



**Geometry and Topology of
Complex Networks**

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TRUSTEES OF THE UNIVERSITY OF PENNSYLVANIA**

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Geometry and Topology of Complex Networks

Abstract

The current vision of many military leaders and policy makers about network-centric technology portrays a future in which networked systems provide unprecedented levels of performance, robustness, and efficiency. Unfortunately, there are fundamental, basic obstacles to realizing this vision. Some of these obstacles stems from a lack of a fundamental theory and our lack of comprehensive understanding of the mathematics of interconnected technological, information, and social networks.

To address these challenges, we embarked on a multi-investigator, multi-disciplinary project on mathematics of networks and networked systems, that addresses fundamental issues from topology to geometry of networks. This multi-year, multidisciplinary effort, led to training of a large group of students and postdocs, involved in academic research activity upon completion of their studies. Aside from many theses and research papers, the research has led to fundamental understanding along multiple domains. The highlights include the following developments: 1) Spectral methods in networks: a) A theory of random walks on directed networks b) A theory of random walks on simplicial complexes

2) Discrete Dirichlet problems, Heat kernel PageRank, and PageRank on simplicial Complexes 3) Developed concept of quantum state transfer on graphs 4) Developed new notions of curvature on graphs 5) New algorithms for distributed optimization using spectral graph theory 6) new algorithms for sensor and actuator placement using spectral methods and sparsification theory.

1 Graph Theory and Spectral Methods in Networks

1.1 Quantum state transfer on graphs

We have studied quantum state transfer on graphs which involves analysis of states that can be achieved by the solution

$$U(t) = e^{it(A-V)}$$

to the Schrödinger equation on a graph, where A is the adjacency matrix of the graph and V gives an energy potential on the graph. We say that there is *perfect state transfer* between vertices u and v of a graph if there is some time t for which

$$U(t)e_u = \lambda e_v$$

where e_u and e_v are indicator vectors for the vertices u and v respectively, and λ is a complex number of modulus 1. Perfect state transfer in graphs has application to the transfer of qubits in quantum information systems (see [2]). Perfect state transfer is closely related to *quantum tunneling* in graphs. This is studied in [12] and approaches the question of state transfer from an asymptotic

point of view, allowing entries of V to become arbitrarily large. In [2], a version of this question is also studied in the absence of the potential. While perfect state transfer is in general a rare phenomenon, we have found instances of graphs where perfect state transfer occurs for specific potentials. In addition, [2] raises the question of whether or not there are potentials on path graphs that give perfect state transfer. We have answered this question in the negative.

Theorem 1 ([9]). *For paths of length greater than 3, there is no choice of potential V for which there is perfect state transfer in the path.*

We have also shown that there are infinitely many graphs where choosing the potential correctly can induce perfect state transfer between a pair of vertices. In particular, this occurs when there are *twin vertices*, i.e. vertices whose neighborhoods are identical.

Theorem 2 ([9]). *Let u and v be vertices of a graph that have identical neighborhoods. Then there is some potential V for which there is perfect state transfer between u and v .*

Since perfect state transfer is difficult to achieve, we consider a relaxation that has been referred to as *pretty good state transfer* (see [2]). We say that there is pretty good state transfer between vertices u and v if, for every $\epsilon > 0$ there is some time t such that $|U(t)_{u,v}| > 1 - \epsilon$. In contrast to the theorem above concerning paths, we have shown that for any path, a potential can be chosen to induce pretty good state transfer between the endpoints of a path. Indeed, this is a specific case of the more general result that this can be done in any graph that has an involution (i.e. a symmetry of order 2).

Theorem 3 ([10]). *Given any graph with an involution and any pair of vertices u and v that are interchanged by the involution, there is some choice of potential V such that there is pretty good state transfer between u and v .*

Furthermore, we have studied perfect and pretty good state transfer in perturbations of strongly regular graphs in [3]. A strongly regular graph with parameters (n, k, a, c) is a k -regular graph in which every pair of adjacent vertices has a common neighbors, and every pair of non-adjacent vertices has c common neighbors. Strongly regular graphs are a widely studied class of graphs. They have nice spectral properties in the sense that their adjacency matrix has only 3 distinct eigenvalues: the regularity k , and two others, θ and τ , that can be expressed explicitly in terms of the parameters n, k, a , and c , and are often integers. We proved the following about strongly regular graphs in [3].

Theorem 4 ([3]). *Let G be a strongly regular graph with eigenvalues k, θ, τ . If $k \equiv \theta \equiv \tau \pmod{4}$, then for some edge weight β and some potential γ , there is perfect state transfer between two vertices u and v of G after placing potential γ on u and v , and adding a weighted edge with weight β between u and v .*

Theorem 5 ([3]). *Let G be any strongly regular graph. For any pair of vertices u, v , there is some choice of potential γ or edge weight β as above such that there is pretty good state transfer from u to v .*

Future work in this area includes investigation of perfect and pretty good state transfer in graphs where there is no symmetry or regularity. It is also of interest to investigate further the time at which perfect or pretty good state transfer occurs.

1.2 Graph curvature and homology

One major theme in graph theory in recent years is the application of ideas and techniques from geometry to graphs. In particular, there are many notions of curvature for graphs that have been inspired by curvature from the study of smooth manifolds. One such notion, called the *Bakry-Émery curvature* is defined in terms of a discrete version of the Bochner formula from geometry (see [1]).

In particular, given a graph $G = (V, E)$, we have a Laplace operator Δ on the space of functions $f : V \rightarrow \mathbb{R}$ given by

$$\Delta f(x) = \frac{1}{d_x} \sum_{y \sim x} (f(y) - f(x)).$$

We further define a Γ -calculus on a graph via the following operators.

$$\begin{aligned} \Gamma(f, g) &= \frac{1}{2}(\Delta(fg) - f\Delta g - g\Delta f) \\ \Gamma_2(f, g) &= \frac{1}{2}(\Delta\Gamma(f, g) - \Gamma(f, \Delta g) - \Gamma(g, \Delta f)). \end{aligned}$$

We say that a graph G satisfies the *curvature dimension inequality* $CD(K, n)$ for $K \in \mathbb{R}$ and $n \in \mathbb{Z}_+$ if for all $f : V \rightarrow \mathbb{R}$,

$$\Gamma_2(f, f) \geq \frac{1}{n}(\Delta f)^2 + K \cdot \Gamma(f, f).$$

In addition to curvature, much work has been done to develop various notions of homology on graphs. In [5], a homology theory, called the *path homology*, for graphs is given based on a chain complex arising from paths in a graph. This homology theory is shown to be compatible with a corresponding notion of homotopy for graphs [4]. We prove an important connection between this homology and the Bakry-Émery curvature in [11].

Theorem 6 ([11]). *If a finite graph has positive Bakry-Émery curvature at every vertex, then its first path homology group is trivial.*

This theorem is a discrete analogue of a classical result of geometry due to Bochner. We also have an analogous connection between curvature and graph homotopy.

Theorem 7. *If a finite graph has positive Bakry-Émery curvature at every vertex, then its fundamental group is finite.*

1.3 Strong embeddings

Building on work from [7] we have developed two conjectures in [6] involving strong embeddings of graphs into surfaces. A strong embedding of a graph is an embedding of the graph into some orientable surface such that the boundary of each face of the embedding is a simple cycle. Two graphs are said to be 2-isomorphic if there is a bijection between their edge sets that preserves cycles.

Conjecture 1. *Let G and H be 2-connected graphs on n vertices, let I_G denote the set of minimal strong embeddings of the graph G , let G_i^* denote the dual graph of G with respect to the embedding i , and let $L(G)$ denote the Laplacian matrix of G . Then the following are equivalent:*

1. *There is a matrix $M \in GL(n, \mathbb{Z})$ such that $M^T L(G) M = L(H)$.*
2. *G and H are 2-isomorphic.*
3. *There exists a bijection $\phi : I_G \rightarrow I_H$ such that G_i^* is isomorphic to $H_{\phi(i)}^*$, for every $i \in I_G$.*

This conjecture relates the arithmetic of quadratic forms defined by the Laplacian of a graph to the combinatorics of the 2-isomorphism class of the graph, and to the geometry of the set of strong embeddings of the graph. It is known that conditions (1) and (2) are equivalent ([13]). Furthermore, we have proven that (2) and (3) are equivalent for planar graphs. We have gathered numerous experimental evidence that the conjecture is true in general as well.

Our second conjecture gives a lower bound on the number of strong embeddings of a graph. Given an orientation on the edges of a graph G , let B be the vertex-edge incidence matrix associated with that orientation, and let P_B be the Newton polytope defined by the columns of B . Two orientations are considered equivalent if one can be obtained from the other by a graph isomorphism, or the reversal of a path that preserves the volume of P_B .

Conjecture 2. *The number of equivalence classes of orientations that achieve the minimum volume of the polytope P_B is a lower bound on the number of minimal strong embeddings of the graph.*

In particular, this conjecture implies that a minimal strong embedding exists. Thus, if true, this conjecture would imply the strong embedding conjecture—a major unsolved problem in graph theory (see for example [8]). We have gathered numerous numerical evidence that this conjecture is true, and we believe that this may be a good way to approach the strong embedding conjecture.

1.4 Spectral methods in graphs and networks

One of the powerful tools for analyzing information network to deduce the crucial properties and structures of a graph/network from its graph spectrum. In particular, many robust local algorithms are based on random walks and

therefore eigenvalues play a main role in quantitative analysis of algorithms for dynamic complex networks.

Previously most of the research on random walks concerns reversible Markov chains, i.e., random walks on undirected graphs. Relatively little is known for random walks on directed graphs, although numerous applications involve directed graphs. For example, one of the main approaches for establishing the pagerank is to use random walks in the Web graph (which is directed). One of the main threads of this project is to develop spectral graph theory for directed graphs and directed random walks.

In addition to using eigenvalues to control local clustering/discrepancy in a graph, we have developed methods to explore the behavior eigenfunctions. The notion of curvature for graphs combined with eigenvalues can be used to control the “stretches” of edges with endpoints specified by eigenfunctions.

We have also developed semi-group spectral methods that can be used to analyze interactive games such as hypergraph coloring games or voter models. Under certain “memoryless” restrictions, we can use semigroup spectral methods to explicitly determine the spectrum of the state graph and therefore determining the appropriate cut-off time for approximation or simulation.

The published work in the above areas is listed below.

- Discrepancy inequalities for directed graphs , *Discrete Applied Mathematics*, 176 (2014) 30-42, (with F. Kenter).
- Harnack inequalities for graphs with non-negative Ricci curvature , *J. Math. Anal. Appl.* 415 (2014), 25 – 32, (with Yong Lin and S.-T. Yau).
- A chapter titled “Spectral Graph Theory”, to appear in *Handbook of Linear Algebra*, CCR Press, (with Steve Butler).
- A note on an alternating upper bound for random walks on semigroups , *Discrete Applied Mathematics*, 176 (2014), 24-29, (with J. Hughes).
- Single processor scheduling with time restrictions, *J. of Scheduling*, 17 (2014), 399-403, (with O. Braun and R. Graham).
- From quasirandom graphs to graph limits and graphlets , *Advances in Applied Math.* 56 (2014), 135-174.
- Edge flipping in the complete graphs, *Advances in Appl. Math.* 69 (2015), 46-64, (with S. Butler, J. Cummings and R. L. Graham).
- Extreme values of the stationary distribution of random walks on directed graphs , *Advance in Applied Math.*, 81, (2016), 128-155, (with Sinan Aksoy and Xing Peng).
- A generalized Alon-Boppana bound and weak Ramanujan graphs, *Electronic Journal of Combinatorics*, 23, (2016), Paper #3.4, (20 pages).
- On the discrepancy of circular sequences of reals, *J. Number Theory*, 164 (2016), 52-65, (with R. L. Graham).

- Curvature aspects of graphs, Proc. of AMS, 145 (2017), 2033–2042, (with F. Bauer, Y. Lin and Y. Liu).
- Decomposition of random graphs into complete bipartite graphs , SIAM J. Discrete Math., 30, no. 1, (2016), 296–310, (with X. Peng).
- Worst-case analysis of the LPT algorithm for single processor scheduling with time restrictions, OR Spectrum, 38(2), (2016), 531–540, (with O. Braun and R. L. Graham).
- The matrix cover polynomial, J. Combinatorics, 7 (2016), 375–412, (with R. L. Graham).
- The spectral gap of graphs arising from substring reversals, The Electronic J. of Combinatorics, 24, no. 3 (2017), #P.3.4, (with J. Tobin)
- A strong Harnack inequality for graphs, Comm. Analysis and Geometry, 25, no. 3, (2017), 557–588, (with S.-T. Yau).

1.5 Heat kernel PageRank and local algorithms

Computing local clusters are of certain application-specific interests, such as detecting communities in social networks or groups of interacting proteins in biological networks. When the graph models the computer network itself, detecting local clusters can help identify communication bottlenecks, where one set of well-connected nodes is separated from another by a small number of links. Further, being able to identify the clusters quickly prevents bottlenecks from developing as the network grows.

In 1998, Brin and Page introduced the notion of PageRank for Google’s Web search algorithm. Since then, PageRank has become a valuable tool for examining the correlations of pairs of vertices (or pairs of subsets) in any given graph and has been used for local algorithms for finding clusters in graphs.

we consider a modified version of PageRank called heat kernel pagerank. Like PageRank, the heat kernel pagerank has two parameters, a seed, and a heat constant or temperature. The heat kernel pagerank can be expressed as an exponential sum of random walks from the seed, scaled by the temperature while the (usual) pagerank can be viewed as a geometric sum of random walks. There are certain advantages in using heat kernel pagerank for improving the time and performance of local algorithms. Here we list published papers in this area.

- Solving linear systems with boundary conditions using heat kernel pagerank Internet Mathematics, 11 (2015), 449-471 (with Olivia Simpson)..
- Distributed algorithms for finding local clusters using heat kernel pagerank, WAW, 2015, LNCS 9479, Springer, 177–189, (with O. Simpson).
- A brief survey of PageRank algorithms, IEEE Transactions on Network Science and Engineering, 1, No. 1, (2015), 38–42.

- Computing heat kernel pagerank and a local clustering algorithms, Combinatorial Algorithms, Lecture Notes in Comput. Sci., 8986, Springer, Cham, 2015, 110-121, and the journal version will appear in European Journal of Combinatorics, (with O. Simpson).
- Complex Networks, a chapter in Handbook of Graph Theory, (eds. J. L. Gross et. al), 1456–1476, CRC Press, 2014, (with A. Bonato).
- Ranking and sparsifying a connection graph, in Internet Mathematics, 10 (2014), 87–115 (with M. Kempton and W. Zhao).
- A local clustering algorithm for connection graphs , Internet Math., 11 (2015), 333-351 (with Mark Kempton).
- Multi-commodity allocation for dynamic demands using PageRank vectors , Internet Mathematics, 10 (2014), 49–65 (with P. Horn and J. Hughes)..
- Hypergraph coloring games and voter models , Internet Mathematics, 10 (2014), 66–86 (with Alex Tsiatas)..
- A semi-supervised heat kernel pagerank MBO algorithm for data classification , preprint (with Ekaterina Merkurjev and Andrea L. Bertozzi)

1.6 Optimal Sensor and Actuator Selection in Large Networks

Power grids, transportation systems, brain neural circuits and social networks are just a few of the large- scale and complex dynamical systems that have drawn the attention of control scientists, since their vast size and interconnectivity necessitate novel control techniques with regard to: 1. tasks that are collective, 2. new cost constraints, e.g., with respect to the number of used sensors and actuators, the level of control effort (input power), and the available bandwidth. Over the past 5 years we have developed new techniques for the analysis and the development of novel control techniques with regard to point 2., as follows:

1. Minimal actuator placement with bounds on control effort: We address the problem of minimal actuator placement in linear systems so that the volume of the set of states reachable with one unit or less of input energy is lower bounded by a desired value. First, following the recent work of Olshevsky, we prove that this is NP-hard. Then, we provide an efficient algorithm which, for a given range of problem parameters, approximates up to a multiplicative factor of $O(\log n)$, n being the network size, any optimal actuator set that meets the same energy criteria; this is the best approximation factor one can achieve in polynomial time, in the worst case. Moreover, the algorithm uses a perturbed version of the involved control energy metric, which we prove to be supermodular. Next, we focus on the related problem of cardinality-constrained actuator placement for minimum control effort, where the optimal actuator set is selected to

maximize the volume of the set of states reachable with one unit or less of input energy. While this is also an NP-hard problem, we use our proposed algorithm to efficiently approximate its solutions as well.

2. resilient submodular maximization over matroid constraints: Applications in control, robotics, and optimization motivate the design of systems by selecting system elements, such as actuators, sensors, or data, subject to complex design constraints that require the system elements not only to be a few in number, but also, to satisfy heterogeneity or global interdependency constraints; in particular, matroid constraints. However, in failure-prone and adversarial environments, sensors get attacked; actuators fail; data get deleted. Thence, traditional matroid-constrained design paradigms become insufficient and, in contrast, resilient matroid-constrained designs against attacks, failures, or deletions become important. In general, resilient matroid-constrained design problems are computationally hard. Also, even though they often involve objective functions that are monotone and (possibly) submodular, no scalable approximation algorithms are known for their solution. In this paper, we provide the first algorithm, that achieves the following characteristics: system-wide resiliency, i.e., the algorithm is valid for any number of denial-of-service attacks, deletions, or failures; minimal running time, i.e., the algorithm terminates with the same running time as state-of-the-art algorithms for (non-resilient) matroid-constrained optimization; and provable approximation performance, i.e., the algorithm guarantees for monotone objective functions a solution close to the optimal. We quantify the algorithm's approximation performance using a notion of curvature for monotone (not necessarily submodular) set functions. Finally, we support our theoretical analyses with numerical experiments, by considering a control-aware sensor selection scenario, namely, sensing-constrained robot navigation.
3. Sensing-constrained LQG control: Linear-Quadratic-Gaussian (LQG) control is concerned with the design of an optimal controller and estimator for linear Gaussian systems with imperfect state information. Standard LQG assumes the set of sensor measurements, to be fed to the estimator, to be given. However, in many problems, arising in networked systems and robotics, one may not be able to use all the available sensors, due to power or payload constraints, or may be interested in using the smallest subset of sensors that guarantees the attainment of a desired control goal. In this paper, we introduce the sensing-constrained LQG control problem, in which one has to jointly design sensing, estimation, and control, under given constraints on the resources spent for sensing. We focus on the realistic case in which the sensing strategy has to be selected among a finite set of possible sensing modalities. While the computation of the optimal sensing strategy is intractable, we present the first scalable algorithm that computes a nearoptimal sensing strategy with provable suboptimality guarantees. To this end, we show that a separation principle

holds, which allows the design of sensing, estimation, and control policies in isolation. We conclude the paper by discussing two applications of sensing-constrained LQG control, namely, sensing-constrained formation control and resource-constrained robot navigation.

4. Hardness of minimal reachability problems: we considered the problem of choosing which nodes of a linear dynamical system should be actuated so that the state transfer from the system's initial condition to a given final state is possible. Assuming a standard complexity hypothesis, we show that this problem cannot be efficiently solved or approximated in polynomial, or even quasi-polynomial, time.

1.7 Exploiting Graph Topology for Distributed Optimization

In this line of research over the past few years, we have demonstrated that by exploiting the geometry and topology of underlying networks, decentralized optimization is capable of addressing our growing computational demands by exploiting the power of coordinated data processing. In particular, we propose an exact distributed Newton method for two important challenges in large-scale optimization: Network Flow and Empirical Risk Minimization. The key observation behind our method is related to the symmetric diagonal dominant structure of the Hessian of dual functions correspondent to the aforementioned problems. Consequently, one can calculate the Newton direction by solving symmetric diagonal dominant (SDD) systems in a decentralized fashion.

We first proposed a fully distributed SDD solver based on a recursive approximation of SDD matrix inverses with a collection of specifically structured distributed matrices. To improve the precision of the algorithm, we then apply Richardson Preconditioners arriving at an efficient algorithm capable of approximating the solution of SDD system with any arbitrary precision. Our second fully distributed SDD solver significantly improves the computational performance of the first algorithm by utilizing Chebyshev polynomials for an approximation of the SDD matrix inverse. The particular choice of Chebyshev polynomials is motivated by their extremal properties and their recursive relation. We then explore mixed strategies for solving SDD systems by slightly relaxing the decentralization requirements. Roughly speaking, by allowing for one computer to aggregate some particular information from all others, one can gain quite surprising computational benefits. The key idea is to construct a spectral sparsifier of the underlying graph of computers by using local communication between them. Finally, we apply these solvers for calculating the Newton direction for the dual function of Network Flow and Empirical Risk Minimization. On the theoretical side, we establish quadratic convergence rate for our algorithms surpassing all existing techniques. On the empirical side, we verify our superior performance in a set of extensive numerical simulations.

1.8 Sparsification and sensor/actuator selection

For the problem of actuator selection for linear dynamical systems, we have developed a framework to design a sparse actuator/sensor schedule for a given large scale linear system with guaranteed performance bounds using deterministic polynomial-time and randomized approximately linear-time algorithms. We first introduced systemic controllability metrics for linear dynamical systems that are monotone, convex, and homogeneous with respect to the controllability Gramian. We show that several popular and widely used optimization criteria in the literature belong to this class of controllability metrics. Our main result is to provide a polynomial-time actuator schedule that on average selects only a constant number of actuators at each time step, independent of the dimension, to furnish a guaranteed approximation of the controllability/observability metrics in comparison to when all actuators/sensors are in use. We illustrate the effectiveness of our theoretical findings via several numerical simulations using benchmark examples.

2 Students/postdocs supported

This grant has been used to partially support the following students (UCSD):

- Josh Tobin
- Sinan Aksoy
- Olivia Simpson
- Jay Cummings
- Franklin Kenter
- Jake Hughes
- Mark Kempton

At Penn, the grant has supported the following students:

- Rasul Tutunov (Now Research at Prowler, UK)
- Amin Rahimian (Now Postdoc at MIT)
- Vassilis Tzoumas (Finished PhD and will start postdoc at MIT in July)
- Shaudi Mahdavi Hosseini

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- Milad Siami (currently postdoc at MIT)
- Amir Ajorlou

This grant has been used to partially support the following postdocs (UCSD):

- Sam Peng
- Stephen Young
- Ekaterina Merkurjev
- Kamalika Chaudhuri

3 Honors

Here is a list of notable honors received during this period:

For Fan Chung Graham:

- Fellow of American Mathematics Society 2013.
- Fellow of SIAM 2015.
- Academician, Academic Sinica, 2016.
- Honorary Degree, University of Waterloo 2017.

For Ali Jadbabaie

- Vannevar Bush fellowship from office of Secretary of Defense, 2016
- IEEE Fellow,
- JR East Professorship at MIT
- Finalist, Best Paper Award, IEEE Conference on Decision and Control, 2016

4 List of Papers

R. Hod, A. Huang, M. Kempton and S.-T. Yau. Strong embeddings and 2-isomorphism, *Notices of the International Congress of Chinese Mathematicians*, 4 (2016) no. 2, 5–13.

M. Kempton, G. Lippner, S.-T. Yau. Perfect state transfer on graphs with a potential *Quantum Information and Computing*, 17 (2017) no. 3&4, 303–327.

M. Kempton, G. Lippner, S.-T. Yau. Pretty good quantum state transfer in symmetric spin networks via magnetic field *Quantum Information Processing*, 16 (2017), no. 210, 1-23.

M. Kempton, F. Münch, S.-T. Yau. Relationships between cycles spaces, gain graphs, coverings, path homology, and graph curvature. Submitted, pre-print: <https://arxiv.org/abs/1710.01264>.

C. Godsil, K. Guo, M. Kempton, G. Lippner. Perfect state transfer on strongly regular graphs with an edge perturbation. In preparation, pre-print: <https://arxiv.org/abs/1710.02181>.

R. Tutunov, H. Bou Ammar, A. Jadbabaie, “A Distributed Newton Method for Large Scale Consensus Optimization,” *IEEE Transactions on Automatic Control*, conditionally accepted

R. Tutunov, H. Bou Ammar, A. Jadbabaie, “Fast, accurate distributed Newton methods for network flow optimization,” Minor Revision, *SIAM Journal of Optimization*

V. Tzoumas, G. J. Pappas, **A. Jadbabaie**, “Minimal Actuator Placement with Bounds on Control Effort,” *IEEE Transactions on Control of Network Systems*, March 2016, pp. 66-78.

A. Jadbabaie, A. Olshevsky, M. Siami, “Deterministic and Randomized Actuator Scheduling With Guaranteed Performance Bounds,” submitted to *IEEE Transactions on Automatic Control*, 2018

V. Tzoumas, L. Carlone, GJ Pappas, A Jadbabaie, “Control and sensing code-sign,” submitted to *IEEE Transactions on Automatic Control*, submitted.

A Jadbabaie, A Olshevsky, GJ Pappas, V Tzoumas, “minimal reachability is hard to approximate,” *IEEE Transactions on Automatic Control*, accepted for publication, 2018

V. Tzoumas, L. Carlone, GJ Pappas, A Jadbabaie, “sensing-constrained LQG control,” IEEE Conference on Decision and Control, 2018, submitted.

V Tzoumas, NA Atanasov, A Jadbabaie, GJ Pappas, “Scheduling nonlinear sensors for stochastic process estimation,” *American Control Conference*, 2017

V Tzoumas, K Gatsis, A Jadbabaie, GJ Pappas, “Resilient monotone submodular function maximization,” *IEEE Conference on Decision and Control*, 2017, **student best paper award finalist**.

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Geometry and Topology of Complex Networks

Grant/Contract Number

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FA9550-13-1-0097

Principal Investigator Name

The full name of the principal investigator on the grant or contract.

Ali Jadbabaie

Program Officer

The AFOSR Program Officer currently assigned to the award

James Lawton

Reporting Period Start Date

03/01/2013

Reporting Period End Date

02/28/2018

Abstract

The current vision of many military leaders and policy makers about network-centric technology portrays a future in which networked systems provide unprecedented levels of performance, robustness, and efficiency. Unfortunately, there are fundamental, basic obstacles to realizing this vision. Some of these obstacles stems from a lack of a fundamental theory and our lack of comprehensive understanding of the mathematics of interconnected technological, information, and social networks.

To address these challenges, we embarked on a multi-investigator, multidisciplinary project on mathematics of networks and networked systems, that addresses fundamental issues from topology to geometry of networks. This multi-year, multidisciplinary effort, led to training of a large group of students and postdocs, involved in academic research activity upon completion of their studies. Aside from many theses and research papers, the research has led to fundamental understanding along multiple domains. The highlights include the following developments:

- 1) Spectral methods in networks:
- a) A theory of random walks on directed networks

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b) A theory of random walks on simplicial complexes

2) Discrete Dirichlet problems, Heat kernel PageRank, and PageRank on simplicial Complexes

3) Developed concept of quantum state transfer on graphs

4) Developed new notions of curvature on graphs

5) New algorithms for distributed optimization using spectral graph theory

6) new algorithms for sensor and actuator placement using spectral methods and sparsification theory.

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M. Kempton, G. Lippner, S.-T. Yau. Perfect state transfer on graphs with a potential {it Quantum Information and Computing}, 17 (2017) no. 3&4, 303--327.

M. Kempton, G. Lippner, S.-T. Yau. Pretty good quantum state transfer in symmetric spin networks via magnetic field {it Quantum Information Processing}, 16 (2017), no. 210, 1-23.

M. Kempton, F. Munch, S.-T. Yau. Relationships between cycles spaces, gain graphs, coverings, path homology, and graph curvature. Submitted, pre-print: [verb+https://arxiv.org/abs/1710.01264](https://arxiv.org/abs/1710.01264)+

C. Godsil, K. Guo, M. Kempton, G. Lippner. Perfect state transfer on strongly regular graphs with an edge perturbation. In preparation, pre-print: [verb+https://arxiv.org/abs/1710.02181](https://arxiv.org/abs/1710.02181)+

A non-backtracking P'olya's theorem. {it Journal of Combinatorics}, 9 (2018), no. 2, 327-343.

R. Tutunov, H. Bou Ammar, { A. Jadbabaie}, ``A Distributed Newton Method for Large Scale Consensus Optimization," {it IEEE Transactions on Automatic Control,} conditionally accepted

R. Tutunov, H. Bou Ammar, {A. Jadbabaie}, ``Fast, accurate distributed Newton methods for network flow optimization," Minor Revision, {it SIAM Journal of Optimization}

V. Tzoumas, G. J. Pappas, {bf A. Jadbabaie}, ``Minimal Actuator Placement with Bounds on Control Effort," {it IEEE Transactions on Control of Network Systems}, March 2016, pp. 66-78.

A. Jadbabaie, A. Olshevsky, M. Siami, `` Deterministic and Randomized Actuator Scheduling With Guaranteed Performance Bounds," submitted to {it IEEE Transactions on Automatic Control,} 2018

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V. Tzoumas, L. Carlone, GJ Pappas, A Jadbabaie, ``Control and sensing codesign," submitted to {it IEEE Transactions on Automatic Control,} submitted.

A Jadbabaie, A Olshevsky, GJ Pappas, V Tzoumas, ``minimal reachability is hard to approximate," {it t IEEE Transactions on Automatic Control,} accepted for publication, 2018

V. Tzoumas, L. Carlone, GJ Pappas, A Jadbabaie, ``sensing-constrained LQG control," IEEE Conference on Decision and Control, 2018, submitted.

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V Tzoumas, K Gatsis, A Jadbabaie, GJ Pappas, ``Resilient monotone submodular function maximization," {it IEEE Conference on Decision and Control,} Finalist, student best paper award

R Tutunov, H. BoumAmmar, E Eaton Safe policy search for lifelong reinforcement learning with sublinear regret, ICML 2015

R Tutunov, H. BoumAmmar, A. jadbabaie , An exact distributed newton method for reinforcement learning, IEEE Conference on Decision and Control, 2016

New discoveries, inventions, or patent disclosures:

Do you have any discoveries, inventions, or patent disclosures to report for this period?

No

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Changes in research objectives (if any):

Added a few new thrusts as outlined in abstract

Change in AFOSR Program Officer, if any:

None

Extensions granted or milestones slipped, if any:

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Research Objectives

Technical Summary

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