



**Optimization and Parameter Characterization
for Rotating Scatter Mask Designs**

THESIS

Robert J. Olesen, 2 Lt, USAF
AFIT-ENP-MS-18-M-091

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OPTIMIZATION AND PARAMETER CHARACTERIZATION FOR ROTATING
SCATTER MASK DESIGNS

THESIS

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in Partial Fulfillment of the Requirements for the
Degree of Master of Science in Nuclear Engineering

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Abstract

The Rotating Scatter Mask system, which consists of a NaI detector, plexiglass mask, and a small motor, is a low cost directional radiation detection system with a nearly 4π field-of-view over a broad range of photon energies. However, the current mask design is limited by similarities in the directional modes of the detector response, causing potential misidentification errors when locating a source. A new class of RSM designs were simulated using MCNP and compared to the current mask design using the modal assurance criterion to characterize differentiability between directional modes. These masks were shown to successfully decouple the angular components of the source's direction, improving the average criterion value by up to 83% and limiting the directional uncertainty to the order of the physical system's angular resolution. Correlation between the geometry of the mask and the detector response for this new class of designs also presents an improved method for determining source direction, potentially enabling imaging capabilities. Finally, the new designs drastically improved the system's efficiency, reducing the time to identify the source by up to two orders of magnitude.

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To Mom, Dad, and My Brothers

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This work would not have been possible without the tremendous support and guidance from my advisor and committee. First and foremost, I would like to convey my sincerest appreciation to my research advisor, LTC O'Day. Thank you for keeping me grounded and focused while still aiming to challenge myself. Your encouragement and wisdom has directed me towards a successful path as both researcher and officer.

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Robert J. Olesen

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OPTIMIZATION AND PARAMETER CHARACTERIZATION FOR ROTATING SCATTER MASK DESIGNS

I. Introduction

1.1 Background

The capability to detect and image a radioactive source would have significant impacts in many applications, ranging from peaceful medical isotope production to national security monitoring. However, for a number of reasons, the physics of radiation transport and interactions make imaging of radioactive sources difficult. Firstly, radiation is ubiquitous in nature as a result of naturally occurring radioactive materials and cosmic rays present in the environment. These sources produce a background that varies depending on the location and potentially obscures signals from manufactured sources. Shielding of the detector from the source through the environment, whether natural or man-made, also limits detection.

Radiation manifests itself in a variety of forms, each with its own difficulties. Heavy charged particles (HCPs), such as protons and alpha particles, interact readily with matter and thus have an extremely short stopping range, on the order of microns in solids. This short range makes shielding these sources simple and detection difficult. Lightly charged particles (LCPs) such as electrons are generally more penetrating on order of centimeters but are similarly difficult to detect and their readiness to scatter in solids complicates imaging.

Neutral particles, including neutrons and photons, generally penetrate further than HCPs and LCPs of similar energies, making detection feasible for realistic ap-

plications. Sources emitting these particles, which may penetrate solids tens of centimeters before interacting, provide the highest probability for detection and the best opportunity for imaging the radioactive source.

While radioactive sources can emit any of these three types of radiation, most sources can be identified through characteristic gammas that are generated directly or indirectly from the source. The detection of energetic photons (energies from tens to thousands of keV) has become trivial with the development of photo-scintillating and solid-state detectors. The efficacy of any of these detection systems depends on their ability to distinguish the source from the background environment. Among other factors, this capability depends on the type, strength, and distance of the source, the material properties of the media between the source and detector, the background environment present, the geometric and material properties of the detector, and the characteristics of the data acquisition.

The ability to distinguish source radiation from background becomes even more critical in imaging systems, which require not only detection of particles but determining the origin of those particles. Imaging systems integrating commercially-available detectors designed for neutral particles have been used to provide directional information of the source. However, these systems are often too large or expensive for practical applications. Furthermore, many of the designs are limited in their spatial resolution, field of view, detection efficiency, or a combination of the three.

The Rotating Scatter Mask (RSM) is a novel concept first proposed by Jack Fitzgerald in 2015 that was designed to be a transportable and inexpensive system that provided accurate and rapid directional detection capabilities [1]. The RSM system is composed of a polymethacrylate mask that rotates about a singular 3 inch x 3 inch NaI(Tl) scintillation detector. The NaI(Tl) detector provides good intrinsic efficiency, good resolution, and is relatively inexpensive. The polymethacrylate mask

is designed such that as it rotates about the detector, a response signal is generated that is unique to every source direction. To reduce directional biasing, the average mass of mask material between the detector and source over a full rotation was designed to be the same regardless of source position.

Since 2015, the original RSM design by Fitzgerald has been tested and validated in experimental and computational simulations [2–4]. The results have been a success; the RSM has been used to determine the direction of a single Cs-137 source by using the full energy peaks (FEP) associated with the capture of the 662 keV gammas emitted by the source. However, the tests conducted were limited to ideal scenarios and initial results suggest that more improvements can be made on the RSM design. This research aims to optimize the physical geometry of the mask to provide faster and more reliable directional capabilities to the RSM system. Such a system has the potential for far-reaching impacts in the radiation detection field of research.

1.2 Motivation

Adding directional and imaging capabilities to detection systems greatly improves their functionality. One application that warrants extensive discussion is the search of an orphan source. An orphan source is defined as a radiation-emitting material that has been moved from its original, proper location. Whether unintentional through mismanagement or more malevolent with illegal smuggling, the orphan source poses substantial security and health risks if left undetected.

In 1983, 50 Mexican citizens who were working as scrap yard workers in Juarez received a dose ranging 390-635 rem when a medical therapy machine containing 450 Ci of Co-60 was dismantled [5]. A full month had passed after the incident until the accidental exposure was discovered, after a truck containing contaminated steel was detected in Los Alamos, New Mexico. By this time, an unknown number of

additional people could have been further exposed. Another incident occurred in 1987 when a teletherapy machine containing 1375 Ci of Cs-137 resulted in the deaths of four individuals and hospitalization of twenty-one people from acute radiation sickness when the machine was abandoned in a local junkyard [5].

For example, the simplest and most widely-used technique of surveying an area with a standard, non-directional handheld detector (such as Geiger counters) requires the user to physically move the detector through the environment. An increase in the count rate indicates that the user is moving closer to the source. This process is inherently inefficient, requiring a time-consuming trial-and-error approach. Making the detector directionally sensitive has shown to significantly increase the search efficiency [6].

The Department of Defense (DoD) has a definite and immediate need for an inexpensive, compact gamma imaging system like the RSM [7–9]. The size of the RSM system allows it to be readily deployable, with the potential for scalability to accommodate mission requirements. The simplicity of the system requires no specialist and enables operation by any airman, soldier, or monitoring team. The use of commercially-available detectors and a simple retrofitted plastic mask minimizes the cost of the RSM system, allowing for rapid widespread deployment. Mounting of the RSM system provides versatile gamma imaging functionality to unmanned aerial vehicles (UAVs), rotary winged aircraft, wheeled and tracked vehicles, and seaborne vessels, allowing them to support non-proliferation, counter-proliferation, and consequence management missions. For example, vehicles RSM-equipped vehicles could map the radioactive field and pinpoint areas of interest in the event of the detonation of an improvised nuclear device or aide in the search for special nuclear material, medical, or industrial radioisotopes.

Similarly, the RSM can aid other governmental and civilian organizations [10].

Of largest relevance to the civilian sector and the Department of Homeland Security is the employment of the RSM as a portal system in areas such as loading docks that process hundreds of container shipments daily. A small array of RSMs can be installed at these locations to prescreen shipments at the border. Domestically, the same system can be mounted at truck weigh stations to monitor the trafficking of sources within the US. Waste treatment facilities, such as the scrapyards from the 1983 and 1987 incidents, could use the RSM system to identify what materials should be set aside for special containment and processing.

1.3 Problem

A variety of directional imaging systems for radiation detection have been manufactured and tested, each with inherent limitations and constraints. An ideal imaging system would have three requirements: high angular resolution, which is the ability to precisely locate the direction of the source; large field-of-view (FOV), so that the system can be employed with minimal knowledge of the source location and environment; and a high efficiency, minimizing the time to locate the source [11]. Most fielded or tested systems suffer limitations in at least one of these categories and are comprised of a large and complex system of instruments [11, 12].

The RSM was designed by Fitzgerald in 2015 to remedy the issues from these systems [1]. An extension of rotation modulation technology, the RSM employs a polymethacrylate mask that surrounds a singular NaI(Tl) photoscintillation detector. Specifically, the RSM has demonstrated the following capabilities:

1. A field of view of nearly 4π (surrounding the detector)
2. Angular resolution to within 5°
3. Significant size and cost reduction over previous imaging systems

4. Reasonable efficiency over a range of photon energies.

While a marked improvement in directionally-sensitive detector technology, the RSM is not without limitations. As with most other imaging techniques, the use of an attenuating material effectively reduces the detection efficiency by reducing the number of photons incident on the detector. Even though the RSM has a reasonable efficiency, it has been shown that different mask geometries increase the overall efficiency of the RSM without a reduction in resolution [3].

Geometry also plays a large role in the performance of the system, since it is the varying thickness (and thus varying attenuation) that generates the angular dependency. The current design features degenerate regions where multiple directional modes share a similar response, resulting in potential misidentification of the source direction [2–4]. This problem is separate from angular resolution, which is a variance about the actual source location.

The current library lookup method also poses limitations in accurately identifying source directions for more complex environments. Apart from potential misidentification from two degenerate response curves, the RSM can only match sources to those within the reference database. For singular point sources, this is not an issue as the curves can readily be generated from radiation transport codes. Different energy sources introduce some complexity, as the macroscopic cross section of the polymethacrylate is energy dependent, but the shape of the response curves remains relatively unaffected. Therefore, the RSM is able to identify multiple source directions as long as each source energy is unique and that the peaks are distinguishable from each other and the background [4].

With multiple sources at the same energy, the current methodology begins to fail. The total response curve generated from multiple sources would be a convolution of two or more single point source response curves, weighted by the relative flux reaching

the detector for each source. Even with two sources, the complexity of attempting to deconvolve the signals drastically increases; with three or more sources, separating the individual response curves would be impossible for practical applications. Perhaps the most complex are distributed sources, with each point along the distribution acting as its own discrete point source.

These issues highlight the necessity for a better RSM design. The first study of this optimization problem, conducted by Dr. Darren Holland, focused on varying the geometry so that the response curves were as unique as possible, minimizing the overlap between any two response curves [3]. This research is an expansion of that study through a basic optimization and parameter study of RSM geometries and analysis techniques. Improved analysis techniques are also explored to replace the library database methodology.

1.4 Hypothesis

In spherical coordinates, a sources location is given by three variables: distance, polar angle, and azimuthal angle. A single RSM cannot determine the distance of the source since the strength of the signal is also dependent on the source activity; thus, a single RSM can only identify the direction of a source through its polar and azimuthal angles. It is possible to reduce this to just one variable by decoupling either angle. This can be achieved by creating an identifiable feature present in the response curve using the geometry of the mask. Decoupling simplifies the optimization and identification process as both angles become independent from each other and separate analysis techniques can be used for each. This process should significantly improve the distinguishability of the directional modes, improving the overall performance of the system.

1.5 Methods

This research simulates the response curves of various RSM designs through the Monte Carlo N-Particle (MCNP) transport code developed by Los Alamos National Laboratory [13]. MCNP simulations have been verified through the GEANT4 radiation transport code, as well as previous experimental work with the RSM [2]. Within MCNP, the geometric, material, and source properties are specified by the user. To simulate the detector response, an F8 pulse height tally was created for the region defining the detector in MCNP. The geometries tested serve as a basis for various parameter and optimization studies as well as a reference for future iterations of the physical RSM system.

1.6 RSM and Simulation Design

The spherical coordinate system is used to describe the RSM geometry and the source location. The mathematical conventions for the polar angle, ϕ , and azimuthal angle, θ , have been adopted, as demonstrated by Figure 1 [14]. The detector is oriented so that its central axis aligns with the z-axis. The polymethacrylate mask is designed to fit over the detector and rotate 360° in the azimuthal direction, with its central axis also on the z-axis. The direction of the source is defined by its own (ϕ, θ) coordinates.

The RSM assembly is composed of four primary units: 1) the stationary NaI(Tl) photoscintillation detector, 2) a stationary aluminum sleeve that houses the detector, 3) the polymethacrylate RSM surrounding the detector and aluminum sleeve, and 4) the base, electric motor, and belt system that drives the rotation of the RSM. Figure 2 shows a cross-section of a simplified RSM system showing these four components. The key component that transforms the NaI(Tl) detector into a directionally sensitive system is the RSM itself. The thickness of material varies as the mask rotates in θ ,

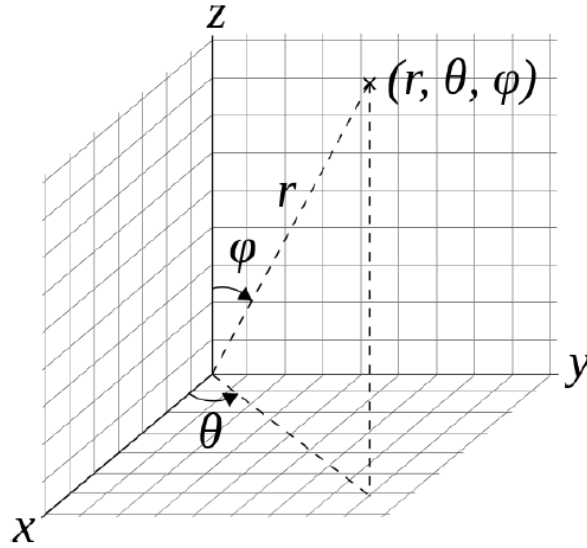


Figure 1. Spherical coordinate system used for the RSM geometry and source direction. The central axis of the detector and mask was aligned to the z-axis such that the mask rotated about θ . The center of the detector corresponded to the origin of the coordinate system. [14]

producing a response curve in the form of the number of counts in the FEP a function of rotation angle θ . The mask is designed such that variation of thickness is also dependent on ϕ , creating unique response curves for each source ϕ .

For the optimization and simulation of various geometries, a convenient mathematical representation to describe the three-dimensional shape of the RSM was developed. This representation, called the design matrix, is shown in Figure 3 as a two-dimensional array. The value of the elements identifies the thickness of material while the elements location in the array gives the (ϕ, θ) direction for the edge of the mask.

1.7 Research Contributions

The original concept and design of the RSM as a directional gamma detection system came from the research by Jack Fitzgerald [1]. Manufacturing of the RSM and

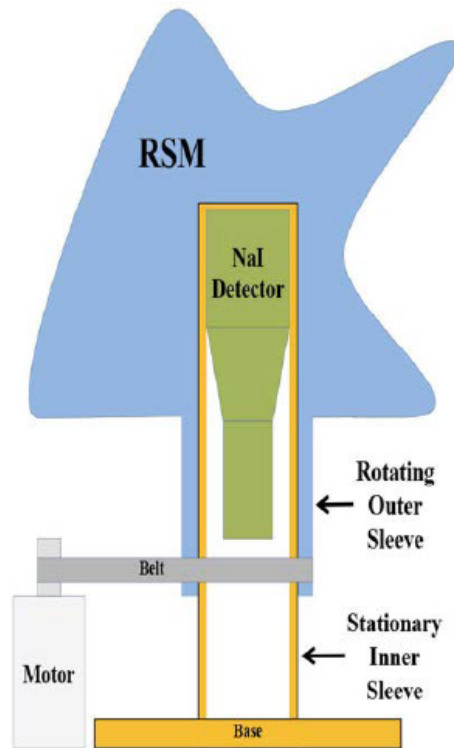


Figure 2. Graphic cross-section showing the major components of the RSM assembly: 1) NaI detector; 2) aluminum sleeve; 3) polymethacrylate RSM; and 4) base, motor, and belt that drives the rotation of the RSM. For the simulations, only the first 3 components were modeled. [1]

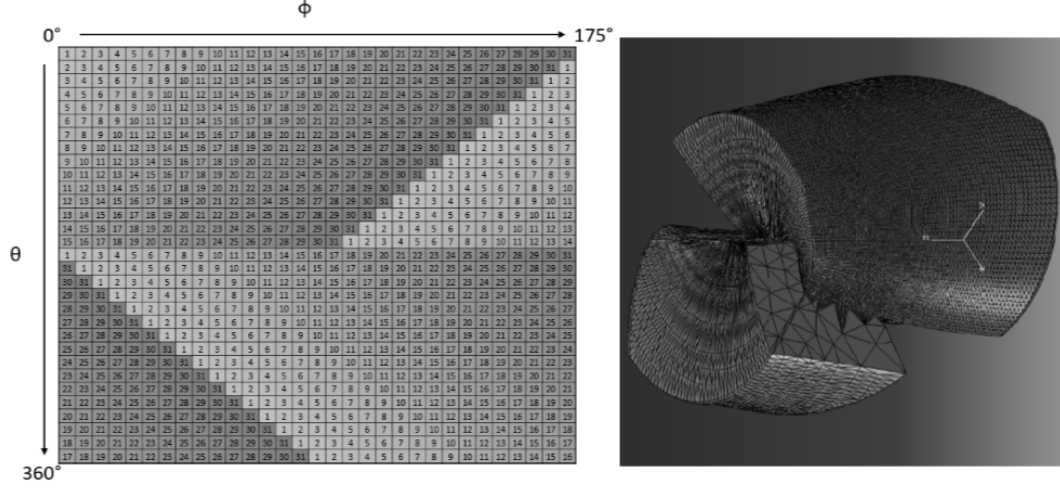


Figure 3. 3D representation of the Fitzgerald RSM (right). A discrete form of the corresponding design matrix is also shown (left). The location of an element within the matrix corresponds to the directional vector pointing towards the surface of the mask, whose length is proportional to the elements magnitude. [3]

initial testing was conducted by Major Christopher Charles. GEANT4 and MCNP validation and verification was performed by a subsequent researcher (Julie Logan) by comparing the simulations to data collected by herself and Major Charles [2]. Logan also developed an algorithm for determining the direction of the source, which serves as the basis for the optimization studies conducted. The MCNP simulations were conducted by a collaborator, Dr. Darren Holland, who also worked on the initial optimization of the RSM design [3]. The code and designs he developed form the foundations for this research. The primary researchers contributions to the development of the RSM include:

1. A software package to automate the simulation and analysis of RSM geometries
2. Increased design parameter studies using MCNP
3. Comparison of new geometries to the original Fitzgerald and optimized Holland designs
4. Characterizing and optimzing the performance of the RSM as a directional

gamma detection system through synthesis and analysis of data.

II. Theory

2.1 Photon Interactions in Matter

Photon interactions in matter occur through three primary mechanisms: the photoelectric effect, the Compton effect, and pair production [15]. In the photoelectric effect, an incident photon is absorbed by a bound, orbital electron within the material. The electron gains the photons energy and is sufficiently excited to become free of the atom. The energy required to free the electron from the bound states is dependent on the type of atom; any excess energy is transferred to the kinetic energy of the electron.

Compton scattering is similar to the photoelectric effect, except that the photon is not completely absorbed. Instead, the electron absorbs a partial amount of the incident photons energy and a second photon is emitted nearly simultaneously, such that it seems as if the photon “scatters” off the electron. The electron is freed from its orbit while the secondary photon is scattered away in a direction obeying the conservation of momentum. Conservation of energy dictates that the energy of the scattered photon and electron must equal the incident photon energy minus the binding energy of the electron. In most cases, the photon scatters off an outer-orbital electron so that the binding energy of the electron can be ignored [16]. Coupled with conservation of momentum, the energy of the scattered photon, E'_γ , can be found with the following equation:

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2} (1 - \cos(\theta))}, \quad (1)$$

where E_γ is the incident photon energy, m_e is the electron rest mass, c is the speed of light, and θ is the scattered photon angle.

Pair production does not require a target electron for the photon to interact with, but it does require the presence of another particle, typically a nucleus. In this interaction, photons of sufficiently high energy can be transformed into an electron-positron pair. It is important to note that the photon is not absorbed by the secondary particle; it is simply the presence of it that allows the electromagnetic energy of the photon to be converted into matter. Since the rest masses of the electron-positron pair are 0.511 MeV each, pair production can only occur for photons with energy of at least 1.022 MeV. Any excess energy is imparted to the kinetic energies of the electron-positron pair.

2.2 Interaction Probability

While classical concepts such as conservation of energy and momentum were used to develop the formulas such as Equation 1, the interactions themselves are described through quantum mechanics. As a result, it is impossible to determine exactly if and when an interaction between any specific particles will occur. At best, one can only calculate the probability of an interaction occurring. This stochastic nature is represented by the reactions cross section, which is dependent on both photon energy and material characteristics.

Specifically, the probability of an interaction with a single atom is known as the microscopic cross section [16]. Since the interactions mentioned occur with the orbital electrons of the atom or the nucleus itself, the probability of interaction is dependent on how tightly bound the electron is, how many there are surrounding the atom, or the size of the nucleus. That is, the microscopic cross section is closely related to the atomic number, Z , of the atom. At a given photon energy, it has been shown that the microscopic cross section for photoelectric absorption generally increases with $\sim Z^{7/2}$, scattering increases proportionally with Z , while pair production increases

with Z^2 [15].

The microscopic cross section is also energy dependent on the incident photon. Besides pair production, most reactions occur at any energy level with varying probability. For any given material, the photoelectric effect dominates for low energy photons up to several hundred keV. Compton scattering tends to dominate interactions in the intermediary photon energies up to a few MeV, at which point pair production begins to dominate. The total microscopic cross section is defined as the sum of the individual reaction microscopic cross section and is the total probability of any interaction at a given energy.

The microscopic cross section has units of length squared. When multiplied by the atom number density of the material, the units become inverse length and the value represents the probability of interaction per unit distance as the photon transverses through the material. This quantity is known as the macroscopic cross section, μ . An interesting feature of the macroscopic cross section is that its inverse is the mean free path of the photon, or the average distance a photon will travel before interacting [15]. Since the macroscopic cross section is dependent on the number density, and thus weight density, of the material, the mass attenuation coefficient is commonly reported over the macroscopic cross section. This is just the macroscopic cross section divided by the weight density, ρ , of the material. The mass attenuation coefficient, symbolized by $\left(\frac{\mu}{\rho}\right)$ removes the density dependency and has units of area per mass. Figure 4 shows a typical mass attenuation coefficient using iron as the material.

As photons pass through a material, some interact while others continue unperturbed. Since the number of atoms the photon must pass by is large ($\sim 10^{20}$), while the probability of interacting with any single atom is low ($\sim 10^{-20}$), the remaining number of photons left unaffected is given by a Poisson distribution [18]:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x}, \quad (2)$$

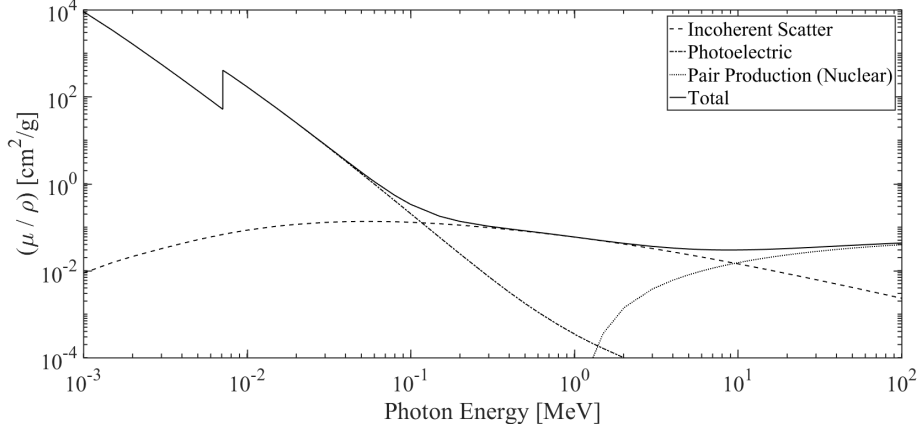


Figure 4. Mass attenuation coefficients for photon interactions in Fe-54, as a function of photon energy. Photoelectric absorption dominates at energies around 100 keV and below. Incoherent scattering (primarily through the Compton effect) dominates around 100 keV 10 MeV. Pair production occurs starting around 1 MeV but only dominates at energies above 10 MeV. [17]

where I_0 is the incident photon intensity, I is the intensity of unchanged photons, $\left(\frac{\mu}{\rho}\right)$ is the mass attenuation coefficient, ρ is the density of material, and x is the path length of the photons [15].

2.3 Photon Detection

In the reactions previously described, the photon imparts its energy into the material, either by exciting electrons or generating electron-positron pairs. Most systems detect incident photons indirectly by measuring the energy the photon deposited into the material through these secondary particles. The number of interactions that occur in the material is simply

$$I' = I_0 - I = I_0 \left(1 - e^{-\left(\frac{\mu}{\rho}\right)\rho x}\right), \quad (3)$$

where I' is the intensity of interactions that took place within a distance x of the material and I is the intensity of photons that passed freely through from Equation

2.

Equation 3 suggests that an ideal detector has a large mass attenuation coefficient or high density so that the maximum number of photons interact within the detector material. The same principle applies to shielding, which is why lead is often used in shielding materials due to its high Z (large cross section) and density. Once the photon has deposited its energy into the detector, it is then converted into an electronic signal that can be post processed, usually as a voltage pulse whose amplitude is proportional to the energy of the incident photon. For pair production, the emitted positron undergoes many coulomb interactions before it eventually annihilates with an electron in the detector. The annihilation event produces two 511 keV photons, moving in opposite directions of each other (since the positron loses enough energy that both it and the electron can be considered at rest before annihilation [15]). The 511 keV photons proceed to interact with the detector either through photoelectric absorption or Compton scattering; the presence of a 511 keV photon FEP indicates that pair production has occurred.

There are a number of methods by which detectors convert the photon energy into an electrical signal. Two prominent detectors used are semiconductor and photoscintillation detectors [15]. Semiconductor detectors, such as high-purity germanium (HPGe) detectors, take advantage of the band-gap within the semiconductor, exciting electrons from the valence to the conduction band and creating a small current. These detectors have some of the best energy resolutions commercially available, but due to their small size, they are limited in efficiency [15]. Random thermal excitations are also usually enough to excite the electrons, so the semiconducting material must be cryogenically cooled, increasing the cost and size of the system.

Photoscintillation detectors contain a material that creates a visible light pulse when struck by a gamma photon. The scintillating material is itself transparent to the

visible light, so that the light pulse can be converted to a current and voltage pulse using a photocathode. While their resolution is not as high as semiconductor detectors, they do not require sophisticated equipment to function and the scintillating material is much easier to manufacture, meaning larger detectors with better efficiencies can be produced [15]. They also function over a wide range of photon energies, making them suitable in many applications. Particularly, inorganic scintillators such as the NaI(Tl) have become standard equipment in gamma radiation detection due to their high efficiency, good resolution, and low cost [15]. These characteristics are also why the NaI(Tl) detector was chosen for the RSM system.

2.4 Inorganic Scintillation Spectroscopy

The NaI(Tl) is an inorganic photoscintillation detector composed of a sodium iodide crystal doped with thallium. The crystal has an electronic band structure similar to that of a semiconductor, containing a valance and conduction band and an associated band gap. When a photon interacts through photoelectric absorption or Compton scattering within the crystal, the high energy electrons created proceed to excite and ionize the surrounding atoms, producing lower energy electrons. Some of these electrons will have enough energy to go into the conduction band, creating electron-hole pairs. By this method, a single gamma can produce many electron-hole pairs [19].

These electron-hole pairs eventually transition to the thallium activation site, which has an energy level just below the conduction band and one above the valence band of the bulk crystal. This results in a thallium atom in an excited state with a hole in its ground state. The electron quickly de-excites into the ground state, emitting a characteristic photon in the process (for thallium, the emitted photon is near the blue end of the visible light spectrum [19]). The crystal is transparent

to the emitted light, so the photon passes through the material until it is collected at a photocathode, where the photon is converted into a cascade of electrons using the photoelectric effect. These electrons are then passed through a photomultiplier tube, in which the electrons are accelerated through a series of dynodes that create a cascade of an increasing number of electrons. At the end of the photomultiplier tube, the amplified current generated from the electrons is collected and turned into a voltage pulse for processing. Figure 5 shows a graphic of this process from energy deposition to voltage pulse.

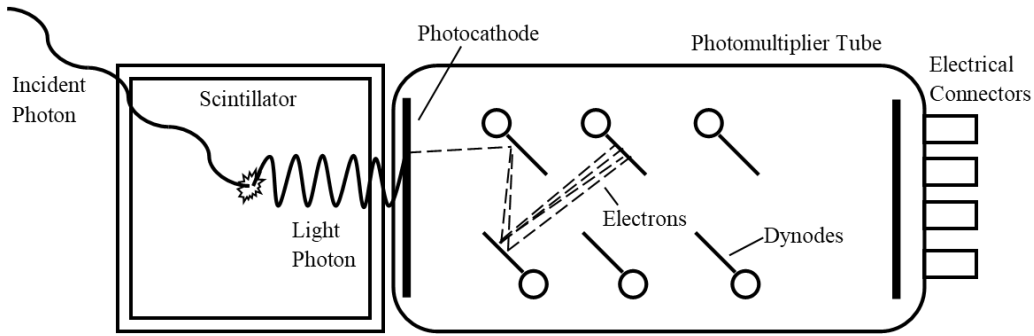


Figure 5. Graphical representation of the interactions occurring in a photoscintillation detector and photomultiplier tube. The scintillator converts the incident photon into a light pulse, which is then converted into an electrical signal via the photocathode and photomultiplier tube. The strength of the signal is proportional to the energy of the incident photon.

Since higher energy gammas create more electron-hole pairs, and thus more scintillations in the crystal, the height of the pulse is proportional to the energy deposited by the gamma. Figure 6 shows the mass attenuation coefficient for a NaI crystal, where photoelectric absorption dominates until around 300 keV. From 300 keV to a few MeV, Compton scattering is the primary interaction mechanism before pair production begins to dominate. In the testing of the RSM, a single Cs-137 source was used to create the response curves. The decay scheme for Cs-137 is pictured in Figure 7. The primary mode of decay first involves a beta emission as the Cs-137 decays into a meta-stable state of Ba-137. The Ba-137 quickly relaxes to the ground

state, releasing a 662 keV gamma in the process.

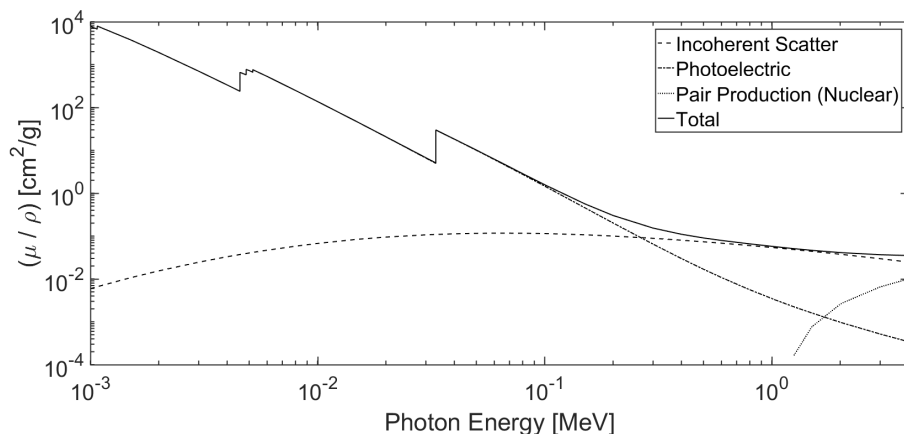


Figure 6. Mass attenuation coefficients for photon interactions in a homogenous crystal of NaI. Photoelectric absorption dominates at the low-level energies, up to a few hundred keV, at which incoherent scatter dominates up to a few MeV. The Thallium concentration within the physical detector is small enough such that its effect on the mass attenuation is negligible, and was thus ignored for the simulations. [17]

Using Figures 6 and 7, it is possible to predict what the recorded energy spectrum of a Cs-137 source by a NaI(Tl) detector would look like. Although the spectrum will be unique based on the environment, detector type, and source, the common features are outlined below:

1. Full Energy Peak (FEP): The primary feature when identifying the source type. In this region, the incoming photon deposits nearly all of its energy through the photoelectric effect, creating a large spike in the spectrum. For an ideal detector, this spike would occur exactly at the photons energy. However, due to the stochastic nature in the production of electron-hole pairs, electrons at the photocathode, and photomultiplier tube, the FEP undergoes Gaussian broadening [15]. The amount of broadening compared to the energy of the FEP is known as the detectors energy resolution.
2. Compton Continuum: Equation 1 showed the resulting energy of a photon after Compton scattering. Since any secondary particles are assumed to be

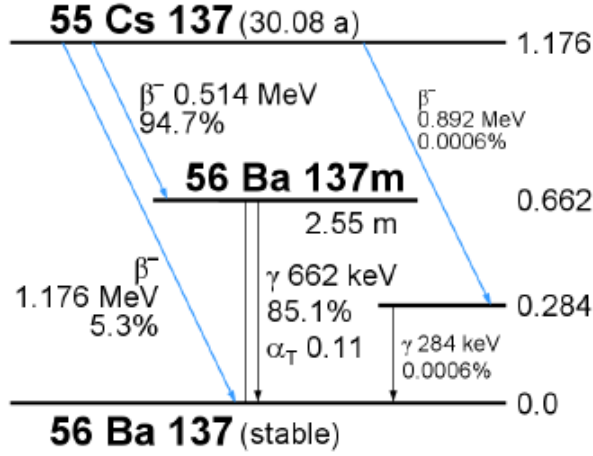


Figure 7. Decay scheme for a Cs-137 source. The primary decay mode occurs when the Cs-137 decays into a meta-stable state of Ba-137, which quickly relaxes into its ground state, emitting a 662 keV in the process. This mode accounts for roughly 80% of all Cs-137 decays. [20]

absorbed in the detector, the deposited energy is simply the change in photon energy. The scattering angle of the photon is determined by a probability density function known as the Klein-Nishina Distribution; the distribution for a 662 keV photon is plotted in Figure 8 [21]. While the photons are predominately forward-scattered, all angles are possible. The spectrum of energy deposited from scattered photons in the detector is known as the Compton continuum.

3. Compton Edge: Equation 1 shows that the maximum energy transfer occurs when $\theta = 180^\circ$. For a 662 keV photon, this results in an energy change of approximately 478 keV. This energy marks the upper limit of the Compton Continuum, at which point there is a drastic decrease in counts up until the FEP. This region between the Compton edge and FEP is known as the Compton valley [15].
4. Backscatter Peak: This feature is a result of interactions surrounding the system rather than in the detector itself. Gammas scattered off surrounding materials

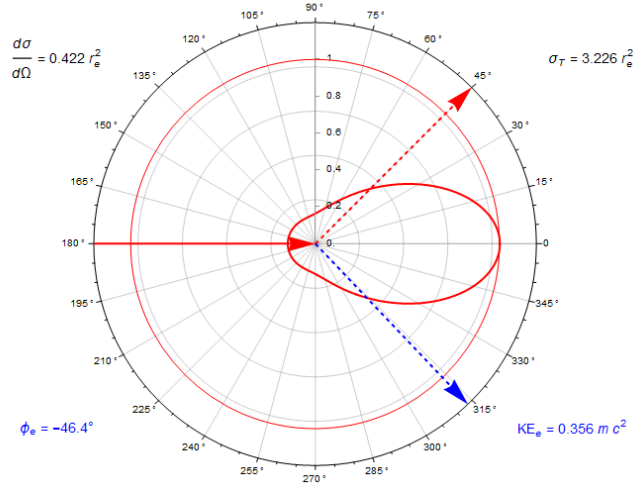


Figure 8. Graphic showing the differential cross-section for the various angles in a Compton scattering event for a 662 keV photon. This plot shows that the scattered photons are forward-peaked, with most of the probability occurring within 30° of the incident photon direction (solid red). An example showing the direction and energy of a freed electron (dashed blue) from a 45° scattered photon (dashed red) is also shown. [21]

can be absorbed in the detector, increasing the number of counts in the Compton continuum. The backscatter peak occurs as gammas scatter off a material behind the detector and then absorbed by the detector. This forms a peak around 184 keV for 662 keV photons.

It is important to note that the broadening mentioned in the FEP occurs for all these phenomena [15]. Figure 9 is a graphical representation showing the difference between an ideal and an actual energy spectrum from an NaI(Tl) detector. A rough approximation for the ratio of counts in the FEP to the Compton continuum can be calculated by the ratio of absorption-to-scatter probabilities in Figure 6. At 662 keV, the ratio is roughly 1:8.

In the RSM system, the detector is surrounded by the polymethacrylate mask. The mask serves primarily to attenuate the photons before they reach the detector in order to change the number of counts in the FEP as it rotates. Figure 10 plots the

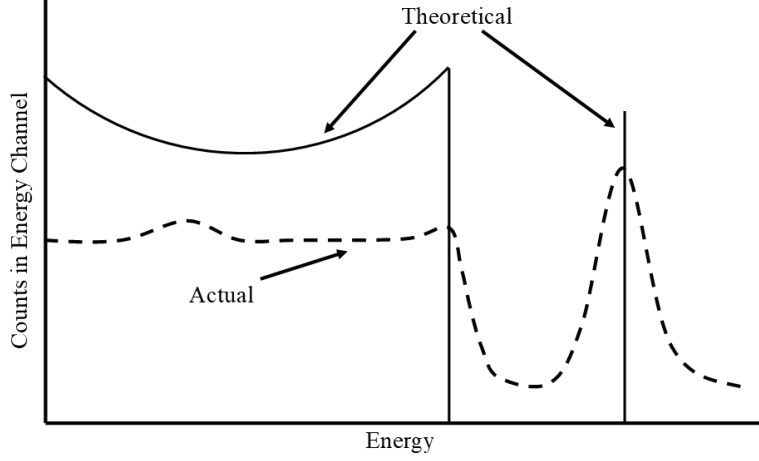


Figure 9. Comparison of an ideal (theoretical) and experimental (actual) photon spectrum from a photoscintillation detector. The Gaussian broadening occurs from the statistical nature of the generation of electron-hole pairs in the detector and electrons in the photomultiplier tube.

mass attenuation coefficient for polymethacrylate. At 662 keV, Compton scattering is the principal mechanism for attenuation. The number of counts detected decreases overall due to the attenuation, but the scattering from the mask causes more counts in the Compton continuum, decreasing the FEP-to-Compton continuum ratio.

2.5 Neutral Particle Transport Methodologies

There are two schools of thought for modeling the transport of neutral particles. The first method is a deterministic approach in which the numerical solution to the Boltzmann transport equation is solved. This equation is simply a conservation law that takes into account the different means of production and loss of particles. An advantage of this approach is that a global solution is found, so that the particle flux can be calculated anywhere in the model [22]. However, due the complexity of most models, an analytical solution is impossible to calculate. This requires a discretization of time, energy, angular direction, and spatial coordinates as well as simplified geometries and averaging of cross sections using Legendre polynomials.

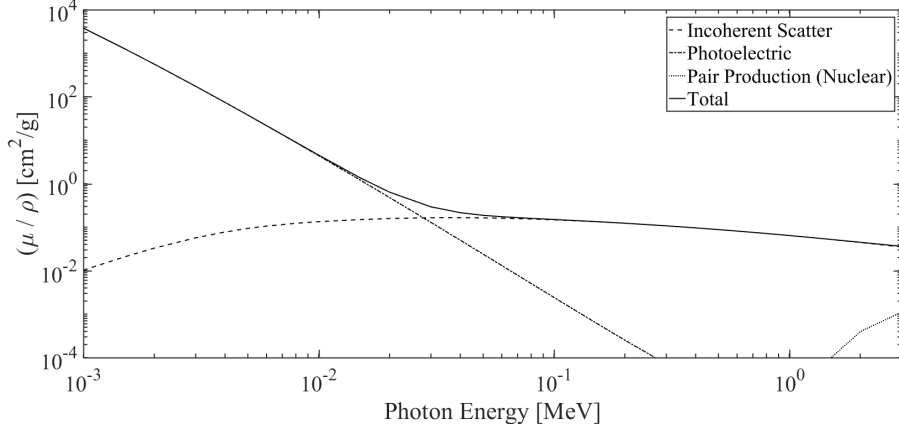


Figure 10. Mass attenuation coefficients for photon interactions in the polymethacrylate RSM material. Incoherent scattering dominates over a wider range compared to the NaI detector, starting at tens of keV and extending to tens of MeV. [17]

The second method is a stochastic approach through a Monte-Carlo process which simulates the probabilistic nature of radiation interaction. Sampling of various probability density functions describing the path length of a particle, the type of interaction, and the resulting particles after interaction allows one to predict the expected behavior of a model. As with any Monte-Carlo process, the law of large numbers governs the statistics of the model so that enough samples must be drawn to ensure the simulated response matches the expected behavior. Unlike the deterministic approach, Monte-Carlo methods can only calculate values at discrete locations and boundaries. At this expense, Monte-Carlo simulations are typically easier to compute and run faster than deterministic models, especially in complex scenarios [22].

2.6 Gamma Ray Imaging

The detection methods mentioned in the previous sections can only give information about the intensity and energy of photons incident on a stand-alone detector; they do not provide information on where those photons originated. Imaging systems enable the user to access both energy spectral and directional information of a ra-

radioactive source. A number of concepts have been tested and developed to achieve this goal; it is worth discussing these concepts, and their limitations, before examining the impact of the RSM in this field of study.

Collimators form the foundation for the most basic imaging systems. In this method, shielding is placed around the detector. An aperture is included in the shielding so that only radiation from a specific direction can be detected; while this results in good angular resolution, most of the radiation is absorbed by the shielding, limiting both efficiency and FOV. Increasing the number of apertures increases the detectors efficiency, but there is no significant improvement to the FOV [11].

Coded aperture detectors are a form of collimator imaging. An array of shielding material is placed in front of an array of detector so that as radiation passes through the shielding, a radiation “shadow” falls on the detector. The source location can be identified through deconvolution or correlation of the shadows image. The Large-Area Coded-Aperture Gamma-Ray Imager employed this technique with an array of lead blocks placed strategically in front of 57 NaI detectors [12]. The benefit of this system was that it was able to distinguish background noise from a low-level radioactive source (on the order of a millicurie) at distances up to 80 m. However, it could only image sources that were in line with the coded aperture and detectors.

The SuperMISTI is a hybrid imaging system that combines the high energy resolution of HPGe detectors with the imaging capabilities of coded aperture arrays [23]. An array of 48 HPGe are used to detect and identify a radioactive source, while a coded-aperture system of 78 NaI detectors and 6 in x 6 in x 2 in lead blocks is used to locate the direction of the source. While the SuperMISTI is able to locate sources ranging up to hundreds of meters, it faces the same FOV limitations inherent to coded apertures. The number of detectors also ran the cost of the project to over 1,000,000 USD for a single system.

In rotation modulation collimators (RMCs), two or more slit collimators are rotated in front of a standard detector [24]. This reduction from an array to a singular detector is a significant improvement in portability compared to the previous two examples. The rotation of the collimators produces a modulation in the recorded energy spectra, from which the source location is extracted from. An RMC developed at AFIT by Kowash achieved an angular resolution of 0.5° , although still limited to a small 9° FOV [25].

These examples demonstrate the push for a gamma-ray imager without a collimator- or aperture-based design. One concept that has become widely used is the Compton imager (frequently called a Compton camera or Compton telescope). Compton imagers rely on the Compton scattering of photons to identify the direction of the source [11]. By measuring the energy of the scattered photon and electron, the imager is able to determine the photons scatter angle (an application of Equation 1). The first Compton imager to successfully demonstrate this capability was the COMPTEL instrument, designed as a space-based gamma-ray observatory [26]. The benefit of these devices is that they provided a nearly 4π steradian FOV, a good angular resolution, and are the only imaging devices currently capable of completely removing background signals [11]. Theoretically, the absence of a collimator allows for better detection efficiencies, but modern imagers use gaseous and liquid scintillators to improve scalability at the expense of poor efficiency. The drawbacks also include the cost and complexity of the instruments.

2.7 Literature Review

The RSM uses the principle that rotating a mask of varying thickness around a detector produces an energy spectrum that is a function of the mask rotation. This concept has its origins in the RMC designed by Kowash in 2014 [25]. Using rotating

plastic discs containing parallel slits as the varying attenuating material, Kowash was able to show that the spectrum detected was dependent on the direction of the photon source relative to the detector. Shortly after, the RMC was modified to successfully identify neutron sources. The major drawback of the system, though, was that it was limited to an extremely narrow $17^\circ - 35^\circ$ FOV.

Fitzgerald designed the RSM to solve the issues regarding FOV and angular resolution. Replacing the plastic discs with a hollow mask designed to surround the detector, Fitzgerald was able to prove computationally that the RSM should work at a near full 4π FOV [1]. This was later verified experimentally by Charles and Logan through a modified set-up of Kowashs RMC assembly, replacing the collimator with Fitzgeralds mask design [2]. Logan also validated and verified computational simulations of the RSM assembly through GEANT4 and MCNP by comparing to the experimentally collected data for a Cs-137 source. Through their work, Charles and Logan developed the first generation of RSM technology, complete with data acquisition and processing.

However, the RSM developed was only a proof-of-principal and not an optimal solution. The design for the mask was chosen by Fitzgerald because it met the requirements of creating unique response curves based on source location, but it is not the only design that meets this requirement. The work of Holland tackled the optimization problem: which mask geometry is the best for identifying source location? Holland developed a series of RSM designs based on eigenvector problems and orthogonal basis sets and tested them through MCNP [3]. While his research concluded that it is impossible to have a perfect design, the masks he created were all shown to outperform the Fitzgerald design. This study uses the techniques developed by Holland as a foundation for the optimization of the RSM design while expanding on the concepts suggested in his studies.

III. Design and Simulation Methodologies

3.1 RSM Geometry

The RSM geometries were created using a modified version of the voxel method developed by Holland. With this method, a user-generated two-dimensional design matrix for the RSM was converted into a three-dimensional mesh, with each point of the mesh corresponding to a node of a hexahedral voxel. The design matrix contained the following parameters:

1. The number of rows, m , in the design matrix corresponded to the number of voxels over a full rotation in θ . That is, it defined the angular width of the voxels in θ , given by $360^\circ/m$.
2. The number of columns, n , in the design matrix similarly defined the angular width of the voxels in ϕ . The maximum extent of ϕ was chosen as 170° to account for the electronics and mechanical components behind the detector. Therefore, the angular width was defined as $170^\circ/n$.
3. The value of a specific element corresponded to the relative thickness of the mask for that voxel. During conversion to the mesh, the values were normalized by dividing by the maximum of the design matrix and then multiplying by a scaling factor chosen by the user. Since these values represent a thickness, all elements were real, non-negative values.

The flexibility of this technique improves upon previous optimization studies, which were limited to design matrices of specific sizes [3]. With this method, any $m \times n$ matrix could be converted into a 3D model. An example of a design matrix used is shown in Figure 11.

2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 11. Design matrix for the Wide Wall RSM concept. The values are normalized to the largest value within the matrix and then adjusted with a scaling factor to create the voxels thickness. The size of the matrix defines the geometrical resolution of the voxels. The mask geometry spans a full 360° rotation in θ and $0^\circ - 170^\circ$ in ϕ .

To provide structural support to the RSM, as well as a connection for the motor, the mask was fitted over a hollow cylindrical shell of the same polymethacrylate material. The thickness of the shell was 1 cm and the inner radius was designed to be 4 cm in order to fit snugly over the aluminum sheath surrounding the detector, creating a consistent contact as the mask rotates. During manufacturing of these masks, the RSM and shell could be created as a single homogeneous unit.

Calculations of the nodal positions were performed in MatLab, a matrix-based programming language developed by MathWorks [27]. The mesh was converted into an Abaqus input file format so that MCNP could correctly generate the mesh in the simulations. A graphic user-interface (GUI) was developed to simplify the process of generating multiple designs. In the GUI, the user is able to create an MCNP input deck and RSM mesh file by uploading a design matrix and entering parameters such as detector size, shell thickness, and scaling factor for the mask. A 3D model of the RSM is also generated for visual verification. Figure 12 is a screenshot of the program, using the design matrix of Figure 11 for the model.

3.2 Design Space

A total of nine RSM designs were evaluated, including the original Fitzgerald mask. These designs can be divided into two classes based on the two design solutions developed in Hollands studies [3]. Masks that are designed to minimize the overlap between all response curve combinations fall into the Eigen Class designs, named after the mass-spring eigenvector approach used by Holland to create the column vectors of the design matrix. The classifying feature of Eigen Class designs is that the directional coordinates (ϕ, θ) are determined simultaneously from the response curves by comparing all θ shifts of the curve to a library database of curves for $\theta = 0^\circ$. The shift required for the best fit yields the θ angle of the source while the

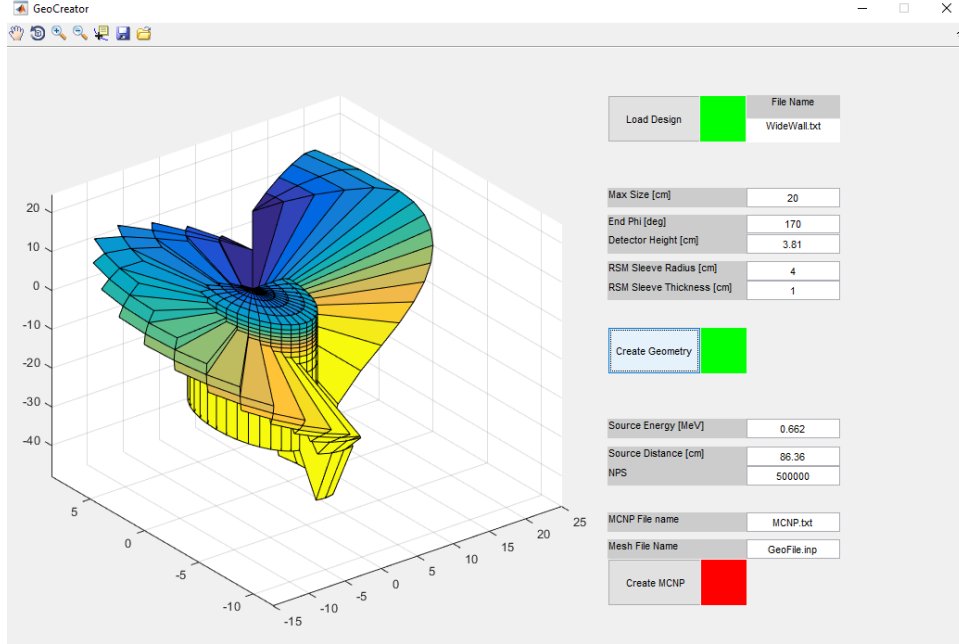


Figure 12. GUI interface developed in MatLab to aid in developing RSM design concepts. The user loads a design matrix and inputs values such as the scaling factor, detector size, and angular extent of the mask. A 3D model is generated for verification before an MCNP input deck and geometry mesh file are generated.

polar angle is determined by the ϕ of the best fit library curve. By this definition, the original Fitzgerald design is included in this class.

The second class of masks are those that decouple the angular components through some means. The easiest method to decouple is including a wall of material along the mask that drastically reduces the number of counts at a specific θ [3]. This wall should occur at the same position over a rotation for every ϕ direction, so that the θ of the source can be identified by the location of the sudden reduction in counts. These designs are referred to as the Spartan Class designs due to the characteristic shape of the wall resembling the plume of a Spartan warrior's helmet.

The Spartan Class has a number of advantages over Eigen Class designs. First, decoupling drastically reduces the number of comparisons that must be made. Since the library database of curves contains the responses from $\theta = 0^\circ$, and θ is already

known, the response curve can be “re-aligned” to $\theta = 0^\circ$ so that the matching algorithm only has to compare the shapes of the various Φ modes. As an example, a library containing response curves for 10 Φ modes, with 100 data points over a full θ rotation, would only require 10 comparisons to be made with the Spartan Class designs, whereas Eigen Class designs must compare the 100 θ -shifted possibilities for each Φ mode, for a total of 1,000 comparisons. Thus, there is an increase in computational efficiency for these designs.

Second, Spartan Class designs provide more flexibility in the design process. Geometries that are better suited for mechanical balancing can be chosen since the design matrix is not limited to specific eigenvector solutions. The variability in designing Spartan Class RSMs leads to their final advantage: they are easier to conceptualize and lead to better parameter studies. It is not intuitive, for example, how changing the spring constant in Hollands mass-spring eigenvector approach correlates to a change in the response curve. Similar techniques involved in Eigen Class designs are extremely sensitive to their input parameters, with their effect difficult to predict [3]. However, Equation 2 predicts that a larger or wider wall should have a greater attenuation than a smaller wall in a Spartan Class design. This provides a good starting point to conduct optimization and parameter studies of various designs through slight variations in the designs that have a predictable effect.

For these reasons, the Spartan Class was chosen as the focus for this optimization study. Figure 13 shows all nine RSM designs simulated; only 2 of the 9 do not fall in the Spartan Class. The original Fitzgerald design was included as a baseline comparison. The other Eigen Class design, the Mace, used the design matrix of the RSM by the same name tested by Holland. His studies showed that the Mace design had the best performance among the geometries tested and was included as the standard to beat. The design matrices of the nine designs are included in Appendix

A. The 7 Spartan designs contained a wall of material to act as the decoupling tool. For 6 of the 7 designs, the wall was 10° wide and centered at $\theta = 5^\circ$. The only exception was the Wide Wall RSM, which contained a wall twice as wide and centered at $\theta = 10^\circ$.

The 7 new designs were chosen primarily due to their simplicity as part of the parameter characterization. Mace II, Spaced Binary, and Modal Binary are based off Holland’s eigenvector approach. The Mace II design is identical to Holland’s Mace, except the 10° wall was included (seen as an additional row vector in the design matrix). The wall was set to be twice as large as any other element in the Mace design. Therefore, once scaled, the off-wall elements in the Mace II design were half as large as corresponding elements in the Mace design. The Modal and Spaced Binary designs were chosen to have column vectors (sans wall) that were completely orthogonal. The different vectors were chosen as part of the parameter characterization to understand how the spacing of the eigenvectors affects the resolution of the RSM.

The other 4 Spartan Class designs are similar in geometry: along a single column in the design matrix, there are only two areas of non-zero values. The first refers to the standard wall of the Spartan Class. The second forms a diagonal in the design matrix; in the 3D model, it forms what looks like a second wall spiraling down the mask. This feature is referred to as the fin of the mask. These were designed with a single concept in mind: producing a response curve that contained just two “pulses”, one corresponding to the wall and the other to the fin. The Binary and Tall Wall are almost identical, however the Binary design has a much finer resolution in the geometry. The Tall Wall also contains a gap between the wall and fin. These differences were chosen to study the theoretical limit of resolution for the two pulses. The Tall Wall was also chosen, along with the Wide Wall and Large Fin, to characterize the shape of the pulses based on the size of the wall and fin as part of a basic parameter

study. The Wide Wall contained a wall twice as wide as the Tall Wall design (spanning 20° in θ) while the Large Fin had a fin section twice as large as the Tall Wall (so that the fin was the same thickness as the wall).

These 7 designs represent a reasonable sample of Spartan Class designs, exemplifying the primary features of the class. Combined with the Fitzgerald and Mace RSM, the nine designs form a solid basis to study RSM characteristics.

3.3 RSM Simulation

Initially, it would seem that Equations 2 and 3 fully describes the photon transport problem: photons passing through a voxel into the detector would be attenuated by an amount governed by Equation 2, with x being the thickness of the voxel. The number of detections would then be governed by Equation 3, where x would be the thickness of the detector. Using this concept, one could generate the full response matrix by applying the equations to every element in the design matrix. However, this would require that the photons only interact with a single voxel of the mask before hitting the detector. This is true for a point detector, but reality dictates that the detector must take up some volume. Larger detectors are actually more desirable due to their higher geometric efficiency. Specifically, the detector used in previous RSM studies was a standard 3 in x 3 in NaI(Tl) detector. This means that multiple voxels were in line-of-sight between the source and detector, with many elements of the design matrix affecting the attenuation of the photons. Compton scattering within the mask similarly complicates the spectrum of gammas incident on the detector. In lieu of calculating the complex physics of photon transport, a Monte-Carlo based simulation code was employed to generate the theoretical detector response curves (DRCs).

The code chosen to model the RSM designs was the Los Alamos radiation transport code, MCNP, due to its relative ease-of-use and vast library of cross-sectional

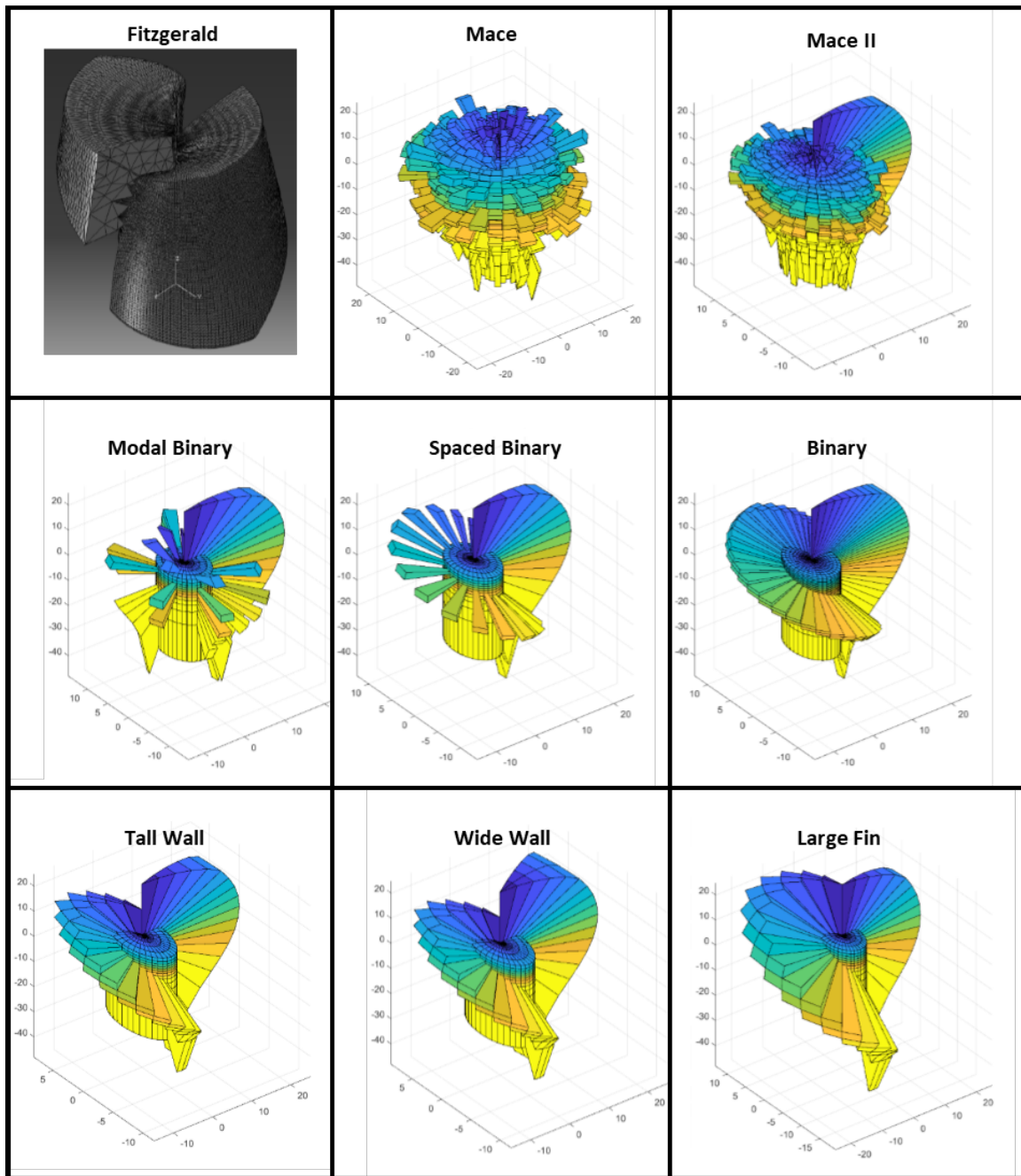


Figure 13. Designs chosen for the optimization and parameter studies. The original Fitzgerald design was used as a base-line comparison while the Mace was the same design from Hollands study [3]. The eight new designs were chosen in accordance to recommendations from his work, with considerations for manufacturing the masks.

data and physics packages [13]. First developed in the late 1940s, MCNP has become one of the most recognized radiation transport codes, with decades of support through validation and verification. While MCNP was used by Fitzgerald and Holland, Logan was able to verify and validate their results through another transport code, GEANT4. She also demonstrated that Hollands MCNP and her GEANT4 simulations matched experimental DRCs collected from the Fitzgerald RSM assembly developed by Charles [2].

For this research, the detector was modeled in MCNP by a right circular cylinder, 7.62 cm in height and diameter, composed of a homogeneous mixture of NaI (the concentration of thallium in the actual detector has negligible effect on the reaction cross sections). A 0.14 cm cylindrical shell of 2024 aluminum alloy surrounded the detector, extending an additional 47 cm below, to simulate the aluminum sleeve. Due to the complexity of the RSM geometries, the mask was modeled using an unstructured mesh, as described by the previous section. The mask was composed of a homogeneous mixture of polymethacrylate. A standard atmospheric composition acted as the background medium. An example MCNP input deck is given in Appendix B, with a typical cross section of the geometry shown in Figure 14.

A Cs-137 source was modeled through a monoenergetic beam of 500,000 662 keV particles at a fixed distance of 86.36 cm from the center of the detector, following Logans and Hollands models. Rather than rotating the masks unstructured mesh, a nontrivial task in MCNP, the source was rotated about the detector to model the rotation of the RSM. To improve computational efficiency, the source was limited to a 5.7° cone directed towards the origin, enough to just cover the detector at every source position. This variance reduction technique assumes that the downscattered photons within the mask have little impact on the FEP of the detected energy spectrum as only the photons with a line-of-sight from source-to-detector could fully deposit their

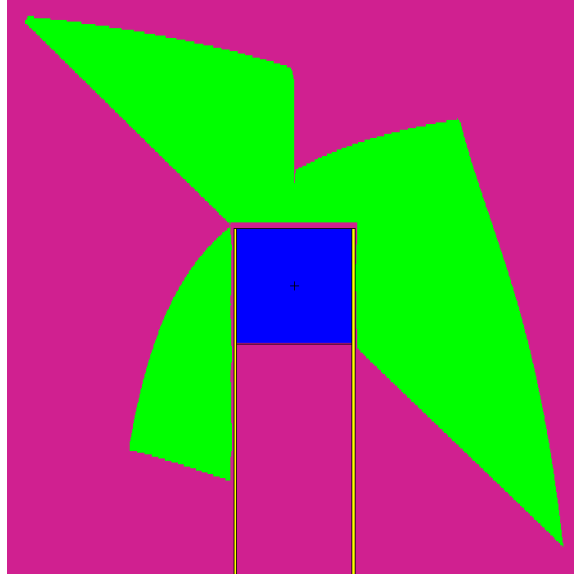


Figure 14. MCNP cross-section of the Fitzgerald design along the xz plane. The figure shows the polymethacrylate RSM (green), thin aluminum sleeve (yellow), NaI detector (blue), and the surrounding air (pink). The MCNP simulation was conducted in a full 3D model.

energy within the FEP.

To model the spectra generated by the detector, MCNPs f8 pulse height tally was used. With this tally, the energy deposited by the photon within the cell defining the NaI crystal was assumed to be collected locally (that is, no secondary particles escaped the detector). For each particle history, the amount of energy deposited was calculated and placed in a histogram containing 1 keV-wide bins from 100 to 700 keV. MCNP also provided the relative uncertainty in the number of pulses in each bin based on its stochastic algorithms.

3.4 Optimization Parameters

As with any optimization study, figures of merit (FOM) provide the basis to compare performance. For this research, the modal assurance criterion (MAC), sensitivity, and normalized count rate values were used as the FOM. The MAC is a value de-

scribing the degree of consistency between any two modal shapes [28]. Both Holland and Logan used the MAC value to compare two response curves for consistency. The MAC, M , for any two response curves is determined by

$$M = \frac{(\mathbf{u} \cdot \mathbf{v})^2}{(uv)^2}, \quad (4)$$

where $\mathbf{u}$ and $\mathbf{v}$ are the reduced response curves of any two curves, represented as a 1D array of values. The response curves were reduced by subtracting the mean number from the response curve for each ϕ location. Reducing the curves does not change the shape of the response and normalizes them so that the MAC value is bound between zero (two modes are completely orthogonal) and one (two modes have the exact same shape).

The sensitivity represents the signal strength of the response caused by the rotation of the mask. The sensitivity, S , of a response curve is defined by

$$S = \frac{N_{max}}{N_{min}}, \quad (5)$$

where N_{max} is the maximum counts detected while N_{min} is the minimum number of counts detected in a single response curve. The normalized count rate measures the efficiency of the design and is given by the mean number of counts detected over a single rotation. Using these definitions, an optimal design would have response curves that have low M values between each other (unique modal shapes), high sensitivity (modal shapes are easy to distinguish), and high normalized count rates (lower uncertainty).

IV. Results and Analysis

4.1 Variance Reduction and Model Verification

Figure 15 shows an example energy spectrum produced by MCNP, using the input deck given in Appendix B. The number of detections were normalized by the total number of source particles emitted by the simulated source (500,000 for this study). It should be noted that the spectrum in the figure is similar to the spectrum of an ideal detector presented in Figure 9. This is expected as the broadening is a result of stochastic processes in the formation and collection of the scintillating light produced by the crystal, which cannot be modeled in MCNP. The f8 tally used is only a measure of the total energy deposited by a single photon in the detector, so instead of a Gaussian distribution, the FEP is constrained to a single pulse at exactly 662 keV.

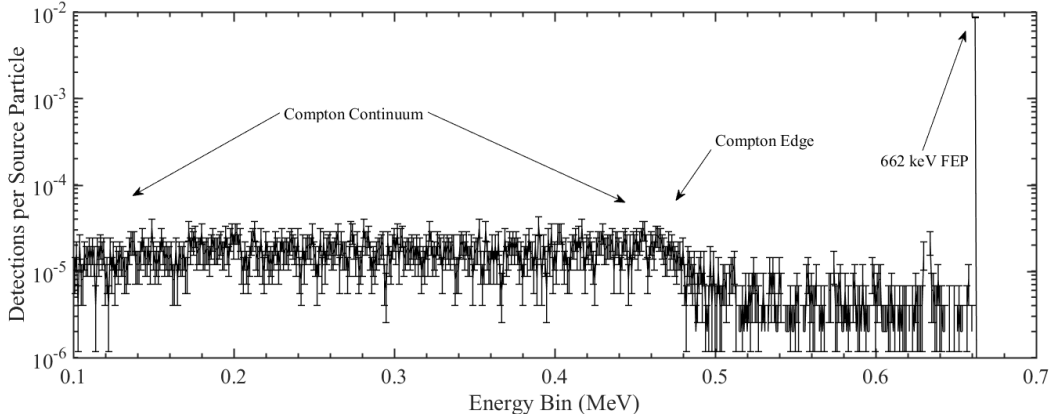


Figure 15. Deposited photon energy spectrum for the NaI detector, generated in MCNP using 500,000 source particles and a source location at $\phi = 0^\circ$. Notably present is the FEP at exactly 662 keV, with a relative uncertainty of less than 2%. Limiting the sources to a smaller cone angle significantly improved computational efficiency.

The reduction of the source cone to 5.7° was a deviation from previous simulations that had cones encompassing the entire mask assembly, up to 30° half-angles. The

benefit of this smaller angle is evident in Figure 15 by the small relative uncertainty at the FEP ($< 2\%$). Using the same number of half a million source particles over a smaller source cone angle produced an overall increase in the number of photons that interacted with the detector. The increase in detections reduced the relative uncertainty by an order of magnitude, with no other changes in the simulation besides the distribution of the source particles. The MCNP manual states that tallies with relative uncertainties below 10% can be considered valid; using this variance reduction method, the relative uncertainty at the FEP was less than 1% for all designs at all source locations. Previous simulations required 5 million or more particles to achieve the same results, so this is a marked improvement in computational efficiency [2].

A limitation to this technique is that the full energy spectrum cannot be simulated. There are fewer counts within the Compton continuum, by nearly three orders of magnitude, so the relative uncertainties are much higher using the bin sizes of 1 keV. This can be resolved by summing over the Compton continuum, as the previous studies did, to increase the number counts at the expense of energy resolution. However, using the smaller cone also neglects the Compton scattering interactions that can occur within the mask that increase the number of lower-energy photons. Therefore, the energy spectra produced in this study are not representative of an actual detectors energy spectra.

Fortunately, the full spectrum is not required to accurately reproduce the response curves. The justification for reducing the source distribution was that downscatters from the mask do not contribute significantly to the FEP of the spectrum. And since it is the integral of the FEP that is used for the response curves, there is no requirement to model the Compton continuum or any features besides the FEP itself. Figure 16 and 17 verify this assumption. On the left of Figure 16 is a surface plot of all the response curves collected by Logan with the Fitzgerald RSM device at AFIT.

The graph plots the FEP of the spectrum as a function of θ and ϕ . On the right is the simulated response matrix of the Fitzgerald design, plotting MCNPs f8 tally at 662 keV for each source location.

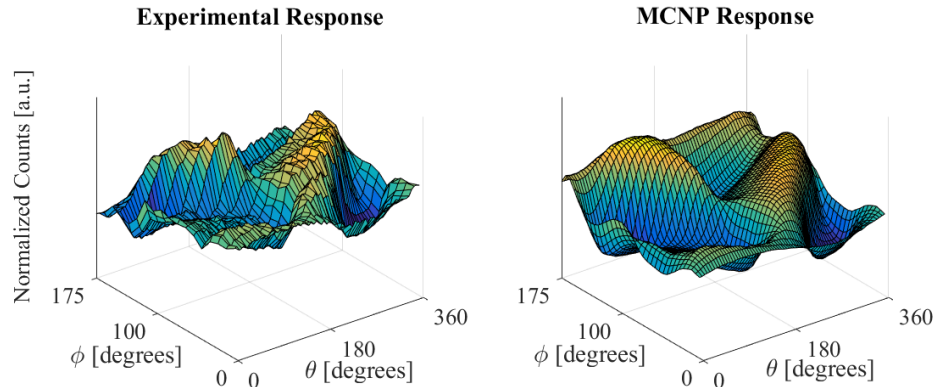


Figure 16. Experimental FEP response curve (left) compared to the MCNP simulated FEP response curve (right). The magnitude differences present for the larger Φ modes between the two plots are due to ignoring the motor, base assembly, and other electronics in the MCNP model, which further attenuates the photons (symmetrically about θ).

Visually, the two are very similar with the same peaks and valleys present. However, there is a decrease in overall counts near the larger Φ modes in the experimental DRC as compared to the simulation. This is most likely due to the effects of the base, motor, and other electronics behind the RSM causing further attenuation and decreasing the number of counts at the larger angles where those systems are in between the source and detector. As the mask rotated in the experiment, the amount of attenuation caused by the electronics and motors should have behaved symmetrically about θ such that the shape of the DRC was preserved. By normalizing the Φ modes by their maximum values, the differences caused by the external systems not modeled should disappear. The result of this normalization is shown in Figure 17. The similarities are more pronounced, with the larger uncertainty in the experimental counts being the primary difference. This is a good visual indication that the variance reduction technique improved computational efficiency while still maintaining

expected results for the FEP counts.

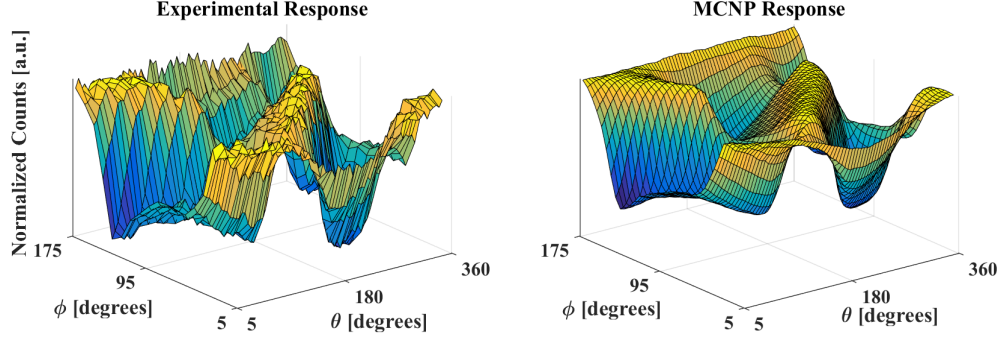


Figure 17. Experimental FEP response curve (left) compared to the MCNP simulated FEP response curve (right) after normalizing each Φ mode by its maximum value. This normalization removes the magnitude differences caused by the systems not modeled and highlights the similarities in the shapes of the modes.

To quantitatively verify this, the MAC values for the corresponding Φ modes were calculated between the experimental and simulated DRCs. If the simulation accurately reproduced the experimental curves, Equation 4 says that M should be close to 1 for all the modes. These MAC values are plotted in Figure 18. All 18 modes fell well within one standard deviation of a MAC value of 1, meaning the two DRCs were statistically consistent. This verifies that the model accurately reproduced expected values with the variance reduction technique, thus greatly improving computational efficiency.

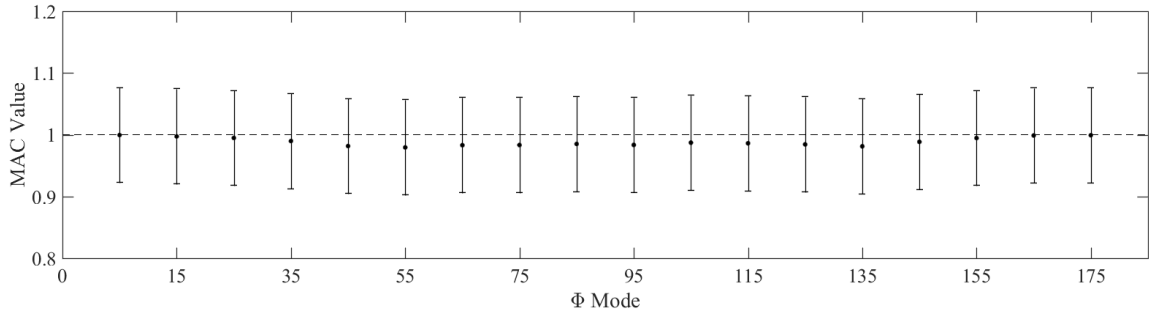


Figure 18. MAC values comparing the Φ modes of the experimental and simulated response curves in Figures 16 and 17. All modes fall well within one standard deviation of a MAC value of 1, meaning the two DRCs are statistically consistent.

4.2 Design Comparisons

Using MCNP, the FEP counts within the detector as a function of source location were calculated for every RSM design. Following precedent, the source location was incremented by 5° for both angles. This created a library of 34 response curves with ϕ ranging from $5^\circ - 170^\circ$ and 72 data points over a full rotation in θ for each response curve. Figure 19 plots the combination of response curves as a surface plot (counts as a function of ϕ and θ) for four of the nine designs. A comprehensive list of all nine designs' surface plots is given in Appendix A.

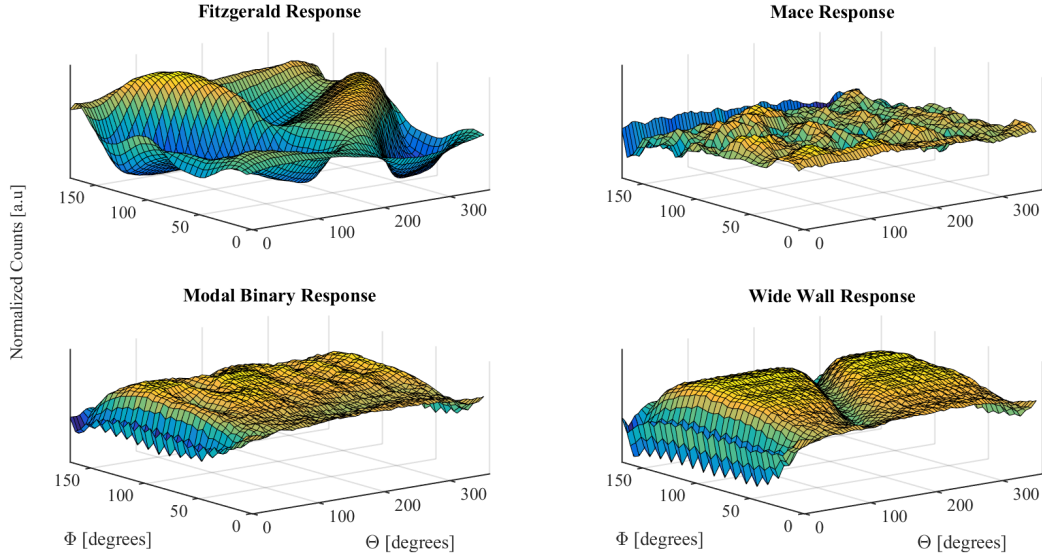


Figure 19. DRCs for four example RSM design concept. Note the Fitzgerald and Wide Wall designs have defining features within the DRC (containing well-defined peaks and valleys) while the other two contain more variance in the responses (indicative of better MAC values between the Φ modes).

The MAC values between the response curves of each design were calculated to determine the uniqueness of curves shape based on the sources location in ϕ . Note that the response curves were only compared to curves from the same design; there is no practical information gained from comparing response curves from different designs. Also, to account for a shift in θ of the source position, all possible MAC values for

two curves were found by incrementally shifting one curve by 5° while maintaining the other in a constant position. With these shifts, two response curves had a total of 72 possible MAC values. The maximum of these 72 values showed the largest amount of overlap between the two curves. This maximum MAC value, which will be referred to as M_{max} , was calculated for every combination of response curves.

The results are plotted in Figure 20 as a bar graph using the same response curves from Figure 19 (all M_{max} bar graphs can be found in Appendix A). As the relative uncertainty in the MCNP counts at the FEP were all less than 5%, the uncertainty in the MAC value calculations were on the same order, with most having maximum errors of about 5%. Since these errors are negligible, they are omitted from the MAC value plots and ignored throughout this discussion (this is only due to the high order of precision from the MCNP simulations; in a realistic scenario, the uncertainty from the detection rate will have a larger impact, whose effects are beyond the scope of this project).

A few features of these plots are worth discussion. First, the x and y values are the two ϕ directions being compared, written as the Φ mode. Equation 4 shows that comparing $\mathbf{u}$ to $\mathbf{v}$ is equivalent to comparing $\mathbf{v}$ to $\mathbf{u}$, so the bar graph should be symmetric about the diagonal. However, the upper triangle was omitted from the plots to remove redundant information. Second, the diagonal for every design has a M_{max} value of 1. This was expected since the curve was being compared to itself. When calculating the average values for M_{max} , the diagonal was also omitted since it does not provide any extra information on the uniqueness of the shapes for the Φ modes.

Third, the graph for the Fitzgerald design shows many regions where the off-diagonal terms approached a M_{max} value of 1. These represent degenerate regions where two separate Φ modes closely resemble each other, introducing potential errors

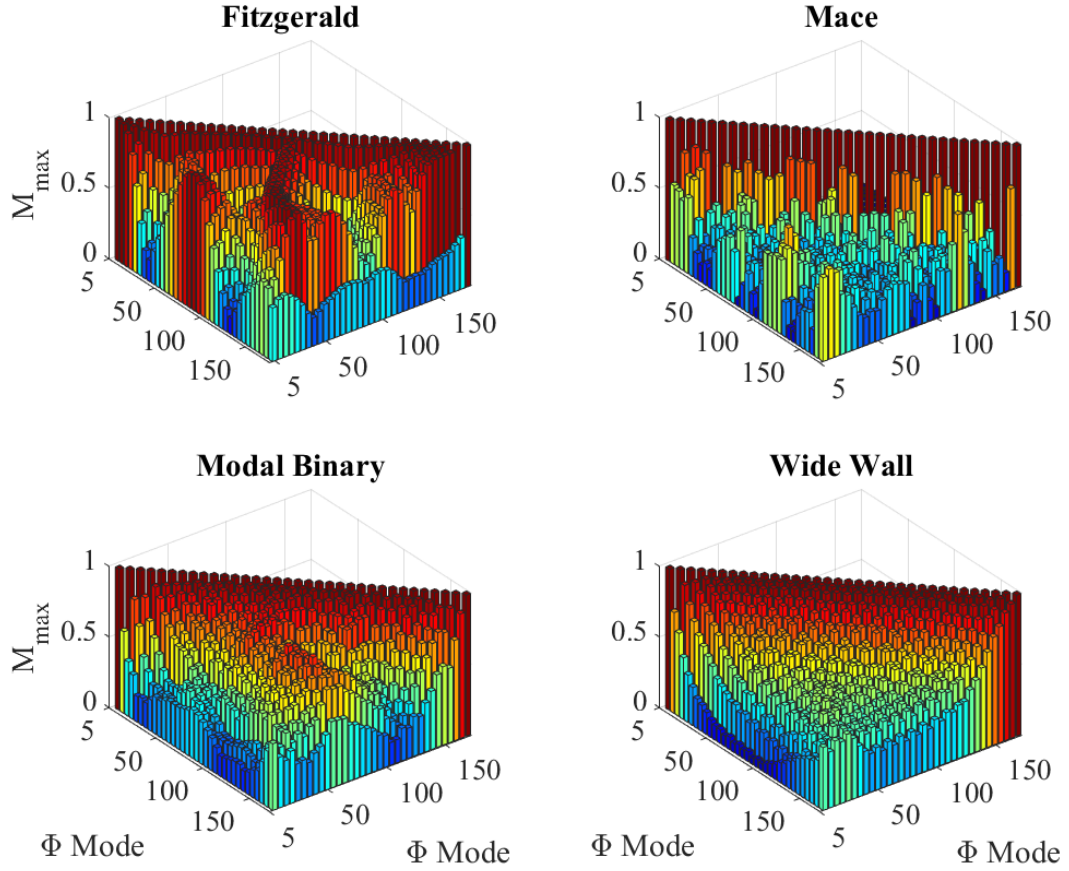


Figure 20. Plots showing the maximum MAC values between the Φ modes for the four designs in Figure 19. The MAC values are calculated for every shifted-combination of Φ modes to account for an unknown starting position for the source, with areas close to 1 indicating regions of degeneracy.

in identifying the actual ϕ of a source. However, every one of the 8 other designs showed improvement in reducing the values of these off-diagonal terms. Therefore, a simple visual comparison of the M_{max} plots shows that the new designs all outperform the Fitzgerald design in terms of producing more independent response curves as a function of source ϕ , reducing the chances of an erroneous identification for the direction of the source.

Table 1. Comparing average reponse of RSM designs using M_{max} , normalized count rates, C , and sensitivity, S .

Design	M_{max}	$C(\times 10^{-3})$	S
Fitzgerald	0.636	6.69	2.527
Mace	0.314	6.59	1.430
Mace II	0.526	2.48	1.667
Modal Binary	0.598	8.64	1.627
Spaced Binary	0.580	8.73	1.534
Binary	0.428	10.96	1.544
Tall Wall	0.486	8.69	1.636
Wide Wall	0.577	8.53	2.572
Large Fin	0.510	8.50	2.087

A more quantitative comparison of the performance for the nine designs is given in Table 1, which plots the average values for M_{max} , normalized count rate, C , and sensitivity S over all Φ modes. The M_{max} values for each design were calculated from the mean of the off-diagonal terms of the MAC plots in Figure 20 and Appendix A. The count rates were normalized through the standard MCNP normalization; that is, the value represents the number of detections that occurred in the detector per source particle simulated. For example, 6.69×10^{-3} particles were detected (on average for every source position) per particle emitted in the simulated 5.7° source cone with Fitzgerald’s RSM design. That means approximately 3,340 out of the 500,000 simulated particles were detected within the FEP. As C is directly proportional to the RSM’s efficiency, this category was used to measure and compare the efficiency

of each design. The values reported in the table are the average count rates over all source directions for each design.

The sensitivity was calculated for each Φ mode using Equation 5 and then averaged over the number of modes to give the results shown in Table 1. For all three FOM, taking the mean of a large number of data points (each with low uncertainties) reduced the relative uncertainty for every value to less than 0.1%. This amount of uncertainty is less than the precision presented in the table, so the errors are omitted and ignored throughout this discussion. It should be noted that this is only allowed due to the high precision of the MCNP simulation as a result of the variance reduction technique discussed in 4.1. Therefore, the FOM discussed in this and subsequent sections are theoretical limits for the RSM, given very little uncertainty in the detection measurements. A discussion of the impacts on uncertainty in the experimental environment is presented in 4.6.

Table 1 is consistent with Holland’s findings. Primarily, these results show that *all designs* improved on Fitzgerald’s design by lowering the degree of consistency between the various Φ modes. Unlike the Fitzgerald design, which was limited in both angular resolution and in the presence of degenerate regions, these new designs feature lower M_{max} and much fewer degenerate regions, therefore becoming primarily limited to only angular resolution. 6 of the 8 designs also showed increased efficiencies, with only the Mace and Mace II designs having smaller count rates. However, the Mace RSM only had a slight decrease in efficiency while supporting the largest improvement in lowering M_{max} (reducing the average value by 51%. The next best design, Binary, only had a 33% change.).

The only area where the Fitzgerald design consistently outperformed the others was in sensitivity; only the Wide Wall design had a larger average. With lower values, the other 7 designs are more susceptible to random noise within the DRC as compared

to the Fitzgerald and Wide Wall RSM, given the same number of detections over a full rotation. Unlike the normalized count rate, which measured the efficiency of the RSM on a unit time basis, the sensitivity is a measure of the efficiency on a per-count basis. The significance of the normalized count rate and sensitivity are thus dependent on the specific application of the RSM, of which that discussion is beyond the scope of this research. For this study, it is sufficient to state that low M_{max} values are the most important FOM for an RSM due to its primary application of a directionally-sensitive detector, with C and S having equal, but lower, weights.

With this argument, the Mace design is a clear winner in terms of the best design of the 9 simulated. This is evidence that the eigenvector approach is perhaps the optimal approach when designing an RSM that has both angular components of the source's direction coupled. However, the Spartan Class RSMs were designed specifically to test the concept of decoupling one of the angular components, so it is not surprising that these designs underperformed using the analysis that the Mace RSM was optimized for. In the next sections, the impact of decoupling the angles on RSM performance is tested and discussed.

4.3 Spartan Analysis

One of the advantages of the Spartan Class designs, and the reason for the focus of this study on those designs, was their potential ability to decouple θ . This ability was ignored in the analysis of the previous section in order to get a baseline comparison to the Fitzgerald and Mace RSMs, as well as verify Holland's findings. What Table 1 showed was that the Mace was the best design using the old method for comparing response curves. That is, it was the best at reducing the overlap of its curves over all shifted combinations. However, the concept of the Spartan design was that the θ could be identified by the sharp decrease in counts caused by the wall. This concept

is shown in Figure 21, which uses the simulated $\phi = 110^\circ$ response curve from the Modal Binary design as an example.

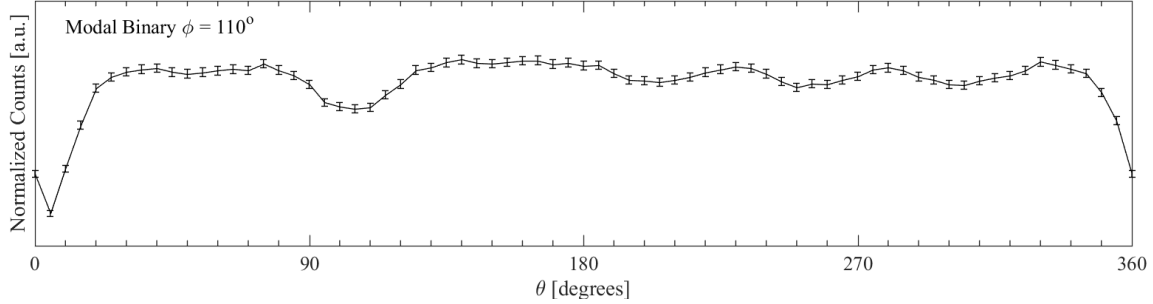


Figure 21. An example simulated response curve for the Modal Binary design concept with a source at $\phi = 110^\circ$. The underlying feature to all the Spartan-class designs was the large valley centered around $\theta = 5^\circ$ (10° for the Wide Wall design) for all Φ modes. This region is referred to as the wall valley.

The minimum number of detections occurred at $\theta = 5^\circ$, when the center of the wall was in line with the source. This behavior, referred to as the wall valley, is exactly what was predicted and sought after, as it allows for the determination of the θ by calculating where the absolute minimum number of counts occurs along a rotation. For example, if source was located at $(\phi, \theta) = (110^\circ, 90^\circ)$, the response curve would be the same as Figure 21, just shifted by 85° in θ so that the minimum occurs at 90° instead of 5° . The determination of θ from a singular point along the DRC is what was referred to as angular decoupling.

The relative size of the wall to the rest of the mask affects the location and size of the valley; Figure 22 gives an example of how changing the height and angular width of the wall affects the DRC. The plot shows the response curves from $\phi = 110^\circ$ for three different designs. All three designs used a 20 cm thick wall. For the Tall Wall design, denoted by the solid line, the wall had an angular width of 10° while the Wide Wall design, marked by the dashed line, had a 20° angular width. The Large Fin design, shown by the dotted line, had a wall the same size as the Tall Wall, except

that the other, non-zero, portions of the mask (referred to as the fin) were also 20 cm. The Wide Wall was designed to determine the effect of increasing the size of the wall, while the Large Fin was a study on the limitations of identifying the valley from the rest of the response curve. The Tall Wall served as a control. It should be noted that all other Spartan Class designs used the same wall design from the Tall Wall RSM.

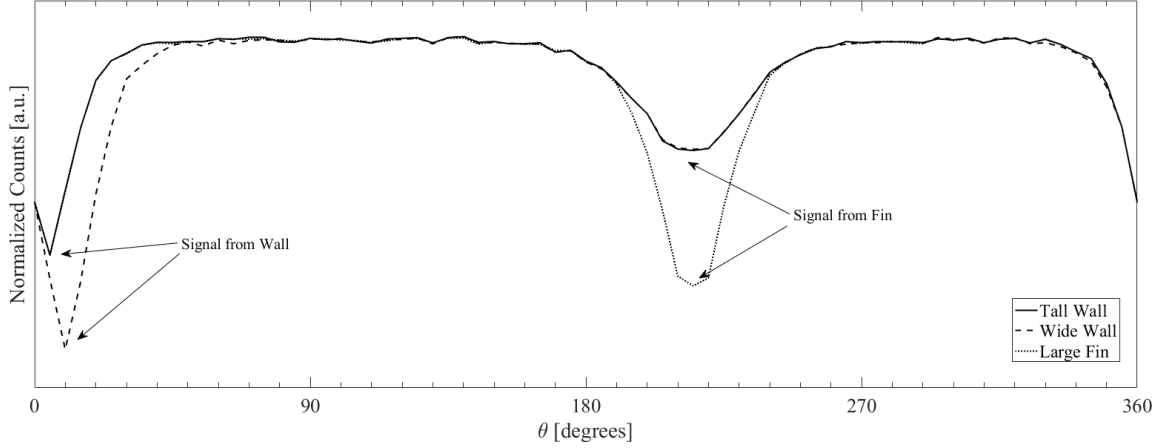


Figure 22. Plot showing the effects of the wall and fin size on the corresponding valleys (error bars omitted for clarity). The DRCs were generated with a source at $\phi = 110^\circ$. As expected, increasing the width of the wall attenuated more material and shifted the minimum of the wall valley. Increasing the length of the fin similarly increase attenuation. Also important to note is that the two responses were independent; changing the wall had no effect on the fin valley.

As expected, increasing the walls angular width to 20° not only decreased the total number of counts in the valley by introducing more attenuating material, but shifted the minimum 10° , exactly aligned with the center of the wall. Without changing the fin, the second pulse remained unaffected. The opposite was achieved by the Large Fin design. While the walls valley was the same as the Tall Walls response (as it should), the signal from the fin was drastically reduced. In fact, the wall did not contain the global minimum for the response, the absolute minimum occurred later along the rotation at the fin. This alludes to potential identification issues when trying to decouple θ using the wall if it is roughly the same size as the rest of the

mask. For the Large Fin design, this was not an issue as only one other location along the rotation of the masks geometry had a non-zero thickness, so the wall valley was still discernible from the rest of the DRC.

The plots from Figure 22 were not exclusive to those three designs at that specific ϕ angle; all Spartan Class designs showed similar correlation between the source and wall valley θ position. The only limitations were for the response curves for the low ϕ angles. Figure 23 shows the first 10 response curves of the Tall Wall design, up to $\phi = 50^\circ$. Up until $\phi = 30^\circ$, the shape of the valley is inconsistent; minor fluctuations occur near the minimum and the location of the valley center moves over a wide range of θ values. This was most likely due to the geometry of the mask and the partitioning of space with the polar and azimuthal angles. Although every mask voxel covered the same solid angle, the actual volume of the voxel changed as a function of ϕ and θ . Particularly, the voxels aligned close to the axis of rotation had the smallest volumes, providing the least amount of attenuating material. Along with the geometry of the detector crystal, this could account for the reduced effect of the wall and mask at these low angles.

At $\phi = 30^\circ$ and above, the valley was easily distinguishable for all Spartan Class designs. This meant that the majority of response curves contained enough information to determine the source θ without having to compare the curve to a library. Therefore, it is possible to realign most of the DRCs to a reference θ before comparing to a library database of DRCs (thus decoupling it from ϕ); in this case reducing the number of comparisons by two orders of magnitude from 2,448 to just 34. Since no shifted comparisons must be done, M_{max} would then just be the MAC value of the unshifted DRC calculated by MCNP (Figure 19). Comparing DRCs in this method is referred to as Spartan Analysis.

Comparing the just the unshifted DRCs eliminated the degeneracies caused by θ

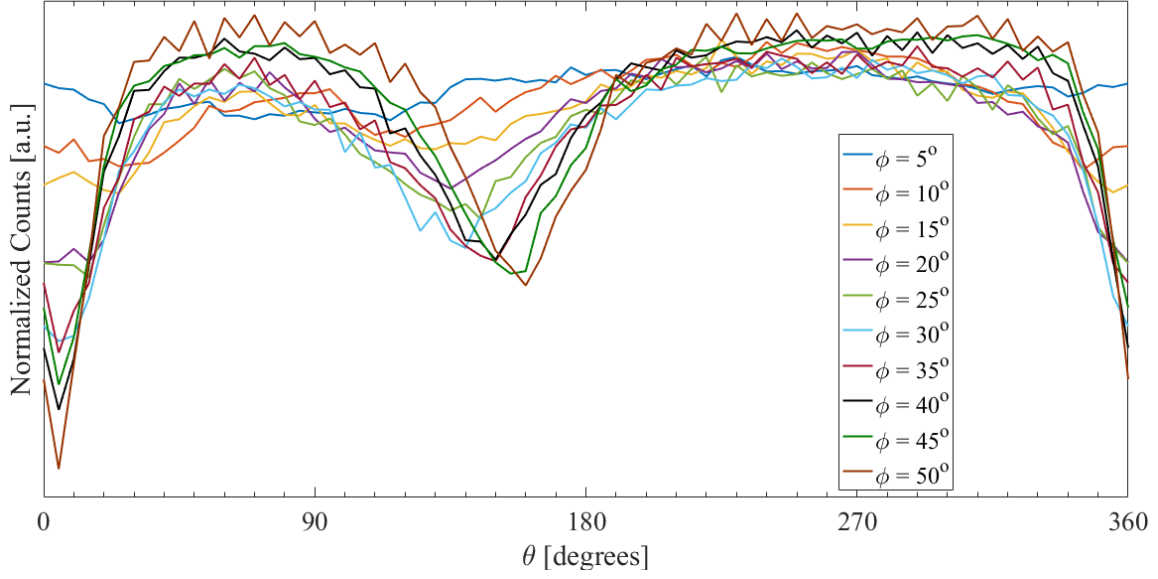


Figure 23. DRCs from Φ modes up to 50° for the Tall Wall design concept (error bars omitted for clarity). The wall valleys are not distinguishable for modes below 30° ; this limitation was consistent for all designs due to the geometry of the masks at these low angles. Therefore, the Spartan Analysis could only precisely be done for Φ modes between $30^\circ - 170^\circ$.

shifts of the Φ modes, reducing the mean M_{max} . Figure 24 demonstrates how the Spartan Analysis technique can greatly improve RSM performance. The figure plots the original M_{max} from 4.2 on the left and the new M_{max} on the right for Φ modes assuming θ is decoupled and no comparisons need to be made for combinations of shifted Φ modes. The results are clear: Spartan Analysis greatly improved performance by reducing the average M_{max} from 0.510 to 0.177, a 72% increase in performance over the Fitzgerald design!

The bar chart on the right in Figure 24 represents an ideal RSM that is limited by resolution rather than degenerate regions. Along a single Φ mode, as one moves down a column or across a row (i.e. decreasing or increasing ϕ), M_{max} steadily declines until it reduces to zero, indicating the modes are completely orthogonal. And unlike the M_{max} plot for the Fitzgerald RSM in Figure 20, which contained regions far

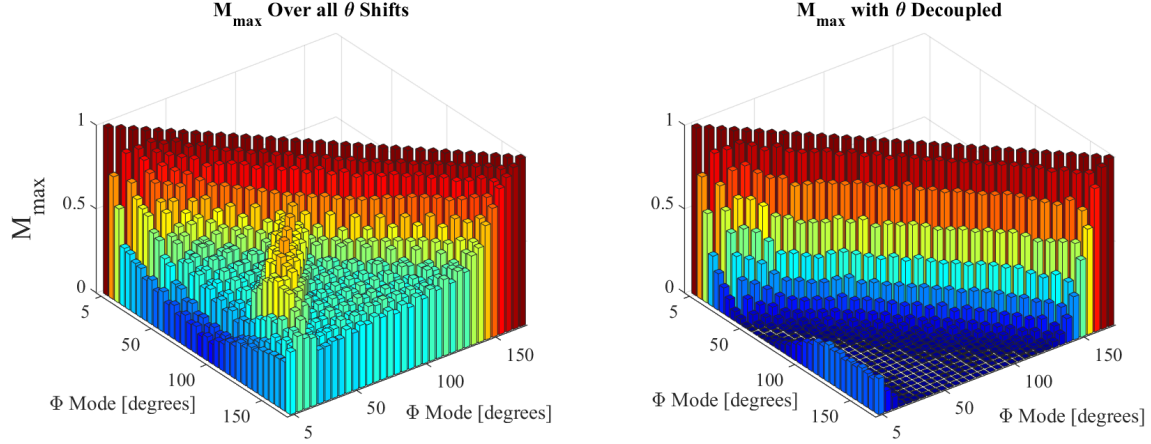


Figure 24. $M_{\max}$ plots showing the effect of θ decoupling for the Large Fin RSM design. The decoupled solution (right) shows a drastic reduction in MAC values over the original method (left), with the ideal performance of $M_{\max}$ reducing to zero for distant Φ modes.

off the diagonal with values close to 1, the regions with high $M_{\max}$ in Figure 24 are centered about the diagonal. In application, this means that instead of misidentifying a source's direction by completely guessing the wrong direction, this technique will ideally only be off by a few degrees in ϕ from the true direction. This is what is meant by the RSM being limited to only angular resolution.

Table 2. FOM values assuming θ decoupling on the 7 Spartan Class designs. This method showed slight improvement in $M_{\max}$.

Design	$M_{\max}$	$C(\times 10^{-3})$	S
Fitzgerald	0.636	6.69	2.527
Mace	0.314	6.59	1.430
Mace II	0.504	2.48	1.667
Modal Binary	0.580	8.64	1.627
Spaced Binary	0.546	8.73	1.534
Binary	0.339	10.96	1.544
Tall Wall	0.347	8.69	1.636
Wide Wall	0.564	8.53	2.572
Large Fin	0.177	8.50	2.087

Table 2 shows the results from applying the Spartan Analysis to the 7 Spartan

Class designs. Note that this technique does not change the average normalized count rates and sensitivity values, as these are only dependent on the scale of the signals from the DRCs. It is also interesting to note that while the Spartan Analysis improved all M_{max} for the Spartan Class designs, none improved to the extent of the Large Fin. Most only showed minimum improvement while the Binary and Tall Wall designs approached values close to that of the Mace. With just this technique, it seems that the best design is the Large Fin, through a combination of significantly lower M_{max} values than any other design and good detection efficiency, S . However, why the Binary, Tall Wall, and Wide Wall designs did not perform as well as the Large Fin, when they all have similar geometries and DRCs, warranted further investigation. The next section discusses the discovery of an inherent bias using Spartan geometries and the classic MAC value comparisons and an approach to further improve RSM performance.

4.4 Bias Reduction

The Spartan Analysis showed that decoupling θ improved the functionality of the RSM by limiting the MAC value comparisons to their unshifted Φ modes. However, only one design, the Large Fin, showed significant improvement. Figure 25 reveals why the Spartan Analysis technique did not have a large impact for most other designs. In the figure, sample DRCs of the Wide Wall RSM, over the available $30^\circ \leq \phi \leq 170^\circ$ range that the wall valley was identifiable, are shown. The figure shows that even in the unshifted positions, the average M_{max} was still relatively high at 0.5639. The wall valley maintained a constant shape for modes $\phi \geq 30^\circ$; the minimum M_{max} for any of these modes was around 0.4. In other words, the wall valley introduced a bias within the MAC value calculations by creating a region about $\theta = 5^\circ$ whose shape was consistent in all Φ modes, setting a lower limit on the M_{max} . This explains

why most of designs showed only minor improvement through the Spartan Analysis; the largest consistency between any two modes occurred at or around when the wall valleys of the modes were aligned, which corresponded to the unshifted positions. Since the largest consistency occurred near these unshifted Φ modes, removing the shifts from the calculation only minimally affected the M_{max} .

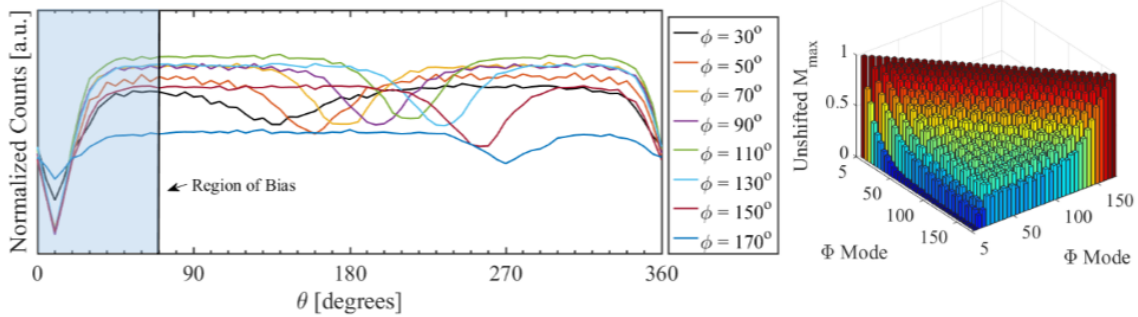


Figure 25. Plots showing the bias region inherent to the Spartan-class design. The wall valley region of the Wide Wall design, highlighted in blue, is key to decoupling the angles, but introduces a consistency between all modes. This has an effect of introducing a bias, with M_{max} having a minimum of about 0.4 (right).

The Large Fin design improved the most due to its geometry and corresponding DRCs. The Binary, Tall Wall, Wide Wall, and Large Wall DRCs were unique in that they contained not only a wall valley, but a second valley produced by the fin (this is discussed in more detail in the next section). For the Large Fin design, the wall valley was approximately equal in size as the fin valley, so that the wall valley did not have as large of a weight in the MAC calculations. Coincidentally, the two valleys gave the Large Fin DRCs an almost sinusoidal response in θ , allowing for more orthogonality in the Φ modes.

In order to improve the performance of all the Spartan Designs, a bias reduction technique was applied to by removing the portion of the DRC corresponding to the wall valley and then calculating the MAC from the remaining portion of the DRC. It should be noted that this form of bias reduction could only be used because θ

was decoupled. Without a signature response such as the wall valley, there would be no way to determine which portion of the response curve should be removed to reduce the bias. The signature also has to occur at the same location in all response curves so that they can be realigned to a reference location. The Spartan Class was developed specifically for this concept; this form of bias reduction cannot be applied to the Fitzgerald and Mace RSM as they would require *a priori* knowledge of the sources ϕ direction.

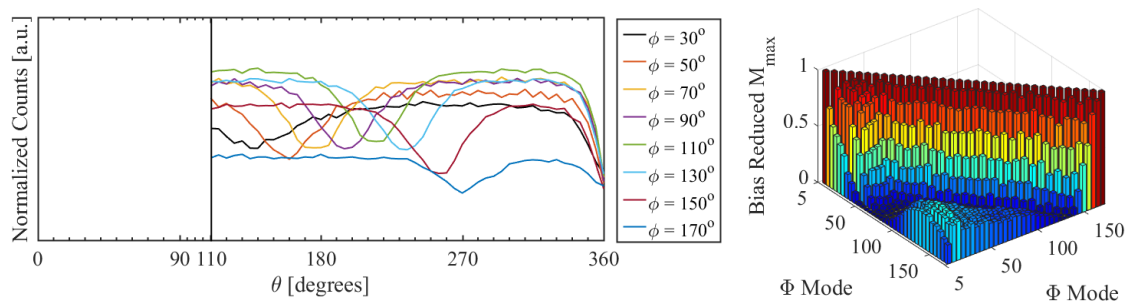


Figure 26. The effect of removing the wall valley region on calculating M_{max} of the Φ modes. By analyzing the portion of the DRC that was intentionally designed to be unique, the bias from the wall valley was removed.

Figure 26 shows the effect of removing the bias region for the Wide Wall design, which decreased the mean M_{max} to 0.2388. To determine the bias region, an iterative process was used to determine the section of the response curves to remove that produced the lowest average M_{max} . The results of that process are shown in Table 3, which gives the range of θ in the DRCs used for the 7 Spartan Class designs.

An alternative approach for this bias reduction is proposed: the Spartan Analysis technique is based on the θ and ϕ independence of a source's direction. A consequence of decoupling the angular components is that the DRC can be expressed as a linear combination of responses from different source positions. For the Spartan Class designs, this means that the DRC can be expressed as a superposition of the wall response and a response from a mask with the exact same geometry but without the

Table 3. Comparing average reponse of RSM designs using the Spartan Analysis technique.

Design	θ Range	Old $M_{\max}$	New $M_{\max}$	Improvement
Mace II	$45^\circ - 345^\circ$	0.504	0.106	79.0%
Modal Binary	$25^\circ - 335^\circ$	0.580	0.166	71.4%
Spaced Binary	$20^\circ - 350^\circ$	0.546	0.180	67.1%
Binary	$35^\circ - 330^\circ$	0.339	0.218	35.8%
Tall Wall	$115^\circ - 360^\circ$	0.347	0.234	32.6%
Wide Wall	$110^\circ - 360^\circ$	0.564	0.239	57.6%
Large Fin	$15^\circ - 360^\circ$	0.189	0.177	6.7%

wall. Deconvolution of the DRC could then allow θ and ϕ by comparing the independent signals. This method has the added benefit that information about ϕ is not lost by discarding portions of the DRC close to the wall valley. Thus geometries can exist next to the wall and not be obscured by the wall valley in the DRC (this issue is discussed in the next section as a limiting factor in an alternate direction identification approach). However, the iterative process was used due to its simplicity as a proof-of-principle in decoupling θ and ϕ .

Table 4. Comparing average reponse of RSM designs after bias reduction.

Design	$M_{\max}$	$C(\times 10^{-3})$	S
Fitzgerald	0.636	6.69	2.527
Mace	0.314	6.59	1.430
Mace II	0.106	2.48	1.667
Modal Binary	0.166	8.64	1.627
Spaced Binary	0.180	8.73	1.534
Binary	0.218	10.96	1.544
Tall Wall	0.234	8.69	1.636
Wide Wall	0.239	8.53	2.572
Large Fin	0.177	8.50	2.087

The results of the bias reduction are shown in Table 4. As before, the sensitivity and normalized count rates were unaffected. The most striking feature of the table is the drastic reduction of $M_{\max}$ for all 7 Spartan Class designs, with improvements

ranging from 62.5% – 83.4%. The best performing Spartan Design was the Mace II, which combined the angular decoupling with the eigenvector approach developed by Holland. However, Mace II was also the only Spartan design that has both a lower sensitivity value and normalized count rate than the Fitzgerald design. The combination of these two means that in order to achieve accurate results, the Mace II would require much larger dwell times than the other RSM. All other Spartan RSMs showed drastic improvement in minimizing M_{max} while increasing detection efficiency. Although they perform worse on average on a per-count basis as compared to the Fitzgerald design, the increased normalized count rates mean they will generally require less dwell time (again, the correlation between sensitivity and count rate not as simplistic as this suggests and is reserved for future studies).

The strength of the Spartan Class designs can now be fully discussed. Table 4 shows that Spartan Class designs, using the appropriate analysis techniques, provide the best opportunity for creating independent Φ modes and limiting the probability of misidentifying a source’s direction. The M_{max} plots after bias reduction demonstrate that Spartan Class designs are limited primarily angular resolution on the order of the geometric resolution of the RSM geometry. Finally, Table 5 demonstrates the mechanical advantage of the Spartan designs. The table gives the volumes for the 9 designs tested. Note that the Spartan Class designs have the smallest volumes. This explains the high efficiencies of the Spartan Class designs as there was less attenuating material (apart from Mace II which had a volume similar to that of the Fitzgerald design). These smaller volumes are also beneficial based on considerations of material available, manufacturing capabilities, and cost restrictions for applications of an RSM system.

The Spartan Class designs are not without limitations. The primary restriction on these designs is in regards to their largest assumption: *decoupling* θ . The entire

Table 5. Volumes of the RSM designs tested.

	Design								
	Fitzgerald	Mace	Mace II	Modal Binary	Spaced Binary	Binary	Tall Wall	Wide Wall	Large Fin
Volume [x 10 ³ cm ³]	10.58	24.366	9.426	3.742	2.948	3.682	3.757	5.684	6.676
Rank (low to high)	8	9	7	3	1	2	4	5	6

premise of these designs rest on the ability to distinguish θ from the DRC through the wall valley or a similar signal. That signal must be both distinguishable from the rest of the DRC and precise enough to determine θ . In realistic scenarios, there will be some uncertainty in determining the minimum of the valley, restricting the precision of the Spartan Class designs. Should the decoupling not be enough to determine θ , none of the techniques discussed will apply. In that scenario, the M_{max} must be compared, at which point the Eigen Class RSMs are much more suitable. A Spartan RSM must guarantee the decoupling of the angular components in order to be feasible.

4.5 Geometric Correlation

Table 4 showed that the new designs were better than the Fitzgerald design based on the original library look-up method. The Spartan Analysis and bias reduction may improve the database comparisons, but there are still limitations inherent with this technique. Four of the RSMs tested were built with these limitations in mind, in an attempt to create alternate methods of calculating the direction of the source. These designs were the Binary, Tall Wall, Wide Wall, and Large Fin.

The geometries of these designs were unique in that they only contained a wall (as all the Spartan Class designs) and a spiral fin that wrapped around the detector. The Wide Walls design matrix was shown in Figure 11; the wall was created from the first two rows while the spiral fin was a result of the bi-diagonal identity matrix

offset from the wall. The other three designs had similar design matrices (Appendix A). Since there were only two locations where the masks thickness was non-zero, the response curves of these designs were similar in that they contained just two “pulses”: one at the wall and one at the fin (Figure 21). These two pulses are referred to as the wall valley and fin valley, respectively.

Using this knowledge, it was possible to determine (ϕ, θ) of the source entirely by the locations of the valleys in the response curves. First, it was assumed that θ could be calculated by the same process as the previous section, so that the response curve could be realigned to its unshifted reference point. From there, the ϕ value would be calculated by the distance of the wall and fin valleys. For the four geometries mentioned, the distance between the wall and fin linearly increased as a function of ϕ , so the distances between the two valleys should have also exhibited a linear correlation. The same limitations from the Spartan Analysis technique applied; in order to sufficiently identify the wall and fin valley, only the modes with $\phi \geq 30$ deg gave well-defined, distinguishable valleys.

Figure 27 shows the actual correlation of distance between the two valleys as a function of ϕ for the four designs. In order to better locate the minimum, the fin valley was smoothed by fitting it to a Gaussian and then extracting the minimum value from the fitted Gaussian equation. Note that the Gaussian does not describe the actual physics involved, as the analytical solution is a complex series of integrals as the path length of the photons changes throughout the rotation, but it does take advantage of the symmetric properties that the pulses exhibit to precisely determine where the pulse is centered. The distance between the valleys was then plotted against the Φ mode of the curve, yielding the data points in Figure 27. A simple linear regression analysis was applied to fit the data points to a linear curve; the fit and its equation is overlaid as a solid line.

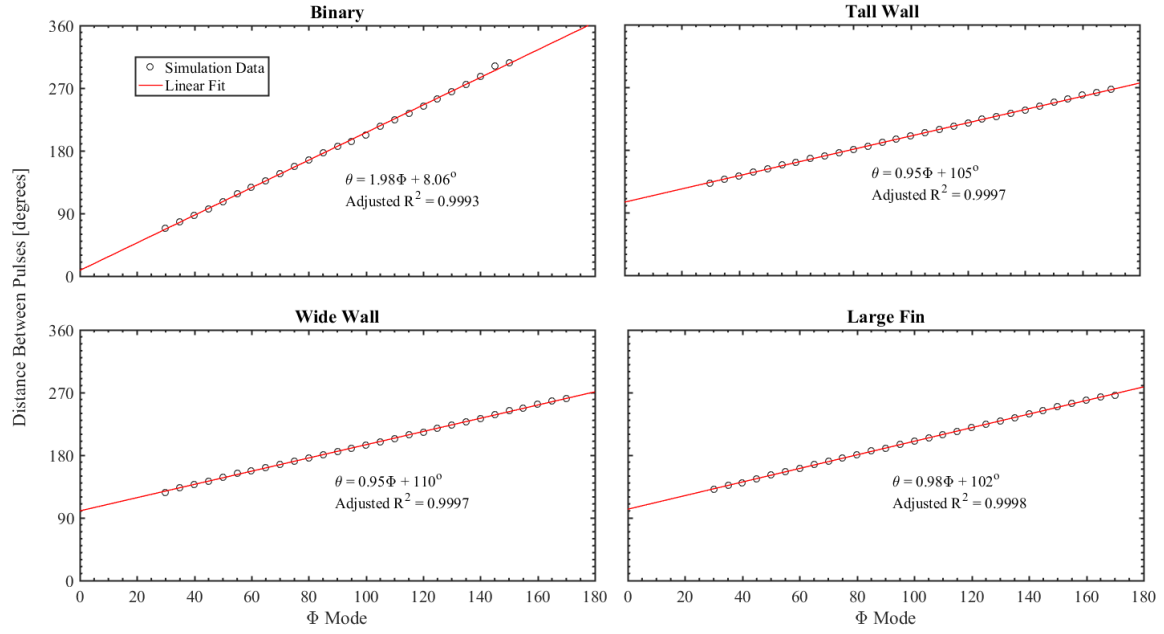


Figure 27. Linear fit of the distance between the wall valley minimum and fin valley minimum for the 4 designs containing the appropriate geometry and DRCs. As before, the valleys were only distinguishable for modes above 30° , but for the modes above the angle, the distance followed a well defined linear relationship corresponding to their geometrical differences of the wall and fin.

The figure verifies the linear correlation, as all four fits had coefficients of determination, R^2 , over 99.9%. Although R^2 is not a rigorous statistical test, it is sufficient in scenarios such as this where there are many data points and only two terms in the regression model (as evident by the large adjusted R^2). Figure 28 plots the residuals from the fit and observed. No apparent structures were present in the residual plots, so the adjusted R^2 is a good indication of correlation. Moreover, the equation for the fitted curves matched precisely what was predicted using just geometry of the mask and assuming that the pulses were aligned with the center of the wall and fin.

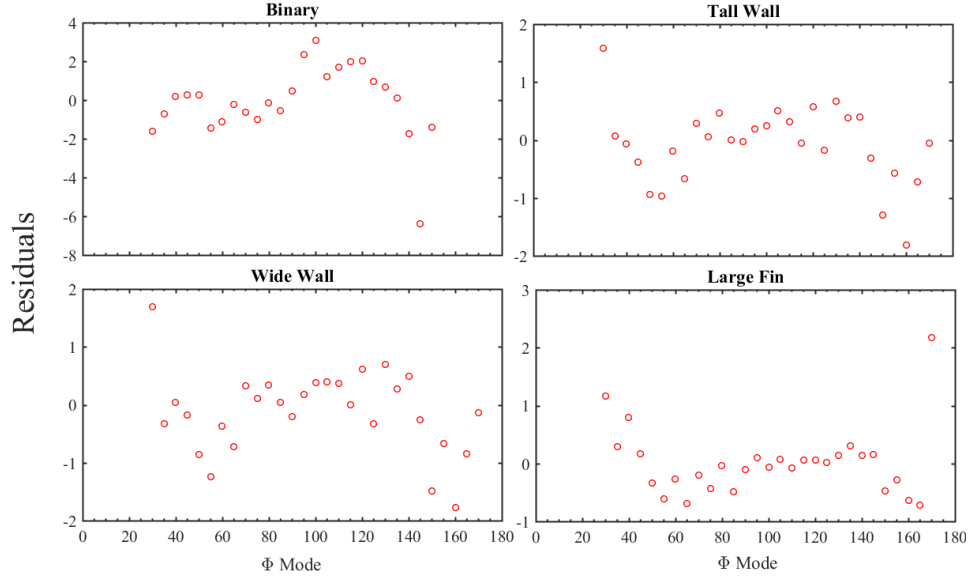


Figure 28. Residuals of the data and fits from Figure 27. The lack of any structure in the residuals verifies that the adjusted R^2 accurately shows correlation, with the random noise about zero caused by the uncertainties from the MCNP simulation.

With these new designs, the direction of the source could be identified in the following manner. θ is determined by the location of the minimum for the wall valley (specifically its shift from 10° for the Wide Wall or 5° for the other 3 designs). The distance between the two valleys would then be substituted in the equations in Figure

27 to yield ϕ . The primary limitation of this method is ensuring enough counts are detected to precisely identify the two valleys and their minima. This also requires that the two valleys are distinguishable from one another. The Binary design had further limitations in that two valleys overlapped at the extremes of ϕ . This was caused by the absolute distance between the wall and fin being too close (this limitation was observed early on and is the reason why the other three geometries contained a minimum angular distance between the wall and fin). As a result, the Binary design was restricted to a range of $30^\circ \leq \phi \leq 150^\circ$.

The benefit of this technique is that it presents a different method for identifying multiple sources. As a demonstration, Figure 29 plots a theoretical response curve of the Tall Wall RSM from two Cs-137 source, one at $(\phi = 50^\circ, \theta = 5^\circ)$ and the other at $(\phi = 100^\circ, \theta = 50^\circ)$. This was done by simply adding the two individual response curves to each other (simulating two sources of equal activity and distance from the detector). As expected, four pulses were present; each source contributed two pulses to the overall response. This is an example of how the number of pulses can readily indicate how many sources there are. Distributed sources would have a similar response, effectively “broadening” the two pulses.

However, Figure 29 also demonstrates the limitation of this method in determining the direction of multiple sources. Specifically, there are four total combinations of wall-fin valley pairs. There are thus four potential solutions to the directions of the sources, indicating that this method introduces degenerate modes *when there is more than once source*. The issue is exacerbated when one considers that the strength of the valley signals are related to the activity and distance of the source, introducing further uncertainties. The figure also shows the resolution issue: the fin valley is not as sharply peaked as the wall valley. Therefore, the measurements within the fin valley must be precise enough to accurately determine the distance between the two

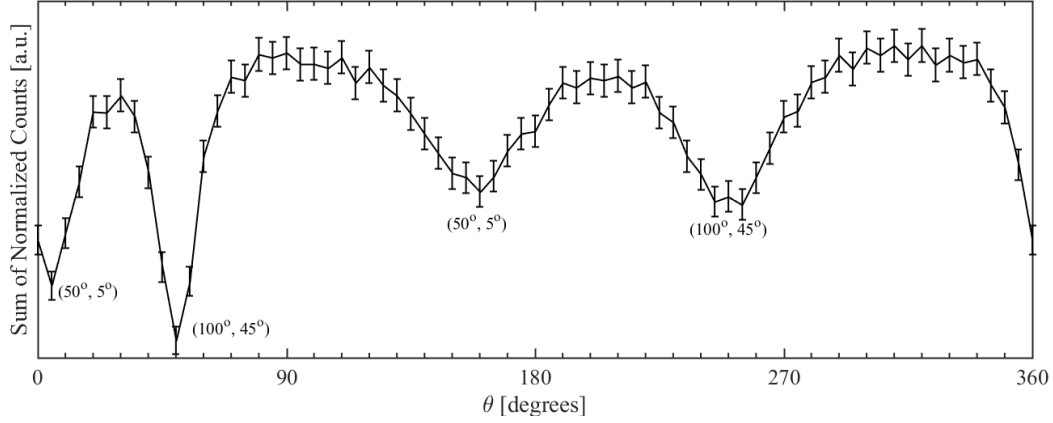


Figure 29. Theoretical response of two equidistant, equal activity Cs-137 sources for the Tall Wall RSM. While the number of pulses are indicative of the number of sources, this does introduce a degeneracy. Specifically, the wall and fin valleys have 4 possible combinations and thus 4 solutions, limiting the accuracy and precision of these designs in complex source environments.

to use in the equations of 27.

4.6 Response Curve Analytics

The experimental data for the Fitzgerald design was collected using a 6 μCi Cs-137 over a 23-hour period for each Φ mode. The equivalent run time would be larger or smaller for the new designs, based on whether their efficiencies were lower or higher, respectively. An approximation for their equivalent run time, t , was calculated using the ratio of their normalized counts:

$$t = 23 \left(\frac{6.693 \times 10^{-3}}{C} \right), \quad (6)$$

where C is the normalized count rate of the design given in Table 4 and 6.693×10^{-3} is the normalized count rate from the Fitzgerald design. Therefore, an RSM with twice the number of average counts would take half as long to detect the same number of photons. The results are shown in Table 6. As mentioned previously, the Mace

and Mace II had lower efficiencies, resulting in equivalent run times of 23.3 and 62.0 hours, respectively (the long run time required for the Mace II design greatly limits its practicality for real-time data acquisition). The other seven designs exhibited lower run times due to their improved efficiencies; the shortest of 14 hours for the Binary design.

Table 6. Comparing dwell times of the various RSM designs. Other than Mace II, the Spartan Class shows higher efficiencies and thus shorter dwell time to achieve the same uncertainty level of the Fitzgerald and Mace designs.

Design	C $\times 10^{-3}$	Fractional Improvement	Equivalent Dwell Time [hr]
Fitzgerald	6.69	0.0%	23.0
Mace	6.59	-1.5%	23.3
Mace II	2.48	-62.9%	62.0
Modal Binary	8.64	29.1%	17.8
Spaced Binary	8.73	30.5%	17.6
Binary	10.96	63.8%	14.0
Tall Wall	8.69	29.8%	17.7
Wide Wall	8.53	27.4%	18.1
Large Fin	8.50	27.1%	18.1

The 23-hour run time was necessary to lower the relative uncertainty in the shape of the response curve so that the MAC value comparisons would be both precise and accurate. However, the alternative method discussed in the previous section only required two data points to determine the sources location. The run time, then, would have to be long enough just so those two points could be accurately determined. Since the ratio of the number of counts at the two minima is known (through the MCNP response curves), it is possible to determine the optimal number of detections required.

Figure 30 represents a typical signal for the two-valley response curves of the Binary, Tall Wall, Wide Wall, and Large Fin designs. Let N_1 be the absolute minimum number of counts and N_2 be the minimum of the smaller pulse. Since the detection

of photons is governed by Poisson statistics, the uncertainty at each point is simply the square root of N_1 and N_2 . To ensure that the two points are at least s standard deviations from each other requires that

$$N_2 - N_1 \geq s \left(\sqrt{N_1} + \sqrt{N_2} \right), \quad (7)$$

where the term in parenthesis on the right is sum of the two standard deviations. Substituting in the ratio $r = N_2/N_1$ and solving Equation 7 for N_1 yields the following inequality:

$$N_1 \geq \left(\frac{s}{\sqrt{r} - 1} \right)^2. \quad (8)$$

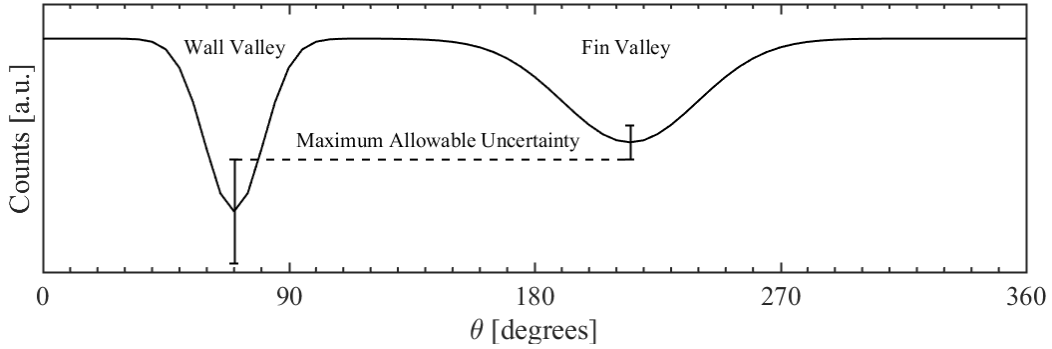


Figure 30. Graphical representation showing the maximum allowable uncertainty in order to accurately distinguish the wall and fin valley. The same principle was applied to determine the maximum allowable uncertainty to distinguish the fin valley from the average maximum response.

Equation 8 for one, two, and three standard deviations is plotted in Figure 31 for ratios between 1 and 2.5. As the ratio increases, the number required at the minimum decreases. The plot also features vertical lines at four r values to represent the average ratios of the four designs, as calculated from the simulated curves. In accordance with the limitations of identifying the pulses, the ratios were calculated for the Φ modes 30° and above (and maximum of 150° for the Binary design). Three performed similarly with approximately equal ratios, but the Wide Wall design had

the highest average ratio of 2.05. The equivalent number of detections required are given in Table 7.

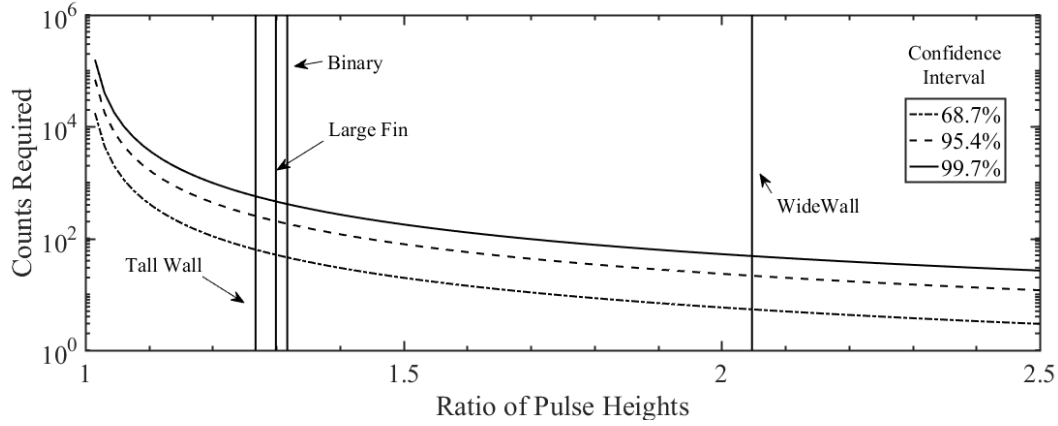


Figure 31. Counts required in order to distinguish the wall and fin valleys to confidence intervals of 1,2, and 3 standard deviations. The average ratio of the two valleys counts were used and are marked by the vertical lines.

Table 7. Values pulled from Figure 31 at the vertical lines marking the average valley ratios. Note that at the highest ratio of 2.05 for the Wide Wall RSM, only 48 counts are needed to distinguish the two valleys. At these low values, the largest uncertainty comes from the precision in determining the absolute minima of the valleys (i.e. angular resolution).

Design	Average Ratio	Counts Required		
		s=1	s=2	s=3
Binary	1.32	45	180	406
Tall Wall	1.27	62	248	559
Wide Wall	2.05	5	21	48
Large Fin	1.30	51	204	458

At three standard deviations, the Wide Wall design required only 50 counts. Statistically, this means that when the minimum number of counts in the response curve reaches 50 for that design, on average there will be 99.7% certainty that the two valleys will be correctly identified on which corresponds to their respective angular

component (i.e. which corresponds to the shift in θ and which determines ϕ). This reflects a significant improvement in efficiency compared to the average 10^3 detections required using the old method of MAC comparisons. At these low numbers, the limiting factor becomes the uncertainty within the two valleys in precisely identifying the locations of the minima. However, this is an uncertainty in the angular resolution, which is inherent to every RSM design. This can also be resolved by letting the RSM run until a minimum of 100 counts have been collected at all angles, reducing the relative uncertainty in the number of counts to a maximum of 10% everywhere.

The approximate time to achieve the counts given in Table 7 were calculated as a function of source activity for the Large Fin, Tall Wall, and Wide Wall designs and is shown in Figure 32. The time was calculated assuming the same source configuration from Logan’s research and using the average count rates from Table 6. Even at a 99.7% confidence interval, the RSM designs decreased the expected dwell time required compared to the time taken in previous research (shown by the star). The Wide Wall design showed the most improved, significantly reducing the time by two orders of magnitude. This shows the strong advantage of these RSMs: they significantly improved the capability to determine the direction of a source at a fraction of the time it would take for the Fitzgerald and Mace designs. These approximate dwell times approach real-time detection capabilities; further research should determine actual times required and applicability in more complex source environments.

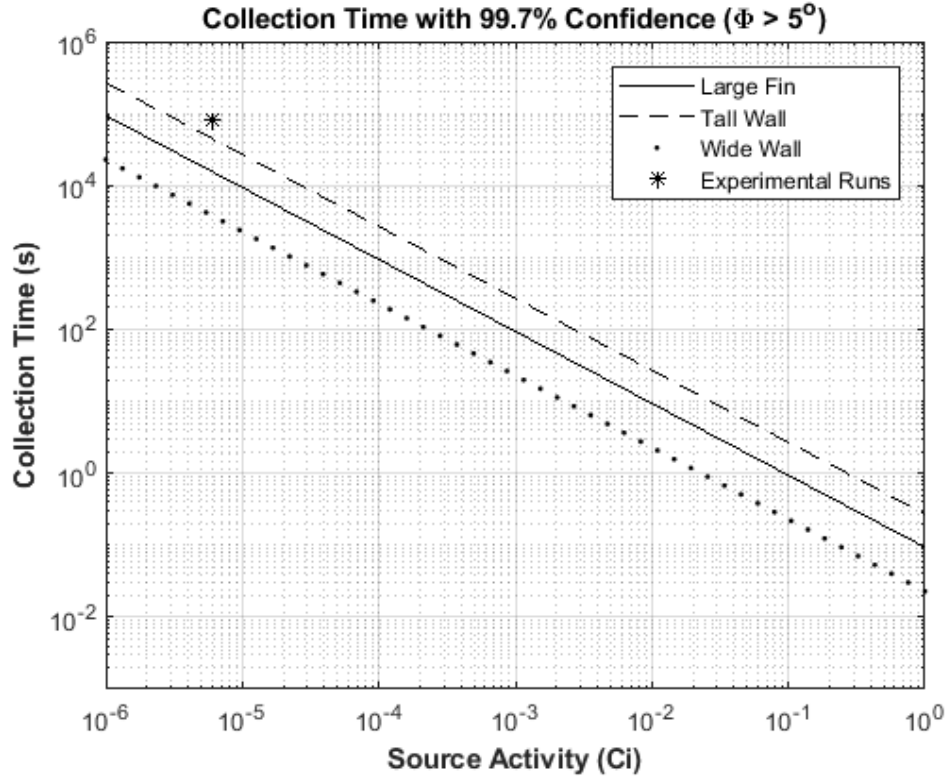


Figure 32. Approximate time required in order to distinguish the wall and fin valleys at a 99.7% confidence, as a function of source activity, using the experimental configuration of the source used for the Fitzgerald RSM. The new designs have an expected dwell time less than that of the Fitzgerald RSM (given by the star), while the Wide Wall RSM reduced the time by two orders of magnitude.

V. Conclusions

5.1 Summary

Previous studies have shown that the RSM concept works as a directionally-sensitive gamma detection system. Early in development, it was concluded that the effectiveness of any RSM is determined by the specific geometry of the mask. Fitzgerald’s preliminary optimization studies produced the first successful RSM design, capable of identifying the direction of a Cs-137 source over a near 4π space. Holland greatly expanded on the optimization of RSM designs, developing a mathematical model for the foundations of the true “optimal” design. Holland also suggested a different approach for RSM designs, one capable of independently determining the angular components of a source’s direction.

This research serves as a proof-of-principle for these designs, referred in this work as the Spartan Class RSMs. Simulations of a sample set of Spartan Class designs were conducted in MCNP in order to collect a theoretical library of DRCs. Decoupling the angular components by producing characteristic signatures in the DRCs were shown to greatly improve RSM performance, assuming the appropriate conditions were met. First, the characteristic signature must have been distinguishable from the rest of the DRC, either in shape or signal strength. Second, the signature must have provided information regarding one of the angular components while being independent of the other. This research showed that a wall of attenuating material produced a characteristic signature meeting both these conditions in the form of a narrow region in the DRC, independent of a source’s polar angle, with a sharp reduction in count rate.

The primary achievements of this work are:

- A significant improvement in computational efficiency for producing DRCs of

the FEP through geometrical variance reduction in MCNP

- Development of a software package for developing, testing, and analyzing RSM designs
- Proof-of-Principle of angular decoupling through the Spartan Class RSMs
- Development of alternative identification algorithms for Spartan Class designs.

5.2 Benefits and Limitations

Spartan Class designs have many desirable features. First, the decoupling of angular components simplifies the design process. Instead of having to minimize consistency between all shifted combinations of DRCs, the RSM must only minimize the smaller set of DRCs at a reference azimuthal angle. This decreases the number of calculations to be made and reduces the overall consistency of modes for a given RSM design. An added benefit of decoupling is that, generally, less material is required to produce the DRCs. This results in not only lower costs and smaller sizes, but increased efficiency, reducing the dwell time required to identify a source by up to two orders of magnitude. Finally, there exists a specific set of Spartan Class RSMs that may provide alternative approaches to identifying a source's direction. By reducing the complexity of the analysis, these designs may provide a solution to the complex source environment problem.

Spartan Class designs also have restrictions and limitations. First, their effectiveness is entirely based on their ability to decouple the angles. An RSM that cannot accurately and precisely do so will not benefit from the techniques mentioned in this research. There are also potential issues with the Spartan Class designs proposed in complex source environments. The designs suggested use a pulse to determine the azimuthal angle of a source. Multiple sources (at the same energy) produce multiple

pulses, potentially obscuring information in the DRCs. Therefore, deconvolution of the responses is still required for the Spartan RSMs. For the specific set of designs mentioned in 4.5, the multiple source problem introduces potential degeneracies in the DRCs, further limiting their performance.

5.3 Recommendations for Future Work

The RSM is still a new concept. There are many areas yet to be explored, and the success of this research promotes further work into RSM design optimization and characterization. As a continuation of this study, potential avenues of research are:

- Conduct simulations to study the imaging potential within the Compton regime, rather than just the FEP. If this is possible, it should reduce the dwell time for the RSM and provide another merit to consider in RSM design optimization.
- Test complex source environments, either through simulation or experimentation, to determine the practicality of the RSM in non-ideal conditions.
- Optimize the Spartan Design based on the wall (and fin) valley requirements. There should be an ideal size that balances detection efficiency with signal strength.
- Compare simulated responses to expected observed responses. Determining better approximations for dwell time, precision, and accuracy of the RSMs will provide more insight into the optimization process.
- Conduct a thorough optimization study in terms of not only size and geometry, but material and density.

Other researchers, such as Condon, have worked on areas regarding the RSM not discussed in detail in this work. The following recommendations are for areas related

to the RSM but not relating to the optimization study:

- Study the performance of the RSM as a directionally-sensitive neutron detector. This area has the greatest potential for future research, as only minor studies have been proposed.
- Develop an experimental algorithm that provides real-time information to the user. Those interested in the experimental efforts of the RSM are referred to the work of Condon, who has improved on the set-up developed by Charles and Logan.
- Analyze the behavior of the RSM using different radiation detectors. While NaI(Tl) has proven to be effective, there may be other options better suited for different radiation environments.

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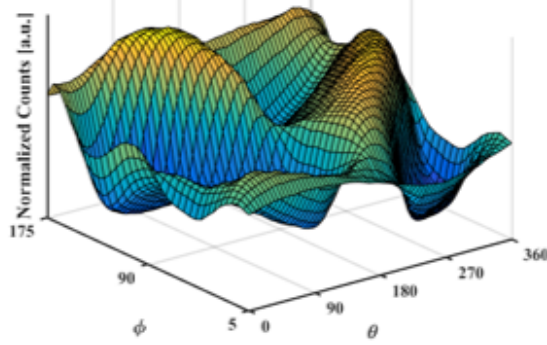
Appendix A. RSM Design Simulation Results

FITZGERALD

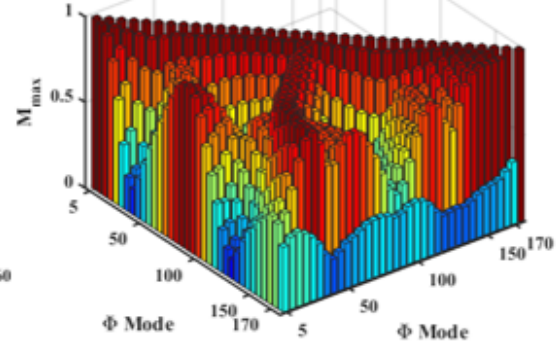
Design Matrix

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Detector Response Curves



$M_{\max}$ Over All Shifts

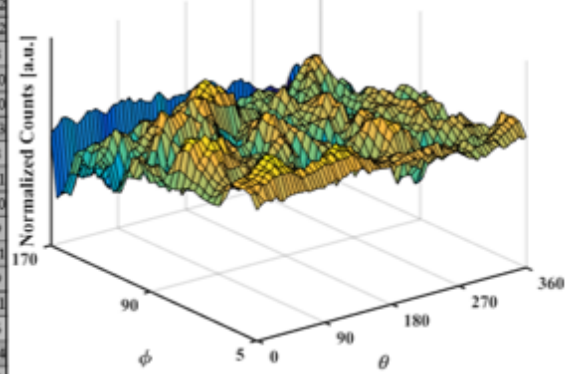


MACE

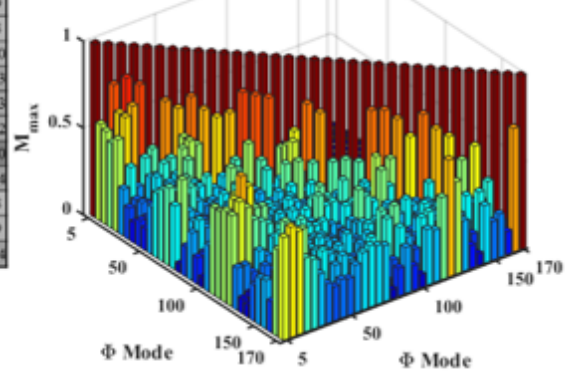
Design Matrix

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Detector Response Curves



$M_{\max}$ Over All Shifts

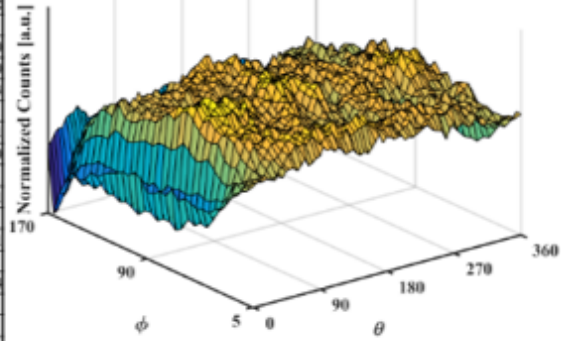


MACE II

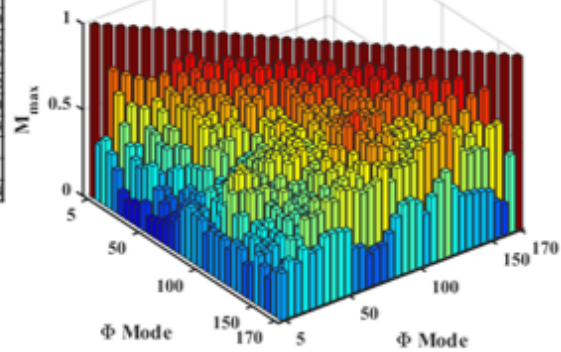
Design Matrix

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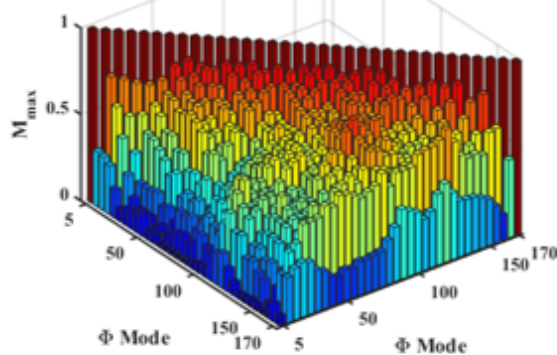
Detector Response Curves



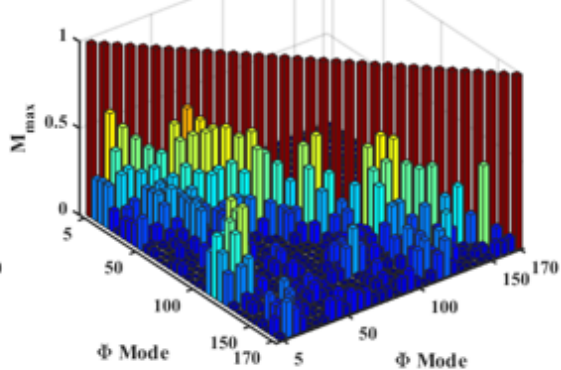
M_{max} Over All Shifts



M_{max} with Spartan Analysis



M_{max} with Bias Reduction

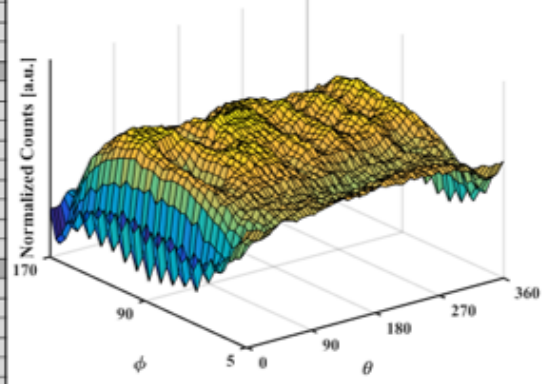


MODAL BINARY

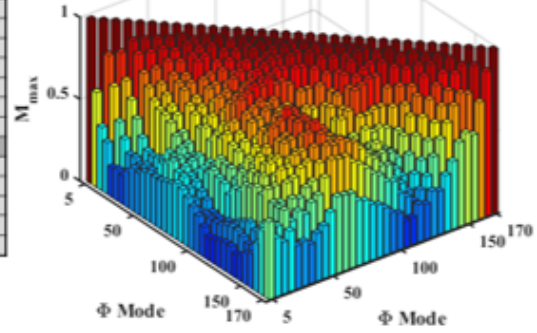
Design Matrix

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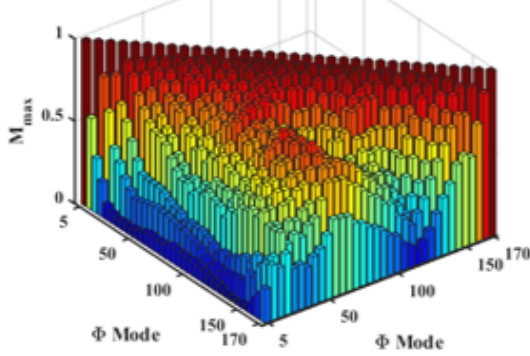
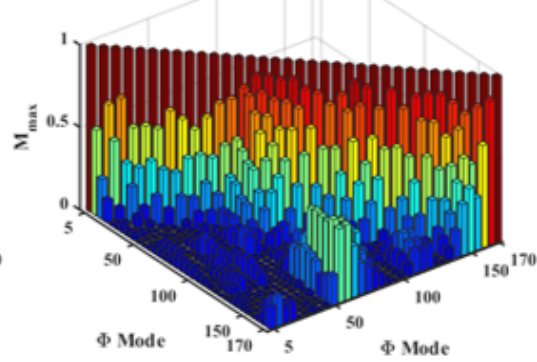
Detector Response Curves



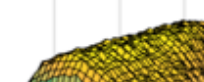
M_{max} Over All Shifts



M_{max} with Spartan Analysis

M_{max} with Bias Reduction

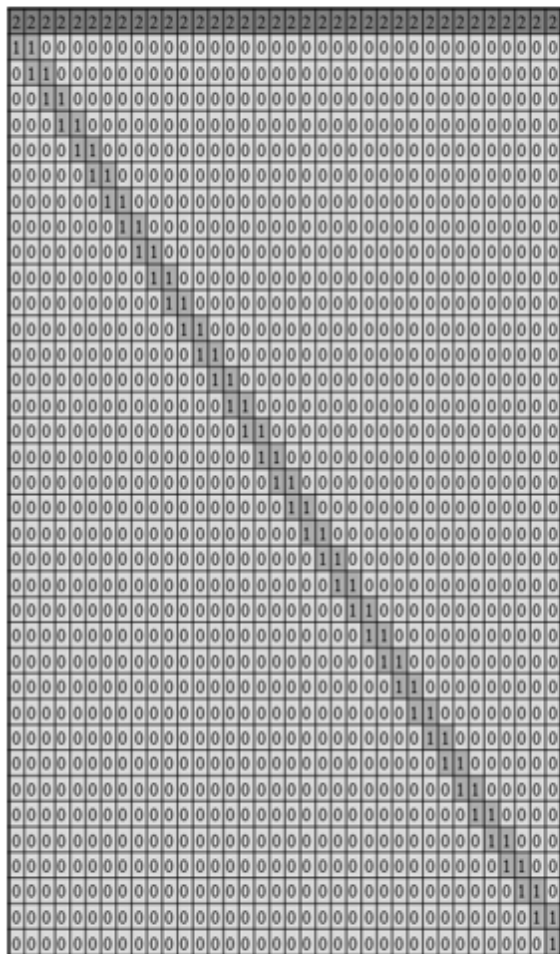
Design Matrix

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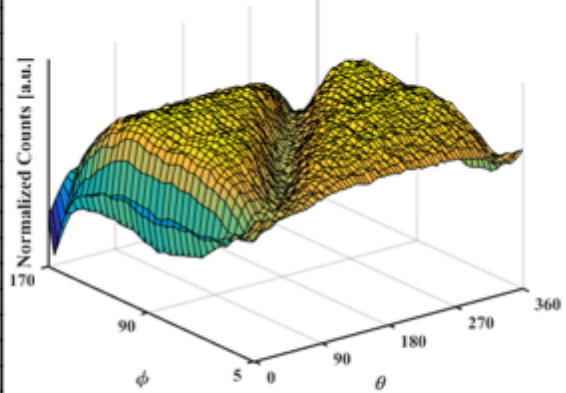
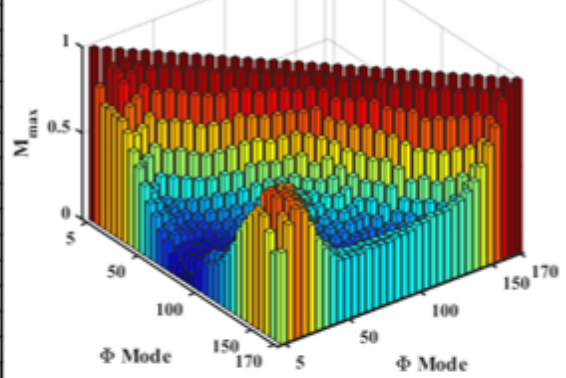
A 3D surface plot showing the normalized counts (in arbitrary units, a.u.) as a function of the azimuthal angle θ and the polar angle ϕ . The vertical axis represents 'Normalized Counts [a.u.]' with a scale from 0 to 170. The horizontal axis represents θ with values 0, 90, 180, 270, and 360. The depth axis represents ϕ with values 0, 5, and 90. The surface shows a complex, wavy structure with a prominent peak reaching approximately 170 counts at $\theta \approx 180^\circ$ and $\phi \approx 5^\circ$.

BINARY

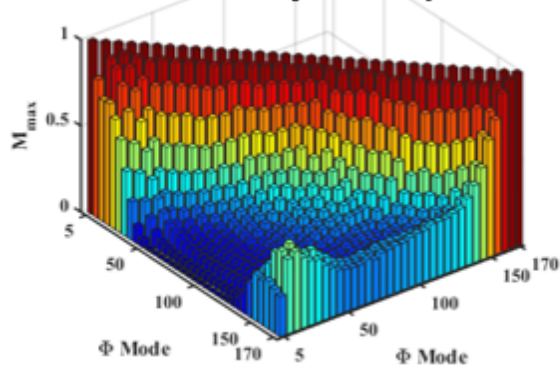
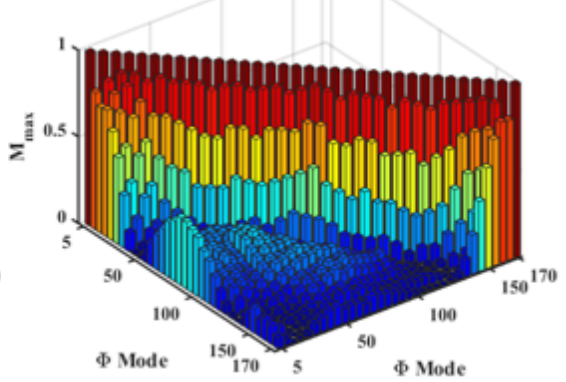
Design Matrix



Detector Response Curves

M_{max} Over All Shifts

M_{max} with Spartan Analysis

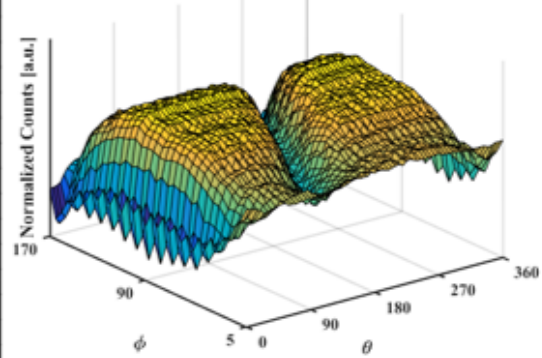
M_{max} with Bias Reduction

TALL WALL

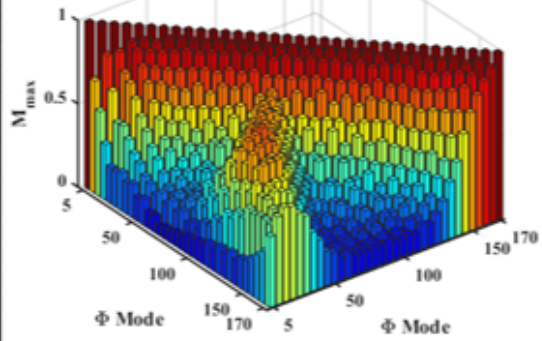
Design Matrix

[illegible]

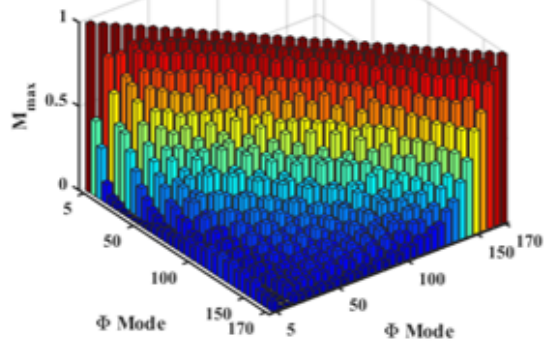
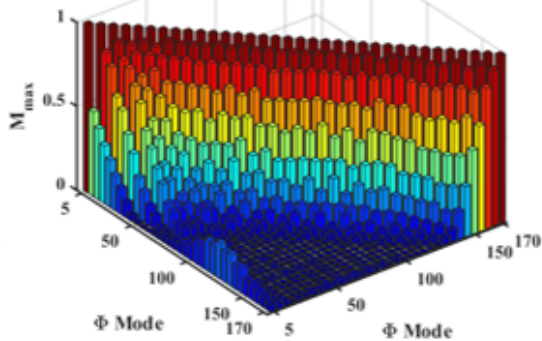
Detector Response Curves



M_{max} Over All Shifts



M_{max} with Spartan Analysis

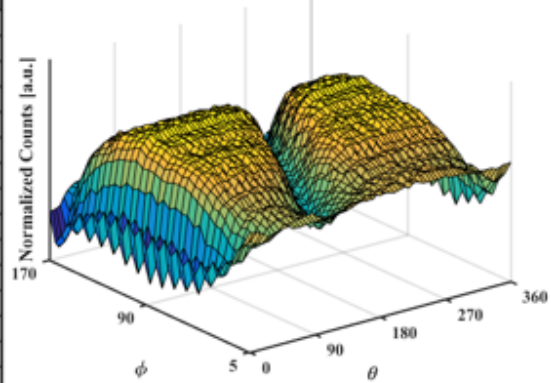
M_{max} with Bias Reduction

WIDE WALL

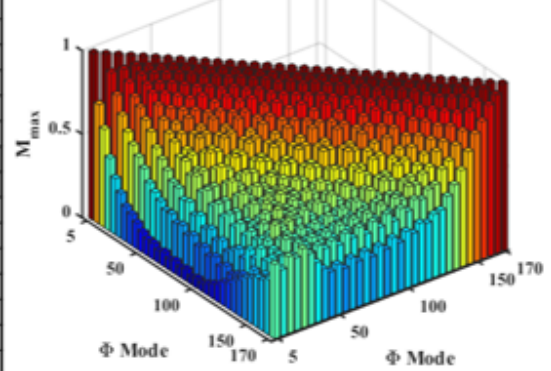
Design Matrix

[illegible]

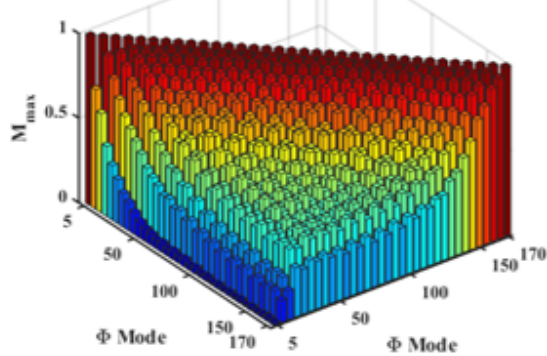
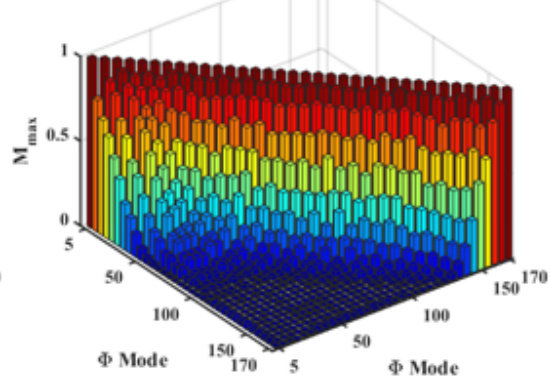
Detector Response Curves



M_{max} Over All Shifts



M_{max} with Spartan Analysis

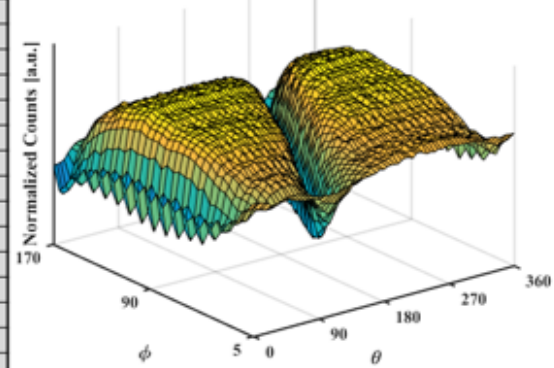
M_{max} with Bias Reduction

LARGE FIN

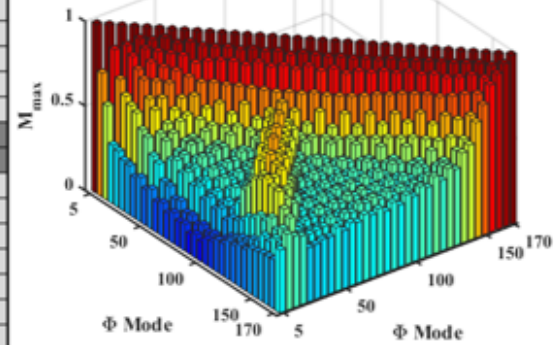
Design Matrix

[illegible]

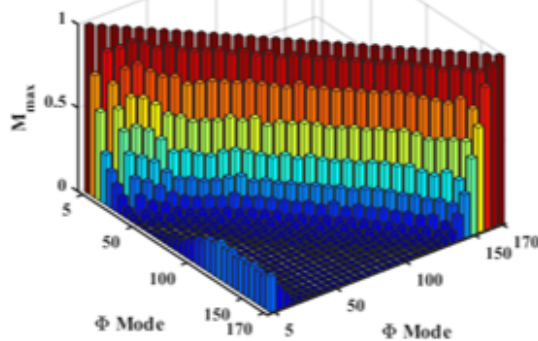
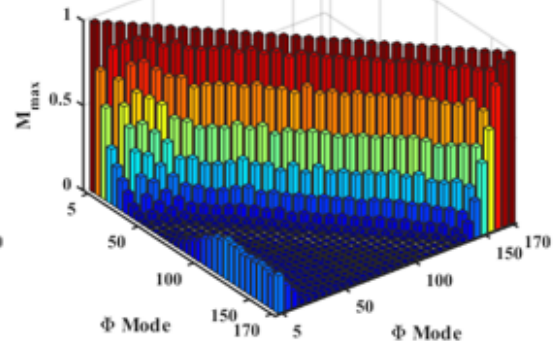
Detector Response Curves



M_{max} Over All Shifts



M_{max} with Spartan Analysis

M_{max} with Bias Reduction

Appendix B. Example MCNP Input Deck

```

1 MCNP Master File for Fitzgerald Design
2 c RSM Cells
3 700 30 -1.190000 0 u=1 imp:p=1
4 701 10 -0.001205 0 u=1 imp:p=1
5 c Environment Cells
6 702 0 -554 552 fill=1 imp:p=1 $ RSM cell
7 703 10 -0.001205 554 -550 imp:p=1 $ inside of sphere, outside of RSM
8 501 0 550 imp:p=0 $ ignore p out of sphere
9 502 20 -3.667000 -551 imp:p=1 $ nai detector
10 503 21 -2.780000 -552 553 imp:p=1 $ aluminum sleeve
11 504 10 -0.001205 551 -553 imp:p=1 $ Air inside sleeve
12
13 c Surfaces
14 550 so 87.360000 $ sphere around everything
15 551 rcc 0 0 -3.810000 0 0 7.620000 3.810000 $ detector
16 552 rcc 0 0 -51.1900 0 0 55.000000 4.000000 $ 1/8 inch sleeve outside
17 553 rcc 0 0 -51.1900 0 0 55.000000 3.810000 $ 1/8 inch sleeve inside
18 554 so 70.000 $ Sphere holding RSM
19
20 c DATA CARDS
21 embed1 meshgeo=abaqus
22 mgeoio=DrillGeo.inp $ Unstructured mesh abaqus file
23 meeout=temp.o
24 length= 1.00000E+00 $ scale units to cm
25 background= 701 $ inferred background cell
26 matcell= 1 700 $ mcnp-abaqus material linking
27 mode p $ p only
28 nps 500000 $ number of histories
29 c material
30 m10 6000 -0.000124 $ air via pnnl
31 7000 -0.755268 $ 7014 for neutrons
32 8000 -0.231781 $ 8016 for neutrons
33 18000 -0.012827
34 m20 11000 -0.153373 $ nai detector via pnnl - na 11023 for n
35 53000 -0.846627 $ 53127 for neutrons
36 m21 12000 -0.015 $ aluminum sleeve (assume 2024)
37 13000 -0.927 $ 13027 for neutrons
38 14000 -0.00283
39 22000 -0.00085
40 24000 -0.00057
41 25000 -0.006 $ 25055 for neutrons
42 26000 -0.00283
43 29000 -0.0435
44 30000 -0.00142
45 m30 1000 -0.080538 $ lucite/acrylic vi pnnl - h 1001 for ne
46 6000 -0.599848
47 8000 -0.319614 $ 8016 for neutrons
48 c source
49 sdef erg 0.662 pos 0.000000 0.000000 86.360000
50 vec -0.000000 -0.000000 -86.360000 dir=dl
51 sil -1 0.99500 1 $ 0.861961 for Compton analysis
52 spl 0 0 1
53 f8:p 502 $ detector response
54 e8 0 1e-5 0.001 0.1 599i 0.7 $ energy bins

```