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Unified Theory and Algorithm for Solving Challenging Problems in Mathematical Physics and Complex Systems with Applications

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14. ABSTRACT
Supported by this AOARD grant, the PI and his post-doctor and co-workers have successfully developed/improved a breakthrough canonical duality theory and its associated algorithms for solving a large class of challenging problems in mathematical physics and complex systems. Within one year, he has published 1 book by Springer, 1 journal special issue (Springer), and about 29 papers (11 are journal papers). The most significant achievement is the solution to the well-known knapsack problem, which is listed as one of 21 NP-complete problems in computer science. By using the canonical duality theory, this 0-1 integer programming problem can be equivalently converted to a non-smooth concave maximization problem with only one dual variable, which can be solved very easily, therefore, this so-called NP-complete problem can be obtained analytically via this canonical dual solution. Application to computational physics leads to a powerful deterministic algorithm for solving the most challenging bi-level mixed integer programming problem in structural topology optimization.

15. SUBJECT TERMS
NP-hardness, nonlinear dynamics, chaos, AOARD

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PI: David Y Gao, Alex Rubinov Professor of Mathematics, *Federation University Australia*
Title: A Breakthrough Theory and Algorithms for Solving Chaotic Dynamics and NP-Hard Problems in Mathematical Physics and Complex Systems
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Other Research Collaborators:

1. Dr. Eldar Hajilarov, Federation University
2. Dr. Vittorio Latorre, University of Rome.

Accomplishments/New Findings:
Supported by this AOARD grant**,** the PI and his post-doctor and co-workers have successfully developed/improved a breakthrough canonical duality theory and its associated algorithms for solving a large class of challenging problems in mathematical physics and complex systems. Within one year, he has published **1** book by Springer, **1** journal special issue (Springer), and about **29** papers (11 are journal papers).
The most significant achievement is the solution to the well-known **knapsack problem**, which is listed as one of 21 NP-complete problems in computer science. By using the canonical duality theory, this 0-1 integer programming problem can be equivalently converted to a non-smooth concave maximization problem with only one dual variable, which can be solved very easily, therefore, this so-called NP-complete problem can be obtained analytically via this canonical dual solution. Application to computational physics leads to **a powerful deterministic algorithm** for solving the most challenging bi-level mixed integer programming problem in structural topology optimization. Research results can be directly applied for **light-weight design** of aircrafts, fight jets, and much more (see Figures 1 and 2).
Another important achievement of this project is a revolutionarily new global optimal method for solving chaotic dynamical systems. Instead of traditional linear iteration methods, a nonlinear dynamical system is first formed as a global optimization problem via the least squares method. Then using the canonical duality theory, this nonconvex minimization problem can converted as a concave maximization problem in dual space, which can be solved easily via the well-developed convex optimization techniques. If this canonical dual has a unique solution, the nonlinear system is not chaotic. Otherwise, the primal global optimization problem is NP-hard, and in this case, the nonlinear system has chaotic solutions. Therefore, the connection between chaos in complex systems and the NP-hardness in computer science is revealed for the first time.
Remaining significant achievements include a unified model for multi-scale complex systems, a unified methodology for solving a class of fully nonlinear partial differential equations, and a unified algorithm for solving a class of so-called NP-Hard problems in decision science and global optimization.
The projects proposed in the proposal are fully completed.

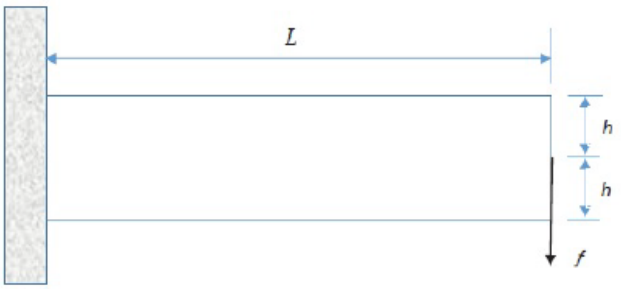

Fig. 1. Design domain for a long cantilever beam with external load

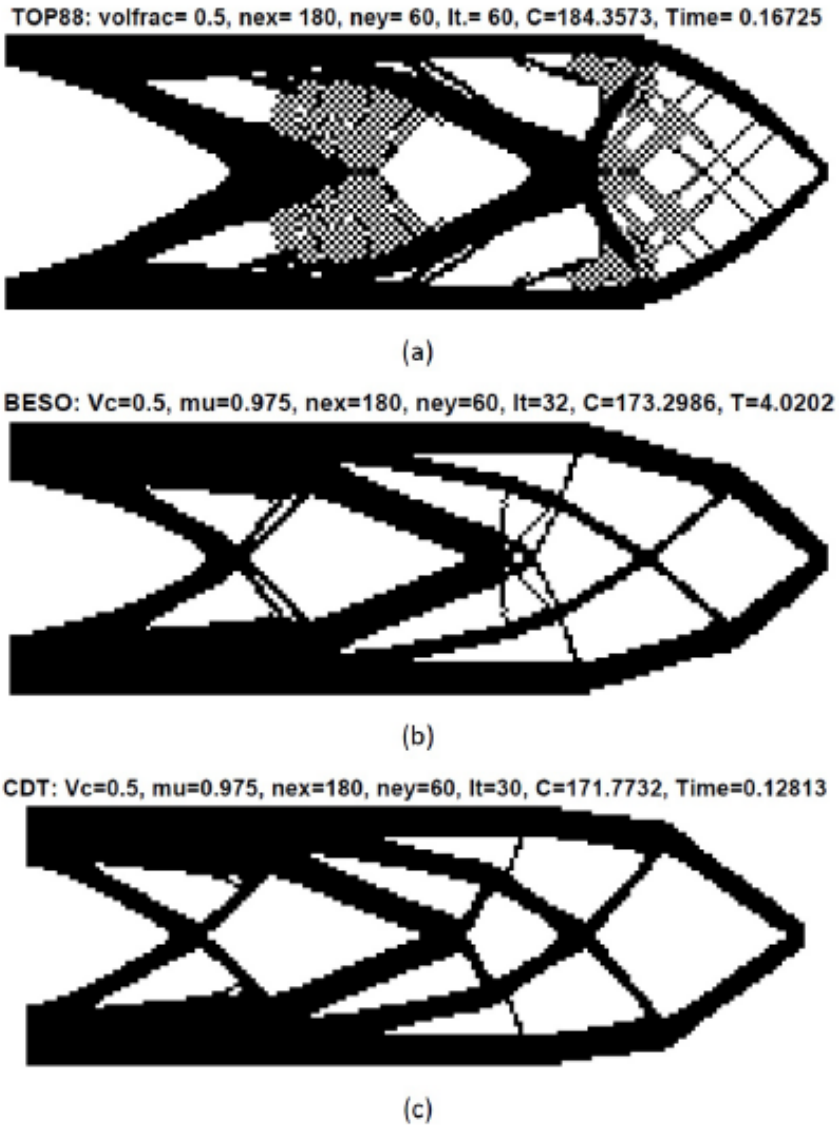

Fig. 2. Topology optimization results obtained by SIMP (a), BESO (b), and the canonical duality theory CDT (c).

Co-sponsored by Federation University, this AFOSR grant has been used to support one senior research fellow, one PhD student, several international visitors, and international research activities.

Keynote, Plenary, and Invited Lectures at International Conferences.

1. Invited Lecture, Symposia on Intelligent Technologies for Advancing and Safeguarding Australia. 15 August 2017, Deakin University, Geelong, Australia.
2. Invited Lecture, 12th International Symposium on Health Informatics and Management, 2-3 June, 2017, National Chiao Tung University, Taiwan.
3. Plenary Lecture, International Conference in Mathematics Trends and Developments 2017 (ICMTD-17), 28 – 30 Dec. 2017, Cairo, Egypt.
4. Plenary Lecture, Frontier Forum on Intelligent Control and Decision Optimization, 23-25 August, 2017, Central South University, Changsha, China
5. Plenary Lecturer, The 2nd International Conference and Summer School on Numerical Computations: Theory and Algorithms, 19 – 25 June 2016 Club Med Resort "Napitia" Pizzo Calabro, Calabria, Italy.
6. Keynote Lecturer, International Symposium of Heterogeneous Materials Mechanics, June 4-6, 2016, Chongqing, China.
7. Invited Lecturer, The 10th International Conference on Scientific Computing and Applications, June 7-10, 2016, Toronto, Ontario

Contributions within Discipline:
The canonical duality methodology can be used for modeling multi-scale complex systems. The canonical duality theory produces analytical solutions to a class of fully nonlinear partial differential equations in large deformation solid mechanics, post-buckling of thin-walled structures and phase transitions problems. The algorithm can be used for solving a large class of challenging problems in mathematical physics and complex systems.

Contributions to Other Disciplines:
Canonical duality theory has been used successfully in neural networks optimization, sensor communication systems, filter design, signal processing, machine learning, chaotic dynamical systems, decision making, supply chain, scheduling problems, and computational mechanics, light-weight structural design, etc.

Publications:

Books/Special Issue Published:

1. Gao, DY, Latorre, V. and Ruan, N. (2017) *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, 377pp.
2. Floudas, C. and Gao, DY, (2016). Special issue of 3rd World Congress of Global Optimization, Journal of Global Optimization, Springer, Volume 64, Issue 3, March 2016

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Book Chapters and Papers in Refereed Proceedings

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1. Gao, DY. On Unified Modeling in Multi-Scale Optimization and Canonical Duality-Triality Theory, http://arxiv.org/abs/1605.05534
2. Gao, DY. Duality in G. Saccomandi's Challenge on Analytical Solutions to Anti-plane Shear Problem in Finite Elasticity, http://arxiv.org/abs/1511.03374
3. Latorre, V. and Gao, DY. Half-Quadratic regularization and canonical duality theory in image/signal process.
4. Ali, E. and Gao, DY. On SDP Method for Solving Canonical Dual Problem in Post Buckling of Large Deformed Elastic Beam.
5. Ruan, N. and Gao, DY. On Modelling and Complete Solutions to General Fixpoint Problems in Multi-Scale Systems with Applications