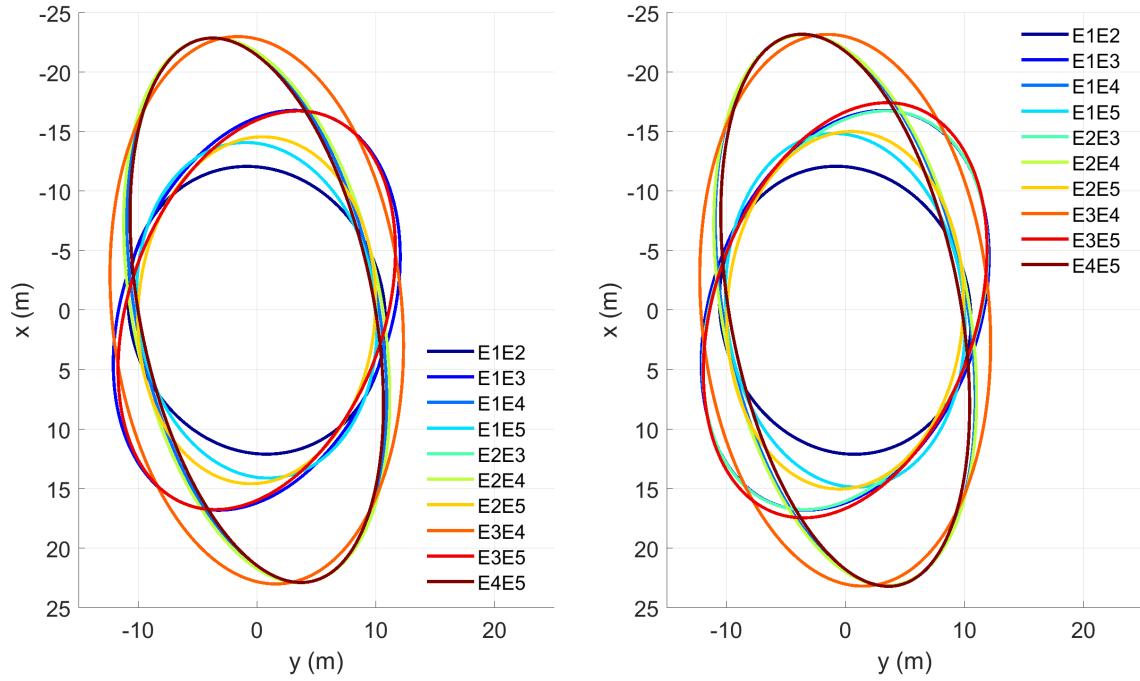


Figure 56. The MRE of each multistatic set in Scenario 7 when there is no position uncertainty.

The error-free MREs in Figure 56 serve as the reference for the upcoming analysis.

4.9.2 Results.

The average MREs of all 10 emitter sets are displayed side by side with the reference MREs determined with no uncertainty in Figure 57.



(a) Reference MREs of 10 emitter sets in Scenario 7 (b) The average MREs of 10 emitter sets in Scenario 7

Figure 57. Average MREs of 10 emitter sets compared with the corresponding reference MREs.

Comparing Figure 57b with Figure 57a, we observe that the averaged MREs closely resemble the reference MREs, suggesting that MRE measures, on average, do not significantly deviate from the reference values. As we proceed to rank the emitter sets, we can select one or more MRE measures as metrics. For example, if we choose EMRA as the primary metric, based on the EMRA data shown in Figure 58, the top five rankings are Set E1E2, E1E5, E2E5, E2E3, and E1E3, in order of ascending average EMRA.

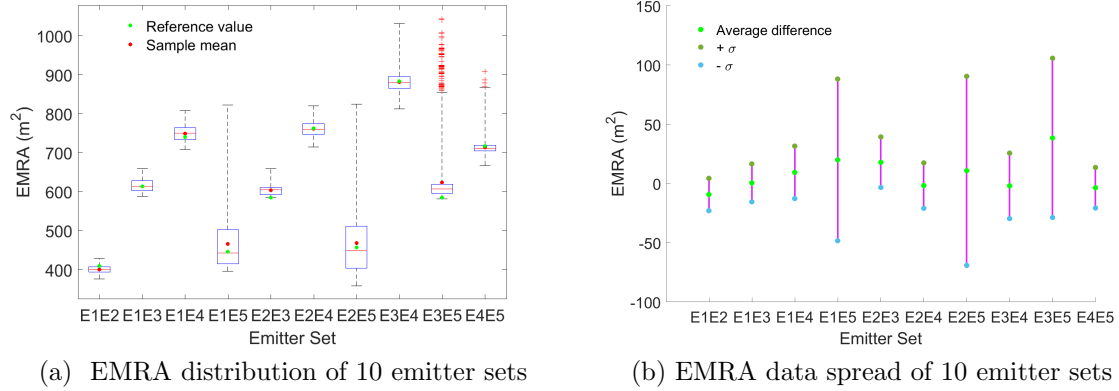


Figure 58. Comparisons of EMRA data of 10 emitter sets.

However, the effects of transmitter uncertainty cause significant amounts of spread in EMRA data of Set E1E5 and Set E2E5, as seen in Figure 58b. If EMRA data spread is an important metric, the top five rankings can be revised to Set E1E2, E2E3, E1E3, E1E4 and E2E4, for example. Note that this is just one of many possible ways of ranking the emitter sets. As other MRE measures such as the eccentricity are considered as metrics, the rankings of the emitter sets might change significantly. Figure 59a shows the distribution of eccentricity data of all 10 emitter sets.

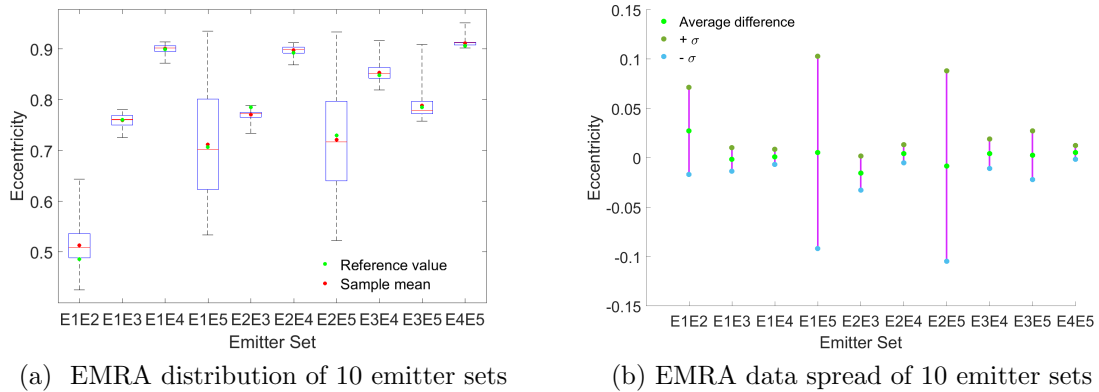


Figure 59. Comparisons of eccentricity data of 10 emitter sets.

Based on the eccentricity sample means on Figure 59a, the top five rankings (having smallest eccentricity) are Set E1E2, E1E5, E2E5, E1E3, and E2E3. However, when

examining the eccentricity data spread in Figures 59a and 59b, we notice significantly large amounts of spread in eccentricity data of Set E1E5 and E2E5, the second and third highest rankings in eccentricity. Consequently, taking the eccentricity data spread into account, the original top five rankings may be changed to, for example, Set E1E2, E1E3, E2E3, E3E5 and E3E4.

In the above examples, Sets E1E5 and E2E5 perform well on average but do poorly when data spread is taken into account. Note that both sets include Emitter 5. Beset by a large bistatic angle and a high level of position uncertainty, Emitter 5 fails to produce better multistatic resolution despite the advantages of having the highest center frequency and the largest bandwidth, compared to the other emitters (see Table 6).

4.9.3 Key Findings.

To sum up our observations on the results from Scenario 7, we find that:

- In the presence of position uncertainty, emitters with high center frequencies and large bandwidths are no longer guaranteed to yield better multistatic resolution.
- Large bistatic angles and high levels of position uncertainty cause significantly large amounts of MRE data spread, which may affect the emitter rankings.

4.10 Additional Thoughts on the ISLR

For a bistatic case, it is traditionally established that lower ISLRs indicate better resolution since the ISLR is the ratio of the reflectivity magnitude of the sidelobes to that of the mainlobe [13, 14]. The 3dB contour of a bistatic PSF is expected to have no pixel below the 3dB threshold. As the resolution degrades, the mainlobe of the bistatic PSF is lower and widened at the same time, while the sidelobes rise up. Thus

it is expected that as the resolution gets worse, we should see a larger 3dB contour and a higher ISLR.

However, in the multistatic case, depending on how the multistatic PSF is combined, the ISLR's behaviors may vary, especially in the presence of large amounts of transmitter position uncertainty. Examples of a multistatic PSF combined using the GLRT and that formed by non-coherently summing the same bistatic PSFs are shown in Figures 60a and 60b, respectively.

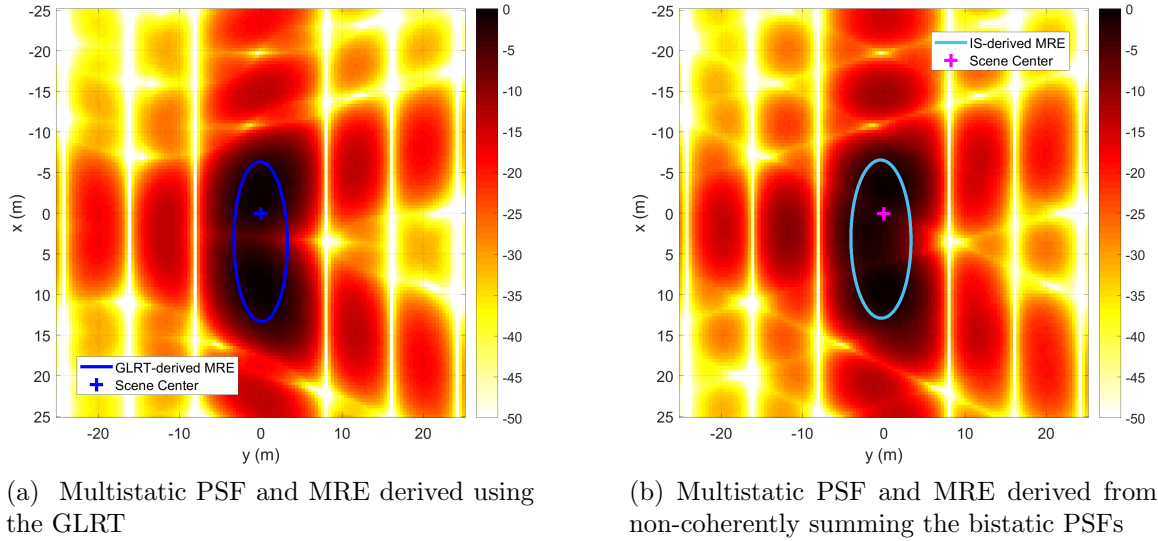
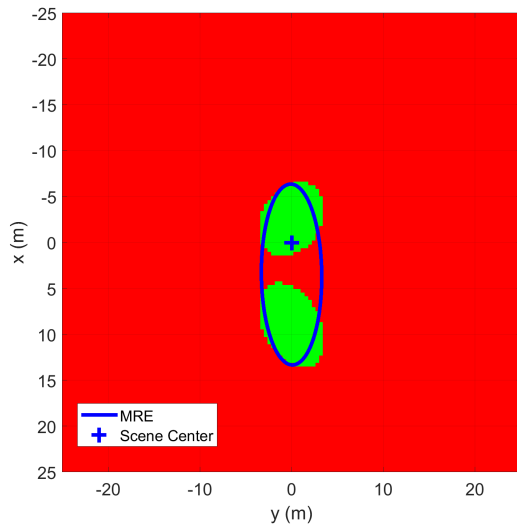
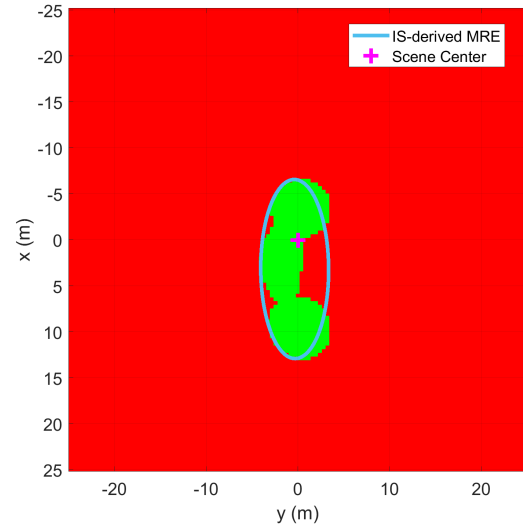


Figure 60. Examples of MRE derived using the GLRT and non-coherent summing methods with large transmitter position error variance. In both cases, the MRE contains some pixels below the 3dB threshold.

Both multistatic PSFs in Figures 60a and 60b are corrupted with 100 m^2 position error variance. In both figures, some pixels with reflectivity magnitude below the 3dB threshold are included within the MRE. The phenomenon is more readily observable in Figures 61a and 61b, which are the two-color versions of Figures 60a and 60b, respectively. In Figures 61a and 61b, pixels above the 3dB threshold are painted green and those below the threshold are painted red.



(a) Alternate view of multistatic PSF and MRE derived using the GLRT



(b) Alternate view of multistatic PSF and MRE derived from non-coherently summing the bistatic PSFs

Figure 61. Alternate view of the multistatic PSF examples seen in Figures 60a and 60b. In both cases, the pixels below the 3dB threshold (red) appear inside the MRE.

Although computing the ISLR does not involve determining the resolution ellipse, only pixels above the 3dB threshold are expected to be within the 3dB contour of the bistatic PSF. Without position uncertainty, only pixels outside of the contour are integrated to make up the numerator of (28). However, due to the separation of the bistatic PSFs, the multistatic resolution ellipse may contain pixels below the 3dB threshold, especially in cases of large position error variances, as illustrated in Figures 60 and 61. These pixels get integrated into the numerator part of the ISLR, which contradicts the conventional meaning of the integrated sidelobe ratio.

The above observations explain some of the peculiarities seen in the ISLR data, which exhibits opposing trends to those of the MRE data on many occasions. Nonetheless, we elect to present the ISLR data for cases in which a pattern is observable.

V. Conclusions and Recommendations

5.1 Key Conclusions

The results in Chapter IV indicate that the MRE data and the ISLR data exhibit discernible patterns. Insights from the results from the scenarios are summarized below:

- In the presence of transmitter position uncertainty, the average value of each MRE measure is different from the error-free value. The difference increases with growing transmitter position uncertainty. The same observations apply to the ISLR.
- As the transmitter position uncertainty increases, the MRE grows in size and eccentricity. The data spread of each MRE measure increases with growing position uncertainty. The extent to which the multistatic resolution is affected is dependent on the bistatic pair with predominantly larger position error variance. The larger the difference in position error variance, the more pronounced the effects.
- With or without transmitter position uncertainty, the MRE's area, shape and orientation, on average, are dependent on the bistatic angles of the individual bistatic pairs. For the same amount of transmitter position uncertainty, the larger the bistatic angles, the more data spread observed on MRE measures including EMRA, semi-major axis, and linear eccentricity. Furthermore, the MRE is highly likely to be quite elongated with large bistatic angles.
- Judging by the MRE measures, the extent to which transmitter position uncertainty affects the multistatic resolution is dependent on the transmitter with

predominantly larger bistatic angle. The larger the difference in bistatic angle, the stronger the dependency and the more obvious the effects.

- The MRE can function as a multistatic resolution metric for up to a finite amount of transmitter velocity measurement error. Within these limits, the MRE grows in area and elongates in the cross-range direction (y-direction) as the transmitter's velocity error increases.
- The MRE can function as a multistatic resolution metric for up to a certain amount of transmitter acceleration measurement error. Within this error range, as the acceleration measurement error increases, the MRE grows in area, expanding in the cross-range direction. Due to the combined blurring effects of the quadratic and cubic components of the differential range error, transmitters with large acceleration measurement errors may severely degrade the multistatic resolution.
- The ISLR is still capable of capturing the effects of velocity and acceleration measurement errors, at least in the range of errors simulated.
- As the center frequencies of individual bistatic pairs increase, the MRE on average becomes smaller and less eccentric with decreasing tilt angle. This phenomenon is observable with or without transmitter position uncertainty. Judging by the EMRA, transmitter position uncertainty effects are more prominent (resulting in more data spread) at lower frequencies.
- With or without transmitter uncertainty, the MRE improves on average, as the bandwidth increases. For the same amount of transmitter position uncertainty, multistatic sets with larger bandwidths are less affected if EMRA is the primary metric.

- In the presence of position uncertainty, including emitters with high center frequency and large bandwidth into the multistatic set does not necessarily produce better resolution. When comparing emitters of opportunity, one should be mindful that large bistatic angles and high levels of position uncertainty cause significantly large amounts of MRE data spread, which may affect the emitter rankings.
- Due to the spatial shifting of the individual bistatic PSFs, especially under the influence of significant levels of position uncertainty, the ISLR of the multistatic PSF no longer corresponds to the ratio of the reflectivity of the sidelobes (outside the MRE) to that of the mainlobe (inside the MRE); the MRE may include pixels below the 3dB threshold. As the result, a more robust metric is needed.

5.2 Significance of the Research

Using MRE measures and the ISLR as metrics, we are able to simulate and characterize the effects of transmitter position uncertainty due to position, velocity, and acceleration measurement errors. Specifically, we gain some insights into how position uncertainty affects the resolution performance of radar configurations of various geometries, center frequencies and bandwidths. The knowledge learned and the methodology developed in this work can be used to lay the groundwork for more complex and realistic investigations in the future. More immediately, the methods of this work can be applied to the emitter selection problem described in [5] in at least two ways: by simulating the emitters' position uncertainty, one can estimate a range of possible EMRA values and use the sample mean for the EMRA objective function; alternatively, one or more of the MRE measures and/or the amount of data spread can be used as a new selection criterion.

5.3 Recommendations for Future Studies

The results of this work can be extended in the following ways

- Increase the complexity of the radar, adding more receivers and transmitters
- Extend the investigation to explore how radar position uncertainty affects radar configurations of different waveforms, power levels and antenna patterns
- Randomize velocity and acceleration measurement errors and characterize the effects
- Set up a more complex emitters of opportunity selection problem, including mobile emitters with various power levels and waveforms
- Devise new ways to quantify the randomness so that meaningful comparisons can be made among data sets.

Additional insights into these areas will enable us to form a more complete picture of what to expect in non-ideal operational conditions, paving the way to advance problems such as emitters of opportunity selection and PMSAR flight path planning to the next levels.

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14. ABSTRACT PMSAR research efforts such as emitters of opportunity selection thus far have not considered radar position uncertainty. Seeking to fill this knowledge gap, we run Monte Carlo trials to simulate transmitter position uncertainty and examine the effects on the multistatic resolution of radar sets of different geometries, center frequencies and bandwidths. By monitoring the 3dB contour of the multistatic point spread function (PSF), we find that the multistatic resolution worsens on average and the data spread grows with increasing levels of radar position uncertainty. We observe a robust correlation between the effects of position uncertainty and the bistatic angles. Furthermore, we find that the same amount of position uncertainty has less effects on transmitters with high frequencies and large bandwidths; however, including these transmitters does not guarantee better multistatic resolution, especially with high levels of position uncertainty. The findings and methods in this work can be immediately applied to enhance objective functions for the emitters of opportunity selection problem, producing more realistic rankings that account for non-ideal conditions.						
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