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14. ABSTRACT A rail plasma actuator (RailPac) is a plasma based device with a high control authority. A directional flow control is initiated with a motion of a high-speed arc (~100 m/s) that propagates due to the Lorentz force. The report summarizes the important results of a three year investigation into the relevance of a RailPac for stall alleviation on a retreating blade. The experiments include arc dynamics measurement, control authority assessment, induced velocity measurement, and stall alleviation demonstration. The results show that a RailPac is ideally suited for a retreating blade application and has the sufficient control authority to influence the flow at full scale retreating conditions.					
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ABSTRACT

A rail plasma actuator (RailPac) is a plasma based device with a high control authority. A directional flow control is initiated with a motion of a high-speed arc (~100 m/s) that propagates due to the Lorentz force. The report summarizes the important results of a three year investigation into the relevance of a RailPac for stall alleviation on a retreating blade. The experiments include arc dynamics measurement, control authority assessment, induced velocity measurement, and stall alleviation demonstration. The results show that a RailPac is ideally suited for a rotorcraft application and has the sufficient control authority to influence the flow at full-scale rotorcraft conditions. The RailPac characteristics discovered in this study warrants its implementation on a full-scale rotorcraft for further study.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

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RailPac: A Rail Plasma Actuator for High Authority Aerodynamic Flow Control

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Department of Aerospace Engineering and Engineering Mechanics
The University of Texas at Austin, Austin, Texas, 78712

I. Publications

a. Peer-reviewed journals

- i. Choi Y-J., Gray M., Sirohi J., and Raja L. L., "Effects of Anchoring and Arc Structure on the Control Authority of a Rail Plasma Actuator," *J. Phys. D: Appl. Phys.*, Accepted for publication on July 7th, 2017.

b. Non-peer-reviewed journals

- i. Choi Y-J., Gray M., Sirohi J., and Raja L. L., "Measurement of velocity induced by a propagating arc magneto-hydrodynamic plasma actuator," *55th AIAA Aerospace Sciences Meeting* (Grapevine, TX), 2017, AIAA 2017-0156.
- ii. Gray M., Choi Y-J., Sirohi J., and Raja L. L., "Experimental investigation of the internal structure and dynamic behavior of the rail plasma actuator arc," *55th AIAA Aerospace Sciences Meeting* (Grapevine, TX), 2017, AIAA 2017-0158.

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- i. Choi Y-J., Gray M., Sirohi J., and Raja L. L., "Measurement of velocity induced by a propagating arc magneto-hydrodynamic plasma actuator," *55th AIAA Aerospace Sciences Meeting* (Grapevine, TX), 2017, AIAA 2017-0156.
- ii. Gray M., Choi Y-J., Sirohi J., and Raja L. L., "Experimental investigation of the internal structure and dynamic behavior of the rail plasma actuator arc," *55th AIAA Aerospace Sciences Meeting* (Grapevine, TX), 2017, AIAA 2017-0158.

d. Honors and awards

- i. American Helicopter Society Forum 73 Chair Award (Jayant Sirohi)
- ii. Bruton Fellowship (Young-Joon Choi)

II. Personnel

a. Graduate students (2)

- i. Young-Joon Choi
- ii. Miles Gray

b. Faculty (2)

- i. Laxminarayan L. Raja
- ii. Jayant Sirohi

c. Masters degrees awarded (1)

- i. M. S. in Aerospace Engineering (Young-Joon Choi)

III. Scientific Progress and Accomplishments

a. Induced velocity measurements

Successful measurement of the induced velocity around a propagating arc was accomplished with conventional and Schlieren-based particle image velocimetry (PIV). Although the full flow field was difficult to obtain due to the high arc temperature that vaporized the seed

particles, a novel application of Schlieren velocimetry resolved this issue by relying on natural turbulence in the flow to determine its velocity. Measurements show that a RailPac can energize the boundary layer through compression and entrainment, and generate large-scale circulation that is favorable for preventing separation of the boundary layer.

b. Stall-alleviation demonstration

A prototype RailPac was matured into a flow actuator on a VR-12 wing. The modified wing was tested in a wind tunnel at low Reynolds numbers ($\sim 45,000$ and $\sim 90,000$). The wing was mounted on force balances to measure lift, and the actuator was characterized with electrical measurements and high-speed imaging. Particle image velocimetry (PIV) was used to measure the flow over the passive and actuated airfoil. The PIV measurements illustrate the flow reattachment process of a RailPac and demonstrate the actuator's efficacy over a wide range of angles of attack. In addition, a single RailPac operation results in $\sim 40\%$ improvement in post-stall lift and $\sim 4^\circ$ increase in stall angle compared to the passive airfoil with a RailPac. Compared to a clean airfoil, a single RailPac operation produces $\sim 20\%$ improvement in post-stall lift and $\sim 2^\circ$ increase in stall angle.

RailPac: A Rail Plasma Actuator for High Authority Aerodynamic Flow Control

Laxminarayan L. Raja and Jayant Sirohi

July 22, 2017

Executive Summary

The future of rotorcraft is a fleet of faster, quieter, and more efficient army helicopters capable of delivering payloads at a range beyond the limits of today's technology. These agile vehicles can counter the ever growing threats from both conventional and unconventional foes. The development of a next generation of helicopters remains a great challenge due to the fundamental issues posed by the rotorcraft's unique operating environment. One such example is the retreating blade stall on a rotor blade. At high flight speed, a helicopter blade encounters time-varying change in its angle-of-attack and experiences dynamic stall on the retreating side. The subsequent increase in torsional loads, as well as a decrease in rotor propulsive efficiency due to the loss of lift generating ability, define the upper limit of lift, thrust, and aero-elastic performance of a rotorcraft.

The army and its partnered laboratories have invested in numerous devices to address the retreating blade stall. In particular, active actuators have been favorably demonstrated for stall alleviation. These devices impart momentum to the flow at arbitrary instances and generate localized flow disturbances that cascade into large-scale flow structures favorable for stall suppression. The performance of an active actuator is defined by its control authority (momentum imparted to the flow) and bandwidth (maximum frequency of repetitive actuation). These expressions gauge how much influence an actuator has on the flow field. The high Reynolds and Mach numbers of a full-scale rotorcraft require actuators with high control authority to be utilized. In addition, the fast rotation speed of rotor blades means that the actuators with high bandwidth are required.

The development of a high control authority actuator with a wide bandwidth has been achieved with a rail plasma actuator (RAilPac). This device addresses the bandwidth limitation by relying on an electrical discharge to transfer momentum to the flow. The actuator can be mounted conformal to the blade surface and is volumetrically compact, thereby exerting minimal effect on the blade's passive performance and structural integrity. In addition, the lack of moving parts permits its reliable operation in the dusty and highly vibratory environment of a combat helicopter.

A RailPac consists of parallel electrodes with an arc situated in the gap. A large current flows through the electrodes and generates a self-induced magnetic field. The arc is accelerated by the Lorentz force that arises as a result of coupling between the current through the

arc and the magnetic field. This principle is quite similar to that of a railgun that accelerates the armature with the same forcing. The Lorentz force can be derived from first principles such that there are no theoretical limits to its value. As such, there are no known upper bound of the RailPac actuation authority.

The objective of this study is to demonstrate the relevance of a RailPac for suppression of dynamic stall on helicopters. A prototype RailPac is built with two different electrode geometries, flush-mounted rails and free-floating rods, to examine the effect of electrode design on its flow control abilities. Further study is undertaken with the prototypes consisting of electrodes with varying separation distances. Various experiments are conducted on the prototypes, and include arc dynamics measurements, actuation authority measurements, transit velocity modeling, and induced velocity measurements. Each topic provide a deeper understanding of the actuation mechanism of a RailPac. Finally, a prototype RailPac is matured into a working actuator on a rotor blade and stall suppression is demonstrated with a wind tunnel experiment.

The momentum transfer in a RailPac is initiated through the arc and its motion. Hence, identifying the arc structures relevant to flow control and discovering methods to improve the arcs transit velocity are crucial to developing an actuator with high control authority and efficiency. Experiments were performed with high-speed imaging and electrical measurements to fully understand the arc dynamics and structure. The earlier studies on a RailPac attributed limitations of its control authority on the instabilities in the arc motion. These studies captured the asymmetric motion of the arc roots, with the anode root anchoring to a particular position, while the cathode root transits smoothly along the electrodes. This investigation first focused on minimizing the "anchoring" phenomenon by adjusting the electrode shape (rails vs. rods) and separation distance. It was observed that anchoring is not present if the rod electrodes are free-floating. For the rail configuration, narrowing the gap width to 2 mm permits smooth transit of the arc. The high-speed images of the arc motion revealed that the lack of anchoring can be attributed to the direct interaction between the field-enhancement regions near the anode and cathode roots. This can be achieved by removing the insulating gap between the electrodes (as in the case of rod configuration) or reducing the separation distance between the electrodes. The transit images also showed that the arc in a rail configuration is brighter compared to rod electrodes and exhibit substantial deflection of the arc away from the surface. This arc structure suggests that a large volume of air is affected by the arc transit.

The actuation authority of a RailPac was quantified with transient force and impulse measurements. The transient force was measured with a force measurement stand and using deconvolution procedure to reconstruct the force from the test stand response. Impulse measurement was conducted on a low-natural frequency test stand that allowed the impulse to be measured directly from the maximum displacement of the test stand. The results clearly demonstrated the large control authority of a RailPac. This is seen from the large and scalable body force delivering a peak force and impulse of 0.6 N and 0.45 mN-s, respectively. The force scales quadratically with the arc current and the impulse scales linearly with the discharge energy. These scaling laws reveal that a RailPac can be operated in a wide range of flight envelope, and its actuation authority can be tailored towards a specific flow condition simply by changing its electrical parameters. Methods to further increase the control authority were determined by changing the electrode shape and separation dis-

tance. The control authority was assessed in terms of the electrical input by calculating the inductance gradient. This scalar quantifies the impulse delivered per electrical input (i. e. the action integral as defined by integral over the square of the arc current), such that the electromechanical effect of the actuator can be analyzed. Narrowly spaced electrodes suppresses anchoring and the inductance gradient (as well as the impulse delivered) increases by $\sim 21\%$ compared to the anchored case. In addition, the impulse per unit width increases with decreasing gap width, such that the actuator's impact on the flow is increased, even though the actuator's area is decreased. The narrow gap increases the transit velocity from ~ 40 m/s to ~ 100 m/s. The measurements clearly show that an effective flow control in a full-scale rotorcraft is possible due to the high control authority of a RailPac. Furthermore, this study provide much needed guidance on the RailPac design.

The effect of RailPac on a quiescent air was studied with particle image velocimetry (PIV) to identify the flow structures favorable for separation control. This study used a RailPac with rod electrodes and projected a laser sheet at the center of the gap. The flow induced by the arc motion was captured with a 2-D PIV.. The measurements revealed that the front of the arc pushes air through compression. A high temperature wake region is formed behind the arc and the air is entrained into this region. Unfortunately, conventional PIV alone was inadequate for a RailPac study. The high arc temperature vaporized the seed particles, such that the flow field behind the arc could not be captured fully. An alternate method based on Schlieren velocimetry was applied in this region. This method relies on the motion of density gradients in the flow to measure the flow velocity. The results revealed that the thermal effect of an arc is confined to a small region due to the arc's propagation velocity. In addition, a high-velocity (~ 20 m/s) flow is generated within the arc wake. The initial push provides mechanism to remove low-momentum air away from the separated boundary layer. The entrainment effect can replace the convected air with the free-stream flow, thereby energizing the boundary layer. These phenomena show that the flow field generated by a moving arc is highly effective for separation control.

A prototype RailPac was redeveloped into a flow actuator on a VR-12 wing. The modified wing was tested in a wind tunnel at low Reynolds numbers ($Re \sim 45,000$ and $90,000$). The airfoil was mounted on force balances to measure lift, and the actuator was characterized with electrical measurements and high-speed imaging. Particle image velocimetry (PIV) was used to measure the flow over the passive and actuated airfoil. The study shows that the RailPac improves the aerodynamic performance of a wing. The PIV measurements illustrate the flow reattachment process of a RailPac and demonstrate the actuator's efficacy over a wide range of angles of attack. In addition, a single RailPac operation results in $\sim 40\%$ improvement in post-stall lift and $\sim 4^\circ$ increase in stall angle compared to the passive airfoil with a RailPac. Compared to a clean airfoil, a single RailPac operation results in $\sim 20\%$ improvement in post-stall lift and $\sim 2^\circ$ increase in stall angle. These results affirm that stall can be effectively manage on a rotor blade by a RailPac.

In conclusion, a RailPac is a highly capable actuator with large control authority and warrants continued study. In particular, a demonstration of a RailPac on a blade mounted in a rotorcraft test stand is recommended to study the actuator in the context of the true flow conditions encountered in a full-scale helicopters. The effect of actuation frequency should also be investigated to fully leverage the natural instabilities of the flow for flow control. The design of a RailPac can be further enhanced to multiply the actuation authority

compared to the results obtained in this study. A design based on an augmented railgun can further enhance the Lorentz force generated by a RailPac. In the present analysis, the RailPac represents a single turn inductor, whereby the arc current flows from one rail, jumps across the gap through the arc, and returns to the power supply via the opposite rail. By introducing additional loops of wire beneath the rails or rods, the inductance gradient (thereby the control authority) of a RailPac geometry can be increased.

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1 Introduction

The army envisions new rotorcraft vehicles with increased range, speed, payload, reliability, efficiency, and stealth capabilities. However, the current state-of-the-art faces limits on the maximum flight speed due to dynamic stall, and innovative solutions are required to realize the future concepts. It is well known that dynamic stall occurs at high flight speed, as the advancing and retreating blades experience sudden change in the incident velocity, control inputs, blade deformation, and dynamic inflow [1]. These phenomena contribute to a time-varying change in the angle-of-attack that ultimately leads to flow separation and undesirable effect on the aerodynamic loads.

Efforts by several authors illuminated the flow phenomenon responsible for dynamic stall. As the blade undergoes a sudden change in its angle-of-attack beyond the static stall angle, the flow separation is delayed and a large lift is generated above the static limit. However, the flow attachment is only temporary, and the first sign of dynamic stall is distinguished by a generation of a leading edge vortex. The vortex grows in strength and begins to convect over the blade surface due to the free-stream velocity. The motion of this vortex causes a rapid change in pressure distribution on the blade surface, and results in large pitching moment. Although the initial increase in lift is desirable, the subsequent torsional loads result in a sudden increase in pitch link loads, which appears as vibratory loads, as well as a decrease in rotor propulsive efficiency due to the loss of lift generating ability. Indeed, the problem of dynamic stall defines the upper limit of lift, propulsive, and aero-elastic performance of a rotorcraft.

A device that can alleviate retreating blade stall is a crucial enabling technology for the next generation of helicopters. Several efforts have been made to control the vortex structure and suppress its formation or convection, in order to minimize the change in the moment loads exerted on the blade. One such example is the use of passive devices, such as leading edge slats [2] and vortex generators [3]. The advantage of these concepts is that they are easy to maintain and requires no external input to operate. However, passive devices produce increased drag on the blade when flow separation is not present, such that their design must be tailored to a specific flight condition. Therefore, the passive concepts alone do not fully address the army's need for a versatile rotorcraft vehicle with wide range of performance envelopes.

1.1 Active Actuators

Recent advances in unsteady aerodynamics yielded new methods to control the large-scale instabilities in the flow with a localized disturbance. Studies by Greenblatt and Wygnanski [4] revealed that the frequency of actuation is just as important as the magnitude of the momentum injected into the flow. Indeed, the effectiveness of an actuator hinges on its ability to excite the flow in the time scale of natural instabilities. This discovery led to the renewed interest in actuators that can be turned on or off at any arbitrary time scales. Active actuators differ from their passive counter-parts because the devices inject momentum to control flow structures. In addition, the devices can be actuated at arbitrary instances, such that their effect on the passive performance of an aerodynamic body can be minimized.

Numerous devices have been proposed with a wide array of mechanisms to inject mo-

momentum into the flow, as reviewed by Cattafesta [5]. The ability of these devices to influence the flow-field depends largely on the maximum flux of momentum as well as the frequency of injection. As such, Greenblatt and Wygnanski [4] proposed the following non-dimensional coefficients that affect the actuators influence on the flow. To gauge the actuation authority of different designs, they defined the momentum coefficient (C_μ) as the ratio of the momentum generated by the actuator (J) to the free stream momentum ($q_\infty L$).

$$C_\mu = \frac{J}{q_\infty L} \quad (1)$$

Note that q_∞ is the dynamic pressure of the free-stream and L is the characteristic length. The reduced frequency (F^+) also has a large impact on the flow,

$$F^+ = \frac{TL}{U_\infty} \quad (2)$$

This timescale parameter is the ratio of actuation time (T) to the convection time defined by the free-stream velocity (U_∞). The goal of any active actuator is to maximize the momentum coefficient, while maintaining a wide bandwidth (wide range of operational F^+) to control the flow at high Reynolds and Mach number regimes.

The discovery of a robust, volumetrically compact, high-bandwidth, and high control authority actuator is still a subject of active research. These devices include fluidic, mechanical, and thermal actuators with different mechanisms to achieve large momentum coupling to the flow. The classic examples of fluidic actuators include boundary suction [6] and blowing [7], as demonstrated on F-16XL [8]. Pulsed jet [9] is an additional example capable of higher jet velocity and faster actuation time than the steady systems. Even faster response times ($\sim$ kHz) have been demonstrated on synthetic jet devices [10]. These actuators are zero net mass flux devices and rely on the piezoelectric or electrodynamic motion of a diaphragm within the cavity to drive a periodic jet.

Mechanical actuators encompass actuated flap that actively modify the wing geometry for flow control application. Piezoelectric flaps have been favorably demonstrated for separation control [11]. The device consists of piezo-actuators that cause oscillatory bending of the shims to generate vortical structures in the flow for boundary layer control. Active flaps have been studied in the context of swashplateless rotor system [12] with the experimental demonstration performed by Sikorsky [13]. The results showed that a significant noise and vibration reduction can be achieved with active flaps. A modified version of this device that utilize piezo-stack actuators for flap control was proposed to increase the actuation bandwidth (albeit with lower actuation amplitude) [14].

Thermal (combustion based) actuators have been proposed for flow control at high Reynolds and Mach number environments. The devices utilize a periodic combustion of fuel-air mixture to impinge a high-speed jet into the flow [15, 16]. A more recent study by Matalanis et. al. [17] demonstrated a combustion powered actuator for dynamic stall suppression on a rotorcraft blade. The results revealed an average of 11% improvement in the post-stall lift coefficient at Mach numbers of ~ 0.4 .

Although numerous devices have been favorably demonstrated in academic studies, the implementation of an actuator for full-scale rotorcraft is challenging. The active actuators require an external source (electrical, mechanical, or hydrodynamic power) that typically

adds complexity to an existing system. The high-frequency motion found in many of these devices reduces the system reliability and increases the maintenance requirements. The devices are also intrusive and require modifications to the internal and surface configuration of an aerodynamic body that impose adverse effect on the structural integrity and the passive performance. The most significant drawback is the bandwidth limitation. The operation of fluidic and thermal actuators are limited to <1 kHz due to the time-scale associated with the fluid or combustion cycle. The use of moving parts in mechanical actuators, and trade-off between the frequency of actuation and the actuator stroke means that the effective frequency of operation is limited to ~ 100 Hz.

1.2 Plasma Actuator

The plasma actuators have been developed to address the limitations associated with the mechanical, fluidic, and thermal actuators. These devices tackle the complexity and bandwidth limitations by replacing mechanically driven parts with an electrical discharge. The plasma actuators transfer momentum to the flow through a direct electrical discharge into a partially ionized air. They are very attractive for stall alleviation because they can be mounted conformal to the airfoil profile, exerting minimal effect on its passive performance. As such, plasma actuator is fail-safe since the wing will simply revert to a purely passive condition in the event of an actuator failure. The lack of moving parts allows bandwidth on the order of several kHz without penalty on the reliability of the existing system. In addition, plasma actuators are volumetrically compact, and its required power can be supplied from the rotorcraft's electrical system.

Plasma actuators differ by the types of discharge and forcing generated by the actuator. The dielectric barrier discharge (DBD) actuators are the most common device in study, and several authors [18–22] have achieved flow separation control over an airfoil with a DBD actuator. These reports document improvement in aerodynamic efficiency over a wide range of Reynolds numbers (Re) and angles of attack (α). Other studies have examined the efficiency of DBD actuators for vortex control, boundary layer manipulation, and transition delay as reviewed by Moreau [23], Caruana [24], and Cattafesta [5]. Despite the promising results, the application of DBD actuators for full-scale flow control remains elusive due to their limited control authority. Studies have shown that the efficiency of DBD for external flow control is diminished considerably for flow velocities in excess of ~ 10 m/s as a result of low momentum delivered by the actuator and the strong pressure gradient imposed by the flow field [25]. The low induced velocity is attributed to the viscous effects from the wall that counteract the thrust produced by a DBD actuator [25–27]. The charge build-up on the dielectric surface changes the discharge characteristics and results in a "memory effect", whereby the propagation velocity of subsequent discharge is diminished by the charge deposited from the past discharges. As a consequence of these physical limitations, experimental demonstrations of DBD actuators have been limited to Reynolds number and Mach number below 1×10^6 and 0.1, respectively.

The deficiencies of DBD actuators resulted in the development of arc filament actuators for flow control at higher Reynolds and Mach numbers. These devices enhance the actuation authority with a higher discharge current in a small cavity built into the aerodynamic surface. The arc discharge leads to a rapid energy deposition into a localized region of the flow. The

sudden heating of a nearby fluid generates strong pressure disturbances that can be utilized for high-speed flow control. A spark jet actuator is an example with a high actuation ability [28]. Shang and Surzhikov [29] provide an overview of the hypersonic flow control achieved using this type of plasma actuator. Further study by Utkin et. al. [30] demonstrated ability of the actuator to control a Mach 1.3 jet with high actuation rate (up to 200 kHz). Despite the promising effects demonstrated by the arc filament actuators, its need for a sub-surface cavity can compromise the structural integrity of a rotorcraft blade. Also, the actuation area is limited due to its small orifice. It can be concluded that the plasma actuators have the sufficient bandwidth to influence the flow at fast time scales. However, the concepts based on EHD forcing and arc discharge lack either the control authority or demand impractical modifications unsuited for full-scale application. Clearly, a novel approach is required to implement the plasma actuators for practical applications.

1.3 Magnetohydrodynamic Plasma Actuator

A magnetohydrodynamic (MHD) actuator is one such concept that demonstrated ability to control high-speed flow with minimal modification to the existing aerodynamic body. These devices combine the thermal effect of an arc filament with the mechanical effect from its motion to increase the actuation authority and area. The device typically consists of an external magnetic field generated by a permanent magnet or a solenoid with a current discharged at a localized region in the flow. The Lorentz force directly acts on the flow through this discharge region and allows a large body force to be generated without any theoretical limitation on its maximum value. Hence, higher Reynolds and Mach number flows can be controlled in theory compared to the EHD actuators.

The use of MHD actuators for flow control was experimentally demonstrated by Weier et. al. [31]. This study immersed a hydrofoil in an electrolyte tunnel, with the electrodes and permanent magnet embedded in the leading edge. The results revealed that flow separation can be suppressed with periodic Lorentz forcing in the Reynolds number ranges of 10^4 to 10^5 . A more practical application of a MHD actuator at higher flow speed was demonstrated by Zaidi et. al. [32]. Their device generated a low current arc in low pressure air and utilized a permanent magnet to impart a Lorentz force on the arc. The arc motion transferred directional momentum to the surrounding air through the snowplow effect, thereby demonstrating that the mechanical effect of a moving arc can theoretically be tailored towards directional flow control. A further study by Balcon et. al. [33] demonstrated the same concept in atmospheric air. This study modified a glidarc setup with an external magnetic field placed between the electrodes. Their measurements captured the induced velocity of a moving arc and identified that the induced flow is caused by the pressure disturbance of a moving arc as well as the thermal convection exerted by Joule heating. Although these studies have provided much needed insight behind the MHD actuators, the requirement of a strong external magnetic field, limited Lorentz force due to small arc current, and the scalability issues associated with a static magnetic field have prevented their full-scale implementation.

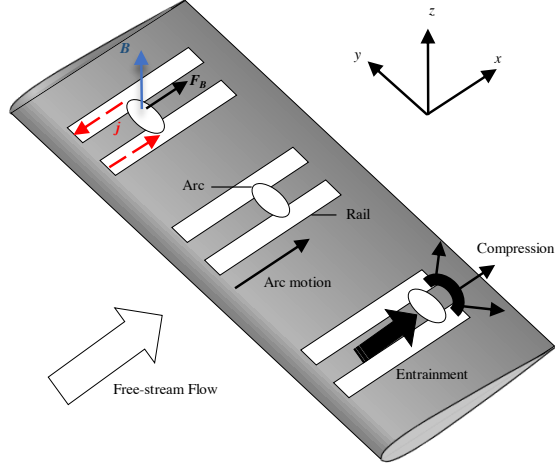


Figure 1: A schematic of a RailPac mounted on an aerodynamic surface.

1.4 Rail Plasma Actuator

A more recent study by a group at the University of Texas at Austin addressed the practical challenges associated with the past MHD concepts by basing their actuator design on a railgun. The Rail Plasma Actuator (RAilPac) shown in figure 1 differs from the past MHD concepts in that it relies on a higher current ($\sim \text{kA}$ as opposed to $\sim \text{A}$) and a self-induced magnetic field to increase the arc velocity to 100 m/s [34]. A RailPac consists of parallel rails flush mounted to the aerodynamic surface and a thermal arc that bridges the gap between the rails. The current through the rails generates a magnetic field ($\vec{B}$) that couples with the arc current density ($\vec{j}$) to impart a Lorentz force ($\vec{F}_B$). The arc accelerates along the rails and its motion imparts momentum to the surrounding air through compression and entrainment effects.

The replacement of a static magnetic field with an induced one in particular provides greater scalability to the actuator. With a simple increase in the arc current, the induced magnetic field and the current density can be augmented, thereby magnifying the Lorentz force exerted on the flow. In addition, Lorentz force can be derived from first principles and does not experience any of the self-limiting effects observed in EHD actuators. The attributes of plasma actuators attractive for active flow control are preserved by this concept. These benefits include lack of moving parts, conformal mounting to an aerodynamic surface, increased actuation area from the arc motion, compact volume, and low mass. Thus, RailPac is ideally suited for dynamic stall alleviation on a rotorcraft blade.

2 Problem Statement and Overview

The objective of this study is to develop, characterize, and understand the RailPac for high-authority flow control for dynamic stall suppression in a rotorcraft blade. A preliminary research on a prototype conducted by Pafford et. al. [34] produced a proof-of-concept RailPac. The work presented expands upon their research to gain deeper insight behind the actuation mechanism of a RailPac, methods to increase its actuation authority, and understand the interaction between a moving arc and the flow.

The report is organized into the following sections: 1) arc dynamics measurements, 2) actuation authority measurements, 3) transit velocity modeling, 3) induced velocity measurements, and 4) stall alleviation demonstration. Note that the sequential order of these topics is deliberate, with each topic providing a deeper understanding of the actuation mechanism of a RailPac. The first topic delves into the physics of an arc discharge because the momentum transfer is initiated through the arc and its motion. Hence, identifying the arc structures relevant to flow control and discovering methods to improve the arcs transit velocity are crucial to developing an actuator with high control authority and efficiency. Experiments are performed with high-speed imaging and electrical measurements to fully understand the arc dynamics and structure.

Once the arc has been characterized, the impact of the various properties of the arc on the RailPac actuation authority is assessed using transient force and impulse measurements. Recall that the momentum coefficient is one of defining characteristics of an active actuator, as the actuators influence on the flow depends on the maximum achievable C_μ . The force and impulse measurements are crucial since the value of momentum coefficient requires a priori knowledge of the momentum transferred to the flow by the actuator.

The momentum coefficient alone does not provide a full description of an actuator. The active actuators, in general, perturb the flow in a localized region to generate large-scale flow structures favorable for flow control. This phenomenon implies that the actuators effect can be amplified by the natural instabilities of the flow. Therefore, it is important to capture the flow motion induced by a moving arc in order to assess the momentum coefficient in context of fluid dynamics. Measurements include Particle Image Velocimetry (PIV) and Schlieren Velocimetry to capture the velocity induced by a moving arc in a bench-scale setup.

The final results of this paper implements the RailPac on a rotorcraft blade to demonstrate static stall alleviation. The lessons learned from the arc dynamics, actuation authority, and induced velocity measurements are implemented to mature the bench-top setup into a practical actuator on a rotorcraft blade. The actuator is assessed through PIV and force measurements to demonstrate a RailPac for flow separation control.

3 Prototype RailPac

A prototype RailPac was constructed based on the study by Pafford et. al. [34]. A RailPac consists of a pulsed power supply connected to a pair of electrodes. The power supply generates an arc between the electrodes that mediate the momentum transfer from a RailPac to the surrounding air.

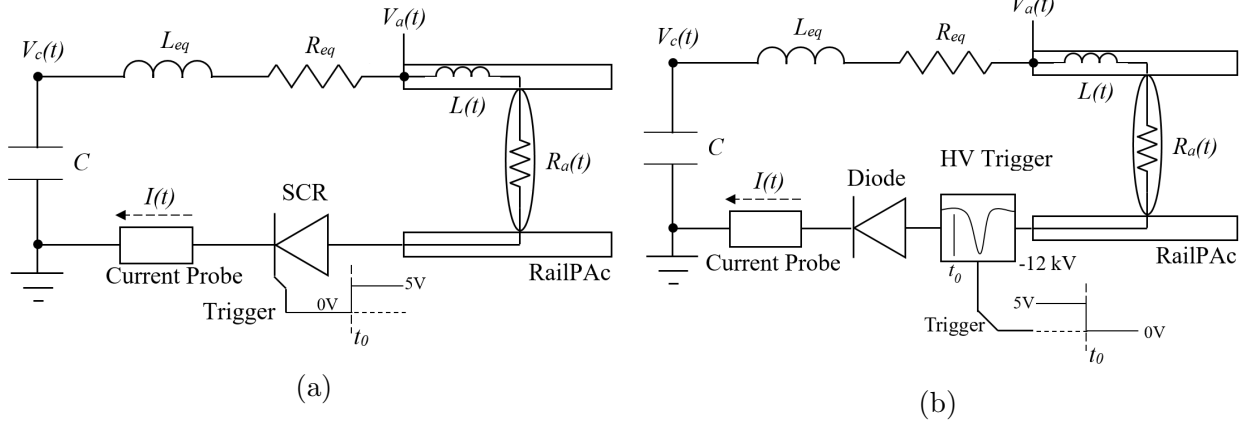


Figure 2: Electrical schematic of a RailPac with (a) Silicon controlled rectifier (SCR) and (b) high-voltage (HV) trigger configurations.

3.1 RailPac Circuit

An actuation by a RailPac begins with the formation of an arc across its electrodes. An arc initiation is a process by which charge carriers are released into the neutral air for electrical conduction [35]. The initial ionization can be achieved by various methods, although only two methods have been employed in a RailPac. The first method bridges the gap between the electrodes with a thin metal wire. When a current is supplied, the metal wire vaporizes and a weakly conducting channel is formed at its location. This method is useful for studying the RailPac in a bench scale setting because it is simple to install and the initiation location can be controlled.

The second method relies on the avalanche breakdown of an air gap between the electrodes. If the electric field across the gap exceeds the dielectric strength of the air (3×10^6 V/m) [36], a single free-electron can cause a cascade of ionizing collisions. The net effect of the collisions is the formation of a conducting channel in an air gap. This initiation method is preferred for a RailPac because it is non-intrusive and allows repetitive discharges at fast time scales. Unfortunately, avalanche breakdown is a stochastic process and the breakdown location cannot be predicted. One method to control the initiation location is to modify the electrode geometry at a localized region to increase the probability of the discharge. The control over the location is achieved by placing an element that enhances the electric field on the surface of the electrode or decreasing the gap distance at a desired initiation location. The practical implementation of these changes is challenging because the field enhancement element can be eroded and modifications to the electrode geometry can adversely affect the transit characteristics of an arc. The preferred method of initiation is to place a graphite trail between the electrodes. The presence of a carbon surface reduces the breakdown voltage of the air, such that an arc can be repeatedly discharged at the same location.

Note that the arc initiation by a fuse-wire is simple to install and does not require a high-voltage pulse. Such method is ideally suited for mounting a RailPac to a test stand (for example, a force balance for control authority measurement), since the volumetric constraint prevents heavy shielding of wires to withstand the breakdown voltage. On the other hand, the

arc initiation by an avalanche breakdown is a practical method for implementing a RailPac for flow control application. Thus, two types of RailPac power supply are in use. The fuse-wire initiation is implemented via the silicone controlled rectifier (SCR) configuration. In the SCR setup (figure 2a), an aluminum fuse wire bridges the gap between the rails, and the capacitor discharge is impeded by the SCR. When a trigger voltage is sent to the SCR at an initiation time (t_0), current discharges through the circuit and a spark is formed by the vaporization of an aluminum fuse wire.

The arc initiation by an avalanche breakdown is implemented by a high-voltage (HV) trigger (figure 2b). The HV setup replaces the SCR with an automobile ignition system (Mallory HyFire). When a trigger voltage is sent, the HV circuit momentarily lowers the cathode to -12 kV, and a spark is formed along the graphite trail. A diode placed between the current probe and the trigger circuit protects the capacitor bank from the -12 kV pulse.

The two trigger mechanisms provide a conducting channel in a RailPac and allow the main capacitor bank (with capacitance C) to discharge. The current rapidly rises to ~ 1 kA and a spark develops into an arc. The capacitor is charged using an external power supply (Xantrex XHR 600) to different initial voltages ($V_{ci} = 160$ V to 250 V) to control the current as well as the energy delivered to the arc.

Regardless of the trigger mechanism, the RailPac circuit can be modeled as a second order non-linear differential equation of the form,

$$\ddot{q} + \frac{R_{eq} + R_a(\dot{q})}{L_{eq} + L_R + L_a(t)}\dot{q} + \frac{1}{(L_{eq} + L_R + L_a(t))C}q = 0 \quad (3)$$

Where q is the charge. The total resistance of the RailPac circuit is the sum of fixed circuit resistance (R_{eq}) and variable arc resistance (R_a). The total inductance is sum of fixed inductance (L_{eq}) and variable RailPac inductance (L_a) as a result of arc motion and deformation. The variable RailPac inductance can be obtained by multiplying the inductance gradient of the RailPac with the arc location along the electrodes. The change in the total inductance due to the arc motion ($L_{eq} + L_a$) is small compared to the fixed inductance of the circuit and therefore can be neglected.

High voltage probes were used to measure the capacitor voltage (V_c) and rail voltage (V_a). The arc current (I) was measured using a Hall effect current probe (Tamura L06P800-05), or a Pearson probe for current exceeding 1.1 kA.

The variable arc resistance (R_a) was measured by dividing the rail voltage by the arc current as a function of time. The total energy delivered by the capacitor (E) was computed based on the initial (V_{ci}) and final (V_{cf}) capacitor voltages.

$$E = \frac{C}{2}(V_{ci}^2 - V_{cf}^2) \quad (4)$$

The arc energy (E_a) was obtained by multiplying the arc voltage with the arc current and integrating the product over the arc transit duration. Note that the sum of the arc energy and energy dissipated by the circuit resistance is equal to the total energy.

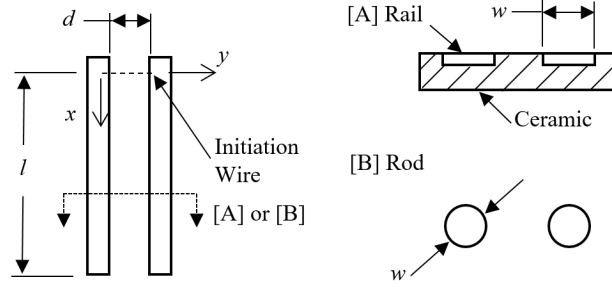


Figure 3: RailPac configuration for rod and rail cases.

Table 1: Summary of RailPac configurations and dimensions.

Case	Shape	d [mm]	w [mm]	l [mm]
A13	Rail	12.7	6.35	127
A02	Rail	2.0	6.35	127
B13	Rod	12.7	3.175	127
B06	Rod	6.4	3.175	127
B02	Rod	2.0	3.175	127

3.2 Electrodes

Different RailPac designs are employed to study the effect of electrode shape and spacing on the arc transit and actuation authority. Figure 3 describes the RailPac geometry for rail and rod electrodes. In the rail case, parallel copper bars are flush mounted to a ceramic surface. For the rod case, tungsten rods are mounted in mid-air in a second test setup.

Rail or rod electrodes (152 mm length) are spaced at different distances as tabulated in table 1. The rod diameter (3.175 mm) as well as the rail width (6.35 mm) are denoted by w . The rails have a thickness of 0.56 mm. Note that each RailPac configuration is labeled with a distinct identifier. For example, a RailPac consisting of flush-mounted rails that are spaced 12.7 mm apart is labeled A13. Similarly, a RailPac comprised of parallel rods separated by 2.0 mm is labeled B02. Each configuration will be referred to by its identifier for clarity.

As shown in figure 3, aluminum fuse wire or graphite trail situated 25 mm away from one end of the electrode initiates the arc at a consistent location. The initiation location and the electrode length limited the arc transit distance (l) to 127 mm.

4 Arc Dynamics Measurement

A moving arc experiencing Lorentz force undergoes significant transformation in arc shape arc root attachment characteristics as observed by several authors [37–39]. Pafford et. al. [34] also identified changes in the arc shape during its transit and hypothesized that the arc morphology affects the mechanism of interaction between a moving arc and the surrounding

air. High-speed images in this study show that the arc roots, or regions where the arc attaches to the electrode surfaces, behave differently on the anode and cathode. Although the motion of anode root along the electrode can be characterized as discrete jumps, the cathode root moves smoothly along the electrode. This phenomenon was defined by Gray et. al. [40,41] as the "anchoring" behavior. Because anchoring may adversely impact the actuation authority and efficiency of a RailPac, an in-depth study is warranted on this phenomenon.

A detailed study into the anchoring behavior was conducted by Gray et. al. [41] that investigated some mechanisms to suppress anchoring in the arc transit. By examining the internal structure of a propagating arc, they attributed anchoring to the absence of interaction between the cathode and anode roots. Their high-speed images identified the arc as two distinct jets issuing from the cathode and anode roots. When the flush mounted rails are spaced closely together (~ 2 to ~ 3 mm), each jet is directed towards the opposite arc root, and a smooth transit of the anode root can be observed. However, if the rails are spaced far apart (~ 12 mm), the jets form vertical columns that behave independently from one another. These observations reveal that the presence of a dielectric gap in a flush mounted case prevents strong interaction between the two arc roots. The anchoring phenomenon can be suppressed if the interaction is enhanced by minimizing the separation distance between the arc roots.

This section explores the effect of anchoring and electrode geometry on the arc morphology and electrical characteristics. Anchoring can be controlled with the shape of the electrode cross-section and the gap width between electrodes. In one case, smooth transit is achieved by constructing a RailPac out of free-floating rods, as shown in figure 3 with [B] configuration, that remove the insulating gap to increase the interaction between the arc jets. In another case, the gap spacing of the configuration shown in figure 3 with [A] configuration is decreased to a small distance until smooth transit of the anode root is achieved. High-speed imaging and electrical measurements are performed on arcs of different shapes and transit characteristics to characterize the dynamics of a moving arc.

4.1 Experimental Setup

The arc dynamics measurement relies on a RailPac power supply with a SCR configuration (figure 2b). The discharge duration is controlled by placing different capacitor and inductor into the circuit. A circuit with long discharge (half-cycle duration ~ 5 ms) is constructed out of a capacitor bank ($C_{eq} = 21$ mF) and an external inductor ($L_{eq} = 110$ μ H). A circuit with short discharge (half-cycle duration ~ 1 ms) is built with a single capacitor ($C_{eq} = 3.55$ mF) and an external inductor ($L_{eq} = 12.4$ μ H). Two different circuits are necessary because the distance that an arc travels depends on the electrode shape (i. e. rails vs. rods) and spacing. The capacitor is charged to various initial voltages (~ 140 V to ~ 250 V).

4.1.1 High-speed Imaging

Two Phantom Miro M-310 cameras were used to capture the front view and top view of the arc transit. Lenses with 100 mm focal length and f -stop of $f/30$ were installed to reduce image saturation from plasma emission. The videos were captured at 20,000 fps with exposure time of 1 μ s.

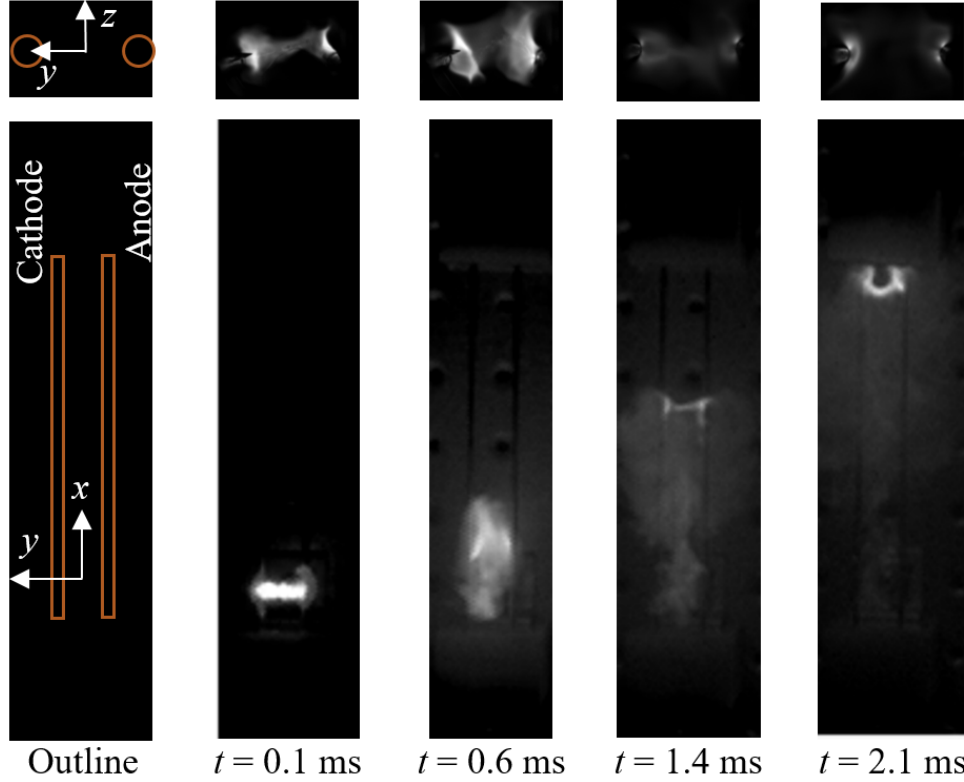


Figure 4: Images of the plasma armature transit. Selected views for $t=0.1$ ms, 0.6 ms, 1.4 ms, and 2.1 ms referenced from the discharge initiation are shown. The muzzle view provides the vertical dimension of the arc, while the top view shows the location of the arc along the rods. The orange outline indicate the profile of the rods.

4.2 High-speed Imaging Results

The high-speed images of the arc transit for rod and rail configurations gave insight into the effects of electrode spacing and geometry on the arc morphology. The arc is readily distinguished from the background as a bright filament bridging the gap between the electrodes. First, the arc was captured at various stages of its transit with a long-discharge circuit. The discharge time of this circuit was sufficient to allow an arc in the rail and rod configurations (A13 and B13) to travel the length of the electrodes (figures 4 and 6).

4.2.1 Rod Configuration

High-speed images for B13 configuration (rods spaced $d = 12.7$ mm) shown in figure 4 reveal a single filament arc moving at high-speed along the electrodes. The arc shape changes at different stages of its transit. The arc luminosity is high in the initial stage ($t < 0.25$ ms), and the arc extends to large distances above and below the electrodes. Note also that the arc luminosity is highest near the electrodes due to electric field enhancement near the electrode surfaces [42]. The arc, if viewed from above, can be described as a linear filament in the initial transit images ($t=0.1$ ms). The arc shape subsequently deforms into an inverted U-shape ($t = 0.6$ ms) and a U-shape ($t=2.1$ ms) as a result of changes in the Lorentz and

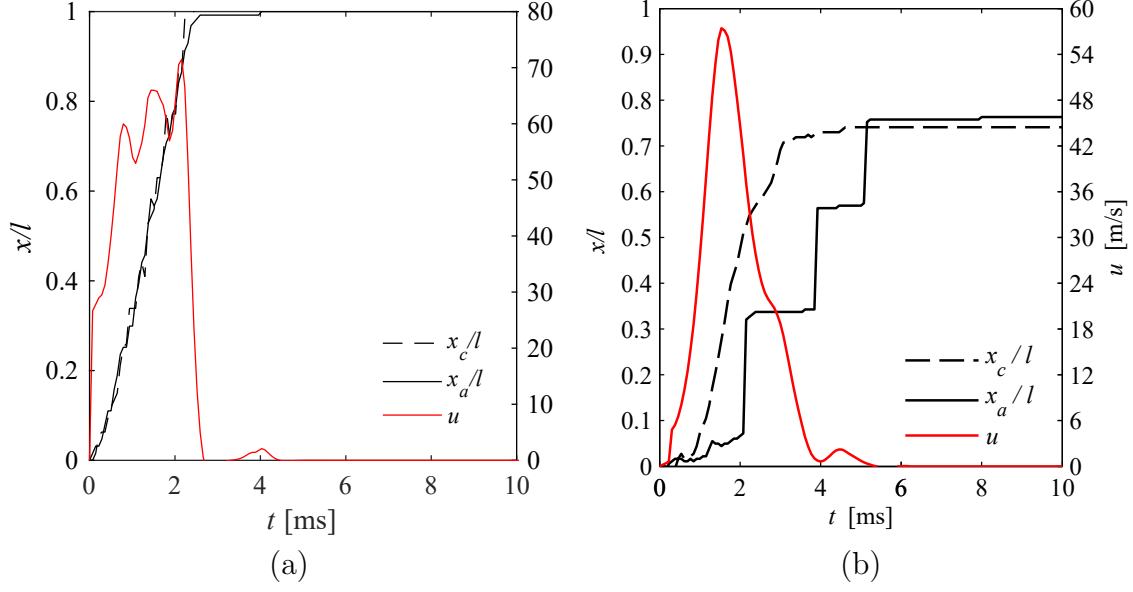


Figure 5: (a) Position of the arc root on the cathode side (x_c) and anode side (x_a) for rods spaced 12.7 mm (B13). The transit velocity (u) is found by numerical differentiation of x_c after applying a smoothing spline to the position. (b) Same analysis applied for rails spaced 12.7 mm (A13).

aerodynamic forces acting on the arc.

The displacement of anode and cathode roots are shown in figure 5a. The position represents the location of highest intensity along each electrode in the propagation direction. Note that the displacement of the arc is nearly linear due to the constant transit velocity of the arc. The arc decelerates at ($t \sim 2.5$ ms) because of the finite length of the electrodes. The arc reaches a peak transit velocity of ~ 70 m/s. The result shows that the discharge duration is too long for the electrode length.

4.2.2 Rail Configuration

The arc in rail electrodes is shown in figure 6. At the initial stage of transit ($t = 0.1$ ms), the arc can also be described as a single filament arc similar to the rod configuration. As the current through the arc increases, the arc increases in intensity ($t=0.9$ ms) and begins to transit along the rails. The muzzle view shows the arc height during this stage of transit. The arc continues to transit and reduces in luminosity and height due to reduction in the arc current. At $t=13$ ms, the arc extinguishes. The images show that a region of diffuse plasma vapor surrounds the arc. This vapor is distinguishable from the arc by the change in intensity between the arc region and the vapor. It has been shown by Parker [43] and Ray [44] that the plasma vapor does not conduct current across the rails. The arc height on average was ~ 2 cm. The deflection is a result of flush mounted rails because the ceramic in the gap prevents the arc from forming near the RailPac surface. The physical mechanism behind the deflection was studied by Gray et. al. [40]. They proposed that the initial deflection of the arc away from the insulating gap is provided by gas expansion due to Ohmic heating.

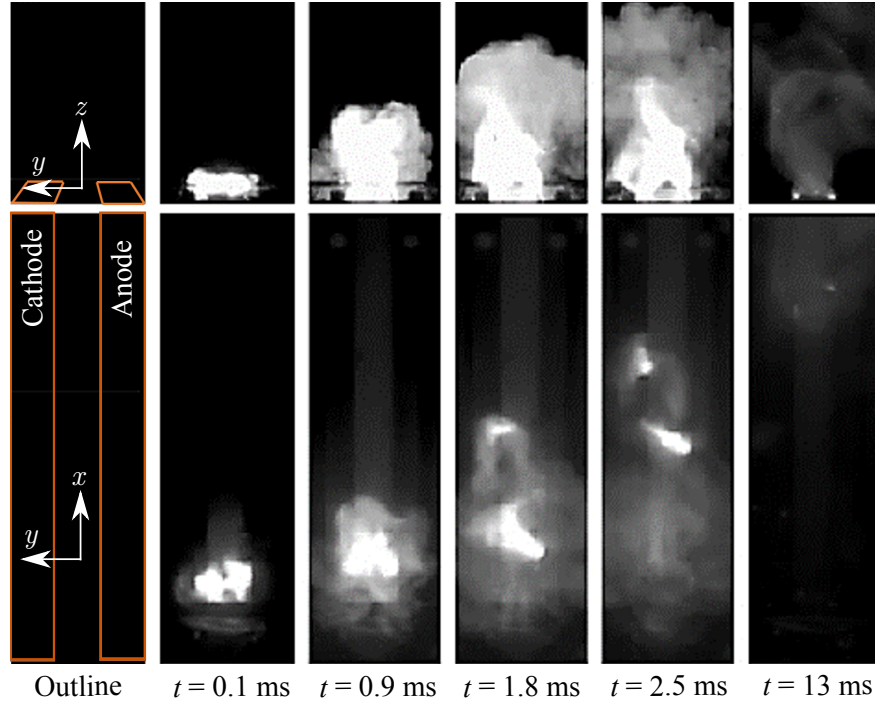


Figure 6: Images of the arc transit. Selected views for $t=0.1$ ms, 0.9 ms, 1.8 ms, 2.5 ms, and 13 ms referenced from the discharge initiation are shown. The muzzle view provides the vertical dimension of the arc, while the top view shows the location of the arc along the rail. The rectangles indicate the outlines of the rails.

This initial distortion results in an arc current above the electrode plane and a component of magnetic field collinear to the propagation direction forms. The interaction of the arc current with this magnetic field results in an expansion of the arc above the insulating gap.

As shown in figure 6, the transit characteristics of cathode and anode roots are different. While the cathode root transits smoothly along the rails, the anode root moves by sporadic jumps. The anode root attaches to the rail surface until it jumps to a new position closer to the cathode root.

The difference in the root motion is explained by the processes required to extract electrons and ions to sustain the arc. In the cathode region, the extraction of electrons is driven by Schottky emission [38], which imposes both local electric field ($4 \times 10^7 \text{ Vm}^{-1}$) [45] and electrode surface temperature requirements for the existence of cathode root in a new location. The field enhanced thermionic emission decreases the work function required (4.7 eV to 4.4 eV) to sustain the armature current, allowing the cathode root to easily transit from one location to another.

On the other hand, the extraction of ions from the anode region requires evaporation of heavier species. Study by Lindmayer showed that the motion of anode root is influenced by the heavy ion evaporation [46]. The temperature requirement for heavy ion evaporation is higher than the extraction of electron. Indeed, the RailPac anode showed marked evaporation of rail material at the anode root locations. It is evident from the rail topology that the electrode surface temperature required for the vaporization is localized at the anode root and is not present elsewhere on the anode surface. Thus, the anode root is anchored to a particular location to achieve the required temperature condition and its motion requires a secondary breakdown to occur at a new location on the anode.

The secondary breakdown is initiated by the increase in relative distance between the anode and cathode roots. Because the armature resistance scales linearly with the armature length, the voltage drop between cathode and anode increases due to the lengthening of the armature [47–49]. At certain critical voltage, the anode root forms at a new location near the cathode root to minimize the armature resistance. Since the anode movement requires a breakdown of gap close to the cathode location, the motion appears as disparate jumps.

The position of each root as a function of the transit time is shown in figure 5b. Due to the jumps in the anode root transit, the cathode root position was differentiated numerically to obtain the transit velocity. A peak velocity of 56 m/s was obtained with the arc dissipating at $x/l = 0.75$. Thus, the transit distance of an arc depends on the electrode configuration. Unlike the rod configuration, the arc in the rail configuration does not reach the end of the electrodes even with the long-discharge circuit.

The roots take a longer time to reach the end of the electrodes than the rod configuration. The delay is attributed to two effects due to the surface. First, the reduced velocity of the cathode root in the rail configuration compared to the rod configuration shows that the arc transit is affected by the drag force from the surface. Second, the reduced velocity of the anode root is attributed to anchoring as evidenced by the initial delay in its propagation as well as the timing of the last jump ($t \sim 5 \text{ ms}$).

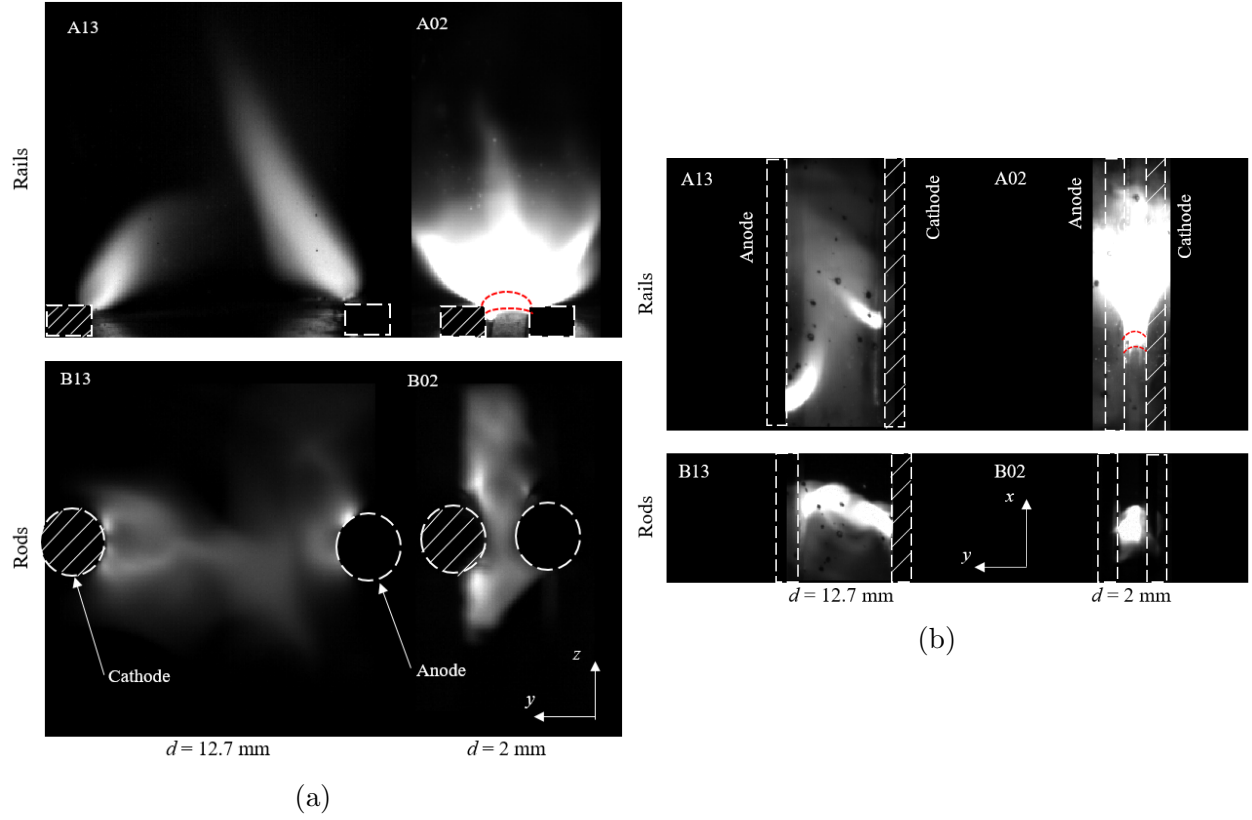


Figure 7: Comparison of arc shapes for A02, A13, B02, and B13 configurations (rod and rail electrodes with 2 mm and 12.7 mm gap distance) at $t = 0.5$ ms, referenced from the arc initiation. Initial capacitor voltage for all cases is 240 V. (a) Front view and (b) top view of the arc. Note for A02, red line is provided to readily distinguish the current carrying portion of the arc.

4.2.3 Electrode Spacing

The effect of electrode spacing on the arc morphology was studied with a short-discharge circuit. Figure 7 provides the image of the arc for rail and rod electrodes with different gap widths. The front view of the arc for B02 and B13 configurations (rods with $d=2$ mm and 12.7 mm) reveals that the extent of the arc above and below the electrodes depends on the electrode spacing. While both configurations host a single filament arc that bridges the gap between the rods, the arc becomes brighter and its extent above and below the electrodes is more significant if the rods are spaced 2 mm apart. The changes in the arc morphology are attributed to the direct interaction between field enhancement regions if the gap width is decreased.

The front view of the arc for A02 and A13 configurations (rails with $d=2$ mm and 12.7 mm) shows that anchoring in rail is suppressed by decreasing the gap width. This is attributed to the interaction between field enhancement regions, also observed in the rod electrodes. When the anode root is anchored, the two arc roots produce distinct jets without a visible filament that connects the two jets. If the gap spacing is decreased as demonstrated in A02 configuration (rails with $d=2$ mm), the two jets merge into a fan-shaped structure. Unlike the rod electrodes, however, the arc appears much brighter and displays substantial upward deflection. The red outline provides approximate extent of the arc that carries the current.

The images clearly show that the arc shape, and therefore the spatial distribution of current density, is sensitive to the electrode geometry. Since the region of highest induced magnetic field exists near the electrodes, it can be argued that a single filament arc will experience the largest volumetric Lorentz force in the region close to the electrodes. If the arc can be confined to this region, one can expect an increase in the actuation authority of a RailPac.

4.3 Electrical Measurement Results

The changes in the arc morphology affect the electrical characteristics of an arc, which can be captured with the measurement of current and voltage waveforms. The measurements were first conducted on a long-discharge circuit, since the longer duration of discharge allows the effect of anchoring to be captured fully. Afterwards, the measurements were repeated on a short-discharge circuit, in order to ensure that only the electrical properties of the arc during its transit is characterized.

The electrical waveform of an arc discharge for rods spaced 12.7 mm (B13 configuration) is shown in figure 8a. The exponential decay of the capacitor voltage and pulse shape of the arc current are consistent with the second-order RLC circuit in series. Although the half-cycle duration of the long-discharge circuit is ~ 5 ms, the discharge is prolonged to ~ 6 ms due to the arc. A peak current of ~ 1.3 kA is measured at the initial capacitor voltage of ~ 250 V. The shape of the current and capacitor waveform is relatively insensitive to the arc morphology, but the arc voltage is highly responsive. The arc voltage is initially zero because the electrodes are shorted by an aluminum fuse wire. At $t=0$, the fuse-wire vaporizes and a spike in the arc voltage is observed. An initiation energy of ~ 15 J was obtained by integrating the product of arc voltage and current over the vaporization duration ($t < 0.1$

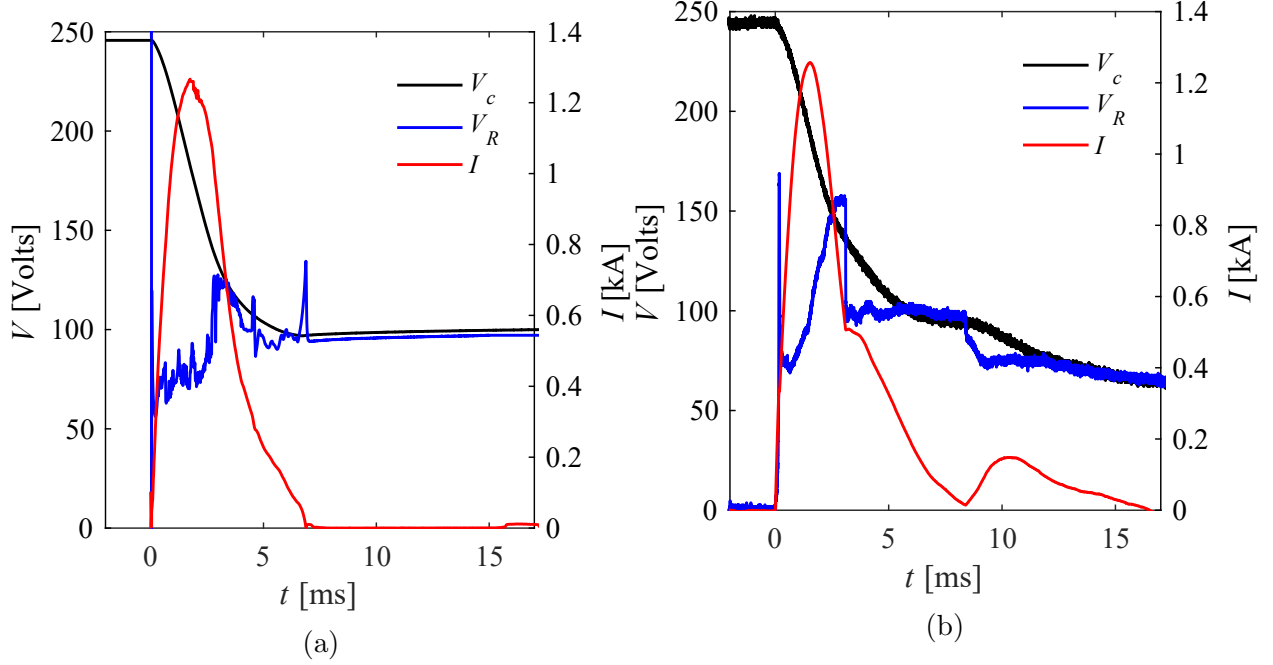


Figure 8: Electrical waveform showing the capacitor voltage (V_c), rail voltage (V_R), and arc current (I) for (a) rod and (b) rail electrode configurations with 12.7 mm gap width (B13 and A13 configurations). A long-discharge circuit is used to sustain the arc.

ms). As the spark transitions into an arc discharge, the arc voltage is decreased to ~ 70 V. This is due to the fact that the arc discharge is characterized by low potential drop across the discharge region. As the arc motion begins, the changes in the arc morphology impose oscillations in the arc voltage. However, the arc voltage is nominally constant throughout the arc transit. When the arc reaches the end of the electrodes ($t \sim 2.2$ ms), the arc voltage suddenly increases to ~ 120 V, and the waveform begins to follow the capacitor voltage. The voltage increase occurs because the arc elongates at the end of the electrodes. When the arc reaches the end, the roots remain attached even though the center of the arc continues to propagate forward. The arc resistance and the rail voltage increase as a result.

The measurements conducted on rail electrodes spaced 12.7 mm (A13) are shown in figure 8b. Although the current and capacitor voltage have a similar waveform as those of the rod electrodes, several transitions that coincide with anchoring are visible. Overall, the discharge duration is prolonged to ~ 15 ms due to the effect of anchoring, while the peak current of ~ 1.3 kA is similar to that of the rod electrodes. The transient spike that coincides with the vaporization of the fuse wire is consistent with that of the rod configuration, and the arc voltage also reaches ~ 70 V after the arc formation. However, a gradual rise in the arc voltage from $t = 2$ ms to $t = 4$ ms is observed as a result of anchoring behavior. When the anode root anchors and the cathode root transits forward, the relative distance between the two roots is increased. The resulting elongation of the arc increases its resistance, and the rise in the arc voltage is observed.

The arc voltage cannot continue to increase indefinitely. By Kirchhoff's law, the sum of the arc voltage, the voltage drop in the resistive and inductive elements in the circuit must

equal the capacitor voltage. If the arc voltage exceeds the limit set by the Kirchhoff's law, the arc discharge cannot be sustained, and a new arc is formed as the anode root jumps to a location close to the cathode. When this jump occurs, the relative distance between the roots is decreased and the arc resistance is lowered. This is evident from the discontinuous drop in the arc voltage at $t = 4$ ms and $t = 8$ ms. Note that the current also has a discontinuity in its slope as a result of the reduced arc resistance.

While the measurements on long-discharge circuit gave insight into the effect of anchoring on the electrical properties of an arc, the results from rod electrodes does not capture the same physics as the rail electrodes. This is due to the finite length of the electrodes, whereby the electrical properties of an arc is greatly changed by the end effects of the electrode. Thus, the discharge time must be carefully chosen to ensure that the arc remains within the electrode length, and additional electrical measurements are carried out with a short-discharge circuit.

4.3.1 Rod Configuration

The results obtained from rod electrodes provide insight into the effect of electrode spacing on the arc resistance. Although the effect of arc shape on its resistance is evident from the current and voltage waveform, the effect of arc current on the resistance is not readily observed. The resistance of an arc during its transit can be obtained by dividing the arc voltage by the current. The arc resistance (figure 9a) decreases with increasing arc current due to the negative differential resistance of the arc [42]. The arc resistance was modeled using a factorial form,

$$R_a = \frac{a}{I + b} \quad (5)$$

Where, a and b are curve fit parameters. This form was assumed based on the observation that the rail voltage (product of current and arc resistance) does not vary significantly over the discharge duration shortly after the arc initiation ($t > 0.14$ ms). For the B13 configuration shown in figure 9a ($d = 12.7$ mm rod and $V_{ci} = 240$ V), the arc resistance was calculated from the current and rail voltage after $t > 0.14$ ms. Then, the curve fit coefficients of $a = 61 \pm 2$ Ω -kA and $b = 2 \pm 3$ kA were obtained.

The measured arc resistances for different rail and rod geometries cannot be compared directly because their value depends on the arc current. Instead, equation (5) was curve fit to all test cases and the arc resistance at 1 kA was interpolated (figure 9b). It can be seen that R_a is independent of arc and electrode shape (i. e. rod and rail cross section), but depends on electrode spacing and anchoring.

As shown in figure 9b, the arc resistance increases with the electrode spacing. It was expected from the spectroscopy measurements by Gray et. al. [40] that the arc resistance would be linearly proportional to the arc length because their measurements showed that the arc temperature, and by extension the arc resistivity, does not vary significantly as a function of the arc current. The non-linearity in the plot of arc resistance vs. gap distance is due to the fact that the gap distance is not a true measure of the arc length. Indeed, the high-speed images of the arc reveal that the arc maintains a U-shape along the plane formed by the parallel rods, such that the actual arc length is longer than the gap width.

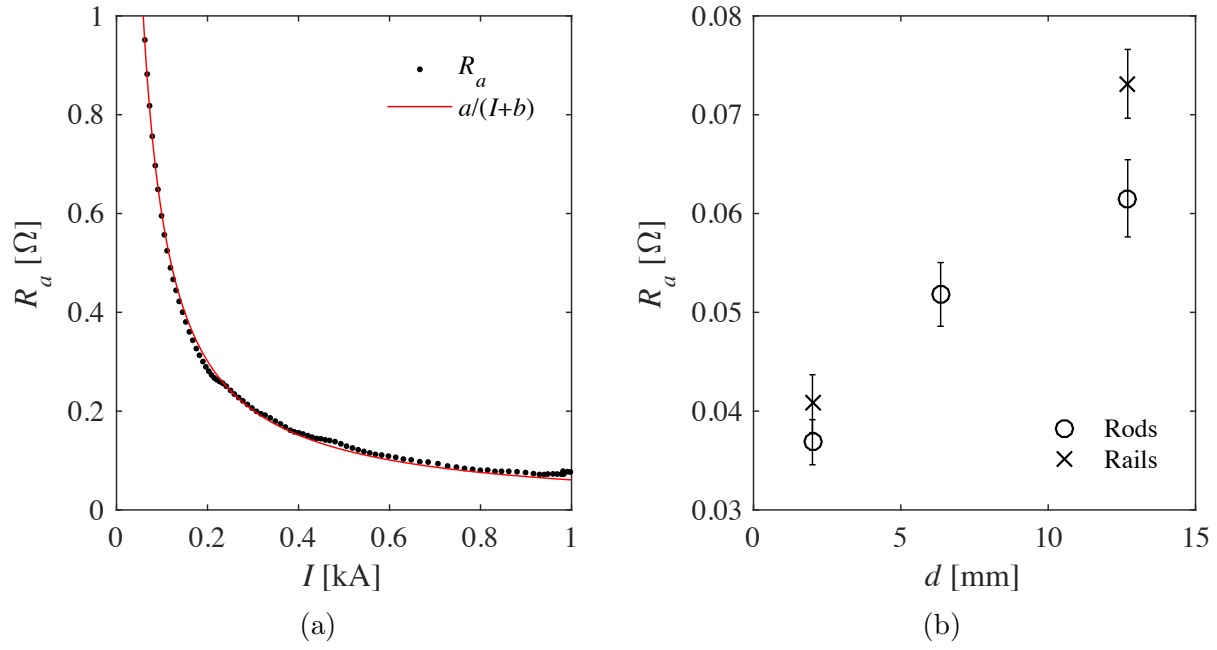


Figure 9: (a) Arc resistance (R_a) as a function of arc current (I) for B13 configuration (rods spaced 12.7 mm apart) at initial capacitor voltage of 240 V. (b) Arc resistance (at $I = 1$ kA) as a function of electrode spacing (d).

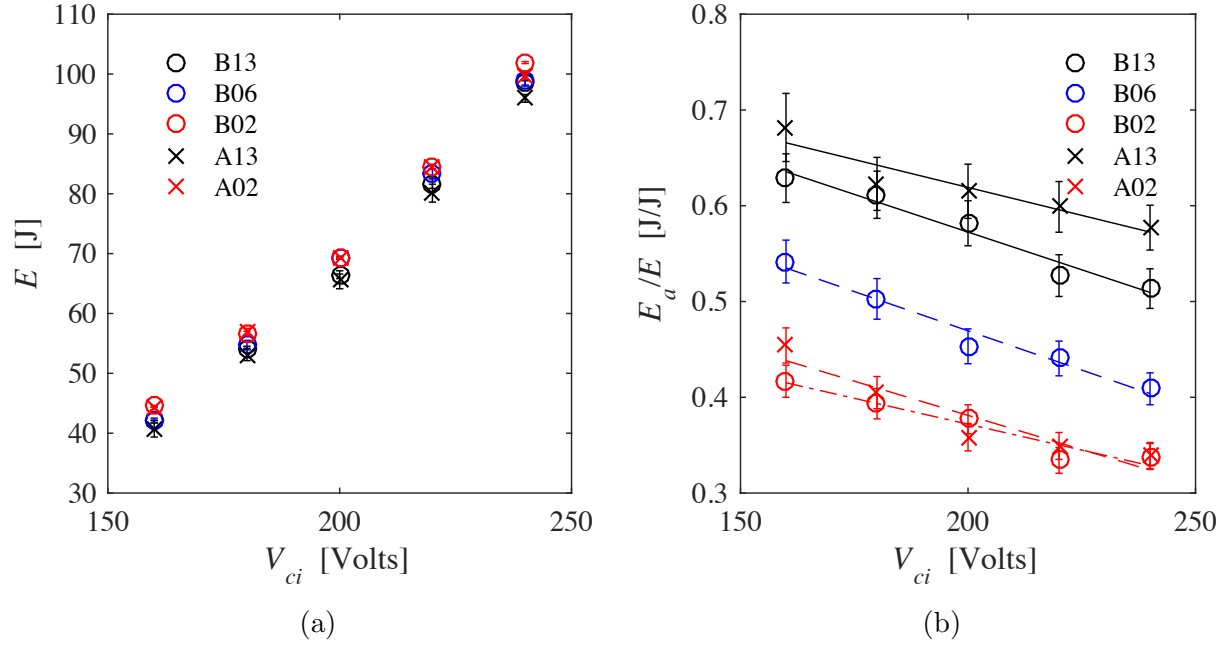


Figure 10: (a) Total energy discharged from the capacitor (E) and (b) energy dissipated by the arc (E_a) as a function of initial capacitor voltage (V_{ci}). The curves in (b) are linear regression lines.

As shown in figure 10a, the total energy discharged by the capacitor is linearly proportional to the initial capacitor voltage and relatively insensitive to the RailPac geometry. Larger discharge energy is achieved for narrowly spaced rods compared to the widely spaced rods. Highest total energy discharged by the capacitor was ~ 100 J at V_{ci} of 240 V, which represents almost 100% of the initial capacitor energy.

The ratio of arc energy to total energy is shown as a function of the initial capacitor voltage in figure 10b. Note that the energy ratio is sensitive to the electrode spacing. For example, at V_{ci} of 160 V, almost 70% of total energy is dissipated by the arc at large electrode spacing ($d = 12.7$ mm), while only 40% is dissipated by the arc at small electrode spacing ($d = 2$ mm). This is explained by the fact that the increase in electrode spacing results in longer arc length, such that the total power dissipated increases.

The peak current increases with initial capacitor voltage because the initial charge stored in the capacitor increases. In comparison to the non-linear arc resistance, the resistance of passive elements of the RailPac circuit is a constant value that is independent of the arc current. The power dissipated by both the arc and the passive elements scales quadratically with the arc current. However, the power dissipated by the arc scales at a reduced rate compared to the passive elements due to the relationship between resistance and current shown in figure 9a. The difference in dissipated power causes the proportion of the energy consumed by the arc to decrease in comparison to that of the passive elements in the RailPac circuit as the current is increased. Since it is desirable to have most of the capacitor energy be delivered to the arc, the energy ratio reveals that a large arc current is inefficient in terms of transferring energy into the arc. Although the Lorentz force increases quadratically with the arc current, the increase in actuation authority comes at a cost of reduced efficiency.

4.3.2 Rail Configuration

The effect of ceramic gap and anchoring on the electrical properties of the arc can be determined from the electrical measurements on the RailPac with rail electrodes. The arc resistances for the rail configurations (figure 9b) increase with the gap distance in a similar manner to the rod configurations. The arc resistance for rail configuration is higher than that of the rod due to elongation of the arc along the $\hat{e}_z$ -direction as a result of upward deflection of the arc away from the ceramic surface. The difference in arc resistance between rail and rod configurations is larger for the wide gap ($d=12.7$ mm) compared to the narrow gap ($d = 2$ mm). The difference is attributed to anchoring and asymmetric transit of the arc roots that cause further elongation of the arc.

Figure 10a shows that the total discharge energies for the rail configurations are equivalent to those of the rod configurations. However, the energy ratio (figure 10b) is sensitive to anchoring. Note that if anchoring is not present (A02 and B02 - rail and rod electrodes with $d=2$ mm), the energy ratio does not vary significantly between rail and rod configurations. However, if anchoring is present (A13 and B13 - rail and rod electrodes with $d=12.7$ mm), the rail configuration has higher energy ratio than the rod configuration. The difference is again attributed to the elongation of the arc due to asymmetric transit.

4.4 Summary and Conclusion

In summary, the dynamics of a moving arc was characterized with high-speed imaging and electrical measurements. The images captured the effects of anchoring and changes in arc morphology during the transit. A RailPac with the electrodes flush-mounted to the ceramic surface (referred to as the rail configuration) hosts the anode root that "anchors" to the electrode and its motion appears as discrete jumps. If the gap width is decreased to ~ 2 mm, anchoring is suppressed and the arc transits as a single filament arc. By arranging the electrodes in free-floating rod configuration, anchoring is prevented irrespective of the gap width.

The RailPac with rod electrodes produce a single filament arc which varies its shape throughout the transit. The change in the arc shape is attributed to the variations in aerodynamic and Lorentz forces acting on the arc. Note that the force variation is expected due to the changes in relative velocity between the arc and the neutral gas. To gain insight into the effect of different electrical input (arc current) and RailPac design (electrode spacing), the efficiency of a RailPac was analyzed with the energy ratio (defined as the ratio of arc energy to the total energy discharged by the capacitor). In the absence of anchoring, the energy ratio decreases with larger arc current. Hence, larger arc current comes at a cost of reduced efficiency, since the proportion of energy dissipated by the arc compared to the total energy decreases with increasing arc current.

The rail electrodes produce an arc with a significant upward deflection with distinct jets emanating from the roots. This deflection is attributed to the presence of an insulating gap between the electrodes. The arc continues its upward motion due to the thermal expansion and the Lorentz force. If the electrode width is reduced to ~ 2 mm, the distinct jets collapse into a single filament arc that transits without anchoring. Therefore, it is clear from the high-speed images that the formation of jets is related to anchoring, although deeper insight into the physical mechanism requires further study. The comparison of arc resistance, as well as arc energy, for rail and rod configurations demonstrates that the anchoring behavior increases the energy dissipated by the arc.

In conclusion, a RailPac impacts a large volume of air as evidenced by the significant upward deflection of the arc. While anchoring becomes an issue if the rails are flush mounted to the aerodynamic surface, the problem can be mitigated by decreasing the gap width to 2 mm and enhancing the interaction between the cathode and anode roots. Electrical measurements show that anchoring prolongs the discharge, even though the arc is stationary for most of the transit. Hence, anchoring reduces the efficiency of the RailPac. These insights demonstrate that a RailPac must be designed with narrowly spaced rails.

5 Actuation Authority Measurement

The actuation authority of an actuator is defined by the momentum delivered to the flow. In the case of a RailPac, the electrical energy is converted to a kinetic energy of the arc, and the motion in turn transfer momentum. Hence, a RailPac is an electromechanical device that drives the conversion process by the self-induced Lorentz force, which is defined by the Lorentz force law,

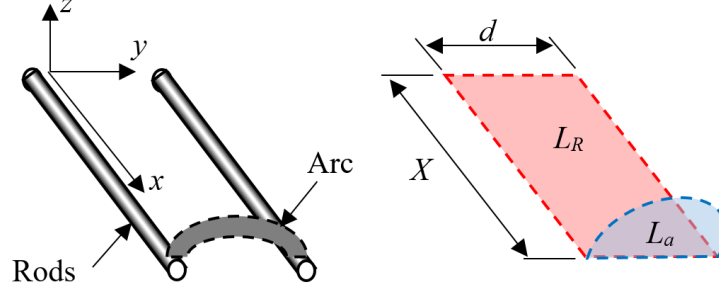


Figure 11: Inductance decomposition into sum of contribution from rod geometry (L_R) and arc shape (L_a).

$$\vec{f}_B = \vec{j} \times \vec{B} \quad (6)$$

Where, $\vec{f}_B$ is the volumetric Lorentz force. The volume integral of this expression over the arc gives the total force. Although the actuation authority of a RailPac is given by the Lorentz force, the experimental measurement of this term is not trivial because $\vec{j}$ within the arc and $\vec{B}$ generated by the RailPac are difficult to obtain analytically or experimentally. Instead, Lorentz force can be expressed in terms of inductance by relating the virtual work done by Lorentz force (δW) to the change in the magnetic potential energy of the RailPac (δU) [50],

$$\delta W = \delta U \quad (7)$$

Consider a deformable arc situated at some X location along the electrodes (figure 11). The total Lorentz force acting on the arc can be evaluated at an X location by integrating the volumetric force over the arc length and arc cross section (A),

$$F_B(X) = \int_0^l \hat{X} \cdot \vec{f}_B A d\zeta \quad (8)$$

By considering the a variation in coordinate, δX , along the propagation direction, the virtual work done by the Lorentz force is given by,

$$\delta W = F_B(X) \delta X \quad (9)$$

Note from figure 11 that the discharge current from a RailPac power supply forms a single loop inductor with inductance, L , consisting of current loop through the electrodes and the arc. Let us decompose the inductance into two terms: inductance contribution from electrode geometry (L_R) and contribution from arc shape (L_a). Then, the total magnetic energy can be expressed in terms of arc current (I) by,

$$U = \frac{1}{2} (L_R + L_a) I^2 \quad (10)$$

The change in magnetic energy due to arc motion and deformation can be expressed as,

$$\delta U = U(X + \delta X) - U(X) \quad (11)$$

Such that the Lorentz force in the propagation direction is given by,

$$F_B(X) = \frac{1}{2} \left(\frac{\partial L_R}{\partial X} + \frac{\partial L_a}{\partial X} \right) I^2 \quad (12)$$

The term $\frac{\partial L_R}{\partial X}$ is known as the inductance gradient based on the electrode geometry. This constant is commonly used in railgun studies to estimate the Lorentz force based on the armature current [51].

The armature of a railgun is a solid body of constant geometry, while a RailPac has deformable plasma arc as an armature. Due to changes in the arc shape during its transit, the inductance gradient of a RailPac has an additional term, $\frac{\partial L_a}{\partial X}$. This term describes the changes in the RailPac force due to elongation and contraction of the arc. Hence, the RailPac inductance gradient is comprised of time invariant term ($\frac{\partial L_R}{\partial X}$) and time variant term ($\frac{\partial L_a}{\partial X}$) that complicates the modeling of RailPac inductance gradient.

A theoretical value of the inductance gradient was proposed by Sitaraman and Raja [52], who made number of assumptions to integrate equation (6). First, the rails are treated as an ideal conductors of infinite length, spaced a distance d apart. Second, the armature has a finite length δ along the x -direction and a height h_a in the $\hat{e}_z$ -direction above the RailPac surface. Third, a uniform current density of magnitude j acts solely along the $\hat{e}_y$ -direction. Finally, a magnetic field of magnitude B_z exists in the $\hat{e}_z$ - direction between the rails. With these assumptions, equation 6 can be volumetrically integrated over the arc volume as,

$$F_B = h_a j d \int_{-\delta/2}^{\delta/2} B_z \, dx \quad (13)$$

The magnetic field is given by Ampere's law

$$\nabla \times \vec{B} = \mu_0 \vec{j} \quad (14)$$

The boundary condition for equation (14) can be found by noting that the rails conduct current from the pulsed power supply upto the armature. We therefore assume no current exists in the rails in the x -direction for $x > \delta/2$, which leads to zero induced magnetic field for $x > \delta/2$, neglecting end effects. Thus, a suitable boundary condition for equation (14) is

$$B_z(\delta/2) = 0 \quad (15)$$

With the boundary condition and equation (14), it can be shown that B_z within the armature decreases linearly from $x = -\delta/2$ to $x = \delta/2$. By using the definition of current density and the solution to equation (14), the Lorentz force can be expressed as

$$F_B = \frac{1}{2} \frac{\mu_0 d}{h_a} I^2 = \frac{1}{2} L' I^2 \quad (16)$$

Note that this theoretical analysis gives an analytical expression of the inductance gradient in terms of the electrode dimension and the arc shape. A nominal inductance gradient ($\bar{L}'$) can be obtained experimentally by integrating equation (12) in time,

$$J_B = \int_0^T F_B dt = \frac{1}{2} \bar{L}' \int_0^T I^2 dt \quad (17)$$

Equation (17) shows that the impulse delivered by the RailPac is linearly proportional to the action integral ($\int_0^T I^2 dt$) [53] scaled by the time averaged inductance gradient. Thus, impulse is a key performance metric that can quantify both the control authority and the inductance gradient. Studies on pulsed plasma devices that act on a similar time scale ($\sim$ ms) as the RailPac (such as pulsed DBD actuators [54] and pulsed plasma thrusters [55–57]) have also quantified this metric to gauge the control authority. While the impulse measurement is useful, the time history of the force applied to the surrounding air yields greater insight into the role of arc structure and motion on the control authority. However, it is challenging to measure a small transient force due to contamination of the measurement by the structural response of the actuator supports. Several studies [58–60] have shown that the structural modes can be removed from the transient force measurement using deconvolution. This process uses an impulse response function to formulate an inverse problem that reconstructs the force from the structural response. However, the force measurements using deconvolution that have been reported in the literature are limited to large impact forces compared to that expected from the RailPac.

Two experimental methods are employed to study the control authority of a RailPac. First, the transient force produced by a RailPac is analyzed by mounting a prototype RailPac on a force test stand. In addition, an independent measurement of the total impulse is conducted to verify the force measurement and gain insight into the role of arc morphology and transit characteristics on the actuation authority.

5.1 Experimental Setup

Actuation authority measurements utilize the same electrical and prototype setup as the arc dynamics measurements. The power supply is comprised of SCR trigger (figure 2a) with long-discharge and short-discharge capacitor and inductor combinations. Experimental setup consists of both the rod and rail configurations of varying dimensions, as summarized in table 1.

Transient force measurements are conducted with a long-discharge circuit with a rail configuration. A short-discharge circuit cannot be used because the discharge time is too short ($t < 1$ ms). Due to the bandwidth limitation of the deconvolution method, the time history of such force cannot be reconstructed. Also, the measurement cannot be performed on a rod configuration with a long-discharge circuit because the finite length of the electrodes truncate the transit distance.

Impulse measurements are conducted on a long-discharge circuit with a rail configuration to verify the transient force measurement. Additional measurements are performed on short-discharge circuit with rod and rail configurations to examine the effect of anchoring on the control authority of a RailPac.

5.1.1 Transient Force Measurement

The transient force of the prototype RailPac was measured on a test stand approximating a single degree of freedom (SDOF) structure. For an accurate measurement without contamination by the structural response, the natural frequency of the test stand must be much higher than the bandwidth of the impulsive forcing. However, the increased stiffness required for the high natural frequency would result in poor force sensitivity. Therefore, in a practical test setup, the measurement will consist of the transient force as well as the structural response of the test stand. In our experiments, the time history of the unknown force is deconvoluted from the known test stand response [58]. Recall that the response of a SDOF structure to an arbitrary forcing $f(t)$ is given by the convolution integral

$$y(t) = \int_0^t g(t - \tau)f(\tau)d\tau = g(t) * f(t) \quad (18)$$

The response $y(t)$ can be the displacement of the test stand, or, when multiplied by a constant calibration factor, the force measured by loadcells on which the structure is mounted. In equation (18), $g(t)$ is the unit impulse response of the structure given by

$$g(t) = \frac{1}{m_{eq}\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t) \quad (19)$$

where m_{eq} is the equivalent mass of the structure (in this case, the test stand), ω_d and ω_n are damped and undamped natural frequencies, and ζ is the damping ratio. The unit impulse response function, is found experimentally by applying a known impulse using an instrumented impulse hammer, and measuring the test stand response. Inoue et. al. [58] proposed that a good estimate of the unit impulse response function can be obtained by Fourier deconvolution if an exponential window function is applied to the measured response. The exponential window is defined as

$$w_e(t) = e^{-\gamma t} \quad (20)$$

The optimal γ for a signal without noise is $2\pi/T$ [61], where T is the total duration of the response measurement. For noisy signals, a smaller γ gives better results [58]; for this paper, $\gamma = 0.4\pi/T$ was chosen. The test stand response can be derived with a known force and unit impulse response function by discretizing the integral in equation (18) to obtain a system of linear equations

$$y(n) = \sum_{k=0}^N \alpha_k f(k)g(n - k) \quad (21)$$

where α_k are the weights of a chosen quadrature method and n is the index of discrete samples. If the sampling interval is small, Riemann quadrature can be used to simplify the expression with $\alpha_k = 1$ for all k . The linear equations describing the convolution operator can then be written in matrix form as

$$\vec{y} = G\vec{f} \quad (22)$$

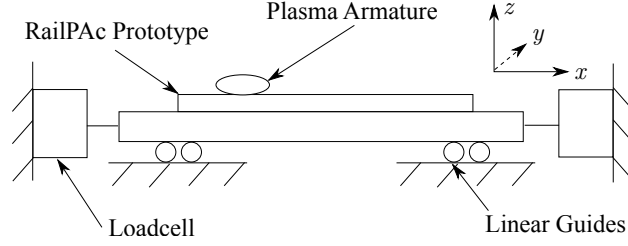


Figure 12: Force test stand designed to measure transient force in x -direction. The RailPac prototype is mounted on linear guides and the test stand displacement is measured using two loadcells.

The inverse problem to solve for the transient force from the measured response and the unit impulse response function is given by

$$\vec{f} = G^{-1}\vec{y} \quad (23)$$

Ideally, equation (23) is sufficient to reconstruct the applied force from the measured test stand response. However, any noise in the measured response will result in large oscillations in the deconvoluted force [60]. The effect of noise on deconvolution can be minimized by using regularization, which imposes additional constraints on the inverse problem. Hansen [59] used Tikhonov regularization to stabilize deconvolution by minimizing the residual ε given by

$$\varepsilon = |G\vec{f} - \vec{y}|^2 + \lambda^2|\vec{f}|^2 \quad (24)$$

The constant λ in equation (24) is called a Tikhonov filter factor. Tikhonov regularization solves the deconvolution problem in a least squares sense with $\lambda = 0$ giving the least squares formulation. A suitable choice of λ can smooth out the effect of noise without distorting the desired signal.

The RailPac prototype was placed on the force measurement stand (figure 12), consisting of an Aluminum honeycomb plate mounted on a pair of linear guides, attached to a load cell (Omegadyne LCMFD-20N) on each end. These load cells can measure ± 20 N with nonlinearity less than 3 mN. The linear guides constrain the motion of the test stand to the direction of the armature transit, such that the test stand acts as a SDOF structure. Two load cells were used to increase the stiffness of the test stand so that it has a natural frequency at ~ 300 Hz. The loadcell signals measured from this test stand were deconvoluted to obtain the transient force.

The validation procedure began by characterizing the test stand with an impulse responses function. Tektronix PCB-4461 impulse hammer was used to obtain the unit impulse response function. It has a sensitivity of 2.25 ± 0.53 mV/N with a resonant frequency at 22 kHz. A steel impact tip was mounted on the impulse hammer to excite the test stand with a short duration (impact duration, $T_h \sim 1$ ms) force, F_I . The output of the load cells is the resulting structural response, S_L . Note that S_L is $y(t)$ in equation (18) scaled by a constant calibration factor.

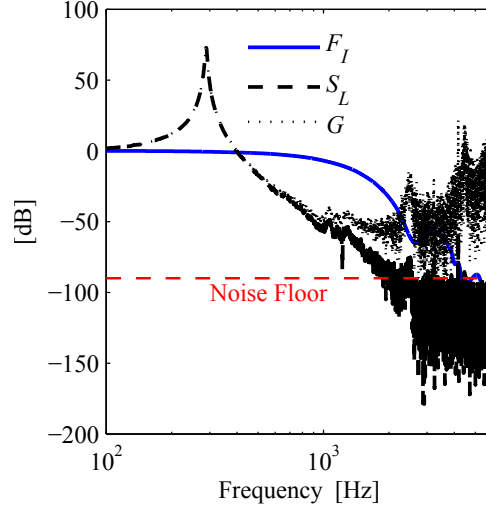


Figure 13: Comparison of power spectrum of the applied forcing (F_I), measured structural response (S_L), and impulse response function ($G(f)$) obtained via deconvolution. For $f \geq 2$ kHz, S_L is dominated by noise and $G(f)$ no longer contains useful information about the structure.

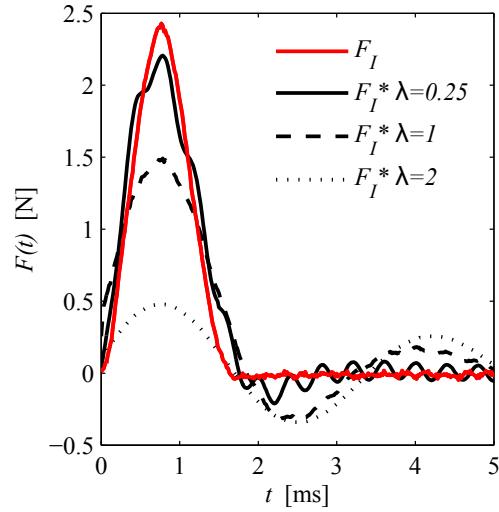


Figure 14: Comparison of the reconstructed forcing F_I^* to the applied forcing F_I for different values of λ .

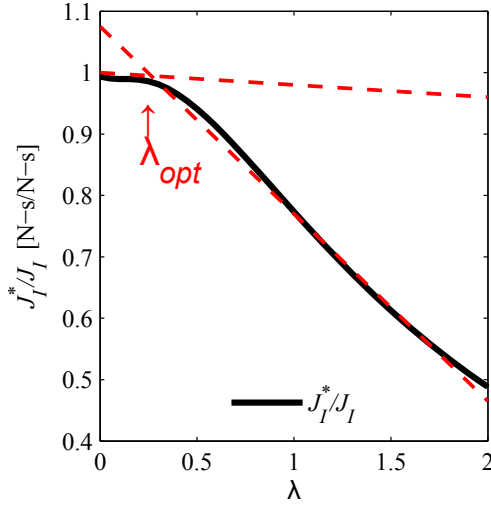


Figure 15: Ratio of the impulse of the reconstructed force (J_I^*) and the impulse of the applied force (J_I). The optimal regularization parameter, λ_{opt} , is found from the transition point at $\lambda = 0.25$

Figure 13 shows the power spectrum of the applied force, test stand response and the unit impulse response function obtained by deconvolution of the test stand response and the applied force. Note that the applied force is bandwidth limited to f_c of ~ 2 kHz, above which the hammer no longer has sufficient energy to elicit a test stand response. As a result, the loadcell signal decays to the noise floor for $f \geq f_c$. The unit impulse response function no longer contains useful information about the test stand response for $f \geq f_c$, which limits the maximum resolvable forcing bandwidth to around 2 kHz. In other words, deconvolution can only reconstruct a forcing of duration greater than 0.5 ms.

The deconvolution methodology using Tikhonov regularization was verified by using the impulse hammer with a soft tip to apply an impulse of duration ~ 1.5 ms (representative of RailPac forcing), and comparing the reconstructed force with the actual applied force. The effect of different values of λ on the reconstructed force F_I^* can be seen in figure 14, where the applied force F_I is measured by the impulse hammer. High values of λ reduce high-frequency oscillations in the reconstructed force, but also decrease the total impulse. Figure 15 shows the effect of λ on the ratio of the reconstructed impulse J_I^* , i.e. the time integral of the reconstructed force (F_I^*), to the applied impulse J_I , i.e. the time integral of the applied force measured by the impulse hammer (F_I). The optimum value of $\lambda_{opt} = 0.25$ is chosen based on the change of slope of this curve. In general, deconvolution captures the shape of the applied force except at the instants of large dF_I/dt . This is a limitation of regularization since the smoothing condition limits the bandwidth of the force that can be reconstructed.

5.1.2 Impulse Measurement

A low natural frequency test stand shown in figure 16 was built to characterize a RailPac impulse. This compact pendulum [62] consists of an upper pendulum and an inverted lower

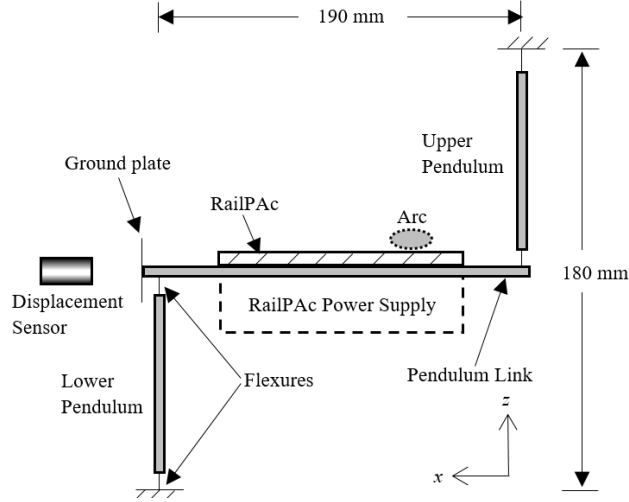


Figure 16: Impulse test stand schematic.

pendulum mechanically coupled via a rigid pendulum link (aluminum honeycomb plate). A RailPac is mounted on top of the rigid link, while a RailPac power supply is situated below the link. Brass flexures act as hinges of the upper pendulum and conductive paths to charge the capacitor bank from an external power supply. The plastic flexures of the lower pendulum reduce the flexure stiffness and electrically isolate the test stand from the base. The test stand displacement was measured using a capacitive displacement sensor (capaNCDT 6110 CS05) with 0.5 mm range and 75 nm resolution. A copper ground plate was mounted at the lower pendulum end of the rigid link and served as a target plane for the displacement sensor.

When the RailPac arc experiences Lorentz force, an equal and opposite reaction force will be imparted on the RailPac and the impulse test stand. However, if electrical wires carry current in and out of the impulse test stand, the reaction force experienced by the test stand does not always equal the Lorentz force experienced by the arc. The measurement error introduced by improper arrangement of electrical wires led to several studies [63–65] that attempted to characterize the reaction force from the Lorentz force, but obtained conflicting results. The issue can be circumvented by mounting the entire RailPac assembly on a test stand. If the setup is electrically isolated within the impulse test stand, only the reaction force from the arc will be imparted on the test stand.

The impulse applied to a test stand can be measured from the impulse response function 19. If the damping ratio is sufficiently small, the maximum displacement can be approximated as,

$$\delta_{max} \approx \frac{J}{m_{eq}\omega_d} \quad (25)$$

Equation (25) shows that the maximum displacement of the test stand is linearly proportional to the impulse delivered. Thus, if the proportionality coefficient ($m_{eq}\omega_d$) is known, then the impulse can be measured from the maximum displacement of the test stand.

The natural frequency of the compact pendulum test stand can be made arbitrarily small

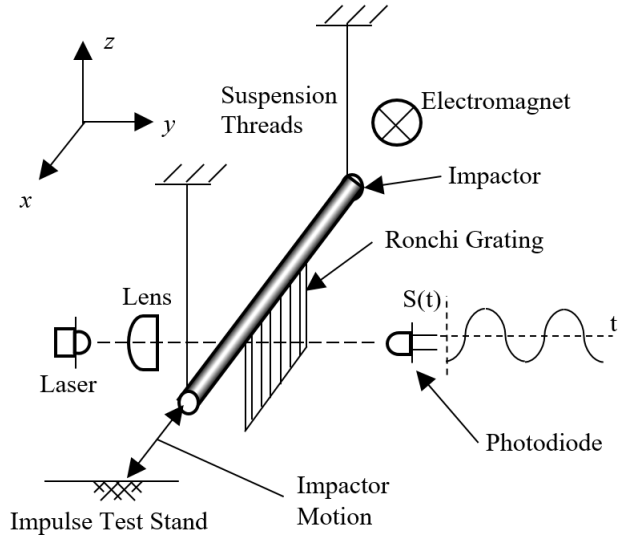


Figure 17: Impulse test stand calibration setup.

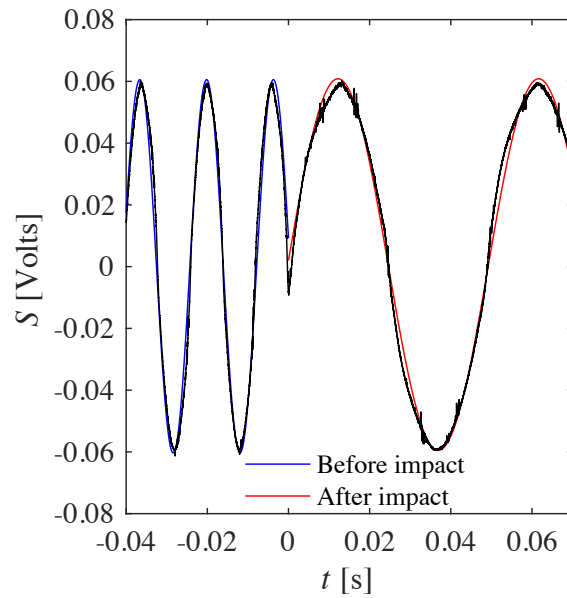


Figure 18: Photodiode signal (S) due to the impactor motion. Time (t) is referenced from the moment of impact with the impulse test stand.

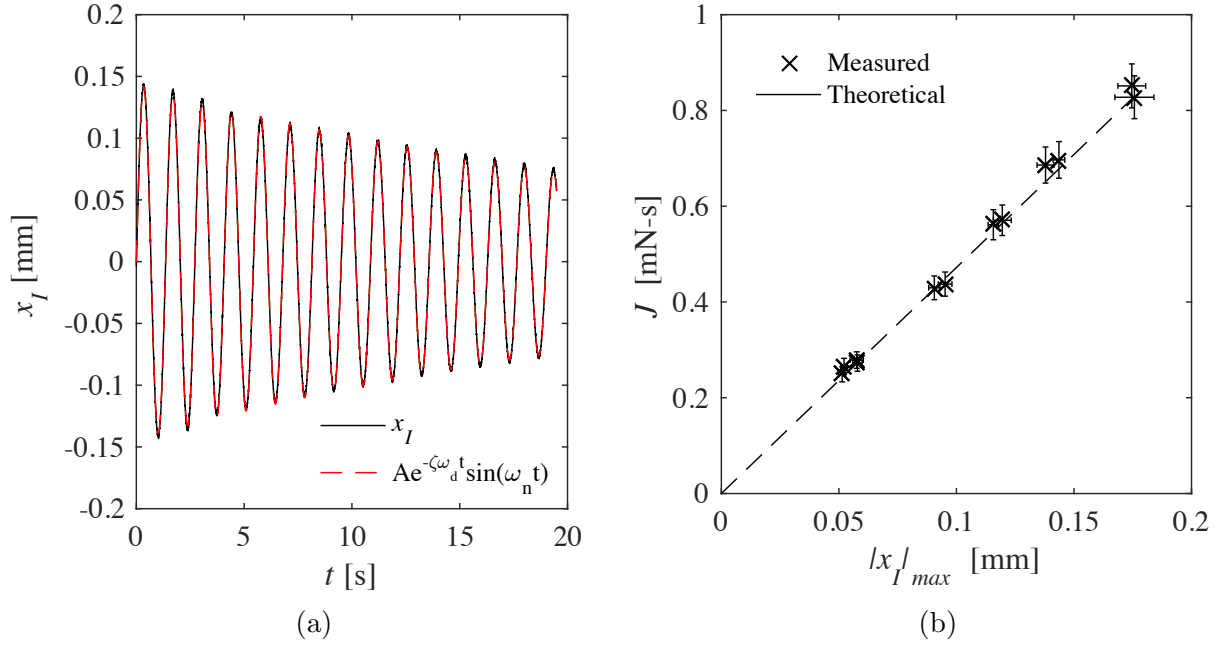


Figure 19: (a) Impulse test stand response to the impactor strike and (b) calibration result for B13 (rod electrodes spaced 12.7 mm apart).

by shifting the center of mass of the rigid link towards the lower pendulum. For this test stand, the center of mass was set by distributing the RailPac power supply mass (572.4 g) towards the lower pendulum. This mass was larger than the equivalent mass contribution from the pendulum link and arms (310.3 g) as well as the RailPac mass (80 ~ 100 g). Note that the exact RailPac mass varied dependent upon the particular rail or rod geometry and the natural frequency was approximately 0.7 Hz.

The impulse test stand was calibrated using an impact pendulum setup shown in figure 17. An aluminum impactor with steel impact tips at its ends was suspended from two nylon threads. Initially, the horizontal pendulum was displaced from its equilibrium position using an electromagnet. When the electromagnet current was interrupted, the impactor was released and subsequently collided inelastically with the pendulum link.

The impulse delivered by the impactor was measured based on the velocity of the impactor before and after the strike. The impact speed was found with an optical method illustrated in figure 17. A laser diode and photodiode were situated at opposite sides of the impactor with the optical path perpendicular to the impactor motion. A Ronchi grating (50 line-pair/in) was mounted below the impactor and positioned such that the grating plane intersected the beam in the perpendicular direction. The impactor motion periodically interrupted the beam received by the photodiode, resulting in the signal shown in figure 18.

The speed change of the impactor before and after its collision with the impulse test stand was computed as follows. First, the impact time was determined from the discontinuity in the photodiode signal (figure 18 at $t=0$). Then, a segment of the photodiode signal before the impact was curve fitted using a sine function. The process was repeated for a segment of the photodiode signal after the impact. The frequency of the photodiode signal before and after

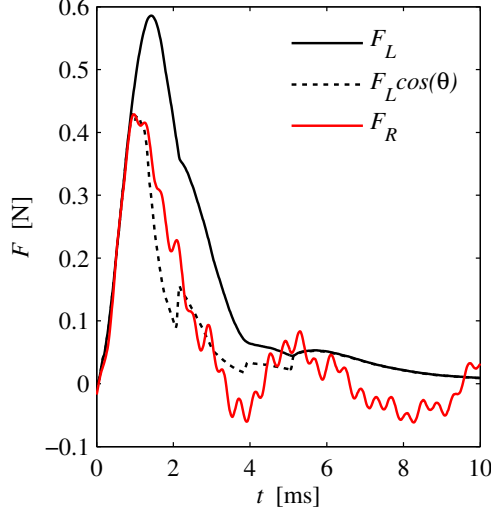


Figure 20: Comparison of the theoretical Lorentz force (F_L), corrected theoretical Lorentz force based on armature orientation ($F_L \cos \theta$), and the reconstructed RailPac force (F_R).

the impact was computed from the curve fit parameter and multiplied with the line spacing of the Ronchi grating to determine the impactor speed. The difference of the velocity before and after the impact was multiplied by the total impactor mass (10.8 g) and the impulse delivered by the impactor was found. The impulse uncertainty was computed based on the measurement uncertainty of impactor mass, curve fit uncertainty of the frequency, and uncertainty in the impactor orientation relative to the impulse test stand ($\pm 1^\circ$).

The displacement of the test stand in response to the impactor strike (figure 19a) was measured simultaneously with the photodiode signal. The impulse response (equation (19)) was fit to the test stand displacement to determine the modal parameters of the test stand.

The maximum displacement of the test stand was found from the amplitude of the curve fit on the test stand displacement, and the maximum displacement was compared to the impulse delivered by the impactor (figure 19b). The slope of the linear fit gave the calibration factor of 4.81 ± 0.13 mN-s/mm (95% confidence based on the uncertainty of the curve fit parameter). This value compares well with the theoretical calibration factor ($m_{eq}\omega_d$) of 4.72 mN-s/mm calculated with the RailPac mass (109.2 g). Because different rail and rod geometries slightly changed the test stand center of mass, the calibration step was repeated for each rail and rod configuration.

5.2 Force Measurement Result

The measured transient force generated by the RailPac was compared to the theoretical Lorentz force (figure 20) calculated using equation (16), where L' was calculated using the average arc height (2 cm). While the assumption of constant arc height is not reflective of the physical phenomenon, this simplifying assumption was used by Hariswaran et. al. [52] to successfully simulate the RailPac transit. Note that the arc transit excites a higher frequency (transverse bending of honeycomb platform) mode at 2.5 kHz. This was filtered out from the test stand response using a third order polynomial Savitzky-Golay filter with 101 bin

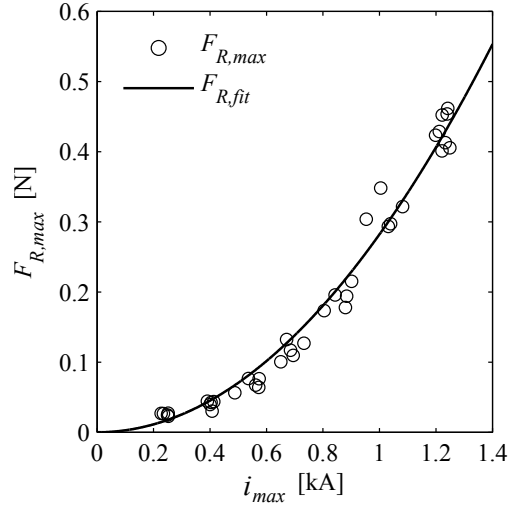


Figure 21: Maximum measured reaction force ($F_{R,max}$). A quadratic fit ($F_{R,fit}$) is provided to determine the scaling of the body force with respect to the armature current (i_{max}).

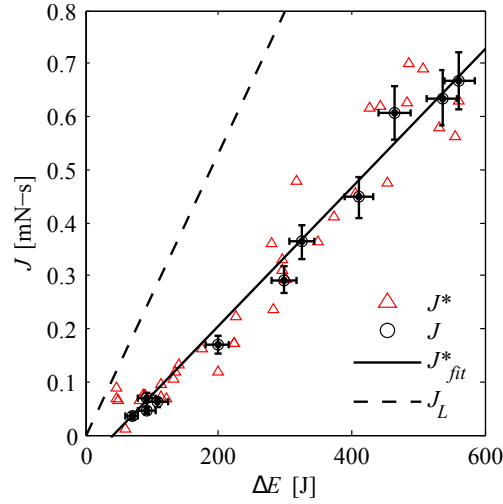


Figure 22: Comparison between the impulse obtained by transient force measurement (J^*) and direct impulse measurement (J). A linear regression of J^* is shown to determine scaling of impulse with respect to the capacitor energy expenditure (ΔE). The theoretical impulse (J_L) is provided for comparison.

size, before reconstructing the transient force.

The theoretical Lorentz force in figure 20 matches the transient force until $t \sim 1$ ms, after which the theoretical force overpredicts the measured force. The discrepancy begins when the arc roots no longer transit together. Recall that when the cathode root transits ahead of the anode root, the component of current density along the $\hat{e}_y$ -direction scales by $\cos \theta$. If the magnetic field between the rails is assumed to be uniform along the $\hat{e}_z$ -direction, then the Lorentz force, as defined in equation (16), must also scale by $\cos \theta$. The corrected theoretical prediction, $F_L \cos \theta$, accounting for the arc orientation closely matches the measured force. Such effect is also seen in Railgun studies, whereby the geometry of the armature (analogous to an arc in a RailPac) affects the Lorentz force. Finite element analysis performed by Kim et. al. [66] and Nie et. al. [67] has demonstrated that the inductance gradient is closely related to the distribution of current density in the electrodes and the armature. Thus, it is expected that the arc morphology has an impact on the Lorentz force.

In addition, there are several assumptions inherent in the theoretical model that are not satisfied in the experimental setup. First, the study by Mercier et. al. [39] showed that the current path between the electrodes is not always linear. Second, the plasma armature after the initial acceleration may not be in local thermal equilibrium. These two effects directly contradict the uniform current density and armature shape assumptions needed to derive equation (16) and may account for some of the discrepancies between the theoretical prediction and measurement.

The theoretical and measured forces suggest that most of the momentum transfer occurred before $t = 4$ ms. The velocity and transit characteristics in figure 5b corroborate this observation since all of the armature motion also occurred before this time. On the other hand, the electrical measurements in figure 8b show that the capacitor bank continues to discharge energy to sustain the armature well after 4 ms. While the energy expenditure up to this point results in a Lorentz force the subsequent energy expenditure results in little or no force. Therefore, the energy expenditure of RailPac can be reduced by stopping its operation at ~ 4 ms without a substantial penalty to the actuator control authority.

The RailPac scaling laws were examined by comparing the force measurements to the electrical measurements. The force scaling law was found by relating the peak force with peak current. As predicted by equation (16), the measured peak force (figure 21) scales quadratically with respect to the peak current.

The impulse delivered to the surrounding air, J^* , was obtained by integrating the measured transient force with respect to time. Second, a direct impulse measurement (J) was performed with the impulse test stand. The results (figure 22) obtained via two methodologies show good agreement. The measurements affirm that RailPac impulse is linearly proportional to the capacitor energy expenditure. However, the measured impulse (J and J^*) was substantially smaller than the theoretical impulse (J_L). This is ascribed to the inadequacy of the model since it does not take into account the difference between the cathode and anode transit behavior.

5.3 Impulse Measurement Result

The effect of arc shape and electrode geometry on the actuation authority of the RailPac was studied with impulse measurements (figure 23). The total impulse scales linearly with

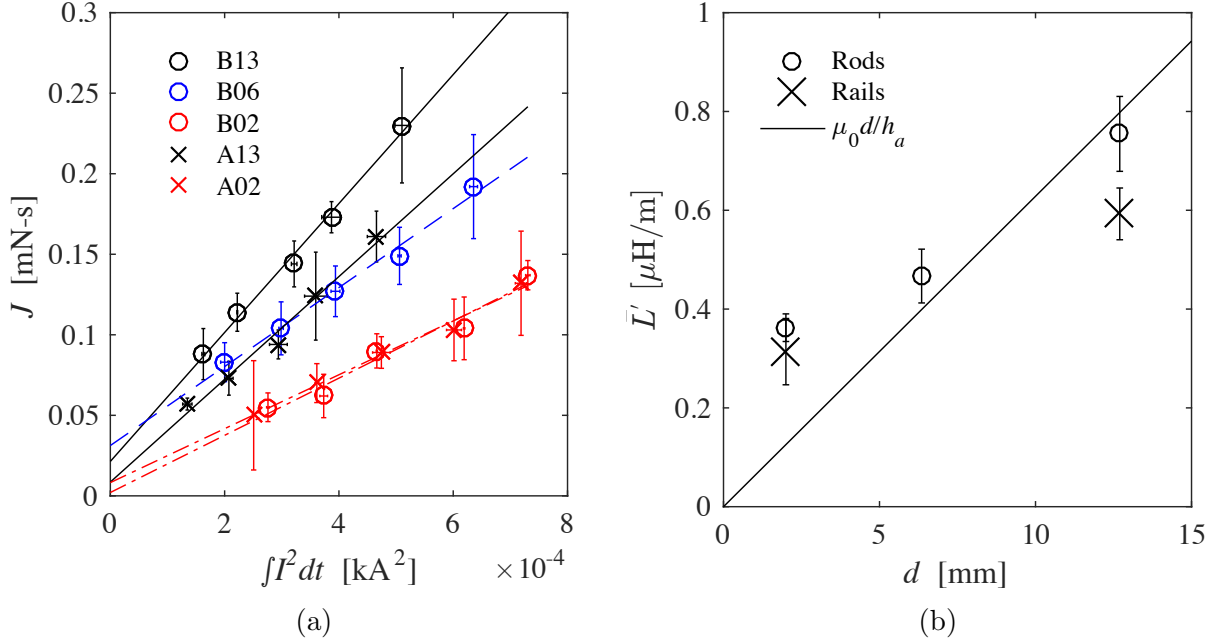


Figure 23: (a) RailPAC impulse (J) as a function of action integral and (b) inductance gradient as a function of gap spacing (d) for rail and rod cases. The curves in (a) are linear regression lines.

the action integral (figure 23a) and the inductance gradient can be determined from its slope (figure 23b).

5.3.1 Rod Electrodes

As shown in figure 23a, the impulse produced by rod electrodes increases with the electrode gap. If the total impulse is divided by the electrode spacing, the highest impulse per unit length for B13 configuration (rods spaced $d=12.7$ mm) is 18 mN-s/m. At the same action integral value, the impulse per unit length for B07 and B02 configurations (rods spaced $d=6.35$ mm and $d=2$ mm) are 23 mN-s/m and 46 mN-s/m, respectively. Therefore, the impulse per unit length increases with decreasing gap width, even though the total impulse decreases with decreasing gap width. These trends are consistent with the fact that the magnitude of magnetic field in the space between the electrodes is inversely proportional to the distance away from the electrode, such that the contribution from volumetric body force near the electrodes is higher than the contribution from the center of the electrode gap.

The actuation authority can be characterized by examining the inductance gradient in the context of electrode gap (figure 23b). If rods have narrow gap ($d = 2$ mm), the inductance gradient is $0.35 \mu\text{H}/\text{m}$, while at wider gap ($d = 12.7$ mm), the inductance gradient is $0.75 \mu\text{H}/\text{m}$. Therefore, the inductance gradient increases with increasing gap width. Note also that the theoretical inductance gradient (equation 16) matches the measured inductance gradient for the rod configuration, unlike the discrepancy shown for the anchored case (figure 23). Unfortunately, the model is not applicable for all gap spacing, especially for $d = 2$ mm

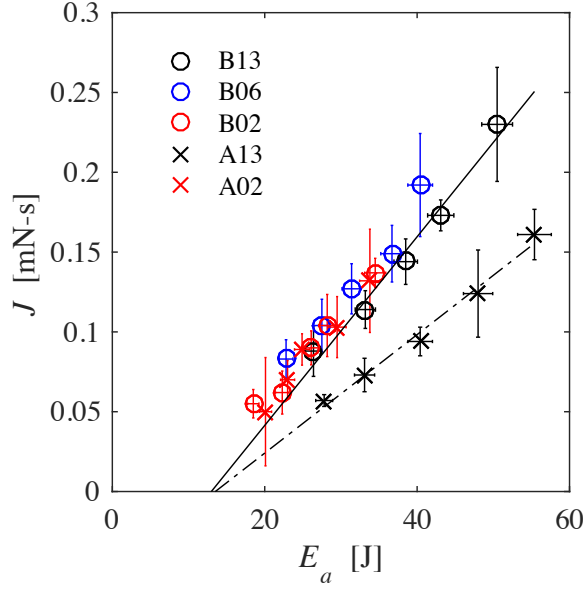


Figure 24: RailPac scaling law comparing impulse (J) produced for a given energy dissipated by the arc (E_a). The curves are linear regression lines.

configuration.

When the impulse is viewed as a function of arc energy, the result is invariant with respect to the gap distance (figure 24). All the results for rod electrodes collapse into a linear line. Note that the slope of impulse vs. arc energy is related to the following proportionality constant,

$$\frac{J}{E_a} \sim \frac{\bar{L}'}{R_a} \quad (26)$$

This relationship was obtained by recognizing that both the impulse and arc energy scale linearly with the action integral. Note that the two proportionality constants, R_a and $\bar{L}'$, represent the arc energy generated for a given electrical input and proportion of the arc energy expended to generate impulse, respectively. As shown in figures 9b and 23b, these values have a weak non-linear relationship with respect to the gap distance. The results show that scaling law does not vary significantly for different gap spacing, if the electrode and arc shapes are held constant.

5.3.2 Rail Electrodes

The effect of anchoring on the RailPac actuation authority can be examined if the measured inductance gradients (figure 23b) between rail and rod geometries are compared at the same gap distance. Consider the impulse generated by A13 and B13 configurations (rails and rods spaced 12.7 mm). The inductance gradient for rail electrodes is $0.59 \mu\text{H/m}$, which is 21% less than the actuation authority of the rod electrodes. If anchoring is suppressed in the rail configuration by moving the electrodes close together to 2 mm (A02), then the measured inductance gradient is $0.33 \mu\text{H/m}$, which is only about 6% smaller than that of the rod

configuration. Thus, the actuation authority of a RailPac can be improved if anchoring can be suppressed.

Figure 24 shows that the RailPac with rail electrodes with small spacing (A02) has a same scaling law as that of the rod electrodes. This trend is readily explained from the high-speed images of the arc. Recall that the narrow rails force the arc to be in proximity of the electrodes and suppress anchoring. If this smooth transit and arc shape can be maintained, the scaling law is invariant to the electrode geometry. If anchoring is present; however, the impulse per unit energy decreases to about half of the smooth transit cases. The high-speed images clearly show that the transit behavior and the arc shape for A13 configuration (rail with 12.7 mm gap spacing) is substantially different than those of other configurations. The arc morphology, therefore, has an impact on the impulse that a RailPac produces for a given energy input. In particular, anchoring is undesirable for designing an efficient actuator capable of exerting high actuation authority.

5.4 Summary and Conclusion

The effects of anchoring and arc morphology on the RailPac actuation authority were studied in this section. A RailPac with rail and rod electrodes were mounted on a force and an impulse test stand to evaluate the effect of anchoring on the actuation authority and efficiency of a RailPac.

A peak force of 0.4 N delivering an impulse of 0.68 mN-s were measured and demonstrated the high actuation authority of a RailPac. In addition, the force scaled with the square of the current and impulse scaled linearly with the discharge energy, suggesting that the RailPac control authority can be tailored towards a specific flow condition simply by changing the electrical parameters.

The effect of anchoring and arc morphology was studied in greater detail with additional impulse measurements. The measurements on rod electrodes showed that the inductance gradient increases with increasing gap width, but the impulse per gap width decreases. When the impulse measurements are compared to the arc energy, the impulse produced for a given arc energy is invariant with respect to the electrode spacing, if smooth transit can be maintained. In comparison, the impulse produced by the RailPac with rail electrodes is less than that of the rod electrodes. In particular, anchoring in rail electrodes reduces the inductance gradient of a RailPac by 21%. In addition, the impulse produced per arc energy decreases in comparison to the smooth transit behavior. The degradation in actuation authority of rail compared to rod configurations can be reduced by decreasing the rail gap and achieving a smooth transit of the arc in a manner similar to the rod configuration.

It can be concluded from the study that the actuation authority of a RailPac must not be increased with wider electrode spacing. This solution results in reduced efficiency due to resistive losses and anchoring. Instead, a maximum electrode gap that achieves smooth transit of the arc should be used to maximize the efficiency and actuation authority of a RailPac.

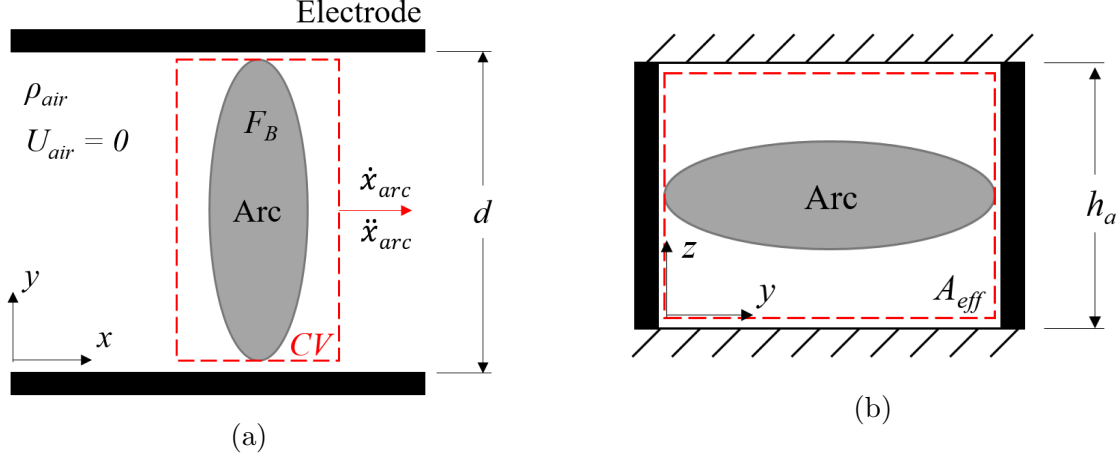


Figure 25: A schematic of a moving control volume as viewed from the lab reference frame. The arc is moving within a confined chamber of quiescent air. (a) Top and (b) front view of an arc in motion.

6 Arc Transit Velocity

The study of arc motion is crucial for understanding the actuation mechanism of a RailPac because the device transfers momentum to the surrounding air through the motion of the arc. While the arc dynamics and actuation authority measurements gave insight into the effect of arc morphology and anchoring on a RailPac, how the measured data relate to the transit velocity of the arc is unknown. One method to gain insight behind the arc transit is by formulating a semi-empirical model that captures the transit velocity. If the physical meaning of the terms within the model are known, then the coupling mechanism between a moving arc and the surrounding air can be identified.

6.1 Methodology

A snowplow model was developed to predict the arc motion in a device that utilize a Lorentz force for arc acceleration. This formulation is popular for simulating a pulse plasma thruster (PPT) [57], magnetoplasmadynamic thruster (MPDT) [68], and plasma accelerators [69,70] due to its computational simplicity and extensive validation with the experimental data. The model can be derived from the conservation equations in integral form. Consider a control volume that accelerates with the arc, as shown in figure 25. Consider an arc moving with velocity ($\dot{x}$) and acceleration ($\ddot{x}$) due to the Lorentz force. The setup contains a quiescent air in room temperature condition. The control volume (CV) moves with the arc and the entirety of the arc is contained within the volume.

Consider the continuity equation for the CV,

$$\frac{\partial}{\partial t} \int_{CV} \rho dV = \frac{dm_{arc}}{dt} = - \int_{CS} \rho_{air} (\vec{V}_{rel} \cdot \hat{n}) dS \quad (27)$$

The mass of the arc (m_{arc}) is assumed to be the volume integral of the density over the control volume, and the relative velocity ($\vec{V}_{rel}$) is defined as the fluid velocity with respect to

the arc velocity. The surface integral over the control surface (CS) is simplified by assuming that the arc behaves as a solid piston. Any mass entering the control volume is immediately accelerated to the arc velocity and compressed into a thin shock ahead of the arc, and no mass leaves the arc behind its transit. With this assumption, the continuity equation (equation 27) simplifies to,

$$\dot{x}_{arc} = \rho_{air} \dot{x}_{arc} A_{eff} \quad (28)$$

The effective area (A_{eff}) of the arc is defined as the surface area of the control surface oriented in the yz -plane (product of h_a and d). The arc height (h_a) can be derived from the definition of current density and simplifying the arc shape into a circular cylinder,

$$h_a = 2\sqrt{\frac{I}{\sigma E \pi}} \quad (29)$$

Note that the conductivity (σ) and electric field (E) of an arc is a function of the arc temperature. The values ($\sigma = 6000 \Omega^{-1}\text{m}^{-1}$ and $E = 200 \text{ Vm}^{-1}$) are obtained from the experimental values [35].

The momentum conservation equation is given by,

$$\vec{F}_B + \vec{F}_S - \int_{CV} \ddot{x}_{arc} \rho dV = \frac{\partial}{\partial t} \int_{CV} \vec{V}_{rel} \rho dV + \int_{CS} \rho \vec{V}_{rel} (\vec{V}_{rel} \cdot n dS) \quad (30)$$

We assume that the Lorentz force is much larger than the surface forces, F_S (i. e. viscous drag force from the surface and pressure). In addition, the arc is assumed to be a solid body, with the species within the arc all moving with the arc velocity. Thus, the term $\frac{\partial}{\partial t} \int_{CV} \vec{V}_{rel} \rho dV$ is assumed to be zero. With these assumptions, the $\hat{e}_x$ -component of the momentum equation is simplified to,

$$m_{arc} \ddot{x}_{arc} + \dot{m}_{arc} \dot{x}_{arc} = F_B \quad (31)$$

Equation (31) shows that the Lorentz force increases the inertia of the arc. The momentum is transferred between the air and the arc by the added mass term ($\dot{m}_{arc} \dot{x}_{arc}$). Thus, snowplow model assumes that the RailPac actuation is driven by the inelastic collision between a moving arc and the surrounding air, similarly to a piston.

The assumption of "piston" effect to describe a moving arc is problematic for a RailPac. This is due to the fact that a RailPac operates in an environment unlike that of the PPT, MPDT, and plasma accelerator devices. For example, a RailPac operates in a higher pressure (atmospheric) compared to near-vacuum condition, with the arc discharge occurring in an open air as opposed to a confined chamber. The transit velocity of a RailPac is lower ~ 100 m/s compared to $\sim \text{km/s}$ found on other devices. These differences cast doubt on whether the arc in a RailPac can be modeled as a solid piston. A method to adapt the model to the RailPac condition is to prescribe a porosity term (γ) that limits the mass added to the arc.

Note from figure 26a that the piston assumption in a snowplow model prevents any mass from leaving the arc behind the transit. If the model is adjusted by introducing (γ), a proportion of added mass is permitted to leave the control volume (figure 26b). The continuity equation for this model is given by,

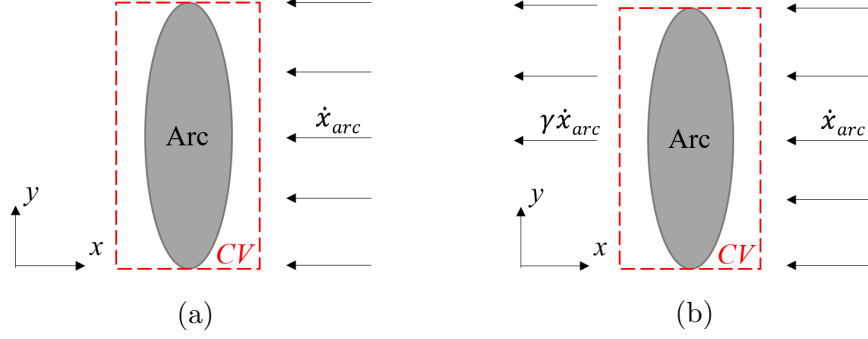


Figure 26: A moving control volume viewed from the reference frame of an accelerating arc. (a) Control volume used in the standard snowplow model and (b) adjusted snowplow model for a RailPac simulation.

$$\dot{m}_{arc} = \rho_{air} A_{eff} (1 - \gamma) \dot{x}_{arc} \quad (32)$$

And $\hat{e}_x$ -component of momentum equation,

$$m_{arc} \ddot{x}_{arc} + \rho_{air} A_{eff} \dot{x}_{arc}^2 (1 - \gamma^2) = F_B \quad (33)$$

Equation (33) can be rewritten to obtain the maximum velocity directly without integration. If the acceleration is assumed to be zero, than the resulting equation is given by,

$$\dot{x}_{arc} = 0.5 \left(\frac{L'}{\rho_{air} d (1 - \gamma^2)} \right)^{0.5} (\sigma E \pi)^{0.25} I^{0.75} \quad (34)$$

The time history of the arc motion can be simulated by integrating equations (32) and (33) with the given parameters (I , L' , ρ_{air} , d , γ , σ , and E). Note that the unknowns are x_{arc} , $\dot{x}_{arc}$, and m_{arc} . The solution requires zero initial conditions for x_{arc} and $\dot{x}_{arc}$, as well as a non-zero value for the initial mass of the arc (m_0).

6.2 Results

The arc transit was simulated by integrating equations (32) and (33). Since the initial mass and porosity of the arc are unknown, these parameters were adjusted to fit the transit measurement. Figure 27 shows the transit simulation for rail electrodes spaced 12.7 mm (A13) and figure 28 is for rod electrodes spaced 12.7 mm (B13). The parameters of the simulation are summarized in table 2. The arc current is obtained from the experimental data.

With a suitable choice for m_0 , the initial velocity of the arc can be matched closely to the measurement. This parameter represents the mass of the arc in the immediate aftermath of the arc formation. Note that the initial mass for the rail configuration is significantly less than that of the rod configuration. This is due to the ceramic gap that reduces the initial acceleration of the rail configuration compared to the rod electrodes. By adjusting γ to 0.89, the peak velocity of the arc in both rail and rod configuration can be matched to the

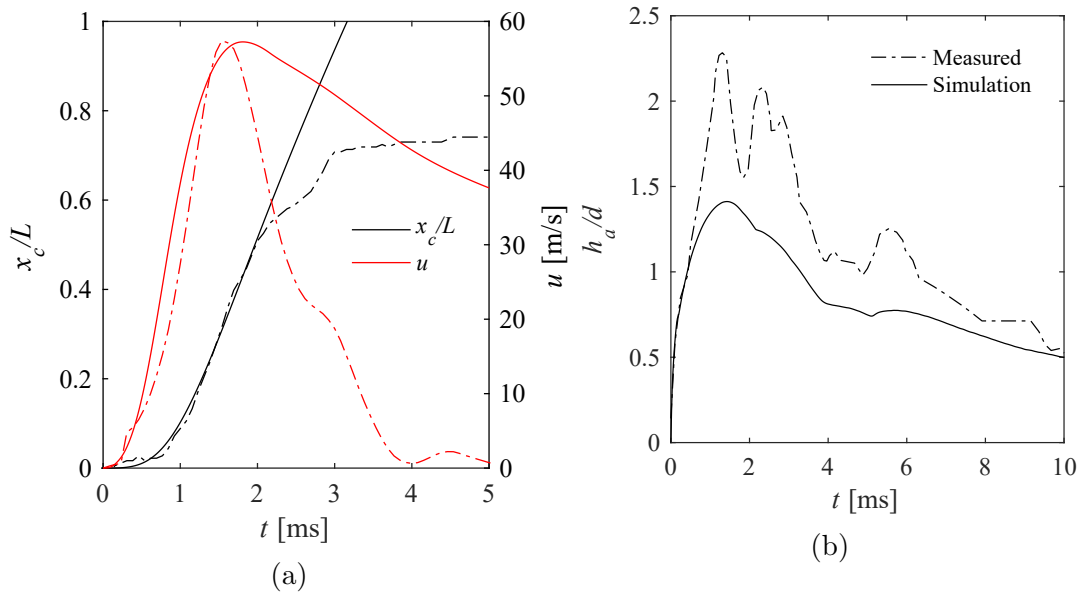


Figure 27: Arc position and velocity as a function of time for rail electrodes spaced 12.7 mm (B13) with a long discharge circuit. Data obtained from measurement (dotted line) and simulation (solid line).

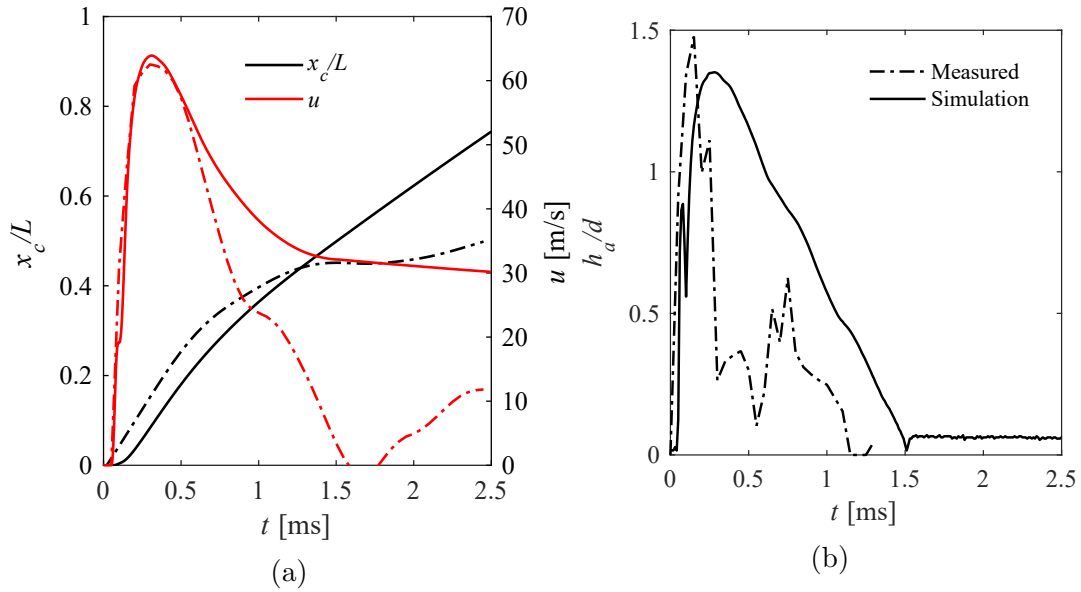


Figure 28: Arc position and velocity as a function of time for rod electrodes spaced 12.7 mm (B13) with a short discharge circuit. Data obtained from measurement (dotted line) and simulation (solid line).

Table 2: Parameters of arc transit simulation

Parameter	Rail	Rod	Unit
L'	0.59	0.75	$\mu\text{H}-\text{m}^{-1}$
ρ_{air}	1.225	1.225	$\text{kg}-\text{m}^{-3}$
d	0.0127	0.0127	m
γ	0.89	0.89	
σ	6000	6000	$\Omega^{-1}\text{m}^{-1}$
E	200	200	$\text{V}-\text{m}^{-1}$
m_0	3×10^{-6}	1×10^{-7}	kg

experimental data. The agreement shows that the snowplow model is useful for simulating the transit velocity in the acceleration phase.

The model fails when the Lorentz force is not sufficient enough to accelerate the arc because a large deviation from the measured values is observed as the arc begins to decelerate. In both the rail and rod electrodes, the model predicts a much smaller deceleration than the measured data. While the measured transit shows the arc eventually anchoring on the electrodes, the simulated arc continues its transit due to the high transit velocity. Since the total distance that the arc travels along the electrodes is different, the actuation area of a RailPac is not accurately captured by the model.

The discrepancy is attributed to the limitations of the snowplow model. First, the snowplow model does not accurately reflect the physics behind the motion of the arc roots. Recall that the Lorentz force equation for a particle states,

$$\vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \quad (35)$$

Now consider an ionized specie with a charge (q) that is extracted from the electrode via a thermionic emission. The initial velocity of this particle is zero, since the electrode is stationary. The particle begins to accelerate to the drift velocity of the arc due to the strong E -field in the cathode and anode fall regions. The acceleration of the particle from the rest reveals that the Lorentz force acting on the particle is initially zero and becomes significant as the particle attains the drift velocity. As the Lorentz force begins to deflect the particles in the transit direction, a significant heating of the electrode ahead of the arc root begins to occur. This temperature rise in the electrode shifts the source of thermionic emission in the direction of transit, and a motion of the arc root is seen in the images. The results reveal that if the Lorentz force is strong, the arc root moves rapidly with the bulk motion of the arc. But, if the Lorentz force begins to decay, the thermal processes occurring near the arc root is important and begin to limit the transit velocity of the arc.

The second issue with the snowplow model deals with the accretion of mass. The snowplow effect is validated for an accelerating arc, but its validity for a decelerating arc is questionable. Indeed, the reduction in the height of the arc shown in figures 27b and 28b suggest that the arc should also lose its mass at the end of the transit. Unfortunately, this effect is not captured by the snowplow model.

The third problem arises from the neglected term ($\frac{\partial}{\partial t} \int_{CV} \vec{V}_{rel} \rho dV$) in the momentum

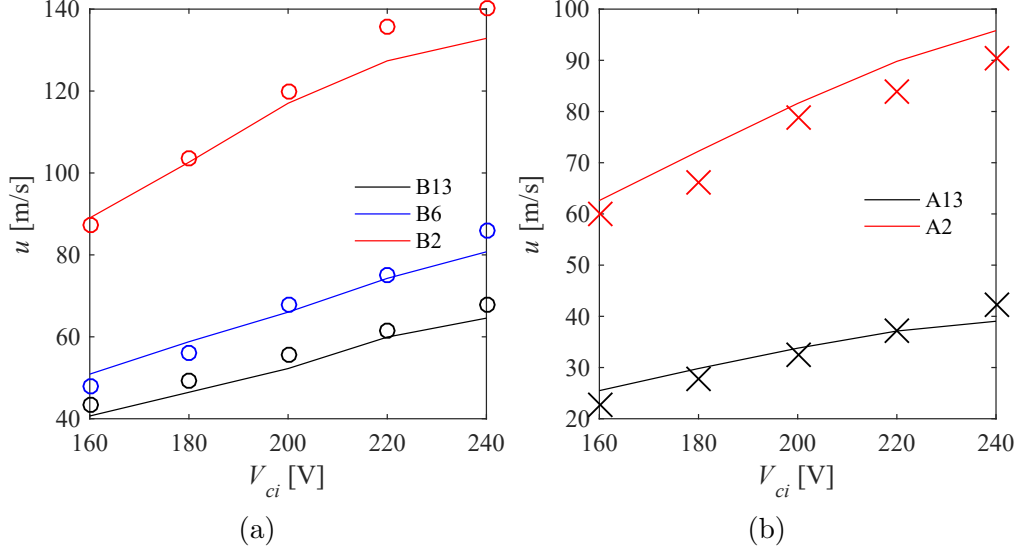


Figure 29: Simulation (line) and measurement (scatter) of the arc velocity as a function of the initial capacitor voltage (V_{ci}). (a) Rod electrodes and (b) rail electrodes. Measurements are an average of several runs (~ 10)

equation (equation (31)). If the arc begins to decelerate due to the reduction in the Lorentz force, the bulk motion of the arc (that is the volume where the current density is high) is different from the gas velocity within the arc. As a consequence, the arc cannot be treated as a "piston" and the relative velocity between the gas velocity and the arc is not negligible. The effect of relative velocity is clearly demonstrated in the case of glidarcs [47, 48, 71, 72], whereby the motion of a low-current arc is achieved with an external flow. In this setup, a nozzle is placed near the initiation location that creates a jet between the electrodes. The arc accelerates to $\sim 90\%$ of the jet velocity, and continues to transit until it reaches the end of the electrodes. A similar effect must also occur in the RailPAc, when the Lorentz force is no longer the dominant term. As the arc decelerates, the momentum is exchanged to the flow, not by the snowplow term, but due to the relative velocity between the arc and the gas.

These three issues prevent an accurate modeling of the arc motion, especially in the deceleration phase. In addition, the model requires an initial mass of the arc that is not easily obtained from the experiments. The model, however, is useful for predicting the maximum transit velocity of the arc, if equation (34) is used.

The velocity measurements in figure 29 are obtained by fitting a linear line on the arc position and taking its slope. The simulation results are obtained with the parameters summarized in table 3. A good agreement with the experimental values are obtained if γ is adjusted properly. The model correctly predicts that the transit velocity increases with respect to the decreasing gap distance and higher current (achieved with higher initial capacitor). This is due to the fact that the Lorentz force per gap width increases with decreasing gap distance, and the total Lorentz force scales quadratically with the current.

Note that a smaller γ is used to model the average of several transit velocities in the rail

Table 3: Parameters of velocity trend simulation

Parameter	A2	A13	B2	B6	B13	Unit
L'	0.32	0.59	0.35	0.47	0.75	$\mu\text{H}-\text{m}^{-1}$
ρ_{air}	1.225	1.225	1.225	1.225	1.225	$\text{kg}-\text{m}^{-3}$
d	2	12.7	2	6.35	12.7	mm
γ	0.80	0.80	0.89	0.89	0.89	
σ	6000	6000	6000	6000	6000	$\Omega^{-1}\text{m}^{-1}$
E	200	200	200	200	200	$\text{V}-\text{m}^{-1}$

configuration compared to the rod configuration. The change is attributed to the difference in arc morphology between the rail and rod configurations. The upward ($\hat{e}_z$) deflection of the arc observed in the rail configuration (figure 7) shows that the arc in rail configuration affects larger volume of air than the rod electrodes, such that a different γ must be used to describe interaction between the arc and the surrounding air. The change in γ is not attributed to the viscous drag from the surface. This is due to the fact that the inductance gradient is computed from the reaction force on the test stand, which is the the difference of the Lorentz force and the surface drag. Thus, the inductance gradient already includes the effect of surface.

6.3 Summary and Conclusion

A modified snowplow model was formulated to simulate a moving arc and validated with the experimental data. The time history of the arc velocity and position were simulated by adjusting the initial mass and porosity of the arc (defined by the constant γ) to match the experiment. While the model agrees well with the experimental data for the accelerating phase of the transit, the model under-predicts the arc deceleration as the Lorentz force is reduced. Consequently, the transit distance of the arc and the velocity during the deceleration phase is much higher than the measured values due to the limitation of the model.

The model was rewritten in terms of the maximum transit velocity, which showed that the transit velocity is a function of various parameters of the arc. By adjusting γ to an appropriate value for each electrode shape (i. e. rail vs rod electrodes), a good agreement with the experimental data is obtained. For a given gap width, the transit velocity of the rod electrodes was higher than the rail electrodes due to the changes in arc morphology. The upward deflection of the arc suggest that the arc in rail electrodes affects larger volume of air than the rod electrodes.

It can be concluded that the snowplow term is the mechanism of momentum exchange between an accelerating arc and the surrounding air. Unfortunately, the large value of γ suggests that most of the air does not undergo inelastic collision with the arc, such that the momentum coupling between an accelerating arc and the air is weak. However, the arc reaches a high velocity (maximum of ~ 100 m/s) and dissipates its inertia during its rapid deceleration. If the studies on glid arcs is applied, then a decelerating arc transfers momentum through the relative velocity between the arc and the gas. Hence, the total

momentum transfer is much greater than the snowplow term.

7 Induced Velocity Measurement

The measurements so far concentrated on characterizing the arc. These include the force and impulse measurements to quantify the control authority of a RailPac, and modeling of the arc motion. However, it is unknown how the transit velocity of the arc relates to the induced velocity in the gas. Although the mechanical effects of a moving arc is hypothesized via the formation of compression and entrainment regions, a formal study to capture these effects have not been conducted. In addition to the pressure perturbations, high arc temperature of $\sim 10,000$ K measured by Gray et al. [40] on a RailPac suggests that thermal actuation is just as important as pressure disturbance in imparting momentum to the surrounding air. Indeed, induced velocity measurements on Glidarcs [33] under an externally imposed magnetic field (~ 1 T) illustrated thermal and pressure mechanisms evidenced by the formation of thermal convection layer and entrainment region around a propagating arc. Unfortunately, this study was limited to low arc current (40 mA) and low propagation velocity (10 m/s) contrary to the RailPac operating conditions ($\sim$ kA arc current and ~ 100 m/s propagation velocity). To date, no conclusive study has been performed to experimentally characterize the momentum exchange mechanisms of a propagating arc in the RailPac operating regime.

Capturing the velocity induced by the motion of a propagating arc under self-induced Lorentz force is challenging. While PIV has been used successfully to measure the induced velocity in other plasma actuators, the high arc temperature as well as intense radiation emitted by a thermal arc pose a unique difficulty in characterizing RailPac induced velocity. Past effort [34] has failed to obtain complete the flow field description around a RailPac due to vaporization of seed particles near the arc and the arc wake.

Recent studies on seedless PIV [73, 74] worked around the seeding issue by using natural refractive index gradients in the flow as seed particles. In the case of highly turbulent or thermal flows, the flow is naturally seeded by the distribution of coherent turbulent structures of varying scales that advect at the local convection velocity of the fluid. These turbulent structures can be captured using Schlieren technique and the local velocity of the fluid can be inferred from their motion between successive frames with a PIV algorithm.

One drawback of seedless PIV is the integrating property of Schlieren techniques across the flowfield. The image of turbulent structures captured by Schlieren techniques is spatially averaged along the light ray between the light source and the density gradient. If the flow exhibits substantial variation in velocity along the light ray, the measured velocity will deviate from the planar velocity measurement performed with a conventional PIV. This integrating effect can be minimized with the use of Focusing Schlieren technique [73, 75], which consists of a source grid, lens, and cutoff grid. This method differs from a conventional Schlieren system by using a broad array of multiple light sources rather than a single point source. A second grid is used to partially block the image of the light sources projected by a Schlieren lens. In this manner, the system can be considered as a superposition of multiple Schlieren systems, with each source and cutoff grid pair viewing different regions of the flow. By placing a density gradient between the source and Schlieren lens, the image of density gradient can be formed at a conjugate location opposite the Schlieren lens. The Schlieren

lens aperture limits the depth of field and the error due to flow variation along the optical path length can be minimized.

The proposed method applies seedless PIV in the arc wake of the RailPac, since the flow is highly turbulent with significant thermal mixing occurring throughout the region. In the measurement volume where the seed particles are present, a conventional PIV is used to determine the induced velocity. The complete flow field induced by a propagating arc can be analyzed with both the seedless and conventional PIV techniques.

7.1 Experimental Setup

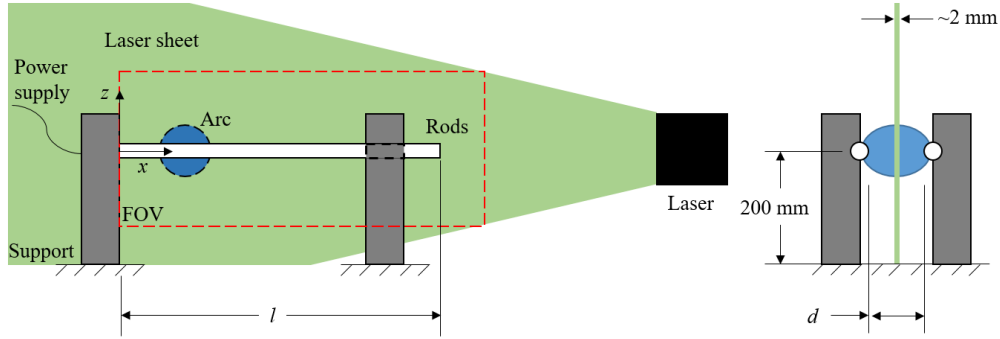


Figure 30: A RailPac prototype placed in a two dimensional PIV measurement setup.

The experiments were conducted on a prototype RailPac placed in two different optical measurement configurations: (a) PIV experimental setup consisting of a camera, a pulsed laser sheet, as well as a seeding chamber and (b) Focusing Schlieren setup. The prototype is a rod configuration with the electrodes spaced 12.7 mm apart (B13 configuration) shown in figure 30. They are suspended 200 mm above a table to minimize the interaction of the arc with the wall. The rods were connected to a long-discharge circuit, and the arc is initiated via a HV trigger.

7.1.1 Particle Imaging Velocimetry

The induced velocity was measured using PIV (figure 30). Glycol seed particles with manufacturer specified diameter and density of $1\text{ }\mu\text{m}$ and 1.1 g/cm^3 were uniformly distributed within the plexiglass chamber. A pulsed laser sheet (29 mJ/pulse, 400 ns pulse duration) illuminated the seed particles, and the particle motion was captured using a Phantom Miro M-310 camera with a 100 mm lens (f-stop 2.8). The field of view was $1024\text{ px} \times 632\text{ px}$ ($270\text{ mm} \times 165\text{ mm}$), and the frame rate was 5 kHz. The captured frame was subdivided into smaller regions called interrogation windows. Then, the ensemble motion of the particles in each interrogation window was inferred by cross-correlating the successive images.

PIV processing was performed with a commercial software (DaVis 8.3). The initial interrogation window size was set to $64\text{ px} \times 64\text{ px}$ with a 50% overlap and was refined to $32\text{ px} \times 32\text{ px}$ with a 75% overlap. The validity of the computed vectors were evaluated with the signal to noise ratio (SNR). If the SNR was below 1.4, the vector was discarded and its value was interpolated with the average of the nearest neighbors.

The sources of uncertainty associated with the PIV measurements include particle tracking error, plane alignment error, scaling error, and particle displacement uncertainty from the correlation algorithm [76]. The particle tracking error was found with the convection time scale defined by $\tau = l/U$, where U is the arc propagation velocity. Based on the maximum expected propagation velocity of 100 m/s, the tracking error was less than 1%. The plane alignment error was computed based on the alignment error of the camera CCD array plane with the axis of the rods ($\pm 5^\circ$). At 100 m/s, the expected error was $\pm 0.4\%$. The scaling error was found based on the precision of the ruler used for calibration. The particle displacement uncertainty of the correlation algorithm was found by measuring the velocity of the quiescent air and computing its variance. An uncertainty based on the 95% confidence interval was ± 0.5 m/s, such that the displacement uncertainty was dominant.

7.1.2 Focusing Schlieren

The passage of a high temperature arc results in vaporization of seed particles in the arc wake. Induced velocity cannot be determined in this region with conventional PIV due to the absence of seed particles. However, the arc wake region is characterized by highly turbulent flow with significant thermal density gradients that can be visualized using Focusing Schlieren technique. A seedless velocimetry can be performed on the Focusing Schlieren images to determine the induced velocity in the arc wake.

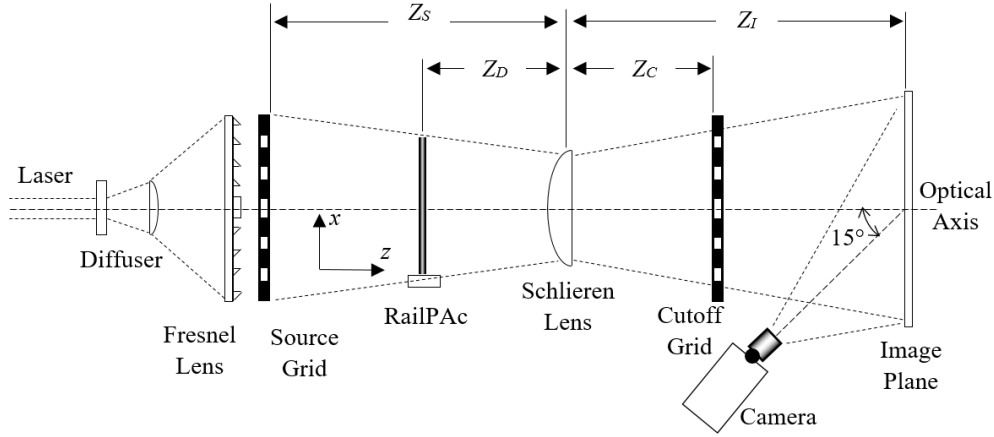


Figure 31: A schematic of a Focusing Schlieren setup.

As shown in figure 31, Focusing Schlieren setup is comprised of a light source (same pulsed laser used for PIV setup), Fresnel lens, source grid, cutoff grid, and an image plane. The design of this system was based on the approach suggested by Weinstein [75]. A collimated laser beam encounters a diffuser and a lens that expands the beam to fill a Fresnel lens. The Fresnel lens focuses the light to fill a Schlieren lens (focal length $f=600$ mm and $A=125$ mm aperture diameter) and the light is projected on to an image plane. The source grid and cutoff grid consist of parallel vertical stripes (oriented in the $\hat{e}_y$ -axis direction) that alternate between opaque and transparent segments. Cutoff grid had opaque stripes that were 0.8 mm wide ($2a$) and transparent segments with 3.7 mm width (b). Source grid was a magnified

photographic negative of the cutoff grid. These grids were made out of Lexan sheet with a painted cover. A laser cutter traced out the grid pattern on the cover, and the transparent sections were obtained by peeling the cover off of the Lexan sheet. The cutoff grid was mounted on a translation and rotation stage for fine adjustment of its position. This setup allowed easy adjustment of the grid to maintain 50% cutoff requirement.

Table 4: Relevant dimensions of a Focusing Schlieren setup

Parameter	Dimension [mm]
Z_S	2100
Z_I	2280
Z_D	815
Z_C	840

The relevant dimensions of the Focusing Schlieren are labelled in figure 31 and tabulated in Table 4. Note that two optical conjugate pairs exist in this setup: one between a pair of grids and another between density gradient image (RailPac) and the image plane. A high speed camera was focused on the image plane at an angle 15° off from the optical axis. A Scheimpflug adaptor mounted between the camera and camera lens (100 mm focal length) allowed the entire image plane to be in focus despite the rotation. Perspective distortion of the camera was corrected by calibrating the camera with a pinhole model. Images were taken at sampling rate of 3 kHz with two laser pulses separated by 100 μ s.

The sensitivity of Focusing Schlieren is given by [75],

$$\epsilon = \frac{aZ_S}{Z_C(Z_S - Z_D)} \quad (36)$$

Such that the sensitivity of the system was 16 arcsec. The depth of field of the system is defined by Weinstein [75] as,

$$DOF \propto \frac{Z_D}{A} \quad (37)$$

Where, the proportionality constant is related to a length scale of the flow structure.

The seedless velocimetry was performed as follows. Focusing Schlieren technique was used to obtain images of density gradients present in the flow. At each time step, two images of the density gradient were obtained separated by time between two laser pulses. PIV algorithm developed by LaVision was applied to the gray-scale images to track the displacement of density gradients present in the flow by comparing the motion of turbulent eddies between two laser pulses. An interrogation window size of 64 px $\times$ 64 px were used with 75% overlap. The flow velocity was computed based on the displacement and the time interval between successive laser pulses.

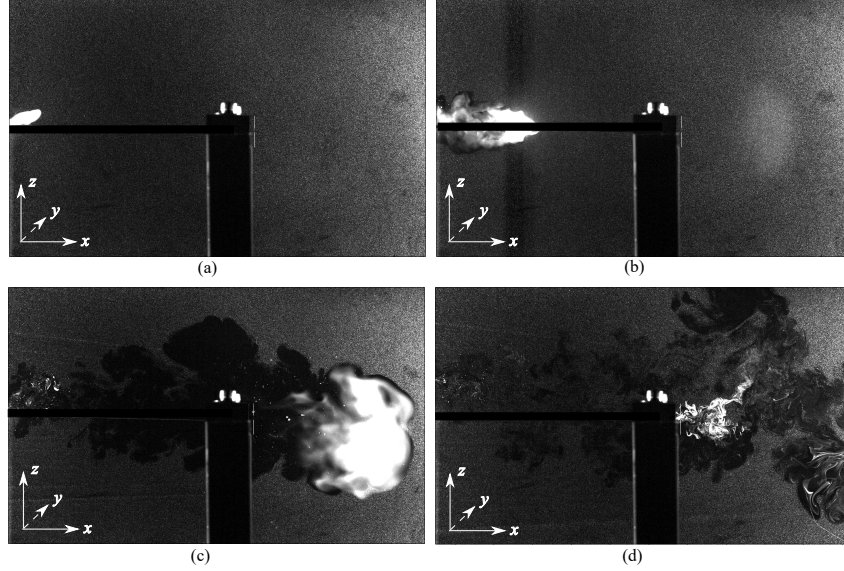


Figure 32: Propagation modes of the arc captured at the initial capacitor voltage of 250 V. The transit time is referenced from the spark formation time: (a) $t=0.6$ ms, (b) $t=1.4$ ms, (c) $t=9.4$ ms, (d) $t=48.4$ ms. The background of the arc is the image of particles used for PIV measurements.

7.2 Results

7.2.1 Particle Imaging Velocimetry

The arc is distinguishable from the background of the PIV video as a luminous region behind the rods. As shown in figure 32a, the initial transit of the arc is characterized by a small propagating arc. As the arc current increases, the dimension of the arc also increases (figure 32b). In addition, the high propagation velocity of the arc causes the arc shape to transform into an aerodynamic shape. A trailing wake forms, and the arc sheds luminous vapor into this region. When the arc reaches the end of the rods, the arc elongates from the rods (figure 32c). This process is characterized by the enlargement of the arc, as evidenced by the growth of the luminous region in the image. The change in the size of the arc is a result of the deceleration of the arc. When the arc velocity decreases, the dilatation effect due to high arc temperature is no longer confined to a small layer surrounding the arc. The arc then thermally expands as a result.

The passage of the arc vaporizes the seed particles in its path, and the wake (devoid of seed particles) shows a clear contrast with the surrounding air. After some time, the entrainment of nearby seed particles and metal vapor from the rod occupied the void in the wake (figure 32d). These observations demonstrate the capability of a propagating arc to disturb a quiescent air. In addition, the motion of seed particles, as well as the motion of metal vapors, hint at the formation of a strong jet in the aftermath of the arc motion.

The induced velocity measured with PIV shows a strong effect of compression and entrainment in the vicinity of a propagating arc. As seen in figures 33 and 34, a large region of air upstream of the arc experiences a radial flow disturbance due to compression effect. If a

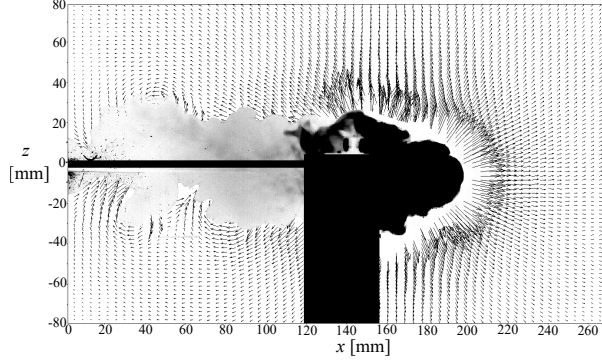


Figure 33: Induced velocity measurement with PIV, with vectors representing the magnitude and direction of the induced velocity. The measurement was performed at $V_c=250$ V at $t \sim 2$ ms. The color inverted image of the arc and the experimental setup is superimposed in the region where the velocity vector could not be obtained. Note that the absence of velocity vectors in the arc and its wake is due to the vaporization of seed particles resulting from the high arc temperature.

reference frame that moves with the arc is considered, then the radial flow field disturbance will result in the deflection of incident streamlines around the arc.

The velocity profile in the $\hat{e}_x$ -direction (u) for the compression region in front of the arc (figure 34a) has a maximum u of ~ 10 m/s, which was substantially lower than the arc velocity of ~ 50 m/s. This is attributed to the low coupling parameter (γ) seen in the arc transit analysis. The low u and v speeds in front of the arc suggest that the arc does not maintain a no mass flux condition at its boundary.

A reverse flow region develops behind the arc above the wake. This effect results in negative u velocity profile shown in region [B] of figure 34a. Sumner et. al. [77] observed a similar reverse flow region in their study of an impulsively started cylinder. They noted that the region of reverse flow around the wake is explained by the temporal development of a recirculating zone. This phenomenon is a result of low pressure wake in the immediate downstream location of the arc that develops a positive velocity within the wake. Meanwhile, a reverse flow region forms above the wake. The discrepancy in the direction of velocities results in a formation of a shear layer that prevents an immediate entrainment of the surrounding fluid. As a result of this shear layer, a large counter-rotating recirculation zones develops behind the arc.

The arc vaporizes the seed particles in its path and prevents the correlation algorithm from determining the velocity vectors at certain regions of the flow. The problem is especially prevalent when the arc reaches the end of the rods, where the thermal expansion and convection of the arc wake causes the particle void region to grow. While the entrainment of surrounding air leads to a gradual increase in seed particles in the wake, the random variations in the rate of entrainment and the extent of the wake prevented repeatable measurements of the induced velocity in the immediate aftermath of the transit. After $t \sim 25$ ms, a sufficient mixing has occurred that allows the velocity vectors to be determined in most locations in the view plane (figure 35). Note that the aftermath of the arc transit is characterized by a formation of a high momentum jet which is seen to convect downstream.

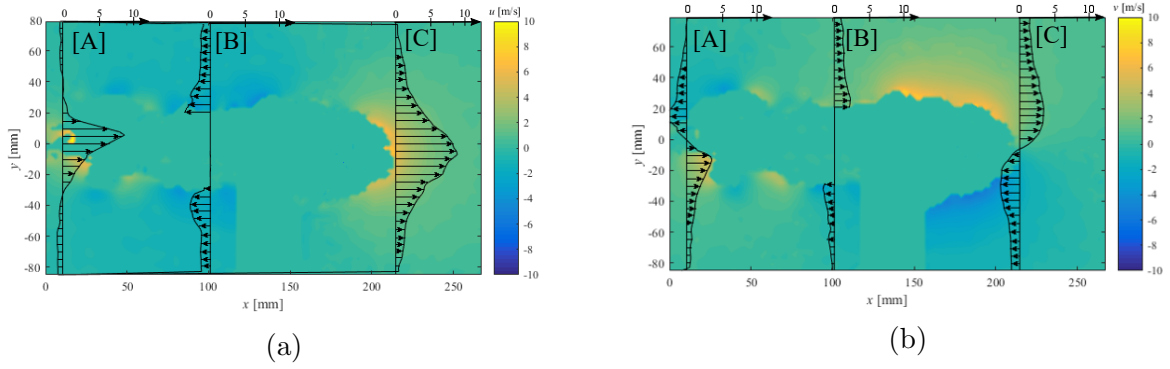


Figure 34: Induced velocity measurement with PIV, showing (a) the velocity component in the x -direction (u) and (b) the velocity component in the y -direction (v). The measurement was performed at $V_c=250$ V at $t \sim 2$ ms. Three velocity profiles are shown, depicting [A] entrainment, [B] reverse flow, and [C] compression effects. For region [B], the velocity profile cannot be obtained within the wake due to the vaporization of seed particles.

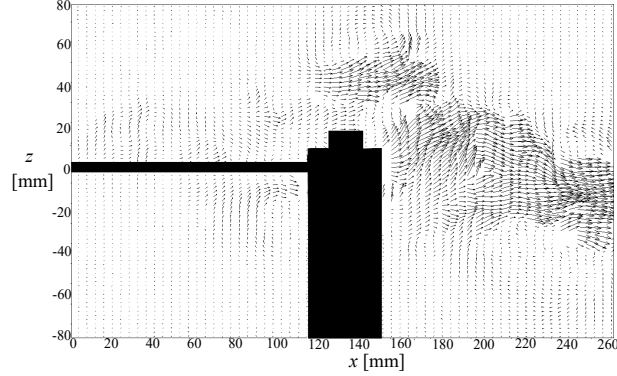


Figure 35: Induced velocity measurement with PIV, with vectors representing the magnitude and direction of the induced velocity. The measurement were performed at $V_c=250$ V at $t \sim 25$ ms. Note that the passage of the arc formed a strong jet that convects downstream in the direction of propagation.

The transient jet is a result of impulse generated by the Lorentz force.

7.2.2 Focusing Schlieren

A high temperature of the arc introduces thermal variations on the surrounding neutral air that become visible as density gradients in the Focusing Schlieren images. The initiation of the arc forms a weak pressure wave that radially propagates away from the initiation location (figure 36, $t = 0$ ms). The high propagation speed of the arc confines the thermal disturbance to a small region enveloping the arc and its wake. Note that the particle void region observed in the PIV images (figure 33) has approximately the same size as the size of thermal gradient region observed in the Focusing Schlieren images. This match reveals that the particle motion observed in the PIV images is not a result of thermal expansion due to arc heating, but a result of pressure and viscous effects.

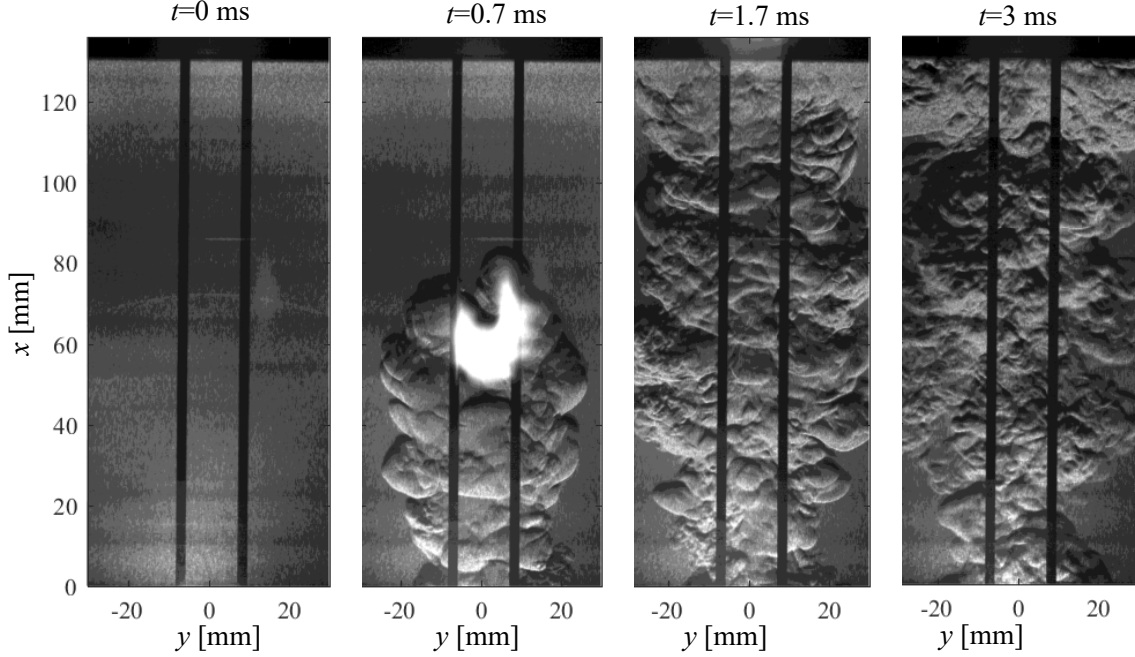


Figure 36: Focusing Schlieren images for $V_c=240$ V.

The outline of thermal wake behind the arc is marked by ridges visible as alternating pattern of positive and negative density gradients. The ridges coincide with the approximate locations where the neutral air begins to entrain into the arc wake region. The mixing between the low density air in the arc wake and high density air disseminate thermal gradient into small scale structures. This is clearly seen as uneven distribution of turbulent structures in space and time. For example, the scale of turbulent structure in the immediate vicinity of the arc (figure 36, $t = 0.7$ ms) can be characterized as ~ 20 mm, based on the distance between alternating bands of density gradient along the $\hat{e}_x$ -direction. This length decreases in the successive images ($t = 3$ and 10 ms) to ~ 1 mm, revealing the turbulent nature of the entrainment process.

The depth of field was tested by moving the RailPac to different distances (ΔZ_d) away from the focal plane of the setup. As shown in figure 37, density gradients are imaged even when the RailPac is translated a substantial distance ($\Delta Z_d = -100$ mm) towards the Schlieren lens, albeit with a significant blur. As the RailPac setup approaches the focal plane, the density gradients become sharper, with small scale eddies clearly visible when the RailPac is situated exactly at the focal plane of the Focusing Schlieren setup. This focusing ability reduces the integrating nature of the seedless velocimetry techniques because the limited size of the PIV interrogation window forced PIV to reject large scale (blurred) features of the image and focus on the motion of small scale features.

The comparison of successive Focusing Schlieren images allows the naked eye to detect the motion of turbulent eddies after the passage of the arc. The small turbulent structures are able to maintain their coherent features throughout multiple camera frames and allow the PIV algorithms to quantify their motion. Unfortunately, the uneven variations in the size of turbulent eddies prevent the PIV algorithm from obtaining velocity vectors throughout

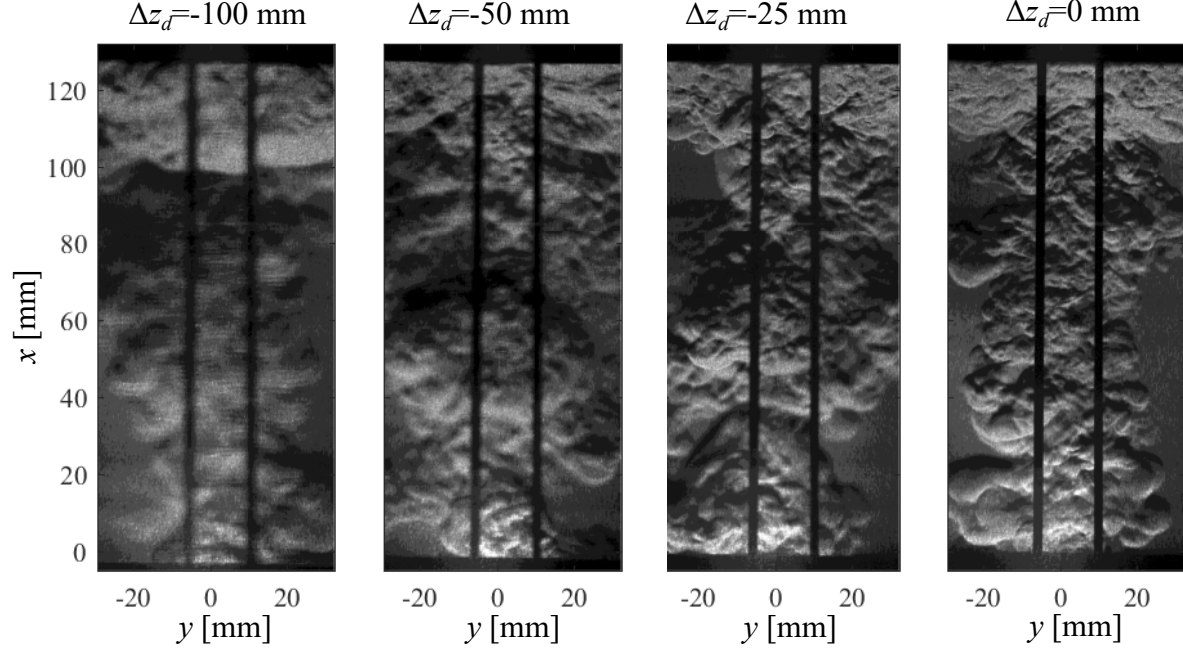


Figure 37: Depth of field demonstration viewing the RailPac thermal density gradients at different test setup distance (ΔZ_d) away from the focal plane

the measurement region. As shown in figure 38, the velocity vectors computed at $t = 2$ ms and $t = 17$ ms reveal intermittent regions where the induced velocity is close to zero. This error is a result of large scale turbulent structure that are characterized by a large region of uniform density outlined by sharp density gradient. Due to the small interrogation size, PIV tracks the individual deformation of large turbulent eddies along its outline, rather than their overall convection motion. Therefore, seedless PIV has an optimal range of length scales over which the motion of turbulent eddies can be characterized as Lagrangian flow descriptors. In the context of induced velocity measurement on the RailPac, this length scale requirement limits the measurement time to within a certain window. Velocity measurement cannot be made reliably during the arc motion due to the large length scale of the thermal disturbance. In addition, velocity measurements cannot be made at substantial time past the arc motion as a result of diffusion process that decrease the sharpness of the density gradient. It was found that $t > \sim 2$ ms and $t < \sim 30$ ms gave reasonable number of velocity vector results for the RailPac induced velocity measurements.

The induced velocity in the arc wake is not directed uniformly along the arc propagation direction. The regions in close proximity to the arc experience large reverse flow due to thermal expansion from the arc heating. This phenomenon delays the entrainment process and decreases the induced velocity along the propagation direction. Note also that the induced velocity is not uniform across $\hat{e}_y$ -direction. This variation is not systematic and is a consequence of turbulent eddy deformation. To minimize the effect of the random motion, the induced velocity was averaged along the $\hat{e}_y$ -direction for each x location along the rods. The maximum induced velocity at each time was found and averaged over six results at each V_c (figure 39).

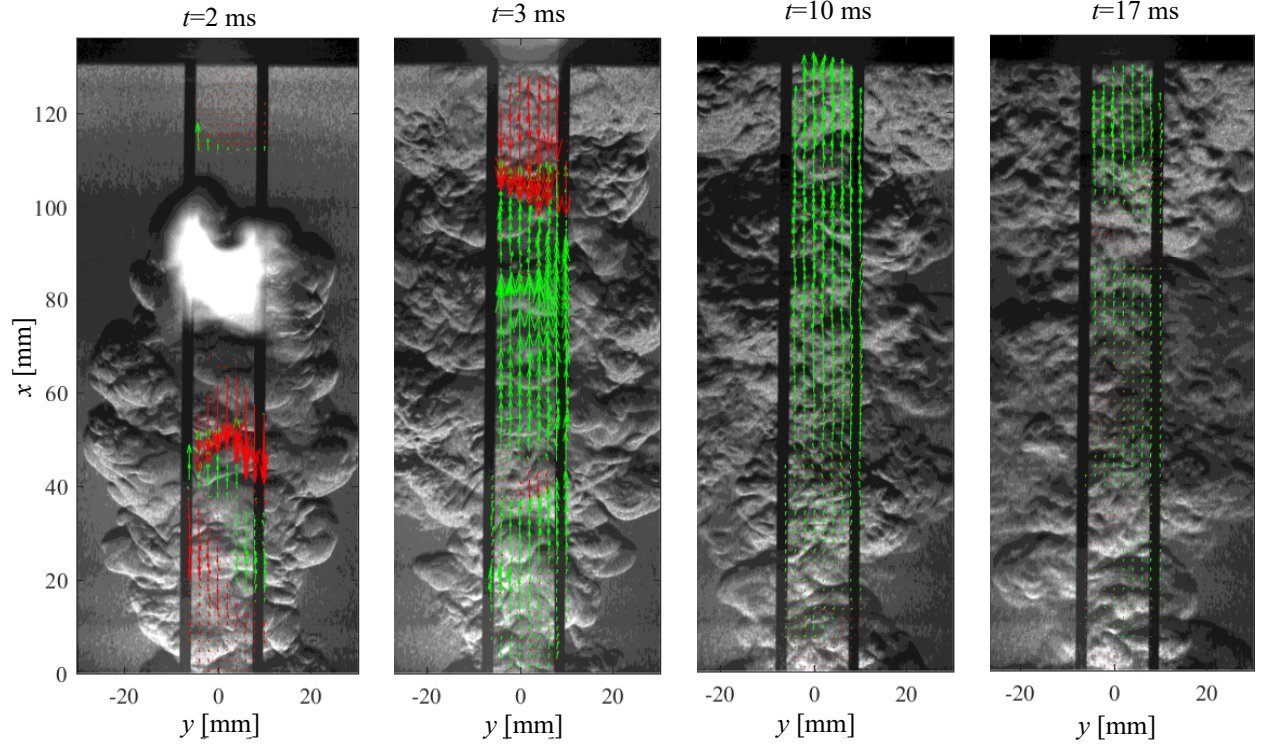


Figure 38: Secondary PIV results on Focusing Schlieren images of $V_c=240$ V case. Note that green vectors refer to velocity directed along the propagation direction, while red vectors indicate reverse flow.

The large fluctuations in the maximum induced velocity measurement near the arc initiation ($t = 0$ ms) are a result of the deformation of the turbulent structure rather than the overall convection motion. The trend of the data suggests that the induced velocity in general increases with increased initial capacitor voltage, especially at short time after the arc transit. The induced velocity, however, appears to converge irrespective of the initial capacitor voltage ~ 15 ms after the transit. The convergence can be explained by the lack of body force or pressure gradient in the flow some time after the arc transit, such that the velocity measured is the action of viscous diffusion, rather than forced convection.

A direct one-to-one comparison of the induced velocity between the PIV and seedless PIV techniques could not be performed in this paper. Highly transient nature of the induced flow requires that the two techniques be used simultaneously to obtain a one to one comparison, rather than conducting the experiments in separate setups. In addition, the regions where PIV cannot obtain transient velocity are most optimal in terms of turbulent length scales and sharpness of density gradient images for seedless PIV. The transient jet shown in figure 35 is not observed in seedless PIV as a result of limited field of view. The maximum directed convection velocity in the arc wake region based on seedless PIV is ~ 20 m/s, while the measured maximum entrainment velocity is ~ 10 m/s. Further work is required to verify whether the discrepancy is a physical phenomenon or a result of measurement error.

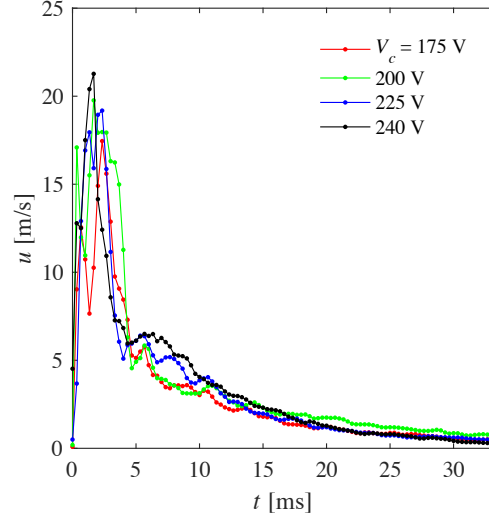


Figure 39: Maximum induced velocity (u) measured along the rods at each time (t).

7.3 Summary and Conclusion

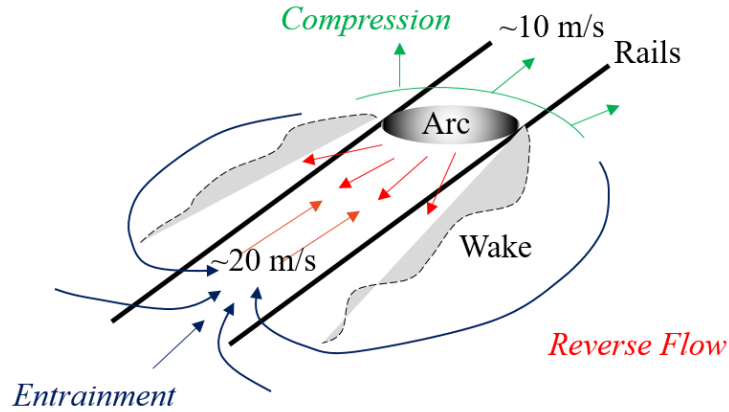


Figure 40: A schematic of the flow-field around a moving arc.

A summary of the measurement is provided as a schematic in figure 40. Focusing Schlieren revealed that the passage of the arc results in a highly stratified flow, separated by a low density arc wake and high density neutral air. The development of shear layer from the difference in induced velocity between the arc wake and the surrounding air confines the dilatation effect to a small layer surrounding the arc and its wake. The mixing of this shear layer is facilitated by turbulent entrainment of surrounding neutral air into the arc wake.

Induced velocity measurements with PIV captured the compression and entrainment effects, as well as the development of a strong jet in the aftermath of the arc transit. A large region in front of the arc transit experiences pushing due to compression. The arc transit forms large counter-rotating recirculation zones that aid in the entrainment of surrounding fluid into the arc wake. The impulse delivered by a moving arc result in the formation of a

transient jet downstream of the rods.

In addition to PIV, seedless PIV based on Focusing Schlieren technique was applied to the arc wake to visualize the flow field in the absence of seed particles. The motion of the arc induce convection velocity ~ 20 m/s in the direction of propagation within the arc wake, despite the initial reverse flow resulting from the thermal expansion from arc heating. A low induced velocity continue to propagate along the rods ~ 15 ms after the arc transit.

While seedless PIV quantified the induced velocity based on the motion of turbulent structures in the arc wake, further experimental verification is required to quantify the measurement error inherent to this method. In particular, the ambiguity in the motion between convection and deformation of turbulent eddies must be resolved with a different correlation algorithm, which will greatly enhance the robustness of the seedless PIV technique.

8 Stall Alleviation Demonstration

Measurements conducted on a prototype RailPac in a bench-top experiment provided much needed insight behind a moving arc and its interaction with the surrounding air. In particular, the actuation authority of a RailPac and the velocity induced by a moving arc were characterized. However, the bench-top studies have been conducted in quiescent air with a uniform pressure field, which is contrary to the flow condition around an aerodynamic body. If the boundary layer over a wing is attached, the pressure and viscous forces near the wall enforce the Kutta condition, allowing the flow to follow the contour of the wing. A pressure gradient is generated, and the surface integral of the pressure distribution is the origin of lift around a wing. If a strong adverse pressure gradient is generated on a wing, the boundary layer is no longer attached, and the Kutta condition cannot be satisfied. This results in a precipitous drop in the lift-to-drag ratio, and the aerodynamic efficiency of a wing is lost. It can be argued that the a goal of an actuator is to adjust the pressure gradient around a wing, such that the Kutta condition is maintained over a large flight envelope. Thus, it is necessary to examine if the control authority of a RailPac is sufficient to change the pressure gradient over a wing.

The objective of this section is to study a RailPac in the context of separation control. This is achieved by mounting a prototype RailPac on a VR-12 wing and conducting the experiment at various Reynolds number (Re) and angles-of-attack (α). As the operating condition is varied from pre-stall to post-stall regimes, measurements are conducted as the RailPac is operated over the wing. Electrical measurements and high-speed imaging are employed to characterized the arc. In addition, PIV gives insight into the reattachment process. Lift measurements are conducted to quantify the improvement in the aerodynamic performance of a VR-12 wing.

8.1 Experimental Setup

The experiments were conducted in an open test section and a closed circuit wind tunnel (figure 41) at the University of Texas at Austin. The maximum free-stream velocity is 20 m/s and the test section measures 0.56 m by 0.91 m. Two mounting arms with force balances position the wing at the center of the test chamber. The force balances are constructed out

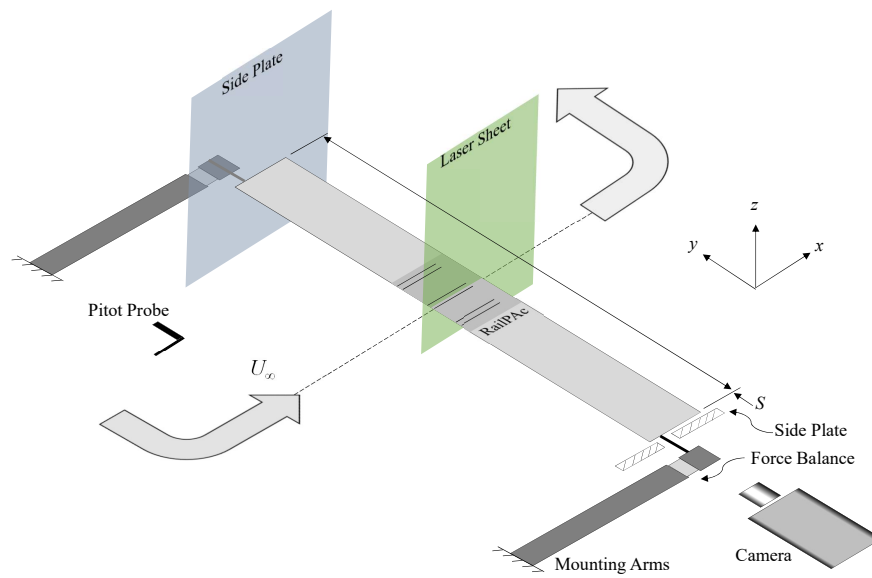


Figure 41: A wind tunnel schematic showing the VR-12 test article, PIV, and force balance setup.

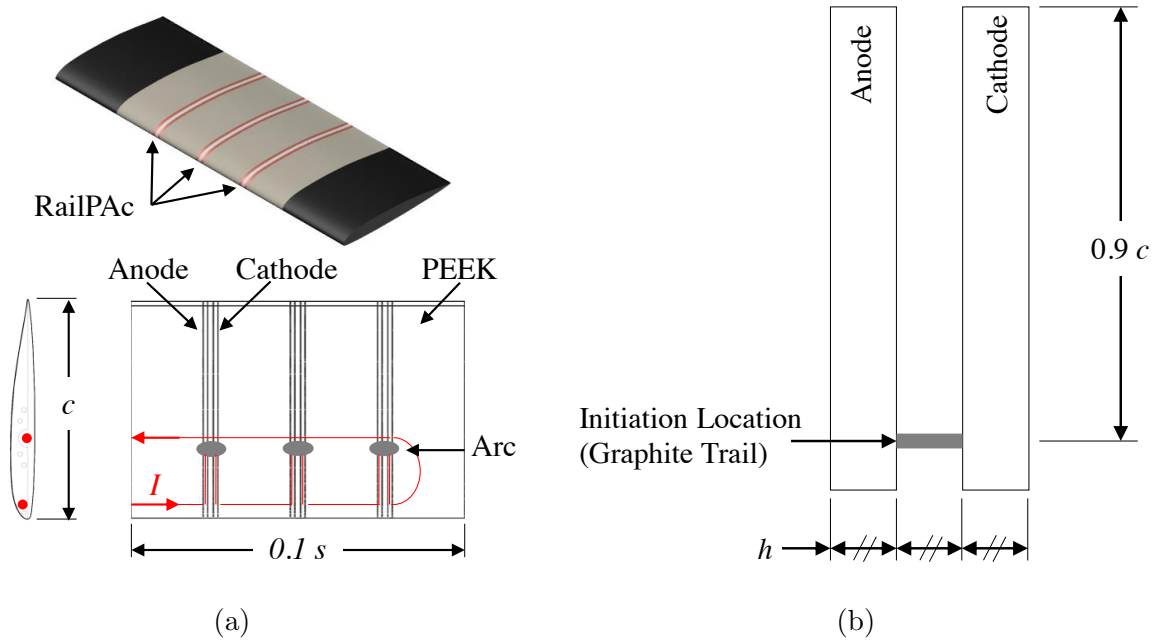


Figure 42: (a) A schematic of the RailPac array mounted on a VR-12 wing and (b) rail configuration of a single RailPac.

of strain gauges mounted on brass flexures that are only sensitive to the force applied in the lift direction (z -direction). Note that the two side plates (polycarbonate sheets) at the ends of the wing prevent formation of wing tip vortices. The side plates are structurally decoupled from the wing, such that the aerodynamic forces acting on the side plates are not measured by the force balances.

The test article is a VR-12 wing (chord (c) = 79 mm and span (s) = 1020 mm) comprised of a foam core with carbon fiber layers (figure 42). Note that the three RailPacs are mounted along the span direction (y -direction) in a notch built into the mid-span location. This array covers $\sim 10\%$ of the wing span. Each RailPac consists of copper anode and Elkonite cathode rails (2 mm wide with 2 mm spacing between rails) flush mounted to the top surface of the wing. The rails permit the arc to transit from a location close to the leading edge ($y/c \sim 10\%$) all the way to the trailing edge. A ceramic insert is present between the rails to electrically isolate the anode from the cathode. The PEEK inserts separate each RailPac from one another with a 25.4 mm gap. Electrical wires run underneath the PEEK inserts to connect the cathode of one RailPac to the anode of the adjacent RailPac (i. e., the RailPacs are connected electrically in series). A notch built into the foam core allows the RailPac power supply to connect to the RailPac array from the mounting end of the wing. Figure 42a shows the top view and side view of the current flow.

The free-stream velocity was measured using a pitot-static probe. The Reynolds number based on chord were 45,000 and 90,000, corresponding to the free-stream velocities of 10 m/s and 20 m/s, respectively. The angle of attack (α), measured with an inclinometer attached to the wing spar, can be varied from 0° to 30° .

8.1.1 Flow Visualization

Both qualitative and quantitative flow visualizations were performed by introducing Glycol seed particles ($1\mu\text{m}$ diameter and $\rho/\rho_a \sim 1.1$) into the wind tunnel. The motion of the seed particles was captured in the xz -plane using a Phantom Miro M-310 camera fitted with 100 mm lens. The framing rate was 4000 fps and viewing window was 2 by 1.5 chord lengths. A particle imaging velocimetry (PIV) was performed by projecting a ~ 2 mm thick laser sheet on the top surface of the wing with the laser sheet centered on the mid-span RailPac gap (Fig. 41). The sheet was formed by redirecting a 29 mJ/pulse Nd:YLF laser beam from the ground to the top of the windtunnel via system of lenses and mirrors. The laser was pulsed synchronously with the camera frames.

8.1.2 RailPac Circuit

The RailPac circuit is comprised of a HV trigger and a short-discharge circuit. The HV pulse creates a breakdown along the graphite trail on the outermost RailPac gap, eventually leading to a sequential break down of all RailPac gaps in the array. Note that the length of the electrodes is shortened compared to the bench-top experiments due to the airfoil chord. Thus, the external inductor is removed from the circuit to ensure that the arc does not reach the end and damage the electrodes.

8.1.3 Transit Video

The arc transit motion was captured from above using a Phantom Miro M-310 camera. A high f-stop ($f/30$) lens with 100 mm focal length and small exposure time ($1\mu\text{s}$) prevented image saturation from the high intensity arc. A band-pass optical filter centered at the wavelength of 650 nm filtered out the PIV laser (532 nm). The camera frame rate was 20,000 fps, and the camera was oriented such that all three RailPacs could be captured simultaneously. The arc position as a function of transit time was found by locating the region of highest intensity along the rail gap. The results characterized the arc velocity as well as changes in the arc shape.

8.2 Results

The experimental results were obtained by operating the wind tunnel at two different velocities. Table 5 summarizes the RailPac operating conditions based on the actuation authority experiments conducted in the bench-top experiments. The momentum coefficient (C_μ) was defined as,

$$C_\mu = \frac{|F_B|_{max}}{\rho U_\infty^2 hc} = \frac{0.5 L' I_{max}^2}{\rho U_\infty^2 hc} \quad (38)$$

Where, the max current measured was 1.4 kA. The actuation time scale, τ_a was found by normalizing the discharge duration by the convection time.

The test matrix (Table 6) summarizes the free-stream velocity and angles of attack explored in this study.

Table 5: Summary of the experimental conditions.

Parameter	$Re \sim 45,000$	$Re \sim 90,000$
U_∞	10 m/s	20 m/s
T_∞	20°C	22°C
P_∞	106.7 kPa	106.2 kPa
C_μ	15	4
τ_a	0.13	0.25

Table 6: Summary of the free-stream velocity and Reynolds numbers.

α	$Re \sim 45,000$	$Re \sim 90,000$
0°	X	X
4°	X	X
8°	X	X
10°	X	X
12°	X	X
14°		X
17°	X	X
22°	X	X
26°	X	X
30°	X	

8.2.1 Transit Video

The arc structure and motion were characterized using high-speed images of the arc transit (figure 43). The arc structure can be described as a luminous filament bridging the gap between the rails. The luminosity is not uniform across the gap with highest luminosity found near the electrode attachment points (arc roots). Note that the arc is not confined in the rail gap region and extends beyond the span-wise width of the RailPac. This structure reveals that the arc interacts with a large volume of air, even on the wing area not directly covered by the RailPac.

The cathode root is observed to transit upto $x/c \sim 0.7$, such that the arc transit covers $\sim 60\%$ of the wing chord. The transit velocity was obtained by tracking the position of highest arc luminosity along the rail. The average arc transit velocities were 54 ± 1 m/s and 58 ± 1 m/s for $U_\infty = 10$ m/s and 20 m/s, respectively. These average velocities were obtained with 90 RailPac operations for each free-stream velocity. Although the increase in free-stream velocity leads to a small increase in transit velocity, the effect is small compared to the variation of individual velocity measurement, which was found to be ± 10 m/s using two standard deviation interval. The transit velocity did not vary with respect to the angle of attack, revealing that the arc transit is insensitive to the surface pressure distribution of the wing.

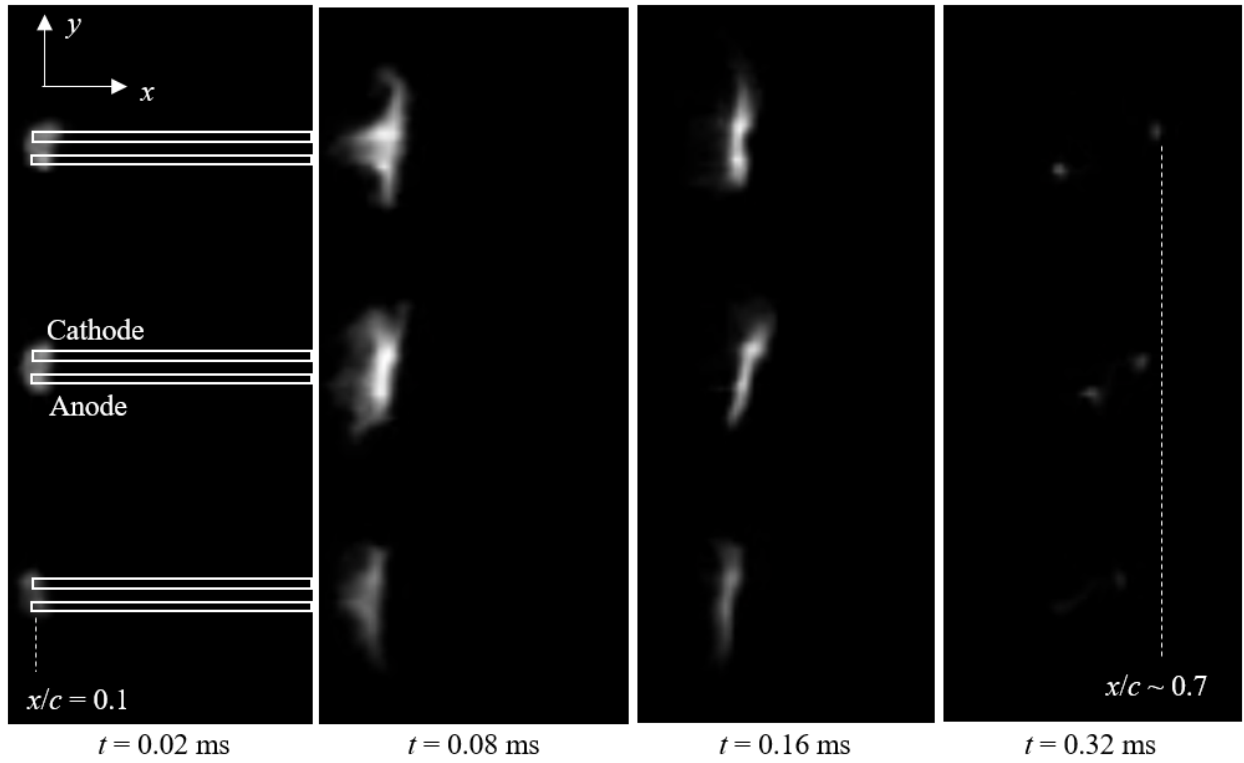


Figure 43: High-speed images of the arc transit. Time t is referenced from the arc initiation time at $t = 0$ ms.

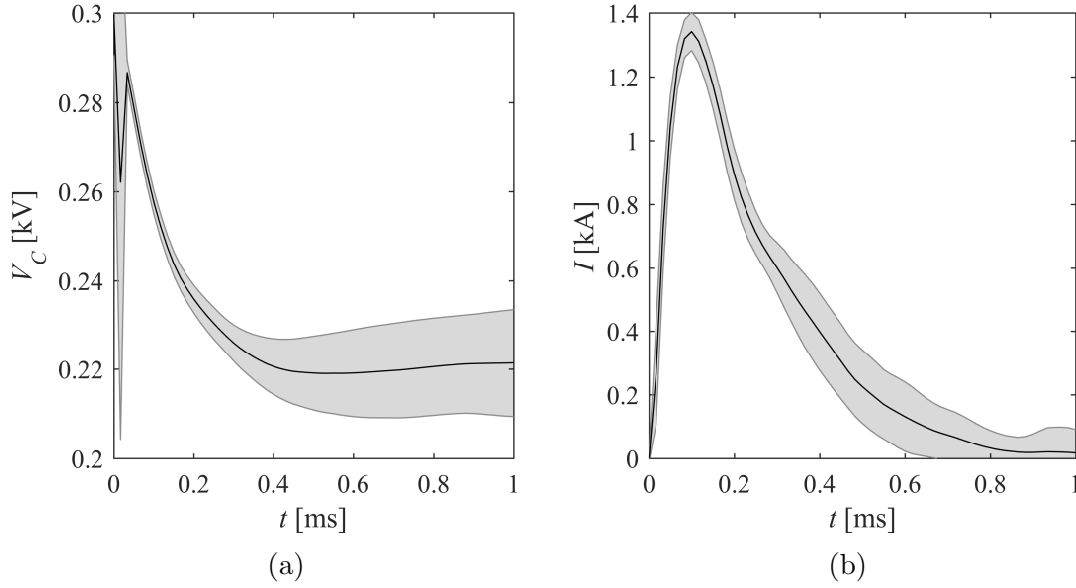


Figure 44: Mean electrical measurement on a RailPac power supply showing (a) capacitor voltage (V_C) and (b) discharge current (I) as a function of time. The uncertainty bounds represent one standard deviation of mean measurement.

8.2.2 Electrical Measurements

The electrical measurements (figure 44) show that the arc current is delivered via a short duration pulse (~ 0.5 ms). The peak current is 1.3 kA at the initial capacitor voltage of 300 V. The instantaneous power can be computed by multiplying the capacitor voltage with the arc current. The peak electrical power is 350 kW. Although instantaneous power is large, the total energy discharged is 22 J per RailPac actuator per pulse. Although the initial capacitor voltage is constant throughout the experiment, the discharge voltage and current varies between each run. In particular, the variation between experiments becomes more significant towards the end of the arc transit, in comparison to the time shortly after the arc initiation. It can be concluded that the electrical characteristics of the arc are repeatable if the arc motion is coherent (shown by initial transit stages in figure 43), while the measurement is not repeatable if the arc exhibits anchoring.

8.2.3 Flow Visualization

The high intensity of arc precludes measurement of velocity vectors during the arc transit using PIV. However, the effect of a moving arc on the separated flow can be qualitatively assessed from the particle images. The passive wing (figure 45a) is in deep stall with separation occurring at the leading edge. A shear layer forms between the free-stream region and the separated region. The RailPac generates an arc that extends to a significant vertical distance above the wing surface (figure 45b). The high temperature of the arc vaporizes the seed particles causing dark patches to appear above the wing (figure 45c). These patches advect at the local convection velocity and move upward due to thermal convection. Free-

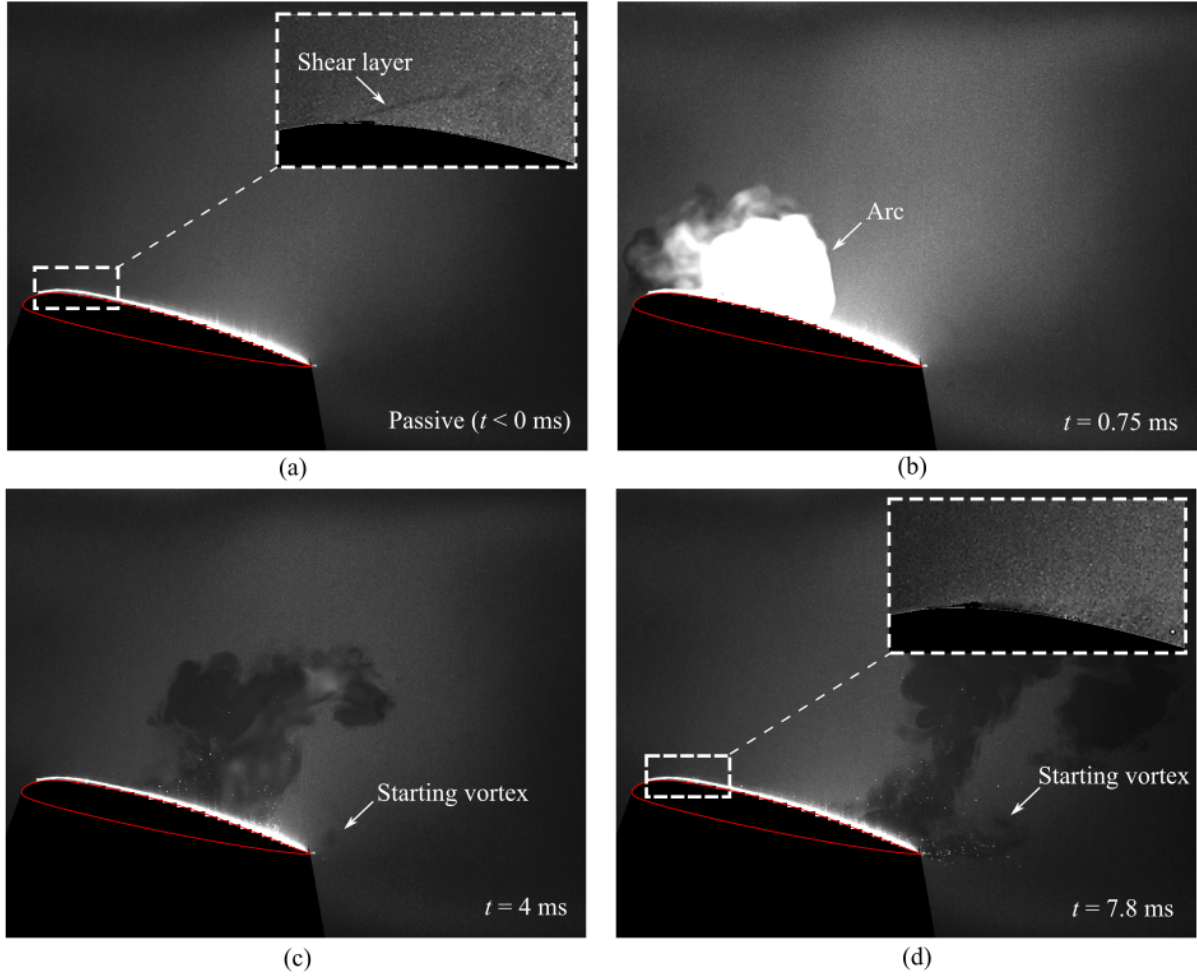


Figure 45: Side view of the arc transit and seed particles ($Re = 45,000$ and $\alpha = 12^\circ$)

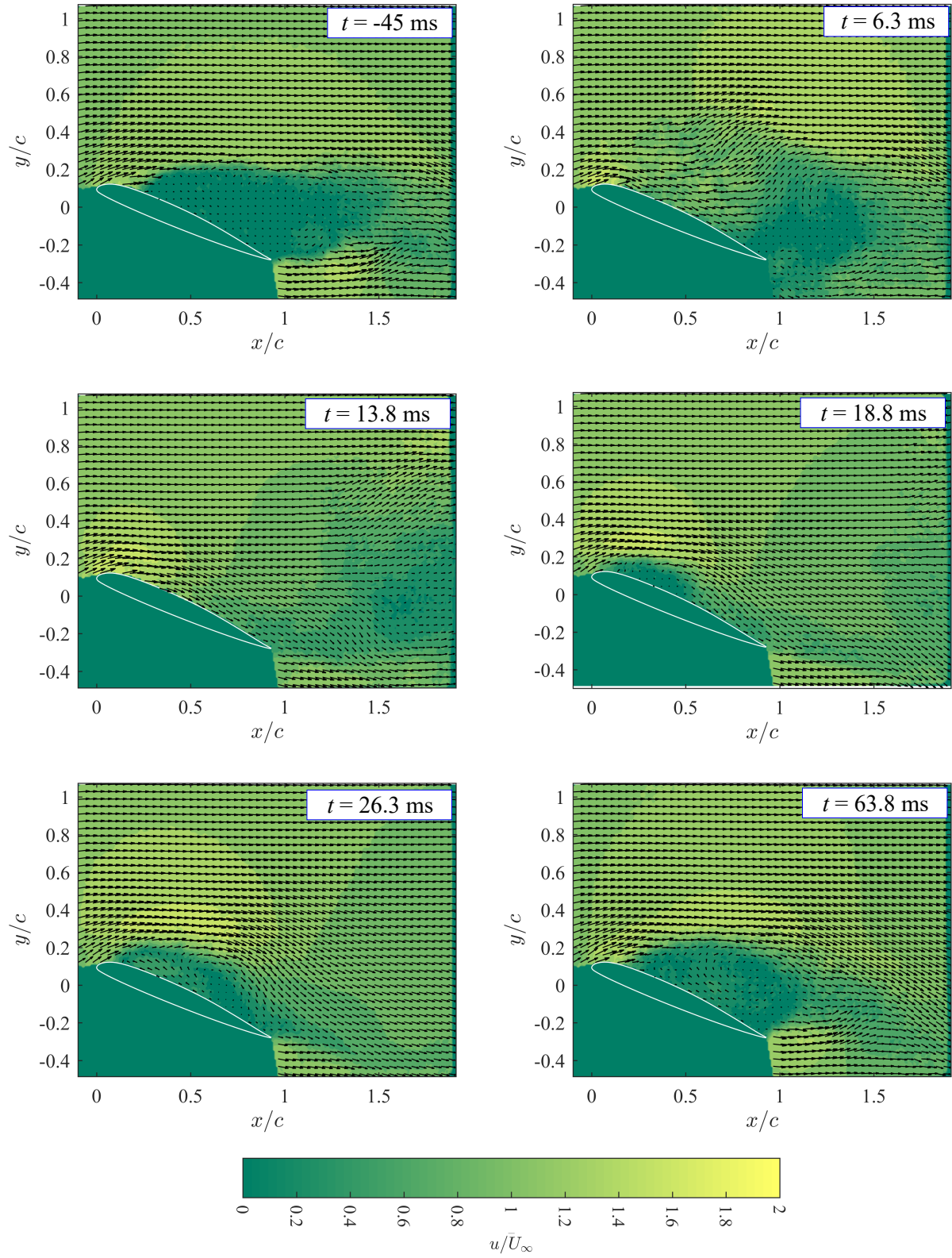


Figure 46: Velocity fields for $U_\infty = 10$ m/s and $\alpha=22^\circ$ obtained with moving averages of 10 successive frames.

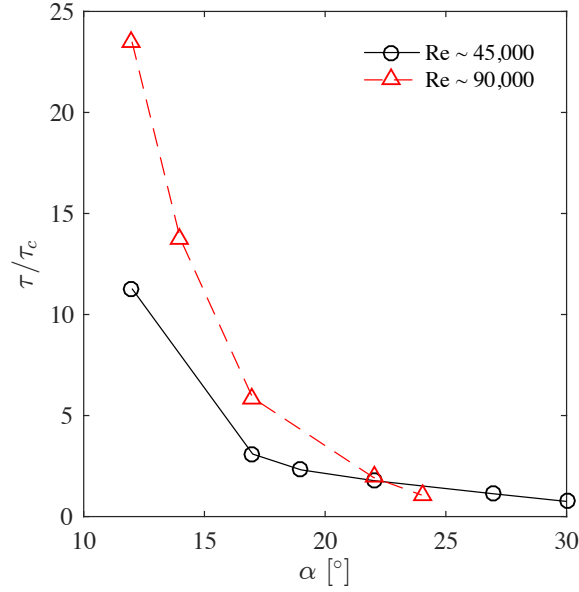


Figure 47: Reattachment time (τ) normalized by convection time (τ_c) for different Reynolds numbers (Re) and angles of attack (α).

stream fluid is entrained behind the patches and the separation above the entire wing is suppressed (figure 45d). This process is similar to the dynamic reattachment observed by Greenblatt et. al. [4]. They proposed that active flow actuators reattach flow by removing low momentum fluid from the surface and replacing the lost fluid with the free-stream fluid. In the case of RailPac, the entrainment of free-stream flow is due to the combined effects of Lorentz force and the high-temperature wake behind the arc. This process energizes the boundary layer and promotes reattachment. The formation of a starting vortex signifies the recovery of the bound circulation.

The motion of the arc on the wing surface induces large flow-field disturbance as a result of flow attachment and separation. The change in the velocity field over the wing is captured using a high-speed PIV. As shown in figure 46, the wing is initially in deep stall with the flow separating near the leading edge. The separation region is characterized by a large vortex above the trailing edge and a reverse flow that extend over the upper surface. Shortly after the actuation, the free-stream flow is entrained into the separated region, and the reattachment occurs. At $t = 13.8$ ms, the flow is attached over the wing and a large increase in measured velocity is observed at the location of maximum thickness. As a result of short actuation time, the wing enters into stall. The high adverse pressure gradient leads to the development of a separation bubble at the location of maximum thickness. A vortex near the leading edge begins to grow and continue promote flow reattachment at the trailing end of the wing. The vortex eventually separates from the leading edge, convects downstream, and the wing enters into deep stall. The unsteady flow persists for ~ 100 ms in the form of large scale vortices that shed at the end of the wing. These structures lead to wake oscillations that decrease over time, and the wing resumes its passive condition. These remarkable phenomena are observed at all post-stall angles of attack, albeit at different time

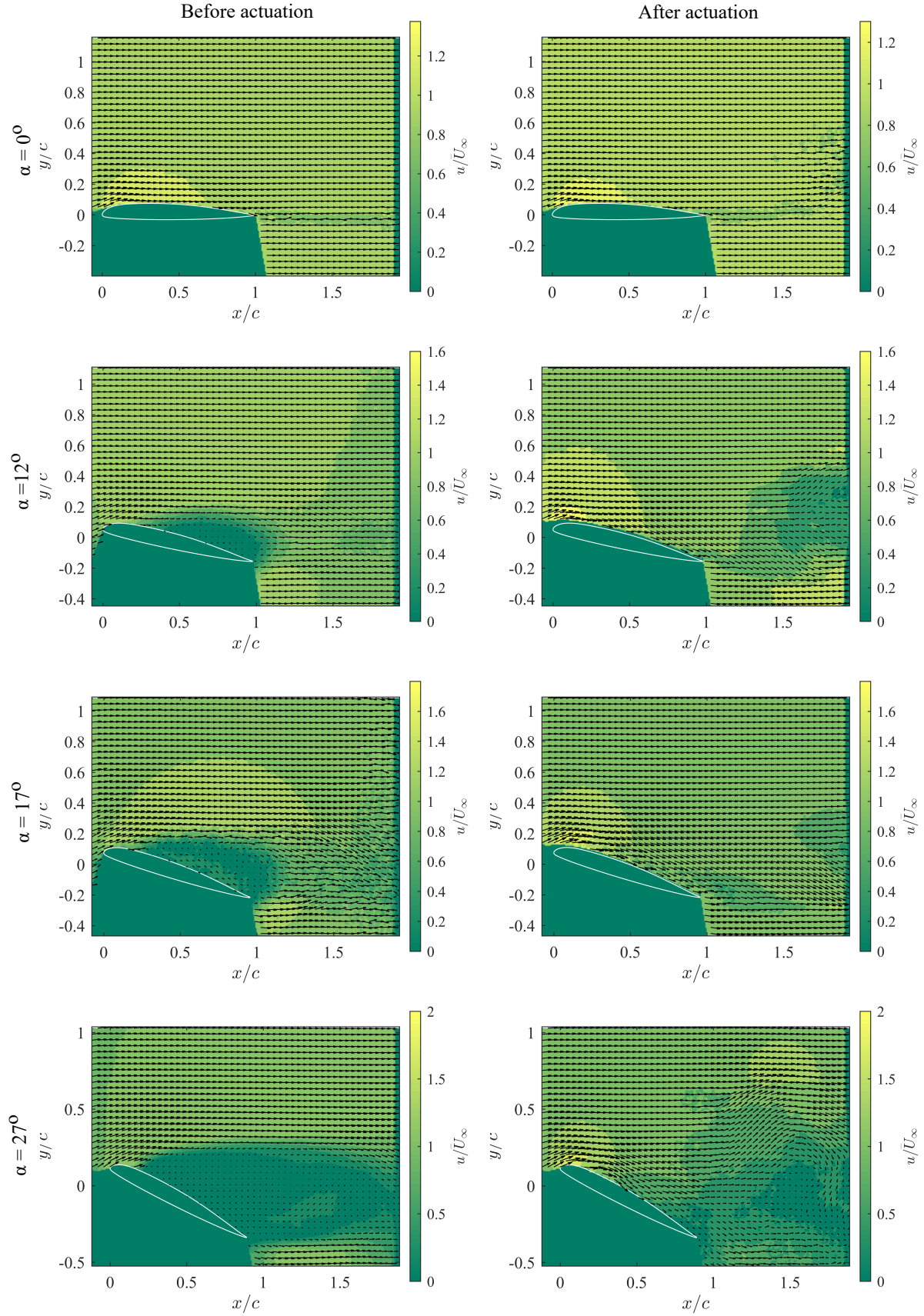


Figure 48: PIV measurement before actuation and one time scale after actuation at $Re \sim 45,000$.

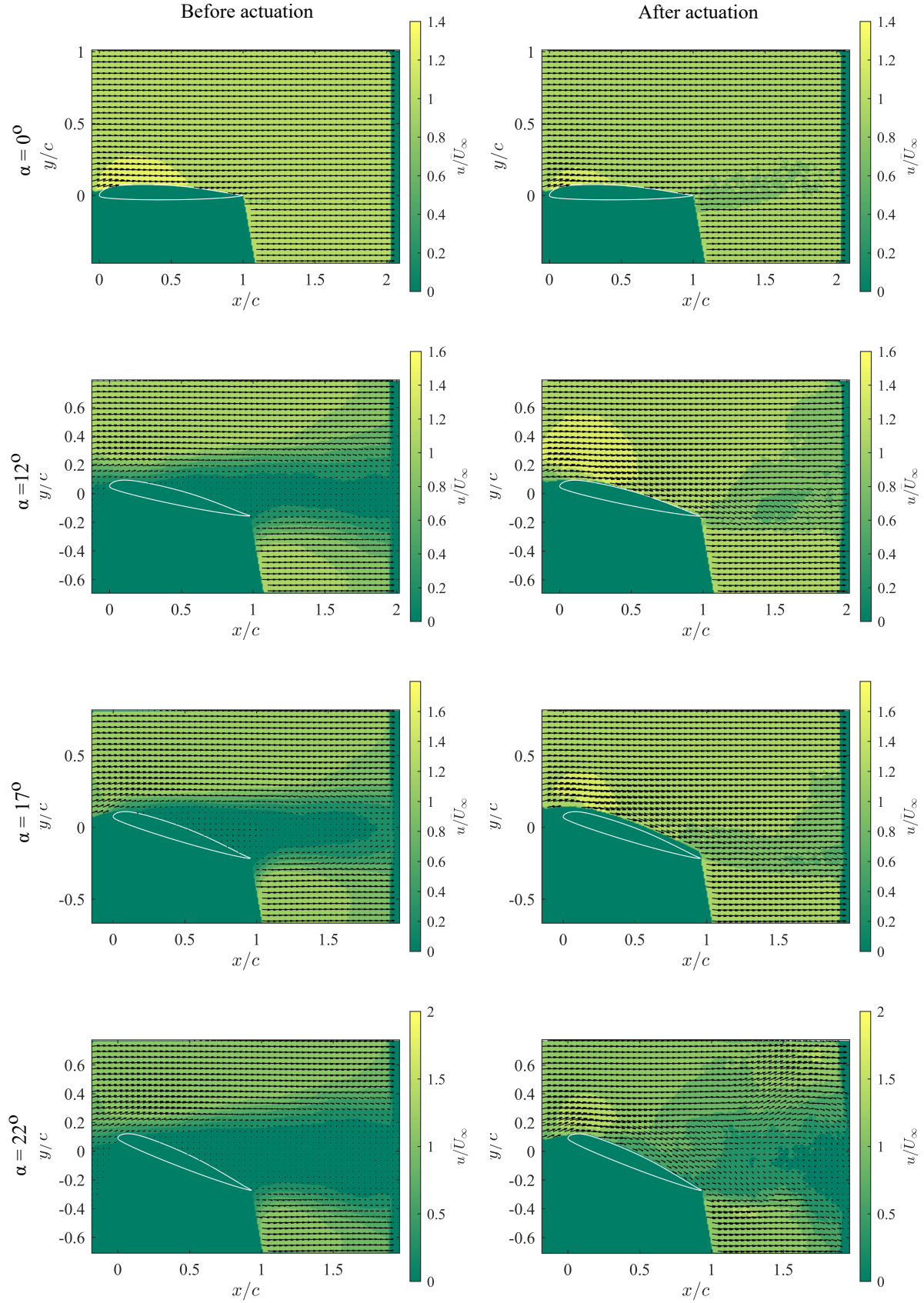


Figure 49: PIV measurement before actuation and one time scale after actuation at $Re \sim 90,000$

scales.

The flow over the wing remains reattached for a finite duration (referred to as reattachment time, τ). The reattachment time was measured by examining the changes in the shear layer (figure 45a). The passive wing develops a shear layer with a particular angle with the wing surface. After actuation, the angle decreases until the shear layer is suppressed due to flow reattachment. After some time, the flow separation causes the angle to increase until the shear layer resumes its passive condition. The duration between the RailPac initiation and the instance at which the shear layer returns to its passive condition is defined as the reattachment time. These times were obtained using high-speed images of the seed particles at various angles of attack and Reynolds numbers (figure 47).

The reattachment time decreases with increasing angle of attack; however, it is insensitive to the Reynolds number. If the reattachment time is normalized by the convection time (figure 47), the normalized time is smaller for lower Reynolds number than for the higher Reynolds number. In fact, the normalized time for $Re \sim 45,000$ is half that of the $Re \sim 90,000$ due to the two-fold difference in the convection time scales. These observations reveal that the reattachment time is strongly influenced by the strength of the adverse pressure gradient, and insensitive to the free-stream condition for low Reynolds number flows. Note that the strength of adverse pressure gradient rises dramatically with increasing angle of attack, while the pressure gradient does not differ significantly between the two Reynolds numbers. Hence, the RailPac actuation depends strongly on the pressure distribution over the wing surface at low Reynolds number regimes.

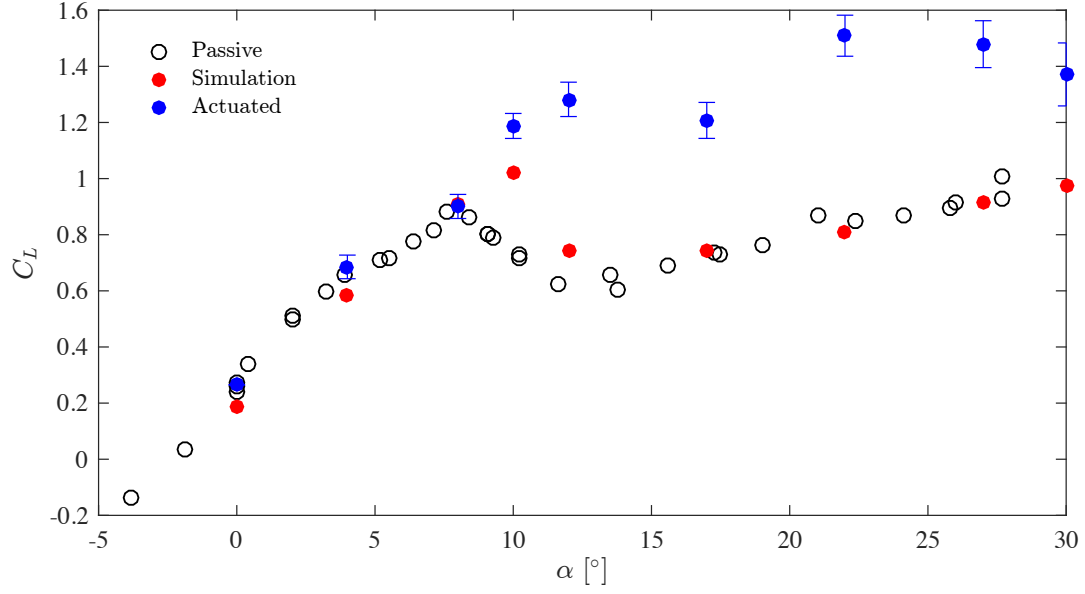
The RailPac is able to achieve flow reattachment at various angles of attack without exerting adverse effects on the passive wing. Figures 48 and 49 show the flow over the passive and actuated wing at different angles of attack for $Re \sim 45,000$ and $Re \sim 90,000$, respectively. At pre-stall angles of attack, the high-temperature fluid introduced by the moving arc momentarily disturbs the wing wake. The flow over the wing remains attached albeit with small reduction in the velocity magnitude. This velocity change is attributed to the rail vaporization that ejects large particles into the flow. A bias error from the non-uniform sizes of the seed particles manifests as a small change in the measured velocity before and after actuation at pre-stall conditions.

The RailPac can reattach flow at high angles of attack, although full reattachment of the flow over the entire wing surface is possible for angles of attack upto $\alpha \sim 17^\circ$. At higher angles of attack, the flow reattachment occurs over a portion of the wing before the onset of stall. At all angles of attack, a significant increase in the flow velocity over the wing is observed in addition to the reduction in the size of the wake. These observations reveal that the RailPac actuation at post-stall angles of attack results in lift enhancement and drag reduction.

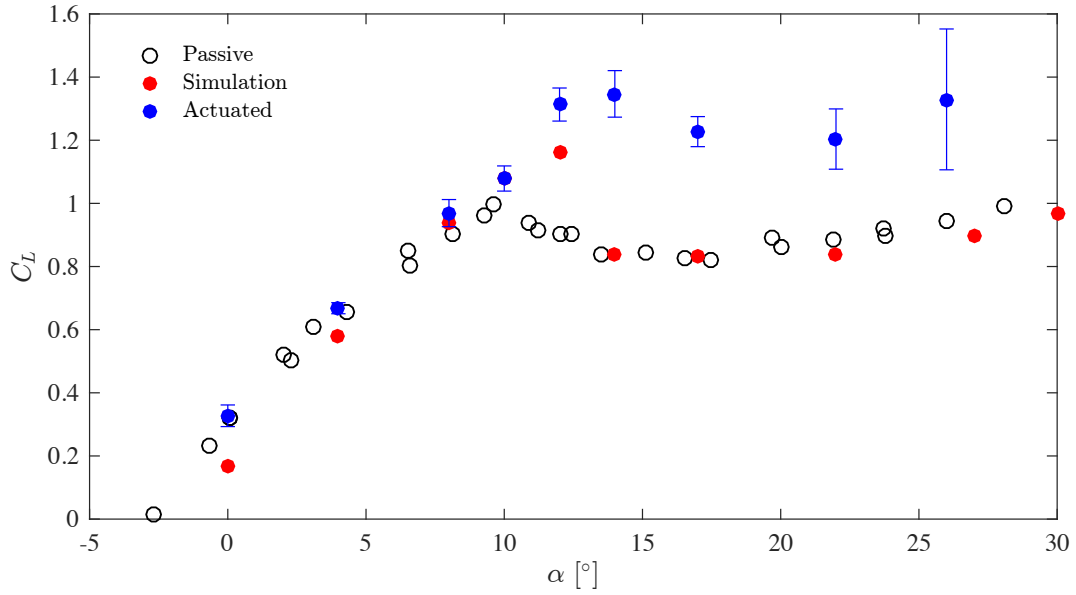
8.2.4 Lift Measurement

The maximum change in the loadcell signal ($|F_R|_{max}$) was found for each angle of attack and Reynolds number. The change in the coefficient of lift is defined as,

$$\Delta C_L = \frac{|F_R|_{max}}{0.5\rho U_0^2 cs} \quad (39)$$



(a)



(b)

Figure 50: The passive and actuated lift characteristics of a VR-12 wing with a RailPac operating at (a) $Re \sim 45,000$ and (b) $Re \sim 90,000$.

This value was added to the passive coefficient of lift to quantify the increase in aerodynamic performance due to RailPac actuation. Figure 50 provide both passive and actuated lift characteristics of a RailPac modified wing for different Reynolds numbers.

Note that the numerical simulation predicts stall angles of 10° and 12° for $Re \sim 45,000$ and $90,000$, respectively. The measured lift coefficients reveal an earlier transition to stall at angles of 8° and 10° for $Re \sim 45,000$ and $90,000$, respectively, with a very gradual drop in C_L at the post stall angles. The smaller stall angle for the experimental result is attributed to the installation of the rails on the top surface of the wing. Due to the manufacturing tolerance, a small ledge present between the wing surface and the rails prompted an earlier separation of the boundary layer. The difference in C_L after the stall angles between the numerical simulation and the experiment is explained by the non-uniform free stream velocity. Because the tunnel speed at the edges of the wind-tunnel was lower, the Reynolds number of the wing varied along the span. The wing experienced a progressive stall that began from the outboard edge of the wing to the mid-span as the angle of attack was increased. Since the loadcell measured the integrated force along the span, a precipitous drop in lift after the stall angle was not observed. Despite the differences between the experimental and simulation results near the stall angle, a good agreement in C_L is observed at all other angles of attack

At the angles of attack below the stall angle, the lift characteristics of the actuated wing do not vary significantly from those of the passive configuration. In contrast, a remarkable improvement in lift characteristics at post-stall angles of attack can be observed. The stall angles for the actuated wing are 12° and 14° for $Re \sim 45,000$ and $90,000$, respectively. These values represent 2° increase in stall angle compared to the clean wing and 4° increase over the passive wing with a RailPac. The maximum C_L for the actuated configuration are ~ 1.3 and ~ 1.4 for $Re \sim 45,000$ and $90,000$, respectively. Thus, $\sim 20\%$ increase over the clean wing and $\sim 40\%$ increase over the passive wing with a RailPac are observed. Lift improvement continues beyond the stall angles, with RailPac delivering a remarkable performance improvement at all angles of attack tested in this study.

8.3 Summary and Conclusion

In summary, a prototype RailPac was mounted on a VR-12 airfoil operating at $Re \sim 45,000$ and $90,000$. The actuator was characterized with high-speed imaging and electrical measurements. The images revealed an arc lasting ~ 1 ms traveling at a velocity of ~ 54 m/s and 58 m/s at the two Reynolds numbers tested. The transit velocity was insensitive to the angle of attack. The electrical measurements captured a peak current of 1.3 kA with each actuator consuming ~ 22 J per operation.

Particle image velocimetry (PIV) captured the changes in the flow over the airfoil before and after the actuation. The images of the seed particles captured the formation and convection of a high-temperature wake behind the arc, entrainment of free-stream fluid, and (for post-stall angles of attack) reattachment of the flow over the airfoil. These observations demonstrate that the flow separation is suppressed by the entrainment effect due to Lorentz force and buoyancy effect that removes the low momentum flow away from the airfoil surface. The comparison of PIV results before and after actuation showed that the RailPac is capable of suppressing flow separation at different angles of attack. A full reattachment of the flow was possible at angles of attack up to $\sim 17^\circ$, while partial reattachment was observed

at higher values. The reattachment time was found to be sensitive to the angle of attack, but insensitive to the Reynolds number.

While the PIV images gave qualitative description of the flow response to the RailPac actuation, the measurements from the force balance quantified the improvement in the aerodynamic performance. At both Reynolds numbers, the stall angle was increased by 4° compared to the passive airfoil with RailPac and 2° over the clean airfoil. The RailPac increased the maximum lift coefficient by $\sim 40\%$ compared to the passive airfoil with RailPac and $\sim 20\%$ over the clean airfoil. The results demonstrate that the RailPac is effective in suppressing flow separation over the airfoil at high angles of attack.

9 Conclusion and Recommendations

In summary, the study investigated the relevance of a RailPac for alleviating dynamic stall on a rotorcraft. A prototype RailPac was developed and characterized with arc dynamics, control authority, and induced velocity measurements. In addition, a prototype was matured into a flow actuator on a rotorcraft wing and demonstrated separation control abilities of a RailPac in atmospheric air.

The earlier studies on a RailPac attributed limitations of its control authority on the instabilities in the arc motion. These studies captured the asymmetric motion of the arc roots, with the anode root anchoring to a particular position, while the cathode root transits smoothly along the electrodes. The investigation first focused on minimizing the "anchoring" phenomenon by adjusting the electrode shape (rails vs. rods) and separation distance. Anchoring is not present if the rod electrodes are free-floating. For the rail configuration, narrowing the gap width to 2 mm permits smooth transit of the arc. The high-speed images of the arc motion reveal that the lack of anchoring is due to the direct interaction between the field-enhancement regions near the anode and cathode roots. The interaction can be enhanced by removing the insulating gap between the electrodes (as in the case of rod configuration) or reducing the separation distance between the electrodes. The transit images also show that the arc in a rail configuration is brighter compared to rod electrodes and exhibit substantial deflection of the arc away from the surface. This arc structure suggests that a large volume of air is affected by the arc transit.

An effective flow control over a rotorcraft blade requires a high control authority actuator capable of coupling large momentum to the flow. If the actuator delivers small momentum, the effect on the flow is negligible due to the the high Reynolds and Mach number environment encountered by a full-scale helicopter. Thus, the relevance of a RailPac for rotorcraft application was demonstrated with the actuation authority measurements. The RailPac prototypes (rail and rod electrodes) were mounted on force and impulse test stands to measure the transient force and impulse delivered by the actuator. The results clearly demonstrate the large control authority of a RailPac. This is seen from the large and scalable body force delivering a peak force and impulse of 0.6 N and 0.45 mN-s, respectively. Methods to further increase the control authority were determined by changing the electrode shape and separation distance. By designing a RailPac capable of sustaining a coherent arc throughout the transit, the actuation authority can be increased by $\sim 21\%$ compared to the anchored case. In addition, the narrow gap increased the transit velocity from ~ 40 m/s to ~ 100 m/s. The

measurements clearly shows that effective flow control in a full-scale rotorcraft is possible due to the high control authority of a RailPac.

The effect of RailPac on quiescent air was studied with PIV to identify flow structures favorable for separation control. The induced flow was measured with conventional and Schlieren velocimetry. The measurements reveal that the front of the arc pushes air through compression effect. A high temperature wake is formed behind the arc and the air is entrained into this region. In addition, the thermal effect of an arc is confined to a small region due to the arc's propagation velocity. The initial push provides mechanism to remove low-momentum air away from the separated boundary layer. The entrainment effect can replace the convected air with the free-stream flow, thereby energizing the boundary layer. These phenomena show that the flow field generated by a moving arc is highly effective for separation control.

A prototype RailPac was redeveloped into a flow actuator on a VR-12 wing. The modified wing was tested in a wind tunnel at low Reynolds numbers ($Re \sim 45,000$ and $90,000$). The wing was mounted on force balances to measure lift, and the actuator was characterized with electrical measurements and high-speed imaging. Particle image velocimetry (PIV) was used to measure the flow over the passive and actuated airfoil. The study shows that the RailPac can improve the aerodynamic performance of a wing. The PIV measurements illustrate the flow reattachment process of a RailPac and demonstrate the actuator's efficacy over a wide range of angles of attack. In addition, a single RailPac operation results in $\sim 40\%$ improvement in post-stall lift and $\sim 4^\circ$ increase in stall angle compared to the passive airfoil with a RailPac. Compared to a clean airfoil, a single RailPac operation results in $\sim 20\%$ improvement in post-stall lift and $\sim 2^\circ$ increase in stall angle.

In conclusion, a RailPac is a highly capable actuator with large control authority and warrants continued study. In particular, a demonstration of a RailPac on a blade mounted in a rotorcraft test stand is recommended to study the actuator in the context of the flow field encountered in a full-scale rotorcrafts. The effect of actuation frequency should also be investigated to fully leverage the natural instabilities of the flow for flow control. The design of a RailPac can be further enhanced to multiply the actuation authority compared to the results obtained in this study. A design based on an augmented railgun can increase the Lorentz force generated by a RailPac. In the present analysis, the RailPac represents a single turn inductor, whereby the arc current flows from one rail, jumps across the gap through the arc, and returns to the power supply via the opposite rail. By introducing additional loops of wire beneath the rails or rods, the inductance gradient (thereby the control authority) of a RailPac geometry can be increased.

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